Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.2/46-1.1.3.2-d

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May 18, 2024 Compiled on May 18, 2024 at 7:58pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**573**]. This is test number [46].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (573)	0.00 (0)
Mathematica	100.00 (573)	0.00 (0)
Sympy	98.95 (567)	1.05 (6)
Maple	98.60 (565)	1.40 (8)
Fricas	94.94 (544)	5.06 (29)
Maxima	87.96 (504)	12.04 (69)
Reduce	87.96 (504)	12.04 (69)
Giac	84.64 (485)	15.36 (88)
Mupad	84.29 (483)	15.71 (90)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

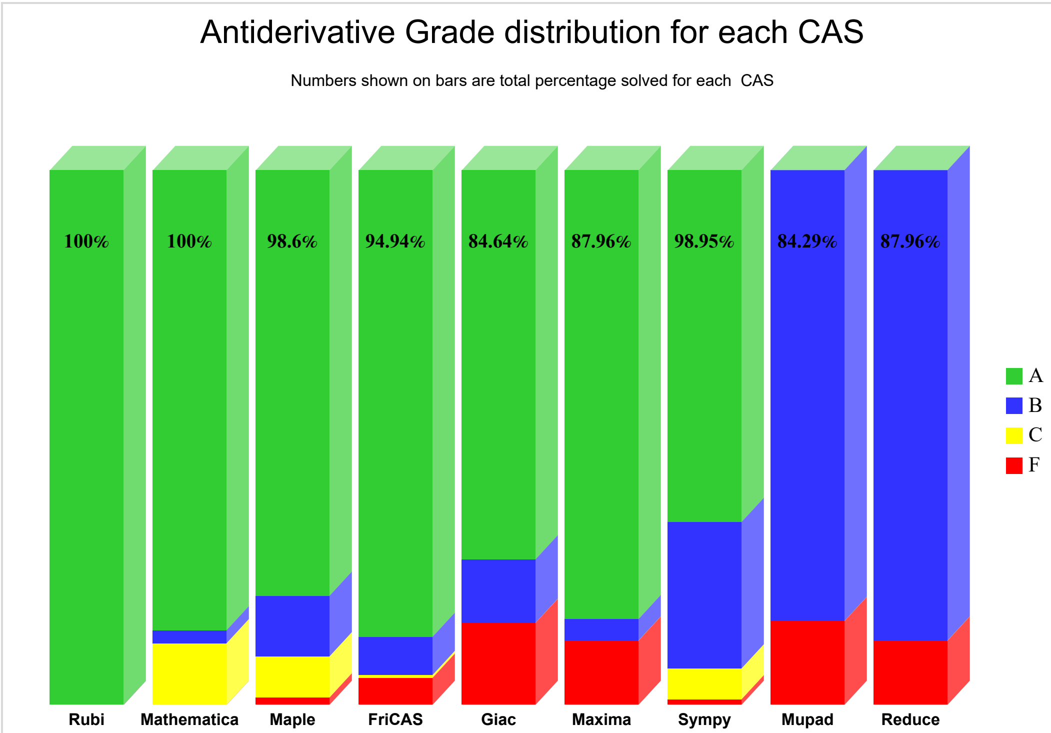
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

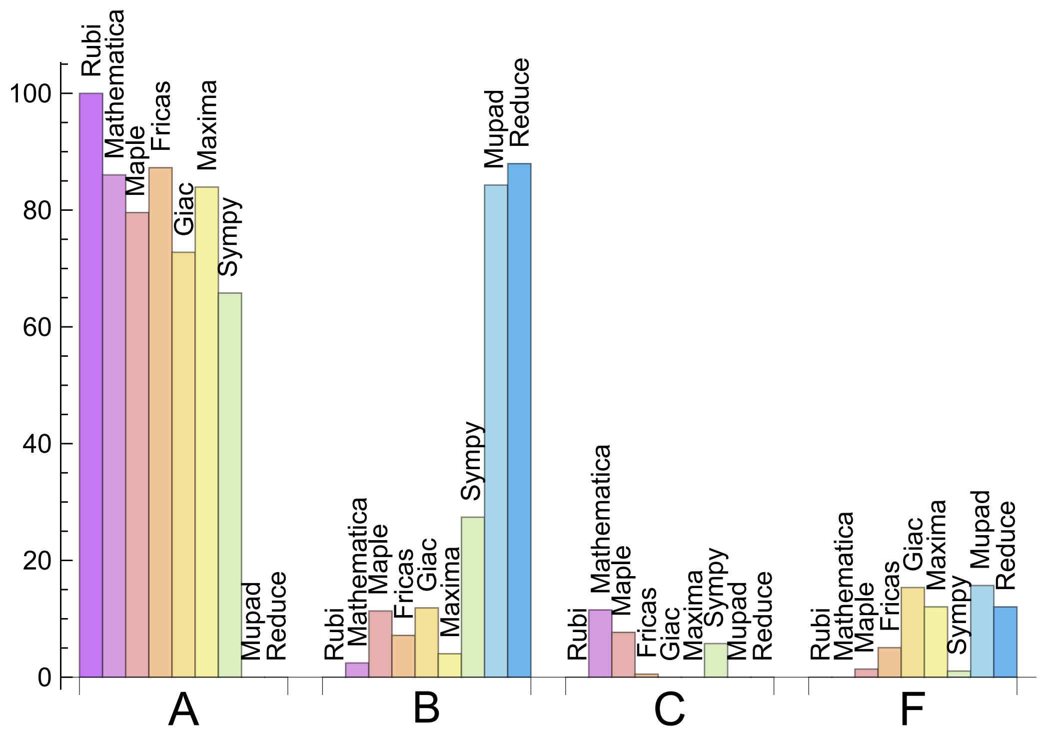
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Fricas	87.260	7.155	0.524	5.061
Mathematica	86.038	2.443	11.518	0.000
Maxima	83.944	4.014	0.000	12.042
Maple	79.581	11.344	7.679	1.396
Giac	72.775	11.867	0.000	15.358
Sympy	65.794	27.400	5.759	1.047
Mupad	0.000	84.293	0.000	15.707
Reduce	0.000	87.958	0.000	12.042

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	6	0.00	100.00	0.00
Maple	8	100.00	0.00	0.00
Fricas	29	100.00	0.00	0.00
Maxima	69	100.00	0.00	0.00
Reduce	69	100.00	0.00	0.00
Giac	88	88.64	0.00	11.36
Mupad	90	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Fricas	0.08
Giac	0.13
Reduce	0.23
Maple	0.28
Rubi	0.34
Mupad	0.42
Mathematica	1.61
Sympy	3.79

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	50.64	0.95	40.00	0.85
Mathematica	56.41	0.95	53.00	0.99
Maxima	59.46	1.02	52.00	0.91
Giac	67.07	1.24	55.00	0.94
Reduce	78.06	1.40	61.00	1.10
Rubi	91.37	1.04	61.00	1.00
Fricas	125.19	1.60	71.50	1.21
Maple	142.92	1.14	53.00	0.91
Sympy	254.61	3.34	53.00	1.07

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

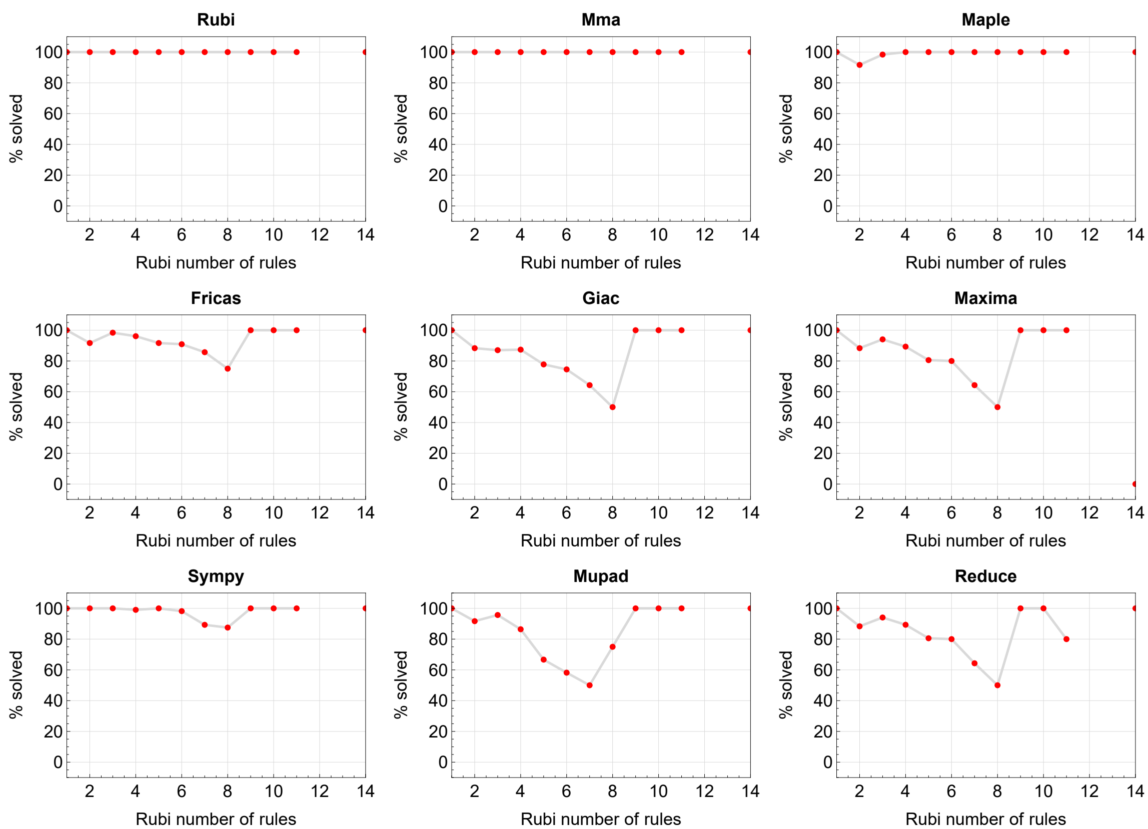


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

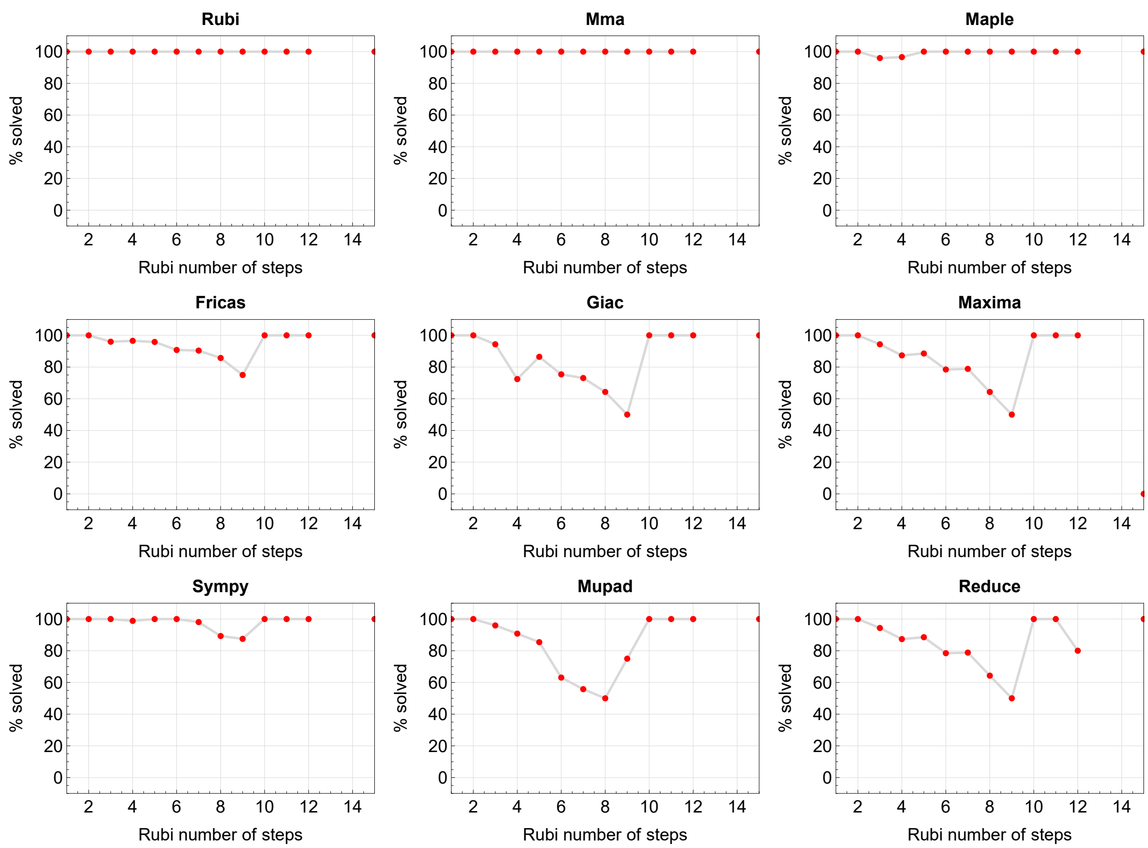


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

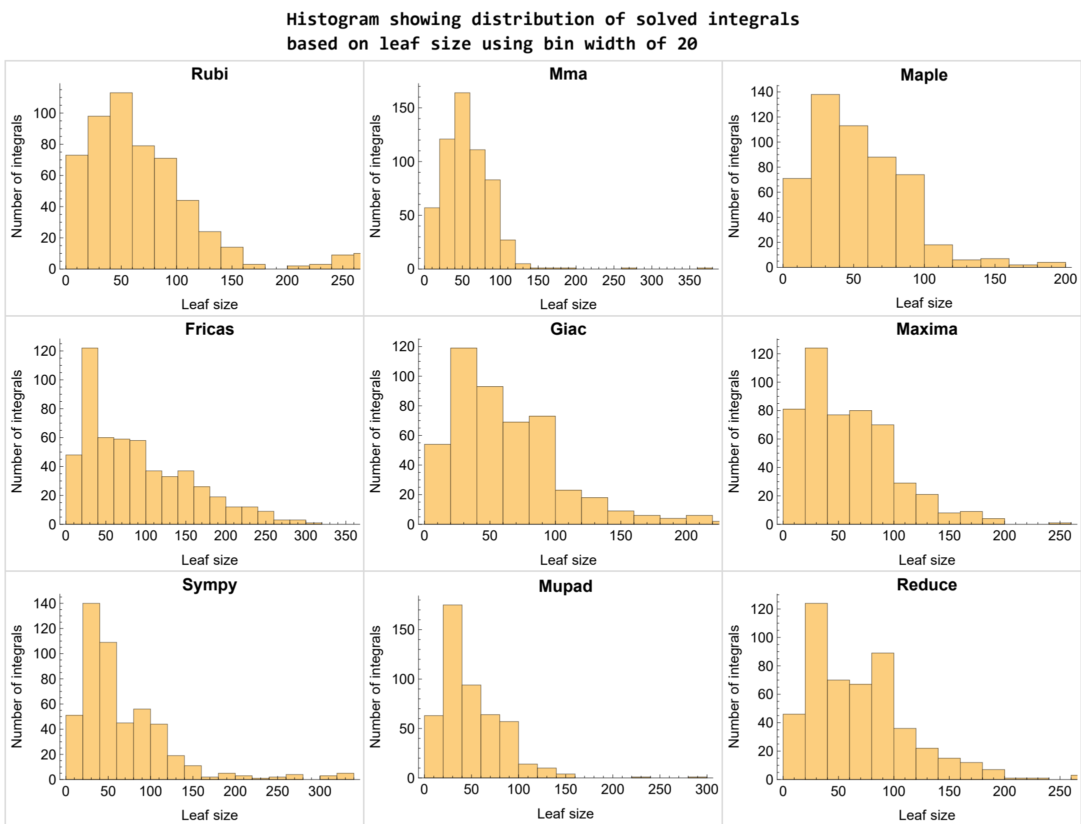


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

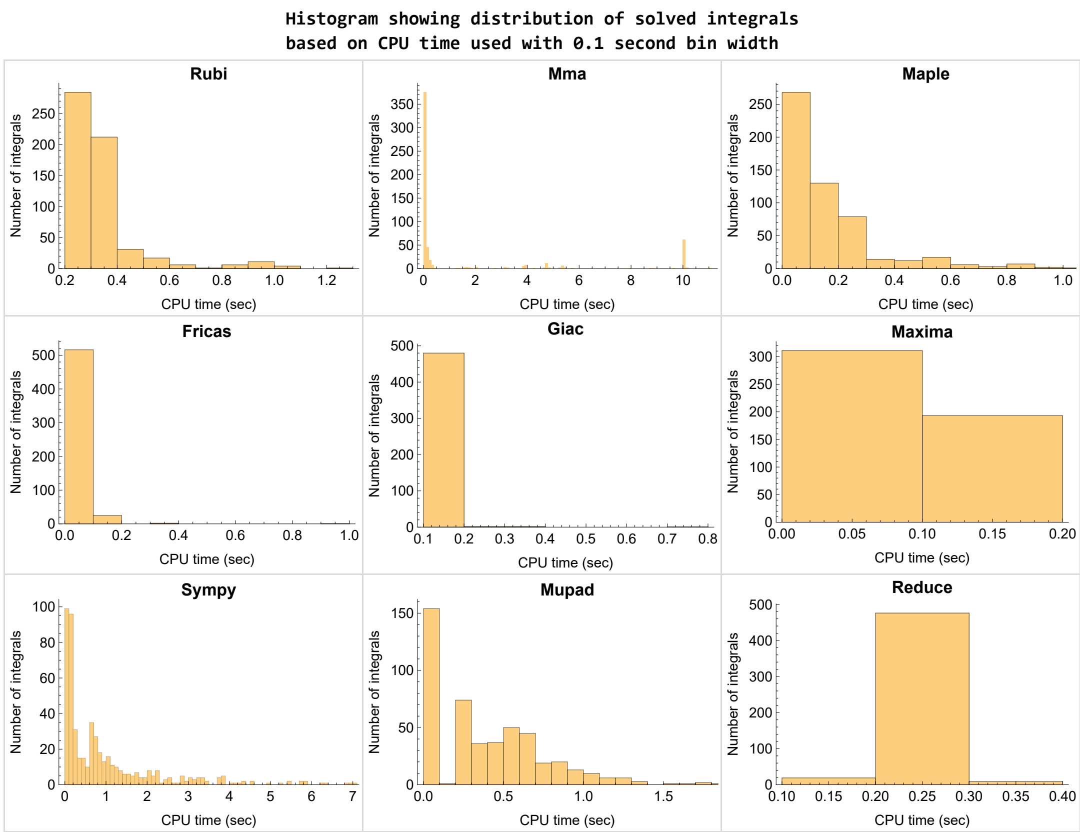


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

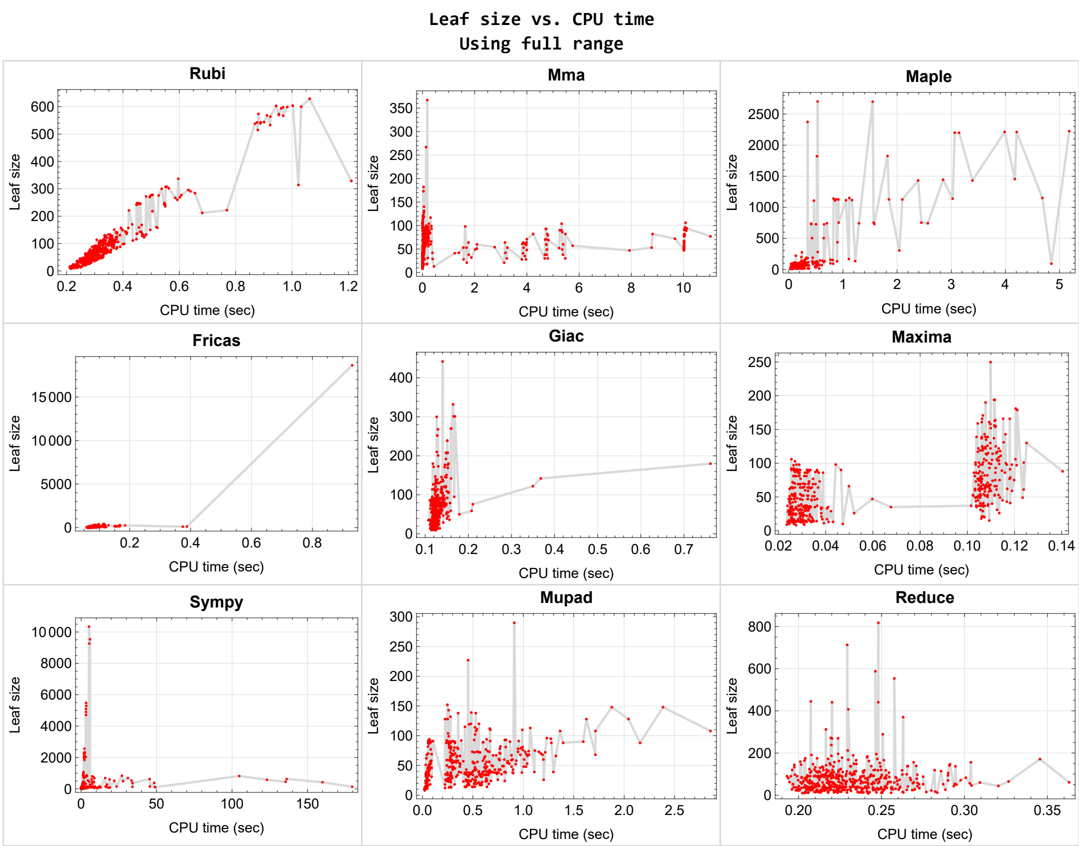


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {467, 468, 469, 470, 471, 472, 493, 494, 495, 496, 497, 498, 499, 513, 514, 515, 516, 517, 518, 519, 522, 524, 530, 532, 538, 540, 548}

Mathematica {}

Maple $\{1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima **abs_integrate** was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See **https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/** for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error **Exception raised: NotImplementedError**.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function **LeafSize** was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user **slelievre** at **https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/**

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

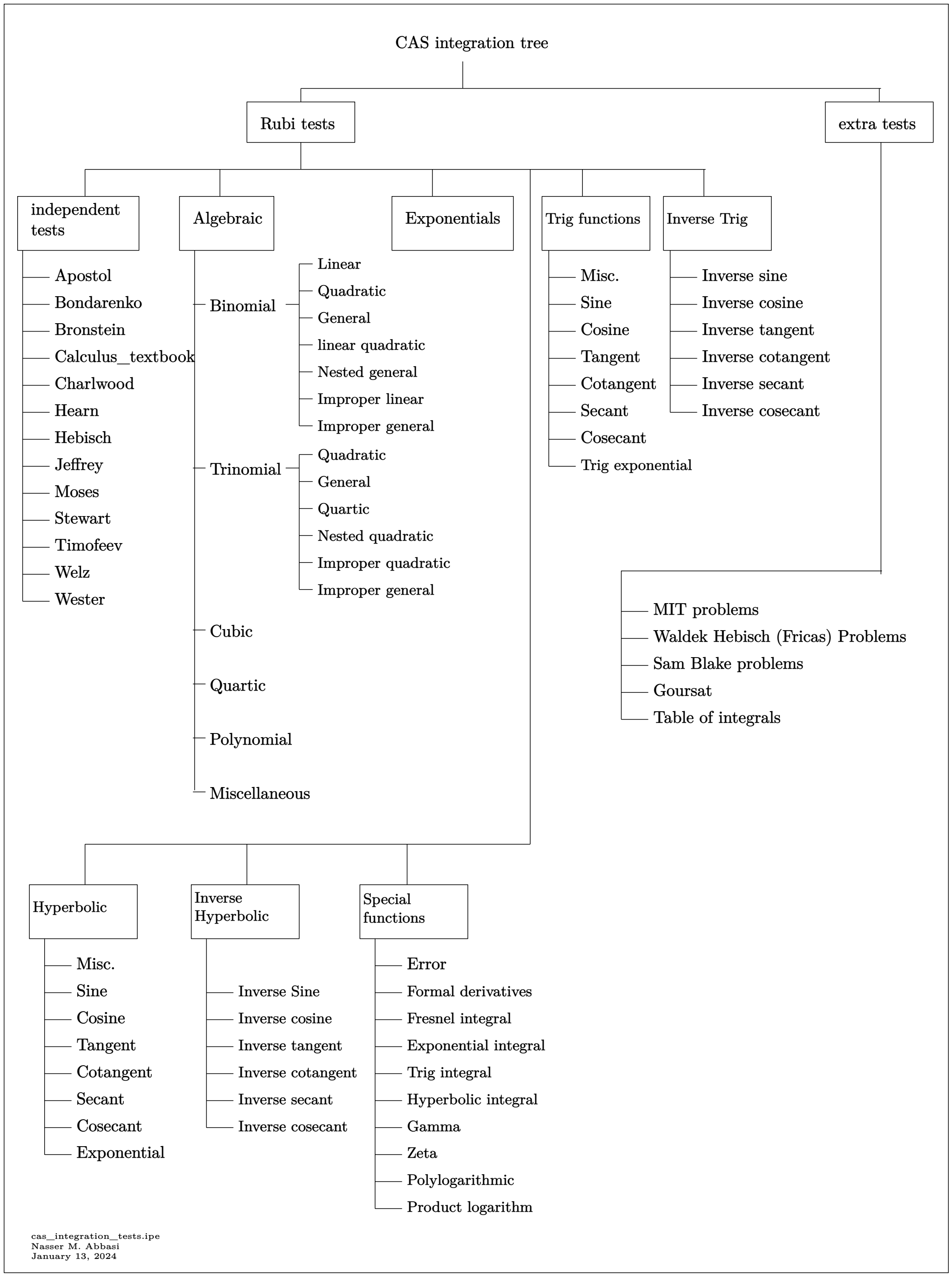
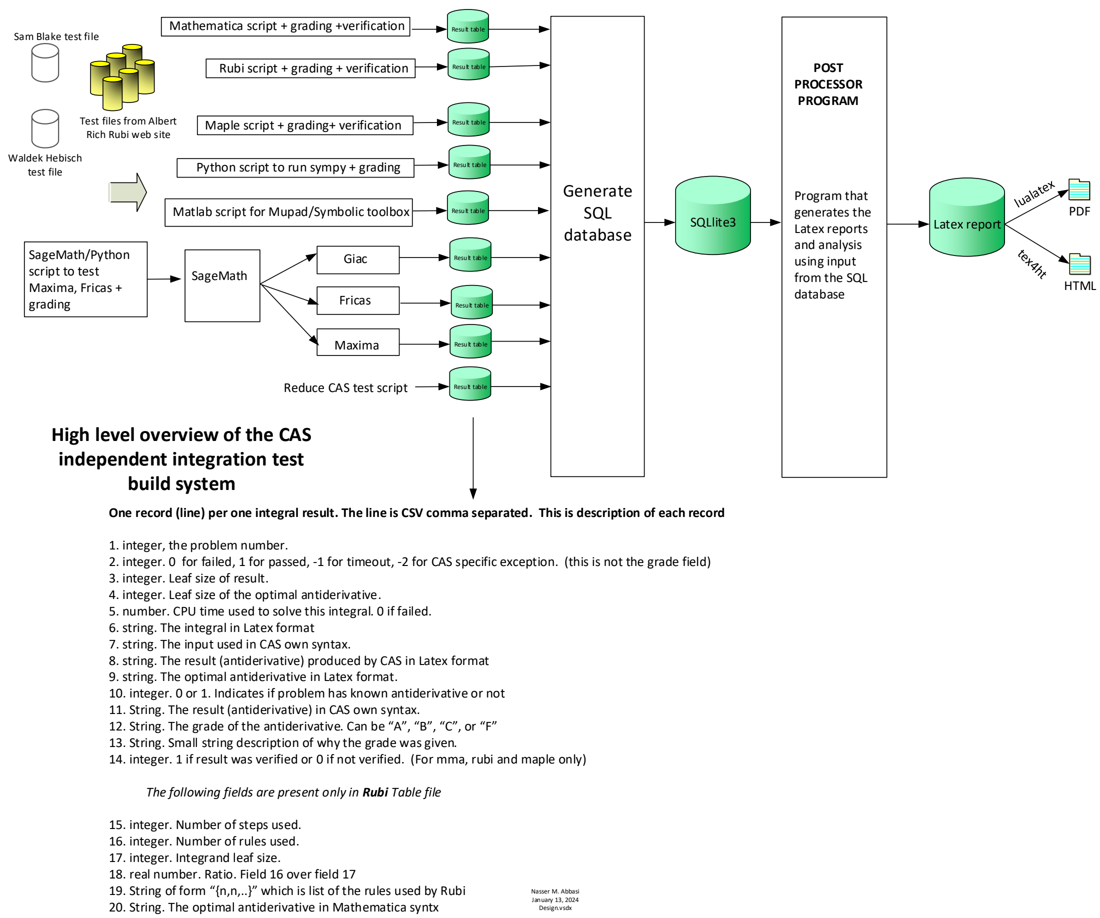


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 252, 253, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455,

456, 457, 458, 459, 460, 473, 474, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 500, 501, 502, 503, 504, 505, 506, 520, 521, 522, 523, 524, 529, 530, 531, 532, 537, 538, 539, 540, 545, 546, 548, 553, 554, 555, 556, 561, 562, 563, 564, 569, 572, 573 }

B grade { 33, 43, 44, 55, 56, 292, 377, 387, 388, 389, 482, 547, 570, 571 }

C grade { 245, 246, 254, 255, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 526, 527, 528, 533, 534, 535, 536, 541, 542, 543, 544, 549, 550, 551, 552, 557, 558, 559, 560, 565, 566, 567, 568 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 386, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 425, 426, 427, 428, 429, 430, 431, 440, 441, 442, 443, 444, 445, 454, 455, 457, 458, 459, 460, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 485, 486, 500, 501, 503, 504, 505, 506, 521, 522, 524, 529,

530, 531, 532, 537, 538, 539, 540, 545, 546, 553, 554, 555, 556, 562, 564 }

B grade { 33, 43, 44, 55, 56, 146, 147, 178, 186, 187, 196, 197, 292, 377, 385, 387, 388, 389, 456, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 481, 482, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 523, 547, 548, 561, 563 }

C grade { 432, 433, 434, 435, 436, 437, 438, 439, 446, 447, 448, 449, 450, 451, 452, 453, 520, 525, 526, 527, 528, 533, 534, 535, 536, 541, 542, 543, 544, 549, 550, 551, 552, 557, 558, 559, 560, 565, 566, 567, 568, 569, 572, 573 }

F normal fail { 419, 420, 421, 422, 423, 424, 570, 571 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 409, 410, 411, 412, 413, 415, 416, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 457, 458, 459, 460, 464, 465, 466, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 483, 484, 485, 486, 490,

491, 492, 496, 497, 498, 499, 500, 504, 505, 506, 510, 511, 512, 516, 517, 518, 519, 521, 522, 523, 524, 525, 527, 528, 529, 530, 531, 532, 535, 536, 537, 538, 539, 540, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 564, 565, 566, 567, 568, 571 }

B grade { 28, 33, 43, 44, 45, 55, 56, 93, 158, 169, 197, 198, 215, 223, 252, 287, 292, 338, 341, 365, 373, 377, 388, 392, 393, 406, 407, 414, 417, 418, 455, 456, 476, 482, 501, 502, 503, 555, 563, 570, 572 }

C grade { 520, 569, 573 }

F normal fail { 419, 420, 421, 422, 423, 424, 461, 462, 463, 467, 468, 469, 487, 488, 489, 493, 494, 495, 507, 508, 509, 513, 514, 515, 526, 533, 534, 541, 542 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 367, 368, 369, 370, 371, 373, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 500, 501, 502, 503, 504, 505, 506, 520, 521, 522, 523, 529, 530, 531, 537, 538, 539, 545, 546, 547, 553, 554, 555, 556, 561, 562, 563, 564, 569, 570, 571, 572 }

B grade { 28, 43, 44, 45, 56, 209, 210, 211, 218, 219, 227, 228, 287, 364, 366, 372, 374, 377, 388, 524, 532, 540, 548 }

C grade { }

F normal fail { 419, 420, 421, 422, 423, 424, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 526, 527, 528, 533, 534, 535, 536, 541, 542, 543, 544, 549, 550, 551, 552, 557, 558, 559, 560, 565, 566, 567, 568, 573 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 153, 154, 155, 164, 165, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 189, 194, 201, 204, 205, 208, 209, 210, 211, 216, 217, 218, 219, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 356, 358, 359, 360, 361, 362, 364, 366, 367, 368, 370, 372, 374, 375, 376, 380, 381, 382, 383, 384, 386, 390, 391, 392, 393, 394, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 408, 409, 411, 412, 413, 414, 415, 416, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 473, 474, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 500, 501, 503, 520, 521, 522, 523, 524, 529, 530, 532, 538, 540, 545, 546, 547, 548, 553, 554, 555, 556, 561, 562, 563, 564, 569, 572 }

B grade { 28, 43, 44, 45, 56, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 169, 170, 171, 172, 173, 178, 186, 187, 188, 195, 196, 197, 198, 199, 206, 207, 212, 213, 214, 215, 220, 221, 222, 223, 234, 287, 292, 354, 355, 357, 363, 365, 369, 371, 373, 377, 378, 379, 385, 387, 388, 389, 395, 396, 401, 410, 417, 418, 531, 537, 539, 573 }

C grade { }

F normal fail { 190, 191, 192, 193, 200, 202, 203, 419, 420, 421, 422, 423, 424, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 526, 527, 528, 533, 534, 535, 536, 541, 542, 543, 544, 549, 550, 551, 552, 557, 558, 559, 560, 565, 566, 567, 568 }

F(-1) timedout fail { }

F(-2) exception fail { 147, 156, 157, 166, 167, 168, 482, 502, 570, 571 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 212, 213, 214, 215, 220, 221, 222, 223, 229, 230, 231, 232, 233, 238, 239, 240, 241, 242, 247, 248, 249, 250, 251, 252, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 359, 361, 362, 363, 364, 365, 367, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 417, 418, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 464, 469, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 490, 495, 500, 501, 502, 503, 504, 505, 506, 510, 515, 520, 521, 523, 526, 527, 529, 531, 534, 535, 537, 539, 542, 543, 545, 546, 547, 550, 551, 553, 554, 555, 556, 558, 559, 561, 562, 563, 564, 566, 567, 569, 570, 571, 572, 573 }

C grade { }

F normal fail { }

F(-1) timedout fail { 208, 209, 210, 211, 216, 217, 218, 219, 224, 225, 226, 227, 228, 234, 235, 236, 237, 243, 244, 245, 246, 253, 254, 255, 358, 360, 366, 368, 374, 402, 403, 415, 416, 419, 420, 421, 422, 423, 424, 461, 462, 463, 465, 466, 467, 468, 470, 471, 472, 487, 488, 489, 491, 492, 493, 494, 496, 497, 498, 499, 507, 508, 509, 511, 512, 513, 514, 516, 517, 518, 519, 522, 524, 525, 528, 530, 532, 533, 536, 538, 540, 541, 544, 548, 549, 552, 557, 560, 565, 568 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 124, 143, 144, 145, 146, 153, 154, 155, 156, 157, 164, 165, 166, 167, 168, 174, 175, 176, 177, 178, 179, 184, 185, 186, 188, 189, 206, 208, 209, 210, 211, 216, 217, 218, 219, 224, 225, 226, 227, 228, 232, 233, 234, 235, 236, 237, 241, 242, 244, 245, 246, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 377, 383, 384, 385, 386, 388, 390, 391, 394, 399, 400, 402, 403, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 480, 481, 482, 483, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 529, 530, 531, 532, 537, 538, 539, 540, 545, 546, 547, 548, 553, 554, 556, 569, 570, 572, 573 }

B grade { 17, 21, 28, 33, 43, 44, 45, 55, 56, 57, 93, 94, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 169, 170, 171, 172, 173, 180, 181, 182, 183, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 212, 213, 214, 215, 221, 222, 223, 229, 230, 231, 238, 239, 240, 243, 247, 248, 249, 250, 251, 252, 253, 254, 271, 275, 287, 292, 312, 313, 314, 315, 316, 317, 330, 331, 338, 347, 352, 357, 365, 373, 378, 379, 380, 381, 382, 392, 393, 395, 396, 397, 398, 401, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 456, 457, 458, 459, 460, 476, 477, 478, 479, 484, 485, 486, 502, 505, 506, 555, 561, 562, 563, 564 }

C grade { 387, 389, 419, 420, 421, 422, 423, 424, 525, 526, 527, 528, 533, 534, 535, 536, 541, 542, 543, 544, 549, 550, 551, 552, 557, 558, 559, 560, 565, 566, 567, 568, 571 }

F normal fail { }

F(-1) timedout fail { 133, 140, 141, 142, 220, 255 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 500, 501, 502, 503, 504, 505, 506, 520, 521, 522, 523, 524, 529, 530, 531, 532, 537, 538, 539, 540, 545, 546, 547, 548, 553, 554, 555, 556, 561, 562, 563, 564, 570, 571, 572, 573 }

C grade { }

F normal fail { 419, 420, 421, 422, 423, 424, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 526, 527, 528, 533, 534, 535, 536, 541, 542, 543, 544, 549, 550, 551, 552, 557, 558, 559, 560, 565, 566, 567, 568, 569 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.244	0.002	0.043	0.025	0.062	0.019	0.123	0.247	0.026

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.240	0.002	0.043	0.027	0.079	0.019	0.123	0.236	0.025

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.244	0.002	0.040	0.027	0.059	0.020	0.126	0.239	0.024

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.241	0.002	0.038	0.032	0.062	0.024	0.118	0.229	0.024

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.237	0.001	0.045	0.027	0.102	0.030	0.126	0.218	0.023

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	10	10	8	10	10	10
N.S.	1	1.00	0.86	0.79	0.71	0.71	0.57	0.71	0.71	0.71
time (sec)	N/A	0.216	0.000	0.037	0.027	0.064	0.027	0.114	0.202	0.019

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	9	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00	1.00
time (sec)	N/A	0.215	0.001	0.027	0.027	0.067	0.048	0.124	0.242	0.023

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	7	12	13	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.64	1.09	1.18	1.00
time (sec)	N/A	0.235	0.002	0.034	0.039	0.064	0.049	0.116	0.238	0.037

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	15	15	12	14	11	12	13	13	11
N.S.	1	0.94	0.94	0.75	0.88	0.69	0.75	0.81	0.81	0.69
time (sec)	N/A	0.237	0.002	0.058	0.025	0.061	0.059	0.119	0.221	0.025

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	14	13	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.82	0.76	0.76	0.76
time (sec)	N/A	0.241	0.002	0.059	0.026	0.065	0.056	0.124	0.206	0.027

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	14	13	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.82	0.76	0.76	0.76
time (sec)	N/A	0.239	0.002	0.061	0.025	0.066	0.078	0.120	0.263	0.029

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	14	13	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.82	0.76	0.76	0.76
time (sec)	N/A	0.242	0.002	0.063	0.026	0.062	0.082	0.121	0.213	0.029

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	14	13	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.82	0.76	0.76	0.76
time (sec)	N/A	0.240	0.002	0.062	0.024	0.063	0.072	0.120	0.204	0.032

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.262	0.002	0.077	0.025	0.062	0.026	0.111	0.238	0.258

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.262	0.002	0.077	0.027	0.065	0.033	0.118	0.245	0.036

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.261	0.002	0.072	0.025	0.069	0.029	0.125	0.211	0.036

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	20	21	20
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	1.43	1.50	1.43
time (sec)	N/A	0.212	0.001	0.069	0.029	0.064	0.046	0.114	0.216	0.034

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	20	21	20	20
N.S.	1	1.00	1.00	0.95	0.91	0.91	0.91	0.95	0.91	0.91
time (sec)	N/A	0.267	0.001	0.086	0.026	0.065	0.050	0.122	0.224	0.035

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	24	17	21	24	20
N.S.	1	1.00	1.00	1.05	1.00	1.20	0.85	1.05	1.20	1.00
time (sec)	N/A	0.267	0.003	0.079	0.034	0.066	0.057	0.121	0.214	0.036

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	21	26	22	22	26	23
N.S.	1	1.00	1.00	0.96	0.88	1.08	0.92	0.92	1.08	0.96
time (sec)	N/A	0.259	0.004	0.066	0.038	0.065	0.072	0.124	0.248	0.253

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	23	14	22	24	14	24	22
N.S.	1	1.00	1.62	1.44	0.88	1.38	1.50	0.88	1.50	1.38
time (sec)	N/A	0.214	0.006	0.064	0.025	0.064	0.082	0.121	0.216	0.040

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.267	0.003	0.065	0.026	0.060	0.093	0.126	0.269	0.039

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.258	0.006	0.064	0.029	0.062	0.132	0.121	0.207	0.039

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.87	0.80	0.80	0.80
time (sec)	N/A	0.260	0.003	0.066	0.026	0.082	0.100	0.130	0.213	0.041

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.281	0.002	0.049	0.031	0.062	0.025	0.113	0.214	0.046

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.81
time (sec)	N/A	0.276	0.003	0.045	0.035	0.075	0.022	0.126	0.202	0.045

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	35	34	34	36	34	35	34
N.S.	1	1.00	1.33	1.17	1.13	1.13	1.20	1.13	1.17	1.13
time (sec)	N/A	0.269	0.002	0.044	0.032	0.087	0.028	0.117	0.253	0.043

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	31	32	31
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	2.21	2.29	2.21
time (sec)	N/A	0.218	0.001	0.039	0.025	0.063	0.024	0.118	0.224	0.043

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	34	32	31	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.97	0.91	0.89	0.89
time (sec)	N/A	0.265	0.003	0.046	0.030	0.066	0.050	0.117	0.206	0.039

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	31	33	36	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.91	0.97	1.06	0.94
time (sec)	N/A	0.274	0.004	0.053	0.038	0.094	0.063	0.124	0.228	0.043

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	31	37	32	31	37	32
N.S.	1	1.00	1.00	0.97	0.94	1.12	0.97	0.94	1.12	0.97
time (sec)	N/A	0.268	0.003	0.048	0.025	0.086	0.084	0.122	0.203	0.035

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	37	36	35	37	34
N.S.	1	1.00	1.00	0.92	0.92	1.00	0.97	0.95	1.00	0.92
time (sec)	N/A	0.276	0.004	0.051	0.033	0.070	0.099	0.112	0.198	0.250

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	39	34	14	33	36	14	35	33
N.S.	1	1.00	2.44	2.12	0.88	2.06	2.25	0.88	2.19	2.06
time (sec)	N/A	0.223	0.004	0.042	0.030	0.061	0.110	0.117	0.238	0.034

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	41	35	35	35	37	35	35	34
N.S.	1	1.06	1.21	1.03	1.03	1.03	1.09	1.03	1.03	1.00
time (sec)	N/A	0.240	0.005	0.042	0.032	0.067	0.118	0.120	0.218	0.033

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.278	0.004	0.039	0.032	0.068	0.134	0.121	0.223	0.034

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.279	0.003	0.040	0.034	0.063	0.151	0.114	0.256	0.034

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.272	0.005	0.040	0.028	0.068	0.149	0.119	0.208	0.044

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	104	90	90	90
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.98	0.85	0.85	0.85
time (sec)	N/A	0.383	0.003	0.059	0.029	0.061	0.033	0.122	0.212	0.287

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	105	90	90	90
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85	0.85
time (sec)	N/A	0.374	0.003	0.054	0.033	0.064	0.035	0.117	0.243	0.262

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	91	90	90	105	90	90	90
N.S.	1	1.00	1.08	0.93	0.92	0.92	1.07	0.92	0.92	0.92
time (sec)	N/A	0.368	0.002	0.053	0.029	0.063	0.030	0.116	0.212	0.057

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	104	91	90	90	104	90	90	90
N.S.	1	1.00	1.28	1.12	1.11	1.11	1.28	1.11	1.11	1.11
time (sec)	N/A	0.342	0.003	0.054	0.027	0.064	0.030	0.128	0.224	0.059

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	106	91	90	90	105	90	90	90
N.S.	1	1.00	1.66	1.42	1.41	1.41	1.64	1.41	1.41	1.41
time (sec)	N/A	0.310	0.003	0.068	0.026	0.064	0.042	0.128	0.238	0.276

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	102	91	90	90	102	90	90	90
N.S.	1	1.00	2.17	1.94	1.91	1.91	2.17	1.91	1.91	1.91
time (sec)	N/A	0.288	0.002	0.053	0.046	0.065	0.033	0.121	0.195	0.058

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	104	91	90	90	104	90	90	90
N.S.	1	1.00	3.47	3.03	3.00	3.00	3.47	3.00	3.00	3.00
time (sec)	N/A	0.252	0.002	0.054	0.036	0.066	0.036	0.120	0.225	0.058

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	86	86	94	86	87	86
N.S.	1	1.00	1.00	0.93	6.14	6.14	6.71	6.14	6.21	6.14
time (sec)	N/A	0.211	0.001	0.047	0.029	0.064	0.034	0.122	0.250	0.265

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	87	86	86	100	87	86	86
N.S.	1	1.00	1.00	0.89	0.88	0.88	1.02	0.89	0.88	0.88
time (sec)	N/A	0.332	0.003	0.132	0.037	0.069	0.086	0.123	0.223	0.263

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	87	92	95	88	92	87
N.S.	1	1.00	1.00	0.93	0.92	0.97	1.00	0.93	0.97	0.92
time (sec)	N/A	0.345	0.007	0.060	0.027	0.072	0.098	0.117	0.219	0.067

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	86	92	97	87	92	88
N.S.	1	1.00	1.00	0.93	0.91	0.97	1.02	0.92	0.97	0.93
time (sec)	N/A	0.344	0.004	0.055	0.028	0.070	0.131	0.120	0.210	0.264

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	88	86	92	95	87	92	88
N.S.	1	1.00	1.00	0.95	0.92	0.99	1.02	0.94	0.99	0.95
time (sec)	N/A	0.348	0.008	0.057	0.039	0.068	0.129	0.121	0.245	0.263

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	88	92	97	89	92	88
N.S.	1	1.00	1.00	0.93	0.93	0.97	1.02	0.94	0.97	0.93
time (sec)	N/A	0.347	0.004	0.060	0.027	0.063	0.162	0.115	0.220	0.278

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	88	88	92	95	89	92	88
N.S.	1	1.00	1.00	0.95	0.95	0.99	1.02	0.96	0.99	0.95
time (sec)	N/A	0.360	0.008	0.056	0.036	0.065	0.210	0.122	0.212	0.056

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	88	88	92	95	89	92	88
N.S.	1	1.00	1.00	0.93	0.93	0.97	1.00	0.94	0.97	0.93
time (sec)	N/A	0.359	0.004	0.056	0.033	0.066	0.240	0.119	0.244	0.236

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	87	87	92	94	88	92	92
N.S.	1	1.00	1.00	0.93	0.93	0.98	1.00	0.94	0.98	0.98
time (sec)	N/A	0.360	0.004	0.056	0.036	0.068	0.277	0.122	0.216	0.241

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	89	92	95	90	92	89
N.S.	1	1.00	1.00	0.89	0.89	0.92	0.95	0.90	0.92	0.89
time (sec)	N/A	0.360	0.004	0.043	0.028	0.067	0.325	0.121	0.221	0.072

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	96	89	14	88	95	14	90	88
N.S.	1	1.00	6.00	5.56	0.88	5.50	5.94	0.88	5.62	5.50
time (sec)	N/A	0.224	0.008	0.046	0.031	0.093	0.364	0.124	0.218	0.262

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	104	90	90	90	97	90	90	90
N.S.	1	1.06	3.06	2.65	2.65	2.65	2.85	2.65	2.65	2.65
time (sec)	N/A	0.237	0.004	0.049	0.027	0.067	0.484	0.121	0.207	0.080

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	53	62	102	90	90	90	97	90	90	90
N.S.	1	1.17	1.92	1.70	1.70	1.70	1.83	1.70	1.70	1.70
time (sec)	N/A	0.267	0.006	0.049	0.027	0.062	0.414	0.121	0.199	0.266

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	88	106	90	90	90	97	90	90	90
N.S.	1	1.22	1.47	1.25	1.25	1.25	1.35	1.25	1.25	1.25
time (sec)	N/A	0.290	0.004	0.048	0.027	0.064	0.404	0.123	0.206	0.080

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	91	114	104	90	90	90	97	90	90	90
N.S.	1	1.25	1.14	0.99	0.99	0.99	1.07	0.99	0.99	0.99
time (sec)	N/A	0.307	0.007	0.046	0.033	0.063	0.436	0.125	0.226	0.084

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	106	106	90	90	90	97	90	90	90
N.S.	1	0.96	0.96	0.82	0.82	0.82	0.88	0.82	0.82	0.82
time (sec)	N/A	0.362	0.004	0.046	0.027	0.064	0.459	0.117	0.220	0.078

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	129	106	106	90	90	90	97	90	90	89
N.S.	1	0.82	0.82	0.70	0.70	0.70	0.75	0.70	0.70	0.69
time (sec)	N/A	0.367	0.006	0.049	0.032	0.070	0.517	0.125	0.210	0.080

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	63	64	63	61	65	63	62
N.S.	1	1.00	1.00	0.90	0.91	0.90	0.87	0.93	0.90	0.89
time (sec)	N/A	0.337	0.004	0.049	0.029	0.070	0.068	0.112	0.249	0.037

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	52	51
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.91	0.89
time (sec)	N/A	0.308	0.004	0.138	0.027	0.064	0.062	0.122	0.197	0.032

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	41	40
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.93	0.91
time (sec)	N/A	0.296	0.004	0.044	0.026	0.065	0.066	0.121	0.205	0.043

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94	0.94
time (sec)	N/A	0.267	0.003	0.047	0.024	0.066	0.056	0.119	0.235	0.043

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	17	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	0.94	1.00
time (sec)	N/A	0.257	0.003	0.041	0.025	0.068	0.055	0.113	0.225	0.224

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	1.00
time (sec)	N/A	0.224	0.001	0.029	0.047	0.069	0.025	0.114	0.204	0.022

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	18	14	13	16	10	14	15	15
N.S.	1	1.00	1.38	1.08	1.00	1.23	0.77	1.08	1.15	1.15
time (sec)	N/A	0.224	0.004	0.049	0.029	0.080	0.080	0.121	0.250	0.050

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	28	26	28	26	19	30	26	25
N.S.	1	1.27	1.27	1.18	1.27	1.18	0.86	1.36	1.18	1.14
time (sec)	N/A	0.271	0.004	0.042	0.024	0.067	0.097	0.118	0.289	0.064

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	42	42	41	40	41	31	45	43	38
N.S.	1	1.20	1.20	1.17	1.14	1.17	0.89	1.29	1.23	1.09
time (sec)	N/A	0.299	0.004	0.043	0.025	0.071	0.100	0.117	0.294	0.248

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	56	56	53	51	54	44	56	54	48
N.S.	1	1.17	1.17	1.10	1.06	1.12	0.92	1.17	1.12	1.00
time (sec)	N/A	0.321	0.005	0.045	0.025	0.072	0.114	0.120	0.294	0.068

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	68	68	63	62	65	56	67	65	60
N.S.	1	1.11	1.11	1.03	1.02	1.07	0.92	1.10	1.07	0.98
time (sec)	N/A	0.356	0.004	0.046	0.025	0.066	0.127	0.124	0.327	0.285

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	82	82	75	73	76	68	78	76	70
N.S.	1	1.11	1.11	1.01	0.99	1.03	0.92	1.05	1.03	0.95
time (sec)	N/A	0.340	0.005	0.052	0.037	0.066	0.141	0.123	0.248	0.290

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	89	92	107	99	95	105	94
N.S.	1	1.00	0.90	0.91	0.94	1.09	1.01	0.97	1.07	0.96
time (sec)	N/A	0.400	0.015	0.056	0.025	0.095	0.135	0.127	0.290	0.069

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	78	82	96	78	85	94	83
N.S.	1	1.00	0.95	0.96	1.01	1.19	0.96	1.05	1.16	1.02
time (sec)	N/A	0.377	0.016	0.056	0.029	0.071	0.122	0.122	0.290	0.264

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	67	70	85	71	73	83	72
N.S.	1	1.00	0.92	0.93	0.97	1.18	0.99	1.01	1.15	1.00
time (sec)	N/A	0.350	0.012	0.053	0.030	0.065	0.110	0.126	0.301	0.274

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	62	71	62
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.22	1.07
time (sec)	N/A	0.330	0.015	0.053	0.033	0.067	0.097	0.121	0.298	0.039

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	48	60	50
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.30	1.09
time (sec)	N/A	0.307	0.011	0.049	0.024	0.070	0.093	0.121	0.309	0.241

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	34	46	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.39	1.09
time (sec)	N/A	0.279	0.009	0.047	0.032	0.065	0.087	0.121	0.304	0.044

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	33	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.43	1.00
time (sec)	N/A	0.270	0.005	0.044	0.025	0.063	0.062	0.122	0.273	0.041

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	13	13	10	13	12	12
N.S.	1	1.00	0.92	1.00	1.00	1.00	0.77	1.00	0.92	0.92
time (sec)	N/A	0.222	0.001	0.045	0.024	0.065	0.073	0.125	0.284	0.224

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	44	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	1.52	0.90
time (sec)	N/A	0.280	0.008	0.044	0.025	0.074	0.107	0.113	0.320	0.243

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	42	35	43	45	63	37	45	70	41
N.S.	1	1.08	0.90	1.10	1.15	1.62	0.95	1.15	1.79	1.05
time (sec)	N/A	0.306	0.028	0.054	0.034	0.076	0.144	0.119	0.290	0.066

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	58	53	57	64	86	54	64	86	57
N.S.	1	1.09	1.00	1.08	1.21	1.62	1.02	1.21	1.62	1.08
time (sec)	N/A	0.329	0.041	0.051	0.025	0.069	0.150	0.118	0.215	0.309

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	69	66	68	73	95	66	73	97	69
N.S.	1	1.11	1.06	1.10	1.18	1.53	1.06	1.18	1.56	1.11
time (sec)	N/A	0.342	0.054	0.051	0.033	0.069	0.245	0.117	0.198	0.091

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	79	79	86	108	80	86	108	79
N.S.	1	1.06	1.00	1.00	1.09	1.37	1.01	1.09	1.37	1.00
time (sec)	N/A	0.367	0.041	0.054	0.032	0.074	0.214	0.111	0.211	0.278

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	90	103	129	109	95	131	104
N.S.	1	1.00	0.86	0.91	1.04	1.30	1.10	0.96	1.32	1.05
time (sec)	N/A	0.405	0.050	0.060	0.027	0.068	0.185	0.116	0.215	0.253

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	73	79	91	117	92	83	119	92
N.S.	1	1.00	0.85	0.92	1.06	1.36	1.07	0.97	1.38	1.07
time (sec)	N/A	0.379	0.035	0.056	0.028	0.066	0.236	0.126	0.215	0.058

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	68	81	107	85	73	109	82
N.S.	1	1.00	0.82	0.88	1.05	1.39	1.10	0.95	1.42	1.06
time (sec)	N/A	0.358	0.031	0.057	0.028	0.071	0.162	0.122	0.213	0.055

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	57	69	95	70	61	97	70
N.S.	1	1.00	0.78	0.89	1.08	1.48	1.09	0.95	1.52	1.09
time (sec)	N/A	0.340	0.025	0.055	0.029	0.076	0.182	0.117	0.272	0.056

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	45	57	83	58	44	85	59
N.S.	1	1.00	0.80	0.90	1.14	1.66	1.16	0.88	1.70	1.18
time (sec)	N/A	0.297	0.029	0.052	0.035	0.064	0.149	0.121	0.205	0.263

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	36	48	61	46	37	71	46
N.S.	1	1.00	0.80	0.88	1.17	1.49	1.12	0.90	1.73	1.12
time (sec)	N/A	0.286	0.011	0.043	0.028	0.064	0.127	0.122	0.200	0.055

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	20	15	14	32	32	14	26	32
N.S.	1	1.00	1.25	0.94	0.88	2.00	2.00	0.88	1.62	2.00
time (sec)	N/A	0.218	0.005	0.052	0.026	0.063	0.113	0.120	0.227	0.034

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	23	26
N.S.	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.64	1.86
time (sec)	N/A	0.215	0.002	0.038	0.030	0.060	0.098	0.122	0.221	0.228

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	43	37	41	51	80	46	43	97	47
N.S.	1	0.90	0.77	0.85	1.06	1.67	0.96	0.90	2.02	0.98
time (sec)	N/A	0.291	0.020	0.044	0.034	0.071	0.184	0.122	0.223	0.262

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	53	56	69	109	66	60	119	63
N.S.	1	0.97	0.90	0.95	1.17	1.85	1.12	1.02	2.02	1.07
time (sec)	N/A	0.317	0.035	0.055	0.028	0.071	0.204	0.121	0.224	0.277

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	68	73	86	130	78	73	138	79
N.S.	1	1.06	0.94	1.01	1.19	1.81	1.08	1.01	1.92	1.10
time (sec)	N/A	0.339	0.033	0.054	0.031	0.087	0.227	0.116	0.244	0.278

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	79	84	97	141	92	86	149	91
N.S.	1	1.05	0.93	0.99	1.14	1.66	1.08	1.01	1.75	1.07
time (sec)	N/A	0.361	0.046	0.057	0.026	0.075	0.234	0.122	0.233	0.104

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	15	18	19	13	15	13
N.S.	1	1.00	0.81	0.67	0.71	0.86	0.90	0.62	0.71	0.62
time (sec)	N/A	0.234	0.011	0.179	0.025	0.067	0.289	0.119	0.210	0.040

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	15	16	19	13	13	13
N.S.	1	1.00	0.81	0.67	0.71	0.76	0.90	0.62	0.62	0.62
time (sec)	N/A	0.233	0.011	0.182	0.033	0.082	0.168	0.120	0.213	0.032

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	13	12	17	13	11	12
N.S.	1	1.00	0.84	0.68	0.68	0.63	0.89	0.68	0.58	0.63
time (sec)	N/A	0.232	0.010	0.063	0.030	0.087	0.092	0.120	0.256	0.032

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	13	13	13	12	15	13	13	11
N.S.	1	1.00	0.76	0.76	0.76	0.71	0.88	0.76	0.76	0.65
time (sec)	N/A	0.234	0.013	0.077	0.029	0.067	0.168	0.120	0.223	0.035

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	12	13	11	19	11	17	13
N.S.	1	1.00	0.79	0.63	0.68	0.58	1.00	0.58	0.89	0.68
time (sec)	N/A	0.229	0.013	0.073	0.043	0.067	0.198	0.121	0.223	0.032

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	13	20	13	17	13
N.S.	1	1.00	0.81	0.67	0.62	0.62	0.95	0.62	0.81	0.62
time (sec)	N/A	0.228	0.014	0.072	0.030	0.068	0.279	0.121	0.226	0.033

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	26	27	34	24	24	24
N.S.	1	1.00	0.78	0.69	0.72	0.75	0.94	0.67	0.67	0.67
time (sec)	N/A	0.247	0.014	0.181	0.030	0.069	0.338	0.112	0.199	0.252

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	24	26	24	32	24	23	24
N.S.	1	1.00	0.82	0.71	0.76	0.71	0.94	0.71	0.68	0.71
time (sec)	N/A	0.252	0.015	0.073	0.038	0.083	0.233	0.121	0.198	0.040

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	24	25	23	31	24	24	24
N.S.	1	1.00	0.88	0.75	0.78	0.72	0.97	0.75	0.75	0.75
time (sec)	N/A	0.253	0.018	0.072	0.027	0.084	0.205	0.115	0.246	0.041

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	24	24	31	23	28	24
N.S.	1	1.00	0.81	0.78	0.75	0.75	0.97	0.72	0.88	0.75
time (sec)	N/A	0.249	0.020	0.079	0.025	0.070	0.203	0.119	0.257	0.235

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	24	34	24	28	24
N.S.	1	1.00	0.82	0.74	0.71	0.71	1.00	0.71	0.82	0.71
time (sec)	N/A	0.254	0.018	0.082	0.033	0.075	0.251	0.129	0.215	0.032

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	24	36	24	28	24
N.S.	1	1.00	0.78	0.69	0.67	0.67	1.00	0.67	0.78	0.67
time (sec)	N/A	0.246	0.018	0.076	0.026	0.070	0.465	0.121	0.224	0.033

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	35	37	35	46	35	34	35
N.S.	1	1.00	0.83	0.74	0.79	0.74	0.98	0.74	0.72	0.74
time (sec)	N/A	0.268	0.016	0.122	0.029	0.067	0.386	0.124	0.208	0.050

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	36	34	44	35	35	35
N.S.	1	1.00	0.87	0.78	0.80	0.76	0.98	0.78	0.78	0.78
time (sec)	N/A	0.264	0.020	0.135	0.026	0.091	0.301	0.115	0.200	0.059

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	38	35	36	34	46	34	38	35
N.S.	1	1.00	0.81	0.74	0.77	0.72	0.98	0.72	0.81	0.74
time (sec)	N/A	0.277	0.021	0.140	0.033	0.090	0.231	0.121	0.226	0.239

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	36	35	35	44	34	39	35
N.S.	1	1.00	0.87	0.80	0.78	0.78	0.98	0.76	0.87	0.78
time (sec)	N/A	0.283	0.022	0.137	0.025	0.074	0.214	0.123	0.215	0.044

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	36	35	35	48	35	39	35
N.S.	1	1.00	0.83	0.77	0.74	0.74	1.02	0.74	0.83	0.74
time (sec)	N/A	0.276	0.020	0.134	0.025	0.065	0.309	0.126	0.231	0.228

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	35	51	35	39	35
N.S.	1	1.00	0.76	0.71	0.69	0.69	1.00	0.69	0.76	0.69
time (sec)	N/A	0.276	0.022	0.135	0.032	0.066	0.356	0.128	0.235	0.040

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	97	72	64	65	153	138	70	68	59
N.S.	1	1.17	0.87	0.77	0.78	1.84	1.66	0.84	0.82	0.71
time (sec)	N/A	0.320	0.062	0.368	0.107	0.072	3.867	0.121	0.238	0.241

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	78	61	53	54	132	122	59	55	48
N.S.	1	1.15	0.90	0.78	0.79	1.94	1.79	0.87	0.81	0.71
time (sec)	N/A	0.283	0.047	0.156	0.106	0.075	1.170	0.130	0.202	0.060

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	49	41	41	103	107	45	39	37
N.S.	1	1.11	0.92	0.77	0.77	1.94	2.02	0.85	0.74	0.70
time (sec)	N/A	0.272	0.040	0.148	0.118	0.093	0.406	0.126	0.204	0.059

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	85	88	31	29	28
N.S.	1	1.00	1.00	0.80	0.78	2.12	2.20	0.78	0.72	0.70
time (sec)	N/A	0.252	0.024	0.142	0.106	0.077	0.422	0.124	0.256	0.048

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	73	18	25	19
N.S.	1	1.00	1.00	0.66	0.62	2.34	2.52	0.62	0.86	0.66
time (sec)	N/A	0.239	0.017	0.117	0.106	0.071	0.705	0.118	0.211	0.048

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	87	85	31	33	28
N.S.	1	1.00	1.00	0.80	0.78	2.18	2.12	0.78	0.82	0.70
time (sec)	N/A	0.255	0.027	0.152	0.113	0.078	1.880	0.123	0.241	0.251

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	48	42	43	113	107	41	46	38
N.S.	1	1.11	0.91	0.79	0.81	2.13	2.02	0.77	0.87	0.72
time (sec)	N/A	0.274	0.042	0.158	0.117	0.082	5.751	0.124	0.220	0.282

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	78	61	53	54	139	126	52	61	49
N.S.	1	1.15	0.90	0.78	0.79	2.04	1.85	0.76	0.90	0.72
time (sec)	N/A	0.295	0.056	0.165	0.107	0.080	16.877	0.124	0.207	0.064

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	122	90	78	88	209	495	88	112	80
N.S.	1	1.21	0.89	0.77	0.87	2.07	4.90	0.87	1.11	0.79
time (sec)	N/A	0.336	0.093	0.207	0.105	0.085	27.779	0.120	0.211	0.265

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	103	79	70	77	188	442	76	99	68
N.S.	1	1.18	0.91	0.80	0.89	2.16	5.08	0.87	1.14	0.78
time (sec)	N/A	0.315	0.091	0.198	0.106	0.083	8.428	0.124	0.219	0.043

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	84	68	55	66	161	389	65	84	58
N.S.	1	1.15	0.93	0.75	0.90	2.21	5.33	0.89	1.15	0.79
time (sec)	N/A	0.298	0.084	0.194	0.106	0.084	2.247	0.123	0.221	0.272

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	65	54	47	52	134	332	46	67	46
N.S.	1	1.14	0.95	0.82	0.91	2.35	5.82	0.81	1.18	0.81
time (sec)	N/A	0.278	0.076	0.176	0.105	0.078	2.524	0.125	0.203	0.075

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	39	115	269	36	60	34
N.S.	1	1.00	1.00	0.80	0.85	2.50	5.85	0.78	1.30	0.74
time (sec)	N/A	0.262	0.058	0.136	0.106	0.079	5.778	0.122	0.258	0.253

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	39	116	277	35	59	33
N.S.	1	1.00	1.00	0.80	0.87	2.58	6.16	0.78	1.31	0.73
time (sec)	N/A	0.261	0.052	0.132	0.109	0.085	17.146	0.123	0.208	0.044

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	54	47	52	143	384	49	71	48
N.S.	1	1.07	0.90	0.78	0.87	2.38	6.40	0.82	1.18	0.80
time (sec)	N/A	0.267	0.075	0.187	0.105	0.080	48.444	0.117	0.205	0.282

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	83	68	55	65	179	452	58	92	58
N.S.	1	1.12	0.92	0.74	0.88	2.42	6.11	0.78	1.24	0.78
time (sec)	N/A	0.304	0.080	0.197	0.108	0.083	135.480	0.116	0.221	0.333

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	102	79	69	77	205	0	70	107	70
N.S.	1	1.13	0.88	0.77	0.86	2.28	0.00	0.78	1.19	0.78
time (sec)	N/A	0.318	0.087	0.204	0.104	0.087	0.000	0.125	0.239	0.323

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	130	92	77	99	254	835	88	151	91
N.S.	1	1.16	0.82	0.69	0.88	2.27	7.46	0.79	1.35	0.81
time (sec)	N/A	0.334	0.126	0.250	0.110	0.080	27.004	0.120	0.219	0.304

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	111	81	65	88	227	762	77	136	81
N.S.	1	1.12	0.82	0.66	0.89	2.29	7.70	0.78	1.37	0.82
time (sec)	N/A	0.320	0.128	0.231	0.140	0.081	8.194	0.119	0.249	0.285

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	92	70	56	75	200	683	59	120	69
N.S.	1	1.10	0.83	0.67	0.89	2.38	8.13	0.70	1.43	0.82
time (sec)	N/A	0.303	0.115	0.235	0.120	0.088	8.780	0.123	0.196	0.298

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	59	50	62	185	605	47	113	58
N.S.	1	1.03	0.83	0.70	0.87	2.61	8.52	0.66	1.59	0.82
time (sec)	N/A	0.287	0.109	0.145	0.108	0.083	18.835	0.125	0.229	0.299

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	72	61	52	66	186	627	52	110	57
N.S.	1	0.99	0.84	0.71	0.90	2.55	8.59	0.71	1.51	0.78
time (sec)	N/A	0.283	0.098	0.151	0.112	0.093	45.421	0.121	0.224	0.078

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	59	59	63	186	632	47	113	57
N.S.	1	1.03	0.84	0.84	0.90	2.66	9.03	0.67	1.61	0.81
time (sec)	N/A	0.283	0.078	0.137	0.109	0.081	136.177	0.121	0.212	0.301

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	91	70	56	75	209	0	59	124	70
N.S.	1	1.03	0.80	0.64	0.85	2.38	0.00	0.67	1.41	0.80
time (sec)	N/A	0.299	0.160	0.211	0.114	0.080	0.000	0.130	0.207	0.323

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	110	81	66	89	245	0	71	144	80
N.S.	1	1.07	0.79	0.64	0.86	2.38	0.00	0.69	1.40	0.78
time (sec)	N/A	0.322	0.119	0.227	0.115	0.089	0.000	0.116	0.219	0.318

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	129	92	76	99	271	0	80	159	92
N.S.	1	1.11	0.79	0.66	0.85	2.34	0.00	0.69	1.37	0.79
time (sec)	N/A	0.339	0.133	0.230	0.113	0.083	0.000	0.125	0.201	0.385

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	81	108	166	178	153	106	95	91
N.S.	1	1.09	0.69	0.92	1.42	1.52	1.31	0.91	0.81	0.78
time (sec)	N/A	0.357	0.108	0.254	0.115	0.088	23.512	0.138	0.232	0.954

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	70	97	137	157	122	92	76	74
N.S.	1	1.04	0.75	1.04	1.47	1.69	1.31	0.99	0.82	0.80
time (sec)	N/A	0.323	0.090	0.130	0.111	0.083	5.866	0.132	0.249	0.647

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	57	84	100	130	97	76	56	54
N.S.	1	0.97	0.83	1.22	1.45	1.88	1.41	1.10	0.81	0.78
time (sec)	N/A	0.287	0.066	0.124	0.109	0.086	2.426	0.129	0.237	0.366

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	72	50	104	42	62	37	58
N.S.	1	1.00	1.00	1.85	1.28	2.67	1.08	1.59	0.95	1.49
time (sec)	N/A	0.259	0.045	0.117	0.109	0.088	1.172	0.129	0.225	0.351

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	70	49	90	68	0	43	31
N.S.	1	1.00	1.00	1.79	1.26	2.31	1.74	0.00	1.10	0.79
time (sec)	N/A	0.272	0.034	0.138	0.105	0.081	0.957	0.000	0.210	0.588

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	15	14	24	41	83	40	22
N.S.	1	1.00	1.11	0.83	0.78	1.33	2.28	4.61	2.22	1.22
time (sec)	N/A	0.227	0.019	0.256	0.025	0.077	0.721	0.141	0.261	0.592

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	31	30	38	304	115	61	35
N.S.	1	1.00	1.11	0.82	0.79	1.00	8.00	3.03	1.61	0.92
time (sec)	N/A	0.279	0.030	0.109	0.025	0.086	0.958	0.139	0.240	0.654

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	42	47	49	899	146	80	70
N.S.	1	1.00	0.90	0.71	0.80	0.83	15.24	2.47	1.36	1.19
time (sec)	N/A	0.300	0.033	0.115	0.036	0.075	1.238	0.146	0.252	0.770

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	64	53	64	60	2297	177	99	90
N.S.	1	1.00	0.80	0.66	0.80	0.75	28.71	2.21	1.24	1.12
time (sec)	N/A	0.325	0.035	0.124	0.027	0.082	1.875	0.146	0.234	0.867

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	64	81	71	5095	208	118	110
N.S.	1	1.00	0.74	0.63	0.80	0.70	50.45	2.06	1.17	1.09
time (sec)	N/A	0.347	0.039	0.127	0.026	0.082	3.204	0.148	0.248	0.990

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	81	108	166	179	153	104	95	89
N.S.	1	1.05	0.70	0.94	1.44	1.56	1.33	0.90	0.83	0.77
time (sec)	N/A	0.343	0.107	0.089	0.118	0.091	16.213	0.139	0.216	0.605

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	97	135	156	124	91	76	72
N.S.	1	1.00	0.77	1.07	1.48	1.71	1.36	1.00	0.84	0.79
time (sec)	N/A	0.314	0.096	0.135	0.108	0.090	4.973	0.124	0.224	0.565

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	65	54	83	98	135	75	77	57	52
N.S.	1	0.97	0.81	1.24	1.46	2.01	1.12	1.15	0.85	0.78
time (sec)	N/A	0.277	0.089	0.147	0.104	0.090	2.184	0.133	0.269	0.375

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	46	78	63	105	92	0	59	34
N.S.	1	1.05	0.84	1.42	1.15	1.91	1.67	0.00	1.07	0.62
time (sec)	N/A	0.282	0.049	0.133	0.105	0.080	2.010	0.000	0.225	0.438

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	58	50	79	61	115	71	0	55	43
N.S.	1	1.05	0.91	1.44	1.11	2.09	1.29	0.00	1.00	0.78
time (sec)	N/A	0.285	0.067	0.136	0.112	0.085	1.725	0.000	0.221	0.717

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	15	14	35	65	145	61	24
N.S.	1	1.00	1.11	0.83	0.78	1.94	3.61	8.06	3.39	1.33
time (sec)	N/A	0.222	0.022	0.255	0.028	0.069	0.791	0.149	0.219	0.816

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	30	49	360	177	80	68
N.S.	1	1.00	1.00	0.82	0.79	1.29	9.47	4.66	2.11	1.79
time (sec)	N/A	0.273	0.033	0.120	0.025	0.070	1.076	0.152	0.253	0.970

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	42	47	60	986	208	99	88
N.S.	1	1.00	0.83	0.71	0.80	1.02	16.71	3.53	1.68	1.49
time (sec)	N/A	0.294	0.037	0.125	0.025	0.072	1.533	0.152	0.216	1.279

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	53	64	71	2297	239	118	108
N.S.	1	1.00	0.75	0.66	0.80	0.89	28.71	2.99	1.48	1.35
time (sec)	N/A	0.320	0.042	0.132	0.030	0.068	2.295	0.151	0.240	1.365

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	71	64	81	82	5289	270	137	128
N.S.	1	1.00	0.70	0.63	0.80	0.81	52.37	2.67	1.36	1.27
time (sec)	N/A	0.337	0.045	0.135	0.032	0.070	3.456	0.159	0.295	1.626

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	82	75	98	93	10344	301	156	148
N.S.	1	1.00	0.67	0.61	0.80	0.76	84.79	2.47	1.28	1.21
time (sec)	N/A	0.375	0.048	0.148	0.029	0.073	5.245	0.169	0.269	1.877

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	108	164	178	155	105	95	89
N.S.	1	1.00	0.70	0.94	1.43	1.55	1.35	0.91	0.83	0.77
time (sec)	N/A	0.331	0.108	0.202	0.112	0.081	9.211	0.140	0.214	0.598

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	66	94	131	157	102	91	76	72
N.S.	1	0.98	0.73	1.03	1.44	1.73	1.12	1.00	0.84	0.79
time (sec)	N/A	0.310	0.100	0.167	0.115	0.085	3.722	0.137	0.233	0.571

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	83	64	93	115	135	126	0	82	72
N.S.	1	0.97	0.74	1.08	1.34	1.57	1.47	0.00	0.95	0.84
time (sec)	N/A	0.304	0.091	0.157	0.109	0.078	3.334	0.000	0.237	0.514

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	64	94	78	144	99	0	84	34
N.S.	1	1.03	0.86	1.27	1.05	1.95	1.34	0.00	1.14	0.46
time (sec)	N/A	0.295	0.086	0.153	0.105	0.086	2.869	0.000	0.224	0.616

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	76	63	92	75	147	97	0	86	60
N.S.	1	1.04	0.86	1.26	1.03	2.01	1.33	0.00	1.18	0.82
time (sec)	N/A	0.299	0.086	0.155	0.112	0.091	3.827	0.000	0.224	1.008

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	15	14	46	80	207	80	68
N.S.	1	1.00	1.11	0.83	0.78	2.56	4.44	11.50	4.44	3.78
time (sec)	N/A	0.221	0.024	0.292	0.026	0.072	0.532	0.150	0.220	1.191

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	30	60	416	239	99	88
N.S.	1	1.00	1.00	0.82	0.79	1.58	10.95	6.29	2.61	2.32
time (sec)	N/A	0.276	0.034	0.142	0.033	0.067	1.214	0.151	0.265	1.395

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	42	47	71	1073	270	118	108
N.S.	1	1.00	0.83	0.71	0.80	1.20	18.19	4.58	2.00	1.83
time (sec)	N/A	0.289	0.037	0.150	0.032	0.066	1.536	0.161	0.253	1.715

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	53	64	82	2562	301	137	128
N.S.	1	1.00	0.75	0.66	0.80	1.02	32.02	3.76	1.71	1.60
time (sec)	N/A	0.315	0.043	0.168	0.033	0.067	2.205	0.166	0.266	2.043

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	71	64	81	93	5482	332	156	148
N.S.	1	1.00	0.70	0.63	0.80	0.92	54.28	3.29	1.54	1.47
time (sec)	N/A	0.337	0.046	0.184	0.027	0.083	3.305	0.164	0.304	2.386

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	133	81	108	166	179	155	120	95	94
N.S.	1	1.11	0.68	0.90	1.38	1.49	1.29	1.00	0.79	0.78
time (sec)	N/A	0.348	0.110	0.263	0.106	0.090	45.568	0.138	0.224	0.636

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	103	69	97	137	156	128	104	76	77
N.S.	1	1.07	0.72	1.01	1.43	1.62	1.33	1.08	0.79	0.80
time (sec)	N/A	0.317	0.094	0.254	0.106	0.098	9.264	0.139	0.231	0.628

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	73	59	86	104	135	100	88	57	57
N.S.	1	1.01	0.82	1.19	1.44	1.88	1.39	1.22	0.79	0.79
time (sec)	N/A	0.290	0.073	0.242	0.113	0.078	3.056	0.141	0.234	0.404

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	71	67	103	44	69	38	66
N.S.	1	1.00	1.00	1.65	1.56	2.40	1.02	1.60	0.88	1.53
time (sec)	N/A	0.254	0.049	0.237	0.109	0.080	1.466	0.144	0.223	0.372

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	119	37	65	22	47	25	19
N.S.	1	1.00	1.08	4.76	1.48	2.60	0.88	1.88	1.00	0.76
time (sec)	N/A	0.239	0.023	0.247	0.109	0.078	0.699	0.130	0.228	0.583

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	14	16	22	27	23	14
N.S.	1	1.00	1.12	0.94	0.88	1.00	1.38	1.69	1.44	0.88
time (sec)	N/A	0.212	0.016	0.353	0.027	0.072	0.308	0.129	0.201	0.493

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	31	30	27	248	53	40	23
N.S.	1	1.00	0.86	0.86	0.83	0.75	6.89	1.47	1.11	0.64
time (sec)	N/A	0.272	0.025	0.238	0.025	0.071	0.878	0.133	0.229	0.473

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	42	42	47	38	813	82	61	54
N.S.	1	1.00	0.74	0.74	0.82	0.67	14.26	1.44	1.07	0.95
time (sec)	N/A	0.301	0.030	0.249	0.025	0.070	1.312	0.133	0.241	0.505

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	53	53	64	49	2164	111	80	73
N.S.	1	1.00	0.70	0.70	0.84	0.64	28.47	1.46	1.05	0.96
time (sec)	N/A	0.309	0.036	0.252	0.032	0.070	1.897	0.138	0.223	0.494

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	64	64	81	60	4901	140	99	93
N.S.	1	1.00	0.65	0.65	0.82	0.61	49.51	1.41	1.00	0.94
time (sec)	N/A	0.351	0.040	0.256	0.029	0.068	3.205	0.140	0.223	0.512

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	128	87	140	154	212	133	151	98	93
N.S.	1	1.09	0.74	1.20	1.32	1.81	1.14	1.29	0.84	0.79
time (sec)	N/A	0.361	0.143	0.289	0.112	0.078	16.512	0.149	0.263	0.816

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	98	76	129	122	191	105	135	85	73
N.S.	1	1.05	0.82	1.39	1.31	2.05	1.13	1.45	0.91	0.78
time (sec)	N/A	0.329	0.111	0.273	0.105	0.082	4.398	0.142	0.221	0.549

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	67	57	116	85	161	71	114	68	34
N.S.	1	1.12	0.95	1.93	1.42	2.68	1.18	1.90	1.13	0.57
time (sec)	N/A	0.301	0.093	0.263	0.105	0.088	2.058	0.136	0.227	0.465

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	50	198	52	133	148	86	56	34
N.S.	1	1.00	1.19	4.71	1.24	3.17	3.52	2.05	1.33	0.81
time (sec)	N/A	0.272	0.065	0.233	0.103	0.096	1.146	0.132	0.207	0.635

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	15	14	24	20	47	30	14
N.S.	1	1.00	1.62	0.94	0.88	1.50	1.25	2.94	1.88	0.88
time (sec)	N/A	0.222	0.023	0.354	0.035	0.080	0.353	0.131	0.245	0.519

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	29	30	31	42	33	39	36
N.S.	1	1.00	0.91	0.85	0.88	0.91	1.24	0.97	1.15	1.06
time (sec)	N/A	0.282	0.031	0.259	0.032	0.075	0.417	0.131	0.207	0.487

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	42	47	48	457	0	55	43
N.S.	1	1.00	0.89	0.76	0.85	0.87	8.31	0.00	1.00	0.78
time (sec)	N/A	0.300	0.037	0.270	0.029	0.070	1.296	0.000	0.209	0.531

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	58	51	64	59	2032	0	70	52
N.S.	1	1.00	0.78	0.69	0.86	0.80	27.46	0.00	0.95	0.70
time (sec)	N/A	0.328	0.043	0.277	0.030	0.069	2.042	0.000	0.247	0.622

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	71	64	81	72	4707	0	83	91
N.S.	1	1.00	0.75	0.67	0.85	0.76	49.55	0.00	0.87	0.96
time (sec)	N/A	0.343	0.050	0.299	0.031	0.067	3.261	0.000	0.250	0.671

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	82	75	98	83	9534	0	96	112
N.S.	1	1.00	0.71	0.65	0.84	0.72	82.19	0.00	0.83	0.97
time (sec)	N/A	0.376	0.050	0.290	0.044	0.070	5.866	0.000	0.225	0.667

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	151	98	181	171	278	532	208	166	113
N.S.	1	1.09	0.71	1.31	1.24	2.01	3.86	1.51	1.20	0.82
time (sec)	N/A	0.369	0.145	0.348	0.107	0.082	33.420	0.150	0.235	1.068

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	121	87	170	139	257	464	192	153	93
N.S.	1	1.06	0.76	1.49	1.22	2.25	4.07	1.68	1.34	0.82
time (sec)	N/A	0.354	0.127	0.322	0.106	0.130	7.527	0.148	0.249	0.828

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	90	72	157	101	230	774	171	136	34
N.S.	1	1.14	0.91	1.99	1.28	2.91	9.80	2.16	1.72	0.43
time (sec)	N/A	0.319	0.094	0.293	0.124	0.083	3.377	0.145	0.221	0.652

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	66	60	274	62	202	700	146	94	49
N.S.	1	1.10	1.00	4.57	1.03	3.37	11.67	2.43	1.57	0.82
time (sec)	N/A	0.305	0.076	0.243	0.117	0.087	1.928	0.138	0.253	0.804

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	15	14	37	39	100	52	14
N.S.	1	1.00	1.11	0.83	0.78	2.06	2.17	5.56	2.89	0.78
time (sec)	N/A	0.223	0.023	0.375	0.028	0.072	0.566	0.130	0.218	0.639

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	31	30	47	82	75	61	25
N.S.	1	1.00	1.00	0.86	0.83	1.31	2.28	2.08	1.69	0.69
time (sec)	N/A	0.285	0.039	0.241	0.030	0.083	0.670	0.130	0.269	0.527

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	46	42	47	55	136	0	74	40
N.S.	1	1.00	0.84	0.76	0.85	1.00	2.47	0.00	1.35	0.73
time (sec)	N/A	0.324	0.036	0.273	0.025	0.073	0.738	0.000	0.217	0.560

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	60	53	64	70	187	66	92	54
N.S.	1	1.00	0.79	0.70	0.84	0.92	2.46	0.87	1.21	0.71
time (sec)	N/A	0.317	0.044	0.280	0.034	0.076	0.810	0.137	0.217	0.516

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	71	64	81	83	2032	0	107	65
N.S.	1	1.00	0.73	0.66	0.84	0.86	20.95	0.00	1.10	0.67
time (sec)	N/A	0.352	0.050	0.294	0.027	0.077	3.107	0.000	0.207	0.620

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	82	75	98	94	9263	0	120	110
N.S.	1	1.00	0.71	0.65	0.84	0.81	79.85	0.00	1.03	0.95
time (sec)	N/A	0.377	0.049	0.294	0.028	0.075	5.423	0.000	0.246	0.673

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	64	53	69	60	518	131	52	58
N.S.	1	1.12	0.64	0.53	0.69	0.60	5.18	1.31	0.52	0.58
time (sec)	N/A	0.359	3.860	0.207	0.029	0.069	21.970	0.128	0.233	0.930

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	53	42	52	49	314	108	41	47
N.S.	1	1.08	0.72	0.57	0.70	0.66	4.24	1.46	0.55	0.64
time (sec)	N/A	0.315	3.874	0.202	0.031	0.069	7.359	0.123	0.219	0.614

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	31	35	37	65	81	29	36
N.S.	1	1.00	0.85	0.65	0.73	0.77	1.35	1.69	0.60	0.75
time (sec)	N/A	0.262	3.851	0.201	0.033	0.076	2.237	0.121	0.248	0.565

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	23	17	24	39	50	16	22
N.S.	1	1.00	1.22	1.00	0.74	1.04	1.70	2.17	0.70	0.96
time (sec)	N/A	0.228	3.806	0.086	0.030	0.072	1.096	0.121	0.212	0.520

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	50	61	50	59	105	68	67	38	0
N.S.	1	1.02	1.24	1.02	1.20	2.14	1.39	1.37	0.78	0.00
time (sec)	N/A	0.274	3.824	0.228	0.110	0.087	1.251	0.125	0.206	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	56	61	54	77	125	44	42	51	0
N.S.	1	1.12	1.22	1.08	1.54	2.50	0.88	0.84	1.02	0.00
time (sec)	N/A	0.277	3.924	0.260	0.118	0.084	1.790	0.134	0.211	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	85	82	69	118	151	97	72	72	0
N.S.	1	1.06	1.02	0.86	1.48	1.89	1.21	0.90	0.90	0.00
time (sec)	N/A	0.317	4.236	0.264	0.114	0.081	4.180	0.143	0.268	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	117	89	81	161	177	122	80	90	0
N.S.	1	1.10	0.84	0.76	1.52	1.67	1.15	0.75	0.85	0.00
time (sec)	N/A	0.360	10.106	0.254	0.109	0.087	12.375	0.141	0.239	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	60	53	69	71	585	210	63	67
N.S.	1	1.12	0.60	0.53	0.69	0.71	5.85	2.10	0.63	0.67
time (sec)	N/A	0.370	4.742	0.216	0.031	0.072	123.130	0.127	0.208	0.700

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	49	42	52	60	369	173	52	56
N.S.	1	1.08	0.66	0.57	0.70	0.81	4.99	2.34	0.70	0.76
time (sec)	N/A	0.322	4.763	0.204	0.033	0.074	34.042	0.131	0.197	0.663

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	31	35	48	88	136	40	45
N.S.	1	1.00	0.79	0.65	0.73	1.00	1.83	2.83	0.83	0.94
time (sec)	N/A	0.274	4.687	0.200	0.031	0.082	11.894	0.125	0.236	0.625

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	30	23	17	35	63	89	27	34
N.S.	1	1.00	1.30	1.00	0.74	1.52	2.74	3.87	1.17	1.48
time (sec)	N/A	0.235	4.683	0.089	0.030	0.072	4.258	0.121	0.196	0.609

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	62	74	120	71	85	51	0
N.S.	1	1.00	0.96	0.85	1.01	1.64	0.97	1.16	0.70	0.00
time (sec)	N/A	0.314	4.702	0.230	0.111	0.087	3.130	0.121	0.260	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	78	69	70	93	134	92	56	59	0
N.S.	1	1.11	0.99	1.00	1.33	1.91	1.31	0.80	0.84	0.00
time (sec)	N/A	0.308	4.824	0.253	0.109	0.085	2.868	0.134	0.217	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	79	67	117	156	76	70	73	0
N.S.	1	1.06	1.03	0.87	1.52	2.03	0.99	0.91	0.95	0.00
time (sec)	N/A	0.302	4.801	0.258	0.104	0.087	3.001	0.144	0.236	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	111	93	81	159	177	124	80	90	0
N.S.	1	1.08	0.90	0.79	1.54	1.72	1.20	0.78	0.87	0.00
time (sec)	N/A	0.339	4.740	0.263	0.104	0.083	6.843	0.145	0.209	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	60	53	69	82	0	300	74	78
N.S.	1	1.12	0.60	0.53	0.69	0.82	0.00	3.00	0.74	0.78
time (sec)	N/A	0.352	5.390	0.261	0.035	0.074	0.000	0.127	0.204	0.767

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	49	42	52	71	423	252	63	67
N.S.	1	1.08	0.66	0.57	0.70	0.96	5.72	3.41	0.85	0.91
time (sec)	N/A	0.304	5.350	0.244	0.026	0.072	159.967	0.128	0.219	0.730

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	31	35	59	114	201	51	56
N.S.	1	1.00	0.79	0.65	0.73	1.23	2.38	4.19	1.06	1.17
time (sec)	N/A	0.259	5.380	0.244	0.068	0.072	48.824	0.130	0.242	0.661

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	30	23	17	46	88	140	38	45
N.S.	1	1.00	1.30	1.00	0.74	2.00	3.83	6.09	1.65	1.96
time (sec)	N/A	0.222	5.471	0.135	0.028	0.074	19.061	0.125	0.222	0.628

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	96	82	81	91	146	97	99	72	0
N.S.	1	0.97	0.83	0.82	0.92	1.47	0.98	1.00	0.73	0.00
time (sec)	N/A	0.328	5.474	0.237	0.118	0.081	12.993	0.128	0.212	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	104	86	82	111	158	99	73	78	0
N.S.	1	1.06	0.88	0.84	1.13	1.61	1.01	0.74	0.80	0.00
time (sec)	N/A	0.328	5.357	0.271	0.114	0.077	8.285	0.149	0.271	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	104	90	80	137	165	126	88	84	0
N.S.	1	1.02	0.88	0.78	1.34	1.62	1.24	0.86	0.82	0.00
time (sec)	N/A	0.327	5.259	0.279	0.106	0.080	7.084	0.145	0.212	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	90	78	156	178	104	75	90	0
N.S.	1	1.08	0.90	0.78	1.56	1.78	1.04	0.75	0.90	0.00
time (sec)	N/A	0.316	5.322	0.271	0.111	0.083	7.328	0.143	0.214	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	104	92	194	199	155	109	107	0
N.S.	1	1.09	0.83	0.73	1.54	1.58	1.23	0.87	0.85	0.00
time (sec)	N/A	0.379	5.331	0.273	0.111	0.080	12.249	0.150	0.220	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	144	64	61	86	60	692	76	52	61
N.S.	1	1.14	0.51	0.48	0.68	0.48	5.49	0.60	0.41	0.48
time (sec)	N/A	0.408	3.160	0.120	0.032	0.073	15.952	0.126	0.273	0.758

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	112	53	50	69	49	452	64	41	50
N.S.	1	1.12	0.53	0.50	0.69	0.49	4.52	0.64	0.41	0.50
time (sec)	N/A	0.361	3.253	0.119	0.036	0.075	5.453	0.126	0.219	0.686

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	42	39	52	38	260	52	30	39
N.S.	1	1.08	0.57	0.53	0.70	0.51	3.51	0.70	0.41	0.53
time (sec)	N/A	0.313	3.177	0.115	0.030	0.089	1.721	0.124	0.223	0.677

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	30	27	34	26	42	38	18	28
N.S.	1	1.00	0.62	0.56	0.71	0.54	0.88	0.79	0.38	0.58
time (sec)	N/A	0.263	3.230	0.115	0.038	0.069	0.771	0.126	0.218	0.632

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	17	19	17	27	11	17
N.S.	1	1.00	1.00	0.95	0.81	0.90	0.81	1.29	0.52	0.81
time (sec)	N/A	0.225	3.130	0.088	0.027	0.068	0.695	0.125	0.250	0.586

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	39	44	74	24	45	31	0
N.S.	1	1.00	1.07	1.30	1.47	2.47	0.80	1.50	1.03	0.00
time (sec)	N/A	0.241	1.872	0.230	0.104	0.080	1.344	0.125	0.239	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	59	54	55	80	125	44	48	51	0
N.S.	1	1.13	1.04	1.06	1.54	2.40	0.85	0.92	0.98	0.00
time (sec)	N/A	0.283	2.765	0.245	0.106	0.079	3.775	0.131	0.211	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	91	82	74	122	155	102	73	73	0
N.S.	1	1.10	0.99	0.89	1.47	1.87	1.23	0.88	0.88	0.00
time (sec)	N/A	0.317	8.814	0.258	0.106	0.086	11.171	0.136	0.242	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	123	106	86	161	177	129	76	90	0
N.S.	1	1.13	0.97	0.79	1.48	1.62	1.18	0.70	0.83	0.00
time (sec)	N/A	0.354	10.076	0.251	0.106	0.096	33.998	0.141	0.214	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	140	71	64	90	70	614	101	54	67
N.S.	1	1.16	0.59	0.53	0.74	0.58	5.07	0.83	0.45	0.55
time (sec)	N/A	0.414	3.998	0.151	0.027	0.071	6.920	0.123	0.226	0.883

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	108	59	52	72	58	320	79	42	59
N.S.	1	1.14	0.62	0.55	0.76	0.61	3.37	0.83	0.44	0.62
time (sec)	N/A	0.360	3.975	0.152	0.026	0.071	2.082	0.125	0.212	0.765

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	76	48	41	55	47	206	70	31	49
N.S.	1	1.10	0.70	0.59	0.80	0.68	2.99	1.01	0.45	0.71
time (sec)	N/A	0.312	3.996	0.149	0.027	0.074	1.346	0.122	0.245	0.698

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	44	35	30	36	36	39	48	20	37
N.S.	1	1.02	0.81	0.70	0.84	0.84	0.91	1.12	0.47	0.86
time (sec)	N/A	0.266	3.917	0.138	0.032	0.074	1.483	0.128	0.223	0.663

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	28	23	17	27	19	27	13	25
N.S.	1	1.00	1.33	1.10	0.81	1.29	0.90	1.29	0.62	1.19
time (sec)	N/A	0.234	3.966	0.117	0.033	0.067	2.243	0.121	0.218	0.638

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	64	53	62	142	146	77	57	0
N.S.	1	1.02	1.23	1.02	1.19	2.73	2.81	1.48	1.10	0.00
time (sec)	N/A	0.286	3.987	0.241	0.106	0.078	6.390	0.130	0.212	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	86	72	61	101	183	73	72	73	0
N.S.	1	1.16	0.97	0.82	1.36	2.47	0.99	0.97	0.99	0.00
time (sec)	N/A	0.338	9.671	0.276	0.103	0.081	18.200	0.140	0.261	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	118	56	78	143	221	107	92	92	0
N.S.	1	1.13	0.54	0.75	1.38	2.12	1.03	0.88	0.88	0.00
time (sec)	N/A	0.363	10.012	0.279	0.107	0.082	49.130	0.144	0.229	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	150	56	89	181	243	134	107	103	0
N.S.	1	1.15	0.43	0.68	1.39	1.87	1.03	0.82	0.79	0.00
time (sec)	N/A	0.398	10.014	0.284	0.121	0.087	179.624	0.154	0.233	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	168	82	75	106	92	799	108	72	78
N.S.	1	1.15	0.56	0.51	0.73	0.63	5.47	0.74	0.49	0.53
time (sec)	N/A	0.467	4.781	0.163	0.025	0.085	7.414	0.125	0.247	0.948

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	136	71	64	89	81	536	98	61	81
N.S.	1	1.13	0.59	0.53	0.74	0.68	4.47	0.82	0.51	0.68
time (sec)	N/A	0.413	4.740	0.165	0.027	0.085	4.523	0.129	0.218	0.825

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	59	52	71	69	320	77	49	71
N.S.	1	1.13	0.64	0.57	0.77	0.75	3.48	0.84	0.53	0.77
time (sec)	N/A	0.360	4.747	0.161	0.026	0.081	3.095	0.114	0.215	0.764

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	74	49	42	52	59	151	62	39	43
N.S.	1	1.06	0.70	0.60	0.74	0.84	2.16	0.89	0.56	0.61
time (sec)	N/A	0.304	4.765	0.154	0.032	0.076	3.853	0.127	0.210	0.756

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	46	38	31	31	48	94	35	28	49
N.S.	1	0.98	0.81	0.66	0.66	1.02	2.00	0.74	0.60	1.04
time (sec)	N/A	0.265	4.754	0.082	0.027	0.077	6.273	0.124	0.270	0.793

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	30	23	17	38	42	27	20	38
N.S.	1	1.00	1.30	1.00	0.74	1.65	1.83	1.17	0.87	1.65
time (sec)	N/A	0.224	4.800	0.079	0.026	0.067	10.628	0.120	0.223	0.707

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	94	85	74	209	697	88	121	0
N.S.	1	1.08	1.25	1.13	0.99	2.79	9.29	1.17	1.61	0.00
time (sec)	N/A	0.311	10.126	0.139	0.105	0.079	30.966	0.134	0.217	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	117	56	90	119	253	818	77	147	0
N.S.	1	1.18	0.57	0.91	1.20	2.56	8.26	0.78	1.48	0.00
time (sec)	N/A	0.356	10.015	0.303	0.105	0.094	104.780	0.149	0.209	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	149	56	100	163	287	0	105	166	0
N.S.	1	1.16	0.43	0.78	1.26	2.22	0.00	0.81	1.29	0.00
time (sec)	N/A	0.399	10.014	0.321	0.112	0.091	0.000	0.154	0.251	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.246	0.002	0.247	0.027	0.065	0.024	0.117	0.210	0.028

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.248	0.002	0.220	0.034	0.063	0.024	0.126	0.238	0.025

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.244	0.002	0.246	0.026	0.065	0.023	0.119	0.241	0.024

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	14	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.82	0.76
time (sec)	N/A	0.243	0.002	0.229	0.041	0.069	0.022	0.118	0.243	0.025

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.239	0.000	0.080	0.031	0.063	0.019	0.118	0.224	0.020

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	12	11	11
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	0.92	0.85	0.85
time (sec)	N/A	0.241	0.003	0.073	0.024	0.062	0.047	0.120	0.206	0.028

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	13	10
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.30	1.00
time (sec)	N/A	0.225	0.000	0.052	0.025	0.068	0.048	0.115	0.237	0.029

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	17	11
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	1.31	0.85
time (sec)	N/A	0.251	0.003	0.062	0.030	0.066	0.058	0.125	0.225	0.048

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	1.00	0.87
time (sec)	N/A	0.247	0.002	0.060	0.026	0.060	0.062	0.123	0.263	0.031

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	17	17	14	14	13	14	13	15	13
N.S.	1	1.06	1.06	0.88	0.88	0.81	0.88	0.81	0.94	0.81
time (sec)	N/A	0.236	0.002	0.059	0.029	0.068	0.071	0.128	0.210	0.037

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.238	0.002	0.059	0.031	0.070	0.071	0.122	0.236	0.031

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.234	0.002	0.063	0.024	0.061	0.082	0.119	0.212	0.031

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.234	0.002	0.068	0.024	0.066	0.100	0.120	0.204	0.032

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.269	0.001	0.238	0.025	0.067	0.024	0.116	0.216	0.268

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.271	0.001	0.220	0.025	0.069	0.028	0.119	0.260	0.037

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	24	24	24	24	25	24
N.S.	1	1.00	1.00	0.94	1.50	1.50	1.50	1.50	1.56	1.50
time (sec)	N/A	0.223	0.002	0.239	0.029	0.062	0.028	0.123	0.228	0.037

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	23	22	24	21	20	22	21	21
N.S.	1	1.30	1.00	0.96	1.04	0.91	0.87	0.96	0.91	0.91
time (sec)	N/A	0.268	0.001	0.185	0.027	0.070	0.057	0.120	0.214	0.035

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	24	27	24	24	27	23
N.S.	1	1.04	1.00	0.89	0.89	1.00	0.89	0.89	1.00	0.85
time (sec)	N/A	0.268	0.005	0.081	0.025	0.062	0.063	0.118	0.209	0.266

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	30	24	23	26	28	24	34	28	24
N.S.	1	1.25	1.00	0.96	1.08	1.17	1.00	1.42	1.17	1.00
time (sec)	N/A	0.276	0.001	0.064	0.052	0.064	0.092	0.116	0.235	0.051

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	25	14	24	26	24	26	26
N.S.	1	1.00	1.88	1.56	0.88	1.50	1.62	1.50	1.62	1.62
time (sec)	N/A	0.223	0.001	0.080	0.028	0.065	0.147	0.122	0.201	0.041

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.286	0.001	0.069	0.025	0.065	0.184	0.123	0.207	0.044

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.287	0.001	0.070	0.026	0.063	0.127	0.125	0.270	0.043

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.277	0.001	0.237	0.026	0.062	0.022	0.124	0.231	0.040

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.267	0.001	0.043	0.030	0.070	0.024	0.120	0.207	0.036

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	25	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	1.04	0.92
time (sec)	N/A	0.267	0.001	0.051	0.032	0.063	0.048	0.116	0.205	0.041

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	26	22	22	26	24
N.S.	1	1.00	1.00	0.96	0.91	1.13	0.96	0.96	1.13	1.04
time (sec)	N/A	0.260	0.004	0.049	0.033	0.067	0.079	0.119	0.217	0.031

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.93	0.89
time (sec)	N/A	0.270	0.001	0.039	0.030	0.075	0.101	0.113	0.216	0.041

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.272	0.001	0.039	0.029	0.064	0.106	0.126	0.227	0.040

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.271	0.001	0.040	0.030	0.067	0.137	0.126	0.216	0.041

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	39	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.296	0.002	0.145	0.025	0.071	0.025	0.119	0.242	0.049

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	43	36	35	35	37	35	37	35
N.S.	1	1.12	1.26	1.06	1.03	1.03	1.09	1.03	1.09	1.03
time (sec)	N/A	0.284	0.002	0.128	0.025	0.066	0.031	0.119	0.226	0.050

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	35	35	37	35	36	35
N.S.	1	1.00	1.00	0.94	2.19	2.19	2.31	2.19	2.25	2.19
time (sec)	N/A	0.235	0.002	0.135	0.032	0.059	0.028	0.124	0.226	0.049

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	36	33	37	36	33	33
N.S.	1	1.10	1.00	0.87	0.92	0.85	0.95	0.92	0.85	0.85
time (sec)	N/A	0.298	0.004	0.057	0.030	0.065	0.055	0.109	0.207	0.040

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	35	36	38	37	35	38	34
N.S.	1	1.05	1.00	0.88	0.90	0.95	0.92	0.88	0.95	0.85
time (sec)	N/A	0.302	0.006	0.108	0.030	0.061	0.073	0.123	0.241	0.042

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	41	40	35	37	39	37	36	39	37
N.S.	1	1.02	1.00	0.88	0.92	0.98	0.92	0.90	0.98	0.92
time (sec)	N/A	0.300	0.005	0.055	0.031	0.065	0.116	0.120	0.225	0.039

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	39	39	37	47	39	36
N.S.	1	1.10	1.00	0.87	1.00	1.00	0.95	1.21	1.00	0.92
time (sec)	N/A	0.302	0.004	0.042	0.030	0.062	0.168	0.121	0.222	0.279

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	43	36	14	35	37	35	37	37
N.S.	1	1.00	2.69	2.25	0.88	2.19	2.31	2.19	2.31	2.31
time (sec)	N/A	0.232	0.005	0.045	0.028	0.065	0.176	0.117	0.210	0.035

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	44	43	36	37	37	39	37	37	37
N.S.	1	1.29	1.26	1.06	1.09	1.09	1.15	1.09	1.09	1.09
time (sec)	N/A	0.269	0.004	0.047	0.031	0.062	0.159	0.129	0.236	0.035

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	37	37	39	37	37	37
N.S.	1	1.09	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.327	0.004	0.046	0.024	0.064	0.173	0.116	0.245	0.037

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.301	0.002	0.135	0.041	0.075	0.028	0.110	0.219	0.048

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	35	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	1.00	0.89
time (sec)	N/A	0.280	0.001	0.043	0.028	0.068	0.024	0.118	0.240	0.046

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	36	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	1.06	0.94
time (sec)	N/A	0.282	0.004	0.053	0.029	0.061	0.060	0.116	0.225	0.050

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36	36
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97	0.97
time (sec)	N/A	0.286	0.004	0.052	0.027	0.058	0.081	0.112	0.204	0.043

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	37	34	33	37	34
N.S.	1	1.00	1.00	0.97	0.94	1.09	1.00	0.97	1.09	1.00
time (sec)	N/A	0.282	0.004	0.052	0.026	0.062	0.110	0.124	0.197	0.282

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	37	35
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.95	0.90
time (sec)	N/A	0.287	0.003	0.053	0.028	0.065	0.162	0.119	0.256	0.035

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.285	0.004	0.048	0.032	0.060	0.158	0.119	0.216	0.035

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.286	0.006	0.045	0.032	0.066	0.183	0.123	0.216	0.038

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	53	46	46	45	44	47	45	45
N.S.	1	0.98	1.00	0.87	0.87	0.85	0.83	0.89	0.85	0.85
time (sec)	N/A	0.310	0.004	0.069	0.030	0.065	0.099	0.119	0.225	0.056

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	40	34	34	33	32	35	33	33
N.S.	1	0.98	1.00	0.85	0.85	0.82	0.80	0.88	0.82	0.82
time (sec)	N/A	0.289	0.005	0.120	0.038	0.060	0.077	0.126	0.247	0.276

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	22	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	0.81	0.81
time (sec)	N/A	0.274	0.004	0.057	0.030	0.064	0.086	0.125	0.200	0.044

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.87
time (sec)	N/A	0.228	0.002	0.033	0.029	0.068	0.058	0.119	0.204	0.263

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	22	14	13	18	15	24	20	18
N.S.	1	1.00	1.47	0.93	0.87	1.20	1.00	1.60	1.33	1.20
time (sec)	N/A	0.230	0.005	0.051	0.026	0.069	0.112	0.114	0.241	0.076

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	36	35	32	33	33	31	43	33	31
N.S.	1	1.33	1.30	1.19	1.22	1.22	1.15	1.59	1.22	1.15
time (sec)	N/A	0.299	0.006	0.047	0.033	0.071	0.154	0.133	0.245	0.335

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	50	49	44	47	45	42	57	47	46
N.S.	1	1.25	1.22	1.10	1.18	1.12	1.05	1.42	1.18	1.15
time (sec)	N/A	0.310	0.006	0.048	0.027	0.071	0.191	0.121	0.222	0.315

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	62	54
N.S.	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.91	0.79
time (sec)	N/A	0.327	0.020	0.159	0.105	0.077	0.117	0.124	0.206	0.293

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	51	43
N.S.	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.93	0.78
time (sec)	N/A	0.306	0.017	0.060	0.104	0.076	0.111	0.124	0.210	0.071

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	37	32
N.S.	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.88	0.76
time (sec)	N/A	0.286	0.015	0.059	0.107	0.080	0.104	0.116	0.250	0.287

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.84	0.74
time (sec)	N/A	0.252	0.008	0.052	0.114	0.075	0.078	0.124	0.205	0.043

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	23	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.96	0.67
time (sec)	N/A	0.233	0.004	0.039	0.109	0.071	0.102	0.122	0.221	0.052

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	30	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.88	0.76
time (sec)	N/A	0.254	0.010	0.075	0.107	0.072	0.127	0.121	0.250	0.279

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	51	43	39	40	106	87	40	43	37
N.S.	1	1.19	1.00	0.91	0.93	2.47	2.02	0.93	1.00	0.86
time (sec)	N/A	0.276	0.016	0.057	0.110	0.073	0.167	0.120	0.227	0.057

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	68	58	52	52	132	100	52	56	48
N.S.	1	1.17	1.00	0.90	0.90	2.28	1.72	0.90	0.97	0.83
time (sec)	N/A	0.283	0.019	0.088	0.108	0.076	0.172	0.123	0.208	0.302

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	60	63	65	81	66	80	81	68
N.S.	1	0.97	0.86	0.90	0.93	1.16	0.94	1.14	1.16	0.97
time (sec)	N/A	0.330	0.017	0.064	0.031	0.070	0.164	0.126	0.194	0.290

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	56	49	54	54	70	53	55	70	57
N.S.	1	0.98	0.86	0.95	0.95	1.23	0.93	0.96	1.23	1.00
time (sec)	N/A	0.311	0.013	0.145	0.029	0.069	0.155	0.114	0.234	0.281

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	38	41	43	56	39	41	57	45
N.S.	1	0.98	0.86	0.93	0.98	1.27	0.89	0.93	1.30	1.02
time (sec)	N/A	0.294	0.014	0.056	0.025	0.065	0.162	0.121	0.246	0.050

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	27	30	32	35	29	32	44	29
N.S.	1	0.94	0.82	0.91	0.97	1.06	0.88	0.97	1.33	0.88
time (sec)	N/A	0.285	0.007	0.046	0.028	0.062	0.124	0.123	0.213	0.266

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	17	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	1.06	0.88
time (sec)	N/A	0.228	0.002	0.050	0.024	0.066	0.078	0.115	0.214	0.028

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	39	33	35	37	47	34	47	59	34
N.S.	1	1.18	1.00	1.06	1.12	1.42	1.03	1.42	1.79	1.03
time (sec)	N/A	0.302	0.011	0.052	0.036	0.070	0.190	0.120	0.242	0.064

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	52	41	54	52	73	51	51	81	51
N.S.	1	1.21	0.95	1.26	1.21	1.70	1.19	1.19	1.88	1.19
time (sec)	N/A	0.318	0.027	0.058	0.026	0.078	0.198	0.123	0.205	0.071

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	68	57	65	70	90	68	86	94	67
N.S.	1	1.21	1.02	1.16	1.25	1.61	1.21	1.54	1.68	1.20
time (sec)	N/A	0.342	0.040	0.062	0.027	0.071	0.292	0.127	0.205	0.298

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	82	74	82	212	134	84	107	77
N.S.	1	1.08	0.93	0.84	0.93	2.41	1.52	0.95	1.22	0.88
time (sec)	N/A	0.352	0.039	0.073	0.109	0.080	0.195	0.123	0.255	0.279

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	71	65	71	190	124	73	96	66
N.S.	1	1.05	0.91	0.83	0.91	2.44	1.59	0.94	1.23	0.85
time (sec)	N/A	0.334	0.035	0.066	0.109	0.077	0.178	0.113	0.250	0.279

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	60	53	59	164	107	61	83	56
N.S.	1	1.06	0.92	0.82	0.91	2.52	1.65	0.94	1.28	0.86
time (sec)	N/A	0.317	0.031	0.062	0.105	0.074	0.181	0.120	0.213	0.080

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	58	51	42	45	136	83	42	69	43
N.S.	1	1.14	1.00	0.82	0.88	2.67	1.63	0.82	1.35	0.84
time (sec)	N/A	0.267	0.023	0.062	0.115	0.070	0.150	0.119	0.227	0.301

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	62	33
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	1.38	0.73
time (sec)	N/A	0.253	0.015	0.053	0.107	0.073	0.131	0.127	0.195	0.278

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	61	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	1.36	0.73
time (sec)	N/A	0.247	0.017	0.048	0.104	0.078	0.133	0.107	0.220	0.049

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	54	45	49	136	92	47	72	44
N.S.	1	1.13	1.00	0.83	0.91	2.52	1.70	0.87	1.33	0.81
time (sec)	N/A	0.282	0.029	0.065	0.103	0.072	0.188	0.120	0.207	0.298

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	67	55	64	172	114	59	88	58
N.S.	1	1.16	1.00	0.82	0.96	2.57	1.70	0.88	1.31	0.87
time (sec)	N/A	0.301	0.027	0.072	0.105	0.079	0.219	0.122	0.217	0.088

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	71	76	89	115	92	92	119	90
N.S.	1	1.02	0.82	0.87	1.02	1.32	1.06	1.06	1.37	1.03
time (sec)	N/A	0.368	0.037	0.075	0.028	0.073	0.265	0.124	0.245	0.321

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	58	65	77	103	78	69	107	78
N.S.	1	1.03	0.78	0.88	1.04	1.39	1.05	0.93	1.45	1.05
time (sec)	N/A	0.346	0.033	0.160	0.029	0.063	0.286	0.121	0.231	0.351

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	62	48	54	66	91	68	53	95	68
N.S.	1	0.95	0.74	0.83	1.02	1.40	1.05	0.82	1.46	1.05
time (sec)	N/A	0.332	0.041	0.062	0.050	0.067	0.198	0.124	0.214	0.312

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	51	39	42	55	69	53	42	81	52
N.S.	1	1.04	0.80	0.86	1.12	1.41	1.08	0.86	1.65	1.06
time (sec)	N/A	0.317	0.012	0.049	0.027	0.067	0.217	0.118	0.242	0.067

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	15	14	36	36	22	28	37
N.S.	1	1.00	1.50	0.94	0.88	2.25	2.25	1.38	1.75	2.31
time (sec)	N/A	0.231	0.006	0.059	0.026	0.061	0.159	0.120	0.251	0.037

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	26	26	27	14	25	28
N.S.	1	1.00	1.00	0.94	1.62	1.62	1.69	0.88	1.56	1.75
time (sec)	N/A	0.227	0.002	0.043	0.029	0.059	0.135	0.112	0.229	0.255

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	55	43	46	60	90	56	59	109	56
N.S.	1	1.10	0.86	0.92	1.20	1.80	1.12	1.18	2.18	1.12
time (sec)	N/A	0.321	0.022	0.061	0.031	0.071	0.269	0.126	0.221	0.300

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	59	66	77	119	80	82	133	75
N.S.	1	1.06	0.91	1.02	1.18	1.83	1.23	1.26	2.05	1.15
time (sec)	N/A	0.349	0.042	0.066	0.035	0.072	0.360	0.115	0.197	0.310

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	88	74	77	92	134	90	80	148	88
N.S.	1	1.19	1.00	1.04	1.24	1.81	1.22	1.08	2.00	1.19
time (sec)	N/A	0.390	0.034	0.070	0.026	0.066	0.406	0.120	0.235	0.322

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	109	88	74	93	256	144	84	145	87
N.S.	1	1.14	0.92	0.77	0.97	2.67	1.50	0.88	1.51	0.91
time (sec)	N/A	0.368	0.034	0.078	0.112	0.083	0.281	0.112	0.202	0.060

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	96	77	62	82	230	133	73	132	77
N.S.	1	1.13	0.91	0.73	0.96	2.71	1.56	0.86	1.55	0.91
time (sec)	N/A	0.359	0.038	0.070	0.107	0.089	0.250	0.119	0.210	0.082

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	66	51	68	202	107	54	118	64
N.S.	1	1.20	0.93	0.72	0.96	2.85	1.51	0.76	1.66	0.90
time (sec)	N/A	0.305	0.034	0.066	0.108	0.073	0.259	0.117	0.262	0.338

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	72	55	47	59	188	110	45	113	56
N.S.	1	1.12	0.86	0.73	0.92	2.94	1.72	0.70	1.77	0.88
time (sec)	N/A	0.288	0.032	0.058	0.119	0.074	0.235	0.120	0.239	0.329

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	58	49	62	190	110	50	110	55
N.S.	1	1.08	0.89	0.75	0.95	2.92	1.69	0.77	1.69	0.85
time (sec)	N/A	0.286	0.021	0.061	0.106	0.071	0.176	0.126	0.227	0.303

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	70	55	57	58	188	105	45	113	55
N.S.	1	1.13	0.89	0.92	0.94	3.03	1.69	0.73	1.82	0.89
time (sec)	N/A	0.279	0.024	0.053	0.107	0.082	0.187	0.123	0.208	0.294

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	88	68	54	71	202	116	57	121	66
N.S.	1	1.22	0.94	0.75	0.99	2.81	1.61	0.79	1.68	0.92
time (sec)	N/A	0.304	0.029	0.098	0.106	0.074	0.260	0.125	0.210	0.343

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	105	79	64	86	238	138	71	137	80
N.S.	1	1.21	0.91	0.74	0.99	2.74	1.59	0.82	1.57	0.92
time (sec)	N/A	0.333	0.033	0.100	0.106	0.080	0.307	0.125	0.232	0.346

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	94	78	100	152	92	69	60	54
N.S.	1	1.03	1.32	1.10	1.41	2.14	1.30	0.97	0.85	0.76
time (sec)	N/A	0.298	0.199	0.124	0.109	0.081	2.549	0.133	0.239	0.844

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	26	23	17	27	41	27	20	23
N.S.	1	1.00	1.24	1.10	0.81	1.29	1.95	1.29	0.95	1.10
time (sec)	N/A	0.234	0.004	0.062	0.041	0.073	0.493	0.119	0.223	0.405

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	45	58	62	53	127	41	52	40	35
N.S.	1	0.96	1.23	1.32	1.13	2.70	0.87	1.11	0.85	0.74
time (sec)	N/A	0.262	0.048	0.068	0.102	0.090	1.044	0.136	0.241	0.631

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	63	53	102	56	69	60	55
N.S.	1	1.00	1.48	1.50	1.26	2.43	1.33	1.64	1.43	1.31
time (sec)	N/A	0.268	0.037	0.056	0.113	0.080	0.877	0.119	0.221	0.335

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	43	62	60	49	109	56	61	43	30
N.S.	1	1.13	1.63	1.58	1.29	2.87	1.47	1.61	1.13	0.79
time (sec)	N/A	0.282	0.062	0.074	0.104	0.086	0.910	0.139	0.200	0.525

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	72	73	123	42	47	79	39
N.S.	1	1.00	1.20	1.44	1.46	2.46	0.84	0.94	1.58	0.78
time (sec)	N/A	0.267	0.059	0.080	0.106	0.083	1.096	0.142	0.204	0.652

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	42	63	42	24
N.S.	1	1.00	1.00	0.83	0.78	1.56	2.33	3.50	2.33	1.33
time (sec)	N/A	0.231	0.051	0.146	0.032	0.076	0.582	0.138	0.272	0.478

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	76	86	114	152	92	78	100	0
N.S.	1	1.04	1.03	1.16	1.54	2.05	1.24	1.05	1.35	0.00
time (sec)	N/A	0.317	0.092	0.089	0.108	0.089	2.186	0.141	0.220	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	69	77	98	157	70	68	61	52
N.S.	1	1.03	1.00	1.12	1.42	2.28	1.01	0.99	0.88	0.75
time (sec)	N/A	0.299	0.082	0.079	0.119	0.089	2.027	0.126	0.214	0.925

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	74	78	68	123	78	89	79	0
N.S.	1	0.98	1.14	1.20	1.05	1.89	1.20	1.37	1.22	0.00
time (sec)	N/A	0.304	0.048	0.066	0.105	0.085	1.412	0.127	0.204	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	64	86	71	66	129	88	79	62	48
N.S.	1	1.03	1.39	1.15	1.06	2.08	1.42	1.27	1.00	0.77
time (sec)	N/A	0.287	0.136	0.083	0.108	0.092	1.598	0.158	0.245	0.884

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	79	86	86	136	88	63	91	36
N.S.	1	1.03	1.22	1.32	1.32	2.09	1.35	0.97	1.40	0.55
time (sec)	N/A	0.285	0.068	0.086	0.104	0.082	1.677	0.136	0.223	0.588

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	62	78	71	61	141	78	122	58	42
N.S.	1	1.15	1.44	1.31	1.13	2.61	1.44	2.26	1.07	0.78
time (sec)	N/A	0.283	0.093	0.084	0.124	0.082	1.625	0.350	0.248	0.665

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	74	70	84	113	157	71	76	101	39
N.S.	1	1.04	0.99	1.18	1.59	2.21	1.00	1.07	1.42	0.55
time (sec)	N/A	0.282	0.108	0.097	0.108	0.083	1.708	0.135	0.233	0.821

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	15	14	39	68	92	63	26
N.S.	1	1.00	1.56	0.83	0.78	2.17	3.78	5.11	3.50	1.44
time (sec)	N/A	0.220	0.064	0.157	0.025	0.073	0.625	0.151	0.232	0.747

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	89	98	155	178	119	88	120	0
N.S.	1	1.06	0.94	1.03	1.63	1.87	1.25	0.93	1.26	0.00
time (sec)	N/A	0.332	0.117	0.092	0.112	0.086	3.886	0.144	0.220	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	89	92	85	115	157	117	95	85	72
N.S.	1	1.01	1.05	0.97	1.31	1.78	1.33	1.08	0.97	0.82
time (sec)	N/A	0.314	0.099	0.109	0.112	0.082	2.903	0.167	0.202	1.217

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	79	107	105	159	112	81	112	0
N.S.	1	1.00	0.86	1.16	1.14	1.73	1.22	0.88	1.22	0.00
time (sec)	N/A	0.330	0.090	0.108	0.110	0.090	2.540	0.135	0.234	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	90	86	81	170	112	142	87	66
N.S.	1	1.01	1.11	1.06	1.00	2.10	1.38	1.75	1.07	0.81
time (sec)	N/A	0.311	0.224	0.101	0.106	0.081	2.962	0.368	0.243	1.041

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	94	100	130	166	117	96	114	36
N.S.	1	1.00	1.03	1.10	1.43	1.82	1.29	1.05	1.25	0.40
time (sec)	N/A	0.317	0.093	0.099	0.110	0.088	2.709	0.141	0.235	0.737

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	89	84	75	169	105	180	89	60
N.S.	1	1.11	1.24	1.17	1.04	2.35	1.46	2.50	1.24	0.83
time (sec)	N/A	0.316	0.114	0.097	0.106	0.082	2.834	0.761	0.218	0.893

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	98	81	95	152	179	99	83	120	39
N.S.	1	1.07	0.88	1.03	1.65	1.95	1.08	0.90	1.30	0.42
time (sec)	N/A	0.323	0.128	0.102	0.110	0.082	2.870	0.141	0.250	1.110

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	15	14	50	88	121	82	68
N.S.	1	1.00	1.56	0.83	0.78	2.78	4.89	6.72	4.56	3.78
time (sec)	N/A	0.227	0.081	0.187	0.036	0.080	0.615	0.144	0.205	1.057

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	125	100	109	190	200	150	119	139	0
N.S.	1	1.08	0.86	0.94	1.64	1.72	1.29	1.03	1.20	0.00
time (sec)	N/A	0.379	0.147	0.122	0.108	0.094	7.616	0.148	0.271	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	97	86	104	157	95	79	61	57
N.S.	1	1.07	1.31	1.16	1.41	2.12	1.28	1.07	0.82	0.77
time (sec)	N/A	0.301	0.157	0.083	0.108	0.086	3.265	0.128	0.201	0.843

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	81	66	68	126	42	59	41	38
N.S.	1	0.98	1.62	1.32	1.36	2.52	0.84	1.18	0.82	0.76
time (sec)	N/A	0.280	0.125	0.078	0.109	0.086	1.293	0.117	0.240	0.725

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	50	46	37	80	17	38	25	18
N.S.	1	1.00	2.08	1.92	1.54	3.33	0.71	1.58	1.04	0.75
time (sec)	N/A	0.258	0.013	0.056	0.105	0.076	0.667	0.123	0.219	0.571

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	26	34	23	14
N.S.	1	1.00	1.00	0.94	0.88	1.12	1.62	2.12	1.44	0.88
time (sec)	N/A	0.233	0.033	0.148	0.030	0.073	0.338	0.132	0.211	0.421

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	31	35	29	31	231	59	42	25
N.S.	1	1.14	0.89	1.00	0.83	0.89	6.60	1.69	1.20	0.71
time (sec)	N/A	0.283	0.048	0.072	0.031	0.072	0.868	0.128	0.222	0.455

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	42	46	47	42	750	81	63	56
N.S.	1	1.07	0.74	0.81	0.82	0.74	13.16	1.42	1.11	0.98
time (sec)	N/A	0.305	0.057	0.073	0.025	0.078	1.584	0.134	0.256	0.499

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	53	57	63	53	1969	107	82	73
N.S.	1	1.07	0.71	0.76	0.84	0.71	26.25	1.43	1.09	0.97
time (sec)	N/A	0.338	0.062	0.076	0.034	0.080	1.914	0.136	0.267	0.540

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	72	40	45	50	40	279	64	34	38
N.S.	1	1.09	0.61	0.68	0.76	0.61	4.23	0.97	0.52	0.58
time (sec)	N/A	0.308	0.028	0.070	0.027	0.085	0.769	0.124	0.261	0.755

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	33	32	28	46	48	22	25
N.S.	1	1.00	0.67	0.79	0.76	0.67	1.10	1.14	0.52	0.60
time (sec)	N/A	0.262	0.026	0.071	0.026	0.075	0.543	0.126	0.280	0.636

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	23	14	18	17	28	12	39
N.S.	1	1.00	1.00	1.44	0.88	1.12	1.06	1.75	0.75	2.44
time (sec)	N/A	0.222	0.004	0.058	0.031	0.073	0.435	0.122	0.242	0.362

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	50	52	41	73	19	46	53	23
N.S.	1	1.00	1.79	1.86	1.46	2.61	0.68	1.64	1.89	0.82
time (sec)	N/A	0.253	0.026	0.061	0.106	0.081	0.659	0.122	0.228	0.626

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	72	73	76	122	42	52	79	60
N.S.	1	1.00	1.36	1.38	1.43	2.30	0.79	0.98	1.49	1.13
time (sec)	N/A	0.291	0.051	0.078	0.105	0.084	1.381	0.126	0.240	0.811

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	56	50	21	88	46	45	15	21
N.S.	1	1.00	2.07	1.85	0.78	3.26	1.70	1.67	0.56	0.78
time (sec)	N/A	0.278	0.023	0.115	0.107	0.089	0.665	0.129	0.277	0.562

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	22	50	52	41	69	20	46	53	17
N.S.	1	1.10	2.50	2.60	2.05	3.45	1.00	2.30	2.65	0.85
time (sec)	N/A	0.237	0.026	0.105	0.107	0.078	0.625	0.128	0.254	0.464

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	22	54	63	20	72	48	42	28	21
N.S.	1	1.10	2.70	3.15	1.00	3.60	2.40	2.10	1.40	1.05
time (sec)	N/A	0.242	0.027	0.112	0.107	0.077	0.791	0.120	0.252	0.450

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	104	97	87	122	221	100	96	137	75
N.S.	1	1.09	1.02	0.92	1.28	2.33	1.05	1.01	1.44	0.79
time (sec)	N/A	0.333	0.226	0.097	0.113	0.093	4.050	0.126	0.231	1.114

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	83	74	86	192	71	74	114	53
N.S.	1	1.06	1.20	1.07	1.25	2.78	1.03	1.07	1.65	0.77
time (sec)	N/A	0.300	0.177	0.092	0.112	0.108	1.909	0.133	0.286	0.861

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	71	63	52	163	187	57	88	33
N.S.	1	1.12	1.73	1.54	1.27	3.98	4.56	1.39	2.15	0.80
time (sec)	N/A	0.284	0.086	0.068	0.108	0.081	1.070	0.133	0.237	0.575

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	29	26	18	39	18
N.S.	1	1.00	1.00	0.93	0.87	1.93	1.73	1.20	2.60	1.20
time (sec)	N/A	0.227	0.030	0.148	0.034	0.072	0.396	0.128	0.252	0.463

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	28	33	30	37	48	58	56	36
N.S.	1	1.12	0.82	0.97	0.88	1.09	1.41	1.71	1.65	1.06
time (sec)	N/A	0.280	0.047	0.083	0.026	0.085	0.498	0.133	0.303	0.428

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	42	46	46	54	423	114	81	47
N.S.	1	1.09	0.78	0.85	0.85	1.00	7.83	2.11	1.50	0.87
time (sec)	N/A	0.305	0.063	0.090	0.034	0.074	1.514	0.138	0.259	0.521

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	53	55	63	63	1844	169	102	56
N.S.	1	1.07	0.73	0.75	0.86	0.86	25.26	2.32	1.40	0.77
time (sec)	N/A	0.331	0.074	0.097	0.026	0.076	2.244	0.153	0.231	0.635

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	98	52	56	69	62	337	86	53	48
N.S.	1	1.15	0.61	0.66	0.81	0.73	3.96	1.01	0.62	0.56
time (sec)	N/A	0.356	0.032	0.100	0.033	0.076	0.935	0.125	0.222	1.027

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	68	41	45	53	51	219	76	42	38
N.S.	1	1.11	0.67	0.74	0.87	0.84	3.59	1.25	0.69	0.62
time (sec)	N/A	0.312	0.031	0.092	0.026	0.074	0.715	0.125	0.253	0.894

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	27	33	32	39	42	51	30	36
N.S.	1	1.06	0.77	0.94	0.91	1.11	1.20	1.46	0.86	1.03
time (sec)	N/A	0.262	0.026	0.083	0.026	0.087	0.544	0.123	0.248	0.418

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	25	17	29	20	28	22	24
N.S.	1	1.00	1.00	1.32	0.89	1.53	1.05	1.47	1.16	1.26
time (sec)	N/A	0.229	0.004	0.066	0.026	0.068	0.510	0.126	0.251	0.434

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	65	58	149	184	79	136	40
N.S.	1	1.00	1.21	1.38	1.23	3.17	3.91	1.68	2.89	0.85
time (sec)	N/A	0.284	0.038	0.066	0.109	0.080	1.235	0.123	0.214	0.633

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	74	81	97	184	73	80	172	0
N.S.	1	1.08	1.04	1.14	1.37	2.59	1.03	1.13	2.42	0.00
time (sec)	N/A	0.321	0.065	0.095	0.121	0.082	2.200	0.133	0.247	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	107	88	94	139	222	102	99	195	0
N.S.	1	1.13	0.93	0.99	1.46	2.34	1.07	1.04	2.05	0.00
time (sec)	N/A	0.352	0.100	0.102	0.108	0.093	4.551	0.143	0.247	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	127	117	98	139	287	432	114	212	95
N.S.	1	1.09	1.01	0.84	1.20	2.47	3.72	0.98	1.83	0.82
time (sec)	N/A	0.360	0.309	0.145	0.104	0.168	7.420	0.129	0.230	1.273

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	104	85	101	260	819	92	189	73
N.S.	1	1.09	1.18	0.97	1.15	2.95	9.31	1.05	2.15	0.83
time (sec)	N/A	0.330	0.245	0.121	0.105	0.117	3.159	0.133	0.233	0.947

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	70	93	73	62	232	743	73	138	47
N.S.	1	1.19	1.58	1.24	1.05	3.93	12.59	1.24	2.34	0.80
time (sec)	N/A	0.294	0.217	0.076	0.103	0.086	1.967	0.131	0.266	0.630

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	15	14	41	48	21	68	14
N.S.	1	1.00	1.56	0.83	0.78	2.28	2.67	1.17	3.78	0.78
time (sec)	N/A	0.220	0.056	0.151	0.034	0.073	0.633	0.129	0.235	0.498

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	39	35	29	53	94	35	84	27
N.S.	1	1.14	1.11	1.00	0.83	1.51	2.69	1.00	2.40	0.77
time (sec)	N/A	0.269	0.048	0.066	0.042	0.075	0.784	0.125	0.217	0.496

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	49	46	47	61	153	76	100	42
N.S.	1	1.07	0.89	0.84	0.85	1.11	2.78	1.38	1.82	0.76
time (sec)	N/A	0.292	0.066	0.095	0.027	0.078	1.097	0.136	0.229	0.520

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	80	60	57	64	76	201	133	124	58
N.S.	1	1.05	0.79	0.75	0.84	1.00	2.64	1.75	1.63	0.76
time (sec)	N/A	0.314	0.071	0.108	0.029	0.091	1.147	0.136	0.225	0.599

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	95	59	56	71	73	337	85	64	81
N.S.	1	1.16	0.72	0.68	0.87	0.89	4.11	1.04	0.78	0.99
time (sec)	N/A	0.314	0.033	0.113	0.033	0.085	0.994	0.128	0.224	0.993

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	66	49	46	51	63	163	68	54	43
N.S.	1	1.14	0.84	0.79	0.88	1.09	2.81	1.17	0.93	0.74
time (sec)	N/A	0.290	0.030	0.101	0.035	0.083	0.811	0.123	0.244	0.516

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	38	35	33	52	105	39	43	34
N.S.	1	0.98	0.88	0.81	0.77	1.21	2.44	0.91	1.00	0.79
time (sec)	N/A	0.275	0.029	0.069	0.031	0.084	0.837	0.119	0.252	0.500

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	28	25	17	40	48	29	33	17
N.S.	1	1.00	1.33	1.19	0.81	1.90	2.29	1.38	1.57	0.81
time (sec)	N/A	0.229	0.005	0.063	0.034	0.084	0.785	0.123	0.218	0.446

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	80	77	73	220	740	93	238	0
N.S.	1	1.09	1.18	1.13	1.07	3.24	10.88	1.37	3.50	0.00
time (sec)	N/A	0.314	0.067	0.069	0.104	0.088	2.092	0.130	0.220	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	107	96	92	117	254	864	85	272	0
N.S.	1	1.13	1.01	0.97	1.23	2.67	9.09	0.89	2.86	0.00
time (sec)	N/A	0.359	0.084	0.134	0.113	0.095	3.784	0.141	0.219	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	21	31	21	12	17
N.S.	1	1.00	1.00	0.77	0.69	1.62	2.38	1.62	0.92	1.31
time (sec)	N/A	0.217	0.438	0.074	0.031	0.077	0.584	0.118	0.250	0.459

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	21	10	9	26	48	23	24	19
N.S.	1	1.00	1.62	0.77	0.69	2.00	3.69	1.77	1.85	1.46
time (sec)	N/A	0.211	1.696	0.079	0.023	0.073	0.404	0.129	0.208	0.658

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	63	0	0	0	56	0	37	0
N.S.	1	1.00	1.43	0.00	0.00	0.00	1.27	0.00	0.84	0.00
time (sec)	N/A	0.291	0.175	0.000	0.000	0.000	2.609	0.000	0.269	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	58	0	0	0	56	0	19	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	1.27	0.00	0.43	0.00
time (sec)	N/A	0.287	0.122	0.000	0.000	0.000	0.924	0.000	0.245	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	64	0	0	0	54	0	19	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	1.23	0.00	0.43	0.00
time (sec)	N/A	0.292	0.127	0.000	0.000	0.000	0.814	0.000	0.208	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	67	0	0	0	54	0	34	0
N.S.	1	1.00	1.52	0.00	0.00	0.00	1.23	0.00	0.77	0.00
time (sec)	N/A	0.293	0.237	0.000	0.000	0.000	1.310	0.000	0.242	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	71	0	0	0	54	0	148	0
N.S.	1	1.00	1.61	0.00	0.00	0.00	1.23	0.00	3.36	0.00
time (sec)	N/A	0.293	0.037	0.000	0.000	0.000	8.797	0.000	0.213	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	0	0	0	60	0	158	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.86	0.00	2.26	0.00
time (sec)	N/A	0.327	0.035	0.000	0.000	0.000	10.914	0.000	0.237	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	53	46	46	45	44	47	73	45
N.S.	1	0.98	1.00	0.87	0.87	0.85	0.83	0.89	1.38	0.85
time (sec)	N/A	0.318	0.005	0.059	0.035	0.068	0.115	0.129	0.227	0.321

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	40	34	34	33	32	35	61	33
N.S.	1	0.98	1.00	0.85	0.85	0.82	0.80	0.88	1.52	0.82
time (sec)	N/A	0.306	0.005	0.116	0.026	0.066	0.118	0.126	0.203	0.062

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	48	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	1.78	0.81
time (sec)	N/A	0.281	0.005	0.049	0.031	0.061	0.100	0.132	0.212	0.282

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	37	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	2.47	0.87
time (sec)	N/A	0.241	0.003	0.036	0.025	0.060	0.079	0.119	0.232	0.285

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	22	14	13	18	15	22	45	18
N.S.	1	1.00	1.47	0.93	0.87	1.20	1.00	1.47	3.00	1.20
time (sec)	N/A	0.224	0.006	0.052	0.031	0.069	0.137	0.119	0.233	0.318

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	36	35	32	33	33	31	42	61	31
N.S.	1	1.33	1.30	1.19	1.22	1.22	1.15	1.56	2.26	1.15
time (sec)	N/A	0.298	0.006	0.053	0.027	0.064	0.180	0.119	0.229	0.090

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	50	49	44	47	45	42	55	78	46
N.S.	1	1.25	1.22	1.10	1.18	1.12	1.05	1.38	1.95	1.15
time (sec)	N/A	0.318	0.007	0.054	0.032	0.066	0.303	0.125	0.217	0.091

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	122	48	125	140	44	132	100	143
N.S.	1	1.00	0.90	0.35	0.92	1.03	0.32	0.97	0.74	1.05
time (sec)	N/A	0.468	0.026	0.079	0.106	0.078	0.151	0.126	0.219	0.259

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	120	46	122	108	37	129	90	119
N.S.	1	1.00	0.91	0.35	0.92	0.82	0.28	0.98	0.68	0.90
time (sec)	N/A	0.471	0.016	0.080	0.110	0.076	0.128	0.125	0.193	0.461

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	133	111	37	109	123	32	114	83	120
N.S.	1	1.07	0.90	0.30	0.88	0.99	0.26	0.92	0.67	0.97
time (sec)	N/A	0.482	0.015	0.083	0.113	0.074	0.089	0.116	0.202	0.479

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	123	108	34	106	106	22	111	80	114
N.S.	1	1.03	0.91	0.29	0.89	0.89	0.18	0.93	0.67	0.96
time (sec)	N/A	0.475	0.012	0.077	0.107	0.073	0.107	0.123	0.204	0.519

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	89	27	98	304	24	112	67	111
N.S.	1	1.01	0.77	0.23	0.85	2.64	0.21	0.97	0.58	0.97
time (sec)	N/A	0.448	0.008	0.063	0.117	0.087	0.083	0.124	0.259	0.540

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	111	89	27	98	299	20	112	69	99
N.S.	1	0.97	0.77	0.23	0.85	2.60	0.17	0.97	0.60	0.86
time (sec)	N/A	0.435	0.009	0.061	0.106	0.085	0.081	0.127	0.227	0.260

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	131	114	53	106	103	29	121	86	102
N.S.	1	1.07	0.93	0.43	0.87	0.84	0.24	0.99	0.70	0.84
time (sec)	N/A	0.480	0.014	0.090	0.120	0.073	0.109	0.124	0.213	0.522

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	119	54	106	143	32	115	98	128
N.S.	1	1.03	0.96	0.44	0.85	1.15	0.26	0.93	0.79	1.03
time (sec)	N/A	0.468	0.015	0.089	0.113	0.077	0.117	0.121	0.244	0.249

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	49	52	53	70	53	54	130	56
N.S.	1	1.00	0.88	0.93	0.95	1.25	0.95	0.96	2.32	1.00
time (sec)	N/A	0.328	0.013	0.191	0.026	0.070	0.167	0.129	0.213	0.314

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	43	38	41	43	56	42	41	115	45
N.S.	1	0.93	0.83	0.89	0.93	1.22	0.91	0.89	2.50	0.98
time (sec)	N/A	0.311	0.012	0.059	0.027	0.064	0.173	0.127	0.218	0.053

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	27	30	32	35	29	32	97	29
N.S.	1	0.94	0.82	0.91	0.97	1.06	0.88	0.97	2.94	0.88
time (sec)	N/A	0.288	0.007	0.044	0.026	0.066	0.140	0.114	0.215	0.294

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	17	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	1.06	0.88
time (sec)	N/A	0.229	0.004	0.052	0.025	0.064	0.105	0.122	0.282	0.280

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	39	33	35	37	47	34	45	114	34
N.S.	1	1.18	1.00	1.06	1.12	1.42	1.03	1.36	3.45	1.03
time (sec)	N/A	0.294	0.010	0.054	0.032	0.122	0.213	0.124	0.218	0.348

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	52	41	55	53	73	54	51	142	53
N.S.	1	1.13	0.89	1.20	1.15	1.59	1.17	1.11	3.09	1.15
time (sec)	N/A	0.330	0.029	0.061	0.030	0.089	0.286	0.119	0.217	0.082

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	140	64	143	158	65	147	191	152
N.S.	1	1.03	0.91	0.42	0.93	1.03	0.42	0.95	1.24	0.99
time (sec)	N/A	0.457	0.056	0.090	0.116	0.074	0.195	0.123	0.208	0.247

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	160	131	55	130	163	58	132	184	139
N.S.	1	1.09	0.89	0.37	0.88	1.11	0.39	0.90	1.25	0.95
time (sec)	N/A	0.519	0.053	0.105	0.125	0.080	0.168	0.125	0.242	0.484

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	127	50	125	146	48	127	180	132
N.S.	1	1.07	0.91	0.36	0.89	1.04	0.34	0.91	1.29	0.94
time (sec)	N/A	0.497	0.050	0.097	0.112	0.087	0.182	0.120	0.241	0.277

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	143	119	45	116	400	44	132	162	138
N.S.	1	1.05	0.88	0.33	0.85	2.94	0.32	0.97	1.19	1.01
time (sec)	N/A	0.461	0.050	0.085	0.112	0.117	0.144	0.125	0.242	0.529

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	136	118	43	114	391	39	130	175	108
N.S.	1	1.01	0.88	0.32	0.85	2.92	0.29	0.97	1.31	0.81
time (sec)	N/A	0.451	0.045	0.076	0.111	0.112	0.153	0.131	0.211	0.532

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	143	119	48	124	386	44	129	162	138
N.S.	1	1.05	0.88	0.35	0.91	2.84	0.32	0.95	1.19	1.01
time (sec)	N/A	0.465	0.042	0.076	0.108	0.098	0.136	0.124	0.224	0.353

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	136	118	46	122	389	39	127	175	128
N.S.	1	1.01	0.88	0.34	0.91	2.90	0.29	0.95	1.31	0.96
time (sec)	N/A	0.449	0.043	0.074	0.108	0.098	0.164	0.119	0.204	0.224

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	158	131	73	126	146	56	139	187	120
N.S.	1	1.09	0.90	0.50	0.87	1.01	0.39	0.96	1.29	0.83
time (sec)	N/A	0.522	0.058	0.093	0.109	0.088	0.198	0.128	0.220	0.560

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	93	92	100	183	100	55	85	76
N.S.	1	1.03	1.31	1.30	1.41	2.58	1.41	0.77	1.20	1.07
time (sec)	N/A	0.300	0.344	4.849	0.116	0.166	2.406	0.150	0.258	0.997

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	45	68	71	53	158	48	39	62	56
N.S.	1	0.96	1.45	1.51	1.13	3.36	1.02	0.83	1.32	1.19
time (sec)	N/A	0.275	0.152	0.565	0.109	0.166	1.165	0.150	0.254	0.845

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	76	67	49	139	76	36	66	31
N.S.	1	1.00	1.77	1.56	1.14	3.23	1.77	0.84	1.53	0.72
time (sec)	N/A	0.279	0.170	0.612	0.114	0.150	0.945	0.156	0.229	0.580

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	46	14	27	24
N.S.	1	1.00	1.00	0.83	0.78	1.56	2.56	0.78	1.50	1.33
time (sec)	N/A	0.233	0.164	0.176	0.029	0.080	0.710	0.125	0.219	0.679

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	41	35	30	42	313	29	41	37
N.S.	1	1.11	1.08	0.92	0.79	1.11	8.24	0.76	1.08	0.97
time (sec)	N/A	0.288	1.235	0.105	0.029	0.084	1.046	0.131	0.233	0.900

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	53	46	47	53	913	43	52	70
N.S.	1	1.07	0.90	0.78	0.80	0.90	15.47	0.73	0.88	1.19
time (sec)	N/A	0.299	1.682	0.119	0.029	0.073	1.454	0.127	0.244	1.175

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	64	57	64	64	2317	57	63	90
N.S.	1	1.05	0.80	0.71	0.80	0.80	28.96	0.71	0.79	1.12
time (sec)	N/A	0.329	1.762	0.118	0.037	0.072	2.180	0.130	0.234	1.594

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	302	96	753	0	0	48	0	85	0
N.S.	1	1.04	0.33	2.59	0.00	0.00	0.16	0.00	0.29	0.00
time (sec)	N/A	0.563	10.055	1.566	0.000	0.000	0.891	0.000	0.358	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	272	80	739	0	0	48	0	64	0
N.S.	1	1.02	0.30	2.77	0.00	0.00	0.18	0.00	0.24	0.00
time (sec)	N/A	0.482	10.027	0.413	0.000	0.000	0.939	0.000	0.304	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	49	725	0	0	44	0	40	0
N.S.	1	1.00	0.20	3.00	0.00	0.00	0.18	0.00	0.17	0.00
time (sec)	N/A	0.448	10.010	0.652	0.000	0.000	0.667	0.000	0.301	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	51	726	0	42	39	0	54	40
N.S.	1	1.00	0.21	2.99	0.00	0.17	0.16	0.00	0.22	0.16
time (sec)	N/A	0.452	10.012	0.504	0.000	0.078	0.633	0.000	0.423	0.610

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	270	51	742	0	59	41	0	54	0
N.S.	1	1.01	0.19	2.78	0.00	0.22	0.15	0.00	0.20	0.00
time (sec)	N/A	0.495	10.013	0.704	0.000	0.072	0.691	0.000	0.590	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	300	51	753	0	70	41	0	54	0
N.S.	1	1.03	0.18	2.59	0.00	0.24	0.14	0.00	0.19	0.00
time (sec)	N/A	0.537	10.013	0.922	0.000	0.076	0.812	0.000	0.835	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	594	80	1127	0	0	48	0	66	0
N.S.	1	1.06	0.14	2.00	0.00	0.00	0.09	0.00	0.12	0.00
time (sec)	N/A	0.962	10.023	0.874	0.000	0.000	0.800	0.000	0.327	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	564	51	1109	0	0	48	0	42	0
N.S.	1	1.05	0.09	2.06	0.00	0.00	0.09	0.00	0.08	0.00
time (sec)	N/A	0.923	10.011	0.851	0.000	0.000	0.652	0.000	0.337	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	533	47	1107	0	0	42	0	48	38
N.S.	1	1.05	0.09	2.18	0.00	0.00	0.08	0.00	0.09	0.07
time (sec)	N/A	0.922	10.010	0.464	0.000	0.000	0.591	0.000	0.388	0.420

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	538	51	1125	0	53	41	0	54	0
N.S.	1	1.04	0.10	2.18	0.00	0.10	0.08	0.00	0.10	0.00
time (sec)	N/A	0.869	10.011	0.917	0.000	0.069	0.622	0.000	0.442	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	568	51	1138	0	66	41	0	54	0
N.S.	1	1.05	0.09	2.10	0.00	0.12	0.08	0.00	0.10	0.00
time (sec)	N/A	0.911	10.012	0.826	0.000	0.069	0.781	0.000	0.647	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	565	598	51	1149	0	77	41	0	54	0
N.S.	1	1.06	0.09	2.03	0.00	0.14	0.07	0.00	0.10	0.00
time (sec)	N/A	0.968	10.012	1.118	0.000	0.068	1.013	0.000	0.980	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	82	91	98	187	76	51	86	74
N.S.	1	1.03	1.19	1.32	1.42	2.71	1.10	0.74	1.25	1.07
time (sec)	N/A	0.309	0.206	0.503	0.106	0.152	2.073	0.149	0.226	1.052

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	64	72	84	66	159	100	55	84	69
N.S.	1	1.05	1.18	1.38	1.08	2.61	1.64	0.90	1.38	1.13
time (sec)	N/A	0.306	0.232	0.542	0.112	0.155	1.777	0.153	0.252	1.030

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	62	87	86	61	172	83	50	88	43
N.S.	1	1.05	1.47	1.46	1.03	2.92	1.41	0.85	1.49	0.73
time (sec)	N/A	0.303	0.222	0.447	0.105	0.165	1.486	0.179	0.275	0.897

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	15	14	39	71	14	41	26
N.S.	1	1.00	1.56	0.83	0.78	2.17	3.94	0.78	2.28	1.44
time (sec)	N/A	0.236	1.576	0.151	0.025	0.073	0.689	0.126	0.220	1.202

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	35	30	53	371	29	52	68
N.S.	1	1.11	1.00	0.92	0.79	1.39	9.76	0.76	1.37	1.79
time (sec)	N/A	0.293	1.770	0.085	0.024	0.116	1.088	0.142	0.275	1.715

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	49	46	47	64	1001	43	63	88
N.S.	1	1.07	0.83	0.78	0.80	1.08	16.97	0.73	1.07	1.49
time (sec)	N/A	0.311	2.015	0.092	0.025	0.081	1.611	0.132	0.232	2.158

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	60	57	64	75	2317	57	74	108
N.S.	1	1.05	0.75	0.71	0.80	0.94	28.96	0.71	0.92	1.35
time (sec)	N/A	0.329	2.087	0.130	0.026	0.076	2.485	0.137	0.231	2.856

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	98	91	104	187	102	59	86	79
N.S.	1	1.07	1.32	1.23	1.41	2.53	1.38	0.80	1.16	1.07
time (sec)	N/A	0.306	0.211	0.819	0.113	0.162	2.890	0.159	0.255	0.976

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	82	79	68	157	49	45	62	59
N.S.	1	0.98	1.64	1.58	1.36	3.14	0.98	0.90	1.24	1.18
time (sec)	N/A	0.285	0.176	0.513	0.107	0.165	1.470	0.153	0.255	0.882

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	60	59	37	110	24	0	43	19
N.S.	1	1.00	2.22	2.19	1.37	4.07	0.89	0.00	1.59	0.70
time (sec)	N/A	0.265	0.247	0.322	0.106	0.157	0.692	0.000	0.215	0.535

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	18	29	14	20	14
N.S.	1	1.00	1.00	0.83	0.78	1.00	1.61	0.78	1.11	0.78
time (sec)	N/A	0.230	0.165	0.562	0.025	0.074	0.396	0.118	0.232	0.434

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	31	35	30	31	255	30	30	25
N.S.	1	1.05	0.82	0.92	0.79	0.82	6.71	0.79	0.79	0.66
time (sec)	N/A	0.290	0.204	0.487	0.025	0.075	1.065	0.130	0.259	0.519

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	42	46	47	42	824	47	41	56
N.S.	1	1.03	0.71	0.78	0.80	0.71	13.97	0.80	0.69	0.95
time (sec)	N/A	0.312	1.396	0.487	0.025	0.081	1.616	0.132	0.266	0.545

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	57	64	53	2183	61	52	73
N.S.	1	1.00	0.66	0.71	0.80	0.66	27.29	0.76	0.65	0.91
time (sec)	N/A	0.330	1.563	0.503	0.026	0.070	2.136	0.131	0.287	0.663

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	308	91	753	0	0	46	0	85	0
N.S.	1	1.05	0.31	2.56	0.00	0.00	0.16	0.00	0.29	0.00
time (sec)	N/A	0.555	10.023	2.448	0.000	0.000	0.754	0.000	0.428	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	278	80	742	0	0	46	0	64	0
N.S.	1	1.03	0.30	2.75	0.00	0.00	0.17	0.00	0.24	0.00
time (sec)	N/A	0.505	10.023	1.296	0.000	0.000	0.692	0.000	0.347	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	64	727	0	0	42	0	45	0
N.S.	1	1.00	0.26	2.93	0.00	0.00	0.17	0.00	0.18	0.00
time (sec)	N/A	0.454	10.024	0.414	0.000	0.000	0.695	0.000	0.323	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	49	437	0	16	37	0	24	40
N.S.	1	1.00	0.22	1.98	0.00	0.07	0.17	0.00	0.11	0.18
time (sec)	N/A	0.421	10.016	0.898	0.000	0.069	0.548	0.000	0.248	0.502

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	51	732	0	43	39	0	26	0
N.S.	1	1.00	0.21	2.98	0.00	0.17	0.16	0.00	0.11	0.00
time (sec)	N/A	0.462	10.013	1.579	0.000	0.074	0.653	0.000	0.316	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	276	51	742	0	59	39	0	26	0
N.S.	1	1.02	0.19	2.75	0.00	0.22	0.14	0.00	0.10	0.00
time (sec)	N/A	0.527	10.012	2.568	0.000	0.074	0.795	0.000	0.314	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	600	79	1125	0	0	46	0	66	0
N.S.	1	1.06	0.14	1.99	0.00	0.00	0.08	0.00	0.12	0.00
time (sec)	N/A	1.033	10.020	2.097	0.000	0.000	0.680	0.000	0.386	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	570	62	1113	0	0	46	0	47	0
N.S.	1	1.05	0.11	2.05	0.00	0.00	0.08	0.00	0.09	0.00
time (sec)	N/A	0.953	10.024	1.056	0.000	0.000	0.636	0.000	0.339	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	539	49	2374	0	0	41	0	23	38
N.S.	1	1.05	0.10	4.63	0.00	0.00	0.08	0.00	0.04	0.07
time (sec)	N/A	0.887	10.018	0.347	0.000	0.000	0.619	0.000	0.276	0.382

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	515	49	1115	0	24	39	0	26	0
N.S.	1	1.05	0.10	2.27	0.00	0.05	0.08	0.00	0.05	0.00
time (sec)	N/A	0.879	10.013	1.167	0.000	0.070	0.652	0.000	0.274	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	544	51	1127	0	52	39	0	26	0
N.S.	1	1.05	0.10	2.17	0.00	0.10	0.08	0.00	0.05	0.00
time (sec)	N/A	0.901	10.012	1.849	0.000	0.074	0.737	0.000	0.267	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	574	51	1138	0	66	39	0	26	0
N.S.	1	1.06	0.09	2.09	0.00	0.12	0.07	0.00	0.05	0.00
time (sec)	N/A	0.881	10.012	3.026	0.000	0.078	0.889	0.000	0.382	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	604	51	1149	0	77	39	0	26	0
N.S.	1	1.06	0.09	2.02	0.00	0.14	0.07	0.00	0.05	0.00
time (sec)	N/A	1.003	10.014	4.683	0.000	0.077	1.087	0.000	0.407	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	104	98	96	122	251	110	76	171	96
N.S.	1	1.09	1.03	1.01	1.28	2.64	1.16	0.80	1.80	1.01
time (sec)	N/A	0.329	1.637	0.769	0.107	0.184	4.317	0.210	0.345	1.232

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	73	84	83	86	223	73	59	146	73
N.S.	1	1.14	1.31	1.30	1.34	3.48	1.14	0.92	2.28	1.14
time (sec)	N/A	0.306	0.294	0.531	0.103	0.165	2.003	0.208	0.291	1.018

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	70	52	194	187	0	119	34
N.S.	1	1.00	1.61	1.52	1.13	4.22	4.07	0.00	2.59	0.74
time (sec)	N/A	0.275	0.226	0.345	0.112	0.168	1.217	0.000	0.282	0.619

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	30	27	14	25	14
N.S.	1	1.00	1.00	0.83	0.78	1.67	1.50	0.78	1.39	0.78
time (sec)	N/A	0.231	0.176	0.138	0.029	0.075	0.483	0.134	0.265	0.427

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	33	30	37	51	0	39	31
N.S.	1	1.00	0.76	0.87	0.79	0.97	1.34	0.00	1.03	0.82
time (sec)	N/A	0.286	0.386	0.500	0.032	0.085	0.702	0.000	0.234	0.454

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	46	47	54	466	0	50	62
N.S.	1	1.00	0.71	0.78	0.80	0.92	7.90	0.00	0.85	1.05
time (sec)	N/A	0.307	0.379	0.519	0.060	0.079	1.663	0.000	0.295	0.611

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	51	55	64	63	2048	0	61	82
N.S.	1	1.00	0.65	0.71	0.82	0.81	26.26	0.00	0.78	1.05
time (sec)	N/A	0.339	2.081	0.514	0.033	0.081	2.533	0.000	0.363	0.773

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	337	91	1454	0	0	46	0	170	0
N.S.	1	1.07	0.29	4.62	0.00	0.00	0.15	0.00	0.54	0.00
time (sec)	N/A	0.596	10.038	4.174	0.000	0.000	0.838	0.000	0.716	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	307	80	1443	0	0	46	0	149	0
N.S.	1	1.05	0.27	4.96	0.00	0.00	0.16	0.00	0.51	0.00
time (sec)	N/A	0.551	10.041	2.850	0.000	0.000	0.812	0.000	0.626	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	277	67	1431	0	0	42	0	128	0
N.S.	1	1.03	0.25	5.32	0.00	0.00	0.16	0.00	0.48	0.00
time (sec)	N/A	0.501	10.022	2.389	0.000	0.000	0.656	0.000	0.550	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	57	1822	0	59	37	0	107	40
N.S.	1	1.00	0.23	7.35	0.00	0.24	0.15	0.00	0.43	0.16
time (sec)	N/A	0.451	10.020	0.520	0.000	0.084	0.754	0.000	0.511	0.668

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	58	1825	0	58	39	0	35	0
N.S.	1	1.00	0.24	7.45	0.00	0.24	0.16	0.00	0.14	0.00
time (sec)	N/A	0.461	10.015	1.823	0.000	0.072	0.774	0.000	0.296	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	275	54	1431	0	75	39	0	37	0
N.S.	1	1.03	0.20	5.36	0.00	0.28	0.15	0.00	0.14	0.00
time (sec)	N/A	0.498	10.014	3.392	0.000	0.103	0.982	0.000	0.377	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	629	78	2211	0	0	46	0	150	0
N.S.	1	1.07	0.13	3.77	0.00	0.00	0.08	0.00	0.26	0.00
time (sec)	N/A	1.063	10.036	4.208	0.000	0.000	0.875	0.000	0.644	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	599	65	2199	0	0	46	0	129	0
N.S.	1	1.06	0.12	3.91	0.00	0.00	0.08	0.00	0.23	0.00
time (sec)	N/A	0.983	10.024	3.145	0.000	0.000	0.654	0.000	0.558	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	567	53	2700	0	0	41	0	110	38
N.S.	1	1.05	0.10	5.01	0.00	0.00	0.08	0.00	0.20	0.07
time (sec)	N/A	0.969	10.015	1.549	0.000	0.000	0.617	0.000	0.561	0.464

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	543	52	2703	0	65	39	0	34	0
N.S.	1	1.04	0.10	5.20	0.00	0.12	0.08	0.00	0.07	0.00
time (sec)	N/A	0.889	10.013	0.532	0.000	0.078	0.675	0.000	0.329	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	543	52	2200	0	64	39	0	37	0
N.S.	1	1.05	0.10	4.26	0.00	0.12	0.08	0.00	0.07	0.00
time (sec)	N/A	0.876	10.013	3.064	0.000	0.073	0.942	0.000	0.316	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	573	54	2213	0	86	39	0	37	0
N.S.	1	1.06	0.10	4.09	0.00	0.16	0.07	0.00	0.07	0.00
time (sec)	N/A	0.954	10.014	3.986	0.000	0.076	1.196	0.000	0.392	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	565	603	54	2224	0	99	39	0	37	0
N.S.	1	1.07	0.10	3.94	0.00	0.18	0.07	0.00	0.07	0.00
time (sec)	N/A	0.943	10.016	5.179	0.000	0.079	1.352	0.000	0.493	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	212	173	34	179	107	22	172	144	48
N.S.	1	1.53	1.24	0.24	1.29	0.77	0.16	1.24	1.04	0.35
time (sec)	N/A	0.681	0.037	0.074	0.121	0.097	0.098	0.120	0.197	0.370

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	45	62	68	53	118	44	41	168	35
N.S.	1	0.96	1.32	1.45	1.13	2.51	0.94	0.87	3.57	0.74
time (sec)	N/A	0.278	0.128	0.151	0.112	0.086	1.292	0.120	0.230	0.783

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	46	66	65	60	118	66	36	178	0
N.S.	1	0.94	1.35	1.33	1.22	2.41	1.35	0.73	3.63	0.00
time (sec)	N/A	0.319	0.038	0.080	0.103	0.079	0.920	0.116	0.226	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	72	65	49	100	66	60	108	31
N.S.	1	1.00	1.67	1.51	1.14	2.33	1.53	1.40	2.51	0.72
time (sec)	N/A	0.296	0.121	0.138	0.123	0.082	1.175	0.131	0.217	0.538

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	60	74	79	136	46	43	289	0
N.S.	1	1.08	1.20	1.48	1.58	2.72	0.92	0.86	5.78	0.00
time (sec)	N/A	0.310	0.051	0.122	0.118	0.075	1.141	0.123	0.251	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	109	93	107	0	47	44	0	35	0
N.S.	1	1.02	0.87	1.00	0.00	0.44	0.41	0.00	0.33	0.00
time (sec)	N/A	0.336	4.720	0.535	0.000	0.081	0.639	0.000	0.254	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	236	47	130	0	0	42	0	41	38
N.S.	1	1.05	0.21	0.58	0.00	0.00	0.19	0.00	0.18	0.17
time (sec)	N/A	0.550	7.928	0.517	0.000	0.000	0.632	0.000	0.243	0.441

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	109	51	107	0	46	39	0	45	39
N.S.	1	1.02	0.48	1.00	0.00	0.43	0.36	0.00	0.42	0.36
time (sec)	N/A	0.326	10.011	0.461	0.000	0.077	0.616	0.000	0.215	0.555

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	243	51	143	0	91	41	0	46	0
N.S.	1	1.03	0.22	0.61	0.00	0.39	0.17	0.00	0.19	0.00
time (sec)	N/A	0.547	10.013	0.575	0.000	0.077	0.696	0.000	0.267	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	81	75	66	120	95	78	270	48
N.S.	1	1.00	1.27	1.17	1.03	1.88	1.48	1.22	4.22	0.75
time (sec)	N/A	0.298	0.192	0.138	0.109	0.088	1.791	0.143	0.224	1.004

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	71	79	85	95	149	95	57	441	0
N.S.	1	1.01	1.13	1.21	1.36	2.13	1.36	0.81	6.30	0.00
time (sec)	N/A	0.338	0.079	0.143	0.105	0.097	1.733	0.123	0.220	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	62	83	76	61	132	80	122	269	43
N.S.	1	1.05	1.41	1.29	1.03	2.24	1.36	2.07	4.56	0.73
time (sec)	N/A	0.306	0.171	0.140	0.105	0.086	1.550	0.148	0.220	0.771

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	78	70	86	119	166	75	70	554	0
N.S.	1	1.10	0.99	1.21	1.68	2.34	1.06	0.99	7.80	0.00
time (sec)	N/A	0.337	0.125	0.150	0.106	0.084	1.820	0.127	0.258	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	131	52	117	0	0	44	0	62	0
N.S.	1	1.03	0.41	0.92	0.00	0.00	0.35	0.00	0.49	0.00
time (sec)	N/A	0.389	10.013	0.484	0.000	0.000	0.764	0.000	0.253	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	260	52	140	0	0	42	0	61	38
N.S.	1	1.04	0.21	0.56	0.00	0.00	0.17	0.00	0.24	0.15
time (sec)	N/A	0.593	10.011	0.622	0.000	0.000	0.700	0.000	0.284	0.658

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	133	52	117	0	58	39	0	62	39
N.S.	1	1.06	0.41	0.93	0.00	0.46	0.31	0.00	0.49	0.31
time (sec)	N/A	0.355	10.012	0.645	0.000	0.085	0.883	0.000	0.274	0.813

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	267	52	156	0	108	41	0	63	0
N.S.	1	1.04	0.20	0.61	0.00	0.42	0.16	0.00	0.25	0.00
time (sec)	N/A	0.586	10.012	0.770	0.000	0.075	0.766	0.000	0.306	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	96	90	81	161	112	142	445	66
N.S.	1	1.01	1.19	1.11	1.00	1.99	1.38	1.75	5.49	0.81
time (sec)	N/A	0.305	0.221	0.201	0.103	0.088	3.424	0.160	0.207	1.323

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	95	94	103	139	175	124	88	713	0
N.S.	1	0.99	0.98	1.07	1.45	1.82	1.29	0.92	7.43	0.00
time (sec)	N/A	0.342	0.092	0.167	0.109	0.079	3.042	0.122	0.229	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	82	89	75	160	107	180	441	60
N.S.	1	1.04	1.06	1.16	0.97	2.08	1.39	2.34	5.73	0.78
time (sec)	N/A	0.329	0.237	0.167	0.105	0.086	3.307	0.154	0.248	1.199

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	102	81	97	158	188	102	75	818	0
N.S.	1	1.11	0.88	1.05	1.72	2.04	1.11	0.82	8.89	0.00
time (sec)	N/A	0.358	0.157	0.188	0.110	0.081	3.764	0.121	0.248	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	54	131	0	0	44	0	81	0
N.S.	1	1.04	0.36	0.87	0.00	0.00	0.29	0.00	0.54	0.00
time (sec)	N/A	0.419	10.013	0.662	0.000	0.000	1.360	0.000	0.264	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	284	54	151	0	0	42	0	82	38
N.S.	1	1.03	0.19	0.55	0.00	0.00	0.15	0.00	0.30	0.14
time (sec)	N/A	0.656	10.012	0.812	0.000	0.000	1.086	0.000	0.331	0.986

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	157	54	130	0	73	39	0	67	39
N.S.	1	1.07	0.37	0.88	0.00	0.50	0.27	0.00	0.46	0.27
time (sec)	N/A	0.393	10.013	0.891	0.000	0.115	1.294	0.000	0.354	1.298

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	291	54	167	0	119	41	0	82	0
N.S.	1	1.05	0.19	0.60	0.00	0.43	0.15	0.00	0.29	0.00
time (sec)	N/A	0.640	10.013	1.110	0.000	0.109	1.145	0.000	0.381	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	76	70	68	117	46	44	172	38
N.S.	1	0.98	1.52	1.40	1.36	2.34	0.92	0.88	3.44	0.76
time (sec)	N/A	0.282	0.142	0.123	0.110	0.089	2.257	0.126	0.218	0.825

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	25	17	21	19	14	43	17
N.S.	1	1.00	1.00	1.19	0.81	1.00	0.90	0.67	2.05	0.81
time (sec)	N/A	0.234	0.017	0.062	0.030	0.067	0.609	0.128	0.208	0.569

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	56	49	37	71	20	25	27	19
N.S.	1	1.00	2.07	1.81	1.37	2.63	0.74	0.93	1.00	0.70
time (sec)	N/A	0.266	0.221	0.085	0.104	0.081	0.870	0.119	0.200	0.513

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	52	52	45	83	22	23	58	0
N.S.	1	1.00	1.73	1.73	1.50	2.77	0.73	0.77	1.93	0.00
time (sec)	N/A	0.278	0.027	0.083	0.105	0.078	0.879	0.123	0.272	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	112	64	111	0	51	42	0	39	0
N.S.	1	1.02	0.58	1.01	0.00	0.46	0.38	0.00	0.35	0.00
time (sec)	N/A	0.361	10.026	0.438	0.000	0.077	0.738	0.000	0.239	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	242	49	113	0	75	41	0	23	43
N.S.	1	1.05	0.21	0.49	0.00	0.32	0.18	0.00	0.10	0.19
time (sec)	N/A	0.550	10.015	0.315	0.000	0.076	0.629	0.000	0.217	0.435

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	90	77	86	0	29	37	0	20	39
N.S.	1	1.02	0.88	0.98	0.00	0.33	0.42	0.00	0.23	0.44
time (sec)	N/A	0.298	11.032	0.270	0.000	0.078	0.795	0.000	0.225	0.495

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	218	49	133	0	72	39	0	24	0
N.S.	1	1.03	0.23	0.63	0.00	0.34	0.18	0.00	0.11	0.00
time (sec)	N/A	0.505	10.011	0.435	0.000	0.074	0.839	0.000	0.230	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	78	79	86	183	75	55	312	53
N.S.	1	1.06	1.13	1.14	1.25	2.65	1.09	0.80	4.52	0.77
time (sec)	N/A	0.298	0.216	0.146	0.116	0.087	2.395	0.126	0.217	1.038

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	34	36	42	42	33	68	24
N.S.	1	1.00	0.75	0.85	0.90	1.05	1.05	0.82	1.70	0.60
time (sec)	N/A	0.266	0.028	0.101	0.033	0.069	0.783	0.123	0.218	0.639

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	68	67	52	154	187	43	141	34
N.S.	1	1.00	1.48	1.46	1.13	3.35	4.07	0.93	3.07	0.74
time (sec)	N/A	0.276	0.168	0.096	0.106	0.083	1.382	0.122	0.291	0.565

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	25	17	31	22	14	46	26
N.S.	1	1.00	1.00	1.19	0.81	1.48	1.05	0.67	2.19	1.24
time (sec)	N/A	0.226	0.018	0.062	0.039	0.079	0.741	0.123	0.223	0.431

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	141	67	133	0	81	42	0	116	0
N.S.	1	1.08	0.51	1.02	0.00	0.62	0.32	0.00	0.89	0.00
time (sec)	N/A	0.380	10.018	1.228	0.000	0.087	0.785	0.000	0.238	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	269	53	188	0	113	41	0	106	43
N.S.	1	1.04	0.21	0.73	0.00	0.44	0.16	0.00	0.41	0.17
time (sec)	N/A	0.602	8.777	0.441	0.000	0.089	0.768	0.000	0.227	0.515

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	112	57	113	0	68	37	0	97	39
N.S.	1	1.02	0.52	1.03	0.00	0.62	0.34	0.00	0.88	0.35
time (sec)	N/A	0.335	5.751	0.359	0.000	0.085	1.089	0.000	0.254	0.599

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	248	52	187	0	102	39	0	34	0
N.S.	1	1.03	0.22	0.78	0.00	0.42	0.16	0.00	0.14	0.00
time (sec)	N/A	0.547	5.233	0.323	0.000	0.083	1.077	0.000	0.205	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	99	140	101	251	819	69	588	73
N.S.	1	1.09	1.12	1.59	1.15	2.85	9.31	0.78	6.68	0.83
time (sec)	N/A	0.329	0.340	0.236	0.107	0.120	3.900	0.137	0.246	1.154

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	49	46	54	65	163	49	196	38
N.S.	1	1.08	0.77	0.72	0.84	1.02	2.55	0.77	3.06	0.59
time (sec)	N/A	0.323	0.033	0.151	0.026	0.080	1.108	0.119	0.224	0.739

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	70	88	221	62	223	743	55	407	48
N.S.	1	1.09	1.38	3.45	0.97	3.48	11.61	0.86	6.36	0.75
time (sec)	N/A	0.311	0.262	0.131	0.110	0.100	2.269	0.137	0.230	0.638

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	44	38	35	33	54	105	24	164	36
N.S.	1	1.02	0.88	0.81	0.77	1.26	2.44	0.56	3.81	0.84
time (sec)	N/A	0.284	0.027	0.092	0.040	0.075	1.035	0.128	0.204	0.479

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	170	94	304	0	114	42	0	221	0
N.S.	1	1.12	0.62	2.00	0.00	0.75	0.28	0.00	1.45	0.00
time (sec)	N/A	0.433	10.051	2.042	0.000	0.087	1.005	0.000	0.253	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	296	81	503	0	157	41	0	214	43
N.S.	1	1.07	0.29	1.82	0.00	0.57	0.15	0.00	0.77	0.16
time (sec)	N/A	0.632	10.037	0.645	0.000	0.089	0.945	0.000	0.299	0.532

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	141	82	279	0	103	37	0	121	39
N.S.	1	1.08	0.63	2.13	0.00	0.79	0.28	0.00	0.92	0.30
time (sec)	N/A	0.376	10.032	0.534	0.000	0.084	1.238	0.000	0.257	0.689

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	277	77	503	0	149	39	0	197	0
N.S.	1	1.06	0.29	1.92	0.00	0.57	0.15	0.00	0.75	0.00
time (sec)	N/A	0.607	10.026	0.430	0.000	0.086	1.191	0.000	0.253	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	329	267	34	250	18648	22	268	20	290
N.S.	1	1.06	0.86	0.11	0.81	60.15	0.07	0.86	0.06	0.94
time (sec)	N/A	1.211	0.137	0.169	0.110	0.931	0.144	0.130	0.237	0.910

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	60	0	37	102	24	0	47	19
N.S.	1	1.00	2.22	0.00	1.37	3.78	0.89	0.00	1.74	0.70
time (sec)	N/A	0.264	0.314	0.000	0.102	0.374	0.809	0.000	0.204	0.570

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	A	C	F(-2)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	66	0	21	111	60	0	63	21
N.S.	1	1.00	2.28	0.00	0.72	3.83	2.07	0.00	2.17	0.72
time (sec)	N/A	0.284	0.334	0.000	0.106	0.388	0.908	0.000	0.207	0.552

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	222	182	34	194	212	22	180	157	227
N.S.	1	1.43	1.17	0.22	1.25	1.37	0.14	1.16	1.01	1.46
time (sec)	N/A	0.769	0.039	0.179	0.112	0.092	0.125	0.118	0.222	0.452

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	314	367	34	0	232	22	442	370	115
N.S.	1	1.46	1.71	0.16	0.00	1.08	0.10	2.06	1.72	0.53
time (sec)	N/A	1.023	0.181	0.101	0.000	0.089	0.254	0.140	0.263	0.421

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [573] had the largest ratio of [1.5555600000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	11	0.182
2	A	2	2	1.00	11	0.182
3	A	2	2	1.00	11	0.182
4	A	2	2	1.00	11	0.182
5	A	2	2	1.00	11	0.182
6	A	2	2	1.00	9	0.222
7	A	1	1	1.00	7	0.143
8	A	2	2	1.00	11	0.182
9	A	2	2	0.94	11	0.182
10	A	2	2	1.00	11	0.182
11	A	2	2	1.00	11	0.182
12	A	2	2	1.00	11	0.182
13	A	2	2	1.00	11	0.182
14	A	3	3	1.00	13	0.231
15	A	3	3	1.00	13	0.231
16	A	3	3	1.00	13	0.231
17	A	2	2	1.00	13	0.154
18	A	3	3	1.00	11	0.273
19	A	3	3	1.00	9	0.333
20	A	3	3	1.00	13	0.231
21	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	13	0.231
23	A	3	3	1.00	13	0.231
24	A	3	3	1.00	13	0.231
25	A	3	3	1.00	13	0.231
26	A	3	3	1.00	13	0.231
27	A	3	3	1.00	13	0.231
28	A	2	2	1.00	13	0.154
29	A	3	3	1.00	13	0.231
30	A	3	3	1.00	11	0.273
31	A	3	3	1.00	9	0.333
32	A	3	3	1.00	13	0.231
33	A	1	1	1.00	13	0.077
34	A	3	3	1.06	13	0.231
35	A	3	3	1.00	13	0.231
36	A	3	3	1.00	13	0.231
37	A	3	3	1.00	13	0.231
38	A	3	3	1.00	13	0.231
39	A	3	3	1.00	13	0.231
40	A	3	3	1.00	13	0.231
41	A	3	3	1.00	13	0.231
42	A	3	3	1.00	13	0.231
43	A	3	3	1.00	13	0.231
44	A	3	3	1.00	13	0.231
45	A	2	2	1.00	13	0.154
46	A	3	3	1.00	13	0.231
47	A	3	3	1.00	13	0.231
48	A	3	3	1.00	13	0.231
49	A	3	3	1.00	13	0.231
50	A	3	3	1.00	13	0.231
51	A	3	3	1.00	13	0.231
52	A	3	3	1.00	11	0.273
53	A	3	3	1.00	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	3	1.00	13	0.231
55	A	1	1	1.00	13	0.077
56	A	3	3	1.06	13	0.231
57	A	4	4	1.17	13	0.308
58	A	5	5	1.22	13	0.385
59	A	6	6	1.25	13	0.462
60	A	3	3	0.96	13	0.231
61	A	3	3	0.82	13	0.231
62	A	3	3	1.00	13	0.231
63	A	3	3	1.00	13	0.231
64	A	3	3	1.00	13	0.231
65	A	3	3	1.00	11	0.273
66	A	3	3	1.00	9	0.333
67	A	2	2	1.00	13	0.154
68	A	1	1	1.00	13	0.077
69	A	3	3	1.27	13	0.231
70	A	3	3	1.20	13	0.231
71	A	3	3	1.17	13	0.231
72	A	3	3	1.11	13	0.231
73	A	3	3	1.11	13	0.231
74	A	3	3	1.00	13	0.231
75	A	3	3	1.00	13	0.231
76	A	3	3	1.00	13	0.231
77	A	3	3	1.00	13	0.231
78	A	3	3	1.00	11	0.273
79	A	3	3	1.00	9	0.333
80	A	3	3	1.00	13	0.231
81	A	1	1	1.00	13	0.077
82	A	3	3	1.00	13	0.231
83	A	3	3	1.08	13	0.231
84	A	3	3	1.09	13	0.231
85	A	3	3	1.11	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.06	13	0.231
87	A	3	3	1.00	13	0.231
88	A	3	3	1.00	13	0.231
89	A	3	3	1.00	13	0.231
90	A	3	3	1.00	11	0.273
91	A	3	3	1.00	9	0.333
92	A	3	3	1.00	13	0.231
93	A	1	1	1.00	13	0.077
94	A	2	2	1.00	13	0.154
95	A	3	3	0.90	13	0.231
96	A	3	3	0.97	13	0.231
97	A	3	3	1.06	13	0.231
98	A	3	3	1.05	13	0.231
99	A	2	2	1.00	13	0.154
100	A	2	2	1.00	13	0.154
101	A	2	2	1.00	13	0.154
102	A	2	2	1.00	13	0.154
103	A	2	2	1.00	13	0.154
104	A	2	2	1.00	13	0.154
105	A	3	3	1.00	15	0.200
106	A	3	3	1.00	15	0.200
107	A	3	3	1.00	15	0.200
108	A	3	3	1.00	15	0.200
109	A	3	3	1.00	15	0.200
110	A	3	3	1.00	15	0.200
111	A	3	3	1.00	15	0.200
112	A	3	3	1.00	15	0.200
113	A	3	3	1.00	15	0.200
114	A	3	3	1.00	15	0.200
115	A	3	3	1.00	15	0.200
116	A	3	3	1.00	15	0.200
117	A	8	7	1.17	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.15	15	0.400
119	A	6	5	1.11	15	0.333
120	A	5	4	1.00	15	0.267
121	A	4	3	1.00	15	0.200
122	A	5	4	1.00	15	0.267
123	A	6	5	1.11	15	0.333
124	A	7	6	1.15	15	0.400
125	A	9	8	1.21	15	0.533
126	A	8	7	1.18	15	0.467
127	A	7	6	1.15	15	0.400
128	A	6	5	1.14	15	0.333
129	A	5	4	1.00	15	0.267
130	A	5	4	1.00	15	0.267
131	A	6	5	1.07	15	0.333
132	A	7	6	1.12	15	0.400
133	A	8	7	1.13	15	0.467
134	A	9	8	1.16	15	0.533
135	A	8	7	1.12	15	0.467
136	A	7	6	1.10	15	0.400
137	A	6	5	1.03	15	0.333
138	A	6	5	0.99	15	0.333
139	A	6	5	1.03	15	0.333
140	A	7	6	1.03	15	0.400
141	A	8	7	1.07	15	0.467
142	A	9	8	1.11	15	0.533
143	A	8	7	1.09	15	0.467
144	A	7	6	1.04	15	0.400
145	A	6	5	0.97	13	0.385
146	A	5	4	1.00	11	0.364
147	A	5	4	1.00	15	0.267
148	A	1	1	1.00	15	0.067
149	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	3	1.00	15	0.200
151	A	4	3	1.00	15	0.200
152	A	4	3	1.00	15	0.200
153	A	8	7	1.05	15	0.467
154	A	7	6	1.00	15	0.400
155	A	6	5	0.97	13	0.385
156	A	6	5	1.05	11	0.455
157	A	6	5	1.05	15	0.333
158	A	1	1	1.00	15	0.067
159	A	4	3	1.00	15	0.200
160	A	4	3	1.00	15	0.200
161	A	4	3	1.00	15	0.200
162	A	4	3	1.00	15	0.200
163	A	4	3	1.00	15	0.200
164	A	8	7	1.00	15	0.467
165	A	7	6	0.98	15	0.400
166	A	7	6	0.97	13	0.462
167	A	7	6	1.03	11	0.545
168	A	7	6	1.04	15	0.400
169	A	1	1	1.00	15	0.067
170	A	4	3	1.00	15	0.200
171	A	4	3	1.00	15	0.200
172	A	4	3	1.00	15	0.200
173	A	4	3	1.00	15	0.200
174	A	8	7	1.11	15	0.467
175	A	7	6	1.07	15	0.400
176	A	6	5	1.01	13	0.385
177	A	5	4	1.00	11	0.364
178	A	4	3	1.00	15	0.200
179	A	1	1	1.00	15	0.067
180	A	4	3	1.00	15	0.200
181	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	3	1.00	15	0.200
183	A	4	3	1.00	15	0.200
184	A	8	7	1.09	15	0.467
185	A	7	6	1.05	13	0.462
186	A	6	5	1.12	11	0.455
187	A	5	4	1.00	15	0.267
188	A	1	1	1.00	15	0.067
189	A	4	3	1.00	15	0.200
190	A	4	3	1.00	15	0.200
191	A	4	3	1.00	15	0.200
192	A	4	3	1.00	15	0.200
193	A	4	3	1.00	15	0.200
194	A	9	8	1.09	15	0.533
195	A	8	7	1.06	13	0.538
196	A	7	6	1.14	11	0.545
197	A	6	5	1.10	15	0.333
198	A	1	1	1.00	15	0.067
199	A	4	3	1.00	15	0.200
200	A	4	3	1.00	15	0.200
201	A	4	3	1.00	15	0.200
202	A	4	3	1.00	15	0.200
203	A	4	3	1.00	15	0.200
204	A	4	4	1.12	17	0.235
205	A	3	3	1.08	17	0.176
206	A	2	2	1.00	17	0.118
207	A	1	1	1.00	17	0.059
208	A	5	4	1.02	17	0.235
209	A	5	4	1.12	17	0.235
210	A	6	5	1.06	17	0.294
211	A	7	6	1.10	17	0.353
212	A	4	4	1.12	17	0.235
213	A	3	3	1.08	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	1.00	17	0.118
215	A	1	1	1.00	17	0.059
216	A	6	5	1.00	17	0.294
217	A	6	5	1.11	17	0.294
218	A	6	5	1.06	17	0.294
219	A	7	6	1.08	17	0.353
220	A	4	4	1.12	17	0.235
221	A	3	3	1.08	17	0.176
222	A	2	2	1.00	17	0.118
223	A	1	1	1.00	17	0.059
224	A	7	6	0.97	17	0.353
225	A	7	6	1.06	17	0.353
226	A	7	6	1.02	17	0.353
227	A	7	6	1.08	17	0.353
228	A	8	7	1.09	17	0.412
229	A	5	5	1.14	17	0.294
230	A	4	4	1.12	17	0.235
231	A	3	3	1.08	17	0.176
232	A	2	2	1.00	17	0.118
233	A	1	1	1.00	17	0.059
234	A	4	3	1.00	17	0.176
235	A	5	4	1.13	17	0.235
236	A	6	5	1.10	17	0.294
237	A	7	6	1.13	17	0.353
238	A	5	5	1.16	17	0.294
239	A	4	4	1.14	17	0.235
240	A	3	3	1.10	17	0.176
241	A	2	2	1.02	17	0.118
242	A	1	1	1.00	17	0.059
243	A	5	4	1.02	17	0.235
244	A	6	5	1.16	17	0.294
245	A	7	6	1.13	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	7	1.15	17	0.412
247	A	6	6	1.15	17	0.353
248	A	5	5	1.13	17	0.294
249	A	4	4	1.13	17	0.235
250	A	3	3	1.06	17	0.176
251	A	2	2	0.98	17	0.118
252	A	1	1	1.00	17	0.059
253	A	6	5	1.08	17	0.294
254	A	7	6	1.18	17	0.353
255	A	8	7	1.16	17	0.412
256	A	2	2	1.00	11	0.182
257	A	2	2	1.00	11	0.182
258	A	2	2	1.00	11	0.182
259	A	2	2	1.00	11	0.182
260	A	2	2	1.00	11	0.182
261	A	2	2	1.00	9	0.222
262	A	1	1	1.00	7	0.143
263	A	2	2	1.00	11	0.182
264	A	2	2	1.00	11	0.182
265	A	2	2	1.06	11	0.182
266	A	2	2	1.00	11	0.182
267	A	2	2	1.00	11	0.182
268	A	2	2	1.00	11	0.182
269	A	5	4	1.13	13	0.308
270	A	5	4	1.13	13	0.308
271	A	2	2	1.00	13	0.154
272	A	5	4	1.30	13	0.308
273	A	5	4	1.04	11	0.364
274	A	5	4	1.25	13	0.308
275	A	1	1	1.00	13	0.077
276	A	5	4	1.13	13	0.308
277	A	5	4	1.13	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	3	3	1.00	13	0.231
279	A	3	3	1.00	13	0.231
280	A	3	3	1.00	13	0.231
281	A	3	3	1.00	9	0.333
282	A	3	3	1.00	13	0.231
283	A	3	3	1.00	13	0.231
284	A	3	3	1.00	13	0.231
285	A	5	4	1.09	13	0.308
286	A	5	4	1.12	13	0.308
287	A	2	2	1.00	13	0.154
288	A	5	4	1.10	13	0.308
289	A	5	4	1.05	13	0.308
290	A	5	4	1.02	11	0.364
291	A	5	4	1.10	13	0.308
292	A	1	1	1.00	13	0.077
293	A	5	4	1.29	13	0.308
294	A	5	4	1.09	13	0.308
295	A	3	3	1.00	13	0.231
296	A	3	3	1.00	13	0.231
297	A	3	3	1.00	13	0.231
298	A	3	3	1.00	13	0.231
299	A	3	3	1.00	9	0.333
300	A	3	3	1.00	13	0.231
301	A	3	3	1.00	13	0.231
302	A	3	3	1.00	13	0.231
303	A	5	4	0.98	13	0.308
304	A	5	4	0.98	13	0.308
305	A	5	4	0.96	11	0.364
306	A	2	2	1.00	13	0.154
307	A	1	1	1.00	13	0.077
308	A	5	4	1.33	13	0.308
309	A	5	4	1.25	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	3	3	1.00	13	0.231
311	A	3	3	1.00	13	0.231
312	A	3	3	1.00	13	0.231
313	A	3	3	1.00	9	0.333
314	A	2	2	1.00	13	0.154
315	A	3	3	1.00	13	0.231
316	A	4	4	1.19	13	0.308
317	A	5	5	1.17	13	0.385
318	A	5	4	0.97	13	0.308
319	A	5	4	0.98	13	0.308
320	A	5	4	0.98	11	0.364
321	A	5	4	0.94	13	0.308
322	A	1	1	1.00	13	0.077
323	A	5	4	1.18	13	0.308
324	A	5	4	1.21	13	0.308
325	A	5	4	1.21	13	0.308
326	A	4	4	1.08	13	0.308
327	A	4	4	1.05	13	0.308
328	A	4	4	1.06	13	0.308
329	A	4	4	1.14	9	0.444
330	A	3	3	1.00	13	0.231
331	A	3	3	1.00	13	0.231
332	A	4	4	1.13	13	0.308
333	A	5	5	1.16	13	0.385
334	A	5	4	1.02	13	0.308
335	A	5	4	1.03	13	0.308
336	A	5	4	0.95	11	0.364
337	A	5	4	1.04	13	0.308
338	A	1	1	1.00	13	0.077
339	A	2	2	1.00	13	0.154
340	A	5	4	1.10	13	0.308
341	A	5	4	1.06	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	5	4	1.19	13	0.308
343	A	5	5	1.14	13	0.385
344	A	5	5	1.13	13	0.385
345	A	5	5	1.20	9	0.556
346	A	4	4	1.12	13	0.308
347	A	4	4	1.08	13	0.308
348	A	4	4	1.13	13	0.308
349	A	5	5	1.22	13	0.385
350	A	6	6	1.21	13	0.462
351	A	6	5	1.03	15	0.333
352	A	1	1	1.00	15	0.067
353	A	5	4	0.96	13	0.308
354	A	5	4	1.00	11	0.364
355	A	5	4	1.13	15	0.267
356	A	5	4	1.00	15	0.267
357	A	1	1	1.00	15	0.067
358	A	6	5	1.04	15	0.333
359	A	6	5	1.03	15	0.333
360	A	6	5	0.98	15	0.333
361	A	6	5	1.03	13	0.385
362	A	6	5	1.03	11	0.455
363	A	6	5	1.15	15	0.333
364	A	6	5	1.04	15	0.333
365	A	1	1	1.00	15	0.067
366	A	7	6	1.06	15	0.400
367	A	7	6	1.01	15	0.400
368	A	7	6	1.00	15	0.400
369	A	7	6	1.01	13	0.462
370	A	7	6	1.00	11	0.545
371	A	7	6	1.11	15	0.400
372	A	7	6	1.07	15	0.400
373	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	8	7	1.08	15	0.467
375	A	6	5	1.07	15	0.333
376	A	5	4	0.98	13	0.308
377	A	4	3	1.00	15	0.200
378	A	1	1	1.00	15	0.067
379	A	4	3	1.14	15	0.200
380	A	4	3	1.07	15	0.200
381	A	4	3	1.07	15	0.200
382	A	3	3	1.09	15	0.200
383	A	2	2	1.00	15	0.133
384	A	1	1	1.00	11	0.091
385	A	4	3	1.00	15	0.200
386	A	5	4	1.00	15	0.267
387	A	4	3	1.00	17	0.176
388	A	3	2	1.10	15	0.133
389	A	3	2	1.10	16	0.125
390	A	7	6	1.09	15	0.400
391	A	6	5	1.06	13	0.385
392	A	5	4	1.12	15	0.267
393	A	1	1	1.00	15	0.067
394	A	4	3	1.12	15	0.200
395	A	4	3	1.09	15	0.200
396	A	4	3	1.07	15	0.200
397	A	6	5	1.15	15	0.333
398	A	5	4	1.11	15	0.267
399	A	4	3	1.06	11	0.273
400	A	1	1	1.00	15	0.067
401	A	5	4	1.00	15	0.267
402	A	6	5	1.08	15	0.333
403	A	7	6	1.13	15	0.400
404	A	8	7	1.09	15	0.467
405	A	7	6	1.09	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	6	5	1.19	15	0.333
407	A	1	1	1.00	15	0.067
408	A	4	3	1.14	15	0.200
409	A	4	3	1.07	15	0.200
410	A	4	3	1.05	15	0.200
411	A	6	5	1.16	15	0.333
412	A	5	4	1.14	11	0.364
413	A	2	2	0.98	15	0.133
414	A	1	1	1.00	15	0.067
415	A	6	5	1.09	15	0.333
416	A	7	6	1.13	15	0.400
417	A	1	1	1.00	13	0.077
418	A	1	1	1.00	13	0.077
419	A	3	2	1.00	17	0.118
420	A	3	2	1.00	17	0.118
421	A	3	2	1.00	17	0.118
422	A	3	2	1.00	17	0.118
423	A	3	2	1.00	15	0.133
424	A	4	3	1.00	15	0.200
425	A	5	4	0.98	13	0.308
426	A	5	4	0.98	13	0.308
427	A	5	4	0.96	13	0.308
428	A	2	2	1.00	13	0.154
429	A	1	1	1.00	13	0.077
430	A	5	4	1.33	13	0.308
431	A	5	4	1.25	13	0.308
432	A	3	3	1.00	13	0.231
433	A	3	3	1.00	13	0.231
434	A	11	10	1.07	11	0.909
435	A	11	10	1.03	9	1.111
436	A	10	9	1.01	13	0.692
437	A	10	9	0.97	13	0.692

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	11	10	1.07	13	0.769
439	A	11	10	1.03	13	0.769
440	A	5	4	1.00	13	0.308
441	A	5	4	0.93	13	0.308
442	A	5	4	0.94	13	0.308
443	A	1	1	1.00	13	0.077
444	A	5	4	1.18	13	0.308
445	A	5	4	1.13	13	0.308
446	A	4	4	1.03	13	0.308
447	A	12	11	1.09	11	1.000
448	A	12	11	1.07	9	1.222
449	A	11	10	1.05	13	0.769
450	A	11	10	1.01	13	0.769
451	A	11	10	1.05	13	0.769
452	A	11	10	1.01	13	0.769
453	A	12	11	1.09	13	0.846
454	A	6	5	1.03	15	0.333
455	A	5	4	0.96	15	0.267
456	A	5	4	1.00	15	0.267
457	A	1	1	1.00	15	0.067
458	A	4	3	1.11	15	0.200
459	A	4	3	1.07	15	0.200
460	A	4	3	1.05	15	0.200
461	A	6	5	1.04	15	0.333
462	A	5	4	1.02	15	0.267
463	A	4	3	1.00	13	0.231
464	A	4	3	1.00	15	0.200
465	A	5	4	1.01	15	0.267
466	A	6	5	1.03	15	0.333
467	A	8	7	1.06	15	0.467
468	A	7	6	1.05	15	0.400
469	A	6	5	1.05	11	0.455

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	6	5	1.04	15	0.333
471	A	7	6	1.05	15	0.400
472	A	8	7	1.06	15	0.467
473	A	6	5	1.03	15	0.333
474	A	6	5	1.05	15	0.333
475	A	6	5	1.05	15	0.333
476	A	1	1	1.00	15	0.067
477	A	4	3	1.11	15	0.200
478	A	4	3	1.07	15	0.200
479	A	4	3	1.05	15	0.200
480	A	6	5	1.07	15	0.333
481	A	5	4	0.98	15	0.267
482	A	4	3	1.00	15	0.200
483	A	1	1	1.00	15	0.067
484	A	4	3	1.05	15	0.200
485	A	4	3	1.03	15	0.200
486	A	4	3	1.00	15	0.200
487	A	6	5	1.05	15	0.333
488	A	5	4	1.03	15	0.267
489	A	4	3	1.00	13	0.231
490	A	3	2	1.00	15	0.133
491	A	4	3	1.00	15	0.200
492	A	5	4	1.02	15	0.267
493	A	8	7	1.06	15	0.467
494	A	7	6	1.05	15	0.400
495	A	6	5	1.05	11	0.455
496	A	5	4	1.05	15	0.267
497	A	6	5	1.05	15	0.333
498	A	7	6	1.06	15	0.400
499	A	8	7	1.06	15	0.467
500	A	7	6	1.09	15	0.400
501	A	6	5	1.14	15	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	5	4	1.00	15	0.267
503	A	1	1	1.00	15	0.067
504	A	4	3	1.00	15	0.200
505	A	4	3	1.00	15	0.200
506	A	4	3	1.00	15	0.200
507	A	7	6	1.07	15	0.400
508	A	6	5	1.05	15	0.333
509	A	5	4	1.03	13	0.308
510	A	4	3	1.00	15	0.200
511	A	4	3	1.00	15	0.200
512	A	5	4	1.03	15	0.267
513	A	9	8	1.07	15	0.533
514	A	8	7	1.06	15	0.467
515	A	7	6	1.05	11	0.545
516	A	6	5	1.04	15	0.333
517	A	6	5	1.05	15	0.333
518	A	7	6	1.06	15	0.400
519	A	8	7	1.07	15	0.467
520	A	11	10	1.53	9	1.111
521	A	5	4	0.96	15	0.267
522	A	6	5	0.94	13	0.385
523	A	5	4	1.00	15	0.267
524	A	6	5	1.08	15	0.333
525	A	4	3	1.02	15	0.200
526	A	7	6	1.05	11	0.545
527	A	4	3	1.02	15	0.200
528	A	7	6	1.03	15	0.400
529	A	6	5	1.00	15	0.333
530	A	7	6	1.01	13	0.462
531	A	6	5	1.05	15	0.333
532	A	7	6	1.10	15	0.400
533	A	5	4	1.03	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
534	A	8	7	1.04	11	0.636
535	A	5	4	1.06	15	0.267
536	A	8	7	1.04	15	0.467
537	A	7	6	1.01	15	0.400
538	A	8	7	0.99	13	0.538
539	A	7	6	1.04	15	0.400
540	A	8	7	1.11	15	0.467
541	A	6	5	1.04	15	0.333
542	A	9	8	1.03	11	0.727
543	A	6	5	1.07	15	0.333
544	A	9	8	1.05	15	0.533
545	A	5	4	0.98	15	0.267
546	A	1	1	1.00	13	0.077
547	A	4	3	1.00	15	0.200
548	A	5	4	1.00	15	0.267
549	A	4	3	1.02	15	0.200
550	A	7	6	1.05	11	0.545
551	A	3	2	1.02	15	0.133
552	A	6	5	1.03	15	0.333
553	A	6	5	1.06	15	0.333
554	A	2	2	1.00	13	0.154
555	A	5	4	1.00	15	0.267
556	A	1	1	1.00	15	0.067
557	A	5	4	1.08	15	0.267
558	A	8	7	1.04	11	0.636
559	A	4	3	1.02	15	0.200
560	A	7	6	1.03	15	0.400
561	A	7	6	1.09	15	0.400
562	A	3	3	1.08	13	0.231
563	A	6	5	1.09	15	0.333
564	A	2	2	1.02	15	0.133
565	A	6	5	1.12	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	9	8	1.07	11	0.727
567	A	5	4	1.08	15	0.267
568	A	8	7	1.06	15	0.467
569	A	12	11	1.06	9	1.222
570	A	4	3	1.00	15	0.200
571	A	4	3	1.00	17	0.176
572	A	12	11	1.43	9	1.222
573	A	15	14	1.46	9	1.556

CHAPTER 3

Listing of integrals

3.1 $\int \left(a+\frac{b}{x}\right) x^6 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x}\right) x^6 \, dx = \frac{bx^6}{6}+\frac{ax^7}{7}$$

output
```
1/6*b*x^6+1/7*a*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) x^6 \, dx = \frac{bx^6}{6}+\frac{ax^7}{7}$$

input
```
Integrate[(a + b/x)*x^6,x]
```

output
```
(b*x^6)/6 + (a*x^7)/7
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+\frac{b}{x}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^6+b x^5\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^7}{7}+\frac{b x^6}{6}$$

input
```
Int[(a + b/x)*x^6,x]
```

output
```
(b*x^6)/6 + (a*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^{6}(6ax+7b)}{42}$	14
default	$\frac{1}{6} b\,x^{6}+\frac{1}{7} a\,x^{7}$	14
norman	$\frac{1}{6} b\,x^{6}+\frac{1}{7} a\,x^{7}$	14
risch	$\frac{1}{6} b\,x^{6}+\frac{1}{7} a\,x^{7}$	14
parallelrisch	$\frac{1}{6} b\,x^{6}+\frac{1}{7} a\,x^{7}$	14
orering	$\frac{x^{7}(6ax+7b)\left(a+\frac{b}{x}\right)}{42ax+42b}$	28

input
```
int((a+b/x)*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/42*x^6*(6*a*x+7*b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^6\,dx=\frac{1}{7}\,ax^7+\frac{1}{6}\,bx^6$$

input
```
integrate((a+b/x)*x^6,x, algorithm="fricas")
```

output
```
1/7*a*x^7 + 1/6*b*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x^6\,dx=\frac{ax^7}{7}+\frac{bx^6}{6}$$

input
```
integrate((a+b/x)*x**6,x)
```

output
```
a*x**7/7 + b*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^6\,dx=\frac{1}{7}\,ax^7+\frac{1}{6}\,bx^6$$

input
```
integrate((a+b/x)*x^6,x, algorithm="maxima")
```

output
```
1/7*a*x^7 + 1/6*b*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^6\,dx=\frac{1}{7}\,ax^7+\frac{1}{6}\,bx^6$$

input
```
integrate((a+b/x)*x^6,x, algorithm="giac")
```

output
```
1/7*a*x^7 + 1/6*b*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^6 \, dx = \frac{x^6 \, (7 \, b + 6 \, a \, x)}{42}$$

input
```
int(x^6*(a + b/x),x)
```

output
```
(x^6*(7*b + 6*a*x))/42
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^6 \, dx = \frac{x^6(6ax + 7b)}{42}$$

input
```
int((a+b/x)*x^6,x)
```

output
```
(x**6*(6*a*x + 7*b))/42
```

3.2 $\int \left(a+\frac{b}{x}\right) x^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x}\right) x^5 \, dx=\frac{b x^5}{5}+\frac{a x^6}{6}$$

output
```
1/5*b*x^5+1/6*a*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) x^5 \, dx=\frac{b x^5}{5}+\frac{a x^6}{6}$$

input
```
Integrate[(a + b/x)*x^5,x]
```

output
```
(b*x^5)/5 + (a*x^6)/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{x}\right) \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(a x^5+b x^4\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a x^6}{6}+\frac{b x^5}{5}$$

input
```
Int[(a + b/x)*x^5,x]
```

output
```
(b*x^5)/5 + (a*x^6)/6
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^5(5ax+6b)}{30}$	14
default	$\frac{1}{5}b\,x^5+\frac{1}{6}a\,x^6$	14
norman	$\frac{1}{5}b\,x^5+\frac{1}{6}a\,x^6$	14
risch	$\frac{1}{5}b\,x^5+\frac{1}{6}a\,x^6$	14
parallelrisch	$\frac{1}{5}b\,x^5+\frac{1}{6}a\,x^6$	14
orering	$\frac{x^6(5ax+6b)\left(a+\frac{b}{x}\right)}{30ax+30b}$	28

input
```
int((a+b/x)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^5*(5*a*x+6*b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^5\,dx=\frac{1}{6}\,ax^6+\frac{1}{5}\,bx^5$$

input
```
integrate((a+b/x)*x^5,x, algorithm="fricas")
```

output
```
1/6*a*x^6 + 1/5*b*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)x^5\,dx=\frac{ax^6}{6}+\frac{bx^5}{5}$$

input
```
integrate((a+b/x)*x**5,x)
```

output
```
a*x**6/6 + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^5\,dx=\frac{1}{6}\,ax^6+\frac{1}{5}\,bx^5$$

input
```
integrate((a+b/x)*x^5,x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/5*b*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^5\,dx=\frac{1}{6}\,ax^6+\frac{1}{5}\,bx^5$$

input
```
integrate((a+b/x)*x^5,x, algorithm="giac")
```

output
```
1/6*a*x^6 + 1/5*b*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^5 \, dx = \frac{x^5\,(6\,b+5\,a\,x)}{30}$$

input
```
int(x^5*(a + b/x),x)
```

output
```
(x^5*(6*b + 5*a*x))/30
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^5 \, dx = \frac{x^5(5ax+6b)}{30}$$

input
```
int((a+b/x)*x^5,x)
```

output
```
(x**5*(5*a*x + 6*b))/30
```

3.3 $\int \left(a+\frac{b}{x}\right) x^4 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x}\right) x^4 \, dx = \frac{bx^4}{4}+\frac{ax^5}{5}$$

output
```
1/4*b*x^4+1/5*a*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) x^4 \, dx = \frac{bx^4}{4}+\frac{ax^5}{5}$$

input
```
Integrate[(a + b/x)*x^4,x]
```

output
```
(b*x^4)/4 + (a*x^5)/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^4+b x^3\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^5}{5}+\frac{b x^4}{4}$$

input
```
Int[(a + b/x)*x^4,x]
```

output
```
(b*x^4)/4 + (a*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^{4}(4ax+5b)}{20}$	14
default	$\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	14
norman	$\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	14
risch	$\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	14
parallelrisch	$\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	14
orering	$\frac{x^{5}(4ax+5b)\left(a+\frac{b}{x}\right)}{20ax+20b}$	28

input
```
int((a+b/x)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/20*x^4*(4*a*x+5*b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int\left(a+\frac{b}{x}\right)x^4\,dx=\frac{1}{5}\,ax^5+\frac{1}{4}\,bx^4$$

input
```
integrate((a+b/x)*x^4,x, algorithm="fricas")
```

output
```
1/5*a*x^5 + 1/4*b*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)x^4\,dx=\frac{ax^5}{5}+\frac{bx^4}{4}$$

input
```
integrate((a+b/x)*x**4,x)
```

output
```
a*x**5/5 + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^4\,dx=\frac{1}{5}\,ax^5+\frac{1}{4}\,bx^4$$

input
```
integrate((a+b/x)*x^4,x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/4*b*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^4\,dx=\frac{1}{5}\,ax^5+\frac{1}{4}\,bx^4$$

input
```
integrate((a+b/x)*x^4,x, algorithm="giac")
```

output
```
1/5*a*x^5 + 1/4*b*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^4 \, dx = \frac{x^4 \, (5 \, b + 4 \, a \, x)}{20}$$

input
```
int(x^4*(a + b/x),x)
```

output
```
(x^4*(5*b + 4*a*x))/20
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^4 \, dx = \frac{x^4(4ax+5b)}{20}$$

input
```
int((a+b/x)*x^4,x)
```

output
```
(x**4*(4*a*x + 5*b))/20
```

3.4 $\int \left(a+\frac{b}{x}\right) x^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x}\right) x^3 \, dx = \frac{bx^3}{3}+\frac{ax^4}{4}$$

output
```
1/3*b*x^3+1/4*a*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) x^3 \, dx = \frac{bx^3}{3}+\frac{ax^4}{4}$$

input
```
Integrate[(a + b/x)*x^3,x]
```

output
```
(b*x^3)/3 + (a*x^4)/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x}\right) \, dx$$

$$\Big\downarrow \ 802$$

$$\int \left(a x^3+b x^2\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a x^4}{4}+\frac{b x^3}{3}$$

input
```
Int[(a + b/x)*x^3,x]
```

output
```
(b*x^3)/3 + (a*x^4)/4
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^3(3ax+4b)}{12}$	14
default	$\frac{1}{3}b\,x^3+\frac{1}{4}a\,x^4$	14
norman	$\frac{1}{3}b\,x^3+\frac{1}{4}a\,x^4$	14
risch	$\frac{1}{3}b\,x^3+\frac{1}{4}a\,x^4$	14
parallelrisch	$\frac{1}{3}b\,x^3+\frac{1}{4}a\,x^4$	14
orering	$\frac{x^4(3ax+4b)\left(a+\frac{b}{x}\right)}{12ax+12b}$	28

input
```
int((a+b/x)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(3*a*x+4*b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^3\,dx=\frac{1}{4}\,ax^4+\frac{1}{3}\,bx^3$$

input
```
integrate((a+b/x)*x^3,x, algorithm="fricas")
```

output
```
1/4*a*x^4 + 1/3*b*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x^3\,dx = \frac{ax^4}{4}+\frac{bx^3}{3}$$

input
```
integrate((a+b/x)*x**3,x)
```

output
```
a*x**4/4 + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^3\,dx = \frac{1}{4}\,ax^4+\frac{1}{3}\,bx^3$$

input
```
integrate((a+b/x)*x^3,x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/3*b*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^3\,dx = \frac{1}{4}\,ax^4+\frac{1}{3}\,bx^3$$

input
```
integrate((a+b/x)*x^3,x, algorithm="giac")
```

output
```
1/4*a*x^4 + 1/3*b*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^3\,dx=\frac{x^3\,\left(4\,b+3\,a\,x\right)}{12}$$

input
```
int(x^3*(a + b/x),x)
```

output
```
(x^3*(4*b + 3*a*x))/12
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right) x^3\,dx=\frac{x^3(3ax+4b)}{12}$$

input
```
int((a+b/x)*x^3,x)
```

output
```
(x**3*(3*a*x + 4*b))/12
```

3.5 $\int \left(a+\frac{b}{x}\right) x^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x}\right) x^2 \, dx=\frac{b x^2}{2}+\frac{a x^3}{3}$$

output
```
1/2*b*x^2+1/3*a*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) x^2 \, dx=\frac{b x^2}{2}+\frac{a x^3}{3}$$

input
```
Integrate[(a + b/x)*x^2,x]
```

output
```
(b*x^2)/2 + (a*x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{x}\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^2+bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^2}{2}$$

input
```
Int[(a + b/x)*x^2,x]
```

output
```
(b*x^2)/2 + (a*x^3)/3
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^2(2ax+3b)}{6}$	14
default	$\frac{1}{2}b\,x^2+\frac{1}{3}a\,x^3$	14
norman	$\frac{1}{2}b\,x^2+\frac{1}{3}a\,x^3$	14
risch	$\frac{1}{2}b\,x^2+\frac{1}{3}a\,x^3$	14
parallelrisch	$\frac{1}{2}b\,x^2+\frac{1}{3}a\,x^3$	14
orering	$\frac{x^3(2ax+3b)\left(a+\frac{b}{x}\right)}{6ax+6b}$	28

input
```
int((a+b/x)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2*(2*a*x+3*b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int\left(a+\frac{b}{x}\right)x^2\,dx=\frac{1}{3}\,ax^3+\frac{1}{2}\,bx^2$$

input
```
integrate((a+b/x)*x^2,x, algorithm="fricas")
```

output
```
1/3*a*x^3 + 1/2*b*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)x^2\,dx=\frac{ax^3}{3}+\frac{bx^2}{2}$$

input
```
integrate((a+b/x)*x**2,x)
```

output
```
a*x**3/3 + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^2\,dx=\frac{1}{3}\,ax^3+\frac{1}{2}\,bx^2$$

input
```
integrate((a+b/x)*x^2,x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/2*b*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^2\,dx=\frac{1}{3}\,ax^3+\frac{1}{2}\,bx^2$$

input
```
integrate((a+b/x)*x^2,x, algorithm="giac")
```

output
```
1/3*a*x^3 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^2\,dx=\frac{x^2\,(3\,b+2\,a\,x)}{6}$$

input
```
int(x^2*(a + b/x),x)
```

output
```
(x^2*(3*b + 2*a*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^2\,dx=\frac{x^2(2ax+3b)}{6}$$

input
```
int((a+b/x)*x^2,x)
```

output
```
(x**2*(2*a*x + 3*b))/6
```

3.6 $\int \left(a+\frac{b}{x}\right) x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \left(a+\frac{b}{x}\right) x \, dx = \frac{(b+ax)^2}{2a}$$

output
```
1/2*(a*x+b)^2/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x}\right) x \, dx = bx + \frac{ax^2}{2}$$

input
```
Integrate[(a + b/x)*x,x]
```

output
```
b*x + (a*x^2)/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {802, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x}\right) dx$$

$$\Big\downarrow \text{802}$$

$$\int (ax+b)dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(ax+b)^2}{2a}$$

input
```
Int[(a + b/x)*x,x]
```

output
```
(b + a*x)^2/(2*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x(ax+2b)}{2}$	11
default	$\frac{1}{2}a\,x^2+bx$	11
norman	$\frac{1}{2}a\,x^2+bx$	11
risch	$\frac{1}{2}a\,x^2+bx$	11
parallelrisch	$\frac{1}{2}a\,x^2+bx$	11
parts	$\frac{1}{2}a\,x^2+bx$	11
orering	$\frac{x^2(ax+2b)\left(a+\frac{b}{x}\right)}{2ax+2b}$	27

input
```
int((a+b/x)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(a*x+2*b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int\left(a+\frac{b}{x}\right)x\,dx=\frac{1}{2}\,ax^2+bx$$

input
```
integrate((a+b/x)*x,x, algorithm="fricas")
```

output
```
1/2*a*x^2 + b*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \left(a+\frac{b}{x}\right) x\,dx = \frac{ax^2}{2} + bx$$

input
```
integrate((a+b/x)*x,x)
```

output
```
a*x**2/2 + b*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x\,dx = \frac{1}{2}\,ax^2 + bx$$

input
```
integrate((a+b/x)*x,x, algorithm="maxima")
```

output
```
1/2*a*x^2 + b*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x\,dx = \frac{1}{2}\,ax^2 + bx$$

input
```
integrate((a+b/x)*x,x, algorithm="giac")
```

output
```
1/2*a*x^2 + b*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x \, dx = \frac{a\,x^2}{2} + b\,x$$

input
```
int(x*(a + b/x),x)
```

output
```
b*x + (a*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x \, dx = \frac{x(ax+2b)}{2}$$

input
```
int((a+b/x)*x,x)
```

output
```
(x*(a*x + 2*b))/2
```

3.7 $\int \left(a+\frac{b}{x}\right) dx$

Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \left(a+\frac{b}{x}\right) dx = ax + b\log(x)$$

output
```
a*x+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) dx = ax + b\log(x)$$

input
```
Integrate[a + b/x,x]
```

output
```
a*x + b*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$ax+b\log(x)$$

input
```
Int[a + b/x,x]
```

output
```
a*x + b*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$ax+b\ln\left(x\right)$	9
norman	$ax+b\ln\left(x\right)$	9
risch	$ax+b\ln\left(x\right)$	9
parallelrisch	$ax+b\ln\left(x\right)$	9

input
```
int(a+b/x,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) dx = ax + b\log\left(x\right)$$

input
```
integrate(a+b/x,x, algorithm="fricas")
```

output
```
a*x + b*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x}\right) dx = ax + b\log\left(x\right)$$

input
```
integrate(a+b/x,x)
```

output
```
a*x + b*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) dx = ax + b\log\left(x\right)$$

input
```
integrate(a+b/x,x, algorithm="maxima")
```

output
```
a*x + b*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \left(a+\frac{b}{x}\right) dx = ax + b\log(|x|)$$

input
```
integrate(a+b/x,x, algorithm="giac")
```

output
```
a*x + b*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) dx = a\,x + b\,\ln(x)$$

input
```
int(a + b/x,x)
```

output
```
a*x + b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right) dx = \log(x)\,b + ax$$

input
```
int(a+b/x,x)
```

output
```
log(x)*b + a*x
```

3.8 $\int \frac{a+\frac{b}{x}}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{a+\frac{b}{x}}{x} dx = -\frac{b}{x} + a\log(x)$$

output
```
-b/x+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x} dx = -\frac{b}{x} + a\log(x)$$

input
```
Integrate[(a + b/x)/x,x]
```

output
```
-(b/x) + a*Log[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x}+\frac{b}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a\log(x)-\frac{b}{x}$$

input
```
Int[(a + b/x)/x,x]
```

output
```
-(b/x) + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{b}{x}+a\ln\left(x\right)$	12
norman	$-\frac{b}{x}+a\ln\left(x\right)$	12
risch	$-\frac{b}{x}+a\ln\left(x\right)$	12
parallelrisch	$\frac{a\ln(x)x-b}{x}$	14

input
```
int((a+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
-b/x+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{a+\frac{b}{x}}{x}\,dx=\frac{ax\log\left(x\right)-b}{x}$$

input
```
integrate((a+b/x)/x,x, algorithm="fricas")
```

output
```
(a*x*log(x) - b)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{a+\frac{b}{x}}{x}\,dx=a\log\left(x\right)-\frac{b}{x}$$

input
```
integrate((a+b/x)/x,x)
```

output
```
a*log(x) - b/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x}\,dx = a\log(x) - \frac{b}{x}$$

input
```
integrate((a+b/x)/x,x, algorithm="maxima")
```

output
```
a*log(x) - b/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{a+\frac{b}{x}}{x}\,dx = a\log(|x|) - \frac{b}{x}$$

input
```
integrate((a+b/x)/x,x, algorithm="giac")
```

output
```
a*log(abs(x)) - b/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x}\,dx = a\,\ln(x) - \frac{b}{x}$$

input
```
int((a + b/x)/x,x)
```

output
```
a*log(x) - b/x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{a+\frac{b}{x}}{x}\,dx=\frac{\log(x)\,ax-b}{x}$$

input
```
int((a+b/x)/x,x)
```

output
```
(log(x)*a*x - b)/x
```

3.9 $\int \frac{a+\frac{b}{x}}{x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{a+\frac{b}{x}}{x^2} \, dx = -\frac{\left(a+\frac{b}{x}\right)^2}{2b}$$

output
```
-1/2*(a+b/x)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+\frac{b}{x}}{x^2} \, dx = -\frac{b}{2x^2} - \frac{a}{x}$$

input
```
Integrate[(a + b/x)/x^2,x]
```

output
```
-1/2*b/x^2 - a/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x^2} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^2}+\frac{b}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{x}-\frac{b}{2x^2}$$

input
```
Int[(a + b/x)/x^2,x]
```

output
```
-1/2*b/x^2 - a/x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{2ax+b}{2x^2}$	12
norman	$\frac{-ax-\frac{b}{2}}{x^2}$	13
risch	$\frac{-ax-\frac{b}{2}}{x^2}$	13
default	$-\frac{b}{2x^2}-\frac{a}{x}$	14
parallelrisch	$\frac{-2ax-b}{2x^2}$	14
orering	$-\frac{(2ax+b)\left(a+\frac{b}{x}\right)}{2x(ax+b)}$	26

input
```
int((a+b/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a*x+b)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{a+\frac{b}{x}}{x^2}\,dx=-\frac{2\,ax+b}{2\,x^2}$$

input
```
integrate((a+b/x)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*x + b)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{a+\frac{b}{x}}{x^2}\,dx = \frac{-2ax-b}{2x^2}$$

input
```
integrate((a+b/x)/x**2,x)
```

output
```
(-2*a*x - b)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x}}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^2}{2\,b}$$

input
```
integrate((a+b/x)/x^2,x, algorithm="maxima")
```

output
```
-1/2*(a + b/x)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x}}{x^2}\,dx = -\frac{a}{x} - \frac{b}{2\,x^2}$$

input
```
integrate((a+b/x)/x^2,x, algorithm="giac")
```

output
```
-a/x - 1/2*b/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{a+\frac{b}{x}}{x^2} \, dx = -\frac{b+2\,a\,x}{2\,x^2}$$

input
```
int((a + b/x)/x^2,x)
```

output
```
-(b + 2*a*x)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x}}{x^2} \, dx = \frac{-2ax-b}{2x^2}$$

input
```
int((a+b/x)/x^2,x)
```

output
```
( - 2*a*x - b)/(2*x**2)
```

3.10 $\int \frac{a+\frac{b}{x}}{x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+\frac{b}{x}}{x^3} \, dx = -\frac{b}{3x^3} - \frac{a}{2x^2}$$

output
```
-1/3*b/x^3-1/2*a/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x^3} \, dx = -\frac{b}{3x^3} - \frac{a}{2x^2}$$

input
```
Integrate[(a + b/x)/x^3,x]
```

output
```
-1/3*b/x^3 - a/(2*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}-\frac{b}{3x^3}$$

input
```
Int[(a + b/x)/x^3,x]
```

output
```
-1/3*b/x^3 - a/(2*x^2)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{ax}{2}-\frac{b}{3}}{x^3}$	13
risch	$\frac{-\frac{ax}{2}-\frac{b}{3}}{x^3}$	13
gosper	$-\frac{3ax+2b}{6x^3}$	14
default	$-\frac{b}{3x^3}-\frac{a}{2x^2}$	14
parallelrisch	$\frac{-3ax-2b}{6x^3}$	14
orering	$-\frac{(3ax+2b)\left(a+\frac{b}{x}\right)}{6x^2(ax+b)}$	28

input
```
int((a+b/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a*x-1/3*b)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^3}\,dx = -\frac{3\,ax+2\,b}{6\,x^3}$$

input
```
integrate((a+b/x)/x^3,x, algorithm="fricas")
```

output
```
-1/6*(3*a*x + 2*b)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+\frac{b}{x}}{x^3}\,dx = \frac{-3ax-2b}{6x^3}$$

input
```
integrate((a+b/x)/x**3,x)
```

output
```
(-3*a*x - 2*b)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^3}\,dx = -\frac{3\,ax+2\,b}{6\,x^3}$$

input
```
integrate((a+b/x)/x^3,x, algorithm="maxima")
```

output
```
-1/6*(3*a*x + 2*b)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^3}\,dx = -\frac{3\,ax+2\,b}{6\,x^3}$$

input
```
integrate((a+b/x)/x^3,x, algorithm="giac")
```

output
```
-1/6*(3*a*x + 2*b)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^3}\,dx = -\frac{2\,b+3\,a\,x}{6\,x^3}$$

input
```
int((a + b/x)/x^3,x)
```

output
```
-(2*b + 3*a*x)/(6*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^3}\,dx = \frac{-3ax-2b}{6x^3}$$

input
```
int((a+b/x)/x^3,x)
```

output
```
( - 3*a*x - 2*b)/(6*x**3)
```

3.11 $\int \frac{a+\frac{b}{x}}{x^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+\frac{b}{x}}{x^4} \, dx = -\frac{b}{4x^4} - \frac{a}{3x^3}$$

output
```
-1/4*b/x^4-1/3*a/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x^4} \, dx = -\frac{b}{4x^4} - \frac{a}{3x^3}$$

input
```
Integrate[(a + b/x)/x^4,x]
```

output
```
-1/4*b/x^4 - a/(3*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x^4}\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^4}+\frac{b}{x^5}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{3x^3}-\frac{b}{4x^4}$$

input
```
Int[(a + b/x)/x^4,x]
```

output
```
-1/4*b/x^4 - a/(3*x^3)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{ax}{3}-\frac{b}{4}}{x^4}$	13
risch	$\frac{-\frac{ax}{3}-\frac{b}{4}}{x^4}$	13
gosper	$-\frac{4ax+3b}{12x^4}$	14
default	$-\frac{b}{4x^4}-\frac{a}{3x^3}$	14
parallelrisch	$\frac{-4ax-3b}{12x^4}$	14
orering	$-\frac{(4ax+3b)\left(a+\frac{b}{x}\right)}{12x^3(ax+b)}$	28

input
```
int((a+b/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*a*x-1/4*b)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^4}\,dx=-\frac{4\,ax+3\,b}{12\,x^4}$$

input
```
integrate((a+b/x)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(4*a*x + 3*b)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+\frac{b}{x}}{x^4}\,dx = \frac{-4ax-3b}{12x^4}$$

input
```
integrate((a+b/x)/x**4,x)
```

output
```
(-4*a*x - 3*b)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^4}\,dx = -\frac{4\,ax+3\,b}{12\,x^4}$$

input
```
integrate((a+b/x)/x^4,x, algorithm="maxima")
```

output
```
-1/12*(4*a*x + 3*b)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^4}\,dx = -\frac{4\,ax+3\,b}{12\,x^4}$$

input
```
integrate((a+b/x)/x^4,x, algorithm="giac")
```

output
```
-1/12*(4*a*x + 3*b)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^4}\,dx=-\frac{3\,b+4\,a\,x}{12\,x^4}$$

input
```
int((a + b/x)/x^4,x)
```

output
```
-(3*b + 4*a*x)/(12*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^4}\,dx=\frac{-4ax-3b}{12x^4}$$

input
```
int((a+b/x)/x^4,x)
```

output
```
( - 4*a*x - 3*b)/(12*x**4)
```

3.12 $\int \frac{a+\frac{b}{x}}{x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+\frac{b}{x}}{x^5} dx = -\frac{b}{5x^5} - \frac{a}{4x^4}$$

output
```
-1/5*b/x^5-1/4*a/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x^5} dx = -\frac{b}{5x^5} - \frac{a}{4x^4}$$

input
```
Integrate[(a + b/x)/x^5,x]
```

output
```
-1/5*b/x^5 - a/(4*x^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x^5} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^5}+\frac{b}{x^6}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{4x^4}-\frac{b}{5x^5}$$

input
```
Int[(a + b/x)/x^5,x]
```

output
```
-1/5*b/x^5 - a/(4*x^4)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{ax}{4}-\frac{b}{5}}{x^{5}}$	13
risch	$\frac{-\frac{ax}{4}-\frac{b}{5}}{x^{5}}$	13
gosper	$-\frac{5ax+4b}{20x^{5}}$	14
default	$-\frac{b}{5x^{5}}-\frac{a}{4x^{4}}$	14
parallelrisch	$\frac{-5ax-4b}{20x^{5}}$	14
orering	$-\frac{(5ax+4b)\left(a+\frac{b}{x}\right)}{20x^{4}(ax+b)}$	28

input
```
int((a+b/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*a*x-1/5*b)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^5}\,dx=-\frac{5\,ax+4\,b}{20\,x^5}$$

input
```
integrate((a+b/x)/x^5,x, algorithm="fricas")
```

output
```
-1/20*(5*a*x + 4*b)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+\frac{b}{x}}{x^5}\,dx = \frac{-5ax-4b}{20x^5}$$

input
```
integrate((a+b/x)/x**5,x)
```

output
```
(-5*a*x - 4*b)/(20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^5}\,dx = -\frac{5\,ax+4\,b}{20\,x^5}$$

input
```
integrate((a+b/x)/x^5,x, algorithm="maxima")
```

output
```
-1/20*(5*a*x + 4*b)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^5}\,dx = -\frac{5\,ax+4\,b}{20\,x^5}$$

input
```
integrate((a+b/x)/x^5,x, algorithm="giac")
```

output
```
-1/20*(5*a*x + 4*b)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^5}\,dx=-\frac{4\,b+5\,a\,x}{20\,x^5}$$

input
```
int((a + b/x)/x^5,x)
```

output
```
-(4*b + 5*a*x)/(20*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^5}\,dx=\frac{-5ax-4b}{20x^5}$$

input
```
int((a+b/x)/x^5,x)
```

output
```
( - 5*a*x - 4*b)/(20*x**5)
```

3.13 $\int \frac{a+\frac{b}{x}}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+\frac{b}{x}}{x^6} dx = -\frac{b}{6x^6} - \frac{a}{5x^5}$$

output
```
-1/6*b/x^6-1/5*a/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x^6} dx = -\frac{b}{6x^6} - \frac{a}{5x^5}$$

input
```
Integrate[(a + b/x)/x^6,x]
```

output
```
-1/6*b/x^6 - a/(5*x^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x^6} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^6}+\frac{b}{x^7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{5x^5}-\frac{b}{6x^6}$$

input
```
Int[(a + b/x)/x^6,x]
```

output
```
-1/6*b/x^6 - a/(5*x^5)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
norman	$\frac{-\frac{ax}{5}-\frac{b}{6}}{x^6}$	13
risch	$\frac{-\frac{ax}{5}-\frac{b}{6}}{x^6}$	13
gosper	$-\frac{6ax+5b}{30x^6}$	14
default	$-\frac{b}{6x^6}-\frac{a}{5x^5}$	14
parallelrisch	$\frac{-6ax-5b}{30x^6}$	14
orering	$-\frac{(6ax+5b)\left(a+\frac{b}{x}\right)}{30x^5(ax+b)}$	28

input
```
int((a+b/x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-1/5*a*x-1/6*b)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^6}\,dx=-\frac{6\,ax+5\,b}{30\,x^6}$$

input
```
integrate((a+b/x)/x^6,x, algorithm="fricas")
```

output
```
-1/30*(6*a*x + 5*b)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a + \frac{b}{x}}{x^6}\, dx = \frac{-6ax - 5b}{30x^6}$$

input
```
integrate((a+b/x)/x**6,x)
```

output
```
(-6*a*x - 5*b)/(30*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a + \frac{b}{x}}{x^6}\, dx = -\frac{6\,ax + 5\,b}{30\,x^6}$$

input
```
integrate((a+b/x)/x^6,x, algorithm="maxima")
```

output
```
-1/30*(6*a*x + 5*b)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a + \frac{b}{x}}{x^6}\, dx = -\frac{6\,ax + 5\,b}{30\,x^6}$$

input
```
integrate((a+b/x)/x^6,x, algorithm="giac")
```

output
```
-1/30*(6*a*x + 5*b)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^6}\,dx = -\frac{5\,b+6\,a\,x}{30\,x^6}$$

input
```
int((a + b/x)/x^6,x)
```

output
```
-(5*b + 6*a*x)/(30*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{x^6}\,dx = \frac{-6ax-5b}{30x^6}$$

input
```
int((a+b/x)/x^6,x)
```

output
```
( - 6*a*x - 5*b)/(30*x**6)
```

3.14 $\int \left(a+\frac{b}{x}\right)^2 x^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x}\right)^2 x^5 \, dx = \frac{b^2x^4}{4}+\frac{2}{5}abx^5+\frac{a^2x^6}{6}$$

output
```
1/4*b^2*x^4+2/5*a*b*x^5+1/6*a^2*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 x^5 \, dx = \frac{b^2x^4}{4}+\frac{2}{5}abx^5+\frac{a^2x^6}{6}$$

input
```
Integrate[(a + b/x)^2*x^5,x]
```

output
```
(b^2*x^4)/4 + (2*a*b*x^5)/5 + (a^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{x}\right)^2 \, dx$$

$$\Big\downarrow \text{795}$$

$$\int x^3 (a x+b)^2 dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2 x^5+2 a b x^4+b^2 x^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 x^6}{6}+\frac{2}{5} a b x^5+\frac{b^2 x^4}{4}$$

input
```
Int[(a + b/x)^2*x^5,x]
```

output
```
(b^2*x^4)/4 + (2*a*b*x^5)/5 + (a^2*x^6)/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{4}\left(10 a^{2} x^{2}+24 a b x+15 b^{2}\right)}{60}$	25
default	$\frac{1}{4} b^{2} x^{4}+\frac{2}{5} a b\, x^{5}+\frac{1}{6} a^{2} x^{6}$	25
risch	$\frac{1}{4} b^{2} x^{4}+\frac{2}{5} a b\, x^{5}+\frac{1}{6} a^{2} x^{6}$	25
parallelrisch	$\frac{1}{4} b^{2} x^{4}+\frac{2}{5} a b\, x^{5}+\frac{1}{6} a^{2} x^{6}$	25
norman	$\frac{\frac{1}{6} a^{2} x^{7}+\frac{1}{4} b^{2} x^{5}+\frac{2}{5} a b\, x^{6}}{x}$	29
orering	$\frac{x^{6}\left(10 a^{2} x^{2}+24 a b x+15 b^{2}\right)\left(a+\frac{b}{x}\right)^{2}}{60\left(a x+b\right)^{2}}$	41

input
```
int((a+b/x)^2*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^4*(10*a^2*x^2+24*a*b*x+15*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^5\,dx=\frac{1}{6}\,a^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,b^2x^4$$

input
```
integrate((a+b/x)^2*x^5,x, algorithm="fricas")
```

output
```
1/6*a^2*x^6 + 2/5*a*b*x^5 + 1/4*b^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x}\right)^2 x^5\,dx=\frac{a^2x^6}{6}+\frac{2abx^5}{5}+\frac{b^2x^4}{4}$$

input
```
integrate((a+b/x)**2*x**5,x)
```

output
```
a**2*x**6/6 + 2*a*b*x**5/5 + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^5\,dx=\frac{1}{6}\,a^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,b^2x^4$$

input
```
integrate((a+b/x)^2*x^5,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 + 2/5*a*b*x^5 + 1/4*b^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^5\,dx=\frac{1}{6}\,a^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,b^2x^4$$

input
```
integrate((a+b/x)^2*x^5,x, algorithm="giac")
```

output
```
1/6*a^2*x^6 + 2/5*a*b*x^5 + 1/4*b^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^5\,dx=\frac{a^2\,x^6}{6}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^4}{4}$$

input
```
int(x^5*(a + b/x)^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*x^4)/4 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^5\,dx=\frac{x^4(10a^2x^2+24abx+15b^2)}{60}$$

input
```
int((a+b/x)^2*x^5,x)
```

output
```
(x**4*(10*a**2*x**2 + 24*a*b*x + 15*b**2))/60
```

3.15 $\int \left(a+\frac{b}{x}\right)^2 x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x}\right)^2 x^4 \, dx = \frac{b^2x^3}{3}+\frac{1}{2}abx^4+\frac{a^2x^5}{5}$$

output
```
1/3*b^2*x^3+1/2*a*b*x^4+1/5*a^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 x^4 \, dx = \frac{b^2x^3}{3}+\frac{1}{2}abx^4+\frac{a^2x^5}{5}$$

input
```
Integrate[(a + b/x)^2*x^4,x]
```

output
```
(b^2*x^3)/3 + (a*b*x^4)/2 + (a^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x}\right)^2 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^2 (a x+b)^2 dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2 x^4+2 a b x^3+b^2 x^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 x^5}{5}+\frac{1}{2} a b x^4+\frac{b^2 x^3}{3}$$

input
```
Int[(a + b/x)^2*x^4,x]
```

output
```
(b^2*x^3)/3 + (a*b*x^4)/2 + (a^2*x^5)/5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{3}\left(6a^{2}x^{2}+15abx+10b^{2}\right)}{30}$	25
default	$\frac{1}{3}b^{2}x^{3}+\frac{1}{2}ab\,x^{4}+\frac{1}{5}a^{2}x^{5}$	25
risch	$\frac{1}{3}b^{2}x^{3}+\frac{1}{2}ab\,x^{4}+\frac{1}{5}a^{2}x^{5}$	25
parallelrisch	$\frac{1}{3}b^{2}x^{3}+\frac{1}{2}ab\,x^{4}+\frac{1}{5}a^{2}x^{5}$	25
norman	$\frac{\frac{1}{5}a^{2}x^{6}+\frac{1}{3}b^{2}x^{4}+\frac{1}{2}ab\,x^{5}}{x}$	29
orering	$\frac{x^{5}\left(6a^{2}x^{2}+15abx+10b^{2}\right)\left(a+\frac{b}{x}\right)^{2}}{30\left(ax+b\right)^{2}}$	41

input
```
int((a+b/x)^2*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^3*(6*a^2*x^2+15*a*b*x+10*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^4\,dx=\frac{1}{5}\,a^{2}x^{5}+\frac{1}{2}\,abx^{4}+\frac{1}{3}\,b^{2}x^{3}$$

input
```
integrate((a+b/x)^2*x^4,x, algorithm="fricas")
```

output
```
1/5*a^2*x^5 + 1/2*a*b*x^4 + 1/3*b^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^4\,dx=\frac{a^2x^5}{5}+\frac{abx^4}{2}+\frac{b^2x^3}{3}$$

input
```
integrate((a+b/x)**2*x**4,x)
```

output
```
a**2*x**5/5 + a*b*x**4/2 + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^4\,dx=\frac{1}{5}\,a^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,b^2x^3$$

input
```
integrate((a+b/x)^2*x^4,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 + 1/2*a*b*x^4 + 1/3*b^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^4\,dx=\frac{1}{5}\,a^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,b^2x^3$$

input
```
integrate((a+b/x)^2*x^4,x, algorithm="giac")
```

output
```
1/5*a^2*x^5 + 1/2*a*b*x^4 + 1/3*b^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^4 \, dx = \frac{a^2 \, x^5}{5} + \frac{a \, b \, x^4}{2} + \frac{b^2 \, x^3}{3}$$

input
```
int(x^4*(a + b/x)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^3)/3 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^4 \, dx = \frac{x^3(6a^2x^2 + 15abx + 10b^2)}{30}$$

input
```
int((a+b/x)^2*x^4,x)
```

output
```
(x**3*(6*a**2*x**2 + 15*a*b*x + 10*b**2))/30
```

3.16 $\int \left(a+\frac{b}{x}\right)^2 x^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx=\frac{b^2x^2}{2}+\frac{2}{3}abx^3+\frac{a^2x^4}{4}$$

output
```
1/2*b^2*x^2+2/3*a*b*x^3+1/4*a^2*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx=\frac{b^2x^2}{2}+\frac{2}{3}abx^3+\frac{a^2x^4}{4}$$

input
```
Integrate[(a + b/x)^2*x^3,x]
```

output
```
(b^2*x^2)/2 + (2*a*b*x^3)/3 + (a^2*x^4)/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{x}\right)^2dx$$
$$\Big\downarrow \text{795}$$
$$\int x(ax+b)^2dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(a^2x^3+2abx^2+b^2x\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2x^4}{4}+\frac{2}{3}abx^3+\frac{b^2x^2}{2}$$

input
```
Int[(a + b/x)^2*x^3,x]
```

output
```
(b^2*x^2)/2 + (2*a*b*x^3)/3 + (a^2*x^4)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^2(3a^2x^2+8abx+6b^2)}{12}$	25
default	$\frac{1}{2}b^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}a^2x^4$	25
risch	$\frac{1}{2}b^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}a^2x^4$	25
parallelrisch	$\frac{1}{2}b^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}a^2x^4$	25
norman	$\frac{\frac{1}{4}a^2x^5+\frac{1}{2}b^2x^3+\frac{2}{3}ab\,x^4}{x}$	29
orering	$\frac{x^4(3a^2x^2+8abx+6b^2)\left(a+\frac{b}{x}\right)^2}{12(ax+b)^2}$	41

input
```
int((a+b/x)^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*a^2*x^2+8*a*b*x+6*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx = \frac{1}{4}\,a^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,b^2x^2$$

input
```
integrate((a+b/x)^2*x^3,x, algorithm="fricas")
```

output
```
1/4*a^2*x^4 + 2/3*a*b*x^3 + 1/2*b^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx = \frac{a^2x^4}{4}+\frac{2abx^3}{3}+\frac{b^2x^2}{2}$$

input
```
integrate((a+b/x)**2*x**3,x)
```

output
```
a**2*x**4/4 + 2*a*b*x**3/3 + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx = \frac{1}{4}\,a^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,b^2x^2$$

input
```
integrate((a+b/x)^2*x^3,x, algorithm="maxima")
```

output
```
1/4*a^2*x^4 + 2/3*a*b*x^3 + 1/2*b^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx = \frac{1}{4}\,a^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,b^2x^2$$

input
```
integrate((a+b/x)^2*x^3,x, algorithm="giac")
```

output
```
1/4*a^2*x^4 + 2/3*a*b*x^3 + 1/2*b^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx=\frac{a^2\,x^4}{4}+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^2}{2}$$

input
```
int(x^3*(a + b/x)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^2)/2 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^2 x^3\,dx=\frac{x^2(3a^2x^2+8abx+6b^2)}{12}$$

input
```
int((a+b/x)^2*x^3,x)
```

output
```
(x**2*(3*a**2*x**2 + 8*a*b*x + 6*b**2))/12
```

3.17 $\int \left(a+\frac{b}{x}\right)^2 x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \left(a+\frac{b}{x}\right)^2 x^2 \, dx = \frac{(b+ax)^3}{3a}$$

output
```
1/3*(a*x+b)^3/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 x^2 \, dx = \frac{(b+ax)^3}{3a}$$

input
```
Integrate[(a + b/x)^2*x^2,x]
```

output
```
(b + a*x)^3/(3*a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x}\right)^2 dx$$

$$\Big\downarrow \text{795}$$

$$\int (ax+b)^2 dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(ax+b)^3}{3a}$$

input
```
Int[(a + b/x)^2*x^2,x]
```

output
```
(b + a*x)^3/(3*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(ax+b)^3}{3a}$	13
parallelrisch	$\frac{1}{3}a^2x^3+ab\,x^2+b^2x$	21
gosper	$\frac{x(a^2x^2+3abx+3b^2)}{3}$	22
norman	$\frac{b^2x^2+ab\,x^3+\frac{1}{3}a^2x^4}{x}$	27
risch	$\frac{a^2x^3}{3}+ab\,x^2+b^2x+\frac{b^3}{3a}$	29
orering	$\frac{x^3(a^2x^2+3abx+3b^2)\left(a+\frac{b}{x}\right)^2}{3(ax+b)^2}$	40

input
```
int((a+b/x)^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x+b)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int\left(a+\frac{b}{x}\right)^2x^2\,dx=\frac{1}{3}\,a^2x^3+abx^2+b^2x$$

input
```
integrate((a+b/x)^2*x^2,x, algorithm="fricas")
```

output
```
1/3*a^2*x^3 + a*b*x^2 + b^2*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x}\right)^2 x^2\,dx = \frac{a^2x^3}{3}+abx^2+b^2x$$

input
```
integrate((a+b/x)**2*x**2,x)
```

output
```
a**2*x**3/3 + a*b*x**2 + b**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a+\frac{b}{x}\right)^2 x^2\,dx = \frac{1}{3}\,a^2x^3+abx^2+b^2x$$

input
```
integrate((a+b/x)^2*x^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + a*b*x^2 + b^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a+\frac{b}{x}\right)^2 x^2\,dx = \frac{1}{3}\,a^2x^3+abx^2+b^2x$$

input
```
integrate((a+b/x)^2*x^2,x, algorithm="giac")
```

output
```
1/3*a^2*x^3 + a*b*x^2 + b^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \left(a+\frac{b}{x}\right)^2 x^2 \, dx = \frac{a^2 x^3}{3} + a\, b\, x^2 + b^2\, x$$

input
```
int(x^2*(a + b/x)^2,x)
```

output
```
b^2*x + (a^2*x^3)/3 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{b}{x}\right)^2 x^2 \, dx = \frac{x(a^2x^2+3abx+3b^2)}{3}$$

input
```
int((a+b/x)^2*x^2,x)
```

output
```
(x*(a**2*x**2 + 3*a*b*x + 3*b**2))/3
```

3.18 $\int \left(a+\frac{b}{x}\right)^2 x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \left(a+\frac{b}{x}\right)^2 x \, dx = 2abx+\frac{a^2x^2}{2}+b^2\log(x)$$

output
```
2*a*b*x+1/2*a^2*x^2+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 x \, dx = 2abx+\frac{a^2x^2}{2}+b^2\log(x)$$

input
```
Integrate[(a + b/x)^2*x,x]
```

output
```
2*a*b*x + (a^2*x^2)/2 + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x}\right)^2dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^2}{x}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2x+2ab+\frac{b^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^2}{2}+2abx+b^2\log(x)$$

input
```
Int[(a + b/x)^2*x,x]
```

output
```
2*a*b*x + (a^2*x^2)/2 + b^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$2abx+\frac{a^{2}x^{2}}{2}+b^{2}\ln\left(x\right)$	21
risch	$2abx+\frac{a^{2}x^{2}}{2}+b^{2}\ln\left(x\right)$	21
parallelrisch	$2abx+\frac{a^{2}x^{2}}{2}+b^{2}\ln\left(x\right)$	21
norman	$\frac{\frac{1}{2}a^{2}x^{3}+2ab\,x^{2}}{x}+b^{2}\ln\left(x\right)$	28

input
```
int((a+b/x)^2*x,x,method=_RETURNVERBOSE)
```

output
```
2*a*b*x+1/2*a^2*x^2+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int\left(a+\frac{b}{x}\right)^{2}x\,dx=\frac{1}{2}\,a^{2}x^{2}+2\,abx+b^{2}\log\left(x\right)$$

input
```
integrate((a+b/x)^2*x,x, algorithm="fricas")
```

output
```
1/2*a^2*x^2 + 2*a*b*x + b^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^2 x\,dx=\frac{a^2x^2}{2}+2abx+b^2\log{(x)}$$

input
```
integrate((a+b/x)**2*x,x)
```

output
```
a**2*x**2/2 + 2*a*b*x + b**2*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^2 x\,dx=\frac{1}{2}\,a^2x^2+2\,abx+b^2\log\left(x\right)$$

input
```
integrate((a+b/x)^2*x,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 2*a*b*x + b^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^2 x\,dx=\frac{1}{2}\,a^2x^2+2\,abx+b^2\log\left(|x|\right)$$

input
```
integrate((a+b/x)^2*x,x, algorithm="giac")
```

output
```
1/2*a^2*x^2 + 2*a*b*x + b^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^2 x\,dx = b^2\,\ln\left(x\right)+\frac{a^2\,x^2}{2}+2\,a\,b\,x$$

input
```
int(x*(a + b/x)^2,x)
```

output
```
b^2*log(x) + (a^2*x^2)/2 + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^2 x\,dx = \log(x)\,b^2+\frac{a^2x^2}{2}+2abx$$

input
```
int((a+b/x)^2*x,x)
```

output
```
(2*log(x)*b**2 + a**2*x**2 + 4*a*b*x)/2
```

3.19 $\int \left(a+\frac{b}{x}\right)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int \left(a+\frac{b}{x}\right)^2 \, dx = -\frac{b^2}{x}+a^2x+2ab\log(x)$$

output
```
-b^2/x+a^2*x+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 \, dx = -\frac{b^2}{x}+a^2x+2ab\log(x)$$

input
```
Integrate[(a + b/x)^2,x]
```

output
```
-(b^2/x) + a^2*x + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^2 dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{(ax+b)^2}{x^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2+\frac{2ab}{x}+\frac{b^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+2ab\log(x)-\frac{b^2}{x}$$

input
```
Int[(a + b/x)^2,x]
```

output
```
-(b^2/x) + a^2*x + 2*a*b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{b^2}{x}+a^2x+2ab\ln\left(x\right)$	21
risch	$-\frac{b^2}{x}+a^2x+2ab\ln\left(x\right)$	21
norman	$\frac{a^2x^2-b^2}{x}+2ab\ln\left(x\right)$	25
parallelrisch	$\frac{2ab\ln(x)x+a^2x^2-b^2}{x}$	25

input
```
int((a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2/x+a^2*x+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x}\right)^2\,dx=\frac{a^2x^2+2\,abx\log\left(x\right)-b^2}{x}$$

input
```
integrate((a+b/x)^2,x, algorithm="fricas")
```

output
```
(a^2*x^2 + 2*a*b*x*log(x) - b^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^2 dx = a^2x + 2ab\log(x) - \frac{b^2}{x}$$

input
```
integrate((a+b/x)**2,x)
```

output
```
a**2*x + 2*a*b*log(x) - b**2/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 dx = a^2x + 2\,ab\log(x) - \frac{b^2}{x}$$

input
```
integrate((a+b/x)^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*a*b*log(x) - b^2/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x}\right)^2 dx = a^2x + 2\,ab\log(|x|) - \frac{b^2}{x}$$

input
```
integrate((a+b/x)^2,x, algorithm="giac")
```

output
```
a^2*x + 2*a*b*log(abs(x)) - b^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^2 dx = a^2\,x-\frac{b^2}{x}+2\,a\,b\,\ln\left(x\right)$$

input
```
int((a + b/x)^2,x)
```

output
```
a^2*x - b^2/x + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x}\right)^2 dx = \frac{2\log(x)\,abx+a^2x^2-b^2}{x}$$

input
```
int((a+b/x)^2,x)
```

output
```
(2*log(x)*a*b*x + a**2*x**2 - b**2)/x
```

3.20 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx=-\frac{b^2}{2x^2}-\frac{2ab}{x}+a^2\log(x)$$

output
```
-1/2*b^2/x^2-2*a*b/x+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx=-\frac{b^2}{2x^2}-\frac{2ab}{x}+a^2\log(x)$$

input
```
Integrate[(a + b/x)^2/x,x]
```

output
```
-1/2*b^2/x^2 - (2*a*b)/x + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x} \, dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{(ax+b)^2}{x^3}dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{a^2}{x}+\frac{2ab}{x^2}+\frac{b^2}{x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$a^2\log(x)-\frac{2ab}{x}-\frac{b^2}{2x^2}$$

input
```
Int[(a + b/x)^2/x,x]
```

output
```
-1/2*b^2/x^2 - (2*a*b)/x + a^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{b^2}{2x^2}-\frac{2ab}{x}+a^2\ln(x)$	23
norman	$\frac{-\frac{1}{2}b^2-2abx}{x^2}+a^2\ln(x)$	23
risch	$\frac{-\frac{1}{2}b^2-2abx}{x^2}+a^2\ln(x)$	23
parallelrisch	$\frac{2a^2\ln(x)x^2-4abx-b^2}{2x^2}$	27

input
```
int((a+b/x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/x^2-2*a*b/x+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx=\frac{2\,a^2x^2\log(x)-4\,abx-b^2}{2\,x^2}$$

input
```
integrate((a+b/x)^2/x,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*x^2*log(x) - 4*a*b*x - b^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx = a^2\log\left(x\right) + \frac{-4abx - b^2}{2x^2}$$

input
```
integrate((a+b/x)**2/x,x)
```

output
```
a**2*log(x) + (-4*a*b*x - b**2)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx = a^2\log\left(x\right) - \frac{4\,abx + b^2}{2\,x^2}$$

input
```
integrate((a+b/x)^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) - 1/2*(4*a*b*x + b^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx = a^2\log\left(|x|\right) - \frac{4\,abx + b^2}{2\,x^2}$$

input
```
integrate((a+b/x)^2/x,x, algorithm="giac")
```

output
```
a^2*log(abs(x)) - 1/2*(4*a*b*x + b^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx = a^2\,\ln\left(x\right)-\frac{\frac{b^2}{2}+2\,a\,x\,b}{x^2}$$

input
```
int((a + b/x)^2/x,x)
```

output
```
a^2*log(x) - (b^2/2 + 2*a*b*x)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x}\,dx = \frac{2\log(x)\,a^2x^2-4abx-b^2}{2x^2}$$

input
```
int((a+b/x)^2/x,x)
```

output
```
(2*log(x)*a**2*x**2 - 4*a*b*x - b**2)/(2*x**2)
```

3.21 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx=-\frac{\left(a+\frac{b}{x}\right)^3}{3b}$$

output
```
-1/3*(a+b/x)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx=-\frac{b^2}{3x^3}-\frac{ab}{x^2}-\frac{a^2}{x}$$

input
```
Integrate[(a + b/x)^2/x^2,x]
```

output
```
-1/3*b^2/x^3 - (a*b)/x^2 - a^2/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{\left(a+\frac{b}{x}\right)^3}{3b}$$

input
```
Int[(a + b/x)^2/x^2,x]
```

output
```
-1/3*(a + b/x)^3/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

method	result	size
gosper	$-\frac{3a^2x^2+3abx+b^2}{3x^3}$	23
norman	$\frac{-a^2x^2-abx-\frac{1}{3}b^2}{x^3}$	24
risch	$\frac{-a^2x^2-abx-\frac{1}{3}b^2}{x^3}$	24
default	$-\frac{b^2}{3x^3}-\frac{ab}{x^2}-\frac{a^2}{x}$	25
parallelrisch	$\frac{-3a^2x^2-3abx-b^2}{3x^3}$	25
orering	$-\frac{(3a^2x^2+3abx+b^2)\left(a+\frac{b}{x}\right)^2}{3x(ax+b)^2}$	39

input
```
int((a+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*a^2*x^2+3*a*b*x+b^2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx = -\frac{3\,a^2x^2+3\,abx+b^2}{3\,x^3}$$

input
```
integrate((a+b/x)^2/x^2,x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*x^2 + 3*a*b*x + b^2)/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx = \frac{-3a^2x^2-3abx-b^2}{3x^3}$$

input
```
integrate((a+b/x)**2/x**2,x)
```

output
```
(-3*a**2*x**2 - 3*a*b*x - b**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^3}{3\,b}$$

input
```
integrate((a+b/x)^2/x^2,x, algorithm="maxima")
```

output
```
-1/3*(a + b/x)^3/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^3}{3\,b}$$

input
```
integrate((a+b/x)^2/x^2,x, algorithm="giac")
```

output
```
-1/3*(a + b/x)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx = -\frac{a^2\,x^2+a\,b\,x+\frac{b^2}{3}}{x^3}$$

input
```
int((a + b/x)^2/x^2,x)
```

output
```
-(b^2/3 + a^2*x^2 + a*b*x)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^2}\,dx = \frac{-3a^2x^2-3abx-b^2}{3x^3}$$

input
```
int((a+b/x)^2/x^2,x)
```

output
```
( - 3*a**2*x**2 - 3*a*b*x - b**2)/(3*x**3)
```

3.22 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3} dx = -\frac{b^2}{4x^4} - \frac{2ab}{3x^3} - \frac{a^2}{2x^2}$$

output
```
-1/4*b^2/x^4-2/3*a*b/x^3-1/2*a^2/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3} dx = -\frac{b^2}{4x^4} - \frac{2ab}{3x^3} - \frac{a^2}{2x^2}$$

input
```
Integrate[(a + b/x)^2/x^3,x]
```

output
```
-1/4*b^2/x^4 - (2*a*b)/(3*x^3) - a^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^2}{x^5}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^4}+\frac{b^2}{x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2x^2}-\frac{2ab}{3x^3}-\frac{b^2}{4x^4}$$

input
```
Int[(a + b/x)^2/x^3,x]
```

output
```
-1/4*b^2/x^4 - (2*a*b)/(3*x^3) - a^2/(2*x^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{2}a^2x^2-\frac{2}{3}abx-\frac{1}{4}b^2}{x^4}$	24
risch	$\frac{-\frac{1}{2}a^2x^2-\frac{2}{3}abx-\frac{1}{4}b^2}{x^4}$	24
gosper	$-\frac{6a^2x^2+8abx+3b^2}{12x^4}$	25
default	$-\frac{b^2}{4x^4}-\frac{2ab}{3x^3}-\frac{a^2}{2x^2}$	25
parallelrisch	$\frac{-6a^2x^2-8abx-3b^2}{12x^4}$	25
orering	$-\frac{(6a^2x^2+8abx+3b^2)\left(a+\frac{b}{x}\right)^2}{12x^2(ax+b)^2}$	41

input
```
int((a+b/x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a^2*x^2-2/3*a*b*x-1/4*b^2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx=-\frac{6\,a^2x^2+8\,abx+3\,b^2}{12\,x^4}$$

input
```
integrate((a+b/x)^2/x^3,x, algorithm="fricas")
```

output
```
-1/12*(6*a^2*x^2 + 8*a*b*x + 3*b^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx=\frac{-6a^2x^2-8abx-3b^2}{12x^4}$$

input
```
integrate((a+b/x)**2/x**3,x)
```

output
```
(-6*a**2*x**2 - 8*a*b*x - 3*b**2)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx=-\frac{6\,a^2x^2+8\,abx+3\,b^2}{12\,x^4}$$

input
```
integrate((a+b/x)^2/x^3,x, algorithm="maxima")
```

output
```
-1/12*(6*a^2*x^2 + 8*a*b*x + 3*b^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx=-\frac{6\,a^2x^2+8\,abx+3\,b^2}{12\,x^4}$$

input
```
integrate((a+b/x)^2/x^3,x, algorithm="giac")
```

output
```
-1/12*(6*a^2*x^2 + 8*a*b*x + 3*b^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx = -\frac{\frac{a^2\,x^2}{2}+\frac{2\,a\,b\,x}{3}+\frac{b^2}{4}}{x^4}$$

input
```
int((a + b/x)^2/x^3,x)
```

output
```
-(b^2/4 + (a^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^3}\,dx = \frac{-6a^2x^2-8abx-3b^2}{12x^4}$$

input
```
int((a+b/x)^2/x^3,x)
```

output
```
( - 6*a**2*x**2 - 8*a*b*x - 3*b**2)/(12*x**4)
```

3.23 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=-\frac{b^2}{5x^5}-\frac{ab}{2x^4}-\frac{a^2}{3x^3}$$

output
```
-1/5*b^2/x^5-1/2*a*b/x^4-1/3*a^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=-\frac{b^2}{5x^5}-\frac{ab}{2x^4}-\frac{a^2}{3x^3}$$

input
```
Integrate[(a + b/x)^2/x^4,x]
```

output
```
-1/5*b^2/x^5 - (a*b)/(2*x^4) - a^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^2}{x^6}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^4}+\frac{2ab}{x^5}+\frac{b^2}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{3x^3}-\frac{ab}{2x^4}-\frac{b^2}{5x^5}$$

input
```
Int[(a + b/x)^2/x^4,x]
```

output
```
-1/5*b^2/x^5 - (a*b)/(2*x^4) - a^2/(3*x^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{3}a^2x^2-\frac{1}{2}abx-\frac{1}{5}b^2}{x^5}$	24
risch	$\frac{-\frac{1}{3}a^2x^2-\frac{1}{2}abx-\frac{1}{5}b^2}{x^5}$	24
gosper	$-\frac{10a^2x^2+15abx+6b^2}{30x^5}$	25
default	$-\frac{b^2}{5x^5}-\frac{ab}{2x^4}-\frac{a^2}{3x^3}$	25
parallelrisch	$\frac{-10a^2x^2-15abx-6b^2}{30x^5}$	25
orering	$-\frac{(10a^2x^2+15abx+6b^2)\left(a+\frac{b}{x}\right)^2}{30x^3(ax+b)^2}$	41

input
```
int((a+b/x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*a^2*x^2-1/2*a*b*x-1/5*b^2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=-\frac{10\,a^2x^2+15\,abx+6\,b^2}{30\,x^5}$$

input
```
integrate((a+b/x)^2/x^4,x, algorithm="fricas")
```

output
```
-1/30*(10*a^2*x^2 + 15*a*b*x + 6*b^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=\frac{-10a^2x^2-15abx-6b^2}{30x^5}$$

input
```
integrate((a+b/x)**2/x**4,x)
```

output
```
(-10*a**2*x**2 - 15*a*b*x - 6*b**2)/(30*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=-\frac{10\,a^2x^2+15\,abx+6\,b^2}{30\,x^5}$$

input
```
integrate((a+b/x)^2/x^4,x, algorithm="maxima")
```

output
```
-1/30*(10*a^2*x^2 + 15*a*b*x + 6*b^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=-\frac{10\,a^2x^2+15\,abx+6\,b^2}{30\,x^5}$$

input
```
integrate((a+b/x)^2/x^4,x, algorithm="giac")
```

output
```
-1/30*(10*a^2*x^2 + 15*a*b*x + 6*b^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=-\frac{\frac{a^2\,x^2}{3}+\frac{a\,b\,x}{2}+\frac{b^2}{5}}{x^5}$$

input
```
int((a + b/x)^2/x^4,x)
```

output
```
-(b^2/5 + (a^2*x^2)/3 + (a*b*x)/2)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^4}\,dx=\frac{-10a^2x^2-15abx-6b^2}{30x^5}$$

input
```
int((a+b/x)^2/x^4,x)
```

output
```
( - 10*a**2*x**2 - 15*a*b*x - 6*b**2)/(30*x**5)
```

3.24 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5} \, dx = -\frac{b^2}{6x^6} - \frac{2ab}{5x^5} - \frac{a^2}{4x^4}$$

output
```
-1/6*b^2/x^6-2/5*a*b/x^5-1/4*a^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5} \, dx = -\frac{b^2}{6x^6} - \frac{2ab}{5x^5} - \frac{a^2}{4x^4}$$

input
```
Integrate[(a + b/x)^2/x^5,x]
```

output
```
-1/6*b^2/x^6 - (2*a*b)/(5*x^5) - a^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^2}{x^7}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^6}+\frac{b^2}{x^7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{4x^4}-\frac{2ab}{5x^5}-\frac{b^2}{6x^6}$$

input
```
Int[(a + b/x)^2/x^5,x]
```

output
```
-1/6*b^2/x^6 - (2*a*b)/(5*x^5) - a^2/(4*x^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{-\frac{1}{4}a^2x^2-\frac{2}{5}abx-\frac{1}{6}b^2}{x^6}$	24
risch	$\frac{-\frac{1}{4}a^2x^2-\frac{2}{5}abx-\frac{1}{6}b^2}{x^6}$	24
gosper	$-\frac{15a^2x^2+24abx+10b^2}{60x^6}$	25
default	$-\frac{b^2}{6x^6}-\frac{2ab}{5x^5}-\frac{a^2}{4x^4}$	25
parallelrisch	$\frac{-15a^2x^2-24abx-10b^2}{60x^6}$	25
orering	$-\frac{(15a^2x^2+24abx+10b^2)\left(a+\frac{b}{x}\right)^2}{60x^4(ax+b)^2}$	41

input
```
int((a+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*a^2*x^2-2/5*a*b*x-1/6*b^2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx=-\frac{15\,a^2x^2+24\,abx+10\,b^2}{60\,x^6}$$

input
```
integrate((a+b/x)^2/x^5,x, algorithm="fricas")
```

output
```
-1/60*(15*a^2*x^2 + 24*a*b*x + 10*b^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx = \frac{-15a^2x^2-24abx-10b^2}{60x^6}$$

input
```
integrate((a+b/x)**2/x**5,x)
```

output
```
(-15*a**2*x**2 - 24*a*b*x - 10*b**2)/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx = -\frac{15\,a^2x^2+24\,abx+10\,b^2}{60\,x^6}$$

input
```
integrate((a+b/x)^2/x^5,x, algorithm="maxima")
```

output
```
-1/60*(15*a^2*x^2 + 24*a*b*x + 10*b^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx = -\frac{15\,a^2x^2+24\,abx+10\,b^2}{60\,x^6}$$

input
```
integrate((a+b/x)^2/x^5,x, algorithm="giac")
```

output
```
-1/60*(15*a^2*x^2 + 24*a*b*x + 10*b^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx = -\frac{\frac{a^2\,x^2}{4}+\frac{2\,a\,b\,x}{5}+\frac{b^2}{6}}{x^6}$$

input
```
int((a + b/x)^2/x^5,x)
```

output
```
-(b^2/6 + (a^2*x^2)/4 + (2*a*b*x)/5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^5}\,dx = \frac{-15a^2x^2-24abx-10b^2}{60x^6}$$

input
```
int((a+b/x)^2/x^5,x)
```

output
```
( - 15*a**2*x**2 - 24*a*b*x - 10*b**2)/(60*x**6)
```

3.25 $\int \left(a+\frac{b}{x}\right)^3 x^6 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \left(a+\frac{b}{x}\right)^3 x^6 \, dx=\frac{b^3x^4}{4}+\frac{3}{5}ab^2x^5+\frac{1}{2}a^2bx^6+\frac{a^3x^7}{7}$$

output
```
1/4*b^3*x^4+3/5*a*b^2*x^5+1/2*a^2*b*x^6+1/7*a^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^3 x^6 \, dx=\frac{b^3x^4}{4}+\frac{3}{5}ab^2x^5+\frac{1}{2}a^2bx^6+\frac{a^3x^7}{7}$$

input
```
Integrate[(a + b/x)^3*x^6,x]
```

output
```
(b^3*x^4)/4 + (3*a*b^2*x^5)/5 + (a^2*b*x^6)/2 + (a^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^3 (a x+b)^3 dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3 x^6+3 a^2 b x^5+3 a b^2 x^4+b^3 x^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3 x^7}{7}+\frac{1}{2} a^2 b x^6+\frac{3}{5} a b^2 x^5+\frac{b^3 x^4}{4}$$

input
```
Int[(a + b/x)^3*x^6,x]
```

output
```
(b^3*x^4)/4 + (3*a*b^2*x^5)/5 + (a^2*b*x^6)/2 + (a^3*x^7)/7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{4} \left(20 a^{3} x^{3}+70 a^{2} b \,x^{2}+84 a \,b^{2} x +35 b^{3}\right)}{140}$	36
default	$\frac{1}{4} b^{3} x^{4}+\frac{3}{5} a \,b^{2} x^{5}+\frac{1}{2} a^{2} b \,x^{6}+\frac{1}{7} a^{3} x^{7}$	36
risch	$\frac{1}{4} b^{3} x^{4}+\frac{3}{5} a \,b^{2} x^{5}+\frac{1}{2} a^{2} b \,x^{6}+\frac{1}{7} a^{3} x^{7}$	36
parallelrisch	$\frac{1}{4} b^{3} x^{4}+\frac{3}{5} a \,b^{2} x^{5}+\frac{1}{2} a^{2} b \,x^{6}+\frac{1}{7} a^{3} x^{7}$	36
norman	$\frac{\frac{1}{7} a^{3} x^{9}+\frac{1}{4} b^{3} x^{6}+\frac{3}{5} a \,b^{2} x^{7}+\frac{1}{2} a^{2} b \,x^{8}}{x^{2}}$	40
orering	$\frac{x^{7} \left(20 a^{3} x^{3}+70 a^{2} b \,x^{2}+84 a \,b^{2} x +35 b^{3}\right) \left(a +\frac{b}{x}\right)^{3}}{140 \left(a x +b \right)^{3}}$	52

input
```
int((a+b/x)^3*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/140*x^4*(20*a^3*x^3+70*a^2*b*x^2+84*a*b^2*x+35*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^6\,dx=\frac{1}{7}\,a^{3}x^{7}+\frac{1}{2}\,a^{2}bx^{6}+\frac{3}{5}\,ab^{2}x^{5}+\frac{1}{4}\,b^{3}x^{4}$$

input
```
integrate((a+b/x)^3*x^6,x, algorithm="fricas")
```

output
```
1/7*a^3*x^7 + 1/2*a^2*b*x^6 + 3/5*a*b^2*x^5 + 1/4*b^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x}\right)^3 x^6\,dx = \frac{a^3x^7}{7}+\frac{a^2bx^6}{2}+\frac{3ab^2x^5}{5}+\frac{b^3x^4}{4}$$

input
```
integrate((a+b/x)**3*x**6,x)
```

output
```
a**3*x**7/7 + a**2*b*x**6/2 + 3*a*b**2*x**5/5 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^6\,dx = \frac{1}{7}\,a^3x^7+\frac{1}{2}\,a^2bx^6+\frac{3}{5}\,ab^2x^5+\frac{1}{4}\,b^3x^4$$

input
```
integrate((a+b/x)^3*x^6,x, algorithm="maxima")
```

output
```
1/7*a^3*x^7 + 1/2*a^2*b*x^6 + 3/5*a*b^2*x^5 + 1/4*b^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^6\,dx = \frac{1}{7}\,a^3x^7+\frac{1}{2}\,a^2bx^6+\frac{3}{5}\,ab^2x^5+\frac{1}{4}\,b^3x^4$$

input
```
integrate((a+b/x)^3*x^6,x, algorithm="giac")
```

output
```
1/7*a^3*x^7 + 1/2*a^2*b*x^6 + 3/5*a*b^2*x^5 + 1/4*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^6\,dx=\frac{a^3\,x^7}{7}+\frac{a^2\,b\,x^6}{2}+\frac{3\,a\,b^2\,x^5}{5}+\frac{b^3\,x^4}{4}$$

input
```
int(x^6*(a + b/x)^3,x)
```

output
```
(a^3*x^7)/7 + (b^3*x^4)/4 + (3*a*b^2*x^5)/5 + (a^2*b*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^6\,dx=\frac{x^4(20a^3x^3+70a^2b\,x^2+84a\,b^2x+35b^3)}{140}$$

input
```
int((a+b/x)^3*x^6,x)
```

output
```
(x**4*(20*a**3*x**3 + 70*a**2*b*x**2 + 84*a*b**2*x + 35*b**3))/140
```

3.26 $\int \left(a+\frac{b}{x}\right)^3 x^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \left(a+\frac{b}{x}\right)^3 x^5 \, dx = \frac{b^3x^3}{3}+\frac{3}{4}ab^2x^4+\frac{3}{5}a^2bx^5+\frac{a^3x^6}{6}$$

output
```
1/3*b^3*x^3+3/4*a*b^2*x^4+3/5*a^2*b*x^5+1/6*a^3*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^3 x^5 \, dx = \frac{b^3x^3}{3}+\frac{3}{4}ab^2x^4+\frac{3}{5}a^2bx^5+\frac{a^3x^6}{6}$$

input
```
Integrate[(a + b/x)^3*x^5,x]
```

output
```
(b^3*x^3)/3 + (3*a*b^2*x^4)/4 + (3*a^2*b*x^5)/5 + (a^3*x^6)/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^2 (a x+b)^3dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3 x^5+3 a^2 b x^4+3 a b^2 x^3+b^3 x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3 x^6}{6}+\frac{3}{5} a^2 b x^5+\frac{3}{4} a b^2 x^4+\frac{b^3 x^3}{3}$$

input
```
Int[(a + b/x)^3*x^5,x]
```

output
```
(b^3*x^3)/3 + (3*a*b^2*x^4)/4 + (3*a^2*b*x^5)/5 + (a^3*x^6)/6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{3}\left(10 a^{3} x^{3}+36 a^{2} b \,x^{2}+45 a \,b^{2} x +20 b^{3}\right)}{60}$	36
default	$\frac{1}{3} b^{3} x^{3}+\frac{3}{4} a \,b^{2} x^{4}+\frac{3}{5} a^{2} b \,x^{5}+\frac{1}{6} a^{3} x^{6}$	36
risch	$\frac{1}{3} b^{3} x^{3}+\frac{3}{4} a \,b^{2} x^{4}+\frac{3}{5} a^{2} b \,x^{5}+\frac{1}{6} a^{3} x^{6}$	36
parallelrisch	$\frac{1}{3} b^{3} x^{3}+\frac{3}{4} a \,b^{2} x^{4}+\frac{3}{5} a^{2} b \,x^{5}+\frac{1}{6} a^{3} x^{6}$	36
norman	$\frac{\frac{1}{6} a^{3} x^{8}+\frac{1}{3} b^{3} x^{5}+\frac{3}{4} a \,b^{2} x^{6}+\frac{3}{5} a^{2} b \,x^{7}}{x^{2}}$	40
orering	$\frac{x^{6}\left(10 a^{3} x^{3}+36 a^{2} b \,x^{2}+45 a \,b^{2} x +20 b^{3}\right)\left(a +\frac{b}{x}\right)^{3}}{60 \left(a x +b \right)^{3}}$	52

input
```
int((a+b/x)^3*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(10*a^3*x^3+36*a^2*b*x^2+45*a*b^2*x+20*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^5\,dx=\frac{1}{6}\,a^3x^6+\frac{3}{5}\,a^2bx^5+\frac{3}{4}\,ab^2x^4+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x)^3*x^5,x, algorithm="fricas")
```

output
```
1/6*a^3*x^6 + 3/5*a^2*b*x^5 + 3/4*a*b^2*x^4 + 1/3*b^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^3 x^5\,dx=\frac{a^3x^6}{6}+\frac{3a^2bx^5}{5}+\frac{3ab^2x^4}{4}+\frac{b^3x^3}{3}$$

input
```
integrate((a+b/x)**3*x**5,x)
```

output
```
a**3*x**6/6 + 3*a**2*b*x**5/5 + 3*a*b**2*x**4/4 + b**3*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^5\,dx=\frac{1}{6}\,a^3x^6+\frac{3}{5}\,a^2bx^5+\frac{3}{4}\,ab^2x^4+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x)^3*x^5,x, algorithm="maxima")
```

output
```
1/6*a^3*x^6 + 3/5*a^2*b*x^5 + 3/4*a*b^2*x^4 + 1/3*b^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^5\,dx=\frac{1}{6}\,a^3x^6+\frac{3}{5}\,a^2bx^5+\frac{3}{4}\,ab^2x^4+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x)^3*x^5,x, algorithm="giac")
```

output
```
1/6*a^3*x^6 + 3/5*a^2*b*x^5 + 3/4*a*b^2*x^4 + 1/3*b^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^5\,dx=\frac{a^3\,x^6}{6}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^3}{3}$$

input
```
int(x^5*(a + b/x)^3,x)
```

output
```
(a^3*x^6)/6 + (b^3*x^3)/3 + (3*a*b^2*x^4)/4 + (3*a^2*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 x^5\,dx=\frac{x^3(10a^3x^3+36a^2b\,x^2+45a\,b^2x+20b^3)}{60}$$

input
```
int((a+b/x)^3*x^5,x)
```

output
```
(x**3*(10*a**3*x**3 + 36*a**2*b*x**2 + 45*a*b**2*x + 20*b**3))/60
```

3.27 $\int \left(a+\frac{b}{x}\right)^3 x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x}\right)^3 x^4 \, dx = -\frac{b(b+ax)^4}{4a^2}+\frac{(b+ax)^5}{5a^2}$$

output
```
-1/4*b*(a*x+b)^4/a^2+1/5*(a*x+b)^5/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \left(a+\frac{b}{x}\right)^3 x^4 \, dx = \frac{b^3x^2}{2}+ab^2x^3+\frac{3}{4}a^2bx^4+\frac{a^3x^5}{5}$$

input
```
Integrate[(a + b/x)^3*x^4,x]
```

output
```
(b^3*x^2)/2 + a*b^2*x^3 + (3*a^2*b*x^4)/4 + (a^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int x(ax+b)^3 dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(ax+b)^4}{a}-\frac{b(ax+b)^3}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ax+b)^5}{5a^2}-\frac{b(ax+b)^4}{4a^2}$$

input
```
Int[(a + b/x)^3*x^4,x]
```

output
```
-1/4*(b*(b + a*x)^4)/a^2 + (b + a*x)^5/(5*a^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{1}{5}a^3x^5+\frac{3}{4}a^2b\,x^4+a\,b^2x^3+\frac{1}{2}b^3x^2$	35
risch	$\frac{1}{5}a^3x^5+\frac{3}{4}a^2b\,x^4+a\,b^2x^3+\frac{1}{2}b^3x^2$	35
parallelrisch	$\frac{1}{5}a^3x^5+\frac{3}{4}a^2b\,x^4+a\,b^2x^3+\frac{1}{2}b^3x^2$	35
gosper	$\frac{x^2(4a^3x^3+15a^2b\,x^2+20a\,b^2x+10b^3)}{20}$	36
norman	$\frac{a\,b^2x^5+\frac{1}{5}a^3x^7+\frac{1}{2}b^3x^4+\frac{3}{4}a^2b\,x^6}{x^2}$	39
orering	$\frac{x^5(4a^3x^3+15a^2b\,x^2+20a\,b^2x+10b^3)\left(a+\frac{b}{x}\right)^3}{20(ax+b)^3}$	52

input
```
int((a+b/x)^3*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3*x^5+3/4*a^2*b*x^4+a*b^2*x^3+1/2*b^3*x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x}\right)^3 x^4\,dx=\frac{1}{5}\,a^3x^5+\frac{3}{4}\,a^2bx^4+ab^2x^3+\frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x)^3*x^4,x, algorithm="fricas")
```

output
```
1/5*a^3*x^5 + 3/4*a^2*b*x^4 + a*b^2*x^3 + 1/2*b^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x}\right)^3 x^4\,dx = \frac{a^3x^5}{5} + \frac{3a^2bx^4}{4} + ab^2x^3 + \frac{b^3x^2}{2}$$

input
```
integrate((a+b/x)**3*x**4,x)
```

output
```
a**3*x**5/5 + 3*a**2*b*x**4/4 + a*b**2*x**3 + b**3*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x}\right)^3 x^4\,dx = \frac{1}{5}\,a^3x^5 + \frac{3}{4}\,a^2bx^4 + ab^2x^3 + \frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x)^3*x^4,x, algorithm="maxima")
```

output
```
1/5*a^3*x^5 + 3/4*a^2*b*x^4 + a*b^2*x^3 + 1/2*b^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x}\right)^3 x^4\,dx = \frac{1}{5}\,a^3x^5 + \frac{3}{4}\,a^2bx^4 + ab^2x^3 + \frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x)^3*x^4,x, algorithm="giac")
```

output
```
1/5*a^3*x^5 + 3/4*a^2*b*x^4 + a*b^2*x^3 + 1/2*b^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x}\right)^3 x^4\,dx=\frac{a^3\,x^5}{5}+\frac{3\,a^2\,b\,x^4}{4}+a\,b^2\,x^3+\frac{b^3\,x^2}{2}$$

input
```
int(x^4*(a + b/x)^3,x)
```

output
```
(a^3*x^5)/5 + (b^3*x^2)/2 + a*b^2*x^3 + (3*a^2*b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x}\right)^3 x^4\,dx=\frac{x^2(4a^3x^3+15a^2b\,x^2+20a\,b^2x+10b^3)}{20}$$

input
```
int((a+b/x)^3*x^4,x)
```

output
```
(x**2*(4*a**3*x**3 + 15*a**2*b*x**2 + 20*a*b**2*x + 10*b**3))/20
```

3.28 $\int \left(a+\frac{b}{x}\right)^3 x^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx=\frac{(b+ax)^4}{4a}$$

output
```
1/4*(a*x+b)^4/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx=\frac{(b+ax)^4}{4a}$$

input
```
Integrate[(a + b/x)^3*x^3,x]
```

output
```
(b + a*x)^4/(4*a)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \ 795$$

$$\int (ax+b)^3 dx$$

$$\Big\downarrow \ 17$$

$$\frac{(ax+b)^4}{4a}$$

input
```
Int[(a + b/x)^3*x^3,x]
```

output
```
(b + a*x)^4/(4*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{(ax+b)^4}{4a}$	13
parallelrisch	$\frac{1}{4}a^3x^4 + a^2b\,x^3 + \frac{3}{2}a\,b^2x^2 + b^3x$	32
gosper	$\frac{x(a^3x^3+4a^2b\,x^2+6a\,b^2x+4b^3)}{4}$	33
norman	$\frac{b^3x^3+a^2b\,x^5+\frac{1}{4}a^3x^6+\frac{3}{2}a\,b^2x^4}{x^2}$	38
risch	$\frac{a^3x^4}{4} + a^2b\,x^3 + \frac{3a\,b^2x^2}{2} + b^3x + \frac{b^4}{4a}$	40
orering	$\frac{x^4(a^3x^3+4a^2b\,x^2+6a\,b^2x+4b^3)\left(a+\frac{b}{x}\right)^3}{4(ax+b)^3}$	51

input
```
int((a+b/x)^3*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*x+b)^4/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx = \frac{1}{4}\,a^3x^4 + a^2bx^3 + \frac{3}{2}\,ab^2x^2 + b^3x$$

input
```
integrate((a+b/x)^3*x^3,x, algorithm="fricas")
```

output
```
1/4*a^3*x^4 + a^2*b*x^3 + 3/2*a*b^2*x^2 + b^3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx = \frac{a^3x^4}{4} + a^2bx^3 + \frac{3ab^2x^2}{2} + b^3x$$

input
```
integrate((a+b/x)**3*x**3,x)
```

output
```
a**3*x**4/4 + a**2*b*x**3 + 3*a*b**2*x**2/2 + b**3*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx = \frac{1}{4}\,a^3x^4 + a^2bx^3 + \frac{3}{2}\,ab^2x^2 + b^3x$$

input
```
integrate((a+b/x)^3*x^3,x, algorithm="maxima")
```

output
```
1/4*a^3*x^4 + a^2*b*x^3 + 3/2*a*b^2*x^2 + b^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx = \frac{1}{4}\,a^3x^4 + a^2bx^3 + \frac{3}{2}\,ab^2x^2 + b^3x$$

input
```
integrate((a+b/x)^3*x^3,x, algorithm="giac")
```

output
```
1/4*a^3*x^4 + a^2*b*x^3 + 3/2*a*b^2*x^2 + b^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx=\frac{a^3\,x^4}{4}+a^2\,b\,x^3+\frac{3\,a\,b^2\,x^2}{2}+b^3\,x$$

input
```
int(x^3*(a + b/x)^3,x)
```

output
```
b^3*x + (a^3*x^4)/4 + (3*a*b^2*x^2)/2 + a^2*b*x^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \left(a+\frac{b}{x}\right)^3 x^3\,dx=\frac{x(a^3x^3+4a^2b\,x^2+6a\,b^2x+4b^3)}{4}$$

input
```
int((a+b/x)^3*x^3,x)
```

output
```
(x*(a**3*x**3 + 4*a**2*b*x**2 + 6*a*b**2*x + 4*b**3))/4
```

3.29 $\int \left(a+\frac{b}{x}\right)^3 x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \left(a+\frac{b}{x}\right)^3 x^2 \, dx = 3ab^2x+\frac{3}{2}a^2bx^2+\frac{a^3x^3}{3}+b^3\log(x)$$

output
```
3*a*b^2*x+3/2*a^2*b*x^2+1/3*a^3*x^3+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^3 x^2 \, dx = 3ab^2x+\frac{3}{2}a^2bx^2+\frac{a^3x^3}{3}+b^3\log(x)$$

input
```
Integrate[(a + b/x)^3*x^2,x]
```

output
```
3*a*b^2*x + (3*a^2*b*x^2)/2 + (a^3*x^3)/3 + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x}\right)^3 dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{(ax+b)^3}{x}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(a^3x^2+3a^2bx+3ab^2+\frac{b^3}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^3x^3}{3}+\frac{3}{2}a^2bx^2+3ab^2x+b^3\log(x)$$

input
```
Int[(a + b/x)^3*x^2,x]
```

output
```
3*a*b^2*x + (3*a^2*b*x^2)/2 + (a^3*x^3)/3 + b^3*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$3a\,b^{2}x+\frac{3a^{2}b\,x^{2}}{2}+\frac{a^{3}x^{3}}{3}+b^{3}\ln\left(x\right)$	32
risch	$3a\,b^{2}x+\frac{3a^{2}b\,x^{2}}{2}+\frac{a^{3}x^{3}}{3}+b^{3}\ln\left(x\right)$	32
parallelrisch	$3a\,b^{2}x+\frac{3a^{2}b\,x^{2}}{2}+\frac{a^{3}x^{3}}{3}+b^{3}\ln\left(x\right)$	32
norman	$\frac{\frac{1}{3}a^{3}x^{5}+3a\,b^{2}x^{3}+\frac{3}{2}a^{2}b\,x^{4}}{x^{2}}+b^{3}\ln\left(x\right)$	39

input
```
int((a+b/x)^3*x^2,x,method=_RETURNVERBOSE)
```

output
```
3*a*b^2*x+3/2*a^2*b*x^2+1/3*a^3*x^3+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int\left(a+\frac{b}{x}\right)^{3}x^{2}\,dx=\frac{1}{3}\,a^{3}x^{3}+\frac{3}{2}\,a^{2}bx^{2}+3\,ab^{2}x+b^{3}\log\left(x\right)$$

input
```
integrate((a+b/x)^3*x^2,x, algorithm="fricas")
```

output
```
1/3*a^3*x^3 + 3/2*a^2*b*x^2 + 3*a*b^2*x + b^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^3 x^2\,dx=\frac{a^3x^3}{3}+\frac{3a^2bx^2}{2}+3ab^2x+b^3\log{(x)}$$

input
```
integrate((a+b/x)**3*x**2,x)
```

output
```
a**3*x**3/3 + 3*a**2*b*x**2/2 + 3*a*b**2*x + b**3*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x}\right)^3 x^2\,dx=\frac{1}{3}\,a^3x^3+\frac{3}{2}\,a^2bx^2+3\,ab^2x+b^3\log\left(x\right)$$

input
```
integrate((a+b/x)^3*x^2,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 + 3/2*a^2*b*x^2 + 3*a*b^2*x + b^3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^3 x^2\,dx=\frac{1}{3}\,a^3x^3+\frac{3}{2}\,a^2bx^2+3\,ab^2x+b^3\log\left(|x|\right)$$

input
```
integrate((a+b/x)^3*x^2,x, algorithm="giac")
```

output
```
1/3*a^3*x^3 + 3/2*a^2*b*x^2 + 3*a*b^2*x + b^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x}\right)^3 x^2\,dx = b^3\,\ln\left(x\right)+\frac{a^3\,x^3}{3}+\frac{3\,a^2\,b\,x^2}{2}+3\,a\,b^2\,x$$

input
```
int(x^2*(a + b/x)^3,x)
```

output
```
b^3*log(x) + (a^3*x^3)/3 + (3*a^2*b*x^2)/2 + 3*a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x}\right)^3 x^2\,dx = \log(x)\,b^3+\frac{a^3x^3}{3}+\frac{3a^2b\,x^2}{2}+3a\,b^2x$$

input
```
int((a+b/x)^3*x^2,x)
```

output
```
(6*log(x)*b**3 + 2*a**3*x**3 + 9*a**2*b*x**2 + 18*a*b**2*x)/6
```

3.30 $\int \left(a+\frac{b}{x}\right)^3 x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \left(a+\frac{b}{x}\right)^3 x \, dx = -\frac{b^3}{x}+3a^2bx+\frac{a^3x^2}{2}+3ab^2\log(x)$$

output
```
-b^3/x+3*a^2*b*x+1/2*a^3*x^2+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^3 x \, dx = -\frac{b^3}{x}+3a^2bx+\frac{a^3x^2}{2}+3ab^2\log(x)$$

input
```
Integrate[(a + b/x)^3*x,x]
```

output
```
-(b^3/x) + 3*a^2*b*x + (a^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^3}{x^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3x+3a^2b+\frac{3ab^2}{x}+\frac{b^3}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^2}{2}+3a^2bx+3ab^2\log(x)-\frac{b^3}{x}$$

input
```
Int[(a + b/x)^3*x,x]
```

output
```
-(b^3/x) + 3*a^2*b*x + (a^3*x^2)/2 + 3*a*b^2*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{b^3}{x}+3a^2bx+\frac{a^3x^2}{2}+3a\,b^2\ln\left(x\right)$	33
risch	$-\frac{b^3}{x}+3a^2bx+\frac{a^3x^2}{2}+3a\,b^2\ln\left(x\right)$	33
parallelrisch	$\frac{a^3x^3+6a\,b^2\ln\left(x\right)x+6a^2b\,x^2-2b^3}{2x}$	37
norman	$\frac{\frac{1}{2}a^3x^4-b^3x+3a^2b\,x^3}{x^2}+3a\,b^2\ln\left(x\right)$	38

input
```
int((a+b/x)^3*x,x,method=_RETURNVERBOSE)
```

output
```
-b^3/x+3*a^2*b*x+1/2*a^3*x^2+3*a*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int\left(a+\frac{b}{x}\right)^3x\,dx=\frac{a^3x^3+6\,a^2bx^2+6\,ab^2x\log\left(x\right)-2\,b^3}{2\,x}$$

input
```
integrate((a+b/x)^3*x,x, algorithm="fricas")
```

output
```
1/2*(a^3*x^3 + 6*a^2*b*x^2 + 6*a*b^2*x*log(x) - 2*b^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^3 x\,dx = \frac{a^3x^2}{2} + 3a^2bx + 3ab^2\log{(x)} - \frac{b^3}{x}$$

input
```
integrate((a+b/x)**3*x,x)
```

output
```
a**3*x**2/2 + 3*a**2*b*x + 3*a*b**2*log(x) - b**3/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^3 x\,dx = \frac{1}{2}\,a^3x^2 + 3\,a^2bx + 3\,ab^2\log{(x)} - \frac{b^3}{x}$$

input
```
integrate((a+b/x)^3*x,x, algorithm="maxima")
```

output
```
1/2*a^3*x^2 + 3*a^2*b*x + 3*a*b^2*log(x) - b^3/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^3 x\,dx = \frac{1}{2}\,a^3x^2 + 3\,a^2bx + 3\,ab^2\log{(|x|)} - \frac{b^3}{x}$$

input
```
integrate((a+b/x)^3*x,x, algorithm="giac")
```

output
```
1/2*a^3*x^2 + 3*a^2*b*x + 3*a*b^2*log(abs(x)) - b^3/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^3 x\,dx = \frac{a^3\,x^2}{2} - \frac{b^3}{x} + 3\,a\,b^2\,\ln\left(x\right) + 3\,a^2\,b\,x$$

input
```
int(x*(a + b/x)^3,x)
```

output
```
(a^3*x^2)/2 - b^3/x + 3*a*b^2*log(x) + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x}\right)^3 x\,dx = \frac{6\log(x)\,a\,b^2x + a^3x^3 + 6a^2b\,x^2 - 2b^3}{2x}$$

input
```
int((a+b/x)^3*x,x)
```

output
```
(6*log(x)*a*b**2*x + a**3*x**3 + 6*a**2*b*x**2 - 2*b**3)/(2*x)
```

3.31 $\int \left(a+\frac{b}{x}\right)^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \left(a+\frac{b}{x}\right)^3 \, dx=-\frac{b^3}{2x^2}-\frac{3ab^2}{x}+a^3x+3a^2b\log(x)$$

output
```
-1/2*b^3/x^2-3*a*b^2/x+a^3*x+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^3 \, dx=-\frac{b^3}{2x^2}-\frac{3ab^2}{x}+a^3x+3a^2b\log(x)$$

input
```
Integrate[(a + b/x)^3,x]
```

output
```
-1/2*b^3/x^2 - (3*a*b^2)/x + a^3*x + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{(ax+b)^3}{x^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3+\frac{3a^2b}{x}+\frac{3ab^2}{x^2}+\frac{b^3}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+3a^2b\log(x)-\frac{3ab^2}{x}-\frac{b^3}{2x^2}$$

input
```
Int[(a + b/x)^3,x]
```

output
```
-1/2*b^3/x^2 - (3*a*b^2)/x + a^3*x + 3*a^2*b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{b^3}{2x^2}-\frac{3a\,b^2}{x}+a^3x+3a^2b\ln\left(x\right)$	32
risch	$a^3x+\frac{-3a\,b^2x-\frac{1}{2}b^3}{x^2}+3a^2b\ln\left(x\right)$	32
norman	$\frac{a^3x^3-\frac{1}{2}b^3-3a\,b^2x}{x^2}+3a^2b\ln\left(x\right)$	34
parallelrisch	$\frac{6a^2b\ln(x)x^2+2a^3x^3-6a\,b^2x-b^3}{2x^2}$	38

input
```
int((a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/x^2-3*a*b^2/x+a^3*x+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int\left(a+\frac{b}{x}\right)^3\,dx=\frac{2\,a^3x^3+6\,a^2bx^2\log\left(x\right)-6\,ab^2x-b^3}{2\,x^2}$$

input
```
integrate((a+b/x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^3*x^3 + 6*a^2*b*x^2*log(x) - 6*a*b^2*x - b^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^3 dx = a^3x + 3a^2b\log(x) + \frac{-6ab^2x - b^3}{2x^2}$$

input
```
integrate((a+b/x)**3,x)
```

output
```
a**3*x + 3*a**2*b*log(x) + (-6*a*b**2*x - b**3)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^3 dx = a^3x + 3\,a^2b\log(x) - \frac{3\,ab^2}{x} - \frac{b^3}{2\,x^2}$$

input
```
integrate((a+b/x)^3,x, algorithm="maxima")
```

output
```
a^3*x + 3*a^2*b*log(x) - 3*a*b^2/x - 1/2*b^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^3 dx = a^3x + 3\,a^2b\log(|x|) - \frac{6\,ab^2x + b^3}{2\,x^2}$$

input
```
integrate((a+b/x)^3,x, algorithm="giac")
```

output
```
a^3*x + 3*a^2*b*log(abs(x)) - 1/2*(6*a*b^2*x + b^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^3 dx = a^3 x - \frac{\frac{b^3}{2} + 3 a x b^2}{x^2} + 3 a^2 b \ln(x)$$

input
```
int((a + b/x)^3,x)
```

output
```
a^3*x - (b^3/2 + 3*a*b^2*x)/x^2 + 3*a^2*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \left(a+\frac{b}{x}\right)^3 dx = \frac{6 \log(x) a^2 b x^2 + 2a^3 x^3 - 6a b^2 x - b^3}{2x^2}$$

input
```
int((a+b/x)^3,x)
```

output
```
(6*log(x)*a**2*b*x**2 + 2*a**3*x**3 - 6*a*b**2*x - b**3)/(2*x**2)
```

3.32 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x} \, dx = -\frac{b^3}{3x^3} - \frac{3ab^2}{2x^2} - \frac{3a^2b}{x} + a^3 \log(x)$$

output
```
-1/3*b^3/x^3-3/2*a*b^2/x^2-3*a^2*b/x+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x} \, dx = -\frac{b^3}{3x^3} - \frac{3ab^2}{2x^2} - \frac{3a^2b}{x} + a^3 \log(x)$$

input
```
Integrate[(a + b/x)^3/x,x]
```

output
```
-1/3*b^3/x^3 - (3*a*b^2)/(2*x^2) - (3*a^2*b)/x + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^3}{x^4}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{a^3}{x}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x^3}+\frac{b^3}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3\log(x)-\frac{3a^2b}{x}-\frac{3ab^2}{2x^2}-\frac{b^3}{3x^3}$$

input
```
Int[(a + b/x)^3/x,x]
```

output
```
-1/3*b^3/x^3 - (3*a*b^2)/(2*x^2) - (3*a^2*b)/x + a^3*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{b^3}{3x^3}-\frac{3a\,b^2}{2x^2}-\frac{3a^2b}{x}+a^3\ln\left(x\right)$	34
norman	$\frac{-\frac{1}{3}b^3-\frac{3}{2}a\,b^2x-3a^2b\,x^2}{x^3}+a^3\ln\left(x\right)$	34
risch	$\frac{-\frac{1}{3}b^3-\frac{3}{2}a\,b^2x-3a^2b\,x^2}{x^3}+a^3\ln\left(x\right)$	34
parallelrisch	$\frac{6a^3\ln(x)x^3-18a^2b\,x^2-9a\,b^2x-2b^3}{6x^3}$	38

input
```
int((a+b/x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^3/x^3-3/2*a*b^2/x^2-3*a^2*b/x+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int\frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx=\frac{6\,a^3x^3\log\left(x\right)-18\,a^2bx^2-9\,ab^2x-2\,b^3}{6\,x^3}$$

input
```
integrate((a+b/x)^3/x,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*x^3*log(x) - 18*a^2*b*x^2 - 9*a*b^2*x - 2*b^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx = a^3\log\left(x\right) + \frac{-18a^2bx^2 - 9ab^2x - 2b^3}{6x^3}$$

input
```
integrate((a+b/x)**3/x,x)
```

output
```
a**3*log(x) + (-18*a**2*b*x**2 - 9*a*b**2*x - 2*b**3)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx = a^3\log\left(x\right) - \frac{18\,a^2bx^2 + 9\,ab^2x + 2\,b^3}{6\,x^3}$$

input
```
integrate((a+b/x)^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) - 1/6*(18*a^2*b*x^2 + 9*a*b^2*x + 2*b^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx = a^3\log\left(|x|\right) - \frac{18\,a^2bx^2 + 9\,ab^2x + 2\,b^3}{6\,x^3}$$

input
```
integrate((a+b/x)^3/x,x, algorithm="giac")
```

output
```
a^3*log(abs(x)) - 1/6*(18*a^2*b*x^2 + 9*a*b^2*x + 2*b^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx = a^3\,\ln\left(x\right)-\frac{3\,a^2\,b\,x^2+\frac{3\,a\,b^2\,x}{2}+\frac{b^3}{3}}{x^3}$$

input
```
int((a + b/x)^3/x,x)
```

output
```
a^3*log(x) - (b^3/3 + 3*a^2*b*x^2 + (3*a*b^2*x)/2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x}\,dx = \frac{6\log(x)\,a^3x^3-18a^2b\,x^2-9a\,b^2x-2b^3}{6x^3}$$

input
```
int((a+b/x)^3/x,x)
```

output
```
(6*log(x)*a**3*x**3 - 18*a**2*b*x**2 - 9*a*b**2*x - 2*b**3)/(6*x**3)
```

3.33 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx=-\frac{\left(a+\frac{b}{x}\right)^4}{4b}$$

output
```
-1/4*(a+b/x)^4/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx=-\frac{b^3}{4x^4}-\frac{ab^2}{x^3}-\frac{3a^2b}{2x^2}-\frac{a^3}{x}$$

input
```
Integrate[(a + b/x)^3/x^2,x]
```

output
```
-1/4*b^3/x^4 - (a*b^2)/x^3 - (3*a^2*b)/(2*x^2) - a^3/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2} \, dx$$
$$\Big\downarrow \text{793}$$
$$-\frac{\left(a+\frac{b}{x}\right)^4}{4b}$$

input
```
Int[(a + b/x)^3/x^2,x]
```

output
```
-1/4*(a + b/x)^4/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

method	result	size
gosper	$-\frac{4a^{3}x^{3}+6a^{2}b\,x^{2}+4a\,b^{2}x+b^{3}}{4x^{4}}$	34
norman	$\frac{-a^{3}x^{3}-\frac{3}{2}a^{2}b\,x^{2}-a\,b^{2}x-\frac{1}{4}b^{3}}{x^{4}}$	35
risch	$\frac{-a^{3}x^{3}-\frac{3}{2}a^{2}b\,x^{2}-a\,b^{2}x-\frac{1}{4}b^{3}}{x^{4}}$	35
default	$-\frac{a\,b^{2}}{x^{3}}-\frac{3a^{2}b}{2x^{2}}-\frac{b^{3}}{4x^{4}}-\frac{a^{3}}{x}$	36
parallelrisch	$\frac{-4a^{3}x^{3}-6a^{2}b\,x^{2}-4a\,b^{2}x-b^{3}}{4x^{4}}$	36
orering	$-\frac{\left(4a^{3}x^{3}+6a^{2}b\,x^{2}+4a\,b^{2}x+b^{3}\right)\left(a+\frac{b}{x}\right)^{3}}{4x\left(ax+b\right)^{3}}$	50

input
```
int((a+b/x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*a^3*x^3+6*a^2*b*x^2+4*a*b^2*x+b^3)/x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx = -\frac{4\,a^3x^3+6\,a^2bx^2+4\,ab^2x+b^3}{4\,x^4}$$

input
```
integrate((a+b/x)^3/x^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a^3*x^3 + 6*a^2*b*x^2 + 4*a*b^2*x + b^3)/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx = \frac{-4a^3x^3-6a^2bx^2-4ab^2x-b^3}{4x^4}$$

input
```
integrate((a+b/x)**3/x**2,x)
```

output
```
(-4*a**3*x**3 - 6*a**2*b*x**2 - 4*a*b**2*x - b**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^4}{4\,b}$$

input
```
integrate((a+b/x)^3/x^2,x, algorithm="maxima")
```

output
```
-1/4*(a + b/x)^4/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^4}{4\,b}$$

input
```
integrate((a+b/x)^3/x^2,x, algorithm="giac")
```

output
```
-1/4*(a + b/x)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx = -\frac{a^3\,x^3+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x+\frac{b^3}{4}}{x^4}$$

input
```
int((a + b/x)^3/x^2,x)
```

output
```
-(b^3/4 + a^3*x^3 + (3*a^2*b*x^2)/2 + a*b^2*x)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^2}\,dx = \frac{-4a^3x^3-6a^2b\,x^2-4a\,b^2x-b^3}{4x^4}$$

input
```
int((a+b/x)^3/x^2,x)
```

output
```
( - 4*a**3*x**3 - 6*a**2*b*x**2 - 4*a*b**2*x - b**3)/(4*x**4)
```

3.34 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3} dx = \frac{a\left(a+\frac{b}{x}\right)^4}{4b^2} - \frac{\left(a+\frac{b}{x}\right)^5}{5b^2}$$

output
```
1/4*a*(a+b/x)^4/b^2-1/5*(a+b/x)^5/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3} dx = -\frac{b^3}{5x^5} - \frac{3ab^2}{4x^4} - \frac{a^2b}{x^3} - \frac{a^3}{2x^2}$$

input
```
Integrate[(a + b/x)^3/x^3,x]
```

output
```
-1/5*b^3/x^5 - (3*a*b^2)/(4*x^4) - (a^2*b)/x^3 - a^3/(2*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{(ax+b)^3}{x^6}dx$$
$$\Big\downarrow \text{55}$$
$$-\frac{a\int \frac{(b+ax)^3}{x^5}dx}{5b}-\frac{(ax+b)^4}{5bx^5}$$
$$\Big\downarrow \text{48}$$
$$\frac{a(ax+b)^4}{20b^2x^4}-\frac{(ax+b)^4}{5bx^5}$$

input
```
Int[(a + b/x)^3/x^3,x]
```

output
```
-1/5*(b + a*x)^4/(b*x^5) + (a*(b + a*x)^4)/(20*b^2*x^4)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
norman	$\frac{-\frac{1}{2}a^{3}x^{3}-a^{2}b\,x^{2}-\frac{3}{4}a\,b^{2}x-\frac{1}{5}b^{3}}{x^{5}}$	35
risch	$\frac{-\frac{1}{2}a^{3}x^{3}-a^{2}b\,x^{2}-\frac{3}{4}a\,b^{2}x-\frac{1}{5}b^{3}}{x^{5}}$	35
gosper	$-\frac{10a^{3}x^{3}+20a^{2}b\,x^{2}+15a\,b^{2}x+4b^{3}}{20x^{5}}$	36
default	$-\frac{a^{2}b}{x^{3}}-\frac{b^{3}}{5x^{5}}-\frac{a^{3}}{2x^{2}}-\frac{3a\,b^{2}}{4x^{4}}$	36
parallelrisch	$\frac{-10a^{3}x^{3}-20a^{2}b\,x^{2}-15a\,b^{2}x-4b^{3}}{20x^{5}}$	36
orering	$-\frac{\left(10a^{3}x^{3}+20a^{2}b\,x^{2}+15a\,b^{2}x+4b^{3}\right)\left(a+\frac{b}{x}\right)^{3}}{20x^{2}\left(ax+b\right)^{3}}$	52

input
```
int((a+b/x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a^3*x^3-a^2*b*x^2-3/4*a*b^2*x-1/5*b^3)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx = -\frac{10\,a^3x^3+20\,a^2bx^2+15\,ab^2x+4\,b^3}{20\,x^5}$$

input
```
integrate((a+b/x)^3/x^3,x, algorithm="fricas")
```

output
```
-1/20*(10*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x + 4*b^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx = \frac{-10a^3x^3-20a^2bx^2-15ab^2x-4b^3}{20x^5}$$

input
```
integrate((a+b/x)**3/x**3,x)
```

output
```
(-10*a**3*x**3 - 20*a**2*b*x**2 - 15*a*b**2*x - 4*b**3)/(20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx = -\frac{10\,a^3x^3+20\,a^2bx^2+15\,ab^2x+4\,b^3}{20\,x^5}$$

input
```
integrate((a+b/x)^3/x^3,x, algorithm="maxima")
```

output
```
-1/20*(10*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x + 4*b^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx = -\frac{10\,a^3x^3+20\,a^2bx^2+15\,ab^2x+4\,b^3}{20\,x^5}$$

input
```
integrate((a+b/x)^3/x^3,x, algorithm="giac")
```

output
```
-1/20*(10*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x + 4*b^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx = -\frac{\frac{a^3\,x^3}{2}+a^2\,b\,x^2+\frac{3\,a\,b^2\,x}{4}+\frac{b^3}{5}}{x^5}$$

input
```
int((a + b/x)^3/x^3,x)
```

output
```
-(b^3/5 + (a^3*x^3)/2 + a^2*b*x^2 + (3*a*b^2*x)/4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^3}\,dx = \frac{-10a^3x^3-20a^2b\,x^2-15a\,b^2x-4b^3}{20x^5}$$

input
```
int((a+b/x)^3/x^3,x)
```

output
```
( - 10*a**3*x**3 - 20*a**2*b*x**2 - 15*a*b**2*x - 4*b**3)/(20*x**5)
```

3.35 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx = -\frac{b^3}{6x^6}-\frac{3ab^2}{5x^5}-\frac{3a^2b}{4x^4}-\frac{a^3}{3x^3}$$

output
```
-1/6*b^3/x^6-3/5*a*b^2/x^5-3/4*a^2*b/x^4-1/3*a^3/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx = -\frac{b^3}{6x^6}-\frac{3ab^2}{5x^5}-\frac{3a^2b}{4x^4}-\frac{a^3}{3x^3}$$

input
```
Integrate[(a + b/x)^3/x^4,x]
```

output
```
-1/6*b^3/x^6 - (3*a*b^2)/(5*x^5) - (3*a^2*b)/(4*x^4) - a^3/(3*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{(ax+b)^3}{x^7}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^5}+\frac{3ab^2}{x^6}+\frac{b^3}{x^7}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{5x^5}-\frac{b^3}{6x^6}$$

input
```
Int[(a + b/x)^3/x^4,x]
```

output
```
-1/6*b^3/x^6 - (3*a*b^2)/(5*x^5) - (3*a^2*b)/(4*x^4) - a^3/(3*x^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{-\frac{1}{3}a^3x^3-\frac{3}{4}a^2b\,x^2-\frac{3}{5}a\,b^2x-\frac{1}{6}b^3}{x^6}$	35
risch	$\frac{-\frac{1}{3}a^3x^3-\frac{3}{4}a^2b\,x^2-\frac{3}{5}a\,b^2x-\frac{1}{6}b^3}{x^6}$	35
gosper	$-\frac{20a^3x^3+45a^2b\,x^2+36a\,b^2x+10b^3}{60x^6}$	36
default	$-\frac{b^3}{6x^6}-\frac{3a\,b^2}{5x^5}-\frac{3a^2b}{4x^4}-\frac{a^3}{3x^3}$	36
parallelrisch	$\frac{-20a^3x^3-45a^2b\,x^2-36a\,b^2x-10b^3}{60x^6}$	36
orering	$-\frac{\left(20a^3x^3+45a^2b\,x^2+36a\,b^2x+10b^3\right)\left(a+\frac{b}{x}\right)^3}{60x^3\left(ax+b\right)^3}$	52

input
```
int((a+b/x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*a^3*x^3-3/4*a^2*b*x^2-3/5*a*b^2*x-1/6*b^3)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx = -\frac{20\,a^3x^3+45\,a^2bx^2+36\,ab^2x+10\,b^3}{60\,x^6}$$

input
```
integrate((a+b/x)^3/x^4,x, algorithm="fricas")
```

output
```
-1/60*(20*a^3*x^3 + 45*a^2*b*x^2 + 36*a*b^2*x + 10*b^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx = \frac{-20a^3x^3 - 45a^2bx^2 - 36ab^2x - 10b^3}{60x^6}$$

input
```
integrate((a+b/x)**3/x**4,x)
```

output
```
(-20*a**3*x**3 - 45*a**2*b*x**2 - 36*a*b**2*x - 10*b**3)/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx = -\frac{20\,a^3x^3 + 45\,a^2bx^2 + 36\,ab^2x + 10\,b^3}{60\,x^6}$$

input
```
integrate((a+b/x)^3/x^4,x, algorithm="maxima")
```

output
```
-1/60*(20*a^3*x^3 + 45*a^2*b*x^2 + 36*a*b^2*x + 10*b^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx = -\frac{20\,a^3x^3 + 45\,a^2bx^2 + 36\,ab^2x + 10\,b^3}{60\,x^6}$$

input
```
integrate((a+b/x)^3/x^4,x, algorithm="giac")
```

output
```
-1/60*(20*a^3*x^3 + 45*a^2*b*x^2 + 36*a*b^2*x + 10*b^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx=-\frac{\frac{a^3\,x^3}{3}+\frac{3\,a^2\,b\,x^2}{4}+\frac{3\,a\,b^2\,x}{5}+\frac{b^3}{6}}{x^6}$$

input
```
int((a + b/x)^3/x^4,x)
```

output
```
-(b^3/6 + (a^3*x^3)/3 + (3*a^2*b*x^2)/4 + (3*a*b^2*x)/5)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^4}\,dx=\frac{-20a^3x^3-45a^2b\,x^2-36a\,b^2x-10b^3}{60x^6}$$

input
```
int((a+b/x)^3/x^4,x)
```

output
```
( - 20*a**3*x**3 - 45*a**2*b*x**2 - 36*a*b**2*x - 10*b**3)/(60*x**6)
```

3.36 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5} dx = -\frac{b^3}{7x^7} - \frac{ab^2}{2x^6} - \frac{3a^2b}{5x^5} - \frac{a^3}{4x^4}$$

output
```
-1/7*b^3/x^7-1/2*a*b^2/x^6-3/5*a^2*b/x^5-1/4*a^3/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5} dx = -\frac{b^3}{7x^7} - \frac{ab^2}{2x^6} - \frac{3a^2b}{5x^5} - \frac{a^3}{4x^4}$$

input
```
Integrate[(a + b/x)^3/x^5,x]
```

output
```
-1/7*b^3/x^7 - (a*b^2)/(2*x^6) - (3*a^2*b)/(5*x^5) - a^3/(4*x^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{(ax+b)^3}{x^8}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^3}{x^5}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^7}+\frac{b^3}{x^8}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{5x^5}-\frac{ab^2}{2x^6}-\frac{b^3}{7x^7}$$

input
```
Int[(a + b/x)^3/x^5,x]
```

output
```
-1/7*b^3/x^7 - (a*b^2)/(2*x^6) - (3*a^2*b)/(5*x^5) - a^3/(4*x^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{-\frac{1}{4}a^{3}x^{3}-\frac{3}{5}a^{2}b\,x^{2}-\frac{1}{2}a\,b^{2}x-\frac{1}{7}b^{3}}{x^{7}}$	35
risch	$\frac{-\frac{1}{4}a^{3}x^{3}-\frac{3}{5}a^{2}b\,x^{2}-\frac{1}{2}a\,b^{2}x-\frac{1}{7}b^{3}}{x^{7}}$	35
gosper	$-\frac{35a^{3}x^{3}+84a^{2}b\,x^{2}+70a\,b^{2}x+20b^{3}}{140x^{7}}$	36
default	$-\frac{b^{3}}{7x^{7}}-\frac{a\,b^{2}}{2x^{6}}-\frac{3a^{2}b}{5x^{5}}-\frac{a^{3}}{4x^{4}}$	36
parallelrisch	$\frac{-35a^{3}x^{3}-84a^{2}b\,x^{2}-70a\,b^{2}x-20b^{3}}{140x^{7}}$	36
orering	$-\frac{\left(35a^{3}x^{3}+84a^{2}b\,x^{2}+70a\,b^{2}x+20b^{3}\right)\left(a+\frac{b}{x}\right)^{3}}{140x^{4}\left(ax+b\right)^{3}}$	52

input
```
int((a+b/x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*a^3*x^3-3/5*a^2*b*x^2-1/2*a*b^2*x-1/7*b^3)/x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx=-\frac{35\,a^3x^3+84\,a^2bx^2+70\,ab^2x+20\,b^3}{140\,x^7}$$

input
```
integrate((a+b/x)^3/x^5,x, algorithm="fricas")
```

output
```
-1/140*(35*a^3*x^3 + 84*a^2*b*x^2 + 70*a*b^2*x + 20*b^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx=\frac{-35a^3x^3-84a^2bx^2-70ab^2x-20b^3}{140x^7}$$

input
```
integrate((a+b/x)**3/x**5,x)
```

output
```
(-35*a**3*x**3 - 84*a**2*b*x**2 - 70*a*b**2*x - 20*b**3)/(140*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx=-\frac{35\,a^3x^3+84\,a^2bx^2+70\,ab^2x+20\,b^3}{140\,x^7}$$

input
```
integrate((a+b/x)^3/x^5,x, algorithm="maxima")
```

output
```
-1/140*(35*a^3*x^3 + 84*a^2*b*x^2 + 70*a*b^2*x + 20*b^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx=-\frac{35\,a^3x^3+84\,a^2bx^2+70\,ab^2x+20\,b^3}{140\,x^7}$$

input
```
integrate((a+b/x)^3/x^5,x, algorithm="giac")
```

output
```
-1/140*(35*a^3*x^3 + 84*a^2*b*x^2 + 70*a*b^2*x + 20*b^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx=-\frac{\frac{a^3\,x^3}{4}+\frac{3\,a^2\,b\,x^2}{5}+\frac{a\,b^2\,x}{2}+\frac{b^3}{7}}{x^7}$$

input
```
int((a + b/x)^3/x^5,x)
```

output
```
-(b^3/7 + (a^3*x^3)/4 + (3*a^2*b*x^2)/5 + (a*b^2*x)/2)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^5}\,dx=\frac{-35a^3x^3-84a^2b\,x^2-70a\,b^2x-20b^3}{140x^7}$$

input
```
int((a+b/x)^3/x^5,x)
```

output
```
( - 35*a**3*x**3 - 84*a**2*b*x**2 - 70*a*b**2*x - 20*b**3)/(140*x**7)
```

3.37 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx=-\frac{b^3}{8x^8}-\frac{3ab^2}{7x^7}-\frac{a^2b}{2x^6}-\frac{a^3}{5x^5}$$

output
```
-1/8*b^3/x^8-3/7*a*b^2/x^7-1/2*a^2*b/x^6-1/5*a^3/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx=-\frac{b^3}{8x^8}-\frac{3ab^2}{7x^7}-\frac{a^2b}{2x^6}-\frac{a^3}{5x^5}$$

input
```
Integrate[(a + b/x)^3/x^6,x]
```

output
```
-1/8*b^3/x^8 - (3*a*b^2)/(7*x^7) - (a^2*b)/(2*x^6) - a^3/(5*x^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6} dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^3}{x^9} dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^3}{x^6}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^8}+\frac{b^3}{x^9}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{5x^5}-\frac{a^2b}{2x^6}-\frac{3ab^2}{7x^7}-\frac{b^3}{8x^8}$$

input
```
Int[(a + b/x)^3/x^6,x]
```

output
```
-1/8*b^3/x^8 - (3*a*b^2)/(7*x^7) - (a^2*b)/(2*x^6) - a^3/(5*x^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{-\frac{1}{5}a^3x^3-\frac{1}{2}a^2b\,x^2-\frac{3}{7}a\,b^2x-\frac{1}{8}b^3}{x^8}$	35
risch	$\frac{-\frac{1}{5}a^3x^3-\frac{1}{2}a^2b\,x^2-\frac{3}{7}a\,b^2x-\frac{1}{8}b^3}{x^8}$	35
gosper	$-\frac{56a^3x^3+140a^2b\,x^2+120a\,b^2x+35b^3}{280x^8}$	36
default	$-\frac{b^3}{8x^8}-\frac{3a\,b^2}{7x^7}-\frac{a^2b}{2x^6}-\frac{a^3}{5x^5}$	36
parallelrisch	$\frac{-56a^3x^3-140a^2b\,x^2-120a\,b^2x-35b^3}{280x^8}$	36
orering	$-\frac{\left(56a^3x^3+140a^2b\,x^2+120a\,b^2x+35b^3\right)\left(a+\frac{b}{x}\right)^3}{280x^5\left(ax+b\right)^3}$	52

input
```
int((a+b/x)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-1/5*a^3*x^3-1/2*a^2*b*x^2-3/7*a*b^2*x-1/8*b^3)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx=-\frac{56\,a^3x^3+140\,a^2bx^2+120\,ab^2x+35\,b^3}{280\,x^8}$$

input
```
integrate((a+b/x)^3/x^6,x, algorithm="fricas")
```

output
```
-1/280*(56*a^3*x^3 + 140*a^2*b*x^2 + 120*a*b^2*x + 35*b^3)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx = \frac{-56a^3x^3 - 140a^2bx^2 - 120ab^2x - 35b^3}{280x^8}$$

input
```
integrate((a+b/x)**3/x**6,x)
```

output
```
(-56*a**3*x**3 - 140*a**2*b*x**2 - 120*a*b**2*x - 35*b**3)/(280*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx = -\frac{56\,a^3x^3 + 140\,a^2bx^2 + 120\,ab^2x + 35\,b^3}{280\,x^8}$$

input
```
integrate((a+b/x)^3/x^6,x, algorithm="maxima")
```

output
```
-1/280*(56*a^3*x^3 + 140*a^2*b*x^2 + 120*a*b^2*x + 35*b^3)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx = -\frac{56\,a^3x^3 + 140\,a^2bx^2 + 120\,ab^2x + 35\,b^3}{280\,x^8}$$

input
```
integrate((a+b/x)^3/x^6,x, algorithm="giac")
```

output
```
-1/280*(56*a^3*x^3 + 140*a^2*b*x^2 + 120*a*b^2*x + 35*b^3)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx=-\frac{\frac{a^3\,x^3}{5}+\frac{a^2\,b\,x^2}{2}+\frac{3\,a\,b^2\,x}{7}+\frac{b^3}{8}}{x^8}$$

input
```
int((a + b/x)^3/x^6,x)
```

output
```
-(b^3/8 + (a^3*x^3)/5 + (a^2*b*x^2)/2 + (3*a*b^2*x)/7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^6}\,dx=\frac{-56a^3x^3-140a^2b\,x^2-120a\,b^2x-35b^3}{280x^8}$$

input
```
int((a+b/x)^3/x^6,x)
```

output
```
( - 56*a**3*x**3 - 140*a**2*b*x**2 - 120*a*b**2*x - 35*b**3)/(280*x**8)
```

3.38 $\int \left(a+\frac{b}{x}\right)^8 x^{16} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \left(a+\frac{b}{x}\right)^8 x^{16} \, dx = \frac{b^8x^9}{9}+\frac{4}{5}ab^7x^{10}+\frac{28}{11}a^2b^6x^{11}+\frac{14}{3}a^3b^5x^{12}+\frac{70}{13}a^4b^4x^{13} +4a^5b^3x^{14}+\frac{28}{15}a^6b^2x^{15}+\frac{1}{2}a^7bx^{16}+\frac{a^8x^{17}}{17}$$

output
```
1/9*b^8*x^9+4/5*a*b^7*x^10+28/11*a^2*b^6*x^11+14/3*a^3*b^5*x^12+70/13*a^4*
b^4*x^13+4*a^5*b^3*x^14+28/15*a^6*b^2*x^15+1/2*a^7*b*x^16+1/17*a^8*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^{16} \, dx = \frac{b^8x^9}{9}+\frac{4}{5}ab^7x^{10}+\frac{28}{11}a^2b^6x^{11}+\frac{14}{3}a^3b^5x^{12}+\frac{70}{13}a^4b^4x^{13} +4a^5b^3x^{14}+\frac{28}{15}a^6b^2x^{15}+\frac{1}{2}a^7bx^{16}+\frac{a^8x^{17}}{17}$$

input
```
Integrate[(a + b/x)^8*x^16,x]
```

output
```
(b^8*x^9)/9 + (4*a*b^7*x^10)/5 + (28*a^2*b^6*x^11)/11 + (14*a^3*b^5*x^12)/
3 + (70*a^4*b^4*x^13)/13 + 4*a^5*b^3*x^14 + (28*a^6*b^2*x^15)/15 + (a^7*b*
x^16)/2 + (a^8*x^17)/17
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{16}\left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^8(ax+b)^8dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^8x^{16}+8a^7bx^{15}+28a^6b^2x^{14}+56a^5b^3x^{13}+70a^4b^4x^{12}+56a^3b^5x^{11}+28a^2b^6x^{10}+8ab^7x^9+b^8x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^{17}}{17}+\frac{1}{2}a^7bx^{16}+\frac{28}{15}a^6b^2x^{15}+4a^5b^3x^{14}+\frac{70}{13}a^4b^4x^{13}+\frac{14}{3}a^3b^5x^{12}+\frac{28}{11}a^2b^6x^{11}+\frac{4}{5}ab^7x^{10}+\frac{b^8x^9}{9}$$

input
```
Int[(a + b/x)^8*x^16,x]
```

output
```
(b^8*x^9)/9 + (4*a*b^7*x^10)/5 + (28*a^2*b^6*x^11)/11 + (14*a^3*b^5*x^12)/
3 + (70*a^4*b^4*x^13)/13 + 4*a^5*b^3*x^14 + (28*a^6*b^2*x^15)/15 + (a^7*b*
x^16)/2 + (a^8*x^17)/17
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{x^{9}\left(12870 a^{8} x^{8}+109395 a^{7} b\,x^{7}+408408 a^{6} b^{2} x^{6}+875160 a^{5} b^{3} x^{5}+1178100 a^{4} x^{4} b^{4}+1021020 a^{3} b^{5} x^{3}+556920 a^{2} b^{6} x^{2}+175032 a\,b^{7}\right.}{218790}$
default	$\frac{1}{9} b^{8} x^{9}+\frac{4}{5} a\,b^{7} x^{10}+\frac{28}{11} a^{2} b^{6} x^{11}+\frac{14}{3} a^{3} b^{5} x^{12}+\frac{70}{13} a^{4} b^{4} x^{13}+4 a^{5} b^{3} x^{14}+\frac{28}{15} a^{6} b^{2} x^{15}+\frac{1}{2} a^{7} b\,x^{16}$
risch	$\frac{1}{9} b^{8} x^{9}+\frac{4}{5} a\,b^{7} x^{10}+\frac{28}{11} a^{2} b^{6} x^{11}+\frac{14}{3} a^{3} b^{5} x^{12}+\frac{70}{13} a^{4} b^{4} x^{13}+4 a^{5} b^{3} x^{14}+\frac{28}{15} a^{6} b^{2} x^{15}+\frac{1}{2} a^{7} b\,x^{16}$
parallelrisch	$\frac{1}{9} b^{8} x^{9}+\frac{4}{5} a\,b^{7} x^{10}+\frac{28}{11} a^{2} b^{6} x^{11}+\frac{14}{3} a^{3} b^{5} x^{12}+\frac{70}{13} a^{4} b^{4} x^{13}+4 a^{5} b^{3} x^{14}+\frac{28}{15} a^{6} b^{2} x^{15}+\frac{1}{2} a^{7} b\,x^{16}$
norman	$\frac{\frac{1}{17} a^{8} x^{24}+\frac{1}{9} b^{8} x^{16}+\frac{4}{5} a\,b^{7} x^{17}+\frac{28}{11} a^{2} b^{6} x^{18}+\frac{14}{3} a^{3} b^{5} x^{19}+\frac{70}{13} a^{4} b^{4} x^{20}+4 a^{5} b^{3} x^{21}+\frac{28}{15} a^{6} b^{2} x^{22}+\frac{1}{2} a^{7} b\,x^{23}}{x^{7}}$
orering	$\frac{x^{17}\left(12870 a^{8} x^{8}+109395 a^{7} b\,x^{7}+408408 a^{6} b^{2} x^{6}+875160 a^{5} b^{3} x^{5}+1178100 a^{4} x^{4} b^{4}+1021020 a^{3} b^{5} x^{3}+556920 a^{2} b^{6} x^{2}+175032 a\,b\right.}{218790 \left(a x +b \right)^{8}}$

input
```
int((a+b/x)^8*x^16,x,method=_RETURNVERBOSE)
```

output
```
1/218790*x^9*(12870*a^8*x^8+109395*a^7*b*x^7+408408*a^6*b^2*x^6+875160*a^5
*b^3*x^5+1178100*a^4*b^4*x^4+1021020*a^3*b^5*x^3+556920*a^2*b^6*x^2+175032
*a*b^7*x+24310*b^8)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx = \frac{1}{17}\,a^8x^{17}+\frac{1}{2}\,a^7bx^{16}+\frac{28}{15}\,a^6b^2x^{15}+4\,a^5b^3x^{14}+\frac{70}{13}\,a^4b^4x^{13}+\frac{14}{3}\,a^3b^5x^{12}+\frac{28}{11}\,a^2b^6x^{11}+\frac{4}{5}\,ab^7x^{10}+\frac{1}{9}\,b^8x^9$$

input
```
integrate((a+b/x)^8*x^16,x, algorithm="fricas")
```

output
```
1/17*a^8*x^17 + 1/2*a^7*b*x^16 + 28/15*a^6*b^2*x^15 + 4*a^5*b^3*x^14 + 70/
13*a^4*b^4*x^13 + 14/3*a^3*b^5*x^12 + 28/11*a^2*b^6*x^11 + 4/5*a*b^7*x^10
+ 1/9*b^8*x^9
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx = \frac{a^8x^{17}}{17}+\frac{a^7bx^{16}}{2}+\frac{28a^6b^2x^{15}}{15}+4a^5b^3x^{14}+\frac{70a^4b^4x^{13}}{13}+\frac{14a^3b^5x^{12}}{3}+\frac{28a^2b^6x^{11}}{11}+\frac{4ab^7x^{10}}{5}+\frac{b^8x^9}{9}$$

input
```
integrate((a+b/x)**8*x**16,x)
```

output
```
a**8*x**17/17 + a**7*b*x**16/2 + 28*a**6*b**2*x**15/15 + 4*a**5*b**3*x**14
 + 70*a**4*b**4*x**13/13 + 14*a**3*b**5*x**12/3 + 28*a**2*b**6*x**11/11 +
4*a*b**7*x**10/5 + b**8*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx = \frac{1}{17}\,a^8x^{17}+\frac{1}{2}\,a^7bx^{16}+\frac{28}{15}\,a^6b^2x^{15}+4\,a^5b^3x^{14}+\frac{70}{13}\,a^4b^4x^{13}+\frac{14}{3}\,a^3b^5x^{12}+\frac{28}{11}\,a^2b^6x^{11}+\frac{4}{5}\,ab^7x^{10}+\frac{1}{9}\,b^8x^9$$

input
```
integrate((a+b/x)^8*x^16,x, algorithm="maxima")
```

output
```
1/17*a^8*x^17 + 1/2*a^7*b*x^16 + 28/15*a^6*b^2*x^15 + 4*a^5*b^3*x^14 + 70/
13*a^4*b^4*x^13 + 14/3*a^3*b^5*x^12 + 28/11*a^2*b^6*x^11 + 4/5*a*b^7*x^10
+ 1/9*b^8*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx = \frac{1}{17}\,a^8x^{17}+\frac{1}{2}\,a^7bx^{16}+\frac{28}{15}\,a^6b^2x^{15}+4\,a^5b^3x^{14}+\frac{70}{13}\,a^4b^4x^{13}+\frac{14}{3}\,a^3b^5x^{12}+\frac{28}{11}\,a^2b^6x^{11}+\frac{4}{5}\,ab^7x^{10}+\frac{1}{9}\,b^8x^9$$

input
```
integrate((a+b/x)^8*x^16,x, algorithm="giac")
```

output
```
1/17*a^8*x^17 + 1/2*a^7*b*x^16 + 28/15*a^6*b^2*x^15 + 4*a^5*b^3*x^14 + 70/
13*a^4*b^4*x^13 + 14/3*a^3*b^5*x^12 + 28/11*a^2*b^6*x^11 + 4/5*a*b^7*x^10
+ 1/9*b^8*x^9
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx=\frac{a^8\,x^{17}}{17}+\frac{a^7\,b\,x^{16}}{2}+\frac{28\,a^6\,b^2\,x^{15}}{15}+4\,a^5\,b^3\,x^{14}+\frac{70\,a^4\,b^4\,x^{13}}{13}+\frac{14\,a^3\,b^5\,x^{12}}{3}+\frac{28\,a^2\,b^6\,x^{11}}{11}+\frac{4\,a\,b^7\,x^{10}}{5}+\frac{b^8\,x^9}{9}$$

input
```
int(x^16*(a + b/x)^8,x)
```

output
```
(a^8*x^17)/17 + (b^8*x^9)/9 + (4*a*b^7*x^10)/5 + (a^7*b*x^16)/2 + (28*a^2*
b^6*x^11)/11 + (14*a^3*b^5*x^12)/3 + (70*a^4*b^4*x^13)/13 + 4*a^5*b^3*x^14
 + (28*a^6*b^2*x^15)/15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{16}\,dx =\frac{x^9(12870a^8x^8+109395a^7b\,x^7+408408a^6b^2x^6+875160a^5b^3x^5+1178100a^4b^4x^4+1021020a^3b^5x^3+556}{218790}$$

input
```
int((a+b/x)^8*x^16,x)
```

output
```
(x**9*(12870*a**8*x**8 + 109395*a**7*b*x**7 + 408408*a**6*b**2*x**6 + 8751
60*a**5*b**3*x**5 + 1178100*a**4*b**4*x**4 + 1021020*a**3*b**5*x**3 + 5569
20*a**2*b**6*x**2 + 175032*a*b**7*x + 24310*b**8))/218790
```

3.39 $\int \left(a+\frac{b}{x}\right)^8 x^{15} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \left(a+\frac{b}{x}\right)^8 x^{15} \, dx = \frac{b^8x^8}{8}+\frac{8}{9}ab^7x^9+\frac{14}{5}a^2b^6x^{10}+\frac{56}{11}a^3b^5x^{11}+\frac{35}{6}a^4b^4x^{12} +\frac{56}{13}a^5b^3x^{13}+2a^6b^2x^{14}+\frac{8}{15}a^7bx^{15}+\frac{a^8x^{16}}{16}$$

output
```
1/8*b^8*x^8+8/9*a*b^7*x^9+14/5*a^2*b^6*x^10+56/11*a^3*b^5*x^11+35/6*a^4*b^
4*x^12+56/13*a^5*b^3*x^13+2*a^6*b^2*x^14+8/15*a^7*b*x^15+1/16*a^8*x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^{15} \, dx = \frac{b^8x^8}{8}+\frac{8}{9}ab^7x^9+\frac{14}{5}a^2b^6x^{10}+\frac{56}{11}a^3b^5x^{11}+\frac{35}{6}a^4b^4x^{12} +\frac{56}{13}a^5b^3x^{13}+2a^6b^2x^{14}+\frac{8}{15}a^7bx^{15}+\frac{a^8x^{16}}{16}$$

input
```
Integrate[(a + b/x)^8*x^15,x]
```

output
```
(b^8*x^8)/8 + (8*a*b^7*x^9)/9 + (14*a^2*b^6*x^10)/5 + (56*a^3*b^5*x^11)/11
 + (35*a^4*b^4*x^12)/6 + (56*a^5*b^3*x^13)/13 + 2*a^6*b^2*x^14 + (8*a^7*b*
x^15)/15 + (a^8*x^16)/16
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{15}\left(a+\frac{b}{x}\right)^8dx$$

$$\Big\downarrow \text{ 795}$$

$$\int x^7(ax+b)^8dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^8x^{15}+8a^7bx^{14}+28a^6b^2x^{13}+56a^5b^3x^{12}+70a^4b^4x^{11}+56a^3b^5x^{10}+28a^2b^6x^9+8ab^7x^8+b^8x^7\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^8x^{16}}{16}+\frac{8}{15}a^7bx^{15}+2a^6b^2x^{14}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{6}a^4b^4x^{12}+\frac{56}{11}a^3b^5x^{11}+\frac{14}{5}a^2b^6x^{10}+\frac{8}{9}ab^7x^9+\frac{b^8x^8}{8}$$

input
```
Int[(a + b/x)^8*x^15,x]
```

output
```
(b^8*x^8)/8 + (8*a*b^7*x^9)/9 + (14*a^2*b^6*x^10)/5 + (56*a^3*b^5*x^11)/11
 + (35*a^4*b^4*x^12)/6 + (56*a^5*b^3*x^13)/13 + 2*a^6*b^2*x^14 + (8*a^7*b*
x^15)/15 + (a^8*x^16)/16
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{x^{8}\left(6435a^{8}x^{8}+54912a^{7}b\,x^{7}+205920a^{6}b^{2}x^{6}+443520a^{5}b^{3}x^{5}+600600a^{4}x^{4}b^{4}+524160a^{3}b^{5}x^{3}+288288a^{2}b^{6}x^{2}+91520a\,b^{7}x+12\right.}{102960}$
default	$\frac{1}{8}b^{8}x^{8}+\frac{8}{9}a\,b^{7}x^{9}+\frac{14}{5}a^{2}b^{6}x^{10}+\frac{56}{11}a^{3}b^{5}x^{11}+\frac{35}{6}a^{4}b^{4}x^{12}+\frac{56}{13}a^{5}b^{3}x^{13}+2a^{6}b^{2}x^{14}+\frac{8}{15}a^{7}b\,x^{15}$
risch	$\frac{1}{8}b^{8}x^{8}+\frac{8}{9}a\,b^{7}x^{9}+\frac{14}{5}a^{2}b^{6}x^{10}+\frac{56}{11}a^{3}b^{5}x^{11}+\frac{35}{6}a^{4}b^{4}x^{12}+\frac{56}{13}a^{5}b^{3}x^{13}+2a^{6}b^{2}x^{14}+\frac{8}{15}a^{7}b\,x^{15}$
parallelrisch	$\frac{1}{8}b^{8}x^{8}+\frac{8}{9}a\,b^{7}x^{9}+\frac{14}{5}a^{2}b^{6}x^{10}+\frac{56}{11}a^{3}b^{5}x^{11}+\frac{35}{6}a^{4}b^{4}x^{12}+\frac{56}{13}a^{5}b^{3}x^{13}+2a^{6}b^{2}x^{14}+\frac{8}{15}a^{7}b\,x^{15}$
norman	$\frac{\frac{1}{16}a^{8}x^{23}+\frac{1}{8}b^{8}x^{15}+\frac{8}{9}a\,b^{7}x^{16}+\frac{14}{5}a^{2}b^{6}x^{17}+\frac{56}{11}a^{3}b^{5}x^{18}+\frac{35}{6}a^{4}b^{4}x^{19}+\frac{56}{13}a^{5}b^{3}x^{20}+2a^{6}b^{2}x^{21}+\frac{8}{15}a^{7}b\,x^{22}}{x^{7}}$
orering	$\frac{x^{16}\left(6435a^{8}x^{8}+54912a^{7}b\,x^{7}+205920a^{6}b^{2}x^{6}+443520a^{5}b^{3}x^{5}+600600a^{4}x^{4}b^{4}+524160a^{3}b^{5}x^{3}+288288a^{2}b^{6}x^{2}+91520a\,b^{7}x+1\right.}{102960\left(ax+b\right)^{8}}$

input
```
int((a+b/x)^8*x^15,x,method=_RETURNVERBOSE)
```

output
```
1/102960*x^8*(6435*a^8*x^8+54912*a^7*b*x^7+205920*a^6*b^2*x^6+443520*a^5*b
^3*x^5+600600*a^4*b^4*x^4+524160*a^3*b^5*x^3+288288*a^2*b^6*x^2+91520*a*b^
7*x+12870*b^8)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx = \frac{1}{16}\,a^8x^{16}+\frac{8}{15}\,a^7bx^{15}+2\,a^6b^2x^{14}+\frac{56}{13}\,a^5b^3x^{13}+\frac{35}{6}\,a^4b^4x^{12}+\frac{56}{11}\,a^3b^5x^{11}+\frac{14}{5}\,a^2b^6x^{10}+\frac{8}{9}\,ab^7x^9+\frac{1}{8}\,b^8x^8$$

input
```
integrate((a+b/x)^8*x^15,x, algorithm="fricas")
```

output
```
1/16*a^8*x^16 + 8/15*a^7*b*x^15 + 2*a^6*b^2*x^14 + 56/13*a^5*b^3*x^13 + 35
/6*a^4*b^4*x^12 + 56/11*a^3*b^5*x^11 + 14/5*a^2*b^6*x^10 + 8/9*a*b^7*x^9 +
 1/8*b^8*x^8
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx = \frac{a^8x^{16}}{16}+\frac{8a^7bx^{15}}{15}+2a^6b^2x^{14}+\frac{56a^5b^3x^{13}}{13}+\frac{35a^4b^4x^{12}}{6}+\frac{56a^3b^5x^{11}}{11}+\frac{14a^2b^6x^{10}}{5}+\frac{8ab^7x^9}{9}+\frac{b^8x^8}{8}$$

input
```
integrate((a+b/x)**8*x**15,x)
```

output
```
a**8*x**16/16 + 8*a**7*b*x**15/15 + 2*a**6*b**2*x**14 + 56*a**5*b**3*x**13
/13 + 35*a**4*b**4*x**12/6 + 56*a**3*b**5*x**11/11 + 14*a**2*b**6*x**10/5
+ 8*a*b**7*x**9/9 + b**8*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx = \frac{1}{16}\,a^8x^{16} + \frac{8}{15}\,a^7bx^{15} + 2\,a^6b^2x^{14} + \frac{56}{13}\,a^5b^3x^{13} + \frac{35}{6}\,a^4b^4x^{12} + \frac{56}{11}\,a^3b^5x^{11} + \frac{14}{5}\,a^2b^6x^{10} + \frac{8}{9}\,ab^7x^9 + \frac{1}{8}\,b^8x^8$$

input
```
integrate((a+b/x)^8*x^15,x, algorithm="maxima")
```

output
```
1/16*a^8*x^16 + 8/15*a^7*b*x^15 + 2*a^6*b^2*x^14 + 56/13*a^5*b^3*x^13 + 35
/6*a^4*b^4*x^12 + 56/11*a^3*b^5*x^11 + 14/5*a^2*b^6*x^10 + 8/9*a*b^7*x^9 +
 1/8*b^8*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx = \frac{1}{16}\,a^8x^{16} + \frac{8}{15}\,a^7bx^{15} + 2\,a^6b^2x^{14} + \frac{56}{13}\,a^5b^3x^{13} + \frac{35}{6}\,a^4b^4x^{12} + \frac{56}{11}\,a^3b^5x^{11} + \frac{14}{5}\,a^2b^6x^{10} + \frac{8}{9}\,ab^7x^9 + \frac{1}{8}\,b^8x^8$$

input
```
integrate((a+b/x)^8*x^15,x, algorithm="giac")
```

output
```
1/16*a^8*x^16 + 8/15*a^7*b*x^15 + 2*a^6*b^2*x^14 + 56/13*a^5*b^3*x^13 + 35
/6*a^4*b^4*x^12 + 56/11*a^3*b^5*x^11 + 14/5*a^2*b^6*x^10 + 8/9*a*b^7*x^9 +
 1/8*b^8*x^8
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx=\frac{a^8\,x^{16}}{16}+\frac{8\,a^7\,b\,x^{15}}{15}+2\,a^6\,b^2\,x^{14}+\frac{56\,a^5\,b^3\,x^{13}}{13}+\frac{35\,a^4\,b^4\,x^{12}}{6}+\frac{56\,a^3\,b^5\,x^{11}}{11}+\frac{14\,a^2\,b^6\,x^{10}}{5}+\frac{8\,a\,b^7\,x^9}{9}+\frac{b^8\,x^8}{8}$$

input
```
int(x^15*(a + b/x)^8,x)
```

output
```
(a^8*x^16)/16 + (b^8*x^8)/8 + (8*a*b^7*x^9)/9 + (8*a^7*b*x^15)/15 + (14*a^
2*b^6*x^10)/5 + (56*a^3*b^5*x^11)/11 + (35*a^4*b^4*x^12)/6 + (56*a^5*b^3*x
^13)/13 + 2*a^6*b^2*x^14
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^8 x^{15}\,dx = \frac{x^8(6435a^8x^8+54912a^7b\,x^7+205920a^6b^2x^6+443520a^5b^3x^5+600600a^4b^4x^4+524160a^3b^5x^3+288288a}{102960}$$

input
```
int((a+b/x)^8*x^15,x)
```

output
```
(x**8*(6435*a**8*x**8 + 54912*a**7*b*x**7 + 205920*a**6*b**2*x**6 + 443520
*a**5*b**3*x**5 + 600600*a**4*b**4*x**4 + 524160*a**3*b**5*x**3 + 288288*a
**2*b**6*x**2 + 91520*a*b**7*x + 12870*b**8))/102960
```

3.40 $\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx = -\frac{b^5(b+ax)^9}{9a^6}+\frac{b^4(b+ax)^{10}}{2a^6}-\frac{10b^3(b+ax)^{11}}{11a^6}+\frac{5b^2(b+ax)^{12}}{6a^6}-\frac{5b(b+ax)^{13}}{13a^6}+\frac{(b+ax)^{14}}{14a^6}$$

output
```
-1/9*b^5*(a*x+b)^9/a^6+1/2*b^4*(a*x+b)^10/a^6-10/11*b^3*(a*x+b)^11/a^6+5/6
*b^2*(a*x+b)^12/a^6-5/13*b*(a*x+b)^13/a^6+1/14*(a*x+b)^14/a^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx = \frac{b^8x^6}{6}+\frac{8}{7}ab^7x^7+\frac{7}{2}a^2b^6x^8+\frac{56}{9}a^3b^5x^9+7a^4b^4x^{10}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{3}a^6b^2x^{12}+\frac{8}{13}a^7bx^{13}+\frac{a^8x^{14}}{14}$$

input
```
Integrate[(a + b/x)^8*x^13,x]
```

output
```
(b^8*x^6)/6 + (8*a*b^7*x^7)/7 + (7*a^2*b^6*x^8)/2 + (56*a^3*b^5*x^9)/9 + 7
*a^4*b^4*x^10 + (56*a^5*b^3*x^11)/11 + (7*a^6*b^2*x^12)/3 + (8*a^7*b*x^13)
/13 + (a^8*x^14)/14
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}\left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^5(ax+b)^8dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^5(ax+b)^8}{a^5}+\frac{5b^4(ax+b)^9}{a^5}-\frac{10b^3(ax+b)^{10}}{a^5}+\frac{10b^2(ax+b)^{11}}{a^5}+\frac{(ax+b)^{13}}{a^5}-\frac{5b(ax+b)^{12}}{a^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^5(ax+b)^9}{9a^6}+\frac{b^4(ax+b)^{10}}{2a^6}-\frac{10b^3(ax+b)^{11}}{11a^6}+\frac{5b^2(ax+b)^{12}}{6a^6}+\frac{(ax+b)^{14}}{14a^6}-\frac{5b(ax+b)^{13}}{13a^6}$$

input
```
Int[(a + b/x)^8*x^13,x]
```

output
```
-1/9*(b^5*(b + a*x)^9)/a^6 + (b^4*(b + a*x)^10)/(2*a^6) - (10*b^3*(b + a*x
)^11)/(11*a^6) + (5*b^2*(b + a*x)^12)/(6*a^6) - (5*b*(b + a*x)^13)/(13*a^6
) + (b + a*x)^14/(14*a^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{x^6(1287a^8x^8+11088a^7b\,x^7+42042a^6b^2x^6+91728a^5b^3x^5+126126a^4x^4b^4+112112a^3b^5x^3+63063a^2b^6x^2+20592a\,b^7x+3003b}{18018}$
default	$\frac{1}{6}b^8x^6+\frac{8}{7}a\,x^7b^7+\frac{7}{2}a^2b^6x^8+\frac{56}{9}a^3b^5x^9+7a^4b^4x^{10}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{3}a^6b^2x^{12}+\frac{8}{13}a^7b\,x^{13}+\frac{1}{14}$
risch	$\frac{1}{6}b^8x^6+\frac{8}{7}a\,x^7b^7+\frac{7}{2}a^2b^6x^8+\frac{56}{9}a^3b^5x^9+7a^4b^4x^{10}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{3}a^6b^2x^{12}+\frac{8}{13}a^7b\,x^{13}+\frac{1}{14}$
parallelrisch	$\frac{1}{6}b^8x^6+\frac{8}{7}a\,x^7b^7+\frac{7}{2}a^2b^6x^8+\frac{56}{9}a^3b^5x^9+7a^4b^4x^{10}+\frac{56}{11}a^5b^3x^{11}+\frac{7}{3}a^6b^2x^{12}+\frac{8}{13}a^7b\,x^{13}+\frac{1}{14}$
norman	$\frac{\frac{1}{14}a^8x^{21}+\frac{1}{6}b^8x^{13}+\frac{8}{7}a\,b^7x^{14}+\frac{7}{2}a^2b^6x^{15}+\frac{56}{9}a^3b^5x^{16}+7a^4b^4x^{17}+\frac{56}{11}a^5b^3x^{18}+\frac{7}{3}a^6b^2x^{19}+\frac{8}{13}a^7b\,x^{20}}{x^7}$
orering	$\frac{x^{14}(1287a^8x^8+11088a^7b\,x^7+42042a^6b^2x^6+91728a^5b^3x^5+126126a^4x^4b^4+112112a^3b^5x^3+63063a^2b^6x^2+20592a\,b^7x+3003}{18018(ax+b)^8}$

input
```
int((a+b/x)^8*x^13,x,method=_RETURNVERBOSE)
```

output
```
1/18018*x^6*(1287*a^8*x^8+11088*a^7*b*x^7+42042*a^6*b^2*x^6+91728*a^5*b^3*
x^5+126126*a^4*b^4*x^4+112112*a^3*b^5*x^3+63063*a^2*b^6*x^2+20592*a*b^7*x+
3003*b^8)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx = \frac{1}{14}\,a^8x^{14}+\frac{8}{13}\,a^7bx^{13}+\frac{7}{3}\,a^6b^2x^{12}+\frac{56}{11}\,a^5b^3x^{11} + 7\,a^4b^4x^{10}+\frac{56}{9}\,a^3b^5x^9+\frac{7}{2}\,a^2b^6x^8+\frac{8}{7}\,ab^7x^7+\frac{1}{6}\,b^8x^6$$

input
```
integrate((a+b/x)^8*x^13,x, algorithm="fricas")
```

output
```
1/14*a^8*x^14 + 8/13*a^7*b*x^13 + 7/3*a^6*b^2*x^12 + 56/11*a^5*b^3*x^11 +
7*a^4*b^4*x^10 + 56/9*a^3*b^5*x^9 + 7/2*a^2*b^6*x^8 + 8/7*a*b^7*x^7 + 1/6*
b^8*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx = \frac{a^8x^{14}}{14}+\frac{8a^7bx^{13}}{13}+\frac{7a^6b^2x^{12}}{3}+\frac{56a^5b^3x^{11}}{11} + 7a^4b^4x^{10}+\frac{56a^3b^5x^9}{9}+\frac{7a^2b^6x^8}{2}+\frac{8ab^7x^7}{7}+\frac{b^8x^6}{6}$$

input
```
integrate((a+b/x)**8*x**13,x)
```

output
```
a**8*x**14/14 + 8*a**7*b*x**13/13 + 7*a**6*b**2*x**12/3 + 56*a**5*b**3*x**
11/11 + 7*a**4*b**4*x**10 + 56*a**3*b**5*x**9/9 + 7*a**2*b**6*x**8/2 + 8*a
*b**7*x**7/7 + b**8*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx=\frac{1}{14}\,a^8x^{14}+\frac{8}{13}\,a^7bx^{13}+\frac{7}{3}\,a^6b^2x^{12}+\frac{56}{11}\,a^5b^3x^{11}+7\,a^4b^4x^{10}+\frac{56}{9}\,a^3b^5x^9+\frac{7}{2}\,a^2b^6x^8+\frac{8}{7}\,ab^7x^7+\frac{1}{6}\,b^8x^6$$

input
```
integrate((a+b/x)^8*x^13,x, algorithm="maxima")
```

output
```
1/14*a^8*x^14 + 8/13*a^7*b*x^13 + 7/3*a^6*b^2*x^12 + 56/11*a^5*b^3*x^11 +
7*a^4*b^4*x^10 + 56/9*a^3*b^5*x^9 + 7/2*a^2*b^6*x^8 + 8/7*a*b^7*x^7 + 1/6*
b^8*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx=\frac{1}{14}\,a^8x^{14}+\frac{8}{13}\,a^7bx^{13}+\frac{7}{3}\,a^6b^2x^{12}+\frac{56}{11}\,a^5b^3x^{11}+7\,a^4b^4x^{10}+\frac{56}{9}\,a^3b^5x^9+\frac{7}{2}\,a^2b^6x^8+\frac{8}{7}\,ab^7x^7+\frac{1}{6}\,b^8x^6$$

input
```
integrate((a+b/x)^8*x^13,x, algorithm="giac")
```

output
```
1/14*a^8*x^14 + 8/13*a^7*b*x^13 + 7/3*a^6*b^2*x^12 + 56/11*a^5*b^3*x^11 +
7*a^4*b^4*x^10 + 56/9*a^3*b^5*x^9 + 7/2*a^2*b^6*x^8 + 8/7*a*b^7*x^7 + 1/6*
b^8*x^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx = \frac{a^8\,x^{14}}{14}+\frac{8\,a^7\,b\,x^{13}}{13}+\frac{7\,a^6\,b^2\,x^{12}}{3}+\frac{56\,a^5\,b^3\,x^{11}}{11} + 7\,a^4\,b^4\,x^{10}+\frac{56\,a^3\,b^5\,x^9}{9}+\frac{7\,a^2\,b^6\,x^8}{2}+\frac{8\,a\,b^7\,x^7}{7}+\frac{b^8\,x^6}{6}$$

input
```
int(x^13*(a + b/x)^8,x)
```

output
```
(a^8*x^14)/14 + (b^8*x^6)/6 + (8*a*b^7*x^7)/7 + (8*a^7*b*x^13)/13 + (7*a^2
*b^6*x^8)/2 + (56*a^3*b^5*x^9)/9 + 7*a^4*b^4*x^10 + (56*a^5*b^3*x^11)/11 +
 (7*a^6*b^2*x^12)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^{13}\,dx = \frac{x^6(1287a^8x^8+11088a^7b\,x^7+42042a^6b^2x^6+91728a^5b^3x^5+126126a^4b^4x^4+112112a^3b^5x^3+63063a^2b^6}{18018}$$

input
```
int((a+b/x)^8*x^13,x)
```

output
```
(x**6*(1287*a**8*x**8 + 11088*a**7*b*x**7 + 42042*a**6*b**2*x**6 + 91728*a
**5*b**3*x**5 + 126126*a**4*b**4*x**4 + 112112*a**3*b**5*x**3 + 63063*a**2
*b**6*x**2 + 20592*a*b**7*x + 3003*b**8))/18018
```

3.41 $\int \left(a+\frac{b}{x}\right)^8 x^{12} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \left(a+\frac{b}{x}\right)^8 x^{12} \, dx = \frac{b^4(b+ax)^9}{9a^5}-\frac{2b^3(b+ax)^{10}}{5a^5}+\frac{6b^2(b+ax)^{11}}{11a^5}-\frac{b(b+ax)^{12}}{3a^5}+\frac{(b+ax)^{13}}{13a^5}$$

output
```
1/9*b^4*(a*x+b)^9/a^5-2/5*b^3*(a*x+b)^10/a^5+6/11*b^2*(a*x+b)^11/a^5-1/3*b
*(a*x+b)^12/a^5+1/13*(a*x+b)^13/a^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int \left(a+\frac{b}{x}\right)^8 x^{12} \, dx = \frac{b^8x^5}{5}+\frac{4}{3}ab^7x^6+4a^2b^6x^7+7a^3b^5x^8+\frac{70}{9}a^4b^4x^9+\frac{28}{5}a^5b^3x^{10}+\frac{28}{11}a^6b^2x^{11}+\frac{2}{3}a^7bx^{12}+\frac{a^8x^{13}}{13}$$

input
```
Integrate[(a + b/x)^8*x^12,x]
```

output
```
(b^8*x^5)/5 + (4*a*b^7*x^6)/3 + 4*a^2*b^6*x^7 + 7*a^3*b^5*x^8 + (70*a^4*b^
4*x^9)/9 + (28*a^5*b^3*x^10)/5 + (28*a^6*b^2*x^11)/11 + (2*a^7*b*x^12)/3 +
 (a^8*x^13)/13
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^4(ax+b)^8dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{b^4(ax+b)^8}{a^4}-\frac{4b^3(ax+b)^9}{a^4}+\frac{6b^2(ax+b)^{10}}{a^4}+\frac{(ax+b)^{12}}{a^4}-\frac{4b(ax+b)^{11}}{a^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^4(ax+b)^9}{9a^5}-\frac{2b^3(ax+b)^{10}}{5a^5}+\frac{6b^2(ax+b)^{11}}{11a^5}+\frac{(ax+b)^{13}}{13a^5}-\frac{b(ax+b)^{12}}{3a^5}$$

input
```
Int[(a + b/x)^8*x^12,x]
```

output
```
(b^4*(b + a*x)^9)/(9*a^5) - (2*b^3*(b + a*x)^10)/(5*a^5) + (6*b^2*(b + a*x
)^11)/(11*a^5) - (b*(b + a*x)^12)/(3*a^5) + (b + a*x)^13/(13*a^5)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

method	result
gosper	$\frac{x^{5}\left(495a^{8}x^{8}+4290a^{7}b\,x^{7}+16380a^{6}b^{2}x^{6}+36036a^{5}b^{3}x^{5}+50050a^{4}x^{4}b^{4}+45045a^{3}b^{5}x^{3}+25740a^{2}b^{6}x^{2}+8580a\,b^{7}x+1287b^{8}\right)}{6435}$
default	$\frac{1}{13}a^{8}x^{13}+\frac{2}{3}a^{7}b\,x^{12}+\frac{28}{11}a^{6}b^{2}x^{11}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{9}a^{4}b^{4}x^{9}+7a^{3}b^{5}x^{8}+4a^{2}b^{6}x^{7}+\frac{4}{3}a\,b^{7}x^{6}+\frac{1}{5}$
risch	$\frac{1}{13}a^{8}x^{13}+\frac{2}{3}a^{7}b\,x^{12}+\frac{28}{11}a^{6}b^{2}x^{11}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{9}a^{4}b^{4}x^{9}+7a^{3}b^{5}x^{8}+4a^{2}b^{6}x^{7}+\frac{4}{3}a\,b^{7}x^{6}+\frac{1}{5}$
parallelrisch	$\frac{1}{13}a^{8}x^{13}+\frac{2}{3}a^{7}b\,x^{12}+\frac{28}{11}a^{6}b^{2}x^{11}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{9}a^{4}b^{4}x^{9}+7a^{3}b^{5}x^{8}+4a^{2}b^{6}x^{7}+\frac{4}{3}a\,b^{7}x^{6}+\frac{1}{5}$
norman	$\frac{\frac{1}{13}a^{8}x^{20}+\frac{1}{5}b^{8}x^{12}+\frac{4}{3}a\,b^{7}x^{13}+4a^{2}b^{6}x^{14}+7a^{3}b^{5}x^{15}+\frac{70}{9}a^{4}b^{4}x^{16}+\frac{28}{5}a^{5}b^{3}x^{17}+\frac{28}{11}a^{6}b^{2}x^{18}+\frac{2}{3}a^{7}b\,x^{19}}{x^{7}}$
orering	$\frac{x^{13}\left(495a^{8}x^{8}+4290a^{7}b\,x^{7}+16380a^{6}b^{2}x^{6}+36036a^{5}b^{3}x^{5}+50050a^{4}x^{4}b^{4}+45045a^{3}b^{5}x^{3}+25740a^{2}b^{6}x^{2}+8580a\,b^{7}x+1287b^{8}\right)\left(a\right.}{6435(ax+b)^{8}}$

input
```
int((a+b/x)^8*x^12,x,method=_RETURNVERBOSE)
```

output
```
1/6435*x^5*(495*a^8*x^8+4290*a^7*b*x^7+16380*a^6*b^2*x^6+36036*a^5*b^3*x^5
+50050*a^4*b^4*x^4+45045*a^3*b^5*x^3+25740*a^2*b^6*x^2+8580*a*b^7*x+1287*b
^8)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx = \frac{1}{13}\,a^8x^{13} + \frac{2}{3}\,a^7bx^{12} + \frac{28}{11}\,a^6b^2x^{11} + \frac{28}{5}\,a^5b^3x^{10} + \frac{70}{9}\,a^4b^4x^9 + 7\,a^3b^5x^8 + 4\,a^2b^6x^7 + \frac{4}{3}\,ab^7x^6 + \frac{1}{5}\,b^8x^5$$

input
```
integrate((a+b/x)^8*x^12,x, algorithm="fricas")
```

output
```
1/13*a^8*x^13 + 2/3*a^7*b*x^12 + 28/11*a^6*b^2*x^11 + 28/5*a^5*b^3*x^10 +
70/9*a^4*b^4*x^9 + 7*a^3*b^5*x^8 + 4*a^2*b^6*x^7 + 4/3*a*b^7*x^6 + 1/5*b^8
*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.28

$$\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx = \frac{a^8x^{13}}{13} + \frac{2a^7bx^{12}}{3} + \frac{28a^6b^2x^{11}}{11} + \frac{28a^5b^3x^{10}}{5} + \frac{70a^4b^4x^9}{9} + 7a^3b^5x^8 + 4a^2b^6x^7 + \frac{4ab^7x^6}{3} + \frac{b^8x^5}{5}$$

input
```
integrate((a+b/x)**8*x**12,x)
```

output
```
a**8*x**13/13 + 2*a**7*b*x**12/3 + 28*a**6*b**2*x**11/11 + 28*a**5*b**3*x*
*10/5 + 70*a**4*b**4*x**9/9 + 7*a**3*b**5*x**8 + 4*a**2*b**6*x**7 + 4*a*b*
*7*x**6/3 + b**8*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx = \frac{1}{13}\,a^8x^{13} + \frac{2}{3}\,a^7bx^{12} + \frac{28}{11}\,a^6b^2x^{11} + \frac{28}{5}\,a^5b^3x^{10} + \frac{70}{9}\,a^4b^4x^9 + 7\,a^3b^5x^8 + 4\,a^2b^6x^7 + \frac{4}{3}\,ab^7x^6 + \frac{1}{5}\,b^8x^5$$

input
```
integrate((a+b/x)^8*x^12,x, algorithm="maxima")
```

output
```
1/13*a^8*x^13 + 2/3*a^7*b*x^12 + 28/11*a^6*b^2*x^11 + 28/5*a^5*b^3*x^10 +
70/9*a^4*b^4*x^9 + 7*a^3*b^5*x^8 + 4*a^2*b^6*x^7 + 4/3*a*b^7*x^6 + 1/5*b^8
*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx = \frac{1}{13}\,a^8x^{13} + \frac{2}{3}\,a^7bx^{12} + \frac{28}{11}\,a^6b^2x^{11} + \frac{28}{5}\,a^5b^3x^{10} + \frac{70}{9}\,a^4b^4x^9 + 7\,a^3b^5x^8 + 4\,a^2b^6x^7 + \frac{4}{3}\,ab^7x^6 + \frac{1}{5}\,b^8x^5$$

input
```
integrate((a+b/x)^8*x^12,x, algorithm="giac")
```

output
```
1/13*a^8*x^13 + 2/3*a^7*b*x^12 + 28/11*a^6*b^2*x^11 + 28/5*a^5*b^3*x^10 +
70/9*a^4*b^4*x^9 + 7*a^3*b^5*x^8 + 4*a^2*b^6*x^7 + 4/3*a*b^7*x^6 + 1/5*b^8
*x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx = \frac{a^8\,x^{13}}{13}+\frac{2\,a^7\,b\,x^{12}}{3}+\frac{28\,a^6\,b^2\,x^{11}}{11}+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{70\,a^4\,b^4\,x^9}{9}+7\,a^3\,b^5\,x^8+4\,a^2\,b^6\,x^7+\frac{4\,a\,b^7\,x^6}{3}+\frac{b^8\,x^5}{5}$$

input
```
int(x^12*(a + b/x)^8,x)
```

output
```
(a^8*x^13)/13 + (b^8*x^5)/5 + (4*a*b^7*x^6)/3 + (2*a^7*b*x^12)/3 + 4*a^2*b
^6*x^7 + 7*a^3*b^5*x^8 + (70*a^4*b^4*x^9)/9 + (28*a^5*b^3*x^10)/5 + (28*a^
6*b^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x}\right)^8 x^{12}\,dx = \frac{x^5(495a^8x^8+4290a^7b\,x^7+16380a^6b^2x^6+36036a^5b^3x^5+50050a^4b^4x^4+45045a^3b^5x^3+25740a^2b^6x^2+}{6435}$$

input
```
int((a+b/x)^8*x^12,x)
```

output
```
(x**5*(495*a**8*x**8 + 4290*a**7*b*x**7 + 16380*a**6*b**2*x**6 + 36036*a**
5*b**3*x**5 + 50050*a**4*b**4*x**4 + 45045*a**3*b**5*x**3 + 25740*a**2*b**
6*x**2 + 8580*a*b**7*x + 1287*b**8))/6435
```

3.42 $\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx=-\frac{b^3(b+ax)^9}{9a^4}+\frac{3b^2(b+ax)^{10}}{10a^4}-\frac{3b(b+ax)^{11}}{11a^4}+\frac{(b+ax)^{12}}{12a^4}$$

output
```
-1/9*b^3*(a*x+b)^9/a^4+3/10*b^2*(a*x+b)^10/a^4-3/11*b*(a*x+b)^11/a^4+1/12*
(a*x+b)^12/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx=\frac{b^8x^4}{4}+\frac{8}{5}ab^7x^5+\frac{14}{3}a^2b^6x^6+8a^3b^5x^7+\frac{35}{4}a^4b^4x^8+\frac{56}{9}a^5b^3x^9+\frac{14}{5}a^6b^2x^{10}+\frac{8}{11}a^7bx^{11}+\frac{a^8x^{12}}{12}$$

input
```
Integrate[(a + b/x)^8*x^11,x]
```

output
```
(b^8*x^4)/4 + (8*a*b^7*x^5)/5 + (14*a^2*b^6*x^6)/3 + 8*a^3*b^5*x^7 + (35*a
^4*b^4*x^8)/4 + (56*a^5*b^3*x^9)/9 + (14*a^6*b^2*x^10)/5 + (8*a^7*b*x^11)/
11 + (a^8*x^12)/12
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+\frac{b}{x}\right)^8dx$$

$$\Big\downarrow \text{795}$$

$$\int x^3(ax+b)^8dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^3(ax+b)^8}{a^3}+\frac{3b^2(ax+b)^9}{a^3}+\frac{(ax+b)^{11}}{a^3}-\frac{3b(ax+b)^{10}}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3(ax+b)^9}{9a^4}+\frac{3b^2(ax+b)^{10}}{10a^4}+\frac{(ax+b)^{12}}{12a^4}-\frac{3b(ax+b)^{11}}{11a^4}$$

input
```
Int[(a + b/x)^8*x^11,x]
```

output
```
-1/9*(b^3*(b + a*x)^9)/a^4 + (3*b^2*(b + a*x)^10)/(10*a^4) - (3*b*(b + a*x
)^11)/(11*a^4) + (b + a*x)^12/(12*a^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

method	result
gosper	$\frac{x^{4}\left(165a^{8}x^{8}+1440a^{7}b\,x^{7}+5544a^{6}b^{2}x^{6}+12320a^{5}b^{3}x^{5}+17325a^{4}x^{4}b^{4}+15840a^{3}b^{5}x^{3}+9240a^{2}b^{6}x^{2}+3168a\,b^{7}x+495b^{8}\right)}{1980}$
default	$\frac{1}{12}a^{8}x^{12}+\frac{8}{11}a^{7}b\,x^{11}+\frac{14}{5}a^{6}b^{2}x^{10}+\frac{56}{9}a^{5}b^{3}x^{9}+\frac{35}{4}a^{4}b^{4}x^{8}+8a^{3}b^{5}x^{7}+\frac{14}{3}a^{2}x^{6}b^{6}+\frac{8}{5}a\,b^{7}x^{5}+$
risch	$\frac{1}{12}a^{8}x^{12}+\frac{8}{11}a^{7}b\,x^{11}+\frac{14}{5}a^{6}b^{2}x^{10}+\frac{56}{9}a^{5}b^{3}x^{9}+\frac{35}{4}a^{4}b^{4}x^{8}+8a^{3}b^{5}x^{7}+\frac{14}{3}a^{2}x^{6}b^{6}+\frac{8}{5}a\,b^{7}x^{5}+$
parallelrisch	$\frac{1}{12}a^{8}x^{12}+\frac{8}{11}a^{7}b\,x^{11}+\frac{14}{5}a^{6}b^{2}x^{10}+\frac{56}{9}a^{5}b^{3}x^{9}+\frac{35}{4}a^{4}b^{4}x^{8}+8a^{3}b^{5}x^{7}+\frac{14}{3}a^{2}x^{6}b^{6}+\frac{8}{5}a\,b^{7}x^{5}+$
norman	$\frac{\frac{1}{12}a^{8}x^{19}+\frac{1}{4}b^{8}x^{11}+\frac{8}{5}a\,b^{7}x^{12}+\frac{14}{3}a^{2}b^{6}x^{13}+8a^{3}b^{5}x^{14}+\frac{35}{4}a^{4}b^{4}x^{15}+\frac{56}{9}a^{5}b^{3}x^{16}+\frac{14}{5}a^{6}b^{2}x^{17}+\frac{8}{11}a^{7}b\,x^{18}}{x^{7}}$
orering	$\frac{x^{12}\left(165a^{8}x^{8}+1440a^{7}b\,x^{7}+5544a^{6}b^{2}x^{6}+12320a^{5}b^{3}x^{5}+17325a^{4}x^{4}b^{4}+15840a^{3}b^{5}x^{3}+9240a^{2}b^{6}x^{2}+3168a\,b^{7}x+495b^{8}\right)\left(a+\frac{b}{x}\right.}{1980(ax+b)^{8}}$

input
```
int((a+b/x)^8*x^11,x,method=_RETURNVERBOSE)
```

output
```
1/1980*x^4*(165*a^8*x^8+1440*a^7*b*x^7+5544*a^6*b^2*x^6+12320*a^5*b^3*x^5+
17325*a^4*b^4*x^4+15840*a^3*b^5*x^3+9240*a^2*b^6*x^2+3168*a*b^7*x+495*b^8)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx = \frac{1}{12}\,a^8x^{12} + \frac{8}{11}\,a^7bx^{11} + \frac{14}{5}\,a^6b^2x^{10} + \frac{56}{9}\,a^5b^3x^9 + \frac{35}{4}\,a^4b^4x^8 + 8\,a^3b^5x^7 + \frac{14}{3}\,a^2b^6x^6 + \frac{8}{5}\,ab^7x^5 + \frac{1}{4}\,b^8x^4$$

input
```
integrate((a+b/x)^8*x^11,x, algorithm="fricas")
```

output
```
1/12*a^8*x^12 + 8/11*a^7*b*x^11 + 14/5*a^6*b^2*x^10 + 56/9*a^5*b^3*x^9 + 3
5/4*a^4*b^4*x^8 + 8*a^3*b^5*x^7 + 14/3*a^2*b^6*x^6 + 8/5*a*b^7*x^5 + 1/4*b
^8*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.64

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx = \frac{a^8x^{12}}{12} + \frac{8a^7bx^{11}}{11} + \frac{14a^6b^2x^{10}}{5} + \frac{56a^5b^3x^9}{9} + \frac{35a^4b^4x^8}{4} + 8a^3b^5x^7 + \frac{14a^2b^6x^6}{3} + \frac{8ab^7x^5}{5} + \frac{b^8x^4}{4}$$

input
```
integrate((a+b/x)**8*x**11,x)
```

output
```
a**8*x**12/12 + 8*a**7*b*x**11/11 + 14*a**6*b**2*x**10/5 + 56*a**5*b**3*x*
*9/9 + 35*a**4*b**4*x**8/4 + 8*a**3*b**5*x**7 + 14*a**2*b**6*x**6/3 + 8*a*
b**7*x**5/5 + b**8*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx = \frac{1}{12}\,a^8x^{12}+\frac{8}{11}\,a^7bx^{11}+\frac{14}{5}\,a^6b^2x^{10}+\frac{56}{9}\,a^5b^3x^9 + \frac{35}{4}\,a^4b^4x^8+8\,a^3b^5x^7+\frac{14}{3}\,a^2b^6x^6+\frac{8}{5}\,ab^7x^5+\frac{1}{4}\,b^8x^4$$

input
```
integrate((a+b/x)^8*x^11,x, algorithm="maxima")
```

output
```
1/12*a^8*x^12 + 8/11*a^7*b*x^11 + 14/5*a^6*b^2*x^10 + 56/9*a^5*b^3*x^9 + 3
5/4*a^4*b^4*x^8 + 8*a^3*b^5*x^7 + 14/3*a^2*b^6*x^6 + 8/5*a*b^7*x^5 + 1/4*b
^8*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx = \frac{1}{12}\,a^8x^{12}+\frac{8}{11}\,a^7bx^{11}+\frac{14}{5}\,a^6b^2x^{10}+\frac{56}{9}\,a^5b^3x^9 + \frac{35}{4}\,a^4b^4x^8+8\,a^3b^5x^7+\frac{14}{3}\,a^2b^6x^6+\frac{8}{5}\,ab^7x^5+\frac{1}{4}\,b^8x^4$$

input
```
integrate((a+b/x)^8*x^11,x, algorithm="giac")
```

output
```
1/12*a^8*x^12 + 8/11*a^7*b*x^11 + 14/5*a^6*b^2*x^10 + 56/9*a^5*b^3*x^9 + 3
5/4*a^4*b^4*x^8 + 8*a^3*b^5*x^7 + 14/3*a^2*b^6*x^6 + 8/5*a*b^7*x^5 + 1/4*b
^8*x^4
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx = \frac{a^8\,x^{12}}{12}+\frac{8\,a^7\,b\,x^{11}}{11}+\frac{14\,a^6\,b^2\,x^{10}}{5}+\frac{56\,a^5\,b^3\,x^9}{9}+\frac{35\,a^4\,b^4\,x^8}{4}+8\,a^3\,b^5\,x^7+\frac{14\,a^2\,b^6\,x^6}{3}+\frac{8\,a\,b^7\,x^5}{5}+\frac{b^8\,x^4}{4}$$

input
```
int(x^11*(a + b/x)^8,x)
```

output
```
(a^8*x^12)/12 + (b^8*x^4)/4 + (8*a*b^7*x^5)/5 + (8*a^7*b*x^11)/11 + (14*a^
2*b^6*x^6)/3 + 8*a^3*b^5*x^7 + (35*a^4*b^4*x^8)/4 + (56*a^5*b^3*x^9)/9 + (
14*a^6*b^2*x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \left(a+\frac{b}{x}\right)^8 x^{11}\,dx = \frac{x^4(165a^8x^8+1440a^7b\,x^7+5544a^6b^2x^6+12320a^5b^3x^5+17325a^4b^4x^4+15840a^3b^5x^3+9240a^2b^6x^2+31}{1980}$$

input
```
int((a+b/x)^8*x^11,x)
```

output
```
(x**4*(165*a**8*x**8 + 1440*a**7*b*x**7 + 5544*a**6*b**2*x**6 + 12320*a**5
*b**3*x**5 + 17325*a**4*b**4*x**4 + 15840*a**3*b**5*x**3 + 9240*a**2*b**6*
x**2 + 3168*a*b**7*x + 495*b**8))/1980
```

3.43 $\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx=\frac{b^2(b+ax)^9}{9a^3}-\frac{b(b+ax)^{10}}{5a^3}+\frac{(b+ax)^{11}}{11a^3}$$

output
```
1/9*b^2*(a*x+b)^9/a^3-1/5*b*(a*x+b)^10/a^3+1/11*(a*x+b)^11/a^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 102 vs. 2(47) = 94.

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.17

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx=\frac{b^8x^3}{3}+2ab^7x^4+\frac{28}{5}a^2b^6x^5+\frac{28}{3}a^3b^5x^6+10a^4b^4x^7+7a^5b^3x^8+\frac{28}{9}a^6b^2x^9+\frac{4}{5}a^7bx^{10}+\frac{a^8x^{11}}{11}$$

input
```
Integrate[(a + b/x)^8*x^10,x]
```

output
```
(b^8*x^3)/3 + 2*a*b^7*x^4 + (28*a^2*b^6*x^5)/5 + (28*a^3*b^5*x^6)/3 + 10*a
^4*b^4*x^7 + 7*a^5*b^3*x^8 + (28*a^6*b^2*x^9)/9 + (4*a^7*b*x^10)/5 + (a^8*
x^11)/11
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^2(ax+b)^8dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{b^2(ax+b)^8}{a^2}+\frac{(ax+b)^{10}}{a^2}-\frac{2b(ax+b)^9}{a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(ax+b)^9}{9a^3}+\frac{(ax+b)^{11}}{11a^3}-\frac{b(ax+b)^{10}}{5a^3}$$

input
```
Int[(a + b/x)^8*x^10,x]
```

output
```
(b^2*(b + a*x)^9)/(9*a^3) - (b*(b + a*x)^10)/(5*a^3) + (b + a*x)^11/(11*a^
3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

method	result
gosper	$\frac{x^{3}\left(45a^{8}x^{8}+396a^{7}b\,x^{7}+1540a^{6}b^{2}x^{6}+3465a^{5}b^{3}x^{5}+4950a^{4}x^{4}b^{4}+4620a^{3}b^{5}x^{3}+2772a^{2}b^{6}x^{2}+990a\,b^{7}x+165b^{8}\right)}{495}$
default	$\frac{1}{11}a^{8}x^{11}+\frac{4}{5}a^{7}b\,x^{10}+\frac{28}{9}a^{6}b^{2}x^{9}+7a^{5}b^{3}x^{8}+10a^{4}b^{4}x^{7}+\frac{28}{3}a^{3}b^{5}x^{6}+\frac{28}{5}a^{2}b^{6}x^{5}+2a\,b^{7}x^{4}+\frac{1}{3}$
risch	$\frac{1}{11}a^{8}x^{11}+\frac{4}{5}a^{7}b\,x^{10}+\frac{28}{9}a^{6}b^{2}x^{9}+7a^{5}b^{3}x^{8}+10a^{4}b^{4}x^{7}+\frac{28}{3}a^{3}b^{5}x^{6}+\frac{28}{5}a^{2}b^{6}x^{5}+2a\,b^{7}x^{4}+\frac{1}{3}$
parallelrisch	$\frac{1}{11}a^{8}x^{11}+\frac{4}{5}a^{7}b\,x^{10}+\frac{28}{9}a^{6}b^{2}x^{9}+7a^{5}b^{3}x^{8}+10a^{4}b^{4}x^{7}+\frac{28}{3}a^{3}b^{5}x^{6}+\frac{28}{5}a^{2}b^{6}x^{5}+2a\,b^{7}x^{4}+\frac{1}{3}$
norman	$\frac{\frac{1}{11}a^{8}x^{18}+\frac{1}{3}b^{8}x^{10}+2a\,b^{7}x^{11}+\frac{28}{5}a^{2}b^{6}x^{12}+\frac{28}{3}a^{3}b^{5}x^{13}+10a^{4}b^{4}x^{14}+7a^{5}b^{3}x^{15}+\frac{28}{9}a^{6}b^{2}x^{16}+\frac{4}{5}a^{7}b\,x^{17}}{x^{7}}$
orering	$\frac{x^{11}\left(45a^{8}x^{8}+396a^{7}b\,x^{7}+1540a^{6}b^{2}x^{6}+3465a^{5}b^{3}x^{5}+4950a^{4}x^{4}b^{4}+4620a^{3}b^{5}x^{3}+2772a^{2}b^{6}x^{2}+990a\,b^{7}x+165b^{8}\right)\left(a+\frac{b}{x}\right)^{8}}{495\left(ax+b\right)^{8}}$

input
```
int((a+b/x)^8*x^10,x,method=_RETURNVERBOSE)
```

output
```
1/495*x^3*(45*a^8*x^8+396*a^7*b*x^7+1540*a^6*b^2*x^6+3465*a^5*b^3*x^5+4950
*a^4*b^4*x^4+4620*a^3*b^5*x^3+2772*a^2*b^6*x^2+990*a*b^7*x+165*b^8)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx = \frac{1}{11}\,a^8x^{11} + \frac{4}{5}\,a^7bx^{10} + \frac{28}{9}\,a^6b^2x^9 + 7\,a^5b^3x^8 + 10\,a^4b^4x^7 + \frac{28}{3}\,a^3b^5x^6 + \frac{28}{5}\,a^2b^6x^5 + 2\,ab^7x^4 + \frac{1}{3}\,b^8x^3$$

input
```
integrate((a+b/x)^8*x^10,x, algorithm="fricas")
```

output
```
1/11*a^8*x^11 + 4/5*a^7*b*x^10 + 28/9*a^6*b^2*x^9 + 7*a^5*b^3*x^8 + 10*a^4
*b^4*x^7 + 28/3*a^3*b^5*x^6 + 28/5*a^2*b^6*x^5 + 2*a*b^7*x^4 + 1/3*b^8*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.17

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx = \frac{a^8x^{11}}{11} + \frac{4a^7bx^{10}}{5} + \frac{28a^6b^2x^9}{9} + 7a^5b^3x^8 + 10a^4b^4x^7 + \frac{28a^3b^5x^6}{3} + \frac{28a^2b^6x^5}{5} + 2ab^7x^4 + \frac{b^8x^3}{3}$$

input
```
integrate((a+b/x)**8*x**10,x)
```

output
```
a**8*x**11/11 + 4*a**7*b*x**10/5 + 28*a**6*b**2*x**9/9 + 7*a**5*b**3*x**8
+ 10*a**4*b**4*x**7 + 28*a**3*b**5*x**6/3 + 28*a**2*b**6*x**5/5 + 2*a*b**7
*x**4 + b**8*x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx = \frac{1}{11}\,a^8x^{11} + \frac{4}{5}\,a^7bx^{10} + \frac{28}{9}\,a^6b^2x^9 + 7\,a^5b^3x^8 + 10\,a^4b^4x^7 + \frac{28}{3}\,a^3b^5x^6 + \frac{28}{5}\,a^2b^6x^5 + 2\,ab^7x^4 + \frac{1}{3}\,b^8x^3$$

input
```
integrate((a+b/x)^8*x^10,x, algorithm="maxima")
```

output
```
1/11*a^8*x^11 + 4/5*a^7*b*x^10 + 28/9*a^6*b^2*x^9 + 7*a^5*b^3*x^8 + 10*a^4
*b^4*x^7 + 28/3*a^3*b^5*x^6 + 28/5*a^2*b^6*x^5 + 2*a*b^7*x^4 + 1/3*b^8*x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx = \frac{1}{11}\,a^8x^{11} + \frac{4}{5}\,a^7bx^{10} + \frac{28}{9}\,a^6b^2x^9 + 7\,a^5b^3x^8 + 10\,a^4b^4x^7 + \frac{28}{3}\,a^3b^5x^6 + \frac{28}{5}\,a^2b^6x^5 + 2\,ab^7x^4 + \frac{1}{3}\,b^8x^3$$

input
```
integrate((a+b/x)^8*x^10,x, algorithm="giac")
```

output
```
1/11*a^8*x^11 + 4/5*a^7*b*x^10 + 28/9*a^6*b^2*x^9 + 7*a^5*b^3*x^8 + 10*a^4
*b^4*x^7 + 28/3*a^3*b^5*x^6 + 28/5*a^2*b^6*x^5 + 2*a*b^7*x^4 + 1/3*b^8*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx = \frac{a^8\,x^{11}}{11}+\frac{4\,a^7\,b\,x^{10}}{5}+\frac{28\,a^6\,b^2\,x^9}{9}+7\,a^5\,b^3\,x^8+10\,a^4\,b^4\,x^7+\frac{28\,a^3\,b^5\,x^6}{3}+\frac{28\,a^2\,b^6\,x^5}{5}+2\,a\,b^7\,x^4+\frac{b^8\,x^3}{3}$$

input
```
int(x^10*(a + b/x)^8,x)
```

output
```
(a^8*x^11)/11 + (b^8*x^3)/3 + 2*a*b^7*x^4 + (4*a^7*b*x^10)/5 + (28*a^2*b^6
*x^5)/5 + (28*a^3*b^5*x^6)/3 + 10*a^4*b^4*x^7 + 7*a^5*b^3*x^8 + (28*a^6*b^
2*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \left(a+\frac{b}{x}\right)^8 x^{10}\,dx = \frac{x^3(45a^8x^8+396a^7b\,x^7+1540a^6b^2x^6+3465a^5b^3x^5+4950a^4b^4x^4+4620a^3b^5x^3+2772a^2b^6x^2+990a\,b^7x}{495}$$

input
```
int((a+b/x)^8*x^10,x)
```

output
```
(x**3*(45*a**8*x**8 + 396*a**7*b*x**7 + 1540*a**6*b**2*x**6 + 3465*a**5*b*
*3*x**5 + 4950*a**4*b**4*x**4 + 4620*a**3*b**5*x**3 + 2772*a**2*b**6*x**2
+ 990*a*b**7*x + 165*b**8))/495
```

3.44 $\int \left(a+\frac{b}{x}\right)^8 x^9 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x}\right)^8 x^9 \, dx=-\frac{b(b+ax)^9}{9a^2}+\frac{(b+ax)^{10}}{10a^2}$$

output
```
-1/9*b*(a*x+b)^9/a^2+1/10*(a*x+b)^10/a^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(30) = 60.

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.47

$$\int \left(a+\frac{b}{x}\right)^8 x^9 \, dx=\frac{b^8x^2}{2}+\frac{8}{3}ab^7x^3+7a^2b^6x^4+\frac{56}{5}a^3b^5x^5+\frac{35}{3}a^4b^4x^6+8a^5b^3x^7+\frac{7}{2}a^6b^2x^8+\frac{8}{9}a^7bx^9+\frac{a^8x^{10}}{10}$$

input
```
Integrate[(a + b/x)^8*x^9,x]
```

output
```
(b^8*x^2)/2 + (8*a*b^7*x^3)/3 + 7*a^2*b^6*x^4 + (56*a^3*b^5*x^5)/5 + (35*a
^4*b^4*x^6)/3 + 8*a^5*b^3*x^7 + (7*a^6*b^2*x^8)/2 + (8*a^7*b*x^9)/9 + (a^8
*x^10)/10
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int x(ax+b)^8dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{(ax+b)^9}{a}-\frac{b(ax+b)^8}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ax+b)^{10}}{10a^2}-\frac{b(ax+b)^9}{9a^2}$$

input
```
Int[(a + b/x)^8*x^9,x]
```

output
```
-1/9*(b*(b + a*x)^9)/a^2 + (b + a*x)^10/(10*a^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.03

method	result
gosper	$\frac{x^{2}\left(9a^{8}x^{8}+80a^{7}b\,x^{7}+315a^{6}b^{2}x^{6}+720a^{5}b^{3}x^{5}+1050a^{4}x^{4}b^{4}+1008a^{3}b^{5}x^{3}+630a^{2}b^{6}x^{2}+240a\,b^{7}x+45b^{8}\right)}{90}$
default	$\frac{1}{10}a^{8}x^{10}+\frac{8}{9}a^{7}b\,x^{9}+\frac{7}{2}a^{6}b^{2}x^{8}+8a^{5}b^{3}x^{7}+\frac{35}{3}a^{4}b^{4}x^{6}+\frac{56}{5}a^{3}x^{5}b^{5}+7a^{2}b^{6}x^{4}+\frac{8}{3}a\,b^{7}x^{3}+\frac{1}{2}b^{8}x$
risch	$\frac{1}{10}a^{8}x^{10}+\frac{8}{9}a^{7}b\,x^{9}+\frac{7}{2}a^{6}b^{2}x^{8}+8a^{5}b^{3}x^{7}+\frac{35}{3}a^{4}b^{4}x^{6}+\frac{56}{5}a^{3}x^{5}b^{5}+7a^{2}b^{6}x^{4}+\frac{8}{3}a\,b^{7}x^{3}+\frac{1}{2}b^{8}x$
parallelrisch	$\frac{1}{10}a^{8}x^{10}+\frac{8}{9}a^{7}b\,x^{9}+\frac{7}{2}a^{6}b^{2}x^{8}+8a^{5}b^{3}x^{7}+\frac{35}{3}a^{4}b^{4}x^{6}+\frac{56}{5}a^{3}x^{5}b^{5}+7a^{2}b^{6}x^{4}+\frac{8}{3}a\,b^{7}x^{3}+\frac{1}{2}b^{8}x$
norman	$\frac{\frac{1}{10}a^{8}x^{17}+\frac{1}{2}b^{8}x^{9}+\frac{8}{3}a\,b^{7}x^{10}+7a^{2}b^{6}x^{11}+\frac{56}{5}a^{3}b^{5}x^{12}+\frac{35}{3}a^{4}b^{4}x^{13}+8a^{5}b^{3}x^{14}+\frac{7}{2}a^{6}b^{2}x^{15}+\frac{8}{9}a^{7}b\,x^{16}}{x^{7}}$
orering	$\frac{x^{10}\left(9a^{8}x^{8}+80a^{7}b\,x^{7}+315a^{6}b^{2}x^{6}+720a^{5}b^{3}x^{5}+1050a^{4}x^{4}b^{4}+1008a^{3}b^{5}x^{3}+630a^{2}b^{6}x^{2}+240a\,b^{7}x+45b^{8}\right)\left(a+\frac{b}{x}\right)^{8}}{90\left(ax+b\right)^{8}}$

input
```
int((a+b/x)^8*x^9,x,method=_RETURNVERBOSE)
```

output
```
1/90*x^2*(9*a^8*x^8+80*a^7*b*x^7+315*a^6*b^2*x^6+720*a^5*b^3*x^5+1050*a^4*
b^4*x^4+1008*a^3*b^5*x^3+630*a^2*b^6*x^2+240*a*b^7*x+45*b^8)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.00

$$\int \left(a+\frac{b}{x}\right)^8 x^9\,dx = \frac{1}{10}\,a^8x^{10}+\frac{8}{9}\,a^7bx^9+\frac{7}{2}\,a^6b^2x^8+8\,a^5b^3x^7+\frac{35}{3}\,a^4b^4x^6+\frac{56}{5}\,a^3b^5x^5+7\,a^2b^6x^4+\frac{8}{3}\,ab^7x^3+\frac{1}{2}\,b^8x^2$$

input
```
integrate((a+b/x)^8*x^9,x, algorithm="fricas")
```

output
```
1/10*a^8*x^10 + 8/9*a^7*b*x^9 + 7/2*a^6*b^2*x^8 + 8*a^5*b^3*x^7 + 35/3*a^4
*b^4*x^6 + 56/5*a^3*b^5*x^5 + 7*a^2*b^6*x^4 + 8/3*a*b^7*x^3 + 1/2*b^8*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.47

$$\int \left(a+\frac{b}{x}\right)^8 x^9\,dx = \frac{a^8x^{10}}{10}+\frac{8a^7bx^9}{9}+\frac{7a^6b^2x^8}{2}+8a^5b^3x^7+\frac{35a^4b^4x^6}{3}+\frac{56a^3b^5x^5}{5}+7a^2b^6x^4+\frac{8ab^7x^3}{3}+\frac{b^8x^2}{2}$$

input
```
integrate((a+b/x)**8*x**9,x)
```

output
```
a**8*x**10/10 + 8*a**7*b*x**9/9 + 7*a**6*b**2*x**8/2 + 8*a**5*b**3*x**7 +
35*a**4*b**4*x**6/3 + 56*a**3*b**5*x**5/5 + 7*a**2*b**6*x**4 + 8*a*b**7*x*
*3/3 + b**8*x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(26) = 52.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.00

$$\int \left(a+\frac{b}{x}\right)^8 x^9\,dx = \frac{1}{10}\,a^8x^{10} + \frac{8}{9}\,a^7bx^9 + \frac{7}{2}\,a^6b^2x^8 + 8\,a^5b^3x^7 + \frac{35}{3}\,a^4b^4x^6 + \frac{56}{5}\,a^3b^5x^5 + 7\,a^2b^6x^4 + \frac{8}{3}\,ab^7x^3 + \frac{1}{2}\,b^8x^2$$

input
```
integrate((a+b/x)^8*x^9,x, algorithm="maxima")
```

output
```
1/10*a^8*x^10 + 8/9*a^7*b*x^9 + 7/2*a^6*b^2*x^8 + 8*a^5*b^3*x^7 + 35/3*a^4
*b^4*x^6 + 56/5*a^3*b^5*x^5 + 7*a^2*b^6*x^4 + 8/3*a*b^7*x^3 + 1/2*b^8*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.00

$$\int \left(a+\frac{b}{x}\right)^8 x^9\,dx = \frac{1}{10}\,a^8x^{10} + \frac{8}{9}\,a^7bx^9 + \frac{7}{2}\,a^6b^2x^8 + 8\,a^5b^3x^7 + \frac{35}{3}\,a^4b^4x^6 + \frac{56}{5}\,a^3b^5x^5 + 7\,a^2b^6x^4 + \frac{8}{3}\,ab^7x^3 + \frac{1}{2}\,b^8x^2$$

input
```
integrate((a+b/x)^8*x^9,x, algorithm="giac")
```

output
```
1/10*a^8*x^10 + 8/9*a^7*b*x^9 + 7/2*a^6*b^2*x^8 + 8*a^5*b^3*x^7 + 35/3*a^4
*b^4*x^6 + 56/5*a^3*b^5*x^5 + 7*a^2*b^6*x^4 + 8/3*a*b^7*x^3 + 1/2*b^8*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.00

$$\int \left(a+\frac{b}{x}\right)^8 x^9\,dx = \frac{a^8\,x^{10}}{10}+\frac{8\,a^7\,b\,x^9}{9}+\frac{7\,a^6\,b^2\,x^8}{2}+8\,a^5\,b^3\,x^7+\frac{35\,a^4\,b^4\,x^6}{3}+\frac{56\,a^3\,b^5\,x^5}{5}+7\,a^2\,b^6\,x^4+\frac{8\,a\,b^7\,x^3}{3}+\frac{b^8\,x^2}{2}$$

input
```
int(x^9*(a + b/x)^8,x)
```

output
```
(a^8*x^10)/10 + (b^8*x^2)/2 + (8*a*b^7*x^3)/3 + (8*a^7*b*x^9)/9 + 7*a^2*b^
6*x^4 + (56*a^3*b^5*x^5)/5 + (35*a^4*b^4*x^6)/3 + 8*a^5*b^3*x^7 + (7*a^6*b
^2*x^8)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.00

$$\int \left(a+\frac{b}{x}\right)^8 x^9\,dx = \frac{x^2(9a^8x^8+80a^7b\,x^7+315a^6b^2x^6+720a^5b^3x^5+1050a^4b^4x^4+1008a^3b^5x^3+630a^2b^6x^2+240a\,b^7x+45}{90}$$

input
```
int((a+b/x)^8*x^9,x)
```

output
```
(x**2*(9*a**8*x**8 + 80*a**7*b*x**7 + 315*a**6*b**2*x**6 + 720*a**5*b**3*x
**5 + 1050*a**4*b**4*x**4 + 1008*a**3*b**5*x**3 + 630*a**2*b**6*x**2 + 240
*a*b**7*x + 45*b**8))/90
```

3.45 $\int \left(a+\frac{b}{x}\right)^8 x^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \left(a+\frac{b}{x}\right)^8 x^8 \, dx = \frac{(b+ax)^9}{9a}$$

output
```
1/9*(a*x+b)^9/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^8 \, dx = \frac{(b+ax)^9}{9a}$$

input
```
Integrate[(a + b/x)^8*x^8,x]
```

output
```
(b + a*x)^9/(9*a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {795, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int (ax+b)^8 dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(ax+b)^9}{9a}$$

input
```
Int[(a + b/x)^8*x^8,x]
```

output
```
(b + a*x)^9/(9*a)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
default	$\frac{(ax+b)^9}{9a}$
parallelrisch	$\frac{1}{9}a^8x^9+a^7b\,x^8+4a^6b^2x^7+\frac{28}{3}a^5b^3x^6+14a^4b^4x^5+14a^3b^5x^4+\frac{28}{3}a^2b^6x^3+4a\,b^7x^2+b^8x$
gosper	$\frac{x\left(a^8x^8+9a^7b\,x^7+36a^6b^2x^6+84a^5b^3x^5+126a^4x^4b^4+126a^3b^5x^3+84a^2b^6x^2+36a\,b^7x+9b^8\right)}{9}$
norman	$\frac{b^8x^8+a^7b\,x^{15}+\frac{1}{9}a^8x^{16}+4a\,b^7x^9+\frac{28}{3}a^2b^6x^{10}+14a^3b^5x^{11}+14a^4b^4x^{12}+\frac{28}{3}a^5b^3x^{13}+4a^6b^2x^{14}}{x^7}$
risch	$\frac{a^8x^9}{9}+a^7b\,x^8+4a^6b^2x^7+\frac{28a^5b^3x^6}{3}+14a^4b^4x^5+14a^3b^5x^4+\frac{28a^2b^6x^3}{3}+4a\,b^7x^2+b^8x+\frac{b^9}{9a}$
orering	$\frac{x^9\left(a^8x^8+9a^7b\,x^7+36a^6b^2x^6+84a^5b^3x^5+126a^4x^4b^4+126a^3b^5x^3+84a^2b^6x^2+36a\,b^7x+9b^8\right)\left(a+\frac{b}{x}\right)^8}{9(ax+b)^8}$

input
```
int((a+b/x)^8*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/9*(a*x+b)^9/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(12) = 24$.

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 6.14

$$\int\left(a+\frac{b}{x}\right)^8x^8\,dx=\frac{1}{9}\,a^8x^9+a^7bx^8+4\,a^6b^2x^7+\frac{28}{3}\,a^5b^3x^6+14\,a^4b^4x^5+14\,a^3b^5x^4+\frac{28}{3}\,a^2b^6x^3+4\,ab^7x^2+b^8x$$

input
```
integrate((a+b/x)^8*x^8,x, algorithm="fricas")
```

output
```
1/9*a^8*x^9 + a^7*b*x^8 + 4*a^6*b^2*x^7 + 28/3*a^5*b^3*x^6 + 14*a^4*b^4*x^
5 + 14*a^3*b^5*x^4 + 28/3*a^2*b^6*x^3 + 4*a*b^7*x^2 + b^8*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 6.71

$$\int \left(a+\frac{b}{x}\right)^8 x^8\,dx = \frac{a^8x^9}{9} + a^7bx^8 + 4a^6b^2x^7 + \frac{28a^5b^3x^6}{3} + 14a^4b^4x^5 + 14a^3b^5x^4 + \frac{28a^2b^6x^3}{3} + 4ab^7x^2 + b^8x$$

input
```
integrate((a+b/x)**8*x**8,x)
```

output
```
a**8*x**9/9 + a**7*b*x**8 + 4*a**6*b**2*x**7 + 28*a**5*b**3*x**6/3 + 14*a*
*4*b**4*x**5 + 14*a**3*b**5*x**4 + 28*a**2*b**6*x**3/3 + 4*a*b**7*x**2 + b
**8*x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 6.14

$$\int \left(a+\frac{b}{x}\right)^8 x^8\,dx = \frac{1}{9}\,a^8x^9 + a^7bx^8 + 4\,a^6b^2x^7 + \frac{28}{3}\,a^5b^3x^6 + 14\,a^4b^4x^5 + 14\,a^3b^5x^4 + \frac{28}{3}\,a^2b^6x^3 + 4\,ab^7x^2 + b^8x$$

input
```
integrate((a+b/x)^8*x^8,x, algorithm="maxima")
```

output
```
1/9*a^8*x^9 + a^7*b*x^8 + 4*a^6*b^2*x^7 + 28/3*a^5*b^3*x^6 + 14*a^4*b^4*x^
5 + 14*a^3*b^5*x^4 + 28/3*a^2*b^6*x^3 + 4*a*b^7*x^2 + b^8*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 6.14

$$\int \left(a+\frac{b}{x}\right)^8 x^8\,dx = \frac{1}{9}\,a^8x^9 + a^7bx^8 + 4\,a^6b^2x^7 + \frac{28}{3}\,a^5b^3x^6 + 14\,a^4b^4x^5 + 14\,a^3b^5x^4 + \frac{28}{3}\,a^2b^6x^3 + 4\,ab^7x^2 + b^8x$$

input
```
integrate((a+b/x)^8*x^8,x, algorithm="giac")
```

output
```
1/9*a^8*x^9 + a^7*b*x^8 + 4*a^6*b^2*x^7 + 28/3*a^5*b^3*x^6 + 14*a^4*b^4*x^
5 + 14*a^3*b^5*x^4 + 28/3*a^2*b^6*x^3 + 4*a*b^7*x^2 + b^8*x
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 6.14

$$\int \left(a+\frac{b}{x}\right)^8 x^8\,dx = \frac{a^8\,x^9}{9} + a^7\,b\,x^8 + 4\,a^6\,b^2\,x^7 + \frac{28\,a^5\,b^3\,x^6}{3} + 14\,a^4\,b^4\,x^5 + 14\,a^3\,b^5\,x^4 + \frac{28\,a^2\,b^6\,x^3}{3} + 4\,a\,b^7\,x^2 + b^8\,x$$

input
```
int(x^8*(a + b/x)^8,x)
```

output
```
b^8*x + (a^8*x^9)/9 + 4*a*b^7*x^2 + a^7*b*x^8 + (28*a^2*b^6*x^3)/3 + 14*a^
3*b^5*x^4 + 14*a^4*b^4*x^5 + (28*a^5*b^3*x^6)/3 + 4*a^6*b^2*x^7
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.21

$$\int \left(a+\frac{b}{x}\right)^8 x^8\,dx = \frac{x(a^8x^8+9a^7b\,x^7+36a^6b^2x^6+84a^5b^3x^5+126a^4b^4x^4+126a^3b^5x^3+84a^2b^6x^2+36a\,b^7x+9b^8)}{9}$$

input
```
int((a+b/x)^8*x^8,x)
```

output
```
(x*(a**8*x**8 + 9*a**7*b*x**7 + 36*a**6*b**2*x**6 + 84*a**5*b**3*x**5 + 12
6*a**4*b**4*x**4 + 126*a**3*b**5*x**3 + 84*a**2*b**6*x**2 + 36*a*b**7*x +
9*b**8))/9
```

3.46 $\int \left(a+\frac{b}{x}\right)^8 x^7 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \left(a+\frac{b}{x}\right)^8 x^7 \, dx = 8ab^7x+14a^2b^6x^2+\frac{56}{3}a^3b^5x^3+\frac{35}{2}a^4b^4x^4+\frac{56}{5}a^5b^3x^5+\frac{14}{3}a^6b^2x^6+\frac{8}{7}a^7bx^7+\frac{a^8x^8}{8}+b^8\log(x)$$

output
```
8*a*b^7*x+14*a^2*b^6*x^2+56/3*a^3*b^5*x^3+35/2*a^4*b^4*x^4+56/5*a^5*b^3*x^
5+14/3*a^6*b^2*x^6+8/7*a^7*b*x^7+1/8*a^8*x^8+b^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^7 \, dx = 8ab^7x+14a^2b^6x^2+\frac{56}{3}a^3b^5x^3+\frac{35}{2}a^4b^4x^4+\frac{56}{5}a^5b^3x^5+\frac{14}{3}a^6b^2x^6+\frac{8}{7}a^7bx^7+\frac{a^8x^8}{8}+b^8\log(x)$$

input
```
Integrate[(a + b/x)^8*x^7,x]
```

output
```
8*a*b^7*x + 14*a^2*b^6*x^2 + (56*a^3*b^5*x^3)/3 + (35*a^4*b^4*x^4)/2 + (56
*a^5*b^3*x^5)/5 + (14*a^6*b^2*x^6)/3 + (8*a^7*b*x^7)/7 + (a^8*x^8)/8 + b^8
*Log[x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^8x^7+8a^7bx^6+28a^6b^2x^5+56a^5b^3x^4+70a^4b^4x^3+56a^3b^5x^2+28a^2b^6x+8ab^7+\frac{b^8}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^8}{8}+\frac{8}{7}a^7bx^7+\frac{14}{3}a^6b^2x^6+\frac{56}{5}a^5b^3x^5+\frac{35}{2}a^4b^4x^4+\frac{56}{3}a^3b^5x^3+14a^2b^6x^2+8ab^7x+b^8\log(x)$$

input
```
Int[(a + b/x)^8*x^7,x]
```

output

```
8*a*b^7*x + 14*a^2*b^6*x^2 + (56*a^3*b^5*x^3)/3 + (35*a^4*b^4*x^4)/2 + (56
*a^5*b^3*x^5)/5 + (14*a^6*b^2*x^6)/3 + (8*a^7*b*x^7)/7 + (a^8*x^8)/8 + b^8
*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

method	result	siz
default	$8a\,b^7x+14a^2b^6x^2+\frac{56a^3b^5x^3}{3}+\frac{35a^4x^4b^4}{2}+\frac{56a^5b^3x^5}{5}+\frac{14a^6b^2x^6}{3}+\frac{8a^7b\,x^7}{7}+\frac{a^8x^8}{8}+b^8\ln(x)$	87
risch	$8a\,b^7x+14a^2b^6x^2+\frac{56a^3b^5x^3}{3}+\frac{35a^4x^4b^4}{2}+\frac{56a^5b^3x^5}{5}+\frac{14a^6b^2x^6}{3}+\frac{8a^7b\,x^7}{7}+\frac{a^8x^8}{8}+b^8\ln(x)$	87
parallelrisch	$8a\,b^7x+14a^2b^6x^2+\frac{56a^3b^5x^3}{3}+\frac{35a^4x^4b^4}{2}+\frac{56a^5b^3x^5}{5}+\frac{14a^6b^2x^6}{3}+\frac{8a^7b\,x^7}{7}+\frac{a^8x^8}{8}+b^8\ln(x)$	87
norman	$\frac{\frac{1}{8}a^8x^{15}+8a\,b^7x^8+14a^2b^6x^9+\frac{56}{3}a^3b^5x^{10}+\frac{35}{2}a^4b^4x^{11}+\frac{56}{5}a^5b^3x^{12}+\frac{14}{3}a^6b^2x^{13}+\frac{8}{7}a^7b\,x^{14}}{x^7}+b^8\ln(x)$	94

input
```
int((a+b/x)^8*x^7,x,method=_RETURNVERBOSE)
```

output
```
8*a*b^7*x+14*a^2*b^6*x^2+56/3*a^3*b^5*x^3+35/2*a^4*x^4*b^4+56/5*a^5*b^3*x^
5+14/3*a^6*b^2*x^6+8/7*a^7*b*x^7+1/8*a^8*x^8+b^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x}\right)^8 x^7\,dx = \frac{1}{8}\,a^8x^8 + \frac{8}{7}\,a^7bx^7 + \frac{14}{3}\,a^6b^2x^6 + \frac{56}{5}\,a^5b^3x^5 + \frac{35}{2}\,a^4b^4x^4 + \frac{56}{3}\,a^3b^5x^3 + 14\,a^2b^6x^2 + 8\,ab^7x + b^8\log\left(x\right)$$

input
```
integrate((a+b/x)^8*x^7,x, algorithm="fricas")
```

output
```
1/8*a^8*x^8 + 8/7*a^7*b*x^7 + 14/3*a^6*b^2*x^6 + 56/5*a^5*b^3*x^5 + 35/2*a
^4*b^4*x^4 + 56/3*a^3*b^5*x^3 + 14*a^2*b^6*x^2 + 8*a*b^7*x + b^8*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x}\right)^8 x^7\,dx = \frac{a^8x^8}{8} + \frac{8a^7bx^7}{7} + \frac{14a^6b^2x^6}{3} + \frac{56a^5b^3x^5}{5} + \frac{35a^4b^4x^4}{2} + \frac{56a^3b^5x^3}{3} + 14a^2b^6x^2 + 8ab^7x + b^8\log\left(x\right)$$

input
```
integrate((a+b/x)**8*x**7,x)
```

output
```
a**8*x**8/8 + 8*a**7*b*x**7/7 + 14*a**6*b**2*x**6/3 + 56*a**5*b**3*x**5/5
+ 35*a**4*b**4*x**4/2 + 56*a**3*b**5*x**3/3 + 14*a**2*b**6*x**2 + 8*a*b**7
*x + b**8*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x}\right)^8 x^7\,dx = \frac{1}{8}\,a^8x^8 + \frac{8}{7}\,a^7bx^7 + \frac{14}{3}\,a^6b^2x^6 + \frac{56}{5}\,a^5b^3x^5 + \frac{35}{2}\,a^4b^4x^4 + \frac{56}{3}\,a^3b^5x^3 + 14\,a^2b^6x^2 + 8\,ab^7x + b^8\log\left(x\right)$$

input
```
integrate((a+b/x)^8*x^7,x, algorithm="maxima")
```

output
```
1/8*a^8*x^8 + 8/7*a^7*b*x^7 + 14/3*a^6*b^2*x^6 + 56/5*a^5*b^3*x^5 + 35/2*a
^4*b^4*x^4 + 56/3*a^3*b^5*x^3 + 14*a^2*b^6*x^2 + 8*a*b^7*x + b^8*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x}\right)^8 x^7\,dx = \frac{1}{8}\,a^8x^8 + \frac{8}{7}\,a^7bx^7 + \frac{14}{3}\,a^6b^2x^6 + \frac{56}{5}\,a^5b^3x^5 + \frac{35}{2}\,a^4b^4x^4 + \frac{56}{3}\,a^3b^5x^3 + 14\,a^2b^6x^2 + 8\,ab^7x + b^8\log\left(|x|\right)$$

input
```
integrate((a+b/x)^8*x^7,x, algorithm="giac")
```

output
```
1/8*a^8*x^8 + 8/7*a^7*b*x^7 + 14/3*a^6*b^2*x^6 + 56/5*a^5*b^3*x^5 + 35/2*a
^4*b^4*x^4 + 56/3*a^3*b^5*x^3 + 14*a^2*b^6*x^2 + 8*a*b^7*x + b^8*log(abs(x
))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x}\right)^8 x^7\,dx = b^8\,\ln(x)+\frac{a^8\,x^8}{8}+\frac{8\,a^7\,b\,x^7}{7}+14\,a^2\,b^6\,x^2+\frac{56\,a^3\,b^5\,x^3}{3}+\frac{35\,a^4\,b^4\,x^4}{2}+\frac{56\,a^5\,b^3\,x^5}{5}+\frac{14\,a^6\,b^2\,x^6}{3}+8\,a\,b^7\,x$$

input
```
int(x^7*(a + b/x)^8,x)
```

output
```
b^8*log(x) + (a^8*x^8)/8 + (8*a^7*b*x^7)/7 + 14*a^2*b^6*x^2 + (56*a^3*b^5*
x^3)/3 + (35*a^4*b^4*x^4)/2 + (56*a^5*b^3*x^5)/5 + (14*a^6*b^2*x^6)/3 + 8*
a*b^7*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x}\right)^8 x^7\,dx = \log(x)\,b^8+\frac{a^8x^8}{8}+\frac{8a^7b\,x^7}{7}+\frac{14a^6b^2x^6}{3}+\frac{56a^5b^3x^5}{5}+\frac{35a^4b^4x^4}{2}+\frac{56a^3b^5x^3}{3}+14a^2b^6x^2+8a\,b^7x$$

input
```
int((a+b/x)^8*x^7,x)
```

output
```
(840*log(x)*b**8 + 105*a**8*x**8 + 960*a**7*b*x**7 + 3920*a**6*b**2*x**6 +
 9408*a**5*b**3*x**5 + 14700*a**4*b**4*x**4 + 15680*a**3*b**5*x**3 + 11760
*a**2*b**6*x**2 + 6720*a*b**7*x)/840
```

3.47 $\int \left(a+\frac{b}{x}\right)^8 x^6 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \left(a+\frac{b}{x}\right)^8 x^6 \, dx = -\frac{b^8}{x}+28a^2b^6x+28a^3b^5x^2+\frac{70}{3}a^4b^4x^3+14a^5b^3x^4+\frac{28}{5}a^6b^2x^5+\frac{4}{3}a^7bx^6+\frac{a^8x^7}{7}+8ab^7\log(x)$$

output
```
-b^8/x+28*a^2*b^6*x+28*a^3*b^5*x^2+70/3*a^4*b^4*x^3+14*a^5*b^3*x^4+28/5*a^
6*b^2*x^5+4/3*a^7*b*x^6+1/7*a^8*x^7+8*a*b^7*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^6 \, dx = -\frac{b^8}{x}+28a^2b^6x+28a^3b^5x^2+\frac{70}{3}a^4b^4x^3+14a^5b^3x^4+\frac{28}{5}a^6b^2x^5+\frac{4}{3}a^7bx^6+\frac{a^8x^7}{7}+8ab^7\log(x)$$

input
```
Integrate[(a + b/x)^8*x^6,x]
```

output
```
-(b^8/x) + 28*a^2*b^6*x + 28*a^3*b^5*x^2 + (70*a^4*b^4*x^3)/3 + 14*a^5*b^3
*x^4 + (28*a^6*b^2*x^5)/5 + (4*a^7*b*x^6)/3 + (a^8*x^7)/7 + 8*a*b^7*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{(ax+b)^8}{x^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^8x^6+8a^7bx^5+28a^6b^2x^4+56a^5b^3x^3+70a^4b^4x^2+56a^3b^5x+28a^2b^6+\frac{8ab^7}{x}+\frac{b^8}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^8x^7}{7}+\frac{4}{3}a^7bx^6+\frac{28}{5}a^6b^2x^5+14a^5b^3x^4+\frac{70}{3}a^4b^4x^3+28a^3b^5x^2+28a^2b^6x+8ab^7\log(x)-\frac{b^8}{x}$$

input
```
Int[(a + b/x)^8*x^6,x]
```

output

```
-(b^8/x) + 28*a^2*b^6*x + 28*a^3*b^5*x^2 + (70*a^4*b^4*x^3)/3 + 14*a^5*b^3
*x^4 + (28*a^6*b^2*x^5)/5 + (4*a^7*b*x^6)/3 + (a^8*x^7)/7 + 8*a*b^7*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result
default	$-\frac{b^8}{x}+28a^2b^6x+28a^3b^5x^2+\frac{70a^4b^4x^3}{3}+14a^5b^3x^4+\frac{28a^6b^2x^5}{5}+\frac{4a^7b\,x^6}{3}+\frac{a^8x^7}{7}+8a\,b^7\ln\left(x\right)$
risch	$-\frac{b^8}{x}+28a^2b^6x+28a^3b^5x^2+\frac{70a^4b^4x^3}{3}+14a^5b^3x^4+\frac{28a^6b^2x^5}{5}+\frac{4a^7b\,x^6}{3}+\frac{a^8x^7}{7}+8a\,b^7\ln\left(x\right)$
parallelrisch	$\frac{15a^8x^8+140a^7b\,x^7+588a^6b^2x^6+1470a^5b^3x^5+2450a^4x^4b^4+2940a^3b^5x^3+840a\,b^7\ln\left(x\right)x+2940a^2b^6x^2-105b^8}{105x}$
norman	$\frac{\frac{1}{7}a^8x^{14}-b^8x^6+28a^2b^6x^8+28a^3b^5x^9+\frac{70}{3}a^4b^4x^{10}+14a^5b^3x^{11}+\frac{28}{5}a^6b^2x^{12}+\frac{4}{3}a^7b\,x^{13}}{x^7}+8a\,b^7\ln\left(x\right)$

input
```
int((a+b/x)^8*x^6,x,method=_RETURNVERBOSE)
```

output
```
-b^8/x+28*a^2*b^6*x+28*a^3*b^5*x^2+70/3*a^4*b^4*x^3+14*a^5*b^3*x^4+28/5*a^
6*b^2*x^5+4/3*a^7*b*x^6+1/7*a^8*x^7+8*a*b^7*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x^6\,dx = \frac{15\,a^8x^8+140\,a^7bx^7+588\,a^6b^2x^6+1470\,a^5b^3x^5+2450\,a^4b^4x^4+2940\,a^3b^5x^3+2940\,a^2b^6x^2+840\,ab^7x\,\mathrm{l}}{105\,x}$$

input
```
integrate((a+b/x)^8*x^6,x, algorithm="fricas")
```

output
```
1/105*(15*a^8*x^8 + 140*a^7*b*x^7 + 588*a^6*b^2*x^6 + 1470*a^5*b^3*x^5 + 2
450*a^4*b^4*x^4 + 2940*a^3*b^5*x^3 + 2940*a^2*b^6*x^2 + 840*a*b^7*x*log(x)
 - 105*b^8)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^6\,dx = \frac{a^8x^7}{7}+\frac{4a^7bx^6}{3}+\frac{28a^6b^2x^5}{5}+14a^5b^3x^4+\frac{70a^4b^4x^3}{3} + 28a^3b^5x^2+28a^2b^6x+8ab^7\log(x)-\frac{b^8}{x}$$

input
```
integrate((a+b/x)**8*x**6,x)
```

output
```
a**8*x**7/7 + 4*a**7*b*x**6/3 + 28*a**6*b**2*x**5/5 + 14*a**5*b**3*x**4 +
70*a**4*b**4*x**3/3 + 28*a**3*b**5*x**2 + 28*a**2*b**6*x + 8*a*b**7*log(x)
 - b**8/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^6\,dx = \frac{1}{7}\,a^8x^7 + \frac{4}{3}\,a^7bx^6 + \frac{28}{5}\,a^6b^2x^5 + 14\,a^5b^3x^4 + \frac{70}{3}\,a^4b^4x^3 + 28\,a^3b^5x^2 + 28\,a^2b^6x + 8\,ab^7\log\left(x\right) - \frac{b^8}{x}$$

input
```
integrate((a+b/x)^8*x^6,x, algorithm="maxima")
```

output
```
1/7*a^8*x^7 + 4/3*a^7*b*x^6 + 28/5*a^6*b^2*x^5 + 14*a^5*b^3*x^4 + 70/3*a^4
*b^4*x^3 + 28*a^3*b^5*x^2 + 28*a^2*b^6*x + 8*a*b^7*log(x) - b^8/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{x}\right)^8 x^6\,dx = \frac{1}{7}\,a^8x^7 + \frac{4}{3}\,a^7bx^6 + \frac{28}{5}\,a^6b^2x^5 + 14\,a^5b^3x^4 + \frac{70}{3}\,a^4b^4x^3 + 28\,a^3b^5x^2 + 28\,a^2b^6x + 8\,ab^7\log\left(|x|\right) - \frac{b^8}{x}$$

input
```
integrate((a+b/x)^8*x^6,x, algorithm="giac")
```

output
```
1/7*a^8*x^7 + 4/3*a^7*b*x^6 + 28/5*a^6*b^2*x^5 + 14*a^5*b^3*x^4 + 70/3*a^4
*b^4*x^3 + 28*a^3*b^5*x^2 + 28*a^2*b^6*x + 8*a*b^7*log(abs(x)) - b^8/x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^6\,dx=\frac{a^8\,x^7}{7}-\frac{b^8}{x}+28\,a^2\,b^6\,x+\frac{4\,a^7\,b\,x^6}{3}+8\,a\,b^7\,\ln\left(x\right)+28\,a^3\,b^5\,x^2+\frac{70\,a^4\,b^4\,x^3}{3}+14\,a^5\,b^3\,x^4+\frac{28\,a^6\,b^2\,x^5}{5}$$

input
```
int(x^6*(a + b/x)^8,x)
```

output
```
(a^8*x^7)/7 - b^8/x + 28*a^2*b^6*x + (4*a^7*b*x^6)/3 + 8*a*b^7*log(x) + 28
*a^3*b^5*x^2 + (70*a^4*b^4*x^3)/3 + 14*a^5*b^3*x^4 + (28*a^6*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x^6\,dx$$
$$=\frac{840\log(x)\,a\,b^7x+15a^8x^8+140a^7b\,x^7+588a^6b^2x^6+1470a^5b^3x^5+2450a^4b^4x^4+2940a^3b^5x^3+2940a^2b}{105x}$$

input
```
int((a+b/x)^8*x^6,x)
```

output
```
(840*log(x)*a*b**7*x + 15*a**8*x**8 + 140*a**7*b*x**7 + 588*a**6*b**2*x**6
 + 1470*a**5*b**3*x**5 + 2450*a**4*b**4*x**4 + 2940*a**3*b**5*x**3 + 2940*
a**2*b**6*x**2 - 105*b**8)/(105*x)
```

3.48 $\int \left(a+\frac{b}{x}\right)^8 x^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \left(a+\frac{b}{x}\right)^8 x^5 \, dx = -\frac{b^8}{2x^2} - \frac{8ab^7}{x} + 56a^3b^5x + 35a^4b^4x^2 + \frac{56}{3}a^5b^3x^3 + 7a^6b^2x^4 + \frac{8}{5}a^7bx^5 + \frac{a^8x^6}{6} + 28a^2b^6\log(x)$$

output
```
-1/2*b^8/x^2-8*a*b^7/x+56*a^3*b^5*x+35*a^4*b^4*x^2+56/3*a^5*b^3*x^3+7*a^6*
b^2*x^4+8/5*a^7*b*x^5+1/6*a^8*x^6+28*a^2*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^5 \, dx = -\frac{b^8}{2x^2} - \frac{8ab^7}{x} + 56a^3b^5x + 35a^4b^4x^2 + \frac{56}{3}a^5b^3x^3 + 7a^6b^2x^4 + \frac{8}{5}a^7bx^5 + \frac{a^8x^6}{6} + 28a^2b^6\log(x)$$

input
```
Integrate[(a + b/x)^8*x^5,x]
```

output
```
-1/2*b^8/x^2 - (8*a*b^7)/x + 56*a^3*b^5*x + 35*a^4*b^4*x^2 + (56*a^5*b^3*x
^3)/3 + 7*a^6*b^2*x^4 + (8*a^7*b*x^5)/5 + (a^8*x^6)/6 + 28*a^2*b^6*Log[x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^8x^5+8a^7bx^4+28a^6b^2x^3+56a^5b^3x^2+70a^4b^4x+56a^3b^5+\frac{28a^2b^6}{x}+\frac{8ab^7}{x^2}+\frac{b^8}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^6}{6}+\frac{8}{5}a^7bx^5+7a^6b^2x^4+\frac{56}{3}a^5b^3x^3+35a^4b^4x^2+56a^3b^5x+28a^2b^6\log(x)-\frac{8ab^7}{x}-\frac{b^8}{2x^2}$$

input
```
Int[(a + b/x)^8*x^5,x]
```

output
```
-1/2*b^8/x^2 - (8*a*b^7)/x + 56*a^3*b^5*x + 35*a^4*b^4*x^2 + (56*a^5*b^3*x
^3)/3 + 7*a^6*b^2*x^4 + (8*a^7*b*x^5)/5 + (a^8*x^6)/6 + 28*a^2*b^6*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result	s
default	$-\frac{b^8}{2x^2}-\frac{8ab^7}{x}+56a^3b^5x+35a^4b^4x^2+\frac{56a^5b^3x^3}{3}+7a^6b^2x^4+\frac{8a^7bx^5}{5}+\frac{a^8x^6}{6}+28a^2b^6\ln(x)$	8
risch	$\frac{a^8x^6}{6}+\frac{8a^7bx^5}{5}+7a^6b^2x^4+\frac{56a^5b^3x^3}{3}+35a^4b^4x^2+56a^3b^5x+\frac{-8ab^7x-\frac{1}{2}b^8}{x^2}+28a^2b^6\ln(x)$	8
parallelrisch	$\frac{5a^8x^8+48a^7bx^7+210a^6b^2x^6+560a^5b^3x^5+1050a^4x^4b^4+840a^2b^6\ln(x)x^2+1680a^3b^5x^3-240ab^7x-15b^8}{30x^2}$	9
norman	$\frac{\frac{1}{6}a^8x^{13}-\frac{1}{2}b^8x^5-8ab^7x^6+56a^3b^5x^8+35a^4b^4x^9+\frac{56}{3}a^5b^3x^{10}+7a^6b^2x^{11}+\frac{8}{5}a^7bx^{12}}{x^7}+28a^2b^6\ln(x)$	9

input
```
int((a+b/x)^8*x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^8/x^2-8*a*b^7/x+56*a^3*b^5*x+35*a^4*b^4*x^2+56/3*a^5*b^3*x^3+7*a^6*
b^2*x^4+8/5*a^7*b*x^5+1/6*a^8*x^6+28*a^2*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x^5\,dx = \frac{5\,a^8x^8+48\,a^7bx^7+210\,a^6b^2x^6+560\,a^5b^3x^5+1050\,a^4b^4x^4+1680\,a^3b^5x^3+840\,a^2b^6x^2\log\left(x\right)-240\,ab^7}{30\,x^2}$$

input
```
integrate((a+b/x)^8*x^5,x, algorithm="fricas")
```

output
```
1/30*(5*a^8*x^8 + 48*a^7*b*x^7 + 210*a^6*b^2*x^6 + 560*a^5*b^3*x^5 + 1050*
a^4*b^4*x^4 + 1680*a^3*b^5*x^3 + 840*a^2*b^6*x^2*log(x) - 240*a*b^7*x - 15
*b^8)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x}\right)^8 x^5\,dx = \frac{a^8x^6}{6}+\frac{8a^7bx^5}{5}+7a^6b^2x^4+\frac{56a^5b^3x^3}{3}+35a^4b^4x^2 + 56a^3b^5x+28a^2b^6\log\left(x\right)+\frac{-16ab^7x-b^8}{2x^2}$$

input
```
integrate((a+b/x)**8*x**5,x)
```

output
```
a**8*x**6/6 + 8*a**7*b*x**5/5 + 7*a**6*b**2*x**4 + 56*a**5*b**3*x**3/3 + 3
5*a**4*b**4*x**2 + 56*a**3*b**5*x + 28*a**2*b**6*log(x) + (-16*a*b**7*x -
b**8)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^8 x^5\,dx = \frac{1}{6}\,a^8x^6 + \frac{8}{5}\,a^7bx^5 + 7\,a^6b^2x^4 + \frac{56}{3}\,a^5b^3x^3 + 35\,a^4b^4x^2 + 56\,a^3b^5x + 28\,a^2b^6\log\left(x\right) - \frac{16\,ab^7x + b^8}{2\,x^2}$$

input
```
integrate((a+b/x)^8*x^5,x, algorithm="maxima")
```

output
```
1/6*a^8*x^6 + 8/5*a^7*b*x^5 + 7*a^6*b^2*x^4 + 56/3*a^5*b^3*x^3 + 35*a^4*b^
4*x^2 + 56*a^3*b^5*x + 28*a^2*b^6*log(x) - 1/2*(16*a*b^7*x + b^8)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^5\,dx = \frac{1}{6}\,a^8x^6 + \frac{8}{5}\,a^7bx^5 + 7\,a^6b^2x^4 + \frac{56}{3}\,a^5b^3x^3 + 35\,a^4b^4x^2 + 56\,a^3b^5x + 28\,a^2b^6\log\left(|x|\right) - \frac{16\,ab^7x + b^8}{2\,x^2}$$

input
```
integrate((a+b/x)^8*x^5,x, algorithm="giac")
```

output
```
1/6*a^8*x^6 + 8/5*a^7*b*x^5 + 7*a^6*b^2*x^4 + 56/3*a^5*b^3*x^3 + 35*a^4*b^
4*x^2 + 56*a^3*b^5*x + 28*a^2*b^6*log(abs(x)) - 1/2*(16*a*b^7*x + b^8)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{x}\right)^8 x^5\,dx = \frac{a^8\,x^6}{6}-\frac{\frac{b^8}{2}+8\,a\,x\,b^7}{x^2}+56\,a^3\,b^5\,x+\frac{8\,a^7\,b\,x^5}{5}+35\,a^4\,b^4\,x^2+\frac{56\,a^5\,b^3\,x^3}{3}+7\,a^6\,b^2\,x^4+28\,a^2\,b^6\,\ln\left(x\right)$$

input
```
int(x^5*(a + b/x)^8,x)
```

output
```
(a^8*x^6)/6 - (b^8/2 + 8*a*b^7*x)/x^2 + 56*a^3*b^5*x + (8*a^7*b*x^5)/5 + 3
5*a^4*b^4*x^2 + (56*a^5*b^3*x^3)/3 + 7*a^6*b^2*x^4 + 28*a^2*b^6*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x^5\,dx = \frac{840\log(x)\,a^2b^6x^2+5a^8x^8+48a^7b\,x^7+210a^6b^2x^6+560a^5b^3x^5+1050a^4b^4x^4+1680a^3b^5x^3-240a\,b^7x-}{30x^2}$$

input
```
int((a+b/x)^8*x^5,x)
```

output
```
(840*log(x)*a**2*b**6*x**2 + 5*a**8*x**8 + 48*a**7*b*x**7 + 210*a**6*b**2*
x**6 + 560*a**5*b**3*x**5 + 1050*a**4*b**4*x**4 + 1680*a**3*b**5*x**3 - 24
0*a*b**7*x - 15*b**8)/(30*x**2)
```

3.49 $\int \left(a+\frac{b}{x}\right)^8 x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \left(a+\frac{b}{x}\right)^8 x^4 \, dx = -\frac{b^8}{3x^3}-\frac{4ab^7}{x^2}-\frac{28a^2b^6}{x}+70a^4b^4x+28a^5b^3x^2+\frac{28}{3}a^6b^2x^3+2a^7bx^4+\frac{a^8x^5}{5}+56a^3b^5\log(x)$$

output
```
-1/3*b^8/x^3-4*a*b^7/x^2-28*a^2*b^6/x+70*a^4*b^4*x+28*a^5*b^3*x^2+28/3*a^6
*b^2*x^3+2*a^7*b*x^4+1/5*a^8*x^5+56*a^3*b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^4 \, dx = -\frac{b^8}{3x^3}-\frac{4ab^7}{x^2}-\frac{28a^2b^6}{x}+70a^4b^4x+28a^5b^3x^2+\frac{28}{3}a^6b^2x^3+2a^7bx^4+\frac{a^8x^5}{5}+56a^3b^5\log(x)$$

input
```
Integrate[(a + b/x)^8*x^4,x]
```

output
```
-1/3*b^8/x^3 - (4*a*b^7)/x^2 - (28*a^2*b^6)/x + 70*a^4*b^4*x + 28*a^5*b^3*
x^2 + (28*a^6*b^2*x^3)/3 + 2*a^7*b*x^4 + (a^8*x^5)/5 + 56*a^3*b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x}\right)^8 \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^4}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^8x^4+8a^7bx^3+28a^6b^2x^2+56a^5b^3x+70a^4b^4+\frac{56a^3b^5}{x}+\frac{28a^2b^6}{x^2}+\frac{8ab^7}{x^3}+\frac{b^8}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^5}{5}+2a^7bx^4+\frac{28}{3}a^6b^2x^3+28a^5b^3x^2+70a^4b^4x+56a^3b^5\log(x)-\frac{28a^2b^6}{x}-\frac{4ab^7}{x^2}-\frac{b^8}{3x^3}$$

input
```
Int[(a + b/x)^8*x^4,x]
```

output
```
-1/3*b^8/x^3 - (4*a*b^7)/x^2 - (28*a^2*b^6)/x + 70*a^4*b^4*x + 28*a^5*b^3*
x^2 + (28*a^6*b^2*x^3)/3 + 2*a^7*b*x^4 + (a^8*x^5)/5 + 56*a^3*b^5*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result	siz
default	$-\frac{b^8}{3x^3}-\frac{4a\,b^7}{x^2}-\frac{28a^2b^6}{x}+70a^4b^4x+28a^5b^3x^2+\frac{28a^6b^2x^3}{3}+2a^7b\,x^4+\frac{a^8x^5}{5}+56a^3b^5\ln(x)$	88
risch	$\frac{a^8x^5}{5}+2a^7b\,x^4+\frac{28a^6b^2x^3}{3}+28a^5b^3x^2+70a^4b^4x+\frac{-28a^2b^6x^2-4a\,b^7x-\frac{1}{3}b^8}{x^3}+56a^3b^5\ln(x)$	88
parallelrisch	$\frac{3a^8x^8+30a^7b\,x^7+140a^6b^2x^6+420a^5b^3x^5+840a^3b^5\ln(x)x^3+1050a^4x^4b^4-420a^2b^6x^2-60a\,b^7x-5b^8}{15x^3}$	93
norman	$\frac{\frac{1}{5}a^8x^{12}-\frac{1}{3}b^8x^4-4a\,b^7x^5-28a^2x^6b^6+70a^4b^4x^8+28a^5b^3x^9+\frac{28}{3}a^6b^2x^{10}+2a^7b\,x^{11}}{x^7}+56a^3b^5\ln(x)$	95

input
```
int((a+b/x)^8*x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^8/x^3-4*a*b^7/x^2-28*a^2*b^6/x+70*a^4*b^4*x+28*a^5*b^3*x^2+28/3*a^6
*b^2*x^3+2*a^7*b*x^4+1/5*a^8*x^5+56*a^3*b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x}\right)^8 x^4\,dx = \frac{3\,a^8x^8+30\,a^7bx^7+140\,a^6b^2x^6+420\,a^5b^3x^5+1050\,a^4b^4x^4+840\,a^3b^5x^3\log\left(x\right)-420\,a^2b^6x^2-60\,ab^7x-}{15\,x^3}$$

input
```
integrate((a+b/x)^8*x^4,x, algorithm="fricas")
```

output
```
1/15*(3*a^8*x^8 + 30*a^7*b*x^7 + 140*a^6*b^2*x^6 + 420*a^5*b^3*x^5 + 1050*
a^4*b^4*x^4 + 840*a^3*b^5*x^3*log(x) - 420*a^2*b^6*x^2 - 60*a*b^7*x - 5*b^
8)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x}\right)^8 x^4\,dx = \frac{a^8x^5}{5}+2a^7bx^4+\frac{28a^6b^2x^3}{3}+28a^5b^3x^2+70a^4b^4x + 56a^3b^5\log\left(x\right)+\frac{-84a^2b^6x^2-12ab^7x-b^8}{3x^3}$$

input
```
integrate((a+b/x)**8*x**4,x)
```

output
```
a**8*x**5/5 + 2*a**7*b*x**4 + 28*a**6*b**2*x**3/3 + 28*a**5*b**3*x**2 + 70
*a**4*b**4*x + 56*a**3*b**5*log(x) + (-84*a**2*b**6*x**2 - 12*a*b**7*x - b
**8)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^8 x^4\,dx = \frac{1}{5}\,a^8x^5 + 2\,a^7bx^4 + \frac{28}{3}\,a^6b^2x^3 + 28\,a^5b^3x^2 + 70\,a^4b^4x + 56\,a^3b^5\log\left(x\right) - \frac{84\,a^2b^6x^2 + 12\,ab^7x + b^8}{3\,x^3}$$

input
```
integrate((a+b/x)^8*x^4,x, algorithm="maxima")
```

output
```
1/5*a^8*x^5 + 2*a^7*b*x^4 + 28/3*a^6*b^2*x^3 + 28*a^5*b^3*x^2 + 70*a^4*b^4
*x + 56*a^3*b^5*log(x) - 1/3*(84*a^2*b^6*x^2 + 12*a*b^7*x + b^8)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^8 x^4\,dx = \frac{1}{5}\,a^8x^5 + 2\,a^7bx^4 + \frac{28}{3}\,a^6b^2x^3 + 28\,a^5b^3x^2 + 70\,a^4b^4x + 56\,a^3b^5\log\left(|x|\right) - \frac{84\,a^2b^6x^2 + 12\,ab^7x + b^8}{3\,x^3}$$

input
```
integrate((a+b/x)^8*x^4,x, algorithm="giac")
```

output
```
1/5*a^8*x^5 + 2*a^7*b*x^4 + 28/3*a^6*b^2*x^3 + 28*a^5*b^3*x^2 + 70*a^4*b^4
*x + 56*a^3*b^5*log(abs(x)) - 1/3*(84*a^2*b^6*x^2 + 12*a*b^7*x + b^8)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^8 x^4\,dx=\frac{a^8\,x^5}{5}-\frac{28\,a^2\,b^6\,x^2+4\,a\,b^7\,x+\frac{b^8}{3}}{x^3}+70\,a^4\,b^4\,x+2\,a^7\,b\,x^4+28\,a^5\,b^3\,x^2+\frac{28\,a^6\,b^2\,x^3}{3}+56\,a^3\,b^5\,\ln\left(x\right)$$

input
```
int(x^4*(a + b/x)^8,x)
```

output
```
(a^8*x^5)/5 - (b^8/3 + 28*a^2*b^6*x^2 + 4*a*b^7*x)/x^3 + 70*a^4*b^4*x + 2*
a^7*b*x^4 + 28*a^5*b^3*x^2 + (28*a^6*b^2*x^3)/3 + 56*a^3*b^5*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x}\right)^8 x^4\,dx = \frac{840\log(x)\,a^3b^5x^3+3a^8x^8+30a^7b\,x^7+140a^6b^2x^6+420a^5b^3x^5+1050a^4b^4x^4-420a^2b^6x^2-60a\,b^7x-5}{15x^3}$$

input
```
int((a+b/x)^8*x^4,x)
```

output
```
(840*log(x)*a**3*b**5*x**3 + 3*a**8*x**8 + 30*a**7*b*x**7 + 140*a**6*b**2*
x**6 + 420*a**5*b**3*x**5 + 1050*a**4*b**4*x**4 - 420*a**2*b**6*x**2 - 60*
a*b**7*x - 5*b**8)/(15*x**3)
```

3.50 $\int \left(a+\frac{b}{x}\right)^8 x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \left(a+\frac{b}{x}\right)^8 x^3 \, dx = -\frac{b^8}{4x^4}-\frac{8ab^7}{3x^3}-\frac{14a^2b^6}{x^2}-\frac{56a^3b^5}{x}+56a^5b^3x+14a^6b^2x^2+\frac{8}{3}a^7bx^3+\frac{a^8x^4}{4}+70a^4b^4\log(x)$$

output
```
-1/4*b^8/x^4-8/3*a*b^7/x^3-14*a^2*b^6/x^2-56*a^3*b^5/x+56*a^5*b^3*x+14*a^6
*b^2*x^2+8/3*a^7*b*x^3+1/4*a^8*x^4+70*a^4*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^3 \, dx = -\frac{b^8}{4x^4}-\frac{8ab^7}{3x^3}-\frac{14a^2b^6}{x^2}-\frac{56a^3b^5}{x}+56a^5b^3x+14a^6b^2x^2+\frac{8}{3}a^7bx^3+\frac{a^8x^4}{4}+70a^4b^4\log(x)$$

input
```
Integrate[(a + b/x)^8*x^3,x]
```

output
```
-1/4*b^8/x^4 - (8*a*b^7)/(3*x^3) - (14*a^2*b^6)/x^2 - (56*a^3*b^5)/x + 56*
a^5*b^3*x + 14*a^6*b^2*x^2 + (8*a^7*b*x^3)/3 + (a^8*x^4)/4 + 70*a^4*b^4*Lo
g[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x}\right)^8 dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{(ax+b)^8}{x^5}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(a^8x^3+8a^7bx^2+28a^6b^2x+56a^5b^3+\frac{70a^4b^4}{x}+\frac{56a^3b^5}{x^2}+\frac{28a^2b^6}{x^3}+\frac{8ab^7}{x^4}+\frac{b^8}{x^5}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^8x^4}{4}+\frac{8}{3}a^7bx^3+14a^6b^2x^2+56a^5b^3x+70a^4b^4\log(x)-\frac{56a^3b^5}{x}-\frac{14a^2b^6}{x^2}-\frac{8ab^7}{3x^3}-\frac{b^8}{4x^4}$$

input
```
Int[(a + b/x)^8*x^3,x]
```

output

```
-1/4*b^8/x^4 - (8*a*b^7)/(3*x^3) - (14*a^2*b^6)/x^2 - (56*a^3*b^5)/x + 56*
a^5*b^3*x + 14*a^6*b^2*x^2 + (8*a^7*b*x^3)/3 + (a^8*x^4)/4 + 70*a^4*b^4*Lo
g[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{b^{8}}{4x^{4}}-\frac{8a\,b^{7}}{3x^{3}}-\frac{14a^{2}b^{6}}{x^{2}}-\frac{56a^{3}b^{5}}{x}+56a^{5}b^{3}x+14a^{6}x^{2}b^{2}+\frac{8a^{7}b\,x^{3}}{3}+\frac{a^{8}x^{4}}{4}+70a^{4}b^{4}\ln\left(x\right)$	88
risch	$\frac{a^{8}x^{4}}{4}+\frac{8a^{7}b\,x^{3}}{3}+14a^{6}x^{2}b^{2}+56a^{5}b^{3}x+\frac{-56a^{3}b^{5}x^{3}-14a^{2}b^{6}x^{2}-\frac{8}{3}a\,b^{7}x-\frac{1}{4}b^{8}}{x^{4}}+70a^{4}b^{4}\ln\left(x\right)$	88
parallelrisch	$\frac{3a^{8}x^{8}+32a^{7}b\,x^{7}+168a^{6}b^{2}x^{6}+840a^{4}b^{4}\ln\left(x\right)x^{4}+672a^{5}b^{3}x^{5}-672a^{3}b^{5}x^{3}-168a^{2}b^{6}x^{2}-32a\,b^{7}x-3b^{8}}{12x^{4}}$	93
norman	$\frac{\frac{1}{4}a^{8}x^{11}-\frac{1}{4}b^{8}x^{3}-\frac{8}{3}a\,b^{7}x^{4}-14a^{2}b^{6}x^{5}-56a^{3}b^{5}x^{6}+56a^{5}b^{3}x^{8}+14a^{6}b^{2}x^{9}+\frac{8}{3}a^{7}b\,x^{10}}{x^{7}}+70a^{4}b^{4}\ln\left(x\right)$	95

input
```
int((a+b/x)^8*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^8/x^4-8/3*a*b^7/x^3-14*a^2*b^6/x^2-56*a^3*b^5/x+56*a^5*b^3*x+14*a^6
*x^2*b^2+8/3*a^7*b*x^3+1/4*a^8*x^4+70*a^4*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x^3\,dx = \frac{3\,a^8x^8+32\,a^7bx^7+168\,a^6b^2x^6+672\,a^5b^3x^5+840\,a^4b^4x^4\log\left(x\right)-672\,a^3b^5x^3-168\,a^2b^6x^2-32\,ab^7x-}{12\,x^4}$$

input
```
integrate((a+b/x)^8*x^3,x, algorithm="fricas")
```

output
```
1/12*(3*a^8*x^8 + 32*a^7*b*x^7 + 168*a^6*b^2*x^6 + 672*a^5*b^3*x^5 + 840*a
^4*b^4*x^4*log(x) - 672*a^3*b^5*x^3 - 168*a^2*b^6*x^2 - 32*a*b^7*x - 3*b^8
)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x}\right)^8 x^3\,dx = \frac{a^8x^4}{4}+\frac{8a^7bx^3}{3}+14a^6b^2x^2+56a^5b^3x+70a^4b^4\log\left(x\right) + \frac{-672a^3b^5x^3-168a^2b^6x^2-32ab^7x-3b^8}{12x^4}$$

input
```
integrate((a+b/x)**8*x**3,x)
```

output
```
a**8*x**4/4 + 8*a**7*b*x**3/3 + 14*a**6*b**2*x**2 + 56*a**5*b**3*x + 70*a*
*4*b**4*log(x) + (-672*a**3*b**5*x**3 - 168*a**2*b**6*x**2 - 32*a*b**7*x -
 3*b**8)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{x}\right)^8 x^3\,dx = \frac{1}{4}\,a^8x^4 + \frac{8}{3}\,a^7bx^3 + 14\,a^6b^2x^2 + 56\,a^5b^3x + 70\,a^4b^4\log\left(x\right) - \frac{672\,a^3b^5x^3 + 168\,a^2b^6x^2 + 32\,ab^7x + 3\,b^8}{12\,x^4}$$

input
```
integrate((a+b/x)^8*x^3,x, algorithm="maxima")
```

output
```
1/4*a^8*x^4 + 8/3*a^7*b*x^3 + 14*a^6*b^2*x^2 + 56*a^5*b^3*x + 70*a^4*b^4*l
og(x) - 1/12*(672*a^3*b^5*x^3 + 168*a^2*b^6*x^2 + 32*a*b^7*x + 3*b^8)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^8 x^3\,dx = \frac{1}{4}\,a^8x^4 + \frac{8}{3}\,a^7bx^3 + 14\,a^6b^2x^2 + 56\,a^5b^3x + 70\,a^4b^4\log\left(|x|\right) - \frac{672\,a^3b^5x^3 + 168\,a^2b^6x^2 + 32\,ab^7x + 3\,b^8}{12\,x^4}$$

input
```
integrate((a+b/x)^8*x^3,x, algorithm="giac")
```

output
```
1/4*a^8*x^4 + 8/3*a^7*b*x^3 + 14*a^6*b^2*x^2 + 56*a^5*b^3*x + 70*a^4*b^4*l
og(abs(x)) - 1/12*(672*a^3*b^5*x^3 + 168*a^2*b^6*x^2 + 32*a*b^7*x + 3*b^8)
/x^4
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{x}\right)^8 x^3\,dx = \frac{a^8\,x^4}{4}-\frac{56\,a^3\,b^5\,x^3+14\,a^2\,b^6\,x^2+\frac{8\,a\,b^7\,x}{3}+\frac{b^8}{4}}{x^4} + 56\,a^5\,b^3\,x+\frac{8\,a^7\,b\,x^3}{3}+14\,a^6\,b^2\,x^2+70\,a^4\,b^4\,\ln\left(x\right)$$

input
```
int(x^3*(a + b/x)^8,x)
```

output
```
(a^8*x^4)/4 - (b^8/4 + 14*a^2*b^6*x^2 + 56*a^3*b^5*x^3 + (8*a*b^7*x)/3)/x^
4 + 56*a^5*b^3*x + (8*a^7*b*x^3)/3 + 14*a^6*b^2*x^2 + 70*a^4*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x^3\,dx = \frac{840\log(x)\,a^4b^4x^4+3a^8x^8+32a^7b\,x^7+168a^6b^2x^6+672a^5b^3x^5-672a^3b^5x^3-168a^2b^6x^2-32a\,b^7x-3b}{12x^4}$$

input
```
int((a+b/x)^8*x^3,x)
```

output
```
(840*log(x)*a**4*b**4*x**4 + 3*a**8*x**8 + 32*a**7*b*x**7 + 168*a**6*b**2*
x**6 + 672*a**5*b**3*x**5 - 672*a**3*b**5*x**3 - 168*a**2*b**6*x**2 - 32*a
*b**7*x - 3*b**8)/(12*x**4)
```

3.51 $\int \left(a+\frac{b}{x}\right)^8 x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \left(a+\frac{b}{x}\right)^8 x^2 \, dx = -\frac{b^8}{5x^5}-\frac{2ab^7}{x^4}-\frac{28a^2b^6}{3x^3}-\frac{28a^3b^5}{x^2}-\frac{70a^4b^4}{x} + 28a^6b^2x+4a^7bx^2+\frac{a^8x^3}{3}+56a^5b^3\log(x)$$

output
```
-1/5*b^8/x^5-2*a*b^7/x^4-28/3*a^2*b^6/x^3-28*a^3*b^5/x^2-70*a^4*b^4/x+28*a
^6*b^2*x+4*a^7*b*x^2+1/3*a^8*x^3+56*a^5*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x^2 \, dx = -\frac{b^8}{5x^5}-\frac{2ab^7}{x^4}-\frac{28a^2b^6}{3x^3}-\frac{28a^3b^5}{x^2}-\frac{70a^4b^4}{x} + 28a^6b^2x+4a^7bx^2+\frac{a^8x^3}{3}+56a^5b^3\log(x)$$

input
```
Integrate[(a + b/x)^8*x^2,x]
```

output
```
-1/5*b^8/x^5 - (2*a*b^7)/x^4 - (28*a^2*b^6)/(3*x^3) - (28*a^3*b^5)/x^2 - (
70*a^4*b^4)/x + 28*a^6*b^2*x + 4*a^7*b*x^2 + (a^8*x^3)/3 + 56*a^5*b^3*Log[
x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x}\right)^8 \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^6}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^8x^2+8a^7bx+28a^6b^2+\frac{56a^5b^3}{x}+\frac{70a^4b^4}{x^2}+\frac{56a^3b^5}{x^3}+\frac{28a^2b^6}{x^4}+\frac{8ab^7}{x^5}+\frac{b^8}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^3}{3}+4a^7bx^2+28a^6b^2x+56a^5b^3\log(x)-\frac{70a^4b^4}{x}-\frac{28a^3b^5}{x^2}-\frac{28a^2b^6}{3x^3}-\frac{2ab^7}{x^4}-\frac{b^8}{5x^5}$$

input
```
Int[(a + b/x)^8*x^2,x]
```

output
```
-1/5*b^8/x^5 - (2*a*b^7)/x^4 - (28*a^2*b^6)/(3*x^3) - (28*a^3*b^5)/x^2 - (
70*a^4*b^4)/x + 28*a^6*b^2*x + 4*a^7*b*x^2 + (a^8*x^3)/3 + 56*a^5*b^3*Log[
x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{b^{8}}{5x^{5}}-\frac{2a\,b^{7}}{x^{4}}-\frac{28a^{2}b^{6}}{3x^{3}}-\frac{28a^{3}b^{5}}{x^{2}}-\frac{70a^{4}b^{4}}{x}+28a^{6}b^{2}x+4a^{7}b\,x^{2}+\frac{a^{8}x^{3}}{3}+56a^{5}b^{3}\ln\left(x\right)$	88
risch	$\frac{a^{8}x^{3}}{3}+4a^{7}b\,x^{2}+28a^{6}b^{2}x+\frac{-70a^{4}x^{4}b^{4}-28a^{3}b^{5}x^{3}-\frac{28}{3}a^{2}b^{6}x^{2}-2a\,b^{7}x-\frac{1}{5}b^{8}}{x^{5}}+56a^{5}b^{3}\ln\left(x\right)$	88
parallelrisch	$\frac{5a^{8}x^{8}+60a^{7}b\,x^{7}+840a^{5}b^{3}\ln(x)x^{5}+420a^{6}b^{2}x^{6}-1050a^{4}x^{4}b^{4}-420a^{3}b^{5}x^{3}-140a^{2}b^{6}x^{2}-30a\,b^{7}x-3b^{8}}{15x^{5}}$	93
norman	$\frac{\frac{1}{3}a^{8}x^{10}-\frac{1}{5}b^{8}x^{2}-2a\,b^{7}x^{3}-\frac{28}{3}a^{2}b^{6}x^{4}-28a^{3}x^{5}b^{5}-70a^{4}b^{4}x^{6}+28a^{6}b^{2}x^{8}+4a^{7}b\,x^{9}}{x^{7}}+56a^{5}b^{3}\ln\left(x\right)$	95

input
```
int((a+b/x)^8*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^8/x^5-2*a*b^7/x^4-28/3*a^2*b^6/x^3-28*a^3*b^5/x^2-70*a^4*b^4/x+28*a
^6*b^2*x+4*a^7*b*x^2+1/3*a^8*x^3+56*a^5*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x}\right)^8 x^2\,dx = \frac{5\,a^8x^8 + 60\,a^7bx^7 + 420\,a^6b^2x^6 + 840\,a^5b^3x^5\log\left(x\right) - 1050\,a^4b^4x^4 - 420\,a^3b^5x^3 - 140\,a^2b^6x^2 - 30\,ab^7x -}{15\,x^5}$$

input
```
integrate((a+b/x)^8*x^2,x, algorithm="fricas")
```

output
```
1/15*(5*a^8*x^8 + 60*a^7*b*x^7 + 420*a^6*b^2*x^6 + 840*a^5*b^3*x^5*log(x)
- 1050*a^4*b^4*x^4 - 420*a^3*b^5*x^3 - 140*a^2*b^6*x^2 - 30*a*b^7*x - 3*b^
8)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x}\right)^8 x^2\,dx = \frac{a^8x^3}{3} + 4a^7bx^2 + 28a^6b^2x + 56a^5b^3\log\left(x\right) + \frac{-1050a^4b^4x^4 - 420a^3b^5x^3 - 140a^2b^6x^2 - 30ab^7x - 3b^8}{15x^5}$$

input
```
integrate((a+b/x)**8*x**2,x)
```

output
```
a**8*x**3/3 + 4*a**7*b*x**2 + 28*a**6*b**2*x + 56*a**5*b**3*log(x) + (-105
0*a**4*b**4*x**4 - 420*a**3*b**5*x**3 - 140*a**2*b**6*x**2 - 30*a*b**7*x -
 3*b**8)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^8 x^2\,dx = \frac{1}{3}\,a^8x^3+4\,a^7bx^2+28\,a^6b^2x+56\,a^5b^3\log\left(x\right) - \frac{1050\,a^4b^4x^4+420\,a^3b^5x^3+140\,a^2b^6x^2+30\,ab^7x+3\,b^8}{15\,x^5}$$

input
```
integrate((a+b/x)^8*x^2,x, algorithm="maxima")
```

output
```
1/3*a^8*x^3 + 4*a^7*b*x^2 + 28*a^6*b^2*x + 56*a^5*b^3*log(x) - 1/15*(1050*
a^4*b^4*x^4 + 420*a^3*b^5*x^3 + 140*a^2*b^6*x^2 + 30*a*b^7*x + 3*b^8)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x}\right)^8 x^2\,dx = \frac{1}{3}\,a^8x^3+4\,a^7bx^2+28\,a^6b^2x+56\,a^5b^3\log\left(|x|\right) - \frac{1050\,a^4b^4x^4+420\,a^3b^5x^3+140\,a^2b^6x^2+30\,ab^7x+3\,b^8}{15\,x^5}$$

input
```
integrate((a+b/x)^8*x^2,x, algorithm="giac")
```

output
```
1/3*a^8*x^3 + 4*a^7*b*x^2 + 28*a^6*b^2*x + 56*a^5*b^3*log(abs(x)) - 1/15*(
1050*a^4*b^4*x^4 + 420*a^3*b^5*x^3 + 140*a^2*b^6*x^2 + 30*a*b^7*x + 3*b^8)
/x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^8 x^2\,dx=\frac{a^8\,x^3}{3}-\frac{70\,a^4\,b^4\,x^4+28\,a^3\,b^5\,x^3+\frac{28\,a^2\,b^6\,x^2}{3}+2\,a\,b^7\,x+\frac{b^8}{5}}{x^5}+28\,a^6\,b^2\,x+4\,a^7\,b\,x^2+56\,a^5\,b^3\,\ln\left(x\right)$$

input
```
int(x^2*(a + b/x)^8,x)
```

output
```
(a^8*x^3)/3 - (b^8/5 + (28*a^2*b^6*x^2)/3 + 28*a^3*b^5*x^3 + 70*a^4*b^4*x^
4 + 2*a*b^7*x)/x^5 + 28*a^6*b^2*x + 4*a^7*b*x^2 + 56*a^5*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x}\right)^8 x^2\,dx=\frac{840\log(x)\,a^5b^3x^5+5a^8x^8+60a^7b\,x^7+420a^6b^2x^6-1050a^4b^4x^4-420a^3b^5x^3-140a^2b^6x^2-30a\,b^7x-3}{15x^5}$$

input
```
int((a+b/x)^8*x^2,x)
```

output
```
(840*log(x)*a**5*b**3*x**5 + 5*a**8*x**8 + 60*a**7*b*x**7 + 420*a**6*b**2*
x**6 - 1050*a**4*b**4*x**4 - 420*a**3*b**5*x**3 - 140*a**2*b**6*x**2 - 30*
a*b**7*x - 3*b**8)/(15*x**5)
```

3.52 $\int \left(a+\frac{b}{x}\right)^8 x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 95

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx=-\frac{b^8}{6x^6}-\frac{8ab^7}{5x^5}-\frac{7a^2b^6}{x^4}-\frac{56a^3b^5}{3x^3}-\frac{35a^4b^4}{x^2}-\frac{56a^5b^3}{x}+8a^7bx+\frac{a^8x^2}{2}+28a^6b^2\log(x)$$

output
```
-1/6*b^8/x^6-8/5*a*b^7/x^5-7*a^2*b^6/x^4-56/3*a^3*b^5/x^3-35*a^4*b^4/x^2-5
6*a^5*b^3/x+8*a^7*b*x+1/2*a^8*x^2+28*a^6*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx=-\frac{b^8}{6x^6}-\frac{8ab^7}{5x^5}-\frac{7a^2b^6}{x^4}-\frac{56a^3b^5}{3x^3}-\frac{35a^4b^4}{x^2}-\frac{56a^5b^3}{x}+8a^7bx+\frac{a^8x^2}{2}+28a^6b^2\log(x)$$

input
```
Integrate[(a + b/x)^8*x,x]
```

output
```
-1/6*b^8/x^6 - (8*a*b^7)/(5*x^5) - (7*a^2*b^6)/x^4 - (56*a^3*b^5)/(3*x^3)
- (35*a^4*b^4)/x^2 - (56*a^5*b^3)/x + 8*a^7*b*x + (a^8*x^2)/2 + 28*a^6*b^2
*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^7}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^8x+8a^7b+\frac{28a^6b^2}{x}+\frac{56a^5b^3}{x^2}+\frac{70a^4b^4}{x^3}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^5}+\frac{8ab^7}{x^6}+\frac{b^8}{x^7}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^2}{2}+8a^7bx+28a^6b^2\log(x)-\frac{56a^5b^3}{x}-\frac{35a^4b^4}{x^2}-\frac{56a^3b^5}{3x^3}-\frac{7a^2b^6}{x^4}-\frac{8ab^7}{5x^5}-\frac{b^8}{6x^6}$$

input
```
Int[(a + b/x)^8*x,x]
```

output
```
-1/6*b^8/x^6 - (8*a*b^7)/(5*x^5) - (7*a^2*b^6)/x^4 - (56*a^3*b^5)/(3*x^3)
- (35*a^4*b^4)/x^2 - (56*a^5*b^3)/x + 8*a^7*b*x + (a^8*x^2)/2 + 28*a^6*b^2
*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{b^8}{6x^6}-\frac{8a\,b^7}{5x^5}-\frac{7a^2b^6}{x^4}-\frac{56a^3b^5}{3x^3}-\frac{35a^4b^4}{x^2}-\frac{56a^5b^3}{x}+8a^7xb+\frac{a^8x^2}{2}+28a^6b^2\ln\left(x\right)$	88
risch	$\frac{a^8x^2}{2}+8a^7xb+\frac{-56a^5b^3x^5-35a^4x^4b^4-\frac{56}{3}a^3b^5x^3-7a^2b^6x^2-\frac{8}{5}a\,b^7x-\frac{1}{6}b^8}{x^6}+28a^6b^2\ln\left(x\right)$	88
norman	$\frac{\frac{1}{2}a^8x^9-\frac{1}{6}b^8x-\frac{8}{5}a\,b^7x^2-7a^2b^6x^3-\frac{56}{3}a^3b^5x^4-35a^4b^4x^5-56a^5b^3x^6+8a^7b\,x^8}{x^7}+28a^6b^2\ln\left(x\right)$	93
parallelrisch	$\frac{15a^8x^8+840a^6b^2\ln(x)x^6+240a^7b\,x^7-1680a^5b^3x^5-1050a^4x^4b^4-560a^3b^5x^3-210a^2b^6x^2-48a\,b^7x-5b^8}{30x^6}$	93

input
```
int((a+b/x)^8*x,x,method=_RETURNVERBOSE)
```

output
```
-1/6*b^8/x^6-8/5*a*b^7/x^5-7*a^2*b^6/x^4-56/3*a^3*b^5/x^3-35*a^4*b^4/x^2-5
6*a^5*b^3/x+8*a^7*x*b+1/2*a^8*x^2+28*a^6*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx = \frac{15\,a^8x^8+240\,a^7bx^7+840\,a^6b^2x^6\log\left(x\right)-1680\,a^5b^3x^5-1050\,a^4b^4x^4-560\,a^3b^5x^3-210\,a^2b^6x^2-48\,ab}{30\,x^6}$$

input
```
integrate((a+b/x)^8*x,x, algorithm="fricas")
```

output
```
1/30*(15*a^8*x^8 + 240*a^7*b*x^7 + 840*a^6*b^2*x^6*log(x) - 1680*a^5*b^3*x
^5 - 1050*a^4*b^4*x^4 - 560*a^3*b^5*x^3 - 210*a^2*b^6*x^2 - 48*a*b^7*x - 5
*b^8)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx = \frac{a^8x^2}{2}+8a^7bx+28a^6b^2\log\left(x\right) + \frac{-1680a^5b^3x^5-1050a^4b^4x^4-560a^3b^5x^3-210a^2b^6x^2-48ab^7x-5b^8}{30x^6}$$

input
```
integrate((a+b/x)**8*x,x)
```

output
```
a**8*x**2/2 + 8*a**7*b*x + 28*a**6*b**2*log(x) + (-1680*a**5*b**3*x**5 - 1
050*a**4*b**4*x**4 - 560*a**3*b**5*x**3 - 210*a**2*b**6*x**2 - 48*a*b**7*x
 - 5*b**8)/(30*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx = \frac{1}{2}\,a^8x^2+8\,a^7bx+28\,a^6b^2\log\left(x\right) - \frac{1680\,a^5b^3x^5+1050\,a^4b^4x^4+560\,a^3b^5x^3+210\,a^2b^6x^2+48\,ab^7x+5\,b^8}{30\,x^6}$$

input
```
integrate((a+b/x)^8*x,x, algorithm="maxima")
```

output
```
1/2*a^8*x^2 + 8*a^7*b*x + 28*a^6*b^2*log(x) - 1/30*(1680*a^5*b^3*x^5 + 105
0*a^4*b^4*x^4 + 560*a^3*b^5*x^3 + 210*a^2*b^6*x^2 + 48*a*b^7*x + 5*b^8)/x^
6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx = \frac{1}{2}\,a^8x^2+8\,a^7bx+28\,a^6b^2\log\left(|x|\right) - \frac{1680\,a^5b^3x^5+1050\,a^4b^4x^4+560\,a^3b^5x^3+210\,a^2b^6x^2+48\,ab^7x+5\,b^8}{30\,x^6}$$

input
```
integrate((a+b/x)^8*x,x, algorithm="giac")
```

output
```
1/2*a^8*x^2 + 8*a^7*b*x + 28*a^6*b^2*log(abs(x)) - 1/30*(1680*a^5*b^3*x^5
+ 1050*a^4*b^4*x^4 + 560*a^3*b^5*x^3 + 210*a^2*b^6*x^2 + 48*a*b^7*x + 5*b^
8)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx = \frac{a^8\,x^2}{2} - \frac{56\,a^5\,b^3\,x^5 + 35\,a^4\,b^4\,x^4 + \frac{56\,a^3\,b^5\,x^3}{3} + 7\,a^2\,b^6\,x^2 + \frac{8\,a\,b^7\,x}{5} + \frac{b^8}{6}}{x^6} + 28\,a^6\,b^2\,\ln(x) + 8\,a^7\,b\,x$$

input
```
int(x*(a + b/x)^8,x)
```

output
```
(a^8*x^2)/2 - (b^8/6 + 7*a^2*b^6*x^2 + (56*a^3*b^5*x^3)/3 + 35*a^4*b^4*x^4
 + 56*a^5*b^3*x^5 + (8*a*b^7*x)/5)/x^6 + 28*a^6*b^2*log(x) + 8*a^7*b*x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^8 x\,dx = \frac{840\log(x)\,a^6b^2x^6 + 15a^8x^8 + 240a^7b\,x^7 - 1680a^5b^3x^5 - 1050a^4b^4x^4 - 560a^3b^5x^3 - 210a^2b^6x^2 - 48a\,b^7x}{30x^6}$$

input
```
int((a+b/x)^8*x,x)
```

output
```
(840*log(x)*a**6*b**2*x**6 + 15*a**8*x**8 + 240*a**7*b*x**7 - 1680*a**5*b*
*3*x**5 - 1050*a**4*b**4*x**4 - 560*a**3*b**5*x**3 - 210*a**2*b**6*x**2 -
48*a*b**7*x - 5*b**8)/(30*x**6)
```

3.53 $\int \left(a+\frac{b}{x}\right)^8 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 94

$$\int \left(a+\frac{b}{x}\right)^8 \, dx = -\frac{b^8}{7x^7}-\frac{4ab^7}{3x^6}-\frac{28a^2b^6}{5x^5}-\frac{14a^3b^5}{x^4}-\frac{70a^4b^4}{3x^3}-\frac{28a^5b^3}{x^2}-\frac{28a^6b^2}{x}+a^8x+8a^7b\log(x)$$

output
```
-1/7*b^8/x^7-4/3*a*b^7/x^6-28/5*a^2*b^6/x^5-14*a^3*b^5/x^4-70/3*a^4*b^4/x^
3-28*a^5*b^3/x^2-28*a^6*b^2/x+a^8*x+8*a^7*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 \, dx = -\frac{b^8}{7x^7}-\frac{4ab^7}{3x^6}-\frac{28a^2b^6}{5x^5}-\frac{14a^3b^5}{x^4}-\frac{70a^4b^4}{3x^3}-\frac{28a^5b^3}{x^2}-\frac{28a^6b^2}{x}+a^8x+8a^7b\log(x)$$

input
```
Integrate[(a + b/x)^8,x]
```

output
```
-1/7*b^8/x^7 - (4*a*b^7)/(3*x^6) - (28*a^2*b^6)/(5*x^5) - (14*a^3*b^5)/x^4
 - (70*a^4*b^4)/(3*x^3) - (28*a^5*b^3)/x^2 - (28*a^6*b^2)/x + a^8*x + 8*a^
7*b*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^8 dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{(ax+b)^8}{x^8}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^8+\frac{8a^7b}{x}+\frac{28a^6b^2}{x^2}+\frac{56a^5b^3}{x^3}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^5}+\frac{28a^2b^6}{x^6}+\frac{8ab^7}{x^7}+\frac{b^8}{x^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^8x+8a^7b\log(x)-\frac{28a^6b^2}{x}-\frac{28a^5b^3}{x^2}-\frac{70a^4b^4}{3x^3}-\frac{14a^3b^5}{x^4}-\frac{28a^2b^6}{5x^5}-\frac{4ab^7}{3x^6}-\frac{b^8}{7x^7}$$

input
```
Int[(a + b/x)^8,x]
```

output
```
-1/7*b^8/x^7 - (4*a*b^7)/(3*x^6) - (28*a^2*b^6)/(5*x^5) - (14*a^3*b^5)/x^4
 - (70*a^4*b^4)/(3*x^3) - (28*a^5*b^3)/x^2 - (28*a^6*b^2)/x + a^8*x + 8*a^
7*b*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{b^8}{7x^7}-\frac{4a\,b^7}{3x^6}-\frac{28a^2b^6}{5x^5}-\frac{14a^3b^5}{x^4}-\frac{70a^4b^4}{3x^3}-\frac{28a^5b^3}{x^2}-\frac{28a^6b^2}{x}+a^8x+8a^7b\ln\left(x\right)$	87
risch	$a^8x+\frac{-28a^6b^2x^6-28a^5b^3x^5-\frac{70}{3}a^4x^4b^4-14a^3b^5x^3-\frac{28}{5}a^2b^6x^2-\frac{4}{3}a\,b^7x-\frac{1}{7}b^8}{x^7}+8a^7b\ln\left(x\right)$	87
norman	$\frac{a^8x^8-\frac{1}{7}b^8-\frac{4}{3}a\,b^7x-\frac{28}{5}a^2b^6x^2-14a^3b^5x^3-\frac{70}{3}a^4x^4b^4-28a^5b^3x^5-28a^6b^2x^6}{x^7}+8a^7b\ln\left(x\right)$	89
parallelrisch	$\frac{840a^7b\ln(x)x^7+105a^8x^8-2940a^6b^2x^6-2940a^5b^3x^5-2450a^4x^4b^4-1470a^3b^5x^3-588a^2b^6x^2-140a\,b^7x-15b^8}{105x^7}$	93

input
```
int((a+b/x)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b^8/x^7-4/3*a*b^7/x^6-28/5*a^2*b^6/x^5-14*a^3*b^5/x^4-70/3*a^4*b^4/x^
3-28*a^5*b^3/x^2-28*a^6*b^2/x+a^8*x+8*a^7*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^8 dx$$
$$=\frac{105\,a^8x^8+840\,a^7bx^7\log\left(x\right)-2940\,a^6b^2x^6-2940\,a^5b^3x^5-2450\,a^4b^4x^4-1470\,a^3b^5x^3-588\,a^2b^6x^2-1}{105\,x^7}$$

input
```
integrate((a+b/x)^8,x, algorithm="fricas")
```

output
```
1/105*(105*a^8*x^8 + 840*a^7*b*x^7*log(x) - 2940*a^6*b^2*x^6 - 2940*a^5*b^
3*x^5 - 2450*a^4*b^4*x^4 - 1470*a^3*b^5*x^3 - 588*a^2*b^6*x^2 - 140*a*b^7*
x - 15*b^8)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^8 dx = a^8x+8a^7b\log\left(x\right)$$
$$+\frac{-2940a^6b^2x^6-2940a^5b^3x^5-2450a^4b^4x^4-1470a^3b^5x^3-588a^2b^6x^2-140ab^7x-15b^8}{105x^7}$$

input
```
integrate((a+b/x)**8,x)
```

output
```
a**8*x + 8*a**7*b*log(x) + (-2940*a**6*b**2*x**6 - 2940*a**5*b**3*x**5 - 2
450*a**4*b**4*x**4 - 1470*a**3*b**5*x**3 - 588*a**2*b**6*x**2 - 140*a*b**7
*x - 15*b**8)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{x}\right)^8 dx = a^8x + 8\,a^7b\log\left(x\right) - \frac{2940\,a^6b^2x^6 + 2940\,a^5b^3x^5 + 2450\,a^4b^4x^4 + 1470\,a^3b^5x^3 + 588\,a^2b^6x^2 + 140\,ab^7x + 15\,b^8}{105\,x^7}$$

input
```
integrate((a+b/x)^8,x, algorithm="maxima")
```

output
```
a^8*x + 8*a^7*b*log(x) - 1/105*(2940*a^6*b^2*x^6 + 2940*a^5*b^3*x^5 + 2450
*a^4*b^4*x^4 + 1470*a^3*b^5*x^3 + 588*a^2*b^6*x^2 + 140*a*b^7*x + 15*b^8)/
x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^8 dx = a^8x + 8\,a^7b\log\left(|x|\right) - \frac{2940\,a^6b^2x^6 + 2940\,a^5b^3x^5 + 2450\,a^4b^4x^4 + 1470\,a^3b^5x^3 + 588\,a^2b^6x^2 + 140\,ab^7x + 15\,b^8}{105\,x^7}$$

input
```
integrate((a+b/x)^8,x, algorithm="giac")
```

output
```
a^8*x + 8*a^7*b*log(abs(x)) - 1/105*(2940*a^6*b^2*x^6 + 2940*a^5*b^3*x^5 +
 2450*a^4*b^4*x^4 + 1470*a^3*b^5*x^3 + 588*a^2*b^6*x^2 + 140*a*b^7*x + 15*
b^8)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^8 dx = -\frac{\frac{b^8}{7}-a^8\,x^8+\frac{28\,a^2\,b^6\,x^2}{5}+14\,a^3\,b^5\,x^3+\frac{70\,a^4\,b^4\,x^4}{3}+28\,a^5\,b^3\,x^5+28\,a^6\,b^2\,x^6+\frac{4\,a\,b^7\,x}{3}-8\,a^7\,b\,x^7\,\ln\left(x\right)}{x^7}$$

input
```
int((a + b/x)^8,x)
```

output
```
-(b^8/7 - a^8*x^8 + (28*a^2*b^6*x^2)/5 + 14*a^3*b^5*x^3 + (70*a^4*b^4*x^4)
/3 + 28*a^5*b^3*x^5 + 28*a^6*b^2*x^6 + (4*a*b^7*x)/3 - 8*a^7*b*x^7*log(x))
/x^7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^8 dx = \frac{840\log(x)\,a^7b\,x^7+105a^8x^8-2940a^6b^2x^6-2940a^5b^3x^5-2450a^4b^4x^4-1470a^3b^5x^3-588a^2b^6x^2-140a}{105x^7}$$

input
```
int((a+b/x)^8,x)
```

output
```
(840*log(x)*a**7*b*x**7 + 105*a**8*x**8 - 2940*a**6*b**2*x**6 - 2940*a**5*
b**3*x**5 - 2450*a**4*b**4*x**4 - 1470*a**3*b**5*x**3 - 588*a**2*b**6*x**2
 - 140*a*b**7*x - 15*b**8)/(105*x**7)
```

3.54 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x} dx = -\frac{b^8}{8x^8}-\frac{8ab^7}{7x^7}-\frac{14a^2b^6}{3x^6}-\frac{56a^3b^5}{5x^5}-\frac{35a^4b^4}{2x^4}-\frac{56a^5b^3}{3x^3}-\frac{14a^6b^2}{x^2}-\frac{8a^7b}{x}+a^8\log(x)$$

output
```
-1/8*b^8/x^8-8/7*a*b^7/x^7-14/3*a^2*b^6/x^6-56/5*a^3*b^5/x^5-35/2*a^4*b^4/
x^4-56/3*a^5*b^3/x^3-14*a^6*b^2/x^2-8*a^7*b/x+a^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x} dx = -\frac{b^8}{8x^8}-\frac{8ab^7}{7x^7}-\frac{14a^2b^6}{3x^6}-\frac{56a^3b^5}{5x^5}-\frac{35a^4b^4}{2x^4}-\frac{56a^5b^3}{3x^3}-\frac{14a^6b^2}{x^2}-\frac{8a^7b}{x}+a^8\log(x)$$

input
```
Integrate[(a + b/x)^8/x,x]
```

output
```
-1/8*b^8/x^8 - (8*a*b^7)/(7*x^7) - (14*a^2*b^6)/(3*x^6) - (56*a^3*b^5)/(5*
x^5) - (35*a^4*b^4)/(2*x^4) - (56*a^5*b^3)/(3*x^3) - (14*a^6*b^2)/x^2 - (8
*a^7*b)/x + a^8*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x}dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{(ax+b)^8}{x^9}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{a^8}{x}+\frac{8a^7b}{x^2}+\frac{28a^6b^2}{x^3}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^5}+\frac{56a^3b^5}{x^6}+\frac{28a^2b^6}{x^7}+\frac{8ab^7}{x^8}+\frac{b^8}{x^9}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a^8\log(x)-\frac{8a^7b}{x}-\frac{14a^6b^2}{x^2}-\frac{56a^5b^3}{3x^3}-\frac{35a^4b^4}{2x^4}-\frac{56a^3b^5}{5x^5}-\frac{14a^2b^6}{3x^6}-\frac{8ab^7}{7x^7}-\frac{b^8}{8x^8}$$

input
```
Int[(a + b/x)^8/x,x]
```

output

```
-1/8*b^8/x^8 - (8*a*b^7)/(7*x^7) - (14*a^2*b^6)/(3*x^6) - (56*a^3*b^5)/(5*
x^5) - (35*a^4*b^4)/(2*x^4) - (56*a^5*b^3)/(3*x^3) - (14*a^6*b^2)/x^2 - (8
*a^7*b)/x + a^8*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result	siz
default	$-\frac{b^{8}}{8x^{8}}-\frac{8a\,b^{7}}{7x^{7}}-\frac{14a^{2}b^{6}}{3x^{6}}-\frac{56a^{3}b^{5}}{5x^{5}}-\frac{35a^{4}b^{4}}{2x^{4}}-\frac{56a^{5}b^{3}}{3x^{3}}-\frac{14a^{6}b^{2}}{x^{2}}-\frac{8a^{7}b}{x}+a^{8}\ln\left(x\right)$	89
norman	$\frac{-\frac{1}{8}b^{8}-\frac{8}{7}a\,b^{7}x-\frac{14}{3}a^{2}b^{6}x^{2}-\frac{56}{5}a^{3}b^{5}x^{3}-\frac{35}{2}a^{4}x^{4}b^{4}-\frac{56}{3}a^{5}b^{3}x^{5}-14a^{6}b^{2}x^{6}-8a^{7}b\,x^{7}}{x^{8}}+a^{8}\ln\left(x\right)$	89
risch	$\frac{-\frac{1}{8}b^{8}-\frac{8}{7}a\,b^{7}x-\frac{14}{3}a^{2}b^{6}x^{2}-\frac{56}{5}a^{3}b^{5}x^{3}-\frac{35}{2}a^{4}x^{4}b^{4}-\frac{56}{3}a^{5}b^{3}x^{5}-14a^{6}b^{2}x^{6}-8a^{7}b\,x^{7}}{x^{8}}+a^{8}\ln\left(x\right)$	89
parallelrisch	$\frac{840a^{8}\ln\left(x\right)x^{8}-6720a^{7}b\,x^{7}-11760a^{6}b^{2}x^{6}-15680a^{5}b^{3}x^{5}-14700a^{4}x^{4}b^{4}-9408a^{3}b^{5}x^{3}-3920a^{2}b^{6}x^{2}-960a\,b^{7}x-105b^{8}}{840x^{8}}$	93

input
```
int((a+b/x)^8/x,x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^8/x^8-8/7*a*b^7/x^7-14/3*a^2*b^6/x^6-56/5*a^3*b^5/x^5-35/2*a^4/x^4*
b^4-56/3*a^5*b^3/x^3-14*a^6*b^2/x^2-8*a^7*b/x+a^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx = \frac{840\,a^8x^8\log\left(x\right)-6720\,a^7bx^7-11760\,a^6b^2x^6-15680\,a^5b^3x^5-14700\,a^4b^4x^4-9408\,a^3b^5x^3-3920\,a^2b^6x}{840\,x^8}$$

input
```
integrate((a+b/x)^8/x,x, algorithm="fricas")
```

output
```
1/840*(840*a^8*x^8*log(x) - 6720*a^7*b*x^7 - 11760*a^6*b^2*x^6 - 15680*a^5
*b^3*x^5 - 14700*a^4*b^4*x^4 - 9408*a^3*b^5*x^3 - 3920*a^2*b^6*x^2 - 960*a
*b^7*x - 105*b^8)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx = a^8\log\left(x\right) + \frac{-6720a^7bx^7-11760a^6b^2x^6-15680a^5b^3x^5-14700a^4b^4x^4-9408a^3b^5x^3-3920a^2b^6x^2-960ab^7x-1}{840x^8}$$

input
```
integrate((a+b/x)**8/x,x)
```

output
```
a**8*log(x) + (-6720*a**7*b*x**7 - 11760*a**6*b**2*x**6 - 15680*a**5*b**3*
x**5 - 14700*a**4*b**4*x**4 - 9408*a**3*b**5*x**3 - 3920*a**2*b**6*x**2 -
960*a*b**7*x - 105*b**8)/(840*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx = a^8\log\left(x\right) - \frac{6720\,a^7bx^7 + 11760\,a^6b^2x^6 + 15680\,a^5b^3x^5 + 14700\,a^4b^4x^4 + 9408\,a^3b^5x^3 + 3920\,a^2b^6x^2 + 960\,ab^7x +}{840\,x^8}$$

input
```
integrate((a+b/x)^8/x,x, algorithm="maxima")
```

output
```
a^8*log(x) - 1/840*(6720*a^7*b*x^7 + 11760*a^6*b^2*x^6 + 15680*a^5*b^3*x^5
 + 14700*a^4*b^4*x^4 + 9408*a^3*b^5*x^3 + 3920*a^2*b^6*x^2 + 960*a*b^7*x +
 105*b^8)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx = a^8\log\left(|x|\right) - \frac{6720\,a^7bx^7 + 11760\,a^6b^2x^6 + 15680\,a^5b^3x^5 + 14700\,a^4b^4x^4 + 9408\,a^3b^5x^3 + 3920\,a^2b^6x^2 + 960\,ab^7x +}{840\,x^8}$$

input
```
integrate((a+b/x)^8/x,x, algorithm="giac")
```

output
```
a^8*log(abs(x)) - 1/840*(6720*a^7*b*x^7 + 11760*a^6*b^2*x^6 + 15680*a^5*b^
3*x^5 + 14700*a^4*b^4*x^4 + 9408*a^3*b^5*x^3 + 3920*a^2*b^6*x^2 + 960*a*b^
7*x + 105*b^8)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx = a^8\,\ln(x) - \frac{8\,a^7\,b\,x^7+14\,a^6\,b^2\,x^6+\frac{56\,a^5\,b^3\,x^5}{3}+\frac{35\,a^4\,b^4\,x^4}{2}+\frac{56\,a^3\,b^5\,x^3}{5}+\frac{14\,a^2\,b^6\,x^2}{3}+\frac{8\,a\,b^7\,x}{7}+\frac{b^8}{8}}{x^8}$$

input
```
int((a + b/x)^8/x,x)
```

output
```
a^8*log(x) - (b^8/8 + 8*a^7*b*x^7 + (14*a^2*b^6*x^2)/3 + (56*a^3*b^5*x^3)/
5 + (35*a^4*b^4*x^4)/2 + (56*a^5*b^3*x^5)/3 + 14*a^6*b^2*x^6 + (8*a*b^7*x)
/7)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x}\,dx = \frac{840\log(x)\,a^8x^8-6720a^7b\,x^7-11760a^6b^2x^6-15680a^5b^3x^5-14700a^4b^4x^4-9408a^3b^5x^3-3920a^2b^6x^2}{840x^8}$$

input
```
int((a+b/x)^8/x,x)
```

output
```
(840*log(x)*a**8*x**8 - 6720*a**7*b*x**7 - 11760*a**6*b**2*x**6 - 15680*a*
*5*b**3*x**5 - 14700*a**4*b**4*x**4 - 9408*a**3*b**5*x**3 - 3920*a**2*b**6
*x**2 - 960*a*b**7*x - 105*b**8)/(840*x**8)
```

3.55 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2} dx = -\frac{\left(a+\frac{b}{x}\right)^9}{9b}$$

output
```
-1/9*(a+b/x)^9/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 6.00

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2} dx = -\frac{b^8}{9x^9} - \frac{ab^7}{x^8} - \frac{4a^2b^6}{x^7} - \frac{28a^3b^5}{3x^6} - \frac{14a^4b^4}{x^5} - \frac{14a^5b^3}{x^4} - \frac{28a^6b^2}{3x^3} - \frac{4a^7b}{x^2} - \frac{a^8}{x}$$

input
```
Integrate[(a + b/x)^8/x^2,x]
```

output
```
-1/9*b^8/x^9 - (a*b^7)/x^8 - (4*a^2*b^6)/x^7 - (28*a^3*b^5)/(3*x^6) - (14*
a^4*b^4)/x^5 - (14*a^5*b^3)/x^4 - (28*a^6*b^2)/(3*x^3) - (4*a^7*b)/x^2 - a
^8/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{\left(a+\frac{b}{x}\right)^9}{9b}$$

input
```
Int[(a + b/x)^8/x^2,x]
```

output
```
-1/9*(a + b/x)^9/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 5.56

method	result	size
gosper	$-\frac{9a^8x^8+36a^7b\,x^7+84a^6b^2x^6+126a^5b^3x^5+126a^4x^4b^4+84a^3b^5x^3+36a^2b^6x^2+9a\,b^7x+b^8}{9x^9}$	89
norman	$\frac{-a^8x^8-4a^7b\,x^7-\frac{28}{3}a^6b^2x^6-14a^5b^3x^5-14a^4x^4b^4-\frac{28}{3}a^3b^5x^3-4a^2b^6x^2-a\,b^7x-\frac{1}{9}b^8}{x^9}$	90
risch	$\frac{-a^8x^8-4a^7b\,x^7-\frac{28}{3}a^6b^2x^6-14a^5b^3x^5-14a^4x^4b^4-\frac{28}{3}a^3b^5x^3-4a^2b^6x^2-a\,b^7x-\frac{1}{9}b^8}{x^9}$	90
default	$-\frac{28a^6b^2}{3x^3}-\frac{14a^4b^4}{x^5}-\frac{4a^7b}{x^2}-\frac{4a^2b^6}{x^7}-\frac{14a^5b^3}{x^4}-\frac{a\,b^7}{x^8}-\frac{a^8}{x}-\frac{28a^3b^5}{3x^6}-\frac{b^8}{9x^9}$	91
parallelrisch	$\frac{-9a^8x^8-36a^7b\,x^7-84a^6b^2x^6-126a^5b^3x^5-126a^4x^4b^4-84a^3b^5x^3-36a^2b^6x^2-9a\,b^7x-b^8}{9x^9}$	91
orering	$-\frac{(9a^8x^8+36a^7b\,x^7+84a^6b^2x^6+126a^5b^3x^5+126a^4x^4b^4+84a^3b^5x^3+36a^2b^6x^2+9a\,b^7x+b^8)\left(a+\frac{b}{x}\right)^8}{9x(ax+b)^8}$	105

input
```
int((a+b/x)^8/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(9*a^8*x^8+36*a^7*b*x^7+84*a^6*b^2*x^6+126*a^5*b^3*x^5+126*a^4*b^4*x^
4+84*a^3*b^5*x^3+36*a^2*b^6*x^2+9*a*b^7*x+b^8)/x^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx = -\frac{9\,a^8x^8+36\,a^7bx^7+84\,a^6b^2x^6+126\,a^5b^3x^5+126\,a^4b^4x^4+84\,a^3b^5x^3+36\,a^2b^6x^2+9\,ab^7x+b^8}{9\,x^9}$$

input
```
integrate((a+b/x)^8/x^2,x, algorithm="fricas")
```

output
```
-1/9*(9*a^8*x^8 + 36*a^7*b*x^7 + 84*a^6*b^2*x^6 + 126*a^5*b^3*x^5 + 126*a^
4*b^4*x^4 + 84*a^3*b^5*x^3 + 36*a^2*b^6*x^2 + 9*a*b^7*x + b^8)/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(10) = 20$.

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 5.94

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx = \frac{-9a^8x^8 - 36a^7bx^7 - 84a^6b^2x^6 - 126a^5b^3x^5 - 126a^4b^4x^4 - 84a^3b^5x^3 - 36a^2b^6x^2 - 9ab^7x - b^8}{9x^9}$$

input
```
integrate((a+b/x)**8/x**2,x)
```

output
```
(-9*a**8*x**8 - 36*a**7*b*x**7 - 84*a**6*b**2*x**6 - 126*a**5*b**3*x**5 -
126*a**4*b**4*x**4 - 84*a**3*b**5*x**3 - 36*a**2*b**6*x**2 - 9*a*b**7*x -
b**8)/(9*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^9}{9\,b}$$

input
```
integrate((a+b/x)^8/x^2,x, algorithm="maxima")
```

output
```
-1/9*(a + b/x)^9/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx = -\frac{\left(a+\frac{b}{x}\right)^9}{9\,b}$$

input
```
integrate((a+b/x)^8/x^2,x, algorithm="giac")
```

output
```
-1/9*(a + b/x)^9/b
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 5.50

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx = -\frac{a^8\,x^8+4\,a^7\,b\,x^7+\frac{28\,a^6\,b^2\,x^6}{3}+14\,a^5\,b^3\,x^5+14\,a^4\,b^4\,x^4+\frac{28\,a^3\,b^5\,x^3}{3}+4\,a^2\,b^6\,x^2+a\,b^7\,x+\frac{b^8}{9}}{x^9}$$

input
```
int((a + b/x)^8/x^2,x)
```

output
```
-(b^8/9 + a^8*x^8 + 4*a^7*b*x^7 + 4*a^2*b^6*x^2 + (28*a^3*b^5*x^3)/3 + 14*
a^4*b^4*x^4 + 14*a^5*b^3*x^5 + (28*a^6*b^2*x^6)/3 + a*b^7*x)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.62

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^2}\,dx = \frac{-9a^8x^8-36a^7b\,x^7-84a^6b^2x^6-126a^5b^3x^5-126a^4b^4x^4-84a^3b^5x^3-36a^2b^6x^2-9a\,b^7x-b^8}{9x^9}$$

input
```
int((a+b/x)^8/x^2,x)
```

output
```
( - 9*a**8*x**8 - 36*a**7*b*x**7 - 84*a**6*b**2*x**6 - 126*a**5*b**3*x**5
- 126*a**4*b**4*x**4 - 84*a**3*b**5*x**3 - 36*a**2*b**6*x**2 - 9*a*b**7*x
- b**8)/(9*x**9)
```

3.56 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx=\frac{a\left(a+\frac{b}{x}\right)^9}{9b^2}-\frac{\left(a+\frac{b}{x}\right)^{10}}{10b^2}$$

output
```
1/9*a*(a+b/x)^9/b^2-1/10*(a+b/x)^10/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(34) = 68.

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx=-\frac{b^8}{10x^{10}}-\frac{8ab^7}{9x^9}-\frac{7a^2b^6}{2x^8}-\frac{8a^3b^5}{x^7}-\frac{35a^4b^4}{3x^6}-\frac{56a^5b^3}{5x^5}-\frac{7a^6b^2}{x^4}-\frac{8a^7b}{3x^3}-\frac{a^8}{2x^2}$$

input
```
Integrate[(a + b/x)^8/x^3,x]
```

output
```
-1/10*b^8/x^10 - (8*a*b^7)/(9*x^9) - (7*a^2*b^6)/(2*x^8) - (8*a^3*b^5)/x^7
 - (35*a^4*b^4)/(3*x^6) - (56*a^5*b^3)/(5*x^5) - (7*a^6*b^2)/x^4 - (8*a^7*
b)/(3*x^3) - a^8/(2*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3} \, dx$$
$$\Big\downarrow \; 795$$
$$\int \frac{(ax+b)^8}{x^{11}}dx$$
$$\Big\downarrow \; 55$$
$$-\frac{a\int \frac{(b+ax)^8}{x^{10}}dx}{10b}-\frac{(ax+b)^9}{10bx^{10}}$$
$$\Big\downarrow \; 48$$
$$\frac{a(ax+b)^9}{90b^2x^9}-\frac{(ax+b)^9}{10bx^{10}}$$

input
```
Int[(a + b/x)^8/x^3,x]
```

output
```
-1/10*(b + a*x)^9/(b*x^10) + (a*(b + a*x)^9)/(90*b^2*x^9)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(30) = 60$.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

method	result	size
norman	$\frac{-\frac{1}{2}a^{8}x^{8}-\frac{8}{3}a^{7}b\,x^{7}-7a^{6}b^{2}x^{6}-\frac{56}{5}a^{5}b^{3}x^{5}-\frac{35}{3}a^{4}x^{4}b^{4}-8a^{3}b^{5}x^{3}-\frac{7}{2}a^{2}b^{6}x^{2}-\frac{8}{9}a\,b^{7}x-\frac{1}{10}b^{8}}{x^{10}}$	90
risch	$\frac{-\frac{1}{2}a^{8}x^{8}-\frac{8}{3}a^{7}b\,x^{7}-7a^{6}b^{2}x^{6}-\frac{56}{5}a^{5}b^{3}x^{5}-\frac{35}{3}a^{4}x^{4}b^{4}-8a^{3}b^{5}x^{3}-\frac{7}{2}a^{2}b^{6}x^{2}-\frac{8}{9}a\,b^{7}x-\frac{1}{10}b^{8}}{x^{10}}$	90
gosper	$-\frac{45a^{8}x^{8}+240a^{7}b\,x^{7}+630a^{6}b^{2}x^{6}+1008a^{5}b^{3}x^{5}+1050a^{4}x^{4}b^{4}+720a^{3}b^{5}x^{3}+315a^{2}b^{6}x^{2}+80a\,b^{7}x+9b^{8}}{90x^{10}}$	91
default	$-\frac{8a^{7}b}{3x^{3}}-\frac{56a^{5}b^{3}}{5x^{5}}-\frac{a^{8}}{2x^{2}}-\frac{8a^{3}b^{5}}{x^{7}}-\frac{7a^{6}b^{2}}{x^{4}}-\frac{7a^{2}b^{6}}{2x^{8}}-\frac{b^{8}}{10x^{10}}-\frac{35a^{4}b^{4}}{3x^{6}}-\frac{8a\,b^{7}}{9x^{9}}$	91
parallelrisch	$\frac{-45a^{8}x^{8}-240a^{7}b\,x^{7}-630a^{6}b^{2}x^{6}-1008a^{5}b^{3}x^{5}-1050a^{4}x^{4}b^{4}-720a^{3}b^{5}x^{3}-315a^{2}b^{6}x^{2}-80a\,b^{7}x-9b^{8}}{90x^{10}}$	91
orering	$-\frac{\left(45a^{8}x^{8}+240a^{7}b\,x^{7}+630a^{6}b^{2}x^{6}+1008a^{5}b^{3}x^{5}+1050a^{4}x^{4}b^{4}+720a^{3}b^{5}x^{3}+315a^{2}b^{6}x^{2}+80a\,b^{7}x+9b^{8}\right)\left(a+\frac{b}{x}\right)^{8}}{90x^{2}\left(ax+b\right)^{8}}$	107

input
```
int((a+b/x)^8/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a^8*x^8-8/3*a^7*b*x^7-7*a^6*b^2*x^6-56/5*a^5*b^3*x^5-35/3*a^4*x^4*b^
4-8*a^3*b^5*x^3-7/2*a^2*b^6*x^2-8/9*a*b^7*x-1/10*b^8)/x^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx = -\frac{45\,a^8x^8 + 240\,a^7bx^7 + 630\,a^6b^2x^6 + 1008\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 720\,a^3b^5x^3 + 315\,a^2b^6x^2 + 80\,ab^7x +}{90\,x^{10}}$$

input
```
integrate((a+b/x)^8/x^3,x, algorithm="fricas")
```

output
```
-1/90*(45*a^8*x^8 + 240*a^7*b*x^7 + 630*a^6*b^2*x^6 + 1008*a^5*b^3*x^5 + 1
050*a^4*b^4*x^4 + 720*a^3*b^5*x^3 + 315*a^2*b^6*x^2 + 80*a*b^7*x + 9*b^8)/
x^10
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(24) = 48$.

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.85

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx = \frac{-45a^8x^8 - 240a^7bx^7 - 630a^6b^2x^6 - 1008a^5b^3x^5 - 1050a^4b^4x^4 - 720a^3b^5x^3 - 315a^2b^6x^2 - 80ab^7x - 9b^8}{90x^{10}}$$

input
```
integrate((a+b/x)**8/x**3,x)
```

output
```
(-45*a**8*x**8 - 240*a**7*b*x**7 - 630*a**6*b**2*x**6 - 1008*a**5*b**3*x**
5 - 1050*a**4*b**4*x**4 - 720*a**3*b**5*x**3 - 315*a**2*b**6*x**2 - 80*a*b
**7*x - 9*b**8)/(90*x**10)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx = -\frac{45\,a^8x^8 + 240\,a^7bx^7 + 630\,a^6b^2x^6 + 1008\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 720\,a^3b^5x^3 + 315\,a^2b^6x^2 + 80\,ab^7x +}{90\,x^{10}}$$

input
```
integrate((a+b/x)^8/x^3,x, algorithm="maxima")
```

output
```
-1/90*(45*a^8*x^8 + 240*a^7*b*x^7 + 630*a^6*b^2*x^6 + 1008*a^5*b^3*x^5 + 1
050*a^4*b^4*x^4 + 720*a^3*b^5*x^3 + 315*a^2*b^6*x^2 + 80*a*b^7*x + 9*b^8)/
x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx = -\frac{45\,a^8x^8 + 240\,a^7bx^7 + 630\,a^6b^2x^6 + 1008\,a^5b^3x^5 + 1050\,a^4b^4x^4 + 720\,a^3b^5x^3 + 315\,a^2b^6x^2 + 80\,ab^7x +}{90\,x^{10}}$$

input
```
integrate((a+b/x)^8/x^3,x, algorithm="giac")
```

output
```
-1/90*(45*a^8*x^8 + 240*a^7*b*x^7 + 630*a^6*b^2*x^6 + 1008*a^5*b^3*x^5 + 1
050*a^4*b^4*x^4 + 720*a^3*b^5*x^3 + 315*a^2*b^6*x^2 + 80*a*b^7*x + 9*b^8)/
x^10
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx = -\frac{\frac{a^8\,x^8}{2}+\frac{8\,a^7\,b\,x^7}{3}+7\,a^6\,b^2\,x^6+\frac{56\,a^5\,b^3\,x^5}{5}+\frac{35\,a^4\,b^4\,x^4}{3}+8\,a^3\,b^5\,x^3+\frac{7\,a^2\,b^6\,x^2}{2}+\frac{8\,a\,b^7\,x}{9}+\frac{b^8}{10}}{x^{10}}$$

input
```
int((a + b/x)^8/x^3,x)
```

output
```
-(b^8/10 + (a^8*x^8)/2 + (8*a^7*b*x^7)/3 + (7*a^2*b^6*x^2)/2 + 8*a^3*b^5*x
^3 + (35*a^4*b^4*x^4)/3 + (56*a^5*b^3*x^5)/5 + 7*a^6*b^2*x^6 + (8*a*b^7*x)
/9)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^3}\,dx = \frac{-45a^8x^8-240a^7b\,x^7-630a^6b^2x^6-1008a^5b^3x^5-1050a^4b^4x^4-720a^3b^5x^3-315a^2b^6x^2-80a\,b^7x-9b}{90x^{10}}$$

input
```
int((a+b/x)^8/x^3,x)
```

output
```
( - 45*a**8*x**8 - 240*a**7*b*x**7 - 630*a**6*b**2*x**6 - 1008*a**5*b**3*x
**5 - 1050*a**4*b**4*x**4 - 720*a**3*b**5*x**3 - 315*a**2*b**6*x**2 - 80*a
*b**7*x - 9*b**8)/(90*x**10)
```

3.57 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = -\frac{a^2\left(a+\frac{b}{x}\right)^9}{9b^3} + \frac{a\left(a+\frac{b}{x}\right)^{10}}{5b^3} - \frac{\left(a+\frac{b}{x}\right)^{11}}{11b^3}$$

output
```
-1/9*a^2*(a+b/x)^9/b^3+1/5*a*(a+b/x)^10/b^3-1/11*(a+b/x)^11/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = -\frac{b^8}{11x^{11}} - \frac{4ab^7}{5x^{10}} - \frac{28a^2b^6}{9x^9} - \frac{7a^3b^5}{x^8} - \frac{10a^4b^4}{x^7} - \frac{28a^5b^3}{3x^6} - \frac{28a^6b^2}{5x^5} - \frac{2a^7b}{x^4} - \frac{a^8}{3x^3}$$

input
```
Integrate[(a + b/x)^8/x^4,x]
```

output
```
-1/11*b^8/x^11 - (4*a*b^7)/(5*x^10) - (28*a^2*b^6)/(9*x^9) - (7*a^3*b^5)/x
^8 - (10*a^4*b^4)/x^7 - (28*a^5*b^3)/(3*x^6) - (28*a^6*b^2)/(5*x^5) - (2*a
^7*b)/x^4 - a^8/(3*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^{12}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{2a\int \frac{(b+ax)^8}{x^{11}}dx}{11b}-\frac{(ax+b)^9}{11bx^{11}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{2a\left(-\frac{a\int \frac{(b+ax)^8}{x^{10}}dx}{10b}-\frac{(ax+b)^9}{10bx^{10}}\right)}{11b}-\frac{(ax+b)^9}{11bx^{11}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{2a\left(\frac{a(ax+b)^9}{90b^2x^9}-\frac{(ax+b)^9}{10bx^{10}}\right)}{11b}-\frac{(ax+b)^9}{11bx^{11}}$$

input
```
Int[(a + b/x)^8/x^4,x]
```

output
```
-1/11*(b + a*x)^9/(b*x^11) - (2*a*(-1/10*(b + a*x)^9/(b*x^10) + (a*(b + a*
x)^9)/(90*b^2*x^9)))/(11*b)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

method	result	s
norman	$\frac{-\frac{1}{3}a^{8}x^{8}-2a^{7}b\,x^{7}-\frac{28}{5}a^{6}b^{2}x^{6}-\frac{28}{3}a^{5}b^{3}x^{5}-10a^{4}x^{4}b^{4}-7a^{3}b^{5}x^{3}-\frac{28}{9}a^{2}b^{6}x^{2}-\frac{4}{5}a\,b^{7}x-\frac{1}{11}b^{8}}{x^{11}}$	9
risch	$\frac{-\frac{1}{3}a^{8}x^{8}-2a^{7}b\,x^{7}-\frac{28}{5}a^{6}b^{2}x^{6}-\frac{28}{3}a^{5}b^{3}x^{5}-10a^{4}x^{4}b^{4}-7a^{3}b^{5}x^{3}-\frac{28}{9}a^{2}b^{6}x^{2}-\frac{4}{5}a\,b^{7}x-\frac{1}{11}b^{8}}{x^{11}}$	9
gosper	$-\frac{165a^{8}x^{8}+990a^{7}b\,x^{7}+2772a^{6}b^{2}x^{6}+4620a^{5}b^{3}x^{5}+4950a^{4}x^{4}b^{4}+3465a^{3}b^{5}x^{3}+1540a^{2}b^{6}x^{2}+396a\,b^{7}x+45b^{8}}{495x^{11}}$	9
default	$-\frac{a^{8}}{3x^{3}}-\frac{28a^{6}b^{2}}{5x^{5}}-\frac{b^{8}}{11x^{11}}-\frac{10a^{4}b^{4}}{x^{7}}-\frac{2a^{7}b}{x^{4}}-\frac{7a^{3}b^{5}}{x^{8}}-\frac{4a\,b^{7}}{5x^{10}}-\frac{28a^{5}b^{3}}{3x^{6}}-\frac{28a^{2}b^{6}}{9x^{9}}$	9
parallelrisch	$\frac{-165a^{8}x^{8}-990a^{7}b\,x^{7}-2772a^{6}b^{2}x^{6}-4620a^{5}b^{3}x^{5}-4950a^{4}x^{4}b^{4}-3465a^{3}b^{5}x^{3}-1540a^{2}b^{6}x^{2}-396a\,b^{7}x-45b^{8}}{495x^{11}}$	9
orering	$-\frac{\left(165a^{8}x^{8}+990a^{7}b\,x^{7}+2772a^{6}b^{2}x^{6}+4620a^{5}b^{3}x^{5}+4950a^{4}x^{4}b^{4}+3465a^{3}b^{5}x^{3}+1540a^{2}b^{6}x^{2}+396a\,b^{7}x+45b^{8}\right)\left(a+\frac{b}{x}\right)^{8}}{495x^{3}\left(ax+b\right)^{8}}$	1

input
```
int((a+b/x)^8/x^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*a^8*x^8-2*a^7*b*x^7-28/5*a^6*b^2*x^6-28/3*a^5*b^3*x^5-10*a^4*x^4*b^4
-7*a^3*b^5*x^3-28/9*a^2*b^6*x^2-4/5*a*b^7*x-1/11*b^8)/x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = -\frac{165\,a^8x^8 + 990\,a^7bx^7 + 2772\,a^6b^2x^6 + 4620\,a^5b^3x^5 + 4950\,a^4b^4x^4 + 3465\,a^3b^5x^3 + 1540\,a^2b^6x^2 + 396\,a}{495\,x^{11}}$$

input
```
integrate((a+b/x)^8/x^4,x, algorithm="fricas")
```

output
```
-1/495*(165*a^8*x^8 + 990*a^7*b*x^7 + 2772*a^6*b^2*x^6 + 4620*a^5*b^3*x^5
+ 4950*a^4*b^4*x^4 + 3465*a^3*b^5*x^3 + 1540*a^2*b^6*x^2 + 396*a*b^7*x + 4
5*b^8)/x^11
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = \frac{-165a^8x^8 - 990a^7bx^7 - 2772a^6b^2x^6 - 4620a^5b^3x^5 - 4950a^4b^4x^4 - 3465a^3b^5x^3 - 1540a^2b^6x^2 - 396ab^7x}{495x^{11}}$$

input
```
integrate((a+b/x)**8/x**4,x)
```

output
```
(-165*a**8*x**8 - 990*a**7*b*x**7 - 2772*a**6*b**2*x**6 - 4620*a**5*b**3*x
**5 - 4950*a**4*b**4*x**4 - 3465*a**3*b**5*x**3 - 1540*a**2*b**6*x**2 - 39
6*a*b**7*x - 45*b**8)/(495*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = -\frac{165\,a^8x^8+990\,a^7bx^7+2772\,a^6b^2x^6+4620\,a^5b^3x^5+4950\,a^4b^4x^4+3465\,a^3b^5x^3+1540\,a^2b^6x^2+396\,a}{495\,x^{11}}$$

input
```
integrate((a+b/x)^8/x^4,x, algorithm="maxima")
```

output
```
-1/495*(165*a^8*x^8 + 990*a^7*b*x^7 + 2772*a^6*b^2*x^6 + 4620*a^5*b^3*x^5
+ 4950*a^4*b^4*x^4 + 3465*a^3*b^5*x^3 + 1540*a^2*b^6*x^2 + 396*a*b^7*x + 4
5*b^8)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = -\frac{165\,a^8x^8+990\,a^7bx^7+2772\,a^6b^2x^6+4620\,a^5b^3x^5+4950\,a^4b^4x^4+3465\,a^3b^5x^3+1540\,a^2b^6x^2+396\,a}{495\,x^{11}}$$

input
```
integrate((a+b/x)^8/x^4,x, algorithm="giac")
```

output
```
-1/495*(165*a^8*x^8 + 990*a^7*b*x^7 + 2772*a^6*b^2*x^6 + 4620*a^5*b^3*x^5
+ 4950*a^4*b^4*x^4 + 3465*a^3*b^5*x^3 + 1540*a^2*b^6*x^2 + 396*a*b^7*x + 4
5*b^8)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = -\frac{\frac{a^8\,x^8}{3}+2\,a^7\,b\,x^7+\frac{28\,a^6\,b^2\,x^6}{5}+\frac{28\,a^5\,b^3\,x^5}{3}+10\,a^4\,b^4\,x^4+7\,a^3\,b^5\,x^3+\frac{28\,a^2\,b^6\,x^2}{9}+\frac{4\,a\,b^7\,x}{5}+\frac{b^8}{11}}{x^{11}}$$

input
```
int((a + b/x)^8/x^4,x)
```

output
```
-(b^8/11 + (a^8*x^8)/3 + 2*a^7*b*x^7 + (28*a^2*b^6*x^2)/9 + 7*a^3*b^5*x^3
+ 10*a^4*b^4*x^4 + (28*a^5*b^3*x^5)/3 + (28*a^6*b^2*x^6)/5 + (4*a*b^7*x)/5
)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^4}\,dx = \frac{-165a^8x^8-990a^7b\,x^7-2772a^6b^2x^6-4620a^5b^3x^5-4950a^4b^4x^4-3465a^3b^5x^3-1540a^2b^6x^2-396a\,b^7x}{495x^{11}}$$

input
```
int((a+b/x)^8/x^4,x)
```

output
```
( - 165*a**8*x**8 - 990*a**7*b*x**7 - 2772*a**6*b**2*x**6 - 4620*a**5*b**3
*x**5 - 4950*a**4*b**4*x**4 - 3465*a**3*b**5*x**3 - 1540*a**2*b**6*x**2 -
396*a*b**7*x - 45*b**8)/(495*x**11)
```

3.58 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5} dx = \frac{a^3\left(a+\frac{b}{x}\right)^9}{9b^4} - \frac{3a^2\left(a+\frac{b}{x}\right)^{10}}{10b^4} + \frac{3a\left(a+\frac{b}{x}\right)^{11}}{11b^4} - \frac{\left(a+\frac{b}{x}\right)^{12}}{12b^4}$$

output
```
1/9*a^3*(a+b/x)^9/b^4-3/10*a^2*(a+b/x)^10/b^4+3/11*a*(a+b/x)^11/b^4-1/12*(
a+b/x)^12/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5} dx = -\frac{b^8}{12x^{12}} - \frac{8ab^7}{11x^{11}} - \frac{14a^2b^6}{5x^{10}} - \frac{56a^3b^5}{9x^9} - \frac{35a^4b^4}{4x^8} - \frac{8a^5b^3}{x^7} - \frac{14a^6b^2}{3x^6} - \frac{8a^7b}{5x^5} - \frac{a^8}{4x^4}$$

input
```
Integrate[(a + b/x)^8/x^5,x]
```

output
```
-1/12*b^8/x^12 - (8*a*b^7)/(11*x^11) - (14*a^2*b^6)/(5*x^10) - (56*a^3*b^5
)/(9*x^9) - (35*a^4*b^4)/(4*x^8) - (8*a^5*b^3)/x^7 - (14*a^6*b^2)/(3*x^6)
- (8*a^7*b)/(5*x^5) - a^8/(4*x^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {795, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^{13}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{a\int \frac{(b+ax)^8}{x^{12}}dx}{4b}-\frac{(ax+b)^9}{12bx^{12}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{a\left(-\frac{2a\int \frac{(b+ax)^8}{x^{11}}dx}{11b}-\frac{(ax+b)^9}{11bx^{11}}\right)}{4b}-\frac{(ax+b)^9}{12bx^{12}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{a\left(-\frac{2a\left(-\frac{a\int \frac{(b+ax)^8}{x^{10}}dx}{10b}-\frac{(ax+b)^9}{10bx^{10}}\right)}{11b}-\frac{(ax+b)^9}{11bx^{11}}\right)}{4b}-\frac{(ax+b)^9}{12bx^{12}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{a\left(-\frac{2a\left(\frac{a(ax+b)^9}{90b^2x^9}-\frac{(ax+b)^9}{10bx^{10}}\right)}{11b}-\frac{(ax+b)^9}{11bx^{11}}\right)}{4b}-\frac{(ax+b)^9}{12bx^{12}}$$

input
```
Int[(a + b/x)^8/x^5,x]
```

output
```
-1/12*(b + a*x)^9/(b*x^12) - (a*(-1/11*(b + a*x)^9/(b*x^11) - (2*a*(-1/10*
(b + a*x)^9/(b*x^10) + (a*(b + a*x)^9)/(90*b^2*x^9)))/(11*b)))/(4*b)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

method	result
norman	$\frac{-\frac{1}{4}a^8x^8-\frac{8}{5}a^7b\,x^7-\frac{14}{3}a^6b^2x^6-8a^5b^3x^5-\frac{35}{4}a^4x^4b^4-\frac{56}{9}a^3b^5x^3-\frac{14}{5}a^2b^6x^2-\frac{8}{11}a\,b^7x-\frac{1}{12}b^8}{x^{12}}$
risch	$\frac{-\frac{1}{4}a^8x^8-\frac{8}{5}a^7b\,x^7-\frac{14}{3}a^6b^2x^6-8a^5b^3x^5-\frac{35}{4}a^4x^4b^4-\frac{56}{9}a^3b^5x^3-\frac{14}{5}a^2b^6x^2-\frac{8}{11}a\,b^7x-\frac{1}{12}b^8}{x^{12}}$
gosper	$-\frac{495a^8x^8+3168a^7b\,x^7+9240a^6b^2x^6+15840a^5b^3x^5+17325a^4x^4b^4+12320a^3b^5x^3+5544a^2b^6x^2+1440a\,b^7x+165b^8}{1980x^{12}}$
default	$-\frac{8a^7b}{5x^5}-\frac{8a\,b^7}{11x^{11}}-\frac{8a^5b^3}{x^7}-\frac{a^8}{4x^4}-\frac{35a^4b^4}{4x^8}-\frac{14a^2b^6}{5x^{10}}-\frac{14a^6b^2}{3x^6}-\frac{56a^3b^5}{9x^9}-\frac{b^8}{12x^{12}}$
parallelrisch	$\frac{-495a^8x^8-3168a^7b\,x^7-9240a^6b^2x^6-15840a^5b^3x^5-17325a^4x^4b^4-12320a^3b^5x^3-5544a^2b^6x^2-1440a\,b^7x-165b^8}{1980x^{12}}$
orering	$-\frac{\left(495a^8x^8+3168a^7b\,x^7+9240a^6b^2x^6+15840a^5b^3x^5+17325a^4x^4b^4+12320a^3b^5x^3+5544a^2b^6x^2+1440a\,b^7x+165b^8\right)\left(a+\frac{b}{x}\right.}{1980x^4(ax+b)^8}$

input
```
int((a+b/x)^8/x^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*a^8*x^8-8/5*a^7*b*x^7-14/3*a^6*b^2*x^6-8*a^5*b^3*x^5-35/4*a^4*x^4*b^
4-56/9*a^3*b^5*x^3-14/5*a^2*b^6*x^2-8/11*a*b^7*x-1/12*b^8)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx = -\frac{495\,a^8x^8+3168\,a^7bx^7+9240\,a^6b^2x^6+15840\,a^5b^3x^5+17325\,a^4b^4x^4+12320\,a^3b^5x^3+5544\,a^2b^6x^2+}{1980\,x^{12}}$$

input
```
integrate((a+b/x)^8/x^5,x, algorithm="fricas")
```

output
```
-1/1980*(495*a^8*x^8 + 3168*a^7*b*x^7 + 9240*a^6*b^2*x^6 + 15840*a^5*b^3*x
^5 + 17325*a^4*b^4*x^4 + 12320*a^3*b^5*x^3 + 5544*a^2*b^6*x^2 + 1440*a*b^7
*x + 165*b^8)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx = \frac{-495a^8x^8 - 3168a^7bx^7 - 9240a^6b^2x^6 - 15840a^5b^3x^5 - 17325a^4b^4x^4 - 12320a^3b^5x^3 - 5544a^2b^6x^2 - 144}{1980x^{12}}$$

input
```
integrate((a+b/x)**8/x**5,x)
```

output
```
(-495*a**8*x**8 - 3168*a**7*b*x**7 - 9240*a**6*b**2*x**6 - 15840*a**5*b**3
*x**5 - 17325*a**4*b**4*x**4 - 12320*a**3*b**5*x**3 - 5544*a**2*b**6*x**2
- 1440*a*b**7*x - 165*b**8)/(1980*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx = -\frac{495\,a^8x^8 + 3168\,a^7bx^7 + 9240\,a^6b^2x^6 + 15840\,a^5b^3x^5 + 17325\,a^4b^4x^4 + 12320\,a^3b^5x^3 + 5544\,a^2b^6x^2 + }{1980\,x^{12}}$$

input
```
integrate((a+b/x)^8/x^5,x, algorithm="maxima")
```

output
```
-1/1980*(495*a^8*x^8 + 3168*a^7*b*x^7 + 9240*a^6*b^2*x^6 + 15840*a^5*b^3*x
^5 + 17325*a^4*b^4*x^4 + 12320*a^3*b^5*x^3 + 5544*a^2*b^6*x^2 + 1440*a*b^7
*x + 165*b^8)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx = -\frac{495\,a^8x^8+3168\,a^7bx^7+9240\,a^6b^2x^6+15840\,a^5b^3x^5+17325\,a^4b^4x^4+12320\,a^3b^5x^3+5544\,a^2b^6x^2+}{1980\,x^{12}}$$

input
```
integrate((a+b/x)^8/x^5,x, algorithm="giac")
```

output
```
-1/1980*(495*a^8*x^8 + 3168*a^7*b*x^7 + 9240*a^6*b^2*x^6 + 15840*a^5*b^3*x
^5 + 17325*a^4*b^4*x^4 + 12320*a^3*b^5*x^3 + 5544*a^2*b^6*x^2 + 1440*a*b^7
*x + 165*b^8)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx = -\frac{\frac{a^8\,x^8}{4}+\frac{8\,a^7\,b\,x^7}{5}+\frac{14\,a^6\,b^2\,x^6}{3}+8\,a^5\,b^3\,x^5+\frac{35\,a^4\,b^4\,x^4}{4}+\frac{56\,a^3\,b^5\,x^3}{9}+\frac{14\,a^2\,b^6\,x^2}{5}+\frac{8\,a\,b^7\,x}{11}+\frac{b^8}{12}}{x^{12}}$$

input
```
int((a + b/x)^8/x^5,x)
```

output
```
-(b^8/12 + (a^8*x^8)/4 + (8*a^7*b*x^7)/5 + (14*a^2*b^6*x^2)/5 + (56*a^3*b^
5*x^3)/9 + (35*a^4*b^4*x^4)/4 + 8*a^5*b^3*x^5 + (14*a^6*b^2*x^6)/3 + (8*a*
b^7*x)/11)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^5}\,dx$$
$$= \frac{-495a^8x^8 - 3168a^7b\,x^7 - 9240a^6b^2x^6 - 15840a^5b^3x^5 - 17325a^4b^4x^4 - 12320a^3b^5x^3 - 5544a^2b^6x^2 - 144}{1980x^{12}}$$

input
```
int((a+b/x)^8/x^5,x)
```

output
```
( - 495*a**8*x**8 - 3168*a**7*b*x**7 - 9240*a**6*b**2*x**6 - 15840*a**5*b*
*3*x**5 - 17325*a**4*b**4*x**4 - 12320*a**3*b**5*x**3 - 5544*a**2*b**6*x**
2 - 1440*a*b**7*x - 165*b**8)/(1980*x**12)
```

3.59 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx=-\frac{a^4\left(a+\frac{b}{x}\right)^9}{9b^5}+\frac{2a^3\left(a+\frac{b}{x}\right)^{10}}{5b^5}-\frac{6a^2\left(a+\frac{b}{x}\right)^{11}}{11b^5}+\frac{a\left(a+\frac{b}{x}\right)^{12}}{3b^5}-\frac{\left(a+\frac{b}{x}\right)^{13}}{13b^5}$$

output
```
-1/9*a^4*(a+b/x)^9/b^5+2/5*a^3*(a+b/x)^10/b^5-6/11*a^2*(a+b/x)^11/b^5+1/3*
a*(a+b/x)^12/b^5-1/13*(a+b/x)^13/b^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx=-\frac{b^8}{13x^{13}}-\frac{2ab^7}{3x^{12}}-\frac{28a^2b^6}{11x^{11}}-\frac{28a^3b^5}{5x^{10}}-\frac{70a^4b^4}{9x^9}-\frac{7a^5b^3}{x^8}-\frac{4a^6b^2}{x^7}-\frac{4a^7b}{3x^6}-\frac{a^8}{5x^5}$$

input
```
Integrate[(a + b/x)^8/x^6,x]
```

output
```
-1/13*b^8/x^13 - (2*a*b^7)/(3*x^12) - (28*a^2*b^6)/(11*x^11) - (28*a^3*b^5
)/(5*x^10) - (70*a^4*b^4)/(9*x^9) - (7*a^5*b^3)/x^8 - (4*a^6*b^2)/x^7 - (4
*a^7*b)/(3*x^6) - a^8/(5*x^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {795, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^{14}}dx$$

$$\Big\downarrow \text{55}$$

$$-\frac{4a\int \frac{(b+ax)^8}{x^{13}}dx}{13b}-\frac{(ax+b)^9}{13bx^{13}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{4a\left(-\frac{a\int \frac{(b+ax)^8}{x^{12}}dx}{4b}-\frac{(ax+b)^9}{12bx^{12}}\right)}{13b}-\frac{(ax+b)^9}{13bx^{13}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{4a\left(-\frac{a\left(-\frac{2a\int \frac{(b+ax)^8}{x^{11}}dx}{11b}-\frac{(ax+b)^9}{11bx^{11}}\right)}{4b}-\frac{(ax+b)^9}{12bx^{12}}\right)}{13b}-\frac{(ax+b)^9}{13bx^{13}}$$

$$\Big\downarrow \text{55}$$

$$-\frac{4a\left(-\frac{a\left(-\frac{2a\left(-\frac{a\int \frac{(b+ax)^8}{x^{10}}dx}{10b}-\frac{(ax+b)^9}{10bx^{10}}\right)}{11b}-\frac{(ax+b)^9}{11bx^{11}}\right)}{4b}-\frac{(ax+b)^9}{12bx^{12}}\right)}{13b}-\frac{(ax+b)^9}{13bx^{13}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{4a\left(-\frac{a\left(-\frac{2a\left(\frac{a(ax+b)^9}{90b^2x^9}-\frac{(ax+b)^9}{10bx^{10}}\right)}{11b}-\frac{(ax+b)^9}{11bx^{11}}\right)}{4b}-\frac{(ax+b)^9}{12bx^{12}}\right)}{13b}-\frac{(ax+b)^9}{13bx^{13}}$$

input
```
Int[(a + b/x)^8/x^6,x]
```

output
```
-1/13*(b + a*x)^9/(b*x^13) - (4*a*(-1/12*(b + a*x)^9/(b*x^12) - (a*(-1/11*
(b + a*x)^9/(b*x^11) - (2*a*(-1/10*(b + a*x)^9/(b*x^10) + (a*(b + a*x)^9)/
(90*b^2*x^9)))/(11*b)))/(4*b)))/(13*b)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result
norman	$\frac{-\frac{1}{5}a^{8}x^{8}-\frac{4}{3}a^{7}b\,x^{7}-4a^{6}b^{2}x^{6}-7a^{5}b^{3}x^{5}-\frac{70}{9}a^{4}x^{4}b^{4}-\frac{28}{5}a^{3}b^{5}x^{3}-\frac{28}{11}a^{2}b^{6}x^{2}-\frac{2}{3}a\,b^{7}x-\frac{1}{13}b^{8}}{x^{13}}$
risch	$\frac{-\frac{1}{5}a^{8}x^{8}-\frac{4}{3}a^{7}b\,x^{7}-4a^{6}b^{2}x^{6}-7a^{5}b^{3}x^{5}-\frac{70}{9}a^{4}x^{4}b^{4}-\frac{28}{5}a^{3}b^{5}x^{3}-\frac{28}{11}a^{2}b^{6}x^{2}-\frac{2}{3}a\,b^{7}x-\frac{1}{13}b^{8}}{x^{13}}$
gosper	$-\frac{1287a^{8}x^{8}+8580a^{7}b\,x^{7}+25740a^{6}b^{2}x^{6}+45045a^{5}b^{3}x^{5}+50050a^{4}x^{4}b^{4}+36036a^{3}b^{5}x^{3}+16380a^{2}b^{6}x^{2}+4290a\,b^{7}x+495b^{8}}{6435x^{13}}$
default	$-\frac{a^{8}}{5x^{5}}-\frac{28a^{2}b^{6}}{11x^{11}}-\frac{4a^{6}b^{2}}{x^{7}}-\frac{7a^{5}b^{3}}{x^{8}}-\frac{b^{8}}{13x^{13}}-\frac{28a^{3}b^{5}}{5x^{10}}-\frac{4a^{7}b}{3x^{6}}-\frac{70a^{4}b^{4}}{9x^{9}}-\frac{2a\,b^{7}}{3x^{12}}$
parallelrisch	$\frac{-1287a^{8}x^{8}-8580a^{7}b\,x^{7}-25740a^{6}b^{2}x^{6}-45045a^{5}b^{3}x^{5}-50050a^{4}x^{4}b^{4}-36036a^{3}b^{5}x^{3}-16380a^{2}b^{6}x^{2}-4290a\,b^{7}x-495b^{8}}{6435x^{13}}$
orering	$-\frac{\left(1287a^{8}x^{8}+8580a^{7}b\,x^{7}+25740a^{6}b^{2}x^{6}+45045a^{5}b^{3}x^{5}+50050a^{4}x^{4}b^{4}+36036a^{3}b^{5}x^{3}+16380a^{2}b^{6}x^{2}+4290a\,b^{7}x+495b^{8}\right)\left(a-\right.}{6435x^{5}(ax+b)^{8}}$

input
```
int((a+b/x)^8/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-1/5*a^8*x^8-4/3*a^7*b*x^7-4*a^6*b^2*x^6-7*a^5*b^3*x^5-70/9*a^4*x^4*b^4-2
8/5*a^3*b^5*x^3-28/11*a^2*b^6*x^2-2/3*a*b^7*x-1/13*b^8)/x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx = -\frac{1287\,a^8x^8+8580\,a^7bx^7+25740\,a^6b^2x^6+45045\,a^5b^3x^5+50050\,a^4b^4x^4+36036\,a^3b^5x^3+16380\,a^2b^6x^2}{6435\,x^{13}}$$

input
```
integrate((a+b/x)^8/x^6,x, algorithm="fricas")
```

output
```
-1/6435*(1287*a^8*x^8 + 8580*a^7*b*x^7 + 25740*a^6*b^2*x^6 + 45045*a^5*b^3
*x^5 + 50050*a^4*b^4*x^4 + 36036*a^3*b^5*x^3 + 16380*a^2*b^6*x^2 + 4290*a*
b^7*x + 495*b^8)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx = \frac{-1287a^8x^8 - 8580a^7bx^7 - 25740a^6b^2x^6 - 45045a^5b^3x^5 - 50050a^4b^4x^4 - 36036a^3b^5x^3 - 16380a^2b^6x^2 - }{6435x^{13}}$$

input
```
integrate((a+b/x)**8/x**6,x)
```

output
```
(-1287*a**8*x**8 - 8580*a**7*b*x**7 - 25740*a**6*b**2*x**6 - 45045*a**5*b*
*3*x**5 - 50050*a**4*b**4*x**4 - 36036*a**3*b**5*x**3 - 16380*a**2*b**6*x*
*2 - 4290*a*b**7*x - 495*b**8)/(6435*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx = -\frac{1287\,a^8x^8 + 8580\,a^7bx^7 + 25740\,a^6b^2x^6 + 45045\,a^5b^3x^5 + 50050\,a^4b^4x^4 + 36036\,a^3b^5x^3 + 16380\,a^2b^6x^2}{6435\,x^{13}}$$

input
```
integrate((a+b/x)^8/x^6,x, algorithm="maxima")
```

output
```
-1/6435*(1287*a^8*x^8 + 8580*a^7*b*x^7 + 25740*a^6*b^2*x^6 + 45045*a^5*b^3
*x^5 + 50050*a^4*b^4*x^4 + 36036*a^3*b^5*x^3 + 16380*a^2*b^6*x^2 + 4290*a*
b^7*x + 495*b^8)/x^13
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx = -\frac{1287\,a^8x^8 + 8580\,a^7bx^7 + 25740\,a^6b^2x^6 + 45045\,a^5b^3x^5 + 50050\,a^4b^4x^4 + 36036\,a^3b^5x^3 + 16380\,a^2b^6x^2}{6435\,x^{13}}$$

input
```
integrate((a+b/x)^8/x^6,x, algorithm="giac")
```

output
```
-1/6435*(1287*a^8*x^8 + 8580*a^7*b*x^7 + 25740*a^6*b^2*x^6 + 45045*a^5*b^3
*x^5 + 50050*a^4*b^4*x^4 + 36036*a^3*b^5*x^3 + 16380*a^2*b^6*x^2 + 4290*a*
b^7*x + 495*b^8)/x^13
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx = -\frac{\frac{a^8\,x^8}{5}+\frac{4\,a^7\,b\,x^7}{3}+4\,a^6\,b^2\,x^6+7\,a^5\,b^3\,x^5+\frac{70\,a^4\,b^4\,x^4}{9}+\frac{28\,a^3\,b^5\,x^3}{5}+\frac{28\,a^2\,b^6\,x^2}{11}+\frac{2\,a\,b^7\,x}{3}+\frac{b^8}{13}}{x^{13}}$$

input
```
int((a + b/x)^8/x^6,x)
```

output
```
-(b^8/13 + (a^8*x^8)/5 + (4*a^7*b*x^7)/3 + (28*a^2*b^6*x^2)/11 + (28*a^3*b
^5*x^3)/5 + (70*a^4*b^4*x^4)/9 + 7*a^5*b^3*x^5 + 4*a^6*b^2*x^6 + (2*a*b^7*
x)/3)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^6}\,dx = \frac{-1287a^8x^8 - 8580a^7b\,x^7 - 25740a^6b^2x^6 - 45045a^5b^3x^5 - 50050a^4b^4x^4 - 36036a^3b^5x^3 - 16380a^2b^6x^2 -}{6435x^{13}}$$

input
```
int((a+b/x)^8/x^6,x)
```

output
```
( - 1287*a**8*x**8 - 8580*a**7*b*x**7 - 25740*a**6*b**2*x**6 - 45045*a**5*
b**3*x**5 - 50050*a**4*b**4*x**4 - 36036*a**3*b**5*x**3 - 16380*a**2*b**6*
x**2 - 4290*a*b**7*x - 495*b**8)/(6435*x**13)
```

3.60 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx=\frac{a^5\left(a+\frac{b}{x}\right)^9}{9b^6}-\frac{a^4\left(a+\frac{b}{x}\right)^{10}}{2b^6}+\frac{10a^3\left(a+\frac{b}{x}\right)^{11}}{11b^6}-\frac{5a^2\left(a+\frac{b}{x}\right)^{12}}{6b^6}+\frac{5a\left(a+\frac{b}{x}\right)^{13}}{13b^6}-\frac{\left(a+\frac{b}{x}\right)^{14}}{14b^6}$$

output
```
1/9*a^5*(a+b/x)^9/b^6-1/2*a^4*(a+b/x)^10/b^6+10/11*a^3*(a+b/x)^11/b^6-5/6*
a^2*(a+b/x)^12/b^6+5/13*a*(a+b/x)^13/b^6-1/14*(a+b/x)^14/b^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx=-\frac{b^8}{14x^{14}}-\frac{8ab^7}{13x^{13}}-\frac{7a^2b^6}{3x^{12}}-\frac{56a^3b^5}{11x^{11}}-\frac{7a^4b^4}{x^{10}}-\frac{56a^5b^3}{9x^9}-\frac{7a^6b^2}{2x^8}-\frac{8a^7b}{7x^7}-\frac{a^8}{6x^6}$$

input
```
Integrate[(a + b/x)^8/x^7,x]
```

output
```
-1/14*b^8/x^14 - (8*a*b^7)/(13*x^13) - (7*a^2*b^6)/(3*x^12) - (56*a^3*b^5)
/(11*x^11) - (7*a^4*b^4)/x^10 - (56*a^5*b^3)/(9*x^9) - (7*a^6*b^2)/(2*x^8)
 - (8*a^7*b)/(7*x^7) - a^8/(6*x^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^8}{x^{15}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^8}{x^7}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^9}+\frac{56a^5b^3}{x^{10}}+\frac{70a^4b^4}{x^{11}}+\frac{56a^3b^5}{x^{12}}+\frac{28a^2b^6}{x^{13}}+\frac{8ab^7}{x^{14}}+\frac{b^8}{x^{15}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{6x^6}-\frac{8a^7b}{7x^7}-\frac{7a^6b^2}{2x^8}-\frac{56a^5b^3}{9x^9}-\frac{7a^4b^4}{x^{10}}-\frac{56a^3b^5}{11x^{11}}-\frac{7a^2b^6}{3x^{12}}-\frac{8ab^7}{13x^{13}}-\frac{b^8}{14x^{14}}$$

input
```
Int[(a + b/x)^8/x^7,x]
```

output

```
-1/14*b^8/x^14 - (8*a*b^7)/(13*x^13) - (7*a^2*b^6)/(3*x^12) - (56*a^3*b^5)
/(11*x^11) - (7*a^4*b^4)/x^10 - (56*a^5*b^3)/(9*x^9) - (7*a^6*b^2)/(2*x^8)
 - (8*a^7*b)/(7*x^7) - a^8/(6*x^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

method	result
norman	$\frac{-\frac{1}{14}b^8-\frac{8}{13}a\,b^7x-\frac{7}{3}a^2b^6x^2-\frac{56}{11}a^3b^5x^3-7a^4x^4b^4-\frac{56}{9}a^5b^3x^5-\frac{7}{2}a^6b^2x^6-\frac{8}{7}a^7b\,x^7-\frac{1}{6}a^8x^8}{x^{14}}$
risch	$\frac{-\frac{1}{14}b^8-\frac{8}{13}a\,b^7x-\frac{7}{3}a^2b^6x^2-\frac{56}{11}a^3b^5x^3-7a^4x^4b^4-\frac{56}{9}a^5b^3x^5-\frac{7}{2}a^6b^2x^6-\frac{8}{7}a^7b\,x^7-\frac{1}{6}a^8x^8}{x^{14}}$
gosper	$-\frac{3003a^8x^8+20592a^7b\,x^7+63063a^6b^2x^6+112112a^5b^3x^5+126126a^4x^4b^4+91728a^3b^5x^3+42042a^2b^6x^2+11088a\,b^7x+1287b^8}{18018x^{14}}$
default	$-\frac{56a^3b^5}{11x^{11}}-\frac{8a^7b}{7x^7}-\frac{7a^6b^2}{2x^8}-\frac{8a\,b^7}{13x^{13}}-\frac{7a^4b^4}{x^{10}}-\frac{b^8}{14x^{14}}-\frac{a^8}{6x^6}-\frac{56a^5b^3}{9x^9}-\frac{7a^2b^6}{3x^{12}}$
parallelrisch	$\frac{-3003a^8x^8-20592a^7b\,x^7-63063a^6b^2x^6-112112a^5b^3x^5-126126a^4x^4b^4-91728a^3b^5x^3-42042a^2b^6x^2-11088a\,b^7x-1287b^8}{18018x^{14}}$
orering	$-\frac{(3003a^8x^8+20592a^7b\,x^7+63063a^6b^2x^6+112112a^5b^3x^5+126126a^4x^4b^4+91728a^3b^5x^3+42042a^2b^6x^2+11088a\,b^7x+1287b}{18018x^6(ax+b)^8}$

input
```
int((a+b/x)^8/x^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/14*b^8-8/13*a*b^7*x-7/3*a^2*b^6*x^2-56/11*a^3*b^5*x^3-7*a^4*x^4*b^4-56
/9*a^5*b^3*x^5-7/2*a^6*b^2*x^6-8/7*a^7*b*x^7-1/6*a^8*x^8)/x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx = -\frac{3003\,a^8x^8+20592\,a^7bx^7+63063\,a^6b^2x^6+112112\,a^5b^3x^5+126126\,a^4b^4x^4+91728\,a^3b^5x^3+42042\,a^2b}{18018\,x^{14}}$$

input
```
integrate((a+b/x)^8/x^7,x, algorithm="fricas")
```

output
```
-1/18018*(3003*a^8*x^8 + 20592*a^7*b*x^7 + 63063*a^6*b^2*x^6 + 112112*a^5*
b^3*x^5 + 126126*a^4*b^4*x^4 + 91728*a^3*b^5*x^3 + 42042*a^2*b^6*x^2 + 110
88*a*b^7*x + 1287*b^8)/x^14
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx = \frac{-3003a^8x^8-20592a^7bx^7-63063a^6b^2x^6-112112a^5b^3x^5-126126a^4b^4x^4-91728a^3b^5x^3-42042a^2b^6x}{18018x^{14}}$$

input
```
integrate((a+b/x)**8/x**7,x)
```

output
```
(-3003*a**8*x**8 - 20592*a**7*b*x**7 - 63063*a**6*b**2*x**6 - 112112*a**5*
b**3*x**5 - 126126*a**4*b**4*x**4 - 91728*a**3*b**5*x**3 - 42042*a**2*b**6
*x**2 - 11088*a*b**7*x - 1287*b**8)/(18018*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx = -\frac{3003\,a^8x^8+20592\,a^7bx^7+63063\,a^6b^2x^6+112112\,a^5b^3x^5+126126\,a^4b^4x^4+91728\,a^3b^5x^3+42042\,a^2b}{18018\,x^{14}}$$

input
```
integrate((a+b/x)^8/x^7,x, algorithm="maxima")
```

output
```
-1/18018*(3003*a^8*x^8 + 20592*a^7*b*x^7 + 63063*a^6*b^2*x^6 + 112112*a^5*
b^3*x^5 + 126126*a^4*b^4*x^4 + 91728*a^3*b^5*x^3 + 42042*a^2*b^6*x^2 + 110
88*a*b^7*x + 1287*b^8)/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx = -\frac{3003\,a^8x^8+20592\,a^7bx^7+63063\,a^6b^2x^6+112112\,a^5b^3x^5+126126\,a^4b^4x^4+91728\,a^3b^5x^3+42042\,a^2b}{18018\,x^{14}}$$

input
```
integrate((a+b/x)^8/x^7,x, algorithm="giac")
```

output
```
-1/18018*(3003*a^8*x^8 + 20592*a^7*b*x^7 + 63063*a^6*b^2*x^6 + 112112*a^5*
b^3*x^5 + 126126*a^4*b^4*x^4 + 91728*a^3*b^5*x^3 + 42042*a^2*b^6*x^2 + 110
88*a*b^7*x + 1287*b^8)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx = -\frac{\frac{a^8\,x^8}{6}+\frac{8\,a^7\,b\,x^7}{7}+\frac{7\,a^6\,b^2\,x^6}{2}+\frac{56\,a^5\,b^3\,x^5}{9}+7\,a^4\,b^4\,x^4+\frac{56\,a^3\,b^5\,x^3}{11}+\frac{7\,a^2\,b^6\,x^2}{3}+\frac{8\,a\,b^7\,x}{13}+\frac{b^8}{14}}{x^{14}}$$

input
```
int((a + b/x)^8/x^7,x)
```

output
```
-(b^8/14 + (a^8*x^8)/6 + (8*a^7*b*x^7)/7 + (7*a^2*b^6*x^2)/3 + (56*a^3*b^5
*x^3)/11 + 7*a^4*b^4*x^4 + (56*a^5*b^3*x^5)/9 + (7*a^6*b^2*x^6)/2 + (8*a*b
^7*x)/13)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^7}\,dx = \frac{-3003a^8x^8-20592a^7b\,x^7-63063a^6b^2x^6-112112a^5b^3x^5-126126a^4b^4x^4-91728a^3b^5x^3-42042a^2b^6x}{18018x^{14}}$$

input
```
int((a+b/x)^8/x^7,x)
```

output
```
( - 3003*a**8*x**8 - 20592*a**7*b*x**7 - 63063*a**6*b**2*x**6 - 112112*a**
5*b**3*x**5 - 126126*a**4*b**4*x**4 - 91728*a**3*b**5*x**3 - 42042*a**2*b*
*6*x**2 - 11088*a*b**7*x - 1287*b**8)/(18018*x**14)
```

3.61 $\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 129

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8} dx = -\frac{a^6\left(a+\frac{b}{x}\right)^9}{9b^7}+\frac{3a^5\left(a+\frac{b}{x}\right)^{10}}{5b^7}-\frac{15a^4\left(a+\frac{b}{x}\right)^{11}}{11b^7}+\frac{5a^3\left(a+\frac{b}{x}\right)^{12}}{3b^7}-\frac{15a^2\left(a+\frac{b}{x}\right)^{13}}{13b^7}+\frac{3a\left(a+\frac{b}{x}\right)^{14}}{7b^7}-\frac{\left(a+\frac{b}{x}\right)^{15}}{15b^7}$$

output
```
-1/9*a^6*(a+b/x)^9/b^7+3/5*a^5*(a+b/x)^10/b^7-15/11*a^4*(a+b/x)^11/b^7+5/3
*a^3*(a+b/x)^12/b^7-15/13*a^2*(a+b/x)^13/b^7+3/7*a*(a+b/x)^14/b^7-1/15*(a+
b/x)^15/b^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8} dx = -\frac{b^8}{15x^{15}}-\frac{4ab^7}{7x^{14}}-\frac{28a^2b^6}{13x^{13}}-\frac{14a^3b^5}{3x^{12}}-\frac{70a^4b^4}{11x^{11}}-\frac{28a^5b^3}{5x^{10}}-\frac{28a^6b^2}{9x^9}-\frac{a^7b}{x^8}-\frac{a^8}{7x^7}$$

input
```
Integrate[(a + b/x)^8/x^8,x]
```

output
```
-1/15*b^8/x^15 - (4*a*b^7)/(7*x^14) - (28*a^2*b^6)/(13*x^13) - (14*a^3*b^5
)/(3*x^12) - (70*a^4*b^4)/(11*x^11) - (28*a^5*b^3)/(5*x^10) - (28*a^6*b^2)
/(9*x^9) - (a^7*b)/x^8 - a^8/(7*x^7)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8} \, dx$$
$$\Big\downarrow \; 795$$
$$\int \frac{(ax+b)^8}{x^{16}}dx$$
$$\Big\downarrow \; 53$$
$$\int \left(\frac{a^8}{x^8}+\frac{8a^7b}{x^9}+\frac{28a^6b^2}{x^{10}}+\frac{56a^5b^3}{x^{11}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{13}}+\frac{28a^2b^6}{x^{14}}+\frac{8ab^7}{x^{15}}+\frac{b^8}{x^{16}}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{a^8}{7x^7}-\frac{a^7b}{x^8}-\frac{28a^6b^2}{9x^9}-\frac{28a^5b^3}{5x^{10}}-\frac{70a^4b^4}{11x^{11}}-\frac{14a^3b^5}{3x^{12}}-\frac{28a^2b^6}{13x^{13}}-\frac{4ab^7}{7x^{14}}-\frac{b^8}{15x^{15}}$$

input
```
Int[(a + b/x)^8/x^8,x]
```

output

```
-1/15*b^8/x^15 - (4*a*b^7)/(7*x^14) - (28*a^2*b^6)/(13*x^13) - (14*a^3*b^5
)/(3*x^12) - (70*a^4*b^4)/(11*x^11) - (28*a^5*b^3)/(5*x^10) - (28*a^6*b^2)
/(9*x^9) - (a^7*b)/x^8 - a^8/(7*x^7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

method	result
norman	$\frac{-\frac{1}{15}b^8-\frac{4}{7}a\,b^7x-\frac{28}{13}a^2b^6x^2-\frac{14}{3}a^3b^5x^3-\frac{70}{11}a^4x^4b^4-\frac{28}{5}a^5b^3x^5-\frac{28}{9}a^6b^2x^6-a^7b\,x^7-\frac{1}{7}a^8x^8}{x^{15}}$
risch	$\frac{-\frac{1}{15}b^8-\frac{4}{7}a\,b^7x-\frac{28}{13}a^2b^6x^2-\frac{14}{3}a^3b^5x^3-\frac{70}{11}a^4x^4b^4-\frac{28}{5}a^5b^3x^5-\frac{28}{9}a^6b^2x^6-a^7b\,x^7-\frac{1}{7}a^8x^8}{x^{15}}$
gosper	$-\frac{6435a^8x^8+45045a^7b\,x^7+140140a^6b^2x^6+252252a^5b^3x^5+286650a^4x^4b^4+210210a^3b^5x^3+97020a^2b^6x^2+25740a\,b^7x+3003}{45045x^{15}}$
default	$-\frac{70a^4b^4}{11x^{11}}-\frac{b^8}{15x^{15}}-\frac{a^8}{7x^7}-\frac{a^7b}{x^8}-\frac{28a^2b^6}{13x^{13}}-\frac{28a^5b^3}{5x^{10}}-\frac{4a\,b^7}{7x^{14}}-\frac{28a^6b^2}{9x^9}-\frac{14a^3b^5}{3x^{12}}$
parallelrisch	$\frac{-6435a^8x^8-45045a^7b\,x^7-140140a^6b^2x^6-252252a^5b^3x^5-286650a^4x^4b^4-210210a^3b^5x^3-97020a^2b^6x^2-25740a\,b^7x-3003}{45045x^{15}}$
orering	$-\frac{(6435a^8x^8+45045a^7b\,x^7+140140a^6b^2x^6+252252a^5b^3x^5+286650a^4x^4b^4+210210a^3b^5x^3+97020a^2b^6x^2+25740a\,b^7x+300}{45045x^7(ax+b)^8}$

input
```
int((a+b/x)^8/x^8,x,method=_RETURNVERBOSE)
```

output
```
(-1/15*b^8-4/7*a*b^7*x-28/13*a^2*b^6*x^2-14/3*a^3*b^5*x^3-70/11*a^4*x^4*b^
4-28/5*a^5*b^3*x^5-28/9*a^6*b^2*x^6-a^7*b*x^7-1/7*a^8*x^8)/x^15
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx = -\frac{6435\,a^8x^8 + 45045\,a^7bx^7 + 140140\,a^6b^2x^6 + 252252\,a^5b^3x^5 + 286650\,a^4b^4x^4 + 210210\,a^3b^5x^3 + 97020\,a^2b^6x^2 + 25740\,ab^7x + 3003\,b^8}{45045\,x^{15}}$$

input
```
integrate((a+b/x)^8/x^8,x, algorithm="fricas")
```

output
```
-1/45045*(6435*a^8*x^8 + 45045*a^7*b*x^7 + 140140*a^6*b^2*x^6 + 252252*a^5
*b^3*x^5 + 286650*a^4*b^4*x^4 + 210210*a^3*b^5*x^3 + 97020*a^2*b^6*x^2 + 2
5740*a*b^7*x + 3003*b^8)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx = \frac{-6435a^8x^8 - 45045a^7bx^7 - 140140a^6b^2x^6 - 252252a^5b^3x^5 - 286650a^4b^4x^4 - 210210a^3b^5x^3 - 97020a^2b^6x^2 - 25740ab^7x - 3003b^8}{45045x^{15}}$$

input
```
integrate((a+b/x)**8/x**8,x)
```

output
```
(-6435*a**8*x**8 - 45045*a**7*b*x**7 - 140140*a**6*b**2*x**6 - 252252*a**5
*b**3*x**5 - 286650*a**4*b**4*x**4 - 210210*a**3*b**5*x**3 - 97020*a**2*b*
*6*x**2 - 25740*a*b**7*x - 3003*b**8)/(45045*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx = -\frac{6435\,a^8x^8 + 45045\,a^7bx^7 + 140140\,a^6b^2x^6 + 252252\,a^5b^3x^5 + 286650\,a^4b^4x^4 + 210210\,a^3b^5x^3 + 97020}{45045\,x^{15}}$$

input
```
integrate((a+b/x)^8/x^8,x, algorithm="maxima")
```

output
```
-1/45045*(6435*a^8*x^8 + 45045*a^7*b*x^7 + 140140*a^6*b^2*x^6 + 252252*a^5
*b^3*x^5 + 286650*a^4*b^4*x^4 + 210210*a^3*b^5*x^3 + 97020*a^2*b^6*x^2 + 2
5740*a*b^7*x + 3003*b^8)/x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx = -\frac{6435\,a^8x^8 + 45045\,a^7bx^7 + 140140\,a^6b^2x^6 + 252252\,a^5b^3x^5 + 286650\,a^4b^4x^4 + 210210\,a^3b^5x^3 + 97020}{45045\,x^{15}}$$

input
```
integrate((a+b/x)^8/x^8,x, algorithm="giac")
```

output
```
-1/45045*(6435*a^8*x^8 + 45045*a^7*b*x^7 + 140140*a^6*b^2*x^6 + 252252*a^5
*b^3*x^5 + 286650*a^4*b^4*x^4 + 210210*a^3*b^5*x^3 + 97020*a^2*b^6*x^2 + 2
5740*a*b^7*x + 3003*b^8)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx = -\frac{\frac{a^8\,x^8}{7}+a^7\,b\,x^7+\frac{28\,a^6\,b^2\,x^6}{9}+\frac{28\,a^5\,b^3\,x^5}{5}+\frac{70\,a^4\,b^4\,x^4}{11}+\frac{14\,a^3\,b^5\,x^3}{3}+\frac{28\,a^2\,b^6\,x^2}{13}+\frac{4\,a\,b^7\,x}{7}+\frac{b^8}{15}}{x^{15}}$$

input
```
int((a + b/x)^8/x^8,x)
```

output
```
-(b^8/15 + (a^8*x^8)/7 + a^7*b*x^7 + (28*a^2*b^6*x^2)/13 + (14*a^3*b^5*x^3
)/3 + (70*a^4*b^4*x^4)/11 + (28*a^5*b^3*x^5)/5 + (28*a^6*b^2*x^6)/9 + (4*a
*b^7*x)/7)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+\frac{b}{x}\right)^8}{x^8}\,dx = \frac{-6435a^8x^8-45045a^7b\,x^7-140140a^6b^2x^6-252252a^5b^3x^5-286650a^4b^4x^4-210210a^3b^5x^3-97020a^2b}{45045x^{15}}$$

input
```
int((a+b/x)^8/x^8,x)
```

output
```
( - 6435*a**8*x**8 - 45045*a**7*b*x**7 - 140140*a**6*b**2*x**6 - 252252*a*
*5*b**3*x**5 - 286650*a**4*b**4*x**4 - 210210*a**3*b**5*x**3 - 97020*a**2*
b**6*x**2 - 25740*a*b**7*x - 3003*b**8)/(45045*x**15)
```

3.62 $\int \frac{x^4}{a+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx=\frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^4}{4a^2}+\frac{x^5}{5a}-\frac{b^5\log(b+ax)}{a^6}$$

output
```
b^4*x/a^5-1/2*b^3*x^2/a^4+1/3*b^2*x^3/a^3-1/4*b*x^4/a^2+1/5*x^5/a-b^5*ln(a
*x+b)/a^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx=\frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^4}{4a^2}+\frac{x^5}{5a}-\frac{b^5\log(b+ax)}{a^6}$$

input
```
Integrate[x^4/(a + b/x),x]
```

output
```
(b^4*x)/a^5 - (b^3*x^2)/(2*a^4) + (b^2*x^3)/(3*a^3) - (b*x^4)/(4*a^2) + x^
5/(5*a) - (b^5*Log[b + a*x])/a^6
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^5}{ax+b}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^5}{a^5(ax+b)}+\frac{b^4}{a^5}-\frac{b^3x}{a^4}+\frac{b^2x^2}{a^3}-\frac{bx^3}{a^2}+\frac{x^4}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^5\log(ax+b)}{a^6}+\frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^4}{4a^2}+\frac{x^5}{5a}$$

input
```
Int[x^4/(a + b/x),x]
```

output
```
(b^4*x)/a^5 - (b^3*x^2)/(2*a^4) + (b^2*x^3)/(3*a^3) - (b*x^4)/(4*a^2) + x^
5/(5*a) - (b^5*Log[b + a*x])/a^6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\frac{1}{5}a^4x^5-\frac{1}{4}a^3b\,x^4+\frac{1}{3}a^2b^2x^3-\frac{1}{2}a\,b^3x^2+b^4x}{a^5}-\frac{b^5\ln(ax+b)}{a^6}$	63
norman	$\frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{b\,x^4}{4a^2}+\frac{x^5}{5a}-\frac{b^5\ln(ax+b)}{a^6}$	63
risch	$\frac{b^4x}{a^5}-\frac{b^3x^2}{2a^4}+\frac{b^2x^3}{3a^3}-\frac{b\,x^4}{4a^2}+\frac{x^5}{5a}-\frac{b^5\ln(ax+b)}{a^6}$	63
parallelrisch	$-\frac{-12a^5x^5+15a^4b\,x^4-20a^3b^2x^3+30a^2b^3x^2+60b^5\ln(ax+b)-60b^4xa}{60a^6}$	64

input
```
int(x^4/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^4*x^5-1/4*a^3*b*x^4+1/3*a^2*b^2*x^3-1/2*a*b^3*x^2+b^4*x)-b^5*
ln(a*x+b)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx=\frac{12\,a^5x^5-15\,a^4bx^4+20\,a^3b^2x^3-30\,a^2b^3x^2+60\,ab^4x-60\,b^5\log{(ax+b)}}{60\,a^6}$$

input
```
integrate(x^4/(a+b/x),x, algorithm="fricas")
```

output
```
1/60*(12*a^5*x^5 - 15*a^4*b*x^4 + 20*a^3*b^2*x^3 - 30*a^2*b^3*x^2 + 60*a*b
^4*x - 60*b^5*log(a*x + b))/a^6
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx = \frac{x^5}{5a} - \frac{bx^4}{4a^2} + \frac{b^2x^3}{3a^3} - \frac{b^3x^2}{2a^4} + \frac{b^4x}{a^5} - \frac{b^5\log{(ax+b)}}{a^6}$$

input
```
integrate(x**4/(a+b/x),x)
```

output
```
x**5/(5*a) - b*x**4/(4*a**2) + b**2*x**3/(3*a**3) - b**3*x**2/(2*a**4) + b
**4*x/a**5 - b**5*log(a*x + b)/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx = -\frac{b^5\log{(ax+b)}}{a^6} + \frac{12\,a^4x^5 - 15\,a^3bx^4 + 20\,a^2b^2x^3 - 30\,ab^3x^2 + 60\,b^4x}{60\,a^5}$$

input
```
integrate(x^4/(a+b/x),x, algorithm="maxima")
```

output
```
-b^5*log(a*x + b)/a^6 + 1/60*(12*a^4*x^5 - 15*a^3*b*x^4 + 20*a^2*b^2*x^3 -
 30*a*b^3*x^2 + 60*b^4*x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx = -\frac{b^5\log{(|ax+b|)}}{a^6} + \frac{12\,a^4x^5 - 15\,a^3bx^4 + 20\,a^2b^2x^3 - 30\,ab^3x^2 + 60\,b^4x}{60\,a^5}$$

input
```
integrate(x^4/(a+b/x),x, algorithm="giac")
```

output
```
-b^5*log(abs(a*x + b))/a^6 + 1/60*(12*a^4*x^5 - 15*a^3*b*x^4 + 20*a^2*b^2*
x^3 - 30*a*b^3*x^2 + 60*b^4*x)/a^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx=\frac{x^5}{5\,a}-\frac{b^5\,\ln\left(b+a\,x\right)}{a^6}-\frac{b\,x^4}{4\,a^2}+\frac{b^4\,x}{a^5}+\frac{b^2\,x^3}{3\,a^3}-\frac{b^3\,x^2}{2\,a^4}$$

input
```
int(x^4/(a + b/x),x)
```

output
```
x^5/(5*a) - (b^5*log(b + a*x))/a^6 - (b*x^4)/(4*a^2) + (b^4*x)/a^5 + (b^2*
x^3)/(3*a^3) - (b^3*x^2)/(2*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{a+\frac{b}{x}}\,dx=\frac{-60\log(ax+b)\,b^5+12a^5x^5-15a^4b\,x^4+20a^3b^2x^3-30a^2b^3x^2+60a\,b^4x}{60a^6}$$

input
```
int(x^4/(a+b/x),x)
```

output
```
( - 60*log(a*x + b)*b**5 + 12*a**5*x**5 - 15*a**4*b*x**4 + 20*a**3*b**2*x*
*3 - 30*a**2*b**3*x**2 + 60*a*b**4*x)/(60*a**6)
```

3.63 $\int \frac{x^3}{a+\frac{b}{x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{x^3}{a+\frac{b}{x}} dx = -\frac{b^3x}{a^4} + \frac{b^2x^2}{2a^3} - \frac{bx^3}{3a^2} + \frac{x^4}{4a} + \frac{b^4\log(b+ax)}{a^5}$$

output
```
-b^3*x/a^4+1/2*b^2*x^2/a^3-1/3*b*x^3/a^2+1/4*x^4/a+b^4*ln(a*x+b)/a^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a+\frac{b}{x}} dx = -\frac{b^3x}{a^4} + \frac{b^2x^2}{2a^3} - \frac{bx^3}{3a^2} + \frac{x^4}{4a} + \frac{b^4\log(b+ax)}{a^5}$$

input
```
Integrate[x^3/(a + b/x),x]
```

output
```
-((b^3*x)/a^4) + (b^2*x^2)/(2*a^3) - (b*x^3)/(3*a^2) + x^4/(4*a) + (b^4*Lo
g[b + a*x])/a^5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^4}{ax+b}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{b^4}{a^4(ax+b)}-\frac{b^3}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{a^2}+\frac{x^3}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^4\log(ax+b)}{a^5}-\frac{b^3x}{a^4}+\frac{b^2x^2}{2a^3}-\frac{bx^3}{3a^2}+\frac{x^4}{4a}$$

input
```
Int[x^3/(a + b/x),x]
```

output
```
-((b^3*x)/a^4) + (b^2*x^2)/(2*a^3) - (b*x^3)/(3*a^2) + x^4/(4*a) + (b^4*Lo
g[b + a*x])/a^5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\frac{1}{4}a^3x^4-\frac{1}{3}a^2b\,x^3+\frac{1}{2}a\,b^2x^2-b^3x}{a^4}+\frac{b^4\ln(ax+b)}{a^5}$	52
norman	$-\frac{x\,b^3}{a^4}+\frac{b^2x^2}{2a^3}-\frac{b\,x^3}{3a^2}+\frac{x^4}{4a}+\frac{b^4\ln(ax+b)}{a^5}$	52
risch	$-\frac{x\,b^3}{a^4}+\frac{b^2x^2}{2a^3}-\frac{b\,x^3}{3a^2}+\frac{x^4}{4a}+\frac{b^4\ln(ax+b)}{a^5}$	52
parallelrisch	$\frac{3a^4x^4-4a^3b\,x^3+6a^2b^2x^2+12b^4\ln(ax+b)-12a\,b^3x}{12a^5}$	53

input
```
int(x^3/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/4*a^3*x^4-1/3*a^2*b*x^3+1/2*a*b^2*x^2-b^3*x)+b^4*ln(a*x+b)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a+\frac{b}{x}}\,dx=\frac{3\,a^4x^4-4\,a^3bx^3+6\,a^2b^2x^2-12\,ab^3x+12\,b^4\log{(ax+b)}}{12\,a^5}$$

input
```
integrate(x^3/(a+b/x),x, algorithm="fricas")
```

output
```
1/12*(3*a^4*x^4 - 4*a^3*b*x^3 + 6*a^2*b^2*x^2 - 12*a*b^3*x + 12*b^4*log(a*
x + b))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{a+\frac{b}{x}}\,dx = \frac{x^4}{4a} - \frac{bx^3}{3a^2} + \frac{b^2x^2}{2a^3} - \frac{b^3x}{a^4} + \frac{b^4\log{(ax+b)}}{a^5}$$

input
```
integrate(x**3/(a+b/x),x)
```

output
```
x**4/(4*a) - b*x**3/(3*a**2) + b**2*x**2/(2*a**3) - b**3*x/a**4 + b**4*log
(a*x + b)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a+\frac{b}{x}}\,dx = \frac{b^4\log{(ax+b)}}{a^5} + \frac{3\,a^3x^4 - 4\,a^2bx^3 + 6\,ab^2x^2 - 12\,b^3x}{12\,a^4}$$

input
```
integrate(x^3/(a+b/x),x, algorithm="maxima")
```

output
```
b^4*log(a*x + b)/a^5 + 1/12*(3*a^3*x^4 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 12*b^
3*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+\frac{b}{x}}\,dx = \frac{b^4\log{(|ax+b|)}}{a^5} + \frac{3\,a^3x^4 - 4\,a^2bx^3 + 6\,ab^2x^2 - 12\,b^3x}{12\,a^4}$$

input
```
integrate(x^3/(a+b/x),x, algorithm="giac")
```

output
```
b^4*log(abs(a*x + b))/a^5 + 1/12*(3*a^3*x^4 - 4*a^2*b*x^3 + 6*a*b^2*x^2 -
12*b^3*x)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{a+\frac{b}{x}}\,dx = \frac{x^4}{4\,a} + \frac{b^4\,\ln\left(b+a\,x\right)}{a^5} - \frac{b\,x^3}{3\,a^2} - \frac{b^3\,x}{a^4} + \frac{b^2\,x^2}{2\,a^3}$$

input
```
int(x^3/(a + b/x),x)
```

output
```
x^4/(4*a) + (b^4*log(b + a*x))/a^5 - (b*x^3)/(3*a^2) - (b^3*x)/a^4 + (b^2*
x^2)/(2*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a+\frac{b}{x}}\,dx = \frac{12\log(ax+b)\,b^4 + 3a^4x^4 - 4a^3b\,x^3 + 6a^2b^2x^2 - 12a\,b^3x}{12a^5}$$

input
```
int(x^3/(a+b/x),x)
```

output
```
(12*log(a*x + b)*b**4 + 3*a**4*x**4 - 4*a**3*b*x**3 + 6*a**2*b**2*x**2 - 1
2*a*b**3*x)/(12*a**5)
```

3.64 $\int \frac{x^2}{a+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx=\frac{b^2x}{a^3}-\frac{bx^2}{2a^2}+\frac{x^3}{3a}-\frac{b^3\log(b+ax)}{a^4}$$

output
```
b^2*x/a^3-1/2*b*x^2/a^2+1/3*x^3/a-b^3*ln(a*x+b)/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx=\frac{b^2x}{a^3}-\frac{bx^2}{2a^2}+\frac{x^3}{3a}-\frac{b^3\log(b+ax)}{a^4}$$

input
```
Integrate[x^2/(a + b/x),x]
```

output
```
(b^2*x)/a^3 - (b*x^2)/(2*a^2) + x^3/(3*a) - (b^3*Log[b + a*x])/a^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^3}{ax+b}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^3}{a^3(ax+b)}+\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3\log(ax+b)}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{2a^2}+\frac{x^3}{3a}$$

input
```
Int[x^2/(a + b/x),x]
```

output
```
(b^2*x)/a^3 - (b*x^2)/(2*a^2) + x^3/(3*a) - (b^3*Log[b + a*x])/a^4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{3}a^2x^3-\frac{1}{2}ab\,x^2+b^2x}{a^3}-\frac{b^3\ln(ax+b)}{a^4}$	41
norman	$\frac{b^2x}{a^3}-\frac{b\,x^2}{2a^2}+\frac{x^3}{3a}-\frac{b^3\ln(ax+b)}{a^4}$	41
risch	$\frac{b^2x}{a^3}-\frac{b\,x^2}{2a^2}+\frac{x^3}{3a}-\frac{b^3\ln(ax+b)}{a^4}$	41
parallelrisch	$-\frac{-2a^3x^3+3a^2b\,x^2+6b^3\ln(ax+b)-6a\,b^2x}{6a^4}$	42

input
```
int(x^2/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*a^2*x^3-1/2*a*b*x^2+b^2*x)-b^3*ln(a*x+b)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx=\frac{2\,a^3x^3-3\,a^2bx^2+6\,ab^2x-6\,b^3\log\left(ax+b\right)}{6\,a^4}$$

input
```
integrate(x^2/(a+b/x),x, algorithm="fricas")
```

output
```
1/6*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*b^3*log(a*x + b))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx = \frac{x^3}{3a} - \frac{bx^2}{2a^2} + \frac{b^2x}{a^3} - \frac{b^3\log{(ax+b)}}{a^4}$$

input
```
integrate(x**2/(a+b/x),x)
```

output
```
x**3/(3*a) - b*x**2/(2*a**2) + b**2*x/a**3 - b**3*log(a*x + b)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx = -\frac{b^3\log{(ax+b)}}{a^4} + \frac{2\,a^2x^3 - 3\,abx^2 + 6\,b^2x}{6\,a^3}$$

input
```
integrate(x^2/(a+b/x),x, algorithm="maxima")
```

output
```
-b^3*log(a*x + b)/a^4 + 1/6*(2*a^2*x^3 - 3*a*b*x^2 + 6*b^2*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx = -\frac{b^3\log{(|ax+b|)}}{a^4} + \frac{2\,a^2x^3 - 3\,abx^2 + 6\,b^2x}{6\,a^3}$$

input
```
integrate(x^2/(a+b/x),x, algorithm="giac")
```

output
```
-b^3*log(abs(a*x + b))/a^4 + 1/6*(2*a^2*x^3 - 3*a*b*x^2 + 6*b^2*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx = \frac{x^3}{3\,a} - \frac{b^3\,\ln\left(b+a\,x\right)}{a^4} - \frac{b\,x^2}{2\,a^2} + \frac{b^2\,x}{a^3}$$

input
```
int(x^2/(a + b/x),x)
```

output
```
x^3/(3*a) - (b^3*log(b + a*x))/a^4 - (b*x^2)/(2*a^2) + (b^2*x)/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{a+\frac{b}{x}}\,dx = \frac{-6\log(ax+b)\,b^3+2a^3x^3-3a^2b\,x^2+6a\,b^2x}{6a^4}$$

input
```
int(x^2/(a+b/x),x)
```

output
```
( - 6*log(a*x + b)*b**3 + 2*a**3*x**3 - 3*a**2*b*x**2 + 6*a*b**2*x)/(6*a**
4)
```

3.65 $\int \frac{x}{a+\frac{b}{x}} dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{x}{a+\frac{b}{x}} dx = -\frac{bx}{a^2} + \frac{x^2}{2a} + \frac{b^2 \log(b+ax)}{a^3}$$

output
```
-b*x/a^2+1/2*x^2/a+b^2*ln(a*x+b)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+\frac{b}{x}} dx = -\frac{bx}{a^2} + \frac{x^2}{2a} + \frac{b^2 \log(b+ax)}{a^3}$$

input
```
Integrate[x/(a + b/x),x]
```

output
```
-((b*x)/a^2) + x^2/(2*a) + (b^2*Log[b + a*x])/a^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^2}{ax+b}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{b^2}{a^2(ax+b)}-\frac{b}{a^2}+\frac{x}{a}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^2\log(ax+b)}{a^3}-\frac{bx}{a^2}+\frac{x^2}{2a}$$

input
```
Int[x/(a + b/x),x]
```

output
```
-((b*x)/a^2) + x^2/(2*a) + (b^2*Log[b + a*x])/a^3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\frac{1}{2}a\,x^2-bx}{a^2}+\frac{b^2\ln(ax+b)}{a^3}$	30
norman	$-\frac{bx}{a^2}+\frac{x^2}{2a}+\frac{b^2\ln(ax+b)}{a^3}$	30
risch	$-\frac{bx}{a^2}+\frac{x^2}{2a}+\frac{b^2\ln(ax+b)}{a^3}$	30
parallelrisch	$\frac{a^2x^2+2b^2\ln(ax+b)-2abx}{2a^3}$	30

input
```
int(x/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*a*x^2-b*x)+b^2*ln(a*x+b)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+\frac{b}{x}}\,dx=\frac{a^2x^2-2\,abx+2\,b^2\log\left(ax+b\right)}{2\,a^3}$$

input
```
integrate(x/(a+b/x),x, algorithm="fricas")
```

output
```
1/2*(a^2*x^2 - 2*a*b*x + 2*b^2*log(a*x + b))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x}{a+\frac{b}{x}}\,dx=\frac{x^2}{2a}-\frac{bx}{a^2}+\frac{b^2\log{(ax+b)}}{a^3}$$

input
```
integrate(x/(a+b/x),x)
```

output
```
x**2/(2*a) - b*x/a**2 + b**2*log(a*x + b)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+\frac{b}{x}}\,dx=\frac{b^2\log{(ax+b)}}{a^3}+\frac{ax^2-2\,bx}{2\,a^2}$$

input
```
integrate(x/(a+b/x),x, algorithm="maxima")
```

output
```
b^2*log(a*x + b)/a^3 + 1/2*(a*x^2 - 2*b*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x}{a+\frac{b}{x}}\,dx=\frac{b^2\log{(|ax+b|)}}{a^3}+\frac{ax^2-2\,bx}{2\,a^2}$$

input
```
integrate(x/(a+b/x),x, algorithm="giac")
```

output
```
b^2*log(abs(a*x + b))/a^3 + 1/2*(a*x^2 - 2*b*x)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+\frac{b}{x}}\,dx=\frac{2\,b^2\,\ln\left(b+a\,x\right)+a^2\,x^2-2\,a\,b\,x}{2\,a^3}$$

input
```
int(x/(a + b/x),x)
```

output
```
(2*b^2*log(b + a*x) + a^2*x^2 - 2*a*b*x)/(2*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+\frac{b}{x}}\,dx=\frac{2\log(ax+b)\,b^2+a^2x^2-2abx}{2a^3}$$

input
```
int(x/(a+b/x),x)
```

output
```
(2*log(a*x + b)*b**2 + a**2*x**2 - 2*a*b*x)/(2*a**3)
```

3.66 $\int \frac{1}{a+\frac{b}{x}} dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int \frac{1}{a+\frac{b}{x}} dx = \frac{x}{a} - \frac{b\log(b+ax)}{a^2}$$

output
```
x/a-b*ln(a*x+b)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+\frac{b}{x}} dx = \frac{x}{a} - \frac{b\log(b+ax)}{a^2}$$

input
```
Integrate[(a + b/x)^(-1),x]
```

output
```
x/a - (b*Log[b + a*x])/a^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{x}{ax+b}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{a}-\frac{b}{a(ax+b)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{a}-\frac{b\log(ax+b)}{a^2}$$

input
```
Int[(a + b/x)^(-1),x]
```

output
```
x/a - (b*Log[b + a*x])/a^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{a}-\frac{b\ln(ax+b)}{a^2}$	19
norman	$\frac{x}{a}-\frac{b\ln(ax+b)}{a^2}$	19
risch	$\frac{x}{a}-\frac{b\ln(ax+b)}{a^2}$	19
parallelrisch	$-\frac{b\ln(ax+b)-ax}{a^2}$	19

input
```
int(1/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
x/a-b*ln(a*x+b)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+\frac{b}{x}}\,dx=\frac{ax-b\log(ax+b)}{a^2}$$

input
```
integrate(1/(a+b/x),x, algorithm="fricas")
```

output
```
(a*x - b*log(a*x + b))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{a+\frac{b}{x}}\,dx=\frac{x}{a}-\frac{b\log{(ax+b)}}{a^2}$$

input
```
integrate(1/(a+b/x),x)
```

output
```
x/a - b*log(a*x + b)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+\frac{b}{x}}\,dx=\frac{x}{a}-\frac{b\log{(ax+b)}}{a^2}$$

input
```
integrate(1/(a+b/x),x, algorithm="maxima")
```

output
```
x/a - b*log(a*x + b)/a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{a+\frac{b}{x}}\,dx=\frac{x}{a}-\frac{b\log{(|ax+b|)}}{a^2}$$

input
```
integrate(1/(a+b/x),x, algorithm="giac")
```

output
```
x/a - b*log(abs(a*x + b))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+\frac{b}{x}}\,dx = -\frac{b\,\ln\left(b+a\,x\right)-a\,x}{a^2}$$

input
```
int(1/(a + b/x),x)
```

output
```
-(b*log(b + a*x) - a*x)/a^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+\frac{b}{x}}\,dx = \frac{-\log(ax+b)\,b+ax}{a^2}$$

input
```
int(1/(a+b/x),x)
```

output
```
( - log(a*x + b)*b + a*x)/a**2
```

3.67 $\int \frac{1}{\left(a+\frac{b}{x}\right)x} dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x} dx = \frac{\log(b+ax)}{a}$$

output
```
ln(a*x+b)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x} dx = \frac{\log(b+ax)}{a}$$

input
```
Integrate[1/((a + b/x)*x),x]
```

output
```
Log[b + a*x]/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x}\right)}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{ax+b}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(ax+b)}{a}$$

input
```
Int[1/((a + b/x)*x),x]
```

output
```
Log[b + a*x]/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(ax+b)}{a}$	11
norman	$\frac{\ln(ax+b)}{a}$	11
risch	$\frac{\ln(ax+b)}{a}$	11
parallelrisch	$\frac{\ln(ax+b)}{a}$	11

input
```
int(1/(a+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(a*x+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx = \frac{\log\left(ax+b\right)}{a}$$

input
```
integrate(1/(a+b/x)/x,x, algorithm="fricas")
```

output
```
log(a*x + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx = \frac{\log\left(ax+b\right)}{a}$$

input
```
integrate(1/(a+b/x)/x,x)
```

output
```
log(a*x + b)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx=\frac{\log\left(ax+b\right)}{a}$$

input
```
integrate(1/(a+b/x)/x,x, algorithm="maxima")
```

output
```
log(a*x + b)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx=\frac{\log\left(|ax+b|\right)}{a}$$

input
```
integrate(1/(a+b/x)/x,x, algorithm="giac")
```

output
```
log(abs(a*x + b))/a
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx=\frac{\ln\left(b+a\,x\right)}{a}$$

input
```
int(1/(x*(a + b/x)),x)
```

output
```
log(b + a*x)/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx=\frac{\log(ax+b)}{a}$$

input
```
int(1/(a+b/x)/x,x)
```

output
```
log(a*x + b)/a
```

3.68 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2} dx = -\frac{\log\left(a+\frac{b}{x}\right)}{b}$$

output
```
-ln(a+b/x)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2} dx = \frac{\log(x)}{b} - \frac{\log(b+ax)}{b}$$

input
```
Integrate[1/((a + b/x)*x^2),x]
```

output
```
Log[x]/b - Log[b + a*x]/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{x}\right)}dx$$

$$\Big\downarrow \text{ 792}$$

$$-\frac{\log\left(a+\frac{b}{x}\right)}{b}$$

input
```
Int[1/((a + b/x)*x^2),x]
```

output
```
-(Log[a + b/x]/b)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln\left(a+\frac{b}{x}\right)}{b}$	14
parallelrisch	$\frac{\ln(x)-\ln(ax+b)}{b}$	16
default	$\frac{\ln(x)}{b}-\frac{\ln(ax+b)}{b}$	19
norman	$\frac{\ln(x)}{b}-\frac{\ln(ax+b)}{b}$	19
risch	$\frac{\ln(-x)}{b}-\frac{\ln(ax+b)}{b}$	21

input
```
int(1/(a+b/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b/x)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx=-\frac{\log\left(ax+b\right)-\log\left(x\right)}{b}$$

input
```
integrate(1/(a+b/x)/x^2,x, algorithm="fricas")
```

output
```
-(log(a*x + b) - log(x))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx=\frac{\log\left(x\right)-\log\left(x+\frac{b}{a}\right)}{b}$$

input
```
integrate(1/(a+b/x)/x**2,x)
```

output
```
(log(x) - log(x + b/a))/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx=-\frac{\log\left(a+\frac{b}{x}\right)}{b}$$

input
```
integrate(1/(a+b/x)/x^2,x, algorithm="maxima")
```

output
```
-log(a + b/x)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx=-\frac{\log\left(\left|a+\frac{b}{x}\right|\right)}{b}$$

input
```
integrate(1/(a+b/x)/x^2,x, algorithm="giac")
```

output
```
-log(abs(a + b/x))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b}$$

input
```
int(1/(x^2*(a + b/x)),x)
```

output
```
-(2*atanh((2*a*x)/b + 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^2}\,dx = \frac{-\log(ax+b)+\log(x)}{b}$$

input
```
int(1/(a+b/x)/x^2,x)
```

output
```
( - log(a*x + b) + log(x))/b
```

3.69 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx = -\frac{1}{bx}+\frac{a\log\left(a+\frac{b}{x}\right)}{b^2}$$

output
```
-1/b/x+a*ln(a+b/x)/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx = -\frac{1}{bx}-\frac{a\log(x)}{b^2}+\frac{a\log(b+ax)}{b^2}$$

input
```
Integrate[1/((a + b/x)*x^3),x]
```

output
```
-(1/(b*x)) - (a*Log[x])/b^2 + (a*Log[b + a*x])/b^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{x}\right)}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{1}{x^2(ax+b)}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{a^2}{b^2(ax+b)}-\frac{a}{b^2x}+\frac{1}{bx^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a\log(x)}{b^2}+\frac{a\log(ax+b)}{b^2}-\frac{1}{bx}$$

input
```
Int[1/((a + b/x)*x^3),x]
```

output
```
-(1/(b*x)) - (a*Log[x])/b^2 + (a*Log[b + a*x])/b^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

method	result	size
parallelrisch	$-\frac{a\ln(x)x-a\ln(ax+b)x+b}{b^2x}$	26
default	$-\frac{1}{xb}-\frac{a\ln(x)}{b^2}+\frac{a\ln(ax+b)}{b^2}$	29
norman	$-\frac{1}{xb}-\frac{a\ln(x)}{b^2}+\frac{a\ln(ax+b)}{b^2}$	29
risch	$-\frac{1}{xb}-\frac{a\ln(x)}{b^2}+\frac{a\ln(-ax-b)}{b^2}$	32

input
```
int(1/(a+b/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(a*ln(x)*x-a*ln(a*x+b)*x+b)/b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx = \frac{ax\log\left(ax+b\right)-ax\log\left(x\right)-b}{b^2x}$$

input
```
integrate(1/(a+b/x)/x^3,x, algorithm="fricas")
```

output
```
(a*x*log(a*x + b) - a*x*log(x) - b)/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx = \frac{a\left(-\log\left(x\right)+\log\left(x+\frac{b}{a}\right)\right)}{b^2} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x)/x**3,x)
```

output
```
a*(-log(x) + log(x + b/a))/b**2 - 1/(b*x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx = \frac{a\log\left(ax+b\right)}{b^2} - \frac{a\log\left(x\right)}{b^2} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x)/x^3,x, algorithm="maxima")
```

output
```
a*log(a*x + b)/b^2 - a*log(x)/b^2 - 1/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx = \frac{a\log\left(|ax+b|\right)}{b^2} - \frac{a\log\left(|x|\right)}{b^2} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x)/x^3,x, algorithm="giac")
```

output
```
a*log(abs(a*x + b))/b^2 - a*log(abs(x))/b^2 - 1/(b*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx=\frac{2\,a\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^2}-\frac{1}{b\,x}$$

input
```
int(1/(x^3*(a + b/x)),x)
```

output
```
(2*a*atanh((2*a*x)/b + 1))/b^2 - 1/(b*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^3}\,dx=\frac{\log(ax+b)\,ax-\log(x)\,ax-b}{b^2x}$$

input
```
int(1/(a+b/x)/x^3,x)
```

output
```
(log(a*x + b)*a*x - log(x)*a*x - b)/(b**2*x)
```

3.70 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx=-\frac{1}{2bx^2}+\frac{a}{b^2x}-\frac{a^2\log\left(a+\frac{b}{x}\right)}{b^3}$$

output
```
-1/2/b/x^2+a/b^2/x-a^2*ln(a+b/x)/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx=-\frac{1}{2bx^2}+\frac{a}{b^2x}+\frac{a^2\log(x)}{b^3}-\frac{a^2\log(b+ax)}{b^3}$$

input
```
Integrate[1/((a + b/x)*x^4),x]
```

output
```
-1/2*1/(b*x^2) + a/(b^2*x) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x])/b^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x}\right)} \, dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{x^3(ax+b)}dx$$

$$\Big\downarrow \ 54$$

$$\int \left(-\frac{a^3}{b^3(ax+b)}+\frac{a^2}{b^3x}-\frac{a}{b^2x^2}+\frac{1}{bx^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\log(x)}{b^3}-\frac{a^2\log(ax+b)}{b^3}+\frac{a}{b^2x}-\frac{1}{2bx^2}$$

input
```
Int[1/((a + b/x)*x^4),x]
```

output
```
-1/2*1/(b*x^2) + a/(b^2*x) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x])/b^3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{1}{2b\,x^2}+\frac{a^2\ln(x)}{b^3}+\frac{a}{b^2x}-\frac{a^2\ln(ax+b)}{b^3}$	41
risch	$\frac{\frac{ax}{b^2}-\frac{1}{2b}}{x^2}+\frac{a^2\ln(-x)}{b^3}-\frac{a^2\ln(ax+b)}{b^3}$	43
norman	$\frac{\frac{a\,x^2}{b^2}-\frac{x}{2b}}{x^3}+\frac{a^2\ln(x)}{b^3}-\frac{a^2\ln(ax+b)}{b^3}$	44
parallelrisch	$\frac{2a^2\ln(x)x^2-2a^2\ln(ax+b)x^2+2abx-b^2}{2b^3x^2}$	44

input
```
int(1/(a+b/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/x^2+a^2/b^3*ln(x)+a/b^2/x-a^2/b^3*ln(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx=-\frac{2\,a^2x^2\log\left(ax+b\right)-2\,a^2x^2\log\left(x\right)-2\,abx+b^2}{2\,b^3x^2}$$

input
```
integrate(1/(a+b/x)/x^4,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*x^2*log(a*x + b) - 2*a^2*x^2*log(x) - 2*a*b*x + b^2)/(b^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx = \frac{a^2\left(\log\left(x\right)-\log\left(x+\frac{b}{a}\right)\right)}{b^3}+\frac{2ax-b}{2b^2x^2}$$

input
```
integrate(1/(a+b/x)/x**4,x)
```

output
```
a**2*(log(x) - log(x + b/a))/b**3 + (2*a*x - b)/(2*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx = -\frac{a^2\log\left(ax+b\right)}{b^3}+\frac{a^2\log\left(x\right)}{b^3}+\frac{2\,ax-b}{2\,b^2x^2}$$

input
```
integrate(1/(a+b/x)/x^4,x, algorithm="maxima")
```

output
```
-a^2*log(a*x + b)/b^3 + a^2*log(x)/b^3 + 1/2*(2*a*x - b)/(b^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx = -\frac{a^2\log\left(|ax+b|\right)}{b^3}+\frac{a^2\log\left(|x|\right)}{b^3}+\frac{2\,abx-b^2}{2\,b^3x^2}$$

input
```
integrate(1/(a+b/x)/x^4,x, algorithm="giac")
```

output
```
-a^2*log(abs(a*x + b))/b^3 + a^2*log(abs(x))/b^3 + 1/2*(2*a*b*x - b^2)/(b^
3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx = -\frac{\frac{b^2}{2}-a\,b\,x}{b^3\,x^2}-\frac{2\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^3}$$

input
```
int(1/(x^4*(a + b/x)),x)
```

output
```
- (b^2/2 - a*b*x)/(b^3*x^2) - (2*a^2*atanh((2*a*x)/b + 1))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^4}\,dx = \frac{-2\log(ax+b)\,a^2x^2+2\log(x)\,a^2x^2+2abx-b^2}{2b^3x^2}$$

input
```
int(1/(a+b/x)/x^4,x)
```

output
```
( - 2*log(a*x + b)*a**2*x**2 + 2*log(x)*a**2*x**2 + 2*a*b*x - b**2)/(2*b**
3*x**2)
```

3.71 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx=-\frac{1}{3bx^3}+\frac{a}{2b^2x^2}-\frac{a^2}{b^3x}+\frac{a^3\log\left(a+\frac{b}{x}\right)}{b^4}$$

output
```
-1/3/b/x^3+1/2*a/b^2/x^2-a^2/b^3/x+a^3*ln(a+b/x)/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx=-\frac{1}{3bx^3}+\frac{a}{2b^2x^2}-\frac{a^2}{b^3x}-\frac{a^3\log(x)}{b^4}+\frac{a^3\log(b+ax)}{b^4}$$

input
```
Integrate[1/((a + b/x)*x^5),x]
```

output
```
-1/3*1/(b*x^3) + a/(2*b^2*x^2) - a^2/(b^3*x) - (a^3*Log[x])/b^4 + (a^3*Log
[b + a*x])/b^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x}\right)} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^4(ax+b)}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{a^4}{b^4(ax+b)}-\frac{a^3}{b^4x}+\frac{a^2}{b^3x^2}-\frac{a}{b^2x^3}+\frac{1}{bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3 \log(x)}{b^4}+\frac{a^3 \log(ax+b)}{b^4}-\frac{a^2}{b^3x}+\frac{a}{2b^2x^2}-\frac{1}{3bx^3}$$

input
```
Int[1/((a + b/x)*x^5),x]
```

output
```
-1/3*1/(b*x^3) + a/(2*b^2*x^2) - a^2/(b^3*x) - (a^3*Log[x])/b^4 + (a^3*Log
[b + a*x])/b^4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{1}{3b\,x^{3}}-\frac{a^{2}}{b^{3}x}+\frac{a}{2b^{2}x^{2}}-\frac{a^{3}\ln\left(x\right)}{b^{4}}+\frac{a^{3}\ln\left(ax+b\right)}{b^{4}}$	53
parallelrisch	$-\frac{6a^{3}\ln\left(x\right)x^{3}-6a^{3}\ln\left(ax+b\right)x^{3}+6a^{2}b\,x^{2}-3a\,b^{2}x+2b^{3}}{6b^{4}x^{3}}$	55
norman	$\frac{-\frac{x}{3b}+\frac{a\,x^{2}}{2b^{2}}-\frac{a^{2}x^{3}}{b^{3}}}{x^{4}}+\frac{a^{3}\ln\left(ax+b\right)}{b^{4}}-\frac{a^{3}\ln\left(x\right)}{b^{4}}$	56
risch	$\frac{-\frac{a^{2}x^{2}}{b^{3}}+\frac{ax}{2b^{2}}-\frac{1}{3b}}{x^{3}}+\frac{a^{3}\ln\left(-ax-b\right)}{b^{4}}-\frac{a^{3}\ln\left(x\right)}{b^{4}}$	56

input
```
int(1/(a+b/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/x^3-a^2/b^3/x+1/2*a/b^2/x^2-a^3/b^4*ln(x)+a^3/b^4*ln(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx=\frac{6\,a^3x^3\log\left(ax+b\right)-6\,a^3x^3\log\left(x\right)-6\,a^2bx^2+3\,ab^2x-2\,b^3}{6\,b^4x^3}$$

input
```
integrate(1/(a+b/x)/x^5,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*x^3*log(a*x + b) - 6*a^3*x^3*log(x) - 6*a^2*b*x^2 + 3*a*b^2*x -
 2*b^3)/(b^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx = \frac{a^3\left(-\log\left(x\right)+\log\left(x+\frac{b}{a}\right)\right)}{b^4} + \frac{-6a^2x^2+3abx-2b^2}{6b^3x^3}$$

input
```
integrate(1/(a+b/x)/x**5,x)
```

output
```
a**3*(-log(x) + log(x + b/a))/b**4 + (-6*a**2*x**2 + 3*a*b*x - 2*b**2)/(6*
b**3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx = \frac{a^3\log\left(ax+b\right)}{b^4} - \frac{a^3\log\left(x\right)}{b^4} - \frac{6\,a^2x^2-3\,abx+2\,b^2}{6\,b^3x^3}$$

input
```
integrate(1/(a+b/x)/x^5,x, algorithm="maxima")
```

output
```
a^3*log(a*x + b)/b^4 - a^3*log(x)/b^4 - 1/6*(6*a^2*x^2 - 3*a*b*x + 2*b^2)/
(b^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx = \frac{a^3\log\left(|ax+b|\right)}{b^4} - \frac{a^3\log\left(|x|\right)}{b^4} - \frac{6\,a^2bx^2-3\,ab^2x+2\,b^3}{6\,b^4x^3}$$

input
```
integrate(1/(a+b/x)/x^5,x, algorithm="giac")
```

output
```
a^3*log(abs(a*x + b))/b^4 - a^3*log(abs(x))/b^4 - 1/6*(6*a^2*b*x^2 - 3*a*b
^2*x + 2*b^3)/(b^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx = \frac{2\,a^3\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^4} - \frac{a^2\,b\,x^2-\frac{a\,b^2\,x}{2}+\frac{b^3}{3}}{b^4\,x^3}$$

input
```
int(1/(x^5*(a + b/x)),x)
```

output
```
(2*a^3*atanh((2*a*x)/b + 1))/b^4 - (b^3/3 + a^2*b*x^2 - (a*b^2*x)/2)/(b^4*
x^3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^5}\,dx = \frac{6\log(ax+b)\,a^3x^3-6\log(x)\,a^3x^3-6a^2b\,x^2+3a\,b^2x-2b^3}{6b^4x^3}$$

input
```
int(1/(a+b/x)/x^5,x)
```

output
```
(6*log(a*x + b)*a**3*x**3 - 6*log(x)*a**3*x**3 - 6*a**2*b*x**2 + 3*a*b**2*
x - 2*b**3)/(6*b**4*x**3)
```

3.72 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx=-\frac{1}{4bx^4}+\frac{a}{3b^2x^3}-\frac{a^2}{2b^3x^2}+\frac{a^3}{b^4x}-\frac{a^4\log\left(a+\frac{b}{x}\right)}{b^5}$$

output
```
-1/4/b/x^4+1/3*a/b^2/x^3-1/2*a^2/b^3/x^2+a^3/b^4/x-a^4*ln(a+b/x)/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx=-\frac{1}{4bx^4}+\frac{a}{3b^2x^3}-\frac{a^2}{2b^3x^2}+\frac{a^3}{b^4x}+\frac{a^4\log(x)}{b^5}-\frac{a^4\log(b+ax)}{b^5}$$

input
```
Integrate[1/((a + b/x)*x^6),x]
```

output
```
-1/4*1/(b*x^4) + a/(3*b^2*x^3) - a^2/(2*b^3*x^2) + a^3/(b^4*x) + (a^4*Log[
x])/b^5 - (a^4*Log[b + a*x])/b^5
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(a+\frac{b}{x}\right)}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^5(ax+b)}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{a^5}{b^5(ax+b)}+\frac{a^4}{b^5x}-\frac{a^3}{b^4x^2}+\frac{a^2}{b^3x^3}-\frac{a}{b^2x^4}+\frac{1}{bx^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\log(x)}{b^5}-\frac{a^4\log(ax+b)}{b^5}+\frac{a^3}{b^4x}-\frac{a^2}{2b^3x^2}+\frac{a}{3b^2x^3}-\frac{1}{4bx^4}$$

input
```
Int[1/((a + b/x)*x^6),x]
```

output
```
-1/4*1/(b*x^4) + a/(3*b^2*x^3) - a^2/(2*b^3*x^2) + a^3/(b^4*x) + (a^4*Log[
x])/b^5 - (a^4*Log[b + a*x])/b^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{1}{4b\,x^{4}}-\frac{a^{2}}{2b^{3}x^{2}}+\frac{a^{4}\ln\left(x\right)}{b^{5}}+\frac{a}{3b^{2}x^{3}}+\frac{a^{3}}{b^{4}x}-\frac{a^{4}\ln\left(ax+b\right)}{b^{5}}$	63
risch	$\frac{\frac{a^{3}x^{3}}{b^{4}}-\frac{a^{2}x^{2}}{2b^{3}}+\frac{ax}{3b^{2}}-\frac{1}{4b}}{x^{4}}+\frac{a^{4}\ln\left(-x\right)}{b^{5}}-\frac{a^{4}\ln\left(ax+b\right)}{b^{5}}$	65
norman	$\frac{\frac{a^{3}x^{4}}{b^{4}}-\frac{x}{4b}+\frac{a\,x^{2}}{3b^{2}}-\frac{a^{2}x^{3}}{2b^{3}}}{x^{5}}+\frac{a^{4}\ln\left(x\right)}{b^{5}}-\frac{a^{4}\ln\left(ax+b\right)}{b^{5}}$	66
parallelrisch	$\frac{12a^{4}\ln\left(x\right)x^{4}-12a^{4}\ln\left(ax+b\right)x^{4}+12a^{3}b\,x^{3}-6a^{2}b^{2}x^{2}+4a\,b^{3}x-3b^{4}}{12b^{5}x^{4}}$	66

input
```
int(1/(a+b/x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/x^4-1/2*a^2/b^3/x^2+a^4/b^5*ln(x)+1/3*a/b^2/x^3+a^3/b^4/x-a^4/b^5*l
n(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{6}}\,dx = -\frac{12\,a^{4}x^{4}\log\left(ax+b\right)-12\,a^{4}x^{4}\log\left(x\right)-12\,a^{3}bx^{3}+6\,a^{2}b^{2}x^{2}-4\,ab^{3}x+3\,b^{4}}{12\,b^{5}x^{4}}$$

input
```
integrate(1/(a+b/x)/x^6,x, algorithm="fricas")
```

output
```
-1/12*(12*a^4*x^4*log(a*x + b) - 12*a^4*x^4*log(x) - 12*a^3*b*x^3 + 6*a^2*
b^2*x^2 - 4*a*b^3*x + 3*b^4)/(b^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx=\frac{a^4\left(\log\left(x\right)-\log\left(x+\frac{b}{a}\right)\right)}{b^5}+\frac{12a^3x^3-6a^2bx^2+4ab^2x-3b^3}{12b^4x^4}$$

input
```
integrate(1/(a+b/x)/x**6,x)
```

output
```
a**4*(log(x) - log(x + b/a))/b**5 + (12*a**3*x**3 - 6*a**2*b*x**2 + 4*a*b*
*2*x - 3*b**3)/(12*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx=-\frac{a^4\log\left(ax+b\right)}{b^5}+\frac{a^4\log\left(x\right)}{b^5}+\frac{12\,a^3x^3-6\,a^2bx^2+4\,ab^2x-3\,b^3}{12\,b^4x^4}$$

input
```
integrate(1/(a+b/x)/x^6,x, algorithm="maxima")
```

output
```
-a^4*log(a*x + b)/b^5 + a^4*log(x)/b^5 + 1/12*(12*a^3*x^3 - 6*a^2*b*x^2 +
4*a*b^2*x - 3*b^3)/(b^4*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx=-\frac{a^4\log\left(|ax+b|\right)}{b^5}+\frac{a^4\log\left(|x|\right)}{b^5}+\frac{12\,a^3bx^3-6\,a^2b^2x^2+4\,ab^3x-3\,b^4}{12\,b^5x^4}$$

input
```
integrate(1/(a+b/x)/x^6,x, algorithm="giac")
```

output
```
-a^4*log(abs(a*x + b))/b^5 + a^4*log(abs(x))/b^5 + 1/12*(12*a^3*b*x^3 - 6*
a^2*b^2*x^2 + 4*a*b^3*x - 3*b^4)/(b^5*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx = -\frac{-a^3\,b\,x^3+\frac{a^2\,b^2\,x^2}{2}-\frac{a\,b^3\,x}{3}+\frac{b^4}{4}}{b^5\,x^4}-\frac{2\,a^4\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^5}$$

input
```
int(1/(x^6*(a + b/x)),x)
```

output
```
- (b^4/4 - a^3*b*x^3 + (a^2*b^2*x^2)/2 - (a*b^3*x)/3)/(b^5*x^4) - (2*a^4*a
tanh((2*a*x)/b + 1))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^6}\,dx = \frac{-12\log(ax+b)\,a^4x^4+12\log(x)\,a^4x^4+12a^3b\,x^3-6a^2b^2x^2+4a\,b^3x-3b^4}{12b^5x^4}$$

input
```
int(1/(a+b/x)/x^6,x)
```

output
```
( - 12*log(a*x + b)*a**4*x**4 + 12*log(x)*a**4*x**4 + 12*a**3*b*x**3 - 6*a
**2*b**2*x**2 + 4*a*b**3*x - 3*b**4)/(12*b**5*x**4)
```

3.73 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx=-\frac{1}{5bx^5}+\frac{a}{4b^2x^4}-\frac{a^2}{3b^3x^3}+\frac{a^3}{2b^4x^2}-\frac{a^4}{b^5x}+\frac{a^5\log\left(a+\frac{b}{x}\right)}{b^6}$$

output
```
-1/5/b/x^5+1/4*a/b^2/x^4-1/3*a^2/b^3/x^3+1/2*a^3/b^4/x^2-a^4/b^5/x+a^5*ln(
a+b/x)/b^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx=-\frac{1}{5bx^5}+\frac{a}{4b^2x^4}-\frac{a^2}{3b^3x^3}+\frac{a^3}{2b^4x^2}-\frac{a^4}{b^5x}-\frac{a^5\log(x)}{b^6}+\frac{a^5\log(b+ax)}{b^6}$$

input
```
Integrate[1/((a + b/x)*x^7),x]
```

output
```
-1/5*1/(b*x^5) + a/(4*b^2*x^4) - a^2/(3*b^3*x^3) + a^3/(2*b^4*x^2) - a^4/(
b^5*x) - (a^5*Log[x])/b^6 + (a^5*Log[b + a*x])/b^6
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x}\right)} \, dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{1}{x^6(ax+b)}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{a^6}{b^6(ax+b)}-\frac{a^5}{b^6x}+\frac{a^4}{b^5x^2}-\frac{a^3}{b^4x^3}+\frac{a^2}{b^3x^4}-\frac{a}{b^2x^5}+\frac{1}{bx^6}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a^5\log(x)}{b^6}+\frac{a^5\log(ax+b)}{b^6}-\frac{a^4}{b^5x}+\frac{a^3}{2b^4x^2}-\frac{a^2}{3b^3x^3}+\frac{a}{4b^2x^4}-\frac{1}{5bx^5}$$

input
```
Int[1/((a + b/x)*x^7),x]
```

output
```
-1/5*1/(b*x^5) + a/(4*b^2*x^4) - a^2/(3*b^3*x^3) + a^3/(2*b^4*x^2) - a^4/(
b^5*x) - (a^5*Log[x])/b^6 + (a^5*Log[b + a*x])/b^6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{1}{5 b \,x^{5}}-\frac{a^{2}}{3 b^{3} x^{3}}-\frac{a^{4}}{b^{5} x}+\frac{a}{4 b^{2} x^{4}}+\frac{a^{3}}{2 b^{4} x^{2}}-\frac{a^{5} \ln \left(x \right)}{b^{6}}+\frac{a^{5} \ln \left(a x +b \right)}{b^{6}}$	75
parallelrisch	$-\frac{60 a^{5} \ln \left(x \right) x^{5}-60 a^{5} \ln \left(a x +b \right) x^{5}+60 a^{4} b \,x^{4}-30 a^{3} b^{2} x^{3}+20 a^{2} b^{3} x^{2}-15 b^{4} x a +12 b^{5}}{60 b^{6} x^{5}}$	77
norman	$\frac{-\frac{x}{5 b}+\frac{a \,x^{2}}{4 b^{2}}-\frac{a^{2} x^{3}}{3 b^{3}}+\frac{a^{3} x^{4}}{2 b^{4}}-\frac{a^{4} x^{5}}{b^{5}}}{x^{6}}+\frac{a^{5} \ln \left(a x +b \right)}{b^{6}}-\frac{a^{5} \ln \left(x \right)}{b^{6}}$	78
risch	$\frac{-\frac{a^{4} x^{4}}{b^{5}}+\frac{a^{3} x^{3}}{2 b^{4}}-\frac{a^{2} x^{2}}{3 b^{3}}+\frac{a x}{4 b^{2}}-\frac{1}{5 b}}{x^{5}}+\frac{a^{5} \ln \left(-a x -b \right)}{b^{6}}-\frac{a^{5} \ln \left(x \right)}{b^{6}}$	78

input
```
int(1/(a+b/x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b/x^5-1/3*a^2/b^3/x^3-a^4/b^5/x+1/4*a/b^2/x^4+1/2*a^3/b^4/x^2-a^5/b^6
*ln(x)+a^5/b^6*ln(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right) x^7} \, dx = \frac{60 \, a^{5} x^{5} \log \left(a x + b\right) - 60 \, a^{5} x^{5} \log \left(x\right) - 60 \, a^{4} b x^{4} + 30 \, a^{3} b^{2} x^{3} - 20 \, a^{2} b^{3} x^{2} + 15 \, a b^{4} x - 12 \, b^{5}}{60 \, b^{6} x^{5}}$$

input
```
integrate(1/(a+b/x)/x^7,x, algorithm="fricas")
```

output
```
1/60*(60*a^5*x^5*log(a*x + b) - 60*a^5*x^5*log(x) - 60*a^4*b*x^4 + 30*a^3*
b^2*x^3 - 20*a^2*b^3*x^2 + 15*a*b^4*x - 12*b^5)/(b^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx = \frac{a^5\left(-\log\left(x\right)+\log\left(x+\frac{b}{a}\right)\right)}{b^6} + \frac{-60a^4x^4+30a^3bx^3-20a^2b^2x^2+15ab^3x-12b^4}{60b^5x^5}$$

input
```
integrate(1/(a+b/x)/x**7,x)
```

output
```
a**5*(-log(x) + log(x + b/a))/b**6 + (-60*a**4*x**4 + 30*a**3*b*x**3 - 20*
a**2*b**2*x**2 + 15*a*b**3*x - 12*b**4)/(60*b**5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx = \frac{a^5\log\left(ax+b\right)}{b^6} - \frac{a^5\log\left(x\right)}{b^6} - \frac{60\,a^4x^4-30\,a^3bx^3+20\,a^2b^2x^2-15\,ab^3x+12\,b^4}{60\,b^5x^5}$$

input
```
integrate(1/(a+b/x)/x^7,x, algorithm="maxima")
```

output
```
a^5*log(a*x + b)/b^6 - a^5*log(x)/b^6 - 1/60*(60*a^4*x^4 - 30*a^3*b*x^3 +
20*a^2*b^2*x^2 - 15*a*b^3*x + 12*b^4)/(b^5*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx = \frac{a^5\log\left(|ax+b|\right)}{b^6} - \frac{a^5\log\left(|x|\right)}{b^6} - \frac{60\,a^4bx^4 - 30\,a^3b^2x^3 + 20\,a^2b^3x^2 - 15\,ab^4x + 12\,b^5}{60\,b^6x^5}$$

input
```
integrate(1/(a+b/x)/x^7,x, algorithm="giac")
```

output
```
a^5*log(abs(a*x + b))/b^6 - a^5*log(abs(x))/b^6 - 1/60*(60*a^4*b*x^4 - 30*
a^3*b^2*x^3 + 20*a^2*b^3*x^2 - 15*a*b^4*x + 12*b^5)/(b^6*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx = \frac{2\,a^5\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^6} - \frac{a^4\,b\,x^4 - \frac{a^3\,b^2\,x^3}{2} + \frac{a^2\,b^3\,x^2}{3} - \frac{a\,b^4\,x}{4} + \frac{b^5}{5}}{b^6\,x^5}$$

input
```
int(1/(x^7*(a + b/x)),x)
```

output
```
(2*a^5*atanh((2*a*x)/b + 1))/b^6 - (b^5/5 + a^4*b*x^4 + (a^2*b^3*x^2)/3 -
(a^3*b^2*x^3)/2 - (a*b^4*x)/4)/(b^6*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^7}\,dx = \frac{60\log(ax+b)\,a^5x^5 - 60\log(x)\,a^5x^5 - 60a^4b\,x^4 + 30a^3b^2x^3 - 20a^2b^3x^2 + 15a\,b^4x - 12b^5}{60b^6x^5}$$

input
```
int(1/(a+b/x)/x^7,x)
```

output
```
(60*log(a*x + b)*a**5*x**5 - 60*log(x)*a**5*x**5 - 60*a**4*b*x**4 + 30*a**
3*b**2*x**3 - 20*a**2*b**3*x**2 + 15*a*b**4*x - 12*b**5)/(60*b**6*x**5)
```

3.74 $\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2} \, dx = -\frac{6b^5x}{a^7}+\frac{5b^4x^2}{2a^6}-\frac{3b^3x^3}{3a^5}+\frac{3b^2x^4}{4a^4}-\frac{2bx^5}{5a^3}+\frac{x^6}{6a^2}+\frac{b^7}{a^8(b+ax)}+\frac{7b^6\log(b+ax)}{a^8}$$

output
```
-6*b^5*x/a^7+5/2*b^4*x^2/a^6-4/3*b^3*x^3/a^5+3/4*b^2*x^4/a^4-2/5*b*x^5/a^3
+1/6*x^6/a^2+b^7/a^8/(a*x+b)+7*b^6*ln(a*x+b)/a^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2} \, dx = \frac{-360ab^5x+150a^2b^4x^2-80a^3b^3x^3+45a^4b^2x^4-24a^5bx^5+10a^6x^6+\frac{60b^7}{b+ax}+420b^6\log(b+ax)}{60a^8}$$

input
```
Integrate[x^5/(a + b/x)^2,x]
```

output
```
(-360*a*b^5*x + 150*a^2*b^4*x^2 - 80*a^3*b^3*x^3 + 45*a^4*b^2*x^4 - 24*a^5
*b*x^5 + 10*a^6*x^6 + (60*b^7)/(b + a*x) + 420*b^6*Log[b + a*x])/(60*a^8)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^7}{(ax+b)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^7}{a^7(ax+b)^2}+\frac{7b^6}{a^7(ax+b)}-\frac{6b^5}{a^7}+\frac{5b^4x}{a^6}-\frac{4b^3x^2}{a^5}+\frac{3b^2x^3}{a^4}-\frac{2bx^4}{a^3}+\frac{x^5}{a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^7}{a^8(ax+b)}+\frac{7b^6\log(ax+b)}{a^8}-\frac{6b^5x}{a^7}+\frac{5b^4x^2}{2a^6}-\frac{4b^3x^3}{3a^5}+\frac{3b^2x^4}{4a^4}-\frac{2bx^5}{5a^3}+\frac{x^6}{6a^2}$$

input
```
Int[x^5/(a + b/x)^2,x]
```

output
```
(-6*b^5*x)/a^7 + (5*b^4*x^2)/(2*a^6) - (4*b^3*x^3)/(3*a^5) + (3*b^2*x^4)/(
4*a^4) - (2*b*x^5)/(5*a^3) + x^6/(6*a^2) + b^7/(a^8*(b + a*x)) + (7*b^6*Lo
g[b + a*x])/a^8
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\frac{1}{6}a^5x^6-\frac{2}{5}a^4b\,x^5+\frac{3}{4}a^3b^2x^4-\frac{4}{3}a^2b^3x^3+\frac{5}{2}b^4x^2a-6b^5x}{a^7}+\frac{b^7}{a^8(ax+b)}+\frac{7b^6\ln(ax+b)}{a^8}$	89
risch	$-\frac{6b^5x}{a^7}+\frac{5b^4x^2}{2a^6}-\frac{4b^3x^3}{3a^5}+\frac{3b^2x^4}{4a^4}-\frac{2b\,x^5}{5a^3}+\frac{x^6}{6a^2}+\frac{b^7}{a^8(ax+b)}+\frac{7b^6\ln(ax+b)}{a^8}$	89
norman	$\frac{\frac{7b^7}{a^8}+\frac{x^7}{6a}-\frac{7b\,x^6}{30a^2}+\frac{7b^2x^5}{20a^3}-\frac{7b^3x^4}{12a^4}+\frac{7b^4x^3}{6a^5}-\frac{7b^5x^2}{2a^6}}{ax+b}+\frac{7b^6\ln(ax+b)}{a^8}$	94
parallelrisch	$\frac{10a^7x^7-14a^6b\,x^6+21a^5b^2x^5-35a^4b^3x^4+70b^4x^3a^3+420\ln(ax+b)xa\,b^6-210b^5x^2a^2+420\ln(ax+b)b^7+420b^7}{60a^8(ax+b)}$	104

input
```
int(x^5/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^7*(1/6*a^5*x^6-2/5*a^4*b*x^5+3/4*a^3*b^2*x^4-4/3*a^2*b^3*x^3+5/2*b^4*x
^2*a-6*b^5*x)+b^7/a^8/(a*x+b)+7*b^6*ln(a*x+b)/a^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{10\,a^7x^7 - 14\,a^6bx^6 + 21\,a^5b^2x^5 - 35\,a^4b^3x^4 + 70\,a^3b^4x^3 - 210\,a^2b^5x^2 - 360\,ab^6x + 60\,b^7 + 420\,(ab^6x + b}{60\,(a^9x + a^8b)}$$

input
```
integrate(x^5/(a+b/x)^2,x, algorithm="fricas")
```

output
```
1/60*(10*a^7*x^7 - 14*a^6*b*x^6 + 21*a^5*b^2*x^5 - 35*a^4*b^3*x^4 + 70*a^3
*b^4*x^3 - 210*a^2*b^5*x^2 - 360*a*b^6*x + 60*b^7 + 420*(a*b^6*x + b^7)*lo
g(a*x + b))/(a^9*x + a^8*b)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{b^7}{a^9x + a^8b} + \frac{x^6}{6a^2} - \frac{2bx^5}{5a^3} + \frac{3b^2x^4}{4a^4} - \frac{4b^3x^3}{3a^5} + \frac{5b^4x^2}{2a^6} - \frac{6b^5x}{a^7} + \frac{7b^6\log(ax+b)}{a^8}$$

input
```
integrate(x**5/(a+b/x)**2,x)
```

output
```
b**7/(a**9*x + a**8*b) + x**6/(6*a**2) - 2*b*x**5/(5*a**3) + 3*b**2*x**4/(
4*a**4) - 4*b**3*x**3/(3*a**5) + 5*b**4*x**2/(2*a**6) - 6*b**5*x/a**7 + 7*
b**6*log(a*x + b)/a**8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{b^7}{a^9x+a^8b} + \frac{7\,b^6\log\left(ax+b\right)}{a^8} + \frac{10\,a^5x^6 - 24\,a^4bx^5 + 45\,a^3b^2x^4 - 80\,a^2b^3x^3 + 150\,ab^4x^2 - 360\,b^5x}{60\,a^7}$$

input
```
integrate(x^5/(a+b/x)^2,x, algorithm="maxima")
```

output
```
b^7/(a^9*x + a^8*b) + 7*b^6*log(a*x + b)/a^8 + 1/60*(10*a^5*x^6 - 24*a^4*b
*x^5 + 45*a^3*b^2*x^4 - 80*a^2*b^3*x^3 + 150*a*b^4*x^2 - 360*b^5*x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{7\,b^6\log\left(|ax+b|\right)}{a^8} + \frac{b^7}{(ax+b)a^8} + \frac{10\,a^{10}x^6 - 24\,a^9bx^5 + 45\,a^8b^2x^4 - 80\,a^7b^3x^3 + 150\,a^6b^4x^2 - 360\,a^5b^5x}{60\,a^{12}}$$

input
```
integrate(x^5/(a+b/x)^2,x, algorithm="giac")
```

output
```
7*b^6*log(abs(a*x + b))/a^8 + b^7/((a*x + b)*a^8) + 1/60*(10*a^10*x^6 - 24
*a^9*b*x^5 + 45*a^8*b^2*x^4 - 80*a^7*b^3*x^3 + 150*a^6*b^4*x^2 - 360*a^5*b
^5*x)/a^12
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{x^6}{6\,a^2}+\frac{7\,b^6\,\ln\left(b+a\,x\right)}{a^8}-\frac{2\,b\,x^5}{5\,a^3}-\frac{6\,b^5\,x}{a^7}+\frac{3\,b^2\,x^4}{4\,a^4}-\frac{4\,b^3\,x^3}{3\,a^5}+\frac{5\,b^4\,x^2}{2\,a^6}+\frac{b^7}{a\,\left(x\,a^8+b\,a^7\right)}$$

input
```
int(x^5/(a + b/x)^2,x)
```

output
```
x^6/(6*a^2) + (7*b^6*log(b + a*x))/a^8 - (2*b*x^5)/(5*a^3) - (6*b^5*x)/a^7
 + (3*b^2*x^4)/(4*a^4) - (4*b^3*x^3)/(3*a^5) + (5*b^4*x^2)/(2*a^6) + b^7/(
a*(a^7*b + a^8*x))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{420\log(ax+b)\,a\,b^6x+420\log(ax+b)\,b^7+10a^7x^7-14a^6b\,x^6+21a^5b^2x^5-35a^4b^3x^4+70a^3b^4x^3-210}{60a^8\,(ax+b)}$$

input
```
int(x^5/(a+b/x)^2,x)
```

output
```
(420*log(a*x + b)*a*b**6*x + 420*log(a*x + b)*b**7 + 10*a**7*x**7 - 14*a**
6*b*x**6 + 21*a**5*b**2*x**5 - 35*a**4*b**3*x**4 + 70*a**3*b**4*x**3 - 210
*a**2*b**5*x**2 - 420*a*b**6*x)/(60*a**8*(a*x + b))
```

3.75 $\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2} \, dx = \frac{5b^4x}{a^6} - \frac{2b^3x^2}{a^5} + \frac{b^2x^3}{a^4} - \frac{bx^4}{2a^3} + \frac{x^5}{5a^2} - \frac{b^6}{a^7(b+ax)} - \frac{6b^5\log(b+ax)}{a^7}$$

output
```
5*b^4*x/a^6-2*b^3*x^2/a^5+b^2*x^3/a^4-1/2*b*x^4/a^3+1/5*x^5/a^2-b^6/a^7/(a
*x+b)-6*b^5*ln(a*x+b)/a^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2} \, dx = \frac{50ab^4x - 20a^2b^3x^2 + 10a^3b^2x^3 - 5a^4bx^4 + 2a^5x^5 - \frac{10b^6}{b+ax} - 60b^5\log(b+ax)}{10a^7}$$

input
```
Integrate[x^4/(a + b/x)^2,x]
```

output
```
(50*a*b^4*x - 20*a^2*b^3*x^2 + 10*a^3*b^2*x^3 - 5*a^4*b*x^4 + 2*a^5*x^5 -
(10*b^6)/(b + a*x) - 60*b^5*Log[b + a*x])/(10*a^7)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{x^6}{(ax+b)^2}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{b^6}{a^6(ax+b)^2}-\frac{6b^5}{a^6(ax+b)}+\frac{5b^4}{a^6}-\frac{4b^3x}{a^5}+\frac{3b^2x^2}{a^4}-\frac{2bx^3}{a^3}+\frac{x^4}{a^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{b^6}{a^7(ax+b)}-\frac{6b^5\log(ax+b)}{a^7}+\frac{5b^4x}{a^6}-\frac{2b^3x^2}{a^5}+\frac{b^2x^3}{a^4}-\frac{bx^4}{2a^3}+\frac{x^5}{5a^2}$$

input
```
Int[x^4/(a + b/x)^2,x]
```

output
```
(5*b^4*x)/a^6 - (2*b^3*x^2)/a^5 + (b^2*x^3)/a^4 - (b*x^4)/(2*a^3) + x^5/(5
*a^2) - b^6/(a^7*(b + a*x)) - (6*b^5*Log[b + a*x])/a^7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\frac{1}{5}a^4x^5-\frac{1}{2}a^3b\,x^4+a^2b^2x^3-2a\,b^3x^2+5b^4x}{a^6}-\frac{b^6}{a^7(ax+b)}-\frac{6b^5\ln(ax+b)}{a^7}$	78
risch	$\frac{5b^4x}{a^6}-\frac{2b^3x^2}{a^5}+\frac{b^2x^3}{a^4}-\frac{b\,x^4}{2a^3}+\frac{x^5}{5a^2}-\frac{b^6}{a^7(ax+b)}-\frac{6b^5\ln(ax+b)}{a^7}$	78
norman	$\frac{-\frac{6b^6}{a^7}+\frac{x^6}{5a}-\frac{3b\,x^5}{10a^2}+\frac{b^2x^4}{2a^3}-\frac{b^3x^3}{a^4}+\frac{3b^4x^2}{a^5}}{ax+b}-\frac{6b^5\ln(ax+b)}{a^7}$	83
parallelrisch	$-\frac{-2a^6x^6+3a^5b\,x^5-5a^4b^2x^4+10a^3x^3b^3+60\ln(ax+b)xa\,b^5-30b^4x^2a^2+60\ln(ax+b)b^6+60b^6}{10a^7(ax+b)}$	93

input
```
int(x^4/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*(1/5*a^4*x^5-1/2*a^3*b*x^4+a^2*b^2*x^3-2*a*b^3*x^2+5*b^4*x)-b^6/a^7/
(a*x+b)-6*b^5*ln(a*x+b)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2} dx = \frac{2 a^6 x^6 - 3 a^5 b x^5 + 5 a^4 b^2 x^4 - 10 a^3 b^3 x^3 + 30 a^2 b^4 x^2 + 50 a b^5 x - 10 b^6 - 60 (a b^5 x + b^6) \log(ax+b)}{10 (a^8 x + a^7 b)}$$

input
```
integrate(x^4/(a+b/x)^2,x, algorithm="fricas")
```

output
```
1/10*(2*a^6*x^6 - 3*a^5*b*x^5 + 5*a^4*b^2*x^4 - 10*a^3*b^3*x^3 + 30*a^2*b^
4*x^2 + 50*a*b^5*x - 10*b^6 - 60*(a*b^5*x + b^6)*log(a*x + b))/(a^8*x + a^
7*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2} dx = -\frac{b^6}{a^8 x + a^7 b} + \frac{x^5}{5a^2} - \frac{b x^4}{2a^3} + \frac{b^2 x^3}{a^4} - \frac{2b^3 x^2}{a^5} + \frac{5b^4 x}{a^6} - \frac{6b^5 \log(ax+b)}{a^7}$$

input
```
integrate(x**4/(a+b/x)**2,x)
```

output
```
-b**6/(a**8*x + a**7*b) + x**5/(5*a**2) - b*x**4/(2*a**3) + b**2*x**3/a**4
 - 2*b**3*x**2/a**5 + 5*b**4*x/a**6 - 6*b**5*log(a*x + b)/a**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{b^6}{a^8x+a^7b} - \frac{6\,b^5\log\left(ax+b\right)}{a^7} + \frac{2\,a^4x^5 - 5\,a^3bx^4 + 10\,a^2b^2x^3 - 20\,ab^3x^2 + 50\,b^4x}{10\,a^6}$$

input
```
integrate(x^4/(a+b/x)^2,x, algorithm="maxima")
```

output
```
-b^6/(a^8*x + a^7*b) - 6*b^5*log(a*x + b)/a^7 + 1/10*(2*a^4*x^5 - 5*a^3*b*
x^4 + 10*a^2*b^2*x^3 - 20*a*b^3*x^2 + 50*b^4*x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{6\,b^5\log\left(|ax+b|\right)}{a^7} - \frac{b^6}{(ax+b)a^7} + \frac{2\,a^8x^5 - 5\,a^7bx^4 + 10\,a^6b^2x^3 - 20\,a^5b^3x^2 + 50\,a^4b^4x}{10\,a^{10}}$$

input
```
integrate(x^4/(a+b/x)^2,x, algorithm="giac")
```

output
```
-6*b^5*log(abs(a*x + b))/a^7 - b^6/((a*x + b)*a^7) + 1/10*(2*a^8*x^5 - 5*a
^7*b*x^4 + 10*a^6*b^2*x^3 - 20*a^5*b^3*x^2 + 50*a^4*b^4*x)/a^10
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{x^5}{5\,a^2}-\frac{6\,b^5\,\ln\left(b+a\,x\right)}{a^7}-\frac{b\,x^4}{2\,a^3}+\frac{5\,b^4\,x}{a^6}+\frac{b^2\,x^3}{a^4}-\frac{2\,b^3\,x^2}{a^5}-\frac{b^6}{a\,\left(x\,a^7+b\,a^6\right)}$$

input
```
int(x^4/(a + b/x)^2,x)
```

output
```
x^5/(5*a^2) - (6*b^5*log(b + a*x))/a^7 - (b*x^4)/(2*a^3) + (5*b^4*x)/a^6 +
 (b^2*x^3)/a^4 - (2*b^3*x^2)/a^5 - b^6/(a*(a^6*b + a^7*x))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{-60\log(ax+b)\,a\,b^5x-60\log(ax+b)\,b^6+2a^6x^6-3a^5b\,x^5+5a^4b^2x^4-10a^3b^3x^3+30a^2b^4x^2+60a\,b^5x}{10a^7\,(ax+b)}$$

input
```
int(x^4/(a+b/x)^2,x)
```

output
```
( - 60*log(a*x + b)*a*b**5*x - 60*log(a*x + b)*b**6 + 2*a**6*x**6 - 3*a**5
*b*x**5 + 5*a**4*b**2*x**4 - 10*a**3*b**3*x**3 + 30*a**2*b**4*x**2 + 60*a*
b**5*x)/(10*a**7*(a*x + b))
```

3.76 $\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2} \, dx = -\frac{4b^3x}{a^5}+\frac{3b^2x^2}{2a^4}-\frac{2bx^3}{3a^3}+\frac{x^4}{4a^2}+\frac{b^5}{a^6(b+ax)}+\frac{5b^4\log(b+ax)}{a^6}$$

output
```
-4*b^3*x/a^5+3/2*b^2*x^2/a^4-2/3*b*x^3/a^3+1/4*x^4/a^2+b^5/a^6/(a*x+b)+5*b
^4*ln(a*x+b)/a^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2} \, dx = \frac{-48ab^3x+18a^2b^2x^2-8a^3bx^3+3a^4x^4+\frac{12b^5}{b+ax}+60b^4\log(b+ax)}{12a^6}$$

input
```
Integrate[x^3/(a + b/x)^2,x]
```

output
```
(-48*a*b^3*x + 18*a^2*b^2*x^2 - 8*a^3*b*x^3 + 3*a^4*x^4 + (12*b^5)/(b + a*
x) + 60*b^4*Log[b + a*x])/(12*a^6)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^5}{(ax+b)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^5}{a^5(ax+b)^2}+\frac{5b^4}{a^5(ax+b)}-\frac{4b^3}{a^5}+\frac{3b^2x}{a^4}-\frac{2bx^2}{a^3}+\frac{x^3}{a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^5}{a^6(ax+b)}+\frac{5b^4\log(ax+b)}{a^6}-\frac{4b^3x}{a^5}+\frac{3b^2x^2}{2a^4}-\frac{2bx^3}{3a^3}+\frac{x^4}{4a^2}$$

input
```
Int[x^3/(a + b/x)^2,x]
```

output
```
(-4*b^3*x)/a^5 + (3*b^2*x^2)/(2*a^4) - (2*b*x^3)/(3*a^3) + x^4/(4*a^2) + b
^5/(a^6*(b + a*x)) + (5*b^4*Log[b + a*x])/a^6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{4}a^3x^4-\frac{2}{3}a^2b\,x^3+\frac{3}{2}a\,b^2x^2-4b^3x}{a^5}+\frac{b^5}{a^6(ax+b)}+\frac{5b^4\ln(ax+b)}{a^6}$	67
risch	$-\frac{4b^3x}{a^5}+\frac{3b^2x^2}{2a^4}-\frac{2b\,x^3}{3a^3}+\frac{x^4}{4a^2}+\frac{b^5}{a^6(ax+b)}+\frac{5b^4\ln(ax+b)}{a^6}$	67
norman	$\frac{\frac{5b^5}{a^6}+\frac{x^5}{4a}-\frac{5b\,x^4}{12a^2}+\frac{5b^2x^3}{6a^3}-\frac{5b^3x^2}{2a^4}}{ax+b}+\frac{5b^4\ln(ax+b)}{a^6}$	72
parallelrisch	$\frac{3a^5x^5-5a^4b\,x^4+10a^3b^2x^3+60\ln(ax+b)xa\,b^4-30a^2b^3x^2+60b^5\ln(ax+b)+60b^5}{12a^6(ax+b)}$	82

input
```
int(x^3/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/4*a^3*x^4-2/3*a^2*b*x^3+3/2*a*b^2*x^2-4*b^3*x)+b^5/a^6/(a*x+b)+5*
b^4*ln(a*x+b)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int\frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx$$
$$=\frac{3\,a^5x^5-5\,a^4bx^4+10\,a^3b^2x^3-30\,a^2b^3x^2-48\,ab^4x+12\,b^5+60\,(ab^4x+b^5)\log{(ax+b)}}{12\,(a^7x+a^6b)}$$

input
```
integrate(x^3/(a+b/x)^2,x, algorithm="fricas")
```

output
```
1/12*(3*a^5*x^5 - 5*a^4*b*x^4 + 10*a^3*b^2*x^3 - 30*a^2*b^3*x^2 - 48*a*b^4
*x + 12*b^5 + 60*(a*b^4*x + b^5)*log(a*x + b))/(a^7*x + a^6*b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{b^5}{a^7x+a^6b}+\frac{x^4}{4a^2}-\frac{2bx^3}{3a^3}+\frac{3b^2x^2}{2a^4}-\frac{4b^3x}{a^5}+\frac{5b^4\log\left(ax+b\right)}{a^6}$$

input
```
integrate(x**3/(a+b/x)**2,x)
```

output
```
b**5/(a**7*x + a**6*b) + x**4/(4*a**2) - 2*b*x**3/(3*a**3) + 3*b**2*x**2/(
2*a**4) - 4*b**3*x/a**5 + 5*b**4*log(a*x + b)/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{b^5}{a^7x+a^6b}+\frac{5\,b^4\log\left(ax+b\right)}{a^6}+\frac{3\,a^3x^4-8\,a^2bx^3+18\,ab^2x^2-48\,b^3x}{12\,a^5}$$

input
```
integrate(x^3/(a+b/x)^2,x, algorithm="maxima")
```

output
```
b^5/(a^7*x + a^6*b) + 5*b^4*log(a*x + b)/a^6 + 1/12*(3*a^3*x^4 - 8*a^2*b*x
^3 + 18*a*b^2*x^2 - 48*b^3*x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{5\,b^4\log\left(|ax+b|\right)}{a^6} + \frac{b^5}{(ax+b)a^6} + \frac{3\,a^6x^4 - 8\,a^5bx^3 + 18\,a^4b^2x^2 - 48\,a^3b^3x}{12\,a^8}$$

input
```
integrate(x^3/(a+b/x)^2,x, algorithm="giac")
```

output
```
5*b^4*log(abs(a*x + b))/a^6 + b^5/((a*x + b)*a^6) + 1/12*(3*a^6*x^4 - 8*a^
5*b*x^3 + 18*a^4*b^2*x^2 - 48*a^3*b^3*x)/a^8
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{x^4}{4\,a^2} + \frac{5\,b^4\,\ln\left(b+a\,x\right)}{a^6} - \frac{2\,b\,x^3}{3\,a^3} - \frac{4\,b^3\,x}{a^5} + \frac{3\,b^2\,x^2}{2\,a^4} + \frac{b^5}{a\,\left(x\,a^6+b\,a^5\right)}$$

input
```
int(x^3/(a + b/x)^2,x)
```

output
```
x^4/(4*a^2) + (5*b^4*log(b + a*x))/a^6 - (2*b*x^3)/(3*a^3) - (4*b^3*x)/a^5
 + (3*b^2*x^2)/(2*a^4) + b^5/(a*(a^5*b + a^6*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{60\log(ax+b)\,a\,b^4x + 60\log(ax+b)\,b^5 + 3a^5x^5 - 5a^4b\,x^4 + 10a^3b^2x^3 - 30a^2b^3x^2 - 60a\,b^4x}{12a^6\,(ax+b)}$$

input
```
int(x^3/(a+b/x)^2,x)
```

output
```
(60*log(a*x + b)*a*b**4*x + 60*log(a*x + b)*b**5 + 3*a**5*x**5 - 5*a**4*b*
x**4 + 10*a**3*b**2*x**3 - 30*a**2*b**3*x**2 - 60*a*b**4*x)/(12*a**6*(a*x
+ b))
```

3.77 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{3b^2x}{a^4}-\frac{bx^2}{a^3}+\frac{x^3}{3a^2}-\frac{b^4}{a^5(b+ax)}-\frac{4b^3\log(b+ax)}{a^5}$$

output
```
3*b^2*x/a^4-b*x^2/a^3+1/3*x^3/a^2-b^4/a^5/(a*x+b)-4*b^3*ln(a*x+b)/a^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{9ab^2x-3a^2bx^2+a^3x^3-\frac{3b^4}{b+ax}-12b^3\log(b+ax)}{3a^5}$$

input
```
Integrate[x^2/(a + b/x)^2,x]
```

output
```
(9*a*b^2*x - 3*a^2*b*x^2 + a^3*x^3 - (3*b^4)/(b + a*x) - 12*b^3*Log[b + a*
x])/(3*a^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^4}{(ax+b)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{b^4}{a^4(ax+b)^2}-\frac{4b^3}{a^4(ax+b)}+\frac{3b^2}{a^4}-\frac{2bx}{a^3}+\frac{x^2}{a^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^4}{a^5(ax+b)}-\frac{4b^3\log(ax+b)}{a^5}+\frac{3b^2x}{a^4}-\frac{bx^2}{a^3}+\frac{x^3}{3a^2}$$

input
```
Int[x^2/(a + b/x)^2,x]
```

output
```
(3*b^2*x)/a^4 - (b*x^2)/a^3 + x^3/(3*a^2) - b^4/(a^5*(b + a*x)) - (4*b^3*L
og[b + a*x])/a^5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\frac{1}{3}a^2x^3-ab\,x^2+3b^2x}{a^4}-\frac{b^4}{a^5(ax+b)}-\frac{4b^3\ln(ax+b)}{a^5}$	57
risch	$\frac{3b^2x}{a^4}-\frac{b\,x^2}{a^3}+\frac{x^3}{3a^2}-\frac{b^4}{a^5(ax+b)}-\frac{4b^3\ln(ax+b)}{a^5}$	57
norman	$\frac{-\frac{4b^4}{a^5}+\frac{x^4}{3a}-\frac{2b\,x^3}{3a^2}+\frac{2b^2x^2}{a^3}}{ax+b}-\frac{4b^3\ln(ax+b)}{a^5}$	61
parallelrisch	$-\frac{-a^4x^4+2a^3b\,x^3+12\ln(ax+b)xa\,b^3-6a^2b^2x^2+12b^4\ln(ax+b)+12b^4}{3a^5(ax+b)}$	71

input
```
int(x^2/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/3*a^2*x^3-a*b*x^2+3*b^2*x)-b^4/a^5/(a*x+b)-4*b^3*ln(a*x+b)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{a^4x^4-2\,a^3bx^3+6\,a^2b^2x^2+9\,ab^3x-3\,b^4-12\,(ab^3x+b^4)\log\left(ax+b\right)}{3\,(a^6x+a^5b)}$$

input
```
integrate(x^2/(a+b/x)^2,x, algorithm="fricas")
```

output
```
1/3*(a^4*x^4 - 2*a^3*b*x^3 + 6*a^2*b^2*x^2 + 9*a*b^3*x - 3*b^4 - 12*(a*b^3
*x + b^4)*log(a*x + b))/(a^6*x + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{b^4}{a^6x+a^5b}+\frac{x^3}{3a^2}-\frac{bx^2}{a^3}+\frac{3b^2x}{a^4}-\frac{4b^3\log\left(ax+b\right)}{a^5}$$

input
```
integrate(x**2/(a+b/x)**2,x)
```

output
```
-b**4/(a**6*x + a**5*b) + x**3/(3*a**2) - b*x**2/a**3 + 3*b**2*x/a**4 - 4*
b**3*log(a*x + b)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{b^4}{a^6x+a^5b}-\frac{4\,b^3\log\left(ax+b\right)}{a^5}+\frac{a^2x^3-3\,abx^2+9\,b^2x}{3\,a^4}$$

input
```
integrate(x^2/(a+b/x)^2,x, algorithm="maxima")
```

output
```
-b^4/(a^6*x + a^5*b) - 4*b^3*log(a*x + b)/a^5 + 1/3*(a^2*x^3 - 3*a*b*x^2 +
 9*b^2*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{4\,b^3\log\left(|ax+b|\right)}{a^5}-\frac{b^4}{(ax+b)a^5}+\frac{a^4x^3-3\,a^3bx^2+9\,a^2b^2x}{3\,a^6}$$

input
```
integrate(x^2/(a+b/x)^2,x, algorithm="giac")
```

output
```
-4*b^3*log(abs(a*x + b))/a^5 - b^4/((a*x + b)*a^5) + 1/3*(a^4*x^3 - 3*a^3*
b*x^2 + 9*a^2*b^2*x)/a^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{x^3}{3\,a^2}-\frac{4\,b^3\,\ln\left(b+a\,x\right)}{a^5}-\frac{b\,x^2}{a^3}+\frac{3\,b^2\,x}{a^4}-\frac{b^4}{a\,\left(x\,a^5+b\,a^4\right)}$$

input
```
int(x^2/(a + b/x)^2,x)
```

output
```
x^3/(3*a^2) - (4*b^3*log(b + a*x))/a^5 - (b*x^2)/a^3 + (3*b^2*x)/a^4 - b^4
/(a*(a^4*b + a^5*x))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{-12\log(ax+b)\,a\,b^3x-12\log(ax+b)\,b^4+a^4x^4-2a^3b\,x^3+6a^2b^2x^2+12a\,b^3x}{3a^5\,(ax+b)}$$

input
```
int(x^2/(a+b/x)^2,x)
```

output
```
( - 12*log(a*x + b)*a*b**3*x - 12*log(a*x + b)*b**4 + a**4*x**4 - 2*a**3*b
*x**3 + 6*a**2*b**2*x**2 + 12*a*b**3*x)/(3*a**5*(a*x + b))
```

3.78 $\int \frac{x}{\left(a+\frac{b}{x}\right)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2} dx = -\frac{2bx}{a^3} + \frac{x^2}{2a^2} + \frac{b^3}{a^4(b+ax)} + \frac{3b^2\log(b+ax)}{a^4}$$

output
```
-2*b*x/a^3+1/2*x^2/a^2+b^3/a^4/(a*x+b)+3*b^2*ln(a*x+b)/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2} dx = \frac{-4abx + a^2x^2 + \frac{2b^3}{b+ax} + 6b^2\log(b+ax)}{2a^4}$$

input
```
Integrate[x/(a + b/x)^2,x]
```

output
```
(-4*a*b*x + a^2*x^2 + (2*b^3)/(b + a*x) + 6*b^2*Log[b + a*x])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^3}{(ax+b)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^3}{a^3(ax+b)^2}+\frac{3b^2}{a^3(ax+b)}-\frac{2b}{a^3}+\frac{x}{a^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3}{a^4(ax+b)}+\frac{3b^2\log(ax+b)}{a^4}-\frac{2bx}{a^3}+\frac{x^2}{2a^2}$$

input
```
Int[x/(a + b/x)^2,x]
```

output
```
(-2*b*x)/a^3 + x^2/(2*a^2) + b^3/(a^4*(b + a*x)) + (3*b^2*Log[b + a*x])/a^
4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\frac{1}{2}a\,x^{2}-2bx}{a^{3}}+\frac{b^{3}}{a^{4}(ax+b)}+\frac{3b^{2}\ln(ax+b)}{a^{4}}$	45
risch	$-\frac{2bx}{a^{3}}+\frac{x^{2}}{2a^{2}}+\frac{b^{3}}{a^{4}(ax+b)}+\frac{3b^{2}\ln(ax+b)}{a^{4}}$	45
norman	$\frac{\frac{3b^{3}}{a^{4}}+\frac{x^{3}}{2a}-\frac{3b\,x^{2}}{2a^{2}}}{ax+b}+\frac{3b^{2}\ln(ax+b)}{a^{4}}$	50
parallelrisch	$\frac{a^{3}x^{3}+6\ln(ax+b)xa\,b^{2}-3a^{2}b\,x^{2}+6b^{3}\ln(ax+b)+6b^{3}}{2a^{4}(ax+b)}$	59

input
```
int(x/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/2*a*x^2-2*b*x)+b^3/a^4/(a*x+b)+3*b^2*ln(a*x+b)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{a^3x^3-3\,a^2bx^2-4\,ab^2x+2\,b^3+6\,(ab^2x+b^3)\log{(ax+b)}}{2\,(a^5x+a^4b)}$$

input
```
integrate(x/(a+b/x)^2,x, algorithm="fricas")
```

output
```
1/2*(a^3*x^3 - 3*a^2*b*x^2 - 4*a*b^2*x + 2*b^3 + 6*(a*b^2*x + b^3)*log(a*x
 + b))/(a^5*x + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{b^3}{a^5x+a^4b}+\frac{x^2}{2a^2}-\frac{2bx}{a^3}+\frac{3b^2\log\left(ax+b\right)}{a^4}$$

input
```
integrate(x/(a+b/x)**2,x)
```

output
```
b**3/(a**5*x + a**4*b) + x**2/(2*a**2) - 2*b*x/a**3 + 3*b**2*log(a*x + b)/
a**4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{b^3}{a^5x+a^4b}+\frac{3\,b^2\log\left(ax+b\right)}{a^4}+\frac{ax^2-4\,bx}{2\,a^3}$$

input
```
integrate(x/(a+b/x)^2,x, algorithm="maxima")
```

output
```
b^3/(a^5*x + a^4*b) + 3*b^2*log(a*x + b)/a^4 + 1/2*(a*x^2 - 4*b*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{3\,b^2\log\left(|ax+b|\right)}{a^4}+\frac{b^3}{(ax+b)a^4}+\frac{a^2x^2-4\,abx}{2\,a^4}$$

input
```
integrate(x/(a+b/x)^2,x, algorithm="giac")
```

output
```
3*b^2*log(abs(a*x + b))/a^4 + b^3/((a*x + b)*a^4) + 1/2*(a^2*x^2 - 4*a*b*x
)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{x^2}{2\,a^2}+\frac{3\,b^2\,\ln\left(b+a\,x\right)}{a^4}+\frac{b^3}{a\,\left(x\,a^4+b\,a^3\right)}-\frac{2\,b\,x}{a^3}$$

input
```
int(x/(a + b/x)^2,x)
```

output
```
x^2/(2*a^2) + (3*b^2*log(b + a*x))/a^4 + b^3/(a*(a^3*b + a^4*x)) - (2*b*x)
/a^3
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{6\log(ax+b)\,a\,b^2x+6\log(ax+b)\,b^3+a^3x^3-3a^2b\,x^2-6a\,b^2x}{2a^4\,(ax+b)}$$

input
```
int(x/(a+b/x)^2,x)
```

output
```
(6*log(a*x + b)*a*b**2*x + 6*log(a*x + b)*b**3 + a**3*x**3 - 3*a**2*b*x**2
 - 6*a*b**2*x)/(2*a**4*(a*x + b))
```

3.79 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2} dx = \frac{x}{a^2} - \frac{b^2}{a^3(b+ax)} - \frac{2b\log(b+ax)}{a^3}$$

output
```
x/a^2-b^2/a^3/(a*x+b)-2*b*ln(a*x+b)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2} dx = \frac{ax - \frac{b^2}{b+ax} - 2b\log(b+ax)}{a^3}$$

input
```
Integrate[(a + b/x)^(-2),x]
```

output
```
(a*x - b^2/(b + a*x) - 2*b*Log[b + a*x])/a^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2} \, dx$$

$$\Big\downarrow \text{ 772}$$

$$\int \frac{x^2}{(ax+b)^2}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{b^2}{a^2(ax+b)^2}-\frac{2b}{a^2(ax+b)}+\frac{1}{a^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^2}{a^3(ax+b)}-\frac{2b\log(ax+b)}{a^3}+\frac{x}{a^2}$$

input
```
Int[(a + b/x)^(-2),x]
```

output
```
x/a^2 - b^2/(a^3*(b + a*x)) - (2*b*Log[b + a*x])/a^3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{a^2}-\frac{b^2}{a^3(ax+b)}-\frac{2b\ln(ax+b)}{a^3}$	34
risch	$\frac{x}{a^2}-\frac{b^2}{a^3(ax+b)}-\frac{2b\ln(ax+b)}{a^3}$	34
norman	$\frac{\frac{x^2}{a}-\frac{2b^2}{a^3}}{ax+b}-\frac{2b\ln(ax+b)}{a^3}$	38
parallelrisch	$-\frac{2\ln(ax+b)xab-a^2x^2+2b^2\ln(ax+b)+2b^2}{a^3(ax+b)}$	49

input
```
int(1/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*x-b^2/a^3/(a*x+b)-2*b*ln(a*x+b)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{a^2x^2+abx-b^2-2\left(abx+b^2\right)\log\left(ax+b\right)}{a^4x+a^3b}$$

input
```
integrate(1/(a+b/x)^2,x, algorithm="fricas")
```

output
```
(a^2*x^2 + a*b*x - b^2 - 2*(a*b*x + b^2)*log(a*x + b))/(a^4*x + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{b^2}{a^4x+a^3b}+\frac{x}{a^2}-\frac{2b\log\left(ax+b\right)}{a^3}$$

input
```
integrate(1/(a+b/x)**2,x)
```

output
```
-b**2/(a**4*x + a**3*b) + x/a**2 - 2*b*log(a*x + b)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{b^2}{a^4x+a^3b}+\frac{x}{a^2}-\frac{2\,b\log\left(ax+b\right)}{a^3}$$

input
```
integrate(1/(a+b/x)^2,x, algorithm="maxima")
```

output
```
-b^2/(a^4*x + a^3*b) + x/a^2 - 2*b*log(a*x + b)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{x}{a^2}-\frac{2\,b\log\left(|ax+b|\right)}{a^3}-\frac{b^2}{(ax+b)a^3}$$

input
```
integrate(1/(a+b/x)^2,x, algorithm="giac")
```

output
```
x/a^2 - 2*b*log(abs(a*x + b))/a^3 - b^2/((a*x + b)*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{x}{a^2} - \frac{b^2}{x\,a^4+b\,a^3} - \frac{2\,b\,\ln\left(b+a\,x\right)}{a^3}$$

input
```
int(1/(a + b/x)^2,x)
```

output
```
x/a^2 - b^2/(a^3*b + a^4*x) - (2*b*log(b + a*x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{-2\log(ax+b)\,abx - 2\log(ax+b)\,b^2 + a^2x^2 + 2abx}{a^3\,(ax+b)}$$

input
```
int(1/(a+b/x)^2,x)
```

output
```
( - 2*log(a*x + b)*a*b*x - 2*log(a*x + b)*b**2 + a**2*x**2 + 2*a*b*x)/(a**
3*(a*x + b))
```

3.80 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x} \, dx = \frac{b}{a^2(b+ax)} + \frac{\log(b+ax)}{a^2}$$

output
```
b/a^2/(a*x+b)+ln(a*x+b)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x} \, dx = \frac{\frac{b}{b+ax} + \log(b+ax)}{a^2}$$

input
```
Integrate[1/((a + b/x)^2*x),x]
```

output
```
(b/(b + a*x) + Log[b + a*x])/a^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x}{(ax+b)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{a(ax+b)}-\frac{b}{a(ax+b)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b}{a^2(ax+b)}+\frac{\log(ax+b)}{a^2}$$

input
```
Int[1/((a + b/x)^2*x),x]
```

output
```
b/(a^2*(b + a*x)) + Log[b + a*x]/a^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{b}{a^2(ax+b)}+\frac{\ln(ax+b)}{a^2}$	24
norman	$\frac{b}{a^2(ax+b)}+\frac{\ln(ax+b)}{a^2}$	24
risch	$\frac{b}{a^2(ax+b)}+\frac{\ln(ax+b)}{a^2}$	24
parallelrisch	$\frac{a\ln(ax+b)x+b\ln(ax+b)+b}{a^2(ax+b)}$	31

input
```
int(1/(a+b/x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
b/a^2/(a*x+b)+ln(a*x+b)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\,dx=\frac{(ax+b)\log(ax+b)+b}{a^3x+a^2b}$$

input
```
integrate(1/(a+b/x)^2/x,x, algorithm="fricas")
```

output
```
((a*x + b)*log(a*x + b) + b)/(a^3*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\,dx = \frac{b}{a^3x+a^2b}+\frac{\log\left(ax+b\right)}{a^2}$$

input
```
integrate(1/(a+b/x)**2/x,x)
```

output
```
b/(a**3*x + a**2*b) + log(a*x + b)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\,dx = \frac{b}{a^3x+a^2b}+\frac{\log\left(ax+b\right)}{a^2}$$

input
```
integrate(1/(a+b/x)^2/x,x, algorithm="maxima")
```

output
```
b/(a^3*x + a^2*b) + log(a*x + b)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\,dx = \frac{\log\left(|ax+b|\right)}{a^2}+\frac{b}{(ax+b)a^2}$$

input
```
integrate(1/(a+b/x)^2/x,x, algorithm="giac")
```

output
```
log(abs(a*x + b))/a^2 + b/((a*x + b)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\, dx = \frac{\ln\left(b+a\,x\right)}{a^2}+\frac{b}{a^2\,\left(b+a\,x\right)}$$

input
```
int(1/(x*(a + b/x)^2),x)
```

output
```
log(b + a*x)/a^2 + b/(a^2*(b + a*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x}\, dx = \frac{\log(ax+b)\,ax+\log(ax+b)\,b-ax}{a^2\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x,x)
```

output
```
(log(a*x + b)*a*x + log(a*x + b)*b - a*x)/(a**2*(a*x + b))
```

3.81 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} \, dx = \frac{1}{b\left(a+\frac{b}{x}\right)}$$

output
```
1/b/(a+b/x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} \, dx = -\frac{1}{a(b+ax)}$$

input
```
Integrate[1/((a + b/x)^2*x^2),x]
```

output
```
-(1/(a*(b + a*x)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x}\right)^2} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{1}{b \left(a+\frac{b}{x}\right)}$$

input
```
Int[1/((a + b/x)^2*x^2),x]
```

output
```
1/(b*(a + b/x))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{1}{(ax+b)a}$	13
default	$-\frac{1}{(ax+b)a}$	13
norman	$\frac{x}{b(ax+b)}$	13
risch	$-\frac{1}{(ax+b)a}$	13
parallelrisch	$-\frac{1}{(ax+b)a}$	13
derivativedivides	$\frac{1}{b\left(a+\frac{b}{x}\right)}$	14
orering	$-\frac{ax+b}{a\left(a+\frac{b}{x}\right)^{2}x^{2}}$	23

input
```
int(1/(a+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*x+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2}\, dx = -\frac{1}{a^2x + ab}$$

input
```
integrate(1/(a+b/x)^2/x^2,x, algorithm="fricas")
```

output
```
-1/(a^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} dx = -\frac{1}{a^2x+ab}$$

input
```
integrate(1/(a+b/x)**2/x**2,x)
```

output
```
-1/(a**2*x + a*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} dx = \frac{1}{\left(a+\frac{b}{x}\right)b}$$

input
```
integrate(1/(a+b/x)^2/x^2,x, algorithm="maxima")
```

output
```
1/((a + b/x)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} dx = \frac{1}{\left(a+\frac{b}{x}\right)b}$$

input
```
integrate(1/(a+b/x)^2/x^2,x, algorithm="giac")
```

output
```
1/((a + b/x)*b)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} \, dx = -\frac{1}{a\,\left(b+a\,x\right)}$$

input
```
int(1/(x^2*(a + b/x)^2),x)
```

output
```
-1/(a*(b + a*x))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^2} \, dx = \frac{x}{b\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^2,x)
```

output
```
x/(b*(a*x + b))
```

3.82 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3} \, dx=-\frac{a}{b^2 \left(a+\frac{b}{x}\right)}-\frac{\log \left(a+\frac{b}{x}\right)}{b^2}$$

output
```
-a/b^2/(a+b/x)-ln(a+b/x)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3} \, dx=\frac{\frac{b}{b+a x}+\log (x)-\log (b+a x)}{b^2}$$

input
```
Integrate[1/((a + b/x)^2*x^3),x]
```

output
```
(b/(b + a*x) + Log[x] - Log[b + a*x])/b^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x}\right)^2} \, dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{x(ax+b)^2}dx$$

$$\Big\downarrow \ 54$$

$$\int \left(-\frac{a}{b^2(ax+b)}-\frac{a}{b(ax+b)^2}+\frac{1}{b^2x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\log(ax+b)}{b^2}+\frac{1}{b(ax+b)}+\frac{\log(x)}{b^2}$$

input
```
Int[1/((a + b/x)^2*x^3),x]
```

output
```
1/(b*(b + a*x)) + Log[x]/b^2 - Log[b + a*x]/b^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{\ln(x)}{b^2}-\frac{\ln(ax+b)}{b^2}+\frac{1}{b(ax+b)}$	30
risch	$\frac{1}{b(ax+b)}-\frac{\ln(ax+b)}{b^2}+\frac{\ln(-x)}{b^2}$	32
norman	$-\frac{ax}{b^2(ax+b)}+\frac{\ln(x)}{b^2}-\frac{\ln(ax+b)}{b^2}$	33
parallelrisch	$\frac{a\ln(x)x-a\ln(ax+b)x+b\ln(x)-b\ln(ax+b)-ax}{b^2(ax+b)}$	45

input
```
int(1/(a+b/x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*ln(x)-1/b^2*ln(a*x+b)+1/b/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\,dx=-\frac{(ax+b)\log(ax+b)-(ax+b)\log(x)-b}{ab^2x+b^3}$$

input
```
integrate(1/(a+b/x)^2/x^3,x, algorithm="fricas")
```

output
```
-((a*x + b)*log(a*x + b) - (a*x + b)*log(x) - b)/(a*b^2*x + b^3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\,dx = \frac{1}{abx+b^2} + \frac{\log(x) - \log\left(x+\frac{b}{a}\right)}{b^2}$$

input
```
integrate(1/(a+b/x)**2/x**3,x)
```

output
```
1/(a*b*x + b**2) + (log(x) - log(x + b/a))/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\,dx = \frac{1}{abx+b^2} - \frac{\log(ax+b)}{b^2} + \frac{\log(x)}{b^2}$$

input
```
integrate(1/(a+b/x)^2/x^3,x, algorithm="maxima")
```

output
```
1/(a*b*x + b^2) - log(a*x + b)/b^2 + log(x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\,dx = -\frac{\log(|ax+b|)}{b^2} + \frac{\log(|x|)}{b^2} + \frac{1}{(ax+b)b}$$

input
```
integrate(1/(a+b/x)^2/x^3,x, algorithm="giac")
```

output
```
-log(abs(a*x + b))/b^2 + log(abs(x))/b^2 + 1/((a*x + b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\, dx = \frac{1}{b^2 + a\,x\,b} - \frac{2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b} + 1\right)}{b^2}$$

input
```
int(1/(x^3*(a + b/x)^2),x)
```

output
```
1/(b^2 + a*b*x) - (2*atanh((2*a*x)/b + 1))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^3}\, dx = \frac{-\log(ax+b)\,ax - \log(ax+b)\,b + \log(x)\,ax + \log(x)\,b - ax}{b^2\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^3,x)
```

output
```
( - log(a*x + b)*a*x - log(a*x + b)*b + log(x)*a*x + log(x)*b - a*x)/(b**2
*(a*x + b))
```

3.83 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4} dx = \frac{a^2}{b^3\left(a+\frac{b}{x}\right)} - \frac{1}{b^2 x} + \frac{2a\log\left(a+\frac{b}{x}\right)}{b^3}$$

output
```
a^2/b^3/(a+b/x)-1/b^2/x+2*a*ln(a+b/x)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4} dx = -\frac{b\left(\frac{1}{x}+\frac{a}{b+ax}\right)+2a\log(x)-2a\log(b+ax)}{b^3}$$

input
```
Integrate[1/((a + b/x)^2*x^4),x]
```

output
```
-((b*(x^(-1) + a/(b + a*x)) + 2*a*Log[x] - 2*a*Log[b + a*x])/b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x}\right)^2} \, dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{1}{x^2(ax+b)^2}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{2a^2}{b^3(ax+b)}+\frac{a^2}{b^2(ax+b)^2}-\frac{2a}{b^3x}+\frac{1}{b^2x^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2a\log(x)}{b^3}+\frac{2a\log(ax+b)}{b^3}-\frac{a}{b^2(ax+b)}-\frac{1}{b^2x}$$

input
```
Int[1/((a + b/x)^2*x^4),x]
```

output
```
-(1/(b^2*x)) - a/(b^2*(b + a*x)) - (2*a*Log[x])/b^3 + (2*a*Log[b + a*x])/b
^3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{1}{b^{2}x}-\frac{2a\ln(x)}{b^{3}}-\frac{a}{b^{2}(ax+b)}+\frac{2a\ln(ax+b)}{b^{3}}$	43
risch	$\frac{-\frac{2ax}{b^{2}}-\frac{1}{b}}{x(ax+b)}+\frac{2a\ln(-ax-b)}{b^{3}}-\frac{2a\ln(x)}{b^{3}}$	49
norman	$\frac{\frac{2a^{2}x^{4}}{b^{3}}-\frac{x^{2}}{b}}{x^{3}(ax+b)}-\frac{2a\ln(x)}{b^{3}}+\frac{2a\ln(ax+b)}{b^{3}}$	53
parallelrisch	$-\frac{2a^{2}\ln(x)x^{2}-2a^{2}\ln(ax+b)x^{2}+2ab\ln(x)x-2\ln(ax+b)xab-2a^{2}x^{2}+b^{2}}{b^{3}x(ax+b)}$	70

input
```
int(1/(a+b/x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/x-2/b^3*a*ln(x)-a/b^2/(a*x+b)+2/b^3*a*ln(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int\frac{1}{\left(a+\frac{b}{x}\right)^{2}x^{4}}\,dx=-\frac{2\,abx+b^{2}-2\,(a^{2}x^{2}+abx)\log\left(ax+b\right)+2\,(a^{2}x^{2}+abx)\log\left(x\right)}{ab^{3}x^{2}+b^{4}x}$$

input
```
integrate(1/(a+b/x)^2/x^4,x, algorithm="fricas")
```

output
```
-(2*a*b*x + b^2 - 2*(a^2*x^2 + a*b*x)*log(a*x + b) + 2*(a^2*x^2 + a*b*x)*l
og(x))/(a*b^3*x^2 + b^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx = \frac{2a\left(-\log\left(x\right)+\log\left(x+\frac{b}{a}\right)\right)}{b^3} + \frac{-2ax-b}{ab^2x^2+b^3x}$$

input
```
integrate(1/(a+b/x)**2/x**4,x)
```

output
```
2*a*(-log(x) + log(x + b/a))/b**3 + (-2*a*x - b)/(a*b**2*x**2 + b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx = -\frac{2\,ax+b}{ab^2x^2+b^3x} + \frac{2\,a\log\left(ax+b\right)}{b^3} - \frac{2\,a\log\left(x\right)}{b^3}$$

input
```
integrate(1/(a+b/x)^2/x^4,x, algorithm="maxima")
```

output
```
-(2*a*x + b)/(a*b^2*x^2 + b^3*x) + 2*a*log(a*x + b)/b^3 - 2*a*log(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx = \frac{2\,a\log\left(|ax+b|\right)}{b^3} - \frac{2\,a\log\left(|x|\right)}{b^3} - \frac{2\,ax+b}{(ax^2+bx)b^2}$$

input
```
integrate(1/(a+b/x)^2/x^4,x, algorithm="giac")
```

output
```
2*a*log(abs(a*x + b))/b^3 - 2*a*log(abs(x))/b^3 - (2*a*x + b)/((a*x^2 + b*
x)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx = \frac{4\,a\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^3}-\frac{\frac{1}{b}+\frac{2\,a\,x}{b^2}}{a\,x^2+b\,x}$$

input
```
int(1/(x^4*(a + b/x)^2),x)
```

output
```
(4*a*atanh((2*a*x)/b + 1))/b^3 - (1/b + (2*a*x)/b^2)/(b*x + a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^4}\,dx = \frac{2\log(ax+b)\,a^2x^2+2\log(ax+b)\,abx-2\log(x)\,a^2x^2-2\log(x)\,abx+2a^2x^2-b^2}{b^3x\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^4,x)
```

output
```
(2*log(a*x + b)*a**2*x**2 + 2*log(a*x + b)*a*b*x - 2*log(x)*a**2*x**2 - 2*
log(x)*a*b*x + 2*a**2*x**2 - b**2)/(b**3*x*(a*x + b))
```

3.84 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5} dx = -\frac{a^3}{b^4\left(a+\frac{b}{x}\right)} - \frac{1}{2b^2x^2} + \frac{2a}{b^3x} - \frac{3a^2\log\left(a+\frac{b}{x}\right)}{b^4}$$

output
```
-a^3/b^4/(a+b/x)-1/2/b^2/x^2+2*a/b^3/x-3*a^2*ln(a+b/x)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5} dx = \frac{b\left(-\frac{b}{x^2}+\frac{4a}{x}+\frac{2a^2}{b+ax}\right)+6a^2\log(x)-6a^2\log(b+ax)}{2b^4}$$

input
```
Integrate[1/((a + b/x)^2*x^5),x]
```

output
```
(b*(-(b/x^2) + (4*a)/x + (2*a^2)/(b + a*x)) + 6*a^2*Log[x] - 6*a^2*Log[b +
 a*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^3(ax+b)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{3a^3}{b^4(ax+b)}-\frac{a^3}{b^3(ax+b)^2}+\frac{3a^2}{b^4x}-\frac{2a}{b^3x^2}+\frac{1}{b^2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2\log(x)}{b^4}-\frac{3a^2\log(ax+b)}{b^4}+\frac{a^2}{b^3(ax+b)}+\frac{2a}{b^3x}-\frac{1}{2b^2x^2}$$

input
```
Int[1/((a + b/x)^2*x^5),x]
```

output
```
-1/2*1/(b^2*x^2) + (2*a)/(b^3*x) + a^2/(b^3*(b + a*x)) + (3*a^2*Log[x])/b^
4 - (3*a^2*Log[b + a*x])/b^4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{1}{2b^2x^2}+\frac{2a}{b^3x}+\frac{3a^2\ln(x)}{b^4}-\frac{3a^2\ln(ax+b)}{b^4}+\frac{a^2}{b^3(ax+b)}$	57
risch	$\frac{\frac{3a^2x^2}{b^3}+\frac{3ax}{2b^2}-\frac{1}{2b}}{(ax+b)x^2}+\frac{3a^2\ln(-x)}{b^4}-\frac{3a^2\ln(ax+b)}{b^4}$	63
norman	$\frac{-\frac{x^2}{2b}+\frac{3a\,x^3}{2b^2}-\frac{3a^3x^5}{b^4}}{x^4(ax+b)}+\frac{3a^2\ln(x)}{b^4}-\frac{3a^2\ln(ax+b)}{b^4}$	66
parallelrisch	$\frac{6a^3\ln(x)x^3-6a^3\ln(ax+b)x^3+6a^2b\ln(x)x^2-6\ln(ax+b)x^2a^2b-6a^3x^3+3a\,b^2x-b^3}{2b^4x^2(ax+b)}$	87

input
```
int(1/(a+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2/x^2+2*a/b^3/x+3*a^2/b^4*ln(x)-3*a^2/b^4*ln(a*x+b)+a^2/b^3/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2x^5}\,dx = \frac{6\,a^2bx^2+3\,ab^2x-b^3-6\,(a^3x^3+a^2bx^2)\log(ax+b)+6\,(a^3x^3+a^2bx^2)\log(x)}{2\,(ab^4x^3+b^5x^2)}$$

input
```
integrate(1/(a+b/x)^2/x^5,x, algorithm="fricas")
```

output
```
1/2*(6*a^2*b*x^2 + 3*a*b^2*x - b^3 - 6*(a^3*x^3 + a^2*b*x^2)*log(a*x + b)
+ 6*(a^3*x^3 + a^2*b*x^2)*log(x))/(a*b^4*x^3 + b^5*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5} \, dx = \frac{3a^2\left(\log\left(x\right)-\log\left(x+\frac{b}{a}\right)\right)}{b^4} + \frac{6a^2x^2+3abx-b^2}{2ab^3x^3+2b^4x^2}$$

input
```
integrate(1/(a+b/x)**2/x**5,x)
```

output
```
3*a**2*(log(x) - log(x + b/a))/b**4 + (6*a**2*x**2 + 3*a*b*x - b**2)/(2*a*
b**3*x**3 + 2*b**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5} \, dx = \frac{6\,a^2x^2+3\,abx-b^2}{2\,(ab^3x^3+b^4x^2)} - \frac{3\,a^2\log\left(ax+b\right)}{b^4} + \frac{3\,a^2\log\left(x\right)}{b^4}$$

input
```
integrate(1/(a+b/x)^2/x^5,x, algorithm="maxima")
```

output
```
1/2*(6*a^2*x^2 + 3*a*b*x - b^2)/(a*b^3*x^3 + b^4*x^2) - 3*a^2*log(a*x + b)
/b^4 + 3*a^2*log(x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5}\,dx = -\frac{3\,a^2\log\left(|ax+b|\right)}{b^4} + \frac{3\,a^2\log\left(|x|\right)}{b^4} + \frac{6\,a^2bx^2+3\,ab^2x-b^3}{2\,(ax+b)b^4x^2}$$

input
```
integrate(1/(a+b/x)^2/x^5,x, algorithm="giac")
```

output
```
-3*a^2*log(abs(a*x + b))/b^4 + 3*a^2*log(abs(x))/b^4 + 1/2*(6*a^2*b*x^2 +
3*a*b^2*x - b^3)/((a*x + b)*b^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5}\,dx = \frac{\frac{3\,a^2\,x^2}{b^3}-\frac{1}{2\,b}+\frac{3\,a\,x}{2\,b^2}}{a\,x^3+b\,x^2} - \frac{6\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^4}$$

input
```
int(1/(x^5*(a + b/x)^2),x)
```

output
```
((3*a^2*x^2)/b^3 - 1/(2*b) + (3*a*x)/(2*b^2))/(a*x^3 + b*x^2) - (6*a^2*ata
nh((2*a*x)/b + 1))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^5}\,dx = \frac{-6\log(ax+b)\,a^3x^3-6\log(ax+b)\,a^2b\,x^2+6\log(x)\,a^3x^3+6\log(x)\,a^2b\,x^2-6a^3x^3+3a\,b^2x-b^3}{2b^4x^2\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^5,x)
```

output
```
( - 6*log(a*x + b)*a**3*x**3 - 6*log(a*x + b)*a**2*b*x**2 + 6*log(x)*a**3*
x**3 + 6*log(x)*a**2*b*x**2 - 6*a**3*x**3 + 3*a*b**2*x - b**3)/(2*b**4*x**
2*(a*x + b))
```

3.85 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6} \, dx = \frac{a^4}{b^5\left(a+\frac{b}{x}\right)}-\frac{1}{3b^2x^3}+\frac{a}{b^3x^2}-\frac{3a^2}{b^4x}+\frac{4a^3\log\left(a+\frac{b}{x}\right)}{b^5}$$

output
```
a^4/b^5/(a+b/x)-1/3/b^2/x^3+a/b^3/x^2-3*a^2/b^4/x+4*a^3*ln(a+b/x)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6} \, dx = -\frac{\frac{b(b^3-2ab^2x+6a^2bx^2+12a^3x^3)}{x^3(b+ax)}+12a^3\log(x)-12a^3\log(b+ax)}{3b^5}$$

input
```
Integrate[1/((a + b/x)^2*x^6),x]
```

output
```
-1/3*((b*(b^3 - 2*a*b^2*x + 6*a^2*b*x^2 + 12*a^3*x^3))/(x^3*(b + a*x)) + 1
2*a^3*Log[x] - 12*a^3*Log[b + a*x])/b^5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x}\right)^2} \, dx$$
$$\Big\downarrow \; 795$$
$$\int \frac{1}{x^4(ax+b)^2}dx$$
$$\Big\downarrow \; 54$$
$$\int \left(\frac{4a^4}{b^5(ax+b)}+\frac{a^4}{b^4(ax+b)^2}-\frac{4a^3}{b^5x}+\frac{3a^2}{b^4x^2}-\frac{2a}{b^3x^3}+\frac{1}{b^2x^4}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{4a^3\log(x)}{b^5}+\frac{4a^3\log(ax+b)}{b^5}-\frac{a^3}{b^4(ax+b)}-\frac{3a^2}{b^4x}+\frac{a}{b^3x^2}-\frac{1}{3b^2x^3}$$

input
```
Int[1/((a + b/x)^2*x^6),x]
```

output
```
-1/3*1/(b^2*x^3) + a/(b^3*x^2) - (3*a^2)/(b^4*x) - a^3/(b^4*(b + a*x)) - (
4*a^3*Log[x])/b^5 + (4*a^3*Log[b + a*x])/b^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{1}{3b^{2}x^{3}}+\frac{a}{b^{3}x^{2}}-\frac{3a^{2}}{b^{4}x}-\frac{4a^{3}\ln\left(x\right)}{b^{5}}-\frac{a^{3}}{b^{4}\left(ax+b\right)}+\frac{4a^{3}\ln\left(ax+b\right)}{b^{5}}$	68
risch	$\frac{-\frac{4a^{3}x^{3}}{b^{4}}-\frac{2a^{2}x^{2}}{b^{3}}+\frac{2ax}{3b^{2}}-\frac{1}{3b}}{x^{3}\left(ax+b\right)}-\frac{4a^{3}\ln\left(x\right)}{b^{5}}+\frac{4a^{3}\ln\left(-ax-b\right)}{b^{5}}$	75
norman	$\frac{\frac{4a^{4}x^{6}}{b^{5}}-\frac{x^{2}}{3b}+\frac{2a\,x^{3}}{3b^{2}}-\frac{2a^{2}x^{4}}{b^{3}}}{\left(ax+b\right)x^{5}}-\frac{4a^{3}\ln\left(x\right)}{b^{5}}+\frac{4a^{3}\ln\left(ax+b\right)}{b^{5}}$	77
parallelrisch	$-\frac{12a^{4}\ln\left(x\right)x^{4}-12a^{4}\ln\left(ax+b\right)x^{4}+12\ln\left(x\right)x^{3}a^{3}b-12\ln\left(ax+b\right)x^{3}a^{3}b-12a^{4}x^{4}+6a^{2}b^{2}x^{2}-2a\,b^{3}x+b^{4}}{3b^{5}x^{3}\left(ax+b\right)}$	96

input
```
int(1/(a+b/x)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2/x^3+a/b^3/x^2-3*a^2/b^4/x-4/b^5*a^3*ln(x)-a^3/b^4/(a*x+b)+4/b^5*a
^3*ln(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2x^6}\,dx = -\frac{12\,a^{3}bx^{3}+6\,a^{2}b^{2}x^{2}-2\,ab^{3}x+b^{4}-12\,\left(a^{4}x^{4}+a^{3}bx^{3}\right)\log\left(ax+b\right)+12\,\left(a^{4}x^{4}+a^{3}bx^{3}\right)\log\left(x\right)}{3\,\left(ab^{5}x^{4}+b^{6}x^{3}\right)}$$

input
```
integrate(1/(a+b/x)^2/x^6,x, algorithm="fricas")
```

output
```
-1/3*(12*a^3*b*x^3 + 6*a^2*b^2*x^2 - 2*a*b^3*x + b^4 - 12*(a^4*x^4 + a^3*b
*x^3)*log(a*x + b) + 12*(a^4*x^4 + a^3*b*x^3)*log(x))/(a*b^5*x^4 + b^6*x^3
)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx = \frac{4a^3\left(-\log\left(x\right)+\log\left(x+\frac{b}{a}\right)\right)}{b^5} + \frac{-12a^3x^3-6a^2bx^2+2ab^2x-b^3}{3ab^4x^4+3b^5x^3}$$

input
```
integrate(1/(a+b/x)**2/x**6,x)
```

output
```
4*a**3*(-log(x) + log(x + b/a))/b**5 + (-12*a**3*x**3 - 6*a**2*b*x**2 + 2*
a*b**2*x - b**3)/(3*a*b**4*x**4 + 3*b**5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx = -\frac{12\,a^3x^3+6\,a^2bx^2-2\,ab^2x+b^3}{3\left(ab^4x^4+b^5x^3\right)} + \frac{4\,a^3\log\left(ax+b\right)}{b^5} - \frac{4\,a^3\log\left(x\right)}{b^5}$$

input
```
integrate(1/(a+b/x)^2/x^6,x, algorithm="maxima")
```

output
```
-1/3*(12*a^3*x^3 + 6*a^2*b*x^2 - 2*a*b^2*x + b^3)/(a*b^4*x^4 + b^5*x^3) +
4*a^3*log(a*x + b)/b^5 - 4*a^3*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx = \frac{4\,a^3\log\left(|ax+b|\right)}{b^5} - \frac{4\,a^3\log\left(|x|\right)}{b^5} - \frac{12\,a^3bx^3+6\,a^2b^2x^2-2\,ab^3x+b^4}{3\,(ax+b)b^5x^3}$$

input

```
integrate(1/(a+b/x)^2/x^6,x, algorithm="giac")
```

output

```
4*a^3*log(abs(a*x + b))/b^5 - 4*a^3*log(abs(x))/b^5 - 1/3*(12*a^3*b*x^3 +
6*a^2*b^2*x^2 - 2*a*b^3*x + b^4)/((a*x + b)*b^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx = \frac{8\,a^3\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^5} - \frac{\frac{1}{3\,b}+\frac{2\,a^2\,x^2}{b^3}+\frac{4\,a^3\,x^3}{b^4}-\frac{2\,a\,x}{3\,b^2}}{a\,x^4+b\,x^3}$$

input

```
int(1/(x^6*(a + b/x)^2),x)
```

output

```
(8*a^3*atanh((2*a*x)/b + 1))/b^5 - (1/(3*b) + (2*a^2*x^2)/b^3 + (4*a^3*x^3
)/b^4 - (2*a*x)/(3*b^2))/(a*x^4 + b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^6}\,dx$$
$$= \frac{12\log(ax+b)\,a^4x^4+12\log(ax+b)\,a^3b\,x^3-12\log(x)\,a^4x^4-12\log(x)\,a^3b\,x^3+12a^4x^4-6a^2b^2x^2+2a}{3b^5x^3\,(ax+b)}$$

input

```
int(1/(a+b/x)^2/x^6,x)
```

output
```
(12*log(a*x + b)*a**4*x**4 + 12*log(a*x + b)*a**3*b*x**3 - 12*log(x)*a**4*
x**4 - 12*log(x)*a**3*b*x**3 + 12*a**4*x**4 - 6*a**2*b**2*x**2 + 2*a*b**3*
x - b**4)/(3*b**5*x**3*(a*x + b))
```

3.86 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7} \, dx = -\frac{a^5}{b^6\left(a+\frac{b}{x}\right)} - \frac{1}{4b^2x^4} + \frac{2a}{3b^3x^3} - \frac{3a^2}{2b^4x^2} + \frac{4a^3}{b^5x} - \frac{5a^4\log\left(a+\frac{b}{x}\right)}{b^6}$$

output
```
-a^5/b^6/(a+b/x)-1/4/b^2/x^4+2/3*a/b^3/x^3-3/2*a^2/b^4/x^2+4*a^3/b^5/x-5*a
^4*ln(a+b/x)/b^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7} \, dx = \frac{\frac{b(-3b^4+5ab^3x-10a^2b^2x^2+30a^3bx^3+60a^4x^4)}{x^4(b+ax)} + 60a^4\log(x) - 60a^4\log(b+ax)}{12b^6}$$

input
```
Integrate[1/((a + b/x)^2*x^7),x]
```

output
```
((b*(-3*b^4 + 5*a*b^3*x - 10*a^2*b^2*x^2 + 30*a^3*b*x^3 + 60*a^4*x^4))/(x^
4*(b + a*x)) + 60*a^4*Log[x] - 60*a^4*Log[b + a*x])/(12*b^6)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^5(ax+b)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(-\frac{5a^5}{b^6(ax+b)}-\frac{a^5}{b^5(ax+b)^2}+\frac{5a^4}{b^6x}-\frac{4a^3}{b^5x^2}+\frac{3a^2}{b^4x^3}-\frac{2a}{b^3x^4}+\frac{1}{b^2x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5a^4\log(x)}{b^6}-\frac{5a^4\log(ax+b)}{b^6}+\frac{a^4}{b^5(ax+b)}+\frac{4a^3}{b^5x}-\frac{3a^2}{2b^4x^2}+\frac{2a}{3b^3x^3}-\frac{1}{4b^2x^4}$$

input
```
Int[1/((a + b/x)^2*x^7),x]
```

output
```
-1/4*1/(b^2*x^4) + (2*a)/(3*b^3*x^3) - (3*a^2)/(2*b^4*x^2) + (4*a^3)/(b^5*
x) + a^4/(b^5*(b + a*x)) + (5*a^4*Log[x])/b^6 - (5*a^4*Log[b + a*x])/b^6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{1}{4b^2x^4}-\frac{3a^2}{2b^4x^2}+\frac{5a^4\ln(x)}{b^6}+\frac{4a^3}{b^5x}+\frac{2a}{3b^3x^3}-\frac{5a^4\ln(ax+b)}{b^6}+\frac{a^4}{b^5(ax+b)}$	79
risch	$\frac{\frac{5a^4x^4}{b^5}+\frac{5a^3x^3}{2b^4}-\frac{5a^2x^2}{6b^3}+\frac{5ax}{12b^2}-\frac{1}{4b}}{x^4(ax+b)}-\frac{5a^4\ln(ax+b)}{b^6}+\frac{5a^4\ln(-x)}{b^6}$	85
norman	$\frac{-\frac{x^2}{4b}+\frac{5a\,x^3}{12b^2}-\frac{5a^2x^4}{6b^3}+\frac{5a^3x^5}{2b^4}-\frac{5a^5x^7}{b^6}}{x^6(ax+b)}+\frac{5a^4\ln(x)}{b^6}-\frac{5a^4\ln(ax+b)}{b^6}$	88
parallelrisch	$\frac{60a^5\ln(x)x^5-60a^5\ln(ax+b)x^5+60\ln(x)x^4a^4b-60\ln(ax+b)x^4a^4b-60a^5x^5+30a^3b^2x^3-10a^2b^3x^2+5b^4xa-3b^5}{12b^6x^4(ax+b)}$	109

input
```
int(1/(a+b/x)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2/x^4-3/2*a^2/b^4/x^2+5*a^4/b^6*ln(x)+4*a^3/b^5/x+2/3*a/b^3/x^3-5*a
^4/b^6*ln(a*x+b)+a^4/b^5/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2x^7}\,dx$$
$$=\frac{60\,a^4bx^4+30\,a^3b^2x^3-10\,a^2b^3x^2+5\,ab^4x-3\,b^5-60\,(a^5x^5+a^4bx^4)\log\left(ax+b\right)+60\,(a^5x^5+a^4bx^4)\log}{12\,(ab^6x^5+b^7x^4)}$$

input
```
integrate(1/(a+b/x)^2/x^7,x, algorithm="fricas")
```

output
```
1/12*(60*a^4*b*x^4 + 30*a^3*b^2*x^3 - 10*a^2*b^3*x^2 + 5*a*b^4*x - 3*b^5 -
 60*(a^5*x^5 + a^4*b*x^4)*log(a*x + b) + 60*(a^5*x^5 + a^4*b*x^4)*log(x))/
(a*b^6*x^5 + b^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx = \frac{5a^4\left(\log(x)-\log\left(x+\frac{b}{a}\right)\right)}{b^6} + \frac{60a^4x^4+30a^3bx^3-10a^2b^2x^2+5ab^3x-3b^4}{12ab^5x^5+12b^6x^4}$$

input
```
integrate(1/(a+b/x)**2/x**7,x)
```

output
```
5*a**4*(log(x) - log(x + b/a))/b**6 + (60*a**4*x**4 + 30*a**3*b*x**3 - 10*
a**2*b**2*x**2 + 5*a*b**3*x - 3*b**4)/(12*a*b**5*x**5 + 12*b**6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx = \frac{60\,a^4x^4+30\,a^3bx^3-10\,a^2b^2x^2+5\,ab^3x-3\,b^4}{12\,(ab^5x^5+b^6x^4)} - \frac{5\,a^4\log(ax+b)}{b^6} + \frac{5\,a^4\log(x)}{b^6}$$

input
```
integrate(1/(a+b/x)^2/x^7,x, algorithm="maxima")
```

output
```
1/12*(60*a^4*x^4 + 30*a^3*b*x^3 - 10*a^2*b^2*x^2 + 5*a*b^3*x - 3*b^4)/(a*b
^5*x^5 + b^6*x^4) - 5*a^4*log(a*x + b)/b^6 + 5*a^4*log(x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx = -\frac{5\,a^4 \log\left(|ax+b|\right)}{b^6} + \frac{5\,a^4 \log\left(|x|\right)}{b^6} + \frac{60\,a^4bx^4 + 30\,a^3b^2x^3 - 10\,a^2b^3x^2 + 5\,ab^4x - 3\,b^5}{12\,(ax+b)b^6x^4}$$

input
```
integrate(1/(a+b/x)^2/x^7,x, algorithm="giac")
```

output
```
-5*a^4*log(abs(a*x + b))/b^6 + 5*a^4*log(abs(x))/b^6 + 1/12*(60*a^4*b*x^4
+ 30*a^3*b^2*x^3 - 10*a^2*b^3*x^2 + 5*a*b^4*x - 3*b^5)/((a*x + b)*b^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx = \frac{\frac{5\,a^3\,x^3}{2\,b^4} - \frac{5\,a^2\,x^2}{6\,b^3} - \frac{1}{4\,b} + \frac{5\,a^4\,x^4}{b^5} + \frac{5\,a\,x}{12\,b^2}}{a\,x^5 + b\,x^4} - \frac{10\,a^4\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^6}$$

input
```
int(1/(x^7*(a + b/x)^2),x)
```

output
```
((5*a^3*x^3)/(2*b^4) - (5*a^2*x^2)/(6*b^3) - 1/(4*b) + (5*a^4*x^4)/b^5 + (
5*a*x)/(12*b^2))/(a*x^5 + b*x^4) - (10*a^4*atanh((2*a*x)/b + 1))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^7}\,dx = \frac{-60\log(ax+b)\,a^5x^5 - 60\log(ax+b)\,a^4b\,x^4 + 60\log(x)\,a^5x^5 + 60\log(x)\,a^4b\,x^4 - 60a^5x^5 + 30a^3b^2x^3 -}{12b^6x^4\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^7,x)
```

output
```
( - 60*log(a*x + b)*a**5*x**5 - 60*log(a*x + b)*a**4*b*x**4 + 60*log(x)*a*
*5*x**5 + 60*log(x)*a**4*b*x**4 - 60*a**5*x**5 + 30*a**3*b**2*x**3 - 10*a*
*2*b**3*x**2 + 5*a*b**4*x - 3*b**5)/(12*b**6*x**4*(a*x + b))
```

3.87 $\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3} dx = \frac{15b^4x}{a^7} - \frac{5b^3x^2}{a^6} + \frac{2b^2x^3}{a^5} - \frac{3bx^4}{4a^4} + \frac{x^5}{5a^3} + \frac{b^7}{2a^8(b+ax)^2} - \frac{7b^6}{a^8(b+ax)} - \frac{21b^5\log(b+ax)}{a^8}$$

output
```
15*b^4*x/a^7-5*b^3*x^2/a^6+2*b^2*x^3/a^5-3/4*b*x^4/a^4+1/5*x^5/a^3+1/2*b^7
/a^8/(a*x+b)^2-7*b^6/a^8/(a*x+b)-21*b^5*ln(a*x+b)/a^8
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3} dx = \frac{300ab^4x - 100a^2b^3x^2 + 40a^3b^2x^3 - 15a^4bx^4 + 4a^5x^5 - \frac{10b^6(13b+14ax)}{(b+ax)^2} - 420b^5\log(b+ax)}{20a^8}$$

input
```
Integrate[x^4/(a + b/x)^3,x]
```

output
```
(300*a*b^4*x - 100*a^2*b^3*x^2 + 40*a^3*b^2*x^3 - 15*a^4*b*x^4 + 4*a^5*x^5
 - (10*b^6*(13*b + 14*a*x))/(b + a*x)^2 - 420*b^5*Log[b + a*x])/(20*a^8)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^7}{(ax+b)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^7}{a^7(ax+b)^3}+\frac{7b^6}{a^7(ax+b)^2}-\frac{21b^5}{a^7(ax+b)}+\frac{15b^4}{a^7}-\frac{10b^3x}{a^6}+\frac{6b^2x^2}{a^5}-\frac{3bx^3}{a^4}+\frac{x^4}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^7}{2a^8(ax+b)^2}-\frac{7b^6}{a^8(ax+b)}-\frac{21b^5\log(ax+b)}{a^8}+\frac{15b^4x}{a^7}-\frac{5b^3x^2}{a^6}+\frac{2b^2x^3}{a^5}-\frac{3bx^4}{4a^4}+\frac{x^5}{5a^3}$$

input
```
Int[x^4/(a + b/x)^3,x]
```

output
```
(15*b^4*x)/a^7 - (5*b^3*x^2)/a^6 + (2*b^2*x^3)/a^5 - (3*b*x^4)/(4*a^4) + x
^5/(5*a^3) + b^7/(2*a^8*(b + a*x)^2) - (7*b^6)/(a^8*(b + a*x)) - (21*b^5*L
og[b + a*x])/a^8
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result
risch	$\frac{x^{5}}{5a^{3}}-\frac{3b\,x^{4}}{4a^{4}}+\frac{2b^{2}x^{3}}{a^{5}}-\frac{5b^{3}x^{2}}{a^{6}}+\frac{15b^{4}x}{a^{7}}+\frac{-7b^{6}x-\frac{13b^{7}}{2a}}{a^{7}\left(ax+b\right)^{2}}-\frac{21b^{5}\ln\left(ax+b\right)}{a^{8}}$
norman	$\frac{\frac{x^{7}}{5a}-\frac{7b\,x^{6}}{20a^{2}}+\frac{7b^{2}x^{5}}{10a^{3}}-\frac{63b^{7}}{2a^{8}}-\frac{7b^{3}x^{4}}{4a^{4}}+\frac{7b^{4}x^{3}}{a^{5}}-\frac{42b^{6}x}{a^{7}}}{\left(ax+b\right)^{2}}-\frac{21b^{5}\ln\left(ax+b\right)}{a^{8}}$
default	$\frac{\frac{1}{5}a^{4}x^{5}-\frac{3}{4}a^{3}b\,x^{4}+2a^{2}b^{2}x^{3}-5a\,b^{3}x^{2}+15b^{4}x}{a^{7}}-\frac{7b^{6}}{a^{8}\left(ax+b\right)}-\frac{21b^{5}\ln\left(ax+b\right)}{a^{8}}+\frac{b^{7}}{2a^{8}\left(ax+b\right)^{2}}$
parallelrisch	$-\frac{-4a^{7}x^{7}+7a^{6}b\,x^{6}-14a^{5}b^{2}x^{5}+35a^{4}b^{3}x^{4}+420\ln\left(ax+b\right)x^{2}a^{2}b^{5}-140b^{4}x^{3}a^{3}+840\ln\left(ax+b\right)xa\,b^{6}+420\ln\left(ax+b\right)b^{7}+840b^{6}xa}{20a^{8}\left(ax+b\right)^{2}}$

input
```
int(x^4/(a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5/a^3*x^5-3/4*b*x^4/a^4+2*b^2*x^3/a^5-5/a^6*b^3*x^2+15*b^4*x/a^7+(-7*b^6
*x-13/2*b^7/a)/a^7/(a*x+b)^2-21*b^5*ln(a*x+b)/a^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx$$
$$= \frac{4\,a^7x^7 - 7\,a^6bx^6 + 14\,a^5b^2x^5 - 35\,a^4b^3x^4 + 140\,a^3b^4x^3 + 500\,a^2b^5x^2 + 160\,ab^6x - 130\,b^7 - 420\,(a^2b^5x^2 +}{20\,(a^{10}x^2 + 2\,a^9bx + a^8b^2)}$$

input
```
integrate(x^4/(a+b/x)^3,x, algorithm="fricas")
```

output
```
1/20*(4*a^7*x^7 - 7*a^6*b*x^6 + 14*a^5*b^2*x^5 - 35*a^4*b^3*x^4 + 140*a^3*
b^4*x^3 + 500*a^2*b^5*x^2 + 160*a*b^6*x - 130*b^7 - 420*(a^2*b^5*x^2 + 2*a
*b^6*x + b^7)*log(a*x + b))/(a^10*x^2 + 2*a^9*b*x + a^8*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{-14ab^6x - 13b^7}{2a^{10}x^2 + 4a^9bx + 2a^8b^2} + \frac{x^5}{5a^3} - \frac{3bx^4}{4a^4} + \frac{2b^2x^3}{a^5} - \frac{5b^3x^2}{a^6} + \frac{15b^4x}{a^7} - \frac{21b^5\log(ax+b)}{a^8}$$

input
```
integrate(x**4/(a+b/x)**3,x)
```

output
```
(-14*a*b**6*x - 13*b**7)/(2*a**10*x**2 + 4*a**9*b*x + 2*a**8*b**2) + x**5/
(5*a**3) - 3*b*x**4/(4*a**4) + 2*b**2*x**3/a**5 - 5*b**3*x**2/a**6 + 15*b*
*4*x/a**7 - 21*b**5*log(a*x + b)/a**8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{14\,ab^6x+13\,b^7}{2\,(a^{10}x^2+2\,a^9bx+a^8b^2)} - \frac{21\,b^5\log{(ax+b)}}{a^8} + \frac{4\,a^4x^5-15\,a^3bx^4+40\,a^2b^2x^3-100\,ab^3x^2+300\,b^4x}{20\,a^7}$$

input
```
integrate(x^4/(a+b/x)^3,x, algorithm="maxima")
```

output
```
-1/2*(14*a*b^6*x + 13*b^7)/(a^10*x^2 + 2*a^9*b*x + a^8*b^2) - 21*b^5*log(a
*x + b)/a^8 + 1/20*(4*a^4*x^5 - 15*a^3*b*x^4 + 40*a^2*b^2*x^3 - 100*a*b^3*
x^2 + 300*b^4*x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{21\,b^5\log{(|ax+b|)}}{a^8} - \frac{14\,ab^6x+13\,b^7}{2\,(ax+b)^2a^8} + \frac{4\,a^{12}x^5-15\,a^{11}bx^4+40\,a^{10}b^2x^3-100\,a^9b^3x^2+300\,a^8b^4x}{20\,a^{15}}$$

input
```
integrate(x^4/(a+b/x)^3,x, algorithm="giac")
```

output
```
-21*b^5*log(abs(a*x + b))/a^8 - 1/2*(14*a*b^6*x + 13*b^7)/((a*x + b)^2*a^8
) + 1/20*(4*a^12*x^5 - 15*a^11*b*x^4 + 40*a^10*b^2*x^3 - 100*a^9*b^3*x^2 +
 300*a^8*b^4*x)/a^15
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{x^5}{5\,a^3}-\frac{7\,b^6\,x+\frac{13\,b^7}{2\,a}}{a^9\,x^2+2\,a^8\,b\,x+a^7\,b^2}-\frac{21\,b^5\,\ln\left(b+a\,x\right)}{a^8}-\frac{3\,b\,x^4}{4\,a^4}+\frac{15\,b^4\,x}{a^7}+\frac{2\,b^2\,x^3}{a^5}-\frac{5\,b^3\,x^2}{a^6}$$

input
```
int(x^4/(a + b/x)^3,x)
```

output
```
x^5/(5*a^3) - (7*b^6*x + (13*b^7)/(2*a))/(a^7*b^2 + a^9*x^2 + 2*a^8*b*x) -
 (21*b^5*log(b + a*x))/a^8 - (3*b*x^4)/(4*a^4) + (15*b^4*x)/a^7 + (2*b^2*x
^3)/a^5 - (5*b^3*x^2)/a^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

$$\int \frac{x^4}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{-420\log(ax+b)\,a^2b^5x^2-840\log(ax+b)\,a\,b^6x-420\log(ax+b)\,b^7+4a^7x^7-7a^6b\,x^6+14a^5b^2x^5-35}{20a^8\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(x^4/(a+b/x)^3,x)
```

output
```
( - 420*log(a*x + b)*a**2*b**5*x**2 - 840*log(a*x + b)*a*b**6*x - 420*log(
a*x + b)*b**7 + 4*a**7*x**7 - 7*a**6*b*x**6 + 14*a**5*b**2*x**5 - 35*a**4*
b**3*x**4 + 140*a**3*b**4*x**3 + 420*a**2*b**5*x**2 - 210*b**7)/(20*a**8*(
a**2*x**2 + 2*a*b*x + b**2))
```

3.88 $\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{10b^3x}{a^6}+\frac{3b^2x^2}{a^5}-\frac{bx^3}{a^4}+\frac{x^4}{4a^3}-\frac{b^6}{2a^7(b+ax)^2}+\frac{6b^5}{a^7(b+ax)}+\frac{15b^4\log(b+ax)}{a^7}$$

output
```
-10*b^3*x/a^6+3*b^2*x^2/a^5-b*x^3/a^4+1/4*x^4/a^3-1/2*b^6/a^7/(a*x+b)^2+6*
b^5/a^7/(a*x+b)+15*b^4*ln(a*x+b)/a^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{-40ab^3x+12a^2b^2x^2-4a^3bx^3+a^4x^4+\frac{2b^5(11b+12ax)}{(b+ax)^2}+60b^4\log(b+ax)}{4a^7}$$

input
```
Integrate[x^3/(a + b/x)^3,x]
```

output
```
(-40*a*b^3*x + 12*a^2*b^2*x^2 - 4*a^3*b*x^3 + a^4*x^4 + (2*b^5*(11*b + 12*
a*x))/(b + a*x)^2 + 60*b^4*Log[b + a*x])/(4*a^7)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^6}{(ax+b)^3}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{b^6}{a^6(ax+b)^3}-\frac{6b^5}{a^6(ax+b)^2}+\frac{15b^4}{a^6(ax+b)}-\frac{10b^3}{a^6}+\frac{6b^2x}{a^5}-\frac{3bx^2}{a^4}+\frac{x^3}{a^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^6}{2a^7(ax+b)^2}+\frac{6b^5}{a^7(ax+b)}+\frac{15b^4\log(ax+b)}{a^7}-\frac{10b^3x}{a^6}+\frac{3b^2x^2}{a^5}-\frac{bx^3}{a^4}+\frac{x^4}{4a^3}$$

input
```
Int[x^3/(a + b/x)^3,x]
```

output
```
(-10*b^3*x)/a^6 + (3*b^2*x^2)/a^5 - (b*x^3)/a^4 + x^4/(4*a^3) - b^6/(2*a^7
*(b + a*x)^2) + (6*b^5)/(a^7*(b + a*x)) + (15*b^4*Log[b + a*x])/a^7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{x^4}{4a^3}-\frac{b\,x^3}{a^4}+\frac{3b^2x^2}{a^5}-\frac{10b^3x}{a^6}+\frac{6b^5x+\frac{11b^6}{2a}}{a^6(ax+b)^2}+\frac{15b^4\ln(ax+b)}{a^7}$	79
norman	$\frac{\frac{x^6}{4a}-\frac{b\,x^5}{2a^2}+\frac{5b^2x^4}{4a^3}+\frac{45b^6}{2a^7}-\frac{5b^3x^3}{a^4}+\frac{30b^5x}{a^6}}{(ax+b)^2}+\frac{15b^4\ln(ax+b)}{a^7}$	81
default	$\frac{\frac{1}{4}a^3x^4-a^2b\,x^3+3a\,b^2x^2-10b^3x}{a^6}+\frac{6b^5}{a^7(ax+b)}+\frac{15b^4\ln(ax+b)}{a^7}-\frac{b^6}{2a^7(ax+b)^2}$	83
parallelrisch	$\frac{a^6x^6-2a^5b\,x^5+5a^4b^2x^4+60\ln(ax+b)x^2a^2b^4-20a^3x^3b^3+120\ln(ax+b)xa\,b^5+60\ln(ax+b)b^6+120b^5xa+90b^6}{4a^7(ax+b)^2}$	105

input
```
int(x^3/(a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/a^3-b*x^3/a^4+3*b^2*x^2/a^5-10*b^3*x/a^6+(6*b^5*x+11/2*b^6/a)/a^6/
(a*x+b)^2+15*b^4*ln(a*x+b)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{a^6x^6-2\,a^5bx^5+5\,a^4b^2x^4-20\,a^3b^3x^3-68\,a^2b^4x^2-16\,ab^5x+22\,b^6+60\,(a^2b^4x^2+2\,ab^5x+b^6)\log(ax}{4\,(a^9x^2+2\,a^8bx+a^7b^2)}$$

input
```
integrate(x^3/(a+b/x)^3,x, algorithm="fricas")
```

output
```
1/4*(a^6*x^6 - 2*a^5*b*x^5 + 5*a^4*b^2*x^4 - 20*a^3*b^3*x^3 - 68*a^2*b^4*x
^2 - 16*a*b^5*x + 22*b^6 + 60*(a^2*b^4*x^2 + 2*a*b^5*x + b^6)*log(a*x + b)
)/(a^9*x^2 + 2*a^8*b*x + a^7*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{12ab^5x+11b^6}{2a^9x^2+4a^8bx+2a^7b^2}+\frac{x^4}{4a^3}-\frac{bx^3}{a^4}+\frac{3b^2x^2}{a^5}-\frac{10b^3x}{a^6}+\frac{15b^4\log(ax+b)}{a^7}$$

input
```
integrate(x**3/(a+b/x)**3,x)
```

output
```
(12*a*b**5*x + 11*b**6)/(2*a**9*x**2 + 4*a**8*b*x + 2*a**7*b**2) + x**4/(4
*a**3) - b*x**3/a**4 + 3*b**2*x**2/a**5 - 10*b**3*x/a**6 + 15*b**4*log(a*x
 + b)/a**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{12\,ab^5x+11\,b^6}{2\,(a^9x^2+2\,a^8bx+a^7b^2)} + \frac{15\,b^4\log\left(ax+b\right)}{a^7} + \frac{a^3x^4-4\,a^2bx^3+12\,ab^2x^2-40\,b^3x}{4\,a^6}$$

input
```
integrate(x^3/(a+b/x)^3,x, algorithm="maxima")
```

output
```
1/2*(12*a*b^5*x + 11*b^6)/(a^9*x^2 + 2*a^8*b*x + a^7*b^2) + 15*b^4*log(a*x
 + b)/a^7 + 1/4*(a^3*x^4 - 4*a^2*b*x^3 + 12*a*b^2*x^2 - 40*b^3*x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{15\,b^4\log\left(|ax+b|\right)}{a^7} + \frac{12\,ab^5x+11\,b^6}{2\,(ax+b)^2a^7} + \frac{a^9x^4-4\,a^8bx^3+12\,a^7b^2x^2-40\,a^6b^3x}{4\,a^{12}}$$

input
```
integrate(x^3/(a+b/x)^3,x, algorithm="giac")
```

output
```
15*b^4*log(abs(a*x + b))/a^7 + 1/2*(12*a*b^5*x + 11*b^6)/((a*x + b)^2*a^7)
 + 1/4*(a^9*x^4 - 4*a^8*b*x^3 + 12*a^7*b^2*x^2 - 40*a^6*b^3*x)/a^12
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{6\,b^5\,x+\frac{11\,b^6}{2\,a}}{a^8\,x^2+2\,a^7\,b\,x+a^6\,b^2}+\frac{x^4}{4\,a^3} + \frac{15\,b^4\,\ln\left(b+a\,x\right)}{a^7}-\frac{b\,x^3}{a^4}-\frac{10\,b^3\,x}{a^6}+\frac{3\,b^2\,x^2}{a^5}$$

input
```
int(x^3/(a + b/x)^3,x)
```

output
```
(6*b^5*x + (11*b^6)/(2*a))/(a^6*b^2 + a^8*x^2 + 2*a^7*b*x) + x^4/(4*a^3) +
 (15*b^4*log(b + a*x))/a^7 - (b*x^3)/a^4 - (10*b^3*x)/a^6 + (3*b^2*x^2)/a^
5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{60\log(ax+b)\,a^2b^4x^2+120\log(ax+b)\,a\,b^5x+60\log(ax+b)\,b^6+a^6x^6-2a^5b\,x^5+5a^4b^2x^4-20a^3b^3x^3}{4a^7\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(x^3/(a+b/x)^3,x)
```

output
```
(60*log(a*x + b)*a**2*b**4*x**2 + 120*log(a*x + b)*a*b**5*x + 60*log(a*x +
 b)*b**6 + a**6*x**6 - 2*a**5*b*x**5 + 5*a**4*b**2*x**4 - 20*a**3*b**3*x**
3 - 60*a**2*b**4*x**2 + 30*b**6)/(4*a**7*(a**2*x**2 + 2*a*b*x + b**2))
```

3.89 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3} dx = \frac{6b^2x}{a^5} - \frac{3bx^2}{2a^4} + \frac{x^3}{3a^3} + \frac{b^5}{2a^6(b+ax)^2} - \frac{5b^4}{a^6(b+ax)} - \frac{10b^3\log(b+ax)}{a^6}$$

output
```
6*b^2*x/a^5-3/2*b*x^2/a^4+1/3*x^3/a^3+1/2*b^5/a^6/(a*x+b)^2-5*b^4/a^6/(a*x
+b)-10*b^3*ln(a*x+b)/a^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3} dx = \frac{36ab^2x - 9a^2bx^2 + 2a^3x^3 - \frac{3b^4(9b+10ax)}{(b+ax)^2} - 60b^3\log(b+ax)}{6a^6}$$

input
```
Integrate[x^2/(a + b/x)^3,x]
```

output
```
(36*a*b^2*x - 9*a^2*b*x^2 + 2*a^3*x^3 - (3*b^4*(9*b + 10*a*x))/(b + a*x)^2
 - 60*b^3*Log[b + a*x])/(6*a^6)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^5}{(ax+b)^3}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(-\frac{b^5}{a^5(ax+b)^3}+\frac{5b^4}{a^5(ax+b)^2}-\frac{10b^3}{a^5(ax+b)}+\frac{6b^2}{a^5}-\frac{3bx}{a^4}+\frac{x^2}{a^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^5}{2a^6(ax+b)^2}-\frac{5b^4}{a^6(ax+b)}-\frac{10b^3\log(ax+b)}{a^6}+\frac{6b^2x}{a^5}-\frac{3bx^2}{2a^4}+\frac{x^3}{3a^3}$$

input
```
Int[x^2/(a + b/x)^3,x]
```

output
```
(6*b^2*x)/a^5 - (3*b*x^2)/(2*a^4) + x^3/(3*a^3) + b^5/(2*a^6*(b + a*x)^2)
- (5*b^4)/(a^6*(b + a*x)) - (10*b^3*Log[b + a*x])/a^6
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{x^{3}}{3a^{3}}-\frac{3b\,x^{2}}{2a^{4}}+\frac{6b^{2}x}{a^{5}}+\frac{-5b^{4}x-\frac{9b^{5}}{2a}}{a^{5}\left(ax+b\right)^{2}}-\frac{10b^{3}\ln\left(ax+b\right)}{a^{6}}$	68
norman	$\frac{-\frac{15b^{5}}{a^{6}}+\frac{x^{5}}{3a}-\frac{5b\,x^{4}}{6a^{2}}+\frac{10b^{2}x^{3}}{3a^{3}}-\frac{20b^{4}x}{a^{5}}}{\left(ax+b\right)^{2}}-\frac{10b^{3}\ln\left(ax+b\right)}{a^{6}}$	70
default	$\frac{\frac{1}{3}a^{2}x^{3}-\frac{3}{2}ab\,x^{2}+6b^{2}x}{a^{5}}-\frac{5b^{4}}{a^{6}\left(ax+b\right)}-\frac{10b^{3}\ln\left(ax+b\right)}{a^{6}}+\frac{b^{5}}{2a^{6}\left(ax+b\right)^{2}}$	72
parallelrisch	$-\frac{-2a^{5}x^{5}+5a^{4}b\,x^{4}+60\ln\left(ax+b\right)x^{2}a^{2}b^{3}-20a^{3}b^{2}x^{3}+120\ln\left(ax+b\right)xa\,b^{4}+60b^{5}\ln\left(ax+b\right)+120b^{4}xa+90b^{5}}{6a^{6}\left(ax+b\right)^{2}}$	95

input
```
int(x^2/(a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/a^3-3/2*b*x^2/a^4+6*b^2*x/a^5+(-5*b^4*x-9/2*b^5/a)/a^5/(a*x+b)^2-1
0*b^3*ln(a*x+b)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int\frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{2\,a^5x^5-5\,a^4bx^4+20\,a^3b^2x^3+63\,a^2b^3x^2+6\,ab^4x-27\,b^5-60\,(a^2b^3x^2+2\,ab^4x+b^5)\log\left(ax+b\right)}{6\,(a^8x^2+2\,a^7bx+a^6b^2)}$$

input
```
integrate(x^2/(a+b/x)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a^5*x^5 - 5*a^4*b*x^4 + 20*a^3*b^2*x^3 + 63*a^2*b^3*x^2 + 6*a*b^4*x
 - 27*b^5 - 60*(a^2*b^3*x^2 + 2*a*b^4*x + b^5)*log(a*x + b))/(a^8*x^2 + 2*
a^7*b*x + a^6*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{-10ab^4x-9b^5}{2a^8x^2+4a^7bx+2a^6b^2}+\frac{x^3}{3a^3}-\frac{3bx^2}{2a^4}+\frac{6b^2x}{a^5}-\frac{10b^3\log\left(ax+b\right)}{a^6}$$

input
```
integrate(x**2/(a+b/x)**3,x)
```

output
```
(-10*a*b**4*x - 9*b**5)/(2*a**8*x**2 + 4*a**7*b*x + 2*a**6*b**2) + x**3/(3
*a**3) - 3*b*x**2/(2*a**4) + 6*b**2*x/a**5 - 10*b**3*log(a*x + b)/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{10\,ab^4x+9\,b^5}{2\,(a^8x^2+2\,a^7bx+a^6b^2)}-\frac{10\,b^3\log\left(ax+b\right)}{a^6}+\frac{2\,a^2x^3-9\,abx^2+36\,b^2x}{6\,a^5}$$

input
```
integrate(x^2/(a+b/x)^3,x, algorithm="maxima")
```

output
```
-1/2*(10*a*b^4*x + 9*b^5)/(a^8*x^2 + 2*a^7*b*x + a^6*b^2) - 10*b^3*log(a*x
 + b)/a^6 + 1/6*(2*a^2*x^3 - 9*a*b*x^2 + 36*b^2*x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{10\,b^3\log\left(|ax+b|\right)}{a^6} - \frac{10\,ab^4x+9\,b^5}{2\,(ax+b)^2a^6} + \frac{2\,a^6x^3-9\,a^5bx^2+36\,a^4b^2x}{6\,a^9}$$

input
```
integrate(x^2/(a+b/x)^3,x, algorithm="giac")
```

output
```
-10*b^3*log(abs(a*x + b))/a^6 - 1/2*(10*a*b^4*x + 9*b^5)/((a*x + b)^2*a^6)
 + 1/6*(2*a^6*x^3 - 9*a^5*b*x^2 + 36*a^4*b^2*x)/a^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{x^3}{3\,a^3} - \frac{5\,b^4\,x+\frac{9\,b^5}{2\,a}}{a^7\,x^2+2\,a^6\,b\,x+a^5\,b^2} - \frac{10\,b^3\,\ln\left(b+a\,x\right)}{a^6} - \frac{3\,b\,x^2}{2\,a^4} + \frac{6\,b^2\,x}{a^5}$$

input
```
int(x^2/(a + b/x)^3,x)
```

output
```
x^3/(3*a^3) - (5*b^4*x + (9*b^5)/(2*a))/(a^5*b^2 + a^7*x^2 + 2*a^6*b*x) -
(10*b^3*log(b + a*x))/a^6 - (3*b*x^2)/(2*a^4) + (6*b^2*x)/a^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{-60\log(ax+b)\,a^2b^3x^2-120\log(ax+b)\,a\,b^4x-60\log(ax+b)\,b^5+2a^5x^5-5a^4b\,x^4+20a^3b^2x^3+60a^2}{6a^6\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(x^2/(a+b/x)^3,x)
```

output

```
( - 60*log(a*x + b)*a**2*b**3*x**2 - 120*log(a*x + b)*a*b**4*x - 60*log(a*
x + b)*b**5 + 2*a**5*x**5 - 5*a**4*b*x**4 + 20*a**3*b**2*x**3 + 60*a**2*b*
*3*x**2 - 30*b**5)/(6*a**6*(a**2*x**2 + 2*a*b*x + b**2))
```

3.90 $\int \frac{x}{\left(a+\frac{b}{x}\right)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 64

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3} dx = -\frac{3bx}{a^4}+\frac{x^2}{2a^3}-\frac{b^4}{2a^5(b+ax)^2}+\frac{4b^3}{a^5(b+ax)}+\frac{6b^2\log(b+ax)}{a^5}$$

output
```
-3*b*x/a^4+1/2*x^2/a^3-1/2*b^4/a^5/(a*x+b)^2+4*b^3/a^5/(a*x+b)+6*b^2*ln(a*
x+b)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3} dx = \frac{-6abx+a^2x^2+\frac{b^3(7b+8ax)}{(b+ax)^2}+12b^2\log(b+ax)}{2a^5}$$

input
```
Integrate[x/(a + b/x)^3,x]
```

output
```
(-6*a*b*x + a^2*x^2 + (b^3*(7*b + 8*a*x))/(b + a*x)^2 + 12*b^2*Log[b + a*x
])/(2*a^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^4}{(ax+b)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{b^4}{a^4(ax+b)^3}-\frac{4b^3}{a^4(ax+b)^2}+\frac{6b^2}{a^4(ax+b)}-\frac{3b}{a^4}+\frac{x}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^4}{2a^5(ax+b)^2}+\frac{4b^3}{a^5(ax+b)}+\frac{6b^2\log(ax+b)}{a^5}-\frac{3bx}{a^4}+\frac{x^2}{2a^3}$$

input
```
Int[x/(a + b/x)^3,x]
```

output
```
(-3*b*x)/a^4 + x^2/(2*a^3) - b^4/(2*a^5*(b + a*x)^2) + (4*b^3)/(a^5*(b + a
*x)) + (6*b^2*Log[b + a*x])/a^5
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x^{2}}{2a^{3}}-\frac{3bx}{a^{4}}+\frac{4b^{3}x+\frac{7b^{4}}{2a}}{a^{4}\left(ax+b\right)^{2}}+\frac{6b^{2}\ln\left(ax+b\right)}{a^{5}}$	57
norman	$\frac{\frac{9b^{4}}{a^{5}}+\frac{x^{4}}{2a}-\frac{2b\,x^{3}}{a^{2}}+\frac{12x\,b^{3}}{a^{4}}}{\left(ax+b\right)^{2}}+\frac{6b^{2}\ln\left(ax+b\right)}{a^{5}}$	59
default	$\frac{\frac{1}{2}a\,x^{2}-3bx}{a^{4}}+\frac{4b^{3}}{a^{5}\left(ax+b\right)}+\frac{6b^{2}\ln\left(ax+b\right)}{a^{5}}-\frac{b^{4}}{2a^{5}\left(ax+b\right)^{2}}$	61
parallelrisch	$\frac{a^{4}x^{4}+12\ln\left(ax+b\right)x^{2}a^{2}b^{2}-4a^{3}b\,x^{3}+24\ln\left(ax+b\right)xa\,b^{3}+12b^{4}\ln\left(ax+b\right)+24a\,b^{3}x+18b^{4}}{2a^{5}\left(ax+b\right)^{2}}$	83

input
```
int(x/(a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a^3-3/a^4*b*x+(4*b^3*x+7/2*b^4/a)/a^4/(a*x+b)^2+6*b^2*ln(a*x+b)/a^
5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.48

$$\int\frac{x}{\left(a+\frac{b}{x}\right)^{3}}\,dx=\frac{a^{4}x^{4}-4\,a^{3}bx^{3}-11\,a^{2}b^{2}x^{2}+2\,ab^{3}x+7\,b^{4}+12\,\left(a^{2}b^{2}x^{2}+2\,ab^{3}x+b^{4}\right)\log\left(ax+b\right)}{2\,\left(a^{7}x^{2}+2\,a^{6}bx+a^{5}b^{2}\right)}$$

input
```
integrate(x/(a+b/x)^3,x, algorithm="fricas")
```

output
```
1/2*(a^4*x^4 - 4*a^3*b*x^3 - 11*a^2*b^2*x^2 + 2*a*b^3*x + 7*b^4 + 12*(a^2*
b^2*x^2 + 2*a*b^3*x + b^4)*log(a*x + b))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{8ab^3x+7b^4}{2a^7x^2+4a^6bx+2a^5b^2} + \frac{x^2}{2a^3} - \frac{3bx}{a^4} + \frac{6b^2\log\left(ax+b\right)}{a^5}$$

input
```
integrate(x/(a+b/x)**3,x)
```

output
```
(8*a*b**3*x + 7*b**4)/(2*a**7*x**2 + 4*a**6*b*x + 2*a**5*b**2) + x**2/(2*a
**3) - 3*b*x/a**4 + 6*b**2*log(a*x + b)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{8\,ab^3x+7\,b^4}{2\left(a^7x^2+2\,a^6bx+a^5b^2\right)} + \frac{6\,b^2\log\left(ax+b\right)}{a^5} + \frac{ax^2-6\,bx}{2\,a^4}$$

input
```
integrate(x/(a+b/x)^3,x, algorithm="maxima")
```

output
```
1/2*(8*a*b^3*x + 7*b^4)/(a^7*x^2 + 2*a^6*b*x + a^5*b^2) + 6*b^2*log(a*x +
b)/a^5 + 1/2*(a*x^2 - 6*b*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{6\,b^2\log\left(|ax+b|\right)}{a^5}+\frac{a^3x^2-6\,a^2bx}{2\,a^6}+\frac{8\,ab^3x+7\,b^4}{2\,(ax+b)^2a^5}$$

input
```
integrate(x/(a+b/x)^3,x, algorithm="giac")
```

output
```
6*b^2*log(abs(a*x + b))/a^5 + 1/2*(a^3*x^2 - 6*a^2*b*x)/a^6 + 1/2*(8*a*b^3
*x + 7*b^4)/((a*x + b)^2*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{4\,b^3\,x+\frac{7\,b^4}{2\,a}}{a^6\,x^2+2\,a^5\,b\,x+a^4\,b^2}+\frac{x^2}{2\,a^3}+\frac{6\,b^2\,\ln\left(b+a\,x\right)}{a^5}-\frac{3\,b\,x}{a^4}$$

input
```
int(x/(a + b/x)^3,x)
```

output
```
(4*b^3*x + (7*b^4)/(2*a))/(a^4*b^2 + a^6*x^2 + 2*a^5*b*x) + x^2/(2*a^3) +
(6*b^2*log(b + a*x))/a^5 - (3*b*x)/a^4
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{12\log(ax+b)\,a^2b^2x^2+24\log(ax+b)\,a\,b^3x+12\log(ax+b)\,b^4+a^4x^4-4a^3b\,x^3-12a^2b^2x^2+6b^4}{2a^5\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(x/(a+b/x)^3,x)
```

output
```
(12*log(a*x + b)*a**2*b**2*x**2 + 24*log(a*x + b)*a*b**3*x + 12*log(a*x +
b)*b**4 + a**4*x**4 - 4*a**3*b*x**3 - 12*a**2*b**2*x**2 + 6*b**4)/(2*a**5*
(a**2*x**2 + 2*a*b*x + b**2))
```

3.91 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 50

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{x}{a^3}+\frac{b^3}{2a^4(b+ax)^2}-\frac{3b^2}{a^4(b+ax)}-\frac{3b\log(b+ax)}{a^4}$$

output
```
x/a^3+1/2*b^3/a^4/(a*x+b)^2-3*b^2/a^4/(a*x+b)-3*b*ln(a*x+b)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx=-\frac{-2ax+\frac{b^2(5b+6ax)}{(b+ax)^2}+6b\log(b+ax)}{2a^4}$$

input
```
Integrate[(a + b/x)^(-3),x]
```

output
```
-1/2*(-2*a*x + (b^2*(5*b + 6*a*x))/(b + a*x)^2 + 6*b*Log[b + a*x])/a^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{x^3}{(ax+b)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{b^3}{a^3(ax+b)^3}+\frac{3b^2}{a^3(ax+b)^2}-\frac{3b}{a^3(ax+b)}+\frac{1}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3}{2a^4(ax+b)^2}-\frac{3b^2}{a^4(ax+b)}-\frac{3b\log(ax+b)}{a^4}+\frac{x}{a^3}$$

input
```
Int[(a + b/x)^(-3),x]
```

output
```
x/a^3 + b^3/(2*a^4*(b + a*x)^2) - (3*b^2)/(a^4*(b + a*x)) - (3*b*Log[b + a
*x])/a^4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{x}{a^3}+\frac{-3b^2x-\frac{5b^3}{2a}}{a^3(ax+b)^2}-\frac{3b\ln(ax+b)}{a^4}$	45
norman	$\frac{\frac{x^3}{a}-\frac{9b^3}{2a^4}-\frac{6b^2x}{a^3}}{(ax+b)^2}-\frac{3b\ln(ax+b)}{a^4}$	47
default	$\frac{x}{a^3}+\frac{b^3}{2a^4(ax+b)^2}-\frac{3b^2}{a^4(ax+b)}-\frac{3b\ln(ax+b)}{a^4}$	49
parallelrisch	$-\frac{6\ln(ax+b)x^2a^2b-2a^3x^3+12\ln(ax+b)xa\,b^2+6b^3\ln(ax+b)+12a\,b^2x+9b^3}{2a^4(ax+b)^2}$	73

input
```
int(1/(a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
x/a^3+(-3*b^2*x-5/2*b^3/a)/a^3/(a*x+b)^2-3*b*ln(a*x+b)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{2\,a^3x^3+4\,a^2bx^2-4\,ab^2x-5\,b^3-6\,(a^2bx^2+2\,ab^2x+b^3)\log(ax+b)}{2\,(a^6x^2+2\,a^5bx+a^4b^2)}$$

input
```
integrate(1/(a+b/x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^3*x^3 + 4*a^2*b*x^2 - 4*a*b^2*x - 5*b^3 - 6*(a^2*b*x^2 + 2*a*b^2*
x + b^3)*log(a*x + b))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{-6ab^2x-5b^3}{2a^6x^2+4a^5bx+2a^4b^2}+\frac{x}{a^3}-\frac{3b\log\left(ax+b\right)}{a^4}$$

input
```
integrate(1/(a+b/x)**3,x)
```

output
```
(-6*a*b**2*x - 5*b**3)/(2*a**6*x**2 + 4*a**5*b*x + 2*a**4*b**2) + x/a**3 -
 3*b*log(a*x + b)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{6\,ab^2x+5\,b^3}{2\,(a^6x^2+2\,a^5bx+a^4b^2)}+\frac{x}{a^3}-\frac{3\,b\log\left(ax+b\right)}{a^4}$$

input
```
integrate(1/(a+b/x)^3,x, algorithm="maxima")
```

output
```
-1/2*(6*a*b^2*x + 5*b^3)/(a^6*x^2 + 2*a^5*b*x + a^4*b^2) + x/a^3 - 3*b*log
(a*x + b)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{x}{a^3}-\frac{3\,b\log\left(|ax+b|\right)}{a^4}-\frac{6\,ab^2x+5\,b^3}{2\,(ax+b)^2a^4}$$

input
```
integrate(1/(a+b/x)^3,x, algorithm="giac")
```

output
```
x/a^3 - 3*b*log(abs(a*x + b))/a^4 - 1/2*(6*a*b^2*x + 5*b^3)/((a*x + b)^2*a
^4)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx=\frac{x}{a^3}-\frac{3\,b^2\,x+\frac{5\,b^3}{2\,a}}{a^5\,x^2+2\,a^4\,b\,x+a^3\,b^2}-\frac{3\,b\,\ln\left(b+a\,x\right)}{a^4}$$

input
```
int(1/(a + b/x)^3,x)
```

output
```
x/a^3 - (3*b^2*x + (5*b^3)/(2*a))/(a^3*b^2 + a^5*x^2 + 2*a^4*b*x) - (3*b*l
og(b + a*x))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{-6\log(ax+b)\,a^2b\,x^2-12\log(ax+b)\,a\,b^2x-6\log(ax+b)\,b^3+2a^3x^3+6a^2b\,x^2-3b^3}{2a^4\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3,x)
```

output
```
( - 6*log(a*x + b)*a**2*b*x**2 - 12*log(a*x + b)*a*b**2*x - 6*log(a*x + b)
*b**3 + 2*a**3*x**3 + 6*a**2*b*x**2 - 3*b**3)/(2*a**4*(a**2*x**2 + 2*a*b*x
 + b**2))
```

3.92 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x} \, dx = -\frac{b^2}{2a^3(b+ax)^2} + \frac{2b}{a^3(b+ax)} + \frac{\log(b+ax)}{a^3}$$

output
```
-1/2*b^2/a^3/(a*x+b)^2+2*b/a^3/(a*x+b)+ln(a*x+b)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x} \, dx = \frac{\frac{b(3b+4ax)}{(b+ax)^2} + 2\log(b+ax)}{2a^3}$$

input
```
Integrate[1/((a + b/x)^3*x),x]
```

output
```
((b*(3*b + 4*a*x))/(b + a*x)^2 + 2*Log[b + a*x])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^2}{(ax+b)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{b^2}{a^2(ax+b)^3}-\frac{2b}{a^2(ax+b)^2}+\frac{1}{a^2(ax+b)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2}{2a^3(ax+b)^2}+\frac{2b}{a^3(ax+b)}+\frac{\log(ax+b)}{a^3}$$

input
```
Int[1/((a + b/x)^3*x),x]
```

output
```
-1/2*b^2/(a^3*(b + a*x)^2) + (2*b)/(a^3*(b + a*x)) + Log[b + a*x]/a^3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
norman	$\frac{\frac{3b^2}{2a^3}+\frac{2bx}{a^2}}{(ax+b)^2}+\frac{\ln(ax+b)}{a^3}$	36
risch	$\frac{\frac{3b^2}{2a^3}+\frac{2bx}{a^2}}{(ax+b)^2}+\frac{\ln(ax+b)}{a^3}$	36
default	$-\frac{b^2}{2a^3(ax+b)^2}+\frac{2b}{a^3(ax+b)}+\frac{\ln(ax+b)}{a^3}$	40
parallelrisch	$\frac{2a^2\ln(ax+b)x^2+4\ln(ax+b)xab+2b^2\ln(ax+b)+4abx+3b^2}{2a^3(ax+b)^2}$	60

input
```
int(1/(a+b/x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(3/2*b^2/a^3+2*b/a^2*x)/(a*x+b)^2+ln(a*x+b)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3x}\,dx=\frac{4\,abx+3\,b^2+2\,(a^2x^2+2\,abx+b^2)\log{(ax+b)}}{2\,(a^5x^2+2\,a^4bx+a^3b^2)}$$

input
```
integrate(1/(a+b/x)^3/x,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*x + 3*b^2 + 2*(a^2*x^2 + 2*a*b*x + b^2)*log(a*x + b))/(a^5*x^2
+ 2*a^4*b*x + a^3*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx = \frac{4abx+3b^2}{2a^5x^2+4a^4bx+2a^3b^2}+\frac{\log\left(ax+b\right)}{a^3}$$

input
```
integrate(1/(a+b/x)**3/x,x)
```

output
```
(4*a*b*x + 3*b**2)/(2*a**5*x**2 + 4*a**4*b*x + 2*a**3*b**2) + log(a*x + b)
/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx = \frac{4\,abx+3\,b^2}{2\left(a^5x^2+2\,a^4bx+a^3b^2\right)}+\frac{\log\left(ax+b\right)}{a^3}$$

input
```
integrate(1/(a+b/x)^3/x,x, algorithm="maxima")
```

output
```
1/2*(4*a*b*x + 3*b^2)/(a^5*x^2 + 2*a^4*b*x + a^3*b^2) + log(a*x + b)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx = \frac{\log\left(|ax+b|\right)}{a^3}+\frac{4\,bx+\frac{3\,b^2}{a}}{2\left(ax+b\right)^2a^2}$$

input
```
integrate(1/(a+b/x)^3/x,x, algorithm="giac")
```

output
```
log(abs(a*x + b))/a^3 + 1/2*(4*b*x + 3*b^2/a)/((a*x + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx = \frac{\ln\left(b+a\,x\right)}{a^3}+\frac{\frac{3\,b^2}{2\,a^3}+\frac{2\,b\,x}{a^2}}{a^2\,x^2+2\,a\,b\,x+b^2}$$

input
```
int(1/(x*(a + b/x)^3),x)
```

output
```
log(b + a*x)/a^3 + ((3*b^2)/(2*a^3) + (2*b*x)/a^2)/(b^2 + a^2*x^2 + 2*a*b*
x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x}\,dx = \frac{2\log(ax+b)\,a^2x^2+4\log(ax+b)\,abx+2\log(ax+b)\,b^2-2a^2x^2+b^2}{2a^3\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x,x)
```

output
```
(2*log(a*x + b)*a**2*x**2 + 4*log(a*x + b)*a*b*x + 2*log(a*x + b)*b**2 - 2
*a**2*x**2 + b**2)/(2*a**3*(a**2*x**2 + 2*a*b*x + b**2))
```

3.93 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2} \, dx = \frac{1}{2b\left(a+\frac{b}{x}\right)^2}$$

output
```
1/2/b/(a+b/x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2} \, dx = -\frac{b+2ax}{2a^2(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^3*x^2),x]
```

output
```
-1/2*(b + 2*a*x)/(a^2*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{2b \left(a+\frac{b}{x}\right)^2}$$

input
```
Int[1/((a + b/x)^3*x^2),x]
```

output
```
1/(2*b*(a + b/x)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{1}{2b\left(a+\frac{b}{x}\right)^{2}}$	15
gosper	$-\frac{2ax+b}{2(ax+b)^{2}a^{2}}$	19
parallelrisch	$\frac{-2ax-b}{2a^{2}(ax+b)^{2}}$	21
risch	$\frac{-\frac{x}{a}-\frac{b}{2a^{2}}}{(ax+b)^{2}}$	22
default	$-\frac{1}{(ax+b)a^{2}}+\frac{b}{2a^{2}(ax+b)^{2}}$	27
norman	$\frac{-\frac{x^{2}}{a}-\frac{bx}{2a^{2}}}{x(ax+b)^{2}}$	28
orering	$-\frac{(2ax+b)(ax+b)}{2a^{2}x^{3}\left(a+\frac{b}{x}\right)^{3}}$	29

input
```
int(1/(a+b/x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(a+b/x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\,dx = -\frac{2\,ax+b}{2\,(a^4x^2+2\,a^3bx+a^2b^2)}$$

input
```
integrate(1/(a+b/x)^3/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*x + b)/(a^4*x^2 + 2*a^3*b*x + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\,dx = \frac{-2ax-b}{2a^4x^2+4a^3bx+2a^2b^2}$$

input
```
integrate(1/(a+b/x)**3/x**2,x)
```

output
```
(-2*a*x - b)/(2*a**4*x**2 + 4*a**3*b*x + 2*a**2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\,dx = \frac{1}{2\left(a+\frac{b}{x}\right)^2 b}$$

input
```
integrate(1/(a+b/x)^3/x^2,x, algorithm="maxima")
```

output
```
1/2/((a + b/x)^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\,dx = \frac{1}{2\left(a+\frac{b}{x}\right)^2 b}$$

input
```
integrate(1/(a+b/x)^3/x^2,x, algorithm="giac")
```

output
```
1/2/((a + b/x)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\, dx = -\frac{\frac{b}{2\,a^2}+\frac{x}{a}}{a^2\,x^2+2\,a\,b\,x+b^2}$$

input
```
int(1/(x^2*(a + b/x)^3),x)
```

output
```
-(b/(2*a^2) + x/a)/(b^2 + a^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^2}\, dx = \frac{x^2}{2b\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^2,x)
```

output
```
x**2/(2*b*(a**2*x**2 + 2*a*b*x + b**2))
```

3.94 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3} \, dx = -\frac{1}{2a(b+ax)^2}$$

output
```
-1/2/a/(a*x+b)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3} \, dx = -\frac{1}{2a(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^3*x^3),x]
```

output
```
-1/2*1/(a*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{(ax+b)^3}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{2a(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*x^3),x]
```

output
```
-1/2*1/(a*(b + a*x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{2a(ax+b)^{2}}$	13
default	$-\frac{1}{2a(ax+b)^{2}}$	13
norman	$-\frac{1}{2a(ax+b)^{2}}$	13
risch	$-\frac{1}{2a(ax+b)^{2}}$	13
parallelrisch	$-\frac{1}{2a(ax+b)^{2}}$	13
orering	$-\frac{ax+b}{2a\left(a+\frac{b}{x}\right)^{3} x^{3}}$	23

input
```
int(1/(a+b/x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/(a*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{1}{2\,(a^3x^2+2\,a^2bx+ab^2)}$$

input
```
integrate(1/(a+b/x)^3/x^3,x, algorithm="fricas")
```

output
```
-1/2/(a^3*x^2 + 2*a^2*b*x + a*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{1}{2a^3x^2 + 4a^2bx + 2ab^2}$$

input
```
integrate(1/(a+b/x)**3/x**3,x)
```

output
```
-1/(2*a**3*x**2 + 4*a**2*b*x + 2*a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{1}{2\,(a^3x^2 + 2\,a^2bx + ab^2)}$$

input
```
integrate(1/(a+b/x)^3/x^3,x, algorithm="maxima")
```

output
```
-1/2/(a^3*x^2 + 2*a^2*b*x + a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{1}{2\,(ax+b)^2a}$$

input
```
integrate(1/(a+b/x)^3/x^3,x, algorithm="giac")
```

output
```
-1/2/((a*x + b)^2*a)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{1}{2\,a^3\,x^2+4\,a^2\,b\,x+2\,a\,b^2}$$

input
```
int(1/(x^3*(a + b/x)^3),x)
```

output
```
-1/(2*a*b^2 + 2*a^3*x^2 + 4*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^3}\,dx = -\frac{1}{2a\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^3,x)
```

output
```
( - 1)/(2*a*(a**2*x**2 + 2*a*b*x + b**2))
```

3.95 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4} \, dx = \frac{a^2}{2b^3\left(a+\frac{b}{x}\right)^2} - \frac{2a}{b^3\left(a+\frac{b}{x}\right)} - \frac{\log\left(a+\frac{b}{x}\right)}{b^3}$$

output
```
1/2*a^2/b^3/(a+b/x)^2-2*a/b^3/(a+b/x)-ln(a+b/x)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4} \, dx = \frac{\frac{b(3b+2ax)}{(b+ax)^2} + 2\log(x) - 2\log(b+ax)}{2b^3}$$

input
```
Integrate[1/((a + b/x)^3*x^4),x]
```

output
```
((b*(3*b + 2*a*x))/(b + a*x)^2 + 2*Log[x] - 2*Log[b + a*x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x(ax+b)^3}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(-\frac{a}{b^3(ax+b)}-\frac{a}{b^2(ax+b)^2}-\frac{a}{b(ax+b)^3}+\frac{1}{b^3x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(ax+b)}{b^3}+\frac{1}{b^2(ax+b)}+\frac{1}{2b(ax+b)^2}+\frac{\log(x)}{b^3}$$

input
```
Int[1/((a + b/x)^3*x^4),x]
```

output
```
1/(2*b*(b + a*x)^2) + 1/(b^2*(b + a*x)) + Log[x]/b^3 - Log[b + a*x]/b^3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\frac{ax}{b^2}+\frac{3}{2b}}{(ax+b)^2}-\frac{\ln(ax+b)}{b^3}+\frac{\ln(-x)}{b^3}$	41
default	$\frac{\ln(x)}{b^3}-\frac{\ln(ax+b)}{b^3}+\frac{1}{b^2(ax+b)}+\frac{1}{2b(ax+b)^2}$	42
norman	$\frac{-\frac{2a\,x^4}{b^2}-\frac{3a^2x^5}{2b^3}}{x^3(ax+b)^2}+\frac{\ln(x)}{b^3}-\frac{\ln(ax+b)}{b^3}$	51
parallelrisch	$\frac{2a^2\ln(x)x^2-2a^2\ln(ax+b)x^2+4ab\ln(x)x-4\ln(ax+b)xab-3a^2x^2+2b^2\ln(x)-2b^2\ln(ax+b)-4abx}{2b^3(ax+b)^2}$	87

input
```
int(1/(a+b/x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
(a*x/b^2+3/2/b)/(a*x+b)^2-1/b^3*ln(a*x+b)+1/b^3*ln(-x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3x^4}\,dx = \frac{2\,abx+3\,b^2-2\,(a^2x^2+2\,abx+b^2)\log(ax+b)+2\,(a^2x^2+2\,abx+b^2)\log(x)}{2\,(a^2b^3x^2+2\,ab^4x+b^5)}$$

input
```
integrate(1/(a+b/x)^3/x^4,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x + 3*b^2 - 2*(a^2*x^2 + 2*a*b*x + b^2)*log(a*x + b) + 2*(a^2*x
^2 + 2*a*b*x + b^2)*log(x))/(a^2*b^3*x^2 + 2*a*b^4*x + b^5)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4} \, dx = \frac{2ax+3b}{2a^2b^2x^2+4ab^3x+2b^4} + \frac{\log(x) - \log\left(x+\frac{b}{a}\right)}{b^3}$$

input
```
integrate(1/(a+b/x)**3/x**4,x)
```

output
```
(2*a*x + 3*b)/(2*a**2*b**2*x**2 + 4*a*b**3*x + 2*b**4) + (log(x) - log(x +
 b/a))/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4} \, dx = \frac{2\,ax+3\,b}{2\,(a^2b^2x^2+2\,ab^3x+b^4)} - \frac{\log(ax+b)}{b^3} + \frac{\log(x)}{b^3}$$

input
```
integrate(1/(a+b/x)^3/x^4,x, algorithm="maxima")
```

output
```
1/2*(2*a*x + 3*b)/(a^2*b^2*x^2 + 2*a*b^3*x + b^4) - log(a*x + b)/b^3 + log
(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4} \, dx = -\frac{\log(|ax+b|)}{b^3} + \frac{\log(|x|)}{b^3} + \frac{2\,abx+3\,b^2}{2\,(ax+b)^2b^3}$$

input
```
integrate(1/(a+b/x)^3/x^4,x, algorithm="giac")
```

output
```
-log(abs(a*x + b))/b^3 + log(abs(x))/b^3 + 1/2*(2*a*b*x + 3*b^2)/((a*x + b
)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4}\,dx = \frac{\frac{3}{2\,b}+\frac{a\,x}{b^2}}{a^2\,x^2+2\,a\,b\,x+b^2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^3}$$

input
```
int(1/(x^4*(a + b/x)^3),x)
```

output
```
(3/(2*b) + (a*x)/b^2)/(b^2 + a^2*x^2 + 2*a*b*x) - (2*atanh((2*a*x)/b + 1))
/b^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.02

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^4}\,dx$$
$$= \frac{-2\log(ax+b)\,a^2x^2-4\log(ax+b)\,abx-2\log(ax+b)\,b^2+2\log(x)\,a^2x^2+4\log(x)\,abx+2\log(x)\,b^2-}{2b^3\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^4,x)
```

output
```
( - 2*log(a*x + b)*a**2*x**2 - 4*log(a*x + b)*a*b*x - 2*log(a*x + b)*b**2
+ 2*log(x)*a**2*x**2 + 4*log(x)*a*b*x + 2*log(x)*b**2 - a**2*x**2 + 2*b**2
)/(2*b**3*(a**2*x**2 + 2*a*b*x + b**2))
```

3.96 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5} \, dx = -\frac{a^3}{2b^4\left(a+\frac{b}{x}\right)^2}+\frac{3a^2}{b^4\left(a+\frac{b}{x}\right)}-\frac{1}{b^3x}+\frac{3a\log\left(a+\frac{b}{x}\right)}{b^4}$$

output
```
-1/2*a^3/b^4/(a+b/x)^2+3*a^2/b^4/(a+b/x)-1/b^3/x+3*a*ln(a+b/x)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5} \, dx = -\frac{\frac{b(2b^2+9abx+6a^2x^2)}{x(b+ax)^2}+6a\log(x)-6a\log(b+ax)}{2b^4}$$

input
```
Integrate[1/((a + b/x)^3*x^5),x]
```

output
```
-1/2*((b*(2*b^2 + 9*a*b*x + 6*a^2*x^2))/(x*(b + a*x)^2) + 6*a*Log[x] - 6*a
*Log[b + a*x])/b^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{x^2 (ax+b)^3}dx$$

$$\Big\downarrow \ 54$$

$$\int \left(\frac{3a^2}{b^4(ax+b)}+\frac{2a^2}{b^3(ax+b)^2}+\frac{a^2}{b^2(ax+b)^3}-\frac{3a}{b^4x}+\frac{1}{b^3x^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3a\log(x)}{b^4}+\frac{3a\log(ax+b)}{b^4}-\frac{2a}{b^3(ax+b)}-\frac{a}{2b^2(ax+b)^2}-\frac{1}{b^3x}$$

input
```
Int[1/((a + b/x)^3*x^5),x]
```

output
```
-(1/(b^3*x)) - a/(2*b^2*(b + a*x)^2) - (2*a)/(b^3*(b + a*x)) - (3*a*Log[x]
)/b^4 + (3*a*Log[b + a*x])/b^4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

method	result
default	$-\frac{1}{b^{3}x}-\frac{3a\ln(x)}{b^{4}}-\frac{a}{2b^{2}(ax+b)^{2}}+\frac{3a\ln(ax+b)}{b^{4}}-\frac{2a}{b^{3}(ax+b)}$
risch	$\frac{-\frac{3a^{2}x^{2}}{b^{3}}-\frac{9ax}{2b^{2}}-\frac{1}{b}}{x(ax+b)^{2}}-\frac{3a\ln(x)}{b^{4}}+\frac{3a\ln(-ax-b)}{b^{4}}$
norman	$\frac{-\frac{x^{3}}{b}+\frac{6a^{2}x^{5}}{b^{3}}+\frac{9a^{3}x^{6}}{2b^{4}}}{x^{4}(ax+b)^{2}}-\frac{3a\ln(x)}{b^{4}}+\frac{3a\ln(ax+b)}{b^{4}}$
parallelrisch	$-\frac{6a^{3}\ln(x)x^{3}-6a^{3}\ln(ax+b)x^{3}+12a^{2}b\ln(x)x^{2}-12\ln(ax+b)x^{2}a^{2}b-9a^{3}x^{3}+6a\,b^{2}\ln(x)x-6\ln(ax+b)xa\,b^{2}-12a^{2}b\,x^{2}+2b^{3}}{2b^{4}x(ax+b)^{2}}$

input
```
int(1/(a+b/x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b^3/x-3/b^4*a*ln(x)-1/2*a/b^2/(a*x+b)^2+3/b^4*a*ln(a*x+b)-2*a/b^3/(a*x+
b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3}x^{5}}\,dx = -\frac{6\,a^{2}bx^{2}+9\,ab^{2}x+2\,b^{3}-6\,(a^{3}x^{3}+2\,a^{2}bx^{2}+ab^{2}x)\log\left(ax+b\right)+6\,(a^{3}x^{3}+2\,a^{2}bx^{2}+ab^{2}x)\log\left(x\right)}{2\,(a^{2}b^{4}x^{3}+2\,ab^{5}x^{2}+b^{6}x)}$$

input
```
integrate(1/(a+b/x)^3/x^5,x, algorithm="fricas")
```

output
```
-1/2*(6*a^2*b*x^2 + 9*a*b^2*x + 2*b^3 - 6*(a^3*x^3 + 2*a^2*b*x^2 + a*b^2*x
)*log(a*x + b) + 6*(a^3*x^3 + 2*a^2*b*x^2 + a*b^2*x)*log(x))/(a^2*b^4*x^3
+ 2*a*b^5*x^2 + b^6*x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5}\,dx = \frac{3a\left(-\log\left(x\right)+\log\left(x+\frac{b}{a}\right)\right)}{b^4} + \frac{-6a^2x^2-9abx-2b^2}{2a^2b^3x^3+4ab^4x^2+2b^5x}$$

input
```
integrate(1/(a+b/x)**3/x**5,x)
```

output
```
3*a*(-log(x) + log(x + b/a))/b**4 + (-6*a**2*x**2 - 9*a*b*x - 2*b**2)/(2*a
**2*b**3*x**3 + 4*a*b**4*x**2 + 2*b**5*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5}\,dx = -\frac{6\,a^2x^2+9\,abx+2\,b^2}{2\,(a^2b^3x^3+2\,ab^4x^2+b^5x)} + \frac{3\,a\log\left(ax+b\right)}{b^4} - \frac{3\,a\log\left(x\right)}{b^4}$$

input
```
integrate(1/(a+b/x)^3/x^5,x, algorithm="maxima")
```

output
```
-1/2*(6*a^2*x^2 + 9*a*b*x + 2*b^2)/(a^2*b^3*x^3 + 2*a*b^4*x^2 + b^5*x) + 3
*a*log(a*x + b)/b^4 - 3*a*log(x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5} \, dx = \frac{3 \, a \log \left(|ax+b|\right)}{b^4} - \frac{3 \, a \log \left(|x|\right)}{b^4} - \frac{6 \, a^2 b x^2 + 9 \, a b^2 x + 2 \, b^3}{2 \, (ax+b)^2 b^4 x}$$

input
```
integrate(1/(a+b/x)^3/x^5,x, algorithm="giac")
```

output
```
3*a*log(abs(a*x + b))/b^4 - 3*a*log(abs(x))/b^4 - 1/2*(6*a^2*b*x^2 + 9*a*b
^2*x + 2*b^3)/((a*x + b)^2*b^4*x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5} \, dx = \frac{6 \, a \, \mathrm{atanh}\left(\frac{2 \, a \, x}{b}+1\right)}{b^4} - \frac{\frac{1}{b} + \frac{3 \, a^2 \, x^2}{b^3} + \frac{9 \, a \, x}{2 \, b^2}}{a^2 \, x^3 + 2 \, a \, b \, x^2 + b^2 \, x}$$

input
```
int(1/(x^5*(a + b/x)^3),x)
```

output
```
(6*a*atanh((2*a*x)/b + 1))/b^4 - (1/b + (3*a^2*x^2)/b^3 + (9*a*x)/(2*b^2))
/(b^2*x + a^2*x^3 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.02

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^5} \, dx$$
$$= \frac{6 \log(ax+b) \, a^3 x^3 + 12 \log(ax+b) \, a^2 b \, x^2 + 6 \log(ax+b) \, a \, b^2 x - 6 \log(x) \, a^3 x^3 - 12 \log(x) \, a^2 b \, x^2 - 6 \log}{2b^4 x \left(a^2 x^2 + 2abx + b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^5,x)
```

output
```
(6*log(a*x + b)*a**3*x**3 + 12*log(a*x + b)*a**2*b*x**2 + 6*log(a*x + b)*a
*b**2*x - 6*log(x)*a**3*x**3 - 12*log(x)*a**2*b*x**2 - 6*log(x)*a*b**2*x +
 3*a**3*x**3 - 6*a*b**2*x - 2*b**3)/(2*b**4*x*(a**2*x**2 + 2*a*b*x + b**2)
)
```

3.97 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6} \, dx = \frac{a^4}{2b^5\left(a+\frac{b}{x}\right)^2} - \frac{4a^3}{b^5\left(a+\frac{b}{x}\right)} - \frac{1}{2b^3x^2} + \frac{3a}{b^4x} - \frac{6a^2\log\left(a+\frac{b}{x}\right)}{b^5}$$

output
```
1/2*a^4/b^5/(a+b/x)^2-4*a^3/b^5/(a+b/x)-1/2/b^3/x^2+3*a/b^4/x-6*a^2*ln(a+b
/x)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6} \, dx = \frac{\frac{b(-b^3+4ab^2x+18a^2bx^2+12a^3x^3)}{x^2(b+ax)^2} + 12a^2\log(x) - 12a^2\log(b+ax)}{2b^5}$$

input
```
Integrate[1/((a + b/x)^3*x^6),x]
```

output
```
((b*(-b^3 + 4*a*b^2*x + 18*a^2*b*x^2 + 12*a^3*x^3))/(x^2*(b + a*x)^2) + 12
*a^2*Log[x] - 12*a^2*Log[b + a*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{1}{x^3 (ax+b)^3} dx$$

$$\Big\downarrow \; 54$$

$$\int \left(-\frac{6a^3}{b^5(ax+b)} - \frac{3a^3}{b^4(ax+b)^2} - \frac{a^3}{b^3(ax+b)^3} + \frac{6a^2}{b^5 x} - \frac{3a}{b^4 x^2} + \frac{1}{b^3 x^3}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{6a^2 \log(x)}{b^5} - \frac{6a^2 \log(ax+b)}{b^5} + \frac{3a^2}{b^4(ax+b)} + \frac{a^2}{2b^3(ax+b)^2} + \frac{3a}{b^4 x} - \frac{1}{2b^3 x^2}$$

input
```
Int[1/((a + b/x)^3*x^6),x]
```

output
```
-1/2*1/(b^3*x^2) + (3*a)/(b^4*x) + a^2/(2*b^3*(b + a*x)^2) + (3*a^2)/(b^4*
(b + a*x)) + (6*a^2*Log[x])/b^5 - (6*a^2*Log[b + a*x])/b^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

method	result
default	$-\frac{1}{2b^3x^2}+\frac{3a}{b^4x}+\frac{6a^2\ln(x)}{b^5}-\frac{6a^2\ln(ax+b)}{b^5}+\frac{3a^2}{b^4(ax+b)}+\frac{a^2}{2b^3(ax+b)^2}$
risch	$\frac{\frac{6a^3x^3}{b^4}+\frac{9a^2x^2}{b^3}+\frac{2ax}{b^2}-\frac{1}{2b}}{x^2(ax+b)^2}+\frac{6a^2\ln(-x)}{b^5}-\frac{6a^2\ln(ax+b)}{b^5}$
norman	$\frac{-\frac{x^3}{2b}+\frac{2a\,x^4}{b^2}-\frac{12a^3x^6}{b^4}-\frac{9a^4x^7}{b^5}}{x^5(ax+b)^2}+\frac{6a^2\ln(x)}{b^5}-\frac{6a^2\ln(ax+b)}{b^5}$
parallelrisch	$\frac{12\ln(x)x^4a^6-12\ln(ax+b)x^4a^6+24\ln(x)x^3a^5b-24\ln(ax+b)x^3a^5b+12\ln(x)x^2a^4b^2-12\ln(ax+b)x^2a^4b^2+12a^5b\,x^3+18a^4x^2}{2a^2b^5x^2(ax+b)^2}$

input
```
int(1/(a+b/x)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3/x^2+3*a/b^4/x+6/b^5*a^2*ln(x)-6/b^5*a^2*ln(a*x+b)+3*a^2/b^4/(a*x+
b)+1/2*a^2/b^3/(a*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx$$
$$=\frac{12\,a^3bx^3+18\,a^2b^2x^2+4\,ab^3x-b^4-12\,(a^4x^4+2\,a^3bx^3+a^2b^2x^2)\log(ax+b)+12\,(a^4x^4+2\,a^3bx^3+a^2}{2\,(a^2b^5x^4+2\,ab^6x^3+b^7x^2)}$$

input
```
integrate(1/(a+b/x)^3/x^6,x, algorithm="fricas")
```

output
```
1/2*(12*a^3*b*x^3 + 18*a^2*b^2*x^2 + 4*a*b^3*x - b^4 - 12*(a^4*x^4 + 2*a^3
*b*x^3 + a^2*b^2*x^2)*log(a*x + b) + 12*(a^4*x^4 + 2*a^3*b*x^3 + a^2*b^2*x
^2)*log(x))/(a^2*b^5*x^4 + 2*a*b^6*x^3 + b^7*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx = \frac{6a^2\left(\log\left(x\right)-\log\left(x+\frac{b}{a}\right)\right)}{b^5} + \frac{12a^3x^3+18a^2bx^2+4ab^2x-b^3}{2a^2b^4x^4+4ab^5x^3+2b^6x^2}$$

input
```
integrate(1/(a+b/x)**3/x**6,x)
```

output
```
6*a**2*(log(x) - log(x + b/a))/b**5 + (12*a**3*x**3 + 18*a**2*b*x**2 + 4*a
*b**2*x - b**3)/(2*a**2*b**4*x**4 + 4*a*b**5*x**3 + 2*b**6*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx = \frac{12\,a^3x^3+18\,a^2bx^2+4\,ab^2x-b^3}{2\,(a^2b^4x^4+2\,ab^5x^3+b^6x^2)} - \frac{6\,a^2\log\left(ax+b\right)}{b^5} + \frac{6\,a^2\log\left(x\right)}{b^5}$$

input
```
integrate(1/(a+b/x)^3/x^6,x, algorithm="maxima")
```

output
```
1/2*(12*a^3*x^3 + 18*a^2*b*x^2 + 4*a*b^2*x - b^3)/(a^2*b^4*x^4 + 2*a*b^5*x
^3 + b^6*x^2) - 6*a^2*log(a*x + b)/b^5 + 6*a^2*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx = -\frac{6\,a^2 \log\left(|ax+b|\right)}{b^5} + \frac{6\,a^2 \log\left(|x|\right)}{b^5} + \frac{12\,a^3x^3 + 18\,a^2bx^2 + 4\,ab^2x - b^3}{2\,\left(ax^2+bx\right)^2 b^4}$$

input
```
integrate(1/(a+b/x)^3/x^6,x, algorithm="giac")
```

output
```
-6*a^2*log(abs(a*x + b))/b^5 + 6*a^2*log(abs(x))/b^5 + 1/2*(12*a^3*x^3 + 1
8*a^2*b*x^2 + 4*a*b^2*x - b^3)/((a*x^2 + b*x)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx = \frac{\frac{9\,a^2\,x^2}{b^3} - \frac{1}{2\,b} + \frac{6\,a^3\,x^3}{b^4} + \frac{2\,a\,x}{b^2}}{a^2\,x^4 + 2\,a\,b\,x^3 + b^2\,x^2} - \frac{12\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^5}$$

input
```
int(1/(x^6*(a + b/x)^3),x)
```

output
```
((9*a^2*x^2)/b^3 - 1/(2*b) + (6*a^3*x^3)/b^4 + (2*a*x)/b^2)/(a^2*x^4 + b^2
*x^2 + 2*a*b*x^3) - (12*a^2*atanh((2*a*x)/b + 1))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^6}\,dx = \frac{-12\log(ax+b)\,a^4x^4 - 24\log(ax+b)\,a^3b\,x^3 - 12\log(ax+b)\,a^2b^2x^2 + 12\log(x)\,a^4x^4 + 24\log(x)\,a^3b\,x^3}{2b^5x^2\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^6,x)
```

output
```
( - 12*log(a*x + b)*a**4*x**4 - 24*log(a*x + b)*a**3*b*x**3 - 12*log(a*x +
 b)*a**2*b**2*x**2 + 12*log(x)*a**4*x**4 + 24*log(x)*a**3*b*x**3 + 12*log(
x)*a**2*b**2*x**2 - 6*a**4*x**4 + 12*a**2*b**2*x**2 + 4*a*b**3*x - b**4)/(
2*b**5*x**2*(a**2*x**2 + 2*a*b*x + b**2))
```

3.98 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7} \, dx = -\frac{a^5}{2b^6\left(a+\frac{b}{x}\right)^2}+\frac{5a^4}{b^6\left(a+\frac{b}{x}\right)}-\frac{1}{3b^3x^3}+\frac{3a}{2b^4x^2}-\frac{6a^2}{b^5x}+\frac{10a^3\log\left(a+\frac{b}{x}\right)}{b^6}$$

output
```
-1/2*a^5/b^6/(a+b/x)^2+5*a^4/b^6/(a+b/x)-1/3/b^3/x^3+3/2*a/b^4/x^2-6*a^2/b
^5/x+10*a^3*ln(a+b/x)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7} \, dx = -\frac{\frac{b(2b^4-5ab^3x+20a^2b^2x^2+90a^3bx^3+60a^4x^4)}{x^3(b+ax)^2}+60a^3\log(x)-60a^3\log(b+ax)}{6b^6}$$

input
```
Integrate[1/((a + b/x)^3*x^7),x]
```

output
```
-1/6*((b*(2*b^4 - 5*a*b^3*x + 20*a^2*b^2*x^2 + 90*a^3*b*x^3 + 60*a^4*x^4))
/(x^3*(b + a*x)^2) + 60*a^3*Log[x] - 60*a^3*Log[b + a*x])/b^6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{x^4(ax+b)^3}dx$$

$$\Big\downarrow \ 54$$

$$\int \left(\frac{10a^4}{b^6(ax+b)}+\frac{4a^4}{b^5(ax+b)^2}+\frac{a^4}{b^4(ax+b)^3}-\frac{10a^3}{b^6x}+\frac{6a^2}{b^5x^2}-\frac{3a}{b^4x^3}+\frac{1}{b^3x^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{10a^3\log(x)}{b^6}+\frac{10a^3\log(ax+b)}{b^6}-\frac{4a^3}{b^5(ax+b)}-\frac{a^3}{2b^4(ax+b)^2}-\frac{6a^2}{b^5x}+\frac{3a}{2b^4x^2}-\frac{1}{3b^3x^3}$$

input
```
Int[1/((a + b/x)^3*x^7),x]
```

output
```
-1/3*1/(b^3*x^3) + (3*a)/(2*b^4*x^2) - (6*a^2)/(b^5*x) - a^3/(2*b^4*(b + a
*x)^2) - (4*a^3)/(b^5*(b + a*x)) - (10*a^3*Log[x])/b^6 + (10*a^3*Log[b + a
*x])/b^6
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

method	result
default	$-\frac{1}{3b^{3}x^{3}}-\frac{10a^{3}\ln(x)}{b^{6}}-\frac{6a^{2}}{b^{5}x}+\frac{3a}{2b^{4}x^{2}}-\frac{a^{3}}{2b^{4}(ax+b)^{2}}+\frac{10a^{3}\ln(ax+b)}{b^{6}}-\frac{4a^{3}}{b^{5}(ax+b)}$
risch	$\frac{-\frac{10a^{4}x^{4}}{b^{5}}-\frac{15a^{3}x^{3}}{b^{4}}-\frac{10a^{2}x^{2}}{3b^{3}}+\frac{5ax}{6b^{2}}-\frac{1}{3b}}{x^{3}(ax+b)^{2}}+\frac{10a^{3}\ln(-ax-b)}{b^{6}}-\frac{10a^{3}\ln(x)}{b^{6}}$
norman	$\frac{\frac{15a^{5}x^{8}}{b^{6}}-\frac{x^{3}}{3b}+\frac{5a\,x^{4}}{6b^{2}}-\frac{10a^{2}x^{5}}{3b^{3}}+\frac{20a^{4}x^{7}}{b^{5}}}{(ax+b)^{2}x^{6}}-\frac{10a^{3}\ln(x)}{b^{6}}+\frac{10a^{3}\ln(ax+b)}{b^{6}}$
parallelrisch	$-\frac{60a^{5}\ln(x)x^{5}-60a^{5}\ln(ax+b)x^{5}+120\ln(x)x^{4}a^{4}b-120\ln(ax+b)x^{4}a^{4}b-90a^{5}x^{5}+60\ln(x)x^{3}a^{3}b^{2}-60\ln(ax+b)x^{3}a^{3}b^{2}-120}{6b^{6}(ax+b)^{2}x^{3}}$

input
```
int(1/(a+b/x)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^3/x^3-10/b^6*a^3*ln(x)-6*a^2/b^5/x+3/2*a/b^4/x^2-1/2*a^3/b^4/(a*x+b
)^2+10/b^6*a^3*ln(a*x+b)-4*a^3/b^5/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx = -\frac{60\,a^4bx^4 + 90\,a^3b^2x^3 + 20\,a^2b^3x^2 - 5\,ab^4x + 2\,b^5 - 60\,(a^5x^5 + 2\,a^4bx^4 + a^3b^2x^3)\log\left(ax+b\right) + 60\,(a^5x}{6\,(a^2b^6x^5 + 2\,ab^7x^4 + b^8x^3)}$$

input
```
integrate(1/(a+b/x)^3/x^7,x, algorithm="fricas")
```

output
```
-1/6*(60*a^4*b*x^4 + 90*a^3*b^2*x^3 + 20*a^2*b^3*x^2 - 5*a*b^4*x + 2*b^5 -
 60*(a^5*x^5 + 2*a^4*b*x^4 + a^3*b^2*x^3)*log(a*x + b) + 60*(a^5*x^5 + 2*a
^4*b*x^4 + a^3*b^2*x^3)*log(x))/(a^2*b^6*x^5 + 2*a*b^7*x^4 + b^8*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx = \frac{10a^3\left(-\log\left(x\right)+\log\left(x+\frac{b}{a}\right)\right)}{b^6} + \frac{-60a^4x^4 - 90a^3bx^3 - 20a^2b^2x^2 + 5ab^3x - 2b^4}{6a^2b^5x^5 + 12ab^6x^4 + 6b^7x^3}$$

input
```
integrate(1/(a+b/x)**3/x**7,x)
```

output
```
10*a**3*(-log(x) + log(x + b/a))/b**6 + (-60*a**4*x**4 - 90*a**3*b*x**3 -
20*a**2*b**2*x**2 + 5*a*b**3*x - 2*b**4)/(6*a**2*b**5*x**5 + 12*a*b**6*x**
4 + 6*b**7*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx = -\frac{60\,a^4x^4+90\,a^3bx^3+20\,a^2b^2x^2-5\,ab^3x+2\,b^4}{6\,\left(a^2b^5x^5+2\,ab^6x^4+b^7x^3\right)} + \frac{10\,a^3\log\left(ax+b\right)}{b^6} - \frac{10\,a^3\log\left(x\right)}{b^6}$$

input
```
integrate(1/(a+b/x)^3/x^7,x, algorithm="maxima")
```

output
```
-1/6*(60*a^4*x^4 + 90*a^3*b*x^3 + 20*a^2*b^2*x^2 - 5*a*b^3*x + 2*b^4)/(a^2
*b^5*x^5 + 2*a*b^6*x^4 + b^7*x^3) + 10*a^3*log(a*x + b)/b^6 - 10*a^3*log(x
)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx = \frac{10\,a^3\log\left(|ax+b|\right)}{b^6} - \frac{10\,a^3\log\left(|x|\right)}{b^6} - \frac{60\,a^4bx^4+90\,a^3b^2x^3+20\,a^2b^3x^2-5\,ab^4x+2\,b^5}{6\,\left(ax+b\right)^2b^6x^3}$$

input
```
integrate(1/(a+b/x)^3/x^7,x, algorithm="giac")
```

output
```
10*a^3*log(abs(a*x + b))/b^6 - 10*a^3*log(abs(x))/b^6 - 1/6*(60*a^4*b*x^4
+ 90*a^3*b^2*x^3 + 20*a^2*b^3*x^2 - 5*a*b^4*x + 2*b^5)/((a*x + b)^2*b^6*x^
3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx = \frac{20\,a^3\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^6} - \frac{\frac{1}{3\,b}+\frac{10\,a^2\,x^2}{3\,b^3}+\frac{15\,a^3\,x^3}{b^4}+\frac{10\,a^4\,x^4}{b^5}-\frac{5\,a\,x}{6\,b^2}}{a^2\,x^5+2\,a\,b\,x^4+b^2\,x^3}$$

input
```
int(1/(x^7*(a + b/x)^3),x)
```

output
```
(20*a^3*atanh((2*a*x)/b + 1))/b^6 - (1/(3*b) + (10*a^2*x^2)/(3*b^3) + (15*
a^3*x^3)/b^4 + (10*a^4*x^4)/b^5 - (5*a*x)/(6*b^2))/(a^2*x^5 + b^2*x^3 + 2*
a*b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^7}\,dx$$
$$= \frac{60\log(ax+b)\,a^5x^5+120\log(ax+b)\,a^4b\,x^4+60\log(ax+b)\,a^3b^2x^3-60\log(x)\,a^5x^5-120\log(x)\,a^4b\,x^4}{6b^6x^3\,(a^2x^2+2abx+b^2)}$$

input
```
int(1/(a+b/x)^3/x^7,x)
```

output
```
(60*log(a*x + b)*a**5*x**5 + 120*log(a*x + b)*a**4*b*x**4 + 60*log(a*x + b
)*a**3*b**2*x**3 - 60*log(x)*a**5*x**5 - 120*log(x)*a**4*b*x**4 - 60*log(x
)*a**3*b**2*x**3 + 30*a**5*x**5 - 60*a**3*b**2*x**3 - 20*a**2*b**3*x**2 +
5*a*b**4*x - 2*b**5)/(6*b**6*x**3*(a**2*x**2 + 2*a*b*x + b**2))
```

3.99 $\int \left(a+\frac{b}{x}\right) x^{5/2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \left(a+\frac{b}{x}\right) x^{5/2} \, dx=\frac{2}{5} b x^{5/2}+\frac{2}{7} a x^{7/2}$$

output
```
2/5*b*x^(5/2)+2/7*a*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right) x^{5/2} \, dx=\frac{2}{35} x^{5/2} (7 b+5 a x)$$

input
```
Integrate[(a + b/x)*x^(5/2),x]
```

output
```
(2*x^(5/2)*(7*b + 5*a*x))/35
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+\frac{b}{x}\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{5/2}+bx^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}ax^{7/2}+\frac{2}{5}bx^{5/2}$$

input
```
Int[(a + b/x)*x^(5/2),x]
```

output
```
(2*b*x^(5/2))/5 + (2*a*x^(7/2))/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2(5ax+7b)x^{\frac{5}{2}}}{35}$	14
derivativedivides	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2a\,x^{\frac{7}{2}}}{7}$	14
default	$\frac{2b\,x^{\frac{5}{2}}}{5}+\frac{2a\,x^{\frac{7}{2}}}{7}$	14
trager	$\frac{2(5ax+7b)x^{\frac{5}{2}}}{35}$	14
risch	$\frac{2(5ax+7b)x^{\frac{5}{2}}}{35}$	14
orering	$\frac{2(5ax+7b)x^{\frac{7}{2}}\left(a+\frac{b}{x}\right)}{35(ax+b)}$	28

input
```
int((a+b/x)*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(5*a*x+7*b)*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x}\right)x^{5/2}\,dx=\frac{2}{35}\left(5\,ax^3+7\,bx^2\right)\sqrt{x}$$

input
```
integrate((a+b/x)*x^(5/2),x, algorithm="fricas")
```

output
```
2/35*(5*a*x^3 + 7*b*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x}\right)x^{5/2}\,dx=\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{5}{2}}}{5}$$

input
```
integrate((a+b/x)*x**(5/2),x)
```

output
```
2*a*x**(7/2)/7 + 2*b*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)x^{5/2}\,dx=\frac{2}{35}\left(5\,a+\frac{7\,b}{x}\right)x^{\frac{7}{2}}$$

input
```
integrate((a+b/x)*x^(5/2),x, algorithm="maxima")
```

output
```
2/35*(5*a + 7*b/x)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{x}\right)x^{5/2}\,dx=\frac{2}{7}\,ax^{\frac{7}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

input
```
integrate((a+b/x)*x^(5/2),x, algorithm="giac")
```

output
```
2/7*a*x^(7/2) + 2/5*b*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{x}\right) x^{5/2}\,dx=\frac{2\,x^{5/2}\,\left(7\,b+5\,a\,x\right)}{35}$$

input
```
int(x^(5/2)*(a + b/x),x)
```

output
```
(2*x^(5/2)*(7*b + 5*a*x))/35
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x^{5/2}\,dx=\frac{2\sqrt{x}\,x^2(5ax+7b)}{35}$$

input
```
int((a+b/x)*x^(5/2),x)
```

output
```
(2*sqrt(x)*x**2*(5*a*x + 7*b))/35
```

3.100 $\int \left(a+\frac{b}{x}\right) x^{3/2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \left(a+\frac{b}{x}\right) x^{3/2} \, dx = \frac{2}{3}bx^{3/2} + \frac{2}{5}ax^{5/2}$$

output
```
2/3*b*x^(3/2)+2/5*a*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right) x^{3/2} \, dx = \frac{2}{15}x^{3/2}(5b+3ax)$$

input
```
Integrate[(a + b/x)*x^(3/2),x]
```

output
```
(2*x^(3/2)*(5*b + 3*a*x))/15
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+\frac{b}{x}\right) dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{3/2}+b\sqrt{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}ax^{5/2}+\frac{2}{3}bx^{3/2}$$

input
```
Int[(a + b/x)*x^(3/2),x]
```

output
```
(2*b*x^(3/2))/3 + (2*a*x^(5/2))/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2(3ax+5b)x^{\frac{3}{2}}}{15}$	14
derivativedivides	$\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2a\,x^{\frac{5}{2}}}{5}$	14
default	$\frac{2b\,x^{\frac{3}{2}}}{3}+\frac{2a\,x^{\frac{5}{2}}}{5}$	14
trager	$\frac{2(3ax+5b)x^{\frac{3}{2}}}{15}$	14
risch	$\frac{2(3ax+5b)x^{\frac{3}{2}}}{15}$	14
orering	$\frac{2x^{\frac{5}{2}}(3ax+5b)\left(a+\frac{b}{x}\right)}{15(ax+b)}$	28

input
```
int((a+b/x)*x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*a*x+5*b)*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)x^{3/2}\,dx=\frac{2}{15}\left(3\,ax^2+5\,bx\right)\sqrt{x}$$

input
```
integrate((a+b/x)*x^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*a*x^2 + 5*b*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x}\right) x^{3/2}\,dx = \frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{3}{2}}}{3}$$

input
```
integrate((a+b/x)*x**(3/2),x)
```

output
```
2*a*x**(5/2)/5 + 2*b*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right) x^{3/2}\,dx = \frac{2}{15}\left(3\,a + \frac{5\,b}{x}\right)x^{\frac{5}{2}}$$

input
```
integrate((a+b/x)*x^(3/2),x, algorithm="maxima")
```

output
```
2/15*(3*a + 5*b/x)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{x}\right) x^{3/2}\,dx = \frac{2}{5}\,ax^{\frac{5}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}}$$

input
```
integrate((a+b/x)*x^(3/2),x, algorithm="giac")
```

output
```
2/5*a*x^(5/2) + 2/3*b*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{x}\right) x^{3/2}\, dx = \frac{2\, x^{3/2}\, (5\, b+3\, a\, x)}{15}$$

input
```
int(x^(3/2)*(a + b/x),x)
```

output
```
(2*x^(3/2)*(5*b + 3*a*x))/15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{x}\right) x^{3/2}\, dx = \frac{2\sqrt{x}\, x(3ax+5b)}{15}$$

input
```
int((a+b/x)*x^(3/2),x)
```

output
```
(2*sqrt(x)*x*(3*a*x + 5*b))/15
```

3.101 $\int \left(a+\frac{b}{x}\right) \sqrt{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+\frac{b}{x}\right) \sqrt{x} \, dx = 2b\sqrt{x}+\frac{2}{3}ax^{3/2}$$

output
```
2*b*x^(1/2)+2/3*a*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x}\right) \sqrt{x} \, dx = \frac{2}{3}\sqrt{x}(3b+ax)$$

input
```
Integrate[(a + b/x)*Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(3*b + a*x))/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+\frac{b}{x}\right) dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a\sqrt{x}+\frac{b}{\sqrt{x}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}ax^{3/2}+2b\sqrt{x}$$

input
```
Int[(a + b/x)*Sqrt[x],x]
```

output
```
2*b*Sqrt[x] + (2*a*x^(3/2))/3
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{2(ax+3b)\sqrt{x}}{3}$	13
trager	$\left(\frac{2ax}{3}+2b\right)\sqrt{x}$	13
risch	$\frac{2(ax+3b)\sqrt{x}}{3}$	13
derivativedivides	$2b\sqrt{x}+\frac{2a\,x^{\frac{3}{2}}}{3}$	14
default	$2b\sqrt{x}+\frac{2a\,x^{\frac{3}{2}}}{3}$	14
orering	$\frac{2(ax+3b)x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)}{3(ax+b)}$	27

input
```
int((a+b/x)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x+3*b)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int\left(a+\frac{b}{x}\right)\sqrt{x}\,dx=\frac{2}{3}\,(ax+3\,b)\sqrt{x}$$

input
```
integrate((a+b/x)*x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a*x + 3*b)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x}\right)\sqrt{x}\,dx=\frac{2ax^{\frac{3}{2}}}{3}+2b\sqrt{x}$$

input
```
integrate((a+b/x)*x**(1/2),x)
```

output
```
2*a*x**(3/2)/3 + 2*b*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{x}\right)\sqrt{x}\,dx=\frac{2}{3}\left(a+\frac{3\,b}{x}\right)x^{\frac{3}{2}}$$

input
```
integrate((a+b/x)*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a + 3*b/x)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{x}\right)\sqrt{x}\,dx=\frac{2}{3}\,ax^{\frac{3}{2}}+2\,b\sqrt{x}$$

input
```
integrate((a+b/x)*x^(1/2),x, algorithm="giac")
```

output
```
2/3*a*x^(3/2) + 2*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \left(a+\frac{b}{x}\right)\sqrt{x}\,dx=\frac{2\,\sqrt{x}\,(3\,b+a\,x)}{3}$$

input
```
int(x^(1/2)*(a + b/x),x)
```

output
```
(2*x^(1/2)*(3*b + a*x))/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \left(a+\frac{b}{x}\right)\sqrt{x}\,dx=\frac{2\sqrt{x}\,(ax+3b)}{3}$$

input
```
int((a+b/x)*x^(1/2),x)
```

output
```
(2*sqrt(x)*(a*x + 3*b))/3
```

3.102 $\int \frac{a+\frac{b}{x}}{\sqrt{x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}} dx = -\frac{2b}{\sqrt{x}} + 2a\sqrt{x}$$

output
```
-2*b/x^(1/2)+2*a*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}} dx = -\frac{2(b-ax)}{\sqrt{x}}$$

input
```
Integrate[(a + b/x)/Sqrt[x],x]
```

output
```
(-2*(b - a*x))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{\sqrt{x}}+\frac{b}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a\sqrt{x}-\frac{2b}{\sqrt{x}}$$

input
```
Int[(a + b/x)/Sqrt[x],x]
```

output
```
(-2*b)/Sqrt[x] + 2*a*Sqrt[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{2ax-2b}{\sqrt{x}}$	13
trager	$\frac{2ax-2b}{\sqrt{x}}$	13
risch	$\frac{2ax-2b}{\sqrt{x}}$	13
derivativedivides	$-\frac{2b}{\sqrt{x}}+2a\sqrt{x}$	14
default	$-\frac{2b}{\sqrt{x}}+2a\sqrt{x}$	14
orering	$\frac{2\sqrt{x}\,(ax-b)\left(a+\frac{b}{x}\right)}{ax+b}$	27

input
```
int((a+b/x)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x-b)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx=\frac{2\,(ax-b)}{\sqrt{x}}$$

input
```
integrate((a+b/x)/x^(1/2),x, algorithm="fricas")
```

output
```
2*(a*x - b)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx = 2a\sqrt{x} - \frac{2b}{\sqrt{x}}$$

input
```
integrate((a+b/x)/x**(1/2),x)
```

output
```
2*a*sqrt(x) - 2*b/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx = 2\,a\sqrt{x} - \frac{2\,b}{\sqrt{x}}$$

input
```
integrate((a+b/x)/x^(1/2),x, algorithm="maxima")
```

output
```
2*a*sqrt(x) - 2*b/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}}\,dx = 2\,a\sqrt{x} - \frac{2\,b}{\sqrt{x}}$$

input
```
integrate((a+b/x)/x^(1/2),x, algorithm="giac")
```

output
```
2*a*sqrt(x) - 2*b/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}} \, dx = -\frac{2\,(b-a\,x)}{\sqrt{x}}$$

input
```
int((a + b/x)/x^(1/2),x)
```

output
```
-(2*(b - a*x))/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{x}}{\sqrt{x}} \, dx = \frac{2ax-2b}{\sqrt{x}}$$

input
```
int((a+b/x)/x^(1/2),x)
```

output
```
(2*(a*x - b))/sqrt(x)
```

3.103 $\int \frac{a+\frac{b}{x}}{x^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+\frac{b}{x}}{x^{3/2}} \, dx = -\frac{2b}{3x^{3/2}} - \frac{2a}{\sqrt{x}}$$

output
```
-2/3*b/x^(3/2)-2*a/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+\frac{b}{x}}{x^{3/2}} \, dx = -\frac{2(b+3ax)}{3x^{3/2}}$$

input
```
Integrate[(a + b/x)/x^(3/2),x]
```

output
```
(-2*(b + 3*a*x))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^{3/2}}+\frac{b}{x^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a}{\sqrt{x}}-\frac{2b}{3x^{3/2}}$$

input
```
Int[(a + b/x)/x^(3/2),x]
```

output
```
(-2*b)/(3*x^(3/2)) - (2*a)/Sqrt[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2(3ax+b)}{3x^{\frac{3}{2}}}$	12
trager	$-\frac{2(3ax+b)}{3x^{\frac{3}{2}}}$	12
risch	$-\frac{2(3ax+b)}{3x^{\frac{3}{2}}}$	12
derivativedivides	$-\frac{2b}{3x^{\frac{3}{2}}}-\frac{2a}{\sqrt{x}}$	14
default	$-\frac{2b}{3x^{\frac{3}{2}}}-\frac{2a}{\sqrt{x}}$	14
orering	$-\frac{2(3ax+b)\left(a+\frac{b}{x}\right)}{3\sqrt{x}\,(ax+b)}$	26

input
```
int((a+b/x)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*a*x+b)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx=-\frac{2\,(3\,ax+b)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)/x^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*a*x + b)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx = -\frac{2a}{\sqrt{x}} - \frac{2b}{3x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)/x**(3/2),x)
```

output
```
-2*a/sqrt(x) - 2*b/(3*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx = -\frac{2\,a}{\sqrt{x}} - \frac{2\,b}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)/x^(3/2),x, algorithm="maxima")
```

output
```
-2*a/sqrt(x) - 2/3*b/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx = -\frac{2\,(3\,ax+b)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)/x^(3/2),x, algorithm="giac")
```

output
```
-2/3*(3*a*x + b)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx = -\frac{2\,b+6\,a\,x}{3\,x^{3/2}}$$

input
```
int((a + b/x)/x^(3/2),x)
```

output
```
-(2*b + 6*a*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+\frac{b}{x}}{x^{3/2}}\,dx = \frac{-2ax-\frac{2b}{3}}{\sqrt{x}\,x}$$

input
```
int((a+b/x)/x^(3/2),x)
```

output
```
(2*( - 3*a*x - b))/(3*sqrt(x)*x)
```

3.104 $\int \frac{a+\frac{b}{x}}{x^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{a+\frac{b}{x}}{x^{5/2}} \, dx = -\frac{2b}{5x^{5/2}} - \frac{2a}{3x^{3/2}}$$

output
```
-2/5*b/x^(5/2)-2/3*a/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x}}{x^{5/2}} \, dx = -\frac{2(3b+5ax)}{15x^{5/2}}$$

input
```
Integrate[(a + b/x)/x^(5/2),x]
```

output
```
(-2*(3*b + 5*a*x))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 802$$

$$\int \left(\frac{a}{x^{5/2}}+\frac{b}{x^{7/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{5x^{5/2}}$$

input
```
Int[(a + b/x)/x^(5/2),x]
```

output
```
(-2*b)/(5*x^(5/2)) - (2*a)/(3*x^(3/2))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2(5ax+3b)}{15x^{\frac{5}{2}}}$	14
derivativedivides	$-\frac{2b}{5x^{\frac{5}{2}}}-\frac{2a}{3x^{\frac{3}{2}}}$	14
default	$-\frac{2b}{5x^{\frac{5}{2}}}-\frac{2a}{3x^{\frac{3}{2}}}$	14
trager	$-\frac{2(5ax+3b)}{15x^{\frac{5}{2}}}$	14
risch	$-\frac{2(5ax+3b)}{15x^{\frac{5}{2}}}$	14
orering	$-\frac{2(5ax+3b)\left(a+\frac{b}{x}\right)}{15x^{\frac{3}{2}}(ax+b)}$	28

input
```
int((a+b/x)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(5*a*x+3*b)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx = -\frac{2\,(5\,ax+3\,b)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(5*a*x + 3*b)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx = -\frac{2a}{3x^{\frac{3}{2}}} - \frac{2b}{5x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)/x**(5/2),x)
```

output
```
-2*a/(3*x**(3/2)) - 2*b/(5*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx = -\frac{2\,a}{3\,x^{\frac{3}{2}}} - \frac{2\,b}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*a/x^(3/2) - 2/5*b/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx = -\frac{2\,(5\,ax+3\,b)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)/x^(5/2),x, algorithm="giac")
```

output
```
-2/15*(5*a*x + 3*b)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx = -\frac{6\,b+10\,a\,x}{15\,x^{5/2}}$$

input
```
int((a + b/x)/x^(5/2),x)
```

output
```
-(6*b + 10*a*x)/(15*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x}}{x^{5/2}}\,dx = \frac{-\frac{2ax}{3}-\frac{2b}{5}}{\sqrt{x}\,x^2}$$

input
```
int((a+b/x)/x^(5/2),x)
```

output
```
(2*( - 5*a*x - 3*b))/(15*sqrt(x)*x**2)
```

3.105 $\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx=\frac{2}{3}b^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}a^2x^{7/2}$$

output
```
2/3*b^2*x^(3/2)+4/5*a*b*x^(5/2)+2/7*a^2*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx=\frac{2}{105}x^{3/2}\left(35b^2+42abx+15a^2x^2\right)$$

input
```
Integrate[(a + b/x)^2*x^(5/2),x]
```

output
```
(2*x^(3/2)*(35*b^2 + 42*a*b*x + 15*a^2*x^2))/105
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+\frac{b}{x}\right)^2dx$$
$$\Big\downarrow \text{795}$$
$$\int \sqrt{x}(ax+b)^2dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(a^2x^{5/2}+2abx^{3/2}+b^2\sqrt{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{7}a^2x^{7/2}+\frac{4}{5}abx^{5/2}+\frac{2}{3}b^2x^{3/2}$$

input
```
Int[(a + b/x)^2*x^(5/2),x]
```

output
```
(2*b^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*a^2*x^(7/2))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{2\left(15a^{2}x^{2}+42abx+35b^{2}\right)x^{\frac{3}{2}}}{105}$	25
derivativedivides	$\frac{2b^{2}x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2a^{2}x^{\frac{7}{2}}}{7}$	25
default	$\frac{2b^{2}x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2a^{2}x^{\frac{7}{2}}}{7}$	25
trager	$\frac{2\left(15a^{2}x^{2}+42abx+35b^{2}\right)x^{\frac{3}{2}}}{105}$	25
risch	$\frac{2\left(15a^{2}x^{2}+42abx+35b^{2}\right)x^{\frac{3}{2}}}{105}$	25
orering	$\frac{2x^{\frac{7}{2}}\left(15a^{2}x^{2}+42abx+35b^{2}\right)\left(a+\frac{b}{x}\right)^{2}}{105(ax+b)^{2}}$	41

input
```
int((a+b/x)^2*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*a^2*x^2+42*a*b*x+35*b^2)*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx=\frac{2}{105}\left(15\,a^2x^3+42\,abx^2+35\,b^2x\right)\sqrt{x}$$

input
```
integrate((a+b/x)^2*x^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*a^2*x^3 + 42*a*b*x^2 + 35*b^2*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx = \frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{3}{2}}}{3}$$

input
```
integrate((a+b/x)**2*x**(5/2),x)
```

output
```
2*a**2*x**(7/2)/7 + 4*a*b*x**(5/2)/5 + 2*b**2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx = \frac{2}{105}\left(15\,a^2+\frac{42\,ab}{x}+\frac{35\,b^2}{x^2}\right)x^{\frac{7}{2}}$$

input
```
integrate((a+b/x)^2*x^(5/2),x, algorithm="maxima")
```

output
```
2/105*(15*a^2 + 42*a*b/x + 35*b^2/x^2)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx = \frac{2}{7}\,a^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,b^2x^{\frac{3}{2}}$$

input
```
integrate((a+b/x)^2*x^(5/2),x, algorithm="giac")
```

output
```
2/7*a^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*b^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx=\frac{2\,x^{3/2}\,\left(15\,a^2\,x^2+42\,a\,b\,x+35\,b^2\right)}{105}$$

input
```
int(x^(5/2)*(a + b/x)^2,x)
```

output
```
(2*x^(3/2)*(35*b^2 + 15*a^2*x^2 + 42*a*b*x))/105
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \left(a+\frac{b}{x}\right)^2 x^{5/2}\,dx=\frac{2\sqrt{x}\,x(15a^2x^2+42abx+35b^2)}{105}$$

input
```
int((a+b/x)^2*x^(5/2),x)
```

output
```
(2*sqrt(x)*x*(15*a**2*x**2 + 42*a*b*x + 35*b**2))/105
```

3.106 $\int \left(a+\frac{b}{x}\right)^2 x^{3/2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2} \, dx = 2b^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}a^2x^{5/2}$$

output
```
2*b^2*x^(1/2)+4/3*a*b*x^(3/2)+2/5*a^2*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2} \, dx = \frac{2}{15}\sqrt{x}\left(15b^2+10abx+3a^2x^2\right)$$

input
```
Integrate[(a + b/x)^2*x^(3/2),x]
```

output
```
(2*Sqrt[x]*(15*b^2 + 10*a*b*x + 3*a^2*x^2))/15
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+\frac{b}{x}\right)^2 dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{(ax+b)^2}{\sqrt{x}}dx$$
$$\Big\downarrow \text{ 53}$$
$$\int \left(a^2x^{3/2}+2ab\sqrt{x}+\frac{b^2}{\sqrt{x}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2}{5}a^2x^{5/2}+\frac{4}{3}abx^{3/2}+2b^2\sqrt{x}$$

input
```
Int[(a + b/x)^2*x^(3/2),x]
```

output
```
2*b^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*a^2*x^(5/2))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
trager	$\left(\frac{2}{5}a^{2}x^{2}+\frac{4}{3}abx+2b^{2}\right)\sqrt{x}$	24
gosper	$\frac{2\left(3a^{2}x^{2}+10abx+15b^{2}\right)\sqrt{x}}{15}$	25
derivativedivides	$2b^{2}\sqrt{x}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{2a^{2}x^{\frac{5}{2}}}{5}$	25
default	$2b^{2}\sqrt{x}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{2a^{2}x^{\frac{5}{2}}}{5}$	25
risch	$\frac{2\left(3a^{2}x^{2}+10abx+15b^{2}\right)\sqrt{x}}{15}$	25
orering	$\frac{2\left(3a^{2}x^{2}+10abx+15b^{2}\right)x^{\frac{5}{2}}\left(a+\frac{b}{x}\right)^{2}}{15\left(ax+b\right)^{2}}$	41

input
```
int((a+b/x)^2*x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5*a^2*x^2+4/3*a*b*x+2*b^2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx=\frac{2}{15}\left(3\,a^2x^2+10\,abx+15\,b^2\right)\sqrt{x}$$

input
```
integrate((a+b/x)^2*x^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*x^2 + 10*a*b*x + 15*b^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx = \frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{3}{2}}}{3}+2b^2\sqrt{x}$$

input
```
integrate((a+b/x)**2*x**(3/2),x)
```

output
```
2*a**2*x**(5/2)/5 + 4*a*b*x**(3/2)/3 + 2*b**2*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx = \frac{2}{15}\left(3\,a^2+\frac{10\,ab}{x}+\frac{15\,b^2}{x^2}\right)x^{\frac{5}{2}}$$

input
```
integrate((a+b/x)^2*x^(3/2),x, algorithm="maxima")
```

output
```
2/15*(3*a^2 + 10*a*b/x + 15*b^2/x^2)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx = \frac{2}{5}\,a^2x^{\frac{5}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}+2\,b^2\sqrt{x}$$

input
```
integrate((a+b/x)^2*x^(3/2),x, algorithm="giac")
```

output
```
2/5*a^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*b^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx=\frac{2\,\sqrt{x}\,\left(3\,a^2\,x^2+10\,a\,b\,x+15\,b^2\right)}{15}$$

input
```
int(x^(3/2)*(a + b/x)^2,x)
```

output
```
(2*x^(1/2)*(15*b^2 + 3*a^2*x^2 + 10*a*b*x))/15
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{x}\right)^2 x^{3/2}\,dx=\frac{2\sqrt{x}\,(3a^2x^2+10abx+15b^2)}{15}$$

input
```
int((a+b/x)^2*x^(3/2),x)
```

output
```
(2*sqrt(x)*(3*a**2*x**2 + 10*a*b*x + 15*b**2))/15
```

3.107 $\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\, dx = -\frac{2b^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}a^2x^{3/2}$$

output
```
-2*b^2/x^(1/2)+4*a*b*x^(1/2)+2/3*a^2*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\, dx = -\frac{2(3b^2-6abx-a^2x^2)}{3\sqrt{x}}$$

input
```
Integrate[(a + b/x)^2*Sqrt[x],x]
```

output
```
(-2*(3*b^2 - 6*a*b*x - a^2*x^2))/(3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+\frac{b}{x}\right)^2 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^2}{x^{3/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2\sqrt{x}+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2x^{3/2}+4ab\sqrt{x}-\frac{2b^2}{\sqrt{x}}$$

input
```
Int[(a + b/x)^2*Sqrt[x],x]
```

output
```
(-2*b^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*a^2*x^(3/2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{-2b^2+\frac{2}{3}a^2x^2+4abx}{\sqrt{x}}$	24
trager	$\frac{-2b^2+\frac{2}{3}a^2x^2+4abx}{\sqrt{x}}$	24
risch	$\frac{-2b^2+\frac{2}{3}a^2x^2+4abx}{\sqrt{x}}$	24
derivativedivides	$-\frac{2b^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2a^2x^{\frac{3}{2}}}{3}$	25
default	$-\frac{2b^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2a^2x^{\frac{3}{2}}}{3}$	25
orering	$\frac{2(a^2x^2+6abx-3b^2)x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^2}{3(ax+b)^2}$	40

input
```
int((a+b/x)^2*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a^2*x^2+6*a*b*x-3*b^2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \left(a+\frac{b}{x}\right)^2\sqrt{x}\,dx=\frac{2\,(a^2x^2+6\,abx-3\,b^2)}{3\,\sqrt{x}}$$

input
```
integrate((a+b/x)^2*x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 + 6*a*b*x - 3*b^2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^2\sqrt{x}\,dx=\frac{2a^2x^{\frac{3}{2}}}{3}+4ab\sqrt{x}-\frac{2b^2}{\sqrt{x}}$$

input
```
integrate((a+b/x)**2*x**(1/2),x)
```

output
```
2*a**2*x**(3/2)/3 + 4*a*b*sqrt(x) - 2*b**2/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x}\right)^2\sqrt{x}\,dx=\frac{2}{3}\left(a^2+\frac{6\,ab}{x}\right)x^{\frac{3}{2}}-\frac{2\,b^2}{\sqrt{x}}$$

input
```
integrate((a+b/x)^2*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a^2 + 6*a*b/x)*x^(3/2) - 2*b^2/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x}\right)^2\sqrt{x}\,dx=\frac{2}{3}\,a^2x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,b^2}{\sqrt{x}}$$

input
```
integrate((a+b/x)^2*x^(1/2),x, algorithm="giac")
```

output
```
2/3*a^2*x^(3/2) + 4*a*b*sqrt(x) - 2*b^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\,dx=\frac{2\,a^2\,x^2+12\,a\,b\,x-6\,b^2}{3\,\sqrt{x}}$$

input
```
int(x^(1/2)*(a + b/x)^2,x)
```

output
```
(2*a^2*x^2 - 6*b^2 + 12*a*b*x)/(3*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x}\right)^2 \sqrt{x}\,dx=\frac{\frac{2}{3}a^2x^2+4abx-2b^2}{\sqrt{x}}$$

input
```
int((a+b/x)^2*x^(1/2),x)
```

output
```
(2*(a**2*x**2 + 6*a*b*x - 3*b**2))/(3*sqrt(x))
```

3.108 $\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx=-\frac{2b^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2a^2\sqrt{x}$$

output
```
-2/3*b^2/x^(3/2)-4*a*b/x^(1/2)+2*a^2*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx=-\frac{2(b^2+6abx-3a^2x^2)}{3x^{3/2}}$$

input
```
Integrate[(a + b/x)^2/Sqrt[x],x]
```

output
```
(-2*(b^2 + 6*a*b*x - 3*a^2*x^2))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^2}{x^{5/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^2}{\sqrt{x}}+\frac{2ab}{x^{3/2}}+\frac{b^2}{x^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^2\sqrt{x}-\frac{4ab}{\sqrt{x}}-\frac{2b^2}{3x^{3/2}}$$

input
```
Int[(a + b/x)^2/Sqrt[x],x]
```

output
```
(-2*b^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*a^2*Sqrt[x]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{-4abx-\frac{2}{3}b^2+2a^2x^2}{x^{\frac{3}{2}}}$	25
derivativedivides	$-\frac{2b^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2a^2\sqrt{x}$	25
default	$-\frac{2b^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2a^2\sqrt{x}$	25
trager	$\frac{-4abx-\frac{2}{3}b^2+2a^2x^2}{x^{\frac{3}{2}}}$	25
risch	$\frac{-4abx-\frac{2}{3}b^2+2a^2x^2}{x^{\frac{3}{2}}}$	25
orering	$\frac{2(3a^2x^2-6abx-b^2)\sqrt{x}\left(a+\frac{b}{x}\right)^2}{3(ax+b)^2}$	41

input
```
int((a+b/x)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*a^2*x^2-6*a*b*x-b^2)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx = \frac{2\,(3\,a^2x^2-6\,abx-b^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*a^2*x^2 - 6*a*b*x - b^2)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x} - \frac{4ab}{\sqrt{x}} - \frac{2b^2}{3x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)**2/x**(1/2),x)
```

output
```
2*a**2*sqrt(x) - 4*a*b/sqrt(x) - 2*b**2/(3*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx = 2\,a^2\sqrt{x} - \frac{4\,ab}{\sqrt{x}} - \frac{2\,b^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2*a^2*sqrt(x) - 4*a*b/sqrt(x) - 2/3*b^2/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx = 2\,a^2\sqrt{x} - \frac{2\,(6\,abx + b^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)^2/x^(1/2),x, algorithm="giac")
```

output
```
2*a^2*sqrt(x) - 2/3*(6*a*b*x + b^2)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx = -\frac{-6\,a^2\,x^2+12\,a\,b\,x+2\,b^2}{3\,x^{3/2}}$$

input
```
int((a + b/x)^2/x^(1/2),x)
```

output
```
-(2*b^2 - 6*a^2*x^2 + 12*a*b*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{\sqrt{x}}\,dx = \frac{2a^2x^2-4abx-\frac{2}{3}b^2}{\sqrt{x}\,x}$$

input
```
int((a+b/x)^2/x^(1/2),x)
```

output
```
(2*(3*a**2*x**2 - 6*a*b*x - b**2))/(3*sqrt(x)*x)
```

3.109 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx=-\frac{2b^2}{5x^{5/2}}-\frac{4ab}{3x^{3/2}}-\frac{2a^2}{\sqrt{x}}$$

output
```
-2/5*b^2/x^(5/2)-4/3*a*b/x^(3/2)-2*a^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx=-\frac{2(3b^2+10abx+15a^2x^2)}{15x^{5/2}}$$

input
```
Integrate[(a + b/x)^2/x^(3/2),x]
```

output
```
(-2*(3*b^2 + 10*a*b*x + 15*a^2*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{(ax+b)^2}{x^{7/2}}dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{a^2}{x^{3/2}}+\frac{2ab}{x^{5/2}}+\frac{b^2}{x^{7/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2}{\sqrt{x}}-\frac{4ab}{3x^{3/2}}-\frac{2b^2}{5x^{5/2}}$$

input
```
Int[(a + b/x)^2/x^(3/2),x]
```

output
```
(-2*b^2)/(5*x^(5/2)) - (4*a*b)/(3*x^(3/2)) - (2*a^2)/Sqrt[x]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{2(15a^2x^2+10abx+3b^2)}{15x^{\frac{5}{2}}}$	25
derivativedivides	$-\frac{2b^2}{5x^{\frac{5}{2}}}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{2a^2}{\sqrt{x}}$	25
default	$-\frac{2b^2}{5x^{\frac{5}{2}}}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{2a^2}{\sqrt{x}}$	25
trager	$-\frac{2(15a^2x^2+10abx+3b^2)}{15x^{\frac{5}{2}}}$	25
risch	$-\frac{2(15a^2x^2+10abx+3b^2)}{15x^{\frac{5}{2}}}$	25
orering	$-\frac{2(15a^2x^2+10abx+3b^2)\left(a+\frac{b}{x}\right)^2}{15\sqrt{x}\,(ax+b)^2}$	41

input
```
int((a+b/x)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*a^2*x^2+10*a*b*x+3*b^2)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx = -\frac{2\,(15\,a^2x^2+10\,abx+3\,b^2)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)^2/x^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(15*a^2*x^2 + 10*a*b*x + 3*b^2)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx=-\frac{2a^2}{\sqrt{x}}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{2b^2}{5x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)**2/x**(3/2),x)
```

output
```
-2*a**2/sqrt(x) - 4*a*b/(3*x**(3/2)) - 2*b**2/(5*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx=-\frac{2\,a^2}{\sqrt{x}}-\frac{4\,ab}{3\,x^{\frac{3}{2}}}-\frac{2\,b^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)^2/x^(3/2),x, algorithm="maxima")
```

output
```
-2*a^2/sqrt(x) - 4/3*a*b/x^(3/2) - 2/5*b^2/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx=-\frac{2\,(15\,a^2x^2+10\,abx+3\,b^2)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)^2/x^(3/2),x, algorithm="giac")
```

output
```
-2/15*(15*a^2*x^2 + 10*a*b*x + 3*b^2)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx=-\frac{30\,a^2\,x^2+20\,a\,b\,x+6\,b^2}{15\,x^{5/2}}$$

input
```
int((a + b/x)^2/x^(3/2),x)
```

output
```
-(6*b^2 + 30*a^2*x^2 + 20*a*b*x)/(15*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{3/2}}\,dx=\frac{-2a^2x^2-\frac{4}{3}abx-\frac{2}{5}b^2}{\sqrt{x}\,x^2}$$

input
```
int((a+b/x)^2/x^(3/2),x)
```

output
```
(2*( - 15*a**2*x**2 - 10*a*b*x - 3*b**2))/(15*sqrt(x)*x**2)
```

3.110 $\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx = -\frac{2b^2}{7x^{7/2}}-\frac{4ab}{5x^{5/2}}-\frac{2a^2}{3x^{3/2}}$$

output
```
-2/7*b^2/x^(7/2)-4/5*a*b/x^(5/2)-2/3*a^2/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx = -\frac{2(15b^2+42abx+35a^2x^2)}{105x^{7/2}}$$

input
```
Integrate[(a + b/x)^2/x^(5/2),x]
```

output
```
(-2*(15*b^2 + 42*a*b*x + 35*a^2*x^2))/(105*x^(7/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}} \, dx$$
$$\Big\downarrow \ 795$$
$$\int \frac{(ax+b)^2}{x^{9/2}}dx$$
$$\Big\downarrow \ 53$$
$$\int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{x^{7/2}}+\frac{b^2}{x^{9/2}}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{5x^{5/2}}-\frac{2b^2}{7x^{7/2}}$$

input
```
Int[(a + b/x)^2/x^(5/2),x]
```

output
```
(-2*b^2)/(7*x^(7/2)) - (4*a*b)/(5*x^(5/2)) - (2*a^2)/(3*x^(3/2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{2\left(35a^{2}x^{2}+42abx+15b^{2}\right)}{105x^{\frac{7}{2}}}$	25
derivativedivides	$-\frac{2b^{2}}{7x^{\frac{7}{2}}}-\frac{4ab}{5x^{\frac{5}{2}}}-\frac{2a^{2}}{3x^{\frac{3}{2}}}$	25
default	$-\frac{2b^{2}}{7x^{\frac{7}{2}}}-\frac{4ab}{5x^{\frac{5}{2}}}-\frac{2a^{2}}{3x^{\frac{3}{2}}}$	25
trager	$-\frac{2\left(35a^{2}x^{2}+42abx+15b^{2}\right)}{105x^{\frac{7}{2}}}$	25
risch	$-\frac{2\left(35a^{2}x^{2}+42abx+15b^{2}\right)}{105x^{\frac{7}{2}}}$	25
orering	$-\frac{2\left(35a^{2}x^{2}+42abx+15b^{2}\right)\left(a+\frac{b}{x}\right)^{2}}{105x^{\frac{3}{2}}\left(ax+b\right)^{2}}$	41

input
```
int((a+b/x)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(35*a^2*x^2+42*a*b*x+15*b^2)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx=-\frac{2\,\left(35\,a^{2}x^{2}+42\,abx+15\,b^{2}\right)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)^2/x^(5/2),x, algorithm="fricas")
```

output
```
-2/105*(35*a^2*x^2 + 42*a*b*x + 15*b^2)/x^(7/2)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx = -\frac{2a^2}{3x^{\frac{3}{2}}} - \frac{4ab}{5x^{\frac{5}{2}}} - \frac{2b^2}{7x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)**2/x**(5/2),x)
```

output
```
-2*a**2/(3*x**(3/2)) - 4*a*b/(5*x**(5/2)) - 2*b**2/(7*x**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx = -\frac{2\,a^2}{3\,x^{\frac{3}{2}}} - \frac{4\,ab}{5\,x^{\frac{5}{2}}} - \frac{2\,b^2}{7\,x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)^2/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*a^2/x^(3/2) - 4/5*a*b/x^(5/2) - 2/7*b^2/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx = -\frac{2\,(35\,a^2x^2 + 42\,abx + 15\,b^2)}{105\,x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)^2/x^(5/2),x, algorithm="giac")
```

output
```
-2/105*(35*a^2*x^2 + 42*a*b*x + 15*b^2)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx=-\frac{70\,a^2\,x^2+84\,a\,b\,x+30\,b^2}{105\,x^{7/2}}$$

input
```
int((a + b/x)^2/x^(5/2),x)
```

output
```
-(30*b^2 + 70*a^2*x^2 + 84*a*b*x)/(105*x^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^2}{x^{5/2}}\,dx=\frac{-\frac{2}{3}a^2x^2-\frac{4}{5}abx-\frac{2}{7}b^2}{\sqrt{x}\,x^3}$$

input
```
int((a+b/x)^2/x^(5/2),x)
```

output
```
(2*( - 35*a**2*x**2 - 42*a*b*x - 15*b**2))/(105*sqrt(x)*x**3)
```

3.111 $\int \left(a+\frac{b}{x}\right)^3 x^{5/2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+\frac{b}{x}\right)^3 x^{5/2} \, dx = 2b^3\sqrt{x}+2ab^2x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{7}a^3x^{7/2}$$

output
```
2*b^3*x^(1/2)+2*a*b^2*x^(3/2)+6/5*a^2*b*x^(5/2)+2/7*a^3*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x}\right)^3 x^{5/2} \, dx = \frac{2}{35}\sqrt{x}\left(35b^3+35ab^2x+21a^2bx^2+5a^3x^3\right)$$

input
```
Integrate[(a + b/x)^3*x^(5/2),x]
```

output
```
(2*Sqrt[x]*(35*b^3 + 35*a*b^2*x + 21*a^2*b*x^2 + 5*a^3*x^3))/35
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^3}{\sqrt{x}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^3x^{5/2}+3a^2bx^{3/2}+3ab^2\sqrt{x}+\frac{b^3}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{5}a^2bx^{5/2}+2ab^2x^{3/2}+2b^3\sqrt{x}$$

input
```
Int[(a + b/x)^3*x^(5/2),x]
```

output
```
2*b^3*Sqrt[x] + 2*a*b^2*x^(3/2) + (6*a^2*b*x^(5/2))/5 + (2*a^3*x^(7/2))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
trager	$\left(\frac{2}{7}a^{3}x^{3}+\frac{6}{5}a^{2}b\,x^{2}+2a\,b^{2}x+2b^{3}\right)\sqrt{x}$	35
gosper	$\frac{2\left(5a^{3}x^{3}+21a^{2}b\,x^{2}+35a\,b^{2}x+35b^{3}\right)\sqrt{x}}{35}$	36
derivativedivides	$2b^{3}\sqrt{x}+2a\,b^{2}x^{\frac{3}{2}}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{2a^{3}x^{\frac{7}{2}}}{7}$	36
default	$2b^{3}\sqrt{x}+2a\,b^{2}x^{\frac{3}{2}}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{2a^{3}x^{\frac{7}{2}}}{7}$	36
risch	$\frac{2\left(5a^{3}x^{3}+21a^{2}b\,x^{2}+35a\,b^{2}x+35b^{3}\right)\sqrt{x}}{35}$	36
orering	$\frac{2\left(5a^{3}x^{3}+21a^{2}b\,x^{2}+35a\,b^{2}x+35b^{3}\right)x^{\frac{7}{2}}\left(a+\frac{b}{x}\right)^{3}}{35\left(ax+b\right)^{3}}$	52

input
```
int((a+b/x)^3*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7*a^3*x^3+6/5*a^2*b*x^2+2*a*b^2*x+2*b^3)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int\left(a+\frac{b}{x}\right)^{3}x^{5/2}\,dx=\frac{2}{35}\left(5\,a^{3}x^{3}+21\,a^{2}bx^{2}+35\,ab^{2}x+35\,b^{3}\right)\sqrt{x}$$

input
```
integrate((a+b/x)^3*x^(5/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^3*x^3 + 21*a^2*b*x^2 + 35*a*b^2*x + 35*b^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx=\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2bx^{\frac{5}{2}}}{5}+2ab^2x^{\frac{3}{2}}+2b^3\sqrt{x}$$

input
```
integrate((a+b/x)**3*x**(5/2),x)
```

output
```
2*a**3*x**(7/2)/7 + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(3/2) + 2*b**3*sqrt(
x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx=\frac{2}{35}\left(5\,a^3+\frac{21\,a^2b}{x}+\frac{35\,ab^2}{x^2}+\frac{35\,b^3}{x^3}\right)x^{\frac{7}{2}}$$

input
```
integrate((a+b/x)^3*x^(5/2),x, algorithm="maxima")
```

output
```
2/35*(5*a^3 + 21*a^2*b/x + 35*a*b^2/x^2 + 35*b^3/x^3)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx=\frac{2}{7}\,a^3x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+2\,ab^2x^{\frac{3}{2}}+2\,b^3\sqrt{x}$$

input
```
integrate((a+b/x)^3*x^(5/2),x, algorithm="giac")
```

output
```
2/7*a^3*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2*a*b^2*x^(3/2) + 2*b^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx = \frac{2\,a^3\,x^{7/2}}{7} + 2\,b^3\,\sqrt{x} + 2\,a\,b^2\,x^{3/2} + \frac{6\,a^2\,b\,x^{5/2}}{5}$$

input
```
int(x^(5/2)*(a + b/x)^3,x)
```

output
```
(2*a^3*x^(7/2))/7 + 2*b^3*x^(1/2) + 2*a*b^2*x^(3/2) + (6*a^2*b*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \left(a+\frac{b}{x}\right)^3 x^{5/2}\,dx = \frac{2\sqrt{x}\,(5a^3x^3 + 21a^2b\,x^2 + 35a\,b^2x + 35b^3)}{35}$$

input
```
int((a+b/x)^3*x^(5/2),x)
```

output
```
(2*sqrt(x)*(5*a**3*x**3 + 21*a**2*b*x**2 + 35*a*b**2*x + 35*b**3))/35
```

3.112 $\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx=-\frac{2b^3}{\sqrt{x}}+6ab^2\sqrt{x}+2a^2bx^{3/2}+\frac{2}{5}a^3x^{5/2}$$

output
```
-2*b^3/x^(1/2)+6*a*b^2*x^(1/2)+2*a^2*b*x^(3/2)+2/5*a^3*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx=-\frac{2(5b^3-15ab^2x-5a^2bx^2-a^3x^3)}{5\sqrt{x}}$$

input
```
Integrate[(a + b/x)^3*x^(3/2),x]
```

output
```
(-2*(5*b^3 - 15*a*b^2*x - 5*a^2*b*x^2 - a^3*x^3))/(5*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^3}{x^{3/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^3x^{3/2}+3a^2b\sqrt{x}+\frac{3ab^2}{\sqrt{x}}+\frac{b^3}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^3x^{5/2}+2a^2bx^{3/2}+6ab^2\sqrt{x}-\frac{2b^3}{\sqrt{x}}$$

input
```
Int[(a + b/x)^3*x^(3/2),x]
```

output
```
(-2*b^3)/Sqrt[x] + 6*a*b^2*Sqrt[x] + 2*a^2*b*x^(3/2) + (2*a^3*x^(5/2))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{-2b^{3}+\frac{2}{5}a^{3}x^{3}+2a^{2}b\,x^{2}+6a\,b^{2}x}{\sqrt{x}}$	35
trager	$\frac{-2b^{3}+\frac{2}{5}a^{3}x^{3}+2a^{2}b\,x^{2}+6a\,b^{2}x}{\sqrt{x}}$	35
risch	$\frac{-2b^{3}+\frac{2}{5}a^{3}x^{3}+2a^{2}b\,x^{2}+6a\,b^{2}x}{\sqrt{x}}$	35
derivativedivides	$-\frac{2b^{3}}{\sqrt{x}}+6a\,b^{2}\sqrt{x}+2a^{2}b\,x^{\frac{3}{2}}+\frac{2a^{3}x^{\frac{5}{2}}}{5}$	36
default	$-\frac{2b^{3}}{\sqrt{x}}+6a\,b^{2}\sqrt{x}+2a^{2}b\,x^{\frac{3}{2}}+\frac{2a^{3}x^{\frac{5}{2}}}{5}$	36
orering	$\frac{2\left(a^{3}x^{3}+5a^{2}b\,x^{2}+15a\,b^{2}x-5b^{3}\right)x^{\frac{5}{2}}\left(a+\frac{b}{x}\right)^{3}}{5\left(ax+b\right)^{3}}$	51

input
```
int((a+b/x)^3*x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a^3*x^3+5*a^2*b*x^2+15*a*b^2*x-5*b^3)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)^{3}x^{3/2}\,dx=\frac{2\left(a^{3}x^{3}+5\,a^{2}bx^{2}+15\,ab^{2}x-5\,b^{3}\right)}{5\,\sqrt{x}}$$

input
```
integrate((a+b/x)^3*x^(3/2),x, algorithm="fricas")
```

output
```
2/5*(a^3*x^3 + 5*a^2*b*x^2 + 15*a*b^2*x - 5*b^3)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx=\frac{2a^3x^{\frac{5}{2}}}{5}+2a^2bx^{\frac{3}{2}}+6ab^2\sqrt{x}-\frac{2b^3}{\sqrt{x}}$$

input
```
integrate((a+b/x)**3*x**(3/2),x)
```

output
```
2*a**3*x**(5/2)/5 + 2*a**2*b*x**(3/2) + 6*a*b**2*sqrt(x) - 2*b**3/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx=\frac{2}{5}\left(a^3+\frac{5\,a^2b}{x}+\frac{15\,ab^2}{x^2}\right)x^{\frac{5}{2}}-\frac{2\,b^3}{\sqrt{x}}$$

input
```
integrate((a+b/x)^3*x^(3/2),x, algorithm="maxima")
```

output
```
2/5*(a^3 + 5*a^2*b/x + 15*a*b^2/x^2)*x^(5/2) - 2*b^3/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx=\frac{2}{5}\,a^3x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+6\,ab^2\sqrt{x}-\frac{2\,b^3}{\sqrt{x}}$$

input
```
integrate((a+b/x)^3*x^(3/2),x, algorithm="giac")
```

output
```
2/5*a^3*x^(5/2) + 2*a^2*b*x^(3/2) + 6*a*b^2*sqrt(x) - 2*b^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx=\frac{2\,a^3\,x^{5/2}}{5}-\frac{2\,b^3}{\sqrt{x}}+6\,a\,b^2\,\sqrt{x}+2\,a^2\,b\,x^{3/2}$$

input
```
int(x^(3/2)*(a + b/x)^3,x)
```

output
```
(2*a^3*x^(5/2))/5 - (2*b^3)/x^(1/2) + 6*a*b^2*x^(1/2) + 2*a^2*b*x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x}\right)^3 x^{3/2}\,dx=\frac{\frac{2}{5}a^3x^3+2a^2b\,x^2+6a\,b^2x-2b^3}{\sqrt{x}}$$

input
```
int((a+b/x)^3*x^(3/2),x)
```

output
```
(2*(a**3*x**3 + 5*a**2*b*x**2 + 15*a*b**2*x - 5*b**3))/(5*sqrt(x))
```

3.113 $\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\,dx=-\frac{2b^3}{3x^{3/2}}-\frac{6ab^2}{\sqrt{x}}+6a^2b\sqrt{x}+\frac{2}{3}a^3x^{3/2}$$

output
```
-2/3*b^3/x^(3/2)-6*a*b^2/x^(1/2)+6*a^2*b*x^(1/2)+2/3*a^3*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\,dx=\frac{2(-b^3-9ab^2x+9a^2bx^2+a^3x^3)}{3x^{3/2}}$$

input
```
Integrate[(a + b/x)^3*Sqrt[x],x]
```

output
```
(2*(-b^3 - 9*a*b^2*x + 9*a^2*b*x^2 + a^3*x^3))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+\frac{b}{x}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^3}{x^{5/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^3\sqrt{x}+\frac{3a^2b}{\sqrt{x}}+\frac{3ab^2}{x^{3/2}}+\frac{b^3}{x^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3x^{3/2}+6a^2b\sqrt{x}-\frac{6ab^2}{\sqrt{x}}-\frac{2b^3}{3x^{3/2}}$$

input
```
Int[(a + b/x)^3*Sqrt[x],x]
```

output
```
(-2*b^3)/(3*x^(3/2)) - (6*a*b^2)/Sqrt[x] + 6*a^2*b*Sqrt[x] + (2*a^3*x^(3/2
))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{-6 a \,b^{2} x -\frac{2}{3} b^{3}+\frac{2}{3} a^{3} x^{3}+6 a^{2} b \,x^{2}}{x^{\frac{3}{2}}}$	35
trager	$\frac{-6 a \,b^{2} x -\frac{2}{3} b^{3}+\frac{2}{3} a^{3} x^{3}+6 a^{2} b \,x^{2}}{x^{\frac{3}{2}}}$	35
risch	$\frac{-6 a \,b^{2} x -\frac{2}{3} b^{3}+\frac{2}{3} a^{3} x^{3}+6 a^{2} b \,x^{2}}{x^{\frac{3}{2}}}$	35
derivativedivides	$-\frac{2 b^{3}}{3 x^{\frac{3}{2}}}-\frac{6 a \,b^{2}}{\sqrt{x}}+6 a^{2} b \sqrt{x}+\frac{2 a^{3} x^{\frac{3}{2}}}{3}$	36
default	$-\frac{2 b^{3}}{3 x^{\frac{3}{2}}}-\frac{6 a \,b^{2}}{\sqrt{x}}+6 a^{2} b \sqrt{x}+\frac{2 a^{3} x^{\frac{3}{2}}}{3}$	36
orering	$\frac{2 \left(a^{3} x^{3}+9 a^{2} b \,x^{2}-9 a \,b^{2} x -b^{3}\right) x^{\frac{3}{2}} \left(a +\frac{b}{x}\right)^{3}}{3 \left(a x +b \right)^{3}}$	51

input
```
int((a+b/x)^3*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a^3*x^3+9*a^2*b*x^2-9*a*b^2*x-b^3)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\, dx = \frac{2 \,{\left(a^{3} x^{3} + 9 \, a^{2} b x^{2} - 9 \, a b^{2} x - b^{3}\right)}}{3 \, x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)^3*x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^3*x^3 + 9*a^2*b*x^2 - 9*a*b^2*x - b^3)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^3\sqrt{x}\,dx=\frac{2a^3x^{\frac{3}{2}}}{3}+6a^2b\sqrt{x}-\frac{6ab^2}{\sqrt{x}}-\frac{2b^3}{3x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)**3*x**(1/2),x)
```

output
```
2*a**3*x**(3/2)/3 + 6*a**2*b*sqrt(x) - 6*a*b**2/sqrt(x) - 2*b**3/(3*x**(3/
2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{b}{x}\right)^3\sqrt{x}\,dx=-\frac{6\,ab^2}{\sqrt{x}}+\frac{2}{3}\left(a^3+\frac{9\,a^2b}{x}\right)x^{\frac{3}{2}}-\frac{2\,b^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)^3*x^(1/2),x, algorithm="maxima")
```

output
```
-6*a*b^2/sqrt(x) + 2/3*(a^3 + 9*a^2*b/x)*x^(3/2) - 2/3*b^3/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \left(a+\frac{b}{x}\right)^3\sqrt{x}\,dx=\frac{2}{3}\,a^3x^{\frac{3}{2}}+6\,a^2b\sqrt{x}-\frac{2\,(9\,ab^2x+b^3)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)^3*x^(1/2),x, algorithm="giac")
```

output
```
2/3*a^3*x^(3/2) + 6*a^2*b*sqrt(x) - 2/3*(9*a*b^2*x + b^3)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\,dx = -\frac{-2\,a^3\,x^3-18\,a^2\,b\,x^2+18\,a\,b^2\,x+2\,b^3}{3\,x^{3/2}}$$

input
```
int(x^(1/2)*(a + b/x)^3,x)
```

output
```
-(2*b^3 - 2*a^3*x^3 - 18*a^2*b*x^2 + 18*a*b^2*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^3 \sqrt{x}\,dx = \frac{\frac{2}{3}a^3x^3+6a^2b\,x^2-6a\,b^2x-\frac{2}{3}b^3}{\sqrt{x}\,x}$$

input
```
int((a+b/x)^3*x^(1/2),x)
```

output
```
(2*(a**3*x**3 + 9*a**2*b*x**2 - 9*a*b**2*x - b**3))/(3*sqrt(x)*x)
```

3.114 $\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}} \, dx = -\frac{2b^3}{5x^{5/2}} - \frac{2ab^2}{x^{3/2}} - \frac{6a^2b}{\sqrt{x}} + 2a^3\sqrt{x}$$

output
```
-2/5*b^3/x^(5/2)-2*a*b^2/x^(3/2)-6*a^2*b/x^(1/2)+2*a^3*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}} \, dx = \frac{2(-b^3 - 5ab^2x - 15a^2bx^2 + 5a^3x^3)}{5x^{5/2}}$$

input
```
Integrate[(a + b/x)^3/Sqrt[x],x]
```

output
```
(2*(-b^3 - 5*a*b^2*x - 15*a^2*b*x^2 + 5*a^3*x^3))/(5*x^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax+b)^3}{x^{7/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{a^3}{\sqrt{x}}+\frac{3a^2b}{x^{3/2}}+\frac{3ab^2}{x^{5/2}}+\frac{b^3}{x^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^3\sqrt{x}-\frac{6a^2b}{\sqrt{x}}-\frac{2ab^2}{x^{3/2}}-\frac{2b^3}{5x^{5/2}}$$

input
```
Int[(a + b/x)^3/Sqrt[x],x]
```

output
```
(-2*b^3)/(5*x^(5/2)) - (2*a*b^2)/x^(3/2) - (6*a^2*b)/Sqrt[x] + 2*a^3*Sqrt[
x]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{-6a^2b\,x^2-\frac{2}{5}b^3-2a\,b^2x+2a^3x^3}{x^{\frac{5}{2}}}$	36
derivativedivides	$-\frac{2b^3}{5x^{\frac{5}{2}}}-\frac{2a\,b^2}{x^{\frac{3}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a^3\sqrt{x}$	36
default	$-\frac{2b^3}{5x^{\frac{5}{2}}}-\frac{2a\,b^2}{x^{\frac{3}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a^3\sqrt{x}$	36
trager	$\frac{-6a^2b\,x^2-\frac{2}{5}b^3-2a\,b^2x+2a^3x^3}{x^{\frac{5}{2}}}$	36
risch	$\frac{-6a^2b\,x^2-\frac{2}{5}b^3-2a\,b^2x+2a^3x^3}{x^{\frac{5}{2}}}$	36
orering	$\frac{2(5a^3x^3-15a^2b\,x^2-5a\,b^2x-b^3)\sqrt{x}\left(a+\frac{b}{x}\right)^3}{5(ax+b)^3}$	52

input
```
int((a+b/x)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(5*a^3*x^3-15*a^2*b*x^2-5*a*b^2*x-b^3)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx = \frac{2\,(5\,a^3x^3-15\,a^2bx^2-5\,ab^2x-b^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/5*(5*a^3*x^3 - 15*a^2*b*x^2 - 5*a*b^2*x - b^3)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x} - \frac{6a^2b}{\sqrt{x}} - \frac{2ab^2}{x^{\frac{3}{2}}} - \frac{2b^3}{5x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)**3/x**(1/2),x)
```

output
```
2*a**3*sqrt(x) - 6*a**2*b/sqrt(x) - 2*a*b**2/x**(3/2) - 2*b**3/(5*x**(5/2)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx = 2\,a^3\sqrt{x} - \frac{6\,a^2b}{\sqrt{x}} - \frac{2\,ab^2}{x^{\frac{3}{2}}} - \frac{2\,b^3}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2*a^3*sqrt(x) - 6*a^2*b/sqrt(x) - 2*a*b^2/x^(3/2) - 2/5*b^3/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx = 2\,a^3\sqrt{x} - \frac{2\,(15\,a^2bx^2 + 5\,ab^2x + b^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)^3/x^(1/2),x, algorithm="giac")
```

output
```
2*a^3*sqrt(x) - 2/5*(15*a^2*b*x^2 + 5*a*b^2*x + b^3)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx=-\frac{-10\,a^3\,x^3+30\,a^2\,b\,x^2+10\,a\,b^2\,x+2\,b^3}{5\,x^{5/2}}$$

input
```
int((a + b/x)^3/x^(1/2),x)
```

output
```
-(2*b^3 - 10*a^3*x^3 + 30*a^2*b*x^2 + 10*a*b^2*x)/(5*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{\sqrt{x}}\,dx=\frac{2a^3x^3-6a^2b\,x^2-2a\,b^2x-\frac{2}{5}b^3}{\sqrt{x}\,x^2}$$

input
```
int((a+b/x)^3/x^(1/2),x)
```

output
```
(2*(5*a**3*x**3 - 15*a**2*b*x**2 - 5*a*b**2*x - b**3))/(5*sqrt(x)*x**2)
```

3.115 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}} \, dx = -\frac{2b^3}{7x^{7/2}}-\frac{6ab^2}{5x^{5/2}}-\frac{2a^2b}{x^{3/2}}-\frac{2a^3}{\sqrt{x}}$$

output
```
-2/7*b^3/x^(7/2)-6/5*a*b^2/x^(5/2)-2*a^2*b/x^(3/2)-2*a^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}} \, dx = -\frac{2(5b^3+21ab^2x+35a^2bx^2+35a^3x^3)}{35x^{7/2}}$$

input
```
Integrate[(a + b/x)^3/x^(3/2),x]
```

output
```
(-2*(5*b^3 + 21*a*b^2*x + 35*a^2*b*x^2 + 35*a^3*x^3))/(35*x^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{(ax+b)^3}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^3}{x^{3/2}}+\frac{3a^2b}{x^{5/2}}+\frac{3ab^2}{x^{7/2}}+\frac{b^3}{x^{9/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3}{\sqrt{x}}-\frac{2a^2b}{x^{3/2}}-\frac{6ab^2}{5x^{5/2}}-\frac{2b^3}{7x^{7/2}}$$

input
```
Int[(a + b/x)^3/x^(3/2),x]
```

output
```
(-2*b^3)/(7*x^(7/2)) - (6*a*b^2)/(5*x^(5/2)) - (2*a^2*b)/x^(3/2) - (2*a^3)
/Sqrt[x]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(35a^3x^3+35a^2b\,x^2+21a\,b^2x+5b^3)}{35x^{\frac{7}{2}}}$	36
derivativedivides	$-\frac{2b^3}{7x^{\frac{7}{2}}}-\frac{6a\,b^2}{5x^{\frac{5}{2}}}-\frac{2a^2b}{x^{\frac{3}{2}}}-\frac{2a^3}{\sqrt{x}}$	36
default	$-\frac{2b^3}{7x^{\frac{7}{2}}}-\frac{6a\,b^2}{5x^{\frac{5}{2}}}-\frac{2a^2b}{x^{\frac{3}{2}}}-\frac{2a^3}{\sqrt{x}}$	36
trager	$-\frac{2(35a^3x^3+35a^2b\,x^2+21a\,b^2x+5b^3)}{35x^{\frac{7}{2}}}$	36
risch	$-\frac{2(35a^3x^3+35a^2b\,x^2+21a\,b^2x+5b^3)}{35x^{\frac{7}{2}}}$	36
orering	$-\frac{2(35a^3x^3+35a^2b\,x^2+21a\,b^2x+5b^3)\left(a+\frac{b}{x}\right)^3}{35\sqrt{x}\,(ax+b)^3}$	52

input
```
int((a+b/x)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(35*a^3*x^3+35*a^2*b*x^2+21*a*b^2*x+5*b^3)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx=-\frac{2\left(35\,a^3x^3+35\,a^2bx^2+21\,ab^2x+5\,b^3\right)}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)^3/x^(3/2),x, algorithm="fricas")
```

output
```
-2/35*(35*a^3*x^3 + 35*a^2*b*x^2 + 21*a*b^2*x + 5*b^3)/x^(7/2)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx=-\frac{2a^3}{\sqrt{x}}-\frac{2a^2b}{x^{\frac{3}{2}}}-\frac{6ab^2}{5x^{\frac{5}{2}}}-\frac{2b^3}{7x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)**3/x**(3/2),x)
```

output
```
-2*a**3/sqrt(x) - 2*a**2*b/x**(3/2) - 6*a*b**2/(5*x**(5/2)) - 2*b**3/(7*x**
*(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx=-\frac{2\,a^3}{\sqrt{x}}-\frac{2\,a^2b}{x^{\frac{3}{2}}}-\frac{6\,ab^2}{5\,x^{\frac{5}{2}}}-\frac{2\,b^3}{7\,x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)^3/x^(3/2),x, algorithm="maxima")
```

output
```
-2*a^3/sqrt(x) - 2*a^2*b/x^(3/2) - 6/5*a*b^2/x^(5/2) - 2/7*b^3/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx=-\frac{2\left(35\,a^3x^3+35\,a^2bx^2+21\,ab^2x+5\,b^3\right)}{35\,x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)^3/x^(3/2),x, algorithm="giac")
```

output
```
-2/35*(35*a^3*x^3 + 35*a^2*b*x^2 + 21*a*b^2*x + 5*b^3)/x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx=-\frac{70\,a^3\,x^3+70\,a^2\,b\,x^2+42\,a\,b^2\,x+10\,b^3}{35\,x^{7/2}}$$

input
```
int((a + b/x)^3/x^(3/2),x)
```

output
```
-(10*b^3 + 70*a^3*x^3 + 70*a^2*b*x^2 + 42*a*b^2*x)/(35*x^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{3/2}}\,dx=\frac{-2a^3x^3-2a^2b\,x^2-\frac{6}{5}a\,b^2x-\frac{2}{7}b^3}{\sqrt{x}\,x^3}$$

input
```
int((a+b/x)^3/x^(3/2),x)
```

output
```
(2*( - 35*a**3*x**3 - 35*a**2*b*x**2 - 21*a*b**2*x - 5*b**3))/(35*sqrt(x)*
x**3)
```

3.116 $\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}} \, dx = -\frac{2b^3}{9x^{9/2}}-\frac{6ab^2}{7x^{7/2}}-\frac{6a^2b}{5x^{5/2}}-\frac{2a^3}{3x^{3/2}}$$

output
```
-2/9*b^3/x^(9/2)-6/7*a*b^2/x^(7/2)-6/5*a^2*b/x^(5/2)-2/3*a^3/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}} \, dx = -\frac{2(35b^3+135ab^2x+189a^2bx^2+105a^3x^3)}{315x^{9/2}}$$

input
```
Integrate[(a + b/x)^3/x^(5/2),x]
```

output
```
(-2*(35*b^3 + 135*a*b^2*x + 189*a^2*b*x^2 + 105*a^3*x^3))/(315*x^(9/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {795, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{(ax+b)^3}{x^{11/2}}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{x^{7/2}}+\frac{3ab^2}{x^{9/2}}+\frac{b^3}{x^{11/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{5x^{5/2}}-\frac{6ab^2}{7x^{7/2}}-\frac{2b^3}{9x^{9/2}}$$

input
```
Int[(a + b/x)^3/x^(5/2),x]
```

output
```
(-2*b^3)/(9*x^(9/2)) - (6*a*b^2)/(7*x^(7/2)) - (6*a^2*b)/(5*x^(5/2)) - (2*
a^3)/(3*x^(3/2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{2\left(105a^{3}x^{3}+189a^{2}b\,x^{2}+135a\,b^{2}x+35b^{3}\right)}{315x^{\frac{9}{2}}}$	36
derivativedivides	$-\frac{2b^{3}}{9x^{\frac{9}{2}}}-\frac{6a\,b^{2}}{7x^{\frac{7}{2}}}-\frac{6a^{2}b}{5x^{\frac{5}{2}}}-\frac{2a^{3}}{3x^{\frac{3}{2}}}$	36
default	$-\frac{2b^{3}}{9x^{\frac{9}{2}}}-\frac{6a\,b^{2}}{7x^{\frac{7}{2}}}-\frac{6a^{2}b}{5x^{\frac{5}{2}}}-\frac{2a^{3}}{3x^{\frac{3}{2}}}$	36
trager	$-\frac{2\left(105a^{3}x^{3}+189a^{2}b\,x^{2}+135a\,b^{2}x+35b^{3}\right)}{315x^{\frac{9}{2}}}$	36
risch	$-\frac{2\left(105a^{3}x^{3}+189a^{2}b\,x^{2}+135a\,b^{2}x+35b^{3}\right)}{315x^{\frac{9}{2}}}$	36
orering	$-\frac{2\left(105a^{3}x^{3}+189a^{2}b\,x^{2}+135a\,b^{2}x+35b^{3}\right)\left(a+\frac{b}{x}\right)^{3}}{315x^{\frac{3}{2}}\left(ax+b\right)^{3}}$	52

input
```
int((a+b/x)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(105*a^3*x^3+189*a^2*b*x^2+135*a*b^2*x+35*b^3)/x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+\frac{b}{x}\right)^{3}}{x^{5/2}}\,dx=-\frac{2\,\left(105\,a^{3}x^{3}+189\,a^{2}bx^{2}+135\,ab^{2}x+35\,b^{3}\right)}{315\,x^{\frac{9}{2}}}$$

input
```
integrate((a+b/x)^3/x^(5/2),x, algorithm="fricas")
```

output
```
-2/315*(105*a^3*x^3 + 189*a^2*b*x^2 + 135*a*b^2*x + 35*b^3)/x^(9/2)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx = -\frac{2a^3}{3x^{\frac{3}{2}}} - \frac{6a^2b}{5x^{\frac{5}{2}}} - \frac{6ab^2}{7x^{\frac{7}{2}}} - \frac{2b^3}{9x^{\frac{9}{2}}}$$

input
```
integrate((a+b/x)**3/x**(5/2),x)
```

output
```
-2*a**3/(3*x**(3/2)) - 6*a**2*b/(5*x**(5/2)) - 6*a*b**2/(7*x**(7/2)) - 2*b
**3/(9*x**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx = -\frac{2\,a^3}{3\,x^{\frac{3}{2}}} - \frac{6\,a^2b}{5\,x^{\frac{5}{2}}} - \frac{6\,ab^2}{7\,x^{\frac{7}{2}}} - \frac{2\,b^3}{9\,x^{\frac{9}{2}}}$$

input
```
integrate((a+b/x)^3/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*a^3/x^(3/2) - 6/5*a^2*b/x^(5/2) - 6/7*a*b^2/x^(7/2) - 2/9*b^3/x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx = -\frac{2\,(105\,a^3x^3 + 189\,a^2bx^2 + 135\,ab^2x + 35\,b^3)}{315\,x^{\frac{9}{2}}}$$

input
```
integrate((a+b/x)^3/x^(5/2),x, algorithm="giac")
```

output
```
-2/315*(105*a^3*x^3 + 189*a^2*b*x^2 + 135*a*b^2*x + 35*b^3)/x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx = -\frac{210\,a^3\,x^3+378\,a^2\,b\,x^2+270\,a\,b^2\,x+70\,b^3}{315\,x^{9/2}}$$

input
```
int((a + b/x)^3/x^(5/2),x)
```

output
```
-(70*b^3 + 210*a^3*x^3 + 378*a^2*b*x^2 + 270*a*b^2*x)/(315*x^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{x}\right)^3}{x^{5/2}}\,dx = \frac{-\frac{2}{3}a^3x^3-\frac{6}{5}a^2b\,x^2-\frac{6}{7}a\,b^2x-\frac{2}{9}b^3}{\sqrt{x}\,x^4}$$

input
```
int((a+b/x)^3/x^(5/2),x)
```

output
```
(2*( - 105*a**3*x**3 - 189*a**2*b*x**2 - 135*a*b**2*x - 35*b**3))/(315*sqr
t(x)*x**4)
```

3.117 $\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx=-\frac{2b^3\sqrt{x}}{a^4}+\frac{2b^2x^{3/2}}{3a^3}-\frac{2bx^{5/2}}{5a^2}+\frac{2x^{7/2}}{7a}+\frac{2b^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

output
```
-2*b^3*x^(1/2)/a^4+2/3*b^2*x^(3/2)/a^3-2/5*b*x^(5/2)/a^2+2/7*x^(7/2)/a+2*b
^(7/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx=\frac{2\sqrt{x}(-105b^3+35ab^2x-21a^2bx^2+15a^3x^3)}{105a^4}+\frac{2b^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

input
```
Integrate[x^(5/2)/(a + b/x),x]
```

output
```
(2*Sqrt[x]*(-105*b^3 + 35*a*b^2*x - 21*a^2*b*x^2 + 15*a^3*x^3))/(105*a^4)
+ (2*b^(7/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(9/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {795, 60, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{7/2}}{ax+b}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/2}}{7a}-\frac{b\int \frac{x^{5/2}}{b+ax}dx}{a}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\int \frac{x^{3/2}}{b+ax}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\int \frac{\sqrt{x}}{b+ax}dx}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int\frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}\right)}{a}\right)}{a}$$

input
```
Int[x^(5/2)/(a + b/x),x]
```

output
```
(2*x^(7/2))/(7*a) - (b*((2*x^(5/2))/(5*a) - (b*((2*x^(3/2))/(3*a) - (b*((2
*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)))/a))/
a))/a
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2\left(15 a^{3} x^{3}-21 a^{2} b \,x^{2}+35 a \,b^{2} x -105 b^{3}\right) \sqrt{x}}{105 a^{4}}+\frac{2 b^{4} \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{a^{4} \sqrt{a b}}$	64
derivativedivides	$\frac{\frac{2 a^{3} x^{\frac{7}{2}}}{7}-\frac{2 a^{2} b \,x^{\frac{5}{2}}}{5}+\frac{2 a \,b^{2} x^{\frac{3}{2}}}{3}-2 b^{3} \sqrt{x}}{a^{4}}+\frac{2 b^{4} \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{a^{4} \sqrt{a b}}$	66
default	$\frac{\frac{2 a^{3} x^{\frac{7}{2}}}{7}-\frac{2 a^{2} b \,x^{\frac{5}{2}}}{5}+\frac{2 a \,b^{2} x^{\frac{3}{2}}}{3}-2 b^{3} \sqrt{x}}{a^{4}}+\frac{2 b^{4} \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{a^{4} \sqrt{a b}}$	66

input
```
int(x^(5/2)/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*a^3*x^3-21*a^2*b*x^2+35*a*b^2*x-105*b^3)*x^(1/2)/a^4+2*b^4/a^4/(
a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.84

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx = \left[\frac{105\,b^3\sqrt{-\frac{b}{a}}\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(15\,a^3x^3-21\,a^2bx^2+35\,ab^2x-105\,b^3)\sqrt{x}}{105\,a^4}, \frac{2\left(105\right.}{}\right.$$

input
```
integrate(x^(5/2)/(a+b/x),x, algorithm="fricas")
```

output
```
[1/105*(105*b^3*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b
)) + 2*(15*a^3*x^3 - 21*a^2*b*x^2 + 35*a*b^2*x - 105*b^3)*sqrt(x))/a^4, 2/
105*(105*b^3*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) + (15*a^3*x^3 - 21*a^
2*b*x^2 + 35*a*b^2*x - 105*b^3)*sqrt(x))/a^4]
```

Sympy [A] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx = \begin{cases} \tilde{\infty}x^{\frac{9}{2}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{9}{2}}}{9b} & \text{for } a=0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0 \\ \frac{2x^{\frac{7}{2}}}{7a} - \frac{2bx^{\frac{5}{2}}}{5a^2} + \frac{2b^2x^{\frac{3}{2}}}{3a^3} - \frac{2b^3\sqrt{x}}{a^4} + \frac{b^4\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a^5\sqrt{-\frac{b}{a}}} - \frac{b^4\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a^5\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(5/2)/(a+b/x),x)
```

output
```
Piecewise((zoo*x**(9/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*b), Eq(a, 0)
), (2*x**(7/2)/(7*a), Eq(b, 0)), (2*x**(7/2)/(7*a) - 2*b*x**(5/2)/(5*a**2)
 + 2*b**2*x**(3/2)/(3*a**3) - 2*b**3*sqrt(x)/a**4 + b**4*log(sqrt(x) - sqr
t(-b/a))/(a**5*sqrt(-b/a)) - b**4*log(sqrt(x) + sqrt(-b/a))/(a**5*sqrt(-b/
a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx = \frac{2\left(15\,a^3 - \frac{21\,a^2b}{x} + \frac{35\,ab^2}{x^2} - \frac{105\,b^3}{x^3}\right)x^{\frac{7}{2}}}{105\,a^4} - \frac{2\,b^4\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a^4}$$

input
```
integrate(x^(5/2)/(a+b/x),x, algorithm="maxima")
```

output
```
2/105*(15*a^3 - 21*a^2*b/x + 35*a*b^2/x^2 - 105*b^3/x^3)*x^(7/2)/a^4 - 2*b
^4*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx = \frac{2\,b^4\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{2\left(15\,a^6x^{\frac{7}{2}} - 21\,a^5bx^{\frac{5}{2}} + 35\,a^4b^2x^{\frac{3}{2}} - 105\,a^3b^3\sqrt{x}\right)}{105\,a^7}$$

input
```
integrate(x^(5/2)/(a+b/x),x, algorithm="giac")
```

output
```
2*b^4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/105*(15*a^6*x^(7/2)
- 21*a^5*b*x^(5/2) + 35*a^4*b^2*x^(3/2) - 105*a^3*b^3*sqrt(x))/a^7
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.71

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx = \frac{2\,x^{7/2}}{7\,a} - \frac{2\,b\,x^{5/2}}{5\,a^2} + \frac{2\,b^2\,x^{3/2}}{3\,a^3} - \frac{2\,b^3\,\sqrt{x}}{a^4} + \frac{2\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

input
```
int(x^(5/2)/(a + b/x),x)
```

output
```
(2*x^(7/2))/(7*a) - (2*b*x^(5/2))/(5*a^2) + (2*b^2*x^(3/2))/(3*a^3) - (2*b
^3*x^(1/2))/a^4 + (2*b^(7/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}}{a+\frac{b}{x}}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^3 + \frac{2\sqrt{x}\,a^4x^3}{7} - \frac{2\sqrt{x}\,a^3b\,x^2}{5} + \frac{2\sqrt{x}\,a^2b^2x}{3} - 2\sqrt{x}\,a\,b^3}{a^5}$$

input
```
int(x^(5/2)/(a+b/x),x)
```

output
```
(2*(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**3 + 15*sqrt
(x)*a**4*x**3 - 21*sqrt(x)*a**3*b*x**2 + 35*sqrt(x)*a**2*b**2*x - 105*sqrt
(x)*a*b**3))/(105*a**5)
```

3.118 $\int \frac{x^{3/2}}{a+\frac{b}{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{x^{3/2}}{a+\frac{b}{x}} \, dx = \frac{2b^2\sqrt{x}}{a^3} - \frac{2bx^{3/2}}{3a^2} + \frac{2x^{5/2}}{5a} - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

output
```
2*b^2*x^(1/2)/a^3-2/3*b*x^(3/2)/a^2+2/5*x^(5/2)/a-2*b^(5/2)*arctan(a^(1/2)
*x^(1/2)/b^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^{3/2}}{a+\frac{b}{x}} \, dx = \frac{2\sqrt{x}(15b^2 - 5abx + 3a^2x^2)}{15a^3} - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

input
```
Integrate[x^(3/2)/(a + b/x),x]
```

output
```
(2*Sqrt[x]*(15*b^2 - 5*a*b*x + 3*a^2*x^2))/(15*a^3) - (2*b^(5/2)*ArcTan[(S
qrt[a]*Sqrt[x])/Sqrt[b]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {795, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{5/2}}{ax+b}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}}{5a}-\frac{b\int \frac{x^{3/2}}{b+ax}dx}{a}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\int \frac{\sqrt{x}}{b+ax}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int \frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}\right)}{a}$$

input
```
Int[x^(3/2)/(a + b/x),x]
```

output
```
(2*x^(5/2))/(5*a) - (b*((2*x^(3/2))/(3*a) - (b*((2*Sqrt[x])/a - (2*Sqrt[b]
*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)))/a))/a
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{2(3a^2x^2-5abx+15b^2)\sqrt{x}}{15a^3}-\frac{2b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2a^2x^{\frac{5}{2}}}{5}-\frac{2ab\,x^{\frac{3}{2}}}{3}+2b^2\sqrt{x}}{a^3}-\frac{2b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	54
default	$\frac{\frac{2a^2x^{\frac{5}{2}}}{5}-\frac{2ab\,x^{\frac{3}{2}}}{3}+2b^2\sqrt{x}}{a^3}-\frac{2b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	54

input
```
int(x^(3/2)/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*a^2*x^2-5*a*b*x+15*b^2)*x^(1/2)/a^3-2*b^3/a^3/(a*b)^(1/2)*arctan(a
*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.94

$$\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx = \left[\frac{15\,b^2\sqrt{-\frac{b}{a}}\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(3\,a^2x^2-5\,abx+15\,b^2)\sqrt{x}}{15\,a^3}, -\frac{2\left(15\,b^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)-(3\,a^2x^2-5\,abx+15\,b^2)\sqrt{x}\right)}{15\,a^3}\right]$$

input
```
integrate(x^(3/2)/(a+b/x),x, algorithm="fricas")
```

output
```
[1/15*(15*b^2*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b))
 + 2*(3*a^2*x^2 - 5*a*b*x + 15*b^2)*sqrt(x))/a^3, -2/15*(15*b^2*sqrt(b/a)*
arctan(a*sqrt(x)*sqrt(b/a)/b) - (3*a^2*x^2 - 5*a*b*x + 15*b^2)*sqrt(x))/a^
3]
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.79

$$\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx = \begin{cases} \tilde{\infty} x^{\frac{7}{2}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{for } a=0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for } b=0 \\ \frac{2x^{\frac{5}{2}}}{5a} - \frac{2bx^{\frac{3}{2}}}{3a^2} + \frac{2b^2\sqrt{x}}{a^3} - \frac{b^3 \log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a^4\sqrt{-\frac{b}{a}}} + \frac{b^3 \log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a^4\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(3/2)/(a+b/x),x)
```

output
```
Piecewise((zoo*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*b), Eq(a, 0)
), (2*x**(5/2)/(5*a), Eq(b, 0)), (2*x**(5/2)/(5*a) - 2*b*x**(3/2)/(3*a**2)
 + 2*b**2*sqrt(x)/a**3 - b**3*log(sqrt(x) - sqrt(-b/a))/(a**4*sqrt(-b/a))
+ b**3*log(sqrt(x) + sqrt(-b/a))/(a**4*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx = \frac{2\left(3a^2 - \frac{5ab}{x} + \frac{15b^2}{x^2}\right)x^{\frac{5}{2}}}{15a^3} + \frac{2b^3 \arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a^3}$$

input
```
integrate(x^(3/2)/(a+b/x),x, algorithm="maxima")
```

output
```
2/15*(3*a^2 - 5*a*b/x + 15*b^2/x^2)*x^(5/2)/a^3 + 2*b^3*arctan(b/(sqrt(a*b
)*sqrt(x)))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx = -\frac{2\,b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{2\left(3\,a^4x^{\frac{5}{2}} - 5\,a^3bx^{\frac{3}{2}} + 15\,a^2b^2\sqrt{x}\right)}{15\,a^5}$$

input
```
integrate(x^(3/2)/(a+b/x),x, algorithm="giac")
```

output
```
-2*b^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + 2/15*(3*a^4*x^(5/2) -
 5*a^3*b*x^(3/2) + 15*a^2*b^2*sqrt(x))/a^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx = \frac{2\,x^{5/2}}{5\,a} - \frac{2\,b\,x^{3/2}}{3\,a^2} + \frac{2\,b^2\,\sqrt{x}}{a^3} - \frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

input
```
int(x^(3/2)/(a + b/x),x)
```

output
```
(2*x^(5/2))/(5*a) - (2*b*x^(3/2))/(3*a^2) + (2*b^2*x^(1/2))/a^3 - (2*b^(5/
2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{a+\frac{b}{x}}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^2 + \frac{2\sqrt{x}\,a^3x^2}{5} - \frac{2\sqrt{x}\,a^2bx}{3} + 2\sqrt{x}\,a\,b^2}{a^4}$$

input
```
int(x^(3/2)/(a+b/x),x)
```

output
```
(2*( - 15*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**2 + 3*sqr
t(x)*a**3*x**2 - 5*sqrt(x)*a**2*b*x + 15*sqrt(x)*a*b**2))/(15*a**4)
```

3.119 $\int \frac{\sqrt{x}}{a+\frac{b}{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}} \, dx = -\frac{2b\sqrt{x}}{a^2} + \frac{2x^{3/2}}{3a} + \frac{2b^{3/2} \arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

output
```
-2*b*x^(1/2)/a^2+2/3*x^(3/2)/a+2*b^(3/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a
^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}} \, dx = \frac{2\sqrt{x}(-3b+ax)}{3a^2} + \frac{2b^{3/2} \arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

input
```
Integrate[Sqrt[x]/(a + b/x),x]
```

output
```
(2*Sqrt[x]*(-3*b + a*x))/(3*a^2) + (2*b^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqr
t[b]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {795, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{3/2}}{ax+b}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{3/2}}{3a}-\frac{b\int \frac{\sqrt{x}}{b+ax}dx}{a}$$

$$\Big\downarrow \text{60}$$

$$\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int \frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}$$

input
```
Int[Sqrt[x]/(a + b/x),x]
```

output
```
(2*x^(3/2))/(3*a) - (b*((2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x]
)/Sqrt[b]])/a^(3/2)))/a
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2(ax-3b)\sqrt{x}}{3a^2}+\frac{2b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	41
derivativedivides	$\frac{\frac{2a\,x^{\frac{3}{2}}}{3}-2b\sqrt{x}}{a^2}+\frac{2b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	44
default	$\frac{\frac{2a\,x^{\frac{3}{2}}}{3}-2b\sqrt{x}}{a^2}+\frac{2b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	44

input
```
int(x^(1/2)/(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x-3*b)*x^(1/2)/a^2+2*b^2/a^2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx = \left[\frac{3\,b\sqrt{-\frac{b}{a}}\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(ax-3\,b)\sqrt{x}}{3\,a^2}, \frac{2\left(3\,b\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)+(ax-3\,b)\sqrt{x}\right)}{3\,a^2}\right]$$

input
```
integrate(x^(1/2)/(a+b/x),x, algorithm="fricas")
```

output
```
[1/3*(3*b*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b)) + 2
*(a*x - 3*b)*sqrt(x))/a^2, 2/3*(3*b*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b
) + (a*x - 3*b)*sqrt(x))/a^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(49) = 98$.

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx = \begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for } a=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} - \frac{2b\sqrt{x}}{a^2} + \frac{b^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a^3\sqrt{-\frac{b}{a}}} - \frac{b^2\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a^3\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(a+b/x),x)
```

output
```
Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b), Eq(a, 0)
), (2*x**(3/2)/(3*a), Eq(b, 0)), (2*x**(3/2)/(3*a) - 2*b*sqrt(x)/a**2 + b*
*2*log(sqrt(x) - sqrt(-b/a))/(a**3*sqrt(-b/a)) - b**2*log(sqrt(x) + sqrt(-
b/a))/(a**3*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx = \frac{2\left(a-\frac{3b}{x}\right)x^{\frac{3}{2}}}{3a^2} - \frac{2b^2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a^2}$$

input
```
integrate(x^(1/2)/(a+b/x),x, algorithm="maxima")
```

output
```
2/3*(a - 3*b/x)*x^(3/2)/a^2 - 2*b^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*
b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx = \frac{2b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\left(a^2x^{\frac{3}{2}}-3ab\sqrt{x}\right)}{3a^3}$$

input
```
integrate(x^(1/2)/(a+b/x),x, algorithm="giac")
```

output
```
2*b^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(a^2*x^(3/2) - 3*a
*b*sqrt(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx = \frac{2\,x^{3/2}}{3\,a} - \frac{2\,b\,\sqrt{x}}{a^2} + \frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

input
```
int(x^(1/2)/(a + b/x),x)
```

output
```
(2*x^(3/2))/(3*a) - (2*b*x^(1/2))/a^2 + (2*b^(3/2)*atan((a^(1/2)*x^(1/2))/
b^(1/2)))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}}{a+\frac{b}{x}}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b + \frac{2\sqrt{x}\,a^2x}{3} - 2\sqrt{x}\,ab}{a^3}$$

input
```
int(x^(1/2)/(a+b/x),x)
```

output
```
(2*(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b + sqrt(x)*a**2
*x - 3*sqrt(x)*a*b))/(3*a**3)
```

3.120 $\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \frac{2\sqrt{x}}{a} - \frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

output
```
2*x^(1/2)/a-2*b^(1/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \frac{2\sqrt{x}}{a} - \frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

input
```
Integrate[1/((a + b/x)*Sqrt[x]),x]
```

output
```
(2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+\frac{b}{x}\right)}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\sqrt{x}}{ax+b}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{x}}{a}-\frac{2b\int \frac{1}{b+ax}d\sqrt{x}}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

input
```
Int[1/((a + b/x)*Sqrt[x]),x]
```

output
```
(2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{a}-\frac{2b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{a}-\frac{2b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{a}-\frac{2b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32

input
```
int(1/(a+b/x)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/a-2*b/a/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,\sqrt{x}}{a},\ -\frac{2\left(\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)-\sqrt{x}\right)}{a}\right]$$

input
```
integrate(1/(a+b/x)/x^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x + b)) + 2*sqrt(x)
)/a, -2*(sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) - sqrt(x))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0\wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0 \\ \frac{2\sqrt{x}}{a} & \text{for } b=0 \\ \frac{2\sqrt{x}}{a}-\frac{b\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a^2\sqrt{-\frac{b}{a}}}+\frac{b\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a^2\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)/x**(1/2),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0)
), (2*sqrt(x)/a, Eq(b, 0)), (2*sqrt(x)/a - b*log(sqrt(x) - sqrt(-b/a))/(a*
*2*sqrt(-b/a)) + b*log(sqrt(x) + sqrt(-b/a))/(a**2*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \frac{2\,b\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a} + \frac{2\,\sqrt{x}}{a}$$

input
```
integrate(1/(a+b/x)/x^(1/2),x, algorithm="maxima")
```

output
```
2*b*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a) + 2*sqrt(x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = -\frac{2\,b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{2\,\sqrt{x}}{a}$$

input
```
integrate(1/(a+b/x)/x^(1/2),x, algorithm="giac")
```

output
```
-2*b*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + 2*sqrt(x)/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \frac{2\sqrt{x}}{a} - \frac{2\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}$$

input
```
int(1/(x^(1/2)*(a + b/x)),x)
```

output
```
(2*x^(1/2))/a - (2*b^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a+\frac{b}{x}\right)\sqrt{x}}\,dx = \frac{-2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)+2\sqrt{x}\,a}{a^2}$$

input
```
int(1/(a+b/x)/x^(1/2),x)
```

output
```
(2*( - sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a))) + sqrt(x)*a))/a
**2
```

3.121 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
2*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx=\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[1/((a + b/x)*x^(3/2)),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {795, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+\frac{b}{x}\right)} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{\sqrt{x}(ax+b)}dx$$

$$\Big\downarrow \text{73}$$

$$2\int \frac{1}{b+ax}d\sqrt{x}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[1/((a + b/x)*x^(3/2)),x]
```

output
```
(2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

input
```
int(1/(a+b/x)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab}\right]$$

input
```
integrate(1/(a+b/x)/x^(3/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqr
t(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ \frac{\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} - \frac{\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)/x**(3/2),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b, Eq(a, 0)), (-2
/(a*sqrt(x)), Eq(b, 0)), (log(sqrt(x) - sqrt(-b/a))/(a*sqrt(-b/a)) - log(s
qrt(x) + sqrt(-b/a))/(a*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx = -\frac{2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a+b/x)/x^(3/2),x, algorithm="maxima")
```

output
```
-2*arctan(b/(sqrt(a*b)*sqrt(x)))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx=\frac{2\,\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a+b/x)/x^(3/2),x, algorithm="giac")
```

output
```
2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx=\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(x^(3/2)*(a + b/x)),x)
```

output
```
(2*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{3/2}}\,dx=\frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/(a+b/x)/x^(3/2),x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.122 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx=-\frac{2}{b\sqrt{x}}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{3/2}}$$

output
```
-2/b/x^(1/2)+2*a^(1/2)*arctan(1/a^(1/2)/x^(1/2)*b^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx=-\frac{2}{b\sqrt{x}}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
Integrate[1/((a + b/x)*x^(5/2)),x]
```

output
```
-2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(a+\frac{b}{x}\right)}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{3/2}(ax+b)}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{a\int \frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2a\int \frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}$$

input
```
Int[1/((a + b/x)*x^(5/2)),x]
```

output
```
-2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{2a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}-\frac{2}{\sqrt{x}\,b}$	32
default	$-\frac{2a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}-\frac{2}{\sqrt{x}\,b}$	32
risch	$-\frac{2a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}-\frac{2}{\sqrt{x}\,b}$	32

input
```
int(1/(a+b/x)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a/b/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))-2/x^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx = \left[\frac{x\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\sqrt{x}}{bx},\right.$$
$$\left.-\frac{2\left(x\sqrt{\frac{a}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{a}{b}}\right)+\sqrt{x}\right)}{bx}\right]$$

input
```
integrate(1/(a+b/x)/x^(5/2),x, algorithm="fricas")
```

output
```
[(x*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a/b) - b)/(a*x + b)) - 2*sqrt(
x))/(b*x), -2*(x*sqrt(a/b)*arctan(sqrt(x)*sqrt(a/b)) + sqrt(x))/(b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(36) = 72.

Time = 1.88 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ a=0\wedge b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\ a=0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for}\ b=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{b\sqrt{-\frac{b}{a}}}+\frac{\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{b\sqrt{-\frac{b}{a}}}-\frac{2}{b\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)/x**(5/2),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)),
(-2/(3*a*x**(3/2)), Eq(b, 0)), (-log(sqrt(x) - sqrt(-b/a))/(b*sqrt(-b/a))
+ log(sqrt(x) + sqrt(-b/a))/(b*sqrt(-b/a)) - 2/(b*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx=\frac{2\,a\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}b}-\frac{2}{b\sqrt{x}}$$

input
```
integrate(1/(a+b/x)/x^(5/2),x, algorithm="maxima")
```

output
```
2*a*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b) - 2/(b*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx=-\frac{2\,a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b}-\frac{2}{b\sqrt{x}}$$

input
```
integrate(1/(a+b/x)/x^(5/2),x, algorithm="giac")
```

output
```
-2*a*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - 2/(b*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx=-\frac{2}{b\sqrt{x}}-\frac{2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
int(1/(x^(5/2)*(a + b/x)),x)
```

output
```
- 2/(b*x^(1/2)) - (2*a^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{5/2}}\,dx=\frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)-2b}{\sqrt{x}\,b^2}$$

input
```
int(1/(a+b/x)/x^(5/2),x)
```

output
```
( - 2*(sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a))) + b))/(
sqrt(x)*b**2)
```

3.123 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = -\frac{2}{3bx^{3/2}}+\frac{2a}{b^2\sqrt{x}}-\frac{2a^{3/2}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{5/2}}$$

output
```
-2/3/b/x^(3/2)+2*a/b^2/x^(1/2)-2*a^(3/2)*arctan(1/a^(1/2)/x^(1/2)*b^(1/2))
/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = -\frac{2(b-3ax)}{3b^2x^{3/2}}+\frac{2a^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

input
```
Integrate[1/((a + b/x)*x^(7/2)),x]
```

output
```
(-2*(b - 3*a*x))/(3*b^2*x^(3/2)) + (2*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqr
t[b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {795, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2} \left(a+\frac{b}{x}\right)} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{5/2}(ax+b)}dx$$

$$\Big\downarrow \text{61}$$

$$-\frac{a\int \frac{1}{x^{3/2}(b+ax)}dx}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{a\left(-\frac{a\int \frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{a\left(-\frac{2a\int \frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}$$

input
```
Int[1/((a + b/x)*x^(7/2)),x]
```

output
```
-2/(3*b*x^(3/2)) - (a*(-2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x]
)/Sqrt[b]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{2ax-\frac{2b}{3}}{b^2x^{\frac{3}{2}}}+\frac{2a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	42
derivativedivides	$\frac{2a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}-\frac{2}{3b\,x^{\frac{3}{2}}}+\frac{2a}{\sqrt{x}\,b^2}$	43
default	$\frac{2a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}-\frac{2}{3b\,x^{\frac{3}{2}}}+\frac{2a}{\sqrt{x}\,b^2}$	43

input
```
int(1/(a+b/x)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*a*x-b)/b^2/x^(3/2)+2*a^2/b^2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.13

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = \left[\frac{3\,ax^2\sqrt{-\frac{a}{b}}\log\left(\frac{ax+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)+2\,(3\,ax-b)\sqrt{x}}{3\,b^2x^2}, \frac{2\,\left(3\,ax^2\sqrt{\frac{a}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{a}{b}}\right)+\right.}{3\,b^2x^2}\right.$$

input
```
integrate(1/(a+b/x)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/3*(3*a*x^2*sqrt(-a/b)*log((a*x + 2*b*sqrt(x)*sqrt(-a/b) - b)/(a*x + b))
 + 2*(3*a*x - b)*sqrt(x))/(b^2*x^2), 2/3*(3*a*x^2*sqrt(a/b)*arctan(sqrt(x)
*sqrt(a/b)) + (3*a*x - b)*sqrt(x))/(b^2*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(49) = 98$.

Time = 5.75 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a=0 \wedge b=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for}\ a=0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\ b=0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{b^2\sqrt{-\frac{b}{a}}} - \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{b^2\sqrt{-\frac{b}{a}}} + \frac{2a}{b^2\sqrt{x}} - \frac{2}{3bx^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)/x**(7/2),x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0
)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (a*log(sqrt(x) - sqrt(-b/a))/(b**2*sqrt
(-b/a)) - a*log(sqrt(x) + sqrt(-b/a))/(b**2*sqrt(-b/a)) + 2*a/(b**2*sqrt(x
)) - 2/(3*b*x**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = -\frac{2\,a^2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}b^2} + \frac{2\left(\frac{3\,a}{\sqrt{x}} - \frac{b}{x^{\frac{3}{2}}}\right)}{3\,b^2}$$

input
```
integrate(1/(a+b/x)/x^(7/2),x, algorithm="maxima")
```

output
```
-2*a^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^2) + 2/3*(3*a/sqrt(x) -
b/x^(3/2))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = \frac{2\,a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,(3\,ax-b)}{3\,b^2x^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)/x^(7/2),x, algorithm="giac")
```

output
```
2*a^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(3*a*x - b)/(b^2*x
^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = \frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}} - \frac{\frac{2}{3\,b}-\frac{2\,a\,x}{b^2}}{x^{3/2}}$$

input
```
int(1/(x^(7/2)*(a + b/x)),x)
```

output
```
(2*a^(3/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(5/2) - (2/(3*b) - (2*a*x)/b
^2)/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{7/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)ax+2abx-\frac{2b^2}{3}}{\sqrt{x}\,b^3x}$$

input
```
int(1/(a+b/x)/x^(7/2),x)
```

output
```
(2*(3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*x + 3*
a*b*x - b**2))/(3*sqrt(x)*b**3*x)
```

3.124 $\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx=-\frac{2}{5bx^{5/2}}+\frac{2a}{3b^2x^{3/2}}-\frac{2a^2}{b^3\sqrt{x}}+\frac{2a^{5/2}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{7/2}}$$

output
```
-2/5/b/x^(5/2)+2/3*a/b^2/x^(3/2)-2*a^2/b^3/x^(1/2)+2*a^(5/2)*arctan(1/a^(1
/2)/x^(1/2)*b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx=-\frac{2(3b^2-5abx+15a^2x^2)}{15b^3x^{5/2}}-\frac{2a^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
Integrate[1/((a + b/x)*x^(9/2)),x]
```

output
```
(-2*(3*b^2 - 5*a*b*x + 15*a^2*x^2))/(15*b^3*x^(5/2)) - (2*a^(5/2)*ArcTan[(
Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {795, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\left(a+\frac{b}{x}\right)}dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{x^{7/2}(ax+b)}dx$$

$$\Big\downarrow \ 61$$

$$-\frac{a\int \frac{1}{x^{5/2}(b+ax)}dx}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \ 61$$

$$-\frac{a\left(-\frac{a\int \frac{1}{x^{3/2}(b+ax)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \ 61$$

$$-\frac{a\left(-\frac{a\left(-\frac{a\int \frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \ 73$$

$$-\frac{a\left(-\frac{a\left(-\frac{2a\int \frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

$$\Big\downarrow \ 218$$

$$-\frac{a\left(-\frac{a\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}$$

input
```
Int[1/((a + b/x)*x^(9/2)),x]
```

output
```
-2/(5*b*x^(5/2)) - (a*(-2/(3*b*x^(3/2)) - (a*(-2/(b*Sqrt[x]) - (2*Sqrt[a]*
ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/b))/b
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{2(15a^2x^2-5abx+3b^2)}{15b^3x^{\frac{5}{2}}}-\frac{2a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$-\frac{2}{5b\,x^{\frac{5}{2}}}-\frac{2a^2}{b^3\sqrt{x}}+\frac{2a}{3b^2x^{\frac{3}{2}}}-\frac{2a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$-\frac{2}{5b\,x^{\frac{5}{2}}}-\frac{2a^2}{b^3\sqrt{x}}+\frac{2a}{3b^2x^{\frac{3}{2}}}-\frac{2a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

input
```
int(1/(a+b/x)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(15*a^2*x^2-5*a*b*x+3*b^2)/b^3/x^(5/2)-2*a^3/b^3/(a*b)^(1/2)*arctan(
a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.04

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx = \left[\frac{15\,a^2x^3\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\,(15\,a^2x^2-5\,abx+3\,b^2)\sqrt{x}}{15\,b^3x^3}, -\frac{2\left(15\,a^2x^3\sqrt{\frac{a}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{a}{b}}\right)+(15\,a^2x^2-5\,abx+3\,b^2)\sqrt{x}\right)}{15\,b^3x^3}\right]$$

input
```
integrate(1/(a+b/x)/x^(9/2),x, algorithm="fricas")
```

output
```
[1/15*(15*a^2*x^3*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a/b) - b)/(a*x +
 b)) - 2*(15*a^2*x^2 - 5*a*b*x + 3*b^2)*sqrt(x))/(b^3*x^3), -2/15*(15*a^2*
x^3*sqrt(a/b)*arctan(sqrt(x)*sqrt(a/b)) + (15*a^2*x^2 - 5*a*b*x + 3*b^2)*s
qrt(x))/(b^3*x^3)]
```

Sympy [A] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0 \\ -\frac{2}{7ax^{\frac{7}{2}}} & \text{for } b=0 \\ -\frac{a^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{b^3\sqrt{-\frac{b}{a}}} + \frac{a^2\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{b^3\sqrt{-\frac{b}{a}}} - \frac{2a^2}{b^3\sqrt{x}} + \frac{2a}{3b^2x^{\frac{3}{2}}} - \frac{2}{5bx^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)/x**(9/2),x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0
)), (-2/(7*a*x**(7/2)), Eq(b, 0)), (-a**2*log(sqrt(x) - sqrt(-b/a))/(b**3*
sqrt(-b/a)) + a**2*log(sqrt(x) + sqrt(-b/a))/(b**3*sqrt(-b/a)) - 2*a**2/(b
**3*sqrt(x)) + 2*a/(3*b**2*x**(3/2)) - 2/(5*b*x**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx = \frac{2\,a^3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}b^3} - \frac{2\left(\frac{15\,a^2}{\sqrt{x}} - \frac{5\,ab}{x^{\frac{3}{2}}} + \frac{3\,b^2}{x^{\frac{5}{2}}}\right)}{15\,b^3}$$

input
```
integrate(1/(a+b/x)/x^(9/2),x, algorithm="maxima")
```

output
```
2*a^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^3) - 2/15*(15*a^2/sqrt(x)
 - 5*a*b/x^(3/2) + 3*b^2/x^(5/2))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx = -\frac{2\,a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{2\,(15\,a^2x^2 - 5\,abx + 3\,b^2)}{15\,b^3x^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)/x^(9/2),x, algorithm="giac")
```

output
```
-2*a^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - 2/15*(15*a^2*x^2 - 5*
a*b*x + 3*b^2)/(b^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx = -\frac{\frac{2}{5\,b}+\frac{2\,a^2\,x^2}{b^3}-\frac{2\,a\,x}{3\,b^2}}{x^{5/2}} - \frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
int(1/(x^(9/2)*(a + b/x)),x)
```

output
```
- (2/(5*b) + (2*a^2*x^2)/b^3 - (2*a*x)/(3*b^2))/x^(5/2) - (2*a^(5/2)*atan(
(a^(1/2)*x^(1/2))/b^(1/2)))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)x^{9/2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2x^2 - 2a^2b\,x^2 + \frac{2a\,b^2x}{3} - \frac{2b^3}{5}}{\sqrt{x}\,b^4x^2}$$

input
```
int(1/(a+b/x)/x^(9/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2
*x**2 - 15*a**2*b*x**2 + 5*a*b**2*x - 3*b**3))/(15*sqrt(x)*b**4*x**2)
```

3.125 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{8b^3\sqrt{x}}{a^5}+\frac{2b^2x^{3/2}}{a^4}-\frac{4bx^{5/2}}{5a^3}+\frac{2x^{7/2}}{7a^2}-\frac{b^4\sqrt{x}}{a^5(b+ax)}+\frac{9b^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{11/2}}$$

output
```
-8*b^3*x^(1/2)/a^5+2*b^2*x^(3/2)/a^4-4/5*b*x^(5/2)/a^3+2/7*x^(7/2)/a^2-b^4
*x^(1/2)/a^5/(a*x+b)+9*b^(7/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{\sqrt{x}(-315b^4-210ab^3x+42a^2b^2x^2-18a^3bx^3+10a^4x^4)}{35a^5(b+ax)}+\frac{9b^{7/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{11/2}}$$

input
```
Integrate[x^(5/2)/(a + b/x)^2,x]
```

output
```
(Sqrt[x]*(-315*b^4 - 210*a*b^3*x + 42*a^2*b^2*x^2 - 18*a^3*b*x^3 + 10*a^4*
x^4))/(35*a^5*(b + a*x)) + (9*b^(7/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a
^(11/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {795, 51, 60, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{9/2}}{(ax+b)^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{9\int \frac{x^{7/2}}{b+ax}dx}{2a}-\frac{x^{9/2}}{a(ax+b)}$$

$$\Big\downarrow \text{60}$$

$$\frac{9\left(\frac{2x^{7/2}}{7a}-\frac{b\int \frac{x^{5/2}}{b+ax}dx}{a}\right)}{2a}-\frac{x^{9/2}}{a(ax+b)}$$

$$\Big\downarrow \text{60}$$

$$\frac{9\left(\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\int \frac{x^{3/2}}{b+ax}dx}{a}\right)}{a}\right)}{2a}-\frac{x^{9/2}}{a(ax+b)}$$

$$\Big\downarrow \ 60$$

$$\frac{9\left(\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\int \frac{\sqrt{x}}{b+ax}dx}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{9/2}}{a(ax+b)}$$

$$\Big\downarrow \ 60$$

$$\frac{9\left(\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{9/2}}{a(ax+b)}$$

$$\Big\downarrow \ 73$$

$$\frac{9\left(\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int \frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{9/2}}{a(ax+b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{9\left(\frac{2x^{7/2}}{7a}-\frac{b\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{9/2}}{a(ax+b)}$$

input
```
Int[x^(5/2)/(a + b/x)^2,x]
```

output
```
-(x^(9/2)/(a*(b + a*x))) + (9*((2*x^(7/2))/(7*a) - (b*((2*x^(5/2))/(5*a) -
 (b*((2*x^(3/2))/(3*a) - (b*((2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sq
rt[x])/Sqrt[b]])/a^(3/2)))/a))/a))/a))/(2*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2\left(5a^{3}x^{3}-14a^{2}b\,x^{2}+35a\,b^{2}x-140b^{3}\right)\sqrt{x}}{35a^{5}}+\frac{b^{4}\left(-\frac{\sqrt{x}}{ax+b}+\frac{9\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^{5}}$	78
derivativedivides	$\frac{\frac{2a^{3}x^{\frac{7}{2}}}{7}-\frac{4a^{2}b\,x^{\frac{5}{2}}}{5}+2a\,b^{2}x^{\frac{3}{2}}-8b^{3}\sqrt{x}}{a^{5}}+\frac{2b^{4}\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{9\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{5}}$	80
default	$\frac{\frac{2a^{3}x^{\frac{7}{2}}}{7}-\frac{4a^{2}b\,x^{\frac{5}{2}}}{5}+2a\,b^{2}x^{\frac{3}{2}}-8b^{3}\sqrt{x}}{a^{5}}+\frac{2b^{4}\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{9\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{5}}$	80

input
```
int(x^(5/2)/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/35*(5*a^3*x^3-14*a^2*b*x^2+35*a*b^2*x-140*b^3)*x^(1/2)/a^5+1/a^5*b^4*(-x
^(1/2)/(a*x+b)+9/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.07

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \left[\frac{315\,(ab^3x+b^4)\sqrt{-\frac{b}{a}}\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(10\,a^4x^4-18\,a^3bx^3+42\,a^2b^2x^2-210\,a}{70\,(a^6x+a^5b)}\right.$$

input
```
integrate(x^(5/2)/(a+b/x)^2,x, algorithm="fricas")
```

output
```
[1/70*(315*(a*b^3*x + b^4)*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) -
b)/(a*x + b)) + 2*(10*a^4*x^4 - 18*a^3*b*x^3 + 42*a^2*b^2*x^2 - 210*a*b^3*
x - 315*b^4)*sqrt(x))/(a^6*x + a^5*b), 1/35*(315*(a*b^3*x + b^4)*sqrt(b/a)
*arctan(a*sqrt(x)*sqrt(b/a)/b) + (10*a^4*x^4 - 18*a^3*b*x^3 + 42*a^2*b^2*x
^2 - 210*a*b^3*x - 315*b^4)*sqrt(x))/(a^6*x + a^5*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(97) = 194$.

Time = 27.78 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.90

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{11}{2}} \\ \frac{2x^{\frac{11}{2}}}{11b^2} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ \frac{20a^5x^{\frac{9}{2}}\sqrt{-\frac{b}{a}}}{70a^7x\sqrt{-\frac{b}{a}}+70a^6b\sqrt{-\frac{b}{a}}} - \frac{36a^4bx^{\frac{7}{2}}\sqrt{-\frac{b}{a}}}{70a^7x\sqrt{-\frac{b}{a}}+70a^6b\sqrt{-\frac{b}{a}}} + \frac{84a^3b^2x^{\frac{5}{2}}\sqrt{-\frac{b}{a}}}{70a^7x\sqrt{-\frac{b}{a}}+70a^6b\sqrt{-\frac{b}{a}}} - \frac{420a^2b^3x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{70a^7x\sqrt{-\frac{b}{a}}+70a^6b\sqrt{-\frac{b}{a}}} \end{cases}$$

input
```
integrate(x**(5/2)/(a+b/x)**2,x)
```

output
```
Piecewise((zoo*x**(11/2), Eq(a, 0) & Eq(b, 0)), (2*x**(11/2)/(11*b**2), Eq
(a, 0)), (2*x**(7/2)/(7*a**2), Eq(b, 0)), (20*a**5*x**(9/2)*sqrt(-b/a)/(70
*a**7*x*sqrt(-b/a) + 70*a**6*b*sqrt(-b/a)) - 36*a**4*b*x**(7/2)*sqrt(-b/a)
/(70*a**7*x*sqrt(-b/a) + 70*a**6*b*sqrt(-b/a)) + 84*a**3*b**2*x**(5/2)*sqr
t(-b/a)/(70*a**7*x*sqrt(-b/a) + 70*a**6*b*sqrt(-b/a)) - 420*a**2*b**3*x**(
3/2)*sqrt(-b/a)/(70*a**7*x*sqrt(-b/a) + 70*a**6*b*sqrt(-b/a)) - 630*a*b**4
*sqrt(x)*sqrt(-b/a)/(70*a**7*x*sqrt(-b/a) + 70*a**6*b*sqrt(-b/a)) + 315*a*
b**4*x*log(sqrt(x) - sqrt(-b/a))/(70*a**7*x*sqrt(-b/a) + 70*a**6*b*sqrt(-b
/a)) - 315*a*b**4*x*log(sqrt(x) + sqrt(-b/a))/(70*a**7*x*sqrt(-b/a) + 70*a
**6*b*sqrt(-b/a)) + 315*b**5*log(sqrt(x) - sqrt(-b/a))/(70*a**7*x*sqrt(-b/
a) + 70*a**6*b*sqrt(-b/a)) - 315*b**5*log(sqrt(x) + sqrt(-b/a))/(70*a**7*x
*sqrt(-b/a) + 70*a**6*b*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{10\,a^4 - \frac{18\,a^3b}{x} + \frac{42\,a^2b^2}{x^2} - \frac{210\,ab^3}{x^3} - \frac{315\,b^4}{x^4}}{35\left(\frac{a^6}{x^{\frac{7}{2}}} + \frac{a^5b}{x^{\frac{9}{2}}}\right)} - \frac{9\,b^4\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a^5}$$

input
```
integrate(x^(5/2)/(a+b/x)^2,x, algorithm="maxima")
```

output
```
1/35*(10*a^4 - 18*a^3*b/x + 42*a^2*b^2/x^2 - 210*a*b^3/x^3 - 315*b^4/x^4)/
(a^6/x^(7/2) + a^5*b/x^(9/2)) - 9*b^4*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(
a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{9\,b^4\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{b^4\sqrt{x}}{(ax+b)a^5} + \frac{2\left(5\,a^{12}x^{\frac{7}{2}} - 14\,a^{11}bx^{\frac{5}{2}} + 35\,a^{10}b^2x^{\frac{3}{2}} - 140\,a^9b^3\sqrt{x}\right)}{35\,a^{14}}$$

input
```
integrate(x^(5/2)/(a+b/x)^2,x, algorithm="giac")
```

output
```
9*b^4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) - b^4*sqrt(x)/((a*x + b)
*a^5) + 2/35*(5*a^12*x^(7/2) - 14*a^11*b*x^(5/2) + 35*a^10*b^2*x^(3/2) - 1
40*a^9*b^3*sqrt(x))/a^14
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{2\,x^{7/2}}{7\,a^2}-\frac{4\,b\,x^{5/2}}{5\,a^3}+\frac{2\,b^2\,x^{3/2}}{a^4}-\frac{8\,b^3\,\sqrt{x}}{a^5}-\frac{b^4\,\sqrt{x}}{x\,a^6+b\,a^5}+\frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{11/2}}$$

input
```
int(x^(5/2)/(a + b/x)^2,x)
```

output
```
(2*x^(7/2))/(7*a^2) - (4*b*x^(5/2))/(5*a^3) + (2*b^2*x^(3/2))/a^4 - (8*b^3
*x^(1/2))/a^5 - (b^4*x^(1/2))/(a^5*b + a^6*x) + (9*b^(7/2)*atan((a^(1/2)*x
^(1/2))/b^(1/2)))/a^(11/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a\,b^3x+315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^4+10\sqrt{x}\,a^5x^4-18\sqrt{x}\,a^4b\,x^3+}{35a^6\,(ax+b)}$$

input
```
int(x^(5/2)/(a+b/x)^2,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b**3*x + 315*sq
rt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**4 + 10*sqrt(x)*a**5*x
**4 - 18*sqrt(x)*a**4*b*x**3 + 42*sqrt(x)*a**3*b**2*x**2 - 210*sqrt(x)*a**
2*b**3*x - 315*sqrt(x)*a*b**4)/(35*a**6*(a*x + b))
```

3.126 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2} dx = \frac{6b^2\sqrt{x}}{a^4} - \frac{4bx^{3/2}}{3a^3} + \frac{2x^{5/2}}{5a^2} + \frac{b^3\sqrt{x}}{a^4(b+ax)} - \frac{7b^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

output
```
6*b^2*x^(1/2)/a^4-4/3*b*x^(3/2)/a^3+2/5*x^(5/2)/a^2+b^3*x^(1/2)/a^4/(a*x+b
)-7*b^(5/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2} dx = \frac{\sqrt{x}(105b^3+70ab^2x-14a^2bx^2+6a^3x^3)}{15a^4(b+ax)} - \frac{7b^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

input
```
Integrate[x^(3/2)/(a + b/x)^2,x]
```

output
```
(Sqrt[x]*(105*b^3 + 70*a*b^2*x - 14*a^2*b*x^2 + 6*a^3*x^3))/(15*a^4*(b + a
*x)) - (7*b^(5/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(9/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {795, 51, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{7/2}}{(ax+b)^2}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{7\int \frac{x^{5/2}}{b+ax}dx}{2a}-\frac{x^{7/2}}{a(ax+b)}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\int \frac{x^{3/2}}{b+ax}dx}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\int \frac{\sqrt{x}}{b+ax}dx}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int\frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}$$

$$\Big\downarrow \; 73$$

$$\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int\frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}$$

$$\Big\downarrow \; 218$$

$$\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}$$

input
```
Int[x^(3/2)/(a + b/x)^2,x]
```

output
```
-(x^(7/2)/(a*(b + a*x))) + (7*((2*x^(5/2))/(5*a) - (b*((2*x^(3/2))/(3*a) -
 (b*((2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)
))/a))/a))/(2*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\frac{2a^2x^{\frac{5}{2}}}{5}-\frac{4abx^{\frac{3}{2}}}{3}+6b^2\sqrt{x}}{a^4}-\frac{2b^3\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{7\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}$	70
default	$\frac{\frac{2a^2x^{\frac{5}{2}}}{5}-\frac{4abx^{\frac{3}{2}}}{3}+6b^2\sqrt{x}}{a^4}-\frac{2b^3\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{7\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}$	70
risch	$\frac{2(3a^2x^2-10abx+45b^2)\sqrt{x}}{15a^4}+\frac{b^3\sqrt{x}}{a^4(ax+b)}-\frac{7b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}$	70

input
```
int(x^(3/2)/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/a^4*(1/5*a^2*x^(5/2)-2/3*a*b*x^(3/2)+3*b^2*x^(1/2))-2*b^3/a^4*(-1/2*x^(1
/2)/(a*x+b)+7/2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.16

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx=\left[\frac{105\,(ab^2x+b^3)\sqrt{-\frac{b}{a}}\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(6\,a^3x^3-14\,a^2bx^2+70\,ab^2x+105\,b^3)\sqrt{x}}{30\,(a^5x+a^4b)}\right.$$
$$\left.-\frac{105\,(ab^2x+b^3)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)-(6\,a^3x^3-14\,a^2bx^2+70\,ab^2x+105\,b^3)\sqrt{x}}{15\,(a^5x+a^4b)}\right]$$

input
```
integrate(x^(3/2)/(a+b/x)^2,x, algorithm="fricas")
```

output
```
[1/30*(105*(a*b^2*x + b^3)*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) -
b)/(a*x + b)) + 2*(6*a^3*x^3 - 14*a^2*b*x^2 + 70*a*b^2*x + 105*b^3)*sqrt(x
))/(a^5*x + a^4*b), -1/15*(105*(a*b^2*x + b^3)*sqrt(b/a)*arctan(a*sqrt(x)*
sqrt(b/a)/b) - (6*a^3*x^3 - 14*a^2*b*x^2 + 70*a*b^2*x + 105*b^3)*sqrt(x))/
(a^5*x + a^4*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(83) = 166$.

Time = 8.43 (sec) , antiderivative size = 442, normalized size of antiderivative = 5.08

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{9}{2}} \\ \frac{2x^{\frac{9}{2}}}{9b^2} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ \frac{12a^4x^{\frac{7}{2}}\sqrt{-\frac{b}{a}}}{30a^6x\sqrt{-\frac{b}{a}}+30a^5b\sqrt{-\frac{b}{a}}} - \frac{28a^3bx^{\frac{5}{2}}\sqrt{-\frac{b}{a}}}{30a^6x\sqrt{-\frac{b}{a}}+30a^5b\sqrt{-\frac{b}{a}}} + \frac{140a^2b^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{30a^6x\sqrt{-\frac{b}{a}}+30a^5b\sqrt{-\frac{b}{a}}} + \frac{210ab^3\sqrt{x}\sqrt{-\frac{b}{a}}}{30a^6x\sqrt{-\frac{b}{a}}+30a^5b\sqrt{-\frac{b}{a}}} - \end{cases}$$

input
```
integrate(x**(3/2)/(a+b/x)**2,x)
```

output
```
Piecewise((zoo*x**(9/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*b**2), Eq(a,
 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0)), (12*a**4*x**(7/2)*sqrt(-b/a)/(30*a*
*6*x*sqrt(-b/a) + 30*a**5*b*sqrt(-b/a)) - 28*a**3*b*x**(5/2)*sqrt(-b/a)/(3
0*a**6*x*sqrt(-b/a) + 30*a**5*b*sqrt(-b/a)) + 140*a**2*b**2*x**(3/2)*sqrt(
-b/a)/(30*a**6*x*sqrt(-b/a) + 30*a**5*b*sqrt(-b/a)) + 210*a*b**3*sqrt(x)*s
qrt(-b/a)/(30*a**6*x*sqrt(-b/a) + 30*a**5*b*sqrt(-b/a)) - 105*a*b**3*x*log
(sqrt(x) - sqrt(-b/a))/(30*a**6*x*sqrt(-b/a) + 30*a**5*b*sqrt(-b/a)) + 105
*a*b**3*x*log(sqrt(x) + sqrt(-b/a))/(30*a**6*x*sqrt(-b/a) + 30*a**5*b*sqrt
(-b/a)) - 105*b**4*log(sqrt(x) - sqrt(-b/a))/(30*a**6*x*sqrt(-b/a) + 30*a*
*5*b*sqrt(-b/a)) + 105*b**4*log(sqrt(x) + sqrt(-b/a))/(30*a**6*x*sqrt(-b/a
) + 30*a**5*b*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{6\,a^3 - \frac{14\,a^2b}{x} + \frac{70\,ab^2}{x^2} + \frac{105\,b^3}{x^3}}{15\left(\frac{a^5}{x^{\frac{5}{2}}} + \frac{a^4b}{x^{\frac{7}{2}}}\right)} + \frac{7\,b^3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a^4}$$

input
```
integrate(x^(3/2)/(a+b/x)^2,x, algorithm="maxima")
```

output
```
1/15*(6*a^3 - 14*a^2*b/x + 70*a*b^2/x^2 + 105*b^3/x^3)/(a^5/x^(5/2) + a^4*
b/x^(7/2)) + 7*b^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = -\frac{7\,b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{b^3\sqrt{x}}{(ax+b)a^4} + \frac{2\left(3\,a^8x^{\frac{5}{2}} - 10\,a^7bx^{\frac{3}{2}} + 45\,a^6b^2\sqrt{x}\right)}{15\,a^{10}}$$

input
```
integrate(x^(3/2)/(a+b/x)^2,x, algorithm="giac")
```

output
```
-7*b^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + b^3*sqrt(x)/((a*x + b
)*a^4) + 2/15*(3*a^8*x^(5/2) - 10*a^7*b*x^(3/2) + 45*a^6*b^2*sqrt(x))/a^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.78

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{2\,x^{5/2}}{5\,a^2} - \frac{4\,b\,x^{3/2}}{3\,a^3} + \frac{6\,b^2\,\sqrt{x}}{a^4} + \frac{b^3\,\sqrt{x}}{x\,a^5 + b\,a^4} - \frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{9/2}}$$

input
```
int(x^(3/2)/(a + b/x)^2,x)
```

output
```
(2*x^(5/2))/(5*a^2) - (4*b*x^(3/2))/(3*a^3) + (6*b^2*x^(1/2))/a^4 + (b^3*x
^(1/2))/(a^4*b + a^5*x) - (7*b^(5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(9
/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x - 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^3 + 6\sqrt{x}\,a^4x^3 - 14\sqrt{x}\,a^3b\,x^2 +}{15a^5\left(ax+b\right)}$$

input
```
int(x^(3/2)/(a+b/x)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b**2*x - 105
*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**3 + 6*sqrt(x)*a**4
*x**3 - 14*sqrt(x)*a**3*b*x**2 + 70*sqrt(x)*a**2*b**2*x + 105*sqrt(x)*a*b*
*3)/(15*a**5*(a*x + b))
```

3.127 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2} \, dx = -\frac{4b\sqrt{x}}{a^3} + \frac{2x^{3/2}}{3a^2} - \frac{b^2\sqrt{x}}{a^3(b+ax)} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

output
```
-4*b*x^(1/2)/a^3+2/3*x^(3/2)/a^2-b^2*x^(1/2)/a^3/(a*x+b)+5*b^(3/2)*arctan(
a^(1/2)*x^(1/2)/b^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2} \, dx = \frac{\sqrt{x}(-15b^2-10abx+2a^2x^2)}{3a^3(b+ax)} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

input
```
Integrate[Sqrt[x]/(a + b/x)^2,x]
```

output
```
(Sqrt[x]*(-15*b^2 - 10*a*b*x + 2*a^2*x^2))/(3*a^3*(b + a*x)) + (5*b^(3/2)*
ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {795, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2} dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^{5/2}}{(ax+b)^2}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{5\int \frac{x^{3/2}}{b+ax}dx}{2a}-\frac{x^{5/2}}{a(ax+b)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\int \frac{\sqrt{x}}{b+ax}dx}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int\frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}$$

$$\Big\downarrow \ 218$$

$$\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}$$

input
```
Int[Sqrt[x]/(a + b/x)^2,x]
```

output
```
-(x^(5/2)/(a*(b + a*x))) + (5*((2*x^(3/2))/(3*a) - (b*((2*Sqrt[x])/a - (2*
Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)))/a))/(2*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2(ax-6b)\sqrt{x}}{3a^3}+\frac{b^2\left(-\frac{\sqrt{x}}{ax+b}+\frac{5\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^3}$	55
derivativedivides	$\frac{\frac{2a\,x^{\frac{3}{2}}}{3}-4b\sqrt{x}}{a^3}+\frac{2b^2\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{5\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	59
default	$\frac{\frac{2a\,x^{\frac{3}{2}}}{3}-4b\sqrt{x}}{a^3}+\frac{2b^2\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{5\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	59

input
```
int(x^(1/2)/(a+b/x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x-6*b)*x^(1/2)/a^3+1/a^3*b^2*(-x^(1/2)/(a*x+b)+5/(a*b)^(1/2)*arctan
(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx$$
$$= \left[\frac{15\,(abx+b^2)\sqrt{-\frac{b}{a}}\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(2\,a^2x^2-10\,abx-15\,b^2)\sqrt{x}}{6\,(a^4x+a^3b)}, \frac{15\,(abx+b^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a}{-}\right.}{3}\right.$$

input
```
integrate(x^(1/2)/(a+b/x)^2,x, algorithm="fricas")
```

output
```
[1/6*(15*(a*b*x + b^2)*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)*sqrt(-b/a) - b)/(
a*x + b)) + 2*(2*a^2*x^2 - 10*a*b*x - 15*b^2)*sqrt(x))/(a^4*x + a^3*b), 1/
3*(15*(a*b*x + b^2)*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) + (2*a^2*x^2 -
 10*a*b*x - 15*b^2)*sqrt(x))/(a^4*x + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(68) = 136.

Time = 2.25 (sec) , antiderivative size = 389, normalized size of antiderivative = 5.33

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx$$
$$= \begin{cases} \tilde{\infty}x^{\frac{7}{2}} \\ \frac{2x^{\frac{7}{2}}}{7b^2} \\ \frac{2x^{\frac{3}{2}}}{3a^2} \\ \frac{4a^3x^{\frac{5}{2}}\sqrt{-\frac{b}{a}}}{6a^5x\sqrt{-\frac{b}{a}}+6a^4b\sqrt{-\frac{b}{a}}} - \frac{20a^2bx^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{6a^5x\sqrt{-\frac{b}{a}}+6a^4b\sqrt{-\frac{b}{a}}} - \frac{30ab^2\sqrt{x}\sqrt{-\frac{b}{a}}}{6a^5x\sqrt{-\frac{b}{a}}+6a^4b\sqrt{-\frac{b}{a}}} + \frac{15ab^2x\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{6a^5x\sqrt{-\frac{b}{a}}+6a^4b\sqrt{-\frac{b}{a}}} - \frac{15ab^2x\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{6a^5x\sqrt{-\frac{b}{a}}+6a^4b\sqrt{-\frac{b}{a}}} \end{cases}$$

input
```
integrate(x**(1/2)/(a+b/x)**2,x)
```

output
```
Piecewise((zoo*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*b**2), Eq(a,
 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0)), (4*a**3*x**(5/2)*sqrt(-b/a)/(6*a**5
*x*sqrt(-b/a) + 6*a**4*b*sqrt(-b/a)) - 20*a**2*b*x**(3/2)*sqrt(-b/a)/(6*a*
*5*x*sqrt(-b/a) + 6*a**4*b*sqrt(-b/a)) - 30*a*b**2*sqrt(x)*sqrt(-b/a)/(6*a
**5*x*sqrt(-b/a) + 6*a**4*b*sqrt(-b/a)) + 15*a*b**2*x*log(sqrt(x) - sqrt(-
b/a))/(6*a**5*x*sqrt(-b/a) + 6*a**4*b*sqrt(-b/a)) - 15*a*b**2*x*log(sqrt(x
) + sqrt(-b/a))/(6*a**5*x*sqrt(-b/a) + 6*a**4*b*sqrt(-b/a)) + 15*b**3*log(
sqrt(x) - sqrt(-b/a))/(6*a**5*x*sqrt(-b/a) + 6*a**4*b*sqrt(-b/a)) - 15*b**
3*log(sqrt(x) + sqrt(-b/a))/(6*a**5*x*sqrt(-b/a) + 6*a**4*b*sqrt(-b/a)), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{2\,a^2-\frac{10\,ab}{x}-\frac{15\,b^2}{x^2}}{3\left(\frac{a^4}{x^{\frac{3}{2}}}+\frac{a^3b}{x^{\frac{5}{2}}}\right)}-\frac{5\,b^2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a^3}$$

input
```
integrate(x^(1/2)/(a+b/x)^2,x, algorithm="maxima")
```

output
```
1/3*(2*a^2 - 10*a*b/x - 15*b^2/x^2)/(a^4/x^(3/2) + a^3*b/x^(5/2)) - 5*b^2*
arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{5\,b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^3}-\frac{b^2\sqrt{x}}{(ax+b)a^3}+\frac{2\left(a^4x^{\frac{3}{2}}-6\,a^3b\sqrt{x}\right)}{3\,a^6}$$

input
```
integrate(x^(1/2)/(a+b/x)^2,x, algorithm="giac")
```

output
```
5*b^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - b^2*sqrt(x)/((a*x + b)
*a^3) + 2/3*(a^4*x^(3/2) - 6*a^3*b*sqrt(x))/a^6
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx=\frac{2\,x^{3/2}}{3\,a^2}-\frac{4\,b\,\sqrt{x}}{a^3}-\frac{b^2\,\sqrt{x}}{x\,a^4+b\,a^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{7/2}}$$

input
```
int(x^(1/2)/(a + b/x)^2,x)
```

output
```
(2*x^(3/2))/(3*a^2) - (4*b*x^(1/2))/a^3 - (b^2*x^(1/2))/(a^3*b + a^4*x) +
(5*b^(3/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^2}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)abx+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^2+2\sqrt{x}\,a^3x^2-10\sqrt{x}\,a^2bx-15\sqrt{x}\,a\,b^2}{3a^4\left(ax+b\right)}$$

input
```
int(x^(1/2)/(a+b/x)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b*x + 15*sqrt(b)
*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**2 + 2*sqrt(x)*a**3*x**2 -
10*sqrt(x)*a**2*b*x - 15*sqrt(x)*a*b**2)/(3*a**4*(a*x + b))
```

3.128 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 \sqrt{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 \sqrt{x}} \, dx = \frac{2\sqrt{x}}{a^2} + \frac{b\sqrt{x}}{a^2(b+ax)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

output
```
2*x^(1/2)/a^2+b*x^(1/2)/a^2/(a*x+b)-3*b^(1/2)*arctan(a^(1/2)*x^(1/2)/b^(1/
2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 \sqrt{x}} \, dx = \frac{\sqrt{x}(3b+2ax)}{a^2(b+ax)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

input
```
Integrate[1/((a + b/x)^2*Sqrt[x]),x]
```

output
```
(Sqrt[x]*(3*b + 2*a*x))/(a^2*(b + a*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[
x])/Sqrt[b]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {795, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{x^{3/2}}{(ax+b)^2}dx$$

$$\Big\downarrow \ 51$$

$$\frac{3\int \frac{\sqrt{x}}{b+ax}dx}{2a}-\frac{x^{3/2}}{a(ax+b)}$$

$$\Big\downarrow \ 60$$

$$\frac{3\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{2a}-\frac{x^{3/2}}{a(ax+b)}$$

$$\Big\downarrow \ 73$$

$$\frac{3\left(\frac{2\sqrt{x}}{a}-\frac{2b\int \frac{1}{b+ax}d\sqrt{x}}{a}\right)}{2a}-\frac{x^{3/2}}{a(ax+b)}$$

$$\Big\downarrow \ 218$$

$$\frac{3\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{x^{3/2}}{a(ax+b)}$$

input
```
Int[1/((a + b/x)^2*Sqrt[x]),x]
```

output
```
-(x^(3/2)/(a*(b + a*x))) + (3*((2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*
Sqrt[x])/Sqrt[b]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{a^2}-\frac{2b\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}$	47
default	$\frac{2\sqrt{x}}{a^2}-\frac{2b\left(-\frac{\sqrt{x}}{2(ax+b)}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2}$	47
risch	$\frac{2\sqrt{x}}{a^2}+\frac{b\sqrt{x}}{a^2(ax+b)}-\frac{3b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	47

input
```
int(1/(a+b/x)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/a^2-2*b/a^2*(-1/2*x^(1/2)/(a*x+b)+3/2/(a*b)^(1/2)*arctan(a*x^(1/
2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.35

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx = \left[\frac{3\,(ax+b)\sqrt{-\frac{b}{a}}\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(2\,ax+3\,b)\sqrt{x}}{2\,(a^3x+a^2b)},\ -\frac{3\,(ax+b)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)-(2\,ax+3\,b)\sqrt{x}}{a^3x+a^2b}\right]$$

input
```
integrate(1/(a+b/x)^2/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*x + b)*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(-b/a) - b)/(a*x +
 b)) + 2*(2*a*x + 3*b)*sqrt(x))/(a^3*x + a^2*b), -(3*(a*x + b)*sqrt(b/a)*a
rctan(a*sqrt(x)*sqrt(b/a)/b) - (2*a*x + 3*b)*sqrt(x))/(a^3*x + a^2*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(53) = 106$.

Time = 2.52 (sec) , antiderivative size = 332, normalized size of antiderivative = 5.82

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx = \begin{cases} \tilde{\infty}x^{\frac{5}{2}} \\ \frac{2x^{\frac{5}{2}}}{5b^2} \\ \frac{2\sqrt{x}}{a^2} \\ \frac{4a^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{2a^4x\sqrt{-\frac{b}{a}}+2a^3b\sqrt{-\frac{b}{a}}} + \frac{6ab\sqrt{x}\sqrt{-\frac{b}{a}}}{2a^4x\sqrt{-\frac{b}{a}}+2a^3b\sqrt{-\frac{b}{a}}} - \frac{3abx\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{2a^4x\sqrt{-\frac{b}{a}}+2a^3b\sqrt{-\frac{b}{a}}} + \frac{3abx\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{2a^4x\sqrt{-\frac{b}{a}}+2a^3b\sqrt{-\frac{b}{a}}} - \frac{3b^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{2a^4x\sqrt{-\frac{b}{a}}+2a^3b\sqrt{-\frac{b}{a}}} + \end{cases}$$

input
```
integrate(1/(a+b/x)**2/x**(1/2),x)
```

output
```
Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b**2), Eq(a,
 0)), (2*sqrt(x)/a**2, Eq(b, 0)), (4*a**2*x**(3/2)*sqrt(-b/a)/(2*a**4*x*sq
rt(-b/a) + 2*a**3*b*sqrt(-b/a)) + 6*a*b*sqrt(x)*sqrt(-b/a)/(2*a**4*x*sqrt(
-b/a) + 2*a**3*b*sqrt(-b/a)) - 3*a*b*x*log(sqrt(x) - sqrt(-b/a))/(2*a**4*x
*sqrt(-b/a) + 2*a**3*b*sqrt(-b/a)) + 3*a*b*x*log(sqrt(x) + sqrt(-b/a))/(2*
a**4*x*sqrt(-b/a) + 2*a**3*b*sqrt(-b/a)) - 3*b**2*log(sqrt(x) - sqrt(-b/a)
)/(2*a**4*x*sqrt(-b/a) + 2*a**3*b*sqrt(-b/a)) + 3*b**2*log(sqrt(x) + sqrt(
-b/a))/(2*a**4*x*sqrt(-b/a) + 2*a**3*b*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx = \frac{2\,a+\frac{3\,b}{x}}{\frac{a^3}{\sqrt{x}}+\frac{a^2b}{x^{\frac{3}{2}}}} + \frac{3\,b\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a^2}$$

input
```
integrate(1/(a+b/x)^2/x^(1/2),x, algorithm="maxima")
```

output
```
(2*a + 3*b/x)/(a^3/sqrt(x) + a^2*b/x^(3/2)) + 3*b*arctan(b/(sqrt(a*b)*sqrt
(x)))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx = -\frac{3\,b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\,\sqrt{x}}{a^2} + \frac{b\sqrt{x}}{(ax+b)a^2}$$

input
```
integrate(1/(a+b/x)^2/x^(1/2),x, algorithm="giac")
```

output
```
-3*b*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*sqrt(x)/a^2 + b*sqrt(
x)/((a*x + b)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx = \frac{2\,\sqrt{x}}{a^2} + \frac{b\,\sqrt{x}}{x\,a^3+b\,a^2} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{5/2}}$$

input
```
int(1/(x^(1/2)*(a + b/x)^2),x)
```

output
```
(2*x^(1/2))/a^2 + (b*x^(1/2))/(a^2*b + a^3*x) - (3*b^(1/2)*atan((a^(1/2)*x
^(1/2))/b^(1/2)))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2\sqrt{x}}\,dx$$
$$=\frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)ax-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b+2\sqrt{x}\,a^2x+3\sqrt{x}\,ab}{a^3\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^(1/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*x - 3*sqrt(b)*
sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b + 2*sqrt(x)*a**2*x + 3*sqrt(
x)*a*b)/(a**3*(a*x + b))
```

3.129 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}} \, dx = -\frac{\sqrt{x}}{a(b+ax)} + \frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}$$

output
```
-x^(1/2)/a/(a*x+b)+arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}} \, dx = -\frac{\sqrt{x}}{a(b+ax)} + \frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}$$

input
```
Integrate[1/((a + b/x)^2*x^(3/2)),x]
```

output
```
-(Sqrt[x]/(a*(b + a*x))) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(a^(3/2)*Sqrt
[b])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+\frac{b}{x}\right)^2} \, dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{\sqrt{x}}{(ax+b)^2}dx$$
$$\Big\downarrow \text{ 51}$$
$$\frac{\int \frac{1}{\sqrt{x}(b+ax)}dx}{2a}-\frac{\sqrt{x}}{a(ax+b)}$$
$$\Big\downarrow \text{ 73}$$
$$\frac{\int \frac{1}{b+ax}d\sqrt{x}}{a}-\frac{\sqrt{x}}{a(ax+b)}$$
$$\Big\downarrow \text{ 218}$$
$$\frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}-\frac{\sqrt{x}}{a(ax+b)}$$

input
```
Int[1/((a + b/x)^2*x^(3/2)),x]
```

output
```
-(Sqrt[x]/(a*(b + a*x))) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(a^(3/2)*Sqrt
[b])
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{a(ax+b)}+\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37
default	$-\frac{\sqrt{x}}{a(ax+b)}+\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37

input
```
int(1/(a+b/x)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)/a/(a*x+b)+1/a/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\, dx = \left[-\frac{2\, ab\sqrt{x} + \sqrt{-ab}(ax+b)\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{2\,(a^3bx+a^2b^2)}, -\frac{ab\sqrt{x}+\sqrt{ab}(ax+b)\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{a^3bx+a^2b^2}\right]$$

input
```
integrate(1/(a+b/x)^2/x^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(a*x + b)*log((a*x - b - 2*sqrt(-a*b)*sq
rt(x))/(a*x + b)))/(a^3*b*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(a*x + b
)*arctan(sqrt(a*b)/(a*sqrt(x))))/(a^3*b*x + a^2*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(37) = 74$.

Time = 5.78 (sec) , antiderivative size = 269, normalized size of antiderivative = 5.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\, dx = \begin{cases} \tilde{\infty} x^{\frac{3}{2}} \\ \frac{2x^{\frac{3}{2}}}{3b^2} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{2a\sqrt{x}\sqrt{-\frac{b}{a}}}{2a^3x\sqrt{-\frac{b}{a}}+2a^2b\sqrt{-\frac{b}{a}}} + \frac{ax\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{2a^3x\sqrt{-\frac{b}{a}}+2a^2b\sqrt{-\frac{b}{a}}} - \frac{ax\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{2a^3x\sqrt{-\frac{b}{a}}+2a^2b\sqrt{-\frac{b}{a}}} + \frac{b\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{2a^3x\sqrt{-\frac{b}{a}}+2a^2b\sqrt{-\frac{b}{a}}} - \end{cases}$$

input
```
integrate(1/(a+b/x)**2/x**(3/2),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b**2), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-2*a*sqrt(x)*sqrt(-b/a)/(2*a**3*x*sq
rt(-b/a) + 2*a**2*b*sqrt(-b/a)) + a*x*log(sqrt(x) - sqrt(-b/a))/(2*a**3*x*
sqrt(-b/a) + 2*a**2*b*sqrt(-b/a)) - a*x*log(sqrt(x) + sqrt(-b/a))/(2*a**3*
x*sqrt(-b/a) + 2*a**2*b*sqrt(-b/a)) + b*log(sqrt(x) - sqrt(-b/a))/(2*a**3*
x*sqrt(-b/a) + 2*a**2*b*sqrt(-b/a)) - b*log(sqrt(x) + sqrt(-b/a))/(2*a**3*
x*sqrt(-b/a) + 2*a**2*b*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\,dx = -\frac{\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}a} - \frac{1}{\left(a^2+\frac{ab}{x}\right)\sqrt{x}}$$

input
```
integrate(1/(a+b/x)^2/x^(3/2),x, algorithm="maxima")
```

output
```
-arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a) - 1/((a^2 + a*b/x)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\,dx = \frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{\sqrt{x}}{(ax+b)a}$$

input
```
integrate(1/(a+b/x)^2/x^(3/2),x, algorithm="giac")
```

output
```
arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - sqrt(x)/((a*x + b)*a)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\,\sqrt{b}} - \frac{\sqrt{x}}{a\,(b+a\,x)}$$

input
```
int(1/(x^(3/2)*(a + b/x)^2),x)
```

output
```
atan((a^(1/2)*x^(1/2))/b^(1/2))/(a^(3/2)*b^(1/2)) - x^(1/2)/(a*(b + a*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)ax + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b - \sqrt{x}\,ab}{a^2b\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*x + sqrt(b)*sqrt(a)
*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b - sqrt(x)*a*b)/(a**2*b*(a*x + b))
```

3.130 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}} \, dx = \frac{\sqrt{x}}{b(b+ax)} + \frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a} b^{3/2}}$$

output
```
x^(1/2)/b/(a*x+b)+arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}} \, dx = \frac{\sqrt{x}}{b(b+ax)} + \frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a} b^{3/2}}$$

input
```
Integrate[1/((a + b/x)^2*x^(5/2)),x]
```

output
```
Sqrt[x]/(b*(b + a*x)) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(Sqrt[a]*b^(3/2)
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(a+\frac{b}{x}\right)^2} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{\sqrt{x}(ax+b)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{\int \frac{1}{\sqrt{x}(b+ax)}dx}{2b}+\frac{\sqrt{x}}{b(ax+b)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{b+ax}d\sqrt{x}}{b}+\frac{\sqrt{x}}{b(ax+b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}b^{3/2}}+\frac{\sqrt{x}}{b(ax+b)}$$

input
```
Int[1/((a + b/x)^2*x^(5/2)),x]
```

output
```
Sqrt[x]/(b*(b + a*x)) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]/(Sqrt[a]*b^(3/2)
)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{x}}{b(ax+b)}+\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	36
default	$\frac{\sqrt{x}}{b(ax+b)}+\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	36

input
```
int(1/(a+b/x)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1/2)/b/(a*x+b)+1/b/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.58

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx = \left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}(ax+b)\log\left(\frac{ax-b-2\,\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{2\,(a^2b^2x+ab^3)}, \frac{ab\sqrt{x}-\sqrt{ab}(ax+b)\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{a^2b^2x+ab^3}\right]$$

input
```
integrate(1/(a+b/x)^2/x^(5/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(a*x + b)*log((a*x - b - 2*sqrt(-a*b)*sqr
t(x))/(a*x + b)))/(a^2*b^2*x + a*b^3), (a*b*sqrt(x) - sqrt(a*b)*(a*x + b)*
arctan(sqrt(a*b)/(a*sqrt(x))))/(a^2*b^2*x + a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(37) = 74$.

Time = 17.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 6.16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2\sqrt{x}}{b^2} \\ -\frac{2}{3a^2x^{\frac{3}{2}}} \\ \frac{2a\sqrt{x}\sqrt{-\frac{b}{a}}}{2a^2bx\sqrt{-\frac{b}{a}}+2ab^2\sqrt{-\frac{b}{a}}} + \frac{ax\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{2a^2bx\sqrt{-\frac{b}{a}}+2ab^2\sqrt{-\frac{b}{a}}} - \frac{ax\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{2a^2bx\sqrt{-\frac{b}{a}}+2ab^2\sqrt{-\frac{b}{a}}} + \frac{b\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{2a^2bx\sqrt{-\frac{b}{a}}+2ab^2\sqrt{-\frac{b}{a}}} \end{cases}$$

input
```
integrate(1/(a+b/x)**2/x**(5/2),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**2, Eq(a, 0)),
(-2/(3*a**2*x**(3/2)), Eq(b, 0)), (2*a*sqrt(x)*sqrt(-b/a)/(2*a**2*b*x*sqrt
(-b/a) + 2*a*b**2*sqrt(-b/a)) + a*x*log(sqrt(x) - sqrt(-b/a))/(2*a**2*b*x*
sqrt(-b/a) + 2*a*b**2*sqrt(-b/a)) - a*x*log(sqrt(x) + sqrt(-b/a))/(2*a**2*
b*x*sqrt(-b/a) + 2*a*b**2*sqrt(-b/a)) + b*log(sqrt(x) - sqrt(-b/a))/(2*a**
2*b*x*sqrt(-b/a) + 2*a*b**2*sqrt(-b/a)) - b*log(sqrt(x) + sqrt(-b/a))/(2*a
**2*b*x*sqrt(-b/a) + 2*a*b**2*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx = -\frac{\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}b} + \frac{1}{\left(ab+\frac{b^2}{x}\right)\sqrt{x}}$$

input
```
integrate(1/(a+b/x)^2/x^(5/2),x, algorithm="maxima")
```

output
```
-arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b) + 1/((a*b + b^2/x)*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx = \frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{\sqrt{x}}{(ax+b)b}$$

input
```
integrate(1/(a+b/x)^2/x^(5/2),x, algorithm="giac")
```

output
```
arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + sqrt(x)/((a*x + b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\,dx = \frac{\sqrt{x}}{b\,(b+a\,x)} + \frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\,b^{3/2}}$$

input
```
int(1/(x^(5/2)*(a + b/x)^2),x)
```

output
```
x^(1/2)/(b*(b + a*x)) + atan((a^(1/2)*x^(1/2))/b^(1/2))/(a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{5/2}}\, dx = \frac{\sqrt{b}\,\sqrt{a}\, atan\left(\frac{\sqrt{x}\, a}{\sqrt{b}\,\sqrt{a}}\right) ax+\sqrt{b}\,\sqrt{a}\, atan\left(\frac{\sqrt{x}\, a}{\sqrt{b}\,\sqrt{a}}\right) b+\sqrt{x}\, ab}{a\, b^2\, (ax+b)}$$

input
```
int(1/(a+b/x)^2/x^(5/2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*x + sqrt(b)*sqrt(a)
*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b + sqrt(x)*a*b)/(a*b**2*(a*x + b))
```

3.131 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}} \, dx = -\frac{2}{b^2\sqrt{x}} - \frac{a}{b^2\left(a+\frac{b}{x}\right)\sqrt{x}} + \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{5/2}}$$

output
```
-2/b^2/x^(1/2)-a/b^2/(a+b/x)/x^(1/2)+3*a^(1/2)*arctan(1/a^(1/2)/x^(1/2)*b^
(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}} \, dx = \frac{-2b-3ax}{b^2\sqrt{x}(b+ax)} - \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

input
```
Integrate[1/((a + b/x)^2*x^(7/2)),x]
```

output
```
(-2*b - 3*a*x)/(b^2*Sqrt[x]*(b + a*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x
])/Sqrt[b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {795, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2} \left(a+\frac{b}{x}\right)^2} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{3/2}(ax+b)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{3\int \frac{1}{x^{3/2}(b+ax)}dx}{2b}+\frac{1}{b\sqrt{x}(ax+b)}$$

$$\Big\downarrow \text{61}$$

$$\frac{3\left(-\frac{a\int \frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{2b}+\frac{1}{b\sqrt{x}(ax+b)}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\left(-\frac{2a\int \frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{2b}+\frac{1}{b\sqrt{x}(ax+b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{2b}+\frac{1}{b\sqrt{x}(ax+b)}$$

input
```
Int[1/((a + b/x)^2*x^(7/2)),x]
```

output
```
1/(b*Sqrt[x]*(b + a*x)) + (3*(-2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*
Sqrt[x])/Sqrt[b]])/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2}{\sqrt{x}\, b^{2}}-\frac{2a\left(\frac{\sqrt{x}}{2ax+2b}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}$	47
default	$-\frac{2}{\sqrt{x}\, b^{2}}-\frac{2a\left(\frac{\sqrt{x}}{2ax+2b}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}$	47
risch	$-\frac{2}{\sqrt{x}\, b^{2}}-\frac{a\sqrt{x}}{b^{2}(ax+b)}-\frac{3a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$	48

input
```
int(1/(a+b/x)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x^(1/2)/b^2-2*a/b^2*(1/2*x^(1/2)/(a*x+b)+3/2/(a*b)^(1/2)*arctan(a*x^(1/
2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{2} x^{7/2}}\,dx=\left[\frac{3\,(ax^{2}+bx)\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\,(3\,ax+2\,b)\sqrt{x}}{2\,(ab^{2}x^{2}+b^{3}x)},\right.$$
$$\left.-\frac{3\,(ax^{2}+bx)\sqrt{\frac{a}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{a}{b}}\right)+(3\,ax+2\,b)\sqrt{x}}{ab^{2}x^{2}+b^{3}x}\right]$$

input
```
integrate(1/(a+b/x)^2/x^(7/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*x^2 + b*x)*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a/b) - b)/(a
*x + b)) - 2*(3*a*x + 2*b)*sqrt(x))/(a*b^2*x^2 + b^3*x), -(3*(a*x^2 + b*x)
*sqrt(a/b)*arctan(sqrt(x)*sqrt(a/b)) + (3*a*x + 2*b)*sqrt(x))/(a*b^2*x^2 +
 b^3*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(53) = 106$.

Time = 48.44 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.40

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ -\frac{2}{b^2\sqrt{x}} \\ -\frac{2}{5a^2x^{\frac{5}{2}}} \\ -\frac{3ax^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{2ab^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}+2b^3\sqrt{x}\sqrt{-\frac{b}{a}}} + \frac{3ax^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{2ab^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}+2b^3\sqrt{x}\sqrt{-\frac{b}{a}}} - \frac{6ax\sqrt{-\frac{b}{a}}}{2ab^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}+2b^3\sqrt{x}\sqrt{-\frac{b}{a}}} - \frac{3b\sqrt{x}\log}{2ab^2x^{\frac{3}{2}}\sqrt{-}} \end{cases}$$

input
```
integrate(1/(a+b/x)**2/x**(7/2),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**2*sqrt(x)), Eq(a, 0)
), (-2/(5*a**2*x**(5/2)), Eq(b, 0)), (-3*a*x**(3/2)*log(sqrt(x) - sqrt(-b/
a))/(2*a*b**2*x**(3/2)*sqrt(-b/a) + 2*b**3*sqrt(x)*sqrt(-b/a)) + 3*a*x**(3
/2)*log(sqrt(x) + sqrt(-b/a))/(2*a*b**2*x**(3/2)*sqrt(-b/a) + 2*b**3*sqrt(
x)*sqrt(-b/a)) - 6*a*x*sqrt(-b/a)/(2*a*b**2*x**(3/2)*sqrt(-b/a) + 2*b**3*s
qrt(x)*sqrt(-b/a)) - 3*b*sqrt(x)*log(sqrt(x) - sqrt(-b/a))/(2*a*b**2*x**(3
/2)*sqrt(-b/a) + 2*b**3*sqrt(x)*sqrt(-b/a)) + 3*b*sqrt(x)*log(sqrt(x) + sq
rt(-b/a))/(2*a*b**2*x**(3/2)*sqrt(-b/a) + 2*b**3*sqrt(x)*sqrt(-b/a)) - 4*b
*sqrt(-b/a)/(2*a*b**2*x**(3/2)*sqrt(-b/a) + 2*b**3*sqrt(x)*sqrt(-b/a)), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx = -\frac{a}{\left(ab^2+\frac{b^3}{x}\right)\sqrt{x}} + \frac{3\,a\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}b^2} - \frac{2}{b^2\sqrt{x}}$$

input
```
integrate(1/(a+b/x)^2/x^(7/2),x, algorithm="maxima")
```

output
```
-a/((a*b^2 + b^3/x)*sqrt(x)) + 3*a*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b
)*b^2) - 2/(b^2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx = -\frac{3\,a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} - \frac{3\,ax+2\,b}{\left(ax^{\frac{3}{2}}+b\sqrt{x}\right)b^2}$$

input
```
integrate(1/(a+b/x)^2/x^(7/2),x, algorithm="giac")
```

output
```
-3*a*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - (3*a*x + 2*b)/((a*x^(3/
2) + b*sqrt(x))*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx = -\frac{\frac{2}{b}+\frac{3\,a\,x}{b^2}}{a\,x^{3/2}+b\,\sqrt{x}} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

input
```
int(1/(x^(7/2)*(a + b/x)^2),x)
```

output
```
- (2/b + (3*a*x)/b^2)/(a*x^(3/2) + b*x^(1/2)) - (3*a^(1/2)*atan((a^(1/2)*x
^(1/2))/b^(1/2)))/b^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{7/2}}\,dx = \frac{-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)ax-3\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b-3abx-2b^2}{\sqrt{x}\,b^3\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^(7/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*x - 3*
sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b - 3*a*b*x -
2*b**2)/(sqrt(x)*b**3*(a*x + b))
```

3.132 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}} \, dx = -\frac{2}{3b^2x^{3/2}} + \frac{4a}{b^3\sqrt{x}} + \frac{a^2}{b^3\left(a+\frac{b}{x}\right)\sqrt{x}} - \frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{7/2}}$$

output
```
-2/3/b^2/x^(3/2)+4*a/b^3/x^(1/2)+a^2/b^3/(a+b/x)/x^(1/2)-5*a^(3/2)*arctan(
1/a^(1/2)/x^(1/2)*b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}} \, dx = \frac{-2b^2+10abx+15a^2x^2}{3b^3x^{3/2}(b+ax)} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
Integrate[1/((a + b/x)^2*x^(9/2)),x]
```

output
```
(-2*b^2 + 10*a*b*x + 15*a^2*x^2)/(3*b^3*x^(3/2)*(b + a*x)) + (5*a^(3/2)*Ar
cTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {795, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{5/2}(ax+b)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{5\int \frac{1}{x^{5/2}(b+ax)}dx}{2b}+\frac{1}{bx^{3/2}(ax+b)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(-\frac{a\int \frac{1}{x^{3/2}(b+ax)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(-\frac{a\left(-\frac{a\int \frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(-\frac{a\left(-\frac{2a\int \frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(-\frac{a\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}$$

input
```
Int[1/((a + b/x)^2*x^(9/2)),x]
```

output
```
1/(b*x^(3/2)*(b + a*x)) + (5*(-2/(3*b*x^(3/2)) - (a*(-2/(b*Sqrt[x]) - (2*S
qrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/b))/(2*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{4ax-\frac{2b}{3}}{b^3x^{\frac{3}{2}}}+\frac{a^2\left(\frac{\sqrt{x}}{ax+b}+\frac{5\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^3}$	55
derivativedivides	$\frac{2a^2\left(\frac{\sqrt{x}}{2ax+2b}+\frac{5\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}-\frac{2}{3b^2x^{\frac{3}{2}}}+\frac{4a}{b^3\sqrt{x}}$	58
default	$\frac{2a^2\left(\frac{\sqrt{x}}{2ax+2b}+\frac{5\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}-\frac{2}{3b^2x^{\frac{3}{2}}}+\frac{4a}{b^3\sqrt{x}}$	58

input
```
int(1/(a+b/x)^2/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(6*a*x-b)/b^3/x^(3/2)+1/b^3*a^2*(x^(1/2)/(a*x+b)+5/(a*b)^(1/2)*arctan(
a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.42

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx = \left[\frac{15\,(a^2x^3+abx^2)\sqrt{-\frac{a}{b}}\log\left(\frac{ax+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)+2\,(15\,a^2x^2+10\,abx-2\,b^2)\sqrt{x}}{6\,(ab^3x^3+b^4x^2)}, \frac{15\,(\ldots$$

input
```
integrate(1/(a+b/x)^2/x^(9/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a^2*x^3 + a*b*x^2)*sqrt(-a/b)*log((a*x + 2*b*sqrt(x)*sqrt(-a/b)
- b)/(a*x + b)) + 2*(15*a^2*x^2 + 10*a*b*x - 2*b^2)*sqrt(x))/(a*b^3*x^3 +
b^4*x^2), 1/3*(15*(a^2*x^3 + a*b*x^2)*sqrt(a/b)*arctan(sqrt(x)*sqrt(a/b))
+ (15*a^2*x^2 + 10*a*b*x - 2*b^2)*sqrt(x))/(a*b^3*x^3 + b^4*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(68) = 136$.

Time = 135.48 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.11

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ -\frac{2}{3b^2x^{\frac{3}{2}}} \\ -\frac{2}{7a^2x^{\frac{7}{2}}} \\ \frac{15a^2x^{\frac{5}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{6ab^3x^{\frac{5}{2}}\sqrt{-\frac{b}{a}}+6b^4x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}} - \frac{15a^2x^{\frac{5}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{b}{a}}\right)}{6ab^3x^{\frac{5}{2}}\sqrt{-\frac{b}{a}}+6b^4x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}} + \frac{30a^2x^2\sqrt{-\frac{b}{a}}}{6ab^3x^{\frac{5}{2}}\sqrt{-\frac{b}{a}}+6b^4x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}} + \frac{15abx^{\frac{3}{2}}\log\left(\right.}{6ab^3x^{\frac{5}{2}}\sqrt{-\frac{b}{a}}\right.} \end{cases}$$

input
```
integrate(1/(a+b/x)**2/x**(9/2),x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**2*x**(3/2)), Eq(a
, 0)), (-2/(7*a**2*x**(7/2)), Eq(b, 0)), (15*a**2*x**(5/2)*log(sqrt(x) - s
qrt(-b/a))/(6*a*b**3*x**(5/2)*sqrt(-b/a) + 6*b**4*x**(3/2)*sqrt(-b/a)) - 1
5*a**2*x**(5/2)*log(sqrt(x) + sqrt(-b/a))/(6*a*b**3*x**(5/2)*sqrt(-b/a) +
6*b**4*x**(3/2)*sqrt(-b/a)) + 30*a**2*x**2*sqrt(-b/a)/(6*a*b**3*x**(5/2)*s
qrt(-b/a) + 6*b**4*x**(3/2)*sqrt(-b/a)) + 15*a*b*x**(3/2)*log(sqrt(x) - sq
rt(-b/a))/(6*a*b**3*x**(5/2)*sqrt(-b/a) + 6*b**4*x**(3/2)*sqrt(-b/a)) - 15
*a*b*x**(3/2)*log(sqrt(x) + sqrt(-b/a))/(6*a*b**3*x**(5/2)*sqrt(-b/a) + 6*
b**4*x**(3/2)*sqrt(-b/a)) + 20*a*b*x*sqrt(-b/a)/(6*a*b**3*x**(5/2)*sqrt(-b
/a) + 6*b**4*x**(3/2)*sqrt(-b/a)) - 4*b**2*sqrt(-b/a)/(6*a*b**3*x**(5/2)*s
qrt(-b/a) + 6*b**4*x**(3/2)*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx=\frac{a^2}{\left(ab^3+\frac{b^4}{x}\right)\sqrt{x}}-\frac{5\,a^2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}b^3}+\frac{2\left(\frac{6\,a}{\sqrt{x}}-\frac{b}{x^{\frac{3}{2}}}\right)}{3\,b^3}$$

input
```
integrate(1/(a+b/x)^2/x^(9/2),x, algorithm="maxima")
```

output
```
a^2/((a*b^3 + b^4/x)*sqrt(x)) - 5*a^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(
a*b)*b^3) + 2/3*(6*a/sqrt(x) - b/x^(3/2))/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx=\frac{5\,a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}+\frac{a^2\sqrt{x}}{(ax+b)b^3}+\frac{2\,(6\,ax-b)}{3\,b^3x^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^2/x^(9/2),x, algorithm="giac")
```

output
```
5*a^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + a^2*sqrt(x)/((a*x + b)
*b^3) + 2/3*(6*a*x - b)/(b^3*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx=\frac{\frac{5\,a^2\,x^2}{b^3}-\frac{2}{3\,b}+\frac{10\,a\,x}{3\,b^2}}{a\,x^{5/2}+b\,x^{3/2}}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
int(1/(x^(9/2)*(a + b/x)^2),x)
```

output
```
((5*a^2*x^2)/b^3 - 2/(3*b) + (10*a*x)/(3*b^2))/(a*x^(5/2) + b*x^(3/2)) + (
5*a^(3/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{9/2}}\,dx = \frac{15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2x^2+15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)abx+15a^2b\,x^2+10a\,b^2}{3\sqrt{x}\,b^4x\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^(9/2),x)
```

output
```
(15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*x**2
+ 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b*x + 1
5*a**2*b*x**2 + 10*a*b**2*x - 2*b**3)/(3*sqrt(x)*b**4*x*(a*x + b))
```

3.133 $\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\, dx = -\frac{2}{5b^2x^{5/2}} + \frac{4a}{3b^3x^{3/2}} - \frac{6a^2}{b^4\sqrt{x}} - \frac{a^3}{b^4\left(a+\frac{b}{x}\right)\sqrt{x}} + \frac{7a^{5/2}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{9/2}}$$

output
```
-2/5/b^2/x^(5/2)+4/3*a/b^3/x^(3/2)-6*a^2/b^4/x^(1/2)-a^3/b^4/(a+b/x)/x^(1/
2)+7*a^(5/2)*arctan(1/a^(1/2)/x^(1/2)*b^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\, dx = \frac{-6b^3+14ab^2x-70a^2bx^2-105a^3x^3}{15b^4x^{5/2}(b+ax)} - \frac{7a^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

input
```
Integrate[1/((a + b/x)^2*x^(11/2)),x]
```

output
```
(-6*b^3 + 14*a*b^2*x - 70*a^2*b*x^2 - 105*a^3*x^3)/(15*b^4*x^(5/2)*(b + a*
x)) - (7*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {795, 52, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/2}\left(a+\frac{b}{x}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{7/2}(ax+b)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{7\int \frac{1}{x^{7/2}(b+ax)}dx}{2b}+\frac{1}{bx^{5/2}(ax+b)}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(-\frac{a\int \frac{1}{x^{5/2}(b+ax)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(-\frac{a\left(-\frac{a\int \frac{1}{x^{3/2}(b+ax)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(-\frac{a\left(-\frac{a\left(-\frac{a\int \frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}$$

$$\Big\downarrow \text{73}$$

$$\frac{7\left(-\frac{a\left(-\frac{a\left(-\frac{2a\int \frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}$$

$$\Big\downarrow \text{218}$$

$$\frac{7\left(-\frac{a\left(-\frac{a\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}$$

input
```
Int[1/((a + b/x)^2*x^(11/2)),x]
```

output
```
1/(b*x^(5/2)*(b + a*x)) + (7*(-2/(5*b*x^(5/2)) - (a*(-2/(3*b*x^(3/2)) - (a
*(-2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)))
/b))/b))/(2*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2a^3\left(\frac{\sqrt{x}}{2ax+2b}+\frac{7\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}-\frac{2}{5b^2x^{\frac{5}{2}}}-\frac{6a^2}{b^4\sqrt{x}}+\frac{4a}{3b^3x^{\frac{3}{2}}}$	69
default	$-\frac{2a^3\left(\frac{\sqrt{x}}{2ax+2b}+\frac{7\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}-\frac{2}{5b^2x^{\frac{5}{2}}}-\frac{6a^2}{b^4\sqrt{x}}+\frac{4a}{3b^3x^{\frac{3}{2}}}$	69
risch	$-\frac{2(45a^2x^2-10abx+3b^2)}{15b^4x^{\frac{5}{2}}}-\frac{a^3\sqrt{x}}{b^4(ax+b)}-\frac{7a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{b^4\sqrt{ab}}$	71

input
```
int(1/(a+b/x)^2/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a^3/b^4*(1/2*x^(1/2)/(a*x+b)+7/2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/
2)))-2/5/b^2/x^(5/2)-6*a^2/b^4/x^(1/2)+4/3*a/b^3/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx = \left[\frac{105\,(a^3x^4+a^2bx^3)\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\,(105\,a^3x^3+70\,a^2bx^2-14\,ab^2x}{30\,(ab^4x^4+b^5x^3)}\right.$$

$$\left.-\frac{105\,(a^3x^4+a^2bx^3)\sqrt{\frac{a}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{a}{b}}\right)+(105\,a^3x^3+70\,a^2bx^2-14\,ab^2x+6\,b^3)\sqrt{x}}{15\,(ab^4x^4+b^5x^3)}\right]$$

input
```
integrate(1/(a+b/x)^2/x^(11/2),x, algorithm="fricas")
```

output
```
[1/30*(105*(a^3*x^4 + a^2*b*x^3)*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*sqrt(-a
/b) - b)/(a*x + b)) - 2*(105*a^3*x^3 + 70*a^2*b*x^2 - 14*a*b^2*x + 6*b^3)*
sqrt(x))/(a*b^4*x^4 + b^5*x^3), -1/15*(105*(a^3*x^4 + a^2*b*x^3)*sqrt(a/b)
*arctan(sqrt(x)*sqrt(a/b)) + (105*a^3*x^3 + 70*a^2*b*x^2 - 14*a*b^2*x + 6*
b^3)*sqrt(x))/(a*b^4*x^4 + b^5*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b/x)**2/x**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx=-\frac{a^3}{\left(ab^4+\frac{b^5}{x}\right)\sqrt{x}}+\frac{7\,a^3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{\sqrt{ab}b^4}-\frac{2\left(\frac{45\,a^2}{\sqrt{x}}-\frac{10\,ab}{x^{\frac{3}{2}}}+\frac{3\,b^2}{x^{\frac{5}{2}}}\right)}{15\,b^4}$$

input
```
integrate(1/(a+b/x)^2/x^(11/2),x, algorithm="maxima")
```

output
```
-a^3/((a*b^4 + b^5/x)*sqrt(x)) + 7*a^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt
(a*b)*b^4) - 2/15*(45*a^2/sqrt(x) - 10*a*b/x^(3/2) + 3*b^2/x^(5/2))/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx = -\frac{7\,a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} - \frac{a^3\sqrt{x}}{(ax+b)b^4} - \frac{2\,(45\,a^2x^2-10\,abx+3\,b^2)}{15\,b^4x^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)^2/x^(11/2),x, algorithm="giac")
```

output
```
-7*a^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - a^3*sqrt(x)/((a*x + b
)*b^4) - 2/15*(45*a^2*x^2 - 10*a*b*x + 3*b^2)/(b^4*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx = -\frac{\frac{2}{5\,b}+\frac{14\,a^2\,x^2}{3\,b^3}+\frac{7\,a^3\,x^3}{b^4}-\frac{14\,a\,x}{15\,b^2}}{a\,x^{7/2}+b\,x^{5/2}} - \frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

input
```
int(1/(x^(11/2)*(a + b/x)^2),x)
```

output
```
- (2/(5*b) + (14*a^2*x^2)/(3*b^3) + (7*a^3*x^3)/b^4 - (14*a*x)/(15*b^2))/(
a*x^(7/2) + b*x^(5/2)) - (7*a^(5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/b^(9/
2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^2 x^{11/2}}\,dx = \frac{-105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^3x^3 - 105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2 - 105a^3b\,x^3}{15\sqrt{x}\,b^5x^2\,(ax+b)}$$

input
```
int(1/(a+b/x)^2/x^(11/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**3*x
**3 - 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2
*b*x**2 - 105*a**3*b*x**3 - 70*a**2*b**2*x**2 + 14*a*b**3*x - 6*b**4)/(15*
sqrt(x)*b**5*x**2*(a*x + b))
```

3.134 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3} dx = \frac{12b^2\sqrt{x}}{a^5} - \frac{2bx^{3/2}}{a^4} + \frac{2x^{5/2}}{5a^3} - \frac{b^4\sqrt{x}}{2a^5(b+ax)^2} + \frac{17b^3\sqrt{x}}{4a^5(b+ax)} - \frac{63b^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{11/2}}$$

output
```
12*b^2*x^(1/2)/a^5-2*b*x^(3/2)/a^4+2/5*x^(5/2)/a^3-1/2*b^4*x^(1/2)/a^5/(a*
x+b)^2+17/4*b^3*x^(1/2)/a^5/(a*x+b)-63/4*b^(5/2)*arctan(a^(1/2)*x^(1/2)/b^
(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3} dx = \frac{\sqrt{x}(315b^4+525ab^3x+168a^2b^2x^2-24a^3bx^3+8a^4x^4)}{20a^5(b+ax)^2} - \frac{63b^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{11/2}}$$

input
```
Integrate[x^(3/2)/(a + b/x)^3,x]
```

output
```
(Sqrt[x]*(315*b^4 + 525*a*b^3*x + 168*a^2*b^2*x^2 - 24*a^3*b*x^3 + 8*a^4*x
^4))/(20*a^5*(b + a*x)^2) - (63*b^(5/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])
/(4*a^(11/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {795, 51, 51, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{x^{9/2}}{(ax+b)^3}dx$$

$$\Big\downarrow \; 51$$

$$\frac{9\int \frac{x^{7/2}}{(b+ax)^2}dx}{4a}-\frac{x^{9/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \; 51$$

$$\frac{9\left(\frac{7\int \frac{x^{5/2}}{b+ax}dx}{2a}-\frac{x^{7/2}}{a(ax+b)}\right)}{4a}-\frac{x^{9/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \; 60$$

$$\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\int \frac{x^{3/2}}{b+ax}dx}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}\right)}{4a}-\frac{x^{9/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\int\frac{\sqrt{x}}{b+ax}dx}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}\right)}{4a}-\frac{x^{9/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int\frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}\right)}{4a}-\frac{x^{9/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int\frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}\right)}{4a}-\frac{x^{9/2}}{2a(ax+b)^2}$$

$$\downarrow \text{218}$$

$$\frac{9\left(\frac{7\left(\frac{2x^{5/2}}{5a}-\frac{b\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}\right)}{a}\right)}{2a}-\frac{x^{7/2}}{a(ax+b)}\right)}{4a}-\frac{x^{9/2}}{2a(ax+b)^2}$$

input
```
Int[x^(3/2)/(a + b/x)^3,x]
```

output
```
-1/2*x^(9/2)/(a*(b + a*x)^2) + (9*(-(x^(7/2)/(a*(b + a*x))) + (7*((2*x^(5/
2))/(5*a) - (b*((2*x^(3/2))/(3*a) - (b*((2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[
(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)))/a))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{2\left(a^{2} x^{2}-5 a b x +30 b^{2}\right) \sqrt{x}}{5 a^{5}}-\frac{b^{3} \left(\frac{-\frac{17 a \,x^{\frac{3}{2}}}{4}-\frac{15 b \sqrt{x}}{4}}{\left(a x +b \right)^{2}}+\frac{63 \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{4 \sqrt{a b}}\right)}{a^{5}}$	77
derivativedivides	$\frac{\frac{2 a^{2} x^{\frac{5}{2}}}{5}-2 a b \,x^{\frac{3}{2}}+12 b^{2} \sqrt{x}}{a^{5}}-\frac{2 b^{3} \left(\frac{-\frac{17 a \,x^{\frac{3}{2}}}{8}-\frac{15 b \sqrt{x}}{8}}{\left(a x +b \right)^{2}}+\frac{63 \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{a^{5}}$	79
default	$\frac{\frac{2 a^{2} x^{\frac{5}{2}}}{5}-2 a b \,x^{\frac{3}{2}}+12 b^{2} \sqrt{x}}{a^{5}}-\frac{2 b^{3} \left(\frac{-\frac{17 a \,x^{\frac{3}{2}}}{8}-\frac{15 b \sqrt{x}}{8}}{\left(a x +b \right)^{2}}+\frac{63 \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{a^{5}}$	79

input
```
int(x^(3/2)/(a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*(a^2*x^2-5*a*b*x+30*b^2)*x^(1/2)/a^5-b^3/a^5*(2*(-17/8*a*x^(3/2)-15/8*
b*x^(1/2))/(a*x+b)^2+63/4/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.27

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx = \left[\frac{315\,(a^2b^2x^2+2\,ab^3x+b^4)\sqrt{-\frac{b}{a}}\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(8\,a^4x^4-24\,a^3bx^3+168\,a^2b}{40\,(a^7x^2+2\,a^6bx+a^5b^2)}\right.$$

$$-\frac{315\,(a^2b^2x^2+2\,ab^3x+b^4)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)-(8\,a^4x^4-24\,a^3bx^3+168\,a^2b^2x^2+525\,ab^3x+315\,b^4)}{20\,(a^7x^2+2\,a^6bx+a^5b^2)}$$

input
```
integrate(x^(3/2)/(a+b/x)^3,x, algorithm="fricas")
```

output
```
[1/40*(315*(a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(-b/a)*log((a*x - 2*a*sqrt(
x)*sqrt(-b/a) - b)/(a*x + b)) + 2*(8*a^4*x^4 - 24*a^3*b*x^3 + 168*a^2*b^2*
x^2 + 525*a*b^3*x + 315*b^4)*sqrt(x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2), -1/
20*(315*(a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/
a)/b) - (8*a^4*x^4 - 24*a^3*b*x^3 + 168*a^2*b^2*x^2 + 525*a*b^3*x + 315*b^
4)*sqrt(x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(107) = 214$.

Time = 27.00 (sec) , antiderivative size = 835, normalized size of antiderivative = 7.46

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)/(a+b/x)**3,x)
```

output
```
Piecewise((zoo*x**(11/2), Eq(a, 0) & Eq(b, 0)), (2*x**(11/2)/(11*b**3), Eq
(a, 0)), (2*x**(5/2)/(5*a**3), Eq(b, 0)), (16*a**5*x**(9/2)*sqrt(-b/a)/(40
*a**8*x**2*sqrt(-b/a) + 80*a**7*b*x*sqrt(-b/a) + 40*a**6*b**2*sqrt(-b/a))
- 48*a**4*b*x**(7/2)*sqrt(-b/a)/(40*a**8*x**2*sqrt(-b/a) + 80*a**7*b*x*sqr
t(-b/a) + 40*a**6*b**2*sqrt(-b/a)) + 336*a**3*b**2*x**(5/2)*sqrt(-b/a)/(40
*a**8*x**2*sqrt(-b/a) + 80*a**7*b*x*sqrt(-b/a) + 40*a**6*b**2*sqrt(-b/a))
+ 1050*a**2*b**3*x**(3/2)*sqrt(-b/a)/(40*a**8*x**2*sqrt(-b/a) + 80*a**7*b*
x*sqrt(-b/a) + 40*a**6*b**2*sqrt(-b/a)) - 315*a**2*b**3*x**2*log(sqrt(x) -
 sqrt(-b/a))/(40*a**8*x**2*sqrt(-b/a) + 80*a**7*b*x*sqrt(-b/a) + 40*a**6*b
**2*sqrt(-b/a)) + 315*a**2*b**3*x**2*log(sqrt(x) + sqrt(-b/a))/(40*a**8*x*
*2*sqrt(-b/a) + 80*a**7*b*x*sqrt(-b/a) + 40*a**6*b**2*sqrt(-b/a)) + 630*a*
b**4*sqrt(x)*sqrt(-b/a)/(40*a**8*x**2*sqrt(-b/a) + 80*a**7*b*x*sqrt(-b/a)
+ 40*a**6*b**2*sqrt(-b/a)) - 630*a*b**4*x*log(sqrt(x) - sqrt(-b/a))/(40*a*
*8*x**2*sqrt(-b/a) + 80*a**7*b*x*sqrt(-b/a) + 40*a**6*b**2*sqrt(-b/a)) + 6
30*a*b**4*x*log(sqrt(x) + sqrt(-b/a))/(40*a**8*x**2*sqrt(-b/a) + 80*a**7*b
*x*sqrt(-b/a) + 40*a**6*b**2*sqrt(-b/a)) - 315*b**5*log(sqrt(x) - sqrt(-b/
a))/(40*a**8*x**2*sqrt(-b/a) + 80*a**7*b*x*sqrt(-b/a) + 40*a**6*b**2*sqrt(
-b/a)) + 315*b**5*log(sqrt(x) + sqrt(-b/a))/(40*a**8*x**2*sqrt(-b/a) + 80*
a**7*b*x*sqrt(-b/a) + 40*a**6*b**2*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{8\,a^4 - \frac{24\,a^3b}{x} + \frac{168\,a^2b^2}{x^2} + \frac{525\,ab^3}{x^3} + \frac{315\,b^4}{x^4}}{20\left(\frac{a^7}{x^{\frac{5}{2}}} + \frac{2\,a^6b}{x^{\frac{7}{2}}} + \frac{a^5b^2}{x^{\frac{9}{2}}}\right)} + \frac{63\,b^3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\,\sqrt{ab}a^5}$$

input
```
integrate(x^(3/2)/(a+b/x)^3,x, algorithm="maxima")
```

output
```
1/20*(8*a^4 - 24*a^3*b/x + 168*a^2*b^2/x^2 + 525*a*b^3/x^3 + 315*b^4/x^4)/
(a^7/x^(5/2) + 2*a^6*b/x^(7/2) + a^5*b^2/x^(9/2)) + 63/4*b^3*arctan(b/(sqr
t(a*b)*sqrt(x)))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx = -\frac{63\,b^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^5}+\frac{17\,ab^3x^{\frac{3}{2}}+15\,b^4\sqrt{x}}{4\,(ax+b)^2a^5}+\frac{2\left(a^{12}x^{\frac{5}{2}}-5\,a^{11}bx^{\frac{3}{2}}+30\,a^{10}b^2\sqrt{x}\right)}{5\,a^{15}}$$

input
```
integrate(x^(3/2)/(a+b/x)^3,x, algorithm="giac")
```

output
```
-63/4*b^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/4*(17*a*b^3*x^(3
/2) + 15*b^4*sqrt(x))/((a*x + b)^2*a^5) + 2/5*(a^12*x^(5/2) - 5*a^11*b*x^(
3/2) + 30*a^10*b^2*sqrt(x))/a^15
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{\frac{15\,b^4\,\sqrt{x}}{4}+\frac{17\,a\,b^3\,x^{3/2}}{4}}{a^7\,x^2+2\,a^6\,b\,x+a^5\,b^2}+\frac{2\,x^{5/2}}{5\,a^3}-\frac{2\,b\,x^{3/2}}{a^4}+\frac{12\,b^2\,\sqrt{x}}{a^5}-\frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{11/2}}$$

input
```
int(x^(3/2)/(a + b/x)^3,x)
```

output
```
((15*b^4*x^(1/2))/4 + (17*a*b^3*x^(3/2))/4)/(a^5*b^2 + a^7*x^2 + 2*a^6*b*x
) + (2*x^(5/2))/(5*a^3) - (2*b*x^(3/2))/a^4 + (12*b^2*x^(1/2))/a^5 - (63*b
^(5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.35

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^2 - 630\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a\,b^3x - 315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right.}{20a^6\left(a^2x^2+2ab\right.}$$

input
```
int(x^(3/2)/(a+b/x)^3,x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**2
 - 630*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b**3*x - 315*
sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**4 + 8*sqrt(x)*a**5*
x**4 - 24*sqrt(x)*a**4*b*x**3 + 168*sqrt(x)*a**3*b**2*x**2 + 525*sqrt(x)*a
**2*b**3*x + 315*sqrt(x)*a*b**4)/(20*a**6*(a**2*x**2 + 2*a*b*x + b**2))
```

3.135 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3} dx = -\frac{6b\sqrt{x}}{a^4} + \frac{2x^{3/2}}{3a^3} + \frac{b^3\sqrt{x}}{2a^4(b+ax)^2} - \frac{13b^2\sqrt{x}}{4a^4(b+ax)} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{9/2}}$$

output
```
-6*b*x^(1/2)/a^4+2/3*x^(3/2)/a^3+1/2*b^3*x^(1/2)/a^4/(a*x+b)^2-13/4*b^2*x^
(1/2)/a^4/(a*x+b)+35/4*b^(3/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3} dx = \frac{\sqrt{x}(-105b^3 - 175ab^2x - 56a^2bx^2 + 8a^3x^3)}{12a^4(b+ax)^2} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{9/2}}$$

input
```
Integrate[Sqrt[x]/(a + b/x)^3,x]
```

output
```
(Sqrt[x]*(-105*b^3 - 175*a*b^2*x - 56*a^2*b*x^2 + 8*a^3*x^3))/(12*a^4*(b +
 a*x)^2) + (35*b^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {795, 51, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{7/2}}{(ax+b)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{7\int \frac{x^{5/2}}{(b+ax)^2}dx}{4a}-\frac{x^{7/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{7\left(\frac{5\int \frac{x^{3/2}}{b+ax}dx}{2a}-\frac{x^{5/2}}{a(ax+b)}\right)}{4a}-\frac{x^{7/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\int \frac{\sqrt{x}}{b+ax}dx}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}\right)}{4a}-\frac{x^{7/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}\right)}{4a}-\frac{x^{7/2}}{2a(ax+b)^2}$$

$\Big\downarrow$ 73

$$\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2b\int \frac{1}{b+ax}d\sqrt{x}}{a}\right)}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}\right)}{4a}-\frac{x^{7/2}}{2a(ax+b)^2}$$

$\Big\downarrow$ 218

$$\frac{7\left(\frac{5\left(\frac{2x^{3/2}}{3a}-\frac{b\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{a}\right)}{2a}-\frac{x^{5/2}}{a(ax+b)}\right)}{4a}-\frac{x^{7/2}}{2a(ax+b)^2}$$

input
```
Int[Sqrt[x]/(a + b/x)^3,x]
```

output
```
-1/2*x^(7/2)/(a*(b + a*x)^2) + (7*(-(x^(5/2)/(a*(b + a*x))) + (5*((2*x^(3/
2))/(3*a) - (b*((2*Sqrt[x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b
]])/a^(3/2)))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{2(ax-9b)\sqrt{x}}{3a^4}+\frac{b^2\left(\frac{-\frac{13a\,x^{\frac{3}{2}}}{4}-\frac{11b\sqrt{x}}{4}}{(ax+b)^2}+\frac{35\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^4}$	65
derivativedivides	$\frac{\frac{2a\,x^{\frac{3}{2}}}{3}-6b\sqrt{x}}{a^4}+\frac{2b^2\left(\frac{-\frac{13a\,x^{\frac{3}{2}}}{8}-\frac{11b\sqrt{x}}{8}}{(ax+b)^2}+\frac{35\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	68
default	$\frac{\frac{2a\,x^{\frac{3}{2}}}{3}-6b\sqrt{x}}{a^4}+\frac{2b^2\left(\frac{-\frac{13a\,x^{\frac{3}{2}}}{8}-\frac{11b\sqrt{x}}{8}}{(ax+b)^2}+\frac{35\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	68

input
```
int(x^(1/2)/(a+b/x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x-9*b)*x^(1/2)/a^4+b^2/a^4*(2*(-13/8*a*x^(3/2)-11/8*b*x^(1/2))/(a*x
+b)^2+35/4/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx$$
$$= \left[\frac{105\,(a^2bx^2+2\,ab^2x+b^3)\sqrt{-\frac{b}{a}}\log\left(\frac{ax+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(8\,a^3x^3-56\,a^2bx^2-175\,ab^2x-105\,b^3)\sqrt{x}}{24\,(a^6x^2+2\,a^5bx+a^4b^2)},\right.$$

input
```
integrate(x^(1/2)/(a+b/x)^3,x, algorithm="fricas")
```

output
```
[1/24*(105*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(-b/a)*log((a*x + 2*a*sqrt(x)
*sqrt(-b/a) - b)/(a*x + b)) + 2*(8*a^3*x^3 - 56*a^2*b*x^2 - 175*a*b^2*x -
105*b^3)*sqrt(x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/12*(105*(a^2*b*x^2 +
2*a*b^2*x + b^3)*sqrt(b/a)*arctan(a*sqrt(x)*sqrt(b/a)/b) + (8*a^3*x^3 - 56
*a^2*b*x^2 - 175*a*b^2*x - 105*b^3)*sqrt(x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^
2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(94) = 188$.

Time = 8.19 (sec) , antiderivative size = 762, normalized size of antiderivative = 7.70

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)/(a+b/x)**3,x)
```

output
```
Piecewise((zoo*x**(9/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*b**3), Eq(a,
 0)), (2*x**(3/2)/(3*a**3), Eq(b, 0)), (16*a**4*x**(7/2)*sqrt(-b/a)/(24*a*
*7*x**2*sqrt(-b/a) + 48*a**6*b*x*sqrt(-b/a) + 24*a**5*b**2*sqrt(-b/a)) - 1
12*a**3*b*x**(5/2)*sqrt(-b/a)/(24*a**7*x**2*sqrt(-b/a) + 48*a**6*b*x*sqrt(
-b/a) + 24*a**5*b**2*sqrt(-b/a)) - 350*a**2*b**2*x**(3/2)*sqrt(-b/a)/(24*a
**7*x**2*sqrt(-b/a) + 48*a**6*b*x*sqrt(-b/a) + 24*a**5*b**2*sqrt(-b/a)) +
105*a**2*b**2*x**2*log(sqrt(x) - sqrt(-b/a))/(24*a**7*x**2*sqrt(-b/a) + 48
*a**6*b*x*sqrt(-b/a) + 24*a**5*b**2*sqrt(-b/a)) - 105*a**2*b**2*x**2*log(s
qrt(x) + sqrt(-b/a))/(24*a**7*x**2*sqrt(-b/a) + 48*a**6*b*x*sqrt(-b/a) + 2
4*a**5*b**2*sqrt(-b/a)) - 210*a*b**3*sqrt(x)*sqrt(-b/a)/(24*a**7*x**2*sqrt
(-b/a) + 48*a**6*b*x*sqrt(-b/a) + 24*a**5*b**2*sqrt(-b/a)) + 210*a*b**3*x*
log(sqrt(x) - sqrt(-b/a))/(24*a**7*x**2*sqrt(-b/a) + 48*a**6*b*x*sqrt(-b/a
) + 24*a**5*b**2*sqrt(-b/a)) - 210*a*b**3*x*log(sqrt(x) + sqrt(-b/a))/(24*
a**7*x**2*sqrt(-b/a) + 48*a**6*b*x*sqrt(-b/a) + 24*a**5*b**2*sqrt(-b/a)) +
 105*b**4*log(sqrt(x) - sqrt(-b/a))/(24*a**7*x**2*sqrt(-b/a) + 48*a**6*b*x
*sqrt(-b/a) + 24*a**5*b**2*sqrt(-b/a)) - 105*b**4*log(sqrt(x) + sqrt(-b/a)
)/(24*a**7*x**2*sqrt(-b/a) + 48*a**6*b*x*sqrt(-b/a) + 24*a**5*b**2*sqrt(-b
/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{8\,a^3-\frac{56\,a^2b}{x}-\frac{175\,ab^2}{x^2}-\frac{105\,b^3}{x^3}}{12\left(\frac{a^6}{x^{\frac{3}{2}}}+\frac{2\,a^5b}{x^{\frac{5}{2}}}+\frac{a^4b^2}{x^{\frac{7}{2}}}\right)}-\frac{35\,b^2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\,\sqrt{ab}a^4}$$

input
```
integrate(x^(1/2)/(a+b/x)^3,x, algorithm="maxima")
```

output
```
1/12*(8*a^3 - 56*a^2*b/x - 175*a*b^2/x^2 - 105*b^3/x^3)/(a^6/x^(3/2) + 2*a
^5*b/x^(5/2) + a^4*b^2/x^(7/2)) - 35/4*b^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(
sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{35\,b^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^4}-\frac{13\,ab^2x^{\frac{3}{2}}+11\,b^3\sqrt{x}}{4\,(ax+b)^2a^4}+\frac{2\left(a^6x^{\frac{3}{2}}-9\,a^5b\sqrt{x}\right)}{3\,a^9}$$

input
```
integrate(x^(1/2)/(a+b/x)^3,x, algorithm="giac")
```

output
```
35/4*b^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/4*(13*a*b^2*x^(3/
2) + 11*b^3*sqrt(x))/((a*x + b)^2*a^4) + 2/3*(a^6*x^(3/2) - 9*a^5*b*sqrt(x
))/a^9
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx = \frac{2\,x^{3/2}}{3\,a^3} - \frac{\frac{11\,b^3\,\sqrt{x}}{4}+\frac{13\,a\,b^2\,x^{3/2}}{4}}{a^6\,x^2+2\,a^5\,b\,x+a^4\,b^2} - \frac{6\,b\,\sqrt{x}}{a^4} + \frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{9/2}}$$

input
```
int(x^(1/2)/(a + b/x)^3,x)
```

output
```
(2*x^(3/2))/(3*a^3) - ((11*b^3*x^(1/2))/4 + (13*a*b^2*x^(3/2))/4)/(a^4*b^2
 + a^6*x^2 + 2*a^5*b*x) - (6*b*x^(1/2))/a^4 + (35*b^(3/2)*atan((a^(1/2)*x^
(1/2))/b^(1/2)))/(4*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^3}\,dx$$
$$= \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2+210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x+105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^3+8\sqrt{x}\,a^4x^3 -}{12a^5\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(x^(1/2)/(a+b/x)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 210
*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b**2*x + 105*sqrt(b
)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**3 + 8*sqrt(x)*a**4*x**3 -
 56*sqrt(x)*a**3*b*x**2 - 175*sqrt(x)*a**2*b**2*x - 105*sqrt(x)*a*b**3)/(1
2*a**5*(a**2*x**2 + 2*a*b*x + b**2))
```

3.136 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 \sqrt{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 \sqrt{x}} \, dx = \frac{2\sqrt{x}}{a^3} - \frac{b^2\sqrt{x}}{2a^3(b+ax)^2} + \frac{9b\sqrt{x}}{4a^3(b+ax)} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{7/2}}$$

output
```
2*x^(1/2)/a^3-1/2*b^2*x^(1/2)/a^3/(a*x+b)^2+9/4*b*x^(1/2)/a^3/(a*x+b)-15/4
*b^(1/2)*arctan(a^(1/2)*x^(1/2)/b^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 \sqrt{x}} \, dx = \frac{\sqrt{x}(15b^2+25abx+8a^2x^2)}{4a^3(b+ax)^2} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{7/2}}$$

input
```
Integrate[1/((a + b/x)^3*Sqrt[x]),x]
```

output
```
(Sqrt[x]*(15*b^2 + 25*a*b*x + 8*a^2*x^2))/(4*a^3*(b + a*x)^2) - (15*Sqrt[b
]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {795, 51, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^{5/2}}{(ax+b)^3}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{5\int \frac{x^{3/2}}{(b+ax)^2}dx}{4a}-\frac{x^{5/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{x}}{b+ax}dx}{2a}-\frac{x^{3/2}}{a(ax+b)}\right)}{4a}-\frac{x^{5/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{a}-\frac{b\int \frac{1}{\sqrt{x}(b+ax)}dx}{a}\right)}{2a}-\frac{x^{3/2}}{a(ax+b)}\right)}{4a}-\frac{x^{5/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{a}-\frac{2b\int \frac{1}{b+ax}d\sqrt{x}}{a}\right)}{2a}-\frac{x^{3/2}}{a(ax+b)}\right)}{4a}-\frac{x^{5/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{5\left(\frac{3\left(\frac{2\sqrt{x}}{a}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{x^{3/2}}{a(ax+b)}\right)}{4a}-\frac{x^{5/2}}{2a(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*Sqrt[x]),x]
```

output
```
-1/2*x^(5/2)/(a*(b + a*x)^2) + (5*(-(x^(3/2)/(a*(b + a*x))) + (3*((2*Sqrt[
x])/a - (2*Sqrt[b]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/a^(3/2)))/(2*a)))/(4
*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{a^3}-\frac{2b\left(\frac{-\frac{9a\,x^{\frac{3}{2}}}{8}-\frac{7b\sqrt{x}}{8}}{(ax+b)^2}+\frac{15\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}$	56
default	$\frac{2\sqrt{x}}{a^3}-\frac{2b\left(\frac{-\frac{9a\,x^{\frac{3}{2}}}{8}-\frac{7b\sqrt{x}}{8}}{(ax+b)^2}+\frac{15\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^3}$	56
risch	$\frac{2\sqrt{x}}{a^3}-\frac{b\left(\frac{-\frac{9a\,x^{\frac{3}{2}}}{4}-\frac{7b\sqrt{x}}{4}}{(ax+b)^2}+\frac{15\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^3}$	57

input
```
int(1/(a+b/x)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/a^3-2/a^3*b*((-9/8*a*x^(3/2)-7/8*b*x^(1/2))/(a*x+b)^2+15/8/(a*b)
^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3\sqrt{x}}\,dx = \left[\frac{15\,(a^2x^2+2\,abx+b^2)\sqrt{-\frac{b}{a}}\log\left(\frac{ax-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-b}{ax+b}\right)+2\,(8\,a^2x^2+25\,abx+15\,b^2)\sqrt{x}}{8\,(a^5x^2+2\,a^4bx+a^3b^2)},\right.$$
$$\left.-\frac{15\,(a^2x^2+2\,abx+b^2)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{x}\sqrt{\frac{b}{a}}}{b}\right)-(8\,a^2x^2+25\,abx+15\,b^2)\sqrt{x}}{4\,(a^5x^2+2\,a^4bx+a^3b^2)}\right]$$

input
```
integrate(1/(a+b/x)^3/x^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-b/a)*log((a*x - 2*a*sqrt(x)*sqrt(
-b/a) - b)/(a*x + b)) + 2*(8*a^2*x^2 + 25*a*b*x + 15*b^2)*sqrt(x))/(a^5*x^
2 + 2*a^4*b*x + a^3*b^2), -1/4*(15*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(b/a)*arc
tan(a*sqrt(x)*sqrt(b/a)/b) - (8*a^2*x^2 + 25*a*b*x + 15*b^2)*sqrt(x))/(a^5
*x^2 + 2*a^4*b*x + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(78) = 156$.

Time = 8.78 (sec) , antiderivative size = 683, normalized size of antiderivative = 8.13

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3\sqrt{x}}\,dx = \begin{cases} \tilde{\infty}x^{\frac{7}{2}} \\ \frac{2x^{\frac{7}{2}}}{7b^3} \\ \frac{2\sqrt{x}}{a^3} \\ \frac{16a^3x^{\frac{5}{2}}\sqrt{-\frac{b}{a}}}{8a^6x^2\sqrt{-\frac{b}{a}}+16a^5bx\sqrt{-\frac{b}{a}}+8a^4b^2\sqrt{-\frac{b}{a}}} + \frac{50a^2bx^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{8a^6x^2\sqrt{-\frac{b}{a}}+16a^5bx\sqrt{-\frac{b}{a}}+8a^4b^2\sqrt{-\frac{b}{a}}} - \frac{15a^2bx^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{8a^6x^2\sqrt{-\frac{b}{a}}+16a^5bx\sqrt{-\frac{b}{a}}+8a^4b^2\sqrt{-\frac{b}{a}}} + \frac{}{8a^6x} \end{cases}$$

input
```
integrate(1/(a+b/x)**3/x**(1/2),x)
```

output
```
Piecewise((zoo*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*b**3), Eq(a,
 0)), (2*sqrt(x)/a**3, Eq(b, 0)), (16*a**3*x**(5/2)*sqrt(-b/a)/(8*a**6*x**
2*sqrt(-b/a) + 16*a**5*b*x*sqrt(-b/a) + 8*a**4*b**2*sqrt(-b/a)) + 50*a**2*
b*x**(3/2)*sqrt(-b/a)/(8*a**6*x**2*sqrt(-b/a) + 16*a**5*b*x*sqrt(-b/a) + 8
*a**4*b**2*sqrt(-b/a)) - 15*a**2*b*x**2*log(sqrt(x) - sqrt(-b/a))/(8*a**6*
x**2*sqrt(-b/a) + 16*a**5*b*x*sqrt(-b/a) + 8*a**4*b**2*sqrt(-b/a)) + 15*a*
*2*b*x**2*log(sqrt(x) + sqrt(-b/a))/(8*a**6*x**2*sqrt(-b/a) + 16*a**5*b*x*
sqrt(-b/a) + 8*a**4*b**2*sqrt(-b/a)) + 30*a*b**2*sqrt(x)*sqrt(-b/a)/(8*a**
6*x**2*sqrt(-b/a) + 16*a**5*b*x*sqrt(-b/a) + 8*a**4*b**2*sqrt(-b/a)) - 30*
a*b**2*x*log(sqrt(x) - sqrt(-b/a))/(8*a**6*x**2*sqrt(-b/a) + 16*a**5*b*x*s
qrt(-b/a) + 8*a**4*b**2*sqrt(-b/a)) + 30*a*b**2*x*log(sqrt(x) + sqrt(-b/a)
)/(8*a**6*x**2*sqrt(-b/a) + 16*a**5*b*x*sqrt(-b/a) + 8*a**4*b**2*sqrt(-b/a
)) - 15*b**3*log(sqrt(x) - sqrt(-b/a))/(8*a**6*x**2*sqrt(-b/a) + 16*a**5*b
*x*sqrt(-b/a) + 8*a**4*b**2*sqrt(-b/a)) + 15*b**3*log(sqrt(x) + sqrt(-b/a)
)/(8*a**6*x**2*sqrt(-b/a) + 16*a**5*b*x*sqrt(-b/a) + 8*a**4*b**2*sqrt(-b/a
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3\sqrt{x}}\,dx = \frac{8\,a^2+\frac{25\,ab}{x}+\frac{15\,b^2}{x^2}}{4\left(\frac{a^5}{\sqrt{x}}+\frac{2\,a^4b}{x^{\frac{3}{2}}}+\frac{a^3b^2}{x^{\frac{5}{2}}}\right)}+\frac{15\,b\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\sqrt{ab}a^3}$$

input
```
integrate(1/(a+b/x)^3/x^(1/2),x, algorithm="maxima")
```

output
```
1/4*(8*a^2 + 25*a*b/x + 15*b^2/x^2)/(a^5/sqrt(x) + 2*a^4*b/x^(3/2) + a^3*b
^2/x^(5/2)) + 15/4*b*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3\sqrt{x}}\,dx=-\frac{15\,b\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}+\frac{2\,\sqrt{x}}{a^3}+\frac{9\,abx^{\frac{3}{2}}+7\,b^2\sqrt{x}}{4\,(ax+b)^2a^3}$$

input
```
integrate(1/(a+b/x)^3/x^(1/2),x, algorithm="giac")
```

output
```
-15/4*b*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + 2*sqrt(x)/a^3 + 1/4*
(9*a*b*x^(3/2) + 7*b^2*sqrt(x))/((a*x + b)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3\sqrt{x}}\,dx=\frac{\frac{7\,b^2\,\sqrt{x}}{4}+\frac{9\,a\,b\,x^{3/2}}{4}}{a^5\,x^2+2\,a^4\,b\,x+a^3\,b^2}+\frac{2\,\sqrt{x}}{a^3}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{7/2}}$$

input
```
int(1/(x^(1/2)*(a + b/x)^3),x)
```

output
```
((7*b^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^3*b^2 + a^5*x^2 + 2*a^4*b*x) +
(2*x^(1/2))/a^3 - (15*b^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3\sqrt{x}}\,dx$$
$$=\frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2x^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)abx-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^2+8\sqrt{x}\,a^3x^2+25}{4a^4\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^(1/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*x**2 - 30*
sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b*x - 15*sqrt(b)*sqr
t(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**2 + 8*sqrt(x)*a**3*x**2 + 25*s
qrt(x)*a**2*b*x + 15*sqrt(x)*a*b**2)/(4*a**4*(a**2*x**2 + 2*a*b*x + b**2))
```

3.137 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}} \, dx = \frac{b\sqrt{x}}{2a^2(b+ax)^2} - \frac{5\sqrt{x}}{4a^2(b+ax)} + \frac{3\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{5/2}\sqrt{b}}$$

output
```
1/2*b*x^(1/2)/a^2/(a*x+b)^2-5/4*x^(1/2)/a^2/(a*x+b)+3/4*arctan(a^(1/2)*x^(
1/2)/b^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}} \, dx = -\frac{\sqrt{x}(3b+5ax)}{4a^2(b+ax)^2} + \frac{3\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{5/2}\sqrt{b}}$$

input
```
Integrate[1/((a + b/x)^3*x^(3/2)),x]
```

output
```
-1/4*(Sqrt[x]*(3*b + 5*a*x))/(a^2*(b + a*x)^2) + (3*ArcTan[(Sqrt[a]*Sqrt[x
])/Sqrt[b]])/(4*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {795, 51, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{3/2}}{(ax+b)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{3\int \frac{\sqrt{x}}{(b+ax)^2}dx}{4a}-\frac{x^{3/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(b+ax)}dx}{2a}-\frac{\sqrt{x}}{a(ax+b)}\right)}{4a}-\frac{x^{3/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\left(\frac{\int \frac{1}{b+ax}d\sqrt{x}}{a}-\frac{\sqrt{x}}{a(ax+b)}\right)}{4a}-\frac{x^{3/2}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{b}}-\frac{\sqrt{x}}{a(ax+b)}\right)}{4a}-\frac{x^{3/2}}{2a(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*x^(3/2)),x]
```

output
```
-1/2*x^(3/2)/(a*(b + a*x)^2) + (3*(-(Sqrt[x]/(a*(b + a*x))) + ArcTan[(Sqrt
[a]*Sqrt[x])/Sqrt[b]]/(a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{4a}-\frac{3b\sqrt{x}}{4a^{2}}}{(ax+b)^{2}}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4a^{2}\sqrt{ab}}$	50
default	$\frac{-\frac{5x^{\frac{3}{2}}}{4a}-\frac{3b\sqrt{x}}{4a^{2}}}{(ax+b)^{2}}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4a^{2}\sqrt{ab}}$	50

input
```
int(1/(a+b/x)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-5/8/a*x^(3/2)-3/8*b*x^(1/2)/a^2)/(a*x+b)^2+3/4/a^2/(a*b)^(1/2)*arctan(
a*x^(1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.61

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx = \left[-\frac{3\,(a^2x^2+2\,abx+b^2)\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)+2\,(5\,a^2bx+3\,ab^2)\sqrt{x}}{8\,(a^5bx^2+2\,a^4b^2x+a^3b^3)}, -\frac{3\,(a^2x^2+2\,abx+b^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)+(5\,a^2bx+3\,ab^2)\sqrt{x}}{4\,(a^5bx^2+2\,a^4b^2x+a^3b^3)}\right]$$

input
```
integrate(1/(a+b/x)^3/x^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*
sqrt(x))/(a*x + b)) + 2*(5*a^2*b*x + 3*a*b^2)*sqrt(x))/(a^5*b*x^2 + 2*a^4*
b^2*x + a^3*b^3), -1/4*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(a*b)*arctan(sqrt(
a*b)/(a*sqrt(x))) + (5*a^2*b*x + 3*a*b^2)*sqrt(x))/(a^5*b*x^2 + 2*a^4*b^2*
x + a^3*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(65) = 130.

Time = 18.83 (sec) , antiderivative size = 605, normalized size of antiderivative = 8.52

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx = \begin{cases} \tilde{\infty} x^{\frac{5}{2}} \\ \frac{2x^{\frac{5}{2}}}{5b^3} \\ -\frac{2}{a^3\sqrt{x}} \\ -\frac{10a^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{8a^5x^2\sqrt{-\frac{b}{a}}+16a^4bx\sqrt{-\frac{b}{a}}+8a^3b^2\sqrt{-\frac{b}{a}}} + \frac{3a^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{8a^5x^2\sqrt{-\frac{b}{a}}+16a^4bx\sqrt{-\frac{b}{a}}+8a^3b^2\sqrt{-\frac{b}{a}}} - \frac{3a^2x^2\log\left(\right.}{8a^5x^2\sqrt{-\frac{b}{a}}+16a^4b} \end{cases}$$

input
```
integrate(1/(a+b/x)**3/x**(3/2),x)
```

output
```
Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b**3), Eq(a,
 0)), (-2/(a**3*sqrt(x)), Eq(b, 0)), (-10*a**2*x**(3/2)*sqrt(-b/a)/(8*a**5
*x**2*sqrt(-b/a) + 16*a**4*b*x*sqrt(-b/a) + 8*a**3*b**2*sqrt(-b/a)) + 3*a*
*2*x**2*log(sqrt(x) - sqrt(-b/a))/(8*a**5*x**2*sqrt(-b/a) + 16*a**4*b*x*sq
rt(-b/a) + 8*a**3*b**2*sqrt(-b/a)) - 3*a**2*x**2*log(sqrt(x) + sqrt(-b/a))
/(8*a**5*x**2*sqrt(-b/a) + 16*a**4*b*x*sqrt(-b/a) + 8*a**3*b**2*sqrt(-b/a)
) - 6*a*b*sqrt(x)*sqrt(-b/a)/(8*a**5*x**2*sqrt(-b/a) + 16*a**4*b*x*sqrt(-b
/a) + 8*a**3*b**2*sqrt(-b/a)) + 6*a*b*x*log(sqrt(x) - sqrt(-b/a))/(8*a**5*
x**2*sqrt(-b/a) + 16*a**4*b*x*sqrt(-b/a) + 8*a**3*b**2*sqrt(-b/a)) - 6*a*b
*x*log(sqrt(x) + sqrt(-b/a))/(8*a**5*x**2*sqrt(-b/a) + 16*a**4*b*x*sqrt(-b
/a) + 8*a**3*b**2*sqrt(-b/a)) + 3*b**2*log(sqrt(x) - sqrt(-b/a))/(8*a**5*x
**2*sqrt(-b/a) + 16*a**4*b*x*sqrt(-b/a) + 8*a**3*b**2*sqrt(-b/a)) - 3*b**2
*log(sqrt(x) + sqrt(-b/a))/(8*a**5*x**2*sqrt(-b/a) + 16*a**4*b*x*sqrt(-b/a
) + 8*a**3*b**2*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx = -\frac{\frac{5\,a}{\sqrt{x}}+\frac{3\,b}{x^{\frac{3}{2}}}}{4\,\left(a^4+\frac{2\,a^3b}{x}+\frac{a^2b^2}{x^2}\right)} - \frac{3\,\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\,\sqrt{ab}a^2}$$

input
```
integrate(1/(a+b/x)^3/x^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(5*a/sqrt(x) + 3*b/x^(3/2))/(a^4 + 2*a^3*b/x + a^2*b^2/x^2) - 3/4*arc
tan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx = \frac{3\,\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^2} - \frac{5\,ax^{\frac{3}{2}}+3\,b\sqrt{x}}{4\,(ax+b)^2a^2}$$

input
```
integrate(1/(a+b/x)^3/x^(3/2),x, algorithm="giac")
```

output
```
3/4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/4*(5*a*x^(3/2) + 3*b*s
qrt(x))/((a*x + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{5/2}\,\sqrt{b}} - \frac{\frac{5\,x^{3/2}}{4\,a}+\frac{3\,b\,\sqrt{x}}{4\,a^2}}{a^2\,x^2+2\,a\,b\,x+b^2}$$

input
```
int(1/(x^(3/2)*(a + b/x)^3),x)
```

output
```
(3*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(5/2)*b^(1/2)) - ((5*x^(3/2))/(4*
a) + (3*b*x^(1/2))/(4*a^2))/(b^2 + a^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{3/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2x^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)abx+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^2-5}{4a^3b\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^(3/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*x**2 + 6*sqrt(
b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b*x + 3*sqrt(b)*sqrt(a)*a
tan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**2 - 5*sqrt(x)*a**2*b*x - 3*sqrt(x)*a
*b**2)/(4*a**3*b*(a**2*x**2 + 2*a*b*x + b**2))
```

3.138 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}} dx = -\frac{\sqrt{x}}{2a(b+ax)^2} + \frac{\sqrt{x}}{4ab(b+ax)} + \frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2}}$$

output
```
-1/2*x^(1/2)/a/(a*x+b)^2+1/4*x^(1/2)/a/b/(a*x+b)+1/4*arctan(a^(1/2)*x^(1/2
)/b^(1/2))/a^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}} dx = \frac{\sqrt{x}(-b+ax)}{4ab(b+ax)^2} + \frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2}}$$

input
```
Integrate[1/((a + b/x)^3*x^(5/2)),x]
```

output
```
(Sqrt[x]*(-b + a*x))/(4*a*b*(b + a*x)^2) + ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b
]]/(4*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {795, 51, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\sqrt{x}}{(ax+b)^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{\int \frac{1}{\sqrt{x}(b+ax)^2}dx}{4a}-\frac{\sqrt{x}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{\int \frac{1}{\sqrt{x}(b+ax)}dx}{2b}+\frac{\sqrt{x}}{b(ax+b)}}{4a}-\frac{\sqrt{x}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\int \frac{1}{b+ax}d\sqrt{x}}{b}+\frac{\sqrt{x}}{b(ax+b)}}{4a}-\frac{\sqrt{x}}{2a(ax+b)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}b^{3/2}}+\frac{\sqrt{x}}{b(ax+b)}}{4a}-\frac{\sqrt{x}}{2a(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*x^(5/2)),x]
```

output
```
-1/2*Sqrt[x]/(a*(b + a*x)^2) + (Sqrt[x]/(b*(b + a*x)) + ArcTan[(Sqrt[a]*Sq
rt[x])/Sqrt[b]]/(Sqrt[a]*b^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{3}{2}}}{4b}-\frac{\sqrt{x}}{4a}}{(ax+b)^2}+\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	52
default	$\frac{\frac{x^{\frac{3}{2}}}{4b}-\frac{\sqrt{x}}{4a}}{(ax+b)^2}+\frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	52

input
```
int(1/(a+b/x)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*x^(3/2)/b-1/8*x^(1/2)/a)/(a*x+b)^2+1/4/a/b/(a*b)^(1/2)*arctan(a*x^(
1/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.55

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx = \left[-\frac{(a^2x^2+2\,abx+b^2)\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)-2\,(a^2bx-ab^2)\sqrt{x}}{8\,(a^4b^2x^2+2\,a^3b^3x+a^2b^4)}, -\frac{(a^2x^2+2\,abx+b^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)-(a^2bx-ab^2)\sqrt{x}}{4\,(a^4b^2x^2+2\,a^3b^3x+a^2b^4)}\right]$$

input
```
integrate(1/(a+b/x)^3/x^(5/2),x, algorithm="fricas")
```

output
```
[-1/8*((a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sq
rt(x))/(a*x + b)) - 2*(a^2*b*x - a*b^2)*sqrt(x))/(a^4*b^2*x^2 + 2*a^3*b^3*
x + a^2*b^4), -1/4*((a^2*x^2 + 2*a*b*x + b^2)*sqrt(a*b)*arctan(sqrt(a*b)/(
a*sqrt(x))) - (a^2*b*x - a*b^2)*sqrt(x))/(a^4*b^2*x^2 + 2*a^3*b^3*x + a^2*
b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(58) = 116$.

Time = 45.42 (sec) , antiderivative size = 627, normalized size of antiderivative = 8.59

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} \\ \frac{2x^{\frac{3}{2}}}{3b^3} \\ -\frac{2}{3a^3x^{\frac{3}{2}}} \\ \frac{2a^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{8a^4bx^2\sqrt{-\frac{b}{a}}+16a^3b^2x\sqrt{-\frac{b}{a}}+8a^2b^3\sqrt{-\frac{b}{a}}} + \frac{a^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{8a^4bx^2\sqrt{-\frac{b}{a}}+16a^3b^2x\sqrt{-\frac{b}{a}}+8a^2b^3\sqrt{-\frac{b}{a}}} - \frac{a^2x^2\log}{8a^4bx^2\sqrt{-\frac{b}{a}}+16a} \end{cases}$$

input
```
integrate(1/(a+b/x)**3/x**(5/2),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b**3), Eq(a,
 0)), (-2/(3*a**3*x**(3/2)), Eq(b, 0)), (2*a**2*x**(3/2)*sqrt(-b/a)/(8*a**
4*b*x**2*sqrt(-b/a) + 16*a**3*b**2*x*sqrt(-b/a) + 8*a**2*b**3*sqrt(-b/a))
+ a**2*x**2*log(sqrt(x) - sqrt(-b/a))/(8*a**4*b*x**2*sqrt(-b/a) + 16*a**3*
b**2*x*sqrt(-b/a) + 8*a**2*b**3*sqrt(-b/a)) - a**2*x**2*log(sqrt(x) + sqrt
(-b/a))/(8*a**4*b*x**2*sqrt(-b/a) + 16*a**3*b**2*x*sqrt(-b/a) + 8*a**2*b**
3*sqrt(-b/a)) - 2*a*b*sqrt(x)*sqrt(-b/a)/(8*a**4*b*x**2*sqrt(-b/a) + 16*a*
*3*b**2*x*sqrt(-b/a) + 8*a**2*b**3*sqrt(-b/a)) + 2*a*b*x*log(sqrt(x) - sqr
t(-b/a))/(8*a**4*b*x**2*sqrt(-b/a) + 16*a**3*b**2*x*sqrt(-b/a) + 8*a**2*b*
*3*sqrt(-b/a)) - 2*a*b*x*log(sqrt(x) + sqrt(-b/a))/(8*a**4*b*x**2*sqrt(-b/
a) + 16*a**3*b**2*x*sqrt(-b/a) + 8*a**2*b**3*sqrt(-b/a)) + b**2*log(sqrt(x
) - sqrt(-b/a))/(8*a**4*b*x**2*sqrt(-b/a) + 16*a**3*b**2*x*sqrt(-b/a) + 8*
a**2*b**3*sqrt(-b/a)) - b**2*log(sqrt(x) + sqrt(-b/a))/(8*a**4*b*x**2*sqrt
(-b/a) + 16*a**3*b**2*x*sqrt(-b/a) + 8*a**2*b**3*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx = \frac{\frac{a}{\sqrt{x}}-\frac{b}{x^{\frac{3}{2}}}}{4\left(a^3b+\frac{2\,a^2b^2}{x}+\frac{ab^3}{x^2}\right)}-\frac{\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\sqrt{ab}ab}$$

input
```
integrate(1/(a+b/x)^3/x^(5/2),x, algorithm="maxima")
```

output
```
1/4*(a/sqrt(x) - b/x^(3/2))/(a^3*b + 2*a^2*b^2/x + a*b^3/x^2) - 1/4*arctan
(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx = \frac{\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}ab} + \frac{ax^{\frac{3}{2}} - b\sqrt{x}}{4\left(ax+b\right)^2 ab}$$

input
```
integrate(1/(a+b/x)^3/x^(5/2),x, algorithm="giac")
```

output
```
1/4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(a*x^(3/2) - b*sqrt(
x))/((a*x + b)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,a^{3/2}\,b^{3/2}} - \frac{\frac{\sqrt{x}}{4\,a} - \frac{x^{3/2}}{4\,b}}{a^2\,x^2 + 2\,a\,b\,x + b^2}$$

input
```
int(1/(x^(5/2)*(a + b/x)^3),x)
```

output
```
atan((a^(1/2)*x^(1/2))/b^(1/2))/(4*a^(3/2)*b^(3/2)) - (x^(1/2)/(4*a) - x^(
3/2)/(4*b))/(b^2 + a^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{5/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2x^2 + 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)abx + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^2 + \sqrt{x}}{4a^2b^2\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^(5/2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*x**2 + 2*sqrt(b)
*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b*x + sqrt(b)*sqrt(a)*atan(
(sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**2 + sqrt(x)*a**2*b*x - sqrt(x)*a*b**2)/(
4*a**2*b**2*(a**2*x**2 + 2*a*b*x + b**2))
```

3.139 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx=\frac{\sqrt{x}}{2b(b+ax)^2}+\frac{3\sqrt{x}}{4b^2(b+ax)}+\frac{3\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{a}b^{5/2}}$$

output
```
1/2*x^(1/2)/b/(a*x+b)^2+3/4*x^(1/2)/b^2/(a*x+b)+3/4*arctan(a^(1/2)*x^(1/2)
/b^(1/2))/a^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx=\frac{\sqrt{x}(5b+3ax)}{4b^2(b+ax)^2}+\frac{3\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{a}b^{5/2}}$$

input
```
Integrate[1/((a + b/x)^3*x^(7/2)),x]
```

output
```
(Sqrt[x]*(5*b + 3*a*x))/(4*b^2*(b + a*x)^2) + (3*ArcTan[(Sqrt[a]*Sqrt[x])/
Sqrt[b]])/(4*Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {795, 52, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(a+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{\sqrt{x}(ax+b)^3}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{3\int \frac{1}{\sqrt{x}(b+ax)^2}dx}{4b}+\frac{\sqrt{x}}{2b(ax+b)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{x}(b+ax)}dx}{2b}+\frac{\sqrt{x}}{b(ax+b)}\right)}{4b}+\frac{\sqrt{x}}{2b(ax+b)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{3\left(\frac{\int \frac{1}{b+ax}d\sqrt{x}}{b}+\frac{\sqrt{x}}{b(ax+b)}\right)}{4b}+\frac{\sqrt{x}}{2b(ax+b)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}b^{3/2}}+\frac{\sqrt{x}}{b(ax+b)}\right)}{4b}+\frac{\sqrt{x}}{2b(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*x^(7/2)),x]
```

output
```
Sqrt[x]/(2*b*(b + a*x)^2) + (3*(Sqrt[x]/(b*(b + a*x)) + ArcTan[(Sqrt[a]*Sq
rt[x])/Sqrt[b]]/(Sqrt[a]*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2b(ax+b)^2}+\frac{\frac{3\sqrt{x}}{4b(ax+b)}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4b\sqrt{ab}}}{b}$	59
default	$\frac{\sqrt{x}}{2b(ax+b)^2}+\frac{\frac{3\sqrt{x}}{4b(ax+b)}+\frac{3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4b\sqrt{ab}}}{b}$	59

input
```
int(1/(a+b/x)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)/b/(a*x+b)^2+3/2/b*(1/2*x^(1/2)/b/(a*x+b)+1/2/b/(a*b)^(1/2)*arc
tan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.66

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx = \left[-\frac{3\left(a^2x^2+2\,abx+b^2\right)\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)-2\left(3\,a^2bx+5\,ab^2\right)\sqrt{x}}{8\left(a^3b^3x^2+2\,a^2b^4x+ab^5\right)},\right.$$
$$\left.-\frac{3\left(a^2x^2+2\,abx+b^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)-\left(3\,a^2bx+5\,ab^2\right)\sqrt{x}}{4\left(a^3b^3x^2+2\,a^2b^4x+ab^5\right)}\right]$$

input
```
integrate(1/(a+b/x)^3/x^(7/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*
sqrt(x))/(a*x + b)) - 2*(3*a^2*b*x + 5*a*b^2)*sqrt(x))/(a^3*b^3*x^2 + 2*a^
2*b^4*x + a*b^5), -1/4*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(a*b)*arctan(sqrt(
a*b)/(a*sqrt(x))) - (3*a^2*b*x + 5*a*b^2)*sqrt(x))/(a^3*b^3*x^2 + 2*a^2*b^
4*x + a*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(61) = 122.

Time = 136.18 (sec) , antiderivative size = 632, normalized size of antiderivative = 9.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2\sqrt{x}}{b^3} \\ -\frac{2}{5a^3x^{\frac{5}{2}}} \\ \frac{6a^2x^{\frac{3}{2}}\sqrt{-\frac{b}{a}}}{8a^3b^2x^2\sqrt{-\frac{b}{a}}+16a^2b^3x\sqrt{-\frac{b}{a}}+8ab^4\sqrt{-\frac{b}{a}}} + \frac{3a^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{a}}\right)}{8a^3b^2x^2\sqrt{-\frac{b}{a}}+16a^2b^3x\sqrt{-\frac{b}{a}}+8ab^4\sqrt{-\frac{b}{a}}} - \frac{3a^2x^2\log}{8a^3b^2x^2\sqrt{-\frac{b}{a}}+16} \end{cases}$$

input
```
integrate(1/(a+b/x)**3/x**(7/2),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**3, Eq(a, 0)),
(-2/(5*a**3*x**(5/2)), Eq(b, 0)), (6*a**2*x**(3/2)*sqrt(-b/a)/(8*a**3*b**2
*x**2*sqrt(-b/a) + 16*a**2*b**3*x*sqrt(-b/a) + 8*a*b**4*sqrt(-b/a)) + 3*a*
*2*x**2*log(sqrt(x) - sqrt(-b/a))/(8*a**3*b**2*x**2*sqrt(-b/a) + 16*a**2*b
**3*x*sqrt(-b/a) + 8*a*b**4*sqrt(-b/a)) - 3*a**2*x**2*log(sqrt(x) + sqrt(-
b/a))/(8*a**3*b**2*x**2*sqrt(-b/a) + 16*a**2*b**3*x*sqrt(-b/a) + 8*a*b**4*
sqrt(-b/a)) + 10*a*b*sqrt(x)*sqrt(-b/a)/(8*a**3*b**2*x**2*sqrt(-b/a) + 16*
a**2*b**3*x*sqrt(-b/a) + 8*a*b**4*sqrt(-b/a)) + 6*a*b*x*log(sqrt(x) - sqrt
(-b/a))/(8*a**3*b**2*x**2*sqrt(-b/a) + 16*a**2*b**3*x*sqrt(-b/a) + 8*a*b**
4*sqrt(-b/a)) - 6*a*b*x*log(sqrt(x) + sqrt(-b/a))/(8*a**3*b**2*x**2*sqrt(-
b/a) + 16*a**2*b**3*x*sqrt(-b/a) + 8*a*b**4*sqrt(-b/a)) + 3*b**2*log(sqrt(
x) - sqrt(-b/a))/(8*a**3*b**2*x**2*sqrt(-b/a) + 16*a**2*b**3*x*sqrt(-b/a)
+ 8*a*b**4*sqrt(-b/a)) - 3*b**2*log(sqrt(x) + sqrt(-b/a))/(8*a**3*b**2*x**
2*sqrt(-b/a) + 16*a**2*b**3*x*sqrt(-b/a) + 8*a*b**4*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx = \frac{\frac{3\,a}{\sqrt{x}}+\frac{5\,b}{x^{\frac{3}{2}}}}{4\left(a^2b^2+\frac{2\,ab^3}{x}+\frac{b^4}{x^2}\right)} - \frac{3\,\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\sqrt{ab}b^2}$$

input
```
integrate(1/(a+b/x)^3/x^(7/2),x, algorithm="maxima")
```

output
```
1/4*(3*a/sqrt(x) + 5*b/x^(3/2))/(a^2*b^2 + 2*a*b^3/x + b^4/x^2) - 3/4*arct
an(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx = \frac{3\,\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}b^2} + \frac{3\,ax^{\frac{3}{2}}+5\,b\sqrt{x}}{4\,(ax+b)^2b^2}$$

input
```
integrate(1/(a+b/x)^3/x^(7/2),x, algorithm="giac")
```

output
```
3/4*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/4*(3*a*x^(3/2) + 5*b*s
qrt(x))/((a*x + b)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx = \frac{\frac{5\sqrt{x}}{4\,b}+\frac{3\,a\,x^{3/2}}{4\,b^2}}{a^2\,x^2+2\,a\,b\,x+b^2}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,\sqrt{a}\,b^{5/2}}$$

input
```
int(1/(x^(7/2)*(a + b/x)^3),x)
```

output
```
((5*x^(1/2))/(4*b) + (3*a*x^(3/2))/(4*b^2))/(b^2 + a^2*x^2 + 2*a*b*x) + (3
*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*a^(1/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{7/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2x^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)abx+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)b^2+3}{4a\,b^3\left(a^2x^2+2abx+b^2\right)}$$

input
```
int(1/(a+b/x)^3/x^(7/2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*x**2 + 6*sqrt(
b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b*x + 3*sqrt(b)*sqrt(a)*a
tan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**2 + 3*sqrt(x)*a**2*b*x + 5*sqrt(x)*a
*b**2)/(4*a*b**3*(a**2*x**2 + 2*a*b*x + b**2))
```

3.140 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx=-\frac{2}{b^3\sqrt{x}}+\frac{a^2}{2b^3\left(a+\frac{b}{x}\right)^2\sqrt{x}}-\frac{9a}{4b^3\left(a+\frac{b}{x}\right)\sqrt{x}}+\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{7/2}}$$

output
```
-2/b^3/x^(1/2)+1/2*a^2/b^3/(a+b/x)^2/x^(1/2)-9/4*a/b^3/(a+b/x)/x^(1/2)+15/
4*a^(1/2)*arctan(1/a^(1/2)/x^(1/2)*b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx=\frac{-8b^2-25abx-15a^2x^2}{4b^3\sqrt{x}(b+ax)^2}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4b^{7/2}}$$

input
```
Integrate[1/((a + b/x)^3*x^(9/2)),x]
```

output
```
(-8*b^2 - 25*a*b*x - 15*a^2*x^2)/(4*b^3*Sqrt[x]*(b + a*x)^2) - (15*Sqrt[a]
*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {795, 52, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2} \left(a+\frac{b}{x}\right)^3} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^{3/2} (a x+b)^3}dx$$

$$\Big\downarrow \text{ 52}$$

$$\frac{5 \int \frac{1}{x^{3/2} (b+a x)^2}dx}{4 b}+\frac{1}{2 b \sqrt{x} (a x+b)^2}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{5 \left(\frac{3 \int \frac{1}{x^{3/2} (b+a x)}dx}{2 b}+\frac{1}{b \sqrt{x} (a x+b)}\right)}{4 b}+\frac{1}{2 b \sqrt{x} (a x+b)^2}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{5 \left(\frac{3 \left(-\frac{a \int \frac{1}{\sqrt{x} (b+a x)}dx}{b}-\frac{2}{b \sqrt{x}}\right)}{2 b}+\frac{1}{b \sqrt{x} (a x+b)}\right)}{4 b}+\frac{1}{2 b \sqrt{x} (a x+b)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{5 \left(\frac{3 \left(-\frac{2 a \int \frac{1}{b+a x}d\sqrt{x}}{b}-\frac{2}{b \sqrt{x}}\right)}{2 b}+\frac{1}{b \sqrt{x} (a x+b)}\right)}{4 b}+\frac{1}{2 b \sqrt{x} (a x+b)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{5\left(\frac{3\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{2b}+\frac{1}{b\sqrt{x}(ax+b)}\right)}{4b}+\frac{1}{2b\sqrt{x}(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*x^(9/2)),x]
```

output
```
1/(2*b*Sqrt[x]*(b + a*x)^2) + (5*(1/(b*Sqrt[x]*(b + a*x)) + (3*(-2/(b*Sqrt
[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/(2*b)))/(4*
b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$-\frac{2}{b^{3}\sqrt{x}}-\frac{2a\left(\frac{\frac{7a\,x^{\frac{3}{2}}}{8}+\frac{9b\sqrt{x}}{8}}{(ax+b)^{2}}+\frac{15\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{3}}$	56
default	$-\frac{2}{b^{3}\sqrt{x}}-\frac{2a\left(\frac{\frac{7a\,x^{\frac{3}{2}}}{8}+\frac{9b\sqrt{x}}{8}}{(ax+b)^{2}}+\frac{15\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{3}}$	56
risch	$-\frac{2}{b^{3}\sqrt{x}}-\frac{a\left(\frac{\frac{7a\,x^{\frac{3}{2}}}{4}+\frac{9b\sqrt{x}}{4}}{(ax+b)^{2}}+\frac{15\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^{3}}$	57

input
```
int(1/(a+b/x)^3/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^3/x^(1/2)-2/b^3*a*((7/8*a*x^(3/2)+9/8*b*x^(1/2))/(a*x+b)^2+15/8/(a*b)
^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3}x^{9/2}}\,dx=\left[\frac{15\left(a^{2}x^{3}+2\,abx^{2}+b^{2}x\right)\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\left(15\,a^{2}x^{2}+25\,abx+8\,b^{2}\right)\sqrt{x}}{8\left(a^{2}b^{3}x^{3}+2\,ab^{4}x^{2}+b^{5}x\right)}\right.$$

$$\left.-\frac{15\left(a^{2}x^{3}+2\,abx^{2}+b^{2}x\right)\sqrt{\frac{a}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{a}{b}}\right)+\left(15\,a^{2}x^{2}+25\,abx+8\,b^{2}\right)\sqrt{x}}{4\left(a^{2}b^{3}x^{3}+2\,ab^{4}x^{2}+b^{5}x\right)}\right]$$

input
```
integrate(1/(a+b/x)^3/x^(9/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(a^2*x^3 + 2*a*b*x^2 + b^2*x)*sqrt(-a/b)*log((a*x - 2*b*sqrt(x)*s
qrt(-a/b) - b)/(a*x + b)) - 2*(15*a^2*x^2 + 25*a*b*x + 8*b^2)*sqrt(x))/(a^
2*b^3*x^3 + 2*a*b^4*x^2 + b^5*x), -1/4*(15*(a^2*x^3 + 2*a*b*x^2 + b^2*x)*s
qrt(a/b)*arctan(sqrt(x)*sqrt(a/b)) + (15*a^2*x^2 + 25*a*b*x + 8*b^2)*sqrt(
x))/(a^2*b^3*x^3 + 2*a*b^4*x^2 + b^5*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**3/x**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx = -\frac{\frac{7a^2}{\sqrt{x}}+\frac{9ab}{x^{\frac{3}{2}}}}{4\left(a^2b^3+\frac{2ab^4}{x}+\frac{b^5}{x^2}\right)}+\frac{15a\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\sqrt{ab}b^3}-\frac{2}{b^3\sqrt{x}}$$

input
```
integrate(1/(a+b/x)^3/x^(9/2),x, algorithm="maxima")
```

output
```
-1/4*(7*a^2/sqrt(x) + 9*a*b/x^(3/2))/(a^2*b^3 + 2*a*b^4/x + b^5/x^2) + 15/
4*a*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^3) - 2/(b^3*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx = -\frac{15\,a\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^3} - \frac{2}{b^3\sqrt{x}} - \frac{7\,a^2x^{\frac{3}{2}} + 9\,ab\sqrt{x}}{4\,(ax+b)^2b^3}$$

input
```
integrate(1/(a+b/x)^3/x^(9/2),x, algorithm="giac")
```

output
```
-15/4*a*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - 2/(b^3*sqrt(x)) - 1/
4*(7*a^2*x^(3/2) + 9*a*b*sqrt(x))/((a*x + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx = -\frac{\frac{2}{b} + \frac{15\,a^2\,x^2}{4\,b^3} + \frac{25\,a\,x}{4\,b^2}}{a^2\,x^{5/2} + b^2\,\sqrt{x} + 2\,a\,b\,x^{3/2}} - \frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,b^{7/2}}$$

input
```
int(1/(x^(9/2)*(a + b/x)^3),x)
```

output
```
- (2/b + (15*a^2*x^2)/(4*b^3) + (25*a*x)/(4*b^2))/(a^2*x^(5/2) + b^2*x^(1/
2) + 2*a*b*x^(3/2)) - (15*a^(1/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*b^(7
/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{9/2}}\,dx = \frac{-15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2x^2 - 30\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)abx - 15\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,at}{4\sqrt{x}\,b^4\,(a^2x^2+2abx+b^2)}$$

input
```
int(1/(a+b/x)^3/x^(9/2),x)
```

output
```
( - 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*x*
*2 - 30*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b*x
- 15*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*b**2 - 15
*a**2*b*x**2 - 25*a*b**2*x - 8*b**3)/(4*sqrt(x)*b**4*(a**2*x**2 + 2*a*b*x
+ b**2))
```

3.141 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx=-\frac{2}{3b^3x^{3/2}}+\frac{6a}{b^4\sqrt{x}}-\frac{a^3}{2b^4\left(a+\frac{b}{x}\right)^2\sqrt{x}}+\frac{13a^2}{4b^4\left(a+\frac{b}{x}\right)\sqrt{x}}-\frac{35a^{3/2}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{9/2}}$$

output
```
-2/3/b^3/x^(3/2)+6*a/b^4/x^(1/2)-1/2*a^3/b^4/(a+b/x)^2/x^(1/2)+13/4*a^2/b^
4/(a+b/x)/x^(1/2)-35/4*a^(3/2)*arctan(1/a^(1/2)/x^(1/2)*b^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx=\frac{-8b^3+56ab^2x+175a^2bx^2+105a^3x^3}{12b^4x^{3/2}(b+ax)^2}+\frac{35a^{3/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4b^{9/2}}$$

input
```
Integrate[1/((a + b/x)^3*x^(11/2)),x]
```

output
```
(-8*b^3 + 56*a*b^2*x + 175*a^2*b*x^2 + 105*a^3*x^3)/(12*b^4*x^(3/2)*(b + a
*x)^2) + (35*a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {795, 52, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/2}\left(a+\frac{b}{x}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{5/2}(ax+b)^3}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{7\int \frac{1}{x^{5/2}(b+ax)^2}dx}{4b}+\frac{1}{2bx^{3/2}(ax+b)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{7\left(\frac{5\int \frac{1}{x^{5/2}(b+ax)}dx}{2b}+\frac{1}{bx^{3/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{3/2}(ax+b)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(\frac{5\left(-\frac{a\int \frac{1}{x^{3/2}(b+ax)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{3/2}(ax+b)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{7\left(\frac{5\left(-\frac{a\left(-\frac{a\int\frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{3/2}(ax+b)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{7\left(\frac{5\left(-\frac{a\left(-\frac{2a\int\frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{3/2}(ax+b)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{7\left(\frac{5\left(-\frac{a\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{3/2}(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*x^(11/2)),x]
```

output
```
1/(2*b*x^(3/2)*(b + a*x)^2) + (7*(1/(b*x^(3/2)*(b + a*x)) + (5*(-2/(3*b*x^
(3/2)) - (a*(-2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]]
)/b^(3/2)))/b))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{6ax-\frac{2b}{3}}{b^4x^{\frac{3}{2}}}+\frac{a^2\left(\frac{\frac{11a\,x^{\frac{3}{2}}}{4}+\frac{13b\sqrt{x}}{4}}{(ax+b)^2}+\frac{35\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^4}$	66
derivativedivides	$-\frac{2}{3b^3x^{\frac{3}{2}}}+\frac{6a}{b^4\sqrt{x}}+\frac{2a^2\left(\frac{\frac{11a\,x^{\frac{3}{2}}}{8}+\frac{13b\sqrt{x}}{8}}{(ax+b)^2}+\frac{35\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	67
default	$-\frac{2}{3b^3x^{\frac{3}{2}}}+\frac{6a}{b^4\sqrt{x}}+\frac{2a^2\left(\frac{\frac{11a\,x^{\frac{3}{2}}}{8}+\frac{13b\sqrt{x}}{8}}{(ax+b)^2}+\frac{35\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	67

input
```
int(1/(a+b/x)^3/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(9*a*x-b)/b^4/x^(3/2)+a^2/b^4*(2*(11/8*a*x^(3/2)+13/8*b*x^(1/2))/(a*x+
b)^2+35/4/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx = \left[\frac{105\left(a^3x^4+2\,a^2bx^3+ab^2x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{ax+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)+2\left(105\,a^3x^3+175\,a^2bx\right.}{24\left(a^2b^4x^4+2\,ab^5x^3+b^6x^2\right)}\right.$$

input
```
integrate(1/(a+b/x)^3/x^(11/2),x, algorithm="fricas")
```

output
```
[1/24*(105*(a^3*x^4 + 2*a^2*b*x^3 + a*b^2*x^2)*sqrt(-a/b)*log((a*x + 2*b*s
qrt(x)*sqrt(-a/b) - b)/(a*x + b)) + 2*(105*a^3*x^3 + 175*a^2*b*x^2 + 56*a*
b^2*x - 8*b^3)*sqrt(x))/(a^2*b^4*x^4 + 2*a*b^5*x^3 + b^6*x^2), 1/12*(105*(
a^3*x^4 + 2*a^2*b*x^3 + a*b^2*x^2)*sqrt(a/b)*arctan(sqrt(x)*sqrt(a/b)) + (
105*a^3*x^3 + 175*a^2*b*x^2 + 56*a*b^2*x - 8*b^3)*sqrt(x))/(a^2*b^4*x^4 +
2*a*b^5*x^3 + b^6*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**3/x**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx = \frac{\frac{11\,a^3}{\sqrt{x}}+\frac{13\,a^2b}{x^{\frac{3}{2}}}}{4\left(a^2b^4+\frac{2\,ab^5}{x}+\frac{b^6}{x^2}\right)} - \frac{35\,a^2\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\sqrt{ab}b^4} + \frac{2\left(\frac{9\,a}{\sqrt{x}}-\frac{b}{x^{\frac{3}{2}}}\right)}{3\,b^4}$$

input
```
integrate(1/(a+b/x)^3/x^(11/2),x, algorithm="maxima")
```

output
```
1/4*(11*a^3/sqrt(x) + 13*a^2*b/x^(3/2))/(a^2*b^4 + 2*a*b^5/x + b^6/x^2) -
35/4*a^2*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^4) + 2/3*(9*a/sqrt(x)
- b/x^(3/2))/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx = \frac{35\,a^2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^4} + \frac{2\,(9\,ax-b)}{3\,b^4x^{\frac{3}{2}}} + \frac{11\,a^3x^{\frac{3}{2}}+13\,a^2b\sqrt{x}}{4\,(ax+b)^2b^4}$$

input
```
integrate(1/(a+b/x)^3/x^(11/2),x, algorithm="giac")
```

output
```
35/4*a^2*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(9*a*x - b)/(b^
4*x^(3/2)) + 1/4*(11*a^3*x^(3/2) + 13*a^2*b*sqrt(x))/((a*x + b)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx = \frac{\frac{175\,a^2\,x^2}{12\,b^3}-\frac{2}{3\,b}+\frac{35\,a^3\,x^3}{4\,b^4}+\frac{14\,a\,x}{3\,b^2}}{a^2\,x^{7/2}+b^2\,x^{3/2}+2\,a\,b\,x^{5/2}} + \frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,b^{9/2}}$$

input
```
int(1/(x^(11/2)*(a + b/x)^3),x)
```

output
```
((175*a^2*x^2)/(12*b^3) - 2/(3*b) + (35*a^3*x^3)/(4*b^4) + (14*a*x)/(3*b^2
))/(a^2*x^(7/2) + b^2*x^(3/2) + 2*a*b*x^(5/2)) + (35*a^(3/2)*atan((a^(1/2)
*x^(1/2))/b^(1/2)))/(4*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{11/2}}\,dx = \frac{105\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^3x^3+210\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2+105\sqrt{x}\,\sqrt{b}\,\sqrt{}}{12\sqrt{x}\,b^5x\,(a^2x^2+2abx+}$$

input
```
int(1/(a+b/x)^3/x^(11/2),x)
```

output
```
(105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**3*x**3
 + 210*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**2*b*
x**2 + 105*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a*b
**2*x + 105*a**3*b*x**3 + 175*a**2*b**2*x**2 + 56*a*b**3*x - 8*b**4)/(12*s
qrt(x)*b**5*x*(a**2*x**2 + 2*a*b*x + b**2))
```

3.142 $\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}} \, dx = -\frac{2}{5b^3x^{5/2}}+\frac{2a}{b^4x^{3/2}}-\frac{12a^2}{b^5\sqrt{x}}+\frac{a^4}{2b^5\left(a+\frac{b}{x}\right)^2\sqrt{x}}-\frac{17a^3}{4b^5\left(a+\frac{b}{x}\right)\sqrt{x}}+\frac{63a^{5/2}\arctan\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{11/2}}$$

output
```
-2/5/b^3/x^(5/2)+2*a/b^4/x^(3/2)-12*a^2/b^5/x^(1/2)+1/2*a^4/b^5/(a+b/x)^2/
x^(1/2)-17/4*a^3/b^5/(a+b/x)/x^(1/2)+63/4*a^(5/2)*arctan(1/a^(1/2)/x^(1/2)
*b^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}} \, dx = \frac{-8b^4+24ab^3x-168a^2b^2x^2-525a^3bx^3-315a^4x^4}{20b^5x^{5/2}(b+ax)^2}-\frac{63a^{5/2}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4b^{11/2}}$$

input
```
Integrate[1/((a + b/x)^3*x^(13/2)),x]
```

output
```
(-8*b^4 + 24*a*b^3*x - 168*a^2*b^2*x^2 - 525*a^3*b*x^3 - 315*a^4*x^4)/(20*
b^5*x^(5/2)*(b + a*x)^2) - (63*a^(5/2)*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/
(4*b^(11/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {795, 52, 52, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13/2} \left(a+\frac{b}{x}\right)^3} \, dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{1}{x^{7/2}(ax+b)^3}dx$$
$$\Big\downarrow \text{52}$$
$$\frac{9\int \frac{1}{x^{7/2}(b+ax)^2}dx}{4b}+\frac{1}{2bx^{5/2}(ax+b)^2}$$
$$\Big\downarrow \text{52}$$
$$\frac{9\left(\frac{7\int \frac{1}{x^{7/2}(b+ax)}dx}{2b}+\frac{1}{bx^{5/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{5/2}(ax+b)^2}$$
$$\Big\downarrow \text{61}$$
$$\frac{9\left(\frac{7\left(-\frac{a\int \frac{1}{x^{5/2}(b+ax)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{5/2}(ax+b)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{9\left(\frac{7\left(-\frac{a\left(-\frac{a\int \frac{1}{x^{3/2}(b+ax)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{5/2}(ax+b)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{9\left(\frac{7\left(-\frac{a\left(-\frac{a\left(-\frac{a\int \frac{1}{\sqrt{x}(b+ax)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{5/2}(ax+b)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{9\left(\frac{7\left(-\frac{a\left(-\frac{a\left(-\frac{2a\int \frac{1}{b+ax}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{5/2}(ax+b)^2}$$

$$\downarrow \color{blue}{218}$$

$$\frac{9\left(\frac{7\left(-\frac{a\left(-\frac{a\left(-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2b}+\frac{1}{bx^{5/2}(ax+b)}\right)}{4b}+\frac{1}{2bx^{5/2}(ax+b)^2}$$

input
```
Int[1/((a + b/x)^3*x^(13/2)),x]
```

output
```
1/(2*b*x^(5/2)*(b + a*x)^2) + (9*(1/(b*x^(5/2)*(b + a*x)) + (7*(-2/(5*b*x^
(5/2)) - (a*(-2/(3*b*x^(3/2)) - (a*(-2/(b*Sqrt[x]) - (2*Sqrt[a]*ArcTan[(Sq
rt[a]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/b))/b))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{2\left(30 a^{2} x^{2}-5 a b x+b^{2}\right)}{5 b^{5} x^{\frac{5}{2}}}-\frac{a^{3} \left(\frac{\frac{15 a \,x^{\frac{3}{2}}}{4}+\frac{17 b \sqrt{x}}{4}}{\left(a x +b \right)^{2}}+\frac{63 \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{4 \sqrt{a b}}\right)}{b^{5}}$	76
derivativedivides	$-\frac{2}{5 b^{3} x^{\frac{5}{2}}}-\frac{12 a^{2}}{b^{5} \sqrt{x}}+\frac{2 a}{b^{4} x^{\frac{3}{2}}}-\frac{2 a^{3} \left(\frac{\frac{15 a \,x^{\frac{3}{2}}}{8}+\frac{17 b \sqrt{x}}{8}}{\left(a x +b \right)^{2}}+\frac{63 \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{b^{5}}$	78
default	$-\frac{2}{5 b^{3} x^{\frac{5}{2}}}-\frac{12 a^{2}}{b^{5} \sqrt{x}}+\frac{2 a}{b^{4} x^{\frac{3}{2}}}-\frac{2 a^{3} \left(\frac{\frac{15 a \,x^{\frac{3}{2}}}{8}+\frac{17 b \sqrt{x}}{8}}{\left(a x +b \right)^{2}}+\frac{63 \arctan \left(\frac{a \sqrt{x}}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{b^{5}}$	78

input
```
int(1/(a+b/x)^3/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(30*a^2*x^2-5*a*b*x+b^2)/b^5/x^(5/2)-1/b^5*a^3*(2*(15/8*a*x^(3/2)+17/
8*b*x^(1/2))/(a*x+b)^2+63/4/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx = \left[\frac{315\left(a^4x^5+2\,a^3bx^4+a^2b^2x^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{ax-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-b}{ax+b}\right)-2\left(315\,a^4x^4+525\,a^3b\right.}{40\left(a^2b^5x^5+2\,ab^6x^4+b^7x^3\right)}\right.$$

$$-\frac{315\left(a^4x^5+2\,a^3bx^4+a^2b^2x^3\right)\sqrt{\frac{a}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{a}{b}}\right)+\left(315\,a^4x^4+525\,a^3bx^3+168\,a^2b^2x^2-24\,ab^3x+8\,b\right.}{20\left(a^2b^5x^5+2\,ab^6x^4+b^7x^3\right)}$$

input
```
integrate(1/(a+b/x)^3/x^(13/2),x, algorithm="fricas")
```

output
```
[1/40*(315*(a^4*x^5 + 2*a^3*b*x^4 + a^2*b^2*x^3)*sqrt(-a/b)*log((a*x - 2*b
*sqrt(x)*sqrt(-a/b) - b)/(a*x + b)) - 2*(315*a^4*x^4 + 525*a^3*b*x^3 + 168
*a^2*b^2*x^2 - 24*a*b^3*x + 8*b^4)*sqrt(x))/(a^2*b^5*x^5 + 2*a*b^6*x^4 + b
^7*x^3), -1/20*(315*(a^4*x^5 + 2*a^3*b*x^4 + a^2*b^2*x^3)*sqrt(a/b)*arctan
(sqrt(x)*sqrt(a/b)) + (315*a^4*x^4 + 525*a^3*b*x^3 + 168*a^2*b^2*x^2 - 24*
a*b^3*x + 8*b^4)*sqrt(x))/(a^2*b^5*x^5 + 2*a*b^6*x^4 + b^7*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**3/x**(13/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx = -\frac{\frac{15\,a^4}{\sqrt{x}}+\frac{17\,a^3b}{x^{\frac{3}{2}}}}{4\,\left(a^2b^5+\frac{2\,ab^6}{x}+\frac{b^7}{x^2}\right)} + \frac{63\,a^3\arctan\left(\frac{b}{\sqrt{ab}\sqrt{x}}\right)}{4\,\sqrt{ab}b^5} - \frac{2\,\left(\frac{30\,a^2}{\sqrt{x}}-\frac{5\,ab}{x^{\frac{3}{2}}}+\frac{b^2}{x^{\frac{5}{2}}}\right)}{5\,b^5}$$

input
```
integrate(1/(a+b/x)^3/x^(13/2),x, algorithm="maxima")
```

output
```
-1/4*(15*a^4/sqrt(x) + 17*a^3*b/x^(3/2))/(a^2*b^5 + 2*a*b^6/x + b^7/x^2) +
 63/4*a^3*arctan(b/(sqrt(a*b)*sqrt(x)))/(sqrt(a*b)*b^5) - 2/5*(30*a^2/sqrt
(x) - 5*a*b/x^(3/2) + b^2/x^(5/2))/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx = -\frac{63\,a^3\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}b^5} - \frac{15\,a^4x^{\frac{3}{2}}+17\,a^3b\sqrt{x}}{4\,(ax+b)^2b^5} - \frac{2\,(30\,a^2x^2-5\,abx+b^2)}{5\,b^5x^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)^3/x^(13/2),x, algorithm="giac")
```

output
```
-63/4*a^3*arctan(a*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/4*(15*a^4*x^(3/2
) + 17*a^3*b*sqrt(x))/((a*x + b)^2*b^5) - 2/5*(30*a^2*x^2 - 5*a*b*x + b^2)
/(b^5*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx = -\frac{\frac{2}{5\,b}+\frac{42\,a^2\,x^2}{5\,b^3}+\frac{105\,a^3\,x^3}{4\,b^4}+\frac{63\,a^4\,x^4}{4\,b^5}-\frac{6\,a\,x}{5\,b^2}}{a^2\,x^{9/2}+b^2\,x^{5/2}+2\,a\,b\,x^{7/2}}-\frac{63\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{4\,b^{11/2}}$$

input
```
int(1/(x^(13/2)*(a + b/x)^3),x)
```

output
```
- (2/(5*b) + (42*a^2*x^2)/(5*b^3) + (105*a^3*x^3)/(4*b^4) + (63*a^4*x^4)/(
4*b^5) - (6*a*x)/(5*b^2))/(a^2*x^(9/2) + b^2*x^(5/2) + 2*a*b*x^(7/2)) - (6
3*a^(5/2)*atan((a^(1/2)*x^(1/2))/b^(1/2)))/(4*b^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^3 x^{13/2}}\,dx = \frac{-315\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^4x^4-630\sqrt{x}\,\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\sqrt{x}\,a}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^3-315\sqrt{x}\,\sqrt{b}}{20\sqrt{x}\,b^6x^2\,(a^2x^2}$$

input
```
int(1/(a+b/x)^3/x^(13/2),x)
```

output
```
( - 315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**4*x
**4 - 630*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*a**3
*b*x**3 - 315*sqrt(x)*sqrt(b)*sqrt(a)*atan((sqrt(x)*a)/(sqrt(b)*sqrt(a)))*
a**2*b**2*x**2 - 315*a**4*b*x**4 - 525*a**3*b**2*x**3 - 168*a**2*b**3*x**2
 + 24*a*b**4*x - 8*b**5)/(20*sqrt(x)*b**6*x**2*(a**2*x**2 + 2*a*b*x + b**2
))
```

3.143 $\int \sqrt{a+\frac{b}{x}}x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \sqrt{a+\frac{b}{x}}x^3\,dx=\frac{5b^3\sqrt{a+\frac{b}{x}}x}{64a^3}-\frac{5b^2\sqrt{a+\frac{b}{x}}x^2}{96a^2}+\frac{b\sqrt{a+\frac{b}{x}}x^3}{24a}+\frac{1}{4}\sqrt{a+\frac{b}{x}}x^4-\frac{5b^4\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{7/2}}$$

output
```
5/64*b^3*(a+b/x)^(1/2)*x/a^3-5/96*b^2*(a+b/x)^(1/2)*x^2/a^2+1/24*b*(a+b/x)
^(1/2)*x^3/a+1/4*(a+b/x)^(1/2)*x^4-5/64*b^4*arctanh((a+b/x)^(1/2)/a^(1/2))
/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int \sqrt{a+\frac{b}{x}}x^3\,dx=\frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(15b^3-10ab^2x+8a^2bx^2+48a^3x^3)-15b^4\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{192a^{7/2}}$$

input
```
Integrate[Sqrt[a + b/x]*x^3,x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(15*b^3 - 10*a*b^2*x + 8*a^2*b*x^2 + 48*a^3*x^3)
- 15*b^4*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(192*a^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \sqrt{a+\frac{b}{x}}x^5d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}-\frac{1}{8}b\int \frac{x^4}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}-\frac{1}{8}b\left(-\frac{5b\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}-\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}-\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}-\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x}}-\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)$$

input

```
Int[Sqrt[a + b/x]*x^3,x]
```

output
```
(Sqrt[a + b/x]*x^4)/4 - (b*(-1/3*(Sqrt[a + b/x]*x^3)/a - (5*b*(-1/2*(Sqrt[
a + b/x]*x^2)/a - (3*b*(-((Sqrt[a + b/x]*x)/a) + (b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a)))/8
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

method	result
risch	$\frac{(48a^3x^3+8a^2b\,x^2-10a\,b^2x+15b^3)x\sqrt{\frac{ax+b}{x}}}{192a^3}-\frac{5b^4\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{128a^{\frac{7}{2}}(ax+b)}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(96x(a\,x^2+bx)^{\frac{3}{2}}a^{\frac{7}{2}}-80a^{\frac{5}{2}}(a\,x^2+bx)^{\frac{3}{2}}b+60a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b^2x+30a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b^3-15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^4\right)}{384\sqrt{x(ax+b)}\,a^{\frac{9}{2}}}$

input
```
int((a+b/x)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*a^3*x^3+8*a^2*b*x^2-10*a*b^2*x+15*b^3)*x/a^3*((a*x+b)/x)^(1/2)-5
/128*b^4/a^(7/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/
2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int\sqrt{a+\frac{b}{x}}x^3\,dx$$

$$=\left[\frac{15\sqrt{a}b^4\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(48\,a^4x^4+8\,a^3bx^3-10\,a^2b^2x^2+15\,ab^3x\right)\sqrt{\frac{ax+b}{x}}}{384\,a^4},\ \frac{15\sqrt{-a}b\cdots}{\cdots}\right.$$

input
```
integrate((a+b/x)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
[1/384*(15*sqrt(a)*b^4*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*
(48*a^4*x^4 + 8*a^3*b*x^3 - 10*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((a*x + b)/x)
)/a^4, 1/192*(15*sqrt(-a)*b^4*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b
)) + (48*a^4*x^4 + 8*a^3*b*x^3 - 10*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((a*x +
b)/x))/a^4]
```

Sympy [A] (verification not implemented)

Time = 23.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.31

$$\int \sqrt{a+\frac{b}{x}}x^3\,dx = \frac{ax^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{7\sqrt{b}x^{\frac{7}{2}}}{24\sqrt{\frac{ax}{b}+1}}-\frac{b^{\frac{3}{2}}x^{\frac{5}{2}}}{96a\sqrt{\frac{ax}{b}+1}}+\frac{5b^{\frac{5}{2}}x^{\frac{3}{2}}}{192a^2\sqrt{\frac{ax}{b}+1}}+\frac{5b^{\frac{7}{2}}\sqrt{x}}{64a^3\sqrt{\frac{ax}{b}+1}}-\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)**(1/2)*x**3,x)
```

output
```
a*x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) + 7*sqrt(b)*x**(7/2)/(24*sqrt(a*x/b
 + 1)) - b**(3/2)*x**(5/2)/(96*a*sqrt(a*x/b + 1)) + 5*b**(5/2)*x**(3/2)/(1
92*a**2*sqrt(a*x/b + 1)) + 5*b**(7/2)*sqrt(x)/(64*a**3*sqrt(a*x/b + 1)) -
5*b**4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.42

$$\int \sqrt{a+\frac{b}{x}}x^3\,dx = \frac{5\,b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{7}{2}}}+\frac{15\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4-55\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4+73\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4+15\sqrt{a+\frac{b}{x}}a^3b^4}{192\left(\left(a+\frac{b}{x}\right)^4a^3-4\left(a+\frac{b}{x}\right)^3a^4+6\left(a+\frac{b}{x}\right)^2a^5-4\left(a+\frac{b}{x}\right)a^6+a^7\right)}$$

input
```
integrate((a+b/x)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
5/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2)
 + 1/192*(15*(a + b/x)^(7/2)*b^4 - 55*(a + b/x)^(5/2)*a*b^4 + 73*(a + b/x)
^(3/2)*a^2*b^4 + 15*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a^3 - 4*(a + b/x)^
3*a^4 + 6*(a + b/x)^2*a^5 - 4*(a + b/x)*a^6 + a^7)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \sqrt{a+\frac{b}{x}}x^3\,dx = \frac{5\,b^4\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{128\,a^{\frac{7}{2}}} - \frac{5\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{128\,a^{\frac{7}{2}}} + \frac{1}{192}\sqrt{ax^2+bx}\left(2\left(4\left(6\,x\operatorname{sgn}(x)+\frac{b\operatorname{sgn}(x)}{a}\right)x-\frac{5\,b^2\operatorname{sgn}(x)}{a^2}\right)x+\frac{15\,b^3\operatorname{sgn}(x)}{a^3}\right)$$

input
```
integrate((a+b/x)^(1/2)*x^3,x, algorithm="giac")
```

output
```
5/128*b^4*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/a
^(7/2) - 5/128*b^4*log(abs(b))*sgn(x)/a^(7/2) + 1/192*sqrt(a*x^2 + b*x)*(2
*(4*(6*x*sgn(x) + b*sgn(x)/a)*x - 5*b^2*sgn(x)/a^2)*x + 15*b^3*sgn(x)/a^3)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \sqrt{a+\frac{b}{x}}x^3\,dx = \frac{5\,x^4\,\sqrt{a+\frac{b}{x}}}{64} + \frac{73\,x^4\,{\left(a+\frac{b}{x}\right)}^{3/2}}{192\,a} - \frac{55\,x^4\,{\left(a+\frac{b}{x}\right)}^{5/2}}{192\,a^2} + \frac{5\,x^4\,{\left(a+\frac{b}{x}\right)}^{7/2}}{64\,a^3} + \frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{64\,a^{7/2}}$$

input
```
int(x^3*(a + b/x)^(1/2),x)
```

output
```
(5*x^4*(a + b/x)^(1/2))/64 + (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)/(
64*a^(7/2)) + (73*x^4*(a + b/x)^(3/2))/(192*a) - (55*x^4*(a + b/x)^(5/2))/
(192*a^2) + (5*x^4*(a + b/x)^(7/2))/(64*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int \sqrt{a+\frac{b}{x}}x^3\,dx = \frac{48\sqrt{x}\,\sqrt{ax+b}\,a^4x^3+8\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^2-10\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x+15\sqrt{x}\,\sqrt{ax+b}\,a\,b^3-15\sqrt{a}\log\left(\frac{\sqrt{ax}}{}\right.}{192a^4}$$

input
```
int((a+b/x)^(1/2)*x^3,x)
```

output
```
(48*sqrt(x)*sqrt(a*x + b)*a**4*x**3 + 8*sqrt(x)*sqrt(a*x + b)*a**3*b*x**2
- 10*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x + 15*sqrt(x)*sqrt(a*x + b)*a*b**3 -
 15*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**4)/(192*a**4
)
```

3.144 $\int \sqrt{a+\frac{b}{x}}x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \sqrt{a+\frac{b}{x}}x^2\,dx=-\frac{b^2\sqrt{a+\frac{b}{x}}x}{8a^2}+\frac{b\sqrt{a+\frac{b}{x}}x^2}{12a}+\frac{1}{3}\sqrt{a+\frac{b}{x}}x^3+\frac{b^3\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/8*b^2*(a+b/x)^(1/2)*x/a^2+1/12*b*(a+b/x)^(1/2)*x^2/a+1/3*(a+b/x)^(1/2)*
x^3+1/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int \sqrt{a+\frac{b}{x}}x^2\,dx=\frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(-3b^2+2abx+8a^2x^2)+3b^3\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{24a^{5/2}}$$

input
```
Integrate[Sqrt[a + b/x]*x^2,x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(-3*b^2 + 2*a*b*x + 8*a^2*x^2) + 3*b^3*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(24*a^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \sqrt{a+\frac{b}{x}}x^4d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}-\frac{1}{6}b\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}-\frac{1}{6}b\left(-\frac{3b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}-\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}-\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}-\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)$$

input
```
Int[Sqrt[a + b/x]*x^2,x]
```

output
```
(Sqrt[a + b/x]*x^3)/3 - (b*(-1/2*(Sqrt[a + b/x]*x^2)/a - (3*b*(-((Sqrt[a +
 b/x]*x)/a) + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/(4*a)))/6
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{(8a^2x^2+2abx-3b^2)x\sqrt{\frac{ax+b}{x}}}{24a^2}+\frac{b^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{16a^{\frac{5}{2}}(ax+b)}$	97
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(16(a\,x^2+bx)^{\frac{3}{2}}a^{\frac{5}{2}}-12\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}bx-6\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b^2+3\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^3\right)}{48\sqrt{x(ax+b)}\,a^{\frac{7}{2}}}$	115

input
```
int((a+b/x)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2+2*a*b*x-3*b^2)*x/a^2*((a*x+b)/x)^(1/2)+1/16/a^(5/2)*b^3*ln
((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2
)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.69

$$\int\sqrt{a+\frac{b}{x}}x^2\,dx$$
$$=\left[\frac{3\sqrt{a}b^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(8\,a^3x^3+2\,a^2bx^2-3\,ab^2x)\sqrt{\frac{ax+b}{x}}}{48\,a^3},\right.$$
$$\left.-\frac{3\sqrt{-a}b^3\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(8\,a^3x^3+2\,a^2bx^2-3\,ab^2x)\sqrt{\frac{ax+b}{x}}}{24\,a^3}\right]$$

input
```
integrate((a+b/x)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(8
*a^3*x^3 + 2*a^2*b*x^2 - 3*a*b^2*x)*sqrt((a*x + b)/x))/a^3, -1/24*(3*sqrt(
-a)*b^3*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (8*a^3*x^3 + 2*a^
2*b*x^2 - 3*a*b^2*x)*sqrt((a*x + b)/x))/a^3]
```

Sympy [A] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \sqrt{a+\frac{b}{x}}x^2\,dx = \frac{ax^{\frac{7}{2}}}{3\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{5\sqrt{b}x^{\frac{5}{2}}}{12\sqrt{\frac{ax}{b}+1}} - \frac{b^{\frac{3}{2}}x^{\frac{3}{2}}}{24a\sqrt{\frac{ax}{b}+1}} - \frac{b^{\frac{5}{2}}\sqrt{x}}{8a^2\sqrt{\frac{ax}{b}+1}} + \frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)**(1/2)*x**2,x)
```

output
```
a*x**(7/2)/(3*sqrt(b)*sqrt(a*x/b + 1)) + 5*sqrt(b)*x**(5/2)/(12*sqrt(a*x/b
 + 1)) - b**(3/2)*x**(3/2)/(24*a*sqrt(a*x/b + 1)) - b**(5/2)*sqrt(x)/(8*a*
*2*sqrt(a*x/b + 1)) + b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(8*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.47

$$\int \sqrt{a+\frac{b}{x}}x^2\,dx = -\frac{b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}} - \frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3 - 8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3 - 3\sqrt{a+\frac{b}{x}}a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3a^2 - 3\left(a+\frac{b}{x}\right)^2a^3 + 3\left(a+\frac{b}{x}\right)a^4 - a^5\right)}$$

input
```
integrate((a+b/x)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
-1/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
 - 1/24*(3*(a + b/x)^(5/2)*b^3 - 8*(a + b/x)^(3/2)*a*b^3 - 3*sqrt(a + b/x)
*a^2*b^3)/((a + b/x)^3*a^2 - 3*(a + b/x)^2*a^3 + 3*(a + b/x)*a^4 - a^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \sqrt{a+\frac{b}{x}}x^2\,dx = -\frac{b^3\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{16\,a^{\frac{5}{2}}}+\frac{b^3\log\left(|b|\right)\operatorname{sgn}(x)}{16\,a^{\frac{5}{2}}}+\frac{1}{24}\sqrt{ax^2+bx}\left(2\left(4\,x\operatorname{sgn}(x)+\frac{b\operatorname{sgn}(x)}{a}\right)x-\frac{3\,b^2\operatorname{sgn}(x)}{a^2}\right)$$

input
```
integrate((a+b/x)^(1/2)*x^2,x, algorithm="giac")
```

output
```
-1/16*b^3*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/a
^(5/2) + 1/16*b^3*log(abs(b))*sgn(x)/a^(5/2) + 1/24*sqrt(a*x^2 + b*x)*(2*(
4*x*sgn(x) + b*sgn(x)/a)*x - 3*b^2*sgn(x)/a^2)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \sqrt{a+\frac{b}{x}}x^2\,dx = \frac{x^3\sqrt{a+\frac{b}{x}}}{8}+\frac{x^3\left(a+\frac{b}{x}\right)^{3/2}}{3\,a}-\frac{x^3\left(a+\frac{b}{x}\right)^{5/2}}{8\,a^2}-\frac{b^3\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{5/2}}$$

input
```
int(x^2*(a + b/x)^(1/2),x)
```

output
```
(x^3*(a + b/x)^(1/2))/8 - (b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*1i)/(8*a
^(5/2)) + (x^3*(a + b/x)^(3/2))/(3*a) - (x^3*(a + b/x)^(5/2))/(8*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \sqrt{a+\frac{b}{x}}x^2\,dx$$
$$=\frac{8\sqrt{x}\,\sqrt{ax+b}\,a^3x^2+2\sqrt{x}\,\sqrt{ax+b}\,a^2bx-3\sqrt{x}\,\sqrt{ax+b}\,a\,b^2+3\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^3}{24a^3}$$

input
```
int((a+b/x)^(1/2)*x^2,x)
```

output
```
(8*sqrt(x)*sqrt(a*x + b)*a**3*x**2 + 2*sqrt(x)*sqrt(a*x + b)*a**2*b*x - 3*
sqrt(x)*sqrt(a*x + b)*a*b**2 + 3*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt
(a))/sqrt(b))*b**3)/(24*a**3)
```

3.145 $\int \sqrt{a+\frac{b}{x}}x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \sqrt{a+\frac{b}{x}}x\,dx=\frac{b\sqrt{a+\frac{b}{x}}x}{4a}+\frac{1}{2}\sqrt{a+\frac{b}{x}}x^2-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
1/4*b*(a+b/x)^(1/2)*x/a+1/2*(a+b/x)^(1/2)*x^2-1/4*b^2*arctanh((a+b/x)^(1/2
)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \sqrt{a+\frac{b}{x}}x\,dx=\frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(b+2ax)-b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[Sqrt[a + b/x]*x,x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(b + 2*a*x) - b^2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \sqrt{a+\frac{b}{x}}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}-\frac{1}{4}b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}-\frac{1}{4}b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}-\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}-\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)$$

input
```
Int[Sqrt[a + b/x]*x,x]
```

output
```
(Sqrt[a + b/x]*x^2)/2 - (b*(-((Sqrt[a + b/x]*x)/a) + (b*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/a^(3/2)))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

method	result	size
risch	$\frac{(2ax+b)x\sqrt{\frac{ax+b}{x}}}{4a}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{8a^{\frac{3}{2}}(ax+b)}$	84
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}x+2\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}b-b^{2}\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\right)}{8\sqrt{x(ax+b)}\,a^{\frac{5}{2}}}$	96

input
```
int((a+b/x)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x+b)*x/a*((a*x+b)/x)^(1/2)-1/8*b^2/a^(3/2)*ln((1/2*b+a*x)/a^(1/2)
+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\int\sqrt{a+\frac{b}{x}}x\,dx$$
$$=\left[\frac{\sqrt{a}b^{2}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(2\,a^{2}x^{2}+abx)\sqrt{\frac{ax+b}{x}}}{8\,a^{2}},\ \frac{\sqrt{-a}b^{2}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+(2\,a^{2}x^{2}}{4\,a^{2}}\right.$$

input
```
integrate((a+b/x)^(1/2)*x,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b^2*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(2*a^
2*x^2 + a*b*x)*sqrt((a*x + b)/x))/a^2, 1/4*(sqrt(-a)*b^2*arctan(sqrt(-a)*x
*sqrt((a*x + b)/x)/(a*x + b)) + (2*a^2*x^2 + a*b*x)*sqrt((a*x + b)/x))/a^2
]
```

Sympy [A] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \sqrt{a+\frac{b}{x}}x\,dx=\frac{ax^{\frac{5}{2}}}{2\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}x^{\frac{3}{2}}}{4\sqrt{\frac{ax}{b}+1}}+\frac{b^{\frac{3}{2}}\sqrt{x}}{4a\sqrt{\frac{ax}{b}+1}}-\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)**(1/2)*x,x)
```

output
```
a*x**(5/2)/(2*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*x**(3/2)/(4*sqrt(a*x/b
+ 1)) + b**(3/2)*sqrt(x)/(4*a*sqrt(a*x/b + 1)) - b**2*asinh(sqrt(a)*sqrt(x
)/sqrt(b))/(4*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \sqrt{a+\frac{b}{x}}x\,dx=\frac{b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}+\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2+\sqrt{a+\frac{b}{x}}ab^2}{4\left(\left(a+\frac{b}{x}\right)^2a-2\left(a+\frac{b}{x}\right)a^2+a^3\right)}$$

input
```
integrate((a+b/x)^(1/2)*x,x, algorithm="maxima")
```

output
```
1/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) +
 1/4*((a + b/x)^(3/2)*b^2 + sqrt(a + b/x)*a*b^2)/((a + b/x)^2*a - 2*(a + b
/x)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \sqrt{a+\frac{b}{x}}x\,dx = -\frac{b^2\log(|b|)\,\mathrm{sgn}(x)}{8\,a^{\frac{3}{2}}} + \frac{1}{8}\left(2\,\sqrt{ax^2+bx}\left(2\,x+\frac{b}{a}\right)+\frac{b^2\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{a^{\frac{3}{2}}}\right)\mathrm{sgn}(x)$$

input
```
integrate((a+b/x)^(1/2)*x,x, algorithm="giac")
```

output
```
-1/8*b^2*log(abs(b))*sgn(x)/a^(3/2) + 1/8*(2*sqrt(a*x^2 + b*x)*(2*x + b/a)
 + b^2*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/a^(3/2))*sg
n(x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \sqrt{a+\frac{b}{x}}x\,dx = \frac{x^2\,\sqrt{a+\frac{b}{x}}}{4} - \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{3/2}} + \frac{x^2\,\left(a+\frac{b}{x}\right)^{3/2}}{4\,a}$$

input
```
int(x*(a + b/x)^(1/2),x)
```

output
```
(x^2*(a + b/x)^(1/2))/4 - (b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(3/2))
 + (x^2*(a + b/x)^(3/2))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \sqrt{a+\frac{b}{x}}x\,dx=\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^{2}x+\sqrt{x}\,\sqrt{ax+b}\,ab-\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^{2}}{4a^{2}}$$

input
```
int((a+b/x)^(1/2)*x,x)
```

output
```
(2*sqrt(x)*sqrt(a*x + b)*a**2*x + sqrt(x)*sqrt(a*x + b)*a*b - sqrt(a)*log(
(sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**2)/(4*a**2)
```

3.146 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \sqrt{a+\frac{b}{x}}\,dx=\sqrt{a+\frac{b}{x}}x+\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
(a+b/x)^(1/2)*x+b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sqrt{a+\frac{b}{x}}\,dx=\sqrt{a+\frac{b}{x}}x+\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b/x],x]
```

output
```
Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {773, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \sqrt{a+\frac{b}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\sqrt{a+\frac{b}{x}}-\frac{1}{2}b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$x\sqrt{a+\frac{b}{x}}-\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}$$

$$\Big\downarrow \text{221}$$

$$\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}$$

input
```
Int[Sqrt[a + b/x],x]
```

output
```
Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

method	result	size
risch	$\sqrt{\frac{ax+b}{x}}\, x+\frac{b \ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right) \sqrt{\frac{ax+b}{x}}\, \sqrt{x(ax+b)}}{2\sqrt{a}\,(ax+b)}$	72
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x\left(2\sqrt{a\,x^{2}+bx}\, \sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\, \sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\, \sqrt{a}}$	74

input
```
int((a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a*x+b)/x)^(1/2)*x+1/2*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2
)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.67

$$\int \sqrt{a+\frac{b}{x}}\,dx = \left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a}, \frac{ax\sqrt{\frac{ax+b}{x}}-\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)}{a}\right]$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*x*s
qrt((a*x + b)/x)/(a*x + b)))/a]
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}+\frac{b\,\text{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate((a+b/x)**(1/2),x)
```

output
```
sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \sqrt{a+\frac{b}{x}}\,dx = \sqrt{a+\frac{b}{x}}x - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqr
t(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int \sqrt{a+\frac{b}{x}}\,dx = -\frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{2\sqrt{a}} + \frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\sqrt{a}} + \sqrt{ax^2+bx}\operatorname{sgn}(x)$$

input
```
integrate((a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \sqrt{a+\frac{b}{x}}\,dx = x\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}} + \frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\,\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

input
```
int((a + b/x)^(1/2),x)
```

output
```
x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
 a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{a+\frac{b}{x}}\,dx=\frac{\sqrt{x}\,\sqrt{ax+b}\,a+\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b}{a}$$

input
```
int((a+b/x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a*x + b)*a + sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/s
qrt(b))*b)/a
```

3.147 $\int \frac{\sqrt{a+\frac{b}{x}}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x} \, dx = -2\sqrt{a+\frac{b}{x}} + 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*(a+b/x)^(1/2)+2*a^(1/2)*arctanh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x} \, dx = -2\sqrt{a+\frac{b}{x}} + 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b/x]/x,x]
```

output
```
-2*Sqrt[a + b/x] + 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x} dx$$

$$\Big\downarrow \text{798}$$

$$-\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$-a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-2\sqrt{a+\frac{b}{x}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}-2\sqrt{a+\frac{b}{x}}$$

$$\Big\downarrow \text{221}$$

$$2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}$$

input
```
Int[Sqrt[a + b/x]/x,x]
```

output
```
-2*Sqrt[a + b/x] + 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(31) = 62$.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.79

method	result	size
risch	$-2\sqrt{\frac{ax+b}{x}}+\frac{\sqrt{a}\,\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{ax+b}$	70
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-2\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}x^{2}-\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}+2\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{x\sqrt{x(ax+b)}\,b\sqrt{a}}$	103

input
```
int((a+b/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*((a*x+b)/x)^(1/2)+a^(1/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a
*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx = \left[\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,\sqrt{\frac{ax+b}{x}},\right.$$
$$\left.-2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-2\,\sqrt{\frac{ax+b}{x}}\right]$$

input
```
integrate((a+b/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*sqrt((a*x + b)
/x), -2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - 2*sqrt((
a*x + b)/x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.

Time = 0.96 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx = 2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{2a\sqrt{x}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{2\sqrt{b}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate((a+b/x)**(1/2)/x,x)
```

output
```
2*sqrt(a)*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a*sqrt(x)/(sqrt(b)*sqrt(a*x/b
 + 1)) - 2*sqrt(b)/(sqrt(x)*sqrt(a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx = -\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}$$

input
```
integrate((a+b/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
-sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 2*sqrt
(a + b/x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx = 2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}$$

input
```
int((a + b/x)^(1/2)/x,x)
```

output
```
2*a^(1/2)*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*(a + b/x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x}\,dx = \frac{-2\sqrt{x}\,\sqrt{ax+b}+2\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)x-2\sqrt{a}\,x}{x}$$

input
```
int((a+b/x)^(1/2)/x,x)
```

output
```
(2*( - sqrt(x)*sqrt(a*x + b) + sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a
))/sqrt(b))*x - sqrt(a)*x))/x
```

3.148 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2} dx = -\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

output
```
-2/3*(a+b/x)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2} dx = -\frac{2\left(\frac{b+ax}{x}\right)^{3/2}}{3b}$$

input
```
Integrate[Sqrt[a + b/x]/x^2,x]
```

output
```
(-2*((b + a*x)/x)^(3/2))/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}dx$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

input
```
Int[Sqrt[a + b/x]/x^2,x]
```

output
```
(-2*(a + b/x)^(3/2))/(3*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b}$	15
orering	$-\frac{2(ax+b)\sqrt{a+\frac{b}{x}}}{3xb}$	23
gosper	$-\frac{2(ax+b)\sqrt{\frac{ax+b}{x}}}{3xb}$	25
risch	$-\frac{2(ax+b)\sqrt{\frac{ax+b}{x}}}{3xb}$	25
trager	$-\frac{2(ax+b)\sqrt{-\frac{-ax-b}{x}}}{3xb}$	29
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(a\,x^{2}+bx\right)^{\frac{3}{2}}}{3x^{2}\sqrt{x(ax+b)}\,b}$	40

input
```
int((a+b/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a+b/x)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx = -\frac{2\,(ax+b)\sqrt{\frac{ax+b}{x}}}{3\,bx}$$

input
```
integrate((a+b/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2/3*(a*x + b)*sqrt((a*x + b)/x)/(b*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.72 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx = -\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{3b} - \frac{2\sqrt{a}\sqrt{1+\frac{b}{ax}}}{3x}$$

input
```
integrate((a+b/x)**(1/2)/x**2,x)
```

output
```
-2*a**(3/2)*sqrt(1 + b/(a*x))/(3*b) - 2*sqrt(a)*sqrt(1 + b/(a*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((a+b/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2/3*(a + b/x)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.61

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx = \frac{2\left(3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2 a\mathrm{sgn}(x)+3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b\mathrm{sgn}(x)+b^2\mathrm{sgn}(x)\right)}{3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3}$$

input
```
integrate((a+b/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*sgn(x) + 3*(sqrt(a)*x - sqrt(a*
x^2 + b*x))*sqrt(a)*b*sgn(x) + b^2*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))
^3
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}\,(b+a\,x)}{3\,b\,x}$$

input
```
int((a + b/x)^(1/2)/x^2,x)
```

output
```
-(2*(a + b/x)^(1/2)*(b + a*x))/(3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}\,dx = \frac{-2\sqrt{x}\,\sqrt{ax+b}\,ax-2\sqrt{x}\,\sqrt{ax+b}\,b-2\sqrt{a}\,a\,x^2}{3b\,x^2}$$

input
```
int((a+b/x)^(1/2)/x^2,x)
```

output
```
( - 2*(sqrt(x)*sqrt(a*x + b)*a*x + sqrt(x)*sqrt(a*x + b)*b + sqrt(a)*a*x**
2))/(3*b*x**2)
```

3.149 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3} dx = \frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{3b^2} - \frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}$$

output
```
2/3*a*(a+b/x)^(3/2)/b^2-2/5*(a+b/x)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3} dx = \frac{2\sqrt{\frac{b+ax}{x}}(-3b^2 - abx + 2a^2x^2)}{15b^2x^2}$$

input
```
Integrate[Sqrt[a + b/x]/x^3,x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-3*b^2 - a*b*x + 2*a^2*x^2))/(15*b^2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{3/2}}{b}-\frac{a\sqrt{a+\frac{b}{x}}}{b}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}$$

input
```
Int[Sqrt[a + b/x]/x^3,x]
```

output
```
(2*a*(a + b/x)^(3/2))/(3*b^2) - (2*(a + b/x)^(5/2))/(5*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{2(2ax-3b)(ax+b)\sqrt{a+\frac{b}{x}}}{15b^2x^2}$	31
gosper	$\frac{2(ax+b)(2ax-3b)\sqrt{\frac{ax+b}{x}}}{15b^2x^2}$	33
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,(2a^2x^2-abx-3b^2)}{15x^2b^2}$	39
trager	$\frac{2(2a^2x^2-abx-3b^2)\sqrt{-\frac{-ax-b}{x}}}{15x^2b^2}$	43
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^2+bx)^{\frac{3}{2}}(2ax-3b)}{15x^3\sqrt{x(ax+b)}\,b^2}$	48

input
```
int((a+b/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*(2*a*x-3*b)/b^2/x^2*(a*x+b)*(a+b/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx = \frac{2\,(2\,a^2x^2-abx-3\,b^2)\sqrt{\frac{ax+b}{x}}}{15\,b^2x^2}$$

input
```
integrate((a+b/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/15*(2*a^2*x^2 - a*b*x - 3*b^2)*sqrt((a*x + b)/x)/(b^2*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(31) = 62.

Time = 0.96 (sec) , antiderivative size = 304, normalized size of antiderivative = 8.00

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx = {} & \frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} \\ & - \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} \\ & - \frac{4a^6bx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{4a^5b^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((a+b/x)**(1/2)/x**3,x)
```

output
```
4*a**(11/2)*b**(3/2)*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*x**2*sqrt(a*x/b + 1)/(15*a**
(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(7/2)*x*s
qrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6
*a**(5/2)*b**(9/2)*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2
)*b**4*x**(5/2)) - 4*a**6*b*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5
/2)*b**4*x**(5/2)) - 4*a**5*b**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^2} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{3\,b^2}$$

input
```
integrate((a+b/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2/5*(a + b/x)^(5/2)/b^2 + 2/3*(a + b/x)^(3/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(30) = 60.

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.03

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx$$
$$= \frac{2\left(15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3 a^{\frac{3}{2}}\mathrm{sgn}(x) + 25\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2 ab\mathrm{sgn}(x) + 15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^2}{15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^5}$$

input
```
integrate((a+b/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*sgn(x) + 25*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^2*a*b*sgn(x) + 15*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqr
t(a)*b^2*sgn(x) + 3*b^3*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^5
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}\,(-2\,a^2\,x^2+a\,b\,x+3\,b^2)}{15\,b^2\,x^2}$$

input
```
int((a + b/x)^(1/2)/x^3,x)
```

output
```
-(2*(a + b/x)^(1/2)*(3*b^2 - 2*a^2*x^2 + a*b*x))/(15*b^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}\,dx = \frac{\frac{4\sqrt{x}\,\sqrt{ax+b}\,a^2x^2}{15}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,abx}{15}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^2}{5}-\frac{4\sqrt{a}\,a^2x^3}{15}}{b^2x^3}$$

input
```
int((a+b/x)^(1/2)/x^3,x)
```

output
```
(2*(2*sqrt(x)*sqrt(a*x + b)*a**2*x**2 - sqrt(x)*sqrt(a*x + b)*a*b*x - 3*sq
rt(x)*sqrt(a*x + b)*b**2 - 2*sqrt(a)*a**2*x**3))/(15*b**2*x**3)
```

3.150 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4} dx = -\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}$$

output
```
-2/3*a^2*(a+b/x)^(3/2)/b^3+4/5*a*(a+b/x)^(5/2)/b^3-2/7*(a+b/x)^(7/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4} dx = -\frac{2\sqrt{\frac{b+ax}{x}}(15b^3+3ab^2x-4a^2bx^2+8a^3x^3)}{105b^3x^3}$$

input
```
Integrate[Sqrt[a + b/x]/x^4,x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(15*b^3 + 3*a*b^2*x - 4*a^2*b*x^2 + 8*a^3*x^3))/(105
*b^3*x^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{5/2}}{b^2}-\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{b^2}+\frac{a^2\sqrt{a+\frac{b}{x}}}{b^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}$$

input
```
Int[Sqrt[a + b/x]/x^4,x]
```

output
```
(-2*a^2*(a + b/x)^(3/2))/(3*b^3) + (4*a*(a + b/x)^(5/2))/(5*b^3) - (2*(a +
 b/x)^(7/2))/(7*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

method	result	size
orering	$-\frac{2(8a^2x^2-12abx+15b^2)(ax+b)\sqrt{a+\frac{b}{x}}}{105b^3x^3}$	42
gosper	$-\frac{2(ax+b)(8a^2x^2-12abx+15b^2)\sqrt{\frac{ax+b}{x}}}{105b^3x^3}$	44
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(8a^3x^3-4a^2b\,x^2+3a\,b^2x+15b^3)}{105x^3b^3}$	50
trager	$-\frac{2(8a^3x^3-4a^2b\,x^2+3a\,b^2x+15b^3)\sqrt{-\frac{-ax-b}{x}}}{105x^3b^3}$	54
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^2+bx)^{\frac{3}{2}}(8a^2x^2-12abx+15b^2)}{105x^4\sqrt{x(ax+b)}\,b^3}$	59

input
```
int((a+b/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/105*(8*a^2*x^2-12*a*b*x+15*b^2)/b^3/x^3*(a*x+b)*(a+b/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx = -\frac{2\,(8\,a^3x^3-4\,a^2bx^2+3\,ab^2x+15\,b^3)\sqrt{\frac{ax+b}{x}}}{105\,b^3x^3}$$

input
```
integrate((a+b/x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/105*(8*a^3*x^3 - 4*a^2*b*x^2 + 3*a*b^2*x + 15*b^3)*sqrt((a*x + b)/x)/(b
^3*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(49) = 98.

Time = 1.24 (sec) , antiderivative size = 899, normalized size of antiderivative = 15.24

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(1/2)/x**4,x)
```

output
```
-16*a**(19/2)*b**(9/2)*x***6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2)
+ 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)
*b**10*x**(7/2)) - 40*a**(17/2)*b**(11/2)*x***5*sqrt(a*x/b + 1)/(105*a**(13
/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(
9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(13/2)*x***4*sqrt(a*x
/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(13/2)*b**(
15/2)*x***3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b
**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2))
- 100*a**(11/2)*b**(17/2)*x***2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/
2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7
/2)*b**10*x**(7/2)) - 96*a**(9/2)*b**(19/2)*x*sqrt(a*x/b + 1)/(105*a**(13/
2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9
/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(7/2)*b**(21/2)*sqrt(a*x/b + 1)
/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/
2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**10*b**4*x**(13/2)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2
)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**5*x**(11/2)/(1
05*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*
b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**6*x**(9/2)/(1...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^3}+\frac{4\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{5\,b^3}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}{3\,b^3}$$

input
```
integrate((a+b/x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)/b^3 + 4/5*(a + b/x)^(5/2)*a/b^3 - 2/3*(a + b/x)^(3/2)
*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(47) = 94$.

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx$$
$$= \frac{2\left(140\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^4 a^2\mathrm{sgn}(x)+315\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3 a^{\frac{3}{2}}b\mathrm{sgn}(x)+273\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2}{105\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^7}$$

input
```
integrate((a+b/x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
2/105*(140*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*sgn(x) + 315*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^3*a^(3/2)*b*sgn(x) + 273*(sqrt(a)*x - sqrt(a*x^2 + b*x
))^2*a*b^2*sgn(x) + 105*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*sgn(x)
 + 15*b^4*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^7
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx = \frac{8\,a^2\,\sqrt{a+\frac{b}{x}}}{105\,b^2\,x} - \frac{16\,a^3\,\sqrt{a+\frac{b}{x}}}{105\,b^3} - \frac{2\,a\,\sqrt{a+\frac{b}{x}}}{35\,b\,x^2} - \frac{2\,\sqrt{a+\frac{b}{x}}}{7\,x^3}$$

input
```
int((a + b/x)^(1/2)/x^4,x)
```

output
```
(8*a^2*(a + b/x)^(1/2))/(105*b^2*x) - (16*a^3*(a + b/x)^(1/2))/(105*b^3) -
 (2*a*(a + b/x)^(1/2))/(35*b*x^2) - (2*(a + b/x)^(1/2))/(7*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^4}\,dx = \frac{-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^3x^3}{105}+\frac{8\sqrt{x}\,\sqrt{ax+b}\,a^2b\,x^2}{105}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a\,b^2x}{35}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^3}{7}+\frac{16\sqrt{a}\,a^3x^4}{105}}{b^3x^4}$$

input
```
int((a+b/x)^(1/2)/x^4,x)
```

output
```
(2*( - 8*sqrt(x)*sqrt(a*x + b)*a**3*x**3 + 4*sqrt(x)*sqrt(a*x + b)*a**2*b*
x**2 - 3*sqrt(x)*sqrt(a*x + b)*a*b**2*x - 15*sqrt(x)*sqrt(a*x + b)*b**3 +
8*sqrt(a)*a**3*x**4))/(105*b**3*x**4)
```

3.151 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5} dx = \frac{2a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^4} - \frac{6a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4} + \frac{6a\left(a+\frac{b}{x}\right)^{7/2}}{7b^4} - \frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^4}$$

output
```
2/3*a^3*(a+b/x)^(3/2)/b^4-6/5*a^2*(a+b/x)^(5/2)/b^4+6/7*a*(a+b/x)^(7/2)/b^
4-2/9*(a+b/x)^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5} dx = \frac{2\sqrt{\frac{b+ax}{x}}(-35b^4 - 5ab^3x + 6a^2b^2x^2 - 8a^3bx^3 + 16a^4x^4)}{315b^4x^4}$$

input
```
Integrate[Sqrt[a + b/x]/x^5,x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-35*b^4 - 5*a*b^3*x + 6*a^2*b^2*x^2 - 8*a^3*b*x^3 +
16*a^4*x^4))/(315*b^4*x^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{7/2}}{b^3}-\frac{3a\left(a+\frac{b}{x}\right)^{5/2}}{b^3}+\frac{3a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^3}-\frac{a^3\sqrt{a+\frac{b}{x}}}{b^3}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}-\frac{6a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}$$

input
```
Int[Sqrt[a + b/x]/x^5,x]
```

output
```
(2*a^3*(a + b/x)^(3/2))/(3*b^4) - (6*a^2*(a + b/x)^(5/2))/(5*b^4) + (6*a*(
a + b/x)^(7/2))/(7*b^4) - (2*(a + b/x)^(9/2))/(9*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

method	result	size
orering	$\frac{2(16a^3x^3-24a^2b\,x^2+30a\,b^2x-35b^3)(ax+b)\sqrt{a+\frac{b}{x}}}{315b^4x^4}$	53
gosper	$\frac{2(ax+b)(16a^3x^3-24a^2b\,x^2+30a\,b^2x-35b^3)\sqrt{\frac{ax+b}{x}}}{315b^4x^4}$	55
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,(16a^4x^4-8a^3b\,x^3+6a^2b^2x^2-5a\,b^3x-35b^4)}{315x^4b^4}$	61
trager	$\frac{2(16a^4x^4-8a^3b\,x^3+6a^2b^2x^2-5a\,b^3x-35b^4)\sqrt{-\frac{-ax-b}{x}}}{315x^4b^4}$	65
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^2+bx)^{\frac{3}{2}}(16a^3x^3-24a^2b\,x^2+30a\,b^2x-35b^3)}{315x^5\sqrt{x(ax+b)}\,b^4}$	70

input
```
int((a+b/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/315*(16*a^3*x^3-24*a^2*b*x^2+30*a*b^2*x-35*b^3)/b^4/x^4*(a*x+b)*(a+b/x)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx = \frac{2\,(16\,a^4x^4 - 8\,a^3bx^3 + 6\,a^2b^2x^2 - 5\,ab^3x - 35\,b^4)\sqrt{\frac{ax+b}{x}}}{315\,b^4x^4}$$

input
```
integrate((a+b/x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
2/315*(16*a^4*x^4 - 8*a^3*b*x^3 + 6*a^2*b^2*x^2 - 5*a*b^3*x - 35*b^4)*sqrt
((a*x + b)/x)/(b^4*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. 2(68) = 136.

Time = 1.88 (sec) , antiderivative size = 2297, normalized size of antiderivative = 28.71

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(1/2)/x**5,x)
```

output
```
32*a**(29/2)*b**(23/2)*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300
*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/
2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**(25/2
)*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**
16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(1
5/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 3
15*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(27/2)*x**8*sqrt(a*x/b + 1)
/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a*
*(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*
b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(
9/2)) + 462*a**(23/2)*b**(29/2)*x**7*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*
x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2
) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890
*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 210*a**(21/2)*
b**(31/2)*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(1
9/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**
18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(1
1/2) + 315*a**(9/2)*b**21*x**(9/2)) - 378*a**(19/2)*b**(33/2)*x**5*sqrt(a*
x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^4} + \frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^4} - \frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{5\,b^4} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3}{3\,b^4}$$

input
```
integrate((a+b/x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x)^(9/2)/b^4 + 6/7*(a + b/x)^(7/2)*a/b^4 - 6/5*(a + b/x)^(5/2)
*a^2/b^4 + 2/3*(a + b/x)^(3/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx = \frac{2\left(630\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^5 a^{\frac{5}{2}}\mathrm{sgn}(x)+1764\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^4 a^2 b\mathrm{sgn}(x)+1995\left(\sqrt{a}x-\sqrt{ax^2+bx}\right.\right.}{315\left(\sqrt{a}x\right.}$$

input
```
integrate((a+b/x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
2/315*(630*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*sgn(x) + 1764*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^4*a^2*b*sgn(x) + 1995*(sqrt(a)*x - sqrt(a*x^2 + b
*x))^3*a^(3/2)*b^2*sgn(x) + 1125*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*s
gn(x) + 315*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*sgn(x) + 35*b^5*sg
n(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^9
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx = \frac{32\,a^4\,\sqrt{a+\frac{b}{x}}}{315\,b^4} - \frac{2\,\sqrt{a+\frac{b}{x}}}{9\,x^4} - \frac{2\,a\,\sqrt{a+\frac{b}{x}}}{63\,b\,x^3} + \frac{4\,a^2\,\sqrt{a+\frac{b}{x}}}{105\,b^2\,x^2} - \frac{16\,a^3\,\sqrt{a+\frac{b}{x}}}{315\,b^3\,x}$$

input
```
int((a + b/x)^(1/2)/x^5,x)
```

output
```
(32*a^4*(a + b/x)^(1/2))/(315*b^4) - (2*(a + b/x)^(1/2))/(9*x^4) - (2*a*(a
 + b/x)^(1/2))/(63*b*x^3) + (4*a^2*(a + b/x)^(1/2))/(105*b^2*x^2) - (16*a^
3*(a + b/x)^(1/2))/(315*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^5}\,dx = \frac{\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^4x^4}{315}-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^3}{315}+\frac{4\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x^2}{105}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a\,b^3x}{63}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^4}{9}-\frac{32\sqrt{a}\,a^4x^5}{315}}{b^4x^5}$$

input
```
int((a+b/x)^(1/2)/x^5,x)
```

output
```
(2*(16*sqrt(x)*sqrt(a*x + b)*a**4*x**4 - 8*sqrt(x)*sqrt(a*x + b)*a**3*b*x*
*3 + 6*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x**2 - 5*sqrt(x)*sqrt(a*x + b)*a*b*
*3*x - 35*sqrt(x)*sqrt(a*x + b)*b**4 - 16*sqrt(a)*a**4*x**5))/(315*b**4*x*
*5)
```

3.152 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6} \, dx = -\frac{2a^4\left(a+\frac{b}{x}\right)^{3/2}}{3b^5} + \frac{8a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^5} - \frac{12a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5} + \frac{8a\left(a+\frac{b}{x}\right)^{9/2}}{9b^5} - \frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}$$

output
```
-2/3*a^4*(a+b/x)^(3/2)/b^5+8/5*a^3*(a+b/x)^(5/2)/b^5-12/7*a^2*(a+b/x)^(7/2
)/b^5+8/9*a*(a+b/x)^(9/2)/b^5-2/11*(a+b/x)^(11/2)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6} \, dx = -\frac{2\sqrt{\frac{b+ax}{x}}(315b^5+35ab^4x-40a^2b^3x^2+48a^3b^2x^3-64a^4bx^4+128a^5x^5)}{3465b^5x^5}$$

input
```
Integrate[Sqrt[a + b/x]/x^6,x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(315*b^5 + 35*a*b^4*x - 40*a^2*b^3*x^2 + 48*a^3*b^2*
x^3 - 64*a^4*b*x^4 + 128*a^5*x^5))/(3465*b^5*x^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}}}{x^4} d\frac{1}{x}$$

$$\Big\downarrow \text{ 53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{9/2}}{b^4}-\frac{4a\left(a+\frac{b}{x}\right)^{7/2}}{b^4}+\frac{6a^2\left(a+\frac{b}{x}\right)^{5/2}}{b^4}-\frac{4a^3\left(a+\frac{b}{x}\right)^{3/2}}{b^4}+\frac{a^4\sqrt{a+\frac{b}{x}}}{b^4}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^4\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}$$

input
```
Int[Sqrt[a + b/x]/x^6,x]
```

output
```
(-2*a^4*(a + b/x)^(3/2))/(3*b^5) + (8*a^3*(a + b/x)^(5/2))/(5*b^5) - (12*a
^2*(a + b/x)^(7/2))/(7*b^5) + (8*a*(a + b/x)^(9/2))/(9*b^5) - (2*(a + b/x)
^(11/2))/(11*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result	size
orering	$-\frac{2\left(128a^{4}x^{4}-192a^{3}b\,x^{3}+240a^{2}b^{2}x^{2}-280a\,b^{3}x+315b^{4}\right)\left(ax+b\right)\sqrt{a+\frac{b}{x}}}{3465b^{5}x^{5}}$	64
gosper	$-\frac{2\left(ax+b\right)\left(128a^{4}x^{4}-192a^{3}b\,x^{3}+240a^{2}b^{2}x^{2}-280a\,b^{3}x+315b^{4}\right)\sqrt{\frac{ax+b}{x}}}{3465b^{5}x^{5}}$	66
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(128a^{5}x^{5}-64a^{4}b\,x^{4}+48a^{3}b^{2}x^{3}-40a^{2}b^{3}x^{2}+35b^{4}xa+315b^{5}\right)}{3465x^{5}b^{5}}$	72
trager	$-\frac{2\left(128a^{5}x^{5}-64a^{4}b\,x^{4}+48a^{3}b^{2}x^{3}-40a^{2}b^{3}x^{2}+35b^{4}xa+315b^{5}\right)\sqrt{-\frac{-ax-b}{x}}}{3465x^{5}b^{5}}$	76
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\left(128a^{4}x^{4}-192a^{3}b\,x^{3}+240a^{2}b^{2}x^{2}-280a\,b^{3}x+315b^{4}\right)}{3465x^{6}\sqrt{x\left(ax+b\right)}\,b^{5}}$	81

input
```
int((a+b/x)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/3465*(128*a^4*x^4-192*a^3*b*x^3+240*a^2*b^2*x^2-280*a*b^3*x+315*b^4)/b^
5/x^5*(a*x+b)*(a+b/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx = -\frac{2\,(128\,a^5x^5-64\,a^4bx^4+48\,a^3b^2x^3-40\,a^2b^3x^2+35\,ab^4x+315\,b^5)\sqrt{\frac{ax+b}{x}}}{3465\,b^5x^5}$$

input
```
integrate((a+b/x)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/3465*(128*a^5*x^5 - 64*a^4*b*x^4 + 48*a^3*b^2*x^3 - 40*a^2*b^3*x^2 + 35
*a*b^4*x + 315*b^5)*sqrt((a*x + b)/x)/(b^5*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5095 vs. $2(87) = 174$.

Time = 3.20 (sec) , antiderivative size = 5095, normalized size of antiderivative = 50.45

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(1/2)/x**6,x)
```

output
```
-256*a**(41/2)*b**(49/2)*x**15*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(3
1/2) + 34650*a**(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2)
+ 415800*a**(25/2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 87
3180*a**(21/2)*b**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800
*a**(17/2)*b**36*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(
13/2)*b**38*x**(13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 2432*a**(39/2)*b
**(51/2)*x**14*sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**
(29/2)*b**30*x**(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/
2)*b**32*x**(25/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b
**34*x**(21/2) + 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36
*x**(17/2) + 155925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(
13/2) + 3465*a**(11/2)*b**39*x**(11/2)) - 10336*a**(37/2)*b**(53/2)*x**13*
sqrt(a*x/b + 1)/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x*
*(29/2) + 155925*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25
/2) + 727650*a**(23/2)*b**33*x**(23/2) + 873180*a**(21/2)*b**34*x**(21/2)
+ 727650*a**(19/2)*b**35*x**(19/2) + 415800*a**(17/2)*b**36*x**(17/2) + 15
5925*a**(15/2)*b**37*x**(15/2) + 34650*a**(13/2)*b**38*x**(13/2) + 3465*a*
*(11/2)*b**39*x**(11/2)) - 25840*a**(35/2)*b**(55/2)*x**12*sqrt(a*x/b + 1)
/(3465*a**(31/2)*b**29*x**(31/2) + 34650*a**(29/2)*b**30*x**(29/2) + 15592
5*a**(27/2)*b**31*x**(27/2) + 415800*a**(25/2)*b**32*x**(25/2) + 727650...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx = -\frac{2\,\left(a+\frac{b}{x}\right)^{\frac{11}{2}}}{11\,b^5}+\frac{8\,\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a}{9\,b^5}-\frac{12\,\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^2}{7\,b^5}+\frac{8\,\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3}{5\,b^5}-\frac{2\,\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}{3\,b^5}$$

input
```
integrate((a+b/x)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-2/11*(a + b/x)^(11/2)/b^5 + 8/9*(a + b/x)^(9/2)*a/b^5 - 12/7*(a + b/x)^(7
/2)*a^2/b^5 + 8/5*(a + b/x)^(5/2)*a^3/b^5 - 2/3*(a + b/x)^(3/2)*a^4/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx = \frac{2\left(11088\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^6a^3\mathrm{sgn}(x)+36960\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^5a^{\frac{5}{2}}b\mathrm{sgn}(x)+51480\left(\sqrt{a}x-\sqrt{ax^2}\right.\right.}{}$$

input
```
integrate((a+b/x)^(1/2)/x^6,x, algorithm="giac")
```

output
```
2/3465*(11088*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*sgn(x) + 36960*(sqrt(a
)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b*sgn(x) + 51480*(sqrt(a)*x - sqrt(a*x^
2 + b*x))^4*a^2*b^2*sgn(x) + 38115*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/
2)*b^3*sgn(x) + 15785*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^4*sgn(x) + 346
5*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^5*sgn(x) + 315*b^6*sgn(x))/(sq
rt(a)*x - sqrt(a*x^2 + b*x))^11
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx = \frac{16\,a^2\sqrt{a+\frac{b}{x}}}{693\,b^2\,x^3} - \frac{256\,a^5\sqrt{a+\frac{b}{x}}}{3465\,b^5} - \frac{2\,a\sqrt{a+\frac{b}{x}}}{99\,b\,x^4} - \frac{2\sqrt{a+\frac{b}{x}}}{11\,x^5} - \frac{32\,a^3\sqrt{a+\frac{b}{x}}}{1155\,b^3\,x^2} + \frac{128\,a^4\sqrt{a+\frac{b}{x}}}{3465\,b^4\,x}$$

input
```
int((a + b/x)^(1/2)/x^6,x)
```

output
```
(16*a^2*(a + b/x)^(1/2))/(693*b^2*x^3) - (256*a^5*(a + b/x)^(1/2))/(3465*b
^5) - (2*a*(a + b/x)^(1/2))/(99*b*x^4) - (2*(a + b/x)^(1/2))/(11*x^5) - (3
2*a^3*(a + b/x)^(1/2))/(1155*b^3*x^2) + (128*a^4*(a + b/x)^(1/2))/(3465*b^
4*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^6}\,dx = \frac{-\frac{256\sqrt{x}\,\sqrt{ax+b}\,a^5x^5}{3465}+\frac{128\sqrt{x}\,\sqrt{ax+b}\,a^4b\,x^4}{3465}-\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^3b^2x^3}{1155}+\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^2b^3x^2}{693}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a\,b^4x}{99}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^5}{11}}{b^5x^6}$$

input
```
int((a+b/x)^(1/2)/x^6,x)
```

output
```
(2*( - 128*sqrt(x)*sqrt(a*x + b)*a**5*x**5 + 64*sqrt(x)*sqrt(a*x + b)*a**4
*b*x**4 - 48*sqrt(x)*sqrt(a*x + b)*a**3*b**2*x**3 + 40*sqrt(x)*sqrt(a*x +
b)*a**2*b**3*x**2 - 35*sqrt(x)*sqrt(a*x + b)*a*b**4*x - 315*sqrt(x)*sqrt(a
*x + b)*b**5 + 128*sqrt(a)*a**5*x**6))/(3465*b**5*x**6)
```

3.153 $\int \left(a+\frac{b}{x}\right)^{3/2} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3 \, dx = -\frac{3b^3\sqrt{a+\frac{b}{x}}x}{64a^2}+\frac{b^2\sqrt{a+\frac{b}{x}}x^2}{32a}+\frac{3}{8}b\sqrt{a+\frac{b}{x}}x^3+\frac{1}{4}a\sqrt{a+\frac{b}{x}}x^4+\frac{3b^4\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{5/2}}$$

output

```
-3/64*b^3*(a+b/x)^(1/2)*x/a^2+1/32*b^2*(a+b/x)^(1/2)*x^2/a+3/8*b*(a+b/x)^(
1/2)*x^3+1/4*a*(a+b/x)^(1/2)*x^4+3/64*b^4*arctanh((a+b/x)^(1/2)/a^(1/2))/a
^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3 \, dx = \frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(-3b^3+2ab^2x+24a^2bx^2+16a^3x^3)+3b^4\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[(a + b/x)^(3/2)*x^3,x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(-3*b^3 + 2*a*b^2*x + 24*a^2*b*x^2 + 16*a^3*x^3)
+ 3*b^4*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(64*a^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 798}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2}x^5d\frac{1}{x}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{8}b\int \sqrt{a+\frac{b}{x}}x^4d\frac{1}{x}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{8}b\left(\frac{1}{6}b\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)-\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)-\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)-\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)-\frac{1}{3}x^3\sqrt{a+\frac{b}{x}}\right)$$

input
```
Int[(a + b/x)^(3/2)*x^3,x]
```

output
```
((a + b/x)^(3/2)*x^4)/4 - (3*b*(-1/3*(Sqrt[a + b/x]*x^3) + (b*(-1/2*(Sqrt[
a + b/x]*x^2)/a - (3*b*(-((Sqrt[a + b/x]*x)/a) + (b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]])/a^(3/2)))/(4*a)))/6))/8
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

method	result
risch	$\frac{\left(16 a^{3} x^{3}+24 a^{2} b\,x^{2}+2 a\,b^{2} x -3 b^{3}\right) x \sqrt{\frac{a x +b}{x}}}{64 a^{2}}+\frac{3 b^{4} \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a\,x^{2}+b x}\right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x \left(a x +b \right)}}{128 a^{\frac{5}{2}} \left(a x +b \right)}$
default	$\frac{\sqrt{\frac{a x +b}{x}}\, x \left(32 x \left(a\,x^{2}+b x \right)^{\frac{3}{2}} a^{\frac{7}{2}}+16 a^{\frac{5}{2}} \left(a\,x^{2}+b x \right)^{\frac{3}{2}} b -12 a^{\frac{5}{2}} \sqrt{a\,x^{2}+b x}\, b^{2} x -6 a^{\frac{3}{2}} \sqrt{a\,x^{2}+b x}\, b^{3}+3 \ln \left(\frac{2 \sqrt{a\,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a\,b^{4}\right)}{128 a^{\frac{7}{2}} \sqrt{x \left(a x +b \right)}}$

input
```
int((a+b/x)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*a^3*x^3+24*a^2*b*x^2+2*a*b^2*x-3*b^3)*x/a^2*((a*x+b)/x)^(1/2)+3/1
28*b^4/a^(5/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)
*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx = \left[\frac{3\,\sqrt{a}b^4\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(16\,a^4x^4+24\,a^3bx^3+2\,a^2b^2x^2-3\,ab^3x)\sqrt{\frac{ax+b}{x}}}{128\,a^3}, -\frac{3\,\sqrt{-a}b^4\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(16\,a^4x^4+24\,a^3bx^3+2\,a^2b^2x^2-3\,ab^3x)\sqrt{\frac{ax+b}{x}}}{64\,a^3}\right]$$

input
```
integrate((a+b/x)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
[1/128*(3*sqrt(a)*b^4*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(
16*a^4*x^4 + 24*a^3*b*x^3 + 2*a^2*b^2*x^2 - 3*a*b^3*x)*sqrt((a*x + b)/x))/
a^3, -1/64*(3*sqrt(-a)*b^4*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b))
- (16*a^4*x^4 + 24*a^3*b*x^3 + 2*a^2*b^2*x^2 - 3*a*b^3*x)*sqrt((a*x + b)/x
))/a^3]
```

Sympy [A] (verification not implemented)

Time = 16.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx = \frac{a^2x^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{5a\sqrt{b}x^{\frac{7}{2}}}{8\sqrt{\frac{ax}{b}+1}} + \frac{13b^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{\frac{ax}{b}+1}} - \frac{b^{\frac{5}{2}}x^{\frac{3}{2}}}{64a\sqrt{\frac{ax}{b}+1}} - \frac{3b^{\frac{7}{2}}\sqrt{x}}{64a^2\sqrt{\frac{ax}{b}+1}} + \frac{3b^4\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{5}{2}}}$$

input
```
integrate((a+b/x)**(3/2)*x**3,x)
```

output
```
a**2*x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) + 5*a*sqrt(b)*x**(7/2)/(8*sqrt(a
*x/b + 1)) + 13*b**(3/2)*x**(5/2)/(32*sqrt(a*x/b + 1)) - b**(5/2)*x**(3/2)
/(64*a*sqrt(a*x/b + 1)) - 3*b**(7/2)*sqrt(x)/(64*a**2*sqrt(a*x/b + 1)) + 3
*b**4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx = -\frac{3\,b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{5}{2}}} - \frac{3\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4 - 11\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4 - 11\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4 + 3\sqrt{a+\frac{b}{x}}a^3b^4}{64\left(\left(a+\frac{b}{x}\right)^4a^2 - 4\left(a+\frac{b}{x}\right)^3a^3 + 6\left(a+\frac{b}{x}\right)^2a^4 - 4\left(a+\frac{b}{x}\right)a^5 + a^6\right)}$$

input
```
integrate((a+b/x)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
-3/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2
) - 1/64*(3*(a + b/x)^(7/2)*b^4 - 11*(a + b/x)^(5/2)*a*b^4 - 11*(a + b/x)^
(3/2)*a^2*b^4 + 3*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a^2 - 4*(a + b/x)^3*
a^3 + 6*(a + b/x)^2*a^4 - 4*(a + b/x)*a^5 + a^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx = -\frac{3\,b^4\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{128\,a^{\frac{5}{2}}}+\frac{3\,b^4\log\left(|b|\right)\operatorname{sgn}(x)}{128\,a^{\frac{5}{2}}} + \frac{1}{64}\sqrt{ax^2+bx}\left(2\left(4\left(2\,ax\operatorname{sgn}(x)+3\,b\operatorname{sgn}(x)\right)x+\frac{b^2\operatorname{sgn}(x)}{a}\right)x-\frac{3\,b^3\operatorname{sgn}(x)}{a^2}\right)$$

input
```
integrate((a+b/x)^(3/2)*x^3,x, algorithm="giac")
```

output
```
-3/128*b^4*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/
a^(5/2) + 3/128*b^4*log(abs(b))*sgn(x)/a^(5/2) + 1/64*sqrt(a*x^2 + b*x)*(2
*(4*(2*a*x*sgn(x) + 3*b*sgn(x))*x + b^2*sgn(x)/a)*x - 3*b^3*sgn(x)/a^2)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx = \frac{11\,x^4\,\left(a+\frac{b}{x}\right)^{3/2}}{64}-\frac{3\,a\,x^4\,\sqrt{a+\frac{b}{x}}}{64} + \frac{11\,x^4\,\left(a+\frac{b}{x}\right)^{5/2}}{64\,a}-\frac{3\,x^4\,\left(a+\frac{b}{x}\right)^{7/2}}{64\,a^2}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{64\,a^{5/2}}$$

input
```
int(x^3*(a + b/x)^(3/2),x)
```

output
```
(11*x^4*(a + b/x)^(3/2))/64 - (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*3i)/
(64*a^(5/2)) - (3*a*x^4*(a + b/x)^(1/2))/64 + (11*x^4*(a + b/x)^(5/2))/(64
*a) - (3*x^4*(a + b/x)^(7/2))/(64*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^3\,dx=\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^4x^3+24\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^2+2\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x-3\sqrt{x}\,\sqrt{ax+b}\,a\,b^3+3}{64a^3}$$

input
```
int((a+b/x)^(3/2)*x^3,x)
```

output
```
(16*sqrt(x)*sqrt(a*x + b)*a**4*x**3 + 24*sqrt(x)*sqrt(a*x + b)*a**3*b*x**2
 + 2*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x - 3*sqrt(x)*sqrt(a*x + b)*a*b**3 +
3*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**4)/(64*a**3)
```

3.154 $\int \left(a+\frac{b}{x}\right)^{3/2} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^2 \, dx=\frac{b^2\sqrt{a+\frac{b}{x}}x}{8a}+\frac{7}{12}b\sqrt{a+\frac{b}{x}}x^2+\frac{1}{3}a\sqrt{a+\frac{b}{x}}x^3-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
1/8*b^2*(a+b/x)^(1/2)*x/a+7/12*b*(a+b/x)^(1/2)*x^2+1/3*a*(a+b/x)^(1/2)*x^3
-1/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^2 \, dx = \frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(3b^2+14abx+8a^2x^2)-3b^3\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{24a^{3/2}}$$

input
```
Integrate[(a + b/x)^(3/2)*x^2,x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(3*b^2 + 14*a*b*x + 8*a^2*x^2) - 3*b^3*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(24*a^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x}\right)^{3/2} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x}\right)^{3/2}-\frac{1}{2}b\int \sqrt{a+\frac{b}{x}}x^3 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x}\right)^{3/2}-\frac{1}{2}b\left(\frac{1}{4}b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x}\right)^{3/2}-\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)-\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x}\right)^{3/2}-\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)-\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x}\right)^{3/2}-\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)-\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}\right)$$

input
```
Int[(a + b/x)^(3/2)*x^2,x]
```

output
```
((a + b/x)^(3/2)*x^3)/3 - (b*(-1/2*(Sqrt[a + b/x]*x^2) + (b*(-((Sqrt[a + b
/x]*x)/a) + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/4))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{(8a^2x^2+14abx+3b^2)x\sqrt{\frac{ax+b}{x}}}{24a}-\frac{b^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{16a^{\frac{3}{2}}(ax+b)}$	97
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(16(a\,x^2+bx)^{\frac{3}{2}}a^{\frac{5}{2}}+12\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}bx+6\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b^2-3\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^3\right)}{48a^{\frac{5}{2}}\sqrt{x(ax+b)}}$	115

input
```
int((a+b/x)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2+14*a*b*x+3*b^2)*x/a*((a*x+b)/x)^(1/2)-1/16*b^3/a^(3/2)*ln(
(1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

$$\int\left(a+\frac{b}{x}\right)^{3/2}x^2\,dx=\left[\frac{3\,\sqrt{a}b^3\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(8\,a^3x^3+14\,a^2bx^2+3\,ab^2x)\sqrt{\frac{ax+b}{x}}}{48\,a^2},\frac{3\,\sqrt{-a}b^3\,a}{}\right.$$

input
```
integrate((a+b/x)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(8
*a^3*x^3 + 14*a^2*b*x^2 + 3*a*b^2*x)*sqrt((a*x + b)/x))/a^2, 1/24*(3*sqrt(
-a)*b^3*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (8*a^3*x^3 + 14*a
^2*b*x^2 + 3*a*b^2*x)*sqrt((a*x + b)/x))/a^2]
```

Sympy [A] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x}\right)^{3/2}x^2\,dx=\frac{a^2x^{\frac{7}{2}}}{3\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{11a\sqrt{b}x^{\frac{5}{2}}}{12\sqrt{\frac{ax}{b}+1}}+\frac{17b^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{\frac{ax}{b}+1}}+\frac{b^{\frac{5}{2}}\sqrt{x}}{8a\sqrt{\frac{ax}{b}+1}}-\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)**(3/2)*x**2,x)
```

output
```
a**2*x**(7/2)/(3*sqrt(b)*sqrt(a*x/b + 1)) + 11*a*sqrt(b)*x**(5/2)/(12*sqrt
(a*x/b + 1)) + 17*b**(3/2)*x**(3/2)/(24*sqrt(a*x/b + 1)) + b**(5/2)*sqrt(x
)/(8*a*sqrt(a*x/b + 1)) - b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(8*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.48

$$\int \left(a+\frac{b}{x}\right)^{3/2}x^2\,dx=\frac{b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}}+\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3+8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3-3\sqrt{a+\frac{b}{x}}a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3a-3\left(a+\frac{b}{x}\right)^2a^2+3\left(a+\frac{b}{x}\right)a^3-a^4\right)}$$

input
```
integrate((a+b/x)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
1/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
+ 1/24*(3*(a + b/x)^(5/2)*b^3 + 8*(a + b/x)^(3/2)*a*b^3 - 3*sqrt(a + b/x)*
a^2*b^3)/((a + b/x)^3*a - 3*(a + b/x)^2*a^2 + 3*(a + b/x)*a^3 - a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^2\, dx = \frac{b^3 \log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{16\, a^{\frac{3}{2}}} - \frac{b^3\log(|b|)\operatorname{sgn}(x)}{16\, a^{\frac{3}{2}}} + \frac{1}{24}\sqrt{ax^2+bx}\left(2\left(4\, ax\operatorname{sgn}(x)+7\, b\operatorname{sgn}(x)\right)x + \frac{3\, b^2\operatorname{sgn}(x)}{a}\right)$$

input
```
integrate((a+b/x)^(3/2)*x^2,x, algorithm="giac")
```

output
```
1/16*b^3*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/a^
(3/2) - 1/16*b^3*log(abs(b))*sgn(x)/a^(3/2) + 1/24*sqrt(a*x^2 + b*x)*(2*(4
*a*x*sgn(x) + 7*b*sgn(x))*x + 3*b^2*sgn(x)/a)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^2\, dx = \frac{x^3\left(a+\frac{b}{x}\right)^{3/2}}{3} - \frac{a\, x^3\sqrt{a+\frac{b}{x}}}{8} + \frac{x^3\left(a+\frac{b}{x}\right)^{5/2}}{8\, a} + \frac{b^3\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\, 1\mathrm{i}}{\sqrt{a}}\right)\, 1\mathrm{i}}{8\, a^{3/2}}$$

input
```
int(x^2*(a + b/x)^(3/2),x)
```

output
```
(x^3*(a + b/x)^(3/2))/3 + (b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*1i)/(8*a
^(3/2)) - (a*x^3*(a + b/x)^(1/2))/8 + (x^3*(a + b/x)^(5/2))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^2\,dx = \frac{8\sqrt{x}\,\sqrt{ax+b}\,a^3x^2+14\sqrt{x}\,\sqrt{ax+b}\,a^2bx+3\sqrt{x}\,\sqrt{ax+b}\,a\,b^2-3\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^3}{24a^2}$$

input
```
int((a+b/x)^(3/2)*x^2,x)
```

output
```
(8*sqrt(x)*sqrt(a*x + b)*a**3*x**2 + 14*sqrt(x)*sqrt(a*x + b)*a**2*b*x + 3
*sqrt(x)*sqrt(a*x + b)*a*b**2 - 3*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqr
t(a))/sqrt(b))*b**3)/(24*a**2)
```

3.155 $\int \left(a+\frac{b}{x}\right)^{3/2} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \left(a+\frac{b}{x}\right)^{3/2} x \, dx = \frac{5}{4}b\sqrt{a+\frac{b}{x}}x+\frac{1}{2}a\sqrt{a+\frac{b}{x}}x^2+\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
5/4*b*(a+b/x)^(1/2)*x+1/2*a*(a+b/x)^(1/2)*x^2+3/4*b^2*arctanh((a+b/x)^(1/2
)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^{3/2} x \, dx = \frac{1}{4}\left(\sqrt{a+\frac{b}{x}}x(5b+2ax)+\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)*x,x]
```

output
```
(Sqrt[a + b/x]*x*(5*b + 2*a*x) + (3*b^2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sq
rt[a])/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {798, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{4}b\int \sqrt{a+\frac{b}{x}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{4}b\left(\frac{1}{2}b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-x\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{4}b\left(\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}-x\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{4}b\left(x\left(-\sqrt{a+\frac{b}{x}}\right)-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Int[(a + b/x)^(3/2)*x,x]
```

output
```
((a + b/x)^(3/2)*x^2)/2 - (3*b*(-(Sqrt[a + b/x]*x) - (b*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/Sqrt[a]))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

method	result	size
risch	$\frac{(2ax+5b)x\sqrt{\frac{ax+b}{x}}}{4}+\frac{3b^{2}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{8\sqrt{a}\,(ax+b)}$	83
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}x+10\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}b+3b^{2}\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\right)}{8\sqrt{x(ax+b)}\,a^{\frac{3}{2}}}$	96

input
```
int((a+b/x)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x+5*b)*x*((a*x+b)/x)^(1/2)+3/8*b^2*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+
b*x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.01

$$\int\left(a+\frac{b}{x}\right)^{3/2}x\,dx=\left[\frac{3\sqrt{a}b^{2}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(2\,a^{2}x^{2}+5\,abx\right)\sqrt{\frac{ax+b}{x}}}{8\,a},\ -\frac{3\sqrt{-a}b^{2}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-\left(2\,a^{2}x^{2}+5\,abx\right)\sqrt{\frac{ax+b}{x}}}{4\,a}\right]$$

input
```
integrate((a+b/x)^(3/2)*x,x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(2*
a^2*x^2 + 5*a*b*x)*sqrt((a*x + b)/x))/a, -1/4*(3*sqrt(-a)*b^2*arctan(sqrt(
-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (2*a^2*x^2 + 5*a*b*x)*sqrt((a*x + b)/
x))/a]
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx = \frac{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{2} + \frac{5b^{\frac{3}{2}}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{4} + \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{a}}$$

input
```
integrate((a+b/x)**(3/2)*x,x)
```

output
```
a*sqrt(b)*x**(3/2)*sqrt(a*x/b + 1)/2 + 5*b**(3/2)*sqrt(x)*sqrt(a*x/b + 1)/
4 + 3*b**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(4*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.46

$$\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx = -\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,\sqrt{a}} + \frac{5\,\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2 - 3\,\sqrt{a+\frac{b}{x}}ab^2}{4\left(\left(a+\frac{b}{x}\right)^2 - 2\,\left(a+\frac{b}{x}\right)a + a^2\right)}$$

input
```
integrate((a+b/x)^(3/2)*x,x, algorithm="maxima")
```

output
```
-3/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a)
+ 1/4*(5*(a + b/x)^(3/2)*b^2 - 3*sqrt(a + b/x)*a*b^2)/((a + b/x)^2 - 2*(a
+ b/x)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx = -\frac{3\,b^2\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{8\,\sqrt{a}} + \frac{3\,b^2\log\left(|b|\right)\operatorname{sgn}(x)}{8\,\sqrt{a}} + \frac{1}{4}\,\sqrt{ax^2+bx}\left(2\,ax\operatorname{sgn}(x)+5\,b\operatorname{sgn}(x)\right)$$

input
```
integrate((a+b/x)^(3/2)*x,x, algorithm="giac")
```

output
```
-3/8*b^2*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/sq
rt(a) + 3/8*b^2*log(abs(b))*sgn(x)/sqrt(a) + 1/4*sqrt(a*x^2 + b*x)*(2*a*x*
sgn(x) + 5*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx = \frac{5\,x^2\left(a+\frac{b}{x}\right)^{3/2}}{4} + \frac{3\,b^2\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,\sqrt{a}} - \frac{3\,a\,x^2\,\sqrt{a+\frac{b}{x}}}{4}$$

input
```
int(x*(a + b/x)^(3/2),x)
```

output
```
(5*x^2*(a + b/x)^(3/2))/4 + (3*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(1
/2)) - (3*a*x^2*(a + b/x)^(1/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^{3/2} x\,dx = \frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2x+5\sqrt{x}\,\sqrt{ax+b}\,ab+3\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2}{4a}$$

input
```
int((a+b/x)^(3/2)*x,x)
```

output
```
(2*sqrt(x)*sqrt(a*x + b)*a**2*x + 5*sqrt(x)*sqrt(a*x + b)*a*b + 3*sqrt(a)*
log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**2)/(4*a)
```

3.156 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 55

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx=-2b\sqrt{a+\frac{b}{x}}+a\sqrt{a+\frac{b}{x}}x+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*b*(a+b/x)^(1/2)+a*(a+b/x)^(1/2)*x+3*a^(1/2)*b*arctanh((a+b/x)^(1/2)/a^(
1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx=\sqrt{a+\frac{b}{x}}(-2b+ax)+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2),x]
```

output
```
Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {773, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx$$

$$\Big\downarrow \ 773$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \ 51$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}$$

$$\Big\downarrow \ 60$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \ 73$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \ 221$$

$$x\left(a+\frac{b}{x}\right)^{3/2}-\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)$$

input
```
Int[(a + b/x)^(3/2),x]
```

output
```
(a + b/x)^(3/2)*x - (3*b*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]]))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

method	result	size
risch	$(ax-2b)\sqrt{\frac{ax+b}{x}}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	78
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}+6\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}x^{2}-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2x\sqrt{x(ax+b)}\,\sqrt{a}}$	100

input
```
int((a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x-2*b)*((a*x+b)/x)^(1/2)+3/2*a^(1/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*
x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int\left(a+\frac{b}{x}\right)^{3/2}dx=\left[\frac{3}{2}\sqrt{a}b\log\left(2\,ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}},-3\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b
)*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*
x + b)) + (a*x - 2*b)*sqrt((a*x + b)/x)]
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = 3\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate((a+b/x)**(3/2),x)
```

output
```
3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a
*x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(
a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \sqrt{a+\frac{b}{x}}ax-\frac{3}{2}\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x}}b$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = -\frac{2\,x\,\left(a+\frac{b}{x}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^{3/2}}$$

input
```
int((a + b/x)^(3/2),x)
```

output
```
-(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1
)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x}\right)^{3/2} dx = \frac{4\sqrt{x}\,\sqrt{ax+b}\,ax-8\sqrt{x}\,\sqrt{ax+b}\,b+12\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)bx-9\sqrt{a}\,bx}{4x}$$

input
```
int((a+b/x)^(3/2),x)
```

output
```
(4*sqrt(x)*sqrt(a*x + b)*a*x - 8*sqrt(x)*sqrt(a*x + b)*b + 12*sqrt(a)*log(
(sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b*x - 9*sqrt(a)*b*x)/(4*x)
```

3.157 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x} dx = -2a\sqrt{a+\frac{b}{x}} - \frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2} + 2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*a*(a+b/x)^(1/2)-2/3*(a+b/x)^(3/2)+2*a^(3/2)*arctanh((a+b/x)^(1/2)/a^(1/
2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x} dx = -\frac{2\sqrt{a+\frac{b}{x}}(b+4ax)}{3x} + 2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(3/2)/x,x]
```

output
```
(-2*Sqrt[a + b/x]*(b + 4*a*x))/(3*x) + 2*a^(3/2)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x} dx$$

$$\Big\downarrow \text{798}$$

$$-\int \left(a+\frac{b}{x}\right)^{3/2} x d\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$-a \int \sqrt{a+\frac{b}{x}} x d\frac{1}{x}-\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}$$

$$\Big\downarrow \text{60}$$

$$-a \left(a \int \frac{x}{\sqrt{a+\frac{b}{x}}} d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}$$

$$\Big\downarrow \text{73}$$

$$-a \left(\frac{2a \int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}} d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}$$

$$\Big\downarrow \text{221}$$

$$-a \left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}$$

input
```
Int[(a + b/x)^(3/2)/x,x]
```

output
```
(-2*(a + b/x)^(3/2))/3 - a*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{2(4ax+b)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{a^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{ax+b}$	79
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-6\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}x^{3}-3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}b\,x^{3}+6\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}x+2\left(a\,x^{2}+bx\right)^{\frac{3}{2}}b\sqrt{a}\right)}{3x^{2}\sqrt{x(ax+b)}\,b\sqrt{a}}$	123

input
```
int((a+b/x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(4*a*x+b)/x*((a*x+b)/x)^(1/2)+a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b
*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.09

$$\int\frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx=\left[\frac{3\,a^{\frac{3}{2}}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(4\,ax+b)\sqrt{\frac{ax+b}{x}}}{3\,x},\right.$$
$$\left.-\frac{2\left(3\,\sqrt{-a}ax\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+(4\,ax+b)\sqrt{\frac{ax+b}{x}}\right)}{3\,x}\right]$$

input
```
integrate((a+b/x)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(4*a*
x + b)*sqrt((a*x + b)/x))/x, -2/3*(3*sqrt(-a)*a*x*arctan(sqrt(-a)*x*sqrt((
a*x + b)/x)/(a*x + b)) + (4*a*x + b)*sqrt((a*x + b)/x))/x]
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx = -\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{3} - a^{\frac{3}{2}}\log\left(\frac{b}{ax}\right) + 2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax}}+1\right) - \frac{2\sqrt{a}b\sqrt{1+\frac{b}{ax}}}{3x}$$

input
```
integrate((a+b/x)**(3/2)/x,x)
```

output
```
-8*a**(3/2)*sqrt(1 + b/(a*x))/3 - a**(3/2)*log(b/(a*x)) + 2*a**(3/2)*log(s
qrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b*sqrt(1 + b/(a*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx = -a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - \frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}} - 2\sqrt{a+\frac{b}{x}}a$$

input
```
integrate((a+b/x)^(3/2)/x,x, algorithm="maxima")
```

output
```
-a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 2/3*(a
 + b/x)^(3/2) - 2*sqrt(a + b/x)*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx = 2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\,a\,\sqrt{a+\frac{b}{x}}-\frac{2\,\left(a+\frac{b}{x}\right)^{3/2}}{3}$$

input
```
int((a + b/x)^(3/2)/x,x)
```

output
```
2*a^(3/2)*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*a*(a + b/x)^(1/2) - (2*(a + b
/x)^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}\,dx = \frac{-\frac{8\sqrt{x}\,\sqrt{ax+b}\,ax}{3}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b}{3}+2\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a\,x^2}{x^2}$$

input
```
int((a+b/x)^(3/2)/x,x)
```

output
```
(2*( - 4*sqrt(x)*sqrt(a*x + b)*a*x - sqrt(x)*sqrt(a*x + b)*b + 3*sqrt(a)*l
og((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*a*x**2))/(3*x**2)
```

3.158 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2} dx = -\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

output
```
-2/5*(a+b/x)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2} dx = -\frac{2\left(\frac{b+ax}{x}\right)^{5/2}}{5b}$$

input
```
Integrate[(a + b/x)^(3/2)/x^2,x]
```

output
```
(-2*((b + a*x)/x)^(5/2))/(5*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \; 793$$

$$-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

input
```
Int[(a + b/x)^(3/2)/x^2,x]
```

output
```
(-2*(a + b/x)^(5/2))/(5*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5b}$	15
orering	$-\frac{2(ax+b)\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{5xb}$	23
gosper	$-\frac{2(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{5xb}$	25
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(a^2x^2+2abx+b^2)}{5x^2b}$	36
trager	$-\frac{2(a^2x^2+2abx+b^2)\sqrt{-\frac{-ax-b}{x}}}{5x^2b}$	40
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^2+bx)^{\frac{3}{2}}(ax+b)}{5x^3b\sqrt{x(ax+b)}}$	45

input
```
int((a+b/x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(a+b/x)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\left(a^2x^2+2\,abx+b^2\right)\sqrt{\frac{ax+b}{x}}}{5\,bx^2}$$

input
```
integrate((a+b/x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-2/5*(a^2*x^2 + 2*a*b*x + b^2)*sqrt((a*x + b)/x)/(b*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(14) = 28.

Time = 0.79 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.61

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}{5b} - \frac{4a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{5x} - \frac{2\sqrt{a}b\sqrt{1+\frac{b}{ax}}}{5x^2}$$

input
```
integrate((a+b/x)**(3/2)/x**2,x)
```

output
```
-2*a**(5/2)*sqrt(1 + b/(a*x))/(5*b) - 4*a**(3/2)*sqrt(1 + b/(a*x))/(5*x) -
 2*sqrt(a)*b*sqrt(1 + b/(a*x))/(5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate((a+b/x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-2/5*(a + b/x)^(5/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(14) = 28.

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 8.06

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx = \frac{2\left(5\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^4 a^2\mathrm{sgn}(x)+10\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3 a^{\frac{3}{2}}b\,\mathrm{sgn}(x)+10\left(\sqrt{a}x-\right.\right.}{5\left(\sqrt{a}x-\sqrt{ax^2+\right.}}$$

input
```
integrate((a+b/x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
2/5*(5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*sgn(x) + 10*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^3*a^(3/2)*b*sgn(x) + 10*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a
*b^2*sgn(x) + 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*sgn(x) + b^4*s
gn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^5
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx = -\frac{2\,\sqrt{a+\frac{b}{x}}\,{\left(b+a\,x\right)}^2}{5\,b\,x^2}$$

input
```
int((a + b/x)^(3/2)/x^2,x)
```

output
```
-(2*(a + b/x)^(1/2)*(b + a*x)^2)/(5*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2x^2}{5}-\frac{4\sqrt{x}\,\sqrt{ax+b}\,abx}{5}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^2}{5}-\frac{2\sqrt{a}\,a^2x^3}{5}}{b\,x^3}$$

input
```
int((a+b/x)^(3/2)/x^2,x)
```

output
```
(2*( - sqrt(x)*sqrt(a*x + b)*a**2*x**2 - 2*sqrt(x)*sqrt(a*x + b)*a*b*x - s
qrt(x)*sqrt(a*x + b)*b**2 - sqrt(a)*a**2*x**3))/(5*b*x**3)
```

3.159 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3} dx = \frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{5b^2} - \frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}$$

output
```
2/5*a*(a+b/x)^(5/2)/b^2-2/7*(a+b/x)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3} dx = \frac{2(b+ax)^2\sqrt{\frac{b+ax}{x}}(-5b+2ax)}{35b^2x^3}$$

input
```
Integrate[(a + b/x)^(3/2)/x^3,x]
```

output
```
(2*(b + a*x)^2*Sqrt[(b + a*x)/x]*(-5*b + 2*a*x))/(35*b^2*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{5/2}}{b}-\frac{a\left(a+\frac{b}{x}\right)^{3/2}}{b}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{5b^2}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}$$

input
```
Int[(a + b/x)^(3/2)/x^3,x]
```

output
```
(2*a*(a + b/x)^(5/2))/(5*b^2) - (2*(a + b/x)^(7/2))/(7*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{2(2ax-5b)(ax+b)\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{35b^{2}x^{2}}$	31
gosper	$\frac{2(ax+b)(2ax-5b)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{35b^{2}x^{2}}$	33
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,(2a^{3}x^{3}-a^{2}b\,x^{2}-8a\,b^{2}x-5b^{3})}{35x^{3}b^{2}}$	50
trager	$\frac{2(2a^{3}x^{3}-a^{2}b\,x^{2}-8a\,b^{2}x-5b^{3})\sqrt{-\frac{-ax-b}{x}}}{35x^{3}b^{2}}$	54
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^{2}+bx)^{\frac{3}{2}}(2a^{2}x^{2}-3abx-5b^{2})}{35x^{4}b^{2}\sqrt{x(ax+b)}}$	59

input
```
int((a+b/x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/35*(2*a*x-5*b)/b^2/x^2*(a*x+b)*(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3}}\,dx=\frac{2\,(2\,a^{3}x^{3}-a^{2}bx^{2}-8\,ab^{2}x-5\,b^{3})\sqrt{\frac{ax+b}{x}}}{35\,b^{2}x^{3}}$$

input
```
integrate((a+b/x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
2/35*(2*a^3*x^3 - a^2*b*x^2 - 8*a*b^2*x - 5*b^3)*sqrt((a*x + b)/x)/(b^2*x^
3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(31) = 62$.

Time = 1.08 (sec) , antiderivative size = 360, normalized size of antiderivative = 9.47

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx = \frac{4a^{\frac{15}{2}}b^{\frac{3}{2}}x^4\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}} + \frac{2a^{\frac{13}{2}}b^{\frac{5}{2}}x^3\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}} - \frac{18a^{\frac{11}{2}}b^{\frac{7}{2}}x^2\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}} - \frac{26a^{\frac{9}{2}}b^{\frac{9}{2}}x\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}} - \frac{10a^{\frac{7}{2}}b^{\frac{11}{2}}\sqrt{\frac{ax}{b}+1}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}} - \frac{4a^8bx^{\frac{9}{2}}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}} - \frac{4a^7b^2x^{\frac{7}{2}}}{35a^{\frac{9}{2}}b^3x^{\frac{9}{2}}+35a^{\frac{7}{2}}b^4x^{\frac{7}{2}}}$$

input
```
integrate((a+b/x)**(3/2)/x**3,x)
```

output
```
4*a**(15/2)*b**(3/2)*x**4*sqrt(a*x/b + 1)/(35*a**(9/2)*b**3*x**(9/2) + 35*
a**(7/2)*b**4*x**(7/2)) + 2*a**(13/2)*b**(5/2)*x**3*sqrt(a*x/b + 1)/(35*a*
*(9/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2)) - 18*a**(11/2)*b**(7/2)*
x**2*sqrt(a*x/b + 1)/(35*a**(9/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2
)) - 26*a**(9/2)*b**(9/2)*x*sqrt(a*x/b + 1)/(35*a**(9/2)*b**3*x**(9/2) + 3
5*a**(7/2)*b**4*x**(7/2)) - 10*a**(7/2)*b**(11/2)*sqrt(a*x/b + 1)/(35*a**(
9/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2)) - 4*a**8*b*x**(9/2)/(35*a*
*(9/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2)) - 4*a**7*b**2*x**(7/2)/(
35*a**(9/2)*b**3*x**(9/2) + 35*a**(7/2)*b**4*x**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^2} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{5\,b^2}$$

input
```
integrate((a+b/x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)/b^2 + 2/5*(a + b/x)^(5/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(30) = 60$.

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.66

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx = \frac{2\left(35\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^5 a^{\frac{5}{2}}\operatorname{sgn}(x)+105\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^4 a^2 b\operatorname{sgn}(x)+140\left(\sqrt{a}x\right.\right.}{}$$

input
```
integrate((a+b/x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*sgn(x) + 105*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^4*a^2*b*sgn(x) + 140*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^3*a^(3/2)*b^2*sgn(x) + 98*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*sgn(x)
+ 35*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*sgn(x) + 5*b^5*sgn(x))/(s
qrt(a)*x - sqrt(a*x^2 + b*x))^7
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx = \frac{4\,a^3\,\sqrt{a+\frac{b}{x}}}{35\,b^2}-\frac{2\,b\,\sqrt{a+\frac{b}{x}}}{7\,x^3}-\frac{16\,a\,\sqrt{a+\frac{b}{x}}}{35\,x^2}-\frac{2\,a^2\,\sqrt{a+\frac{b}{x}}}{35\,b\,x}$$

input
```
int((a + b/x)^(3/2)/x^3,x)
```

output
```
(4*a^3*(a + b/x)^(1/2))/(35*b^2) - (2*b*(a + b/x)^(1/2))/(7*x^3) - (16*a*(
a + b/x)^(1/2))/(35*x^2) - (2*a^2*(a + b/x)^(1/2))/(35*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}\,dx = \frac{\frac{4\sqrt{x}\,\sqrt{ax+b}\,a^3x^3}{35}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2b\,x^2}{35}-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a\,b^2x}{35}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^3}{7}-\frac{4\sqrt{a}\,a^3x^4}{35}}{b^2x^4}$$

input
```
int((a+b/x)^(3/2)/x^3,x)
```

output
```
(2*(2*sqrt(x)*sqrt(a*x + b)*a**3*x**3 - sqrt(x)*sqrt(a*x + b)*a**2*b*x**2
- 8*sqrt(x)*sqrt(a*x + b)*a*b**2*x - 5*sqrt(x)*sqrt(a*x + b)*b**3 - 2*sqrt
(a)*a**3*x**4))/(35*b**2*x**4)
```

3.160 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4} dx = -\frac{2a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3} + \frac{4a\left(a+\frac{b}{x}\right)^{7/2}}{7b^3} - \frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}$$

output
```
-2/5*a^2*(a+b/x)^(5/2)/b^3+4/7*a*(a+b/x)^(7/2)/b^3-2/9*(a+b/x)^(9/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4} dx = -\frac{2(b+ax)^2\sqrt{\frac{b+ax}{x}}(35b^2-20abx+8a^2x^2)}{315b^3x^4}$$

input
```
Integrate[(a + b/x)^(3/2)/x^4,x]
```

output
```
(-2*(b + a*x)^2*Sqrt[(b + a*x)/x]*(35*b^2 - 20*a*b*x + 8*a^2*x^2))/(315*b^
3*x^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{7/2}}{b^2}-\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{b^2}+\frac{a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^2}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}$$

input
```
Int[(a + b/x)^(3/2)/x^4,x]
```

output
```
(-2*a^2*(a + b/x)^(5/2))/(5*b^3) + (4*a*(a + b/x)^(7/2))/(7*b^3) - (2*(a +
 b/x)^(9/2))/(9*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

method	result	size
orering	$-\frac{2\left(8a^{2}x^{2}-20abx+35b^{2}\right)\left(ax+b\right)\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	42
gosper	$-\frac{2\left(ax+b\right)\left(8a^{2}x^{2}-20abx+35b^{2}\right)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	44
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(8a^{4}x^{4}-4a^{3}b\,x^{3}+3a^{2}b^{2}x^{2}+50a\,b^{3}x+35b^{4}\right)}{315x^{4}b^{3}}$	61
trager	$-\frac{2\left(8a^{4}x^{4}-4a^{3}b\,x^{3}+3a^{2}b^{2}x^{2}+50a\,b^{3}x+35b^{4}\right)\sqrt{-\frac{-ax-b}{x}}}{315x^{4}b^{3}}$	65
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\left(8a^{3}x^{3}-12a^{2}b\,x^{2}+15a\,b^{2}x+35b^{3}\right)}{315x^{5}b^{3}\sqrt{x\left(ax+b\right)}}$	70

input
```
int((a+b/x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/315*(8*a^2*x^2-20*a*b*x+35*b^2)/b^3/x^3*(a*x+b)*(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx = -\frac{2\left(8\,a^4x^4-4\,a^3bx^3+3\,a^2b^2x^2+50\,ab^3x+35\,b^4\right)\sqrt{\frac{ax+b}{x}}}{315\,b^3x^4}$$

input
```
integrate((a+b/x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-2/315*(8*a^4*x^4 - 4*a^3*b*x^3 + 3*a^2*b^2*x^2 + 50*a*b^3*x + 35*b^4)*sqr
t((a*x + b)/x)/(b^3*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. 2(49) = 98.

Time = 1.53 (sec) , antiderivative size = 986, normalized size of antiderivative = 16.71

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(3/2)/x**4,x)
```

output
```
-16*a**(23/2)*b**(9/2)*x**7*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2)
+ 945*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/
2)*b**10*x**(9/2)) - 40*a**(21/2)*b**(11/2)*x**6*sqrt(a*x/b + 1)/(315*a**(
15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x
**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) - 30*a**(19/2)*b**(13/2)*x**5*sqrt
(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) +
 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) - 110*a**(17/
2)*b**(15/2)*x**4*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(
13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x
**(9/2)) - 380*a**(15/2)*b**(17/2)*x**3*sqrt(a*x/b + 1)/(315*a**(15/2)*b**
7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2)
+ 315*a**(9/2)*b**10*x**(9/2)) - 516*a**(13/2)*b**(19/2)*x**2*sqrt(a*x/b +
 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*x**(13/2) + 945*a**
(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) - 310*a**(11/2)*b**(2
1/2)*x*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*a**(13/2)*b**8*
x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**10*x**(9/2)) -
70*a**(9/2)*b**(23/2)*sqrt(a*x/b + 1)/(315*a**(15/2)*b**7*x**(15/2) + 945*
a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)*b**
10*x**(9/2)) + 16*a**12*b**4*x**(15/2)/(315*a**(15/2)*b**7*x**(15/2) + 945
*a**(13/2)*b**8*x**(13/2) + 945*a**(11/2)*b**9*x**(11/2) + 315*a**(9/2)...
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^3} + \frac{4\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^3} - \frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{5\,b^3}$$

input
```
integrate((a+b/x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x)^(9/2)/b^3 + 4/7*(a + b/x)^(7/2)*a/b^3 - 2/5*(a + b/x)^(5/2)
*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(47) = 94$.

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.53

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx = \frac{2\left(420\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^6 a^3\operatorname{sgn}(x)+1575\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^5 a^{\frac{5}{2}}b\operatorname{sgn}(x)+2583\left(\right.\right.}{}$$

input
```
integrate((a+b/x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
2/315*(420*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*sgn(x) + 1575*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^5*a^(5/2)*b*sgn(x) + 2583*(sqrt(a)*x - sqrt(a*x^2 + b
*x))^4*a^2*b^2*sgn(x) + 2310*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^3
*sgn(x) + 1170*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^4*sgn(x) + 315*(sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^5*sgn(x) + 35*b^6*sgn(x))/(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^9
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx = \frac{8\,a^3\,\sqrt{a+\frac{b}{x}}}{315\,b^2\,x} - \frac{2\,b\,\sqrt{a+\frac{b}{x}}}{9\,x^4} - \frac{16\,a^4\,\sqrt{a+\frac{b}{x}}}{315\,b^3} - \frac{2\,a^2\,\sqrt{a+\frac{b}{x}}}{105\,b\,x^2} - \frac{20\,a\,\sqrt{a+\frac{b}{x}}}{63\,x^3}$$

input
```
int((a + b/x)^(3/2)/x^4,x)
```

output
```
(8*a^3*(a + b/x)^(1/2))/(315*b^2*x) - (2*b*(a + b/x)^(1/2))/(9*x^4) - (16*
a^4*(a + b/x)^(1/2))/(315*b^3) - (2*a^2*(a + b/x)^(1/2))/(105*b*x^2) - (20
*a*(a + b/x)^(1/2))/(63*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}\,dx = \frac{-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^4x^4}{315}+\frac{8\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^3}{315}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x^2}{105}-\frac{20\sqrt{x}\,\sqrt{ax+b}\,a\,b^3x}{63}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^4}{9}+16}{b^3x^5}$$

input

```
int((a+b/x)^(3/2)/x^4,x)
```

output

```
(2*( - 8*sqrt(x)*sqrt(a*x + b)*a**4*x**4 + 4*sqrt(x)*sqrt(a*x + b)*a**3*b*
x**3 - 3*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x**2 - 50*sqrt(x)*sqrt(a*x + b)*a
*b**3*x - 35*sqrt(x)*sqrt(a*x + b)*b**4 + 8*sqrt(a)*a**4*x**5))/(315*b**3*
x**5)
```

3.161 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5} dx = \frac{2a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^4} - \frac{6a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4} + \frac{2a\left(a+\frac{b}{x}\right)^{9/2}}{3b^4} - \frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}$$

output
```
2/5*a^3*(a+b/x)^(5/2)/b^4-6/7*a^2*(a+b/x)^(7/2)/b^4+2/3*a*(a+b/x)^(9/2)/b^
4-2/11*(a+b/x)^(11/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5} dx = \frac{2(b+ax)^2\sqrt{\frac{b+ax}{x}}(-105b^3+70ab^2x-40a^2bx^2+16a^3x^3)}{1155b^4x^5}$$

input
```
Integrate[(a + b/x)^(3/2)/x^5,x]
```

output
```
(2*(b + a*x)^2*Sqrt[(b + a*x)/x]*(-105*b^3 + 70*a*b^2*x - 40*a^2*b*x^2 + 1
6*a^3*x^3))/(1155*b^4*x^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{9/2}}{b^3}-\frac{3a\left(a+\frac{b}{x}\right)^{7/2}}{b^3}+\frac{3a^2\left(a+\frac{b}{x}\right)^{5/2}}{b^3}-\frac{a^3\left(a+\frac{b}{x}\right)^{3/2}}{b^3}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}-\frac{6a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}$$

input
```
Int[(a + b/x)^(3/2)/x^5,x]
```

output
```
(2*a^3*(a + b/x)^(5/2))/(5*b^4) - (6*a^2*(a + b/x)^(7/2))/(7*b^4) + (2*a*(
a + b/x)^(9/2))/(3*b^4) - (2*(a + b/x)^(11/2))/(11*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

method	result	size
orering	$\frac{2(16a^3x^3-40a^2b\,x^2+70a\,b^2x-105b^3)(ax+b)\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{1155b^4x^4}$	53
gosper	$\frac{2(ax+b)(16a^3x^3-40a^2b\,x^2+70a\,b^2x-105b^3)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{1155b^4x^4}$	55
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,(16a^5x^5-8a^4b\,x^4+6a^3b^2x^3-5a^2b^3x^2-140b^4xa-105b^5)}{1155x^5b^4}$	72
trager	$\frac{2(16a^5x^5-8a^4b\,x^4+6a^3b^2x^3-5a^2b^3x^2-140b^4xa-105b^5)\sqrt{-\frac{-ax-b}{x}}}{1155x^5b^4}$	76
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^2+bx)^{\frac{3}{2}}(16a^4x^4-24a^3b\,x^3+30a^2b^2x^2-35a\,b^3x-105b^4)}{1155x^6b^4\sqrt{x(ax+b)}}$	81

input
```
int((a+b/x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/1155*(16*a^3*x^3-40*a^2*b*x^2+70*a*b^2*x-105*b^3)/b^4/x^4*(a*x+b)*(a+b/x
)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx = \frac{2\,(16\,a^5x^5-8\,a^4bx^4+6\,a^3b^2x^3-5\,a^2b^3x^2-140\,ab^4x-105\,b^5)\sqrt{\frac{ax+b}{x}}}{1155\,b^4x^5}$$

input
```
integrate((a+b/x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
2/1155*(16*a^5*x^5 - 8*a^4*b*x^4 + 6*a^3*b^2*x^3 - 5*a^2*b^3*x^2 - 140*a*b
^4*x - 105*b^5)*sqrt((a*x + b)/x)/(b^4*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. 2(68) = 136.

Time = 2.29 (sec) , antiderivative size = 2297, normalized size of antiderivative = 28.71

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(3/2)/x**5,x)
```

output
```
32*a**(33/2)*b**(23/2)*x**11*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/
2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23
100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**
(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) + 176*a**(31/2)*b
**(25/2)*x**10*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(
21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*
b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x
**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) + 396*a**(29/2)*b**(27/2)*x**9*
sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**
(21/2) + 17325*a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2)
 + 17325*a**(15/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155
*a**(11/2)*b**21*x**(11/2)) + 462*a**(27/2)*b**(29/2)*x**8*sqrt(a*x/b + 1)
/(1155*a**(23/2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*
a**(19/2)*b**17*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15
/2)*b**19*x**(15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**2
1*x**(11/2)) - 1848*a**(23/2)*b**(33/2)*x**6*sqrt(a*x/b + 1)/(1155*a**(23/
2)*b**15*x**(23/2) + 6930*a**(21/2)*b**16*x**(21/2) + 17325*a**(19/2)*b**1
7*x**(19/2) + 23100*a**(17/2)*b**18*x**(17/2) + 17325*a**(15/2)*b**19*x**(
15/2) + 6930*a**(13/2)*b**20*x**(13/2) + 1155*a**(11/2)*b**21*x**(11/2)) -
 5544*a**(21/2)*b**(35/2)*x**5*sqrt(a*x/b + 1)/(1155*a**(23/2)*b**15*x*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{11}{2}}}{11\,b^4} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a}{3\,b^4} - \frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^2}{7\,b^4} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3}{5\,b^4}$$

input
```
integrate((a+b/x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-2/11*(a + b/x)^(11/2)/b^4 + 2/3*(a + b/x)^(9/2)*a/b^4 - 6/7*(a + b/x)^(7/
2)*a^2/b^4 + 2/5*(a + b/x)^(5/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.99

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx = \frac{2\left(2310\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^7 a^{\frac{7}{2}}\operatorname{sgn}(x)+10164\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^6 a^3 b\operatorname{sgn}(x)+196\right.}{}$$

input
```
integrate((a+b/x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
2/1155*(2310*(sqrt(a)*x - sqrt(a*x^2 + b*x))^7*a^(7/2)*sgn(x) + 10164*(sqr
t(a)*x - sqrt(a*x^2 + b*x))^6*a^3*b*sgn(x) + 19635*(sqrt(a)*x - sqrt(a*x^2
 + b*x))^5*a^(5/2)*b^2*sgn(x) + 21285*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^
2*b^3*sgn(x) + 13860*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^4*sgn(x)
+ 5390*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^5*sgn(x) + 1155*(sqrt(a)*x -
sqrt(a*x^2 + b*x))*sqrt(a)*b^6*sgn(x) + 105*b^7*sgn(x))/(sqrt(a)*x - sqrt(
a*x^2 + b*x))^11
```

Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx = \frac{32\,a^5\,\sqrt{a+\frac{b}{x}}}{1155\,b^4} - \frac{2\,b\,\sqrt{a+\frac{b}{x}}}{11\,x^5} - \frac{8\,a\,\sqrt{a+\frac{b}{x}}}{33\,x^4} - \frac{2\,a^2\,\sqrt{a+\frac{b}{x}}}{231\,b\,x^3} + \frac{4\,a^3\,\sqrt{a+\frac{b}{x}}}{385\,b^2\,x^2} - \frac{16\,a^4\,\sqrt{a+\frac{b}{x}}}{1155\,b^3\,x}$$

input
```
int((a + b/x)^(3/2)/x^5,x)
```

output
```
(32*a^5*(a + b/x)^(1/2))/(1155*b^4) - (2*b*(a + b/x)^(1/2))/(11*x^5) - (8*
a*(a + b/x)^(1/2))/(33*x^4) - (2*a^2*(a + b/x)^(1/2))/(231*b*x^3) + (4*a^3
*(a + b/x)^(1/2))/(385*b^2*x^2) - (16*a^4*(a + b/x)^(1/2))/(1155*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}\,dx=\frac{\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^5x^5}{1155}-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^4b\,x^4}{1155}+\frac{4\sqrt{x}\,\sqrt{ax+b}\,a^3b^2x^3}{385}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2b^3x^2}{231}-\frac{8\sqrt{x}\,\sqrt{ax+b}\,a\,b^4x}{33}-}{b^4x^6}$$

input
```
int((a+b/x)^(3/2)/x^5,x)
```

output
```
(2*(16*sqrt(x)*sqrt(a*x + b)*a**5*x**5 - 8*sqrt(x)*sqrt(a*x + b)*a**4*b*x*
*4 + 6*sqrt(x)*sqrt(a*x + b)*a**3*b**2*x**3 - 5*sqrt(x)*sqrt(a*x + b)*a**2
*b**3*x**2 - 140*sqrt(x)*sqrt(a*x + b)*a*b**4*x - 105*sqrt(x)*sqrt(a*x + b
)*b**5 - 16*sqrt(a)*a**5*x**6))/(1155*b**4*x**6)
```

3.162 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6} dx = -\frac{2a^4\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^5} -\frac{4a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}$$

output
```
-2/5*a^4*(a+b/x)^(5/2)/b^5+8/7*a^3*(a+b/x)^(7/2)/b^5-4/3*a^2*(a+b/x)^(9/2)
/b^5+8/11*a*(a+b/x)^(11/2)/b^5-2/13*(a+b/x)^(13/2)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6} dx = -\frac{2(b+ax)^2\sqrt{\frac{b+ax}{x}}(1155b^4-840ab^3x+560a^2b^2x^2-320a^3bx^3+128a^4x^4)}{15015b^5x^6}$$

input
```
Integrate[(a + b/x)^(3/2)/x^6,x]
```

output
```
(-2*(b + a*x)^2*Sqrt[(b + a*x)/x]*(1155*b^4 - 840*a*b^3*x + 560*a^2*b^2*x^
2 - 320*a^3*b*x^3 + 128*a^4*x^4))/(15015*b^5*x^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{11/2}}{b^4}-\frac{4a\left(a+\frac{b}{x}\right)^{9/2}}{b^4}+\frac{6a^2\left(a+\frac{b}{x}\right)^{7/2}}{b^4}-\frac{4a^3\left(a+\frac{b}{x}\right)^{5/2}}{b^4}+\frac{a^4\left(a+\frac{b}{x}\right)^{3/2}}{b^4}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{4a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^5}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}$$

input
```
Int[(a + b/x)^(3/2)/x^6,x]
```

output
```
(-2*a^4*(a + b/x)^(5/2))/(5*b^5) + (8*a^3*(a + b/x)^(7/2))/(7*b^5) - (4*a^
2*(a + b/x)^(9/2))/(3*b^5) + (8*a*(a + b/x)^(11/2))/(11*b^5) - (2*(a + b/x
)^(13/2))/(13*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result	size
orering	$-\frac{2(128a^4x^4-320a^3b\,x^3+560a^2b^2x^2-840a\,b^3x+1155b^4)(ax+b)\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{15015b^5x^5}$	64
gosper	$-\frac{2(ax+b)(128a^4x^4-320a^3b\,x^3+560a^2b^2x^2-840a\,b^3x+1155b^4)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{15015b^5x^5}$	66
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(128a^6x^6-64a^5b\,x^5+48a^4b^2x^4-40a^3x^3b^3+35b^4x^2a^2+1470b^5xa+1155b^6)}{15015x^6b^5}$	83
trager	$-\frac{2(128a^6x^6-64a^5b\,x^5+48a^4b^2x^4-40a^3x^3b^3+35b^4x^2a^2+1470b^5xa+1155b^6)\sqrt{-\frac{-ax-b}{x}}}{15015x^6b^5}$	87
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^2+bx)^{\frac{3}{2}}(128a^5x^5-192a^4b\,x^4+240a^3b^2x^3-280a^2b^3x^2+315b^4xa+1155b^5)}{15015x^7b^5\sqrt{x(ax+b)}}$	92

input
```
int((a+b/x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/15015*(128*a^4*x^4-320*a^3*b*x^3+560*a^2*b^2*x^2-840*a*b^3*x+1155*b^4)/
b^5/x^5*(a*x+b)*(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx = -\frac{2\,(128\,a^6x^6-64\,a^5bx^5+48\,a^4b^2x^4-40\,a^3b^3x^3+35\,a^2b^4x^2+1470\,ab^5x+1155\,b^6)\sqrt{\frac{ax+b}{x}}}{15015\,b^5x^6}$$

input
```
integrate((a+b/x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-2/15015*(128*a^6*x^6 - 64*a^5*b*x^5 + 48*a^4*b^2*x^4 - 40*a^3*b^3*x^3 + 3
5*a^2*b^4*x^2 + 1470*a*b^5*x + 1155*b^6)*sqrt((a*x + b)/x)/(b^5*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5289 vs. 2(87) = 174.

Time = 3.46 (sec) , antiderivative size = 5289, normalized size of antiderivative = 52.37

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(3/2)/x**6,x)
```

output
```
-256*a**(45/2)*b**(49/2)*x**16*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(
33/2) + 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2
) + 1801800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2)
+ 3783780*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) +
1801800*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150
150*a**(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 2432*a*
*(43/2)*b**(51/2)*x**15*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) +
 150150*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 180
1800*a**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 37837
80*a**(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800
*a**(19/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**
(15/2)*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 10336*a**(41/2
)*b**(53/2)*x**14*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 15015
0*a**(31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a
**(27/2)*b**32*x**(27/2) + 3153150*a**(25/2)*b**33*x**(25/2) + 3783780*a**
(23/2)*b**34*x**(23/2) + 3153150*a**(21/2)*b**35*x**(21/2) + 1801800*a**(1
9/2)*b**36*x**(19/2) + 675675*a**(17/2)*b**37*x**(17/2) + 150150*a**(15/2)
*b**38*x**(15/2) + 15015*a**(13/2)*b**39*x**(13/2)) - 25840*a**(39/2)*b**(
55/2)*x**13*sqrt(a*x/b + 1)/(15015*a**(33/2)*b**29*x**(33/2) + 150150*a**(
31/2)*b**30*x**(31/2) + 675675*a**(29/2)*b**31*x**(29/2) + 1801800*a**(...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{13}{2}}}{13\,b^5} + \frac{8\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a}{11\,b^5} - \frac{4\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^2}{3\,b^5} + \frac{8\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^3}{7\,b^5} - \frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^4}{5\,b^5}$$

input
```
integrate((a+b/x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-2/13*(a + b/x)^(13/2)/b^5 + 8/11*(a + b/x)^(11/2)*a/b^5 - 4/3*(a + b/x)^(
9/2)*a^2/b^5 + 8/7*(a + b/x)^(7/2)*a^3/b^5 - 2/5*(a + b/x)^(5/2)*a^4/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(81) = 162$.

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.67

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx = \frac{2\left(48048\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^8 a^4 \mathrm{sgn}(x)+240240\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^7 a^{\frac{7}{2}} b\,\mathrm{sgn}(x)+53\right.}{}$$

input
```
integrate((a+b/x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
2/15015*(48048*(sqrt(a)*x - sqrt(a*x^2 + b*x))^8*a^4*sgn(x) + 240240*(sqrt
(a)*x - sqrt(a*x^2 + b*x))^7*a^(7/2)*b*sgn(x) + 531960*(sqrt(a)*x - sqrt(a
*x^2 + b*x))^6*a^3*b^2*sgn(x) + 675675*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a
^(5/2)*b^3*sgn(x) + 535535*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^4*sgn(x
) + 270270*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^5*sgn(x) + 84630*(s
qrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^6*sgn(x) + 15015*(sqrt(a)*x - sqrt(a*x
^2 + b*x))*sqrt(a)*b^7*sgn(x) + 1155*b^8*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^13
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx = \frac{16\,a^3\sqrt{a+\frac{b}{x}}}{3003\,b^2\,x^3} - \frac{2\,b\sqrt{a+\frac{b}{x}}}{13\,x^6} - \frac{256\,a^6\sqrt{a+\frac{b}{x}}}{15015\,b^5} - \frac{2\,a^2\sqrt{a+\frac{b}{x}}}{429\,b\,x^4} - \frac{28\,a\sqrt{a+\frac{b}{x}}}{143\,x^5} - \frac{32\,a^4\sqrt{a+\frac{b}{x}}}{5005\,b^3\,x^2} + \frac{128\,a^5\sqrt{a+\frac{b}{x}}}{15015\,b^4\,x}$$

input
```
int((a + b/x)^(3/2)/x^6,x)
```

output
```
(16*a^3*(a + b/x)^(1/2))/(3003*b^2*x^3) - (2*b*(a + b/x)^(1/2))/(13*x^6) -
 (256*a^6*(a + b/x)^(1/2))/(15015*b^5) - (2*a^2*(a + b/x)^(1/2))/(429*b*x^
4) - (28*a*(a + b/x)^(1/2))/(143*x^5) - (32*a^4*(a + b/x)^(1/2))/(5005*b^3
*x^2) + (128*a^5*(a + b/x)^(1/2))/(15015*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^6}\,dx = \frac{-\frac{256\sqrt{x}\,\sqrt{ax+b}\,a^6x^6}{15015}+\frac{128\sqrt{x}\,\sqrt{ax+b}\,a^5b\,x^5}{15015}-\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^4b^2x^4}{5005}+\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^3b^3x^3}{3003}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2}{429}}{b^5x^7}$$

input
```
int((a+b/x)^(3/2)/x^6,x)
```

output
```
(2*( - 128*sqrt(x)*sqrt(a*x + b)*a**6*x**6 + 64*sqrt(x)*sqrt(a*x + b)*a**5
*b*x**5 - 48*sqrt(x)*sqrt(a*x + b)*a**4*b**2*x**4 + 40*sqrt(x)*sqrt(a*x +
b)*a**3*b**3*x**3 - 35*sqrt(x)*sqrt(a*x + b)*a**2*b**4*x**2 - 1470*sqrt(x)
*sqrt(a*x + b)*a*b**5*x - 1155*sqrt(x)*sqrt(a*x + b)*b**6 + 128*sqrt(a)*a*
*6*x**7))/(15015*b**5*x**7)
```

3.163 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7} \, dx = \frac{2a^5\left(a+\frac{b}{x}\right)^{5/2}}{5b^6} - \frac{10a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^6} + \frac{20a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^6} - \frac{20a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^6} + \frac{10a\left(a+\frac{b}{x}\right)^{13/2}}{13b^6} - \frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^6}$$

output
```
2/5*a^5*(a+b/x)^(5/2)/b^6-10/7*a^4*(a+b/x)^(7/2)/b^6+20/9*a^3*(a+b/x)^(9/2
)/b^6-20/11*a^2*(a+b/x)^(11/2)/b^6+10/13*a*(a+b/x)^(13/2)/b^6-2/15*(a+b/x)
^(15/2)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7} \, dx = \frac{2(b+ax)^2\sqrt{\frac{b+ax}{x}}\left(-3003b^5+2310ab^4x-1680a^2b^3x^2+1120a^3b^2x^3-640a^4bx^4+256a\right.}{45045b^6x^7}$$

input
```
Integrate[(a + b/x)^(3/2)/x^7,x]
```

output
```
(2*(b + a*x)^2*Sqrt[(b + a*x)/x]*(-3003*b^5 + 2310*a*b^4*x - 1680*a^2*b^3*
x^2 + 1120*a^3*b^2*x^3 - 640*a^4*b*x^4 + 256*a^5*x^5))/(45045*b^6*x^7)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^5}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{13/2}}{b^5}-\frac{5a\left(a+\frac{b}{x}\right)^{11/2}}{b^5}+\frac{10a^2\left(a+\frac{b}{x}\right)^{9/2}}{b^5}-\frac{10a^3\left(a+\frac{b}{x}\right)^{7/2}}{b^5}+\frac{5a^4\left(a+\frac{b}{x}\right)^{5/2}}{b^5}-\frac{a^5\left(a+\frac{b}{x}\right)^{3/2}}{b^5}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^5\left(a+\frac{b}{x}\right)^{5/2}}{5b^6}-\frac{10a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}+\frac{20a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}-\frac{20a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^6}-\frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^6}+\frac{10a\left(a+\frac{b}{x}\right)^{13/2}}{13b^6}$$

input
```
Int[(a + b/x)^(3/2)/x^7,x]
```

output
```
(2*a^5*(a + b/x)^(5/2))/(5*b^6) - (10*a^4*(a + b/x)^(7/2))/(7*b^6) + (20*a
^3*(a + b/x)^(9/2))/(9*b^6) - (20*a^2*(a + b/x)^(11/2))/(11*b^6) + (10*a*(
a + b/x)^(13/2))/(13*b^6) - (2*(a + b/x)^(15/2))/(15*b^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

method	result	size
orering	$\frac{2(256a^5x^5-640a^4b\,x^4+1120a^3b^2x^3-1680a^2b^3x^2+2310b^4xa-3003b^5)(ax+b)\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{45045b^6x^6}$	75
gosper	$\frac{2(ax+b)(256a^5x^5-640a^4b\,x^4+1120a^3b^2x^3-1680a^2b^3x^2+2310b^4xa-3003b^5)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{45045b^6x^6}$	77
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,(256a^7x^7-128a^6b\,x^6+96a^5b^2x^5-80a^4b^3x^4+70b^4x^3a^3-63b^5x^2a^2-3696b^6xa-3003b^7)}{45045x^7b^6}$	94
trager	$\frac{2(256a^7x^7-128a^6b\,x^6+96a^5b^2x^5-80a^4b^3x^4+70b^4x^3a^3-63b^5x^2a^2-3696b^6xa-3003b^7)\sqrt{-\frac{-ax-b}{x}}}{45045x^7b^6}$	98
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^2+bx)^{\frac{3}{2}}(256a^6x^6-384a^5b\,x^5+480a^4b^2x^4-560a^3x^3b^3+630b^4x^2a^2-693b^5xa-3003b^6)}{45045x^8b^6\sqrt{x(ax+b)}}$	103

input
```
int((a+b/x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
2/45045*(256*a^5*x^5-640*a^4*b*x^4+1120*a^3*b^2*x^3-1680*a^2*b^3*x^2+2310*
a*b^4*x-3003*b^5)/b^6/x^6*(a*x+b)*(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx = \frac{2\,(256\,a^7x^7 - 128\,a^6bx^6 + 96\,a^5b^2x^5 - 80\,a^4b^3x^4 + 70\,a^3b^4x^3 - 63\,a^2b^5x^2 - 3696\,ab^6x -}{45045\,b^6x^7}$$

input
```
integrate((a+b/x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
2/45045*(256*a^7*x^7 - 128*a^6*b*x^6 + 96*a^5*b^2*x^5 - 80*a^4*b^3*x^4 + 7
0*a^3*b^4*x^3 - 63*a^2*b^5*x^2 - 3696*a*b^6*x - 3003*b^7)*sqrt((a*x + b)/x
)/(b^6*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10344 vs. 2(105) = 210.

Time = 5.24 (sec) , antiderivative size = 10344, normalized size of antiderivative = 84.79

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(3/2)/x**7,x)
```

output

```
512*a**(59/2)*b**(91/2)*x**22*sqrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(4
5/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2
) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2)*b**55*x**(37/2
) + 135270135*a**(35/2)*b**56*x**(35/2) + 225450225*a**(33/2)*b**57*x**(33
/2) + 289864575*a**(31/2)*b**58*x**(31/2) + 289864575*a**(29/2)*b**59*x**(
29/2) + 225450225*a**(27/2)*b**60*x**(27/2) + 135270135*a**(25/2)*b**61*x*
*(25/2) + 61486425*a**(23/2)*b**62*x**(23/2) + 20495475*a**(21/2)*b**63*x*
*(21/2) + 4729725*a**(19/2)*b**64*x**(19/2) + 675675*a**(17/2)*b**65*x**(1
7/2) + 45045*a**(15/2)*b**66*x**(15/2)) + 7424*a**(57/2)*b**(93/2)*x**21*s
qrt(a*x/b + 1)/(45045*a**(45/2)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x
**(43/2) + 4729725*a**(41/2)*b**53*x**(41/2) + 20495475*a**(39/2)*b**54*x*
*(39/2) + 61486425*a**(37/2)*b**55*x**(37/2) + 135270135*a**(35/2)*b**56*x
**(35/2) + 225450225*a**(33/2)*b**57*x**(33/2) + 289864575*a**(31/2)*b**58
*x**(31/2) + 289864575*a**(29/2)*b**59*x**(29/2) + 225450225*a**(27/2)*b**
60*x**(27/2) + 135270135*a**(25/2)*b**61*x**(25/2) + 61486425*a**(23/2)*b*
*62*x**(23/2) + 20495475*a**(21/2)*b**63*x**(21/2) + 4729725*a**(19/2)*b**
64*x**(19/2) + 675675*a**(17/2)*b**65*x**(17/2) + 45045*a**(15/2)*b**66*x*
*(15/2)) + 50112*a**(55/2)*b**(95/2)*x**20*sqrt(a*x/b + 1)/(45045*a**(45/2
)*b**51*x**(45/2) + 675675*a**(43/2)*b**52*x**(43/2) + 4729725*a**(41/2)*b
**53*x**(41/2) + 20495475*a**(39/2)*b**54*x**(39/2) + 61486425*a**(37/2...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{15}{2}}}{15\,b^6}+\frac{10\left(a+\frac{b}{x}\right)^{\frac{13}{2}}a}{13\,b^6}-\frac{20\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a^2}{11\,b^6}+\frac{20\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^3}{9\,b^6}-\frac{10\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4}{7\,b^6}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^5}{5\,b^6}$$

input

```
integrate((a+b/x)^(3/2)/x^7,x, algorithm="maxima")
```

output

```
-2/15*(a + b/x)^(15/2)/b^6 + 10/13*(a + b/x)^(13/2)*a/b^6 - 20/11*(a + b/x
)^(11/2)*a^2/b^6 + 20/9*(a + b/x)^(9/2)*a^3/b^6 - 10/7*(a + b/x)^(7/2)*a^4
/b^6 + 2/5*(a + b/x)^(5/2)*a^5/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(98) = 196$.

Time = 0.17 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.47

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx = \frac{2\left(240240\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^9 a^{\frac{9}{2}}\mathrm{sgn}(x)+1338480\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^8 a^4 b\,\mathrm{sgn}(x)+\right.}{}$$

input
```
integrate((a+b/x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
2/45045*(240240*(sqrt(a)*x - sqrt(a*x^2 + b*x))^9*a^(9/2)*sgn(x) + 1338480
*(sqrt(a)*x - sqrt(a*x^2 + b*x))^8*a^4*b*sgn(x) + 3333330*(sqrt(a)*x - sqr
t(a*x^2 + b*x))^7*a^(7/2)*b^2*sgn(x) + 4844840*(sqrt(a)*x - sqrt(a*x^2 + b
*x))^6*a^3*b^3*sgn(x) + 4513509*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*
b^4*sgn(x) + 2788695*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^5*sgn(x) + 11
41140*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^6*sgn(x) + 297990*(sqrt(
a)*x - sqrt(a*x^2 + b*x))^2*a*b^7*sgn(x) + 45045*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))*sqrt(a)*b^8*sgn(x) + 3003*b^9*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x
))^15
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx = \frac{512\,a^7\sqrt{a+\frac{b}{x}}}{45045\,b^6} - \frac{2\,b\sqrt{a+\frac{b}{x}}}{15\,x^7} - \frac{32\,a\sqrt{a+\frac{b}{x}}}{195\,x^6} - \frac{2\,a^2\sqrt{a+\frac{b}{x}}}{715\,b\,x^5} + \frac{4\,a^3\sqrt{a+\frac{b}{x}}}{1287\,b^2\,x^4} - \frac{32\,a^4\sqrt{a+\frac{b}{x}}}{9009\,b^3\,x^3} + \frac{64\,a^5\sqrt{a+\frac{b}{x}}}{15015\,b^4\,x^2} - \frac{256\,a^6\sqrt{a+\frac{b}{x}}}{45045\,b^5\,x}$$

input
```
int((a + b/x)^(3/2)/x^7,x)
```

output
```
(512*a^7*(a + b/x)^(1/2))/(45045*b^6) - (2*b*(a + b/x)^(1/2))/(15*x^7) - (
32*a*(a + b/x)^(1/2))/(195*x^6) - (2*a^2*(a + b/x)^(1/2))/(715*b*x^5) + (4
*a^3*(a + b/x)^(1/2))/(1287*b^2*x^4) - (32*a^4*(a + b/x)^(1/2))/(9009*b^3*
x^3) + (64*a^5*(a + b/x)^(1/2))/(15015*b^4*x^2) - (256*a^6*(a + b/x)^(1/2)
)/(45045*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^7}\,dx = \frac{\frac{512\sqrt{x}\,\sqrt{ax+b}\,a^7x^7}{45045}-\frac{256\sqrt{x}\,\sqrt{ax+b}\,a^6b\,x^6}{45045}+\frac{64\sqrt{x}\,\sqrt{ax+b}\,a^5b^2x^5}{15015}-\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^4b^3x^4}{9009}+\frac{4\sqrt{x}\,\sqrt{ax+b}\,a^3b^4}{1287}}{b^6x^8}$$

input
```
int((a+b/x)^(3/2)/x^7,x)
```

output
```
(2*(256*sqrt(x)*sqrt(a*x + b)*a**7*x**7 - 128*sqrt(x)*sqrt(a*x + b)*a**6*b
*x**6 + 96*sqrt(x)*sqrt(a*x + b)*a**5*b**2*x**5 - 80*sqrt(x)*sqrt(a*x + b)
*a**4*b**3*x**4 + 70*sqrt(x)*sqrt(a*x + b)*a**3*b**4*x**3 - 63*sqrt(x)*sqr
t(a*x + b)*a**2*b**5*x**2 - 3696*sqrt(x)*sqrt(a*x + b)*a*b**6*x - 3003*sqr
t(x)*sqrt(a*x + b)*b**7 - 256*sqrt(a)*a**7*x**8))/(45045*b**6*x**8)
```

3.164 $\int \left(a+\frac{b}{x}\right)^{5/2} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3 \, dx = \frac{5b^3\sqrt{a+\frac{b}{x}}x}{64a}+\frac{59}{96}b^2\sqrt{a+\frac{b}{x}}x^2+\frac{17}{24}ab\sqrt{a+\frac{b}{x}}x^3+\frac{1}{4}a^2\sqrt{a+\frac{b}{x}}x^4-\frac{5b^4\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{3/2}}$$

output
```
5/64*b^3*(a+b/x)^(1/2)*x/a+59/96*b^2*(a+b/x)^(1/2)*x^2+17/24*a*b*(a+b/x)^(
1/2)*x^3+1/4*a^2*(a+b/x)^(1/2)*x^4-5/64*b^4*arctanh((a+b/x)^(1/2)/a^(1/2))
/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3 \, dx = \frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(15b^3+118ab^2x+136a^2bx^2+48a^3x^3)-15b^4\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{192a^{3/2}}$$

input
```
Integrate[(a + b/x)^(5/2)*x^3,x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(15*b^3 + 118*a*b^2*x + 136*a^2*b*x^2 + 48*a^3*x^
3) - 15*b^4*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(192*a^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x}\right)^{5/2} dx$$

$$\Big\downarrow \text{798}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2} x^5 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4} x^4 \left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{8} b \int \left(a+\frac{b}{x}\right)^{3/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4} x^4 \left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{8} b \left(\frac{1}{2} b \int \sqrt{a+\frac{b}{x}} x^3 d\frac{1}{x}-\frac{1}{3} x^3 \left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4} x^4 \left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{8} b \left(\frac{1}{2} b \left(\frac{1}{4} b \int \frac{x^2}{\sqrt{a+\frac{b}{x}}} d\frac{1}{x}-\frac{1}{2} x^2 \sqrt{a+\frac{b}{x}}\right)-\frac{1}{3} x^3 \left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)-\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)-\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}x^4\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)-\frac{1}{2}x^2\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(5/2)*x^3,x]
```

output
```
((a + b/x)^(5/2)*x^4)/4 - (5*b*(-1/3*((a + b/x)^(3/2)*x^3) + (b*(-1/2*(Sqr
t[a + b/x]*x^2) + (b*(-((Sqrt[a + b/x]*x)/a) + (b*ArcTanh[Sqrt[a + b/x]/Sq
rt[a]])/a^(3/2)))/4))/2))/8
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

method	result
risch	$\frac{(48a^3x^3+136a^2b\,x^2+118a\,b^2x+15b^3)x\sqrt{\frac{ax+b}{x}}}{192a}-\frac{5b^4\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{128a^{\frac{3}{2}}(ax+b)}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-96x(a\,x^2+bx)^{\frac{3}{2}}a^{\frac{7}{2}}-176a^{\frac{5}{2}}(a\,x^2+bx)^{\frac{3}{2}}b-60a^{\frac{5}{2}}\sqrt{a\,x^2+bx}\,b^2x-30a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b^3+15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a}{384a^{\frac{5}{2}}\sqrt{x(ax+b)}}$

input
```
int((a+b/x)^(5/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*a^3*x^3+136*a^2*b*x^2+118*a*b^2*x+15*b^3)*x/a*((a*x+b)/x)^(1/2)-
5/128*b^4/a^(3/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1
/2)*(x*(a*x+b))^(1/2)/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.55

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx = \left[\frac{15\,\sqrt{a}b^4\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(48\,a^4x^4+136\,a^3bx^3+118\,a^2b^2x^2+15\,ab^3x)}{384\,a^2}\right.$$

input
```
integrate((a+b/x)^(5/2)*x^3,x, algorithm="fricas")
```

output
```
[1/384*(15*sqrt(a)*b^4*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*
(48*a^4*x^4 + 136*a^3*b*x^3 + 118*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((a*x + b)
/x))/a^2, 1/192*(15*sqrt(-a)*b^4*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x
+ b)) + (48*a^4*x^4 + 136*a^3*b*x^3 + 118*a^2*b^2*x^2 + 15*a*b^3*x)*sqrt((
a*x + b)/x))/a^2]
```

Sympy [A] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx = \frac{a^3x^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{23a^2\sqrt{b}x^{\frac{7}{2}}}{24\sqrt{\frac{ax}{b}+1}} + \frac{127ab^{\frac{3}{2}}x^{\frac{5}{2}}}{96\sqrt{\frac{ax}{b}+1}} + \frac{133b^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{\frac{ax}{b}+1}} + \frac{5b^{\frac{7}{2}}\sqrt{x}}{64a\sqrt{\frac{ax}{b}+1}} - \frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{3}{2}}}$$

input
```
integrate((a+b/x)**(5/2)*x**3,x)
```

output
```
a**3*x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) + 23*a**2*sqrt(b)*x**(7/2)/(24*s
qrt(a*x/b + 1)) + 127*a*b**(3/2)*x**(5/2)/(96*sqrt(a*x/b + 1)) + 133*b**(5
/2)*x**(3/2)/(192*sqrt(a*x/b + 1)) + 5*b**(7/2)*sqrt(x)/(64*a*sqrt(a*x/b +
 1)) - 5*b**4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.43

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx = \frac{5\,b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{3}{2}}} + \frac{15\,\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4 + 73\,\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4 - 55\,\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4 + 15\,\sqrt{a+\frac{b}{x}}a^3b^4}{192\,\left(\left(a+\frac{b}{x}\right)^4 a - 4\,\left(a+\frac{b}{x}\right)^3 a^2 + 6\,\left(a+\frac{b}{x}\right)^2 a^3 - 4\,\left(a+\frac{b}{x}\right)a^4 + a^5\right)}$$

input
```
integrate((a+b/x)^(5/2)*x^3,x, algorithm="maxima")
```

output
```
5/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
 + 1/192*(15*(a + b/x)^(7/2)*b^4 + 73*(a + b/x)^(5/2)*a*b^4 - 55*(a + b/x)
^(3/2)*a^2*b^4 + 15*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a - 4*(a + b/x)^3*
a^2 + 6*(a + b/x)^2*a^3 - 4*(a + b/x)*a^4 + a^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx = \frac{5\,b^4\log\left(\left|2\,\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\mathrm{sgn}(x)}{128\,a^{\frac{3}{2}}} - \frac{5\,b^4\log\left(|b|\right)\mathrm{sgn}(x)}{128\,a^{\frac{3}{2}}} + \frac{1}{192}\sqrt{ax^2+bx}\left(\frac{15\,b^3\mathrm{sgn}(x)}{a} + 2\,\left(59\,b^2\mathrm{sgn}(x) + 4\,\left(6\,a^2x\mathrm{sgn}(x) + 17\,ab\mathrm{sgn}(x)\right)x\right)x\right)$$

input
```
integrate((a+b/x)^(5/2)*x^3,x, algorithm="giac")
```

output
```
5/128*b^4*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/a
^(3/2) - 5/128*b^4*log(abs(b))*sgn(x)/a^(3/2) + 1/192*sqrt(a*x^2 + b*x)*(1
5*b^3*sgn(x)/a + 2*(59*b^2*sgn(x) + 4*(6*a^2*x*sgn(x) + 17*a*b*sgn(x))*x)*
x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx = \frac{73\,x^4\left(a+\frac{b}{x}\right)^{5/2}}{192} - \frac{55\,a\,x^4\left(a+\frac{b}{x}\right)^{3/2}}{192} + \frac{5\,a^2\,x^4\sqrt{a+\frac{b}{x}}}{64} + \frac{5\,x^4\left(a+\frac{b}{x}\right)^{7/2}}{64\,a} + \frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{64\,a^{3/2}}$$

input
```
int(x^3*(a + b/x)^(5/2),x)
```

output
```
(73*x^4*(a + b/x)^(5/2))/192 + (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)
/(64*a^(3/2)) - (55*a*x^4*(a + b/x)^(3/2))/192 + (5*a^2*x^4*(a + b/x)^(1/2
))/64 + (5*x^4*(a + b/x)^(7/2))/(64*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^3\,dx = \frac{48\sqrt{x}\,\sqrt{ax+b}\,a^4x^3+136\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^2+118\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x+15\sqrt{x}\,\sqrt{ax+b}\,a\,b}{192a^2}$$

input
```
int((a+b/x)^(5/2)*x^3,x)
```

output
```
(48*sqrt(x)*sqrt(a*x + b)*a**4*x**3 + 136*sqrt(x)*sqrt(a*x + b)*a**3*b*x**
2 + 118*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x + 15*sqrt(x)*sqrt(a*x + b)*a*b**
3 - 15*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**4)/(192*a
**2)
```

3.165 $\int \left(a+\frac{b}{x}\right)^{5/2} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2 \, dx = \frac{11}{8} b^2 \sqrt{a+\frac{b}{x}} x + \frac{13}{12} a b \sqrt{a+\frac{b}{x}} x^2 + \frac{1}{3} a^2 \sqrt{a+\frac{b}{x}} x^3 + \frac{5 b^3 \operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8 \sqrt{a}}$$

output
```
11/8*b^2*(a+b/x)^(1/2)*x+13/12*a*b*(a+b/x)^(1/2)*x^2+1/3*a^2*(a+b/x)^(1/2)
*x^3+5/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2 \, dx = \frac{1}{24} \sqrt{a+\frac{b}{x}} x \left(33 b^2 + 26 a b x + 8 a^2 x^2\right) + \frac{5 b^3 \operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8 \sqrt{a}}$$

input
```
Integrate[(a + b/x)^(5/2)*x^2,x]
```

output
```
(Sqrt[a + b/x]*x*(33*b^2 + 26*a*b*x + 8*a^2*x^2))/24 + (5*b^3*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x}\right)^{5/2} dx$$

$$\Big\downarrow \text{798}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{6}b\int \left(a+\frac{b}{x}\right)^{3/2} x^3 d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{6}b\left(\frac{3}{4}b\int \sqrt{a+\frac{b}{x}}x^2 d\frac{1}{x}-\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}-x\sqrt{a+\frac{b}{x}}\right)-\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{6}b\left(\frac{3}{4}b\left(\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}-x\sqrt{a+\frac{b}{x}}\right)-\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{6}b\left(\frac{3}{4}b\left(x\left(-\sqrt{a+\frac{b}{x}}\right)-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(5/2)*x^2,x]
```

output
```
((a + b/x)^(5/2)*x^3)/3 - (5*b*(-1/2*((a + b/x)^(3/2)*x^2) + (3*b*(-(Sqrt[
a + b/x]*x) - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]))/4))/6
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{(8a^2x^2+26abx+33b^2)x\sqrt{\frac{ax+b}{x}}}{24}+\frac{5b^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{16\sqrt{a}\,(ax+b)}$	94
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(16\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}+36\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}bx+66\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b^2+15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^3\right)}{48\sqrt{x(ax+b)}\,a^{\frac{3}{2}}}$	115

input
```
int((a+b/x)^(5/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2+26*a*b*x+33*b^2)*x*((a*x+b)/x)^(1/2)+5/16*b^3*ln((1/2*b+a*
x)/a^(1/2)+(a*x^2+b*x)^(1/2))/a^(1/2)*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/
(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.73

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx = \left[\frac{15\,\sqrt{a}b^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(8\,a^3x^3+26\,a^2bx^2+33\,ab^2x)\sqrt{\frac{ax+b}{x}}}{48\,a},\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^3\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(8\,a^3x^3+26\,a^2bx^2+33\,ab^2x)\sqrt{\frac{ax+b}{x}}}{24\,a}\right]$$

input
```
integrate((a+b/x)^(5/2)*x^2,x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(
8*a^3*x^3 + 26*a^2*b*x^2 + 33*a*b^2*x)*sqrt((a*x + b)/x))/a, -1/24*(15*sqr
t(-a)*b^3*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (8*a^3*x^3 + 26
*a^2*b*x^2 + 33*a*b^2*x)*sqrt((a*x + b)/x))/a]
```

Sympy [A] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx = \frac{a^2\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{3} + \frac{13ab^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{12} + \frac{11b^{\frac{5}{2}}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{8} + \frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{8\sqrt{a}}$$

input
```
integrate((a+b/x)**(5/2)*x**2,x)
```

output
```
a**2*sqrt(b)*x**(5/2)*sqrt(a*x/b + 1)/3 + 13*a*b**(3/2)*x**(3/2)*sqrt(a*x/
b + 1)/12 + 11*b**(5/2)*sqrt(x)*sqrt(a*x/b + 1)/8 + 5*b**3*asinh(sqrt(a)*s
qrt(x)/sqrt(b))/(8*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.44

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx = -\frac{5\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,\sqrt{a}} + \frac{33\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3 - 40\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3 + 15\,\sqrt{a+\frac{b}{x}}a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3 - 3\left(a+\frac{b}{x}\right)^2a + 3\left(a+\frac{b}{x}\right)a^2 - a^3\right)}$$

input
```
integrate((a+b/x)^(5/2)*x^2,x, algorithm="maxima")
```

output
```
-5/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a)
 + 1/24*(33*(a + b/x)^(5/2)*b^3 - 40*(a + b/x)^(3/2)*a*b^3 + 15*sqrt(a + b
/x)*a^2*b^3)/((a + b/x)^3 - 3*(a + b/x)^2*a + 3*(a + b/x)*a^2 - a^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx = -\frac{5\,b^3\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)\operatorname{sgn}(x)}{16\sqrt{a}} + \frac{5\,b^3\log\left(|b|\right)\operatorname{sgn}(x)}{16\sqrt{a}} + \frac{1}{24}\sqrt{ax^2+bx}\left(33\,b^2\operatorname{sgn}(x)+2\left(4\,a^2x\operatorname{sgn}(x)+13\,ab\operatorname{sgn}(x)\right)x\right)$$

input
```
integrate((a+b/x)^(5/2)*x^2,x, algorithm="giac")
```

output
```
-5/16*b^3*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))*sgn(x)/s
qrt(a) + 5/16*b^3*log(abs(b))*sgn(x)/sqrt(a) + 1/24*sqrt(a*x^2 + b*x)*(33*
b^2*sgn(x) + 2*(4*a^2*x*sgn(x) + 13*a*b*sgn(x))*x)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx = \frac{11\,x^3\left(a+\frac{b}{x}\right)^{5/2}}{8} - \frac{5\,a\,x^3\left(a+\frac{b}{x}\right)^{3/2}}{3} + \frac{5\,a^2\,x^3\sqrt{a+\frac{b}{x}}}{8} - \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{8\sqrt{a}}$$

input
```
int(x^2*(a + b/x)^(5/2),x)
```

output
```
(11*x^3*(a + b/x)^(5/2))/8 - (b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)/(
8*a^(1/2)) - (5*a*x^3*(a + b/x)^(3/2))/3 + (5*a^2*x^3*(a + b/x)^(1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^2\,dx = \frac{8\sqrt{x}\,\sqrt{ax+b}\,a^3x^2+26\sqrt{x}\,\sqrt{ax+b}\,a^2bx+33\sqrt{x}\,\sqrt{ax+b}\,a\,b^2+15\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)}{24a}$$

input
```
int((a+b/x)^(5/2)*x^2,x)
```

output
```
(8*sqrt(x)*sqrt(a*x + b)*a**3*x**2 + 26*sqrt(x)*sqrt(a*x + b)*a**2*b*x + 3
3*sqrt(x)*sqrt(a*x + b)*a*b**2 + 15*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*s
qrt(a))/sqrt(b))*b**3)/(24*a)
```

3.166 $\int \left(a+\frac{b}{x}\right)^{5/2} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \left(a+\frac{b}{x}\right)^{5/2} x \, dx = -2b^2\sqrt{a+\frac{b}{x}}+\frac{9}{4}ab\sqrt{a+\frac{b}{x}}x+\frac{1}{2}a^2\sqrt{a+\frac{b}{x}}x^2+\frac{15}{4}\sqrt{a}b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*b^2*(a+b/x)^(1/2)+9/4*a*b*(a+b/x)^(1/2)*x+1/2*a^2*(a+b/x)^(1/2)*x^2+15/
4*a^(1/2)*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^{5/2} x \, dx = \frac{1}{4}\left(\sqrt{a+\frac{b}{x}}\left(-8b^2+9abx+2a^2x^2\right)+15\sqrt{a}b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)$$

input
```
Integrate[(a + b/x)^(5/2)*x,x]
```

output
```
(Sqrt[a + b/x]*(-8*b^2 + 9*a*b*x + 2*a^2*x^2) + 15*Sqrt[a]*b^2*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]])/4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {798, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{4}b\int \left(a+\frac{b}{x}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{4}b\left(\frac{3}{2}b\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}-x\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{4}b\left(\frac{3}{2}b\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)-x\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)-x\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}x^2\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)-x\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(5/2)*x,x]
```

output
```
((a + b/x)^(5/2)*x^2)/2 - (5*b*(-((a + b/x)^(3/2)*x) + (3*b*(2*Sqrt[a + b/
x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]))/2))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{\left(2 a^{2} x^{2}+9 a b x -8 b^{2}\right) \sqrt{\frac{a x +b}{x}}}{4}+\frac{15 \sqrt{a}\, b^{2} \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a \,x^{2}+b x}\right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x \left(a x +b \right)}}{8 \left(a x +b \right)}$	93
default	$-\frac{\sqrt{\frac{a x +b}{x}}\, \left(-4 \sqrt{a \,x^{2}+b x}\, a^{\frac{7}{2}} x^{3}-34 \sqrt{a \,x^{2}+b x}\, a^{\frac{5}{2}} b \,x^{2}-15 a^{2} \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) b^{2} x^{2}+16 \left(a \,x^{2}+b x \right)^{\frac{3}{2}} a^{\frac{3}{2}} b \right)}{8 x \sqrt{x \left(a x +b \right)}\, a^{\frac{3}{2}}}$	125

input
```
int((a+b/x)^(5/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a^2*x^2+9*a*b*x-8*b^2)*((a*x+b)/x)^(1/2)+15/8*a^(1/2)*b^2*ln((1/2*b
+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+
b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.57

$$\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx = \left[\frac{15}{8}\,\sqrt{a}b^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right) + \frac{1}{4}\left(2\,a^2x^2+9\,abx-8\,b^2\right)\sqrt{\frac{ax+b}{x}}, -\frac{15}{4}\,\sqrt{-a}b^2\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right) + \frac{1}{4}\left(2\,a^2x^2+9\,abx-8\,b^2\right)\sqrt{\frac{ax+b}{x}}\right]$$

input
```
integrate((a+b/x)^(5/2)*x,x, algorithm="fricas")
```

output
```
[15/8*sqrt(a)*b^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 1/4*(2*
a^2*x^2 + 9*a*b*x - 8*b^2)*sqrt((a*x + b)/x), -15/4*sqrt(-a)*b^2*arctan(sq
rt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + 1/4*(2*a^2*x^2 + 9*a*b*x - 8*b^2)*
sqrt((a*x + b)/x)]
```

Sympy [A] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.47

$$\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx = \frac{15\sqrt{a}b^2 \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4} + \frac{a^3x^{\frac{5}{2}}}{2\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{11a^2\sqrt{b}x^{\frac{3}{2}}}{4\sqrt{\frac{ax}{b}+1}} + \frac{ab^{\frac{3}{2}}\sqrt{x}}{4\sqrt{\frac{ax}{b}+1}} - \frac{2b^{\frac{5}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate((a+b/x)**(5/2)*x,x)
```

output
```
15*sqrt(a)*b**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/4 + a**3*x**(5/2)/(2*sqrt(b
)*sqrt(a*x/b + 1)) + 11*a**2*sqrt(b)*x**(3/2)/(4*sqrt(a*x/b + 1)) + a*b**(
3/2)*sqrt(x)/(4*sqrt(a*x/b + 1)) - 2*b**(5/2)/(sqrt(x)*sqrt(a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx = -\frac{15}{8}\sqrt{a}b^2 \log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}b^2 + \frac{9\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^2 - 7\sqrt{a+\frac{b}{x}}a^2b^2}{4\left(\left(a+\frac{b}{x}\right)^2 - 2\left(a+\frac{b}{x}\right)a + a^2\right)}$$

input
```
integrate((a+b/x)^(5/2)*x,x, algorithm="maxima")
```

output
```
-15/8*sqrt(a)*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))
 - 2*sqrt(a + b/x)*b^2 + 1/4*(9*(a + b/x)^(3/2)*a*b^2 - 7*sqrt(a + b/x)*a^
2*b^2)/((a + b/x)^2 - 2*(a + b/x)*a + a^2)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)*x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx = \frac{9\,a\,x^2\,\left(a+\frac{b}{x}\right)^{3/2}}{4} - 2\,b^2\,\sqrt{a+\frac{b}{x}} - \frac{7\,a^2\,x^2\,\sqrt{a+\frac{b}{x}}}{4} - \frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{4}$$

input
```
int(x*(a + b/x)^(5/2),x)
```

output
```
(9*a*x^2*(a + b/x)^(3/2))/4 - (a^(1/2)*b^2*atan(((a + b/x)^(1/2)*1i)/a^(1/
2))*15i)/4 - 2*b^2*(a + b/x)^(1/2) - (7*a^2*x^2*(a + b/x)^(1/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x}\right)^{5/2} x\,dx = \frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2x^2+9\sqrt{x}\,\sqrt{ax+b}\,abx-8\sqrt{x}\,\sqrt{ax+b}\,b^2+15\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2x-}{4x}$$

input
```
int((a+b/x)^(5/2)*x,x)
```

output
```
(2*sqrt(x)*sqrt(a*x + b)*a**2*x**2 + 9*sqrt(x)*sqrt(a*x + b)*a*b*x - 8*sqr
t(x)*sqrt(a*x + b)*b**2 + 15*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))
/sqrt(b))*b**2*x - 10*sqrt(a)*b**2*x)/(4*x)
```

3.167 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = -4ab\sqrt{a+\frac{b}{x}} - \frac{2}{3}b\left(a+\frac{b}{x}\right)^{3/2} + a^2\sqrt{a+\frac{b}{x}}x + 5a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-4*a*b*(a+b/x)^(1/2)-2/3*b*(a+b/x)^(3/2)+a^2*(a+b/x)^(1/2)*x+5*a^(3/2)*b*a
rctanh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \frac{\sqrt{a+\frac{b}{x}}(-2b^2-14abx+3a^2x^2)}{3x} + 5a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {773, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{51}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{2}b\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(5/2),x]
```

output
```
(a + b/x)^(5/2)*x - (5*b*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2*S
qrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{(3a^2x^2-14abx-2b^2)\sqrt{\frac{ax+b}{x}}}{3x}+\frac{5a^{\frac{3}{2}}b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{2(ax+b)}$	94
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^3+30\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}x^3-24\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}x-4\left(a\,x^2+bx\right)^{\frac{3}{2}}b\sqrt{a}\right)}{6x^2\sqrt{x(ax+b)}\,\sqrt{a}}$	120

input
```
int((a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a^2*x^2-14*a*b*x-2*b^2)/x*((a*x+b)/x)^(1/2)+5/2*a^(3/2)*b*ln((1/2*b
+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+
b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.95

$$\int\left(a+\frac{b}{x}\right)^{5/2}dx=\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{6\,x},\right.$$
$$\left.-\frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(3\,a^2x^2-14\,abx-2\,b^2)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3
*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x
*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (3*a^2*x^2 - 14*a*b*x -
2*b^2)*sqrt((a*x + b)/x))/x]
```

Sympy [A] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}} - \frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3} - \frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2} + 5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right) - \frac{2\sqrt{a}b^2\sqrt{1+\frac{b}{ax}}}{3x}$$

input
```
integrate((a+b/x)**(5/2),x)
```

output
```
a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3
/2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)
*b**2*sqrt(1 + b/(a*x))/(3*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \sqrt{a+\frac{b}{x}}a^2x - \frac{5}{2}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - \frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b - 4\sqrt{a+\frac{b}{x}}ab$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.46

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = -\frac{2\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}$$

input
```
int((a + b/x)^(5/2),x)
```

output
```
-(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
 + 1)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x}\right)^{5/2} dx = \frac{6\sqrt{x}\,\sqrt{ax+b}\,a^2x^2-28\sqrt{x}\,\sqrt{ax+b}\,abx-4\sqrt{x}\,\sqrt{ax+b}\,b^2+30\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)ab\,x^2 \cdots}{6x^2}$$

input
```
int((a+b/x)^(5/2),x)
```

output
```
(6*sqrt(x)*sqrt(a*x + b)*a**2*x**2 - 28*sqrt(x)*sqrt(a*x + b)*a*b*x - 4*sq
rt(x)*sqrt(a*x + b)*b**2 + 30*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a)
)/sqrt(b))*a*b*x**2 + 5*sqrt(a)*a*b*x**2)/(6*x**2)
```

3.168 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx = -2a^2\sqrt{a+\frac{b}{x}} - \frac{2}{3}a\left(a+\frac{b}{x}\right)^{3/2} - \frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2} + 2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

output
```
-2*a^2*(a+b/x)^(1/2)-2/3*a*(a+b/x)^(3/2)-2/5*(a+b/x)^(5/2)+2*a^(5/2)*arcta
nh((a+b/x)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}\,(3b^2+11abx+23a^2x^2)}{15x^2} + 2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/x)^(5/2)/x,x]
```

output
```
(-2*Sqrt[a + b/x]*(3*b^2 + 11*a*b*x + 23*a^2*x^2))/(15*x^2) + 2*a^(5/2)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \left(a+\frac{b}{x}\right)^{5/2}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$-a\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{x}-\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$-a\left(a\int \sqrt{a+\frac{b}{x}}xd\frac{1}{x}+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}$$

$$\Big\downarrow \text{60}$$

$$-a\left(a\left(a\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}$$

$$\Big\downarrow \text{73}$$

$$-a\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}+2\sqrt{a+\frac{b}{x}}\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}$$

$$\Big\downarrow \text{221}$$

$$-a\left(a\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x}\right)^{5/2}$$

input
```
Int[(a + b/x)^(5/2)/x,x]
```

output
```
(-2*(a + b/x)^(5/2))/5 - a*((2*(a + b/x)^(3/2))/3 + a*(2*Sqrt[a + b/x] - 2
*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{2(23a^2x^2+11abx+3b^2)\sqrt{\frac{ax+b}{x}}}{15x^2}+\frac{a^{\frac{5}{2}}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x(ax+b)}}{ax+b}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-30\sqrt{a\,x^2+bx}\,a^{\frac{7}{2}}x^4-15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b\,x^4+30\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}x^2+16a^{\frac{3}{2}}\left(a\,x^2+bx\right)^{\frac{3}{2}}bx+6\left(a\,x^2+bx\right)^{\frac{3}{2}}\right.}{15x^3b\sqrt{x(ax+b)}\,\sqrt{a}}$

input
```
int((a+b/x)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(23*a^2*x^2+11*a*b*x+3*b^2)/x^2*((a*x+b)/x)^(1/2)+a^(5/2)*ln((1/2*b+
a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))*((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)/(a*x+b
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.01

$$\int\frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx=\left[\frac{15\,a^{\frac{5}{2}}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(23\,a^2x^2+11\,abx+3\,b^2)\sqrt{\frac{ax+b}{x}}}{15\,x^2},\right.$$
$$\left.-\frac{2\left(15\,\sqrt{-a}a^2x^2\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+(23\,a^2x^2+11\,abx+3\,b^2)\sqrt{\frac{ax+b}{x}}\right)}{15\,x^2}\right]$$

input
```
integrate((a+b/x)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*a^(5/2)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(
23*a^2*x^2 + 11*a*b*x + 3*b^2)*sqrt((a*x + b)/x))/x^2, -2/15*(15*sqrt(-a)*
a^2*x^2*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (23*a^2*x^2 + 11*
a*b*x + 3*b^2)*sqrt((a*x + b)/x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx = -\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}{15} - a^{\frac{5}{2}}\log\left(\frac{b}{ax}\right) + 2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{b}{ax}}+1\right) - \frac{22a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{15x} - \frac{2\sqrt{a}b^2\sqrt{1+\frac{b}{ax}}}{5x^2}$$

input
```
integrate((a+b/x)**(5/2)/x,x)
```

output
```
-46*a**(5/2)*sqrt(1 + b/(a*x))/15 - a**(5/2)*log(b/(a*x)) + 2*a**(5/2)*log
(sqrt(1 + b/(a*x)) + 1) - 22*a**(3/2)*b*sqrt(1 + b/(a*x))/(15*x) - 2*sqrt(
a)*b**2*sqrt(1 + b/(a*x))/(5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx = -a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - \frac{2}{5}\left(a+\frac{b}{x}\right)^{\frac{5}{2}} - \frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a - 2\sqrt{a+\frac{b}{x}}a^2$$

input
```
integrate((a+b/x)^(5/2)/x,x, algorithm="maxima")
```

output
```
-a^(5/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 2/5*(a
 + b/x)^(5/2) - 2/3*(a + b/x)^(3/2)*a - 2*sqrt(a + b/x)*a^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/x)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx = -\frac{2\,a\,\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{2\,\left(a+\frac{b}{x}\right)^{5/2}}{5} - 2\,a^2\,\sqrt{a+\frac{b}{x}}-a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

input
```
int((a + b/x)^(5/2)/x,x)
```

output
```
- a^(5/2)*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*(a + b/x)^(3/2))/3
- (2*(a + b/x)^(5/2))/5 - 2*a^2*(a + b/x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}\,dx = \frac{-\frac{46\sqrt{x}\,\sqrt{ax+b}\,a^2x^2}{15}-\frac{22\sqrt{x}\,\sqrt{ax+b}\,abx}{15}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^2}{5}+2\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a^2x^3+\frac{2\sqrt{a}\,a^2x^3}{3}}{x^3}$$

input
```
int((a+b/x)^(5/2)/x,x)
```

output
```
(2*( - 23*sqrt(x)*sqrt(a*x + b)*a**2*x**2 - 11*sqrt(x)*sqrt(a*x + b)*a*b*x
 - 3*sqrt(x)*sqrt(a*x + b)*b**2 + 15*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*
sqrt(a))/sqrt(b))*a**2*x**3 + 5*sqrt(a)*a**2*x**3))/(15*x**3)
```

3.169 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2} \, dx=-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

output
```
-2/7*(a+b/x)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2} \, dx=-\frac{2\left(\frac{b+ax}{x}\right)^{7/2}}{7b}$$

input
```
Integrate[(a + b/x)^(5/2)/x^2,x]
```

output
```
(-2*((b + a*x)/x)^(7/2))/(7*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

input
```
Int[(a + b/x)^(5/2)/x^2,x]
```

output
```
(-2*(a + b/x)^(7/2))/(7*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7b}$	15
orering	$-\frac{2(ax+b)\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{7xb}$	23
gosper	$-\frac{2(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{7xb}$	25
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(a^{3}x^{3}+3a^{2}b\,x^{2}+3a\,b^{2}x+b^{3})}{7x^{3}b}$	47
trager	$-\frac{2(a^{3}x^{3}+3a^{2}b\,x^{2}+3a\,b^{2}x+b^{3})\sqrt{-\frac{-ax-b}{x}}}{7x^{3}b}$	51
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,(a\,x^{2}+bx)^{\frac{3}{2}}\,(a^{2}x^{2}+2abx+b^{2})}{7x^{4}b\sqrt{x(ax+b)}}$	56

input
```
int((a+b/x)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(a+b/x)^(7/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx=-\frac{2\,(a^3x^3+3\,a^2bx^2+3\,ab^2x+b^3)\sqrt{\frac{ax+b}{x}}}{7\,bx^3}$$

input
```
integrate((a+b/x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
-2/7*(a^3*x^3 + 3*a^2*b*x^2 + 3*a*b^2*x + b^3)*sqrt((a*x + b)/x)/(b*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(14) = 28$.

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.44

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx = \begin{cases} -\frac{2a^3\sqrt{a+\frac{b}{x}}}{7b} - \frac{6a^2\sqrt{a+\frac{b}{x}}}{7x} - \frac{6ab\sqrt{a+\frac{b}{x}}}{7x^2} - \frac{2b^2\sqrt{a+\frac{b}{x}}}{7x^3} & \text{for } b \neq 0 \\ -\frac{a^{\frac{5}{2}}}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x)**(5/2)/x**2,x)
```

output
```
Piecewise((-2*a**3*sqrt(a + b/x)/(7*b) - 6*a**2*sqrt(a + b/x)/(7*x) - 6*a*
b*sqrt(a + b/x)/(7*x**2) - 2*b**2*sqrt(a + b/x)/(7*x**3), Ne(b, 0)), (-a**
(5/2)/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b}$$

input
```
integrate((a+b/x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(14) = 28.

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 11.50

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx = \frac{2\left(7\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^6 a^3\mathrm{sgn}(x)+21\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^5 a^{\frac{5}{2}}b\mathrm{sgn}(x)+35\left(\sqrt{a}x-\right.\right.}{}$$

input
```
integrate((a+b/x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
2/7*(7*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*sgn(x) + 21*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^5*a^(5/2)*b*sgn(x) + 35*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a
^2*b^2*sgn(x) + 35*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^3*sgn(x) +
21*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^4*sgn(x) + 7*(sqrt(a)*x - sqrt(a*
x^2 + b*x))*sqrt(a)*b^5*sgn(x) + b^6*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x
))^7
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx = -\frac{2\,a^3\,\sqrt{a+\frac{b}{x}}}{7\,b}-\frac{6\,a^2\,\sqrt{a+\frac{b}{x}}}{7\,x}-\frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{7\,x^3}-\frac{6\,a\,b\,\sqrt{a+\frac{b}{x}}}{7\,x^2}$$

input
```
int((a + b/x)^(5/2)/x^2,x)
```

output
```
- (2*a^3*(a + b/x)^(1/2))/(7*b) - (6*a^2*(a + b/x)^(1/2))/(7*x) - (2*b^2*(
a + b/x)^(1/2))/(7*x^3) - (6*a*b*(a + b/x)^(1/2))/(7*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.44

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^3x^3}{7}-\frac{6\sqrt{x}\,\sqrt{ax+b}\,a^2b\,x^2}{7}-\frac{6\sqrt{x}\,\sqrt{ax+b}\,a\,b^2x}{7}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^3}{7}-\frac{2\sqrt{a}\,a^3x^4}{7}}{b\,x^4}$$

input
```
int((a+b/x)^(5/2)/x^2,x)
```

output
```
(2*( - sqrt(x)*sqrt(a*x + b)*a**3*x**3 - 3*sqrt(x)*sqrt(a*x + b)*a**2*b*x*
*2 - 3*sqrt(x)*sqrt(a*x + b)*a*b**2*x - sqrt(x)*sqrt(a*x + b)*b**3 - sqrt(
a)*a**3*x**4))/(7*b*x**4)
```

3.170 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3} dx = \frac{2a\left(a+\frac{b}{x}\right)^{7/2}}{7b^2} - \frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^2}$$

output
```
2/7*a*(a+b/x)^(7/2)/b^2-2/9*(a+b/x)^(9/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3} dx = \frac{2(b+ax)^3\sqrt{\frac{b+ax}{x}}(-7b+2ax)}{63b^2x^4}$$

input
```
Integrate[(a + b/x)^(5/2)/x^3,x]
```

output
```
(2*(b + a*x)^3*Sqrt[(b + a*x)/x]*(-7*b + 2*a*x))/(63*b^2*x^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{ 798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{7/2}}{b}-\frac{a\left(a+\frac{b}{x}\right)^{5/2}}{b}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a\left(a+\frac{b}{x}\right)^{7/2}}{7b^2}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^2}$$

input
```
Int[(a + b/x)^(5/2)/x^3,x]
```

output
```
(2*a*(a + b/x)^(7/2))/(7*b^2) - (2*(a + b/x)^(9/2))/(9*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result	size
orering	$\frac{2(2ax-7b)(ax+b)\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{63b^{2}x^{2}}$	31
gosper	$\frac{2(ax+b)(2ax-7b)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{63b^{2}x^{2}}$	33
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\left(2a^{4}x^{4}-a^{3}b\,x^{3}-15a^{2}b^{2}x^{2}-19a\,b^{3}x-7b^{4}\right)}{63x^{4}b^{2}}$	61
trager	$\frac{2\left(2a^{4}x^{4}-a^{3}b\,x^{3}-15a^{2}b^{2}x^{2}-19a\,b^{3}x-7b^{4}\right)\sqrt{-\frac{-ax-b}{x}}}{63x^{4}b^{2}}$	65
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\left(2a^{3}x^{3}-3a^{2}b\,x^{2}-12a\,b^{2}x-7b^{3}\right)}{63x^{5}b^{2}\sqrt{x(ax+b)}}$	70

input
```
int((a+b/x)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/63*(2*a*x-7*b)/b^2/x^2*(a*x+b)*(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3}}\,dx=\frac{2\left(2\,a^{4}x^{4}-a^{3}bx^{3}-15\,a^{2}b^{2}x^{2}-19\,ab^{3}x-7\,b^{4}\right)\sqrt{\frac{ax+b}{x}}}{63\,b^{2}x^{4}}$$

input
```
integrate((a+b/x)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
2/63*(2*a^4*x^4 - a^3*b*x^3 - 15*a^2*b^2*x^2 - 19*a*b^3*x - 7*b^4)*sqrt((a
*x + b)/x)/(b^2*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(31) = 62$.

Time = 1.21 (sec) , antiderivative size = 416, normalized size of antiderivative = 10.95

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx = \frac{4a^{\frac{19}{2}}b^{\frac{3}{2}}x^5\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}} + \frac{2a^{\frac{17}{2}}b^{\frac{5}{2}}x^4\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}} - \frac{32a^{\frac{15}{2}}b^{\frac{7}{2}}x^3\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}} - \frac{68a^{\frac{13}{2}}b^{\frac{9}{2}}x^2\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}} - \frac{52a^{\frac{11}{2}}b^{\frac{11}{2}}x\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}} - \frac{14a^{\frac{9}{2}}b^{\frac{13}{2}}\sqrt{\frac{ax}{b}+1}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}} - \frac{4a^{10}bx^{\frac{11}{2}}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}} - \frac{4a^9b^2x^{\frac{9}{2}}}{63a^{\frac{11}{2}}b^3x^{\frac{11}{2}}+63a^{\frac{9}{2}}b^4x^{\frac{9}{2}}}$$

input
```
integrate((a+b/x)**(5/2)/x**3,x)
```

output
```
4*a**(19/2)*b**(3/2)*x**5*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x**(11/2) + 6
3*a**(9/2)*b**4*x**(9/2)) + 2*a**(17/2)*b**(5/2)*x**4*sqrt(a*x/b + 1)/(63*
a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**4*x**(9/2)) - 32*a**(15/2)*b**(7
/2)*x**3*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**4*x
**(9/2)) - 68*a**(13/2)*b**(9/2)*x**2*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x
**(11/2) + 63*a**(9/2)*b**4*x**(9/2)) - 52*a**(11/2)*b**(11/2)*x*sqrt(a*x/
b + 1)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*b**4*x**(9/2)) - 14*a**(
9/2)*b**(13/2)*sqrt(a*x/b + 1)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**(9/2)*
b**4*x**(9/2)) - 4*a**10*b*x**(11/2)/(63*a**(11/2)*b**3*x**(11/2) + 63*a**
(9/2)*b**4*x**(9/2)) - 4*a**9*b**2*x**(9/2)/(63*a**(11/2)*b**3*x**(11/2) +
 63*a**(9/2)*b**4*x**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^2}+\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^2}$$

input
```
integrate((a+b/x)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x)^(9/2)/b^2 + 2/7*(a + b/x)^(7/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(30) = 60.

Time = 0.15 (sec) , antiderivative size = 239, normalized size of antiderivative = 6.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx = \frac{2\left(63\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^7a^{\frac{7}{2}}\mathrm{sgn}(x)+273\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^6a^3b\,\mathrm{sgn}(x)+567\left(\sqrt{a}x\right.\right.}{}$$

input
```
integrate((a+b/x)^(5/2)/x^3,x, algorithm="giac")
```

output
```
2/63*(63*(sqrt(a)*x - sqrt(a*x^2 + b*x))^7*a^(7/2)*sgn(x) + 273*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))^6*a^3*b*sgn(x) + 567*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^5*a^(5/2)*b^2*sgn(x) + 693*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^3*sgn(
x) + 525*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^4*sgn(x) + 243*(sqrt(
a)*x - sqrt(a*x^2 + b*x))^2*a*b^5*sgn(x) + 63*(sqrt(a)*x - sqrt(a*x^2 + b*
x))*sqrt(a)*b^6*sgn(x) + 7*b^7*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^9
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx = \frac{4\,a^4\,\sqrt{a+\frac{b}{x}}}{63\,b^2} - \frac{10\,a^2\,\sqrt{a+\frac{b}{x}}}{21\,x^2} - \frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{9\,x^4} - \frac{2\,a^3\,\sqrt{a+\frac{b}{x}}}{63\,b\,x} - \frac{38\,a\,b\,\sqrt{a+\frac{b}{x}}}{63\,x^3}$$

input
```
int((a + b/x)^(5/2)/x^3,x)
```

output
```
(4*a^4*(a + b/x)^(1/2))/(63*b^2) - (10*a^2*(a + b/x)^(1/2))/(21*x^2) - (2*
b^2*(a + b/x)^(1/2))/(9*x^4) - (2*a^3*(a + b/x)^(1/2))/(63*b*x) - (38*a*b*
(a + b/x)^(1/2))/(63*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.61

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}\,dx = \frac{\frac{4\sqrt{x}\,\sqrt{ax+b}\,a^4x^4}{63} - \frac{2\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^3}{63} - \frac{10\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x^2}{21} - \frac{38\sqrt{x}\,\sqrt{ax+b}\,a\,b^3x}{63} - \frac{2\sqrt{x}\,\sqrt{ax+b}\,b^4}{9} - \cdots}{b^2x^5}$$

input
```
int((a+b/x)^(5/2)/x^3,x)
```

output
```
(2*(2*sqrt(x)*sqrt(a*x + b)*a**4*x**4 - sqrt(x)*sqrt(a*x + b)*a**3*b*x**3
- 15*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x**2 - 19*sqrt(x)*sqrt(a*x + b)*a*b**
3*x - 7*sqrt(x)*sqrt(a*x + b)*b**4 - 2*sqrt(a)*a**4*x**5))/(63*b**2*x**5)
```

3.171 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4} dx = -\frac{2a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3} + \frac{4a\left(a+\frac{b}{x}\right)^{9/2}}{9b^3} - \frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^3}$$

output
```
-2/7*a^2*(a+b/x)^(7/2)/b^3+4/9*a*(a+b/x)^(9/2)/b^3-2/11*(a+b/x)^(11/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4} dx = -\frac{2(b+ax)^3\sqrt{\frac{b+ax}{x}}(63b^2-28abx+8a^2x^2)}{693b^3x^5}$$

input
```
Integrate[(a + b/x)^(5/2)/x^4,x]
```

output
```
(-2*(b + a*x)^3*Sqrt[(b + a*x)/x]*(63*b^2 - 28*a*b*x + 8*a^2*x^2))/(693*b^
3*x^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{9/2}}{b^2}-\frac{2a\left(a+\frac{b}{x}\right)^{7/2}}{b^2}+\frac{a^2\left(a+\frac{b}{x}\right)^{5/2}}{b^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2\left(a+\frac{b}{x}\right)^{7/2}}{7b^3}-\frac{2\left(a+\frac{b}{x}\right)^{11/2}}{11b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{9/2}}{9b^3}$$

input
```
Int[(a + b/x)^(5/2)/x^4,x]
```

output
```
(-2*a^2*(a + b/x)^(7/2))/(7*b^3) + (4*a*(a + b/x)^(9/2))/(9*b^3) - (2*(a +
 b/x)^(11/2))/(11*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

method	result	size
orering	$-\frac{2\left(8a^{2}x^{2}-28abx+63b^{2}\right)\left(ax+b\right)\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{693b^{3}x^{3}}$	42
gosper	$-\frac{2\left(ax+b\right)\left(8a^{2}x^{2}-28abx+63b^{2}\right)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{693b^{3}x^{3}}$	44
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(8a^{5}x^{5}-4a^{4}b\,x^{4}+3a^{3}b^{2}x^{3}+113a^{2}b^{3}x^{2}+161b^{4}xa+63b^{5}\right)}{693x^{5}b^{3}}$	72
trager	$-\frac{2\left(8a^{5}x^{5}-4a^{4}b\,x^{4}+3a^{3}b^{2}x^{3}+113a^{2}b^{3}x^{2}+161b^{4}xa+63b^{5}\right)\sqrt{-\frac{-ax-b}{x}}}{693x^{5}b^{3}}$	76
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\left(8a^{4}x^{4}-12a^{3}b\,x^{3}+15a^{2}b^{2}x^{2}+98a\,b^{3}x+63b^{4}\right)}{693x^{6}b^{3}\sqrt{x\left(ax+b\right)}}$	81

input
```
int((a+b/x)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/693*(8*a^2*x^2-28*a*b*x+63*b^2)/b^3/x^3*(a*x+b)*(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx = -\frac{2\left(8\,a^5x^5-4\,a^4bx^4+3\,a^3b^2x^3+113\,a^2b^3x^2+161\,ab^4x+63\,b^5\right)\sqrt{\frac{ax+b}{x}}}{693\,b^3x^5}$$

input
```
integrate((a+b/x)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
-2/693*(8*a^5*x^5 - 4*a^4*b*x^4 + 3*a^3*b^2*x^3 + 113*a^2*b^3*x^2 + 161*a*
b^4*x + 63*b^5)*sqrt((a*x + b)/x)/(b^3*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. 2(49) = 98.

Time = 1.54 (sec) , antiderivative size = 1073, normalized size of antiderivative = 18.19

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(5/2)/x**4,x)
```

output
```
-16*a**(27/2)*b**(9/2)*x***8*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2)
+ 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(
11/2)*b**10*x**(11/2)) - 40*a**(25/2)*b**(11/2)*x***7*sqrt(a*x/b + 1)/(693*
a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*
b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) - 30*a**(23/2)*b**(13/2)*x
**6*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x*
*(15/2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) -
 236*a**(21/2)*b**(15/2)*x***5*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2
) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a*
*(11/2)*b**10*x**(11/2)) - 1010*a**(19/2)*b**(17/2)*x***4*sqrt(a*x/b + 1)/(
693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13
/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) - 1776*a**(17/2)*b**(1
9/2)*x***3*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b
**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11
/2)) - 1570*a**(15/2)*b**(21/2)*x***2*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x
**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**(13/2)*b**9*x**(13/2) +
 693*a**(11/2)*b**10*x**(11/2)) - 700*a**(13/2)*b**(23/2)*x*sqrt(a*x/b + 1
)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b**8*x**(15/2) + 2079*a**
(13/2)*b**9*x**(13/2) + 693*a**(11/2)*b**10*x**(11/2)) - 126*a**(11/2)*b**
(25/2)*sqrt(a*x/b + 1)/(693*a**(17/2)*b**7*x**(17/2) + 2079*a**(15/2)*b...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{11}{2}}}{11\,b^3} + \frac{4\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a}{9\,b^3} - \frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^2}{7\,b^3}$$

input
```
integrate((a+b/x)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
-2/11*(a + b/x)^(11/2)/b^3 + 4/9*(a + b/x)^(9/2)*a/b^3 - 2/7*(a + b/x)^(7/
2)*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.58

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx = \frac{2\left(924\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^8 a^4\mathrm{sgn}(x)+4851\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^7 a^{\frac{7}{2}}b\mathrm{sgn}(x)+11781\right.}{}$$

input
```
integrate((a+b/x)^(5/2)/x^4,x, algorithm="giac")
```

output
```
2/693*(924*(sqrt(a)*x - sqrt(a*x^2 + b*x))^8*a^4*sgn(x) + 4851*(sqrt(a)*x
- sqrt(a*x^2 + b*x))^7*a^(7/2)*b*sgn(x) + 11781*(sqrt(a)*x - sqrt(a*x^2 +
b*x))^6*a^3*b^2*sgn(x) + 16863*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b
^3*sgn(x) + 15345*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^4*sgn(x) + 9009*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^5*sgn(x) + 3311*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a*b^6*sgn(x) + 693*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqr
t(a)*b^7*sgn(x) + 63*b^8*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^11
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx = \frac{8\,a^4\,\sqrt{a+\frac{b}{x}}}{693\,b^2\,x} - \frac{226\,a^2\,\sqrt{a+\frac{b}{x}}}{693\,x^3} - \frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{11\,x^5} - \frac{2\,a^3\,\sqrt{a+\frac{b}{x}}}{231\,b\,x^2} - \frac{16\,a^5\,\sqrt{a+\frac{b}{x}}}{693\,b^3} - \frac{46\,a\,b\,\sqrt{a+\frac{b}{x}}}{99\,x^4}$$

input
```
int((a + b/x)^(5/2)/x^4,x)
```

output
```
(8*a^4*(a + b/x)^(1/2))/(693*b^2*x) - (226*a^2*(a + b/x)^(1/2))/(693*x^3)
- (2*b^2*(a + b/x)^(1/2))/(11*x^5) - (2*a^3*(a + b/x)^(1/2))/(231*b*x^2) -
 (16*a^5*(a + b/x)^(1/2))/(693*b^3) - (46*a*b*(a + b/x)^(1/2))/(99*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}\,dx = \frac{-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^5x^5}{693}+\frac{8\sqrt{x}\,\sqrt{ax+b}\,a^4b\,x^4}{693}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^3b^2x^3}{231}-\frac{226\sqrt{x}\,\sqrt{ax+b}\,a^2b^3x^2}{693}-\frac{46\sqrt{x}\,\sqrt{ax+b}\,a\,b^4x}{99}}{b^3x^6}$$

input
```
int((a+b/x)^(5/2)/x^4,x)
```

output
```
(2*( - 8*sqrt(x)*sqrt(a*x + b)*a**5*x**5 + 4*sqrt(x)*sqrt(a*x + b)*a**4*b*
x**4 - 3*sqrt(x)*sqrt(a*x + b)*a**3*b**2*x**3 - 113*sqrt(x)*sqrt(a*x + b)*
a**2*b**3*x**2 - 161*sqrt(x)*sqrt(a*x + b)*a*b**4*x - 63*sqrt(x)*sqrt(a*x
+ b)*b**5 + 8*sqrt(a)*a**5*x**6))/(693*b**3*x**6)
```

3.172 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5} \, dx = \frac{2a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^4} - \frac{2a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^4} + \frac{6a\left(a+\frac{b}{x}\right)^{11/2}}{11b^4} - \frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^4}$$

output
```
2/7*a^3*(a+b/x)^(7/2)/b^4-2/3*a^2*(a+b/x)^(9/2)/b^4+6/11*a*(a+b/x)^(11/2)/
b^4-2/13*(a+b/x)^(13/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5} \, dx = \frac{2(b+ax)^3\sqrt{\frac{b+ax}{x}}(-231b^3+126ab^2x-56a^2bx^2+16a^3x^3)}{3003b^4x^6}$$

input
```
Integrate[(a + b/x)^(5/2)/x^5,x]
```

output
```
(2*(b + a*x)^3*Sqrt[(b + a*x)/x]*(-231*b^3 + 126*a*b^2*x - 56*a^2*b*x^2 +
16*a^3*x^3))/(3003*b^4*x^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{11/2}}{b^3}-\frac{3a\left(a+\frac{b}{x}\right)^{9/2}}{b^3}+\frac{3a^2\left(a+\frac{b}{x}\right)^{7/2}}{b^3}-\frac{a^3\left(a+\frac{b}{x}\right)^{5/2}}{b^3}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}-\frac{2a^2\left(a+\frac{b}{x}\right)^{9/2}}{3b^4}-\frac{2\left(a+\frac{b}{x}\right)^{13/2}}{13b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{11/2}}{11b^4}$$

input
```
Int[(a + b/x)^(5/2)/x^5,x]
```

output
```
(2*a^3*(a + b/x)^(7/2))/(7*b^4) - (2*a^2*(a + b/x)^(9/2))/(3*b^4) + (6*a*(
a + b/x)^(11/2))/(11*b^4) - (2*(a + b/x)^(13/2))/(13*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

method	result	size
orering	$\frac{2\left(16 a^{3} x^{3}-56 a^{2} b\,x^{2}+126 a\,b^{2} x-231 b^{3}\right)\left(a x+b\right)\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{3003 b^{4} x^{4}}$	53
gosper	$\frac{2\left(a x+b\right)\left(16 a^{3} x^{3}-56 a^{2} b\,x^{2}+126 a\,b^{2} x-231 b^{3}\right)\left(\frac{a x+b}{x}\right)^{\frac{5}{2}}}{3003 b^{4} x^{4}}$	55
risch	$\frac{2\sqrt{\frac{a x+b}{x}}\,\left(16 a^{6} x^{6}-8 a^{5} b\,x^{5}+6 a^{4} b^{2} x^{4}-5 a^{3} x^{3} b^{3}-371 b^{4} x^{2} a^{2}-567 b^{5} x a-231 b^{6}\right)}{3003 x^{6} b^{4}}$	83
trager	$\frac{2\left(16 a^{6} x^{6}-8 a^{5} b\,x^{5}+6 a^{4} b^{2} x^{4}-5 a^{3} x^{3} b^{3}-371 b^{4} x^{2} a^{2}-567 b^{5} x a-231 b^{6}\right)\sqrt{-\frac{-a x-b}{x}}}{3003 x^{6} b^{4}}$	87
default	$\frac{2\sqrt{\frac{a x+b}{x}}\,\left(a\,x^{2}+b x\right)^{\frac{3}{2}}\left(16 a^{5} x^{5}-24 a^{4} b\,x^{4}+30 a^{3} b^{2} x^{3}-35 a^{2} b^{3} x^{2}-336 b^{4} x a-231 b^{5}\right)}{3003 x^{7} b^{4}\sqrt{x\left(a x+b\right)}}$	92

input
```
int((a+b/x)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/3003*(16*a^3*x^3-56*a^2*b*x^2+126*a*b^2*x-231*b^3)/b^4/x^4*(a*x+b)*(a+b/
x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx = \frac{2\,(16\,a^6x^6-8\,a^5bx^5+6\,a^4b^2x^4-5\,a^3b^3x^3-371\,a^2b^4x^2-567\,ab^5x-231\,b^6)\sqrt{\frac{ax+b}{x}}}{3003\,b^4x^6}$$

input
```
integrate((a+b/x)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
2/3003*(16*a^6*x^6 - 8*a^5*b*x^5 + 6*a^4*b^2*x^4 - 5*a^3*b^3*x^3 - 371*a^2
*b^4*x^2 - 567*a*b^5*x - 231*b^6)*sqrt((a*x + b)/x)/(b^4*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2562 vs. 2(68) = 136.

Time = 2.20 (sec) , antiderivative size = 2562, normalized size of antiderivative = 32.02

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(5/2)/x**5,x)
```

output
```
32*a**(37/2)*b**(23/2)*x**12*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/
2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 6
0060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a
**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) + 176*a**(35/2)
*b**(25/2)*x**11*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a
**(23/2)*b**16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/
2)*b**18*x**(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**
20*x**(15/2) + 3003*a**(13/2)*b**21*x**(13/2)) + 396*a**(33/2)*b**(27/2)*x
**10*sqrt(a*x/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**
16*x**(23/2) + 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**
(19/2) + 45045*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2)
 + 3003*a**(13/2)*b**21*x**(13/2)) + 462*a**(31/2)*b**(29/2)*x**9*sqrt(a*x
/b + 1)/(3003*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2)
+ 45045*a**(21/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 4504
5*a**(17/2)*b**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(1
3/2)*b**21*x**(13/2)) - 462*a**(29/2)*b**(31/2)*x**8*sqrt(a*x/b + 1)/(3003
*a**(25/2)*b**15*x**(25/2) + 18018*a**(23/2)*b**16*x**(23/2) + 45045*a**(2
1/2)*b**17*x**(21/2) + 60060*a**(19/2)*b**18*x**(19/2) + 45045*a**(17/2)*b
**19*x**(17/2) + 18018*a**(15/2)*b**20*x**(15/2) + 3003*a**(13/2)*b**21*x*
*(13/2)) - 5544*a**(27/2)*b**(33/2)*x**7*sqrt(a*x/b + 1)/(3003*a**(25/2...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{13}{2}}}{13\,b^4} + \frac{6\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a}{11\,b^4} - \frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^2}{3\,b^4} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^3}{7\,b^4}$$

input
```
integrate((a+b/x)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
-2/13*(a + b/x)^(13/2)/b^4 + 6/11*(a + b/x)^(11/2)*a/b^4 - 2/3*(a + b/x)^(
9/2)*a^2/b^4 + 2/7*(a + b/x)^(7/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(64) = 128$.

Time = 0.17 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.76

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx = \frac{2\left(6006\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^9 a^{\frac{9}{2}}\operatorname{sgn}(x)+36036\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^8 a^4 b\operatorname{sgn}(x)+990\right.}{}$$

input
```
integrate((a+b/x)^(5/2)/x^5,x, algorithm="giac")
```

output
```
2/3003*(6006*(sqrt(a)*x - sqrt(a*x^2 + b*x))^9*a^(9/2)*sgn(x) + 36036*(sqr
t(a)*x - sqrt(a*x^2 + b*x))^8*a^4*b*sgn(x) + 99099*(sqrt(a)*x - sqrt(a*x^2
 + b*x))^7*a^(7/2)*b^2*sgn(x) + 161733*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a
^3*b^3*sgn(x) + 171171*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b^4*sgn(x
) + 121121*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^5*sgn(x) + 57057*(sqrt(
a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^6*sgn(x) + 17199*(sqrt(a)*x - sqrt(a
*x^2 + b*x))^2*a*b^7*sgn(x) + 3003*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)
*b^8*sgn(x) + 231*b^9*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^13
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx = \frac{32\,a^6\,\sqrt{a+\frac{b}{x}}}{3003\,b^4} - \frac{106\,a^2\,\sqrt{a+\frac{b}{x}}}{429\,x^4} - \frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{13\,x^6} - \frac{10\,a^3\,\sqrt{a+\frac{b}{x}}}{3003\,b\,x^3} + \frac{4\,a^4\,\sqrt{a+\frac{b}{x}}}{1001\,b^2\,x^2} - \frac{16\,a^5\,\sqrt{a+\frac{b}{x}}}{3003\,b^3\,x} - \frac{54\,a\,b\,\sqrt{a+\frac{b}{x}}}{143\,x^5}$$

input
```
int((a + b/x)^(5/2)/x^5,x)
```

output
```
(32*a^6*(a + b/x)^(1/2))/(3003*b^4) - (106*a^2*(a + b/x)^(1/2))/(429*x^4)
- (2*b^2*(a + b/x)^(1/2))/(13*x^6) - (10*a^3*(a + b/x)^(1/2))/(3003*b*x^3)
 + (4*a^4*(a + b/x)^(1/2))/(1001*b^2*x^2) - (16*a^5*(a + b/x)^(1/2))/(3003
*b^3*x) - (54*a*b*(a + b/x)^(1/2))/(143*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^5}\,dx = \frac{\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^6x^6}{3003} - \frac{16\sqrt{x}\,\sqrt{ax+b}\,a^5b\,x^5}{3003} + \frac{4\sqrt{x}\,\sqrt{ax+b}\,a^4b^2x^4}{1001} - \frac{10\sqrt{x}\,\sqrt{ax+b}\,a^3b^3x^3}{3003} - \frac{106\sqrt{x}\,\sqrt{ax+b}\,a^2b^4x}{429}}{b^4x^7}$$

input
```
int((a+b/x)^(5/2)/x^5,x)
```

output
```
(2*(16*sqrt(x)*sqrt(a*x + b)*a**6*x**6 - 8*sqrt(x)*sqrt(a*x + b)*a**5*b*x*
*5 + 6*sqrt(x)*sqrt(a*x + b)*a**4*b**2*x**4 - 5*sqrt(x)*sqrt(a*x + b)*a**3
*b**3*x**3 - 371*sqrt(x)*sqrt(a*x + b)*a**2*b**4*x**2 - 567*sqrt(x)*sqrt(a
*x + b)*a*b**5*x - 231*sqrt(x)*sqrt(a*x + b)*b**6 - 16*sqrt(a)*a**6*x**7))
/(3003*b**4*x**7)
```

3.173 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6} dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6} dx = -\frac{2a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^5} + \frac{8a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^5} - \frac{12a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5} + \frac{8a\left(a+\frac{b}{x}\right)^{13/2}}{13b^5} - \frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^5}$$

output
```
-2/7*a^4*(a+b/x)^(7/2)/b^5+8/9*a^3*(a+b/x)^(9/2)/b^5-12/11*a^2*(a+b/x)^(11
/2)/b^5+8/13*a*(a+b/x)^(13/2)/b^5-2/15*(a+b/x)^(15/2)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6} dx = -\frac{2(b+ax)^3\sqrt{\frac{b+ax}{x}}\left(3003b^4-1848ab^3x+1008a^2b^2x^2-448a^3bx^3+128a^4x^4\right)}{45045b^5x^7}$$

input
```
Integrate[(a + b/x)^(5/2)/x^6,x]
```

output
```
(-2*(b + a*x)^3*Sqrt[(b + a*x)/x]*(3003*b^4 - 1848*a*b^3*x + 1008*a^2*b^2*
x^2 - 448*a^3*b*x^3 + 128*a^4*x^4))/(45045*b^5*x^7)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\left(a+\frac{b}{x}\right)^{13/2}}{b^4}-\frac{4a\left(a+\frac{b}{x}\right)^{11/2}}{b^4}+\frac{6a^2\left(a+\frac{b}{x}\right)^{9/2}}{b^4}-\frac{4a^3\left(a+\frac{b}{x}\right)^{7/2}}{b^4}+\frac{a^4\left(a+\frac{b}{x}\right)^{5/2}}{b^4}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{11/2}}{11b^5}-\frac{2\left(a+\frac{b}{x}\right)^{15/2}}{15b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{13/2}}{13b^5}$$

input
```
Int[(a + b/x)^(5/2)/x^6,x]
```

output
```
(-2*a^4*(a + b/x)^(7/2))/(7*b^5) + (8*a^3*(a + b/x)^(9/2))/(9*b^5) - (12*a
^2*(a + b/x)^(11/2))/(11*b^5) + (8*a*(a + b/x)^(13/2))/(13*b^5) - (2*(a +
b/x)^(15/2))/(15*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

method	result	size
orering	$-\frac{2\left(128a^{4}x^{4}-448a^{3}b\,x^{3}+1008a^{2}b^{2}x^{2}-1848a\,b^{3}x+3003b^{4}\right)\left(ax+b\right)\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{45045b^{5}x^{5}}$	64
gosper	$-\frac{2\left(ax+b\right)\left(128a^{4}x^{4}-448a^{3}b\,x^{3}+1008a^{2}b^{2}x^{2}-1848a\,b^{3}x+3003b^{4}\right)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{45045b^{5}x^{5}}$	66
risch	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(128a^{7}x^{7}-64a^{6}b\,x^{6}+48a^{5}b^{2}x^{5}-40a^{4}b^{3}x^{4}+35b^{4}x^{3}a^{3}+4473b^{5}x^{2}a^{2}+7161b^{6}xa+3003b^{7}\right)}{45045x^{7}b^{5}}$	94
trager	$-\frac{2\left(128a^{7}x^{7}-64a^{6}b\,x^{6}+48a^{5}b^{2}x^{5}-40a^{4}b^{3}x^{4}+35b^{4}x^{3}a^{3}+4473b^{5}x^{2}a^{2}+7161b^{6}xa+3003b^{7}\right)\sqrt{-\frac{-ax-b}{x}}}{45045x^{7}b^{5}}$	98
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\left(128a^{6}x^{6}-192a^{5}b\,x^{5}+240a^{4}b^{2}x^{4}-280a^{3}x^{3}b^{3}+315b^{4}x^{2}a^{2}+4158b^{5}xa+3003b^{6}\right)}{45045x^{8}b^{5}\sqrt{x\left(ax+b\right)}}$	103

input
```
int((a+b/x)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/45045*(128*a^4*x^4-448*a^3*b*x^3+1008*a^2*b^2*x^2-1848*a*b^3*x+3003*b^4
)/b^5/x^5*(a*x+b)*(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx = -\frac{2\,(128\,a^7x^7 - 64\,a^6bx^6 + 48\,a^5b^2x^5 - 40\,a^4b^3x^4 + 35\,a^3b^4x^3 + 4473\,a^2b^5x^2 + 7161\,ab^6x + 3003\,b^7)\sqrt{\frac{ax+b}{x}}}{45045\,b^5x^7}$$

input
```
integrate((a+b/x)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
-2/45045*(128*a^7*x^7 - 64*a^6*b*x^6 + 48*a^5*b^2*x^5 - 40*a^4*b^3*x^4 + 3
5*a^3*b^4*x^3 + 4473*a^2*b^5*x^2 + 7161*a*b^6*x + 3003*b^7)*sqrt((a*x + b)
/x)/(b^5*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5482 vs. 2(87) = 174.

Time = 3.30 (sec) , antiderivative size = 5482, normalized size of antiderivative = 54.28

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x)**(5/2)/x**6,x)
```

output
```
-256*a**(49/2)*b**(49/2)*x**17*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(
35/2) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/
2) + 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2)
 + 11351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2)
+ 5405400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) +
450450*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 2432
*a**(47/2)*b**(51/2)*x**16*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2
) + 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) +
 5405400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 1
1351340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 54
05400*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 4504
50*a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 10336*a*
*(45/2)*b**(53/2)*x**15*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) +
 450450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 54
05400*a**(29/2)*b**32*x**(29/2) + 9459450*a**(27/2)*b**33*x**(27/2) + 1135
1340*a**(25/2)*b**34*x**(25/2) + 9459450*a**(23/2)*b**35*x**(23/2) + 54054
00*a**(21/2)*b**36*x**(21/2) + 2027025*a**(19/2)*b**37*x**(19/2) + 450450*
a**(17/2)*b**38*x**(17/2) + 45045*a**(15/2)*b**39*x**(15/2)) - 25840*a**(4
3/2)*b**(55/2)*x**14*sqrt(a*x/b + 1)/(45045*a**(35/2)*b**29*x**(35/2) + 45
0450*a**(33/2)*b**30*x**(33/2) + 2027025*a**(31/2)*b**31*x**(31/2) + 54...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{15}{2}}}{15\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{13}{2}}a}{13\,b^5} - \frac{12\left(a+\frac{b}{x}\right)^{\frac{11}{2}}a^2}{11\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^3}{9\,b^5}-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4}{7\,b^5}$$

input
```
integrate((a+b/x)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
-2/15*(a + b/x)^(15/2)/b^5 + 8/13*(a + b/x)^(13/2)*a/b^5 - 12/11*(a + b/x)
^(11/2)*a^2/b^5 + 8/9*(a + b/x)^(9/2)*a^3/b^5 - 2/7*(a + b/x)^(7/2)*a^4/b^
5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(81) = 162$.

Time = 0.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.29

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx = \frac{2\left(144144\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^{10}a^5\mathrm{sgn}(x)+960960\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^9a^{\frac{9}{2}}b\mathrm{sgn}(x)+\right.}{}$$

input
```
integrate((a+b/x)^(5/2)/x^6,x, algorithm="giac")
```

output
```
2/45045*(144144*(sqrt(a)*x - sqrt(a*x^2 + b*x))^10*a^5*sgn(x) + 960960*(sq
rt(a)*x - sqrt(a*x^2 + b*x))^9*a^(9/2)*b*sgn(x) + 2934360*(sqrt(a)*x - sqr
t(a*x^2 + b*x))^8*a^4*b^2*sgn(x) + 5360355*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^7*a^(7/2)*b^3*sgn(x) + 6451445*(sqrt(a)*x - sqrt(a*x^2 + b*x))^6*a^3*b^4*
sgn(x) + 5324319*(sqrt(a)*x - sqrt(a*x^2 + b*x))^5*a^(5/2)*b^5*sgn(x) + 30
42585*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2*b^6*sgn(x) + 1186185*(sqrt(a)*
x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b^7*sgn(x) + 301455*(sqrt(a)*x - sqrt(a*x
^2 + b*x))^2*a*b^8*sgn(x) + 45045*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*
b^9*sgn(x) + 3003*b^10*sgn(x))/(sqrt(a)*x - sqrt(a*x^2 + b*x))^15
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx = \frac{16\,a^4\,\sqrt{a+\frac{b}{x}}}{9009\,b^2\,x^3}-\frac{142\,a^2\,\sqrt{a+\frac{b}{x}}}{715\,x^5}-\frac{2\,b^2\,\sqrt{a+\frac{b}{x}}}{15\,x^7}-\frac{2\,a^3\,\sqrt{a+\frac{b}{x}}}{1287\,b\,x^4}-\frac{256\,a^7\,\sqrt{a+\frac{b}{x}}}{45045\,b^5}-\frac{32\,a^5\,\sqrt{a+\frac{b}{x}}}{15015\,b^3\,x^2}+\frac{128\,a^6\,\sqrt{a+\frac{b}{x}}}{45045\,b^4\,x}-\frac{62\,a\,b\,\sqrt{a+\frac{b}{x}}}{195\,x^6}$$

input
```
int((a + b/x)^(5/2)/x^6,x)
```

output

```
(16*a^4*(a + b/x)^(1/2))/(9009*b^2*x^3) - (142*a^2*(a + b/x)^(1/2))/(715*x
^5) - (2*b^2*(a + b/x)^(1/2))/(15*x^7) - (2*a^3*(a + b/x)^(1/2))/(1287*b*x
^4) - (256*a^7*(a + b/x)^(1/2))/(45045*b^5) - (32*a^5*(a + b/x)^(1/2))/(15
015*b^3*x^2) + (128*a^6*(a + b/x)^(1/2))/(45045*b^4*x) - (62*a*b*(a + b/x)
^(1/2))/(195*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^6}\,dx = \frac{-\frac{256\sqrt{x}\,\sqrt{ax+b}\,a^7x^7}{45045}+\frac{128\sqrt{x}\,\sqrt{ax+b}\,a^6b\,x^6}{45045}-\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^5b^2x^5}{15015}+\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^4b^3x^4}{9009}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,a^3}{1287}}{b^5x^8}$$

input

```
int((a+b/x)^(5/2)/x^6,x)
```

output

```
(2*( - 128*sqrt(x)*sqrt(a*x + b)*a**7*x**7 + 64*sqrt(x)*sqrt(a*x + b)*a**6
*b*x**6 - 48*sqrt(x)*sqrt(a*x + b)*a**5*b**2*x**5 + 40*sqrt(x)*sqrt(a*x +
b)*a**4*b**3*x**4 - 35*sqrt(x)*sqrt(a*x + b)*a**3*b**4*x**3 - 4473*sqrt(x)
*sqrt(a*x + b)*a**2*b**5*x**2 - 7161*sqrt(x)*sqrt(a*x + b)*a*b**6*x - 3003
*sqrt(x)*sqrt(a*x + b)*b**7 + 128*sqrt(a)*a**7*x**8))/(45045*b**5*x**8)
```

3.174 $\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{35b^3\sqrt{a+\frac{b}{x}}x}{64a^4}+\frac{35b^2\sqrt{a+\frac{b}{x}}x^2}{96a^3}-\frac{7b\sqrt{a+\frac{b}{x}}x^3}{24a^2}+\frac{\sqrt{a+\frac{b}{x}}x^4}{4a}+\frac{35b^4\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{64a^{9/2}}$$

output
```
-35/64*b^3*(a+b/x)^(1/2)*x/a^4+35/96*b^2*(a+b/x)^(1/2)*x^2/a^3-7/24*b*(a+b
/x)^(1/2)*x^3/a^2+1/4*(a+b/x)^(1/2)*x^4/a+35/64*b^4*arctanh((a+b/x)^(1/2)/
a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(-105b^3+70ab^2x-56a^2bx^2+48a^3x^3)+105b^4\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{192a^{9/2}}$$

input
```
Integrate[x^3/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(-105*b^3 + 70*a*b^2*x - 56*a^2*b*x^2 + 48*a^3*x^
3) + 105*b^4*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(192*a^(9/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 52, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x^5}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{7b\int \frac{x^4}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{8a}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

$$\Big\downarrow \text{52}$$

$$\frac{7b\left(-\frac{5b\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)}{8a}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

$$\Big\downarrow \text{52}$$

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)}{8a}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)}{8a}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)}{8a}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}-\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}\right)}{8a}+\frac{x^4\sqrt{a+\frac{b}{x}}}{4a}$$

input
```
Int[x^3/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x^4)/(4*a) + (7*b*(-1/3*(Sqrt[a + b/x]*x^3)/a - (5*b*(-1/2*
(Sqrt[a + b/x]*x^2)/a - (3*b*(-((Sqrt[a + b/x]*x)/a) + (b*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

method	result
risch	$\frac{(48a^3x^3-56a^2b\,x^2+70a\,b^2x-105b^3)(ax+b)}{192a^4\sqrt{\frac{ax+b}{x}}}+\frac{35b^4\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{128a^{\frac{9}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-96x(a\,x^2+bx)^{\frac{3}{2}}a^{\frac{7}{2}}+208a^{\frac{5}{2}}(a\,x^2+bx)^{\frac{3}{2}}b-348\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b^2x+384\sqrt{x(ax+b)}\,a^{\frac{3}{2}}b^3-174a^{\frac{3}{2}}\sqrt{a\,x^2+bx}\,b^3-192\ln\right.}{384\sqrt{x(ax+b)}\,a^{\frac{11}{2}}}$

input
```
int(x^3/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*a^3*x^3-56*a^2*b*x^2+70*a*b^2*x-105*b^3)*(a*x+b)/a^4/((a*x+b)/x)
^(1/2)+35/128*b^4/a^(9/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/x/((a*
x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.49

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx = \left[\frac{105\,\sqrt{a}b^4\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(48\,a^4x^4-56\,a^3bx^3+70\,a^2b^2x^2-105\,ab^3x)\sqrt{\frac{ax+b}{x}}}{384\,a^5},\right.$$
$$\left.-\frac{105\,\sqrt{-a}b^4\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(48\,a^4x^4-56\,a^3bx^3+70\,a^2b^2x^2-105\,ab^3x)\sqrt{\frac{ax+b}{x}}}{192\,a^5}\right]$$

input
```
integrate(x^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(105*sqrt(a)*b^4*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2
*(48*a^4*x^4 - 56*a^3*b*x^3 + 70*a^2*b^2*x^2 - 105*a*b^3*x)*sqrt((a*x + b)
/x))/a^5, -1/192*(105*sqrt(-a)*b^4*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*
x + b)) - (48*a^4*x^4 - 56*a^3*b*x^3 + 70*a^2*b^2*x^2 - 105*a*b^3*x)*sqrt(
(a*x + b)/x))/a^5]
```

Sympy [A] (verification not implemented)

Time = 45.57 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{x^{\frac{9}{2}}}{4\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{\sqrt{b}x^{\frac{7}{2}}}{24a\sqrt{\frac{ax}{b}+1}} + \frac{7b^{\frac{3}{2}}x^{\frac{5}{2}}}{96a^2\sqrt{\frac{ax}{b}+1}} - \frac{35b^{\frac{5}{2}}x^{\frac{3}{2}}}{192a^3\sqrt{\frac{ax}{b}+1}} - \frac{35b^{\frac{7}{2}}\sqrt{x}}{64a^4\sqrt{\frac{ax}{b}+1}} + \frac{35b^4\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{64a^{\frac{9}{2}}}$$

input
```
integrate(x**3/(a+b/x)**(1/2),x)
```

output
```
x**(9/2)/(4*sqrt(b)*sqrt(a*x/b + 1)) - sqrt(b)*x**(7/2)/(24*a*sqrt(a*x/b +
 1)) + 7*b**(3/2)*x**(5/2)/(96*a**2*sqrt(a*x/b + 1)) - 35*b**(5/2)*x**(3/2
)/(192*a**3*sqrt(a*x/b + 1)) - 35*b**(7/2)*sqrt(x)/(64*a**4*sqrt(a*x/b + 1
)) + 35*b**4*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(64*a**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{35\,b^4\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{128\,a^{\frac{9}{2}}} - \frac{105\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4-385\left(a+\frac{b}{x}\right)^{\frac{5}{2}}ab^4+511\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2b^4-279\sqrt{a+\frac{b}{x}}a^3b^4}{192\left(\left(a+\frac{b}{x}\right)^4a^4-4\left(a+\frac{b}{x}\right)^3a^5+6\left(a+\frac{b}{x}\right)^2a^6-4\left(a+\frac{b}{x}\right)a^7+a^8\right)}$$

input
```
integrate(x^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
-35/128*b^4*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(9/
2) - 1/192*(105*(a + b/x)^(7/2)*b^4 - 385*(a + b/x)^(5/2)*a*b^4 + 511*(a +
 b/x)^(3/2)*a^2*b^4 - 279*sqrt(a + b/x)*a^3*b^4)/((a + b/x)^4*a^4 - 4*(a +
 b/x)^3*a^5 + 6*(a + b/x)^2*a^6 - 4*(a + b/x)*a^7 + a^8)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{192}\sqrt{ax^2+bx}\left(2\left(4\,x\left(\frac{6\,x}{a\mathrm{sgn}\left(x\right)}-\frac{7\,b}{a^2\mathrm{sgn}\left(x\right)}\right)+\frac{35\,b^2}{a^3\mathrm{sgn}\left(x\right)}\right)x-\frac{105\,b^3}{a^4\mathrm{sgn}\left(x\right)}\right) + \frac{35\,b^4\log\left(|b|\right)\mathrm{sgn}(x)}{128\,a^{\frac{9}{2}}} - \frac{35\,b^4\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{128\,a^{\frac{9}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(a*x^2 + b*x)*(2*(4*x*(6*x/(a*sgn(x)) - 7*b/(a^2*sgn(x))) + 35*b
^2/(a^3*sgn(x)))*x - 105*b^3/(a^4*sgn(x))) + 35/128*b^4*log(abs(b))*sgn(x)
/a^(9/2) - 35/128*b^4*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) +
b))/(a^(9/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{93\,x^4\,\sqrt{a+\frac{b}{x}}}{64\,a}-\frac{511\,x^4\,{\left(a+\frac{b}{x}\right)}^{3/2}}{192\,a^2}+\frac{385\,x^4\,{\left(a+\frac{b}{x}\right)}^{5/2}}{192\,a^3}-\frac{35\,x^4\,{\left(a+\frac{b}{x}\right)}^{7/2}}{64\,a^4}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,35{}\mathrm{i}}{64\,a^{9/2}}$$

input
```
int(x^3/(a + b/x)^(1/2),x)
```

output
```
(93*x^4*(a + b/x)^(1/2))/(64*a) - (b^4*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*
35i)/(64*a^(9/2)) - (511*x^4*(a + b/x)^(3/2))/(192*a^2) + (385*x^4*(a + b/
x)^(5/2))/(192*a^3) - (35*x^4*(a + b/x)^(7/2))/(64*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$=\frac{48\sqrt{x}\,\sqrt{ax+b}\,a^4x^3-56\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^2+70\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x-105\sqrt{x}\,\sqrt{ax+b}\,a\,b^3+105\sqrt{a}\log\Big(}{192a^5}$$

input
```
int(x^3/(a+b/x)^(1/2),x)
```

output
```
(48*sqrt(x)*sqrt(a*x + b)*a**4*x**3 - 56*sqrt(x)*sqrt(a*x + b)*a**3*b*x**2
 + 70*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x - 105*sqrt(x)*sqrt(a*x + b)*a*b**3
 + 105*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**4)/(192*a
**5)
```

3.175 $\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{5b^2\sqrt{a+\frac{b}{x}}x}{8a^3}-\frac{5b\sqrt{a+\frac{b}{x}}x^2}{12a^2}+\frac{\sqrt{a+\frac{b}{x}}x^3}{3a}-\frac{5b^3\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
5/8*b^2*(a+b/x)^(1/2)*x/a^3-5/12*b*(a+b/x)^(1/2)*x^2/a^2+1/3*(a+b/x)^(1/2)
*x^3/a-5/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(15b^2-10abx+8a^2x^2)}{24a^3}-\frac{5b^3\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{7/2}}$$

input
```
Integrate[x^2/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^2 - 10*a*b*x + 8*a^2*x^2))/(24*a^3) - (5*b^3*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x^4}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{6a}+\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\left(-\frac{3b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}+\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}+\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}$$

$$\Big\downarrow \text{73}$$

$$\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}+\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}$$

$$\Big\downarrow \text{221}$$

$$\frac{5b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}-\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}\right)}{6a}+\frac{x^3\sqrt{a+\frac{b}{x}}}{3a}$$

input
```
Int[x^2/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x^3)/(3*a) + (5*b*(-1/2*(Sqrt[a + b/x]*x^2)/a - (3*b*(-((Sq
rt[a + b/x]*x)/a) + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/(4*a)))/(
6*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01

method	result
risch	$\frac{\left(8 a^{2} x^{2}-10 a b x+15 b^{2}\right) \left(a x+b\right)}{24 a^{3} \sqrt{\frac{a x+b}{x}}}-\frac{5 b^{3} \ln \left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a \,x^{2}+b x}\right) \sqrt{x \left(a x+b\right)}}{16 a^{\frac{7}{2}} x \sqrt{\frac{a x+b}{x}}}$
default	$\frac{\sqrt{\frac{a x+b}{x}}\, x \left(16 \left(a \,x^{2}+b x\right)^{\frac{3}{2}} a^{\frac{5}{2}}-36 \sqrt{a \,x^{2}+b x}\, a^{\frac{5}{2}} b x+48 \sqrt{x \left(a x+b\right)}\, a^{\frac{3}{2}} b^{2}-24 \ln \left(\frac{2 \sqrt{x \left(a x+b\right)}\, \sqrt{a}+2 a x+b}{2 \sqrt{a}}\right) a \,b^{3}-18 \sqrt{a \,x^{2}+b x}\, a^{\frac{3}{2}} b^{2}+9 \ln \cdots}{48 a^{\frac{9}{2}} \sqrt{x \left(a x+b\right)}}$

input
```
int(x^2/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2-10*a*b*x+15*b^2)*(a*x+b)/a^3/((a*x+b)/x)^(1/2)-5/16*b^3/a^
(7/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*
x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$= \left[\frac{15\,\sqrt{a}b^3\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(8\,a^3x^3-10\,a^2bx^2+15\,ab^2x)\sqrt{\frac{ax+b}{x}}}{48\,a^4}, \frac{15\,\sqrt{-a}b^3\arctan\left(\cdots\right.}{\cdots}\right.$$

input
```
integrate(x^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(
8*a^3*x^3 - 10*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/a^4, 1/24*(15*sq
rt(-a)*b^3*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (8*a^3*x^3 - 1
0*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/a^4]
```

Sympy [A] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{x^{\frac{7}{2}}}{3\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{\sqrt{b}x^{\frac{5}{2}}}{12a\sqrt{\frac{ax}{b}+1}} + \frac{5b^{\frac{3}{2}}x^{\frac{3}{2}}}{24a^2\sqrt{\frac{ax}{b}+1}} + \frac{5b^{\frac{5}{2}}\sqrt{x}}{8a^3\sqrt{\frac{ax}{b}+1}} - \frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{7}{2}}}$$

input
```
integrate(x**2/(a+b/x)**(1/2),x)
```

output
```
x**(7/2)/(3*sqrt(b)*sqrt(a*x/b + 1)) - sqrt(b)*x**(5/2)/(12*a*sqrt(a*x/b +
 1)) + 5*b**(3/2)*x**(3/2)/(24*a**2*sqrt(a*x/b + 1)) + 5*b**(5/2)*sqrt(x)/
(8*a**3*sqrt(a*x/b + 1)) - 5*b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(8*a**(7/
2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{5\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}} + \frac{15\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3 - 40\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ab^3 + 33\sqrt{a+\frac{b}{x}}a^2b^3}{24\left(\left(a+\frac{b}{x}\right)^3a^3 - 3\left(a+\frac{b}{x}\right)^2a^4 + 3\left(a+\frac{b}{x}\right)a^5 - a^6\right)}$$

input
```
integrate(x^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
5/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2)
+ 1/24*(15*(a + b/x)^(5/2)*b^3 - 40*(a + b/x)^(3/2)*a*b^3 + 33*sqrt(a + b/
x)*a^2*b^3)/((a + b/x)^3*a^3 - 3*(a + b/x)^2*a^4 + 3*(a + b/x)*a^5 - a^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{24}\sqrt{ax^2+bx}\left(2\,x\left(\frac{4\,x}{a\mathrm{sgn}\left(x\right)}-\frac{5\,b}{a^2\mathrm{sgn}\left(x\right)}\right)+\frac{15\,b^2}{a^3\mathrm{sgn}\left(x\right)}\right)-\frac{5\,b^3\log\left(|b|\right)\mathrm{sgn}(x)}{16\,a^{\frac{7}{2}}}+\frac{5\,b^3\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{16\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*x^2 + b*x)*(2*x*(4*x/(a*sgn(x)) - 5*b/(a^2*sgn(x))) + 15*b^2/(
a^3*sgn(x))) - 5/16*b^3*log(abs(b))*sgn(x)/a^(7/2) + 5/16*b^3*log(abs(2*(s
qrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{11\,x^3\,\sqrt{a+\frac{b}{x}}}{8\,a}-\frac{5\,x^3\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3\,a^2}+\frac{5\,x^3\,{\left(a+\frac{b}{x}\right)}^{5/2}}{8\,a^3}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{8\,a^{7/2}}$$

input
```
int(x^2/(a + b/x)^(1/2),x)
```

output
```
(b^3*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^(7/2)) + (11*x^3*(a + b/x
)^(1/2))/(8*a) - (5*x^3*(a + b/x)^(3/2))/(3*a^2) + (5*x^3*(a + b/x)^(5/2))
/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}\,dx$$
$$= \frac{8\sqrt{x}\,\sqrt{ax+b}\,a^3x^2-10\sqrt{x}\,\sqrt{ax+b}\,a^2bx+15\sqrt{x}\,\sqrt{ax+b}\,a\,b^2-15\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^3}{24a^4}$$

input
```
int(x^2/(a+b/x)^(1/2),x)
```

output
```
(8*sqrt(x)*sqrt(a*x + b)*a**3*x**2 - 10*sqrt(x)*sqrt(a*x + b)*a**2*b*x + 1
5*sqrt(x)*sqrt(a*x + b)*a*b**2 - 15*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*s
qrt(a))/sqrt(b))*b**3)/(24*a**4)
```

3.176 $\int \frac{x}{\sqrt{a+\frac{b}{x}}} dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\, dx = -\frac{3b\sqrt{a+\frac{b}{x}}x}{4a^2}+\frac{\sqrt{a+\frac{b}{x}}x^2}{2a}+\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
-3/4*b*(a+b/x)^(1/2)*x/a^2+1/2*(a+b/x)^(1/2)*x^2/a+3/4*b^2*arctanh((a+b/x)
^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\, dx = \frac{\sqrt{a}\sqrt{a+\frac{b}{x}}x(-3b+2ax)+3b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[x/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a]*Sqrt[a + b/x]*x*(-3*b + 2*a*x) + 3*b^2*ArcTanh[Sqrt[a + b/x]/Sqrt
[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x^3}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{4a}+\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\left(-\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}+\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}$$

$$\Big\downarrow \text{73}$$

$$\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}+\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x\sqrt{a+\frac{b}{x}}}{a}\right)}{4a}+\frac{x^2\sqrt{a+\frac{b}{x}}}{2a}$$

input
```
Int[x/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x^2)/(2*a) + (3*b*(-((Sqrt[a + b/x]*x)/a) + (b*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/a^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

method	result
risch	$\frac{(2ax-3b)(ax+b)}{4a^2\sqrt{\frac{ax+b}{x}}}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)\sqrt{x(ax+b)}}{8a^{\frac{5}{2}}x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(4\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}x-8a^{\frac{3}{2}}\sqrt{x(ax+b)}\,b+2\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b-b^2\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a+4a\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^2\right.}{8\sqrt{x(ax+b)}\,a^{\frac{7}{2}}}$

input
```
int(x/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x-3*b)*(a*x+b)/a^2/((a*x+b)/x)^(1/2)+3/8*b^2/a^(5/2)*ln((1/2*b+a*
x)/a^(1/2)+(a*x^2+b*x)^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.88

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx=\left[\frac{3\sqrt{a}b^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(2\,a^2x^2-3\,abx\right)\sqrt{\frac{ax+b}{x}}}{8\,a^3},\right.$$
$$\left.-\frac{3\sqrt{-a}b^2\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-\left(2\,a^2x^2-3\,abx\right)\sqrt{\frac{ax+b}{x}}}{4\,a^3}\right]$$

input
```
integrate(x/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(2*
a^2*x^2 - 3*a*b*x)*sqrt((a*x + b)/x))/a^3, -1/4*(3*sqrt(-a)*b^2*arctan(sqr
t(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) - (2*a^2*x^2 - 3*a*b*x)*sqrt((a*x + b
)/x))/a^3]
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{x^{\frac{5}{2}}}{2\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{\sqrt{b}x^{\frac{3}{2}}}{4a\sqrt{\frac{ax}{b}+1}} - \frac{3b^{\frac{3}{2}}\sqrt{x}}{4a^2\sqrt{\frac{ax}{b}+1}} + \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}}$$

input
```
integrate(x/(a+b/x)**(1/2),x)
```

output
```
x**(5/2)/(2*sqrt(b)*sqrt(a*x/b + 1)) - sqrt(b)*x**(3/2)/(4*a*sqrt(a*x/b +
1)) - 3*b**(3/2)*sqrt(x)/(4*a**2*sqrt(a*x/b + 1)) + 3*b**2*asinh(sqrt(a)*s
qrt(x)/sqrt(b))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.44

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}} - \frac{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2 - 5\sqrt{a+\frac{b}{x}}ab^2}{4\left(\left(a+\frac{b}{x}\right)^2a^2 - 2\left(a+\frac{b}{x}\right)a^3 + a^4\right)}$$

input
```
integrate(x/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
-3/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
- 1/4*(3*(a + b/x)^(3/2)*b^2 - 5*sqrt(a + b/x)*a*b^2)/((a + b/x)^2*a^2 - 2
*(a + b/x)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{1}{4}\sqrt{ax^2+bx}\left(\frac{2\,x}{a\mathrm{sgn}\,(x)} - \frac{3\,b}{a^2\mathrm{sgn}\,(x)}\right) + \frac{3\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{8\,a^{\frac{5}{2}}} - \frac{3\,b^2\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{8\,a^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*x^2 + b*x)*(2*x/(a*sgn(x)) - 3*b/(a^2*sgn(x))) + 3/8*b^2*log(ab
s(b))*sgn(x)/a^(5/2) - 3/8*b^2*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*s
qrt(a) + b))/(a^(5/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{5/2}} + \frac{5\,x^2\,\sqrt{a+\frac{b}{x}}}{4\,a} - \frac{3\,x^2\,\left(a+\frac{b}{x}\right)^{3/2}}{4\,a^2}$$

input
```
int(x/(a + b/x)^(1/2),x)
```

output
```
(3*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(5/2)) + (5*x^2*(a + b/x)^(1/2
))/(4*a) - (3*x^2*(a + b/x)^(3/2))/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\sqrt{x}\,\sqrt{ax+b}\,a^2x-3\sqrt{x}\,\sqrt{ax+b}\,ab+3\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2}{4a^3}$$

input
```
int(x/(a+b/x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a*x + b)*a**2*x - 3*sqrt(x)*sqrt(a*x + b)*a*b + 3*sqrt(a)*
log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b**2)/(4*a**3)
```

3.177 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(a+b/x)^(1/2)*x/a-b*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x}{a}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/Sqrt[a + b/x],x]
```

output
```
(Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {773, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{b\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{2a}+\frac{x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{a}+\frac{x\sqrt{a+\frac{b}{x}}}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input

```
Int[1/Sqrt[a + b/x],x]
```

output

```
(Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(2\sqrt{x(ax+b)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\,a^{\frac{3}{2}}}$	71
risch	$\frac{ax+b}{a\sqrt{\frac{ax+b}{x}}}-\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)\sqrt{x(ax+b)}}{2a^{\frac{3}{2}}x\sqrt{\frac{ax+b}{x}}}$	75

input
```
int(1/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*(2*(x*(a*x+b))^(1/2)*a^(1/2)-b*ln(1/2*(2*(x*(a*x+b
))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \left[\frac{2\,ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}b\log\left(2\,ax - 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}} + \sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)}{a^2}\right]$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*x
*sqrt((a*x + b)/x)/(a*x + b)))/a^2]
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)**(1/2),x)
```

output
```
sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3
/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/
(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\,a^{\frac{3}{2}}} + \frac{b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)} + \frac{\sqrt{ax^2+bx}}{a\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) + b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,x\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x} + \frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

input
```
int(1/(a + b/x)^(1/2),x)
```

output
```
(2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)
*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{\sqrt{x}\,\sqrt{ax+b}\,a-\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b}{a^2}$$

input
```
int(1/(a+b/x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(a*x + b)*a - sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/s
qrt(b))*b)/a**2
```

3.178 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{\frac{b+ax}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x),x]
```

output
```
(2*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b/x]*x),x]
```

output
```
(2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.76

method	result	size
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)-2\sqrt{x(ax+b)}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right)}{2\sqrt{x(ax+b)}\,b\sqrt{a}}$	119

input
```
int(1/(a+b/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*
x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))-2*(x*(a*x+b))^(1/2)*a^(1/2)+b*ln(1/2*(2
*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/(x*(a*x+b))^(1/2)/b/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx = \left[\frac{\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{\sqrt{a}}, -\frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)}{a}\right]$$

input
```
integrate(1/(a+b/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)/sqrt(a), -2*sqrt(-a)*arcta
n(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b))/a]
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx = \frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate(1/(a+b/x)**(1/2)/x,x)
```

output
```
2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/(a+b/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
-log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx = \frac{\log\left(|b|\right)\operatorname{sgn}(x)}{\sqrt{a}} - \frac{\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{\sqrt{a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x,x, algorithm="giac")
```

output
```
log(abs(b))*sgn(x)/sqrt(a) - log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqr
t(a) + b))/(sqrt(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x}\,dx = \frac{2\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b/x)^(1/2)),x)
```

output
```
(2*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x}\,dx = \frac{2\sqrt{a}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)}{a}$$

input
```
int(1/(a+b/x)^(1/2)/x,x)
```

output
```
(2*sqrt(a)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)))/a
```

3.179 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2} \, dx = -\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

output
```
-2*(a+b/x)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2} \, dx = -\frac{2\sqrt{\frac{b+ax}{x}}}{b}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^2),x]
```

output
```
(-2*Sqrt[(b + a*x)/x])/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

input
```
Int[1/(Sqrt[a + b/x]*x^2),x]
```

output
```
(-2*Sqrt[a + b/x])/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2\sqrt{a+\frac{b}{x}}}{b}$
trager	$-\frac{2\sqrt{-\frac{-ax-b}{x}}}{b}$
orering	$-\frac{2(ax+b)}{xb\sqrt{a+\frac{b}{x}}}$
gosper	$-\frac{2(ax+b)}{xb\sqrt{\frac{ax+b}{x}}}$
risch	$-\frac{2(ax+b)}{xb\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(2\sqrt{x(ax+b)}\,a^{\frac{3}{2}}x^{2}+2\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}x^{2}+\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}-\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)ab\,x^{2}-\right.}{2x\sqrt{x(ax+b)}\,b^{2}\sqrt{a}}$

input
```
int(1/(a+b/x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(a+b/x)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}\,dx = -\frac{2\sqrt{\frac{ax+b}{x}}}{b}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2*sqrt((a*x + b)/x)/b
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}\,dx = \begin{cases} -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{for } b\neq 0 \\ -\frac{1}{\sqrt{a}x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**2,x)
```

output
```
Piecewise((-2*sqrt(a + b/x)/b, Ne(b, 0)), (-1/(sqrt(a)*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt(a + b/x)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}\,dx = \frac{2}{(\sqrt{a}x-\sqrt{ax^2+bx})\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2/((sqrt(a)*x - sqrt(a*x^2 + b*x))*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}}{b}$$

input
```
int(1/(x^2*(a + b/x)^(1/2)),x)
```

output
```
-(2*(a + b/x)^(1/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}\,dx = \frac{-2\sqrt{x}\,\sqrt{ax+b}-2\sqrt{a}\,x}{bx}$$

input
```
int(1/(a+b/x)^(1/2)/x^2,x)
```

output
```
( - 2*(sqrt(x)*sqrt(a*x + b) + sqrt(a)*x))/(b*x)
```

3.180 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3} dx = \frac{2a\sqrt{a+\frac{b}{x}}}{b^2} - \frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}$$

output
```
2*a*(a+b/x)^(1/2)/b^2-2/3*(a+b/x)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3} dx = \frac{2\sqrt{\frac{b+ax}{x}}(-b+2ax)}{3b^2x}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^3),x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-b + 2*a*x))/(3*b^2*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{\sqrt{a+\frac{b}{x}}}{b}-\frac{a}{b\sqrt{a+\frac{b}{x}}}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\sqrt{a+\frac{b}{x}}}{b^2}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^2}$$

input
```
Int[1/(Sqrt[a + b/x]*x^3),x]
```

output
```
(2*a*Sqrt[a + b/x])/b^2 - (2*(a + b/x)^(3/2))/(3*b^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result
orering	$\frac{2(2ax-b)(ax+b)}{3b^2x^2\sqrt{a+\frac{b}{x}}}$
trager	$\frac{2(2ax-b)\sqrt{-\frac{-ax-b}{x}}}{3x\,b^2}$
gosper	$\frac{2(ax+b)(2ax-b)}{3x^2b^2\sqrt{\frac{ax+b}{x}}}$
risch	$\frac{2(ax+b)(2ax-b)}{3x^2b^2\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(6\sqrt{x(ax+b)}\,a^{\frac{5}{2}}x^3+6\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}x^3+3\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^3-3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^3-12(a\right.}{6x^2\sqrt{x(ax+b)}\,b^3\sqrt{a}}$

input
```
int(1/(a+b/x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(2*a*x-b)/b^2/x^2*(a*x+b)/(a+b/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}\,dx=\frac{2\,(2\,ax-b)\sqrt{\frac{ax+b}{x}}}{3\,b^2x}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/3*(2*a*x - b)*sqrt((a*x + b)/x)/(b^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(29) = 58.

Time = 0.88 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.89

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}\,dx=\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^4bx^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^3b^2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**3,x)
```

output
```
4*a**(7/2)*b**(3/2)*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**
(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*x*sqrt(a*x/b + 1)/(3*a**(5/2)*b
**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*sqrt(a*x/b
+ 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*a**4*b*x**(
5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*a**3*b**2*x
**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3\,b^2} + \frac{2\sqrt{a+\frac{b}{x}}a}{b^2}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2/3*(a + b/x)^(3/2)/b^2 + 2*sqrt(a + b/x)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}\,dx = \frac{2\left(3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)}{3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3\operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)/((sqrt(a)*x - sqrt(a*x
^2 + b*x))^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}\,(b-2\,a\,x)}{3\,b^2\,x}$$

input
```
int(1/(x^3*(a + b/x)^(1/2)),x)
```

output
```
-(2*(a + b/x)^(1/2)*(b - 2*a*x))/(3*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}\,dx = \frac{\frac{4\sqrt{x}\,\sqrt{ax+b}\,ax}{3} - \frac{2\sqrt{x}\,\sqrt{ax+b}\,b}{3} - \frac{4\sqrt{a}\,a\,x^2}{3}}{b^2x^2}$$

input
```
int(1/(a+b/x)^(1/2)/x^3,x)
```

output
```
(2*(2*sqrt(x)*sqrt(a*x + b)*a*x - sqrt(x)*sqrt(a*x + b)*b - 2*sqrt(a)*a*x*
*2))/(3*b**2*x**2)
```

3.181 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4} dx = -\frac{2a^2\sqrt{a+\frac{b}{x}}}{b^3} + \frac{4a\left(a+\frac{b}{x}\right)^{3/2}}{3b^3} - \frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}$$

output
```
-2*a^2*(a+b/x)^(1/2)/b^3+4/3*a*(a+b/x)^(3/2)/b^3-2/5*(a+b/x)^(5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4} dx = -\frac{2\sqrt{\frac{b+ax}{x}}(3b^2-4abx+8a^2x^2)}{15b^3x^2}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^4),x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(3*b^2 - 4*a*b*x + 8*a^2*x^2))/(15*b^3*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{a^2}{b^2\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}a}{b^2}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{b^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2\sqrt{a+\frac{b}{x}}}{b^3}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^3}+\frac{4a\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}$$

input
```
Int[1/(Sqrt[a + b/x]*x^4),x]
```

output
```
(-2*a^2*Sqrt[a + b/x])/b^3 + (4*a*(a + b/x)^(3/2))/(3*b^3) - (2*(a + b/x)^
(5/2))/(5*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
orering	$-\frac{2\left(8a^{2}x^{2}-4abx+3b^{2}\right)\left(ax+b\right)}{15b^{3}x^{3}\sqrt{a+\frac{b}{x}}}$
trager	$-\frac{2\left(8a^{2}x^{2}-4abx+3b^{2}\right)\sqrt{-\frac{-ax-b}{x}}}{15x^{2}b^{3}}$
gosper	$-\frac{2\left(ax+b\right)\left(8a^{2}x^{2}-4abx+3b^{2}\right)}{15x^{3}b^{3}\sqrt{\frac{ax+b}{x}}}$
risch	$-\frac{2\left(ax+b\right)\left(8a^{2}x^{2}-4abx+3b^{2}\right)}{15x^{3}b^{3}\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(30\sqrt{x(ax+b)}\,a^{\frac{7}{2}}x^{4}+30\sqrt{a\,x^{2}+bx}\,a^{\frac{7}{2}}x^{4}+15\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}b\,x^{4}-15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}b\,x^{4}-60(}{30x^{3}\sqrt{x(ax+b)}\,b^{4}\sqrt{a}}$

input
```
int(1/(a+b/x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(8*a^2*x^2-4*a*b*x+3*b^2)/b^3/x^3*(a*x+b)/(a+b/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4}\,dx = -\frac{2\,(8\,a^2x^2-4\,abx+3\,b^2)\sqrt{\frac{ax+b}{x}}}{15\,b^3x^2}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/15*(8*a^2*x^2 - 4*a*b*x + 3*b^2)*sqrt((a*x + b)/x)/(b^3*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(48) = 96.

Time = 1.31 (sec) , antiderivative size = 813, normalized size of antiderivative = 14.26

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**4,x)
```

output
```
-16*a**(15/2)*b**(9/2)*x**5*sqrt(a*x/b + 1)/(15*a**(11/2)*b**7*x**(11/2) +
 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10
*x**(5/2)) - 40*a**(13/2)*b**(11/2)*x**4*sqrt(a*x/b + 1)/(15*a**(11/2)*b**
7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2) + 15*a
**(5/2)*b**10*x**(5/2)) - 30*a**(11/2)*b**(13/2)*x**3*sqrt(a*x/b + 1)/(15*
a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x*
*(7/2) + 15*a**(5/2)*b**10*x**(5/2)) - 10*a**(9/2)*b**(15/2)*x**2*sqrt(a*x
/b + 1)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(
7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) - 10*a**(7/2)*b**(17/2)*x
*sqrt(a*x/b + 1)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2)
+ 45*a**(7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) - 6*a**(5/2)*b**
(19/2)*sqrt(a*x/b + 1)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**
(9/2) + 45*a**(7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) + 16*a**8*
b**4*x**(11/2)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) +
45*a**(7/2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) + 48*a**7*b**5*x**
(9/2)/(15*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/
2)*b**9*x**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) + 48*a**6*b**6*x**(7/2)/(15
*a**(11/2)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x
**(7/2) + 15*a**(5/2)*b**10*x**(5/2)) + 16*a**5*b**7*x**(5/2)/(15*a**(11/2
)*b**7*x**(11/2) + 45*a**(9/2)*b**8*x**(9/2) + 45*a**(7/2)*b**9*x**(7/2...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^3} + \frac{4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{3\,b^3} - \frac{2\sqrt{a+\frac{b}{x}}a^2}{b^3}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-2/5*(a + b/x)^(5/2)/b^3 + 4/3*(a + b/x)^(3/2)*a/b^3 - 2*sqrt(a + b/x)*a^2
/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4}\,dx = \frac{2\left(20\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2a+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b+3\,b^2\right)}{15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^5\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
2/15*(20*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a + 15*(sqrt(a)*x - sqrt(a*x^2
+ b*x))*sqrt(a)*b + 3*b^2)/((sqrt(a)*x - sqrt(a*x^2 + b*x))^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4}\,dx = -\frac{6\,b^2\,\sqrt{a+\frac{b}{x}}+16\,a^2\,x^2\,\sqrt{a+\frac{b}{x}}-8\,a\,b\,x\,\sqrt{a+\frac{b}{x}}}{15\,b^3\,x^2}$$

input
```
int(1/(x^4*(a + b/x)^(1/2)),x)
```

output
```
-(6*b^2*(a + b/x)^(1/2) + 16*a^2*x^2*(a + b/x)^(1/2) - 8*a*b*x*(a + b/x)^(
1/2))/(15*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4}\,dx = \frac{-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^2x^2}{15}+\frac{8\sqrt{x}\,\sqrt{ax+b}\,abx}{15}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^2}{5}+\frac{16\sqrt{a}\,a^2x^3}{15}}{b^3x^3}$$

input
```
int(1/(a+b/x)^(1/2)/x^4,x)
```

output
```
(2*( - 8*sqrt(x)*sqrt(a*x + b)*a**2*x**2 + 4*sqrt(x)*sqrt(a*x + b)*a*b*x -
 3*sqrt(x)*sqrt(a*x + b)*b**2 + 8*sqrt(a)*a**2*x**3))/(15*b**3*x**3)
```

3.182 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5} dx = \frac{2a^3\sqrt{a+\frac{b}{x}}}{b^4} - \frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^4} + \frac{6a\left(a+\frac{b}{x}\right)^{5/2}}{5b^4} - \frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}$$

output
```
2*a^3*(a+b/x)^(1/2)/b^4-2*a^2*(a+b/x)^(3/2)/b^4+6/5*a*(a+b/x)^(5/2)/b^4-2/
7*(a+b/x)^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5} dx = \frac{2\sqrt{\frac{b+ax}{x}}(-5b^3+6ab^2x-8a^2bx^2+16a^3x^3)}{35b^4x^3}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^5),x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-5*b^3 + 6*a*b^2*x - 8*a^2*b*x^2 + 16*a^3*x^3))/(35*
b^4*x^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(-\frac{a^3}{b^3\sqrt{a+\frac{b}{x}}}+\frac{3\sqrt{a+\frac{b}{x}}a^2}{b^3}-\frac{3\left(a+\frac{b}{x}\right)^{3/2}a}{b^3}+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{b^3}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\sqrt{a+\frac{b}{x}}}{b^4}-\frac{2a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^4}+\frac{6a\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}$$

input
```
Int[1/(Sqrt[a + b/x]*x^5),x]
```

output
```
(2*a^3*Sqrt[a + b/x])/b^4 - (2*a^2*(a + b/x)^(3/2))/b^4 + (6*a*(a + b/x)^(
5/2))/(5*b^4) - (2*(a + b/x)^(7/2))/(7*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result
orering	$\frac{2\left(16a^{3}x^{3}-8a^{2}b\,x^{2}+6a\,b^{2}x-5b^{3}\right)\left(ax+b\right)}{35b^{4}x^{4}\sqrt{a+\frac{b}{x}}}$
trager	$\frac{2\left(16a^{3}x^{3}-8a^{2}b\,x^{2}+6a\,b^{2}x-5b^{3}\right)\sqrt{-\frac{-ax-b}{x}}}{35x^{3}b^{4}}$
gosper	$\frac{2\left(ax+b\right)\left(16a^{3}x^{3}-8a^{2}b\,x^{2}+6a\,b^{2}x-5b^{3}\right)}{35x^{4}b^{4}\sqrt{\frac{ax+b}{x}}}$
risch	$\frac{2\left(ax+b\right)\left(16a^{3}x^{3}-8a^{2}b\,x^{2}+6a\,b^{2}x-5b^{3}\right)}{35x^{4}b^{4}\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(70\sqrt{x\left(ax+b\right)}\,a^{\frac{9}{2}}x^{5}+70\sqrt{a\,x^{2}+bx}\,a^{\frac{9}{2}}x^{5}+35\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{4}b\,x^{5}-35\ln\left(\frac{2\sqrt{x\left(ax+b\right)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{4}b\,x^{5}-\right.}{70x^{4}\sqrt{x\left(ax+b\right)}\,b^{5}\sqrt{a}}$

input
```
int(1/(a+b/x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/35*(16*a^3*x^3-8*a^2*b*x^2+6*a*b^2*x-5*b^3)/b^4/x^4*(a*x+b)/(a+b/x)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5}\,dx = \frac{2\,(16\,a^3x^3 - 8\,a^2bx^2 + 6\,ab^2x - 5\,b^3)\sqrt{\frac{ax+b}{x}}}{35\,b^4x^3}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
2/35*(16*a^3*x^3 - 8*a^2*b*x^2 + 6*a*b^2*x - 5*b^3)*sqrt((a*x + b)/x)/(b^4
*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2164 vs. 2(65) = 130.

Time = 1.90 (sec) , antiderivative size = 2164, normalized size of antiderivative = 28.47

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**5,x)
```

output
```
32*a**(25/2)*b**(23/2)*x**9*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2)
+ 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(
13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20
*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) + 176*a**(23/2)*b**(25/2)*x**8*sqr
t(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2)
 + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**
(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x
**(7/2)) + 396*a**(21/2)*b**(27/2)*x**7*sqrt(a*x/b + 1)/(35*a**(19/2)*b**1
5*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2
) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a*
*(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) + 462*a**(19/2)*b**(29
/2)*x**6*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**
16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/
2) + 525*a**(11/2)*b**19*x**(11/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(
7/2)*b**21*x**(7/2)) + 280*a**(17/2)*b**(31/2)*x**5*sqrt(a*x/b + 1)/(35*a*
*(19/2)*b**15*x**(19/2) + 210*a**(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b*
*17*x**(15/2) + 700*a**(13/2)*b**18*x**(13/2) + 525*a**(11/2)*b**19*x**(11
/2) + 210*a**(9/2)*b**20*x**(9/2) + 35*a**(7/2)*b**21*x**(7/2)) + 42*a**(1
5/2)*b**(33/2)*x**4*sqrt(a*x/b + 1)/(35*a**(19/2)*b**15*x**(19/2) + 210*a*
*(17/2)*b**16*x**(17/2) + 525*a**(15/2)*b**17*x**(15/2) + 700*a**(13/2)...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\,b^4} + \frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{5\,b^4} - \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}{b^4} + \frac{2\sqrt{a+\frac{b}{x}}a^3}{b^4}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)/b^4 + 6/5*(a + b/x)^(5/2)*a/b^4 - 2*(a + b/x)^(3/2)*a
^2/b^4 + 2*sqrt(a + b/x)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5}\,dx = \frac{2\left(70\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^{\frac{3}{2}}+84\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab+35\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^2+5\,b^3\right)}{35\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^7\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
2/35*(70*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2) + 84*(sqrt(a)*x - sqrt(
a*x^2 + b*x))^2*a*b + 35*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^2 + 5*b
^3)/((sqrt(a)*x - sqrt(a*x^2 + b*x))^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5}\,dx = \frac{32\,a^3\,\sqrt{a+\frac{b}{x}}}{35\,b^4}-\frac{2\,\sqrt{a+\frac{b}{x}}}{7\,b\,x^3}+\frac{12\,a\,\sqrt{a+\frac{b}{x}}}{35\,b^2\,x^2}-\frac{16\,a^2\,\sqrt{a+\frac{b}{x}}}{35\,b^3\,x}$$

input
```
int(1/(x^5*(a + b/x)^(1/2)),x)
```

output
```
(32*a^3*(a + b/x)^(1/2))/(35*b^4) - (2*(a + b/x)^(1/2))/(7*b*x^3) + (12*a*
(a + b/x)^(1/2))/(35*b^2*x^2) - (16*a^2*(a + b/x)^(1/2))/(35*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^5}\,dx$$
$$=\frac{\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^3x^3}{35}-\frac{16\sqrt{x}\,\sqrt{ax+b}\,a^2b\,x^2}{35}+\frac{12\sqrt{x}\,\sqrt{ax+b}\,a\,b^2x}{35}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^3}{7}-\frac{32\sqrt{a}\,a^3x^4}{35}}{b^4x^4}$$

input
```
int(1/(a+b/x)^(1/2)/x^5,x)
```

output
```
(2*(16*sqrt(x)*sqrt(a*x + b)*a**3*x**3 - 8*sqrt(x)*sqrt(a*x + b)*a**2*b*x*
*2 + 6*sqrt(x)*sqrt(a*x + b)*a*b**2*x - 5*sqrt(x)*sqrt(a*x + b)*b**3 - 16*
sqrt(a)*a**3*x**4))/(35*b**4*x**4)
```

3.183 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx = -\frac{2a^4\sqrt{a+\frac{b}{x}}}{b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^5} - \frac{12a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}$$

output
```
-2*a^4*(a+b/x)^(1/2)/b^5+8/3*a^3*(a+b/x)^(3/2)/b^5-12/5*a^2*(a+b/x)^(5/2)/
b^5+8/7*a*(a+b/x)^(7/2)/b^5-2/9*(a+b/x)^(9/2)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx = -\frac{2\sqrt{\frac{b+ax}{x}}(35b^4-40ab^3x+48a^2b^2x^2-64a^3bx^3+128a^4x^4)}{315b^5x^4}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^6),x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(35*b^4 - 40*a*b^3*x + 48*a^2*b^2*x^2 - 64*a^3*b*x^3
 + 128*a^4*x^4))/(315*b^5*x^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x}}x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{a^4}{b^4\sqrt{a+\frac{b}{x}}}-\frac{4\sqrt{a+\frac{b}{x}}a^3}{b^4}+\frac{6\left(a+\frac{b}{x}\right)^{3/2}a^2}{b^4}-\frac{4\left(a+\frac{b}{x}\right)^{5/2}a}{b^4}+\frac{\left(a+\frac{b}{x}\right)^{7/2}}{b^4}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^4\sqrt{a+\frac{b}{x}}}{b^5}+\frac{8a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^5}-\frac{12a^2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}$$

input
```
Int[1/(Sqrt[a + b/x]*x^6),x]
```

output
```
(-2*a^4*Sqrt[a + b/x])/b^5 + (8*a^3*(a + b/x)^(3/2))/(3*b^5) - (12*a^2*(a
+ b/x)^(5/2))/(5*b^5) + (8*a*(a + b/x)^(7/2))/(7*b^5) - (2*(a + b/x)^(9/2)
)/(9*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result
orering	$-\frac{2\left(128a^{4}x^{4}-64a^{3}b\,x^{3}+48a^{2}b^{2}x^{2}-40a\,b^{3}x+35b^{4}\right)\left(ax+b\right)}{315b^{5}x^{5}\sqrt{a+\frac{b}{x}}}$
trager	$-\frac{2\left(128a^{4}x^{4}-64a^{3}b\,x^{3}+48a^{2}b^{2}x^{2}-40a\,b^{3}x+35b^{4}\right)\sqrt{-\frac{-ax-b}{x}}}{315x^{4}b^{5}}$
gosper	$-\frac{2\left(ax+b\right)\left(128a^{4}x^{4}-64a^{3}b\,x^{3}+48a^{2}b^{2}x^{2}-40a\,b^{3}x+35b^{4}\right)}{315x^{5}b^{5}\sqrt{\frac{ax+b}{x}}}$
risch	$-\frac{2\left(ax+b\right)\left(128a^{4}x^{4}-64a^{3}b\,x^{3}+48a^{2}b^{2}x^{2}-40a\,b^{3}x+35b^{4}\right)}{315x^{5}b^{5}\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\left(630\sqrt{x(ax+b)}\,a^{\frac{11}{2}}x^{6}+630\sqrt{a\,x^{2}+bx}\,a^{\frac{11}{2}}x^{6}+315\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{5}b\,x^{6}-315\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{5}b\,x}{630x^{5}\sqrt{x(ax}}$

input
```
int(1/(a+b/x)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/315*(128*a^4*x^4-64*a^3*b*x^3+48*a^2*b^2*x^2-40*a*b^3*x+35*b^4)/b^5/x^5
*(a*x+b)/(a+b/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx = -\frac{2\left(128\,a^4x^4-64\,a^3bx^3+48\,a^2b^2x^2-40\,ab^3x+35\,b^4\right)\sqrt{\frac{ax+b}{x}}}{315\,b^5x^4}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/315*(128*a^4*x^4 - 64*a^3*b*x^3 + 48*a^2*b^2*x^2 - 40*a*b^3*x + 35*b^4)
*sqrt((a*x + b)/x)/(b^5*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4901 vs. 2(85) = 170.

Time = 3.20 (sec) , antiderivative size = 4901, normalized size of antiderivative = 49.51

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**6,x)
```

output
```
-256*a**(37/2)*b**(49/2)*x**14*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29
/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 3
7800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a
**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/
2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**3
8*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 2432*a**(35/2)*b**(51/2)*x**1
3*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x*
*(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2
) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66
150*a**(17/2)*b**35*x**(17/2) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a*
*(13/2)*b**37*x**(13/2) + 3150*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b*
*39*x**(9/2)) - 10336*a**(33/2)*b**(53/2)*x**12*sqrt(a*x/b + 1)/(315*a**(2
9/2)*b**29*x**(29/2) + 3150*a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b*
*31*x**(25/2) + 37800*a**(23/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x*
*(21/2) + 79380*a**(19/2)*b**34*x**(19/2) + 66150*a**(17/2)*b**35*x**(17/2
) + 37800*a**(15/2)*b**36*x**(15/2) + 14175*a**(13/2)*b**37*x**(13/2) + 31
50*a**(11/2)*b**38*x**(11/2) + 315*a**(9/2)*b**39*x**(9/2)) - 25840*a**(31
/2)*b**(55/2)*x**11*sqrt(a*x/b + 1)/(315*a**(29/2)*b**29*x**(29/2) + 3150*
a**(27/2)*b**30*x**(27/2) + 14175*a**(25/2)*b**31*x**(25/2) + 37800*a**(23
/2)*b**32*x**(23/2) + 66150*a**(21/2)*b**33*x**(21/2) + 79380*a**(19/2)...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^5}-\frac{12\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{5\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3}{3\,b^5}-\frac{2\sqrt{a+\frac{b}{x}}a^4}{b^5}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x)^(9/2)/b^5 + 8/7*(a + b/x)^(7/2)*a/b^5 - 12/5*(a + b/x)^(5/2
)*a^2/b^5 + 8/3*(a + b/x)^(3/2)*a^3/b^5 - 2*sqrt(a + b/x)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx$$
$$=\frac{2\left(1008\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^4a^2+1680\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^3a^{\frac{3}{2}}b+1080\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2+315\right.}{315\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^9\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^6,x, algorithm="giac")
```

output
```
2/315*(1008*(sqrt(a)*x - sqrt(a*x^2 + b*x))^4*a^2 + 1680*(sqrt(a)*x - sqrt
(a*x^2 + b*x))^3*a^(3/2)*b + 1080*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2
+ 315*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3 + 35*b^4)/((sqrt(a)*x -
sqrt(a*x^2 + b*x))^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^6}\,dx=\frac{16\,a\,\sqrt{a+\frac{b}{x}}}{63\,b^2\,x^3}-\frac{2\,\sqrt{a+\frac{b}{x}}}{9\,b\,x^4}-\frac{256\,a^4\,\sqrt{a+\frac{b}{x}}}{315\,b^5}-\frac{32\,a^2\,\sqrt{a+\frac{b}{x}}}{105\,b^3\,x^2}+\frac{128\,a^3\,\sqrt{a+\frac{b}{x}}}{315\,b^4\,x}$$

input
```
int(1/(x^6*(a + b/x)^(1/2)),x)
```

output
```
(16*a*(a + b/x)^(1/2))/(63*b^2*x^3) - (2*(a + b/x)^(1/2))/(9*b*x^4) - (256
*a^4*(a + b/x)^(1/2))/(315*b^5) - (32*a^2*(a + b/x)^(1/2))/(105*b^3*x^2) +
 (128*a^3*(a + b/x)^(1/2))/(315*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^6}\,dx$$
$$= \frac{-\frac{256\sqrt{x}\,\sqrt{ax+b}\,a^4x^4}{315}+\frac{128\sqrt{x}\,\sqrt{ax+b}\,a^3b\,x^3}{315}-\frac{32\sqrt{x}\,\sqrt{ax+b}\,a^2b^2x^2}{105}+\frac{16\sqrt{x}\,\sqrt{ax+b}\,a\,b^3x}{63}-\frac{2\sqrt{x}\,\sqrt{ax+b}\,b^4}{9}+\frac{256\sqrt{a}\,a^4x^5}{315}}{b^5x^5}$$

input
```
int(1/(a+b/x)^(1/2)/x^6,x)
```

output
```
(2*( - 128*sqrt(x)*sqrt(a*x + b)*a**4*x**4 + 64*sqrt(x)*sqrt(a*x + b)*a**3
*b*x**3 - 48*sqrt(x)*sqrt(a*x + b)*a**2*b**2*x**2 + 40*sqrt(x)*sqrt(a*x +
b)*a*b**3*x - 35*sqrt(x)*sqrt(a*x + b)*b**4 + 128*sqrt(a)*a**4*x**5))/(315
*b**5*x**5)
```

3.184 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{35b^3}{8a^4\sqrt{a+\frac{b}{x}}}+\frac{35b^2x}{24a^3\sqrt{a+\frac{b}{x}}}-\frac{7bx^2}{12a^2\sqrt{a+\frac{b}{x}}}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}-\frac{35b^3\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{9/2}}$$

output
```
35/8*b^3/a^4/(a+b/x)^(1/2)+35/24*b^2*x/a^3/(a+b/x)^(1/2)-7/12*b*x^2/a^2/(a
+b/x)^(1/2)+1/3*x^3/a/(a+b/x)^(1/2)-35/8*b^3*arctanh((a+b/x)^(1/2)/a^(1/2)
)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\,x(105b^3+35ab^2x-14a^2bx^2+8a^3x^3)}{24a^4(b+ax)}-\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{9/2}}$$

input

```
Integrate[x^2/(a + b/x)^(3/2),x]
```

output

```
(Sqrt[a + b/x]*x*(105*b^3 + 35*a*b^2*x - 14*a^2*b*x^2 + 8*a^3*x^3))/(24*a^
4*(b + a*x)) - (35*b^3*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(8*a^(9/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {798, 52, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \; 798$$

$$-\int \frac{x^4}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \; 52$$

$$\frac{7b\int \frac{x^3}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{6a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \; 52$$

$$\frac{7b\left(-\frac{5b\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{4a}-\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}\right)}{6a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}$$

$\downarrow$ 52

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}-\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}\right)}{6a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}$$

$\downarrow$ 61

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}-\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}\right)}{6a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}$$

$\downarrow$ 73

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}-\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}\right)}{6a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}$$

$\downarrow$ 221

$$\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}-\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}\right)}{6a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[x^2/(a + b/x)^(3/2),x]
```

output
```
x^3/(3*a*Sqrt[a + b/x]) + (7*b*(-1/2*x^2/(a*Sqrt[a + b/x]) - (5*b*(-(x/(a*
Sqrt[a + b/x])) - (3*b*(2/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

method	result
risch	$\frac{(8a^2x^2-22abx+57b^2)(ax+b)}{24a^4\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{35b^3\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a x^2+bx}\right)}{16a^{\frac{9}{2}}}+\frac{2b^3\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^5\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$\sqrt{\frac{ax+b}{x}}\,x\left(16(a x^2+bx)^{\frac{3}{2}}a^{\frac{9}{2}}x^2-60\sqrt{a x^2+bx}\,a^{\frac{9}{2}}bx^3+32(a x^2+bx)^{\frac{3}{2}}a^{\frac{7}{2}}bx-150\sqrt{a x^2+bx}\,a^{\frac{7}{2}}b^2x^2+240a^{\frac{7}{2}}\sqrt{x(ax+b)}\,b^2x^2-120a\right.$

input
```
int(x^2/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2-22*a*b*x+57*b^2)*(a*x+b)/a^4/((a*x+b)/x)^(1/2)+(-35/16*b^3
/a^(9/2)*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))+2*b^3/a^5/(x+b/a)*(a*(x
+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.81

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[\frac{105\,(ab^3x+b^4)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(8\,a^4x^4-14\,a^3bx^3+35\,a^2b^2x^2 \cdots}{48\,(a^6x+a^5b)}\right.$$

input
```
integrate(x^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(105*(a*b^3*x + b^4)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/
x) + b) + 2*(8*a^4*x^4 - 14*a^3*b*x^3 + 35*a^2*b^2*x^2 + 105*a*b^3*x)*sqrt
((a*x + b)/x))/(a^6*x + a^5*b), 1/24*(105*(a*b^3*x + b^4)*sqrt(-a)*arctan(
sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (8*a^4*x^4 - 14*a^3*b*x^3 + 35*a
^2*b^2*x^2 + 105*a*b^3*x)*sqrt((a*x + b)/x))/(a^6*x + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 16.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{x^{\frac{7}{2}}}{3a\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{7\sqrt{b}x^{\frac{5}{2}}}{12a^2\sqrt{\frac{ax}{b}+1}} + \frac{35b^{\frac{3}{2}}x^{\frac{3}{2}}}{24a^3\sqrt{\frac{ax}{b}+1}} + \frac{35b^{\frac{5}{2}}\sqrt{x}}{8a^4\sqrt{\frac{ax}{b}+1}} - \frac{35b^3\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{8a^{\frac{9}{2}}}$$

input
```
integrate(x**2/(a+b/x)**(3/2),x)
```

output
```
x**(7/2)/(3*a*sqrt(b)*sqrt(a*x/b + 1)) - 7*sqrt(b)*x**(5/2)/(12*a**2*sqrt(
a*x/b + 1)) + 35*b**(3/2)*x**(3/2)/(24*a**3*sqrt(a*x/b + 1)) + 35*b**(5/2)
*sqrt(x)/(8*a**4*sqrt(a*x/b + 1)) - 35*b**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))
/(8*a**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{105\left(a+\frac{b}{x}\right)^3 b^3 - 280\left(a+\frac{b}{x}\right)^2 ab^3 + 231\left(a+\frac{b}{x}\right)a^2b^3 - 48\,a^3b^3}{24\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4 - 3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^5 + 3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^6 - \sqrt{a+\frac{b}{x}}\,a^7\right)} + \frac{35\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{9}{2}}}$$

input
```
integrate(x^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
1/24*(105*(a + b/x)^3*b^3 - 280*(a + b/x)^2*a*b^3 + 231*(a + b/x)*a^2*b^3
- 48*a^3*b^3)/((a + b/x)^(7/2)*a^4 - 3*(a + b/x)^(5/2)*a^5 + 3*(a + b/x)^(
3/2)*a^6 - sqrt(a + b/x)*a^7) + 35/16*b^3*log((sqrt(a + b/x) - sqrt(a))/(s
qrt(a + b/x) + sqrt(a)))/a^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.29

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{1}{24}\sqrt{ax^2+bx}\left(2\,x\left(\frac{4\,x}{a^2\mathrm{sgn}\,(x)} - \frac{11\,b}{a^3\mathrm{sgn}\,(x)}\right) + \frac{57\,b^2}{a^4\mathrm{sgn}\,(x)}\right) + \frac{35\,b^3\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{16\,a^{\frac{9}{2}}\mathrm{sgn}\,(x)} + \frac{2\,b^4}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^{\frac{9}{2}}\mathrm{sgn}\,(x)} - \frac{(35\,b^3\log\left(|b|\right)+32\,b^3)\mathrm{sgn}(x)}{16\,a^{\frac{9}{2}}}$$

input
```
integrate(x^2/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*x^2 + b*x)*(2*x*(4*x/(a^2*sgn(x)) - 11*b/(a^3*sgn(x))) + 57*b^
2/(a^4*sgn(x))) + 35/16*b^3*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt
(a) + b))/(a^(9/2)*sgn(x)) + 2*b^4/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(
a) + b)*a^(9/2)*sgn(x)) - 1/16*(35*b^3*log(abs(b)) + 32*b^3)*sgn(x)/a^(9/2
)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{35\,b^3}{8\,a^4\,\sqrt{a+\frac{b}{x}}}-\frac{35\,b^3\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8\,a^{9/2}}+\frac{x^3}{3\,a\,\sqrt{a+\frac{b}{x}}}-\frac{7\,b\,x^2}{12\,a^2\,\sqrt{a+\frac{b}{x}}}+\frac{35\,b^2\,x}{24\,a^3\,\sqrt{a+\frac{b}{x}}}$$

input
```
int(x^2/(a + b/x)^(3/2),x)
```

output
```
(35*b^3)/(8*a^4*(a + b/x)^(1/2)) - (35*b^3*atanh((a + b/x)^(1/2)/a^(1/2)))
/(8*a^(9/2)) + x^3/(3*a*(a + b/x)^(1/2)) - (7*b*x^2)/(12*a^2*(a + b/x)^(1/
2)) + (35*b^2*x)/(24*a^3*(a + b/x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{-840\sqrt{a}\,\sqrt{ax+b}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^3+525\sqrt{a}\,\sqrt{ax+b}\,b^3+64\sqrt{x}\,a^4x^3-112\sqrt{x}\,a^3b}{192\sqrt{ax+b}\,a^5}$$

input
```
int(x^2/(a+b/x)^(3/2),x)
```

output
```
( - 840*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b
))*b**3 + 525*sqrt(a)*sqrt(a*x + b)*b**3 + 64*sqrt(x)*a**4*x**3 - 112*sqrt
(x)*a**3*b*x**2 + 280*sqrt(x)*a**2*b**2*x + 840*sqrt(x)*a*b**3)/(192*sqrt(
a*x + b)*a**5)
```

3.185 $\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}} dx = -\frac{15b^2}{4a^3\sqrt{a+\frac{b}{x}}} - \frac{5bx}{4a^2\sqrt{a+\frac{b}{x}}} + \frac{x^2}{2a\sqrt{a+\frac{b}{x}}} + \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
-15/4*b^2/a^3/(a+b/x)^(1/2)-5/4*b*x/a^2/(a+b/x)^(1/2)+1/2*x^2/a/(a+b/x)^(1
/2)+15/4*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}} dx = \frac{\sqrt{a+\frac{b}{x}}x(-15b^2-5abx+2a^2x^2)}{4a^3(b+ax)} + \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[x/(a + b/x)^(3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(-15*b^2 - 5*a*b*x + 2*a^2*x^2))/(4*a^3*(b + a*x)) + (15*
b^2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {798, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x^3}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{4a}+\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\left(-\frac{3b\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}+\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(-\frac{3b\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}+\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \ 73$$

$$\frac{5b\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}+\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \ 221$$

$$\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{x}{a\sqrt{a+\frac{b}{x}}}\right)}{4a}+\frac{x^2}{2a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[x/(a + b/x)^(3/2),x]
```

output
```
x^2/(2*a*Sqrt[a + b/x]) + (5*b*(-(x/(a*Sqrt[a + b/x])) - (3*b*(2/(a*Sqrt[a
 + b/x]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.39

method	result
risch	$\frac{(2ax-7b)(ax+b)}{4a^{3}\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{15b^{2}\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{8a^{\frac{7}{2}}}-\frac{2b^{2}\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{a^{4}\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$\sqrt{\frac{ax+b}{x}}\,x\left(4\sqrt{a\,x^{2}+bx}\,a^{\frac{9}{2}}x^{3}-32\sqrt{x(ax+b)}\,a^{\frac{7}{2}}b\,x^{2}+10\sqrt{a\,x^{2}+bx}\,a^{\frac{7}{2}}b\,x^{2}+16a^{3}\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^{2}x^{2}+16(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}\right.$

input
```
int(x/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x-7*b)*(a*x+b)/a^3/((a*x+b)/x)^(1/2)+(15/8*b^2/a^(7/2)*ln((1/2*b+
a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))-2*b^2/a^4/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^
(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.05

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \left[\frac{15\,(ab^2x+b^3)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(2\,a^3x^3-5\,a^2bx^2-15\,ab^2x)\sqrt{\frac{ax+b}{x}}}{8\,(a^5x+a^4b)}, -\frac{15\,(ab^2x+b^3)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(2\,a^3x^3-5\,a^2bx^2-15\,ab^2x)\sqrt{\frac{ax+b}{x}}}{4\,(a^5x+a^4b)}\right]$$

input
```
integrate(x/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(a*b^2*x + b^3)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b) + 2*(2*a^3*x^3 - 5*a^2*b*x^2 - 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^5*x
 + a^4*b), -1/4*(15*(a*b^2*x + b^3)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x +
 b)/x)/(a*x + b)) - (2*a^3*x^3 - 5*a^2*b*x^2 - 15*a*b^2*x)*sqrt((a*x + b)/
x))/(a^5*x + a^4*b)]
```

Sympy [A] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{x^{\frac{5}{2}}}{2a\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{5\sqrt{b}x^{\frac{3}{2}}}{4a^2\sqrt{\frac{ax}{b}+1}} - \frac{15b^{\frac{3}{2}}\sqrt{x}}{4a^3\sqrt{\frac{ax}{b}+1}} + \frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{4a^{\frac{7}{2}}}$$

input
```
integrate(x/(a+b/x)**(3/2),x)
```

output
```
x**(5/2)/(2*a*sqrt(b)*sqrt(a*x/b + 1)) - 5*sqrt(b)*x**(3/2)/(4*a**2*sqrt(a
*x/b + 1)) - 15*b**(3/2)*sqrt(x)/(4*a**3*sqrt(a*x/b + 1)) + 15*b**2*asinh(
sqrt(a)*sqrt(x)/sqrt(b))/(4*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{15\left(a+\frac{b}{x}\right)^{2}b^{2}-25\left(a+\frac{b}{x}\right)ab^{2}+8\,a^{2}b^{2}}{4\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^{3}-2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{4}+\sqrt{a+\frac{b}{x}}a^{5}\right)}-\frac{15\,b^{2}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

input
```
integrate(x/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(15*(a + b/x)^2*b^2 - 25*(a + b/x)*a*b^2 + 8*a^2*b^2)/((a + b/x)^(5/2
)*a^3 - 2*(a + b/x)^(3/2)*a^4 + sqrt(a + b/x)*a^5) - 15/8*b^2*log((sqrt(a
+ b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= \frac{1}{4}\sqrt{ax^{2}+bx}\left(\frac{2\,x}{a^{2}\mathrm{sgn}\left(x\right)}-\frac{7\,b}{a^{3}\mathrm{sgn}\left(x\right)}\right)\\ &\quad-\frac{15\,b^{2}\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right|\right)}{8\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}\\ &\quad+\frac{\left(15\,b^{2}\log\left(\left|b\right|\right)+16\,b^{2}\right)\mathrm{sgn}(x)}{8\,a^{\frac{7}{2}}}-\frac{2\,b^{3}}{\left(\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right)a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}\end{aligned}$$

input
```
integrate(x/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*x^2 + b*x)*(2*x/(a^2*sgn(x)) - 7*b/(a^3*sgn(x))) - 15/8*b^2*log
(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + 1/
8*(15*b^2*log(abs(b)) + 16*b^2)*sgn(x)/a^(7/2) - 2*b^3/(((sqrt(a)*x - sqrt
(a*x^2 + b*x))*sqrt(a) + b)*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{15\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{7/2}}-\frac{15\,b^2}{4\,a^3\,\sqrt{a+\frac{b}{x}}}+\frac{x^2}{2\,a\,\sqrt{a+\frac{b}{x}}}-\frac{5\,b\,x}{4\,a^2\,\sqrt{a+\frac{b}{x}}}$$

input
```
int(x/(a + b/x)^(3/2),x)
```

output
```
(15*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(7/2)) - (15*b^2)/(4*a^3*(a +
 b/x)^(1/2)) + x^2/(2*a*(a + b/x)^(1/2)) - (5*b*x)/(4*a^2*(a + b/x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{15\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2-10\sqrt{a}\,\sqrt{ax+b}\,b^2+2\sqrt{x}\,a^3x^2-5\sqrt{x}\,a^2bx-15}{4\sqrt{ax+b}\,a^4}$$

input
```
int(x/(a+b/x)^(3/2),x)
```

output
```
(15*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*b
**2 - 10*sqrt(a)*sqrt(a*x + b)*b**2 + 2*sqrt(x)*a**3*x**2 - 5*sqrt(x)*a**2
*b*x - 15*sqrt(x)*a*b**2)/(4*sqrt(a*x + b)*a**4)
```

3.186 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}-\frac{3b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
3*b/a^2/(a+b/x)^(1/2)+x/a/(a+b/x)^(1/2)-3*b*arctanh((a+b/x)^(1/2)/a^(1/2))
/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(3b+ax)}{a^2(b+ax)}-\frac{3b\,\mathrm{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(a + b/x)^(-3/2),x]
```

output
```
(Sqrt[a + b/x]*x*(3*b + a*x))/(a^2*(b + a*x)) - (3*b*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {773, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{61}$$

$$\frac{3b\left(\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{73}$$

$$\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[(a + b/x)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b/x]) + (3*b*(2/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

method	result
risch	$\frac{ax+b}{a^2\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{3b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{2a^{\frac{5}{2}}}+\frac{2b\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{a^3\left(x+\frac{b}{a}\right)}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^2b\,x^2-6\sqrt{x(ax+b)}\,a^{\frac{5}{2}}x^2+6\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\,b^2x+4a^{\frac{3}{2}}\left(x(ax+b)\right)^{\frac{3}{2}}-12\sqrt{x}\right.}{2a^{\frac{5}{2}}\sqrt{x(ax+b)}\,(ax+b)^2}$

input
```
int(1/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(a*x+b)/((a*x+b)/x)^(1/2)+(-3/2*b/a^(5/2)*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))+2*b/a^3/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.68

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\left[\frac{3\,(abx+b^2)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(a^2x^2+3\,abx)\sqrt{\frac{ax+b}{x}}}{2\,(a^4x+a^3b)},\frac{3\,(abx+b^2)}{}\right.$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
b) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x +
 b^2)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (a^2*x^2 +
 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}} - \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)**(3/2),x)
```

output
```
x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b
+ 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3} + \frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*
log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(50) = 100$.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{(3\,b\log\left(|b|\right)+4\,b)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}} + \frac{3\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)} + \frac{\sqrt{ax^2+bx}}{a^2\mathrm{sgn}\left(x\right)} + \frac{2\,b^2}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b*log(abs(b)) + 4*b)*sgn(x)/a^(5/2) + 3/2*b*log(abs(2*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a^
2*sgn(x)) + 2*b^2/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*a^(5/2)*s
gn(x))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\,x\left(\frac{a\,x}{b}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(1/(a + b/x)^(3/2),x)
```

output
```
(2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x
)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{-12\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b+9\sqrt{a}\,\sqrt{ax+b}\,b+4\sqrt{x}\,a^2x+12\sqrt{x}\,ab}{4\sqrt{ax+b}\,a^3}$$

input

```
int(1/(a+b/x)^(3/2),x)
```

output

```
( - 12*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)
)*b + 9*sqrt(a)*sqrt(a*x + b)*b + 4*sqrt(x)*a**2*x + 12*sqrt(x)*a*b)/(4*sq
rt(a*x + b)*a**3)
```

3.187 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x} \, dx = -\frac{2}{a\sqrt{a+\frac{b}{x}}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-2/a/(a+b/x)^(1/2)+2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x} \, dx = -\frac{2\sqrt{a+\frac{b}{x}}x}{a(b+ax)} + \frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x),x]
```

output
```
(-2*Sqrt[a + b/x]*x)/(a*(b + a*x)) + (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^
(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}-\frac{2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}-\frac{2}{a\sqrt{a+\frac{b}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[1/((a + b/x)^(3/2)*x),x]
```

output
```
-2/(a*Sqrt[a + b/x]) + (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(34) = 68$.

Time = 0.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 4.71

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(-2\sqrt{x(ax+b)}\, a^{\frac{5}{2}} x^2+\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right) a^2 b\, x^2+2a^{\frac{3}{2}} (x(ax+b))^{\frac{3}{2}}-4\sqrt{x(ax+b)}\, a^{\frac{3}{2}} bx+2\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax\right.\right.}{2\sqrt{a}}}{a^{\frac{3}{2}}\sqrt{x(ax+b)}\, b(ax+b)^2}$

input
```
int(1/(a+b/x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
((a*x+b)/x)^(1/2)*x/a^(3/2)*(-2*(x*(a*x+b))^(1/2)*a^(5/2)*x^2+ln(1/2*(2*(x
*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b*x^2+2*a^(3/2)*(x*(a*x+b))^
(3/2)-4*(x*(a*x+b))^(1/2)*a^(3/2)*b*x+2*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*a*b^2*x-2*(x*(a*x+b))^(1/2)*a^(1/2)*b^2+ln(1/2*(2*(x*(
a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3)/(x*(a*x+b))^(1/2)/b/(a*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x}\,dx = \left[-\frac{2\,ax\sqrt{\frac{ax+b}{x}} - (ax+b)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{a^3x+a^2b},\right.$$
$$\left.-\frac{2\left(ax\sqrt{\frac{ax+b}{x}}+(ax+b)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)\right)}{a^3x+a^2b}\right]$$

input
```
integrate(1/(a+b/x)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-(2*a*x*sqrt((a*x + b)/x) - (a*x + b)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b))/(a^3*x + a^2*b), -2*(a*x*sqrt((a*x + b)/x) + (a*x + b
)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)))/(a^3*x + a^2*b)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(32) = 64$.

Time = 1.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.52

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x}\,dx = -\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} - \frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} + \frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} - \frac{a^2b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} + \frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}$$

input
```
integrate(1/(a+b/x)**(3/2)/x,x)
```

output
```
-2*a**3*x*sqrt(1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x)
)/(a**(9/2)*x + a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2
)*x + a**(7/2)*b) - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2
*b*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{a+\frac{b}{x}}a}$$

input
```
integrate(1/(a+b/x)^(3/2)/x,x, algorithm="maxima")
```

output
```
-log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) - 2/(sqr
t(a + b/x)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.05

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x} \, dx = \frac{(\log(|b|)+2)\text{sgn}(x)}{a^{\frac{3}{2}}} - \frac{\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{a^{\frac{3}{2}}\text{sgn}(x)} - \frac{2\,b}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)a^{\frac{3}{2}}\text{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x,x, algorithm="giac")
```

output
```
(log(abs(b)) + 2)*sgn(x)/a^(3/2) - log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x
))*sqrt(a) + b))/(a^(3/2)*sgn(x)) - 2*b/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*
sqrt(a) + b)*a^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x} \, dx = \frac{2\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2}{a\,\sqrt{a+\frac{b}{x}}}$$

input
```
int(1/(x*(a + b/x)^(3/2)),x)
```

output
```
(2*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - 2/(a*(a + b/x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x}\, dx = \frac{2\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)-2\sqrt{a}\,\sqrt{ax+b}-2\sqrt{x}\,a}{\sqrt{ax+b}\,a^2}$$

input
```
int(1/(a+b/x)^(3/2)/x,x)
```

output
```
(2*(sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)) -
 sqrt(a)*sqrt(a*x + b) - sqrt(x)*a))/(sqrt(a*x + b)*a**2)
```

3.188 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2} \, dx = \frac{2}{b\sqrt{a+\frac{b}{x}}}$$

output
```
2/b/(a+b/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2} \, dx = \frac{2x\sqrt{\frac{b+ax}{x}}}{b(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^2),x]
```

output
```
(2*x*Sqrt[(b + a*x)/x])/(b*(b + a*x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x}\right)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 793}$$
$$\frac{2}{b\sqrt{a+\frac{b}{x}}}$$

input
```
Int[1/((a + b/x)^(3/2)*x^2),x]
```

output
```
2/(b*Sqrt[a + b/x])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2}{b\sqrt{a+\frac{b}{x}}}$
orering	$\frac{2ax+2b}{xb\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$
gosper	$\frac{2ax+2b}{xb\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}$
trager	$\frac{2x\sqrt{-\frac{-ax-b}{x}}}{b(ax+b)}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-2\sqrt{x(ax+b)}\,a^{\frac{5}{2}}x^{2}-2\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}x^{2}-\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{2}b\,x^{2}+\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\right.}{}$

input
```
int(1/(a+b/x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/b/(a+b/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^2}\,dx=\frac{2\,x\sqrt{\frac{ax+b}{x}}}{abx+b^2}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2*x*sqrt((a*x + b)/x)/(a*b*x + b^2)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^2}\,dx = \begin{cases} \frac{2}{b\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{a^{\frac{3}{2}}x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**2,x)
```

output
```
Piecewise((2/(b*sqrt(a + b/x)), Ne(b, 0)), (-1/(a**(3/2)*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^2}\,dx = \frac{2}{\sqrt{a+\frac{b}{x}}\,b}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2/(sqrt(a + b/x)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^2}\,dx = -\frac{2\,\mathrm{sgn}(x)}{\sqrt{a}b} + \frac{2}{\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)\sqrt{a}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
-2*sgn(x)/(sqrt(a)*b) + 2/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)*s
qrt(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^2}\,dx=\frac{2}{b\sqrt{a+\frac{b}{x}}}$$

input
```
int(1/(x^2*(a + b/x)^(3/2)),x)
```

output
```
2/(b*(a + b/x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^2}\,dx=\frac{2\sqrt{a}\,\sqrt{ax+b}+2\sqrt{x}\,a}{\sqrt{ax+b}\,ab}$$

input
```
int(1/(a+b/x)^(3/2)/x^2,x)
```

output
```
(2*(sqrt(a)*sqrt(a*x + b) + sqrt(x)*a))/(sqrt(a*x + b)*a*b)
```

3.189 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} \, dx = -\frac{2a}{b^2 \sqrt{a+\frac{b}{x}}} - \frac{2\sqrt{a+\frac{b}{x}}}{b^2}$$

output
```
-2*a/b^2/(a+b/x)^(1/2)-2*(a+b/x)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} \, dx = -\frac{2\sqrt{\frac{b+ax}{x}}(b+2ax)}{b^2(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^3),x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(b + 2*a*x))/(b^2*(b + a*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x} d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{1}{b\sqrt{a+\frac{b}{x}}}-\frac{a}{b \left(a+\frac{b}{x}\right)^{3/2}}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a}{b^2\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^2}$$

input
```
Int[1/((a + b/x)^(3/2)*x^3),x]
```

output
```
(-2*a)/(b^2*Sqrt[a + b/x]) - (2*Sqrt[a + b/x])/b^2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
orering	$-\frac{2(2ax+b)(ax+b)}{b^{2}x^{2}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$	29
gosper	$-\frac{2(ax+b)(2ax+b)}{x^{2}b^{2}\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}$	31
trager	$-\frac{2(2ax+b)\sqrt{-\frac{-ax-b}{x}}}{b^{2}(ax+b)}$	34
risch	$-\frac{2(ax+b)}{b^{2}x\sqrt{\frac{ax+b}{x}}}-\frac{2a}{b^{2}\sqrt{\frac{ax+b}{x}}}$	43
default	$\frac{2\sqrt{\frac{ax+b}{x}}\left((x(ax+b))^{\frac{3}{2}}a^{2}x^{2}-(a\,x^{2}+bx)^{\frac{3}{2}}a^{2}x^{2}-2(a\,x^{2}+bx)^{\frac{3}{2}}abx-(a\,x^{2}+bx)^{\frac{3}{2}}b^{2}\right)}{x\sqrt{x(ax+b)}\,b^{3}(ax+b)^{2}}$	104

input
```
int(1/(a+b/x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(2*a*x+b)/b^2/x^2*(a*x+b)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} \, dx = -\frac{2\,(2\,ax+b)\sqrt{\frac{ax+b}{x}}}{ab^2x+b^3}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-2*(2*a*x + b)*sqrt((a*x + b)/x)/(a*b^2*x + b^3)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} \, dx = \begin{cases} -\frac{4a}{b^2\sqrt{a+\frac{b}{x}}} - \frac{2}{bx\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{2a^{\frac{3}{2}}x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**3,x)
```

output
```
Piecewise((-4*a/(b**2*sqrt(a + b/x)) - 2/(b*x*sqrt(a + b/x)), Ne(b, 0)), (
-1/(2*a**(3/2)*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} \, dx = -\frac{2\sqrt{a+\frac{b}{x}}}{b^2} - \frac{2\,a}{\sqrt{a+\frac{b}{x}}b^2}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-2*sqrt(a + b/x)/b^2 - 2*a/(sqrt(a + b/x)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^3}\,dx=-\frac{2\left(\frac{2\,ax}{b^2\mathrm{sgn}(x)}+\frac{1}{b\mathrm{sgn}(x)}\right)}{\sqrt{ax^2+bx}}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-2*(2*a*x/(b^2*sgn(x)) + 1/(b*sgn(x)))/sqrt(a*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^3}\,dx=-\frac{x\,\sqrt{a+\frac{b}{x}}\,\left(\frac{2}{b}+\frac{4\,a\,x}{b^2}\right)}{a\,x^2+b\,x}$$

input
```
int(1/(x^3*(a + b/x)^(3/2)),x)
```

output
```
-(x*(a + b/x)^(1/2)*(2/b + (4*a*x)/b^2))/(b*x + a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} \, dx = \frac{-4\sqrt{a}\,\sqrt{ax+b}\,x - 4\sqrt{x}\,ax - 2\sqrt{x}\,b}{\sqrt{ax+b}\,b^2x}$$

input
```
int(1/(a+b/x)^(3/2)/x^3,x)
```

output
```
(2*( - 2*sqrt(a)*sqrt(a*x + b)*x - 2*sqrt(x)*a*x - sqrt(x)*b))/(sqrt(a*x +
 b)*b**2*x)
```

3.190 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^4}\,dx=\frac{2a^2}{b^3\sqrt{a+\frac{b}{x}}}+\frac{4a\sqrt{a+\frac{b}{x}}}{b^3}-\frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^3}$$

output
```
2*a^2/b^3/(a+b/x)^(1/2)+4*a*(a+b/x)^(1/2)/b^3-2/3*(a+b/x)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^4}\,dx=\frac{2\sqrt{\frac{b+ax}{x}}\left(-b^2+4abx+8a^2x^2\right)}{3b^3x(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^4),x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-b^2 + 4*a*b*x + 8*a^2*x^2))/(3*b^3*x*(b + a*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{a^2}{b^2 \left(a+\frac{b}{x}\right)^{3/2}}-\frac{2 a}{b^2 \sqrt{a+\frac{b}{x}}}+\frac{\sqrt{a+\frac{b}{x}}}{b^2}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2 a^2}{b^3 \sqrt{a+\frac{b}{x}}}+\frac{4 a \sqrt{a+\frac{b}{x}}}{b^3}-\frac{2 \left(a+\frac{b}{x}\right)^{3/2}}{3 b^3}$$

input

```
Int[1/((a + b/x)^(3/2)*x^4),x]
```

output

```
(2*a^2)/(b^3*Sqrt[a + b/x]) + (4*a*Sqrt[a + b/x])/b^3 - (2*(a + b/x)^(3/2)
)/(3*b^3)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result
orering	$\frac{2\left(8a^{2}x^{2}+4abx-b^{2}\right)\left(ax+b\right)}{3b^{3}x^{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$
gosper	$\frac{2\left(ax+b\right)\left(8a^{2}x^{2}+4abx-b^{2}\right)}{3x^{3}b^{3}\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}$
trager	$\frac{2\left(8a^{2}x^{2}+4abx-b^{2}\right)\sqrt{-\frac{-ax-b}{x}}}{3x\,b^{3}\left(ax+b\right)}$
risch	$\frac{2\left(ax+b\right)\left(5ax-b\right)}{3b^{3}x^{2}\sqrt{\frac{ax+b}{x}}}+\frac{2a^{2}}{b^{3}\sqrt{\frac{ax+b}{x}}}$
default	$\sqrt{\frac{ax+b}{x}}\,\left(-6\sqrt{a\,x^{2}+bx}\,a^{\frac{9}{2}}x^{5}-3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{4}b\,x^{5}-6\sqrt{x\left(ax+b\right)}\,a^{\frac{9}{2}}x^{5}+3\ln\left(\frac{2\sqrt{x\left(ax+b\right)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{4}b\,x^{5}+24(a\,x$

input
```
int(1/(a+b/x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3*(8*a^2*x^2+4*a*b*x-b^2)/b^3/x^3*(a*x+b)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4}\,dx=\frac{2\,(8\,a^2x^2+4\,abx-b^2)\sqrt{\frac{ax+b}{x}}}{3\,(ab^3x^2+b^4x)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
2/3*(8*a^2*x^2 + 4*a*b*x - b^2)*sqrt((a*x + b)/x)/(a*b^3*x^2 + b^4*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(46) = 92.

Time = 1.30 (sec) , antiderivative size = 457, normalized size of antiderivative = 8.31

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4}\,dx &= \frac{16a^{\frac{9}{2}}b^{\frac{7}{2}}x^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}\\ &+\frac{24a^{\frac{7}{2}}b^{\frac{9}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}+\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}\\ &-\frac{2a^{\frac{3}{2}}b^{\frac{13}{2}}\sqrt{\frac{ax}{b}+1}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}-\frac{16a^5b^3x^{\frac{7}{2}}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}\\ &-\frac{32a^4b^4x^{\frac{5}{2}}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}-\frac{16a^3b^5x^{\frac{3}{2}}}{3a^{\frac{7}{2}}b^6x^{\frac{7}{2}}+6a^{\frac{5}{2}}b^7x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^8x^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**4,x)
```

output
```
16*a**(9/2)*b**(7/2)*x**3*sqrt(a*x/b + 1)/(3*a**(7/2)*b**6*x**(7/2) + 6*a*
*(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2)) + 24*a**(7/2)*b**(9/2)*x*
*2*sqrt(a*x/b + 1)/(3*a**(7/2)*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) +
3*a**(3/2)*b**8*x**(3/2)) + 6*a**(5/2)*b**(11/2)*x*sqrt(a*x/b + 1)/(3*a**(
7/2)*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2))
- 2*a**(3/2)*b**(13/2)*sqrt(a*x/b + 1)/(3*a**(7/2)*b**6*x**(7/2) + 6*a**(5
/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2)) - 16*a**5*b**3*x**(7/2)/(3*a
**(7/2)*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2
)) - 32*a**4*b**4*x**(5/2)/(3*a**(7/2)*b**6*x**(7/2) + 6*a**(5/2)*b**7*x**
(5/2) + 3*a**(3/2)*b**8*x**(3/2)) - 16*a**3*b**5*x**(3/2)/(3*a**(7/2)*b**6
*x**(7/2) + 6*a**(5/2)*b**7*x**(5/2) + 3*a**(3/2)*b**8*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4} \, dx = -\frac{2 \left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3 \, b^3} + \frac{4 \sqrt{a+\frac{b}{x}} a}{b^3} + \frac{2 \, a^2}{\sqrt{a+\frac{b}{x}} b^3}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-2/3*(a + b/x)^(3/2)/b^3 + 4*sqrt(a + b/x)*a/b^3 + 2*a^2/(sqrt(a + b/x)*b^
3)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4} \, dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(1/(a+b/x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x)^(3/2)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4} \, dx = \frac{2\sqrt{a+\frac{b}{x}}\,(8\,a^2\,x^2+4\,a\,b\,x-b^2)}{3\,b^3\,x\,(b+a\,x)}$$

input
```
int(1/(x^4*(a + b/x)^(3/2)),x)
```

output
```
(2*(a + b/x)^(1/2)*(8*a^2*x^2 - b^2 + 4*a*b*x))/(3*b^3*x*(b + a*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4} \, dx = \frac{-\frac{16\sqrt{a}\,\sqrt{ax+b}\,a\,x^2}{3}+\frac{16\sqrt{x}\,a^2x^2}{3}+\frac{8\sqrt{x}\,abx}{3}-\frac{2\sqrt{x}\,b^2}{3}}{\sqrt{ax+b}\,b^3x^2}$$

input
```
int(1/(a+b/x)^(3/2)/x^4,x)
```

output
```
(2*( - 8*sqrt(a)*sqrt(a*x + b)*a*x**2 + 8*sqrt(x)*a**2*x**2 + 4*sqrt(x)*a*
b*x - sqrt(x)*b**2))/(3*sqrt(a*x + b)*b**3*x**2)
```

3.191 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5} \, dx = -\frac{2a^3}{b^4\sqrt{a+\frac{b}{x}}}-\frac{6a^2\sqrt{a+\frac{b}{x}}}{b^4}+\frac{2a\left(a+\frac{b}{x}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^4}$$

output
```
-2*a^3/b^4/(a+b/x)^(1/2)-6*a^2*(a+b/x)^(1/2)/b^4+2*a*(a+b/x)^(3/2)/b^4-2/5
*(a+b/x)^(5/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5} \, dx = -\frac{2\sqrt{\frac{b+ax}{x}}(b^3-2ab^2x+8a^2bx^2+16a^3x^3)}{5b^4x^2(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^5),x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(b^3 - 2*a*b^2*x + 8*a^2*b*x^2 + 16*a^3*x^3))/(5*b^4
*x^2*(b + a*x))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 798$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} d\frac{1}{x}$$

$$\Big\downarrow \ 53$$

$$-\int \left(-\frac{a^3}{b^3 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{3 a^2}{b^3 \sqrt{a+\frac{b}{x}}}-\frac{3 \sqrt{a+\frac{b}{x}} a}{b^3}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{b^3}\right) d\frac{1}{x}$$

$$\Big\downarrow \ 2009$$

$$-\frac{2 a^3}{b^4 \sqrt{a+\frac{b}{x}}}-\frac{6 a^2 \sqrt{a+\frac{b}{x}}}{b^4}+\frac{2 a \left(a+\frac{b}{x}\right)^{3/2}}{b^4}-\frac{2 \left(a+\frac{b}{x}\right)^{5/2}}{5 b^4}$$

input
```
Int[1/((a + b/x)^(3/2)*x^5),x]
```

output
```
(-2*a^3)/(b^4*Sqrt[a + b/x]) - (6*a^2*Sqrt[a + b/x])/b^4 + (2*a*(a + b/x)^
(3/2))/b^4 - (2*(a + b/x)^(5/2))/(5*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

method	result
orering	$-\frac{2\left(16a^3x^3+8a^2b\,x^2-2a\,b^2x+b^3\right)(ax+b)}{5b^4x^4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$
gosper	$-\frac{2(ax+b)\left(16a^3x^3+8a^2b\,x^2-2a\,b^2x+b^3\right)}{5x^4b^4\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}$
trager	$-\frac{2\left(16a^3x^3+8a^2b\,x^2-2a\,b^2x+b^3\right)\sqrt{-\frac{-ax-b}{x}}}{5x^2b^4(ax+b)}$
risch	$-\frac{2(ax+b)\left(11a^2x^2-3abx+b^2\right)}{5b^4x^3\sqrt{\frac{ax+b}{x}}}-\frac{2a^3}{b^4\sqrt{\frac{ax+b}{x}}}$
default	$\sqrt{\frac{ax+b}{x}}\left(10\sqrt{x(ax+b)}\,a^{\frac{11}{2}}x^6+10\sqrt{a\,x^2+bx}\,a^{\frac{11}{2}}x^6+5\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^5b\,x^6-5\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^5b\,x^6+10a\right.$

input
```
int(1/(a+b/x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(16*a^3*x^3+8*a^2*b*x^2-2*a*b^2*x+b^3)/b^4/x^4*(a*x+b)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\, dx = -\frac{2\,(16\,a^3x^3+8\,a^2bx^2-2\,ab^2x+b^3)\sqrt{\frac{ax+b}{x}}}{5\,(ab^4x^3+b^5x^2)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-2/5*(16*a^3*x^3 + 8*a^2*b*x^2 - 2*a*b^2*x + b^3)*sqrt((a*x + b)/x)/(a*b^4
*x^3 + b^5*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2032 vs. 2(63) = 126.

Time = 2.04 (sec) , antiderivative size = 2032, normalized size of antiderivative = 27.46

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**5,x)
```

output
```
-32*a**(21/2)*b**(23/2)*x**8*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x**(17/2)
+ 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11
/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7
/2) + 5*a**(5/2)*b**21*x**(5/2)) - 176*a**(19/2)*b**(25/2)*x**7*sqrt(a*x/b
 + 1)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**
(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19
*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) - 396*
a**(17/2)*b**(27/2)*x**6*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x**(17/2) + 30
*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*
b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2)
+ 5*a**(5/2)*b**21*x**(5/2)) - 462*a**(15/2)*b**(29/2)*x**5*sqrt(a*x/b + 1
)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/
2)*b**17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**
(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*a**(5/2)*b**21*x**(5/2)) - 290*a**(
13/2)*b**(31/2)*x**4*sqrt(a*x/b + 1)/(5*a**(17/2)*b**15*x**(17/2) + 30*a**
(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b**17*x**(13/2) + 100*a**(11/2)*b**1
8*x**(11/2) + 75*a**(9/2)*b**19*x**(9/2) + 30*a**(7/2)*b**20*x**(7/2) + 5*
a**(5/2)*b**21*x**(5/2)) - 92*a**(11/2)*b**(33/2)*x**3*sqrt(a*x/b + 1)/(5*
a**(17/2)*b**15*x**(17/2) + 30*a**(15/2)*b**16*x**(15/2) + 75*a**(13/2)*b*
*17*x**(13/2) + 100*a**(11/2)*b**18*x**(11/2) + 75*a**(9/2)*b**19*x**(9...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^4} + \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{b^4} - \frac{6\sqrt{a+\frac{b}{x}}\,a^2}{b^4} - \frac{2\,a^3}{\sqrt{a+\frac{b}{x}}\,b^4}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-2/5*(a + b/x)^(5/2)/b^4 + 2*(a + b/x)^(3/2)*a/b^4 - 6*sqrt(a + b/x)*a^2/b
^4 - 2*a^3/(sqrt(a + b/x)*b^4)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}} x^5}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x)^(3/2)*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}\,(16\,a^3\,x^3+8\,a^2\,b\,x^2-2\,a\,b^2\,x+b^3)}{5\,b^4\,x^2\,(b+a\,x)}$$

input
```
int(1/(x^5*(a + b/x)^(3/2)),x)
```

output
```
-(2*(a + b/x)^(1/2)*(b^3 + 16*a^3*x^3 + 8*a^2*b*x^2 - 2*a*b^2*x))/(5*b^4*x
^2*(b + a*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}\,dx = \frac{\frac{32\sqrt{a}\,\sqrt{ax+b}\,a^2x^3}{5}-\frac{32\sqrt{x}\,a^3x^3}{5}-\frac{16\sqrt{x}\,a^2bx^2}{5}+\frac{4\sqrt{x}\,a\,b^2x}{5}-\frac{2\sqrt{x}\,b^3}{5}}{\sqrt{ax+b}\,b^4x^3}$$

input
```
int(1/(a+b/x)^(3/2)/x^5,x)
```

output
```
(2*(16*sqrt(a)*sqrt(a*x + b)*a**2*x**3 - 16*sqrt(x)*a**3*x**3 - 8*sqrt(x)*
a**2*b*x**2 + 2*sqrt(x)*a*b**2*x - sqrt(x)*b**3))/(5*sqrt(a*x + b)*b**4*x*
*3)
```

3.192 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6} \, dx = \frac{2a^4}{b^5\sqrt{a+\frac{b}{x}}}+\frac{8a^3\sqrt{a+\frac{b}{x}}}{b^5} - \frac{4a^2\left(a+\frac{b}{x}\right)^{3/2}}{b^5}+\frac{8a\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^5}$$

output
```
2*a^4/b^5/(a+b/x)^(1/2)+8*a^3*(a+b/x)^(1/2)/b^5-4*a^2*(a+b/x)^(3/2)/b^5+8/
5*a*(a+b/x)^(5/2)/b^5-2/7*(a+b/x)^(7/2)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6} \, dx = \frac{2\sqrt{\frac{b+ax}{x}}(-5b^4+8ab^3x-16a^2b^2x^2+64a^3bx^3+128a^4x^4)}{35b^5x^3(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^6),x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-5*b^4 + 8*a*b^3*x - 16*a^2*b^2*x^2 + 64*a^3*b*x^3 +
 128*a^4*x^4))/(35*b^5*x^3*(b + a*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \ 798$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^4}d\frac{1}{x}$$

$$\Big\downarrow \ 53$$

$$-\int \left(\frac{a^4}{b^4 \left(a+\frac{b}{x}\right)^{3/2}}-\frac{4 a^3}{b^4 \sqrt{a+\frac{b}{x}}}+\frac{6 \sqrt{a+\frac{b}{x}} a^2}{b^4}-\frac{4 \left(a+\frac{b}{x}\right)^{3/2} a}{b^4}+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{b^4}\right)d\frac{1}{x}$$

$$\Big\downarrow \ 2009$$

$$\frac{2 a^4}{b^5 \sqrt{a+\frac{b}{x}}}+\frac{8 a^3 \sqrt{a+\frac{b}{x}}}{b^5}-\frac{4 a^2 \left(a+\frac{b}{x}\right)^{3/2}}{b^5}+\frac{8 a \left(a+\frac{b}{x}\right)^{5/2}}{5 b^5}-\frac{2 \left(a+\frac{b}{x}\right)^{7/2}}{7 b^5}$$

input
```
Int[1/((a + b/x)^(3/2)*x^6),x]
```

output
```
(2*a^4)/(b^5*Sqrt[a + b/x]) + (8*a^3*Sqrt[a + b/x])/b^5 - (4*a^2*(a + b/x)
^(3/2))/b^5 + (8*a*(a + b/x)^(5/2))/(5*b^5) - (2*(a + b/x)^(7/2))/(7*b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

method	result
orering	$\frac{2\left(128 a^{4} x^{4}+64 a^{3} b \,x^{3}-16 a^{2} b^{2} x^{2}+8 a \,b^{3} x -5 b^{4}\right) \left(a x +b \right)}{35 b^{5} x^{5} \left(a +\frac{b}{x}\right)^{\frac{3}{2}}}$
gosper	$\frac{2 \left(a x +b \right) \left(128 a^{4} x^{4}+64 a^{3} b \,x^{3}-16 a^{2} b^{2} x^{2}+8 a \,b^{3} x -5 b^{4}\right)}{35 x^{5} b^{5} \left(\frac{a x +b}{x}\right)^{\frac{3}{2}}}$
trager	$\frac{2 \left(128 a^{4} x^{4}+64 a^{3} b \,x^{3}-16 a^{2} b^{2} x^{2}+8 a \,b^{3} x -5 b^{4}\right) \sqrt{-\frac{-a x -b}{x}}}{35 x^{3} b^{5} \left(a x +b \right)}$
risch	$\frac{2 \left(a x +b \right) \left(93 a^{3} x^{3}-29 a^{2} b \,x^{2}+13 a \,b^{2} x -5 b^{3}\right)}{35 b^{5} x^{4} \sqrt{\frac{a x +b}{x}}}+\frac{2 a^{4}}{b^{5} \sqrt{\frac{a x +b}{x}}}$
default	$-\frac{\sqrt{\frac{a x +b}{x}}\, \left(210 \sqrt{x \left(a x +b \right)}\, a^{\frac{13}{2}} x^{7}+210 \sqrt{a \,x^{2}+b x}\, a^{\frac{13}{2}} x^{7}+105 \ln \left(\frac{2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a^{6} b \,x^{7}-105 \ln \left(\frac{2 \sqrt{x \left(a x +b \right)}\, \sqrt{a}+2 a x +b}{2 \sqrt{a}}\right) a^{6}\right.}{}$

input
```
int(1/(a+b/x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
2/35*(128*a^4*x^4+64*a^3*b*x^3-16*a^2*b^2*x^2+8*a*b^3*x-5*b^4)/b^5/x^5*(a*
x+b)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\, dx = \frac{2\,(128\,a^4x^4+64\,a^3bx^3-16\,a^2b^2x^2+8\,ab^3x-5\,b^4)\sqrt{\frac{ax+b}{x}}}{35\,(ab^5x^4+b^6x^3)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
2/35*(128*a^4*x^4 + 64*a^3*b*x^3 - 16*a^2*b^2*x^2 + 8*a*b^3*x - 5*b^4)*sqr
t((a*x + b)/x)/(a*b^5*x^4 + b^6*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4707 vs. 2(82) = 164.

Time = 3.26 (sec) , antiderivative size = 4707, normalized size of antiderivative = 49.55

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**6,x)
```

output
```
256*a**(33/2)*b**(49/2)*x**13*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2
) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*
a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2
)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*
x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) +
 35*a**(7/2)*b**39*x**(7/2)) + 2432*a**(31/2)*b**(51/2)*x**12*sqrt(a*x/b +
 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a
**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)
*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x
**(15/2) + 4200*a**(13/2)*b**36*x**(13/2) + 1575*a**(11/2)*b**37*x**(11/2)
 + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7/2)*b**39*x**(7/2)) + 10336*a**(2
9/2)*b**(53/2)*x**11*sqrt(a*x/b + 1)/(35*a**(27/2)*b**29*x**(27/2) + 350*a
**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)*b**31*x**(23/2) + 4200*a**(21/2)
*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x**(19/2) + 8820*a**(17/2)*b**34*x
**(17/2) + 7350*a**(15/2)*b**35*x**(15/2) + 4200*a**(13/2)*b**36*x**(13/2)
 + 1575*a**(11/2)*b**37*x**(11/2) + 350*a**(9/2)*b**38*x**(9/2) + 35*a**(7
/2)*b**39*x**(7/2)) + 25840*a**(27/2)*b**(55/2)*x**10*sqrt(a*x/b + 1)/(35*
a**(27/2)*b**29*x**(27/2) + 350*a**(25/2)*b**30*x**(25/2) + 1575*a**(23/2)
*b**31*x**(23/2) + 4200*a**(21/2)*b**32*x**(21/2) + 7350*a**(19/2)*b**33*x
**(19/2) + 8820*a**(17/2)*b**34*x**(17/2) + 7350*a**(15/2)*b**35*x**(15...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\, dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7\, b^5} + \frac{8\left(a+\frac{b}{x}\right)^{\frac{5}{2}} a}{5\, b^5} - \frac{4\left(a+\frac{b}{x}\right)^{\frac{3}{2}} a^2}{b^5} + \frac{8\sqrt{a+\frac{b}{x}}\, a^3}{b^5} + \frac{2\, a^4}{\sqrt{a+\frac{b}{x}}\, b^5}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)/b^5 + 8/5*(a + b/x)^(5/2)*a/b^5 - 4*(a + b/x)^(3/2)*a
^2/b^5 + 8*sqrt(a + b/x)*a^3/b^5 + 2*a^4/(sqrt(a + b/x)*b^5)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}} x^6}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x)^(3/2)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\,dx = \frac{\sqrt{a+\frac{b}{x}}\left(\frac{186\,a^3}{35\,b^4}+\frac{256\,a^4\,x}{35\,b^5}\right)}{b+a\,x} - \frac{2\sqrt{a+\frac{b}{x}}}{7\,b^2\,x^3} + \frac{26\,a\sqrt{a+\frac{b}{x}}}{35\,b^3\,x^2} - \frac{58\,a^2\sqrt{a+\frac{b}{x}}}{35\,b^4\,x}$$

input
```
int(1/(x^6*(a + b/x)^(3/2)),x)
```

output
```
((a + b/x)^(1/2)*((186*a^3)/(35*b^4) + (256*a^4*x)/(35*b^5)))/(b + a*x) -
(2*(a + b/x)^(1/2))/(7*b^2*x^3) + (26*a*(a + b/x)^(1/2))/(35*b^3*x^2) - (5
8*a^2*(a + b/x)^(1/2))/(35*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^6}\,dx = \frac{-\frac{256\sqrt{a}\,\sqrt{ax+b}\,a^3x^4}{35}+\frac{256\sqrt{x}\,a^4x^4}{35}+\frac{128\sqrt{x}\,a^3b\,x^3}{35}-\frac{32\sqrt{x}\,a^2b^2x^2}{35}+\frac{16\sqrt{x}\,a\,b^3x}{35}-\frac{2\sqrt{x}\,b^4}{7}}{\sqrt{ax+b}\,b^5x^4}$$

input
```
int(1/(a+b/x)^(3/2)/x^6,x)
```

output
```
(2*( - 128*sqrt(a)*sqrt(a*x + b)*a**3*x**4 + 128*sqrt(x)*a**4*x**4 + 64*sq
rt(x)*a**3*b*x**3 - 16*sqrt(x)*a**2*b**2*x**2 + 8*sqrt(x)*a*b**3*x - 5*sqr
t(x)*b**4))/(35*sqrt(a*x + b)*b**5*x**4)
```

3.193 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7} \, dx = -\frac{2a^5}{b^6\sqrt{a+\frac{b}{x}}} - \frac{10a^4\sqrt{a+\frac{b}{x}}}{b^6} + \frac{20a^3\left(a+\frac{b}{x}\right)^{3/2}}{3b^6} - \frac{4a^2\left(a+\frac{b}{x}\right)^{5/2}}{b^6} + \frac{10a\left(a+\frac{b}{x}\right)^{7/2}}{7b^6} - \frac{2\left(a+\frac{b}{x}\right)^{9/2}}{9b^6}$$

output
```
-2*a^5/b^6/(a+b/x)^(1/2)-10*a^4*(a+b/x)^(1/2)/b^6+20/3*a^3*(a+b/x)^(3/2)/b
^6-4*a^2*(a+b/x)^(5/2)/b^6+10/7*a*(a+b/x)^(7/2)/b^6-2/9*(a+b/x)^(9/2)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7} \, dx = -\frac{2\sqrt{\frac{b+ax}{x}}\left(7b^5-10ab^4x+16a^2b^3x^2-32a^3b^2x^3+128a^4bx^4+256a^5x^5\right)}{63b^6x^4(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^7),x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(7*b^5 - 10*a*b^4*x + 16*a^2*b^3*x^2 - 32*a^3*b^2*x^
3 + 128*a^4*b*x^4 + 256*a^5*x^5))/(63*b^6*x^4*(b + a*x))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^5}d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(-\frac{a^5}{b^5 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{5 a^4}{b^5 \sqrt{a+\frac{b}{x}}}-\frac{10 \sqrt{a+\frac{b}{x}} a^3}{b^5}+\frac{10 \left(a+\frac{b}{x}\right)^{3/2} a^2}{b^5}-\frac{5 \left(a+\frac{b}{x}\right)^{5/2} a}{b^5}+\frac{\left(a+\frac{b}{x}\right)^{7/2}}{b^5}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2 a^5}{b^6 \sqrt{a+\frac{b}{x}}}-\frac{10 a^4 \sqrt{a+\frac{b}{x}}}{b^6}+\frac{20 a^3 \left(a+\frac{b}{x}\right)^{3/2}}{3 b^6}-\frac{4 a^2 \left(a+\frac{b}{x}\right)^{5/2}}{b^6}+\frac{10 a \left(a+\frac{b}{x}\right)^{7/2}}{7 b^6}-\frac{2 \left(a+\frac{b}{x}\right)^{9/2}}{9 b^6}$$

input
```
Int[1/((a + b/x)^(3/2)*x^7),x]
```

output
```
(-2*a^5)/(b^6*Sqrt[a + b/x]) - (10*a^4*Sqrt[a + b/x])/b^6 + (20*a^3*(a + b
/x)^(3/2))/(3*b^6) - (4*a^2*(a + b/x)^(5/2))/b^6 + (10*a*(a + b/x)^(7/2))/
(7*b^6) - (2*(a + b/x)^(9/2))/(9*b^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

method	result
orering	$-\frac{2\left(256 a^{5} x^{5}+128 a^{4} b\,x^{4}-32 a^{3} b^{2} x^{3}+16 a^{2} b^{3} x^{2}-10 b^{4} x a+7 b^{5}\right) \left(a x+b\right)}{63 b^{6} x^{6} \left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$
gosper	$-\frac{2 \left(a x+b\right) \left(256 a^{5} x^{5}+128 a^{4} b\,x^{4}-32 a^{3} b^{2} x^{3}+16 a^{2} b^{3} x^{2}-10 b^{4} x a+7 b^{5}\right)}{63 x^{6} b^{6} \left(\frac{a x+b}{x}\right)^{\frac{3}{2}}}$
trager	$-\frac{2\left(256 a^{5} x^{5}+128 a^{4} b\,x^{4}-32 a^{3} b^{2} x^{3}+16 a^{2} b^{3} x^{2}-10 b^{4} x a+7 b^{5}\right) \sqrt{-\frac{-a x-b}{x}}}{63 x^{4} b^{6} \left(a x+b\right)}$
risch	$-\frac{2 \left(a x+b\right) \left(193 a^{4} x^{4}-65 a^{3} b\,x^{3}+33 a^{2} b^{2} x^{2}-17 a\,b^{3} x+7 b^{4}\right)}{63 b^{6} x^{5} \sqrt{\frac{a x+b}{x}}}-\frac{2 a^{5}}{b^{6} \sqrt{\frac{a x+b}{x}}}$
default	$-\frac{2 \sqrt{\frac{a x+b}{x}}\, \left(-126 \sqrt{a\,x^{2}+b x}\, a^{\frac{15}{2}} x^{8}+63 \ln \left(\frac{2 \sqrt{x \left(a x+b\right)}\, \sqrt{a}+2 a x+b}{2 \sqrt{a}}\right) a^{7} b\,x^{8}-126 a^{\frac{15}{2}} \sqrt{x \left(a x+b\right)}\, x^{8}-63 \ln \left(\frac{2 \sqrt{a\,x^{2}+b x}\, \sqrt{a}+2 a x+b}{2 \sqrt{a}}\right) a\right.}{}$

input
```
int(1/(a+b/x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/63*(256*a^5*x^5+128*a^4*b*x^4-32*a^3*b^2*x^3+16*a^2*b^3*x^2-10*a*b^4*x+
7*b^5)/b^6/x^6*(a*x+b)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7} dx = -\frac{2 \left(256 a^5 x^5+128 a^4 b x^4-32 a^3 b^2 x^3+16 a^2 b^3 x^2-10 a b^4 x+7 b^5\right) \sqrt{\frac{a x+b}{x}}}{63 \left(a b^6 x^5+b^7 x^4\right)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-2/63*(256*a^5*x^5 + 128*a^4*b*x^4 - 32*a^3*b^2*x^3 + 16*a^2*b^3*x^2 - 10*
a*b^4*x + 7*b^5)*sqrt((a*x + b)/x)/(a*b^6*x^5 + b^7*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9534 vs. 2(100) = 200.

Time = 5.87 (sec) , antiderivative size = 9534, normalized size of antiderivative = 82.19

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7} dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**7,x)
```

output
```
-512*a**(47/2)*b**(91/2)*x**19*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/
2) + 945*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 2866
5*a**(33/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**
(29/2)*b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/
2)*b**58*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b
**60*x**(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*
x**(17/2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/
2) + 945*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 7424*a*
*(45/2)*b**(93/2)*x**18*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 94
5*a**(37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(3
3/2)*b**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*
b**56*x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**5
8*x**(25/2) + 405405*a**(23/2)*b**59*x**(23/2) + 315315*a**(21/2)*b**60*x*
*(21/2) + 189189*a**(19/2)*b**61*x**(19/2) + 85995*a**(17/2)*b**62*x**(17/
2) + 28665*a**(15/2)*b**63*x**(15/2) + 6615*a**(13/2)*b**64*x**(13/2) + 94
5*a**(11/2)*b**65*x**(11/2) + 63*a**(9/2)*b**66*x**(9/2)) - 50112*a**(43/2
)*b**(95/2)*x**17*sqrt(a*x/b + 1)/(63*a**(39/2)*b**51*x**(39/2) + 945*a**(
37/2)*b**52*x**(37/2) + 6615*a**(35/2)*b**53*x**(35/2) + 28665*a**(33/2)*b
**54*x**(33/2) + 85995*a**(31/2)*b**55*x**(31/2) + 189189*a**(29/2)*b**56*
x**(29/2) + 315315*a**(27/2)*b**57*x**(27/2) + 405405*a**(25/2)*b**58*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^7}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{9\,b^6}+\frac{10\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{7\,b^6} - \frac{4\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^2}{b^6}+\frac{20\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3}{3\,b^6}-\frac{10\sqrt{a+\frac{b}{x}}a^4}{b^6}-\frac{2\,a^5}{\sqrt{a+\frac{b}{x}}b^6}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x)^(9/2)/b^6 + 10/7*(a + b/x)^(7/2)*a/b^6 - 4*(a + b/x)^(5/2)*
a^2/b^6 + 20/3*(a + b/x)^(3/2)*a^3/b^6 - 10*sqrt(a + b/x)*a^4/b^6 - 2*a^5/
(sqrt(a + b/x)*b^6)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7}\,dx=\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}} x^7}\,dx$$

input
```
integrate(1/(a+b/x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x)^(3/2)*x^7), x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7}\,dx=\frac{34\,a\,\sqrt{a+\frac{b}{x}}}{63\,b^3\,x^3}-\frac{2\,\sqrt{a+\frac{b}{x}}}{9\,b^2\,x^4}-\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{386\,a^4}{63\,b^5}+\frac{512\,a^5\,x}{63\,b^6}\right)}{b+a\,x}-\frac{22\,a^2\,\sqrt{a+\frac{b}{x}}}{21\,b^4\,x^2}+\frac{130\,a^3\,\sqrt{a+\frac{b}{x}}}{63\,b^5\,x}$$

input
```
int(1/(x^7*(a + b/x)^(3/2)),x)
```

output
```
(34*a*(a + b/x)^(1/2))/(63*b^3*x^3) - (2*(a + b/x)^(1/2))/(9*b^2*x^4) - ((
a + b/x)^(1/2)*((386*a^4)/(63*b^5) + (512*a^5*x)/(63*b^6)))/(b + a*x) - (2
2*a^2*(a + b/x)^(1/2))/(21*b^4*x^2) + (130*a^3*(a + b/x)^(1/2))/(63*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^7}\,dx=\frac{\frac{512\sqrt{a}\,\sqrt{ax+b}\,a^4x^5}{63}-\frac{512\sqrt{x}\,a^5x^5}{63}-\frac{256\sqrt{x}\,a^4b\,x^4}{63}+\frac{64\sqrt{x}\,a^3b^2x^3}{63}-\frac{32\sqrt{x}\,a^2b^3x^2}{63}+\frac{20\sqrt{x}\,a\,b^4x}{63}-\frac{2\sqrt{x}}{9}}{\sqrt{ax+b}\,b^6x^5}$$

input
```
int(1/(a+b/x)^(3/2)/x^7,x)
```

output
```
(2*(256*sqrt(a)*sqrt(a*x + b)*a**4*x**5 - 256*sqrt(x)*a**5*x**5 - 128*sqrt
(x)*a**4*b*x**4 + 32*sqrt(x)*a**3*b**2*x**3 - 16*sqrt(x)*a**2*b**3*x**2 +
10*sqrt(x)*a*b**4*x - 7*sqrt(x)*b**5))/(63*sqrt(a*x + b)*b**6*x**5)
```

3.194 $\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 138

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{35b^3}{8a^4\left(a+\frac{b}{x}\right)^{3/2}}+\frac{105b^3}{8a^5\sqrt{a+\frac{b}{x}}}+\frac{21b^2x}{8a^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{3bx^2}{4a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{105b^3\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{11/2}}$$

output

```
35/8*b^3/a^4/(a+b/x)^(3/2)+105/8*b^3/a^5/(a+b/x)^(1/2)+21/8*b^2*x/a^3/(a+b
/x)^(3/2)-3/4*b*x^2/a^2/(a+b/x)^(3/2)+1/3*x^3/a/(a+b/x)^(3/2)-105/8*b^3*ar
ctanh((a+b/x)^(1/2)/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x(315b^4+420ab^3x+63a^2b^2x^2-18a^3bx^3+8a^4x^4)}{24a^5(b+ax)^2} - \frac{105b^3\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8a^{11/2}}$$

input
```
Integrate[x^2/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(315*b^4 + 420*a*b^3*x + 63*a^2*b^2*x^2 - 18*a^3*b*x^3 +
8*a^4*x^4))/(24*a^5*(b + a*x)^2) - (105*b^3*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/(8*a^(11/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {798, 52, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x^4}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{3b\int \frac{x^3}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow 52$$

$$\frac{3b\left(-\frac{7b\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{4a}-\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow 52$$

$$\frac{3b\left(-\frac{7b\left(-\frac{5b\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}-\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow 61$$

$$\frac{3b\left(-\frac{7b\left(-\frac{5b\left(\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}-\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow 61$$

$$\frac{3b\left(-\frac{7b\left(-\frac{5b\left(\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}-\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$\Big\downarrow$ 73

$$\frac{3b\left(-\frac{7b\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}-\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{3b\left(-\frac{7b\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}-\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x^3}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[x^2/(a + b/x)^(5/2),x]
```

output
```
x^3/(3*a*(a + b/x)^(3/2)) + (3*b*(-1/2*x^2/(a*(a + b/x)^(3/2)) - (7*b*(-(x
/(a*(a + b/x)^(3/2))) - (5*b*(2/(3*a*(a + b/x)^(3/2)) + (2/(a*Sqrt[a + b/x
]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/a))/(2*a)))/(4*a)))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

method	result
risch	$\frac{\left(8 a^{2} x^{2}-34 a b x+123 b^{2}\right)\left(a x+b\right)}{24 a^{5} \sqrt{\frac{a x+b}{x}}}+\frac{\left(-\frac{105 b^{3} \ln\left(\frac{\frac{b}{2}+a x}{\sqrt{a}}+\sqrt{a\,x^{2}+b x}\right)}{16 a^{\frac{11}{2}}}+\frac{26 b^{3} \sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3 a^{6}\left(x+\frac{b}{a}\right)}-\frac{2 b^{4} \sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3 a^{7}\left(x+\frac{b}{a}\right)^{2}}\right) \sqrt{x(ax-}{x \sqrt{\frac{a x+b}{x}}}$
default	$-\frac{\sqrt{\frac{a x+b}{x}}\, x \left(-16\left(a\,x^{2}+b x\right)^{\frac{3}{2}} a^{\frac{11}{2}} x^{3}+84\sqrt{a\,x^{2}+b x}\, a^{\frac{11}{2}} b\,x^{4}-672\sqrt{x(ax+b)}\, a^{\frac{9}{2}} b^{2} x^{3}-48\left(a\,x^{2}+b x\right)^{\frac{3}{2}} a^{\frac{9}{2}} b\,x^{2}+294\sqrt{a\,x^{2}+b x}\, a^{\frac{9}{2}} b^{2} x}{}$

input
```
int(x^2/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*a^2*x^2-34*a*b*x+123*b^2)*(a*x+b)/a^5/((a*x+b)/x)^(1/2)+(-105/16/a
^(11/2)*b^3*ln((1/2*b+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))+26/3/a^6*b^3/(x+b/a)
*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-2/3/a^7*b^4/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a
))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.01

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[\frac{315\,(a^2b^3x^2+2\,ab^4x+b^5)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(8\,a^5x^5-18\,a^4bx^4+}{48\,(a^8x^2+2\,a^7bx+a^6b^2)}\right.$$

input
```
integrate(x^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(315*(a^2*b^3*x^2 + 2*a*b^4*x + b^5)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(8*a^5*x^5 - 18*a^4*b*x^4 + 63*a^3*b^2*x^3 + 4
20*a^2*b^3*x^2 + 315*a*b^4*x)*sqrt((a*x + b)/x))/(a^8*x^2 + 2*a^7*b*x + a^
6*b^2), 1/24*(315*(a^2*b^3*x^2 + 2*a*b^4*x + b^5)*sqrt(-a)*arctan(sqrt(-a)
*x*sqrt((a*x + b)/x)/(a*x + b)) + (8*a^5*x^5 - 18*a^4*b*x^4 + 63*a^3*b^2*x
^3 + 420*a^2*b^3*x^2 + 315*a*b^4*x)*sqrt((a*x + b)/x))/(a^8*x^2 + 2*a^7*b*
x + a^6*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(119) = 238$.

Time = 33.42 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.86

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{8a^{\frac{133}{2}}b^{128}x^{72}}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}} - \frac{18a^{\frac{131}{2}}b^{129}x^{71}}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}} + \frac{63a^{\frac{129}{2}}b^{130}x^{70}}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}} + \frac{420a^{\frac{127}{2}}b^{131}x^{69}}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}} + \frac{315a^{\frac{125}{2}}b^{132}x^{68}}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}} - \frac{315a^{63}b^{\frac{263}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}} - \frac{315a^{62}b^{\frac{265}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{24a^{\frac{137}{2}}b^{\frac{257}{2}}x^{\frac{137}{2}}\sqrt{\frac{ax}{b}+1}+24a^{\frac{135}{2}}b^{\frac{259}{2}}x^{\frac{135}{2}}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate(x**2/(a+b/x)**(5/2),x)
```

output
```
8*a**(133/2)*b**128*x**72/(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b
+ 1) + 24*a**(135/2)*b**(259/2)*x**(135/2)*sqrt(a*x/b + 1)) - 18*a**(131/2
)*b**129*x**71/(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a
**(135/2)*b**(259/2)*x**(135/2)*sqrt(a*x/b + 1)) + 63*a**(129/2)*b**130*x*
*70/(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b
**(259/2)*x**(135/2)*sqrt(a*x/b + 1)) + 420*a**(127/2)*b**131*x**69/(24*a*
*(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b**(259/2)*
x**(135/2)*sqrt(a*x/b + 1)) + 315*a**(125/2)*b**132*x**68/(24*a**(137/2)*b
**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b**(259/2)*x**(135/2)
*sqrt(a*x/b + 1)) - 315*a**63*b**(263/2)*x**(137/2)*sqrt(a*x/b + 1)*asinh(
sqrt(a)*sqrt(x)/sqrt(b))/(24*a**(137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b +
 1) + 24*a**(135/2)*b**(259/2)*x**(135/2)*sqrt(a*x/b + 1)) - 315*a**62*b**
(265/2)*x**(135/2)*sqrt(a*x/b + 1)*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(24*a**(
137/2)*b**(257/2)*x**(137/2)*sqrt(a*x/b + 1) + 24*a**(135/2)*b**(259/2)*x*
*(135/2)*sqrt(a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{315\left(a+\frac{b}{x}\right)^4 b^3 - 840\left(a+\frac{b}{x}\right)^3 ab^3 + 693\left(a+\frac{b}{x}\right)^2 a^2b^3 - 144\left(a+\frac{b}{x}\right)a^3b^3 - 16\,a^4b^3}{24\left(\left(a+\frac{b}{x}\right)^{\frac{9}{2}}a^5 - 3\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^6 + 3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^7 - \left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^8\right)} + \frac{105\,b^3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{16\,a^{\frac{11}{2}}}$$

input
```
integrate(x^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/24*(315*(a + b/x)^4*b^3 - 840*(a + b/x)^3*a*b^3 + 693*(a + b/x)^2*a^2*b^
3 - 144*(a + b/x)*a^3*b^3 - 16*a^4*b^3)/((a + b/x)^(9/2)*a^5 - 3*(a + b/x)
^(7/2)*a^6 + 3*(a + b/x)^(5/2)*a^7 - (a + b/x)^(3/2)*a^8) + 105/16*b^3*log
((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.51

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{1}{24}\sqrt{ax^2+bx}\left(2\,x\left(\frac{4\,x}{a^3\mathrm{sgn}\left(x\right)}-\frac{17\,b}{a^4\mathrm{sgn}\left(x\right)}\right)+\frac{123\,b^2}{a^5\mathrm{sgn}\left(x\right)}\right) + \frac{105\,b^3\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{16\,a^{\frac{11}{2}}\mathrm{sgn}\left(x\right)} - \frac{(315\,b^3\log\left(|b|\right)+416\,b^3)\mathrm{sgn}(x)}{48\,a^{\frac{11}{2}}} + \frac{2\left(15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^4+27\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^5+13\,b^6\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{11}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*x^2 + b*x)*(2*x*(4*x/(a^3*sgn(x)) - 17*b/(a^4*sgn(x))) + 123*b
^2/(a^5*sgn(x))) + 105/16*b^3*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sq
rt(a) + b))/(a^(11/2)*sgn(x)) - 1/48*(315*b^3*log(abs(b)) + 416*b^3)*sgn(x
)/a^(11/2) + 2/3*(15*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^4 + 27*(sqrt(a)
*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^5 + 13*b^6)/(((sqrt(a)*x - sqrt(a*x^2 +
b*x))*sqrt(a) + b)^3*a^(11/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{35\,b^3}{2\,a^4\left(a+\frac{b}{x}\right)^{3/2}} - \frac{105\,b^3\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{8\,a^{11/2}} + \frac{x^3}{3\,a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{3\,b\,x^2}{4\,a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{21\,b^2\,x}{8\,a^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{105\,b^4}{8\,a^5\,x\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(x^2/(a + b/x)^(5/2),x)
```

output
```
(35*b^3)/(2*a^4*(a + b/x)^(3/2)) - (105*b^3*atanh((a + b/x)^(1/2)/a^(1/2))
)/(8*a^(11/2)) + x^3/(3*a*(a + b/x)^(3/2)) - (3*b*x^2)/(4*a^2*(a + b/x)^(3
/2)) + (21*b^2*x)/(8*a^3*(a + b/x)^(3/2)) + (105*b^4)/(8*a^5*x*(a + b/x)^(
3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{-2520\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a\,b^3x-2520\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^4-}{}$$

input
```
int(x^2/(a+b/x)^(5/2),x)
```

output
```
( - 2520*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(
b))*a*b**3*x - 2520*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqr
t(a))/sqrt(b))*b**4 - 567*sqrt(a)*sqrt(a*x + b)*a*b**3*x - 567*sqrt(a)*sqr
t(a*x + b)*b**4 + 64*sqrt(x)*a**5*x**4 - 144*sqrt(x)*a**4*b*x**3 + 504*sqr
t(x)*a**3*b**2*x**2 + 3360*sqrt(x)*a**2*b**3*x + 2520*sqrt(x)*a*b**4)/(192
*sqrt(a*x + b)*a**6*(a*x + b))
```

3.195 $\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 114

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{35b^2}{12a^3\left(a+\frac{b}{x}\right)^{3/2}} - \frac{35b^2}{4a^4\sqrt{a+\frac{b}{x}}} - \frac{7bx}{4a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{35b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{9/2}}$$

output
```
-35/12*b^2/a^3/(a+b/x)^(3/2)-35/4*b^2/a^4/(a+b/x)^(1/2)-7/4*b*x/a^2/(a+b/x
)^(3/2)+1/2*x^2/a/(a+b/x)^(3/2)+35/4*b^2*arctanh((a+b/x)^(1/2)/a^(1/2))/a^
(9/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}x(-105b^3-140ab^2x-21a^2bx^2+6a^3x^3)}{12a^4(b+ax)^2} + \frac{35b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4a^{9/2}}$$

input
```
Integrate[x/(a + b/x)^(5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(-105*b^3 - 140*a*b^2*x - 21*a^2*b*x^2 + 6*a^3*x^3))/(12*
a^4*(b + a*x)^2) + (35*b^2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*a^(9/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {798, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x^3}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{7b\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{4a}+\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{52}$$

$$\frac{7b\left(-\frac{5b\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}+\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{7b\left(-\frac{5b\left(\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}+\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{7b\left(-\frac{5b\left(\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}+\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{7b\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}+\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{7b\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}-\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{4a}+\frac{x^2}{2a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[x/(a + b/x)^(5/2),x]
```

output
```
x^2/(2*a*(a + b/x)^(3/2)) + (7*b*(-(x/(a*(a + b/x)^(3/2))) - (5*b*(2/(3*a*
(a + b/x)^(3/2)) + (2/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
])/a^(3/2))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

method	result
risch	$\frac{(2ax-11b)(ax+b)}{4a^4\sqrt{\frac{ax+b}{x}}}+\frac{\left(\frac{35b^2\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx}\right)}{8a^{\frac{9}{2}}}-\frac{20b^2\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3a^5\left(x+\frac{b}{a}\right)}+\frac{2b^3\sqrt{a\left(x+\frac{b}{a}\right)^2-b\left(x+\frac{b}{a}\right)}}{3a^6\left(x+\frac{b}{a}\right)^2}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$\frac{\sqrt{\frac{ax+b}{x}}\,x\left(12\sqrt{a\,x^2+bx}\,a^{\frac{11}{2}}x^4-216\sqrt{x(ax+b)}\,a^{\frac{9}{2}}b\,x^3+42\sqrt{a\,x^2+bx}\,a^{\frac{9}{2}}b\,x^3+108a^4\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)b^2x^3+144(x(ax+b\right.}{}$

input
```
int(x/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*a*x-11*b)*(a*x+b)/a^4/((a*x+b)/x)^(1/2)+(35/8/a^(9/2)*b^2*ln((1/2*b
+a*x)/a^(1/2)+(a*x^2+b*x)^(1/2))-20/3/a^5*b^2/(x+b/a)*(a*(x+b/a)^2-b*(x+b/
a))^(1/2)+2/3/a^6*b^3/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/
x)^(1/2)*(x*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.25

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[\frac{105\,(a^2b^2x^2+2\,ab^3x+b^4)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(6\,a^4x^4-21\,a^3bx^3-\ldots}{24\,(a^7x^2+2\,a^6bx+a^5b^2)}\right.$$

$$-\frac{105\,(a^2b^2x^2+2\,ab^3x+b^4)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-(6\,a^4x^4-21\,a^3bx^3-140\,a^2b^2x^2-105\,ab^3x)\sqrt{\frac{ax\ldots}{}}}{12\,(a^7x^2+2\,a^6bx+a^5b^2)}$$

input
```
integrate(x/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(105*(a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(6*a^4*x^4 - 21*a^3*b*x^3 - 140*a^2*b^2*x^2 -
105*a*b^3*x)*sqrt((a*x + b)/x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2), -1/12*(10
5*(a^2*b^2*x^2 + 2*a*b^3*x + b^4)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b
)/x)/(a*x + b)) - (6*a^4*x^4 - 21*a^3*b*x^3 - 140*a^2*b^2*x^2 - 105*a*b^3*
x)*sqrt((a*x + b)/x))/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(97) = 194.

Time = 7.53 (sec) , antiderivative size = 464, normalized size of antiderivative = 4.07

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{6a^{\frac{89}{2}}b^{75}x^{49}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}} - \frac{21a^{\frac{87}{2}}b^{76}x^{48}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}} - \frac{140a^{\frac{85}{2}}b^{77}x^{47}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}} - \frac{105a^{\frac{83}{2}}b^{78}x^{46}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}} + \frac{105a^{42}b^{\frac{155}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}} + \frac{105a^{41}b^{\frac{157}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{93}{2}}\sqrt{\frac{ax}{b}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{91}{2}}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate(x/(a+b/x)**(5/2),x)
```

output
```
6*a**(89/2)*b**75*x**49/(12*a**(93/2)*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1)
 + 12*a**(91/2)*b**(153/2)*x**(91/2)*sqrt(a*x/b + 1)) - 21*a**(87/2)*b**76
*x**48/(12*a**(93/2)*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b
**(153/2)*x**(91/2)*sqrt(a*x/b + 1)) - 140*a**(85/2)*b**77*x**47/(12*a**(9
3/2)*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b**(153/2)*x**(91
/2)*sqrt(a*x/b + 1)) - 105*a**(83/2)*b**78*x**46/(12*a**(93/2)*b**(151/2)*
x**(93/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b**(153/2)*x**(91/2)*sqrt(a*x/b +
 1)) + 105*a**42*b**(155/2)*x**(93/2)*sqrt(a*x/b + 1)*asinh(sqrt(a)*sqrt(x
)/sqrt(b))/(12*a**(93/2)*b**(151/2)*x**(93/2)*sqrt(a*x/b + 1) + 12*a**(91/
2)*b**(153/2)*x**(91/2)*sqrt(a*x/b + 1)) + 105*a**41*b**(157/2)*x**(91/2)*
sqrt(a*x/b + 1)*asinh(sqrt(a)*sqrt(x)/sqrt(b))/(12*a**(93/2)*b**(151/2)*x*
*(93/2)*sqrt(a*x/b + 1) + 12*a**(91/2)*b**(153/2)*x**(91/2)*sqrt(a*x/b + 1
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.22

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{105\left(a+\frac{b}{x}\right)^{3}b^{2}-175\left(a+\frac{b}{x}\right)^{2}ab^{2}+56\left(a+\frac{b}{x}\right)a^{2}b^{2}+8\,a^{3}b^{2}}{12\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^{4}-2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^{5}+\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{6}\right)} - \frac{35\,b^{2}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{8\,a^{\frac{9}{2}}}$$

input
```
integrate(x/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(105*(a + b/x)^3*b^2 - 175*(a + b/x)^2*a*b^2 + 56*(a + b/x)*a^2*b^2
+ 8*a^3*b^2)/((a + b/x)^(7/2)*a^4 - 2*(a + b/x)^(5/2)*a^5 + (a + b/x)^(3/2
)*a^6) - 35/8*b^2*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))
/a^(9/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(90) = 180$.

Time = 0.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.68

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{1}{4}\sqrt{ax^{2}+bx}\left(\frac{2\,x}{a^{3}\mathrm{sgn}\left(x\right)}-\frac{11\,b}{a^{4}\mathrm{sgn}\left(x\right)}\right) - \frac{35\,b^{2}\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right|\right)}{8\,a^{\frac{9}{2}}\mathrm{sgn}\left(x\right)} + \frac{5\left(21\,b^{2}\log\left(\left|b\right|\right)+32\,b^{2}\right)\mathrm{sgn}(x)}{24\,a^{\frac{9}{2}}} - \frac{2\left(12\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)^{2}ab^{3}+21\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}b^{4}+10\,b^{5}\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^{2}+bx}\right)\sqrt{a}+b\right)^{3}a^{\frac{9}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*x^2 + b*x)*(2*x/(a^3*sgn(x)) - 11*b/(a^4*sgn(x))) - 35/8*b^2*lo
g(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(9/2)*sgn(x)) + 5
/24*(21*b^2*log(abs(b)) + 32*b^2)*sgn(x)/a^(9/2) - 2/3*(12*(sqrt(a)*x - sq
rt(a*x^2 + b*x))^2*a*b^3 + 21*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4
+ 10*b^5)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(9/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4\,a^{9/2}}-\frac{35\,b^2}{3\,a^3\,\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x^2}{2\,a\,\left(a+\frac{b}{x}\right)^{3/2}}-\frac{35\,b^3}{4\,a^4\,x\,\left(a+\frac{b}{x}\right)^{3/2}}-\frac{7\,b\,x}{4\,a^2\,\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(x/(a + b/x)^(5/2),x)
```

output
```
(35*b^2*atanh((a + b/x)^(1/2)/a^(1/2)))/(4*a^(9/2)) - (35*b^2)/(3*a^3*(a +
 b/x)^(3/2)) + x^2/(2*a*(a + b/x)^(3/2)) - (35*b^3)/(4*a^4*x*(a + b/x)^(3/
2)) - (7*b*x)/(4*a^2*(a + b/x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.34

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{840\sqrt{a}\,\sqrt{ax+b}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)a\,b^2x+840\sqrt{a}\,\sqrt{ax+b}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^3+175\sqrt{}}{96\sqrt{a}}$$

input
```
int(x/(a+b/x)^(5/2),x)
```

output
```
(840*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))*
a*b**2*x + 840*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))
/sqrt(b))*b**3 + 175*sqrt(a)*sqrt(a*x + b)*a*b**2*x + 175*sqrt(a)*sqrt(a*x
 + b)*b**3 + 48*sqrt(x)*a**4*x**3 - 168*sqrt(x)*a**3*b*x**2 - 1120*sqrt(x)
*a**2*b**2*x - 840*sqrt(x)*a*b**3)/(96*sqrt(a*x + b)*a**5*(a*x + b))
```

3.196 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
5/3*b/a^2/(a+b/x)^(3/2)+5*b/a^3/(a+b/x)^(1/2)+x/a/(a+b/x)^(3/2)-5*b*arctan
h((a+b/x)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}x(15b^2+20abx+3a^2x^2)}{3a^3(b+ax)^2}-\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(a + b/x)^(-5/2),x]
```

output
```
(Sqrt[a + b/x]*x*(15*b^2 + 20*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x)^2) - (5
*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {773, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{52}$$

$$\frac{5b\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$\frac{5b\left(\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

$\Big\downarrow$ 221

$$\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}\right)}{2a}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[(a + b/x)^(-5/2),x]
```

output
```
x/(a*(a + b/x)^(3/2)) + (5*b*(2/(3*a*(a + b/x)^(3/2)) + (2/(a*Sqrt[a + b/x
]) - (2*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2))/a))/(2*a)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(65) = 130$.

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.99

method	result
risch	$\frac{ax+b}{a^{3}\sqrt{\frac{ax+b}{x}}}+\frac{\left(-\frac{5b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx}\right)}{2a^{\frac{7}{2}}}+\frac{14b\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3a^{4}\left(x+\frac{b}{a}\right)}-\frac{2b^{2}\sqrt{a\left(x+\frac{b}{a}\right)^{2}-b\left(x+\frac{b}{a}\right)}}{3a^{5}\left(x+\frac{b}{a}\right)^{2}}\right)\sqrt{x(ax+b)}}{x\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,x\left(-30\sqrt{x(ax+b)}\,a^{\frac{7}{2}}x^{3}+15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{3}b\,x^{3}+24(x(ax+b))^{\frac{3}{2}}a^{\frac{5}{2}}x-90\sqrt{x(ax+b)}\,a^{\frac{5}{2}}b\,x^{2}+45\ln\left(\frac{2\sqrt{x(\cdots}\right.\right.}{}$

input
```
int(1/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(a*x+b)/((a*x+b)/x)^(1/2)+(-5/2/a^(7/2)*b*ln((1/2*b+a*x)/a^(1/2)+(a*
x^2+b*x)^(1/2))+14/3/a^4*b/(x+b/a)*(a*(x+b/a)^2-b*(x+b/a))^(1/2)-2/3/a^5*b
^2/(x+b/a)^2*(a*(x+b/a)^2-b*(x+b/a))^(1/2))/x/((a*x+b)/x)^(1/2)*(x*(a*x+b)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \left[\frac{15\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^3x^3+20\,a^2bx^2+1\right.}{6\left(a^6x^2+2\,a^5bx+a^4b^2\right)}\right.$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x
+ b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x +
 b^3)*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (3*a^3*x^3
 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^
4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(66) = 132$.

Time = 3.38 (sec) , antiderivative size = 774, normalized size of antiderivative = 9.80

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(5/2),x)
```

output
```
6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 1
8*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))
/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b
*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt
(1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*
b**2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3)
+ 45*a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2
+ 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1
+ b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x
**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a
**14*b**3*x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) +
 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(
33/2)*b**3) + 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*
b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(
1 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b
**2*x + 6*a**(33/2)*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{15\left(a+\frac{b}{x}\right)^2 b - 10\left(a+\frac{b}{x}\right)ab - 2\,a^2 b}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3 - \left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4\right)} + \frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
 (a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x)
 + sqrt(a)))/a^(7/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(65) = 130$.

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{(15\,b\log\left(|b|\right)+28\,b)\mathrm{sgn}(x)}{6\,a^{\frac{7}{2}}} + \frac{5\,b\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)} + \frac{\sqrt{ax^2+bx}}{a^3\mathrm{sgn}\left(x\right)} + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab^2+15\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^3+7\,b^4\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(15*b*log(abs(b)) + 28*b)*sgn(x)/a^(7/2) + 5/2*b*log(abs(2*(sqrt(a)*x
 - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(a^(7/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(
a^3*sgn(x)) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2 + 15*(sqrt(a)
*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3 + 7*b^4)/(((sqrt(a)*x - sqrt(a*x^2 + b
*x))*sqrt(a) + b)^3*a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\,x\,\left(\frac{a\,x}{b}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,\left(a+\frac{b}{x}\right)^{5/2}}$$

input
```
int(1/(a + b/x)^(5/2),x)
```

output
```
(2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x
)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{-30\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)abx - 30\sqrt{a}\,\sqrt{ax+b}\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right)b^2 - 5\sqrt{a}\,\sqrt{}}{6\sqrt{ax+b}\,a^4\,(ax+b}$$

input
```
int(1/(a+b/x)^(5/2),x)
```

output
```
( - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b)
)*a*b*x - 30*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/s
qrt(b))*b**2 - 5*sqrt(a)*sqrt(a*x + b)*a*b*x - 5*sqrt(a)*sqrt(a*x + b)*b**
2 + 6*sqrt(x)*a**3*x**2 + 40*sqrt(x)*a**2*b*x + 30*sqrt(x)*a*b**2)/(6*sqrt
(a*x + b)*a**4*(a*x + b))
```

3.197 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x} \, dx = -\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2}{a^2\sqrt{a+\frac{b}{x}}} + \frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-2/3/a/(a+b/x)^(3/2)-2/a^2/(a+b/x)^(1/2)+2*arctanh((a+b/x)^(1/2)/a^(1/2))/
a^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x} \, dx = -\frac{2\sqrt{a+\frac{b}{x}}x(3b+4ax)}{3a^2(b+ax)^2} + \frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x),x]
```

output
```
(-2*Sqrt[a + b/x]*x*(3*b + 4*a*x))/(3*a^2*(b + a*x)^2) + (2*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{x}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\int \frac{x}{\left(a+\frac{b}{x}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{61}$$

$$-\frac{\frac{\int \frac{x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}-\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{2\int \frac{1}{\frac{1}{bx^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{x}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x}}}}{a}-\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{2}{a\sqrt{a+\frac{b}{x}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}-\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[1/((a + b/x)^(5/2)*x),x]
```

output
```
-2/(3*a*(a + b/x)^(3/2)) - (2/(a*Sqrt[a + b/x]) - (2*ArcTanh[Sqrt[a + b/x]
/Sqrt[a]])/a^(3/2))/a
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(48) = 96$.

Time = 0.24 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.57

method	result
default	$\frac{\sqrt{\frac{ax+b}{x}}\, x \left(-6\sqrt{x(ax+b)}\, a^{\frac{7}{2}} x^{3}+3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right) a^{3} b\, x^{3}+6(x(ax+b))^{\frac{3}{2}} a^{\frac{5}{2}} x-18\sqrt{x(ax+b)}\, a^{\frac{5}{2}} b\, x^{2}+9\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{}}{2\sqrt{a}}\right.}{3a}$

input
```
int(1/(a+b/x)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*x+b)/x)^(1/2)*x/a^(5/2)*(-6*(x*(a*x+b))^(1/2)*a^(7/2)*x^3+3*ln(1/2
*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b*x^3+6*(x*(a*x+b))^(3
/2)*a^(5/2)*x-18*(x*(a*x+b))^(1/2)*a^(5/2)*b*x^2+9*ln(1/2*(2*(x*(a*x+b))^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^2*x^2+4*b*a^(3/2)*(x*(a*x+b))^(3/2)-1
8*(x*(a*x+b))^(1/2)*a^(3/2)*b^2*x+9*ln(1/2*(2*(x*(a*x+b))^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*a*b^3*x-6*(x*(a*x+b))^(1/2)*a^(1/2)*b^3+3*ln(1/2*(2*(x*(a*
x+b))^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^4)/(x*(a*x+b))^(1/2)/b/(a*x+b)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(48) = 96$.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.37

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x}\, dx = \left[\frac{3\left(a^{2}x^{2}+2\,abx+b^{2}\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(4\,a^{2}x^{2}+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^{5}x^{2}+2\,a^{4}bx+a^{3}b^{2}\right)},\right.$$
$$\left.-\frac{2\left(3\left(a^{2}x^{2}+2\,abx+b^{2}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+\left(4\,a^{2}x^{2}+3\,abx\right)\sqrt{\frac{ax+b}{x}}\right)}{3\left(a^{5}x^{2}+2\,a^{4}bx+a^{3}b^{2}\right)}\right]$$

input
```
integrate(1/(a+b/x)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*
x + b)/x) + b) - 2*(4*a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^5*x^2 + 2*a
^4*b*x + a^3*b^2), -2/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-a)*arctan(sqrt(
-a)*x*sqrt((a*x + b)/x)/(a*x + b)) + (4*a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/
x))/(a^5*x^2 + 2*a^4*b*x + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(48) = 96$.

Time = 1.93 (sec) , antiderivative size = 700, normalized size of antiderivative = 11.67

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x}\, dx = & -\frac{8a^7x^3\sqrt{1+\frac{b}{ax}}}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& - \frac{3a^7x^3\log\left(\frac{b}{ax}\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& + \frac{6a^7x^3\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& - \frac{14a^6bx^2\sqrt{1+\frac{b}{ax}}}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& - \frac{9a^6bx^2\log\left(\frac{b}{ax}\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& + \frac{18a^6bx^2\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& - \frac{6a^5b^2x\sqrt{1+\frac{b}{ax}}}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& - \frac{9a^5b^2x\log\left(\frac{b}{ax}\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& + \frac{18a^5b^2x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& - \frac{3a^4b^3\log\left(\frac{b}{ax}\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3} \\
& + \frac{6a^4b^3\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{3a^{\frac{19}{2}}x^3+9a^{\frac{17}{2}}bx^2+9a^{\frac{15}{2}}b^2x+3a^{\frac{13}{2}}b^3}
\end{aligned}
$$

input
```
integrate(1/(a+b/x)**(5/2)/x,x)
```

output
```
-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*
a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*log(b/(a*x))/(3*a**(19/
2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 6*
a**7*x**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**
2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqrt(1 + b/(a*
x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13
/2)*b**3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x
**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*log(sqrt(1 +
 b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x
 + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 +
 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5*b**2
*x*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*
x + 3*a**(13/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(1
9/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) -
3*a**4*b**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15
/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x)) + 1)/(3
*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b*
*3)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{2\left(4\,a+\frac{3\,b}{x}\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x,x, algorithm="maxima")
```

output
```
-log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2) - 2/3*(4
*a + 3*b/x)/((a + b/x)^(3/2)*a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(48) = 96$.

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x}\,dx = \frac{(3\log(|b|)+8)\mathrm{sgn}(x)}{3\,a^{\frac{5}{2}}} - \frac{\log\left(\left|2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right|\right)}{a^{\frac{5}{2}}\mathrm{sgn}(x)} - \frac{2\left(6\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2ab+9\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b^2+4\,b^3\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3a^{\frac{5}{2}}\mathrm{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x,x, algorithm="giac")
```

output
```
1/3*(3*log(abs(b)) + 8)*sgn(x)/a^(5/2) - log(abs(2*(sqrt(a)*x - sqrt(a*x^2
 + b*x))*sqrt(a) + b))/(a^(5/2)*sgn(x)) - 2/3*(6*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))^2*a*b + 9*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^2 + 4*b^3)/(((s
qrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*a^(5/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2\left(a+\frac{b}{x}\right)}{a^2}+\frac{2}{3\,a}}{\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(1/(x*(a + b/x)^(5/2)),x)
```

output
```
(2*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*(a + b/x))/a^2 + 2/(3*a))
/(a + b/x)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x}\,dx = \frac{2\sqrt{a}\,\sqrt{ax+b}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right) ax + 2\sqrt{a}\,\sqrt{ax+b}\,\log\left(\frac{\sqrt{ax+b}+\sqrt{x}\,\sqrt{a}}{\sqrt{b}}\right) b - \frac{8\sqrt{x}\,a^2 x}{3} - 2}{\sqrt{ax+b}\,a^3\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x,x)
```

output
```
(2*(3*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(b))
*a*x + 3*sqrt(a)*sqrt(a*x + b)*log((sqrt(a*x + b) + sqrt(x)*sqrt(a))/sqrt(
b))*b - 4*sqrt(x)*a**2*x - 3*sqrt(x)*a*b))/(3*sqrt(a*x + b)*a**3*(a*x + b)
)
```

3.198 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2} \, dx = \frac{2}{3b \left(a+\frac{b}{x}\right)^{3/2}}$$

output
```
2/3/b/(a+b/x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2} \, dx = \frac{2}{3b \left(\frac{b+ax}{x}\right)^{3/2}}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^2),x]
```

output
```
2/(3*b*((b + a*x)/x)^(3/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{2}{3b \left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[1/((a + b/x)^(5/2)*x^2),x]
```

output
```
2/(3*b*(a + b/x)^(3/2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2}{3b\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$	15
orering	$\frac{\frac{2ax}{3}+\frac{2b}{3}}{xb\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$	23
gosper	$\frac{\frac{2ax}{3}+\frac{2b}{3}}{xb\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$	25
default	$\frac{2x^{2}\sqrt{\frac{ax+b}{x}}}{3(ax+b)^{2}b}$	27
trager	$\frac{2x^{2}\sqrt{-\frac{-ax-b}{x}}}{3(ax+b)^{2}b}$	31

input
```
int(1/(a+b/x)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/b/(a+b/x)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^2}\,dx = \frac{2\,x^2\sqrt{\frac{ax+b}{x}}}{3\,(a^2bx^2+2\,ab^2x+b^3)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
2/3*x^2*sqrt((a*x + b)/x)/(a^2*b*x^2 + 2*a*b^2*x + b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2}\,dx = \begin{cases} \frac{2}{3ab\sqrt{a+\frac{b}{x}}+\frac{3b^2\sqrt{a+\frac{b}{x}}}{x}} & \text{for}\ b \neq 0 \\ -\frac{1}{a^{\frac{5}{2}}x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**2,x)
```

output
```
Piecewise((2/(3*a*b*sqrt(a + b/x) + 3*b**2*sqrt(a + b/x)/x), Ne(b, 0)), (-
1/(a**(5/2)*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2}\,dx = \frac{2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}} b}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
2/3/((a + b/x)^(3/2)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.56

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2}\,dx = -\frac{2\,\mathrm{sgn}(x)}{3\,a^{\frac{3}{2}}b} + \frac{2\left(3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)^2 a + 3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}b + b^2\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3 a^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
-2/3*sgn(x)/(a^(3/2)*b) + 2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a + 3*(
sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b + b^2)/(((sqrt(a)*x - sqrt(a*x^2
+ b*x))*sqrt(a) + b)^3*a^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^2}\,dx=\frac{2}{3\,b\,\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(1/(x^2*(a + b/x)^(5/2)),x)
```

output
```
2/(3*b*(a + b/x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.89

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^2}\,dx=\frac{\frac{2\sqrt{a}\,\sqrt{ax+b}\,ax}{3}+\frac{2\sqrt{a}\,\sqrt{ax+b}\,b}{3}+\frac{2\sqrt{x}\,a^2x}{3}}{\sqrt{ax+b}\,a^2b\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^2,x)
```

output
```
(2*(sqrt(a)*sqrt(a*x + b)*a*x + sqrt(a)*sqrt(a*x + b)*b + sqrt(x)*a**2*x))
/(3*sqrt(a*x + b)*a**2*b*(a*x + b))
```

3.199 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^3}\,dx=-\frac{2a}{3b^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2}{b^2\sqrt{a+\frac{b}{x}}}$$

output
```
-2/3*a/b^2/(a+b/x)^(3/2)+2/b^2/(a+b/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^3}\,dx=\frac{2x\sqrt{\frac{b+ax}{x}}(3b+2ax)}{3b^2(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^3),x]
```

output
```
(2*x*Sqrt[(b + a*x)/x]*(3*b + 2*a*x))/(3*b^2*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x}\right)^{5/2}}dx$$
$$\Big\downarrow \text{798}$$
$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}d\frac{1}{x}$$
$$\Big\downarrow \text{53}$$
$$-\int \left(\frac{1}{b \left(a+\frac{b}{x}\right)^{3/2}}-\frac{a}{b \left(a+\frac{b}{x}\right)^{5/2}}\right)d\frac{1}{x}$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{b^2 \sqrt{a+\frac{b}{x}}}-\frac{2 a}{3 b^2 \left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[1/((a + b/x)^(5/2)*x^3),x]
```

output
```
(-2*a)/(3*b^2*(a + b/x)^(3/2)) + 2/(b^2*Sqrt[a + b/x])
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result
orering	$\frac{2(2ax+3b)(ax+b)}{3b^2x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$
gosper	$\frac{2(ax+b)(2ax+3b)}{3x^2b^2\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$
trager	$\frac{2x(2ax+3b)\sqrt{-\frac{-ax-b}{x}}}{3b^2(ax+b)^2}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\, x\left(-6\sqrt{x(ax+b)}\, a^{\frac{7}{2}} x^3-3\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b\,x^3-6\sqrt{a\,x^2+bx}\,a^{\frac{7}{2}}x^3+3\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^3b\,x^3+12\right.}{}$

input
```
int(1/(a+b/x)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(2*a*x+3*b)/b^2/x^2*(a*x+b)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\, dx = \frac{2\left(2\,ax^2+3\,bx\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^2b^2x^2+2\,ab^3x+b^4\right)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
2/3*(2*a*x^2 + 3*b*x)*sqrt((a*x + b)/x)/(a^2*b^2*x^2 + 2*a*b^3*x + b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(29) = 58$.

Time = 0.67 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx = \begin{cases} \frac{4ax}{3ab^2x\sqrt{a+\frac{b}{x}}+3b^3\sqrt{a+\frac{b}{x}}} + \frac{6b}{3ab^2x\sqrt{a+\frac{b}{x}}+3b^3\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{2a^{\frac{5}{2}}x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**3,x)
```

output
```
Piecewise((4*a*x/(3*a*b**2*x*sqrt(a + b/x) + 3*b**3*sqrt(a + b/x)) + 6*b/(
3*a*b**2*x*sqrt(a + b/x) + 3*b**3*sqrt(a + b/x)), Ne(b, 0)), (-1/(2*a**(5/
2)*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx = \frac{2}{\sqrt{a+\frac{b}{x}}b^2} - \frac{2\,a}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
2/(sqrt(a + b/x)*b^2) - 2/3*a/((a + b/x)^(3/2)*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.08

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx = -\frac{4\,\mathrm{sgn}(x)}{3\,\sqrt{a}b^2} + \frac{2\left(3\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+2\,b\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}+b\right)^3\sqrt{a}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^3,x, algorithm="giac")
```

output
```
-4/3*sgn(x)/(sqrt(a)*b^2) + 2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)
 + 2*b)/(((sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b)^3*sqrt(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx = \frac{6\,b+4\,a\,x}{3\,b^2\,x\,\left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
int(1/(x^3*(a + b/x)^(5/2)),x)
```

output
```
(6*b + 4*a*x)/(3*b^2*x*(a + b/x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3}\,dx = \frac{-\frac{4\sqrt{a}\,\sqrt{ax+b}\,ax}{3}-\frac{4\sqrt{a}\,\sqrt{ax+b}\,b}{3}+\frac{4\sqrt{x}\,a^2x}{3}+2\sqrt{x}\,ab}{\sqrt{ax+b}\,a\,b^2\left(ax+b\right)}$$

input
```
int(1/(a+b/x)^(5/2)/x^3,x)
```

output
```
(2*( - 2*sqrt(a)*sqrt(a*x + b)*a*x - 2*sqrt(a)*sqrt(a*x + b)*b + 2*sqrt(x)
*a**2*x + 3*sqrt(x)*a*b))/(3*sqrt(a*x + b)*a*b**2*(a*x + b))
```

3.200 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx=\frac{2a^2}{3b^3\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4a}{b^3\sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^3}$$

output
```
2/3*a^2/b^3/(a+b/x)^(3/2)-4*a/b^3/(a+b/x)^(1/2)-2*(a+b/x)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx=-\frac{2\sqrt{\frac{b+ax}{x}}\left(3b^2+12abx+8a^2x^2\right)}{3b^3(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^4),x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(3*b^2 + 12*a*b*x + 8*a^2*x^2))/(3*b^3*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{a^2}{b^2 \left(a+\frac{b}{x}\right)^{5/2}}-\frac{2a}{b^2 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{1}{b^2 \sqrt{a+\frac{b}{x}}}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2}{3b^3 \left(a+\frac{b}{x}\right)^{3/2}}-\frac{4a}{b^3 \sqrt{a+\frac{b}{x}}}-\frac{2\sqrt{a+\frac{b}{x}}}{b^3}$$

input
```
Int[1/((a + b/x)^(5/2)*x^4),x]
```

output
```
(2*a^2)/(3*b^3*(a + b/x)^(3/2)) - (4*a)/(b^3*Sqrt[a + b/x]) - (2*Sqrt[a +
b/x])/b^3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result
orering	$-\frac{2\left(8a^{2}x^{2}+12abx+3b^{2}\right)\left(ax+b\right)}{3b^{3}x^{3}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$
gosper	$-\frac{2\left(ax+b\right)\left(8a^{2}x^{2}+12abx+3b^{2}\right)}{3x^{3}b^{3}\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(8a^{2}x^{2}+12abx+3b^{2}\right)\sqrt{-\frac{-ax-b}{x}}}{3b^{3}\left(ax+b\right)^{2}}$
risch	$-\frac{2\left(ax+b\right)}{b^{3}x\sqrt{\frac{ax+b}{x}}}-\frac{2a\left(5ax+6b\right)}{3\left(ax+b\right)b^{3}\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(6\sqrt{a\,x^{2}+bx}\,a^{\frac{9}{2}}x^{5}+3\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{4}b\,x^{5}+6a^{\frac{9}{2}}\sqrt{x\left(ax+b\right)}\,x^{5}-3\ln\left(\frac{2\sqrt{x\left(ax+b\right)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{4}b\,x^{5}+12(a\right.}{}$

input
```
int(1/(a+b/x)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(8*a^2*x^2+12*a*b*x+3*b^2)/b^3/x^3*(a*x+b)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx = -\frac{2\left(8\,a^2x^2+12\,abx+3\,b^2\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^2b^3x^2+2\,ab^4x+b^5\right)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
-2/3*(8*a^2*x^2 + 12*a*b*x + 3*b^2)*sqrt((a*x + b)/x)/(a^2*b^3*x^2 + 2*a*b
^4*x + b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(46) = 92.

Time = 0.74 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.47

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx = \begin{cases} -\frac{16a^2x^2}{3ab^3x^2\sqrt{a+\frac{b}{x}}+3b^4x\sqrt{a+\frac{b}{x}}} - \frac{24abx}{3ab^3x^2\sqrt{a+\frac{b}{x}}+3b^4x\sqrt{a+\frac{b}{x}}} - \frac{6b^2}{3ab^3x^2\sqrt{a+\frac{b}{x}}+3b^4x\sqrt{a+\frac{b}{x}}} & \text{for } b \neq 0 \\ -\frac{1}{3a^{\frac{5}{2}}x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**4,x)
```

output
```
Piecewise((-16*a**2*x**2/(3*a*b**3*x**2*sqrt(a + b/x) + 3*b**4*x*sqrt(a +
b/x)) - 24*a*b*x/(3*a*b**3*x**2*sqrt(a + b/x) + 3*b**4*x*sqrt(a + b/x)) -
6*b**2/(3*a*b**3*x**2*sqrt(a + b/x) + 3*b**4*x*sqrt(a + b/x)), Ne(b, 0)),
(-1/(3*a**(5/2)*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}}{b^3} - \frac{4\,a}{\sqrt{a+\frac{b}{x}}b^3} + \frac{2\,a^2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^3}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
-2*sqrt(a + b/x)/b^3 - 4*a/(sqrt(a + b/x)*b^3) + 2/3*a^2/((a + b/x)^(3/2)*
b^3)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate(1/(a+b/x)^(5/2)/x^4,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x)^(5/2)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^4}\,dx = -\frac{2\sqrt{a+\frac{b}{x}}\,(8\,a^2\,x^2+12\,a\,b\,x+3\,b^2)}{3\,b^3\,(b+a\,x)^2}$$

input
```
int(1/(x^4*(a + b/x)^(5/2)),x)
```

output
```
-(2*(a + b/x)^(1/2)*(3*b^2 + 8*a^2*x^2 + 12*a*b*x))/(3*b^3*(b + a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^4} \, dx = \frac{\frac{16\sqrt{a}\,\sqrt{ax+b}\,a\,x^2}{3}+\frac{16\sqrt{a}\,\sqrt{ax+b}\,bx}{3}-\frac{16\sqrt{x}\,a^2x^2}{3}-8\sqrt{x}\,abx-2\sqrt{x}\,b^2}{\sqrt{ax+b}\,b^3x\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^4,x)
```

output
```
(2*(8*sqrt(a)*sqrt(a*x + b)*a*x**2 + 8*sqrt(a)*sqrt(a*x + b)*b*x - 8*sqrt(
x)*a**2*x**2 - 12*sqrt(x)*a*b*x - 3*sqrt(x)*b**2))/(3*sqrt(a*x + b)*b**3*x
*(a*x + b))
```

3.201 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5} \, dx = -\frac{2a^3}{3b^4 \left(a+\frac{b}{x}\right)^{3/2}} + \frac{6a^2}{b^4 \sqrt{a+\frac{b}{x}}} + \frac{6a\sqrt{a+\frac{b}{x}}}{b^4} - \frac{2\left(a+\frac{b}{x}\right)^{3/2}}{3b^4}$$

output
```
-2/3*a^3/b^4/(a+b/x)^(3/2)+6*a^2/b^4/(a+b/x)^(1/2)+6*a*(a+b/x)^(1/2)/b^4-2
/3*(a+b/x)^(3/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5} \, dx = \frac{2\sqrt{\frac{b+ax}{x}}(-b^3 + 6ab^2x + 24a^2bx^2 + 16a^3x^3)}{3b^4x(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^5),x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-b^3 + 6*a*b^2*x + 24*a^2*b*x^2 + 16*a^3*x^3))/(3*b^
4*x*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3} d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(-\frac{a^3}{b^3 \left(a+\frac{b}{x}\right)^{5/2}}+\frac{3 a^2}{b^3 \left(a+\frac{b}{x}\right)^{3/2}}-\frac{3 a}{b^3 \sqrt{a+\frac{b}{x}}}+\frac{\sqrt{a+\frac{b}{x}}}{b^3}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2 a^3}{3 b^4 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{6 a^2}{b^4 \sqrt{a+\frac{b}{x}}}+\frac{6 a \sqrt{a+\frac{b}{x}}}{b^4}-\frac{2 \left(a+\frac{b}{x}\right)^{3/2}}{3 b^4}$$

input
```
Int[1/((a + b/x)^(5/2)*x^5),x]
```

output
```
(-2*a^3)/(3*b^4*(a + b/x)^(3/2)) + (6*a^2)/(b^4*Sqrt[a + b/x]) + (6*a*Sqrt
[a + b/x])/b^4 - (2*(a + b/x)^(3/2))/(3*b^4)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result
orering	$\frac{2(16a^3x^3+24a^2bx^2+6ab^2x-b^3)(ax+b)}{3b^4x^4\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$
gosper	$\frac{2(ax+b)(16a^3x^3+24a^2bx^2+6ab^2x-b^3)}{3x^4b^4\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$
trager	$\frac{2(2ax+b)(8a^2x^2+8abx-b^2)\sqrt{-\frac{-ax-b}{x}}}{3x\,b^4(ax+b)^2}$
risch	$\frac{2(ax+b)(8ax-b)}{3b^4x^2\sqrt{\frac{ax+b}{x}}}+\frac{2a^2(8ax+9b)}{3(ax+b)b^4\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\left(9(x(ax+b))^{\frac{3}{2}}a^4x^4-9(ax^2+bx)^{\frac{3}{2}}a^4x^4+10(x(ax+b))^{\frac{3}{2}}a^3bx^3-26(ax^2+bx)^{\frac{3}{2}}a^3bx^3-24(ax^2+bx)^{\frac{3}{2}}a^2b^2x^2-6(ax^2-\right.}{3x^2\sqrt{x(ax+b)}\,b^5(ax+b)^3}$

input
```
int(1/(a+b/x)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/3*(16*a^3*x^3+24*a^2*b*x^2+6*a*b^2*x-b^3)/b^4/x^4*(a*x+b)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\, dx = \frac{2\,(16\,a^3x^3 + 24\,a^2bx^2 + 6\,ab^2x - b^3)\sqrt{\frac{ax+b}{x}}}{3\,(a^2b^4x^3 + 2\,ab^5x^2 + b^6x)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
2/3*(16*a^3*x^3 + 24*a^2*b*x^2 + 6*a*b^2*x - b^3)*sqrt((a*x + b)/x)/(a^2*b
^4*x^3 + 2*a*b^5*x^2 + b^6*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(65) = 130.

Time = 0.81 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\, dx = \begin{cases} \frac{32a^3x^3}{3ab^4x^3\sqrt{a+\frac{b}{x}}+3b^5x^2\sqrt{a+\frac{b}{x}}} + \frac{48a^2bx^2}{3ab^4x^3\sqrt{a+\frac{b}{x}}+3b^5x^2\sqrt{a+\frac{b}{x}}} + \frac{12ab^2x}{3ab^4x^3\sqrt{a+\frac{b}{x}}+3b^5x^2\sqrt{a+\frac{b}{x}}} - \cdots \\ -\frac{1}{4a^{\frac{5}{2}}x^4} \end{cases}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**5,x)
```

output
```
Piecewise(((32*a**3*x**3/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a
+ b/x)) + 48*a**2*b*x**2/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a
 + b/x)) + 12*a*b**2*x/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a +
 b/x)) - 2*b**3/(3*a*b**4*x**3*sqrt(a + b/x) + 3*b**5*x**2*sqrt(a + b/x)),
 Ne(b, 0)), (-1/(4*a**(5/2)*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3\,b^4}+\frac{6\sqrt{a+\frac{b}{x}}a}{b^4}+\frac{6\,a^2}{\sqrt{a+\frac{b}{x}}b^4}-\frac{2\,a^3}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^4}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
-2/3*(a + b/x)^(3/2)/b^4 + 6*sqrt(a + b/x)*a/b^4 + 6*a^2/(sqrt(a + b/x)*b^
4) - 2/3*a^3/((a + b/x)^(3/2)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\,dx = \frac{2\left(2\left(4\,x\left(\frac{2\,a^3x}{b^4\text{sgn}(x)}+\frac{3\,a^2}{b^3\text{sgn}(x)}\right)+\frac{3\,a}{b^2\text{sgn}(x)}\right)x-\frac{1}{b\text{sgn}(x)}\right)}{3\,(ax^2+bx)^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^5,x, algorithm="giac")
```

output
```
2/3*(2*(4*x*(2*a^3*x/(b^4*sgn(x)) + 3*a^2/(b^3*sgn(x))) + 3*a/(b^2*sgn(x))
)*x - 1/(b*sgn(x)))/(a*x^2 + b*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\left(16\,a^3\,x^3+24\,a^2\,b\,x^2+6\,a\,b^2\,x-b^3\right)}{3\,b^4\,x\,{\left(b+a\,x\right)}^2}$$

input
```
int(1/(x^5*(a + b/x)^(5/2)),x)
```

output
```
(2*(a + b/x)^(1/2)*(16*a^3*x^3 - b^3 + 24*a^2*b*x^2 + 6*a*b^2*x))/(3*b^4*x
*(b + a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5}\,dx = \frac{-\frac{32\sqrt{a}\,\sqrt{ax+b}\,a^2x^3}{3}-\frac{32\sqrt{a}\,\sqrt{ax+b}\,ab\,x^2}{3}+\frac{32\sqrt{x}\,a^3x^3}{3}+16\sqrt{x}\,a^2b\,x^2+4\sqrt{x}\,a\,b^2x-\frac{2\sqrt{x}\,b^3}{3}}{\sqrt{ax+b}\,b^4x^2\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^5,x)
```

output
```
(2*( - 16*sqrt(a)*sqrt(a*x + b)*a**2*x**3 - 16*sqrt(a)*sqrt(a*x + b)*a*b*x
**2 + 16*sqrt(x)*a**3*x**3 + 24*sqrt(x)*a**2*b*x**2 + 6*sqrt(x)*a*b**2*x -
 sqrt(x)*b**3))/(3*sqrt(a*x + b)*b**4*x**2*(a*x + b))
```

3.202 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6} \, dx = \frac{2a^4}{3b^5\left(a+\frac{b}{x}\right)^{3/2}} - \frac{8a^3}{b^5\sqrt{a+\frac{b}{x}}} - \frac{12a^2\sqrt{a+\frac{b}{x}}}{b^5} + \frac{8a\left(a+\frac{b}{x}\right)^{3/2}}{3b^5} - \frac{2\left(a+\frac{b}{x}\right)^{5/2}}{5b^5}$$

output
```
2/3*a^4/b^5/(a+b/x)^(3/2)-8*a^3/b^5/(a+b/x)^(1/2)-12*a^2*(a+b/x)^(1/2)/b^5
+8/3*a*(a+b/x)^(3/2)/b^5-2/5*(a+b/x)^(5/2)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6} \, dx = -\frac{2\sqrt{\frac{b+ax}{x}}\left(3b^4-8ab^3x+48a^2b^2x^2+192a^3bx^3+128a^4x^4\right)}{15b^5x^2(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^6),x]
```

output
```
(-2*Sqrt[(b + a*x)/x]*(3*b^4 - 8*a*b^3*x + 48*a^2*b^2*x^2 + 192*a^3*b*x^3
+ 128*a^4*x^4))/(15*b^5*x^2*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^4} d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(\frac{a^4}{b^4 \left(a+\frac{b}{x}\right)^{5/2}}-\frac{4 a^3}{b^4 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{6 a^2}{b^4 \sqrt{a+\frac{b}{x}}}-\frac{4 \sqrt{a+\frac{b}{x}} a}{b^4}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{b^4}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2 a^4}{3 b^5 \left(a+\frac{b}{x}\right)^{3/2}}-\frac{8 a^3}{b^5 \sqrt{a+\frac{b}{x}}}-\frac{12 a^2 \sqrt{a+\frac{b}{x}}}{b^5}+\frac{8 a \left(a+\frac{b}{x}\right)^{3/2}}{3 b^5}-\frac{2 \left(a+\frac{b}{x}\right)^{5/2}}{5 b^5}$$

input
```
Int[1/((a + b/x)^(5/2)*x^6),x]
```

output
```
(2*a^4)/(3*b^5*(a + b/x)^(3/2)) - (8*a^3)/(b^5*Sqrt[a + b/x]) - (12*a^2*Sq
rt[a + b/x])/b^5 + (8*a*(a + b/x)^(3/2))/(3*b^5) - (2*(a + b/x)^(5/2))/(5*
b^5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
orering	$-\frac{2\left(128a^4x^4+192a^3b\,x^3+48a^2b^2x^2-8a\,b^3x+3b^4\right)(ax+b)}{15b^5x^5\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$
gosper	$-\frac{2(ax+b)\left(128a^4x^4+192a^3b\,x^3+48a^2b^2x^2-8a\,b^3x+3b^4\right)}{15x^5b^5\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(128a^4x^4+192a^3b\,x^3+48a^2b^2x^2-8a\,b^3x+3b^4\right)\sqrt{-\frac{-ax-b}{x}}}{15x^2b^5(ax+b)^2}$
risch	$-\frac{2(ax+b)\left(73a^2x^2-14abx+3b^2\right)}{15b^5x^3\sqrt{\frac{ax+b}{x}}}-\frac{2a^3(11ax+12b)}{3(ax+b)b^5\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-30\sqrt{a\,x^2+bx}\,a^{\frac{13}{2}}x^7+15\ln\left(\frac{2\sqrt{x(ax+b)}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^6b\,x^7-30\sqrt{x(ax+b)}\,a^{\frac{13}{2}}x^7-15\ln\left(\frac{2\sqrt{a\,x^2+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^6b\,x\right.}{}$

input
```
int(1/(a+b/x)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(128*a^4*x^4+192*a^3*b*x^3+48*a^2*b^2*x^2-8*a*b^3*x+3*b^4)/b^5/x^5*(
a*x+b)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6} \, dx = -\frac{2 \, {\left(128 \, a^{4} x^{4} + 192 \, a^{3} b x^{3} + 48 \, a^{2} b^{2} x^{2} - 8 \, a b^{3} x + 3 \, b^{4}\right)} \sqrt{\frac{a x + b}{x}}}{15 \, {\left(a^{2} b^{5} x^{4} + 2 \, a b^{6} x^{3} + b^{7} x^{2}\right)}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
-2/15*(128*a^4*x^4 + 192*a^3*b*x^3 + 48*a^2*b^2*x^2 - 8*a*b^3*x + 3*b^4)*s
qrt((a*x + b)/x)/(a^2*b^5*x^4 + 2*a*b^6*x^3 + b^7*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2032 vs. 2(83) = 166.

Time = 3.11 (sec) , antiderivative size = 2032, normalized size of antiderivative = 20.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**6,x)
```

output
```
-256*a**(21/2)*b**(33/2)*x**8*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2
) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**
(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x
**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 1408*a**(19/2)*b**(35/2)*x**7*sqrt
(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) +
 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9
/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/
2)) - 3168*a**(17/2)*b**(37/2)*x**6*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x*
*(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 3
00*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b
**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 3696*a**(15/2)*b**(39/2)*x**
5*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*x**(1
5/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/2) + 225
*a**(9/2)*b**25*x**(9/2) + 90*a**(7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*
x**(5/2)) - 2310*a**(13/2)*b**(41/2)*x**4*sqrt(a*x/b + 1)/(15*a**(17/2)*b*
*21*x**(17/2) + 90*a**(15/2)*b**22*x**(15/2) + 225*a**(13/2)*b**23*x**(13/
2) + 300*a**(11/2)*b**24*x**(11/2) + 225*a**(9/2)*b**25*x**(9/2) + 90*a**(
7/2)*b**26*x**(7/2) + 15*a**(5/2)*b**27*x**(5/2)) - 696*a**(11/2)*b**(43/2
)*x**3*sqrt(a*x/b + 1)/(15*a**(17/2)*b**21*x**(17/2) + 90*a**(15/2)*b**22*
x**(15/2) + 225*a**(13/2)*b**23*x**(13/2) + 300*a**(11/2)*b**24*x**(11/...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^6}\,dx = -\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{5\,b^5}+\frac{8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{3\,b^5} - \frac{12\,\sqrt{a+\frac{b}{x}}a^2}{b^5}-\frac{8\,a^3}{\sqrt{a+\frac{b}{x}}b^5}+\frac{2\,a^4}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^5}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
-2/5*(a + b/x)^(5/2)/b^5 + 8/3*(a + b/x)^(3/2)*a/b^5 - 12*sqrt(a + b/x)*a^
2/b^5 - 8*a^3/(sqrt(a + b/x)*b^5) + 2/3*a^4/((a + b/x)^(3/2)*b^5)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6}\, dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}} x^6}\, dx$$

input
```
integrate(1/(a+b/x)^(5/2)/x^6,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x)^(5/2)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6}\, dx = -\frac{2\sqrt{a+\frac{b}{x}}\,\left(128\,a^4\,x^4+192\,a^3\,b\,x^3+48\,a^2\,b^2\,x^2-8\,a\,b^3\,x+3\,b^4\right)}{15\,b^5\,x^2\,{\left(b+a\,x\right)}^2}$$

input
```
int(1/(x^6*(a + b/x)^(5/2)),x)
```

output
```
-(2*(a + b/x)^(1/2)*(3*b^4 + 128*a^4*x^4 + 192*a^3*b*x^3 + 48*a^2*b^2*x^2
- 8*a*b^3*x))/(15*b^5*x^2*(b + a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^6}\, dx = \frac{\frac{256\sqrt{a}\,\sqrt{ax+b}\,a^3x^4}{15}+\frac{256\sqrt{a}\,\sqrt{ax+b}\,a^2b\,x^3}{15}-\frac{256\sqrt{x}\,a^4x^4}{15}-\frac{128\sqrt{x}\,a^3b\,x^3}{5}-\frac{32\sqrt{x}\,a^2b^2x^2}{5}+\frac{16\sqrt{x}\,a\,b^3x}{15}}{\sqrt{ax+b}\,b^5x^3\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^6,x)
```

output
```
(2*(128*sqrt(a)*sqrt(a*x + b)*a**3*x**4 + 128*sqrt(a)*sqrt(a*x + b)*a**2*b
*x**3 - 128*sqrt(x)*a**4*x**4 - 192*sqrt(x)*a**3*b*x**3 - 48*sqrt(x)*a**2*
b**2*x**2 + 8*sqrt(x)*a*b**3*x - 3*sqrt(x)*b**4))/(15*sqrt(a*x + b)*b**5*x
**3*(a*x + b))
```

3.203 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^7}\,dx=-\frac{2a^5}{3b^6\left(a+\frac{b}{x}\right)^{3/2}}+\frac{10a^4}{b^6\sqrt{a+\frac{b}{x}}}+\frac{20a^3\sqrt{a+\frac{b}{x}}}{b^6}-\frac{20a^2\left(a+\frac{b}{x}\right)^{3/2}}{3b^6}+\frac{2a\left(a+\frac{b}{x}\right)^{5/2}}{b^6}-\frac{2\left(a+\frac{b}{x}\right)^{7/2}}{7b^6}$$

output
```
-2/3*a^5/b^6/(a+b/x)^(3/2)+10*a^4/b^6/(a+b/x)^(1/2)+20*a^3*(a+b/x)^(1/2)/b
^6-20/3*a^2*(a+b/x)^(3/2)/b^6+2*a*(a+b/x)^(5/2)/b^6-2/7*(a+b/x)^(7/2)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^7}\,dx=\frac{2\sqrt{\frac{b+ax}{x}}\left(-3b^5+6ab^4x-16a^2b^3x^2+96a^3b^2x^3+384a^4bx^4+256a^5x^5\right)}{21b^6x^3(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^7),x]
```

output
```
(2*Sqrt[(b + a*x)/x]*(-3*b^5 + 6*a*b^4*x - 16*a^2*b^3*x^2 + 96*a^3*b^2*x^3
 + 384*a^4*b*x^4 + 256*a^5*x^5))/(21*b^6*x^3*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^5} d\frac{1}{x}$$

$$\Big\downarrow \text{53}$$

$$-\int \left(-\frac{a^5}{b^5 \left(a+\frac{b}{x}\right)^{5/2}}+\frac{5 a^4}{b^5 \left(a+\frac{b}{x}\right)^{3/2}}-\frac{10 a^3}{b^5 \sqrt{a+\frac{b}{x}}}+\frac{10 \sqrt{a+\frac{b}{x}} a^2}{b^5}-\frac{5 \left(a+\frac{b}{x}\right)^{3/2} a}{b^5}+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{b^5}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2 a^5}{3 b^6 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{10 a^4}{b^6 \sqrt{a+\frac{b}{x}}}+\frac{20 a^3 \sqrt{a+\frac{b}{x}}}{b^6}-\frac{20 a^2 \left(a+\frac{b}{x}\right)^{3/2}}{3 b^6}+\frac{2 a \left(a+\frac{b}{x}\right)^{5/2}}{b^6}-\frac{2 \left(a+\frac{b}{x}\right)^{7/2}}{7 b^6}$$

input
```
Int[1/((a + b/x)^(5/2)*x^7),x]
```

output
```
(-2*a^5)/(3*b^6*(a + b/x)^(3/2)) + (10*a^4)/(b^6*Sqrt[a + b/x]) + (20*a^3*
Sqrt[a + b/x])/b^6 - (20*a^2*(a + b/x)^(3/2))/(3*b^6) + (2*a*(a + b/x)^(5/
2))/b^6 - (2*(a + b/x)^(7/2))/(7*b^6)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

method	result
orering	$\frac{2\left(256a^{5}x^{5}+384a^{4}b\,x^{4}+96a^{3}b^{2}x^{3}-16a^{2}b^{3}x^{2}+6b^{4}xa-3b^{5}\right)\left(ax+b\right)}{21b^{6}x^{6}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$
gosper	$\frac{2\left(ax+b\right)\left(256a^{5}x^{5}+384a^{4}b\,x^{4}+96a^{3}b^{2}x^{3}-16a^{2}b^{3}x^{2}+6b^{4}xa-3b^{5}\right)}{21x^{6}b^{6}\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$
trager	$\frac{2\left(256a^{5}x^{5}+384a^{4}b\,x^{4}+96a^{3}b^{2}x^{3}-16a^{2}b^{3}x^{2}+6b^{4}xa-3b^{5}\right)\sqrt{-\frac{-ax-b}{x}}}{21x^{3}b^{6}\left(ax+b\right)^{2}}$
risch	$\frac{2\left(ax+b\right)\left(158a^{3}x^{3}-37a^{2}b\,x^{2}+12a\,b^{2}x-3b^{3}\right)}{21b^{6}x^{4}\sqrt{\frac{ax+b}{x}}}+\frac{2a^{4}\left(14ax+15b\right)}{3\left(ax+b\right)b^{6}\sqrt{\frac{ax+b}{x}}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(210a^{\frac{15}{2}}\sqrt{x\left(ax+b\right)}\,x^{8}+105\ln\left(\frac{2\sqrt{a\,x^{2}+bx}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a^{7}b\,x^{8}-840\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{13}{2}}x^{6}+420a^{\frac{13}{2}}\left(x\left(ax+b\right)\right)^{\frac{3}{2}}x^{6}-315\ln\left(\right.\right.}{}$

input
```
int(1/(a+b/x)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
2/21*(256*a^5*x^5+384*a^4*b*x^4+96*a^3*b^2*x^3-16*a^2*b^3*x^2+6*a*b^4*x-3*
b^5)/b^6/x^6*(a*x+b)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\,dx = \frac{2\,(256\,a^5x^5 + 384\,a^4bx^4 + 96\,a^3b^2x^3 - 16\,a^2b^3x^2 + 6\,ab^4x - 3\,b^5)\sqrt{\frac{ax+b}{x}}}{21\,(a^2b^6x^5 + 2\,ab^7x^4 + b^8x^3)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
2/21*(256*a^5*x^5 + 384*a^4*b*x^4 + 96*a^3*b^2*x^3 - 16*a^2*b^3*x^2 + 6*a*
b^4*x - 3*b^5)*sqrt((a*x + b)/x)/(a^2*b^6*x^5 + 2*a*b^7*x^4 + b^8*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9263 vs. 2(100) = 200.

Time = 5.42 (sec) , antiderivative size = 9263, normalized size of antiderivative = 79.85

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**7,x)
```

output
```
512*a**(43/2)*b**(91/2)*x**18*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2
) + 315*a**(35/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*
a**(31/2)*b**54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27
/2)*b**56*x**(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*
b**58*x**(23/2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**6
0*x**(19/2) + 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(
15/2) + 9555*a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) +
315*a**(9/2)*b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 7424*a**(41/2)
*b**(93/2)*x**17*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(3
5/2)*b**52*x**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**
54*x**(31/2) + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**
(27/2) + 105105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/
2) + 135135*a**(21/2)*b**59*x**(21/2) + 105105*a**(19/2)*b**60*x**(19/2) +
 63063*a**(17/2)*b**61*x**(17/2) + 28665*a**(15/2)*b**62*x**(15/2) + 9555*
a**(13/2)*b**63*x**(13/2) + 2205*a**(11/2)*b**64*x**(11/2) + 315*a**(9/2)*
b**65*x**(9/2) + 21*a**(7/2)*b**66*x**(7/2)) + 50112*a**(39/2)*b**(95/2)*x
**16*sqrt(a*x/b + 1)/(21*a**(37/2)*b**51*x**(37/2) + 315*a**(35/2)*b**52*x
**(35/2) + 2205*a**(33/2)*b**53*x**(33/2) + 9555*a**(31/2)*b**54*x**(31/2)
 + 28665*a**(29/2)*b**55*x**(29/2) + 63063*a**(27/2)*b**56*x**(27/2) + 105
105*a**(25/2)*b**57*x**(25/2) + 135135*a**(23/2)*b**58*x**(23/2) + 1351...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7} \, dx = -\frac{2 \left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{7 b^6} + \frac{2 \left(a+\frac{b}{x}\right)^{\frac{5}{2}} a}{b^6} - \frac{20 \left(a+\frac{b}{x}\right)^{\frac{3}{2}} a^2}{3 b^6} + \frac{20 \sqrt{a+\frac{b}{x}} a^3}{b^6} + \frac{10 a^4}{\sqrt{a+\frac{b}{x}} b^6} - \frac{2 a^5}{3 \left(a+\frac{b}{x}\right)^{\frac{3}{2}} b^6}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
-2/7*(a + b/x)^(7/2)/b^6 + 2*(a + b/x)^(5/2)*a/b^6 - 20/3*(a + b/x)^(3/2)*
a^2/b^6 + 20*sqrt(a + b/x)*a^3/b^6 + 10*a^4/(sqrt(a + b/x)*b^6) - 2/3*a^5/
((a + b/x)^(3/2)*b^6)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\, dx = \int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}} x^7}\, dx$$

input
```
integrate(1/(a+b/x)^(5/2)/x^7,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x)^(5/2)*x^7), x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\, dx = \frac{\sqrt{a+\frac{b}{x}}\left(\frac{256\,a^3}{21\,b^5}+\frac{512\,a^4\,x}{21\,b^6}\right)}{b+a\,x} - \frac{2\,\sqrt{a+\frac{b}{x}}}{7\,b^3\,x^3} + \frac{8\,a\,\sqrt{a+\frac{b}{x}}}{7\,b^4\,x^2} - \frac{\sqrt{a+\frac{b}{x}}\left(\frac{74\,a^2}{21\,b^3}+\frac{88\,a^3\,x}{21\,b^4}\right)}{x\,(b+a\,x)^2}$$

input
```
int(1/(x^7*(a + b/x)^(5/2)),x)
```

output
```
((a + b/x)^(1/2)*((256*a^3)/(21*b^5) + (512*a^4*x)/(21*b^6)))/(b + a*x) -
(2*(a + b/x)^(1/2))/(7*b^3*x^3) + (8*a*(a + b/x)^(1/2))/(7*b^4*x^2) - ((a
+ b/x)^(1/2)*((74*a^2)/(21*b^3) + (88*a^3*x)/(21*b^4)))/(x*(b + a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^7}\, dx = \frac{-\frac{512\sqrt{a}\,\sqrt{ax+b}\,a^4x^5}{21} - \frac{512\sqrt{a}\,\sqrt{ax+b}\,a^3b\,x^4}{21} + \frac{512\sqrt{x}\,a^5x^5}{21} + \frac{256\sqrt{x}\,a^4b\,x^4}{7} + \frac{64\sqrt{x}\,a^3b^2x^3}{7} - \frac{32\sqrt{x}\,a^2b}{21}}{\sqrt{ax+b}\,b^6x^4\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^7,x)
```

output
```
(2*( - 256*sqrt(a)*sqrt(a*x + b)*a**4*x**5 - 256*sqrt(a)*sqrt(a*x + b)*a**
3*b*x**4 + 256*sqrt(x)*a**5*x**5 + 384*sqrt(x)*a**4*b*x**4 + 96*sqrt(x)*a*
*3*b**2*x**3 - 16*sqrt(x)*a**2*b**3*x**2 + 6*sqrt(x)*a*b**4*x - 3*sqrt(x)*
b**5))/(21*sqrt(a*x + b)*b**6*x**4*(a*x + b))
```

3.204 $\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx=-\frac{32b^3\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}{315a^4}+\frac{16b^2\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{105a^3}-\frac{4b\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{21a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{9/2}}{9a}$$

output
```
-32/315*b^3*(a+b/x)^(3/2)*x^(3/2)/a^4+16/105*b^2*(a+b/x)^(3/2)*x^(5/2)/a^3
-4/21*b*(a+b/x)^(3/2)*x^(7/2)/a^2+2/9*(a+b/x)^(3/2)*x^(9/2)/a
```

Mathematica [A] (verified)

Time = 3.86 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-16b^4+8ab^3x-6a^2b^2x^2+5a^3bx^3+35a^4x^4)}{315a^4}$$

input
```
Integrate[Sqrt[a + b/x]*x^(7/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-16*b^4 + 8*a*b^3*x - 6*a^2*b^2*x^2 + 5*a^3*b*x^
3 + 35*a^4*x^4))/(315*a^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}}{9a}-\frac{2b\int \sqrt{a+\frac{b}{x}}x^{5/2}dx}{3a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}}{9a}-\frac{2b\left(\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}-\frac{4b\int \sqrt{a+\frac{b}{x}}x^{3/2}dx}{7a}\right)}{3a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}}{9a}-\frac{2b\left(\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}-\frac{4b\left(\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{2b\int \sqrt{a+\frac{b}{x}}\sqrt{x}dx}{5a}\right)}{7a}\right)}{3a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}}{9a}-\frac{2b\left(\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}-\frac{4b\left(\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{4bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{15a^2}\right)}{7a}\right)}{3a}$$

input
```
Int[Sqrt[a + b/x]*x^(7/2),x]
```

output
```
(2*(a + b/x)^(3/2)*x^(9/2))/(9*a) - (2*b*((2*(a + b/x)^(3/2)*x^(7/2))/(7*a
) - (4*b*((-4*b*(a + b/x)^(3/2)*x^(3/2))/(15*a^2) + (2*(a + b/x)^(3/2)*x^(
5/2))/(5*a)))/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.53

method	result	size
orering	$\frac{2\sqrt{a+\frac{b}{x}}\,\sqrt{x}\,(ax+b)\left(35a^{3}x^{3}-30a^{2}b\,x^{2}+24a\,b^{2}x-16b^{3}\right)}{315a^{4}}$	53
gosper	$\frac{2(ax+b)\left(35a^{3}x^{3}-30a^{2}b\,x^{2}+24a\,b^{2}x-16b^{3}\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{315a^{4}}$	55
default	$\frac{2(ax+b)\left(35a^{3}x^{3}-30a^{2}b\,x^{2}+24a\,b^{2}x-16b^{3}\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{315a^{4}}$	55
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(35a^{4}x^{4}+5a^{3}b\,x^{3}-6a^{2}b^{2}x^{2}+8a\,b^{3}x-16b^{4}\right)}{315a^{4}}$	61

input
```
int((a+b/x)^(1/2)*x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(a+b/x)^(1/2)*x^(1/2)*(a*x+b)*(35*a^3*x^3-30*a^2*b*x^2+24*a*b^2*x-16
*b^3)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx=\frac{2\,(35\,a^4x^4+5\,a^3bx^3-6\,a^2b^2x^2+8\,ab^3x-16\,b^4)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{315\,a^4}$$

input
```
integrate((a+b/x)^(1/2)*x^(7/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^4*x^4 + 5*a^3*b*x^3 - 6*a^2*b^2*x^2 + 8*a*b^3*x - 16*b^4)*sqrt
(x)*sqrt((a*x + b)/x)/a^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(87) = 174.

Time = 21.97 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.18

$$\begin{aligned}\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx &= \frac{70a^7b^{\frac{19}{2}}x^7\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}} \\
&+ \frac{220a^6b^{\frac{21}{2}}x^6\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}} \\
&+ \frac{228a^5b^{\frac{23}{2}}x^5\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}} \\
&+ \frac{80a^4b^{\frac{25}{2}}x^4\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}} \\
&- \frac{10a^3b^{\frac{27}{2}}x^3\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}} \\
&- \frac{60a^2b^{\frac{29}{2}}x^2\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}} \\
&- \frac{80ab^{\frac{31}{2}}x\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}} \\
&- \frac{32b^{\frac{33}{2}}\sqrt{\frac{ax}{b}+1}}{315a^7b^9x^3+945a^6b^{10}x^2+945a^5b^{11}x+315a^4b^{12}}\end{aligned}$$

input
```
integrate((a+b/x)**(1/2)*x**(7/2),x)
```

output
```
70*a**7*b**(19/2)*x**7*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**1
0*x**2 + 945*a**5*b**11*x + 315*a**4*b**12) + 220*a**6*b**(21/2)*x**6*sqrt
(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**10*x**2 + 945*a**5*b**11*x +
 315*a**4*b**12) + 228*a**5*b**(23/2)*x**5*sqrt(a*x/b + 1)/(315*a**7*b**9*
x**3 + 945*a**6*b**10*x**2 + 945*a**5*b**11*x + 315*a**4*b**12) + 80*a**4*
b**(25/2)*x**4*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**10*x**2 +
 945*a**5*b**11*x + 315*a**4*b**12) - 10*a**3*b**(27/2)*x**3*sqrt(a*x/b +
1)/(315*a**7*b**9*x**3 + 945*a**6*b**10*x**2 + 945*a**5*b**11*x + 315*a**4
*b**12) - 60*a**2*b**(29/2)*x**2*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945
*a**6*b**10*x**2 + 945*a**5*b**11*x + 315*a**4*b**12) - 80*a*b**(31/2)*x*s
qrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 945*a**6*b**10*x**2 + 945*a**5*b**11*
x + 315*a**4*b**12) - 32*b**(33/2)*sqrt(a*x/b + 1)/(315*a**7*b**9*x**3 + 9
45*a**6*b**10*x**2 + 945*a**5*b**11*x + 315*a**4*b**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx=\frac{2\left(35\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}}-135\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}}+189\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^2x^{\frac{5}{2}}-105\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^3x^{\frac{3}{2}}\right)}{315\,a^4}$$

input
```
integrate((a+b/x)^(1/2)*x^(7/2),x, algorithm="maxima")
```

output
```
2/315*(35*(a + b/x)^(9/2)*x^(9/2) - 135*(a + b/x)^(7/2)*b*x^(7/2) + 189*(a
 + b/x)^(5/2)*b^2*x^(5/2) - 105*(a + b/x)^(3/2)*b^3*x^(3/2))/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx = \frac{32\,b^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,a^4} + \frac{2\left(\frac{9\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2-35\,\sqrt{ax+b}b^3\right)b\mathrm{sgn}(x)}{a^3}+\frac{\left(35\,(ax+b)^{\frac{9}{2}}-180\,(ax+b)^{\frac{7}{2}}b+378\,(ax+b)^{\frac{5}{2}}b^2-420\,(ax+b)^{\frac{3}{2}}b^3+31\right.}{a^3}}{315\,a}$$

input
```
integrate((a+b/x)^(1/2)*x^(7/2),x, algorithm="giac")
```

output
```
32/315*b^(9/2)*sgn(x)/a^4 + 2/315*(9*(5*(a*x + b)^(7/2) - 21*(a*x + b)^(5/
2)*b + 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*b*sgn(x)/a^3 + (35*(
a*x + b)^(9/2) - 180*(a*x + b)^(7/2)*b + 378*(a*x + b)^(5/2)*b^2 - 420*(a*
x + b)^(3/2)*b^3 + 315*sqrt(a*x + b)*b^4)*sgn(x)/a^3)/a
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{9/2}}{9}+\frac{2\,b\,x^{7/2}}{63\,a}-\frac{4\,b^2\,x^{5/2}}{105\,a^2}+\frac{16\,b^3\,x^{3/2}}{315\,a^3}-\frac{32\,b^4\,\sqrt{x}}{315\,a^4}\right)$$

input
```
int(x^(7/2)*(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((2*x^(9/2))/9 + (2*b*x^(7/2))/(63*a) - (4*b^2*x^(5/2))/(1
05*a^2) + (16*b^3*x^(3/2))/(315*a^3) - (32*b^4*x^(1/2))/(315*a^4))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.52

$$\int \sqrt{a+\frac{b}{x}}x^{7/2}\,dx = \frac{2\sqrt{ax+b}\,(35a^4x^4+5a^3b\,x^3-6a^2b^2x^2+8a\,b^3x-16b^4)}{315a^4}$$

input
```
int((a+b/x)^(1/2)*x^(7/2),x)
```

output
```
(2*sqrt(a*x + b)*(35*a**4*x**4 + 5*a**3*b*x**3 - 6*a**2*b**2*x**2 + 8*a*b*
*3*x - 16*b**4))/(315*a**4)
```

3.205 $\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=\frac{16b^2\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}{105a^3}-\frac{8b\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}{35a^2}+\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}{7a}$$

output
```
16/105*b^2*(a+b/x)^(3/2)*x^(3/2)/a^3-8/35*b*(a+b/x)^(3/2)*x^(5/2)/a^2+2/7*
(a+b/x)^(3/2)*x^(7/2)/a
```

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(8b^3-4ab^2x+3a^2bx^2+15a^3x^3)}{105a^3}$$

input
```
Integrate[Sqrt[a + b/x]*x^(5/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(8*b^3 - 4*a*b^2*x + 3*a^2*b*x^2 + 15*a^3*x^3))/(
105*a^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}-\frac{4b\int \sqrt{a+\frac{b}{x}}x^{3/2}dx}{7a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}-\frac{4b\left(\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{2b\int \sqrt{a+\frac{b}{x}}\sqrt{x}dx}{5a}\right)}{7a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}{7a}-\frac{4b\left(\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{4bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{15a^2}\right)}{7a}$$

input
```
Int[Sqrt[a + b/x]*x^(5/2),x]
```

output
```
(2*(a + b/x)^(3/2)*x^(7/2))/(7*a) - (4*b*((-4*b*(a + b/x)^(3/2)*x^(3/2))/(
15*a^2) + (2*(a + b/x)^(3/2)*x^(5/2))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

method	result	size
orering	$\frac{2\sqrt{a+\frac{b}{x}}\,\sqrt{x}\,(ax+b)\left(15a^{2}x^{2}-12abx+8b^{2}\right)}{105a^{3}}$	42
gosper	$\frac{2(ax+b)\left(15a^{2}x^{2}-12abx+8b^{2}\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{105a^{3}}$	44
default	$\frac{2(ax+b)\left(15a^{2}x^{2}-12abx+8b^{2}\right)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{105a^{3}}$	44
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(15a^{3}x^{3}+3a^{2}b\,x^{2}-4a\,b^{2}x+8b^{3}\right)}{105a^{3}}$	50

input
```
int((a+b/x)^(1/2)*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(a+b/x)^(1/2)*x^(1/2)*(a*x+b)*(15*a^2*x^2-12*a*b*x+8*b^2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=\frac{2\,(15\,a^3x^3+3\,a^2bx^2-4\,ab^2x+8\,b^3)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{105\,a^3}$$

input
```
integrate((a+b/x)^(1/2)*x^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*a^3*x^3 + 3*a^2*b*x^2 - 4*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b
)/x)/a^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(63) = 126.

Time = 7.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.24

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=\frac{30a^5b^{\frac{9}{2}}x^5\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{66a^4b^{\frac{11}{2}}x^4\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{34a^3b^{\frac{13}{2}}x^3\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{6a^2b^{\frac{15}{2}}x^2\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{24ab^{\frac{17}{2}}x\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}+\frac{16b^{\frac{19}{2}}\sqrt{\frac{ax}{b}+1}}{105a^5b^4x^2+210a^4b^5x+105a^3b^6}$$

input
```
integrate((a+b/x)**(1/2)*x**(5/2),x)
```

output
```
30*a**5*b**(9/2)*x**5*sqrt(a*x/b + 1)/(105*a**5*b**4*x**2 + 210*a**4*b**5*
x + 105*a**3*b**6) + 66*a**4*b**(11/2)*x**4*sqrt(a*x/b + 1)/(105*a**5*b**4
*x**2 + 210*a**4*b**5*x + 105*a**3*b**6) + 34*a**3*b**(13/2)*x**3*sqrt(a*x
/b + 1)/(105*a**5*b**4*x**2 + 210*a**4*b**5*x + 105*a**3*b**6) + 6*a**2*b*
*(15/2)*x**2*sqrt(a*x/b + 1)/(105*a**5*b**4*x**2 + 210*a**4*b**5*x + 105*a
**3*b**6) + 24*a*b**(17/2)*x*sqrt(a*x/b + 1)/(105*a**5*b**4*x**2 + 210*a**
4*b**5*x + 105*a**3*b**6) + 16*b**(19/2)*sqrt(a*x/b + 1)/(105*a**5*b**4*x*
*2 + 210*a**4*b**5*x + 105*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=\frac{2\left(15\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}-42\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}}+35\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}\right)}{105\,a^3}$$

input
```
integrate((a+b/x)^(1/2)*x^(5/2),x, algorithm="maxima")
```

output
```
2/105*(15*(a + b/x)^(7/2)*x^(7/2) - 42*(a + b/x)^(5/2)*b*x^(5/2) + 35*(a +
 b/x)^(3/2)*b^2*x^(3/2))/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=-\frac{16\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,a^3}+\frac{2\left(\frac{7\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b+15\,\sqrt{ax+b}b^2\right)b\mathrm{sgn}(x)}{a^2}+\frac{3\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2-35\,\sqrt{ax+b}b^3\right)\mathrm{sgn}(x)}{a^2}\right)}{105\,a}$$

input
```
integrate((a+b/x)^(1/2)*x^(5/2),x, algorithm="giac")
```

output
```
-16/105*b^(7/2)*sgn(x)/a^3 + 2/105*(7*(3*(a*x + b)^(5/2) - 10*(a*x + b)^(3
/2)*b + 15*sqrt(a*x + b)*b^2)*b*sgn(x)/a^2 + 3*(5*(a*x + b)^(7/2) - 21*(a*
x + b)^(5/2)*b + 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*sgn(x)/a^2
)/a
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{7/2}}{7}+\frac{2\,b\,x^{5/2}}{35\,a}-\frac{8\,b^2\,x^{3/2}}{105\,a^2}+\frac{16\,b^3\,\sqrt{x}}{105\,a^3}\right)$$

input
```
int(x^(5/2)*(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((2*x^(7/2))/7 + (2*b*x^(5/2))/(35*a) - (8*b^2*x^(3/2))/(1
05*a^2) + (16*b^3*x^(1/2))/(105*a^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \sqrt{a+\frac{b}{x}}x^{5/2}\,dx=\frac{2\sqrt{ax+b}\,(15a^3x^3+3a^2b\,x^2-4a\,b^2x+8b^3)}{105a^3}$$

input
```
int((a+b/x)^(1/2)*x^(5/2),x)
```

output
```
(2*sqrt(a*x + b)*(15*a**3*x**3 + 3*a**2*b*x**2 - 4*a*b**2*x + 8*b**3))/(10
5*a**3)
```

3.206 $\int \sqrt{a+\frac{b}{x}} x^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \sqrt{a+\frac{b}{x}} x^{3/2} \, dx = -\frac{4b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}}{15a^2} + \frac{2\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}}{5a}$$

output
```
-4/15*b*(a+b/x)^(3/2)*x^(3/2)/a^2+2/5*(a+b/x)^(3/2)*x^(5/2)/a
```

Mathematica [A] (verified)

Time = 3.85 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \sqrt{a+\frac{b}{x}} x^{3/2} \, dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-2b^2+abx+3a^2x^2)}{15a^2}$$

input
```
Integrate[Sqrt[a + b/x]*x^(3/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-2*b^2 + a*b*x + 3*a^2*x^2))/(15*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\sqrt{a+\frac{b}{x}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{2b\int \sqrt{a+\frac{b}{x}}\sqrt{x}dx}{5a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}{5a}-\frac{4bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{15a^2}$$

input
```
Int[Sqrt[a + b/x]*x^(3/2),x]
```

output
```
(-4*b*(a + b/x)^(3/2)*x^(3/2))/(15*a^2) + (2*(a + b/x)^(3/2)*x^(5/2))/(5*a
)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

method	result	size
orering	$\frac{2\sqrt{a+\frac{b}{x}}\,\sqrt{x}\,(ax+b)(3ax-2b)}{15a^2}$	31
gosper	$\frac{2(ax+b)(3ax-2b)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{15a^2}$	33
default	$\frac{2(ax+b)(3ax-2b)\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}{15a^2}$	33
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(3a^2x^2+abx-2b^2)}{15a^2}$	38

input
```
int((a+b/x)^(1/2)*x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(a+b/x)^(1/2)*x^(1/2)*(a*x+b)*(3*a*x-2*b)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

$$\int \sqrt{a+\frac{b}{x}}x^{3/2}\,dx = \frac{2\,(3\,a^2x^2+abx-2\,b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{15\,a^2}$$

input
```
integrate((a+b/x)^(1/2)*x^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*x^2 + a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x)/a^2
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int \sqrt{a+\frac{b}{x}}x^{3/2}\,dx = \frac{2\sqrt{b}x^2\sqrt{\frac{ax}{b}+1}}{5} + \frac{2b^{\frac{3}{2}}x\sqrt{\frac{ax}{b}+1}}{15a} - \frac{4b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{15a^2}$$

input
```
integrate((a+b/x)**(1/2)*x**(3/2),x)
```

output
```
2*sqrt(b)*x**2*sqrt(a*x/b + 1)/5 + 2*b**(3/2)*x*sqrt(a*x/b + 1)/(15*a) - 4
*b**(5/2)*sqrt(a*x/b + 1)/(15*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \sqrt{a+\frac{b}{x}}x^{3/2}\,dx = \frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}} - 5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}}\right)}{15\,a^2}$$

input
```
integrate((a+b/x)^(1/2)*x^(3/2),x, algorithm="maxima")
```

output
```
2/15*(3*(a + b/x)^(5/2)*x^(5/2) - 5*(a + b/x)^(3/2)*b*x^(3/2))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(36) = 72.

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \sqrt{a+\frac{b}{x}}x^{3/2}\,dx = \frac{4\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,a^2} + \frac{2\left(\frac{5\left((ax+b)^{\frac{3}{2}}-3\sqrt{ax+b}b\right)b\mathrm{sgn}(x)}{a} + \frac{\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b+15\,\sqrt{ax+b}b^2\right)\mathrm{sgn}(x)}{a}\right)}{15\,a}$$

input
```
integrate((a+b/x)^(1/2)*x^(3/2),x, algorithm="giac")
```

output
```
4/15*b^(5/2)*sgn(x)/a^2 + 2/15*(5*((a*x + b)^(3/2) - 3*sqrt(a*x + b)*b)*b*
sgn(x)/a + (3*(a*x + b)^(5/2) - 10*(a*x + b)^(3/2)*b + 15*sqrt(a*x + b)*b^
2)*sgn(x)/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \sqrt{a+\frac{b}{x}}x^{3/2}\,dx=\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{5/2}}{5}+\frac{2\,b\,x^{3/2}}{15\,a}-\frac{4\,b^2\,\sqrt{x}}{15\,a^2}\right)$$

input
```
int(x^(3/2)*(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((2*x^(5/2))/5 + (2*b*x^(3/2))/(15*a) - (4*b^2*x^(1/2))/(1
5*a^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \sqrt{a+\frac{b}{x}}x^{3/2}\,dx=\frac{2\sqrt{ax+b}\,(3a^2x^2+abx-2b^2)}{15a^2}$$

input
```
int((a+b/x)^(1/2)*x^(3/2),x)
```

output
```
(2*sqrt(a*x + b)*(3*a**2*x**2 + a*b*x - 2*b**2))/(15*a**2)
```

3.207 $\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx=\frac{2\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}{3a}$$

output
```
2/3*(a+b/x)^(3/2)*x^(3/2)/a
```

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)}{3a}$$

input
```
Integrate[Sqrt[a + b/x]*Sqrt[x],x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x))/(3*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{a+\frac{b}{x}}\,dx$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}{3a}$$

input
```
Int[Sqrt[a + b/x]*Sqrt[x],x]
```

output
```
(2*(a + b/x)^(3/2)*x^(3/2))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
orering	$\frac{2\sqrt{a+\frac{b}{x}}\,\sqrt{x}\,(ax+b)}{3a}$	23
gosper	$\frac{2(ax+b)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{3a}$	25
default	$\frac{2(ax+b)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{3a}$	25
risch	$\frac{2(ax+b)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{3a}$	25

input
```
int((a+b/x)^(1/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a+b/x)^(1/2)*x^(1/2)*(a*x+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx = \frac{2\,(ax+b)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\,a}$$

input
```
integrate((a+b/x)^(1/2)*x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a*x + b)*sqrt(x)*sqrt((a*x + b)/x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 1.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx = \frac{2\sqrt{b}x\sqrt{\frac{ax}{b}+1}}{3} + \frac{2b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3a}$$

input
```
integrate((a+b/x)**(1/2)*x**(1/2),x)
```

output
```
2*sqrt(b)*x*sqrt(a*x/b + 1)/3 + 2*b**(3/2)*sqrt(a*x/b + 1)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx = \frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{3\,a}$$

input
```
integrate((a+b/x)^(1/2)*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a + b/x)^(3/2)*x^(3/2)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx = -\frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,a} + \frac{2\left(3\,\sqrt{ax+b}b\mathrm{sgn}(x)+\left((ax+b)^{\frac{3}{2}}-3\,\sqrt{ax+b}b\right)\mathrm{sgn}(x)\right)}{3\,a}$$

input
```
integrate((a+b/x)^(1/2)*x^(1/2),x, algorithm="giac")
```

output
```
-2/3*b^(3/2)*sgn(x)/a + 2/3*(3*sqrt(a*x + b)*b*sgn(x) + ((a*x + b)^(3/2) -
 3*sqrt(a*x + b)*b)*sgn(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx=\frac{2\,\sqrt{x}\,\sqrt{a+\frac{b}{x}}\,\left(b+a\,x\right)}{3\,a}$$

input
```
int(x^(1/2)*(a + b/x)^(1/2),x)
```

output
```
(2*x^(1/2)*(a + b/x)^(1/2)*(b + a*x))/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \sqrt{a+\frac{b}{x}}\sqrt{x}\,dx=\frac{2\sqrt{ax+b}\,(ax+b)}{3a}$$

input
```
int((a+b/x)^(1/2)*x^(1/2),x)
```

output
```
(2*sqrt(a*x + b)*(a*x + b))/(3*a)
```

3.208 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}} \, dx = 2\sqrt{a+\frac{b}{x}}\sqrt{x} - 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

output
```
2*(a+b/x)^(1/2)*x^(1/2)-2*b^(1/2)*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 3.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}} \, dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(2\sqrt{b+ax} - 2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)\right)}{\sqrt{b+ax}}$$

input
```
Integrate[Sqrt[a + b/x]/Sqrt[x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*(2*Sqrt[b + a*x] - 2*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/
Sqrt[b]]))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {860, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \sqrt{a+\frac{b}{x}}xd\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{247}$$

$$-2\left(b\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(b\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)$$

input
```
Int[Sqrt[a + b/x]/Sqrt[x],x]
```

output
```
-2*(-(Sqrt[a + b/x]*Sqrt[x]) + Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt
[x])])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-\sqrt{ax+b}\right)}{\sqrt{ax+b}}$	50

input
```
int((a+b/x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((a*x+b)/x)^(1/2)*x^(1/2)*(b^(1/2)*arctanh((a*x+b)^(1/2)/b^(1/2))-(a*x+
b)^(1/2))/(a*x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx = \left[\sqrt{b}\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right) + 2\sqrt{x}\sqrt{\frac{ax+b}{x}}, 2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+2\sqrt{x}\sqrt{\frac{ax+b}{x}}\right]$$

input
```
integrate((a+b/x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2*sqrt
(x)*sqrt((a*x + b)/x), 2*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x
)/(a*x + b)) + 2*sqrt(x)*sqrt((a*x + b)/x)]
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx = \frac{2\sqrt{a}\sqrt{x}}{\sqrt{1+\frac{b}{ax}}} - 2\sqrt{b}\,\text{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right) + \frac{2b}{\sqrt{a}\sqrt{x}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate((a+b/x)**(1/2)/x**(1/2),x)
```

output
```
2*sqrt(a)*sqrt(x)/sqrt(1 + b/(a*x)) - 2*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*sqr
t(x))) + 2*b/(sqrt(a)*sqrt(x)*sqrt(1 + b/(a*x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx = \sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)+2\sqrt{a+\frac{b}{x}}\sqrt{x}$$

input
```
integrate((a+b/x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqr
t(b))) + 2*sqrt(a + b/x)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx = \frac{2\,b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+2\sqrt{ax+b}\operatorname{sgn}(x) - \frac{2\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

input
```
integrate((a+b/x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
2*b*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(a*x + b)*sgn(x
) - 2*(b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx$$

input
```
int((a + b/x)^(1/2)/x^(1/2),x)
```

output
```
int((a + b/x)^(1/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}}\,dx=2\sqrt{ax+b}+\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)-\sqrt{b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)$$

input
```
int((a+b/x)^(1/2)/x^(1/2),x)
```

output
```
2*sqrt(a*x + b) + sqrt(b)*log(sqrt(a*x + b) - sqrt(b)) - sqrt(b)*log(sqrt(
a*x + b) + sqrt(b))
```

3.209 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx = -\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{x}} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{\sqrt{b}}$$

output
```
-(a+b/x)^(1/2)/x^(1/2)-a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 3.92 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx = -\frac{\sqrt{a+\frac{b}{x}}\left(\sqrt{b+ax}+\frac{ax\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{\sqrt{b}}\right)}{\sqrt{x}\sqrt{b+ax}}$$

input
```
Integrate[Sqrt[a + b/x]/x^(3/2),x]
```

output
```
-((Sqrt[a + b/x]*(Sqrt[b + a*x] + (a*x*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/Sqr
t[b]))/(Sqrt[x]*Sqrt[b + a*x]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {860, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 860}$$

$$-2\int \sqrt{a+\frac{b}{x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 211}$$

$$-2\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-2\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-2\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)$$

input
```
Int[Sqrt[a + b/x]/x^(3/2),x]
```

output
```
-2*(Sqrt[a + b/x]/(2*Sqrt[x]) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])
])/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)ax+\sqrt{ax+b}\,\sqrt{b}\right)}{\sqrt{x}\,\sqrt{ax+b}\,\sqrt{b}}$	54
risch	$-\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{x}}-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{\sqrt{b}\,\sqrt{ax+b}}$	57

input
```
int((a+b/x)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x+b)/x)^(1/2)*(arctanh((a*x+b)^(1/2)/b^(1/2))*a*x+(a*x+b)^(1/2)*b^(1/
2))/x^(1/2)/(a*x+b)^(1/2)/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx = \left[\frac{a\sqrt{b}x\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)-2b\sqrt{x}\sqrt{\frac{ax+b}{x}}}{2bx}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ax+b}\right)-b\sqrt{x}\sqrt{}}{bx}\right.$$

input
```
integrate((a+b/x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*x*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x)
 - 2*b*sqrt(x)*sqrt((a*x + b)/x))/(b*x), (a*sqrt(-b)*x*arctan(sqrt(-b)*sqr
t(x)*sqrt((a*x + b)/x)/(a*x + b)) - b*sqrt(x)*sqrt((a*x + b)/x))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx = -\frac{\sqrt{a}\sqrt{1+\frac{b}{ax}}}{\sqrt{x}} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{b}}$$

input
```
integrate((a+b/x)**(1/2)/x**(3/2),x)
```

output
```
-sqrt(a)*sqrt(1 + b/(a*x))/sqrt(x) - a*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/sq
rt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(38) = 76.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx = \frac{a\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{2\sqrt{b}} - \frac{\sqrt{a+\frac{b}{x}}a\sqrt{x}}{\left(a+\frac{b}{x}\right)x-b}$$

input
```
integrate((a+b/x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
1/2*a*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(
b)))/sqrt(b) - sqrt(a + b/x)*a*sqrt(x)/((a + b/x)*x - b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx = \left(\frac{\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{\sqrt{ax+b}\,\mathrm{sgn}(x)}{ax}\right)a$$

input
```
integrate((a+b/x)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
(arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) - sqrt(a*x + b)*sgn(x)/(a*
x))*a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx$$

input
```
int((a + b/x)^(1/2)/x^(3/2),x)
```

output
```
int((a + b/x)^(1/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{3/2}}\,dx=\frac{-2\sqrt{ax+b}\,b+\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)ax-\sqrt{b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)ax}{2bx}$$

input
```
int((a+b/x)^(1/2)/x^(3/2),x)
```

output
```
( - 2*sqrt(a*x + b)*b + sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a*x - sqrt(b)
*log(sqrt(a*x + b) + sqrt(b))*a*x)/(2*b*x)
```

3.210 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx=-\frac{\sqrt{a+\frac{b}{x}}}{2x^{3/2}}-\frac{a\sqrt{a+\frac{b}{x}}}{4b\sqrt{x}}+\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4b^{3/2}}$$

output
```
-1/2*(a+b/x)^(1/2)/x^(3/2)-1/4*a*(a+b/x)^(1/2)/b/x^(1/2)+1/4*a^2*arctanh(b
^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{(-2b-ax)\sqrt{b+ax}}{4bx^2}+\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{4b^{3/2}}\right)}{\sqrt{b+ax}}$$

input
```
Integrate[Sqrt[a + b/x]/x^(5/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*(((-2*b - a*x)*Sqrt[b + a*x])/(4*b*x^2) + (a^2*ArcT
anh[Sqrt[b + a*x]/Sqrt[b]])/(4*b^(3/2))))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {860, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{860}$$

$$-2 \int \frac{\sqrt{a+\frac{b}{x}}}{x} d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{248}$$

$$-2 \left(\frac{1}{4} a \int \frac{1}{\sqrt{a+\frac{b}{x}}x} d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2 \left(\frac{1}{4} a \left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a \int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2 \left(\frac{1}{4} a \left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a \int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)$$

input
```
Int[Sqrt[a + b/x]/x^(5/2),x]
```

output
```
-2*(Sqrt[a + b/x]/(4*x^(3/2)) + (a*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTa
nh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*b^(3/2))))/4)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{(ax+2b)\sqrt{\frac{ax+b}{x}}}{4x^{\frac{3}{2}}b}+\frac{a^{2}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{4b^{\frac{3}{2}}\sqrt{ax+b}}$	69
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^{2}x^{2}+2\sqrt{ax+b}\,b^{\frac{3}{2}}+ax\sqrt{ax+b}\,\sqrt{b}\right)}{4x^{\frac{3}{2}}b^{\frac{3}{2}}\sqrt{ax+b}}$	73

input
```
int((a+b/x)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a*x+2*b)/x^(3/2)/b*((a*x+b)/x)^(1/2)+1/4/b^(3/2)*a^2*arctanh((a*x+b)
^(1/2)/b^(1/2))*((a*x+b)/x)^(1/2)/(a*x+b)^(1/2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.89

$$\int\frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx=\left[\frac{a^{2}\sqrt{b}x^{2}\log\left(\frac{ax+2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(abx+2\,b^{2})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{8\,b^{2}x^{2}},\right.$$
$$\left.-\frac{a^{2}\sqrt{-b}x^{2}\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+(abx+2\,b^{2})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{4\,b^{2}x^{2}}\right]$$

input
```
integrate((a+b/x)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/8*(a^2*sqrt(b)*x^2*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b
)/x) - 2*(a*b*x + 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x^2), -1/4*(a^2*s
qrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (a*b*x
+ 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx = -\frac{a^{\frac{3}{2}}}{4b\sqrt{x}\sqrt{1+\frac{b}{ax}}} - \frac{3\sqrt{a}}{4x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} + \frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{\frac{3}{2}}} - \frac{b}{2\sqrt{a}x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate((a+b/x)**(1/2)/x**(5/2),x)
```

output
```
-a**(3/2)/(4*b*sqrt(x)*sqrt(1 + b/(a*x))) - 3*sqrt(a)/(4*x**(3/2)*sqrt(1 +
 b/(a*x))) + a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*b**(3/2)) - b/(2*sqr
t(a)*x**(5/2)*sqrt(1 + b/(a*x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(58) = 116.

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx = -\frac{a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{3}{2}}} - \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}} + \sqrt{a+\frac{b}{x}}a^2b\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2bx^2 - 2\left(a+\frac{b}{x}\right)b^2x + b^3\right)}$$

input
```
integrate((a+b/x)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-1/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sq
rt(b)))/b^(3/2) - 1/4*((a + b/x)^(3/2)*a^2*x^(3/2) + sqrt(a + b/x)*a^2*b*s
qrt(x))/((a + b/x)^2*b*x^2 - 2*(a + b/x)*b^2*x + b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx = -\frac{\frac{a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}b}+\frac{(ax+b)^{\frac{3}{2}}a^3\mathrm{sgn}(x)+\sqrt{ax+b}a^3b\mathrm{sgn}(x)}{a^2bx^2}}{4a}$$

input
```
integrate((a+b/x)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/4*(a^3*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + ((a*x + b)^
(3/2)*a^3*sgn(x) + sqrt(a*x + b)*a^3*b*sgn(x))/(a^2*b*x^2))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx = \int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx$$

input
```
int((a + b/x)^(1/2)/x^(5/2),x)
```

output
```
int((a + b/x)^(1/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{5/2}}\,dx = \frac{-2\sqrt{ax+b}\,abx-4\sqrt{ax+b}\,b^2-\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^2x^2+\sqrt{b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)}{8b^2x^2}$$

input
```
int((a+b/x)^(1/2)/x^(5/2),x)
```

output
```
( - 2*sqrt(a*x + b)*a*b*x - 4*sqrt(a*x + b)*b**2 - sqrt(b)*log(sqrt(a*x +
b) - sqrt(b))*a**2*x**2 + sqrt(b)*log(sqrt(a*x + b) + sqrt(b))*a**2*x**2)/
(8*b**2*x**2)
```

3.211 $\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 106

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx=-\frac{\sqrt{a+\frac{b}{x}}}{3x^{5/2}}-\frac{a\sqrt{a+\frac{b}{x}}}{12bx^{3/2}}+\frac{a^2\sqrt{a+\frac{b}{x}}}{8b^2\sqrt{x}}-\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{8b^{5/2}}$$

output
```
-1/3*(a+b/x)^(1/2)/x^(5/2)-1/12*a*(a+b/x)^(1/2)/b/x^(3/2)+1/8*a^2*(a+b/x)^
(1/2)/b^2/x^(1/2)-1/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 10.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\left(\frac{\sqrt{b}(-8b^2-2abx+3a^2x^2)}{x^{5/2}}-\frac{3a^{5/2}\text{arcsinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{1+\frac{b}{ax}}}\right)}{24b^{5/2}}$$

input
```
Integrate[Sqrt[a + b/x]/x^(7/2),x]
```

output
```
(Sqrt[a + b/x]*((Sqrt[b]*(-8*b^2 - 2*a*b*x + 3*a^2*x^2))/x^(5/2) - (3*a^(5
/2)*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/Sqrt[1 + b/(a*x)]))/(24*b^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {860, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{\sqrt{a+\frac{b}{x}}}{x^2}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{248}$$

$$-2\left(\frac{1}{6}a\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{6x^{5/2}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{1}{6}a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\int \frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}}{4b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{6x^{5/2}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{1}{6}a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{4b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{6x^{5/2}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{1}{6}a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{4b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{6x^{5/2}}\right)$$

$\downarrow$ 219

$$-2\left(\frac{1}{6}a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{6x^{5/2}}\right)$$

input
```
Int[Sqrt[a + b/x]/x^(7/2),x]
```

output
```
-2*(Sqrt[a + b/x]/(6*x^(5/2)) + (a*(Sqrt[a + b/x]/(4*b*x^(3/2)) - (3*a*(Sq
rt[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(
2*b^(3/2))))/(4*b)))/6)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{\left(3a^{2}x^{2}-2abx-8b^{2}\right)\sqrt{\frac{ax+b}{x}}}{24x^{\frac{5}{2}}b^{2}}-\frac{a^{3}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{8b^{\frac{5}{2}}\sqrt{ax+b}}$	81
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(3\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^{3}x^{3}-3a^{2}x^{2}\sqrt{ax+b}\,\sqrt{b}+2a\,b^{\frac{3}{2}}x\sqrt{ax+b}+8\sqrt{ax+b}\,b^{\frac{5}{2}}\right)}{24x^{\frac{5}{2}}b^{\frac{5}{2}}\sqrt{ax+b}}$	92

input
```
int((a+b/x)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(3*a^2*x^2-2*a*b*x-8*b^2)/x^(5/2)/b^2*((a*x+b)/x)^(1/2)-1/8*a^3/b^(5/
2)*arctanh((a*x+b)^(1/2)/b^(1/2))*((a*x+b)/x)^(1/2)/(a*x+b)^(1/2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx = \left[\frac{3\,a^{3}\sqrt{b}x^{3}\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,(3\,a^{2}bx^{2}-2\,ab^{2}x-8\,b^{3})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{48\,b^{3}x^{3}}, \frac{3\,a^{3}\sqrt{-b}x^{3}\,a}{}\right.$$

input
```
integrate((a+b/x)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/48*(3*a^3*sqrt(b)*x^3*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
2*b)/x) + 2*(3*a^2*b*x^2 - 2*a*b^2*x - 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(
b^3*x^3), 1/24*(3*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/
x)/(a*x + b)) + (3*a^2*b*x^2 - 2*a*b^2*x - 8*b^3)*sqrt(x)*sqrt((a*x + b)/x
))/(b^3*x^3)]
```

Sympy [A] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx = \frac{a^{\frac{5}{2}}}{8b^2\sqrt{x}\sqrt{1+\frac{b}{ax}}} + \frac{a^{\frac{3}{2}}}{24bx^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} - \frac{5\sqrt{a}}{12x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}} - \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{8b^{\frac{5}{2}}} - \frac{b}{3\sqrt{a}x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate((a+b/x)**(1/2)/x**(7/2),x)
```

output
```
a**(5/2)/(8*b**2*sqrt(x)*sqrt(1 + b/(a*x))) + a**(3/2)/(24*b*x**(3/2)*sqrt
(1 + b/(a*x))) - 5*sqrt(a)/(12*x**(5/2)*sqrt(1 + b/(a*x))) - a**3*asinh(sq
rt(b)/(sqrt(a)*sqrt(x)))/(8*b**(5/2)) - b/(3*sqrt(a)*x**(7/2)*sqrt(1 + b/(
a*x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(78) = 156$.

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx = \frac{a^3 \log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{5}{2}}} + \frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}} - 8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}} - 3\sqrt{a+\frac{b}{x}}a^3b^2\sqrt{x}}{24\left(\left(a+\frac{b}{x}\right)^3b^2x^3 - 3\left(a+\frac{b}{x}\right)^2b^3x^2 + 3\left(a+\frac{b}{x}\right)b^4x - b^5\right)}$$

input
```
integrate((a+b/x)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
1/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sq
rt(b)))/b^(5/2) + 1/24*(3*(a + b/x)^(5/2)*a^3*x^(5/2) - 8*(a + b/x)^(3/2)*
a^3*b*x^(3/2) - 3*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*b^2*x^3 - 3*
(a + b/x)^2*b^3*x^2 + 3*(a + b/x)*b^4*x - b^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx = \frac{1}{24}\,a^3\left(\frac{3\,\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b^2} + \frac{3\,(ax+b)^{\frac{5}{2}}\operatorname{sgn}(x) - 8\,(ax+b)^{\frac{3}{2}}b\operatorname{sgn}(x) - 3\,\sqrt{ax+b}b}{a^3b^2x^3}\right.$$

input
```
integrate((a+b/x)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
1/24*a^3*(3*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b^2) + (3*(a*x
 + b)^(5/2)*sgn(x) - 8*(a*x + b)^(3/2)*b*sgn(x) - 3*sqrt(a*x + b)*b^2*sgn(
x))/(a^3*b^2*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx=\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx$$

input
```
int((a + b/x)^(1/2)/x^(7/2),x)
```

output
```
int((a + b/x)^(1/2)/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+\frac{b}{x}}}{x^{7/2}}\,dx=\frac{6\sqrt{ax+b}\,a^2b\,x^2-4\sqrt{ax+b}\,a\,b^2x-16\sqrt{ax+b}\,b^3+3\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^3x^3-3\surd}{48b^3x^3}$$

input
```
int((a+b/x)^(1/2)/x^(7/2),x)
```

output
```
(6*sqrt(a*x + b)*a**2*b*x**2 - 4*sqrt(a*x + b)*a*b**2*x - 16*sqrt(a*x + b)
*b**3 + 3*sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a**3*x**3 - 3*sqrt(b)*log(s
qrt(a*x + b) + sqrt(b))*a**3*x**3)/(48*b**3*x**3)
```

3.212 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2} \, dx = -\frac{32b^3\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}{1155a^4} + \frac{16b^2\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}}{231a^3} - \frac{4b\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}{33a^2} + \frac{2\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}}{11a}$$

output
```
-32/1155*b^3*(a+b/x)^(5/2)*x^(5/2)/a^4+16/231*b^2*(a+b/x)^(5/2)*x^(7/2)/a^
3-4/33*b*(a+b/x)^(5/2)*x^(9/2)/a^2+2/11*(a+b/x)^(5/2)*x^(11/2)/a
```

Mathematica [A] (verified)

Time = 4.74 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2} \, dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^2 \left(-16b^3+40ab^2x-70a^2bx^2+105a^3x^3\right)}{1155a^4}$$

input
```
Integrate[(a + b/x)^(3/2)*x^(9/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^2*(-16*b^3 + 40*a*b^2*x - 70*a^2*b*x^2
+ 105*a^3*x^3))/(1155*a^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{9/2}\left(a+\frac{b}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}}{11a}-\frac{6b\int\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}dx}{11a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}}{11a}-\frac{6b\left(\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}-\frac{4b\int\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}dx}{9a}\right)}{11a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}}{11a}-\frac{6b\left(\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}-\frac{4b\left(\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{2b\int\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}dx}{7a}\right)}{9a}\right)}{11a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}}{11a}-\frac{6b\left(\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}-\frac{4b\left(\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{4bx^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{35a^2}\right)}{9a}\right)}{11a}$$

input
```
Int[(a + b/x)^(3/2)*x^(9/2),x]
```

output
```
(2*(a + b/x)^(5/2)*x^(11/2))/(11*a) - (6*b*((2*(a + b/x)^(5/2)*x^(9/2))/(9
*a) - (4*b*((-4*b*(a + b/x)^(5/2)*x^(5/2))/(35*a^2) + (2*(a + b/x)^(5/2)*x
^(7/2))/(7*a)))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.53

method	result	size
orering	$\frac{2\left(105 a^{3} x^{3}-70 a^{2} b\,x^{2}+40 a\,b^{2} x-16 b^{3}\right) x^{\frac{3}{2}} \left(a x+b\right) \left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{1155 a^{4}}$	53
gosper	$\frac{2\left(a x+b\right) \left(105 a^{3} x^{3}-70 a^{2} b\,x^{2}+40 a\,b^{2} x-16 b^{3}\right) x^{\frac{3}{2}} \left(\frac{a x+b}{x}\right)^{\frac{3}{2}}}{1155 a^{4}}$	55
default	$\frac{2 \sqrt{\frac{a x+b}{x}}\, \sqrt{x}\, \left(a x+b\right)^{2} \left(105 a^{3} x^{3}-70 a^{2} b\,x^{2}+40 a\,b^{2} x-16 b^{3}\right)}{1155 a^{4}}$	57
risch	$\frac{2 \sqrt{\frac{a x+b}{x}}\, \sqrt{x}\, \left(105 a^{5} x^{5}+140 a^{4} b\,x^{4}+5 a^{3} b^{2} x^{3}-6 a^{2} b^{3} x^{2}+8 b^{4} x a-16 b^{5}\right)}{1155 a^{4}}$	72

input
```
int((a+b/x)^(3/2)*x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(105*a^3*x^3-70*a^2*b*x^2+40*a*b^2*x-16*b^3)/a^4*x^(3/2)*(a*x+b)*(a
+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx=\frac{2\,(105\,a^5x^5+140\,a^4bx^4+5\,a^3b^2x^3-6\,a^2b^3x^2+8\,ab^4x-16\,b^5)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{1155\,a^4}$$

input
```
integrate((a+b/x)^(3/2)*x^(9/2),x, algorithm="fricas")
```

output
```
2/1155*(105*a^5*x^5 + 140*a^4*b*x^4 + 5*a^3*b^2*x^3 - 6*a^2*b^3*x^2 + 8*a*
b^4*x - 16*b^5)*sqrt(x)*sqrt((a*x + b)/x)/a^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(87) = 174$.

Time = 123.13 (sec) , antiderivative size = 585, normalized size of antiderivative = 5.85

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx = \frac{210a^8b^{\frac{19}{2}}x^8\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} + \frac{910a^7b^{\frac{21}{2}}x^7\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} + \frac{1480a^6b^{\frac{23}{2}}x^6\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} + \frac{1068a^5b^{\frac{25}{2}}x^5\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} + \frac{290a^4b^{\frac{27}{2}}x^4\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} - \frac{10a^3b^{\frac{29}{2}}x^3\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} - \frac{60a^2b^{\frac{31}{2}}x^2\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} - \frac{80ab^{\frac{33}{2}}x\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}} - \frac{32b^{\frac{35}{2}}\sqrt{\frac{ax}{b}+1}}{1155a^7b^9x^3+3465a^6b^{10}x^2+3465a^5b^{11}x+1155a^4b^{12}}$$

input
```
integrate((a+b/x)**(3/2)*x**(9/2),x)
```

output
```
210*a**8*b**(19/2)*x**8*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b
**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) + 910*a**7*b**(21/2)*x**7
*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b
**11*x + 1155*a**4*b**12) + 1480*a**6*b**(23/2)*x**6*sqrt(a*x/b + 1)/(1155
*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**
12) + 1068*a**5*b**(25/2)*x**5*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465
*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) + 290*a**4*b**(27/
2)*x**4*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465
*a**5*b**11*x + 1155*a**4*b**12) - 10*a**3*b**(29/2)*x**3*sqrt(a*x/b + 1)/
(1155*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**
4*b**12) - 60*a**2*b**(31/2)*x**2*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3
465*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12) - 80*a*b**(33/2
)*x*sqrt(a*x/b + 1)/(1155*a**7*b**9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**
5*b**11*x + 1155*a**4*b**12) - 32*b**(35/2)*sqrt(a*x/b + 1)/(1155*a**7*b**
9*x**3 + 3465*a**6*b**10*x**2 + 3465*a**5*b**11*x + 1155*a**4*b**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx=\frac{2\left(105\left(a+\frac{b}{x}\right)^{\frac{11}{2}}x^{\frac{11}{2}}-385\left(a+\frac{b}{x}\right)^{\frac{9}{2}}bx^{\frac{9}{2}}+495\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^2x^{\frac{7}{2}}-231\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3x^{\frac{5}{2}}\right)}{1155\,a^4}$$

input
```
integrate((a+b/x)^(3/2)*x^(9/2),x, algorithm="maxima")
```

output
```
2/1155*(105*(a + b/x)^(11/2)*x^(11/2) - 385*(a + b/x)^(9/2)*b*x^(9/2) + 49
5*(a + b/x)^(7/2)*b^2*x^(7/2) - 231*(a + b/x)^(5/2)*b^3*x^(5/2))/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.10

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx = \frac{32\,b^{\frac{11}{2}}\,\mathrm{sgn}(x)}{1155\,a^4} + \frac{2\left(\frac{99\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2-35\,\sqrt{ax+b}b^3\right)b^2\mathrm{sgn}(x)}{a^3}+\frac{22\left(35\,(ax+b)^{\frac{9}{2}}-180\,(ax+b)^{\frac{7}{2}}b+378\,(ax+b)^{\frac{5}{2}}b^2-420\,(ax+b)^{\frac{3}{2}}b\right.}{a^3}\right.}{3465}$$

input
```
integrate((a+b/x)^(3/2)*x^(9/2),x, algorithm="giac")
```

output
```
32/1155*b^(11/2)*sgn(x)/a^4 + 2/3465*(99*(5*(a*x + b)^(7/2) - 21*(a*x + b)
^(5/2)*b + 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*b^2*sgn(x)/a^3 +
 22*(35*(a*x + b)^(9/2) - 180*(a*x + b)^(7/2)*b + 378*(a*x + b)^(5/2)*b^2
- 420*(a*x + b)^(3/2)*b^3 + 315*sqrt(a*x + b)*b^4)*b*sgn(x)/a^3 + 5*(63*(a
*x + b)^(11/2) - 385*(a*x + b)^(9/2)*b + 990*(a*x + b)^(7/2)*b^2 - 1386*(a
*x + b)^(5/2)*b^3 + 1155*(a*x + b)^(3/2)*b^4 - 693*sqrt(a*x + b)*b^5)*sgn(
x)/a^3)/a
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,a\,x^{11/2}}{11}+\frac{8\,b\,x^{9/2}}{33}+\frac{2\,b^2\,x^{7/2}}{231\,a}-\frac{4\,b^3\,x^{5/2}}{385\,a^2}+\frac{16\,b^4\,x^{3/2}}{1155\,a^3}-\frac{32\,b^5\,\sqrt{x}}{1155\,a^4}\right)$$

input
```
int(x^(9/2)*(a + b/x)^(3/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a*x^(11/2))/11 + (8*b*x^(9/2))/33 + (2*b^2*x^(7/2))/(2
31*a) - (4*b^3*x^(5/2))/(385*a^2) + (16*b^4*x^(3/2))/(1155*a^3) - (32*b^5*
x^(1/2))/(1155*a^4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.63

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{9/2}\,dx = \frac{2\sqrt{ax+b}\,(105a^5x^5+140a^4b\,x^4+5a^3b^2x^3-6a^2b^3x^2+8a\,b^4x-16b^5)}{1155a^4}$$

input
```
int((a+b/x)^(3/2)*x^(9/2),x)
```

output
```
(2*sqrt(a*x + b)*(105*a**5*x**5 + 140*a**4*b*x**4 + 5*a**3*b**2*x**3 - 6*a
**2*b**3*x**2 + 8*a*b**4*x - 16*b**5))/(1155*a**4)
```

3.213 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx=\frac{16b^2\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}{315a^3}-\frac{8b\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}{63a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}{9a}$$

output
```
16/315*b^2*(a+b/x)^(5/2)*x^(5/2)/a^3-8/63*b*(a+b/x)^(5/2)*x^(7/2)/a^2+2/9*
(a+b/x)^(5/2)*x^(9/2)/a
```

Mathematica [A] (verified)

Time = 4.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^2\left(8b^2-20abx+35a^2x^2\right)}{315a^3}$$

input
```
Integrate[(a + b/x)^(3/2)*x^(7/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^2*(8*b^2 - 20*a*b*x + 35*a^2*x^2))/(315
*a^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}-\frac{4b\int \left(a+\frac{b}{x}\right)^{3/2}x^{5/2}dx}{9a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}-\frac{4b\left(\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{2b\int \left(a+\frac{b}{x}\right)^{3/2}x^{3/2}dx}{7a}\right)}{9a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}{9a}-\frac{4b\left(\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{4bx^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{35a^2}\right)}{9a}$$

input
```
Int[(a + b/x)^(3/2)*x^(7/2),x]
```

output
```
(2*(a + b/x)^(5/2)*x^(9/2))/(9*a) - (4*b*((-4*b*(a + b/x)^(5/2)*x^(5/2))/(
35*a^2) + (2*(a + b/x)^(5/2)*x^(7/2))/(7*a)))/(9*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

method	result	size
orering	$\frac{2(35a^2x^2-20abx+8b^2)(ax+b)x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{315a^3}$	42
gosper	$\frac{2(ax+b)(35a^2x^2-20abx+8b^2)x^{\frac{3}{2}}\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{315a^3}$	44
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(ax+b)^2(35a^2x^2-20abx+8b^2)}{315a^3}$	46
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(35a^4x^4+50a^3b\,x^3+3a^2b^2x^2-4a\,b^3x+8b^4)}{315a^3}$	61

input
```
int((a+b/x)^(3/2)*x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(35*a^2*x^2-20*a*b*x+8*b^2)/a^3*(a*x+b)*x^(3/2)*(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx = \frac{2\,(35\,a^4x^4+50\,a^3bx^3+3\,a^2b^2x^2-4\,ab^3x+8\,b^4)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{315\,a^3}$$

input
```
integrate((a+b/x)^(3/2)*x^(7/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^4*x^4 + 50*a^3*b*x^3 + 3*a^2*b^2*x^2 - 4*a*b^3*x + 8*b^4)*sqrt
(x)*sqrt((a*x + b)/x)/a^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(63) = 126$.

Time = 34.04 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.99

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx &= \frac{70a^6b^{\frac{9}{2}}x^6\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6} \\ &+ \frac{240a^5b^{\frac{11}{2}}x^5\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6} + \frac{276a^4b^{\frac{13}{2}}x^4\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6} \\ &+ \frac{104a^3b^{\frac{15}{2}}x^3\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6} + \frac{6a^2b^{\frac{17}{2}}x^2\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6} \\ &+ \frac{24ab^{\frac{19}{2}}x\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6} + \frac{16b^{\frac{21}{2}}\sqrt{\frac{ax}{b}+1}}{315a^5b^4x^2+630a^4b^5x+315a^3b^6}\end{aligned}$$

input
```
integrate((a+b/x)**(3/2)*x**(7/2),x)
```

output
```
70*a**6*b**(9/2)*x**6*sqrt(a*x/b + 1)/(315*a**5*b**4*x**2 + 630*a**4*b**5*
x + 315*a**3*b**6) + 240*a**5*b**(11/2)*x**5*sqrt(a*x/b + 1)/(315*a**5*b**
4*x**2 + 630*a**4*b**5*x + 315*a**3*b**6) + 276*a**4*b**(13/2)*x**4*sqrt(a
*x/b + 1)/(315*a**5*b**4*x**2 + 630*a**4*b**5*x + 315*a**3*b**6) + 104*a**
3*b**(15/2)*x**3*sqrt(a*x/b + 1)/(315*a**5*b**4*x**2 + 630*a**4*b**5*x + 3
15*a**3*b**6) + 6*a**2*b**(17/2)*x**2*sqrt(a*x/b + 1)/(315*a**5*b**4*x**2
+ 630*a**4*b**5*x + 315*a**3*b**6) + 24*a*b**(19/2)*x*sqrt(a*x/b + 1)/(315
*a**5*b**4*x**2 + 630*a**4*b**5*x + 315*a**3*b**6) + 16*b**(21/2)*sqrt(a*x
/b + 1)/(315*a**5*b**4*x**2 + 630*a**4*b**5*x + 315*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx = \frac{2\left(35\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}} - 90\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}} + 63\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^2x^{\frac{5}{2}}\right)}{315\,a^3}$$

input
```
integrate((a+b/x)^(3/2)*x^(7/2),x, algorithm="maxima")
```

output
```
2/315*(35*(a + b/x)^(9/2)*x^(9/2) - 90*(a + b/x)^(7/2)*b*x^(7/2) + 63*(a +
 b/x)^(5/2)*b^2*x^(5/2))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.34

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx = -\frac{16\,b^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,a^3} + \frac{2\left(\frac{21\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b+15\,\sqrt{ax+b}\,b^2\right)b^2\mathrm{sgn}(x)}{a^2} + \frac{18\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2-35\,\sqrt{ax+b}\,b^3\right)b\,\mathrm{sgn}(x)}{a^2} + \frac{\left(35\,(ax \cdots}{\cdots}\right.}{315\,a}$$

input
```
integrate((a+b/x)^(3/2)*x^(7/2),x, algorithm="giac")
```

output
```
-16/315*b^(9/2)*sgn(x)/a^3 + 2/315*(21*(3*(a*x + b)^(5/2) - 10*(a*x + b)^(
3/2)*b + 15*sqrt(a*x + b)*b^2)*b^2*sgn(x)/a^2 + 18*(5*(a*x + b)^(7/2) - 21
*(a*x + b)^(5/2)*b + 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*b*sgn(
x)/a^2 + (35*(a*x + b)^(9/2) - 180*(a*x + b)^(7/2)*b + 378*(a*x + b)^(5/2)
*b^2 - 420*(a*x + b)^(3/2)*b^3 + 315*sqrt(a*x + b)*b^4)*sgn(x)/a^2)/a
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx=\sqrt{a+\frac{b}{x}}\left(\frac{2\,a\,x^{9/2}}{9}+\frac{20\,b\,x^{7/2}}{63}+\frac{2\,b^2\,x^{5/2}}{105\,a}-\frac{8\,b^3\,x^{3/2}}{315\,a^2}+\frac{16\,b^4\,\sqrt{x}}{315\,a^3}\right)$$

input
```
int(x^(7/2)*(a + b/x)^(3/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a*x^(9/2))/9 + (20*b*x^(7/2))/63 + (2*b^2*x^(5/2))/(10
5*a) - (8*b^3*x^(3/2))/(315*a^2) + (16*b^4*x^(1/2))/(315*a^3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{7/2}\,dx=\frac{2\sqrt{ax+b}\,(35a^4x^4+50a^3b\,x^3+3a^2b^2x^2-4a\,b^3x+8b^4)}{315a^3}$$

input
```
int((a+b/x)^(3/2)*x^(7/2),x)
```

output
```
(2*sqrt(a*x + b)*(35*a**4*x**4 + 50*a**3*b*x**3 + 3*a**2*b**2*x**2 - 4*a*b
**3*x + 8*b**4))/(315*a**3)
```

3.214 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx=-\frac{4b\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}{35a^2}+\frac{2\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}}{7a}$$

output
```
-4/35*b*(a+b/x)^(5/2)*x^(5/2)/a^2+2/7*(a+b/x)^(5/2)*x^(7/2)/a
```

Mathematica [A] (verified)

Time = 4.69 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^2(-2b+5ax)}{35a^2}$$

input
```
Integrate[(a + b/x)^(3/2)*x^(5/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^2*(-2*b + 5*a*x))/(35*a^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{2b\int \left(a+\frac{b}{x}\right)^{3/2}x^{3/2}dx}{7a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}{7a}-\frac{4bx^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{35a^2}$$

input
```
Int[(a + b/x)^(3/2)*x^(5/2),x]
```

output
```
(-4*b*(a + b/x)^(5/2)*x^(5/2))/(35*a^2) + (2*(a + b/x)^(5/2)*x^(7/2))/(7*a
)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

method	result	size
orering	$\frac{2(5ax-2b)(ax+b)x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{35a^{2}}$	31
gosper	$\frac{2(ax+b)(5ax-2b)x^{\frac{3}{2}}\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}{35a^{2}}$	33
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(ax+b)^{2}(5ax-2b)}{35a^{2}}$	35
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(5a^{3}x^{3}+8a^{2}b\,x^{2}+a\,b^{2}x-2b^{3})}{35a^{2}}$	49

input
```
int((a+b/x)^(3/2)*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(5*a*x-2*b)/a^2*(a*x+b)*x^(3/2)*(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx=\frac{2\,(5\,a^{3}x^{3}+8\,a^{2}bx^{2}+ab^{2}x-2\,b^{3})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{35\,a^{2}}$$

input
```
integrate((a+b/x)^(3/2)*x^(5/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^3*x^3 + 8*a^2*b*x^2 + a*b^2*x - 2*b^3)*sqrt(x)*sqrt((a*x + b)/x)
/a^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(39) = 78.

Time = 11.89 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.83

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx = \frac{2a\sqrt{b}x^3\sqrt{\frac{ax}{b}+1}}{7} + \frac{16b^{\frac{3}{2}}x^2\sqrt{\frac{ax}{b}+1}}{35} + \frac{2b^{\frac{5}{2}}x\sqrt{\frac{ax}{b}+1}}{35a} - \frac{4b^{\frac{7}{2}}\sqrt{\frac{ax}{b}+1}}{35a^2}$$

input
```
integrate((a+b/x)**(3/2)*x**(5/2),x)
```

output
```
2*a*sqrt(b)*x**3*sqrt(a*x/b + 1)/7 + 16*b**(3/2)*x**2*sqrt(a*x/b + 1)/35 +
 2*b**(5/2)*x*sqrt(a*x/b + 1)/(35*a) - 4*b**(7/2)*sqrt(a*x/b + 1)/(35*a**2
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx = \frac{2\left(5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}} - 7\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}}\right)}{35\,a^2}$$

input
```
integrate((a+b/x)^(3/2)*x^(5/2),x, algorithm="maxima")
```

output
```
2/35*(5*(a + b/x)^(7/2)*x^(7/2) - 7*(a + b/x)^(5/2)*b*x^(5/2))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx = \frac{4\,b^{\frac{7}{2}}\,\mathrm{sgn}(x)}{35\,a^2} + \frac{2\left(\frac{35\left((ax+b)^{\frac{3}{2}}-3\,\sqrt{ax+b}b\right)b^2\mathrm{sgn}(x)}{a}+\frac{14\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b+15\,\sqrt{ax+b}b^2\right)b\mathrm{sgn}(x)}{a}+\frac{3\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}\right.}{a}\right.}{105\,a}$$

input
```
integrate((a+b/x)^(3/2)*x^(5/2),x, algorithm="giac")
```

output
```
4/35*b^(7/2)*sgn(x)/a^2 + 2/105*(35*((a*x + b)^(3/2) - 3*sqrt(a*x + b)*b)*
b^2*sgn(x)/a + 14*(3*(a*x + b)^(5/2) - 10*(a*x + b)^(3/2)*b + 15*sqrt(a*x
+ b)*b^2)*b*sgn(x)/a + 3*(5*(a*x + b)^(7/2) - 21*(a*x + b)^(5/2)*b + 35*(a
*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*sgn(x)/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,a\,x^{7/2}}{7}+\frac{16\,b\,x^{5/2}}{35}+\frac{2\,b^2\,x^{3/2}}{35\,a}-\frac{4\,b^3\,\sqrt{x}}{35\,a^2}\right)$$

input
```
int(x^(5/2)*(a + b/x)^(3/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a*x^(7/2))/7 + (16*b*x^(5/2))/35 + (2*b^2*x^(3/2))/(35
*a) - (4*b^3*x^(1/2))/(35*a^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{5/2}\,dx = \frac{2\sqrt{ax+b}\,(5a^3x^3+8a^2b\,x^2+a\,b^2x-2b^3)}{35a^2}$$

input
```
int((a+b/x)^(3/2)*x^(5/2),x)
```

output
```
(2*sqrt(a*x + b)*(5*a**3*x**3 + 8*a**2*b*x**2 + a*b**2*x - 2*b**3))/(35*a*
*2)
```

3.215 $\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2} \, dx=\frac{2\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}{5a}$$

output
```
2/5*(a+b/x)^(5/2)*x^(5/2)/a
```

Mathematica [A] (verified)

Time = 4.68 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2} \, dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^2}{5a}$$

input
```
Integrate[(a + b/x)^(3/2)*x^(3/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^2)/(5*a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\,dx$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}}{5a}$$

input
```
Int[(a + b/x)^(3/2)*x^(3/2),x]
```

output
```
(2*(a + b/x)^(5/2)*x^(5/2))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
orering	$\frac{2(ax+b)x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{5a}$	23
gosper	$\frac{2(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}{5a}$	25
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(ax+b)^{2}}{5a}$	27
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(a^{2}x^{2}+2abx+b^{2})}{5a}$	36

input
```
int((a+b/x)^(3/2)*x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/a*(a*x+b)*x^(3/2)*(a+b/x)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\,dx = \frac{2\,(a^2x^2+2\,abx+b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{5\,a}$$

input
```
integrate((a+b/x)^(3/2)*x^(3/2),x, algorithm="fricas")
```

output
```
2/5*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(x)*sqrt((a*x + b)/x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 4.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\,dx = \frac{2a\sqrt{b}x^2\sqrt{\frac{ax}{b}+1}}{5} + \frac{4b^{\frac{3}{2}}x\sqrt{\frac{ax}{b}+1}}{5} + \frac{2b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{5a}$$

input
```
integrate((a+b/x)**(3/2)*x**(3/2),x)
```

output
```
2*a*sqrt(b)*x**2*sqrt(a*x/b + 1)/5 + 4*b**(3/2)*x*sqrt(a*x/b + 1)/5 + 2*b*
*(5/2)*sqrt(a*x/b + 1)/(5*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\,dx = \frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}{5\,a}$$

input
```
integrate((a+b/x)^(3/2)*x^(3/2),x, algorithm="maxima")
```

output
```
2/5*(a + b/x)^(5/2)*x^(5/2)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\,dx = -\frac{2\,b^{\frac{5}{2}}\operatorname{sgn}(x)}{5\,a} + \frac{2\left(15\sqrt{ax+b}b^2\operatorname{sgn}(x) + 10\left((ax+b)^{\frac{3}{2}} - 3\sqrt{ax+b}b\right)b\operatorname{sgn}(x) + \left(3\,(ax+b)^{\frac{5}{2}} - 10\,(ax+b)^{\frac{3}{2}}b + 15\sqrt{ax}\right.\right.}{15\,a}$$

input
```
integrate((a+b/x)^(3/2)*x^(3/2),x, algorithm="giac")
```

output
```
-2/5*b^(5/2)*sgn(x)/a + 2/15*(15*sqrt(a*x + b)*b^2*sgn(x) + 10*((a*x + b)^
(3/2) - 3*sqrt(a*x + b)*b)*b*sgn(x) + (3*(a*x + b)^(5/2) - 10*(a*x + b)^(3
/2)*b + 15*sqrt(a*x + b)*b^2)*sgn(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\, dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,a\,x^{5/2}}{5}+\frac{4\,b\,x^{3/2}}{5}+\frac{2\,b^2\,\sqrt{x}}{5\,a}\right)$$

input
```
int(x^(3/2)*(a + b/x)^(3/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a*x^(5/2))/5 + (4*b*x^(3/2))/5 + (2*b^2*x^(1/2))/(5*a)
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x}\right)^{3/2} x^{3/2}\, dx = \frac{2\sqrt{ax+b}\,(a^2x^2+2abx+b^2)}{5a}$$

input
```
int((a+b/x)^(3/2)*x^(3/2),x)
```

output
```
(2*sqrt(a*x + b)*(a**2*x**2 + 2*a*b*x + b**2))/(5*a)
```

3.216 $\int \left(a+\frac{b}{x}\right)^{3/2} \sqrt{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \left(a+\frac{b}{x}\right)^{3/2} \sqrt{x}\,dx=\frac{8}{3}b\sqrt{a+\frac{b}{x}}\sqrt{x}+\frac{2}{3}a\sqrt{a+\frac{b}{x}}x^{3/2}-2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

output
```
8/3*b*(a+b/x)^(1/2)*x^(1/2)+2/3*a*(a+b/x)^(1/2)*x^(3/2)-2*b^(3/2)*arctanh(
b^(1/2)/(a+b/x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 4.70 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x}\right)^{3/2} \sqrt{x}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{2}{3}\sqrt{b+ax}(4b+ax)-2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(3/2)*Sqrt[x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((2*Sqrt[b + a*x]*(4*b + a*x))/3 - 2*b^(3/2)*ArcTan
h[Sqrt[b + a*x]/Sqrt[b]]))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {860, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \left(a+\frac{b}{x}\right)^{3/2}x^2d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{247}$$

$$-2\left(b\int \sqrt{a+\frac{b}{x}}xd\frac{1}{\sqrt{x}}-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{247}$$

$$-2\left(b\left(b\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(b\left(b\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(b\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(3/2)*Sqrt[x],x]
```

output
```
-2*(-1/3*((a + b/x)^(3/2)*x^(3/2)) + b*(-(Sqrt[a + b/x]*Sqrt[x]) + Sqrt[b]
*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(-3b^{\frac{3}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)+ax\sqrt{ax+b}+4b\sqrt{ax+b}\right)}{3\sqrt{ax+b}}$	62

input
```
int((a+b/x)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((a*x+b)/x)^(1/2)*x^(1/2)*(-3*b^(3/2)*arctanh((a*x+b)^(1/2)/b^(1/2))+a
*x*(a*x+b)^(1/2)+4*b*(a*x+b)^(1/2))/(a*x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}\,dx = \left[b^{\frac{3}{2}}\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+\frac{2}{3}(ax+4b)\sqrt{x}\sqrt{\frac{ax+b}{x}},\ 2\sqrt{-b}b\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+\frac{2}{3}(ax+4b)\sqrt{x}\sqrt{\frac{ax+b}{x}}\right]$$

input
```
integrate((a+b/x)^(3/2)*x^(1/2),x, algorithm="fricas")
```

output
```
[b^(3/2)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2/3*(a
*x + 4*b)*sqrt(x)*sqrt((a*x + b)/x), 2*sqrt(-b)*b*arctan(sqrt(-b)*sqrt(x)*
sqrt((a*x + b)/x)/(a*x + b)) + 2/3*(a*x + 4*b)*sqrt(x)*sqrt((a*x + b)/x)]
```

Sympy [A] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}\,dx = \frac{2a\sqrt{b}x\sqrt{\frac{ax}{b}+1}}{3}+\frac{8b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3}+b^{\frac{3}{2}}\log\left(\frac{ax}{b}\right)-2b^{\frac{3}{2}}\log\left(\sqrt{\frac{ax}{b}+1}+1\right)$$

input
```
integrate((a+b/x)**(3/2)*x**(1/2),x)
```

output
```
2*a*sqrt(b)*x*sqrt(a*x/b + 1)/3 + 8*b**(3/2)*sqrt(a*x/b + 1)/3 + b**(3/2)*
log(a*x/b) - 2*b**(3/2)*log(sqrt(a*x/b + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}\,dx=\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}+b^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)+2\sqrt{a+\frac{b}{x}}b\sqrt{x}$$

input
```
integrate((a+b/x)^(3/2)*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a + b/x)^(3/2)*x^(3/2) + b^(3/2)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b)
)/(sqrt(a + b/x)*sqrt(x) + sqrt(b))) + 2*sqrt(a + b/x)*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int \left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}\,dx=\frac{2\,b^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+\frac{2}{3}\,(ax+b)^{\frac{3}{2}}\mathrm{sgn}(x)+2\sqrt{ax+b}b\mathrm{sgn}(x)-\frac{2\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}b^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\sqrt{-b}}$$

input
```
integrate((a+b/x)^(3/2)*x^(1/2),x, algorithm="giac")
```

output
```
2*b^2*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2/3*(a*x + b)^(3/2)
*sgn(x) + 2*sqrt(a*x + b)*b*sgn(x) - 2/3*(3*b^2*arctan(sqrt(b)/sqrt(-b)) +
 4*sqrt(-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^{3/2} \sqrt{x}\,dx = \int \sqrt{x}\left(a+\frac{b}{x}\right)^{3/2} dx$$

input
```
int(x^(1/2)*(a + b/x)^(3/2),x)
```

output
```
int(x^(1/2)*(a + b/x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x}\right)^{3/2} \sqrt{x}\,dx = \frac{2\sqrt{ax+b}\,ax}{3} + \frac{8\sqrt{ax+b}\,b}{3} + \sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)b - \sqrt{b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)b$$

input
```
int((a+b/x)^(3/2)*x^(1/2),x)
```

output
```
(2*sqrt(a*x + b)*a*x + 8*sqrt(a*x + b)*b + 3*sqrt(b)*log(sqrt(a*x + b) - s
qrt(b))*b - 3*sqrt(b)*log(sqrt(a*x + b) + sqrt(b))*b)/3
```

3.217 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx=-\frac{b\sqrt{a+\frac{b}{x}}}{\sqrt{x}}+2a\sqrt{a+\frac{b}{x}}\sqrt{x}-3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

output
```
-b*(a+b/x)^(1/2)/x^(1/2)+2*a*(a+b/x)^(1/2)*x^(1/2)-3*a*b^(1/2)*arctanh(b^(
1/2)/(a+b/x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx=-\frac{\sqrt{a+\frac{b}{x}}\left((b-2ax)\sqrt{b+ax}+3a\sqrt{b}x\operatorname{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)\right)}{\sqrt{x}\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(3/2)/Sqrt[x],x]
```

output
```
-((Sqrt[a + b/x]*((b - 2*a*x)*Sqrt[b + a*x] + 3*a*Sqrt[b]*x*ArcTanh[Sqrt[b
 + a*x]/Sqrt[b]]))/(Sqrt[x]*Sqrt[b + a*x]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {860, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{247}$$

$$-2\left(3b\int \sqrt{a+\frac{b}{x}}d\frac{1}{\sqrt{x}}-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$-2\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)$$

input
```
Int[(a + b/x)^(3/2)/Sqrt[x],x]
```

output
```
-2*(-((a + b/x)^(3/2)*Sqrt[x]) + 3*b*(Sqrt[a + b/x]/(2*Sqrt[x]) + (a*ArcTa
nh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*Sqrt[b])))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(\sqrt{ax+b}\,b^{\frac{3}{2}}-2ax\sqrt{ax+b}\,\sqrt{b}+3\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)abx\right)}{\sqrt{x}\,\sqrt{ax+b}\,\sqrt{b}}$	70
risch	$-\frac{b\sqrt{\frac{ax+b}{x}}}{\sqrt{x}}+\frac{a\left(4\sqrt{ax+b}-6\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{2\sqrt{ax+b}}$	70

input
```
int((a+b/x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x+b)/x)^(1/2)*((a*x+b)^(1/2)*b^(3/2)-2*a*x*(a*x+b)^(1/2)*b^(1/2)+3*ar
ctanh((a*x+b)^(1/2)/b^(1/2))*a*b*x)/x^(1/2)/(a*x+b)^(1/2)/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.91

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx = \left[\frac{3\,a\sqrt{b}x\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,(2\,ax-b)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{2\,x}, \frac{3\,a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{}}{ax+b}\right.}{}\right.$$

input
```
integrate((a+b/x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(3*a*sqrt(b)*x*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/
x) + 2*(2*a*x - b)*sqrt(x)*sqrt((a*x + b)/x))/x, (3*a*sqrt(-b)*x*arctan(sq
rt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (2*a*x - b)*sqrt(x)*sqrt((a*
x + b)/x))/x]
```

Sympy [A] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2a^{\frac{3}{2}}\sqrt{x}}{\sqrt{1+\frac{b}{ax}}} + \frac{\sqrt{a}b}{\sqrt{x}\sqrt{1+\frac{b}{ax}}} - 3a\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right) - \frac{b^2}{\sqrt{a}x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate((a+b/x)**(3/2)/x**(1/2),x)
```

output
```
2*a**(3/2)*sqrt(x)/sqrt(1 + b/(a*x)) + sqrt(a)*b/(sqrt(x)*sqrt(1 + b/(a*x)
)) - 3*a*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*sqrt(x))) - b**2/(sqrt(a)*x**(3/2)
*sqrt(1 + b/(a*x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx = \frac{3}{2}a\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right) + 2\sqrt{a+\frac{b}{x}}a\sqrt{x} - \frac{\sqrt{a+\frac{b}{x}}ab\sqrt{x}}{\left(a+\frac{b}{x}\right)x-b}$$

input
```
integrate((a+b/x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
3/2*a*sqrt(b)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x)
 + sqrt(b))) + 2*sqrt(a + b/x)*a*sqrt(x) - sqrt(a + b/x)*a*b*sqrt(x)/((a +
 b/x)*x - b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx = \left(\frac{3\,b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+2\,\sqrt{ax+b}\operatorname{sgn}(x)-\frac{\sqrt{ax+b}b\operatorname{sgn}(x)}{ax}\right)a$$

input
```
integrate((a+b/x)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
(3*b*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(a*x + b)*sgn(
x) - sqrt(a*x + b)*b*sgn(x)/(a*x))*a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx$$

input
```
int((a + b/x)^(3/2)/x^(1/2),x)
```

output
```
int((a + b/x)^(3/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\sqrt{x}}\,dx = \frac{4\sqrt{ax+b}\,ax-2\sqrt{ax+b}\,b+3\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)ax-3\sqrt{b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)a}{2x}$$

input
```
int((a+b/x)^(3/2)/x^(1/2),x)
```

output
```
(4*sqrt(a*x + b)*a*x - 2*sqrt(a*x + b)*b + 3*sqrt(b)*log(sqrt(a*x + b) - s
qrt(b))*a*x - 3*sqrt(b)*log(sqrt(a*x + b) + sqrt(b))*a*x)/(2*x)
```

3.218 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}} dx = -\frac{3a\sqrt{a+\frac{b}{x}}}{4\sqrt{x}} - \frac{\left(a+\frac{b}{x}\right)^{3/2}}{2\sqrt{x}} - \frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4\sqrt{b}}$$

output
```
-3/4*a*(a+b/x)^(1/2)/x^(1/2)-1/2*(a+b/x)^(3/2)/x^(1/2)-3/4*a^2*arctanh(b^(
1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 4.80 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}} dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{(-2b-5ax)\sqrt{b+ax}}{4x^2} - \frac{3a^2\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{4\sqrt{b}}\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(3/2)/x^(3/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*(((-2*b - 5*a*x)*Sqrt[b + a*x])/(4*x^2) - (3*a^2*Ar
cTanh[Sqrt[b + a*x]/Sqrt[b]])/(4*Sqrt[b])))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {860, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 860}$$

$$-2\int \left(a+\frac{b}{x}\right)^{3/2}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 211}$$

$$-2\left(\frac{3}{4}a\int \sqrt{a+\frac{b}{x}}d\frac{1}{\sqrt{x}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 211}$$

$$-2\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-2\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-2\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)$$

input
```
Int[(a + b/x)^(3/2)/x^(3/2),x]
```

output
```
-2*((a + b/x)^(3/2)/(4*Sqrt[x]) + (3*a*(Sqrt[a + b/x]/(2*Sqrt[x]) + (a*Arc
Tanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*Sqrt[b])))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{(5ax+2b)\sqrt{\frac{ax+b}{x}}}{4x^{\frac{3}{2}}}-\frac{3a^{2}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{4\sqrt{b}\,\sqrt{ax+b}}$	67
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(3\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^{2}x^{2}+5ax\sqrt{ax+b}\,\sqrt{b}+2\sqrt{ax+b}\,b^{\frac{3}{2}}\right)}{4x^{\frac{3}{2}}\sqrt{ax+b}\,\sqrt{b}}$	74

input
```
int((a+b/x)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(5*a*x+2*b)/x^(3/2)*((a*x+b)/x)^(1/2)-3/4*a^2/b^(1/2)*arctanh((a*x+b)
^(1/2)/b^(1/2))*((a*x+b)/x)^(1/2)/(a*x+b)^(1/2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.03

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx = \left[\frac{3\,a^2\sqrt{b}x^2\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(5\,abx+2\,b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{8\,bx^2}, \frac{3\,a^2\sqrt{-b}x^2\arctan\left(\right.}{}\right.$$

input
```
integrate((a+b/x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*x^2*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2
*b)/x) - 2*(5*a*b*x + 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b*x^2), 1/4*(3*a^
2*sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) - (5*a
*b*x + 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx = -\frac{5a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}{4\sqrt{x}}-\frac{\sqrt{a}b\sqrt{1+\frac{b}{ax}}}{2x^{\frac{3}{2}}}-\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4\sqrt{b}}$$

input
```
integrate((a+b/x)**(3/2)/x**(3/2),x)
```

output
```
-5*a**(3/2)*sqrt(1 + b/(a*x))/(4*sqrt(x)) - sqrt(a)*b*sqrt(1 + b/(a*x))/(2
*x**(3/2)) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.52

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx = \frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{8\,\sqrt{b}} - \frac{5\,\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}} - 3\,\sqrt{a+\frac{b}{x}}a^2b\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2x^2 - 2\left(a+\frac{b}{x}\right)bx + b^2\right)}$$

input
```
integrate((a+b/x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
3/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqr
t(b)))/sqrt(b) - 1/4*(5*(a + b/x)^(3/2)*a^2*x^(3/2) - 3*sqrt(a + b/x)*a^2*
b*sqrt(x))/((a + b/x)^2*x^2 - 2*(a + b/x)*b*x + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx = \frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{5\,(ax+b)^{\frac{3}{2}}a^3\mathrm{sgn}(x) - 3\,\sqrt{ax+b}a^3b\mathrm{sgn}(x)}{a^2x^2}}{4\,a}$$

input
```
integrate((a+b/x)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
1/4*(3*a^3*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (5*(a*x + b)^(
3/2)*a^3*sgn(x) - 3*sqrt(a*x + b)*a^3*b*sgn(x))/(a^2*x^2))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx$$

input
```
int((a + b/x)^(3/2)/x^(3/2),x)
```

output
```
int((a + b/x)^(3/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{3/2}}\,dx=\frac{-10\sqrt{ax+b}\,abx-4\sqrt{ax+b}\,b^2+3\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^2x^2-3\sqrt{b}\log\left(\sqrt{ax+b}-\right.}{8b\,x^2}$$

input
```
int((a+b/x)^(3/2)/x^(3/2),x)
```

output
```
( - 10*sqrt(a*x + b)*a*b*x - 4*sqrt(a*x + b)*b**2 + 3*sqrt(b)*log(sqrt(a*x
 + b) - sqrt(b))*a**2*x**2 - 3*sqrt(b)*log(sqrt(a*x + b) + sqrt(b))*a**2*x
**2)/(8*b*x**2)
```

3.219 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx=-\frac{a\sqrt{a+\frac{b}{x}}}{4x^{3/2}}-\frac{\left(a+\frac{b}{x}\right)^{3/2}}{3x^{3/2}}-\frac{a^2\sqrt{a+\frac{b}{x}}}{8b\sqrt{x}}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{8b^{3/2}}$$

output
```
-1/4*a*(a+b/x)^(1/2)/x^(3/2)-1/3*(a+b/x)^(3/2)/x^(3/2)-1/8*a^2*(a+b/x)^(1/
2)/b/x^(1/2)+1/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 4.74 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{\sqrt{b+ax}(-8b^2-14abx-3a^2x^2)}{24bx^3}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{8b^{3/2}}\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(3/2)/x^(5/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((Sqrt[b + a*x]*(-8*b^2 - 14*a*b*x - 3*a^2*x^2))/(2
4*b*x^3) + (a^3*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/(8*b^(3/2))))/Sqrt[b + a*x
]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {860, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{248}$$

$$-2\left(\frac{1}{2}a\int \frac{\sqrt{a+\frac{b}{x}}}{x}d\frac{1}{\sqrt{x}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)$$

$$\Big\downarrow \text{248}$$

$$-2\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)$$

input
```
Int[(a + b/x)^(3/2)/x^(5/2),x]
```

output
```
-2*((a + b/x)^(3/2)/(6*x^(3/2)) + (a*(Sqrt[a + b/x]/(4*x^(3/2)) + (a*(Sqrt
[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*
b^(3/2))))/4))/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{\left(3 a^{2} x^{2}+14 a b x +8 b^{2}\right) \sqrt{\frac{a x +b}{x}}}{24 x^{\frac{5}{2}} b}+\frac{a^{3} \operatorname{arctanh}\left(\frac{\sqrt{a x +b}}{\sqrt{b}}\right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x}}{8 b^{\frac{3}{2}} \sqrt{a x +b}}$	81
default	$-\frac{\sqrt{\frac{a x +b}{x}}\, \left(-3 \,\operatorname{arctanh}\left(\frac{\sqrt{a x +b}}{\sqrt{b}}\right) a^{3} x^{3}+8 \sqrt{a x +b}\, b^{\frac{5}{2}}+14 a \,b^{\frac{3}{2}} x \sqrt{a x +b}+3 a^{2} x^{2} \sqrt{a x +b}\, \sqrt{b}\right)}{24 x^{\frac{5}{2}} b^{\frac{3}{2}} \sqrt{a x +b}}$	92

input
```
int((a+b/x)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(3*a^2*x^2+14*a*b*x+8*b^2)/x^(5/2)/b*((a*x+b)/x)^(1/2)+1/8/b^(3/2)*a
^3*arctanh((a*x+b)^(1/2)/b^(1/2))*((a*x+b)/x)^(1/2)/(a*x+b)^(1/2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.72

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx = \left[\frac{3\,a^3\sqrt{b}x^3\log\left(\frac{ax+2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(3\,a^2bx^2+14\,ab^2x+8\,b^3)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{48\,b^2x^3}, -\frac{3\,a^3\sqrt{-}}{}\right.$$

input
```
integrate((a+b/x)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*a^3*sqrt(b)*x^3*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
2*b)/x) - 2*(3*a^2*b*x^2 + 14*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x))/
(b^2*x^3), -1/24*(3*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b
)/x)/(a*x + b)) + (3*a^2*b*x^2 + 14*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b
)/x))/(b^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx = -\frac{a^{\frac{5}{2}}}{8b\sqrt{x}\sqrt{1+\frac{b}{ax}}} - \frac{17a^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} - \frac{11\sqrt{a}b}{12x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}} + \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{8b^{\frac{3}{2}}} - \frac{b^2}{3\sqrt{a}x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate((a+b/x)**(3/2)/x**(5/2),x)
```

output
```
-a**(5/2)/(8*b*sqrt(x)*sqrt(1 + b/(a*x))) - 17*a**(3/2)/(24*x**(3/2)*sqrt(
1 + b/(a*x))) - 11*sqrt(a)*b/(12*x**(5/2)*sqrt(1 + b/(a*x))) + a**3*asinh(
sqrt(b)/(sqrt(a)*sqrt(x)))/(8*b**(3/2)) - b**2/(3*sqrt(a)*x**(7/2)*sqrt(1
+ b/(a*x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(75) = 150$.

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx = -\frac{a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{3}{2}}} - \frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}}+8\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}}-3\sqrt{a+\frac{b}{x}}a^3b^2\sqrt{x}}{24\left(\left(a+\frac{b}{x}\right)^3bx^3-3\left(a+\frac{b}{x}\right)^2b^2x^2+3\left(a+\frac{b}{x}\right)b^3x-b^4\right)}$$

input
```
integrate((a+b/x)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-1/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + s
qrt(b)))/b^(3/2) - 1/24*(3*(a + b/x)^(5/2)*a^3*x^(5/2) + 8*(a + b/x)^(3/2)
*a^3*b*x^(3/2) - 3*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*b*x^3 - 3*(
a + b/x)^2*b^2*x^2 + 3*(a + b/x)*b^3*x - b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx = -\frac{1}{24}\,a^3\left(\frac{3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b}+\frac{3\,(ax+b)^{\frac{5}{2}}\operatorname{sgn}(x)+8\,(ax+b)^{\frac{3}{2}}b\operatorname{sgn}(x)-3\sqrt{ax+b}b^2\operatorname{sgn}(x)}{a^3bx^3}\right)$$

input
```
integrate((a+b/x)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/24*a^3*(3*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (3*(a*x
+ b)^(5/2)*sgn(x) + 8*(a*x + b)^(3/2)*b*sgn(x) - 3*sqrt(a*x + b)*b^2*sgn(x
))/(a^3*b*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx$$

input
```
int((a + b/x)^(3/2)/x^(5/2),x)
```

output
```
int((a + b/x)^(3/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x^{5/2}}\,dx=\frac{-6\sqrt{ax+b}\,a^2b\,x^2-28\sqrt{ax+b}\,a\,b^2x-16\sqrt{ax+b}\,b^3-3\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^3x^3}{48b^2x^3}$$

input
```
int((a+b/x)^(3/2)/x^(5/2),x)
```

output
```
( - 6*sqrt(a*x + b)*a**2*b*x**2 - 28*sqrt(a*x + b)*a*b**2*x - 16*sqrt(a*x
+ b)*b**3 - 3*sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a**3*x**3 + 3*sqrt(b)*l
og(sqrt(a*x + b) + sqrt(b))*a**3*x**3)/(48*b**2*x**3)
```

3.220 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx = -\frac{32b^3\left(a+\frac{b}{x}\right)^{7/2} x^{7/2}}{3003a^4} + \frac{16b^2\left(a+\frac{b}{x}\right)^{7/2} x^{9/2}}{429a^3} - \frac{12b\left(a+\frac{b}{x}\right)^{7/2} x^{11/2}}{143a^2} + \frac{2\left(a+\frac{b}{x}\right)^{7/2} x^{13/2}}{13a}$$

output
```
-32/3003*b^3*(a+b/x)^(7/2)*x^(7/2)/a^4+16/429*b^2*(a+b/x)^(7/2)*x^(9/2)/a^
3-12/143*b*(a+b/x)^(7/2)*x^(11/2)/a^2+2/13*(a+b/x)^(7/2)*x^(13/2)/a
```

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^3\left(-16b^3+56ab^2x-126a^2bx^2+231a^3x^3\right)}{3003a^4}$$

input
```
Integrate[(a + b/x)^(5/2)*x^(11/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^3*(-16*b^3 + 56*a*b^2*x - 126*a^2*b*x^2
 + 231*a^3*x^3))/(3003*a^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11/2}\left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{13/2}\left(a+\frac{b}{x}\right)^{7/2}}{13a}-\frac{6b\int \left(a+\frac{b}{x}\right)^{5/2}x^{9/2}dx}{13a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{13/2}\left(a+\frac{b}{x}\right)^{7/2}}{13a}-\frac{6b\left(\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}-\frac{4b\int \left(a+\frac{b}{x}\right)^{5/2}x^{7/2}dx}{11a}\right)}{13a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{13/2}\left(a+\frac{b}{x}\right)^{7/2}}{13a}-\frac{6b\left(\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}-\frac{4b\left(\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{2b\int \left(a+\frac{b}{x}\right)^{5/2}x^{5/2}dx}{9a}\right)}{11a}\right)}{13a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{2x^{13/2}\left(a+\frac{b}{x}\right)^{7/2}}{13a}-\frac{6b\left(\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}-\frac{4b\left(\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{4bx^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{63a^2}\right)}{11a}\right)}{13a}$$

input
```
Int[(a + b/x)^(5/2)*x^(11/2),x]
```

output
```
(2*(a + b/x)^(7/2)*x^(13/2))/(13*a) - (6*b*((2*(a + b/x)^(7/2)*x^(11/2))/(
11*a) - (4*b*((-4*b*(a + b/x)^(7/2)*x^(7/2))/(63*a^2) + (2*(a + b/x)^(7/2)
*x^(9/2))/(9*a)))/(11*a)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.53

method	result	size
orering	$\frac{2\left(231 a^{3} x^{3}-126 a^{2} b \,x^{2}+56 a \,b^{2} x -16 b^{3}\right) x^{\frac{5}{2}} \left(a x +b \right) \left(a +\frac{b}{x}\right)^{\frac{5}{2}}}{3003 a^{4}}$	53
gosper	$\frac{2 \left(a x +b \right) \left(231 a^{3} x^{3}-126 a^{2} b \,x^{2}+56 a \,b^{2} x -16 b^{3}\right) x^{\frac{5}{2}} \left(\frac{a x +b}{x}\right)^{\frac{5}{2}}}{3003 a^{4}}$	55
default	$\frac{2 \sqrt{\frac{a x +b}{x}}\, \sqrt{x}\, \left(a x +b \right)^{3} \left(231 a^{3} x^{3}-126 a^{2} b \,x^{2}+56 a \,b^{2} x -16 b^{3}\right)}{3003 a^{4}}$	57
risch	$\frac{2 \sqrt{\frac{a x +b}{x}}\, \sqrt{x}\, \left(231 a^{6} x^{6}+567 a^{5} b \,x^{5}+371 a^{4} b^{2} x^{4}+5 a^{3} x^{3} b^{3}-6 b^{4} x^{2} a^{2}+8 b^{5} x a -16 b^{6}\right)}{3003 a^{4}}$	83

input
```
int((a+b/x)^(5/2)*x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/3003*(231*a^3*x^3-126*a^2*b*x^2+56*a*b^2*x-16*b^3)/a^4*x^(5/2)*(a*x+b)*(
a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\, dx = \frac{2\,(231\,a^6x^6+567\,a^5bx^5+371\,a^4b^2x^4+5\,a^3b^3x^3-6\,a^2b^4x^2+8\,ab^5x-16\,b^6)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3003\,a^4}$$

input
```
integrate((a+b/x)^(5/2)*x^(11/2),x, algorithm="fricas")
```

output
```
2/3003*(231*a^6*x^6 + 567*a^5*b*x^5 + 371*a^4*b^2*x^4 + 5*a^3*b^3*x^3 - 6*
a^2*b^4*x^2 + 8*a*b^5*x - 16*b^6)*sqrt(x)*sqrt((a*x + b)/x)/a^4
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\, dx = \text{Timed out}$$

input
```
integrate((a+b/x)**(5/2)*x**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx=\frac{2\left(231\left(a+\frac{b}{x}\right)^{\frac{13}{2}}x^{\frac{13}{2}}-819\left(a+\frac{b}{x}\right)^{\frac{11}{2}}bx^{\frac{11}{2}}+1001\left(a+\frac{b}{x}\right)^{\frac{9}{2}}b^2x^{\frac{9}{2}}-429\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^3x^{\frac{7}{2}}\right)}{3003\,a^4}$$

input
```
integrate((a+b/x)^(5/2)*x^(11/2),x, algorithm="maxima")
```

output
```
2/3003*(231*(a + b/x)^(13/2)*x^(13/2) - 819*(a + b/x)^(11/2)*b*x^(11/2) +
1001*(a + b/x)^(9/2)*b^2*x^(9/2) - 429*(a + b/x)^(7/2)*b^3*x^(7/2))/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(76) = 152.

Time = 0.13 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.00

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx=\frac{32\,b^{\frac{13}{2}}\operatorname{sgn}(x)}{3003\,a^4}+\frac{2\left(\frac{429\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2-35\,\sqrt{ax+b}b^3\right)b^3\operatorname{sgn}(x)}{a^3}+\frac{143\left(35\,(ax+b)^{\frac{9}{2}}-180\,(ax+b)^{\frac{7}{2}}b+378\,(ax+b)^{\frac{5}{2}}b^2-420\,(ax+b)^{\frac{3}{2}}\right.}{a^3}\right.}{}$$

input
```
integrate((a+b/x)^(5/2)*x^(11/2),x, algorithm="giac")
```

output
```
32/3003*b^(13/2)*sgn(x)/a^4 + 2/15015*(429*(5*(a*x + b)^(7/2) - 21*(a*x +
b)^(5/2)*b + 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*b^3*sgn(x)/a^3
 + 143*(35*(a*x + b)^(9/2) - 180*(a*x + b)^(7/2)*b + 378*(a*x + b)^(5/2)*b
^2 - 420*(a*x + b)^(3/2)*b^3 + 315*sqrt(a*x + b)*b^4)*b^2*sgn(x)/a^3 + 65*
(63*(a*x + b)^(11/2) - 385*(a*x + b)^(9/2)*b + 990*(a*x + b)^(7/2)*b^2 - 1
386*(a*x + b)^(5/2)*b^3 + 1155*(a*x + b)^(3/2)*b^4 - 693*sqrt(a*x + b)*b^5
)*b*sgn(x)/a^3 + 5*(231*(a*x + b)^(13/2) - 1638*(a*x + b)^(11/2)*b + 5005*
(a*x + b)^(9/2)*b^2 - 8580*(a*x + b)^(7/2)*b^3 + 9009*(a*x + b)^(5/2)*b^4
- 6006*(a*x + b)^(3/2)*b^5 + 3003*sqrt(a*x + b)*b^6)*sgn(x)/a^3)/a
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx=\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,a^2\,x^{13/2}}{13}+\frac{106\,b^2\,x^{9/2}}{429}+\frac{10\,b^3\,x^{7/2}}{3003\,a}-\frac{4\,b^4\,x^{5/2}}{1001\,a^2}+\frac{16\,b^5\,x^{3/2}}{3003\,a^3}-\frac{32\,b^6\,\sqrt{x}}{3003\,a^4}+\frac{54\,a\,b\,x}{143}\right.$$

input
```
int(x^(11/2)*(a + b/x)^(5/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a^2*x^(13/2))/13 + (106*b^2*x^(9/2))/429 + (10*b^3*x^(
7/2))/(3003*a) - (4*b^4*x^(5/2))/(1001*a^2) + (16*b^5*x^(3/2))/(3003*a^3)
- (32*b^6*x^(1/2))/(3003*a^4) + (54*a*b*x^(11/2))/143)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{11/2}\,dx=\frac{2\sqrt{ax+b}\,(231a^6x^6+567a^5b\,x^5+371a^4b^2x^4+5a^3b^3x^3-6a^2b^4x^2+8a\,b^5x-16b^6)}{3003a^4}$$

input
```
int((a+b/x)^(5/2)*x^(11/2),x)
```

output
```
(2*sqrt(a*x + b)*(231*a**6*x**6 + 567*a**5*b*x**5 + 371*a**4*b**2*x**4 + 5
*a**3*b**3*x**3 - 6*a**2*b**4*x**2 + 8*a*b**5*x - 16*b**6))/(3003*a**4)
```

3.221 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2} \, dx = \frac{16b^2\left(a+\frac{b}{x}\right)^{7/2} x^{7/2}}{693a^3} - \frac{8b\left(a+\frac{b}{x}\right)^{7/2} x^{9/2}}{99a^2} + \frac{2\left(a+\frac{b}{x}\right)^{7/2} x^{11/2}}{11a}$$

output
```
16/693*b^2*(a+b/x)^(7/2)*x^(7/2)/a^3-8/99*b*(a+b/x)^(7/2)*x^(9/2)/a^2+2/11
*(a+b/x)^(7/2)*x^(11/2)/a
```

Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2} \, dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^3 \left(8b^2-28abx+63a^2x^2\right)}{693a^3}$$

input
```
Integrate[(a + b/x)^(5/2)*x^(9/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^3*(8*b^2 - 28*a*b*x + 63*a^2*x^2))/(693
*a^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}-\frac{4b\int\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}dx}{11a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}-\frac{4b\left(\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{2b\int\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}dx}{9a}\right)}{11a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{11/2}\left(a+\frac{b}{x}\right)^{7/2}}{11a}-\frac{4b\left(\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{4bx^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{63a^2}\right)}{11a}$$

input
```
Int[(a + b/x)^(5/2)*x^(9/2),x]
```

output
```
(2*(a + b/x)^(7/2)*x^(11/2))/(11*a) - (4*b*((-4*b*(a + b/x)^(7/2)*x^(7/2))
/(63*a^2) + (2*(a + b/x)^(7/2)*x^(9/2))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

method	result	size
orering	$\frac{2\left(63 a^{2} x^{2}-28 a b x +8 b^{2}\right)\left(a x +b \right) x^{\frac{5}{2}} \left(a +\frac{b}{x}\right)^{\frac{5}{2}}}{693 a^{3}}$	42
gosper	$\frac{2\left(a x +b \right)\left(63 a^{2} x^{2}-28 a b x +8 b^{2}\right) x^{\frac{5}{2}} \left(\frac{a x +b}{x}\right)^{\frac{5}{2}}}{693 a^{3}}$	44
default	$\frac{2 \sqrt{\frac{a x +b}{x}}\, \sqrt{x}\, \left(a x +b \right)^{3} \left(63 a^{2} x^{2}-28 a b x +8 b^{2}\right)}{693 a^{3}}$	46
risch	$\frac{2 \sqrt{\frac{a x +b}{x}}\, \sqrt{x}\, \left(63 a^{5} x^{5}+161 a^{4} b \,x^{4}+113 a^{3} b^{2} x^{3}+3 a^{2} b^{3} x^{2}-4 b^{4} x a +8 b^{5}\right)}{693 a^{3}}$	72

input
```
int((a+b/x)^(5/2)*x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/693*(63*a^2*x^2-28*a*b*x+8*b^2)/a^3*(a*x+b)*x^(5/2)*(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx = \frac{2\,(63\,a^5x^5+161\,a^4bx^4+113\,a^3b^2x^3+3\,a^2b^3x^2-4\,ab^4x+8\,b^5)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{693\,a^3}$$

input
```
integrate((a+b/x)^(5/2)*x^(9/2),x, algorithm="fricas")
```

output
```
2/693*(63*a^5*x^5 + 161*a^4*b*x^4 + 113*a^3*b^2*x^3 + 3*a^2*b^3*x^2 - 4*a*
b^4*x + 8*b^5)*sqrt(x)*sqrt((a*x + b)/x)/a^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(63) = 126.

Time = 159.97 (sec) , antiderivative size = 423, normalized size of antiderivative = 5.72

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx = \frac{126a^7b^{\frac{9}{2}}x^7\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6} + \frac{574a^6b^{\frac{11}{2}}x^6\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6} + \frac{996a^5b^{\frac{13}{2}}x^5\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6} + \frac{780a^4b^{\frac{15}{2}}x^4\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6} + \frac{230a^3b^{\frac{17}{2}}x^3\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6} + \frac{6a^2b^{\frac{19}{2}}x^2\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6} + \frac{24ab^{\frac{21}{2}}x\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6} + \frac{16b^{\frac{23}{2}}\sqrt{\frac{ax}{b}+1}}{693a^5b^4x^2+1386a^4b^5x+693a^3b^6}$$

input
```
integrate((a+b/x)**(5/2)*x**(9/2),x)
```

output
```
126*a**7*b**(9/2)*x**7*sqrt(a*x/b + 1)/(693*a**5*b**4*x**2 + 1386*a**4*b**
5*x + 693*a**3*b**6) + 574*a**6*b**(11/2)*x**6*sqrt(a*x/b + 1)/(693*a**5*b
**4*x**2 + 1386*a**4*b**5*x + 693*a**3*b**6) + 996*a**5*b**(13/2)*x**5*sqr
t(a*x/b + 1)/(693*a**5*b**4*x**2 + 1386*a**4*b**5*x + 693*a**3*b**6) + 780
*a**4*b**(15/2)*x**4*sqrt(a*x/b + 1)/(693*a**5*b**4*x**2 + 1386*a**4*b**5*
x + 693*a**3*b**6) + 230*a**3*b**(17/2)*x**3*sqrt(a*x/b + 1)/(693*a**5*b**
4*x**2 + 1386*a**4*b**5*x + 693*a**3*b**6) + 6*a**2*b**(19/2)*x**2*sqrt(a*
x/b + 1)/(693*a**5*b**4*x**2 + 1386*a**4*b**5*x + 693*a**3*b**6) + 24*a*b*
*(21/2)*x*sqrt(a*x/b + 1)/(693*a**5*b**4*x**2 + 1386*a**4*b**5*x + 693*a**
3*b**6) + 16*b**(23/2)*sqrt(a*x/b + 1)/(693*a**5*b**4*x**2 + 1386*a**4*b**
5*x + 693*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx = \frac{2\left(63\left(a+\frac{b}{x}\right)^{\frac{11}{2}}x^{\frac{11}{2}} - 154\left(a+\frac{b}{x}\right)^{\frac{9}{2}}bx^{\frac{9}{2}} + 99\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^2x^{\frac{7}{2}}\right)}{693\,a^3}$$

input
```
integrate((a+b/x)^(5/2)*x^(9/2),x, algorithm="maxima")
```

output
```
2/693*(63*(a + b/x)^(11/2)*x^(11/2) - 154*(a + b/x)^(9/2)*b*x^(9/2) + 99*(
a + b/x)^(7/2)*b^2*x^(7/2))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.41

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx = -\frac{16\,b^{\frac{11}{2}}\operatorname{sgn}(x)}{693\,a^3} + \frac{2\left(\frac{231\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b+15\,\sqrt{ax+b}b^2\right)b^3\operatorname{sgn}(x)}{a^2} + \frac{297\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+b)^{\frac{3}{2}}b^2-35\,\sqrt{ax+b}b^3\right)b^2\operatorname{sgn}(x)}{a^2} + \frac{33\,(\cdots}{}\right.}{}$$

input
```
integrate((a+b/x)^(5/2)*x^(9/2),x, algorithm="giac")
```

output
```
-16/693*b^(11/2)*sgn(x)/a^3 + 2/3465*(231*(3*(a*x + b)^(5/2) - 10*(a*x + b
)^(3/2)*b + 15*sqrt(a*x + b)*b^2)*b^3*sgn(x)/a^2 + 297*(5*(a*x + b)^(7/2)
- 21*(a*x + b)^(5/2)*b + 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*b^
2*sgn(x)/a^2 + 33*(35*(a*x + b)^(9/2) - 180*(a*x + b)^(7/2)*b + 378*(a*x +
 b)^(5/2)*b^2 - 420*(a*x + b)^(3/2)*b^3 + 315*sqrt(a*x + b)*b^4)*b*sgn(x)/
a^2 + 5*(63*(a*x + b)^(11/2) - 385*(a*x + b)^(9/2)*b + 990*(a*x + b)^(7/2)
*b^2 - 1386*(a*x + b)^(5/2)*b^3 + 1155*(a*x + b)^(3/2)*b^4 - 693*sqrt(a*x
+ b)*b^5)*sgn(x)/a^2)/a
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx=\sqrt{a+\frac{b}{x}}\left(\frac{2\,a^2\,x^{11/2}}{11}+\frac{226\,b^2\,x^{7/2}}{693}+\frac{2\,b^3\,x^{5/2}}{231\,a}-\frac{8\,b^4\,x^{3/2}}{693\,a^2}+\frac{16\,b^5\,\sqrt{x}}{693\,a^3}+\frac{46\,a\,b\,x^{9/2}}{99}\right)$$

input
```
int(x^(9/2)*(a + b/x)^(5/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a^2*x^(11/2))/11 + (226*b^2*x^(7/2))/693 + (2*b^3*x^(5
/2))/(231*a) - (8*b^4*x^(3/2))/(693*a^2) + (16*b^5*x^(1/2))/(693*a^3) + (4
6*a*b*x^(9/2))/99)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{9/2}\,dx=\frac{2\sqrt{ax+b}\,(63a^5x^5+161a^4b\,x^4+113a^3b^2x^3+3a^2b^3x^2-4a\,b^4x+8b^5)}{693a^3}$$

input
```
int((a+b/x)^(5/2)*x^(9/2),x)
```

output
```
(2*sqrt(a*x + b)*(63*a**5*x**5 + 161*a**4*b*x**4 + 113*a**3*b**2*x**3 + 3*
a**2*b**3*x**2 - 4*a*b**4*x + 8*b**5))/(693*a**3)
```

3.222 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx=-\frac{4b\left(a+\frac{b}{x}\right)^{7/2}x^{7/2}}{63a^2}+\frac{2\left(a+\frac{b}{x}\right)^{7/2}x^{9/2}}{9a}$$

output
```
-4/63*b*(a+b/x)^(7/2)*x^(7/2)/a^2+2/9*(a+b/x)^(7/2)*x^(9/2)/a
```

Mathematica [A] (verified)

Time = 5.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^3(-2b+7ax)}{63a^2}$$

input
```
Integrate[(a + b/x)^(5/2)*x^(7/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^3*(-2*b + 7*a*x))/(63*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{2b\int\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}dx}{9a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{9/2}\left(a+\frac{b}{x}\right)^{7/2}}{9a}-\frac{4bx^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{63a^2}$$

input
```
Int[(a + b/x)^(5/2)*x^(7/2),x]
```

output
```
(-4*b*(a + b/x)^(7/2)*x^(7/2))/(63*a^2) + (2*(a + b/x)^(7/2)*x^(9/2))/(9*a
)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

method	result	size
orering	$\frac{2(7ax-2b)(ax+b)x^{\frac{5}{2}}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{63a^{2}}$	31
gosper	$\frac{2(ax+b)(7ax-2b)x^{\frac{5}{2}}\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}{63a^{2}}$	33
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(ax+b)^{3}(7ax-2b)}{63a^{2}}$	35
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(7a^{4}x^{4}+19a^{3}b\,x^{3}+15a^{2}b^{2}x^{2}+a\,b^{3}x-2b^{4})}{63a^{2}}$	60

input
```
int((a+b/x)^(5/2)*x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/63*(7*a*x-2*b)/a^2*(a*x+b)*x^(5/2)*(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \left(a+\frac{b}{x}\right)^{5/2}x^{7/2}\,dx=\frac{2\,(7\,a^{4}x^{4}+19\,a^{3}bx^{3}+15\,a^{2}b^{2}x^{2}+ab^{3}x-2\,b^{4})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{63\,a^{2}}$$

input
```
integrate((a+b/x)^(5/2)*x^(7/2),x, algorithm="fricas")
```

output
```
2/63*(7*a^4*x^4 + 19*a^3*b*x^3 + 15*a^2*b^2*x^2 + a*b^3*x - 2*b^4)*sqrt(x)
*sqrt((a*x + b)/x)/a^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(39) = 78$.

Time = 48.82 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx = \frac{2a^2\sqrt{b}x^4\sqrt{\frac{ax}{b}+1}}{9} + \frac{38ab^{\frac{3}{2}}x^3\sqrt{\frac{ax}{b}+1}}{63} + \frac{10b^{\frac{5}{2}}x^2\sqrt{\frac{ax}{b}+1}}{21} + \frac{2b^{\frac{7}{2}}x\sqrt{\frac{ax}{b}+1}}{63a} - \frac{4b^{\frac{9}{2}}\sqrt{\frac{ax}{b}+1}}{63a^2}$$

input
```
integrate((a+b/x)**(5/2)*x**(7/2),x)
```

output
```
2*a**2*sqrt(b)*x**4*sqrt(a*x/b + 1)/9 + 38*a*b**(3/2)*x**3*sqrt(a*x/b + 1)
/63 + 10*b**(5/2)*x**2*sqrt(a*x/b + 1)/21 + 2*b**(7/2)*x*sqrt(a*x/b + 1)/(
63*a) - 4*b**(9/2)*sqrt(a*x/b + 1)/(63*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx = \frac{2\left(7\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}} - 9\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}}\right)}{63\,a^2}$$

input
```
integrate((a+b/x)^(5/2)*x^(7/2),x, algorithm="maxima")
```

output
```
2/63*(7*(a + b/x)^(9/2)*x^(9/2) - 9*(a + b/x)^(7/2)*b*x^(7/2))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.19

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx = \frac{4\,b^{\frac{9}{2}}\operatorname{sgn}(x)}{63\,a^2} + \frac{2\left(\frac{105\left((ax+b)^{\frac{3}{2}}-3\sqrt{ax+b}b\right)b^3\operatorname{sgn}(x)}{a} + \frac{63\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b+15\,\sqrt{ax+b}b^2\right)b^2\operatorname{sgn}(x)}{a} + \frac{27\left(5\,(ax+b)^{\frac{7}{2}}-21\,(ax+b)^{\frac{5}{2}}b+35\,(ax+\cdots\right.}{a}\right.}{315\,a}$$

input
```
integrate((a+b/x)^(5/2)*x^(7/2),x, algorithm="giac")
```

output
```
4/63*b^(9/2)*sgn(x)/a^2 + 2/315*(105*((a*x + b)^(3/2) - 3*sqrt(a*x + b)*b)
*b^3*sgn(x)/a + 63*(3*(a*x + b)^(5/2) - 10*(a*x + b)^(3/2)*b + 15*sqrt(a*x
 + b)*b^2)*b^2*sgn(x)/a + 27*(5*(a*x + b)^(7/2) - 21*(a*x + b)^(5/2)*b + 3
5*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*b*sgn(x)/a + (35*(a*x + b)^(
9/2) - 180*(a*x + b)^(7/2)*b + 378*(a*x + b)^(5/2)*b^2 - 420*(a*x + b)^(3/
2)*b^3 + 315*sqrt(a*x + b)*b^4)*sgn(x)/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,a^2\,x^{9/2}}{9}+\frac{10\,b^2\,x^{5/2}}{21}+\frac{2\,b^3\,x^{3/2}}{63\,a}-\frac{4\,b^4\,\sqrt{x}}{63\,a^2}+\frac{38\,a\,b\,x^{7/2}}{63}\right)$$

input
```
int(x^(7/2)*(a + b/x)^(5/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a^2*x^(9/2))/9 + (10*b^2*x^(5/2))/21 + (2*b^3*x^(3/2))
/(63*a) - (4*b^4*x^(1/2))/(63*a^2) + (38*a*b*x^(7/2))/63)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{7/2}\,dx = \frac{2\sqrt{ax+b}\,(7a^4x^4+19a^3b\,x^3+15a^2b^2x^2+a\,b^3x-2b^4)}{63a^2}$$

input
```
int((a+b/x)^(5/2)*x^(7/2),x)
```

output
```
(2*sqrt(a*x + b)*(7*a**4*x**4 + 19*a**3*b*x**3 + 15*a**2*b**2*x**2 + a*b**
3*x - 2*b**4))/(63*a**2)
```

3.223 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx=\frac{2\left(a+\frac{b}{x}\right)^{7/2} x^{7/2}}{7a}$$

output
```
2/7*(a+b/x)^(7/2)*x^(7/2)/a
```

Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(b+ax)^3}{7a}$$

input
```
Integrate[(a + b/x)^(5/2)*x^(5/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(b + a*x)^3)/(7*a)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{7/2}\left(a+\frac{b}{x}\right)^{7/2}}{7a}$$

input
```
Int[(a + b/x)^(5/2)*x^(5/2),x]
```

output
```
(2*(a + b/x)^(7/2)*x^(7/2))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
orering	$\frac{2(ax+b)x^{\frac{5}{2}}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{7a}$	23
gosper	$\frac{2(ax+b)\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}{7a}$	25
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(ax+b)^{3}}{7a}$	27
risch	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(a^{3}x^{3}+3a^{2}b\,x^{2}+3a\,b^{2}x+b^{3}\right)}{7a}$	47

input
```
int((a+b/x)^(5/2)*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/a*(a*x+b)*x^(5/2)*(a+b/x)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx = \frac{2\,(a^3x^3+3\,a^2bx^2+3\,ab^2x+b^3)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{7\,a}$$

input
```
integrate((a+b/x)^(5/2)*x^(5/2),x, algorithm="fricas")
```

output
```
2/7*(a^3*x^3 + 3*a^2*b*x^2 + 3*a*b^2*x + b^3)*sqrt(x)*sqrt((a*x + b)/x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(17) = 34.

Time = 19.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.83

$$\int \left(a+\frac{b}{x}\right)^{5/2}x^{5/2}\,dx=\frac{2a^2\sqrt{b}x^3\sqrt{\frac{ax}{b}+1}}{7}+\frac{6ab^{\frac{3}{2}}x^2\sqrt{\frac{ax}{b}+1}}{7}+\frac{6b^{\frac{5}{2}}x\sqrt{\frac{ax}{b}+1}}{7}+\frac{2b^{\frac{7}{2}}\sqrt{\frac{ax}{b}+1}}{7a}$$

input
```
integrate((a+b/x)**(5/2)*x**(5/2),x)
```

output
```
2*a**2*sqrt(b)*x**3*sqrt(a*x/b + 1)/7 + 6*a*b**(3/2)*x**2*sqrt(a*x/b + 1)/
7 + 6*b**(5/2)*x*sqrt(a*x/b + 1)/7 + 2*b**(7/2)*sqrt(a*x/b + 1)/(7*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^{5/2}x^{5/2}\,dx=\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}}}{7\,a}$$

input
```
integrate((a+b/x)^(5/2)*x^(5/2),x, algorithm="maxima")
```

output
```
2/7*(a + b/x)^(7/2)*x^(7/2)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.09

$$\int \left(a+\frac{b}{x}\right)^{5/2}x^{5/2}\,dx=-\frac{2\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{7\,a}+\frac{2\left(35\sqrt{ax+b}b^3\mathrm{sgn}(x)+35\left((ax+b)^{\frac{3}{2}}-3\sqrt{ax+b}b\right)b^2\mathrm{sgn}(x)+7\left(3\,(ax+b)^{\frac{5}{2}}-10\,(ax+b)^{\frac{3}{2}}b+15\sqrt{\cdots}\right.\right.}{35\,a}$$

input
```
integrate((a+b/x)^(5/2)*x^(5/2),x, algorithm="giac")
```

output
```
-2/7*b^(7/2)*sgn(x)/a + 2/35*(35*sqrt(a*x + b)*b^3*sgn(x) + 35*((a*x + b)^
(3/2) - 3*sqrt(a*x + b)*b)*b^2*sgn(x) + 7*(3*(a*x + b)^(5/2) - 10*(a*x + b
)^(3/2)*b + 15*sqrt(a*x + b)*b^2)*b*sgn(x) + (5*(a*x + b)^(7/2) - 21*(a*x
+ b)^(5/2)*b + 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)*sgn(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx=\sqrt{a+\frac{b}{x}}\left(\frac{2\,a^2\,x^{7/2}}{7}+\frac{6\,b^2\,x^{3/2}}{7}+\frac{2\,b^3\,\sqrt{x}}{7\,a}+\frac{6\,a\,b\,x^{5/2}}{7}\right)$$

input
```
int(x^(5/2)*(a + b/x)^(5/2),x)
```

output
```
(a + b/x)^(1/2)*((2*a^2*x^(7/2))/7 + (6*b^2*x^(3/2))/7 + (2*b^3*x^(1/2))/(
7*a) + (6*a*b*x^(5/2))/7)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{5/2}\,dx=\frac{2\sqrt{ax+b}\,(a^3x^3+3a^2b\,x^2+3a\,b^2x+b^3)}{7a}$$

input
```
int((a+b/x)^(5/2)*x^(5/2),x)
```

output
```
(2*sqrt(a*x + b)*(a**3*x**3 + 3*a**2*b*x**2 + 3*a*b**2*x + b**3))/(7*a)
```

3.224 $\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx=\frac{46}{15}b^2\sqrt{a+\frac{b}{x}}\sqrt{x}+\frac{22}{15}ab\sqrt{a+\frac{b}{x}}x^{3/2}+\frac{2}{5}a^2\sqrt{a+\frac{b}{x}}x^{5/2}-2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

output
```
46/15*b^2*(a+b/x)^(1/2)*x^(1/2)+22/15*a*b*(a+b/x)^(1/2)*x^(3/2)+2/5*a^2*(a
+b/x)^(1/2)*x^(5/2)-2*b^(5/2)*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{2}{15}\sqrt{b+ax}(23b^2+11abx+3a^2x^2)-2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(5/2)*x^(3/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((2*Sqrt[b + a*x]*(23*b^2 + 11*a*b*x + 3*a^2*x^2))/
15 - 2*b^(5/2)*ArcTanh[Sqrt[b + a*x]/Sqrt[b]]))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {860, 247, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \left(a+\frac{b}{x}\right)^{5/2}x^3d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{247}$$

$$-2\left(b\int \left(a+\frac{b}{x}\right)^{3/2}x^2d\frac{1}{\sqrt{x}}-\frac{1}{5}x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{247}$$

$$-2\left(b\left(b\int \sqrt{a+\frac{b}{x}}xd\frac{1}{\sqrt{x}}-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{5}x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{247}$$

$$-2\left(b\left(b\left(b\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{5}x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(b\left(b\left(b\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{5}x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(b\left(b\left(\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)-\sqrt{x}\sqrt{a+\frac{b}{x}}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{5}x^{5/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

input
```
Int[(a + b/x)^(5/2)*x^(3/2),x]
```

output
```
-2*(-1/5*((a + b/x)^(5/2)*x^(5/2)) + b*(-1/3*((a + b/x)^(3/2)*x^(3/2)) + b
*(-(Sqrt[a + b/x]*Sqrt[x]) + Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x
])])))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(-3a^2x^2\sqrt{ax+b}+15b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)-11abx\sqrt{ax+b}-23b^2\sqrt{ax+b}\right)}{15\sqrt{ax+b}}$	81

input
```
int((a+b/x)^(5/2)*x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((a*x+b)/x)^(1/2)*x^(1/2)*(-3*a^2*x^2*(a*x+b)^(1/2)+15*b^(5/2)*arcta
nh((a*x+b)^(1/2)/b^(1/2))-11*a*b*x*(a*x+b)^(1/2)-23*b^2*(a*x+b)^(1/2))/(a*
x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.47

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx=\left[b^{\frac{5}{2}}\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+\frac{2}{15}\left(3\,a^2x^2+11\,abx+23\,b^2\right)\sqrt{x}\sqrt{\frac{ax+b}{x}},2\,\sqrt{-b}\right.$$

input
```
integrate((a+b/x)^(5/2)*x^(3/2),x, algorithm="fricas")
```

output
```
[b^(5/2)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2/15*(
3*a^2*x^2 + 11*a*b*x + 23*b^2)*sqrt(x)*sqrt((a*x + b)/x), 2*sqrt(-b)*b^2*a
rctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + 2/15*(3*a^2*x^2 + 11
*a*b*x + 23*b^2)*sqrt(x)*sqrt((a*x + b)/x)]
```

Sympy [A] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\, dx = \frac{2a^2\sqrt{b}x^2\sqrt{\frac{ax}{b}+1}}{5} + \frac{22ab^{\frac{3}{2}}x\sqrt{\frac{ax}{b}+1}}{15} + \frac{46b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{15} + b^{\frac{5}{2}}\log\left(\frac{ax}{b}\right) - 2b^{\frac{5}{2}}\log\left(\sqrt{\frac{ax}{b}+1}+1\right)$$

input
```
integrate((a+b/x)**(5/2)*x**(3/2),x)
```

output
```
2*a**2*sqrt(b)*x**2*sqrt(a*x/b + 1)/5 + 22*a*b**(3/2)*x*sqrt(a*x/b + 1)/15
 + 46*b**(5/2)*sqrt(a*x/b + 1)/15 + b**(5/2)*log(a*x/b) - 2*b**(5/2)*log(s
qrt(a*x/b + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\, dx = \frac{2}{5}\left(a+\frac{b}{x}\right)^{\frac{5}{2}} x^{\frac{5}{2}} + \frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}} b x^{\frac{3}{2}} + b^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right) + 2\sqrt{a+\frac{b}{x}}b^2\sqrt{x}$$

input
```
integrate((a+b/x)^(5/2)*x^(3/2),x, algorithm="maxima")
```

output
```
2/5*(a + b/x)^(5/2)*x^(5/2) + 2/3*(a + b/x)^(3/2)*b*x^(3/2) + b^(5/2)*log(
(sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b))) + 2*s
qrt(a + b/x)*b^2*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx = \frac{2\,b^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{2}{5}\,(ax+b)^{\frac{5}{2}}\operatorname{sgn}(x) + \frac{2}{3}\,(ax+b)^{\frac{3}{2}}b\operatorname{sgn}(x) + 2\,\sqrt{ax+b}b^2\operatorname{sgn}(x) - \frac{2\left(15\,b^3\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+23\,\sqrt{-b}b^{\frac{5}{2}}\right)\operatorname{sgn}(x)}{15\,\sqrt{-b}}$$

input
```
integrate((a+b/x)^(5/2)*x^(3/2),x, algorithm="giac")
```

output
```
2*b^3*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2/5*(a*x + b)^(5/2)
*sgn(x) + 2/3*(a*x + b)^(3/2)*b*sgn(x) + 2*sqrt(a*x + b)*b^2*sgn(x) - 2/15
*(15*b^3*arctan(sqrt(b)/sqrt(-b)) + 23*sqrt(-b)*b^(5/2))*sgn(x)/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx = \int x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}\,dx$$

input
```
int(x^(3/2)*(a + b/x)^(5/2),x)
```

output
```
int(x^(3/2)*(a + b/x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x}\right)^{5/2} x^{3/2}\,dx = \frac{2\sqrt{ax+b}\,a^2x^2}{5}+\frac{22\sqrt{ax+b}\,abx}{15} + \frac{46\sqrt{ax+b}\,b^2}{15}+\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)b^2-\sqrt{b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)b^2$$

input
```
int((a+b/x)^(5/2)*x^(3/2),x)
```

output
```
(6*sqrt(a*x + b)*a**2*x**2 + 22*sqrt(a*x + b)*a*b*x + 46*sqrt(a*x + b)*b**
2 + 15*sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*b**2 - 15*sqrt(b)*log(sqrt(a*x
 + b) + sqrt(b))*b**2)/15
```

3.225 $\int \left(a+\frac{b}{x}\right)^{5/2} \sqrt{x}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \left(a+\frac{b}{x}\right)^{5/2} \sqrt{x}\, dx = -\frac{b^2\sqrt{a+\frac{b}{x}}}{\sqrt{x}}+\frac{14}{3}ab\sqrt{a+\frac{b}{x}}\sqrt{x} + \frac{2}{3}a^2\sqrt{a+\frac{b}{x}}x^{3/2}-5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

output
```
-b^2*(a+b/x)^(1/2)/x^(1/2)+14/3*a*b*(a+b/x)^(1/2)*x^(1/2)+2/3*a^2*(a+b/x)^
(1/2)*x^(3/2)-5*a*b^(3/2)*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x}\right)^{5/2} \sqrt{x}\, dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{\sqrt{b+ax}(-3b^2+14abx+2a^2x^2)}{3x}-5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(5/2)*Sqrt[x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((Sqrt[b + a*x]*(-3*b^2 + 14*a*b*x + 2*a^2*x^2))/(3
*x) - 5*a*b^(3/2)*ArcTanh[Sqrt[b + a*x]/Sqrt[b]]))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {860, 247, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \left(a+\frac{b}{x}\right)^{5/2}x^2d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{247}$$

$$-2\left(\frac{5}{3}b\int \left(a+\frac{b}{x}\right)^{3/2}xd\frac{1}{\sqrt{x}}-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{247}$$

$$-2\left(\frac{5}{3}b\left(3b\int \sqrt{a+\frac{b}{x}}d\frac{1}{\sqrt{x}}-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{211}$$

$$-2\left(\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{5}{3}b\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}\right)-\frac{1}{3}x^{3/2}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

input
```
Int[(a + b/x)^(5/2)*Sqrt[x],x]
```

output
```
-2*(-1/3*((a + b/x)^(5/2)*x^(3/2)) + (5*b*(-((a + b/x)^(3/2)*Sqrt[x]) + 3*
b*(Sqrt[a + b/x]/(2*Sqrt[x]) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])]
)/(2*Sqrt[b]))))/3)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{b^{2}\sqrt{\frac{ax+b}{x}}}{\sqrt{x}}+\frac{a\left(\frac{4(ax+b)^{\frac{3}{2}}}{3}+8b\sqrt{ax+b}-10b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{2\sqrt{ax+b}}$	82
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-2a^{2}x^{2}\sqrt{ax+b}\,\sqrt{b}+15\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a\,b^{2}x-14a\,b^{\frac{3}{2}}x\sqrt{ax+b}+3\sqrt{ax+b}\,b^{\frac{5}{2}}\right)}{3\sqrt{x}\,\sqrt{ax+b}\,\sqrt{b}}$	91

input
```
int((a+b/x)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-b^2/x^(1/2)*((a*x+b)/x)^(1/2)+1/2*a*(4/3*(a*x+b)^(3/2)+8*b*(a*x+b)^(1/2)-
10*b^(3/2)*arctanh((a*x+b)^(1/2)/b^(1/2)))*((a*x+b)/x)^(1/2)/(a*x+b)^(1/2)
*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.61

$$\int\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx=\left[\frac{15\,ab^{\frac{3}{2}}x\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,(2\,a^{2}x^{2}+14\,abx-3\,b^{2})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{6\,x},\ \frac{15\,a\sqrt{-b}bx\operatorname{arc}

output
```
[1/6*(15*a*b^(3/2)*x*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)
/x) + 2*(2*a^2*x^2 + 14*a*b*x - 3*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x, 1/3*(
15*a*sqrt(-b)*b*x*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (
2*a^2*x^2 + 14*a*b*x - 3*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x]
```

Sympy [A] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx = \frac{2a^2\sqrt{b}x\sqrt{\frac{ax}{b}+1}}{3} + \frac{14ab^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3} + \frac{5ab^{\frac{3}{2}}\log\left(\frac{ax}{b}\right)}{2} - 5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{ax}{b}+1}+1\right) - \frac{b^{\frac{5}{2}}\sqrt{\frac{ax}{b}+1}}{x}$$

input
```
integrate((a+b/x)**(5/2)*x**(1/2),x)
```

output
```
2*a**2*sqrt(b)*x*sqrt(a*x/b + 1)/3 + 14*a*b**(3/2)*sqrt(a*x/b + 1)/3 + 5*a
*b**(3/2)*log(a*x/b)/2 - 5*a*b**(3/2)*log(sqrt(a*x/b + 1) + 1) - b**(5/2)*
sqrt(a*x/b + 1)/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx = \frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}ax^{\frac{3}{2}} + \frac{5}{2}ab^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right) + 4\sqrt{a+\frac{b}{x}}ab\sqrt{x} - \frac{\sqrt{a+\frac{b}{x}}ab^2\sqrt{x}}{(a+\frac{b}{x})x-b}$$

input
```
integrate((a+b/x)^(5/2)*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a + b/x)^(3/2)*a*x^(3/2) + 5/2*a*b^(3/2)*log((sqrt(a + b/x)*sqrt(x) -
 sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b))) + 4*sqrt(a + b/x)*a*b*sqrt(x)
 - sqrt(a + b/x)*a*b^2*sqrt(x)/((a + b/x)*x - b)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx=\frac{1}{3}\left(\frac{15\,b^2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+2\,(ax+b)^{\frac{3}{2}}\mathrm{sgn}(x)+12\,\sqrt{ax+b}b\mathrm{sgn}(x)-\frac{3\,\sqrt{ax+b}b\ldots}{ax\ldots}\right.$$

input
```
integrate((a+b/x)^(5/2)*x^(1/2),x, algorithm="giac")
```

output
```
1/3*(15*b^2*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*(a*x + b)^(
3/2)*sgn(x) + 12*sqrt(a*x + b)*b*sgn(x) - 3*sqrt(a*x + b)*b^2*sgn(x)/(a*x)
)*a
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx=\int \sqrt{x}\,\left(a+\frac{b}{x}\right)^{5/2}\,dx$$

input
```
int(x^(1/2)*(a + b/x)^(5/2),x)
```

output
```
int(x^(1/2)*(a + b/x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}\,dx=\frac{4\sqrt{ax+b}\,a^2x^2+28\sqrt{ax+b}\,abx-6\sqrt{ax+b}\,b^2+15\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)abx-15\sqrt{b}\ldots}{6x}$$

input
```
int((a+b/x)^(5/2)*x^(1/2),x)
```

output
```
(4*sqrt(a*x + b)*a**2*x**2 + 28*sqrt(a*x + b)*a*b*x - 6*sqrt(a*x + b)*b**2
 + 15*sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a*b*x - 15*sqrt(b)*log(sqrt(a*x
 + b) + sqrt(b))*a*b*x)/(6*x)
```

3.226 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx=-\frac{b^2\sqrt{a+\frac{b}{x}}}{2x^{3/2}}-\frac{9ab\sqrt{a+\frac{b}{x}}}{4\sqrt{x}}+2a^2\sqrt{a+\frac{b}{x}}\sqrt{x}-\frac{15}{4}a^2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)$$

output
```
-1/2*b^2*(a+b/x)^(1/2)/x^(3/2)-9/4*a*b*(a+b/x)^(1/2)/x^(1/2)+2*a^2*(a+b/x)
^(1/2)*x^(1/2)-15/4*a^2*b^(1/2)*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 5.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{\sqrt{b+ax}(-2b^2-9abx+8a^2x^2)}{4x^2}-\frac{15}{4}a^2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(5/2)/Sqrt[x],x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((Sqrt[b + a*x]*(-2*b^2 - 9*a*b*x + 8*a^2*x^2))/(4*
x^2) - (15*a^2*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/4))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {860, 247, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 860}$$

$$-2\int \left(a+\frac{b}{x}\right)^{5/2}xd\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 247}$$

$$-2\left(5b\int \left(a+\frac{b}{x}\right)^{3/2}d\frac{1}{\sqrt{x}}-\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{ 211}$$

$$-2\left(5b\left(\frac{3}{4}a\int \sqrt{a+\frac{b}{x}}d\frac{1}{\sqrt{x}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{ 211}$$

$$-2\left(5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-2\left(5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-2\left(5b\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)-\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}\right)$$

input
```
Int[(a + b/x)^(5/2)/Sqrt[x],x]
```

output
```
-2*(-((a + b/x)^(5/2)*Sqrt[x]) + 5*b*((a + b/x)^(3/2)/(4*Sqrt[x]) + (3*a*(
Sqrt[a + b/x]/(2*Sqrt[x]) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(
2*Sqrt[b])))/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{b(9ax+2b)\sqrt{\frac{ax+b}{x}}}{4x^{\frac{3}{2}}}+\frac{a^{2}\left(16\sqrt{ax+b}-30\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{8\sqrt{ax+b}}$	80
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(15\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^{2}b\,x^{2}-8a^{2}x^{2}\sqrt{ax+b}\,\sqrt{b}+9a\,b^{\frac{3}{2}}x\sqrt{ax+b}+2\sqrt{ax+b}\,b^{\frac{5}{2}}\right)}{4x^{\frac{3}{2}}\sqrt{ax+b}\,\sqrt{b}}$	93

input
```
int((a+b/x)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*(9*a*x+2*b)/x^(3/2)*((a*x+b)/x)^(1/2)+1/8*a^2*(16*(a*x+b)^(1/2)-30*
b^(1/2)*arctanh((a*x+b)^(1/2)/b^(1/2)))*((a*x+b)/x)^(1/2)/(a*x+b)^(1/2)*x^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx=\left[\frac{15\,a^{2}\sqrt{b}x^{2}\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,(8\,a^{2}x^{2}-9\,abx-2\,b^{2})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{8\,x^{2}},\frac{15\,a^{2}\sqrt{-b}x\right.$$

input
```
integrate((a+b/x)^(5/2)/x^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(15*a^2*sqrt(b)*x^2*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
2*b)/x) + 2*(8*a^2*x^2 - 9*a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x^2,
1/4*(15*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x +
b)) + (8*a^2*x^2 - 9*a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/x^2]
```

Sympy [A] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx = \frac{2a^{\frac{5}{2}}\sqrt{x}}{\sqrt{1+\frac{b}{ax}}} - \frac{a^{\frac{3}{2}}b}{4\sqrt{x}\sqrt{1+\frac{b}{ax}}} - \frac{11\sqrt{a}b^2}{4x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} - \frac{15a^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4} - \frac{b^3}{2\sqrt{a}x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate((a+b/x)**(5/2)/x**(1/2),x)
```

output
```
2*a**(5/2)*sqrt(x)/sqrt(1 + b/(a*x)) - a**(3/2)*b/(4*sqrt(x)*sqrt(1 + b/(a
*x))) - 11*sqrt(a)*b**2/(4*x**(3/2)*sqrt(1 + b/(a*x))) - 15*a**2*sqrt(b)*a
sinh(sqrt(b)/(sqrt(a)*sqrt(x)))/4 - b**3/(2*sqrt(a)*x**(5/2)*sqrt(1 + b/(a
*x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx = \frac{15}{8}a^2\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right) + 2\sqrt{a+\frac{b}{x}}a^2\sqrt{x} - \frac{9\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2bx^{\frac{3}{2}} - 7\sqrt{a+\frac{b}{x}}a^2b^2\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2x^2 - 2\left(a+\frac{b}{x}\right)bx + b^2\right)}$$

input
```
integrate((a+b/x)^(5/2)/x^(1/2),x, algorithm="maxima")
```

output
```
15/8*a^2*sqrt(b)*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt
(x) + sqrt(b))) + 2*sqrt(a + b/x)*a^2*sqrt(x) - 1/4*(9*(a + b/x)^(3/2)*a^2
*b*x^(3/2) - 7*sqrt(a + b/x)*a^2*b^2*sqrt(x))/((a + b/x)^2*x^2 - 2*(a + b/
x)*b*x + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx = \frac{\frac{15\,a^3b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} + 8\sqrt{ax+b}a^3\mathrm{sgn}(x) - \frac{9\,(ax+b)^{\frac{3}{2}}\,a^3b\mathrm{sgn}(x) - 7\sqrt{ax+b}a^3b^2\mathrm{sgn}(x)}{a^2x^2}}{4\,a}$$

input
```
integrate((a+b/x)^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
1/4*(15*a^3*b*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 8*sqrt(a*x
+ b)*a^3*sgn(x) - (9*(a*x + b)^(3/2)*a^3*b*sgn(x) - 7*sqrt(a*x + b)*a^3*b^
2*sgn(x))/(a^2*x^2))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx$$

input
```
int((a + b/x)^(5/2)/x^(1/2),x)
```

output
```
int((a + b/x)^(5/2)/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\sqrt{x}}\,dx = \frac{16\sqrt{ax+b}\,a^2x^2-18\sqrt{ax+b}\,abx-4\sqrt{ax+b}\,b^2+15\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^2x^2-1}{8x^2}$$

input
```
int((a+b/x)^(5/2)/x^(1/2),x)
```

output
```
(16*sqrt(a*x + b)*a**2*x**2 - 18*sqrt(a*x + b)*a*b*x - 4*sqrt(a*x + b)*b**
2 + 15*sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a**2*x**2 - 15*sqrt(b)*log(sqr
t(a*x + b) + sqrt(b))*a**2*x**2)/(8*x**2)
```

3.227 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx=-\frac{5a^2\sqrt{a+\frac{b}{x}}}{8\sqrt{x}}-\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{12\sqrt{x}}-\frac{\left(a+\frac{b}{x}\right)^{5/2}}{3\sqrt{x}}-\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{8\sqrt{b}}$$

output
```
-5/8*a^2*(a+b/x)^(1/2)/x^(1/2)-5/12*a*(a+b/x)^(3/2)/x^(1/2)-1/3*(a+b/x)^(5
/2)/x^(1/2)-5/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 5.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{\sqrt{b+ax}(-8b^2-26abx-33a^2x^2)}{24x^3}-\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{8\sqrt{b}}\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(5/2)/x^(3/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((Sqrt[b + a*x]*(-8*b^2 - 26*a*b*x - 33*a^2*x^2))/(
24*x^3) - (5*a^3*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/(8*Sqrt[b])))/Sqrt[b + a*
x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {860, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \left(a+\frac{b}{x}\right)^{5/2}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{211}$$

$$-2\left(\frac{5}{6}a\int \left(a+\frac{b}{x}\right)^{3/2}d\frac{1}{\sqrt{x}}+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{6\sqrt{x}}\right)$$

$$\Big\downarrow \text{211}$$

$$-2\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{a+\frac{b}{x}}d\frac{1}{\sqrt{x}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{6\sqrt{x}}\right)$$

$$\Big\downarrow \text{211}$$

$$-2\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{6\sqrt{x}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{6\sqrt{x}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x}}}{2\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{4\sqrt{x}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{6\sqrt{x}}\right)$$

input
```
Int[(a + b/x)^(5/2)/x^(3/2),x]
```

output
```
-2*((a + b/x)^(5/2)/(6*Sqrt[x]) + (5*a*((a + b/x)^(3/2)/(4*Sqrt[x]) + (3*a
*(Sqrt[a + b/x]/(2*Sqrt[x]) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])
/(2*Sqrt[b])))/4))/6)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{(33a^2x^2+26abx+8b^2)\sqrt{\frac{ax+b}{x}}}{24x^{\frac{5}{2}}}-\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{8\sqrt{b}\,\sqrt{ax+b}}$	78
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(15\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^3x^3+33a^2x^2\sqrt{ax+b}\,\sqrt{b}+26a\,b^{\frac{3}{2}}x\sqrt{ax+b}+8\sqrt{ax+b}\,b^{\frac{5}{2}}\right)}{24x^{\frac{5}{2}}\sqrt{ax+b}\,\sqrt{b}}$	92

input
```
int((a+b/x)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(33*a^2*x^2+26*a*b*x+8*b^2)/x^(5/2)*((a*x+b)/x)^(1/2)-5/8*a^3/b^(1/2
)*arctanh((a*x+b)^(1/2)/b^(1/2))*((a*x+b)/x)^(1/2)/(a*x+b)^(1/2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx = \left[\frac{15\,a^3\sqrt{b}x^3\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(33\,a^2bx^2+26\,ab^2x+8\,b^3)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{48\,bx^3}, \frac{15\,a^3\sqrt{}}{}\right.$$

input
```
integrate((a+b/x)^(5/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*x^3*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
 2*b)/x) - 2*(33*a^2*b*x^2 + 26*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x)
)/(b*x^3), 1/24*(15*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b
)/x)/(a*x + b)) - (33*a^2*b*x^2 + 26*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x +
b)/x))/(b*x^3)]
```

Sympy [A] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx = -\frac{11a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}{8\sqrt{x}} - \frac{13a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{12x^{\frac{3}{2}}} - \frac{\sqrt{a}b^2\sqrt{1+\frac{b}{ax}}}{3x^{\frac{5}{2}}} - \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{8\sqrt{b}}$$

input
```
integrate((a+b/x)**(5/2)/x**(3/2),x)
```

output
```
-11*a**(5/2)*sqrt(1 + b/(a*x))/(8*sqrt(x)) - 13*a**(3/2)*b*sqrt(1 + b/(a*x
))/(12*x**(3/2)) - sqrt(a)*b**2*sqrt(1 + b/(a*x))/(3*x**(5/2)) - 5*a**3*as
inh(sqrt(b)/(sqrt(a)*sqrt(x)))/(8*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(72) = 144.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.56

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx = \frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{16\,\sqrt{b}} - \frac{33\,\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}} - 40\,\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}} + 15\,\sqrt{a+\frac{b}{x}}a^3b^2\sqrt{x}}{24\,\left(\left(a+\frac{b}{x}\right)^3x^3 - 3\,\left(a+\frac{b}{x}\right)^2bx^2 + 3\,\left(a+\frac{b}{x}\right)b^2x - b^3\right)}$$

input
```
integrate((a+b/x)^(5/2)/x^(3/2),x, algorithm="maxima")
```

output
```
5/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sq
rt(b)))/sqrt(b) - 1/24*(33*(a + b/x)^(5/2)*a^3*x^(5/2) - 40*(a + b/x)^(3/2
)*a^3*b*x^(3/2) + 15*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*x^3 - 3*(
a + b/x)^2*b*x^2 + 3*(a + b/x)*b^2*x - b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx = \frac{1}{24}\left(\frac{15\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{33\,(ax+b)^{\frac{5}{2}}\operatorname{sgn}(x) - 40\,(ax+b)^{\frac{3}{2}}b\operatorname{sgn}(x) + 15\,\sqrt{ax}}{a^3x^3}\right.$$

input
```
integrate((a+b/x)^(5/2)/x^(3/2),x, algorithm="giac")
```

output
```
1/24*(15*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (33*(a*x + b)^(5
/2)*sgn(x) - 40*(a*x + b)^(3/2)*b*sgn(x) + 15*sqrt(a*x + b)*b^2*sgn(x))/(a
^3*x^3))*a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx$$

input
```
int((a + b/x)^(5/2)/x^(3/2),x)
```

output
```
int((a + b/x)^(5/2)/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{3/2}}\,dx = \frac{-66\sqrt{ax+b}\,a^2b\,x^2 - 52\sqrt{ax+b}\,a\,b^2x - 16\sqrt{ax+b}\,b^3 + 15\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^3}{48b\,x^3}$$

input
```
int((a+b/x)^(5/2)/x^(3/2),x)
```

output
```
( - 66*sqrt(a*x + b)*a**2*b*x**2 - 52*sqrt(a*x + b)*a*b**2*x - 16*sqrt(a*x
 + b)*b**3 + 15*sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a**3*x**3 - 15*sqrt(b
)*log(sqrt(a*x + b) + sqrt(b))*a**3*x**3)/(48*b*x**3)
```

3.228 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx=-\frac{5a^2\sqrt{a+\frac{b}{x}}}{32x^{3/2}}-\frac{5a\left(a+\frac{b}{x}\right)^{3/2}}{24x^{3/2}}-\frac{\left(a+\frac{b}{x}\right)^{5/2}}{4x^{3/2}}-\frac{5a^3\sqrt{a+\frac{b}{x}}}{64b\sqrt{x}}+\frac{5a^4\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{64b^{3/2}}$$

output
```
-5/32*a^2*(a+b/x)^(1/2)/x^(3/2)-5/24*a*(a+b/x)^(3/2)/x^(3/2)-1/4*(a+b/x)^(
5/2)/x^(3/2)-5/64*a^3*(a+b/x)^(1/2)/b/x^(1/2)+5/64*a^4*arctanh(b^(1/2)/(a+
b/x)^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 5.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{\sqrt{b+ax}\left(-48b^3-136ab^2x-118a^2bx^2-15a^3x^3\right)}{192bx^4}+\frac{5a^4\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{64b^{3/2}}\right)}{\sqrt{b+ax}}$$

input
```
Integrate[(a + b/x)^(5/2)/x^(5/2),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((Sqrt[b + a*x]*(-48*b^3 - 136*a*b^2*x - 118*a^2*b*
x^2 - 15*a^3*x^3))/(192*b*x^4) + (5*a^4*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/(6
4*b^(3/2))))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {860, 248, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{248}$$

$$-2\left(\frac{5}{8}a\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{x}d\frac{1}{\sqrt{x}}+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{8x^{3/2}}\right)$$

$$\Big\downarrow \text{248}$$

$$-2\left(\frac{5}{8}a\left(\frac{1}{2}a\int \frac{\sqrt{a+\frac{b}{x}}}{x}d\frac{1}{\sqrt{x}}+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{8x^{3/2}}\right)$$

$$\Big\downarrow \text{248}$$

$$-2\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{8x^{3/2}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{8x^{3/2}}\right)$$

$$\Big\downarrow \; 224$$

$$-2\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{8x^{3/2}}\right)$$

$$\Big\downarrow \; 219$$

$$-2\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)+\frac{\sqrt{a+\frac{b}{x}}}{4x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{3/2}}{6x^{3/2}}\right)+\frac{\left(a+\frac{b}{x}\right)^{5/2}}{8x^{3/2}}\right)$$

input
```
Int[(a + b/x)^(5/2)/x^(5/2),x]
```

output
```
-2*((a + b/x)^(5/2)/(8*x^(3/2)) + (5*a*((a + b/x)^(3/2)/(6*x^(3/2)) + (a*(
Sqrt[a + b/x]/(4*x^(3/2)) + (a*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTanh[S
qrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*b^(3/2))))/4))/2))/8)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{\left(15 a^{3} x^{3}+118 a^{2} b\, x^{2}+136 a\, b^{2} x+48 b^{3}\right) \sqrt{\frac{a x+b}{x}}}{192 x^{\frac{7}{2}} b}+\frac{5 a^{4} \operatorname{arctanh}\left(\frac{\sqrt{a x+b}}{\sqrt{b}}\right) \sqrt{\frac{a x+b}{x}}\, \sqrt{x}}{64 b^{\frac{3}{2}} \sqrt{a x+b}}$	92
default	$-\frac{\sqrt{\frac{a x+b}{x}}\, \left(-15 \operatorname{arctanh}\left(\frac{\sqrt{a x+b}}{\sqrt{b}}\right) a^{4} x^{4}+48 \sqrt{a x+b}\, b^{\frac{7}{2}}+136 a\, b^{\frac{5}{2}} x \sqrt{a x+b}+118 a^{2} b^{\frac{3}{2}} x^{2} \sqrt{a x+b}+15 a^{3} x^{3} \sqrt{a x+b}\, \sqrt{b}\right)}{192 x^{\frac{7}{2}} b^{\frac{3}{2}} \sqrt{a x+b}}$	110

input
```
int((a+b/x)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(15*a^3*x^3+118*a^2*b*x^2+136*a*b^2*x+48*b^3)/x^(7/2)/b*((a*x+b)/x)
^(1/2)+5/64/b^(3/2)*a^4*arctanh((a*x+b)^(1/2)/b^(1/2))*((a*x+b)/x)^(1/2)/(
a*x+b)^(1/2)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx = \left[\frac{15\,a^4\sqrt{b}x^4\log\left(\frac{ax+2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\left(15\,a^3bx^3+118\,a^2b^2x^2+136\,ab^3x+48\,b^4\right)\sqrt{}}{384\,b^2x^4}\right.$$

input
```
integrate((a+b/x)^(5/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(15*a^4*sqrt(b)*x^4*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x)
+ 2*b)/x) - 2*(15*a^3*b*x^3 + 118*a^2*b^2*x^2 + 136*a*b^3*x + 48*b^4)*sqrt
(x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/192*(15*a^4*sqrt(-b)*x^4*arctan(sqrt(
-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (15*a^3*b*x^3 + 118*a^2*b^2*x^2
 + 136*a*b^3*x + 48*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

Sympy [A] (verification not implemented)

Time = 12.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx = -\frac{5a^{\frac{7}{2}}}{64b\sqrt{x}\sqrt{1+\frac{b}{ax}}}-\frac{133a^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{127a^{\frac{3}{2}}b}{96x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}-\frac{23\sqrt{a}b^2}{24x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}+\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{64b^{\frac{3}{2}}}-\frac{b^3}{4\sqrt{a}x^{\frac{9}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate((a+b/x)**(5/2)/x**(5/2),x)
```

output
```
-5*a**(7/2)/(64*b*sqrt(x)*sqrt(1 + b/(a*x))) - 133*a**(5/2)/(192*x**(3/2)*
sqrt(1 + b/(a*x))) - 127*a**(3/2)*b/(96*x**(5/2)*sqrt(1 + b/(a*x))) - 23*s
qrt(a)*b**2/(24*x**(7/2)*sqrt(1 + b/(a*x))) + 5*a**4*asinh(sqrt(b)/(sqrt(a
)*sqrt(x)))/(64*b**(3/2)) - b**3/(4*sqrt(a)*x**(9/2)*sqrt(1 + b/(a*x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx = -\frac{5\,a^4\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{128\,b^{\frac{3}{2}}} - \frac{15\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a^4x^{\frac{7}{2}} + 73\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^4bx^{\frac{5}{2}} - 55\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4b^2x^{\frac{3}{2}} + 15\sqrt{a+\frac{b}{x}}a^4b^3\sqrt{x}}{192\left(\left(a+\frac{b}{x}\right)^4bx^4 - 4\left(a+\frac{b}{x}\right)^3b^2x^3 + 6\left(a+\frac{b}{x}\right)^2b^3x^2 - 4\left(a+\frac{b}{x}\right)b^4x + b^5\right)}$$

input
```
integrate((a+b/x)^(5/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-5/128*a^4*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) +
sqrt(b)))/b^(3/2) - 1/192*(15*(a + b/x)^(7/2)*a^4*x^(7/2) + 73*(a + b/x)^(
5/2)*a^4*b*x^(5/2) - 55*(a + b/x)^(3/2)*a^4*b^2*x^(3/2) + 15*sqrt(a + b/x)
*a^4*b^3*sqrt(x))/((a + b/x)^4*b*x^4 - 4*(a + b/x)^3*b^2*x^3 + 6*(a + b/x)
^2*b^3*x^2 - 4*(a + b/x)*b^4*x + b^5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx = -\frac{\frac{15\,a^5\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b} + \frac{15\,(ax+b)^{\frac{7}{2}}a^5\operatorname{sgn}(x)+73\,(ax+b)^{\frac{5}{2}}a^5b\operatorname{sgn}(x)-55\,(ax+b)^{\frac{3}{2}}a^5b^2\operatorname{sgn}(x)+15\,\sqrt{ax+b}a^5b^3\operatorname{sgn}(x)}{a^4bx^4}}{192\,a}$$

input
```
integrate((a+b/x)^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
-1/192*(15*a^5*arctan(sqrt(a*x + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (15*(a
*x + b)^(7/2)*a^5*sgn(x) + 73*(a*x + b)^(5/2)*a^5*b*sgn(x) - 55*(a*x + b)^
(3/2)*a^5*b^2*sgn(x) + 15*sqrt(a*x + b)*a^5*b^3*sgn(x))/(a^4*b*x^4))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx$$

input
```
int((a + b/x)^(5/2)/x^(5/2),x)
```

output
```
int((a + b/x)^(5/2)/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{x^{5/2}}\,dx=\frac{-30\sqrt{ax+b}\,a^3b\,x^3-236\sqrt{ax+b}\,a^2b^2x^2-272\sqrt{ax+b}\,a\,b^3x-96\sqrt{ax+b}\,b^4-15\sqrt{b}}{384b^2x^4}$$

input
```
int((a+b/x)^(5/2)/x^(5/2),x)
```

output
```
( - 30*sqrt(a*x + b)*a**3*b*x**3 - 236*sqrt(a*x + b)*a**2*b**2*x**2 - 272*
sqrt(a*x + b)*a*b**3*x - 96*sqrt(a*x + b)*b**4 - 15*sqrt(b)*log(sqrt(a*x +
 b) - sqrt(b))*a**4*x**4 + 15*sqrt(b)*log(sqrt(a*x + b) + sqrt(b))*a**4*x**
*4)/(384*b**2*x**4)
```

3.229 $\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{256b^4\sqrt{a+\frac{b}{x}}\sqrt{x}}{315a^5}-\frac{128b^3\sqrt{a+\frac{b}{x}}x^{3/2}}{315a^4}+\frac{32b^2\sqrt{a+\frac{b}{x}}x^{5/2}}{105a^3}-\frac{16b\sqrt{a+\frac{b}{x}}x^{7/2}}{63a^2}+\frac{2\sqrt{a+\frac{b}{x}}x^{9/2}}{9a}$$

output
```
256/315*b^4*(a+b/x)^(1/2)*x^(1/2)/a^5-128/315*b^3*(a+b/x)^(1/2)*x^(3/2)/a^
4+32/105*b^2*(a+b/x)^(1/2)*x^(5/2)/a^3-16/63*b*(a+b/x)^(1/2)*x^(7/2)/a^2+2
/9*(a+b/x)^(1/2)*x^(9/2)/a
```

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(128b^4-64ab^3x+48a^2b^2x^2-40a^3bx^3+35a^4x^4)}{315a^5}$$

input
```
Integrate[x^(7/2)/Sqrt[a + b/x],x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(128*b^4 - 64*a*b^3*x + 48*a^2*b^2*x^2 - 40*a^3*b
*x^3 + 35*a^4*x^4))/(315*a^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {803, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{9/2}\sqrt{a+\frac{b}{x}}}{9a}-\frac{8b\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}dx}{9a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{9/2}\sqrt{a+\frac{b}{x}}}{9a}-\frac{8b\left(\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}dx}{7a}\right)}{9a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{9/2}\sqrt{a+\frac{b}{x}}}{9a}-\frac{8b\left(\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\left(\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}dx}{5a}\right)}{7a}\right)}{9a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{9/2}\sqrt{a+\frac{b}{x}}}{9a}-\frac{8b\left(\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\left(\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\left(\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{2b\int\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}dx}{3a}\right)}{5a}\right)}{7a}\right)}{9a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{2x^{9/2}\sqrt{a+\frac{b}{x}}}{9a}-\frac{8b\left(\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\left(\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\left(\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{4b\sqrt{x}\sqrt{a+\frac{b}{x}}}{3a^2}\right)}{5a}\right)}{7a}\right)}{9a}$$

input
```
Int[x^(7/2)/Sqrt[a + b/x],x]
```

output
```
(2*Sqrt[a + b/x]*x^(9/2))/(9*a) - (8*b*((2*Sqrt[a + b/x]*x^(7/2))/(7*a) -
(6*b*((2*Sqrt[a + b/x]*x^(5/2))/(5*a) - (4*b*((-4*b*Sqrt[a + b/x]*Sqrt[x])
/(3*a^2) + (2*Sqrt[a + b/x]*x^(3/2))/(3*a)))/(5*a)))/(7*a)))/(9*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(35a^4x^4-40a^3b\,x^3+48a^2b^2x^2-64a\,b^3x+128b^4)}{315a^5}$	61
orering	$\frac{2(35a^4x^4-40a^3b\,x^3+48a^2b^2x^2-64a\,b^3x+128b^4)(ax+b)}{315a^5\sqrt{x}\,\sqrt{a+\frac{b}{x}}}$	64
gosper	$\frac{2(ax+b)(35a^4x^4-40a^3b\,x^3+48a^2b^2x^2-64a\,b^3x+128b^4)}{315a^5\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	66
risch	$\frac{2(ax+b)(35a^4x^4-40a^3b\,x^3+48a^2b^2x^2-64a\,b^3x+128b^4)}{315a^5\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	66

input
```
int(x^(7/2)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*((a*x+b)/x)^(1/2)*x^(1/2)*(35*a^4*x^4-40*a^3*b*x^3+48*a^2*b^2*x^2-64
*a*b^3*x+128*b^4)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,(35\,a^4x^4-40\,a^3bx^3+48\,a^2b^2x^2-64\,ab^3x+128\,b^4)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{315\,a^5}$$

input
```
integrate(x^(7/2)/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*a^4*x^4 - 40*a^3*b*x^3 + 48*a^2*b^2*x^2 - 64*a*b^3*x + 128*b^4)*
sqrt(x)*sqrt((a*x + b)/x)/a^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(110) = 220$.

Time = 15.95 (sec) , antiderivative size = 692, normalized size of antiderivative = 5.49

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{70a^8b^{\frac{33}{2}}x^8\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{200a^7b^{\frac{35}{2}}x^7\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{196a^6b^{\frac{37}{2}}x^6\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{56a^5b^{\frac{39}{2}}x^5\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{70a^4b^{\frac{41}{2}}x^4\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{560a^3b^{\frac{43}{2}}x^3\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{1120a^2b^{\frac{45}{2}}x^2\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{896ab^{\frac{47}{2}}x\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}} + \frac{256b^{\frac{49}{2}}\sqrt{\frac{ax}{b}+1}}{315a^9b^{16}x^4+1260a^8b^{17}x^3+1890a^7b^{18}x^2+1260a^6b^{19}x+315a^5b^{20}}$$

input
```
integrate(x**(7/2)/(a+b/x)**(1/2),x)
```

output
```
70*a**8*b**(33/2)*x**8*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b*
*17*x**3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 20
0*a**7*b**(35/2)*x**7*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**
17*x**3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 196
*a**6*b**(37/2)*x**6*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**1
7*x**3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 56*a
**5*b**(39/2)*x**5*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*
x**3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 70*a**
4*b**(41/2)*x**4*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x*
*3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 560*a**3
*b**(43/2)*x**3*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**
3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 1120*a**2
*b**(45/2)*x**2*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**
3 + 1890*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 896*a*b**
(47/2)*x*sqrt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 + 189
0*a**7*b**18*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20) + 256*b**(49/2)*sq
rt(a*x/b + 1)/(315*a**9*b**16*x**4 + 1260*a**8*b**17*x**3 + 1890*a**7*b**1
8*x**2 + 1260*a**6*b**19*x + 315*a**5*b**20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.68

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\left(35\left(a+\frac{b}{x}\right)^{\frac{9}{2}}x^{\frac{9}{2}} - 180\left(a+\frac{b}{x}\right)^{\frac{7}{2}}bx^{\frac{7}{2}} + 378\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^2x^{\frac{5}{2}} - 420\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^3x^{\frac{3}{2}} + 315\sqrt{a}\right.}{315\,a^5}$$

input
```
integrate(x^(7/2)/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(a + b/x)^(9/2)*x^(9/2) - 180*(a + b/x)^(7/2)*b*x^(7/2) + 378*(a
 + b/x)^(5/2)*b^2*x^(5/2) - 420*(a + b/x)^(3/2)*b^3*x^(3/2) + 315*sqrt(a +
 b/x)*b^4*sqrt(x))/a^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{256\,b^{\frac{9}{2}}\operatorname{sgn}(x)}{315\,a^5} + \frac{2\left(35\,(ax+b)^{\frac{9}{2}} - 180\,(ax+b)^{\frac{7}{2}}b + 378\,(ax+b)^{\frac{5}{2}}b^2 - 420\,(ax+b)^{\frac{3}{2}}b^3 + 315\,\sqrt{ax+b}b^4\right)}{315\,a^5\operatorname{sgn}(x)}$$

input
```
integrate(x^(7/2)/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-256/315*b^(9/2)*sgn(x)/a^5 + 2/315*(35*(a*x + b)^(9/2) - 180*(a*x + b)^(7
/2)*b + 378*(a*x + b)^(5/2)*b^2 - 420*(a*x + b)^(3/2)*b^3 + 315*sqrt(a*x +
 b)*b^4)/(a^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.48

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{9/2}}{9\,a} - \frac{16\,b\,x^{7/2}}{63\,a^2} + \frac{32\,b^2\,x^{5/2}}{105\,a^3} - \frac{128\,b^3\,x^{3/2}}{315\,a^4} + \frac{256\,b^4\,\sqrt{x}}{315\,a^5}\right)$$

input
```
int(x^(7/2)/(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((2*x^(9/2))/(9*a) - (16*b*x^(7/2))/(63*a^2) + (32*b^2*x^(
5/2))/(105*a^3) - (128*b^3*x^(3/2))/(315*a^4) + (256*b^4*x^(1/2))/(315*a^5
))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \frac{x^{7/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\sqrt{ax+b}\,(35a^4x^4-40a^3b\,x^3+48a^2b^2x^2-64a\,b^3x+128b^4)}{315a^5}$$

input
```
int(x^(7/2)/(a+b/x)^(1/2),x)
```

output
```
(2*sqrt(a*x + b)*(35*a**4*x**4 - 40*a**3*b*x**3 + 48*a**2*b**2*x**2 - 64*a
*b**3*x + 128*b**4))/(315*a**5)
```

3.230 $\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{32b^3\sqrt{a+\frac{b}{x}}\sqrt{x}}{35a^4}+\frac{16b^2\sqrt{a+\frac{b}{x}}x^{3/2}}{35a^3}-\frac{12b\sqrt{a+\frac{b}{x}}x^{5/2}}{35a^2}+\frac{2\sqrt{a+\frac{b}{x}}x^{7/2}}{7a}$$

output
```
-32/35*b^3*(a+b/x)^(1/2)*x^(1/2)/a^4+16/35*b^2*(a+b/x)^(1/2)*x^(3/2)/a^3-1
2/35*b*(a+b/x)^(1/2)*x^(5/2)/a^2+2/7*(a+b/x)^(1/2)*x^(7/2)/a
```

Mathematica [A] (verified)

Time = 3.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.53

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-16b^3+8ab^2x-6a^2bx^2+5a^3x^3)}{35a^4}$$

input
```
Integrate[x^(5/2)/Sqrt[a + b/x],x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-16*b^3 + 8*a*b^2*x - 6*a^2*b*x^2 + 5*a^3*x^3))/
(35*a^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}dx}{7a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\left(\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}dx}{5a}\right)}{7a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\left(\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\left(\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{2b\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}dx}{3a}\right)}{5a}\right)}{7a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{7/2}\sqrt{a+\frac{b}{x}}}{7a}-\frac{6b\left(\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\left(\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{4b\sqrt{x}\sqrt{a+\frac{b}{x}}}{3a^2}\right)}{5a}\right)}{7a}$$

input
```
Int[x^(5/2)/Sqrt[a + b/x],x]
```

output
```
(2*Sqrt[a + b/x]*x^(7/2))/(7*a) - (6*b*((2*Sqrt[a + b/x]*x^(5/2))/(5*a) -
(4*b*((-4*b*Sqrt[a + b/x]*Sqrt[x])/(3*a^2) + (2*Sqrt[a + b/x]*x^(3/2))/(3*
a)))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(5a^{3}x^{3}-6a^{2}b\,x^{2}+8a\,b^{2}x-16b^{3}\right)}{35a^{4}}$	50
orering	$\frac{2\left(5a^{3}x^{3}-6a^{2}b\,x^{2}+8a\,b^{2}x-16b^{3}\right)\left(ax+b\right)}{35a^{4}\sqrt{x}\,\sqrt{a+\frac{b}{x}}}$	53
gosper	$\frac{2\left(ax+b\right)\left(5a^{3}x^{3}-6a^{2}b\,x^{2}+8a\,b^{2}x-16b^{3}\right)}{35a^{4}\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	55
risch	$\frac{2\left(ax+b\right)\left(5a^{3}x^{3}-6a^{2}b\,x^{2}+8a\,b^{2}x-16b^{3}\right)}{35a^{4}\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	55

input
```
int(x^(5/2)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*((a*x+b)/x)^(1/2)*x^(1/2)*(5*a^3*x^3-6*a^2*b*x^2+8*a*b^2*x-16*b^3)/a^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\left(5\,a^3x^3-6\,a^2bx^2+8\,ab^2x-16\,b^3\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{35\,a^4}$$

input
```
integrate(x^(5/2)/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^3*x^3 - 6*a^2*b*x^2 + 8*a*b^2*x - 16*b^3)*sqrt(x)*sqrt((a*x + b)
/x)/a^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(87) = 174.

Time = 5.45 (sec) , antiderivative size = 452, normalized size of antiderivative = 4.52

$$\begin{aligned}\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx &= \frac{10a^6b^{\frac{19}{2}}x^6\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}} \\ &+ \frac{18a^5b^{\frac{21}{2}}x^5\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}} \\ &+ \frac{10a^4b^{\frac{23}{2}}x^4\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}} \\ &- \frac{10a^3b^{\frac{25}{2}}x^3\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}} \\ &- \frac{60a^2b^{\frac{27}{2}}x^2\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}} \\ &- \frac{80ab^{\frac{29}{2}}x\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}} \\ &- \frac{32b^{\frac{31}{2}}\sqrt{\frac{ax}{b}+1}}{35a^7b^9x^3+105a^6b^{10}x^2+105a^5b^{11}x+35a^4b^{12}}\end{aligned}$$

input
```
integrate(x**(5/2)/(a+b/x)**(1/2),x)
```

output
```
10*a**6*b**(19/2)*x**6*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10
*x**2 + 105*a**5*b**11*x + 35*a**4*b**12) + 18*a**5*b**(21/2)*x**5*sqrt(a*
x/b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**2 + 105*a**5*b**11*x + 35*
a**4*b**12) + 10*a**4*b**(23/2)*x**4*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 +
105*a**6*b**10*x**2 + 105*a**5*b**11*x + 35*a**4*b**12) - 10*a**3*b**(25/2
)*x**3*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**2 + 105*a**5
*b**11*x + 35*a**4*b**12) - 60*a**2*b**(27/2)*x**2*sqrt(a*x/b + 1)/(35*a**
7*b**9*x**3 + 105*a**6*b**10*x**2 + 105*a**5*b**11*x + 35*a**4*b**12) - 80
*a*b**(29/2)*x*sqrt(a*x/b + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**2 +
105*a**5*b**11*x + 35*a**4*b**12) - 32*b**(31/2)*sqrt(a*x/b + 1)/(35*a**7*
b**9*x**3 + 105*a**6*b**10*x**2 + 105*a**5*b**11*x + 35*a**4*b**12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\left(5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}} - 21\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}} + 35\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}} - 35\sqrt{a+\frac{b}{x}}b^3\sqrt{x}\right)}{35\,a^4}$$

input
```
integrate(x^(5/2)/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*(a + b/x)^(7/2)*x^(7/2) - 21*(a + b/x)^(5/2)*b*x^(5/2) + 35*(a + b
/x)^(3/2)*b^2*x^(3/2) - 35*sqrt(a + b/x)*b^3*sqrt(x))/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{32\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,a^4} + \frac{2\left(5\,(ax+b)^{\frac{7}{2}} - 21\,(ax+b)^{\frac{5}{2}}b + 35\,(ax+b)^{\frac{3}{2}}b^2 - 35\,\sqrt{ax+b}b^3\right)}{35\,a^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^(5/2)/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
32/35*b^(7/2)*sgn(x)/a^4 + 2/35*(5*(a*x + b)^(7/2) - 21*(a*x + b)^(5/2)*b
+ 35*(a*x + b)^(3/2)*b^2 - 35*sqrt(a*x + b)*b^3)/(a^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{7/2}}{7\,a}-\frac{12\,b\,x^{5/2}}{35\,a^2}+\frac{16\,b^2\,x^{3/2}}{35\,a^3}-\frac{32\,b^3\,\sqrt{x}}{35\,a^4}\right)$$

input
```
int(x^(5/2)/(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((2*x^(7/2))/(7*a) - (12*b*x^(5/2))/(35*a^2) + (16*b^2*x^(
3/2))/(35*a^3) - (32*b^3*x^(1/2))/(35*a^4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{x^{5/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\sqrt{ax+b}\,(5a^3x^3-6a^2b\,x^2+8a\,b^2x-16b^3)}{35a^4}$$

input
```
int(x^(5/2)/(a+b/x)^(1/2),x)
```

output
```
(2*sqrt(a*x + b)*(5*a**3*x**3 - 6*a**2*b*x**2 + 8*a*b**2*x - 16*b**3))/(35
*a**4)
```

3.231 $\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{16b^2\sqrt{a+\frac{b}{x}}\sqrt{x}}{15a^3}-\frac{8b\sqrt{a+\frac{b}{x}}x^{3/2}}{15a^2}+\frac{2\sqrt{a+\frac{b}{x}}x^{5/2}}{5a}$$

output
```
16/15*b^2*(a+b/x)^(1/2)*x^(1/2)/a^3-8/15*b*(a+b/x)^(1/2)*x^(3/2)/a^2+2/5*(
a+b/x)^(1/2)*x^(5/2)/a
```

Mathematica [A] (verified)

Time = 3.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(8b^2-4abx+3a^2x^2)}{15a^3}$$

input
```
Integrate[x^(3/2)/Sqrt[a + b/x],x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(8*b^2 - 4*a*b*x + 3*a^2*x^2))/(15*a^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}dx}{5a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\left(\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{2b\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{2x^{5/2}\sqrt{a+\frac{b}{x}}}{5a}-\frac{4b\left(\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{4b\sqrt{x}\sqrt{a+\frac{b}{x}}}{3a^2}\right)}{5a}$$

input
```
Int[x^(3/2)/Sqrt[a + b/x],x]
```

output
```
(2*Sqrt[a + b/x]*x^(5/2))/(5*a) - (4*b*((-4*b*Sqrt[a + b/x]*Sqrt[x])/(3*a^
2) + (2*Sqrt[a + b/x]*x^(3/2))/(3*a)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(3a^{2}x^{2}-4abx+8b^{2}\right)}{15a^{3}}$	39
orering	$\frac{2\left(3a^{2}x^{2}-4abx+8b^{2}\right)\left(ax+b\right)}{15a^{3}\sqrt{x}\,\sqrt{a+\frac{b}{x}}}$	42
gosper	$\frac{2\left(ax+b\right)\left(3a^{2}x^{2}-4abx+8b^{2}\right)}{15a^{3}\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	44
risch	$\frac{2\left(ax+b\right)\left(3a^{2}x^{2}-4abx+8b^{2}\right)}{15a^{3}\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	44

input
```
int(x^(3/2)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*((a*x+b)/x)^(1/2)*x^(1/2)*(3*a^2*x^2-4*a*b*x+8*b^2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,(3\,a^2x^2-4\,abx+8\,b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{15\,a^3}$$

input
```
integrate(x^(3/2)/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^2*x^2 - 4*a*b*x + 8*b^2)*sqrt(x)*sqrt((a*x + b)/x)/a^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(63) = 126.

Time = 1.72 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.51

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{6a^4b^{\frac{9}{2}}x^4\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6} + \frac{4a^3b^{\frac{11}{2}}x^3\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6} + \frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6} + \frac{24ab^{\frac{15}{2}}x\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6} + \frac{16b^{\frac{17}{2}}\sqrt{\frac{ax}{b}+1}}{15a^5b^4x^2+30a^4b^5x+15a^3b^6}$$

input
```
integrate(x**(3/2)/(a+b/x)**(1/2),x)
```

output
```
6*a**4*b**(9/2)*x**4*sqrt(a*x/b + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x +
 15*a**3*b**6) + 4*a**3*b**(11/2)*x**3*sqrt(a*x/b + 1)/(15*a**5*b**4*x**2
+ 30*a**4*b**5*x + 15*a**3*b**6) + 6*a**2*b**(13/2)*x**2*sqrt(a*x/b + 1)/(
15*a**5*b**4*x**2 + 30*a**4*b**5*x + 15*a**3*b**6) + 24*a*b**(15/2)*x*sqrt
(a*x/b + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x + 15*a**3*b**6) + 16*b**(1
7/2)*sqrt(a*x/b + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x + 15*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}} - 10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}} + 15\sqrt{a+\frac{b}{x}}b^2\sqrt{x}\right)}{15\,a^3}$$

input
```
integrate(x^(3/2)/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*(a + b/x)^(5/2)*x^(5/2) - 10*(a + b/x)^(3/2)*b*x^(3/2) + 15*sqrt(a
 + b/x)*b^2*sqrt(x))/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx = -\frac{16\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,a^3} + \frac{2\left(3\,(ax+b)^{\frac{5}{2}} - 10\,(ax+b)^{\frac{3}{2}}b + 15\sqrt{ax+b}b^2\right)}{15\,a^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^(3/2)/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
-16/15*b^(5/2)*sgn(x)/a^3 + 2/15*(3*(a*x + b)^(5/2) - 10*(a*x + b)^(3/2)*b
 + 15*sqrt(a*x + b)*b^2)/(a^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx = \sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{5/2}}{5\,a} - \frac{8\,b\,x^{3/2}}{15\,a^2} + \frac{16\,b^2\,\sqrt{x}}{15\,a^3}\right)$$

input
```
int(x^(3/2)/(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((2*x^(5/2))/(5*a) - (8*b*x^(3/2))/(15*a^2) + (16*b^2*x^(1
/2))/(15*a^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.41

$$\int \frac{x^{3/2}}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{2\sqrt{ax+b}\,(3a^2x^2-4abx+8b^2)}{15a^3}$$

input
```
int(x^(3/2)/(a+b/x)^(1/2),x)
```

output
```
(2*sqrt(a*x + b)*(3*a**2*x**2 - 4*a*b*x + 8*b**2))/(15*a**3)
```

3.232 $\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}} dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}} dx = -\frac{4b\sqrt{a+\frac{b}{x}}\sqrt{x}}{3a^2} + \frac{2\sqrt{a+\frac{b}{x}}x^{3/2}}{3a}$$

output
```
-4/3*b*(a+b/x)^(1/2)*x^(1/2)/a^2+2/3*(a+b/x)^(1/2)*x^(3/2)/a
```

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}} dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-2b+ax)}{3a^2}$$

input
```
Integrate[Sqrt[x]/Sqrt[a + b/x],x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-2*b + a*x))/(3*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{2b\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}dx}{3a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{2x^{3/2}\sqrt{a+\frac{b}{x}}}{3a}-\frac{4b\sqrt{x}\sqrt{a+\frac{b}{x}}}{3a^2}$$

input
```
Int[Sqrt[x]/Sqrt[a + b/x],x]
```

output
```
(-4*b*Sqrt[a + b/x]*Sqrt[x])/(3*a^2) + (2*Sqrt[a + b/x]*x^(3/2))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(ax-2b)}{3a^{2}}$	27
orering	$\frac{2(ax-2b)(ax+b)}{3a^{2}\sqrt{x}\,\sqrt{a+\frac{b}{x}}}$	30
gosper	$\frac{2(ax+b)(ax-2b)}{3a^{2}\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	32
risch	$\frac{2(ax+b)(ax-2b)}{3a^{2}\sqrt{x}\,\sqrt{\frac{ax+b}{x}}}$	32

input
```
int(x^(1/2)/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((a*x+b)/x)^(1/2)*x^(1/2)*(a*x-2*b)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\,(ax-2\,b)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\,a^{2}}$$

input
```
integrate(x^(1/2)/(a+b/x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a*x - 2*b)*sqrt(x)*sqrt((a*x + b)/x)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\sqrt{b}x\sqrt{\frac{ax}{b}+1}}{3a} - \frac{4b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}{3a^2}$$

input
```
integrate(x**(1/2)/(a+b/x)**(1/2),x)
```

output
```
2*sqrt(b)*x*sqrt(a*x/b + 1)/(3*a) - 4*b**(3/2)*sqrt(a*x/b + 1)/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}} - 3\sqrt{a+\frac{b}{x}}b\sqrt{x}\right)}{3\,a^2}$$

input
```
integrate(x^(1/2)/(a+b/x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*((a + b/x)^(3/2)*x^(3/2) - 3*sqrt(a + b/x)*b*sqrt(x))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx = \frac{4\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,a^2} + \frac{2\left((ax+b)^{\frac{3}{2}} - 3\sqrt{ax+b}b\right)}{3\,a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^(1/2)/(a+b/x)^(1/2),x, algorithm="giac")
```

output
```
4/3*b^(3/2)*sgn(x)/a^2 + 2/3*((a*x + b)^(3/2) - 3*sqrt(a*x + b)*b)/(a^2*sg
n(x))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx=\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{3/2}}{3\,a}-\frac{4\,b\,\sqrt{x}}{3\,a^2}\right)$$

input
```
int(x^(1/2)/(a + b/x)^(1/2),x)
```

output
```
(a + b/x)^(1/2)*((2*x^(3/2))/(3*a) - (4*b*x^(1/2))/(3*a^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{x}}{\sqrt{a+\frac{b}{x}}}\,dx=\frac{2\sqrt{ax+b}\,(ax-2b)}{3a^2}$$

input
```
int(x^(1/2)/(a+b/x)^(1/2),x)
```

output
```
(2*sqrt(a*x + b)*(a*x - 2*b))/(3*a**2)
```

3.233 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a}$$

output
```
2*(a+b/x)^(1/2)*x^(1/2)/a
```

Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a}$$

input
```
Integrate[1/(Sqrt[a + b/x]*Sqrt[x]),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x])/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{796}$$

$$\frac{2\sqrt{x}\sqrt{a+\frac{b}{x}}}{a}$$

input
```
Int[1/(Sqrt[a + b/x]*Sqrt[x]),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x])/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{a}$	20
orering	$\frac{2ax+2b}{a\sqrt{a+\frac{b}{x}}\,\sqrt{x}}$	23
gosper	$\frac{2ax+2b}{a\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	25
risch	$\frac{2ax+2b}{a\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	25

input
```
int(1/(a+b/x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((a*x+b)/x)^(1/2)*x^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx=\frac{2\,\sqrt{x}\sqrt{\frac{ax+b}{x}}}{a}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x)*sqrt((a*x + b)/x)/a
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx=\frac{2\sqrt{b}\sqrt{\frac{ax}{b}+1}}{a}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**(1/2),x)
```

output
```
2*sqrt(b)*sqrt(a*x/b + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a + b/x)*sqrt(x)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx = -\frac{2\sqrt{b}\mathrm{sgn}(x)}{a} + \frac{2\sqrt{ax+b}}{a\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(b)*sgn(x)/a + 2*sqrt(a*x + b)/(a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx = \frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}}{a}$$

input
```
int(1/(x^(1/2)*(a + b/x)^(1/2)),x)
```

output
```
(2*x^(1/2)*(a + b/x)^(1/2))/a
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\,dx = \frac{2\sqrt{ax+b}}{a}$$

input
```
int(1/(a+b/x)^(1/2)/x^(1/2),x)
```

output
```
(2*sqrt(a*x + b))/a
```

3.234 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{\sqrt{b}}$$

output
```
-2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{x}\sqrt{\frac{b+ax}{x}}}{\sqrt{b}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^(3/2)),x]
```

output
```
(-2*ArcTanh[(Sqrt[x]*Sqrt[(b + a*x)/x])/Sqrt[b]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {860, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \ 860$$

$$-2\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \ 224$$

$$-2\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}$$

$$\Big\downarrow \ 219$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[a + b/x]*x^(3/2)),x]
```

output
```
(-2*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

method	result	size
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{\sqrt{ax+b}\,\sqrt{b}}$	39

input
```
int(1/(a+b/x)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((a*x+b)/x)^(1/2)*x^(1/2)/(a*x+b)^(1/2)/b^(1/2)*arctanh((a*x+b)^(1/2)/b
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx = \left[\frac{\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)}{\sqrt{b}}, \frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ax+b}\right)}{b}\right]$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x)/sqrt(b), 2*sqrt(
-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b))/b]
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx = -\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**(3/2),x)
```

output
```
-2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/s
qrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*arctan(sqrt(a*x + b)/sqrt(
-b))/(sqrt(-b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{3/2}}\,dx = \int \frac{1}{x^{3/2}\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
int(1/(x^(3/2)*(a + b/x)^(1/2)),x)
```

output
```
int(1/(x^(3/2)*(a + b/x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\,x^{3/2}}\,dx = \frac{\sqrt{b}\left(\log\left(\sqrt{ax+b}-\sqrt{b}\right)-\log\left(\sqrt{ax+b}+\sqrt{b}\right)\right)}{b}$$

input
```
int(1/(a+b/x)^(1/2)/x^(3/2),x)
```

output
```
(sqrt(b)*(log(sqrt(a*x + b) - sqrt(b)) - log(sqrt(a*x + b) + sqrt(b))))/b
```

3.235 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx=-\frac{\sqrt{a+\frac{b}{x}}}{b\sqrt{x}}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^{3/2}}$$

output
```
-(a+b/x)^(1/2)/b/x^(1/2)+a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx=-\frac{\sqrt{a+\frac{b}{x}}}{b\sqrt{x}}+\frac{a\text{arctanh}\left(\frac{\sqrt{x}\sqrt{\frac{b+ax}{x}}}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^(5/2)),x]
```

output
```
-(Sqrt[a + b/x]/(b*Sqrt[x])) + (a*ArcTanh[(Sqrt[x]*Sqrt[(b + a*x)/x])/Sqrt
[b]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {860, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{1}{\sqrt{a+\frac{b}{x}x}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)$$

input
```
Int[1/(Sqrt[a + b/x]*x^(5/2)),x]
```

output
```
-2*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x
])])/(2*b^(3/2)))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)ax+\sqrt{ax+b}\,\sqrt{b}\right)}{\sqrt{x}\,b^{\frac{3}{2}}\sqrt{ax+b}}$	55
risch	$-\frac{ax+b}{b\,x^{\frac{3}{2}}\sqrt{\frac{ax+b}{x}}}+\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}}{b^{\frac{3}{2}}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	64

input
```
int(1/(a+b/x)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x+b)/x)^(1/2)*(-arctanh((a*x+b)^(1/2)/b^(1/2))*a*x+(a*x+b)^(1/2)*b^(1
/2))/x^(1/2)/b^(3/2)/(a*x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx = \left[\frac{a\sqrt{b}x\log\left(\frac{ax+2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)-2b\sqrt{x}\sqrt{\frac{ax+b}{x}}}{2b^2x}, -\frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+b\sqrt{x}\sqrt{\frac{ax+b}{x}}}{b^2x}\right]$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(b)*x*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x)
 - 2*b*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x), -(a*sqrt(-b)*x*arctan(sqrt(-b)*
sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + b*sqrt(x)*sqrt((a*x + b)/x))/(b^2*x
)]
```

Sympy [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx = -\frac{\sqrt{a}\sqrt{1+\frac{b}{ax}}}{b\sqrt{x}}+\frac{a\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**(5/2),x)
```

output
```
-sqrt(a)*sqrt(1 + b/(a*x))/(b*sqrt(x)) + a*asinh(sqrt(b)/(sqrt(a)*sqrt(x))
)/b**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx = -\frac{\sqrt{a+\frac{b}{x}}a\sqrt{x}}{\left(a+\frac{b}{x}\right)bx-b^2} - \frac{a\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-sqrt(a + b/x)*a*sqrt(x)/((a + b/x)*b*x - b^2) - 1/2*a*log((sqrt(a + b/x)*
sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx = -\frac{a\left(\frac{\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\sqrt{ax+b}}{abx}\right)}{\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
-a*(arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x + b)/(a*b*x))/s
gn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
int(1/(x^(5/2)*(a + b/x)^(1/2)),x)
```

output
```
int(1/(x^(5/2)*(a + b/x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{5/2}}\,dx = \frac{-2\sqrt{ax+b}\,b-\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)ax+\sqrt{b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)ax}{2b^2x}$$

input
```
int(1/(a+b/x)^(1/2)/x^(5/2),x)
```

output
```
( - 2*sqrt(a*x + b)*b - sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a*x + sqrt(b)
*log(sqrt(a*x + b) + sqrt(b))*a*x)/(2*b**2*x)
```

3.236 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx=-\frac{\sqrt{a+\frac{b}{x}}}{2bx^{3/2}}+\frac{3a\sqrt{a+\frac{b}{x}}}{4b^2\sqrt{x}}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4b^{5/2}}$$

output
```
-1/2*(a+b/x)^(1/2)/b/x^(3/2)+3/4*a*(a+b/x)^(1/2)/b^2/x^(1/2)-3/4*a^2*arcta
nh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 8.81 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{\sqrt{b+ax}(-2b+3ax)}{4b^2x^2}-\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{4b^{5/2}}\right)}{\sqrt{b+ax}}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^(7/2)),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*((Sqrt[b + a*x]*(-2*b + 3*a*x))/(4*b^2*x^2) - (3*a^
2*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/(4*b^(5/2))))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {860, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{ 860}$$

$$-2\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 262}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\int \frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}}{4b}\right)$$

$$\Big\downarrow \text{ 262}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{4b}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{4b}\right)$$

$$\downarrow \text{219}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{4b}\right)$$

input
```
Int[1/(Sqrt[a + b/x]*x^(7/2)),x]
```

output
```
-2*(Sqrt[a + b/x]/(4*b*x^(3/2)) - (3*a*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*A
rcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*b^(3/2))))/(4*b))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(3\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^{2}x^{2}-3ax\sqrt{ax+b}\,\sqrt{b}+2\sqrt{ax+b}\,b^{\frac{3}{2}}\right)}{4x^{\frac{3}{2}}b^{\frac{5}{2}}\sqrt{ax+b}}$	74
risch	$\frac{(ax+b)(3ax-2b)}{4b^{2}x^{\frac{5}{2}}\sqrt{\frac{ax+b}{x}}}-\frac{3a^{2}\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}}{4b^{\frac{5}{2}}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	75

input
```
int(1/(a+b/x)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x+b)/x)^(1/2)/x^(3/2)/b^(5/2)*(3*arctanh((a*x+b)^(1/2)/b^(1/2))*a
^2*x^2-3*a*x*(a*x+b)^(1/2)*b^(1/2)+2*(a*x+b)^(1/2)*b^(3/2))/(a*x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx=\left[\frac{3\,a^{2}\sqrt{b}x^{2}\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,(3\,abx-2\,b^{2})\sqrt{x}\sqrt{\frac{ax+b}{x}}}{8\,b^{3}x^{2}},\ \frac{3\,a^{2}\sqrt{-b}x^{2}\arctan\left(\right.}{}\right.$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*x^2*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2
*b)/x) + 2*(3*a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b^3*x^2), 1/4*(3*
a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (3
*a*b*x - 2*b^2)*sqrt(x)*sqrt((a*x + b)/x))/(b^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 11.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx = \frac{3a^{\frac{3}{2}}}{4b^2\sqrt{x}\sqrt{1+\frac{b}{ax}}} + \frac{\sqrt{a}}{4bx^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} - \frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{\frac{5}{2}}} - \frac{1}{2\sqrt{a}x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**(7/2),x)
```

output
```
3*a**(3/2)/(4*b**2*sqrt(x)*sqrt(1 + b/(a*x))) + sqrt(a)/(4*b*x**(3/2)*sqrt
(1 + b/(a*x))) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*b**(5/2)) - 1/
(2*sqrt(a)*x**(5/2)*sqrt(1 + b/(a*x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx = \frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{5}{2}}} + \frac{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}} - 5\sqrt{a+\frac{b}{x}}a^2b\sqrt{x}}{4\left(\left(a+\frac{b}{x}\right)^2b^2x^2 - 2\left(a+\frac{b}{x}\right)b^3x + b^4\right)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
3/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqr
t(b)))/b^(5/2) + 1/4*(3*(a + b/x)^(3/2)*a^2*x^(3/2) - 5*sqrt(a + b/x)*a^2*
b*sqrt(x))/((a + b/x)^2*b^2*x^2 - 2*(a + b/x)*b^3*x + b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx = \frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\,(ax+b)^{\frac{3}{2}}a^3-5\,\sqrt{ax+b}a^3b}{a^2b^2x^2}}{4\,a\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
1/4*(3*a^3*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*(a*x + b)^(3
/2)*a^3 - 5*sqrt(a*x + b)*a^3*b)/(a^2*b^2*x^2))/(a*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
int(1/(x^(7/2)*(a + b/x)^(1/2)),x)
```

output
```
int(1/(x^(7/2)*(a + b/x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{7/2}}\,dx = \frac{6\sqrt{ax+b}\,abx-4\sqrt{ax+b}\,b^2+3\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^2x^2-3\sqrt{b}\log\left(\sqrt{ax+b}+\right.}{8b^3x^2}$$

input
```
int(1/(a+b/x)^(1/2)/x^(7/2),x)
```

output
```
(6*sqrt(a*x + b)*a*b*x - 4*sqrt(a*x + b)*b**2 + 3*sqrt(b)*log(sqrt(a*x + b
) - sqrt(b))*a**2*x**2 - 3*sqrt(b)*log(sqrt(a*x + b) + sqrt(b))*a**2*x**2)
/(8*b**3*x**2)
```

3.237 $\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx=-\frac{\sqrt{a+\frac{b}{x}}}{3bx^{5/2}}+\frac{5a\sqrt{a+\frac{b}{x}}}{12b^2x^{3/2}}-\frac{5a^2\sqrt{a+\frac{b}{x}}}{8b^3\sqrt{x}}+\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{8b^{7/2}}$$

output
```
-1/3*(a+b/x)^(1/2)/b/x^(5/2)+5/12*a*(a+b/x)^(1/2)/b^2/x^(3/2)-5/8*a^2*(a+b
/x)^(1/2)/b^3/x^(1/2)+5/8*a^3*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx=\frac{-\sqrt{b}(8b^3-2ab^2x+5a^2bx^2+15a^3x^3)+15a^{7/2}\sqrt{1+\frac{b}{ax}}x^{7/2}\text{arcsinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{24b^{7/2}\sqrt{a+\frac{b}{x}}x^{7/2}}$$

input
```
Integrate[1/(Sqrt[a + b/x]*x^(9/2)),x]
```

output
```
(-(Sqrt[b]*(8*b^3 - 2*a*b^2*x + 5*a^2*b*x^2 + 15*a^3*x^3)) + 15*a^(7/2)*Sq
rt[1 + b/(a*x)]*x^(7/2)*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])])/(24*b^(7/2)*Sq
rt[a + b/x]*x^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {860, 262, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2}\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\int \frac{1}{\sqrt{a+\frac{b}{x}}x^2}d\frac{1}{\sqrt{x}}}{6b}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\int \frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}}{4b}\right)}{6b}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{4b}\right)}{6b}\right)$$

$\big\downarrow$ 224

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{4b}\right)}{6b}\right)$$

$\big\downarrow$ 219

$$-2\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{4b}\right)}{6b}\right)$$

input
```
Int[1/(Sqrt[a + b/x]*x^(9/2)),x]
```

output
```
-2*(Sqrt[a + b/x]/(6*b*x^(5/2)) - (5*a*(Sqrt[a + b/x]/(4*b*x^(3/2)) - (3*a
*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])
])/(2*b^(3/2))))/(4*b)))/(6*b))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{(ax+b)(15a^2x^2-10abx+8b^2)}{24b^3x^{\frac{7}{2}}\sqrt{\frac{ax+b}{x}}}+\frac{5a^3\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}}{8b^{\frac{7}{2}}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	86
default	$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-15\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^3x^3+8\sqrt{ax+b}\,b^{\frac{5}{2}}-10a\,b^{\frac{3}{2}}x\sqrt{ax+b}+15a^2x^2\sqrt{ax+b}\,\sqrt{b}\right)}{24x^{\frac{5}{2}}b^{\frac{7}{2}}\sqrt{ax+b}}$	92

input
```
int(1/(a+b/x)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(a*x+b)*(15*a^2*x^2-10*a*b*x+8*b^2)/b^3/x^(7/2)/((a*x+b)/x)^(1/2)+5/
8*a^3/b^(7/2)*arctanh((a*x+b)^(1/2)/b^(1/2))/((a*x+b)/x)^(1/2)/x^(1/2)*(a*
x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx = \left[\frac{15\,a^3\sqrt{b}x^3\log\left(\frac{ax+2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(15\,a^2bx^2-10\,ab^2x+8\,b^3)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{48\,b^4x^3}, -\frac{15\,a^3\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ax+b}\right)+(15\,a^2bx^2-10\,ab^2x+8\,b^3)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{24\,b^4x^3}\right]$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*x^3*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) +
 2*b)/x) - 2*(15*a^2*b*x^2 - 10*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x + b)/x)
)/(b^4*x^3), -1/24*(15*a^3*sqrt(-b)*x^3*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x
+ b)/x)/(a*x + b)) + (15*a^2*b*x^2 - 10*a*b^2*x + 8*b^3)*sqrt(x)*sqrt((a*x
 + b)/x))/(b^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 34.00 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx = -\frac{5a^{\frac{5}{2}}}{8b^3\sqrt{x}\sqrt{1+\frac{b}{ax}}} - \frac{5a^{\frac{3}{2}}}{24b^2x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} + \frac{\sqrt{a}}{12bx^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}} + \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{8b^{\frac{7}{2}}} - \frac{1}{3\sqrt{a}x^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate(1/(a+b/x)**(1/2)/x**(9/2),x)
```

output
```
-5*a**(5/2)/(8*b**3*sqrt(x)*sqrt(1 + b/(a*x))) - 5*a**(3/2)/(24*b**2*x**(3
/2)*sqrt(1 + b/(a*x))) + sqrt(a)/(12*b*x**(5/2)*sqrt(1 + b/(a*x))) + 5*a**
3*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(8*b**(7/2)) - 1/(3*sqrt(a)*x**(7/2)*sq
rt(1 + b/(a*x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx = -\frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{7}{2}}} - \frac{15\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3x^{\frac{5}{2}} - 40\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3bx^{\frac{3}{2}} + 33\sqrt{a+\frac{b}{x}}a^3b^2\sqrt{x}}{24\left(\left(a+\frac{b}{x}\right)^3b^3x^3 - 3\left(a+\frac{b}{x}\right)^2b^4x^2 + 3\left(a+\frac{b}{x}\right)b^5x - b^6\right)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-5/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + s
qrt(b)))/b^(7/2) - 1/24*(15*(a + b/x)^(5/2)*a^3*x^(5/2) - 40*(a + b/x)^(3/
2)*a^3*b*x^(3/2) + 33*sqrt(a + b/x)*a^3*b^2*sqrt(x))/((a + b/x)^3*b^3*x^3
- 3*(a + b/x)^2*b^4*x^2 + 3*(a + b/x)*b^5*x - b^6)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx = -\frac{a^3\left(\frac{15\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{15(ax+b)^{\frac{5}{2}}-40(ax+b)^{\frac{3}{2}}b+33\sqrt{ax+b}b^2}{a^3b^3x^3}\right)}{24\,\mathrm{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
-1/24*a^3*(15*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^3) + (15*(a*x + b
)^(5/2) - 40*(a*x + b)^(3/2)*b + 33*sqrt(a*x + b)*b^2)/(a^3*b^3*x^3))/sgn(
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx = \int \frac{1}{x^{9/2}\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
int(1/(x^(9/2)*(a + b/x)^(1/2)),x)
```

output
```
int(1/(x^(9/2)*(a + b/x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}x^{9/2}}\,dx = \frac{-30\sqrt{ax+b}\,a^2b\,x^2+20\sqrt{ax+b}\,a\,b^2x-16\sqrt{ax+b}\,b^3-15\sqrt{b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a}{48b^4x^3}$$

input
```
int(1/(a+b/x)^(1/2)/x^(9/2),x)
```

output
```
( - 30*sqrt(a*x + b)*a**2*b*x**2 + 20*sqrt(a*x + b)*a*b**2*x - 16*sqrt(a*x
 + b)*b**3 - 15*sqrt(b)*log(sqrt(a*x + b) - sqrt(b))*a**3*x**3 + 15*sqrt(b
)*log(sqrt(a*x + b) + sqrt(b))*a**3*x**3)/(48*b**4*x**3)
```

3.238 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{256b^3\sqrt{a+\frac{b}{x}}\sqrt{x}}{35a^5} + \frac{128b^2\sqrt{a+\frac{b}{x}}x^{3/2}}{35a^4} - \frac{96b\sqrt{a+\frac{b}{x}}x^{5/2}}{35a^3} - \frac{2x^{7/2}}{a\sqrt{a+\frac{b}{x}}} + \frac{16\sqrt{a+\frac{b}{x}}x^{7/2}}{7a^2}$$

output
```
-256/35*b^3*(a+b/x)^(1/2)*x^(1/2)/a^5+128/35*b^2*(a+b/x)^(1/2)*x^(3/2)/a^4
-96/35*b*(a+b/x)^(1/2)*x^(5/2)/a^3-2*x^(7/2)/a/(a+b/x)^(1/2)+16/7*(a+b/x)^
(1/2)*x^(7/2)/a^2
```

Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-128b^4-64ab^3x+16a^2b^2x^2-8a^3bx^3+5a^4x^4)}{35a^5(b+ax)}$$

input
```
Integrate[x^(5/2)/(a + b/x)^(3/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-128*b^4 - 64*a*b^3*x + 16*a^2*b^2*x^2 - 8*a^3*b
*x^3 + 5*a^4*x^4))/(35*a^5*(b + a*x))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {803, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}-\frac{8b\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}dx}{7a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}-\frac{8b\left(\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}dx}{5a}\right)}{7a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}-\frac{8b\left(\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\left(\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\int\frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}dx}{3a}\right)}{5a}\right)}{7a}$$

$\downarrow$ 803

$$\frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}-\frac{8b\left(\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\left(\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\left(\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}-\frac{2b\int\frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}dx}{a}\right)}{3a}\right)}{5a}\right)}{7a}$$

$\downarrow$ 796

$$\frac{2x^{7/2}}{7a\sqrt{a+\frac{b}{x}}}-\frac{8b\left(\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\left(\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\left(\frac{4b}{a^2\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}\right)}{3a}\right)}{5a}\right)}{7a}$$

input
```
Int[x^(5/2)/(a + b/x)^(3/2),x]
```

output
```
(2*x^(7/2))/(7*a*Sqrt[a + b/x]) - (8*b*((2*x^(5/2))/(5*a*Sqrt[a + b/x]) -
(6*b*((-4*b*((4*b)/(a^2*Sqrt[a + b/x]*Sqrt[x]) + (2*Sqrt[x])/(a*Sqrt[a + b
/x])))/(3*a) + (2*x^(3/2))/(3*a*Sqrt[a + b/x])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

method	result	size
orering	$\frac{2(5a^4x^4-8a^3b\,x^3+16a^2b^2x^2-64a\,b^3x-128b^4)(ax+b)}{35a^5x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$	64
gosper	$\frac{2(ax+b)(5a^4x^4-8a^3b\,x^3+16a^2b^2x^2-64a\,b^3x-128b^4)}{35a^5x^{\frac{3}{2}}\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}$	66
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(5a^4x^4-8a^3b\,x^3+16a^2b^2x^2-64a\,b^3x-128b^4)}{35(ax+b)a^5}$	68
risch	$\frac{2(5a^3x^3-13a^2b\,x^2+29a\,b^2x-93b^3)(ax+b)}{35a^5\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}-\frac{2b^4}{a^5\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	78

input
```
int(x^(5/2)/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(5*a^4*x^4-8*a^3*b*x^3+16*a^2*b^2*x^2-64*a*b^3*x-128*b^4)/a^5/x^(3/2)
*(a*x+b)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2\,(5\,a^4x^4-8\,a^3bx^3+16\,a^2b^2x^2-64\,ab^3x-128\,b^4)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{35\,(a^6x+a^5b)}$$

input
```
integrate(x^(5/2)/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*a^4*x^4 - 8*a^3*b*x^3 + 16*a^2*b^2*x^2 - 64*a*b^3*x - 128*b^4)*sqr
t(x)*sqrt((a*x + b)/x)/(a^6*x + a^5*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(105) = 210.

Time = 6.92 (sec) , antiderivative size = 614, normalized size of antiderivative = 5.07

$$\begin{aligned}\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= \frac{10a^7b^{\frac{33}{2}}x^7\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\\
&+\frac{14a^6b^{\frac{35}{2}}x^6\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\\
&+\frac{14a^5b^{\frac{37}{2}}x^5\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\\
&-\frac{70a^4b^{\frac{39}{2}}x^4\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\\
&-\frac{560a^3b^{\frac{41}{2}}x^3\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\\
&-\frac{1120a^2b^{\frac{43}{2}}x^2\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\\
&-\frac{896ab^{\frac{45}{2}}x\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\\
&-\frac{256b^{\frac{47}{2}}\sqrt{\frac{ax}{b}+1}}{35a^9b^{16}x^4+140a^8b^{17}x^3+210a^7b^{18}x^2+140a^6b^{19}x+35a^5b^{20}}\end{aligned}$$

input
```
integrate(x**(5/2)/(a+b/x)**(3/2),x)
```

output
```
10*a**7*b**(33/2)*x**7*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**1
7*x**3 + 210*a**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20) + 14*a**6
*b**(35/2)*x**6*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3
+ 210*a**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20) + 14*a**5*b**(37
/2)*x**5*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a
**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20) - 70*a**4*b**(39/2)*x**
4*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b**
18*x**2 + 140*a**6*b**19*x + 35*a**5*b**20) - 560*a**3*b**(41/2)*x**3*sqrt
(a*x/b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b**18*x**
2 + 140*a**6*b**19*x + 35*a**5*b**20) - 1120*a**2*b**(43/2)*x**2*sqrt(a*x/
b + 1)/(35*a**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b**18*x**2 + 1
40*a**6*b**19*x + 35*a**5*b**20) - 896*a*b**(45/2)*x*sqrt(a*x/b + 1)/(35*a
**9*b**16*x**4 + 140*a**8*b**17*x**3 + 210*a**7*b**18*x**2 + 140*a**6*b**1
9*x + 35*a**5*b**20) - 256*b**(47/2)*sqrt(a*x/b + 1)/(35*a**9*b**16*x**4 +
 140*a**8*b**17*x**3 + 210*a**7*b**18*x**2 + 140*a**6*b**19*x + 35*a**5*b*
*20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{2\,b^4}{\sqrt{a+\frac{b}{x}}a^5\sqrt{x}} + \frac{2\left(5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}} - 28\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}} + 70\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}} - 140\sqrt{a+\frac{b}{x}}b^3\sqrt{x}\right)}{35\,a^5}$$

input
```
integrate(x^(5/2)/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
-2*b^4/(sqrt(a + b/x)*a^5*sqrt(x)) + 2/35*(5*(a + b/x)^(7/2)*x^(7/2) - 28*
(a + b/x)^(5/2)*b*x^(5/2) + 70*(a + b/x)^(3/2)*b^2*x^(3/2) - 140*sqrt(a +
b/x)*b^3*sqrt(x))/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{256\,b^{\frac{7}{2}}\operatorname{sgn}(x)}{35\,a^5} - \frac{2\left(\frac{35\,b^4}{\sqrt{ax+b}a\operatorname{sgn}(x)} - \frac{5\,(ax+b)^{\frac{7}{2}}a^6-28\,(ax+b)^{\frac{5}{2}}a^6b+70\,(ax+b)^{\frac{3}{2}}a^6b^2-140\,\sqrt{ax+b}a^6b^3}{a^7\operatorname{sgn}(x)}\right)}{35\,a^4}$$

input
```
integrate(x^(5/2)/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
256/35*b^(7/2)*sgn(x)/a^5 - 2/35*(35*b^4/(sqrt(a*x + b)*a*sgn(x)) - (5*(a*
x + b)^(7/2)*a^6 - 28*(a*x + b)^(5/2)*a^6*b + 70*(a*x + b)^(3/2)*a^6*b^2 -
 140*sqrt(a*x + b)*a^6*b^3)/(a^7*sgn(x)))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{2\,\sqrt{x}\,\sqrt{\frac{b+a\,x}{x}}\,\left(-5\,a^4\,x^4+8\,a^3\,b\,x^3-16\,a^2\,b^2\,x^2+64\,a\,b^3\,x+128\,b^4\right)}{35\,a^5\,\left(b+a\,x\right)}$$

input
```
int(x^(5/2)/(a + b/x)^(3/2),x)
```

output
```
-(2*x^(1/2)*((b + a*x)/x)^(1/2)*(128*b^4 - 5*a^4*x^4 + 8*a^3*b*x^3 - 16*a^
2*b^2*x^2 + 64*a*b^3*x))/(35*a^5*(b + a*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\frac{2}{7}a^4x^4-\frac{16}{35}a^3b\,x^3+\frac{32}{35}a^2b^2x^2-\frac{128}{35}a\,b^3x-\frac{256}{35}b^4}{\sqrt{ax+b}\,a^5}$$

input

```
int(x^(5/2)/(a+b/x)^(3/2),x)
```

output

```
(2*(5*a**4*x**4 - 8*a**3*b*x**3 + 16*a**2*b**2*x**2 - 64*a*b**3*x - 128*b*
*4))/(35*sqrt(a*x + b)*a**5)
```

3.239 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{32b^2\sqrt{a+\frac{b}{x}}\sqrt{x}}{5a^4}-\frac{16b\sqrt{a+\frac{b}{x}}x^{3/2}}{5a^3}-\frac{2x^{5/2}}{a\sqrt{a+\frac{b}{x}}}+\frac{12\sqrt{a+\frac{b}{x}}x^{5/2}}{5a^2}$$

output
```
32/5*b^2*(a+b/x)^(1/2)*x^(1/2)/a^4-16/5*b*(a+b/x)^(1/2)*x^(3/2)/a^3-2*x^(5
/2)/a/(a+b/x)^(1/2)+12/5*(a+b/x)^(1/2)*x^(5/2)/a^2
```

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(16b^3+8ab^2x-2a^2bx^2+a^3x^3)}{5a^4(b+ax)}$$

input
```
Integrate[x^(3/2)/(a + b/x)^(3/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(16*b^3 + 8*a*b^2*x - 2*a^2*b*x^2 + a^3*x^3))/(5*
a^4*(b + a*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}dx}{5a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\left(\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\left(\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\left(\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}-\frac{2b\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}dx}{a}\right)}{3a}\right)}{5a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{5/2}}{5a\sqrt{a+\frac{b}{x}}}-\frac{6b\left(\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\left(\frac{4b}{a^2\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}\right)}{3a}\right)}{5a}$$

input
```
Int[x^(3/2)/(a + b/x)^(3/2),x]
```

output
```
(2*x^(5/2))/(5*a*Sqrt[a + b/x]) - (6*b*((-4*b*((4*b)/(a^2*Sqrt[a + b/x]*Sq
rt[x]) + (2*Sqrt[x])/(a*Sqrt[a + b/x])))/(3*a) + (2*x^(3/2))/(3*a*Sqrt[a +
 b/x])))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.55

method	result	size
orering	$\frac{2(a^3x^3-2a^2b\,x^2+8a\,b^2x+16b^3)(ax+b)}{5a^4x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$	52
gosper	$\frac{2(ax+b)(a^3x^3-2a^2b\,x^2+8a\,b^2x+16b^3)}{5a^4x^{\frac{3}{2}}\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}$	54
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(a^3x^3-2a^2b\,x^2+8a\,b^2x+16b^3)}{5(ax+b)a^4}$	56
risch	$\frac{2(a^2x^2-3abx+11b^2)(ax+b)}{5a^4\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}+\frac{2b^3}{a^4\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	66

input
```
int(x^(3/2)/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a^3*x^3-2*a^2*b*x^2+8*a*b^2*x+16*b^3)/a^4/x^(3/2)*(a*x+b)/(a+b/x)^(3/
2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\,(a^3x^3-2\,a^2bx^2+8\,ab^2x+16\,b^3)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{5\,(a^5x+a^4b)}$$

input
```
integrate(x^(3/2)/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
2/5*(a^3*x^3 - 2*a^2*b*x^2 + 8*a*b^2*x + 16*b^3)*sqrt(x)*sqrt((a*x + b)/x)
/(a^5*x + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(82) = 164$.

Time = 2.08 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.37

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2a^5b^{\frac{19}{2}}x^5\sqrt{\frac{ax}{b}+1}}{5a^7b^9x^3+15a^6b^{10}x^2+15a^5b^{11}x+5a^4b^{12}} + \frac{10a^3b^{\frac{23}{2}}x^3\sqrt{\frac{ax}{b}+1}}{5a^7b^9x^3+15a^6b^{10}x^2+15a^5b^{11}x+5a^4b^{12}} + \frac{60a^2b^{\frac{25}{2}}x^2\sqrt{\frac{ax}{b}+1}}{5a^7b^9x^3+15a^6b^{10}x^2+15a^5b^{11}x+5a^4b^{12}} + \frac{80ab^{\frac{27}{2}}x\sqrt{\frac{ax}{b}+1}}{5a^7b^9x^3+15a^6b^{10}x^2+15a^5b^{11}x+5a^4b^{12}} + \frac{32b^{\frac{29}{2}}\sqrt{\frac{ax}{b}+1}}{5a^7b^9x^3+15a^6b^{10}x^2+15a^5b^{11}x+5a^4b^{12}}$$

input
```
integrate(x**(3/2)/(a+b/x)**(3/2),x)
```

output
```
2*a**5*b**(19/2)*x**5*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**5*b**11*x + 5*a**4*b**12) + 10*a**3*b**(23/2)*x**3*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**5*b**11*x + 5*a**4*b**12) + 60*a**2*b**(25/2)*x**2*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**5*b**11*x + 5*a**4*b**12) + 80*a*b**(27/2)*x*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**5*b**11*x + 5*a**4*b**12) + 32*b**(29/2)*sqrt(a*x/b + 1)/(5*a**7*b**9*x**3 + 15*a**6*b**10*x**2 + 15*a**5*b**11*x + 5*a**4*b**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\,b^3}{\sqrt{a+\frac{b}{x}}a^4\sqrt{x}} + \frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}} - 5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}} + 15\sqrt{a+\frac{b}{x}}b^2\sqrt{x}\right)}{5\,a^4}$$

input
```
integrate(x^(3/2)/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
2*b^3/(sqrt(a + b/x)*a^4*sqrt(x)) + 2/5*((a + b/x)^(5/2)*x^(5/2) - 5*(a +
b/x)^(3/2)*b*x^(3/2) + 15*sqrt(a + b/x)*b^2*sqrt(x))/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{32\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,a^4} + \frac{2\,b^3}{\sqrt{ax+b}a^4\mathrm{sgn}\left(x\right)} + \frac{2\left(\left(ax+b\right)^{\frac{5}{2}}a^{16} - 5\left(ax+b\right)^{\frac{3}{2}}a^{16}b + 15\sqrt{ax+b}a^{16}b^2\right)}{5\,a^{20}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^(3/2)/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
-32/5*b^(5/2)*sgn(x)/a^4 + 2*b^3/(sqrt(a*x + b)*a^4*sgn(x)) + 2/5*((a*x +
b)^(5/2)*a^16 - 5*(a*x + b)^(3/2)*a^16*b + 15*sqrt(a*x + b)*a^16*b^2)/(a^2
0*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{7/2}}{5\,a^2} - \frac{4\,b\,x^{5/2}}{5\,a^3} + \frac{16\,b^2\,x^{3/2}}{5\,a^4} + \frac{32\,b^3\,\sqrt{x}}{5\,a^5}\right)}{x+\frac{b}{a}}$$

input
```
int(x^(3/2)/(a + b/x)^(3/2),x)
```

output
```
((a + b/x)^(1/2)*((2*x^(7/2))/(5*a^2) - (4*b*x^(5/2))/(5*a^3) + (16*b^2*x^
(3/2))/(5*a^4) + (32*b^3*x^(1/2))/(5*a^5)))/(x + b/a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.44

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\frac{2}{5}a^3x^3-\frac{4}{5}a^2b\,x^2+\frac{16}{5}a\,b^2x+\frac{32}{5}b^3}{\sqrt{ax+b}\,a^4}$$

input
```
int(x^(3/2)/(a+b/x)^(3/2),x)
```

output
```
(2*(a**3*x**3 - 2*a**2*b*x**2 + 8*a*b**2*x + 16*b**3))/(5*sqrt(a*x + b)*a*
*4)
```

3.240 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{16b\sqrt{a+\frac{b}{x}}\sqrt{x}}{3a^3} - \frac{2x^{3/2}}{a\sqrt{a+\frac{b}{x}}} + \frac{8\sqrt{a+\frac{b}{x}}x^{3/2}}{3a^2}$$

output
```
-16/3*b*(a+b/x)^(1/2)*x^(1/2)/a^3-2*x^(3/2)/a/(a+b/x)^(1/2)+8/3*(a+b/x)^(1
/2)*x^(3/2)/a^2
```

Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-8b^2-4abx+a^2x^2)}{3a^3(b+ax)}$$

input
```
Integrate[Sqrt[x]/(a + b/x)^(3/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-8*b^2 - 4*a*b*x + a^2*x^2))/(3*a^3*(b + a*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 803$$

$$\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}dx}{3a}$$

$$\Big\downarrow \ 803$$

$$\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\left(\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}-\frac{2b\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}dx}{a}\right)}{3a}$$

$$\Big\downarrow \ 796$$

$$\frac{2x^{3/2}}{3a\sqrt{a+\frac{b}{x}}}-\frac{4b\left(\frac{4b}{a^2\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}\right)}{3a}$$

input
```
Int[Sqrt[x]/(a + b/x)^(3/2),x]
```

output
```
(-4*b*((4*b)/(a^2*Sqrt[a + b/x]*Sqrt[x]) + (2*Sqrt[x])/(a*Sqrt[a + b/x])))
/(3*a) + (2*x^(3/2))/(3*a*Sqrt[a + b/x])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

method	result	size
orering	$\frac{2\left(a^{2} x^{2}-4 a b x -8 b^{2}\right) \left(a x +b \right)}{3 a^{3} x^{\frac{3}{2}} \left(a +\frac{b}{x}\right)^{\frac{3}{2}}}$	41
gosper	$\frac{2 \left(a x +b \right) \left(a^{2} x^{2}-4 a b x -8 b^{2}\right)}{3 a^{3} x^{\frac{3}{2}} \left(\frac{a x +b}{x}\right)^{\frac{3}{2}}}$	43
default	$\frac{2 \sqrt{\frac{a x +b}{x}}\, \sqrt{x}\, \left(a^{2} x^{2}-4 a b x -8 b^{2}\right)}{3 \left(a x +b \right) a^{3}}$	45
risch	$\frac{2 \left(a x -5 b \right) \left(a x +b \right)}{3 a^{3} \sqrt{\frac{a x +b}{x}}\, \sqrt{x}}-\frac{2 b^{2}}{a^{3} \sqrt{\frac{a x +b}{x}}\, \sqrt{x}}$	55

input
```
int(x^(1/2)/(a+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a^2*x^2-4*a*b*x-8*b^2)/a^3*(a*x+b)/x^(3/2)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2\,(a^2x^2-4\,abx-8\,b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\,(a^4x+a^3b)}$$

input
```
integrate(x^(1/2)/(a+b/x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a^2*x^2 - 4*a*b*x - 8*b^2)*sqrt(x)*sqrt((a*x + b)/x)/(a^4*x + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(58) = 116.

Time = 1.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.99

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx=\frac{2a^3b^{\frac{9}{2}}x^3\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}-\frac{6a^2b^{\frac{11}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}-\frac{24ab^{\frac{13}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}-\frac{16b^{\frac{15}{2}}\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}$$

input
```
integrate(x**(1/2)/(a+b/x)**(3/2),x)
```

output
```
2*a**3*b**(9/2)*x**3*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x + 3
*a**3*b**6) - 6*a**2*b**(11/2)*x**2*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*
a**4*b**5*x + 3*a**3*b**6) - 24*a*b**(13/2)*x*sqrt(a*x/b + 1)/(3*a**5*b**4
*x**2 + 6*a**4*b**5*x + 3*a**3*b**6) - 16*b**(15/2)*sqrt(a*x/b + 1)/(3*a**
5*b**4*x**2 + 6*a**4*b**5*x + 3*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}-6\,\sqrt{a+\frac{b}{x}}b\sqrt{x}\right)}{3\,a^3}-\frac{2\,b^2}{\sqrt{a+\frac{b}{x}}a^3\sqrt{x}}$$

input
```
integrate(x^(1/2)/(a+b/x)^(3/2),x, algorithm="maxima")
```

output
```
2/3*((a + b/x)^(3/2)*x^(3/2) - 6*sqrt(a + b/x)*b*sqrt(x))/a^3 - 2*b^2/(sqr
t(a + b/x)*a^3*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{16\,b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,a^3}-\frac{2\left(\frac{3\,b^2}{\sqrt{ax+b}a\operatorname{sgn}(x)}-\frac{(ax+b)^{\frac{3}{2}}a^2-6\,\sqrt{ax+b}a^2b}{a^3\operatorname{sgn}(x)}\right)}{3\,a^2}$$

input
```
integrate(x^(1/2)/(a+b/x)^(3/2),x, algorithm="giac")
```

output
```
16/3*b^(3/2)*sgn(x)/a^3 - 2/3*(3*b^2/(sqrt(a*x + b)*a*sgn(x)) - ((a*x + b)
^(3/2)*a^2 - 6*sqrt(a*x + b)*a^2*b)/(a^3*sgn(x)))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\frac{\sqrt{a+\frac{b}{x}}\left(\frac{8\,b\,x^{3/2}}{3\,a^3}-\frac{2\,x^{5/2}}{3\,a^2}+\frac{16\,b^2\,\sqrt{x}}{3\,a^4}\right)}{x+\frac{b}{a}}$$

input
```
int(x^(1/2)/(a + b/x)^(3/2),x)
```

output
```
-((a + b/x)^(1/2)*((8*b*x^(3/2))/(3*a^3) - (2*x^(5/2))/(3*a^2) + (16*b^2*x
^(1/2))/(3*a^4)))/(x + b/a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = \frac{\frac{2}{3}a^2x^2 - \frac{8}{3}abx - \frac{16}{3}b^2}{\sqrt{ax+b}\,a^3}$$

input
```
int(x^(1/2)/(a+b/x)^(3/2),x)
```

output
```
(2*(a**2*x**2 - 4*a*b*x - 8*b**2))/(3*sqrt(a*x + b)*a**3)
```

3.241 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx=-\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}+\frac{4\sqrt{a+\frac{b}{x}}\sqrt{x}}{a^2}$$

output
```
-2*x^(1/2)/a/(a+b/x)^(1/2)+4*(a+b/x)^(1/2)*x^(1/2)/a^2
```

Mathematica [A] (verified)

Time = 3.92 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(2b+ax)}{a^2(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*Sqrt[x]),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(2*b + a*x))/(a^2*(b + a*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}-\frac{2b\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}dx}{a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{4b}{a^2\sqrt{x}\sqrt{a+\frac{b}{x}}}+\frac{2\sqrt{x}}{a\sqrt{a+\frac{b}{x}}}$$

input
```
Int[1/((a + b/x)^(3/2)*Sqrt[x]),x]
```

output
```
(4*b)/(a^2*Sqrt[a + b/x]*Sqrt[x]) + (2*Sqrt[x])/(a*Sqrt[a + b/x])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	size
orering	$\frac{2(ax+2b)(ax+b)}{a^{2}x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$	30
gosper	$\frac{2(ax+b)(ax+2b)}{a^{2}x^{\frac{3}{2}}\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}}$	32
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(ax+2b)}{(ax+b)a^{2}}$	34
risch	$\frac{2ax+2b}{a^{2}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}+\frac{2b}{a^{2}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	46

input
```
int(1/(a+b/x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*x+2*b)/a^2*(a*x+b)/x^(3/2)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx=\frac{2\,(ax+2\,b)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{a^{3}x+a^{2}b}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2*(a*x + 2*b)*sqrt(x)*sqrt((a*x + b)/x)/(a^3*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx = \frac{2x}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{4\sqrt{b}}{a^2\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**(1/2),x)
```

output
```
2*x/(a*sqrt(b)*sqrt(a*x/b + 1)) + 4*sqrt(b)/(a**2*sqrt(a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a^2} + \frac{2b}{\sqrt{a+\frac{b}{x}}a^2\sqrt{x}}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a + b/x)*sqrt(x)/a^2 + 2*b/(sqrt(a + b/x)*a^2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx = \frac{2\left(\frac{\sqrt{ax+b}}{a\mathrm{sgn}(x)} + \frac{b}{\sqrt{ax+b}a\mathrm{sgn}(x)}\right)}{a} - \frac{4\sqrt{b}\mathrm{sgn}(x)}{a^2}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
2*(sqrt(a*x + b)/(a*sgn(x)) + b/(sqrt(a*x + b)*a*sgn(x)))/a - 4*sqrt(b)*sg
n(x)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx=\frac{\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{3/2}}{a^2}+\frac{4\,b\,\sqrt{x}}{a^3}\right)}{x+\frac{b}{a}}$$

input
```
int(1/(x^(1/2)*(a + b/x)^(3/2)),x)
```

output
```
((a + b/x)^(1/2)*((2*x^(3/2))/a^2 + (4*b*x^(1/2))/a^3))/(x + b/a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.47

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}}\,dx=\frac{2ax+4b}{\sqrt{ax+b}\,a^2}$$

input
```
int(1/(a+b/x)^(3/2)/x^(1/2),x)
```

output
```
(2*(a*x + 2*b))/(sqrt(a*x + b)*a**2)
```

3.242 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} \, dx = -\frac{2}{a\sqrt{a+\frac{b}{x}}\sqrt{x}}$$

output
```
-2/a/(a+b/x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 3.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} \, dx = -\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^(3/2)),x]
```

output
```
(-2*Sqrt[a + b/x]*Sqrt[x])/(a*(b + a*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+\frac{b}{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 796$$

$$-\frac{2}{a\sqrt{x}\sqrt{a+\frac{b}{x}}}$$

input
```
Int[1/((a + b/x)^(3/2)*x^(3/2)),x]
```

output
```
-2/(a*Sqrt[a + b/x]*Sqrt[x])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
orering	$-\frac{2(ax+b)}{a\,x^{\frac{3}{2}}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}$	23
gosper	$-\frac{2(ax+b)}{a\left(\frac{ax+b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}$	25
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{(ax+b)a}$	27

input
```
int(1/(a+b/x)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a*(a*x+b)/x^(3/2)/(a+b/x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}\,dx=-\frac{2\sqrt{x}\sqrt{\frac{ax+b}{x}}}{a^2x+ab}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(x)*sqrt((a*x + b)/x)/(a^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}\,dx=-\frac{2}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**(3/2),x)
```

output
```
-2/(a*sqrt(b)*sqrt(a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} \, dx = -\frac{2}{\sqrt{a+\frac{b}{x}} a\sqrt{x}}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(a + b/x)*a*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} \, dx = \frac{2\,\mathrm{sgn}(x)}{a\sqrt{b}} - \frac{2}{\sqrt{ax+b}a\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
2*sgn(x)/(a*sqrt(b)) - 2/(sqrt(a*x + b)*a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} \, dx = -\frac{2\,\sqrt{x}\,\sqrt{a+\frac{b}{x}}}{x\,a^2+b\,a}$$

input
```
int(1/(x^(3/2)*(a + b/x)^(3/2)),x)
```

output
```
-(2*x^(1/2)*(a + b/x)^(1/2))/(a*b + a^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}}\, dx = -\frac{2}{\sqrt{ax+b}\, a}$$

input
```
int(1/(a+b/x)^(3/2)/x^(3/2),x)
```

output
```
( - 2)/(sqrt(a*x + b)*a)
```

3.243 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} \, dx = \frac{2}{b\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^{3/2}}$$

output
```
2/b/(a+b/x)^(1/2)/x^(1/2)-2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} \, dx = \frac{\sqrt{a+\frac{b}{x}}\sqrt{x}\left(\frac{2}{b\sqrt{b+ax}} - \frac{2\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{b^{3/2}}\right)}{\sqrt{b+ax}}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^(5/2)),x]
```

output
```
(Sqrt[a + b/x]*Sqrt[x]*(2/(b*Sqrt[b + a*x]) - (2*ArcTanh[Sqrt[b + a*x]/Sqr
t[b]])/b^(3/2)))/Sqrt[b + a*x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {860, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{252}$$

$$-2\left(\frac{\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{b}-\frac{1}{b\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{b}-\frac{1}{b\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{3/2}}-\frac{1}{b\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)$$

input
```
Int[1/((a + b/x)^(3/2)*x^(5/2)),x]
```

output
```
-2*(-(1/(b*Sqrt[a + b/x]*Sqrt[x])) + ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x
])]/b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}-\sqrt{b}\right)}{b^{\frac{3}{2}}\left(ax+b\right)}$	53

input
```
int(1/(a+b/x)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((a*x+b)/x)^(1/2)*x^(1/2)/b^(3/2)*(arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+
b)^(1/2)-b^(1/2))/(a*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.73

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}}\,dx = \left[\frac{(ax+b)\sqrt{b}\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2b}{x}\right)+2b\sqrt{x}\sqrt{\frac{ax+b}{x}}}{ab^2x+b^3}, \frac{2\left((ax+b)\sqrt{-b}\arctan\left(\right.\right.}{ab^2}\right.$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[((a*x + b)*sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/
x) + 2*b*sqrt(x)*sqrt((a*x + b)/x))/(a*b^2*x + b^3), 2*((a*x + b)*sqrt(-b)
*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + b*sqrt(x)*sqrt((a*
x + b)/x))/(a*b^2*x + b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(42) = 84.

Time = 6.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}}\,dx = \frac{ab^2x\log\left(\frac{ax}{b}\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}} - \frac{2ab^2x\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}} + \frac{2b^3\sqrt{\frac{ax}{b}+1}}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}} + \frac{b^3\log\left(\frac{ax}{b}\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}} - \frac{2b^3\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{ab^{\frac{7}{2}}x+b^{\frac{9}{2}}}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**(5/2),x)
```

output
```
a*b**2*x*log(a*x/b)/(a*b**(7/2)*x + b**(9/2)) - 2*a*b**2*x*log(sqrt(a*x/b
+ 1) + 1)/(a*b**(7/2)*x + b**(9/2)) + 2*b**3*sqrt(a*x/b + 1)/(a*b**(7/2)*x
 + b**(9/2)) + b**3*log(a*x/b)/(a*b**(7/2)*x + b**(9/2)) - 2*b**3*log(sqrt
(a*x/b + 1) + 1)/(a*b**(7/2)*x + b**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{b^{\frac{3}{2}}} + \frac{2}{\sqrt{a+\frac{b}{x}}b\sqrt{x}}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b
^(3/2) + 2/(sqrt(a + b/x)*b*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}}\,dx = -\frac{2\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)\mathrm{sgn}(x)}{\sqrt{-b}b^{\frac{3}{2}}} + \frac{2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b\,\mathrm{sgn}\left(x\right)} + \frac{2}{\sqrt{ax+b}b\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
-2*(sqrt(b)*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b))*sgn(x)/(sqrt(-b)*b^(3/2))
 + 2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b*sgn(x)) + 2/(sqrt(a*x + b)
*b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}\,dx=\int \frac{1}{x^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(5/2)*(a + b/x)^(3/2)),x)
```

output
```
int(1/(x^(5/2)*(a + b/x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{5/2}}\,dx=\frac{\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)-\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)+2b}{\sqrt{ax+b}\,b^2}$$

input
```
int(1/(a+b/x)^(3/2)/x^(5/2),x)
```

output
```
(sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b)) - sqrt(b)*sqrt(a*x + b
)*log(sqrt(a*x + b) + sqrt(b)) + 2*b)/(sqrt(a*x + b)*b**2)
```

3.244 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} \, dx = \frac{2}{b\sqrt{a+\frac{b}{x}}x^{3/2}} - \frac{3\sqrt{a+\frac{b}{x}}}{b^2\sqrt{x}} + \frac{3a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^{5/2}}$$

output
```
2/b/(a+b/x)^(1/2)/x^(3/2)-3*(a+b/x)^(1/2)/b^2/x^(1/2)+3*a*arctanh(b^(1/2)/
(a+b/x)^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 9.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} \, dx = \frac{\sqrt{a+\frac{b}{x}}\left(-\sqrt{b}(b+3ax)+3ax\sqrt{b+ax}\text{arctanh}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)\right)}{b^{5/2}\sqrt{x}(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^(7/2)),x]
```

output
```
(Sqrt[a + b/x]*(-(Sqrt[b]*(b + 3*a*x)) + 3*a*x*Sqrt[b + a*x]*ArcTanh[Sqrt[
b + a*x]/Sqrt[b]]))/(b^(5/2)*Sqrt[x]*(b + a*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {860, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^2}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{252}$$

$$-2\left(\frac{3\int \frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}}{b}-\frac{1}{bx^{3/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{b}-\frac{1}{bx^{3/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int \frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{b}-\frac{1}{bx^{3/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{b}-\frac{1}{bx^{3/2}\sqrt{a+\frac{b}{x}}}\right)$$

input
```
Int[1/((a + b/x)^(3/2)*x^(7/2)),x]
```

output
```
-2*(-(1/(b*Sqrt[a + b/x]*x^(3/2))) + (3*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*
ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*b^(3/2))))/b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-3\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}\,ax+3xa\sqrt{b}+b^{\frac{3}{2}}\right)}{\sqrt{x}\,(ax+b)b^{\frac{5}{2}}}$	61
risch	$-\frac{ax+b}{b^{2}x^{\frac{3}{2}}\sqrt{\frac{ax+b}{x}}}-\frac{a\left(\frac{4}{\sqrt{ax+b}}-\frac{6\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{ax+b}}{2b^{2}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	80

input
```
int(1/(a+b/x)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x+b)/x)^(1/2)*(-3*arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+b)^(1/2)*a*x+3*
x*a*b^(1/2)+b^(3/2))/x^(1/2)/(a*x+b)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.47

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}\,dx=\left[\frac{3\,(a^2x^2+abx)\sqrt{b}\log\left(\frac{ax+2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(3\,abx+b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{2\,(ab^3x^2+b^4x)},-\frac{3\,(a^2x^2}{}\right.$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/2*(3*(a^2*x^2 + a*b*x)*sqrt(b)*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt((a*x +
 b)/x) + 2*b)/x) - 2*(3*a*b*x + b^2)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^3*x^2
 + b^4*x), -(3*(a^2*x^2 + a*b*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*
x + b)/x)/(a*x + b)) + (3*a*b*x + b^2)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^3*x
^2 + b^4*x)]
```

Sympy [A] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}\,dx = -\frac{3\sqrt{a}}{b^2\sqrt{x}\sqrt{1+\frac{b}{ax}}}+\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{b^{\frac{5}{2}}}-\frac{1}{\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**(7/2),x)
```

output
```
-3*sqrt(a)/(b**2*sqrt(x)*sqrt(1 + b/(a*x))) + 3*a*asinh(sqrt(b)/(sqrt(a)*s
qrt(x)))/b**(5/2) - 1/(sqrt(a)*b*x**(3/2)*sqrt(1 + b/(a*x)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{7/2}}\,dx = -\frac{3\left(a+\frac{b}{x}\right)ax-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}-\sqrt{a+\frac{b}{x}}b^3\sqrt{x}}-\frac{3\,a\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(7/2),x, algorithm="maxima")
```

output
```
-(3*(a + b/x)*a*x - 2*a*b)/((a + b/x)^(3/2)*b^2*x^(3/2) - sqrt(a + b/x)*b^
3*sqrt(x)) - 3/2*a*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sq
rt(x) + sqrt(b)))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}}\,dx = -\frac{3\,a\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2\mathrm{sgn}\left(x\right)} - \frac{3\,(ax+b)a-2\,ab}{\left((ax+b)^{\frac{3}{2}}-\sqrt{ax+b}b\right)b^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
-3*a*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x)) - (3*(a*x + b)*a
 - 2*a*b)/(((a*x + b)^(3/2) - sqrt(a*x + b)*b)*b^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}}\,dx = \int \frac{1}{x^{7/2}\,\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(7/2)*(a + b/x)^(3/2)),x)
```

output
```
int(1/(x^(7/2)*(a + b/x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}}\,dx = \frac{-3\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)ax+3\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)ax-6}{2\sqrt{ax+b}\,b^3x}$$

input
```
int(1/(a+b/x)^(3/2)/x^(7/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a*x + 3*sqrt(b)*s
qrt(a*x + b)*log(sqrt(a*x + b) + sqrt(b))*a*x - 6*a*b*x - 2*b**2)/(2*sqrt(
a*x + b)*b**3*x)
```

3.245 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}} dx = \frac{2}{b\sqrt{a+\frac{b}{x}}x^{5/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{2b^2x^{3/2}} + \frac{15a\sqrt{a+\frac{b}{x}}}{4b^3\sqrt{x}} - \frac{15a^2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4b^{7/2}}$$

output
```
2/b/(a+b/x)^(1/2)/x^(5/2)-5/2*(a+b/x)^(1/2)/b^2/x^(3/2)+15/4*a*(a+b/x)^(1/
2)/b^3/x^(1/2)-15/4*a^2*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.54

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}} dx = -\frac{2\sqrt{1+\frac{b}{ax}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{2},\frac{9}{2},-\frac{b}{ax}\right)}{7a\sqrt{a+\frac{b}{x}}x^{7/2}}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^(9/2)),x]
```

output
```
(-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[3/2, 7/2, 9/2, -(b/(a*x))])/(7*a*S
qrt[a + b/x]*x^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {860, 252, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2} \left(a+\frac{b}{x}\right)^{3/2}} \, dx$$

$$\Big\downarrow \; 860$$

$$-2 \int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3} d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \; 252$$

$$-2 \left(\frac{5 \int \frac{1}{\sqrt{a+\frac{b}{x}} x^2} d\frac{1}{\sqrt{x}}}{b}-\frac{1}{b x^{5/2} \sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \; 262$$

$$-2 \left(\frac{5 \left(\frac{\sqrt{a+\frac{b}{x}}}{4 b x^{3/2}}-\frac{3 a \int \frac{1}{\sqrt{a+\frac{b}{x}} x} d\frac{1}{\sqrt{x}}}{4 b}\right)}{b}-\frac{1}{b x^{5/2} \sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \; 262$$

$$-2\left(\frac{5\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{4b}\right)}{b}-\frac{1}{bx^{5/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{5\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{4b}\right)}{b}-\frac{1}{bx^{5/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{5\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{4b}\right)}{b}-\frac{1}{bx^{5/2}\sqrt{a+\frac{b}{x}}}\right)$$

input
```
Int[1/((a + b/x)^(3/2)*x^(9/2)),x]
```

output
```
-2*(-(1/(b*Sqrt[a + b/x]*x^(5/2))) + (5*(Sqrt[a + b/x]/(4*b*x^(3/2)) - (3*
a*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x]
)])/(2*b^(3/2))))/(4*b)))/b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(15\sqrt{ax+b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^{2}x^{2}-5b^{\frac{3}{2}}ax-15a^{2}x^{2}\sqrt{b}+2b^{\frac{5}{2}}\right)}{4x^{\frac{3}{2}}(ax+b)b^{\frac{7}{2}}}$	78
risch	$\frac{(ax+b)(7ax-2b)}{4b^{3}x^{\frac{5}{2}}\sqrt{\frac{ax+b}{x}}}+\frac{a^{2}\left(\frac{16}{\sqrt{ax+b}}-\frac{30\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{ax+b}}{8b^{3}\sqrt{\frac{ax+b}{x}}\sqrt{x}}$	90

input
```
int(1/(a+b/x)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x+b)/x)^(1/2)/x^(3/2)*(15*(a*x+b)^(1/2)*arctanh((a*x+b)^(1/2)/b^(
1/2))*a^2*x^2-5*b^(3/2)*a*x-15*a^2*x^2*b^(1/2)+2*b^(5/2))/(a*x+b)/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.12

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{9/2}}\,dx = \left[\frac{15\,(a^{3}x^{3}+a^{2}bx^{2})\sqrt{b}\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,(15\,a^{2}bx^{2}+5\,ab^{2}x-2\,b^{3})\sqrt{x}}{8\,(ab^{4}x^{3}+b^{5}x^{2})}\right.$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(9/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(a^3*x^3 + a^2*b*x^2)*sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqrt((
a*x + b)/x) + 2*b)/x) + 2*(15*a^2*b*x^2 + 5*a*b^2*x - 2*b^3)*sqrt(x)*sqrt(
(a*x + b)/x))/(a*b^4*x^3 + b^5*x^2), 1/4*(15*(a^3*x^3 + a^2*b*x^2)*sqrt(-b
)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (15*a^2*b*x^2 + 5
*a*b^2*x - 2*b^3)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^4*x^3 + b^5*x^2)]
```

Sympy [A] (verification not implemented)

Time = 49.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx = \frac{15a^{\frac{3}{2}}}{4b^3\sqrt{x}\sqrt{1+\frac{b}{ax}}} + \frac{5\sqrt{a}}{4b^2x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} - \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{4b^{\frac{7}{2}}} - \frac{1}{2\sqrt{a}bx^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**(9/2),x)
```

output
```
15*a**(3/2)/(4*b**3*sqrt(x)*sqrt(1 + b/(a*x))) + 5*sqrt(a)/(4*b**2*x**(3/2
)*sqrt(1 + b/(a*x))) - 15*a**2*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(4*b**(7/2
)) - 1/(2*sqrt(a)*b*x**(5/2)*sqrt(1 + b/(a*x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx = \frac{15\left(a+\frac{b}{x}\right)^2 a^2x^2 - 25\left(a+\frac{b}{x}\right)a^2bx + 8\,a^2b^2}{4\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^3x^{\frac{5}{2}} - 2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^4x^{\frac{3}{2}} + \sqrt{a+\frac{b}{x}}b^5\sqrt{x}\right)} + \frac{15\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(9/2),x, algorithm="maxima")
```

output
```
1/4*(15*(a + b/x)^2*a^2*x^2 - 25*(a + b/x)*a^2*b*x + 8*a^2*b^2)/((a + b/x)
^(5/2)*b^3*x^(5/2) - 2*(a + b/x)^(3/2)*b^4*x^(3/2) + sqrt(a + b/x)*b^5*sqr
t(x)) + 15/8*a^2*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt
(x) + sqrt(b)))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx = \frac{15\,a^2 \arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{4\,\sqrt{-b}b^3 \mathrm{sgn}\,(x)} + \frac{2\,a^2}{\sqrt{ax+b}b^3 \mathrm{sgn}\,(x)} + \frac{7\,(ax+b)^{\frac{3}{2}}a^2 - 9\,\sqrt{ax+b}a^2b}{4\,a^2b^3x^2 \mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
15/4*a^2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn(x)) + 2*a^2/(sqr
t(a*x + b)*b^3*sgn(x)) + 1/4*(7*(a*x + b)^(3/2)*a^2 - 9*sqrt(a*x + b)*a^2*
b)/(a^2*b^3*x^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx = \int \frac{1}{x^{9/2}\,\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(9/2)*(a + b/x)^(3/2)),x)
```

output
```
int(1/(x^(9/2)*(a + b/x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{9/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^2x^2 - 15\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)a^2x^2}{8\sqrt{ax+b}\,b^4x^2}$$

input
```
int(1/(a+b/x)^(3/2)/x^(9/2),x)
```

output
```
(15*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a**2*x**2 - 15*sqrt
(b)*sqrt(a*x + b)*log(sqrt(a*x + b) + sqrt(b))*a**2*x**2 + 30*a**2*b*x**2
+ 10*a*b**2*x - 4*b**3)/(8*sqrt(a*x + b)*b**4*x**2)
```

3.246 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}} \, dx = \frac{2}{b\sqrt{a+\frac{b}{x}}\,x^{7/2}} - \frac{7\sqrt{a+\frac{b}{x}}}{3b^2x^{5/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{12b^3x^{3/2}} - \frac{35a^2\sqrt{a+\frac{b}{x}}}{8b^4\sqrt{x}} + \frac{35a^3\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{8b^{9/2}}$$

output
```
2/b/(a+b/x)^(1/2)/x^(7/2)-7/3*(a+b/x)^(1/2)/b^2/x^(5/2)+35/12*a*(a+b/x)^(1
/2)/b^3/x^(3/2)-35/8*a^2*(a+b/x)^(1/2)/b^4/x^(1/2)+35/8*a^3*arctanh(b^(1/2
)/(a+b/x)^(1/2)/x^(1/2))/b^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}}\,dx = -\frac{2\sqrt{1+\frac{b}{ax}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{2},\frac{11}{2},-\frac{b}{ax}\right)}{9a\sqrt{a+\frac{b}{x}}x^{9/2}}$$

input
```
Integrate[1/((a + b/x)^(3/2)*x^(11/2)),x]
```

output
```
(-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[3/2, 9/2, 11/2, -(b/(a*x))])/(9*a*
Sqrt[a + b/x]*x^(9/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {860, 252, 262, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/2}\left(a+\frac{b}{x}\right)^{3/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^4}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{252}$$

$$-2\left(\frac{7\int \frac{1}{\sqrt{a+\frac{b}{x}}x^3}d\frac{1}{\sqrt{x}}}{b}-\frac{1}{bx^{7/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{7\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\int\frac{1}{\sqrt{a+\frac{b}{x}}x^2}d\frac{1}{\sqrt{x}}}{6b}\right)}{b}-\frac{1}{bx^{7/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{7\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\int\frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}}{4b}\right)}{6b}\right)}{b}-\frac{1}{bx^{7/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{7\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{4b}\right)}{6b}\right)}{b}-\frac{1}{bx^{7/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \text{224}$$

$$-2\left(\frac{7\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{4b}\right)}{6b}\right)}{b}-\frac{1}{bx^{7/2}\sqrt{a+\frac{b}{x}}}\right)$$

$$\Big\downarrow \; 219$$

$$-2\left(\frac{7\left(\frac{\sqrt{a+\frac{b}{x}}}{6bx^{5/2}}-\frac{5a\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{4b}\right)}{6b}\right)}{b}-\frac{1}{bx^{7/2}\sqrt{a+\frac{b}{x}}}\right)$$

input
```
Int[1/((a + b/x)^(3/2)*x^(11/2)),x]
```

output
```
-2*(-(1/(b*Sqrt[a + b/x]*x^(7/2))) + (7*(Sqrt[a + b/x]/(6*b*x^(5/2)) - (5*
a*(Sqrt[a + b/x]/(4*b*x^(3/2)) - (3*a*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*Ar
cTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*b^(3/2))))/(4*b)))/(6*b)))/b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(-105\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}\,a^3x^3-14b^{\frac{5}{2}}ax+35a^2b^{\frac{3}{2}}x^2+105a^3x^3\sqrt{b}+8b^{\frac{7}{2}}\right)}{24x^{\frac{5}{2}}(ax+b)b^{\frac{9}{2}}}$	89
risch	$-\frac{(ax+b)(57a^2x^2-22abx+8b^2)}{24b^4x^{\frac{7}{2}}\sqrt{\frac{ax+b}{x}}}-\frac{a^3\left(\frac{32}{\sqrt{ax+b}}-\frac{70\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{ax+b}}{16b^4\sqrt{\frac{ax+b}{x}}\sqrt{x}}$	101

input
```
int(1/(a+b/x)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*((a*x+b)/x)^(1/2)*(-105*arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+b)^(1/2)
*a^3*x^3-14*b^(5/2)*a*x+35*a^2*b^(3/2)*x^2+105*a^3*x^3*b^(1/2)+8*b^(7/2))/
x^(5/2)/(a*x+b)/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{11/2}}\,dx = \left[\frac{105\,(a^4x^4+a^3bx^3)\sqrt{b}\log\left(\frac{ax+2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(105\,a^3bx^3+35\,a^2b^2x^2-14}{48\,(ab^5x^4+b^6x^3)}\right.$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(11/2),x, algorithm="fricas")
```

output
```
[1/48*(105*(a^4*x^4 + a^3*b*x^3)*sqrt(b)*log((a*x + 2*sqrt(b)*sqrt(x)*sqrt
((a*x + b)/x) + 2*b)/x) - 2*(105*a^3*b*x^3 + 35*a^2*b^2*x^2 - 14*a*b^3*x +
 8*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(a*b^5*x^4 + b^6*x^3), -1/24*(105*(a^4*
x^4 + a^3*b*x^3)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x +
 b)) + (105*a^3*b*x^3 + 35*a^2*b^2*x^2 - 14*a*b^3*x + 8*b^4)*sqrt(x)*sqrt(
(a*x + b)/x))/(a*b^5*x^4 + b^6*x^3)]
```

Sympy [A] (verification not implemented)

Time = 179.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}}\,dx = -\frac{35a^{\frac{5}{2}}}{8b^4\sqrt{x}\sqrt{1+\frac{b}{ax}}} - \frac{35a^{\frac{3}{2}}}{24b^3x^{\frac{3}{2}}\sqrt{1+\frac{b}{ax}}} + \frac{7\sqrt{a}}{12b^2x^{\frac{5}{2}}\sqrt{1+\frac{b}{ax}}} + \frac{35a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)}{8b^{\frac{9}{2}}} - \frac{1}{3\sqrt{a}bx^{\frac{7}{2}}\sqrt{1+\frac{b}{ax}}}$$

input
```
integrate(1/(a+b/x)**(3/2)/x**(11/2),x)
```

output
```
-35*a**(5/2)/(8*b**4*sqrt(x)*sqrt(1 + b/(a*x))) - 35*a**(3/2)/(24*b**3*x**
(3/2)*sqrt(1 + b/(a*x))) + 7*sqrt(a)/(12*b**2*x**(5/2)*sqrt(1 + b/(a*x)))
+ 35*a**3*asinh(sqrt(b)/(sqrt(a)*sqrt(x)))/(8*b**(9/2)) - 1/(3*sqrt(a)*b*x
**(7/2)*sqrt(1 + b/(a*x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}}\,dx = -\frac{105\left(a+\frac{b}{x}\right)^3a^3x^3 - 280\left(a+\frac{b}{x}\right)^2a^3bx^2 + 231\left(a+\frac{b}{x}\right)a^3b^2x - 48\,a^3b^3}{24\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4x^{\frac{7}{2}} - 3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^5x^{\frac{5}{2}} + 3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^6x^{\frac{3}{2}} - \sqrt{a+\frac{b}{x}}\,b^7\sqrt{x}\right)} - \frac{35\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{16\,b^{\frac{9}{2}}}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(11/2),x, algorithm="maxima")
```

output
```
-1/24*(105*(a + b/x)^3*a^3*x^3 - 280*(a + b/x)^2*a^3*b*x^2 + 231*(a + b/x)
*a^3*b^2*x - 48*a^3*b^3)/((a + b/x)^(7/2)*b^4*x^(7/2) - 3*(a + b/x)^(5/2)*
b^5*x^(5/2) + 3*(a + b/x)^(3/2)*b^6*x^(3/2) - sqrt(a + b/x)*b^7*sqrt(x)) -
 35/16*a^3*log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) +
sqrt(b)))/b^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{11/2}}\,dx = -\frac{35\,a^3\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}b^4\text{sgn}\left(x\right)} - \frac{2\,a^3}{\sqrt{ax+b}b^4\text{sgn}\left(x\right)} - \frac{57\,(ax+b)^{\frac{5}{2}}a^3 - 136\,(ax+b)^{\frac{3}{2}}a^3b + 87\,\sqrt{ax+b}a^3b^2}{24\,a^3b^4x^3\text{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(3/2)/x^(11/2),x, algorithm="giac")
```

output
```
-35/8*a^3*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^4*sgn(x)) - 2*a^3/(sq
rt(a*x + b)*b^4*sgn(x)) - 1/24*(57*(a*x + b)^(5/2)*a^3 - 136*(a*x + b)^(3/
2)*a^3*b + 87*sqrt(a*x + b)*a^3*b^2)/(a^3*b^4*x^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}x^{11/2}}\,dx = \int \frac{1}{x^{11/2}\,\left(a+\frac{b}{x}\right)^{3/2}}\,dx$$

input
```
int(1/(x^(11/2)*(a + b/x)^(3/2)),x)
```

output
```
int(1/(x^(11/2)*(a + b/x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^{11/2}}\,dx = \frac{-105\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^3x^3+105\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}+\sqrt{b}\right)}{48\sqrt{ax+b}\,b^5x^3}$$

input
```
int(1/(a+b/x)^(3/2)/x^(11/2),x)
```

output
```
( - 105*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a**3*x**3 + 105
*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) + sqrt(b))*a**3*x**3 - 210*a**3*b
*x**3 - 70*a**2*b**2*x**2 + 28*a*b**3*x - 16*b**4)/(48*sqrt(a*x + b)*b**5*
x**3)
```

3.247 $\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 146

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{512b^3\sqrt{a+\frac{b}{x}}\sqrt{x}}{21a^6}+\frac{256b^2\sqrt{a+\frac{b}{x}}x^{3/2}}{21a^5} - \frac{64b\sqrt{a+\frac{b}{x}}x^{5/2}}{7a^4}-\frac{2x^{7/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{20x^{7/2}}{3a^2\sqrt{a+\frac{b}{x}}}+\frac{160\sqrt{a+\frac{b}{x}}x^{7/2}}{21a^3}$$

output
```
-512/21*b^3*(a+b/x)^(1/2)*x^(1/2)/a^6+256/21*b^2*(a+b/x)^(1/2)*x^(3/2)/a^5
-64/7*b*(a+b/x)^(1/2)*x^(5/2)/a^4-2/3*x^(7/2)/a/(a+b/x)^(3/2)-20/3*x^(7/2)
/a^2/(a+b/x)^(1/2)+160/21*(a+b/x)^(1/2)*x^(7/2)/a^3
```

Mathematica [A] (verified)

Time = 4.78 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-256b^5-384ab^4x-96a^2b^3x^2+16a^3b^2x^3-6a^4bx^4+3a^5x^5)}{21a^6(b+ax)^2}$$

input
```
Integrate[x^(5/2)/(a + b/x)^(5/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-256*b^5 - 384*a*b^4*x - 96*a^2*b^3*x^2 + 16*a^3
*b^2*x^3 - 6*a^4*b*x^4 + 3*a^5*x^5))/(21*a^6*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {803, 803, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{10b\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}dx}{7a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{10b\left(\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}dx}{5a}\right)}{7a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{10b\left(\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\int\frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}dx}{a}\right)}{5a}\right)}{7a}$$

$\Big\downarrow$ 803

$$\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{10b\left(\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\int\frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}dx}{a}\right)}{a}\right)}{5a}\right)}{7a}$$

$\Big\downarrow$ 803

$$\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{10b\left(\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(\frac{2b\int\frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}dx}{a}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}\right)}{a}\right)}{5a}\right)}{7a}$$

$$\downarrow \text{796}$$

$$\frac{2x^{7/2}}{7a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{10b\left(\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(-\frac{4b}{3a^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}\right)}{a}\right)}{5a}\right)}{7a}$$

input
```
Int[x^(5/2)/(a + b/x)^(5/2),x]
```

output
```
(2*x^(7/2))/(7*a*(a + b/x)^(3/2)) - (10*b*((2*x^(5/2))/(5*a*(a + b/x)^(3/2
)) - (8*b*((-2*b*((-4*b*((-4*b)/(3*a^2*(a + b/x)^(3/2)*x^(3/2)) - 2/(a*(a
+ b/x)^(3/2)*Sqrt[x])))/a + (2*Sqrt[x])/(a*(a + b/x)^(3/2))))/a + (2*x^(3/
2))/(3*a*(a + b/x)^(3/2))))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.51

method	result	size
orering	$\frac{2\left(3a^{5}x^{5}-6a^{4}b\,x^{4}+16a^{3}b^{2}x^{3}-96a^{2}b^{3}x^{2}-384b^{4}xa-256b^{5}\right)\left(ax+b\right)}{21a^{6}x^{\frac{5}{2}}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$	75
gosper	$\frac{2\left(ax+b\right)\left(3a^{5}x^{5}-6a^{4}b\,x^{4}+16a^{3}b^{2}x^{3}-96a^{2}b^{3}x^{2}-384b^{4}xa-256b^{5}\right)}{21a^{6}x^{\frac{5}{2}}\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$	77
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(3a^{5}x^{5}-6a^{4}b\,x^{4}+16a^{3}b^{2}x^{3}-96a^{2}b^{3}x^{2}-384b^{4}xa-256b^{5}\right)}{21\left(ax+b\right)^{2}a^{6}}$	79
risch	$\frac{2\left(3a^{3}x^{3}-12a^{2}b\,x^{2}+37a\,b^{2}x-158b^{3}\right)\left(ax+b\right)}{21a^{6}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}-\frac{2b^{4}\left(15ax+14b\right)}{3a^{6}\left(ax+b\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	93

input
```
int(x^(5/2)/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*(3*a^5*x^5-6*a^4*b*x^4+16*a^3*b^2*x^3-96*a^2*b^3*x^2-384*a*b^4*x-256*
b^5)/a^6/x^(5/2)*(a*x+b)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\left(3\,a^5x^5-6\,a^4bx^4+16\,a^3b^2x^3-96\,a^2b^3x^2-384\,ab^4x-256\,b^5\right)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{21\left(a^8x^2+2\,a^7bx+a^6b^2\right)}$$

input
```
integrate(x^(5/2)/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^5*x^5 - 6*a^4*b*x^4 + 16*a^3*b^2*x^3 - 96*a^2*b^3*x^2 - 384*a*b^
4*x - 256*b^5)*sqrt(x)*sqrt((a*x + b)/x)/(a^8*x^2 + 2*a^7*b*x + a^6*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(128) = 256.

Time = 7.41 (sec) , antiderivative size = 799, normalized size of antiderivative = 5.47

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{6a^8b^{\frac{51}{2}}x^8\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$+\frac{6a^7b^{\frac{53}{2}}x^7\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$+\frac{14a^6b^{\frac{55}{2}}x^6\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$-\frac{126a^5b^{\frac{57}{2}}x^5\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$-\frac{1260a^4b^{\frac{59}{2}}x^4\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$-\frac{3360a^3b^{\frac{61}{2}}x^3\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$-\frac{4032a^2b^{\frac{63}{2}}x^2\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$-\frac{2304ab^{\frac{65}{2}}x\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$
$$-\frac{512b^{\frac{67}{2}}\sqrt{\frac{ax}{b}+1}}{21a^{11}b^{25}x^5+105a^{10}b^{26}x^4+210a^9b^{27}x^3+210a^8b^{28}x^2+105a^7b^{29}x+21a^6b^{30}}$$

input
```
integrate(x**(5/2)/(a+b/x)**(5/2),x)
```

output
```
6*a**8*b**(51/2)*x**8*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**
26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 2
1*a**6*b**30) + 6*a**7*b**(53/2)*x**7*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5
 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*
a**7*b**29*x + 21*a**6*b**30) + 14*a**6*b**(55/2)*x**6*sqrt(a*x/b + 1)/(21
*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*
b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30) - 126*a**5*b**(57/2)*x**5*s
qrt(a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**2
7*x**3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30) - 1260*a*
*4*b**(59/2)*x**4*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**26*x
**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 21*a*
*6*b**30) - 3360*a**3*b**(61/2)*x**3*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5
+ 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8*b**28*x**2 + 105*a
**7*b**29*x + 21*a**6*b**30) - 4032*a**2*b**(63/2)*x**2*sqrt(a*x/b + 1)/(2
1*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**3 + 210*a**8
*b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30) - 2304*a*b**(65/2)*x*sqrt(
a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x*
*3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30) - 512*b**(67/
2)*sqrt(a*x/b + 1)/(21*a**11*b**25*x**5 + 105*a**10*b**26*x**4 + 210*a**9*
b**27*x**3 + 210*a**8*b**28*x**2 + 105*a**7*b**29*x + 21*a**6*b**30)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{7}{2}}x^{\frac{7}{2}} - 21\left(a+\frac{b}{x}\right)^{\frac{5}{2}}bx^{\frac{5}{2}} + 70\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}} - 210\sqrt{a+\frac{b}{x}}b^3\sqrt{x}\right)}{21\,a^6} - \frac{2\left(15\left(a+\frac{b}{x}\right)b^4x - b^5\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^6x^{\frac{3}{2}}}$$

input
```
integrate(x^(5/2)/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
2/21*(3*(a + b/x)^(7/2)*x^(7/2) - 21*(a + b/x)^(5/2)*b*x^(5/2) + 70*(a + b
/x)^(3/2)*b^2*x^(3/2) - 210*sqrt(a + b/x)*b^3*sqrt(x))/a^6 - 2/3*(15*(a +
b/x)*b^4*x - b^5)/((a + b/x)^(3/2)*a^6*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{512\,b^{\frac{7}{2}}\mathrm{sgn}(x)}{21\,a^6} - \frac{2\,(15\,(ax+b)b^4-b^5)}{3\,(ax+b)^{\frac{3}{2}}a^6\mathrm{sgn}\,(x)} + \frac{2\left(3\,(ax+b)^{\frac{7}{2}}a^{36}-21\,(ax+b)^{\frac{5}{2}}a^{36}b+70\,(ax+b)^{\frac{3}{2}}a^{36}b^2-210\,\sqrt{ax+b}a^{36}b^3\right)}{21\,a^{42}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^(5/2)/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
512/21*b^(7/2)*sgn(x)/a^6 - 2/3*(15*(a*x + b)*b^4 - b^5)/((a*x + b)^(3/2)*
a^6*sgn(x)) + 2/21*(3*(a*x + b)^(7/2)*a^36 - 21*(a*x + b)^(5/2)*a^36*b + 7
0*(a*x + b)^(3/2)*a^36*b^2 - 210*sqrt(a*x + b)*a^36*b^3)/(a^42*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.53

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{2\,\sqrt{x}\,\sqrt{\frac{b+a\,x}{x}}\,\left(-3\,a^5\,x^5+6\,a^4\,b\,x^4-16\,a^3\,b^2\,x^3+96\,a^2\,b^3\,x^2+384\,a\,b^4\,x+256\,b^5\right)}{21\,a^6\,{\left(b+a\,x\right)}^2}$$

input
```
int(x^(5/2)/(a + b/x)^(5/2),x)
```

output
```
-(2*x^(1/2)*((b + a*x)/x)^(1/2)*(256*b^5 - 3*a^5*x^5 + 6*a^4*b*x^4 + 96*a^
2*b^3*x^2 - 16*a^3*b^2*x^3 + 384*a*b^4*x))/(21*a^6*(b + a*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \frac{x^{5/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2}{7}a^5x^5 - \frac{4}{7}a^4b\,x^4 + \frac{32}{21}a^3b^2x^3 - \frac{64}{7}a^2b^3x^2 - \frac{256}{7}a\,b^4x - \frac{512}{21}b^5}{\sqrt{ax+b}\,a^6\,(ax+b)}$$

input
```
int(x^(5/2)/(a+b/x)^(5/2),x)
```

output
```
(2*(3*a**5*x**5 - 6*a**4*b*x**4 + 16*a**3*b**2*x**3 - 96*a**2*b**3*x**2 -
384*a*b**4*x - 256*b**5))/(21*sqrt(a*x + b)*a**6*(a*x + b))
```

3.248 $\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{256b^2\sqrt{a+\frac{b}{x}}\sqrt{x}}{15a^5} - \frac{128b\sqrt{a+\frac{b}{x}}x^{3/2}}{15a^4} - \frac{2x^{5/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{16x^{5/2}}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{32\sqrt{a+\frac{b}{x}}x^{5/2}}{5a^3}$$

output
```
256/15*b^2*(a+b/x)^(1/2)*x^(1/2)/a^5-128/15*b*(a+b/x)^(1/2)*x^(3/2)/a^4-2/
3*x^(5/2)/a/(a+b/x)^(3/2)-16/3*x^(5/2)/a^2/(a+b/x)^(1/2)+32/5*(a+b/x)^(1/2
)*x^(5/2)/a^3
```

Mathematica [A] (verified)

Time = 4.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(128b^4+192ab^3x+48a^2b^2x^2-8a^3bx^3+3a^4x^4)}{15a^5(b+ax)^2}$$

input
```
Integrate[x^(3/2)/(a + b/x)^(5/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(128*b^4 + 192*a*b^3*x + 48*a^2*b^2*x^2 - 8*a^3*b
*x^3 + 3*a^4*x^4))/(15*a^5*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {803, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}dx}{5a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}dx}{a}\right)}{5a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\int\frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}dx}{a}\right)}{a}\right)}{5a}$$

$\Big\downarrow$ 803

$$\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(\frac{2b\int\frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}dx}{a}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}\right)}{a}\right)}{5a}$$

$\Big\downarrow$ 796

$$\frac{2x^{5/2}}{5a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8b\left(\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(-\frac{4b}{3a^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}\right)}{a}\right)}{5a}$$

input

```
Int[x^(3/2)/(a + b/x)^(5/2),x]
```

output

```
(2*x^(5/2))/(5*a*(a + b/x)^(3/2)) - (8*b*((-2*b*((-4*b*((-4*b)/(3*a^2*(a +
 b/x)^(3/2)*x^(3/2)) - 2/(a*(a + b/x)^(3/2)*Sqrt[x])))/a + (2*Sqrt[x])/(a*
(a + b/x)^(3/2))))/a + (2*x^(3/2))/(3*a*(a + b/x)^(3/2))))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

method	result	size
orering	$\frac{2\left(3 a^{4} x^{4}-8 a^{3} b\,x^{3}+48 a^{2} b^{2} x^{2}+192 a\,b^{3} x +128 b^{4}\right) \left(a x +b \right)}{15 a^{5} x^{\frac{5}{2}} \left(a +\frac{b}{x}\right)^{\frac{5}{2}}}$	64
gosper	$\frac{2 \left(a x +b \right) \left(3 a^{4} x^{4}-8 a^{3} b\,x^{3}+48 a^{2} b^{2} x^{2}+192 a\,b^{3} x +128 b^{4}\right)}{15 a^{5} x^{\frac{5}{2}} \left(\frac{a x +b}{x}\right)^{\frac{5}{2}}}$	66
default	$\frac{2 \sqrt{\frac{a x +b}{x}}\, \sqrt{x}\, \left(3 a^{4} x^{4}-8 a^{3} b\,x^{3}+48 a^{2} b^{2} x^{2}+192 a\,b^{3} x +128 b^{4}\right)}{15 \left(a x +b \right)^{2} a^{5}}$	68
risch	$\frac{2 \left(3 a^{2} x^{2}-14 a b x +73 b^{2}\right) \left(a x +b \right)}{15 a^{5} \sqrt{\frac{a x +b}{x}}\, \sqrt{x}}+\frac{2 b^{3} \left(12 a x +11 b \right)}{3 a^{5} \left(a x +b \right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x}}$	82

input
```
int(x^(3/2)/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*a^4*x^4-8*a^3*b*x^3+48*a^2*b^2*x^2+192*a*b^3*x+128*b^4)/a^5/x^(5/2
)*(a*x+b)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\,(3\,a^4x^4-8\,a^3bx^3+48\,a^2b^2x^2+192\,ab^3x+128\,b^4)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{15\,(a^7x^2+2\,a^6bx+a^5b^2)}$$

input
```
integrate(x^(3/2)/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*a^4*x^4 - 8*a^3*b*x^3 + 48*a^2*b^2*x^2 + 192*a*b^3*x + 128*b^4)*sq
rt(x)*sqrt((a*x + b)/x)/(a^7*x^2 + 2*a^6*b*x + a^5*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(104) = 208.

Time = 4.52 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.47

$$\begin{aligned}\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= \frac{6a^6b^{\frac{33}{2}}x^6\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}} \\ &- \frac{4a^5b^{\frac{35}{2}}x^5\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}} \\ &+ \frac{70a^4b^{\frac{37}{2}}x^4\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}} \\ &+ \frac{560a^3b^{\frac{39}{2}}x^3\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}} \\ &+ \frac{1120a^2b^{\frac{41}{2}}x^2\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}} \\ &+ \frac{896ab^{\frac{43}{2}}x\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}} \\ &+ \frac{256b^{\frac{45}{2}}\sqrt{\frac{ax}{b}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^3+90a^7b^{18}x^2+60a^6b^{19}x+15a^5b^{20}}\end{aligned}$$

input
```
integrate(x**(3/2)/(a+b/x)**(5/2),x)
```

output
```
6*a**6*b**(33/2)*x**6*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*
x**3 + 90*a**7*b**18*x**2 + 60*a**6*b**19*x + 15*a**5*b**20) - 4*a**5*b**(
35/2)*x**5*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a
**7*b**18*x**2 + 60*a**6*b**19*x + 15*a**5*b**20) + 70*a**4*b**(37/2)*x**4
*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a**7*b**18*
x**2 + 60*a**6*b**19*x + 15*a**5*b**20) + 560*a**3*b**(39/2)*x**3*sqrt(a*x
/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a**7*b**18*x**2 + 60
*a**6*b**19*x + 15*a**5*b**20) + 1120*a**2*b**(41/2)*x**2*sqrt(a*x/b + 1)/
(15*a**9*b**16*x**4 + 60*a**8*b**17*x**3 + 90*a**7*b**18*x**2 + 60*a**6*b*
*19*x + 15*a**5*b**20) + 896*a*b**(43/2)*x*sqrt(a*x/b + 1)/(15*a**9*b**16*
x**4 + 60*a**8*b**17*x**3 + 90*a**7*b**18*x**2 + 60*a**6*b**19*x + 15*a**5
*b**20) + 256*b**(45/2)*sqrt(a*x/b + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**1
7*x**3 + 90*a**7*b**18*x**2 + 60*a**6*b**19*x + 15*a**5*b**20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\left(3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}} - 20\left(a+\frac{b}{x}\right)^{\frac{3}{2}}bx^{\frac{3}{2}} + 90\sqrt{a+\frac{b}{x}}\,b^2\sqrt{x}\right)}{15\,a^5} + \frac{2\left(12\left(a+\frac{b}{x}\right)b^3x - b^4\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^5x^{\frac{3}{2}}}$$

input
```
integrate(x^(3/2)/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
2/15*(3*(a + b/x)^(5/2)*x^(5/2) - 20*(a + b/x)^(3/2)*b*x^(3/2) + 90*sqrt(a
 + b/x)*b^2*sqrt(x))/a^5 + 2/3*(12*(a + b/x)*b^3*x - b^4)/((a + b/x)^(3/2)
*a^5*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{256\,b^{\frac{5}{2}}\operatorname{sgn}(x)}{15\,a^5} + \frac{2\left(\frac{5\left(12\,(ax+b)b^3-b^4\right)}{(ax+b)^{\frac{3}{2}}\,a\operatorname{sgn}(x)} + \frac{3\,(ax+b)^{\frac{5}{2}}\,a^4-20\,(ax+b)^{\frac{3}{2}}\,a^4b+90\,\sqrt{ax+b}a^4b^2}{a^5\operatorname{sgn}(x)}\right)}{15\,a^4}$$

input
```
integrate(x^(3/2)/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
-256/15*b^(5/2)*sgn(x)/a^5 + 2/15*(5*(12*(a*x + b)*b^3 - b^4)/((a*x + b)^(
3/2)*a*sgn(x)) + (3*(a*x + b)^(5/2)*a^4 - 20*(a*x + b)^(3/2)*a^4*b + 90*sq
rt(a*x + b)*a^4*b^2)/(a^5*sgn(x)))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x}}\left(\frac{2\,x^{9/2}}{5\,a^3} - \frac{16\,b\,x^{7/2}}{15\,a^4} + \frac{32\,b^2\,x^{5/2}}{5\,a^5} + \frac{128\,b^3\,x^{3/2}}{5\,a^6} + \frac{256\,b^4\,\sqrt{x}}{15\,a^7}\right)}{x^2 + \frac{b^2}{a^2} + \frac{2\,b\,x}{a}}$$

input
```
int(x^(3/2)/(a + b/x)^(5/2),x)
```

output
```
((a + b/x)^(1/2)*((2*x^(9/2))/(5*a^3) - (16*b*x^(7/2))/(15*a^4) + (32*b^2*
x^(5/2))/(5*a^5) + (128*b^3*x^(3/2))/(5*a^6) + (256*b^4*x^(1/2))/(15*a^7))
)/(x^2 + b^2/a^2 + (2*b*x)/a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2}{5}a^4x^4-\frac{16}{15}a^3b\,x^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a\,b^3x+\frac{256}{15}b^4}{\sqrt{ax+b}\,a^5\,(ax+b)}$$

input
```
int(x^(3/2)/(a+b/x)^(5/2),x)
```

output
```
(2*(3*a**4*x**4 - 8*a**3*b*x**3 + 48*a**2*b**2*x**2 + 192*a*b**3*x + 128*b
**4))/(15*sqrt(a*x + b)*a**5*(a*x + b))
```

3.249 $\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{32b\sqrt{a+\frac{b}{x}}\sqrt{x}}{3a^4} - \frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{4x^{3/2}}{a^2\sqrt{a+\frac{b}{x}}} + \frac{16\sqrt{a+\frac{b}{x}}\,x^{3/2}}{3a^3}$$

output
```
-32/3*b*(a+b/x)^(1/2)*x^(1/2)/a^4-2/3*x^(3/2)/a/(a+b/x)^(3/2)-4*x^(3/2)/a^
2/(a+b/x)^(1/2)+16/3*(a+b/x)^(1/2)*x^(3/2)/a^3
```

Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(-16b^3-24ab^2x-6a^2bx^2+a^3x^3)}{3a^4(b+ax)^2}$$

input
```
Integrate[Sqrt[x]/(a + b/x)^(5/2),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(-16*b^3 - 24*a*b^2*x - 6*a^2*b*x^2 + a^3*x^3))/(
3*a^4*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}dx}{a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{803}$$

$$\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(\frac{2b\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}dx}{a}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}\right)}{a}$$

$$\Big\downarrow \text{796}$$

$$\frac{2x^{3/2}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2b\left(\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(-\frac{4b}{3a^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}\right)}{a}$$

input
```
Int[Sqrt[x]/(a + b/x)^(5/2),x]
```

output
```
(-2*b*((-4*b*((-4*b)/(3*a^2*(a + b/x)^(3/2)*x^(3/2)) - 2/(a*(a + b/x)^(3/2
)*Sqrt[x])))/a + (2*Sqrt[x])/(a*(a + b/x)^(3/2))))/a + (2*x^(3/2))/(3*a*(a
 + b/x)^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
orering	$\frac{2\left(a^{3} x^{3}-6 a^{2} b \,x^{2}-24 a \,b^{2} x -16 b^{3}\right) \left(a x +b \right)}{3 a^{4} x^{\frac{5}{2}} \left(a +\frac{b}{x}\right)^{\frac{5}{2}}}$	52
gosper	$\frac{2 \left(a x +b \right) \left(a^{3} x^{3}-6 a^{2} b \,x^{2}-24 a \,b^{2} x -16 b^{3}\right)}{3 a^{4} x^{\frac{5}{2}} \left(\frac{a x +b}{x}\right)^{\frac{5}{2}}}$	54
default	$\frac{2 \sqrt{\frac{a x +b}{x}}\, \sqrt{x}\, \left(a^{3} x^{3}-6 a^{2} b \,x^{2}-24 a \,b^{2} x -16 b^{3}\right)}{3 \left(a x +b \right)^{2} a^{4}}$	56
risch	$\frac{2 \left(a x -8 b \right) \left(a x +b \right)}{3 a^{4} \sqrt{\frac{a x +b}{x}}\, \sqrt{x}}-\frac{2 b^{2} \left(9 a x +8 b \right)}{3 a^{4} \left(a x +b \right) \sqrt{\frac{a x +b}{x}}\, \sqrt{x}}$	70

input
```
int(x^(1/2)/(a+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a^3*x^3-6*a^2*b*x^2-24*a*b^2*x-16*b^3)/a^4/x^(5/2)*(a*x+b)/(a+b/x)^(5
/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\,{\left(a^{3} x^{3} - 6\,a^{2} b x^{2} - 24\,a b^{2} x - 16\,b^{3}\right)} \sqrt{x} \sqrt{\frac{a x + b}{x}}}{3\,{\left(a^{6} x^{2} + 2\,a^{5} b x + a^{4} b^{2}\right)}}$$

input
```
integrate(x^(1/2)/(a+b/x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(a^3*x^3 - 6*a^2*b*x^2 - 24*a*b^2*x - 16*b^3)*sqrt(x)*sqrt((a*x + b)/x
)/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(78) = 156$.

Time = 3.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.48

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2a^4b^{\frac{19}{2}}x^4\sqrt{\frac{ax}{b}+1}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b^{11}x+3a^4b^{12}} - \frac{10a^3b^{\frac{21}{2}}x^3\sqrt{\frac{ax}{b}+1}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b^{11}x+3a^4b^{12}} - \frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b^{11}x+3a^4b^{12}} - \frac{80ab^{\frac{25}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b^{11}x+3a^4b^{12}} - \frac{32b^{\frac{27}{2}}\sqrt{\frac{ax}{b}+1}}{3a^7b^9x^3+9a^6b^{10}x^2+9a^5b^{11}x+3a^4b^{12}}$$

input
```
integrate(x**(1/2)/(a+b/x)**(5/2),x)
```

output
```
2*a**4*b**(19/2)*x**4*sqrt(a*x/b + 1)/(3*a**7*b**9*x**3 + 9*a**6*b**10*x**
2 + 9*a**5*b**11*x + 3*a**4*b**12) - 10*a**3*b**(21/2)*x**3*sqrt(a*x/b + 1
)/(3*a**7*b**9*x**3 + 9*a**6*b**10*x**2 + 9*a**5*b**11*x + 3*a**4*b**12) -
 60*a**2*b**(23/2)*x**2*sqrt(a*x/b + 1)/(3*a**7*b**9*x**3 + 9*a**6*b**10*x
**2 + 9*a**5*b**11*x + 3*a**4*b**12) - 80*a*b**(25/2)*x*sqrt(a*x/b + 1)/(3
*a**7*b**9*x**3 + 9*a**6*b**10*x**2 + 9*a**5*b**11*x + 3*a**4*b**12) - 32*
b**(27/2)*sqrt(a*x/b + 1)/(3*a**7*b**9*x**3 + 9*a**6*b**10*x**2 + 9*a**5*b
**11*x + 3*a**4*b**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{2\left(\left(a+\frac{b}{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}-9\sqrt{a+\frac{b}{x}}b\sqrt{x}\right)}{3\,a^4} - \frac{2\left(9\left(a+\frac{b}{x}\right)b^2x-b^3\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4x^{\frac{3}{2}}}$$

input
```
integrate(x^(1/2)/(a+b/x)^(5/2),x, algorithm="maxima")
```

output
```
2/3*((a + b/x)^(3/2)*x^(3/2) - 9*sqrt(a + b/x)*b*sqrt(x))/a^4 - 2/3*(9*(a
+ b/x)*b^2*x - b^3)/((a + b/x)^(3/2)*a^4*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{32\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,a^4} - \frac{2\,(9\,(ax+b)b^2-b^3)}{3\,(ax+b)^{\frac{3}{2}}a^4\mathrm{sgn}\,(x)} + \frac{2\left((ax+b)^{\frac{3}{2}}a^8-9\,\sqrt{ax+b}a^8b\right)}{3\,a^{12}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^(1/2)/(a+b/x)^(5/2),x, algorithm="giac")
```

output
```
32/3*b^(3/2)*sgn(x)/a^4 - 2/3*(9*(a*x + b)*b^2 - b^3)/((a*x + b)^(3/2)*a^4
*sgn(x)) + 2/3*((a*x + b)^(3/2)*a^8 - 9*sqrt(a*x + b)*a^8*b)/(a^12*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\frac{\sqrt{a+\frac{b}{x}}\left(\frac{4\,b\,x^{5/2}}{a^4}-\frac{2\,x^{7/2}}{3\,a^3}+\frac{16\,b^2\,x^{3/2}}{a^5}+\frac{32\,b^3\,\sqrt{x}}{3\,a^6}\right)}{x^2+\frac{b^2}{a^2}+\frac{2\,b\,x}{a}}$$

input
```
int(x^(1/2)/(a + b/x)^(5/2),x)
```

output
```
-((a + b/x)^(1/2)*((4*b*x^(5/2))/a^4 - (2*x^(7/2))/(3*a^3) + (16*b^2*x^(3/
2))/a^5 + (32*b^3*x^(1/2))/(3*a^6)))/(x^2 + b^2/a^2 + (2*b*x)/a)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = \frac{\frac{2}{3}a^3x^3-4a^2b\,x^2-16a\,b^2x-\frac{32}{3}b^3}{\sqrt{ax+b}\,a^4\,(ax+b)}$$

input
```
int(x^(1/2)/(a+b/x)^(5/2),x)
```

output
```
(2*(a**3*x**3 - 6*a**2*b*x**2 - 24*a*b**2*x - 16*b**3))/(3*sqrt(a*x + b)*a
**4*(a*x + b))
```

3.250 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx=-\frac{2\sqrt{x}}{3a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{8\sqrt{x}}{3a^2\sqrt{a+\frac{b}{x}}}+\frac{16\sqrt{a+\frac{b}{x}}\sqrt{x}}{3a^3}$$

output
```
-2/3*x^(1/2)/a/(a+b/x)^(3/2)-8/3*x^(1/2)/a^2/(a+b/x)^(1/2)+16/3*(a+b/x)^(1
/2)*x^(1/2)/a^3
```

Mathematica [A] (verified)

Time = 4.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(8b^2+12abx+3a^2x^2)}{3a^3(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*Sqrt[x]),x]
```

output
```
(2*Sqrt[a + b/x]*Sqrt[x]*(8*b^2 + 12*a*b*x + 3*a^2*x^2))/(3*a^3*(b + a*x)^
2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}dx}{a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(\frac{2b\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}dx}{a}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{2\sqrt{x}}{a\left(a+\frac{b}{x}\right)^{3/2}}-\frac{4b\left(-\frac{4b}{3a^2x^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}-\frac{2}{a\sqrt{x}\left(a+\frac{b}{x}\right)^{3/2}}\right)}{a}$$

input
```
Int[1/((a + b/x)^(5/2)*Sqrt[x]),x]
```

output
```
(-4*b*((-4*b)/(3*a^2*(a + b/x)^(3/2)*x^(3/2)) - 2/(a*(a + b/x)^(3/2)*Sqrt[
x])))/a + (2*Sqrt[x])/(a*(a + b/x)^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

method	result	size
orering	$\frac{2\left(3 a^{2} x^{2}+12 a b x+8 b^{2}\right)\left(a x+b\right)}{3 a^{3} x^{\frac{5}{2}} \left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$	42
gosper	$\frac{2\left(a x+b\right)\left(3 a^{2} x^{2}+12 a b x+8 b^{2}\right)}{3 a^{3} x^{\frac{5}{2}} \left(\frac{a x+b}{x}\right)^{\frac{5}{2}}}$	44
default	$\frac{2 \sqrt{\frac{a x+b}{x}}\, \sqrt{x}\, \left(3 a^{2} x^{2}+12 a b x+8 b^{2}\right)}{3 \left(a x+b\right)^{2} a^{3}}$	46
risch	$\frac{2 a x+2 b}{a^{3} \sqrt{\frac{a x+b}{x}}\, \sqrt{x}}+\frac{2 b \left(6 a x+5 b\right)}{3 a^{3} \left(a x+b\right) \sqrt{\frac{a x+b}{x}}\, \sqrt{x}}$	61

input
```
int(1/(a+b/x)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*a^2*x^2+12*a*b*x+8*b^2)/a^3*(a*x+b)/x^(5/2)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx = \frac{2\,(3\,a^2x^2+12\,abx+8\,b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\,(a^5x^2+2\,a^4bx+a^3b^2)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(3*a^2*x^2 + 12*a*b*x + 8*b^2)*sqrt(x)*sqrt((a*x + b)/x)/(a^5*x^2 + 2*
a^4*b*x + a^3*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(58) = 116.

Time = 3.85 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.16

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx = \frac{6a^2b^{\frac{9}{2}}x^2\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6} + \frac{24ab^{\frac{11}{2}}x\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6} + \frac{16b^{\frac{13}{2}}\sqrt{\frac{ax}{b}+1}}{3a^5b^4x^2+6a^4b^5x+3a^3b^6}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**(1/2),x)
```

output
```
6*a**2*b**(9/2)*x**2*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x + 3
*a**3*b**6) + 24*a*b**(11/2)*x*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6*a**4*
b**5*x + 3*a**3*b**6) + 16*b**(13/2)*sqrt(a*x/b + 1)/(3*a**5*b**4*x**2 + 6
*a**4*b**5*x + 3*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx=\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{a^3}+\frac{2\left(6\left(a+\frac{b}{x}\right)bx-b^2\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^3x^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a + b/x)*sqrt(x)/a^3 + 2/3*(6*(a + b/x)*b*x - b^2)/((a + b/x)^(3/2)
*a^3*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx=\frac{2\left(\frac{3\sqrt{ax+b}}{a\mathrm{sgn}(x)}+\frac{6(ax+b)b-b^2}{(ax+b)^{\frac{3}{2}}a\mathrm{sgn}(x)}\right)}{3\,a^2}-\frac{16\sqrt{b}\mathrm{sgn}(x)}{3\,a^3}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*sqrt(a*x + b)/(a*sgn(x)) + (6*(a*x + b)*b - b^2)/((a*x + b)^(3/2)*a
*sgn(x)))/a^2 - 16/3*sqrt(b)*sgn(x)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx=\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}\,\left(3\,a^2\,x^2+12\,a\,b\,x+8\,b^2\right)}{3\,a^3\,{\left(b+a\,x\right)}^2}$$

input
```
int(1/(x^(1/2)*(a + b/x)^(5/2)),x)
```

output
```
(2*x^(1/2)*(a + b/x)^(1/2)*(8*b^2 + 3*a^2*x^2 + 12*a*b*x))/(3*a^3*(b + a*x
)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\sqrt{x}}\,dx=\frac{2a^2x^2+8abx+\frac{16}{3}b^2}{\sqrt{ax+b}\,a^3\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^(1/2),x)
```

output
```
(2*(3*a**2*x**2 + 12*a*b*x + 8*b**2))/(3*sqrt(a*x + b)*a**3*(a*x + b))
```

3.251 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}} \, dx = -\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}\sqrt{x}} - \frac{4}{3a^2\sqrt{a+\frac{b}{x}}\sqrt{x}}$$

output
```
-2/3/a/(a+b/x)^(3/2)/x^(1/2)-4/3/a^2/(a+b/x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}} \, dx = -\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}(2b+3ax)}{3a^2(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^(3/2)),x]
```

output
```
(-2*Sqrt[a + b/x]*Sqrt[x]*(2*b + 3*a*x))/(3*a^2*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2b \int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}}dx}{a}-\frac{2}{a \sqrt{x} \left(a+\frac{b}{x}\right)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{4b}{3a^2 x^{3/2} \left(a+\frac{b}{x}\right)^{3/2}}-\frac{2}{a \sqrt{x} \left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[1/((a + b/x)^(5/2)*x^(3/2)),x]
```

output
```
(-4*b)/(3*a^2*(a + b/x)^(3/2)*x^(3/2)) - 2/(a*(a + b/x)^(3/2)*Sqrt[x])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

method	result	size
orering	$-\frac{2(3ax+2b)(ax+b)}{3a^{2}x^{\frac{5}{2}}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$	31
gosper	$-\frac{2(3ax+2b)(ax+b)}{3a^{2}x^{\frac{5}{2}}\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}}$	33
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,(3ax+2b)}{3(ax+b)^{2}a^{2}}$	35

input
```
int(1/(a+b/x)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*a*x+2*b)/a^2*(a*x+b)/x^(5/2)/(a+b/x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{3/2}}\,dx = -\frac{2\,(3\,ax+2\,b)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\,(a^{4}x^{2}+2\,a^{3}bx+a^{2}b^{2})}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*a*x + 2*b)*sqrt(x)*sqrt((a*x + b)/x)/(a^4*x^2 + 2*a^3*b*x + a^2*b^
2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.

Time = 6.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}}\, dx = -\frac{6ax}{3a^3\sqrt{b}x\sqrt{\frac{ax}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}} - \frac{4b}{3a^3\sqrt{b}x\sqrt{\frac{ax}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**(3/2),x)
```

output
```
-6*a*x/(3*a**3*sqrt(b)*x*sqrt(a*x/b + 1) + 3*a**2*b**(3/2)*sqrt(a*x/b + 1)
) - 4*b/(3*a**3*sqrt(b)*x*sqrt(a*x/b + 1) + 3*a**2*b**(3/2)*sqrt(a*x/b + 1
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}}\, dx = -\frac{2\left(3\left(a+\frac{b}{x}\right)x-b\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(3/2),x, algorithm="maxima")
```

output
```
-2/3*(3*(a + b/x)*x - b)/((a + b/x)^(3/2)*a^2*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}}\,dx = \frac{4\,\mathrm{sgn}(x)}{3\,a^2\sqrt{b}} - \frac{2\,(3\,ax+2\,b)}{3\,(ax+b)^{\frac{3}{2}}a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(3/2),x, algorithm="giac")
```

output
```
4/3*sgn(x)/(a^2*sqrt(b)) - 2/3*(3*a*x + 2*b)/((a*x + b)^(3/2)*a^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}}\,dx = -\frac{\sqrt{a+\frac{b}{x}}\,\left(\frac{2\,x^{3/2}}{a^3}+\frac{4\,b\,\sqrt{x}}{3\,a^4}\right)}{x^2+\frac{b^2}{a^2}+\frac{2\,b\,x}{a}}$$

input
```
int(1/(x^(3/2)*(a + b/x)^(5/2)),x)
```

output
```
-((a + b/x)^(1/2)*((2*x^(3/2))/a^3 + (4*b*x^(1/2))/(3*a^4)))/(x^2 + b^2/a^
2 + (2*b*x)/a)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{3/2}}\,dx = \frac{-2ax-\frac{4b}{3}}{\sqrt{ax+b}\,a^2\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^(3/2),x)
```

output
```
(2*( - 3*a*x - 2*b))/(3*sqrt(a*x + b)*a**2*(a*x + b))
```

3.252 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx=-\frac{2}{3a\left(a+\frac{b}{x}\right)^{3/2}x^{3/2}}$$

output
```
-2/3/a/(a+b/x)^(3/2)/x^(3/2)
```

Mathematica [A] (verified)

Time = 4.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx=-\frac{2\sqrt{a+\frac{b}{x}}\sqrt{x}}{3a(b+ax)^2}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^(5/2)),x]
```

output
```
(-2*Sqrt[a + b/x]*Sqrt[x])/(3*a*(b + a*x)^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{2}{3 a x^{3/2} \left(a+\frac{b}{x}\right)^{3/2}}$$

input
```
Int[1/((a + b/x)^(5/2)*x^(5/2)),x]
```

output
```
-2/(3*a*(a + b/x)^(3/2)*x^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
orering	$-\frac{2(ax+b)}{3a\,x^{\frac{5}{2}}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}$	23
gosper	$-\frac{2(ax+b)}{3a\left(\frac{ax+b}{x}\right)^{\frac{5}{2}}x^{\frac{5}{2}}}$	25
default	$-\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}{3(ax+b)^{2}a}$	27

input
```
int(1/(a+b/x)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a*(a*x+b)/x^(5/2)/(a+b/x)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx = -\frac{2\,\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\,(a^3x^2+2\,a^2bx+ab^2)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(x)*sqrt((a*x + b)/x)/(a^3*x^2 + 2*a^2*b*x + a*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 10.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{5/2}}\,dx = -\frac{2}{3a^2\sqrt{b}x\sqrt{\frac{ax}{b}+1}+3ab^{\frac{3}{2}}\sqrt{\frac{ax}{b}+1}}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**(5/2),x)
```

output
```
-2/(3*a**2*sqrt(b)*x*sqrt(a*x/b + 1) + 3*a*b**(3/2)*sqrt(a*x/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}} \, dx = -\frac{2}{3 \left(a+\frac{b}{x}\right)^{\frac{3}{2}} a x^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3/((a + b/x)^(3/2)*a*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}} \, dx = \frac{2 \operatorname{sgn}(x)}{3 \, a b^{\frac{3}{2}}} - \frac{2}{3 \, (a x + b)^{\frac{3}{2}} a \operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*sgn(x)/(a*b^(3/2)) - 2/3/((a*x + b)^(3/2)*a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}} \, dx = -\frac{2\sqrt{x}\,\sqrt{a+\frac{b}{x}}}{3\,\left(a^3\,x^2+2\,a^2\,b\,x+a\,b^2\right)}$$

input
```
int(1/(x^(5/2)*(a + b/x)^(5/2)),x)
```

output
```
-(2*x^(1/2)*(a + b/x)^(1/2))/(3*(a*b^2 + a^3*x^2 + 2*a^2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{5/2}} \, dx = -\frac{2}{3\sqrt{ax+b}\,a\,(ax+b)}$$

input
```
int(1/(a+b/x)^(5/2)/x^(5/2),x)
```

output
```
( - 2)/(3*sqrt(a*x + b)*a*(a*x + b))
```

3.253 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}} dx = \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{3/2}} + \frac{2}{b^2\sqrt{a+\frac{b}{x}}\sqrt{x}} - \frac{2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^{5/2}}$$

output
```
2/3/b/(a+b/x)^(3/2)/x^(3/2)+2/b^2/(a+b/x)^(1/2)/x^(1/2)-2*arctanh(b^(1/2)/
(a+b/x)^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}} dx = \frac{2\left(\sqrt{b}\sqrt{x}(4b+3ax) - 3\sqrt{a}\sqrt{1+\frac{b}{ax}}x(b+ax)\text{arcsinh}\left(\frac{\sqrt{b}}{\sqrt{a}\sqrt{x}}\right)\right)}{3b^{5/2}\sqrt{a+\frac{b}{x}}x(b+ax)}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^(7/2)),x]
```

output
```
(2*(Sqrt[b]*Sqrt[x]*(4*b + 3*a*x) - 3*Sqrt[a]*Sqrt[1 + b/(a*x)]*x*(b + a*x
)*ArcSinh[Sqrt[b]/(Sqrt[a]*Sqrt[x])]))/(3*b^(5/2)*Sqrt[a + b/x]*x*(b + a*x
))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {860, 252, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2} \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 860}$$

$$-2 \int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^2} d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{ 252}$$

$$-2 \left(\frac{\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x} d\frac{1}{\sqrt{x}}}{b} - \frac{1}{3 b x^{3/2} \left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 252}$$

$$-2 \left(\frac{\frac{\int \frac{1}{\sqrt{a+\frac{b}{x}}} d\frac{1}{\sqrt{x}}}{b} - \frac{1}{b \sqrt{x} \sqrt{a+\frac{b}{x}}}}{b} - \frac{1}{3 b x^{3/2} \left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 224}$$

$$-2 \left(\frac{\frac{\int \frac{1}{1-\frac{b}{x}} d\frac{1}{\sqrt{a+\frac{b}{x}} \sqrt{x}}}{b} - \frac{1}{b \sqrt{x} \sqrt{a+\frac{b}{x}}}}{b} - \frac{1}{3 b x^{3/2} \left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-2\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{b^{3/2}}-\frac{1}{b\sqrt{x}\sqrt{a+\frac{b}{x}}}}{b}-\frac{1}{3bx^{3/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

input
```
Int[1/((a + b/x)^(5/2)*x^(7/2)),x]
```

output
```
-2*(-1/3*1/(b*(a + b/x)^(3/2)*x^(3/2)) + (-(1/(b*Sqrt[a + b/x]*Sqrt[x])) +
 ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])]/b^(3/2))/b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{2\sqrt{\frac{ax+b}{x}}\,\sqrt{x}\,\left(-3\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}\,ax+4b^{\frac{3}{2}}+3xa\sqrt{b}-3\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)b\sqrt{ax+b}\right)}{3b^{\frac{5}{2}}\,(ax+b)^{2}}$	85

input
```
int(1/(a+b/x)^(5/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((a*x+b)/x)^(1/2)*x^(1/2)*(-3*arctanh((a*x+b)^(1/2)/b^(1/2))*(a*x+b)^(
1/2)*a*x+4*b^(3/2)+3*x*a*b^(1/2)-3*arctanh((a*x+b)^(1/2)/b^(1/2))*b*(a*x+b
)^(1/2))/b^(5/2)/(a*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.79

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}\,dx = \left[\frac{3\,(a^2x^2+2\,abx+b^2)\sqrt{b}\log\left(\frac{ax-2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\,(3\,abx+4\,b^2)\sqrt{x}\sqrt{\frac{ax+b}{x}}}{3\,(a^2b^3x^2+2\,ab^4x+b^5)}, -2\right.$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(b)*log((a*x - 2*sqrt(b)*sqrt(x)*sqr
t((a*x + b)/x) + 2*b)/x) + 2*(3*a*b*x + 4*b^2)*sqrt(x)*sqrt((a*x + b)/x))/
(a^2*b^3*x^2 + 2*a*b^4*x + b^5), 2/3*(3*(a^2*x^2 + 2*a*b*x + b^2)*sqrt(-b)
*arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (3*a*b*x + 4*b^2)*
sqrt(x)*sqrt((a*x + b)/x))/(a^2*b^3*x^2 + 2*a*b^4*x + b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(63) = 126$.

Time = 30.97 (sec) , antiderivative size = 697, normalized size of antiderivative = 9.29

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}\,dx = \frac{3a^3b^4x^3\log\left(\frac{ax}{b}\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} - \frac{6a^3b^4x^3\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} + \frac{6a^2b^5x^2\sqrt{\frac{ax}{b}+1}}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} + \frac{9a^2b^5x^2\log\left(\frac{ax}{b}\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} - \frac{18a^2b^5x^2\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} + \frac{14ab^6x\sqrt{\frac{ax}{b}+1}}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} + \frac{9ab^6x\log\left(\frac{ax}{b}\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} - \frac{18ab^6x\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} + \frac{8b^7\sqrt{\frac{ax}{b}+1}}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} + \frac{3b^7\log\left(\frac{ax}{b}\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}} - \frac{6b^7\log\left(\sqrt{\frac{ax}{b}+1}+1\right)}{3a^3b^{\frac{13}{2}}x^3+9a^2b^{\frac{15}{2}}x^2+9ab^{\frac{17}{2}}x+3b^{\frac{19}{2}}}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**(7/2),x)
```

output
```
3*a**3*b**4*x**3*log(a*x/b)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2
 + 9*a*b**(17/2)*x + 3*b**(19/2)) - 6*a**3*b**4*x**3*log(sqrt(a*x/b + 1) +
 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b
**(19/2)) + 6*a**2*b**5*x**2*sqrt(a*x/b + 1)/(3*a**3*b**(13/2)*x**3 + 9*a*
*2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) + 9*a**2*b**5*x**2*log(
a*x/b)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x +
3*b**(19/2)) - 18*a**2*b**5*x**2*log(sqrt(a*x/b + 1) + 1)/(3*a**3*b**(13/2
)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) + 14*a*b**
6*x*sqrt(a*x/b + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b
**(17/2)*x + 3*b**(19/2)) + 9*a*b**6*x*log(a*x/b)/(3*a**3*b**(13/2)*x**3 +
 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) - 18*a*b**6*x*log(
sqrt(a*x/b + 1) + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*
b**(17/2)*x + 3*b**(19/2)) + 8*b**7*sqrt(a*x/b + 1)/(3*a**3*b**(13/2)*x**3
 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2)) + 3*b**7*log(a*x
/b)/(3*a**3*b**(13/2)*x**3 + 9*a**2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b
**(19/2)) - 6*b**7*log(sqrt(a*x/b + 1) + 1)/(3*a**3*b**(13/2)*x**3 + 9*a**
2*b**(15/2)*x**2 + 9*a*b**(17/2)*x + 3*b**(19/2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{b^{\frac{5}{2}}} + \frac{2\left(3\left(a+\frac{b}{x}\right)x+b\right)}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2x^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(7/2),x, algorithm="maxima")
```

output
```
log((sqrt(a + b/x)*sqrt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b
^(5/2) + 2/3*(3*(a + b/x)*x + b)/((a + b/x)^(3/2)*b^2*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}\,dx = -\frac{2\left(3\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}\right)\mathrm{sgn}(x)}{3\sqrt{-b}b^{\frac{5}{2}}} + \frac{2\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2\mathrm{sgn}(x)} + \frac{2(3ax+4b)}{3(ax+b)^{\frac{3}{2}}b^2\mathrm{sgn}(x)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(7/2),x, algorithm="giac")
```

output
```
-2/3*(3*sqrt(b)*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-b))*sgn(x)/(sqrt(-b)*b^
(5/2)) + 2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x)) + 2/3*(3*a
*x + 4*b)/((a*x + b)^(3/2)*b^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{7/2}}\,dx = \int \frac{1}{x^{7/2}\left(a+\frac{b}{x}\right)^{5/2}}\,dx$$

input
```
int(1/(x^(7/2)*(a + b/x)^(5/2)),x)
```

output
```
int(1/(x^(7/2)*(a + b/x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{7/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)ax+3\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)b-3\sqrt{b}}{3\sqrt{ax+b}}$$

input
```
int(1/(a+b/x)^(5/2)/x^(7/2),x)
```

output
```
(3*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a*x + 3*sqrt(b)*sqrt
(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*b - 3*sqrt(b)*sqrt(a*x + b)*log(sqr
t(a*x + b) + sqrt(b))*a*x - 3*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) + sq
rt(b))*b + 6*a*b*x + 8*b**2)/(3*sqrt(a*x + b)*b**3*(a*x + b))
```

3.254 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{5/2}} + \frac{10}{3b^2\sqrt{a+\frac{b}{x}}\,x^{3/2}} - \frac{5\sqrt{a+\frac{b}{x}}}{b^3\sqrt{x}} + \frac{5a\,\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{b^{7/2}}$$

output
```
2/3/b/(a+b/x)^(3/2)/x^(5/2)+10/3/b^2/(a+b/x)^(1/2)/x^(3/2)-5*(a+b/x)^(1/2)
/b^3/x^(1/2)+5*a*arctanh(b^(1/2)/(a+b/x)^(1/2)/x^(1/2))/b^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = -\frac{2\sqrt{1+\frac{b}{ax}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{7}{2},\frac{9}{2},-\frac{b}{ax}\right)}{7a^2\sqrt{a+\frac{b}{x}}\,x^{7/2}}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^(9/2)),x]
```

output
```
(-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[5/2, 7/2, 9/2, -(b/(a*x))])/(7*a^2
*Sqrt[a + b/x]*x^(7/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {860, 252, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{9/2} \left(a+\frac{b}{x}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{860}$$

$$-2 \int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^3} d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{252}$$

$$-2 \left(\frac{5 \int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^2} d\frac{1}{\sqrt{x}}}{3 b}-\frac{1}{3 b x^{5/2} \left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{252}$$

$$-2 \left(\frac{5 \left(\frac{3 \int \frac{1}{\sqrt{a+\frac{b}{x}} x} d\frac{1}{\sqrt{x}}}{b}-\frac{1}{b x^{3/2} \sqrt{a+\frac{b}{x}}}\right)}{3 b}-\frac{1}{3 b x^{5/2} \left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{5\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{b}-\frac{1}{bx^{3/2}\sqrt{a+\frac{b}{x}}}\right)}{3b}-\frac{1}{3bx^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 224$$

$$-2\left(\frac{5\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{b}-\frac{1}{bx^{3/2}\sqrt{a+\frac{b}{x}}}\right)}{3b}-\frac{1}{3bx^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 219$$

$$-2\left(\frac{5\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{b}-\frac{1}{bx^{3/2}\sqrt{a+\frac{b}{x}}}\right)}{3b}-\frac{1}{3bx^{5/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

input
```
Int[1/((a + b/x)^(5/2)*x^(9/2)),x]
```

output
```
-2*(-1/3*1/(b*(a + b/x)^(3/2)*x^(5/2)) + (5*(-(1/(b*Sqrt[a + b/x]*x^(3/2))
) + (3*(Sqrt[a + b/x]/(2*b*Sqrt[x]) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sq
rt[x])])/(2*b^(3/2))))/b))/(3*b))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{ax+b}{b^{3}x^{\frac{3}{2}}\sqrt{\frac{ax+b}{x}}}-\frac{a\left(-\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{8}{\sqrt{ax+b}}+\frac{4b}{3(ax+b)^{\frac{3}{2}}}\right)\sqrt{ax+b}}{2b^{3}\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$	90
default	$\frac{\sqrt{\frac{ax+b}{x}}\,\left(15\sqrt{ax+b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^{2}x^{2}+15\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)abx\sqrt{ax+b}-15a^{2}x^{2}\sqrt{b}-20b^{\frac{3}{2}}ax-3b^{\frac{5}{2}}\right)}{3\sqrt{x}\,(ax+b)^{2}b^{\frac{7}{2}}}$	102

input
```
int(1/(a+b/x)^(5/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(a*x+b)/x^(3/2)/((a*x+b)/x)^(1/2)-1/2/b^3*a*(-10/b^(1/2)*arctanh((a
*x+b)^(1/2)/b^(1/2))+8/(a*x+b)^(1/2)+4/3*b/(a*x+b)^(3/2))/((a*x+b)/x)^(1/2
)/x^(1/2)*(a*x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.56

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{9/2}}\,dx=\left[\frac{15\,(a^3x^3+2\,a^2bx^2+ab^2x)\sqrt{b}\log\left(\frac{ax+2\,\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)-2\,(15\,a^2bx^2+20\,ab^2x-}{6\,(a^2b^4x^3+2\,ab^5x^2+b^6x)}\right.$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(9/2),x, algorithm="fricas")
```

output
```
[1/6*(15*(a^3*x^3 + 2*a^2*b*x^2 + a*b^2*x)*sqrt(b)*log((a*x + 2*sqrt(b)*sq
rt(x)*sqrt((a*x + b)/x) + 2*b)/x) - 2*(15*a^2*b*x^2 + 20*a*b^2*x + 3*b^3)*
sqrt(x)*sqrt((a*x + b)/x))/(a^2*b^4*x^3 + 2*a*b^5*x^2 + b^6*x), -1/3*(15*(
a^3*x^3 + 2*a^2*b*x^2 + a*b^2*x)*sqrt(-b)*arctan(sqrt(-b)*sqrt(x)*sqrt((a*
x + b)/x)/(a*x + b)) + (15*a^2*b*x^2 + 20*a*b^2*x + 3*b^3)*sqrt(x)*sqrt((a
*x + b)/x))/(a^2*b^4*x^3 + 2*a*b^5*x^2 + b^6*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(85) = 170$.

Time = 104.78 (sec) , antiderivative size = 818, normalized size of antiderivative = 8.26

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**(9/2),x)
```

output
```
-15*a**4*b**13*x**4*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*
x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 30*a**4*b**13*x**4*log(sqrt(
a*x/b + 1) + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**
(37/2)*x**2 + 6*b**(39/2)*x) - 30*a**3*b**14*x**3*sqrt(a*x/b + 1)/(6*a**3*
b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2
)*x) - 45*a**3*b**14*x**3*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(
35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 90*a**3*b**14*x**3*log
(sqrt(a*x/b + 1) + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18
*a*b**(37/2)*x**2 + 6*b**(39/2)*x) - 70*a**2*b**15*x**2*sqrt(a*x/b + 1)/(6
*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b*
*(39/2)*x) - 45*a**2*b**15*x**2*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**
2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 90*a**2*b**15*x*
*2*log(sqrt(a*x/b + 1) + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**
3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) - 46*a*b**16*x*sqrt(a*x/b + 1)/(6
*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b*
*(39/2)*x) - 15*a*b**16*x*log(a*x/b)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(
35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x) + 30*a*b**16*x*log(sqrt(
a*x/b + 1) + 1)/(6*a**3*b**(33/2)*x**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**
(37/2)*x**2 + 6*b**(39/2)*x) - 6*b**17*sqrt(a*x/b + 1)/(6*a**3*b**(33/2)*x
**4 + 18*a**2*b**(35/2)*x**3 + 18*a*b**(37/2)*x**2 + 6*b**(39/2)*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = -\frac{15\left(a+\frac{b}{x}\right)^2 a x^2 - 10\left(a+\frac{b}{x}\right) a b x - 2\,ab^2}{3\left(\left(a+\frac{b}{x}\right)^{\frac{5}{2}} b^3 x^{\frac{5}{2}} - \left(a+\frac{b}{x}\right)^{\frac{3}{2}} b^4 x^{\frac{3}{2}}\right)} - \frac{5\,a\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(9/2),x, algorithm="maxima")
```

output
```
-1/3*(15*(a + b/x)^2*a*x^2 - 10*(a + b/x)*a*b*x - 2*a*b^2)/((a + b/x)^(5/2
)*b^3*x^(5/2) - (a + b/x)^(3/2)*b^4*x^(3/2)) - 5/2*a*log((sqrt(a + b/x)*sq
rt(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = -\frac{5\,a\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3\mathrm{sgn}\left(x\right)} - \frac{2\left(6\left(ax+b\right)a+ab\right)}{3\left(ax+b\right)^{\frac{3}{2}} b^3\mathrm{sgn}\left(x\right)} - \frac{\sqrt{ax+b}}{b^3 x\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(9/2),x, algorithm="giac")
```

output
```
-5*a*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn(x)) - 2/3*(6*(a*x +
b)*a + a*b)/((a*x + b)^(3/2)*b^3*sgn(x)) - sqrt(a*x + b)/(b^3*x*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = \int \frac{1}{x^{9/2}\left(a+\frac{b}{x}\right)^{5/2}}\,dx$$

input
```
int(1/(x^(9/2)*(a + b/x)^(5/2)),x)
```

output
```
int(1/(x^(9/2)*(a + b/x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{9/2}}\,dx = \frac{-15\sqrt{b}\,\sqrt{ax+b}\,\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^2x^2 - 15\sqrt{b}\,\sqrt{ax+b}\,\log\left(\sqrt{ax+b}-\sqrt{b}\right)ab}{}$$

input
```
int(1/(a+b/x)^(5/2)/x^(9/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a**2*x**2 - 15*s
qrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a*b*x + 15*sqrt(b)*sqrt(
a*x + b)*log(sqrt(a*x + b) + sqrt(b))*a**2*x**2 + 15*sqrt(b)*sqrt(a*x + b)
*log(sqrt(a*x + b) + sqrt(b))*a*b*x - 30*a**2*b*x**2 - 40*a*b**2*x - 6*b**
3)/(6*sqrt(a*x + b)*b**4*x*(a*x + b))
```

3.255 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 129

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}} dx = \frac{2}{3b\left(a+\frac{b}{x}\right)^{3/2} x^{7/2}} + \frac{14}{3b^2\sqrt{a+\frac{b}{x}}x^{5/2}} - \frac{35\sqrt{a+\frac{b}{x}}}{6b^3x^{3/2}} + \frac{35a\sqrt{a+\frac{b}{x}}}{4b^4\sqrt{x}} - \frac{35a^2\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}}\right)}{4b^{9/2}}$$

output
```
2/3/b/(a+b/x)^(3/2)/x^(7/2)+14/3/b^2/(a+b/x)^(1/2)/x^(5/2)-35/6*(a+b/x)^(1
/2)/b^3/x^(3/2)+35/4*a*(a+b/x)^(1/2)/b^4/x^(1/2)-35/4*a^2*arctanh(b^(1/2)/
(a+b/x)^(1/2)/x^(1/2))/b^(9/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}} dx = -\frac{2\sqrt{1+\frac{b}{ax}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{9}{2},\frac{11}{2},-\frac{b}{ax}\right)}{9a^2\sqrt{a+\frac{b}{x}}x^{9/2}}$$

input
```
Integrate[1/((a + b/x)^(5/2)*x^(11/2)),x]
```

output
```
(-2*Sqrt[1 + b/(a*x)]*Hypergeometric2F1[5/2, 9/2, 11/2, -(b/(a*x))])/(9*a^
2*Sqrt[a + b/x]*x^(9/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {860, 252, 252, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11/2} \left(a+\frac{b}{x}\right)^{5/2}}dx$$

$$\Big\downarrow \text{860}$$

$$-2 \int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^4}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{252}$$

$$-2 \left(\frac{7 \int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} x^3}d\frac{1}{\sqrt{x}}}{3 b}-\frac{1}{3 b x^{7/2} \left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{252}$$

$$-2 \left(\frac{7 \left(\frac{5 \int \frac{1}{\sqrt{a+\frac{b}{x}} x^2}d\frac{1}{\sqrt{x}}}{b}-\frac{1}{b x^{5/2} \sqrt{a+\frac{b}{x}}}\right)}{3 b}-\frac{1}{3 b x^{7/2} \left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{262}$$

$$-2\left(\frac{7\left(\frac{5\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\int\frac{1}{\sqrt{a+\frac{b}{x}}x}d\frac{1}{\sqrt{x}}}{4b}\right)}{b}-\frac{1}{bx^{5/2}\sqrt{a+\frac{b}{x}}}\right)}{3b}-\frac{1}{3bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$\downarrow$ 262

$$-2\left(\frac{7\left(\frac{5\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{\sqrt{a+\frac{b}{x}}}d\frac{1}{\sqrt{x}}}{2b}\right)}{4b}\right)}{b}-\frac{1}{bx^{5/2}\sqrt{a+\frac{b}{x}}}\right)}{3b}-\frac{1}{3bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

$\downarrow$ 224

$$-2\left(\frac{7\left(\frac{5\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\int\frac{1}{1-\frac{b}{x}}d\frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{x}}}{2b}\right)}{4b}\right)}{b}-\frac{1}{bx^{5/2}\sqrt{a+\frac{b}{x}}}\right)}{3b}-\frac{1}{3bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

↓ 219

$$-2\left(\frac{7\left(\frac{5\left(\frac{\sqrt{a+\frac{b}{x}}}{4bx^{3/2}}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x}}}{2b\sqrt{x}}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{x}\sqrt{a+\frac{b}{x}}}\right)}{2b^{3/2}}\right)}{4b}\right)}{b}-\frac{1}{bx^{5/2}\sqrt{a+\frac{b}{x}}}\right)}{3b}-\frac{1}{3bx^{7/2}\left(a+\frac{b}{x}\right)^{3/2}}\right)$$

input
```
Int[1/((a + b/x)^(5/2)*x^(11/2)),x]
```

output
```
-2*(-1/3*1/(b*(a + b/x)^(3/2)*x^(7/2)) + (7*(-(1/(b*Sqrt[a + b/x]*x^(5/2))
) + (5*(Sqrt[a + b/x]/(4*b*x^(3/2)) - (3*a*(Sqrt[a + b/x]/(2*b*Sqrt[x]) -
(a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x]*Sqrt[x])])/(2*b^(3/2))))/(4*b)))/b))/(3*
b))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 860
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[-k/c   Subst[Int[(a + b/(c^n*x^(k*n)))^p/x^(k*(m + 1
) + 1), x], x, 1/(c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && ILtQ[n, 0]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

method	result
risch	$\frac{(ax+b)(11ax-2b)}{4b^4x^{\frac{5}{2}}\sqrt{\frac{ax+b}{x}}}+\frac{a^2\left(-\frac{70\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{48}{\sqrt{ax+b}}+\frac{16b}{3(ax+b)^{\frac{3}{2}}}\right)\sqrt{ax+b}}{8b^4\sqrt{\frac{ax+b}{x}}\,\sqrt{x}}$
default	$-\frac{\sqrt{\frac{ax+b}{x}}\left(105\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)\sqrt{ax+b}\,a^3x^3+105\,\mathrm{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)a^2b\,x^2\sqrt{ax+b}-105a^3x^3\sqrt{b}-140a^2b^{\frac{3}{2}}x^2-21b^{\frac{5}{2}}ax+6b^{\frac{7}{2}}\right)}{12x^{\frac{3}{2}}(ax+b)^2b^{\frac{9}{2}}}$

input
```
int(1/(a+b/x)^(5/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*x+b)*(11*a*x-2*b)/b^4/x^(5/2)/((a*x+b)/x)^(1/2)+1/8/b^4*a^2*(-70/b^
(1/2)*arctanh((a*x+b)^(1/2)/b^(1/2))+48/(a*x+b)^(1/2)+16/3*b/(a*x+b)^(3/2)
)/((a*x+b)/x)^(1/2)/x^(1/2)*(a*x+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.22

$$\int\frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{11/2}}\,dx=\left[\frac{105\left(a^4x^4+2\,a^3bx^3+a^2b^2x^2\right)\sqrt{b}\log\left(\frac{ax-2\sqrt{b}\sqrt{x}\sqrt{\frac{ax+b}{x}}+2\,b}{x}\right)+2\left(105\,a^3bx^3+140\right.}{24\left(a^2b^5x^4+2\,ab^6x^3+b^7x^2\right)}\right.$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(11/2),x, algorithm="fricas")
```

output
```
[1/24*(105*(a^4*x^4 + 2*a^3*b*x^3 + a^2*b^2*x^2)*sqrt(b)*log((a*x - 2*sqrt
(b)*sqrt(x)*sqrt((a*x + b)/x) + 2*b)/x) + 2*(105*a^3*b*x^3 + 140*a^2*b^2*x
^2 + 21*a*b^3*x - 6*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(a^2*b^5*x^4 + 2*a*b^6
*x^3 + b^7*x^2), 1/12*(105*(a^4*x^4 + 2*a^3*b*x^3 + a^2*b^2*x^2)*sqrt(-b)*
arctan(sqrt(-b)*sqrt(x)*sqrt((a*x + b)/x)/(a*x + b)) + (105*a^3*b*x^3 + 14
0*a^2*b^2*x^2 + 21*a*b^3*x - 6*b^4)*sqrt(x)*sqrt((a*x + b)/x))/(a^2*b^5*x^
4 + 2*a*b^6*x^3 + b^7*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b/x)**(5/2)/x**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}x^{11/2}}\,dx = \frac{105\left(a+\frac{b}{x}\right)^3a^2x^3-175\left(a+\frac{b}{x}\right)^2a^2bx^2+56\left(a+\frac{b}{x}\right)a^2b^2x+8\,a^2b^3}{12\left(\left(a+\frac{b}{x}\right)^{\frac{7}{2}}b^4x^{\frac{7}{2}}-2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}b^5x^{\frac{5}{2}}+\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^6x^{\frac{3}{2}}\right)} + \frac{35\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x}}\sqrt{x}-\sqrt{b}}{\sqrt{a+\frac{b}{x}}\sqrt{x}+\sqrt{b}}\right)}{8\,b^{\frac{9}{2}}}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(11/2),x, algorithm="maxima")
```

output
```
1/12*(105*(a + b/x)^3*a^2*x^3 - 175*(a + b/x)^2*a^2*b*x^2 + 56*(a + b/x)*a
^2*b^2*x + 8*a^2*b^3)/((a + b/x)^(7/2)*b^4*x^(7/2) - 2*(a + b/x)^(5/2)*b^5
*x^(5/2) + (a + b/x)^(3/2)*b^6*x^(3/2)) + 35/8*a^2*log((sqrt(a + b/x)*sqrt
(x) - sqrt(b))/(sqrt(a + b/x)*sqrt(x) + sqrt(b)))/b^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}}\,dx = \frac{35\,a^2 \arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{4\,\sqrt{-b}b^4 \mathrm{sgn}\left(x\right)} + \frac{2\,(9\,(ax+b)a^2+a^2b)}{3\,(ax+b)^{\frac{3}{2}}b^4 \mathrm{sgn}\left(x\right)} + \frac{11\,(ax+b)^{\frac{3}{2}}a^2-13\,\sqrt{ax+b}a^2b}{4\,a^2b^4x^2 \mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x)^(5/2)/x^(11/2),x, algorithm="giac")
```

output
```
35/4*a^2*arctan(sqrt(a*x + b)/sqrt(-b))/(sqrt(-b)*b^4*sgn(x)) + 2/3*(9*(a*
x + b)*a^2 + a^2*b)/((a*x + b)^(3/2)*b^4*sgn(x)) + 1/4*(11*(a*x + b)^(3/2)
*a^2 - 13*sqrt(a*x + b)*a^2*b)/(a^2*b^4*x^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}}\,dx = \int \frac{1}{x^{11/2}\,\left(a+\frac{b}{x}\right)^{5/2}}\,dx$$

input
```
int(1/(x^(11/2)*(a + b/x)^(5/2)),x)
```

output
```
int(1/(x^(11/2)*(a + b/x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} x^{11/2}}\,dx = \frac{105\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a^3x^3+105\sqrt{b}\,\sqrt{ax+b}\log\left(\sqrt{ax+b}-\sqrt{b}\right)a}{}$$

input
```
int(1/(a+b/x)^(5/2)/x^(11/2),x)
```

output
```
(105*sqrt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a**3*x**3 + 105*sq
rt(b)*sqrt(a*x + b)*log(sqrt(a*x + b) - sqrt(b))*a**2*b*x**2 - 105*sqrt(b)
*sqrt(a*x + b)*log(sqrt(a*x + b) + sqrt(b))*a**3*x**3 - 105*sqrt(b)*sqrt(a
*x + b)*log(sqrt(a*x + b) + sqrt(b))*a**2*b*x**2 + 210*a**3*b*x**3 + 280*a
**2*b**2*x**2 + 42*a*b**3*x - 12*b**4)/(24*sqrt(a*x + b)*b**5*x**2*(a*x +
b))
```

3.256 $\int \left(a+\frac{b}{x^2}\right) x^6 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x^2}\right) x^6 \, dx = \frac{b x^5}{5}+\frac{a x^7}{7}$$

output
```
1/5*b*x^5+1/7*a*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) x^6 \, dx = \frac{b x^5}{5}+\frac{a x^7}{7}$$

input
```
Integrate[(a + b/x^2)*x^6,x]
```

output
```
(b*x^5)/5 + (a*x^7)/7
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+\frac{b}{x^2}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^6+b x^4\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^7}{7}+\frac{b x^5}{5}$$

input
```
Int[(a + b/x^2)*x^6,x]
```

output
```
(b*x^5)/5 + (a*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{5}b\,x^{5}+\frac{1}{7}a\,x^{7}$	14
risch	$\frac{1}{5}b\,x^{5}+\frac{1}{7}a\,x^{7}$	14
parallelrisch	$\frac{1}{5}b\,x^{5}+\frac{1}{7}a\,x^{7}$	14
gosper	$\frac{x^{5}\left(5a\,x^{2}+7b\right)}{35}$	16
norman	$\frac{\frac{1}{7}a\,x^{8}+\frac{1}{5}b\,x^{6}}{x}$	18
orering	$\frac{x^{7}\left(5a\,x^{2}+7b\right)\left(a+\frac{b}{x^{2}}\right)}{35a\,x^{2}+35b}$	32

input
```
int((a+b/x^2)*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/5*b*x^5+1/7*a*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right)x^6\,dx=\frac{1}{7}\,ax^7+\frac{1}{5}\,bx^5$$

input
```
integrate((a+b/x^2)*x^6,x, algorithm="fricas")
```

output
```
1/7*a*x^7 + 1/5*b*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x^2}\right) x^6 \, dx = \frac{a x^7}{7} + \frac{b x^5}{5}$$

input
```
integrate((a+b/x**2)*x**6,x)
```

output
```
a*x**7/7 + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^6 \, dx = \frac{1}{7} \, a x^7 + \frac{1}{5} \, b x^5$$

input
```
integrate((a+b/x^2)*x^6,x, algorithm="maxima")
```

output
```
1/7*a*x^7 + 1/5*b*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^6 \, dx = \frac{1}{7} \, a x^7 + \frac{1}{5} \, b x^5$$

input
```
integrate((a+b/x^2)*x^6,x, algorithm="giac")
```

output
```
1/7*a*x^7 + 1/5*b*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^6\,dx = \frac{a\,x^7}{7}+\frac{b\,x^5}{5}$$

input
```
int(x^6*(a + b/x^2),x)
```

output
```
(a*x^7)/7 + (b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right) x^6\,dx = \frac{x^5(5a\,x^2+7b)}{35}$$

input
```
int((a+b/x^2)*x^6,x)
```

output
```
(x**5*(5*a*x**2 + 7*b))/35
```

3.257 $\int \left(a+\frac{b}{x^2}\right) x^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x^2}\right) x^5 \, dx=\frac{b x^4}{4}+\frac{a x^6}{6}$$

output
```
1/4*b*x^4+1/6*a*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) x^5 \, dx=\frac{b x^4}{4}+\frac{a x^6}{6}$$

input
```
Integrate[(a + b/x^2)*x^5,x]
```

output
```
(b*x^4)/4 + (a*x^6)/6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{x^2}\right) \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(a x^5+b x^3\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a x^6}{6}+\frac{b x^4}{4}$$

input
```
Int[(a + b/x^2)*x^5,x]
```

output
```
(b*x^4)/4 + (a*x^6)/6
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{4}b\,x^{4}+\frac{1}{6}a\,x^{6}$	14
risch	$\frac{1}{4}b\,x^{4}+\frac{1}{6}a\,x^{6}$	14
parallelrisch	$\frac{1}{4}b\,x^{4}+\frac{1}{6}a\,x^{6}$	14
gosper	$\frac{x^{4}\left(2a\,x^{2}+3b\right)}{12}$	16
norman	$\frac{\frac{1}{6}a\,x^{7}+\frac{1}{4}b\,x^{5}}{x}$	18
orering	$\frac{x^{6}\left(2a\,x^{2}+3b\right)\left(a+\frac{b}{x^{2}}\right)}{12a\,x^{2}+12b}$	32

input
```
int((a+b/x^2)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*x^4+1/6*a*x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int\left(a+\frac{b}{x^2}\right)x^5\,dx=\frac{1}{6}\,ax^6+\frac{1}{4}\,bx^4$$

input
```
integrate((a+b/x^2)*x^5,x, algorithm="fricas")
```

output
```
1/6*a*x^6 + 1/4*b*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x^2}\right) x^5 \, dx = \frac{ax^6}{6}+\frac{bx^4}{4}$$

input
```
integrate((a+b/x**2)*x**5,x)
```

output
```
a*x**6/6 + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^5 \, dx = \frac{1}{6} \, ax^6+\frac{1}{4} \, bx^4$$

input
```
integrate((a+b/x^2)*x^5,x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/4*b*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^5 \, dx = \frac{1}{6} \, ax^6+\frac{1}{4} \, bx^4$$

input
```
integrate((a+b/x^2)*x^5,x, algorithm="giac")
```

output
```
1/6*a*x^6 + 1/4*b*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^5 \, dx = \frac{a\,x^6}{6} + \frac{b\,x^4}{4}$$

input
```
int(x^5*(a + b/x^2),x)
```

output
```
(a*x^6)/6 + (b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right) x^5 \, dx = \frac{x^4(2a\,x^2+3b)}{12}$$

input
```
int((a+b/x^2)*x^5,x)
```

output
```
(x**4*(2*a*x**2 + 3*b))/12
```

3.258 $\int \left(a+\frac{b}{x^2}\right) x^4 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x^2}\right) x^4 \, dx=\frac{b x^3}{3}+\frac{a x^5}{5}$$

output
```
1/3*b*x^3+1/5*a*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) x^4 \, dx=\frac{b x^3}{3}+\frac{a x^5}{5}$$

input
```
Integrate[(a + b/x^2)*x^4,x]
```

output
```
(b*x^3)/3 + (a*x^5)/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^4+b x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^5}{5}+\frac{b x^3}{3}$$

input
```
Int[(a + b/x^2)*x^4,x]
```

output
```
(b*x^3)/3 + (a*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{3}b\,x^{3}+\frac{1}{5}a\,x^{5}$	14
risch	$\frac{1}{3}b\,x^{3}+\frac{1}{5}a\,x^{5}$	14
parallelrisch	$\frac{1}{3}b\,x^{3}+\frac{1}{5}a\,x^{5}$	14
gosper	$\frac{x^{3}\left(3a\,x^{2}+5b\right)}{15}$	16
norman	$\frac{\frac{1}{5}a\,x^{6}+\frac{1}{3}b\,x^{4}}{x}$	18
orering	$\frac{x^{5}\left(3a\,x^{2}+5b\right)\left(a+\frac{b}{x^{2}}\right)}{15a\,x^{2}+15b}$	32

input
```
int((a+b/x^2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3+1/5*a*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right)x^4\,dx=\frac{1}{5}\,ax^5+\frac{1}{3}\,bx^3$$

input
```
integrate((a+b/x^2)*x^4,x, algorithm="fricas")
```

output
```
1/5*a*x^5 + 1/3*b*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x^2}\right) x^4 \, dx = \frac{a x^5}{5} + \frac{b x^3}{3}$$

input
```
integrate((a+b/x**2)*x**4,x)
```

output
```
a*x**5/5 + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^4 \, dx = \frac{1}{5} a x^5 + \frac{1}{3} b x^3$$

input
```
integrate((a+b/x^2)*x^4,x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/3*b*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^4 \, dx = \frac{1}{5} a x^5 + \frac{1}{3} b x^3$$

input
```
integrate((a+b/x^2)*x^4,x, algorithm="giac")
```

output
```
1/5*a*x^5 + 1/3*b*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right)x^4\,dx=\frac{a\,x^5}{5}+\frac{b\,x^3}{3}$$

input
```
int(x^4*(a + b/x^2),x)
```

output
```
(a*x^5)/5 + (b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right)x^4\,dx=\frac{x^3(3a\,x^2+5b)}{15}$$

input
```
int((a+b/x^2)*x^4,x)
```

output
```
(x**3*(3*a*x**2 + 5*b))/15
```

3.259 $\int \left(a+\frac{b}{x^2}\right) x^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \left(a+\frac{b}{x^2}\right) x^3 \, dx=\frac{b x^2}{2}+\frac{a x^4}{4}$$

output
```
1/2*b*x^2+1/4*a*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) x^3 \, dx=\frac{b x^2}{2}+\frac{a x^4}{4}$$

input
```
Integrate[(a + b/x^2)*x^3,x]
```

output
```
(b*x^2)/2 + (a*x^4)/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^3+b x\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^4}{4}+\frac{b x^2}{2}$$

input
```
Int[(a + b/x^2)*x^3,x]
```

output
```
(b*x^2)/2 + (a*x^4)/4
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{1}{2}b\,x^{2}+\frac{1}{4}a\,x^{4}$	14
gosper	$\frac{x^{2}\left(a\,x^{2}+2b\right)}{4}$	15
default	$\frac{\left(a\,x^{2}+b\right)^{2}}{4a}$	15
norman	$\frac{\frac{1}{4}a\,x^{5}+\frac{1}{2}b\,x^{3}}{x}$	18
risch	$\frac{a\,x^{4}}{4}+\frac{b\,x^{2}}{2}+\frac{b^{2}}{4a}$	22
orering	$\frac{x^{4}\left(a\,x^{2}+2b\right)\left(a+\frac{b}{x^{2}}\right)}{4a\,x^{2}+4b}$	31

input
```
int((a+b/x^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+1/4*a*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right)x^3\,dx=\frac{1}{4}\,ax^4+\frac{1}{2}\,bx^2$$

input
```
integrate((a+b/x^2)*x^3,x, algorithm="fricas")
```

output
```
1/4*a*x^4 + 1/2*b*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{x^2}\right)x^3\,dx=\frac{ax^4}{4}+\frac{bx^2}{2}$$

input
```
integrate((a+b/x**2)*x**3,x)
```

output
```
a*x**4/4 + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right)x^3\,dx=\frac{1}{4}\,ax^4+\frac{1}{2}\,bx^2$$

input
```
integrate((a+b/x^2)*x^3,x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/2*b*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right)x^3\,dx=\frac{1}{4}\,ax^4+\frac{1}{2}\,bx^2$$

input
```
integrate((a+b/x^2)*x^3,x, algorithm="giac")
```

output
```
1/4*a*x^4 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) x^3\,dx=\frac{a\,x^4}{4}+\frac{b\,x^2}{2}$$

input
```
int(x^3*(a + b/x^2),x)
```

output
```
(a*x^4)/4 + (b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x^2}\right) x^3\,dx=\frac{x^2(a\,x^2+2b)}{4}$$

input
```
int((a+b/x^2)*x^3,x)
```

output
```
(x**2*(a*x**2 + 2*b))/4
```

3.260 $\int \left(a+\frac{b}{x^2}\right) x^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \left(a+\frac{b}{x^2}\right) x^2 \, dx = bx+\frac{ax^3}{3}$$

output
```
b*x+1/3*a*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) x^2 \, dx = bx+\frac{ax^3}{3}$$

input
```
Integrate[(a + b/x^2)*x^2,x]
```

output
```
b*x + (a*x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^2}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^2+b\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^3}{3}+b x$$

input
```
Int[(a + b/x^2)*x^2,x]
```

output
```
b*x + (a*x^3)/3
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$bx+\frac{1}{3}a\,x^{3}$	11
risch	$bx+\frac{1}{3}a\,x^{3}$	11
parallelrisch	$bx+\frac{1}{3}a\,x^{3}$	11
parts	$bx+\frac{1}{3}a\,x^{3}$	11
gosper	$\frac{x\left(a\,x^{2}+3b\right)}{3}$	13
norman	$\frac{b\,x^{2}+\frac{1}{3}a\,x^{4}}{x}$	17
orering	$\frac{x^{3}\left(a\,x^{2}+3b\right)\left(a+\frac{b}{x^{2}}\right)}{3a\,x^{2}+3b}$	31

input
```
int((a+b/x^2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
b*x+1/3*a*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int\left(a+\frac{b}{x^{2}}\right)x^{2}\,dx=\frac{1}{3}\,ax^{3}+bx$$

input
```
integrate((a+b/x^2)*x^2,x, algorithm="fricas")
```

output
```
1/3*a*x^3 + b*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \left(a+\frac{b}{x^2}\right) x^2\,dx=\frac{ax^3}{3}+bx$$

input
```
integrate((a+b/x**2)*x**2,x)
```

output
```
a*x**3/3 + b*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right) x^2\,dx=\frac{1}{3}\,ax^3+bx$$

input
```
integrate((a+b/x^2)*x^2,x, algorithm="maxima")
```

output
```
1/3*a*x^3 + b*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right) x^2\,dx=\frac{1}{3}\,ax^3+bx$$

input
```
integrate((a+b/x^2)*x^2,x, algorithm="giac")
```

output
```
1/3*a*x^3 + b*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right)x^2\,dx=\frac{a\,x^3}{3}+b\,x$$

input
```
int(x^2*(a + b/x^2),x)
```

output
```
b*x + (a*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)x^2\,dx=\frac{x(a\,x^2+3b)}{3}$$

input
```
int((a+b/x^2)*x^2,x)
```

output
```
(x*(a*x**2 + 3*b))/3
```

3.261 $\int \left(a+\frac{b}{x^2}\right) x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \left(a+\frac{b}{x^2}\right) x \, dx = \frac{a x^2}{2} + b \log(x)$$

output
```
1/2*a*x^2+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) x \, dx = \frac{a x^2}{2} + b \log(x)$$

input
```
Integrate[(a + b/x^2)*x,x]
```

output
```
(a*x^2)/2 + b*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right) dx$$

$$\Big\downarrow \ 802$$

$$\int \left(ax+\frac{b}{x}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax^2}{2}+b\log(x)$$

input
```
Int[(a + b/x^2)*x,x]
```

output
```
(a*x^2)/2 + b*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{a\,x^2}{2}+b\ln\left(x\right)$	12
norman	$\frac{a\,x^2}{2}+b\ln\left(x\right)$	12
risch	$\frac{a\,x^2}{2}+b\ln\left(x\right)$	12
parallelrisch	$\frac{a\,x^2}{2}+b\ln\left(x\right)$	12

input
```
int((a+b/x^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)x\,dx=\frac{1}{2}\,ax^2+b\log\left(x\right)$$

input
```
integrate((a+b/x^2)*x,x, algorithm="fricas")
```

output
```
1/2*a*x^2 + b*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{b}{x^2}\right)x\,dx=\frac{ax^2}{2}+b\log\left(x\right)$$

input
```
integrate((a+b/x**2)*x,x)
```

output
```
a*x**2/2 + b*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^2}\right)x\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\,b\log\left(x^2\right)$$

input
```
integrate((a+b/x^2)*x,x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*b*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)x\,dx=\frac{1}{2}\,ax^2+b\log\left(|x|\right)$$

input
```
integrate((a+b/x^2)*x,x, algorithm="giac")
```

output
```
1/2*a*x^2 + b*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)x\,dx=\frac{a\,x^2}{2}+b\,\ln\left(x\right)$$

input
```
int(x*(a + b/x^2),x)
```

output
```
(a*x^2)/2 + b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right) x \, dx = \log(x)\, b + \frac{a\, x^2}{2}$$

input
```
int((a+b/x^2)*x,x)
```

output
```
(2*log(x)*b + a*x**2)/2
```

3.262 $\int \left(a+\frac{b}{x^2}\right) dx$

Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \left(a+\frac{b}{x^2}\right) dx = -\frac{b}{x}+ax$$

output
```
-b/x+a*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) dx = -\frac{b}{x}+ax$$

input
```
Integrate[a + b/x^2,x]
```

output
```
-(b/x) + a*x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax-\frac{b}{x}$$

input
```
Int[a + b/x^2,x]
```

output
```
-(b/x) + a*x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{b}{x}+ax$	11
parallelrisch	$-\frac{b}{x}+ax$	11
gosper	$\frac{a\,x^{2}-b}{x}$	14
norman	$\frac{a\,x^{2}-b}{x}$	14
risch	$\frac{a\,x^{2}-b}{x}$	14
orering	$\frac{x\left(a\,x^{2}-b\right)\left(a+\frac{b}{x^{2}}\right)}{a\,x^{2}+b}$	28

input
```
int(a+b/x^2,x,method=_RETURNVERBOSE)
```

output
```
-b/x+a*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \left(a+\frac{b}{x^2}\right)\,dx=\frac{ax^2-b}{x}$$

input
```
integrate(a+b/x^2,x, algorithm="fricas")
```

output
```
(a*x^2 - b)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \left(a+\frac{b}{x^2}\right) dx = ax - \frac{b}{x}$$

input
```
integrate(a+b/x**2,x)
```

output
```
a*x - b/x
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) dx = ax - \frac{b}{x}$$

input
```
integrate(a+b/x^2,x, algorithm="maxima")
```

output
```
a*x - b/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) dx = ax - \frac{b}{x}$$

input
```
integrate(a+b/x^2,x, algorithm="giac")
```

output
```
a*x - b/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)\,dx = a\,x-\frac{b}{x}$$

input
```
int(a + b/x^2,x)
```

output
```
a*x - b/x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \left(a+\frac{b}{x^2}\right)\,dx = \frac{a\,x^2-b}{x}$$

input
```
int(a+b/x^2,x)
```

output
```
(a*x**2 - b)/x
```

3.263 $\int \frac{a+\frac{b}{x^2}}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+\frac{b}{x^2}}{x} \, dx = -\frac{b}{2x^2} + a \log(x)$$

output
```
-1/2*b/x^2+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x^2}}{x} \, dx = -\frac{b}{2x^2} + a \log(x)$$

input
```
Integrate[(a + b/x^2)/x,x]
```

output
```
-1/2*b/x^2 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x}+\frac{b}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a\log(x)-\frac{b}{2x^2}$$

input
```
Int[(a + b/x^2)/x,x]
```

output
```
-1/2*b/x^2 + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{b}{2x^2}+a\ln\left(x\right)$	12
norman	$-\frac{b}{2x^2}+a\ln\left(x\right)$	12
risch	$-\frac{b}{2x^2}+a\ln\left(x\right)$	12
parallelrisch	$\frac{2a\ln(x)x^2-b}{2x^2}$	18

input
```
int((a+b/x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x^2+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+\frac{b}{x^2}}{x}\,dx=\frac{2\,ax^2\log\left(x\right)-b}{2\,x^2}$$

input
```
integrate((a+b/x^2)/x,x, algorithm="fricas")
```

output
```
1/2*(2*a*x^2*log(x) - b)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+\frac{b}{x^2}}{x}\,dx=a\log\left(x\right)-\frac{b}{2x^2}$$

input
```
integrate((a+b/x**2)/x,x)
```

output
```
a*log(x) - b/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+\frac{b}{x^2}}{x}\,dx=\frac{1}{2}\,a\log\left(x^2\right)-\frac{b}{2\,x^2}$$

input
```
integrate((a+b/x^2)/x,x, algorithm="maxima")
```

output
```
1/2*a*log(x^2) - 1/2*b/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{a+\frac{b}{x^2}}{x}\,dx=\frac{1}{2}\,a\log\left(x^2\right)-\frac{ax^2+b}{2\,x^2}$$

input
```
integrate((a+b/x^2)/x,x, algorithm="giac")
```

output
```
1/2*a*log(x^2) - 1/2*(a*x^2 + b)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{x}\,dx=a\,\ln\left(x\right)-\frac{b}{2\,x^2}$$

input
```
int((a + b/x^2)/x,x)
```

output
```
a*log(x) - b/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+\frac{b}{x^2}}{x}\,dx=\frac{2\log(x)\,a\,x^2-b}{2x^2}$$

input
```
int((a+b/x^2)/x,x)
```

output
```
(2*log(x)*a*x**2 - b)/(2*x**2)
```

3.264 $\int \frac{a+\frac{b}{x^2}}{x^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{a+\frac{b}{x^2}}{x^2} \, dx = -\frac{b}{3x^3} - \frac{a}{x}$$

output
```
-1/3*b/x^3-a/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x^2}}{x^2} \, dx = -\frac{b}{3x^3} - \frac{a}{x}$$

input
```
Integrate[(a + b/x^2)/x^2,x]
```

output
```
-1/3*b/x^3 - a/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^2} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^2}+\frac{b}{x^4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{x}-\frac{b}{3 x^3}$$

input
```
Int[(a + b/x^2)/x^2,x]
```

output
```
-1/3*b/x^3 - a/x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{3a\,x^{2}+b}{3x^{3}}$	14
default	$-\frac{b}{3x^{3}}-\frac{a}{x}$	14
norman	$\frac{-a\,x^{2}-\frac{b}{3}}{x^{3}}$	15
risch	$\frac{-a\,x^{2}-\frac{b}{3}}{x^{3}}$	15
parallelrisch	$\frac{-3a\,x^{2}-b}{3x^{3}}$	16
orering	$-\frac{(3a\,x^{2}+b)\left(a+\frac{b}{x^{2}}\right)}{3x(a\,x^{2}+b)}$	30

input
```
int((a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*a*x^2+b)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+\frac{b}{x^2}}{x^2}\,dx=-\frac{3\,ax^2+b}{3\,x^3}$$

input
```
integrate((a+b/x^2)/x^2,x, algorithm="fricas")
```

output
```
-1/3*(3*a*x^2 + b)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+\frac{b}{x^2}}{x^2}\,dx=\frac{-3ax^2-b}{3x^3}$$

input
```
integrate((a+b/x**2)/x**2,x)
```

output
```
(-3*a*x**2 - b)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+\frac{b}{x^2}}{x^2}\,dx=-\frac{3\,ax^2+b}{3\,x^3}$$

input
```
integrate((a+b/x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/3*(3*a*x^2 + b)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+\frac{b}{x^2}}{x^2}\,dx=-\frac{3\,ax^2+b}{3\,x^3}$$

input
```
integrate((a+b/x^2)/x^2,x, algorithm="giac")
```

output
```
-1/3*(3*a*x^2 + b)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+\frac{b}{x^2}}{x^2}\,dx = -\frac{3\,a\,x^2+b}{3\,x^3}$$

input
```
int((a + b/x^2)/x^2,x)
```

output
```
-(b + 3*a*x^2)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x^2}}{x^2}\,dx = \frac{-3a\,x^2-b}{3x^3}$$

input
```
int((a+b/x^2)/x^2,x)
```

output
```
( - 3*a*x**2 - b)/(3*x**3)
```

3.265 $\int \frac{a+\frac{b}{x^2}}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{a+\frac{b}{x^2}}{x^3} dx = -\frac{\left(a+\frac{b}{x^2}\right)^2}{4b}$$

output
```
-1/4*(a+b/x^2)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{a+\frac{b}{x^2}}{x^3} dx = -\frac{b}{4x^4} - \frac{a}{2x^2}$$

input
```
Integrate[(a + b/x^2)/x^3,x]
```

output
```
-1/4*b/x^4 - a/(2*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^3} \, dx$$
$$\Big\downarrow \ 802$$
$$\int \left(\frac{a}{x^3}+\frac{b}{x^5}\right) dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{a}{2x^2}-\frac{b}{4x^4}$$

input
```
Int[(a + b/x^2)/x^3,x]
```

output
```
-1/4*b/x^4 - a/(2*x^2)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{2a\,x^{2}+b}{4x^{4}}$	14
default	$-\frac{a}{2x^{2}}-\frac{b}{4x^{4}}$	14
norman	$\frac{-\frac{a\,x^{2}}{2}-\frac{b}{4}}{x^{4}}$	15
risch	$\frac{-\frac{a\,x^{2}}{2}-\frac{b}{4}}{x^{4}}$	15
parallelrisch	$\frac{-2a\,x^{2}-b}{4x^{4}}$	16
orering	$-\frac{(2a\,x^{2}+b)\left(a+\frac{b}{x^{2}}\right)}{4x^{2}(a\,x^{2}+b)}$	30

input
```
int((a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*a*x^2+b)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{x^3}\,dx=-\frac{2\,ax^2+b}{4\,x^4}$$

input
```
integrate((a+b/x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*x^2 + b)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^3}\,dx = \frac{-2ax^2-b}{4x^4}$$

input
```
integrate((a+b/x**2)/x**3,x)
```

output
```
(-2*a*x**2 - b)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^2}{4\,b}$$

input
```
integrate((a+b/x^2)/x^3,x, algorithm="maxima")
```

output
```
-1/4*(a + b/x^2)^2/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{x^3}\,dx = -\frac{2\,ax^2+b}{4\,x^4}$$

input
```
integrate((a+b/x^2)/x^3,x, algorithm="giac")
```

output
```
-1/4*(2*a*x^2 + b)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{x^3}\,dx = -\frac{2\,a\,x^2+b}{4\,x^4}$$

input
```
int((a + b/x^2)/x^3,x)
```

output
```
-(b + 2*a*x^2)/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+\frac{b}{x^2}}{x^3}\,dx = \frac{-2a\,x^2-b}{4x^4}$$

input
```
int((a+b/x^2)/x^3,x)
```

output
```
( - 2*a*x**2 - b)/(4*x**4)
```

3.266 $\int \frac{a+\frac{b}{x^2}}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+\frac{b}{x^2}}{x^4} \, dx = -\frac{b}{5x^5} - \frac{a}{3x^3}$$

output
```
-1/5*b/x^5-1/3*a/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x^2}}{x^4} \, dx = -\frac{b}{5x^5} - \frac{a}{3x^3}$$

input
```
Integrate[(a + b/x^2)/x^4,x]
```

output
```
-1/5*b/x^5 - a/(3*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{3x^3}-\frac{b}{5x^5}$$

input
```
Int[(a + b/x^2)/x^4,x]
```

output
```
-1/5*b/x^5 - a/(3*x^3)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{b}{5x^5}-\frac{a}{3x^3}$	14
norman	$\frac{-\frac{a\,x^2}{3}-\frac{b}{5}}{x^5}$	15
risch	$\frac{-\frac{a\,x^2}{3}-\frac{b}{5}}{x^5}$	15
gosper	$-\frac{5a\,x^2+3b}{15x^5}$	16
parallelrisch	$\frac{-5a\,x^2-3b}{15x^5}$	16
orering	$-\frac{(5a\,x^2+3b)\left(a+\frac{b}{x^2}\right)}{15x^3(a\,x^2+b)}$	32

input
```
int((a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b/x^5-1/3*a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^4}\,dx=-\frac{5\,ax^2+3\,b}{15\,x^5}$$

input
```
integrate((a+b/x^2)/x^4,x, algorithm="fricas")
```

output
```
-1/15*(5*a*x^2 + 3*b)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a + \frac{b}{x^2}}{x^4}\,dx = \frac{-5ax^2 - 3b}{15x^5}$$

input
```
integrate((a+b/x**2)/x**4,x)
```

output
```
(-5*a*x**2 - 3*b)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a + \frac{b}{x^2}}{x^4}\,dx = -\frac{5\,ax^2 + 3\,b}{15\,x^5}$$

input
```
integrate((a+b/x^2)/x^4,x, algorithm="maxima")
```

output
```
-1/15*(5*a*x^2 + 3*b)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a + \frac{b}{x^2}}{x^4}\,dx = -\frac{5\,ax^2 + 3\,b}{15\,x^5}$$

input
```
integrate((a+b/x^2)/x^4,x, algorithm="giac")
```

output
```
-1/15*(5*a*x^2 + 3*b)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a + \frac{b}{x^2}}{x^4} \, dx = -\frac{5 \, a \, x^2 + 3 \, b}{15 \, x^5}$$

input
```
int((a + b/x^2)/x^4,x)
```

output
```
-(3*b + 5*a*x^2)/(15*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a + \frac{b}{x^2}}{x^4} \, dx = \frac{-5a \, x^2 - 3b}{15x^5}$$

input
```
int((a+b/x^2)/x^4,x)
```

output
```
( - 5*a*x**2 - 3*b)/(15*x**5)
```

3.267 $\int \frac{a+\frac{b}{x^2}}{x^5} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+\frac{b}{x^2}}{x^5} \, dx = -\frac{b}{6x^6} - \frac{a}{4x^4}$$

output
```
-1/6*b/x^6-1/4*a/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x^2}}{x^5} \, dx = -\frac{b}{6x^6} - \frac{a}{4x^4}$$

input
```
Integrate[(a + b/x^2)/x^5,x]
```

output
```
-1/6*b/x^6 - a/(4*x^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^5} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x^7}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{4x^4}-\frac{b}{6x^6}$$

input
```
Int[(a + b/x^2)/x^5,x]
```

output
```
-1/6*b/x^6 - a/(4*x^4)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{b}{6x^{6}}-\frac{a}{4x^{4}}$	14
norman	$\frac{-\frac{a\,x^{2}}{4}-\frac{b}{6}}{x^{6}}$	15
risch	$\frac{-\frac{a\,x^{2}}{4}-\frac{b}{6}}{x^{6}}$	15
gosper	$-\frac{3a\,x^{2}+2b}{12x^{6}}$	16
parallelrisch	$\frac{-3a\,x^{2}-2b}{12x^{6}}$	16
orering	$-\frac{\left(3a\,x^{2}+2b\right)\left(a+\frac{b}{x^{2}}\right)}{12x^{4}\left(a\,x^{2}+b\right)}$	32

input
```
int((a+b/x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6*b/x^6-1/4*a/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^5}\,dx=-\frac{3\,ax^2+2\,b}{12\,x^6}$$

input
```
integrate((a+b/x^2)/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*a*x^2 + 2*b)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^5}\,dx = \frac{-3ax^2-2b}{12x^6}$$

input
```
integrate((a+b/x**2)/x**5,x)
```

output
```
(-3*a*x**2 - 2*b)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^5}\,dx = -\frac{3\,ax^2+2\,b}{12\,x^6}$$

input
```
integrate((a+b/x^2)/x^5,x, algorithm="maxima")
```

output
```
-1/12*(3*a*x^2 + 2*b)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^5}\,dx = -\frac{3\,ax^2+2\,b}{12\,x^6}$$

input
```
integrate((a+b/x^2)/x^5,x, algorithm="giac")
```

output
```
-1/12*(3*a*x^2 + 2*b)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a + \frac{b}{x^2}}{x^5}\,dx = -\frac{3\,a\,x^2 + 2\,b}{12\,x^6}$$

input
```
int((a + b/x^2)/x^5,x)
```

output
```
-(2*b + 3*a*x^2)/(12*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a + \frac{b}{x^2}}{x^5}\,dx = \frac{-3a\,x^2 - 2b}{12x^6}$$

input
```
int((a+b/x^2)/x^5,x)
```

output
```
( - 3*a*x**2 - 2*b)/(12*x**6)
```

3.268 $\int \frac{a+\frac{b}{x^2}}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+\frac{b}{x^2}}{x^6} dx = -\frac{b}{7x^7} - \frac{a}{5x^5}$$

output
```
-1/7*b/x^7-1/5*a/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{x^2}}{x^6} dx = -\frac{b}{7x^7} - \frac{a}{5x^5}$$

input
```
Integrate[(a + b/x^2)/x^6,x]
```

output
```
-1/7*b/x^7 - a/(5*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^6} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^6}+\frac{b}{x^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{5x^5}-\frac{b}{7x^7}$$

input
```
Int[(a + b/x^2)/x^6,x]
```

output
```
-1/7*b/x^7 - a/(5*x^5)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{b}{7x^{7}}-\frac{a}{5x^{5}}$	14
norman	$\frac{-\frac{a\,x^{2}}{5}-\frac{b}{7}}{x^{7}}$	15
risch	$\frac{-\frac{a\,x^{2}}{5}-\frac{b}{7}}{x^{7}}$	15
gosper	$-\frac{7a\,x^{2}+5b}{35x^{7}}$	16
parallelrisch	$\frac{-7a\,x^{2}-5b}{35x^{7}}$	16
orering	$-\frac{\left(7a\,x^{2}+5b\right)\left(a+\frac{b}{x^{2}}\right)}{35x^{5}\left(a\,x^{2}+b\right)}$	32

input
```
int((a+b/x^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b/x^7-1/5*a/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^6}\,dx=-\frac{7\,ax^2+5\,b}{35\,x^7}$$

input
```
integrate((a+b/x^2)/x^6,x, algorithm="fricas")
```

output
```
-1/35*(7*a*x^2 + 5*b)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^6}\,dx = \frac{-7ax^2-5b}{35x^7}$$

input
```
integrate((a+b/x**2)/x**6,x)
```

output
```
(-7*a*x**2 - 5*b)/(35*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^6}\,dx = -\frac{7\,ax^2+5\,b}{35\,x^7}$$

input
```
integrate((a+b/x^2)/x^6,x, algorithm="maxima")
```

output
```
-1/35*(7*a*x^2 + 5*b)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^6}\,dx = -\frac{7\,ax^2+5\,b}{35\,x^7}$$

input
```
integrate((a+b/x^2)/x^6,x, algorithm="giac")
```

output
```
-1/35*(7*a*x^2 + 5*b)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^6}\,dx = -\frac{7\,a\,x^2+5\,b}{35\,x^7}$$

input
```
int((a + b/x^2)/x^6,x)
```

output
```
-(5*b + 7*a*x^2)/(35*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{x^6}\,dx = \frac{-7a\,x^2-5b}{35x^7}$$

input
```
int((a+b/x^2)/x^6,x)
```

output
```
( - 7*a*x**2 - 5*b)/(35*x**7)
```

3.269 $\int \left(a+\frac{b}{x^2}\right)^2 x^9 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9 \, dx = \frac{b^2x^6}{6}+\frac{1}{4}abx^8+\frac{a^2x^{10}}{10}$$

output
```
1/6*b^2*x^6+1/4*a*b*x^8+1/10*a^2*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9 \, dx = \frac{b^2x^6}{6}+\frac{1}{4}abx^8+\frac{a^2x^{10}}{10}$$

input
```
Integrate[(a + b/x^2)^2*x^9,x]
```

output
```
(b^2*x^6)/6 + (a*b*x^8)/4 + (a^2*x^10)/10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+\frac{b}{x^2}\right)^2 \, dx$$

$$\Big\downarrow \; 795$$

$$\int x^5 \left(a x^2+b\right)^2 \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2} \int x^4 \left(a x^2+b\right)^2 \, dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2} \int \left(a^2 x^8+2 a b x^6+b^2 x^4\right) \, dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(\frac{a^2 x^{10}}{5}+\frac{1}{2} a b x^8+\frac{b^2 x^6}{3}\right)$$

input
```
Int[(a + b/x^2)^2*x^9,x]
```

output
```
((b^2*x^6)/3 + (a*b*x^8)/2 + (a^2*x^10)/5)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{6}b^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}a^2x^{10}$	25
risch	$\frac{1}{6}b^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}a^2x^{10}$	25
parallelrisch	$\frac{1}{6}b^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}a^2x^{10}$	25
gosper	$\frac{x^6(6a^2x^4+15ab\,x^2+10b^2)}{60}$	27
norman	$\frac{\frac{1}{10}a^2x^{13}+\frac{1}{6}b^2x^9+\frac{1}{4}ab\,x^{11}}{x^3}$	29
orering	$\frac{x^{10}(6a^2x^4+15ab\,x^2+10b^2)\left(a+\frac{b}{x^2}\right)^2}{60(a\,x^2+b)^2}$	45

input
```
int((a+b/x^2)^2*x^9,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*x^6+1/4*a*b*x^8+1/10*a^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9\,dx=\frac{1}{10}\,a^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,b^2x^6$$

input
```
integrate((a+b/x^2)^2*x^9,x, algorithm="fricas")
```

output
```
1/10*a^2*x^10 + 1/4*a*b*x^8 + 1/6*b^2*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9\,dx=\frac{a^2x^{10}}{10}+\frac{abx^8}{4}+\frac{b^2x^6}{6}$$

input
```
integrate((a+b/x**2)**2*x**9,x)
```

output
```
a**2*x**10/10 + a*b*x**8/4 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9\,dx=\frac{1}{10}\,a^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,b^2x^6$$

input
```
integrate((a+b/x^2)^2*x^9,x, algorithm="maxima")
```

output
```
1/10*a^2*x^10 + 1/4*a*b*x^8 + 1/6*b^2*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9\,dx = \frac{1}{10}\,a^2x^{10} + \frac{1}{4}\,abx^8 + \frac{1}{6}\,b^2x^6$$

input
```
integrate((a+b/x^2)^2*x^9,x, algorithm="giac")
```

output
```
1/10*a^2*x^10 + 1/4*a*b*x^8 + 1/6*b^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9\,dx = \frac{a^2\,x^{10}}{10} + \frac{a\,b\,x^8}{4} + \frac{b^2\,x^6}{6}$$

input
```
int(x^9*(a + b/x^2)^2,x)
```

output
```
(a^2*x^10)/10 + (b^2*x^6)/6 + (a*b*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right)^2 x^9\,dx = \frac{x^6(6a^2x^4 + 15ab\,x^2 + 10b^2)}{60}$$

input
```
int((a+b/x^2)^2*x^9,x)
```

output
```
(x**6*(6*a**2*x**4 + 15*a*b*x**2 + 10*b**2))/60
```

3.270 $\int \left(a+\frac{b}{x^2}\right)^2 x^7 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7 \, dx = \frac{b^2x^4}{4}+\frac{1}{3}abx^6+\frac{a^2x^8}{8}$$

output
```
1/4*b^2*x^4+1/3*a*b*x^6+1/8*a^2*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7 \, dx = \frac{b^2x^4}{4}+\frac{1}{3}abx^6+\frac{a^2x^8}{8}$$

input
```
Integrate[(a + b/x^2)^2*x^7,x]
```

output
```
(b^2*x^4)/4 + (a*b*x^6)/3 + (a^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+\frac{b}{x^2}\right)^2 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^3 \left(a x^2+b\right)^2 dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^2 \left(a x^2+b\right)^2 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(a^2 x^6+2 a b x^4+b^2 x^2\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a^2 x^8}{4}+\frac{2}{3} a b x^6+\frac{b^2 x^4}{2}\right)$$

input
```
Int[(a + b/x^2)^2*x^7,x]
```

output
```
((b^2*x^4)/2 + (2*a*b*x^6)/3 + (a^2*x^8)/4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{4}b^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}a^2x^8$	25
risch	$\frac{1}{4}b^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}a^2x^8$	25
parallelrisch	$\frac{1}{4}b^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}a^2x^8$	25
gosper	$\frac{x^4(3a^2x^4+8ab\,x^2+6b^2)}{24}$	27
norman	$\frac{\frac{1}{8}a^2x^{11}+\frac{1}{4}b^2x^7+\frac{1}{3}ab\,x^9}{x^3}$	29
orering	$\frac{x^8(3a^2x^4+8ab\,x^2+6b^2)\left(a+\frac{b}{x^2}\right)^2}{24(a\,x^2+b)^2}$	45

input
```
int((a+b/x^2)^2*x^7,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*x^4+1/3*a*b*x^6+1/8*a^2*x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7\,dx = \frac{1}{8}\,a^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,b^2x^4$$

input
```
integrate((a+b/x^2)^2*x^7,x, algorithm="fricas")
```

output
```
1/8*a^2*x^8 + 1/3*a*b*x^6 + 1/4*b^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7\,dx = \frac{a^2x^8}{8} + \frac{abx^6}{3} + \frac{b^2x^4}{4}$$

input
```
integrate((a+b/x**2)**2*x**7,x)
```

output
```
a**2*x**8/8 + a*b*x**6/3 + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7\,dx = \frac{1}{8}\,a^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,b^2x^4$$

input
```
integrate((a+b/x^2)^2*x^7,x, algorithm="maxima")
```

output
```
1/8*a^2*x^8 + 1/3*a*b*x^6 + 1/4*b^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7\,dx = \frac{1}{8}\,a^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,b^2x^4$$

input
```
integrate((a+b/x^2)^2*x^7,x, algorithm="giac")
```

output
```
1/8*a^2*x^8 + 1/3*a*b*x^6 + 1/4*b^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7\,dx = \frac{a^2\,x^8}{8} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^4}{4}$$

input
```
int(x^7*(a + b/x^2)^2,x)
```

output
```
(a^2*x^8)/8 + (b^2*x^4)/4 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right)^2 x^7\,dx = \frac{x^4(3a^2x^4 + 8ab\,x^2 + 6b^2)}{24}$$

input
```
int((a+b/x^2)^2*x^7,x)
```

output
```
(x**4*(3*a**2*x**4 + 8*a*b*x**2 + 6*b**2))/24
```

3.271 $\int \left(a+\frac{b}{x^2}\right)^2 x^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5 \, dx = \frac{(b+ax^2)^3}{6a}$$

output
```
1/6*(a*x^2+b)^3/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5 \, dx = \frac{(b+ax^2)^3}{6a}$$

input
```
Integrate[(a + b/x^2)^2*x^5,x]
```

output
```
(b + a*x^2)^3/(6*a)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+\frac{b}{x^2}\right)^2dx$$
$$\Big\downarrow \text{795}$$
$$\int x\left(ax^2+b\right)^2dx$$
$$\Big\downarrow \text{241}$$
$$\frac{\left(ax^2+b\right)^3}{6a}$$

input
```
Int[(a + b/x^2)^2*x^5,x]
```

output
```
(b + a*x^2)^3/(6*a)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(a\,x^{2}+b)^{3}}{6a}$	15
parallelrisch	$\frac{1}{6}a^{2}x^{6}+\frac{1}{2}ab\,x^{4}+\frac{1}{2}b^{2}x^{2}$	25
gosper	$\frac{x^{2}(a^{2}x^{4}+3ab\,x^{2}+3b^{2})}{6}$	26
norman	$\frac{\frac{1}{6}a^{2}x^{9}+\frac{1}{2}b^{2}x^{5}+\frac{1}{2}ab\,x^{7}}{x^{3}}$	29
risch	$\frac{a^{2}x^{6}}{6}+\frac{ab\,x^{4}}{2}+\frac{b^{2}x^{2}}{2}+\frac{b^{3}}{6a}$	33
orering	$\frac{x^{6}(a^{2}x^{4}+3ab\,x^{2}+3b^{2})\left(a+\frac{b}{x^{2}}\right)^{2}}{6(a\,x^{2}+b)^{2}}$	44

input
```
int((a+b/x^2)^2*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x^2+b)^3/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5\,dx = \frac{1}{6}\,a^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,b^2x^2$$

input
```
integrate((a+b/x^2)^2*x^5,x, algorithm="fricas")
```

output
```
1/6*a^2*x^6 + 1/2*a*b*x^4 + 1/2*b^2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5\,dx=\frac{a^2x^6}{6}+\frac{abx^4}{2}+\frac{b^2x^2}{2}$$

input
```
integrate((a+b/x**2)**2*x**5,x)
```

output
```
a**2*x**6/6 + a*b*x**4/2 + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5\,dx=\frac{1}{6}\,a^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,b^2x^2$$

input
```
integrate((a+b/x^2)^2*x^5,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 + 1/2*a*b*x^4 + 1/2*b^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5\,dx=\frac{1}{6}\,a^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,b^2x^2$$

input
```
integrate((a+b/x^2)^2*x^5,x, algorithm="giac")
```

output
```
1/6*a^2*x^6 + 1/2*a*b*x^4 + 1/2*b^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5\,dx=\frac{a^2\,x^6}{6}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^2}{2}$$

input
```
int(x^5*(a + b/x^2)^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*x^2)/2 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x^2}\right)^2 x^5\,dx=\frac{x^2(a^2x^4+3ab\,x^2+3b^2)}{6}$$

input
```
int((a+b/x^2)^2*x^5,x)
```

output
```
(x**2*(a**2*x**4 + 3*a*b*x**2 + 3*b**2))/6
```

3.272 $\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx=abx^2+\frac{a^2x^4}{4}+b^2\log(x)$$

output
```
a*b*x^2+1/4*a^2*x^4+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx=abx^2+\frac{a^2x^4}{4}+b^2\log(x)$$

input
```
Integrate[(a + b/x^2)^2*x^3,x]
```

output
```
a*b*x^2 + (a^2*x^4)/4 + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right)^2 dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{\left(a x^2+b\right)^2}{x}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{\left(a x^2+b\right)^2}{x^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(\frac{b^2}{x^2}+2 a b+a^2 x^2\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{a^2 x^4}{2}+2 a b x^2+b^2 \log \left(x^2\right)\right)$$

input
```
Int[(a + b/x^2)^2*x^3,x]
```

output
```
(2*a*b*x^2 + (a^2*x^4)/2 + b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$ab\,x^{2}+\frac{a^{2} x^{4}}{4}+b^{2} \ln \left(x \right)$	22
parallelrisch	$ab\,x^{2}+\frac{a^{2} x^{4}}{4}+b^{2} \ln \left(x \right)$	22
risch	$\frac{a^{2} x^{4}}{4}+ab\,x^{2}+b^{2}+b^{2} \ln \left(x \right)$	25
norman	$\frac{ab\,x^{5}+\frac{1}{4} a^{2} x^{7}}{x^{3}}+b^{2} \ln \left(x \right)$	27

input
```
int((a+b/x^2)^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
a*b*x^2+1/4*a^2*x^4+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx = \frac{1}{4}\,a^2x^4 + abx^2 + b^2\log\left(x\right)$$

input
```
integrate((a+b/x^2)^2*x^3,x, algorithm="fricas")
```

output
```
1/4*a^2*x^4 + a*b*x^2 + b^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx = \frac{a^2x^4}{4} + abx^2 + b^2\log\left(x\right)$$

input
```
integrate((a+b/x**2)**2*x**3,x)
```

output
```
a**2*x**4/4 + a*b*x**2 + b**2*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx = \frac{1}{4}\,a^2x^4 + abx^2 + \frac{1}{2}\,b^2\log\left(x^2\right)$$

input
```
integrate((a+b/x^2)^2*x^3,x, algorithm="maxima")
```

output
```
1/4*a^2*x^4 + a*b*x^2 + 1/2*b^2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx=\frac{1}{4}\,a^2x^4+abx^2+b^2\log\left(|x|\right)$$

input
```
integrate((a+b/x^2)^2*x^3,x, algorithm="giac")
```

output
```
1/4*a^2*x^4 + a*b*x^2 + b^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx=b^2\,\ln\left(x\right)+\frac{a^2\,x^4}{4}+a\,b\,x^2$$

input
```
int(x^3*(a + b/x^2)^2,x)
```

output
```
b^2*log(x) + (a^2*x^4)/4 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)^2 x^3\,dx=\log(x)\,b^2+\frac{a^2x^4}{4}+ab\,x^2$$

input
```
int((a+b/x^2)^2*x^3,x)
```

output
```
(4*log(x)*b**2 + a**2*x**4 + 4*a*b*x**2)/4
```

3.273 $\int \left(a+\frac{b}{x^2}\right)^2 x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \left(a+\frac{b}{x^2}\right)^2 x \, dx = -\frac{b^2}{2x^2}+\frac{a^2x^2}{2}+2ab\log(x)$$

output
```
-1/2*b^2/x^2+1/2*a^2*x^2+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x \, dx = -\frac{b^2}{2x^2}+\frac{a^2x^2}{2}+2ab\log(x)$$

input
```
Integrate[(a + b/x^2)^2*x,x]
```

output
```
-1/2*b^2/x^2 + (a^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)^2 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(ax^2+b\right)^2}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(ax^2+b\right)^2}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(a^2+\frac{2ba}{x^2}+\frac{b^2}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^2x^2+2ab\log\left(x^2\right)-\frac{b^2}{x^2}\right)$$

input
```
Int[(a + b/x^2)^2*x,x]
```

output
```
(-(b^2/x^2) + a^2*x^2 + 2*a*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{b^2}{2x^2}+\frac{a^2x^2}{2}+2ab\ln\left(x\right)$	24
risch	$-\frac{b^2}{2x^2}+\frac{a^2x^2}{2}+2ab\ln\left(x\right)$	24
norman	$\frac{\frac{1}{2}a^2x^5-\frac{1}{2}b^2x}{x^3}+2ab\ln\left(x\right)$	27
parallelrisch	$\frac{a^2x^4+4ab\ln(x)x^2-b^2}{2x^2}$	28

input
```
int((a+b/x^2)^2*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/x^2+1/2*a^2*x^2+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x\,dx=\frac{a^2x^4+4\,abx^2\log\left(x\right)-b^2}{2\,x^2}$$

input
```
integrate((a+b/x^2)^2*x,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^4 + 4*a*b*x^2*log(x) - b^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)^2 x\,dx=\frac{a^2x^2}{2}+2ab\log\left(x\right)-\frac{b^2}{2x^2}$$

input
```
integrate((a+b/x**2)**2*x,x)
```

output
```
a**2*x**2/2 + 2*a*b*log(x) - b**2/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)^2 x\,dx=\frac{1}{2}\,a^2x^2+ab\log\left(x^2\right)-\frac{b^2}{2\,x^2}$$

input
```
integrate((a+b/x^2)^2*x,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + a*b*log(x^2) - 1/2*b^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)^2 x\,dx=\frac{1}{2}\,a^2x^2+2\,ab\log\left(|x|\right)-\frac{b^2}{2\,x^2}$$

input
```
integrate((a+b/x^2)^2*x,x, algorithm="giac")
```

output
```
1/2*a^2*x^2 + 2*a*b*log(abs(x)) - 1/2*b^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)^2 x\,dx=\frac{a^2\,x^2}{2}-\frac{b^2}{2\,x^2}+2\,a\,b\,\ln\left(x\right)$$

input
```
int(x*(a + b/x^2)^2,x)
```

output
```
(a^2*x^2)/2 - b^2/(2*x^2) + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x\,dx=\frac{4\log(x)\,ab\,x^2+a^2x^4-b^2}{2x^2}$$

input
```
int((a+b/x^2)^2*x,x)
```

output
```
(4*log(x)*a*b*x**2 + a**2*x**4 - b**2)/(2*x**2)
```

3.274 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x} dx = -\frac{b^2}{4x^4} - \frac{ab}{x^2} + a^2 \log(x)$$

output
```
-1/4*b^2/x^4-a*b/x^2+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x} dx = -\frac{b^2}{4x^4} - \frac{ab}{x^2} + a^2 \log(x)$$

input
```
Integrate[(a + b/x^2)^2/x,x]
```

output
```
-1/4*b^2/x^4 - (a*b)/x^2 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(ax^2+b\right)^2}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(ax^2+b\right)^2}{x^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^2}+\frac{2ba}{x^4}+\frac{b^2}{x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^2\log\left(x^2\right)-\frac{2ab}{x^2}-\frac{b^2}{2x^4}\right)$$

input
```
Int[(a + b/x^2)^2/x,x]
```

output
```
(-1/2*b^2/x^4 - (2*a*b)/x^2 + a^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{b^2}{4x^4}-\frac{ab}{x^2}+a^2\ln\left(x\right)$	23
norman	$\frac{-\frac{1}{4}b^2-ab\,x^2}{x^4}+a^2\ln\left(x\right)$	25
risch	$\frac{-\frac{1}{4}b^2-ab\,x^2}{x^4}+a^2\ln\left(x\right)$	25
parallelrisch	$\frac{4a^2\ln\left(x\right)x^4-4ab\,x^2-b^2}{4x^4}$	29

input
```
int((a+b/x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^2/x^4-a*b/x^2+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx=\frac{4\,a^2x^4\log\left(x\right)-4\,abx^2-b^2}{4\,x^4}$$

input
```
integrate((a+b/x^2)^2/x,x, algorithm="fricas")
```

output
```
1/4*(4*a^2*x^4*log(x) - 4*a*b*x^2 - b^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx=a^2\log\left(x\right)+\frac{-4abx^2-b^2}{4x^4}$$

input
```
integrate((a+b/x**2)**2/x,x)
```

output
```
a**2*log(x) + (-4*a*b*x**2 - b**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx=\frac{1}{2}\,a^2\log\left(x^2\right)-\frac{4\,abx^2+b^2}{4\,x^4}$$

input
```
integrate((a+b/x^2)^2/x,x, algorithm="maxima")
```

output
```
1/2*a^2*log(x^2) - 1/4*(4*a*b*x^2 + b^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx = \frac{1}{2}\,a^2\log\left(x^2\right) - \frac{3\,a^2x^4+4\,abx^2+b^2}{4\,x^4}$$

input
```
integrate((a+b/x^2)^2/x,x, algorithm="giac")
```

output
```
1/2*a^2*log(x^2) - 1/4*(3*a^2*x^4 + 4*a*b*x^2 + b^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx = a^2\,\ln\left(x\right) - \frac{\frac{b^2}{4}+a\,b\,x^2}{x^4}$$

input
```
int((a + b/x^2)^2/x,x)
```

output
```
a^2*log(x) - (b^2/4 + a*b*x^2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x}\,dx = \frac{4\log(x)\,a^2x^4-4ab\,x^2-b^2}{4x^4}$$

input
```
int((a+b/x^2)^2/x,x)
```

output
```
(4*log(x)*a**2*x**4 - 4*a*b*x**2 - b**2)/(4*x**4)
```

3.275 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3} dx = -\frac{\left(a+\frac{b}{x^2}\right)^3}{6b}$$

output
```
-1/6*(a+b/x^2)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3} dx = -\frac{b^2}{6x^6} - \frac{ab}{2x^4} - \frac{a^2}{2x^2}$$

input
```
Integrate[(a + b/x^2)^2/x^3,x]
```

output
```
-1/6*b^2/x^6 - (a*b)/(2*x^4) - a^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3} \, dx$$

$$\Big\downarrow \ 793$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^3}{6 b}$$

input
```
Int[(a + b/x^2)^2/x^3,x]
```

output
```
-1/6*(a + b/x^2)^3/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

method	result	size
gosper	$-\frac{3a^2x^4+3ab\,x^2+b^2}{6x^6}$	25
default	$-\frac{a^2}{2x^2}-\frac{ab}{2x^4}-\frac{b^2}{6x^6}$	25
norman	$\frac{-\frac{1}{2}a^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}b^2}{x^6}$	26
risch	$\frac{-\frac{1}{2}a^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}b^2}{x^6}$	26
parallelrisch	$\frac{-3a^2x^4-3ab\,x^2-b^2}{6x^6}$	27
orering	$-\frac{(3a^2x^4+3ab\,x^2+b^2)\left(a+\frac{b}{x^2}\right)^2}{6x^2(a\,x^2+b)^2}$	43

input
```
int((a+b/x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*a^2*x^4+3*a*b*x^2+b^2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx=-\frac{3\,a^2x^4+3\,abx^2+b^2}{6\,x^6}$$

input
```
integrate((a+b/x^2)^2/x^3,x, algorithm="fricas")
```

output
```
-1/6*(3*a^2*x^4 + 3*a*b*x^2 + b^2)/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx=\frac{-3a^2x^4-3abx^2-b^2}{6x^6}$$

input
```
integrate((a+b/x**2)**2/x**3,x)
```

output
```
(-3*a**2*x**4 - 3*a*b*x**2 - b**2)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^3}{6\,b}$$

input
```
integrate((a+b/x^2)^2/x^3,x, algorithm="maxima")
```

output
```
-1/6*(a + b/x^2)^3/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx = -\frac{3\,a^2x^4+3\,abx^2+b^2}{6\,x^6}$$

input
```
integrate((a+b/x^2)^2/x^3,x, algorithm="giac")
```

output
```
-1/6*(3*a^2*x^4 + 3*a*b*x^2 + b^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx=-\frac{\frac{a^2\,x^4}{2}+\frac{a\,b\,x^2}{2}+\frac{b^2}{6}}{x^6}$$

input
```
int((a + b/x^2)^2/x^3,x)
```

output
```
-(b^2/6 + (a^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^3}\,dx=\frac{-3a^2x^4-3ab\,x^2-b^2}{6x^6}$$

input
```
int((a+b/x^2)^2/x^3,x)
```

output
```
( - 3*a**2*x**4 - 3*a*b*x**2 - b**2)/(6*x**6)
```

3.276 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5} dx = -\frac{b^2}{8x^8} - \frac{ab}{3x^6} - \frac{a^2}{4x^4}$$

output
```
-1/8*b^2/x^8-1/3*a*b/x^6-1/4*a^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5} dx = -\frac{b^2}{8x^8} - \frac{ab}{3x^6} - \frac{a^2}{4x^4}$$

input
```
Integrate[(a + b/x^2)^2/x^5,x]
```

output
```
-1/8*b^2/x^8 - (a*b)/(3*x^6) - a^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{\left(ax^2+b\right)^2}{x^9}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{\left(ax^2+b\right)^2}{x^{10}}dx^2$$
$$\Big\downarrow \text{ 53}$$
$$\frac{1}{2}\int \left(\frac{a^2}{x^6}+\frac{2ba}{x^8}+\frac{b^2}{x^{10}}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{a^2}{2x^4}-\frac{2ab}{3x^6}-\frac{b^2}{4x^8}\right)$$

input
```
Int[(a + b/x^2)^2/x^5,x]
```

output
```
(-1/4*b^2/x^8 - (2*a*b)/(3*x^6) - a^2/(2*x^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^{2}}{8x^{8}}-\frac{ab}{3x^{6}}-\frac{a^{2}}{4x^{4}}$	25
norman	$\frac{-\frac{1}{4}a^{2}x^{4}-\frac{1}{3}ab\,x^{2}-\frac{1}{8}b^{2}}{x^{8}}$	26
risch	$\frac{-\frac{1}{4}a^{2}x^{4}-\frac{1}{3}ab\,x^{2}-\frac{1}{8}b^{2}}{x^{8}}$	26
gosper	$-\frac{6a^{2}x^{4}+8ab\,x^{2}+3b^{2}}{24x^{8}}$	27
parallelrisch	$\frac{-6a^{2}x^{4}-8ab\,x^{2}-3b^{2}}{24x^{8}}$	27
orering	$-\frac{\left(6a^{2}x^{4}+8ab\,x^{2}+3b^{2}\right)\left(a+\frac{b}{x^{2}}\right)^{2}}{24x^{4}\left(a\,x^{2}+b\right)^{2}}$	45

input
```
int((a+b/x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^2/x^8-1/3*a*b/x^6-1/4*a^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx = -\frac{6\,a^2x^4+8\,abx^2+3\,b^2}{24\,x^8}$$

input
```
integrate((a+b/x^2)^2/x^5,x, algorithm="fricas")
```

output
```
-1/24*(6*a^2*x^4 + 8*a*b*x^2 + 3*b^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx = \frac{-6a^2x^4-8abx^2-3b^2}{24x^8}$$

input
```
integrate((a+b/x**2)**2/x**5,x)
```

output
```
(-6*a**2*x**4 - 8*a*b*x**2 - 3*b**2)/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx = -\frac{6\,a^2x^4+8\,abx^2+3\,b^2}{24\,x^8}$$

input
```
integrate((a+b/x^2)^2/x^5,x, algorithm="maxima")
```

output
```
-1/24*(6*a^2*x^4 + 8*a*b*x^2 + 3*b^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx = -\frac{6\,a^2x^4+8\,abx^2+3\,b^2}{24\,x^8}$$

input
```
integrate((a+b/x^2)^2/x^5,x, algorithm="giac")
```

output
```
-1/24*(6*a^2*x^4 + 8*a*b*x^2 + 3*b^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx = -\frac{\frac{a^2\,x^4}{4}+\frac{a\,b\,x^2}{3}+\frac{b^2}{8}}{x^8}$$

input
```
int((a + b/x^2)^2/x^5,x)
```

output
```
-(b^2/8 + (a^2*x^4)/4 + (a*b*x^2)/3)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^5}\,dx = \frac{-6a^2x^4-8ab\,x^2-3b^2}{24x^8}$$

input
```
int((a+b/x^2)^2/x^5,x)
```

output
```
( - 6*a**2*x**4 - 8*a*b*x**2 - 3*b**2)/(24*x**8)
```

3.277 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7} dx = -\frac{b^2}{10x^{10}} - \frac{ab}{4x^8} - \frac{a^2}{6x^6}$$

output
```
-1/10*b^2/x^10-1/4*a*b/x^8-1/6*a^2/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7} dx = -\frac{b^2}{10x^{10}} - \frac{ab}{4x^8} - \frac{a^2}{6x^6}$$

input
```
Integrate[(a + b/x^2)^2/x^7,x]
```

output
```
-1/10*b^2/x^10 - (a*b)/(4*x^8) - a^2/(6*x^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7} \, dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{\left(a x^2+b\right)^2}{x^{11}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int \frac{\left(a x^2+b\right)^2}{x^{12}}dx^2$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{2} \int \left(\frac{a^2}{x^8}+\frac{2 b a}{x^{10}}+\frac{b^2}{x^{12}}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2} \left(-\frac{a^2}{3 x^6}-\frac{a b}{2 x^8}-\frac{b^2}{5 x^{10}}\right)$$

input
```
Int[(a + b/x^2)^2/x^7,x]
```

output
```
(-1/5*b^2/x^10 - (a*b)/(2*x^8) - a^2/(3*x^6))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^{2}}{10x^{10}}-\frac{ab}{4x^{8}}-\frac{a^{2}}{6x^{6}}$	25
norman	$\frac{-\frac{1}{6}a^{2}x^{4}-\frac{1}{4}ab\,x^{2}-\frac{1}{10}b^{2}}{x^{10}}$	26
risch	$\frac{-\frac{1}{6}a^{2}x^{4}-\frac{1}{4}ab\,x^{2}-\frac{1}{10}b^{2}}{x^{10}}$	26
gosper	$-\frac{10a^{2}x^{4}+15ab\,x^{2}+6b^{2}}{60x^{10}}$	27
parallelrisch	$\frac{-10a^{2}x^{4}-15ab\,x^{2}-6b^{2}}{60x^{10}}$	27
orering	$-\frac{\left(10a^{2}x^{4}+15ab\,x^{2}+6b^{2}\right)\left(a+\frac{b}{x^{2}}\right)^{2}}{60x^{6}\left(a\,x^{2}+b\right)^{2}}$	45

input
```
int((a+b/x^2)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/10*b^2/x^10-1/4*a*b/x^8-1/6*a^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7}\,dx = -\frac{10\,a^2x^4+15\,abx^2+6\,b^2}{60\,x^{10}}$$

input
```
integrate((a+b/x^2)^2/x^7,x, algorithm="fricas")
```

output
```
-1/60*(10*a^2*x^4 + 15*a*b*x^2 + 6*b^2)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7}\,dx = \frac{-10a^2x^4-15abx^2-6b^2}{60x^{10}}$$

input
```
integrate((a+b/x**2)**2/x**7,x)
```

output
```
(-10*a**2*x**4 - 15*a*b*x**2 - 6*b**2)/(60*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7}\,dx = -\frac{10\,a^2x^4+15\,abx^2+6\,b^2}{60\,x^{10}}$$

input
```
integrate((a+b/x^2)^2/x^7,x, algorithm="maxima")
```

output
```
-1/60*(10*a^2*x^4 + 15*a*b*x^2 + 6*b^2)/x^10
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7}\,dx = -\frac{10\,a^2x^4+15\,abx^2+6\,b^2}{60\,x^{10}}$$

input
```
integrate((a+b/x^2)^2/x^7,x, algorithm="giac")
```

output
```
-1/60*(10*a^2*x^4 + 15*a*b*x^2 + 6*b^2)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7}\,dx = -\frac{\frac{a^2\,x^4}{6}+\frac{a\,b\,x^2}{4}+\frac{b^2}{10}}{x^{10}}$$

input
```
int((a + b/x^2)^2/x^7,x)
```

output
```
-(b^2/10 + (a^2*x^4)/6 + (a*b*x^2)/4)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^7}\,dx = \frac{-10a^2x^4-15ab\,x^2-6b^2}{60x^{10}}$$

input
```
int((a+b/x^2)^2/x^7,x)
```

output
```
( - 10*a**2*x**4 - 15*a*b*x**2 - 6*b**2)/(60*x**10)
```

3.278 $\int \left(a+\frac{b}{x^2}\right)^2 x^6 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+\frac{b}{x^2}\right)^2 x^6 \, dx=\frac{b^2x^3}{3}+\frac{2}{5}abx^5+\frac{a^2x^7}{7}$$

output
```
1/3*b^2*x^3+2/5*a*b*x^5+1/7*a^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x^6 \, dx=\frac{b^2x^3}{3}+\frac{2}{5}abx^5+\frac{a^2x^7}{7}$$

input
```
Integrate[(a + b/x^2)^2*x^6,x]
```

output
```
(b^2*x^3)/3 + (2*a*b*x^5)/5 + (a^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+\frac{b}{x^2}\right)^2 \, dx$$

$$\Big\downarrow \text{795}$$

$$\int x^2 \left(a x^2+b\right)^2 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2 x^6+2 a b x^4+b^2 x^2\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 x^7}{7}+\frac{2}{5} a b x^5+\frac{b^2 x^3}{3}$$

input
```
Int[(a + b/x^2)^2*x^6,x]
```

output
```
(b^2*x^3)/3 + (2*a*b*x^5)/5 + (a^2*x^7)/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{3}b^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}a^2x^7$	25
risch	$\frac{1}{3}b^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}a^2x^7$	25
parallelrisch	$\frac{1}{3}b^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}a^2x^7$	25
gosper	$\frac{x^3(15a^2x^4+42ab\,x^2+35b^2)}{105}$	27
norman	$\frac{\frac{1}{7}a^2x^{10}+\frac{1}{3}b^2x^6+\frac{2}{5}ab\,x^8}{x^3}$	29
orering	$\frac{x^7(15a^2x^4+42ab\,x^2+35b^2)\left(a+\frac{b}{x^2}\right)^2}{105(a\,x^2+b)^2}$	45

input
```
int((a+b/x^2)^2*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*x^3+2/5*a*b*x^5+1/7*a^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int\left(a+\frac{b}{x^2}\right)^2x^6\,dx=\frac{1}{7}\,a^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,b^2x^3$$

input
```
integrate((a+b/x^2)^2*x^6,x, algorithm="fricas")
```

output
```
1/7*a^2*x^7 + 2/5*a*b*x^5 + 1/3*b^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right)^2 x^6\,dx = \frac{a^2x^7}{7}+\frac{2abx^5}{5}+\frac{b^2x^3}{3}$$

input
```
integrate((a+b/x**2)**2*x**6,x)
```

output
```
a**2*x**7/7 + 2*a*b*x**5/5 + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^6\,dx = \frac{1}{7}\,a^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,b^2x^3$$

input
```
integrate((a+b/x^2)^2*x^6,x, algorithm="maxima")
```

output
```
1/7*a^2*x^7 + 2/5*a*b*x^5 + 1/3*b^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^6\,dx = \frac{1}{7}\,a^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,b^2x^3$$

input
```
integrate((a+b/x^2)^2*x^6,x, algorithm="giac")
```

output
```
1/7*a^2*x^7 + 2/5*a*b*x^5 + 1/3*b^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(a+\frac{b}{x^2}\right)^2 x^6\,dx=\frac{a^2\,x^7}{7}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^3}{3}$$

input
```
int(x^6*(a + b/x^2)^2,x)
```

output
```
(a^2*x^7)/7 + (b^2*x^3)/3 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right)^2 x^6\,dx=\frac{x^3(15a^2x^4+42ab\,x^2+35b^2)}{105}$$

input
```
int((a+b/x^2)^2*x^6,x)
```

output
```
(x**3*(15*a**2*x**4 + 42*a*b*x**2 + 35*b**2))/105
```

3.279 $\int \left(a+\frac{b}{x^2}\right)^2 x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4 \, dx = b^2x+\frac{2}{3}abx^3+\frac{a^2x^5}{5}$$

output
```
b^2*x+2/3*a*b*x^3+1/5*a^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4 \, dx = b^2x+\frac{2}{3}abx^3+\frac{a^2x^5}{5}$$

input
```
Integrate[(a + b/x^2)^2*x^4,x]
```

output
```
b^2*x + (2*a*b*x^3)/3 + (a^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right)^2 dx$$

$$\Big\downarrow \text{795}$$

$$\int \left(a x^2+b\right)^2 dx$$

$$\Big\downarrow \text{210}$$

$$\int \left(a^2 x^4+2 a b x^2+b^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 x^5}{5}+\frac{2}{3} a b x^3+b^2 x$$

input
```
Int[(a + b/x^2)^2*x^4,x]
```

output
```
b^2*x + (2*a*b*x^3)/3 + (a^2*x^5)/5
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$b^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}a^2x^5$	22
risch	$b^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}a^2x^5$	22
parallelrisch	$b^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}a^2x^5$	22
gosper	$\frac{x\left(3a^2x^4+10ab\,x^2+15b^2\right)}{15}$	25
norman	$\frac{b^2x^4+\frac{1}{5}a^2x^8+\frac{2}{3}ab\,x^6}{x^3}$	28
orering	$\frac{x^5\left(3a^2x^4+10ab\,x^2+15b^2\right)\left(a+\frac{b}{x^2}\right)^2}{15\left(a\,x^2+b\right)^2}$	45

input
```
int((a+b/x^2)^2*x^4,x,method=_RETURNVERBOSE)
```

output
```
b^2*x+2/3*a*b*x^3+1/5*a^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4\,dx=\frac{1}{5}\,a^2x^5+\frac{2}{3}\,abx^3+b^2x$$

input
```
integrate((a+b/x^2)^2*x^4,x, algorithm="fricas")
```

output
```
1/5*a^2*x^5 + 2/3*a*b*x^3 + b^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4\,dx = \frac{a^2x^5}{5}+\frac{2abx^3}{3}+b^2x$$

input
```
integrate((a+b/x**2)**2*x**4,x)
```

output
```
a**2*x**5/5 + 2*a*b*x**3/3 + b**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4\,dx = \frac{1}{5}\,a^2x^5+\frac{2}{3}\,abx^3+b^2x$$

input
```
integrate((a+b/x^2)^2*x^4,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 + 2/3*a*b*x^3 + b^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4\,dx = \frac{1}{5}\,a^2x^5+\frac{2}{3}\,abx^3+b^2x$$

input
```
integrate((a+b/x^2)^2*x^4,x, algorithm="giac")
```

output
```
1/5*a^2*x^5 + 2/3*a*b*x^3 + b^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4 \, dx = \frac{a^2 x^5}{5} + \frac{2 a b x^3}{3} + b^2 x$$

input
```
int(x^4*(a + b/x^2)^2,x)
```

output
```
b^2*x + (a^2*x^5)/5 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x^2}\right)^2 x^4 \, dx = \frac{x(3a^2x^4 + 10ab\, x^2 + 15b^2)}{15}$$

input
```
int((a+b/x^2)^2*x^4,x)
```

output
```
(x*(3*a**2*x**4 + 10*a*b*x**2 + 15*b**2))/15
```

3.280 $\int \left(a+\frac{b}{x^2}\right)^2 x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2 \, dx = -\frac{b^2}{x}+2abx+\frac{a^2x^3}{3}$$

output
```
-b^2/x+2*a*b*x+1/3*a^2*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2 \, dx = -\frac{b^2}{x}+2abx+\frac{a^2x^3}{3}$$

input
```
Integrate[(a + b/x^2)^2*x^2,x]
```

output
```
-(b^2/x) + 2*a*b*x + (a^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^2}\right)^2 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a x^2+b\right)^2}{x^2}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2 x^2+2 a b+\frac{b^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 x^3}{3}+2 a b x-\frac{b^2}{x}$$

input
```
Int[(a + b/x^2)^2*x^2,x]
```

output
```
-(b^2/x) + 2*a*b*x + (a^2*x^3)/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{b^2}{x}+2abx+\frac{a^2x^3}{3}$	23
risch	$-\frac{b^2}{x}+2abx+\frac{a^2x^3}{3}$	23
gosper	$\frac{a^2x^4+6ab\,x^2-3b^2}{3x}$	26
parallelrisch	$\frac{a^2x^4+6ab\,x^2-3b^2}{3x}$	26
norman	$\frac{\frac{1}{3}a^2x^6-b^2x^2+2ab\,x^4}{x^3}$	29
orering	$\frac{(a^2x^4+6ab\,x^2-3b^2)x^3\left(a+\frac{b}{x^2}\right)^2}{3(a\,x^2+b)^2}$	44

input
```
int((a+b/x^2)^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2/x+2*a*b*x+1/3*a^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2\,dx=\frac{a^2x^4+6\,abx^2-3\,b^2}{3\,x}$$

input
```
integrate((a+b/x^2)^2*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^2*x^4 + 6*a*b*x^2 - 3*b^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2\,dx=\frac{a^2x^3}{3}+2abx-\frac{b^2}{x}$$

input
```
integrate((a+b/x**2)**2*x**2,x)
```

output
```
a**2*x**3/3 + 2*a*b*x - b**2/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2\,dx=\frac{1}{3}\,a^2x^3+2\,abx-\frac{b^2}{x}$$

input
```
integrate((a+b/x^2)^2*x^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 2*a*b*x - b^2/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2\,dx=\frac{1}{3}\,a^2x^3+2\,abx-\frac{b^2}{x}$$

input
```
integrate((a+b/x^2)^2*x^2,x, algorithm="giac")
```

output
```
1/3*a^2*x^3 + 2*a*b*x - b^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2 \, dx = \frac{a^2 \, x^3}{3} - \frac{b^2}{x} + 2 \, a \, b \, x$$

input
```
int(x^2*(a + b/x^2)^2,x)
```

output
```
(a^2*x^3)/3 - b^2/x + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)^2 x^2 \, dx = \frac{a^2x^4 + 6ab \, x^2 - 3b^2}{3x}$$

input
```
int((a+b/x^2)^2*x^2,x)
```

output
```
(a**2*x**4 + 6*a*b*x**2 - 3*b**2)/(3*x)
```

3.281 $\int \left(a+\frac{b}{x^2}\right)^2 dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \left(a+\frac{b}{x^2}\right)^2 dx=-\frac{b^2}{3x^3}-\frac{2ab}{x}+a^2x$$

output
```
-1/3*b^2/x^3-2*a*b/x+a^2*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^2 dx=-\frac{b^2}{3x^3}-\frac{2ab}{x}+a^2x$$

input
```
Integrate[(a + b/x^2)^2,x]
```

output
```
-1/3*b^2/x^3 - (2*a*b)/x + a^2*x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^2 dx$$
$$\Big\downarrow \text{772}$$
$$\int \frac{\left(ax^2+b\right)^2}{x^4}dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(a^2+\frac{2ab}{x^2}+\frac{b^2}{x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a^2x-\frac{2ab}{x}-\frac{b^2}{3x^3}$$

input
```
Int[(a + b/x^2)^2,x]
```

output
```
-1/3*b^2/x^3 - (2*a*b)/x + a^2*x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{b^2}{3x^3}-\frac{2ab}{x}+a^2x$	22
risch	$a^2x+\frac{-2ab\,x^2-\frac{1}{3}b^2}{x^3}$	24
norman	$\frac{a^2x^4-2ab\,x^2-\frac{1}{3}b^2}{x^3}$	25
gosper	$\frac{3a^2x^4-6ab\,x^2-b^2}{3x^3}$	27
parallelrisch	$\frac{3a^2x^4-6ab\,x^2-b^2}{3x^3}$	27
orering	$\frac{(3a^2x^4-6ab\,x^2-b^2)x\left(a+\frac{b}{x^2}\right)^2}{3(a\,x^2+b)^2}$	43

input
```
int((a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2/x^3-2*a*b/x+a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right)^2\,dx=\frac{3\,a^2x^4-6\,abx^2-b^2}{3\,x^3}$$

input
```
integrate((a+b/x^2)^2,x, algorithm="fricas")
```

output
```
1/3*(3*a^2*x^4 - 6*a*b*x^2 - b^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x^2}\right)^2 dx = a^2x + \frac{-6abx^2 - b^2}{3x^3}$$

input
```
integrate((a+b/x**2)**2,x)
```

output
```
a**2*x + (-6*a*b*x**2 - b**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)^2 dx = a^2x - \frac{2\,ab}{x} - \frac{b^2}{3\,x^3}$$

input
```
integrate((a+b/x^2)^2,x, algorithm="maxima")
```

output
```
a^2*x - 2*a*b/x - 1/3*b^2/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x^2}\right)^2 dx = a^2x - \frac{6\,abx^2 + b^2}{3\,x^3}$$

input
```
integrate((a+b/x^2)^2,x, algorithm="giac")
```

output
```
a^2*x - 1/3*(6*a*b*x^2 + b^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)^2 dx = a^2 x - \frac{\frac{b^2}{3} + 2\,a\,b\,x^2}{x^3}$$

input
```
int((a + b/x^2)^2,x)
```

output
```
a^2*x - (b^2/3 + 2*a*b*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right)^2 dx = \frac{3a^2x^4 - 6ab\,x^2 - b^2}{3x^3}$$

input
```
int((a+b/x^2)^2,x)
```

output
```
(3*a**2*x**4 - 6*a*b*x**2 - b**2)/(3*x**3)
```

3.282 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2} dx = -\frac{b^2}{5x^5} - \frac{2ab}{3x^3} - \frac{a^2}{x}$$

output
```
-1/5*b^2/x^5-2/3*a*b/x^3-a^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2} dx = -\frac{b^2}{5x^5} - \frac{2ab}{3x^3} - \frac{a^2}{x}$$

input
```
Integrate[(a + b/x^2)^2/x^2,x]
```

output
```
-1/5*b^2/x^5 - (2*a*b)/(3*x^3) - a^2/x
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{\left(ax^2+b\right)^2}{x^6}dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^2}{x^2}+\frac{2ab}{x^4}+\frac{b^2}{x^6}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2}{x}-\frac{2ab}{3x^3}-\frac{b^2}{5x^5}$$

input
```
Int[(a + b/x^2)^2/x^2,x]
```

output
```
-1/5*b^2/x^5 - (2*a*b)/(3*x^3) - a^2/x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{b^2}{5x^5}-\frac{2ab}{3x^3}-\frac{a^2}{x}$	25
norman	$\frac{-a^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}b^2}{x^5}$	26
risch	$\frac{-a^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}b^2}{x^5}$	26
gosper	$-\frac{15a^2x^4+10ab\,x^2+3b^2}{15x^5}$	27
parallelrisch	$\frac{-15a^2x^4-10ab\,x^2-3b^2}{15x^5}$	27
orering	$-\frac{(15a^2x^4+10ab\,x^2+3b^2)\left(a+\frac{b}{x^2}\right)^2}{15x(a\,x^2+b)^2}$	45

input
```
int((a+b/x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^2/x^5-2/3*a*b/x^3-a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx = -\frac{15\,a^2x^4+10\,abx^2+3\,b^2}{15\,x^5}$$

input
```
integrate((a+b/x^2)^2/x^2,x, algorithm="fricas")
```

output
```
-1/15*(15*a^2*x^4 + 10*a*b*x^2 + 3*b^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx=\frac{-15a^2x^4-10abx^2-3b^2}{15x^5}$$

input
```
integrate((a+b/x**2)**2/x**2,x)
```

output
```
(-15*a**2*x**4 - 10*a*b*x**2 - 3*b**2)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx=-\frac{15\,a^2x^4+10\,abx^2+3\,b^2}{15\,x^5}$$

input
```
integrate((a+b/x^2)^2/x^2,x, algorithm="maxima")
```

output
```
-1/15*(15*a^2*x^4 + 10*a*b*x^2 + 3*b^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx=-\frac{15\,a^2x^4+10\,abx^2+3\,b^2}{15\,x^5}$$

input
```
integrate((a+b/x^2)^2/x^2,x, algorithm="giac")
```

output
```
-1/15*(15*a^2*x^4 + 10*a*b*x^2 + 3*b^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx = -\frac{a^2\,x^4+\frac{2\,a\,b\,x^2}{3}+\frac{b^2}{5}}{x^5}$$

input
```
int((a + b/x^2)^2/x^2,x)
```

output
```
-(b^2/5 + a^2*x^4 + (2*a*b*x^2)/3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^2}\,dx = \frac{-15a^2x^4-10ab\,x^2-3b^2}{15x^5}$$

input
```
int((a+b/x^2)^2/x^2,x)
```

output
```
( - 15*a**2*x**4 - 10*a*b*x**2 - 3*b**2)/(15*x**5)
```

3.283 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4} dx = -\frac{b^2}{7x^7} - \frac{2ab}{5x^5} - \frac{a^2}{3x^3}$$

output
```
-1/7*b^2/x^7-2/5*a*b/x^5-1/3*a^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4} dx = -\frac{b^2}{7x^7} - \frac{2ab}{5x^5} - \frac{a^2}{3x^3}$$

input
```
Integrate[(a + b/x^2)^2/x^4,x]
```

output
```
-1/7*b^2/x^7 - (2*a*b)/(5*x^5) - a^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax^2+b)^2}{x^8}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^4}+\frac{2ab}{x^6}+\frac{b^2}{x^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{3x^3}-\frac{2ab}{5x^5}-\frac{b^2}{7x^7}$$

input
```
Int[(a + b/x^2)^2/x^4,x]
```

output
```
-1/7*b^2/x^7 - (2*a*b)/(5*x^5) - a^2/(3*x^3)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^2}{7x^7}-\frac{2ab}{5x^5}-\frac{a^2}{3x^3}$	25
norman	$\frac{-\frac{1}{3}a^2x^4-\frac{2}{5}ab\,x^2-\frac{1}{7}b^2}{x^7}$	26
risch	$\frac{-\frac{1}{3}a^2x^4-\frac{2}{5}ab\,x^2-\frac{1}{7}b^2}{x^7}$	26
gosper	$-\frac{35a^2x^4+42ab\,x^2+15b^2}{105x^7}$	27
parallelrisch	$\frac{-35a^2x^4-42ab\,x^2-15b^2}{105x^7}$	27
orering	$-\frac{(35a^2x^4+42ab\,x^2+15b^2)\left(a+\frac{b}{x^2}\right)^2}{105x^3(a\,x^2+b)^2}$	45

input
```
int((a+b/x^2)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b^2/x^7-2/5*a*b/x^5-1/3*a^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx=-\frac{35\,a^2x^4+42\,abx^2+15\,b^2}{105\,x^7}$$

input
```
integrate((a+b/x^2)^2/x^4,x, algorithm="fricas")
```

output
```
-1/105*(35*a^2*x^4 + 42*a*b*x^2 + 15*b^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx=\frac{-35a^2x^4-42abx^2-15b^2}{105x^7}$$

input
```
integrate((a+b/x**2)**2/x**4,x)
```

output
```
(-35*a**2*x**4 - 42*a*b*x**2 - 15*b**2)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx=-\frac{35\,a^2x^4+42\,abx^2+15\,b^2}{105\,x^7}$$

input
```
integrate((a+b/x^2)^2/x^4,x, algorithm="maxima")
```

output
```
-1/105*(35*a^2*x^4 + 42*a*b*x^2 + 15*b^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx=-\frac{35\,a^2x^4+42\,abx^2+15\,b^2}{105\,x^7}$$

input
```
integrate((a+b/x^2)^2/x^4,x, algorithm="giac")
```

output
```
-1/105*(35*a^2*x^4 + 42*a*b*x^2 + 15*b^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx=-\frac{\frac{a^2\,x^4}{3}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2}{7}}{x^7}$$

input
```
int((a + b/x^2)^2/x^4,x)
```

output
```
-(b^2/7 + (a^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^4}\,dx=\frac{-35a^2x^4-42ab\,x^2-15b^2}{105x^7}$$

input
```
int((a+b/x^2)^2/x^4,x)
```

output
```
( - 35*a**2*x**4 - 42*a*b*x**2 - 15*b**2)/(105*x**7)
```

3.284 $\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6} dx = -\frac{b^2}{9x^9} - \frac{2ab}{7x^7} - \frac{a^2}{5x^5}$$

output
```
-1/9*b^2/x^9-2/7*a*b/x^7-1/5*a^2/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6} dx = -\frac{b^2}{9x^9} - \frac{2ab}{7x^7} - \frac{a^2}{5x^5}$$

input
```
Integrate[(a + b/x^2)^2/x^6,x]
```

output
```
-1/9*b^2/x^9 - (2*a*b)/(7*x^7) - a^2/(5*x^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{\left(a x^2+b\right)^2}{x^{10}}dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^2}{x^6}+\frac{2 a b}{x^8}+\frac{b^2}{x^{10}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{5 x^5}-\frac{2 a b}{7 x^7}-\frac{b^2}{9 x^9}$$

input
```
Int[(a + b/x^2)^2/x^6,x]
```

output
```
-1/9*b^2/x^9 - (2*a*b)/(7*x^7) - a^2/(5*x^5)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^2}{9x^9}-\frac{2ab}{7x^7}-\frac{a^2}{5x^5}$	25
norman	$\frac{-\frac{1}{5}a^2x^4-\frac{2}{7}ab\,x^2-\frac{1}{9}b^2}{x^9}$	26
risch	$\frac{-\frac{1}{5}a^2x^4-\frac{2}{7}ab\,x^2-\frac{1}{9}b^2}{x^9}$	26
gosper	$-\frac{63a^2x^4+90ab\,x^2+35b^2}{315x^9}$	27
parallelrisch	$\frac{-63a^2x^4-90ab\,x^2-35b^2}{315x^9}$	27
orering	$-\frac{(63a^2x^4+90ab\,x^2+35b^2)\left(a+\frac{b}{x^2}\right)^2}{315x^5(a\,x^2+b)^2}$	45

input
```
int((a+b/x^2)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/9*b^2/x^9-2/7*a*b/x^7-1/5*a^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx=-\frac{63\,a^2x^4+90\,abx^2+35\,b^2}{315\,x^9}$$

input
```
integrate((a+b/x^2)^2/x^6,x, algorithm="fricas")
```

output
```
-1/315*(63*a^2*x^4 + 90*a*b*x^2 + 35*b^2)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx = \frac{-63a^2x^4-90abx^2-35b^2}{315x^9}$$

input
```
integrate((a+b/x**2)**2/x**6,x)
```

output
```
(-63*a**2*x**4 - 90*a*b*x**2 - 35*b**2)/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx = -\frac{63\,a^2x^4+90\,abx^2+35\,b^2}{315\,x^9}$$

input
```
integrate((a+b/x^2)^2/x^6,x, algorithm="maxima")
```

output
```
-1/315*(63*a^2*x^4 + 90*a*b*x^2 + 35*b^2)/x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx = -\frac{63\,a^2x^4+90\,abx^2+35\,b^2}{315\,x^9}$$

input
```
integrate((a+b/x^2)^2/x^6,x, algorithm="giac")
```

output
```
-1/315*(63*a^2*x^4 + 90*a*b*x^2 + 35*b^2)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx=-\frac{\frac{a^2\,x^4}{5}+\frac{2\,a\,b\,x^2}{7}+\frac{b^2}{9}}{x^9}$$

input
```
int((a + b/x^2)^2/x^6,x)
```

output
```
-(b^2/9 + (a^2*x^4)/5 + (2*a*b*x^2)/7)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)^2}{x^6}\,dx=\frac{-63a^2x^4-90ab\,x^2-35b^2}{315x^9}$$

input
```
int((a+b/x^2)^2/x^6,x)
```

output
```
( - 63*a**2*x**4 - 90*a*b*x**2 - 35*b**2)/(315*x**9)
```

3.285 $\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\, dx = \frac{b^3x^6}{6}+\frac{3}{8}ab^2x^8+\frac{3}{10}a^2bx^{10}+\frac{a^3x^{12}}{12}$$

output
```
1/6*b^3*x^6+3/8*a*b^2*x^8+3/10*a^2*b*x^10+1/12*a^3*x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\, dx = \frac{b^3x^6}{6}+\frac{3}{8}ab^2x^8+\frac{3}{10}a^2bx^{10}+\frac{a^3x^{12}}{12}$$

input
```
Integrate[(a + b/x^2)^3*x^11,x]
```

output
```
(b^3*x^6)/6 + (3*a*b^2*x^8)/8 + (3*a^2*b*x^10)/10 + (a^3*x^12)/12
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+\frac{b}{x^2}\right)^3dx$$

$$\Big\downarrow \text{795}$$

$$\int x^5\left(ax^2+b\right)^3dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4\left(ax^2+b\right)^3dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(a^3x^{10}+3a^2bx^8+3ab^2x^6+b^3x^4\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^3x^{12}}{6}+\frac{3}{5}a^2bx^{10}+\frac{3}{4}ab^2x^8+\frac{b^3x^6}{3}\right)$$

input
```
Int[(a + b/x^2)^3*x^11,x]
```

output
```
((b^3*x^6)/3 + (3*a*b^2*x^8)/4 + (3*a^2*b*x^10)/5 + (a^3*x^12)/6)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{6}b^3x^6+\frac{3}{8}a\,b^2x^8+\frac{3}{10}a^2b\,x^{10}+\frac{1}{12}a^3x^{12}$	36
risch	$\frac{1}{6}b^3x^6+\frac{3}{8}a\,b^2x^8+\frac{3}{10}a^2b\,x^{10}+\frac{1}{12}a^3x^{12}$	36
parallelrisch	$\frac{1}{6}b^3x^6+\frac{3}{8}a\,b^2x^8+\frac{3}{10}a^2b\,x^{10}+\frac{1}{12}a^3x^{12}$	36
gosper	$\frac{x^6(10a^3x^6+36a^2b\,x^4+45a\,b^2x^2+20b^3)}{120}$	38
norman	$\frac{\frac{1}{12}a^3x^{17}+\frac{1}{6}b^3x^{11}+\frac{3}{8}a\,b^2x^{13}+\frac{3}{10}a^2b\,x^{15}}{x^5}$	40
orering	$\frac{x^{12}(10a^3x^6+36a^2b\,x^4+45a\,b^2x^2+20b^3)\left(a+\frac{b}{x^2}\right)^3}{120(a\,x^2+b)^3}$	56

input
```
int((a+b/x^2)^3*x^11,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^3*x^6+3/8*a*b^2*x^8+3/10*a^2*b*x^10+1/12*a^3*x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\,dx=\frac{1}{12}\,a^3x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{6}\,b^3x^6$$

input
```
integrate((a+b/x^2)^3*x^11,x, algorithm="fricas")
```

output
```
1/12*a^3*x^12 + 3/10*a^2*b*x^10 + 3/8*a*b^2*x^8 + 1/6*b^3*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\,dx=\frac{a^3x^{12}}{12}+\frac{3a^2bx^{10}}{10}+\frac{3ab^2x^8}{8}+\frac{b^3x^6}{6}$$

input
```
integrate((a+b/x**2)**3*x**11,x)
```

output
```
a**3*x**12/12 + 3*a**2*b*x**10/10 + 3*a*b**2*x**8/8 + b**3*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\,dx=\frac{1}{12}\,a^3x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{6}\,b^3x^6$$

input
```
integrate((a+b/x^2)^3*x^11,x, algorithm="maxima")
```

output
```
1/12*a^3*x^12 + 3/10*a^2*b*x^10 + 3/8*a*b^2*x^8 + 1/6*b^3*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\,dx = \frac{1}{12}\,a^3x^{12}+\frac{3}{10}\,a^2bx^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{6}\,b^3x^6$$

input
```
integrate((a+b/x^2)^3*x^11,x, algorithm="giac")
```

output
```
1/12*a^3*x^12 + 3/10*a^2*b*x^10 + 3/8*a*b^2*x^8 + 1/6*b^3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\,dx = \frac{a^3\,x^{12}}{12}+\frac{3\,a^2\,b\,x^{10}}{10}+\frac{3\,a\,b^2\,x^8}{8}+\frac{b^3\,x^6}{6}$$

input
```
int(x^11*(a + b/x^2)^3,x)
```

output
```
(a^3*x^12)/12 + (b^3*x^6)/6 + (3*a*b^2*x^8)/8 + (3*a^2*b*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x^2}\right)^3 x^{11}\,dx = \frac{x^6(10a^3x^6+36a^2b\,x^4+45a\,b^2x^2+20b^3)}{120}$$

input
```
int((a+b/x^2)^3*x^11,x)
```

output
```
(x**6*(10*a**3*x**6 + 36*a**2*b*x**4 + 45*a*b**2*x**2 + 20*b**3))/120
```

3.286 $\int \left(a+\frac{b}{x^2}\right)^3 x^9 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9 \, dx = -\frac{b(b+ax^2)^4}{8a^2}+\frac{(b+ax^2)^5}{10a^2}$$

output
```
-1/8*b*(a*x^2+b)^4/a^2+1/10*(a*x^2+b)^5/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9 \, dx = \frac{b^3x^4}{4}+\frac{1}{2}ab^2x^6+\frac{3}{8}a^2bx^8+\frac{a^3x^{10}}{10}$$

input
```
Integrate[(a + b/x^2)^3*x^9,x]
```

output
```
(b^3*x^4)/4 + (a*b^2*x^6)/2 + (3*a^2*b*x^8)/8 + (a^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+\frac{b}{x^2}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^3 \left(a x^2+b\right)^3 dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^2 \left(a x^2+b\right)^3 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(\frac{\left(a x^2+b\right)^4}{a}-\frac{b \left(a x^2+b\right)^3}{a}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{\left(a x^2+b\right)^5}{5 a^2}-\frac{b \left(a x^2+b\right)^4}{4 a^2}\right)$$

input
```
Int[(a + b/x^2)^3*x^9,x]
```

output
```
(-1/4*(b*(b + a*x^2)^4)/a^2 + (b + a*x^2)^5/(5*a^2))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{1}{10}a^3x^{10}+\frac{3}{8}a^2b\,x^8+\frac{1}{2}a\,b^2x^6+\frac{1}{4}b^3x^4$	36
risch	$\frac{1}{10}a^3x^{10}+\frac{3}{8}a^2b\,x^8+\frac{1}{2}a\,b^2x^6+\frac{1}{4}b^3x^4$	36
parallelrisch	$\frac{1}{10}a^3x^{10}+\frac{3}{8}a^2b\,x^8+\frac{1}{2}a\,b^2x^6+\frac{1}{4}b^3x^4$	36
gosper	$\frac{x^4(4a^3x^6+15a^2b\,x^4+20a\,b^2x^2+10b^3)}{40}$	38
norman	$\frac{\frac{1}{10}a^3x^{15}+\frac{1}{4}b^3x^9+\frac{1}{2}a\,b^2x^{11}+\frac{3}{8}a^2b\,x^{13}}{x^5}$	40
orering	$\frac{x^{10}(4a^3x^6+15a^2b\,x^4+20a\,b^2x^2+10b^3)\left(a+\frac{b}{x^2}\right)^3}{40(a\,x^2+b)^3}$	56

input
```
int((a+b/x^2)^3*x^9,x,method=_RETURNVERBOSE)
```

output
```
1/10*a^3*x^10+3/8*a^2*b*x^8+1/2*a*b^2*x^6+1/4*b^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9\,dx=\frac{1}{10}\,a^3x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{2}\,ab^2x^6+\frac{1}{4}\,b^3x^4$$

input
```
integrate((a+b/x^2)^3*x^9,x, algorithm="fricas")
```

output
```
1/10*a^3*x^10 + 3/8*a^2*b*x^8 + 1/2*a*b^2*x^6 + 1/4*b^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9\,dx=\frac{a^3x^{10}}{10}+\frac{3a^2bx^8}{8}+\frac{ab^2x^6}{2}+\frac{b^3x^4}{4}$$

input
```
integrate((a+b/x**2)**3*x**9,x)
```

output
```
a**3*x**10/10 + 3*a**2*b*x**8/8 + a*b**2*x**6/2 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9\,dx=\frac{1}{10}\,a^3x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{2}\,ab^2x^6+\frac{1}{4}\,b^3x^4$$

input
```
integrate((a+b/x^2)^3*x^9,x, algorithm="maxima")
```

output
```
1/10*a^3*x^10 + 3/8*a^2*b*x^8 + 1/2*a*b^2*x^6 + 1/4*b^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9\,dx = \frac{1}{10}\,a^3x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{2}\,ab^2x^6+\frac{1}{4}\,b^3x^4$$

input
```
integrate((a+b/x^2)^3*x^9,x, algorithm="giac")
```

output
```
1/10*a^3*x^10 + 3/8*a^2*b*x^8 + 1/2*a*b^2*x^6 + 1/4*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9\,dx = \frac{a^3\,x^{10}}{10}+\frac{3\,a^2\,b\,x^8}{8}+\frac{a\,b^2\,x^6}{2}+\frac{b^3\,x^4}{4}$$

input
```
int(x^9*(a + b/x^2)^3,x)
```

output
```
(a^3*x^10)/10 + (b^3*x^4)/4 + (a*b^2*x^6)/2 + (3*a^2*b*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{b}{x^2}\right)^3 x^9\,dx = \frac{x^4(4a^3x^6+15a^2b\,x^4+20a\,b^2x^2+10b^3)}{40}$$

input
```
int((a+b/x^2)^3*x^9,x)
```

output
```
(x**4*(4*a**3*x**6 + 15*a**2*b*x**4 + 20*a*b**2*x**2 + 10*b**3))/40
```

3.287 $\int \left(a+\frac{b}{x^2}\right)^3 x^7 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7 \, dx=\frac{(b+ax^2)^4}{8a}$$

output
```
1/8*(a*x^2+b)^4/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7 \, dx=\frac{(b+ax^2)^4}{8a}$$

input
```
Integrate[(a + b/x^2)^3*x^7,x]
```

output
```
(b + a*x^2)^4/(8*a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+\frac{b}{x^2}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int x \left(a x^2+b\right)^3 dx$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(a x^2+b\right)^4}{8 a}$$

input
```
Int[(a + b/x^2)^3*x^7,x]
```

output
```
(b + a*x^2)^4/(8*a)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(a x^2+b)^4}{8a}$	15
parallelrisch	$\frac{1}{8}a^3x^8+\frac{1}{2}a^2b x^6+\frac{3}{4}a b^2x^4+\frac{1}{2}b^3x^2$	36
gosper	$\frac{x^2(a^3x^6+4a^2b x^4+6a b^2x^2+4b^3)}{8}$	37
norman	$\frac{\frac{1}{8}a^3x^{13}+\frac{1}{2}b^3x^7+\frac{3}{4}a b^2x^9+\frac{1}{2}a^2b x^{11}}{x^5}$	40
risch	$\frac{a^3x^8}{8}+\frac{a^2b x^6}{2}+\frac{3a b^2x^4}{4}+\frac{b^3x^2}{2}+\frac{b^4}{8a}$	44
orering	$\frac{x^8(a^3x^6+4a^2b x^4+6a b^2x^2+4b^3)\left(a+\frac{b}{x^2}\right)^3}{8(a x^2+b)^3}$	55

input
```
int((a+b/x^2)^3*x^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*(a*x^2+b)^4/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7\,dx=\frac{1}{8}\,a^3x^8+\frac{1}{2}\,a^2bx^6+\frac{3}{4}\,ab^2x^4+\frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x^2)^3*x^7,x, algorithm="fricas")
```

output
```
1/8*a^3*x^8 + 1/2*a^2*b*x^6 + 3/4*a*b^2*x^4 + 1/2*b^3*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7\,dx = \frac{a^3x^8}{8}+\frac{a^2bx^6}{2}+\frac{3ab^2x^4}{4}+\frac{b^3x^2}{2}$$

input
```
integrate((a+b/x**2)**3*x**7,x)
```

output
```
a**3*x**8/8 + a**2*b*x**6/2 + 3*a*b**2*x**4/4 + b**3*x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7\,dx = \frac{1}{8}\,a^3x^8+\frac{1}{2}\,a^2bx^6+\frac{3}{4}\,ab^2x^4+\frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x^2)^3*x^7,x, algorithm="maxima")
```

output
```
1/8*a^3*x^8 + 1/2*a^2*b*x^6 + 3/4*a*b^2*x^4 + 1/2*b^3*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7\,dx = \frac{1}{8}\,a^3x^8+\frac{1}{2}\,a^2bx^6+\frac{3}{4}\,ab^2x^4+\frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x^2)^3*x^7,x, algorithm="giac")
```

output
```
1/8*a^3*x^8 + 1/2*a^2*b*x^6 + 3/4*a*b^2*x^4 + 1/2*b^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7\,dx=\frac{a^3\,x^8}{8}+\frac{a^2\,b\,x^6}{2}+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^2}{2}$$

input
```
int(x^7*(a + b/x^2)^3,x)
```

output
```
(a^3*x^8)/8 + (b^3*x^2)/2 + (3*a*b^2*x^4)/4 + (a^2*b*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \left(a+\frac{b}{x^2}\right)^3 x^7\,dx=\frac{x^2(a^3x^6+4a^2b\,x^4+6a\,b^2x^2+4b^3)}{8}$$

input
```
int((a+b/x^2)^3*x^7,x)
```

output
```
(x**2*(a**3*x**6 + 4*a**2*b*x**4 + 6*a*b**2*x**2 + 4*b**3))/8
```

3.288 $\int \left(a+\frac{b}{x^2}\right)^3 x^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5 \, dx=\frac{3}{2}ab^2x^2+\frac{3}{4}a^2bx^4+\frac{a^3x^6}{6}+b^3\log(x)$$

output
```
3/2*a*b^2*x^2+3/4*a^2*b*x^4+1/6*a^3*x^6+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5 \, dx=\frac{3}{2}ab^2x^2+\frac{3}{4}a^2bx^4+\frac{a^3x^6}{6}+b^3\log(x)$$

input
```
Integrate[(a + b/x^2)^3*x^5,x]
```

output
```
(3*a*b^2*x^2)/2 + (3*a^2*b*x^4)/4 + (a^3*x^6)/6 + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{x^2}\right)^3 \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a x^2+b\right)^3}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{\left(a x^2+b\right)^3}{x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(a^3 x^4+3 a^2 b x^2+3 a b^2+\frac{b^3}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a^3 x^6}{3}+\frac{3}{2} a^2 b x^4+3 a b^2 x^2+b^3 \log \left(x^2\right)\right)$$

input
```
Int[(a + b/x^2)^3*x^5,x]
```

output
```
(3*a*b^2*x^2 + (3*a^2*b*x^4)/2 + (a^3*x^6)/3 + b^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{3a\,b^{2}x^{2}}{2}+\frac{3a^{2}b\,x^{4}}{4}+\frac{a^{3}x^{6}}{6}+b^{3}\ln\left(x\right)$	34
risch	$\frac{3a\,b^{2}x^{2}}{2}+\frac{3a^{2}b\,x^{4}}{4}+\frac{a^{3}x^{6}}{6}+b^{3}\ln\left(x\right)$	34
parallelrisch	$\frac{3a\,b^{2}x^{2}}{2}+\frac{3a^{2}b\,x^{4}}{4}+\frac{a^{3}x^{6}}{6}+b^{3}\ln\left(x\right)$	34
norman	$\frac{\frac{1}{6}a^{3}x^{11}+\frac{3}{2}a\,b^{2}x^{7}+\frac{3}{4}a^{2}b\,x^{9}}{x^{5}}+b^{3}\ln\left(x\right)$	39

input
```
int((a+b/x^2)^3*x^5,x,method=_RETURNVERBOSE)
```

output
```
3/2*a*b^2*x^2+3/4*a^2*b*x^4+1/6*a^3*x^6+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5\,dx=\frac{1}{6}\,a^3x^6+\frac{3}{4}\,a^2bx^4+\frac{3}{2}\,ab^2x^2+b^3\log\left(x\right)$$

input
```
integrate((a+b/x^2)^3*x^5,x, algorithm="fricas")
```

output
```
1/6*a^3*x^6 + 3/4*a^2*b*x^4 + 3/2*a*b^2*x^2 + b^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5\,dx=\frac{a^3x^6}{6}+\frac{3a^2bx^4}{4}+\frac{3ab^2x^2}{2}+b^3\log\left(x\right)$$

input
```
integrate((a+b/x**2)**3*x**5,x)
```

output
```
a**3*x**6/6 + 3*a**2*b*x**4/4 + 3*a*b**2*x**2/2 + b**3*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5\,dx=\frac{1}{6}\,a^3x^6+\frac{3}{4}\,a^2bx^4+\frac{3}{2}\,ab^2x^2+\frac{1}{2}\,b^3\log\left(x^2\right)$$

input
```
integrate((a+b/x^2)^3*x^5,x, algorithm="maxima")
```

output
```
1/6*a^3*x^6 + 3/4*a^2*b*x^4 + 3/2*a*b^2*x^2 + 1/2*b^3*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5\,dx = \frac{1}{6}\,a^3x^6+\frac{3}{4}\,a^2bx^4+\frac{3}{2}\,ab^2x^2+\frac{1}{2}\,b^3\log\left(x^2\right)$$

input
```
integrate((a+b/x^2)^3*x^5,x, algorithm="giac")
```

output
```
1/6*a^3*x^6 + 3/4*a^2*b*x^4 + 3/2*a*b^2*x^2 + 1/2*b^3*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5\,dx = b^3\,\ln\left(x\right)+\frac{a^3\,x^6}{6}+\frac{3\,a\,b^2\,x^2}{2}+\frac{3\,a^2\,b\,x^4}{4}$$

input
```
int(x^5*(a + b/x^2)^3,x)
```

output
```
b^3*log(x) + (a^3*x^6)/6 + (3*a*b^2*x^2)/2 + (3*a^2*b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)^3 x^5\,dx = \log(x)\,b^3+\frac{a^3x^6}{6}+\frac{3a^2b\,x^4}{4}+\frac{3a\,b^2x^2}{2}$$

input
```
int((a+b/x^2)^3*x^5,x)
```

output
```
(12*log(x)*b**3 + 2*a**3*x**6 + 9*a**2*b*x**4 + 18*a*b**2*x**2)/12
```

3.289 $\int \left(a+\frac{b}{x^2}\right)^3 x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3 \, dx = -\frac{b^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{a^3x^4}{4}+3ab^2\log(x)$$

output
```
-1/2*b^3/x^2+3/2*a^2*b*x^2+1/4*a^3*x^4+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3 \, dx = -\frac{b^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{a^3x^4}{4}+3ab^2\log(x)$$

input
```
Integrate[(a + b/x^2)^3*x^3,x]
```

output
```
-1/2*b^3/x^2 + (3*a^2*b*x^2)/2 + (a^3*x^4)/4 + 3*a*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right)^3 dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{\left(a x^2+b\right)^3}{x^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{\left(a x^2+b\right)^3}{x^4}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(x^2 a^3+3 b a^2+\frac{3 b^2 a}{x^2}+\frac{b^3}{x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{a^3 x^4}{2}+3 a^2 b x^2+3 a b^2 \log \left(x^2\right)-\frac{b^3}{x^2}\right)$$

input
```
Int[(a + b/x^2)^3*x^3,x]
```

output
```
(-(b^3/x^2) + 3*a^2*b*x^2 + (a^3*x^4)/2 + 3*a*b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b^{3}}{2 x^{2}}+\frac{3 a^{2} b \,x^{2}}{2}+\frac{a^{3} x^{4}}{4}+3 a \,b^{2} \ln \left(x \right)$	35
parallelrisch	$\frac{a^{3} x^{6}+6 a^{2} b \,x^{4}+12 a \,b^{2} \ln \left(x \right) x^{2}-2 b^{3}}{4 x^{2}}$	39
norman	$\frac{\frac{1}{4} a^{3} x^{9}-\frac{1}{2} b^{3} x^{3}+\frac{3}{2} a^{2} b \,x^{7}}{x^{5}}+3 a \,b^{2} \ln \left(x \right)$	40
risch	$\frac{a^{3} x^{4}}{4}+\frac{3 a^{2} b \,x^{2}}{2}+\frac{9 a \,b^{2}}{4}-\frac{b^{3}}{2 x^{2}}+3 a \,b^{2} \ln \left(x \right)$	41

input
```
int((a+b/x^2)^3*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/x^2+3/2*a^2*b*x^2+1/4*a^3*x^4+3*a*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3\,dx=\frac{a^3x^6+6\,a^2bx^4+12\,ab^2x^2\log\left(x\right)-2\,b^3}{4\,x^2}$$

input
```
integrate((a+b/x^2)^3*x^3,x, algorithm="fricas")
```

output
```
1/4*(a^3*x^6 + 6*a^2*b*x^4 + 12*a*b^2*x^2*log(x) - 2*b^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3\,dx=\frac{a^3x^4}{4}+\frac{3a^2bx^2}{2}+3ab^2\log\left(x\right)-\frac{b^3}{2x^2}$$

input
```
integrate((a+b/x**2)**3*x**3,x)
```

output
```
a**3*x**4/4 + 3*a**2*b*x**2/2 + 3*a*b**2*log(x) - b**3/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3\,dx=\frac{1}{4}\,a^3x^4+\frac{3}{2}\,a^2bx^2+\frac{3}{2}\,ab^2\log\left(x^2\right)-\frac{b^3}{2\,x^2}$$

input
```
integrate((a+b/x^2)^3*x^3,x, algorithm="maxima")
```

output
```
1/4*a^3*x^4 + 3/2*a^2*b*x^2 + 3/2*a*b^2*log(x^2) - 1/2*b^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3\,dx = \frac{1}{4}\,a^3x^4+\frac{3}{2}\,a^2bx^2+3\,ab^2\log\left(|x|\right)-\frac{b^3}{2\,x^2}$$

input
```
integrate((a+b/x^2)^3*x^3,x, algorithm="giac")
```

output
```
1/4*a^3*x^4 + 3/2*a^2*b*x^2 + 3*a*b^2*log(abs(x)) - 1/2*b^3/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3\,dx = \frac{a^3\,x^4}{4}-\frac{b^3}{2\,x^2}+\frac{3\,a^2\,b\,x^2}{2}+3\,a\,b^2\,\ln\left(x\right)$$

input
```
int(x^3*(a + b/x^2)^3,x)
```

output
```
(a^3*x^4)/4 - b^3/(2*x^2) + (3*a^2*b*x^2)/2 + 3*a*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x^2}\right)^3 x^3\,dx = \frac{12\log(x)\,a\,b^2x^2+a^3x^6+6a^2b\,x^4-2b^3}{4x^2}$$

input
```
int((a+b/x^2)^3*x^3,x)
```

output
```
(12*log(x)*a*b**2*x**2 + a**3*x**6 + 6*a**2*b*x**4 - 2*b**3)/(4*x**2)
```

3.290 $\int \left(a+\frac{b}{x^2}\right)^3 x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=-\frac{b^3}{4x^4}-\frac{3ab^2}{2x^2}+\frac{a^3x^2}{2}+3a^2b\log(x)$$

output
```
-1/4*b^3/x^4-3/2*a*b^2/x^2+1/2*a^3*x^2+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=-\frac{b^3}{4x^4}-\frac{3ab^2}{2x^2}+\frac{a^3x^2}{2}+3a^2b\log(x)$$

input
```
Integrate[(a + b/x^2)^3*x,x]
```

output
```
-1/4*b^3/x^4 - (3*a*b^2)/(2*x^2) + (a^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)^3dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(ax^2+b\right)^3}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(ax^2+b\right)^3}{x^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(a^3+\frac{3ba^2}{x^2}+\frac{3b^2a}{x^4}+\frac{b^3}{x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^3x^2+3a^2b\log\left(x^2\right)-\frac{3ab^2}{x^2}-\frac{b^3}{2x^4}\right)$$

input
```
Int[(a + b/x^2)^3*x,x]
```

output
```
(-1/2*b^3/x^4 - (3*a*b^2)/x^2 + a^3*x^2 + 3*a^2*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b^{3}}{4x^{4}}-\frac{3a\,b^{2}}{2x^{2}}+\frac{a^{3}x^{2}}{2}+3a^{2}b\ln\left(x\right)$	35
risch	$\frac{a^{3}x^{2}}{2}+\frac{-\frac{3}{2}a\,b^{2}x^{2}-\frac{1}{4}b^{3}}{x^{4}}+3a^{2}b\ln\left(x\right)$	37
norman	$\frac{\frac{1}{2}a^{3}x^{7}-\frac{1}{4}b^{3}x-\frac{3}{2}a\,b^{2}x^{3}}{x^{5}}+3a^{2}b\ln\left(x\right)$	38
parallelrisch	$\frac{2a^{3}x^{6}+12a^{2}b\ln\left(x\right)x^{4}-6a\,b^{2}x^{2}-b^{3}}{4x^{4}}$	40

input
```
int((a+b/x^2)^3*x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3/x^4-3/2*a*b^2/x^2+1/2*a^3*x^2+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=\frac{2\,a^3x^6+12\,a^2bx^4\log\left(x\right)-6\,ab^2x^2-b^3}{4\,x^4}$$

input
```
integrate((a+b/x^2)^3*x,x, algorithm="fricas")
```

output
```
1/4*(2*a^3*x^6 + 12*a^2*b*x^4*log(x) - 6*a*b^2*x^2 - b^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=\frac{a^3x^2}{2}+3a^2b\log\left(x\right)+\frac{-6ab^2x^2-b^3}{4x^4}$$

input
```
integrate((a+b/x**2)**3*x,x)
```

output
```
a**3*x**2/2 + 3*a**2*b*log(x) + (-6*a*b**2*x**2 - b**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=\frac{1}{2}\,a^3x^2+\frac{3}{2}\,a^2b\log\left(x^2\right)-\frac{6\,ab^2x^2+b^3}{4\,x^4}$$

input
```
integrate((a+b/x^2)^3*x,x, algorithm="maxima")
```

output
```
1/2*a^3*x^2 + 3/2*a^2*b*log(x^2) - 1/4*(6*a*b^2*x^2 + b^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=\frac{1}{2}\,a^3x^2+3\,a^2b\log\left(|x|\right)-\frac{6\,ab^2x^2+b^3}{4\,x^4}$$

input
```
integrate((a+b/x^2)^3*x,x, algorithm="giac")
```

output
```
1/2*a^3*x^2 + 3*a^2*b*log(abs(x)) - 1/4*(6*a*b^2*x^2 + b^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=\frac{a^3\,x^2}{2}-\frac{\frac{b^3}{4}+\frac{3\,a\,b^2\,x^2}{2}}{x^4}+3\,a^2\,b\,\ln\left(x\right)$$

input
```
int(x*(a + b/x^2)^3,x)
```

output
```
(a^3*x^2)/2 - (b^3/4 + (3*a*b^2*x^2)/2)/x^4 + 3*a^2*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x^2}\right)^3 x\,dx=\frac{12\log(x)\,a^2b\,x^4+2a^3x^6-6a\,b^2x^2-b^3}{4x^4}$$

input
```
int((a+b/x^2)^3*x,x)
```

output
```
(12*log(x)*a**2*b*x**4 + 2*a**3*x**6 - 6*a*b**2*x**2 - b**3)/(4*x**4)
```

3.291 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x} dx = -\frac{b^3}{6x^6} - \frac{3ab^2}{4x^4} - \frac{3a^2b}{2x^2} + a^3 \log(x)$$

output
```
-1/6*b^3/x^6-3/4*a*b^2/x^4-3/2*a^2*b/x^2+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x} dx = -\frac{b^3}{6x^6} - \frac{3ab^2}{4x^4} - \frac{3a^2b}{2x^2} + a^3 \log(x)$$

input
```
Integrate[(a + b/x^2)^3/x,x]
```

output
```
-1/6*b^3/x^6 - (3*a*b^2)/(4*x^4) - (3*a^2*b)/(2*x^2) + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{\left(a x^2+b\right)^3}{x^7}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{\left(a x^2+b\right)^3}{x^8}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(\frac{a^3}{x^2}+\frac{3 b a^2}{x^4}+\frac{3 b^2 a}{x^6}+\frac{b^3}{x^8}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(a^3 \log \left(x^2\right)-\frac{3 a^2 b}{x^2}-\frac{3 a b^2}{2 x^4}-\frac{b^3}{3 x^6}\right)$$

input
```
Int[(a + b/x^2)^3/x,x]
```

output
```
(-1/3*b^3/x^6 - (3*a*b^2)/(2*x^4) - (3*a^2*b)/x^2 + a^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{b^{3}}{6x^{6}}-\frac{3a\,b^{2}}{4x^{4}}-\frac{3a^{2}b}{2x^{2}}+a^{3}\ln\left(x\right)$	34
norman	$\frac{-\frac{1}{6}b^{3}-\frac{3}{4}a\,b^{2}x^{2}-\frac{3}{2}a^{2}b\,x^{4}}{x^{6}}+a^{3}\ln\left(x\right)$	36
risch	$\frac{-\frac{1}{6}b^{3}-\frac{3}{4}a\,b^{2}x^{2}-\frac{3}{2}a^{2}b\,x^{4}}{x^{6}}+a^{3}\ln\left(x\right)$	36
parallelrisch	$\frac{12a^{3}\ln\left(x\right)x^{6}-18a^{2}b\,x^{4}-9a\,b^{2}x^{2}-2b^{3}}{12x^{6}}$	40

input
```
int((a+b/x^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/6*b^3/x^6-3/4*a*b^2/x^4-3/2*a^2*b/x^2+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx=\frac{12\,a^3x^6\log\left(x\right)-18\,a^2bx^4-9\,ab^2x^2-2\,b^3}{12\,x^6}$$

input
```
integrate((a+b/x^2)^3/x,x, algorithm="fricas")
```

output
```
1/12*(12*a^3*x^6*log(x) - 18*a^2*b*x^4 - 9*a*b^2*x^2 - 2*b^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx=a^3\log\left(x\right)+\frac{-18a^2bx^4-9ab^2x^2-2b^3}{12x^6}$$

input
```
integrate((a+b/x**2)**3/x,x)
```

output
```
a**3*log(x) + (-18*a**2*b*x**4 - 9*a*b**2*x**2 - 2*b**3)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx=\frac{1}{2}\,a^3\log\left(x^2\right)-\frac{18\,a^2bx^4+9\,ab^2x^2+2\,b^3}{12\,x^6}$$

input
```
integrate((a+b/x^2)^3/x,x, algorithm="maxima")
```

output
```
1/2*a^3*log(x^2) - 1/12*(18*a^2*b*x^4 + 9*a*b^2*x^2 + 2*b^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx=\frac{1}{2}\,a^3\log\left(x^2\right)-\frac{11\,a^3x^6+18\,a^2bx^4+9\,ab^2x^2+2\,b^3}{12\,x^6}$$

input
```
integrate((a+b/x^2)^3/x,x, algorithm="giac")
```

output
```
1/2*a^3*log(x^2) - 1/12*(11*a^3*x^6 + 18*a^2*b*x^4 + 9*a*b^2*x^2 + 2*b^3)/
x^6
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx=a^3\,\ln\left(x\right)-\frac{\frac{3\,a^2\,b\,x^4}{2}+\frac{3\,a\,b^2\,x^2}{4}+\frac{b^3}{6}}{x^6}$$

input
```
int((a + b/x^2)^3/x,x)
```

output
```
a^3*log(x) - (b^3/6 + (3*a*b^2*x^2)/4 + (3*a^2*b*x^4)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x}\,dx=\frac{12\log\left(x\right)a^3x^6-18a^2b\,x^4-9a\,b^2x^2-2b^3}{12x^6}$$

input
```
int((a+b/x^2)^3/x,x)
```

output
```
(12*log(x)*a**3*x**6 - 18*a**2*b*x**4 - 9*a*b**2*x**2 - 2*b**3)/(12*x**6)
```

3.292 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3} dx = -\frac{\left(a+\frac{b}{x^2}\right)^4}{8b}$$

output
```
-1/8*(a+b/x^2)^4/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3} dx = -\frac{b^3}{8x^8} - \frac{ab^2}{2x^6} - \frac{3a^2b}{4x^4} - \frac{a^3}{2x^2}$$

input
```
Integrate[(a + b/x^2)^3/x^3,x]
```

output
```
-1/8*b^3/x^8 - (a*b^2)/(2*x^6) - (3*a^2*b)/(4*x^4) - a^3/(2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3} \, dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^4}{8 b}$$

input
```
Int[(a + b/x^2)^3/x^3,x]
```

output
```
-1/8*(a + b/x^2)^4/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

method	result	size
gosper	$-\frac{4a^3x^6+6a^2b\,x^4+4a\,b^2x^2+b^3}{8x^8}$	36
default	$-\frac{a^3}{2x^2}-\frac{3a^2b}{4x^4}-\frac{b^3}{8x^8}-\frac{a\,b^2}{2x^6}$	36
norman	$\frac{-\frac{1}{2}a^3x^6-\frac{3}{4}a^2b\,x^4-\frac{1}{2}a\,b^2x^2-\frac{1}{8}b^3}{x^8}$	37
risch	$\frac{-\frac{1}{2}a^3x^6-\frac{3}{4}a^2b\,x^4-\frac{1}{2}a\,b^2x^2-\frac{1}{8}b^3}{x^8}$	37
parallelrisch	$\frac{-4a^3x^6-6a^2b\,x^4-4a\,b^2x^2-b^3}{8x^8}$	38
orering	$-\frac{(4a^3x^6+6a^2b\,x^4+4a\,b^2x^2+b^3)\left(a+\frac{b}{x^2}\right)^3}{8x^2(a\,x^2+b)^3}$	54

input
```
int((a+b/x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*a^3*x^6+6*a^2*b*x^4+4*a*b^2*x^2+b^3)/x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx = -\frac{4\,a^3x^6+6\,a^2bx^4+4\,ab^2x^2+b^3}{8\,x^8}$$

input
```
integrate((a+b/x^2)^3/x^3,x, algorithm="fricas")
```

output
```
-1/8*(4*a^3*x^6 + 6*a^2*b*x^4 + 4*a*b^2*x^2 + b^3)/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx = \frac{-4a^3x^6-6a^2bx^4-4ab^2x^2-b^3}{8x^8}$$

input
```
integrate((a+b/x**2)**3/x**3,x)
```

output
```
(-4*a**3*x**6 - 6*a**2*b*x**4 - 4*a*b**2*x**2 - b**3)/(8*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^4}{8\,b}$$

input
```
integrate((a+b/x^2)^3/x^3,x, algorithm="maxima")
```

output
```
-1/8*(a + b/x^2)^4/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx = -\frac{4\,a^3x^6+6\,a^2bx^4+4\,ab^2x^2+b^3}{8\,x^8}$$

input
```
integrate((a+b/x^2)^3/x^3,x, algorithm="giac")
```

output
```
-1/8*(4*a^3*x^6 + 6*a^2*b*x^4 + 4*a*b^2*x^2 + b^3)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx = -\frac{\frac{a^3\,x^6}{2}+\frac{3\,a^2\,b\,x^4}{4}+\frac{a\,b^2\,x^2}{2}+\frac{b^3}{8}}{x^8}$$

input
```
int((a + b/x^2)^3/x^3,x)
```

output
```
-(b^3/8 + (a^3*x^6)/2 + (a*b^2*x^2)/2 + (3*a^2*b*x^4)/4)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^3}\,dx = \frac{-4a^3x^6-6a^2b\,x^4-4a\,b^2x^2-b^3}{8x^8}$$

input
```
int((a+b/x^2)^3/x^3,x)
```

output
```
( - 4*a**3*x**6 - 6*a**2*b*x**4 - 4*a*b**2*x**2 - b**3)/(8*x**8)
```

3.293 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5} dx = \frac{a\left(a+\frac{b}{x^2}\right)^4}{8b^2} - \frac{\left(a+\frac{b}{x^2}\right)^5}{10b^2}$$

output
```
1/8*a*(a+b/x^2)^4/b^2-1/10*(a+b/x^2)^5/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5} dx = -\frac{b^3}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{a^2b}{2x^6} - \frac{a^3}{4x^4}$$

input
```
Integrate[(a + b/x^2)^3/x^5,x]
```

output
```
-1/10*b^3/x^10 - (3*a*b^2)/(8*x^8) - (a^2*b)/(2*x^6) - a^3/(4*x^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5} \, dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{\left(a x^2+b\right)^3}{x^{11}}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2} \int \frac{\left(a x^2+b\right)^3}{x^{12}}dx^2$$

$$\Big\downarrow \; 55$$

$$\frac{1}{2} \left(-\frac{a \int \frac{\left(a x^2+b\right)^3}{x^{10}}dx^2}{5 b}-\frac{\left(a x^2+b\right)^4}{5 b x^{10}}\right)$$

$$\Big\downarrow \; 48$$

$$\frac{1}{2} \left(\frac{a \left(a x^2+b\right)^4}{20 b^2 x^8}-\frac{\left(a x^2+b\right)^4}{5 b x^{10}}\right)$$

input
```
Int[(a + b/x^2)^3/x^5,x]
```

output
```
(-1/5*(b + a*x^2)^4/(b*x^10) + (a*(b + a*x^2)^4)/(20*b^2*x^8))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{a^{3}}{4x^{4}}-\frac{3a\,b^{2}}{8x^{8}}-\frac{b^{3}}{10x^{10}}-\frac{a^{2}b}{2x^{6}}$	36
norman	$\frac{-\frac{1}{4}a^{3}x^{6}-\frac{1}{2}a^{2}b\,x^{4}-\frac{3}{8}a\,b^{2}x^{2}-\frac{1}{10}b^{3}}{x^{10}}$	37
risch	$\frac{-\frac{1}{4}a^{3}x^{6}-\frac{1}{2}a^{2}b\,x^{4}-\frac{3}{8}a\,b^{2}x^{2}-\frac{1}{10}b^{3}}{x^{10}}$	37
gosper	$-\frac{10a^{3}x^{6}+20a^{2}b\,x^{4}+15a\,b^{2}x^{2}+4b^{3}}{40x^{10}}$	38
parallelrisch	$\frac{-10a^{3}x^{6}-20a^{2}b\,x^{4}-15a\,b^{2}x^{2}-4b^{3}}{40x^{10}}$	38
orering	$-\frac{\left(10a^{3}x^{6}+20a^{2}b\,x^{4}+15a\,b^{2}x^{2}+4b^{3}\right)\left(a+\frac{b}{x^{2}}\right)^{3}}{40x^{4}\left(a\,x^{2}+b\right)^{3}}$	56

input
```
int((a+b/x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4-3/8*a*b^2/x^8-1/10*b^3/x^10-1/2*a^2*b/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5} \, dx = -\frac{10 \, a^3 x^6 + 20 \, a^2 b x^4 + 15 \, a b^2 x^2 + 4 \, b^3}{40 \, x^{10}}$$

input
```
integrate((a+b/x^2)^3/x^5,x, algorithm="fricas")
```

output
```
-1/40*(10*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2 + 4*b^3)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5} \, dx = \frac{-10a^3x^6 - 20a^2bx^4 - 15ab^2x^2 - 4b^3}{40x^{10}}$$

input
```
integrate((a+b/x**2)**3/x**5,x)
```

output
```
(-10*a**3*x**6 - 20*a**2*b*x**4 - 15*a*b**2*x**2 - 4*b**3)/(40*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5}\,dx = -\frac{10\,a^3x^6+20\,a^2bx^4+15\,ab^2x^2+4\,b^3}{40\,x^{10}}$$

input
```
integrate((a+b/x^2)^3/x^5,x, algorithm="maxima")
```

output
```
-1/40*(10*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2 + 4*b^3)/x^10
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5}\,dx = -\frac{10\,a^3x^6+20\,a^2bx^4+15\,ab^2x^2+4\,b^3}{40\,x^{10}}$$

input
```
integrate((a+b/x^2)^3/x^5,x, algorithm="giac")
```

output
```
-1/40*(10*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2 + 4*b^3)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5}\,dx = -\frac{\frac{a^3\,x^6}{4}+\frac{a^2\,b\,x^4}{2}+\frac{3\,a\,b^2\,x^2}{8}+\frac{b^3}{10}}{x^{10}}$$

input
```
int((a + b/x^2)^3/x^5,x)
```

output
```
-(b^3/10 + (a^3*x^6)/4 + (3*a*b^2*x^2)/8 + (a^2*b*x^4)/2)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^5}\,dx = \frac{-10a^3x^6-20a^2b\,x^4-15a\,b^2x^2-4b^3}{40x^{10}}$$

input
```
int((a+b/x^2)^3/x^5,x)
```

output
```
( - 10*a**3*x**6 - 20*a**2*b*x**4 - 15*a*b**2*x**2 - 4*b**3)/(40*x**10)
```

3.294 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx=-\frac{b^3}{12x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{3a^2b}{8x^8}-\frac{a^3}{6x^6}$$

output
```
-1/12*b^3/x^12-3/10*a*b^2/x^10-3/8*a^2*b/x^8-1/6*a^3/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx=-\frac{b^3}{12x^{12}}-\frac{3ab^2}{10x^{10}}-\frac{3a^2b}{8x^8}-\frac{a^3}{6x^6}$$

input
```
Integrate[(a + b/x^2)^3/x^7,x]
```

output
```
-1/12*b^3/x^12 - (3*a*b^2)/(10*x^10) - (3*a^2*b)/(8*x^8) - a^3/(6*x^6)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(ax^2+b\right)^3}{x^{13}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(ax^2+b\right)^3}{x^{14}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^3}{x^8}+\frac{3ba^2}{x^{10}}+\frac{3b^2a}{x^{12}}+\frac{b^3}{x^{14}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{3x^6}-\frac{3a^2b}{4x^8}-\frac{3ab^2}{5x^{10}}-\frac{b^3}{6x^{12}}\right)$$

input
```
Int[(a + b/x^2)^3/x^7,x]
```

output
```
(-1/6*b^3/x^12 - (3*a*b^2)/(5*x^10) - (3*a^2*b)/(4*x^8) - a^3/(3*x^6))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{b^3}{12x^{12}}-\frac{3a\,b^2}{10x^{10}}-\frac{3a^2b}{8x^8}-\frac{a^3}{6x^6}$	36
norman	$\frac{-\frac{1}{6}a^3x^6-\frac{3}{8}a^2b\,x^4-\frac{3}{10}a\,b^2x^2-\frac{1}{12}b^3}{x^{12}}$	37
risch	$\frac{-\frac{1}{6}a^3x^6-\frac{3}{8}a^2b\,x^4-\frac{3}{10}a\,b^2x^2-\frac{1}{12}b^3}{x^{12}}$	37
gosper	$-\frac{20a^3x^6+45a^2b\,x^4+36a\,b^2x^2+10b^3}{120x^{12}}$	38
parallelrisch	$\frac{-20a^3x^6-45a^2b\,x^4-36a\,b^2x^2-10b^3}{120x^{12}}$	38
orering	$-\frac{\left(20a^3x^6+45a^2b\,x^4+36a\,b^2x^2+10b^3\right)\left(a+\frac{b}{x^2}\right)^3}{120x^6\left(a\,x^2+b\right)^3}$	56

input
```
int((a+b/x^2)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12*b^3/x^12-3/10*a*b^2/x^10-3/8*a^2*b/x^8-1/6*a^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx = -\frac{20\,a^3x^6+45\,a^2bx^4+36\,ab^2x^2+10\,b^3}{120\,x^{12}}$$

input
```
integrate((a+b/x^2)^3/x^7,x, algorithm="fricas")
```

output
```
-1/120*(20*a^3*x^6 + 45*a^2*b*x^4 + 36*a*b^2*x^2 + 10*b^3)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx = \frac{-20a^3x^6-45a^2bx^4-36ab^2x^2-10b^3}{120x^{12}}$$

input
```
integrate((a+b/x**2)**3/x**7,x)
```

output
```
(-20*a**3*x**6 - 45*a**2*b*x**4 - 36*a*b**2*x**2 - 10*b**3)/(120*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx = -\frac{20\,a^3x^6+45\,a^2bx^4+36\,ab^2x^2+10\,b^3}{120\,x^{12}}$$

input
```
integrate((a+b/x^2)^3/x^7,x, algorithm="maxima")
```

output
```
-1/120*(20*a^3*x^6 + 45*a^2*b*x^4 + 36*a*b^2*x^2 + 10*b^3)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx = -\frac{20\,a^3x^6+45\,a^2bx^4+36\,ab^2x^2+10\,b^3}{120\,x^{12}}$$

input
```
integrate((a+b/x^2)^3/x^7,x, algorithm="giac")
```

output
```
-1/120*(20*a^3*x^6 + 45*a^2*b*x^4 + 36*a*b^2*x^2 + 10*b^3)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx = -\frac{\frac{a^3\,x^6}{6}+\frac{3\,a^2\,b\,x^4}{8}+\frac{3\,a\,b^2\,x^2}{10}+\frac{b^3}{12}}{x^{12}}$$

input
```
int((a + b/x^2)^3/x^7,x)
```

output
```
-(b^3/12 + (a^3*x^6)/6 + (3*a*b^2*x^2)/10 + (3*a^2*b*x^4)/8)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^7}\,dx = \frac{-20a^3x^6-45a^2b\,x^4-36a\,b^2x^2-10b^3}{120x^{12}}$$

input
```
int((a+b/x^2)^3/x^7,x)
```

output
```
( - 20*a**3*x**6 - 45*a**2*b*x**4 - 36*a*b**2*x**2 - 10*b**3)/(120*x**12)
```

3.295 $\int \left(a+\frac{b}{x^2}\right)^3 x^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \left(a+\frac{b}{x^2}\right)^3 x^8 \, dx=\frac{b^3x^3}{3}+\frac{3}{5}ab^2x^5+\frac{3}{7}a^2bx^7+\frac{a^3x^9}{9}$$

output
```
1/3*b^3*x^3+3/5*a*b^2*x^5+3/7*a^2*b*x^7+1/9*a^3*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^8 \, dx=\frac{b^3x^3}{3}+\frac{3}{5}ab^2x^5+\frac{3}{7}a^2bx^7+\frac{a^3x^9}{9}$$

input
```
Integrate[(a + b/x^2)^3*x^8,x]
```

output
```
(b^3*x^3)/3 + (3*a*b^2*x^5)/5 + (3*a^2*b*x^7)/7 + (a^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+\frac{b}{x^2}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^2 \left(a x^2+b\right)^3 dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^3 x^8+3 a^2 b x^6+3 a b^2 x^4+b^3 x^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3 x^9}{9}+\frac{3}{7} a^2 b x^7+\frac{3}{5} a b^2 x^5+\frac{b^3 x^3}{3}$$

input
```
Int[(a + b/x^2)^3*x^8,x]
```

output
```
(b^3*x^3)/3 + (3*a*b^2*x^5)/5 + (3*a^2*b*x^7)/7 + (a^3*x^9)/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{1}{3}b^3x^3+\frac{3}{5}a\,b^2x^5+\frac{3}{7}a^2b\,x^7+\frac{1}{9}a^3x^9$	36
risch	$\frac{1}{3}b^3x^3+\frac{3}{5}a\,b^2x^5+\frac{3}{7}a^2b\,x^7+\frac{1}{9}a^3x^9$	36
parallelrisch	$\frac{1}{3}b^3x^3+\frac{3}{5}a\,b^2x^5+\frac{3}{7}a^2b\,x^7+\frac{1}{9}a^3x^9$	36
gosper	$\frac{x^3(35a^3x^6+135a^2b\,x^4+189a\,b^2x^2+105b^3)}{315}$	38
norman	$\frac{\frac{1}{9}a^3x^{14}+\frac{1}{3}b^3x^8+\frac{3}{5}a\,b^2x^{10}+\frac{3}{7}a^2b\,x^{12}}{x^5}$	40
orering	$\frac{x^9(35a^3x^6+135a^2b\,x^4+189a\,b^2x^2+105b^3)\left(a+\frac{b}{x^2}\right)^3}{315(a\,x^2+b)^3}$	56

input
```
int((a+b/x^2)^3*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^3*x^3+3/5*a*b^2*x^5+3/7*a^2*b*x^7+1/9*a^3*x^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int\left(a+\frac{b}{x^2}\right)^3x^8\,dx=\frac{1}{9}\,a^3x^9+\frac{3}{7}\,a^2bx^7+\frac{3}{5}\,ab^2x^5+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x^2)^3*x^8,x, algorithm="fricas")
```

output
```
1/9*a^3*x^9 + 3/7*a^2*b*x^7 + 3/5*a*b^2*x^5 + 1/3*b^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)^3 x^8\,dx=\frac{a^3x^9}{9}+\frac{3a^2bx^7}{7}+\frac{3ab^2x^5}{5}+\frac{b^3x^3}{3}$$

input
```
integrate((a+b/x**2)**3*x**8,x)
```

output
```
a**3*x**9/9 + 3*a**2*b*x**7/7 + 3*a*b**2*x**5/5 + b**3*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^3 x^8\,dx=\frac{1}{9}\,a^3x^9+\frac{3}{7}\,a^2bx^7+\frac{3}{5}\,ab^2x^5+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x^2)^3*x^8,x, algorithm="maxima")
```

output
```
1/9*a^3*x^9 + 3/7*a^2*b*x^7 + 3/5*a*b^2*x^5 + 1/3*b^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^3 x^8\,dx=\frac{1}{9}\,a^3x^9+\frac{3}{7}\,a^2bx^7+\frac{3}{5}\,ab^2x^5+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x^2)^3*x^8,x, algorithm="giac")
```

output
```
1/9*a^3*x^9 + 3/7*a^2*b*x^7 + 3/5*a*b^2*x^5 + 1/3*b^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^3 x^8\,dx=\frac{a^3\,x^9}{9}+\frac{3\,a^2\,b\,x^7}{7}+\frac{3\,a\,b^2\,x^5}{5}+\frac{b^3\,x^3}{3}$$

input
```
int(x^8*(a + b/x^2)^3,x)
```

output
```
(a^3*x^9)/9 + (b^3*x^3)/3 + (3*a*b^2*x^5)/5 + (3*a^2*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x^2}\right)^3 x^8\,dx=\frac{x^3(35a^3x^6+135a^2b\,x^4+189a\,b^2x^2+105b^3)}{315}$$

input
```
int((a+b/x^2)^3*x^8,x)
```

output
```
(x**3*(35*a**3*x**6 + 135*a**2*b*x**4 + 189*a*b**2*x**2 + 105*b**3))/315
```

3.296 $\int \left(a+\frac{b}{x^2}\right)^3 x^6 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6 \, dx = b^3x + ab^2x^3 + \frac{3}{5}a^2bx^5 + \frac{a^3x^7}{7}$$

output
```
b^3*x+a*b^2*x^3+3/5*a^2*b*x^5+1/7*a^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6 \, dx = b^3x + ab^2x^3 + \frac{3}{5}a^2bx^5 + \frac{a^3x^7}{7}$$

input
```
Integrate[(a + b/x^2)^3*x^6,x]
```

output
```
b^3*x + a*b^2*x^3 + (3*a^2*b*x^5)/5 + (a^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(a+\frac{b}{x^2}\right)^3\,dx$$

$$\Big\downarrow \text{795}$$

$$\int \left(ax^2+b\right)^3dx$$

$$\Big\downarrow \text{210}$$

$$\int \left(a^3x^6+3a^2bx^4+3ab^2x^2+b^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^7}{7}+\frac{3}{5}a^2bx^5+ab^2x^3+b^3x$$

input
```
Int[(a + b/x^2)^3*x^6,x]
```

output
```
b^3*x + a*b^2*x^3 + (3*a^2*b*x^5)/5 + (a^3*x^7)/7
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$b^3x + a\,b^2x^3 + \frac{3}{5}a^2b\,x^5 + \frac{1}{7}a^3x^7$	32
risch	$b^3x + a\,b^2x^3 + \frac{3}{5}a^2b\,x^5 + \frac{1}{7}a^3x^7$	32
parallelrisch	$b^3x + a\,b^2x^3 + \frac{3}{5}a^2b\,x^5 + \frac{1}{7}a^3x^7$	32
gosper	$\frac{x(5a^3x^6+21a^2b\,x^4+35a\,b^2x^2+35b^3)}{35}$	36
norman	$\frac{b^3x^6+a\,b^2x^8+\frac{1}{7}a^3x^{12}+\frac{3}{5}a^2b\,x^{10}}{x^5}$	38
orering	$\frac{x^7(5a^3x^6+21a^2b\,x^4+35a\,b^2x^2+35b^3)\left(a+\frac{b}{x^2}\right)^3}{35(a\,x^2+b)^3}$	56

input
```
int((a+b/x^2)^3*x^6,x,method=_RETURNVERBOSE)
```

output
```
b^3*x+a*b^2*x^3+3/5*a^2*b*x^5+1/7*a^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6\,dx = \frac{1}{7}\,a^3x^7 + \frac{3}{5}\,a^2bx^5 + ab^2x^3 + b^3x$$

input
```
integrate((a+b/x^2)^3*x^6,x, algorithm="fricas")
```

output
```
1/7*a^3*x^7 + 3/5*a^2*b*x^5 + a*b^2*x^3 + b^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6\,dx=\frac{a^3x^7}{7}+\frac{3a^2bx^5}{5}+ab^2x^3+b^3x$$

input
```
integrate((a+b/x**2)**3*x**6,x)
```

output
```
a**3*x**7/7 + 3*a**2*b*x**5/5 + a*b**2*x**3 + b**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6\,dx=\frac{1}{7}\,a^3x^7+\frac{3}{5}\,a^2bx^5+ab^2x^3+b^3x$$

input
```
integrate((a+b/x^2)^3*x^6,x, algorithm="maxima")
```

output
```
1/7*a^3*x^7 + 3/5*a^2*b*x^5 + a*b^2*x^3 + b^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6\,dx=\frac{1}{7}\,a^3x^7+\frac{3}{5}\,a^2bx^5+ab^2x^3+b^3x$$

input
```
integrate((a+b/x^2)^3*x^6,x, algorithm="giac")
```

output
```
1/7*a^3*x^7 + 3/5*a^2*b*x^5 + a*b^2*x^3 + b^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6\,dx = \frac{a^3\,x^7}{7}+\frac{3\,a^2\,b\,x^5}{5}+a\,b^2\,x^3+b^3\,x$$

input
```
int(x^6*(a + b/x^2)^3,x)
```

output
```
b^3*x + (a^3*x^7)/7 + a*b^2*x^3 + (3*a^2*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^6\,dx = \frac{x(5a^3x^6+21a^2b\,x^4+35a\,b^2x^2+35b^3)}{35}$$

input
```
int((a+b/x^2)^3*x^6,x)
```

output
```
(x*(5*a**3*x**6 + 21*a**2*b*x**4 + 35*a*b**2*x**2 + 35*b**3))/35
```

3.297 $\int \left(a+\frac{b}{x^2}\right)^3 x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4 \, dx = -\frac{b^3}{x}+3ab^2x+a^2bx^3+\frac{a^3x^5}{5}$$

output
```
-b^3/x+3*a*b^2*x+a^2*b*x^3+1/5*a^3*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4 \, dx = -\frac{b^3}{x}+3ab^2x+a^2bx^3+\frac{a^3x^5}{5}$$

input
```
Integrate[(a + b/x^2)^3*x^4,x]
```

output
```
-(b^3/x) + 3*a*b^2*x + a^2*b*x^3 + (a^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{(ax^2+b)^3}{x^2}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^3x^4+3a^2bx^2+3ab^2+\frac{b^3}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^5}{5}+a^2bx^3+3ab^2x-\frac{b^3}{x}$$

input
```
Int[(a + b/x^2)^3*x^4,x]
```

output
```
-(b^3/x) + 3*a*b^2*x + a^2*b*x^3 + (a^3*x^5)/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{b^3}{x}+3a\,b^2x+a^2b\,x^3+\frac{a^3x^5}{5}$	33
risch	$-\frac{b^3}{x}+3a\,b^2x+a^2b\,x^3+\frac{a^3x^5}{5}$	33
gosper	$\frac{a^3x^6+5a^2b\,x^4+15a\,b^2x^2-5b^3}{5x}$	37
parallelrisch	$\frac{a^3x^6+5a^2b\,x^4+15a\,b^2x^2-5b^3}{5x}$	37
norman	$\frac{a^2b\,x^8+\frac{1}{5}a^3x^{10}-b^3x^4+3a\,b^2x^6}{x^5}$	39
orering	$\frac{\left(a^3x^6+5a^2b\,x^4+15a\,b^2x^2-5b^3\right)x^5\left(a+\frac{b}{x^2}\right)^3}{5\left(a\,x^2+b\right)^3}$	55

input
```
int((a+b/x^2)^3*x^4,x,method=_RETURNVERBOSE)
```

output
```
-b^3/x+3*a*b^2*x+a^2*b*x^3+1/5*a^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4\,dx=\frac{a^3x^6+5\,a^2bx^4+15\,ab^2x^2-5\,b^3}{5\,x}$$

input
```
integrate((a+b/x^2)^3*x^4,x, algorithm="fricas")
```

output
```
1/5*(a^3*x^6 + 5*a^2*b*x^4 + 15*a*b^2*x^2 - 5*b^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4\,dx=\frac{a^3x^5}{5}+a^2bx^3+3ab^2x-\frac{b^3}{x}$$

input
```
integrate((a+b/x**2)**3*x**4,x)
```

output
```
a**3*x**5/5 + a**2*b*x**3 + 3*a*b**2*x - b**3/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4\,dx=\frac{1}{5}\,a^3x^5+a^2bx^3+3\,ab^2x-\frac{b^3}{x}$$

input
```
integrate((a+b/x^2)^3*x^4,x, algorithm="maxima")
```

output
```
1/5*a^3*x^5 + a^2*b*x^3 + 3*a*b^2*x - b^3/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4\,dx=\frac{1}{5}\,a^3x^5+a^2bx^3+3\,ab^2x-\frac{b^3}{x}$$

input
```
integrate((a+b/x^2)^3*x^4,x, algorithm="giac")
```

output
```
1/5*a^3*x^5 + a^2*b*x^3 + 3*a*b^2*x - b^3/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4\,dx=\frac{a^3\,x^5}{5}-\frac{b^3}{x}+a^2\,b\,x^3+3\,a\,b^2\,x$$

input
```
int(x^4*(a + b/x^2)^3,x)
```

output
```
(a^3*x^5)/5 - b^3/x + a^2*b*x^3 + 3*a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)^3 x^4\,dx=\frac{a^3x^6+5a^2b\,x^4+15a\,b^2x^2-5b^3}{5x}$$

input
```
int((a+b/x^2)^3*x^4,x)
```

output
```
(a**3*x**6 + 5*a**2*b*x**4 + 15*a*b**2*x**2 - 5*b**3)/(5*x)
```

3.298 $\int \left(a+\frac{b}{x^2}\right)^3 x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \left(a+\frac{b}{x^2}\right)^3 x^2 \, dx = -\frac{b^3}{3x^3}-\frac{3ab^2}{x}+3a^2bx+\frac{a^3x^3}{3}$$

output
```
-1/3*b^3/x^3-3*a*b^2/x+3*a^2*b*x+1/3*a^3*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 x^2 \, dx = -\frac{b^3}{3x^3}-\frac{3ab^2}{x}+3a^2bx+\frac{a^3x^3}{3}$$

input
```
Integrate[(a + b/x^2)^3*x^2,x]
```

output
```
-1/3*b^3/x^3 - (3*a*b^2)/x + 3*a^2*b*x + (a^3*x^3)/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^2}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a x^2+b\right)^3}{x^4}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^3 x^2+3 a^2 b+\frac{3 a b^2}{x^2}+\frac{b^3}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3 x^3}{3}+3 a^2 b x-\frac{3 a b^2}{x}-\frac{b^3}{3 x^3}$$

input
```
Int[(a + b/x^2)^3*x^2,x]
```

output
```
-1/3*b^3/x^3 - (3*a*b^2)/x + 3*a^2*b*x + (a^3*x^3)/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{b^3}{3x^3}-\frac{3a\,b^2}{x}+3a^2bx+\frac{a^3x^3}{3}$	34
risch	$\frac{a^3x^3}{3}+3a^2bx+\frac{-3a\,b^2x^2-\frac{1}{3}b^3}{x^3}$	36
gosper	$\frac{a^3x^6+9a^2b\,x^4-9a\,b^2x^2-b^3}{3x^3}$	37
parallelrisch	$\frac{a^3x^6+9a^2b\,x^4-9a\,b^2x^2-b^3}{3x^3}$	37
norman	$\frac{\frac{1}{3}a^3x^8-\frac{1}{3}b^3x^2-3a\,b^2x^4+3a^2b\,x^6}{x^5}$	40
orering	$\frac{(a^3x^6+9a^2b\,x^4-9a\,b^2x^2-b^3)x^3\left(a+\frac{b}{x^2}\right)^3}{3(a\,x^2+b)^3}$	55

input
```
int((a+b/x^2)^3*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^3/x^3-3*a*b^2/x+3*a^2*b*x+1/3*a^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int\left(a+\frac{b}{x^2}\right)^3x^2\,dx=\frac{a^3x^6+9\,a^2bx^4-9\,ab^2x^2-b^3}{3\,x^3}$$

input
```
integrate((a+b/x^2)^3*x^2,x, algorithm="fricas")
```

output
```
1/3*(a^3*x^6 + 9*a^2*b*x^4 - 9*a*b^2*x^2 - b^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx=\frac{a^3x^3}{3}+3a^2bx+\frac{-9ab^2x^2-b^3}{3x^3}$$

input
```
integrate((a+b/x**2)**3*x**2,x)
```

output
```
a**3*x**3/3 + 3*a**2*b*x + (-9*a*b**2*x**2 - b**3)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx=\frac{1}{3}\,a^3x^3+3\,a^2bx-\frac{9\,ab^2x^2+b^3}{3\,x^3}$$

input
```
integrate((a+b/x^2)^3*x^2,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 + 3*a^2*b*x - 1/3*(9*a*b^2*x^2 + b^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx=\frac{1}{3}\,a^3x^3+3\,a^2bx-\frac{9\,ab^2x^2+b^3}{3\,x^3}$$

input
```
integrate((a+b/x^2)^3*x^2,x, algorithm="giac")
```

output
```
1/3*a^3*x^3 + 3*a^2*b*x - 1/3*(9*a*b^2*x^2 + b^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx=\frac{a^3\,x^3}{3}-\frac{\frac{b^3}{3}+3\,a\,b^2\,x^2}{x^3}+3\,a^2\,b\,x$$

input
```
int(x^2*(a + b/x^2)^3,x)
```

output
```
(a^3*x^3)/3 - (b^3/3 + 3*a*b^2*x^2)/x^3 + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x^2}\right)^3 x^2\,dx=\frac{a^3x^6+9a^2b\,x^4-9a\,b^2x^2-b^3}{3x^3}$$

input
```
int((a+b/x^2)^3*x^2,x)
```

output
```
(a**3*x**6 + 9*a**2*b*x**4 - 9*a*b**2*x**2 - b**3)/(3*x**3)
```

3.299 $\int \left(a+\frac{b}{x^2}\right)^3 dx$

Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \left(a+\frac{b}{x^2}\right)^3 dx=-\frac{b^3}{5x^5}-\frac{ab^2}{x^3}-\frac{3a^2b}{x}+a^3x$$

output
```
-1/5*b^3/x^5-a*b^2/x^3-3*a^2*b/x+a^3*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 dx=-\frac{b^3}{5x^5}-\frac{ab^2}{x^3}-\frac{3a^2b}{x}+a^3x$$

input
```
Integrate[(a + b/x^2)^3,x]
```

output
```
-1/5*b^3/x^5 - (a*b^2)/x^3 - (3*a^2*b)/x + a^3*x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {772, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^3 dx$$
$$\Big\downarrow \text{772}$$
$$\int \frac{(ax^2+b)^3}{x^6}dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(a^3+\frac{3a^2b}{x^2}+\frac{3ab^2}{x^4}+\frac{b^3}{x^6}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a^3x-\frac{3a^2b}{x}-\frac{ab^2}{x^3}-\frac{b^3}{5x^5}$$

input
```
Int[(a + b/x^2)^3,x]
```

output
```
-1/5*b^3/x^5 - (a*b^2)/x^3 - (3*a^2*b)/x + a^3*x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{b^{3}}{5x^{5}}-\frac{a\,b^{2}}{x^{3}}-\frac{3a^{2}b}{x}+a^{3}x$	33
risch	$a^{3}x+\frac{-3a^{2}b\,x^{4}-a\,b^{2}x^{2}-\frac{1}{5}b^{3}}{x^{5}}$	35
norman	$\frac{a^{3}x^{6}-3a^{2}b\,x^{4}-a\,b^{2}x^{2}-\frac{1}{5}b^{3}}{x^{5}}$	36
gosper	$\frac{5a^{3}x^{6}-15a^{2}b\,x^{4}-5a\,b^{2}x^{2}-b^{3}}{5x^{5}}$	38
parallelrisch	$\frac{5a^{3}x^{6}-15a^{2}b\,x^{4}-5a\,b^{2}x^{2}-b^{3}}{5x^{5}}$	38
orering	$\frac{\left(5a^{3}x^{6}-15a^{2}b\,x^{4}-5a\,b^{2}x^{2}-b^{3}\right)x\left(a+\frac{b}{x^{2}}\right)^{3}}{5\left(a\,x^{2}+b\right)^{3}}$	54

input
```
int((a+b/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^3/x^5-a*b^2/x^3-3*a^2*b/x+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{b}{x^2}\right)^3\,dx=\frac{5\,a^3x^6-15\,a^2bx^4-5\,ab^2x^2-b^3}{5\,x^5}$$

input
```
integrate((a+b/x^2)^3,x, algorithm="fricas")
```

output
```
1/5*(5*a^3*x^6 - 15*a^2*b*x^4 - 5*a*b^2*x^2 - b^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 dx = a^3x + \frac{-15a^2bx^4 - 5ab^2x^2 - b^3}{5x^5}$$

input
```
integrate((a+b/x**2)**3,x)
```

output
```
a**3*x + (-15*a**2*b*x**4 - 5*a*b**2*x**2 - b**3)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right)^3 dx = a^3x - \frac{3\,a^2b}{x} - \frac{ab^2}{x^3} - \frac{b^3}{5\,x^5}$$

input
```
integrate((a+b/x^2)^3,x, algorithm="maxima")
```

output
```
a^3*x - 3*a^2*b/x - a*b^2/x^3 - 1/5*b^3/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x^2}\right)^3 dx = a^3x - \frac{15\,a^2bx^4 + 5\,ab^2x^2 + b^3}{5\,x^5}$$

input
```
integrate((a+b/x^2)^3,x, algorithm="giac")
```

output
```
a^3*x - 1/5*(15*a^2*b*x^4 + 5*a*b^2*x^2 + b^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^3 dx = a^3\,x - \frac{3\,a^2\,b\,x^4 + a\,b^2\,x^2 + \frac{b^3}{5}}{x^5}$$

input
```
int((a + b/x^2)^3,x)
```

output
```
a^3*x - (b^3/5 + a*b^2*x^2 + 3*a^2*b*x^4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{b}{x^2}\right)^3 dx = \frac{5a^3x^6 - 15a^2b\,x^4 - 5a\,b^2x^2 - b^3}{5x^5}$$

input
```
int((a+b/x^2)^3,x)
```

output
```
(5*a**3*x**6 - 15*a**2*b*x**4 - 5*a*b**2*x**2 - b**3)/(5*x**5)
```

3.300 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2} dx = -\frac{b^3}{7x^7} - \frac{3ab^2}{5x^5} - \frac{a^2b}{x^3} - \frac{a^3}{x}$$

output
```
-1/7*b^3/x^7-3/5*a*b^2/x^5-a^2*b/x^3-a^3/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2} dx = -\frac{b^3}{7x^7} - \frac{3ab^2}{5x^5} - \frac{a^2b}{x^3} - \frac{a^3}{x}$$

input
```
Integrate[(a + b/x^2)^3/x^2,x]
```

output
```
-1/7*b^3/x^7 - (3*a*b^2)/(5*x^5) - (a^2*b)/x^3 - a^3/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2} \, dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{(ax^2+b)^3}{x^8}dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^3}{x^2}+\frac{3a^2b}{x^4}+\frac{3ab^2}{x^6}+\frac{b^3}{x^8}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^3}{x}-\frac{a^2b}{x^3}-\frac{3ab^2}{5x^5}-\frac{b^3}{7x^7}$$

input
```
Int[(a + b/x^2)^3/x^2,x]
```

output
```
-1/7*b^3/x^7 - (3*a*b^2)/(5*x^5) - (a^2*b)/x^3 - a^3/x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{b^3}{7x^7}-\frac{3a\,b^2}{5x^5}-\frac{a^2b}{x^3}-\frac{a^3}{x}$	36
norman	$\frac{-a^3x^6-a^2b\,x^4-\frac{3}{5}a\,b^2x^2-\frac{1}{7}b^3}{x^7}$	37
risch	$\frac{-a^3x^6-a^2b\,x^4-\frac{3}{5}a\,b^2x^2-\frac{1}{7}b^3}{x^7}$	37
gosper	$-\frac{35a^3x^6+35a^2b\,x^4+21a\,b^2x^2+5b^3}{35x^7}$	38
parallelrisch	$\frac{-35a^3x^6-35a^2b\,x^4-21a\,b^2x^2-5b^3}{35x^7}$	38
orering	$-\frac{(35a^3x^6+35a^2b\,x^4+21a\,b^2x^2+5b^3)\left(a+\frac{b}{x^2}\right)^3}{35x(a\,x^2+b)^3}$	56

input
```
int((a+b/x^2)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b^3/x^7-3/5*a*b^2/x^5-a^2*b/x^3-a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx=-\frac{35\,a^3x^6+35\,a^2bx^4+21\,ab^2x^2+5\,b^3}{35\,x^7}$$

input
```
integrate((a+b/x^2)^3/x^2,x, algorithm="fricas")
```

output
```
-1/35*(35*a^3*x^6 + 35*a^2*b*x^4 + 21*a*b^2*x^2 + 5*b^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx = \frac{-35a^3x^6 - 35a^2bx^4 - 21ab^2x^2 - 5b^3}{35x^7}$$

input
```
integrate((a+b/x**2)**3/x**2,x)
```

output
```
(-35*a**3*x**6 - 35*a**2*b*x**4 - 21*a*b**2*x**2 - 5*b**3)/(35*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx = -\frac{35\,a^3x^6 + 35\,a^2bx^4 + 21\,ab^2x^2 + 5\,b^3}{35\,x^7}$$

input
```
integrate((a+b/x^2)^3/x^2,x, algorithm="maxima")
```

output
```
-1/35*(35*a^3*x^6 + 35*a^2*b*x^4 + 21*a*b^2*x^2 + 5*b^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx = -\frac{35\,a^3x^6 + 35\,a^2bx^4 + 21\,ab^2x^2 + 5\,b^3}{35\,x^7}$$

input
```
integrate((a+b/x^2)^3/x^2,x, algorithm="giac")
```

output
```
-1/35*(35*a^3*x^6 + 35*a^2*b*x^4 + 21*a*b^2*x^2 + 5*b^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx = -\frac{a^3\,x^6+a^2\,b\,x^4+\frac{3\,a\,b^2\,x^2}{5}+\frac{b^3}{7}}{x^7}$$

input
```
int((a + b/x^2)^3/x^2,x)
```

output
```
-(b^3/7 + a^3*x^6 + (3*a*b^2*x^2)/5 + a^2*b*x^4)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^2}\,dx = \frac{-35a^3x^6-35a^2b\,x^4-21a\,b^2x^2-5b^3}{35x^7}$$

input
```
int((a+b/x^2)^3/x^2,x)
```

output
```
( - 35*a**3*x**6 - 35*a**2*b*x**4 - 21*a*b**2*x**2 - 5*b**3)/(35*x**7)
```

3.301 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4} dx = -\frac{b^3}{9x^9} - \frac{3ab^2}{7x^7} - \frac{3a^2b}{5x^5} - \frac{a^3}{3x^3}$$

output
```
-1/9*b^3/x^9-3/7*a*b^2/x^7-3/5*a^2*b/x^5-1/3*a^3/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4} dx = -\frac{b^3}{9x^9} - \frac{3ab^2}{7x^7} - \frac{3a^2b}{5x^5} - \frac{a^3}{3x^3}$$

input
```
Integrate[(a + b/x^2)^3/x^4,x]
```

output
```
-1/9*b^3/x^9 - (3*a*b^2)/(7*x^7) - (3*a^2*b)/(5*x^5) - a^3/(3*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(ax^2+b\right)^3}{x^{10}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^8}+\frac{b^3}{x^{10}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{7x^7}-\frac{b^3}{9x^9}$$

input
```
Int[(a + b/x^2)^3/x^4,x]
```

output
```
-1/9*b^3/x^9 - (3*a*b^2)/(7*x^7) - (3*a^2*b)/(5*x^5) - a^3/(3*x^3)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{b^3}{9x^9}-\frac{3a\,b^2}{7x^7}-\frac{3a^2b}{5x^5}-\frac{a^3}{3x^3}$	36
norman	$\frac{-\frac{1}{3}a^3x^6-\frac{3}{5}a^2b\,x^4-\frac{3}{7}a\,b^2x^2-\frac{1}{9}b^3}{x^9}$	37
risch	$\frac{-\frac{1}{3}a^3x^6-\frac{3}{5}a^2b\,x^4-\frac{3}{7}a\,b^2x^2-\frac{1}{9}b^3}{x^9}$	37
gosper	$-\frac{105a^3x^6+189a^2b\,x^4+135a\,b^2x^2+35b^3}{315x^9}$	38
parallelrisch	$\frac{-105a^3x^6-189a^2b\,x^4-135a\,b^2x^2-35b^3}{315x^9}$	38
orering	$-\frac{\left(105a^3x^6+189a^2b\,x^4+135a\,b^2x^2+35b^3\right)\left(a+\frac{b}{x^2}\right)^3}{315x^3\left(a\,x^2+b\right)^3}$	56

input
```
int((a+b/x^2)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/9*b^3/x^9-3/7*a*b^2/x^7-3/5*a^2*b/x^5-1/3*a^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx=-\frac{105\,a^3x^6+189\,a^2bx^4+135\,ab^2x^2+35\,b^3}{315\,x^9}$$

input
```
integrate((a+b/x^2)^3/x^4,x, algorithm="fricas")
```

output
```
-1/315*(105*a^3*x^6 + 189*a^2*b*x^4 + 135*a*b^2*x^2 + 35*b^3)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx = \frac{-105a^3x^6-189a^2bx^4-135ab^2x^2-35b^3}{315x^9}$$

input
```
integrate((a+b/x**2)**3/x**4,x)
```

output
```
(-105*a**3*x**6 - 189*a**2*b*x**4 - 135*a*b**2*x**2 - 35*b**3)/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx = -\frac{105\,a^3x^6+189\,a^2bx^4+135\,ab^2x^2+35\,b^3}{315\,x^9}$$

input
```
integrate((a+b/x^2)^3/x^4,x, algorithm="maxima")
```

output
```
-1/315*(105*a^3*x^6 + 189*a^2*b*x^4 + 135*a*b^2*x^2 + 35*b^3)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx = -\frac{105\,a^3x^6+189\,a^2bx^4+135\,ab^2x^2+35\,b^3}{315\,x^9}$$

input
```
integrate((a+b/x^2)^3/x^4,x, algorithm="giac")
```

output
```
-1/315*(105*a^3*x^6 + 189*a^2*b*x^4 + 135*a*b^2*x^2 + 35*b^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx = -\frac{\frac{a^3\,x^6}{3}+\frac{3\,a^2\,b\,x^4}{5}+\frac{3\,a\,b^2\,x^2}{7}+\frac{b^3}{9}}{x^9}$$

input
```
int((a + b/x^2)^3/x^4,x)
```

output
```
-(b^3/9 + (a^3*x^6)/3 + (3*a*b^2*x^2)/7 + (3*a^2*b*x^4)/5)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^4}\,dx = \frac{-105a^3x^6-189a^2b\,x^4-135a\,b^2x^2-35b^3}{315x^9}$$

input
```
int((a+b/x^2)^3/x^4,x)
```

output
```
( - 105*a**3*x**6 - 189*a**2*b*x**4 - 135*a*b**2*x**2 - 35*b**3)/(315*x**9
)
```

3.302 $\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6} dx = -\frac{b^3}{11x^{11}} - \frac{ab^2}{3x^9} - \frac{3a^2b}{7x^7} - \frac{a^3}{5x^5}$$

output
```
-1/11*b^3/x^11-1/3*a*b^2/x^9-3/7*a^2*b/x^7-1/5*a^3/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6} dx = -\frac{b^3}{11x^{11}} - \frac{ab^2}{3x^9} - \frac{3a^2b}{7x^7} - \frac{a^3}{5x^5}$$

input
```
Integrate[(a + b/x^2)^3/x^6,x]
```

output
```
-1/11*b^3/x^11 - (a*b^2)/(3*x^9) - (3*a^2*b)/(7*x^7) - a^3/(5*x^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(ax^2+b\right)^3}{x^{12}}dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^3}{x^6}+\frac{3a^2b}{x^8}+\frac{3ab^2}{x^{10}}+\frac{b^3}{x^{12}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{7x^7}-\frac{ab^2}{3x^9}-\frac{b^3}{11x^{11}}$$

input
```
Int[(a + b/x^2)^3/x^6,x]
```

output
```
-1/11*b^3/x^11 - (a*b^2)/(3*x^9) - (3*a^2*b)/(7*x^7) - a^3/(5*x^5)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{b^3}{11x^{11}}-\frac{a\,b^2}{3x^9}-\frac{3a^2b}{7x^7}-\frac{a^3}{5x^5}$	36
norman	$\frac{-\frac{1}{5}a^3x^6-\frac{3}{7}a^2b\,x^4-\frac{1}{3}a\,b^2x^2-\frac{1}{11}b^3}{x^{11}}$	37
risch	$\frac{-\frac{1}{5}a^3x^6-\frac{3}{7}a^2b\,x^4-\frac{1}{3}a\,b^2x^2-\frac{1}{11}b^3}{x^{11}}$	37
gosper	$-\frac{231a^3x^6+495a^2b\,x^4+385a\,b^2x^2+105b^3}{1155x^{11}}$	38
parallelrisch	$\frac{-231a^3x^6-495a^2b\,x^4-385a\,b^2x^2-105b^3}{1155x^{11}}$	38
orering	$-\frac{\left(231a^3x^6+495a^2b\,x^4+385a\,b^2x^2+105b^3\right)\left(a+\frac{b}{x^2}\right)^3}{1155x^5\left(a\,x^2+b\right)^3}$	56

input
```
int((a+b/x^2)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/11*b^3/x^11-1/3*a*b^2/x^9-3/7*a^2*b/x^7-1/5*a^3/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx=-\frac{231\,a^3x^6+495\,a^2bx^4+385\,ab^2x^2+105\,b^3}{1155\,x^{11}}$$

input
```
integrate((a+b/x^2)^3/x^6,x, algorithm="fricas")
```

output
```
-1/1155*(231*a^3*x^6 + 495*a^2*b*x^4 + 385*a*b^2*x^2 + 105*b^3)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx = \frac{-231a^3x^6 - 495a^2bx^4 - 385ab^2x^2 - 105b^3}{1155x^{11}}$$

input
```
integrate((a+b/x**2)**3/x**6,x)
```

output
```
(-231*a**3*x**6 - 495*a**2*b*x**4 - 385*a*b**2*x**2 - 105*b**3)/(1155*x**1
1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx = -\frac{231\,a^3x^6 + 495\,a^2bx^4 + 385\,ab^2x^2 + 105\,b^3}{1155\,x^{11}}$$

input
```
integrate((a+b/x^2)^3/x^6,x, algorithm="maxima")
```

output
```
-1/1155*(231*a^3*x^6 + 495*a^2*b*x^4 + 385*a*b^2*x^2 + 105*b^3)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx = -\frac{231\,a^3x^6 + 495\,a^2bx^4 + 385\,ab^2x^2 + 105\,b^3}{1155\,x^{11}}$$

input
```
integrate((a+b/x^2)^3/x^6,x, algorithm="giac")
```

output
```
-1/1155*(231*a^3*x^6 + 495*a^2*b*x^4 + 385*a*b^2*x^2 + 105*b^3)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx = -\frac{\frac{a^3\,x^6}{5}+\frac{3\,a^2\,b\,x^4}{7}+\frac{a\,b^2\,x^2}{3}+\frac{b^3}{11}}{x^{11}}$$

input
```
int((a + b/x^2)^3/x^6,x)
```

output
```
-(b^3/11 + (a^3*x^6)/5 + (a*b^2*x^2)/3 + (3*a^2*b*x^4)/7)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^3}{x^6}\,dx = \frac{-231a^3x^6-495a^2b\,x^4-385a\,b^2x^2-105b^3}{1155x^{11}}$$

input
```
int((a+b/x^2)^3/x^6,x)
```

output
```
( - 231*a**3*x**6 - 495*a**2*b*x**4 - 385*a*b**2*x**2 - 105*b**3)/(1155*x*
*11)
```

3.303 $\int \frac{x^5}{a+\frac{b}{x^2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^5}{a+\frac{b}{x^2}} dx = \frac{b^2x^2}{2a^3} - \frac{bx^4}{4a^2} + \frac{x^6}{6a} - \frac{b^3 \log(b+ax^2)}{2a^4}$$

output
```
1/2*b^2*x^2/a^3-1/4*b*x^4/a^2+1/6*x^6/a-1/2*b^3*ln(a*x^2+b)/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{a+\frac{b}{x^2}} dx = \frac{b^2x^2}{2a^3} - \frac{bx^4}{4a^2} + \frac{x^6}{6a} - \frac{b^3 \log(b+ax^2)}{2a^4}$$

input
```
Integrate[x^5/(a + b/x^2),x]
```

output
```
(b^2*x^2)/(2*a^3) - (b*x^4)/(4*a^2) + x^6/(6*a) - (b^3*Log[b + a*x^2])/(2*
a^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^7}{ax^2+b}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^6}{ax^2+b}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{x^4}{a}-\frac{bx^2}{a^2}+\frac{b^2}{a^3}-\frac{b^3}{a^3\left(ax^2+b\right)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^3\log\left(ax^2+b\right)}{a^4}+\frac{b^2x^2}{a^3}-\frac{bx^4}{2a^2}+\frac{x^6}{3a}\right)$$

input
```
Int[x^5/(a + b/x^2),x]
```

output
```
((b^2*x^2)/a^3 - (b*x^4)/(2*a^2) + x^6/(3*a) - (b^3*Log[b + a*x^2])/a^4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{1}{3}a^2x^6-\frac{1}{2}ab\,x^4+b^2x^2}{2a^3}-\frac{b^3\ln\left(a\,x^2+b\right)}{2a^4}$	46
norman	$\frac{b^2x^2}{2a^3}-\frac{b\,x^4}{4a^2}+\frac{x^6}{6a}-\frac{b^3\ln\left(a\,x^2+b\right)}{2a^4}$	46
risch	$\frac{b^2x^2}{2a^3}-\frac{b\,x^4}{4a^2}+\frac{x^6}{6a}-\frac{b^3\ln\left(a\,x^2+b\right)}{2a^4}$	46
parallelrisch	$-\frac{-2a^3x^6+3a^2b\,x^4-6a\,b^2x^2+6b^3\ln\left(a\,x^2+b\right)}{12a^4}$	46

input
```
int(x^5/(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(1/3*a^2*x^6-1/2*a*b*x^4+b^2*x^2)-1/2*b^3*ln(a*x^2+b)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{a+\frac{b}{x^2}}\,dx = \frac{2\,a^3x^6 - 3\,a^2bx^4 + 6\,ab^2x^2 - 6\,b^3\log\left(ax^2+b\right)}{12\,a^4}$$

input
```
integrate(x^5/(a+b/x^2),x, algorithm="fricas")
```

output
```
1/12*(2*a^3*x^6 - 3*a^2*b*x^4 + 6*a*b^2*x^2 - 6*b^3*log(a*x^2 + b))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{a+\frac{b}{x^2}}\,dx = \frac{x^6}{6a} - \frac{bx^4}{4a^2} + \frac{b^2x^2}{2a^3} - \frac{b^3\log\left(ax^2+b\right)}{2a^4}$$

input
```
integrate(x**5/(a+b/x**2),x)
```

output
```
x**6/(6*a) - b*x**4/(4*a**2) + b**2*x**2/(2*a**3) - b**3*log(a*x**2 + b)/(
2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{a+\frac{b}{x^2}}\,dx = -\frac{b^3\log\left(ax^2+b\right)}{2\,a^4} + \frac{2\,a^2x^6 - 3\,abx^4 + 6\,b^2x^2}{12\,a^3}$$

input
```
integrate(x^5/(a+b/x^2),x, algorithm="maxima")
```

output
```
-1/2*b^3*log(a*x^2 + b)/a^4 + 1/12*(2*a^2*x^6 - 3*a*b*x^4 + 6*b^2*x^2)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{a+\frac{b}{x^2}}\,dx = -\frac{b^3\log\left(|ax^2+b|\right)}{2\,a^4}+\frac{2\,a^2x^6-3\,abx^4+6\,b^2x^2}{12\,a^3}$$

input
```
integrate(x^5/(a+b/x^2),x, algorithm="giac")
```

output
```
-1/2*b^3*log(abs(a*x^2 + b))/a^4 + 1/12*(2*a^2*x^6 - 3*a*b*x^4 + 6*b^2*x^2
)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{a+\frac{b}{x^2}}\,dx = \frac{x^6}{6\,a}-\frac{b\,x^4}{4\,a^2}-\frac{b^3\,\ln\left(a\,x^2+b\right)}{2\,a^4}+\frac{b^2\,x^2}{2\,a^3}$$

input
```
int(x^5/(a + b/x^2),x)
```

output
```
x^6/(6*a) - (b*x^4)/(4*a^2) - (b^3*log(b + a*x^2))/(2*a^4) + (b^2*x^2)/(2*
a^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{a+\frac{b}{x^2}}\,dx = \frac{-6\log(a\,x^2+b)\,b^3+2a^3x^6-3a^2b\,x^4+6a\,b^2x^2}{12a^4}$$

input
```
int(x^5/(a+b/x^2),x)
```

output
```
( - 6*log(a*x**2 + b)*b**3 + 2*a**3*x**6 - 3*a**2*b*x**4 + 6*a*b**2*x**2)/
(12*a**4)
```

3.304 $\int \frac{x^3}{a+\frac{b}{x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx=-\frac{bx^2}{2a^2}+\frac{x^4}{4a}+\frac{b^2\log\left(b+ax^2\right)}{2a^3}$$

output
```
-1/2*b*x^2/a^2+1/4*x^4/a+1/2*b^2*ln(a*x^2+b)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx=-\frac{bx^2}{2a^2}+\frac{x^4}{4a}+\frac{b^2\log\left(b+ax^2\right)}{2a^3}$$

input
```
Integrate[x^3/(a + b/x^2),x]
```

output
```
-1/2*(b*x^2)/a^2 + x^4/(4*a) + (b^2*Log[b + a*x^2])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^5}{ax^2+b}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{ax^2+b}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{b^2}{a^2\left(ax^2+b\right)}-\frac{b}{a^2}+\frac{x^2}{a}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b^2\log\left(ax^2+b\right)}{a^3}-\frac{bx^2}{a^2}+\frac{x^4}{2a}\right)$$

input
```
Int[x^3/(a + b/x^2),x]
```

output
```
(-((b*x^2)/a^2) + x^4/(2*a) + (b^2*Log[b + a*x^2])/a^3)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{a^2x^4-2ab\,x^2+2b^2\ln\left(a\,x^2+b\right)}{4a^3}$	34
norman	$-\frac{b\,x^2}{2a^2}+\frac{x^4}{4a}+\frac{b^2\ln\left(a\,x^2+b\right)}{2a^3}$	35
default	$\frac{\frac{1}{2}a\,x^4-b\,x^2}{2a^2}+\frac{b^2\ln\left(a\,x^2+b\right)}{2a^3}$	36
risch	$\frac{x^4}{4a}-\frac{b\,x^2}{2a^2}+\frac{b^2}{4a^3}+\frac{b^2\ln\left(a\,x^2+b\right)}{2a^3}$	43

input
```
int(x^3/(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^2*x^4-2*a*b*x^2+2*b^2*ln(a*x^2+b))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx = \frac{a^2x^4 - 2\,abx^2 + 2\,b^2\log\left(ax^2+b\right)}{4\,a^3}$$

input
```
integrate(x^3/(a+b/x^2),x, algorithm="fricas")
```

output
```
1/4*(a^2*x^4 - 2*a*b*x^2 + 2*b^2*log(a*x^2 + b))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx = \frac{x^4}{4a} - \frac{bx^2}{2a^2} + \frac{b^2\log\left(ax^2+b\right)}{2a^3}$$

input
```
integrate(x**3/(a+b/x**2),x)
```

output
```
x**4/(4*a) - b*x**2/(2*a**2) + b**2*log(a*x**2 + b)/(2*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx = \frac{b^2\log\left(ax^2+b\right)}{2\,a^3} + \frac{ax^4 - 2\,bx^2}{4\,a^2}$$

input
```
integrate(x^3/(a+b/x^2),x, algorithm="maxima")
```

output
```
1/2*b^2*log(a*x^2 + b)/a^3 + 1/4*(a*x^4 - 2*b*x^2)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx = \frac{b^2\log\left(|ax^2+b|\right)}{2\,a^3} + \frac{ax^4-2\,bx^2}{4\,a^2}$$

input
```
integrate(x^3/(a+b/x^2),x, algorithm="giac")
```

output
```
1/2*b^2*log(abs(a*x^2 + b))/a^3 + 1/4*(a*x^4 - 2*b*x^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx = \frac{2\,b^2\,\ln\left(a\,x^2+b\right)+a^2\,x^4-2\,a\,b\,x^2}{4\,a^3}$$

input
```
int(x^3/(a + b/x^2),x)
```

output
```
(2*b^2*log(b + a*x^2) + a^2*x^4 - 2*a*b*x^2)/(4*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a+\frac{b}{x^2}}\,dx = \frac{2\log(a\,x^2+b)\,b^2+a^2x^4-2ab\,x^2}{4a^3}$$

input
```
int(x^3/(a+b/x^2),x)
```

output
```
(2*log(a*x**2 + b)*b**2 + a**2*x**4 - 2*a*b*x**2)/(4*a**3)
```

3.305 $\int \frac{x}{a+\frac{b}{x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx=\frac{x^2}{2a}-\frac{b\log\left(b+ax^2\right)}{2a^2}$$

output
```
1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx=\frac{x^2}{2a}-\frac{b\log\left(b+ax^2\right)}{2a^2}$$

input
```
Integrate[x/(a + b/x^2),x]
```

output
```
x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^3}{ax^2+b}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{ax^2+b}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{1}{a}-\frac{b}{a(ax^2+b)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{x^2}{a}-\frac{b\log(ax^2+b)}{a^2}\right)$$

input
```
Int[x/(a + b/x^2),x]
```

output
```
(x^2/a - (b*Log[b + a*x^2])/a^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-a\,x^{2}+b\ln\left(a\,x^{2}+b\right)}{2a^{2}}$	23
default	$\frac{x^{2}}{2a}-\frac{b\ln\left(a\,x^{2}+b\right)}{2a^{2}}$	24
norman	$\frac{x^{2}}{2a}-\frac{b\ln\left(a\,x^{2}+b\right)}{2a^{2}}$	24
risch	$\frac{x^{2}}{2a}-\frac{b\ln\left(a\,x^{2}+b\right)}{2a^{2}}$	24

input
```
int(x/(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a*x^2+b*ln(a*x^2+b))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx = \frac{ax^2 - b\log(ax^2+b)}{2\,a^2}$$

input
```
integrate(x/(a+b/x^2),x, algorithm="fricas")
```

output
```
1/2*(a*x^2 - b*log(a*x^2 + b))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx = \frac{x^2}{2a} - \frac{b\log(ax^2+b)}{2a^2}$$

input
```
integrate(x/(a+b/x**2),x)
```

output
```
x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx = \frac{x^2}{2\,a} - \frac{b\log(ax^2+b)}{2\,a^2}$$

input
```
integrate(x/(a+b/x^2),x, algorithm="maxima")
```

output
```
1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx = \frac{x^2}{2\,a} - \frac{b\log\left(|ax^2+b|\right)}{2\,a^2}$$

input
```
integrate(x/(a+b/x^2),x, algorithm="giac")
```

output
```
1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx = -\frac{b\,\ln\left(a\,x^2+b\right)-a\,x^2}{2\,a^2}$$

input
```
int(x/(a + b/x^2),x)
```

output
```
-(b*log(b + a*x^2) - a*x^2)/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+\frac{b}{x^2}}\,dx = \frac{-\log(a\,x^2+b)\,b+a\,x^2}{2a^2}$$

input
```
int(x/(a+b/x^2),x)
```

output
```
( - log(a*x**2 + b)*b + a*x**2)/(2*a**2)
```

3.306 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x} dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x} dx = \frac{\log(b+ax^2)}{2a}$$

output
```
1/2*ln(a*x^2+b)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x} dx = \frac{\log(b+ax^2)}{2a}$$

input
```
Integrate[1/((a + b/x^2)*x),x]
```

output
```
Log[b + a*x^2]/(2*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^2}\right)}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{x}{ax^2+b}dx$$
$$\Big\downarrow \text{ 240}$$
$$\frac{\log\left(ax^2+b\right)}{2a}$$

input
```
Int[1/((a + b/x^2)*x),x]
```

output
```
Log[b + a*x^2]/(2*a)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^2+b)}{2a}$	14
norman	$\frac{\ln(a\,x^2+b)}{2a}$	14
risch	$\frac{\ln(a\,x^2+b)}{2a}$	14
parallelrisch	$\frac{\ln(a\,x^2+b)}{2a}$	14

input
```
int(1/(a+b/x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(a*x^2+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx = \frac{\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(1/(a+b/x^2)/x,x, algorithm="fricas")
```

output
```
1/2*log(a*x^2 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx = \frac{\log\left(ax^2+b\right)}{2a}$$

input
```
integrate(1/(a+b/x**2)/x,x)
```

output
```
log(a*x**2 + b)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx = \frac{\log\left(ax^2+b\right)}{2\,a}$$

input
```
integrate(1/(a+b/x^2)/x,x, algorithm="maxima")
```

output
```
1/2*log(a*x^2 + b)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx = \frac{\log\left(|ax^2+b|\right)}{2\,a}$$

input
```
integrate(1/(a+b/x^2)/x,x, algorithm="giac")
```

output
```
1/2*log(abs(a*x^2 + b))/a
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx = \frac{\ln\left(a\,x^2+b\right)}{2\,a}$$

input
```
int(1/(x*(a + b/x^2)),x)
```

output
```
log(b + a*x^2)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx = \frac{\log(a\,x^2+b)}{2a}$$

input
```
int(1/(a+b/x^2)/x,x)
```

output
```
log(a*x**2 + b)/(2*a)
```

3.307 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = -\frac{\log\left(a+\frac{b}{x^2}\right)}{2b}$$

output
```
-1/2*ln(a+b/x^2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = \frac{\log(x)}{b} - \frac{\log(b+ax^2)}{2b}$$

input
```
Integrate[1/((a + b/x^2)*x^3),x]
```

output
```
Log[x]/b - Log[b + a*x^2]/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x^2}\right)} \, dx$$

$$\Big\downarrow \; 792$$

$$-\frac{\log \left(a+\frac{b}{x^2}\right)}{2 b}$$

input
```
Int[1/((a + b/x^2)*x^3),x]
```

output
```
-1/2*Log[a + b/x^2]/b
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\ln\left(a+\frac{b}{x^2}\right)}{2b}$	14
default	$-\frac{\ln\left(a\,x^2+b\right)}{2b}+\frac{\ln\left(x\right)}{b}$	21
norman	$-\frac{\ln\left(a\,x^2+b\right)}{2b}+\frac{\ln\left(x\right)}{b}$	21
risch	$-\frac{\ln\left(a\,x^2+b\right)}{2b}+\frac{\ln\left(x\right)}{b}$	21
parallelrisch	$\frac{2\ln\left(x\right)-\ln\left(a\,x^2+b\right)}{2b}$	21

input
```
int(1/(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(a+b/x^2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = -\frac{\log\left(ax^2+b\right)-2\,\log\left(x\right)}{2\,b}$$

input
```
integrate(1/(a+b/x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(log(a*x^2 + b) - 2*log(x))/b
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = \frac{\log\left(x\right)}{b} - \frac{\log\left(x^2+\frac{b}{a}\right)}{2b}$$

input
```
integrate(1/(a+b/x**2)/x**3,x)
```

output
```
log(x)/b - log(x**2 + b/a)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = -\frac{\log\left(a+\frac{b}{x^2}\right)}{2\,b}$$

input
```
integrate(1/(a+b/x^2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*log(a + b/x^2)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = \frac{\log\left(x^2\right)}{2\,b} - \frac{\log\left(|ax^2+b|\right)}{2\,b}$$

input
```
integrate(1/(a+b/x^2)/x^3,x, algorithm="giac")
```

output
```
1/2*log(x^2)/b - 1/2*log(abs(a*x^2 + b))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = -\frac{\ln\left(a\,x^2+b\right)-2\,\ln\left(x\right)}{2\,b}$$

input
```
int(1/(x^3*(a + b/x^2)),x)
```

output
```
-(log(b + a*x^2) - 2*log(x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^3}\,dx = \frac{-\log(a\,x^2+b)+2\log(x)}{2b}$$

input
```
int(1/(a+b/x^2)/x^3,x)
```

output
```
( - log(a*x**2 + b) + 2*log(x))/(2*b)
```

3.308 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = -\frac{1}{2bx^2}+\frac{a\log\left(a+\frac{b}{x^2}\right)}{2b^2}$$

output
```
-1/2/b/x^2+1/2*a*ln(a+b/x^2)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = -\frac{1}{2bx^2}-\frac{a\log(x)}{b^2}+\frac{a\log\left(b+ax^2\right)}{2b^2}$$

input
```
Integrate[1/((a + b/x^2)*x^5),x]
```

output
```
-1/2*1/(b*x^2) - (a*Log[x])/b^2 + (a*Log[b + a*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x^2}\right)} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^3 \left(a x^2+b\right)}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a x^2+b\right)}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(\frac{a^2}{b^2 \left(a x^2+b\right)}-\frac{a}{b^2 x^2}+\frac{1}{b x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{a \log \left(x^2\right)}{b^2}+\frac{a \log \left(a x^2+b\right)}{b^2}-\frac{1}{b x^2}\right)$$

input
```
Int[1/((a + b/x^2)*x^5),x]
```

output
```
(-(1/(b*x^2)) - (a*Log[x^2])/b^2 + (a*Log[b + a*x^2])/b^2)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{a \ln \left(a \,x^{2}+b \right)}{2 b^{2}}-\frac{1}{2 b \,x^{2}}-\frac{a \ln \left(x \right)}{b^{2}}$	32
norman	$\frac{a \ln \left(a \,x^{2}+b \right)}{2 b^{2}}-\frac{1}{2 b \,x^{2}}-\frac{a \ln \left(x \right)}{b^{2}}$	32
parallelrisch	$-\frac{2 a \ln \left(x \right) x^{2}-a \ln \left(a \,x^{2}+b \right) x^{2}+b}{2 x^{2} b^{2}}$	33
risch	$-\frac{1}{2 b \,x^{2}}-\frac{a \ln \left(x \right)}{b^{2}}+\frac{a \ln \left(-a \,x^{2}-b \right)}{2 b^{2}}$	35

input
```
int(1/(a+b/x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/b^2*ln(a*x^2+b)-1/2/b/x^2-a/b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = \frac{ax^2\log\left(ax^2+b\right)-2\,ax^2\log\left(x\right)-b}{2\,b^2x^2}$$

input
```
integrate(1/(a+b/x^2)/x^5,x, algorithm="fricas")
```

output
```
1/2*(a*x^2*log(a*x^2 + b) - 2*a*x^2*log(x) - b)/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = -\frac{a\log\left(x\right)}{b^2}+\frac{a\log\left(x^2+\frac{b}{a}\right)}{2b^2}-\frac{1}{2bx^2}$$

input
```
integrate(1/(a+b/x**2)/x**5,x)
```

output
```
-a*log(x)/b**2 + a*log(x**2 + b/a)/(2*b**2) - 1/(2*b*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = \frac{a\log\left(ax^2+b\right)}{2\,b^2}-\frac{a\log\left(x^2\right)}{2\,b^2}-\frac{1}{2\,bx^2}$$

input
```
integrate(1/(a+b/x^2)/x^5,x, algorithm="maxima")
```

output
```
1/2*a*log(a*x^2 + b)/b^2 - 1/2*a*log(x^2)/b^2 - 1/2/(b*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = -\frac{a\log\left(x^2\right)}{2\,b^2}+\frac{a\log\left(|ax^2+b|\right)}{2\,b^2}+\frac{ax^2-b}{2\,b^2x^2}$$

input
```
integrate(1/(a+b/x^2)/x^5,x, algorithm="giac")
```

output
```
-1/2*a*log(x^2)/b^2 + 1/2*a*log(abs(a*x^2 + b))/b^2 + 1/2*(a*x^2 - b)/(b^2
*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = \frac{a\,\ln\left(a\,x^2+b\right)}{2\,b^2}-\frac{1}{2\,b\,x^2}-\frac{a\,\ln\left(x\right)}{b^2}$$

input
```
int(1/(x^5*(a + b/x^2)),x)
```

output
```
(a*log(b + a*x^2))/(2*b^2) - 1/(2*b*x^2) - (a*log(x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^5}\,dx = \frac{\log(a\,x^2+b)\,a\,x^2-2\log(x)\,a\,x^2-b}{2b^2x^2}$$

input
```
int(1/(a+b/x^2)/x^5,x)
```

output
```
(log(a*x**2 + b)*a*x**2 - 2*log(x)*a*x**2 - b)/(2*b**2*x**2)
```

3.309 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx=-\frac{1}{4bx^4}+\frac{a}{2b^2x^2}-\frac{a^2\log\left(a+\frac{b}{x^2}\right)}{2b^3}$$

output
```
-1/4/b/x^4+1/2*a/b^2/x^2-1/2*a^2*ln(a+b/x^2)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx=-\frac{1}{4bx^4}+\frac{a}{2b^2x^2}+\frac{a^2\log(x)}{b^3}-\frac{a^2\log\left(b+ax^2\right)}{2b^3}$$

input
```
Integrate[1/((a + b/x^2)*x^7),x]
```

output
```
-1/4*1/(b*x^4) + a/(2*b^2*x^2) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x^2])/(
2*b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^2}\right)}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{1}{x^5 \left(a x^2+b\right)}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2} \int \frac{1}{x^6 \left(a x^2+b\right)}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$\frac{1}{2} \int \left(-\frac{a^3}{b^3 \left(a x^2+b\right)}+\frac{a^2}{b^3 x^2}-\frac{a}{b^2 x^4}+\frac{1}{b x^6}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2} \left(\frac{a^2 \log \left(x^2\right)}{b^3}-\frac{a^2 \log \left(a x^2+b\right)}{b^3}+\frac{a}{b^2 x^2}-\frac{1}{2 b x^4}\right)$$

input
```
Int[1/((a + b/x^2)*x^7),x]
```

output
```
(-1/2*1/(b*x^4) + a/(b^2*x^2) + (a^2*Log[x^2])/b^3 - (a^2*Log[b + a*x^2])/
b^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{a^{2} \ln \left(a \,x^{2}+b \right)}{2 b^{3}}-\frac{1}{4 b \,x^{4}}+\frac{a^{2} \ln \left(x \right)}{b^{3}}+\frac{a}{2 b^{2} x^{2}}$	44
risch	$\frac{\frac{a \,x^{2}}{2 b^{2}}-\frac{1}{4 b}}{x^{4}}+\frac{a^{2} \ln \left(x \right)}{b^{3}}-\frac{a^{2} \ln \left(a \,x^{2}+b \right)}{2 b^{3}}$	46
parallelrisch	$\frac{4 a^{2} \ln \left(x \right) x^{4}-2 a^{2} \ln \left(a \,x^{2}+b \right) x^{4}+2 a b \,x^{2}-b^{2}}{4 b^{3} x^{4}}$	48
norman	$\frac{-\frac{x^{2}}{4 b}+\frac{a \,x^{4}}{2 b^{2}}}{x^{6}}+\frac{a^{2} \ln \left(x \right)}{b^{3}}-\frac{a^{2} \ln \left(a \,x^{2}+b \right)}{2 b^{3}}$	49

input
```
int(1/(a+b/x^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/b^3*ln(a*x^2+b)-1/4/b/x^4+a^2/b^3*ln(x)+1/2*a/b^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx = -\frac{2\,a^2x^4\log\left(ax^2+b\right)-4\,a^2x^4\log\left(x\right)-2\,abx^2+b^2}{4\,b^3x^4}$$

input
```
integrate(1/(a+b/x^2)/x^7,x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*x^4*log(a*x^2 + b) - 4*a^2*x^4*log(x) - 2*a*b*x^2 + b^2)/(b^3*
x^4)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx = \frac{a^2\log\left(x\right)}{b^3}-\frac{a^2\log\left(x^2+\frac{b}{a}\right)}{2b^3}+\frac{2ax^2-b}{4b^2x^4}$$

input
```
integrate(1/(a+b/x**2)/x**7,x)
```

output
```
a**2*log(x)/b**3 - a**2*log(x**2 + b/a)/(2*b**3) + (2*a*x**2 - b)/(4*b**2*
x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx = -\frac{a^2\log\left(ax^2+b\right)}{2\,b^3}+\frac{a^2\log\left(x^2\right)}{2\,b^3}+\frac{2\,ax^2-b}{4\,b^2x^4}$$

input
```
integrate(1/(a+b/x^2)/x^7,x, algorithm="maxima")
```

output
```
-1/2*a^2*log(a*x^2 + b)/b^3 + 1/2*a^2*log(x^2)/b^3 + 1/4*(2*a*x^2 - b)/(b^
2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx = \frac{a^2\log\left(x^2\right)}{2\,b^3} - \frac{a^2\log\left(|ax^2+b|\right)}{2\,b^3} - \frac{3\,a^2x^4-2\,abx^2+b^2}{4\,b^3x^4}$$

input
```
integrate(1/(a+b/x^2)/x^7,x, algorithm="giac")
```

output
```
1/2*a^2*log(x^2)/b^3 - 1/2*a^2*log(abs(a*x^2 + b))/b^3 - 1/4*(3*a^2*x^4 -
2*a*b*x^2 + b^2)/(b^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx = \frac{a^2\,\ln\left(x\right)}{b^3} - \frac{a^2\,\ln\left(a\,x^2+b\right)}{2\,b^3} - \frac{\frac{1}{4\,b}-\frac{a\,x^2}{2\,b^2}}{x^4}$$

input
```
int(1/(x^7*(a + b/x^2)),x)
```

output
```
(a^2*log(x))/b^3 - (a^2*log(b + a*x^2))/(2*b^3) - (1/(4*b) - (a*x^2)/(2*b^
2))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^7}\,dx = \frac{-2\log(a\,x^2+b)\,a^2x^4+4\log(x)\,a^2x^4+2ab\,x^2-b^2}{4b^3x^4}$$

input
```
int(1/(a+b/x^2)/x^7,x)
```

output
```
( - 2*log(a*x**2 + b)*a**2*x**4 + 4*log(x)*a**2*x**4 + 2*a*b*x**2 - b**2)/
(4*b**3*x**4)
```

3.310 $\int \frac{x^6}{a+\frac{b}{x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{x^6}{a+\frac{b}{x^2}} \, dx = -\frac{b^3 x}{a^4} + \frac{b^2 x^3}{3 a^3} - \frac{b x^5}{5 a^2} + \frac{x^7}{7 a} + \frac{b^{7/2} \arctan\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{a^{9/2}}$$

output
```
-b^3*x/a^4+1/3*b^2*x^3/a^3-1/5*b*x^5/a^2+1/7*x^7/a+b^(7/2)*arctan(a^(1/2)*
x/b^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{a+\frac{b}{x^2}} \, dx = -\frac{b^3 x}{a^4} + \frac{b^2 x^3}{3 a^3} - \frac{b x^5}{5 a^2} + \frac{x^7}{7 a} + \frac{b^{7/2} \arctan\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{a^{9/2}}$$

input
```
Integrate[x^6/(a + b/x^2),x]
```

output
```
-((b^3*x)/a^4) + (b^2*x^3)/(3*a^3) - (b*x^5)/(5*a^2) + x^7/(7*a) + (b^(7/2
)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(9/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^8}{ax^2+b}dx$$

$$\Big\downarrow \text{ 254}$$

$$\int \left(\frac{b^4}{a^4\left(ax^2+b\right)}-\frac{b^3}{a^4}+\frac{b^2x^2}{a^3}-\frac{bx^4}{a^2}+\frac{x^6}{a}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^{7/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{9/2}}-\frac{b^3x}{a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^5}{5a^2}+\frac{x^7}{7a}$$

input
```
Int[x^6/(a + b/x^2),x]
```

output
```
-((b^3*x)/a^4) + (b^2*x^3)/(3*a^3) - (b*x^5)/(5*a^2) + x^7/(7*a) + (b^(7/2
)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(9/2)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\frac{1}{7}a^3x^7-\frac{1}{5}a^2b\,x^5+\frac{1}{3}a\,b^2x^3-b^3x}{a^4}+\frac{b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}$	60
risch	$\frac{x^7}{7a}-\frac{b\,x^5}{5a^2}+\frac{b^2x^3}{3a^3}-\frac{x\,b^3}{a^4}+\frac{\sqrt{-ab}\,b^3\ln\left(-\sqrt{-ab}\,x+b\right)}{2a^5}-\frac{\sqrt{-ab}\,b^3\ln\left(\sqrt{-ab}\,x+b\right)}{2a^5}$	90

input
```
int(x^6/(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/7*a^3*x^7-1/5*a^2*b*x^5+1/3*a*b^2*x^3-b^3*x)+b^4/a^4/(a*b)^(1/2)*
arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int \frac{x^6}{a+\frac{b}{x^2}}\,dx$$
$$=\left[\frac{30\,a^3x^7-42\,a^2bx^5+70\,ab^2x^3+105\,b^3\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)-210\,b^3x}{210\,a^4},\ \frac{15\,a^3x^7-21\,a^2bx^5+35\,a\right.$$

input
```
integrate(x^6/(a+b/x^2),x, algorithm="fricas")
```

output
```
[1/210*(30*a^3*x^7 - 42*a^2*b*x^5 + 70*a*b^2*x^3 + 105*b^3*sqrt(-b/a)*log(
(a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) - 210*b^3*x)/a^4, 1/105*(15*a^
3*x^7 - 21*a^2*b*x^5 + 35*a*b^2*x^3 + 105*b^3*sqrt(b/a)*arctan(a*x*sqrt(b/
a)/b) - 105*b^3*x)/a^4]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.57

$$\int \frac{x^6}{a+\frac{b}{x^2}}\,dx = -\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(-\frac{a^4\sqrt{-\frac{b^7}{a^9}}}{b^3}+x\right)}{2} + \frac{\sqrt{-\frac{b^7}{a^9}}\log\left(\frac{a^4\sqrt{-\frac{b^7}{a^9}}}{b^3}+x\right)}{2} + \frac{x^7}{7a} - \frac{bx^5}{5a^2} + \frac{b^2x^3}{3a^3} - \frac{b^3x}{a^4}$$

input
```
integrate(x**6/(a+b/x**2),x)
```

output
```
-sqrt(-b**7/a**9)*log(-a**4*sqrt(-b**7/a**9)/b**3 + x)/2 + sqrt(-b**7/a**9
)*log(a**4*sqrt(-b**7/a**9)/b**3 + x)/2 + x**7/(7*a) - b*x**5/(5*a**2) + b
**2*x**3/(3*a**3) - b**3*x/a**4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{a+\frac{b}{x^2}}\,dx = \frac{b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{15\,a^3x^7 - 21\,a^2bx^5 + 35\,ab^2x^3 - 105\,b^3x}{105\,a^4}$$

input
```
integrate(x^6/(a+b/x^2),x, algorithm="maxima")
```

output
```
b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(15*a^3*x^7 - 21*a^2*b*x
^5 + 35*a*b^2*x^3 - 105*b^3*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{a+\frac{b}{x^2}}\,dx = \frac{b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{15\,a^6x^7 - 21\,a^5bx^5 + 35\,a^4b^2x^3 - 105\,a^3b^3x}{105\,a^7}$$

input
```
integrate(x^6/(a+b/x^2),x, algorithm="giac")
```

output
```
b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(15*a^6*x^7 - 21*a^5*b*x
^5 + 35*a^4*b^2*x^3 - 105*a^3*b^3*x)/a^7
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{x^6}{a+\frac{b}{x^2}}\,dx = \frac{x^7}{7\,a} - \frac{b\,x^5}{5\,a^2} - \frac{b^3\,x}{a^4} + \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{9/2}} + \frac{b^2\,x^3}{3\,a^3}$$

input
```
int(x^6/(a + b/x^2),x)
```

output
```
x^7/(7*a) - (b*x^5)/(5*a^2) - (b^3*x)/a^4 + (b^(7/2)*atan((a^(1/2)*x)/b^(1
/2)))/a^(9/2) + (b^2*x^3)/(3*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{a+\frac{b}{x^2}}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^3 + 15a^4x^7 - 21a^3b\,x^5 + 35a^2b^2x^3 - 105a\,b^3x}{105a^5}$$

input
```
int(x^6/(a+b/x^2),x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b**3 + 15*a**4*x**7 - 2
1*a**3*b*x**5 + 35*a**2*b**2*x**3 - 105*a*b**3*x)/(105*a**5)
```

3.311 $\int \frac{x^4}{a+\frac{b}{x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{x^4}{a+\frac{b}{x^2}} \, dx = \frac{b^2x}{a^3} - \frac{bx^3}{3a^2} + \frac{x^5}{5a} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{7/2}}$$

output
```
b^2*x/a^3-1/3*b*x^3/a^2+1/5*x^5/a-b^(5/2)*arctan(a^(1/2)*x/b^(1/2))/a^(7/2
)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+\frac{b}{x^2}} \, dx = \frac{b^2x}{a^3} - \frac{bx^3}{3a^2} + \frac{x^5}{5a} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{7/2}}$$

input
```
Integrate[x^4/(a + b/x^2),x]
```

output
```
(b^2*x)/a^3 - (b*x^3)/(3*a^2) + x^5/(5*a) - (b^(5/2)*ArcTan[(Sqrt[a]*x)/Sq
rt[b]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^6}{ax^2+b}dx$$

$$\Big\downarrow \text{254}$$

$$\int \left(-\frac{b^3}{a^3\left(ax^2+b\right)}+\frac{b^2}{a^3}-\frac{bx^2}{a^2}+\frac{x^4}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{7/2}}+\frac{b^2x}{a^3}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}$$

input
```
Int[x^4/(a + b/x^2),x]
```

output
```
(b^2*x)/a^3 - (b*x^3)/(3*a^2) + x^5/(5*a) - (b^(5/2)*ArcTan[(Sqrt[a]*x)/Sq
rt[b]])/a^(7/2)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{5}a^2x^5-\frac{1}{3}ab\,x^3+b^2x}{a^3}-\frac{b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	49
risch	$\frac{x^5}{5a}-\frac{b\,x^3}{3a^2}+\frac{b^2x}{a^3}+\frac{\sqrt{-ab}\,b^2\ln\left(-\sqrt{-ab}\,x-b\right)}{2a^4}-\frac{\sqrt{-ab}\,b^2\ln\left(\sqrt{-ab}\,x-b\right)}{2a^4}$	82

input
```
int(x^4/(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*a^2*x^5-1/3*a*b*x^3+b^2*x)-b^3/a^3/(a*b)^(1/2)*arctan(a*x/(a*b)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int \frac{x^4}{a+\frac{b}{x^2}}\,dx$$
$$= \left[\frac{6\,a^2x^5-10\,abx^3+15\,b^2\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+30\,b^2x}{30\,a^3},\frac{3\,a^2x^5-5\,abx^3-15\,b^2\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right.}{15\,a^3}\right.$$

input
```
integrate(x^4/(a+b/x^2),x, algorithm="fricas")
```

output
```
[1/30*(6*a^2*x^5 - 10*a*b*x^3 + 15*b^2*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(
-b/a) - b)/(a*x^2 + b)) + 30*b^2*x)/a^3, 1/15*(3*a^2*x^5 - 5*a*b*x^3 - 15*
b^2*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) + 15*b^2*x)/a^3]
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

$$\int \frac{x^4}{a+\frac{b}{x^2}}\,dx = \frac{\sqrt{-\frac{b^5}{a^7}}\log\left(-\frac{a^3\sqrt{-\frac{b^5}{a^7}}}{b^2}+x\right)}{2} - \frac{\sqrt{-\frac{b^5}{a^7}}\log\left(\frac{a^3\sqrt{-\frac{b^5}{a^7}}}{b^2}+x\right)}{2} + \frac{x^5}{5a} - \frac{bx^3}{3a^2} + \frac{b^2x}{a^3}$$

input
```
integrate(x**4/(a+b/x**2),x)
```

output
```
sqrt(-b**5/a**7)*log(-a**3*sqrt(-b**5/a**7)/b**2 + x)/2 - sqrt(-b**5/a**7)
*log(a**3*sqrt(-b**5/a**7)/b**2 + x)/2 + x**5/(5*a) - b*x**3/(3*a**2) + b*
*2*x/a**3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{a+\frac{b}{x^2}}\,dx = -\frac{b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{3\,a^2x^5 - 5\,abx^3 + 15\,b^2x}{15\,a^3}$$

input
```
integrate(x^4/(a+b/x^2),x, algorithm="maxima")
```

output
```
-b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(3*a^2*x^5 - 5*a*b*x^3 +
 15*b^2*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+\frac{b}{x^2}}\,dx = -\frac{b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{3\,a^4x^5 - 5\,a^3bx^3 + 15\,a^2b^2x}{15\,a^5}$$

input
```
integrate(x^4/(a+b/x^2),x, algorithm="giac")
```

output
```
-b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(3*a^4*x^5 - 5*a^3*b*x^3
 + 15*a^2*b^2*x)/a^5
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{a+\frac{b}{x^2}}\,dx = \frac{x^5}{5\,a} - \frac{b\,x^3}{3\,a^2} + \frac{b^2\,x}{a^3} - \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{7/2}}$$

input
```
int(x^4/(a + b/x^2),x)
```

output
```
x^5/(5*a) - (b*x^3)/(3*a^2) + (b^2*x)/a^3 - (b^(5/2)*atan((a^(1/2)*x)/b^(1
/2)))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{a+\frac{b}{x^2}}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^2 + 3a^3x^5 - 5a^2b\,x^3 + 15a\,b^2x}{15a^4}$$

input
```
int(x^4/(a+b/x^2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b**2 + 3*a**3*x**5 -
5*a**2*b*x**3 + 15*a*b**2*x)/(15*a**4)
```

3.312 $\int \frac{x^2}{a+\frac{b}{x^2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{x^2}{a+\frac{b}{x^2}} dx = -\frac{bx}{a^2}+\frac{x^3}{3a}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{5/2}}$$

output
```
-b*x/a^2+1/3*x^3/a+b^(3/2)*arctan(a^(1/2)*x/b^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+\frac{b}{x^2}} dx = -\frac{bx}{a^2}+\frac{x^3}{3a}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{5/2}}$$

input
```
Integrate[x^2/(a + b/x^2),x]
```

output
```
-((b*x)/a^2) + x^3/(3*a) + (b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^4}{ax^2+b}dx$$

$$\Big\downarrow \text{254}$$

$$\int \left(\frac{b^2}{a^2(ax^2+b)}-\frac{b}{a^2}+\frac{x^2}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{bx}{a^2}+\frac{x^3}{3a}$$

input
```
Int[x^2/(a + b/x^2),x]
```

output
```
-((b*x)/a^2) + x^3/(3*a) + (b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(5/2)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\frac{1}{3}a\,x^3-bx}{a^2}+\frac{b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	38
risch	$\frac{x^3}{3a}-\frac{bx}{a^2}+\frac{\sqrt{-ab}\,b\ln\left(-\sqrt{-ab}\,x+b\right)}{2a^3}-\frac{\sqrt{-ab}\,b\ln\left(\sqrt{-ab}\,x+b\right)}{2a^3}$	64

input
```
int(x^2/(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/3*a*x^3-b*x)+b^2/a^2/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.36

$$\int \frac{x^2}{a+\frac{b}{x^2}}\,dx = \left[\frac{2\,ax^3+3\,b\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)-6\,bx}{6\,a^2}, \frac{ax^3+3\,b\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)-3\,bx}{3\,a^2}\right]$$

input
```
integrate(x^2/(a+b/x^2),x, algorithm="fricas")
```

output
```
[1/6*(2*a*x^3 + 3*b*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 +
 b)) - 6*b*x)/a^2, 1/3*(a*x^3 + 3*b*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - 3*
b*x)/a^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{a+\frac{b}{x^2}}\,dx = -\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^2\sqrt{-\frac{b^3}{a^5}}}{b}+x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^2\sqrt{-\frac{b^3}{a^5}}}{b}+x\right)}{2} + \frac{x^3}{3a} - \frac{bx}{a^2}$$

input
```
integrate(x**2/(a+b/x**2),x)
```

output
```
-sqrt(-b**3/a**5)*log(-a**2*sqrt(-b**3/a**5)/b + x)/2 + sqrt(-b**3/a**5)*l
og(a**2*sqrt(-b**3/a**5)/b + x)/2 + x**3/(3*a) - b*x/a**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a+\frac{b}{x^2}}\,dx = \frac{b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{ax^3-3\,bx}{3\,a^2}$$

input
```
integrate(x^2/(a+b/x^2),x, algorithm="maxima")
```

output
```
b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(a*x^3 - 3*b*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{a+\frac{b}{x^2}}\,dx = \frac{b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{a^2x^3-3\,abx}{3\,a^3}$$

input
```
integrate(x^2/(a+b/x^2),x, algorithm="giac")
```

output
```
b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(a^2*x^3 - 3*a*b*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{a+\frac{b}{x^2}}\,dx = \frac{x^3}{3\,a} + \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{5/2}} - \frac{b\,x}{a^2}$$

input
```
int(x^2/(a + b/x^2),x)
```

output
```
x^3/(3*a) + (b^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/a^(5/2) - (b*x)/a^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{a+\frac{b}{x^2}}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b + a^2x^3 - 3abx}{3a^3}$$

input
```
int(x^2/(a+b/x^2),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b + a**2*x**3 - 3*a*b*x)/
(3*a**3)
```

3.313 $\int \frac{1}{a+\frac{b}{x^2}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 31

$$\int \frac{1}{a+\frac{b}{x^2}} \, dx = \frac{x}{a} - \frac{\sqrt{b} \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{3/2}}$$

output
```
x/a-b^(1/2)*arctan(a^(1/2)*x/b^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+\frac{b}{x^2}} \, dx = \frac{x}{a} - \frac{\sqrt{b} \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{3/2}}$$

input
```
Integrate[(a + b/x^2)^(-1),x]
```

output
```
x/a - (Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{ 772}$$

$$\int \frac{x^2}{ax^2+b}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{ax^2+b}dx}{a}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{3/2}}$$

input
```
Int[(a + b/x^2)^(-1),x]
```

output
```
x/a - (Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(3/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{a}-\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	27
risch	$\frac{x}{a}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-b\right)}{2a^2}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-b\right)}{2a^2}$	56

input
```
int(1/(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
x/a-b/a/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{1}{a+\frac{b}{x^2}}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+2\,x}{2\,a}, -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)-x}{a}\right]$$

input
```
integrate(1/(a+b/x^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) + 2*x)/a,
 -(sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - x)/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{1}{a+\frac{b}{x^2}}\,dx = \frac{\sqrt{-\frac{b}{a^3}}\log\left(-a\sqrt{-\frac{b}{a^3}}+x\right)}{2} - \frac{\sqrt{-\frac{b}{a^3}}\log\left(a\sqrt{-\frac{b}{a^3}}+x\right)}{2} + \frac{x}{a}$$

input
```
integrate(1/(a+b/x**2),x)
```

output
```
sqrt(-b/a**3)*log(-a*sqrt(-b/a**3) + x)/2 - sqrt(-b/a**3)*log(a*sqrt(-b/a*
*3) + x)/2 + x/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+\frac{b}{x^2}}\,dx = -\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{x}{a}$$

input
```
integrate(1/(a+b/x^2),x, algorithm="maxima")
```

output
```
-b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+\frac{b}{x^2}}\,dx = -\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{x}{a}$$

input
```
integrate(1/(a+b/x^2),x, algorithm="giac")
```

output
```
-b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + x/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{a+\frac{b}{x^2}}\,dx = \frac{x}{a} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a^{3/2}}$$

input
```
int(1/(a + b/x^2),x)
```

output
```
x/a - (b^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+\frac{b}{x^2}}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)+ax}{a^2}$$

input
```
int(1/(a+b/x^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a))) + a*x)/a**2
```

3.314 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx = \frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(a^(1/2)*x/b^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx = \frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[1/((a + b/x^2)*x^2),x]
```

output
```
ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {795, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{ax^2+b}dx$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[1/((a + b/x^2)*x^2),x]
```

output
```
ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln(ax+\sqrt{-ab})}{2\sqrt{-ab}}+\frac{\ln(-ax+\sqrt{-ab})}{2\sqrt{-ab}}$	41

input
```
int(1/(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{ax^2-2\sqrt{-ab}x-b}{ax^2+b}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{b}\right)}{ab}\right]$$

input
```
integrate(1/(a+b/x^2)/x^2,x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/b)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-b\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(b\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(a+b/x**2)/x**2,x)
```

output
```
-sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(b*sqrt(-
1/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx=\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a+b/x^2)/x^2,x, algorithm="maxima")
```

output
```
arctan(a*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx=\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(a+b/x^2)/x^2,x, algorithm="giac")
```

output
```
arctan(a*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(x^2*(a + b/x^2)),x)
```

output
```
atan((a^(1/2)*x)/b^(1/2))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/(a+b/x^2)/x^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.315 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx=-\frac{1}{bx}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{3/2}}$$

output
```
-1/b/x-a^(1/2)*arctan(a^(1/2)*x/b^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx=-\frac{1}{bx}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
Integrate[1/((a + b/x^2)*x^4),x]
```

output
```
-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+\frac{b}{x^2}\right)}dx$$
$$\Big\downarrow \ 795$$
$$\int \frac{1}{x^2\left(ax^2+b\right)}dx$$
$$\Big\downarrow \ 264$$
$$-\frac{a\int \frac{1}{ax^2+b}dx}{b}-\frac{1}{bx}$$
$$\Big\downarrow \ 218$$
$$-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

input
```
Int[1/((a + b/x^2)*x^4),x]
```

output
```
-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{b\sqrt{ab}}-\frac{1}{xb}$	30
risch	$-\frac{1}{xb}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{3}\_Z^{2}+a\right)}\_R\ln\left(\left(3\_R^{2}b^{3}+2a\right)x+b^{2}\_R\right)\right)}{2}$	48

input
```
int(1/(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-a/b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))-1/x/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx=\left[\frac{x\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-2}{2\,bx},-\frac{x\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+1}{bx}\right]$$

input
```
integrate(1/(a+b/x^2)/x^4,x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - 2)/(b
*x), -(x*sqrt(a/b)*arctan(x*sqrt(a/b)) + 1)/(b*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx = \frac{\sqrt{-\frac{a}{b^3}}\log\left(x-\frac{b^2\sqrt{-\frac{a}{b^3}}}{a}\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}}\log\left(x+\frac{b^2\sqrt{-\frac{a}{b^3}}}{a}\right)}{2} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x**2)/x**4,x)
```

output
```
sqrt(-a/b**3)*log(x - b**2*sqrt(-a/b**3)/a)/2 - sqrt(-a/b**3)*log(x + b**2
*sqrt(-a/b**3)/a)/2 - 1/(b*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx = -\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x^2)/x^4,x, algorithm="maxima")
```

output
```
-a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx = -\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x^2)/x^4,x, algorithm="giac")
```

output
```
-a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/(b*x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx = -\frac{1}{b\,x} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
int(1/(x^4*(a + b/x^2)),x)
```

output
```
- 1/(b*x) - (a^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)x - b}{b^2x}$$

input
```
int(1/(a+b/x^2)/x^4,x)
```

output
```
( - (sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*x + b))/(b**2*x)
```

3.316 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = -\frac{1}{3bx^3}+\frac{a}{b^2x}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{5/2}}$$

output
```
-1/3/b/x^3+a/b^2/x+a^(3/2)*arctan(a^(1/2)*x/b^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = -\frac{1}{3bx^3}+\frac{a}{b^2x}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{5/2}}$$

input
```
Integrate[1/((a + b/x^2)*x^6),x]
```

output
```
-1/3*1/(b*x^3) + a/(b^2*x) + (a^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x^2}\right)}dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{1}{x^4 \left(a x^2+b\right)}dx$$
$$\Big\downarrow \text{264}$$
$$-\frac{a \int \frac{1}{x^2 \left(a x^2+b\right)}dx}{b}-\frac{1}{3 b x^3}$$
$$\Big\downarrow \text{264}$$
$$-\frac{a \left(-\frac{a \int \frac{1}{a x^2+b}dx}{b}-\frac{1}{b x}\right)}{b}-\frac{1}{3 b x^3}$$
$$\Big\downarrow \text{218}$$
$$-\frac{a \left(-\frac{\sqrt{a} \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{b x}\right)}{b}-\frac{1}{3 b x^3}$$

input
```
Int[1/((a + b/x^2)*x^6),x]
```

output
```
-1/3*1/(b*x^3) - (a*(-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^
(3/2)))/b
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{a^{2} \arctan\left(\frac{a x}{\sqrt{a b}}\right)}{b^{2} \sqrt{a b}}-\frac{1}{3 b \,x^{3}}+\frac{a}{b^{2} x}$	39
risch	$\frac{\frac{a \,x^{2}}{b^{2}}-\frac{1}{3 b}}{x^{3}}+\frac{\sqrt{-a b}\, a \ln\left(-a x-\sqrt{-a b}\right)}{2 b^{3}}-\frac{\sqrt{-a b}\, a \ln\left(-a x+\sqrt{-a b}\right)}{2 b^{3}}$	70

input
```
int(1/(a+b/x^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
a^2/b^2/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))-1/3/b/x^3+a/b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = \left[\frac{3\,ax^3\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)+6\,ax^2-2\,b}{6\,b^2x^3}, \frac{3\,ax^3\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+3\,ax^2-b}{3\,b^2x^3}\right]$$

input
```
integrate(1/(a+b/x^2)/x^6,x, algorithm="fricas")
```

output
```
[1/6*(3*a*x^3*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) +
 6*a*x^2 - 2*b)/(b^2*x^3), 1/3*(3*a*x^3*sqrt(a/b)*arctan(x*sqrt(a/b)) + 3*
a*x^2 - b)/(b^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(37) = 74.

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = -\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^5}}}{a^2}\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^5}}}{a^2}\right)}{2}+\frac{3ax^2-b}{3b^2x^3}$$

input
```
integrate(1/(a+b/x**2)/x**6,x)
```

output
```
-sqrt(-a**3/b**5)*log(x - b**3*sqrt(-a**3/b**5)/a**2)/2 + sqrt(-a**3/b**5)
*log(x + b**3*sqrt(-a**3/b**5)/a**2)/2 + (3*a*x**2 - b)/(3*b**2*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = \frac{a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{3\,ax^2-b}{3\,b^2x^3}$$

input
```
integrate(1/(a+b/x^2)/x^6,x, algorithm="maxima")
```

output
```
a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(3*a*x^2 - b)/(b^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = \frac{a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{3\,ax^2-b}{3\,b^2x^3}$$

input
```
integrate(1/(a+b/x^2)/x^6,x, algorithm="giac")
```

output
```
a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(3*a*x^2 - b)/(b^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{5/2}} - \frac{\frac{1}{3\,b}-\frac{a\,x^2}{b^2}}{x^3}$$

input
```
int(1/(x^6*(a + b/x^2)),x)
```

output
```
(a^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/b^(5/2) - (1/(3*b) - (a*x^2)/b^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,x^3+3ab\,x^2-b^2}{3b^3x^3}$$

input
```
int(1/(a+b/x^2)/x^6,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*x**3 + 3*a*b*x**2 - b**
2)/(3*b**3*x**3)
```

3.317 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx=-\frac{1}{5bx^5}+\frac{a}{3b^2x^3}-\frac{a^2}{b^3x}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{7/2}}$$

output
```
-1/5/b/x^5+1/3*a/b^2/x^3-a^2/b^3/x-a^(5/2)*arctan(a^(1/2)*x/b^(1/2))/b^(7/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx=-\frac{1}{5bx^5}+\frac{a}{3b^2x^3}-\frac{a^2}{b^3x}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
Integrate[1/((a + b/x^2)*x^8),x]
```

output
```
-1/5*1/(b*x^5) + a/(3*b^2*x^3) - a^2/(b^3*x) - (a^(5/2)*ArcTan[(Sqrt[a]*x)
/Sqrt[b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {795, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a+\frac{b}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^6 \left(a x^2+b\right)}dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{a \int \frac{1}{x^4 \left(a x^2+b\right)}dx}{b}-\frac{1}{5 b x^5}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{a \left(-\frac{a \int \frac{1}{x^2 \left(a x^2+b\right)}dx}{b}-\frac{1}{3 b x^3}\right)}{b}-\frac{1}{5 b x^5}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{a \left(-\frac{a \left(-\frac{a \int \frac{1}{a x^2+b}dx}{b}-\frac{1}{b x}\right)}{b}-\frac{1}{3 b x^3}\right)}{b}-\frac{1}{5 b x^5}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{a \left(-\frac{a \left(-\frac{\sqrt{a} \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{b x}\right)}{b}-\frac{1}{3 b x^3}\right)}{b}-\frac{1}{5 b x^5}$$

input
```
Int[1/((a + b/x^2)*x^8),x]
```

output
```
-1/5*1/(b*x^5) - (a*(-1/3*1/(b*x^3) - (a*(-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sq
rt[a]*x)/Sqrt[b]])/b^(3/2)))/b))/b
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{3}\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}-\frac{1}{5b\,x^{5}}-\frac{a^{2}}{b^{3}x}+\frac{a}{3b^{2}x^{3}}$	52
risch	$\frac{-\frac{a^{2}x^{4}}{b^{3}}+\frac{a\,x^{2}}{3b^{2}}-\frac{1}{5b}}{x^{5}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{7}\_Z^{2}+a^{5}\right)}\_R\ln\left(\left(3\_R^{2}b^{7}+2a^{5}\right)x+a^{2}b^{4}\_R\right)\right)}{2}$	77

input
```
int(1/(a+b/x^2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-a^3/b^3/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))-1/5/b/x^5-a^2/b^3/x+1/3*a/b^2
/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx = \left[\frac{15\,a^2x^5\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-30\,a^2x^4+10\,abx^2-6\,b^2}{30\,b^3x^5},\ -\frac{15\,a^2x^5\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+15\,a^2x^4-5\,abx^2+3\,b^2}{15\,b^3x^5}\right]$$

input
```
integrate(1/(a+b/x^2)/x^8,x, algorithm="fricas")
```

output
```
[1/30*(15*a^2*x^5*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b
)) - 30*a^2*x^4 + 10*a*b*x^2 - 6*b^2)/(b^3*x^5), -1/15*(15*a^2*x^5*sqrt(a/
b)*arctan(x*sqrt(a/b)) + 15*a^2*x^4 - 5*a*b*x^2 + 3*b^2)/(b^3*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx = \frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x-\frac{b^4\sqrt{-\frac{a^5}{b^7}}}{a^3}\right)}{2}-\frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x+\frac{b^4\sqrt{-\frac{a^5}{b^7}}}{a^3}\right)}{2}+\frac{-15a^2x^4+5abx^2-3b^2}{15b^3x^5}$$

input
```
integrate(1/(a+b/x**2)/x**8,x)
```

output
```
sqrt(-a**5/b**7)*log(x - b**4*sqrt(-a**5/b**7)/a**3)/2 - sqrt(-a**5/b**7)*
log(x + b**4*sqrt(-a**5/b**7)/a**3)/2 + (-15*a**2*x**4 + 5*a*b*x**2 - 3*b*
*2)/(15*b**3*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx = -\frac{a^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{15\,a^2x^4 - 5\,abx^2 + 3\,b^2}{15\,b^3x^5}$$

input
```
integrate(1/(a+b/x^2)/x^8,x, algorithm="maxima")
```

output
```
-a^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/15*(15*a^2*x^4 - 5*a*b*x^2
+ 3*b^2)/(b^3*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx = -\frac{a^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{15\,a^2x^4 - 5\,abx^2 + 3\,b^2}{15\,b^3x^5}$$

input
```
integrate(1/(a+b/x^2)/x^8,x, algorithm="giac")
```

output
```
-a^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/15*(15*a^2*x^4 - 5*a*b*x^2
+ 3*b^2)/(b^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx = -\frac{\frac{1}{5\,b} - \frac{a\,x^2}{3\,b^2} + \frac{a^2\,x^4}{b^3}}{x^5} - \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
int(1/(x^8*(a + b/x^2)),x)
```

output
```
- (1/(5*b) - (a*x^2)/(3*b^2) + (a^2*x^4)/b^3)/x^5 - (a^(5/2)*atan((a^(1/2)
*x)/b^(1/2)))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^8}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2x^5 - 15a^2b\,x^4 + 5a\,b^2x^2 - 3b^3}{15b^4x^5}$$

input
```
int(1/(a+b/x^2)/x^8,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*x**5 - 15*a**2*b
*x**4 + 5*a*b**2*x**2 - 3*b**3)/(15*b**4*x**5)
```

3.318 $\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{3b^2x^2}{2a^4} - \frac{bx^4}{2a^3} + \frac{x^6}{6a^2} - \frac{b^4}{2a^5(b+ax^2)} - \frac{2b^3\log(b+ax^2)}{a^5}$$

output
```
3/2*b^2*x^2/a^4-1/2*b*x^4/a^3+1/6*x^6/a^2-1/2*b^4/a^5/(a*x^2+b)-2*b^3*ln(a
*x^2+b)/a^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{9ab^2x^2 - 3a^2bx^4 + a^3x^6 - \frac{3b^4}{b+ax^2} - 12b^3\log(b+ax^2)}{6a^5}$$

input
```
Integrate[x^5/(a + b/x^2)^2,x]
```

output
```
(9*a*b^2*x^2 - 3*a^2*b*x^4 + a^3*x^6 - (3*b^4)/(b + a*x^2) - 12*b^3*Log[b
 + a*x^2])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^9}{(ax^2+b)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^8}{(ax^2+b)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{b^4}{a^4(ax^2+b)^2}-\frac{4b^3}{a^4(ax^2+b)}+\frac{3b^2}{a^4}-\frac{2x^2b}{a^3}+\frac{x^4}{a^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^4}{a^5(ax^2+b)}-\frac{4b^3\log(ax^2+b)}{a^5}+\frac{3b^2x^2}{a^4}-\frac{bx^4}{a^3}+\frac{x^6}{3a^2}\right)$$

input
```
Int[x^5/(a + b/x^2)^2,x]
```

output
```
((3*b^2*x^2)/a^4 - (b*x^4)/a^3 + x^6/(3*a^2) - b^4/(a^5*(b + a*x^2)) - (4*
b^3*Log[b + a*x^2])/a^5)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{3b^2x^2}{2a^4}-\frac{b\,x^4}{2a^3}+\frac{x^6}{6a^2}-\frac{b^4}{2a^5(a\,x^2+b)}-\frac{2b^3\ln\left(a\,x^2+b\right)}{a^5}$	63
norman	$\frac{-\frac{2b^4}{a^5}+\frac{b^2x^4}{a^3}+\frac{x^8}{6a}-\frac{b\,x^6}{3a^2}}{a\,x^2+b}-\frac{2b^3\ln\left(a\,x^2+b\right)}{a^5}$	64
default	$\frac{\frac{1}{6}a^2x^6-\frac{1}{2}ab\,x^4+\frac{3}{2}b^2x^2}{a^4}-\frac{b^3\left(\frac{b}{a\left(a\,x^2+b\right)}+\frac{4\ln\left(a\,x^2+b\right)}{a}\right)}{2a^4}$	66
parallelrisch	$-\frac{-a^4x^8+2a^3b\,x^6-6a^2b^2x^4+12\ln\left(a\,x^2+b\right)x^2a\,b^3+12\ln\left(a\,x^2+b\right)b^4+12b^4}{6a^5\left(a\,x^2+b\right)}$	79

input
```
int(x^5/(a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2/a^4*b^2*x^2-1/2*b*x^4/a^3+1/6*x^6/a^2-1/2*b^4/a^5/(a*x^2+b)-2*b^3*ln(a
*x^2+b)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{a^4x^8 - 2\,a^3bx^6 + 6\,a^2b^2x^4 + 9\,ab^3x^2 - 3\,b^4 - 12\,(ab^3x^2 + b^4)\log\left(ax^2 + b\right)}{6\,(a^6x^2 + a^5b)}$$

input
```
integrate(x^5/(a+b/x^2)^2,x, algorithm="fricas")
```

output
```
1/6*(a^4*x^8 - 2*a^3*b*x^6 + 6*a^2*b^2*x^4 + 9*a*b^3*x^2 - 3*b^4 - 12*(a*b
^3*x^2 + b^4)*log(a*x^2 + b))/(a^6*x^2 + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^4}{2a^6x^2 + 2a^5b} + \frac{x^6}{6a^2} - \frac{bx^4}{2a^3} + \frac{3b^2x^2}{2a^4} - \frac{2b^3\log\left(ax^2 + b\right)}{a^5}$$

input
```
integrate(x**5/(a+b/x**2)**2,x)
```

output
```
-b**4/(2*a**6*x**2 + 2*a**5*b) + x**6/(6*a**2) - b*x**4/(2*a**3) + 3*b**2*
x**2/(2*a**4) - 2*b**3*log(a*x**2 + b)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^4}{2\,(a^6x^2 + a^5b)} - \frac{2\,b^3\log\left(ax^2 + b\right)}{a^5} + \frac{a^2x^6 - 3\,abx^4 + 9\,b^2x^2}{6\,a^4}$$

input
```
integrate(x^5/(a+b/x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b^4/(a^6*x^2 + a^5*b) - 2*b^3*log(a*x^2 + b)/a^5 + 1/6*(a^2*x^6 - 3*a
*b*x^4 + 9*b^2*x^2)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{2\,b^3\log\left(|ax^2+b|\right)}{a^5} + \frac{a^4x^6 - 3\,a^3bx^4 + 9\,a^2b^2x^2}{6\,a^6} + \frac{4\,ab^3x^2 + 3\,b^4}{2\,(ax^2+b)a^5}$$

input
```
integrate(x^5/(a+b/x^2)^2,x, algorithm="giac")
```

output
```
-2*b^3*log(abs(a*x^2 + b))/a^5 + 1/6*(a^4*x^6 - 3*a^3*b*x^4 + 9*a^2*b^2*x^
2)/a^6 + 1/2*(4*a*b^3*x^2 + 3*b^4)/((a*x^2 + b)*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{x^6}{6\,a^2} - \frac{b^4}{2\,a\,\left(a^5\,x^2+b\,a^4\right)} - \frac{b\,x^4}{2\,a^3} - \frac{2\,b^3\,\ln\left(a\,x^2+b\right)}{a^5} + \frac{3\,b^2\,x^2}{2\,a^4}$$

input
```
int(x^5/(a + b/x^2)^2,x)
```

output
```
x^6/(6*a^2) - b^4/(2*a*(a^4*b + a^5*x^2)) - (b*x^4)/(2*a^3) - (2*b^3*log(b
 + a*x^2))/a^5 + (3*b^2*x^2)/(2*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^2}\,dx$$
$$= \frac{-12\log(a\,x^2+b)\,a\,b^3x^2-12\log(a\,x^2+b)\,b^4+a^4x^8-2a^3b\,x^6+6a^2b^2x^4+12a\,b^3x^2}{6a^5\,(a\,x^2+b)}$$

input
```
int(x^5/(a+b/x^2)^2,x)
```

output
```
( - 12*log(a*x**2 + b)*a*b**3*x**2 - 12*log(a*x**2 + b)*b**4 + a**4*x**8 -
 2*a**3*b*x**6 + 6*a**2*b**2*x**4 + 12*a*b**3*x**2)/(6*a**5*(a*x**2 + b))
```

3.319 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2} dx = -\frac{bx^2}{a^3} + \frac{x^4}{4a^2} + \frac{b^3}{2a^4(b+ax^2)} + \frac{3b^2\log(b+ax^2)}{2a^4}$$

output
```
-b*x^2/a^3+1/4*x^4/a^2+1/2*b^3/a^4/(a*x^2+b)+3/2*b^2*ln(a*x^2+b)/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{-4abx^2 + a^2x^4 + \frac{2b^3}{b+ax^2} + 6b^2\log(b+ax^2)}{4a^4}$$

input
```
Integrate[x^3/(a + b/x^2)^2,x]
```

output
```
(-4*a*b*x^2 + a^2*x^4 + (2*b^3)/(b + a*x^2) + 6*b^2*Log[b + a*x^2])/(4*a^4
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^7}{(ax^2+b)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^6}{(ax^2+b)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(-\frac{b^3}{a^3(ax^2+b)^2}+\frac{3b^2}{a^3(ax^2+b)}-\frac{2b}{a^3}+\frac{x^2}{a^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{b^3}{a^4(ax^2+b)}+\frac{3b^2\log(ax^2+b)}{a^4}-\frac{2bx^2}{a^3}+\frac{x^4}{2a^2}\right)$$

input
```
Int[x^3/(a + b/x^2)^2,x]
```

output
```
((-2*b*x^2)/a^3 + x^4/(2*a^2) + b^3/(a^4*(b + a*x^2)) + (3*b^2*Log[b + a*x
^2])/a^4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\left(a\,x^{2}-2b\right)^{2}}{4a^{4}}+\frac{b^{2}\left(\frac{b}{a\left(a\,x^{2}+b\right)}+\frac{3\ln\left(a\,x^{2}+b\right)}{a}\right)}{2a^{3}}$	54
norman	$\frac{\frac{x^{6}}{4a}-\frac{3b\,x^{4}}{4a^{2}}+\frac{3b^{3}}{2a^{4}}}{a\,x^{2}+b}+\frac{3b^{2}\ln\left(a\,x^{2}+b\right)}{2a^{4}}$	54
risch	$\frac{x^{4}}{4a^{2}}-\frac{b\,x^{2}}{a^{3}}+\frac{b^{2}}{a^{4}}+\frac{b^{3}}{2a^{4}\left(a\,x^{2}+b\right)}+\frac{3b^{2}\ln\left(a\,x^{2}+b\right)}{2a^{4}}$	59
parallelrisch	$\frac{a^{3}x^{6}-3a^{2}b\,x^{4}+6\ln\left(a\,x^{2}+b\right)x^{2}a\,b^{2}+6b^{3}\ln\left(a\,x^{2}+b\right)+6b^{3}}{4a^{4}\left(a\,x^{2}+b\right)}$	67

input
```
int(x^3/(a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*x^2-2*b)^2/a^4+1/2*b^2/a^3*(b/a/(a*x^2+b)+3*ln(a*x^2+b)/a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{a^3x^6 - 3\,a^2bx^4 - 4\,ab^2x^2 + 2\,b^3 + 6\,(ab^2x^2 + b^3)\log\left(ax^2+b\right)}{4\,(a^5x^2 + a^4b)}$$

input
```
integrate(x^3/(a+b/x^2)^2,x, algorithm="fricas")
```

output
```
1/4*(a^3*x^6 - 3*a^2*b*x^4 - 4*a*b^2*x^2 + 2*b^3 + 6*(a*b^2*x^2 + b^3)*log
(a*x^2 + b))/(a^5*x^2 + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{b^3}{2a^5x^2 + 2a^4b} + \frac{x^4}{4a^2} - \frac{bx^2}{a^3} + \frac{3b^2\log\left(ax^2+b\right)}{2a^4}$$

input
```
integrate(x**3/(a+b/x**2)**2,x)
```

output
```
b**3/(2*a**5*x**2 + 2*a**4*b) + x**4/(4*a**2) - b*x**2/a**3 + 3*b**2*log(a
*x**2 + b)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{b^3}{2\,(a^5x^2 + a^4b)} + \frac{3\,b^2\log\left(ax^2+b\right)}{2\,a^4} + \frac{ax^4 - 4\,bx^2}{4\,a^3}$$

input
```
integrate(x^3/(a+b/x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^3/(a^5*x^2 + a^4*b) + 3/2*b^2*log(a*x^2 + b)/a^4 + 1/4*(a*x^4 - 4*b*
x^2)/a^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{3\,b^2\log\left(|ax^2+b|\right)}{2\,a^4} + \frac{b^3}{2\,(ax^2+b)a^4} + \frac{a^2x^4-4\,abx^2}{4\,a^4}$$

input
```
integrate(x^3/(a+b/x^2)^2,x, algorithm="giac")
```

output
```
3/2*b^2*log(abs(a*x^2 + b))/a^4 + 1/2*b^3/((a*x^2 + b)*a^4) + 1/4*(a^2*x^4
 - 4*a*b*x^2)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{x^4}{4\,a^2} + \frac{b^3}{2\,a\,\left(a^4\,x^2+b\,a^3\right)} - \frac{b\,x^2}{a^3} + \frac{3\,b^2\,\ln\left(a\,x^2+b\right)}{2\,a^4}$$

input
```
int(x^3/(a + b/x^2)^2,x)
```

output
```
x^4/(4*a^2) + b^3/(2*a*(a^3*b + a^4*x^2)) - (b*x^2)/a^3 + (3*b^2*log(b + a
*x^2))/(2*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{6\log(a\,x^2+b)\,a\,b^2x^2+6\log(a\,x^2+b)\,b^3+a^3x^6-3a^2b\,x^4-6a\,b^2x^2}{4a^4\,(a\,x^2+b)}$$

input
```
int(x^3/(a+b/x^2)^2,x)
```

output
```
(6*log(a*x**2 + b)*a*b**2*x**2 + 6*log(a*x**2 + b)*b**3 + a**3*x**6 - 3*a*
*2*b*x**4 - 6*a*b**2*x**2)/(4*a**4*(a*x**2 + b))
```

3.320 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{x^2}{2a^2} - \frac{b^2}{2a^3(b+ax^2)} - \frac{b\log(b+ax^2)}{a^3}$$

output
```
1/2*x^2/a^2-1/2*b^2/a^3/(a*x^2+b)-b*ln(a*x^2+b)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{ax^2 - \frac{b^2}{b+ax^2} - 2b\log(b+ax^2)}{2a^3}$$

input
```
Integrate[x/(a + b/x^2)^2,x]
```

output
```
(a*x^2 - b^2/(b + a*x^2) - 2*b*Log[b + a*x^2])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{x^5}{(ax^2+b)^2}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^4}{(ax^2+b)^2}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{b^2}{a^2(ax^2+b)^2}-\frac{2b}{a^2(ax^2+b)}+\frac{1}{a^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{b^2}{a^3(ax^2+b)}-\frac{2b\log(ax^2+b)}{a^3}+\frac{x^2}{a^2}\right)$$

input
```
Int[x/(a + b/x^2)^2,x]
```

output
```
(x^2/a^2 - b^2/(a^3*(b + a*x^2)) - (2*b*Log[b + a*x^2])/a^3)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{x^2}{2a^2}-\frac{b^2}{2a^3(a\,x^2+b)}-\frac{b\ln(a\,x^2+b)}{a^3}$	41
norman	$\frac{-\frac{b^2}{a^3}+\frac{x^4}{2a}}{a\,x^2+b}-\frac{b\ln(a\,x^2+b)}{a^3}$	43
default	$\frac{x^2}{2a^2}-\frac{b\left(\frac{b}{a\left(a\,x^2+b\right)}+\frac{2\ln\left(a\,x^2+b\right)}{a}\right)}{2a^2}$	44
parallelrisch	$-\frac{-a^2x^4+2\ln(a\,x^2+b)x^2ab+2b^2\ln(a\,x^2+b)+2b^2}{2a^3(a\,x^2+b)}$	57

input
```
int(x/(a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*x^2-1/2*b^2/a^3/(a*x^2+b)-b*ln(a*x^2+b)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{a^2x^4+abx^2-b^2-2\left(abx^2+b^2\right)\log\left(ax^2+b\right)}{2\left(a^4x^2+a^3b\right)}$$

input
```
integrate(x/(a+b/x^2)^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*x^4 + a*b*x^2 - b^2 - 2*(a*b*x^2 + b^2)*log(a*x^2 + b))/(a^4*x^2
+ a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^2}{2a^4x^2+2a^3b}+\frac{x^2}{2a^2}-\frac{b\log\left(ax^2+b\right)}{a^3}$$

input
```
integrate(x/(a+b/x**2)**2,x)
```

output
```
-b**2/(2*a**4*x**2 + 2*a**3*b) + x**2/(2*a**2) - b*log(a*x**2 + b)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^2}{2\left(a^4x^2+a^3b\right)}+\frac{x^2}{2\,a^2}-\frac{b\log\left(ax^2+b\right)}{a^3}$$

input
```
integrate(x/(a+b/x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b^2/(a^4*x^2 + a^3*b) + 1/2*x^2/a^2 - b*log(a*x^2 + b)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{x^2}{2\,a^2} - \frac{b\log\left(|ax^2+b|\right)}{a^3} - \frac{b^2}{2\,(ax^2+b)a^3}$$

input
```
integrate(x/(a+b/x^2)^2,x, algorithm="giac")
```

output
```
1/2*x^2/a^2 - b*log(abs(a*x^2 + b))/a^3 - 1/2*b^2/((a*x^2 + b)*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{x^2}{2\,a^2} - \frac{b^2}{2\,\left(a^4\,x^2+b\,a^3\right)} - \frac{b\,\ln\left(a\,x^2+b\right)}{a^3}$$

input
```
int(x/(a + b/x^2)^2,x)
```

output
```
x^2/(2*a^2) - b^2/(2*(a^3*b + a^4*x^2)) - (b*log(b + a*x^2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{-2\log(a\,x^2+b)\,ab\,x^2 - 2\log(a\,x^2+b)\,b^2 + a^2x^4 + 2ab\,x^2}{2a^3\,(a\,x^2+b)}$$

input
```
int(x/(a+b/x^2)^2,x)
```

output
```
( - 2*log(a*x**2 + b)*a*b*x**2 - 2*log(a*x**2 + b)*b**2 + a**2*x**4 + 2*a*
b*x**2)/(2*a**3*(a*x**2 + b))
```

3.321 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x} dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x} dx = \frac{b}{2a^2(b+ax^2)} + \frac{\log(b+ax^2)}{2a^2}$$

output
```
1/2*b/a^2/(a*x^2+b)+1/2*ln(a*x^2+b)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x} dx = \frac{\frac{b}{b+ax^2} + \log(b+ax^2)}{2a^2}$$

input
```
Integrate[1/((a + b/x^2)^2*x),x]
```

output
```
(b/(b + a*x^2) + Log[b + a*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^3}{(ax^2+b)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(ax^2+b)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{1}{a(ax^2+b)}-\frac{b}{a(ax^2+b)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b}{a^2(ax^2+b)}+\frac{\log(ax^2+b)}{a^2}\right)$$

input
```
Int[1/((a + b/x^2)^2*x),x]
```

output
```
(b/(a^2*(b + a*x^2)) + Log[b + a*x^2]/a^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b}{2a^2(a\,x^2+b)}+\frac{\ln(a\,x^2+b)}{2a^2}$	30
norman	$\frac{b}{2a^2(a\,x^2+b)}+\frac{\ln(a\,x^2+b)}{2a^2}$	30
risch	$\frac{b}{2a^2(a\,x^2+b)}+\frac{\ln(a\,x^2+b)}{2a^2}$	30
parallelrisch	$\frac{a\ln(a\,x^2+b)x^2+b\ln(a\,x^2+b)+b}{2a^2(a\,x^2+b)}$	40

input
```
int(1/(a+b/x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*b/a^2/(a*x^2+b)+1/2*ln(a*x^2+b)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx = \frac{(ax^2+b)\log\left(ax^2+b\right)+b}{2\left(a^3x^2+a^2b\right)}$$

input
```
integrate(1/(a+b/x^2)^2/x,x, algorithm="fricas")
```

output
```
1/2*((a*x^2 + b)*log(a*x^2 + b) + b)/(a^3*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx = \frac{b}{2a^3x^2+2a^2b}+\frac{\log\left(ax^2+b\right)}{2a^2}$$

input
```
integrate(1/(a+b/x**2)**2/x,x)
```

output
```
b/(2*a**3*x**2 + 2*a**2*b) + log(a*x**2 + b)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx = \frac{b}{2\left(a^3x^2+a^2b\right)}+\frac{\log\left(ax^2+b\right)}{2\,a^2}$$

input
```
integrate(1/(a+b/x^2)^2/x,x, algorithm="maxima")
```

output
```
1/2*b/(a^3*x^2 + a^2*b) + 1/2*log(a*x^2 + b)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx = -\frac{x^2}{2\,(ax^2+b)a} + \frac{\log\left(|ax^2+b|\right)}{2\,a^2}$$

input
```
integrate(1/(a+b/x^2)^2/x,x, algorithm="giac")
```

output
```
-1/2*x^2/((a*x^2 + b)*a) + 1/2*log(abs(a*x^2 + b))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx = \frac{\ln\left(a\,x^2+b\right)}{2\,a^2} + \frac{b}{2\,a^2\,\left(a\,x^2+b\right)}$$

input
```
int(1/(x*(a + b/x^2)^2),x)
```

output
```
log(b + a*x^2)/(2*a^2) + b/(2*a^2*(b + a*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x}\,dx = \frac{\log(a\,x^2+b)\,a\,x^2+\log(a\,x^2+b)\,b-a\,x^2}{2a^2\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^2/x,x)
```

output
```
(log(a*x**2 + b)*a*x**2 + log(a*x**2 + b)*b - a*x**2)/(2*a**2*(a*x**2 + b)
)
```

3.322 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3} \, dx = \frac{1}{2b\left(a+\frac{b}{x^2}\right)}$$

output
```
1/2/b/(a+b/x^2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3} \, dx = -\frac{1}{2a\left(b+ax^2\right)}$$

input
```
Integrate[1/((a + b/x^2)^2*x^3),x]
```

output
```
-1/2*1/(a*(b + a*x^2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x^2}\right)^2} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{2b \left(a+\frac{b}{x^2}\right)}$$

input
```
Int[1/((a + b/x^2)^2*x^3),x]
```

output
```
1/(2*b*(a + b/x^2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{2(a\,x^2+b)a}$	15
derivativedivides	$\frac{1}{2b\left(a+\frac{b}{x^2}\right)}$	15
default	$-\frac{1}{2(a\,x^2+b)a}$	15
norman	$-\frac{1}{2(a\,x^2+b)a}$	15
risch	$-\frac{1}{2(a\,x^2+b)a}$	15
parallelrisch	$-\frac{1}{2(a\,x^2+b)a}$	15
orering	$-\frac{a\,x^2+b}{2a\,x^4\left(a+\frac{b}{x^2}\right)^2}$	25

input
```
int(1/(a+b/x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*x^2+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx = -\frac{1}{2\,(a^2x^2+ab)}$$

input
```
integrate(1/(a+b/x^2)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2/(a^2*x^2 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx = -\frac{1}{2a^2x^2+2ab}$$

input
```
integrate(1/(a+b/x**2)**2/x**3,x)
```

output
```
-1/(2*a**2*x**2 + 2*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx = \frac{1}{2\left(a+\frac{b}{x^2}\right)b}$$

input
```
integrate(1/(a+b/x^2)^2/x^3,x, algorithm="maxima")
```

output
```
1/2/((a + b/x^2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx = -\frac{1}{2\left(ax^2+b\right)a}$$

input
```
integrate(1/(a+b/x^2)^2/x^3,x, algorithm="giac")
```

output
```
-1/2/((a*x^2 + b)*a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx = -\frac{1}{2\,a\,\left(a\,x^2+b\right)}$$

input
```
int(1/(x^3*(a + b/x^2)^2),x)
```

output
```
-1/(2*a*(b + a*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^3}\,dx = \frac{x^2}{2b\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^2/x^3,x)
```

output
```
x**2/(2*b*(a*x**2 + b))
```

3.323 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx = -\frac{a}{2b^2\left(a+\frac{b}{x^2}\right)} - \frac{\log\left(a+\frac{b}{x^2}\right)}{2b^2}$$

output
```
-1/2*a/b^2/(a+b/x^2)-1/2*ln(a+b/x^2)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx = \frac{\frac{b}{b+ax^2} + 2\log(x) - \log\left(b+ax^2\right)}{2b^2}$$

input
```
Integrate[1/((a + b/x^2)^2*x^5),x]
```

output
```
(b/(b + a*x^2) + 2*Log[x] - Log[b + a*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x^2}\right)^2} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x \left(a x^2+b\right)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^2 \left(a x^2+b\right)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(-\frac{a}{b^2 \left(a x^2+b\right)}-\frac{a}{b \left(a x^2+b\right)^2}+\frac{1}{b^2 x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{\log \left(a x^2+b\right)}{b^2}+\frac{1}{b \left(a x^2+b\right)}+\frac{\log \left(x^2\right)}{b^2}\right)$$

input
```
Int[1/((a + b/x^2)^2*x^5),x]
```

output
```
(1/(b*(b + a*x^2)) + Log[x^2]/b^2 - Log[b + a*x^2]/b^2)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
risch	$\frac{1}{2b(a\,x^2+b)}+\frac{\ln(x)}{b^2}-\frac{\ln(a\,x^2+b)}{2b^2}$	35
norman	$-\frac{a\,x^2}{2b^2(a\,x^2+b)}+\frac{\ln(x)}{b^2}-\frac{\ln(a\,x^2+b)}{2b^2}$	39
default	$-\frac{a\left(-\frac{b}{a\left(a\,x^2+b\right)}+\frac{\ln\left(a\,x^2+b\right)}{a}\right)}{2b^2}+\frac{\ln(x)}{b^2}$	42
parallelrisch	$\frac{2a\ln(x)x^2-a\ln(a\,x^2+b)x^2-a\,x^2+2b\ln(x)-b\ln(a\,x^2+b)}{2b^2(a\,x^2+b)}$	60

input
```
int(1/(a+b/x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(a*x^2+b)+1/b^2*ln(x)-1/2/b^2*ln(a*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5} \, dx = -\frac{(ax^2+b)\log(ax^2+b)-2(ax^2+b)\log(x)-b}{2(ab^2x^2+b^3)}$$

input
```
integrate(1/(a+b/x^2)^2/x^5,x, algorithm="fricas")
```

output
```
-1/2*((a*x^2 + b)*log(a*x^2 + b) - 2*(a*x^2 + b)*log(x) - b)/(a*b^2*x^2 +
b^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5} \, dx = \frac{1}{2abx^2+2b^2} + \frac{\log(x)}{b^2} - \frac{\log\left(x^2+\frac{b}{a}\right)}{2b^2}$$

input
```
integrate(1/(a+b/x**2)**2/x**5,x)
```

output
```
1/(2*a*b*x**2 + 2*b**2) + log(x)/b**2 - log(x**2 + b/a)/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5} \, dx = \frac{1}{2(abx^2+b^2)} - \frac{\log(ax^2+b)}{2b^2} + \frac{\log(x^2)}{2b^2}$$

input
```
integrate(1/(a+b/x^2)^2/x^5,x, algorithm="maxima")
```

output
```
1/2/(a*b*x^2 + b^2) - 1/2*log(a*x^2 + b)/b^2 + 1/2*log(x^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx = \frac{\log\left(x^2\right)}{2\,b^2} - \frac{\log\left(|ax^2+b|\right)}{2\,b^2} + \frac{ax^2+2\,b}{2\,(ax^2+b)b^2}$$

input
```
integrate(1/(a+b/x^2)^2/x^5,x, algorithm="giac")
```

output
```
1/2*log(x^2)/b^2 - 1/2*log(abs(a*x^2 + b))/b^2 + 1/2*(a*x^2 + 2*b)/((a*x^2
 + b)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx = \frac{\ln(x)}{b^2} + \frac{1}{2\,b\,\left(a\,x^2+b\right)} - \frac{\ln\left(a\,x^2+b\right)}{2\,b^2}$$

input
```
int(1/(x^5*(a + b/x^2)^2),x)
```

output
```
log(x)/b^2 + 1/(2*b*(b + a*x^2)) - log(b + a*x^2)/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^5}\,dx = \frac{-\log(a\,x^2+b)\,a\,x^2-\log(a\,x^2+b)\,b+2\log(x)\,a\,x^2+2\log(x)\,b-a\,x^2}{2b^2\left(a\,x^2+b\right)}$$

input
```
int(1/(a+b/x^2)^2/x^5,x)
```

output
```
( - log(a*x**2 + b)*a*x**2 - log(a*x**2 + b)*b + 2*log(x)*a*x**2 + 2*log(x
)*b - a*x**2)/(2*b**2*(a*x**2 + b))
```

3.324 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx=\frac{a^2}{2b^3\left(a+\frac{b}{x^2}\right)}-\frac{1}{2b^2x^2}+\frac{a\log\left(a+\frac{b}{x^2}\right)}{b^3}$$

output
```
1/2*a^2/b^3/(a+b/x^2)-1/2/b^2/x^2+a*ln(a+b/x^2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx=-\frac{b\left(\frac{1}{x^2}+\frac{a}{b+ax^2}\right)+4a\log(x)-2a\log\left(b+ax^2\right)}{2b^3}$$

input
```
Integrate[1/((a + b/x^2)^2*x^7),x]
```

output
```
-1/2*(b*(x^(-2) + a/(b + a*x^2)) + 4*a*Log[x] - 2*a*Log[b + a*x^2])/b^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^2}\right)^2} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^3 \left(a x^2+b\right)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a x^2+b\right)^2}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(\frac{2 a^2}{b^3 \left(a x^2+b\right)}+\frac{a^2}{b^2 \left(a x^2+b\right)^2}-\frac{2 a}{b^3 x^2}+\frac{1}{b^2 x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{2 a \log \left(x^2\right)}{b^3}+\frac{2 a \log \left(a x^2+b\right)}{b^3}-\frac{a}{b^2 \left(a x^2+b\right)}-\frac{1}{b^2 x^2}\right)$$

input
```
Int[1/((a + b/x^2)^2*x^7),x]
```

output
```
(-(1/(b^2*x^2)) - a/(b^2*(b + a*x^2)) - (2*a*Log[x^2])/b^3 + (2*a*Log[b +
a*x^2])/b^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{-\frac{x^{4}}{2 b}-\frac{a\,x^{6}}{b^{2}}}{x^{6} \left(a\,x^{2}+b \right)}+\frac{a \ln \left(a\,x^{2}+b \right)}{b^{3}}-\frac{2 a \ln \left(x \right)}{b^{3}}$	54
risch	$\frac{-\frac{a\,x^{2}}{b^{2}}-\frac{1}{2 b}}{\left(a\,x^{2}+b \right) x^{2}}-\frac{2 a \ln \left(x \right)}{b^{3}}+\frac{a \ln \left(-a\,x^{2}-b \right)}{b^{3}}$	54
default	$\frac{a^{2} \left(-\frac{b}{a \left(a\,x^{2}+b \right)}+\frac{2 \ln \left(a\,x^{2}+b \right)}{a}\right)}{2 b^{3}}-\frac{1}{2 b^{2} x^{2}}-\frac{2 a \ln \left(x \right)}{b^{3}}$	55
parallelrisch	$-\frac{4 a^{2} \ln \left(x \right) x^{4}-2 a^{2} \ln \left(a\,x^{2}+b \right) x^{4}-2 a^{2} x^{4}+4 a b \ln \left(x \right) x^{2}-2 \ln \left(a\,x^{2}+b \right) x^{2} a b +b^{2}}{2 b^{3} x^{2} \left(a\,x^{2}+b \right)}$	80

input
```
int(1/(a+b/x^2)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^4/b-a/b^2*x^6)/x^6/(a*x^2+b)+1/b^3*a*ln(a*x^2+b)-2/b^3*a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx = -\frac{2\,abx^2+b^2-2\,(a^2x^4+abx^2)\log\left(ax^2+b\right)+4\,(a^2x^4+abx^2)\log\left(x\right)}{2\,(ab^3x^4+b^4x^2)}$$

input
```
integrate(1/(a+b/x^2)^2/x^7,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*x^2 + b^2 - 2*(a^2*x^4 + a*b*x^2)*log(a*x^2 + b) + 4*(a^2*x^4
+ a*b*x^2)*log(x))/(a*b^3*x^4 + b^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx = -\frac{2a\log\left(x\right)}{b^3}+\frac{a\log\left(x^2+\frac{b}{a}\right)}{b^3}+\frac{-2ax^2-b}{2ab^2x^4+2b^3x^2}$$

input
```
integrate(1/(a+b/x**2)**2/x**7,x)
```

output
```
-2*a*log(x)/b**3 + a*log(x**2 + b/a)/b**3 + (-2*a*x**2 - b)/(2*a*b**2*x**4
 + 2*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx = -\frac{2\,ax^2+b}{2\,(ab^2x^4+b^3x^2)}+\frac{a\log\left(ax^2+b\right)}{b^3}-\frac{a\log\left(x^2\right)}{b^3}$$

input
```
integrate(1/(a+b/x^2)^2/x^7,x, algorithm="maxima")
```

output
```
-1/2*(2*a*x^2 + b)/(a*b^2*x^4 + b^3*x^2) + a*log(a*x^2 + b)/b^3 - a*log(x^
2)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx = -\frac{a\log\left(x^2\right)}{b^3} + \frac{a\log\left(|ax^2+b|\right)}{b^3} - \frac{2\,ax^2+b}{2\,(ax^4+bx^2)b^2}$$

input
```
integrate(1/(a+b/x^2)^2/x^7,x, algorithm="giac")
```

output
```
-a*log(x^2)/b^3 + a*log(abs(a*x^2 + b))/b^3 - 1/2*(2*a*x^2 + b)/((a*x^4 +
b*x^2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx = \frac{a\,\ln\left(a\,x^2+b\right)}{b^3} - \frac{\frac{1}{2\,b}+\frac{a\,x^2}{b^2}}{a\,x^4+b\,x^2} - \frac{2\,a\,\ln\left(x\right)}{b^3}$$

input
```
int(1/(x^7*(a + b/x^2)^2),x)
```

output
```
(a*log(b + a*x^2))/b^3 - (1/(2*b) + (a*x^2)/b^2)/(a*x^4 + b*x^2) - (2*a*lo
g(x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^7}\,dx$$
$$= \frac{2\log(a\,x^2+b)\,a^2x^4+2\log(a\,x^2+b)\,ab\,x^2-4\log(x)\,a^2x^4-4\log(x)\,ab\,x^2+2a^2x^4-b^2}{2b^3x^2\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^2/x^7,x)
```

output
```
(2*log(a*x**2 + b)*a**2*x**4 + 2*log(a*x**2 + b)*a*b*x**2 - 4*log(x)*a**2*
x**4 - 4*log(x)*a*b*x**2 + 2*a**2*x**4 - b**2)/(2*b**3*x**2*(a*x**2 + b))
```

3.325 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9} \, dx = -\frac{a^3}{2b^4\left(a+\frac{b}{x^2}\right)} - \frac{1}{4b^2x^4} + \frac{a}{b^3x^2} - \frac{3a^2\log\left(a+\frac{b}{x^2}\right)}{2b^4}$$

output
```
-1/2*a^3/b^4/(a+b/x^2)-1/4/b^2/x^4+a/b^3/x^2-3/2*a^2*ln(a+b/x^2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9} \, dx = \frac{b\left(-\frac{b}{x^4}+\frac{4a}{x^2}+\frac{2a^2}{b+ax^2}\right)+12a^2\log(x)-6a^2\log\left(b+ax^2\right)}{4b^4}$$

input
```
Integrate[1/((a + b/x^2)^2*x^9),x]
```

output
```
(b*(-(b/x^4) + (4*a)/x^2 + (2*a^2)/(b + a*x^2)) + 12*a^2*Log[x] - 6*a^2*Lo
g[b + a*x^2])/(4*b^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+\frac{b}{x^2}\right)^2} dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^5 \left(ax^2+b\right)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^6 \left(ax^2+b\right)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(-\frac{3a^3}{b^4 \left(ax^2+b\right)}-\frac{a^3}{b^3 \left(ax^2+b\right)^2}+\frac{3a^2}{b^4x^2}-\frac{2a}{b^3x^4}+\frac{1}{b^2x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3a^2 \log \left(x^2\right)}{b^4}-\frac{3a^2 \log \left(ax^2+b\right)}{b^4}+\frac{a^2}{b^3 \left(ax^2+b\right)}+\frac{2a}{b^3x^2}-\frac{1}{2b^2x^4}\right)$$

input
```
Int[1/((a + b/x^2)^2*x^9),x]
```

output
```
(-1/2*1/(b^2*x^4) + (2*a)/(b^3*x^2) + a^2/(b^3*(b + a*x^2)) + (3*a^2*Log[x
^2])/b^4 - (3*a^2*Log[b + a*x^2])/b^4)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{a^{3}\left(-\frac{b}{a\left(a\,x^{2}+b\right)}+\frac{3\ln\left(a\,x^{2}+b\right)}{a}\right)}{2b^{4}}-\frac{1}{4b^{2}x^{4}}+\frac{a}{b^{3}x^{2}}+\frac{3a^{2}\ln\left(x\right)}{b^{4}}$	65
risch	$\frac{\frac{3a^{2}x^{4}}{2b^{3}}+\frac{3a\,x^{2}}{4b^{2}}-\frac{1}{4b}}{\left(a\,x^{2}+b\right)x^{4}}+\frac{3a^{2}\ln\left(x\right)}{b^{4}}-\frac{3a^{2}\ln\left(a\,x^{2}+b\right)}{2b^{4}}$	67
norman	$\frac{-\frac{x^{4}}{4b}+\frac{3a\,x^{6}}{4b^{2}}-\frac{3a^{3}x^{10}}{2b^{4}}}{\left(a\,x^{2}+b\right)x^{8}}+\frac{3a^{2}\ln\left(x\right)}{b^{4}}-\frac{3a^{2}\ln\left(a\,x^{2}+b\right)}{2b^{4}}$	70
parallelrisch	$\frac{12a^{3}\ln\left(x\right)x^{6}-6\ln\left(a\,x^{2}+b\right)x^{6}a^{3}-6a^{3}x^{6}+12a^{2}b\ln\left(x\right)x^{4}-6\ln\left(a\,x^{2}+b\right)x^{4}a^{2}b+3a\,b^{2}x^{2}-b^{3}}{4b^{4}x^{4}\left(a\,x^{2}+b\right)}$	95

input
```
int(1/(a+b/x^2)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/b^4*(-b/a/(a*x^2+b)+3*ln(a*x^2+b)/a)-1/4/b^2/x^4+a/b^3/x^2+3*a^2/
b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9} \, dx = \frac{6\,a^2bx^4 + 3\,ab^2x^2 - b^3 - 6\,(a^3x^6 + a^2bx^4)\log\left(ax^2+b\right) + 12\,(a^3x^6 + a^2bx^4)\log\left(x\right)}{4\,(ab^4x^6 + b^5x^4)}$$

input
```
integrate(1/(a+b/x^2)^2/x^9,x, algorithm="fricas")
```

output
```
1/4*(6*a^2*b*x^4 + 3*a*b^2*x^2 - b^3 - 6*(a^3*x^6 + a^2*b*x^4)*log(a*x^2 +
 b) + 12*(a^3*x^6 + a^2*b*x^4)*log(x))/(a*b^4*x^6 + b^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9} \, dx = \frac{3a^2\log\left(x\right)}{b^4} - \frac{3a^2\log\left(x^2+\frac{b}{a}\right)}{2b^4} + \frac{6a^2x^4+3abx^2-b^2}{4ab^3x^6+4b^4x^4}$$

input
```
integrate(1/(a+b/x**2)**2/x**9,x)
```

output
```
3*a**2*log(x)/b**4 - 3*a**2*log(x**2 + b/a)/(2*b**4) + (6*a**2*x**4 + 3*a*
b*x**2 - b**2)/(4*a*b**3*x**6 + 4*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9} \, dx = \frac{6\,a^2x^4+3\,abx^2-b^2}{4\,(ab^3x^6+b^4x^4)} - \frac{3\,a^2\log\left(ax^2+b\right)}{2\,b^4} + \frac{3\,a^2\log\left(x^2\right)}{2\,b^4}$$

input
```
integrate(1/(a+b/x^2)^2/x^9,x, algorithm="maxima")
```

output
```
1/4*(6*a^2*x^4 + 3*a*b*x^2 - b^2)/(a*b^3*x^6 + b^4*x^4) - 3/2*a^2*log(a*x^
2 + b)/b^4 + 3/2*a^2*log(x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.54

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9}\,dx = \frac{3\,a^2\log\left(x^2\right)}{2\,b^4} - \frac{3\,a^2\log\left(|ax^2+b|\right)}{2\,b^4} + \frac{3\,a^3x^2+4\,a^2b}{2\,(ax^2+b)b^4} - \frac{9\,a^2x^4-4\,abx^2+b^2}{4\,b^4x^4}$$

input
```
integrate(1/(a+b/x^2)^2/x^9,x, algorithm="giac")
```

output
```
3/2*a^2*log(x^2)/b^4 - 3/2*a^2*log(abs(a*x^2 + b))/b^4 + 1/2*(3*a^3*x^2 +
4*a^2*b)/((a*x^2 + b)*b^4) - 1/4*(9*a^2*x^4 - 4*a*b*x^2 + b^2)/(b^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9}\,dx = \frac{\frac{3\,a\,x^2}{4\,b^2}-\frac{1}{4\,b}+\frac{3\,a^2\,x^4}{2\,b^3}}{a\,x^6+b\,x^4} - \frac{3\,a^2\,\ln\left(a\,x^2+b\right)}{2\,b^4} + \frac{3\,a^2\,\ln\left(x\right)}{b^4}$$

input
```
int(1/(x^9*(a + b/x^2)^2),x)
```

output
```
((3*a*x^2)/(4*b^2) - 1/(4*b) + (3*a^2*x^4)/(2*b^3))/(a*x^6 + b*x^4) - (3*a
^2*log(b + a*x^2))/(2*b^4) + (3*a^2*log(x))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^9} \, dx = \frac{-6\log(a\,x^2+b)\,a^3x^6-6\log(a\,x^2+b)\,a^2b\,x^4+12\log(x)\,a^3x^6+12\log(x)\,a^2b\,x^4-6a^3x^6+3a\,b^2x^2-b^3}{4b^4x^4\,(a\,x^2+b)}$$

input

```
int(1/(a+b/x^2)^2/x^9,x)
```

output

```
( - 6*log(a*x**2 + b)*a**3*x**6 - 6*log(a*x**2 + b)*a**2*b*x**4 + 12*log(x
)*a**3*x**6 + 12*log(x)*a**2*b*x**4 - 6*a**3*x**6 + 3*a*b**2*x**2 - b**3)/
(4*b**4*x**4*(a*x**2 + b))
```

3.326 $\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx=-\frac{4b^3x}{a^5}+\frac{b^2x^3}{a^4}-\frac{2bx^5}{5a^3}+\frac{x^7}{7a^2}-\frac{b^4x}{2a^5\left(b+ax^2\right)}+\frac{9b^{7/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{11/2}}$$

output
```
-4*b^3*x/a^5+b^2*x^3/a^4-2/5*b*x^5/a^3+1/7*x^7/a^2-1/2*b^4*x/a^5/(a*x^2+b)
+9/2*b^(7/2)*arctan(a^(1/2)*x/b^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx=\frac{x\left(-280b^3+70ab^2x^2-28a^2bx^4+10a^3x^6-\frac{35b^4}{b+ax^2}\right)}{70a^5}+\frac{9b^{7/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{11/2}}$$

input
```
Integrate[x^6/(a + b/x^2)^2,x]
```

output
```
(x*(-280*b^3 + 70*a*b^2*x^2 - 28*a^2*b*x^4 + 10*a^3*x^6 - (35*b^4)/(b + a*
x^2)))/(70*a^5) + (9*b^(7/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*a^(11/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^{10}}{\left(ax^2+b\right)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{9\int \frac{x^8}{ax^2+b}dx}{2a}-\frac{x^9}{2a\left(ax^2+b\right)}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{9\int \left(\frac{x^6}{a}-\frac{bx^4}{a^2}+\frac{b^2x^2}{a^3}-\frac{b^3}{a^4}+\frac{b^4}{a^4\left(ax^2+b\right)}\right)dx}{2a}-\frac{x^9}{2a\left(ax^2+b\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{9\left(\frac{b^{7/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{9/2}}-\frac{b^3x}{a^4}+\frac{b^2x^3}{3a^3}-\frac{bx^5}{5a^2}+\frac{x^7}{7a}\right)}{2a}-\frac{x^9}{2a\left(ax^2+b\right)}$$

input
```
Int[x^6/(a + b/x^2)^2,x]
```

output
```
-1/2*x^9/(a*(b + a*x^2)) + (9*(-((b^3*x)/a^4) + (b^2*x^3)/(3*a^3) - (b*x^5
)/(5*a^2) + x^7/(7*a) + (b^(7/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(9/2)))/(2
*a)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{1}{7}a^{3}x^{7}-\frac{2}{5}a^{2}b\,x^{5}+a\,b^{2}x^{3}-4b^{3}x}{a^{5}}+\frac{b^{4}\left(-\frac{x}{2\left(a\,x^{2}+b\right)}+\frac{9\arctan\left(\frac{a x}{\sqrt{a b}}\right)}{2\sqrt{a b}}\right)}{a^{5}}$	74
risch	$\frac{x^{7}}{7a^{2}}-\frac{2b\,x^{5}}{5a^{3}}+\frac{b^{2}x^{3}}{a^{4}}-\frac{4b^{3}x}{a^{5}}-\frac{b^{4}x}{2a^{5}(a\,x^{2}+b)}+\frac{9\sqrt{-a b}\,b^{3}\ln\left(-\sqrt{-a b}\,x+b\right)}{4a^{6}}-\frac{9\sqrt{-a b}\,b^{3}\ln\left(\sqrt{-a b}\,x+b\right)}{4a^{6}}$	107

input
```
int(x^6/(a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/7*a^3*x^7-2/5*a^2*b*x^5+a*b^2*x^3-4*b^3*x)+b^4/a^5*(-1/2*x/(a*x^2
+b)+9/2/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.41

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \left[\frac{20\,a^4x^9-36\,a^3bx^7+84\,a^2b^2x^5-420\,ab^3x^3-630\,b^4x+315\,(ab^3x^2+b^4)\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{140\,(a^6x^2+a^5b)}, \; 1 \ldots\right.$$

input
```
integrate(x^6/(a+b/x^2)^2,x, algorithm="fricas")
```

output
```
[1/140*(20*a^4*x^9 - 36*a^3*b*x^7 + 84*a^2*b^2*x^5 - 420*a*b^3*x^3 - 630*b
^4*x + 315*(a*b^3*x^2 + b^4)*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)
/(a*x^2 + b)))/(a^6*x^2 + a^5*b), 1/70*(10*a^4*x^9 - 18*a^3*b*x^7 + 42*a^2
*b^2*x^5 - 210*a*b^3*x^3 - 315*b^4*x + 315*(a*b^3*x^2 + b^4)*sqrt(b/a)*arc
tan(a*x*sqrt(b/a)/b))/(a^6*x^2 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^4x}{2a^6x^2+2a^5b} - \frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(-\frac{a^5\sqrt{-\frac{b^7}{a^{11}}}}{b^3}+x\right)}{4} + \frac{9\sqrt{-\frac{b^7}{a^{11}}}\log\left(\frac{a^5\sqrt{-\frac{b^7}{a^{11}}}}{b^3}+x\right)}{4} + \frac{x^7}{7a^2} - \frac{2bx^5}{5a^3} + \frac{b^2x^3}{a^4} - \frac{4b^3x}{a^5}$$

input
```
integrate(x**6/(a+b/x**2)**2,x)
```

output
```
-b**4*x/(2*a**6*x**2 + 2*a**5*b) - 9*sqrt(-b**7/a**11)*log(-a**5*sqrt(-b**
7/a**11)/b**3 + x)/4 + 9*sqrt(-b**7/a**11)*log(a**5*sqrt(-b**7/a**11)/b**3
 + x)/4 + x**7/(7*a**2) - 2*b*x**5/(5*a**3) + b**2*x**3/a**4 - 4*b**3*x/a*
*5
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^4x}{2\,(a^6x^2+a^5b)} + \frac{9\,b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5} + \frac{5\,a^3x^7-14\,a^2bx^5+35\,ab^2x^3-140\,b^3x}{35\,a^5}$$

input
```
integrate(x^6/(a+b/x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b^4*x/(a^6*x^2 + a^5*b) + 9/2*b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^
5) + 1/35*(5*a^3*x^7 - 14*a^2*b*x^5 + 35*a*b^2*x^3 - 140*b^3*x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{9\,b^4\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5} - \frac{b^4x}{2\,(ax^2+b)a^5} + \frac{5\,a^{12}x^7-14\,a^{11}bx^5+35\,a^{10}b^2x^3-140\,a^9b^3x}{35\,a^{14}}$$

input
```
integrate(x^6/(a+b/x^2)^2,x, algorithm="giac")
```

output
```
9/2*b^4*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/2*b^4*x/((a*x^2 + b)*a^5
) + 1/35*(5*a^12*x^7 - 14*a^11*b*x^5 + 35*a^10*b^2*x^3 - 140*a^9*b^3*x)/a^
14
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx=\frac{x^7}{7\,a^2}-\frac{2\,b\,x^5}{5\,a^3}-\frac{4\,b^3\,x}{a^5}+\frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{11/2}}+\frac{b^2\,x^3}{a^4}-\frac{b^4\,x}{2\,\left(a^6\,x^2+b\,a^5\right)}$$

input
```
int(x^6/(a + b/x^2)^2,x)
```

output
```
x^7/(7*a^2) - (2*b*x^5)/(5*a^3) - (4*b^3*x)/a^5 + (9*b^(7/2)*atan((a^(1/2)
*x)/b^(1/2)))/(2*a^(11/2)) + (b^2*x^3)/a^4 - (b^4*x)/(2*(a^5*b + a^6*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^2}\,dx$$
$$=\frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,b^3x^2+315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^4+10a^5x^9-18a^4b\,x^7+42a^3b^2x^5-210a^2b^3x^3}{70a^6\left(a\,x^2+b\right)}$$

input
```
int(x^6/(a+b/x^2)^2,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b**3*x**2 + 315*sqrt(
b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b**4 + 10*a**5*x**9 - 18*a**4*b*x
**7 + 42*a**3*b**2*x**5 - 210*a**2*b**3*x**3 - 315*a*b**4*x)/(70*a**6*(a*x
**2 + b))
```

3.327 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{3b^2x}{a^4} - \frac{2bx^3}{3a^3} + \frac{x^5}{5a^2} + \frac{b^3x}{2a^4(b+ax^2)} - \frac{7b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{9/2}}$$

output
```
3*b^2*x/a^4-2/3*b*x^3/a^3+1/5*x^5/a^2+1/2*b^3*x/a^4/(a*x^2+b)-7/2*b^(5/2)*
arctan(a^(1/2)*x/b^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{x\left(90b^2 - 20abx^2 + 6a^2x^4 + \frac{15b^3}{b+ax^2}\right)}{30a^4} - \frac{7b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{9/2}}$$

input
```
Integrate[x^4/(a + b/x^2)^2,x]
```

output
```
(x*(90*b^2 - 20*a*b*x^2 + 6*a^2*x^4 + (15*b^3)/(b + a*x^2)))/(30*a^4) - (7
*b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*a^(9/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^8}{\left(ax^2+b\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7\int \frac{x^6}{ax^2+b}dx}{2a}-\frac{x^7}{2a\left(ax^2+b\right)}$$

$$\Big\downarrow \text{254}$$

$$\frac{7\int \left(\frac{x^4}{a}-\frac{bx^2}{a^2}+\frac{b^2}{a^3}-\frac{b^3}{a^3\left(ax^2+b\right)}\right)dx}{2a}-\frac{x^7}{2a\left(ax^2+b\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7\left(-\frac{b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{7/2}}+\frac{b^2x}{a^3}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}\right)}{2a}-\frac{x^7}{2a\left(ax^2+b\right)}$$

input
```
Int[x^4/(a + b/x^2)^2,x]
```

output
```
-1/2*x^7/(a*(b + a*x^2)) + (7*((b^2*x)/a^3 - (b*x^3)/(3*a^2) + x^5/(5*a) -
 (b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(7/2)))/(2*a)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{1}{5}a^2x^5-\frac{2}{3}ab\,x^3+3b^2x}{a^4}-\frac{b^3\left(-\frac{x}{2\left(a\,x^2+b\right)}+\frac{7\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}$	65
risch	$\frac{x^5}{5a^2}-\frac{2b\,x^3}{3a^3}+\frac{3b^2x}{a^4}+\frac{b^3x}{2a^4(a\,x^2+b)}+\frac{7\sqrt{-ab}\,b^2\ln\left(-\sqrt{-ab}\,x-b\right)}{4a^5}-\frac{7\sqrt{-ab}\,b^2\ln\left(\sqrt{-ab}\,x-b\right)}{4a^5}$	101

input
```
int(x^4/(a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/5*a^2*x^5-2/3*a*b*x^3+3*b^2*x)-b^3/a^4*(-1/2*x/(a*x^2+b)+7/2/(a*b
)^(1/2)*arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.44

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \left[\frac{12\,a^3x^7-28\,a^2bx^5+140\,ab^2x^3+210\,b^3x+105\,(ab^2x^2+b^3)\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{60\,(a^5x^2+a^4b)}, \frac{6\,a^3x^7-14\,a^2\cdots}{\cdots}\right.$$

input
```
integrate(x^4/(a+b/x^2)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*a^3*x^7 - 28*a^2*b*x^5 + 140*a*b^2*x^3 + 210*b^3*x + 105*(a*b^2*
x^2 + b^3)*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)))/(a^
5*x^2 + a^4*b), 1/30*(6*a^3*x^7 - 14*a^2*b*x^5 + 70*a*b^2*x^3 + 105*b^3*x
- 105*(a*b^2*x^2 + b^3)*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b))/(a^5*x^2 + a^4*
b)]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{b^3x}{2a^5x^2+2a^4b}+\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^4\sqrt{-\frac{b^5}{a^9}}}{b^2}+x\right)}{4} - \frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^4\sqrt{-\frac{b^5}{a^9}}}{b^2}+x\right)}{4}+\frac{x^5}{5a^2}-\frac{2bx^3}{3a^3}+\frac{3b^2x}{a^4}$$

input
```
integrate(x**4/(a+b/x**2)**2,x)
```

output
```
b**3*x/(2*a**5*x**2 + 2*a**4*b) + 7*sqrt(-b**5/a**9)*log(-a**4*sqrt(-b**5/
a**9)/b**2 + x)/4 - 7*sqrt(-b**5/a**9)*log(a**4*sqrt(-b**5/a**9)/b**2 + x)
/4 + x**5/(5*a**2) - 2*b*x**3/(3*a**3) + 3*b**2*x/a**4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx=\frac{b^3x}{2\,(a^5x^2+a^4b)}-\frac{7\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}+\frac{3\,a^2x^5-10\,abx^3+45\,b^2x}{15\,a^4}$$

input
```
integrate(x^4/(a+b/x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^3*x/(a^5*x^2 + a^4*b) - 7/2*b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4
) + 1/15*(3*a^2*x^5 - 10*a*b*x^3 + 45*b^2*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx=-\frac{7\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}+\frac{b^3x}{2\,(ax^2+b)a^4}+\frac{3\,a^8x^5-10\,a^7bx^3+45\,a^6b^2x}{15\,a^{10}}$$

input
```
integrate(x^4/(a+b/x^2)^2,x, algorithm="giac")
```

output
```
-7/2*b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/2*b^3*x/((a*x^2 + b)*a^
4) + 1/15*(3*a^8*x^5 - 10*a^7*b*x^3 + 45*a^6*b^2*x)/a^10
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx=\frac{x^5}{5\,a^2}-\frac{2\,b\,x^3}{3\,a^3}+\frac{3\,b^2\,x}{a^4}-\frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{9/2}}+\frac{b^3\,x}{2\,\left(a^5\,x^2+b\,a^4\right)}$$

input
```
int(x^4/(a + b/x^2)^2,x)
```

output
```
x^5/(5*a^2) - (2*b*x^3)/(3*a^3) + (3*b^2*x)/a^4 - (7*b^(5/2)*atan((a^(1/2)
*x)/b^(1/2)))/(2*a^(9/2)) + (b^3*x)/(2*(a^4*b + a^5*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^2}\,dx$$
$$= \frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2 - 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^3 + 6a^4x^7 - 14a^3b\,x^5 + 70a^2b^2x^3 + 105a\,b^3x}{30a^5\,(a\,x^2+b)}$$

input
```
int(x^4/(a+b/x^2)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 - 105*sq
rt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b**3 + 6*a**4*x**7 - 14*a**3*b
*x**5 + 70*a**2*b**2*x**3 + 105*a*b**3*x)/(30*a**5*(a*x**2 + b))
```

3.328 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2} dx = -\frac{2bx}{a^3} + \frac{x^3}{3a^2} - \frac{b^2x}{2a^3(b+ax^2)} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{7/2}}$$

output
```
-2*b*x/a^3+1/3*x^3/a^2-1/2*b^2*x/a^3/(a*x^2+b)+5/2*b^(3/2)*arctan(a^(1/2)*
x/b^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2} dx = \frac{x\left(-12b+2ax^2-\frac{3b^2}{b+ax^2}\right)}{6a^3} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{7/2}}$$

input
```
Integrate[x^2/(a + b/x^2)^2,x]
```

output
```
(x*(-12*b + 2*a*x^2 - (3*b^2)/(b + a*x^2)))/(6*a^3) + (5*b^(3/2)*ArcTan[(S
qrt[a]*x)/Sqrt[b]])/(2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^6}{(ax^2+b)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5\int \frac{x^4}{ax^2+b}dx}{2a}-\frac{x^5}{2a\,(ax^2+b)}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{5\int \left(\frac{b^2}{a^2(ax^2+b)}-\frac{b}{a^2}+\frac{x^2}{a}\right)dx}{2a}-\frac{x^5}{2a\,(ax^2+b)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{bx}{a^2}+\frac{x^3}{3a}\right)}{2a}-\frac{x^5}{2a\,(ax^2+b)}$$

input
```
Int[x^2/(a + b/x^2)^2,x]
```

output
```
-1/2*x^5/(a*(b + a*x^2)) + (5*(-((b*x)/a^2) + x^3/(3*a) + (b^(3/2)*ArcTan[
(Sqrt[a]*x)/Sqrt[b]])/a^(5/2)))/(2*a)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\frac{1}{3}a\,x^{3}-2bx}{a^{3}}+\frac{b^{2}\left(-\frac{x}{2\left(a\,x^{2}+b\right)}+\frac{5\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}$	53
risch	$\frac{x^{3}}{3a^{2}}-\frac{2bx}{a^{3}}-\frac{b^{2}x}{2a^{3}(a\,x^{2}+b)}+\frac{5\sqrt{-ab}\,b\ln\left(-\sqrt{-ab}\,x+b\right)}{4a^{4}}-\frac{5\sqrt{-ab}\,b\ln\left(\sqrt{-ab}\,x+b\right)}{4a^{4}}$	82

input
```
int(x^2/(a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/3*a*x^3-2*b*x)+b^2/a^3*(-1/2*x/(a*x^2+b)+5/2/(a*b)^(1/2)*arctan(a
*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.52

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \left[\frac{4\,a^2x^5 - 20\,abx^3 - 30\,b^2x + 15\,(abx^2+b^2)\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{12\,(a^4x^2+a^3b)}, \frac{2\,a^2x^5 - 10\,abx^3 - 15\,b^2x + 15}{6\,(a^4x^2}\right.$$

input
```
integrate(x^2/(a+b/x^2)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a^2*x^5 - 20*a*b*x^3 - 30*b^2*x + 15*(a*b*x^2 + b^2)*sqrt(-b/a)*l
og((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)))/(a^4*x^2 + a^3*b), 1/6*(2*
a^2*x^5 - 10*a*b*x^3 - 15*b^2*x + 15*(a*b*x^2 + b^2)*sqrt(b/a)*arctan(a*x*
sqrt(b/a)/b))/(a^4*x^2 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^2x}{2a^4x^2+2a^3b} - \frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^7}}}{b}+x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^7}}}{b}+x\right)}{4} + \frac{x^3}{3a^2} - \frac{2bx}{a^3}$$

input
```
integrate(x**2/(a+b/x**2)**2,x)
```

output
```
-b**2*x/(2*a**4*x**2 + 2*a**3*b) - 5*sqrt(-b**3/a**7)*log(-a**3*sqrt(-b**3
/a**7)/b + x)/4 + 5*sqrt(-b**3/a**7)*log(a**3*sqrt(-b**3/a**7)/b + x)/4 +
x**3/(3*a**2) - 2*b*x/a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{b^2x}{2\left(a^4x^2+a^3b\right)} + \frac{5\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}a^3} + \frac{ax^3-6\,bx}{3\,a^3}$$

input
```
integrate(x^2/(a+b/x^2)^2,x, algorithm="maxima")
```

output
```
-1/2*b^2*x/(a^4*x^2 + a^3*b) + 5/2*b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^
3) + 1/3*(a*x^3 - 6*b*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{5\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}a^3} - \frac{b^2x}{2\left(ax^2+b\right)a^3} + \frac{a^4x^3-6\,a^3bx}{3\,a^6}$$

input
```
integrate(x^2/(a+b/x^2)^2,x, algorithm="giac")
```

output
```
5/2*b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*b^2*x/((a*x^2 + b)*a^3
) + 1/3*(a^4*x^3 - 6*a^3*b*x)/a^6
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{x^3}{3\,a^2} + \frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{7/2}} - \frac{b^2\,x}{2\,\left(a^4\,x^2+b\,a^3\right)} - \frac{2\,b\,x}{a^3}$$

input
```
int(x^2/(a + b/x^2)^2,x)
```

output
```
x^3/(3*a^2) + (5*b^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/(2*a^(7/2)) - (b^2*x)/
(2*(a^3*b + a^4*x^2)) - (2*b*x)/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^2+2a^3x^5-10a^2b\,x^3-15a\,b^2x}{6a^4\,(a\,x^2+b)}$$

input
```
int(x^2/(a+b/x^2)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 15*sqrt(b)*sq
rt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b**2 + 2*a**3*x**5 - 10*a**2*b*x**3 -
15*a*b**2*x)/(6*a**4*(a*x**2 + b))
```

3.329 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 51

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx=\frac{x}{a^2}+\frac{bx}{2a^2\left(b+ax^2\right)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{5/2}}$$

output
```
x/a^2+1/2*b*x/a^2/(a*x^2+b)-3/2*b^(1/2)*arctan(a^(1/2)*x/b^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx=\frac{x}{a^2}+\frac{bx}{2a^2\left(b+ax^2\right)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{5/2}}$$

input
```
Integrate[(a + b/x^2)^(-2),x]
```

output
```
x/a^2 + (b*x)/(2*a^2*(b + a*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]]
)/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {772, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{x^4}{\left(ax^2+b\right)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3\int \frac{x^2}{ax^2+b}dx}{2a}-\frac{x^3}{2a\left(ax^2+b\right)}$$

$$\Big\downarrow \text{262}$$

$$\frac{3\left(\frac{x}{a}-\frac{b\int \frac{1}{ax^2+b}dx}{a}\right)}{2a}-\frac{x^3}{2a\left(ax^2+b\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{x}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{x^3}{2a\left(ax^2+b\right)}$$

input
```
Int[(a + b/x^2)^(-2),x]
```

output
```
-1/2*x^3/(a*(b + a*x^2)) + (3*(x/a - (Sqrt[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x}{a^{2}}-\frac{b\left(-\frac{x}{2\left(a\,x^{2}+b\right)}+\frac{3\arctan\left(\frac{a x}{\sqrt{a b}}\right)}{2\sqrt{a b}}\right)}{a^{2}}$	42
risch	$\frac{x}{a^{2}}+\frac{b x}{2a^{2}\left(a\,x^{2}+b\right)}+\frac{3\sqrt{-a b}\,\ln\left(-\sqrt{-a b}\,x-b\right)}{4a^{3}}-\frac{3\sqrt{-a b}\,\ln\left(\sqrt{-a b}\,x-b\right)}{4a^{3}}$	72

input
```
int(1/(a+b/x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*x-b/a^2*(-1/2*x/(a*x^2+b)+3/2/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \left[\frac{4\,ax^3+3\,(ax^2+b)\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+6\,bx}{4\,(a^3x^2+a^2b)}, \frac{2\,ax^3-3\,(ax^2+b)\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)+3\,bx}{2\,(a^3x^2+a^2b)}\right]$$

input
```
integrate(1/(a+b/x^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*x^3 + 3*(a*x^2 + b)*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b
)/(a*x^2 + b)) + 6*b*x)/(a^3*x^2 + a^2*b), 1/2*(2*a*x^3 - 3*(a*x^2 + b)*sq
rt(b/a)*arctan(a*x*sqrt(b/a)/b) + 3*b*x)/(a^3*x^2 + a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{bx}{2a^3x^2+2a^2b} + \frac{3\sqrt{-\frac{b}{a^5}}\log\left(-a^2\sqrt{-\frac{b}{a^5}}+x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}}\log\left(a^2\sqrt{-\frac{b}{a^5}}+x\right)}{4} + \frac{x}{a^2}$$

input
```
integrate(1/(a+b/x**2)**2,x)
```

output
```
b*x/(2*a**3*x**2 + 2*a**2*b) + 3*sqrt(-b/a**5)*log(-a**2*sqrt(-b/a**5) + x
)/4 - 3*sqrt(-b/a**5)*log(a**2*sqrt(-b/a**5) + x)/4 + x/a**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{bx}{2\left(a^3x^2+a^2b\right)} - \frac{3\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2} + \frac{x}{a^2}$$

input
```
integrate(1/(a+b/x^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*x/(a^3*x^2 + a^2*b) - 3/2*b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) +
x/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx = -\frac{3\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2} + \frac{x}{a^2} + \frac{bx}{2\left(ax^2+b\right)a^2}$$

input
```
integrate(1/(a+b/x^2)^2,x, algorithm="giac")
```

output
```
-3/2*b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) + x/a^2 + 1/2*b*x/((a*x^2 + b
)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{x}{a^2} + \frac{b\,x}{2\,\left(a^3\,x^2+b\,a^2\right)} - \frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{5/2}}$$

input
```
int(1/(a + b/x^2)^2,x)
```

output
```
x/a^2 + (b*x)/(2*(a^2*b + a^3*x^2)) - (3*b^(1/2)*atan((a^(1/2)*x)/b^(1/2))
)/(2*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,x^2-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b+2a^2x^3+3abx}{2a^3\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*x**2 - 3*sqrt(b)*sqr
t(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b + 2*a**2*x**3 + 3*a*b*x)/(2*a**3*(a*x
**2 + b))
```

3.330 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2} \, dx = -\frac{x}{2a\left(b+ax^2\right)} + \frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
-1/2*x/a/(a*x^2+b)+1/2*arctan(a^(1/2)*x/b^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2} \, dx = -\frac{x}{2a\left(b+ax^2\right)} + \frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[1/((a + b/x^2)^2*x^2),x]
```

output
```
-1/2*x/(a*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x^2}\right)^2} \, dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{x^2}{\left(a x^2+b\right)^2}dx$$
$$\Big\downarrow \text{252}$$
$$\frac{\int \frac{1}{a x^2+b}dx}{2 a}-\frac{x}{2 a \left(a x^2+b\right)}$$
$$\Big\downarrow \text{218}$$
$$\frac{\arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{2 a^{3/2} \sqrt{b}}-\frac{x}{2 a \left(a x^2+b\right)}$$

input
```
Int[1/((a + b/x^2)^2*x^2),x]
```

output
```
-1/2*x/(a*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x}{2a(a\,x^2+b)}+\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$-\frac{x}{2a(a\,x^2+b)}-\frac{\ln\left(ax+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-ax+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	62

input
```
int(1/(a+b/x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/a/(a*x^2+b)+1/2/a/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx = \left[-\frac{2\,abx+(ax^2+b)\sqrt{-ab}\log\left(\frac{ax^2-2\,\sqrt{-ab}x-b}{ax^2+b}\right)}{4\,(a^3bx^2+a^2b^2)},\ -\frac{abx-(ax^2+b)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{b}\right)}{2\,(a^3bx^2+a^2b^2)}\right]$$

input
```
integrate(1/(a+b/x^2)^2/x^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*x + (a*x^2 + b)*sqrt(-a*b)*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(
a*x^2 + b)))/(a^3*b*x^2 + a^2*b^2), -1/2*(a*b*x - (a*x^2 + b)*sqrt(a*b)*ar
ctan(sqrt(a*b)*x/b))/(a^3*b*x^2 + a^2*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx = -\frac{x}{2a^2x^2+2ab} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-ab\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(ab\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(1/(a+b/x**2)**2/x**2,x)
```

output
```
-x/(2*a**2*x**2 + 2*a*b) - sqrt(-1/(a**3*b))*log(-a*b*sqrt(-1/(a**3*b)) +
x)/4 + sqrt(-1/(a**3*b))*log(a*b*sqrt(-1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx = -\frac{x}{2\,(a^2x^2+ab)} + \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a}$$

input
```
integrate(1/(a+b/x^2)^2/x^2,x, algorithm="maxima")
```

output
```
-1/2*x/(a^2*x^2 + a*b) + 1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx = \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}a} - \frac{x}{2\,(ax^2+b)a}$$

input
```
integrate(1/(a+b/x^2)^2/x^2,x, algorithm="giac")
```

output
```
1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*x/((a*x^2 + b)*a)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,a^{3/2}\,\sqrt{b}} - \frac{x}{2\,a\,\left(a\,x^2+b\right)}$$

input
```
int(1/(x^2*(a + b/x^2)^2),x)
```

output
```
atan((a^(1/2)*x)/b^(1/2))/(2*a^(3/2)*b^(1/2)) - x/(2*a*(b + a*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,x^2 + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b - abx}{2a^2b\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^2/x^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*x**2 + sqrt(b)*sqrt(a)*at
an((a*x)/(sqrt(b)*sqrt(a)))*b - a*b*x)/(2*a**2*b*(a*x**2 + b))
```

3.331 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4} \, dx = \frac{x}{2b(b+ax^2)} + \frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}b^{3/2}}$$

output
```
1/2*x/b/(a*x^2+b)+1/2*arctan(a^(1/2)*x/b^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4} \, dx = \frac{x}{2b(b+ax^2)} + \frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}b^{3/2}}$$

input
```
Integrate[1/((a + b/x^2)^2*x^4),x]
```

output
```
x/(2*b*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*Sqrt[a]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {795, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{(ax^2+b)^2}dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\int \frac{1}{ax^2+b}dx}{2b}+\frac{x}{2b(ax^2+b)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}b^{3/2}}+\frac{x}{2b(ax^2+b)}$$

input
```
Int[1/((a + b/x^2)^2*x^4),x]
```

output
```
x/(2*b*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[b]]/(2*Sqrt[a]*b^(3/2))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2b(a\,x^2+b)}+\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2b\sqrt{ab}}$	36
risch	$\frac{x}{2b(a\,x^2+b)}-\frac{\ln\left(ax+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}+\frac{\ln\left(-ax+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}$	62

input
```
int(1/(a+b/x^2)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/b/(a*x^2+b)+1/2/b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2x^4}\,dx$$
$$=\left[\frac{2\,abx-(ax^2+b)\sqrt{-ab}\log\left(\frac{ax^2-2\sqrt{-ab}x-b}{ax^2+b}\right)}{4\,(a^2b^2x^2+ab^3)},\ \frac{abx+(ax^2+b)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{b}\right)}{2\,(a^2b^2x^2+ab^3)}\right]$$

input
```
integrate(1/(a+b/x^2)^2/x^4,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x - (a*x^2 + b)*sqrt(-a*b)*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a
*x^2 + b)))/(a^2*b^2*x^2 + a*b^3), 1/2*(a*b*x + (a*x^2 + b)*sqrt(a*b)*arct
an(sqrt(a*b)*x/b))/(a^2*b^2*x^2 + a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx = \frac{x}{2abx^2+2b^2} - \frac{\sqrt{-\frac{1}{ab^3}}\log\left(-b^2\sqrt{-\frac{1}{ab^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{ab^3}}\log\left(b^2\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

input
```
integrate(1/(a+b/x**2)**2/x**4,x)
```

output
```
x/(2*a*b*x**2 + 2*b**2) - sqrt(-1/(a*b**3))*log(-b**2*sqrt(-1/(a*b**3)) +
x)/4 + sqrt(-1/(a*b**3))*log(b**2*sqrt(-1/(a*b**3)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx = \frac{x}{2\left(abx^2+b^2\right)} + \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}b}$$

input
```
integrate(1/(a+b/x^2)^2/x^4,x, algorithm="maxima")
```

output
```
1/2*x/(a*b*x^2 + b^2) + 1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx = \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}b} + \frac{x}{2\,(ax^2+b)b}$$

input
```
integrate(1/(a+b/x^2)^2/x^4,x, algorithm="giac")
```

output
```
1/2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*x/((a*x^2 + b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx = \frac{x}{2\,b\,\left(a\,x^2+b\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

input
```
int(1/(x^4*(a + b/x^2)^2),x)
```

output
```
x/(2*b*(b + a*x^2)) + atan((a^(1/2)*x)/b^(1/2))/(2*a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^4}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b+abx}{2a\,b^2\left(a\,x^2+b\right)}$$

input
```
int(1/(a+b/x^2)^2/x^4,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*x**2 + sqrt(b)*sqrt(a)*at
an((a*x)/(sqrt(b)*sqrt(a)))*b + a*b*x)/(2*a*b**2*(a*x**2 + b))
```

3.332 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6} \, dx = -\frac{1}{b^2 x}-\frac{a x}{2 b^2 \left(b+a x^2\right)}-\frac{3 \sqrt{a} \arctan\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{2 b^{5/2}}$$

output
```
-1/b^2/x-1/2*a*x/b^2/(a*x^2+b)-3/2*a^(1/2)*arctan(a^(1/2)*x/b^(1/2))/b^(5/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6} \, dx = -\frac{1}{b^2 x}-\frac{a x}{2 b^2 \left(b+a x^2\right)}-\frac{3 \sqrt{a} \arctan\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{2 b^{5/2}}$$

input
```
Integrate[1/((a + b/x^2)^2*x^6),x]
```

output
```
-(1/(b^2*x)) - (a*x)/(2*b^2*(b + a*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[a]*x)/S
qrt[b]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x^2}\right)^2} dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{1}{x^2 \left(ax^2+b\right)^2} dx$$

$$\Big\downarrow \; 253$$

$$\frac{3\int \frac{1}{x^2(ax^2+b)}dx}{2b}+\frac{1}{2bx\left(ax^2+b\right)}$$

$$\Big\downarrow \; 264$$

$$\frac{3\left(-\frac{a\int \frac{1}{ax^2+b}dx}{b}-\frac{1}{bx}\right)}{2b}+\frac{1}{2bx\left(ax^2+b\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{3\left(-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{2b}+\frac{1}{2bx\left(ax^2+b\right)}$$

input
```
Int[1/((a + b/x^2)^2*x^6),x]
```

output
```
1/(2*b*x*(b + a*x^2)) + (3*(-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[
b]])/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a\left(\frac{x}{2a\,x^{2}+2b}+\frac{3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}-\frac{1}{b^{2}x}$	45
risch	$\frac{-\frac{3a\,x^{2}}{2b^{2}}-\frac{1}{b}}{\left(a\,x^{2}+b\right)x}+\frac{3\sqrt{-ab}\,\ln\left(-ax+\sqrt{-ab}\right)}{4b^{3}}-\frac{3\sqrt{-ab}\,\ln\left(-ax-\sqrt{-ab}\right)}{4b^{3}}$	78

input
```
int(1/(a+b/x^2)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-a/b^2*(1/2*x/(a*x^2+b)+3/2/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)))-1/b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.52

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx = \left[-\frac{6\,ax^2 - 3\,(ax^3+bx)\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)+4\,b}{4\,(ab^2x^3+b^3x)}, -\frac{3\,ax^2+3\,(ax^3+bx)\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+2\,b}{2\,(ab^2x^3+b^3x)}\right]$$

input
```
integrate(1/(a+b/x^2)^2/x^6,x, algorithm="fricas")
```

output
```
[-1/4*(6*a*x^2 - 3*(a*x^3 + b*x)*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b)
- b)/(a*x^2 + b)) + 4*b)/(a*b^2*x^3 + b^3*x), -1/2*(3*a*x^2 + 3*(a*x^3 + b
*x)*sqrt(a/b)*arctan(x*sqrt(a/b)) + 2*b)/(a*b^2*x^3 + b^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx = \frac{3\sqrt{-\frac{a}{b^5}}\log\left(x-\frac{b^3\sqrt{-\frac{a}{b^5}}}{a}\right)}{4} - \frac{3\sqrt{-\frac{a}{b^5}}\log\left(x+\frac{b^3\sqrt{-\frac{a}{b^5}}}{a}\right)}{4} + \frac{-3ax^2-2b}{2ab^2x^3+2b^3x}$$

input
```
integrate(1/(a+b/x**2)**2/x**6,x)
```

output
```
3*sqrt(-a/b**5)*log(x - b**3*sqrt(-a/b**5)/a)/4 - 3*sqrt(-a/b**5)*log(x +
b**3*sqrt(-a/b**5)/a)/4 + (-3*a*x**2 - 2*b)/(2*a*b**2*x**3 + 2*b**3*x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx = -\frac{3\,ax^2+2\,b}{2\,(ab^2x^3+b^3x)} - \frac{3\,a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}$$

input
```
integrate(1/(a+b/x^2)^2/x^6,x, algorithm="maxima")
```

output
```
-1/2*(3*a*x^2 + 2*b)/(a*b^2*x^3 + b^3*x) - 3/2*a*arctan(a*x/sqrt(a*b))/(sq
rt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx = -\frac{3\,a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} - \frac{3\,ax^2+2\,b}{2\,(ax^3+bx)b^2}$$

input
```
integrate(1/(a+b/x^2)^2/x^6,x, algorithm="giac")
```

output
```
-3/2*a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(3*a*x^2 + 2*b)/((a*x^3
 + b*x)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx = -\frac{\frac{1}{b}+\frac{3\,a\,x^2}{2\,b^2}}{a\,x^3+b\,x} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,b^{5/2}}$$

input
```
int(1/(x^6*(a + b/x^2)^2),x)
```

output
```
- (1/b + (3*a*x^2)/(2*b^2))/(b*x + a*x^3) - (3*a^(1/2)*atan((a^(1/2)*x)/b^
(1/2)))/(2*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^6}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,x^3-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)bx-3ab\,x^2-2b^2}{2b^3x\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^2/x^6,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*x**3 - 3*sqrt(b)*sqr
t(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b*x - 3*a*b*x**2 - 2*b**2)/(2*b**3*x*(a
*x**2 + b))
```

3.333 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8} \, dx = -\frac{1}{3b^2x^3}+\frac{2a}{b^3x}+\frac{a^2x}{2b^3(b+ax^2)}+\frac{5a^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

output
```
-1/3/b^2/x^3+2*a/b^3/x+1/2*a^2*x/b^3/(a*x^2+b)+5/2*a^(3/2)*arctan(a^(1/2)*
x/b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8} \, dx = -\frac{1}{3b^2x^3}+\frac{2a}{b^3x}+\frac{a^2x}{2b^3(b+ax^2)}+\frac{5a^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2b^{7/2}}$$

input
```
Integrate[1/((a + b/x^2)^2*x^8),x]
```

output
```
-1/3*1/(b^2*x^3) + (2*a)/(b^3*x) + (a^2*x)/(2*b^3*(b + a*x^2)) + (5*a^(3/2
)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {795, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a+\frac{b}{x^2}\right)^2} \, dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{x^4 (ax^2+b)^2}dx$$

$$\Big\downarrow \ 253$$

$$\frac{5\int \frac{1}{x^4(ax^2+b)}dx}{2b}+\frac{1}{2bx^3(ax^2+b)}$$

$$\Big\downarrow \ 264$$

$$\frac{5\left(-\frac{a\int \frac{1}{x^2\left(ax^2+b\right)}dx}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(ax^2+b)}$$

$$\Big\downarrow \ 264$$

$$\frac{5\left(-\frac{a\left(-\frac{a\int \frac{1}{ax^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(ax^2+b)}$$

$$\Big\downarrow \ 218$$

$$\frac{5\left(-\frac{a\left(-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(ax^2+b)}$$

input
```
Int[1/((a + b/x^2)^2*x^8),x]
```

output
```
1/(2*b*x^3*(b + a*x^2)) + (5*(-1/3*1/(b*x^3) - (a*(-(1/(b*x)) - (Sqrt[a]*A
rcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)))/b))/(2*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{a^{2}\left(\frac{x}{2 a \,x^{2}+2 b}+\frac{5 \arctan\left(\frac{a x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^{3}}-\frac{1}{3 b^{2} x^{3}}+\frac{2 a}{b^{3} x}$	55
risch	$\frac{\frac{5 a^{2} x^{4}}{2 b^{3}}+\frac{5 a \,x^{2}}{3 b^{2}}-\frac{1}{3 b}}{\left(a \,x^{2}+b \right) x^{3}}+\frac{5 \sqrt{-a b}\, a \ln\left(-a x -\sqrt{-a b}\right)}{4 b^{4}}-\frac{5 \sqrt{-a b}\, a \ln\left(-a x +\sqrt{-a b}\right)}{4 b^{4}}$	91

input
```
int(1/(a+b/x^2)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
a^2/b^3*(1/2*x/(a*x^2+b)+5/2/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)))-1/3/b^2/
x^3+2*a/b^3/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.57

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx = \left[\frac{30\,a^2x^4+20\,abx^2+15\,(a^2x^5+abx^3)\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-4\,b^2}{12\,(ab^3x^5+b^4x^3)}, \frac{15\,a^2x^4+10\,abx^2+15\,(a^2x^5+\ldots}{6\,(ab^3x^{\ldots}}\right.$$

input
```
integrate(1/(a+b/x^2)^2/x^8,x, algorithm="fricas")
```

output
```
[1/12*(30*a^2*x^4 + 20*a*b*x^2 + 15*(a^2*x^5 + a*b*x^3)*sqrt(-a/b)*log((a*
x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - 4*b^2)/(a*b^3*x^5 + b^4*x^3), 1
/6*(15*a^2*x^4 + 10*a*b*x^2 + 15*(a^2*x^5 + a*b*x^3)*sqrt(a/b)*arctan(x*sq
rt(a/b)) - 2*b^2)/(a*b^3*x^5 + b^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx = -\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^4\sqrt{-\frac{a^3}{b^7}}}{a^2}\right)}{4} + \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^4\sqrt{-\frac{a^3}{b^7}}}{a^2}\right)}{4} + \frac{15a^2x^4+10abx^2-2b^2}{6ab^3x^5+6b^4x^3}$$

input
```
integrate(1/(a+b/x**2)**2/x**8,x)
```

output
```
-5*sqrt(-a**3/b**7)*log(x - b**4*sqrt(-a**3/b**7)/a**2)/4 + 5*sqrt(-a**3/b
**7)*log(x + b**4*sqrt(-a**3/b**7)/a**2)/4 + (15*a**2*x**4 + 10*a*b*x**2 -
 2*b**2)/(6*a*b**3*x**5 + 6*b**4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx = \frac{15\,a^2x^4+10\,abx^2-2\,b^2}{6\,(ab^3x^5+b^4x^3)} + \frac{5\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}$$

input
```
integrate(1/(a+b/x^2)^2/x^8,x, algorithm="maxima")
```

output
```
1/6*(15*a^2*x^4 + 10*a*b*x^2 - 2*b^2)/(a*b^3*x^5 + b^4*x^3) + 5/2*a^2*arct
an(a*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx = \frac{5\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{a^2x}{2\,(ax^2+b)b^3} + \frac{6\,ax^2-b}{3\,b^3x^3}$$

input
```
integrate(1/(a+b/x^2)^2/x^8,x, algorithm="giac")
```

output
```
5/2*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*a^2*x/((a*x^2 + b)*b^3
) + 1/3*(6*a*x^2 - b)/(b^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx = \frac{\frac{5\,a\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{5\,a^2\,x^4}{2\,b^3}}{a\,x^5+b\,x^3}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{2\,b^{7/2}}$$

input
```
int(1/(x^8*(a + b/x^2)^2),x)
```

output
```
((5*a*x^2)/(3*b^2) - 1/(3*b) + (5*a^2*x^4)/(2*b^3))/(a*x^5 + b*x^3) + (5*a
^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/(2*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^2 x^8}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2x^5+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^3+15a^2b\,x^4+10a\,b^2x^2-2b^3}{6b^4x^3\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^2/x^8,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*x**5 + 15*sqrt(b)*s
qrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b*x**3 + 15*a**2*b*x**4 + 10*a*b**2
*x**2 - 2*b**3)/(6*b**4*x**3*(a*x**2 + b))
```

3.334 $\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx=\frac{3b^2x^2}{a^5}-\frac{3bx^4}{4a^4}+\frac{x^6}{6a^3}+\frac{b^5}{4a^6(b+ax^2)^2}-\frac{5b^4}{2a^6(b+ax^2)}-\frac{5b^3\log(b+ax^2)}{a^6}$$

output
```
3*b^2*x^2/a^5-3/4*b*x^4/a^4+1/6*x^6/a^3+1/4*b^5/a^6/(a*x^2+b)^2-5/2*b^4/a^
6/(a*x^2+b)-5*b^3*ln(a*x^2+b)/a^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx=\frac{36ab^2x^2-9a^2bx^4+2a^3x^6-\frac{3b^4(9b+10ax^2)}{(b+ax^2)^2}-60b^3\log(b+ax^2)}{12a^6}$$

input
```
Integrate[x^5/(a + b/x^2)^3,x]
```

output
```
(36*a*b^2*x^2 - 9*a^2*b*x^4 + 2*a^3*x^6 - (3*b^4*(9*b + 10*a*x^2))/(b + a*
x^2)^2 - 60*b^3*Log[b + a*x^2])/(12*a^6)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^{11}}{(ax^2+b)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^{10}}{(ax^2+b)^3}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(-\frac{b^5}{a^5\left(ax^2+b\right)^3}+\frac{5b^4}{a^5\left(ax^2+b\right)^2}-\frac{10b^3}{a^5\left(ax^2+b\right)}+\frac{6b^2}{a^5}-\frac{3x^2b}{a^4}+\frac{x^4}{a^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{b^5}{2a^6\left(ax^2+b\right)^2}-\frac{5b^4}{a^6\left(ax^2+b\right)}-\frac{10b^3\log\left(ax^2+b\right)}{a^6}+\frac{6b^2x^2}{a^5}-\frac{3bx^4}{2a^4}+\frac{x^6}{3a^3}\right)$$

input
```
Int[x^5/(a + b/x^2)^3,x]
```

output
```
((6*b^2*x^2)/a^5 - (3*b*x^4)/(2*a^4) + x^6/(3*a^3) + b^5/(2*a^6*(b + a*x^2
)^2) - (5*b^4)/(a^6*(b + a*x^2)) - (10*b^3*Log[b + a*x^2])/a^6)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

method	result	size
norman	$\frac{\frac{x^{10}}{6a}-\frac{5b\,x^{8}}{12a^{2}}+\frac{5b^{2}x^{6}}{3a^{3}}-\frac{15b^{5}}{2a^{6}}-\frac{10b^{4}x^{2}}{a^{5}}}{\left(a\,x^{2}+b\right)^{2}}-\frac{5b^{3}\ln\left(a\,x^{2}+b\right)}{a^{6}}$	76
risch	$\frac{x^{6}}{6a^{3}}-\frac{3b\,x^{4}}{4a^{4}}+\frac{3b^{2}x^{2}}{a^{5}}+\frac{-\frac{5b^{4}x^{2}}{2}-\frac{9b^{5}}{4a}}{a^{5}\left(a\,x^{2}+b\right)^{2}}-\frac{5b^{3}\ln\left(a\,x^{2}+b\right)}{a^{6}}$	76
default	$\frac{\frac{1}{6}a^{2}x^{6}-\frac{3}{4}ab\,x^{4}+3b^{2}x^{2}}{a^{5}}-\frac{b^{3}\left(\frac{5b}{a\left(a\,x^{2}+b\right)}+\frac{10\ln\left(a\,x^{2}+b\right)}{a}-\frac{b^{2}}{2a\left(a\,x^{2}+b\right)^{2}}\right)}{2a^{5}}$	84
parallelrisch	$-\frac{-2a^{5}x^{10}+5a^{4}b\,x^{8}-20a^{3}b^{2}x^{6}+60\ln\left(a\,x^{2}+b\right)x^{4}a^{2}b^{3}+120\ln\left(a\,x^{2}+b\right)x^{2}a\,b^{4}+120b^{4}x^{2}a+60\ln\left(a\,x^{2}+b\right)b^{5}+90b^{5}}{12a^{6}\left(a\,x^{2}+b\right)^{2}}$	107

input
```
int(x^5/(a+b/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/6/a*x^10-5/12*b/a^2*x^8+5/3*b^2/a^3*x^6-15/2*b^5/a^6-10*b^4/a^5*x^2)/(a
*x^2+b)^2-5*b^3*ln(a*x^2+b)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{2\,a^5x^{10} - 5\,a^4bx^8 + 20\,a^3b^2x^6 + 63\,a^2b^3x^4 + 6\,ab^4x^2 - 27\,b^5 - 60\,(a^2b^3x^4 + 2\,ab^4x^2 + b^5)\log\left(ax^2+b\right)}{12\,(a^8x^4 + 2\,a^7bx^2 + a^6b^2)}$$

input
```
integrate(x^5/(a+b/x^2)^3,x, algorithm="fricas")
```

output
```
1/12*(2*a^5*x^10 - 5*a^4*b*x^8 + 20*a^3*b^2*x^6 + 63*a^2*b^3*x^4 + 6*a*b^4
*x^2 - 27*b^5 - 60*(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5)*log(a*x^2 + b))/(a^8*
x^4 + 2*a^7*b*x^2 + a^6*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{-10ab^4x^2 - 9b^5}{4a^8x^4 + 8a^7bx^2 + 4a^6b^2} + \frac{x^6}{6a^3} - \frac{3bx^4}{4a^4} + \frac{3b^2x^2}{a^5} - \frac{5b^3\log\left(ax^2+b\right)}{a^6}$$

input
```
integrate(x**5/(a+b/x**2)**3,x)
```

output
```
(-10*a*b**4*x**2 - 9*b**5)/(4*a**8*x**4 + 8*a**7*b*x**2 + 4*a**6*b**2) + x
**6/(6*a**3) - 3*b*x**4/(4*a**4) + 3*b**2*x**2/a**5 - 5*b**3*log(a*x**2 +
b)/a**6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx = -\frac{10\,ab^4x^2+9\,b^5}{4\,(a^8x^4+2\,a^7bx^2+a^6b^2)} - \frac{5\,b^3\log\left(ax^2+b\right)}{a^6} + \frac{2\,a^2x^6-9\,abx^4+36\,b^2x^2}{12\,a^5}$$

input
```
integrate(x^5/(a+b/x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(10*a*b^4*x^2 + 9*b^5)/(a^8*x^4 + 2*a^7*b*x^2 + a^6*b^2) - 5*b^3*log(
a*x^2 + b)/a^6 + 1/12*(2*a^2*x^6 - 9*a*b*x^4 + 36*b^2*x^2)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx = -\frac{5\,b^3\log\left(|ax^2+b|\right)}{a^6} + \frac{30\,a^2b^3x^4+50\,ab^4x^2+21\,b^5}{4\,(ax^2+b)^2a^6} + \frac{2\,a^6x^6-9\,a^5bx^4+36\,a^4b^2x^2}{12\,a^9}$$

input
```
integrate(x^5/(a+b/x^2)^3,x, algorithm="giac")
```

output
```
-5*b^3*log(abs(a*x^2 + b))/a^6 + 1/4*(30*a^2*b^3*x^4 + 50*a*b^4*x^2 + 21*b
^5)/((a*x^2 + b)^2*a^6) + 1/12*(2*a^6*x^6 - 9*a^5*b*x^4 + 36*a^4*b^2*x^2)/
a^9
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{x^6}{6\,a^3} - \frac{\frac{9\,b^5}{4\,a}+\frac{5\,b^4\,x^2}{2}}{a^7\,x^4+2\,a^6\,b\,x^2+a^5\,b^2} - \frac{3\,b\,x^4}{4\,a^4} - \frac{5\,b^3\,\ln\left(a\,x^2+b\right)}{a^6} + \frac{3\,b^2\,x^2}{a^5}$$

input
```
int(x^5/(a + b/x^2)^3,x)
```

output
```
x^6/(6*a^3) - ((9*b^5)/(4*a) + (5*b^4*x^2)/2)/(a^5*b^2 + a^7*x^4 + 2*a^6*b
*x^2) - (3*b*x^4)/(4*a^4) - (5*b^3*log(b + a*x^2))/a^6 + (3*b^2*x^2)/a^5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{x^5}{\left(a+\frac{b}{x^2}\right)^3}\,dx$$
$$= \frac{-60\log(a\,x^2+b)\,a^2b^3x^4 - 120\log(a\,x^2+b)\,a\,b^4x^2 - 60\log(a\,x^2+b)\,b^5 + 2a^5x^{10} - 5a^4b\,x^8 + 20a^3b^2x^6 +}{12a^6\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(x^5/(a+b/x^2)^3,x)
```

output
```
( - 60*log(a*x**2 + b)*a**2*b**3*x**4 - 120*log(a*x**2 + b)*a*b**4*x**2 -
60*log(a*x**2 + b)*b**5 + 2*a**5*x**10 - 5*a**4*b*x**8 + 20*a**3*b**2*x**6
 + 60*a**2*b**3*x**4 - 30*b**5)/(12*a**6*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.335 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3} dx = -\frac{3bx^2}{2a^4}+\frac{x^4}{4a^3}-\frac{b^4}{4a^5(b+ax^2)^2}+\frac{2b^3}{a^5(b+ax^2)}+\frac{3b^2\log(b+ax^2)}{a^5}$$

output
```
-3/2*b*x^2/a^4+1/4*x^4/a^3-1/4*b^4/a^5/(a*x^2+b)^2+2*b^3/a^5/(a*x^2+b)+3*b
^2*ln(a*x^2+b)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3} dx = \frac{-6abx^2+a^2x^4+\frac{b^3(7b+8ax^2)}{(b+ax^2)^2}+12b^2\log(b+ax^2)}{4a^5}$$

input
```
Integrate[x^3/(a + b/x^2)^3,x]
```

output
```
(-6*a*b*x^2 + a^2*x^4 + (b^3*(7*b + 8*a*x^2))/(b + a*x^2)^2 + 12*b^2*Log[b
 + a*x^2])/(4*a^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^9}{\left(ax^2+b\right)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^8}{\left(ax^2+b\right)^3}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{b^4}{a^4\left(ax^2+b\right)^3}-\frac{4b^3}{a^4\left(ax^2+b\right)^2}+\frac{6b^2}{a^4\left(ax^2+b\right)}-\frac{3b}{a^4}+\frac{x^2}{a^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^4}{2a^5\left(ax^2+b\right)^2}+\frac{4b^3}{a^5\left(ax^2+b\right)}+\frac{6b^2\log\left(ax^2+b\right)}{a^5}-\frac{3bx^2}{a^4}+\frac{x^4}{2a^3}\right)$$

input
```
Int[x^3/(a + b/x^2)^3,x]
```

output
```
((-3*b*x^2)/a^4 + x^4/(2*a^3) - b^4/(2*a^5*(b + a*x^2)^2) + (4*b^3)/(a^5*(
b + a*x^2)) + (6*b^2*Log[b + a*x^2])/a^5)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result	size
norman	$\frac{\frac{x^{8}}{4a}-\frac{b\,x^{6}}{a^{2}}+\frac{9b^{4}}{2a^{5}}+\frac{6b^{3}x^{2}}{a^{4}}}{\left(a\,x^{2}+b\right)^{2}}+\frac{3b^{2}\ln\left(a\,x^{2}+b\right)}{a^{5}}$	65
default	$\frac{\left(a\,x^{2}-3b\right)^{2}}{4a^{5}}+\frac{b^{2}\left(\frac{4b}{a\left(a\,x^{2}+b\right)}+\frac{6\ln\left(a\,x^{2}+b\right)}{a}-\frac{b^{2}}{2a\left(a\,x^{2}+b\right)^{2}}\right)}{2a^{4}}$	72
risch	$\frac{x^{4}}{4a^{3}}-\frac{3b\,x^{2}}{2a^{4}}+\frac{9b^{2}}{4a^{5}}+\frac{2b^{3}x^{2}+\frac{7b^{4}}{4a}}{a^{4}\left(a\,x^{2}+b\right)^{2}}+\frac{3b^{2}\ln\left(a\,x^{2}+b\right)}{a^{5}}$	73
parallelrisch	$\frac{a^{4}x^{8}-4a^{3}b\,x^{6}+12\ln\left(a\,x^{2}+b\right)x^{4}a^{2}b^{2}+24\ln\left(a\,x^{2}+b\right)x^{2}a\,b^{3}+24a\,b^{3}x^{2}+12\ln\left(a\,x^{2}+b\right)b^{4}+18b^{4}}{4a^{5}\left(a\,x^{2}+b\right)^{2}}$	95

input
```
int(x^3/(a+b/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*x^8-b/a^2*x^6+9/2*b^4/a^5+6*b^3/a^4*x^2)/(a*x^2+b)^2+3*b^2*ln(a*x^2
+b)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{a^4x^8 - 4\,a^3bx^6 - 11\,a^2b^2x^4 + 2\,ab^3x^2 + 7\,b^4 + 12\,(a^2b^2x^4 + 2\,ab^3x^2 + b^4)\log\left(ax^2 + b\right)}{4\,(a^7x^4 + 2\,a^6bx^2 + a^5b^2)}$$

input
```
integrate(x^3/(a+b/x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(a^4*x^8 - 4*a^3*b*x^6 - 11*a^2*b^2*x^4 + 2*a*b^3*x^2 + 7*b^4 + 12*(a^
2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*log(a*x^2 + b))/(a^7*x^4 + 2*a^6*b*x^2 + a^
5*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{8ab^3x^2 + 7b^4}{4a^7x^4 + 8a^6bx^2 + 4a^5b^2} + \frac{x^4}{4a^3} - \frac{3bx^2}{2a^4} + \frac{3b^2\log\left(ax^2 + b\right)}{a^5}$$

input
```
integrate(x**3/(a+b/x**2)**3,x)
```

output
```
(8*a*b**3*x**2 + 7*b**4)/(4*a**7*x**4 + 8*a**6*b*x**2 + 4*a**5*b**2) + x**
4/(4*a**3) - 3*b*x**2/(2*a**4) + 3*b**2*log(a*x**2 + b)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{8\,ab^3x^2+7\,b^4}{4\,(a^7x^4+2\,a^6bx^2+a^5b^2)} + \frac{3\,b^2\log\left(ax^2+b\right)}{a^5} + \frac{ax^4-6\,bx^2}{4\,a^4}$$

input
```
integrate(x^3/(a+b/x^2)^3,x, algorithm="maxima")
```

output
```
1/4*(8*a*b^3*x^2 + 7*b^4)/(a^7*x^4 + 2*a^6*b*x^2 + a^5*b^2) + 3*b^2*log(a*
x^2 + b)/a^5 + 1/4*(a*x^4 - 6*b*x^2)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{3\,b^2\log\left(|ax^2+b|\right)}{a^5} + \frac{a^3x^4-6\,a^2bx^2}{4\,a^6} + \frac{8\,ab^3x^2+7\,b^4}{4\,(ax^2+b)^2a^5}$$

input
```
integrate(x^3/(a+b/x^2)^3,x, algorithm="giac")
```

output
```
3*b^2*log(abs(a*x^2 + b))/a^5 + 1/4*(a^3*x^4 - 6*a^2*b*x^2)/a^6 + 1/4*(8*a
*b^3*x^2 + 7*b^4)/((a*x^2 + b)^2*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{\frac{7\,b^4}{4\,a}+2\,b^3\,x^2}{a^6\,x^4+2\,a^5\,b\,x^2+a^4\,b^2} + \frac{x^4}{4\,a^3} - \frac{3\,b\,x^2}{2\,a^4} + \frac{3\,b^2\,\ln\left(a\,x^2+b\right)}{a^5}$$

input
```
int(x^3/(a + b/x^2)^3,x)
```

output
```
((7*b^4)/(4*a) + 2*b^3*x^2)/(a^4*b^2 + a^6*x^4 + 2*a^5*b*x^2) + x^4/(4*a^3
) - (3*b*x^2)/(2*a^4) + (3*b^2*log(b + a*x^2))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{12\log(a\,x^2+b)\,a^2b^2x^4+24\log(a\,x^2+b)\,a\,b^3x^2+12\log(a\,x^2+b)\,b^4+a^4x^8-4a^3b\,x^6-12a^2b^2x^4+6b^4}{4a^5\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(x^3/(a+b/x^2)^3,x)
```

output
```
(12*log(a*x**2 + b)*a**2*b**2*x**4 + 24*log(a*x**2 + b)*a*b**3*x**2 + 12*l
og(a*x**2 + b)*b**4 + a**4*x**8 - 4*a**3*b*x**6 - 12*a**2*b**2*x**4 + 6*b*
*4)/(4*a**5*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.336 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3} dx = \frac{x^2}{2a^3} + \frac{b^3}{4a^4(b+ax^2)^2} - \frac{3b^2}{2a^4(b+ax^2)} - \frac{3b\log(b+ax^2)}{2a^4}$$

output
```
1/2*x^2/a^3+1/4*b^3/a^4/(a*x^2+b)^2-3/2*b^2/a^4/(a*x^2+b)-3/2*b*ln(a*x^2+b
)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3} dx = -\frac{-2ax^2 + \frac{b^2(5b+6ax^2)}{(b+ax^2)^2} + 6b\log(b+ax^2)}{4a^4}$$

input
```
Integrate[x/(a + b/x^2)^3,x]
```

output
```
-1/4*(-2*a*x^2 + (b^2*(5*b + 6*a*x^2))/(b + a*x^2)^2 + 6*b*Log[b + a*x^2])
/a^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^7}{\left(ax^2+b\right)^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^6}{\left(ax^2+b\right)^3}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(-\frac{b^3}{a^3\left(ax^2+b\right)^3}+\frac{3b^2}{a^3\left(ax^2+b\right)^2}-\frac{3b}{a^3\left(ax^2+b\right)}+\frac{1}{a^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b^3}{2a^4\left(ax^2+b\right)^2}-\frac{3b^2}{a^4\left(ax^2+b\right)}-\frac{3b\log\left(ax^2+b\right)}{a^4}+\frac{x^2}{a^3}\right)$$

input
```
Int[x/(a + b/x^2)^3,x]
```

output
```
(x^2/a^3 + b^3/(2*a^4*(b + a*x^2)^2) - (3*b^2)/(a^4*(b + a*x^2)) - (3*b*Lo
g[b + a*x^2])/a^4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{-\frac{3b^2x^2}{a^3}+\frac{x^6}{2a}-\frac{9b^3}{4a^4}}{(a\,x^2+b)^2}-\frac{3b\ln\left(a\,x^2+b\right)}{2a^4}$	54
risch	$\frac{x^2}{2a^3}+\frac{-\frac{3b^2x^2}{2}-\frac{5b^3}{4a}}{a^3(a\,x^2+b)^2}-\frac{3b\ln\left(a\,x^2+b\right)}{2a^4}$	54
default	$\frac{x^2}{2a^3}-\frac{b\left(\frac{3b}{a\left(a\,x^2+b\right)}+\frac{3\ln\left(a\,x^2+b\right)}{a}-\frac{b^2}{2a\left(a\,x^2+b\right)^2}\right)}{2a^3}$	62
parallelrisch	$-\frac{-2a^3x^6+6\ln\left(a\,x^2+b\right)x^4a^2b+12\ln\left(a\,x^2+b\right)x^2a\,b^2+12a\,b^2x^2+6b^3\ln\left(a\,x^2+b\right)+9b^3}{4a^4(a\,x^2+b)^2}$	85

input
```
int(x/(a+b/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3*b^2/a^3*x^2+1/2/a*x^6-9/4*b^3/a^4)/(a*x^2+b)^2-3/2*b*ln(a*x^2+b)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{2\,a^3x^6+4\,a^2bx^4-4\,ab^2x^2-5\,b^3-6\,(a^2bx^4+2\,ab^2x^2+b^3)\log\left(ax^2+b\right)}{4\,(a^6x^4+2\,a^5bx^2+a^4b^2)}$$

input
```
integrate(x/(a+b/x^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*a^3*x^6 + 4*a^2*b*x^4 - 4*a*b^2*x^2 - 5*b^3 - 6*(a^2*b*x^4 + 2*a*b^
2*x^2 + b^3)*log(a*x^2 + b))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{-6ab^2x^2-5b^3}{4a^6x^4+8a^5bx^2+4a^4b^2}+\frac{x^2}{2a^3}-\frac{3b\log\left(ax^2+b\right)}{2a^4}$$

input
```
integrate(x/(a+b/x**2)**3,x)
```

output
```
(-6*a*b**2*x**2 - 5*b**3)/(4*a**6*x**4 + 8*a**5*b*x**2 + 4*a**4*b**2) + x*
*2/(2*a**3) - 3*b*log(a*x**2 + b)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx = -\frac{6\,ab^2x^2+5\,b^3}{4\,(a^6x^4+2\,a^5bx^2+a^4b^2)}+\frac{x^2}{2\,a^3}-\frac{3\,b\log\left(ax^2+b\right)}{2\,a^4}$$

input
```
integrate(x/(a+b/x^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*a*b^2*x^2 + 5*b^3)/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2) + 1/2*x^2/a^3
 - 3/2*b*log(a*x^2 + b)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{x^2}{2\,a^3} - \frac{3\,b\log\left(|ax^2+b|\right)}{2\,a^4} - \frac{6\,ab^2x^2+5\,b^3}{4\,(ax^2+b)^2a^4}$$

input
```
integrate(x/(a+b/x^2)^3,x, algorithm="giac")
```

output
```
1/2*x^2/a^3 - 3/2*b*log(abs(a*x^2 + b))/a^4 - 1/4*(6*a*b^2*x^2 + 5*b^3)/((
a*x^2 + b)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{x^2}{2\,a^3} - \frac{\frac{5\,b^3}{4\,a}+\frac{3\,b^2\,x^2}{2}}{a^5\,x^4+2\,a^4\,b\,x^2+a^3\,b^2} - \frac{3\,b\,\ln\left(a\,x^2+b\right)}{2\,a^4}$$

input
```
int(x/(a + b/x^2)^3,x)
```

output
```
x^2/(2*a^3) - ((5*b^3)/(4*a) + (3*b^2*x^2)/2)/(a^3*b^2 + a^5*x^4 + 2*a^4*b
*x^2) - (3*b*log(b + a*x^2))/(2*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{-6\log(a\,x^2+b)\,a^2b\,x^4-12\log(a\,x^2+b)\,a\,b^2x^2-6\log(a\,x^2+b)\,b^3+2a^3x^6+6a^2b\,x^4-3b^3}{4a^4\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(x/(a+b/x^2)^3,x)
```

output
```
( - 6*log(a*x**2 + b)*a**2*b*x**4 - 12*log(a*x**2 + b)*a*b**2*x**2 - 6*log
(a*x**2 + b)*b**3 + 2*a**3*x**6 + 6*a**2*b*x**4 - 3*b**3)/(4*a**4*(a**2*x*
*4 + 2*a*b*x**2 + b**2))
```

3.337 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x} \, dx = -\frac{b^2}{4a^3\left(b+ax^2\right)^2}+\frac{b}{a^3\left(b+ax^2\right)}+\frac{\log\left(b+ax^2\right)}{2a^3}$$

output
```
-1/4*b^2/a^3/(a*x^2+b)^2+b/a^3/(a*x^2+b)+1/2*ln(a*x^2+b)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x} \, dx = \frac{\frac{b(3b+4ax^2)}{(b+ax^2)^2}+2\log\left(b+ax^2\right)}{4a^3}$$

input
```
Integrate[1/((a + b/x^2)^3*x),x]
```

output
```
((b*(3*b + 4*a*x^2))/(b + a*x^2)^2 + 2*Log[b + a*x^2])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^2}\right)^3}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{x^5}{\left(ax^2+b\right)^3}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{x^4}{\left(ax^2+b\right)^3}dx^2$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(\frac{b^2}{a^2\left(ax^2+b\right)^3}-\frac{2b}{a^2\left(ax^2+b\right)^2}+\frac{1}{a^2\left(ax^2+b\right)}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{b^2}{2a^3\left(ax^2+b\right)^2}+\frac{2b}{a^3\left(ax^2+b\right)}+\frac{\log\left(ax^2+b\right)}{a^3}\right)$$

input
```
Int[1/((a + b/x^2)^3*x),x]
```

output
```
(-1/2*b^2/(a^3*(b + a*x^2)^2) + (2*b)/(a^3*(b + a*x^2)) + Log[b + a*x^2]/a
^3)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{\frac{b\,x^{2}}{a^{2}}+\frac{3b^{2}}{4a^{3}}}{\left(a\,x^{2}+b\right)^{2}}+\frac{\ln\left(a\,x^{2}+b\right)}{2a^{3}}$	42
risch	$\frac{\frac{b\,x^{2}}{a^{2}}+\frac{3b^{2}}{4a^{3}}}{\left(a\,x^{2}+b\right)^{2}}+\frac{\ln\left(a\,x^{2}+b\right)}{2a^{3}}$	42
default	$-\frac{b^{2}}{4a^{3}\left(a\,x^{2}+b\right)^{2}}+\frac{b}{a^{3}\left(a\,x^{2}+b\right)}+\frac{\ln\left(a\,x^{2}+b\right)}{2a^{3}}$	46
parallelrisch	$\frac{2a^{2}\ln\left(a\,x^{2}+b\right)x^{4}+4\ln\left(a\,x^{2}+b\right)x^{2}ab+4ab\,x^{2}+2b^{2}\ln\left(a\,x^{2}+b\right)+3b^{2}}{4a^{3}\left(a\,x^{2}+b\right)^{2}}$	72

input
```
int(1/(a+b/x^2)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(b/a^2*x^2+3/4*b^2/a^3)/(a*x^2+b)^2+1/2*ln(a*x^2+b)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx = \frac{4\,abx^2+3\,b^2+2\,(a^2x^4+2\,abx^2+b^2)\log\left(ax^2+b\right)}{4\,(a^5x^4+2\,a^4bx^2+a^3b^2)}$$

input
```
integrate(1/(a+b/x^2)^3/x,x, algorithm="fricas")
```

output
```
1/4*(4*a*b*x^2 + 3*b^2 + 2*(a^2*x^4 + 2*a*b*x^2 + b^2)*log(a*x^2 + b))/(a^
5*x^4 + 2*a^4*b*x^2 + a^3*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx = \frac{4abx^2+3b^2}{4a^5x^4+8a^4bx^2+4a^3b^2}+\frac{\log\left(ax^2+b\right)}{2a^3}$$

input
```
integrate(1/(a+b/x**2)**3/x,x)
```

output
```
(4*a*b*x**2 + 3*b**2)/(4*a**5*x**4 + 8*a**4*b*x**2 + 4*a**3*b**2) + log(a*
x**2 + b)/(2*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx = \frac{4\,abx^2+3\,b^2}{4\,(a^5x^4+2\,a^4bx^2+a^3b^2)}+\frac{\log\left(ax^2+b\right)}{2\,a^3}$$

input
```
integrate(1/(a+b/x^2)^3/x,x, algorithm="maxima")
```

output
```
1/4*(4*a*b*x^2 + 3*b^2)/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2) + 1/2*log(a*x^2
+ b)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx=\frac{\log\left(\left|ax^2+b\right|\right)}{2\,a^3}-\frac{3\,ax^4+2\,bx^2}{4\,(ax^2+b)^2a^2}$$

input
```
integrate(1/(a+b/x^2)^3/x,x, algorithm="giac")
```

output
```
1/2*log(abs(a*x^2 + b))/a^3 - 1/4*(3*a*x^4 + 2*b*x^2)/((a*x^2 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx=\frac{\frac{3\,b^2}{4\,a^3}+\frac{b\,x^2}{a^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2}+\frac{\ln\left(a\,x^2+b\right)}{2\,a^3}$$

input
```
int(1/(x*(a + b/x^2)^3),x)
```

output
```
((3*b^2)/(4*a^3) + (b*x^2)/a^2)/(b^2 + a^2*x^4 + 2*a*b*x^2) + log(b + a*x^
2)/(2*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x}\,dx =\frac{2\log(a\,x^2+b)\,a^2x^4+4\log(a\,x^2+b)\,ab\,x^2+2\log(a\,x^2+b)\,b^2-2a^2x^4+b^2}{4a^3\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x,x)
```

output
```
(2*log(a*x**2 + b)*a**2*x**4 + 4*log(a*x**2 + b)*a*b*x**2 + 2*log(a*x**2 +
 b)*b**2 - 2*a**2*x**4 + b**2)/(4*a**3*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.338 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3} \, dx = \frac{1}{4b\left(a+\frac{b}{x^2}\right)^2}$$

output
```
1/4/b/(a+b/x^2)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3} \, dx = -\frac{b+2ax^2}{4a^2\left(b+ax^2\right)^2}$$

input
```
Integrate[1/((a + b/x^2)^3*x^3),x]
```

output
```
-1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x^2}\right)^3} \, dx$$
$$\Big\downarrow \text{ 793}$$
$$\frac{1}{4b\left(a+\frac{b}{x^2}\right)^2}$$

input
```
Int[1/((a + b/x^2)^3*x^3),x]
```

output
```
1/(4*b*(a + b/x^2)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{1}{4b\left(a+\frac{b}{x^2}\right)^2}$	15
gosper	$-\frac{2a\,x^2+b}{4(a\,x^2+b)^2 a^2}$	23
parallelrisch	$\frac{-2a\,x^2-b}{4a^2(a\,x^2+b)^2}$	25
risch	$\frac{-\frac{x^2}{2a}-\frac{b}{4a^2}}{(a\,x^2+b)^2}$	26
default	$-\frac{1}{2a^2(a\,x^2+b)}+\frac{b}{4a^2(a\,x^2+b)^2}$	31
norman	$\frac{-\frac{x^4}{2a}-\frac{b\,x^2}{4a^2}}{(a\,x^2+b)^2 x^2}$	32
orering	$-\frac{(2a\,x^2+b)(a\,x^2+b)}{4a^2x^6\left(a+\frac{b}{x^2}\right)^3}$	33

input
```
int(1/(a+b/x^2)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/b/(a+b/x^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx = -\frac{2\,ax^2+b}{4\,(a^4x^4+2\,a^3bx^2+a^2b^2)}$$

input
```
integrate(1/(a+b/x^2)^3/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx = \frac{-2ax^2-b}{4a^4x^4+8a^3bx^2+4a^2b^2}$$

input
```
integrate(1/(a+b/x**2)**3/x**3,x)
```

output
```
(-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx = \frac{1}{4\left(a+\frac{b}{x^2}\right)^2 b}$$

input
```
integrate(1/(a+b/x^2)^3/x^3,x, algorithm="maxima")
```

output
```
1/4/((a + b/x^2)^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx = -\frac{2\,ax^2+b}{4\,(ax^2+b)^2a^2}$$

input
```
integrate(1/(a+b/x^2)^3/x^3,x, algorithm="giac")
```

output
```
-1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx = -\frac{\frac{b}{4\,a^2}+\frac{x^2}{2\,a}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

input
```
int(1/(x^3*(a + b/x^2)^3),x)
```

output
```
-(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^3}\,dx = \frac{x^4}{4b\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(1/(a+b/x^2)^3/x^3,x)
```

output
```
x**4/(4*b*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.339 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5} \, dx = -\frac{1}{4a\left(b+ax^2\right)^2}$$

output
```
-1/4/a/(a*x^2+b)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5} \, dx = -\frac{1}{4a\left(b+ax^2\right)^2}$$

input
```
Integrate[1/((a + b/x^2)^3*x^5),x]
```

output
```
-1/4*1/(a*(b + a*x^2)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {795, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x^2}\right)^3} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x}{(ax^2+b)^3}dx$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{1}{4a\left(ax^2+b\right)^2}$$

input
```
Int[1/((a + b/x^2)^3*x^5),x]
```

output
```
-1/4*1/(a*(b + a*x^2)^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{4a(a\,x^2+b)^2}$	15
default	$-\frac{1}{4a(a\,x^2+b)^2}$	15
norman	$-\frac{1}{4a(a\,x^2+b)^2}$	15
risch	$-\frac{1}{4a(a\,x^2+b)^2}$	15
parallelrisch	$-\frac{1}{4a(a\,x^2+b)^2}$	15
orering	$-\frac{a\,x^2+b}{4a\,x^6\left(a+\frac{b}{x^2}\right)^3}$	25

input
```
int(1/(a+b/x^2)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/(a*x^2+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5}\,dx = -\frac{1}{4\,(a^3x^4+2\,a^2bx^2+ab^2)}$$

input
```
integrate(1/(a+b/x^2)^3/x^5,x, algorithm="fricas")
```

output
```
-1/4/(a^3*x^4 + 2*a^2*b*x^2 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5}\,dx = -\frac{1}{4a^3x^4+8a^2bx^2+4ab^2}$$

input
```
integrate(1/(a+b/x**2)**3/x**5,x)
```

output
```
-1/(4*a**3*x**4 + 8*a**2*b*x**2 + 4*a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5}\,dx = -\frac{1}{4\,(a^3x^4+2\,a^2bx^2+ab^2)}$$

input
```
integrate(1/(a+b/x^2)^3/x^5,x, algorithm="maxima")
```

output
```
-1/4/(a^3*x^4 + 2*a^2*b*x^2 + a*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5}\,dx = -\frac{1}{4\,(ax^2+b)^2a}$$

input
```
integrate(1/(a+b/x^2)^3/x^5,x, algorithm="giac")
```

output
```
-1/4/((a*x^2 + b)^2*a)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5} \, dx = -\frac{1}{4 \, a^3 \, x^4 + 8 \, a^2 \, b \, x^2 + 4 \, a \, b^2}$$

input
```
int(1/(x^5*(a + b/x^2)^3),x)
```

output
```
-1/(4*a*b^2 + 4*a^3*x^4 + 8*a^2*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^5} \, dx = -\frac{1}{4a \left(a^2 x^4 + 2ab \, x^2 + b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^5,x)
```

output
```
( - 1)/(4*a*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.340 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx = \frac{a^2}{4b^3\left(a+\frac{b}{x^2}\right)^2} - \frac{a}{b^3\left(a+\frac{b}{x^2}\right)} - \frac{\log\left(a+\frac{b}{x^2}\right)}{2b^3}$$

output
```
1/4*a^2/b^3/(a+b/x^2)^2-a/b^3/(a+b/x^2)-1/2*ln(a+b/x^2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx = \frac{\frac{b(3b+2ax^2)}{(b+ax^2)^2} + 4\log(x) - 2\log\left(b+ax^2\right)}{4b^3}$$

input
```
Integrate[1/((a + b/x^2)^3*x^7),x]
```

output
```
((b*(3*b + 2*a*x^2))/(b + a*x^2)^2 + 4*Log[x] - 2*Log[b + a*x^2])/(4*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^2}\right)^3} \, dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{1}{x \left(a x^2+b\right)^3}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int \frac{1}{x^2 \left(a x^2+b\right)^3}dx^2$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{2} \int \left(-\frac{a}{b^3 \left(a x^2+b\right)}-\frac{a}{b^2 \left(a x^2+b\right)^2}-\frac{a}{b \left(a x^2+b\right)^3}+\frac{1}{b^3 x^2}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2} \left(-\frac{\log \left(a x^2+b\right)}{b^3}+\frac{1}{b^2 \left(a x^2+b\right)}+\frac{1}{2 b \left(a x^2+b\right)^2}+\frac{\log \left(x^2\right)}{b^3}\right)$$

input
```
Int[1/((a + b/x^2)^3*x^7),x]
```

output
```
(1/(2*b*(b + a*x^2)^2) + 1/(b^2*(b + a*x^2)) + Log[x^2]/b^3 - Log[b + a*x^
2]/b^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\frac{a\,x^{2}}{2 b^{2}}+\frac{3}{4 b}}{\left(a\,x^{2}+b\right)^{2}}+\frac{\ln \left(x \right)}{b^{3}}-\frac{\ln \left(a\,x^{2}+b \right)}{2 b^{3}}$	46
norman	$\frac{-\frac{a\,x^{8}}{b^{2}}-\frac{3 a^{2} x^{10}}{4 b^{3}}}{\left(a\,x^{2}+b \right)^{2} x^{6}}+\frac{\ln \left(x \right)}{b^{3}}-\frac{\ln \left(a\,x^{2}+b \right)}{2 b^{3}}$	55
default	$-\frac{a \left(-\frac{b}{a \left(a\,x^{2}+b \right)}+\frac{\ln \left(a\,x^{2}+b \right)}{a}-\frac{b^{2}}{2 a \left(a\,x^{2}+b \right)^{2}}\right)}{2 b^{3}}+\frac{\ln \left(x \right)}{b^{3}}$	59
parallelrisch	$\frac{4 a^{2} \ln \left(x \right) x^{4}-2 a^{2} \ln \left(a\,x^{2}+b \right) x^{4}-3 a^{2} x^{4}+8 a b \ln \left(x \right) x^{2}-4 \ln \left(a\,x^{2}+b \right) x^{2} a b-4 a b\,x^{2}+4 b^{2} \ln \left(x \right)-2 b^{2} \ln \left(a\,x^{2}+b \right)}{4 b^{3} \left(a\,x^{2}+b \right)^{2}}$	101

input
```
int(1/(a+b/x^2)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
(1/2*a*x^2/b^2+3/4/b)/(a*x^2+b)^2+1/b^3*ln(x)-1/2/b^3*ln(a*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx = \frac{2\,abx^2+3\,b^2-2\,(a^2x^4+2\,abx^2+b^2)\log\left(ax^2+b\right)+4\,(a^2x^4+2\,abx^2+b^2)\log\left(x\right)}{4\,(a^2b^3x^4+2\,ab^4x^2+b^5)}$$

input
```
integrate(1/(a+b/x^2)^3/x^7,x, algorithm="fricas")
```

output
```
1/4*(2*a*b*x^2 + 3*b^2 - 2*(a^2*x^4 + 2*a*b*x^2 + b^2)*log(a*x^2 + b) + 4*
(a^2*x^4 + 2*a*b*x^2 + b^2)*log(x))/(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx = \frac{2ax^2+3b}{4a^2b^2x^4+8ab^3x^2+4b^4}+\frac{\log\left(x\right)}{b^3}-\frac{\log\left(x^2+\frac{b}{a}\right)}{2b^3}$$

input
```
integrate(1/(a+b/x**2)**3/x**7,x)
```

output
```
(2*a*x**2 + 3*b)/(4*a**2*b**2*x**4 + 8*a*b**3*x**2 + 4*b**4) + log(x)/b**3
 - log(x**2 + b/a)/(2*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx = \frac{2\,ax^2+3\,b}{4\,(a^2b^2x^4+2\,ab^3x^2+b^4)}-\frac{\log\left(ax^2+b\right)}{2\,b^3}+\frac{\log\left(x^2\right)}{2\,b^3}$$

input
```
integrate(1/(a+b/x^2)^3/x^7,x, algorithm="maxima")
```

output
```
1/4*(2*a*x^2 + 3*b)/(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4) - 1/2*log(a*x^2 + b)
/b^3 + 1/2*log(x^2)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx = \frac{\log\left(x^2\right)}{2\,b^3} - \frac{\log\left(|ax^2+b|\right)}{2\,b^3} + \frac{3\,a^2x^4+8\,abx^2+6\,b^2}{4\,(ax^2+b)^2b^3}$$

input
```
integrate(1/(a+b/x^2)^3/x^7,x, algorithm="giac")
```

output
```
1/2*log(x^2)/b^3 - 1/2*log(abs(a*x^2 + b))/b^3 + 1/4*(3*a^2*x^4 + 8*a*b*x^
2 + 6*b^2)/((a*x^2 + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx = \frac{\ln(x)}{b^3} + \frac{\frac{3}{4\,b}+\frac{a\,x^2}{2\,b^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2} - \frac{\ln\left(a\,x^2+b\right)}{2\,b^3}$$

input
```
int(1/(x^7*(a + b/x^2)^3),x)
```

output
```
log(x)/b^3 + (3/(4*b) + (a*x^2)/(2*b^2))/(b^2 + a^2*x^4 + 2*a*b*x^2) - log
(b + a*x^2)/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.18

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^7}\,dx$$
$$= \frac{-2\log(a\,x^2+b)\,a^2x^4-4\log(a\,x^2+b)\,ab\,x^2-2\log(a\,x^2+b)\,b^2+4\log(x)\,a^2x^4+8\log(x)\,ab\,x^2+4\log(}{4b^3\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^7,x)
```

output
```
( - 2*log(a*x**2 + b)*a**2*x**4 - 4*log(a*x**2 + b)*a*b*x**2 - 2*log(a*x**
2 + b)*b**2 + 4*log(x)*a**2*x**4 + 8*log(x)*a*b*x**2 + 4*log(x)*b**2 - a**
2*x**4 + 2*b**2)/(4*b**3*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.341 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9} dx = -\frac{a^3}{4b^4\left(a+\frac{b}{x^2}\right)^2}+\frac{3a^2}{2b^4\left(a+\frac{b}{x^2}\right)}-\frac{1}{2b^3x^2}+\frac{3a\log\left(a+\frac{b}{x^2}\right)}{2b^4}$$

output
```
-1/4*a^3/b^4/(a+b/x^2)^2+3/2*a^2/b^4/(a+b/x^2)-1/2/b^3/x^2+3/2*a*ln(a+b/x^
2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9} dx = -\frac{\frac{b(2b^2+9abx^2+6a^2x^4)}{x^2(b+ax^2)^2}+12a\log(x)-6a\log(b+ax^2)}{4b^4}$$

input
```
Integrate[1/((a + b/x^2)^3*x^9),x]
```

output
```
-1/4*((b*(2*b^2 + 9*a*b*x^2 + 6*a^2*x^4))/(x^2*(b + a*x^2)^2) + 12*a*Log[x
] - 6*a*Log[b + a*x^2])/b^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+\frac{b}{x^2}\right)^3} \, dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{x^3 \left(a x^2+b\right)^3}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a x^2+b\right)^3}dx^2$$

$$\Big\downarrow \ 54$$

$$\frac{1}{2} \int \left(\frac{3 a^2}{b^4 \left(a x^2+b\right)}+\frac{2 a^2}{b^3 \left(a x^2+b\right)^2}+\frac{a^2}{b^2 \left(a x^2+b\right)^3}-\frac{3 a}{b^4 x^2}+\frac{1}{b^3 x^4}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(-\frac{3 a \log \left(x^2\right)}{b^4}+\frac{3 a \log \left(a x^2+b\right)}{b^4}-\frac{2 a}{b^3 \left(a x^2+b\right)}-\frac{a}{2 b^2 \left(a x^2+b\right)^2}-\frac{1}{b^3 x^2}\right)$$

input
```
Int[1/((a + b/x^2)^3*x^9),x]
```

output
```
(-(1/(b^3*x^2)) - a/(2*b^2*(b + a*x^2)^2) - (2*a)/(b^3*(b + a*x^2)) - (3*a
*Log[x^2])/b^4 + (3*a*Log[b + a*x^2])/b^4)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

method	result
risch	$\frac{-\frac{3a^2x^4}{2b^3}-\frac{9a\,x^2}{4b^2}-\frac{1}{2b}}{\left(a\,x^2+b\right)^2x^2}-\frac{3a\ln\left(x\right)}{b^4}+\frac{3a\ln\left(-a\,x^2-b\right)}{2b^4}$
norman	$\frac{\frac{3a^2x^{10}}{b^3}-\frac{x^6}{2b}+\frac{9a^3x^{12}}{4b^4}}{\left(a\,x^2+b\right)^2x^8}-\frac{3a\ln\left(x\right)}{b^4}+\frac{3a\ln\left(a\,x^2+b\right)}{2b^4}$
default	$\frac{a^2\left(-\frac{2b}{a\left(a\,x^2+b\right)}+\frac{3\ln\left(a\,x^2+b\right)}{a}-\frac{b^2}{2a\left(a\,x^2+b\right)^2}\right)}{2b^4}-\frac{1}{2b^3x^2}-\frac{3a\ln\left(x\right)}{b^4}$
parallelrisch	$-\frac{12a^3\ln\left(x\right)x^6-6\ln\left(a\,x^2+b\right)x^6a^3-9a^3x^6+24a^2b\ln\left(x\right)x^4-12\ln\left(a\,x^2+b\right)x^4a^2b-12a^2b\,x^4+12a\,b^2\ln\left(x\right)x^2-6\ln\left(a\,x^2+b\right)x^2}{4b^4x^2\left(a\,x^2+b\right)^2}$

input
```
int(1/(a+b/x^2)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*a^2/b^3*x^4-9/4*a*x^2/b^2-1/2/b)/(a*x^2+b)^2/x^2-3/b^4*a*ln(x)+3/2/b
^4*a*ln(-a*x^2-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx = -\frac{6\,a^2bx^4 + 9\,ab^2x^2 + 2\,b^3 - 6\,(a^3x^6 + 2\,a^2bx^4 + ab^2x^2)\log\left(ax^2 + b\right) + 12\,(a^3x^6 + 2\,a^2bx^4 + ab^2x^2)\log\left(\right.}{4\,(a^2b^4x^6 + 2\,ab^5x^4 + b^6x^2)}$$

input
```
integrate(1/(a+b/x^2)^3/x^9,x, algorithm="fricas")
```

output
```
-1/4*(6*a^2*b*x^4 + 9*a*b^2*x^2 + 2*b^3 - 6*(a^3*x^6 + 2*a^2*b*x^4 + a*b^2
*x^2)*log(a*x^2 + b) + 12*(a^3*x^6 + 2*a^2*b*x^4 + a*b^2*x^2)*log(x))/(a^2
*b^4*x^6 + 2*a*b^5*x^4 + b^6*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx = -\frac{3a\log\left(x\right)}{b^4} + \frac{3a\log\left(x^2+\frac{b}{a}\right)}{2b^4} + \frac{-6a^2x^4 - 9abx^2 - 2b^2}{4a^2b^3x^6 + 8ab^4x^4 + 4b^5x^2}$$

input
```
integrate(1/(a+b/x**2)**3/x**9,x)
```

output
```
-3*a*log(x)/b**4 + 3*a*log(x**2 + b/a)/(2*b**4) + (-6*a**2*x**4 - 9*a*b*x*
*2 - 2*b**2)/(4*a**2*b**3*x**6 + 8*a*b**4*x**4 + 4*b**5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx = -\frac{6\,a^2x^4+9\,abx^2+2\,b^2}{4\,(a^2b^3x^6+2\,ab^4x^4+b^5x^2)}+\frac{3\,a\log\left(ax^2+b\right)}{2\,b^4}-\frac{3\,a\log\left(x^2\right)}{2\,b^4}$$

input
```
integrate(1/(a+b/x^2)^3/x^9,x, algorithm="maxima")
```

output
```
-1/4*(6*a^2*x^4 + 9*a*b*x^2 + 2*b^2)/(a^2*b^3*x^6 + 2*a*b^4*x^4 + b^5*x^2)
 + 3/2*a*log(a*x^2 + b)/b^4 - 3/2*a*log(x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx = -\frac{3\,a\log\left(x^2\right)}{2\,b^4}+\frac{3\,a\log\left(|ax^2+b|\right)}{2\,b^4} - \frac{9\,a^3x^4+22\,a^2bx^2+14\,ab^2}{4\,(ax^2+b)^2b^4}+\frac{3\,ax^2-b}{2\,b^4x^2}$$

input
```
integrate(1/(a+b/x^2)^3/x^9,x, algorithm="giac")
```

output
```
-3/2*a*log(x^2)/b^4 + 3/2*a*log(abs(a*x^2 + b))/b^4 - 1/4*(9*a^3*x^4 + 22*
a^2*b*x^2 + 14*a*b^2)/((a*x^2 + b)^2*b^4) + 1/2*(3*a*x^2 - b)/(b^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx = \frac{3\,a\,\ln\left(a\,x^2+b\right)}{2\,b^4}-\frac{\frac{1}{2\,b}+\frac{9\,a\,x^2}{4\,b^2}+\frac{3\,a^2\,x^4}{2\,b^3}}{a^2\,x^6+2\,a\,b\,x^4+b^2\,x^2}-\frac{3\,a\,\ln\left(x\right)}{b^4}$$

input
```
int(1/(x^9*(a + b/x^2)^3),x)
```

output
```
(3*a*log(b + a*x^2))/(2*b^4) - (1/(2*b) + (9*a*x^2)/(4*b^2) + (3*a^2*x^4)/
(2*b^3))/(a^2*x^6 + b^2*x^2 + 2*a*b*x^4) - (3*a*log(x))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^9}\,dx$$
$$= \frac{6\log(a\,x^2+b)\,a^3x^6+12\log(a\,x^2+b)\,a^2b\,x^4+6\log(a\,x^2+b)\,a\,b^2x^2-12\log(x)\,a^3x^6-24\log(x)\,a^2b\,x^4 -}{4b^4x^2\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^9,x)
```

output
```
(6*log(a*x**2 + b)*a**3*x**6 + 12*log(a*x**2 + b)*a**2*b*x**4 + 6*log(a*x*
*2 + b)*a*b**2*x**2 - 12*log(x)*a**3*x**6 - 24*log(x)*a**2*b*x**4 - 12*log
(x)*a*b**2*x**2 + 3*a**3*x**6 - 6*a*b**2*x**2 - 2*b**3)/(4*b**4*x**2*(a**2
*x**4 + 2*a*b*x**2 + b**2))
```

3.342 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}} \, dx = \frac{a^4}{4b^5\left(a+\frac{b}{x^2}\right)^2} - \frac{2a^3}{b^5\left(a+\frac{b}{x^2}\right)} - \frac{1}{4b^3x^4} + \frac{3a}{2b^4x^2} - \frac{3a^2\log\left(a+\frac{b}{x^2}\right)}{b^5}$$

output
```
1/4*a^4/b^5/(a+b/x^2)^2-2*a^3/b^5/(a+b/x^2)-1/4/b^3/x^4+3/2*a/b^4/x^2-3*a^
2*ln(a+b/x^2)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}} \, dx = \frac{\frac{b(-b^3+4ab^2x^2+18a^2bx^4+12a^3x^6)}{x^4(b+ax^2)^2} + 24a^2\log(x) - 12a^2\log\left(b+ax^2\right)}{4b^5}$$

input
```
Integrate[1/((a + b/x^2)^3*x^11),x]
```

output
```
((b*(-b^3 + 4*a*b^2*x^2 + 18*a^2*b*x^4 + 12*a^3*x^6))/(x^4*(b + a*x^2)^2)
+ 24*a^2*Log[x] - 12*a^2*Log[b + a*x^2])/(4*b^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11} \left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^5 \left(a x^2+b\right)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^6 \left(a x^2+b\right)^3}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(-\frac{6 a^3}{b^5 \left(a x^2+b\right)}-\frac{3 a^3}{b^4 \left(a x^2+b\right)^2}-\frac{a^3}{b^3 \left(a x^2+b\right)^3}+\frac{6 a^2}{b^5 x^2}-\frac{3 a}{b^4 x^4}+\frac{1}{b^3 x^6}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{6 a^2 \log \left(x^2\right)}{b^5}-\frac{6 a^2 \log \left(a x^2+b\right)}{b^5}+\frac{3 a^2}{b^4 \left(a x^2+b\right)}+\frac{a^2}{2 b^3 \left(a x^2+b\right)^2}+\frac{3 a}{b^4 x^2}-\frac{1}{2 b^3 x^4}\right)$$

input
```
Int[1/((a + b/x^2)^3*x^11),x]
```

output
```
(-1/2*1/(b^3*x^4) + (3*a)/(b^4*x^2) + a^2/(2*b^3*(b + a*x^2)^2) + (3*a^2)/
(b^4*(b + a*x^2)) + (6*a^2*Log[x^2])/b^5 - (6*a^2*Log[b + a*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

method	result
risch	$\frac{\frac{3a^{3}x^{6}}{b^{4}}+\frac{9a^{2}x^{4}}{2b^{3}}+\frac{a\,x^{2}}{b^{2}}-\frac{1}{4b}}{\left(a\,x^{2}+b\right)^{2}x^{4}}+\frac{6a^{2}\ln\left(x\right)}{b^{5}}-\frac{3a^{2}\ln\left(a\,x^{2}+b\right)}{b^{5}}$
norman	$\frac{\frac{a\,x^{8}}{b^{2}}-\frac{x^{6}}{4b}-\frac{6a^{3}x^{12}}{b^{4}}-\frac{9a^{4}x^{14}}{2b^{5}}}{\left(a\,x^{2}+b\right)^{2}x^{10}}+\frac{6a^{2}\ln\left(x\right)}{b^{5}}-\frac{3a^{2}\ln\left(a\,x^{2}+b\right)}{b^{5}}$
default	$-\frac{a^{3}\left(-\frac{3b}{a\left(a\,x^{2}+b\right)}+\frac{6\ln\left(a\,x^{2}+b\right)}{a}-\frac{b^{2}}{2a\left(a\,x^{2}+b\right)^{2}}\right)}{2b^{5}}-\frac{1}{4b^{3}x^{4}}+\frac{6a^{2}\ln\left(x\right)}{b^{5}}+\frac{3a}{2b^{4}x^{2}}$
parallelrisch	$\frac{24\ln\left(x\right)x^{8}a^{4}-12\ln\left(a\,x^{2}+b\right)x^{8}a^{4}-18a^{4}x^{8}+48\ln\left(x\right)x^{6}a^{3}b-24\ln\left(a\,x^{2}+b\right)x^{6}a^{3}b-24a^{3}b\,x^{6}+24\ln\left(x\right)x^{4}a^{2}b^{2}-12\ln\left(a\,x^{2}+b\right)x}{4b^{5}x^{4}\left(a\,x^{2}+b\right)^{2}}$

input
```
int(1/(a+b/x^2)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
(3*a^3/b^4*x^6+9/2*a^2/b^3*x^4+a*x^2/b^2-1/4/b)/(a*x^2+b)^2/x^4+6/b^5*a^2*
ln(x)-3/b^5*a^2*ln(a*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx = \frac{12\,a^3bx^6+18\,a^2b^2x^4+4\,ab^3x^2-b^4-12\,(a^4x^8+2\,a^3bx^6+a^2b^2x^4)\log\left(ax^2+b\right)+24\,(a^4x^8+2\,a^3bx^6+}{4\,(a^2b^5x^8+2\,ab^6x^6+b^7x^4)}$$

input
```
integrate(1/(a+b/x^2)^3/x^11,x, algorithm="fricas")
```

output
```
1/4*(12*a^3*b*x^6 + 18*a^2*b^2*x^4 + 4*a*b^3*x^2 - b^4 - 12*(a^4*x^8 + 2*a
^3*b*x^6 + a^2*b^2*x^4)*log(a*x^2 + b) + 24*(a^4*x^8 + 2*a^3*b*x^6 + a^2*b
^2*x^4)*log(x))/(a^2*b^5*x^8 + 2*a*b^6*x^6 + b^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx = \frac{6a^2\log\left(x\right)}{b^5} - \frac{3a^2\log\left(x^2+\frac{b}{a}\right)}{b^5} + \frac{12a^3x^6+18a^2bx^4+4ab^2x^2-b^3}{4a^2b^4x^8+8ab^5x^6+4b^6x^4}$$

input
```
integrate(1/(a+b/x**2)**3/x**11,x)
```

output
```
6*a**2*log(x)/b**5 - 3*a**2*log(x**2 + b/a)/b**5 + (12*a**3*x**6 + 18*a**2
*b*x**4 + 4*a*b**2*x**2 - b**3)/(4*a**2*b**4*x**8 + 8*a*b**5*x**6 + 4*b**6
*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx = \frac{12\,a^3x^6+18\,a^2bx^4+4\,ab^2x^2-b^3}{4\,(a^2b^4x^8+2\,ab^5x^6+b^6x^4)} - \frac{3\,a^2\log\left(ax^2+b\right)}{b^5} + \frac{3\,a^2\log\left(x^2\right)}{b^5}$$

input
```
integrate(1/(a+b/x^2)^3/x^11,x, algorithm="maxima")
```

output
```
1/4*(12*a^3*x^6 + 18*a^2*b*x^4 + 4*a*b^2*x^2 - b^3)/(a^2*b^4*x^8 + 2*a*b^5
*x^6 + b^6*x^4) - 3*a^2*log(a*x^2 + b)/b^5 + 3*a^2*log(x^2)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx = \frac{3\,a^2\log\left(x^2\right)}{b^5} - \frac{3\,a^2\log\left(|ax^2+b|\right)}{b^5} + \frac{12\,a^3x^6+18\,a^2bx^4+4\,ab^2x^2-b^3}{4\,(ax^4+bx^2)^2b^4}$$

input
```
integrate(1/(a+b/x^2)^3/x^11,x, algorithm="giac")
```

output
```
3*a^2*log(x^2)/b^5 - 3*a^2*log(abs(a*x^2 + b))/b^5 + 1/4*(12*a^3*x^6 + 18*
a^2*b*x^4 + 4*a*b^2*x^2 - b^3)/((a*x^4 + b*x^2)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx = \frac{\frac{a\,x^2}{b^2}-\frac{1}{4\,b}+\frac{9\,a^2\,x^4}{2\,b^3}+\frac{3\,a^3\,x^6}{b^4}}{a^2\,x^8+2\,a\,b\,x^6+b^2\,x^4} - \frac{3\,a^2\,\ln\left(a\,x^2+b\right)}{b^5} + \frac{6\,a^2\,\ln\left(x\right)}{b^5}$$

input
```
int(1/(x^11*(a + b/x^2)^3),x)
```

output
```
((a*x^2)/b^2 - 1/(4*b) + (9*a^2*x^4)/(2*b^3) + (3*a^3*x^6)/b^4)/(a^2*x^8 +
 b^2*x^4 + 2*a*b*x^6) - (3*a^2*log(b + a*x^2))/b^5 + (6*a^2*log(x))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{11}}\,dx$$
$$= \frac{-12\log(a\,x^2+b)\,a^4x^8-24\log(a\,x^2+b)\,a^3b\,x^6-12\log(a\,x^2+b)\,a^2b^2x^4+24\log(x)\,a^4x^8+48\log(x)\,a^3}{4b^5x^4\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^11,x)
```

output
```
( - 12*log(a*x**2 + b)*a**4*x**8 - 24*log(a*x**2 + b)*a**3*b*x**6 - 12*log
(a*x**2 + b)*a**2*b**2*x**4 + 24*log(x)*a**4*x**8 + 48*log(x)*a**3*b*x**6
+ 24*log(x)*a**2*b**2*x**4 - 6*a**4*x**8 + 12*a**2*b**2*x**4 + 4*a*b**3*x*
*2 - b**4)/(4*b**5*x**4*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.343 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 96

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx=\frac{6b^2x}{a^5}-\frac{bx^3}{a^4}+\frac{x^5}{5a^3}-\frac{b^4x}{4a^5\left(b+ax^2\right)^2}+\frac{17b^3x}{8a^5\left(b+ax^2\right)}-\frac{63b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{11/2}}$$

output
```
6*b^2*x/a^5-b*x^3/a^4+1/5*x^5/a^3-1/4*b^4*x/a^5/(a*x^2+b)^2+17/8*b^3*x/a^5
/(a*x^2+b)-63/8*b^(5/2)*arctan(a^(1/2)*x/b^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx=\frac{315b^4x+525ab^3x^3+168a^2b^2x^5-24a^3bx^7+8a^4x^9}{40a^5\left(b+ax^2\right)^2}-\frac{63b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{11/2}}$$

input
```
Integrate[x^4/(a + b/x^2)^3,x]
```

output
```
(315*b^4*x + 525*a*b^3*x^3 + 168*a^2*b^2*x^5 - 24*a^3*b*x^7 + 8*a^4*x^9)/(
40*a^5*(b + a*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(11/
2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {795, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{10}}{\left(ax^2+b\right)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{9\int \frac{x^8}{\left(ax^2+b\right)^2}dx}{4a}-\frac{x^9}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{9\left(\frac{7\int \frac{x^6}{ax^2+b}dx}{2a}-\frac{x^7}{2a(ax^2+b)}\right)}{4a}-\frac{x^9}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{254}$$

$$\frac{9\left(\frac{7\int \left(\frac{x^4}{a}-\frac{bx^2}{a^2}+\frac{b^2}{a^3}-\frac{b^3}{a^3\left(ax^2+b\right)}\right)dx}{2a}-\frac{x^7}{2a(ax^2+b)}\right)}{4a}-\frac{x^9}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{9\left(\frac{7\left(-\frac{b^{5/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{7/2}}+\frac{b^2x}{a^3}-\frac{bx^3}{3a^2}+\frac{x^5}{5a}\right)}{2a}-\frac{x^7}{2a(ax^2+b)}\right)}{4a}-\frac{x^9}{4a\left(ax^2+b\right)^2}$$

input
```
Int[x^4/(a + b/x^2)^3,x]
```

output
```
-1/4*x^9/(a*(b + a*x^2)^2) + (9*(-1/2*x^7/(a*(b + a*x^2)) + (7*((b^2*x)/a^
3 - (b*x^3)/(3*a^2) + x^5/(5*a) - (b^(5/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^
(7/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\frac{1}{5}a^2x^5-ab\,x^3+6b^2x}{a^5}-\frac{b^3\left(\frac{-\frac{17}{8}a\,x^3-\frac{15}{8}bx}{\left(a\,x^2+b\right)^2}+\frac{63\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5}$	74
risch	$\frac{x^5}{5a^3}-\frac{b\,x^3}{a^4}+\frac{6b^2x}{a^5}+\frac{\frac{17}{8}a\,x^3b^3+\frac{15}{8}b^4x}{a^5(a\,x^2+b)^2}+\frac{63\sqrt{-ab}\,b^2\ln\left(-\sqrt{-ab}\,x-b\right)}{16a^6}-\frac{63\sqrt{-ab}\,b^2\ln\left(\sqrt{-ab}\,x-b\right)}{16a^6}$	112

input
```
int(x^4/(a+b/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/5*a^2*x^5-a*b*x^3+6*b^2*x)-1/a^5*b^3*((-17/8*a*x^3-15/8*b*x)/(a*x
^2+b)^2+63/8/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.67

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx$$
$$=\left[\frac{16\,a^4x^9-48\,a^3bx^7+336\,a^2b^2x^5+1050\,ab^3x^3+630\,b^4x+315\,(a^2b^2x^4+2\,ab^3x^2+b^4)\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2}{a}\right.}{80\,(a^7x^4+2\,a^6bx^2+a^5b^2)}\right.$$

input
```
integrate(x^4/(a+b/x^2)^3,x, algorithm="fricas")
```

output
```
[1/80*(16*a^4*x^9 - 48*a^3*b*x^7 + 336*a^2*b^2*x^5 + 1050*a*b^3*x^3 + 630*
b^4*x + 315*(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(-b/a)*log((a*x^2 - 2*a*
x*sqrt(-b/a) - b)/(a*x^2 + b)))/(a^7*x^4 + 2*a^6*b*x^2 + a^5*b^2), 1/40*(8
*a^4*x^9 - 24*a^3*b*x^7 + 168*a^2*b^2*x^5 + 525*a*b^3*x^3 + 315*b^4*x - 31
5*(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b))/(a^
7*x^4 + 2*a^6*b*x^2 + a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.50

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^{11}}}}{b^2}+x\right)}{16} - \frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^{11}}}}{b^2}+x\right)}{16} + \frac{17ab^3x^3+15b^4x}{8a^7x^4+16a^6bx^2+8a^5b^2} + \frac{x^5}{5a^3} - \frac{bx^3}{a^4} + \frac{6b^2x}{a^5}$$

input
```
integrate(x**4/(a+b/x**2)**3,x)
```

output
```
63*sqrt(-b**5/a**11)*log(-a**5*sqrt(-b**5/a**11)/b**2 + x)/16 - 63*sqrt(-b
**5/a**11)*log(a**5*sqrt(-b**5/a**11)/b**2 + x)/16 + (17*a*b**3*x**3 + 15*
b**4*x)/(8*a**7*x**4 + 16*a**6*b*x**2 + 8*a**5*b**2) + x**5/(5*a**3) - b*x
**3/a**4 + 6*b**2*x/a**5
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{17\,ab^3x^3+15\,b^4x}{8\,(a^7x^4+2\,a^6bx^2+a^5b^2)} - \frac{63\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5} + \frac{a^2x^5-5\,abx^3+30\,b^2x}{5\,a^5}$$

input
```
integrate(x^4/(a+b/x^2)^3,x, algorithm="maxima")
```

output
```
1/8*(17*a*b^3*x^3 + 15*b^4*x)/(a^7*x^4 + 2*a^6*b*x^2 + a^5*b^2) - 63/8*b^3
*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/5*(a^2*x^5 - 5*a*b*x^3 + 30*b^2
*x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx = -\frac{63\,b^3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5} + \frac{17\,ab^3x^3+15\,b^4x}{8\,(ax^2+b)^2a^5} + \frac{a^{12}x^5-5\,a^{11}bx^3+30\,a^{10}b^2x}{5\,a^{15}}$$

input
```
integrate(x^4/(a+b/x^2)^3,x, algorithm="giac")
```

output
```
-63/8*b^3*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/8*(17*a*b^3*x^3 + 15*b
^4*x)/((a*x^2 + b)^2*a^5) + 1/5*(a^12*x^5 - 5*a^11*b*x^3 + 30*a^10*b^2*x)/
a^15
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{\frac{15\,b^4\,x}{8}+\frac{17\,a\,b^3\,x^3}{8}}{a^7\,x^4+2\,a^6\,b\,x^2+a^5\,b^2} + \frac{x^5}{5\,a^3} - \frac{b\,x^3}{a^4} + \frac{6\,b^2\,x}{a^5} - \frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{11/2}}$$

input
```
int(x^4/(a + b/x^2)^3,x)
```

output
```
((15*b^4*x)/8 + (17*a*b^3*x^3)/8)/(a^5*b^2 + a^7*x^4 + 2*a^6*b*x^2) + x^5/
(5*a^3) - (b*x^3)/a^4 + (6*b^2*x)/a^5 - (63*b^(5/2)*atan((a^(1/2)*x)/b^(1/
2)))/(8*a^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^3}\,dx$$

$$= \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^4 - 630\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,b^3x^2 - 315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^4 + 8a^5x^9 -}{40a^6\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(x^4/(a+b/x^2)^3,x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 - 630
*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b**3*x**2 - 315*sqrt(b)*s
qrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*b**4 + 8*a**5*x**9 - 24*a**4*b*x**7 +
 168*a**3*b**2*x**5 + 525*a**2*b**3*x**3 + 315*a*b**4*x)/(40*a**6*(a**2*x*
*4 + 2*a*b*x**2 + b**2))
```

3.344 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3} dx = -\frac{3bx}{a^4} + \frac{x^3}{3a^3} + \frac{b^3x}{4a^4(b+ax^2)^2} - \frac{13b^2x}{8a^4(b+ax^2)} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{9/2}}$$

output
```
-3*b*x/a^4+1/3*x^3/a^3+1/4*b^3*x/a^4/(a*x^2+b)^2-13/8*b^2*x/a^4/(a*x^2+b)+
35/8*b^(3/2)*arctan(a^(1/2)*x/b^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3} dx = -\frac{105b^3x + 175ab^2x^3 + 56a^2bx^5 - 8a^3x^7}{24a^4(b+ax^2)^2} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{9/2}}$$

input
```
Integrate[x^2/(a + b/x^2)^3,x]
```

output
```
-1/24*(105*b^3*x + 175*a*b^2*x^3 + 56*a^2*b*x^5 - 8*a^3*x^7)/(a^4*(b + a*x
^2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(9/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {795, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^8}{\left(ax^2+b\right)^3}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{7\int \frac{x^6}{\left(ax^2+b\right)^2}dx}{4a}-\frac{x^7}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{7\left(\frac{5\int \frac{x^4}{ax^2+b}dx}{2a}-\frac{x^5}{2a(ax^2+b)}\right)}{4a}-\frac{x^7}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{7\left(\frac{5\int \left(\frac{b^2}{a^2\left(ax^2+b\right)}-\frac{b}{a^2}+\frac{x^2}{a}\right)dx}{2a}-\frac{x^5}{2a(ax^2+b)}\right)}{4a}-\frac{x^7}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{7\left(\frac{5\left(\frac{b^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{5/2}}-\frac{bx}{a^2}+\frac{x^3}{3a}\right)}{2a}-\frac{x^5}{2a(ax^2+b)}\right)}{4a}-\frac{x^7}{4a\left(ax^2+b\right)^2}$$

input
```
Int[x^2/(a + b/x^2)^3,x]
```

output
```
-1/4*x^7/(a*(b + a*x^2)^2) + (7*(-1/2*x^5/(a*(b + a*x^2)) + (5*(-((b*x)/a^
2) + x^3/(3*a) + (b^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(5/2)))/(2*a)))/(
4*a)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{1}{3} a\,x^{3}-3 b x}{a^{4}}+\frac{b^{2} \left(\frac{-\frac{13}{8} a\,x^{3}-\frac{11}{8} b x}{\left(a\,x^{2}+b \right)^{2}}+\frac{35 \arctan \left(\frac{a x}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{a^{4}}$	62
risch	$\frac{x^{3}}{3 a^{3}}-\frac{3 b x}{a^{4}}+\frac{-\frac{13}{8} a\,b^{2} x^{3}-\frac{11}{8} b^{3} x}{a^{4} \left(a\,x^{2}+b \right)^{2}}+\frac{35 \sqrt{-a b}\, b \ln \left(-\sqrt{-a b}\, x+b \right)}{16 a^{5}}-\frac{35 \sqrt{-a b}\, b \ln \left(\sqrt{-a b}\, x+b \right)}{16 a^{5}}$	93

input
```
int(x^2/(a+b/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/3*a*x^3-3*b*x)+1/a^4*b^2*((-13/8*a*x^3-11/8*b*x)/(a*x^2+b)^2+35/8
/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.71

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \left[\frac{16\,a^3x^7 - 112\,a^2bx^5 - 350\,ab^2x^3 - 210\,b^3x + 105\,(a^2bx^4 + 2\,ab^2x^2 + b^3)\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{48\,(a^6x^4 + 2\,a^5bx^2 + a^4b^2)}, \; 8\,a\right.$$

input
```
integrate(x^2/(a+b/x^2)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*a^3*x^7 - 112*a^2*b*x^5 - 350*a*b^2*x^3 - 210*b^3*x + 105*(a^2*b
*x^4 + 2*a*b^2*x^2 + b^3)*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a
*x^2 + b)))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2), 1/24*(8*a^3*x^7 - 56*a^2*b*
x^5 - 175*a*b^2*x^3 - 105*b^3*x + 105*(a^2*b*x^4 + 2*a*b^2*x^2 + b^3)*sqrt
(b/a)*arctan(a*x*sqrt(b/a)/b))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx = -\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^9}}}{b}+x\right)}{16} + \frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^9}}}{b}+x\right)}{16} + \frac{-13ab^2x^3 - 11b^3x}{8a^6x^4 + 16a^5bx^2 + 8a^4b^2} + \frac{x^3}{3a^3} - \frac{3bx}{a^4}$$

input
```
integrate(x**2/(a+b/x**2)**3,x)
```

output
```
-35*sqrt(-b**3/a**9)*log(-a**4*sqrt(-b**3/a**9)/b + x)/16 + 35*sqrt(-b**3/
a**9)*log(a**4*sqrt(-b**3/a**9)/b + x)/16 + (-13*a*b**2*x**3 - 11*b**3*x)/
(8*a**6*x**4 + 16*a**5*b*x**2 + 8*a**4*b**2) + x**3/(3*a**3) - 3*b*x/a**4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx = -\frac{13\,ab^2x^3+11\,b^3x}{8\,(a^6x^4+2\,a^5bx^2+a^4b^2)}+\frac{35\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}+\frac{ax^3-9\,bx}{3\,a^4}$$

input
```
integrate(x^2/(a+b/x^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(13*a*b^2*x^3 + 11*b^3*x)/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2) + 35/8*b^
2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/3*(a*x^3 - 9*b*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{35\,b^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}-\frac{13\,ab^2x^3+11\,b^3x}{8\,(ax^2+b)^2a^4}+\frac{a^6x^3-9\,a^5bx}{3\,a^9}$$

input
```
integrate(x^2/(a+b/x^2)^3,x, algorithm="giac")
```

output
```
35/8*b^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/8*(13*a*b^2*x^3 + 11*b^
3*x)/((a*x^2 + b)^2*a^4) + 1/3*(a^6*x^3 - 9*a^5*b*x)/a^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{x^3}{3\,a^3} - \frac{\frac{11\,b^3\,x}{8} + \frac{13\,a\,b^2\,x^3}{8}}{a^6\,x^4 + 2\,a^5\,b\,x^2 + a^4\,b^2} + \frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{9/2}} - \frac{3\,b\,x}{a^4}$$

input
```
int(x^2/(a + b/x^2)^3,x)
```

output
```
x^3/(3*a^3) - ((11*b^3*x)/8 + (13*a*b^2*x^3)/8)/(a^4*b^2 + a^6*x^4 + 2*a^5
*b*x^2) + (35*b^(3/2)*atan((a^(1/2)*x)/b^(1/2)))/(8*a^(9/2)) - (3*b*x)/a^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^4 + 210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^2 + 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^3 + 8a^4x^7 - 5}{24a^5\left(a^2x^4 + 2ab\,x^2 + b^2\right)}$$

input
```
int(x^2/(a+b/x^2)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**4 + 210*sqrt(
b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**2 + 105*sqrt(b)*sqrt(a)
*atan((a*x)/(sqrt(b)*sqrt(a)))*b**3 + 8*a**4*x**7 - 56*a**3*b*x**5 - 175*a
**2*b**2*x**3 - 105*a*b**3*x)/(24*a**5*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.345 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 71

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3} dx = \frac{x}{a^3} - \frac{b^2 x}{4 a^3 (b+a x^2)^2} + \frac{9 b x}{8 a^3 (b+a x^2)} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8 a^{7/2}}$$

output
```
x/a^3-1/4*b^2*x/a^3/(a*x^2+b)^2+9/8*b*x/a^3/(a*x^2+b)-15/8*b^(1/2)*arctan(
a^(1/2)*x/b^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3} dx = \frac{15 b^2 x + 25 a b x^3 + 8 a^2 x^5}{8 a^3 (b+a x^2)^2} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8 a^{7/2}}$$

input
```
Integrate[(a + b/x^2)^(-3),x]
```

output
```
(15*b^2*x + 25*a*b*x^3 + 8*a^2*x^5)/(8*a^3*(b + a*x^2)^2) - (15*Sqrt[b]*Ar
cTan[(Sqrt[a]*x)/Sqrt[b]])/(8*a^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {772, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{x^6}{(ax^2+b)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^4}{(ax^2+b)^2}dx}{4a}-\frac{x^5}{4a(ax^2+b)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{5\left(\frac{3\int \frac{x^2}{ax^2+b}dx}{2a}-\frac{x^3}{2a(ax^2+b)}\right)}{4a}-\frac{x^5}{4a(ax^2+b)^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{5\left(\frac{3\left(\frac{x}{a}-\frac{b\int \frac{1}{ax^2+b}dx}{a}\right)}{2a}-\frac{x^3}{2a(ax^2+b)}\right)}{4a}-\frac{x^5}{4a(ax^2+b)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(\frac{x}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{a^{3/2}}\right)}{2a}-\frac{x^3}{2a(ax^2+b)}\right)}{4a}-\frac{x^5}{4a(ax^2+b)^2}$$

input
```
Int[(a + b/x^2)^(-3),x]
```

output
```
-1/4*x^5/(a*(b + a*x^2)^2) + (5*(-1/2*x^3/(a*(b + a*x^2)) + (3*(x/a - (Sqr
t[b]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/a^(3/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{x}{a^{3}}-\frac{b\left(\frac{-\frac{9}{8}a\,x^{3}-\frac{7}{8}bx}{\left(a\,x^{2}+b\right)^{2}}+\frac{15\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}$	51
risch	$\frac{x}{a^{3}}+\frac{\frac{9}{8}ab\,x^{3}+\frac{7}{8}b^{2}x}{a^{3}\left(a\,x^{2}+b\right)^{2}}+\frac{15\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-b\right)}{16a^{4}}-\frac{15\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-b\right)}{16a^{4}}$	83

input
```
int(1/(a+b/x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x/a^3-1/a^3*b*((-9/8*a*x^3-7/8*b*x)/(a*x^2+b)^2+15/8/(a*b)^(1/2)*arctan(a*
x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.85

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \left[\frac{16\,a^2x^5+50\,abx^3+30\,b^2x+15\,(a^2x^4+2\,abx^2+b^2)\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)}{16\,(a^5x^4+2\,a^4bx^2+a^3b^2)}, \frac{8\,a^2x^5+25\,abx^3+15\right.$$

input
```
integrate(1/(a+b/x^2)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*a^2*x^5 + 50*a*b*x^3 + 30*b^2*x + 15*(a^2*x^4 + 2*a*b*x^2 + b^2)
*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)))/(a^5*x^4 + 2*
a^4*b*x^2 + a^3*b^2), 1/8*(8*a^2*x^5 + 25*a*b*x^3 + 15*b^2*x - 15*(a^2*x^4
 + 2*a*b*x^2 + b^2)*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b))/(a^5*x^4 + 2*a^4*b*
x^2 + a^3*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx=\frac{15\sqrt{-\frac{b}{a^7}}\log\left(-a^3\sqrt{-\frac{b}{a^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{a^7}}\log\left(a^3\sqrt{-\frac{b}{a^7}}+x\right)}{16}+\frac{9abx^3+7b^2x}{8a^5x^4+16a^4bx^2+8a^3b^2}+\frac{x}{a^3}$$

input
```
integrate(1/(a+b/x**2)**3,x)
```

output
```
15*sqrt(-b/a**7)*log(-a**3*sqrt(-b/a**7) + x)/16 - 15*sqrt(-b/a**7)*log(a*
*3*sqrt(-b/a**7) + x)/16 + (9*a*b*x**3 + 7*b**2*x)/(8*a**5*x**4 + 16*a**4*
b*x**2 + 8*a**3*b**2) + x/a**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx=\frac{9\,abx^3+7\,b^2x}{8\,(a^5x^4+2\,a^4bx^2+a^3b^2)}-\frac{15\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}+\frac{x}{a^3}$$

input
```
integrate(1/(a+b/x^2)^3,x, algorithm="maxima")
```

output
```
1/8*(9*a*b*x^3 + 7*b^2*x)/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2) - 15/8*b*arcta
n(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + x/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx = -\frac{15\,b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3} + \frac{x}{a^3} + \frac{9\,abx^3 + 7\,b^2x}{8\,\left(ax^2+b\right)^2a^3}$$

input
```
integrate(1/(a+b/x^2)^3,x, algorithm="giac")
```

output
```
-15/8*b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^3) + x/a^3 + 1/8*(9*a*b*x^3 + 7
*b^2*x)/((a*x^2 + b)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{\frac{7\,b^2\,x}{8}+\frac{9\,a\,b\,x^3}{8}}{a^5\,x^4+2\,a^4\,b\,x^2+a^3\,b^2} + \frac{x}{a^3} - \frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{7/2}}$$

input
```
int(1/(a + b/x^2)^3,x)
```

output
```
((7*b^2*x)/8 + (9*a*b*x^3)/8)/(a^3*b^2 + a^5*x^4 + 2*a^4*b*x^2) + x/a^3 -
(15*b^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/(8*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2x^4 - 30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2 - 15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^2 + 8a^3x^5 + 25a^2b}{8a^4\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*x**4 - 30*sqrt(b
)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 - 15*sqrt(b)*sqrt(a)*atan
((a*x)/(sqrt(b)*sqrt(a)))*b**2 + 8*a**3*x**5 + 25*a**2*b*x**3 + 15*a*b**2*
x)/(8*a**4*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.346 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2} \, dx=-\frac{x^3}{4a\left(b+ax^2\right)^2}-\frac{3x}{8a^2\left(b+ax^2\right)}+\frac{3\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{5/2}\sqrt{b}}$$

output
```
-1/4*x^3/a/(a*x^2+b)^2-3/8*x/a^2/(a*x^2+b)+3/8*arctan(a^(1/2)*x/b^(1/2))/a
^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2} \, dx=-\frac{3bx+5ax^3}{8a^2\left(b+ax^2\right)^2}+\frac{3\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{5/2}\sqrt{b}}$$

input
```
Integrate[1/((a + b/x^2)^3*x^2),x]
```

output
```
-1/8*(3*b*x + 5*a*x^3)/(a^2*(b + a*x^2)^2) + (3*ArcTan[(Sqrt[a]*x)/Sqrt[b]
])/(8*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x^2}\right)^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^4}{(ax^2+b)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3\int \frac{x^2}{(ax^2+b)^2}dx}{4a}-\frac{x^3}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{3\left(\frac{\int \frac{1}{ax^2+b}dx}{2a}-\frac{x}{2a(ax^2+b)}\right)}{4a}-\frac{x^3}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{x}{2a(ax^2+b)}\right)}{4a}-\frac{x^3}{4a\left(ax^2+b\right)^2}$$

input
```
Int[1/((a + b/x^2)^3*x^2),x]
```

output
```
-1/4*x^3/(a*(b + a*x^2)^2) + (3*(-1/2*x/(a*(b + a*x^2)) + ArcTan[(Sqrt[a]*
x)/Sqrt[b]]/(2*a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{-\frac{5x^3}{8a}-\frac{3bx}{8a^2}}{\left(a\,x^2+b\right)^2}+\frac{3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}$	47
risch	$\frac{-\frac{5x^3}{8a}-\frac{3bx}{8a^2}}{\left(a\,x^2+b\right)^2}-\frac{3\ln\left(ax+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}+\frac{3\ln\left(-ax+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}$	73

input
```
int(1/(a+b/x^2)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-5/8/a*x^3-3/8*b/a^2*x)/(a*x^2+b)^2+3/8/a^2/(a*b)^(1/2)*arctan(a*x/(a*b)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx = \left[-\frac{10\,a^2bx^3+6\,ab^2x+3\,(a^2x^4+2\,abx^2+b^2)\sqrt{-ab}\log\left(\frac{ax^2-2\,\sqrt{-ab}x-b}{ax^2+b}\right)}{16\,(a^5bx^4+2\,a^4b^2x^2+a^3b^3)}, -\frac{5\,a^2bx^3+3\,ab^2x-3\,(a^2x^4+2\,abx^2+b^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{b}\right)}{8\,(a^5bx^4+2\,a^4b^2x^2+a^3b^3)}\right]$$

input
```
integrate(1/(a+b/x^2)^3/x^2,x, algorithm="fricas")
```

output
```
[-1/16*(10*a^2*b*x^3 + 6*a*b^2*x + 3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(-a*b
)*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b)))/(a^5*b*x^4 + 2*a^4*b^2*x^
2 + a^3*b^3), -1/8*(5*a^2*b*x^3 + 3*a*b^2*x - 3*(a^2*x^4 + 2*a*b*x^2 + b^2
)*sqrt(a*b)*arctan(sqrt(a*b)*x/b))/(a^5*b*x^4 + 2*a^4*b^2*x^2 + a^3*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx = -\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(-a^2b\sqrt{-\frac{1}{a^5b}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5b}}\log\left(a^2b\sqrt{-\frac{1}{a^5b}}+x\right)}{16} + \frac{-5ax^3-3bx}{8a^4x^4+16a^3bx^2+8a^2b^2}$$

input
```
integrate(1/(a+b/x**2)**3/x**2,x)
```

output
```
-3*sqrt(-1/(a**5*b))*log(-a**2*b*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a*
*5*b))*log(a**2*b*sqrt(-1/(a**5*b)) + x)/16 + (-5*a*x**3 - 3*b*x)/(8*a**4*
x**4 + 16*a**3*b*x**2 + 8*a**2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx = -\frac{5\,ax^3+3\,bx}{8\,(a^4x^4+2\,a^3bx^2+a^2b^2)} + \frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2}$$

input
```
integrate(1/(a+b/x^2)^3/x^2,x, algorithm="maxima")
```

output
```
-1/8*(5*a*x^3 + 3*b*x)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2) + 3/8*arctan(a*x/
sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx = \frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2} - \frac{5\,ax^3+3\,bx}{8\,(ax^2+b)^2a^2}$$

input
```
integrate(1/(a+b/x^2)^3/x^2,x, algorithm="giac")
```

output
```
3/8*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/8*(5*a*x^3 + 3*b*x)/((a*x^2
+ b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{5/2}\,\sqrt{b}} - \frac{\frac{5\,x^3}{8\,a}+\frac{3\,b\,x}{8\,a^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

input
```
int(1/(x^2*(a + b/x^2)^3),x)
```

output
```
(3*atan((a^(1/2)*x)/b^(1/2)))/(8*a^(5/2)*b^(1/2)) - ((5*x^3)/(8*a) + (3*b*
x)/(8*a^2))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^2}\, dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2x^4+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^2-5a^2b\,x^3-3a\,b^2x}{8a^3b\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*x**4 + 6*sqrt(b)*sqr
t(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((a*x)
/(sqrt(b)*sqrt(a)))*b**2 - 5*a**2*b*x**3 - 3*a*b**2*x)/(8*a**3*b*(a**2*x**
4 + 2*a*b*x**2 + b**2))
```

3.347 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4} dx = -\frac{x}{4a(b+ax^2)^2}+\frac{x}{8ab(b+ax^2)}+\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}}$$

output
```
-1/4*x/a/(a*x^2+b)^2+1/8*x/a/b/(a*x^2+b)+1/8*arctan(a^(1/2)*x/b^(1/2))/a^(
3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4} dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-b+ax^2)}{(b+ax^2)^2}+\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}}$$

input
```
Integrate[1/((a + b/x^2)^3*x^4),x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-b + a*x^2))/(b + a*x^2)^2 + ArcTan[(Sqrt[a]*x)/Sqrt[
b]])/(8*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{x^2}{\left(ax^2+b\right)^3}dx$$

$$\Big\downarrow \ 252$$

$$\frac{\int \frac{1}{\left(ax^2+b\right)^2}dx}{4a}-\frac{x}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{\int \frac{1}{ax^2+b}dx}{2b}+\frac{x}{2b(ax^2+b)}}{4a}-\frac{x}{4a\left(ax^2+b\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}b^{3/2}}+\frac{x}{2b(ax^2+b)}}{4a}-\frac{x}{4a\left(ax^2+b\right)^2}$$

input
```
Int[1/((a + b/x^2)^3*x^4),x]
```

output
```
-1/4*x/(a*(b + a*x^2)^2) + (x/(2*b*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[
b]]/(2*Sqrt[a]*b^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\frac{x^3}{8b}-\frac{x}{8a}}{\left(a\,x^2+b\right)^2}+\frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8ab\sqrt{ab}}$	49
risch	$\frac{\frac{x^3}{8b}-\frac{x}{8a}}{\left(a\,x^2+b\right)^2}-\frac{\ln\left(ax+\sqrt{-ab}\right)}{16\sqrt{-ab}\,ba}+\frac{\ln\left(-ax+\sqrt{-ab}\right)}{16\sqrt{-ab}\,ba}$	78

input
```
int(1/(a+b/x^2)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
(1/8*x^3/b-1/8*x/a)/(a*x^2+b)^2+1/8/a/b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx$$
$$= \left[\frac{2\,a^2bx^3-2\,ab^2x-(a^2x^4+2\,abx^2+b^2)\sqrt{-ab}\log\left(\frac{ax^2-2\,\sqrt{-ab}x-b}{ax^2+b}\right)}{16\,(a^4b^2x^4+2\,a^3b^3x^2+a^2b^4)}, \frac{a^2bx^3-ab^2x+(a^2x^4+2\,abx^2+b^2}{8\,(a^4b^2x^4+2\,a^3b^3x^2+}\right.$$

input
```
integrate(1/(a+b/x^2)^3/x^4,x, algorithm="fricas")
```

output
```
[1/16*(2*a^2*b*x^3 - 2*a*b^2*x - (a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(-a*b)*lo
g((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b)))/(a^4*b^2*x^4 + 2*a^3*b^3*x^2
+ a^2*b^4), 1/8*(a^2*b*x^3 - a*b^2*x + (a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(a*
b)*arctan(sqrt(a*b)*x/b))/(a^4*b^2*x^4 + 2*a^3*b^3*x^2 + a^2*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(51) = 102.

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(-ab^2\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(ab^2\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16} + \frac{ax^3-bx}{8a^3bx^4+16a^2b^2x^2+8ab^3}$$

input
```
integrate(1/(a+b/x**2)**3/x**4,x)
```

output
```
-sqrt(-1/(a**3*b**3))*log(-a*b**2*sqrt(-1/(a**3*b**3)) + x)/16 + sqrt(-1/(
a**3*b**3))*log(a*b**2*sqrt(-1/(a**3*b**3)) + x)/16 + (a*x**3 - b*x)/(8*a*
*3*b*x**4 + 16*a**2*b**2*x**2 + 8*a*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx = \frac{ax^3-bx}{8\,(a^3bx^4+2\,a^2b^2x^2+ab^3)} + \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab}$$

input
```
integrate(1/(a+b/x^2)^3/x^4,x, algorithm="maxima")
```

output
```
1/8*(a*x^3 - b*x)/(a^3*b*x^4 + 2*a^2*b^2*x^2 + a*b^3) + 1/8*arctan(a*x/sqr
t(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx = \frac{\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab} + \frac{ax^3-bx}{8\,(ax^2+b)^2ab}$$

input
```
integrate(1/(a+b/x^2)^3/x^4,x, algorithm="giac")
```

output
```
1/8*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(a*x^3 - b*x)/((a*x^2 + b)
^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,a^{3/2}\,b^{3/2}} - \frac{\frac{x}{8\,a}-\frac{x^3}{8\,b}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

input
```
int(1/(x^4*(a + b/x^2)^3),x)
```

output
```
atan((a^(1/2)*x)/b^(1/2))/(8*a^(3/2)*b^(3/2)) - (x/(8*a) - x^3/(8*b))/(b^2
 + a^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^4}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right) a^2x^4 + 2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right) ab\,x^2 + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right) b^2 + a^2b\,x^3 - a\,b^2x}{8a^2b^2\left(a^2x^4 + 2ab\,x^2 + b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^4,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*x**4 + 2*sqrt(b)*sqrt(
a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + sqrt(b)*sqrt(a)*atan((a*x)/(sq
rt(b)*sqrt(a)))*b**2 + a**2*b*x**3 - a*b**2*x)/(8*a**2*b**2*(a**2*x**4 + 2
*a*b*x**2 + b**2))
```

3.348 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6} \, dx=\frac{x}{4b\left(b+ax^2\right)^2}+\frac{3x}{8b^2\left(b+ax^2\right)}+\frac{3\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8\sqrt{a}b^{5/2}}$$

output
```
1/4*x/b/(a*x^2+b)^2+3/8*x/b^2/(a*x^2+b)+3/8*arctan(a^(1/2)*x/b^(1/2))/a^(1
/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6} \, dx=\frac{5bx+3ax^3}{8b^2\left(b+ax^2\right)^2}+\frac{3\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8\sqrt{a}b^{5/2}}$$

input
```
Integrate[1/((a + b/x^2)^3*x^6),x]
```

output
```
(5*b*x + 3*a*x^3)/(8*b^2*(b + a*x^2)^2) + (3*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/
(8*Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x^2}\right)^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{(ax^2+b)^3}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{1}{(ax^2+b)^2}dx}{4b}+\frac{x}{4b\,(ax^2+b)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{\int \frac{1}{ax^2+b}dx}{2b}+\frac{x}{2b(ax^2+b)}\right)}{4b}+\frac{x}{4b\,(ax^2+b)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}b^{3/2}}+\frac{x}{2b(ax^2+b)}\right)}{4b}+\frac{x}{4b\,(ax^2+b)^2}$$

input
```
Int[1/((a + b/x^2)^3*x^6),x]
```

output
```
x/(4*b*(b + a*x^2)^2) + (3*(x/(2*b*(b + a*x^2)) + ArcTan[(Sqrt[a]*x)/Sqrt[
b]]/(2*Sqrt[a]*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x}{4b(a\,x^{2}+b)^{2}}+\frac{\frac{3x}{8b\left(a\,x^{2}+b\right)}+\frac{3\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{b}$	57
risch	$\frac{\frac{3a\,x^{3}}{8b^{2}}+\frac{5x}{8b}}{(a\,x^{2}+b)^{2}}-\frac{3\ln\left(ax+\sqrt{-ab}\right)}{16\sqrt{-ab}\,b^{2}}+\frac{3\ln\left(-ax+\sqrt{-ab}\right)}{16\sqrt{-ab}\,b^{2}}$	73

input
```
int(1/(a+b/x^2)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/b/(a*x^2+b)^2+3/4/b*(1/2*x/b/(a*x^2+b)+1/2/b/(a*b)^(1/2)*arctan(a*x/
(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.03

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx = \left[\frac{6\,a^2bx^3+10\,ab^2x-3\,(a^2x^4+2\,abx^2+b^2)\sqrt{-ab}\log\left(\frac{ax^2-2\,\sqrt{-ab}x-b}{ax^2+b}\right)}{16\,(a^3b^3x^4+2\,a^2b^4x^2+ab^5)}, \frac{3\,a^2bx^3+5\,ab^2x+3\,(a^2x^4+2\,ab}{8\,(a^3b^3x^4+2\,a^2b}\right.$$

input
```
integrate(1/(a+b/x^2)^3/x^6,x, algorithm="fricas")
```

output
```
[1/16*(6*a^2*b*x^3 + 10*a*b^2*x - 3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(-a*b)
*log((a*x^2 - 2*sqrt(-a*b)*x - b)/(a*x^2 + b)))/(a^3*b^3*x^4 + 2*a^2*b^4*x
^2 + a*b^5), 1/8*(3*a^2*b*x^3 + 5*a*b^2*x + 3*(a^2*x^4 + 2*a*b*x^2 + b^2)*
sqrt(a*b)*arctan(sqrt(a*b)*x/b))/(a^3*b^3*x^4 + 2*a^2*b^4*x^2 + a*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx = -\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(-b^3\sqrt{-\frac{1}{ab^5}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{ab^5}}\log\left(b^3\sqrt{-\frac{1}{ab^5}}+x\right)}{16} + \frac{3ax^3+5bx}{8a^2b^2x^4+16ab^3x^2+8b^4}$$

input
```
integrate(1/(a+b/x**2)**3/x**6,x)
```

output
```
-3*sqrt(-1/(a*b**5))*log(-b**3*sqrt(-1/(a*b**5)) + x)/16 + 3*sqrt(-1/(a*b*
*5))*log(b**3*sqrt(-1/(a*b**5)) + x)/16 + (3*a*x**3 + 5*b*x)/(8*a**2*b**2*
x**4 + 16*a*b**3*x**2 + 8*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx = \frac{3\,ax^3+5\,bx}{8\,(a^2b^2x^4+2\,ab^3x^2+b^4)} + \frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2}$$

input
```
integrate(1/(a+b/x^2)^3/x^6,x, algorithm="maxima")
```

output
```
1/8*(3*a*x^3 + 5*b*x)/(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4) + 3/8*arctan(a*x/s
qrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx = \frac{3\,\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2} + \frac{3\,ax^3+5\,bx}{8\,(ax^2+b)^2b^2}$$

input
```
integrate(1/(a+b/x^2)^3/x^6,x, algorithm="giac")
```

output
```
3/8*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/8*(3*a*x^3 + 5*b*x)/((a*x^2
+ b)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx = \frac{\frac{5\,x}{8\,b}+\frac{3\,a\,x^3}{8\,b^2}}{a^2\,x^4+2\,a\,b\,x^2+b^2} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,\sqrt{a}\,b^{5/2}}$$

input
```
int(1/(x^6*(a + b/x^2)^3),x)
```

output
```
((5*x)/(8*b) + (3*a*x^3)/(8*b^2))/(b^2 + a^2*x^4 + 2*a*b*x^2) + (3*atan((a
^(1/2)*x)/b^(1/2)))/(8*a^(1/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^6}\,dx$$
$$=\frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2x^4+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^2+3a^2b\,x^3+5a\,b^2x}{8a\,b^3\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^6,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*x**4 + 6*sqrt(b)*sqr
t(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((a*x)
/(sqrt(b)*sqrt(a)))*b**2 + 3*a**2*b*x**3 + 5*a*b**2*x)/(8*a*b**3*(a**2*x**
4 + 2*a*b*x**2 + b**2))
```

3.349 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8} \, dx = -\frac{1}{b^3 x} - \frac{ax}{4b^2 (b+ax^2)^2} - \frac{7ax}{8b^3 (b+ax^2)} - \frac{15\sqrt{a} \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8b^{7/2}}$$

output
```
-1/b^3/x-1/4*a*x/b^2/(a*x^2+b)^2-7/8*a*x/b^3/(a*x^2+b)-15/8*a^(1/2)*arctan
(a^(1/2)*x/b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8} \, dx = -\frac{8b^2 + 25abx^2 + 15a^2x^4}{8b^3 x (b+ax^2)^2} - \frac{15\sqrt{a} \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8b^{7/2}}$$

input
```
Integrate[1/((a + b/x^2)^3*x^8),x]
```

output
```
-1/8*(8*b^2 + 25*a*b*x^2 + 15*a^2*x^4)/(b^3*x*(b + a*x^2)^2) - (15*Sqrt[a]
*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {795, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^2 \left(a x^2+b\right)^3}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5 \int \frac{1}{x^2 \left(a x^2+b\right)^2}dx}{4 b}+\frac{1}{4 b x \left(a x^2+b\right)^2}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5 \left(\frac{3 \int \frac{1}{x^2 \left(a x^2+b\right)}dx}{2 b}+\frac{1}{2 b x \left(a x^2+b\right)}\right)}{4 b}+\frac{1}{4 b x \left(a x^2+b\right)^2}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5 \left(\frac{3 \left(-\frac{a \int \frac{1}{a x^2+b}dx}{b}-\frac{1}{b x}\right)}{2 b}+\frac{1}{2 b x \left(a x^2+b\right)}\right)}{4 b}+\frac{1}{4 b x \left(a x^2+b\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{5 \left(\frac{3 \left(-\frac{\sqrt{a} \arctan \left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{b x}\right)}{2 b}+\frac{1}{2 b x \left(a x^2+b\right)}\right)}{4 b}+\frac{1}{4 b x \left(a x^2+b\right)^2}$$

input
```
Int[1/((a + b/x^2)^3*x^8),x]
```

output
```
1/(4*b*x*(b + a*x^2)^2) + (5*(1/(2*b*x*(b + a*x^2)) + (3*(-(1/(b*x)) - (Sq
rt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{a\left(\frac{\frac{7}{8}a\,x^{3}+\frac{9}{8}bx}{\left(a\,x^{2}+b\right)^{2}}+\frac{15\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{3}}-\frac{1}{b^{3}x}$	54
risch	$\frac{-\frac{15a^{2}x^{4}}{8b^{3}}-\frac{25a\,x^{2}}{8b^{2}}-\frac{1}{b}}{x\left(a\,x^{2}+b\right)^{2}}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}\left(b^{7}\_Z^{2}+a\right)}\_R\ln\left(\left(3\_R^{2}b^{7}+2a\right)x+b^{4}\_R\right)\right)}{16}$	79

input
```
int(1/(a+b/x^2)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*a*((7/8*a*x^3+9/8*b*x)/(a*x^2+b)^2+15/8/(a*b)^(1/2)*arctan(a*x/(a*b
)^(1/2)))-1/b^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.81

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\,dx = \left[-\frac{30\,a^2x^4+50\,abx^2-15\,(a^2x^5+2\,abx^3+b^2x)\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)+16\,b^2}{16\,(a^2b^3x^5+2\,ab^4x^3+b^5x)}, -\frac{15\,a^2x^4+25\,abx^2+15\,(a^2x^5+2\,abx^3+b^2x)\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+8\,b^2}{8\,(a^2b^3x^5+2\,ab^4x^3+b^5x)}\right]$$

input
```
integrate(1/(a+b/x^2)^3/x^8,x, algorithm="fricas")
```

output
```
[-1/16*(30*a^2*x^4 + 50*a*b*x^2 - 15*(a^2*x^5 + 2*a*b*x^3 + b^2*x)*sqrt(-a
/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) + 16*b^2)/(a^2*b^3*x^5
 + 2*a*b^4*x^3 + b^5*x), -1/8*(15*a^2*x^4 + 25*a*b*x^2 + 15*(a^2*x^5 + 2*a
*b*x^3 + b^2*x)*sqrt(a/b)*arctan(x*sqrt(a/b)) + 8*b^2)/(a^2*b^3*x^5 + 2*a*
b^4*x^3 + b^5*x)]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\,dx = \frac{15\sqrt{-\frac{a}{b^7}}\log\left(x-\frac{b^4\sqrt{-\frac{a}{b^7}}}{a}\right)}{16} - \frac{15\sqrt{-\frac{a}{b^7}}\log\left(x+\frac{b^4\sqrt{-\frac{a}{b^7}}}{a}\right)}{16} + \frac{-15a^2x^4-25abx^2-8b^2}{8a^2b^3x^5+16ab^4x^3+8b^5x}$$

input
```
integrate(1/(a+b/x**2)**3/x**8,x)
```

output
```
15*sqrt(-a/b**7)*log(x - b**4*sqrt(-a/b**7)/a)/16 - 15*sqrt(-a/b**7)*log(x
 + b**4*sqrt(-a/b**7)/a)/16 + (-15*a**2*x**4 - 25*a*b*x**2 - 8*b**2)/(8*a*
*2*b**3*x**5 + 16*a*b**4*x**3 + 8*b**5*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\,dx = -\frac{15\,a^2x^4+25\,abx^2+8\,b^2}{8\,(a^2b^3x^5+2\,ab^4x^3+b^5x)} - \frac{15\,a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}$$

input
```
integrate(1/(a+b/x^2)^3/x^8,x, algorithm="maxima")
```

output
```
-1/8*(15*a^2*x^4 + 25*a*b*x^2 + 8*b^2)/(a^2*b^3*x^5 + 2*a*b^4*x^3 + b^5*x)
 - 15/8*a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\, dx = -\frac{15\, a \arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} - \frac{7\,a^2x^3+9\,abx}{8\,(ax^2+b)^2b^3} - \frac{1}{b^3x}$$

input
```
integrate(1/(a+b/x^2)^3/x^8,x, algorithm="giac")
```

output
```
-15/8*a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/8*(7*a^2*x^3 + 9*a*b*x)/
((a*x^2 + b)^2*b^3) - 1/(b^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\, dx = -\frac{\frac{1}{b}+\frac{25\,a\,x^2}{8\,b^2}+\frac{15\,a^2\,x^4}{8\,b^3}}{a^2\,x^5+2\,a\,b\,x^3+b^2\,x} - \frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,b^{7/2}}$$

input
```
int(1/(x^8*(a + b/x^2)^3),x)
```

output
```
- (1/b + (25*a*x^2)/(8*b^2) + (15*a^2*x^4)/(8*b^3))/(b^2*x + a^2*x^5 + 2*a
*b*x^3) - (15*a^(1/2)*atan((a^(1/2)*x)/b^(1/2)))/(8*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^8}\, dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2x^5-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^3-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)b^2x-15a^2b\,x^4-2}{8b^4x\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^8,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*x**5 - 30*sqrt(b
)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a*b*x**3 - 15*sqrt(b)*sqrt(a)*atan
((a*x)/(sqrt(b)*sqrt(a)))*b**2*x - 15*a**2*b*x**4 - 25*a*b**2*x**2 - 8*b**
3)/(8*b**4*x*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.350 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = -\frac{1}{3b^3x^3}+\frac{3a}{b^4x}+\frac{a^2x}{4b^3\left(b+ax^2\right)^2}+\frac{11a^2x}{8b^4\left(b+ax^2\right)}+\frac{35a^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

output
```
-1/3/b^3/x^3+3*a/b^4/x+1/4*a^2*x/b^3/(a*x^2+b)^2+11/8*a^2*x/b^4/(a*x^2+b)+
35/8*a^(3/2)*arctan(a^(1/2)*x/b^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = \frac{-8b^3+56ab^2x^2+175a^2bx^4+105a^3x^6}{24b^4x^3\left(b+ax^2\right)^2}+\frac{35a^{3/2}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8b^{9/2}}$$

input
```
Integrate[1/((a + b/x^2)^3*x^10),x]
```

output
```
(-8*b^3 + 56*a*b^2*x^2 + 175*a^2*b*x^4 + 105*a^3*x^6)/(24*b^4*x^3*(b + a*x
^2)^2) + (35*a^(3/2)*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(8*b^(9/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {795, 253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\left(a+\frac{b}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^4\left(ax^2+b\right)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7\int \frac{1}{x^4(ax^2+b)^2}dx}{4b}+\frac{1}{4bx^3\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{7\left(\frac{5\int \frac{1}{x^4\left(ax^2+b\right)}dx}{2b}+\frac{1}{2bx^3(ax^2+b)}\right)}{4b}+\frac{1}{4bx^3\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(\frac{5\left(-\frac{a\int \frac{1}{x^2\left(ax^2+b\right)}dx}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(ax^2+b)}\right)}{4b}+\frac{1}{4bx^3\left(ax^2+b\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(\frac{5\left(-\frac{a\left(-\frac{a\int\frac{1}{ax^2+b}dx}{b}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(ax^2+b)}\right)}{4b}+\frac{1}{4bx^3\left(ax^2+b\right)^2}$$

$\downarrow$ 218

$$\frac{7\left(\frac{5\left(-\frac{a\left(-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}\right)}{b}-\frac{1}{3bx^3}\right)}{2b}+\frac{1}{2bx^3(ax^2+b)}\right)}{4b}+\frac{1}{4bx^3\left(ax^2+b\right)^2}$$

input
```
Int[1/((a + b/x^2)^3*x^10),x]
```

output
```
1/(4*b*x^3*(b + a*x^2)^2) + (7*(1/(2*b*x^3*(b + a*x^2)) + (5*(-1/3*1/(b*x^
3) - (a*(-(1/(b*x)) - (Sqrt[a]*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/b^(3/2)))/b))/
(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{a^{2}\left(\frac{\frac{11}{8}a\,x^{3}+\frac{13}{8}bx}{\left(a\,x^{2}+b\right)^{2}}+\frac{35\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}-\frac{1}{3b^{3}x^{3}}+\frac{3a}{b^{4}x}$	64
risch	$\frac{\frac{35a^{3}x^{6}}{8b^{4}}+\frac{175a^{2}x^{4}}{24b^{3}}+\frac{7a\,x^{2}}{3b^{2}}-\frac{1}{3b}}{\left(a\,x^{2}+b\right)^{2}x^{3}}+\frac{35\left(\sum_{\_R=\operatorname{RootOf}\left(b^{9}\_Z^{2}+a^{3}\right)}\_R\ln\left(\left(3\_R^{2}b^{9}+2a^{3}\right)x-a\,b^{5}\_R\right)\right)}{16}$	96

input
```
int(1/(a+b/x^2)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
a^2/b^4*((11/8*a*x^3+13/8*b*x)/(a*x^2+b)^2+35/8/(a*b)^(1/2)*arctan(a*x/(a*
b)^(1/2)))-1/3/b^3/x^3+3*a/b^4/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.74

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = \left[\frac{210\,a^3x^6+350\,a^2bx^4+112\,ab^2x^2-16\,b^3+105\left(a^3x^7+2\,a^2bx^5+ab^2x^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)}{48\left(a^2b^4x^7+2\,ab^5x^5+b^6x^3\right)},\frac{1}{}\right.$$

input
```
integrate(1/(a+b/x^2)^3/x^10,x, algorithm="fricas")
```

output
```
[1/48*(210*a^3*x^6 + 350*a^2*b*x^4 + 112*a*b^2*x^2 - 16*b^3 + 105*(a^3*x^7
 + 2*a^2*b*x^5 + a*b^2*x^3)*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/
(a*x^2 + b)))/(a^2*b^4*x^7 + 2*a*b^5*x^5 + b^6*x^3), 1/24*(105*a^3*x^6 + 1
75*a^2*b*x^4 + 56*a*b^2*x^2 - 8*b^3 + 105*(a^3*x^7 + 2*a^2*b*x^5 + a*b^2*x
^3)*sqrt(a/b)*arctan(x*sqrt(a/b)))/(a^2*b^4*x^7 + 2*a*b^5*x^5 + b^6*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = -\frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x-\frac{b^5\sqrt{-\frac{a^3}{b^9}}}{a^2}\right)}{16}+\frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x+\frac{b^5\sqrt{-\frac{a^3}{b^9}}}{a^2}\right)}{16} + \frac{105a^3x^6+175a^2bx^4+56ab^2x^2-8b^3}{24a^2b^4x^7+48ab^5x^5+24b^6x^3}$$

input
```
integrate(1/(a+b/x**2)**3/x**10,x)
```

output
```
-35*sqrt(-a**3/b**9)*log(x - b**5*sqrt(-a**3/b**9)/a**2)/16 + 35*sqrt(-a**
3/b**9)*log(x + b**5*sqrt(-a**3/b**9)/a**2)/16 + (105*a**3*x**6 + 175*a**2
*b*x**4 + 56*a*b**2*x**2 - 8*b**3)/(24*a**2*b**4*x**7 + 48*a*b**5*x**5 + 2
4*b**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = \frac{105\,a^3x^6+175\,a^2bx^4+56\,ab^2x^2-8\,b^3}{24\,(a^2b^4x^7+2\,ab^5x^5+b^6x^3)}+\frac{35\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4}$$

input
```
integrate(1/(a+b/x^2)^3/x^10,x, algorithm="maxima")
```

output
```
1/24*(105*a^3*x^6 + 175*a^2*b*x^4 + 56*a*b^2*x^2 - 8*b^3)/(a^2*b^4*x^7 + 2
*a*b^5*x^5 + b^6*x^3) + 35/8*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = \frac{35\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} + \frac{11\,a^3x^3+13\,a^2bx}{8\,(ax^2+b)^2b^4} + \frac{9\,ax^2-b}{3\,b^4x^3}$$

input
```
integrate(1/(a+b/x^2)^3/x^10,x, algorithm="giac")
```

output
```
35/8*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/8*(11*a^3*x^3 + 13*a^2*
b*x)/((a*x^2 + b)^2*b^4) + 1/3*(9*a*x^2 - b)/(b^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx = \frac{\frac{7\,a\,x^2}{3\,b^2}-\frac{1}{3\,b}+\frac{175\,a^2\,x^4}{24\,b^3}+\frac{35\,a^3\,x^6}{8\,b^4}}{a^2\,x^7+2\,a\,b\,x^5+b^2\,x^3} + \frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{8\,b^{9/2}}$$

input
```
int(1/(x^10*(a + b/x^2)^3),x)
```

output
```
((7*a*x^2)/(3*b^2) - 1/(3*b) + (175*a^2*x^4)/(24*b^3) + (35*a^3*x^6)/(8*b^
4))/(a^2*x^7 + b^2*x^3 + 2*a*b*x^5) + (35*a^(3/2)*atan((a^(1/2)*x)/b^(1/2)
))/(8*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^3 x^{10}}\,dx$$
$$= \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^3x^7 + 210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^5 + 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{ax}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^3 + 105a^3b}{24b^5x^3\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^3/x^10,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**3*x**7 + 210*sqrt(b)
*sqrt(a)*atan((a*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**5 + 105*sqrt(b)*sqrt(a)*a
tan((a*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**3 + 105*a**3*b*x**6 + 175*a**2*b**2
*x**4 + 56*a*b**3*x**2 - 8*b**4)/(24*b**5*x**3*(a**2*x**4 + 2*a*b*x**2 + b
**2))
```

3.351 $\int \sqrt{a+\frac{b}{x^2}}x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \sqrt{a+\frac{b}{x^2}}x^3 \, dx = \frac{b\sqrt{a+\frac{b}{x^2}}x^2}{8a}+\frac{1}{4}\sqrt{a+\frac{b}{x^2}}x^4-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
1/8*b*(a+b/x^2)^(1/2)*x^2/a+1/4*(a+b/x^2)^(1/2)*x^4-1/8*b^2*arctanh((a+b/x
^2)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \sqrt{a+\frac{b}{x^2}}x^3 \, dx = \frac{\sqrt{a+\frac{b}{x^2}}x^2(b+2ax^2)}{8a}-\frac{b^2\sqrt{a+\frac{b}{x^2}}x\text{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{4a^{3/2}\sqrt{b+ax^2}}$$

input
```
Integrate[Sqrt[a + b/x^2]*x^3,x]
```

output
```
(Sqrt[a + b/x^2]*x^2*(b + 2*a*x^2))/(8*a) - (b^2*Sqrt[a + b/x^2]*x*ArcTanh
[(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b + a*x^2])])/(4*a^(3/2)*Sqrt[b + a*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \sqrt{a+\frac{b}{x^2}}x^6d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{a+\frac{b}{x^2}}-\frac{1}{4}b\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{a+\frac{b}{x^2}}-\frac{1}{4}b\left(-\frac{b\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{2a}-\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{a+\frac{b}{x^2}}-\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{a}-\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\sqrt{a+\frac{b}{x^2}}-\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)\right)$$

input
```
Int[Sqrt[a + b/x^2]*x^3,x]
```

output
```
((Sqrt[a + b/x^2]*x^4)/2 - (b*(-((Sqrt[a + b/x^2]*x^2)/a) + (b*ArcTanh[Sqr
t[a + b/x^2]/Sqrt[a]])/a^(3/2)))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{x^{2}\left(2a\,x^{2}+b\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{8a}-\frac{b^{2}\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{8a^{\frac{3}{2}}\sqrt{a\,x^{2}+b}}$	78
default	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x\left(-2x\left(a\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{a}+\sqrt{a}\,\sqrt{a\,x^{2}+b}\,b x+\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)b^{2}\right)}{8\sqrt{a\,x^{2}+b}\,a^{\frac{3}{2}}}$	80

input
```
int((a+b/x^2)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^2*(2*a*x^2+b)/a*((a*x^2+b)/x^2)^(1/2)-1/8/a^(3/2)*b^2*ln(a^(1/2)*x+(
a*x^2+b)^(1/2))*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.14

$$\int \sqrt{a+\frac{b}{x^2}}x^3\,dx$$
$$=\left[\frac{\sqrt{a}b^2\log\left(-2\,ax^2+2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\left(2\,a^2x^4+abx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\,a^2},\ \frac{\sqrt{-a}b^2\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\cdots}{8\,a^2}\right.$$

input
```
integrate((a+b/x^2)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
[1/16*(sqrt(a)*b^2*log(-2*a*x^2 + 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b)
 + 2*(2*a^2*x^4 + a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a^2, 1/8*(sqrt(-a)*b^2*a
rctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (2*a^2*x^4 + a*b*x
^2)*sqrt((a*x^2 + b)/x^2))/a^2]
```

Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int \sqrt{a+\frac{b}{x^2}}x^3\,dx = \frac{ax^5}{4\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} + \frac{3\sqrt{b}x^3}{8\sqrt{\frac{ax^2}{b}+1}} + \frac{b^{\frac{3}{2}}x}{8a\sqrt{\frac{ax^2}{b}+1}} - \frac{b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((a+b/x**2)**(1/2)*x**3,x)
```

output
```
a*x**5/(4*sqrt(b)*sqrt(a*x**2/b + 1)) + 3*sqrt(b)*x**3/(8*sqrt(a*x**2/b +
1)) + b**(3/2)*x/(8*a*sqrt(a*x**2/b + 1)) - b**2*asinh(sqrt(a)*x/sqrt(b))/
(8*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \sqrt{a+\frac{b}{x^2}}x^3\,dx = \frac{b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}} + \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2+\sqrt{a+\frac{b}{x^2}}ab^2}{8\left(\left(a+\frac{b}{x^2}\right)^2a-2\left(a+\frac{b}{x^2}\right)a^2+a^3\right)}$$

input
```
integrate((a+b/x^2)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
1/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(3
/2) + 1/8*((a + b/x^2)^(3/2)*b^2 + sqrt(a + b/x^2)*a*b^2)/((a + b/x^2)^2*a
 - 2*(a + b/x^2)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \sqrt{a+\frac{b}{x^2}}x^3\,dx = \frac{1}{8}\sqrt{ax^2+b}\left(2\,x^2\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{a}\right)x + \frac{b^2\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)\mathrm{sgn}(x)}{8\,a^{\frac{3}{2}}} - \frac{b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,a^{\frac{3}{2}}}$$

input
```
integrate((a+b/x^2)^(1/2)*x^3,x, algorithm="giac")
```

output
```
1/8*sqrt(a*x^2 + b)*(2*x^2*sgn(x) + b*sgn(x)/a)*x + 1/8*b^2*log(abs(-sqrt(
a)*x + sqrt(a*x^2 + b)))*sgn(x)/a^(3/2) - 1/16*b^2*log(abs(b))*sgn(x)/a^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \sqrt{a+\frac{b}{x^2}}x^3\,dx = \frac{x^4\sqrt{a+\frac{b}{x^2}}}{8} - \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{3/2}} + \frac{x^4\left(a+\frac{b}{x^2}\right)^{3/2}}{8\,a}$$

input
```
int(x^3*(a + b/x^2)^(1/2),x)
```

output
```
(x^4*(a + b/x^2)^(1/2))/8 - (b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(3
/2)) + (x^4*(a + b/x^2)^(3/2))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \sqrt{a+\frac{b}{x^2}}x^3\,dx = \frac{2\sqrt{a\,x^2+b}\,a^2x^3+\sqrt{a\,x^2+b}\,abx-\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)b^2}{8a^2}$$

input
```
int((a+b/x^2)^(1/2)*x^3,x)
```

output
```
(2*sqrt(a*x**2 + b)*a**2*x**3 + sqrt(a*x**2 + b)*a*b*x - sqrt(a)*log((sqrt
(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b**2)/(8*a**2)
```

3.352 $\int \sqrt{a+\frac{b}{x^2}}x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx=\frac{\left(a+\frac{b}{x^2}\right)^{3/2}x^3}{3a}$$

output
```
1/3*(a+b/x^2)^(3/2)*x^3/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx=\frac{\sqrt{a+\frac{b}{x^2}}x(b+ax^2)}{3a}$$

input
```
Integrate[Sqrt[a + b/x^2]*x^2,x]
```

output
```
(Sqrt[a + b/x^2]*x*(b + a*x^2))/(3*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \ 796$$

$$\frac{x^3\left(a+\frac{b}{x^2}\right)^{3/2}}{3a}$$

input
```
Int[Sqrt[a + b/x^2]*x^2,x]
```

output
```
((a + b/x^2)^(3/2)*x^3)/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
orering	$\frac{(a\,x^2+b)x\sqrt{a+\frac{b}{x^2}}}{3a}$	23
gosper	$\frac{(a\,x^2+b)x\sqrt{\frac{a\,x^2+b}{x^2}}}{3a}$	27
default	$\frac{(a\,x^2+b)x\sqrt{\frac{a\,x^2+b}{x^2}}}{3a}$	27
risch	$\frac{(a\,x^2+b)x\sqrt{\frac{a\,x^2+b}{x^2}}}{3a}$	27
trager	$\frac{(a\,x^2+b)x\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3a}$	31

input
```
int((a+b/x^2)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x^2+b)/a*x*(a+b/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx=\frac{(ax^3+bx)\sqrt{\frac{ax^2+b}{x^2}}}{3\,a}$$

input
```
integrate((a+b/x^2)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
1/3*(a*x^3 + b*x)*sqrt((a*x^2 + b)/x^2)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx = \frac{\sqrt{b}x^2\sqrt{\frac{ax^2}{b}+1}}{3} + \frac{b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a}$$

input
```
integrate((a+b/x**2)**(1/2)*x**2,x)
```

output
```
sqrt(b)*x**2*sqrt(a*x**2/b + 1)/3 + b**(3/2)*sqrt(a*x**2/b + 1)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3}{3\,a}$$

input
```
integrate((a+b/x^2)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*(a + b/x^2)^(3/2)*x^3/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx = \frac{(ax^2+b)^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,a} - \frac{b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,a}$$

input
```
integrate((a+b/x^2)^(1/2)*x^2,x, algorithm="giac")
```

output
```
1/3*(a*x^2 + b)^(3/2)*sgn(x)/a - 1/3*b^(3/2)*sgn(x)/a
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx=\sqrt{a+\frac{b}{x^2}}\left(\frac{x^3}{3}+\frac{b\,x}{3\,a}\right)$$

input
```
int(x^2*(a + b/x^2)^(1/2),x)
```

output
```
(a + b/x^2)^(1/2)*(x^3/3 + (b*x)/(3*a))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sqrt{a+\frac{b}{x^2}}x^2\,dx=\frac{\sqrt{a\,x^2+b}\,(a\,x^2+b)}{3a}$$

input
```
int((a+b/x^2)^(1/2)*x^2,x)
```

output
```
(sqrt(a*x**2 + b)*(a*x**2 + b))/(3*a)
```

3.353 $\int \sqrt{a+\frac{b}{x^2}}x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \sqrt{a+\frac{b}{x^2}}x\,dx=\frac{1}{2}\sqrt{a+\frac{b}{x^2}}x^2+\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
1/2*(a+b/x^2)^(1/2)*x^2+1/2*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \sqrt{a+\frac{b}{x^2}}x\,dx=\frac{1}{2}\sqrt{a+\frac{b}{x^2}}x\left(x-\frac{b\log\left(-\sqrt{a}x+\sqrt{b+ax^2}\right)}{\sqrt{a}\sqrt{b+ax^2}}\right)$$

input
```
Integrate[Sqrt[a + b/x^2]*x,x]
```

output
```
(Sqrt[a + b/x^2]*x*(x - (b*Log[-(Sqrt[a]*x) + Sqrt[b + a*x^2]])/(Sqrt[a]*S
qrt[b + a*x^2])))/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{2}\int \sqrt{a+\frac{b}{x^2}}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(x^2\sqrt{a+\frac{b}{x^2}}-\frac{1}{2}b\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(x^2\sqrt{a+\frac{b}{x^2}}-\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}+x^2\sqrt{a+\frac{b}{x^2}}\right)$$

input
```
Int[Sqrt[a + b/x^2]*x,x]
```

output
```
(Sqrt[a + b/x^2]*x^2 + (b*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/Sqrt[a])/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\, x \left(x \sqrt{a\,x^{2}+b}\, \sqrt{a}+b \ln\left(\sqrt{a}\, x +\sqrt{a\,x^{2}+b}\right)\right)}{2 \sqrt{a\,x^{2}+b}\, \sqrt{a}}$	62
risch	$\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\, x^{2}}{2}+\frac{b \ln\left(\sqrt{a}\, x +\sqrt{a\,x^{2}+b}\right) \sqrt{\frac{a\,x^{2}+b}{x^{2}}}\, x}{2 \sqrt{a}\, \sqrt{a\,x^{2}+b}}$	65

input
```
int((a+b/x^2)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x^2+b)/x^2)^(1/2)*x*(x*(a*x^2+b)^(1/2)*a^(1/2)+b*ln(a^(1/2)*x+(a*x
^2+b)^(1/2)))/(a*x^2+b)^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.70

$$\int \sqrt{a+\frac{b}{x^2}}x\,dx$$
$$=\left[\frac{2\,ax^2\sqrt{\frac{ax^2+b}{x^2}}+\sqrt{a}b\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)}{4\,a},\frac{ax^2\sqrt{\frac{ax^2+b}{x^2}}-\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{2\,a}\right]$$

input
```
integrate((a+b/x^2)^(1/2)*x,x, algorithm="fricas")
```

output
```
[1/4*(2*a*x^2*sqrt((a*x^2 + b)/x^2) + sqrt(a)*b*log(-2*a*x^2 - 2*sqrt(a)*x
^2*sqrt((a*x^2 + b)/x^2) - b))/a, 1/2*(a*x^2*sqrt((a*x^2 + b)/x^2) - sqrt(
-a)*b*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)))/a]
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \sqrt{a+\frac{b}{x^2}}x\,dx=\frac{\sqrt{b}x\sqrt{\frac{ax^2}{b}+1}}{2}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2\sqrt{a}}$$

input
```
integrate((a+b/x**2)**(1/2)*x,x)
```

output
```
sqrt(b)*x*sqrt(a*x**2/b + 1)/2 + b*asinh(sqrt(a)*x/sqrt(b))/(2*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \sqrt{a+\frac{b}{x^2}}x\,dx=\frac{1}{2}\sqrt{a+\frac{b}{x^2}}x^2-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
integrate((a+b/x^2)^(1/2)*x,x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^2)*x^2 - 1/4*b*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a +
b/x^2) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \sqrt{a+\frac{b}{x^2}}x\,dx=\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\sqrt{a}}+\frac{1}{2}\left(\sqrt{ax^2+b}x-\frac{b\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{\sqrt{a}}\right)\operatorname{sgn}(x)$$

input
```
integrate((a+b/x^2)^(1/2)*x,x, algorithm="giac")
```

output
```
1/4*b*log(abs(b))*sgn(x)/sqrt(a) + 1/2*(sqrt(a*x^2 + b)*x - b*log(abs(-sqr
t(a)*x + sqrt(a*x^2 + b)))/sqrt(a))*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \sqrt{a+\frac{b}{x^2}}\,x\,dx=\frac{x^2\,\sqrt{a+\frac{b}{x^2}}}{2}+\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

input
```
int(x*(a + b/x^2)^(1/2),x)
```

output
```
(x^2*(a + b/x^2)^(1/2))/2 + (b*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(2*a^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \sqrt{a+\frac{b}{x^2}}\,x\,dx=\frac{\sqrt{a\,x^2+b}\,ax+\sqrt{a}\,\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)b}{2a}$$

input
```
int((a+b/x^2)^(1/2)*x,x)
```

output
```
(sqrt(a*x**2 + b)*a*x + sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b)
)*b)/(2*a)
```

3.354 $\int \sqrt{a+\frac{b}{x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \sqrt{a+\frac{b}{x^2}}\,dx=\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)$$

output
```
(a+b/x^2)^(1/2)*x-b^(1/2)*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \sqrt{a+\frac{b}{x^2}}\,dx=\sqrt{a+\frac{b}{x^2}}x-\frac{\sqrt{b}\sqrt{a+\frac{b}{x^2}}x\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b+ax^2}}$$

input
```
Integrate[Sqrt[a + b/x^2],x]
```

output
```
Sqrt[a + b/x^2]*x - (Sqrt[b]*Sqrt[a + b/x^2]*x*ArcTanh[Sqrt[b + a*x^2]/Sqr
t[b]])/Sqrt[b + a*x^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {773, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x^2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \sqrt{a+\frac{b}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$x\sqrt{a+\frac{b}{x^2}}-b\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{224}$$

$$x\sqrt{a+\frac{b}{x^2}}-b\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}$$

$$\Big\downarrow \text{219}$$

$$x\sqrt{a+\frac{b}{x^2}}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)$$

input
```
Int[Sqrt[a + b/x^2],x]
```

output
```
Sqrt[a + b/x^2]*x - Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

method	result	size
default	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)-\sqrt{a\,x^{2}+b}\right)}{\sqrt{a\,x^{2}+b}}$	63

input
```
int((a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x^2+b)/x^2)^(1/2)*x*(b^(1/2)*ln(2*(b^(1/2)*(a*x^2+b)^(1/2)+b)/x)-(a*x
^2+b)^(1/2))/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int \sqrt{a+\frac{b}{x^2}}\,dx = \left[x\sqrt{\frac{ax^2+b}{x^2}}+\frac{1}{2}\sqrt{b}\log\left(-\frac{ax^2-2\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right), x\sqrt{\frac{ax^2+b}{x^2}} + \sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)\right]$$

input
```
integrate((a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[x*sqrt((a*x^2 + b)/x^2) + 1/2*sqrt(b)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x
^2 + b)/x^2) + 2*b)/x^2), x*sqrt((a*x^2 + b)/x^2) + sqrt(-b)*arctan(sqrt(-
b)*x*sqrt((a*x^2 + b)/x^2)/b)]
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \sqrt{a+\frac{b}{x^2}}\,dx = \frac{\sqrt{a}x}{\sqrt{1+\frac{b}{ax^2}}} - \sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right) + \frac{b}{\sqrt{a}x\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate((a+b/x**2)**(1/2),x)
```

output
```
sqrt(a)*x/sqrt(1 + b/(a*x**2)) - sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x)) + b/(s
qrt(a)*x*sqrt(1 + b/(a*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \sqrt{a+\frac{b}{x^2}}\,dx=\sqrt{a+\frac{b}{x^2}}x+\frac{1}{2}\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)$$

input
```
integrate((a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x^2)*x + 1/2*sqrt(b)*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a
+ b/x^2)*x + sqrt(b)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \sqrt{a+\frac{b}{x^2}}\,dx=\frac{b\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}+\sqrt{ax^2+b}\operatorname{sgn}(x)-\frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

input
```
integrate((a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
b*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + sqrt(a*x^2 + b)*sgn(x
) - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sgn(x)/sqrt(-b)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \sqrt{a+\frac{b}{x^2}}\,dx = x\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,1\mathrm{i}}{\sqrt{a}\,x}\right)\,\sqrt{a+\frac{b}{x^2}}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b}{a\,x^2}+1}}$$

input
```
int((a + b/x^2)^(1/2),x)
```

output
```
x*(a + b/x^2)^(1/2) + (b^(1/2)*asin((b^(1/2)*1i)/(a^(1/2)*x))*(a + b/x^2)^
(1/2)*1i)/(a^(1/2)*(b/(a*x^2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \sqrt{a+\frac{b}{x^2}}\,dx = \sqrt{a\,x^2+b}+\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right) - \sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x+\sqrt{b}}{\sqrt{b}}\right)$$

input
```
int((a+b/x^2)^(1/2),x)
```

output
```
sqrt(a*x**2 + b) + sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sq
rt(b)) - sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))
```

3.355 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x} dx = -\sqrt{a+\frac{b}{x^2}}+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

output
```
-(a+b/x^2)^(1/2)+a^(1/2)*arctanh((a+b/x^2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x} dx = \sqrt{a+\frac{b}{x^2}}\left(-1+\frac{2\sqrt{a}x\text{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{\sqrt{b+ax^2}}\right)$$

input
```
Integrate[Sqrt[a + b/x^2]/x,x]
```

output
```
Sqrt[a + b/x^2]*(-1 + (2*Sqrt[a]*x*ArcTanh[(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b
+ a*x^2])])/Sqrt[b + a*x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \sqrt{a+\frac{b}{x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}-2\sqrt{a+\frac{b}{x^2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2a\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{b}-2\sqrt{a+\frac{b}{x^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x^2}}\right)$$

input
```
Int[Sqrt[a + b/x^2]/x,x]
```

output
```
(-2*Sqrt[a + b/x^2] + 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

method	result	size
risch	$-\sqrt{\frac{a\,x^{2}+b}{x^{2}}}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{\sqrt{a\,x^{2}+b}}$	60
default	$\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,\left(a^{\frac{3}{2}}\sqrt{a\,x^{2}+b}\,x^{2}-\left(a\,x^{2}+b\right)^{\frac{3}{2}}\sqrt{a}+\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)a b x\right)}{\sqrt{a\,x^{2}+b}\,b\sqrt{a}}$	81

input
```
int((a+b/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*x^2+b)/x^2)^(1/2)+a^(1/2)*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*((a*x^2+b)/x^
2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx = \left[\frac{1}{2}\,\sqrt{a}\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-\sqrt{\frac{ax^2+b}{x^2}},\; -\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-\sqrt{\frac{ax^2+b}{x^2}}\right]$$

input
```
integrate((a+b/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b) - sqr
t((a*x^2 + b)/x^2), -sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a
*x^2 + b)) - sqrt((a*x^2 + b)/x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx = \sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)-\frac{ax}{\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}-\frac{\sqrt{b}}{x\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate((a+b/x**2)**(1/2)/x,x)
```

output
```
sqrt(a)*asinh(sqrt(a)*x/sqrt(b)) - a*x/(sqrt(b)*sqrt(a*x**2/b + 1)) - sqrt
(b)/(x*sqrt(a*x**2/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx = -\frac{1}{2}\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}$$

input
```
integrate((a+b/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))
- sqrt(a + b/x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx = -\frac{1}{2}\sqrt{a}\log\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2\right)\mathrm{sgn}(x)+\frac{2\sqrt{a}b\mathrm{sgn}(x)}{\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b}$$

input
```
integrate((a+b/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 2*sqrt(a)*b*sgn
(x)/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx=\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)-\sqrt{a+\frac{b}{x^2}}$$

input
```
int((a + b/x^2)^(1/2)/x,x)
```

output
```
a^(1/2)*atanh((a + b/x^2)^(1/2)/a^(1/2)) - (a + b/x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x}\,dx=\frac{-\sqrt{a\,x^2+b}+\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)x-\sqrt{a}\,x}{x}$$

input
```
int((a+b/x^2)^(1/2)/x,x)
```

output
```
( - sqrt(a*x**2 + b) + sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))
*x - sqrt(a)*x)/x
```

3.356 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2} dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{2x} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)}{2\sqrt{b}}$$

output
```
-1/2*(a+b/x^2)^(1/2)/x-1/2*a*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2} dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-1-\frac{ax^2\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+ax^2}}\right)}{2x}$$

input
```
Integrate[Sqrt[a + b/x^2]/x^2,x]
```

output
```
(Sqrt[a + b/x^2]*(-1 - (a*x^2*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]])/(Sqrt[b]*S
qrt[b + a*x^2])))/(2*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}-\frac{\sqrt{a+\frac{b}{x^2}}}{2x}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}-\frac{\sqrt{a+\frac{b}{x^2}}}{2x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}-\frac{\sqrt{a+\frac{b}{x^2}}}{2x}$$

input
```
Int[Sqrt[a + b/x^2]/x^2,x]
```

output
```
-1/2*Sqrt[a + b/x^2]/x - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*Sqrt[
b])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{2x}-\frac{a\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{2\sqrt{b}\,\sqrt{a\,x^{2}+b}}$	72
default	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)a\,x^{2}-\sqrt{a\,x^{2}+b}\,a\,x^{2}+\left(a\,x^{2}+b\right)^{\frac{3}{2}}\right)}{2x\sqrt{a\,x^{2}+b}\,b}$	85

input
```
int((a+b/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x^2+b)/x^2)^(1/2)/x-1/2*a/b^(1/2)*ln((2*b+2*b^(1/2)*(a*x^2+b)^(1/
2))/x)*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx = \left[\frac{a\sqrt{b}x\log\left(-\frac{ax^2-2\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)-2b\sqrt{\frac{ax^2+b}{x^2}}}{4bx}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)-b\sqrt{\frac{ax^2+b}{x^2}}}{2bx}\right]$$

input
```
integrate((a+b/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(a*sqrt(b)*x*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x
^2) - 2*b*sqrt((a*x^2 + b)/x^2))/(b*x), 1/2*(a*sqrt(-b)*x*arctan(sqrt(-b)*
x*sqrt((a*x^2 + b)/x^2)/b) - b*sqrt((a*x^2 + b)/x^2))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx = -\frac{\sqrt{a}\sqrt{1+\frac{b}{ax^2}}}{2x} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{2\sqrt{b}}$$

input
```
integrate((a+b/x**2)**(1/2)/x**2,x)
```

output
```
-sqrt(a)*sqrt(1 + b/(a*x**2))/(2*x) - a*asinh(sqrt(b)/(sqrt(a)*x))/(2*sqrt
(b))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\,ax}{2\left(\left(a+\frac{b}{x^2}\right)x^2-b\right)} + \frac{a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{4\sqrt{b}}$$

input
```
integrate((a+b/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(a + b/x^2)*a*x/((a + b/x^2)*x^2 - b) + 1/4*a*log((sqrt(a + b/x^2
)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx = \frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\sqrt{ax^2+b}\operatorname{sgn}(x)}{ax^2}\right)a$$

input
```
integrate((a+b/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*(arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - sqrt(a*x^2 + b)*sg
n(x)/(a*x^2))*a
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{2\,x} - \frac{a\,\ln\left(\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}$$

input
```
int((a + b/x^2)^(1/2)/x^2,x)
```

output
```
- (a + b/x^2)^(1/2)/(2*x) - (a*log((a + b/x^2)^(1/2) + b^(1/2)/x))/(2*b^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}\,dx = \frac{-\sqrt{a\,x^2+b}\,b+\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a\,x^2-\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x+\sqrt{b}}{\sqrt{b}}\right)a\,x^2}{2b\,x^2}$$

input
```
int((a+b/x^2)^(1/2)/x^2,x)
```

output
```
( - sqrt(a*x**2 + b)*b + sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(
b))/sqrt(b))*a*x**2 - sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))
/sqrt(b))*a*x**2)/(2*b*x**2)
```

3.357 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3} dx = -\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b}$$

output
```
-1/3*(a+b/x^2)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3} dx = -\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b}$$

input
```
Integrate[Sqrt[a + b/x^2]/x^3,x]
```

output
```
-1/3*(a + b/x^2)^(3/2)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3} \, dx$$
$$\Big\downarrow \text{793}$$
$$-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b}$$

input
```
Int[Sqrt[a + b/x^2]/x^3,x]
```

output
```
-1/3*(a + b/x^2)^(3/2)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3b}$	15
orering	$-\frac{\left(a\,x^2+b\right)\sqrt{a+\frac{b}{x^2}}}{3x^2b}$	25
gosper	$-\frac{\left(a\,x^2+b\right)\sqrt{\frac{a\,x^2+b}{x^2}}}{3x^2b}$	29
default	$-\frac{\left(a\,x^2+b\right)\sqrt{\frac{a\,x^2+b}{x^2}}}{3x^2b}$	29
risch	$-\frac{\left(a\,x^2+b\right)\sqrt{\frac{a\,x^2+b}{x^2}}}{3x^2b}$	29
trager	$-\frac{\left(a\,x^2+b\right)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3x^2b}$	33

input
```
int((a+b/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a+b/x^2)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{(ax^2+b)\sqrt{\frac{ax^2+b}{x^2}}}{3\,bx^2}$$

input
```
integrate((a+b/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/3*(a*x^2 + b)*sqrt((a*x^2 + b)/x^2)/(b*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{3b} - \frac{\sqrt{a}\sqrt{1+\frac{b}{ax^2}}}{3x^2}$$

input
```
integrate((a+b/x**2)**(1/2)/x**3,x)
```

output
```
-a**(3/2)*sqrt(1 + b/(a*x**2))/(3*b) - sqrt(a)*sqrt(1 + b/(a*x**2))/(3*x**
2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((a+b/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/3*(a + b/x^2)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx = \frac{2\left(3\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4 a^{\frac{3}{2}}\operatorname{sgn}(x) + a^{\frac{3}{2}}b^2\operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^3}$$

input
```
integrate((a+b/x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^(3/2)*sgn(x) + a^(3/2)*b^2*sgn(x)
)/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^3
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\,(a\,x^2+b)}{3\,b\,x^2}$$

input
```
int((a + b/x^2)^(1/2)/x^3,x)
```

output
```
-((a + b/x^2)^(1/2)*(b + a*x^2))/(3*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^3}\,dx = \frac{-\sqrt{a\,x^2+b}\,a\,x^2-\sqrt{a\,x^2+b}\,b-\sqrt{a}\,a\,x^3}{3b\,x^3}$$

input
```
int((a+b/x^2)^(1/2)/x^3,x)
```

output
```
( - (sqrt(a*x**2 + b)*a*x**2 + sqrt(a*x**2 + b)*b + sqrt(a)*a*x**3))/(3*b*
x**3)
```

3.358 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4} dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3} - \frac{a\sqrt{a+\frac{b}{x^2}}}{8bx} + \frac{a^2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)}{8b^{3/2}}$$

output
```
-1/4*(a+b/x^2)^(1/2)/x^3-1/8*a*(a+b/x^2)^(1/2)/b/x+1/8*a^2*arctanh(b^(1/2)
/(a+b/x^2)^(1/2)/x)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4} dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-\sqrt{b}(2b+ax^2) + \frac{a^2x^4\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b+ax^2}}\right)}{8b^{3/2}x^3}$$

input
```
Integrate[Sqrt[a + b/x^2]/x^4,x]
```

output
```
(Sqrt[a + b/x^2]*(-(Sqrt[b]*(2*b + a*x^2)) + (a^2*x^4*ArcTanh[Sqrt[b + a*x
^2]/Sqrt[b]])/Sqrt[b + a*x^2]))/(8*b^(3/2)*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {858, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{248}$$

$$-\frac{1}{4}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x}-\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}$$

$$\Big\downarrow \text{262}$$

$$-\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{2b}\right)-\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{2b}\right)-\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}\right)-\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}$$

input
```
Int[Sqrt[a + b/x^2]/x^4,x]
```

output
```
-1/4*Sqrt[a + b/x^2]/x^3 - (a*(Sqrt[a + b/x^2]/(2*b*x) - (a*ArcTanh[Sqrt[b
]/(Sqrt[a + b/x^2]*x)])/(2*b^(3/2))))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{(a\,x^{2}+2b)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{8x^{3}b}+\frac{a^{2}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{8b^{\frac{3}{2}}\sqrt{a\,x^{2}+b}}$	86
default	$\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,\left(\sqrt{b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)a^{2}x^{4}-\sqrt{a\,x^{2}+b}\,a^{2}x^{4}+(a\,x^{2}+b)^{\frac{3}{2}}a\,x^{2}-2(a\,x^{2}+b)^{\frac{3}{2}}b\right)}{8x^{3}\sqrt{a\,x^{2}+b}\,b^{2}}$	106

input
```
int((a+b/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*x^2+2*b)/x^3/b*((a*x^2+b)/x^2)^(1/2)+1/8/b^(3/2)*a^2*ln((2*b+2*b^(
1/2)*(a*x^2+b)^(1/2))/x)*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx=\left[\frac{a^2\sqrt{b}x^3\log\left(-\frac{ax^2+2\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\left(abx^2+2\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{16\,b^2x^3},\right.$$
$$\left.-\frac{a^2\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)+\left(abx^2+2\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{8\,b^2x^3}\right]$$

input
```
integrate((a+b/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/16*(a^2*sqrt(b)*x^3*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2
*b)/x^2) - 2*(a*b*x^2 + 2*b^2)*sqrt((a*x^2 + b)/x^2))/(b^2*x^3), -1/8*(a^2
*sqrt(-b)*x^3*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b) + (a*b*x^2 + 2*b^
2)*sqrt((a*x^2 + b)/x^2))/(b^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx = -\frac{a^{\frac{3}{2}}}{8bx\sqrt{1+\frac{b}{ax^2}}} - \frac{3\sqrt{a}}{8x^3\sqrt{1+\frac{b}{ax^2}}} + \frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{8b^{\frac{3}{2}}} - \frac{b}{4\sqrt{a}x^5\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate((a+b/x**2)**(1/2)/x**4,x)
```

output
```
-a**(3/2)/(8*b*x*sqrt(1 + b/(a*x**2))) - 3*sqrt(a)/(8*x**3*sqrt(1 + b/(a*x
**2))) + a**2*asinh(sqrt(b)/(sqrt(a)*x))/(8*b**(3/2)) - b/(4*sqrt(a)*x**5*
sqrt(1 + b/(a*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx = -\frac{a^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{16\,b^{\frac{3}{2}}} - \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2x^3+\sqrt{a+\frac{b}{x^2}}a^2bx}{8\left(\left(a+\frac{b}{x^2}\right)^2bx^4-2\left(a+\frac{b}{x^2}\right)b^2x^2+b^3\right)}$$

input
```
integrate((a+b/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/16*a^2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
/b^(3/2) - 1/8*((a + b/x^2)^(3/2)*a^2*x^3 + sqrt(a + b/x^2)*a^2*b*x)/((a +
 b/x^2)^2*b*x^4 - 2*(a + b/x^2)*b^2*x^2 + b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx = -\frac{\frac{a^3 \arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b} + \frac{(ax^2+b)^{\frac{3}{2}}a^3\operatorname{sgn}(x)+\sqrt{ax^2+b}a^3b\operatorname{sgn}(x)}{a^2bx^4}}{8\,a}$$

input
```
integrate((a+b/x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/8*(a^3*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + ((a*x^2 +
 b)^(3/2)*a^3*sgn(x) + sqrt(a*x^2 + b)*a^3*b*sgn(x))/(a^2*b*x^4))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx = \int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx$$

input
```
int((a + b/x^2)^(1/2)/x^4,x)
```

output
```
int((a + b/x^2)^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^4}\,dx = \frac{-\sqrt{a\,x^2+b}\,ab\,x^2 - 2\sqrt{a\,x^2+b}\,b^2 - \sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a^2x^4 + \sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x+\sqrt{b}}{\sqrt{b}}\right)a^2x^4}{8b^2x^4}$$

input
```
int((a+b/x^2)^(1/2)/x^4,x)
```

output
```
( - sqrt(a*x**2 + b)*a*b*x**2 - 2*sqrt(a*x**2 + b)*b**2 - sqrt(b)*log((sqr
t(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a**2*x**4 + sqrt(b)*log((sqr
t(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a**2*x**4)/(8*b**2*x**4)
```

3.359 $\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx=\frac{5}{8}b\sqrt{a+\frac{b}{x^2}}x^2+\frac{1}{4}a\sqrt{a+\frac{b}{x^2}}x^4+\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
5/8*b*(a+b/x^2)^(1/2)*x^2+1/4*a*(a+b/x^2)^(1/2)*x^4+3/8*b^2*arctanh((a+b/x
^2)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx=\frac{1}{8}\sqrt{a+\frac{b}{x^2}}x\left(5bx+2ax^3-\frac{3b^2\log\left(-\sqrt{a}x+\sqrt{b+ax^2}\right)}{\sqrt{a}\sqrt{b+ax^2}}\right)$$

input
```
Integrate[(a + b/x^2)^(3/2)*x^3,x]
```

output
```
(Sqrt[a + b/x^2]*x*(5*b*x + 2*a*x^3 - (3*b^2*Log[-(Sqrt[a]*x) + Sqrt[b + a
*x^2]])/(Sqrt[a]*Sqrt[b + a*x^2])))/8
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{3/2} x^6 d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{4}b\int \sqrt{a+\frac{b}{x^2}}x^4 d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{4}b\left(\frac{1}{2}b\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}-x^2\sqrt{a+\frac{b}{x^2}}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{4}b\left(\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}-x^2\sqrt{a+\frac{b}{x^2}}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{4}b\left(x^2\left(-\sqrt{a+\frac{b}{x^2}}\right)-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\right)$$

input
```
Int[(a + b/x^2)^(3/2)*x^3,x]
```

output
```
(((a + b/x^2)^(3/2)*x^4)/2 - (3*b*(-(Sqrt[a + b/x^2]*x^2) - (b*ArcTanh[Sqr
t[a + b/x^2]/Sqrt[a]])/Sqrt[a]))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{x^{2}\left(2 a \,x^{2}+5 b \right) \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}{8}+\frac{3 b^{2} \ln \left(\sqrt{a}\, x +\sqrt{a \,x^{2}+b}\right) \sqrt{\frac{a \,x^{2}+b}{x^{2}}}\, x}{8 \sqrt{a}\, \sqrt{a \,x^{2}+b}}$	77
default	$\frac{\left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(2 x \left(a \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{a}+3 \sqrt{a}\, \sqrt{a \,x^{2}+b}\, b x +3 \ln \left(\sqrt{a}\, x +\sqrt{a \,x^{2}+b}\right) b^{2}\right)}{8 \left(a \,x^{2}+b \right)^{\frac{3}{2}} \sqrt{a}}$	84

input
```
int((a+b/x^2)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^2*(2*a*x^2+5*b)*((a*x^2+b)/x^2)^(1/2)+3/8*b^2*ln(a^(1/2)*x+(a*x^2+b)
^(1/2))/a^(1/2)*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.28

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx = \left[\frac{3\sqrt{a}b^2\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\,(2\,a^2x^4+5\,abx^2)\sqrt{\frac{ax^2+b}{x^2}}}{16\,a}, -\frac{3\sqrt{-a}b^2\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-(2\,a^2x^4+5\,abx^2)\sqrt{\frac{ax^2+b}{x^2}}}{8\,a}\right]$$

input
```
integrate((a+b/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(a)*b^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) -
b) + 2*(2*a^2*x^4 + 5*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a, -1/8*(3*sqrt(-a)*
b^2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (2*a^2*x^4 +
5*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a]
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx=\frac{a\sqrt{b}x^3\sqrt{\frac{ax^2}{b}+1}}{4}+\frac{5b^{\frac{3}{2}}x\sqrt{\frac{ax^2}{b}+1}}{8}+\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8\sqrt{a}}$$

input
```
integrate((a+b/x**2)**(3/2)*x**3,x)
```

output
```
a*sqrt(b)*x**3*sqrt(a*x**2/b + 1)/4 + 5*b**(3/2)*x*sqrt(a*x**2/b + 1)/8 +
3*b**2*asinh(sqrt(a)*x/sqrt(b))/(8*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx=-\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\,\sqrt{a}}+\frac{5\,\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2-3\,\sqrt{a+\frac{b}{x^2}}ab^2}{8\left(\left(a+\frac{b}{x^2}\right)^2-2\,\left(a+\frac{b}{x^2}\right)a+a^2\right)}$$

input
```
integrate((a+b/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
-3/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/sqr
t(a) + 1/8*(5*(a + b/x^2)^(3/2)*b^2 - 3*sqrt(a + b/x^2)*a*b^2)/((a + b/x^2
)^2 - 2*(a + b/x^2)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx=-\frac{3\,b^2\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)\operatorname{sgn}(x)}{8\,\sqrt{a}}+\frac{3\,b^2\log\left(|b|\right)\operatorname{sgn}(x)}{16\,\sqrt{a}}+\frac{1}{8}\,\left(2\,ax^2\operatorname{sgn}(x)+5\,b\operatorname{sgn}(x)\right)\sqrt{ax^2+b}x$$

input
```
integrate((a+b/x^2)^(3/2)*x^3,x, algorithm="giac")
```

output
```
-3/8*b^2*log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))*sgn(x)/sqrt(a) + 3/16*b^2*
log(abs(b))*sgn(x)/sqrt(a) + 1/8*(2*a*x^2*sgn(x) + 5*b*sgn(x))*sqrt(a*x^2
+ b)*x
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx = \frac{5\,x^4\,\left(a+\frac{b}{x^2}\right)^{3/2}}{8}+\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{3\,a\,x^4\,\sqrt{a+\frac{b}{x^2}}}{8}$$

input
```
int(x^3*(a + b/x^2)^(3/2),x)
```

output
```
(5*x^4*(a + b/x^2)^(3/2))/8 + (3*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*
a^(1/2)) - (3*a*x^4*(a + b/x^2)^(1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^3\,dx = \frac{2\sqrt{a\,x^2+b}\,a^2x^3+5\sqrt{a\,x^2+b}\,abx+3\sqrt{a}\,\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)b^2}{8a}$$

input
```
int((a+b/x^2)^(3/2)*x^3,x)
```

output
```
(2*sqrt(a*x**2 + b)*a**2*x**3 + 5*sqrt(a*x**2 + b)*a*b*x + 3*sqrt(a)*log((
sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b**2)/(8*a)
```

3.360 $\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2 \, dx=\frac{4}{3}b\sqrt{a+\frac{b}{x^2}}x+\frac{1}{3}a\sqrt{a+\frac{b}{x^2}}x^3-b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)$$

output
```
4/3*b*(a+b/x^2)^(1/2)*x+1/3*a*(a+b/x^2)^(1/2)*x^3-b^(3/2)*arctanh(b^(1/2)/
(a+b/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2 \, dx=\frac{\sqrt{a+\frac{b}{x^2}}x\left(\sqrt{b+ax^2}(4b+ax^2)-3b^{3/2}\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)\right)}{3\sqrt{b+ax^2}}$$

input
```
Integrate[(a + b/x^2)^(3/2)*x^2,x]
```

output
```
(Sqrt[a + b/x^2]*x*(Sqrt[b + a*x^2]*(4*b + a*x^2) - 3*b^(3/2)*ArcTanh[Sqrt
[b + a*x^2]/Sqrt[b]]))/(3*Sqrt[b + a*x^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {858, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \left(a+\frac{b}{x^2}\right)^{3/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{3/2}-b\int \sqrt{a+\frac{b}{x^2}}x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{3/2}-b\left(b\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}-x\sqrt{a+\frac{b}{x^2}}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{3/2}-b\left(b\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}-x\sqrt{a+\frac{b}{x^2}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{3/2}-b\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)-x\sqrt{a+\frac{b}{x^2}}\right)$$

input
```
Int[(a + b/x^2)^(3/2)*x^2,x]
```

output
```
((a + b/x^2)^(3/2)*x^3)/3 - b*(-(Sqrt[a + b/x^2]*x) + Sqrt[b]*ArcTanh[Sqrt
[b]/(Sqrt[a + b/x^2]*x)])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(3 b^{\frac{3}{2}} \ln \left(\frac{2 b+2 \sqrt{b}\, \sqrt{a\,x^{2}+b}}{x}\right)-\left(a\,x^{2}+b\right)^{\frac{3}{2}}-3 \sqrt{a\,x^{2}+b}\, b\right)}{3 \left(a\,x^{2}+b\right)^{\frac{3}{2}}}$	78

input
```
int((a+b/x^2)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((a*x^2+b)/x^2)^(3/2)*x^3*(3*b^(3/2)*ln(2*(b^(1/2)*(a*x^2+b)^(1/2)+b)
/x)-(a*x^2+b)^(3/2)-3*(a*x^2+b)^(1/2)*b)/(a*x^2+b)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.89

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx = \left[\frac{1}{2}\,b^{\frac{3}{2}}\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right) + \frac{1}{3}\left(ax^3+4\,bx\right)\sqrt{\frac{ax^2+b}{x^2}}, \sqrt{-b}b\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right) + \frac{1}{3}\left(ax^3+4\,bx\right)\sqrt{\frac{ax^2+b}{x^2}}\right]$$

input
```
integrate((a+b/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
[1/2*b^(3/2)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2) +
 1/3*(a*x^3 + 4*b*x)*sqrt((a*x^2 + b)/x^2), sqrt(-b)*b*arctan(sqrt(-b)*x*s
qrt((a*x^2 + b)/x^2)/b) + 1/3*(a*x^3 + 4*b*x)*sqrt((a*x^2 + b)/x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx = \frac{a\sqrt{b}x^2\sqrt{\frac{ax^2}{b}+1}}{3} + \frac{4b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3} + \frac{b^{\frac{3}{2}}\log\left(\frac{ax^2}{b}\right)}{2} - b^{\frac{3}{2}}\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)$$

input
```
integrate((a+b/x**2)**(3/2)*x**2,x)
```

output
```
a*sqrt(b)*x**2*sqrt(a*x**2/b + 1)/3 + 4*b**(3/2)*sqrt(a*x**2/b + 1)/3 + b*
*(3/2)*log(a*x**2/b)/2 - b**(3/2)*log(sqrt(a*x**2/b + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx = \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}} x^3 + \sqrt{a+\frac{b}{x^2}}\,bx + \frac{1}{2}\,b^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)$$

input
```
integrate((a+b/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*(a + b/x^2)^(3/2)*x^3 + sqrt(a + b/x^2)*b*x + 1/2*b^(3/2)*log((sqrt(a
+ b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx = \frac{b^2\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + \frac{1}{3}\left(ax^2+b\right)^{\frac{3}{2}}\operatorname{sgn}(x) + \sqrt{ax^2+b}\,b\operatorname{sgn}(x) - \frac{\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\,\sqrt{-b}\,b^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-b}}$$

input
```
integrate((a+b/x^2)^(3/2)*x^2,x, algorithm="giac")
```

output
```
b^2*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 1/3*(a*x^2 + b)^(3/
2)*sgn(x) + sqrt(a*x^2 + b)*b*sgn(x) - 1/3*(3*b^2*arctan(sqrt(b)/sqrt(-b))
 + 4*sqrt(-b)*b^(3/2))*sgn(x)/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx=\int x^2 \left(a+\frac{b}{x^2}\right)^{3/2} dx$$

input
```
int(x^2*(a + b/x^2)^(3/2),x)
```

output
```
int(x^2*(a + b/x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2\,dx=\frac{\sqrt{a\,x^2+b}\,a\,x^2}{3}+\frac{4\sqrt{a\,x^2+b}\,b}{3}+\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)b-\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x+\sqrt{b}}{\sqrt{b}}\right)b$$

input
```
int((a+b/x^2)^(3/2)*x^2,x)
```

output
```
(sqrt(a*x**2 + b)*a*x**2 + 4*sqrt(a*x**2 + b)*b + 3*sqrt(b)*log((sqrt(a*x*
*2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*b - 3*sqrt(b)*log((sqrt(a*x**2 + b
) + sqrt(a)*x + sqrt(b))/sqrt(b))*b)/3
```

3.361 $\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx = -b\sqrt{a+\frac{b}{x^2}}+\frac{1}{2}a\sqrt{a+\frac{b}{x^2}}x^2+\frac{3}{2}\sqrt{a}b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

output
```
-b*(a+b/x^2)^(1/2)+1/2*a*(a+b/x^2)^(1/2)*x^2+3/2*a^(1/2)*b*arctanh((a+b/x^
2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\left((-2b+ax^2)\sqrt{b+ax^2}+6\sqrt{a}bx\operatorname{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)\right)}{2\sqrt{b+ax^2}}$$

input
```
Integrate[(a + b/x^2)^(3/2)*x,x]
```

output
```
(Sqrt[a + b/x^2]*((-2*b + a*x^2)*Sqrt[b + a*x^2] + 6*Sqrt[a]*b*x*ArcTanh[(
Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b + a*x^2])]))/(2*Sqrt[b + a*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {798, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{3/2}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{2}b\int \sqrt{a+\frac{b}{x^2}}x^2d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{2}b\left(a\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+2\sqrt{a+\frac{b}{x^2}}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{b}+2\sqrt{a+\frac{b}{x^2}}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{3/2}-\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)\right)\right)$$

input
```
Int[(a + b/x^2)^(3/2)*x,x]
```

output
```
((a + b/x^2)^(3/2)*x^2 - (3*b*(2*Sqrt[a + b/x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b/x^2]/Sqrt[a]]))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

method	result	size
risch	$\frac{\left(a\,x^{2}-2 b\right) \sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{2}+\frac{3 \sqrt{a}\, b \ln \left(\sqrt{a}\, x+\sqrt{a\,x^{2}+b}\right) \sqrt{\frac{a\,x^{2}+b}{x^{2}}}\, x}{2 \sqrt{a\,x^{2}+b}}$	71
default	$\frac{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}} x^{2} \left(2 a^{\frac{3}{2}} \left(a\,x^{2}+b\right)^{\frac{3}{2}} x^{2}+3 a^{\frac{3}{2}} \sqrt{a\,x^{2}+b}\, b\,x^{2}-2 \left(a\,x^{2}+b\right)^{\frac{5}{2}} \sqrt{a}+3 \ln \left(\sqrt{a}\, x+\sqrt{a\,x^{2}+b}\right) a\,b^{2} x\right)}{2 \left(a\,x^{2}+b\right)^{\frac{3}{2}} b \sqrt{a}}$	107

input
```
int((a+b/x^2)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2-2*b)*((a*x^2+b)/x^2)^(1/2)+3/2*a^(1/2)*b*ln(a^(1/2)*x+(a*x^2+b)
^(1/2))*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.08

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx = \left[\frac{3}{4}\,\sqrt{a}b\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right) + \frac{1}{2}\left(ax^2-2\,b\right)\sqrt{\frac{ax^2+b}{x^2}}, -\frac{3}{2}\,\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right) + \frac{1}{2}\left(ax^2-2\,b\right)\sqrt{\frac{ax^2+b}{x^2}}\right]$$

input
```
integrate((a+b/x^2)^(3/2)*x,x, algorithm="fricas")
```

output
```
[3/4*sqrt(a)*b*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b) + 1
/2*(a*x^2 - 2*b)*sqrt((a*x^2 + b)/x^2), -3/2*sqrt(-a)*b*arctan(sqrt(-a)*x^
2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + 1/2*(a*x^2 - 2*b)*sqrt((a*x^2 + b)/
x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx = \frac{3\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2} + \frac{a^2x^3}{2\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} - \frac{a\sqrt{b}x}{2\sqrt{\frac{ax^2}{b}+1}} - \frac{b^{\frac{3}{2}}}{x\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate((a+b/x**2)**(3/2)*x,x)
```

output
```
3*sqrt(a)*b*asinh(sqrt(a)*x/sqrt(b))/2 + a**2*x**3/(2*sqrt(b)*sqrt(a*x**2/
b + 1)) - a*sqrt(b)*x/(2*sqrt(a*x**2/b + 1)) - b**(3/2)/(x*sqrt(a*x**2/b +
 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx = \frac{1}{2}\sqrt{a+\frac{b}{x^2}}ax^2 - \frac{3}{4}\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right) - \sqrt{a+\frac{b}{x^2}}b$$

input
```
integrate((a+b/x^2)^(3/2)*x,x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^2)*a*x^2 - 3/4*sqrt(a)*b*log((sqrt(a + b/x^2) - sqrt(a))/
(sqrt(a + b/x^2) + sqrt(a))) - sqrt(a + b/x^2)*b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx &= \frac{1}{2}\sqrt{ax^2+b}\,ax\operatorname{sgn}(x) \\ &\quad - \frac{3}{4}\sqrt{a}b\log\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2\right)\operatorname{sgn}(x) + \frac{2\sqrt{a}b^2\operatorname{sgn}(x)}{\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b}\end{aligned}$$

input
```
integrate((a+b/x^2)^(3/2)*x,x, algorithm="giac")
```

output
```
1/2*sqrt(a*x^2 + b)*a*x*sgn(x) - 3/4*sqrt(a)*b*log((sqrt(a)*x - sqrt(a*x^2
 + b))^2)*sgn(x) + 2*sqrt(a)*b^2*sgn(x)/((sqrt(a)*x - sqrt(a*x^2 + b))^2 -
 b)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx = \frac{a\,x^2\,\sqrt{a+\frac{b}{x^2}}}{2} - b\,\sqrt{a+\frac{b}{x^2}} + \frac{3\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2}$$

input
```
int(x*(a + b/x^2)^(3/2),x)
```

output
```
(a*x^2*(a + b/x^2)^(1/2))/2 - b*(a + b/x^2)^(1/2) + (3*a^(1/2)*b*atanh((a
+ b/x^2)^(1/2)/a^(1/2)))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} x\,dx = \frac{4\sqrt{a\,x^2+b}\,a\,x^2 - 8\sqrt{a\,x^2+b}\,b + 12\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)bx - 9\sqrt{a}\,bx}{8x}$$

input
```
int((a+b/x^2)^(3/2)*x,x)
```

output
```
(4*sqrt(a*x**2 + b)*a*x**2 - 8*sqrt(a*x**2 + b)*b + 12*sqrt(a)*log((sqrt(a
*x**2 + b) + sqrt(a)*x)/sqrt(b))*b*x - 9*sqrt(a)*b*x)/(8*x)
```

3.362 $\int \left(a+\frac{b}{x^2}\right)^{3/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} dx=-\frac{b\sqrt{a+\frac{b}{x^2}}}{2x}+a\sqrt{a+\frac{b}{x^2}}\,x-\frac{3}{2}a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)$$

output
```
-1/2*b*(a+b/x^2)^(1/2)/x+a*(a+b/x^2)^(1/2)*x-3/2*a*b^(1/2)*arctanh(b^(1/2)
/(a+b/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} dx=-\frac{\sqrt{a+\frac{b}{x^2}}\left((b-2ax^2)\sqrt{b+ax^2}+3a\sqrt{b}x^2\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)\right)}{2x\sqrt{b+ax^2}}$$

input
```
Integrate[(a + b/x^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[a + b/x^2]*((b - 2*a*x^2)*Sqrt[b + a*x^2] + 3*a*Sqrt[b]*x^2*Arc
Tanh[Sqrt[b + a*x^2]/Sqrt[b]]))/(x*Sqrt[b + a*x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {773, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{211}$$

$$x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)$$

$$\Big\downarrow \text{224}$$

$$x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)$$

$$\Big\downarrow \text{219}$$

$$x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)$$

input
```
Int[(a + b/x^2)^(3/2),x]
```

output
```
(a + b/x^2)^(3/2)*x - 3*b*(Sqrt[a + b/x^2]/(2*x) + (a*ArcTanh[Sqrt[b]/(Sqr
t[a + b/x^2]*x)])/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{b\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{2x}+\frac{\left(-\frac{3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)\sqrt{b}\,a}{2}+\sqrt{a\,x^{2}+b}\,a\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{\sqrt{a\,x^{2}+b}}$	86
default	$-\frac{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}x\left(3b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)a\,x^{2}-\left(a\,x^{2}+b\right)^{\frac{3}{2}}a\,x^{2}+\left(a\,x^{2}+b\right)^{\frac{5}{2}}-3\sqrt{a\,x^{2}+b}\,ab\,x^{2}\right)}{2\left(a\,x^{2}+b\right)^{\frac{3}{2}}b}$	100

input
```
int((a+b/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*((a*x^2+b)/x^2)^(1/2)/x+(-3/2*ln((2*b+2*b^(1/2)*(a*x^2+b)^(1/2))/x)
*b^(1/2)*a+(a*x^2+b)^(1/2)*a)*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.09

$$\int\left(a+\frac{b}{x^2}\right)^{3/2}dx=\left[\frac{3\,a\sqrt{b}x\log\left(-\frac{ax^2-2\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+2\left(2\,ax^2-b\right)\sqrt{\frac{ax^2+b}{x^2}}}{4\,x},\frac{3\,a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right.}{2\,x}\right.$$

input
```
integrate((a+b/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(b)*x*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)
/x^2) + 2*(2*a*x^2 - b)*sqrt((a*x^2 + b)/x^2))/x, 1/2*(3*a*sqrt(-b)*x*arct
an(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b) + (2*a*x^2 - b)*sqrt((a*x^2 + b)/x^
2))/x]
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} dx = \frac{a^{\frac{3}{2}}x}{\sqrt{1+\frac{b}{ax^2}}}+\frac{\sqrt{a}b}{2x\sqrt{1+\frac{b}{ax^2}}}-\frac{3a\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{2}-\frac{b^2}{2\sqrt{a}x^3\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate((a+b/x**2)**(3/2),x)
```

output
```
a**(3/2)*x/sqrt(1 + b/(a*x**2)) + sqrt(a)*b/(2*x*sqrt(1 + b/(a*x**2))) - 3
*a*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x))/2 - b**2/(2*sqrt(a)*x**3*sqrt(1 + b/
(a*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} dx = \sqrt{a+\frac{b}{x^2}}ax-\frac{\sqrt{a+\frac{b}{x^2}}abx}{2\left(\left(a+\frac{b}{x^2}\right)x^2-b\right)}+\frac{3}{4}a\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)$$

input
```
integrate((a+b/x^2)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x^2)*a*x - 1/2*sqrt(a + b/x^2)*a*b*x/((a + b/x^2)*x^2 - b) + 3/
4*a*sqrt(b)*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)
))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \left(a + \frac{b}{x^2}\right)^{3/2} dx = \frac{1}{2}\left(\frac{3\,b\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + 2\sqrt{ax^2+b}\operatorname{sgn}(x) - \frac{\sqrt{ax^2+b}b\operatorname{sgn}(x)}{ax^2}\right)a$$

input
```
integrate((a+b/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(3*b*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*sqrt(a*x^2 +
 b)*sgn(x) - sqrt(a*x^2 + b)*b*sgn(x)/(a*x^2))*a
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int \left(a + \frac{b}{x^2}\right)^{3/2} dx = \frac{x\left(a\,x^2+b\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a\,x^2}\right)}{\left(\frac{b}{a}+x^2\right)^{3/2}}$$

input
```
int((a + b/x^2)^(3/2),x)
```

output
```
(x*(b + a*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -b/(a*x^2)))/(b/a + x^2)
^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int \left(a+\frac{b}{x^2}\right)^{3/2} dx = \frac{2\sqrt{a\,x^2+b}\,a\,x^2-\sqrt{a\,x^2+b}\,b+3\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a\,x^2-3\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x+\sqrt{b}}{\sqrt{b}}\right)}{2x^2}$$

input
```
int((a+b/x^2)^(3/2),x)
```

output
```
(2*sqrt(a*x**2 + b)*a*x**2 - sqrt(a*x**2 + b)*b + 3*sqrt(b)*log((sqrt(a*x*
*2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a*x**2 - 3*sqrt(b)*log((sqrt(a*x**
2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a*x**2)/(2*x**2)
```

3.363 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x} \, dx = -a\sqrt{a+\frac{b}{x^2}}-\frac{1}{3}\left(a+\frac{b}{x^2}\right)^{3/2}+a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

output
```
-a*(a+b/x^2)^(1/2)-1/3*(a+b/x^2)^(3/2)+a^(3/2)*arctanh((a+b/x^2)^(1/2)/a^(
1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x} \, dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-b-4ax^2+\frac{6a^{3/2}x^3\text{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{\sqrt{b+ax^2}}\right)}{3x^2}$$

input
```
Integrate[(a + b/x^2)^(3/2)/x,x]
```

output
```
(Sqrt[a + b/x^2]*(-b - 4*a*x^2 + (6*a^(3/2)*x^3*ArcTanh[(Sqrt[a]*x)/(-Sqrt
[b] + Sqrt[b + a*x^2])])/Sqrt[b + a*x^2]))/(3*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\int \sqrt{a+\frac{b}{x^2}}x^2d\frac{1}{x^2}-\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(a\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+2\sqrt{a+\frac{b}{x^2}}\right)-\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-a\left(\frac{2a\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{b}+2\sqrt{a+\frac{b}{x^2}}\right)-\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-a\left(2\sqrt{a+\frac{b}{x^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)\right)-\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

input
```
Int[(a + b/x^2)^(3/2)/x,x]
```

output
```
((-2*(a + b/x^2)^(3/2))/3 - a*(2*Sqrt[a + b/x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b/x^2]/Sqrt[a]]))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

method	result	size
risch	$-\frac{(4a\,x^2+b)\sqrt{\frac{a\,x^2+b}{x^2}}}{3x^2}+\frac{a^{\frac{3}{2}}\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)\sqrt{\frac{a\,x^2+b}{x^2}}\,x}{\sqrt{a\,x^2+b}}$	71
default	$\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}\left(2a^{\frac{5}{2}}(a\,x^2+b)^{\frac{3}{2}}x^4+3a^{\frac{5}{2}}\sqrt{a\,x^2+b}\,b\,x^4-2a^{\frac{3}{2}}(a\,x^2+b)^{\frac{5}{2}}x^2+3\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)a^2b^2x^3-(a\,x^2+b)^{\frac{5}{2}}b\sqrt{a}\right)}{3(a\,x^2+b)^{\frac{3}{2}}b^2\sqrt{a}}$	126

input
```
int((a+b/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*a*x^2+b)/x^2*((a*x^2+b)/x^2)^(1/2)+a^(3/2)*ln(a^(1/2)*x+(a*x^2+b)^
(1/2))*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.61

$$\int\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx=\left[\frac{3\,a^{\frac{3}{2}}x^2\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-2\,(4\,ax^2+b)\sqrt{\frac{ax^2+b}{x^2}}}{6\,x^2},\ -\frac{3\,\sqrt{-a}ax^2\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+(4\,ax^2+b)\sqrt{\frac{ax^2+b}{x^2}}}{3\,x^2}\right]$$

input
```
integrate((a+b/x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/6*(3*a^(3/2)*x^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b
) - 2*(4*a*x^2 + b)*sqrt((a*x^2 + b)/x^2))/x^2, -1/3*(3*sqrt(-a)*a*x^2*arc
tan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (4*a*x^2 + b)*sqrt((
a*x^2 + b)/x^2))/x^2]
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx = -\frac{4a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{3} - \frac{a^{\frac{3}{2}}\log\left(\frac{b}{ax^2}\right)}{2} + a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right) - \frac{\sqrt{a}b\sqrt{1+\frac{b}{ax^2}}}{3x^2}$$

input
```
integrate((a+b/x**2)**(3/2)/x,x)
```

output
```
-4*a**(3/2)*sqrt(1 + b/(a*x**2))/3 - a**(3/2)*log(b/(a*x**2))/2 + a**(3/2)
*log(sqrt(1 + b/(a*x**2)) + 1) - sqrt(a)*b*sqrt(1 + b/(a*x**2))/(3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx = -\frac{1}{2}a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right) - \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}} - \sqrt{a+\frac{b}{x^2}}a$$

input
```
integrate((a+b/x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/2*a^(3/2)*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))
- 1/3*(a + b/x^2)^(3/2) - sqrt(a + b/x^2)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(42) = 84$.

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.26

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx = -\frac{1}{2}\,a^{\frac{3}{2}}\log\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2\right)\mathrm{sgn}(x) + \frac{4\left(3\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4a^{\frac{3}{2}}b\mathrm{sgn}(x)-3\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2a^{\frac{3}{2}}b^2\mathrm{sgn}(x)+2\,a^{\frac{3}{2}}b^3\mathrm{sgn}(x)\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^3}$$

input
```
integrate((a+b/x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
-1/2*a^(3/2)*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 4/3*(3*(sqrt(a)
*x - sqrt(a*x^2 + b))^4*a^(3/2)*b*sgn(x) - 3*(sqrt(a)*x - sqrt(a*x^2 + b))
^2*a^(3/2)*b^2*sgn(x) + 2*a^(3/2)*b^3*sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b
))^2 - b)^3
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx = a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right) - a\sqrt{a+\frac{b}{x^2}} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3}$$

input
```
int((a + b/x^2)^(3/2)/x,x)
```

output
```
a^(3/2)*atanh((a + b/x^2)^(1/2)/a^(1/2)) - a*(a + b/x^2)^(1/2) - (a + b/x^
2)^(3/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x}\,dx = \frac{-4\sqrt{a\,x^2+b}\,a\,x^2-\sqrt{a\,x^2+b}\,b+3\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)a\,x^3}{3x^3}$$

input
```
int((a+b/x^2)^(3/2)/x,x)
```

output
```
( - 4*sqrt(a*x**2 + b)*a*x**2 - sqrt(a*x**2 + b)*b + 3*sqrt(a)*log((sqrt(a
*x**2 + b) + sqrt(a)*x)/sqrt(b))*a*x**3)/(3*x**3)
```

3.364 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2} dx = -\frac{3a\sqrt{a+\frac{b}{x^2}}}{8x} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x} - \frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{8\sqrt{b}}$$

output
```
-3/8*a*(a+b/x^2)^(1/2)/x-1/4*(a+b/x^2)^(3/2)/x-3/8*a^2*arctanh(b^(1/2)/(a+
b/x^2)^(1/2)/x)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2} dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-2b-5ax^2-\frac{3a^2x^4\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+ax^2}}\right)}{8x^3}$$

input
```
Integrate[(a + b/x^2)^(3/2)/x^2,x]
```

output
```
(Sqrt[a + b/x^2]*(-2*b - 5*a*x^2 - (3*a^2*x^4*ArcTanh[Sqrt[b + a*x^2]/Sqrt
[b]])/(Sqrt[b]*Sqrt[b + a*x^2])))/(8*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \ 858$$

$$-\int \left(a+\frac{b}{x^2}\right)^{3/2}d\frac{1}{x}$$

$$\Big\downarrow \ 211$$

$$-\frac{3}{4}a\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}$$

$$\Big\downarrow \ 211$$

$$-\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}$$

$$\Big\downarrow \ 224$$

$$-\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}$$

$$\Big\downarrow \ 219$$

$$-\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}$$

input
```
Int[(a + b/x^2)^(3/2)/x^2,x]
```

output
```
-1/4*(a + b/x^2)^(3/2)/x - (3*a*(Sqrt[a + b/x^2]/(2*x) + (a*ArcTanh[Sqrt[b
]/(Sqrt[a + b/x^2]*x)])/(2*Sqrt[b])))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{\left(5 a \,x^{2}+2 b \right) \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}{8 x^{3}}-\frac{3 a^{2} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{a \,x^{2}+b}}{x}\right) \sqrt{\frac{a \,x^{2}+b}{x^{2}}}\, x}{8 \sqrt{b}\, \sqrt{a \,x^{2}+b}}$	84
default	$-\frac{\left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}} \left(-\left(a \,x^{2}+b \right)^{\frac{3}{2}} a^{2} x^{4}+3 b^{\frac{3}{2}} \ln \left(\frac{2 b +2 \sqrt{b}\, \sqrt{a \,x^{2}+b}}{x}\right) a^{2} x^{4}+\left(a \,x^{2}+b \right)^{\frac{5}{2}} a \,x^{2}-3 \sqrt{a \,x^{2}+b}\, a^{2} b \,x^{4}+2 \left(a \,x^{2}+b \right)^{\frac{5}{2}} b \right)}{8 x \left(a \,x^{2}+b \right)^{\frac{3}{2}} b^{2}}$	125

input
```
int((a+b/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(5*a*x^2+2*b)/x^3*((a*x^2+b)/x^2)^(1/2)-3/8*a^2/b^(1/2)*ln((2*b+2*b^(
1/2)*(a*x^2+b)^(1/2))/x)*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx = \left[\frac{3\,a^2\sqrt{b}x^3\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\,(5\,abx^2+2\,b^2)\sqrt{\frac{ax^2+b}{x^2}}}{16\,bx^3}, \frac{3\,a^2\sqrt{-b}x^3\arctan\left(\right.}{}\right.$$

input
```
integrate((a+b/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(b)*x^3*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) +
 2*b)/x^2) - 2*(5*a*b*x^2 + 2*b^2)*sqrt((a*x^2 + b)/x^2))/(b*x^3), 1/8*(3*
a^2*sqrt(-b)*x^3*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b) - (5*a*b*x^2 +
 2*b^2)*sqrt((a*x^2 + b)/x^2))/(b*x^3)]
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{5a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{8x}-\frac{\sqrt{a}b\sqrt{1+\frac{b}{ax^2}}}{4x^3}-\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{8\sqrt{b}}$$

input
```
integrate((a+b/x**2)**(3/2)/x**2,x)
```

output
```
-5*a**(3/2)*sqrt(1 + b/(a*x**2))/(8*x) - sqrt(a)*b*sqrt(1 + b/(a*x**2))/(4
*x**3) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*x))/(8*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{16\,\sqrt{b}} - \frac{5\,\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2x^3 - 3\,\sqrt{a+\frac{b}{x^2}}a^2bx}{8\,\left(\left(a+\frac{b}{x^2}\right)^2x^4 - 2\,\left(a+\frac{b}{x^2}\right)bx^2 + b^2\right)}$$

input
```
integrate((a+b/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
3/16*a^2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/
sqrt(b) - 1/8*(5*(a + b/x^2)^(3/2)*a^2*x^3 - 3*sqrt(a + b/x^2)*a^2*b*x)/((
a + b/x^2)^2*x^4 - 2*(a + b/x^2)*b*x^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{5\,(ax^2+b)^{\frac{3}{2}}a^3\mathrm{sgn}(x) - 3\,\sqrt{ax^2+b}a^3b\mathrm{sgn}(x)}{a^2x^4}}{8\,a}$$

input
```
integrate((a+b/x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/8*(3*a^3*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (5*(a*x^2 +
b)^(3/2)*a^3*sgn(x) - 3*sqrt(a*x^2 + b)*a^3*b*sgn(x))/(a^2*x^4))/a
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{\left(a\,x^2+b\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b}{a\,x^2}\right)}{x\,\left(\frac{b}{a}+x^2\right)^{3/2}}$$

input
```
int((a + b/x^2)^(3/2)/x^2,x)
```

output
```
-((b + a*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -b/(a*x^2)))/(x*(b/a + x^2
)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{-5\sqrt{a\,x^2+b}\,ab\,x^2-2\sqrt{a\,x^2+b}\,b^2+3\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a^2x^4-3\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+}}{}\right.}{8b\,x^4}$$

input
```
int((a+b/x^2)^(3/2)/x^2,x)
```

output
```
( - 5*sqrt(a*x**2 + b)*a*b*x**2 - 2*sqrt(a*x**2 + b)*b**2 + 3*sqrt(b)*log(
(sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a**2*x**4 - 3*sqrt(b)*lo
g((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a**2*x**4)/(8*b*x**4)
```

3.365 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b}$$

output
```
-1/5*(a+b/x^2)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{3/2} (b+ax^2)}{5bx^2}$$

input
```
Integrate[(a + b/x^2)^(3/2)/x^3,x]
```

output
```
-1/5*((a + b/x^2)^(3/2)*(b + a*x^2))/(b*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b}$$

input
```
Int[(a + b/x^2)^(3/2)/x^3,x]
```

output
```
-1/5*(a + b/x^2)^(5/2)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}{5b}$	15
orering	$-\frac{(a\,x^2+b)\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{5x^2b}$	25
gosper	$-\frac{(a\,x^2+b)\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}{5x^2b}$	29
default	$-\frac{(a\,x^2+b)\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}{5x^2b}$	29
risch	$-\frac{\sqrt{\frac{a\,x^2+b}{x^2}}\,(a^2x^4+2ab\,x^2+b^2)}{5x^4b}$	40
trager	$-\frac{(a^2x^4+2ab\,x^2+b^2)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{5x^4b}$	44

input
```
int((a+b/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a+b/x^2)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{(a^2x^4+2\,abx^2+b^2)\sqrt{\frac{ax^2+b}{x^2}}}{5\,bx^4}$$

input
```
integrate((a+b/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-1/5*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt((a*x^2 + b)/x^2)/(b*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.

Time = 0.63 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^2}}}{5b} - \frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^2}}}{5x^2} - \frac{\sqrt{a}b\sqrt{1+\frac{b}{ax^2}}}{5x^4}$$

input
```
integrate((a+b/x**2)**(3/2)/x**3,x)
```

output
```
-a**(5/2)*sqrt(1 + b/(a*x**2))/(5*b) - 2*a**(3/2)*sqrt(1 + b/(a*x**2))/(5*
x**2) - sqrt(a)*b*sqrt(1 + b/(a*x**2))/(5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate((a+b/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-1/5*(a + b/x^2)^(5/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(14) = 28$.

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 5.11

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{2\left(5\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^8 a^{\frac{5}{2}}\mathrm{sgn}(x) + 10\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4 a^{\frac{5}{2}}b^2\mathrm{sgn}(x) + a^{\frac{5}{2}}b^4\mathrm{sgn}(x)\right)}{5\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^5}$$

input
```
integrate((a+b/x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2/5*(5*(sqrt(a)*x - sqrt(a*x^2 + b))^8*a^(5/2)*sgn(x) + 10*(sqrt(a)*x - sq
rt(a*x^2 + b))^4*a^(5/2)*b^2*sgn(x) + a^(5/2)*b^4*sgn(x))/((sqrt(a)*x - sq
rt(a*x^2 + b))^2 - b)^5
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\,\left(a\,x^2+b\right)^2}{5\,b\,x^4}$$

input
```
int((a + b/x^2)^(3/2)/x^3,x)
```

output
```
-((a + b/x^2)^(1/2)*(b + a*x^2)^2)/(5*b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{-\sqrt{a\,x^2+b}\,a^2x^4-2\sqrt{a\,x^2+b}\,ab\,x^2-\sqrt{a\,x^2+b}\,b^2-\sqrt{a}\,a^2x^5}{5b\,x^5}$$

input
```
int((a+b/x^2)^(3/2)/x^3,x)
```

output
```
( - sqrt(a*x**2 + b)*a**2*x**4 - 2*sqrt(a*x**2 + b)*a*b*x**2 - sqrt(a*x**2
 + b)*b**2 - sqrt(a)*a**2*x**5)/(5*b*x**5)
```

3.366 $\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4} dx = -\frac{a\sqrt{a+\frac{b}{x^2}}}{8x^3} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3} - \frac{a^2\sqrt{a+\frac{b}{x^2}}}{16bx} + \frac{a^3\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{16b^{3/2}}$$

output
```
-1/8*a*(a+b/x^2)^(1/2)/x^3-1/6*(a+b/x^2)^(3/2)/x^3-1/16*a^2*(a+b/x^2)^(1/2
)/b/x+1/16*a^3*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4} dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-\sqrt{b}(8b^2+14abx^2+3a^2x^4)+\frac{3a^3x^6\operatorname{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b+ax^2}}\right)}{48b^{3/2}x^5}$$

input
```
Integrate[(a + b/x^2)^(3/2)/x^4,x]
```

output
```
(Sqrt[a + b/x^2]*(-(Sqrt[b]*(8*b^2 + 14*a*b*x^2 + 3*a^2*x^4)) + (3*a^3*x^6
*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]])/Sqrt[b + a*x^2]))/(48*b^(3/2)*x^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{248}$$

$$-\frac{1}{2}a\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}d\frac{1}{x}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{248}$$

$$-\frac{1}{2}a\left(\frac{1}{4}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{262}$$

$$-\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}$$

$$\Big\downarrow 219$$

$$-\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}\right)+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}$$

input
```
Int[(a + b/x^2)^(3/2)/x^4,x]
```

output
```
-1/6*(a + b/x^2)^(3/2)/x^3 - (a*(Sqrt[a + b/x^2]/(4*x^3) + (a*(Sqrt[a + b/
x^2]/(2*b*x) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*b^(3/2))))/4))/
2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(3a^{2}x^{4}+14ab\,x^{2}+8b^{2}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{48x^{5}b}+\frac{a^{3}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{16b^{\frac{3}{2}}\sqrt{a\,x^{2}+b}}$
default	$\frac{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}\left(3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)b^{\frac{3}{2}}a^{3}x^{6}-\left(a\,x^{2}+b\right)^{\frac{3}{2}}a^{3}x^{6}+\left(a\,x^{2}+b\right)^{\frac{5}{2}}a^{2}x^{4}-3\sqrt{a\,x^{2}+b}\,a^{3}b\,x^{6}+2\left(a\,x^{2}+b\right)^{\frac{5}{2}}ab\,x^{2}-8\left(a\,x^{2}+b\right)^{\frac{}{}}\right.}{48x^{3}\left(a\,x^{2}+b\right)^{\frac{3}{2}}b^{3}}$

input
```
int((a+b/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(3*a^2*x^4+14*a*b*x^2+8*b^2)/x^5/b*((a*x^2+b)/x^2)^(1/2)+1/16/b^(3/2
)*a^3*ln((2*b+2*b^(1/2)*(a*x^2+b)^(1/2))/x)*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2
+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.87

$$\int\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx=\left[\frac{3\,a^3\sqrt{b}x^5\log\left(-\frac{ax^2+2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\left(3\,a^2bx^4+14\,ab^2x^2+8\,b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{96\,b^2x^5},\right.$$
$$\left.-\frac{3\,a^3\sqrt{-b}x^5\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)+\left(3\,a^2bx^4+14\,ab^2x^2+8\,b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{48\,b^2x^5}\right]$$

input
```
integrate((a+b/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(3*a^3*sqrt(b)*x^5*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) +
 2*b)/x^2) - 2*(3*a^2*b*x^4 + 14*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2))
/(b^2*x^5), -1/48*(3*a^3*sqrt(-b)*x^5*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x
^2)/b) + (3*a^2*b*x^4 + 14*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2))/(b^2*
x^5)]
```

Sympy [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{a^{\frac{5}{2}}}{16bx\sqrt{1+\frac{b}{ax^2}}} - \frac{17a^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{b}{ax^2}}} - \frac{11\sqrt{a}b}{24x^5\sqrt{1+\frac{b}{ax^2}}} + \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{16b^{\frac{3}{2}}} - \frac{b^2}{6\sqrt{a}x^7\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate((a+b/x**2)**(3/2)/x**4,x)
```

output
```
-a**(5/2)/(16*b*x*sqrt(1 + b/(a*x**2))) - 17*a**(3/2)/(48*x**3*sqrt(1 + b/
(a*x**2))) - 11*sqrt(a)*b/(24*x**5*sqrt(1 + b/(a*x**2))) + a**3*asinh(sqrt
(b)/(sqrt(a)*x))/(16*b**(3/2)) - b**2/(6*sqrt(a)*x**7*sqrt(1 + b/(a*x**2))
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(75) = 150.

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{a^3\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{32\,b^{\frac{3}{2}}} - \frac{3\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3x^5 + 8\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^3bx^3 - 3\sqrt{a+\frac{b}{x^2}}a^3b^2x}{48\left(\left(a+\frac{b}{x^2}\right)^3bx^6 - 3\left(a+\frac{b}{x^2}\right)^2b^2x^4 + 3\left(a+\frac{b}{x^2}\right)b^3x^2 - b^4\right)}$$

input
```
integrate((a+b/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/32*a^3*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
/b^(3/2) - 1/48*(3*(a + b/x^2)^(5/2)*a^3*x^5 + 8*(a + b/x^2)^(3/2)*a^3*b*x
^3 - 3*sqrt(a + b/x^2)*a^3*b^2*x)/((a + b/x^2)^3*b*x^6 - 3*(a + b/x^2)^2*b
^2*x^4 + 3*(a + b/x^2)*b^3*x^2 - b^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{1}{48}\,a^3\left(\frac{3\,\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}b}+\frac{3\,(ax^2+b)^{\frac{5}{2}}\mathrm{sgn}(x)+8\,(ax^2+b)^{\frac{3}{2}}b\mathrm{sgn}(x)-3\,\sqrt{ax^2+b}b^2\mathrm{sgn}(x)}{a^3bx^6}\right)$$

input
```
integrate((a+b/x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/48*a^3*(3*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (3*(a*
x^2 + b)^(5/2)*sgn(x) + 8*(a*x^2 + b)^(3/2)*b*sgn(x) - 3*sqrt(a*x^2 + b)*b
^2*sgn(x))/(a^3*b*x^6))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a + b/x^2)^(3/2)/x^4,x)
```

output
```
int((a + b/x^2)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^4}\,dx = \frac{-3\sqrt{a\,x^2+b}\,a^2b\,x^4-14\sqrt{a\,x^2+b}\,a\,b^2x^2-8\sqrt{a\,x^2+b}\,b^3-3\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)}{48b^2x^6}$$

input
```
int((a+b/x^2)^(3/2)/x^4,x)
```

output
```
( - 3*sqrt(a*x**2 + b)*a**2*b*x**4 - 14*sqrt(a*x**2 + b)*a*b**2*x**2 - 8*s
qrt(a*x**2 + b)*b**3 - 3*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(
b))/sqrt(b))*a**3*x**6 + 3*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqr
t(b))/sqrt(b))*a**3*x**6)/(48*b**2*x**6)
```

3.367 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3 \, dx = -b^2\sqrt{a+\frac{b}{x^2}}+\frac{9}{8}ab\sqrt{a+\frac{b}{x^2}}x^2 + \frac{1}{4}a^2\sqrt{a+\frac{b}{x^2}}x^4+\frac{15}{8}\sqrt{a}b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

output
```
-b^2*(a+b/x^2)^(1/2)+9/8*a*b*(a+b/x^2)^(1/2)*x^2+1/4*a^2*(a+b/x^2)^(1/2)*x
^4+15/8*a^(1/2)*b^2*arctanh((a+b/x^2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3 \, dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(\sqrt{b+ax^2}(-8b^2+9abx^2+2a^2x^4)-15\sqrt{a}b^2x\log\left(-\sqrt{a}x+\sqrt{b+ax^2}\right)\right)}{8\sqrt{b+ax^2}}$$

input
```
Integrate[(a + b/x^2)^(5/2)*x^3,x]
```

output
```
(Sqrt[a + b/x^2]*(Sqrt[b + a*x^2]*(-8*b^2 + 9*a*b*x^2 + 2*a^2*x^4) - 15*Sq
rt[a]*b^2*x*Log[-(Sqrt[a]*x) + Sqrt[b + a*x^2]]))/(8*Sqrt[b + a*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right)^{5/2} dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2} \int \left(a+\frac{b}{x^2}\right)^{5/2} x^6 d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{4} b \int \left(a+\frac{b}{x^2}\right)^{3/2} x^4 d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{4} b \left(\frac{3}{2} b \int \sqrt{a+\frac{b}{x^2}} x^2 d\frac{1}{x^2}-x^2 \left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{4} b \left(\frac{3}{2} b \left(a \int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}} d\frac{1}{x^2}+2 \sqrt{a+\frac{b}{x^2}}\right)-x^2 \left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{4} b \left(\frac{3}{2} b \left(\frac{2 a \int \frac{1}{\frac{1}{b x^4}-\frac{a}{b}} d\sqrt{a+\frac{b}{x^2}}}{b}+2 \sqrt{a+\frac{b}{x^2}}\right)-x^2 \left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)\right)-x^2\left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

input
```
Int[(a + b/x^2)^(5/2)*x^3,x]
```

output
```
(((a + b/x^2)^(5/2)*x^4)/2 - (5*b*(-((a + b/x^2)^(3/2)*x^2) + (3*b*(2*Sqrt
[a + b/x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]]))/2))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\left(2a^{2}x^{4}+9ab\,x^{2}-8b^{2}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{8}+\frac{15\sqrt{a}\,b^{2}\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{8\sqrt{a\,x^{2}+b}}$
default	$-\frac{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}x^{4}\left(-8a^{\frac{3}{2}}\left(a\,x^{2}+b\right)^{\frac{5}{2}}x^{2}+8\left(a\,x^{2}+b\right)^{\frac{7}{2}}\sqrt{a}-10\left(a\,x^{2}+b\right)^{\frac{3}{2}}a^{\frac{3}{2}}b\,x^{2}-15\sqrt{a\,x^{2}+b}\,a^{\frac{3}{2}}b^{2}x^{2}-15\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)a\,b^{3}x\right)}{8\left(a\,x^{2}+b\right)^{\frac{5}{2}}b\sqrt{a}}$

input
```
int((a+b/x^2)^(5/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a^2*x^4+9*a*b*x^2-8*b^2)*((a*x^2+b)/x^2)^(1/2)+15/8*a^(1/2)*b^2*ln(
a^(1/2)*x+(a*x^2+b)^(1/2))*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.78

$$\int\left(a+\frac{b}{x^2}\right)^{5/2}x^3\,dx=\left[\frac{15}{16}\sqrt{a}b^2\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+\frac{1}{8}\left(2\,a^2x^4+9\,abx^2-8\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}},-\frac{15}{8}\sqrt{-a}b^2\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\frac{1}{8}\left(2\,a^2x^4+9\,abx^2-8\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}\right]$$

input
```
integrate((a+b/x^2)^(5/2)*x^3,x, algorithm="fricas")
```

output
```
[15/16*sqrt(a)*b^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b)
 + 1/8*(2*a^2*x^4 + 9*a*b*x^2 - 8*b^2)*sqrt((a*x^2 + b)/x^2), -15/8*sqrt(-
a)*b^2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + 1/8*(2*a^2
*x^4 + 9*a*b*x^2 - 8*b^2)*sqrt((a*x^2 + b)/x^2)]
```

Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx = \frac{15\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8} + \frac{a^3x^5}{4\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} + \frac{11a^2\sqrt{b}x^3}{8\sqrt{\frac{ax^2}{b}+1}} + \frac{ab^{\frac{3}{2}}x}{8\sqrt{\frac{ax^2}{b}+1}} - \frac{b^{\frac{5}{2}}}{x\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate((a+b/x**2)**(5/2)*x**3,x)
```

output
```
15*sqrt(a)*b**2*asinh(sqrt(a)*x/sqrt(b))/8 + a**3*x**5/(4*sqrt(b)*sqrt(a*x
**2/b + 1)) + 11*a**2*sqrt(b)*x**3/(8*sqrt(a*x**2/b + 1)) + a*b**(3/2)*x/(
8*sqrt(a*x**2/b + 1)) - b**(5/2)/(x*sqrt(a*x**2/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx = -\frac{15}{16}\sqrt{a}b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right) - \sqrt{a+\frac{b}{x^2}}b^2 + \frac{9\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}ab^2 - 7\sqrt{a+\frac{b}{x^2}}a^2b^2}{8\left(\left(a+\frac{b}{x^2}\right)^2 - 2\left(a+\frac{b}{x^2}\right)a + a^2\right)}$$

input
```
integrate((a+b/x^2)^(5/2)*x^3,x, algorithm="maxima")
```

output
```
-15/16*sqrt(a)*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt
(a))) - sqrt(a + b/x^2)*b^2 + 1/8*(9*(a + b/x^2)^(3/2)*a*b^2 - 7*sqrt(a +
b/x^2)*a^2*b^2)/((a + b/x^2)^2 - 2*(a + b/x^2)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx = -\frac{15}{16}\sqrt{a}b^2\log\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2\right)\mathrm{sgn}(x) + \frac{2\sqrt{a}b^3\mathrm{sgn}(x)}{\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b} + \frac{1}{8}\left(2\,a^2x^2\mathrm{sgn}(x)+9\,ab\mathrm{sgn}(x)\right)\sqrt{ax^2+b}x$$

input
```
integrate((a+b/x^2)^(5/2)*x^3,x, algorithm="giac")
```

output
```
-15/16*sqrt(a)*b^2*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 2*sqrt(a)
*b^3*sgn(x)/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b) + 1/8*(2*a^2*x^2*sgn(x)
+ 9*a*b*sgn(x))*sqrt(a*x^2 + b)*x
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx = \frac{9\,a\,x^4\,\left(a+\frac{b}{x^2}\right)^{3/2}}{8} - b^2\sqrt{a+\frac{b}{x^2}} - \frac{7\,a^2\,x^4\,\sqrt{a+\frac{b}{x^2}}}{8} - \frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{8}$$

input
```
int(x^3*(a + b/x^2)^(5/2),x)
```

output
```
(9*a*x^4*(a + b/x^2)^(3/2))/8 - (a^(1/2)*b^2*atan(((a + b/x^2)^(1/2)*1i)/a
^(1/2))*15i)/8 - b^2*(a + b/x^2)^(1/2) - (7*a^2*x^4*(a + b/x^2)^(1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^3\,dx = \frac{2\sqrt{a\,x^2+b}\,a^2x^4+9\sqrt{a\,x^2+b}\,ab\,x^2-8\sqrt{a\,x^2+b}\,b^2+15\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)b^2x-10\sqrt{}}{8x}$$

input
```
int((a+b/x^2)^(5/2)*x^3,x)
```

output
```
(2*sqrt(a*x**2 + b)*a**2*x**4 + 9*sqrt(a*x**2 + b)*a*b*x**2 - 8*sqrt(a*x**
2 + b)*b**2 + 15*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b**2*
x - 10*sqrt(a)*b**2*x)/(8*x)
```

3.368 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2 \, dx = -\frac{b^2\sqrt{a+\frac{b}{x^2}}}{2x}+\frac{7}{3}ab\sqrt{a+\frac{b}{x^2}}x+\frac{1}{3}a^2\sqrt{a+\frac{b}{x^2}}x^3-\frac{5}{2}ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)$$

output
```
-1/2*b^2*(a+b/x^2)^(1/2)/x+7/3*a*b*(a+b/x^2)^(1/2)*x+1/3*a^2*(a+b/x^2)^(1/
2)*x^3-5/2*a*b^(3/2)*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2 \, dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-3b^2+14abx^2+2a^2x^4-\frac{15ab^{3/2}x^2\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b+ax^2}}\right)}{6x}$$

input
```
Integrate[(a + b/x^2)^(5/2)*x^2,x]
```

output
```
(Sqrt[a + b/x^2]*(-3*b^2 + 14*a*b*x^2 + 2*a^2*x^4 - (15*a*b^(3/2)*x^2*ArcT
anh[Sqrt[b + a*x^2]/Sqrt[b]])/Sqrt[b + a*x^2]))/(6*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 247, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^2}\right)^{5/2} dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^2}\right)^{5/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{3}b\int \left(a+\frac{b}{x^2}\right)^{3/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{3}b\left(3b\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x}-x\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)-x\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{3}x^3 \left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)-x\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{3}b\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)-x\left(a+\frac{b}{x^2}\right)^{3/2}\right)$$

input
```
Int[(a + b/x^2)^(5/2)*x^2,x]
```

output
```
((a + b/x^2)^(5/2)*x^3)/3 - (5*b*(-((a + b/x^2)^(3/2)*x) + 3*b*(Sqrt[a + b
/x^2]/(2*x) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*Sqrt[b]))))/3
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

method	result	si
risch	$-\frac{b^{2}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{2x}+\frac{\left(\frac{a^{2}x^{2}\sqrt{a\,x^{2}+b}}{3}+\frac{7ab\sqrt{a\,x^{2}+b}}{3}-\frac{5a\,b^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)}{2}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{\sqrt{a\,x^{2}+b}}$	10
default	$-\frac{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}x^{3}\left(-3\left(a\,x^{2}+b\right)^{\frac{5}{2}}a\,x^{2}+3\left(a\,x^{2}+b\right)^{\frac{7}{2}}-5\left(a\,x^{2}+b\right)^{\frac{3}{2}}ab\,x^{2}+15b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)a\,x^{2}-15\sqrt{a\,x^{2}+b}\,a\,b^{2}x^{2}\right)}{6\left(a\,x^{2}+b\right)^{\frac{5}{2}}b}$	12

input
```
int((a+b/x^2)^(5/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/x*((a*x^2+b)/x^2)^(1/2)+(1/3*a^2*x^2*(a*x^2+b)^(1/2)+7/3*a*b*(a*x
^2+b)^(1/2)-5/2*a*b^(3/2)*ln((2*b+2*b^(1/2)*(a*x^2+b)^(1/2))/x))*((a*x^2+b
)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.73

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx = \left[\frac{15\,ab^{\frac{3}{2}}x\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+2\,(2\,a^2x^4+14\,abx^2-3\,b^2)\sqrt{\frac{ax^2+b}{x^2}}}{12\,x}, \frac{15\,a\sqrt{-b}bx\,\mathrm{ar}\cdots}{\cdots}\right.$$

input
```
integrate((a+b/x^2)^(5/2)*x^2,x, algorithm="fricas")
```

output
```
[1/12*(15*a*b^(3/2)*x*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*
b)/x^2) + 2*(2*a^2*x^4 + 14*a*b*x^2 - 3*b^2)*sqrt((a*x^2 + b)/x^2))/x, 1/6
*(15*a*sqrt(-b)*b*x*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b) + (2*a^2*x^
4 + 14*a*b*x^2 - 3*b^2)*sqrt((a*x^2 + b)/x^2))/x]
```

Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx = \frac{a^2\sqrt{b}x^2\sqrt{\frac{ax^2}{b}+1}}{3} + \frac{7ab^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3} + \frac{5ab^{\frac{3}{2}}\log\left(\frac{ax^2}{b}\right)}{4} - \frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{2} - \frac{b^{\frac{5}{2}}\sqrt{\frac{ax^2}{b}+1}}{2x^2}$$

input
```
integrate((a+b/x**2)**(5/2)*x**2,x)
```

output
```
a**2*sqrt(b)*x**2*sqrt(a*x**2/b + 1)/3 + 7*a*b**(3/2)*sqrt(a*x**2/b + 1)/3
 + 5*a*b**(3/2)*log(a*x**2/b)/4 - 5*a*b**(3/2)*log(sqrt(a*x**2/b + 1) + 1)
/2 - b**(5/2)*sqrt(a*x**2/b + 1)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx = \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}} ax^3 + 2\sqrt{a+\frac{b}{x^2}}abx - \frac{\sqrt{a+\frac{b}{x^2}}ab^2x}{2\left(\left(a+\frac{b}{x^2}\right)x^2-b\right)} + \frac{5}{4}ab^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)$$

input
```
integrate((a+b/x^2)^(5/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*(a + b/x^2)^(3/2)*a*x^3 + 2*sqrt(a + b/x^2)*a*b*x - 1/2*sqrt(a + b/x^2
)*a*b^2*x/((a + b/x^2)*x^2 - b) + 5/4*a*b^(3/2)*log((sqrt(a + b/x^2)*x - s
qrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx = \frac{1}{6}\left(\frac{15\,b^2\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} + 2\left(ax^2+b\right)^{\frac{3}{2}}\operatorname{sgn}(x) + 12\sqrt{ax^2+b}\,b\operatorname{sgn}(x) - \frac{3\sqrt{ax^2}}{}\right.$$

input
```
integrate((a+b/x^2)^(5/2)*x^2,x, algorithm="giac")
```

output
```
1/6*(15*b^2*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 2*(a*x^2 +
b)^(3/2)*sgn(x) + 12*sqrt(a*x^2 + b)*b*sgn(x) - 3*sqrt(a*x^2 + b)*b^2*sgn(
x)/(a*x^2))*a
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx = \int x^2\left(a+\frac{b}{x^2}\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b/x^2)^(5/2),x)
```

output
```
int(x^2*(a + b/x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2\,dx = \frac{2\sqrt{a\,x^2+b}\,a^2x^4 + 14\sqrt{a\,x^2+b}\,ab\,x^2 - 3\sqrt{a\,x^2+b}\,b^2 + 15\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)ab\,x^2}{6x^2}$$

input
```
int((a+b/x^2)^(5/2)*x^2,x)
```

output
```
(2*sqrt(a*x**2 + b)*a**2*x**4 + 14*sqrt(a*x**2 + b)*a*b*x**2 - 3*sqrt(a*x*
*2 + b)*b**2 + 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqr
t(b))*a*b*x**2 - 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/s
qrt(b))*a*b*x**2)/(6*x**2)
```

3.369 $\int \left(a+\frac{b}{x^2}\right)^{5/2} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x \, dx = -2ab\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}b\left(a+\frac{b}{x^2}\right)^{3/2} + \frac{1}{2}a^2\sqrt{a+\frac{b}{x^2}}x^2 + \frac{5}{2}a^{3/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

output
```
-2*a*b*(a+b/x^2)^(1/2)-1/3*b*(a+b/x^2)^(3/2)+1/2*a^2*(a+b/x^2)^(1/2)*x^2+5
/2*a^(3/2)*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x \, dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-2b^2-14abx^2+3a^2x^4+\frac{30a^{3/2}bx^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{\sqrt{b+ax^2}}\right)}{6x^2}$$

input
```
Integrate[(a + b/x^2)^(5/2)*x,x]
```

output
```
(Sqrt[a + b/x^2]*(-2*b^2 - 14*a*b*x^2 + 3*a^2*x^4 + (30*a^(3/2)*b*x^3*ArcT
anh[(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b + a*x^2])])/Sqrt[b + a*x^2]))/(6*x^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {798, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)^{5/2}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int\left(a+\frac{b}{x^2}\right)^{5/2}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{2}b\int\left(a+\frac{b}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{2}b\left(a\int\sqrt{a+\frac{b}{x^2}}x^2d\frac{1}{x^2}+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{2}b\left(a\left(a\int\frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+2\sqrt{a+\frac{b}{x^2}}\right)+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{2}b\left(a\left(\frac{2a\int\frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{b}+2\sqrt{a+\frac{b}{x^2}}\right)+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(x^2\left(a+\frac{b}{x^2}\right)^{5/2}-\frac{5}{2}b\left(a\left(2\sqrt{a+\frac{b}{x^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)\right)$$

input
```
Int[(a + b/x^2)^(5/2)*x,x]
```

output
```
((a + b/x^2)^(5/2)*x^2 - (5*b*((2*(a + b/x^2)^(3/2))/3 + a*(2*Sqrt[a + b/x
^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\left(3a^{2}x^{4}-14ab\,x^{2}-2b^{2}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}{6x^{2}}+\frac{5a^{\frac{3}{2}}b\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}{2\sqrt{a\,x^{2}+b}}$
default	$-\frac{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}x^{2}\left(-8\left(a\,x^{2}+b\right)^{\frac{5}{2}}a^{\frac{5}{2}}x^{4}+8\left(a\,x^{2}+b\right)^{\frac{7}{2}}a^{\frac{3}{2}}x^{2}-10\left(a\,x^{2}+b\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\,x^{4}-15\sqrt{a\,x^{2}+b}\,a^{\frac{5}{2}}b^{2}x^{4}+2\left(a\,x^{2}+b\right)^{\frac{7}{2}}b\sqrt{a}-15\ln\left(\sqrt{a}\right.}{6\left(a\,x^{2}+b\right)^{\frac{5}{2}}b^{2}\sqrt{a}}$

input
```
int((a+b/x^2)^(5/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*a^2*x^4-14*a*b*x^2-2*b^2)/x^2*((a*x^2+b)/x^2)^(1/2)+5/2*a^(3/2)*b*l
n(a^(1/2)*x+(a*x^2+b)^(1/2))*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int\left(a+\frac{b}{x^2}\right)^{5/2}x\,dx=\left[\frac{15\,a^{\frac{3}{2}}bx^{2}\log\left(-2\,ax^{2}-2\,\sqrt{a}x^{2}\sqrt{\frac{ax^{2}+b}{x^{2}}}-b\right)+2\left(3\,a^{2}x^{4}-14\,abx^{2}-2\,b^{2}\right)\sqrt{\frac{ax^{2}+b}{x^{2}}}}{12\,x^{2}},\right.$$
$$\left.-\frac{15\sqrt{-a}abx^{2}\arctan\left(\frac{\sqrt{-a}x^{2}\sqrt{\frac{ax^{2}+b}{x^{2}}}}{ax^{2}+b}\right)-\left(3\,a^{2}x^{4}-14\,abx^{2}-2\,b^{2}\right)\sqrt{\frac{ax^{2}+b}{x^{2}}}}{6\,x^{2}}\right]$$

input
```
integrate((a+b/x^2)^(5/2)*x,x, algorithm="fricas")
```

output
```
[1/12*(15*a^(3/2)*b*x^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2)
 - b) + 2*(3*a^2*x^4 - 14*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^2, -1/
6*(15*sqrt(-a)*a*b*x^2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 +
b)) - (3*a^2*x^4 - 14*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^2]
```

Sympy [A] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx = \frac{a^{\frac{5}{2}}x^2\sqrt{1+\frac{b}{ax^2}}}{2} - \frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^2}}}{3} - \frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax^2}\right)}{4} + \frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)}{2} - \frac{\sqrt{a}b^2\sqrt{1+\frac{b}{ax^2}}}{3x^2}$$

input
```
integrate((a+b/x**2)**(5/2)*x,x)
```

output
```
a**(5/2)*x**2*sqrt(1 + b/(a*x**2))/2 - 7*a**(3/2)*b*sqrt(1 + b/(a*x**2))/3
 - 5*a**(3/2)*b*log(b/(a*x**2))/4 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x**2))
+ 1)/2 - sqrt(a)*b**2*sqrt(1 + b/(a*x**2))/(3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx = \frac{1}{2}\sqrt{a+\frac{b}{x^2}}a^2x^2 - \frac{5}{4}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right) - \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b - 2\sqrt{a+\frac{b}{x^2}}ab$$

input
```
integrate((a+b/x^2)^(5/2)*x,x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^2)*a^2*x^2 - 5/4*a^(3/2)*b*log((sqrt(a + b/x^2) - sqrt(a)
)/(sqrt(a + b/x^2) + sqrt(a))) - 1/3*(a + b/x^2)^(3/2)*b - 2*sqrt(a + b/x^
2)*a*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(63) = 126$.

Time = 0.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.75

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx = \frac{1}{2}\sqrt{ax^2+b}a^2x\operatorname{sgn}(x) - \frac{5}{4}a^{\frac{3}{2}}b\log\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(9\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4a^{\frac{3}{2}}b^2\operatorname{sgn}(x) - 12\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2a^{\frac{3}{2}}b^3\operatorname{sgn}(x) + 7a^{\frac{3}{2}}b^4\operatorname{sgn}(x)\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^3}$$

input
```
integrate((a+b/x^2)^(5/2)*x,x, algorithm="giac")
```

output
```
1/2*sqrt(a*x^2 + b)*a^2*x*sgn(x) - 5/4*a^(3/2)*b*log((sqrt(a)*x - sqrt(a*x
^2 + b))^2)*sgn(x) + 2/3*(9*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^(3/2)*b^2*sg
n(x) - 12*(sqrt(a)*x - sqrt(a*x^2 + b))^2*a^(3/2)*b^3*sgn(x) + 7*a^(3/2)*b
^4*sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^3
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx = \frac{a^2\,x^2\,\sqrt{a+\frac{b}{x^2}}}{2} - \frac{b\,{\left(a+\frac{b}{x^2}\right)}^{3/2}}{3} - 2\,a\,b\,\sqrt{a+\frac{b}{x^2}} - \frac{a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}$$

input
```
int(x*(a + b/x^2)^(5/2),x)
```

output
```
(a^2*x^2*(a + b/x^2)^(1/2))/2 - (b*(a + b/x^2)^(3/2))/3 - (a^(3/2)*b*atan(
((a + b/x^2)^(1/2)*1i)/a^(1/2))*5i)/2 - 2*a*b*(a + b/x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} x\,dx = \frac{6\sqrt{a\,x^2+b}\,a^2x^4-28\sqrt{a\,x^2+b}\,ab\,x^2-4\sqrt{a\,x^2+b}\,b^2+30\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)ab\,x^3+5\sqrt{}}{12x^3}$$

input
```
int((a+b/x^2)^(5/2)*x,x)
```

output
```
(6*sqrt(a*x**2 + b)*a**2*x**4 - 28*sqrt(a*x**2 + b)*a*b*x**2 - 4*sqrt(a*x*
*2 + b)*b**2 + 30*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*a*b*
x**3 + 5*sqrt(a)*a*b*x**3)/(12*x**3)
```

3.370 $\int \left(a+\frac{b}{x^2}\right)^{5/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx = -\frac{b^2\sqrt{a+\frac{b}{x^2}}}{4x^3} - \frac{9ab\sqrt{a+\frac{b}{x^2}}}{8x} + a^2\sqrt{a+\frac{b}{x^2}}x - \frac{15}{8}a^2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)$$

output

```
-1/4*b^2*(a+b/x^2)^(1/2)/x^3-9/8*a*b*(a+b/x^2)^(1/2)/x+a^2*(a+b/x^2)^(1/2)
*x-15/8*a^2*b^(1/2)*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx = -\frac{\sqrt{a+\frac{b}{x^2}}\left(\sqrt{b+ax^2}(2b^2+9abx^2-8a^2x^4)+15a^2\sqrt{b}x^4\operatorname{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)\right)}{8x^3\sqrt{b+ax^2}}$$

input
```
Integrate[(a + b/x^2)^(5/2),x]
```

output
```
-1/8*(Sqrt[a + b/x^2]*(Sqrt[b + a*x^2]*(2*b^2 + 9*a*b*x^2 - 8*a^2*x^4) + 1
5*a^2*Sqrt[b]*x^4*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]]))/(x^3*Sqrt[b + a*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {773, 247, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx$$

$$\Big\downarrow \ 773$$

$$-\int \left(a+\frac{b}{x^2}\right)^{5/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \ 247$$

$$x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\int \left(a+\frac{b}{x^2}\right)^{3/2} d\frac{1}{x}$$

$$\Big\downarrow \ 211$$

$$x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x}+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right)$$

$$\Big\downarrow \ 211$$

$$x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right)$$

$$\Big\downarrow \ 224$$

$$x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right)$$

$$\Big\downarrow \text{219}$$

$$x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right)$$

input
```
Int[(a + b/x^2)^(5/2),x]
```

output
```
(a + b/x^2)^(5/2)*x - 5*b*((a + b/x^2)^(3/2)/(4*x) + (3*a*(Sqrt[a + b/x^2]
/(2*x) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*Sqrt[b])))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{b(9a\,x^2+2b)\sqrt{\frac{a\,x^2+b}{x^2}}}{8x^3}+\frac{\left(-\frac{15\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^2+b}}{x}\right)\sqrt{b}\,a^2}{8}+\sqrt{a\,x^2+b}\,a^2\right)\sqrt{\frac{a\,x^2+b}{x^2}}\,x}{\sqrt{a\,x^2+b}}$
default	$-\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}x\left(-3(a\,x^2+b)^{\frac{5}{2}}a^2x^4+15\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^2+b}}{x}\right)b^{\frac{5}{2}}a^2x^4+3(a\,x^2+b)^{\frac{7}{2}}a\,x^2-5(a\,x^2+b)^{\frac{3}{2}}a^2b\,x^4-15\sqrt{a\,x^2+b}\,a^2b^2x^4+\right.}{8(a\,x^2+b)^{\frac{5}{2}}b^2}$

input
```
int((a+b/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*b*(9*a*x^2+2*b)/x^3*((a*x^2+b)/x^2)^(1/2)+(-15/8*ln((2*b+2*b^(1/2)*(a
*x^2+b)^(1/2))/x)*b^(1/2)*a^2+(a*x^2+b)^(1/2)*a^2)*((a*x^2+b)/x^2)^(1/2)*x
/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.82

$$\int \left(a+\frac{b}{x^2}\right)^{5/2}\,dx=\left[\frac{15\,a^2\sqrt{b}x^3\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+2\,(8\,a^2x^4-9\,abx^2-2\,b^2)\sqrt{\frac{ax^2+b}{x^2}}}{16\,x^3},\ \frac{15\,a^2\sqrt{-b}x^3\,\mathrm{ar}}{}\right.$$

input
```
integrate((a+b/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(15*a^2*sqrt(b)*x^3*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2)
+ 2*b)/x^2) + 2*(8*a^2*x^4 - 9*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^3
, 1/8*(15*a^2*sqrt(-b)*x^3*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b) + (8
*a^2*x^4 - 9*a*b*x^2 - 2*b^2)*sqrt((a*x^2 + b)/x^2))/x^3]
```

Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.29

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx = \frac{a^{\frac{5}{2}}x}{\sqrt{1+\frac{b}{ax^2}}} - \frac{a^{\frac{3}{2}}b}{8x\sqrt{1+\frac{b}{ax^2}}} - \frac{11\sqrt{a}b^2}{8x^3\sqrt{1+\frac{b}{ax^2}}} - \frac{15a^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{8} - \frac{b^3}{4\sqrt{a}x^5\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate((a+b/x**2)**(5/2),x)
```

output
```
a**(5/2)*x/sqrt(1 + b/(a*x**2)) - a**(3/2)*b/(8*x*sqrt(1 + b/(a*x**2))) -
11*sqrt(a)*b**2/(8*x**3*sqrt(1 + b/(a*x**2))) - 15*a**2*sqrt(b)*asinh(sqrt
(b)/(sqrt(a)*x))/8 - b**3/(4*sqrt(a)*x**5*sqrt(1 + b/(a*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx = \sqrt{a+\frac{b}{x^2}}a^2x + \frac{15}{16}a^2\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right) - \frac{9\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2bx^3 - 7\sqrt{a+\frac{b}{x^2}}a^2b^2x}{8\left(\left(a+\frac{b}{x^2}\right)^2x^4 - 2\left(a+\frac{b}{x^2}\right)bx^2 + b^2\right)}$$

input
```
integrate((a+b/x^2)^(5/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x^2)*a^2*x + 15/16*a^2*sqrt(b)*log((sqrt(a + b/x^2)*x - sqrt(b)
)/(sqrt(a + b/x^2)*x + sqrt(b))) - 1/8*(9*(a + b/x^2)^(3/2)*a^2*b*x^3 - 7*
sqrt(a + b/x^2)*a^2*b^2*x)/((a + b/x^2)^2*x^4 - 2*(a + b/x^2)*b*x^2 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx = \frac{\frac{15\,a^3b\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}+8\,\sqrt{ax^2+b}a^3\mathrm{sgn}(x)-\frac{9\,(ax^2+b)^{\frac{3}{2}}a^3b\mathrm{sgn}(x)-7\,\sqrt{ax^2+b}a^3b^2\mathrm{sgn}(x)}{a^2x^4}}{8\,a}$$

input
```
integrate((a+b/x^2)^(5/2),x, algorithm="giac")
```

output
```
1/8*(15*a^3*b*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) + 8*sqrt(a*
x^2 + b)*a^3*sgn(x) - (9*(a*x^2 + b)^(3/2)*a^3*b*sgn(x) - 7*sqrt(a*x^2 + b
)*a^3*b^2*sgn(x))/(a^2*x^4))/a
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx = \frac{x\,(a\,x^2+b)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b}{a\,x^2}\right)}{\left(\frac{b}{a}+x^2\right)^{5/2}}$$

input
```
int((a + b/x^2)^(5/2),x)
```

output
```
(x*(b + a*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -b/(a*x^2)))/(b/a + x^2)
^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)^{5/2} dx = \frac{8\sqrt{a\,x^2+b}\,a^2x^4 - 9\sqrt{a\,x^2+b}\,ab\,x^2 - 2\sqrt{a\,x^2+b}\,b^2 + 15\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a^2x^4 - 15}{8x^4}$$

input
```
int((a+b/x^2)^(5/2),x)
```

output
```
(8*sqrt(a*x**2 + b)*a**2*x**4 - 9*sqrt(a*x**2 + b)*a*b*x**2 - 2*sqrt(a*x**
2 + b)*b**2 + 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt
(b))*a**2*x**4 - 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/s
qrt(b))*a**2*x**4)/(8*x**4)
```

3.371 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x} dx = -a^2\sqrt{a+\frac{b}{x^2}} - \frac{1}{3}a\left(a+\frac{b}{x^2}\right)^{3/2} - \frac{1}{5}\left(a+\frac{b}{x^2}\right)^{5/2} + a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)$$

output
```
-a^2*(a+b/x^2)^(1/2)-1/3*a*(a+b/x^2)^(3/2)-1/5*(a+b/x^2)^(5/2)+a^(5/2)*arc
tanh((a+b/x^2)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.24

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x} dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-3b^2-11abx^2-23a^2x^4+\frac{30a^{5/2}x^5\text{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{\sqrt{b+ax^2}}\right)}{15x^4}$$

input
```
Integrate[(a + b/x^2)^(5/2)/x,x]
```

output
```
(Sqrt[a + b/x^2]*(-3*b^2 - 11*a*b*x^2 - 23*a^2*x^4 + (30*a^(5/2)*x^5*ArcTa
nh[(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b + a*x^2])])/Sqrt[b + a*x^2]))/(15*x^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{5/2}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\int \left(a+\frac{b}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}-\frac{2}{5}\left(a+\frac{b}{x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(a\int \sqrt{a+\frac{b}{x^2}}x^2d\frac{1}{x^2}+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(a\left(a\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+2\sqrt{a+\frac{b}{x^2}}\right)+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-a\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{b}+2\sqrt{a+\frac{b}{x^2}}\right)+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^2}\right)^{5/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-a\left(a\left(2\sqrt{a+\frac{b}{x^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x^2}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^2}\right)^{5/2}\right)$$

input
```
Int[(a + b/x^2)^(5/2)/x,x]
```

output
```
((-2*(a + b/x^2)^(5/2))/5 - a*((2*(a + b/x^2)^(3/2))/3 + a*(2*Sqrt[a + b/x
^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{(23a^2x^4+11ab\,x^2+3b^2)\sqrt{\frac{a\,x^2+b}{x^2}}}{15x^4}+\frac{a^{\frac{5}{2}}\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)\sqrt{\frac{a\,x^2+b}{x^2}}\,x}{\sqrt{a\,x^2+b}}$
default	$-\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\left(-8(a\,x^2+b)^{\frac{5}{2}}a^{\frac{7}{2}}x^6+8(a\,x^2+b)^{\frac{7}{2}}a^{\frac{5}{2}}x^4-10(a\,x^2+b)^{\frac{3}{2}}a^{\frac{7}{2}}b\,x^6-15\sqrt{a\,x^2+b}\,a^{\frac{7}{2}}b^2x^6+2(a\,x^2+b)^{\frac{7}{2}}a^{\frac{3}{2}}b\,x^2-15\ln\left(\sqrt{a}\right.\right.}{15(a\,x^2+b)^{\frac{5}{2}}b^3\sqrt{a}}$

input
```
int((a+b/x^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(23*a^2*x^4+11*a*b*x^2+3*b^2)/x^4*((a*x^2+b)/x^2)^(1/2)+a^(5/2)*ln(a
^(1/2)*x+(a*x^2+b)^(1/2))*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.35

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx = \left[\frac{15\,a^{\frac{5}{2}}x^4\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-2\,(23\,a^2x^4+11\,abx^2+3\,b^2)\sqrt{\frac{ax^2+b}{x^2}}}{30\,x^4}, -\frac{15\,\sqrt{-a}a^2x^4\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+(23\,a^2x^4+11\,abx^2+3\,b^2)\sqrt{\frac{ax^2+b}{x^2}}}{15\,x^4}\right]$$

input
```
integrate((a+b/x^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/30*(15*a^(5/2)*x^4*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) -
 b) - 2*(23*a^2*x^4 + 11*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2))/x^4, -1/1
5*(15*sqrt(-a)*a^2*x^4*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 +
b)) + (23*a^2*x^4 + 11*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2))/x^4]
```

Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx = -\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^2}}}{15} - \frac{a^{\frac{5}{2}}\log\left(\frac{b}{ax^2}\right)}{2} + a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right) - \frac{11a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^2}}}{15x^2} - \frac{\sqrt{a}b^2\sqrt{1+\frac{b}{ax^2}}}{5x^4}$$

input
```
integrate((a+b/x**2)**(5/2)/x,x)
```

output
```
-23*a**(5/2)*sqrt(1 + b/(a*x**2))/15 - a**(5/2)*log(b/(a*x**2))/2 + a**(5/
2)*log(sqrt(1 + b/(a*x**2)) + 1) - 11*a**(3/2)*b*sqrt(1 + b/(a*x**2))/(15*
x**2) - sqrt(a)*b**2*sqrt(1 + b/(a*x**2))/(5*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx = -\frac{1}{2}a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right) - \frac{1}{5}\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}} - \frac{1}{3}\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a - \sqrt{a+\frac{b}{x^2}}\,a^2$$

input
```
integrate((a+b/x^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
-1/2*a^(5/2)*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))
- 1/5*(a + b/x^2)^(5/2) - 1/3*(a + b/x^2)^(3/2)*a - sqrt(a + b/x^2)*a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(56) = 112$.

Time = 0.76 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx = -\frac{1}{2}a^{\frac{5}{2}}\log\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2\right)\mathrm{sgn}(x) + \frac{2\left(45\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^8 a^{\frac{5}{2}}b\mathrm{sgn}(x) - 90\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^6 a^{\frac{5}{2}}b^2\mathrm{sgn}(x) + 140\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4 a^{\frac{5}{2}}b^3\right.}{15\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^5}$$

input
```
integrate((a+b/x^2)^(5/2)/x,x, algorithm="giac")
```

output
```
-1/2*a^(5/2)*log((sqrt(a)*x - sqrt(a*x^2 + b))^2)*sgn(x) + 2/15*(45*(sqrt(
a)*x - sqrt(a*x^2 + b))^8*a^(5/2)*b*sgn(x) - 90*(sqrt(a)*x - sqrt(a*x^2 +
b))^6*a^(5/2)*b^2*sgn(x) + 140*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^(5/2)*b^3
*sgn(x) - 70*(sqrt(a)*x - sqrt(a*x^2 + b))^2*a^(5/2)*b^4*sgn(x) + 23*a^(5/
2)*b^5*sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^5
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx = -\frac{a\left(a+\frac{b}{x^2}\right)^{3/2}}{3} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5} - a^2\sqrt{a+\frac{b}{x^2}} - a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

input
```
int((a + b/x^2)^(5/2)/x,x)
```

output
```
- a^(5/2)*atan(((a + b/x^2)^(1/2)*1i)/a^(1/2))*1i - (a*(a + b/x^2)^(3/2))/
3 - (a + b/x^2)^(5/2)/5 - a^2*(a + b/x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.24

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x}\,dx = \frac{-23\sqrt{a\,x^2+b}\,a^2x^4-11\sqrt{a\,x^2+b}\,ab\,x^2-3\sqrt{a\,x^2+b}\,b^2+15\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)a^2}{15x^5}$$

input

```
int((a+b/x^2)^(5/2)/x,x)
```

output

```
( - 23*sqrt(a*x**2 + b)*a**2*x**4 - 11*sqrt(a*x**2 + b)*a*b*x**2 - 3*sqrt(
a*x**2 + b)*b**2 + 15*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*
a**2*x**5 + 5*sqrt(a)*a**2*x**5)/(15*x**5)
```

3.372 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2} dx = -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{16x}-\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{24x}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}-\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{16\sqrt{b}}$$

output
```
-5/16*a^2*(a+b/x^2)^(1/2)/x-5/24*a*(a+b/x^2)^(3/2)/x-1/6*(a+b/x^2)^(5/2)/x
-5/16*a^3*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2} dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-8b^2-26abx^2-33a^2x^4-\frac{15a^3x^6\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+ax^2}}\right)}{48x^5}$$

input
```
Integrate[(a + b/x^2)^(5/2)/x^2,x]
```

output
```
(Sqrt[a + b/x^2]*(-8*b^2 - 26*a*b*x^2 - 33*a^2*x^4 - (15*a^3*x^6*ArcTanh[S
qrt[b + a*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + a*x^2])))/(48*x^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2} dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^2}\right)^{5/2} d\frac{1}{x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{6} a \int \left(a+\frac{b}{x^2}\right)^{3/2} d\frac{1}{x} - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{6} a \left(\frac{3}{4} a \int \sqrt{a+\frac{b}{x^2}} d\frac{1}{x} + \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right) - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{6} a \left(\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{\sqrt{a+\frac{b}{x^2}}} d\frac{1}{x} + \frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right) + \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right) - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}$$

$$\Big\downarrow \text{224}$$

$$-\frac{5}{6} a \left(\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{1-\frac{b}{x^2}} d\frac{1}{\sqrt{a+\frac{b}{x^2}} x} + \frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right) + \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right) - \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x}$$

input
```
Int[(a + b/x^2)^(5/2)/x^2,x]
```

output
```
-1/6*(a + b/x^2)^(5/2)/x - (5*a*((a + b/x^2)^(3/2)/(4*x) + (3*a*(Sqrt[a +
b/x^2]/(2*x) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*Sqrt[b])))/4))/
6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(33a^2x^4+26ab\,x^2+8b^2)\sqrt{\frac{a\,x^2+b}{x^2}}}{48x^5}-\frac{5a^3\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^2+b}}{x}\right)\sqrt{\frac{a\,x^2+b}{x^2}}\,x}{16\sqrt{b}\,\sqrt{a\,x^2+b}}$
default	$-\frac{\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}\left(-3(a\,x^2+b)^{\frac{5}{2}}a^3x^6+15b^{\frac{5}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^2+b}}{x}\right)a^3x^6+3(a\,x^2+b)^{\frac{7}{2}}a^2x^4-5(a\,x^2+b)^{\frac{3}{2}}a^3b\,x^6-15\sqrt{a\,x^2+b}\,a^3b^2x^6+\right.}{48x(a\,x^2+b)^{\frac{5}{2}}b^3}$

input
```
int((a+b/x^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(33*a^2*x^4+26*a*b*x^2+8*b^2)/x^5*((a*x^2+b)/x^2)^(1/2)-5/16*a^3/b^(
1/2)*ln((2*b+2*b^(1/2)*(a*x^2+b)^(1/2))/x)*((a*x^2+b)/x^2)^(1/2)*x/(a*x^2+
b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx = \left[\frac{15\,a^3\sqrt{b}x^5\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\,(33\,a^2bx^4+26\,ab^2x^2+8\,b^3)\sqrt{\frac{ax^2+b}{x^2}}}{96\,bx^5}, \frac{15\,a^3}{\quad}\right.$$

input
```
integrate((a+b/x^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[1/96*(15*a^3*sqrt(b)*x^5*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2)
+ 2*b)/x^2) - 2*(33*a^2*b*x^4 + 26*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2
))/(b*x^5), 1/48*(15*a^3*sqrt(-b)*x^5*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x
^2)/b) - (33*a^2*b*x^4 + 26*a*b^2*x^2 + 8*b^3)*sqrt((a*x^2 + b)/x^2))/(b*x
^5)]
```

Sympy [A] (verification not implemented)

Time = 2.87 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx = -\frac{11a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^2}}}{16x} - \frac{13a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^2}}}{24x^3} - \frac{\sqrt{a}b^2\sqrt{1+\frac{b}{ax^2}}}{6x^5} - \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{16\sqrt{b}}$$

input
```
integrate((a+b/x**2)**(5/2)/x**2,x)
```

output
```
-11*a**(5/2)*sqrt(1 + b/(a*x**2))/(16*x) - 13*a**(3/2)*b*sqrt(1 + b/(a*x**
2))/(24*x**3) - sqrt(a)*b**2*sqrt(1 + b/(a*x**2))/(6*x**5) - 5*a**3*asinh(
sqrt(b)/(sqrt(a)*x))/(16*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(72) = 144.

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.65

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx = \frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{32\,\sqrt{b}} - \frac{33\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3x^5 - 40\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^3bx^3 + 15\,\sqrt{a+\frac{b}{x^2}}a^3b^2x}{48\left(\left(a+\frac{b}{x^2}\right)^3x^6 - 3\left(a+\frac{b}{x^2}\right)^2bx^4 + 3\left(a+\frac{b}{x^2}\right)b^2x^2 - b^3\right)}$$

input
```
integrate((a+b/x^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
5/32*a^3*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/
sqrt(b) - 1/48*(33*(a + b/x^2)^(5/2)*a^3*x^5 - 40*(a + b/x^2)^(3/2)*a^3*b*
x^3 + 15*sqrt(a + b/x^2)*a^3*b^2*x)/((a + b/x^2)^3*x^6 - 3*(a + b/x^2)^2*b
*x^4 + 3*(a + b/x^2)*b^2*x^2 - b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx = \frac{1}{48}\left(\frac{15\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{33\left(ax^2+b\right)^{\frac{5}{2}}\operatorname{sgn}(x) - 40\left(ax^2+b\right)^{\frac{3}{2}}b\operatorname{sgn}(x) + 15\cdots}{a^3x^6}\right.$$

input
```
integrate((a+b/x^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/48*(15*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - (33*(a*x^2 + b
)^(5/2)*sgn(x) - 40*(a*x^2 + b)^(3/2)*b*sgn(x) + 15*sqrt(a*x^2 + b)*b^2*sg
n(x))/(a^3*x^6))*a^3
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx = -\frac{\left(a\,x^2+b\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b}{a\,x^2}\right)}{x\,\left(\frac{b}{a}+x^2\right)^{5/2}}$$

input
```
int((a + b/x^2)^(5/2)/x^2,x)
```

output
```
-((b + a*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -b/(a*x^2)))/(x*(b/a + x^2
)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}\,dx = \frac{-33\sqrt{a\,x^2+b}\,a^2b\,x^4 - 26\sqrt{a\,x^2+b}\,a\,b^2x^2 - 8\sqrt{a\,x^2+b}\,b^3 + 15\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{\cdots}}{\sqrt{b}}\right.\cdots}{48b\,x^6}$$

input
```
int((a+b/x^2)^(5/2)/x^2,x)
```

output
```
( - 33*sqrt(a*x**2 + b)*a**2*b*x**4 - 26*sqrt(a*x**2 + b)*a*b**2*x**2 - 8*
sqrt(a*x**2 + b)*b**3 + 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqr
t(b))/sqrt(b))*a**3*x**6 - 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x +
sqrt(b))/sqrt(b))*a**3*x**6)/(48*b*x**6)
```

3.373 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b}$$

output
```
-1/7*(a+b/x^2)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{5/2} (b+ax^2)}{7bx^2}$$

input
```
Integrate[(a + b/x^2)^(5/2)/x^3,x]
```

output
```
-1/7*((a + b/x^2)^(5/2)*(b + a*x^2))/(b*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b}$$

input
```
Int[(a + b/x^2)^(5/2)/x^3,x]
```

output
```
-1/7*(a + b/x^2)^(7/2)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\left(a+\frac{b}{x^{2}}\right)^{\frac{7}{2}}}{7b}$	15
orering	$-\frac{\left(a\,x^{2}+b\right)\left(a+\frac{b}{x^{2}}\right)^{\frac{5}{2}}}{7x^{2}b}$	25
gosper	$-\frac{\left(a\,x^{2}+b\right)\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}{7x^{2}b}$	29
default	$-\frac{\left(a\,x^{2}+b\right)\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}{7x^{2}b}$	29
risch	$-\frac{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,\left(a^{3}x^{6}+3a^{2}b\,x^{4}+3a\,b^{2}x^{2}+b^{3}\right)}{7x^{6}b}$	51
trager	$-\frac{\left(a^{3}x^{6}+3a^{2}b\,x^{4}+3a\,b^{2}x^{2}+b^{3}\right)\sqrt{-\frac{-a\,x^{2}-b}{x^{2}}}}{7x^{6}b}$	55

input
```
int((a+b/x^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(a+b/x^2)^(7/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx=-\frac{\left(a^3x^6+3\,a^2bx^4+3\,ab^2x^2+b^3\right)\sqrt{\frac{ax^2+b}{x^2}}}{7\,bx^6}$$

input
```
integrate((a+b/x^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
-1/7*(a^3*x^6 + 3*a^2*b*x^4 + 3*a*b^2*x^2 + b^3)*sqrt((a*x^2 + b)/x^2)/(b*
x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.

Time = 0.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx = \begin{cases} -\frac{a^3\sqrt{a+\frac{b}{x^2}}}{7b} - \frac{3a^2\sqrt{a+\frac{b}{x^2}}}{7x^2} - \frac{3ab\sqrt{a+\frac{b}{x^2}}}{7x^4} - \frac{b^2\sqrt{a+\frac{b}{x^2}}}{7x^6} & \text{for } b \neq 0 \\ -\frac{a^{\frac{5}{2}}}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)**(5/2)/x**3,x)
```

output
```
Piecewise((-a**3*sqrt(a + b/x**2)/(7*b) - 3*a**2*sqrt(a + b/x**2)/(7*x**2)
 - 3*a*b*sqrt(a + b/x**2)/(7*x**4) - b**2*sqrt(a + b/x**2)/(7*x**6), Ne(b,
 0)), (-a**(5/2)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}}{7\,b}$$

input
```
integrate((a+b/x^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
-1/7*(a + b/x^2)^(7/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(14) = 28.

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx = \frac{2\left(7\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^{12}a^{\frac{7}{2}}\mathrm{sgn}(x)+35\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^{8}a^{\frac{7}{2}}b^2\mathrm{sgn}(x)+21\left(\sqrt{a}x-\right.\right.}{7\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^7}$$

input
```
integrate((a+b/x^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
2/7*(7*(sqrt(a)*x - sqrt(a*x^2 + b))^12*a^(7/2)*sgn(x) + 35*(sqrt(a)*x - s
qrt(a*x^2 + b))^8*a^(7/2)*b^2*sgn(x) + 21*(sqrt(a)*x - sqrt(a*x^2 + b))^4*
a^(7/2)*b^4*sgn(x) + a^(7/2)*b^6*sgn(x))/((sqrt(a)*x - sqrt(a*x^2 + b))^2
- b)^7
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx = -\frac{a^3\sqrt{a+\frac{b}{x^2}}}{7\,b}-\frac{3\,a^2\sqrt{a+\frac{b}{x^2}}}{7\,x^2}-\frac{b^2\sqrt{a+\frac{b}{x^2}}}{7\,x^6}-\frac{3\,a\,b\sqrt{a+\frac{b}{x^2}}}{7\,x^4}$$

input
```
int((a + b/x^2)^(5/2)/x^3,x)
```

output
```
- (a^3*(a + b/x^2)^(1/2))/(7*b) - (3*a^2*(a + b/x^2)^(1/2))/(7*x^2) - (b^2
*(a + b/x^2)^(1/2))/(7*x^6) - (3*a*b*(a + b/x^2)^(1/2))/(7*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.56

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^3}\,dx = \frac{-\sqrt{a\,x^2+b}\,a^3x^6-3\sqrt{a\,x^2+b}\,a^2b\,x^4-3\sqrt{a\,x^2+b}\,a\,b^2x^2-\sqrt{a\,x^2+b}\,b^3-\sqrt{a}\,a^3x^7}{7b\,x^7}$$

input
```
int((a+b/x^2)^(5/2)/x^3,x)
```

output
```
( - sqrt(a*x**2 + b)*a**3*x**6 - 3*sqrt(a*x**2 + b)*a**2*b*x**4 - 3*sqrt(a
*x**2 + b)*a*b**2*x**2 - sqrt(a*x**2 + b)*b**3 - sqrt(a)*a**3*x**7)/(7*b*x
**7)
```

3.374 $\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx = -\frac{5a^2\sqrt{a+\frac{b}{x^2}}}{64x^3}-\frac{5a\left(a+\frac{b}{x^2}\right)^{3/2}}{48x^3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}-\frac{5a^3\sqrt{a+\frac{b}{x^2}}}{128bx}+\frac{5a^4\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{128b^{3/2}}$$

output

```
-5/64*a^2*(a+b/x^2)^(1/2)/x^3-5/48*a*(a+b/x^2)^(3/2)/x^3-1/8*(a+b/x^2)^(5/
2)/x^3-5/128*a^3*(a+b/x^2)^(1/2)/b/x+5/128*a^4*arctanh(b^(1/2)/(a+b/x^2)^(
1/2)/x)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\left(-\sqrt{b}(48b^3+136ab^2x^2+118a^2bx^4+15a^3x^6)+\frac{15a^4x^8\operatorname{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b+ax^2}}\right)}{384b^{3/2}x^7}$$

input
```
Integrate[(a + b/x^2)^(5/2)/x^4,x]
```

output
```
(Sqrt[a + b/x^2]*(-(Sqrt[b]*(48*b^3 + 136*a*b^2*x^2 + 118*a^2*b*x^4 + 15*a
^3*x^6)) + (15*a^4*x^8*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]])/Sqrt[b + a*x^2]))
/(384*b^(3/2)*x^7)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {858, 248, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{248}$$

$$-\frac{5}{8}a\int \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{x^2}d\frac{1}{x}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

$$\Big\downarrow \text{248}$$

$$-\frac{5}{8}a\left(\frac{1}{2}a\int \frac{\sqrt{a+\frac{b}{x^2}}}{x^2}d\frac{1}{x}+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

$$\Big\downarrow \text{248}$$

$$-\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

$$\Big\downarrow \text{262}$$

$$-\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{2b}\right)+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}\right)+\frac{\sqrt{a+\frac{b}{x^2}}}{4x^3}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{6x^3}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{8x^3}$$

input
```
Int[(a + b/x^2)^(5/2)/x^4,x]
```

output
```
-1/8*(a + b/x^2)^(5/2)/x^3 - (5*a*((a + b/x^2)^(3/2)/(6*x^3) + (a*(Sqrt[a
+ b/x^2]/(4*x^3) + (a*(Sqrt[a + b/x^2]/(2*b*x) - (a*ArcTanh[Sqrt[b]/(Sqrt[
a + b/x^2]*x)])/(2*b^(3/2))))/4))/2))/8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\left(15 a^{3} x^{6}+118 a^{2} b\, x^{4}+136 a\, b^{2} x^{2}+48 b^{3}\right) \sqrt{\frac{a\, x^{2}+b}{x^{2}}}}{384 x^{7} b}+\frac{5 a^{4} \ln \left(\frac{2 b+2 \sqrt{b}\, \sqrt{a\, x^{2}+b}}{x}\right) \sqrt{\frac{a\, x^{2}+b}{x^{2}}}\, x}{128 b^{\frac{3}{2}} \sqrt{a\, x^{2}+b}}$
default	$\frac{\left(\frac{a\, x^{2}+b}{x^{2}}\right)^{\frac{5}{2}} \left(-3 \left(a\, x^{2}+b\right)^{\frac{5}{2}} a^{4} x^{8}+3 \left(a\, x^{2}+b\right)^{\frac{7}{2}} a^{3} x^{6}-5 \left(a\, x^{2}+b\right)^{\frac{3}{2}} a^{4} b\, x^{8}+15 b^{\frac{5}{2}} \ln \left(\frac{2 b+2 \sqrt{b}\, \sqrt{a\, x^{2}+b}}{x}\right) a^{4} x^{8}-15 \sqrt{a\, x^{2}+b}\, a^{4} b^{2} x^{8}+2 (\right.}{384 x^{3} \left(a\, x^{2}+b\right)^{\frac{5}{2}} b^{4}}$

input
```
int((a+b/x^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/384*(15*a^3*x^6+118*a^2*b*x^4+136*a*b^2*x^2+48*b^3)/x^7/b*((a*x^2+b)/x^
2)^(1/2)+5/128/b^(3/2)*a^4*ln((2*b+2*b^(1/2)*(a*x^2+b)^(1/2))/x)*((a*x^2+b
)/x^2)^(1/2)*x/(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx = \left[\frac{15\,a^4\sqrt{b}x^7\log\left(-\frac{ax^2+2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\,(15\,a^3bx^6+118\,a^2b^2x^4+136\,ab^3x^2+48\,b^4)\sqrt{\frac{ax^2+b}{x^2}}}{768\,b^2x^7}, -\frac{15\,a^4\sqrt{-b}x^7\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)+(15\,a^3bx^6+118\,a^2b^2x^4+136\,ab^3x^2+48\,b^4)\sqrt{\frac{ax^2+b}{x^2}}}{384\,b^2x^7}\right]$$

input
```
integrate((a+b/x^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
[1/768*(15*a^4*sqrt(b)*x^7*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2)
 + 2*b)/x^2) - 2*(15*a^3*b*x^6 + 118*a^2*b^2*x^4 + 136*a*b^3*x^2 + 48*b^4)
*sqrt((a*x^2 + b)/x^2))/(b^2*x^7), -1/384*(15*a^4*sqrt(-b)*x^7*arctan(sqrt
(-b)*x*sqrt((a*x^2 + b)/x^2)/b) + (15*a^3*b*x^6 + 118*a^2*b^2*x^4 + 136*a*
b^3*x^2 + 48*b^4)*sqrt((a*x^2 + b)/x^2))/(b^2*x^7)]
```

Sympy [A] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx = -\frac{5a^{\frac{7}{2}}}{128bx\sqrt{1+\frac{b}{ax^2}}}-\frac{133a^{\frac{5}{2}}}{384x^3\sqrt{1+\frac{b}{ax^2}}}-\frac{127a^{\frac{3}{2}}b}{192x^5\sqrt{1+\frac{b}{ax^2}}}-\frac{23\sqrt{a}b^2}{48x^7\sqrt{1+\frac{b}{ax^2}}}+\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{128b^{\frac{3}{2}}}-\frac{b^3}{8\sqrt{a}x^9\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate((a+b/x**2)**(5/2)/x**4,x)
```

output
```
-5*a**(7/2)/(128*b*x*sqrt(1 + b/(a*x**2))) - 133*a**(5/2)/(384*x**3*sqrt(1
 + b/(a*x**2))) - 127*a**(3/2)*b/(192*x**5*sqrt(1 + b/(a*x**2))) - 23*sqrt
(a)*b**2/(48*x**7*sqrt(1 + b/(a*x**2))) + 5*a**4*asinh(sqrt(b)/(sqrt(a)*x)
)/(128*b**(3/2)) - b**3/(8*sqrt(a)*x**9*sqrt(1 + b/(a*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(92) = 184$.

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.64

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx = -\frac{5\,a^4\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{256\,b^{\frac{3}{2}}} - \frac{15\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}a^4x^7+73\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^4bx^5-55\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4b^2x^3+15\sqrt{a+\frac{b}{x^2}}a^4b^3x}{384\left(\left(a+\frac{b}{x^2}\right)^4bx^8-4\left(a+\frac{b}{x^2}\right)^3b^2x^6+6\left(a+\frac{b}{x^2}\right)^2b^3x^4-4\left(a+\frac{b}{x^2}\right)b^4x^2+b^5\right)}$$

input
```
integrate((a+b/x^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
-5/256*a^4*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b))
)/b^(3/2) - 1/384*(15*(a + b/x^2)^(7/2)*a^4*x^7 + 73*(a + b/x^2)^(5/2)*a^4
*b*x^5 - 55*(a + b/x^2)^(3/2)*a^4*b^2*x^3 + 15*sqrt(a + b/x^2)*a^4*b^3*x)/
((a + b/x^2)^4*b*x^8 - 4*(a + b/x^2)^3*b^2*x^6 + 6*(a + b/x^2)^2*b^3*x^4 -
 4*(a + b/x^2)*b^4*x^2 + b^5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx = -\frac{\frac{15\,a^5\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}b}+\frac{15\left(ax^2+b\right)^{\frac{7}{2}}a^5\operatorname{sgn}(x)+73\left(ax^2+b\right)^{\frac{5}{2}}a^5b\operatorname{sgn}(x)-55\left(ax^2+b\right)^{\frac{3}{2}}a^5b^2\operatorname{sgn}(x)+15\sqrt{ax^2+b}a^5b^3\operatorname{sgn}(x)}{a^4bx^8}}{384\,a}$$

input
```
integrate((a+b/x^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
-1/384*(15*a^5*arctan(sqrt(a*x^2 + b)/sqrt(-b))*sgn(x)/(sqrt(-b)*b) + (15*
(a*x^2 + b)^(7/2)*a^5*sgn(x) + 73*(a*x^2 + b)^(5/2)*a^5*b*sgn(x) - 55*(a*x
^2 + b)^(3/2)*a^5*b^2*sgn(x) + 15*sqrt(a*x^2 + b)*a^5*b^3*sgn(x))/(a^4*b*x
^8))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx$$

input
```
int((a + b/x^2)^(5/2)/x^4,x)
```

output
```
int((a + b/x^2)^(5/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{x^4}\,dx=\frac{-15\sqrt{a\,x^2+b}\,a^3b\,x^6-118\sqrt{a\,x^2+b}\,a^2b^2x^4-136\sqrt{a\,x^2+b}\,a\,b^3x^2-48\sqrt{a\,x^2+b}\,b^4-}{384b^2x^8}$$

input
```
int((a+b/x^2)^(5/2)/x^4,x)
```

output
```
( - 15*sqrt(a*x**2 + b)*a**3*b*x**6 - 118*sqrt(a*x**2 + b)*a**2*b**2*x**4
- 136*sqrt(a*x**2 + b)*a*b**3*x**2 - 48*sqrt(a*x**2 + b)*b**4 - 15*sqrt(b)
*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a**4*x**8 + 15*sqrt
(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a**4*x**8)/(384*
b**2*x**8)
```

3.375 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx=-\frac{3b\sqrt{a+\frac{b}{x^2}}\,x^2}{8a^2}+\frac{\sqrt{a+\frac{b}{x^2}}\,x^4}{4a}+\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-3/8*b*(a+b/x^2)^(1/2)*x^2/a^2+1/4*(a+b/x^2)^(1/2)*x^4/a+3/8*b^2*arctanh((
a+b/x^2)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx=\frac{\sqrt{a}x(-3b^2-abx^2+2a^2x^4)+6b^2\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{8a^{5/2}\sqrt{a+\frac{b}{x^2}}\,x}$$

input
```
Integrate[x^3/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[a]*x*(-3*b^2 - a*b*x^2 + 2*a^2*x^4) + 6*b^2*Sqrt[b + a*x^2]*ArcTanh[
(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b + a*x^2])])/(8*a^(5/2)*Sqrt[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^6}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{3b\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{4a}+\frac{x^4\sqrt{a+\frac{b}{x^2}}}{2a}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{3b\left(-\frac{b\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{2a}-\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)}{4a}+\frac{x^4\sqrt{a+\frac{b}{x^2}}}{2a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{a}-\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)}{4a}+\frac{x^4\sqrt{a+\frac{b}{x^2}}}{2a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)}{4a}+\frac{x^4\sqrt{a+\frac{b}{x^2}}}{2a}\right)$$

input
```
Int[x^3/Sqrt[a + b/x^2],x]
```

output
```
((Sqrt[a + b/x^2]*x^4)/(2*a) + (3*b*(-((Sqrt[a + b/x^2]*x^2)/a) + (b*ArcTa
nh[Sqrt[a + b/x^2]/Sqrt[a]])/a^(3/2)))/(4*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

method	result	size
risch	$\frac{(2a\,x^2-3b)\,(a\,x^2+b)}{8a^2\sqrt{\frac{a\,x^2+b}{x^2}}}+\frac{3b^2\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)\sqrt{a\,x^2+b}}{8a^{\frac{5}{2}}\sqrt{\frac{a\,x^2+b}{x^2}}\,x}$	86
default	$\frac{\sqrt{a\,x^2+b}\,\left(2x^3\sqrt{a\,x^2+b}\,a^{\frac{5}{2}}-3a^{\frac{3}{2}}\sqrt{a\,x^2+b}\,bx+3\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)a\,b^2\right)}{8\sqrt{\frac{a\,x^2+b}{x^2}}\,x\,a^{\frac{7}{2}}}$	87

input
```
int(x^3/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a*x^2-3*b)*(a*x^2+b)/a^2/((a*x^2+b)/x^2)^(1/2)+3/8*b^2/a^(5/2)*ln(a
^(1/2)*x+(a*x^2+b)^(1/2))/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx = \left[\frac{3\,\sqrt{a}b^2\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\,(2\,a^2x^4-3\,abx^2)\sqrt{\frac{ax^2+b}{x^2}}}{16\,a^3}, -\frac{3\,\sqrt{-a}b^2\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-(2\,a^2x^4-3\,abx^2)\sqrt{\frac{ax^2+b}{x^2}}}{8\,a^3}\right]$$

input
```
integrate(x^3/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(a)*b^2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) -
b) + 2*(2*a^2*x^4 - 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a^3, -1/8*(3*sqrt(-a
)*b^2*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (2*a^2*x^4
- 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/a^3]
```

Sympy [A] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{x^5}{4\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} - \frac{\sqrt{b}x^3}{8a\sqrt{\frac{ax^2}{b}+1}} - \frac{3b^{\frac{3}{2}}x}{8a^2\sqrt{\frac{ax^2}{b}+1}} + \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{\frac{5}{2}}}$$

input
```
integrate(x**3/(a+b/x**2)**(1/2),x)
```

output
```
x**5/(4*sqrt(b)*sqrt(a*x**2/b + 1)) - sqrt(b)*x**3/(8*a*sqrt(a*x**2/b + 1)
) - 3*b**(3/2)*x/(8*a**2*sqrt(a*x**2/b + 1)) + 3*b**2*asinh(sqrt(a)*x/sqrt
(b))/(8*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}} - \frac{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2 - 5\sqrt{a+\frac{b}{x^2}}\,ab^2}{8\left(\left(a+\frac{b}{x^2}\right)^2a^2 - 2\left(a+\frac{b}{x^2}\right)a^3 + a^4\right)}$$

input
```
integrate(x^3/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(
5/2) - 1/8*(3*(a + b/x^2)^(3/2)*b^2 - 5*sqrt(a + b/x^2)*a*b^2)/((a + b/x^2
)^2*a^2 - 2*(a + b/x^2)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{1}{8}\sqrt{ax^2+b}\,x\left(\frac{2\,x^2}{a\text{sgn}(x)} - \frac{3\,b}{a^2\text{sgn}(x)}\right) + \frac{3\,b^2\log(|b|)\,\text{sgn}(x)}{16\,a^{\frac{5}{2}}} - \frac{3\,b^2\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{8\,a^{\frac{5}{2}}\text{sgn}(x)}$$

input
```
integrate(x^3/(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*x^2 + b)*x*(2*x^2/(a*sgn(x)) - 3*b/(a^2*sgn(x))) + 3/16*b^2*log
(abs(b))*sgn(x)/a^(5/2) - 3/8*b^2*log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))/(
a^(5/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{5/2}} + \frac{5\,x^4\,\sqrt{a+\frac{b}{x^2}}}{8\,a} - \frac{3\,x^4\,\left(a+\frac{b}{x^2}\right)^{3/2}}{8\,a^2}$$

input
```
int(x^3/(a + b/x^2)^(1/2),x)
```

output
```
(3*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(5/2)) + (5*x^4*(a + b/x^2)^
(1/2))/(8*a) - (3*x^4*(a + b/x^2)^(3/2))/(8*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{2\sqrt{a\,x^2+b}\,a^2x^3 - 3\sqrt{a\,x^2+b}\,abx + 3\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)b^2}{8a^3}$$

input
```
int(x^3/(a+b/x^2)^(1/2),x)
```

output
```
(2*sqrt(a*x**2 + b)*a**2*x**3 - 3*sqrt(a*x**2 + b)*a*b*x + 3*sqrt(a)*log((
sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b**2)/(8*a**3)
```

3.376 $\int \frac{x}{\sqrt{a+\frac{b}{x^2}}} dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}} dx = \frac{\sqrt{a+\frac{b}{x^2}}x^2}{2a} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
1/2*(a+b/x^2)^(1/2)*x^2/a-1/2*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}} dx = \frac{\sqrt{a}x(b+ax^2) + 2b\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{b}-\sqrt{b+ax^2}}\right)}{2a^{3/2}\sqrt{a+\frac{b}{x^2}}x}$$

input
```
Integrate[x/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[a]*x*(b + a*x^2) + 2*b*Sqrt[b + a*x^2]*ArcTanh[(Sqrt[a]*x)/(Sqrt[b]
- Sqrt[b + a*x^2])])/(2*a^(3/2)*Sqrt[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{b\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{2a}+\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{a}+\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+\frac{b}{x^2}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[x/Sqrt[a + b/x^2],x]
```

output
```
((Sqrt[a + b/x^2]*x^2)/a - (b*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/a^(3/2))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\sqrt{a\,x^{2}+b}\,\left(x\sqrt{a\,x^{2}+b}\,a^{\frac{3}{2}}-b\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)a\right)}{2\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x\,a^{\frac{5}{2}}}$	66
risch	$\frac{a\,x^{2}+b}{2a\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}-\frac{b\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)\sqrt{a\,x^{2}+b}}{2a^{\frac{3}{2}}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}$	74

input
```
int(x/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2+b)^(1/2)*(x*(a*x^2+b)^(1/2)*a^(3/2)-b*ln(a^(1/2)*x+(a*x^2+b)^(1
/2))*a)/((a*x^2+b)/x^2)^(1/2)/x/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.52

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx = \left[\frac{2\,ax^2\sqrt{\frac{ax^2+b}{x^2}}+\sqrt{a}b\log\left(-2\,ax^2+2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)}{4\,a^2}, \frac{ax^2\sqrt{\frac{ax^2+b}{x^2}}+\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{2\,a^2}\right]$$

input
```
integrate(x/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*x^2*sqrt((a*x^2 + b)/x^2) + sqrt(a)*b*log(-2*a*x^2 + 2*sqrt(a)*x
^2*sqrt((a*x^2 + b)/x^2) - b))/a^2, 1/2*(a*x^2*sqrt((a*x^2 + b)/x^2) + sqr
t(-a)*b*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)))/a^2]
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{b}x\sqrt{\frac{ax^2}{b}+1}}{2a} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate(x/(a+b/x**2)**(1/2),x)
```

output
```
sqrt(b)*x*sqrt(a*x**2/b + 1)/(2*a) - b*asinh(sqrt(a)*x/sqrt(b))/(2*a**(3/2
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\,b}{2\left(\left(a+\frac{b}{x^2}\right)a-a^2\right)} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}$$

input
```
integrate(x/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^2)*b/((a + b/x^2)*a - a^2) + 1/4*b*log((sqrt(a + b/x^2) -
 sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{4\,a^{\frac{3}{2}}} + \frac{\sqrt{ax^2+b}\,x}{2\,a\operatorname{sgn}(x)} + \frac{b\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}$$

input
```
integrate(x/(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*sqrt(a*x^2 + b)*x/(a*sgn(x)) + 1/2
*b*log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))/(a^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{x^2\sqrt{a+\frac{b}{x^2}}}{2\,a} - \frac{b\,\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int(x/(a + b/x^2)^(1/2),x)
```

output
```
(x^2*(a + b/x^2)^(1/2))/(2*a) - (b*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(2*a^
(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{a+\frac{b}{x^2}}}\,dx=\frac{\sqrt{a\,x^2+b}\,ax-\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)b}{2a^2}$$

input
```
int(x/(a+b/x^2)^(1/2),x)
```

output
```
(sqrt(a*x**2 + b)*a*x - sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b)
)*b)/(2*a**2)
```

3.377 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x} dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(24) = 48.

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x} dx = \frac{\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{b+ax^2}}\right)}{\sqrt{a}\sqrt{a+\frac{b}{x^2}}x}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*x),x]
```

output
```
(Sqrt[b + a*x^2]*ArcTanh[(Sqrt[a]*x)/Sqrt[b + a*x^2]])/(Sqrt[a]*Sqrt[a + b
/x^2]*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{x^2}}}dx$$
$$\Big\downarrow \text{798}$$
$$-\frac{1}{2}\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}$$
$$\Big\downarrow \text{73}$$
$$-\frac{\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{b}$$
$$\Big\downarrow \text{221}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b/x^2]*x),x]
```

output
```
ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]]/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{\sqrt{a\,x^{2}+b}\,\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)}{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x\sqrt{a}}$	46

input
```
int(1/(a+b/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)^(1/2)*ln(a^(1/2)*x+(a*x^2+b)^(1/2))/a^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}}x}\,dx = \left[\frac{\log\left(-2\,ax^2 - 2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right)}{2\,\sqrt{a}}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{a} \right]$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b)/sqrt(a), -sqr
t(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b))/a]
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}}x}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}}$$

input
```
integrate(1/(a+b/x**2)**(1/2)/x,x)
```

output
```
asinh(sqrt(a)*x/sqrt(b))/sqrt(a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x}\,dx = \frac{\log(|b|)\,\mathrm{sgn}(x)}{2\sqrt{a}} - \frac{\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{\sqrt{a}\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(x)/sqrt(a) - log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))/(s
qrt(a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b/x^2)^(1/2)),x)
```

output
```
atanh((a + b/x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}\,dx=\frac{\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)}{a}$$

input
```
int(1/(a+b/x^2)^(1/2)/x,x)
```

output
```
(sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b)))/a
```

3.378 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^3} dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

output
```
-(a+b/x^2)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^3} dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*x^3),x]
```

output
```
-(Sqrt[a + b/x^2]/b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

input
```
Int[1/(Sqrt[a + b/x^2]*x^3),x]
```

output
```
-(Sqrt[a + b/x^2]/b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{\sqrt{a+\frac{b}{x^2}}}{b}$	15
trager	$-\frac{\sqrt{-\frac{-a\,x^2-b}{x^2}}}{b}$	23
orering	$-\frac{a\,x^2+b}{x^2 b\sqrt{a+\frac{b}{x^2}}}$	25
gosper	$-\frac{a\,x^2+b}{x^2 b\sqrt{\frac{a\,x^2+b}{x^2}}}$	29
default	$-\frac{a\,x^2+b}{x^2 b\sqrt{\frac{a\,x^2+b}{x^2}}}$	29
risch	$-\frac{a\,x^2+b}{x^2 b\sqrt{\frac{a\,x^2+b}{x^2}}}$	29

input
```
int(1/(a+b/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(a+b/x^2)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^3}\,dx = -\frac{\sqrt{\frac{ax^2+b}{x^2}}}{b}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-sqrt((a*x^2 + b)/x^2)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^3}\,dx = \begin{cases} -\frac{\sqrt{a+\frac{b}{x^2}}}{b} & \text{for } b \neq 0 \\ -\frac{1}{2\sqrt{a}x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**2)**(1/2)/x**3,x)
```

output
```
Piecewise((-sqrt(a + b/x**2)/b, Ne(b, 0)), (-1/(2*sqrt(a)*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^3}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-sqrt(a + b/x^2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^3}\,dx = \frac{2\sqrt{a}}{\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2*sqrt(a)/((((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^3}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{b}$$

input
```
int(1/(x^3*(a + b/x^2)^(1/2)),x)
```

output
```
-(a + b/x^2)^(1/2)/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^3}\,dx = \frac{-\sqrt{a\,x^2+b}-\sqrt{a}\,x}{bx}$$

input
```
int(1/(a+b/x^2)^(1/2)/x^3,x)
```

output
```
( - (sqrt(a*x**2 + b) + sqrt(a)*x))/(b*x)
```

3.379 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5} dx = \frac{a\sqrt{a+\frac{b}{x^2}}}{b^2} - \frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^2}$$

output
```
a*(a+b/x^2)^(1/2)/b^2-1/3*(a+b/x^2)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5} dx = \frac{\sqrt{a+\frac{b}{x^2}}(-b+2ax^2)}{3b^2x^2}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*x^5),x]
```

output
```
(Sqrt[a + b/x^2]*(-b + 2*a*x^2))/(3*b^2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{\sqrt{a+\frac{b}{x^2}}}{b}-\frac{a}{b\sqrt{a+\frac{b}{x^2}}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{a+\frac{b}{x^2}}}{b^2}-\frac{2\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^2}\right)$$

input
```
Int[1/(Sqrt[a + b/x^2]*x^5),x]
```

output
```
((2*a*Sqrt[a + b/x^2])/b^2 - (2*(a + b/x^2)^(3/2))/(3*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
orering	$\frac{(2a\,x^2-b)\,(a\,x^2+b)}{3b^2x^4\sqrt{a+\frac{b}{x^2}}}$	35
trager	$\frac{(2a\,x^2-b)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3x^2b^2}$	36
gosper	$\frac{(a\,x^2+b)\,(2a\,x^2-b)}{3x^4b^2\sqrt{\frac{a\,x^2+b}{x^2}}}$	39
default	$\frac{(a\,x^2+b)\,(2a\,x^2-b)}{3x^4b^2\sqrt{\frac{a\,x^2+b}{x^2}}}$	39
risch	$\frac{(a\,x^2+b)\,(2a\,x^2-b)}{3x^4b^2\sqrt{\frac{a\,x^2+b}{x^2}}}$	39

input
```
int(1/(a+b/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*a*x^2-b)/b^2*(a*x^2+b)/x^4/(a+b/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5}\,dx = \frac{(2\,ax^2-b)\sqrt{\frac{ax^2+b}{x^2}}}{3\,b^2x^2}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/3*(2*a*x^2 - b)*sqrt((a*x^2 + b)/x^2)/(b^2*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(29) = 58.

Time = 0.87 (sec) , antiderivative size = 231, normalized size of antiderivative = 6.60

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5}\,dx = \frac{2a^{\frac{7}{2}}b^{\frac{3}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3} + \frac{a^{\frac{5}{2}}b^{\frac{5}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3} - \frac{a^{\frac{3}{2}}b^{\frac{7}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3} - \frac{2a^4bx^5}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3} - \frac{2a^3b^2x^3}{3a^{\frac{5}{2}}b^3x^5+3a^{\frac{3}{2}}b^4x^3}$$

input
```
integrate(1/(a+b/x**2)**(1/2)/x**5,x)
```

output
```
2*a**(7/2)*b**(3/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**(5/2)*b**3*x**5 + 3*a**(
3/2)*b**4*x**3) + a**(5/2)*b**(5/2)*x**2*sqrt(a*x**2/b + 1)/(3*a**(5/2)*b*
*3*x**5 + 3*a**(3/2)*b**4*x**3) - a**(3/2)*b**(7/2)*sqrt(a*x**2/b + 1)/(3*
a**(5/2)*b**3*x**5 + 3*a**(3/2)*b**4*x**3) - 2*a**4*b*x**5/(3*a**(5/2)*b**
3*x**5 + 3*a**(3/2)*b**4*x**3) - 2*a**3*b**2*x**3/(3*a**(5/2)*b**3*x**5 +
3*a**(3/2)*b**4*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b^2}+\frac{\sqrt{a+\frac{b}{x^2}}a}{b^2}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/3*(a + b/x^2)^(3/2)/b^2 + sqrt(a + b/x^2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5}\,dx = \frac{4\left(3\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)a^{\frac{3}{2}}}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^3\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)*a^(3/2)/(((sqrt(a)*x - sqrt(a*
x^2 + b))^2 - b)^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^5}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\,\left(b-2\,a\,x^2\right)}{3\,b^2\,x^2}$$

input
```
int(1/(x^5*(a + b/x^2)^(1/2)),x)
```

output
```
-((a + b/x^2)^(1/2)*(b - 2*a*x^2))/(3*b^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^5}\,dx = \frac{2\sqrt{a\,x^2+b}\,a\,x^2-\sqrt{a\,x^2+b}\,b-2\sqrt{a}\,a\,x^3}{3b^2x^3}$$

input
```
int(1/(a+b/x^2)^(1/2)/x^5,x)
```

output
```
(2*sqrt(a*x**2 + b)*a*x**2 - sqrt(a*x**2 + b)*b - 2*sqrt(a)*a*x**3)/(3*b**
2*x**3)
```

3.380 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^7}\,dx = -\frac{a^2\sqrt{a+\frac{b}{x^2}}}{b^3}+\frac{2a\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^3}$$

output
```
-a^2*(a+b/x^2)^(1/2)/b^3+2/3*a*(a+b/x^2)^(3/2)/b^3-1/5*(a+b/x^2)^(5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^7}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}(3b^2-4abx^2+8a^2x^4)}{15b^3x^4}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*x^7),x]
```

output
```
-1/15*(Sqrt[a + b/x^2]*(3*b^2 - 4*a*b*x^2 + 8*a^2*x^4))/(b^3*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{a^2}{b^2\sqrt{a+\frac{b}{x^2}}}-\frac{2\sqrt{a+\frac{b}{x^2}}a}{b^2}+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{b^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^2\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{2\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^3}+\frac{4a\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}\right)$$

input
```
Int[1/(Sqrt[a + b/x^2]*x^7),x]
```

output
```
((-2*a^2*Sqrt[a + b/x^2])/b^3 + (4*a*(a + b/x^2)^(3/2))/(3*b^3) - (2*(a +
b/x^2)^(5/2))/(5*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
orering	$-\frac{\left(8 a^{2} x^{4}-4 a b \,x^{2}+3 b^{2}\right) \left(a \,x^{2}+b \right)}{15 b^{3} x^{6} \sqrt{a +\frac{b}{x^{2}}}}$	46
trager	$-\frac{\left(8 a^{2} x^{4}-4 a b \,x^{2}+3 b^{2}\right) \sqrt{-\frac{-a \,x^{2}-b}{x^{2}}}}{15 x^{4} b^{3}}$	47
gosper	$-\frac{\left(a \,x^{2}+b \right) \left(8 a^{2} x^{4}-4 a b \,x^{2}+3 b^{2}\right)}{15 x^{6} b^{3} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}$	50
default	$-\frac{\left(a \,x^{2}+b \right) \left(8 a^{2} x^{4}-4 a b \,x^{2}+3 b^{2}\right)}{15 x^{6} b^{3} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}$	50
risch	$-\frac{\left(a \,x^{2}+b \right) \left(8 a^{2} x^{4}-4 a b \,x^{2}+3 b^{2}\right)}{15 x^{6} b^{3} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}$	50

input
```
int(1/(a+b/x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(8*a^2*x^4-4*a*b*x^2+3*b^2)/b^3*(a*x^2+b)/x^6/(a+b/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^7}\,dx = -\frac{\left(8\,a^2x^4-4\,abx^2+3\,b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{15\,b^3x^4}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/15*(8*a^2*x^4 - 4*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2)/(b^3*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(49) = 98$.

Time = 1.58 (sec) , antiderivative size = 750, normalized size of antiderivative = 13.16

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^7}\,dx = -\frac{8a^{\frac{15}{2}}b^{\frac{9}{2}}x^{10}\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} - \frac{20a^{\frac{13}{2}}b^{\frac{11}{2}}x^8\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} - \frac{15a^{\frac{11}{2}}b^{\frac{13}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} - \frac{5a^{\frac{9}{2}}b^{\frac{15}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} - \frac{5a^{\frac{7}{2}}b^{\frac{17}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} - \frac{3a^{\frac{5}{2}}b^{\frac{19}{2}}\sqrt{\frac{ax^2}{b}+1}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} + \frac{8a^8b^4x^{11}}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} + \frac{24a^7b^5x^9}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} + \frac{24a^6b^6x^7}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5} + \frac{8a^5b^7x^5}{15a^{\frac{11}{2}}b^7x^{11}+45a^{\frac{9}{2}}b^8x^9+45a^{\frac{7}{2}}b^9x^7+15a^{\frac{5}{2}}b^{10}x^5}$$

input
```
integrate(1/(a+b/x**2)**(1/2)/x**7,x)
```

output
```
-8*a**(15/2)*b**(9/2)*x**10*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 +
45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) -
20*a**(13/2)*b**(11/2)*x**8*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 +
45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) -
15*a**(11/2)*b**(13/2)*x**6*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 +
45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) -
5*a**(9/2)*b**(15/2)*x**4*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45
*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) - 5*
a**(7/2)*b**(17/2)*x**2*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45*a
**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) - 3*a*
*(5/2)*b**(19/2)*sqrt(a*x**2/b + 1)/(15*a**(11/2)*b**7*x**11 + 45*a**(9/2)
*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) + 8*a**8*b**4
*x**11/(15*a**(11/2)*b**7*x**11 + 45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9
*x**7 + 15*a**(5/2)*b**10*x**5) + 24*a**7*b**5*x**9/(15*a**(11/2)*b**7*x**
11 + 45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**
5) + 24*a**6*b**6*x**7/(15*a**(11/2)*b**7*x**11 + 45*a**(9/2)*b**8*x**9 +
45*a**(7/2)*b**9*x**7 + 15*a**(5/2)*b**10*x**5) + 8*a**5*b**7*x**5/(15*a**
(11/2)*b**7*x**11 + 45*a**(9/2)*b**8*x**9 + 45*a**(7/2)*b**9*x**7 + 15*a**
(5/2)*b**10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,x^7}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}{5\,b^3} + \frac{2\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a}{3\,b^3} - \frac{\sqrt{a+\frac{b}{x^2}}\,a^2}{b^3}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/5*(a + b/x^2)^(5/2)/b^3 + 2/3*(a + b/x^2)^(3/2)*a/b^3 - sqrt(a + b/x^2)
*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^7}\,dx = \frac{16\left(10\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4-5\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2b+b^2\right)a^{\frac{5}{2}}}{15\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^5\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
16/15*(10*(sqrt(a)*x - sqrt(a*x^2 + b))^4 - 5*(sqrt(a)*x - sqrt(a*x^2 + b)
)^2*b + b^2)*a^(5/2)/(((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^7}\,dx = -\frac{3\,b^2\,\sqrt{a+\frac{b}{x^2}}+8\,a^2\,x^4\,\sqrt{a+\frac{b}{x^2}}-4\,a\,b\,x^2\,\sqrt{a+\frac{b}{x^2}}}{15\,b^3\,x^4}$$

input
```
int(1/(x^7*(a + b/x^2)^(1/2)),x)
```

output
```
-(3*b^2*(a + b/x^2)^(1/2) + 8*a^2*x^4*(a + b/x^2)^(1/2) - 4*a*b*x^2*(a + b
/x^2)^(1/2))/(15*b^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^7}\,dx = \frac{-8\sqrt{a\,x^2+b}\,a^2x^4+4\sqrt{a\,x^2+b}\,ab\,x^2-3\sqrt{a\,x^2+b}\,b^2+8\sqrt{a}\,a^2x^5}{15b^3x^5}$$

input
```
int(1/(a+b/x^2)^(1/2)/x^7,x)
```

output
```
( - 8*sqrt(a*x**2 + b)*a**2*x**4 + 4*sqrt(a*x**2 + b)*a*b*x**2 - 3*sqrt(a*
x**2 + b)*b**2 + 8*sqrt(a)*a**2*x**5)/(15*b**3*x**5)
```

3.381 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9} dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9} dx = \frac{a^3\sqrt{a+\frac{b}{x^2}}}{b^4} - \frac{a^2\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4} + \frac{3a\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4} - \frac{\left(a+\frac{b}{x^2}\right)^{7/2}}{7b^4}$$

output
```
a^3*(a+b/x^2)^(1/2)/b^4-a^2*(a+b/x^2)^(3/2)/b^4+3/5*a*(a+b/x^2)^(5/2)/b^4-
1/7*(a+b/x^2)^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9} dx = \frac{\sqrt{a+\frac{b}{x^2}}(-5b^3+6ab^2x^2-8a^2bx^4+16a^3x^6)}{35b^4x^6}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*x^9),x]
```

output
```
(Sqrt[a + b/x^2]*(-5*b^3 + 6*a*b^2*x^2 - 8*a^2*b*x^4 + 16*a^3*x^6))/(35*b^
4*x^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^6}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(-\frac{a^3}{b^3\sqrt{a+\frac{b}{x^2}}}+\frac{3\sqrt{a+\frac{b}{x^2}}a^2}{b^3}-\frac{3\left(a+\frac{b}{x^2}\right)^{3/2}a}{b^3}+\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{b^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^3\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{2a^2\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x^2}\right)^{7/2}}{7b^4}+\frac{6a\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}\right)$$

input
```
Int[1/(Sqrt[a + b/x^2]*x^9),x]
```

output
```
((2*a^3*Sqrt[a + b/x^2])/b^4 - (2*a^2*(a + b/x^2)^(3/2))/b^4 + (6*a*(a + b
/x^2)^(5/2))/(5*b^4) - (2*(a + b/x^2)^(7/2))/(7*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

method	result	size
orering	$\frac{\left(16a^{3}x^{6}-8a^{2}b\,x^{4}+6a\,b^{2}x^{2}-5b^{3}\right)\left(a\,x^{2}+b\right)}{35b^{4}x^{8}\sqrt{a+\frac{b}{x^{2}}}}$	57
trager	$\frac{\left(16a^{3}x^{6}-8a^{2}b\,x^{4}+6a\,b^{2}x^{2}-5b^{3}\right)\sqrt{-\frac{-a\,x^{2}-b}{x^{2}}}}{35x^{6}b^{4}}$	58
gosper	$\frac{\left(a\,x^{2}+b\right)\left(16a^{3}x^{6}-8a^{2}b\,x^{4}+6a\,b^{2}x^{2}-5b^{3}\right)}{35x^{8}b^{4}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}$	61
default	$\frac{\left(a\,x^{2}+b\right)\left(16a^{3}x^{6}-8a^{2}b\,x^{4}+6a\,b^{2}x^{2}-5b^{3}\right)}{35x^{8}b^{4}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}$	61
risch	$\frac{\left(a\,x^{2}+b\right)\left(16a^{3}x^{6}-8a^{2}b\,x^{4}+6a\,b^{2}x^{2}-5b^{3}\right)}{35x^{8}b^{4}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}$	61

input
```
int(1/(a+b/x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/35*(16*a^3*x^6-8*a^2*b*x^4+6*a*b^2*x^2-5*b^3)/b^4*(a*x^2+b)/x^8/(a+b/x^2
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9}\,dx = \frac{(16\,a^3x^6 - 8\,a^2bx^4 + 6\,ab^2x^2 - 5\,b^3)\sqrt{\frac{ax^2+b}{x^2}}}{35\,b^4x^6}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
1/35*(16*a^3*x^6 - 8*a^2*b*x^4 + 6*a*b^2*x^2 - 5*b^3)*sqrt((a*x^2 + b)/x^2
)/(b^4*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1969 vs. 2(66) = 132.

Time = 1.91 (sec) , antiderivative size = 1969, normalized size of antiderivative = 26.25

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**2)**(1/2)/x**9,x)
```

output
```
16*a**(25/2)*b**(23/2)*x**18*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19
+ 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b*
*18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7
/2)*b**21*x**7) + 88*a**(23/2)*b**(25/2)*x**16*sqrt(a*x**2/b + 1)/(35*a**(
19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15
+ 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**
20*x**9 + 35*a**(7/2)*b**21*x**7) + 198*a**(21/2)*b**(27/2)*x**14*sqrt(a*x
**2/b + 1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**
(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11
 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) + 231*a**(19/2)*b**(2
9/2)*x**12*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19 + 210*a**(17/2)*b*
*16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b**18*x**13 + 525*a*
*(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(7/2)*b**21*x**7) +
140*a**(17/2)*b**(31/2)*x**10*sqrt(a*x**2/b + 1)/(35*a**(19/2)*b**15*x**19
 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15 + 700*a**(13/2)*b
**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**20*x**9 + 35*a**(
7/2)*b**21*x**7) + 21*a**(15/2)*b**(33/2)*x**8*sqrt(a*x**2/b + 1)/(35*a**(
19/2)*b**15*x**19 + 210*a**(17/2)*b**16*x**17 + 525*a**(15/2)*b**17*x**15
+ 700*a**(13/2)*b**18*x**13 + 525*a**(11/2)*b**19*x**11 + 210*a**(9/2)*b**
20*x**9 + 35*a**(7/2)*b**21*x**7) - 42*a**(13/2)*b**(35/2)*x**6*sqrt(a*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}}{7\,b^4} + \frac{3\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a}{5\,b^4} - \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2}{b^4} + \frac{\sqrt{a+\frac{b}{x^2}}a^3}{b^4}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/7*(a + b/x^2)^(7/2)/b^4 + 3/5*(a + b/x^2)^(5/2)*a/b^4 - (a + b/x^2)^(3/
2)*a^2/b^4 + sqrt(a + b/x^2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9}\,dx = \frac{32\left(35\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^6-21\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4b+7\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2b^2-b^3\right)a^{\frac{7}{2}}}{35\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^7\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
32/35*(35*(sqrt(a)*x - sqrt(a*x^2 + b))^6 - 21*(sqrt(a)*x - sqrt(a*x^2 + b
))^4*b + 7*(sqrt(a)*x - sqrt(a*x^2 + b))^2*b^2 - b^3)*a^(7/2)/(((sqrt(a)*x
 - sqrt(a*x^2 + b))^2 - b)^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9}\,dx = \frac{16\,a^3\sqrt{a+\frac{b}{x^2}}}{35\,b^4}-\frac{\sqrt{a+\frac{b}{x^2}}}{7\,b\,x^6}+\frac{6\,a\sqrt{a+\frac{b}{x^2}}}{35\,b^2\,x^4}-\frac{8\,a^2\sqrt{a+\frac{b}{x^2}}}{35\,b^3\,x^2}$$

input
```
int(1/(x^9*(a + b/x^2)^(1/2)),x)
```

output
```
(16*a^3*(a + b/x^2)^(1/2))/(35*b^4) - (a + b/x^2)^(1/2)/(7*b*x^6) + (6*a*(
a + b/x^2)^(1/2))/(35*b^2*x^4) - (8*a^2*(a + b/x^2)^(1/2))/(35*b^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^9}\,dx$$
$$= \frac{16\sqrt{a\,x^2+b}\,a^3x^6-8\sqrt{a\,x^2+b}\,a^2b\,x^4+6\sqrt{a\,x^2+b}\,a\,b^2x^2-5\sqrt{a\,x^2+b}\,b^3-16\sqrt{a}\,a^3x^7}{35b^4x^7}$$

input
```
int(1/(a+b/x^2)^(1/2)/x^9,x)
```

output
```
(16*sqrt(a*x**2 + b)*a**3*x**6 - 8*sqrt(a*x**2 + b)*a**2*b*x**4 + 6*sqrt(a
*x**2 + b)*a*b**2*x**2 - 5*sqrt(a*x**2 + b)*b**3 - 16*sqrt(a)*a**3*x**7)/(
35*b**4*x**7)
```

3.382 $\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}} \, dx = \frac{8b^2\sqrt{a+\frac{b}{x^2}}x}{15a^3} - \frac{4b\sqrt{a+\frac{b}{x^2}}x^3}{15a^2} + \frac{\sqrt{a+\frac{b}{x^2}}x^5}{5a}$$

output
```
8/15*b^2*(a+b/x^2)^(1/2)*x/a^3-4/15*b*(a+b/x^2)^(1/2)*x^3/a^2+1/5*(a+b/x^2
)^(1/2)*x^5/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}} \, dx = \frac{\sqrt{a+\frac{b}{x^2}}x(8b^2 - 4abx^2 + 3a^2x^4)}{15a^3}$$

input
```
Integrate[x^4/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*x*(8*b^2 - 4*a*b*x^2 + 3*a^2*x^4))/(15*a^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{x^5\sqrt{a+\frac{b}{x^2}}}{5a}-\frac{4b\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}dx}{5a}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{x^5\sqrt{a+\frac{b}{x^2}}}{5a}-\frac{4b\left(\frac{x^3\sqrt{a+\frac{b}{x^2}}}{3a}-\frac{2b\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{ 746}$$

$$\frac{x^5\sqrt{a+\frac{b}{x^2}}}{5a}-\frac{4b\left(\frac{x^3\sqrt{a+\frac{b}{x^2}}}{3a}-\frac{2bx\sqrt{a+\frac{b}{x^2}}}{3a^2}\right)}{5a}$$

input
```
Int[x^4/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*x^5)/(5*a) - (4*b*((-2*b*Sqrt[a + b/x^2]*x)/(3*a^2) + (Sq
rt[a + b/x^2]*x^3)/(3*a)))/(5*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

method	result	size
trager	$\frac{(3a^2x^4-4ab\,x^2+8b^2)x\sqrt{-\frac{-a\,x^2-b}{x^2}}}{15a^3}$	45
orering	$\frac{(3a^2x^4-4ab\,x^2+8b^2)(a\,x^2+b)}{15a^3x\sqrt{a+\frac{b}{x^2}}}$	46
gosper	$\frac{(a\,x^2+b)(3a^2x^4-4ab\,x^2+8b^2)}{15a^3x\sqrt{\frac{a\,x^2+b}{x^2}}}$	50
default	$\frac{(a\,x^2+b)(3a^2x^4-4ab\,x^2+8b^2)}{15a^3x\sqrt{\frac{a\,x^2+b}{x^2}}}$	50
risch	$\frac{(a\,x^2+b)(3a^2x^4-4ab\,x^2+8b^2)}{15a^3x\sqrt{\frac{a\,x^2+b}{x^2}}}$	50

input
```
int(x^4/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*a^2*x^4-4*a*b*x^2+8*b^2)*x/a^3*(-(-a*x^2-b)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{(3\,a^2x^5-4\,abx^3+8\,b^2x)\sqrt{\frac{ax^2+b}{x^2}}}{15\,a^3}$$

input
```
integrate(x^4/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*a^2*x^5 - 4*a*b*x^3 + 8*b^2*x)*sqrt((a*x^2 + b)/x^2)/a^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(60) = 120.

Time = 0.77 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.23

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{3a^4b^{\frac{9}{2}}x^8\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6} + \frac{2a^3b^{\frac{11}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6} + \frac{3a^2b^{\frac{13}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6} + \frac{12ab^{\frac{15}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6} + \frac{8b^{\frac{17}{2}}\sqrt{\frac{ax^2}{b}+1}}{15a^5b^4x^4+30a^4b^5x^2+15a^3b^6}$$

input
```
integrate(x**4/(a+b/x**2)**(1/2),x)
```

output
```
3*a**4*b**(9/2)*x**8*sqrt(a*x**2/b + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**2 + 15*a**3*b**6) + 2*a**3*b**(11/2)*x**6*sqrt(a*x**2/b + 1)/(15*a**5*b
**4*x**4 + 30*a**4*b**5*x**2 + 15*a**3*b**6) + 3*a**2*b**(13/2)*x**4*sqrt(
a*x**2/b + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**2 + 15*a**3*b**6) + 12*
a*b**(15/2)*x**2*sqrt(a*x**2/b + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**2
 + 15*a**3*b**6) + 8*b**(17/2)*sqrt(a*x**2/b + 1)/(15*a**5*b**4*x**4 + 30*
a**4*b**5*x**2 + 15*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{3\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}x^5 - 10\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}bx^3 + 15\sqrt{a+\frac{b}{x^2}}b^2x}{15\,a^3}$$

input
```
integrate(x^4/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*(a + b/x^2)^(5/2)*x^5 - 10*(a + b/x^2)^(3/2)*b*x^3 + 15*sqrt(a + b
/x^2)*b^2*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\frac{8\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,a^3} + \frac{\sqrt{ax^2+b}b^2}{a^3\mathrm{sgn}\,(x)} + \frac{3\,(ax^2+b)^{\frac{5}{2}} - 10\,(ax^2+b)^{\frac{3}{2}}b}{15\,a^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4/(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
-8/15*b^(5/2)*sgn(x)/a^3 + sqrt(a*x^2 + b)*b^2/(a^3*sgn(x)) + 1/15*(3*(a*x
^2 + b)^(5/2) - 10*(a*x^2 + b)^(3/2)*b)/(a^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{x^5\sqrt{a+\frac{b}{x^2}}\left(3\,a^2+\frac{8\,b^2}{x^4}-\frac{4\,a\,b}{x^2}\right)}{15\,a^3}$$

input
```
int(x^4/(a + b/x^2)^(1/2),x)
```

output
```
(x^5*(a + b/x^2)^(1/2)*(3*a^2 + (8*b^2)/x^4 - (4*a*b)/x^2))/(15*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{a\,x^2+b}\,(3a^2x^4-4ab\,x^2+8b^2)}{15a^3}$$

input
```
int(x^4/(a+b/x^2)^(1/2),x)
```

output
```
(sqrt(a*x**2 + b)*(3*a**2*x**4 - 4*a*b*x**2 + 8*b**2))/(15*a**3)
```

3.383 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\frac{2b\sqrt{a+\frac{b}{x^2}}\,x}{3a^2} + \frac{\sqrt{a+\frac{b}{x^2}}\,x^3}{3a}$$

output
```
-2/3*b*(a+b/x^2)^(1/2)*x/a^2+1/3*(a+b/x^2)^(1/2)*x^3/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\,x(-2b+ax^2)}{3a^2}$$

input
```
Integrate[x^2/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*x*(-2*b + a*x^2))/(3*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}dx$$
$$\Big\downarrow \text{803}$$
$$\frac{x^3\sqrt{a+\frac{b}{x^2}}}{3a}-\frac{2b\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}dx}{3a}$$
$$\Big\downarrow \text{746}$$
$$\frac{x^3\sqrt{a+\frac{b}{x^2}}}{3a}-\frac{2bx\sqrt{a+\frac{b}{x^2}}}{3a^2}$$

input
```
Int[x^2/Sqrt[a + b/x^2],x]
```

output
```
(-2*b*Sqrt[a + b/x^2]*x)/(3*a^2) + (Sqrt[a + b/x^2]*x^3)/(3*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result	size
trager	$\frac{(a\,x^2-2b)\,x\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3a^2}$	33
orering	$\frac{(a\,x^2-2b)\,(a\,x^2+b)}{3a^2x\sqrt{a+\frac{b}{x^2}}}$	34
gosper	$\frac{(a\,x^2+b)\,(a\,x^2-2b)}{3a^2x\sqrt{\frac{a\,x^2+b}{x^2}}}$	38
default	$\frac{(a\,x^2+b)\,(a\,x^2-2b)}{3a^2x\sqrt{\frac{a\,x^2+b}{x^2}}}$	38
risch	$\frac{(a\,x^2+b)\,(a\,x^2-2b)}{3a^2x\sqrt{\frac{a\,x^2+b}{x^2}}}$	38

input
```
int(x^2/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x^2-2*b)*x/a^2*(-(-a*x^2-b)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{(ax^3-2\,bx)\sqrt{\frac{ax^2+b}{x^2}}}{3\,a^2}$$

input
```
integrate(x^2/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(a*x^3 - 2*b*x)*sqrt((a*x^2 + b)/x^2)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{b}x^2\sqrt{\frac{ax^2}{b}+1}}{3a} - \frac{2b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^2}$$

input
```
integrate(x**2/(a+b/x**2)**(1/2),x)
```

output
```
sqrt(b)*x**2*sqrt(a*x**2/b + 1)/(3*a) - 2*b**(3/2)*sqrt(a*x**2/b + 1)/(3*a
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3 - 3\sqrt{a+\frac{b}{x^2}}\,bx}{3\,a^2}$$

input
```
integrate(x^2/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*((a + b/x^2)^(3/2)*x^3 - 3*sqrt(a + b/x^2)*b*x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{2\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,a^2} + \frac{(ax^2+b)^{\frac{3}{2}}}{3\,a^2\mathrm{sgn}\,(x)} - \frac{\sqrt{ax^2+b}b}{a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^2/(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*b^(3/2)*sgn(x)/a^2 + 1/3*(a*x^2 + b)^(3/2)/(a^2*sgn(x)) - sqrt(a*x^2 +
 b)*b/(a^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{x^3\,\sqrt{a+\frac{b}{x^2}}\,\left(a-\frac{2\,b}{x^2}\right)}{3\,a^2}$$

input
```
int(x^2/(a + b/x^2)^(1/2),x)
```

output
```
(x^3*(a + b/x^2)^(1/2)*(a - (2*b)/x^2))/(3*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{a\,x^2+b}\,(a\,x^2-2b)}{3a^2}$$

input
```
int(x^2/(a+b/x^2)^(1/2),x)
```

output
```
(sqrt(a*x**2 + b)*(a*x**2 - 2*b))/(3*a**2)
```

3.384 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx=\frac{\sqrt{a+\frac{b}{x^2}}x}{a}$$

output
```
(a+b/x^2)^(1/2)*x/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx=\frac{\sqrt{a+\frac{b}{x^2}}x}{a}$$

input
```
Integrate[1/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*x)/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

$$\Big\downarrow \text{746}$$

$$\frac{x\sqrt{a+\frac{b}{x^2}}}{a}$$

input
```
Int[1/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[a + b/x^2]*x)/a
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

method	result	size
trager	$\frac{x\sqrt{-\frac{-a\,x^{2}-b}{x^{2}}}}{a}$	23
orering	$\frac{a\,x^{2}+b}{ax\sqrt{a+\frac{b}{x^{2}}}}$	24
gosper	$\frac{a\,x^{2}+b}{ax\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}$	28
default	$\frac{a\,x^{2}+b}{ax\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}$	28
risch	$\frac{a\,x^{2}+b}{ax\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}$	28

input
```
int(1/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/a*(-(-a*x^2-b)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx=\frac{x\sqrt{\frac{ax^2+b}{x^2}}}{a}$$

input
```
integrate(1/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
x*sqrt((a*x^2 + b)/x^2)/a
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}{a}$$

input
```
integrate(1/(a+b/x**2)**(1/2),x)
```

output
```
sqrt(b)*sqrt(a*x**2/b + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\,x}{a}$$

input
```
integrate(1/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x^2)*x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx = -\frac{\sqrt{b}\mathrm{sgn}(x)}{a} + \frac{\sqrt{ax^2+b}}{a\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*sgn(x)/a + sqrt(a*x^2 + b)/(a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{x\,\sqrt{\frac{a\,x^2}{b}+1}}{\sqrt{a+\frac{b}{x^2}}\,\left(\sqrt{\frac{a\,x^2}{b}+1}+1\right)}$$

input
```
int(1/(a + b/x^2)^(1/2),x)
```

output
```
(x*((a*x^2)/b + 1)^(1/2))/((a + b/x^2)^(1/2)*(((a*x^2)/b + 1)^(1/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{a\,x^2+b}}{a}$$

input
```
int(1/(a+b/x^2)^(1/2),x)
```

output
```
sqrt(a*x**2 + b)/a
```

3.385 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)}{\sqrt{b}}$$

output
```
-arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx=-\frac{\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{a+\frac{b}{x^2}}x}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*x^2),x]
```

output
```
-((Sqrt[b + a*x^2]*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[a + b/x
^2]*x))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {858, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{224}$$

$$-\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[a + b/x^2]*x^2),x]
```

output
```
-(ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)]/Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result	size
default	$-\frac{\sqrt{a\,x^{2}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)}{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x\sqrt{b}}$	52

input
```
int(1/(a+b/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(a*x^2+b)
^(1/2)+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx = \left[\frac{\log\left(-\frac{ax^2-2\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)}{2\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)}{b}\right]$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2)/sqrt(b),
sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b)/b]
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b/x**2)**(1/2)/x**2,x)
```

output
```
-asinh(sqrt(b)/(sqrt(a)*x))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{2\sqrt{b}}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/sqrt(
b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(a*x^2 + b)/sqrt(-b
))/(sqrt(-b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx = -\frac{\ln\left(\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}}{x}\right)}{\sqrt{b}}$$

input
```
int(1/(x^2*(a + b/x^2)^(1/2)),x)
```

output
```
-log((a + b/x^2)^(1/2) + b^(1/2)/x)/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\,dx = \frac{\sqrt{b}\left(\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)-\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x+\sqrt{b}}{\sqrt{b}}\right)\right)}{b}$$

input
```
int(1/(a+b/x^2)^(1/2)/x^2,x)
```

output
```
(sqrt(b)*(log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b)) - log((sqr
t(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))))/b
```

3.386 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4} \, dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)}{2b^{3/2}}$$

output
```
-1/2*(a+b/x^2)^(1/2)/b/x+1/2*a*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4} \, dx = \frac{-\sqrt{b}(b+ax^2)+ax^2\sqrt{b+ax^2}\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{a+\frac{b}{x^2}}x^3}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*x^4),x]
```

output
```
(-(Sqrt[b]*(b + a*x^2)) + a*x^2*Sqrt[b + a*x^2]*ArcTanh[Sqrt[b + a*x^2]/Sq
rt[b]])/(2*b^(3/2)*Sqrt[a + b/x^2]*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {858, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{2b}-\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{2b}-\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}-\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}$$

input
```
Int[1/(Sqrt[a + b/x^2]*x^4),x]
```

output
```
-1/2*Sqrt[a + b/x^2]/(b*x) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*b
^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\sqrt{a\,x^{2}+b}\,\left(-a \ln \left(\frac{2 b +2 \sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right) b\,x^{2}+\sqrt{a\,x^{2}+b}\,b^{\frac{3}{2}}\right)}{2 \sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x^{3} b^{\frac{5}{2}}}$	73
risch	$-\frac{a\,x^{2}+b}{2 b\,x^{3} \sqrt{\frac{a\,x^{2}+b}{x^{2}}}}+\frac{a \ln \left(\frac{2 b +2 \sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right) \sqrt{a\,x^{2}+b}}{2 b^{\frac{3}{2}} \sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}$	84

input
```
int(1/(a+b/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x^2+b)^(1/2)*(-a*ln(2*(b^(1/2)*(a*x^2+b)^(1/2)+b)/x)*b*x^2+(a*x^2+
b)^(1/2)*b^(3/2))/((a*x^2+b)/x^2)^(1/2)/x^3/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}\,dx = \left[\frac{a\sqrt{b}x\log\left(-\frac{ax^2+2\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2b}{x^2}\right)-2b\sqrt{\frac{ax^2+b}{x^2}}}{4b^2x}, -\frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)+b\sqrt{\frac{ax^2+b}{x^2}}}{2b^2x}\right]$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/4*(a*sqrt(b)*x*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x
^2) - 2*b*sqrt((a*x^2 + b)/x^2))/(b^2*x), -1/2*(a*sqrt(-b)*x*arctan(sqrt(-
b)*x*sqrt((a*x^2 + b)/x^2)/b) + b*sqrt((a*x^2 + b)/x^2))/(b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}\,dx = -\frac{\sqrt{a}\sqrt{1+\frac{b}{ax^2}}}{2bx}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{2b^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x**2)**(1/2)/x**4,x)
```

output
```
-sqrt(a)*sqrt(1 + b/(a*x**2))/(2*b*x) + a*asinh(sqrt(b)/(sqrt(a)*x))/(2*b*
*(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}ax}{2\left(\left(a+\frac{b}{x^2}\right)bx^2-b^2\right)} - \frac{a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{4\,b^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/2*sqrt(a + b/x^2)*a*x/((a + b/x^2)*b*x^2 - b^2) - 1/4*a*log((sqrt(a + b
/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}\,dx = -\frac{a\left(\frac{\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\sqrt{ax^2+b}}{abx^2}\right)}{2\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/2*a*(arctan(sqrt(a*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^2 + b)/(a
*b*x^2))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}\,dx = \begin{cases} -\frac{1}{3\sqrt{a}\,x^3} & \text{if } b=0 \\ \frac{a\ln\left(2\sqrt{a+\frac{b}{x^2}}+\frac{2\sqrt{b}}{x}\right)}{2b^{3/2}} - \frac{\sqrt{a+\frac{b}{x^2}}}{2\,b\,x} & \text{if } b\neq 0 \end{cases}$$

input
```
int(1/(x^4*(a + b/x^2)^(1/2)),x)
```

output
```
piecewise(b == 0, -1/(3*a^(1/2)*x^3), b ~= 0, (a*log(2*(a + b/x^2)^(1/2) +
 (2*b^(1/2))/x))/(2*b^(3/2)) - (a + b/x^2)^(1/2)/(2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}\,dx = \frac{-\sqrt{a\,x^2+b}\,b-\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a\,x^2+\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x+\sqrt{b}}{\sqrt{b}}\right)a\,x^2}{2b^2x^2}$$

input
```
int(1/(a+b/x^2)^(1/2)/x^4,x)
```

output
```
( - sqrt(a*x**2 + b)*b - sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(
b))/sqrt(b))*a*x**2 + sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))
/sqrt(b))*a*x**2)/(2*b**2*x**2)
```

3.387 $\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}x} \, dx = -\frac{\arctan\left(\frac{\sqrt{-a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-arctan((-a+b/x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}x} \, dx = \frac{\sqrt{-b+ax^2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{-b+ax^2}}\right)}{\sqrt{a}\sqrt{-a+\frac{b}{x^2}}x}$$

input
```
Integrate[1/(Sqrt[-a + b/x^2]*x),x]
```

output
```
(Sqrt[-b + a*x^2]*ArcTanh[(Sqrt[a]*x)/Sqrt[-b + a*x^2]])/(Sqrt[a]*Sqrt[-a
+ b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\frac{b}{x^2}-a}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^2}{\sqrt{\frac{b}{x^2}-a}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{a}{b}+\frac{1}{bx^4}}d\sqrt{\frac{b}{x^2}-a}}{b}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\arctan\left(\frac{\sqrt{\frac{b}{x^2}-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[-a + b/x^2]*x),x]
```

output
```
-(ArcTan[Sqrt[-a + b/x^2]/Sqrt[a]]/Sqrt[a])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

method	result	size
default	$\frac{\sqrt{-a\,x^{2}+b}\,\arctan\left(\frac{\sqrt{a}\,x}{\sqrt{-a\,x^{2}+b}}\right)}{\sqrt{-\frac{a\,x^{2}-b}{x^{2}}}\,x\sqrt{a}}$	50

input
```
int(1/(-a+b/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/(-(a*x^2-b)/x^2)^(1/2)/x*(-a*x^2+b)^(1/2)/a^(1/2)*arctan(a^(1/2)*x/(-a*x
^2+b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx = \left[-\frac{\sqrt{-a}\log\left(2\,ax^2-2\,\sqrt{-a}x^2\sqrt{-\frac{ax^2-b}{x^2}}-b\right)}{2\,a},\ -\frac{\arctan\left(\frac{\sqrt{a}x^2\sqrt{-\frac{ax^2-b}{x^2}}}{ax^2-b}\right)}{\sqrt{a}}\right]$$

input
```
integrate(1/(-a+b/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a)*log(2*a*x^2 - 2*sqrt(-a)*x^2*sqrt(-(a*x^2 - b)/x^2) - b)/a,
 -arctan(sqrt(a)*x^2*sqrt(-(a*x^2 - b)/x^2)/(a*x^2 - b))/sqrt(a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx = \begin{cases} -\frac{i\,\text{acosh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}} & \text{for } \left|\frac{ax^2}{b}\right| > 1 \\ \frac{\text{asin}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-a+b/x**2)**(1/2)/x,x)
```

output
```
Piecewise((-I*acosh(sqrt(a)*x/sqrt(b))/sqrt(a), Abs(a*x**2/b) > 1), (asin(
sqrt(a)*x/sqrt(b))/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx = -\frac{\arctan\left(\frac{\sqrt{-a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/(-a+b/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-arctan(sqrt(-a + b/x^2)/sqrt(a))/sqrt(a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx = \frac{\log(|b|)\,\mathrm{sgn}(x)}{2\sqrt{-a}} - \frac{\log\left(\left|-\sqrt{-a}x+\sqrt{-ax^2+b}\right|\right)}{\sqrt{-a}\mathrm{sgn}(x)}$$

input
```
integrate(1/(-a+b/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(x)/sqrt(-a) - log(abs(-sqrt(-a)*x + sqrt(-a*x^2 + b)))
/(sqrt(-a)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx = -\frac{\mathrm{atan}\left(\frac{\sqrt{\frac{b}{x^2}-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(b/x^2 - a)^(1/2)),x)
```

output
```
-atan((b/x^2 - a)^(1/2)/a^(1/2))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^2}}\,x}\,dx=\frac{\sqrt{a}\,asin\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{a}$$

input
```
int(1/(-a+b/x^2)^(1/2)/x,x)
```

output
```
(sqrt(a)*asin((sqrt(a)*x)/sqrt(b)))/a
```

3.388 $\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2} dx = -\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

output
```
-arccsch(2^(1/2)*x/b^(1/2))/b^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2} dx = -\frac{\sqrt{b+2x^2}\operatorname{arctanh}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{2+\frac{b}{x^2}}x}$$

input
```
Integrate[1/(Sqrt[2 + b/x^2]*x^2),x]
```

output
```
-((Sqrt[b + 2*x^2]*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[2 + b/x
^2]*x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {858, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\frac{b}{x^2}+2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{\frac{b}{x^2}+2}}d\frac{1}{x}$$

$$\Big\downarrow \text{222}$$

$$-\frac{\text{arcsinh}\left(\frac{\sqrt{b}}{\sqrt{2}x}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[2 + b/x^2]*x^2),x]
```

output
```
-(ArcSinh[Sqrt[b]/(Sqrt[2]*x)]/Sqrt[b])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

method	result	size
default	$-\frac{\sqrt{2x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{2x^2+b}}{x}\right)}{\sqrt{\frac{2x^2+b}{x^2}}\,x\sqrt{b}}$	52

input
```
int(1/(2+b/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/((2*x^2+b)/x^2)^(1/2)/x*(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)
^(1/2)+b)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.45

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2}\,dx = \left[\frac{\log\left(-\frac{x^2-\sqrt{b}x\sqrt{\frac{2\,x^2+b}{x^2}}+b}{x^2}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{2\,x^2+b}{x^2}}}{b}\right)}{b}\right]$$

input
```
integrate(1/(2+b/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-
b)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/b)/b]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}}{2x}\right)}{\sqrt{b}}$$

input
```
integrate(1/(2+b/x**2)**(1/2)/x**2,x)
```

output
```
-asinh(sqrt(2)*sqrt(b)/(2*x))/sqrt(b)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2}\,dx = \frac{\log\left(\frac{x\sqrt{\frac{b}{x^2}+2}-\sqrt{b}}{x\sqrt{\frac{b}{x^2}+2}+\sqrt{b}}\right)}{2\sqrt{b}}$$

input
```
integrate(1/(2+b/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*log((x*sqrt(b/x^2 + 2) - sqrt(b))/(x*sqrt(b/x^2 + 2) + sqrt(b)))/sqrt(
b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{2\,x^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(2+b/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b) + arctan(sqrt(2*x^2 + b)/sqrt(-b
))/(sqrt(-b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2}\,dx = -\frac{\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2\,x}\right)}{\sqrt{b}}$$

input
```
int(1/(x^2*(b/x^2 + 2)^(1/2)),x)
```

output
```
-asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{2+\frac{b}{x^2}}x^2}\,dx = \frac{\sqrt{b}\left(\log\left(\frac{\sqrt{2x^2+b}-\sqrt{b}+\sqrt{2}\,x}{\sqrt{b}}\right)-\log\left(\frac{\sqrt{2x^2+b}+\sqrt{b}+\sqrt{2}\,x}{\sqrt{b}}\right)\right)}{b}$$

input
```
int(1/(2+b/x^2)^(1/2)/x^2,x)
```

output
```
(sqrt(b)*(log((sqrt(b + 2*x**2) - sqrt(b) + sqrt(2)*x)/sqrt(b)) - log((sqr
t(b + 2*x**2) + sqrt(b) + sqrt(2)*x)/sqrt(b))))/b
```

3.389 $\int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2} dx = -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

output
```
-arccsc(2^(1/2)*x/b^(1/2))/b^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(20) = 40.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2} dx = \frac{\sqrt{-b+2x^2}\arctan\left(\frac{\sqrt{-b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{2-\frac{b}{x^2}}x}$$

input
```
Integrate[1/(Sqrt[2 - b/x^2]*x^2),x]
```

output
```
(Sqrt[-b + 2*x^2]*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[2 - b/x^
2]*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {858, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{2-\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{2-\frac{b}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\arcsin\left(\frac{\sqrt{b}}{\sqrt{2}x}\right)}{\sqrt{b}}$$

input
```
Int[1/(Sqrt[2 - b/x^2]*x^2),x]
```

output
```
-(ArcSin[Sqrt[b]/(Sqrt[2]*x)]/Sqrt[b])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

method	result	size
default	$-\frac{\sqrt{2x^2-b}\,\ln\left(\frac{-2b+2\sqrt{-b}\,\sqrt{2x^2-b}}{x}\right)}{\sqrt{-\frac{-2x^2+b}{x^2}}\,x\sqrt{-b}}$	63

input
```
int(1/(2-b/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(-(-2*x^2+b)/x^2)^(1/2)/x*(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(
2*x^2-b)^(1/2)-b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.60

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{x^2-\sqrt{-b}x\sqrt{\frac{2\,x^2-b}{x^2}}-b}{x^2}\right)}{2\,b}, \frac{\arctan\left(\frac{x\sqrt{\frac{2\,x^2-b}{x^2}}}{\sqrt{b}}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/(2-b/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, a
rctan(x*sqrt((2*x^2 - b)/x^2)/sqrt(b))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx = \begin{cases} \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}}{2x}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{b}{x^2}\right| > 2 \\ -\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}}{2x}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2-b/x**2)**(1/2)/x**2,x)
```

output
```
Piecewise((I*acosh(sqrt(2)*sqrt(b)/(2*x))/sqrt(b), Abs(b/x**2) > 2), (-asi
n(sqrt(2)*sqrt(b)/(2*x))/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx = \frac{\arctan\left(\frac{x\sqrt{-\frac{b}{x^2}+2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(2-b/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
arctan(x*sqrt(-b/x^2 + 2)/sqrt(b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)}{\sqrt{b}\operatorname{sgn}(x)}$$

input
```
integrate(1/(2-b/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-arctan(sqrt(-b)/sqrt(b))*sgn(x)/sqrt(b) + arctan(sqrt(2*x^2 - b)/sqrt(b))
/(sqrt(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{-b}}{2\,x}\right)}{\sqrt{-b}}$$

input
```
int(1/(x^2*(2 - b/x^2)^(1/2)),x)
```

output
```
-asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{2-\frac{b}{x^2}}x^2}\,dx = \frac{2\sqrt{b}\,atan\left(\frac{\sqrt{2x^2-b}+\sqrt{2}\,x}{\sqrt{b}}\right)}{b}$$

input
```
int(1/(2-b/x^2)^(1/2)/x^2,x)
```

output
```
(2*sqrt(b)*atan((sqrt( - b + 2*x**2) + sqrt(2)*x)/sqrt(b)))/b
```

3.390 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=-\frac{15b^2}{8a^3\sqrt{a+\frac{b}{x^2}}}-\frac{5bx^2}{8a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^4}{4a\sqrt{a+\frac{b}{x^2}}}+\frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
-15/8*b^2/a^3/(a+b/x^2)^(1/2)-5/8*b*x^2/a^2/(a+b/x^2)^(1/2)+1/4*x^4/a/(a+b
/x^2)^(1/2)+15/8*b^2*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{\sqrt{a}x(-15b^2-5abx^2+2a^2x^4)+30b^2\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{8a^{7/2}\sqrt{a+\frac{b}{x^2}}\,x}$$

input
```
Integrate[x^3/(a + b/x^2)^(3/2),x]
```

output
```
(Sqrt[a]*x*(-15*b^2 - 5*a*b*x^2 + 2*a^2*x^4) + 30*b^2*Sqrt[b + a*x^2]*ArcT
anh[(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b + a*x^2])])/(8*a^(7/2)*Sqrt[a + b/x^2]*
x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{5b\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{4a}+\frac{x^4}{2a\sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{5b\left(-\frac{3b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{2a}-\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{4a}+\frac{x^4}{2a\sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{5b\left(-\frac{3b\left(\frac{\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{2a}-\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{4a}+\frac{x^4}{2a\sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{5b\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{2a}-\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{4a}+\frac{x^4}{2a\sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{4a}+\frac{x^4}{2a\sqrt{a+\frac{b}{x^2}}}\right)$$

input
```
Int[x^3/(a + b/x^2)^(3/2),x]
```

output
```
(x^4/(2*a*Sqrt[a + b/x^2]) + (5*b*(-(x^2/(a*Sqrt[a + b/x^2])) - (3*b*(2/(a
*Sqrt[a + b/x^2]) - (2*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/a^(3/2)))/(2*a)))
/(4*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\left(a\,x^{2}+b\right)\left(2x^{5}a^{\frac{7}{2}}-5a^{\frac{5}{2}}b\,x^{3}-15a^{\frac{3}{2}}b^{2}x+15\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)\sqrt{a\,x^{2}+b}\,a\,b^{2}\right)}{8\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}x^{3}a^{\frac{9}{2}}}$	87
risch	$\frac{\left(2a\,x^{2}-7b\right)\left(a\,x^{2}+b\right)}{8a^{3}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}+\frac{\left(-\frac{b^{2}x}{a^{3}\sqrt{a\,x^{2}+b}}+\frac{15b^{2}\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)}{8a^{\frac{7}{2}}}\right)\sqrt{a\,x^{2}+b}}{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}$	106

input
```
int(x^3/(a+b/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(a*x^2+b)*(2*x^5*a^(7/2)-5*a^(5/2)*b*x^3-15*a^(3/2)*b^2*x+15*ln(a^(1/2
)*x+(a*x^2+b)^(1/2))*(a*x^2+b)^(1/2)*a*b^2)/((a*x^2+b)/x^2)^(3/2)/x^3/a^(9
/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.33

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \left[\frac{15\,(ab^2x^2+b^3)\sqrt{a}\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\,(2\,a^3x^6-5\,a^2bx^4-15\,ab^2x^2)\sqrt{\frac{ax^2+b}{x^2}}}{16\,(a^5x^2+a^4b)}, -\frac{15\,(ab^2x^2+b^3)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-(2\,a^3x^6-5\,a^2bx^4-15\,ab^2x^2)\sqrt{\frac{ax^2+b}{x^2}}}{8\,(a^5x^2+a^4b)}\right]$$

input
```
integrate(x^3/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(15*(a*b^2*x^2 + b^3)*sqrt(a)*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x
^2 + b)/x^2) - b) + 2*(2*a^3*x^6 - 5*a^2*b*x^4 - 15*a*b^2*x^2)*sqrt((a*x^2
 + b)/x^2))/(a^5*x^2 + a^4*b), -1/8*(15*(a*b^2*x^2 + b^3)*sqrt(-a)*arctan(
sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (2*a^3*x^6 - 5*a^2*b*x^4
 - 15*a*b^2*x^2)*sqrt((a*x^2 + b)/x^2))/(a^5*x^2 + a^4*b)]
```

Sympy [A] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{x^5}{4a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} - \frac{5\sqrt{b}x^3}{8a^2\sqrt{\frac{ax^2}{b}+1}} - \frac{15b^{\frac{3}{2}}x}{8a^3\sqrt{\frac{ax^2}{b}+1}} + \frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{8a^{\frac{7}{2}}}$$

input
```
integrate(x**3/(a+b/x**2)**(3/2),x)
```

output
```
x**5/(4*a*sqrt(b)*sqrt(a*x**2/b + 1)) - 5*sqrt(b)*x**3/(8*a**2*sqrt(a*x**2
/b + 1)) - 15*b**(3/2)*x/(8*a**3*sqrt(a*x**2/b + 1)) + 15*b**2*asinh(sqrt(
a)*x/sqrt(b))/(8*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = -\frac{15\left(a+\frac{b}{x^2}\right)^2 b^2 - 25\left(a+\frac{b}{x^2}\right)ab^2 + 8a^2b^2}{8\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3 - 2\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4 + \sqrt{a+\frac{b}{x^2}}\,a^5\right)} - \frac{15\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}$$

input
```
integrate(x^3/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(15*(a + b/x^2)^2*b^2 - 25*(a + b/x^2)*a*b^2 + 8*a^2*b^2)/((a + b/x^2
)^(5/2)*a^3 - 2*(a + b/x^2)^(3/2)*a^4 + sqrt(a + b/x^2)*a^5) - 15/16*b^2*l
og((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{\left(x^2\left(\frac{2\,x^2}{a\mathrm{sgn}(x)} - \frac{5\,b}{a^2\mathrm{sgn}(x)}\right) - \frac{15\,b^2}{a^3\mathrm{sgn}(x)}\right)x}{8\sqrt{ax^2+b}} + \frac{15\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,a^{\frac{7}{2}}} - \frac{15\,b^2\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{8\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(a+b/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(x^2*(2*x^2/(a*sgn(x)) - 5*b/(a^2*sgn(x))) - 15*b^2/(a^3*sgn(x)))*x/sq
rt(a*x^2 + b) + 15/16*b^2*log(abs(b))*sgn(x)/a^(7/2) - 15/8*b^2*log(abs(-s
qrt(a)*x + sqrt(a*x^2 + b)))/(a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{15\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{15\,b^2}{8\,a^3\,\sqrt{a+\frac{b}{x^2}}}+\frac{x^4}{4\,a\,\sqrt{a+\frac{b}{x^2}}}-\frac{5\,b\,x^2}{8\,a^2\,\sqrt{a+\frac{b}{x^2}}}$$

input
```
int(x^3/(a + b/x^2)^(3/2),x)
```

output
```
(15*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - (15*b^2)/(8*a^3*(a
 + b/x^2)^(1/2)) + x^4/(4*a*(a + b/x^2)^(1/2)) - (5*b*x^2)/(8*a^2*(a + b/x
^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{2\sqrt{a\,x^2+b}\,a^3x^5-5\sqrt{a\,x^2+b}\,a^2b\,x^3-15\sqrt{a\,x^2+b}\,a\,b^2x+15\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)a\cdots}{8a^4\left(a\,x^2+b\right)}$$

input
```
int(x^3/(a+b/x^2)^(3/2),x)
```

output
```
(2*sqrt(a*x**2 + b)*a**3*x**5 - 5*sqrt(a*x**2 + b)*a**2*b*x**3 - 15*sqrt(a
*x**2 + b)*a*b**2*x + 15*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b
))*a*b**2*x**2 + 15*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b*
*3 - 10*sqrt(a)*a*b**2*x**2 - 10*sqrt(a)*b**3)/(8*a**4*(a*x**2 + b))
```

3.391 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{3b}{2a^2\sqrt{a+\frac{b}{x^2}}}+\frac{x^2}{2a\sqrt{a+\frac{b}{x^2}}}-\frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
3/2*b/a^2/(a+b/x^2)^(1/2)+1/2*x^2/a/(a+b/x^2)^(1/2)-3/2*b*arctanh((a+b/x^2
)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{\sqrt{a}x(3b+ax^2)+6b\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{b}-\sqrt{b+ax^2}}\right)}{2a^{5/2}\sqrt{a+\frac{b}{x^2}}x}$$

input
```
Integrate[x/(a + b/x^2)^(3/2),x]
```

output
```
(Sqrt[a]*x*(3*b + a*x^2) + 6*b*Sqrt[b + a*x^2]*ArcTanh[(Sqrt[a]*x)/(Sqrt[b
] - Sqrt[b + a*x^2])])/(2*a^(5/2)*Sqrt[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {798, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{3b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{2a}+\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{2a}+\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}\right)}{2a}+\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}+\frac{x^2}{a\sqrt{a+\frac{b}{x^2}}}\right)$$

input
```
Int[x/(a + b/x^2)^(3/2),x]
```

output
```
(x^2/(a*Sqrt[a + b/x^2]) + (3*b*(2/(a*Sqrt[a + b/x^2]) - (2*ArcTanh[Sqrt[a
 + b/x^2]/Sqrt[a]])/a^(3/2)))/(2*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{\left(a\,x^{2}+b\right)\left(-x^{3} a^{\frac{5}{2}}-3 a^{\frac{3}{2}} b x+3 \sqrt{a\,x^{2}+b}\,\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right) a b\right)}{2 \left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}} x^{3} a^{\frac{7}{2}}}$	74
risch	$\frac{a\,x^{2}+b}{2 a^{2} \sqrt{\frac{a\,x^{2}+b}{x^{2}}}}+\frac{\left(\frac{b x}{a^{2} \sqrt{a\,x^{2}+b}}-\frac{3 b \ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)}{2 a^{\frac{5}{2}}}\right) \sqrt{a\,x^{2}+b}}{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\, x}$	91

input
```
int(x/(a+b/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x^2+b)*(-x^3*a^(5/2)-3*a^(3/2)*b*x+3*(a*x^2+b)^(1/2)*ln(a^(1/2)*x+
(a*x^2+b)^(1/2))*a*b)/((a*x^2+b)/x^2)^(3/2)/x^3/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.78

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \left[\frac{3\,(abx^2+b^2)\sqrt{a}\log\left(-2\,ax^2+2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\,(a^2x^4+3\,abx^2)\sqrt{\frac{ax^2+b}{x^2}}}{4\,(a^4x^2+a^3b)}, \frac{3\,(\ldots}{\ldots}\right.$$

input
```
integrate(x/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*b*x^2 + b^2)*sqrt(a)*log(-2*a*x^2 + 2*sqrt(a)*x^2*sqrt((a*x^2 +
 b)/x^2) - b) + 2*(a^2*x^4 + 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/(a^4*x^2 +
a^3*b), 1/2*(3*(a*b*x^2 + b^2)*sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 +
b)/x^2)/(a*x^2 + b)) + (a^2*x^4 + 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/(a^4*x
^2 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{x^3}{2a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} + \frac{3\sqrt{b}x}{2a^2\sqrt{\frac{ax^2}{b}+1}} - \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{2a^{\frac{5}{2}}}$$

input
```
integrate(x/(a+b/x**2)**(3/2),x)
```

output
```
x**3/(2*a*sqrt(b)*sqrt(a*x**2/b + 1)) + 3*sqrt(b)*x/(2*a**2*sqrt(a*x**2/b
+ 1)) - 3*b*asinh(sqrt(a)*x/sqrt(b))/(2*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{3\left(a+\frac{b}{x^2}\right)b-2\,ab}{2\left(\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x^2}}\,a^3\right)}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}$$

input
```
integrate(x/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(3*(a + b/x^2)*b - 2*a*b)/((a + b/x^2)^(3/2)*a^2 - sqrt(a + b/x^2)*a^3
) + 3/4*b*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(
5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{x\left(\frac{x^2}{a\mathrm{sgn}(x)}+\frac{3\,b}{a^2\mathrm{sgn}(x)}\right)}{2\,\sqrt{ax^2+b}} - \frac{3\,b\log\left(|b|\right)\mathrm{sgn}(x)}{4\,a^{\frac{5}{2}}}+\frac{3\,b\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x/(a+b/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*(x^2/(a*sgn(x)) + 3*b/(a^2*sgn(x)))/sqrt(a*x^2 + b) - 3/4*b*log(abs(
b))*sgn(x)/a^(5/2) + 3/2*b*log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))/(a^(5/2)
*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{3\,b}{2\,a^2\,\sqrt{a+\frac{b}{x^2}}}+\frac{x^2}{2\,a\,\sqrt{a+\frac{b}{x^2}}}-\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int(x/(a + b/x^2)^(3/2),x)
```

output
```
(3*b)/(2*a^2*(a + b/x^2)^(1/2)) + x^2/(2*a*(a + b/x^2)^(1/2)) - (3*b*atanh
((a + b/x^2)^(1/2)/a^(1/2)))/(2*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.65

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{4\sqrt{a\,x^2+b}\,a^2x^3+12\sqrt{a\,x^2+b}\,abx-12\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)ab\,x^2-12\sqrt{a}\log\left(\frac{\sqrt{a\,x^2\cdots}}{}\right.}{8a^3\left(a\,x^2+b\right)}$$

input
```
int(x/(a+b/x^2)^(3/2),x)
```

output
```
(4*sqrt(a*x**2 + b)*a**2*x**3 + 12*sqrt(a*x**2 + b)*a*b*x - 12*sqrt(a)*log
((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*a*b*x**2 - 12*sqrt(a)*log((sqrt(a
*x**2 + b) + sqrt(a)*x)/sqrt(b))*b**2 + 9*sqrt(a)*a*b*x**2 + 9*sqrt(a)*b**
2)/(8*a**3*(a*x**2 + b))
```

3.392 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x} \, dx = -\frac{1}{a\sqrt{a+\frac{b}{x^2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-1/a/(a+b/x^2)^(1/2)+arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x} \, dx = \frac{-\sqrt{a}x + 2\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{a^{3/2}\sqrt{a+\frac{b}{x^2}}x}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x),x]
```

output
```
(-(Sqrt[a]*x) + 2*Sqrt[b + a*x^2]*ArcTanh[(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b +
 a*x^2])])/(a^(3/2)*Sqrt[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+\frac{b}{x^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{2} \int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}} d\frac{1}{x^2}$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2} \left(-\frac{\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}} d\frac{1}{x^2}}{a}-\frac{2}{a \sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2} \left(-\frac{2 \int \frac{1}{\frac{1}{b x^4}-\frac{a}{b}} d\sqrt{a+\frac{b}{x^2}}}{a b}-\frac{2}{a \sqrt{a+\frac{b}{x^2}}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2} \left(\frac{2 \text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a \sqrt{a+\frac{b}{x^2}}}\right)$$

input
```
Int[1/((a + b/x^2)^(3/2)*x),x]
```

output
```
(-2/(a*Sqrt[a + b/x^2]) + (2*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/a^(3/2))/2
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

method	result	size
default	$-\frac{\left(a\,x^{2}+b\right)\left(x\,a^{\frac{3}{2}}-\ln\left(\sqrt{a}\,x+\sqrt{a\,x^{2}+b}\right)a\sqrt{a\,x^{2}+b}\right)}{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}x^{3}a^{\frac{5}{2}}}$	63

input
```
int(1/(a+b/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-(a*x^2+b)*(x*a^(3/2)-ln(a^(1/2)*x+(a*x^2+b)^(1/2))*a*(a*x^2+b)^(1/2))/((a
*x^2+b)/x^2)^(3/2)/x^3/a^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.98

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx = \left[-\frac{2\,ax^2\sqrt{\frac{ax^2+b}{x^2}} - (ax^2+b)\sqrt{a}\log\left(-2\,ax^2 - 2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}} - b\right)}{2\,(a^3x^2+a^2b)}, -\frac{ax^2\sqrt{\frac{ax^2+b}{x^2}} + (ax^2+b)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)}{a^3x^2+a^2b}\right]$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/2*(2*a*x^2*sqrt((a*x^2 + b)/x^2) - (a*x^2 + b)*sqrt(a)*log(-2*a*x^2 -
2*sqrt(a)*x^2*sqrt((a*x^2 + b)/x^2) - b))/(a^3*x^2 + a^2*b), -(a*x^2*sqrt(
(a*x^2 + b)/x^2) + (a*x^2 + b)*sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 +
b)/x^2)/(a*x^2 + b)))/(a^3*x^2 + a^2*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(34) = 68$.

Time = 1.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.56

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx = -\frac{2a^3x^2\sqrt{1+\frac{b}{ax^2}}}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b} - \frac{a^3x^2\log\left(\frac{b}{ax^2}\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b} + \frac{2a^3x^2\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b} - \frac{a^2b\log\left(\frac{b}{ax^2}\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b} + \frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax^2}}+1\right)}{2a^{\frac{9}{2}}x^2+2a^{\frac{7}{2}}b}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x,x)
```

output
```
-2*a**3*x**2*sqrt(1 + b/(a*x**2))/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) - a**3*
x**2*log(b/(a*x**2))/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) + 2*a**3*x**2*log(sq
rt(1 + b/(a*x**2)) + 1)/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) - a**2*b*log(b/(a
*x**2))/(2*a**(9/2)*x**2 + 2*a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/(a*x**2
)) + 1)/(2*a**(9/2)*x**2 + 2*a**(7/2)*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}} - \frac{1}{\sqrt{a+\frac{b}{x^2}}a}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(3/2)
- 1/(sqrt(a + b/x^2)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx = \frac{\log\left(|b|\right)\operatorname{sgn}(x)}{2\,a^{\frac{3}{2}}} - \frac{x}{\sqrt{ax^2+b}a\operatorname{sgn}\left(x\right)} - \frac{\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/2*log(abs(b))*sgn(x)/a^(3/2) - x/(sqrt(a*x^2 + b)*a*sgn(x)) - log(abs(-s
qrt(a)*x + sqrt(a*x^2 + b)))/(a^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\sqrt{a+\frac{b}{x^2}}}$$

input
```
int(1/(x*(a + b/x^2)^(3/2)),x)
```

output
```
atanh((a + b/x^2)^(1/2)/a^(1/2))/a^(3/2) - 1/(a*(a + b/x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x}\,dx = \frac{-\sqrt{a\,x^2+b}\,ax+\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)a\,x^2+\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)b-\sqrt{a}\,a\,x^2-}{a^2\left(a\,x^2+b\right)}$$

input
```
int(1/(a+b/x^2)^(3/2)/x,x)
```

output
```
( - sqrt(a*x**2 + b)*a*x + sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt
(b))*a*x**2 + sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b - sqrt
(a)*a*x**2 - sqrt(a)*b)/(a**2*(a*x**2 + b))
```

3.393 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3} dx = \frac{1}{b\sqrt{a+\frac{b}{x^2}}}$$

output
```
1/b/(a+b/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3} dx = \frac{1}{b\sqrt{a+\frac{b}{x^2}}}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^3),x]
```

output
```
1/(b*Sqrt[a + b/x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{b\sqrt{a+\frac{b}{x^2}}}$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^3),x]
```

output
```
1/(b*Sqrt[a + b/x^2])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{1}{b\sqrt{a+\frac{b}{x^{2}}}}$	14
orering	$\frac{a\,x^{2}+b}{x^{2} b \left(a+\frac{b}{x^{2}}\right)^{\frac{3}{2}}}$	24
gosper	$\frac{a\,x^{2}+b}{x^{2} b \left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	28
default	$\frac{a\,x^{2}+b}{x^{2} b \left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	28
trager	$\frac{x^{2} \sqrt{-\frac{-a\,x^{2}-b}{x^{2}}}}{b \left(a\,x^{2}+b\right)}$	34

input
```
int(1/(a+b/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/b/(a+b/x^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\,dx = \frac{x^2\sqrt{\frac{ax^2+b}{x^2}}}{abx^2+b^2}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
x^2*sqrt((a*x^2 + b)/x^2)/(a*b*x^2 + b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\, dx = \begin{cases} \frac{1}{b\sqrt{a+\frac{b}{x^2}}} & \text{for } b \neq 0 \\ -\frac{1}{2a^{\frac{3}{2}}x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**3,x)
```

output
```
Piecewise((1/(b*sqrt(a + b/x**2)), Ne(b, 0)), (-1/(2*a**(3/2)*x**2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\, dx = \frac{1}{\sqrt{a+\frac{b}{x^2}}\,b}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/(sqrt(a + b/x^2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\, dx = \frac{x}{\sqrt{ax^2+b}\,b\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
x/(sqrt(a*x^2 + b)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\,dx = \frac{\sqrt{x^2}}{b\sqrt{a\,x^2+b}}$$

input
```
int(1/(x^3*(a + b/x^2)^(3/2)),x)
```

output
```
(x^2)^(1/2)/(b*(b + a*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^3}\,dx = \frac{\sqrt{a\,x^2+b}\,ax+\sqrt{a}\,a\,x^2+\sqrt{a}\,b}{ab\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^3,x)
```

output
```
(sqrt(a*x**2 + b)*a*x + sqrt(a)*a*x**2 + sqrt(a)*b)/(a*b*(a*x**2 + b))
```

3.394 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^5} \, dx = -\frac{a}{b^2\sqrt{a+\frac{b}{x^2}}} - \frac{\sqrt{a+\frac{b}{x^2}}}{b^2}$$

output
```
-a/b^2/(a+b/x^2)^(1/2)-(a+b/x^2)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^5} \, dx = \frac{-b-2ax^2}{b^2\sqrt{a+\frac{b}{x^2}}x^2}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^5),x]
```

output
```
(-b - 2*a*x^2)/(b^2*Sqrt[a + b/x^2]*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{1}{b\sqrt{a+\frac{b}{x^2}}}-\frac{a}{b\left(a+\frac{b}{x^2}\right)^{3/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a}{b^2\sqrt{a+\frac{b}{x^2}}}-\frac{2\sqrt{a+\frac{b}{x^2}}}{b^2}\right)$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^5),x]
```

output
```
((-2*a)/(b^2*Sqrt[a + b/x^2]) - (2*Sqrt[a + b/x^2])/b^2)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
orering	$-\frac{(2a\,x^2+b)\,(a\,x^2+b)}{b^2x^4\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}$	33
gosper	$-\frac{(a\,x^2+b)\,(2a\,x^2+b)}{x^4b^2\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	37
default	$-\frac{(a\,x^2+b)\,(2a\,x^2+b)}{x^4b^2\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	37
trager	$-\frac{(2a\,x^2+b)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{b^2(a\,x^2+b)}$	40
risch	$-\frac{a\,x^2+b}{b^2x^2\sqrt{\frac{a\,x^2+b}{x^2}}}-\frac{a}{b^2\sqrt{\frac{a\,x^2+b}{x^2}}}$	49

input
```
int(1/(a+b/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-(2*a*x^2+b)/b^2*(a*x^2+b)/x^4/(a+b/x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^5}\,dx = -\frac{(2\,ax^2+b)\sqrt{\frac{ax^2+b}{x^2}}}{ab^2x^2+b^3}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-(2*a*x^2 + b)*sqrt((a*x^2 + b)/x^2)/(a*b^2*x^2 + b^3)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^5}\,dx = \begin{cases} -\frac{2a}{b^2\sqrt{a+\frac{b}{x^2}}} - \frac{1}{bx^2\sqrt{a+\frac{b}{x^2}}} & \text{for } b\neq 0 \\ -\frac{1}{4a^{\frac{3}{2}}x^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**5,x)
```

output
```
Piecewise((-2*a/(b**2*sqrt(a + b/x**2)) - 1/(b*x**2*sqrt(a + b/x**2)), Ne(
b, 0)), (-1/(4*a**(3/2)*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^5}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{b^2} - \frac{a}{\sqrt{a+\frac{b}{x^2}}\,b^2}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-sqrt(a + b/x^2)/b^2 - a/(sqrt(a + b/x^2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^5}\,dx=-\frac{ax}{\sqrt{ax^2+b}b^2\mathrm{sgn}\,(x)}+\frac{2\sqrt{a}}{\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)b\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-a*x/(sqrt(a*x^2 + b)*b^2*sgn(x)) + 2*sqrt(a)/(((sqrt(a)*x - sqrt(a*x^2 +
b))^2 - b)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^5}\,dx=-\frac{x\,\sqrt{a+\frac{b}{x^2}}\,\left(\frac{1}{b}+\frac{2\,a\,x^2}{b^2}\right)}{a\,x^3+b\,x}$$

input
```
int(1/(x^5*(a + b/x^2)^(3/2)),x)
```

output
```
-(x*(a + b/x^2)^(1/2)*(1/b + (2*a*x^2)/b^2))/(b*x + a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^5}\,dx = \frac{-2\sqrt{a\,x^2+b}\,a\,x^2-\sqrt{a\,x^2+b}\,b-2\sqrt{a}\,a\,x^3-2\sqrt{a}\,bx}{b^2x\left(a\,x^2+b\right)}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^5,x)
```

output
```
( - 2*sqrt(a*x**2 + b)*a*x**2 - sqrt(a*x**2 + b)*b - 2*sqrt(a)*a*x**3 - 2*
sqrt(a)*b*x)/(b**2*x*(a*x**2 + b))
```

3.395 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7} \, dx = \frac{a^2}{b^3\sqrt{a+\frac{b}{x^2}}}+\frac{2a\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}$$

output
```
a^2/b^3/(a+b/x^2)^(1/2)+2*a*(a+b/x^2)^(1/2)/b^3-1/3*(a+b/x^2)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7} \, dx = \frac{-b^2+4abx^2+8a^2x^4}{3b^3\sqrt{a+\frac{b}{x^2}}x^4}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^7),x]
```

output
```
(-b^2 + 4*a*b*x^2 + 8*a^2*x^4)/(3*b^3*Sqrt[a + b/x^2]*x^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \; 798$$

$$-\frac{1}{2} \int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4} d\frac{1}{x^2}$$

$$\Big\downarrow \; 53$$

$$-\frac{1}{2} \int \left(\frac{a^2}{b^2 \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{2a}{b^2 \sqrt{a+\frac{b}{x^2}}}+\frac{\sqrt{a+\frac{b}{x^2}}}{b^2}\right) d\frac{1}{x^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(\frac{2a^2}{b^3 \sqrt{a+\frac{b}{x^2}}}+\frac{4a\sqrt{a+\frac{b}{x^2}}}{b^3}-\frac{2\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^3}\right)$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^7),x]
```

output
```
((2*a^2)/(b^3*Sqrt[a + b/x^2]) + (4*a*Sqrt[a + b/x^2])/b^3 - (2*(a + b/x^2
)^(3/2))/(3*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
orering	$\frac{(8a^2x^4+4ab\,x^2-b^2)\,(a\,x^2+b)}{3b^3x^6\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}$	46
gosper	$\frac{(a\,x^2+b)\,(8a^2x^4+4ab\,x^2-b^2)}{3x^6b^3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	50
default	$\frac{(a\,x^2+b)\,(8a^2x^4+4ab\,x^2-b^2)}{3x^6b^3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	50
trager	$\frac{(8a^2x^4+4ab\,x^2-b^2)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3x^2b^3(a\,x^2+b)}$	56
risch	$\frac{(a\,x^2+b)\,(5a\,x^2-b)}{3b^3x^4\sqrt{\frac{a\,x^2+b}{x^2}}}+\frac{a^2}{b^3\sqrt{\frac{a\,x^2+b}{x^2}}}$	60

input
```
int(1/(a+b/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/3*(8*a^2*x^4+4*a*b*x^2-b^2)/b^3*(a*x^2+b)/x^6/(a+b/x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7}\,dx = \frac{\left(8\,a^2x^4+4\,abx^2-b^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(ab^3x^4+b^4x^2\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/3*(8*a^2*x^4 + 4*a*b*x^2 - b^2)*sqrt((a*x^2 + b)/x^2)/(a*b^3*x^4 + b^4*x
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(48) = 96.

Time = 1.51 (sec) , antiderivative size = 423, normalized size of antiderivative = 7.83

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7}\,dx &= \frac{8a^{\frac{9}{2}}b^{\frac{7}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3} \\ &+ \frac{12a^{\frac{7}{2}}b^{\frac{9}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3} + \frac{3a^{\frac{5}{2}}b^{\frac{11}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3} \\ &- \frac{a^{\frac{3}{2}}b^{\frac{13}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3} - \frac{8a^5b^3x^7}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3} \\ &- \frac{16a^4b^4x^5}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3} - \frac{8a^3b^5x^3}{3a^{\frac{7}{2}}b^6x^7+6a^{\frac{5}{2}}b^7x^5+3a^{\frac{3}{2}}b^8x^3}\end{aligned}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**7,x)
```

output
```
8*a**(9/2)*b**(7/2)*x**6*sqrt(a*x**2/b + 1)/(3*a**(7/2)*b**6*x**7 + 6*a**(
5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x**3) + 12*a**(7/2)*b**(9/2)*x**4*sqrt(a*
x**2/b + 1)/(3*a**(7/2)*b**6*x**7 + 6*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b**8
*x**3) + 3*a**(5/2)*b**(11/2)*x**2*sqrt(a*x**2/b + 1)/(3*a**(7/2)*b**6*x**
7 + 6*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x**3) - a**(3/2)*b**(13/2)*sqrt
(a*x**2/b + 1)/(3*a**(7/2)*b**6*x**7 + 6*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b
**8*x**3) - 8*a**5*b**3*x**7/(3*a**(7/2)*b**6*x**7 + 6*a**(5/2)*b**7*x**5
+ 3*a**(3/2)*b**8*x**3) - 16*a**4*b**4*x**5/(3*a**(7/2)*b**6*x**7 + 6*a**(
5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x**3) - 8*a**3*b**5*x**3/(3*a**(7/2)*b**6
*x**7 + 6*a**(5/2)*b**7*x**5 + 3*a**(3/2)*b**8*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b^3} + \frac{2\,\sqrt{a+\frac{b}{x^2}}a}{b^3} + \frac{a^2}{\sqrt{a+\frac{b}{x^2}}b^3}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/3*(a + b/x^2)^(3/2)/b^3 + 2*sqrt(a + b/x^2)*a/b^3 + a^2/(sqrt(a + b/x^2
)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(46) = 92.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.11

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7} \, dx = \frac{a^2 x}{\sqrt{ax^2+b}b^3\mathrm{sgn}\left(x\right)} - \frac{2\left(3\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4 a^{\frac{3}{2}} - 12\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2 a^{\frac{3}{2}} b + 5\,a^{\frac{3}{2}} b^2\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^3 b^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
a^2*x/(sqrt(a*x^2 + b)*b^3*sgn(x)) - 2/3*(3*(sqrt(a)*x - sqrt(a*x^2 + b))^
4*a^(3/2) - 12*(sqrt(a)*x - sqrt(a*x^2 + b))^2*a^(3/2)*b + 5*a^(3/2)*b^2)/
(((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^3*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\,(8\,a^2\,x^4+4\,a\,b\,x^2-b^2)}{3\,b^3\,x^2\,\left(a\,x^2+b\right)}$$

input
```
int(1/(x^7*(a + b/x^2)^(3/2)),x)
```

output
```
((a + b/x^2)^(1/2)*(8*a^2*x^4 - b^2 + 4*a*b*x^2))/(3*b^3*x^2*(b + a*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^7}\,dx = \frac{8\sqrt{a\,x^2+b}\,a^2x^4+4\sqrt{a\,x^2+b}\,ab\,x^2-\sqrt{a\,x^2+b}\,b^2-8\sqrt{a}\,a^2x^5-8\sqrt{a}\,ab\,x^3}{3b^3x^3\left(a\,x^2+b\right)}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^7,x)
```

output
```
(8*sqrt(a*x**2 + b)*a**2*x**4 + 4*sqrt(a*x**2 + b)*a*b*x**2 - sqrt(a*x**2
+ b)*b**2 - 8*sqrt(a)*a**2*x**5 - 8*sqrt(a)*a*b*x**3)/(3*b**3*x**3*(a*x**2
 + b))
```

3.396 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9} \, dx = -\frac{a^3}{b^4\sqrt{a+\frac{b}{x^2}}}-\frac{3a^2\sqrt{a+\frac{b}{x^2}}}{b^4}+\frac{a\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}$$

output
```
-a^3/b^4/(a+b/x^2)^(1/2)-3*a^2*(a+b/x^2)^(1/2)/b^4+a*(a+b/x^2)^(3/2)/b^4-1
/5*(a+b/x^2)^(5/2)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9} \, dx = \frac{-b^3+2ab^2x^2-8a^2bx^4-16a^3x^6}{5b^4\sqrt{a+\frac{b}{x^2}}x^6}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^9),x]
```

output
```
(-b^3 + 2*a*b^2*x^2 - 8*a^2*b*x^4 - 16*a^3*x^6)/(5*b^4*Sqrt[a + b/x^2]*x^6
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+\frac{b}{x^2}\right)^{3/2}} dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2} \int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6} d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2} \int \left(-\frac{a^3}{b^3 \left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{3a^2}{b^3 \sqrt{a+\frac{b}{x^2}}}-\frac{3\sqrt{a+\frac{b}{x^2}}a}{b^3}+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{b^3}\right) d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{2a^3}{b^4 \sqrt{a+\frac{b}{x^2}}}-\frac{6a^2 \sqrt{a+\frac{b}{x^2}}}{b^4}+\frac{2a\left(a+\frac{b}{x^2}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x^2}\right)^{5/2}}{5b^4}\right)$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^9),x]
```

output
```
((-2*a^3)/(b^4*Sqrt[a + b/x^2]) - (6*a^2*Sqrt[a + b/x^2])/b^4 + (2*a*(a +
b/x^2)^(3/2))/b^4 - (2*(a + b/x^2)^(5/2))/(5*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
orering	$-\frac{\left(16 a^{3} x^{6}+8 a^{2} b \,x^{4}-2 a \,b^{2} x^{2}+b^{3}\right) \left(a \,x^{2}+b \right)}{5 b^{4} x^{8} \left(a +\frac{b}{x^{2}}\right)^{\frac{3}{2}}}$	55
gosper	$-\frac{\left(a \,x^{2}+b \right) \left(16 a^{3} x^{6}+8 a^{2} b \,x^{4}-2 a \,b^{2} x^{2}+b^{3}\right)}{5 x^{8} b^{4} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	59
default	$-\frac{\left(a \,x^{2}+b \right) \left(16 a^{3} x^{6}+8 a^{2} b \,x^{4}-2 a \,b^{2} x^{2}+b^{3}\right)}{5 x^{8} b^{4} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	59
trager	$-\frac{\left(16 a^{3} x^{6}+8 a^{2} b \,x^{4}-2 a \,b^{2} x^{2}+b^{3}\right) \sqrt{-\frac{-a \,x^{2}-b}{x^{2}}}}{5 x^{4} b^{4} \left(a \,x^{2}+b \right)}$	65
risch	$-\frac{\left(a \,x^{2}+b \right) \left(11 a^{2} x^{4}-3 a b \,x^{2}+b^{2}\right)}{5 b^{4} x^{6} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}-\frac{a^{3}}{b^{4} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}$	70

input
```
int(1/(a+b/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(16*a^3*x^6+8*a^2*b*x^4-2*a*b^2*x^2+b^3)/b^4*(a*x^2+b)/x^8/(a+b/x^2)^
(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9}\, dx = -\frac{\left(16\, a^3 x^6 + 8\, a^2 b x^4 - 2\, a b^2 x^2 + b^3\right)\sqrt{\frac{a x^2+b}{x^2}}}{5\,\left(a b^4 x^6 + b^5 x^4\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-1/5*(16*a^3*x^6 + 8*a^2*b*x^4 - 2*a*b^2*x^2 + b^3)*sqrt((a*x^2 + b)/x^2)/
(a*b^4*x^6 + b^5*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1844 vs. 2(65) = 130.

Time = 2.24 (sec) , antiderivative size = 1844, normalized size of antiderivative = 25.26

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**9,x)
```

output
```
-16*a**(21/2)*b**(23/2)*x**16*sqrt(a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17
+ 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**1
8*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**
21*x**5) - 88*a**(19/2)*b**(25/2)*x**14*sqrt(a*x**2/b + 1)/(5*a**(17/2)*b*
*15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13 + 100*a**(
11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a*
*(5/2)*b**21*x**5) - 198*a**(17/2)*b**(27/2)*x**12*sqrt(a*x**2/b + 1)/(5*a
**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*b**17*x**13
 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(7/2)*b**20*
x**7 + 5*a**(5/2)*b**21*x**5) - 231*a**(15/2)*b**(29/2)*x**10*sqrt(a*x**2/
b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*a**(13/2)*
b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9 + 30*a**(
7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) - 145*a**(13/2)*b**(31/2)*x**8*sq
rt(a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x**15 + 75*
a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b**19*x**9
 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) - 46*a**(11/2)*b**(33/2
)*x**6*sqrt(a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2)*b**16*x*
*15 + 75*a**(13/2)*b**17*x**13 + 100*a**(11/2)*b**18*x**11 + 75*a**(9/2)*b
**19*x**9 + 30*a**(7/2)*b**20*x**7 + 5*a**(5/2)*b**21*x**5) - 8*a**(9/2)*b
**(35/2)*x**4*sqrt(a*x**2/b + 1)/(5*a**(17/2)*b**15*x**17 + 30*a**(15/2...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}{5\,b^4} + \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a}{b^4} - \frac{3\,\sqrt{a+\frac{b}{x^2}}a^2}{b^4} - \frac{a^3}{\sqrt{a+\frac{b}{x^2}}b^4}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/5*(a + b/x^2)^(5/2)/b^4 + (a + b/x^2)^(3/2)*a/b^4 - 3*sqrt(a + b/x^2)*a
^2/b^4 - a^3/(sqrt(a + b/x^2)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(63) = 126$.

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.32

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^9}\,dx = -\frac{a^3x}{\sqrt{ax^2+b}b^4\mathrm{sgn}\left(x\right)} + \frac{2\left(5\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^8a^{\frac{5}{2}}-30\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^6a^{\frac{5}{2}}b+80\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4a^{\frac{5}{2}}b^2-50\left(\sqrt{a}x-\sqrt{a\right.}\right.}{5\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^5b^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
-a^3*x/(sqrt(a*x^2 + b)*b^4*sgn(x)) + 2/5*(5*(sqrt(a)*x - sqrt(a*x^2 + b))
^8*a^(5/2) - 30*(sqrt(a)*x - sqrt(a*x^2 + b))^6*a^(5/2)*b + 80*(sqrt(a)*x
- sqrt(a*x^2 + b))^4*a^(5/2)*b^2 - 50*(sqrt(a)*x - sqrt(a*x^2 + b))^2*a^(5
/2)*b^3 + 11*a^(5/2)*b^4)/(((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^5*b^3*sgn
(x))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^9}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\,\left(16\,a^3\,x^6+8\,a^2\,b\,x^4-2\,a\,b^2\,x^2+b^3\right)}{5\,b^4\,x^4\,\left(a\,x^2+b\right)}$$

input
```
int(1/(x^9*(a + b/x^2)^(3/2)),x)
```

output
```
-((a + b/x^2)^(1/2)*(b^3 + 16*a^3*x^6 - 2*a*b^2*x^2 + 8*a^2*b*x^4))/(5*b^4
*x^4*(b + a*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^9}\,dx = \frac{-16\sqrt{a\,x^2+b}\,a^3x^6-8\sqrt{a\,x^2+b}\,a^2b\,x^4+2\sqrt{a\,x^2+b}\,a\,b^2x^2-\sqrt{a\,x^2+b}\,b^3+16\sqrt{a}}{5b^4x^5\left(a\,x^2+b\right)}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^9,x)
```

output
```
( - 16*sqrt(a*x**2 + b)*a**3*x**6 - 8*sqrt(a*x**2 + b)*a**2*b*x**4 + 2*sqr
t(a*x**2 + b)*a*b**2*x**2 - sqrt(a*x**2 + b)*b**3 + 16*sqrt(a)*a**3*x**7 +
 16*sqrt(a)*a**2*b*x**5)/(5*b**4*x**5*(a*x**2 + b))
```

3.397 $\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{16b^2\sqrt{a+\frac{b}{x^2}}\,x}{5a^4}-\frac{8b\sqrt{a+\frac{b}{x^2}}\,x^3}{5a^3}-\frac{x^5}{a\sqrt{a+\frac{b}{x^2}}}+\frac{6\sqrt{a+\frac{b}{x^2}}\,x^5}{5a^2}$$

output
```
16/5*b^2*(a+b/x^2)^(1/2)*x/a^4-8/5*b*(a+b/x^2)^(1/2)*x^3/a^3-x^5/a/(a+b/x^
2)^(1/2)+6/5*(a+b/x^2)^(1/2)*x^5/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{16b^3+8ab^2x^2-2a^2bx^4+a^3x^6}{5a^4\sqrt{a+\frac{b}{x^2}}\,x}$$

input
```
Integrate[x^4/(a + b/x^2)^(3/2),x]
```

output
```
(16*b^3 + 8*a*b^2*x^2 - 2*a^2*b*x^4 + a^3*x^6)/(5*a^4*Sqrt[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {803, 803, 773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}-\frac{6b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx}{5a}$$

$$\Big\downarrow \text{803}$$

$$\frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}-\frac{6b\left(\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}-\frac{4b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx}{3a}\right)}{5a}$$

$$\Big\downarrow \text{773}$$

$$\frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}-\frac{6b\left(\frac{4b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}\right)}{5a}$$

$$\Big\downarrow \text{245}$$

$$\frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}-\frac{6b\left(\frac{4b\left(-\frac{2b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x}{a\sqrt{a+\frac{b}{x^2}}}\right)}{3a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}\right)}{5a}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x^5}{5a\sqrt{a+\frac{b}{x^2}}}-\frac{6b\left(\frac{4b\left(-\frac{2b}{a^2x\sqrt{a+\frac{b}{x^2}}}-\frac{x}{a\sqrt{a+\frac{b}{x^2}}}\right)}{3a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}\right)}{5a}$$

input
```
Int[x^4/(a + b/x^2)^(3/2),x]
```

output
```
x^5/(5*a*Sqrt[a + b/x^2]) - (6*b*(x^3/(3*a*Sqrt[a + b/x^2]) + (4*b*((-2*b)
/(a^2*Sqrt[a + b/x^2]*x) - x/(a*Sqrt[a + b/x^2])))/(3*a)))/(5*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

method	result	size
orering	$\frac{(a^3x^6-2a^2b\,x^4+8a\,b^2x^2+16b^3)(a\,x^2+b)}{5a^4x^3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}$	56
gosper	$\frac{(a\,x^2+b)(a^3x^6-2a^2b\,x^4+8a\,b^2x^2+16b^3)}{5a^4x^3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	60
default	$\frac{(a\,x^2+b)(a^3x^6-2a^2b\,x^4+8a\,b^2x^2+16b^3)}{5a^4x^3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	60
trager	$\frac{(a^3x^6-2a^2b\,x^4+8a\,b^2x^2+16b^3)x\sqrt{-\frac{-a\,x^2-b}{x^2}}}{5a^4(a\,x^2+b)}$	64
risch	$\frac{(a^2x^4-3ab\,x^2+11b^2)(a\,x^2+b)}{5a^4\sqrt{\frac{a\,x^2+b}{x^2}}\,x}+\frac{b^3}{a^4\sqrt{\frac{a\,x^2+b}{x^2}}\,x}$	73

input
```
int(x^4/(a+b/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a^3*x^6-2*a^2*b*x^4+8*a*b^2*x^2+16*b^3)/a^4*(a*x^2+b)/x^3/(a+b/x^2)^(
3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{(a^3x^7-2\,a^2bx^5+8\,ab^2x^3+16\,b^3x)\sqrt{\frac{ax^2+b}{x^2}}}{5\,(a^5x^2+a^4b)}$$

input
```
integrate(x^4/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(a^3*x^7 - 2*a^2*b*x^5 + 8*a*b^2*x^3 + 16*b^3*x)*sqrt((a*x^2 + b)/x^2)
/(a^5*x^2 + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(78) = 156$.

Time = 0.94 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.96

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{a^5b^{\frac{19}{2}}x^{10}\sqrt{\frac{ax^2}{b}+1}}{5a^7b^9x^6+15a^6b^{10}x^4+15a^5b^{11}x^2+5a^4b^{12}} + \frac{5a^3b^{\frac{23}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{5a^7b^9x^6+15a^6b^{10}x^4+15a^5b^{11}x^2+5a^4b^{12}} + \frac{30a^2b^{\frac{25}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{5a^7b^9x^6+15a^6b^{10}x^4+15a^5b^{11}x^2+5a^4b^{12}} + \frac{40ab^{\frac{27}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{5a^7b^9x^6+15a^6b^{10}x^4+15a^5b^{11}x^2+5a^4b^{12}} + \frac{16b^{\frac{29}{2}}\sqrt{\frac{ax^2}{b}+1}}{5a^7b^9x^6+15a^6b^{10}x^4+15a^5b^{11}x^2+5a^4b^{12}}$$

input
```
integrate(x**4/(a+b/x**2)**(3/2),x)
```

output
```
a**5*b**(19/2)*x**10*sqrt(a*x**2/b + 1)/(5*a**7*b**9*x**6 + 15*a**6*b**10*
x**4 + 15*a**5*b**11*x**2 + 5*a**4*b**12) + 5*a**3*b**(23/2)*x**6*sqrt(a*x
**2/b + 1)/(5*a**7*b**9*x**6 + 15*a**6*b**10*x**4 + 15*a**5*b**11*x**2 + 5
*a**4*b**12) + 30*a**2*b**(25/2)*x**4*sqrt(a*x**2/b + 1)/(5*a**7*b**9*x**6
 + 15*a**6*b**10*x**4 + 15*a**5*b**11*x**2 + 5*a**4*b**12) + 40*a*b**(27/2
)*x**2*sqrt(a*x**2/b + 1)/(5*a**7*b**9*x**6 + 15*a**6*b**10*x**4 + 15*a**5
*b**11*x**2 + 5*a**4*b**12) + 16*b**(29/2)*sqrt(a*x**2/b + 1)/(5*a**7*b**9
*x**6 + 15*a**6*b**10*x**4 + 15*a**5*b**11*x**2 + 5*a**4*b**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{b^3}{\sqrt{a+\frac{b}{x^2}}a^4x} + \frac{\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}x^5 - 5\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}bx^3 + 15\sqrt{a+\frac{b}{x^2}}b^2x}{5\,a^4}$$

input
```
integrate(x^4/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

output
```
b^3/(sqrt(a + b/x^2)*a^4*x) + 1/5*((a + b/x^2)^(5/2)*x^5 - 5*(a + b/x^2)^(
3/2)*b*x^3 + 15*sqrt(a + b/x^2)*b^2*x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = -\frac{16\,b^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,a^4} + \frac{b^3}{\sqrt{ax^2+b}a^4\mathrm{sgn}\left(x\right)} + \frac{\left(ax^2+b\right)^{\frac{5}{2}}a^{16} - 5\left(ax^2+b\right)^{\frac{3}{2}}a^{16}b + 15\sqrt{ax^2+b}a^{16}b^2}{5\,a^{20}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^4/(a+b/x^2)^(3/2),x, algorithm="giac")
```

output
```
-16/5*b^(5/2)*sgn(x)/a^4 + b^3/(sqrt(a*x^2 + b)*a^4*sgn(x)) + 1/5*((a*x^2
+ b)^(5/2)*a^16 - 5*(a*x^2 + b)^(3/2)*a^16*b + 15*sqrt(a*x^2 + b)*a^16*b^2
)/(a^20*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.56

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{a^3\,x^6-2\,a^2\,b\,x^4+8\,a\,b^2\,x^2+16\,b^3}{5\,a^4\,x\,\sqrt{a+\frac{b}{x^2}}}$$

input
```
int(x^4/(a + b/x^2)^(3/2),x)
```

output
```
(16*b^3 + a^3*x^6 + 8*a*b^2*x^2 - 2*a^2*b*x^4)/(5*a^4*x*(a + b/x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{\sqrt{a\,x^2+b}\,(a^3x^6-2a^2b\,x^4+8a\,b^2x^2+16b^3)}{5a^4\,(a\,x^2+b)}$$

input
```
int(x^4/(a+b/x^2)^(3/2),x)
```

output
```
(sqrt(a*x**2 + b)*(a**3*x**6 - 2*a**2*b*x**4 + 8*a*b**2*x**2 + 16*b**3))/(
5*a**4*(a*x**2 + b))
```

3.398 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=-\frac{8b\sqrt{a+\frac{b}{x^2}}\,x}{3a^3}-\frac{x^3}{a\sqrt{a+\frac{b}{x^2}}}+\frac{4\sqrt{a+\frac{b}{x^2}}\,x^3}{3a^2}$$

output
```
-8/3*b*(a+b/x^2)^(1/2)*x/a^3-x^3/a/(a+b/x^2)^(1/2)+4/3*(a+b/x^2)^(1/2)*x^3
/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{-8b^2-4abx^2+a^2x^4}{3a^3\sqrt{a+\frac{b}{x^2}}\,x}$$

input
```
Integrate[x^2/(a + b/x^2)^(3/2),x]
```

output
```
(-8*b^2 - 4*a*b*x^2 + a^2*x^4)/(3*a^3*Sqrt[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {803, 773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}-\frac{4b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{773}$$

$$\frac{4b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{245}$$

$$\frac{4b\left(-\frac{2b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}}{a}-\frac{x}{a\sqrt{a+\frac{b}{x^2}}}\right)}{3a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4b\left(-\frac{2b}{a^2x\sqrt{a+\frac{b}{x^2}}}-\frac{x}{a\sqrt{a+\frac{b}{x^2}}}\right)}{3a}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^2}}}$$

input
```
Int[x^2/(a + b/x^2)^(3/2),x]
```

output
```
x^3/(3*a*Sqrt[a + b/x^2]) + (4*b*((-2*b)/(a^2*Sqrt[a + b/x^2]*x) - x/(a*Sq
rt[a + b/x^2])))/(3*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

method	result	size
orering	$\frac{\left(a^{2} x^{4}-4 a b \,x^{2}-8 b^{2}\right) \left(a \,x^{2}+b \right)}{3 a^{3} x^{3} \left(a +\frac{b}{x^{2}}\right)^{\frac{3}{2}}}$	45
gosper	$\frac{\left(a \,x^{2}+b \right) \left(a^{2} x^{4}-4 a b \,x^{2}-8 b^{2}\right)}{3 a^{3} x^{3} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	49
default	$\frac{\left(a \,x^{2}+b \right) \left(a^{2} x^{4}-4 a b \,x^{2}-8 b^{2}\right)}{3 a^{3} x^{3} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	49
trager	$\frac{\left(a^{2} x^{4}-4 a b \,x^{2}-8 b^{2}\right) x \sqrt{-\frac{-a \,x^{2}-b}{x^{2}}}}{3 \left(a \,x^{2}+b \right) a^{3}}$	53
risch	$\frac{\left(a \,x^{2}-5 b \right) \left(a \,x^{2}+b \right)}{3 a^{3} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}\, x}-\frac{b^{2}}{a^{3} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}\, x}$	63

input
```
int(x^2/(a+b/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^2*x^4-4*a*b*x^2-8*b^2)/a^3*(a*x^2+b)/x^3/(a+b/x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{\left(a^2x^5-4\,abx^3-8\,b^2x\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\,\left(a^4x^2+a^3b\right)}$$

input
```
integrate(x^2/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a^2*x^5 - 4*a*b*x^3 - 8*b^2*x)*sqrt((a*x^2 + b)/x^2)/(a^4*x^2 + a^3*b
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(54) = 108$.

Time = 0.72 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.59

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{a^3b^{\frac{9}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6} - \frac{3a^2b^{\frac{11}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6} - \frac{12ab^{\frac{13}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6} - \frac{8b^{\frac{15}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}$$

input
```
integrate(x**2/(a+b/x**2)**(3/2),x)
```

output
```
a**3*b**(9/2)*x**6*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x**4 + 6*a**4*b**5*x**2
 + 3*a**3*b**6) - 3*a**2*b**(11/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x*
*4 + 6*a**4*b**5*x**2 + 3*a**3*b**6) - 12*a*b**(13/2)*x**2*sqrt(a*x**2/b +
 1)/(3*a**5*b**4*x**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6) - 8*b**(15/2)*sqrt
(a*x**2/b + 1)/(3*a**5*b**4*x**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3 - 6\sqrt{a+\frac{b}{x^2}}bx}{3\,a^3} - \frac{b^2}{\sqrt{a+\frac{b}{x^2}}a^3x}$$

input
```
integrate(x^2/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*((a + b/x^2)^(3/2)*x^3 - 6*sqrt(a + b/x^2)*b*x)/a^3 - b^2/(sqrt(a + b/
x^2)*a^3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{8\,b^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,a^3} - \frac{\frac{3\,b^2}{\sqrt{ax^2+b}a\operatorname{sgn}(x)} - \frac{(ax^2+b)^{\frac{3}{2}}a^2-6\,\sqrt{ax^2+b}a^2b}{a^3\operatorname{sgn}(x)}}{3\,a^2}$$

input
```
integrate(x^2/(a+b/x^2)^(3/2),x, algorithm="giac")
```

output
```
8/3*b^(3/2)*sgn(x)/a^3 - 1/3*(3*b^2/(sqrt(a*x^2 + b)*a*sgn(x)) - ((a*x^2 +
 b)^(3/2)*a^2 - 6*sqrt(a*x^2 + b)*a^2*b)/(a^3*sgn(x)))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = -\frac{-a^2\,x^4+4\,a\,b\,x^2+8\,b^2}{3\,a^3\,x\,\sqrt{a+\frac{b}{x^2}}}$$

input
```
int(x^2/(a + b/x^2)^(3/2),x)
```

output
```
-(8*b^2 - a^2*x^4 + 4*a*b*x^2)/(3*a^3*x*(a + b/x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{\sqrt{a\,x^2+b}\,(a^2x^4-4ab\,x^2-8b^2)}{3a^3\,(a\,x^2+b)}$$

input
```
int(x^2/(a+b/x^2)^(3/2),x)
```

output
```
(sqrt(a*x**2 + b)*(a**2*x**4 - 4*a*b*x**2 - 8*b**2))/(3*a**3*(a*x**2 + b))
```

3.399 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=-\frac{x}{a\sqrt{a+\frac{b}{x^2}}}+\frac{2\sqrt{a+\frac{b}{x^2}}\,x}{a^2}$$

output
```
-x/a/(a+b/x^2)^(1/2)+2*(a+b/x^2)^(1/2)*x/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx=\frac{2b+ax^2}{a^2\sqrt{a+\frac{b}{x^2}}\,x}$$

input
```
Integrate[(a + b/x^2)^(-3/2),x]
```

output
```
(2*b + a*x^2)/(a^2*Sqrt[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 773$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \ 245$$

$$\frac{2b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}}{a}+\frac{x}{a\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \ 208$$

$$\frac{2b}{a^2x\sqrt{a+\frac{b}{x^2}}}+\frac{x}{a\sqrt{a+\frac{b}{x^2}}}$$

input
```
Int[(a + b/x^2)^(-3/2),x]
```

output
```
(2*b)/(a^2*Sqrt[a + b/x^2]*x) + x/(a*Sqrt[a + b/x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
orering	$\frac{(a\,x^2+2b)\,(a\,x^2+b)}{a^2x^3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}$	33
gosper	$\frac{(a\,x^2+b)\,(a\,x^2+2b)}{a^2x^3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	37
default	$\frac{(a\,x^2+b)\,(a\,x^2+2b)}{a^2x^3\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{3}{2}}}$	37
trager	$\frac{(a\,x^2+2b)x\sqrt{-\frac{-a\,x^2-b}{x^2}}}{(a\,x^2+b)a^2}$	41
risch	$\frac{a\,x^2+b}{a^2\sqrt{\frac{a\,x^2+b}{x^2}}\,x}+\frac{b}{a^2\sqrt{\frac{a\,x^2+b}{x^2}}\,x}$	50

input
```
int(1/(a+b/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x^2+2*b)/a^2*(a*x^2+b)/x^3/(a+b/x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{(ax^3+2\,bx)\sqrt{\frac{ax^2+b}{x^2}}}{a^3x^2+a^2b}$$

input
```
integrate(1/(a+b/x^2)^(3/2),x, algorithm="fricas")
```

output
```
(a*x^3 + 2*b*x)*sqrt((a*x^2 + b)/x^2)/(a^3*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{x^2}{a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}} + \frac{2\sqrt{b}}{a^2\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate(1/(a+b/x**2)**(3/2),x)
```

output
```
x**2/(a*sqrt(b)*sqrt(a*x**2/b + 1)) + 2*sqrt(b)/(a**2*sqrt(a*x**2/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\,x}{a^2} + \frac{b}{\sqrt{a+\frac{b}{x^2}}\,a^2x}$$

input
```
integrate(1/(a+b/x^2)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x^2)*x/a^2 + b/(sqrt(a + b/x^2)*a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{\frac{\sqrt{ax^2+b}}{a\operatorname{sgn}(x)} + \frac{b}{\sqrt{ax^2+b}\,a\operatorname{sgn}(x)}}{a} - \frac{2\sqrt{b}\operatorname{sgn}(x)}{a^2}$$

input
```
integrate(1/(a+b/x^2)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(a*x^2 + b)/(a*sgn(x)) + b/(sqrt(a*x^2 + b)*a*sgn(x)))/a - 2*sqrt(b)*
sgn(x)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{a^2\,x^4+3\,a\,b\,x^2+2\,b^2}{a^2\,x^3\,\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(1/(a + b/x^2)^(3/2),x)
```

output
```
(2*b^2 + a^2*x^4 + 3*a*b*x^2)/(a^2*x^3*(a + b/x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{\sqrt{a\,x^2+b}\,(a\,x^2+2b)}{a^2\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^(3/2),x)
```

output
```
(sqrt(a*x**2 + b)*(a*x**2 + 2*b))/(a**2*(a*x**2 + b))
```

3.400 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2} \, dx = -\frac{1}{a\sqrt{a+\frac{b}{x^2}}x}$$

output
```
-1/a/(a+b/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2} \, dx = -\frac{1}{a\sqrt{a+\frac{b}{x^2}}x}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^2),x]
```

output
```
-(1/(a*Sqrt[a + b/x^2]*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{1}{a x \sqrt{a+\frac{b}{x^2}}}$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^2),x]
```

output
```
-(1/(a*Sqrt[a + b/x^2]*x))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
orering	$-\frac{a\,x^{2}+b}{a\,x^{3}\left(a+\frac{b}{x^{2}}\right)^{\frac{3}{2}}}$	25
gosper	$-\frac{a\,x^{2}+b}{a\,x^{3}\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	29
default	$-\frac{a\,x^{2}+b}{a\,x^{3}\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}}$	29
trager	$-\frac{x\sqrt{-\frac{-a\,x^{2}-b}{x^{2}}}}{a(a\,x^{2}+b)}$	33

input
```
int(1/(a+b/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*x^2+b)/a/x^3/(a+b/x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^2}\,dx=-\frac{x\sqrt{\frac{ax^2+b}{x^2}}}{a^2x^2+ab}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-x*sqrt((a*x^2 + b)/x^2)/(a^2*x^2 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^2}\,dx=-\frac{1}{a\sqrt{b}\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**2,x)
```

output
```
-1/(a*sqrt(b)*sqrt(a*x**2/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2} \, dx = -\frac{1}{\sqrt{a+\frac{b}{x^2}}ax}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-1/(sqrt(a + b/x^2)*a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2} \, dx = \frac{\text{sgn}(x)}{a\sqrt{b}} - \frac{1}{\sqrt{ax^2+b}a\text{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
sgn(x)/(a*sqrt(b)) - 1/(sqrt(a*x^2 + b)*a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2} \, dx = -\frac{x\,\sqrt{a+\frac{b}{x^2}}}{a\,\left(a\,x^2+b\right)}$$

input
```
int(1/(x^2*(a + b/x^2)^(3/2)),x)
```

output
```
-(x*(a + b/x^2)^(1/2))/(a*(b + a*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^2}\,dx = -\frac{\sqrt{a\,x^2+b}}{a\,(a\,x^2+b)}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^2,x)
```

output
```
( - sqrt(a*x**2 + b))/(a*(a*x**2 + b))
```

3.401 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4} \, dx = \frac{1}{b\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{b^{3/2}}$$

output
```
1/b/(a+b/x^2)^(1/2)/x-arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4} \, dx = \frac{\sqrt{b}-\sqrt{b+ax^2}\operatorname{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{b^{3/2}\sqrt{a+\frac{b}{x^2}}\,x}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^4),x]
```

output
```
(Sqrt[b] - Sqrt[b + a*x^2]*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]])/(b^(3/2)*Sqrt
[a + b/x^2]*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {858, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}-\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}-\frac{\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{b^{3/2}}$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^4),x]
```

output
```
1/(b*Sqrt[a + b/x^2]*x) - ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)]/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\left(a\,x^{2}+b\right)\left(b^{\frac{3}{2}}-\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)b\sqrt{a\,x^{2}+b}\right)}{\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}}x^{3}b^{\frac{5}{2}}}$	65

input
```
int(1/(a+b/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
(a*x^2+b)*(b^(3/2)-ln(2*(b^(1/2)*(a*x^2+b)^(1/2)+b)/x)*b*(a*x^2+b)^(1/2))/
((a*x^2+b)/x^2)^(3/2)/x^3/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.17

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx = \left[\frac{2\,bx\sqrt{\frac{ax^2+b}{x^2}} + (ax^2+b)\sqrt{b}\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)}{2\,(ab^2x^2+b^3)}, \frac{bx\sqrt{\frac{ax^2+b}{x^2}} + (ax^2+b)\sqrt{-}}{ab^2x^2}\right.$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/2*(2*b*x*sqrt((a*x^2 + b)/x^2) + (a*x^2 + b)*sqrt(b)*log(-(a*x^2 - 2*sq
rt(b)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2))/(a*b^2*x^2 + b^3), (b*x*sqrt((a
*x^2 + b)/x^2) + (a*x^2 + b)*sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x
^2)/b))/(a*b^2*x^2 + b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(37) = 74.

Time = 1.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.91

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx = \frac{ab^2x^2\log\left(\frac{ax^2}{b}\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}} - \frac{2ab^2x^2\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}} + \frac{2b^3\sqrt{\frac{ax^2}{b}+1}}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}} + \frac{b^3\log\left(\frac{ax^2}{b}\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}} - \frac{2b^3\log\left(\sqrt{\frac{ax^2}{b}+1}+1\right)}{2ab^{\frac{7}{2}}x^2+2b^{\frac{9}{2}}}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**4,x)
```

output
```
a*b**2*x**2*log(a*x**2/b)/(2*a*b**(7/2)*x**2 + 2*b**(9/2)) - 2*a*b**2*x**2
*log(sqrt(a*x**2/b + 1) + 1)/(2*a*b**(7/2)*x**2 + 2*b**(9/2)) + 2*b**3*sqr
t(a*x**2/b + 1)/(2*a*b**(7/2)*x**2 + 2*b**(9/2)) + b**3*log(a*x**2/b)/(2*a
*b**(7/2)*x**2 + 2*b**(9/2)) - 2*b**3*log(sqrt(a*x**2/b + 1) + 1)/(2*a*b**
(7/2)*x**2 + 2*b**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{2\,b^{\frac{3}{2}}}+\frac{1}{\sqrt{a+\frac{b}{x^2}}bx}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(3/
2) + 1/(sqrt(a + b/x^2)*b*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(39) = 78.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx = -\frac{\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)\mathrm{sgn}(x)}{\sqrt{-b}b^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b\mathrm{sgn}\left(x\right)}+\frac{1}{\sqrt{ax^2+b}b\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-(sqrt(b)*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b))*sgn(x)/(sqrt(-b)*b^(3/2)) +
 arctan(sqrt(a*x^2 + b)/sqrt(-b))/(sqrt(-b)*b*sgn(x)) + 1/(sqrt(a*x^2 + b)
*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx=\frac{1}{b\,x\,\sqrt{a+\frac{b}{x^2}}}-\frac{\ln\left(\sqrt{a+\frac{b}{x^2}}+\frac{\sqrt{b}}{x}\right)}{b^{3/2}}$$

input
```
int(1/(x^4*(a + b/x^2)^(3/2)),x)
```

output
```
1/(b*x*(a + b/x^2)^(1/2)) - log((a + b/x^2)^(1/2) + b^(1/2)/x)/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}\,dx=\frac{\sqrt{a\,x^2+b}\,b+\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a\,x^2+\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)b-\sqrt{b}\log\left(\right.}{b^2\left(a\,x^2+b\right)}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^4,x)
```

output
```
(sqrt(a*x**2 + b)*b + sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))
/sqrt(b))*a*x**2 + sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sq
rt(b))*b - sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a
*x**2 - sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*b)/(
b**2*(a*x**2 + b))
```

3.402 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6} \, dx=\frac{1}{b\sqrt{a+\frac{b}{x^2}}x^3}-\frac{3\sqrt{a+\frac{b}{x^2}}}{2b^2x}+\frac{3a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)}{2b^{5/2}}$$

output
```
1/b/(a+b/x^2)^(1/2)/x^3-3/2*(a+b/x^2)^(1/2)/b^2/x+3/2*a*arctanh(b^(1/2)/(a
+b/x^2)^(1/2)/x)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6} \, dx=\frac{-\sqrt{b}(b+3ax^2)+3ax^2\sqrt{b+ax^2}\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{2b^{5/2}\sqrt{a+\frac{b}{x^2}}x^3}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^6),x]
```

output
```
(-(Sqrt[b]*(b + 3*a*x^2)) + 3*a*x^2*Sqrt[b + a*x^2]*ArcTanh[Sqrt[b + a*x^2
]/Sqrt[b]])/(2*b^(5/2)*Sqrt[a + b/x^2]*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 858$$

$$-\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^4}d\frac{1}{x}$$

$$\Big\downarrow \ 252$$

$$\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}-\frac{3\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x}}{b}$$

$$\Big\downarrow \ 262$$

$$\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}-\frac{3\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{2b}\right)}{b}$$

$$\Big\downarrow \ 224$$

$$\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}-\frac{3\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{2b}\right)}{b}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}-\frac{3\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}\right)}{b}$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^6),x]
```

output
```
1/(b*Sqrt[a + b/x^2]*x^3) - (3*(Sqrt[a + b/x^2]/(2*b*x) - (a*ArcTanh[Sqrt[
b]/(Sqrt[a + b/x^2]*x)])/(2*b^(3/2))))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{(a x^2+b)\left(3\sqrt{a x^2+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a x^2+b}}{x}\right)a b x^2-3x^2 a b^{\frac{3}{2}}-b^{\frac{5}{2}}\right)}{2\left(\frac{a x^2+b}{x^2}\right)^{\frac{3}{2}} x^5 b^{\frac{7}{2}}}$	81
risch	$-\frac{a x^2+b}{2b^2 x^3\sqrt{\frac{a x^2+b}{x^2}}}+\frac{\left(-\frac{a}{b^2\sqrt{a x^2+b}}+\frac{3a\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a x^2+b}}{x}\right)}{2b^{\frac{5}{2}}}\right)\sqrt{a x^2+b}}{\sqrt{\frac{a x^2+b}{x^2}}\,x}$	101

input
```
int(1/(a+b/x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2+b)*(3*(a*x^2+b)^(1/2)*ln(2*(b^(1/2)*(a*x^2+b)^(1/2)+b)/x)*a*b*x
^2-3*x^2*a*b^(3/2)-b^(5/2))/((a*x^2+b)/x^2)^(3/2)/x^5/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.59

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\,dx = \left[\frac{3\,(a^2x^3+abx)\sqrt{b}\log\left(-\frac{ax^2+2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\,(3\,abx^2+b^2)\sqrt{\frac{ax^2+b}{x^2}}}{4\,(ab^3x^3+b^4x)}, -\frac{3\,(a^2x^3+abx)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)+(3\,abx^2+b^2)\sqrt{\frac{ax^2+b}{x^2}}}{2\,(ab^3x^3+b^4x)}\right]$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/4*(3*(a^2*x^3 + a*b*x)*sqrt(b)*log(-(a*x^2 + 2*sqrt(b)*x*sqrt((a*x^2 +
b)/x^2) + 2*b)/x^2) - 2*(3*a*b*x^2 + b^2)*sqrt((a*x^2 + b)/x^2))/(a*b^3*x^
3 + b^4*x), -1/2*(3*(a^2*x^3 + a*b*x)*sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x
^2 + b)/x^2)/b) + (3*a*b*x^2 + b^2)*sqrt((a*x^2 + b)/x^2))/(a*b^3*x^3 + b^
4*x)]
```

Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\, dx = -\frac{3\sqrt{a}}{2b^2x\sqrt{1+\frac{b}{ax^2}}} + \frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{2b^{\frac{5}{2}}} - \frac{1}{2\sqrt{a}bx^3\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**6,x)
```

output
```
-3*sqrt(a)/(2*b**2*x*sqrt(1 + b/(a*x**2))) + 3*a*asinh(sqrt(b)/(sqrt(a)*x)
)/(2*b**(5/2)) - 1/(2*sqrt(a)*b*x**3*sqrt(1 + b/(a*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\, dx = -\frac{3\left(a+\frac{b}{x^2}\right)ax^2 - 2\,ab}{2\left(\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2x^3 - \sqrt{a+\frac{b}{x^2}}\,b^3x\right)} - \frac{3\,a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{4\,b^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-1/2*(3*(a + b/x^2)*a*x^2 - 2*a*b)/((a + b/x^2)^(3/2)*b^2*x^3 - sqrt(a + b
/x^2)*b^3*x) - 3/4*a*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x
+ sqrt(b)))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\,dx = -\frac{3\,a\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{2\,\sqrt{-b}b^2\mathrm{sgn}\left(x\right)} - \frac{3\left(ax^2+b\right)a-2\,ab}{2\left(\left(ax^2+b\right)^{\frac{3}{2}}-\sqrt{ax^2+b}b\right)b^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
-3/2*a*arctan(sqrt(a*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x)) - 1/2*(3*(a*
x^2 + b)*a - 2*a*b)/(((a*x^2 + b)^(3/2) - sqrt(a*x^2 + b)*b)*b^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\,dx = \int \frac{1}{x^6\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$$

input
```
int(1/(x^6*(a + b/x^2)^(3/2)),x)
```

output
```
int(1/(x^6*(a + b/x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.42

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^6}\,dx = \frac{-3\sqrt{a\,x^2+b}\,ab\,x^2-\sqrt{a\,x^2+b}\,b^2-3\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a^2x^4-3\sqrt{b}\log\left(\frac{\sqrt{a\,x}}{}\right.}{2b^3x^2\,(}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^6,x)
```

output
```
( - 3*sqrt(a*x**2 + b)*a*b*x**2 - sqrt(a*x**2 + b)*b**2 - 3*sqrt(b)*log((s
qrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a**2*x**4 - 3*sqrt(b)*log(
(sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a*b*x**2 + 3*sqrt(b)*log
((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a**2*x**4 + 3*sqrt(b)*l
og((sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a*b*x**2)/(2*b**3*x**
2*(a*x**2 + b))
```

3.403 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8} \, dx=\frac{1}{b\sqrt{a+\frac{b}{x^2}}x^5}-\frac{5\sqrt{a+\frac{b}{x^2}}}{4b^2x^3}+\frac{15a\sqrt{a+\frac{b}{x^2}}}{8b^3x}-\frac{15a^2\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x}\right)}{8b^{7/2}}$$

output
```
1/b/(a+b/x^2)^(1/2)/x^5-5/4*(a+b/x^2)^(1/2)/b^2/x^3+15/8*a*(a+b/x^2)^(1/2)
/b^3/x-15/8*a^2*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8} \, dx=\frac{\sqrt{b}(-2b^2+5abx^2+15a^2x^4)-15a^2x^4\sqrt{b+ax^2}\text{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{8b^{7/2}\sqrt{a+\frac{b}{x^2}}x^5}$$

input
```
Integrate[1/((a + b/x^2)^(3/2)*x^8),x]
```

output
```
(Sqrt[b]*(-2*b^2 + 5*a*b*x^2 + 15*a^2*x^4) - 15*a^2*x^4*Sqrt[b + a*x^2]*Ar
cTanh[Sqrt[b + a*x^2]/Sqrt[b]])/(8*b^(7/2)*Sqrt[a + b/x^2]*x^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {858, 252, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(a+\frac{b}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^6}d\frac{1}{x}$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{bx^5\sqrt{a+\frac{b}{x^2}}}-\frac{5\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^4}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{bx^5\sqrt{a+\frac{b}{x^2}}}-\frac{5\left(\frac{\sqrt{a+\frac{b}{x^2}}}{4bx^3}-\frac{3a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x}}{4b}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{bx^5\sqrt{a+\frac{b}{x^2}}}-\frac{5\left(\frac{\sqrt{a+\frac{b}{x^2}}}{4bx^3}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{2b}\right)}{4b}\right)}{b}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{bx^5\sqrt{a+\frac{b}{x^2}}}-\frac{5\left(\frac{\sqrt{a+\frac{b}{x^2}}}{4bx^3}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int\frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{2b}\right)}{4b}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{bx^5\sqrt{a+\frac{b}{x^2}}}-\frac{5\left(\frac{\sqrt{a+\frac{b}{x^2}}}{4bx^3}-\frac{3a\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}\right)}{4b}\right)}{b}$$

input
```
Int[1/((a + b/x^2)^(3/2)*x^8),x]
```

output
```
1/(b*Sqrt[a + b/x^2]*x^5) - (5*(Sqrt[a + b/x^2]/(4*b*x^3) - (3*a*(Sqrt[a +
 b/x^2]/(2*b*x) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*b^(3/2))))/(
4*b)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{\left(a\,x^{2}+b\right)\left(-15 b^{\frac{3}{2}} a^{2} x^{4}+15 \ln\left(\frac{2 b+2 \sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)\sqrt{a\,x^{2}+b}\,a^{2} b\,x^{4}-5 b^{\frac{5}{2}} a\,x^{2}+2 b^{\frac{7}{2}}\right)}{8\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{3}{2}} x^{7} b^{\frac{9}{2}}}$	94
risch	$\frac{\left(a\,x^{2}+b\right)\left(7 a\,x^{2}-2 b\right)}{8 b^{3} x^{5}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}+\frac{\left(\frac{a^{2}}{b^{3}\sqrt{a\,x^{2}+b}}-\frac{15 a^{2} \ln\left(\frac{2 b+2 \sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)}{8 b^{\frac{7}{2}}}\right)\sqrt{a\,x^{2}+b}}{\sqrt{\frac{a\,x^{2}+b}{x^{2}}}\,x}$	114

input
```
int(1/(a+b/x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*x^2+b)*(-15*b^(3/2)*a^2*x^4+15*ln(2*(b^(1/2)*(a*x^2+b)^(1/2)+b)/x)
*(a*x^2+b)^(1/2)*a^2*b*x^4-5*b^(5/2)*a*x^2+2*b^(7/2))/((a*x^2+b)/x^2)^(3/2
)/x^7/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8}\,dx = \left[\frac{15\,(a^3x^5+a^2bx^3)\sqrt{b}\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+2\,(15\,a^2bx^4+5\,ab^2x^2-2\,b^3)\sqrt{}}{16\,(ab^4x^5+b^5x^3)}\right.$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/16*(15*(a^3*x^5 + a^2*b*x^3)*sqrt(b)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt((a*
x^2 + b)/x^2) + 2*b)/x^2) + 2*(15*a^2*b*x^4 + 5*a*b^2*x^2 - 2*b^3)*sqrt((a
*x^2 + b)/x^2))/(a*b^4*x^5 + b^5*x^3), 1/8*(15*(a^3*x^5 + a^2*b*x^3)*sqrt(
-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b) + (15*a^2*b*x^4 + 5*a*b^2*x
^2 - 2*b^3)*sqrt((a*x^2 + b)/x^2))/(a*b^4*x^5 + b^5*x^3)]
```

Sympy [A] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8}\,dx = \frac{15a^{\frac{3}{2}}}{8b^3x\sqrt{1+\frac{b}{ax^2}}}+\frac{5\sqrt{a}}{8b^2x^3\sqrt{1+\frac{b}{ax^2}}} - \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{8b^{\frac{7}{2}}} - \frac{1}{4\sqrt{a}bx^5\sqrt{1+\frac{b}{ax^2}}}$$

input
```
integrate(1/(a+b/x**2)**(3/2)/x**8,x)
```

output
```
15*a**(3/2)/(8*b**3*x*sqrt(1 + b/(a*x**2))) + 5*sqrt(a)/(8*b**2*x**3*sqrt(
1 + b/(a*x**2))) - 15*a**2*asinh(sqrt(b)/(sqrt(a)*x))/(8*b**(7/2)) - 1/(4*
sqrt(a)*b*x**5*sqrt(1 + b/(a*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.46

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^8}\,dx = \frac{15\left(a+\frac{b}{x^2}\right)^2a^2x^4-25\left(a+\frac{b}{x^2}\right)a^2bx^2+8\,a^2b^2}{8\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}b^3x^5-2\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^4x^3+\sqrt{a+\frac{b}{x^2}}\,b^5x\right)} + \frac{15\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}\,x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x+\sqrt{b}}\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
1/8*(15*(a + b/x^2)^2*a^2*x^4 - 25*(a + b/x^2)*a^2*b*x^2 + 8*a^2*b^2)/((a
+ b/x^2)^(5/2)*b^3*x^5 - 2*(a + b/x^2)^(3/2)*b^4*x^3 + sqrt(a + b/x^2)*b^5
*x) + 15/16*a^2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqr
t(b)))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^8}\,dx = \frac{15\,a^2\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{8\sqrt{-b}b^3\mathrm{sgn}\left(x\right)} + \frac{a^2}{\sqrt{ax^2+b}b^3\mathrm{sgn}\left(x\right)} + \frac{7\left(ax^2+b\right)^{\frac{3}{2}}a^2-9\sqrt{ax^2+b}a^2b}{8\,a^2b^3x^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
15/8*a^2*arctan(sqrt(a*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn(x)) + a^2/(sqr
t(a*x^2 + b)*b^3*sgn(x)) + 1/8*(7*(a*x^2 + b)^(3/2)*a^2 - 9*sqrt(a*x^2 + b
)*a^2*b)/(a^2*b^3*x^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8}\,dx = \int \frac{1}{x^8\left(a+\frac{b}{x^2}\right)^{3/2}}\,dx$$

input
```
int(1/(x^8*(a + b/x^2)^(3/2)),x)
```

output
```
int(1/(x^8*(a + b/x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.05

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^8}\,dx = \frac{15\sqrt{a\,x^2+b}\,a^2 b\,x^4+5\sqrt{a\,x^2+b}\,a\,b^2x^2-2\sqrt{a\,x^2+b}\,b^3+15\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right.}{}$$

input
```
int(1/(a+b/x^2)^(3/2)/x^8,x)
```

output
```
(15*sqrt(a*x**2 + b)*a**2*b*x**4 + 5*sqrt(a*x**2 + b)*a*b**2*x**2 - 2*sqrt
(a*x**2 + b)*b**3 + 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b)
)/sqrt(b))*a**3*x**6 + 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqrt
(b))/sqrt(b))*a**2*b*x**4 - 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x +
 sqrt(b))/sqrt(b))*a**3*x**6 - 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*
x + sqrt(b))/sqrt(b))*a**2*b*x**4)/(8*b**4*x**4*(a*x**2 + b))
```

3.404 $\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = -\frac{35b^2}{24a^3\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{35b^2}{8a^4\sqrt{a+\frac{b}{x^2}}} - \frac{7bx^2}{8a^2\left(a+\frac{b}{x^2}\right)^{3/2}} + \frac{x^4}{4a\left(a+\frac{b}{x^2}\right)^{3/2}} + \frac{35b^2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{9/2}}$$

output

```
-35/24*b^2/a^3/(a+b/x^2)^(3/2)-35/8*b^2/a^4/(a+b/x^2)^(1/2)-7/8*b*x^2/a^2/
(a+b/x^2)^(3/2)+1/4*x^4/a/(a+b/x^2)^(3/2)+35/8*b^2*arctanh((a+b/x^2)^(1/2)
/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{\sqrt{a}x(-105b^3-140ab^2x^2-21a^2bx^4+6a^3x^6)+210b^2(b+ax^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{24a^{9/2}\sqrt{a+\frac{b}{x^2}}x\,(b+ax^2)}$$

input
```
Integrate[x^3/(a + b/x^2)^(5/2),x]
```

output
```
(Sqrt[a]*x*(-105*b^3 - 140*a*b^2*x^2 - 21*a^2*b*x^4 + 6*a^3*x^6) + 210*b^2
*(b + a*x^2)^(3/2)*ArcTanh[(Sqrt[a]*x)/(-Sqrt[b] + Sqrt[b + a*x^2])])/(24*
a^(9/2)*Sqrt[a + b/x^2]*x*(b + a*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.467$, Rules used = {798, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^6}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{7b\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x^2}}{4a}+\frac{x^4}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{7b\left(-\frac{5b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x^2}}{2a}-\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{4a}+\frac{x^4}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(\frac{7b\left(-\frac{5b\left(\frac{\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}-\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{4a}+\frac{x^4}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(\frac{7b\left(-\frac{5b\left(\frac{\frac{\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}-\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{4a}+\frac{x^4}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{7b\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}-\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{4a}+\frac{x^4}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{7b\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}-\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{4a}+\frac{x^4}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

input
```
Int[x^3/(a + b/x^2)^(5/2),x]
```

output
```
(x^4/(2*a*(a + b/x^2)^(3/2)) + (7*b*(-(x^2/(a*(a + b/x^2)^(3/2))) - (5*b*(
2/(3*a*(a + b/x^2)^(3/2)) + (2/(a*Sqrt[a + b/x^2]) - (2*ArcTanh[Sqrt[a + b
/x^2]/Sqrt[a]])/a^(3/2))/a))/(2*a)))/(4*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result
default	$\frac{(a\,x^2+b)\left(6x^7a^{\frac{9}{2}}-21a^{\frac{7}{2}}b\,x^5-140a^{\frac{5}{2}}b^2x^3+105\left(a\,x^2+b\right)^{\frac{3}{2}}\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)a\,b^2-105a^{\frac{3}{2}}b^3x\right)}{24\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}x^5a^{\frac{11}{2}}}$
risch	$\frac{(2a\,x^2-11b)(a\,x^2+b)}{8a^4\sqrt{\frac{a\,x^2+b}{x^2}}}+\frac{\left(\frac{35b^2\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)}{8a^{\frac{9}{2}}}+\frac{b^3\sqrt{a\left(x-\frac{\sqrt{-ab}}{a}\right)^2+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{a}\right)}}{12a^5\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{a}\right)^2}-\frac{5b^2\sqrt{a\left(x-\frac{\sqrt{-ab}}{a}\right)^2+2\sqrt{-ab}\left(x-\frac{\sqrt{-a}}{a}\right.}}{3a^5\left(x-\frac{\sqrt{-ab}}{a}\right)}\right.}{\sqrt{\frac{a\,x^2+b}{x^2}}}$

input

```
int(x^3/(a+b/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/24*(a*x^2+b)*(6*x^7*a^(9/2)-21*a^(7/2)*b*x^5-140*a^(5/2)*b^2*x^3+105*(a*
x^2+b)^(3/2)*ln(a^(1/2)*x+(a*x^2+b)^(1/2))*a*b^2-105*a^(3/2)*b^3*x)/((a*x^
2+b)/x^2)^(5/2)/x^5/a^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.47

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \left[\frac{105\,(a^2b^2x^4+2\,ab^3x^2+b^4)\sqrt{a}\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\,(6\,a^4x^8-21\,}{48\,(a^7x^4+2\,a^6bx^2+a^5b^2)}\right.$$

$$-\frac{105\,(a^2b^2x^4+2\,ab^3x^2+b^4)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)-(6\,a^4x^8-21\,a^3bx^6-140\,a^2b^2x^4-105\,ab^3x^2)}{24\,(a^7x^4+2\,a^6bx^2+a^5b^2)}$$

input

```
integrate(x^3/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(105*(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(a)*log(-2*a*x^2 - 2*sqrt
(a)*x^2*sqrt((a*x^2 + b)/x^2) - b) + 2*(6*a^4*x^8 - 21*a^3*b*x^6 - 140*a^2
*b^2*x^4 - 105*a*b^3*x^2)*sqrt((a*x^2 + b)/x^2))/(a^7*x^4 + 2*a^6*b*x^2 +
a^5*b^2), -1/24*(105*(a^2*b^2*x^4 + 2*a*b^3*x^2 + b^4)*sqrt(-a)*arctan(sqr
t(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) - (6*a^4*x^8 - 21*a^3*b*x^6 -
 140*a^2*b^2*x^4 - 105*a*b^3*x^2)*sqrt((a*x^2 + b)/x^2))/(a^7*x^4 + 2*a^6*
b*x^2 + a^5*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(107) = 214$.

Time = 7.42 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.72

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{6a^{\frac{89}{2}}b^{75}x^7}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}} - \frac{21a^{\frac{87}{2}}b^{76}x^5}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}} - \frac{140a^{\frac{85}{2}}b^{77}x^3}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}} - \frac{105a^{\frac{83}{2}}b^{78}x}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}} + \frac{105a^{42}b^{\frac{155}{2}}x^2\sqrt{\frac{ax^2}{b}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}} + \frac{105a^{41}b^{\frac{157}{2}}\sqrt{\frac{ax^2}{b}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{24a^{\frac{93}{2}}b^{\frac{151}{2}}x^2\sqrt{\frac{ax^2}{b}+1}+24a^{\frac{91}{2}}b^{\frac{153}{2}}\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate(x**3/(a+b/x**2)**(5/2),x)
```

output
```
6*a**(89/2)*b**75*x**7/(24*a**(93/2)*b**(151/2)*x**2*sqrt(a*x**2/b + 1) +
24*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1)) - 21*a**(87/2)*b**76*x**5/(24*
a**(93/2)*b**(151/2)*x**2*sqrt(a*x**2/b + 1) + 24*a**(91/2)*b**(153/2)*sqr
t(a*x**2/b + 1)) - 140*a**(85/2)*b**77*x**3/(24*a**(93/2)*b**(151/2)*x**2*
sqrt(a*x**2/b + 1) + 24*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1)) - 105*a**
(83/2)*b**78*x/(24*a**(93/2)*b**(151/2)*x**2*sqrt(a*x**2/b + 1) + 24*a**(9
1/2)*b**(153/2)*sqrt(a*x**2/b + 1)) + 105*a**42*b**(155/2)*x**2*sqrt(a*x**
2/b + 1)*asinh(sqrt(a)*x/sqrt(b))/(24*a**(93/2)*b**(151/2)*x**2*sqrt(a*x**
2/b + 1) + 24*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1)) + 105*a**41*b**(157
/2)*sqrt(a*x**2/b + 1)*asinh(sqrt(a)*x/sqrt(b))/(24*a**(93/2)*b**(151/2)*x
**2*sqrt(a*x**2/b + 1) + 24*a**(91/2)*b**(153/2)*sqrt(a*x**2/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = -\frac{105\left(a+\frac{b}{x^2}\right)^3 b^2 - 175\left(a+\frac{b}{x^2}\right)^2 ab^2 + 56\left(a+\frac{b}{x^2}\right)a^2b^2 + 8\,a^3b^2}{24\left(\left(a+\frac{b}{x^2}\right)^{\frac{7}{2}}a^4 - 2\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^5 + \left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^6\right)} - \frac{35\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{16\,a^{\frac{9}{2}}}$$

input
```
integrate(x^3/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/24*(105*(a + b/x^2)^3*b^2 - 175*(a + b/x^2)^2*a*b^2 + 56*(a + b/x^2)*a^
2*b^2 + 8*a^3*b^2)/((a + b/x^2)^(7/2)*a^4 - 2*(a + b/x^2)^(5/2)*a^5 + (a +
 b/x^2)^(3/2)*a^6) - 35/16*b^2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b
/x^2) + sqrt(a)))/a^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{\left(\left(3\,x^2\left(\frac{2\,x^2}{a\mathrm{sgn}(x)}-\frac{7\,b}{a^2\mathrm{sgn}(x)}\right)-\frac{140\,b^2}{a^3\mathrm{sgn}(x)}\right)x^2-\frac{105\,b^3}{a^4\mathrm{sgn}(x)}\right)x}{24\,(ax^2+b)^{\frac{3}{2}}} + \frac{35\,b^2\log\left(|b|\right)\mathrm{sgn}(x)}{16\,a^{\frac{9}{2}}} - \frac{35\,b^2\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{8\,a^{\frac{9}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3/(a+b/x^2)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*x^2*(2*x^2/(a*sgn(x)) - 7*b/(a^2*sgn(x))) - 140*b^2/(a^3*sgn(x)))
*x^2 - 105*b^3/(a^4*sgn(x)))*x/(a*x^2 + b)^(3/2) + 35/16*b^2*log(abs(b))*s
gn(x)/a^(9/2) - 35/8*b^2*log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))/(a^(9/2)*s
gn(x))
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{8\,a^{9/2}} - \frac{35\,b^2}{6\,a^3\,\left(a+\frac{b}{x^2}\right)^{3/2}} + \frac{x^4}{4\,a\,\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{7\,b\,x^2}{8\,a^2\,\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{35\,b^3}{8\,a^4\,x^2\,\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(x^3/(a + b/x^2)^(5/2),x)
```

output
```
(35*b^2*atanh((a + b/x^2)^(1/2)/a^(1/2)))/(8*a^(9/2)) - (35*b^2)/(6*a^3*(a
 + b/x^2)^(3/2)) + x^4/(4*a*(a + b/x^2)^(3/2)) - (7*b*x^2)/(8*a^2*(a + b/x
^2)^(3/2)) - (35*b^3)/(8*a^4*x^2*(a + b/x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.83

$$\int \frac{x^3}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{48\sqrt{a\,x^2+b}\,a^4x^7-168\sqrt{a\,x^2+b}\,a^3b\,x^5-1120\sqrt{a\,x^2+b}\,a^2b^2x^3-840\sqrt{a\,x^2+b}\,a\,b^3x}{}$$

input
```
int(x^3/(a+b/x^2)^(5/2),x)
```

output
```
(48*sqrt(a*x**2 + b)*a**4*x**7 - 168*sqrt(a*x**2 + b)*a**3*b*x**5 - 1120*s
qrt(a*x**2 + b)*a**2*b**2*x**3 - 840*sqrt(a*x**2 + b)*a*b**3*x + 840*sqrt(
a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*a**2*b**2*x**4 + 1680*sqrt(
a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*a*b**3*x**2 + 840*sqrt(a)*l
og((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b**4 + 175*sqrt(a)*a**2*b**2*x*
*4 + 350*sqrt(a)*a*b**3*x**2 + 175*sqrt(a)*b**4)/(192*a**5*(a**2*x**4 + 2*
a*b*x**2 + b**2))
```

3.405 $\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{5b}{6a^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{5b}{2a^3\sqrt{a+\frac{b}{x^2}}}+\frac{x^2}{2a\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
5/6*b/a^2/(a+b/x^2)^(3/2)+5/2*b/a^3/(a+b/x^2)^(1/2)+1/2*x^2/a/(a+b/x^2)^(3
/2)-5/2*b*arctanh((a+b/x^2)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{\sqrt{a}x(15b^2+20abx^2+3a^2x^4)+30b(b+ax^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{b}-\sqrt{b+ax^2}}\right)}{6a^{7/2}\sqrt{a+\frac{b}{x^2}}x\,(b+ax^2)}$$

input
```
Integrate[x/(a + b/x^2)^(5/2),x]
```

output
```
(Sqrt[a]*x*(15*b^2 + 20*a*b*x^2 + 3*a^2*x^4) + 30*b*(b + a*x^2)^(3/2)*ArcT
anh[(Sqrt[a]*x)/(Sqrt[b] - Sqrt[b + a*x^2])])/(6*a^(7/2)*Sqrt[a + b/x^2]*x
*(b + a*x^2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {798, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^4}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{5b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x^2}}{2a}+\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{5b\left(\frac{\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}+\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{5b\left(\frac{\frac{\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}+\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}+\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{2a}+\frac{x^2}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

input
```
Int[x/(a + b/x^2)^(5/2),x]
```

output
```
(x^2/(a*(a + b/x^2)^(3/2)) + (5*b*(2/(3*a*(a + b/x^2)^(3/2)) + (2/(a*Sqrt[
a + b/x^2]) - (2*ArcTanh[Sqrt[a + b/x^2]/Sqrt[a]])/a^(3/2))/a))/(2*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

method	result
default	$\frac{(a\,x^2+b)\left(3x^5 a^{\frac{7}{2}}+20a^{\frac{5}{2}}b\,x^3+15a^{\frac{3}{2}}b^2x-15\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)(a\,x^2+b)^{\frac{3}{2}}ab\right)}{6\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}x^5a^{\frac{9}{2}}}$
risch	$\frac{a\,x^2+b}{2a^3\sqrt{\frac{a\,x^2+b}{x^2}}}+\frac{\left(-\frac{5b\ln\left(\sqrt{a}\,x+\sqrt{a\,x^2+b}\right)}{2a^{\frac{7}{2}}}-\frac{b^2\sqrt{a\left(x-\frac{\sqrt{-ab}}{a}\right)^2+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{a}\right)}}{12a^4\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{a}\right)^2}+\frac{7b\sqrt{a\left(x-\frac{\sqrt{-ab}}{a}\right)^2+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{a}\right)}}{6a^4\left(x-\frac{\sqrt{-ab}}{a}\right)}+\frac{b^2\sqrt{a}}{}\right.}{\sqrt{\frac{a\,x^2+b}{x^2}}\,x}$

input
```
int(x/(a+b/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x^2+b)*(3*x^5*a^(7/2)+20*a^(5/2)*b*x^3+15*a^(3/2)*b^2*x-15*ln(a^(1/
2)*x+(a*x^2+b)^(1/2))*(a*x^2+b)^(3/2)*a*b)/((a*x^2+b)/x^2)^(5/2)/x^5/a^(9/
2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.95

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \left[\frac{15\,(a^2bx^4+2\,ab^2x^2+b^3)\sqrt{a}\log\left(-2\,ax^2+2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)+2\,(3\,a^3x^6+20\,a^2b}{12\,(a^6x^4+2\,a^5bx^2+a^4b^2)}\right.$$

input
```
integrate(x/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(15*(a^2*b*x^4 + 2*a*b^2*x^2 + b^3)*sqrt(a)*log(-2*a*x^2 + 2*sqrt(a)
*x^2*sqrt((a*x^2 + b)/x^2) - b) + 2*(3*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x
^2)*sqrt((a*x^2 + b)/x^2))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2), 1/6*(15*(a^2
*b*x^4 + 2*a*b^2*x^2 + b^3)*sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/
x^2)/(a*x^2 + b)) + (3*a^3*x^6 + 20*a^2*b*x^4 + 15*a*b^2*x^2)*sqrt((a*x^2
+ b)/x^2))/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(80) = 160$.

Time = 3.16 (sec) , antiderivative size = 819, normalized size of antiderivative = 9.31

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b/x**2)**(5/2),x)
```

output
```
6*a**17*x**8*sqrt(1 + b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4
 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 46*a**16*b*x**6*sqrt(1 +
b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x
**2 + 12*a**(33/2)*b**3) + 15*a**16*b*x**6*log(b/(a*x**2))/(12*a**(39/2)*x
**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) -
30*a**16*b*x**6*log(sqrt(1 + b/(a*x**2)) + 1)/(12*a**(39/2)*x**6 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 70*a**15*b**2
*x**4*sqrt(1 + b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 45*a**15*b**2*x**4*log(b/(a*x**2
))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*
a**(33/2)*b**3) - 90*a**15*b**2*x**4*log(sqrt(1 + b/(a*x**2)) + 1)/(12*a**
(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*
b**3) + 30*a**14*b**3*x**2*sqrt(1 + b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a*
*(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) + 45*a**14*b*
*3*x**2*log(b/(a*x**2))/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(
35/2)*b**2*x**2 + 12*a**(33/2)*b**3) - 90*a**14*b**3*x**2*log(sqrt(1 + b/(
a*x**2)) + 1)/(12*a**(39/2)*x**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2
*x**2 + 12*a**(33/2)*b**3) + 15*a**13*b**4*log(b/(a*x**2))/(12*a**(39/2)*x
**6 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**2 + 12*a**(33/2)*b**3) -
30*a**13*b**4*log(sqrt(1 + b/(a*x**2)) + 1)/(12*a**(39/2)*x**6 + 36*a**...
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{15\left(a+\frac{b}{x^2}\right)^2b-10\left(a+\frac{b}{x^2}\right)ab-2a^2b}{6\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4\right)}+\frac{5b\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$$

input
```
integrate(x/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(15*(a + b/x^2)^2*b - 10*(a + b/x^2)*a*b - 2*a^2*b)/((a + b/x^2)^(5/2)
*a^3 - (a + b/x^2)^(3/2)*a^4) + 5/4*b*log((sqrt(a + b/x^2) - sqrt(a))/(sqr
t(a + b/x^2) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{\left(x^2\left(\frac{3x^2}{a\mathrm{sgn}(x)}+\frac{20b}{a^2\mathrm{sgn}(x)}\right)+\frac{15b^2}{a^3\mathrm{sgn}(x)}\right)x}{6\left(ax^2+b\right)^{\frac{3}{2}}}-\frac{5b\log\left(|b|\right)\mathrm{sgn}(x)}{4a^{\frac{7}{2}}}+\frac{5b\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{2a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x/(a+b/x^2)^(5/2),x, algorithm="giac")
```

output
```
1/6*(x^2*(3*x^2/(a*sgn(x)) + 20*b/(a^2*sgn(x))) + 15*b^2/(a^3*sgn(x)))*x/(
a*x^2 + b)^(3/2) - 5/4*b*log(abs(b))*sgn(x)/a^(7/2) + 5/2*b*log(abs(-sqrt(
a)*x + sqrt(a*x^2 + b)))/(a^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{10\,b}{3\,a^2\,\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{x^2}{2\,a\,\left(a+\frac{b}{x^2}\right)^{3/2}} -\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{2\,a^{7/2}}+\frac{5\,b^2}{2\,a^3\,x^2\,\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(x/(a + b/x^2)^(5/2),x)
```

output
```
(10*b)/(3*a^2*(a + b/x^2)^(3/2)) + x^2/(2*a*(a + b/x^2)^(3/2)) - (5*b*atan
h((a + b/x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) + (5*b^2)/(2*a^3*x^2*(a + b/x^2)
^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \frac{x}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{6\sqrt{a\,x^2+b}\,a^3x^5+40\sqrt{a\,x^2+b}\,a^2b\,x^3+30\sqrt{a\,x^2+b}\,a\,b^2x-30\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right) a}{12}$$

input
```
int(x/(a+b/x^2)^(5/2),x)
```

output
```
(6*sqrt(a*x**2 + b)*a**3*x**5 + 40*sqrt(a*x**2 + b)*a**2*b*x**3 + 30*sqrt(
a*x**2 + b)*a*b**2*x - 30*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(
b))*a**2*b*x**4 - 60*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*a
*b**2*x**2 - 30*sqrt(a)*log((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*b**3 -
 5*sqrt(a)*a**2*b*x**4 - 10*sqrt(a)*a*b**2*x**2 - 5*sqrt(a)*b**3)/(12*a**4
*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.406 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x} \, dx = -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{1}{a^2\sqrt{a+\frac{b}{x^2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-1/3/a/(a+b/x^2)^(3/2)-1/a^2/(a+b/x^2)^(1/2)+arctanh((a+b/x^2)^(1/2)/a^(1/
2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.58

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x} \, dx = \frac{-\sqrt{a}x(3b+4ax^2) + 6(b+ax^2)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a}x}{-\sqrt{b}+\sqrt{b+ax^2}}\right)}{3a^{5/2}\sqrt{a+\frac{b}{x^2}}\,x\,(b+ax^2)}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x),x]
```

output
```
(-(Sqrt[a]*x*(3*b + 4*a*x^2)) + 6*(b + a*x^2)^(3/2)*ArcTanh[(Sqrt[a]*x)/(-
Sqrt[b] + Sqrt[b + a*x^2])])/(3*a^(5/2)*Sqrt[a + b/x^2]*x*(b + a*x^2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{a}-\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{x^2}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}}{a}-\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{2\int \frac{1}{\frac{1}{bx^4}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^2}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^2}}}}{a}-\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{2}{a\sqrt{a+\frac{b}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}-\frac{2}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

input
```
Int[1/((a + b/x^2)^(5/2)*x),x]
```

output
```
(-2/(3*a*(a + b/x^2)^(3/2)) - (2/(a*Sqrt[a + b/x^2]) - (2*ArcTanh[Sqrt[a +
 b/x^2]/Sqrt[a]])/a^(3/2))/a)/2
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\left(a\,x^{2}+b\right)\left(4 x^{3} a^{\frac{5}{2}}+3 a^{\frac{3}{2}} b x-3 \ln\left(\sqrt{a}\, x+\sqrt{a\,x^{2}+b}\right)\left(a\,x^{2}+b\right)^{\frac{3}{2}} a\right)}{3\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}} x^{5} a^{\frac{7}{2}}}$	73

input
```
int(1/(a+b/x^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*x^2+b)*(4*x^3*a^(5/2)+3*a^(3/2)*b*x-3*ln(a^(1/2)*x+(a*x^2+b)^(1/2)
)*(a*x^2+b)^(3/2)*a)/((a*x^2+b)/x^2)^(5/2)/x^5/a^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(47) = 94.

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x}\,dx = \left[\frac{3\left(a^2x^4+2\,abx^2+b^2\right)\sqrt{a}\log\left(-2\,ax^2-2\,\sqrt{a}x^2\sqrt{\frac{ax^2+b}{x^2}}-b\right)-2\left(4\,a^2x^4+3\,abx^2\right)}{6\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}\right.$$

$$\left.-\frac{3\left(a^2x^4+2\,abx^2+b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x^2\sqrt{\frac{ax^2+b}{x^2}}}{ax^2+b}\right)+\left(4\,a^2x^4+3\,abx^2\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}\right]$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(a)*log(-2*a*x^2 - 2*sqrt(a)*x^2*s
qrt((a*x^2 + b)/x^2) - b) - 2*(4*a^2*x^4 + 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2
))/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2), -1/3*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)*
sqrt(-a)*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + b)/x^2)/(a*x^2 + b)) + (4*a^2*x
^4 + 3*a*b*x^2)*sqrt((a*x^2 + b)/x^2))/(a^5*x^4 + 2*a^4*b*x^2 + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(51) = 102$.

Time = 1.97 (sec) , antiderivative size = 743, normalized size of antiderivative = 12.59

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x} \, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x,x)
```

output
```
-8*a**7*x**6*sqrt(1 + b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4
+ 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 3*a**7*x**6*log(b/(a*x**2))
/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(
13/2)*b**3) + 6*a**7*x**6*log(sqrt(1 + b/(a*x**2)) + 1)/(6*a**(19/2)*x**6
+ 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 14*a*
*6*b*x**4*sqrt(1 + b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 1
8*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 9*a**6*b*x**4*log(b/(a*x**2))/
(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(1
3/2)*b**3) + 18*a**6*b*x**4*log(sqrt(1 + b/(a*x**2)) + 1)/(6*a**(19/2)*x**
6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 6*a
**5*b**2*x**2*sqrt(1 + b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4
 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) - 9*a**5*b**2*x**2*log(b/(a*
x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 +
6*a**(13/2)*b**3) + 18*a**5*b**2*x**2*log(sqrt(1 + b/(a*x**2)) + 1)/(6*a**
(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b
**3) - 3*a**4*b**3*log(b/(a*x**2))/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4
 + 18*a**(15/2)*b**2*x**2 + 6*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b
/(a*x**2)) + 1)/(6*a**(19/2)*x**6 + 18*a**(17/2)*b*x**4 + 18*a**(15/2)*b**
2*x**2 + 6*a**(13/2)*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^2}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}} - \frac{4\,a+\frac{3\,b}{x^2}}{3\,\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
-1/2*log((sqrt(a + b/x^2) - sqrt(a))/(sqrt(a + b/x^2) + sqrt(a)))/a^(5/2)
- 1/3*(4*a + 3*b/x^2)/((a + b/x^2)^(3/2)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x}\,dx = -\frac{x\left(\frac{4\,x^2}{a\mathrm{sgn}(x)}+\frac{3\,b}{a^2\mathrm{sgn}(x)}\right)}{3\,(ax^2+b)^{\frac{3}{2}}} + \frac{\log\left(|b|\right)\mathrm{sgn}(x)}{2\,a^{\frac{5}{2}}} - \frac{\log\left(\left|-\sqrt{a}x+\sqrt{ax^2+b}\right|\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x,x, algorithm="giac")
```

output
```
-1/3*x*(4*x^2/(a*sgn(x)) + 3*b/(a^2*sgn(x)))/(a*x^2 + b)^(3/2) + 1/2*log(a
bs(b))*sgn(x)/a^(5/2) - log(abs(-sqrt(a)*x + sqrt(a*x^2 + b)))/(a^(5/2)*sg
n(x))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{a+\frac{b}{x^2}}{a^2}+\frac{1}{3\,a}}{\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(1/(x*(a + b/x^2)^(5/2)),x)
```

output
```
atanh((a + b/x^2)^(1/2)/a^(1/2))/a^(5/2) - ((a + b/x^2)/a^2 + 1/(3*a))/(a
+ b/x^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x}\,dx = \frac{-4\sqrt{a\,x^2+b}\,a^2x^3 - 3\sqrt{a\,x^2+b}\,abx + 3\sqrt{a}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right)a^2x^4 + 6\sqrt{a}\log\Big(}{3a^3\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^(5/2)/x,x)
```

output
```
( - 4*sqrt(a*x**2 + b)*a**2*x**3 - 3*sqrt(a*x**2 + b)*a*b*x + 3*sqrt(a)*lo
g((sqrt(a*x**2 + b) + sqrt(a)*x)/sqrt(b))*a**2*x**4 + 6*sqrt(a)*log((sqrt(
a*x**2 + b) + sqrt(a)*x)/sqrt(b))*a*b*x**2 + 3*sqrt(a)*log((sqrt(a*x**2 +
b) + sqrt(a)*x)/sqrt(b))*b**2)/(3*a**3*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.407 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3} \, dx=\frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2}}$$

output
```
1/3/b/(a+b/x^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3} \, dx=\frac{b+a x^2}{3b\left(a+\frac{b}{x^2}\right)^{5/2} x^2}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^3),x]
```

output
```
(b + a*x^2)/(3*b*(a + b/x^2)^(5/2)*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x^2}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{3b \left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^3),x]
```

output
```
1/(3*b*(a + b/x^2)^(3/2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{1}{3b\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}$	15
orering	$\frac{a\,x^2+b}{3x^2b\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}$	25
gosper	$\frac{a\,x^2+b}{3x^2b\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	29
default	$\frac{a\,x^2+b}{3x^2b\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	29
trager	$\frac{x^4\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3(a\,x^2+b)^2b}$	35

input
```
int(1/(a+b/x^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/b/(a+b/x^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^3}\,dx = \frac{x^4\sqrt{\frac{ax^2+b}{x^2}}}{3\,(a^2bx^4+2\,ab^2x^2+b^3)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
1/3*x^4*sqrt((a*x^2 + b)/x^2)/(a^2*b*x^4 + 2*a*b^2*x^2 + b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(14) = 28$.

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3}\,dx = \begin{cases} \frac{1}{3ab\sqrt{a+\frac{b}{x^2}}+\frac{3b^2\sqrt{a+\frac{b}{x^2}}}{x^2}} & \text{for}\: b \neq 0 \\ -\frac{1}{2a^{\frac{5}{2}}x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**3,x)
```

output
```
Piecewise((1/(3*a*b*sqrt(a + b/x**2) + 3*b**2*sqrt(a + b/x**2)/x**2), Ne(b
, 0)), (-1/(2*a**(5/2)*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3}\,dx = \frac{1}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}} b}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
1/3/((a + b/x^2)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3}\,dx = \frac{x^3}{3\left(ax^2+b\right)^{\frac{3}{2}} b\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/3*x^3/((a*x^2 + b)^(3/2)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3} \, dx = \frac{1}{3\, b \left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(1/(x^3*(a + b/x^2)^(5/2)),x)
```

output
```
1/(3*b*(a + b/x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^3} \, dx = \frac{\sqrt{a\, x^2+b}\, a^2 x^3+\sqrt{a}\, a^2 x^4+2 \sqrt{a}\, a b\, x^2+\sqrt{a}\, b^2}{3 a^2 b \left(a^2 x^4+2 a b\, x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^3,x)
```

output
```
(sqrt(a*x**2 + b)*a**2*x**3 + sqrt(a)*a**2*x**4 + 2*sqrt(a)*a*b*x**2 + sqr
t(a)*b**2)/(3*a**2*b*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.408 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5} dx = -\frac{a}{3b^2\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{1}{b^2\sqrt{a+\frac{b}{x^2}}}$$

output
```
-1/3*a/b^2/(a+b/x^2)^(3/2)+1/b^2/(a+b/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5} dx = \frac{(b+ax^2)(3bx+2ax^3)}{3b^2\left(a+\frac{b}{x^2}\right)^{5/2} x^5}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^5),x]
```

output
```
((b + a*x^2)*(3*b*x + 2*a*x^3))/(3*b^2*(a + b/x^2)^(5/2)*x^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{1}{b \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{a}{b \left(a+\frac{b}{x^2}\right)^{5/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{2}{b^2 \sqrt{a+\frac{b}{x^2}}}-\frac{2 a}{3 b^2 \left(a+\frac{b}{x^2}\right)^{3/2}}\right)$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^5),x]
```

output
```
((-2*a)/(3*b^2*(a + b/x^2)^(3/2)) + 2/(b^2*Sqrt[a + b/x^2]))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
orering	$\frac{(2a\,x^2+3b)\,(a\,x^2+b)}{3x^4b^2\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}$	35
gosper	$\frac{(a\,x^2+b)\,(2a\,x^2+3b)}{3x^4b^2\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	39
default	$\frac{(a\,x^2+b)\,(2a\,x^2+3b)}{3x^4b^2\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	39
trager	$\frac{x^2\,(2a\,x^2+3b)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3b^2\,(a\,x^2+b)^2}$	45

input
```
int(1/(a+b/x^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3/x^4*(2*a*x^2+3*b)/b^2*(a*x^2+b)/(a+b/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5}\,dx = \frac{(2\,ax^4+3\,bx^2)\sqrt{\frac{ax^2+b}{x^2}}}{3\,(a^2b^2x^4+2\,ab^3x^2+b^4)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
1/3*(2*a*x^4 + 3*b*x^2)*sqrt((a*x^2 + b)/x^2)/(a^2*b^2*x^4 + 2*a*b^3*x^2 +
 b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(31) = 62.

Time = 0.78 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.69

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5}\,dx = \begin{cases} \frac{2ax^2}{3ab^2x^2\sqrt{a+\frac{b}{x^2}}+3b^3\sqrt{a+\frac{b}{x^2}}} + \frac{3b}{3ab^2x^2\sqrt{a+\frac{b}{x^2}}+3b^3\sqrt{a+\frac{b}{x^2}}} & \text{for}\ b \neq 0 \\ -\frac{1}{4a^{\frac{5}{2}}x^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**5,x)
```

output
```
Piecewise((2*a*x**2/(3*a*b**2*x**2*sqrt(a + b/x**2) + 3*b**3*sqrt(a + b/x*
*2)) + 3*b/(3*a*b**2*x**2*sqrt(a + b/x**2) + 3*b**3*sqrt(a + b/x**2)), Ne(
b, 0)), (-1/(4*a**(5/2)*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^5}\,dx=\frac{1}{\sqrt{a+\frac{b}{x^2}}\,b^2}-\frac{a}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^2}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
1/(sqrt(a + b/x^2)*b^2) - 1/3*a/((a + b/x^2)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^5}\,dx=\frac{x\left(\frac{2\,ax^2}{b^2\mathrm{sgn}(x)}+\frac{3}{b\mathrm{sgn}(x)}\right)}{3\,(ax^2+b)^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/3*x*(2*a*x^2/(b^2*sgn(x)) + 3/(b*sgn(x)))/(a*x^2 + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^5}\,dx=\frac{2\,a\,x^2+3\,b}{3\,b^2\,x^2\,\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(1/(x^5*(a + b/x^2)^(5/2)),x)
```

output
```
(3*b + 2*a*x^2)/(3*b^2*x^2*(a + b/x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.40

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^5}\,dx = \frac{2\sqrt{a\,x^2+b}\,a^2x^3+3\sqrt{a\,x^2+b}\,abx-2\sqrt{a}\,a^2x^4-4\sqrt{a}\,ab\,x^2-2\sqrt{a}\,b^2}{3a\,b^2\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^5,x)
```

output
```
(2*sqrt(a*x**2 + b)*a**2*x**3 + 3*sqrt(a*x**2 + b)*a*b*x - 2*sqrt(a)*a**2*
x**4 - 4*sqrt(a)*a*b*x**2 - 2*sqrt(a)*b**2)/(3*a*b**2*(a**2*x**4 + 2*a*b*x
**2 + b**2))
```

3.409 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7} dx = \frac{a^2}{3b^3\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{2a}{b^3\sqrt{a+\frac{b}{x^2}}} - \frac{\sqrt{a+\frac{b}{x^2}}}{b^3}$$

output
```
1/3*a^2/b^3/(a+b/x^2)^(3/2)-2*a/b^3/(a+b/x^2)^(1/2)-(a+b/x^2)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7} dx = \frac{(b+ax^2)\left(-3b^2-12abx^2-8a^2x^4\right)}{3b^3\left(a+\frac{b}{x^2}\right)^{5/2} x^6}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^7),x]
```

output
```
((b + a*x^2)*(-3*b^2 - 12*a*b*x^2 - 8*a^2*x^4))/(3*b^3*(a + b/x^2)^(5/2)*x
^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^2}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2} \int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4} d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2} \int \left(\frac{a^2}{b^2 \left(a+\frac{b}{x^2}\right)^{5/2}}-\frac{2 a}{b^2 \left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{1}{b^2 \sqrt{a+\frac{b}{x^2}}}\right) d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{2 a^2}{3 b^3 \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{4 a}{b^3 \sqrt{a+\frac{b}{x^2}}}-\frac{2 \sqrt{a+\frac{b}{x^2}}}{b^3}\right)$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^7),x]
```

output
```
((2*a^2)/(3*b^3*(a + b/x^2)^(3/2)) - (4*a)/(b^3*Sqrt[a + b/x^2]) - (2*Sqrt
[a + b/x^2])/b^3)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
orering	$-\frac{\left(8a^{2}x^{4}+12ab\,x^{2}+3b^{2}\right)\left(a\,x^{2}+b\right)}{3b^{3}x^{6}\left(a+\frac{b}{x^{2}}\right)^{\frac{5}{2}}}$	46
gosper	$-\frac{\left(a\,x^{2}+b\right)\left(8a^{2}x^{4}+12ab\,x^{2}+3b^{2}\right)}{3x^{6}b^{3}\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	50
default	$-\frac{\left(a\,x^{2}+b\right)\left(8a^{2}x^{4}+12ab\,x^{2}+3b^{2}\right)}{3x^{6}b^{3}\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	50
trager	$-\frac{\left(8a^{2}x^{4}+12ab\,x^{2}+3b^{2}\right)\sqrt{-\frac{-a\,x^{2}-b}{x^{2}}}}{3b^{3}\left(a\,x^{2}+b\right)^{2}}$	53
risch	$-\frac{a\,x^{2}+b}{b^{3}x^{2}\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}-\frac{\left(a\,x^{2}+b\right)\left(5a\,x^{2}+6b\right)a}{3b^{3}\left(a^{2}x^{4}+2ab\,x^{2}+b^{2}\right)\sqrt{\frac{a\,x^{2}+b}{x^{2}}}}$	86

input
```
int(1/(a+b/x^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(8*a^2*x^4+12*a*b*x^2+3*b^2)/b^3*(a*x^2+b)/x^6/(a+b/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7} \, dx = -\frac{\left(8 a^2 x^4 + 12 a b x^2 + 3 b^2\right) \sqrt{\frac{a x^2 + b}{x^2}}}{3 \left(a^2 b^3 x^4 + 2 a b^4 x^2 + b^5\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
-1/3*(8*a^2*x^4 + 12*a*b*x^2 + 3*b^2)*sqrt((a*x^2 + b)/x^2)/(a^2*b^3*x^4 +
 2*a*b^4*x^2 + b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(48) = 96.

Time = 1.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.78

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7} \, dx = \begin{cases} -\frac{8 a^2 x^4}{3 a b^3 x^4 \sqrt{a+\frac{b}{x^2}} + 3 b^4 x^2 \sqrt{a+\frac{b}{x^2}}} - \frac{12 a b x^2}{3 a b^3 x^4 \sqrt{a+\frac{b}{x^2}} + 3 b^4 x^2 \sqrt{a+\frac{b}{x^2}}} - \frac{3 b^2}{3 a b^3 x^4 \sqrt{a+\frac{b}{x^2}} + 3 b^4 x^2 \sqrt{a+\frac{b}{x^2}}} & \text{f} \\ -\frac{1}{6 a^{\frac{5}{2}} x^6} & \text{o} \end{cases}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**7,x)
```

output
```
Piecewise((-8*a**2*x**4/(3*a*b**3*x**4*sqrt(a + b/x**2) + 3*b**4*x**2*sqrt
(a + b/x**2)) - 12*a*b*x**2/(3*a*b**3*x**4*sqrt(a + b/x**2) + 3*b**4*x**2*
sqrt(a + b/x**2)) - 3*b**2/(3*a*b**3*x**4*sqrt(a + b/x**2) + 3*b**4*x**2*s
qrt(a + b/x**2)), Ne(b, 0)), (-1/(6*a**(5/2)*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^7}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}}{b^3} - \frac{2\,a}{\sqrt{a+\frac{b}{x^2}}b^3} + \frac{a^2}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^3}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
-sqrt(a + b/x^2)/b^3 - 2*a/(sqrt(a + b/x^2)*b^3) + 1/3*a^2/((a + b/x^2)^(3
/2)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^7}\,dx = -\frac{x\left(\frac{5\,a^2x^2}{b^3\mathrm{sgn}(x)}+\frac{6\,a}{b^2\mathrm{sgn}(x)}\right)}{3\left(ax^2+b\right)^{\frac{3}{2}}} + \frac{2\sqrt{a}}{\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)b^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
-1/3*x*(5*a^2*x^2/(b^3*sgn(x)) + 6*a/(b^2*sgn(x)))/(a*x^2 + b)^(3/2) + 2*s
qrt(a)/(((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^7}\,dx = -\frac{\sqrt{a+\frac{b}{x^2}}\left(\frac{8\,a^2\,x^4}{3}+4\,a\,b\,x^2+b^2\right)}{b^3\left(a\,x^2+b\right)^2}$$

input
```
int(1/(x^7*(a + b/x^2)^(5/2)),x)
```

output
```
-((a + b/x^2)^(1/2)*(b^2 + (8*a^2*x^4)/3 + 4*a*b*x^2))/(b^3*(b + a*x^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.82

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^7}\,dx = \frac{-8\sqrt{a\,x^2+b}\,a^2x^4-12\sqrt{a\,x^2+b}\,ab\,x^2-3\sqrt{a\,x^2+b}\,b^2+8\sqrt{a}\,a^2x^5+16\sqrt{a}\,ab\,x^3+}{3b^3x\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^7,x)
```

output
```
( - 8*sqrt(a*x**2 + b)*a**2*x**4 - 12*sqrt(a*x**2 + b)*a*b*x**2 - 3*sqrt(a
*x**2 + b)*b**2 + 8*sqrt(a)*a**2*x**5 + 16*sqrt(a)*a*b*x**3 + 8*sqrt(a)*b*
*2*x)/(3*b**3*x*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.410 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9} \, dx = -\frac{a^3}{3b^4\left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{3a^2}{b^4\sqrt{a+\frac{b}{x^2}}}+\frac{3a\sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{3b^4}$$

output
```
-1/3*a^3/b^4/(a+b/x^2)^(3/2)+3*a^2/b^4/(a+b/x^2)^(1/2)+3*a*(a+b/x^2)^(1/2)
/b^4-1/3*(a+b/x^2)^(3/2)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9} \, dx = \frac{(b+ax^2)\left(-b^3+6ab^2x^2+24a^2bx^4+16a^3x^6\right)}{3b^4\left(a+\frac{b}{x^2}\right)^{5/2}x^8}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^9),x]
```

output
```
((b + a*x^2)*(-b^3 + 6*a*b^2*x^2 + 24*a^2*b*x^4 + 16*a^3*x^6))/(3*b^4*(a +
 b/x^2)^(5/2)*x^8)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 798}$$

$$-\frac{1}{2}\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{1}{2}\int \left(-\frac{a^3}{b^3 \left(a+\frac{b}{x^2}\right)^{5/2}}+\frac{3 a^2}{b^3 \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{3 a}{b^3 \sqrt{a+\frac{b}{x^2}}}+\frac{\sqrt{a+\frac{b}{x^2}}}{b^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{2 a^3}{3 b^4 \left(a+\frac{b}{x^2}\right)^{3/2}}+\frac{6 a^2}{b^4 \sqrt{a+\frac{b}{x^2}}}+\frac{6 a \sqrt{a+\frac{b}{x^2}}}{b^4}-\frac{2 \left(a+\frac{b}{x^2}\right)^{3/2}}{3 b^4}\right)$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^9),x]
```

output
```
((-2*a^3)/(3*b^4*(a + b/x^2)^(3/2)) + (6*a^2)/(b^4*Sqrt[a + b/x^2]) + (6*a
*Sqrt[a + b/x^2])/b^4 - (2*(a + b/x^2)^(3/2))/(3*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result	size
orering	$\frac{\left(16 a^{3} x^{6}+24 a^{2} b \,x^{4}+6 a \,b^{2} x^{2}-b^{3}\right) \left(a \,x^{2}+b \right)}{3 b^{4} x^{8} \left(a +\frac{b}{x^{2}}\right)^{\frac{5}{2}}}$	57
gosper	$\frac{\left(a \,x^{2}+b \right) \left(16 a^{3} x^{6}+24 a^{2} b \,x^{4}+6 a \,b^{2} x^{2}-b^{3}\right)}{3 x^{8} b^{4} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	61
default	$\frac{\left(a \,x^{2}+b \right) \left(16 a^{3} x^{6}+24 a^{2} b \,x^{4}+6 a \,b^{2} x^{2}-b^{3}\right)}{3 x^{8} b^{4} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	61
trager	$\frac{\left(16 a^{3} x^{6}+24 a^{2} b \,x^{4}+6 a \,b^{2} x^{2}-b^{3}\right) \sqrt{-\frac{-a \,x^{2}-b}{x^{2}}}}{3 x^{2} b^{4} \left(a \,x^{2}+b \right)^{2}}$	67
risch	$\frac{\left(a \,x^{2}+b \right) \left(8 a \,x^{2}-b \right)}{3 b^{4} x^{4} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}+\frac{\left(a \,x^{2}+b \right) \left(8 a \,x^{2}+9 b \right) a^{2}}{3 b^{4} \left(a^{2} x^{4}+2 a b \,x^{2}+b^{2}\right) \sqrt{\frac{a \,x^{2}+b}{x^{2}}}}$	98

input
```
int(1/(a+b/x^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/3*(16*a^3*x^6+24*a^2*b*x^4+6*a*b^2*x^2-b^3)/b^4*(a*x^2+b)/x^8/(a+b/x^2)^
(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9}\, dx = \frac{(16\, a^3 x^6 + 24\, a^2 b x^4 + 6\, a b^2 x^2 - b^3)\sqrt{\frac{a x^2+b}{x^2}}}{3\,(a^2 b^4 x^6 + 2\, a b^5 x^4 + b^6 x^2)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
1/3*(16*a^3*x^6 + 24*a^2*b*x^4 + 6*a*b^2*x^2 - b^3)*sqrt((a*x^2 + b)/x^2)/
(a^2*b^4*x^6 + 2*a*b^5*x^4 + b^6*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(68) = 136.

Time = 1.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.64

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9}\, dx = \begin{cases} \frac{16a^3x^6}{3ab^4x^6\sqrt{a+\frac{b}{x^2}}+3b^5x^4\sqrt{a+\frac{b}{x^2}}} + \frac{24a^2bx^4}{3ab^4x^6\sqrt{a+\frac{b}{x^2}}+3b^5x^4\sqrt{a+\frac{b}{x^2}}} + \frac{6ab^2x^2}{3ab^4x^6\sqrt{a+\frac{b}{x^2}}+3b^5x^4\sqrt{a+\frac{b}{x^2}}} - \frac{}{3a\cdots} \\ -\frac{1}{8a^{\frac{5}{2}}x^8} \end{cases}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**9,x)
```

output
```
Piecewise((16*a**3*x**6/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b**5*x**4*sqrt
(a + b/x**2)) + 24*a**2*b*x**4/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b**5*x*
*4*sqrt(a + b/x**2)) + 6*a*b**2*x**2/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b
**5*x**4*sqrt(a + b/x**2)) - b**3/(3*a*b**4*x**6*sqrt(a + b/x**2) + 3*b**5
*x**4*sqrt(a + b/x**2)), Ne(b, 0)), (-1/(8*a**(5/2)*x**8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}}{3\,b^4} + \frac{3\,\sqrt{a+\frac{b}{x^2}}a}{b^4} + \frac{3\,a^2}{\sqrt{a+\frac{b}{x^2}}b^4} - \frac{a^3}{3\,\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}} b^4}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
-1/3*(a + b/x^2)^(3/2)/b^4 + 3*sqrt(a + b/x^2)*a/b^4 + 3*a^2/(sqrt(a + b/x
^2)*b^4) - 1/3*a^3/((a + b/x^2)^(3/2)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(64) = 128.

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9} \, dx = \frac{x\left(\frac{8\,a^3x^2}{b^4\mathrm{sgn}(x)} + \frac{9\,a^2}{b^3\mathrm{sgn}(x)}\right)}{3\,(ax^2+b)^{\frac{3}{2}}} - \frac{4\left(3\,\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^4 a^{\frac{3}{2}} - 9\,\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2 a^{\frac{3}{2}}b + 4\,a^{\frac{3}{2}}b^2\right)}{3\left(\left(\sqrt{a}x-\sqrt{ax^2+b}\right)^2-b\right)^3 b^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^9,x, algorithm="giac")
```

output
```
1/3*x*(8*a^3*x^2/(b^4*sgn(x)) + 9*a^2/(b^3*sgn(x)))/(a*x^2 + b)^(3/2) - 4/
3*(3*(sqrt(a)*x - sqrt(a*x^2 + b))^4*a^(3/2) - 9*(sqrt(a)*x - sqrt(a*x^2 +
 b))^2*a^(3/2)*b + 4*a^(3/2)*b^2)/(((sqrt(a)*x - sqrt(a*x^2 + b))^2 - b)^3
*b^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9}\,dx=\frac{\sqrt{a+\frac{b}{x^2}}\,\left(16\,a^3\,x^6+24\,a^2\,b\,x^4+6\,a\,b^2\,x^2-b^3\right)}{3\,b^4\,x^2\,{\left(a\,x^2+b\right)}^2}$$

input
```
int(1/(x^9*(a + b/x^2)^(5/2)),x)
```

output
```
((a + b/x^2)^(1/2)*(16*a^3*x^6 - b^3 + 6*a*b^2*x^2 + 24*a^2*b*x^4))/(3*b^4
*x^2*(b + a*x^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^9}\,dx=\frac{16\sqrt{a\,x^2+b}\,a^3x^6+24\sqrt{a\,x^2+b}\,a^2b\,x^4+6\sqrt{a\,x^2+b}\,a\,b^2x^2-\sqrt{a\,x^2+b}\,b^3-16\sqrt{a}}{3b^4x^3\left(a^2x^4+2ab\,x^2+b^2\right)}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^9,x)
```

output
```
(16*sqrt(a*x**2 + b)*a**3*x**6 + 24*sqrt(a*x**2 + b)*a**2*b*x**4 + 6*sqrt(
a*x**2 + b)*a*b**2*x**2 - sqrt(a*x**2 + b)*b**3 - 16*sqrt(a)*a**3*x**7 - 3
2*sqrt(a)*a**2*b*x**5 - 16*sqrt(a)*a*b**2*x**3)/(3*b**4*x**3*(a**2*x**4 +
2*a*b*x**2 + b**2))
```

3.411 $\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=-\frac{16b\sqrt{a+\frac{b}{x^2}}\,x}{3a^4}-\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{2x^3}{a^2\sqrt{a+\frac{b}{x^2}}}+\frac{8\sqrt{a+\frac{b}{x^2}}\,x^3}{3a^3}$$

output
```
-16/3*b*(a+b/x^2)^(1/2)*x/a^4-1/3*x^3/a/(a+b/x^2)^(3/2)-2*x^3/a^2/(a+b/x^2
)^(1/2)+8/3*(a+b/x^2)^(1/2)*x^3/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{(b+ax^2)\left(-16b^3-24ab^2x^2-6a^2bx^4+a^3x^6\right)}{3a^4\left(a+\frac{b}{x^2}\right)^{5/2}x^5}$$

input
```
Integrate[x^2/(a + b/x^2)^(5/2),x]
```

output
```
((b + a*x^2)*(-16*b^3 - 24*a*b^2*x^2 - 6*a^2*b*x^4 + a^3*x^6))/(3*a^4*(a +
 b/x^2)^(5/2)*x^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {803, 773, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{2b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}dx}{a}$$

$$\Big\downarrow \text{ 773}$$

$$\frac{2b\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x}}{a}+\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 245}$$

$$\frac{2b\left(-\frac{4b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x}}{a}-\frac{x}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{a}+\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{2b\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a}+\frac{1}{3ax\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{a}-\frac{x}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{a}+\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{2b\left(-\frac{4b\left(\frac{2}{3a^2x\sqrt{a+\frac{b}{x^2}}}+\frac{1}{3ax\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{a}-\frac{x}{a\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{a}+\frac{x^3}{3a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
Int[x^2/(a + b/x^2)^(5/2),x]
```

output
```
x^3/(3*a*(a + b/x^2)^(3/2)) + (2*b*((-4*b*(1/(3*a*(a + b/x^2)^(3/2)*x) + 2
/(3*a^2*Sqrt[a + b/x^2]*x)))/a - x/(a*(a + b/x^2)^(3/2))))/a
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

method	result	size
orering	$\frac{(a^3x^6-6a^2b\,x^4-24a\,b^2x^2-16b^3)(a\,x^2+b)}{3a^4x^5\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}$	56
gosper	$\frac{(a\,x^2+b)(a^3x^6-6a^2b\,x^4-24a\,b^2x^2-16b^3)}{3a^4x^5\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	60
default	$\frac{(a\,x^2+b)(a^3x^6-6a^2b\,x^4-24a\,b^2x^2-16b^3)}{3a^4x^5\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	60
trager	$\frac{x(a^3x^6-6a^2b\,x^4-24a\,b^2x^2-16b^3)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3a^4(a\,x^2+b)^2}$	64
risch	$\frac{(a\,x^2-8b)(a\,x^2+b)}{3a^4\sqrt{\frac{a\,x^2+b}{x^2}}\,x}-\frac{(a\,x^2+b)(9a\,x^2+8b)b^2}{3a^4(a^2x^4+2ab\,x^2+b^2)\sqrt{\frac{a\,x^2+b}{x^2}}\,x}$	100

input
```
int(x^2/(a+b/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a^3*x^6-6*a^2*b*x^4-24*a*b^2*x^2-16*b^3)/a^4*(a*x^2+b)/x^5/(a+b/x^2)^
(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{(a^3x^7-6\,a^2bx^5-24\,ab^2x^3-16\,b^3x)\sqrt{\frac{ax^2+b}{x^2}}}{3\,(a^6x^4+2\,a^5bx^2+a^4b^2)}$$

input
```
integrate(x^2/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(a^3*x^7 - 6*a^2*b*x^5 - 24*a*b^2*x^3 - 16*b^3*x)*sqrt((a*x^2 + b)/x^2
)/(a^6*x^4 + 2*a^5*b*x^2 + a^4*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(75) = 150$.

Time = 0.99 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.11

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{a^4b^{\frac{19}{2}}x^8\sqrt{\frac{ax^2}{b}+1}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{11}x^2+3a^4b^{12}} - \frac{5a^3b^{\frac{21}{2}}x^6\sqrt{\frac{ax^2}{b}+1}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{11}x^2+3a^4b^{12}} - \frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{11}x^2+3a^4b^{12}} - \frac{40ab^{\frac{25}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{11}x^2+3a^4b^{12}} - \frac{16b^{\frac{27}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^7b^9x^6+9a^6b^{10}x^4+9a^5b^{11}x^2+3a^4b^{12}}$$

input
```
integrate(x**2/(a+b/x**2)**(5/2),x)
```

output
```
a**4*b**(19/2)*x**8*sqrt(a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9*a**6*b**10*x*
*4 + 9*a**5*b**11*x**2 + 3*a**4*b**12) - 5*a**3*b**(21/2)*x**6*sqrt(a*x**2
/b + 1)/(3*a**7*b**9*x**6 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**2 + 3*a**4
*b**12) - 30*a**2*b**(23/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9*
a**6*b**10*x**4 + 9*a**5*b**11*x**2 + 3*a**4*b**12) - 40*a*b**(25/2)*x**2*
sqrt(a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x*
*2 + 3*a**4*b**12) - 16*b**(27/2)*sqrt(a*x**2/b + 1)/(3*a**7*b**9*x**6 + 9
*a**6*b**10*x**4 + 9*a**5*b**11*x**2 + 3*a**4*b**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}x^3 - 9\sqrt{a+\frac{b}{x^2}}bx}{3\,a^4} - \frac{9\left(a+\frac{b}{x^2}\right)b^2x^2 - b^3}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^4x^3}$$

input
```
integrate(x^2/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*((a + b/x^2)^(3/2)*x^3 - 9*sqrt(a + b/x^2)*b*x)/a^4 - 1/3*(9*(a + b/x^
2)*b^2*x^2 - b^3)/((a + b/x^2)^(3/2)*a^4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{16\,b^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,a^4} - \frac{9\,(ax^2+b)b^2 - b^3}{3\,(ax^2+b)^{\frac{3}{2}}a^4\mathrm{sgn}\,(x)} + \frac{(ax^2+b)^{\frac{3}{2}}a^8 - 9\sqrt{ax^2+b}a^8b}{3\,a^{12}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^2/(a+b/x^2)^(5/2),x, algorithm="giac")
```

output
```
16/3*b^(3/2)*sgn(x)/a^4 - 1/3*(9*(a*x^2 + b)*b^2 - b^3)/((a*x^2 + b)^(3/2)
*a^4*sgn(x)) + 1/3*((a*x^2 + b)^(3/2)*a^8 - 9*sqrt(a*x^2 + b)*a^8*b)/(a^12
*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{6\,a^2\left(a+\frac{b}{x^2}\right) - 24\,a\left(a+\frac{b}{x^2}\right)^2 + 16\left(a+\frac{b}{x^2}\right)^3 + a^3}{\left(\frac{3\,a^5}{b\,x} - \frac{3\,a^4\left(a+\frac{b}{x^2}\right)}{b\,x}\right)\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(x^2/(a + b/x^2)^(5/2),x)
```

output
```
(6*a^2*(a + b/x^2) - 24*a*(a + b/x^2)^2 + 16*(a + b/x^2)^3 + a^3)/(((3*a^5
)/(b*x) - (3*a^4*(a + b/x^2))/(b*x))*(a + b/x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{\sqrt{a\,x^2+b}\,(a^3x^6-6a^2b\,x^4-24a\,b^2x^2-16b^3)}{3a^4\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(x^2/(a+b/x^2)^(5/2),x)
```

output
```
(sqrt(a*x**2 + b)*(a**3*x**6 - 6*a**2*b*x**4 - 24*a*b**2*x**2 - 16*b**3))/
(3*a**4*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.412 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}} \, dx = -\frac{x}{3a\left(a+\frac{b}{x^2}\right)^{3/2}} - \frac{4x}{3a^2\sqrt{a+\frac{b}{x^2}}} + \frac{8\sqrt{a+\frac{b}{x^2}}\,x}{3a^3}$$

output
```
-1/3*x/a/(a+b/x^2)^(3/2)-4/3*x/a^2/(a+b/x^2)^(1/2)+8/3*(a+b/x^2)^(1/2)*x/a
^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}} \, dx = \frac{(b+ax^2)\left(8b^2+12abx^2+3a^2x^4\right)}{3a^3\left(a+\frac{b}{x^2}\right)^{5/2}x^5}$$

input
```
Integrate[(a + b/x^2)^(-5/2),x]
```

output
```
((b + a*x^2)*(8*b^2 + 12*a*b*x^2 + 3*a^2*x^4))/(3*a^3*(a + b/x^2)^(5/2)*x^
5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {773, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{245}$$

$$\frac{4b\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}d\frac{1}{x}}{a}+\frac{x}{a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4b\left(\frac{2\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}}d\frac{1}{x}}{3a}+\frac{1}{3ax\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{a}+\frac{x}{a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4b\left(\frac{2}{3a^2x\sqrt{a+\frac{b}{x^2}}}+\frac{1}{3ax\left(a+\frac{b}{x^2}\right)^{3/2}}\right)}{a}+\frac{x}{a\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
Int[(a + b/x^2)^(-5/2),x]
```

output
```
(4*b*(1/(3*a*(a + b/x^2)^(3/2)*x) + 2/(3*a^2*Sqrt[a + b/x^2]*x)))/a + x/(a
*(a + b/x^2)^(3/2))
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

method	result	size
orering	$\frac{\left(3 a^{2} x^{4}+12 a b \,x^{2}+8 b^{2}\right) \left(a \,x^{2}+b \right)}{3 a^{3} x^{5} \left(a +\frac{b}{x^{2}}\right)^{\frac{5}{2}}}$	46
gosper	$\frac{\left(a \,x^{2}+b \right) \left(3 a^{2} x^{4}+12 a b \,x^{2}+8 b^{2}\right)}{3 a^{3} x^{5} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	50
default	$\frac{\left(a \,x^{2}+b \right) \left(3 a^{2} x^{4}+12 a b \,x^{2}+8 b^{2}\right)}{3 a^{3} x^{5} \left(\frac{a \,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	50
trager	$\frac{x \left(3 a^{2} x^{4}+12 a b \,x^{2}+8 b^{2}\right) \sqrt{-\frac{-a \,x^{2}-b}{x^{2}}}}{3 a^{3} \left(a \,x^{2}+b \right)^{2}}$	54
risch	$\frac{a \,x^{2}+b}{a^{3} \sqrt{\frac{a \,x^{2}+b}{x^{2}}}\, x}+\frac{\left(a \,x^{2}+b \right) \left(6 a \,x^{2}+5 b \right) b}{3 a^{3} \left(a^{2} x^{4}+2 a b \,x^{2}+b^{2}\right) \sqrt{\frac{a \,x^{2}+b}{x^{2}}}\, x}$	88

input
```
int(1/(a+b/x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*a^2*x^4+12*a*b*x^2+8*b^2)/a^3*(a*x^2+b)/x^5/(a+b/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{\left(3\,a^2x^5+12\,abx^3+8\,b^2x\right)\sqrt{\frac{ax^2+b}{x^2}}}{3\,\left(a^5x^4+2\,a^4bx^2+a^3b^2\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*a^2*x^5 + 12*a*b*x^3 + 8*b^2*x)*sqrt((a*x^2 + b)/x^2)/(a^5*x^4 + 2*
a^4*b*x^2 + a^3*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(51) = 102.

Time = 0.81 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.81

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx=\frac{3a^2b^{\frac{9}{2}}x^4\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}+\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}+\frac{8b^{\frac{13}{2}}\sqrt{\frac{ax^2}{b}+1}}{3a^5b^4x^4+6a^4b^5x^2+3a^3b^6}$$

input
```
integrate(1/(a+b/x**2)**(5/2),x)
```

output
```
3*a**2*b**(9/2)*x**4*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x**4 + 6*a**4*b**5*x*
*2 + 3*a**3*b**6) + 12*a*b**(11/2)*x**2*sqrt(a*x**2/b + 1)/(3*a**5*b**4*x*
*4 + 6*a**4*b**5*x**2 + 3*a**3*b**6) + 8*b**(13/2)*sqrt(a*x**2/b + 1)/(3*a
**5*b**4*x**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x^2}}\,x}{a^3} + \frac{6\left(a+\frac{b}{x^2}\right)bx^2 - b^2}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^3x^3}$$

input
```
integrate(1/(a+b/x^2)^(5/2),x, algorithm="maxima")
```

output
```
sqrt(a + b/x^2)*x/a^3 + 1/3*(6*(a + b/x^2)*b*x^2 - b^2)/((a + b/x^2)^(3/2)
*a^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{\frac{3\sqrt{ax^2+b}}{a\mathrm{sgn}(x)} + \frac{6\,(ax^2+b)b-b^2}{(ax^2+b)^{\frac{3}{2}}\,a\mathrm{sgn}(x)}}{3\,a^2} - \frac{8\,\sqrt{b}\mathrm{sgn}(x)}{3\,a^3}$$

input
```
integrate(1/(a+b/x^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*(3*sqrt(a*x^2 + b)/(a*sgn(x)) + (6*(a*x^2 + b)*b - b^2)/((a*x^2 + b)^(
3/2)*a*sgn(x)))/a^2 - 8/3*sqrt(b)*sgn(x)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{x\left(\frac{a\,x^2}{b}+1\right)^{5/2}\sqrt{x^{10}}\,{}_2\mathrm{F}_1\left(\frac{5}{2},3;\ 4;\ -\frac{a\,x^2}{b}\right)}{6\,(a\,x^2+b)^{5/2}}$$

input
```
int(1/(a + b/x^2)^(5/2),x)
```

output
```
(x*((a*x^2)/b + 1)^(5/2)*(x^10)^(1/2)*hypergeom([5/2, 3], 4, -(a*x^2)/b))/
(6*(b + a*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx = \frac{\sqrt{a\,x^2+b}\,(3a^2x^4+12ab\,x^2+8b^2)}{3a^3\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(1/(a+b/x^2)^(5/2),x)
```

output
```
(sqrt(a*x**2 + b)*(3*a**2*x**4 + 12*a*b*x**2 + 8*b**2))/(3*a**3*(a**2*x**4
 + 2*a*b*x**2 + b**2))
```

3.413 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2} \, dx = -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2} x} - \frac{2}{3a^2\sqrt{a+\frac{b}{x^2}}\,x}$$

output
```
-1/3/a/(a+b/x^2)^(3/2)/x-2/3/a^2/(a+b/x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2} \, dx = \frac{(-2b-3ax^2)(b+ax^2)}{3a^2\left(a+\frac{b}{x^2}\right)^{5/2} x^5}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^2),x]
```

output
```
((-2*b - 3*a*x^2)*(b + a*x^2))/(3*a^2*(a + b/x^2)^(5/2)*x^5)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x^2}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2b \int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4}dx}{a}-\frac{1}{ax \left(a+\frac{b}{x^2}\right)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{2b}{3a^2x^3 \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{1}{ax \left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^2),x]
```

output
```
(-2*b)/(3*a^2*(a + b/x^2)^(3/2)*x^3) - 1/(a*(a + b/x^2)^(3/2)*x)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
orering	$-\frac{(3a\,x^2+2b)(a\,x^2+b)}{3a^2x^5\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}}$	35
gosper	$-\frac{(a\,x^2+b)(3a\,x^2+2b)}{3a^2x^5\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	39
default	$-\frac{(a\,x^2+b)(3a\,x^2+2b)}{3a^2x^5\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}}}$	39
trager	$-\frac{x(3a\,x^2+2b)\sqrt{-\frac{-a\,x^2-b}{x^2}}}{3a^2(a\,x^2+b)^2}$	43

input
```
int(1/(a+b/x^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*a*x^2+2*b)/a^2*(a*x^2+b)/x^5/(a+b/x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^2}\,dx = -\frac{(3\,ax^3+2\,bx)\sqrt{\frac{ax^2+b}{x^2}}}{3\,(a^4x^4+2\,a^3bx^2+a^2b^2)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
-1/3*(3*a*x^3 + 2*b*x)*sqrt((a*x^2 + b)/x^2)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*
b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(36) = 72$.

Time = 0.84 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}\,dx = -\frac{3ax^2}{3a^3\sqrt{b}x^2\sqrt{\frac{ax^2}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}} - \frac{2b}{3a^3\sqrt{b}x^2\sqrt{\frac{ax^2}{b}+1}+3a^2b^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**2,x)
```

output
```
-3*a*x**2/(3*a**3*sqrt(b)*x**2*sqrt(a*x**2/b + 1) + 3*a**2*b**(3/2)*sqrt(a
*x**2/b + 1)) - 2*b/(3*a**3*sqrt(b)*x**2*sqrt(a*x**2/b + 1) + 3*a**2*b**(3
/2)*sqrt(a*x**2/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}\,dx = -\frac{3\left(a+\frac{b}{x^2}\right)x^2-b}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}a^2x^3}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
-1/3*(3*(a + b/x^2)*x^2 - b)/((a + b/x^2)^(3/2)*a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}\, dx = \frac{2\,\mathrm{sgn}(x)}{3\,a^2\sqrt{b}} - \frac{3\,ax^2+2\,b}{3\,(ax^2+b)^{\frac{3}{2}}a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
2/3*sgn(x)/(a^2*sqrt(b)) - 1/3*(3*a*x^2 + 2*b)/((a*x^2 + b)^(3/2)*a^2*sgn(
x))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}\, dx = -\frac{x\,\sqrt{a+\frac{b}{x^2}}\,(3\,a\,x^2+2\,b)}{3\,a^2\,(a\,x^2+b)^2}$$

input
```
int(1/(x^2*(a + b/x^2)^(5/2)),x)
```

output
```
-(x*(a + b/x^2)^(1/2)*(2*b + 3*a*x^2))/(3*a^2*(b + a*x^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^2}\, dx = \frac{\sqrt{a\,x^2+b}\,(-3a\,x^2-2b)}{3a^2\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^2,x)
```

output
```
(sqrt(a*x**2 + b)*( - 3*a*x**2 - 2*b))/(3*a**2*(a**2*x**4 + 2*a*b*x**2 + b
**2))
```

3.414 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4} \, dx = -\frac{1}{3a\left(a+\frac{b}{x^2}\right)^{3/2} x^3}$$

output
```
-1/3/a/(a+b/x^2)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4} \, dx = -\frac{b+ax^2}{3a\left(a+\frac{b}{x^2}\right)^{5/2} x^5}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^4),x]
```

output
```
-1/3*(b + a*x^2)/(a*(a + b/x^2)^(5/2)*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^2}\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 796}$$
$$-\frac{1}{3 a x^3 \left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^4),x]
```

output
```
-1/3*1/(a*(a + b/x^2)^(3/2)*x^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

method	result	size
orering	$-\frac{a\,x^{2}+b}{3a\,x^{5}\left(a+\frac{b}{x^{2}}\right)^{\frac{5}{2}}}$	25
gosper	$-\frac{a\,x^{2}+b}{3a\,x^{5}\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	29
default	$-\frac{a\,x^{2}+b}{3a\,x^{5}\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}}$	29
trager	$-\frac{x\sqrt{-\frac{-a\,x^{2}-b}{x^{2}}}}{3a\left(a\,x^{2}+b\right)^{2}}$	33

input
```
int(1/(a+b/x^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*x^2+b)/a/x^5/(a+b/x^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^4}\,dx = -\frac{x\sqrt{\frac{ax^2+b}{x^2}}}{3\left(a^3x^4+2\,a^2bx^2+ab^2\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*x*sqrt((a*x^2 + b)/x^2)/(a^3*x^4 + 2*a^2*b*x^2 + a*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^4}\,dx = -\frac{1}{3a^2\sqrt{b}x^2\sqrt{\frac{ax^2}{b}+1}+3ab^{\frac{3}{2}}\sqrt{\frac{ax^2}{b}+1}}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**4,x)
```

output
```
-1/(3*a**2*sqrt(b)*x**2*sqrt(a*x**2/b + 1) + 3*a*b**(3/2)*sqrt(a*x**2/b +
1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^4}\,dx = -\frac{1}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}ax^3}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
-1/3/((a + b/x^2)^(3/2)*a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^4}\,dx = \frac{\operatorname{sgn}(x)}{3\,ab^{\frac{3}{2}}} - \frac{1}{3\,(ax^2+b)^{\frac{3}{2}}a\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/3*sgn(x)/(a*b^(3/2)) - 1/3/((a*x^2 + b)^(3/2)*a*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4} \, dx = -\frac{1}{3\,a\,x^3\,\left(a+\frac{b}{x^2}\right)^{3/2}}$$

input
```
int(1/(x^4*(a + b/x^2)^(5/2)),x)
```

output
```
-1/(3*a*x^3*(a + b/x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4} \, dx = -\frac{\sqrt{a\,x^2+b}}{3a\,(a^2x^4+2ab\,x^2+b^2)}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^4,x)
```

output
```
( - sqrt(a*x**2 + b))/(3*a*(a**2*x**4 + 2*a*b*x**2 + b**2))
```

3.415 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6} \, dx = \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2} x^3} + \frac{1}{b^2\sqrt{a+\frac{b}{x^2}}\,x} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{b^{5/2}}$$

output
```
1/3/b/(a+b/x^2)^(3/2)/x^3+1/b^2/(a+b/x^2)^(1/2)/x-arctanh(b^(1/2)/(a+b/x^2
)^(1/2)/x)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6} \, dx = \frac{\sqrt{b}(4b+3ax^2) - 3(b+ax^2)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{3b^{5/2}\sqrt{a+\frac{b}{x^2}}\,x\,(b+ax^2)}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^6),x]
```

output
```
(Sqrt[b]*(4*b + 3*a*x^2) - 3*(b + a*x^2)^(3/2)*ArcTanh[Sqrt[b + a*x^2]/Sqr
t[b]])/(3*b^(5/2)*Sqrt[a + b/x^2]*x*(b + a*x^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {858, 252, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{3bx^3 \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2} x^2}d\frac{1}{x}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{3bx^3 \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{b}-\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}}{b}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{3bx^3 \left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{\frac{\int \frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}\,x}}{b}-\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3bx^3\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{b^{3/2}}-\frac{1}{bx\sqrt{a+\frac{b}{x^2}}}}{b}$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^6),x]
```

output
```
1/(3*b*(a + b/x^2)^(3/2)*x^3) - (-(1/(b*Sqrt[a + b/x^2]*x)) + ArcTanh[Sqrt
[b]/(Sqrt[a + b/x^2]*x)]/b^(3/2))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{\left(a\,x^{2}+b\right)\left(-3x^{2}a\,b^{\frac{3}{2}}+3\left(a\,x^{2}+b\right)^{\frac{3}{2}}\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{2}+b}}{x}\right)b-4b^{\frac{5}{2}}\right)}{3\left(\frac{a\,x^{2}+b}{x^{2}}\right)^{\frac{5}{2}}x^{5}b^{\frac{7}{2}}}$	77

input
```
int(1/(a+b/x^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*x^2+b)*(-3*x^2*a*b^(3/2)+3*(a*x^2+b)^(3/2)*ln(2*(b^(1/2)*(a*x^2+b)
^(1/2)+b)/x)*b-4*b^(5/2))/((a*x^2+b)/x^2)^(5/2)/x^5/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.24

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^6}\,dx = \left[\frac{3\left(a^2x^4+2\,abx^2+b^2\right)\sqrt{b}\log\left(-\frac{ax^2-2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)+2\left(3\,abx^3+4\,b^2x\right)\sqrt{\frac{ax^2+b}{x^2}}}{6\left(a^2b^3x^4+2\,ab^4x^2+b^5\right)},\right.$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)*sqrt(b)*log(-(a*x^2 - 2*sqrt(b)*x*sqrt
((a*x^2 + b)/x^2) + 2*b)/x^2) + 2*(3*a*b*x^3 + 4*b^2*x)*sqrt((a*x^2 + b)/x
^2))/(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5), 1/3*(3*(a^2*x^4 + 2*a*b*x^2 + b^2)
*sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2 + b)/x^2)/b) + (3*a*b*x^3 + 4*b^2*
x)*sqrt((a*x^2 + b)/x^2))/(a^2*b^3*x^4 + 2*a*b^4*x^2 + b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(58) = 116$.

Time = 2.09 (sec) , antiderivative size = 740, normalized size of antiderivative = 10.88

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**6,x)
```

output
```
3*a**3*b**4*x**6*log(a*x**2/b)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*
x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) - 6*a**3*b**4*x**6*log(sqrt(a*x*
*2/b + 1) + 1)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(
17/2)*x**2 + 6*b**(19/2)) + 6*a**2*b**5*x**4*sqrt(a*x**2/b + 1)/(6*a**3*b*
*(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2))
 + 9*a**2*b**5*x**4*log(a*x**2/b)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/
2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) - 18*a**2*b**5*x**4*log(sqrt(
a*x**2/b + 1) + 1)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*
b**(17/2)*x**2 + 6*b**(19/2)) + 14*a*b**6*x**2*sqrt(a*x**2/b + 1)/(6*a**3*
b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2
)) + 9*a*b**6*x**2*log(a*x**2/b)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2
)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) - 18*a*b**6*x**2*log(sqrt(a*x*
*2/b + 1) + 1)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(
17/2)*x**2 + 6*b**(19/2)) + 8*b**7*sqrt(a*x**2/b + 1)/(6*a**3*b**(13/2)*x*
*6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2)) + 3*b**7*
log(a*x**2/b)/(6*a**3*b**(13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(1
7/2)*x**2 + 6*b**(19/2)) - 6*b**7*log(sqrt(a*x**2/b + 1) + 1)/(6*a**3*b**(
13/2)*x**6 + 18*a**2*b**(15/2)*x**4 + 18*a*b**(17/2)*x**2 + 6*b**(19/2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6}\, dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{2\, b^{\frac{5}{2}}} + \frac{3\left(a+\frac{b}{x^2}\right)x^2+b}{3\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}} b^2 x^3}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
1/2*log((sqrt(a + b/x^2)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(5/
2) + 1/3*(3*(a + b/x^2)*x^2 + b)/((a + b/x^2)^(3/2)*b^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6} \, dx = -\frac{\left(3 \sqrt{b} \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 4 \sqrt{-b}\right) \mathrm{sgn}(x)}{3 \sqrt{-b} b^{\frac{5}{2}}} + \frac{\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b} b^2 \mathrm{sgn}\left(x\right)} + \frac{3 \, ax^2 + 4 \, b}{3 \left(ax^2+b\right)^{\frac{3}{2}} b^2 \mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(b)*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-b))*sgn(x)/(sqrt(-b)*b^
(5/2)) + arctan(sqrt(a*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2*sgn(x)) + 1/3*(3*a
*x^2 + 4*b)/((a*x^2 + b)^(3/2)*b^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6} \, dx = \int \frac{1}{x^6 \left(a+\frac{b}{x^2}\right)^{5/2}} \, dx$$

input
```
int(1/(x^6*(a + b/x^2)^(5/2)),x)
```

output
```
int(1/(x^6*(a + b/x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.50

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^6} \, dx = \frac{3\sqrt{a\,x^2+b}\,ab\,x^2+4\sqrt{a\,x^2+b}\,b^2+3\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x-\sqrt{b}}{\sqrt{b}}\right)a^2x^4+6\sqrt{b}\log\left(\frac{\sqrt{a\,x^2}}{}\right.}{}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^6,x)
```

output
```
(3*sqrt(a*x**2 + b)*a*b*x**2 + 4*sqrt(a*x**2 + b)*b**2 + 3*sqrt(b)*log((sq
rt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a**2*x**4 + 6*sqrt(b)*log((
sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*a*b*x**2 + 3*sqrt(b)*log(
(sqrt(a*x**2 + b) + sqrt(a)*x - sqrt(b))/sqrt(b))*b**2 - 3*sqrt(b)*log((sq
rt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a**2*x**4 - 6*sqrt(b)*log((
sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a*b*x**2 - 3*sqrt(b)*log(
(sqrt(a*x**2 + b) + sqrt(a)*x + sqrt(b))/sqrt(b))*b**2)/(3*b**3*(a**2*x**4
 + 2*a*b*x**2 + b**2))
```

3.416 $\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8} \, dx = \frac{1}{3b\left(a+\frac{b}{x^2}\right)^{3/2} x^5} + \frac{5}{3b^2\sqrt{a+\frac{b}{x^2}}\,x^3} - \frac{5\sqrt{a+\frac{b}{x^2}}}{2b^3 x} + \frac{5a\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}\,x}\right)}{2b^{7/2}}$$

output
```
1/3/b/(a+b/x^2)^(3/2)/x^5+5/3/b^2/(a+b/x^2)^(1/2)/x^3-5/2*(a+b/x^2)^(1/2)/
b^3/x+5/2*a*arctanh(b^(1/2)/(a+b/x^2)^(1/2)/x)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8} \, dx = \frac{-\sqrt{b}(3b^2+20abx^2+15a^2x^4)+15ax^2(b+ax^2)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b+ax^2}}{\sqrt{b}}\right)}{6b^{7/2}\sqrt{a+\frac{b}{x^2}}\,x^3\,(b+ax^2)}$$

input
```
Integrate[1/((a + b/x^2)^(5/2)*x^8),x]
```

output
```
(-(Sqrt[b]*(3*b^2 + 20*a*b*x^2 + 15*a^2*x^4)) + 15*a*x^2*(b + a*x^2)^(3/2)
*ArcTanh[Sqrt[b + a*x^2]/Sqrt[b]])/(6*b^(7/2)*Sqrt[a + b/x^2]*x^3*(b + a*x
^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 252, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(a+\frac{b}{x^2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^6}d\frac{1}{x}$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{3bx^5\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{5\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{3/2}x^4}d\frac{1}{x}}{3b}$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{3bx^5\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{5\left(\frac{3\int \frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{x}}{b}-\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}\right)}{3b}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3bx^5\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{5\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x}}{2b}\right)}{b}-\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}\right)}{3b}$$

$$\Big\downarrow \; 224$$

$$\frac{1}{3bx^5\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{5\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\int\frac{1}{1-\frac{b}{x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x}}{2b}\right)}{b}-\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}\right)}{3b}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3bx^5\left(a+\frac{b}{x^2}\right)^{3/2}}-\frac{5\left(\frac{3\left(\frac{\sqrt{a+\frac{b}{x^2}}}{2bx}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a+\frac{b}{x^2}}}\right)}{2b^{3/2}}\right)}{b}-\frac{1}{bx^3\sqrt{a+\frac{b}{x^2}}}\right)}{3b}$$

input
```
Int[1/((a + b/x^2)^(5/2)*x^8),x]
```

output
```
1/(3*b*(a + b/x^2)^(3/2)*x^5) - (5*(-(1/(b*Sqrt[a + b/x^2]*x^3)) + (3*(Sqr
t[a + b/x^2]/(2*b*x) - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x)])/(2*b^(3/2)
)))/b))/(3*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

method	result
default	$-\frac{(a\,x^2+b)\left(15 b^{\frac{3}{2}} a^2 x^4+20 b^{\frac{5}{2}} a\,x^2-15\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^2+b}}{x}\right)(a\,x^2+b)^{\frac{3}{2}} a b\,x^2+3 b^{\frac{7}{2}}\right)}{6\left(\frac{a\,x^2+b}{x^2}\right)^{\frac{5}{2}} x^7 b^{\frac{9}{2}}}$
risch	$-\frac{a\,x^2+b}{2b^3x^3\sqrt{\frac{a\,x^2+b}{x^2}}}+\frac{\left(\frac{5a\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^2+b}}{x}\right)}{2b^{\frac{7}{2}}}-\frac{13a\sqrt{a\left(x-\frac{\sqrt{-ab}}{a}\right)^2+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{a}\right)}}{12b^3\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{a}\right)}+\frac{13a\sqrt{a\left(x+\frac{\sqrt{-ab}}{a}\right)^2-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{a}\right)}}{12b^3\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{a}\right)}\right.}{\sqrt{\frac{a\,x^2+b}{x^2}}\,x}$

input
```
int(1/(a+b/x^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x^2+b)*(15*b^(3/2)*a^2*x^4+20*b^(5/2)*a*x^2-15*ln(2*(b^(1/2)*(a*x^
2+b)^(1/2)+b)/x)*(a*x^2+b)^(3/2)*a*b*x^2+3*b^(7/2))/((a*x^2+b)/x^2)^(5/2)/
x^7/b^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.67

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx = \left[\frac{15\,(a^3x^5+2\,a^2bx^3+ab^2x)\sqrt{b}\log\left(-\frac{ax^2+2\,\sqrt{b}x\sqrt{\frac{ax^2+b}{x^2}}+2\,b}{x^2}\right)-2\,(15\,a^2bx^4+20\,ab^2x}{12\,(a^2b^4x^5+2\,ab^5x^3+b^6x)}\right.$$

$$\left.-\frac{15\,(a^3x^5+2\,a^2bx^3+ab^2x)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{ax^2+b}{x^2}}}{b}\right)+(15\,a^2bx^4+20\,ab^2x^2+3\,b^3)\sqrt{\frac{ax^2+b}{x^2}}}{6\,(a^2b^4x^5+2\,ab^5x^3+b^6x)}\right]$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
[1/12*(15*(a^3*x^5 + 2*a^2*b*x^3 + a*b^2*x)*sqrt(b)*log(-(a*x^2 + 2*sqrt(b
)*x*sqrt((a*x^2 + b)/x^2) + 2*b)/x^2) - 2*(15*a^2*b*x^4 + 20*a*b^2*x^2 + 3
*b^3)*sqrt((a*x^2 + b)/x^2))/(a^2*b^4*x^5 + 2*a*b^5*x^3 + b^6*x), -1/6*(15
*(a^3*x^5 + 2*a^2*b*x^3 + a*b^2*x)*sqrt(-b)*arctan(sqrt(-b)*x*sqrt((a*x^2
+ b)/x^2)/b) + (15*a^2*b*x^4 + 20*a*b^2*x^2 + 3*b^3)*sqrt((a*x^2 + b)/x^2)
)/(a^2*b^4*x^5 + 2*a*b^5*x^3 + b^6*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(85) = 170$.

Time = 3.78 (sec) , antiderivative size = 864, normalized size of antiderivative = 9.09

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**2)**(5/2)/x**8,x)
```

output
```
-15*a**4*b**13*x**8*log(a*x**2/b)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35
/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) + 30*a**4*b**13*x**8*l
og(sqrt(a*x**2/b + 1) + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**
6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) - 30*a**3*b**14*x**6*sqrt(a*x
**2/b + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/
2)*x**4 + 12*b**(39/2)*x**2) - 45*a**3*b**14*x**6*log(a*x**2/b)/(12*a**3*b
**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2
)*x**2) + 90*a**3*b**14*x**6*log(sqrt(a*x**2/b + 1) + 1)/(12*a**3*b**(33/2
)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2)
 - 70*a**2*b**15*x**4*sqrt(a*x**2/b + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2
*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) - 45*a**2*b**15
*x**4*log(a*x**2/b)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*
a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) + 90*a**2*b**15*x**4*log(sqrt(a*x**2
/b + 1) + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(3
7/2)*x**4 + 12*b**(39/2)*x**2) - 46*a*b**16*x**2*sqrt(a*x**2/b + 1)/(12*a*
*3*b**(33/2)*x**8 + 36*a**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(
39/2)*x**2) - 15*a*b**16*x**2*log(a*x**2/b)/(12*a**3*b**(33/2)*x**8 + 36*a
**2*b**(35/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) + 30*a*b**16
*x**2*log(sqrt(a*x**2/b + 1) + 1)/(12*a**3*b**(33/2)*x**8 + 36*a**2*b**(35
/2)*x**6 + 36*a*b**(37/2)*x**4 + 12*b**(39/2)*x**2) - 6*b**17*sqrt(a*x*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2}x^8}\,dx = -\frac{15\left(a+\frac{b}{x^2}\right)^2ax^4-10\left(a+\frac{b}{x^2}\right)abx^2-2\,ab^2}{6\left(\left(a+\frac{b}{x^2}\right)^{\frac{5}{2}}b^3x^5-\left(a+\frac{b}{x^2}\right)^{\frac{3}{2}}b^4x^3\right)}-\frac{5\,a\log\left(\frac{\sqrt{a+\frac{b}{x^2}}x-\sqrt{b}}{\sqrt{a+\frac{b}{x^2}}x+\sqrt{b}}\right)}{4\,b^{\frac{7}{2}}}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
-1/6*(15*(a + b/x^2)^2*a*x^4 - 10*(a + b/x^2)*a*b*x^2 - 2*a*b^2)/((a + b/x
^2)^(5/2)*b^3*x^5 - (a + b/x^2)^(3/2)*b^4*x^3) - 5/4*a*log((sqrt(a + b/x^2
)*x - sqrt(b))/(sqrt(a + b/x^2)*x + sqrt(b)))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx = -\frac{5\,a\arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}b^3\mathrm{sgn}\left(x\right)} - \frac{6\left(ax^2+b\right)a+ab}{3\left(ax^2+b\right)^{\frac{3}{2}}b^3\mathrm{sgn}\left(x\right)} - \frac{\sqrt{ax^2+b}}{2\,b^3x^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(a+b/x^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
-5/2*a*arctan(sqrt(a*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3*sgn(x)) - 1/3*(6*(a*
x^2 + b)*a + a*b)/((a*x^2 + b)^(3/2)*b^3*sgn(x)) - 1/2*sqrt(a*x^2 + b)/(b^
3*x^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx = \int \frac{1}{x^8\left(a+\frac{b}{x^2}\right)^{5/2}}\,dx$$

input
```
int(1/(x^8*(a + b/x^2)^(5/2)),x)
```

output
```
int(1/(x^8*(a + b/x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.86

$$\int \frac{1}{\left(a+\frac{b}{x^2}\right)^{5/2} x^8}\,dx = \frac{-15\sqrt{a\,x^2+b}\,a^2b\,x^4-20\sqrt{a\,x^2+b}\,a\,b^2x^2-3\sqrt{a\,x^2+b}\,b^3-15\sqrt{b}\log\left(\frac{\sqrt{a\,x^2+b}+\sqrt{a}\,x}{\sqrt{b}}\right.}{}$$

input
```
int(1/(a+b/x^2)^(5/2)/x^8,x)
```

output
```
( - 15*sqrt(a*x**2 + b)*a**2*b*x**4 - 20*sqrt(a*x**2 + b)*a*b**2*x**2 - 3*
sqrt(a*x**2 + b)*b**3 - 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x - sqr
t(b))/sqrt(b))*a**3*x**6 - 30*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)*x -
sqrt(b))/sqrt(b))*a**2*b*x**4 - 15*sqrt(b)*log((sqrt(a*x**2 + b) + sqrt(a)
*x - sqrt(b))/sqrt(b))*a*b**2*x**2 + 15*sqrt(b)*log((sqrt(a*x**2 + b) + sq
rt(a)*x + sqrt(b))/sqrt(b))*a**3*x**6 + 30*sqrt(b)*log((sqrt(a*x**2 + b) +
 sqrt(a)*x + sqrt(b))/sqrt(b))*a**2*b*x**4 + 15*sqrt(b)*log((sqrt(a*x**2 +
 b) + sqrt(a)*x + sqrt(b))/sqrt(b))*a*b**2*x**2)/(6*b**4*x**2*(a**2*x**4 +
 2*a*b*x**2 + b**2))
```

3.417 $\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3}{8}\left(1+\frac{1}{x^2}\right)^{4/3}$$

output
```
-3/8*(1+1/x^2)^(4/3)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3}{8}\left(1+\frac{1}{x^2}\right)^{4/3}$$

input
```
Integrate[(1 + x^(-2))^(1/3)/x^3,x]
```

output
```
(-3*(1 + x^(-2))^(4/3))/8
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\frac{1}{x^2}+1}}{x^3} \, dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{4/3}$$

input
```
Int[(1 + x^(-2))^(1/3)/x^3,x]
```

output
```
(-3*(1 + x^(-2))^(4/3))/8
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{3\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}}{8}$	10
default	$-\frac{3\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}}{8}$	10
orering	$-\frac{3(x^2+1)\left(\frac{1}{x^2}+1\right)^{\frac{1}{3}}}{8x^2}$	18
gosper	$-\frac{3(x^2+1)\left(\frac{x^2+1}{x^2}\right)^{\frac{1}{3}}}{8x^2}$	22
trager	$-\frac{3(x^2+1)\left(-\frac{-x^2-1}{x^2}\right)^{\frac{1}{3}}}{8x^2}$	25
risch	$-\frac{3\left(\frac{x^2+1}{x^2}\right)^{\frac{1}{3}}(x^4+2x^2+1)}{8(x^2+1)x^2}$	34

input
```
int((1/x^2+1)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*(1/x^2+1)^(4/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3\,(x^2+1)\left(\frac{x^2+1}{x^2}\right)^{\frac{1}{3}}}{8\,x^2}$$

input
```
integrate((1+1/x^2)^(1/3)/x^3,x, algorithm="fricas")
```

output
```
-3/8*(x^2 + 1)*((x^2 + 1)/x^2)^(1/3)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3\sqrt[3]{1+\frac{1}{x^2}}}{8} - \frac{3\sqrt[3]{1+\frac{1}{x^2}}}{8x^2}$$

input
```
integrate((1+1/x**2)**(1/3)/x**3,x)
```

output
```
-3*(1 + x**(-2))**(1/3)/8 - 3*(1 + x**(-2))**(1/3)/(8*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}$$

input
```
integrate((1+1/x^2)^(1/3)/x^3,x, algorithm="maxima")
```

output
```
-3/8*(1/x^2 + 1)^(4/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3\,(x^2+1)\left(\frac{x^2+1}{x^2}\right)^{\frac{1}{3}}}{8\,x^2}$$

input
```
integrate((1+1/x^2)^(1/3)/x^3,x, algorithm="giac")
```

output
```
-3/8*(x^2 + 1)*((x^2 + 1)/x^2)^(1/3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3\,\left(\frac{1}{x^2}+1\right)^{1/3}\,\left(x^2+1\right)}{8\,x^2}$$

input
```
int((1/x^2 + 1)^(1/3)/x^3,x)
```

output
```
-(3*(1/x^2 + 1)^(1/3)*(x^2 + 1))/(8*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{1+\frac{1}{x^2}}}{x^3}\,dx = -\frac{3(x^2+1)^{\frac{4}{3}}}{8x^{\frac{8}{3}}}$$

input
```
int((1+1/x^2)^(1/3)/x^3,x)
```

output
```
( - 3*(x**2 + 1)**(1/3)*(x**2 + 1))/(8*x**(2/3)*x**2)
```

3.418 $\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3} dx = -\frac{3}{16}\left(1+\frac{1}{x^2}\right)^{8/3}$$

output
```
-3/16*(1+1/x^2)^(8/3)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3} dx = -\frac{3\left(1+\frac{1}{x^2}\right)^{5/3}(1+x^2)}{16x^2}$$

input
```
Integrate[(1 + x^(-2))^(5/3)/x^3,x]
```

output
```
(-3*(1 + x^(-2))^(5/3)*(1 + x^2))/(16*x^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{1}{x^2}+1\right)^{5/3}}{x^3} \, dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{3}{16}\left(\frac{1}{x^2}+1\right)^{8/3}$$

input
```
Int[(1 + x^(-2))^(5/3)/x^3,x]
```

output
```
(-3*(1 + x^(-2))^(8/3))/16
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{3\left(\frac{1}{x^2}+1\right)^{\frac{8}{3}}}{16}$	10
default	$-\frac{3\left(\frac{1}{x^2}+1\right)^{\frac{8}{3}}}{16}$	10
orering	$-\frac{3(x^2+1)\left(\frac{1}{x^2}+1\right)^{\frac{5}{3}}}{16x^2}$	18
gosper	$-\frac{3(x^2+1)\left(\frac{x^2+1}{x^2}\right)^{\frac{5}{3}}}{16x^2}$	22
trager	$-\frac{3(x^4+2x^2+1)\left(-\frac{-x^2-1}{x^2}\right)^{\frac{2}{3}}}{16x^4}$	30
risch	$-\frac{3\left(\frac{x^2+1}{x^2}\right)^{\frac{2}{3}}(x^6+3x^4+3x^2+1)}{16(x^2+1)x^4}$	39

input
```
int((1/x^2+1)^(5/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/16*(1/x^2+1)^(8/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx = -\frac{3\,(x^4+2\,x^2+1)\left(\frac{x^2+1}{x^2}\right)^{\frac{2}{3}}}{16\,x^4}$$

input
```
integrate((1+1/x^2)^(5/3)/x^3,x, algorithm="fricas")
```

output
```
-3/16*(x^4 + 2*x^2 + 1)*((x^2 + 1)/x^2)^(2/3)/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(14) = 28$.

Time = 0.40 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.69

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx = -\frac{3\left(1+\frac{1}{x^2}\right)^{\frac{2}{3}}}{16} - \frac{3\left(1+\frac{1}{x^2}\right)^{\frac{2}{3}}}{8x^2} - \frac{3\left(1+\frac{1}{x^2}\right)^{\frac{2}{3}}}{16x^4}$$

input
```
integrate((1+1/x**2)**(5/3)/x**3,x)
```

output
```
-3*(1 + x**(-2))**(2/3)/16 - 3*(1 + x**(-2))**(2/3)/(8*x**2) - 3*(1 + x**(
-2))**(2/3)/(16*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx = -\frac{3}{16}\left(\frac{1}{x^2}+1\right)^{\frac{8}{3}}$$

input
```
integrate((1+1/x^2)^(5/3)/x^3,x, algorithm="maxima")
```

output
```
-3/16*(1/x^2 + 1)^(8/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx = -\frac{3\,(x^2+1)^2\left(\frac{x^2+1}{x^2}\right)^{\frac{2}{3}}}{16\,x^4}$$

input
```
integrate((1+1/x^2)^(5/3)/x^3,x, algorithm="giac")
```

output
```
-3/16*(x^2 + 1)^2*((x^2 + 1)/x^2)^(2/3)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx = -\frac{3\left(\frac{1}{x^2}+1\right)^{2/3}\left(x^2+1\right)^2}{16\,x^4}$$

input
```
int((1/x^2 + 1)^(5/3)/x^3,x)
```

output
```
-(3*(1/x^2 + 1)^(2/3)*(x^2 + 1)^2)/(16*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{\left(1+\frac{1}{x^2}\right)^{5/3}}{x^3}\,dx = \frac{3(x^2+1)^{\frac{2}{3}}\left(-x^4-2x^2-1\right)}{16x^{\frac{16}{3}}}$$

input
```
int((1+1/x^2)^(5/3)/x^3,x)
```

output
```
(3*(x**2 + 1)**(2/3)*( - x**4 - 2*x**2 - 1))/(16*x**(1/3)*x**5)
```

3.419 $\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx = \frac{(cx)^{1+m} \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{2}(-1-m), \frac{1-m}{2}, -\frac{b}{x^2}\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([-3/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx = \frac{b\sqrt{1+\frac{b}{x^2}}(cx)^m \text{Hypergeometric2F1}\left(-\frac{3}{2}, -1+\frac{m}{2}, \frac{m}{2}, -\frac{x^2}{b}\right)}{(-2+m)x\sqrt{\frac{b+x^2}{b}}}$$

input
```
Integrate[(1 + b/x^2)^(3/2)*(c*x)^m,x]
```

output
```
(b*Sqrt[1 + b/x^2]*(c*x)^m*Hypergeometric2F1[-3/2, -1 + m/2, m/2, -(x^2/b)
])/((-2 + m)*x*Sqrt[(b + x^2)/b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b}{x^2}+1\right)^{3/2} (cx)^m \, dx$$

$$\Big\downarrow \text{862}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+1} (cx)^{m+1} \int \left(\frac{b}{x^2}+1\right)^{3/2} \left(\frac{1}{x}\right)^{-m-2} d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{278}$$

$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(m+1)}$$

input
```
Int[(1 + b/x^2)^(3/2)*(c*x)^m,x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[-3/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(
c*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Maple [F]

$$\int \left(1+\frac{b}{x^2}\right)^{\frac{3}{2}} (cx)^m\,dx$$

input
```
int((1+b/x^2)^(3/2)*(c*x)^m,x)
```

output
```
int((1+b/x^2)^(3/2)*(c*x)^m,x)
```

Fricas [F]

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m\,dx = \int (cx)^m \left(\frac{b}{x^2}+1\right)^{\frac{3}{2}} dx$$

input
```
integrate((1+b/x^2)^(3/2)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((x^2 + b)*(c*x)^m*sqrt((x^2 + b)/x^2)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m\,dx = -\frac{c^m x^{m+1}\Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2} \end{matrix}\middle| \frac{be^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

input
```
integrate((1+b/x**2)**(3/2)*(c*x)**m,x)
```

output
```
-c**m*x**(m + 1)*gamma(-m/2 - 1/2)*hyper((-3/2, -m/2 - 1/2), (1/2 - m/2,),
 b*exp_polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

Maxima [F]

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx = \int (cx)^m \left(\frac{b}{x^2}+1\right)^{\frac{3}{2}} dx$$

input
```
integrate((1+b/x^2)^(3/2)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m*(b/x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx = \int (cx)^m \left(\frac{b}{x^2}+1\right)^{\frac{3}{2}} dx$$

input
```
integrate((1+b/x^2)^(3/2)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*(b/x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m \, dx = \int (c\,x)^m \left(\frac{b}{x^2}+1\right)^{3/2} dx$$

input
```
int((c*x)^m*(b/x^2 + 1)^(3/2),x)
```

output
```
int((c*x)^m*(b/x^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int \left(1+\frac{b}{x^2}\right)^{3/2} (cx)^m\,dx = c^m \left(\left(\int \frac{x^m\sqrt{x^2+b}}{x^3}dx\right)b+\int \frac{x^m\sqrt{x^2+b}}{x}dx\right)$$

input
```
int((1+b/x^2)^(3/2)*(c*x)^m,x)
```

output
```
c**m*(int((x**m*sqrt(b + x**2))/x**3,x)*b + int((x**m*sqrt(b + x**2))/x,x)
)
```

3.420 $\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx=\frac{(cx)^{1+m}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([-1/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx=\frac{\sqrt{1+\frac{b}{x^2}}x(cx)^m\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},1+\frac{m}{2},-\frac{x^2}{b}\right)}{m\sqrt{1+\frac{x^2}{b}}}$$

input
```
Integrate[Sqrt[1 + b/x^2]*(c*x)^m,x]
```

output
```
(Sqrt[1 + b/x^2]*x*(c*x)^m*Hypergeometric2F1[-1/2, m/2, 1 + m/2, -(x^2/b)]
)/(m*Sqrt[1 + x^2/b])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{b}{x^2}+1}(cx)^m\,dx$$

$$\Big\downarrow \; 862$$

$$-\frac{\left(\frac{1}{x}\right)^{m+1}(cx)^{m+1}\int \sqrt{\frac{b}{x^2}+1}\left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \; 278$$

$$\frac{(cx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(m+1)}$$

input
```
Int[Sqrt[1 + b/x^2]*(c*x)^m,x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[-1/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(
c*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Maple [F]

$$\int \sqrt{1+\frac{b}{x^2}}\,(cx)^m\,dx$$

input
```
int((1+b/x^2)^(1/2)*(c*x)^m,x)
```

output
```
int((1+b/x^2)^(1/2)*(c*x)^m,x)
```

Fricas [F]

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx = \int (cx)^m\sqrt{\frac{b}{x^2}+1}\,dx$$

input
```
integrate((1+b/x^2)^(1/2)*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((c*x)^m*sqrt((x^2 + b)/x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx = -\frac{b^{-\frac{m}{2}}b^{\frac{m}{2}+\frac{1}{2}}c^m x^m\Gamma\left(-\frac{m}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{x^2e^{i\pi}}{b}\right)}{2\Gamma\left(1-\frac{m}{2}\right)}$$

input
```
integrate((1+b/x**2)**(1/2)*(c*x)**m,x)
```

output
```
-b**(m/2 + 1/2)*c**m*x**m*gamma(-m/2)*hyper((-1/2, m/2), (m/2 + 1,), x**2*
exp_polar(I*pi)/b)/(2*b**(m/2)*gamma(1 - m/2))
```

Maxima [F]

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx = \int (cx)^m \sqrt{\frac{b}{x^2}+1}\,dx$$

input
```
integrate((1+b/x^2)^(1/2)*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m*sqrt(b/x^2 + 1), x)
```

Giac [F]

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx = \int (cx)^m \sqrt{\frac{b}{x^2}+1}\,dx$$

input
```
integrate((1+b/x^2)^(1/2)*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*sqrt(b/x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx = \int (c\,x)^m \sqrt{\frac{b}{x^2}+1}\,dx$$

input
```
int((c*x)^m*(b/x^2 + 1)^(1/2),x)
```

output
```
int((c*x)^m*(b/x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1+\frac{b}{x^2}}(cx)^m\,dx = c^m\left(\int \frac{x^m\sqrt{x^2+b}}{x}dx\right)$$

input
```
int((1+b/x^2)^(1/2)*(c*x)^m,x)
```

output
```
c**m*int((x**m*sqrt(b + x**2))/x,x)
```

3.421 $\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = \frac{(cx)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([1/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{x^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},1+\frac{2+m}{2},-\frac{x^2}{b}\right)}{(2+m)\sqrt{1+\frac{b}{x^2}}}$$

input
```
Integrate[(c*x)^m/Sqrt[1 + b/x^2],x]
```

output
```
(x*(c*x)^m*Sqrt[1 + x^2/b]*Hypergeometric2F1[1/2, (2 + m)/2, 1 + (2 + m)/2
, -(x^2/b)])/((2 + m)*Sqrt[1 + b/x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{\frac{b}{x^2}+1}}dx$$

$$\Big\downarrow \text{862}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+1}(cx)^{m+1}\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\sqrt{\frac{b}{x^2}+1}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{278}$$

$$\frac{(cx)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(m+1)}$$

input
```
Int[(c*x)^m/Sqrt[1 + b/x^2],x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[1/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(c
*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}dx$$

input
```
int((c*x)^m/(1+b/x^2)^(1/2),x)
```

output
```
int((c*x)^m/(1+b/x^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = \int \frac{(cx)^m}{\sqrt{\frac{b}{x^2}+1}}\,dx$$

input
```
integrate((c*x)^m/(1+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^m*x^2*sqrt((x^2 + b)/x^2)/(x^2 + b), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = -\frac{c^m x^{m+1}\Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2} \end{matrix}\middle| \frac{be^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

input
```
integrate((c*x)**m/(1+b/x**2)**(1/2),x)
```

output
```
-c**m*x**(m + 1)*gamma(-m/2 - 1/2)*hyper((1/2, -m/2 - 1/2), (1/2 - m/2,),
b*exp_polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = \int \frac{(cx)^m}{\sqrt{\frac{b}{x^2}+1}}\,dx$$

input
```
integrate((c*x)^m/(1+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt(b/x^2 + 1), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = \int \frac{(cx)^m}{\sqrt{\frac{b}{x^2}+1}}\,dx$$

input
```
integrate((c*x)^m/(1+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(b/x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = \int \frac{(c\,x)^m}{\sqrt{\frac{b}{x^2}+1}}\,dx$$

input
```
int((c*x)^m/(b/x^2 + 1)^(1/2),x)
```

output
```
int((c*x)^m/(b/x^2 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{1+\frac{b}{x^2}}}\,dx = c^m\left(\int \frac{x^m x}{\sqrt{x^2+b}}dx\right)$$

input
```
int((c*x)^m/(1+b/x^2)^(1/2),x)
```

output
```
c**m*int((x**m*x)/sqrt(b + x**2),x)
```

3.422 $\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{(cx)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([3/2, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = \frac{x^3(cx)^m\sqrt{\frac{b+x^2}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},2+\frac{m}{2},3+\frac{m}{2},-\frac{x^2}{b}\right)}{b(4+m)\sqrt{1+\frac{b}{x^2}}}$$

input
```
Integrate[(c*x)^m/(1 + b/x^2)^(3/2),x]
```

output
```
(x^3*(c*x)^m*Sqrt[(b + x^2)/b]*Hypergeometric2F1[3/2, 2 + m/2, 3 + m/2, -(
x^2/b)])/(b*(4 + m)*Sqrt[1 + b/x^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\left(\frac{b}{x^2}+1\right)^{3/2}}dx$$
$$\Big\downarrow \text{862}$$
$$-\frac{\left(\frac{1}{x}\right)^{m+1}(cx)^{m+1}\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{\left(\frac{b}{x^2}+1\right)^{3/2}}d\frac{1}{x}}{c}$$
$$\Big\downarrow \text{278}$$
$$\frac{(cx)^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-m-1),\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(m+1)}$$

input
```
Int[(c*x)^m/(1 + b/x^2)^(3/2),x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[3/2, (-1 - m)/2, (1 - m)/2, -(b/x^2)])/(c
*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

Maple [F]

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^m/(1+b/x^2)^(3/2),x)
```

output
```
int((c*x)^m/(1+b/x^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = \int \frac{(cx)^m}{\left(\frac{b}{x^2}+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(1+b/x^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x)^m*x^4*sqrt((x^2 + b)/x^2)/(x^4 + 2*b*x^2 + b^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = -\frac{c^m x^{m+1}\Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2} \end{matrix}\middle| \frac{be^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

input
```
integrate((c*x)**m/(1+b/x**2)**(3/2),x)
```

output
```
-c**m*x**(m + 1)*gamma(-m/2 - 1/2)*hyper((3/2, -m/2 - 1/2), (1/2 - m/2,),
b*exp_polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

Maxima [F]

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = \int \frac{(cx)^m}{\left(\frac{b}{x^2}+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(1+b/x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b/x^2 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = \int \frac{(cx)^m}{\left(\frac{b}{x^2}+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(1+b/x^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b/x^2 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = \int \frac{(c\,x)^m}{\left(\frac{b}{x^2}+1\right)^{3/2}}\,dx$$

input
```
int((c*x)^m/(b/x^2 + 1)^(3/2),x)
```

output
```
int((c*x)^m/(b/x^2 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\left(1+\frac{b}{x^2}\right)^{3/2}}\,dx = c^m\left(\int \frac{x^m x^3}{\sqrt{x^2+b}\,b+\sqrt{x^2+b}\,x^2}dx\right)$$

input
```
int((c*x)^m/(1+b/x^2)^(3/2),x)
```

output
```
c**m*int((x**m*x**3)/(sqrt(b + x**2)*b + sqrt(b + x**2)*x**2),x)
```

3.423 $\int \left(1+\frac{b}{x^2}\right)^p (cx)^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m \, dx = \frac{(cx)^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}(-1-m),-p,\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([-p, -1/2-1/2*m],[1/2-1/2*m],-b/x^2)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m \, dx = \frac{\left(1+\frac{b}{x^2}\right)^p x(cx)^m \left(1+\frac{x^2}{b}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+m-2p),-p,1+\frac{1}{2}(1+m-2p),-\frac{x^2}{b}\right)}{1+m-2p}$$

input
```
Integrate[(1 + b/x^2)^p*(c*x)^m,x]
```

output
```
((1 + b/x^2)^p*x*(c*x)^m*Hypergeometric2F1[(1 + m - 2*p)/2, -p, 1 + (1 + m
 - 2*p)/2, -(x^2/b)])/((1 + m - 2*p)*(1 + x^2/b)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{b}{x^2}+1\right)^p (cx)^m \, dx$$

$$\Big\downarrow \text{862}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+1} (cx)^{m+1} \int \left(\frac{b}{x^2}+1\right)^p \left(\frac{1}{x}\right)^{-m-2} d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{278}$$

$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(\frac{1}{2}(-m-1),-p,\frac{1-m}{2},-\frac{b}{x^2}\right)}{c(m+1)}$$

input
```
Int[(1 + b/x^2)^p*(c*x)^m,x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[(-1 - m)/2, -p, (1 - m)/2, -(b/x^2)])/(c*
(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

Maple [F]

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx$$

input
```
int((1+b/x^2)^p*(c*x)^m,x)
```

output
```
int((1+b/x^2)^p*(c*x)^m,x)
```

Fricas [F]

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx = \int (cx)^m \left(\frac{b}{x^2}+1\right)^p\,dx$$

input
```
integrate((1+b/x^2)^p*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((c*x)^m*((x^2 + b)/x^2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx = -\frac{c^m x^{m+1}\Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\left.\begin{matrix} -p, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2} \end{matrix}\right| \frac{be^{i\pi}}{x^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

input
```
integrate((1+b/x**2)**p*(c*x)**m,x)
```

output
```
-c**m*x**(m + 1)*gamma(-m/2 - 1/2)*hyper((-p, -m/2 - 1/2), (1/2 - m/2,), b
*exp_polar(I*pi)/x**2)/(2*gamma(1/2 - m/2))
```

Maxima [F]

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx=\int (cx)^m\left(\frac{b}{x^2}+1\right)^p\,dx$$

input
```
integrate((1+b/x^2)^p*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m*(b/x^2 + 1)^p, x)
```

Giac [F]

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx=\int (cx)^m\left(\frac{b}{x^2}+1\right)^p\,dx$$

input
```
integrate((1+b/x^2)^p*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*(b/x^2 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx=\int (c\,x)^m\left(\frac{b}{x^2}+1\right)^p\,dx$$

input
```
int((c*x)^m*(b/x^2 + 1)^p,x)
```

output
```
int((c*x)^m*(b/x^2 + 1)^p, x)
```

Reduce [F]

$$\int \left(1+\frac{b}{x^2}\right)^p (cx)^m\,dx$$
$$= \frac{c^m \left(x^m (x^2+b)^p x + 2x^{2p} \left(\int \frac{x^m (x^2+b)^p}{x^{2p} bm + x^{2p} b + x^{2p} m\, x^2 + x^{2p} x^2}dx\right) bmp + 2x^{2p} \left(\int \frac{x^m (x^2+b)^p}{x^{2p} bm + x^{2p} b + x^{2p} m\, x^2 + x^{2p} x^2}dx\right) bp\right)}{x^{2p}\,(m+1)}$$

input
```
int((1+b/x^2)^p*(c*x)^m,x)
```

output
```
(c**m*(x**m*(b + x**2)**p*x + 2*x**(2*p)*int((x**m*(b + x**2)**p)/(x**(2*p
)*b*m + x**(2*p)*b + x**(2*p)*m*x**2 + x**(2*p)*x**2),x)*b*m*p + 2*x**(2*p
)*int((x**m*(b + x**2)**p)/(x**(2*p)*b*m + x**(2*p)*b + x**(2*p)*m*x**2 +
x**(2*p)*x**2),x)*b*p))/(x**(2*p)*(m + 1))
```

3.424 $\int \left(a+\frac{b}{x^2}\right)^p (cx)^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m \, dx = \frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} (cx)^{1+m} \text{Hypergeometric2F1}\left(\frac{1}{2}(-1-m),-p,\frac{1-m}{2},-\frac{b}{ax^2}\right)}{c(1+m)}$$

output
```
(a+b/x^2)^p*(c*x)^(1+m)*hypergeom([-p, -1/2-1/2*m],[1/2-1/2*m],-b/a/x^2)/c
/(1+m)/((1+b/a/x^2)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m \, dx = \frac{\left(a+\frac{b}{x^2}\right)^p x(cx)^m \left(1+\frac{ax^2}{b}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}(1+m-2p),-p,1+\frac{1}{2}(1+m-2p),-\frac{ax^2}{b}\right)}{1+m-2p}$$

input
```
Integrate[(a + b/x^2)^p*(c*x)^m,x]
```

output
```
((a + b/x^2)^p*x*(c*x)^m*Hypergeometric2F1[(1 + m - 2*p)/2, -p, 1 + (1 + m
 - 2*p)/2, -((a*x^2)/b)])/((1 + m - 2*p)*(1 + (a*x^2)/b)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {862, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(a+\frac{b}{x^2}\right)^p dx$$

$$\Big\downarrow \text{862}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+1} (cx)^{m+1} \int \left(a+\frac{b}{x^2}\right)^p \left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{279}$$

$$-\frac{\left(\frac{1}{x}\right)^{m+1} (cx)^{m+1} \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{1}{x}\right)^{-m-2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{278}$$

$$\frac{(cx)^{m+1} \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}(-m-1),-p,\frac{1-m}{2},-\frac{b}{ax^2}\right)}{c(m+1)}$$

input
```
Int[(a + b/x^2)^p*(c*x)^m,x]
```

output
```
((a + b/x^2)^p*(c*x)^(1 + m)*Hypergeometric2F1[(-1 - m)/2, -p, (1 - m)/2,
-(b/(a*x^2))])/(c*(1 + m)*(1 + b/(a*x^2))^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx$$

input
```
int((a+b/x^2)^p*(c*x)^m,x)
```

output
```
int((a+b/x^2)^p*(c*x)^m,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx=\int (cx)^m\left(a+\frac{b}{x^2}\right)^p\,dx$$

input
```
integrate((a+b/x^2)^p*(c*x)^m,x, algorithm="fricas")
```

output
```
integral((c*x)^m*((a*x^2 + b)/x^2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.91 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx = -\frac{a^p c^m x^{m+1}\Gamma\left(-\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, -\frac{m}{2}-\frac{1}{2} \\ \frac{1}{2}-\frac{m}{2} \end{matrix}\middle| \frac{be^{i\pi}}{ax^2}\right)}{2\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}$$

input
```
integrate((a+b/x**2)**p*(c*x)**m,x)
```

output
```
-a**p*c**m*x**(m + 1)*gamma(-m/2 - 1/2)*hyper((-p, -m/2 - 1/2), (1/2 - m/2
,), b*exp_polar(I*pi)/(a*x**2))/(2*gamma(1/2 - m/2))
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx = \int (cx)^m \left(a+\frac{b}{x^2}\right)^p\,dx$$

input
```
integrate((a+b/x^2)^p*(c*x)^m,x, algorithm="maxima")
```

output
```
integrate((c*x)^m*(a + b/x^2)^p, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx = \int (cx)^m \left(a+\frac{b}{x^2}\right)^p\,dx$$

input
```
integrate((a+b/x^2)^p*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*(a + b/x^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx=\int (c\,x)^m\,\left(a+\frac{b}{x^2}\right)^p\,dx$$

input
```
int((c*x)^m*(a + b/x^2)^p,x)
```

output
```
int((c*x)^m*(a + b/x^2)^p, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p (cx)^m\,dx = \frac{c^m\left(x^m(a\,x^2+b)^p\,x+2x^{2p}\left(\int \frac{x^m\,(a\,x^2+b)^p}{x^{2p}am\,x^2+x^{2p}a\,x^2+x^{2p}bm+x^{2p}b}dx\right)bmp+2x^{2p}\left(\int \frac{x^m\,(a\,x^2+b)^p}{x^{2p}am\,x^2+x^{2p}a\,x^2+x^{2p}bm+x^{2p}b}dx\right)\right.}{x^{2p}\,(m+1)}$$

input
```
int((a+b/x^2)^p*(c*x)^m,x)
```

output
```
(c**m*(x**m*(a*x**2 + b)**p*x + 2*x**(2*p)*int((x**m*(a*x**2 + b)**p)/(x**
(2*p)*a*m*x**2 + x**(2*p)*a*x**2 + x**(2*p)*b*m + x**(2*p)*b),x)*b*m*p + 2
*x**(2*p)*int((x**m*(a*x**2 + b)**p)/(x**(2*p)*a*m*x**2 + x**(2*p)*a*x**2
+ x**(2*p)*b*m + x**(2*p)*b),x)*b*p))/(x**(2*p)*(m + 1))
```

3.425 $\int \frac{x^8}{a+\frac{b}{x^3}} dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^8}{a+\frac{b}{x^3}} dx = \frac{b^2x^3}{3a^3} - \frac{bx^6}{6a^2} + \frac{x^9}{9a} - \frac{b^3\log(b+ax^3)}{3a^4}$$

output
```
1/3*b^2*x^3/a^3-1/6*b*x^6/a^2+1/9*x^9/a-1/3*b^3*ln(a*x^3+b)/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{a+\frac{b}{x^3}} dx = \frac{b^2x^3}{3a^3} - \frac{bx^6}{6a^2} + \frac{x^9}{9a} - \frac{b^3\log(b+ax^3)}{3a^4}$$

input
```
Integrate[x^8/(a + b/x^3),x]
```

output
```
(b^2*x^3)/(3*a^3) - (b*x^6)/(6*a^2) + x^9/(9*a) - (b^3*Log[b + a*x^3])/(3*
a^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{11}}{ax^3+b}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^9}{ax^3+b}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{x^6}{a}-\frac{bx^3}{a^2}+\frac{b^2}{a^3}-\frac{b^3}{a^3\left(ax^3+b\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{b^3\log\left(ax^3+b\right)}{a^4}+\frac{b^2x^3}{a^3}-\frac{bx^6}{2a^2}+\frac{x^9}{3a}\right)$$

input
```
Int[x^8/(a + b/x^3),x]
```

output
```
((b^2*x^3)/a^3 - (b*x^6)/(2*a^2) + x^9/(3*a) - (b^3*Log[b + a*x^3])/a^4)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{1}{3}a^{2}x^{9}-\frac{1}{2}ab\,x^{6}+b^{2}x^{3}}{3a^{3}}-\frac{b^{3}\ln\left(a\,x^{3}+b\right)}{3a^{4}}$	46
norman	$\frac{b^{2}x^{3}}{3a^{3}}-\frac{b\,x^{6}}{6a^{2}}+\frac{x^{9}}{9a}-\frac{b^{3}\ln\left(a\,x^{3}+b\right)}{3a^{4}}$	46
risch	$\frac{b^{2}x^{3}}{3a^{3}}-\frac{b\,x^{6}}{6a^{2}}+\frac{x^{9}}{9a}-\frac{b^{3}\ln\left(a\,x^{3}+b\right)}{3a^{4}}$	46
parallelrisch	$-\frac{-2a^{3}x^{9}+3a^{2}b\,x^{6}-6a\,b^{2}x^{3}+6b^{3}\ln\left(a\,x^{3}+b\right)}{18a^{4}}$	46

input
```
int(x^8/(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^3*(1/3*a^2*x^9-1/2*a*b*x^6+b^2*x^3)-1/3*b^3*ln(a*x^3+b)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{a+\frac{b}{x^3}}\,dx = \frac{2\,a^3x^9 - 3\,a^2bx^6 + 6\,ab^2x^3 - 6\,b^3\log\left(ax^3+b\right)}{18\,a^4}$$

input
```
integrate(x^8/(a+b/x^3),x, algorithm="fricas")
```

output
```
1/18*(2*a^3*x^9 - 3*a^2*b*x^6 + 6*a*b^2*x^3 - 6*b^3*log(a*x^3 + b))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{a+\frac{b}{x^3}}\,dx = \frac{x^9}{9a} - \frac{bx^6}{6a^2} + \frac{b^2x^3}{3a^3} - \frac{b^3\log\left(ax^3+b\right)}{3a^4}$$

input
```
integrate(x**8/(a+b/x**3),x)
```

output
```
x**9/(9*a) - b*x**6/(6*a**2) + b**2*x**3/(3*a**3) - b**3*log(a*x**3 + b)/(
3*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x^8}{a+\frac{b}{x^3}}\,dx = -\frac{b^3\log\left(ax^3+b\right)}{3\,a^4} + \frac{2\,a^2x^9 - 3\,abx^6 + 6\,b^2x^3}{18\,a^3}$$

input
```
integrate(x^8/(a+b/x^3),x, algorithm="maxima")
```

output
```
-1/3*b^3*log(a*x^3 + b)/a^4 + 1/18*(2*a^2*x^9 - 3*a*b*x^6 + 6*b^2*x^3)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{a+\frac{b}{x^3}}\,dx = -\frac{b^3 \log\left(|ax^3+b|\right)}{3\,a^4} + \frac{2\,a^2x^9 - 3\,abx^6 + 6\,b^2x^3}{18\,a^3}$$

input
```
integrate(x^8/(a+b/x^3),x, algorithm="giac")
```

output
```
-1/3*b^3*log(abs(a*x^3 + b))/a^4 + 1/18*(2*a^2*x^9 - 3*a*b*x^6 + 6*b^2*x^3
)/a^3
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{a+\frac{b}{x^3}}\,dx = \frac{x^9}{9\,a} - \frac{b\,x^6}{6\,a^2} - \frac{b^3\,\ln\left(a\,x^3+b\right)}{3\,a^4} + \frac{b^2\,x^3}{3\,a^3}$$

input
```
int(x^8/(a + b/x^3),x)
```

output
```
x^9/(9*a) - (b*x^6)/(6*a^2) - (b^3*log(b + a*x^3))/(3*a^4) + (b^2*x^3)/(3*
a^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{x^8}{a+\frac{b}{x^3}}\,dx = \frac{-6\log\left(a^{\frac{2}{3}}x^2 - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)b^3 - 6\log\left(a^{\frac{1}{3}}x + b^{\frac{1}{3}}\right)b^3 + 2a^3x^9 - 3a^2b\,x^6 + 6a\,b^2x^3}{18a^4}$$

input
```
int(x^8/(a+b/x^3),x)
```

output
```
( - 6*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b**3 - 6*log(a**
(1/3)*x + b**(1/3))*b**3 + 2*a**3*x**9 - 3*a**2*b*x**6 + 6*a*b**2*x**3)/(1
8*a**4)
```

3.426 $\int \frac{x^5}{a+\frac{b}{x^3}} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^5}{a+\frac{b}{x^3}} dx = -\frac{bx^3}{3a^2} + \frac{x^6}{6a} + \frac{b^2 \log(b+ax^3)}{3a^3}$$

output
```
-1/3*b*x^3/a^2+1/6*x^6/a+1/3*b^2*ln(a*x^3+b)/a^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{a+\frac{b}{x^3}} dx = -\frac{bx^3}{3a^2} + \frac{x^6}{6a} + \frac{b^2 \log(b+ax^3)}{3a^3}$$

input
```
Integrate[x^5/(a + b/x^3),x]
```

output
```
-1/3*(b*x^3)/a^2 + x^6/(6*a) + (b^2*Log[b + a*x^3])/(3*a^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^8}{ax^3+b}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{ax^3+b}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{x^3}{a}-\frac{b}{a^2}+\frac{b^2}{a^2\left(ax^3+b\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{b^2\log\left(ax^3+b\right)}{a^3}-\frac{bx^3}{a^2}+\frac{x^6}{2a}\right)$$

input
```
Int[x^5/(a + b/x^3),x]
```

output
```
(-((b*x^3)/a^2) + x^6/(2*a) + (b^2*Log[b + a*x^3])/a^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{a^{2} x^{6}-2 a b \,x^{3}+2 b^{2} \ln \left(a \,x^{3}+b \right)}{6 a^{3}}$	34
norman	$-\frac{b \,x^{3}}{3 a^{2}}+\frac{x^{6}}{6 a}+\frac{b^{2} \ln \left(a \,x^{3}+b \right)}{3 a^{3}}$	35
default	$\frac{\frac{1}{2} a \,x^{6}-b \,x^{3}}{3 a^{2}}+\frac{b^{2} \ln \left(a \,x^{3}+b \right)}{3 a^{3}}$	36
risch	$\frac{x^{6}}{6 a}-\frac{b \,x^{3}}{3 a^{2}}+\frac{b^{2}}{6 a^{3}}+\frac{b^{2} \ln \left(a \,x^{3}+b \right)}{3 a^{3}}$	43

input
```
int(x^5/(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a^2*x^6-2*a*b*x^3+2*b^2*ln(a*x^3+b))/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{a+\frac{b}{x^3}}\,dx = \frac{a^2x^6 - 2\,abx^3 + 2\,b^2\log\left(ax^3+b\right)}{6\,a^3}$$

input
```
integrate(x^5/(a+b/x^3),x, algorithm="fricas")
```

output
```
1/6*(a^2*x^6 - 2*a*b*x^3 + 2*b^2*log(a*x^3 + b))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{a+\frac{b}{x^3}}\,dx = \frac{x^6}{6a} - \frac{bx^3}{3a^2} + \frac{b^2\log\left(ax^3+b\right)}{3a^3}$$

input
```
integrate(x**5/(a+b/x**3),x)
```

output
```
x**6/(6*a) - b*x**3/(3*a**2) + b**2*log(a*x**3 + b)/(3*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{a+\frac{b}{x^3}}\,dx = \frac{b^2\log\left(ax^3+b\right)}{3\,a^3} + \frac{ax^6 - 2\,bx^3}{6\,a^2}$$

input
```
integrate(x^5/(a+b/x^3),x, algorithm="maxima")
```

output
```
1/3*b^2*log(a*x^3 + b)/a^3 + 1/6*(a*x^6 - 2*b*x^3)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{a+\frac{b}{x^3}}\,dx = \frac{b^2\log\left(|ax^3+b|\right)}{3\,a^3} + \frac{ax^6-2\,bx^3}{6\,a^2}$$

input
```
integrate(x^5/(a+b/x^3),x, algorithm="giac")
```

output
```
1/3*b^2*log(abs(a*x^3 + b))/a^3 + 1/6*(a*x^6 - 2*b*x^3)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{a+\frac{b}{x^3}}\,dx = \frac{2\,b^2\,\ln\left(a\,x^3+b\right)+a^2\,x^6-2\,a\,b\,x^3}{6\,a^3}$$

input
```
int(x^5/(a + b/x^3),x)
```

output
```
(2*b^2*log(b + a*x^3) + a^2*x^6 - 2*a*b*x^3)/(6*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{x^5}{a+\frac{b}{x^3}}\,dx = \frac{2\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)b^2+2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)b^2+a^2x^6-2ab\,x^3}{6a^3}$$

input
```
int(x^5/(a+b/x^3),x)
```

output
```
(2*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b**2 + 2*log(a**(1/
3)*x + b**(1/3))*b**2 + a**2*x**6 - 2*a*b*x**3)/(6*a**3)
```

3.427 $\int \frac{x^2}{a+\frac{b}{x^3}} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^2}{a+\frac{b}{x^3}} dx = \frac{x^3}{3a} - \frac{b\log(b+ax^3)}{3a^2}$$

output
```
1/3*x^3/a-1/3*b*ln(a*x^3+b)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+\frac{b}{x^3}} dx = \frac{x^3}{3a} - \frac{b\log(b+ax^3)}{3a^2}$$

input
```
Integrate[x^2/(a + b/x^3),x]
```

output
```
x^3/(3*a) - (b*Log[b + a*x^3])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^5}{ax^3+b}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{ax^3+b}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{1}{a}-\frac{b}{a\left(ax^3+b\right)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^3}{a}-\frac{b\log\left(ax^3+b\right)}{a^2}\right)$$

input
```
Int[x^2/(a + b/x^3),x]
```

output
```
(x^3/a - (b*Log[b + a*x^3])/a^2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-a\,x^{3}+b\ln\left(a\,x^{3}+b\right)}{3a^{2}}$	23
default	$\frac{x^{3}}{3a}-\frac{b\ln\left(a\,x^{3}+b\right)}{3a^{2}}$	24
norman	$\frac{x^{3}}{3a}-\frac{b\ln\left(a\,x^{3}+b\right)}{3a^{2}}$	24
risch	$\frac{x^{3}}{3a}-\frac{b\ln\left(a\,x^{3}+b\right)}{3a^{2}}$	24

input
```
int(x^2/(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-a*x^3+b*ln(a*x^3+b))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{a+\frac{b}{x^3}}\,dx = \frac{ax^3 - b\log\left(ax^3+b\right)}{3\,a^2}$$

input
```
integrate(x^2/(a+b/x^3),x, algorithm="fricas")
```

output
```
1/3*(a*x^3 - b*log(a*x^3 + b))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{a+\frac{b}{x^3}}\,dx = \frac{x^3}{3a} - \frac{b\log\left(ax^3+b\right)}{3a^2}$$

input
```
integrate(x**2/(a+b/x**3),x)
```

output
```
x**3/(3*a) - b*log(a*x**3 + b)/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{a+\frac{b}{x^3}}\,dx = \frac{x^3}{3\,a} - \frac{b\log\left(ax^3+b\right)}{3\,a^2}$$

input
```
integrate(x^2/(a+b/x^3),x, algorithm="maxima")
```

output
```
1/3*x^3/a - 1/3*b*log(a*x^3 + b)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{a+\frac{b}{x^3}}\,dx = \frac{x^3}{3\,a} - \frac{b\log\left(|ax^3+b|\right)}{3\,a^2}$$

input
```
integrate(x^2/(a+b/x^3),x, algorithm="giac")
```

output
```
1/3*x^3/a - 1/3*b*log(abs(a*x^3 + b))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{a+\frac{b}{x^3}}\,dx = -\frac{b\,\ln\left(a\,x^3+b\right) - a\,x^3}{3\,a^2}$$

input
```
int(x^2/(a + b/x^3),x)
```

output
```
-(b*log(b + a*x^3) - a*x^3)/(3*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{x^2}{a+\frac{b}{x^3}}\,dx = \frac{-\log\left(a^{\frac{2}{3}}x^2 - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)b - \log\left(a^{\frac{1}{3}}x + b^{\frac{1}{3}}\right)b + a\,x^3}{3a^2}$$

input
```
int(x^2/(a+b/x^3),x)
```

output
```
( - log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b - log(a**(1/3)*x
 + b**(1/3))*b + a*x**3)/(3*a**2)
```

3.428 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx = \frac{\log\left(b+ax^3\right)}{3a}$$

output
```
1/3*ln(a*x^3+b)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx = \frac{\log\left(b+ax^3\right)}{3a}$$

input
```
Integrate[1/((a + b/x^3)*x),x]
```

output
```
Log[b + a*x^3]/(3*a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {795, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^3}\right)}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^2}{ax^3+b}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{\log\left(ax^3+b\right)}{3a}$$

input
```
Int[1/((a + b/x^3)*x),x]
```

output
```
Log[b + a*x^3]/(3*a)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(a\,x^3+b)}{3a}$	14
norman	$\frac{\ln(a\,x^3+b)}{3a}$	14
risch	$\frac{\ln(a\,x^3+b)}{3a}$	14
parallelrisch	$\frac{\ln(a\,x^3+b)}{3a}$	14

input
```
int(1/(a+b/x^3)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(a*x^3+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx = \frac{\log\left(ax^3+b\right)}{3\,a}$$

input
```
integrate(1/(a+b/x^3)/x,x, algorithm="fricas")
```

output
```
1/3*log(a*x^3 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx = \frac{\log\left(ax^3+b\right)}{3a}$$

input
```
integrate(1/(a+b/x**3)/x,x)
```

output
```
log(a*x**3 + b)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx = \frac{\log\left(ax^3+b\right)}{3\,a}$$

input
```
integrate(1/(a+b/x^3)/x,x, algorithm="maxima")
```

output
```
1/3*log(a*x^3 + b)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx = \frac{\log\left(|ax^3+b|\right)}{3\,a}$$

input
```
integrate(1/(a+b/x^3)/x,x, algorithm="giac")
```

output
```
1/3*log(abs(a*x^3 + b))/a
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx = \frac{\ln\left(a\,x^3+b\right)}{3\,a}$$

input
```
int(1/(x*(a + b/x^3)),x)
```

output
```
log(b + a*x^3)/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx=\frac{\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)+\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)}{3a}$$

input
```
int(1/(a+b/x^3)/x,x)
```

output
```
(log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3)) + log(a**(1/3)*x + b*
*(1/3)))/(3*a)
```

3.429 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx=-\frac{\log\left(a+\frac{b}{x^3}\right)}{3b}$$

output
```
-1/3*ln(a+b/x^3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx=\frac{\log(x)}{b}-\frac{\log\left(b+ax^3\right)}{3b}$$

input
```
Integrate[1/((a + b/x^3)*x^4),x]
```

output
```
Log[x]/b - Log[b + a*x^3]/(3*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^3}\right)} \, dx$$

$$\Big\downarrow \ 792$$

$$-\frac{\log \left(a+\frac{b}{x^3}\right)}{3 b}$$

input
```
Int[1/((a + b/x^3)*x^4),x]
```

output
```
-1/3*Log[a + b/x^3]/b
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\ln\left(a+\frac{b}{x^3}\right)}{3b}$	14
default	$\frac{\ln(x)}{b}-\frac{\ln(a\,x^3+b)}{3b}$	21
norman	$\frac{\ln(x)}{b}-\frac{\ln(a\,x^3+b)}{3b}$	21
risch	$\frac{\ln(x)}{b}-\frac{\ln(a\,x^3+b)}{3b}$	21
parallelrisch	$\frac{3\ln(x)-\ln(a\,x^3+b)}{3b}$	21

input
```
int(1/(a+b/x^3)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(a+b/x^3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx = -\frac{\log\left(ax^3+b\right)-3\,\log\left(x\right)}{3\,b}$$

input
```
integrate(1/(a+b/x^3)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(log(a*x^3 + b) - 3*log(x))/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx = \frac{\log(x)}{b} - \frac{\log\left(x^3+\frac{b}{a}\right)}{3b}$$

input
```
integrate(1/(a+b/x**3)/x**4,x)
```

output
```
log(x)/b - log(x**3 + b/a)/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx = -\frac{\log\left(a+\frac{b}{x^3}\right)}{3\,b}$$

input
```
integrate(1/(a+b/x^3)/x^4,x, algorithm="maxima")
```

output
```
-1/3*log(a + b/x^3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx = -\frac{\log\left(|ax^3+b|\right)}{3\,b} + \frac{\log\left(|x|\right)}{b}$$

input
```
integrate(1/(a+b/x^3)/x^4,x, algorithm="giac")
```

output
```
-1/3*log(abs(a*x^3 + b))/b + log(abs(x))/b
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx = -\frac{\ln\left(a\,x^3+b\right)-3\,\ln\left(x\right)}{3\,b}$$

input
```
int(1/(x^4*(a + b/x^3)),x)
```

output
```
-(log(b + a*x^3) - 3*log(x))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^4}\,dx = \frac{-\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)-\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)+3\log(x)}{3b}$$

input
```
int(1/(a+b/x^3)/x^4,x)
```

output
```
( - log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3)) - log(a**(1/3)*x +
 b**(1/3)) + 3*log(x))/(3*b)
```

3.430 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = -\frac{1}{3bx^3} + \frac{a\log\left(a+\frac{b}{x^3}\right)}{3b^2}$$

output
```
-1/3/b/x^3+1/3*a*ln(a+b/x^3)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = -\frac{1}{3bx^3} - \frac{a\log(x)}{b^2} + \frac{a\log\left(b+ax^3\right)}{3b^2}$$

input
```
Integrate[1/((a + b/x^3)*x^7),x]
```

output
```
-1/3*1/(b*x^3) - (a*Log[x])/b^2 + (a*Log[b + a*x^3])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^3}\right)}dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{1}{x^4 \left(a x^3+b\right)}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3} \int \frac{1}{x^6 \left(a x^3+b\right)}dx^3$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{3} \int \left(\frac{a^2}{b^2 \left(a x^3+b\right)}-\frac{a}{b^2 x^3}+\frac{1}{b x^6}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3} \left(-\frac{a \log \left(x^3\right)}{b^2}+\frac{a \log \left(a x^3+b\right)}{b^2}-\frac{1}{b x^3}\right)$$

input
```
Int[1/((a + b/x^3)*x^7),x]
```

output
```
(-(1/(b*x^3)) - (a*Log[x^3])/b^2 + (a*Log[b + a*x^3])/b^2)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result	size
default	$-\frac{1}{3b\,x^{3}}-\frac{a\ln\left(x\right)}{b^{2}}+\frac{a\ln\left(a\,x^{3}+b\right)}{3b^{2}}$	32
norman	$-\frac{1}{3b\,x^{3}}-\frac{a\ln\left(x\right)}{b^{2}}+\frac{a\ln\left(a\,x^{3}+b\right)}{3b^{2}}$	32
parallelrisch	$-\frac{3a\ln\left(x\right)x^{3}-a\ln\left(a\,x^{3}+b\right)x^{3}+b}{3b^{2}x^{3}}$	33
risch	$-\frac{1}{3b\,x^{3}}-\frac{a\ln\left(x\right)}{b^{2}}+\frac{a\ln\left(-a\,x^{3}-b\right)}{3b^{2}}$	35

input
```
int(1/(a+b/x^3)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/x^3-a/b^2*ln(x)+1/3*a/b^2*ln(a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = \frac{ax^3\log\left(ax^3+b\right)-3\,ax^3\log\left(x\right)-b}{3\,b^2x^3}$$

input
```
integrate(1/(a+b/x^3)/x^7,x, algorithm="fricas")
```

output
```
1/3*(a*x^3*log(a*x^3 + b) - 3*a*x^3*log(x) - b)/(b^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = -\frac{a\log\left(x\right)}{b^2}+\frac{a\log\left(x^3+\frac{b}{a}\right)}{3b^2}-\frac{1}{3bx^3}$$

input
```
integrate(1/(a+b/x**3)/x**7,x)
```

output
```
-a*log(x)/b**2 + a*log(x**3 + b/a)/(3*b**2) - 1/(3*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = \frac{a\log\left(ax^3+b\right)}{3\,b^2}-\frac{a\log\left(x^3\right)}{3\,b^2}-\frac{1}{3\,bx^3}$$

input
```
integrate(1/(a+b/x^3)/x^7,x, algorithm="maxima")
```

output
```
1/3*a*log(a*x^3 + b)/b^2 - 1/3*a*log(x^3)/b^2 - 1/3/(b*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = \frac{a\log\left(|ax^3+b|\right)}{3\,b^2} - \frac{a\log\left(|x|\right)}{b^2} + \frac{ax^3-b}{3\,b^2x^3}$$

input
```
integrate(1/(a+b/x^3)/x^7,x, algorithm="giac")
```

output
```
1/3*a*log(abs(a*x^3 + b))/b^2 - a*log(abs(x))/b^2 + 1/3*(a*x^3 - b)/(b^2*x
^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = \frac{a\,\ln\left(a\,x^3+b\right)}{3\,b^2} - \frac{1}{3\,b\,x^3} - \frac{a\,\ln\left(x\right)}{b^2}$$

input
```
int(1/(x^7*(a + b/x^3)),x)
```

output
```
(a*log(b + a*x^3))/(3*b^2) - 1/(3*b*x^3) - (a*log(x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^7}\,dx = \frac{\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a\,x^3+\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)a\,x^3-3\log(x)\,a\,x^3-b}{3b^2x^3}$$

input
```
int(1/(a+b/x^3)/x^7,x)
```

output
```
(log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*x**3 + log(a**(1/3)
*x + b**(1/3))*a*x**3 - 3*log(x)*a*x**3 - b)/(3*b**2*x**3)
```

3.431 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx=-\frac{1}{6bx^6}+\frac{a}{3b^2x^3}-\frac{a^2\log\left(a+\frac{b}{x^3}\right)}{3b^3}$$

output
```
-1/6/b/x^6+1/3*a/b^2/x^3-1/3*a^2*ln(a+b/x^3)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx=-\frac{1}{6bx^6}+\frac{a}{3b^2x^3}+\frac{a^2\log(x)}{b^3}-\frac{a^2\log\left(b+ax^3\right)}{3b^3}$$

input
```
Integrate[1/((a + b/x^3)*x^10),x]
```

output
```
-1/6*1/(b*x^6) + a/(3*b^2*x^3) + (a^2*Log[x])/b^3 - (a^2*Log[b + a*x^3])/(
3*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\left(a+\frac{b}{x^3}\right)}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^7\left(ax^3+b\right)}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{1}{x^9\left(ax^3+b\right)}dx^3$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^3\left(ax^3+b\right)}+\frac{a^2}{b^3x^3}-\frac{a}{b^2x^6}+\frac{1}{bx^9}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{a^2\log\left(x^3\right)}{b^3}-\frac{a^2\log\left(ax^3+b\right)}{b^3}+\frac{a}{b^2x^3}-\frac{1}{2bx^6}\right)$$

input
```
Int[1/((a + b/x^3)*x^10),x]
```

output
```
(-1/2*1/(b*x^6) + a/(b^2*x^3) + (a^2*Log[x^3])/b^3 - (a^2*Log[b + a*x^3])/
b^3)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{1}{6x^{6}b}+\frac{a^{2}\ln\left(x\right)}{b^{3}}+\frac{a}{3b^{2}x^{3}}-\frac{a^{2}\ln\left(a\,x^{3}+b\right)}{3b^{3}}$	44
risch	$\frac{\frac{a\,x^{3}}{3b^{2}}-\frac{1}{6b}}{x^{6}}+\frac{a^{2}\ln\left(x\right)}{b^{3}}-\frac{a^{2}\ln\left(a\,x^{3}+b\right)}{3b^{3}}$	46
parallelrisch	$\frac{6a^{2}\ln\left(x\right)x^{6}-2a^{2}\ln\left(a\,x^{3}+b\right)x^{6}+2ab\,x^{3}-b^{2}}{6b^{3}x^{6}}$	48
norman	$\frac{-\frac{x^{3}}{6b}+\frac{a\,x^{6}}{3b^{2}}}{x^{9}}+\frac{a^{2}\ln\left(x\right)}{b^{3}}-\frac{a^{2}\ln\left(a\,x^{3}+b\right)}{3b^{3}}$	49

input
```
int(1/(a+b/x^3)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^6/b+a^2/b^3*ln(x)+1/3*a/b^2/x^3-1/3*a^2/b^3*ln(a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx = -\frac{2\,a^2x^6\log\left(ax^3+b\right)-6\,a^2x^6\log\left(x\right)-2\,abx^3+b^2}{6\,b^3x^6}$$

input
```
integrate(1/(a+b/x^3)/x^10,x, algorithm="fricas")
```

output
```
-1/6*(2*a^2*x^6*log(a*x^3 + b) - 6*a^2*x^6*log(x) - 2*a*b*x^3 + b^2)/(b^3*
x^6)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx = \frac{a^2\log\left(x\right)}{b^3}-\frac{a^2\log\left(x^3+\frac{b}{a}\right)}{3b^3}+\frac{2ax^3-b}{6b^2x^6}$$

input
```
integrate(1/(a+b/x**3)/x**10,x)
```

output
```
a**2*log(x)/b**3 - a**2*log(x**3 + b/a)/(3*b**3) + (2*a*x**3 - b)/(6*b**2*
x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx = -\frac{a^2\log\left(ax^3+b\right)}{3\,b^3}+\frac{a^2\log\left(x^3\right)}{3\,b^3}+\frac{2\,ax^3-b}{6\,b^2x^6}$$

input
```
integrate(1/(a+b/x^3)/x^10,x, algorithm="maxima")
```

output
```
-1/3*a^2*log(a*x^3 + b)/b^3 + 1/3*a^2*log(x^3)/b^3 + 1/6*(2*a*x^3 - b)/(b^
2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx = -\frac{a^2\log\left(|ax^3+b|\right)}{3\,b^3} + \frac{a^2\log\left(|x|\right)}{b^3} - \frac{3\,a^2x^6-2\,abx^3+b^2}{6\,b^3x^6}$$

input
```
integrate(1/(a+b/x^3)/x^10,x, algorithm="giac")
```

output
```
-1/3*a^2*log(abs(a*x^3 + b))/b^3 + a^2*log(abs(x))/b^3 - 1/6*(3*a^2*x^6 -
2*a*b*x^3 + b^2)/(b^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx = \frac{a^2\,\ln\left(x\right)}{b^3} - \frac{a^2\,\ln\left(a\,x^3+b\right)}{3\,b^3} - \frac{\frac{1}{6\,b}-\frac{a\,x^3}{3\,b^2}}{x^6}$$

input
```
int(1/(x^10*(a + b/x^3)),x)
```

output
```
(a^2*log(x))/b^3 - (a^2*log(b + a*x^3))/(3*b^3) - (1/(6*b) - (a*x^3)/(3*b^
2))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^{10}}\,dx = \frac{-2\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a^2x^6-2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)a^2x^6+6\log(x)\,a^2x^6+2ab\,x^3-b^2}{6b^3x^6}$$

input
```
int(1/(a+b/x^3)/x^10,x)
```

output
```
( - 2*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a**2*x**6 - 2*lo
g(a**(1/3)*x + b**(1/3))*a**2*x**6 + 6*log(x)*a**2*x**6 + 2*a*b*x**3 - b**
2)/(6*b**3*x**6)
```

3.432 $\int \frac{x^4}{a+\frac{b}{x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 136

$$\int \frac{x^4}{a+\frac{b}{x^3}} \, dx = -\frac{bx^2}{2a^2}+\frac{x^5}{5a}-\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{8/3}} -\frac{b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3a^{8/3}}+\frac{b^{5/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{8/3}}$$

output
```
-1/2*b*x^2/a^2+1/5*x^5/a-1/3*b^(5/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1
/2)/b^(1/3))*3^(1/2)/a^(8/3)-1/3*b^(5/3)*ln(b^(1/3)+a^(1/3)*x)/a^(8/3)+1/6
*b^(5/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{a+\frac{b}{x^3}} \, dx = \frac{-15a^{2/3}bx^2+6a^{5/3}x^5-10\sqrt{3}b^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-10b^{5/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)+5b^{5/3}\log\left(b^{2/3}-\sqrt[3]{a}\right.}{30a^{8/3}}$$

input
```
Integrate[x^4/(a + b/x^3),x]
```

output
```
(-15*a^(2/3)*b*x^2 + 6*a^(5/3)*x^5 - 10*Sqrt[3]*b^(5/3)*ArcTan[(1 - (2*a^(
1/3)*x)/b^(1/3))/Sqrt[3]] - 10*b^(5/3)*Log[b^(1/3) + a^(1/3)*x] + 5*b^(5/3
)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(30*a^(8/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^7}{ax^3+b}dx$$

$$\Big\downarrow \text{831}$$

$$\int \left(\frac{b^2x}{a^2\left(ax^3+b\right)}-\frac{bx}{a^2}+\frac{x^4}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{8/3}}+\frac{b^{5/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{6a^{8/3}}-\frac{b^{5/3}\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{8/3}}-\frac{bx^2}{2a^2}+\frac{x^5}{5a}$$

input
```
Int[x^4/(a + b/x^3),x]
```

output
```
-1/2*(b*x^2)/a^2 + x^5/(5*a) - (b^(5/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sq
rt[3]*b^(1/3))])/(Sqrt[3]*a^(8/3)) - (b^(5/3)*Log[b^(1/3) + a^(1/3)*x])/(3
*a^(8/3)) + (b^(5/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^
(8/3))
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{x^{5}}{5a}-\frac{b\,x^{2}}{2a^{2}}+\frac{b^{2}\left(\sum_{\_R=\text{RootOf}\left(a\_Z^{3}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3a^{3}}$	48
default	$\frac{\frac{1}{5}a\,x^{5}-\frac{1}{2}b\,x^{2}}{a^{2}}+\frac{\left(-\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)b^{2}}{a^{2}}$	116

input
```
int(x^4/(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
1/5/a*x^5-1/2*b/a^2*x^2+1/3/a^3*b^2*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03

$$\int \frac{x^4}{a+\frac{b}{x^3}}\,dx = \frac{6\,ax^5 - 15\,bx^2 + 10\,\sqrt{3}b\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+\sqrt{3}b}{3\,b}\right) - 5\,b\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^2 - ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}} - b\left(-\right.\right.}{30\,a^2}$$

input
```
integrate(x^4/(a+b/x^3),x, algorithm="fricas")
```

output
```
1/30*(6*a*x^5 - 15*b*x^2 + 10*sqrt(3)*b*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqr
t(3)*a*x*(-b^2/a^2)^(1/3) + sqrt(3)*b)/b) - 5*b*(-b^2/a^2)^(1/3)*log(b*x^2
 - a*x*(-b^2/a^2)^(2/3) - b*(-b^2/a^2)^(1/3)) + 10*b*(-b^2/a^2)^(1/3)*log(
b*x + a*(-b^2/a^2)^(2/3)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{x^4}{a+\frac{b}{x^3}}\,dx = \operatorname{RootSum}\left(27t^3a^8+b^5,\left(t\mapsto t\log\left(\frac{9t^2a^5}{b^3}+x\right)\right)\right)+\frac{x^5}{5a}-\frac{bx^2}{2a^2}$$

input
```
integrate(x**4/(a+b/x**3),x)
```

output
```
RootSum(27*_t**3*a**8 + b**5, Lambda(_t, _t*log(9*_t**2*a**5/b**3 + x))) +
 x**5/(5*a) - b*x**2/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{a+\frac{b}{x^3}}\,dx = \frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{b^2\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{b^2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{2\,ax^5-5\,bx^2}{10\,a^2}$$

input
```
integrate(x^4/(a+b/x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(
b/a)^(1/3)) + 1/6*b^2*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^3*(b/a)^(1
/3)) - 1/3*b^2*log(x + (b/a)^(1/3))/(a^3*(b/a)^(1/3)) + 1/10*(2*a*x^5 - 5*
b*x^2)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{a+\frac{b}{x^3}}\,dx = -\frac{b\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} - \frac{\sqrt{3}(-a^2b)^{\frac{2}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^4} + \frac{(-a^2b)^{\frac{2}{3}}\,b\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^4} + \frac{2\,a^4x^5-5\,a^3bx^2}{10\,a^5}$$

input
```
integrate(x^4/(a+b/x^3),x, algorithm="giac")
```

output
```
-1/3*b*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/a^2 - 1/3*sqrt(3)*(-a^2*b)^
(2/3)*b*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^4 + 1/6*(-
a^2*b)^(2/3)*b*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^4 + 1/10*(2*a^4*
x^5 - 5*a^3*b*x^2)/a^5
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{a+\frac{b}{x^3}}\,dx = \frac{x^5}{5\,a}-\frac{b\,x^2}{2\,a^2}+\frac{{\left(-b\right)}^{5/3}\,\ln\left(\frac{b^4\,x}{a^3}-\frac{{\left(-b\right)}^{13/3}}{a^{10/3}}\right)}{3\,a^{8/3}} - \frac{{\left(-b\right)}^{5/3}\,\ln\left(\frac{b^4\,x}{a^3}-\frac{{\left(-b\right)}^{13/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{a^{10/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}} + \frac{{\left(-b\right)}^{5/3}\,\ln\left(\frac{b^4\,x}{a^3}-\frac{9\,{\left(-b\right)}^{13/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{a^{10/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{8/3}}$$

input
```
int(x^4/(a + b/x^3),x)
```

output
```
x^5/(5*a) - (b*x^2)/(2*a^2) + ((-b)^(5/3)*log((b^4*x)/a^3 - (-b)^(13/3)/a^
(10/3)))/(3*a^(8/3)) - ((-b)^(5/3)*log((b^4*x)/a^3 - ((-b)^(13/3)*((3^(1/2
)*1i)/2 + 1/2)^2)/a^(10/3))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(8/3)) + ((-b)^(5
/3)*log((b^4*x)/a^3 - (9*(-b)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/a^(10/3))*(
(3^(1/2)*1i)/6 - 1/6))/a^(8/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{a+\frac{b}{x^3}}\,dx = \frac{10\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)b^2+6b^{\frac{1}{3}}a^{\frac{5}{3}}x^5-15b^{\frac{4}{3}}a^{\frac{2}{3}}x^2+5\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)b^2-10\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)b^2}{30b^{\frac{1}{3}}a^{\frac{8}{3}}}$$

input
```
int(x^4/(a+b/x^3),x)
```

output
```
(10*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*b**2 + 6*b*
*(1/3)*a**(2/3)*a*x**5 - 15*b**(1/3)*a**(2/3)*b*x**2 + 5*log(a**(2/3)*x**2
 - b**(1/3)*a**(1/3)*x + b**(2/3))*b**2 - 10*log(a**(1/3)*x + b**(1/3))*b*
*2)/(30*b**(1/3)*a**(2/3)*a**2)
```

3.433 $\int \frac{x^3}{a+\frac{b}{x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 132

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx = -\frac{bx}{a^2}+\frac{x^4}{4a}-\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{7/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3a^{7/3}}-\frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{7/3}}$$

output
```
-b*x/a^2+1/4*x^4/a-1/3*b^(4/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^
(1/3))*3^(1/2)/a^(7/3)+1/3*b^(4/3)*ln(b^(1/3)+a^(1/3)*x)/a^(7/3)-1/6*b^(4/
3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx$$

$$= \frac{-12\sqrt[3]{a}bx+3a^{4/3}x^4-4\sqrt{3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)+4b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)-2b^{4/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{12a^{7/3}}$$

input
```
Integrate[x^3/(a + b/x^3),x]
```

output
```
(-12*a^(1/3)*b*x + 3*a^(4/3)*x^4 - 4*Sqrt[3]*b^(4/3)*ArcTan[(1 - (2*a^(1/3
)*x)/b^(1/3))/Sqrt[3]] + 4*b^(4/3)*Log[b^(1/3) + a^(1/3)*x] - 2*b^(4/3)*Lo
g[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(12*a^(7/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {795, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+\frac{b}{x^3}} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^6}{a x^3+b}dx$$

$$\Big\downarrow \text{831}$$

$$\int \left(\frac{b^2}{a^2 \left(a x^3+b\right)}-\frac{b}{a^2}+\frac{x^3}{a}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^{4/3} \arctan \left(\frac{\sqrt[3]{b}-2 \sqrt[3]{a} x}{\sqrt{3} \sqrt[3]{b}}\right)}{\sqrt{3} a^{7/3}}-\frac{b^{4/3} \log \left(a^{2/3} x^2-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3}\right)}{6 a^{7/3}}+\frac{b^{4/3} \log \left(\sqrt[3]{a} x+\sqrt[3]{b}\right)}{3 a^{7/3}}-\frac{b x}{a^2}+\frac{x^4}{4 a}$$

input
```
Int[x^3/(a + b/x^3),x]
```

output
```
-((b*x)/a^2) + x^4/(4*a) - (b^(4/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3
]*b^(1/3))])/(Sqrt[3]*a^(7/3)) + (b^(4/3)*Log[b^(1/3) + a^(1/3)*x])/(3*a^(
7/3)) - (b^(4/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^(7/3
))
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{x^4}{4a}-\frac{bx}{a^2}+\frac{b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^3+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{3a^3}$	46
default	$\frac{\frac{1}{4}a\,x^4-bx}{a^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)b^2}{a^2}$	114

input
```
int(x^3/(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*x^4-b/a^2*x+1/3/a^3*b^2*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx = \frac{3\,ax^4 + 4\,\sqrt{3}b\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right) - 2\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x^2 - x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right) + 4\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x + \left(\frac{b}{a}\right.\right.}{12\,a^2}$$

input
```
integrate(x^3/(a+b/x^3),x, algorithm="fricas")
```

output
```
1/12*(3*a*x^4 + 4*sqrt(3)*b*(b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b/a)^(2
/3) - sqrt(3)*b)/b) - 2*b*(b/a)^(1/3)*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3
)) + 4*b*(b/a)^(1/3)*log(x + (b/a)^(1/3)) - 12*b*x)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.28

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx = \operatorname{RootSum}\left(27t^3a^7 - b^4, \left(t \mapsto t\log\left(\frac{3ta^2}{b} + x\right)\right)\right) + \frac{x^4}{4a} - \frac{bx}{a^2}$$

input
```
integrate(x**3/(a+b/x**3),x)
```

output
```
RootSum(27*_t**3*a**7 - b**4, Lambda(_t, _t*log(3*_t*a**2/b + x))) + x**4/
(4*a) - b*x/a**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx = \frac{\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{b^2\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{b^2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{ax^4-4\,bx}{4\,a^2}$$

input
```
integrate(x^3/(a+b/x^3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(
b/a)^(2/3)) - 1/6*b^2*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^3*(b/a)^(2
/3)) + 1/3*b^2*log(x + (b/a)^(1/3))/(a^3*(b/a)^(2/3)) + 1/4*(a*x^4 - 4*b*x
)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx = -\frac{b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3} + \frac{(-a^2b)^{\frac{1}{3}}\,b\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3} + \frac{a^3x^4-4\,a^2bx}{4\,a^4}$$

input
```
integrate(x^3/(a+b/x^3),x, algorithm="giac")
```

output
```
-1/3*b*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a^2 + 1/3*sqrt(3)*(-a^2*b)^
(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^3 + 1/6*(-
a^2*b)^(1/3)*b*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^3 + 1/4*(a^3*x^4
 - 4*a^2*b*x)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx = \frac{x^4}{4\,a}+\frac{b^{4/3}\,\ln\left(3\,b^2\,x+\frac{3\,b^{7/3}}{a^{1/3}}\right)}{3\,a^{7/3}}-\frac{b\,x}{a^2} - \frac{b^{4/3}\,\ln\left(3\,b^2\,x-\frac{3\,b^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{7/3}} + \frac{b^{4/3}\,\ln\left(3\,b^2\,x+\frac{9\,b^{7/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{1/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{7/3}}$$

input
```
int(x^3/(a + b/x^3),x)
```

output
```
x^4/(4*a) + (b^(4/3)*log(3*b^2*x + (3*b^(7/3))/a^(1/3)))/(3*a^(7/3)) - (b*
x)/a^2 - (b^(4/3)*log(3*b^2*x - (3*b^(7/3)*((3^(1/2)*1i)/2 + 1/2))/a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2))/(3*a^(7/3)) + (b^(4/3)*log(3*b^2*x + (9*b^(7/3)*
((3^(1/2)*1i)/6 - 1/6))/a^(1/3))*((3^(1/2)*1i)/6 - 1/6))/a^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int \frac{x^3}{a+\frac{b}{x^3}}\,dx = \frac{4b^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)+3a^{\frac{4}{3}}x^4-12a^{\frac{1}{3}}bx-2b^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)+4b^{\frac{4}{3}}\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)}{12a^{\frac{7}{3}}}$$

input
```
int(x^3/(a+b/x^3),x)
```

output
```
(4*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*b +
 3*a**(1/3)*a*x**4 - 12*a**(1/3)*b*x - 2*b**(1/3)*log(a**(2/3)*x**2 - b**(
1/3)*a**(1/3)*x + b**(2/3))*b + 4*b**(1/3)*log(a**(1/3)*x + b**(1/3))*b)/(
12*a**(1/3)*a**2)
```

3.434 $\int \frac{x}{a+\frac{b}{x^3}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 124

$$\int \frac{x}{a+\frac{b}{x^3}} \, dx = \frac{x^2}{2a}+\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{5/3}}+\frac{b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3a^{5/3}}-\frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{5/3}}$$

output
```
1/2*x^2/a+1/3*b^(2/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3^
(1/2)/a^(5/3)+1/3*b^(2/3)*ln(b^(1/3)+a^(1/3)*x)/a^(5/3)-1/6*b^(2/3)*ln(b^(
2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{x}{a+\frac{b}{x^3}} \, dx = \frac{3a^{2/3}x^2+2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)+2b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)-b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{5/3}}$$

input
```
Integrate[x/(a + b/x^3),x]
```

output
```
(3*a^(2/3)*x^2 + 2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt
[3]] + 2*b^(2/3)*Log[b^(1/3) + a^(1/3)*x] - b^(2/3)*Log[b^(2/3) - a^(1/3)*
b^(1/3)*x + a^(2/3)*x^2])/(6*a^(5/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {795, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^4}{ax^3+b}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^2}{2a}-\frac{b\int \frac{x}{ax^3+b}dx}{a}$$

$$\Big\downarrow \text{821}$$

$$\frac{x^2}{2a}-\frac{b\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{x^2}{2a}-\frac{b\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}$$

$$\Big\downarrow \; 1142$$

$$\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}$$

$$\Big\downarrow \; 1082$$

$$\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}$$

$$\Big\downarrow \; 217$$

$$\frac{x^2}{2a}-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}$$

$\Big\downarrow$ 1103

$$\frac{x^2}{2a}-\frac{b\left(\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}$$

input
```
Int[x/(a + b/x^3),x]
```

output
```
x^2/(2*a) - (b*(-1/3*Log[b^(1/3) + a^(1/3)*x]/(a^(2/3)*b^(1/3)) + (-((Sqrt
[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) + Log[b^(2/3) -
a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*a^(1/3)*b^(1/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{x^2}{2a}-\frac{b\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^3+b\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3a^2}$	37
default	$\frac{x^2}{2a}-\frac{\left(-\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)b}{a}$	106

input
```
int(x/(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*x^2-1/3/a^2*b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int\frac{x}{a+\frac{b}{x^3}}\,dx=\frac{3\,x^2-2\,\sqrt{3}\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}-\sqrt{3}b}{3\,b}\right)-\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}+b\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(\right.}{6\,a}$$

input
```
integrate(x/(a+b/x^3),x, algorithm="fricas")
```

output
```
1/6*(3*x^2 - 2*sqrt(3)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)
^(1/3) - sqrt(3)*b)/b) - (b^2/a^2)^(1/3)*log(b*x^2 - a*x*(b^2/a^2)^(2/3) +
 b*(b^2/a^2)^(1/3)) + 2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(2/3)))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{x}{a+\frac{b}{x^3}}\,dx = \operatorname{RootSum}\left(27t^3a^5-b^2,\left(t\mapsto t\log\left(\frac{9t^2a^3}{b}+x\right)\right)\right)+\frac{x^2}{2a}$$

input
```
integrate(x/(a+b/x**3),x)
```

output
```
RootSum(27*_t**3*a**5 - b**2, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) + x*
*2/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int \frac{x}{a+\frac{b}{x^3}}\,dx = \frac{x^2}{2\,a}-\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}-\frac{b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(a+b/x^3),x, algorithm="maxima")
```

output
```
1/2*x^2/a - 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/
3))/(a^2*(b/a)^(1/3)) - 1/6*b*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^2*
(b/a)^(1/3)) + 1/3*b*log(x + (b/a)^(1/3))/(a^2*(b/a)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{x}{a+\frac{b}{x^3}}\,dx=\frac{x^2}{2\,a}+\frac{\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}(-a^2b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^3}-\frac{(-a^2b)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^3}$$

input
```
integrate(x/(a+b/x^3),x, algorithm="giac")
```

output
```
1/2*x^2/a + 1/3*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/a + 1/3*sqrt(3)*(-
a^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^3 - 1
/6*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

$$\int \frac{x}{a+\frac{b}{x^3}}\,dx=\frac{x^2}{2\,a}+\frac{b^{2/3}\,\ln\left(\frac{b^{7/3}}{a^{4/3}}+\frac{b^2\,x}{a}\right)}{3\,a^{5/3}}-\frac{b^{2/3}\,\ln\left(\frac{b^2\,x}{a}+\frac{b^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{a^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{5/3}}+\frac{b^{2/3}\,\ln\left(\frac{b^2\,x}{a}+\frac{9\,b^{7/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{a^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{5/3}}$$

input
```
int(x/(a + b/x^3),x)
```

output
```
x^2/(2*a) + (b^(2/3)*log(b^(7/3)/a^(4/3) + (b^2*x)/a))/(3*a^(5/3)) - (b^(2
/3)*log((b^2*x)/a + (b^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/a^(4/3))*((3^(1/2)*
1i)/2 + 1/2))/(3*a^(5/3)) + (b^(2/3)*log((b^2*x)/a + (9*b^(7/3)*((3^(1/2)*
1i)/6 - 1/6)^2)/a^(4/3))*((3^(1/2)*1i)/6 - 1/6))/a^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int \frac{x}{a+\frac{b}{x^3}}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)b+3b^{\frac{1}{3}}a^{\frac{2}{3}}x^2-\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)b+2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)b}{6b^{\frac{1}{3}}a^{\frac{5}{3}}}$$

input
```
int(x/(a+b/x^3),x)
```

output
```
( - 2*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*b + 3*b**
(1/3)*a**(2/3)*x**2 - log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*
b + 2*log(a**(1/3)*x + b**(1/3))*b)/(6*b**(1/3)*a**(2/3)*a)
```

3.435 $\int \frac{1}{a+\frac{b}{x^3}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 119

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx = \frac{x}{a} + \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3a^{4/3}} + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{4/3}}$$

output
```
x/a+1/3*b^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3^(1/2)/
a^(4/3)-1/3*b^(1/3)*ln(b^(1/3)+a^(1/3)*x)/a^(4/3)+1/6*b^(1/3)*ln(b^(2/3)-a
^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx = \frac{6\sqrt[3]{a}x + 2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right) - 2\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right) + \sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{4/3}}$$

input
```
Integrate[(a + b/x^3)^(-1),x]
```

output
```
(6*a^(1/3)*x + 2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3
]] - 2*b^(1/3)*Log[b^(1/3) + a^(1/3)*x] + b^(1/3)*Log[b^(2/3) - a^(1/3)*b^
(1/3)*x + a^(2/3)*x^2])/(6*a^(4/3))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {772, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{ 772}$$

$$\int \frac{x^3}{ax^3+b}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{ax^3+b}dx}{a}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3b^{2/3}}\right)}{a}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}$$

$$\Big\downarrow \; 1142$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}$$

$$\Big\downarrow \; 1082$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}$$

$$\Big\downarrow \; 217$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{a}-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}$$

input
```
Int[(a + b/x^3)^(-1),x]
```

output
```
x/a - (b*(Log[b^(1/3) + a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(2/3) - a^(1/3)*b
^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*b^(2/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{x}{a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\,\_Z^{3}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3a^{2}}$	34
default	$\frac{x}{a}-\frac{\left(\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)b}{a}$	103

input
```
int(1/(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/3/a^2*b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx$$
$$=\frac{2\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+2\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{b}{a}\right.\right.}{6a}$$

input
```
integrate(1/(a+b/x^3),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(-b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b/a)^(2/3) - sqrt(
3)*b)/b) - (-b/a)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3)) + 2*(-b/a
)^(1/3)*log(x - (-b/a)^(1/3)) + 6*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.18

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx = \operatorname{RootSum}\left(27t^3a^4+b,\left(t\mapsto t\log\left(-3ta+x\right)\right)\right)+\frac{x}{a}$$

input
```
integrate(1/(a+b/x**3),x)
```

output
```
RootSum(27*_t**3*a**4 + b, Lambda(_t, _t*log(-3*_t*a + x))) + x/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx = \frac{x}{a} - \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(a+b/x^3),x, algorithm="maxima")
```

output
```
x/a - 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a
^2*(b/a)^(2/3)) + 1/6*b*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a^2*(b/a)^
(2/3)) - 1/3*b*log(x + (b/a)^(1/3))/(a^2*(b/a)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx = \frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{x}{a} - \frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^2} - \frac{(-a^2b)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^2}$$

input
```
integrate(1/(a+b/x^3),x, algorithm="giac")
```

output
```
1/3*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a + x/a - 1/3*sqrt(3)*(-a^2*b)
^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/a^2 - 1/6*(-a
^2*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx = \frac{x}{a} + \frac{(-b)^{1/3}\ln\left((-b)^{4/3}+a^{1/3}\,b\,x\right)}{3\,a^{4/3}} - \frac{(-b)^{1/3}\ln\left(3\,a^{2/3}\,(-b)^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-3\,a\,b\,x\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}} + \frac{(-b)^{1/3}\ln\left(9\,a^{2/3}\,(-b)^{4/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+3\,a\,b\,x\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{a^{4/3}}$$

input
```
int(1/(a + b/x^3),x)
```

output
```
x/a + ((-b)^(1/3)*log((-b)^(4/3) + a^(1/3)*b*x))/(3*a^(4/3)) - ((-b)^(1/3)
*log(3*a^(2/3)*(-b)^(4/3)*((3^(1/2)*1i)/2 + 1/2) - 3*a*b*x)*((3^(1/2)*1i)/
2 + 1/2))/(3*a^(4/3)) + ((-b)^(1/3)*log(9*a^(2/3)*(-b)^(4/3)*((3^(1/2)*1i)
/6 - 1/6) + 3*a*b*x)*((3^(1/2)*1i)/6 - 1/6))/a^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+\frac{b}{x^3}}\,dx$$
$$= \frac{-2b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)+6a^{\frac{1}{3}}x+b^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)-2b^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)}{6a^{\frac{4}{3}}}$$

input
```
int(1/(a+b/x^3),x)
```

output
```
( - 2*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))
+ 6*a**(1/3)*x + b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/
3)) - 2*b**(1/3)*log(a**(1/3)*x + b**(1/3)))/(6*a**(1/3)*a)
```

3.436 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output

```
-1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3^(1/2)/a^(2/3)/b^(
1/3)-1/3*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/b^(1/3)+1/6*ln(b^(2/3)-a^(1/3)*b^(1
/3)*x+a^(2/3)*x^2)/a^(2/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)+\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

input
```
Integrate[1/((a + b/x^3)*x^2),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3) +
a^(1/3)*x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]])/(6*a^(2/3)*b^
(1/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {795, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{x^3}\right)}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x}{ax^3+b}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \ 16$$

$$\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 217$$

$$\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[1/((a + b/x^3)*x^2),x]
```

output
```
-1/3*Log[b^(1/3) + a^(1/3)*x]/(a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) + Log[b^(2/3) - a^(1/3)*b^(1/3)*
x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*a^(1/3)*b^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{3}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{3a}$	27
default	$-\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}$	91

input
```
int(1/(a+b/x^3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.64

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2a^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2(-a^2b)^{\frac{2}{3}}x^2+(-a^2b)^{\frac{1}{3}}b\right)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}-3(-a^2b)^{\frac{2}{3}}x}{ax^3+b}\right)+(-a^2b)^{\frac{2}{3}}\log\left(a^2x\right.}{6a^2b}\right.$$

input
```
integrate(1/(a+b/x^3)/x^2,x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt((-a^2*b)^(1/3)/b)*log((2*a^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a^2*b)^(2/3)*x^2 + (-a^2*b)^(1/3)*b)*sqrt((-a^2*b)^(1/3
)/b) - 3*(-a^2*b)^(2/3)*x)/(a*x^3 + b)) + (-a^2*b)^(2/3)*log(a^2*x^2 + (-a
^2*b)^(1/3)*a*x + (-a^2*b)^(2/3)) - 2*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1
/3)))/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/
3)*(2*a*x + (-a^2*b)^(1/3))*sqrt(-(-a^2*b)^(1/3)/b)/a) + (-a^2*b)^(2/3)*lo
g(a^2*x^2 + (-a^2*b)^(1/3)*a*x + (-a^2*b)^(2/3)) - 2*(-a^2*b)^(2/3)*log(a*
x - (-a^2*b)^(1/3)))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = \operatorname{RootSum}\left(27t^3a^2b+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

input
```
integrate(1/(a+b/x**3)/x**2,x)
```

output
```
RootSum(27*_t**3*a**2*b + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(a+b/x^3)/x^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^(
1/3)) + 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(1/3)) - 1/3*l
og(x + (b/a)^(1/3))/(a*(b/a)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = -\frac{\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b} - \frac{\sqrt{3}(-a^2b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} + \frac{(-a^2b)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a^2b}$$

input
```
integrate(1/(a+b/x^3)/x^2,x, algorithm="giac")
```

output
```
-1/3*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/b - 1/3*sqrt(3)*(-a^2*b)^(2/3
)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a^2*b) + 1/6*(-a^
2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = \frac{\ln\left(a^{1/3}\,x-(-b)^{1/3}\right)}{3\,a^{2/3}\,(-b)^{1/3}} + \frac{\ln\left(a\,x-\frac{a^{2/3}\,(-b)^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,(-b)^{1/3}} - \frac{\ln\left(a\,x-\frac{a^{2/3}\,(-b)^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,(-b)^{1/3}}$$

input
```
int(1/(x^2*(a + b/x^3)),x)
```

output
```
log(a^(1/3)*x - (-b)^(1/3))/(3*a^(2/3)*(-b)^(1/3)) + (log(a*x - (a^(2/3)*(
-b)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a^(2/3)*(-b)^(1/3))
- (log(a*x - (a^(2/3)*(-b)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/
(6*a^(2/3)*(-b)^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)-2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)}{6b^{\frac{1}{3}}a^{\frac{2}{3}}}$$

input
```
int(1/(a+b/x^3)/x^2,x)
```

output
```
(2*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3))) + log(a**(2/
3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3)) - 2*log(a**(1/3)*x + b**(1/3)))/
(6*b**(1/3)*a**(2/3))
```

3.437 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
-1/3*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3^(1/2)/a^(1/3)/b^(
2/3)+1/3*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/b^(2/3)-1/6*ln(b^(2/3)-a^(1/3)*b^(1
/3)*x+a^(2/3)*x^2)/a^(1/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)+\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[1/((a + b/x^3)*x^3),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3
) + a^(1/3)*x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(a^(1/3)*
b^(2/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {795, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{x^3}\right)}\,dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{1}{ax^3+b}dx$$

$$\Big\downarrow \; 750$$

$$\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3b^{2/3}}$$

$$\Big\downarrow \; 16$$

$$\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \; 1103$$

$$\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[1/((a + b/x^3)*x^3),x]
```

output
```
Log[b^(1/3) + a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*
a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(2/3) - a^(1/3)*b^(1/3)*x +
 a^(2/3)*x^2]/(2*a^(1/3)))/(3*b^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{3}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}}{3a}$	27
default	$\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	91

input
```
int(1/(a+b/x^3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.60

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abx^3-3\,(ab^2)^{\frac{1}{3}}bx-b^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(ab^2)^{\frac{2}{3}}x-(ab^2)^{\frac{1}{3}}b\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ax^3+b}\right)-(ab^2)^{\frac{2}{3}}\log\left(abx^2-\right.\cdots}{6\,ab^2}\right.$$

input
```
integrate(1/(a+b/x^3)/x^3,x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*x^3 - 3*(a*b^2)^(1
/3)*b*x - b^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b
)*sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 + b)) - (a*b^2)^(2/3)*log(a*b*x^2 - (a*b^
2)^(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*
(a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) - (a*b^2)^(2
/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log
(a*b*x + (a*b^2)^(2/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = \operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(3tb+x\right)\right)\right)$$

input
```
integrate(1/(a+b/x**3)/x**3,x)
```

output
```
RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(3*_t*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(a+b/x^3)/x^3,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^(
2/3)) - 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(2/3)) + 1/3*l
og(x + (b/a)^(1/3))/(a*(b/a)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = -\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b} + \frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{(-a^2b)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

input
```
integrate(1/(a+b/x^3)/x^3,x, algorithm="giac")
```

output
```
-1/3*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/b + 1/3*sqrt(3)*(-a^2*b)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a*b) + 1/6*(-a^2*
b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = \frac{\ln\left(a^{1/3}\,x+b^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}} + \frac{\ln\left(3\,a^2\,x+\frac{3\,a^{5/3}\,b^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}} - \frac{\ln\left(3\,a^2\,x-\frac{3\,a^{5/3}\,b^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

input
```
int(1/(x^3*(a + b/x^3)),x)
```

output
```
log(a^(1/3)*x + b^(1/3))/(3*a^(1/3)*b^(2/3)) + (log(3*a^2*x + (3*a^(5/3)*b
^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)) - (log(3
*a^2*x - (3*a^(5/3)*b^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(1
/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)+2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(1/(a+b/x^3)/x^3,x)
```

output
```
(b**(1/3)*(2*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3))) -
log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3)) + 2*log(a**(1/3)*x + b
**(1/3))))/(6*a**(1/3)*b)
```

3.438 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 122

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx = -\frac{1}{bx}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{4/3}}+\frac{\sqrt[3]{a}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6b^{4/3}}$$

output
```
-1/b/x+1/3*a^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3^(1/
2)/b^(4/3)+1/3*a^(1/3)*ln(b^(1/3)+a^(1/3)*x)/b^(4/3)-1/6*a^(1/3)*ln(b^(2/3
)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx = \frac{-6\sqrt[3]{b}+2\sqrt{3}\sqrt[3]{a}x\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)+2\sqrt[3]{a}x\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)-\sqrt[3]{a}x\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6b^{4/3}x}$$

input
```
Integrate[1/((a + b/x^3)*x^5),x]
```

output
```
(-6*b^(1/3) + 2*Sqrt[3]*a^(1/3)*x*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[
3]] + 2*a^(1/3)*x*Log[b^(1/3) + a^(1/3)*x] - a^(1/3)*x*Log[b^(2/3) - a^(1/
3)*b^(1/3)*x + a^(2/3)*x^2])/(6*b^(4/3)*x)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {795, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+\frac{b}{x^3}\right)} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^2 \left(a x^3+b\right)}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{a \int \frac{x}{a x^3+b}dx}{b}-\frac{1}{b x}$$

$$\Big\downarrow \text{821}$$

$$-\frac{a \left(\frac{\int \frac{\sqrt[3]{a} x+\sqrt[3]{b}}{a^{2/3} x^2-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3}}dx}{3 \sqrt[3]{a} \sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{a} x+\sqrt[3]{b}}dx}{3 \sqrt[3]{a} \sqrt[3]{b}}\right)}{b}-\frac{1}{b x}$$

$$\Big\downarrow \text{16}$$

$$-\frac{a \left(\frac{\int \frac{\sqrt[3]{a} x+\sqrt[3]{b}}{a^{2/3} x^2-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3}}dx}{3 \sqrt[3]{a} \sqrt[3]{b}}-\frac{\log \left(\sqrt[3]{a} x+\sqrt[3]{b}\right)}{3 a^{2/3} \sqrt[3]{b}}\right)}{b}-\frac{1}{b x}$$

$\Big\downarrow$ 1142

$$-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}$$

$\Big\downarrow$ 25

$$-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}$$

$\Big\downarrow$ 27

$$-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}$$

$\Big\downarrow$ 1082

$$-\frac{a\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}$$

$\Big\downarrow$ 217

$$-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}$$

$\downarrow$ 1103

$$-\frac{a\left(\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}$$

input
```
Int[1/((a + b/x^3)*x^5),x]
```

output
```
-(1/(b*x)) - (a*(-1/3*Log[b^(1/3) + a^(1/3)*x]/(a^(2/3)*b^(1/3)) + (-((Sqr
t[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) + Log[b^(2/3) -
 a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*a^(1/3)*b^(1/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.43

method	result	size
risch	$-\frac{1}{xb}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{4}\_Z^{3}-a\right)}\_R\ln\left(\left(-4\_R^{3}b^{4}+3a\right)x-b^{3}\_R^{2}\right)\right)}{3}$	53
default	$-\frac{1}{xb}-\frac{\left(-\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)a}{b}$	106

input
```
int(1/(a+b/x^3)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/x/b+1/3*sum(_R*ln((-4*_R^3*b^4+3*a)*x-b^3*_R^2),_R=RootOf(_Z^3*b^4-a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx =$$
$$-\frac{2\sqrt{3}x\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(ax^{2}-bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+b\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-2x\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(ax+b\right.}{6\,bx}$$

input
```
integrate(1/(a+b/x^3)/x^5,x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*(a/b)^(1/3)*arctan(2/3*sqrt(3)*x*(a/b)^(1/3) - 1/3*sqrt(
3)) + x*(a/b)^(1/3)*log(a*x^2 - b*x*(a/b)^(2/3) + b*(a/b)^(1/3)) - 2*x*(a/
b)^(1/3)*log(a*x + b*(a/b)^(2/3)) + 6)/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx = \operatorname{RootSum}\left(27t^3b^4 - a, \left(t \mapsto t\log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x**3)/x**5,x)
```

output
```
RootSum(27*_t**3*b**4 - a, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) - 1/(b*
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2 - x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x^3)/x^5,x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(b*(b/a)^
(1/3)) - 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(b*(b/a)^(1/3)) + 1/3*
log(x + (b/a)^(1/3))/(b*(b/a)^(1/3)) - 1/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx = \frac{a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b^2} + \frac{\sqrt{3}(-a^2b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} - \frac{(-a^2b)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab^2} - \frac{1}{bx}$$

input
```
integrate(1/(a+b/x^3)/x^5,x, algorithm="giac")
```

output
```
1/3*a*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/b^2 + 1/3*sqrt(3)*(-a^2*b)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a*b^2) - 1/6*(
-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/(a*b^2) - 1/(b*x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx = \frac{a^{1/3}\,\ln\left(a^{1/3}\,x+b^{1/3}\right)}{3\,b^{4/3}} - \frac{1}{b\,x} - \frac{a^{1/3}\,\ln\left(4\,a^{1/3}\,x-2\,b^{1/3}+\sqrt{3}\,b^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{4/3}} + \frac{a^{1/3}\,\ln\left(4\,a^{1/3}\,x-2\,b^{1/3}-\sqrt{3}\,b^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{4/3}}$$

input
```
int(1/(x^5*(a + b/x^3)),x)
```

output
```
(a^(1/3)*log(a^(1/3)*x + b^(1/3)))/(3*b^(4/3)) - 1/(b*x) - (a^(1/3)*log(3^
(1/2)*b^(1/3)*2i + 4*a^(1/3)*x - 2*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*b^(
4/3)) + (a^(1/3)*log(4*a^(1/3)*x - 3^(1/2)*b^(1/3)*2i - 2*b^(1/3))*((3^(1/
2)*1i)/6 - 1/6))/b^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)ax-6b^{\frac{1}{3}}a^{\frac{2}{3}}-\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)ax+2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)ax}{6b^{\frac{4}{3}}a^{\frac{2}{3}}x}$$

input
```
int(1/(a+b/x^3)/x^5,x)
```

output
```
( - 2*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*x - 6*b
**(1/3)*a**(2/3) - log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*x
 + 2*log(a**(1/3)*x + b**(1/3))*a*x)/(6*b**(1/3)*a**(2/3)*b*x)
```

3.439 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx=-\frac{1}{2bx^2}+\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{5/3}}-\frac{a^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{3b^{5/3}}+\frac{a^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6b^{5/3}}$$

output
```
-1/2/b/x^2+1/3*a^(2/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3
^(1/2)/b^(5/3)-1/3*a^(2/3)*ln(b^(1/3)+a^(1/3)*x)/b^(5/3)+1/6*a^(2/3)*ln(b^
(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx$$

$$=\frac{-3b^{2/3}+2\sqrt{3}a^{2/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-2a^{2/3}x^2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)+a^{2/3}x^2\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}\right.}{6b^{5/3}x^2}$$

input
```
Integrate[1/((a + b/x^3)*x^6),x]
```

output
```
(-3*b^(2/3) + 2*Sqrt[3]*a^(2/3)*x^2*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqr
t[3]] - 2*a^(2/3)*x^2*Log[b^(1/3) + a^(1/3)*x] + a^(2/3)*x^2*Log[b^(2/3) -
 a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*b^(5/3)*x^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {795, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x^3}\right)} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^3 \left(a x^3+b\right)}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{a \int \frac{1}{a x^3+b}dx}{b}-\frac{1}{2 b x^2}$$

$$\Big\downarrow \text{750}$$

$$-\frac{a \left(\frac{\int \frac{2 \sqrt[3]{b}-\sqrt[3]{a} x}{a^{2/3} x^2-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3}}dx}{3 b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a} x+\sqrt[3]{b}}dx}{3 b^{2/3}}\right)}{b}-\frac{1}{2 b x^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{a \left(\frac{\int \frac{2 \sqrt[3]{b}-\sqrt[3]{a} x}{a^{2/3} x^2-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3}}dx}{3 b^{2/3}}+\frac{\log \left(\sqrt[3]{a} x+\sqrt[3]{b}\right)}{3 \sqrt[3]{a} b^{2/3}}\right)}{b}-\frac{1}{2 b x^2}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{1}{2bx^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{1}{2bx^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{1}{2bx^2}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{1}{2bx^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{1}{2bx^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{1}{2bx^2}$$

input
```
Int[1/((a + b/x^3)*x^6),x]
```

output
```
-1/2*1/(b*x^2) - (a*(Log[b^(1/3) + a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + (-((Sq
rt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(2/3)
- a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*b^(2/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{1}{2b\,x^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{5}\_Z^{3}+a^{2}\right)}\_R\ln\left(\left(-4\_R^{3}b^{5}-3a^{2}\right)x-a\,b^{2}\_R\right)\right)}{3}$	54
default	$-\frac{1}{2b\,x^{2}}-\frac{\left(\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)a}{b}$	106

input
```
int(1/(a+b/x^3)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/x^2+1/3*sum(_R*ln((-4*_R^3*b^5-3*a^2)*x-a*b^2*_R),_R=RootOf(_Z^3*b^
5+a^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx$$
$$=\frac{2\sqrt{3}x^2\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-x^2\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(a^2x^2+abx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+b^2\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)+2\,x^2}{6\,bx^2}$$

input
```
integrate(1/(a+b/x^3)/x^6,x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*x^2*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(
2/3) - sqrt(3)*a)/a) - x^2*(-a^2/b^2)^(1/3)*log(a^2*x^2 + a*b*x*(-a^2/b^2)
^(1/3) + b^2*(-a^2/b^2)^(2/3)) + 2*x^2*(-a^2/b^2)^(1/3)*log(a*x - b*(-a^2/
b^2)^(1/3)) - 3)/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx = \operatorname{RootSum}\left(27t^3b^5+a^2,\left(t\mapsto t\log\left(-\frac{3tb^2}{a}+x\right)\right)\right)-\frac{1}{2bx^2}$$

input
```
integrate(1/(a+b/x**3)/x**6,x)
```

output
```
RootSum(27*_t**3*b**5 + a**2, Lambda(_t, _t*log(-3*_t*b**2/a + x))) - 1/(2
*b*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{1}{2\,bx^2}$$

input
```
integrate(1/(a+b/x^3)/x^6,x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(b*(b/a)^
(2/3)) + 1/6*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3))/(b*(b/a)^(2/3)) - 1/3*
log(x + (b/a)^(1/3))/(b*(b/a)^(2/3)) - 1/2/(b*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx = \frac{a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b^2} - \frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,b^2} - \frac{(-a^2b)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,b^2} - \frac{1}{2\,bx^2}$$

input
```
integrate(1/(a+b/x^3)/x^6,x, algorithm="giac")
```

output
```
1/3*a*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/b^2 - 1/3*sqrt(3)*(-a^2*b)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/b^2 - 1/6*(-a^2
*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/3))/b^2 - 1/2/(b*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^6}\,dx = \frac{a^{2/3}\,\ln\left(\left(-b\right)^{7/3}-a^{1/3}\,b^2\,x\right)}{3\,\left(-b\right)^{5/3}} - \frac{1}{2\,b\,x^2} - \frac{a^{2/3}\,\ln\left(3\,a^3\,b^2\,x+3\,a^{8/3}\,\left(-b\right)^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,\left(-b\right)^{5/3}} + \frac{a^{2/3}\,\ln\left(3\,a^3\,b^2\,x-9\,a^{8/3}\,\left(-b\right)^{7/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{\left(-b\right)^{5/3}}$$

input
```
int(1/(x^6*(a + b/x^3)),x)
```

output
```
(a^(2/3)*log((-b)^(7/3) - a^(1/3)*b^2*x))/(3*(-b)^(5/3)) - 1/(2*b*x^2) - (
a^(2/3)*log(3*a^3*b^2*x + 3*a^(8/3)*(-b)^(7/3)*((3^(1/2)*1i)/2 + 1/2))*((3
^(1/2)*1i)/2 + 1/2))/(3*(-b)^(5/3)) + (a^(2/3)*log(3*a^3*b^2*x - 9*a^(8/3)
*(-b)^(7/3)*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6))/(-b)^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right) x^6} \, dx = \frac{-2b^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right) a\, x^2 - 3a^{\frac{1}{3}}b + b^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}x^2 - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}\right) a\, x^2 - 2b^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}x + b^{\frac{1}{3}}\right) a\, x^2}{6a^{\frac{1}{3}}b^2x^2}$$

input
```
int(1/(a+b/x^3)/x^6,x)
```

output
```
( - 2*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*
a*x**2 - 3*a**(1/3)*b + b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x +
 b**(2/3))*a*x**2 - 2*b**(1/3)*log(a**(1/3)*x + b**(1/3))*a*x**2)/(6*a**(1
/3)*b**2*x**2)
```

3.440 $\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2} dx = -\frac{2bx^3}{3a^3} + \frac{x^6}{6a^2} + \frac{b^3}{3a^4(b+ax^3)} + \frac{b^2 \log(b+ax^3)}{a^4}$$

output
```
-2/3*b*x^3/a^3+1/6*x^6/a^2+1/3*b^3/a^4/(a*x^3+b)+b^2*ln(a*x^3+b)/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2} dx = \frac{-4abx^3 + a^2x^6 + \frac{2b^3}{b+ax^3} + 6b^2 \log(b+ax^3)}{6a^4}$$

input
```
Integrate[x^5/(a + b/x^3)^2,x]
```

output
```
(-4*a*b*x^3 + a^2*x^6 + (2*b^3)/(b + a*x^3) + 6*b^2*Log[b + a*x^3])/(6*a^4
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{x^{11}}{\left(ax^3+b\right)^2}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{x^9}{\left(ax^3+b\right)^2}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\int \left(-\frac{b^3}{a^3\left(ax^3+b\right)^2}+\frac{3b^2}{a^3\left(ax^3+b\right)}-\frac{2b}{a^3}+\frac{x^3}{a^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{b^3}{a^4\left(ax^3+b\right)}+\frac{3b^2\log\left(ax^3+b\right)}{a^4}-\frac{2bx^3}{a^3}+\frac{x^6}{2a^2}\right)$$

input
```
Int[x^5/(a + b/x^3)^2,x]
```

output
```
((-2*b*x^3)/a^3 + x^6/(2*a^2) + b^3/(a^4*(b + a*x^3)) + (3*b^2*Log[b + a*x
^3])/a^4)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

method	result	size
norman	$\frac{\frac{b^3}{a^4}+\frac{x^9}{6a}-\frac{b\,x^6}{2a^2}}{a\,x^3+b}+\frac{b^2\ln\left(a\,x^3+b\right)}{a^4}$	52
default	$\frac{\left(a\,x^3-2b\right)^2}{6a^4}+\frac{b^2\left(\frac{b}{a\left(a\,x^3+b\right)}+\frac{3\ln\left(a\,x^3+b\right)}{a}\right)}{3a^3}$	54
risch	$\frac{x^6}{6a^2}-\frac{2b\,x^3}{3a^3}+\frac{2b^2}{3a^4}+\frac{b^3}{3a^4\left(a\,x^3+b\right)}+\frac{b^2\ln\left(a\,x^3+b\right)}{a^4}$	59
parallelrisch	$\frac{a^3x^9-3a^2b\,x^6+6\ln\left(a\,x^3+b\right)x^3a\,b^2+6b^3\ln\left(a\,x^3+b\right)+6b^3}{6a^4\left(a\,x^3+b\right)}$	67

input
```
int(x^5/(a+b/x^3)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^3/a^4+1/6/a*x^9-1/2*b/a^2*x^6)/(a*x^3+b)+b^2*ln(a*x^3+b)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{a^3x^9 - 3\,a^2bx^6 - 4\,ab^2x^3 + 2\,b^3 + 6\,(ab^2x^3 + b^3)\log\left(ax^3 + b\right)}{6\,(a^5x^3 + a^4b)}$$

input
```
integrate(x^5/(a+b/x^3)^2,x, algorithm="fricas")
```

output
```
1/6*(a^3*x^9 - 3*a^2*b*x^6 - 4*a*b^2*x^3 + 2*b^3 + 6*(a*b^2*x^3 + b^3)*log
(a*x^3 + b))/(a^5*x^3 + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{b^3}{3a^5x^3 + 3a^4b} + \frac{x^6}{6a^2} - \frac{2bx^3}{3a^3} + \frac{b^2\log\left(ax^3 + b\right)}{a^4}$$

input
```
integrate(x**5/(a+b/x**3)**2,x)
```

output
```
b**3/(3*a**5*x**3 + 3*a**4*b) + x**6/(6*a**2) - 2*b*x**3/(3*a**3) + b**2*l
og(a*x**3 + b)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{b^3}{3\,(a^5x^3 + a^4b)} + \frac{b^2\log\left(ax^3 + b\right)}{a^4} + \frac{ax^6 - 4\,bx^3}{6\,a^3}$$

input
```
integrate(x^5/(a+b/x^3)^2,x, algorithm="maxima")
```

output
```
1/3*b^3/(a^5*x^3 + a^4*b) + b^2*log(a*x^3 + b)/a^4 + 1/6*(a*x^6 - 4*b*x^3)
/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{b^2\log\left(|ax^3+b|\right)}{a^4} + \frac{b^3}{3\,(ax^3+b)a^4} + \frac{a^2x^6-4\,abx^3}{6\,a^4}$$

input
```
integrate(x^5/(a+b/x^3)^2,x, algorithm="giac")
```

output
```
b^2*log(abs(a*x^3 + b))/a^4 + 1/3*b^3/((a*x^3 + b)*a^4) + 1/6*(a^2*x^6 - 4
*a*b*x^3)/a^4
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{x^6}{6\,a^2} + \frac{b^3}{3\,a\,\left(a^4\,x^3+b\,a^3\right)} - \frac{2\,b\,x^3}{3\,a^3} + \frac{b^2\,\ln\left(a\,x^3+b\right)}{a^4}$$

input
```
int(x^5/(a + b/x^3)^2,x)
```

output
```
x^6/(6*a^2) + b^3/(3*a*(a^3*b + a^4*x^3)) - (2*b*x^3)/(3*a^3) + (b^2*log(b
 + a*x^3))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.32

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^2}\,dx$$
$$= \frac{6\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a\,b^2x^3+6\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)b^3+6\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)a\,b^2x^3+6\log\left(a^{\frac{1}{3}}x\right.}{6a^4\left(a\,x^3+b\right)}$$

input
```
int(x^5/(a+b/x^3)^2,x)
```

output
```
(6*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*b**2*x**3 + 6*log
(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b**3 + 6*log(a**(1/3)*x +
 b**(1/3))*a*b**2*x**3 + 6*log(a**(1/3)*x + b**(1/3))*b**3 + a**3*x**9 - 3
*a**2*b*x**6 - 6*a*b**2*x**3)/(6*a**4*(a*x**3 + b))
```

3.441 $\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2} dx = \frac{x^3}{3a^2} - \frac{b^2}{3a^3(b+ax^3)} - \frac{2b\log(b+ax^3)}{3a^3}$$

output
```
1/3*x^3/a^2-1/3*b^2/a^3/(a*x^3+b)-2/3*b*ln(a*x^3+b)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2} dx = \frac{ax^3 - \frac{b^2}{b+ax^3} - 2b\log(b+ax^3)}{3a^3}$$

input
```
Integrate[x^2/(a + b/x^3)^2,x]
```

output
```
(a*x^3 - b^2/(b + a*x^3) - 2*b*Log[b + a*x^3])/(3*a^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^8}{(ax^3+b)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{(ax^3+b)^2}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{b^2}{a^2(ax^3+b)^2}-\frac{2b}{a^2(ax^3+b)}+\frac{1}{a^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{b^2}{a^3(ax^3+b)}-\frac{2b\log(ax^3+b)}{a^3}+\frac{x^3}{a^2}\right)$$

input
```
Int[x^2/(a + b/x^3)^2,x]
```

output
```
(x^3/a^2 - b^2/(a^3*(b + a*x^3)) - (2*b*Log[b + a*x^3])/a^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x^{3}}{3a^{2}}-\frac{b^{2}}{3a^{3}\left(a\,x^{3}+b\right)}-\frac{2b\ln\left(a\,x^{3}+b\right)}{3a^{3}}$	41
norman	$\frac{\frac{x^{6}}{3a}-\frac{2b^{2}}{3a^{3}}}{a\,x^{3}+b}-\frac{2b\ln\left(a\,x^{3}+b\right)}{3a^{3}}$	43
default	$\frac{x^{3}}{3a^{2}}-\frac{b\left(\frac{b}{a\left(a\,x^{3}+b\right)}+\frac{2\ln\left(a\,x^{3}+b\right)}{a}\right)}{3a^{2}}$	44
parallelrisch	$-\frac{-a^{2}x^{6}+2\ln\left(a\,x^{3}+b\right)x^{3}ab+2b^{2}\ln\left(a\,x^{3}+b\right)+2b^{2}}{3a^{3}\left(a\,x^{3}+b\right)}$	57

input
```
int(x^2/(a+b/x^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*x^3-1/3*b^2/a^3/(a*x^3+b)-2/3*b*ln(a*x^3+b)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2} \, dx = \frac{a^2x^6 + abx^3 - b^2 - 2\left(abx^3 + b^2\right)\log\left(ax^3 + b\right)}{3\left(a^4x^3 + a^3b\right)}$$

input
```
integrate(x^2/(a+b/x^3)^2,x, algorithm="fricas")
```

output
```
1/3*(a^2*x^6 + a*b*x^3 - b^2 - 2*(a*b*x^3 + b^2)*log(a*x^3 + b))/(a^4*x^3
+ a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2} \, dx = -\frac{b^2}{3a^4x^3 + 3a^3b} + \frac{x^3}{3a^2} - \frac{2b\log\left(ax^3 + b\right)}{3a^3}$$

input
```
integrate(x**2/(a+b/x**3)**2,x)
```

output
```
-b**2/(3*a**4*x**3 + 3*a**3*b) + x**3/(3*a**2) - 2*b*log(a*x**3 + b)/(3*a*
*3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2} \, dx = -\frac{b^2}{3\left(a^4x^3 + a^3b\right)} + \frac{x^3}{3\,a^2} - \frac{2\,b\log\left(ax^3 + b\right)}{3\,a^3}$$

input
```
integrate(x^2/(a+b/x^3)^2,x, algorithm="maxima")
```

output
```
-1/3*b^2/(a^4*x^3 + a^3*b) + 1/3*x^3/a^2 - 2/3*b*log(a*x^3 + b)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{x^3}{3\,a^2} - \frac{2\,b\log\left(|ax^3+b|\right)}{3\,a^3} - \frac{b^2}{3\,(ax^3+b)a^3}$$

input
```
integrate(x^2/(a+b/x^3)^2,x, algorithm="giac")
```

output
```
1/3*x^3/a^2 - 2/3*b*log(abs(a*x^3 + b))/a^3 - 1/3*b^2/((a*x^3 + b)*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{x^3}{3\,a^2} - \frac{b^2}{3\,\left(a^4\,x^3+b\,a^3\right)} - \frac{2\,b\,\ln\left(a\,x^3+b\right)}{3\,a^3}$$

input
```
int(x^2/(a + b/x^3)^2,x)
```

output
```
x^3/(3*a^2) - b^2/(3*(a^3*b + a^4*x^3)) - (2*b*log(b + a*x^3))/(3*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{-2\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)ab\,x^3-2\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)b^2-2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)ab\,x^3-2\log\left(a^{\frac{1}{3}}x\right.}{3a^3\left(a\,x^3+b\right)}$$

input
```
int(x^2/(a+b/x^3)^2,x)
```

output
```
( - 2*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*b*x**3 - 2*log
(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b**2 - 2*log(a**(1/3)*x +
 b**(1/3))*a*b*x**3 - 2*log(a**(1/3)*x + b**(1/3))*b**2 + a**2*x**6 + 2*a*
b*x**3)/(3*a**3*(a*x**3 + b))
```

3.442 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x} \, dx = \frac{b}{3a^2 (b+ax^3)} + \frac{\log (b+ax^3)}{3a^2}$$

output
```
1/3*b/a^2/(a*x^3+b)+1/3*ln(a*x^3+b)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x} \, dx = \frac{\frac{b}{b+ax^3} + \log (b+ax^3)}{3a^2}$$

input
```
Integrate[1/((a + b/x^3)^2*x),x]
```

output
```
(b/(b + a*x^3) + Log[b + a*x^3])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^3}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^5}{(ax^3+b)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{(ax^3+b)^2}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{1}{a(ax^3+b)}-\frac{b}{a(ax^3+b)^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{b}{a^2(ax^3+b)}+\frac{\log(ax^3+b)}{a^2}\right)$$

input
```
Int[1/((a + b/x^3)^2*x),x]
```

output
```
(b/(a^2*(b + a*x^3)) + Log[b + a*x^3]/a^2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b}{3a^{2}\left(a\,x^{3}+b\right)}+\frac{\ln\left(a\,x^{3}+b\right)}{3a^{2}}$	30
norman	$\frac{b}{3a^{2}\left(a\,x^{3}+b\right)}+\frac{\ln\left(a\,x^{3}+b\right)}{3a^{2}}$	30
risch	$\frac{b}{3a^{2}\left(a\,x^{3}+b\right)}+\frac{\ln\left(a\,x^{3}+b\right)}{3a^{2}}$	30
parallelrisch	$\frac{a\ln\left(a\,x^{3}+b\right)x^{3}+b\ln\left(a\,x^{3}+b\right)+b}{3a^{2}\left(a\,x^{3}+b\right)}$	40

input
```
int(1/(a+b/x^3)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*b/a^2/(a*x^3+b)+1/3*ln(a*x^3+b)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx = \frac{(ax^3+b)\log(ax^3+b)+b}{3(a^3x^3+a^2b)}$$

input
```
integrate(1/(a+b/x^3)^2/x,x, algorithm="fricas")
```

output
```
1/3*((a*x^3 + b)*log(a*x^3 + b) + b)/(a^3*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx = \frac{b}{3a^3x^3+3a^2b}+\frac{\log(ax^3+b)}{3a^2}$$

input
```
integrate(1/(a+b/x**3)**2/x,x)
```

output
```
b/(3*a**3*x**3 + 3*a**2*b) + log(a*x**3 + b)/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx = \frac{b}{3(a^3x^3+a^2b)}+\frac{\log(ax^3+b)}{3\,a^2}$$

input
```
integrate(1/(a+b/x^3)^2/x,x, algorithm="maxima")
```

output
```
1/3*b/(a^3*x^3 + a^2*b) + 1/3*log(a*x^3 + b)/a^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx = -\frac{x^3}{3\,(ax^3+b)a} + \frac{\log\left(|ax^3+b|\right)}{3\,a^2}$$

input
```
integrate(1/(a+b/x^3)^2/x,x, algorithm="giac")
```

output
```
-1/3*x^3/((a*x^3 + b)*a) + 1/3*log(abs(a*x^3 + b))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx = \frac{\ln\left(a\,x^3+b\right)}{3\,a^2} + \frac{b}{3\,a^2\,\left(a\,x^3+b\right)}$$

input
```
int(1/(x*(a + b/x^3)^2),x)
```

output
```
log(b + a*x^3)/(3*a^2) + b/(3*a^2*(b + a*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.94

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x}\,dx$$
$$= \frac{\log\left(a^{\frac{2}{3}}x^2 - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}\right) a\,x^3 + \log\left(a^{\frac{2}{3}}x^2 - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}\right) b + \log\left(a^{\frac{1}{3}}x + b^{\frac{1}{3}}\right) a\,x^3 + \log\left(a^{\frac{1}{3}}x + b^{\frac{1}{3}}\right) b -}{3a^2\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^2/x,x)
```

output
```
(log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*x**3 + log(a**(2/3)
*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b + log(a**(1/3)*x + b**(1/3))*a*x
**3 + log(a**(1/3)*x + b**(1/3))*b - a*x**3)/(3*a**2*(a*x**3 + b))
```

3.443 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4} dx = \frac{1}{3b\left(a+\frac{b}{x^3}\right)}$$

output
```
1/3/b/(a+b/x^3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4} dx = -\frac{1}{3a\left(b+ax^3\right)}$$

input
```
Integrate[1/((a + b/x^3)^2*x^4),x]
```

output
```
-1/3*1/(a*(b + a*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^3}\right)^2} \, dx$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{3b\left(a+\frac{b}{x^3}\right)}$$

input
```
Int[1/((a + b/x^3)^2*x^4),x]
```

output
```
1/(3*b*(a + b/x^3))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{3(a\,x^3+b)a}$	15
derivativedivides	$\frac{1}{3b\left(a+\frac{b}{x^3}\right)}$	15
default	$-\frac{1}{3(a\,x^3+b)a}$	15
norman	$-\frac{1}{3(a\,x^3+b)a}$	15
risch	$-\frac{1}{3(a\,x^3+b)a}$	15
parallelrisch	$-\frac{1}{3(a\,x^3+b)a}$	15
orering	$-\frac{a\,x^3+b}{3a\,x^6\left(a+\frac{b}{x^3}\right)^2}$	25

input
```
int(1/(a+b/x^3)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a*x^3+b)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4}\,dx = -\frac{1}{3\left(a^2x^3+ab\right)}$$

input
```
integrate(1/(a+b/x^3)^2/x^4,x, algorithm="fricas")
```

output
```
-1/3/(a^2*x^3 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4} \, dx = -\frac{1}{3a^2x^3 + 3ab}$$

input
```
integrate(1/(a+b/x**3)**2/x**4,x)
```

output
```
-1/(3*a**2*x**3 + 3*a*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4} \, dx = \frac{1}{3\left(a+\frac{b}{x^3}\right)b}$$

input
```
integrate(1/(a+b/x^3)^2/x^4,x, algorithm="maxima")
```

output
```
1/3/((a + b/x^3)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4} \, dx = -\frac{1}{3\left(ax^3+b\right)a}$$

input
```
integrate(1/(a+b/x^3)^2/x^4,x, algorithm="giac")
```

output
```
-1/3/((a*x^3 + b)*a)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4}\,dx = -\frac{1}{3\,a\,\left(a\,x^3+b\right)}$$

input
```
int(1/(x^4*(a + b/x^3)^2),x)
```

output
```
-1/(3*a*(b + a*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^4}\,dx = \frac{x^3}{3b\,(a\,x^3+b)}$$

input
```
int(1/(a+b/x^3)^2/x^4,x)
```

output
```
x**3/(3*b*(a*x**3 + b))
```

3.444 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7} \, dx = -\frac{a}{3b^2\left(a+\frac{b}{x^3}\right)}-\frac{\log\left(a+\frac{b}{x^3}\right)}{3b^2}$$

output
```
-1/3*a/b^2/(a+b/x^3)-1/3*ln(a+b/x^3)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7} \, dx = \frac{\frac{b}{b+ax^3}+3\log(x)-\log\left(b+ax^3\right)}{3b^2}$$

input
```
Integrate[1/((a + b/x^3)^2*x^7),x]
```

output
```
(b/(b + a*x^3) + 3*Log[x] - Log[b + a*x^3])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^3}\right)^2} \, dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{1}{x \left(a x^3+b\right)^2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3} \int \frac{1}{x^3 \left(a x^3+b\right)^2}dx^3$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{3} \int \left(-\frac{a}{b^2 \left(a x^3+b\right)}-\frac{a}{b \left(a x^3+b\right)^2}+\frac{1}{b^2 x^3}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3} \left(-\frac{\log \left(a x^3+b\right)}{b^2}+\frac{1}{b \left(a x^3+b\right)}+\frac{\log \left(x^3\right)}{b^2}\right)$$

input
```
Int[1/((a + b/x^3)^2*x^7),x]
```

output
```
(1/(b*(b + a*x^3)) + Log[x^3]/b^2 - Log[b + a*x^3]/b^2)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
risch	$\frac{1}{3b(a\,x^{3}+b)}+\frac{\ln(x)}{b^{2}}-\frac{\ln(a\,x^{3}+b)}{3b^{2}}$	35
norman	$-\frac{a\,x^{3}}{3b^{2}(a\,x^{3}+b)}+\frac{\ln(x)}{b^{2}}-\frac{\ln(a\,x^{3}+b)}{3b^{2}}$	39
default	$\frac{\ln(x)}{b^{2}}-\frac{a\left(-\frac{b}{a(a\,x^{3}+b)}+\frac{\ln(a\,x^{3}+b)}{a}\right)}{3b^{2}}$	42
parallelrisch	$\frac{3a\ln(x)x^{3}-a\ln(a\,x^{3}+b)x^{3}-a\,x^{3}+3b\ln(x)-b\ln(a\,x^{3}+b)}{3b^{2}(a\,x^{3}+b)}$	60

input
```
int(1/(a+b/x^3)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/3/b/(a*x^3+b)+1/b^2*ln(x)-1/3/b^2*ln(a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx = -\frac{(ax^3+b)\log(ax^3+b) - 3\,(ax^3+b)\log(x) - b}{3\,(ab^2x^3+b^3)}$$

input
```
integrate(1/(a+b/x^3)^2/x^7,x, algorithm="fricas")
```

output
```
-1/3*((a*x^3 + b)*log(a*x^3 + b) - 3*(a*x^3 + b)*log(x) - b)/(a*b^2*x^3 +
b^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx = \frac{1}{3abx^3+3b^2} + \frac{\log(x)}{b^2} - \frac{\log\left(x^3+\frac{b}{a}\right)}{3b^2}$$

input
```
integrate(1/(a+b/x**3)**2/x**7,x)
```

output
```
1/(3*a*b*x**3 + 3*b**2) + log(x)/b**2 - log(x**3 + b/a)/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx = \frac{1}{3\,(abx^3+b^2)} - \frac{\log(ax^3+b)}{3\,b^2} + \frac{\log(x^3)}{3\,b^2}$$

input
```
integrate(1/(a+b/x^3)^2/x^7,x, algorithm="maxima")
```

output
```
1/3/(a*b*x^3 + b^2) - 1/3*log(a*x^3 + b)/b^2 + 1/3*log(x^3)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx = -\frac{\log\left(|ax^3+b|\right)}{3\,b^2} + \frac{\log\left(|x|\right)}{b^2} + \frac{ax^3+2\,b}{3\,(ax^3+b)b^2}$$

input
```
integrate(1/(a+b/x^3)^2/x^7,x, algorithm="giac")
```

output
```
-1/3*log(abs(a*x^3 + b))/b^2 + log(abs(x))/b^2 + 1/3*(a*x^3 + 2*b)/((a*x^3
 + b)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx = \frac{\ln(x)}{b^2} + \frac{1}{3\,b\,\left(a\,x^3+b\right)} - \frac{\ln\left(a\,x^3+b\right)}{3\,b^2}$$

input
```
int(1/(x^7*(a + b/x^3)^2),x)
```

output
```
log(x)/b^2 + 1/(3*b*(b + a*x^3)) - log(b + a*x^3)/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.45

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^7}\,dx$$
$$= \frac{-\log\left(a^{\frac{2}{3}}x^2 - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)a\,x^3 - \log\left(a^{\frac{2}{3}}x^2 - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}\right)b - \log\left(a^{\frac{1}{3}}x + b^{\frac{1}{3}}\right)a\,x^3 - \log\left(a^{\frac{1}{3}}x + b^{\frac{1}{3}}\right)b}{3b^2\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^2/x^7,x)
```

output

```
( - log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*x**3 - log(a**(2
/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b - log(a**(1/3)*x + b**(1/3))*
a*x**3 - log(a**(1/3)*x + b**(1/3))*b + 3*log(x)*a*x**3 + 3*log(x)*b - a*x
**3)/(3*b**2*(a*x**3 + b))
```

3.445 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx = \frac{a^2}{3b^3\left(a+\frac{b}{x^3}\right)} - \frac{1}{3b^2x^3} + \frac{2a\log\left(a+\frac{b}{x^3}\right)}{3b^3}$$

output
```
1/3*a^2/b^3/(a+b/x^3)-1/3/b^2/x^3+2/3*a*ln(a+b/x^3)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx = -\frac{b\left(\frac{1}{x^3}+\frac{a}{b+ax^3}\right)+6a\log(x)-2a\log\left(b+ax^3\right)}{3b^3}$$

input
```
Integrate[1/((a + b/x^3)^2*x^10),x]
```

output
```
-1/3*(b*(x^(-3) + a/(b + a*x^3)) + 6*a*Log[x] - 2*a*Log[b + a*x^3])/b^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} \left(a+\frac{b}{x^3}\right)^2} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^4 \left(a x^3+b\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{1}{x^6 \left(a x^3+b\right)^2}dx^3$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3} \int \left(\frac{2 a^2}{b^3 \left(a x^3+b\right)}+\frac{a^2}{b^2 \left(a x^3+b\right)^2}-\frac{2 a}{b^3 x^3}+\frac{1}{b^2 x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2 a \log \left(x^3\right)}{b^3}+\frac{2 a \log \left(a x^3+b\right)}{b^3}-\frac{a}{b^2 \left(a x^3+b\right)}-\frac{1}{b^2 x^3}\right)$$

input
```
Int[1/((a + b/x^3)^2*x^10),x]
```

output
```
(-(1/(b^2*x^3)) - a/(b^2*(b + a*x^3)) - (2*a*Log[x^3])/b^3 + (2*a*Log[b +
a*x^3])/b^3)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{1}{3b^{2}x^{3}}-\frac{2a\ln \left(x \right)}{b^{3}}+\frac{a^{2} \left(-\frac{b}{a \left(a \,x^{3}+b \right)}+\frac{2\ln \left(a \,x^{3}+b \right)}{a}\right)}{3b^{3}}$	55
risch	$\frac{-\frac{2a \,x^{3}}{3b^{2}}-\frac{1}{3b}}{\left(a \,x^{3}+b \right) x^{3}}-\frac{2a\ln \left(x \right)}{b^{3}}+\frac{2a\ln \left(-a \,x^{3}-b \right)}{3b^{3}}$	55
norman	$\frac{-\frac{x^{6}}{3b}+\frac{2a^{2}x^{12}}{3b^{3}}}{x^{9} \left(a \,x^{3}+b \right)}-\frac{2a\ln \left(x \right)}{b^{3}}+\frac{2a\ln \left(a \,x^{3}+b \right)}{3b^{3}}$	57
parallelrisch	$-\frac{6a^{2}\ln \left(x \right) x^{6}-2a^{2}\ln \left(a \,x^{3}+b \right) x^{6}-2a^{2}x^{6}+6\ln \left(x \right) x^{3}ab-2\ln \left(a \,x^{3}+b \right) x^{3}ab+b^{2}}{3b^{3}x^{3} \left(a \,x^{3}+b \right)}$	80

input
```
int(1/(a+b/x^3)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2/x^3-2/b^3*a*ln(x)+1/3*a^2/b^3*(-b/a/(a*x^3+b)+2*ln(a*x^3+b)/a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx = -\frac{2\,abx^3+b^2-2\left(a^2x^6+abx^3\right)\log\left(ax^3+b\right)+6\left(a^2x^6+abx^3\right)\log\left(x\right)}{3\left(ab^3x^6+b^4x^3\right)}$$

input
```
integrate(1/(a+b/x^3)^2/x^10,x, algorithm="fricas")
```

output
```
-1/3*(2*a*b*x^3 + b^2 - 2*(a^2*x^6 + a*b*x^3)*log(a*x^3 + b) + 6*(a^2*x^6
+ a*b*x^3)*log(x))/(a*b^3*x^6 + b^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx = -\frac{2a\log\left(x\right)}{b^3}+\frac{2a\log\left(x^3+\frac{b}{a}\right)}{3b^3}+\frac{-2ax^3-b}{3ab^2x^6+3b^3x^3}$$

input
```
integrate(1/(a+b/x**3)**2/x**10,x)
```

output
```
-2*a*log(x)/b**3 + 2*a*log(x**3 + b/a)/(3*b**3) + (-2*a*x**3 - b)/(3*a*b**
2*x**6 + 3*b**3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx = -\frac{2\,ax^3+b}{3\left(ab^2x^6+b^3x^3\right)}+\frac{2\,a\log\left(ax^3+b\right)}{3\,b^3}-\frac{2\,a\log\left(x^3\right)}{3\,b^3}$$

input
```
integrate(1/(a+b/x^3)^2/x^10,x, algorithm="maxima")
```

output
```
-1/3*(2*a*x^3 + b)/(a*b^2*x^6 + b^3*x^3) + 2/3*a*log(a*x^3 + b)/b^3 - 2/3*
a*log(x^3)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx = \frac{2\,a\log\left(|ax^3+b|\right)}{3\,b^3} - \frac{2\,a\log\left(|x|\right)}{b^3} - \frac{2\,ax^3+b}{3\,(ax^6+bx^3)b^2}$$

input
```
integrate(1/(a+b/x^3)^2/x^10,x, algorithm="giac")
```

output
```
2/3*a*log(abs(a*x^3 + b))/b^3 - 2*a*log(abs(x))/b^3 - 1/3*(2*a*x^3 + b)/((
a*x^6 + b*x^3)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx = \frac{2\,a\,\ln\left(a\,x^3+b\right)}{3\,b^3} - \frac{\frac{1}{3\,b}+\frac{2\,a\,x^3}{3\,b^2}}{a\,x^6+b\,x^3} - \frac{2\,a\,\ln\left(x\right)}{b^3}$$

input
```
int(1/(x^10*(a + b/x^3)^2),x)
```

output
```
(2*a*log(b + a*x^3))/(3*b^3) - (1/(3*b) + (2*a*x^3)/(3*b^2))/(a*x^6 + b*x^
3) - (2*a*log(x))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.09

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^{10}}\,dx$$
$$= \frac{2\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a^2x^6+2\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)ab\,x^3+2\log\left(a^{\frac{1}{3}}x+b^{\frac{1}{3}}\right)a^2x^6+2\log\left(a^{\frac{1}{3}}x\right.}{3b^3x^3\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^2/x^10,x)
```

output
```
(2*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a**2*x**6 + 2*log(a
**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*b*x**3 + 2*log(a**(1/3)*x
 + b**(1/3))*a**2*x**6 + 2*log(a**(1/3)*x + b**(1/3))*a*b*x**3 - 6*log(x)*
a**2*x**6 - 6*log(x)*a*b*x**3 + 2*a**2*x**6 - b**2)/(3*b**3*x**3*(a*x**3 +
 b))
```

3.446 $\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 154

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx=-\frac{2bx}{a^3}+\frac{x^4}{4a^2}-\frac{b^2x}{3a^3(b+ax^3)}-\frac{7b^{4/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}a^{10/3}}+\frac{7b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9a^{10/3}}-\frac{7b^{4/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{18a^{10/3}}$$

output
```
-2*b*x/a^3+1/4*x^4/a^2-1/3*b^2*x/a^3/(a*x^3+b)-7/9*b^(4/3)*arctan(1/3*(b^(
1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3^(1/2)/a^(10/3)+7/9*b^(4/3)*ln(b^(1/3)
+a^(1/3)*x)/a^(10/3)-7/18*b^(4/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2
)/a^(10/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{-72\sqrt[3]{a}bx+9a^{4/3}x^4-\frac{12\sqrt[3]{a}b^2x}{b+ax^3}-28\sqrt{3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)+28b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)-14b^{4/3}\log\left(\right.}{36a^{10/3}}$$

input
```
Integrate[x^3/(a + b/x^3)^2,x]
```

output
```
(-72*a^(1/3)*b*x + 9*a^(4/3)*x^4 - (12*a^(1/3)*b^2*x)/(b + a*x^3) - 28*Sqr
t[3]*b^(4/3)*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] + 28*b^(4/3)*Log[
b^(1/3) + a^(1/3)*x] - 14*b^(4/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3
)*x^2])/(36*a^(10/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 817, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{x^9}{(ax^3+b)^2}dx$$

$$\Big\downarrow \; 817$$

$$\frac{7\int \frac{x^6}{ax^3+b}dx}{3a}-\frac{x^7}{3a(ax^3+b)}$$

$$\Big\downarrow \; 831$$

$$\frac{7\int \left(\frac{x^3}{a}-\frac{b}{a^2}+\frac{b^2}{a^2(ax^3+b)}\right)dx}{3a}-\frac{x^7}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{7\left(-\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{7/3}}-\frac{b^{4/3}\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{6a^{7/3}}+\frac{b^{4/3}\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{7/3}}-\frac{bx}{a^2}+\frac{x^4}{4a}\right)}{3a}-\frac{x^7}{3a\left(ax^3+b\right)}$$

input
```
Int[x^3/(a + b/x^3)^2,x]
```

output
```
-1/3*x^7/(a*(b + a*x^3)) + (7*(-((b*x)/a^2) + x^4/(4*a) - (b^(4/3)*ArcTan[
(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(Sqrt[3]*a^(7/3)) + (b^(4/3)*L
og[b^(1/3) + a^(1/3)*x])/(3*a^(7/3)) - (b^(4/3)*Log[b^(2/3) - a^(1/3)*b^(1
/3)*x + a^(2/3)*x^2])/(6*a^(7/3))))/(3*a)
```

Defintions of rubi rules used

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{x^4}{4a^2}-\frac{2bx}{a^3}-\frac{b^2x}{3a^3(a\,x^3+b)}+\frac{7b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^3+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{9a^4}$	64
default	$\frac{\frac{1}{4}a\,x^4-2bx}{a^3}+\frac{b^2\left(-\frac{x}{3\left(a\,x^3+b\right)}+\frac{7\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{7\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{7\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)}{a^3}$	126

input
```
int(x^3/(a+b/x^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/a^2-2/a^3*b*x-1/3*b^2*x/a^3/(a*x^3+b)+7/9/a^4*b^2*sum(1/_R^2*ln(x-
_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{9\,a^2x^7-63\,abx^4-84\,b^2x+28\,\sqrt{3}(abx^3+b^2)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-14\,(abx^3+b^2)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\right.}{36\,(a^4x^3+a^3b)}$$

input
```
integrate(x^3/(a+b/x^3)^2,x, algorithm="fricas")
```

output
```
1/36*(9*a^2*x^7 - 63*a*b*x^4 - 84*b^2*x + 28*sqrt(3)*(a*b*x^3 + b^2)*(b/a)
^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b/a)^(2/3) - sqrt(3)*b)/b) - 14*(a*b*x^3
 + b^2)*(b/a)^(1/3)*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3)) + 28*(a*b*x^3 +
 b^2)*(b/a)^(1/3)*log(x + (b/a)^(1/3)))/(a^4*x^3 + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.42

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = -\frac{b^2x}{3a^4x^3+3a^3b} + \operatorname{RootSum}\left(729t^3a^{10}-343b^4,\left(t\mapsto t\log\left(\frac{9ta^3}{7b}+x\right)\right)\right) + \frac{x^4}{4a^2}-\frac{2bx}{a^3}$$

input
```
integrate(x**3/(a+b/x**3)**2,x)
```

output
```
-b**2*x/(3*a**4*x**3 + 3*a**3*b) + RootSum(729*_t**3*a**10 - 343*b**4, Lam
bda(_t, _t*log(9*_t*a**3/(7*b) + x))) + x**4/(4*a**2) - 2*b*x/a**3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = -\frac{b^2x}{3\,(a^4x^3+a^3b)} + \frac{7\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{7\,b^2\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{7\,b^2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,a^4\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{ax^4-8\,bx}{4\,a^3}$$

input
```
integrate(x^3/(a+b/x^3)^2,x, algorithm="maxima")
```

output
```
-1/3*b^2*x/(a^4*x^3 + a^3*b) + 7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (
b/a)^(1/3))/(b/a)^(1/3))/(a^4*(b/a)^(2/3)) - 7/18*b^2*log(x^2 - x*(b/a)^(1
/3) + (b/a)^(2/3))/(a^4*(b/a)^(2/3)) + 7/9*b^2*log(x + (b/a)^(1/3))/(a^4*(
b/a)^(2/3)) + 1/4*(a*x^4 - 8*b*x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = -\frac{7\,b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} - \frac{b^2x}{3\,(ax^3+b)a^3} + \frac{7\sqrt{3}(-a^2b)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^4} + \frac{7\,(-a^2b)^{\frac{1}{3}}\,b\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^4} + \frac{a^6x^4-8\,a^5bx}{4\,a^8}$$

input
```
integrate(x^3/(a+b/x^3)^2,x, algorithm="giac")
```

output
```
-7/9*b*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a^3 - 1/3*b^2*x/((a*x^3 + b
)*a^3) + 7/9*sqrt(3)*(-a^2*b)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/
3))/(-b/a)^(1/3))/a^4 + 7/18*(-a^2*b)^(1/3)*b*log(x^2 + x*(-b/a)^(1/3) + (
-b/a)^(2/3))/a^4 + 1/4*(a^6*x^4 - 8*a^5*b*x)/a^8
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{x^4}{4\,a^2}-\frac{b^2\,x}{3\,\left(a^4\,x^3+b\,a^3\right)}+\frac{7\,b^{4/3}\,\ln\left(\frac{7\,b^{7/3}}{a^{4/3}}+\frac{7\,b^2\,x}{a}\right)}{9\,a^{10/3}}-\frac{2\,b\,x}{a^3}+\frac{7\,b^{4/3}\,\ln\left(\frac{7\,b^2\,x}{a}+\frac{7\,b^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{10/3}}-\frac{7\,b^{4/3}\,\ln\left(\frac{7\,b^2\,x}{a}-\frac{7\,b^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

input
```
int(x^3/(a + b/x^3)^2,x)
```

output
```
x^4/(4*a^2) - (b^2*x)/(3*(a^3*b + a^4*x^3)) + (7*b^(4/3)*log((7*b^(7/3))/a
^(4/3) + (7*b^2*x)/a))/(9*a^(10/3)) - (2*b*x)/a^3 + (7*b^(4/3)*log((7*b^2*
x)/a + (7*b^(7/3)*((3^(1/2)*1i)/2 - 1/2))/a^(4/3))*((3^(1/2)*1i)/2 - 1/2))
/(9*a^(10/3)) - (7*b^(4/3)*log((7*b^2*x)/a - (7*b^(7/3)*((3^(1/2)*1i)/2 +
1/2))/a^(4/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.24

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{28b^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)a\,x^3+28b^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)+9a^{\frac{7}{3}}x^7-63a^{\frac{4}{3}}b\,x^4-84a^{\frac{1}{3}}b^2x-14b^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}x^2\right.}{36a^{\frac{10}{3}}\left(a\right.}$$

input
```
int(x^3/(a+b/x^3)^2,x)
```

output
```
(28*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*
b*x**3 + 28*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt
(3)))*b**2 + 9*a**(1/3)*a**2*x**7 - 63*a**(1/3)*a*b*x**4 - 84*a**(1/3)*b**
2*x - 14*b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*b*
x**3 - 14*b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b**
2 + 28*b**(1/3)*log(a**(1/3)*x + b**(1/3))*a*b*x**3 + 28*b**(1/3)*log(a**(
1/3)*x + b**(1/3))*b**2)/(36*a**(1/3)*a**3*(a*x**3 + b))
```

3.447 $\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 147

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2} dx = \frac{x^2}{2a^2} + \frac{bx^2}{3a^2(b+ax^3)} + \frac{5b^{2/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}a^{8/3}} + \frac{5b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9a^{8/3}} - \frac{5b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{18a^{8/3}}$$

output
```
1/2*x^2/a^2+1/3*b*x^2/a^2/(a*x^3+b)+5/9*b^(2/3)*arctan(1/3*(b^(1/3)-2*a^(1
/3)*x)*3^(1/2)/b^(1/3))*3^(1/2)/a^(8/3)+5/9*b^(2/3)*ln(b^(1/3)+a^(1/3)*x)/
a^(8/3)-5/18*b^(2/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2} dx$$
$$= \frac{9a^{2/3}x^2 + \frac{6a^{2/3}bx^2}{b+ax^3} + 10\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right) + 10b^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right) - 5b^{2/3}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{18a^{8/3}}$$

input
```
Integrate[x/(a + b/x^3)^2,x]
```

output
```
(9*a^(2/3)*x^2 + (6*a^(2/3)*b*x^2)/(b + a*x^3) + 10*Sqrt[3]*b^(2/3)*ArcTan
[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] + 10*b^(2/3)*Log[b^(1/3) + a^(1/3)*x
] - 5*b^(2/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(18*a^(8/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {795, 817, 843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2} dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{x^7}{(ax^3+b)^2}dx$$
$$\Big\downarrow \text{ 817}$$
$$\frac{5\int \frac{x^4}{ax^3+b}dx}{3a} - \frac{x^5}{3a(ax^3+b)}$$

$$\Big\downarrow \text{843}$$

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\int \frac{x}{ax^3+b}dx}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{821}$$

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$\big\downarrow$ 25

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$\big\downarrow$ 27

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$\downarrow$ 1082

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$\downarrow$ 217

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

$\big\downarrow$ 1103

$$\frac{5\left(\frac{x^2}{2a}-\frac{b\left(\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}\right)}{3a}-\frac{x^5}{3a\left(ax^3+b\right)}$$

input
```
Int[x/(a + b/x^3)^2,x]
```

output
```
-1/3*x^5/(a*(b + a*x^3)) + (5*(x^2/(2*a) - (b*(-1/3*Log[b^(1/3) + a^(1/3)*
x]/(a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt
[3]])/a^(1/3)) + Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)
))/(3*a^(1/3)*b^(1/3))))/a))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{x^2}{2a^2}+\frac{b\,x^2}{3a^2(a\,x^3+b)}-\frac{5b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^3+b\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{9a^3}$	55
default	$\frac{x^2}{2a^2}-\frac{b\left(-\frac{x^2}{3\left(a\,x^3+b\right)}-\frac{5\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9a\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{a^2}$	120

input
```
int(x/(a+b/x^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*x^2+1/3*b*x^2/a^2/(a*x^3+b)-5/9/a^3*b*sum(1/_R*ln(x-_R),_R=RootOf(
_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx$$
$$=\frac{9\,ax^5+15\,bx^2-10\,\sqrt{3}(ax^3+b)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}-\sqrt{3}b}{3\,b}\right)-5\,(ax^3+b)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b^2}{a^2}\right.\right.}{18\,(a^3x^3+a^2b)}$$

input
```
integrate(x/(a+b/x^3)^2,x, algorithm="fricas")
```

output
```
1/18*(9*a*x^5 + 15*b*x^2 - 10*sqrt(3)*(a*x^3 + b)*(b^2/a^2)^(1/3)*arctan(1
/3*(2*sqrt(3)*a*x*(b^2/a^2)^(1/3) - sqrt(3)*b)/b) - 5*(a*x^3 + b)*(b^2/a^2
)^(1/3)*log(b*x^2 - a*x*(b^2/a^2)^(2/3) + b*(b^2/a^2)^(1/3)) + 10*(a*x^3 +
 b)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(2/3)))/(a^3*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.39

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{bx^2}{3a^3x^3+3a^2b} + \operatorname{RootSum}\left(729t^3a^8-125b^2,\left(t\mapsto t\log\left(\frac{81t^2a^5}{25b}+x\right)\right)\right)+\frac{x^2}{2a^2}$$

input
```
integrate(x/(a+b/x**3)**2,x)
```

output
```
b*x**2/(3*a**3*x**3 + 3*a**2*b) + RootSum(729*_t**3*a**8 - 125*b**2, Lambd
a(_t, _t*log(81*_t**2*a**5/(25*b) + x))) + x**2/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{bx^2}{3\,(a^3x^3+a^2b)} + \frac{x^2}{2\,a^2} - \frac{5\,\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{5\,b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{5\,b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(a+b/x^3)^2,x, algorithm="maxima")
```

output
```
1/3*b*x^2/(a^3*x^3 + a^2*b) + 1/2*x^2/a^2 - 5/9*sqrt(3)*b*arctan(1/3*sqrt(
3)*(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(b/a)^(1/3)) - 5/18*b*log(x^2 - x
*(b/a)^(1/3) + (b/a)^(2/3))/(a^3*(b/a)^(1/3)) + 5/9*b*log(x + (b/a)^(1/3))
/(a^3*(b/a)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{x^2}{2\,a^2} + \frac{bx^2}{3\,(ax^3+b)a^2} + \frac{5\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{5\,\sqrt{3}(-a^2b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^4} - \frac{5\,(-a^2b)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^4}$$

input
```
integrate(x/(a+b/x^3)^2,x, algorithm="giac")
```

output
```
1/2*x^2/a^2 + 1/3*b*x^2/((a*x^3 + b)*a^2) + 5/9*(-b/a)^(2/3)*log(abs(x - (
-b/a)^(1/3)))/a^2 + 5/9*sqrt(3)*(-a^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (
-b/a)^(1/3))/(-b/a)^(1/3))/a^4 - 5/18*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1
/3) + (-b/a)^(2/3))/a^4
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{x^2}{2\,a^2} + \frac{5\,b^{2/3}\,\ln\left(a^{1/3}\,x+b^{1/3}\right)}{9\,a^{8/3}} + \frac{b\,x^2}{3\,\left(a^3\,x^3+b\,a^2\right)} + \frac{5\,b^{2/3}\,\ln\left(\frac{25\,b^2\,x}{9\,a^3}+\frac{25\,b^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9\,a^{10/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}} - \frac{5\,b^{2/3}\,\ln\left(\frac{25\,b^2\,x}{9\,a^3}+\frac{25\,b^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{9\,a^{10/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{8/3}}$$

input
```
int(x/(a + b/x^3)^2,x)
```

output
```
x^2/(2*a^2) + (5*b^(2/3)*log(a^(1/3)*x + b^(1/3)))/(9*a^(8/3)) + (b*x^2)/(
3*(a^2*b + a^3*x^3)) + (5*b^(2/3)*log((25*b^2*x)/(9*a^3) + (25*b^(7/3)*((3
^(1/2)*1i)/2 - 1/2)^2)/(9*a^(10/3)))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)) -
 (5*b^(2/3)*log((25*b^2*x)/(9*a^3) + (25*b^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)
/(9*a^(10/3)))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{-10\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)ab\,x^3-10\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)b^2+9b^{\frac{1}{3}}a^{\frac{5}{3}}x^5+15b^{\frac{4}{3}}a^{\frac{2}{3}}x^2-5\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+\right.}{18b^{\frac{1}{3}}a^{\frac{8}{3}}\left(a\,x^3+\right.}$$

input
```
int(x/(a+b/x^3)^2,x)
```

output
```
( - 10*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*b*x**3
 - 10*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*b**2 + 9*
b**(1/3)*a**(2/3)*a*x**5 + 15*b**(1/3)*a**(2/3)*b*x**2 - 5*log(a**(2/3)*x*
*2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*b*x**3 - 5*log(a**(2/3)*x**2 - b**(
1/3)*a**(1/3)*x + b**(2/3))*b**2 + 10*log(a**(1/3)*x + b**(1/3))*a*b*x**3
+ 10*log(a**(1/3)*x + b**(1/3))*b**2)/(18*b**(1/3)*a**(2/3)*a**2*(a*x**3 +
 b))
```

3.448 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 140

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2} dx = \frac{x}{a^2}+\frac{bx}{3a^2(b+ax^3)}+\frac{4\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}a^{7/3}} -\frac{4\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9a^{7/3}}+\frac{2\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{9a^{7/3}}$$

output
```
x/a^2+1/3*b*x/a^2/(a*x^3+b)+4/9*b^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3
^(1/2)/b^(1/3))*3^(1/2)/a^(7/3)-4/9*b^(1/3)*ln(b^(1/3)+a^(1/3)*x)/a^(7/3)+
2/9*b^(1/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx$$
$$= \frac{9\sqrt[3]{a}x+\frac{3\sqrt[3]{a}bx}{b+ax^3}+4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-4\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)+2\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{9a^{7/3}}$$

input
```
Integrate[(a + b/x^3)^(-2),x]
```

output
```
(9*a^(1/3)*x + (3*a^(1/3)*b*x)/(b + a*x^3) + 4*Sqrt[3]*b^(1/3)*ArcTan[(1 -
 (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 4*b^(1/3)*Log[b^(1/3) + a^(1/3)*x] + 2*
b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(9*a^(7/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {772, 817, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx$$
$$\Big\downarrow \text{772}$$
$$\int \frac{x^6}{(ax^3+b)^2}dx$$
$$\Big\downarrow \text{817}$$
$$\frac{4\int \frac{x^3}{ax^3+b}dx}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{843}$$

$$\frac{4\left(\frac{x}{a}-\frac{b\int \frac{1}{ax^3+b}dx}{a}\right)}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{750}$$

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\downarrow 25$$

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\downarrow 27$$

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\,(ax^3+b)}$$

$\Big\downarrow$ 1103

$$\frac{4\left(\frac{x}{a}-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{3a}-\frac{x^4}{3a\,(ax^3+b)}$$

input
```
Int[(a + b/x^3)^(-2),x]
```

output
```
-1/3*x^4/(a*(b + a*x^3)) + (4*(x/a - (b*(Log[b^(1/3) + a^(1/3)*x]/(3*a^(1/
3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(
1/3)) - Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*b^(
2/3))))/a))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{x}{a^2}+\frac{bx}{3a^2(a\,x^3+b)}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^3+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{9a^3}$	50
default	$\frac{x}{a^2}-\frac{b\left(-\frac{x}{3\left(a\,x^3+b\right)}+\frac{4\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)}{a^2}$	115

input
```
int(1/(a+b/x^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*x+1/3*b*x/a^2/(a*x^3+b)-4/9/a^3*b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3
*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int\frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx$$
$$=\frac{9\,ax^4+4\,\sqrt{3}(ax^3+b)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-2\,(ax^3+b)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right.\right.}{9\,(a^3x^3+a^2b)}$$

input
```
integrate(1/(a+b/x^3)^2,x, algorithm="fricas")
```

output
```
1/9*(9*a*x^4 + 4*sqrt(3)*(a*x^3 + b)*(-b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*
x*(-b/a)^(2/3) - sqrt(3)*b)/b) - 2*(a*x^3 + b)*(-b/a)^(1/3)*log(x^2 + x*(-
b/a)^(1/3) + (-b/a)^(2/3)) + 4*(a*x^3 + b)*(-b/a)^(1/3)*log(x - (-b/a)^(1/
3)) + 12*b*x)/(a^3*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{bx}{3a^3x^3+3a^2b} + \operatorname{RootSum}\left(729t^3a^7+64b,\left(t\mapsto t\log\left(-\frac{9ta^2}{4}+x\right)\right)\right)+\frac{x}{a^2}$$

input
```
integrate(1/(a+b/x**3)**2,x)
```

output
```
b*x/(3*a**3*x**3 + 3*a**2*b) + RootSum(729*_t**3*a**7 + 64*b, Lambda(_t, _
t*log(-9*_t*a**2/4 + x))) + x/a**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{bx}{3\,(a^3x^3+a^2b)}+\frac{x}{a^2}-\frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{2\,b\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{4\,b\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(a+b/x^3)^2,x, algorithm="maxima")
```

output
```
1/3*b*x/(a^3*x^3 + a^2*b) + x/a^2 - 4/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x
- (b/a)^(1/3))/(b/a)^(1/3))/(a^3*(b/a)^(2/3)) + 2/9*b*log(x^2 - x*(b/a)^(1
/3) + (b/a)^(2/3))/(a^3*(b/a)^(2/3)) - 4/9*b*log(x + (b/a)^(1/3))/(a^3*(b/
a)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2}+\frac{x}{a^2}+\frac{bx}{3\,(ax^3+b)a^2}-\frac{4\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^3}-\frac{2\,(-a^2b)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,a^3}$$

input
```
integrate(1/(a+b/x^3)^2,x, algorithm="giac")
```

output
```
4/9*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/a^2 + x/a^2 + 1/3*b*x/((a*x^3
+ b)*a^2) - 4/9*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1
/3))/(-b/a)^(1/3))/a^3 - 2/9*(-a^2*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b
/a)^(2/3))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{x}{a^2}+\frac{4\,(-b)^{1/3}\,\ln\left((-b)^{4/3}+a^{1/3}\,b\,x\right)}{9\,a^{7/3}}+\frac{b\,x}{3\,\left(a^3\,x^3+b\,a^2\right)}-\frac{4\,(-b)^{1/3}\,\ln\left(4\,b\,x-\frac{4\,(-b)^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{7/3}}+\frac{(-b)^{1/3}\,\ln\left(4\,b\,x+\frac{9\,(-b)^{4/3}\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2\mathrm{i}}{9}\right)}{a^{1/3}}\right)\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2\mathrm{i}}{9}\right)}{a^{7/3}}$$

input
```
int(1/(a + b/x^3)^2,x)
```

output
```
x/a^2 + (4*(-b)^(1/3)*log((-b)^(4/3) + a^(1/3)*b*x))/(9*a^(7/3)) + (b*x)/(
3*(a^2*b + a^3*x^3)) - (4*(-b)^(1/3)*log(4*b*x - (4*(-b)^(4/3)*((3^(1/2)*1
i)/2 + 1/2))/a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)) + ((-b)^(1/3)*lo
g(4*b*x + (9*(-b)^(4/3)*((3^(1/2)*2i)/9 - 2/9))/a^(1/3))*((3^(1/2)*2i)/9 -
 2/9))/a^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2}\,dx = \frac{-4b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)a\,x^3-4b^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)+9a^{\frac{4}{3}}x^4+12a^{\frac{1}{3}}bx+2b^{\frac{1}{3}}\log\Bigl(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}}{9a^{\frac{7}{3}}\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^2,x)
```

output
```
( - 4*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*
a*x**3 - 4*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(
3)))*b + 9*a**(1/3)*a*x**4 + 12*a**(1/3)*b*x + 2*b**(1/3)*log(a**(2/3)*x**
2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*x**3 + 2*b**(1/3)*log(a**(2/3)*x**2
- b**(1/3)*a**(1/3)*x + b**(2/3))*b - 4*b**(1/3)*log(a**(1/3)*x + b**(1/3)
)*a*x**3 - 4*b**(1/3)*log(a**(1/3)*x + b**(1/3))*b)/(9*a**(1/3)*a**2*(a*x*
*3 + b))
```

3.449 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 136

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2} dx = -\frac{x^2}{3a(b+ax^3)} - \frac{2\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}} - \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}}$$

output
```
-1/3*x^2/a/(a*x^3+b)-2/9*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))
*3^(1/2)/a^(5/3)/b^(1/3)-2/9*ln(b^(1/3)+a^(1/3)*x)/a^(5/3)/b^(1/3)+1/9*ln(
b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(5/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2} \, dx$$
$$= \frac{-\frac{3a^{2/3}x^2}{b+ax^3} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{\sqrt[3]{b}}}{9a^{5/3}}$$

input
```
Integrate[1/((a + b/x^3)^2*x^2),x]
```

output
```
((-3*a^(2/3)*x^2)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/
3))/Sqrt[3]])/b^(1/3) - (2*Log[b^(1/3) + a^(1/3)*x])/b^(1/3) + Log[b^(2/3)
 - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/b^(1/3))/(9*a^(5/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {795, 817, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{x^3}\right)^2} \, dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{x^4}{(ax^3+b)^2}dx$$
$$\Big\downarrow \text{817}$$
$$\frac{2\int \frac{x}{ax^3+b}dx}{3a} - \frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\left(\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}-\frac{x^2}{3a\left(ax^3+b\right)}$$

input
```
Int[1/((a + b/x^3)^2*x^2),x]
```

output
```
-1/3*x^2/(a*(b + a*x^3)) + (2*(-1/3*Log[b^(1/3) + a^(1/3)*x]/(a^(2/3)*b^(1
/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) +
 Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*a^(1/3)*b^
(1/3))))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.33

method	result	size
risch	$-\frac{x^{2}}{3 a \left(a \,x^{3}+b \right)}+\frac{2 \left(\sum_{\textit{\_R} =\operatorname{RootOf}\left(a \,\textit{\_Z}^{3}+b \right)}\frac{\ln \left(x -\textit{\_R} \right)}{\textit{\_R}}\right)}{9 a^{2}}$	45
default	$-\frac{x^{2}}{3 a \left(a \,x^{3}+b \right)}+\frac{-\frac{2 \ln \left(x +\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9 a \left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\ln \left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x +\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9 a \left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{2 \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 a \left(\frac{b}{a}\right)^{\frac{1}{3}}}}{a}$	114

input
```
int(1/(a+b/x^3)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^2/a/(a*x^3+b)+2/9/a^2*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.94

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx$$

$$= \left[-\frac{3\,a^2bx^2 - 3\sqrt{\frac{1}{3}}(a^2bx^3+ab^2)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,a^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-a^2b)^{\frac{2}{3}}x^2+(-a^2b)^{\frac{1}{3}}b\right)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}-3\,(-a^2b)^{\frac{2}{3}}}{ax^3+b}\right.}{9\,(a^4bx}\right.$$

$$-\frac{3\,a^2bx^2 - 6\sqrt{\frac{1}{3}}(a^2bx^3+ab^2)\sqrt{-\frac{(-a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,ax+(-a^2b)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a^2b)^{\frac{1}{3}}}{b}}}{a}\right) - (ax^3+b)(-a^2b)^{\frac{2}{3}}\,\mathrm{lo}}{9\,(a^4bx^3+a^3b^2)}$$

input
```
integrate(1/(a+b/x^3)^2/x^2,x, algorithm="fricas")
```

output
```
[-1/9*(3*a^2*b*x^2 - 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((-a^2*b)^(1/3)/b
)*log((2*a^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a^2*b)^(2/3)*x^2 + (-a^2
*b)^(1/3)*b)*sqrt((-a^2*b)^(1/3)/b) - 3*(-a^2*b)^(2/3)*x)/(a*x^3 + b)) - (
a*x^3 + b)*(-a^2*b)^(2/3)*log(a^2*x^2 + (-a^2*b)^(1/3)*a*x + (-a^2*b)^(2/3
)) + 2*(a*x^3 + b)*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1/3)))/(a^4*b*x^3 +
a^3*b^2), -1/9*(3*a^2*b*x^2 - 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(-a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*a*x + (-a^2*b)^(1/3))*sqrt(-(-a^2*b)^(1/3)
/b)/a) - (a*x^3 + b)*(-a^2*b)^(2/3)*log(a^2*x^2 + (-a^2*b)^(1/3)*a*x + (-a
^2*b)^(2/3)) + 2*(a*x^3 + b)*(-a^2*b)^(2/3)*log(a*x - (-a^2*b)^(1/3)))/(a^
4*b*x^3 + a^3*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx = -\frac{x^2}{3a^2x^3+3ab} + \operatorname{RootSum}\left(729t^3a^5b+8,\left(t\mapsto t\log\left(\frac{81t^2a^3b}{4}+x\right)\right)\right)$$

input
```
integrate(1/(a+b/x**3)**2/x**2,x)
```

output
```
-x**2/(3*a**2*x**3 + 3*a*b) + RootSum(729*_t**3*a**5*b + 8, Lambda(_t, _t*
log(81*_t**2*a**3*b/4 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx = -\frac{x^2}{3\left(a^2x^3+ab\right)} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(a+b/x^3)^2/x^2,x, algorithm="maxima")
```

output
```
-1/3*x^2/(a^2*x^3 + a*b) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/
3))/(b/a)^(1/3))/(a^2*(b/a)^(1/3)) + 1/9*log(x^2 - x*(b/a)^(1/3) + (b/a)^(
2/3))/(a^2*(b/a)^(1/3)) - 2/9*log(x + (b/a)^(1/3))/(a^2*(b/a)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx = -\frac{x^2}{3\,(ax^3+b)a} - \frac{2\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{2\sqrt{3}(-a^2b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} + \frac{(-a^2b)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,a^3b}$$

input
```
integrate(1/(a+b/x^3)^2/x^2,x, algorithm="giac")
```

output
```
-1/3*x^2/((a*x^3 + b)*a) - 2/9*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/(a*
b) - 2/9*sqrt(3)*(-a^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-
b/a)^(1/3))/(a^3*b) + 1/9*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)
^(2/3))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx = \frac{2\,\ln\left(\frac{4\,b^{1/3}}{9\,{(-a)}^{4/3}}+\frac{4\,x}{9\,a}\right)}{9\,{(-a)}^{5/3}\,b^{1/3}} - \frac{x^2}{3\,a\,\left(a\,x^3+b\right)} + \frac{\ln\left(\frac{4\,x}{9\,a}+\frac{b^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{9\,{(-a)}^{4/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,{(-a)}^{5/3}\,b^{1/3}} - \frac{\ln\left(\frac{4\,x}{9\,a}+\frac{b^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{9\,{(-a)}^{4/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,{(-a)}^{5/3}\,b^{1/3}}$$

input
```
int(1/(x^2*(a + b/x^3)^2),x)
```

output
```
(2*log((4*b^(1/3))/(9*(-a)^(4/3)) + (4*x)/(9*a)))/(9*(-a)^(5/3)*b^(1/3)) -
 x^2/(3*a*(b + a*x^3)) + (log((4*x)/(9*a) + (b^(1/3)*(3^(1/2)*1i - 1)^2)/(
9*(-a)^(4/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(5/3)*b^(1/3)) - (log((4*x)/(9*a)
 + (b^(1/3)*(3^(1/2)*1i + 1)^2)/(9*(-a)^(4/3)))*(3^(1/2)*1i + 1))/(9*(-a)^
(5/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^2}\,dx$$
$$= \frac{2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)a\,x^3+2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)b-3b^{\frac{1}{3}}a^{\frac{2}{3}}x^2+\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a\,x^3+\log\left(a^{\frac{2}{3}}x\right.}{9b^{\frac{1}{3}}a^{\frac{5}{3}}\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^2/x^2,x)
```

output
```
(2*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*x**3 + 2*s
qrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*b - 3*b**(1/3)*a
**(2/3)*x**2 + log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*x**3
+ log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b - 2*log(a**(1/3)*x
 + b**(1/3))*a*x**3 - 2*log(a**(1/3)*x + b**(1/3))*b)/(9*b**(1/3)*a**(2/3)
*a*(a*x**3 + b))
```

3.450 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3} \, dx = -\frac{x}{3a(b+ax^3)} - \frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}a^{4/3}b^{2/3}} + \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9a^{4/3}b^{2/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{18a^{4/3}b^{2/3}}$$

output
```
-1/3*x/a/(a*x^3+b)-1/9*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3
^(1/2)/a^(4/3)/b^(2/3)+1/9*ln(b^(1/3)+a^(1/3)*x)/a^(4/3)/b^(2/3)-1/18*ln(b
^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(4/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3} dx = \frac{-\frac{6\sqrt[3]{a}x}{b+ax^3}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{b^{2/3}}-\frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{b^{2/3}}}{18a^{4/3}}$$

input
```
Integrate[1/((a + b/x^3)^2*x^3),x]
```

output
```
((-6*a^(1/3)*x)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3)
)/Sqrt[3]])/b^(2/3) + (2*Log[b^(1/3) + a^(1/3)*x])/b^(2/3) - Log[b^(2/3) -
 a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/b^(2/3))/(18*a^(4/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {795, 817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{x^3}\right)^2} dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^3}{(ax^3+b)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{\int \frac{1}{ax^3+b}dx}{3a}-\frac{x}{3a(ax^3+b)}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}-\frac{x}{3a\left(ax^3+b\right)}$$

input
```
Int[1/((a + b/x^3)^2*x^3),x]
```

output
```
-1/3*x/(a*(b + a*x^3)) + (Log[b^(1/3) + a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + (
-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(
2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*b^(2/3)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result	size
risch	$-\frac{x}{3a(a\,x^3+b)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(a\_Z^3+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{9a^2}$	43
default	$-\frac{x}{3a(a\,x^3+b)}+\frac{\frac{\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{3a}$	112

input
```
int(1/(a+b/x^3)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x/a/(a*x^3+b)+1/9/a^2*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.92

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx$$

$$= \left[-\frac{6\,ab^2x - 3\,\sqrt{\frac{1}{3}}(a^2bx^3+ab^2)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abx^3-3\,(ab^2)^{\frac{1}{3}}bx-b^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(ab^2)^{\frac{2}{3}}x-(ab^2)^{\frac{1}{3}}b\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ax^3+b}\right)}{18\,(a^3b^2x^3+a\cdots} \right.$$

$$-\frac{6\,ab^2x - 6\,\sqrt{\frac{1}{3}}(a^2bx^3+ab^2)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(ab^2)^{\frac{2}{3}}x-(ab^2)^{\frac{1}{3}}b\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b^2}\right)+(ax^3+b)(ab^2)^{\frac{2}{3}}\log\left(a\cdots\right.}{18\,(a^3b^2x^3+a^2b^3)}$$

input
```
integrate(1/(a+b/x^3)^2/x^3,x, algorithm="fricas")
```

output
```
[-1/18*(6*a*b^2*x - 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(a*b^2)^(1/3)/a)
*log((2*a*b*x^3 - 3*(a*b^2)^(1/3)*b*x - b^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a*
b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 + b)) + (a*
x^3 + b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) -
2*(a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a^3*b^2*x^3 + a^2
*b^3), -1/18*(6*a*b^2*x - 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((a*b^2)^(1/
3)/a)*arctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^
(1/3)/a)/b^2) + (a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x +
(a*b^2)^(1/3)*b) - 2*(a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))
/(a^3*b^2*x^3 + a^2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx = -\frac{x}{3a^2x^3+3ab} + \operatorname{RootSum}\left(729t^3a^4b^2-1,\left(t\mapsto t\log\left(9tab+x\right)\right)\right)$$

input
```
integrate(1/(a+b/x**3)**2/x**3,x)
```

output
```
-x/(3*a**2*x**3 + 3*a*b) + RootSum(729*_t**3*a**4*b**2 - 1, Lambda(_t, _t*
log(9*_t*a*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx = -\frac{x}{3\left(a^2x^3+ab\right)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(a+b/x^3)^2/x^3,x, algorithm="maxima")
```

output
```
-1/3*x/(a^2*x^3 + a*b) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3)
)/(b/a)^(1/3))/(a^2*(b/a)^(2/3)) - 1/18*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2
/3))/(a^2*(b/a)^(2/3)) + 1/9*log(x + (b/a)^(1/3))/(a^2*(b/a)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx = -\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{x}{3\,(ax^3+b)a} + \frac{\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^2b} + \frac{(-a^2b)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^2b}$$

input
```
integrate(1/(a+b/x^3)^2/x^3,x, algorithm="giac")
```

output
```
-1/9*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/(a*b) - 1/3*x/((a*x^3 + b)*a)
 + 1/9*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/
a)^(1/3))/(a^2*b) + 1/18*(-a^2*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^
(2/3))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx = \frac{\ln\left(a^{1/3}\,x+b^{1/3}\right)}{9\,a^{4/3}\,b^{2/3}} - \frac{x}{3\,a\,\left(a\,x^3+b\right)} + \frac{\ln\left(a\,x+\frac{a^{2/3}\,b^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,b^{2/3}} - \frac{\ln\left(a\,x-\frac{a^{2/3}\,b^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,b^{2/3}}$$

input
```
int(1/(x^3*(a + b/x^3)^2),x)
```

output
```
log(a^(1/3)*x + b^(1/3))/(9*a^(4/3)*b^(2/3)) - x/(3*a*(b + a*x^3)) + (log(
a*x + (a^(2/3)*b^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(18*a^(4/3)*
b^(2/3)) - (log(a*x - (a^(2/3)*b^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i +
1))/(18*a^(4/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^3}\,dx$$
$$= \frac{2b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)a\,x^3+2b^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)-6a^{\frac{1}{3}}bx-b^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a\,x^3-b^{\frac{4}{3}}\log}{18a^{\frac{4}{3}}b\,(a\,x^3+b)}$$

input
```
int(1/(a+b/x^3)^2/x^3,x)
```

output
```
(2*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*x
**3 + 2*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3))
)*b - 6*a**(1/3)*b*x - b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x +
b**(2/3))*a*x**3 - b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(
2/3))*b + 2*b**(1/3)*log(a**(1/3)*x + b**(1/3))*a*x**3 + 2*b**(1/3)*log(a*
*(1/3)*x + b**(1/3))*b)/(18*a**(1/3)*a*b*(a*x**3 + b))
```

3.451 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 136

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5} dx = \frac{x^2}{3b(b+ax^3)} - \frac{\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}a^{2/3}b^{4/3}} - \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9a^{2/3}b^{4/3}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}$$

output
```
1/3*x^2/b/(a*x^3+b)-1/9*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*
3^(1/2)/a^(2/3)/b^(4/3)-1/9*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/b^(4/3)+1/18*ln(
b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(2/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5} \, dx = \frac{\frac{6\sqrt[3]{b}x^2}{b+ax^3} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{a^{2/3}} + \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{a^{2/3}}}{18b^{4/3}}$$

input
```
Integrate[1/((a + b/x^3)^2*x^5),x]
```

output
```
((6*b^(1/3)*x^2)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3
))/Sqrt[3]])/a^(2/3) - (2*Log[b^(1/3) + a^(1/3)*x])/a^(2/3) + Log[b^(2/3)
- a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/a^(2/3))/(18*b^(4/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {795, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+\frac{b}{x^3}\right)^2} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x}{\left(ax^3+b\right)^2}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{\int \frac{x}{ax^3+b}dx}{3b} + \frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \; 821$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}+\frac{x^2}{3b\left(ax^3+b\right)}$$

input
```
Int[1/((a + b/x^3)^2*x^5),x]
```

output
```
x^2/(3*b*(b + a*x^3)) + (-1/3*Log[b^(1/3) + a^(1/3)*x]/(a^(2/3)*b^(1/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) + Log[b
^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*a^(1/3)*b^(1/3))
)/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{x^{2}}{3 b \left(a \,x^{3}+b \right)}+\frac{\munderset{\textit{\_R} =\operatorname{RootOf}\left(a \,\textit{\_Z}^{3}+b \right)}{\sum}\frac{\ln \left(x -\textit{\_R} \right)}{\textit{\_R}}}{9 a b}$	48
default	$\frac{x^{2}}{3 b \left(a \,x^{3}+b \right)}+\frac{-\frac{\ln \left(x +\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3 a \left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\ln \left(x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}} x +\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6 a \left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(\frac{2 x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 a \left(\frac{b}{a}\right)^{\frac{1}{3}}}}{3 b}$	114

input
```
int(1/(a+b/x^3)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2/b/(a*x^3+b)+1/9/a/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.84

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx$$
$$= \left[\frac{6\,a^2bx^2 + 3\,\sqrt{\frac{1}{3}}(a^2bx^3+ab^2)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,a^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\,(a^2b)^{\frac{2}{3}}x^2-(a^2b)^{\frac{1}{3}}b\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3\,(a^2b)^{\frac{2}{3}}x}{ax^3+b}\right)+}{18\,(a^3b^2x^3+a^2b}\right.$$

input
```
integrate(1/(a+b/x^3)^2/x^5,x, algorithm="fricas")
```

output
```
[1/18*(6*a^2*b*x^2 + 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(a^2*b)^(2/3)*x^2 - (a^2*b
)^(1/3)*b)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(2/3)*x)/(a*x^3 + b)) + (a*x
^3 + b)*(a^2*b)^(2/3)*log(a^2*x^2 - (a^2*b)^(1/3)*a*x + (a^2*b)^(2/3)) - 2
*(a*x^3 + b)*(a^2*b)^(2/3)*log(a*x + (a^2*b)^(1/3)))/(a^3*b^2*x^3 + a^2*b^
3), 1/18*(6*a^2*b*x^2 - 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((a^2*b)^(1/3)
/b)*arctan(-sqrt(1/3)*(2*a*x - (a^2*b)^(1/3))*sqrt((a^2*b)^(1/3)/b)/a) + (
a*x^3 + b)*(a^2*b)^(2/3)*log(a^2*x^2 - (a^2*b)^(1/3)*a*x + (a^2*b)^(2/3))
- 2*(a*x^3 + b)*(a^2*b)^(2/3)*log(a*x + (a^2*b)^(1/3)))/(a^3*b^2*x^3 + a^2
*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx = \frac{x^2}{3abx^3+3b^2} + \operatorname{RootSum}\left(729t^3a^2b^4+1, \left(t\mapsto t\log\left(81t^2ab^3+x\right)\right)\right)$$

input
```
integrate(1/(a+b/x**3)**2/x**5,x)
```

output
```
x**2/(3*a*b*x**3 + 3*b**2) + RootSum(729*_t**3*a**2*b**4 + 1, Lambda(_t, _
t*log(81*_t**2*a*b**3 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx = \frac{x^2}{3\left(abx^3+b^2\right)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(a+b/x^3)^2/x^5,x, algorithm="maxima")
```

output
```
1/3*x^2/(a*b*x^3 + b^2) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3
))/(b/a)^(1/3))/(a*b*(b/a)^(1/3)) + 1/18*log(x^2 - x*(b/a)^(1/3) + (b/a)^(
2/3))/(a*b*(b/a)^(1/3)) - 1/9*log(x + (b/a)^(1/3))/(a*b*(b/a)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx = \frac{x^2}{3\,(ax^3+b)b} - \frac{\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,b^2} - \frac{\sqrt{3}(-a^2b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{(-a^2b)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2}$$

input
```
integrate(1/(a+b/x^3)^2/x^5,x, algorithm="giac")
```

output
```
1/3*x^2/((a*x^3 + b)*b) - 1/9*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/b^2
- 1/9*sqrt(3)*(-a^2*b)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a
)^(1/3))/(a^2*b^2) + 1/18*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)
^(2/3))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx$$
$$= \frac{x^2}{3\,b\,(a\,x^3+b)} + \frac{(-1)^{1/3}\,\ln\left(\frac{(-1)^{2/3}\,a^{2/3}}{9\,b^{5/3}}+\frac{a\,x}{9\,b^2}\right)}{9\,a^{2/3}\,b^{4/3}}$$
$$-\frac{(-1)^{1/3}\,\ln\left((-1)^{2/3}\,b^{1/3}-2\,a^{1/3}\,x+(-1)^{1/6}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{2/3}\,b^{4/3}}$$
$$+\frac{(-1)^{1/3}\,\ln\left(2\,a^{1/3}\,x-(-1)^{2/3}\,b^{1/3}+(-1)^{1/6}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{2/3}\,b^{4/3}}$$

input
```
int(1/(x^5*(a + b/x^3)^2),x)
```

output
```
x^2/(3*b*(b + a*x^3)) + ((-1)^(1/3)*log(((-1)^(2/3)*a^(2/3))/(9*b^(5/3)) +
 (a*x)/(9*b^2)))/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/3)*log((-1)^(2/3)*b^(1/3)
- 2*a^(1/3)*x + (-1)^(1/6)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(
2/3)*b^(4/3)) + ((-1)^(1/3)*log(2*a^(1/3)*x - (-1)^(2/3)*b^(1/3) + (-1)^(1
/6)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(2/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^5}\,dx$$
$$= \frac{2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)a\,x^3+2\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)b+6b^{\frac{1}{3}}a^{\frac{2}{3}}x^2+\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a\,x^3+\log\left(a^{\frac{2}{3}}x\right.}{18b^{\frac{4}{3}}a^{\frac{2}{3}}\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^2/x^5,x)
```

output
```
(2*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*x**3 + 2*s
qrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*b + 6*b**(1/3)*a
**(2/3)*x**2 + log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*a*x**3
+ log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(2/3))*b - 2*log(a**(1/3)*x
 + b**(1/3))*a*x**3 - 2*log(a**(1/3)*x + b**(1/3))*b)/(18*b**(1/3)*a**(2/3
)*b*(a*x**3 + b))
```

3.452 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6} \, dx = \frac{x}{3b(b+ax^3)} - \frac{2\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{5/3}} + \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9\sqrt[3]{a}b^{5/3}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{9\sqrt[3]{a}b^{5/3}}$$

output
```
1/3*x/b/(a*x^3+b)-2/9*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)*3^(1/2)/b^(1/3))*3^
(1/2)/a^(1/3)/b^(5/3)+2/9*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/b^(5/3)-1/9*ln(b^(
2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6} \, dx = \frac{\frac{3b^{2/3}x}{b+ax^3} - \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}} - \frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{\sqrt[3]{a}}}{9b^{5/3}}$$

input
```
Integrate[1/((a + b/x^3)^2*x^6),x]
```

output
```
((3*b^(2/3)*x)/(b + a*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))
/Sqrt[3]])/a^(1/3) + (2*Log[b^(1/3) + a^(1/3)*x])/a^(1/3) - Log[b^(2/3) -
a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/a^(1/3))/(9*b^(5/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {795, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+\frac{b}{x^3}\right)^2} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{\left(ax^3+b\right)^2} dx$$

$$\Big\downarrow \text{ 749}$$

$$\frac{2\int \frac{1}{ax^3+b}dx}{3b} + \frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2\left(\frac{-\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}+\frac{x}{3b\left(ax^3+b\right)}$$

input

```
Int[1/((a + b/x^3)^2*x^6),x]
```

output

```
x/(3*b*(b + a*x^3)) + (2*(Log[b^(1/3) + a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) + (
-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]])/a^(1/3)) - Log[b^(
2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))/(3*b^(2/3))))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x}{3b(a\,x^3+b)}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^3+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{9ab}$	46
default	$\frac{x}{3b(a\,x^3+b)}+\frac{\frac{2\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9a\left(\frac{b}{a}\right)^{\frac{2}{3}}}}{b}$	112

input
```
int(1/(a+b/x^3)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/3*x/b/(a*x^3+b)+2/9/a/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*a+b))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.90

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx = \left[\frac{3\,ab^2x + 3\,\sqrt{\frac{1}{3}}(a^2bx^3+ab^2)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abx^3-3\,(ab^2)^{\frac{1}{3}}bx-b^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(ab^2)^{\frac{2}{3}}x-(ab^2)^{\frac{1}{3}}b\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}}{ax^3+b}\right) -}{9\,(a^2b^3x^3+ab^4}\right.$$

input
```
integrate(1/(a+b/x^3)^2/x^6,x, algorithm="fricas")
```

output
```
[1/9*(3*a*b^2*x + 3*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt(-(a*b^2)^(1/3)/a)*l
og((2*a*b*x^3 - 3*(a*b^2)^(1/3)*b*x - b^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a*b^
2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 + b)) - (a*x^
3 + b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) + 2*
(a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a^2*b^3*x^3 + a*b^4
), 1/9*(3*a*b^2*x + 6*sqrt(1/3)*(a^2*b*x^3 + a*b^2)*sqrt((a*b^2)^(1/3)/a)*
arctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/
a)/b^2) - (a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x^2 - (a*b^2)^(2/3)*x + (a*b^2
)^(1/3)*b) + 2*(a*x^3 + b)*(a*b^2)^(2/3)*log(a*b*x + (a*b^2)^(2/3)))/(a^2*
b^3*x^3 + a*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx = \frac{x}{3abx^3+3b^2} + \operatorname{RootSum}\left(729t^3ab^5-8,\left(t\mapsto t\log\left(\frac{9tb^2}{2}+x\right)\right)\right)$$

input
```
integrate(1/(a+b/x**3)**2/x**6,x)
```

output
```
x/(3*a*b*x**3 + 3*b**2) + RootSum(729*_t**3*a*b**5 - 8, Lambda(_t, _t*log(
9*_t*b**2/2 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx = \frac{x}{3\,(abx^3+b^2)} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{b}{a}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,ab\left(\frac{b}{a}\right)^{\frac{2}{3}}} + \frac{2\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(a+b/x^3)^2/x^6,x, algorithm="maxima")
```

output
```
1/3*x/(a*b*x^3 + b^2) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (b/a)^(1/3))
/(b/a)^(1/3))/(a*b*(b/a)^(2/3)) - 1/9*log(x^2 - x*(b/a)^(1/3) + (b/a)^(2/3
))/(a*b*(b/a)^(2/3)) + 2/9*log(x + (b/a)^(1/3))/(a*b*(b/a)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx = -\frac{2\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,b^2} + \frac{x}{3\,(ax^3+b)b} + \frac{2\sqrt{3}(-a^2b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,ab^2} + \frac{(-a^2b)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,ab^2}$$

input
```
integrate(1/(a+b/x^3)^2/x^6,x, algorithm="giac")
```

output
```
-2/9*(-b/a)^(1/3)*log(abs(x - (-b/a)^(1/3)))/b^2 + 1/3*x/((a*x^3 + b)*b) +
 2/9*sqrt(3)*(-a^2*b)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)
^(1/3))/(a*b^2) + 1/9*(-a^2*b)^(1/3)*log(x^2 + x*(-b/a)^(1/3) + (-b/a)^(2/
3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx = \frac{x}{3\,b\,\left(a\,x^3+b\right)}+\frac{2\,\ln\left(\frac{2\,a^{5/3}}{b^{2/3}}+\frac{2\,a^2\,x}{b}\right)}{9\,a^{1/3}\,b^{5/3}}+\frac{\ln\left(\frac{2\,a^2\,x}{b}+\frac{a^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{b^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,a^{1/3}\,b^{5/3}}-\frac{\ln\left(\frac{2\,a^2\,x}{b}-\frac{a^{5/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{b^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,a^{1/3}\,b^{5/3}}$$

input
```
int(1/(x^6*(a + b/x^3)^2),x)
```

output
```
x/(3*b*(b + a*x^3)) + (2*log((2*a^(5/3))/b^(2/3) + (2*a^2*x)/b))/(9*a^(1/3
)*b^(5/3)) + (log((2*a^2*x)/b + (a^(5/3)*(3^(1/2)*1i - 1))/b^(2/3))*(3^(1/
2)*1i - 1))/(9*a^(1/3)*b^(5/3)) - (log((2*a^2*x)/b - (a^(5/3)*(3^(1/2)*1i
+ 1))/b^(2/3))*(3^(1/2)*1i + 1))/(9*a^(1/3)*b^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^6}\,dx$$
$$= \frac{2b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)a\,x^3+2b^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)+3a^{\frac{1}{3}}bx-b^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)a\,x^3-b^{\frac{4}{3}}\log}{9a^{\frac{1}{3}}b^2\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^2/x^6,x)
```

output
```
(2*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*x
**3 + 2*b**(1/3)*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3))
)*b + 3*a**(1/3)*b*x - b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x +
b**(2/3))*a*x**3 - b**(1/3)*log(a**(2/3)*x**2 - b**(1/3)*a**(1/3)*x + b**(
2/3))*b + 2*b**(1/3)*log(a**(1/3)*x + b**(1/3))*a*x**3 + 2*b**(1/3)*log(a*
*(1/3)*x + b**(1/3))*b)/(9*a**(1/3)*b**2*(a*x**3 + b))
```

3.453 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 145

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8} \, dx = -\frac{1}{b^2 x} - \frac{a x^2}{3 b^2 (b+a x^3)} + \frac{4 \sqrt[3]{a} \arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3} b^{7/3}} + \frac{4\sqrt[3]{a}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{9 b^{7/3}} - \frac{2\sqrt[3]{a}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{9 b^{7/3}}$$

output
```
-1/b^2/x-1/3*a*x^2/b^2/(a*x^3+b)+4/9*a^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)
*x)*3^(1/2)/b^(1/3))*3^(1/2)/b^(7/3)+4/9*a^(1/3)*ln(b^(1/3)+a^(1/3)*x)/b^(
7/3)-2/9*a^(1/3)*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8}\,dx$$
$$=\frac{-\frac{9\sqrt[3]{b}}{x}-\frac{3a\sqrt[3]{b}x^2}{b+ax^3}+4\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)+4\sqrt[3]{a}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)-2\sqrt[3]{a}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{9b^{7/3}}$$

input
```
Integrate[1/((a + b/x^3)^2*x^8),x]
```

output
```
((-9*b^(1/3))/x - (3*a*b^(1/3)*x^2)/(b + a*x^3) + 4*Sqrt[3]*a^(1/3)*ArcTan
[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] + 4*a^(1/3)*Log[b^(1/3) + a^(1/3)*x]
 - 2*a^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(9*b^(7/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {795, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(a+\frac{b}{x^3}\right)^2}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{1}{x^2\left(ax^3+b\right)^2}dx$$
$$\Big\downarrow \text{ 819}$$
$$\frac{4\int \frac{1}{x^2(ax^3+b)}dx}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{4\left(-\frac{a\int \frac{x}{ax^3+b}dx}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$$\Big\downarrow \text{821}$$

$$\frac{4\left(-\frac{a\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{a}x+\sqrt[3]{b}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{4\left(-\frac{a\left(\frac{\int \frac{\sqrt[3]{a}x+\sqrt[3]{b}}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{4\left(-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$\Big\downarrow$ 25

$$\frac{4\left(-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x\right)}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{2\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$\Big\downarrow$ 27

$$\frac{4\left(-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$$\Big\downarrow \ 1082$$

$$\frac{4\left(-\frac{a\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}-\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{4\left(-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

$\Big\downarrow$ 1103

$$\frac{4\left(-\frac{a\left(\frac{\frac{\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{1}{bx}\right)}{3b}+\frac{1}{3bx\left(ax^3+b\right)}$$

input
```
Int[1/((a + b/x^3)^2*x^8),x]
```

output
```
1/(3*b*x*(b + a*x^3)) + (4*(-(1/(b*x)) - (a*(-1/3*Log[b^(1/3) + a^(1/3)*x]
/(a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*x)/b^(1/3))/Sqrt[3
]])/a^(1/3)) + Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2]/(2*a^(1/3)))
/(3*a^(1/3)*b^(1/3))))/b))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

method	result	size
risch	$\frac{-\frac{4a x^3}{3b^2}-\frac{1}{b}}{x(a x^3+b)}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(b^7\_Z^3-a\right)}\_R\ln\left(\left(-4\_R^3b^7+3a\right)x-b^5\_R^2\right)\right)}{9}$	73
default	$-\frac{1}{b^2x}-\frac{a\left(\frac{x^2}{3a x^3+3b}-\frac{4\ln\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{2\ln\left(x^2-\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9a\left(\frac{b}{a}\right)^{\frac{1}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9a\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{b^2}$	120

input
```
int(1/(a+b/x^3)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
(-4/3*a*x^3/b^2-1/b)/x/(a*x^3+b)+4/9*sum(_R*ln((-4*_R^3*b^7+3*a)*x-b^5*_R^
2),_R=RootOf(_Z^3*b^7-a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2x^8}\,dx = -\frac{12\,ax^3+4\,\sqrt{3}(ax^4+bx)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+2\,(ax^4+bx)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a}{b}\right)^{\frac{2}{3}}+\right.}{9\,(ab^2x^4+b^3x)}$$

input
```
integrate(1/(a+b/x^3)^2/x^8,x, algorithm="fricas")
```

output
```
-1/9*(12*a*x^3 + 4*sqrt(3)*(a*x^4 + b*x)*(a/b)^(1/3)*arctan(2/3*sqrt(3)*x*
(a/b)^(1/3) - 1/3*sqrt(3)) + 2*(a*x^4 + b*x)*(a/b)^(1/3)*log(a*x^2 - b*x*(
a/b)^(2/3) + b*(a/b)^(1/3)) - 4*(a*x^4 + b*x)*(a/b)^(1/3)*log(a*x + b*(a/b
)^(2/3)) + 9*b)/(a*b^2*x^4 + b^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8}\,dx = \frac{-4ax^3-3b}{3ab^2x^4+3b^3x} + \operatorname{RootSum}\left(729t^3b^7-64a, \left(t\mapsto t\log\left(\frac{81t^2b^5}{16a}+x\right)\right)\right)$$

input
```
integrate(1/(a+b/x**3)**2/x**8,x)
```

output
```
(-4*a*x**3 - 3*b)/(3*a*b**2*x**4 + 3*b**3*x) + RootSum(729*_t**3*b**7 - 64
*a, Lambda(_t, _t*log(81*_t**2*b**5/(16*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8}\,dx = -\frac{4\,ax^3+3\,b}{3\,(ab^2x^4+b^3x)} - \frac{4\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,b^2\left(\frac{b}{a}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x^2-x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,b^2\left(\frac{b}{a}\right)^{\frac{1}{3}}} + \frac{4\,\log\left(x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{9\,b^2\left(\frac{b}{a}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/(a+b/x^3)^2/x^8,x, algorithm="maxima")
```

output
```
-1/3*(4*a*x^3 + 3*b)/(a*b^2*x^4 + b^3*x) - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - (b/a)^(1/3))/(b/a)^(1/3))/(b^2*(b/a)^(1/3)) - 2/9*log(x^2 - x*(b/a)
^(1/3) + (b/a)^(2/3))/(b^2*(b/a)^(1/3)) + 4/9*log(x + (b/a)^(1/3))/(b^2*(b
/a)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8}\,dx = \frac{4\,a\left(-\frac{b}{a}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{9\,b^3} + \frac{4\,\sqrt{3}\left(-a^2b\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{9\,ab^3} - \frac{4\,ax^3+3\,b}{3\,(ax^4+bx)b^2} - \frac{2\left(-a^2b\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)}{9\,ab^3}$$

input
```
integrate(1/(a+b/x^3)^2/x^8,x, algorithm="giac")
```

output
```
4/9*a*(-b/a)^(2/3)*log(abs(x - (-b/a)^(1/3)))/b^3 + 4/9*sqrt(3)*(-a^2*b)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-b/a)^(1/3))/(-b/a)^(1/3))/(a*b^3) - 1/3*(
4*a*x^3 + 3*b)/((a*x^4 + b*x)*b^2) - 2/9*(-a^2*b)^(2/3)*log(x^2 + x*(-b/a)
^(1/3) + (-b/a)^(2/3))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8}\,dx = \frac{4\,a^{1/3}\,\ln\left(a^{1/3}\,x+b^{1/3}\right)}{9\,b^{7/3}} - \frac{\frac{1}{b}+\frac{4\,a\,x^3}{3\,b^2}}{a\,x^4+b\,x} - \frac{4\,a^{1/3}\,\ln\left(4\,a^{1/3}\,x-2\,b^{1/3}+\sqrt{3}\,b^{1/3}\,2\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,b^{7/3}} + \frac{a^{1/3}\,\ln\left(4\,a^{1/3}\,x-2\,b^{1/3}-\sqrt{3}\,b^{1/3}\,2\mathrm{i}\right)\left(-\frac{2}{9}+\frac{\sqrt{3}\,2\mathrm{i}}{9}\right)}{b^{7/3}}$$

input
```
int(1/(x^8*(a + b/x^3)^2),x)
```

output
```
(4*a^(1/3)*log(a^(1/3)*x + b^(1/3)))/(9*b^(7/3)) - (1/b + (4*a*x^3)/(3*b^2
))/(b*x + a*x^4) - (4*a^(1/3)*log(3^(1/2)*b^(1/3)*2i + 4*a^(1/3)*x - 2*b^(
1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*b^(7/3)) + (a^(1/3)*log(4*a^(1/3)*x - 3^(
1/2)*b^(1/3)*2i - 2*b^(1/3))*((3^(1/2)*2i)/9 - 2/9))/b^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^2 x^8}\,dx$$
$$= \frac{-4\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)a^2x^4-4\sqrt{3}\,atan\left(\frac{2a^{\frac{1}{3}}x-b^{\frac{1}{3}}}{b^{\frac{1}{3}}\sqrt{3}}\right)abx-12b^{\frac{1}{3}}a^{\frac{5}{3}}x^3-9b^{\frac{4}{3}}a^{\frac{2}{3}}-2\log\left(a^{\frac{2}{3}}x^2-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}\right.}{9b^{\frac{7}{3}}a^{\frac{2}{3}}x\,(a\,x^3+b}$$

input
```
int(1/(a+b/x^3)^2/x^8,x)
```

output
```
( - 4*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a**2*x**4
 - 4*sqrt(3)*atan((2*a**(1/3)*x - b**(1/3))/(b**(1/3)*sqrt(3)))*a*b*x - 12
*b**(1/3)*a**(2/3)*a*x**3 - 9*b**(1/3)*a**(2/3)*b - 2*log(a**(2/3)*x**2 -
b**(1/3)*a**(1/3)*x + b**(2/3))*a**2*x**4 - 2*log(a**(2/3)*x**2 - b**(1/3)
*a**(1/3)*x + b**(2/3))*a*b*x + 4*log(a**(1/3)*x + b**(1/3))*a**2*x**4 + 4
*log(a**(1/3)*x + b**(1/3))*a*b*x)/(9*b**(1/3)*a**(2/3)*b**2*x*(a*x**3 + b
))
```

3.454 $\int \sqrt{a+\frac{b}{x^3}}x^5\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx=\frac{b\sqrt{a+\frac{b}{x^3}}x^3}{12a}+\frac{1}{6}\sqrt{a+\frac{b}{x^3}}x^6-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{12a^{3/2}}$$

output
```
1/12*b*(a+b/x^3)^(1/2)*x^3/a+1/6*(a+b/x^3)^(1/2)*x^6-1/12*b^2*arctanh((a+b
/x^3)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx=\frac{\sqrt{a+\frac{b}{x^3}}x^3(b+2ax^3)}{12a}-\frac{b^2\sqrt{a+\frac{b}{x^3}}x^{3/2}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{12a^{3/2}\sqrt{b+ax^3}}$$

input
```
Integrate[Sqrt[a + b/x^3]*x^5,x]
```

output
```
(Sqrt[a + b/x^3]*x^3*(b + 2*a*x^3))/(12*a) - (b^2*Sqrt[a + b/x^3]*x^(3/2)*
Log[Sqrt[a]*x^(3/2) + Sqrt[b + a*x^3]])/(12*a^(3/2)*Sqrt[b + a*x^3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \sqrt{a+\frac{b}{x^3}}x^9d\frac{1}{x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\sqrt{a+\frac{b}{x^3}}-\frac{1}{4}b\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\sqrt{a+\frac{b}{x^3}}-\frac{1}{4}b\left(-\frac{b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}}{2a}-\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\sqrt{a+\frac{b}{x^3}}-\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{a}-\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\sqrt{a+\frac{b}{x^3}}-\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)\right)$$

input
```
Int[Sqrt[a + b/x^3]*x^5,x]
```

output
```
((Sqrt[a + b/x^3]*x^6)/2 - (b*(-((Sqrt[a + b/x^3]*x^3)/a) + (b*ArcTanh[Sqr
t[a + b/x^3]/Sqrt[a]])/a^(3/2)))/4)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{x^{3}\left(2 a \,x^{3}+b \right) \sqrt{\frac{a \,x^{3}+b}{x^{3}}}}{12 a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x \left(a \,x^{3}+b \right)}}{x^{2} \sqrt{a}}\right) b^{2} \sqrt{\frac{a \,x^{3}+b}{x^{3}}}\, x \sqrt{x \left(a \,x^{3}+b \right)}}{12 a^{\frac{3}{2}} \left(a \,x^{3}+b \right)}$	92
default	$\frac{\sqrt{\frac{a \,x^{3}+b}{x^{3}}}\, x^{2} \left(2 \sqrt{x \left(a \,x^{3}+b \right)}\, a^{\frac{3}{2}} x^{4}+b x \sqrt{x \left(a \,x^{3}+b \right)}\, \sqrt{a}-\operatorname{arctanh}\left(\frac{\sqrt{x \left(a \,x^{3}+b \right)}}{x^{2} \sqrt{a}}\right) b^{2}\right)}{12 \sqrt{x \left(a \,x^{3}+b \right)}\, a^{\frac{3}{2}}}$	94

input
```
int((a+b/x^3)^(1/2)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(2*a*x^3+b)/a*((a*x^3+b)/x^3)^(1/2)-1/12/a^(3/2)*arctanh((x*(a*x^
3+b))^(1/2)/x^2/a^(1/2))*b^2*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(
a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.58

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx$$

$$= \left[\frac{\sqrt{a}b^{2}\log\left(-8\,a^{2}x^{6}-8\,abx^{3}-b^{2}+4\,(2\,ax^{6}+bx^{3})\sqrt{a}\sqrt{\frac{ax^{3}+b}{x^{3}}}\right)+4\,(2\,a^{2}x^{6}+abx^{3})\sqrt{\frac{ax^{3}+b}{x^{3}}}}{48\,a^{2}}, \frac{\sqrt{-a}b^{2}\,\mathrm{ar}\ldots}{\ldots}\right.$$

input
```
integrate((a+b/x^3)^(1/2)*x^5,x, algorithm="fricas")
```

output
```
[1/48*(sqrt(a)*b^2*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 + 4*(2*a*x^6 + b*x^3)*
sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^2*x^6 + a*b*x^3)*sqrt((a*x^3 + b)/
x^3))/a^2, 1/24*(sqrt(-a)*b^2*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt((a*x^
3 + b)/x^3)/(a^2*x^3 + a*b)) + 2*(2*a^2*x^6 + a*b*x^3)*sqrt((a*x^3 + b)/x^
3))/a^2]
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx=\frac{ax^{\frac{15}{2}}}{6\sqrt{b}\sqrt{\frac{ax^3}{b}+1}}+\frac{\sqrt{b}x^{\frac{9}{2}}}{4\sqrt{\frac{ax^3}{b}+1}}+\frac{b^{\frac{3}{2}}x^{\frac{3}{2}}}{12a\sqrt{\frac{ax^3}{b}+1}}-\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{12a^{\frac{3}{2}}}$$

input
```
integrate((a+b/x**3)**(1/2)*x**5,x)
```

output
```
a*x**(15/2)/(6*sqrt(b)*sqrt(a*x**3/b + 1)) + sqrt(b)*x**(9/2)/(4*sqrt(a*x*
*3/b + 1)) + b**(3/2)*x**(3/2)/(12*a*sqrt(a*x**3/b + 1)) - b**2*asinh(sqrt
(a)*x**(3/2)/sqrt(b))/(12*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx=\frac{b^2\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{24\,a^{\frac{3}{2}}}+\frac{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}b^2+\sqrt{a+\frac{b}{x^3}}ab^2}{12\left(\left(a+\frac{b}{x^3}\right)^2a-2\left(a+\frac{b}{x^3}\right)a^2+a^3\right)}$$

input
```
integrate((a+b/x^3)^(1/2)*x^5,x, algorithm="maxima")
```

output
```
1/24*b^2*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(3
/2) + 1/12*((a + b/x^3)^(3/2)*b^2 + sqrt(a + b/x^3)*a*b^2)/((a + b/x^3)^2*
a - 2*(a + b/x^3)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx = \frac{1}{12}\,\sqrt{ax^4+bx}\left(2\,x^3+\frac{b}{a}\right)x+\frac{b^2\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{12\,\sqrt{-a}a}$$

input
```
integrate((a+b/x^3)^(1/2)*x^5,x, algorithm="giac")
```

output
```
1/12*sqrt(a*x^4 + b*x)*(2*x^3 + b/a)*x + 1/12*b^2*arctan(sqrt(a + b/x^3)/s
qrt(-a))/(sqrt(-a)*a)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx = \frac{x^6\,\sqrt{a+\frac{b}{x^3}}}{6}+\frac{b^2\,\ln\left(x^6\,\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)^3\,\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)\right)}{24\,a^{3/2}}+\frac{b\,x^3\,\sqrt{a+\frac{b}{x^3}}}{12\,a}$$

input
```
int(x^5*(a + b/x^3)^(1/2),x)
```

output
```
(x^6*(a + b/x^3)^(1/2))/6 + (b^2*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))^3*(
(a + b/x^3)^(1/2) + a^(1/2))))/(24*a^(3/2)) + (b*x^3*(a + b/x^3)^(1/2))/(1
2*a)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \sqrt{a+\frac{b}{x^3}}x^5\,dx$$
$$=\frac{4\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^4+2\sqrt{x}\,\sqrt{a\,x^3+b}\,abx+\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)b^2-\sqrt{a}\log\left(\sqrt{a\,x^3+b}+\sqrt{x}\right.}{24a^2}$$

input
```
int((a+b/x^3)^(1/2)*x^5,x)
```

output
```
(4*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**4 + 2*sqrt(x)*sqrt(a*x**3 + b)*a*b*x +
 sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*b**2 - sqrt(a)*log(sqrt
(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*b**2)/(24*a**2)
```

3.455 $\int \sqrt{a+\frac{b}{x^3}}x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx=\frac{1}{3}\sqrt{a+\frac{b}{x^3}}x^3+\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
1/3*(a+b/x^3)^(1/2)*x^3+1/3*b*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx=\frac{1}{3}\sqrt{a+\frac{b}{x^3}}x^{3/2}\left(x^{3/2}+\frac{b\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{\sqrt{a}\sqrt{b+ax^3}}\right)$$

input
```
Integrate[Sqrt[a + b/x^3]*x^2,x]
```

output
```
(Sqrt[a + b/x^3]*x^(3/2)*(x^(3/2) + (b*Log[Sqrt[a]*x^(3/2) + Sqrt[b + a*x^
3]])/(Sqrt[a]*Sqrt[b + a*x^3])))/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \sqrt{a+\frac{b}{x^3}}x^6d\frac{1}{x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(x^3\sqrt{a+\frac{b}{x^3}}-\frac{1}{2}b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(x^3\sqrt{a+\frac{b}{x^3}}-\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{\sqrt{a}}+x^3\sqrt{a+\frac{b}{x^3}}\right)$$

input
```
Int[Sqrt[a + b/x^3]*x^2,x]
```

output
```
(Sqrt[a + b/x^3]*x^3 + (b*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/Sqrt[a])/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x^{2}\left(\sqrt{x\left(a\,x^{3}+b\right)}\,x\sqrt{a}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)b\right)}{3\sqrt{x\left(a\,x^{3}+b\right)}\,\sqrt{a}}$	71
risch	$\frac{x^{3}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{3}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)b\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x\sqrt{x\left(a\,x^{3}+b\right)}}{3\sqrt{a}\,\left(a\,x^{3}+b\right)}$	79

input
```
int((a+b/x^3)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*x^3+b)/x^3)^(1/2)*x^2/(x*(a*x^3+b))^(1/2)*((x*(a*x^3+b))^(1/2)*x*a
^(1/2)+arctanh((x*(a*x^3+b))^(1/2)/x^2/a^(1/2))*b)/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(35) = 70$.

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.36

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx = \left[\frac{4\,ax^3\sqrt{\frac{ax^3+b}{x^3}}+\sqrt{a}b\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\,(2\,ax^6+bx^3)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)}{12\,a}, \frac{2\,ax^3\sqrt{\frac{ax^3+b}{x^3}}-\sqrt{-a}b\right.$$

input
```
integrate((a+b/x^3)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a*x^3*sqrt((a*x^3 + b)/x^3) + sqrt(a)*b*log(-8*a^2*x^6 - 8*a*b*x^
3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)))/a, 1/6*(2*a*
x^3*sqrt((a*x^3 + b)/x^3) - sqrt(-a)*b*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*s
qrt((a*x^3 + b)/x^3)/(a^2*x^3 + a*b)))/a]
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx = \frac{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}{3} + \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3\sqrt{a}}$$

input
```
integrate((a+b/x**3)**(1/2)*x**2,x)
```

output
```
sqrt(b)*x**(3/2)*sqrt(a*x**3/b + 1)/3 + b*asinh(sqrt(a)*x**(3/2)/sqrt(b))/
(3*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx = \frac{1}{3}\sqrt{a+\frac{b}{x^3}}x^3 - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{6\sqrt{a}}$$

input
```
integrate((a+b/x^3)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(a + b/x^3)*x^3 - 1/6*b*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a +
b/x^3) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx = \frac{1}{3}\sqrt{ax^4+bx}x - \frac{b\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

input
```
integrate((a+b/x^3)^(1/2)*x^2,x, algorithm="giac")
```

output
```
1/3*sqrt(a*x^4 + b*x)*x - 1/3*b*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx = \frac{x^3\sqrt{a+\frac{b}{x^3}}}{3} + \frac{b\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{6\sqrt{a}}$$

input
```
int(x^2*(a + b/x^3)^(1/2),x)
```

output
```
(x^3*(a + b/x^3)^(1/2))/3 + (b*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))*((a +
 b/x^3)^(1/2) + a^(1/2))^3))/(6*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \sqrt{a+\frac{b}{x^3}}x^2\,dx = \frac{2\sqrt{x}\,\sqrt{a\,x^3+b}\,ax-\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)b+\sqrt{a}\log\left(\sqrt{a\,x^3+b}+\sqrt{x}\,\sqrt{a}\,x\right)b}{6a}$$

input
```
int((a+b/x^3)^(1/2)*x^2,x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b)*a*x - sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*s
qrt(a)*x)*b + sqrt(a)*log(sqrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*b)/(6*a)
```

3.456 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x} \, dx = -\frac{2}{3}\sqrt{a+\frac{b}{x^3}}+\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)$$

output
```
-2/3*(a+b/x^3)^(1/2)+2/3*a^(1/2)*arctanh((a+b/x^3)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x} \, dx = -\frac{2}{3}\sqrt{a+\frac{b}{x^3}}+\frac{2\sqrt{a}\sqrt{a+\frac{b}{x^3}}x^{3/2}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{3\sqrt{b+ax^3}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x,x]
```

output
```
(-2*Sqrt[a + b/x^3])/3 + (2*Sqrt[a]*Sqrt[a + b/x^3]*x^(3/2)*Log[Sqrt[a]*x^
(3/2) + Sqrt[b + a*x^3]])/(3*Sqrt[b + a*x^3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \sqrt{a+\frac{b}{x^3}}x^3d\frac{1}{x^3}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(-a\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}-2\sqrt{a+\frac{b}{x^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{2a\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{b}-2\sqrt{a+\frac{b}{x^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x^3}}\right)$$

input
```
Int[Sqrt[a + b/x^3]/x,x]
```

output
```
(-2*Sqrt[a + b/x^3] + 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 0.61 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{2\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)x^{2}-\sqrt{x\left(a\,x^{3}+b\right)}\right)}{3\sqrt{x\left(a\,x^{3}+b\right)}}$	67
risch	$-\frac{2\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{3}+\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x\sqrt{x\left(a\,x^{3}+b\right)}}{3\left(a\,x^{3}+b\right)}$	75

input
```
int((a+b/x^3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*((a*x^3+b)/x^3)^(1/2)*(a^(1/2)*arctanh((x*(a*x^3+b))^(1/2)/x^2/a^(1/2)
)*x^2-(x*(a*x^3+b))^(1/2))/(x*(a*x^3+b))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(31) = 62$.

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx = \left[\frac{1}{6}\sqrt{a}\log\left(-8\,a^2x^6 - 8\,abx^3 - b^2 - 4\left(2\,ax^6 + bx^3\right)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right) - \frac{2}{3}\sqrt{\frac{ax^3+b}{x^3}}, -\frac{1}{3}\sqrt{-a}\arctan\left(\frac{(2\,ax^3+b)\sqrt{-a}\sqrt{\frac{ax^3+b}{x^3}}}{2\left(a^2x^3+ab\right)}\right) - \frac{2}{3}\sqrt{\frac{ax^3+b}{x^3}}\right]$$

input
```
integrate((a+b/x^3)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/6*sqrt(a)*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a
)*sqrt((a*x^3 + b)/x^3)) - 2/3*sqrt((a*x^3 + b)/x^3), -1/3*sqrt(-a)*arctan
(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt((a*x^3 + b)/x^3)/(a^2*x^3 + a*b)) - 2/3*s
qrt((a*x^3 + b)/x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

Time = 0.94 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx = \frac{2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3} - \frac{2ax^{\frac{3}{2}}}{3\sqrt{b}\sqrt{\frac{ax^3}{b}+1}} - \frac{2\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}$$

input
```
integrate((a+b/x**3)**(1/2)/x,x)
```

output
```
2*sqrt(a)*asinh(sqrt(a)*x**(3/2)/sqrt(b))/3 - 2*a*x**(3/2)/(3*sqrt(b)*sqrt
(a*x**3/b + 1)) - 2*sqrt(b)/(3*x**(3/2)*sqrt(a*x**3/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx = -\frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right) - \frac{2}{3}\sqrt{a+\frac{b}{x^3}}$$

input
```
integrate((a+b/x^3)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/3*sqrt(a)*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))
- 2/3*sqrt(a + b/x^3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx = -\frac{2\,a\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} - \frac{2}{3}\,\sqrt{a+\frac{b}{x^3}}$$

input
```
integrate((a+b/x^3)^(1/2)/x,x, algorithm="giac")
```

output
```
-2/3*a*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt(-a) - 2/3*sqrt(a + b/x^3)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx = \frac{2\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3} - \frac{2\,\sqrt{a+\frac{b}{x^3}}}{3}$$

input
```
int((a + b/x^3)^(1/2)/x,x)
```

output
```
(2*a^(1/2)*atanh((a + b/x^3)^(1/2)/a^(1/2)))/3 - (2*(a + b/x^3)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}\,dx = \frac{-2\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)x+\sqrt{x}\,\sqrt{a}\log\left(\sqrt{a\,x^3+b}+\sqrt{x}\,\sqrt{a}\,x\right)x}{3\sqrt{x}\,x}$$

input
```
int((a+b/x^3)^(1/2)/x,x)
```

output

```
( - 2*sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*sq
rt(a)*x)*x + sqrt(x)*sqrt(a)*log(sqrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*x)/
(3*sqrt(x)*x)
```

3.457 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4} dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b}$$

output
```
-2/9*(a+b/x^3)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4} dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^4,x]
```

output
```
(-2*(a + b/x^3)^(3/2))/(9*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4} \, dx$$

$$\Big\downarrow \ 793$$

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b}$$

input
```
Int[Sqrt[a + b/x^3]/x^4,x]
```

output
```
(-2*(a + b/x^3)^(3/2))/(9*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{2}}}{9b}$	15
orering	$-\frac{2\left(a\,x^{3}+b\right)\sqrt{a+\frac{b}{x^{3}}}}{9x^{3}b}$	25
gosper	$-\frac{2\left(a\,x^{3}+b\right)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{9x^{3}b}$	29
risch	$-\frac{2\left(a\,x^{3}+b\right)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{9x^{3}b}$	29
trager	$-\frac{2\left(a\,x^{3}+b\right)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{9x^{3}b}$	33
default	$-\frac{2\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,\sqrt{a\,x^{4}+bx}\,\left(a\,x^{3}+b\right)}{9x^{3}\sqrt{x\left(a\,x^{3}+b\right)}\,b}$	51

input
```
int((a+b/x^3)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(a+b/x^3)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx=-\frac{2\left(ax^3+b\right)\sqrt{\frac{ax^3+b}{x^3}}}{9\,bx^3}$$

input
```
integrate((a+b/x^3)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/9*(a*x^3 + b)*sqrt((a*x^3 + b)/x^3)/(b*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx = -\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{9b} - \frac{2\sqrt{a}\sqrt{1+\frac{b}{ax^3}}}{9x^3}$$

input
```
integrate((a+b/x**3)**(1/2)/x**4,x)
```

output
```
-2*a**(3/2)*sqrt(1 + b/(a*x**3))/(9*b) - 2*sqrt(a)*sqrt(1 + b/(a*x**3))/(9
*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate((a+b/x^3)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x^3)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate((a+b/x^3)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2/9*(a + b/x^3)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx=-\frac{2\,\sqrt{a+\frac{b}{x^3}}\,\left(a\,x^3+b\right)}{9\,b\,x^3}$$

input
```
int((a + b/x^3)^(1/2)/x^4,x)
```

output
```
-(2*(a + b/x^3)^(1/2)*(b + a*x^3))/(9*b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}\,dx=-\frac{2\sqrt{a\,x^3+b}\,\left(a\,x^3+b\right)}{9\sqrt{x}\,b\,x^4}$$

input
```
int((a+b/x^3)^(1/2)/x^4,x)
```

output
```
( - 2*sqrt(a*x**3 + b)*(a*x**3 + b))/(9*sqrt(x)*b*x**4)
```

3.458 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx=\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2}$$

output
```
2/9*a*(a+b/x^3)^(3/2)/b^2-2/15*(a+b/x^3)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx=-\frac{2\sqrt{a+\frac{b}{x^3}}\,(3b^2+abx^3-2a^2x^6)}{45b^2x^6}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^7,x]
```

output
```
(-2*Sqrt[a + b/x^3]*(3*b^2 + a*b*x^3 - 2*a^2*x^6))/(45*b^2*x^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}dx$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{3}\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}d\frac{1}{x^3}$$

$$\Big\downarrow \ 53$$

$$-\frac{1}{3}\int \left(\frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{b}-\frac{a\sqrt{a+\frac{b}{x^3}}}{b}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^2}\right)$$

input
```
Int[Sqrt[a + b/x^3]/x^7,x]
```

output
```
((2*a*(a + b/x^3)^(3/2))/(3*b^2) - (2*(a + b/x^3)^(5/2))/(5*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
orering	$\frac{2(2a\,x^3-3b)\,(a\,x^3+b)\,\sqrt{a+\frac{b}{x^3}}}{45b^2x^6}$	35
gosper	$\frac{2(a\,x^3+b)\,(2a\,x^3-3b)\,\sqrt{\frac{a\,x^3+b}{x^3}}}{45b^2x^6}$	39
risch	$\frac{2\sqrt{\frac{a\,x^3+b}{x^3}}\,(2a^2x^6-ab\,x^3-3b^2)}{45x^6b^2}$	43
trager	$\frac{2(2a^2x^6-ab\,x^3-3b^2)\,\sqrt{-\frac{-a\,x^3-b}{x^3}}}{45x^6b^2}$	47
default	$\frac{2\sqrt{\frac{a\,x^3+b}{x^3}}\,\sqrt{a\,x^4+bx}\,(2a^2x^6-ab\,x^3-3b^2)}{45x^6\sqrt{x(a\,x^3+b)}\,b^2}$	65

input
```
int((a+b/x^3)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
2/45*(2*a*x^3-3*b)/b^2/x^6*(a*x^3+b)*(a+b/x^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx=\frac{2\,(2\,a^2x^6-abx^3-3\,b^2)\sqrt{\frac{ax^3+b}{x^3}}}{45\,b^2x^6}$$

input
```
integrate((a+b/x^3)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
2/45*(2*a^2*x^6 - a*b*x^3 - 3*b^2)*sqrt((a*x^3 + b)/x^3)/(b^2*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(34) = 68.

Time = 1.05 (sec) , antiderivative size = 313, normalized size of antiderivative = 8.24

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx=\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}x^9\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}\sqrt{\frac{ax^3}{b}+1}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}}-\frac{4a^6bx^{\frac{21}{2}}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}}-\frac{4a^5b^2x^{\frac{15}{2}}}{45a^{\frac{7}{2}}b^3x^{\frac{21}{2}}+45a^{\frac{5}{2}}b^4x^{\frac{15}{2}}}$$

input
```
integrate((a+b/x**3)**(1/2)/x**7,x)
```

output
```
4*a**(11/2)*b**(3/2)*x**9*sqrt(a*x**3/b + 1)/(45*a**(7/2)*b**3*x**(21/2) +
 45*a**(5/2)*b**4*x**(15/2)) + 2*a**(9/2)*b**(5/2)*x**6*sqrt(a*x**3/b + 1)
/(45*a**(7/2)*b**3*x**(21/2) + 45*a**(5/2)*b**4*x**(15/2)) - 8*a**(7/2)*b*
*(7/2)*x**3*sqrt(a*x**3/b + 1)/(45*a**(7/2)*b**3*x**(21/2) + 45*a**(5/2)*b
**4*x**(15/2)) - 6*a**(5/2)*b**(9/2)*sqrt(a*x**3/b + 1)/(45*a**(7/2)*b**3*
x**(21/2) + 45*a**(5/2)*b**4*x**(15/2)) - 4*a**6*b*x**(21/2)/(45*a**(7/2)*
b**3*x**(21/2) + 45*a**(5/2)*b**4*x**(15/2)) - 4*a**5*b**2*x**(15/2)/(45*a
**(7/2)*b**3*x**(21/2) + 45*a**(5/2)*b**4*x**(15/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b^2}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a}{9\,b^2}$$

input
```
integrate((a+b/x^3)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-2/15*(a + b/x^3)^(5/2)/b^2 + 2/9*(a + b/x^3)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx = -\frac{2\left(3\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}-5\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a\right)}{45\,b^2}$$

input
```
integrate((a+b/x^3)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-2/45*(3*(a + b/x^3)^(5/2) - 5*(a + b/x^3)^(3/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}\,(-2\,a^2\,x^6+a\,b\,x^3+3\,b^2)}{45\,b^2\,x^6}$$

input
```
int((a + b/x^3)^(1/2)/x^7,x)
```

output
```
-(2*(a + b/x^3)^(1/2)*(3*b^2 - 2*a^2*x^6 + a*b*x^3))/(45*b^2*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}\,dx = \frac{2\sqrt{a\,x^3+b}\,(2a^2x^6-ab\,x^3-3b^2)}{45\sqrt{x}\,b^2x^7}$$

input
```
int((a+b/x^3)^(1/2)/x^7,x)
```

output
```
(2*sqrt(a*x**3 + b)*(2*a**2*x**6 - a*b*x**3 - 3*b**2))/(45*sqrt(x)*b**2*x*
*7)
```

3.459 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}} dx = -\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}$$

output
```
-2/9*a^2*(a+b/x^3)^(3/2)/b^3+4/15*a*(a+b/x^3)^(5/2)/b^3-2/21*(a+b/x^3)^(7/
2)/b^3
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}} dx = -\frac{2\sqrt{a+\frac{b}{x^3}}(15b^3+3ab^2x^3-4a^2bx^6+8a^3x^9)}{315b^3x^9}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^10,x]
```

output
```
(-2*Sqrt[a + b/x^3]*(15*b^3 + 3*a*b^2*x^3 - 4*a^2*b*x^6 + 8*a^3*x^9))/(315
*b^3*x^9)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(\frac{\left(a+\frac{b}{x^3}\right)^{5/2}}{b^2}-\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{b^2}+\frac{a^2\sqrt{a+\frac{b}{x^3}}}{b^2}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^3}\right)$$

input
```
Int[Sqrt[a + b/x^3]/x^10,x]
```

output
```
((-2*a^2*(a + b/x^3)^(3/2))/(3*b^3) + (4*a*(a + b/x^3)^(5/2))/(5*b^3) - (2
*(a + b/x^3)^(7/2))/(7*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

method	result	size
orering	$-\frac{2\left(8a^{2}x^{6}-12ab\,x^{3}+15b^{2}\right)\left(a\,x^{3}+b\right)\sqrt{a+\frac{b}{x^{3}}}}{315b^{3}x^{9}}$	46
gosper	$-\frac{2\left(a\,x^{3}+b\right)\left(8a^{2}x^{6}-12ab\,x^{3}+15b^{2}\right)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{315b^{3}x^{9}}$	50
risch	$-\frac{2\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,\left(8a^{3}x^{9}-4a^{2}b\,x^{6}+3a\,b^{2}x^{3}+15b^{3}\right)}{315x^{9}b^{3}}$	54
trager	$-\frac{2\left(8a^{3}x^{9}-4a^{2}b\,x^{6}+3a\,b^{2}x^{3}+15b^{3}\right)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{315x^{9}b^{3}}$	58
default	$-\frac{2\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,\sqrt{a\,x^{4}+bx}\,\left(8a^{3}x^{9}-4a^{2}b\,x^{6}+3a\,b^{2}x^{3}+15b^{3}\right)}{315x^{9}\sqrt{x\left(a\,x^{3}+b\right)}\,b^{3}}$	76

input
```
int((a+b/x^3)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-2/315*(8*a^2*x^6-12*a*b*x^3+15*b^2)/b^3/x^9*(a*x^3+b)*(a+b/x^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx = -\frac{2\,(8\,a^3x^9-4\,a^2bx^6+3\,ab^2x^3+15\,b^3)\sqrt{\frac{ax^3+b}{x^3}}}{315\,b^3x^9}$$

input
```
integrate((a+b/x^3)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
-2/315*(8*a^3*x^9 - 4*a^2*b*x^6 + 3*a*b^2*x^3 + 15*b^3)*sqrt((a*x^3 + b)/x
^3)/(b^3*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. 2(54) = 108.

Time = 1.45 (sec) , antiderivative size = 913, normalized size of antiderivative = 15.47

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**3)**(1/2)/x**10,x)
```

output
```
-16*a**(19/2)*b**(9/2)*x**18*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39
/2) + 945*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**
(7/2)*b**10*x**(21/2)) - 40*a**(17/2)*b**(11/2)*x**15*sqrt(a*x**3/b + 1)/(
315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)
*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) - 30*a**(15/2)*b**(13/2)*x
**12*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8
*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) -
 40*a**(13/2)*b**(15/2)*x**9*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39
/2) + 945*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**
(7/2)*b**10*x**(21/2)) - 100*a**(11/2)*b**(17/2)*x**6*sqrt(a*x**3/b + 1)/(
315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)
*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) - 96*a**(9/2)*b**(19/2)*x*
*3*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39/2) + 945*a**(11/2)*b**8*x
**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b**10*x**(21/2)) - 3
0*a**(7/2)*b**(21/2)*sqrt(a*x**3/b + 1)/(315*a**(13/2)*b**7*x**(39/2) + 94
5*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b*
*10*x**(21/2)) + 16*a**10*b**4*x**(39/2)/(315*a**(13/2)*b**7*x**(39/2) + 9
45*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2)*b
**10*x**(21/2)) + 48*a**9*b**5*x**(33/2)/(315*a**(13/2)*b**7*x**(39/2) + 9
45*a**(11/2)*b**8*x**(33/2) + 945*a**(9/2)*b**9*x**(27/2) + 315*a**(7/2...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}}{21\,b^3} + \frac{4\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a}{15\,b^3} - \frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2}{9\,b^3}$$

input
```
integrate((a+b/x^3)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
-2/21*(a + b/x^3)^(7/2)/b^3 + 4/15*(a + b/x^3)^(5/2)*a/b^3 - 2/9*(a + b/x^
3)^(3/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx = -\frac{2\left(15\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}} - 42\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a + 35\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2\right)}{315\,b^3}$$

input
```
integrate((a+b/x^3)^(1/2)/x^10,x, algorithm="giac")
```

output
```
-2/315*(15*(a + b/x^3)^(7/2) - 42*(a + b/x^3)^(5/2)*a + 35*(a + b/x^3)^(3/
2)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx = \frac{8\,a^2\,\sqrt{a+\frac{b}{x^3}}}{315\,b^2\,x^3} - \frac{16\,a^3\,\sqrt{a+\frac{b}{x^3}}}{315\,b^3} - \frac{2\,a\,\sqrt{a+\frac{b}{x^3}}}{105\,b\,x^6} - \frac{2\,\sqrt{a+\frac{b}{x^3}}}{21\,x^9}$$

input
```
int((a + b/x^3)^(1/2)/x^10,x)
```

output
```
(8*a^2*(a + b/x^3)^(1/2))/(315*b^2*x^3) - (16*a^3*(a + b/x^3)^(1/2))/(315*
b^3) - (2*a*(a + b/x^3)^(1/2))/(105*b*x^6) - (2*(a + b/x^3)^(1/2))/(21*x^9
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{10}}\,dx = \frac{2\sqrt{a\,x^3+b}\,(-8a^3x^9+4a^2b\,x^6-3a\,b^2x^3-15b^3)}{315\sqrt{x}\,b^3x^{10}}$$

input
```
int((a+b/x^3)^(1/2)/x^10,x)
```

output
```
(2*sqrt(a*x**3 + b)*( - 8*a**3*x**9 + 4*a**2*b*x**6 - 3*a*b**2*x**3 - 15*b
**3))/(315*sqrt(x)*b**3*x**10)
```

3.460 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}} dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}} dx = \frac{2a^3\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^4} - \frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4} + \frac{2a\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4} - \frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^4}$$

output
```
2/9*a^3*(a+b/x^3)^(3/2)/b^4-2/5*a^2*(a+b/x^3)^(5/2)/b^4+2/7*a*(a+b/x^3)^(7
/2)/b^4-2/27*(a+b/x^3)^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}} dx = -\frac{2\sqrt{a+\frac{b}{x^3}}(35b^4+5ab^3x^3-6a^2b^2x^6+8a^3bx^9-16a^4x^{12})}{945b^4x^{12}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^13,x]
```

output
```
(-2*Sqrt[a + b/x^3]*(35*b^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^6 + 8*a^3*b*x^9 -
16*a^4*x^12))/(945*b^4*x^12)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(\frac{\left(a+\frac{b}{x^3}\right)^{7/2}}{b^3}-\frac{3a\left(a+\frac{b}{x^3}\right)^{5/2}}{b^3}+\frac{3a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{b^3}-\frac{a^3\sqrt{a+\frac{b}{x^3}}}{b^3}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^3\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4}-\frac{6a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{9b^4}+\frac{6a\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}\right)$$

input
```
Int[Sqrt[a + b/x^3]/x^13,x]
```

output
```
((2*a^3*(a + b/x^3)^(3/2))/(3*b^4) - (6*a^2*(a + b/x^3)^(5/2))/(5*b^4) + (
6*a*(a + b/x^3)^(7/2))/(7*b^4) - (2*(a + b/x^3)^(9/2))/(9*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

method	result	size
orering	$\frac{2(16a^3x^9-24a^2b\,x^6+30a\,b^2x^3-35b^3)\,(a\,x^3+b)\,\sqrt{a+\frac{b}{x^3}}}{945b^4x^{12}}$	57
gosper	$\frac{2(a\,x^3+b)\,(16a^3x^9-24a^2b\,x^6+30a\,b^2x^3-35b^3)\,\sqrt{\frac{a\,x^3+b}{x^3}}}{945x^{12}b^4}$	61
risch	$\frac{2\sqrt{\frac{a\,x^3+b}{x^3}}\,(16a^4x^{12}-8a^3b\,x^9+6a^2b^2x^6-5a\,x^3b^3-35b^4)}{945x^{12}b^4}$	65
trager	$\frac{2(16a^4x^{12}-8a^3b\,x^9+6a^2b^2x^6-5a\,x^3b^3-35b^4)\,\sqrt{-\frac{-a\,x^3-b}{x^3}}}{945x^{12}b^4}$	69
default	$\frac{2\sqrt{\frac{a\,x^3+b}{x^3}}\,\sqrt{a\,x^4+bx}\,(16a^4x^{12}-8a^3b\,x^9+6a^2b^2x^6-5a\,x^3b^3-35b^4)}{945x^{12}\sqrt{x(a\,x^3+b)}\,b^4}$	87

input
```
int((a+b/x^3)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
2/945*(16*a^3*x^9-24*a^2*b*x^6+30*a*b^2*x^3-35*b^3)/b^4/x^12*(a*x^3+b)*(a+
b/x^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx = \frac{2\,(16\,a^4x^{12}-8\,a^3bx^9+6\,a^2b^2x^6-5\,ab^3x^3-35\,b^4)\sqrt{\frac{ax^3+b}{x^3}}}{945\,b^4x^{12}}$$

input
```
integrate((a+b/x^3)^(1/2)/x^13,x, algorithm="fricas")
```

output
```
2/945*(16*a^4*x^12 - 8*a^3*b*x^9 + 6*a^2*b^2*x^6 - 5*a*b^3*x^3 - 35*b^4)*s
qrt((a*x^3 + b)/x^3)/(b^4*x^12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2317 vs. 2(75) = 150.

Time = 2.18 (sec) , antiderivative size = 2317, normalized size of antiderivative = 28.96

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**3)**(1/2)/x**13,x)
```

output
```
32*a**(29/2)*b**(23/2)*x**30*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(6
3/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) +
18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a
**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) + 176*a**(27/2)*b
**(25/2)*x**27*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a*
*(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2
)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20
*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2)) + 396*a**(25/2)*b**(27/2)*x**24
*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*
x**(57/2) + 14175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45
/2) + 14175*a**(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 9
45*a**(9/2)*b**21*x**(27/2)) + 462*a**(23/2)*b**(29/2)*x**21*sqrt(a*x**3/b
 + 1)/(945*a**(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14
175*a**(17/2)*b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a*
*(13/2)*b**19*x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b*
*21*x**(27/2)) + 210*a**(21/2)*b**(31/2)*x**18*sqrt(a*x**3/b + 1)/(945*a**
(21/2)*b**15*x**(63/2) + 5670*a**(19/2)*b**16*x**(57/2) + 14175*a**(17/2)*
b**17*x**(51/2) + 18900*a**(15/2)*b**18*x**(45/2) + 14175*a**(13/2)*b**19*
x**(39/2) + 5670*a**(11/2)*b**20*x**(33/2) + 945*a**(9/2)*b**21*x**(27/2))
 - 378*a**(19/2)*b**(33/2)*x**15*sqrt(a*x**3/b + 1)/(945*a**(21/2)*b**1...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}}{27\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a}{7\,b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2}{5\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^3}{9\,b^4}$$

input
```
integrate((a+b/x^3)^(1/2)/x^13,x, algorithm="maxima")
```

output
```
-2/27*(a + b/x^3)^(9/2)/b^4 + 2/7*(a + b/x^3)^(7/2)*a/b^4 - 2/5*(a + b/x^3
)^(5/2)*a^2/b^4 + 2/9*(a + b/x^3)^(3/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx = -\frac{2\left(35\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}-135\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a+189\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2-105\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^3\right)}{945\,b^4}$$

input
```
integrate((a+b/x^3)^(1/2)/x^13,x, algorithm="giac")
```

output
```
-2/945*(35*(a + b/x^3)^(9/2) - 135*(a + b/x^3)^(7/2)*a + 189*(a + b/x^3)^(
5/2)*a^2 - 105*(a + b/x^3)^(3/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx = \frac{32\,a^4\,\sqrt{a+\frac{b}{x^3}}}{945\,b^4}-\frac{2\,\sqrt{a+\frac{b}{x^3}}}{27\,x^{12}}-\frac{2\,a\,\sqrt{a+\frac{b}{x^3}}}{189\,b\,x^9}-\frac{16\,a^3\,\sqrt{a+\frac{b}{x^3}}}{945\,b^3\,x^3}+\frac{4\,a^2\,\sqrt{a+\frac{b}{x^3}}}{315\,b^2\,x^6}$$

input
```
int((a + b/x^3)^(1/2)/x^13,x)
```

output
```
(32*a^4*(a + b/x^3)^(1/2))/(945*b^4) - (2*(a + b/x^3)^(1/2))/(27*x^12) - (
2*a*(a + b/x^3)^(1/2))/(189*b*x^9) - (16*a^3*(a + b/x^3)^(1/2))/(945*b^3*x
^3) + (4*a^2*(a + b/x^3)^(1/2))/(315*b^2*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^{13}}\,dx = \frac{2\sqrt{a\,x^3+b}\,(16a^4x^{12}-8a^3b\,x^9+6a^2b^2x^6-5a\,b^3x^3-35b^4)}{945\sqrt{x}\,b^4x^{13}}$$

input
```
int((a+b/x^3)^(1/2)/x^13,x)
```

output
```
(2*sqrt(a*x**3 + b)*(16*a**4*x**12 - 8*a**3*b*x**9 + 6*a**2*b**2*x**6 - 5*
a*b**3*x**3 - 35*b**4))/(945*sqrt(x)*b**4*x**13)
```

3.461 $\int \sqrt{a+\frac{b}{x^3}}x^7\,dx$

Optimal result

Integrand size = 15, antiderivative size = 291

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx=-\frac{21b^2\sqrt{a+\frac{b}{x^3}}x^2}{320a^2}+\frac{3b\sqrt{a+\frac{b}{x^3}}x^5}{80a}+\frac{1}{8}\sqrt{a+\frac{b}{x^3}}x^8$$
$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4}{320a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-21/320*b^2*(a+b/x^3)^(1/2)*x^2/a^2+3/80*b*(a+b/x^3)^(1/2)*x^5/a+1/8*(a+b/
x^3)^(1/2)*x^8-7/320*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(8/3)*(a^(1/3)+b^
(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/a^2/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.33

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx = \frac{\sqrt{a+\frac{b}{x^3}}x^2\left(\sqrt{1+\frac{ax^3}{b}}(-7b^2+3abx^3+10a^2x^6)+7b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{ax^3}{b}\right)\right)}{80a^2\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]*x^7,x]
```

output
```
(Sqrt[a + b/x^3]*x^2*(Sqrt[1 + (a*x^3)/b]*(-7*b^2 + 3*a*b*x^3 + 10*a^2*x^6
) + 7*b^2*Hypergeometric2F1[-1/2, 1/6, 7/6, -((a*x^3)/b)]))/(80*a^2*Sqrt[1
 + (a*x^3)/b])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {858, 809, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{a+\frac{b}{x^3}}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^3}}x^9d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{8}x^8\sqrt{a+\frac{b}{x^3}}-\frac{3}{16}b\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{8}x^8\sqrt{a+\frac{b}{x^3}}-\frac{3}{16}b\left(-\frac{7b\int\frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{8}x^8\sqrt{a+\frac{b}{x^3}}-\frac{3}{16}b\left(-\frac{7b\left(-\frac{b\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{8}x^8\sqrt{a+\frac{b}{x^3}}-\frac{3}{16}b\left(-\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}\right.$$

input

```
Int[Sqrt[a + b/x^3]*x^7,x]
```

output
```
(Sqrt[a + b/x^3]*x^8)/8 - (3*b*(-1/5*(Sqrt[a + b/x^3]*x^5)/a - (7*b*(-1/2*
(Sqrt[a + b/x^3]*x^2)/a - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)/x)
*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[a + b
/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)/x)^2])))/(10*a)))/16
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 752 vs. $2(224) = 448$.

Time = 1.57 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.59

method	result
risch	$\frac{(40a^2x^6+12ab\,x^3-21b^2)x^2\sqrt{\frac{a\,x^3+b}{x^3}}}{320a^2}+\frac{21b^3\left(\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)}}\left(x-\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((a+b/x^3)^(1/2)*x^7,x,method=_RETURNVERBOSE)
```

output
```
1/320*(40*a^2*x^6+12*a*b*x^3-21*b^2)/a^2*x^2*((a*x^3+b)/x^3)^(1/2)+21/320/
a*b^3*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3
^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/
3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)
^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^
2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)
)/(x-1/a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3))/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^
(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3)))^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)
/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)
)/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*
(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(
1/2)/(a*x^3+b)
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx=\int \sqrt{a+\frac{b}{x^3}}x^7\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^7,x, algorithm="fricas")
```

output
```
integral(x^7*sqrt((a*x^3 + b)/x^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.16

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx=-\frac{\sqrt{a}x^8\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(-\frac{5}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)*x**7,x)
```

output
```
-sqrt(a)*x**8*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*exp_polar(I*pi)/(
a*x**3))/(3*gamma(-5/3))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx=\int \sqrt{a+\frac{b}{x^3}}x^7\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^7,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)*x^7, x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx=\int \sqrt{a+\frac{b}{x^3}}x^7\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^7,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)*x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx=\int x^7\,\sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
int(x^7*(a + b/x^3)^(1/2),x)
```

output
```
int(x^7*(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^7\,dx$$
$$=\frac{80\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^6+24\sqrt{x}\,\sqrt{a\,x^3+b}\,ab\,x^3-42\sqrt{x}\,\sqrt{a\,x^3+b}\,b^2+21\left(\int\frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^4+bx}dx\right)b^3}{640a^2}$$

input
```
int((a+b/x^3)^(1/2)*x^7,x)
```

output
```
(80*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**6 + 24*sqrt(x)*sqrt(a*x**3 + b)*a*b*x
**3 - 42*sqrt(x)*sqrt(a*x**3 + b)*b**2 + 21*int((sqrt(x)*sqrt(a*x**3 + b))
/(a*x**4 + b*x),x)*b**3)/(640*a**2)
```

3.462 $\int \sqrt{a+\frac{b}{x^3}}x^4\,dx$

Optimal result

Integrand size = 15, antiderivative size = 267

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx=\frac{3b\sqrt{a+\frac{b}{x^3}}x^2}{20a}+\frac{1}{5}\sqrt{a+\frac{b}{x^3}}x^5 +\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{}\right.}{20a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
3/20*b*(a+b/x^3)^(1/2)*x^2/a+1/5*(a+b/x^3)^(1/2)*x^5+1/20*3^(3/4)*(1/2*6^(
1/2)+1/2*2^(1/2))*b^(5/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3
)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2
))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/a/(a+
b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx = \frac{\sqrt{a+\frac{b}{x^3}}x^2\left((b+ax^3)\sqrt{1+\frac{ax^3}{b}}-b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{ax^3}{b}\right)\right)}{5a\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]*x^4,x]
```

output
```
(Sqrt[a + b/x^3]*x^2*((b + a*x^3)*Sqrt[1 + (a*x^3)/b] - b*Hypergeometric2F
1[-1/2, 1/6, 7/6, -((a*x^3)/b)]))/(5*a*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {858, 809, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^3}}x^6 d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{5}x^5\sqrt{a+\frac{b}{x^3}}-\frac{3}{10}b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{5}x^5\sqrt{a+\frac{b}{x^3}}-\frac{3}{10}b\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{5}x^5\sqrt{a+\frac{b}{x^3}}-\frac{3}{10}b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\right.$$

input
```
Int[Sqrt[a + b/x^3]*x^4,x]
```

output
```
(Sqrt[a + b/x^3]*x^5)/5 - (3*b*(-1/2*(Sqrt[a + b/x^3]*x^2)/a - (Sqrt[2 + S
qrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/
3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7
- 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/10
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(204) = 408$.

Time = 0.41 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.77

method	result
risch	$\frac{(4a\,x^3+3b)x^2\sqrt{\frac{a\,x^3+b}{x^3}}}{20a}-\frac{3b^2\left(\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)^2\sqrt{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((a+b/x^3)^(1/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*a*x^3+3*b)/a*x^2*((a*x^3+b)/x^3)^(1/2)-3/20*b^2*(1/2/a*(-a^2*b)^(1
/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3
)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)
^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*
(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3))
)^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-a^2*b)^(1
/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-
a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/
2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3))
)^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^
2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
)))^(1/2))*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(a*x^3+b)
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx=\int \sqrt{a+\frac{b}{x^3}}x^4\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^4,x, algorithm="fricas")
```

output
```
integral(x^4*sqrt((a*x^3 + b)/x^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx=-\frac{\sqrt{a}x^5\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)*x**4,x)
```

output
```
-sqrt(a)*x**5*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*exp_polar(I*pi)/(
a*x**3))/(3*gamma(-2/3))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx=\int \sqrt{a+\frac{b}{x^3}}x^4\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)*x^4, x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx=\int \sqrt{a+\frac{b}{x^3}}x^4\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx = \int x^4\,\sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
int(x^4*(a + b/x^3)^(1/2),x)
```

output
```
int(x^4*(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^4\,dx = \frac{8\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,x^3+6\sqrt{x}\,\sqrt{a\,x^3+b}\,b-3\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^4+bx}dx\right)b^2}{40a}$$

input
```
int((a+b/x^3)^(1/2)*x^4,x)
```

output
```
(8*sqrt(x)*sqrt(a*x**3 + b)*a*x**3 + 6*sqrt(x)*sqrt(a*x**3 + b)*b - 3*int(
(sqrt(x)*sqrt(a*x**3 + b))/(a*x**4 + b*x),x)*b**2)/(40*a)
```

3.463 $\int \sqrt{a+\frac{b}{x^3}}x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 242

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx=\frac{1}{2}\sqrt{a+\frac{b}{x^3}}x^2 -\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
1/2*(a+b/x^3)^(1/2)*x^2-1/2*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(a^(
1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.20

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx=\frac{2\sqrt{a+\frac{b}{x^3}}x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{ax^3}{b}\right)}{\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]*x,x]
```

output
```
(2*Sqrt[a + b/x^3]*x^2*Hypergeometric2F1[-1/2, 1/6, 7/6, -((a*x^3)/b)])/Sq
rt[1 + (a*x^3)/b]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {858, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^3}}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^3}}-\frac{3}{4}b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{2}x^2\sqrt{a+\frac{b}{x^3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

input
```
Int[Sqrt[a + b/x^3]*x,x]
```

output
```
(Sqrt[a + b/x^3]*x^2)/2 - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^
(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*Sqrt[a + b/x
^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/
x)^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(183) = 366.

Time = 0.65 (sec) , antiderivative size = 725, normalized size of antiderivative = 3.00

method	result
risch	$\frac{\sqrt{\frac{a x^3+b}{x^3}}\, x^2}{2} + \frac{3b\left(\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a} - \frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)^2\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}\cdots}{a\left(-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}\cdots\right.}}\cdots}{2\left(-\frac{3\left(-\cdots\right.}{\cdots}\right.\cdots}$
default	Expression too large to display

input
```
int((a+b/x^3)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*((a*x^3+b)/x^3)^(1/2)*x^2+3/2*b*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/
(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/
3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^
(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)
/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/
2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^
2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-
a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1
/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*EllipticF(((-3/
2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3
)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-
a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3
^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1
/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))*((a*x^3
+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(a*x^3+b)
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx=\int \sqrt{a+\frac{b}{x^3}}x\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x,x, algorithm="fricas")
```

output
```
integral(x*sqrt((a*x^3 + b)/x^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx=-\frac{\sqrt{a}x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)*x,x)
```

output
```
-sqrt(a)*x**2*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*exp_polar(I*pi)/(a
*x**3))/(3*gamma(1/3))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx = \int \sqrt{a+\frac{b}{x^3}}x\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)*x, x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx = \int \sqrt{a+\frac{b}{x^3}}x\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx = \int x\,\sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
int(x*(a + b/x^3)^(1/2),x)
```

output
```
int(x*(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^3}}x\,dx = \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{2}+\frac{3\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^4+bx}dx\right)b}{4}$$

input
```
int((a+b/x^3)^(1/2)*x,x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b) + 3*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**4 + b
*x),x)*b)/4
```

3.464 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 243

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{5x} - \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{\ldots}}{5\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/5*(a+b/x^3)^(1/2)/x-2/5*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*(a^(1/3)+b^
(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(1/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},-\frac{ax^3}{b}\right)}{5x\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^2,x]
```

output
```
(-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-5/6, -1/2, 1/6, -((a*x^3)/b)])/(5*x
*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {858, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^3}}d\frac{1}{x}$$

$$\Big\downarrow \text{748}$$

$$-\frac{3}{5}a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5x}$$

$$\Big\downarrow \text{759}$$

$$-\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5x}$$

input
```
Int[Sqrt[a + b/x^3]/x^2,x]
```

output
```
(-2*Sqrt[a + b/x^3])/(5*x) - (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(
1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[
a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(184) = 368.

Time = 0.50 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.99

method	result
risch	$-\frac{2\sqrt{\frac{a\,x^3+b}{x^3}}}{5x}+\frac{6a^2\left(\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,\left(-a^2b\right)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{a}\right)^2\sqrt{\frac{\left(-a^2b\right)\cdots}{a\left(-\frac{\left(-a^2b\right)\cdots}{2a}\cdots\right.}}\cdots}{5\left(-\frac{3\left(-\cdots\right.}{}\cdots\right.}$
default	Expression too large to display

input
```
int((a+b/x^3)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*((a*x^3+b)/x^3)^(1/2)/x+6/5*a^2*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x
/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1
/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)
^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1
/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)
+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a
^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/
2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*EllipticF(((-3/2
/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)
+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))*((a*x^3+
b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = -\frac{2\left(3\,a\sqrt{b}x\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)+b\sqrt{\frac{ax^3+b}{x^3}}\right)}{5\,bx}$$

input
```
integrate((a+b/x^3)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2/5*(3*a*sqrt(b)*x*weierstrassPInverse(0, -4*a/b, 1/x) + b*sqrt((a*x^3 +
b)/x^3))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = -\frac{\sqrt{a}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)/x**2,x)
```

output
```
-sqrt(a)*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*exp_polar(I*pi)/(a*x**3))
/(3*x*gamma(4/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = -\frac{\sqrt{a+\frac{b}{x^3}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\ \frac{4}{3};\ -\frac{b}{a\,x^3}\right)}{x\,\sqrt{\frac{b}{a\,x^3}+1}}$$

input
```
int((a + b/x^3)^(1/2)/x^2,x)
```

output
```
-((a + b/x^3)^(1/2)*hypergeom([-1/2, 1/3], 4/3, -b/(a*x^3)))/(x*(b/(a*x^3)
 + 1)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^2}\,dx = \frac{-2\sqrt{a\,x^3+b}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^7+b\,x^4}dx\right)b\,x^2}{2\sqrt{x}\,x^2}$$

input
```
int((a+b/x^3)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(a*x**3 + b) - 3*sqrt(x)*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**7
+ b*x**4),x)*b*x**2)/(2*sqrt(x)*x**2)
```

3.465 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 267

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5} \, dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{55bx} + \frac{4\, 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\, \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right), -7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/11*(a+b/x^3)^(1/2)/x^4-6/55*a*(a+b/x^3)^(1/2)/b/x+4/55*3^(3/4)*(1/2*6^(
1/2)+1/2*2^(1/2))*a^2*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^
(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a
^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(4/3)/(
a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x
)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx=-\frac{2\sqrt{a+\frac{b}{x^3}}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{1}{2},-\frac{5}{6},-\frac{ax^3}{b}\right)}{11x^4\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^5,x]
```

output
```
(-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-11/6, -1/2, -5/6, -((a*x^3)/b)])/(1
1*x^4*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {858, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$-\frac{3}{11}a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}d\frac{1}{x}-\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4}$$

$$\Big\downarrow \text{843}$$

$$-\frac{3}{11}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{2a\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{5b}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4}$$

$\Big\downarrow$ 759

$$-\frac{3}{11}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{11x^4}$$

input
```
Int[Sqrt[a + b/x^3]/x^5,x]
```

output
```
(-2*Sqrt[a + b/x^3])/(11*x^4) - (3*a*((2*Sqrt[a + b/x^3])/(5*b*x) - (4*Sqr
t[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1
/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7
 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/11
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(204) = 408$.

Time = 0.70 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.78

method	result
risch	$-\frac{2(3a x^3+5b)\sqrt{\frac{a x^3+b}{x^3}}}{55x^4 b}-\frac{12a^3\left(\frac{(-a^2b)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)^2 \cdots}{\cdots}$
default	Expression too large to display

input
```
int((a+b/x^3)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/55*(3*a*x^3+5*b)/x^4/b*((a*x^3+b)/x^3)^(1/2)-12/55/b*a^3*(1/2/a*(-a^2*b
)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1
/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1
/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^
(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^
2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(
1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1
/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-a^2*b
)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))
^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x
/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1
/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*
(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3)))^(1/2))*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx = \frac{2\left(6\,a^2\sqrt{b}x^4\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)-\left(3\,abx^3+5\,b^2\right)\sqrt{\frac{ax^3+b}{x^3}}\right)}{55\,b^2x^4}$$

input
```
integrate((a+b/x^3)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
2/55*(6*a^2*sqrt(b)*x^4*weierstrassPInverse(0, -4*a/b, 1/x) - (3*a*b*x^3 +
 5*b^2)*sqrt((a*x^3 + b)/x^3))/(b^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx = -\frac{\sqrt{a}\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3x^4\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)/x**5,x)
```

output
```
-sqrt(a)*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*exp_polar(I*pi)/(a*x**3))
/(3*x**4*gamma(7/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx=\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx$$

input
```
int((a + b/x^3)^(1/2)/x^5,x)
```

output
```
int((a + b/x^3)^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^5}\,dx=\frac{-2\sqrt{a\,x^3+b}-3\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^{10}+b\,x^7}dx\right)b\,x^5}{8\sqrt{x}\,x^5}$$

input
```
int((a+b/x^3)^(1/2)/x^5,x)
```

output
```
( - 2*sqrt(a*x**3 + b) - 3*sqrt(x)*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**10
 + b*x**7),x)*b*x**5)/(8*sqrt(x)*x**5)
```

3.466 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 291

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8} \, dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{187bx^4} + \frac{48a^2\sqrt{a+\frac{b}{x^3}}}{935b^2x} - \frac{32\, 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4}{935b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/17*(a+b/x^3)^(1/2)/x^7-6/187*a*(a+b/x^3)^(1/2)/b/x^4+48/935*a^2*(a+b/x^
3)^(1/2)/b^2/x-32/935*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^3*(a^(1/3)+b^(1/
3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(7/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}\,\text{Hypergeometric2F1}\left(-\frac{17}{6},-\frac{1}{2},-\frac{11}{6},-\frac{ax^3}{b}\right)}{17x^7\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^8,x]
```

output
```
(-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-17/6, -1/2, -11/6, -((a*x^3)/b)])/(
17*x^7*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 811, 843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$-\frac{3}{17}a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}d\frac{1}{x}-\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7}$$

$$\Big\downarrow \text{843}$$

$$-\frac{3}{17}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4}-\frac{8a\int\frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}d\frac{1}{x}}{11b}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7}$$

$$\Big\downarrow \text{843}$$

$$-\frac{3}{17}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4}-\frac{8a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{2a\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{5b}\right)}{11b}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7}$$

$$\Big\downarrow \text{759}$$

$$-\frac{3}{17}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4}-\frac{8a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{11b}\right.$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{17x^7}$$

input
```
Int[Sqrt[a + b/x^3]/x^8,x]
```

output
```
(-2*Sqrt[a + b/x^3])/(17*x^7) - (3*a*((2*Sqrt[a + b/x^3])/(11*b*x^4) - (8*
a*((2*Sqrt[a + b/x^3])/(5*b*x) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)
/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*S
qrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)^2])))/(11*b)))/17
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 752 vs. $2(224) = 448$.

Time = 0.92 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.59

method	result
risch	$\frac{2(24a^2x^6-15ab\,x^3-55b^2)\sqrt{\frac{a\,x^3+b}{x^3}}}{935x^7b^2}+\frac{96a^4\left(\frac{(-a^2b)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)}}\left(x-\left(-\right.\right.}{}$
default	Expression too large to display

input
```
int((a+b/x^3)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
2/935*(24*a^2*x^6-15*a*b*x^3-55*b^2)/x^7/b^2*((a*x^3+b)/x^3)^(1/2)+96/935*
a^4/b^2*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(
1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*
b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-
a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b
)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1
/2)/a*(-a^2*b)^(1/3)))^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))
^(1/2)/(a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = -\frac{2\left(48\,a^3\sqrt{b}x^7\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)-\left(24\,a^2bx^6-15\,ab^2x^3-55\,b^3\right)\sqrt{\frac{ax^3+b}{x^3}}\right)}{935\,b^3x^7}$$

input
```
integrate((a+b/x^3)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-2/935*(48*a^3*sqrt(b)*x^7*weierstrassPInverse(0, -4*a/b, 1/x) - (24*a^2*b
*x^6 - 15*a*b^2*x^3 - 55*b^3)*sqrt((a*x^3 + b)/x^3))/(b^3*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = -\frac{\sqrt{a}\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3x^7\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)/x**8,x)
```

output
```
-sqrt(a)*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*exp_polar(I*pi)/(a*x**3)
)/(3*x**7*gamma(10/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx$$

input
```
int((a + b/x^3)^(1/2)/x^8,x)
```

output
```
int((a + b/x^3)^(1/2)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^8}\,dx = \frac{-2\sqrt{a\,x^3+b}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^{13}+b\,x^{10}}dx\right)b\,x^8}{14\sqrt{x}\,x^8}$$

input
```
int((a+b/x^3)^(1/2)/x^8,x)
```

output
```
( - 2*sqrt(a*x**3 + b) - 3*sqrt(x)*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**13
 + b*x**10),x)*b*x**8)/(14*sqrt(x)*x**8)
```

3.467 $\int \sqrt{a+\frac{b}{x^3}}x^6\,dx$

Optimal result

Integrand size = 15, antiderivative size = 563

$$\begin{aligned}\int \sqrt{a+\frac{b}{x^3}}x^6\,dx &= \frac{15b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{15b^2\sqrt{a+\frac{b}{x^3}}x}{112a^2}+\frac{3b\sqrt{a+\frac{b}{x^3}}x^4}{56a}+\frac{1}{7}\sqrt{a+\frac{b}{x^3}}x^7\\
&\quad-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{224a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\\
&\quad+\frac{5\ 3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\end{aligned}$$

output
```
15/112*b^(7/3)*(a+b/x^3)^(1/2)/a^2/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-15/112*
b^2*(a+b/x^3)^(1/2)*x/a^2+3/56*b*(a+b/x^3)^(1/2)*x^4/a+1/7*(a+b/x^3)^(1/2)
*x^7-15/224*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(7/3)*(a^(1/3)+b^(1/3)/x)*
((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2
)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)/x),I*3^(1/2)+2*I)/a^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x
)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)+5/112*3^(3/4)*b^(7/3)*(a^(1/3)+
b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*2^(1/2)/a^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \sqrt{a+\frac{b}{x^3}}x^6\,dx$$
$$=\frac{\sqrt{a+\frac{b}{x^3}}x^4\left((b+ax^3)\sqrt{1+\frac{ax^3}{b}}-b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{ax^3}{b}\right)\right)}{7a\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]*x^6,x]
```

output
```
(Sqrt[a + b/x^3]*x^4*((b + a*x^3)*Sqrt[1 + (a*x^3)/b] - b*Hypergeometric2F
1[-1/2, 5/6, 11/6, -((a*x^3)/b)]))/(7*a*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {858, 809, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\sqrt{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^3}}x^8d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{7}x^7\sqrt{a+\frac{b}{x^3}}-\frac{3}{14}b\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{7}x^7\sqrt{a+\frac{b}{x^3}}-\frac{3}{14}b\left(-\frac{5b\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{7}x^7\sqrt{a+\frac{b}{x^3}}-\frac{3}{14}b\left(-\frac{5b\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)$$

$$\Big\downarrow \text{832}$$

$$\frac{1}{7}x^7\sqrt{a+\frac{b}{x^3}}-\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)$$

$\big\downarrow$ 759

$$\frac{1}{7}x^7\sqrt{a+\frac{b}{x^3}}-$$

$$\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{2a}\right.}{8a}\right.$$

$\downarrow$ 2416

$$\frac{1}{7}x^7\sqrt{a+\frac{b}{x^3}}-$$

$$\frac{3}{14}b\left(-\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{\sqrt[3]{b}}\right.}{2a}\right.}{8a}\right.$$

input
```
Int[Sqrt[a + b/x^3]*x^6,x]
```

output
```
(Sqrt[a + b/x^3]*x^7)/7 - (3*b*(-1/4*(Sqrt[a + b/x^3]*x^4)/a - (5*b*(-((Sq
rt[a + b/x^3]*x)/a) + (b*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(
1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)
/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/
x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b
^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/
3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)/x)^2])))/(2*a)))/(8*a)))/14
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(421) = 842$.

Time = 0.87 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.00

method	result	size
risch	Expression too large to display	1127
default	Expression too large to display	2799

input
```
int((a+b/x^3)^(1/2)*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/56*x^4*(8*a*x^3+3*b)/a*((a*x^3+b)/x^3)^(1/2)-15/112/a*b^2*(x*(x+1/2/a*(-
a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(
-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2
)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*
b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2
*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))
/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*E
llipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a
*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1
/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)
^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)
/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^
(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-
3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2...
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^6\,dx = \int \sqrt{a+\frac{b}{x^3}}x^6\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^6,x, algorithm="fricas")
```

output
```
integral(x^6*sqrt((a*x^3 + b)/x^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \sqrt{a+\frac{b}{x^3}}x^6\,dx = -\frac{\sqrt{a}x^7\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\ -\frac{4}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(-\frac{4}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)*x**6,x)
```

output
```
-sqrt(a)*x**7*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*exp_polar(I*pi)/(
a*x**3))/(3*gamma(-4/3))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^6\,dx = \int \sqrt{a+\frac{b}{x^3}}x^6\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)*x^6, x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^6\,dx = \int \sqrt{a+\frac{b}{x^3}}x^6\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^3}}x^6\,dx=\int x^6\,\sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
int(x^6*(a + b/x^3)^(1/2),x)
```

output
```
int(x^6*(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^6\,dx=\frac{16\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,x^5+6\sqrt{x}\,\sqrt{a\,x^3+b}\,b\,x^2-15\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a\,x^3+b}dx\right)b^2}{112a}$$

input
```
int((a+b/x^3)^(1/2)*x^6,x)
```

output
```
(16*sqrt(x)*sqrt(a*x**3 + b)*a*x**5 + 6*sqrt(x)*sqrt(a*x**3 + b)*b*x**2 -
15*int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a*x**3 + b),x)*b**2)/(112*a)
```

3.468 $\int \sqrt{a+\frac{b}{x^3}}x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 539

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx = -\frac{3b^{4/3}\sqrt{a+\frac{b}{x^3}}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{3b\sqrt{a+\frac{b}{x^3}}x}{8a}+\frac{1}{4}\sqrt{a+\frac{b}{x^3}}x^4$$
$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-3/8*b^(4/3)*(a+b/x^3)^(1/2)/a/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+3/8*b*(a+b/
x^3)^(1/2)*x/a+1/4*(a+b/x^3)^(1/2)*x^4+3/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/
2))*b^(4/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/a^(2/3)/(a+b/x^3)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
-1/8*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*2^(1/2)/a^
(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx=\frac{2\sqrt{a+\frac{b}{x^3}}x^4\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{ax^3}{b}\right)}{5\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]*x^3,x]
```

output
```
(2*Sqrt[a + b/x^3]*x^4*Hypergeometric2F1[-1/2, 5/6, 11/6, -((a*x^3)/b)])/(
5*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {858, 809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{x^3}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \sqrt{a+\frac{b}{x^3}}x^5d\frac{1}{x}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x^3}}-\frac{3}{8}b\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x^3}}-\frac{3}{8}b\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)$$

$$\Big\downarrow \text{ 832}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x^3}}-\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)$$

$$\Big\downarrow \text{ 759}$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x^3}}-$$

$$\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{2a}\right.$$

$$\downarrow 2416$$

$$\frac{1}{4}x^4\sqrt{a+\frac{b}{x^3}}-\frac{3}{8}b\left(\frac{b\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}}-\frac{2(\cdots}{\cdots}\right)}{2a}\cdots\right.$$

input
```
Int[Sqrt[a + b/x^3]*x^3,x]
```

output
```
(Sqrt[a + b/x^3]*x^4)/4 - (3*b*(-((Sqrt[a + b/x^3]*x)/a) + (b*(((2*Sqrt[a
+ b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^
(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
-7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*S
qrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2
 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(2*a)))/8
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1108 vs. $2(401) = 802$.

Time = 0.85 (sec) , antiderivative size = 1109, normalized size of antiderivative = 2.06

method	result	size
risch	Expression too large to display	1109
default	Expression too large to display	2579

input
```
int((a+b/x^3)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((a*x^3+b)/x^3)^(1/2)*x^4+3/8*b*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)
^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1
/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))
^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1
/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b
)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(
x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*
b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/
3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/
a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(
1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*...
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx=\int \sqrt{a+\frac{b}{x^3}}x^3\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
integral(x^3*sqrt((a*x^3 + b)/x^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx = -\frac{\sqrt{a}x^4\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{4}{3},-\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)*x**3,x)
```

output
```
-sqrt(a)*x**4*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*exp_polar(I*pi)/(
a*x**3))/(3*gamma(-1/3))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx = \int \sqrt{a+\frac{b}{x^3}}x^3\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)*x^3, x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx = \int \sqrt{a+\frac{b}{x^3}}x^3\,dx$$

input
```
integrate((a+b/x^3)^(1/2)*x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx=\int x^3\,\sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
int(x^3*(a + b/x^3)^(1/2),x)
```

output
```
int(x^3*(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^3}}x^3\,dx=\frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x^2}{4}+\frac{3\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a\,x^3+b}dx\right)b}{8}$$

input
```
int((a+b/x^3)^(1/2)*x^3,x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b)*x**2 + 3*int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a*x
**3 + b),x)*b)/8
```

3.469 $\int \sqrt{a+\frac{b}{x^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 507

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = -\frac{3\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}+\sqrt{a+\frac{b}{x^3}}\,x$$
$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-3*b^(1/3)*(a+b/x^3)^(1/2)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+(a+b/x^3)^(1/2)
*x+3/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)/
x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x
)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+
b^(1/3)/x),I*3^(1/2)+2*I)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1
+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)-2^(1/2)*3^(3/4)*a^(1/3)*b^(1/3)*(a^(
1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \sqrt{a+\frac{b}{x^3}}\,dx=-\frac{2\sqrt{a+\frac{b}{x^3}}\,x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},-\frac{ax^3}{b}\right)}{\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3],x]
```

output
```
(-2*Sqrt[a + b/x^3]*x*Hypergeometric2F1[-1/2, -1/6, 5/6, -((a*x^3)/b)])/Sq
rt[1 + (a*x^3)/b]
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {773, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x^3}}\,dx$$

$$\Big\downarrow \ 773$$

$$-\int \sqrt{a+\frac{b}{x^3}}x^2d\frac{1}{x}$$

$$\Big\downarrow \ 809$$

$$x\sqrt{a+\frac{b}{x^3}}-\frac{3}{2}b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}$$

$$\Big\downarrow \ 832$$

$$x\sqrt{a+\frac{b}{x^3}}-\frac{3}{2}b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)$$

$$\Big\downarrow \ 759$$

$$x\sqrt{a+\frac{b}{x^3}}-$$

$$\frac{3}{2}b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(1}{(1}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.$$

$$\Big\downarrow \ 2416$$

$$x\sqrt{a+\frac{b}{x^3}}-\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}}\right.$$

input
```
Int[Sqrt[a + b/x^3],x]
```

output
```
Sqrt[a + b/x^3]*x - (3*b*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(
1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)
/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/
x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b
^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/
3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)/x)^2])))/2
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1106 vs. $2(379) = 758$.

Time = 0.46 (sec) , antiderivative size = 1107, normalized size of antiderivative = 2.18

method	result	size
risch	Expression too large to display	1107
default	Expression too large to display	2864

input
```
int((a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((a*x^3+b)/x^3)^(1/2)*x+3*a*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1
/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3
)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1
/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/
(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/
3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a
*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)
/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*
b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a
^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/...
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = \int \sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
integrate((a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt((a*x^3 + b)/x^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.08

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = -\frac{\sqrt{a}x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2),x)
```

output
```
-sqrt(a)*x*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*exp_polar(I*pi)/(a*x*
*3))/(3*gamma(2/3))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = \int \sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
integrate((a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = \int \sqrt{a+\frac{b}{x^3}}\,dx$$

input
```
integrate((a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.07

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = -\frac{2\,x\,\sqrt{a+\frac{b}{x^3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{6};\ \frac{5}{6};\ -\frac{a\,x^3}{b}\right)}{\sqrt{\frac{a\,x^3}{b}+1}}$$

input
```
int((a + b/x^3)^(1/2),x)
```

output
```
-(2*x*(a + b/x^3)^(1/2)*hypergeom([-1/2, -1/6], 5/6, -(a*x^3)/b))/((a*x^3)
/b + 1)^(1/2)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^3}}\,dx = \frac{2\sqrt{a\,x^3+b}+3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^5+b\,x^2}dx\right)b}{2\sqrt{x}}$$

input
```
int((a+b/x^3)^(1/2),x)
```

output
```
(2*sqrt(a*x**3 + b) + 3*sqrt(x)*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**5 + b
*x**2),x)*b)/(2*sqrt(x))
```

3.470 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 517

$$
\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx &= -\frac{6a\sqrt{a+\frac{b}{x^3}}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2} \\
&\quad+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} \\
&\quad-\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}
\end{aligned}
$$

output
```
-6/7*a*(a+b/x^3)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/7*(a+b/x^
3)^(1/2)/x^2+3/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*(a^(1/3)+b^(1/3
)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)-2/7*2^(1/2)*3^(3/4)*a^(4/3
)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2)
)*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((
1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx=-\frac{2\sqrt{a+\frac{b}{x^3}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},-\frac{ax^3}{b}\right)}{7x^2\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^3,x]
```

output
```
(-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-7/6, -1/2, -1/6, -((a*x^3)/b)])/(7*
x^2*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

$$\Big\downarrow \; 858$$

$$-\int \frac{\sqrt{a+\frac{b}{x^3}}}{x}d\frac{1}{x}$$

$$\Big\downarrow \; 811$$

$$-\frac{3}{7}a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}-\frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2}$$

$$\Big\downarrow \; 832$$

$$-\frac{3}{7}a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2}$$

$$\Big\downarrow \; 759$$

$$-\frac{3}{7}a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2}$$

$$\Big\downarrow \; 2416$$

$$-\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}}\right.$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{7x^2}$$

input

```
Int[Sqrt[a + b/x^3]/x^3,x]
```

output

```
(-2*Sqrt[a + b/x^3])/(7*x^2) - (3*a*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*
Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)/x)^2])))/7
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1124 vs. 2(383) = 766.

Time = 0.92 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.18

method	result	size
risch	Expression too large to display	1125
default	Expression too large to display	3309

input
```
int((a+b/x^3)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(3*a*x^3+b)/x^2/b*((a*x^3+b)/x^3)^(1/2)+6/7/b*a^2*(x*(x+1/2/a*(-a^2*b
)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1
/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)
)*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*
b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-
1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1
/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(
1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1
/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/
2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*Ellipt
icF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^
2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),(
(3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3
)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-
a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)
)+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a
*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1
/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx = \frac{2\left(3\,a\sqrt{b}x^2\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)\right)-b\sqrt{\frac{ax^3+b}{x^3}}\right)}{7\,bx^2}$$

input
```
integrate((a+b/x^3)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2/7*(3*a*sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, 1/x)) - b*sqrt((a*x^3 + b)/x^3))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx = -\frac{\sqrt{a}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)/x**3,x)
```

output
```
-sqrt(a)*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*exp_polar(I*pi)/(a*x**3))
/(3*x**2*gamma(5/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx=\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx$$

input
```
int((a + b/x^3)^(1/2)/x^3,x)
```

output
```
int((a + b/x^3)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^3}\,dx = \frac{-2\sqrt{a\,x^3+b}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^8+b\,x^5}dx\right)b\,x^3}{4\sqrt{x}\,x^3}$$

input
```
int((a+b/x^3)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(a*x**3 + b) - 3*sqrt(x)*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**8
+ b*x**5),x)*b*x**3)/(4*sqrt(x)*x**3)
```

3.471 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6} dx$

Optimal result

Integrand size = 15, antiderivative size = 541

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6} dx = \frac{24a^2\sqrt{a+\frac{b}{x^3}}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{91bx^2}$$
$$- \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$+ \frac{8\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right), -7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
24/91*a^2*(a+b/x^3)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/13*(a+
b/x^3)^(1/2)/x^5-6/91*a*(a+b/x^3)^(1/2)/b/x^2-12/91*3^(1/4)*(1/2*6^(1/2)-1
/2*2^(1/2))*a^(7/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1
/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(
1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(5/3)/(a+
b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^
2)^(1/2)+8/91*2^(1/2)*3^(3/4)*a^(7/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3
)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2
)+2*I)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a
^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{1}{2},-\frac{7}{6},-\frac{ax^3}{b}\right)}{13x^5\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^6,x]
```

output
```
(-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-13/6, -1/2, -7/6, -((a*x^3)/b)])/(1
3*x^5*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$-\frac{3}{13}a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}d\frac{1}{x}-\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5}$$

$$\Big\downarrow \text{843}$$

$$-\frac{3}{13}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{7b}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5}$$

$$\Big\downarrow \text{832}$$

$$-\frac{3}{13}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{7b}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5}$$

$$\Big\downarrow \text{759}$$

$$-\frac{3}{13}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\text{arcsin}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{7b}\right.$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5}$$

$$\downarrow \text{2416}$$

$$-\frac{3}{13}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots}\cdots\right.\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\cdots}}{\sqrt[3]{b}}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{13x^5}$$

input
```
Int[Sqrt[a + b/x^3]/x^6,x]
```

output
```
(-2*Sqrt[a + b/x^3])/(13*x^5) - (3*a*((2*Sqrt[a + b/x^3])/(7*b*x^2) - (4*a
*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2
/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1
 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3)
+ b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/
(7*b)))/13
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(403) = 806$.

Time = 0.83 (sec) , antiderivative size = 1138, normalized size of antiderivative = 2.10

method	result	size
risch	Expression too large to display	1138
default	Expression too large to display	3556

input
```
int((a+b/x^3)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
2/91*(12*a^2*x^6-3*a*b*x^3-7*b^2)/x^5/b^2*((a*x^3+b)/x^3)^(1/2)-24/91*a^3/
b^2*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-
a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3
^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a
^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*
(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*
I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1
/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*
b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((
-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a
^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*
a/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b
)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(
-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(
1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1
/3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(
-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx = -\frac{2\left(12\,a^2\sqrt{b}x^5\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)\right)+\left(3\,abx^3+7\,b^2\right)\sqrt{\frac{ax^3+b}{x^3}}\right)}{91\,b^2x^5}$$

input
```
integrate((a+b/x^3)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/91*(12*a^2*sqrt(b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0
, -4*a/b, 1/x)) + (3*a*b*x^3 + 7*b^2)*sqrt((a*x^3 + b)/x^3))/(b^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx = -\frac{\sqrt{a}\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3x^5\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)/x**6,x)
```

output
```
-sqrt(a)*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*exp_polar(I*pi)/(a*x**3))
/(3*x**5*gamma(8/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx=\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx$$

input
```
int((a + b/x^3)^(1/2)/x^6,x)
```

output
```
int((a + b/x^3)^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^6}\,dx=\frac{-2\sqrt{a\,x^3+b}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^{11}+b\,x^8}dx\right)b\,x^6}{10\sqrt{x}\,x^6}$$

input
```
int((a+b/x^3)^(1/2)/x^6,x)
```

output
```
( - 2*sqrt(a*x**3 + b) - 3*sqrt(x)*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**11
 + b*x**8),x)*b*x**6)/(10*sqrt(x)*x**6)
```

3.472 $\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9} dx$

Optimal result

Integrand size = 15, antiderivative size = 565

$$\begin{aligned}
&\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9} dx \\
&= -\frac{240a^3\sqrt{a+\frac{b}{x^3}}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8} - \frac{6a\sqrt{a+\frac{b}{x^3}}}{247bx^5} + \frac{60a^2\sqrt{a+\frac{b}{x^3}}}{1729b^2x^2} \\
&+ \frac{120\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} \\
&- \frac{80\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}
\end{aligned}$$

output
```
-240/1729*a^3*(a+b/x^3)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/19
*(a+b/x^3)^(1/2)/x^8-6/247*a*(a+b/x^3)^(1/2)/b/x^5+60/1729*a^2*(a+b/x^3)^(
1/2)/b^2/x^2+120/1729*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(10/3)*(a^(1/3)+
b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)-80/1729*2^(1/2)*3^(3
/4)*a^(10/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^
(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(8/3)/(a+b/x^3)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx=-\frac{2\sqrt{a+\frac{b}{x^3}}\,\text{Hypergeometric2F1}\left(-\frac{19}{6},-\frac{1}{2},-\frac{13}{6},-\frac{ax^3}{b}\right)}{19x^8\sqrt{1+\frac{ax^3}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^3]/x^9,x]
```

output
```
(-2*Sqrt[a + b/x^3]*Hypergeometric2F1[-19/6, -1/2, -13/6, -((a*x^3)/b)])/(
19*x^8*Sqrt[1 + (a*x^3)/b])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {858, 811, 843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^7}d\frac{1}{x}$$

$$\Big\downarrow \text{ 811}$$

$$-\frac{3}{19}a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7}d\frac{1}{x}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8}$$

$$\Big\downarrow \text{ 843}$$

$$-\frac{3}{19}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}d\frac{1}{x}}{13b}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8}$$

$$\Big\downarrow \text{ 843}$$

$$-\frac{3}{19}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{7b}\right)}{13b}\right)-\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8}$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{3}{19}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}\right)-$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{3}{19}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{Elli}}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}\right)}{7b}}{13b}\right.$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{19x^8}$$

$\downarrow$ 2416

$$-\frac{3}{19}a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\text{ar}\cdots\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\cdots\right.}}}\cdots\right)}{\sqrt[3]{b}\cdots}\cdots\right)}{\cdots}\cdots\right.$$

input
```
Int[Sqrt[a + b/x^3]/x^9,x]
```

output
```
(-2*Sqrt[a + b/x^3])/(19*x^8) - (3*a*((2*Sqrt[a + b/x^3])/(13*b*x^5) - (10
*a*((2*Sqrt[a + b/x^3])/(7*b*x^2) - (4*a*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((
1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a
^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1
/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]]
)/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(7*b)))/(13*b)))/19
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(423) = 846$.

Time = 1.12 (sec) , antiderivative size = 1149, normalized size of antiderivative = 2.03

method	result	size
risch	Expression too large to display	1149
default	Expression too large to display	3788

input
```
int((a+b/x^3)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-2/1729*(120*a^3*x^9-30*a^2*b*x^6+21*a*b^2*x^3+91*b^3)/x^8/b^3*((a*x^3+b)/
x^3)^(1/2)+240/1729*a^4/b^3*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/
a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1
/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-
a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*
(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1
/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3))
)^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*
b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-
a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)
+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*
b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx = \frac{2\left(120\,a^3\sqrt{b}x^8\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)\right)+\left(30\,a^2bx^6-21\,ab^2x^3-91\,b^3\right)\right.}{1729\,b^3x^8}$$

input
```
integrate((a+b/x^3)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
2/1729*(120*a^3*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierstrassPInverse
(0, -4*a/b, 1/x)) + (30*a^2*b*x^6 - 21*a*b^2*x^3 - 91*b^3)*sqrt((a*x^3 + b
)/x^3))/(b^3*x^8)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx = -\frac{\sqrt{a}\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3x^8\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate((a+b/x**3)**(1/2)/x**9,x)
```

output
```
-sqrt(a)*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*exp_polar(I*pi)/(a*x**3)
)/(3*x**8*gamma(11/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^3)/x^9, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx = \int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

input
```
integrate((a+b/x^3)^(1/2)/x^9,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^3)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx=\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx$$

input
```
int((a + b/x^3)^(1/2)/x^9,x)
```

output
```
int((a + b/x^3)^(1/2)/x^9, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^3}}}{x^9}\,dx=\frac{-2\sqrt{a\,x^3+b}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^{14}+b\,x^{11}}dx\right)b\,x^9}{16\sqrt{x}\,x^9}$$

input
```
int((a+b/x^3)^(1/2)/x^9,x)
```

output
```
( - 2*sqrt(a*x**3 + b) - 3*sqrt(x)*int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**14
 + b*x**11),x)*b*x**9)/(16*sqrt(x)*x**9)
```

3.473 $\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5 \, dx = \frac{5}{12} b \sqrt{a+\frac{b}{x^3}} x^3 + \frac{1}{6} a \sqrt{a+\frac{b}{x^3}} x^6 + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
5/12*b*(a+b/x^3)^(1/2)*x^3+1/6*a*(a+b/x^3)^(1/2)*x^6+1/4*b^2*arctanh((a+b/
x^3)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5 \, dx = \frac{1}{12} \sqrt{a+\frac{b}{x^3}} x^{3/2} \left(x^{3/2} (5b+2ax^3) + \frac{3b^2 \log \left(\sqrt{a} x^{3/2} + \sqrt{b+ax^3}\right)}{\sqrt{a}\sqrt{b+ax^3}}\right)$$

input
```
Integrate[(a + b/x^3)^(3/2)*x^5,x]
```

output
```
(Sqrt[a + b/x^3]*x^(3/2)*(x^(3/2)*(5*b + 2*a*x^3) + (3*b^2*Log[Sqrt[a]*x^(
3/2) + Sqrt[b + a*x^3]])/(Sqrt[a]*Sqrt[b + a*x^3])))/12
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{x^3}\right)^{3/2} dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3} \int \left(a+\frac{b}{x^3}\right)^{3/2} x^9 d\frac{1}{x^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{4} b \int \sqrt{a+\frac{b}{x^3}} x^6 d\frac{1}{x^3}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{4} b \left(\frac{1}{2} b \int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}} d\frac{1}{x^3}-x^3 \sqrt{a+\frac{b}{x^3}}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{4} b \left(\int \frac{1}{\frac{1}{b x^6}-\frac{a}{b}} d\sqrt{a+\frac{b}{x^3}}-x^3 \sqrt{a+\frac{b}{x^3}}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{4} b \left(x^3 \left(-\sqrt{a+\frac{b}{x^3}}\right)-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\right)$$

input
```
Int[(a + b/x^3)^(3/2)*x^5,x]
```

output
```
(((a + b/x^3)^(3/2)*x^6)/2 - (3*b*(-(Sqrt[a + b/x^3]*x^3) - (b*ArcTanh[Sqr
t[a + b/x^3]/Sqrt[a]])/Sqrt[a]))/4)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{x^{3}\left(2 a\,x^{3}+5 b\right) \sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{12}+\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2} \sqrt{a}}\right) \sqrt{\frac{a\,x^{3}+b}{x^{3}}}\, x \sqrt{x\left(a\,x^{3}+b\right)}}{4 \sqrt{a}\,\left(a\,x^{3}+b\right)}$	91
default	$\frac{\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}} x^{5} \left(2 \sqrt{x\left(a\,x^{3}+b\right)}\, a^{\frac{3}{2}} x^{4}+5 b x \sqrt{x\left(a\,x^{3}+b\right)}\, \sqrt{a}+3 \operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2} \sqrt{a}}\right) b^{2}\right)}{12 \left(a\,x^{3}+b\right) \sqrt{x\left(a\,x^{3}+b\right)}\, \sqrt{a}}$	104

input
```
int((a+b/x^3)^(3/2)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(2*a*x^3+5*b)*((a*x^3+b)/x^3)^(1/2)+1/4*b^2/a^(1/2)*arctanh((x*(a
*x^3+b))^(1/2)/x^2/a^(1/2))*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(a
*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.71

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx = \left[\frac{3\,\sqrt{a}b^2\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\,(2\,ax^6+bx^3)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)+4\,(2\,a^2x^6+5\,abx^3)\sqrt{\frac{ax^3+b}{x^3}}}{48\,a}, -\frac{3\,\sqrt{-a}b^2\arctan\left(\frac{(2\,ax^3+b)\sqrt{-a}\sqrt{\frac{ax^3+b}{x^3}}}{2\,(a^2x^3+ab)}\right)-2\,(2\,a^2x^6+5\,abx^3)\sqrt{\frac{ax^3+b}{x^3}}}{24\,a}\right]$$

input
```
integrate((a+b/x^3)^(3/2)*x^5,x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^2*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3
)*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^2*x^6 + 5*a*b*x^3)*sqrt((a*x^3 +
 b)/x^3))/a, -1/24*(3*sqrt(-a)*b^2*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt(
(a*x^3 + b)/x^3)/(a^2*x^3 + a*b)) - 2*(2*a^2*x^6 + 5*a*b*x^3)*sqrt((a*x^3
+ b)/x^3))/a]
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx = \frac{a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{ax^3}{b}+1}}{6} + \frac{5b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}{12} + \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{4\sqrt{a}}$$

input
```
integrate((a+b/x**3)**(3/2)*x**5,x)
```

output
```
a*sqrt(b)*x**(9/2)*sqrt(a*x**3/b + 1)/6 + 5*b**(3/2)*x**(3/2)*sqrt(a*x**3/
b + 1)/12 + b**2*asinh(sqrt(a)*x**(3/2)/sqrt(b))/(4*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx = -\frac{b^2\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{8\sqrt{a}} + \frac{5\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}b^2 - 3\sqrt{a+\frac{b}{x^3}}ab^2}{12\left(\left(a+\frac{b}{x^3}\right)^2 - 2\left(a+\frac{b}{x^3}\right)a + a^2\right)}$$

input
```
integrate((a+b/x^3)^(3/2)*x^5,x, algorithm="maxima")
```

output
```
-1/8*b^2*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/sqrt
(a) + 1/12*(5*(a + b/x^3)^(3/2)*b^2 - 3*sqrt(a + b/x^3)*a*b^2)/((a + b/x^3
)^2 - 2*(a + b/x^3)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx = \frac{1}{12}\sqrt{ax^4+bx}\,(2\,ax^3+5\,b)x - \frac{b^2\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{4\sqrt{-a}}$$

input
```
integrate((a+b/x^3)^(3/2)*x^5,x, algorithm="giac")
```

output
```
1/12*sqrt(a*x^4 + b*x)*(2*a*x^3 + 5*b)*x - 1/4*b^2*arctan(sqrt(a + b/x^3)/
sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx = \frac{a\,x^6\sqrt{a+\frac{b}{x^3}}}{6} + \frac{5\,b\,x^3\sqrt{a+\frac{b}{x^3}}}{12} + \frac{b^2\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{8\sqrt{a}}$$

input
```
int(x^5*(a + b/x^3)^(3/2),x)
```

output
```
(a*x^6*(a + b/x^3)^(1/2))/6 + (5*b*x^3*(a + b/x^3)^(1/2))/12 + (b^2*log(x^
6*((a + b/x^3)^(1/2) - a^(1/2))*((a + b/x^3)^(1/2) + a^(1/2))^3))/(8*a^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^5\,dx = \frac{4\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^4+10\sqrt{x}\,\sqrt{a\,x^3+b}\,abx-3\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)b^2+3\sqrt{a}\log\left(\sqrt{a\,x^3+b}+\sqrt{x}\,\sqrt{a}\,x\right)b^2}{24a}$$

input
```
int((a+b/x^3)^(3/2)*x^5,x)
```

output
```
(4*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**4 + 10*sqrt(x)*sqrt(a*x**3 + b)*a*b*x
- 3*sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*b**2 + 3*sqrt(a)*log
(sqrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*b**2)/(24*a)
```

3.474 $\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2 \, dx = -\frac{2}{3}b\sqrt{a+\frac{b}{x^3}}+\frac{1}{3}a\sqrt{a+\frac{b}{x^3}}x^3+\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)$$

output
```
-2/3*b*(a+b/x^3)^(1/2)+1/3*a*(a+b/x^3)^(1/2)*x^3+a^(1/2)*b*arctanh((a+b/x^
3)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2 \, dx = \frac{1}{3}\sqrt{a+\frac{b}{x^3}}\left(-2b+ax^3+\frac{3\sqrt{a}bx^{3/2}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{\sqrt{b+ax^3}}\right)$$

input
```
Integrate[(a + b/x^3)^(3/2)*x^2,x]
```

output
```
(Sqrt[a + b/x^3]*(-2*b + a*x^3 + (3*Sqrt[a]*b*x^(3/2)*Log[Sqrt[a]*x^(3/2)
+ Sqrt[b + a*x^3]])/Sqrt[b + a*x^3]))/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{x^3}\right)^{3/2}dx$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{3}\int \left(a+\frac{b}{x^3}\right)^{3/2}x^6d\frac{1}{x^3}$$

$$\Big\downarrow \ 51$$

$$\frac{1}{3}\left(x^3\left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{2}b\int \sqrt{a+\frac{b}{x^3}}x^3d\frac{1}{x^3}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{3}\left(x^3\left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{2}b\left(a\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}+2\sqrt{a+\frac{b}{x^3}}\right)\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(x^3\left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{b}+2\sqrt{a+\frac{b}{x^3}}\right)\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(x^3\left(a+\frac{b}{x^3}\right)^{3/2}-\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x^3}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)\right)\right)$$

input
```
Int[(a + b/x^3)^(3/2)*x^2,x]
```

output
```
((a + b/x^3)^(3/2)*x^3 - (3*b*(2*Sqrt[a + b/x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b/x^3]/Sqrt[a]]))/2)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38

method	result	size
risch	$\frac{(a\,x^{3}-2b)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{3}+\frac{\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x\sqrt{x(a\,x^{3}+b)}}{a\,x^{3}+b}$	84
default	$\frac{\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}x^{3}\left(a\,x^{3}\sqrt{x(a\,x^{3}+b)}+3\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)b\,x^{2}-2b\sqrt{x(a\,x^{3}+b)}\right)}{3(a\,x^{3}+b)\sqrt{x(a\,x^{3}+b)}}$	98

input
```
int((a+b/x^3)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x^3-2*b)*((a*x^3+b)/x^3)^(1/2)+a^(1/2)*b*arctanh((x*(a*x^3+b))^(1/2
)/x^2/a^(1/2))*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(a*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.61

$$\int\left(a+\frac{b}{x^3}\right)^{3/2}x^2\,dx=\left[\frac{1}{4}\,\sqrt{a}b\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\left(2\,ax^6+bx^3\right)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)+\frac{1}{3}\left(ax^3-2\,b\right)\sqrt{\frac{ax^3+b}{x^3}},-\frac{1}{2}\,\sqrt{-a}b\arctan\left(\frac{(2\,ax^3+b)\sqrt{-a}\sqrt{\frac{ax^3+b}{x^3}}}{2\left(a^2x^3+ab\right)}\right)+\frac{1}{3}\left(ax^3-2\,b\right)\sqrt{\frac{ax^3+b}{x^3}}\right]$$

input
```
integrate((a+b/x^3)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*b*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt
(a)*sqrt((a*x^3 + b)/x^3)) + 1/3*(a*x^3 - 2*b)*sqrt((a*x^3 + b)/x^3), -1/2
*sqrt(-a)*b*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt((a*x^3 + b)/x^3)/(a^2*x
^3 + a*b)) + 1/3*(a*x^3 - 2*b)*sqrt((a*x^3 + b)/x^3)]
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx = \sqrt{a}b\,\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right) + \frac{a^2x^{\frac{9}{2}}}{3\sqrt{b}\sqrt{\frac{ax^3}{b}+1}} - \frac{a\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{ax^3}{b}+1}} - \frac{2b^{\frac{3}{2}}}{3x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}$$

input
```
integrate((a+b/x**3)**(3/2)*x**2,x)
```

output
```
sqrt(a)*b*asinh(sqrt(a)*x**(3/2)/sqrt(b)) + a**2*x**(9/2)/(3*sqrt(b)*sqrt(
a*x**3/b + 1)) - a*sqrt(b)*x**(3/2)/(3*sqrt(a*x**3/b + 1)) - 2*b**(3/2)/(3
*x**(3/2)*sqrt(a*x**3/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx = \frac{1}{3}\sqrt{a+\frac{b}{x^3}}ax^3 - \frac{1}{2}\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right) - \frac{2}{3}\sqrt{a+\frac{b}{x^3}}b$$

input
```
integrate((a+b/x^3)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(a + b/x^3)*a*x^3 - 1/2*sqrt(a)*b*log((sqrt(a + b/x^3) - sqrt(a))/
(sqrt(a + b/x^3) + sqrt(a))) - 2/3*sqrt(a + b/x^3)*b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx = \frac{1}{3}\,\sqrt{ax^4+bx}ax - \frac{1}{3}\left(\frac{3\,a\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\,\sqrt{a+\frac{b}{x^3}}\right)b$$

input
```
integrate((a+b/x^3)^(3/2)*x^2,x, algorithm="giac")
```

output
```
1/3*sqrt(a*x^4 + b*x)*a*x - 1/3*(3*a*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt
(-a) + 2*sqrt(a + b/x^3))*b
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx = \frac{a\,x^3\,\sqrt{a+\frac{b}{x^3}}}{3} - \frac{2\,b\,\sqrt{a+\frac{b}{x^3}}}{3} + \frac{\sqrt{a}\,b\,\ln\left(x^6\,\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\,\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{2}$$

input
```
int(x^2*(a + b/x^3)^(3/2),x)
```

output
```
(a*x^3*(a + b/x^3)^(1/2))/3 - (2*b*(a + b/x^3)^(1/2))/3 + (a^(1/2)*b*log(x
^6*((a + b/x^3)^(1/2) - a^(1/2))*((a + b/x^3)^(1/2) + a^(1/2))^3))/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38

$$\int \left(a+\frac{b}{x^3}\right)^{3/2} x^2\,dx = \frac{2\sqrt{a\,x^3+b}\,a\,x^3-4\sqrt{a\,x^3+b}\,b-3\sqrt{x}\,\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)bx+3\sqrt{x}\,\sqrt{a}\log\left(\sqrt{}\right.}{6\sqrt{x}\,x}$$

input
```
int((a+b/x^3)^(3/2)*x^2,x)
```

output
```
(2*sqrt(a*x**3 + b)*a*x**3 - 4*sqrt(a*x**3 + b)*b - 3*sqrt(x)*sqrt(a)*log(
sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*b*x + 3*sqrt(x)*sqrt(a)*log(sqrt(a*x
**3 + b) + sqrt(x)*sqrt(a)*x)*b*x)/(6*sqrt(x)*x)
```

3.475 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x} dx = -\frac{2}{3}a\sqrt{a+\frac{b}{x^3}}-\frac{2}{9}\left(a+\frac{b}{x^3}\right)^{3/2}+\frac{2}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)$$

output
```
-2/3*a*(a+b/x^3)^(1/2)-2/9*(a+b/x^3)^(3/2)+2/3*a^(3/2)*arctanh((a+b/x^3)^(
1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x} dx = \frac{2\sqrt{a+\frac{b}{x^3}}\left(-\sqrt{b+ax^3}(b+4ax^3)+3a^{3/2}x^{9/2}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)\right)}{9x^3\sqrt{b+ax^3}}$$

input
```
Integrate[(a + b/x^3)^(3/2)/x,x]
```

output
```
(2*Sqrt[a + b/x^3]*(-(Sqrt[b + a*x^3]*(b + 4*a*x^3)) + 3*a^(3/2)*x^(9/2)*L
og[Sqrt[a]*x^(3/2) + Sqrt[b + a*x^3]]))/(9*x^3*Sqrt[b + a*x^3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \left(a+\frac{b}{x^3}\right)^{3/2} x^3 d\frac{1}{x^3}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(-a\int \sqrt{a+\frac{b}{x^3}}x^3d\frac{1}{x^3}-\frac{2}{3}\left(a+\frac{b}{x^3}\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(-a\left(a\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}+2\sqrt{a+\frac{b}{x^3}}\right)-\frac{2}{3}\left(a+\frac{b}{x^3}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-a\left(\frac{2a\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{b}+2\sqrt{a+\frac{b}{x^3}}\right)-\frac{2}{3}\left(a+\frac{b}{x^3}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-a\left(2\sqrt{a+\frac{b}{x^3}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)\right)-\frac{2}{3}\left(a+\frac{b}{x^3}\right)^{3/2}\right)$$

input
```
Int[(a + b/x^3)^(3/2)/x,x]
```

output
```
((-2*(a + b/x^3)^(3/2))/3 - a*(2*Sqrt[a + b/x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b/x^3]/Sqrt[a]]))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

method	result	size
risch	$-\frac{2(4a\,x^{3}+b)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}{9x^{3}}+\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x\sqrt{x(a\,x^{3}+b)}}{3(a\,x^{3}+b)}$	86
default	$\frac{2\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}\left(3a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)x^{5}-a\sqrt{a\,x^{4}+bx}\,x^{3}-3a\,x^{3}\sqrt{x(a\,x^{3}+b)}-\sqrt{a\,x^{4}+bx}\,b\right)}{9(a\,x^{3}+b)\sqrt{x(a\,x^{3}+b)}}$	112

input
```
int((a+b/x^3)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(4*a*x^3+b)/x^3*((a*x^3+b)/x^3)^(1/2)+2/3*a^(3/2)*arctanh((x*(a*x^3+b
))^(1/2)/x^2/a^(1/2))*((a*x^3+b)/x^3)^(1/2)*x*(x*(a*x^3+b))^(1/2)/(a*x^3+b
)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.92

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx = \left[\frac{3\,a^{\frac{3}{2}}x^{3}\log\left(-8\,a^{2}x^{6}-8\,abx^{3}-b^{2}-4\,(2\,ax^{6}+bx^{3})\sqrt{a}\sqrt{\frac{ax^{3}+b}{x^{3}}}\right)-4\,(4\,ax^{3}+b)\sqrt{\frac{ax^{3}+b}{x^{3}}}}{18\,x^{3}}, -\frac{3\sqrt{-a}ax^{3}\arctan\left(\frac{(2\,ax^{3}+b)\sqrt{-a}\sqrt{\frac{ax^{3}+b}{x^{3}}}}{2\,(a^{2}x^{3}+ab)}\right)+2\,(4\,ax^{3}+b)\sqrt{\frac{ax^{3}+b}{x^{3}}}}{9\,x^{3}}\right]$$

input
```
integrate((a+b/x^3)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/18*(3*a^(3/2)*x^3*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3
)*sqrt(a)*sqrt((a*x^3 + b)/x^3)) - 4*(4*a*x^3 + b)*sqrt((a*x^3 + b)/x^3))/
x^3, -1/9*(3*sqrt(-a)*a*x^3*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt((a*x^3
+ b)/x^3)/(a^2*x^3 + a*b)) + 2*(4*a*x^3 + b)*sqrt((a*x^3 + b)/x^3))/x^3]
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx = -\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{9} - \frac{a^{\frac{3}{2}}\log\left(\frac{b}{ax^3}\right)}{3} + \frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax^3}}+1\right)}{3} - \frac{2\sqrt{a}b\sqrt{1+\frac{b}{ax^3}}}{9x^3}$$

input
```
integrate((a+b/x**3)**(3/2)/x,x)
```

output
```
-8*a**(3/2)*sqrt(1 + b/(a*x**3))/9 - a**(3/2)*log(b/(a*x**3))/3 + 2*a**(3/
2)*log(sqrt(1 + b/(a*x**3)) + 1)/3 - 2*sqrt(a)*b*sqrt(1 + b/(a*x**3))/(9*x
**3)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx = -\frac{1}{3}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right) - \frac{2}{9}\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{a+\frac{b}{x^3}}a$$

input
```
integrate((a+b/x^3)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/3*a^(3/2)*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))
- 2/9*(a + b/x^3)^(3/2) - 2/3*sqrt(a + b/x^3)*a
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx = -\frac{2\,a^2 \arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} - \frac{2}{9}\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{a+\frac{b}{x^3}}a$$

input
```
integrate((a+b/x^3)^(3/2)/x,x, algorithm="giac")
```

output
```
-2/3*a^2*arctan(sqrt(a + b/x^3)/sqrt(-a))/sqrt(-a) - 2/9*(a + b/x^3)^(3/2)
 - 2/3*sqrt(a + b/x^3)*a
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx = \frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3} - \frac{2\,a\,\sqrt{a+\frac{b}{x^3}}}{3} - \frac{2\,\left(a+\frac{b}{x^3}\right)^{3/2}}{9}$$

input
```
int((a + b/x^3)^(3/2)/x,x)
```

output
```
(2*a^(3/2)*atanh((a + b/x^3)^(1/2)/a^(1/2)))/3 - (2*a*(a + b/x^3)^(1/2))/3
 - (2*(a + b/x^3)^(3/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x}\,dx = \frac{-8\sqrt{a\,x^3+b}\,a\,x^3 - 2\sqrt{a\,x^3+b}\,b - 3\sqrt{x}\,\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)a\,x^4 + 3\sqrt{x}\,\sqrt{\cdots}}{9\sqrt{x}\,x^4}$$

input
```
int((a+b/x^3)^(3/2)/x,x)
```

output
```
( - 8*sqrt(a*x**3 + b)*a*x**3 - 2*sqrt(a*x**3 + b)*b - 3*sqrt(x)*sqrt(a)*l
og(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*a*x**4 + 3*sqrt(x)*sqrt(a)*log(sq
rt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*a*x**4)/(9*sqrt(x)*x**4)
```

3.476 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4} dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b}$$

output
```
-2/15*(a+b/x^3)^(5/2)/b
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4} dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}(b+ax^3)}{15bx^3}$$

input
```
Integrate[(a + b/x^3)^(3/2)/x^4,x]
```

output
```
(-2*(a + b/x^3)^(3/2)*(b + a*x^3))/(15*b*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b}$$

input
```
Int[(a + b/x^3)^(3/2)/x^4,x]
```

output
```
(-2*(a + b/x^3)^(5/2))/(15*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\left(a+\frac{b}{x^{3}}\right)^{\frac{5}{2}}}{15b}$	15
orering	$-\frac{2\left(a\,x^{3}+b\right)\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{2}}}{15x^{3}b}$	25
gosper	$-\frac{2\left(a\,x^{3}+b\right)\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}}{15x^{3}b}$	29
risch	$-\frac{2\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,\left(a^{2}x^{6}+2ab\,x^{3}+b^{2}\right)}{15x^{6}b}$	40
trager	$-\frac{2\left(a^{2}x^{6}+2ab\,x^{3}+b^{2}\right)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{15x^{6}b}$	44
default	$-\frac{2\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}\left(a\,x^{3}+b\right)\sqrt{a\,x^{4}+bx}}{15x^{3}b\sqrt{x\left(a\,x^{3}+b\right)}}$	51

input
```
int((a+b/x^3)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(a+b/x^3)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx=-\frac{2\left(a^2x^6+2\,abx^3+b^2\right)\sqrt{\frac{ax^3+b}{x^3}}}{15\,bx^6}$$

input
```
integrate((a+b/x^3)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-2/15*(a^2*x^6 + 2*a*b*x^3 + b^2)*sqrt((a*x^3 + b)/x^3)/(b*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(15) = 30$.

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.94

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx = -\frac{2a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^3}}}{15b} - \frac{4a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^3}}}{15x^3} - \frac{2\sqrt{a}b\sqrt{1+\frac{b}{ax^3}}}{15x^6}$$

input
```
integrate((a+b/x**3)**(3/2)/x**4,x)
```

output
```
-2*a**(5/2)*sqrt(1 + b/(a*x**3))/(15*b) - 4*a**(3/2)*sqrt(1 + b/(a*x**3))/
(15*x**3) - 2*sqrt(a)*b*sqrt(1 + b/(a*x**3))/(15*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b}$$

input
```
integrate((a+b/x^3)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-2/15*(a + b/x^3)^(5/2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b}$$

input
```
integrate((a+b/x^3)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-2/15*(a + b/x^3)^(5/2)/b
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx=-\frac{2\,\sqrt{a+\frac{b}{x^3}}\,{\left(a\,x^3+b\right)}^2}{15\,b\,x^6}$$

input
```
int((a + b/x^3)^(3/2)/x^4,x)
```

output
```
-(2*(a + b/x^3)^(1/2)*(b + a*x^3)^2)/(15*b*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^4}\,dx=\frac{2\sqrt{a\,x^3+b}\,(-a^2x^6-2ab\,x^3-b^2)}{15\sqrt{x}\,b\,x^7}$$

input
```
int((a+b/x^3)^(3/2)/x^4,x)
```

output
```
(2*sqrt(a*x**3 + b)*( - a**2*x**6 - 2*a*b*x**3 - b**2))/(15*sqrt(x)*b*x**7
)
```

3.477 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7} dx = \frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^2} - \frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^2}$$

output
```
2/15*a*(a+b/x^3)^(5/2)/b^2-2/21*(a+b/x^3)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7} dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}(5b-2ax^3)\,(b+ax^3)}{105b^2x^6}$$

input
```
Integrate[(a + b/x^3)^(3/2)/x^7,x]
```

output
```
(-2*(a + b/x^3)^(3/2)*(5*b - 2*a*x^3)*(b + a*x^3))/(105*b^2*x^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3} \int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^3} d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3} \int \left(\frac{\left(a+\frac{b}{x^3}\right)^{5/2}}{b}-\frac{a\left(a+\frac{b}{x^3}\right)^{3/2}}{b}\right) d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^2}\right)$$

input
```
Int[(a + b/x^3)^(3/2)/x^7,x]
```

output
```
((2*a*(a + b/x^3)^(5/2))/(5*b^2) - (2*(a + b/x^3)^(7/2))/(7*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
orering	$\frac{2(2a\,x^3-5b)\,(a\,x^3+b)\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{105b^2x^6}$	35
gosper	$\frac{2(a\,x^3+b)\,(2a\,x^3-5b)\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}}{105b^2x^6}$	39
risch	$\frac{2\sqrt{\frac{a\,x^3+b}{x^3}}\,(2a^3x^9-a^2b\,x^6-8a\,b^2x^3-5b^3)}{105x^9b^2}$	54
trager	$\frac{2(2a^3x^9-a^2b\,x^6-8a\,b^2x^3-5b^3)\sqrt{-\frac{-a\,x^3-b}{x^3}}}{105x^9b^2}$	58
default	$\frac{2\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}(2a^2x^6-3ab\,x^3-5b^2)\sqrt{a\,x^4+bx}}{105x^6b^2\sqrt{x(a\,x^3+b)}}$	65

input
```
int((a+b/x^3)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
2/105*(2*a*x^3-5*b)/b^2/x^6*(a*x^3+b)*(a+b/x^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx = \frac{2\left(2\,a^3x^9 - a^2bx^6 - 8\,ab^2x^3 - 5\,b^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{105\,b^2x^9}$$

input
```
integrate((a+b/x^3)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
2/105*(2*a^3*x^9 - a^2*b*x^6 - 8*a*b^2*x^3 - 5*b^3)*sqrt((a*x^3 + b)/x^3)/
(b^2*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(34) = 68$.

Time = 1.09 (sec) , antiderivative size = 371, normalized size of antiderivative = 9.76

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx = \frac{4a^{\frac{15}{2}}b^{\frac{3}{2}}x^{12}\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}} + \frac{2a^{\frac{13}{2}}b^{\frac{5}{2}}x^9\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}} - \frac{18a^{\frac{11}{2}}b^{\frac{7}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}} - \frac{26a^{\frac{9}{2}}b^{\frac{9}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}} - \frac{10a^{\frac{7}{2}}b^{\frac{11}{2}}\sqrt{\frac{ax^3}{b}+1}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}} - \frac{4a^8bx^{\frac{27}{2}}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}} - \frac{4a^7b^2x^{\frac{21}{2}}}{105a^{\frac{9}{2}}b^3x^{\frac{27}{2}}+105a^{\frac{7}{2}}b^4x^{\frac{21}{2}}}$$

input
```
integrate((a+b/x**3)**(3/2)/x**7,x)
```

output
```
4*a**(15/2)*b**(3/2)*x***12*sqrt(a*x***3/b + 1)/(105*a**(9/2)*b**3*x**(27/2)
 + 105*a**(7/2)*b**4*x**(21/2)) + 2*a**(13/2)*b**(5/2)*x***9*sqrt(a*x***3/b
+ 1)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(7/2)*b**4*x**(21/2)) - 18*a**(
11/2)*b**(7/2)*x***6*sqrt(a*x***3/b + 1)/(105*a**(9/2)*b**3*x**(27/2) + 105*
a**(7/2)*b**4*x**(21/2)) - 26*a**(9/2)*b**(9/2)*x***3*sqrt(a*x***3/b + 1)/(1
05*a**(9/2)*b**3*x**(27/2) + 105*a**(7/2)*b**4*x**(21/2)) - 10*a**(7/2)*b*
*(11/2)*sqrt(a*x***3/b + 1)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(7/2)*b**
4*x**(21/2)) - 4*a**8*b*x**(27/2)/(105*a**(9/2)*b**3*x**(27/2) + 105*a**(7
/2)*b**4*x**(21/2)) - 4*a**7*b**2*x**(21/2)/(105*a**(9/2)*b**3*x**(27/2) +
 105*a**(7/2)*b**4*x**(21/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}}{21\,b^2}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a}{15\,b^2}$$

input
```
integrate((a+b/x^3)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-2/21*(a + b/x^3)^(7/2)/b^2 + 2/15*(a + b/x^3)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx = -\frac{2\left(5\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}-7\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a\right)}{105\,b^2}$$

input
```
integrate((a+b/x^3)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-2/105*(5*(a + b/x^3)^(7/2) - 7*(a + b/x^3)^(5/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx = \frac{4\,a^3\,\sqrt{a+\frac{b}{x^3}}}{105\,b^2}-\frac{2\,b\,\sqrt{a+\frac{b}{x^3}}}{21\,x^9}-\frac{16\,a\,\sqrt{a+\frac{b}{x^3}}}{105\,x^6}-\frac{2\,a^2\,\sqrt{a+\frac{b}{x^3}}}{105\,b\,x^3}$$

input
```
int((a + b/x^3)^(3/2)/x^7,x)
```

output
```
(4*a^3*(a + b/x^3)^(1/2))/(105*b^2) - (2*b*(a + b/x^3)^(1/2))/(21*x^9) - (
16*a*(a + b/x^3)^(1/2))/(105*x^6) - (2*a^2*(a + b/x^3)^(1/2))/(105*b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^7}\,dx = \frac{2\sqrt{a\,x^3+b}\,(2a^3x^9-a^2b\,x^6-8a\,b^2x^3-5b^3)}{105\sqrt{x}\,b^2x^{10}}$$

input
```
int((a+b/x^3)^(3/2)/x^7,x)
```

output
```
(2*sqrt(a*x**3 + b)*(2*a**3*x**9 - a**2*b*x**6 - 8*a*b**2*x**3 - 5*b**3))/
(105*sqrt(x)*b**2*x**10)
```

3.478 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx=-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{27b^3}$$

output
```
-2/15*a^2*(a+b/x^3)^(5/2)/b^3+4/21*a*(a+b/x^3)^(7/2)/b^3-2/27*(a+b/x^3)^(9
/2)/b^3
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx=-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}(b+ax^3)\left(35b^2-20abx^3+8a^2x^6\right)}{945b^3x^9}$$

input
```
Integrate[(a + b/x^3)^(3/2)/x^10,x]
```

output
```
(-2*(a + b/x^3)^(3/2)*(b + a*x^3)*(35*b^2 - 20*a*b*x^3 + 8*a^2*x^6))/(945*
b^3*x^9)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^6}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(\frac{\left(a+\frac{b}{x^3}\right)^{7/2}}{b^2}-\frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{b^2}+\frac{a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{b^2}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{9/2}}{9b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^3}\right)$$

input
```
Int[(a + b/x^3)^(3/2)/x^10,x]
```

output
```
((-2*a^2*(a + b/x^3)^(5/2))/(5*b^3) + (4*a*(a + b/x^3)^(7/2))/(7*b^3) - (2
*(a + b/x^3)^(9/2))/(9*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

method	result	size
orering	$-\frac{2(8a^2x^6-20abx^3+35b^2)(ax^3+b)\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{945b^3x^9}$	46
gosper	$-\frac{2(ax^3+b)(8a^2x^6-20abx^3+35b^2)\left(\frac{ax^3+b}{x^3}\right)^{\frac{3}{2}}}{945b^3x^9}$	50
risch	$-\frac{2\sqrt{\frac{ax^3+b}{x^3}}\,(8a^4x^{12}-4a^3bx^9+3a^2b^2x^6+50ax^3b^3+35b^4)}{945x^{12}b^3}$	65
trager	$-\frac{2(8a^4x^{12}-4a^3bx^9+3a^2b^2x^6+50ax^3b^3+35b^4)\sqrt{-\frac{-ax^3-b}{x^3}}}{945x^{12}b^3}$	69
default	$-\frac{2\left(\frac{ax^3+b}{x^3}\right)^{\frac{3}{2}}(8a^3x^9-12a^2bx^6+15ab^2x^3+35b^3)\sqrt{ax^4+bx}}{945x^9b^3\sqrt{x(ax^3+b)}}$	76

input
```
int((a+b/x^3)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-2/945*(8*a^2*x^6-20*a*b*x^3+35*b^2)/b^3/x^9*(a*x^3+b)*(a+b/x^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx = -\frac{2\,(8\,a^4x^{12}-4\,a^3bx^9+3\,a^2b^2x^6+50\,ab^3x^3+35\,b^4)\sqrt{\frac{ax^3+b}{x^3}}}{945\,b^3x^{12}}$$

input
```
integrate((a+b/x^3)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
-2/945*(8*a^4*x^12 - 4*a^3*b*x^9 + 3*a^2*b^2*x^6 + 50*a*b^3*x^3 + 35*b^4)*
sqrt((a*x^3 + b)/x^3)/(b^3*x^12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(54) = 108$.

Time = 1.61 (sec) , antiderivative size = 1001, normalized size of antiderivative = 16.97

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**3)**(3/2)/x**10,x)
```

output
```
-16*a**(23/2)*b**(9/2)*x**21*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45
/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) + 945*
a**(9/2)*b**10*x**(27/2)) - 40*a**(21/2)*b**(11/2)*x**18*sqrt(a*x**3/b + 1
)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**
(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) - 30*a**(19/2)*b**(1
3/2)*x**15*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/
2)*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**
(27/2)) - 110*a**(17/2)*b**(15/2)*x**12*sqrt(a*x**3/b + 1)/(945*a**(15/2)*
b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(3
3/2) + 945*a**(9/2)*b**10*x**(27/2)) - 380*a**(15/2)*b**(17/2)*x**9*sqrt(a
*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2)
 + 2835*a**(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) - 516*a**
(13/2)*b**(19/2)*x**6*sqrt(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45/2) + 2
835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/2)*b**9*x**(33/2) + 945*a**(9/2
)*b**10*x**(27/2)) - 310*a**(11/2)*b**(21/2)*x**3*sqrt(a*x**3/b + 1)/(945*
a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39/2) + 2835*a**(11/2)*
b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) - 70*a**(9/2)*b**(23/2)*sqr
t(a*x**3/b + 1)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8*x**(39
/2) + 2835*a**(11/2)*b**9*x**(33/2) + 945*a**(9/2)*b**10*x**(27/2)) + 16*a
**12*b**4*x**(45/2)/(945*a**(15/2)*b**7*x**(45/2) + 2835*a**(13/2)*b**8...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}}{27\,b^3}+\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a}{21\,b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2}{15\,b^3}$$

input
```
integrate((a+b/x^3)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
-2/27*(a + b/x^3)^(9/2)/b^3 + 4/21*(a + b/x^3)^(7/2)*a/b^3 - 2/15*(a + b/x
^3)^(5/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx = -\frac{2\left(35\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}} - 90\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a + 63\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^2\right)}{945\,b^3}$$

input
```
integrate((a+b/x^3)^(3/2)/x^10,x, algorithm="giac")
```

output
```
-2/945*(35*(a + b/x^3)^(9/2) - 90*(a + b/x^3)^(7/2)*a + 63*(a + b/x^3)^(5/
2)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx = \frac{8\,a^3\,\sqrt{a+\frac{b}{x^3}}}{945\,b^2\,x^3} - \frac{2\,b\,\sqrt{a+\frac{b}{x^3}}}{27\,x^{12}} - \frac{16\,a^4\,\sqrt{a+\frac{b}{x^3}}}{945\,b^3} - \frac{20\,a\,\sqrt{a+\frac{b}{x^3}}}{189\,x^9} - \frac{2\,a^2\,\sqrt{a+\frac{b}{x^3}}}{315\,b\,x^6}$$

input
```
int((a + b/x^3)^(3/2)/x^10,x)
```

output
```
(8*a^3*(a + b/x^3)^(1/2))/(945*b^2*x^3) - (2*b*(a + b/x^3)^(1/2))/(27*x^12
) - (16*a^4*(a + b/x^3)^(1/2))/(945*b^3) - (20*a*(a + b/x^3)^(1/2))/(189*x
^9) - (2*a^2*(a + b/x^3)^(1/2))/(315*b*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{10}}\,dx = \frac{2\sqrt{a\,x^3+b}\,(-8a^4x^{12}+4a^3b\,x^9-3a^2b^2x^6-50a\,b^3x^3-35b^4)}{945\sqrt{x}\,b^3x^{13}}$$

input
```
int((a+b/x^3)^(3/2)/x^10,x)
```

output
```
(2*sqrt(a*x**3 + b)*( - 8*a**4*x**12 + 4*a**3*b*x**9 - 3*a**2*b**2*x**6 -
50*a*b**3*x**3 - 35*b**4))/(945*sqrt(x)*b**3*x**13)
```

3.479 $\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx=\frac{2a^3\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{9/2}}{9b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{11/2}}{33b^4}$$

output
```
2/15*a^3*(a+b/x^3)^(5/2)/b^4-2/7*a^2*(a+b/x^3)^(7/2)/b^4+2/9*a*(a+b/x^3)^(
9/2)/b^4-2/33*(a+b/x^3)^(11/2)/b^4
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx=-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}(b+ax^3)\left(105b^3-70ab^2x^3+40a^2bx^6-16a^3x^9\right)}{3465b^4x^{12}}$$

input
```
Integrate[(a + b/x^3)^(3/2)/x^13,x]
```

output
```
(-2*(a + b/x^3)^(3/2)*(b + a*x^3)*(105*b^3 - 70*a*b^2*x^3 + 40*a^2*b*x^6 -
 16*a^3*x^9))/(3465*b^4*x^12)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^9}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(\frac{\left(a+\frac{b}{x^3}\right)^{9/2}}{b^3}-\frac{3a\left(a+\frac{b}{x^3}\right)^{7/2}}{b^3}+\frac{3a^2\left(a+\frac{b}{x^3}\right)^{5/2}}{b^3}-\frac{a^3\left(a+\frac{b}{x^3}\right)^{3/2}}{b^3}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^3\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}-\frac{6a^2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{11/2}}{11b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{9/2}}{3b^4}\right)$$

input
```
Int[(a + b/x^3)^(3/2)/x^13,x]
```

output
```
((2*a^3*(a + b/x^3)^(5/2))/(5*b^4) - (6*a^2*(a + b/x^3)^(7/2))/(7*b^4) + (
2*a*(a + b/x^3)^(9/2))/(3*b^4) - (2*(a + b/x^3)^(11/2))/(11*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

method	result	size
orering	$\frac{2(16a^3x^9-40a^2b\,x^6+70a\,b^2x^3-105b^3)(a\,x^3+b)\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{3465b^4x^{12}}$	57
gosper	$\frac{2(a\,x^3+b)(16a^3x^9-40a^2b\,x^6+70a\,b^2x^3-105b^3)\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}}{3465x^{12}b^4}$	61
risch	$\frac{2\sqrt{\frac{a\,x^3+b}{x^3}}\,(16a^5x^{15}-8a^4b\,x^{12}+6a^3b^2x^9-5a^2b^3x^6-140a\,b^4x^3-105b^5)}{3465x^{15}b^4}$	76
trager	$\frac{2(16a^5x^{15}-8a^4b\,x^{12}+6a^3b^2x^9-5a^2b^3x^6-140a\,b^4x^3-105b^5)\sqrt{-\frac{-a\,x^3-b}{x^3}}}{3465x^{15}b^4}$	80
default	$\frac{2\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}(16a^4x^{12}-24a^3b\,x^9+30a^2b^2x^6-35a\,x^3b^3-105b^4)\sqrt{a\,x^4+bx}}{3465x^{12}b^4\sqrt{x(a\,x^3+b)}}$	87

input
```
int((a+b/x^3)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
2/3465*(16*a^3*x^9-40*a^2*b*x^6+70*a*b^2*x^3-105*b^3)/b^4/x^12*(a*x^3+b)*(
a+b/x^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx = \frac{2\,(16\,a^5x^{15}-8\,a^4bx^{12}+6\,a^3b^2x^9-5\,a^2b^3x^6-140\,ab^4x^3-105\,b^5)\sqrt{\frac{ax^3+b}{x^3}}}{3465\,b^4x^{15}}$$

input
```
integrate((a+b/x^3)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
2/3465*(16*a^5*x^15 - 8*a^4*b*x^12 + 6*a^3*b^2*x^9 - 5*a^2*b^3*x^6 - 140*a
*b^4*x^3 - 105*b^5)*sqrt((a*x^3 + b)/x^3)/(b^4*x^15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2317 vs. 2(75) = 150.

Time = 2.49 (sec) , antiderivative size = 2317, normalized size of antiderivative = 28.96

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**3)**(3/2)/x**13,x)
```

output
```
32*a**(33/2)*b**(23/2)*x***33*sqrt(a*x***3/b + 1)/(3465*a**(23/2)*b**15*x**(
69/2) + 20790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2)
+ 69300*a**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 2079
0*a**(13/2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) + 176*a**(31
/2)*b**(25/2)*x***30*sqrt(a*x***3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 2
0790*a**(21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a
**(17/2)*b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/
2)*b**20*x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) + 396*a**(29/2)*b**(2
7/2)*x***27*sqrt(a*x***3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(
21/2)*b**16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*
b**18*x**(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*
x**(39/2) + 3465*a**(11/2)*b**21*x**(33/2)) + 462*a**(27/2)*b**(29/2)*x***2
4*sqrt(a*x***3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**
16*x**(63/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**
(51/2) + 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2)
 + 3465*a**(11/2)*b**21*x**(33/2)) - 1848*a**(23/2)*b**(33/2)*x***18*sqrt(a
*x***3/b + 1)/(3465*a**(23/2)*b**15*x**(69/2) + 20790*a**(21/2)*b**16*x**(6
3/2) + 51975*a**(19/2)*b**17*x**(57/2) + 69300*a**(17/2)*b**18*x**(51/2) +
 51975*a**(15/2)*b**19*x**(45/2) + 20790*a**(13/2)*b**20*x**(39/2) + 3465*
a**(11/2)*b**21*x**(33/2)) - 5544*a**(21/2)*b**(35/2)*x***15*sqrt(a*x***3...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{11}{2}}}{33\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}a}{9\,b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a^2}{7\,b^4}+\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^3}{15\,b^4}$$

input
```
integrate((a+b/x^3)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
-2/33*(a + b/x^3)^(11/2)/b^4 + 2/9*(a + b/x^3)^(9/2)*a/b^4 - 2/7*(a + b/x^
3)^(7/2)*a^2/b^4 + 2/15*(a + b/x^3)^(5/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx = -\frac{2\left(105\left(a+\frac{b}{x^3}\right)^{\frac{11}{2}}-385\left(a+\frac{b}{x^3}\right)^{\frac{9}{2}}a+495\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}a^2-231\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^3\right)}{3465\,b^4}$$

input
```
integrate((a+b/x^3)^(3/2)/x^13,x, algorithm="giac")
```

output
```
-2/3465*(105*(a + b/x^3)^(11/2) - 385*(a + b/x^3)^(9/2)*a + 495*(a + b/x^3
)^(7/2)*a^2 - 231*(a + b/x^3)^(5/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx = \frac{32\,a^5\,\sqrt{a+\frac{b}{x^3}}}{3465\,b^4}-\frac{2\,b\,\sqrt{a+\frac{b}{x^3}}}{33\,x^{15}}-\frac{8\,a\,\sqrt{a+\frac{b}{x^3}}}{99\,x^{12}}-\frac{16\,a^4\,\sqrt{a+\frac{b}{x^3}}}{3465\,b^3\,x^3}+\frac{4\,a^3\,\sqrt{a+\frac{b}{x^3}}}{1155\,b^2\,x^6}-\frac{2\,a^2\,\sqrt{a+\frac{b}{x^3}}}{693\,b\,x^9}$$

input
```
int((a + b/x^3)^(3/2)/x^13,x)
```

output
```
(32*a^5*(a + b/x^3)^(1/2))/(3465*b^4) - (2*b*(a + b/x^3)^(1/2))/(33*x^15)
- (8*a*(a + b/x^3)^(1/2))/(99*x^12) - (16*a^4*(a + b/x^3)^(1/2))/(3465*b^3
*x^3) + (4*a^3*(a + b/x^3)^(1/2))/(1155*b^2*x^6) - (2*a^2*(a + b/x^3)^(1/2
))/(693*b*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{x^{13}}\,dx = \frac{2\sqrt{a\,x^3+b}\,(16a^5x^{15}-8a^4b\,x^{12}+6a^3b^2x^9-5a^2b^3x^6-140a\,b^4x^3-105b^5)}{3465\sqrt{x}\,b^4x^{16}}$$

input
```
int((a+b/x^3)^(3/2)/x^13,x)
```

output
```
(2*sqrt(a*x**3 + b)*(16*a**5*x**15 - 8*a**4*b*x**12 + 6*a**3*b**2*x**9 - 5
*a**2*b**3*x**6 - 140*a*b**4*x**3 - 105*b**5))/(3465*sqrt(x)*b**4*x**16)
```

3.480 $\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx=-\frac{b\sqrt{a+\frac{b}{x^3}}x^3}{4a^2}+\frac{\sqrt{a+\frac{b}{x^3}}x^6}{6a}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
-1/4*b*(a+b/x^3)^(1/2)*x^3/a^2+1/6*(a+b/x^3)^(1/2)*x^6/a+1/4*b^2*arctanh((
a+b/x^3)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{\sqrt{a}x^{3/2}(-3b^2-abx^3+2a^2x^6)+3b^2\sqrt{b+ax^3}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{12a^{5/2}\sqrt{a+\frac{b}{x^3}}x^{3/2}}$$

input
```
Integrate[x^5/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a]*x^(3/2)*(-3*b^2 - a*b*x^3 + 2*a^2*x^6) + 3*b^2*Sqrt[b + a*x^3]*Lo
g[Sqrt[a]*x^(3/2) + Sqrt[b + a*x^3]])/(12*a^(5/2)*Sqrt[a + b/x^3]*x^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{x^9}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{3b\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}}{4a}+\frac{x^6\sqrt{a+\frac{b}{x^3}}}{2a}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{3b\left(-\frac{b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}}{2a}-\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)}{4a}+\frac{x^6\sqrt{a+\frac{b}{x^3}}}{2a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{3b\left(-\frac{\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{a}-\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)}{4a}+\frac{x^6\sqrt{a+\frac{b}{x^3}}}{2a}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)}{4a}+\frac{x^6\sqrt{a+\frac{b}{x^3}}}{2a}\right)$$

input
```
Int[x^5/Sqrt[a + b/x^3],x]
```

output
```
((Sqrt[a + b/x^3]*x^6)/(2*a) + (3*b*(-((Sqrt[a + b/x^3]*x^3)/a) + (b*ArcTa
nh[Sqrt[a + b/x^3]/Sqrt[a]])/a^(3/2)))/(4*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

method	result	size
risch	$\frac{(2a x^3-3b)(a x^3+b)}{12a^2\sqrt{\frac{a x^3+b}{x^3}}}+\frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{x\left(a x^3+b\right)}}{x^2\sqrt{a}}\right)\sqrt{x(a x^3+b)}}{4a^{\frac{5}{2}}x^2\sqrt{\frac{a x^3+b}{x^3}}}$	91
default	$\frac{(a x^3+b)\left(2\sqrt{x(a x^3+b)}\,a^{\frac{7}{2}}x^4-3\sqrt{x(a x^3+b)}\,a^{\frac{5}{2}}bx+3a^2 \operatorname{arctanh}\left(\frac{\sqrt{x\left(a x^3+b\right)}}{x^2\sqrt{a}}\right)b^2\right)}{12\sqrt{\frac{a x^3+b}{x^3}}\,x\sqrt{x(a x^3+b)}\,a^{\frac{9}{2}}}$	105

input
```
int(x^5/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*a*x^3-3*b)/a^2*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)+1/4*b^2/a^(5/2)*arc
tanh((x*(a*x^3+b))^(1/2)/x^2/a^(1/2))/x^2/((a*x^3+b)/x^3)^(1/2)*(x*(a*x^3+
b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.53

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx = \left[\frac{3\sqrt{a}b^2\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\,(2\,ax^6+bx^3)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)+4\,(2\,a^2x^6-3\,abx^3)\sqrt{\frac{ax^3+b}{x^3}}}{48\,a^3}, -\frac{3\sqrt{-a}b^2\arctan\left(\frac{(2\,ax^3+b)\sqrt{-a}\sqrt{\frac{ax^3+b}{x^3}}}{2\,(a^2x^3+ab)}\right)-2\,(2\,a^2x^6-3\,abx^3)\sqrt{\frac{ax^3+b}{x^3}}}{24\,a^3}\right]$$

input
```
integrate(x^5/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^2*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3
)*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^2*x^6 - 3*a*b*x^3)*sqrt((a*x^3 +
 b)/x^3))/a^3, -1/24*(3*sqrt(-a)*b^2*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqr
t((a*x^3 + b)/x^3)/(a^2*x^3 + a*b)) - 2*(2*a^2*x^6 - 3*a*b*x^3)*sqrt((a*x^
3 + b)/x^3))/a^3]
```

Sympy [A] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.38

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{x^{\frac{15}{2}}}{6\sqrt{b}\sqrt{\frac{ax^3}{b}+1}} - \frac{\sqrt{b}x^{\frac{9}{2}}}{12a\sqrt{\frac{ax^3}{b}+1}} - \frac{b^{\frac{3}{2}}x^{\frac{3}{2}}}{4a^2\sqrt{\frac{ax^3}{b}+1}} + \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}}$$

input
```
integrate(x**5/(a+b/x**3)**(1/2),x)
```

output
```
x**(15/2)/(6*sqrt(b)*sqrt(a*x**3/b + 1)) - sqrt(b)*x**(9/2)/(12*a*sqrt(a*x
**3/b + 1)) - b**(3/2)*x**(3/2)/(4*a**2*sqrt(a*x**3/b + 1)) + b**2*asinh(s
qrt(a)*x**(3/2)/sqrt(b))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{b^2\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}} - \frac{3\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}b^2 - 5\,\sqrt{a+\frac{b}{x^3}}ab^2}{12\left(\left(a+\frac{b}{x^3}\right)^2a^2 - 2\left(a+\frac{b}{x^3}\right)a^3 + a^4\right)}$$

input
```
integrate(x^5/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*b^2*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(5
/2) - 1/12*(3*(a + b/x^3)^(3/2)*b^2 - 5*sqrt(a + b/x^3)*a*b^2)/((a + b/x^3
)^2*a^2 - 2*(a + b/x^3)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{1}{12}\,\sqrt{ax^4+bx}\left(\frac{2\,x^3}{a} - \frac{3\,b}{a^2}\right)x - \frac{b^2\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2}$$

input
```
integrate(x^5/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(a*x^4 + b*x)*(2*x^3/a - 3*b/a^2)*x - 1/4*b^2*arctan(sqrt(a + b/x
^3)/sqrt(-a))/(sqrt(-a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{x^6\sqrt{a+\frac{b}{x^3}}}{6\,a} + \frac{b^2\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{8\,a^{5/2}} - \frac{b\,x^3\sqrt{a+\frac{b}{x^3}}}{4\,a^2}$$

input
```
int(x^5/(a + b/x^3)^(1/2),x)
```

output
```
(x^6*(a + b/x^3)^(1/2))/(6*a) + (b^2*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))
*((a + b/x^3)^(1/2) + a^(1/2))^3))/(8*a^(5/2)) - (b*x^3*(a + b/x^3)^(1/2))
/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

$$= \frac{4\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^4-6\sqrt{x}\,\sqrt{a\,x^3+b}\,abx-3\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)b^2+3\sqrt{a}\log\left(\sqrt{a\,x^3+b}+\right.}{24a^3}$$

input
```
int(x^5/(a+b/x^3)^(1/2),x)
```

output
```
(4*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**4 - 6*sqrt(x)*sqrt(a*x**3 + b)*a*b*x -
 3*sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*b**2 + 3*sqrt(a)*log(
sqrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*b**2)/(24*a**3)
```

3.481 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{\sqrt{a+\frac{b}{x^3}}\,x^3}{3a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
1/3*(a+b/x^3)^(1/2)*x^3/a-1/3*b*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{\sqrt{a}x^{3/2}(b+ax^3)-b\sqrt{b+ax^3}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{3a^{3/2}\sqrt{a+\frac{b}{x^3}}\,x^{3/2}}$$

input
```
Integrate[x^2/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a]*x^(3/2)*(b + a*x^3) - b*Sqrt[b + a*x^3]*Log[Sqrt[a]*x^(3/2) + Sqr
t[b + a*x^3]])/(3*a^(3/2)*Sqrt[a + b/x^3]*x^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}}{2a}+\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{a}+\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{a+\frac{b}{x^3}}}{a}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[x^2/Sqrt[a + b/x^3],x]
```

output
```
((Sqrt[a + b/x^3]*x^3)/a - (b*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/a^(3/2))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(38) = 76$.

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
default	$\frac{\left(a\,x^{3}+b\right)\left(\sqrt{x\left(a\,x^{3}+b\right)}\,x\sqrt{a}-\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)b\right)}{3\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x\sqrt{x\left(a\,x^{3}+b\right)}\,a^{\frac{3}{2}}}$	79
risch	$\frac{a\,x^{3}+b}{3a\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)\sqrt{x\left(a\,x^{3}+b\right)}}{3a^{\frac{3}{2}}\,x^{2}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	79

input
```
int(x^2/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)*((x*(a*x^3+b))^(1/2)*x*a^(1/2)-arcta
nh((x*(a*x^3+b))^(1/2)/x^2/a^(1/2))*b)/(x*(a*x^3+b))^(1/2)/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.14

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx = \left[\frac{4\,ax^3\sqrt{\frac{ax^3+b}{x^3}}+\sqrt{a}b\log\left(-8\,a^2x^6-8\,abx^3-b^2+4\,(2\,ax^6+bx^3)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)}{12\,a^2}, \frac{2\,ax^3\sqrt{\frac{ax^3+b}{x^3}}+\sqrt{-a}b\,a\right.$$

input
```
integrate(x^2/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(4*a*x^3*sqrt((a*x^3 + b)/x^3) + sqrt(a)*b*log(-8*a^2*x^6 - 8*a*b*x^
3 - b^2 + 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)))/a^2, 1/6*(2*
a*x^3*sqrt((a*x^3 + b)/x^3) + sqrt(-a)*b*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)
*sqrt((a*x^3 + b)/x^3)/(a^2*x^3 + a*b)))/a^2]
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{ax^3}{b}+1}}{3a} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3a^{\frac{3}{2}}}$$

input
```
integrate(x**2/(a+b/x**3)**(1/2),x)
```

output
```
sqrt(b)*x**(3/2)*sqrt(a*x**3/b + 1)/(3*a) - b*asinh(sqrt(a)*x**(3/2)/sqrt(
b))/(3*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{\sqrt{a+\frac{b}{x^3}}\,b}{3\left(\left(a+\frac{b}{x^3}\right)a-a^2\right)} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{6\,a^{\frac{3}{2}}}$$

input
```
integrate(x^2/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(a + b/x^3)*b/((a + b/x^3)*a - a^2) + 1/6*b*log((sqrt(a + b/x^3) -
 sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{\sqrt{ax^4+bx}\,x}{3\,a} + \frac{b\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a}$$

input
```
integrate(x^2/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(a*x^4 + b*x)*x/a + 1/3*b*arctan(sqrt(a + b/x^3)/sqrt(-a))/(sqrt(-
a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{b\,\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)^3\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)\right)}{6\,a^{3/2}} + \frac{x^3\,\sqrt{a+\frac{b}{x^3}}}{3\,a}$$

input
```
int(x^2/(a + b/x^3)^(1/2),x)
```

output
```
(b*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))^3*((a + b/x^3)^(1/2) + a^(1/2))))
/(6*a^(3/2)) + (x^3*(a + b/x^3)^(1/2))/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{2\sqrt{x}\,\sqrt{a\,x^3+b}\,ax+\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)b-\sqrt{a}\log\left(\sqrt{a\,x^3+b}+\sqrt{x}\,\sqrt{a}\,x\right)b}{6a^2}$$

input
```
int(x^2/(a+b/x^3)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b)*a*x + sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*s
qrt(a)*x)*b - sqrt(a)*log(sqrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*b)/(6*a**2
)
```

3.482 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
2/3*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(27) = 54.

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}\,dx=\frac{2\sqrt{b+ax^3}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{3\sqrt{a}\sqrt{a+\frac{b}{x^3}}x^{3/2}}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x),x]
```

output
```
(2*Sqrt[b + a*x^3]*Log[Sqrt[a]*x^(3/2) + Sqrt[b + a*x^3]])/(3*Sqrt[a]*Sqrt
[a + b/x^3]*x^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{3b}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x),x]
```

output
```
(2*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(19) = 38$.

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

method	result	size
default	$\frac{2\left(a\,x^{3}+b\right) \operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)}{3\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\, x\sqrt{x\left(a\,x^{3}+b\right)}\,\sqrt{a}}$	59

input
```
int(1/(a+b/x^3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/(x*(a*x^3+b))^(1/2)/a^(1/2)*arctanh(
(x*(a*x^3+b))^(1/2)/x^2/a^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx = \left[\frac{\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\,(2\,ax^6+bx^3)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)}{6\,\sqrt{a}},\ -\frac{\sqrt{-a}\arctan\left(\frac{(2\,ax^3+b)\sqrt{-a}\sqrt{\frac{ax^3+b}{x^3}}}{2\,(a^2x^3+ab)}\right)}{3\,a}\right]$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/6*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((
a*x^3 + b)/x^3))/sqrt(a), -1/3*sqrt(-a)*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*
sqrt((a*x^3 + b)/x^3)/(a^2*x^3 + a*b))/a]
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx = \frac{2\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{3\sqrt{a}}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x,x)
```

output
```
2*asinh(sqrt(a)*x**(3/2)/sqrt(b))/(3*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/3*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/sqrt(a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
int(1/(x*(a + b/x^3)^(1/2)),x)
```

output
```
(2*atanh((a + b/x^3)^(1/2)/a^(1/2)))/(3*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}\,dx = \frac{\sqrt{a}\left(-\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)+\log\left(\sqrt{a\,x^3+b}+\sqrt{x}\,\sqrt{a}\,x\right)\right)}{3a}$$

input
```
int(1/(a+b/x^3)^(1/2)/x,x)
```

output
```
(sqrt(a)*( - log(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x) + log(sqrt(a*x**3 +
 b) + sqrt(x)*sqrt(a)*x)))/(3*a)
```

3.483 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4} \, dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$$

output
```
-2/3*(a+b/x^3)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4} \, dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^4),x]
```

output
```
(-2*Sqrt[a + b/x^3])/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{x^3}}}\,dx$$

$$\Big\downarrow \; 793$$

$$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^4),x]
```

output
```
(-2*Sqrt[a + b/x^3])/(3*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\sqrt{a+\frac{b}{x^3}}}{3b}$	15
trager	$-\frac{2\sqrt{-\frac{-a\,x^3-b}{x^3}}}{3b}$	23
orering	$-\frac{2(a\,x^3+b)}{3x^3b\sqrt{a+\frac{b}{x^3}}}$	25
gosper	$-\frac{2(a\,x^3+b)}{3x^3b\sqrt{\frac{a\,x^3+b}{x^3}}}$	29
default	$-\frac{2(a\,x^3+b)}{3x^3b\sqrt{\frac{a\,x^3+b}{x^3}}}$	29
risch	$-\frac{2(a\,x^3+b)}{3x^3b\sqrt{\frac{a\,x^3+b}{x^3}}}$	29

input
```
int(1/(a+b/x^3)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a+b/x^3)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}\,dx = -\frac{2\sqrt{\frac{ax^3+b}{x^3}}}{3\,b}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/3*sqrt((a*x^3 + b)/x^3)/b
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}\,dx = \begin{cases} -\frac{2\sqrt{a+\frac{b}{x^3}}}{3b} & \text{for } b \neq 0 \\ -\frac{1}{3\sqrt{a}x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**4,x)
```

output
```
Piecewise((-2*sqrt(a + b/x**3)/(3*b), Ne(b, 0)), (-1/(3*sqrt(a)*x**3), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,b}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-2/3*sqrt(a + b/x^3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,b}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2/3*sqrt(a + b/x^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,b}$$

input
```
int(1/(x^4*(a + b/x^3)^(1/2)),x)
```

output
```
-(2*(a + b/x^3)^(1/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}\,dx = -\frac{2\sqrt{a\,x^3+b}}{3\sqrt{x}\,bx}$$

input
```
int(1/(a+b/x^3)^(1/2)/x^4,x)
```

output
```
( - 2*sqrt(a*x**3 + b))/(3*sqrt(x)*b*x)
```

3.484 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7} dx = \frac{2a\sqrt{a+\frac{b}{x^3}}}{3b^2} - \frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^2}$$

output
```
2/3*a*(a+b/x^3)^(1/2)/b^2-2/9*(a+b/x^3)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7} dx = \frac{2\sqrt{a+\frac{b}{x^3}}(-b+2ax^3)}{9b^2x^3}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^7),x]
```

output
```
(2*Sqrt[a + b/x^3]*(-b + 2*a*x^3))/(9*b^2*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(\frac{\sqrt{a+\frac{b}{x^3}}}{b}-\frac{a}{b\sqrt{a+\frac{b}{x^3}}}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a\sqrt{a+\frac{b}{x^3}}}{b^2}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^2}\right)$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^7),x]
```

output
```
((2*a*Sqrt[a + b/x^3])/b^2 - (2*(a + b/x^3)^(3/2))/(3*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
orering	$\frac{2(2a\,x^{3}-b)\,(a\,x^{3}+b)}{9b^{2}x^{6}\sqrt{a+\frac{b}{x^{3}}}}$	35
trager	$\frac{2(2a\,x^{3}-b)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{9x^{3}b^{2}}$	36
gosper	$\frac{2(a\,x^{3}+b)\,(2a\,x^{3}-b)}{9x^{6}b^{2}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	39
risch	$\frac{2(a\,x^{3}+b)\,(2a\,x^{3}-b)}{9x^{6}b^{2}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	39
default	$\frac{2(a\,x^{3}+b)\left(3a\,x^{3}\sqrt{x(a\,x^{3}+b)}-a\sqrt{a\,x^{4}+bx}\,x^{3}-\sqrt{a\,x^{4}+bx}\,b\right)}{9\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x^{6}\sqrt{x(a\,x^{3}+b)}\,b^{2}}$	89

input
```
int(1/(a+b/x^3)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
2/9*(2*a*x^3-b)/b^2/x^6*(a*x^3+b)/(a+b/x^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7}\,dx = \frac{2\,(2\,ax^3-b)\sqrt{\frac{ax^3+b}{x^3}}}{9\,b^2x^3}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
2/9*(2*a*x^3 - b)*sqrt((a*x^3 + b)/x^3)/(b^2*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(34) = 68.

Time = 1.06 (sec) , antiderivative size = 255, normalized size of antiderivative = 6.71

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7}\,dx = \frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}} + \frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}} - \frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}} - \frac{4a^4bx^{\frac{15}{2}}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}} - \frac{4a^3b^2x^{\frac{9}{2}}}{9a^{\frac{5}{2}}b^3x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^4x^{\frac{9}{2}}}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**7,x)
```

output
```
4*a**(7/2)*b**(3/2)*x**6*sqrt(a*x**3/b + 1)/(9*a**(5/2)*b**3*x**(15/2) + 9
*a**(3/2)*b**4*x**(9/2)) + 2*a**(5/2)*b**(5/2)*x**3*sqrt(a*x**3/b + 1)/(9*
a**(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(9/2)) - 2*a**(3/2)*b**(7/2)*
sqrt(a*x**3/b + 1)/(9*a**(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(9/2))
- 4*a**4*b*x**(15/2)/(9*a**(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(9/2)
) - 4*a**3*b**2*x**(9/2)/(9*a**(5/2)*b**3*x**(15/2) + 9*a**(3/2)*b**4*x**(
9/2))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^7}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b^2} + \frac{2\sqrt{a+\frac{b}{x^3}}\,a}{3\,b^2}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x^3)^(3/2)/b^2 + 2/3*sqrt(a + b/x^3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^7}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b^2} + \frac{2\sqrt{a+\frac{b}{x^3}}\,a}{3\,b^2}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-2/9*(a + b/x^3)^(3/2)/b^2 + 2/3*sqrt(a + b/x^3)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^7}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}\,(b-2\,a\,x^3)}{9\,b^2\,x^3}$$

input
```
int(1/(x^7*(a + b/x^3)^(1/2)),x)
```

output
```
-(2*(a + b/x^3)^(1/2)*(b - 2*a*x^3))/(9*b^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7}\,dx=\frac{2\sqrt{a\,x^3+b}\,(2a\,x^3-b)}{9\sqrt{x}\,b^2x^4}$$

input
```
int(1/(a+b/x^3)^(1/2)/x^7,x)
```

output
```
(2*sqrt(a*x**3 + b)*(2*a*x**3 - b))/(9*sqrt(x)*b**2*x**4)
```

3.485 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx=-\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{3b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^3}$$

output
```
-2/3*a^2*(a+b/x^3)^(1/2)/b^3+4/9*a*(a+b/x^3)^(3/2)/b^3-2/15*(a+b/x^3)^(5/2
)/b^3
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx=-\frac{2\sqrt{a+\frac{b}{x^3}}(3b^2-4abx^3+8a^2x^6)}{45b^3x^6}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^10),x]
```

output
```
(-2*Sqrt[a + b/x^3]*(3*b^2 - 4*a*b*x^3 + 8*a^2*x^6))/(45*b^3*x^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(\frac{a^2}{b^2\sqrt{a+\frac{b}{x^3}}}-\frac{2\sqrt{a+\frac{b}{x^3}}a}{b^2}+\frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{b^2}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2\sqrt{a+\frac{b}{x^3}}}{b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^3}+\frac{4a\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^3}\right)$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^10),x]
```

output
```
((-2*a^2*Sqrt[a + b/x^3])/b^3 + (4*a*(a + b/x^3)^(3/2))/(3*b^3) - (2*(a +
b/x^3)^(5/2))/(5*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

method	result	size
orering	$-\frac{2\left(8a^{2}x^{6}-4ab\,x^{3}+3b^{2}\right)\left(a\,x^{3}+b\right)}{45b^{3}x^{9}\sqrt{a+\frac{b}{x^{3}}}}$	46
trager	$-\frac{2\left(8a^{2}x^{6}-4ab\,x^{3}+3b^{2}\right)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{45x^{6}b^{3}}$	47
gosper	$-\frac{2\left(a\,x^{3}+b\right)\left(8a^{2}x^{6}-4ab\,x^{3}+3b^{2}\right)}{45x^{9}b^{3}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	50
risch	$-\frac{2\left(a\,x^{3}+b\right)\left(8a^{2}x^{6}-4ab\,x^{3}+3b^{2}\right)}{45x^{9}b^{3}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	50
default	$-\frac{2\left(a\,x^{3}+b\right)\left(15\sqrt{x\left(a\,x^{3}+b\right)}\,a^{2}x^{6}-7a^{2}\sqrt{a\,x^{4}+bx}\,x^{6}-4a\sqrt{a\,x^{4}+bx}\,b\,x^{3}+3\sqrt{a\,x^{4}+bx}\,b^{2}\right)}{45\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x^{9}\sqrt{x\left(a\,x^{3}+b\right)}\,b^{3}}$	113

input
```
int(1/(a+b/x^3)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-2/45*(8*a^2*x^6-4*a*b*x^3+3*b^2)/b^3/x^9*(a*x^3+b)/(a+b/x^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx = -\frac{2\,(8\,a^2x^6-4\,abx^3+3\,b^2)\sqrt{\frac{ax^3+b}{x^3}}}{45\,b^3x^6}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
-2/45*(8*a^2*x^6 - 4*a*b*x^3 + 3*b^2)*sqrt((a*x^3 + b)/x^3)/(b^3*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(54) = 108.

Time = 1.62 (sec) , antiderivative size = 824, normalized size of antiderivative = 13.97

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**10,x)
```

output
```
-16*a**(15/2)*b**(9/2)*x**15*sqrt(a*x**3/b + 1)/(45*a**(11/2)*b**7*x**(33/
2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/
2)*b**10*x**(15/2)) - 40*a**(13/2)*b**(11/2)*x**12*sqrt(a*x**3/b + 1)/(45*
a**(11/2)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9
*x**(21/2) + 45*a**(5/2)*b**10*x**(15/2)) - 30*a**(11/2)*b**(13/2)*x**9*sq
rt(a*x**3/b + 1)/(45*a**(11/2)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2
) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)*b**10*x**(15/2)) - 10*a**(9/
2)*b**(15/2)*x**6*sqrt(a*x**3/b + 1)/(45*a**(11/2)*b**7*x**(33/2) + 135*a*
*(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) + 45*a**(5/2)*b**10*x*
*(15/2)) - 10*a**(7/2)*b**(17/2)*x**3*sqrt(a*x**3/b + 1)/(45*a**(11/2)*b**
7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/2) +
45*a**(5/2)*b**10*x**(15/2)) - 6*a**(5/2)*b**(19/2)*sqrt(a*x**3/b + 1)/(45
*a**(11/2)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**
9*x**(21/2) + 45*a**(5/2)*b**10*x**(15/2)) + 16*a**8*b**4*x**(33/2)/(45*a*
*(11/2)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x
**(21/2) + 45*a**(5/2)*b**10*x**(15/2)) + 48*a**7*b**5*x**(27/2)/(45*a**(1
1/2)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(
21/2) + 45*a**(5/2)*b**10*x**(15/2)) + 48*a**6*b**6*x**(21/2)/(45*a**(11/2
)*b**7*x**(33/2) + 135*a**(9/2)*b**8*x**(27/2) + 135*a**(7/2)*b**9*x**(21/
2) + 45*a**(5/2)*b**10*x**(15/2)) + 16*a**5*b**7*x**(15/2)/(45*a**(11/2...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{10}}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b^3}+\frac{4\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a}{9\,b^3}-\frac{2\sqrt{a+\frac{b}{x^3}}\,a^2}{3\,b^3}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
-2/15*(a + b/x^3)^(5/2)/b^3 + 4/9*(a + b/x^3)^(3/2)*a/b^3 - 2/3*sqrt(a + b
/x^3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}a^2}{3\,b^3} - \frac{2\left(3\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}} - 10\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a\right)}{45\,b^3}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^10,x, algorithm="giac")
```

output
```
-2/3*sqrt(a + b/x^3)*a^2/b^3 - 2/45*(3*(a + b/x^3)^(5/2) - 10*(a + b/x^3)^
(3/2)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx = -\frac{6\,b^2\,\sqrt{a+\frac{b}{x^3}} + 16\,a^2\,x^6\,\sqrt{a+\frac{b}{x^3}} - 8\,a\,b\,x^3\,\sqrt{a+\frac{b}{x^3}}}{45\,b^3\,x^6}$$

input
```
int(1/(x^10*(a + b/x^3)^(1/2)),x)
```

output
```
-(6*b^2*(a + b/x^3)^(1/2) + 16*a^2*x^6*(a + b/x^3)^(1/2) - 8*a*b*x^3*(a +
b/x^3)^(1/2))/(45*b^3*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}\,dx = \frac{2\sqrt{a\,x^3+b}\,(-8a^2x^6+4ab\,x^3-3b^2)}{45\sqrt{x}\,b^3x^7}$$

input
```
int(1/(a+b/x^3)^(1/2)/x^10,x)
```

output
```
(2*sqrt(a*x**3 + b)*( - 8*a**2*x**6 + 4*a*b*x**3 - 3*b**2))/(45*sqrt(x)*b*
*3*x**7)
```

3.486 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{13}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{13}}\,dx=\frac{2a^3\sqrt{a+\frac{b}{x^3}}}{3b^4}-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{21b^4}$$

output
```
2/3*a^3*(a+b/x^3)^(1/2)/b^4-2/3*a^2*(a+b/x^3)^(3/2)/b^4+2/5*a*(a+b/x^3)^(5
/2)/b^4-2/21*(a+b/x^3)^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{13}}\,dx=\frac{2\sqrt{a+\frac{b}{x^3}}(-5b^3+6ab^2x^3-8a^2bx^6+16a^3x^9)}{105b^4x^9}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^13),x]
```

output
```
(2*Sqrt[a + b/x^3]*(-5*b^3 + 6*a*b^2*x^3 - 8*a^2*b*x^6 + 16*a^3*x^9))/(105
*b^4*x^9)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13}\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(-\frac{a^3}{b^3\sqrt{a+\frac{b}{x^3}}}+\frac{3\sqrt{a+\frac{b}{x^3}}a^2}{b^3}-\frac{3\left(a+\frac{b}{x^3}\right)^{3/2}a}{b^3}+\frac{\left(a+\frac{b}{x^3}\right)^{5/2}}{b^3}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^3\sqrt{a+\frac{b}{x^3}}}{b^4}-\frac{2a^2\left(a+\frac{b}{x^3}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{7/2}}{7b^4}+\frac{6a\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}\right)$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^13),x]
```

output
```
((2*a^3*Sqrt[a + b/x^3])/b^4 - (2*a^2*(a + b/x^3)^(3/2))/b^4 + (6*a*(a + b
/x^3)^(5/2))/(5*b^4) - (2*(a + b/x^3)^(7/2))/(7*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

method	result	size
orering	$\frac{2\left(16a^{3}x^{9}-8a^{2}b\,x^{6}+6a\,b^{2}x^{3}-5b^{3}\right)\left(a\,x^{3}+b\right)}{105b^{4}x^{12}\sqrt{a+\frac{b}{x^{3}}}}$	57
trager	$\frac{2\left(16a^{3}x^{9}-8a^{2}b\,x^{6}+6a\,b^{2}x^{3}-5b^{3}\right)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{105x^{9}b^{4}}$	58
gosper	$\frac{2\left(a\,x^{3}+b\right)\left(16a^{3}x^{9}-8a^{2}b\,x^{6}+6a\,b^{2}x^{3}-5b^{3}\right)}{105x^{12}b^{4}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	61
risch	$\frac{2\left(a\,x^{3}+b\right)\left(16a^{3}x^{9}-8a^{2}b\,x^{6}+6a\,b^{2}x^{3}-5b^{3}\right)}{105x^{12}b^{4}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	61
default	$\frac{2\left(a\,x^{3}+b\right)\left(35\sqrt{x\left(a\,x^{3}+b\right)}\,a^{3}x^{9}-19a^{3}\sqrt{a\,x^{4}+bx}\,x^{9}-8a^{2}\sqrt{a\,x^{4}+bx}\,b\,x^{6}+6a\sqrt{a\,x^{4}+bx}\,b^{2}x^{3}-5\sqrt{a\,x^{4}+bx}\,b^{3}\right)}{105\sqrt{\frac{a\,x^{3}+b}{x^{3}}}\,x^{12}\sqrt{x\left(a\,x^{3}+b\right)}\,b^{4}}$	135

input
```
int(1/(a+b/x^3)^(1/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
2/105*(16*a^3*x^9-8*a^2*b*x^6+6*a*b^2*x^3-5*b^3)/b^4/x^12*(a*x^3+b)/(a+b/x
^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{13}}\,dx = \frac{2\,(16\,a^3x^9-8\,a^2bx^6+6\,ab^2x^3-5\,b^3)\sqrt{\frac{ax^3+b}{x^3}}}{105\,b^4x^9}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^13,x, algorithm="fricas")
```

output
```
2/105*(16*a^3*x^9 - 8*a^2*b*x^6 + 6*a*b^2*x^3 - 5*b^3)*sqrt((a*x^3 + b)/x^
3)/(b^4*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(75) = 150$.

Time = 2.14 (sec) , antiderivative size = 2183, normalized size of antiderivative = 27.29

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{13}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**13,x)
```

output
```
32*a**(25/2)*b**(23/2)*x**27*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(5
7/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 21
00*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/
2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) + 176*a**(23/2)*b**(25/
2)*x**24*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)
*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x
**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) +
 105*a**(7/2)*b**21*x**(21/2)) + 396*a**(21/2)*b**(27/2)*x**21*sqrt(a*x**3
/b + 1)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1
575*a**(15/2)*b**17*x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(
11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*
x**(21/2)) + 462*a**(19/2)*b**(29/2)*x**18*sqrt(a*x**3/b + 1)/(105*a**(19/
2)*b**15*x**(57/2) + 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*
x**(45/2) + 2100*a**(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2
) + 630*a**(9/2)*b**20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) + 280*a**
(17/2)*b**(31/2)*x**15*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57/2) +
 630*a**(17/2)*b**16*x**(51/2) + 1575*a**(15/2)*b**17*x**(45/2) + 2100*a**
(13/2)*b**18*x**(39/2) + 1575*a**(11/2)*b**19*x**(33/2) + 630*a**(9/2)*b**
20*x**(27/2) + 105*a**(7/2)*b**21*x**(21/2)) + 42*a**(15/2)*b**(33/2)*x**1
2*sqrt(a*x**3/b + 1)/(105*a**(19/2)*b**15*x**(57/2) + 630*a**(17/2)*b**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{13}}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}}}{21\,b^4} + \frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a}{5\,b^4} - \frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2}{3\,b^4} + \frac{2\sqrt{a+\frac{b}{x^3}}\,a^3}{3\,b^4}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^13,x, algorithm="maxima")
```

output
```
-2/21*(a + b/x^3)^(7/2)/b^4 + 2/5*(a + b/x^3)^(5/2)*a/b^4 - 2/3*(a + b/x^3
)^(3/2)*a^2/b^4 + 2/3*sqrt(a + b/x^3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{13}}\,dx = \frac{2\sqrt{a+\frac{b}{x^3}}a^3}{3\,b^4} - \frac{2\left(5\left(a+\frac{b}{x^3}\right)^{\frac{7}{2}} - 21\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a + 35\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2\right)}{105\,b^4}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^13,x, algorithm="giac")
```

output
```
2/3*sqrt(a + b/x^3)*a^3/b^4 - 2/105*(5*(a + b/x^3)^(7/2) - 21*(a + b/x^3)^
(5/2)*a + 35*(a + b/x^3)^(3/2)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{13}}\,dx = \frac{32\,a^3\,\sqrt{a+\frac{b}{x^3}}}{105\,b^4} - \frac{2\,\sqrt{a+\frac{b}{x^3}}}{21\,b\,x^9} + \frac{4\,a\,\sqrt{a+\frac{b}{x^3}}}{35\,b^2\,x^6} - \frac{16\,a^2\,\sqrt{a+\frac{b}{x^3}}}{105\,b^3\,x^3}$$

input
```
int(1/(x^13*(a + b/x^3)^(1/2)),x)
```

output
```
(32*a^3*(a + b/x^3)^(1/2))/(105*b^4) - (2*(a + b/x^3)^(1/2))/(21*b*x^9) +
(4*a*(a + b/x^3)^(1/2))/(35*b^2*x^6) - (16*a^2*(a + b/x^3)^(1/2))/(105*b^3
*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{13}}\,dx = \frac{2\sqrt{a\,x^3+b}\,(16a^3x^9 - 8a^2b\,x^6 + 6a\,b^2x^3 - 5b^3)}{105\sqrt{x}\,b^4x^{10}}$$

input
```
int(1/(a+b/x^3)^(1/2)/x^13,x)
```

output
```
(2*sqrt(a*x**3 + b)*(16*a**3*x**9 - 8*a**2*b*x**6 + 6*a*b**2*x**3 - 5*b**3
))/(105*sqrt(x)*b**4*x**10)
```

3.487 $\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 294

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{91b^2\sqrt{a+\frac{b}{x^3}}x^2}{320a^3}-\frac{13b\sqrt{a+\frac{b}{x^3}}x^5}{80a^2}+\frac{\sqrt{a+\frac{b}{x^3}}x^8}{8a}$$

$$+\frac{91\sqrt{2+\sqrt{3}}b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}}{320\sqrt[4]{3}a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
91/320*b^2*(a+b/x^3)^(1/2)*x^2/a^3-13/80*b*(a+b/x^3)^(1/2)*x^5/a^2+1/8*(a+
b/x^3)^(1/2)*x^8/a+91/960*(1/2*6^(1/2)+1/2*2^(1/2))*b^(8/3)*(a^(1/3)+b^(1/
3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a^3/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{91b^3+39ab^2x^3-12a^2bx^6+40a^3x^9-91b^3\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)}{320a^3\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[x^7/Sqrt[a + b/x^3],x]
```

output
```
(91*b^3 + 39*a*b^2*x^3 - 12*a^2*b*x^6 + 40*a^3*x^9 - 91*b^3*Sqrt[1 + (a*x^
3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(320*a^3*Sqrt[a + b/
x^3]*x)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 847, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^9}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{13b\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{16a}+\frac{x^8\sqrt{a+\frac{b}{x^3}}}{8a}$$

$\big\downarrow$ 847

$$\frac{13b\left(-\frac{7b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{16a}+\frac{x^8\sqrt{a+\frac{b}{x^3}}}{8a}$$

$\big\downarrow$ 847

$$\frac{13b\left(-\frac{7b\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{16a}+\frac{x^8\sqrt{a+\frac{b}{x^3}}}{8a}$$

$\big\downarrow$ 759

$$\frac{13b\left(-\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{16a}$$

$$\frac{x^8\sqrt{a+\frac{b}{x^3}}}{8a}$$

input
```
Int[x^7/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a + b/x^3]*x^8)/(8*a) + (13*b*(-1/5*(Sqrt[a + b/x^3]*x^5)/a - (7*b*(
-1/2*(Sqrt[a + b/x^3]*x^2)/a - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/
3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[
a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2])))/(10*a)))/(16*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 752 vs. $2(227) = 454$.

Time = 2.45 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.56

method	result
risch	$\frac{(40a^2x^6-52ab\,x^3+91b^2)(a\,x^3+b)}{320a^3x\sqrt{\frac{a\,x^3+b}{x^3}}} - \frac{91b^3\left(\frac{(-a^2b)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{(-a\cdots}{\cdots}\right)\cdots}{\cdots}$
default	Expression too large to display

input
```
int(x^7/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/320*(40*a^2*x^6-52*a*b*x^3+91*b^2)/a^3/x*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)
-91/320*b^3/a^2*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/
3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-
a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(
x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1
/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a
*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(
1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))/x^2/((a*x^3+b)/x^3)^(1/2)*(x*
(a*x^3+b))^(1/2)
```

Fricas [F]

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^7/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^10*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.16

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{x^8\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\Gamma\left(-\frac{5}{3}\right)}$$

input
```
integrate(x**7/(a+b/x**3)**(1/2),x)
```

output
```
-x**8*gamma(-8/3)*hyper((-8/3, 1/2), (-5/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*sqrt(a)*gamma(-5/3))
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^7/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sqrt(a + b/x^3), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^7/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/sqrt(a + b/x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(x^7/(a + b/x^3)^(1/2),x)
```

output
```
int(x^7/(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^7}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{80\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^6-104\sqrt{x}\,\sqrt{a\,x^3+b}\,ab\,x^3+182\sqrt{x}\,\sqrt{a\,x^3+b}\,b^2-91\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^4+bx}dx\right)b^3}{640a^3}$$

input
```
int(x^7/(a+b/x^3)^(1/2),x)
```

output
```
(80*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**6 - 104*sqrt(x)*sqrt(a*x**3 + b)*a*b*
x**3 + 182*sqrt(x)*sqrt(a*x**3 + b)*b**2 - 91*int((sqrt(x)*sqrt(a*x**3 + b
))/(a*x**4 + b*x),x)*b**3)/(640*a**3)
```

3.488 $\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{7b\sqrt{a+\frac{b}{x^3}}x^2}{20a^2}+\frac{\sqrt{a+\frac{b}{x^3}}x^5}{5a} - \frac{7\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{20\sqrt[4]{3}a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-7/20*b*(a+b/x^3)^(1/2)*x^2/a^2+1/5*(a+b/x^3)^(1/2)*x^5/a-7/60*(1/2*6^(1/2
)+1/2*2^(1/2))*b^(5/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b
^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*
a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/
a^2/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1
/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{-7b^2-3abx^3+4a^2x^6+7b^2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)}{20a^2\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[x^4/Sqrt[a + b/x^3],x]
```

output
```
(-7*b^2 - 3*a*b*x^3 + 4*a^2*x^6 + 7*b^2*Sqrt[1 + (a*x^3)/b]*Hypergeometric
2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(20*a^2*Sqrt[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{7b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,d\frac{1}{x}}{10a}+\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}$$

$$\Big\downarrow \text{847}$$

$$\frac{7b\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}+\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}$$

$$\Big\downarrow \text{759}$$

$$\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}+\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}$$

input
```
Int[x^4/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a + b/x^3]*x^5)/(5*a) + (7*b*(-1/2*(Sqrt[a + b/x^3]*x^2)/a - (Sqrt[2
 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a
^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
 -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(10*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(207) = 414$.

Time = 1.30 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.75

method	result
risch	$\frac{(4a\,x^3-7b)(a\,x^3+b)}{20a^2x\sqrt{\frac{a\,x^3+b}{x^3}}}+\frac{7b^2\left(\frac{(-a^2b)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)^2\sqrt{\ldots}}{20a\left(-\frac{3\ldots}{\ldots}\right.}$
default	Expression too large to display

input
```
int(x^4/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*a*x^3-7*b)/a^2/x*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)+7/20*b^2/a*(1/2/a
*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(
-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/
2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))
^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b
)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a
^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*
3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)
^(1/3)))^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a
^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/
3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3)))^(1/2))/x^2/((a*x^3+b)/x^3)^(1/2)*(x*(a*x^3+b))^(1/2)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^4/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^7*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{x^5\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(x**4/(a+b/x**3)**(1/2),x)
```

output
```
-x**5*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*sqrt(a)*gamma(-2/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^4/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(a + b/x^3), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^4/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(a + b/x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(x^4/(a + b/x^3)^(1/2),x)
```

output
```
int(x^4/(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{8\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,x^3-14\sqrt{x}\,\sqrt{a\,x^3+b}\,b+7\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^4+bx}dx\right)b^2}{40a^2}$$

input
```
int(x^4/(a+b/x^3)^(1/2),x)
```

output
```
(8*sqrt(x)*sqrt(a*x**3 + b)*a*x**3 - 14*sqrt(x)*sqrt(a*x**3 + b)*b + 7*int
((sqrt(x)*sqrt(a*x**3 + b))/(a*x**4 + b*x),x)*b**2)/(40*a**2)
```

3.489 $\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 248

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{\sqrt{a+\frac{b}{x^3}}x^2}{2a}+\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
1/2*(a+b/x^3)^(1/2)*x^2/a+1/6*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(a^(1/3)+b
^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a
^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{b+ax^3-b\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)}{2a\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[x/Sqrt[a + b/x^3],x]
```

output
```
(b + a*x^3 - b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x
^3)/b)])/(2*a*Sqrt[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {858, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}+\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}$$

input
```
Int[x/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a + b/x^3]*x^2)/(2*a) + (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3
)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[a
 + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)/x)^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(189) = 378$.

Time = 0.41 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.93

method	result
risch	$\frac{a x^{3}+b}{2 a x \sqrt{\frac{a x^{3}+b}{x^{3}}}}-\frac{b \left(\frac{\left(-a^{2} b\right)^{\frac{1}{3}}}{2 a}-\frac{i \sqrt{3}\, \left(-a^{2} b\right)^{\frac{1}{3}}}{2 a}\right) \sqrt{\frac{\left(-\frac{3 \left(-a^{2} b\right)^{\frac{1}{3}}}{2 a}+\frac{i \sqrt{3}\, \left(-a^{2} b\right)^{\frac{1}{3}}}{2 a}\right) x}{\left(-\frac{\left(-a^{2} b\right)^{\frac{1}{3}}}{2 a}+\frac{i \sqrt{3}\, \left(-a^{2} b\right)^{\frac{1}{3}}}{2 a}\right) \left(x-\frac{\left(-a^{2} b\right)^{\frac{1}{3}}}{a}\right)}}\, \left(x-\frac{\left(-a^{2} b\right)^{\frac{1}{3}}}{a}\right)^{2} \sqrt{\frac{\left(-a^{2} b\right)^{\frac{1}{3}} \cdots}{a \left(-\frac{\left(-a^{2} b\right)^{\frac{1}{3}}}{2 a} \cdots\right.}} \cdots}{2 \left(-\frac{3 \left(-a^{2} b\right)^{\frac{1}{3}}}{2 a}+\cdots\right.}$
default	Expression too large to display

input
```
int(x/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/x*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)-1/2*b*(1/2/a*(-a^2*b)^(1/3)-1/2*I*
3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)
^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-
a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a
*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(
1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)/(-
3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-a^2*b)^(1/3)/(a*x*(
x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*Ellipti
cF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((
3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)
-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))
/x^2/((a*x^3+b)/x^3)^(1/2)*(x*(a*x^3+b))^(1/2)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^4*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx=-\frac{x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\,\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(x/(a+b/x**3)**(1/2),x)
```

output
```
-x**2*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*sqrt(a)*gamma(1/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(a + b/x^3), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(a + b/x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(x/(a + b/x^3)^(1/2),x)
```

output
```
int(x/(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{2\sqrt{x}\,\sqrt{a\,x^3+b}-\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^4+bx}dx\right)b}{4a}$$

input
```
int(x/(a+b/x^3)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b) - int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**4 + b*x
),x)*b)/(4*a)
```

3.490 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 221

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}x^2}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a
^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3
^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*
3^(3/4)/b^(1/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^2}\,dx = \frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)}{\sqrt{a+\frac{b}{x^3}}x}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^2),x]
```

output
```
(2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(Sq
rt[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {858, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{759}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^2),x]
```

output
```
(-2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 -
(a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)
], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(168) = 336$.

Time = 0.90 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.98

method	result
default	$-\frac{4(a x^3+b)\sqrt{-\frac{(i\sqrt{3}-3)xa}{(i\sqrt{3}-1)\left(-ax+(-a^2b)^{\frac{1}{3}}\right)}}\sqrt{\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}+2ax+\left(-a^2b\right)^{\frac{1}{3}}}{(1+i\sqrt{3})\left(-ax+(-a^2b)^{\frac{1}{3}}\right)}}\sqrt{\frac{i\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}-2ax-\left(-a^2b\right)^{\frac{1}{3}}}{(i\sqrt{3}-1)\left(-ax+(-a^2b)^{\frac{1}{3}}\right)}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cdots}{(i\sqrt{3}-\cdots}}\right.}{\sqrt{\frac{a x^3+b}{x^3}}\,xa(-a^2b)^{\frac{1}{3}}\sqrt{x(a x^3+b)}\left(i\sqrt{3}-3\right)\sqrt{\frac{x\left(-ax+(-a^2b)^{\frac{1}{3}}\right)(\cdots}{\cdots}}}$

input
```
int(1/(a+b/x^3)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-4/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a/(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)*x*a/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*
x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(I*3^
(1/2)*a^2*x^2-2*I*(-a^2*b)^(1/3)*3^(1/2)*a*x+I*(-a^2*b)^(2/3)*3^(1/2)-a^2*
x^2+2*(-a^2*b)^(1/3)*a*x-(-a^2*b)^(2/3))/(x*(a*x^3+b))^(1/2)/(I*3^(1/2)-3)
/(1/a^2*x*(-a*x+(-a^2*b)^(1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(
1/3))*(I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^2}\,dx = -\frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},\frac{1}{x}\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-2*weierstrassPInverse(0, -4*a/b, 1/x)/sqrt(b)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^2}\,dx = -\frac{\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**2,x)
```

output
```
-gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(
a)*x*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^2}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^2}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^2}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^2}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^2}\,dx = -\frac{\sqrt{\frac{b}{a\,x^3}+1}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -\frac{b}{a\,x^3}\right)}{x\,\sqrt{a+\frac{b}{x^3}}}$$

input
```
int(1/(x^2*(a + b/x^3)^(1/2)),x)
```

output
```
-((b/(a*x^3) + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -b/(a*x^3)))/(x*(a + b/
x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^2}\,dx = \int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^4+bx}dx$$

input
```
int(1/(a+b/x^3)^(1/2)/x^2,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**4 + b*x),x)
```

3.491 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 246

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5} dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx} + \frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/5*(a+b/x^3)^(1/2)/b/x+4/15*(1/2*6^(1/2)+1/2*2^(1/2))*a*(a^(1/3)+b^(1/3)
/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/
x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)
+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^(4/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5} dx = -\frac{2\sqrt{1+\frac{ax^3}{b}} \text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{ax^3}{b}\right)}{5\sqrt{a+\frac{b}{x^3}}x^4}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^5),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((a*x^3)/b)])/(
5*Sqrt[a + b/x^3]*x^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {858, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{2a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{5b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^5),x]
```

output
```
(-2*Sqrt[a + b/x^3])/(5*b*x) + (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)/x
)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqr
t[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)/x)^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(187) = 374$.

Time = 1.58 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.98

method	result
risch	$-\frac{2(a x^3+b)}{5b x^4\sqrt{\frac{a x^3+b}{x^3}}}-\frac{4a^2\left(\frac{(-a^2b)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)^2\sqrt{\frac{(-}{a\left(-\frac{(}{}\right.}}}{5b\left(-\frac{3(-a^2}{2a}\right.}$
default	Expression too large to display

input
```
int(1/(a+b/x^3)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5/b*(a*x^3+b)/x^4/((a*x^3+b)/x^3)^(1/2)-4/5*a^2/b*(1/2/a*(-a^2*b)^(1/3)
-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x
-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(
x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1
/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a
^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(
1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-a^2*b)^(1/3)
/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2
*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*
EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(
1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b
)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))
^(1/2))/x^2/((a*x^3+b)/x^3)^(1/2)*(x*(a*x^3+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx = \frac{2\left(2\,a\sqrt{b}x\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)-b\sqrt{\frac{ax^3+b}{x^3}}\right)}{5\,b^2x}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
2/5*(2*a*sqrt(b)*x*weierstrassPInverse(0, -4*a/b, 1/x) - b*sqrt((a*x^3 + b
)/x^3))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx = -\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}x^4\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**5,x)
```

output
```
-gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(
a)*x**4*gamma(7/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx=\int \frac{1}{x^5\,\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(1/(x^5*(a + b/x^3)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b/x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^5}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^7+b\,x^4}dx$$

input
```
int(1/(a+b/x^3)^(1/2)/x^5,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**7 + b*x**4),x)
```

3.492 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8} dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4} + \frac{16a\sqrt{a+\frac{b}{x^3}}}{55b^2x} - \frac{32\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/11*(a+b/x^3)^(1/2)/b/x^4+16/55*a*(a+b/x^3)^(1/2)/b^2/x-32/165*(1/2*6^(1
/2)+1/2*2^(1/2))*a^2*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^
(7/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx=-\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{1}{2},-\frac{5}{6},-\frac{ax^3}{b}\right)}{11\sqrt{a+\frac{b}{x^3}}x^7}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^8),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((a*x^3)/b)])
/(11*Sqrt[a + b/x^3]*x^7)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {858, 843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}d\frac{1}{x}$$

$$\Big\downarrow \text{843}$$

$$\frac{8a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}d\frac{1}{x}}{11b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{8a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{2a\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{5b}\right)}{11b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4}$$

$$\Big\downarrow \; 759$$

$$\frac{8a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{11b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{11bx^4}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^8),x]
```

output
```
(-2*Sqrt[a + b/x^3])/(11*b*x^4) + (8*a*((2*Sqrt[a + b/x^3])/(5*b*x) - (4*S
qrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^
(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)],
-7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(11*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(207) = 414$.

Time = 2.57 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.75

method	result
risch	$\frac{2(a\,x^3+b)(8a\,x^3-5b)}{55b^2x^7\sqrt{\frac{a\,x^3+b}{x^3}}}+\frac{32a^3\left(\frac{(-a^2b)^{\frac{1}{3}}}{2a}-\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\sqrt{\frac{\left(-\frac{3(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)x}{\left(-\frac{(-a^2b)^{\frac{1}{3}}}{2a}+\frac{i\sqrt{3}\,(-a^2b)^{\frac{1}{3}}}{2a}\right)\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)}}\left(x-\frac{(-a^2b)^{\frac{1}{3}}}{a}\right)^2\sqrt{\cdots}}{55b^2\left(-\cdots\right)}$
default	Expression too large to display

input
```
int(1/(a+b/x^3)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
2/55*(a*x^3+b)*(8*a*x^3-5*b)/b^2/x^7/((a*x^3+b)/x^3)^(1/2)+32/55*a^3/b^2*(
1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/
3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(
1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1
/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-
a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/
a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3))/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1
/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3)))^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/
a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b
)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3)))^(1/2))/x^2/((a*x^3+b)/x^3)^(1/2)*(x*(a*x^3+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx$$
$$= -\frac{2\left(16\,a^2\sqrt{b}x^4\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)-\left(8\,abx^3-5\,b^2\right)\sqrt{\frac{ax^3+b}{x^3}}\right)}{55\,b^3x^4}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-2/55*(16*a^2*sqrt(b)*x^4*weierstrassPInverse(0, -4*a/b, 1/x) - (8*a*b*x^3
 - 5*b^2)*sqrt((a*x^3 + b)/x^3))/(b^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.14

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx = -\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}x^7\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**8,x)
```

output
```
-gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt
(a)*x**7*gamma(10/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx = \int \frac{1}{x^8\,\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(1/(x^8*(a + b/x^3)^(1/2)),x)
```

output
```
int(1/(x^8*(a + b/x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^8}\,dx = \int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^{10}+b\,x^7}dx$$

input
```
int(1/(a+b/x^3)^(1/2)/x^8,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**10 + b*x**7),x)
```

3.493 $\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 566

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{55b^{7/3}\sqrt{a+\frac{b}{x^3}}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{55b^2\sqrt{a+\frac{b}{x^3}}\,x}{112a^3}-\frac{11b\sqrt{a+\frac{b}{x^3}}\,x^4}{56a^2}+\frac{\sqrt{a+\frac{b}{x^3}}\,x^7}{7a}$$
$$+\frac{55\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{224a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{55b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-55/112*b^(7/3)*(a+b/x^3)^(1/2)/a^3/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+55/112
*b^2*(a+b/x^3)^(1/2)*x/a^3-11/56*b*(a+b/x^3)^(1/2)*x^4/a^2+1/7*(a+b/x^3)^(
1/2)*x^7/a+55/224*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(7/3)*(a^(1/3)+b^(1/
3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)/x),I*3^(1/2)+2*I)/a^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)-55/336*b^(7/3)*(a^(1/3)+b
^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a
^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(8/3)/(a+b/x^3)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{-11b^2x-3abx^4+8a^2x^7+11b^2x\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{ax^3}{b}\right)}{56a^2\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[x^6/Sqrt[a + b/x^3],x]
```

output
```
(-11*b^2*x - 3*a*b*x^4 + 8*a^2*x^7 + 11*b^2*x*Sqrt[1 + (a*x^3)/b]*Hypergeo
metric2F1[1/2, 5/6, 11/6, -((a*x^3)/b)])/(56*a^2*Sqrt[a + b/x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {858, 847, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^8}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{11b\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{14a}+\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}$$

$$\Big\downarrow \text{847}$$

$$\frac{11b\left(-\frac{5b\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{14a}+\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}$$

$$\Big\downarrow \text{847}$$

$$\frac{11b\left(-\frac{5b\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{14a}+\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}$$

$$\Big\downarrow \text{832}$$

$$\frac{11b\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{14a}+\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}$$

$$\Big\downarrow \text{759}$$

$$\frac{11b\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{2a}}{8a}\right)}{14a}$$

$$\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}$$

$\downarrow$ 2416

$$\Bigg(11b - \frac{5b\left(\frac{b\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} }{\sqrt[3]{b}} }{2a}}{8a}$$

input
```
Int[x^6/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a + b/x^3]*x^7)/(7*a) + (11*b*(-1/4*(Sqrt[a + b/x^3]*x^4)/a - (5*b*(
-((Sqrt[a + b/x^3]*x)/a) + (b*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^
(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a
 + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2])))/(2*a)))/(8*a)))/(14*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(424) = 848$.

Time = 2.10 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.99

method	result	size
risch	Expression too large to display	1125
default	Expression too large to display	2806

input
```
int(x^6/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/56*x*(8*a*x^3-11*b)/a^2*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)+55/112*b^2/a^2*(
x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b
)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)
)*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)
^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(
x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-
a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a
^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*
b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1
/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*
EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3))...
```

Fricas [F]

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^6/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^9*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{x^7\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\Gamma\left(-\frac{4}{3}\right)}$$

input
```
integrate(x**6/(a+b/x**3)**(1/2),x)
```

output
```
-x**7*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*sqrt(a)*gamma(-4/3))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^6/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(a + b/x^3), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^6/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(a + b/x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(x^6/(a + b/x^3)^(1/2),x)
```

output
```
int(x^6/(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}\,dx = \frac{16\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,x^5-22\sqrt{x}\,\sqrt{a\,x^3+b}\,b\,x^2+55\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a\,x^3+b}dx\right)b^2}{112a^2}$$

input
```
int(x^6/(a+b/x^3)^(1/2),x)
```

output
```
(16*sqrt(x)*sqrt(a*x**3 + b)*a*x**5 - 22*sqrt(x)*sqrt(a*x**3 + b)*b*x**2 +
 55*int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a*x**3 + b),x)*b**2)/(112*a**2)
```

3.494 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}} dx$

Optimal result

Integrand size = 15, antiderivative size = 542

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}} dx = \frac{5b^{4/3}\sqrt{a+\frac{b}{x^3}}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{5b\sqrt{a+\frac{b}{x^3}}x}{8a^2}+\frac{\sqrt{a+\frac{b}{x^3}}x^4}{4a}$$
$$-\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{16a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$+\frac{5b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
5/8*b^(4/3)*(a+b/x^3)^(1/2)/a^2/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-5/8*b*(a+b
/x^3)^(1/2)*x/a^2+1/4*(a+b/x^3)^(1/2)*x^4/a-5/16*3^(1/4)*(1/2*6^(1/2)-1/2*
2^(1/2))*b^(4/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)
/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3
)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/a^(5/3)/(a+b/x
^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^
(1/2)+5/24*b^(4/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/
3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1
/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*2^(1/2)*3^(3
/4)/a^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{x\left(b+ax^3-b\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{ax^3}{b}\right)\right)}{4a\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[x^3/Sqrt[a + b/x^3],x]
```

output
```
(x*(b + a*x^3 - b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/2, 5/6, 11/6, -(
(a*x^3)/b)]))/(4*a*Sqrt[a + b/x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \; 858$$

$$-\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \; 847$$

$$\frac{5b\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{8a}+\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}$$

$$\Big\downarrow \; 847$$

$$\frac{5b\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}+\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}$$

$$\Big\downarrow \; 832$$

$$\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}+\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}$$

$$\Big\downarrow \; 759$$

$$\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{2a}\right.}{8a}$$

$$\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}$$

$\downarrow$ 2416

$$\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{\sqrt[3]{b}}-\frac{2\left(1-\cdots\right.}{\cdots}}{2a}\cdots\right)}{8a}\cdots$$

$$\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}$$

input

```
Int[x^3/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a + b/x^3]*x^4)/(4*a) + (5*b*(-((Sqrt[a + b/x^3]*x)/a) + (b*(((2*Sqr
t[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqr
t[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 -
 (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x
)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3
])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)
/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(2*a)))/(
8*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(404) = 808$.

Time = 1.06 (sec) , antiderivative size = 1113, normalized size of antiderivative = 2.05

method	result	size
risch	Expression too large to display	1113
default	Expression too large to display	2586

input
```
int(x^3/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*x*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)-5/8*b/a*(x*(x+1/2/a*(-a^2*b)^(1/3)
+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/
2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3
)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a
^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x
-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2
*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-
3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*
(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*
b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^...
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^3/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^6*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{x^4\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(x**3/(a+b/x**3)**(1/2),x)
```

output
```
-x**4*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*sqrt(a)*gamma(-1/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^3/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(a + b/x^3), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(x^3/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(a + b/x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(x^3/(a + b/x^3)^(1/2),x)
```

output
```
int(x^3/(a + b/x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{2\sqrt{x}\,\sqrt{a\,x^3+b}\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a\,x^3+b}dx\right)b}{8a}$$

input
```
int(x^3/(a+b/x^3)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b)*x**2 - 5*int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a*x
**3 + b),x)*b)/(8*a)
```

3.495 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}} dx$

Optimal result

Integrand size = 11, antiderivative size = 513

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}} dx = -\frac{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{\sqrt{a+\frac{b}{x^3}}x}{a}$$
$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{\sqrt{2}\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-b^(1/3)*(a+b/x^3)^(1/2)/a/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+(a+b/x^3)^(1/2)
*x/a+1/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1/3)*(a^(1/3)+b^(1/3)/x)*((a
^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(
1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)/x),I*3^(1/2)+2*I)/a^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)-1/3*2^(1/2)*b^(1/3)*(a^(1/3)+b^(1/
3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{2x\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{ax^3}{b}\right)}{5\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[1/Sqrt[a + b/x^3],x]
```

output
```
(2*x*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/2, 5/6, 11/6, -((a*x^3)/b)])/
(5*Sqrt[a + b/x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {773, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{x\sqrt{a+\frac{b}{x^3}}}{a}-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{832}$$

$$\frac{x\sqrt{a+\frac{b}{x^3}}}{a}-\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}$$

$$\Big\downarrow \text{759}$$

$$\frac{x\sqrt{a+\frac{b}{x^3}}}{a}-\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\ldots\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{2a}$$

$$\Big\downarrow \text{2416}$$

$$\frac{x\sqrt{a+\frac{b}{x^3}}}{a} - \frac{b\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}} - \frac{2\left(1-\sqrt{3}\cdots\right.}{\cdots}\right.}{2a}$$

input
```
Int[1/Sqrt[a + b/x^3],x]
```

output
```
(Sqrt[a + b/x^3]*x)/a - (b*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a
^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/
3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a +
b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) +
 b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(
2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)/x)^2])))/(2*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2373 vs. $2(385) = 770$.

Time = 0.35 (sec) , antiderivative size = 2374, normalized size of antiderivative = 4.63

method	result	size
default	Expression too large to display	2374

input
```
int(1/(a+b/x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/((a*x^3+b)/x^3)^(1/2)/x*(a*x^3+b)/a^2*(-I*(-a^2*b)^(1/3)*3^(1/2)*(-(I*3
^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2
*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^
2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2
*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*a*x^2+2*I*(-a^2*b)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(
1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3
)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ellipti
cE((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x-2*(-a^2*b)^(1/3
)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a
*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*a*x^2+3*(-a^2*b)^(1/3)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/
3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/...
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3*sqrt((a*x^3 + b)/x^3)/(a*x^3 + b), x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx = -\frac{x\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2),x)
```

output
```
-x*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*exp_polar(I*pi)/(a*x**3))/(3*s
qrt(a)*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/x^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a + b/x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx=\frac{2\,x\,\sqrt{\frac{a\,x^3}{b}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{11}{6};\ -\frac{a\,x^3}{b}\right)}{5\,\sqrt{a+\frac{b}{x^3}}}$$

input
```
int(1/(a + b/x^3)^(1/2),x)
```

output
```
(2*x*((a*x^3)/b + 1)^(1/2)*hypergeom([1/2, 5/6], 11/6, -(a*x^3)/b))/(5*(a
+ b/x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a\,x^3+b}dx$$

input
```
int(1/(a+b/x^3)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a*x**3 + b),x)
```

3.496 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 491

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} - \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2*(a+b/x^3)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+3^(1/4)*(1/2*6^
(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/
3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/
2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(2
/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1
/3)/x)^2)^(1/2)-2/3*2^(1/2)*a^(1/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/
x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(
((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+
2*I)*3^(3/4)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3} \, dx = -\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{ax^3}{b}\right)}{\sqrt{a+\frac{b}{x^3}}x^2}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^3),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((a*x^3)/b)])/(
Sqrt[a + b/x^3]*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{x^3}}}\,dx$$

$$\Big\downarrow \; 858$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x}d\frac{1}{x}$$

$$\Big\downarrow \; 832$$

$$\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 759$$

$$\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 2416$$

$$\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}}$$

$$\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^3),x]
```

output
```
-(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2
/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3)) + (2*(
1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3)
 + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(367) = 734$.

Time = 1.17 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.27

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2860

input
```
int(1/(a+b/x^3)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b*(a*x^3+b)/x^2/((a*x^3+b)/x^3)^(1/2)+2*a/b*(x*(x+1/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-
a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2
/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)
+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^
2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-
1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*
b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^
2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-3
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/
3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(
-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*
3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/a
*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*b
)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx = \frac{2\,\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, 1/x))/sqrt(b)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx=-\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**3,x)
```

output
```
-gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(
a)*x**2*gamma(5/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx=\int \frac{1}{x^3\,\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(1/(x^3*(a + b/x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b/x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^5+b\,x^2}dx$$

input
```
int(1/(a+b/x^3)^(1/2)/x^3,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**5 + b*x**2),x)
```

3.497 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 520

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6} \, dx = \frac{8a\sqrt{a+\frac{b}{x^3}}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}+\frac{8\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
8/7*a*(a+b/x^3)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/7*(a+b/x^3
)^(1/2)/b/x^2-4/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*(a^(1/3)+b^(1/
3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)+8/21*2^(1/2)*a^(4/3)*(a^(
1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^(5/3)/(a+b/x^3)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}\,dx = -\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},-\frac{ax^3}{b}\right)}{7\sqrt{a+\frac{b}{x^3}}x^5}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^6),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((a*x^3)/b)])/
(7*Sqrt[a + b/x^3]*x^5)
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{843}$$

$$\frac{4a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{7b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}$$

$$\Big\downarrow \text{832}$$

$$\frac{4a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{7b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{4a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7\cdots}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{7b}$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}$$

$$\Big\downarrow \text{2416}$$

$$\frac{4a\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{\cdots}\right.}{\cdots}\right)}{7b}$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^6),x]
```

output
```
(-2*Sqrt[a + b/x^3])/(7*b*x^2) + (4*a*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1
/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3
)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)
*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(7*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(386) = 772$.

Time = 1.85 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.17

method	result	size
risch	Expression too large to display	1127
default	Expression too large to display	3300

input
```
int(1/(a+b/x^3)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
2/7*(a*x^3+b)*(4*a*x^3-b)/b^2/x^5/((a*x^3+b)/x^3)^(1/2)-8/7*a^2/b^2*(x*(x+
1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/
3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(
-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3
)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(
1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2
/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1
/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b
)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)
^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x
/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1
/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*
(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*Ellip
ticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.10

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6} \, dx$$
$$= -\frac{2 \left(4 a\sqrt{b}x^2 \text{weierstrassZeta}\left(0, -\frac{4 a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4 a}{b}, \frac{1}{x}\right)\right) + b\sqrt{\frac{ax^3+b}{x^3}}\right)}{7 b^2 x^2}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/7*(4*a*sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, 1/x)) + b*sqrt((a*x^3 + b)/x^3))/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6} \, dx = -\frac{\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}x^5\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**6,x)
```

output
```
-gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt(
a)*x**5*gamma(8/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^6), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^6}\,dx = \int \frac{1}{x^6\,\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(1/(x^6*(a + b/x^3)^(1/2)),x)
```

output
```
int(1/(x^6*(a + b/x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^6}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^8+b\,x^5}dx$$

input
```
int(1/(a+b/x^3)^(1/2)/x^6,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**8 + b*x**5),x)
```

3.498 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}x^9}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 544

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx = -\frac{80a^2\sqrt{a+\frac{b}{x^3}}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5} + \frac{20a\sqrt{a+\frac{b}{x^3}}}{91b^2x^2}$$
$$+\frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{91b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{80\sqrt{2}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-80/91*a^2*(a+b/x^3)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/13*(a
+b/x^3)^(1/2)/b/x^5+20/91*a*(a+b/x^3)^(1/2)/b^2/x^2+40/91*3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*a^(7/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3
)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2
))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(8/
3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/
3)/x)^2)^(1/2)-80/273*2^(1/2)*a^(7/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3
)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2
)+2*I)*3^(3/4)/b^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^
(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx = -\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{13}{6},\frac{1}{2},-\frac{7}{6},-\frac{ax^3}{b}\right)}{13\sqrt{a+\frac{b}{x^3}}x^8}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^9),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-13/6, 1/2, -7/6, -((a*x^3)/b)])
/(13*Sqrt[a + b/x^3]*x^8)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\sqrt{a+\frac{b}{x^3}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7}d\frac{1}{x}$$

$$\Big\downarrow \text{843}$$

$$\frac{10a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}d\frac{1}{x}}{13b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}$$

$$\Big\downarrow \text{843}$$

$$\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{7b}\right)}{13b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}$$

$$\Big\downarrow \text{832}$$

$$\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}$$

$$\Big\downarrow \text{759}$$

$$\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{7b}\right.}{13b}$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}$$

$$\downarrow 2416$$

$$\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-\cdots\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\cdots}}{\sqrt[3]{b}}\cdots\right)}{7\cdots}\cdots\right)}{13b\cdots}$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^9),x]
```

output
```
(-2*Sqrt[a + b/x^3])/(13*b*x^5) + (10*a*((2*Sqrt[a + b/x^3])/(7*b*x^2) - (
4*a*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) -
(3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b
^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2
*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/
3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])
))/(7*b)))/(13*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(406) = 812$.

Time = 3.03 (sec) , antiderivative size = 1138, normalized size of antiderivative = 2.09

method	result	size
risch	Expression too large to display	1138
default	Expression too large to display	3547

input
```
int(1/(a+b/x^3)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-2/91*(a*x^3+b)*(40*a^2*x^6-10*a*b*x^3+7*b^2)/b^3/x^8/((a*x^3+b)/x^3)^(1/2
)+80/91*a^3/b^3*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(
1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)
/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)
)/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/
3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b
)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a
*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-
1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)
))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*
b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b
)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx = \frac{2\left(40\,a^2\sqrt{b}x^5\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)\right)+\left(10\,abx^3-7\,b^2\right)\sqrt{\frac{ax^3+b}{x^3}}\right)}{91\,b^3x^5}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
2/91*(40*a^2*sqrt(b)*x^5*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
 -4*a/b, 1/x)) + (10*a*b*x^3 - 7*b^2)*sqrt((a*x^3 + b)/x^3))/(b^3*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx = -\frac{\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}x^8\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**9,x)
```

output
```
-gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sqrt
(a)*x**8*gamma(11/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^9), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^9,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx=\int \frac{1}{x^9\,\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(1/(x^9*(a + b/x^3)^(1/2)),x)
```

output
```
int(1/(x^9*(a + b/x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^9}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^{11}+b\,x^8}dx$$

input
```
int(1/(a+b/x^3)^(1/2)/x^9,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**11 + b*x**8),x)
```

3.499 $\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 568

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx = \frac{1280a^3\sqrt{a+\frac{b}{x^3}}}{1729b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8}+\frac{32a\sqrt{a+\frac{b}{x^3}}}{247b^2x^5}-\frac{320a^2\sqrt{a+\frac{b}{x^3}}}{1729b^3x^2} - \frac{640\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{1729b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} + \frac{1280\sqrt{2}a^{10/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{1729\sqrt[4]{3}b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
1280/1729*a^3*(a+b/x^3)^(1/2)/b^(11/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/1
9*(a+b/x^3)^(1/2)/b/x^8+32/247*a*(a+b/x^3)^(1/2)/b^2/x^5-320/1729*a^2*(a+b
/x^3)^(1/2)/b^3/x^2-640/1729*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(10/3)*(a
^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^
(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)/b^(11/3)/(a+b/x^3)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)+1280/5187*2^
(1/2)*a^(10/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x
)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+
b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^(11/3)
/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx=-\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{19}{6},\frac{1}{2},-\frac{13}{6},-\frac{ax^3}{b}\right)}{19\sqrt{a+\frac{b}{x^3}}x^{11}}$$

input
```
Integrate[1/(Sqrt[a + b/x^3]*x^12),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-19/6, 1/2, -13/6, -((a*x^3)/b)]
)/(19*Sqrt[a + b/x^3]*x^11)
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {858, 843, 843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12}\sqrt{a+\frac{b}{x^3}}}dx$$

$\downarrow$ 858

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{10}}d\frac{1}{x}$$

$\downarrow$ 843

$$\frac{16a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7}d\frac{1}{x}}{19b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8}$$

$\downarrow$ 843

$$\frac{16a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}d\frac{1}{x}}{13b}\right)}{19b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8}$$

$\downarrow$ 843

$$\frac{16a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{7b}\right)}{13b}\right)}{19b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8}$$

$\downarrow$ 832

$$\frac{16a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}\right)}{19b}-\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8}$$

$$\Big\downarrow \ 759$$

$$\frac{16a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{7b}\right.}{13b}\right.}{19b}$$

$$\frac{2\sqrt{a+\frac{b}{x^3}}}{19bx^8}$$

$\downarrow$ 2416

$$16a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right.\cdots}{\left(1+\sqrt{3}\right.\cdots}\cdots\right.\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\cdots}\cdots\right)}{\sqrt[3]{b}\cdots}\cdots\right)}{\cdots}\cdots\right.$$

input
```
Int[1/(Sqrt[a + b/x^3]*x^12),x]
```

output
```
(-2*Sqrt[a + b/x^3])/(19*b*x^8) + (16*a*((2*Sqrt[a + b/x^3])/(13*b*x^5) -
(10*a*((2*Sqrt[a + b/x^3])/(7*b*x^2) - (4*a*(((2*Sqrt[a + b/x^3])/(b^(1/3)
*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)
*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(
b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a
^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3
))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(7*b)))/(13*b)))/(19*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(426) = 852$.

Time = 4.68 (sec) , antiderivative size = 1149, normalized size of antiderivative = 2.02

method	result	size
risch	Expression too large to display	1149
default	Expression too large to display	3779

input
```
int(1/(a+b/x^3)^(1/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
2/1729*(a*x^3+b)*(640*a^3*x^9-160*a^2*b*x^6+112*a*b^2*x^3-91*b^3)/b^4/x^11
/((a*x^3+b)/x^3)^(1/2)-1280/1729*a^4/b^4*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*
3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)
^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(
1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*
b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-
a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(
1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(
-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2
*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b
)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/
(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3
)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.14

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx = -\frac{2\left(640\,a^3\sqrt{b}x^8\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)\right)+\left(160\,a^2bx^6-112\,ab^2x^3+9\right.\right.}{1729\,b^4x^8}$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^12,x, algorithm="fricas")
```

output
```
-2/1729*(640*a^3*sqrt(b)*x^8*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, 1/x)) + (160*a^2*b*x^6 - 112*a*b^2*x^3 + 91*b^3)*sqrt((a*x^3
+ b)/x^3))/(b^4*x^8)
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx = -\frac{\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3\sqrt{a}x^{11}\Gamma\left(\frac{14}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(1/2)/x**12,x)
```

output
```
-gamma(11/3)*hyper((1/2, 11/3), (14/3,), b*exp_polar(I*pi)/(a*x**3))/(3*sq
rt(a)*x**11*gamma(14/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^12,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^12), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^{12}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(1/2)/x^12,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^3)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{12}}\,dx=\int \frac{1}{x^{12}\,\sqrt{a+\frac{b}{x^3}}}\,dx$$

input
```
int(1/(x^12*(a + b/x^3)^(1/2)),x)
```

output
```
int(1/(x^12*(a + b/x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}\,x^{12}}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a\,x^{14}+b\,x^{11}}dx$$

input
```
int(1/(a+b/x^3)^(1/2)/x^12,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a*x**14 + b*x**11),x)
```

3.500 $\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=-\frac{5b^2}{4a^3\sqrt{a+\frac{b}{x^3}}}-\frac{5bx^3}{12a^2\sqrt{a+\frac{b}{x^3}}}+\frac{x^6}{6a\sqrt{a+\frac{b}{x^3}}}+\frac{5b^2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
-5/4*b^2/a^3/(a+b/x^3)^(1/2)-5/12*b*x^3/a^2/(a+b/x^3)^(1/2)+1/6*x^6/a/(a+b
/x^3)^(1/2)+5/4*b^2*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{\sqrt{a}x^{3/2}(-15b^2-5abx^3+2a^2x^6)+15b^2\sqrt{b+ax^3}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{12a^{7/2}\sqrt{a+\frac{b}{x^3}}x^{3/2}}$$

input
```
Integrate[x^5/(a + b/x^3)^(3/2),x]
```

output
```
(Sqrt[a]*x^(3/2)*(-15*b^2 - 5*a*b*x^3 + 2*a^2*x^6) + 15*b^2*Sqrt[b + a*x^3
]*Log[Sqrt[a]*x^(3/2) + Sqrt[b + a*x^3]])/(12*a^(7/2)*Sqrt[a + b/x^3]*x^(3
/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{x^9}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{5b\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x^3}}{4a}+\frac{x^6}{2a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{5b\left(-\frac{3b\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x^3}}{2a}-\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)}{4a}+\frac{x^6}{2a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{5b\left(-\frac{3b\left(\frac{\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^3}}}\right)}{2a}-\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)}{4a}+\frac{x^6}{2a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{5b\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^3}}}\right)}{2a}-\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)}{4a}+\frac{x^6}{2a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x^3}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)}{4a}+\frac{x^6}{2a\sqrt{a+\frac{b}{x^3}}}\right)$$

input
```
Int[x^5/(a + b/x^3)^(3/2),x]
```

output
```
(x^6/(2*a*Sqrt[a + b/x^3]) + (5*b*(-(x^3/(a*Sqrt[a + b/x^3])) - (3*b*(2/(a
*Sqrt[a + b/x^3]) - (2*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/a^(3/2)))/(2*a)))
/(4*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{(a\,x^3+b)\left(2a^{\frac{11}{2}}x^8-5a^{\frac{9}{2}}b\,x^5-15b^2x^2a^{\frac{7}{2}}+15\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^3+b\right)}}{x^2\sqrt{a}}\right)\sqrt{x(a\,x^3+b)}\,a^3b^2\right)}{12\left(\frac{a\,x^3+b}{x^3}\right)^{\frac{3}{2}}x^5a^{\frac{13}{2}}}$	96
risch	$\frac{(2a\,x^3-7b)\,(a\,x^3+b)}{12a^3\sqrt{\frac{a\,x^3+b}{x^3}}}+\frac{b^2\left(\frac{10\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^3+b\right)}}{x^2\sqrt{a}}\right)}{\sqrt{a}}-\frac{16x^2}{3\sqrt{\left(x^3+\frac{b}{a}\right)ax}}\right)\sqrt{x(a\,x^3+b)}}{8a^3x^2\sqrt{\frac{a\,x^3+b}{x^3}}}$	116

input
```
int(x^5/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)*(2*a^(11/2)*x^8-5*a^(9/2)*b*x^5-1
5*b^2*x^2*a^(7/2)+15*arctanh((x*(a*x^3+b))^(1/2)/x^2/a^(1/2))*(x*(a*x^3+b)
)^(1/2)*a^3*b^2)/a^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.64

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \left[\frac{15\left(ab^2x^3+b^3\right)\sqrt{a}\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\left(2\,ax^6+bx^3\right)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)+4\left(2\,a\right.}{48\left(a^5x^3+a^4b\right)}\right.$$
$$\left.-\frac{15\left(ab^2x^3+b^3\right)\sqrt{-a}\arctan\left(\frac{\left(2\,ax^3+b\right)\sqrt{-a}\sqrt{\frac{ax^3+b}{x^3}}}{2\left(a^2x^3+ab\right)}\right)-2\left(2\,a^3x^9-5\,a^2bx^6-15\,ab^2x^3\right)\sqrt{\frac{ax^3+b}{x^3}}}{24\left(a^5x^3+a^4b\right)}\right]$$

input
```
integrate(x^5/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(15*(a*b^2*x^3 + b^3)*sqrt(a)*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 - 4*(
2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(2*a^3*x^9 - 5*a^2*b*x
^6 - 15*a*b^2*x^3)*sqrt((a*x^3 + b)/x^3))/(a^5*x^3 + a^4*b), -1/24*(15*(a*
b^2*x^3 + b^3)*sqrt(-a)*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt((a*x^3 + b)
/x^3)/(a^2*x^3 + a*b)) - 2*(2*a^3*x^9 - 5*a^2*b*x^6 - 15*a*b^2*x^3)*sqrt((
a*x^3 + b)/x^3))/(a^5*x^3 + a^4*b)]
```

Sympy [A] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{x^{\frac{15}{2}}}{6a\sqrt{b}\sqrt{\frac{ax^3}{b}+1}}-\frac{5\sqrt{b}x^{\frac{9}{2}}}{12a^2\sqrt{\frac{ax^3}{b}+1}}-\frac{5b^{\frac{3}{2}}x^{\frac{3}{2}}}{4a^3\sqrt{\frac{ax^3}{b}+1}}+\frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{4a^{\frac{7}{2}}}$$

input
```
integrate(x**5/(a+b/x**3)**(3/2),x)
```

output
```
x**(15/2)/(6*a*sqrt(b)*sqrt(a*x**3/b + 1)) - 5*sqrt(b)*x**(9/2)/(12*a**2*s
qrt(a*x**3/b + 1)) - 5*b**(3/2)*x**(3/2)/(4*a**3*sqrt(a*x**3/b + 1)) + 5*b
**2*asinh(sqrt(a)*x**(3/2)/sqrt(b))/(4*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{15\left(a+\frac{b}{x^3}\right)^2 b^2 - 25\left(a+\frac{b}{x^3}\right)ab^2 + 8\,a^2b^2}{12\left(\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}a^3 - 2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^4 + \sqrt{a+\frac{b}{x^3}}\,a^5\right)} - \frac{5\,b^2\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

input
```
integrate(x^5/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
-1/12*(15*(a + b/x^3)^2*b^2 - 25*(a + b/x^3)*a*b^2 + 8*a^2*b^2)/((a + b/x^
3)^(5/2)*a^3 - 2*(a + b/x^3)^(3/2)*a^4 + sqrt(a + b/x^3)*a^5) - 5/8*b^2*lo
g((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{1}{12}\sqrt{ax^4+bx}\,x\left(\frac{2\,x^3}{a^2} - \frac{7\,b}{a^3}\right) - \frac{5\,b^2\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,a^3} - \frac{2\,b^2}{3\sqrt{a+\frac{b}{x^3}}\,a^3}$$

input
```
integrate(x^5/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
1/12*sqrt(a*x^4 + b*x)*x*(2*x^3/a^2 - 7*b/a^3) - 5/4*b^2*arctan(sqrt(a + b
/x^3)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*b^2/(sqrt(a + b/x^3)*a^3)
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{x^6\sqrt{a+\frac{b}{x^3}}}{6\,a^2} - \frac{2\,b^2}{3\,a^3\sqrt{a+\frac{b}{x^3}}} + \frac{5\,b^2\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)^3\right)}{8\,a^{7/2}} - \frac{7\,b\,x^3\sqrt{a+\frac{b}{x^3}}}{12\,a^3}$$

input
```
int(x^5/(a + b/x^3)^(3/2),x)
```

output
```
(x^6*(a + b/x^3)^(1/2))/(6*a^2) - (2*b^2)/(3*a^3*(a + b/x^3)^(1/2)) + (5*b
^2*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))*((a + b/x^3)^(1/2) + a^(1/2))^3))
/(8*a^(7/2)) - (7*b*x^3*(a + b/x^3)^(1/2))/(12*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.80

$$\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{4\sqrt{x}\,\sqrt{a\,x^3+b}\,a^3x^7 - 10\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2b\,x^4 - 30\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,b^2x - 15\sqrt{a}\log(\sqrt{a}}{}$$

input
```
int(x^5/(a+b/x^3)^(3/2),x)
```

output
```
(4*sqrt(x)*sqrt(a*x**3 + b)*a**3*x**7 - 10*sqrt(x)*sqrt(a*x**3 + b)*a**2*b
*x**4 - 30*sqrt(x)*sqrt(a*x**3 + b)*a*b**2*x - 15*sqrt(a)*log(sqrt(a*x**3
+ b) - sqrt(x)*sqrt(a)*x)*a*b**2*x**3 - 15*sqrt(a)*log(sqrt(a*x**3 + b) -
sqrt(x)*sqrt(a)*x)*b**3 + 15*sqrt(a)*log(sqrt(a*x**3 + b) + sqrt(x)*sqrt(a
)*x)*a*b**2*x**3 + 15*sqrt(a)*log(sqrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*b*
*3)/(24*a**4*(a*x**3 + b))
```

3.501 $\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{b}{a^2\sqrt{a+\frac{b}{x^3}}}+\frac{x^3}{3a\sqrt{a+\frac{b}{x^3}}}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
b/a^2/(a+b/x^3)^(1/2)+1/3*x^3/a/(a+b/x^3)^(1/2)-b*arctanh((a+b/x^3)^(1/2)/
a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{\sqrt{a}x^{3/2}(3b+ax^3)-3b\sqrt{b+ax^3}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)}{3a^{5/2}\sqrt{a+\frac{b}{x^3}}x^{3/2}}$$

input
```
Integrate[x^2/(a + b/x^3)^(3/2),x]
```

output
```
(Sqrt[a]*x^(3/2)*(3*b + a*x^3) - 3*b*Sqrt[b + a*x^3]*Log[Sqrt[a]*x^(3/2) +
 Sqrt[b + a*x^3]])/(3*a^(5/2)*Sqrt[a + b/x^3]*x^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{3b\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x^3}}{2a}+\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{3b\left(\frac{\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^3}}}\right)}{2a}+\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^3}}}\right)}{2a}+\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x^3}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}+\frac{x^3}{a\sqrt{a+\frac{b}{x^3}}}\right)$$

input
```
Int[x^2/(a + b/x^3)^(3/2),x]
```

output
```
(x^3/(a*Sqrt[a + b/x^3]) + (3*b*(2/(a*Sqrt[a + b/x^3]) - (2*ArcTanh[Sqrt[a
 + b/x^3]/Sqrt[a]])/a^(3/2)))/(2*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

method	result	size
default	$-\frac{(a\,x^{3}+b)\left(-x^{5} a^{\frac{7}{2}}-3 a^{\frac{5}{2}} b\,x^{2}+3 \,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2} \sqrt{a}}\right) b\,a^{2} \sqrt{x(a\,x^{3}+b)}\right)}{3\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}} x^{5} a^{\frac{9}{2}}}$	83
risch	$\frac{a\,x^{3}+b}{3 a^{2} \sqrt{\frac{a\,x^{3}+b}{x^{3}}}}-\frac{b\left(\frac{2 \,\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2} \sqrt{a}}\right)}{\sqrt{a}}-\frac{4 x^{2}}{3 \sqrt{\left(x^{3}+\frac{b}{a}\right) a x}}\right) \sqrt{x(a\,x^{3}+b)}}{2 a^{2} x^{2} \sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	104

input
```
int(x^2/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)*(-x^5*a^(7/2)-3*a^(5/2)*b*x^2+3*a
rctanh((x*(a*x^3+b))^(1/2)/x^2/a^(1/2))*b*a^2*(x*(a*x^3+b))^(1/2))/a^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(52) = 104$.

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.48

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \left[\frac{3\left(abx^3+b^2\right)\sqrt{a}\log\left(-8\,a^2x^6-8\,abx^3-b^2+4\left(2\,ax^6+bx^3\right)\sqrt{a}\sqrt{\frac{ax^3+b}{x^3}}\right)+4\left(a^2x^6\right.}{12\left(a^4x^3+a^3b\right)}\right.$$

input
```
integrate(x^2/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*b*x^3 + b^2)*sqrt(a)*log(-8*a^2*x^6 - 8*a*b*x^3 - b^2 + 4*(2*a
*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)) + 4*(a^2*x^6 + 3*a*b*x^3)*sqr
t((a*x^3 + b)/x^3))/(a^4*x^3 + a^3*b), 1/6*(3*(a*b*x^3 + b^2)*sqrt(-a)*arc
tan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt((a*x^3 + b)/x^3)/(a^2*x^3 + a*b)) + 2*
(a^2*x^6 + 3*a*b*x^3)*sqrt((a*x^3 + b)/x^3))/(a^4*x^3 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{x^{\frac{9}{2}}}{3a\sqrt{b}\sqrt{\frac{ax^3}{b}+1}} + \frac{\sqrt{b}x^{\frac{3}{2}}}{a^2\sqrt{\frac{ax^3}{b}+1}} - \frac{b\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

input
```
integrate(x**2/(a+b/x**3)**(3/2),x)
```

output
```
x**(9/2)/(3*a*sqrt(b)*sqrt(a*x**3/b + 1)) + sqrt(b)*x**(3/2)/(a**2*sqrt(a*
x**3/b + 1)) - b*asinh(sqrt(a)*x**(3/2)/sqrt(b))/a**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{3\left(a+\frac{b}{x^3}\right)b-2\,ab}{3\left(\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x^3}}a^3\right)}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

input
```
integrate(x^2/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
1/3*(3*(a + b/x^3)*b - 2*a*b)/((a + b/x^3)^(3/2)*a^2 - sqrt(a + b/x^3)*a^3
) + 1/2*b*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(
5/2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{\sqrt{ax^4+bx}x}{3\,a^2}+\frac{b\arctan\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{2\,b}{3\,\sqrt{a+\frac{b}{x^3}}a^2}$$

input
```
integrate(x^2/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
1/3*sqrt(a*x^4 + b*x)*x/a^2 + b*arctan(sqrt(a + b/x^3)/sqrt(-a))/(sqrt(-a)
*a^2) + 2/3*b/(sqrt(a + b/x^3)*a^2)
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{b}{a^2\sqrt{a+\frac{b}{x^3}}} + \frac{b\ln\left(x^6\left(\sqrt{a+\frac{b}{x^3}}-\sqrt{a}\right)^3\left(\sqrt{a+\frac{b}{x^3}}+\sqrt{a}\right)\right)}{2\,a^{5/2}} + \frac{x^3}{3\,a\sqrt{a+\frac{b}{x^3}}}$$

input
```
int(x^2/(a + b/x^3)^(3/2),x)
```

output
```
b/(a^2*(a + b/x^3)^(1/2)) + (b*log(x^6*((a + b/x^3)^(1/2) - a^(1/2))^3*((a
 + b/x^3)^(1/2) + a^(1/2))))/(2*a^(5/2)) + x^3/(3*a*(a + b/x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^4+6\sqrt{x}\,\sqrt{a\,x^3+b}\,abx+3\sqrt{a}\log\left(\sqrt{a\,x^3+b}-\sqrt{x}\,\sqrt{a}\,x\right)ab\,x^3+3}{}$$

input
```
int(x^2/(a+b/x^3)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**4 + 6*sqrt(x)*sqrt(a*x**3 + b)*a*b*x +
 3*sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*a*b*x**3 + 3*sqrt(a)*
log(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*b**2 - 3*sqrt(a)*log(sqrt(a*x**3
 + b) + sqrt(x)*sqrt(a)*x)*a*b*x**3 - 3*sqrt(a)*log(sqrt(a*x**3 + b) + sqr
t(x)*sqrt(a)*x)*b**2)/(6*a**3*(a*x**3 + b))
```

3.502 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x} \, dx = -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
-2/3/a/(a+b/x^3)^(1/2)+2/3*arctanh((a+b/x^3)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x} \, dx = -\frac{2\left(\sqrt{a}x^{3/2}-\sqrt{b+ax^3}\log\left(\sqrt{a}x^{3/2}+\sqrt{b+ax^3}\right)\right)}{3a^{3/2}\sqrt{a+\frac{b}{x^3}}x^{3/2}}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x),x]
```

output
```
(-2*(Sqrt[a]*x^(3/2) - Sqrt[b + a*x^3]*Log[Sqrt[a]*x^(3/2) + Sqrt[b + a*x^
3]]))/(3*a^(3/2)*Sqrt[a + b/x^3]*x^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x^3}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x^3}}{a}-\frac{2}{a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{2\int \frac{1}{\frac{1}{bx^6}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^3}}}{ab}-\frac{2}{a\sqrt{a+\frac{b}{x^3}}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a+\frac{b}{x^3}}}\right)$$

input
```
Int[1/((a + b/x^3)^(3/2)*x),x]
```

output
```
(-2/(a*Sqrt[a + b/x^3]) + (2*ArcTanh[Sqrt[a + b/x^3]/Sqrt[a]])/a^(3/2))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{2(a\,x^{3}+b)\left(-x^{2}a^{\frac{3}{2}}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(a\,x^{3}+b\right)}}{x^{2}\sqrt{a}}\right)a\sqrt{x(a\,x^{3}+b)}\right)}{3\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}x^{5}a^{\frac{5}{2}}}$	70

input
```
int(1/(a+b/x^3)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)*(-x^2*a^(3/2)+arctanh((x*(a*x^3+b)
)^(1/2)/x^2/a^(1/2))*a*(x*(a*x^3+b))^(1/2))/a^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(34) = 68$.

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.22

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x}\,dx = \left[-\frac{4\,ax^3\sqrt{\frac{ax^3+b}{x^3}}-(ax^3+b)\sqrt{a}\log\left(-8\,a^2x^6-8\,abx^3-b^2-4\,(2\,ax^6+bx^3)\sqrt{a}\sqrt{\frac{a}{}}\right.}{6\,(a^3x^3+a^2b)}\right.$$

$$\left. -\frac{2\,ax^3\sqrt{\frac{ax^3+b}{x^3}}+(ax^3+b)\sqrt{-a}\arctan\left(\frac{(2\,ax^3+b)\sqrt{-a}\sqrt{\frac{ax^3+b}{x^3}}}{2\,(a^2x^3+ab)}\right)}{3\,(a^3x^3+a^2b)}\right]$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/6*(4*a*x^3*sqrt((a*x^3 + b)/x^3) - (a*x^3 + b)*sqrt(a)*log(-8*a^2*x^6
- 8*a*b*x^3 - b^2 - 4*(2*a*x^6 + b*x^3)*sqrt(a)*sqrt((a*x^3 + b)/x^3)))/(a
^3*x^3 + a^2*b), -1/3*(2*a*x^3*sqrt((a*x^3 + b)/x^3) + (a*x^3 + b)*sqrt(-a
)*arctan(1/2*(2*a*x^3 + b)*sqrt(-a)*sqrt((a*x^3 + b)/x^3)/(a^2*x^3 + a*b))
)/(a^3*x^3 + a^2*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(39) = 78$.

Time = 1.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.07

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x}\,dx = -\frac{2a^3x^3\sqrt{1+\frac{b}{ax^3}}}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}-\frac{a^3x^3\log\left(\frac{b}{ax^3}\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}+\frac{2a^3x^3\log\left(\sqrt{1+\frac{b}{ax^3}}+1\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}-\frac{a^2b\log\left(\frac{b}{ax^3}\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}+\frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax^3}}+1\right)}{3a^{\frac{9}{2}}x^3+3a^{\frac{7}{2}}b}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x,x)
```

output
```
-2*a**3*x**3*sqrt(1 + b/(a*x**3))/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) - a**3*
x**3*log(b/(a*x**3))/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) + 2*a**3*x**3*log(sq
rt(1 + b/(a*x**3)) + 1)/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) - a**2*b*log(b/(a
*x**3))/(3*a**(9/2)*x**3 + 3*a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/(a*x**3
)) + 1)/(3*a**(9/2)*x**3 + 3*a**(7/2)*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^3}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^3}}+\sqrt{a}}\right)}{3\,a^{\frac{3}{2}}} - \frac{2}{3\,\sqrt{a+\frac{b}{x^3}}a}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/3*log((sqrt(a + b/x^3) - sqrt(a))/(sqrt(a + b/x^3) + sqrt(a)))/a^(3/2)
- 2/3/(sqrt(a + b/x^3)*a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^3}}}{\sqrt{a}}\right)}{3\,a^{3/2}} - \frac{2}{3\,a\,\sqrt{a+\frac{b}{x^3}}}$$

input
```
int(1/(x*(a + b/x^3)^(3/2)),x)
```

output
```
(2*atanh((a + b/x^3)^(1/2)/a^(1/2)))/(3*a^(3/2)) - 2/(3*a*(a + b/x^3)^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.59

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x}\,dx = \frac{-2\sqrt{x}\,\sqrt{a\,x^3+b}\,ax - \sqrt{a}\log\left(\sqrt{a\,x^3+b} - \sqrt{x}\,\sqrt{a}\,x\right)a\,x^3 - \sqrt{a}\log\left(\sqrt{a\,x^3+b} - \sqrt{x}\right.}{3a^2\,(a\,x^3}$$

input
```
int(1/(a+b/x^3)^(3/2)/x,x)
```

output
```
( - 2*sqrt(x)*sqrt(a*x**3 + b)*a*x - sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x
)*sqrt(a)*x)*a*x**3 - sqrt(a)*log(sqrt(a*x**3 + b) - sqrt(x)*sqrt(a)*x)*b
+ sqrt(a)*log(sqrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*a*x**3 + sqrt(a)*log(s
qrt(a*x**3 + b) + sqrt(x)*sqrt(a)*x)*b)/(3*a**2*(a*x**3 + b))
```

3.503 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4} \, dx = \frac{2}{3b\sqrt{a+\frac{b}{x^3}}}$$

output
```
2/3/b/(a+b/x^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4} \, dx = \frac{2}{3b\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^4),x]
```

output
```
2/(3*b*Sqrt[a + b/x^3])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^3}\right)^{3/2}} \, dx$$
$$\Big\downarrow \ 793$$
$$\frac{2}{3b\sqrt{a+\frac{b}{x^3}}}$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^4),x]
```

output
```
2/(3*b*Sqrt[a + b/x^3])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2}{3b\sqrt{a+\frac{b}{x^{3}}}}$	15
orering	$\frac{\frac{2a\,x^{3}}{3}+\frac{2b}{3}}{x^{3}b\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{2}}}$	25
gosper	$\frac{\frac{2a\,x^{3}}{3}+\frac{2b}{3}}{x^{3}b\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}}$	29
default	$\frac{\frac{2a\,x^{3}}{3}+\frac{2b}{3}}{x^{3}b\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}}$	29
trager	$\frac{2x^{3}\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{3b(a\,x^{3}+b)}$	35

input
```
int(1/(a+b/x^3)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3/b/(a+b/x^3)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

Time $= 0.07$ (sec) , antiderivative size $= 30$, normalized size of antiderivative $= 1.67$

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4}\,dx = \frac{2\,x^3\sqrt{\frac{ax^3+b}{x^3}}}{3\,(abx^3+b^2)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
2/3*x^3*sqrt((a*x^3 + b)/x^3)/(a*b*x^3 + b^2)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4} \, dx = \begin{cases} \frac{2}{3b\sqrt{a+\frac{b}{x^3}}} & \text{for } b \neq 0 \\ -\frac{1}{3a^{\frac{3}{2}}x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**4,x)
```

output
```
Piecewise((2/(3*b*sqrt(a + b/x**3)), Ne(b, 0)), (-1/(3*a**(3/2)*x**3), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4} \, dx = \frac{2}{3\sqrt{a+\frac{b}{x^3}}b}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
2/3/(sqrt(a + b/x^3)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4} \, dx = \frac{2}{3\sqrt{a+\frac{b}{x^3}}b}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^4,x, algorithm="giac")
```

output
```
2/3/(sqrt(a + b/x^3)*b)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4}\,dx = \frac{2}{3\,b\,\sqrt{a+\frac{b}{x^3}}}$$

input
```
int(1/(x^4*(a + b/x^3)^(3/2)),x)
```

output
```
2/(3*b*(a + b/x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^4}\,dx = \frac{2\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{3b\,(a\,x^3+b)}$$

input
```
int(1/(a+b/x^3)^(3/2)/x^4,x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b)*x)/(3*b*(a*x**3 + b))
```

3.504 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7} \, dx = -\frac{2a}{3b^2\sqrt{a+\frac{b}{x^3}}} - \frac{2\sqrt{a+\frac{b}{x^3}}}{3b^2}$$

output
```
-2/3*a/b^2/(a+b/x^3)^(1/2)-2/3*(a+b/x^3)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7} \, dx = -\frac{2(b+2ax^3)}{3b^2\sqrt{a+\frac{b}{x^3}}x^3}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^7),x]
```

output
```
(-2*(b + 2*a*x^3))/(3*b^2*Sqrt[a + b/x^3]*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+\frac{b}{x^3}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3} \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3} d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3} \int \left(\frac{1}{b\sqrt{a+\frac{b}{x^3}}}-\frac{a}{b\left(a+\frac{b}{x^3}\right)^{3/2}}\right) d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2a}{b^2\sqrt{a+\frac{b}{x^3}}}-\frac{2\sqrt{a+\frac{b}{x^3}}}{b^2}\right)$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^7),x]
```

output
```
((-2*a)/(b^2*Sqrt[a + b/x^3]) - (2*Sqrt[a + b/x^3])/b^2)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
orering	$-\frac{2(2a\,x^{3}+b)\left(a\,x^{3}+b\right)}{3b^{2}x^{6}\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{2}}}$	33
gosper	$-\frac{2\left(a\,x^{3}+b\right)\left(2a\,x^{3}+b\right)}{3x^{6}b^{2}\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}}$	37
default	$-\frac{2\left(a\,x^{3}+b\right)\left(2a\,x^{3}+b\right)}{3x^{6}b^{2}\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}}$	37
trager	$-\frac{2(2a\,x^{3}+b)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{3b^{2}\left(a\,x^{3}+b\right)}$	40
risch	$-\frac{2\left(a\,x^{3}+b\right)}{3b^{2}x^{3}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}-\frac{2a}{3b^{2}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	49

input
```
int(1/(a+b/x^3)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(2*a*x^3+b)/b^2/x^6*(a*x^3+b)/(a+b/x^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx = -\frac{2\,(2\,ax^3+b)\sqrt{\frac{ax^3+b}{x^3}}}{3\,(ab^2x^3+b^3)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-2/3*(2*a*x^3 + b)*sqrt((a*x^3 + b)/x^3)/(a*b^2*x^3 + b^3)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx = \begin{cases} -\frac{4a}{3b^2\sqrt{a+\frac{b}{x^3}}} - \frac{2}{3bx^3\sqrt{a+\frac{b}{x^3}}} & \text{for } b \neq 0 \\ -\frac{1}{6a^{\frac{3}{2}}x^6} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**7,x)
```

output
```
Piecewise((-4*a/(3*b**2*sqrt(a + b/x**3)) - 2/(3*b*x**3*sqrt(a + b/x**3)),
 Ne(b, 0)), (-1/(6*a**(3/2)*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}}{3\,b^2} - \frac{2\,a}{3\sqrt{a+\frac{b}{x^3}}b^2}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-2/3*sqrt(a + b/x^3)/b^2 - 2/3*a/(sqrt(a + b/x^3)*b^2)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^7}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^7), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx = -\frac{2\sqrt{a+\frac{b}{x^3}}\,(2\,a\,x^3+b)}{3\,b^2\,(a\,x^3+b)}$$

input
```
int(1/(x^7*(a + b/x^3)^(3/2)),x)
```

output
```
-(2*(a + b/x^3)^(1/2)*(b + 2*a*x^3))/(3*b^2*(b + a*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}\,dx = \frac{2\sqrt{a\,x^3+b}\,(-2a\,x^3-b)}{3\sqrt{x}\,b^2x\,(a\,x^3+b)}$$

input
```
int(1/(a+b/x^3)^(3/2)/x^7,x)
```

output
```
(2*sqrt(a*x**3 + b)*( - 2*a*x**3 - b))/(3*sqrt(x)*b**2*x*(a*x**3 + b))
```

3.505 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}} \, dx = \frac{2a^2}{3b^3\sqrt{a+\frac{b}{x^3}}} + \frac{4a\sqrt{a+\frac{b}{x^3}}}{3b^3} - \frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{9b^3}$$

output
```
2/3*a^2/b^3/(a+b/x^3)^(1/2)+4/3*a*(a+b/x^3)^(1/2)/b^3-2/9*(a+b/x^3)^(3/2)/
b^3
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}} \, dx = \frac{2(-b^2+4abx^3+8a^2x^6)}{9b^3\sqrt{a+\frac{b}{x^3}}x^6}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^10),x]
```

output
```
(2*(-b^2 + 4*a*b*x^3 + 8*a^2*x^6))/(9*b^3*Sqrt[a + b/x^3]*x^6)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3}\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^6}d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3}\int \left(\frac{a^2}{b^2\left(a+\frac{b}{x^3}\right)^{3/2}}-\frac{2a}{b^2\sqrt{a+\frac{b}{x^3}}}+\frac{\sqrt{a+\frac{b}{x^3}}}{b^2}\right)d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^2}{b^3\sqrt{a+\frac{b}{x^3}}}+\frac{4a\sqrt{a+\frac{b}{x^3}}}{b^3}-\frac{2\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^3}\right)$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^10),x]
```

output
```
((2*a^2)/(b^3*Sqrt[a + b/x^3]) + (4*a*Sqrt[a + b/x^3])/b^3 - (2*(a + b/x^3
)^(3/2))/(3*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

method	result	size
orering	$\frac{2\left(8a^{2}x^{6}+4ab\,x^{3}-b^{2}\right)\left(a\,x^{3}+b\right)}{9b^{3}x^{9}\left(a+\frac{b}{x^{3}}\right)^{\frac{3}{2}}}$	46
gosper	$\frac{2\left(a\,x^{3}+b\right)\left(8a^{2}x^{6}+4ab\,x^{3}-b^{2}\right)}{9x^{9}b^{3}\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}}$	50
trager	$\frac{2\left(8a^{2}x^{6}+4ab\,x^{3}-b^{2}\right)\sqrt{-\frac{-a\,x^{3}-b}{x^{3}}}}{9x^{3}b^{3}\left(a\,x^{3}+b\right)}$	56
risch	$\frac{2\left(a\,x^{3}+b\right)\left(5a\,x^{3}-b\right)}{9b^{3}x^{6}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}+\frac{2a^{2}}{3b^{3}\sqrt{\frac{a\,x^{3}+b}{x^{3}}}}$	61
default	$-\frac{2\left(a\,x^{3}+b\right)\left(-9a^{2}x^{7}+\sqrt{x\left(a\,x^{3}+b\right)}\,\sqrt{a\,x^{4}+bx}\,a\,x^{3}-6ab\,x^{4}+\sqrt{x\left(a\,x^{3}+b\right)}\,\sqrt{a\,x^{4}+bx}\,b\right)}{9\left(\frac{a\,x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}x^{10}b^{3}}$	96

input
```
int(1/(a+b/x^3)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
2/9*(8*a^2*x^6+4*a*b*x^3-b^2)/b^3/x^9*(a*x^3+b)/(a+b/x^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx = \frac{2\left(8\,a^2x^6+4\,abx^3-b^2\right)\sqrt{\frac{ax^3+b}{x^3}}}{9\left(ab^3x^6+b^4x^3\right)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
2/9*(8*a^2*x^6 + 4*a*b*x^3 - b^2)*sqrt((a*x^3 + b)/x^3)/(a*b^3*x^6 + b^4*x
^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(54) = 108.

Time = 1.66 (sec) , antiderivative size = 466, normalized size of antiderivative = 7.90

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx &= \frac{16a^{\frac{9}{2}}b^{\frac{7}{2}}x^9\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}} \\ &+ \frac{24a^{\frac{7}{2}}b^{\frac{9}{2}}x^6\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}} + \frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}x^3\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}} \\ &- \frac{2a^{\frac{3}{2}}b^{\frac{13}{2}}\sqrt{\frac{ax^3}{b}+1}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}} - \frac{16a^5b^3x^{\frac{21}{2}}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}} \\ &- \frac{32a^4b^4x^{\frac{15}{2}}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}} - \frac{16a^3b^5x^{\frac{9}{2}}}{9a^{\frac{7}{2}}b^6x^{\frac{21}{2}}+18a^{\frac{5}{2}}b^7x^{\frac{15}{2}}+9a^{\frac{3}{2}}b^8x^{\frac{9}{2}}}\end{aligned}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**10,x)
```

output
```
16*a**(9/2)*b**(7/2)*x**9*sqrt(a*x**3/b + 1)/(9*a**(7/2)*b**6*x**(21/2) +
18*a**(5/2)*b**7*x**(15/2) + 9*a**(3/2)*b**8*x**(9/2)) + 24*a**(7/2)*b**(9
/2)*x**6*sqrt(a*x**3/b + 1)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*
x**(15/2) + 9*a**(3/2)*b**8*x**(9/2)) + 6*a**(5/2)*b**(11/2)*x**3*sqrt(a*x
**3/b + 1)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**(15/2) + 9*a**
(3/2)*b**8*x**(9/2)) - 2*a**(3/2)*b**(13/2)*sqrt(a*x**3/b + 1)/(9*a**(7/2)
*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**(15/2) + 9*a**(3/2)*b**8*x**(9/2)) -
 16*a**5*b**3*x**(21/2)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**(
15/2) + 9*a**(3/2)*b**8*x**(9/2)) - 32*a**4*b**4*x**(15/2)/(9*a**(7/2)*b**
6*x**(21/2) + 18*a**(5/2)*b**7*x**(15/2) + 9*a**(3/2)*b**8*x**(9/2)) - 16*
a**3*b**5*x**(9/2)/(9*a**(7/2)*b**6*x**(21/2) + 18*a**(5/2)*b**7*x**(15/2)
 + 9*a**(3/2)*b**8*x**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9\,b^3}+\frac{4\sqrt{a+\frac{b}{x^3}}\,a}{3\,b^3}+\frac{2\,a^2}{3\sqrt{a+\frac{b}{x^3}}\,b^3}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
-2/9*(a + b/x^3)^(3/2)/b^3 + 4/3*sqrt(a + b/x^3)*a/b^3 + 2/3*a^2/(sqrt(a +
 b/x^3)*b^3)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^{10}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^10,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^10), x)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx = \frac{10\,a\,\sqrt{a+\frac{b}{x^3}}}{9\,b^3} - \frac{2\,\sqrt{a+\frac{b}{x^3}}}{9\,b^2\,x^3} + \frac{2\,a^2\,x^3\,\sqrt{a+\frac{b}{x^3}}}{3\,b^3\,\left(a\,x^3+b\right)}$$

input
```
int(1/(x^10*(a + b/x^3)^(3/2)),x)
```

output
```
(10*a*(a + b/x^3)^(1/2))/(9*b^3) - (2*(a + b/x^3)^(1/2))/(9*b^2*x^3) + (2*
a^2*x^3*(a + b/x^3)^(1/2))/(3*b^3*(b + a*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}\,dx = \frac{2\sqrt{a\,x^3+b}\,(8a^2x^6+4ab\,x^3-b^2)}{9\sqrt{x}\,b^3x^4\,(a\,x^3+b)}$$

input
```
int(1/(a+b/x^3)^(3/2)/x^10,x)
```

output
```
(2*sqrt(a*x**3 + b)*(8*a**2*x**6 + 4*a*b*x**3 - b**2))/(9*sqrt(x)*b**3*x**
4*(a*x**3 + b))
```

3.506 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}} \, dx = -\frac{2a^3}{3b^4\sqrt{a+\frac{b}{x^3}}} - \frac{2a^2\sqrt{a+\frac{b}{x^3}}}{b^4} + \frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{3b^4} - \frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{15b^4}$$

output
```
-2/3*a^3/b^4/(a+b/x^3)^(1/2)-2*a^2*(a+b/x^3)^(1/2)/b^4+2/3*a*(a+b/x^3)^(3/
2)/b^4-2/15*(a+b/x^3)^(5/2)/b^4
```

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}} \, dx = -\frac{2(b^3-2ab^2x^3+8a^2bx^6+16a^3x^9)}{15b^4\sqrt{a+\frac{b}{x^3}}x^9}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^13),x]
```

output
```
(-2*(b^3 - 2*a*b^2*x^3 + 8*a^2*b*x^6 + 16*a^3*x^9))/(15*b^4*Sqrt[a + b/x^3
]*x^9)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13} \left(a+\frac{b}{x^3}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{3} \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9} d\frac{1}{x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{3} \int \left(-\frac{a^3}{b^3 \left(a+\frac{b}{x^3}\right)^{3/2}}+\frac{3a^2}{b^3 \sqrt{a+\frac{b}{x^3}}}-\frac{3\sqrt{a+\frac{b}{x^3}}a}{b^3}+\frac{\left(a+\frac{b}{x^3}\right)^{3/2}}{b^3}\right) d\frac{1}{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2a^3}{b^4 \sqrt{a+\frac{b}{x^3}}}-\frac{6a^2 \sqrt{a+\frac{b}{x^3}}}{b^4}+\frac{2a\left(a+\frac{b}{x^3}\right)^{3/2}}{b^4}-\frac{2\left(a+\frac{b}{x^3}\right)^{5/2}}{5b^4}\right)$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^13),x]
```

output
```
((-2*a^3)/(b^4*Sqrt[a + b/x^3]) - (6*a^2*Sqrt[a + b/x^3])/b^4 + (2*a*(a +
b/x^3)^(3/2))/b^4 - (2*(a + b/x^3)^(5/2))/(5*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

method	result	si
orering	$-\frac{2\left(16 a^{3} x^{9}+8 a^{2} b\, x^{6}-2 a\, b^{2} x^{3}+b^{3}\right) \left(a\, x^{3}+b \right)}{15 b^{4} x^{12} \left(a +\frac{b}{x^{3}}\right)^{\frac{3}{2}}}$	5
gosper	$-\frac{2 \left(a\, x^{3}+b \right) \left(16 a^{3} x^{9}+8 a^{2} b\, x^{6}-2 a\, b^{2} x^{3}+b^{3}\right)}{15 x^{12} b^{4} \left(\frac{a\, x^{3}+b}{x^{3}}\right)^{\frac{3}{2}}}$	59
trager	$-\frac{2\left(16 a^{3} x^{9}+8 a^{2} b\, x^{6}-2 a\, b^{2} x^{3}+b^{3}\right) \sqrt{-\frac{-a\, x^{3}-b}{x^{3}}}}{15 x^{6} b^{4} \left(a\, x^{3}+b \right)}$	6
risch	$-\frac{2 \left(a\, x^{3}+b \right) \left(11 a^{2} x^{6}-3 a b\, x^{3}+b^{2}\right)}{15 b^{4} x^{9} \sqrt{\frac{a\, x^{3}+b}{x^{3}}}}-\frac{2 a^{3}}{3 b^{4} \sqrt{\frac{a\, x^{3}+b}{x^{3}}}}$	70
default	$\frac{2 \left(a\, x^{3}+b \right) \left(-20 a^{3} x^{10}+4 \sqrt{a\, x^{4}+b x}\, \sqrt{x \left(a\, x^{3}+b \right)}\, a^{2} x^{6}-15 a^{2} b\, x^{7}+3 \sqrt{a\, x^{4}+b x}\, \sqrt{x \left(a\, x^{3}+b \right)}\, a b\, x^{3}-\sqrt{a\, x^{4}+b x}\, \sqrt{x \left(a\, x^{3}+b \right)}\, b^{2}\right)}{15 \left(\frac{a\, x^{3}+b}{x^{3}}\right)^{\frac{3}{2}} x^{13} b^{4}}$	1

input
```
int(1/(a+b/x^3)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(16*a^3*x^9+8*a^2*b*x^6-2*a*b^2*x^3+b^3)/b^4/x^12*(a*x^3+b)/(a+b/x^3
)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}}\,dx = -\frac{2\,(16\,a^3x^9+8\,a^2bx^6-2\,ab^2x^3+b^3)\sqrt{\frac{ax^3+b}{x^3}}}{15\,(ab^4x^9+b^5x^6)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
-2/15*(16*a^3*x^9 + 8*a^2*b*x^6 - 2*a*b^2*x^3 + b^3)*sqrt((a*x^3 + b)/x^3)
/(a*b^4*x^9 + b^5*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. 2(73) = 146.

Time = 2.53 (sec) , antiderivative size = 2048, normalized size of antiderivative = 26.26

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**13,x)
```

output
```
-32*a**(21/2)*b**(23/2)*x**24*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(5
1/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*
a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**
20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) - 176*a**(19/2)*b**(25/2)*x**2
1*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x*
*(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) +
225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b
**21*x**(15/2)) - 396*a**(17/2)*b**(27/2)*x**18*sqrt(a*x**3/b + 1)/(15*a**
(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**1
7*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2)
 + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(15/2)) - 462*a**(15
/2)*b**(29/2)*x**15*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15*x**(51/2) + 90*
a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*
b**18*x**(33/2) + 225*a**(9/2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/
2) + 15*a**(5/2)*b**21*x**(15/2)) - 290*a**(13/2)*b**(31/2)*x**12*sqrt(a*x
**3/b + 1)/(15*a**(17/2)*b**15*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) +
225*a**(13/2)*b**17*x**(39/2) + 300*a**(11/2)*b**18*x**(33/2) + 225*a**(9/
2)*b**19*x**(27/2) + 90*a**(7/2)*b**20*x**(21/2) + 15*a**(5/2)*b**21*x**(1
5/2)) - 92*a**(11/2)*b**(33/2)*x**9*sqrt(a*x**3/b + 1)/(15*a**(17/2)*b**15
*x**(51/2) + 90*a**(15/2)*b**16*x**(45/2) + 225*a**(13/2)*b**17*x**(39/...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}} dx = -\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{5}{2}}}{15\,b^4} + \frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} a}{3\,b^4} - \frac{2\sqrt{a+\frac{b}{x^3}}\,a^2}{b^4} - \frac{2\,a^3}{3\sqrt{a+\frac{b}{x^3}}\,b^4}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
-2/15*(a + b/x^3)^(5/2)/b^4 + 2/3*(a + b/x^3)^(3/2)*a/b^4 - 2*sqrt(a + b/x
^3)*a^2/b^4 - 2/3*a^3/(sqrt(a + b/x^3)*b^4)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^{13}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^13,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^13), x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}}\,dx = \frac{2\,a\,\sqrt{a+\frac{b}{x^3}}}{5\,b^3\,x^3} - \frac{2\,\sqrt{a+\frac{b}{x^3}}}{15\,b^2\,x^6} - \frac{22\,a^2\,\sqrt{a+\frac{b}{x^3}}}{15\,b^4} - \frac{2\,a^3\,x^3\,\sqrt{a+\frac{b}{x^3}}}{3\,b^4\,\left(a\,x^3+b\right)}$$

input
```
int(1/(x^13*(a + b/x^3)^(3/2)),x)
```

output
```
(2*a*(a + b/x^3)^(1/2))/(5*b^3*x^3) - (2*(a + b/x^3)^(1/2))/(15*b^2*x^6) -
 (22*a^2*(a + b/x^3)^(1/2))/(15*b^4) - (2*a^3*x^3*(a + b/x^3)^(1/2))/(3*b^
4*(b + a*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{13}}\,dx = \frac{2\sqrt{a\,x^3+b}\,\left(-16a^3x^9-8a^2b\,x^6+2a\,b^2x^3-b^3\right)}{15\sqrt{x}\,b^4x^7\left(a\,x^3+b\right)}$$

input
```
int(1/(a+b/x^3)^(3/2)/x^13,x)
```

output
```
(2*sqrt(a*x**3 + b)*( - 16*a**3*x**9 - 8*a**2*b*x**6 + 2*a*b**2*x**3 - b**
3))/(15*sqrt(x)*b**4*x**7*(a*x**3 + b))
```

3.507 $\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 315

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{1729b^2\sqrt{a+\frac{b}{x^3}}x^2}{960a^4} - \frac{247b\sqrt{a+\frac{b}{x^3}}x^5}{240a^3} - \frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{19\sqrt{a+\frac{b}{x^3}}x^8}{24a^2} + \frac{1729\sqrt{2+\sqrt{3}}b^{8/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{960\sqrt[4]{3}a^4\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
1729/960*b^2*(a+b/x^3)^(1/2)*x^2/a^4-247/240*b*(a+b/x^3)^(1/2)*x^5/a^3-2/3
*x^8/a/(a+b/x^3)^(1/2)+19/24*(a+b/x^3)^(1/2)*x^8/a^2+1729/2880*(1/2*6^(1/2
)+1/2*2^(1/2))*b^(8/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b
^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*
a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/
a^4/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1
/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.29

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{1729b^3+741ab^2x^3-228a^2bx^6+120a^3x^9-1729b^3\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\right.}{960a^4\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[x^7/(a + b/x^3)^(3/2),x]
```

output
```
(1729*b^3 + 741*a*b^2*x^3 - 228*a^2*b*x^6 + 120*a^3*x^9 - 1729*b^3*Sqrt[1
+ (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(960*a^4*Sqrt
[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 819, 847, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{x^9}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{19\int \frac{x^9}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{3a}-\frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \; 847$$

$$-\frac{19\left(-\frac{13b\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{16a}-\frac{x^8\sqrt{a+\frac{b}{x^3}}}{8a}\right)}{3a}-\frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \; 847$$

$$-\frac{19\left(-\frac{13b\left(-\frac{7b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{16a}-\frac{x^8\sqrt{a+\frac{b}{x^3}}}{8a}\right)}{3a}-\frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \; 847$$

$$-\frac{19\left(-\frac{13b\left(-\frac{7b\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{16a}-\frac{x^8\sqrt{a+\frac{b}{x^3}}}{8a}\right)}{3a}-\frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \; 759$$

$$-\frac{19\left(-\frac{13b\left(-\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}-\frac{x^5\sqrt{\cdots}}{5\cdots}\cdots\right)}{16a}\cdots\right)}{3a}$$

$$\frac{2x^8}{3a\sqrt{a+\frac{b}{x^3}}}$$

input
```
Int[x^7/(a + b/x^3)^(3/2),x]
```

output
```
(-2*x^8)/(3*a*Sqrt[a + b/x^3]) - (19*(-1/8*(Sqrt[a + b/x^3]*x^8)/a - (13*b
*(-1/5*(Sqrt[a + b/x^3]*x^5)/a - (7*b*(-1/2*(Sqrt[a + b/x^3]*x^2)/a - (Sqr
t[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 -
 (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x
)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(10*a)))/(16*a)))/(
3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1453 vs. 2(244) = 488.

Time = 4.17 (sec) , antiderivative size = 1454, normalized size of antiderivative = 4.62

method	result	size
risch	Expression too large to display	1454
default	Expression too large to display	2540

input

```
int(x^7/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/320*(40*a^2*x^6-116*a*b*x^3+363*b^2)/a^4/x*(a*x^3+b)/((a*x^3+b)/x^3)^(1/
2)-1/640/a^4*b^3*(2006*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x
-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3
^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^
(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)
/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))
*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)
^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^
(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(
-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))-640*b*(2/3*x/b/((x^3
+b/a)*a*x)^(1/2)+4/3/b*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)...
```

Fricas [F]

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^13*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.15

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{x^8\Gamma\left(-\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{8}{3},\frac{3}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right)}$$

input
```
integrate(x**7/(a+b/x**3)**(3/2),x)
```

output
```
-x**8*gamma(-8/3)*hyper((-8/3, 3/2), (-5/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*a**(3/2)*gamma(-5/3))
```

Maxima [F]

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(a + b/x^3)^(3/2), x)
```

Giac [F]

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^7/(a + b/x^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(x^7/(a + b/x^3)^(3/2),x)
```

output
```
int(x^7/(a + b/x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^7}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{160\sqrt{x}\,\sqrt{a\,x^3+b}\,a^3x^9-304\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2b\,x^6+988\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,b^2x^3+3458\sqrt{x}}{1280a^4\,(a\,x^3}$$

input
```
int(x^7/(a+b/x^3)^(3/2),x)
```

output
```
(160*sqrt(x)*sqrt(a*x**3 + b)*a**3*x**9 - 304*sqrt(x)*sqrt(a*x**3 + b)*a**
2*b*x**6 + 988*sqrt(x)*sqrt(a*x**3 + b)*a*b**2*x**3 + 3458*sqrt(x)*sqrt(a*
x**3 + b)*b**3 - 1729*int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**7 + 2*a*b*x*
*4 + b**2*x),x)*a*b**4*x**3 - 1729*int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x*
*7 + 2*a*b*x**4 + b**2*x),x)*b**5)/(1280*a**4*(a*x**3 + b))
```

3.508 $\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 291

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=-\frac{91b\sqrt{a+\frac{b}{x^3}}x^2}{60a^3}-\frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}}+\frac{13\sqrt{a+\frac{b}{x^3}}x^5}{15a^2}-\frac{91\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{60\sqrt[4]{3}a^3\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output

```
-91/60*b*(a+b/x^3)^(1/2)*x^2/a^3-2/3*x^5/a/(a+b/x^3)^(1/2)+13/15*(a+b/x^3)
^(1/2)*x^5/a^2-91/180*(1/2*6^(1/2)+1/2*2^(1/2))*b^(5/3)*(a^(1/3)+b^(1/3)/x
)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)
^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a^3/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{-91b^2-39abx^3+12a^2x^6+91b^2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)}{60a^3\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[x^4/(a + b/x^3)^(3/2),x]
```

output
```
(-91*b^2 - 39*a*b*x^3 + 12*a^2*x^6 + 91*b^2*Sqrt[1 + (a*x^3)/b]*Hypergeome
tric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])/(60*a^3*Sqrt[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 819, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$-\frac{13\int \frac{x^6}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{3a}-\frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{13\left(-\frac{7b\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{3a}-\frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\big\downarrow$ 847

$$-\frac{13\left(-\frac{7b\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{3a}-\frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\big\downarrow$ 759

$$-\frac{13\left(-\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{10a}-\frac{x^5\sqrt{a+\frac{b}{x^3}}}{5a}\right)}{3a}$$

$$\frac{2x^5}{3a\sqrt{a+\frac{b}{x^3}}}$$

input
```
Int[x^4/(a + b/x^3)^(3/2),x]
```

output
```
(-2*x^5)/(3*a*Sqrt[a + b/x^3]) - (13*(-1/5*(Sqrt[a + b/x^3]*x^5)/a - (7*b*
(-1/2*(Sqrt[a + b/x^3]*x^2)/a - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1
/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt
[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)/x)^2])))/(10*a)))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1442 vs. $2(224) = 448$.

Time = 2.85 (sec) , antiderivative size = 1443, normalized size of antiderivative = 4.96

method	result	size
risch	Expression too large to display	1443
default	Expression too large to display	2302

input
```
int(x^4/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*a*x^3-17*b)/a^3/x*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)+1/40/a^3*b^2*(11
4*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^
(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^
2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)
^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x
-1/a*(-a^2*b)^(1/3)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b
)^(1/3))*a/(-a^2*b)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3)))^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/
(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2
*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-
a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3
^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))-40*b*(2/3*x/b/((x^3+b/a)*a*x)^(1/2)+4/3/b
*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(
1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3)...
```

Fricas [F]

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^10*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=-\frac{x^5\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(x**4/(a+b/x**3)**(3/2),x)
```

output
```
-x**5*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*a**(3/2)*gamma(-2/3))
```

Maxima [F]

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(a + b/x^3)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(a + b/x^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(x^4/(a + b/x^3)^(3/2),x)
```

output
```
int(x^4/(a + b/x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{16\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^6-52\sqrt{x}\,\sqrt{a\,x^3+b}\,ab\,x^3-182\sqrt{x}\,\sqrt{a\,x^3+b}\,b^2+91\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3}}{a^2x^7+2ab\,x^4}\right.}{80a^3\left(a\,x^3+b\right)}$$

input
```
int(x^4/(a+b/x^3)^(3/2),x)
```

output
```
(16*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**6 - 52*sqrt(x)*sqrt(a*x**3 + b)*a*b*x
**3 - 182*sqrt(x)*sqrt(a*x**3 + b)*b**2 + 91*int((sqrt(x)*sqrt(a*x**3 + b)
)/(a**2*x**7 + 2*a*b*x**4 + b**2*x),x)*a*b**3*x**3 + 91*int((sqrt(x)*sqrt(
a*x**3 + b))/(a**2*x**7 + 2*a*b*x**4 + b**2*x),x)*b**4)/(80*a**3*(a*x**3 +
 b))
```

3.509 $\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 269

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}}+\frac{7\sqrt{a+\frac{b}{x^3}}\,x^2}{6a^2} + \frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}a^2\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/3*x^2/a/(a+b/x^3)^(1/2)+7/6*(a+b/x^3)^(1/2)*x^2/a^2+7/18*(1/2*6^(1/2)+1
/2*2^(1/2))*b^(2/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1
/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(
1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a^2
/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{7b+3ax^3-7b\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)}{6a^2\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[x/(a + b/x^3)^(3/2),x]
```

output
```
(7*b + 3*a*x^3 - 7*b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6,
-((a*x^3)/b)])/(6*a^2*Sqrt[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {858, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$-\frac{7\int \frac{x^3}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{3a}-\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{7\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{4a}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{3a}-\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \; 759$$

$$-\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{x^2\sqrt{a+\frac{b}{x^3}}}{2a}\right)}{3a}-\frac{2x^2}{3a\sqrt{a+\frac{b}{x^3}}}$$

input
```
Int[x/(a + b/x^3)^(3/2),x]
```

output
```
(-2*x^2)/(3*a*Sqrt[a + b/x^3]) - (7*(-1/2*(Sqrt[a + b/x^3]*x^2)/a - (Sqrt[
2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (
a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)]
, -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1430 vs. $2(206) = 412$.

Time = 2.39 (sec) , antiderivative size = 1431, normalized size of antiderivative = 5.32

method	result	size
risch	Expression too large to display	1431
default	Expression too large to display	2052

input
```
int(x/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/x*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)-1/4/a^2*b*(10*(1/2/a*(-a^2*b)^(1
/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3
)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)
^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*
(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3))
)^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^
(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(
1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(
-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3
)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-
a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3
^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1
/3)))^(1/2))-4*b*(2/3*x/b/((x^3+b/a)*a*x)^(1/2)+4/3/b*(1/2/a*(-a^2*b)^(1/3
)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(
x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/...
```

Fricas [F]

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^7*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.16

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{x^2\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(x/(a+b/x**3)**(3/2),x)
```

output
```
-x**2*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*exp_polar(I*pi)/(a*x**3))/(
3*a**(3/2)*gamma(1/3))
```

Maxima [F]

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(a + b/x^3)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(a + b/x^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(x/(a + b/x^3)^(3/2),x)
```

output
```
int(x/(a + b/x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{4\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,x^3+14\sqrt{x}\,\sqrt{a\,x^3+b}\,b-7\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^7+2ab\,x^4+b^2x}dx\right)a\,b^2x^3-7\left(\int \frac{\sqrt{x}\,\sqrt{a}}{a^2x^7+2a}\right.}{8a^2\,(a\,x^3+b)}$$

input
```
int(x/(a+b/x^3)^(3/2),x)
```

output
```
(4*sqrt(x)*sqrt(a*x**3 + b)*a*x**3 + 14*sqrt(x)*sqrt(a*x**3 + b)*b - 7*int
((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**7 + 2*a*b*x**4 + b**2*x),x)*a*b**2*x*
*3 - 7*int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**7 + 2*a*b*x**4 + b**2*x),x)
*b**3)/(8*a**2*(a*x**3 + b))
```

3.510 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 248

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2} \, dx = -\frac{2}{3a\sqrt{a+\frac{b}{x^3}}\,x} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-2/3/a/(a+b/x^3)^(1/2)/x-2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)/x)
*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(1/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3
)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\,dx = \frac{2\left(-1+\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)\right)}{3a\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^2),x]
```

output
```
(2*(-1 + Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)
]))/(3*a*Sqrt[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {858, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 749}$$

$$-\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{3a}-\frac{2}{3ax\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}-\frac{2}{3ax\sqrt{a+\frac{b}{x^3}}}$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^2),x]
```

output
```
-2/(3*a*Sqrt[a + b/x^3]*x) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)/x)*Sq
rt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[
a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1821 vs. $2(189) = 378$.

Time = 0.52 (sec) , antiderivative size = 1822, normalized size of antiderivative = 7.35

method	result	size
default	Expression too large to display	1822

input
```
int(1/(a+b/x^3)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^2/(-a^2*b)^(1/3)*(2*I*(-(I*3^(1
/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)
^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*(x*(a*x^3+b))^(1/2)*3^(1/2)*a^2*x^2-4*I*(-(I*3^(1/2)-3)*x*a/(I*3^(1/
2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*
b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^
(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*(a*x^3+b))
^(1/2)*(-a^2*b)^(1/3)*3^(1/2)*a*x+2*I*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-
a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3)
)/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*
a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(x*(a*x^3+b))^(1/2)*(
-a^2*b)^(2/3)*3^(1/2)-2*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\, dx = -\frac{2 \left(bx^2\sqrt{\frac{ax^3+b}{x^3}} + (ax^3+b)\sqrt{b}\text{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right)}{3\,(a^2bx^3+ab^2)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-2/3*(b*x^2*sqrt((a*x^3 + b)/x^3) + (a*x^3 + b)*sqrt(b)*weierstrassPInvers
e(0, -4*a/b, 1/x))/(a^2*b*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\, dx = -\frac{\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3}, \frac{3}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**2,x)
```

output
```
-gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3
/2)*x*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\,dx = -\frac{\left(\frac{b}{a\,x^3}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b}{a\,x^3}\right)}{x\,\left(a+\frac{b}{x^3}\right)^{3/2}}$$

input
```
int(1/(x^2*(a + b/x^3)^(3/2)),x)
```

output
```
-((b/(a*x^3) + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -b/(a*x^3)))/(x*(a + b/
x^3)^(3/2))
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^2}\, dx = \frac{-2\sqrt{x}\,\sqrt{a\,x^3+b}+\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^7+2ab\,x^4+b^2x}dx\right)ab\,x^3+\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^7+2ab\,x^4+b^2x}dx\right)b^2}{2a\,(a\,x^3+b)}$$

input
```
int(1/(a+b/x^3)^(3/2)/x^2,x)
```

output
```
( - 2*sqrt(x)*sqrt(a*x**3 + b) + int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**7
 + 2*a*b*x**4 + b**2*x),x)*a*b*x**3 + int((sqrt(x)*sqrt(a*x**3 + b))/(a**2
*x**7 + 2*a*b*x**4 + b**2*x),x)*b**2)/(2*a*(a*x**3 + b))
```

3.511 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 245

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5} dx = \frac{2}{3b\sqrt{a+\frac{b}{x^3}}x} - \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
2/3/b/(a+b/x^3)^(1/2)/x-4/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)/x)*
((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2
)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^(4/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.24

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5}\,dx = \frac{2+4\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{ax^3}{b}\right)}{3b\sqrt{a+\frac{b}{x^3}}\,x}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^5),x]
```

output
```
(2 + 4*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[1/6, 1/2, 7/6, -((a*x^3)/b)])
/(3*b*Sqrt[a + b/x^3]*x)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {858, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,d\frac{1}{x}$$

$$\Big\downarrow \text{817}$$

$$\frac{2}{3bx\sqrt{a+\frac{b}{x^3}}} - \frac{2\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}\,d\frac{1}{x}}{3b}$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{3bx\sqrt{a+\frac{b}{x^3}}}-\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^5),x]
```

output
```
2/(3*b*Sqrt[a + b/x^3]*x) - (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)/x)*Sqr
t[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[a +
 b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)/x)^2])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1824 vs. $2(186) = 372$.

Time = 1.82 (sec) , antiderivative size = 1825, normalized size of antiderivative = 7.45

method	result	size
default	Expression too large to display	1825

input
```
int(1/(a+b/x^3)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a/(-a^2*b)^(1/3)/b*(4*I*(x*(a*x^3
+b))^(1/2)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a
*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))
/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/
(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*x^2-8*I*(x*(a*x^3+b))^(1/2)*(-a^2*b)
^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(
-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*
3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3
^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*a*x+4*I*(x*(a*x^3+b))^(1/2)*(-a^2*b)^(2/3)*3
^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1
)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))-4*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.24

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5} \, dx = \frac{2 \left(b x^2 \sqrt{\frac{a x^3+b}{x^3}} - 2 \, (a x^3+b) \sqrt{b} \mathrm{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, \frac{1}{x}\right)\right)}{3 \, (a b^2 x^3 + b^3)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
2/3*(b*x^2*sqrt((a*x^3 + b)/x^3) - 2*(a*x^3 + b)*sqrt(b)*weierstrassPInver
se(0, -4*a/b, 1/x))/(a*b^2*x^3 + b^3)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5} \, dx = -\frac{\Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{4}{3}, \frac{3}{2} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b e^{i\pi}}{a x^3}} \right)}{3 a^{\frac{3}{2}} x^4 \Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**5,x)
```

output
```
-gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3
/2)*x**4*gamma(7/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5} \, dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^5} \, dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^5), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5} \, dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^5} \, dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5} \, dx = \int \frac{1}{x^5 \left(a+\frac{b}{x^3}\right)^{3/2}} \, dx$$

input
```
int(1/(x^5*(a + b/x^3)^(3/2)),x)
```

output
```
int(1/(x^5*(a + b/x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^5}\, dx = \int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^7+2ab\,x^4+b^2x} dx$$

input
```
int(1/(a+b/x^3)^(3/2)/x^5,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**7 + 2*a*b*x**4 + b**2*x),x)
```

3.512 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 267

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8} dx = \frac{2}{3b\sqrt{a+\frac{b}{x^3}}x^4} - \frac{16\sqrt{a+\frac{b}{x^3}}}{15b^2x} + \frac{32\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output

```
2/3/b/(a+b/x^3)^(1/2)/x^4-16/15*(a+b/x^3)^(1/2)/b^2/x+32/45*(1/2*6^(1/2)+1
/2*2^(1/2))*a*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)
/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b
^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^(7/3)/(
a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x
)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.20

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8}\,dx = -\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{3}{2},\frac{1}{6},-\frac{ax^3}{b}\right)}{5b\sqrt{a+\frac{b}{x^3}}x^4}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^8),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-5/6, 3/2, 1/6, -((a*x^3)/b)])/(
5*b*Sqrt[a + b/x^3]*x^4)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 817, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^6}d\frac{1}{x}$$

$$\Big\downarrow \text{817}$$

$$\frac{2}{3bx^4\sqrt{a+\frac{b}{x^3}}}-\frac{8\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^3}d\frac{1}{x}}{3b}$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{3bx^4\sqrt{a+\frac{b}{x^3}}}-\frac{8\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{2a\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{5b}\right)}{3b}$$

$\Big\downarrow$ 759

$$\frac{2}{3bx^4\sqrt{a+\frac{b}{x^3}}}-\frac{8\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{5bx}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{3b}$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^8),x]
```

output
```
2/(3*b*Sqrt[a + b/x^3]*x^4) - (8*((2*Sqrt[a + b/x^3])/(5*b*x) - (4*Sqrt[2
+ Sqrt[3]]*a*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*
b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4
*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(3*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1430 vs. $2(204) = 408$.

Time = 3.39 (sec) , antiderivative size = 1431, normalized size of antiderivative = 5.36

method	result	size
risch	Expression too large to display	1431
default	Expression too large to display	2056

input
```
int(1/(a+b/x^3)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/5/b^2*(a*x^3+b)/x^4/((a*x^3+b)/x^3)^(1/2)-1/5/b^2*a*(4*(1/2/a*(-a^2*b)^
(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1
/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*
b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/
a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(
-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3
)))^(1/2)/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b
)^(1/3)/(a*x*(x-1/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))
^(1/2)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x
/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1
/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*
(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3)))^(1/2))+5*b*(2/3*x/b/((x^3+b/a)*a*x)^(1/2)+4/3/b*(1/2/a*(-a^2*b)^(1
/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/
a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.28

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8}\,dx = \frac{2\left(16\,(a^2x^4+abx)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},\frac{1}{x}\right)-(8\,abx^3+3\,b^2)\sqrt{\frac{ax^3+b}{x^3}}\right)}{15\,(ab^3x^4+b^4x)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
2/15*(16*(a^2*x^4 + a*b*x)*sqrt(b)*weierstrassPInverse(0, -4*a/b, 1/x) - (
8*a*b*x^3 + 3*b^2)*sqrt((a*x^3 + b)/x^3))/(a*b^3*x^4 + b^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.15

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8}\, dx = -\frac{\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}} x^7 \Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**8,x)
```

output
```
-gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(
3/2)*x**7*gamma(10/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8}\, dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^8}\, dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^8}\, dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^8}\, dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^8}\,dx=\int \frac{1}{x^8\,\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^8*(a + b/x^3)^(3/2)),x)
```

output
```
int(1/(x^8*(a + b/x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^8}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^{10}+2ab\,x^7+b^2x^4}dx$$

input
```
int(1/(a+b/x^3)^(3/2)/x^8,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**10 + 2*a*b*x**7 + b**2*x**4),x)
```

3.513 $\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 587

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{935b^{7/3}\sqrt{a+\frac{b}{x^3}}}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{935b^2\sqrt{a+\frac{b}{x^3}}\,x}{336a^4} - \frac{187b\sqrt{a+\frac{b}{x^3}}\,x^4}{168a^3} - \frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{17\sqrt{a+\frac{b}{x^3}}\,x^7}{21a^2}$$
$$+\frac{935\sqrt{2-\sqrt{3}}\,b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{224\ 3^{3/4}a^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{935b^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output

```
-935/336*b^(7/3)*(a+b/x^3)^(1/2)/a^4/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+935/3
36*b^2*(a+b/x^3)^(1/2)*x/a^4-187/168*b*(a+b/x^3)^(1/2)*x^4/a^3-2/3*x^7/a/(
a+b/x^3)^(1/2)+17/21*(a+b/x^3)^(1/2)*x^7/a^2+935/672*(1/2*6^(1/2)-1/2*2^(1
/2))*b^(7/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^
(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(1/4)/a^(11/3)/(
a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x
)^2)^(1/2)-935/1008*b^(7/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1
/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1
/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*2^(
1/2)*3^(3/4)/a^(11/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{x\left(187b^2-34abx^3+16a^2x^6-187b^2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{ax^3}{b}\right)\right)}{112a^3\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[x^6/(a + b/x^3)^(3/2),x]
```

output
```
(x*(187*b^2 - 34*a*b*x^3 + 16*a^2*x^6 - 187*b^2*Sqrt[1 + (a*x^3)/b]*Hyperg
eometric2F1[5/6, 3/2, 11/6, -((a*x^3)/b)]))/(112*a^3*Sqrt[a + b/x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {858, 819, 847, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^8}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$-\frac{17\int \frac{x^8}{\sqrt{a+\frac{b}{x^3}}}\,d\frac{1}{x}}{3a}-\frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{17\left(-\frac{11b\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{14a}-\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}\right)}{3a}-\frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 847

$$-\frac{17\left(-\frac{11b\left(-\frac{5b\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{14a}-\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}\right)}{3a}-\frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 847

$$-\frac{17\left(-\frac{11b\left(-\frac{5b\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{14a}-\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}\right)}{3a}-\frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 832

$$-\frac{17\left(-\frac{11b\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{14a}-\frac{x^7\sqrt{a+\frac{b}{x^3}}}{7a}\right)}{3a}-\frac{2x^7}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\downarrow$ 759

$$17\left(-\frac{11b\left(-\frac{5b\left(b\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}\,d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots}\right.\right.\cdots}{\sqrt[4]{3}\,b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\cdots}\right.\right.\cdots}{2a}\cdots\right.\cdots}{8a}\cdots\right.\cdots}{14a}\cdots\right.\cdots$$

↓ 2416

$$
\left(11b\left(-\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7\cdots\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\cdots\right)}{\sqrt[3]{b}}\cdots\right)}{2a\cdots}\cdots\right.\right.
$$

input
```
Int[x^6/(a + b/x^3)^(3/2),x]
```

output
```
(-2*x^7)/(3*a*Sqrt[a + b/x^3]) - (17*(-1/7*(Sqrt[a + b/x^3]*x^7)/a - (11*b
*(-1/4*(Sqrt[a + b/x^3]*x^4)/a - (5*b*(-((Sqrt[a + b/x^3]*x)/a) + (b*(((2*
Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^
2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqr
t[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2
/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(2*a))
)/(8*a)))/(14*a)))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2210 vs. $2(441) = 882$.

Time = 4.21 (sec) , antiderivative size = 2211, normalized size of antiderivative = 3.77

method	result	size
risch	Expression too large to display	2211
default	Expression too large to display	3182

input
```
int(x^6/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/56*x*(8*a*x^3-25*b)/a^3*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)+1/112/a^3*b^2*(2
37*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a
^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^
2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(
-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I
*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/
3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b
)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-
1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^
2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a
/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)
^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-
a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)
)*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*
(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-
1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1...
```

Fricas [F]

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^12*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{x^7\Gamma\left(-\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{3}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right)}$$

input
```
integrate(x**6/(a+b/x**3)**(3/2),x)
```

output
```
-x**7*gamma(-7/3)*hyper((-7/3, 3/2), (-4/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*a**(3/2)*gamma(-4/3))
```

Maxima [F]

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(a + b/x^3)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(a + b/x^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(x^6/(a + b/x^3)^(3/2),x)
```

output
```
int(x^6/(a + b/x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{32\sqrt{x}\,\sqrt{a\,x^3+b}\,a^2x^8-68\sqrt{x}\,\sqrt{a\,x^3+b}\,ab\,x^5+374\sqrt{x}\,\sqrt{a\,x^3+b}\,b^2x^2-935\left(\int \frac{\sqrt{x}\,\sqrt{a}}{a^2x^6+2}\right.}{224a^3\left(a\,x^3+b\right)}$$

input
```
int(x^6/(a+b/x^3)^(3/2),x)
```

output
```
(32*sqrt(x)*sqrt(a*x**3 + b)*a**2*x**8 - 68*sqrt(x)*sqrt(a*x**3 + b)*a*b*x
**5 + 374*sqrt(x)*sqrt(a*x**3 + b)*b**2*x**2 - 935*int((sqrt(x)*sqrt(a*x**
3 + b)*x)/(a**2*x**6 + 2*a*b*x**3 + b**2),x)*a*b**3*x**3 - 935*int((sqrt(x
)*sqrt(a*x**3 + b)*x)/(a**2*x**6 + 2*a*b*x**3 + b**2),x)*b**4)/(224*a**3*(
a*x**3 + b))
```

3.514 $\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 563

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \frac{55b^{4/3}\sqrt{a+\frac{b}{x^3}}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{55b\sqrt{a+\frac{b}{x^3}}x}{24a^3} - \frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{11\sqrt{a+\frac{b}{x^3}}x^4}{12a^2} - \frac{55\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{16\ 3^{3/4}a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} + \frac{55b^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output

```
55/24*b^(4/3)*(a+b/x^3)^(1/2)/a^3/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-55/24*b*
(a+b/x^3)^(1/2)*x/a^3-2/3*x^4/a/(a+b/x^3)^(1/2)+11/12*(a+b/x^3)^(1/2)*x^4/
a^2-55/48*(1/2*6^(1/2)-1/2*2^(1/2))*b^(4/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+
b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*El
lipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*
3^(1/2)+2*I)*3^(1/4)/a^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)+55/72*b^(4/3)*(a^(1/3)+b^(1/3)/x)
*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)/x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.12

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{x\left(-11b+2ax^3+11b\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{ax^3}{b}\right)\right)}{8a^2\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[x^3/(a + b/x^3)^(3/2),x]
```

output
```
(x*(-11*b + 2*a*x^3 + 11*b*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[5/6, 3/2,
 11/6, -((a*x^3)/b)]))/(8*a^2*Sqrt[a + b/x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {858, 819, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{x^5}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{11\int \frac{x^5}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{3a}-\frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{11\left(-\frac{5b\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{3a}-\frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 847

$$-\frac{11\left(-\frac{5b\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{3a}-\frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 832

$$-\frac{11\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{8a}-\frac{x^4\sqrt{a+\frac{b}{x^3}}}{4a}\right)}{3a}-\frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 759

$$-\frac{11\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{2a}\right.}{8a}\right.}{3a}$$

$$\frac{2x^4}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\downarrow$ 2416

$$11 \left(-\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{\sqrt[3]{b}}}{2a}}{8a}\right.$$

input
```
Int[x^3/(a + b/x^3)^(3/2),x]
```

output
```
(-2*x^4)/(3*a*Sqrt[a + b/x^3]) - (11*(-1/4*(Sqrt[a + b/x^3]*x^4)/a - (5*b*
(-((Sqrt[a + b/x^3]*x)/a) + (b*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b
^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[
a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/
3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)^2])))/(2*a)))/(8*a)))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2198 vs. $2(421) = 842$.

Time = 3.14 (sec) , antiderivative size = 2199, normalized size of antiderivative = 3.91

method	result	size
risch	Expression too large to display	2199
default	Expression too large to display	2936

input
```
int(x^3/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a^2*(a*x^3+b)/((a*x^3+b)/x^3)^(1/2)-1/8/a^2*b*(13*(x*(x+1/2/a*(-a^2*
b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x
-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3
^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^
(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)
/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3
/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*Ellip
ticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a
^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),
((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/
3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(
-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2
))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(...
```

Fricas [F]

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^9*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{x^4\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(x**3/(a+b/x**3)**(3/2),x)
```

output
```
-x**4*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*exp_polar(I*pi)/(a*x**3))/
(3*a**(3/2)*gamma(-1/3))
```

Maxima [F]

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(a + b/x^3)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(a + b/x^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + b/x^3)^(3/2),x)
```

output
```
int(x^3/(a + b/x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{4\sqrt{x}\,\sqrt{a\,x^3+b}\,a\,x^5-22\sqrt{x}\,\sqrt{a\,x^3+b}\,b\,x^2+55\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a^2x^6+2ab\,x^3+b^2}dx\right)a\,b^2x^3+55\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a^2x^6+2ab\,x^3+b^2}dx\right)b^3}{16a^2\left(a\,x^3+b\right)}$$

input
```
int(x^3/(a+b/x^3)^(3/2),x)
```

output
```
(4*sqrt(x)*sqrt(a*x**3 + b)*a*x**5 - 22*sqrt(x)*sqrt(a*x**3 + b)*b*x**2 +
55*int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a**2*x**6 + 2*a*b*x**3 + b**2),x)*a*b
**2*x**3 + 55*int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a**2*x**6 + 2*a*b*x**3 + b
**2),x)*b**3)/(16*a**2*(a*x**3 + b))
```

3.515 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 539

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}} dx = -\frac{5\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2x}{3a\sqrt{a+\frac{b}{x^3}}} + \frac{5\sqrt{a+\frac{b}{x^3}}\,x}{3a^2}$$
$$+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{2\,3^{3/4}a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{5\sqrt{2}\sqrt[3]{b}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-5/3*b^(1/3)*(a+b/x^3)^(1/2)/a^2/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/3*x/a/(
a+b/x^3)^(1/2)+5/3*(a+b/x^3)^(1/2)*x/a^2+5/6*(1/2*6^(1/2)-1/2*2^(1/2))*b^(
1/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)
/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(1/4)/a^(5/3)/(a+b/x^3)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2
)-5/9*2^(1/2)*b^(1/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^
(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a
^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a
^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{x-x\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{ax^3}{b}\right)}{a\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[(a + b/x^3)^(-3/2),x]
```

output
```
(x - x*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[5/6, 3/2, 11/6, -((a*x^3)/b)]
)/(a*Sqrt[a + b/x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.545$, Rules used = {773, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$\Big\downarrow$ 773

$$-\int \frac{x^2}{\left(a+\frac{b}{x^3}\right)^{3/2}}d\frac{1}{x}$$

$\Big\downarrow$ 819

$$-\frac{5\int \frac{x^2}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{3a}-\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 847

$$-\frac{5\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{3a}-\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 832

$$-\frac{5\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{2a}-\frac{x\sqrt{a+\frac{b}{x^3}}}{a}\right)}{3a}-\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 759

$$-\frac{5\left(\frac{b\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{2a}\right.}{3a}$$

$$\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}}$$

$\downarrow$ 2416

$$-\frac{5\left(\frac{b\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{\sqrt[3]{b}}-\frac{2\left(1-\cdots\right.}{\cdots}}{2a}\cdots\right)}{3a}$$

$$\frac{2x}{3a\sqrt{a+\frac{b}{x^3}}}$$

input
```
Int[(a + b/x^3)^(-3/2),x]
```

output
```
(-2*x)/(3*a*Sqrt[a + b/x^3]) - (5*(-((Sqrt[a + b/x^3]*x)/a) + (b*(((2*Sqrt
[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt
[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 -
(a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)
], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3]
)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/
x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(2*a)))/(3
*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2699 vs. $2(401) = 802$.

Time = 1.55 (sec) , antiderivative size = 2700, normalized size of antiderivative = 5.01

method	result	size
default	Expression too large to display	2700

input
```
int(1/(a+b/x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^3*(5*I*(x*(a*x^3+b))^(1/2)*(-(I
*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-
a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a
^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*3^(1/2)*a*b-5*I*(x*(a*x^3+b))^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1
/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2
*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)
^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*El
lipticE((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/
3)*3^(1/2)*a*x^2-10*(x*(a*x^3+b))^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/
(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/
3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-
2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*a*x^
2+15*(x*(a*x^3+b))^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(...
```

Fricas [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^6*sqrt((a*x^3 + b)/x^3)/(a^2*x^6 + 2*a*b*x^3 + b^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.08

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = -\frac{x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2),x)
```

output
```
-x*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a
**(3/2)*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^3)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.07

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{2\,x\,{\left(\frac{a\,x^3}{b}+1\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(\frac{3}{2},\frac{11}{6};\ \frac{17}{6};\ -\frac{a\,x^3}{b}\right)}{11\,{\left(a+\frac{b}{x^3}\right)}^{3/2}}$$

input
```
int(1/(a + b/x^3)^(3/2),x)
```

output
```
(2*x*((a*x^3)/b + 1)^(3/2)*hypergeom([3/2, 11/6], 17/6, -(a*x^3)/b))/(11*(
a + b/x^3)^(3/2))
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx=\frac{2\sqrt{x}\,\sqrt{a\,x^{3}+b}\,x^{2}-5\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^{3}+b}\,x}{a^{2}x^{6}+2ab\,x^{3}+b^{2}}dx\right)ab\,x^{3}-5\left(\int \frac{\sqrt{x}\,\sqrt{a\,x^{3}+b}\,x}{a^{2}x^{6}+2ab\,x^{3}+b^{2}}dx\right)b^{2}}{2a\left(a\,x^{3}+b\right)}$$

input
```
int(1/(a+b/x^3)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(a*x**3 + b)*x**2 - 5*int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a**
2*x**6 + 2*a*b*x**3 + b**2),x)*a*b*x**3 - 5*int((sqrt(x)*sqrt(a*x**3 + b)*
x)/(a**2*x**6 + 2*a*b*x**3 + b**2),x)*b**2)/(2*a*(a*x**3 + b))
```

3.516 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 520

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,dx = \frac{2\sqrt{a+\frac{b}{x^3}}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} - \frac{2}{3a\sqrt{a+\frac{b}{x^3}}\,x^2} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)\middle|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} + \frac{2\sqrt{2}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
2/3*(a+b/x^3)^(1/2)/a/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)-2/3/a/(a+b/x
^3)^(1/2)/x^2-1/3*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+
b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*El
lipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*
3^(1/2)+2*I)*3^(1/4)/a^(2/3)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)+2/9*2^(1/2)*(a^(1/3)+b^(1
/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/
3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/b^(2/3)/(a+b/x^3)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.10

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3} \, dx = \frac{2x\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{ax^3}{b}\right)}{5b\sqrt{a+\frac{b}{x^3}}}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^3),x]
```

output
```
(2*x*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[5/6, 3/2, 11/6, -((a*x^3)/b)])/
(5*b*Sqrt[a + b/x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

$\Big\downarrow$ 858

$$-\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x}d\frac{1}{x}$$

$\Big\downarrow$ 819

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{3a}-\frac{2}{3ax^2\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 832

$$\frac{\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}}{3a}-\frac{2}{3ax^2\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 759

$$\frac{\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{3a}$$

$$\frac{2}{3ax^2\sqrt{a+\frac{b}{x^3}}}$$

$\Big\downarrow$ 2416

$$\frac{\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{\cdots}}{\cdots}}{3a}$$

$$\frac{2}{3ax^2\sqrt{a+\frac{b}{x^3}}}$$

input

```
Int[1/((a + b/x^3)^(3/2)*x^3),x]
```

output

```
-2/(3*a*Sqrt[a + b/x^3]*x^2) + (((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b
^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[
a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/
3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)/x)^2]))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2702 vs. $2(388) = 776$.

Time = 0.53 (sec) , antiderivative size = 2703, normalized size of antiderivative = 5.20

method	result	size
default	Expression too large to display	2703

input
```
int(1/(a+b/x^3)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/((a*x^3+b)/x^3)^(3/2)/x^5*(a*x^3+b)/a^2*(-2*I*(x*(a*x^3+b))^(1/2)*(-a^
2*b)^(1/3)*3^(1/2)*a*x^2+2*I*(x*(a*x^3+b))^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^
(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a
^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*
b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*
EllipticE((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*
b-4*(x*(a*x^3+b))^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2)
)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(
1/3))/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*a*x^2+6*(x*(a*x^3+b)
)^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a^2*b)^(1/3)+2*a*x+(-a^2*b)^(1/3))/(1+I*3^(1/2))/(-a*x+(-a^2*b)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a^2*b)^(1/3)-2*a*x-(-a^2*b)^(1/3))/(I*3^(1/2)
-1)/(-a*x+(-a^2*b)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-
1)/(-a*x+(-a^2*b)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*(-a^2*b)^(1/3)*a*x^2-2*I*(x*(a*x^3+b))^(1/2)*3^(1/2
)*a^2*x^3+8*(x*(a*x^3+b))^(1/2)*(-(I*3^(1/2)-3)*x*a/(I*3^(1/2)-1)/(-a*x...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.12

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3} \, dx = -\frac{2 \left(bx \sqrt{\frac{ax^3+b}{x^3}} + (ax^3+b)\sqrt{b}\text{weierstrassZeta}\left(0, -\frac{4a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4a}{b}, \frac{1}{x}\right)\right)\right)}{3\left(a^2bx^3 + ab^2\right)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-2/3*(b*x*sqrt((a*x^3 + b)/x^3) + (a*x^3 + b)*sqrt(b)*weierstrassZeta(0, -
4*a/b, weierstrassPInverse(0, -4*a/b, 1/x)))/(a^2*b*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3} \, dx = -\frac{\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{3} \end{matrix}\right| \frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}} x^2 \Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**3,x)
```

output
```
-gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3
/2)*x**2*gamma(5/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\,dx = \int \frac{1}{x^3 \left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b/x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b/x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^3}\, dx = \int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}\,x}{a^2x^6+2ab\,x^3+b^2}dx$$

input
```
int(1/(a+b/x^3)^(3/2)/x^3,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b)*x)/(a**2*x**6 + 2*a*b*x**3 + b**2),x)
```

3.517 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 517

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6} \, dx = -\frac{8\sqrt{a+\frac{b}{x^3}}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}+\frac{2}{3b\sqrt{a+\frac{b}{x^3}}\,x^2}$$
$$+\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$-\frac{8\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
-8/3*(a+b/x^3)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+2/3/b/(a+b/x^
3)^(1/2)/x^2+4/3*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)/x)*((a
^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(
1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)/x),I*3^(1/2)+2*I)*3^(1/4)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1
/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)-8/9*2^(1/2)*a^(1/3)*(a^(1/
3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2
))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^(5/3)/(a+b/x^3)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.10

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6} \, dx = -\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{3}{2},\frac{5}{6},-\frac{ax^3}{b}\right)}{b\sqrt{a+\frac{b}{x^3}}x^2}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^6),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-1/6, 3/2, 5/6, -((a*x^3)/b)])/(
b*Sqrt[a + b/x^3]*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{817}$$

$$\frac{2}{3bx^2\sqrt{a+\frac{b}{x^3}}}-\frac{4\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{3b}$$

$$\Big\downarrow \text{832}$$

$$\frac{2}{3bx^2\sqrt{a+\frac{b}{x^3}}}-\frac{4\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{3b}$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{3bx^2\sqrt{a+\frac{b}{x^3}}}-\frac{4\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-\cdots}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\cdots\right)}{3b}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2}{3bx^2\sqrt{a+\frac{b}{x^3}}}-\frac{4\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right.}{}\right.}{3b}$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^6),x]
```

output
```
2/(3*b*Sqrt[a + b/x^3]*x^2) - (4*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) +
 b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqr
t[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(
1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)/x)^2])))/(3*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2199 vs. 2(385) = 770.

Time = 3.06 (sec) , antiderivative size = 2200, normalized size of antiderivative = 4.26

method	result	size
risch	Expression too large to display	2200
default	Expression too large to display	2867

input
```
int(1/(a+b/x^3)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*(a*x^3+b)/x^2/((a*x^3+b)/x^3)^(1/2)+1/b^2*a*(2*(x*(x+1/2/a*(-a^2*b)
^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/
2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))
*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b
)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1
/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1
/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2
/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*Ellipti
cF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((
3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)
-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a
^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))
+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*
(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/
2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.12

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx = \frac{2\left(bx\sqrt{\frac{ax^3+b}{x^3}} + 4\,(ax^3+b)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},\right.\right.\right.}{3\,(ab^2x^3+b^3)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
2/3*(b*x*sqrt((a*x^3 + b)/x^3) + 4*(a*x^3 + b)*sqrt(b)*weierstrassZeta(0,
-4*a/b, weierstrassPInverse(0, -4*a/b, 1/x)))/(a*b^2*x^3 + b^3)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx = -\frac{\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**6,x)
```

output
```
-gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(3
/2)*x**5*gamma(8/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^6}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}} x^6}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx = \int \frac{1}{x^6 \left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^6*(a + b/x^3)^(3/2)),x)
```

output
```
int(1/(x^6*(a + b/x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^6}\,dx = \int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^8+2ab\,x^5+b^2x^2}dx$$

input
```
int(1/(a+b/x^3)^(3/2)/x^6,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**8 + 2*a*b*x**5 + b**2*x**2),x)
```

3.518 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9} dx$

Optimal result

Integrand size = 15, antiderivative size = 541

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9} dx = \frac{80a\sqrt{a+\frac{b}{x^3}}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{2}{3b\sqrt{a+\frac{b}{x^3}}x^5} - \frac{20\sqrt{a+\frac{b}{x^3}}}{21b^2x^2} - \frac{40\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{7\,3^{3/4}b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}} + \frac{80\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}b^{8/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output
```
80/21*a*(a+b/x^3)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+2/3/b/(a+b
/x^3)^(1/2)/x^5-20/21*(a+b/x^3)^(1/2)/b^2/x^2-40/21*(1/2*6^(1/2)-1/2*2^(1/
2))*a^(4/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(
1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(1/4)/b^(8/3)/(a+
b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^
2)^(1/2)+80/63*2^(1/2)*a^(4/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a
^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3
^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*
3^(3/4)/b^(8/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)/x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.10

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9} \, dx = -\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{3}{2},-\frac{1}{6},-\frac{ax^3}{b}\right)}{7b\sqrt{a+\frac{b}{x^3}}x^5}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^9),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-7/6, 3/2, -1/6, -((a*x^3)/b)])/
(7*b*Sqrt[a + b/x^3]*x^5)
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 817, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 858$$

$$-\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^7}d\frac{1}{x}$$

$$\Big\downarrow \; 817$$

$$\frac{2}{3bx^5\sqrt{a+\frac{b}{x^3}}}-\frac{10\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}d\frac{1}{x}}{3b}$$

$$\Big\downarrow \; 843$$

$$\frac{2}{3bx^5\sqrt{a+\frac{b}{x^3}}}-\frac{10\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{7b}\right)}{3b}$$

$$\Big\downarrow \; 832$$

$$\frac{2}{3bx^5\sqrt{a+\frac{b}{x^3}}}-\frac{10\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{7b}\right)}{3b}$$

$$\Big\downarrow \; 759$$

$$\frac{2}{3bx^5\sqrt{a+\frac{b}{x^3}}}-$$

$$\frac{10\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{7b}\right.}{3b}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2}{3bx^5\sqrt{a+\frac{b}{x^3}}}-\frac{10\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7\cdots\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\cdots}}{\sqrt[3]{b}}\cdots\right)}{7b\cdots}\right)}{3b\cdots}$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^9),x]
```

output
```
2/(3*b*Sqrt[a + b/x^3]*x^5) - (10*((2*Sqrt[a + b/x^3])/(7*b*x^2) - (4*a*((
(2*Sqrt[a + b/x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)
/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 -
Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b
^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(7*
b)))/(3*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2212 vs. $2(403) = 806$.

Time = 3.99 (sec) , antiderivative size = 2213, normalized size of antiderivative = 4.09

method	result	size
risch	Expression too large to display	2213
default	Expression too large to display	3307

input
```
int(1/(a+b/x^3)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
2/7*(a*x^3+b)*(11*a*x^3-b)/b^3/x^5/((a*x^3+b)/x^3)^(1/2)-1/7/b^3*a^2*(22*(
x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(x+1/2/a*(-a^2*b
)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)
)*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)
^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2
*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(
1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(1/a*(-a^2*b)^(1/3)*(
x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-a^2*b)^(1
/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(((-1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(-a^2*b)^(1/3)+1/a^2*(-
a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*a/(-a
^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/
3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*
b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*(1
/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(1/2/a*(-a^2*b)^(1/3)+
1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^
2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*
EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/
a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3))...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.16

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^9}\,dx =$$

$$-\frac{2\left(40\left(a^2x^5+abx^2\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},\frac{1}{x}\right)\right)+\left(10abx^3+3b^2\right)\sqrt{\frac{ax^3+b}{x^3}}\right)}{21\left(ab^3x^5+b^4x^2\right)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-2/21*(40*(a^2*x^5 + a*b*x^2)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstra
ssPInverse(0, -4*a/b, 1/x)) + (10*a*b*x^3 + 3*b^2)*sqrt((a*x^3 + b)/x^3))/
(a*b^3*x^5 + b^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^9}\,dx = -\frac{\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^8\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**9,x)
```

output
```
-gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a**(
3/2)*x**8*gamma(11/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^9}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^9}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^9), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^9}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^9}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^9,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9}\, dx = \int \frac{1}{x^9 \left(a+\frac{b}{x^3}\right)^{3/2}}\, dx$$

input
```
int(1/(x^9*(a + b/x^3)^(3/2)),x)
```

output
```
int(1/(x^9*(a + b/x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^9}\, dx = \int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^{11}+2ab\,x^8+b^2x^5}dx$$

input
```
int(1/(a+b/x^3)^(3/2)/x^9,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**11 + 2*a*b*x**8 + b**2*x**5),x)
```

3.519 $\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 565

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx = -\frac{1280a^2\sqrt{a+\frac{b}{x^3}}}{273b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)} + \frac{2}{3b\sqrt{a+\frac{b}{x^3}}x^8} - \frac{32\sqrt{a+\frac{b}{x^3}}}{39b^2x^5} + \frac{320a\sqrt{a+\frac{b}{x^3}}}{273b^3x^2}$$
$$+ \frac{640\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right)|-7-4\sqrt{3}\right)}{91\ 3^{3/4}b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$
$$- \frac{1280\sqrt{2}a^{7/3}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}+\frac{b^{2/3}}{x^2}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}\right),-7-4\sqrt{3}\right)}{273\sqrt[4]{3}b^{11/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}$$

output

```
-1280/273*a^2*(a+b/x^3)^(1/2)/b^(11/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)+2/3
/b/(a+b/x^3)^(1/2)/x^8-32/39*(a+b/x^3)^(1/2)/b^2/x^5+320/273*a*(a+b/x^3)^(
1/2)/b^3/x^2+640/273*(1/2*6^(1/2)-1/2*2^(1/2))*a^(7/3)*(a^(1/3)+b^(1/3)/x)
*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^
2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)/x),I*3^(1/2)+2*I)*3^(1/4)/b^(11/3)/(a+b/x^3)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)-1280/819*2^(1/2)*a^(7
/3)*(a^(1/3)+b^(1/3)/x)*((a^(2/3)+b^(2/3)/x^2-a^(1/3)*b^(1/3)/x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)/x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)/x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)/x),I*3^(1/2)+2*I)*3^(3/4)/b^(11/3)/(a+b/x^3)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)/x)/((1+3^(1/2))*a^(1/3)+b^(1/3)/x)^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.10

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx=-\frac{2\sqrt{1+\frac{ax^3}{b}}\,\text{Hypergeometric2F1}\left(-\frac{13}{6},\frac{3}{2},-\frac{7}{6},-\frac{ax^3}{b}\right)}{13b\sqrt{a+\frac{b}{x^3}}\,x^8}$$

input
```
Integrate[1/((a + b/x^3)^(3/2)*x^12),x]
```

output
```
(-2*Sqrt[1 + (a*x^3)/b]*Hypergeometric2F1[-13/6, 3/2, -7/6, -((a*x^3)/b)])
/(13*b*Sqrt[a + b/x^3]*x^8)
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {858, 817, 843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12}\left(a+\frac{b}{x^3}\right)^{3/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{10}}d\frac{1}{x}$$

$$\Big\downarrow \text{817}$$

$$\frac{2}{3bx^8\sqrt{a+\frac{b}{x^3}}}-\frac{16\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7}d\frac{1}{x}}{3b}$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{3bx^8\sqrt{a+\frac{b}{x^3}}}-\frac{16\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\int\frac{1}{\sqrt{a+\frac{b}{x^3}}x^4}d\frac{1}{x}}{13b}\right)}{3b}$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{3bx^8\sqrt{a+\frac{b}{x^3}}}-\frac{16\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\int\frac{1}{\sqrt{a+\frac{b}{x^3}}x}d\frac{1}{x}}{7b}\right)}{13b}\right)}{3b}$$

$$\Big\downarrow \text{832}$$

$$\frac{2}{3bx^8\sqrt{a+\frac{b}{x^3}}}-\frac{16\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}\right)}{3b}$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{3bx^8\sqrt{a+\frac{b}{x^3}}}-\frac{16\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}}{\sqrt{a+\frac{b}{x^3}}}d\frac{1}{x}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\ldots\right)\right)}{\sqrt[4]{3}b^{2/3}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right)}{7b}\right)}{13b}\right)}{3b}$$

$\downarrow$ 2416

$$\frac{2}{3bx^8\sqrt{a+\frac{b}{x^3}}}-$$

$$16\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{13bx^5}-\frac{10a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{7bx^2}-\frac{4a\left(\frac{2\sqrt{a+\frac{b}{x^3}}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)\sqrt{\frac{a^{2/3}-\frac{\sqrt[3]{a}\sqrt[3]{b}}{x}+\frac{b^{2/3}}{x^2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{a+\frac{b}{x^3}}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\frac{\sqrt[3]{b}}{x}\right)^2}}}\right.}{\sqrt[3]{b}}\right.}{}\right.$$

input
```
Int[1/((a + b/x^3)^(3/2)*x^12),x]
```

output
```
2/(3*b*Sqrt[a + b/x^3]*x^8) - (16*((2*Sqrt[a + b/x^3])/(13*b*x^5) - (10*a*
((2*Sqrt[a + b/x^3])/(7*b*x^2) - (4*a*(((2*Sqrt[a + b/x^3])/(b^(1/3)*((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1
/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(b^(1/3
)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)/x)^2]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)
*(a^(1/3) + b^(1/3)/x)*Sqrt[(a^(2/3) + b^(2/3)/x^2 - (a^(1/3)*b^(1/3))/x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)/x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)/x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(2/3)*Sqrt[a + b/x^3]*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)/x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)/x)^2])))/(7*b)))/(13*b)))/(3*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2223 vs. $2(423) = 846$.

Time = 5.18 (sec) , antiderivative size = 2224, normalized size of antiderivative = 3.94

method	result	size
risch	Expression too large to display	2224
default	Expression too large to display	3554

input
```
int(1/(a+b/x^3)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-2/91*(a*x^3+b)*(183*a^2*x^6-23*a*b*x^3+7*b^2)/b^4/x^8/((a*x^3+b)/x^3)^(1/
2)+1/91*a^3/b^4*(366*(x*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(
1/3))*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))+(1/2/a*(-a^2
*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))*((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^
(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^
(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)*(x-1/a*(-a^2*b)^(1/3))^2*(1/a*(-a^2*b
)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(-1/2/a*(-
a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2)
*(1/a*(-a^2*b)^(1/3)*(x+1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3
))/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(x-1/a*(-a^2*b)^
(1/3)))^(1/2)*(((-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/a*(
-a^2*b)^(1/3)+1/a^2*(-a^2*b)^(2/3))/(-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a
*(-a^2*b)^(1/3))*a/(-a^2*b)^(1/3)*EllipticF(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*
3^(1/2)/a*(-a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b
)^(1/3))/(x-1/a*(-a^2*b)^(1/3)))^(1/2),((3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2
)/a*(-a^2*b)^(1/3))*(1/2/a*(-a^2*b)^(1/3)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/
(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/(3/2/a*(-a^2*b)^(1/3
)-1/2*I*3^(1/2)/a*(-a^2*b)^(1/3)))^(1/2))+(1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1
/2)/a*(-a^2*b)^(1/3))*EllipticE(((-3/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-
a^2*b)^(1/3))*x/(-1/2/a*(-a^2*b)^(1/3)+1/2*I*3^(1/2)/a*(-a^2*b)^(1/3))/...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.18

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx = \frac{2\left(640\,(a^3x^8+a^2bx^5)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},\frac{1}{x}\right)\right)\right.}{273\,(ab^4x^8+b^5x^5)}$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
2/273*(640*(a^3*x^8 + a^2*b*x^5)*sqrt(b)*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, 1/x)) + (160*a^2*b*x^6 + 48*a*b^2*x^3 - 21*b^3)*s
qrt((a*x^3 + b)/x^3))/(a*b^4*x^8 + b^5*x^5)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^{12}}\,dx = -\frac{\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{be^{i\pi}}{ax^3}\right)}{3a^{\frac{3}{2}}x^{11}\Gamma\left(\frac{14}{3}\right)}$$

input
```
integrate(1/(a+b/x**3)**(3/2)/x**12,x)
```

output
```
-gamma(11/3)*hyper((3/2, 11/3), (14/3,), b*exp_polar(I*pi)/(a*x**3))/(3*a*
*(3/2)*x**11*gamma(14/3))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^{12}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^{12}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^12), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2}x^{12}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}x^{12}}\,dx$$

input
```
integrate(1/(a+b/x^3)^(3/2)/x^12,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^3)^(3/2)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx=\int \frac{1}{x^{12}\,\left(a+\frac{b}{x^3}\right)^{3/2}}\,dx$$

input
```
int(1/(x^12*(a + b/x^3)^(3/2)),x)
```

output
```
int(1/(x^12*(a + b/x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^3}\right)^{3/2} x^{12}}\,dx=\int \frac{\sqrt{x}\,\sqrt{a\,x^3+b}}{a^2x^{14}+2ab\,x^{11}+b^2x^8}dx$$

input
```
int(1/(a+b/x^3)^(3/2)/x^12,x)
```

output
```
int((sqrt(x)*sqrt(a*x**3 + b))/(a**2*x**14 + 2*a*b*x**11 + b**2*x**8),x)
```

3.520 $\int \frac{1}{a+\frac{b}{x^4}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 139

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx = \frac{x}{a}+\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{b}+\sqrt{a}x^2}\right)}{2\sqrt{2}a^{5/4}}$$

output
```
x/a-1/4*b^(1/4)*arctan(-1+2^(1/2)*a^(1/4)*x/b^(1/4))*2^(1/2)/a^(5/4)-1/4*b
^(1/4)*arctan(1+2^(1/2)*a^(1/4)*x/b^(1/4))*2^(1/2)/a^(5/4)-1/4*b^(1/4)*arc
tanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(b^(1/2)+a^(1/2)*x^2))*2^(1/2)/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx = \frac{8\sqrt[4]{a}x+2\sqrt{2}\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)+\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x\right.}{8a^{5/4}}$$

input
```
Integrate[(a + b/x^4)^(-1),x]
```

output
```
(8*a^(1/4)*x + 2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)] -
 2*Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/b^(1/4)] + Sqrt[2]*b^(1/
4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2] - Sqrt[2]*b^(1/4
)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2])/(8*a^(5/4))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.53, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {772, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{x^4}}dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{x^4}{ax^4+b}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{ax^4+b}dx}{a}$$

$$\Big\downarrow \text{755}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}+\frac{\int \frac{\sqrt{a}x^2+\sqrt{b}}{ax^4+b}dx}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{1476}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{a}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{a}}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \; 1082$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \; 217$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{\sqrt{b}-\sqrt{a}x^2}{ax^4+b}dx}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \; 1479$$

$$\frac{x}{a}-\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x}{\sqrt[4]{a}\left(x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x}{x^2-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{2}\sqrt{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x+\sqrt[4]{b}}{x^2+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx}{2\sqrt{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}x^2+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

input
```
Int[(a + b/x^4)^(-1),x]
```

output
```
x/a - (b*((-(ArcTan[1 - (Sqrt[2]*a^(1/4)*x)/b^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1
/4))) + ArcTan[1 + (Sqrt[2]*a^(1/4)*x)/b^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[a]*x^2
]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[a]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.24

method	result	size
risch	$\frac{x}{a}-\frac{b\left(\sum_{\_R=\operatorname{RootOf}\left(a\_Z^{4}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4a^{2}}$	34
default	$\frac{x}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{b}{a}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{b}{a}}}{x^{2}-\left(\frac{b}{a}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{b}{a}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	108

input
```
int(1/(a+b/x^4),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/4/a^2*b*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx =$$
$$-\frac{a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}+x\right)+i\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}+x\right)-i\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}+x\right)-}{4\,a}$$

input
```
integrate(1/(a+b/x^4),x, algorithm="fricas")
```

output
```
-1/4*(a*(-b/a^5)^(1/4)*log(a*(-b/a^5)^(1/4) + x) + I*a*(-b/a^5)^(1/4)*log(
I*a*(-b/a^5)^(1/4) + x) - I*a*(-b/a^5)^(1/4)*log(-I*a*(-b/a^5)^(1/4) + x)
- a*(-b/a^5)^(1/4)*log(-a*(-b/a^5)^(1/4) + x) - 4*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx = \operatorname{RootSum}\left(256t^4a^5+b,\left(t\mapsto t\log\left(-4ta+x\right)\right)\right)+\frac{x}{a}$$

input
```
integrate(1/(a+b/x**4),x)
```

output
```
RootSum(256*_t**4*a**5 + b, Lambda(_t, _t*log(-4*_t*a + x))) + x/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx = -\frac{\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{a}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{a}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{a}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{b}\right)}{a^{\frac{1}{4}}}-\frac{\sqrt{2}b^{\frac{1}{4}}\log}{}}{8\,a} + \frac{x}{a}$$

input
```
integrate(1/(a+b/x^4),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(2*sqrt(a)*x + sqrt(2)*a^(1/4)*
b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(b)*
arctan(1/2*sqrt(2)*(2*sqrt(a)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*b^(1/4)*log(sqrt(a)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(b))/a^(1/4) - sqrt(2)*b^(1/4)*log(sqrt(a)*x^2 - s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(b))/a^(1/4))/a + x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx = \frac{x}{a} - \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,a^2} - \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{2\left(\frac{b}{a}\right)^{\frac{1}{4}}}\right)}{4\,a^2} - \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{a}}\right)}{8\,a^2} + \frac{\sqrt{2}(a^3b)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{b}{a}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{a}}\right)}{8\,a^2}$$

input
```
integrate(1/(a+b/x^4),x, algorithm="giac")
```

output
```
x/a - 1/4*sqrt(2)*(a^3*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(b/a)^(1
/4))/(b/a)^(1/4))/a^2 - 1/4*sqrt(2)*(a^3*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(b/a)^(1/4))/(b/a)^(1/4))/a^2 - 1/8*sqrt(2)*(a^3*b)^(1/4)*log(x^
2 + sqrt(2)*x*(b/a)^(1/4) + sqrt(b/a))/a^2 + 1/8*sqrt(2)*(a^3*b)^(1/4)*log
(x^2 - sqrt(2)*x*(b/a)^(1/4) + sqrt(b/a))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx=\frac{x}{a}-\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{a^{1/4}\,x}{(-b)^{1/4}}\right)}{2\,a^{5/4}}-\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{a^{1/4}\,x}{(-b)^{1/4}}\right)}{2\,a^{5/4}}$$

input
```
int(1/(a + b/x^4),x)
```

output
```
x/a - ((-b)^(1/4)*atan((a^(1/4)*x)/(-b)^(1/4)))/(2*a^(5/4)) - ((-b)^(1/4)*
atanh((a^(1/4)*x)/(-b)^(1/4)))/(2*a^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{1}{a+\frac{b}{x^4}}\,dx$$
$$=\frac{2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{a}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{a}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}\,x^2+\right.}{8a^2}$$

input
```
int(1/(a+b/x^4),x)
```

output
```
(2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(a)*x
)/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(a)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + b**(1/4)*
a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)*x**2 + sqrt(
b)) - b**(1/4)*a**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)*
x**2 + sqrt(b)) + 8*a*x)/(8*a**2)
```

3.521 $\int \sqrt{a+\frac{b}{x^4}} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \sqrt{a+\frac{b}{x^4}} x^3 \, dx = \frac{1}{4}\sqrt{a+\frac{b}{x^4}} x^4 + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
1/4*(a+b/x^4)^(1/2)*x^4+1/4*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \sqrt{a+\frac{b}{x^4}} x^3 \, dx = \frac{1}{4}\sqrt{a+\frac{b}{x^4}} x^2 \left(x^2 + \frac{b\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{\sqrt{a}\sqrt{b+ax^4}}\right)$$

input
```
Integrate[Sqrt[a + b/x^4]*x^3,x]
```

output
```
(Sqrt[a + b/x^4]*x^2*(x^2 + (b*Log[Sqrt[a]*x^2 + Sqrt[b + a*x^4]])/(Sqrt[a
]*Sqrt[b + a*x^4])))/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{x^4}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \sqrt{a+\frac{b}{x^4}}x^8d\frac{1}{x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(x^4\sqrt{a+\frac{b}{x^4}}-\frac{1}{2}b\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(x^4\sqrt{a+\frac{b}{x^4}}-\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{\sqrt{a}}+x^4\sqrt{a+\frac{b}{x^4}}\right)$$

input
```
Int[Sqrt[a + b/x^4]*x^3,x]
```

output
```
(Sqrt[a + b/x^4]*x^4 + (b*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/Sqrt[a])/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}\left(x^{2}\sqrt{a\,x^{4}+b}\,\sqrt{a}+b\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)\right)}{4\sqrt{a\,x^{4}+b}\,\sqrt{a}}$	68
risch	$\frac{x^{4}\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{4}+\frac{b\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}{4\sqrt{a}\,\sqrt{a\,x^{4}+b}}$	69

input
```
int((a+b/x^4)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((a*x^4+b)/x^4)^(1/2)*x^2*(x^2*(a*x^4+b)^(1/2)*a^(1/2)+b*ln(a^(1/2)*x^
2+(a*x^4+b)^(1/2)))/(a*x^4+b)^(1/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.51

$$\int \sqrt{a+\frac{b}{x^4}}x^3\,dx = \left[\frac{2\,ax^4\sqrt{\frac{ax^4+b}{x^4}}+\sqrt{a}b\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)}{8\,a}, \frac{ax^4\sqrt{\frac{ax^4+b}{x^4}}-\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)}{4\,a}\right]$$

input
```
integrate((a+b/x^4)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
[1/8*(2*a*x^4*sqrt((a*x^4 + b)/x^4) + sqrt(a)*b*log(-2*a*x^4 - 2*sqrt(a)*x
^4*sqrt((a*x^4 + b)/x^4) - b))/a, 1/4*(a*x^4*sqrt((a*x^4 + b)/x^4) - sqrt(
-a)*b*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4)/a))/a]
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \sqrt{a+\frac{b}{x^4}}x^3\,dx = \frac{\sqrt{b}x^2\sqrt{\frac{ax^4}{b}+1}}{4}+\frac{b\,\text{asinh}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{4\sqrt{a}}$$

input
```
integrate((a+b/x**4)**(1/2)*x**3,x)
```

output
```
sqrt(b)*x**2*sqrt(a*x**4/b + 1)/4 + b*asinh(sqrt(a)*x**2/sqrt(b))/(4*sqrt(
a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \sqrt{a+\frac{b}{x^4}}x^3\,dx = \frac{1}{4}\sqrt{a+\frac{b}{x^4}}x^4 - \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8\sqrt{a}}$$

input
```
integrate((a+b/x^4)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
1/4*sqrt(a + b/x^4)*x^4 - 1/8*b*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a +
b/x^4) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \sqrt{a+\frac{b}{x^4}}x^3\,dx = \frac{1}{4}\sqrt{ax^4+b}x^2 - \frac{b\log\left(\left|-\sqrt{a}x^2+\sqrt{ax^4+b}\right|\right)}{4\sqrt{a}}$$

input
```
integrate((a+b/x^4)^(1/2)*x^3,x, algorithm="giac")
```

output
```
1/4*sqrt(a*x^4 + b)*x^2 - 1/4*b*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/s
qrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \sqrt{a+\frac{b}{x^4}}x^3\,dx = \frac{x^4\sqrt{a+\frac{b}{x^4}}}{4} + \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
int(x^3*(a + b/x^4)^(1/2),x)
```

output
```
(x^4*(a + b/x^4)^(1/2))/4 + (b*atanh((a + b/x^4)^(1/2)/a^(1/2)))/(4*a^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.57

$$\int \sqrt{a+\frac{b}{x^4}}x^3\,dx = \frac{2\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)b\,x^2+2\sqrt{a}\,\sqrt{a\,x^4+b}\,a\,x^6+\sqrt{a}\,\sqrt{a\,x^4+b}\,b\,x^2+2\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)}{8\sqrt{a\,x^4+b}\,a\,x^2+8\sqrt{a}\,a\,x^4+4\sqrt{a}\,b}$$

input
```
int((a+b/x^4)^(1/2)*x^3,x)
```

output
```
(2*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))
*b*x**2 + 2*sqrt(a)*sqrt(a*x**4 + b)*a*x**6 + sqrt(a)*sqrt(a*x**4 + b)*b*x
**2 + 2*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a*b*x**4 + log((sqr
t(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*b**2 + 2*a**2*x**8 + 2*a*b*x**4)/(4
*(2*sqrt(a*x**4 + b)*a*x**2 + 2*sqrt(a)*a*x**4 + sqrt(a)*b))
```

3.522 $\int \sqrt{a+\frac{b}{x^4}}x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \sqrt{a+\frac{b}{x^4}}x\,dx = \frac{1}{2}\sqrt{a+\frac{b}{x^4}}x^2-\frac{1}{2}\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2}\right)$$

output
```
1/2*(a+b/x^4)^(1/2)*x^2-1/2*b^(1/2)*arctanh(b^(1/2)/(a+b/x^4)^(1/2)/x^2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \sqrt{a+\frac{b}{x^4}}x\,dx = \frac{\sqrt{a+\frac{b}{x^4}}x^2\left(\sqrt{b+ax^4}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)\right)}{2\sqrt{b+ax^4}}$$

input
```
Integrate[Sqrt[a + b/x^4]*x,x]
```

output
```
(Sqrt[a + b/x^4]*x^2*(Sqrt[b + a*x^4] - Sqrt[b]*ArcTanh[Sqrt[b + a*x^4]/Sq
rt[b]]))/(2*Sqrt[b + a*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {858, 807, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{x^4}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^4}}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}\int \sqrt{a+\frac{b}{x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(x\sqrt{a+\frac{b}{x^2}}-b\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(x\sqrt{a+\frac{b}{x^2}}-b\int \frac{1}{1-\frac{b}{\sqrt{a+\frac{b}{x^2}}x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(x\sqrt{a+\frac{b}{x^2}}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^2}}}\right)\right)$$

input
```
Int[Sqrt[a + b/x^4]*x,x]
```

output
```
(Sqrt[a + b/x^2]*x - Sqrt[b]*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x^2)])/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}\left(\sqrt{b}\,\ln\left(\frac{2 b+2 \sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right)-\sqrt{a\,x^{4}+b}\right)}{2 \sqrt{a\,x^{4}+b}}$	65

input
```
int((a+b/x^4)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x^4+b)/x^4)^(1/2)*x^2*(b^(1/2)*ln(2*(b^(1/2)*(a*x^4+b)^(1/2)+b)/x
^2)-(a*x^4+b)^(1/2))/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.41

$$\int \sqrt{a+\frac{b}{x^4}}x\,dx = \left[\frac{1}{2}x^2\sqrt{\frac{ax^4+b}{x^4}} + \frac{1}{4}\sqrt{b}\log\left(\frac{ax^4-2\sqrt{b}x^2\sqrt{\frac{ax^4+b}{x^4}}+2b}{x^4}\right), \frac{1}{2}x^2\sqrt{\frac{ax^4+b}{x^4}} + \frac{1}{2}\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^2\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)\right]$$

input
```
integrate((a+b/x^4)^(1/2)*x,x, algorithm="fricas")
```

output
```
[1/2*x^2*sqrt((a*x^4 + b)/x^4) + 1/4*sqrt(b)*log((a*x^4 - 2*sqrt(b)*x^2*sq
rt((a*x^4 + b)/x^4) + 2*b)/x^4), 1/2*x^2*sqrt((a*x^4 + b)/x^4) + 1/2*sqrt(
-b)*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b))]
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \sqrt{a+\frac{b}{x^4}}x\,dx = \frac{\sqrt{a}x^2}{2\sqrt{1+\frac{b}{ax^4}}} - \frac{\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x^2}\right)}{2} + \frac{b}{2\sqrt{a}x^2\sqrt{1+\frac{b}{ax^4}}}$$

input
```
integrate((a+b/x**4)**(1/2)*x,x)
```

output
```
sqrt(a)*x**2/(2*sqrt(1 + b/(a*x**4))) - sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x**
2))/2 + b/(2*sqrt(a)*x**2*sqrt(1 + b/(a*x**4)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \sqrt{a+\frac{b}{x^4}}x\,dx = \frac{1}{2}\sqrt{a+\frac{b}{x^4}}x^2 + \frac{1}{4}\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2+\sqrt{b}}\right)$$

input
```
integrate((a+b/x^4)^(1/2)*x,x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^4)*x^2 + 1/4*sqrt(b)*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/
(sqrt(a + b/x^4)*x^2 + sqrt(b)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \sqrt{a+\frac{b}{x^4}}x\,dx = \frac{b\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}} + \frac{1}{2}\sqrt{ax^4+b}$$

input
```
integrate((a+b/x^4)^(1/2)*x,x, algorithm="giac")
```

output
```
1/2*b*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) + 1/2*sqrt(a*x^4 + b)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^4}}x\,dx = \int x\sqrt{a+\frac{b}{x^4}}\,dx$$

input
```
int(x*(a + b/x^4)^(1/2),x)
```

output
```
int(x*(a + b/x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.63

$$\int \sqrt{a+\frac{b}{x^4}}\,x\,dx$$
$$= \frac{\sqrt{a}\,\sqrt{a\,x^4+b}\,x^2+\sqrt{b}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}\right)-\sqrt{b}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}\right)+\sqrt{b}\,\sqrt{a}}{2\sqrt{a\,x^4+b}+2\sqrt{a}\,x^2}$$

input
```
int((a+b/x^4)^(1/2)*x,x)
```

output
```
(sqrt(a)*sqrt(a*x**4 + b)*x**2 + sqrt(b)*sqrt(a*x**4 + b)*log((sqrt(a*x**4
 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b)) - sqrt(b)*sqrt(a*x**4 + b)*log((s
qrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b)) + sqrt(b)*sqrt(a)*log((
sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*x**2 - sqrt(b)*sqrt(a)
*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*x**2 + a*x**4 +
b)/(2*(sqrt(a*x**4 + b) + sqrt(a)*x**2))
```

3.523 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x} \, dx = -\frac{1}{2}\sqrt{a+\frac{b}{x^4}}+\frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)$$

output
```
-1/2*(a+b/x^4)^(1/2)+1/2*a^(1/2)*arctanh((a+b/x^4)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x} \, dx = -\frac{1}{2}\sqrt{a+\frac{b}{x^4}}+\frac{\sqrt{a}\sqrt{a+\frac{b}{x^4}}x^2\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{2\sqrt{b+ax^4}}$$

input
```
Integrate[Sqrt[a + b/x^4]/x,x]
```

output
```
-1/2*Sqrt[a + b/x^4] + (Sqrt[a]*Sqrt[a + b/x^4]*x^2*Log[Sqrt[a]*x^2 + Sqrt
[b + a*x^4]])/(2*Sqrt[b + a*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x} dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \sqrt{a+\frac{b}{x^4}}x^4 d\frac{1}{x^4}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-a\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}-2\sqrt{a+\frac{b}{x^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{2a\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{b}-2\sqrt{a+\frac{b}{x^4}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)-2\sqrt{a+\frac{b}{x^4}}\right)$$

input
```
Int[Sqrt[a + b/x^4]/x,x]
```

output
```
(-2*Sqrt[a + b/x^4] + 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(31) = 62$.

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

method	result	size
risch	$-\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{2}+\frac{\sqrt{a}\,\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}{2\sqrt{a\,x^{4}+b}}$	65
default	$-\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\left(-a\,x^{4}\sqrt{a\,x^{4}+b}-\sqrt{a}\,\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)b\,x^{2}+\left(a\,x^{4}+b\right)^{\frac{3}{2}}\right)}{2\sqrt{a\,x^{4}+b}\,b}$	80

input
```
int((a+b/x^4)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*x^4+b)/x^4)^(1/2)+1/2*a^(1/2)*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*((a
*x^4+b)/x^4)^(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx = \left[\frac{1}{4}\,\sqrt{a}\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-\frac{1}{2}\,\sqrt{\frac{ax^4+b}{x^4}},\ -\frac{1}{2}\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)-\frac{1}{2}\,\sqrt{\frac{ax^4+b}{x^4}}\right]$$

input
```
integrate((a+b/x^4)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b) - 1/2
*sqrt((a*x^4 + b)/x^4), -1/2*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4
)/a) - 1/2*sqrt((a*x^4 + b)/x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx = \frac{\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{2}-\frac{ax^2}{2\sqrt{b}\sqrt{\frac{ax^4}{b}+1}}-\frac{\sqrt{b}}{2x^2\sqrt{\frac{ax^4}{b}+1}}$$

input
```
integrate((a+b/x**4)**(1/2)/x,x)
```

output
```
sqrt(a)*asinh(sqrt(a)*x**2/sqrt(b))/2 - a*x**2/(2*sqrt(b)*sqrt(a*x**4/b +
1)) - sqrt(b)/(2*x**2*sqrt(a*x**4/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx = -\frac{1}{4}\sqrt{a}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}$$

input
```
integrate((a+b/x^4)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/4*sqrt(a)*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))
- 1/2*sqrt(a + b/x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx = -\frac{1}{4}\sqrt{a}\log\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2\right)+\frac{\sqrt{a}b}{\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2-b}$$

input
```
integrate((a+b/x^4)^(1/2)/x,x, algorithm="giac")
```

output
```
-1/4*sqrt(a)*log((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2) + sqrt(a)*b/((sqrt(a)*
x^2 - sqrt(a*x^4 + b))^2 - b)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx = \frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2}-\frac{\sqrt{a+\frac{b}{x^4}}}{2}$$

input
```
int((a + b/x^4)^(1/2)/x,x)
```

output
```
(a^(1/2)*atanh((a + b/x^4)^(1/2)/a^(1/2)))/2 - (a + b/x^4)^(1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.51

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}\,dx$$
$$= \frac{\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)x^2-2\sqrt{a}\,\sqrt{a\,x^4+b}\,x^2+\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a\,x^4-2a\,x^4-b}{2x^2\left(\sqrt{a\,x^4+b}+\sqrt{a}\,x^2\right)}$$

input
```
int((a+b/x^4)^(1/2)/x,x)
```

output
```
(sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*x
**2 - 2*sqrt(a)*sqrt(a*x**4 + b)*x**2 + log((sqrt(a*x**4 + b) + sqrt(a)*x*
*2)/sqrt(b))*a*x**4 - 2*a*x**4 - b)/(2*x**2*(sqrt(a*x**4 + b) + sqrt(a)*x*
*2))
```

3.524 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3} \, dx = -\frac{\sqrt{a+\frac{b}{x^4}}}{4x^2} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2}\right)}{4\sqrt{b}}$$

output
```
-1/4*(a+b/x^4)^(1/2)/x^2-1/4*a*arctanh(b^(1/2)/(a+b/x^4)^(1/2)/x^2)/b^(1/2
)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3} \, dx = \frac{\sqrt{a+\frac{b}{x^4}}\left(-1-\frac{ax^4\text{arctanh}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+ax^4}}\right)}{4x^2}$$

input
```
Integrate[Sqrt[a + b/x^4]/x^3,x]
```

output
```
(Sqrt[a + b/x^4]*(-1 - (a*x^4*ArcTanh[Sqrt[b + a*x^4]/Sqrt[b]])/(Sqrt[b]*S
qrt[b + a*x^4])))/(4*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 807, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3} \, dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^4}}}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(-\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}-\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(-\frac{1}{2}a\int \frac{1}{1-\frac{b}{\sqrt{a+\frac{b}{x^2}}x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}-\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}-\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)$$

input
```
Int[Sqrt[a + b/x^4]/x^3,x]
```

output
```
(-1/2*Sqrt[a + b/x^2]/x^2 - (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x^2)])/(2*
Sqrt[b]))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

method	result	size
risch	$-\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{4x^{2}}-\frac{a\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}{4\sqrt{b}\,\sqrt{a\,x^{4}+b}}$	74
default	$-\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\left(-a\sqrt{a\,x^{4}+b}\,x^{4}\sqrt{b}+a\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right)b\,x^{4}+\left(a\,x^{4}+b\right)^{\frac{3}{2}}\sqrt{b}\right)}{4x^{2}\sqrt{a\,x^{4}+b}\,b^{\frac{3}{2}}}$	90

input
```
int((a+b/x^4)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x^4+b)/x^4)^(1/2)/x^2-1/4*a/b^(1/2)*ln((2*b+2*b^(1/2)*(a*x^4+b)^(
1/2))/x^2)*((a*x^4+b)/x^4)^(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx = \left[\frac{a\sqrt{b}x^2\log\left(\frac{ax^4-2\sqrt{b}x^2\sqrt{\frac{ax^4+b}{x^4}}+2\,b}{x^4}\right)-2\,b\sqrt{\frac{ax^4+b}{x^4}}}{8\,bx^2}, \frac{a\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x^2\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)-b\sqrt{\frac{ax^4+b}{x^4}}}{4\,bx^2}\right]$$

input
```
integrate((a+b/x^4)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(a*sqrt(b)*x^2*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4) + 2*b
)/x^4) - 2*b*sqrt((a*x^4 + b)/x^4))/(b*x^2), 1/4*(a*sqrt(-b)*x^2*arctan(sq
rt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)) - b*sqrt((a*x^4 + b)/x^4))/(
b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx = -\frac{\sqrt{a}\sqrt{1+\frac{b}{ax^4}}}{4x^2} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x^2}\right)}{4\sqrt{b}}$$

input
```
integrate((a+b/x**4)**(1/2)/x**3,x)
```

output
```
-sqrt(a)*sqrt(1 + b/(a*x**4))/(4*x**2) - a*asinh(sqrt(b)/(sqrt(a)*x**2))/(
4*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx = -\frac{\sqrt{a+\frac{b}{x^4}}\,ax^2}{4\left(\left(a+\frac{b}{x^4}\right)x^4-b\right)} + \frac{a\log\left(\frac{\sqrt{a+\frac{b}{x^4}}x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2+\sqrt{b}}\right)}{8\sqrt{b}}$$

input
```
integrate((a+b/x^4)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/4*sqrt(a + b/x^4)*a*x^2/((a + b/x^4)*x^4 - b) + 1/8*a*log((sqrt(a + b/x
^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b)))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx = \frac{1}{4}\,a\left(\frac{\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{ax^4+b}}{ax^4}\right)$$

input
```
integrate((a+b/x^4)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/4*a*(arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x^4 + b)/(a*x^4)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx = \int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx$$

input
```
int((a + b/x^4)^(1/2)/x^3,x)
```

output
```
int((a + b/x^4)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 5.78

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^3}\,dx = \frac{2\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}\right)a\,x^6 - 2\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}\right)a\,x^6 - 2\sqrt{b}\,\sqrt{a\,x^4+b}}{}$$

input
```
int((a+b/x^4)^(1/2)/x^3,x)
```

output

```
(2*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b)
)/sqrt(b))*a*x**6 - 2*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqr
t(a)*x**2 + sqrt(b))/sqrt(b))*a*x**6 - 2*sqrt(b)*sqrt(a*x**4 + b)*a*x**4 -
 sqrt(b)*sqrt(a*x**4 + b)*b - 2*sqrt(b)*sqrt(a)*a*x**6 - 2*sqrt(b)*sqrt(a)
*b*x**2 + 2*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**2*
x**8 + log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a*b*x**4 -
 2*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**2*x**8 - lo
g((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a*b*x**4)/(4*sqrt(b
)*x**4*(2*sqrt(a)*sqrt(a*x**4 + b)*x**2 + 2*a*x**4 + b))
```

3.525 $\int \sqrt{a+\frac{b}{x^4}}x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \sqrt{a+\frac{b}{x^4}}x^2\,dx=\frac{1}{3}\sqrt{a+\frac{b}{x^4}}x^3-\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+\frac{b}{x^4}}}$$

output
```
1/3*(a+b/x^4)^(1/2)*x^3-1/3*b^(3/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1
/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*
2^(1/2))/a^(1/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.72 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \sqrt{a+\frac{b}{x^4}}x^2\,dx$$
$$=\frac{1}{3}\sqrt{a+\frac{b}{x^4}}x^2\left(x-\frac{2ib\sqrt{1+\frac{ax^4}{b}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(b+ax^4)}\right)$$

input
```
Integrate[Sqrt[a + b/x^4]*x^2,x]
```

output
```
(Sqrt[a + b/x^4]*x^2*(x - ((2*I)*b*Sqrt[1 + (a*x^4)/b]*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[a])/Sqrt[b]]*x], -1])/(Sqrt[(I*Sqrt[a])/Sqrt[b]]*(b + a*x^4)
)))/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {858, 809, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{x^4}}\,dx$$
$$\Big\downarrow \text{858}$$
$$-\int \sqrt{a+\frac{b}{x^4}}x^4d\frac{1}{x}$$
$$\Big\downarrow \text{809}$$
$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x^4}}-\frac{2}{3}b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}$$
$$\Big\downarrow \text{761}$$

$$\frac{1}{3}x^3\sqrt{a+\frac{b}{x^4}}-\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[Sqrt[a + b/x^4]*x^2,x]
```

output
```
(Sqrt[a + b/x^4]*x^3)/3 - (b^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2
)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2]
)/(3*a^(1/4)*Sqrt[a + b/x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{3}}{3}+\frac{2b\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}{3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a\,x^{4}+b)}$	107
default	$\frac{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^{5}+2b\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,bx\right)}{3(a\,x^{4}+b)\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$	130

input
```
int((a+b/x^4)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*((a*x^4+b)/x^4)^(1/2)*x^3+2/3*b/(I*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)
/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)*EllipticF(x*
(I*a^(1/2)/b^(1/2))^(1/2),I)*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int\sqrt{a+\frac{b}{x^4}}x^2\,dx=\frac{1}{3}\,x^3\sqrt{\frac{ax^4+b}{x^4}}+\frac{2}{3}\,\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right)|-1)$$

input
```
integrate((a+b/x^4)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
1/3*x^3*sqrt((a*x^4 + b)/x^4) + 2/3*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin
((-b/a)^(1/4)/x), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.41

$$\int \sqrt{a+\frac{b}{x^4}}x^2\,dx = -\frac{\sqrt{a}x^3\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4},-\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((a+b/x**4)**(1/2)*x**2,x)
```

output
```
-sqrt(a)*x**3*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*exp_polar(I*pi)/(a
*x**4))/(4*gamma(1/4))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^4}}x^2\,dx = \int \sqrt{a+\frac{b}{x^4}}x^2\,dx$$

input
```
integrate((a+b/x^4)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^4)*x^2, x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^4}}x^2\,dx = \int \sqrt{a+\frac{b}{x^4}}x^2\,dx$$

input
```
integrate((a+b/x^4)^(1/2)*x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{x^4}}x^2\,dx=\int x^2\,\sqrt{a+\frac{b}{x^4}}\,dx$$

input
```
int(x^2*(a + b/x^4)^(1/2),x)
```

output
```
int(x^2*(a + b/x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^4}}x^2\,dx=\frac{\sqrt{a\,x^4+b}\,x}{3}+\frac{2\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^4+b}dx\right)b}{3}$$

input
```
int((a+b/x^4)^(1/2)*x^2,x)
```

output
```
(sqrt(a*x**4 + b)*x + 2*int(sqrt(a*x**4 + b)/(a*x**4 + b),x)*b)/3
```

3.526 $\int \sqrt{a+\frac{b}{x^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 224

$$\int \sqrt{a+\frac{b}{x^4}}\,dx=-\frac{2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\sqrt{a+\frac{b}{x^4}}\,x+\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt[4]{a}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{a+\frac{b}{x^4}}}$$

output
```
-2*b^(1/2)*(a+b/x^4)^(1/2)/(a^(1/2)+b^(1/2)/x^2)/x+(a+b/x^4)^(1/2)*x+2*a^(
1/4)*b^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^
2)*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))/(a+b/x^4)^(1/2)
-a^(1/4)*b^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2
)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/(a+b/x^4)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.93 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.21

$$\int \sqrt{a+\frac{b}{x^4}}\,dx=-\frac{\sqrt{a+\frac{b}{x^4}}\,x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{ax^4}{b}\right)}{\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^4],x]
```

output
```
-((Sqrt[a + b/x^4]*x*Hypergeometric2F1[-1/2, -1/4, 3/4, -((a*x^4)/b)])/Sqr
t[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {773, 809, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{x^4}}\,dx$$

$$\Big\downarrow \; 773$$

$$-\int \sqrt{a+\frac{b}{x^4}}\,x^2 d\frac{1}{x}$$

$$\Big\downarrow \; 809$$

$$x\sqrt{a+\frac{b}{x^4}}-2b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 834$$

$$x\sqrt{a+\frac{b}{x^4}}-2b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)$$

$$\Big\downarrow \text{27}$$

$$x\sqrt{a+\frac{b}{x^4}}-2b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)$$

$$\Big\downarrow \text{761}$$

$$x\sqrt{a+\frac{b}{x^4}}-2b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)$$

$$\Big\downarrow \text{1510}$$

$$x\sqrt{a+\frac{b}{x^4}}-2b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}}{\sqrt{b}}\right.$$

input
```
Int[Sqrt[a + b/x^4],x]
```

output
```
Sqrt[a + b/x^4]*x - 2*b*(-((-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x))
 + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]
/x^2)*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x
^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqr
t[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3
/4)*Sqrt[a + b/x^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.58

method	result
risch	$-\sqrt{\frac{a\,x^4+b}{x^4}}\,x+\frac{2i\sqrt{a}\,\sqrt{b}\,\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a\,x^4+b)}$
default	$\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\,x\left(2i\sqrt{a}\,\sqrt{b}\,\sqrt{\frac{-i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-2i\sqrt{a}\,\sqrt{b}\,\sqrt{\frac{-i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x\operatorname{EllipticE}\left(x\sqrt{\right.\right.}{(a\,x^4+b)\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int((a+b/x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a*x^4+b)/x^4)^(1/2)*x+2*I*a^(1/2)*b^(1/2)/(I*a^(1/2)/b^(1/2))^(1/2)*(1-
I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)*(El
lipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/
2),I))*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [F]

$$\int \sqrt{a+\frac{b}{x^4}}\,dx=\int \sqrt{a+\frac{b}{x^4}}\,dx$$

input
```
integrate((a+b/x^4)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt((a*x^4 + b)/x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

$$\int \sqrt{a+\frac{b}{x^4}}\,dx = -\frac{\sqrt{a}x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((a+b/x**4)**(1/2),x)
```

output
```
-sqrt(a)*x*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*exp_polar(I*pi)/(a*x*
*4))/(4*gamma(3/4))
```

Maxima [F]

$$\int \sqrt{a+\frac{b}{x^4}}\,dx = \int \sqrt{a+\frac{b}{x^4}}\,dx$$

input
```
integrate((a+b/x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^4), x)
```

Giac [F]

$$\int \sqrt{a+\frac{b}{x^4}}\,dx = \int \sqrt{a+\frac{b}{x^4}}\,dx$$

input
```
integrate((a+b/x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.17

$$\int \sqrt{a+\frac{b}{x^4}}\,dx=-\frac{x\,\sqrt{a+\frac{b}{x^4}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a\,x^4}{b}\right)}{\sqrt{\frac{a\,x^4}{b}+1}}$$

input
```
int((a + b/x^4)^(1/2),x)
```

output
```
-(x*(a + b/x^4)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -(a*x^4)/b))/((a*x^4)/b
 + 1)^(1/2)
```

Reduce [F]

$$\int \sqrt{a+\frac{b}{x^4}}\,dx=\frac{\sqrt{a\,x^4+b}+2\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^6+b\,x^2}dx\right)bx}{x}$$

input
```
int((a+b/x^4)^(1/2),x)
```

output
```
(sqrt(a*x**4 + b) + 2*int(sqrt(a*x**4 + b)/(a*x**6 + b*x**2),x)*b*x)/x
```

3.527 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2} \, dx = -\frac{\sqrt{a+\frac{b}{x^4}}}{3x} - \frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/3*(a+b/x^4)^(1/2)/x-1/3*a^(3/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/
2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2
^(1/2))/b^(1/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx = -\frac{\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{ax^4}{b}\right)}{3x\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^4]/x^2,x]
```

output
```
-1/3*(Sqrt[a + b/x^4]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((a*x^4)/b)])/(x
*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {858, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \sqrt{a+\frac{b}{x^4}}\,d\frac{1}{x}$$

$$\Big\downarrow \text{748}$$

$$-\frac{2}{3}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}-\frac{\sqrt{a+\frac{b}{x^4}}}{3x}$$

$$\Big\downarrow \text{761}$$

$$-\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{3x}$$

input
```
Int[Sqrt[a + b/x^4]/x^2,x]
```

output
```
-1/3*Sqrt[a + b/x^4]/x - (a^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)
^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])
/(3*b^(1/4)*Sqrt[a + b/x^4])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{\sqrt{\frac{a\,x^4+b}{x^4}}}{3x}+\frac{2a\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a\,x^4+b)}$	107
default	$-\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(-2a\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)x^3+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^4+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,b\right)}{3x(a\,x^4+b)\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$	132

input
```
int((a+b/x^4)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((a*x^4+b)/x^4)^(1/2)/x+2/3*a/(I*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/
b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)*EllipticF(x*(
I*a^(1/2)/b^(1/2))^(1/2),I)*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx=-\frac{2\,\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}\,F\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|-1\right)+\sqrt{\frac{ax^4+b}{x^4}}}{3\,x}$$

input
```
integrate((a+b/x^4)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4)), -1) + sq
rt((a*x^4 + b)/x^4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx = -\frac{\sqrt{a}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((a+b/x**4)**(1/2)/x**2,x)
```

output
```
-sqrt(a)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**4))
/(4*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$$

input
```
integrate((a+b/x^4)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^4)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx$$

input
```
integrate((a+b/x^4)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx=-\frac{\sqrt{a\,x^4+b}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\,\sqrt{\frac{b}{a}+x^4}}$$

input
```
int((a + b/x^4)^(1/2)/x^2,x)
```

output
```
-((b + a*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -b/(a*x^4)))/(x*(b/a + x^4
)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}\,dx=\frac{-\sqrt{a\,x^4+b}-2\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^8+b\,x^4}dx\right)b\,x^3}{x^3}$$

input
```
int((a+b/x^4)^(1/2)/x^2,x)
```

output
```
( - sqrt(a*x**4 + b) - 2*int(sqrt(a*x**4 + b)/(a*x**8 + b*x**4),x)*b*x**3)
/x**3
```

3.528 $\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 236

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4} dx = -\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3} - \frac{2a\sqrt{a+\frac{b}{x^4}}}{5\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{2a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}} - \frac{a^{5/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/5*(a+b/x^4)^(1/2)/x^3-2/5*a*(a+b/x^4)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)/x^
2)/x+2/5*a^(5/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2
)/x^2)*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))/b^(3/4)/(a+
b/x^4)^(1/2)-1/5*a^(5/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2
)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/b^
(3/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4} dx = -\frac{\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{ax^4}{b}\right)}{5x^3\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[Sqrt[a + b/x^4]/x^4,x]
```

output
```
-1/5*(Sqrt[a + b/x^4]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^4)/b)])/(
x^3*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4} dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$-\frac{2}{5}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2} d\frac{1}{x} - \frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}$$

$$\Big\downarrow \text{834}$$

$$-\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)-\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)-\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}$$

$$\Big\downarrow \; 761$$

$$-\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)-\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}$$

$$\Big\downarrow \; 1510$$

$$-\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)\right|}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}}{\sqrt{b}}\right.$$

$$\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}$$

input
```
Int[Sqrt[a + b/x^4]/x^4,x]
```

output
```
-1/5*Sqrt[a + b/x^4]/x^3 - (2*a*(-((-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/
x^2)*x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] +
 Sqrt[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt
[a + b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)
^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])
/(2*b^(3/4)*Sqrt[a + b/x^4])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{(2a\,x^4+b)\sqrt{\frac{a\,x^4+b}{x^4}}}{5x^3b}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{5\sqrt{b}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a\,x^4+b)}$
default	$-\frac{\sqrt{\frac{a\,x^4+b}{x^4}}\left(-2ia^{\frac{3}{2}}\sqrt{\frac{-i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x^5b\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+2ia^{\frac{3}{2}}\sqrt{\frac{-i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x^5b\,\mathrm{EllipticE}\left(x\sqrt{\right.\right.}{5x^3(a\,x^4+b)b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int((a+b/x^4)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(2*a*x^4+b)/x^3/b*((a*x^4+b)/x^4)^(1/2)+2/5*I/b^(1/2)*a^(3/2)/(I*a^(1
/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^
2)^(1/2)/(a*x^4+b)*(EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-EllipticE(x*(
I*a^(1/2)/b^(1/2))^(1/2),I))*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx = -\frac{2\,a\sqrt{b}x^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\mid -1)-2\,a\sqrt{b}x^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\mid -1)+(2\,ax^4+b)\sqrt{\ }}{5\,bx^3}$$

input
```
integrate((a+b/x^4)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/5*(2*a*sqrt(b)*x^3*(-a/b)^(3/4)*elliptic_e(arcsin(x*(-a/b)^(1/4)), -1)
- 2*a*sqrt(b)*x^3*(-a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4)), -1) + (2
*a*x^4 + b)*sqrt((a*x^4 + b)/x^4))/(b*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx = -\frac{\sqrt{a}\,\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((a+b/x**4)**(1/2)/x**4,x)
```

output
```
-sqrt(a)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4))
/(4*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx = \int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$$

input
```
integrate((a+b/x^4)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x^4)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx = \int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$$

input
```
integrate((a+b/x^4)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/x^4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx = \int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx$$

input
```
int((a + b/x^4)^(1/2)/x^4,x)
```

output
```
int((a + b/x^4)^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^4}\,dx = \frac{-\sqrt{a\,x^4+b}-2\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{10}+b\,x^6}dx\right)b\,x^5}{3x^5}$$

input
```
int((a+b/x^4)^(1/2)/x^4,x)
```

output
```
( - sqrt(a*x**4 + b) - 2*int(sqrt(a*x**4 + b)/(a*x**10 + b*x**6),x)*b*x**5
)/(3*x**5)
```

3.529 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx=-\frac{1}{2}b\sqrt{a+\frac{b}{x^4}}+\frac{1}{4}a\sqrt{a+\frac{b}{x^4}}x^4+\frac{3}{4}\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)$$

output
```
-1/2*b*(a+b/x^4)^(1/2)+1/4*a*(a+b/x^4)^(1/2)*x^4+3/4*a^(1/2)*b*arctanh((a+
b/x^4)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx=\frac{\sqrt{a+\frac{b}{x^4}}\left((-2b+ax^4)\sqrt{b+ax^4}+3\sqrt{a}bx^2\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)\right)}{4\sqrt{b+ax^4}}$$

input
```
Integrate[(a + b/x^4)^(3/2)*x^3,x]
```

output
```
(Sqrt[a + b/x^4]*((-2*b + a*x^4)*Sqrt[b + a*x^4] + 3*Sqrt[a]*b*x^2*Log[Sqr
t[a]*x^2 + Sqrt[b + a*x^4]]))/(4*Sqrt[b + a*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{x^4}\right)^{3/2}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \left(a+\frac{b}{x^4}\right)^{3/2}x^8d\frac{1}{x^4}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(x^4\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{3}{2}b\int \sqrt{a+\frac{b}{x^4}}x^4d\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(x^4\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{3}{2}b\left(a\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}+2\sqrt{a+\frac{b}{x^4}}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(x^4\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{b}+2\sqrt{a+\frac{b}{x^4}}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(x^4\left(a+\frac{b}{x^4}\right)^{3/2}-\frac{3}{2}b\left(2\sqrt{a+\frac{b}{x^4}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)\right)\right)$$

input
```
Int[(a + b/x^4)^(3/2)*x^3,x]
```

output
```
((a + b/x^4)^(3/2)*x^4 - (3*b*(2*Sqrt[a + b/x^4] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b/x^4]/Sqrt[a]]))/2)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

method	result	size
risch	$\frac{(a\,x^4-2b)\sqrt{\frac{a\,x^4+b}{x^4}}}{4}+\frac{3\sqrt{a}\,b\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{4\sqrt{a\,x^4+b}}$	75
default	$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}} x^4\left(a\,x^4\sqrt{a\,x^4+b}+3\sqrt{a}\,\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)b\,x^2-2b\sqrt{a\,x^4+b}\right)}{4(a\,x^4+b)^{\frac{3}{2}}}$	82

input
```
int((a+b/x^4)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*x^4-2*b)*((a*x^4+b)/x^4)^(1/2)+3/4*a^(1/2)*b*ln(a^(1/2)*x^2+(a*x^4+
b)^(1/2))*((a*x^4+b)/x^4)^(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.88

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx = \left[\frac{3}{8}\,\sqrt{a}b\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+\frac{1}{4}\left(ax^4-2\,b\right)\sqrt{\frac{ax^4+b}{x^4}},-\frac{3}{4}\,\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)+\frac{1}{4}\left(ax^4-2\,b\right)\sqrt{\frac{ax^4+b}{x^4}}\right]$$

input
```
integrate((a+b/x^4)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
[3/8*sqrt(a)*b*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b) + 1
/4*(a*x^4 - 2*b)*sqrt((a*x^4 + b)/x^4), -3/4*sqrt(-a)*b*arctan(sqrt(-a)*sq
rt((a*x^4 + b)/x^4)/a) + 1/4*(a*x^4 - 2*b)*sqrt((a*x^4 + b)/x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.48

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx = \frac{3\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{4} + \frac{a^2x^6}{4\sqrt{b}\sqrt{\frac{ax^4}{b}+1}} - \frac{a\sqrt{b}x^2}{4\sqrt{\frac{ax^4}{b}+1}} - \frac{b^{\frac{3}{2}}}{2x^2\sqrt{\frac{ax^4}{b}+1}}$$

input
```
integrate((a+b/x**4)**(3/2)*x**3,x)
```

output
```
3*sqrt(a)*b*asinh(sqrt(a)*x**2/sqrt(b))/4 + a**2*x**6/(4*sqrt(b)*sqrt(a*x*
*4/b + 1)) - a*sqrt(b)*x**2/(4*sqrt(a*x**4/b + 1)) - b**(3/2)/(2*x**2*sqrt
(a*x**4/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx = \frac{1}{4}\sqrt{a+\frac{b}{x^4}}ax^4 - \frac{3}{8}\sqrt{a}b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right) - \frac{1}{2}\sqrt{a+\frac{b}{x^4}}b$$

input
```
integrate((a+b/x^4)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
1/4*sqrt(a + b/x^4)*a*x^4 - 3/8*sqrt(a)*b*log((sqrt(a + b/x^4) - sqrt(a))/
(sqrt(a + b/x^4) + sqrt(a))) - 1/2*sqrt(a + b/x^4)*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx = \frac{1}{4}\sqrt{ax^4+b}ax^2 - \frac{3}{8}\sqrt{a}b\log\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2\right) + \frac{\sqrt{a}b^2}{\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2-b}$$

input
```
integrate((a+b/x^4)^(3/2)*x^3,x, algorithm="giac")
```

output
```
1/4*sqrt(a*x^4 + b)*a*x^2 - 3/8*sqrt(a)*b*log((sqrt(a)*x^2 - sqrt(a*x^4 +
b))^2) + sqrt(a)*b^2/((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2 - b)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx=\frac{a\,x^4\,\sqrt{a+\frac{b}{x^4}}}{4}-\frac{b\,\sqrt{a+\frac{b}{x^4}}}{2}+\frac{3\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4}$$

input
```
int(x^3*(a + b/x^4)^(3/2),x)
```

output
```
(a*x^4*(a + b/x^4)^(1/2))/4 - (b*(a + b/x^4)^(1/2))/2 + (3*a^(1/2)*b*atanh
((a + b/x^4)^(1/2)/a^(1/2)))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.22

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^3\,dx=\frac{48\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)ab\,x^6+12\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)b^2x^2+16\sqrt{}}{}$$

input
```
int((a+b/x^4)^(3/2)*x^3,x)
```

output
```
(48*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b)
)*a*b*x**6 + 12*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x
**2)/sqrt(b))*b**2*x**2 + 16*sqrt(a)*sqrt(a*x**4 + b)*a**2*x**10 - 56*sqrt
(a)*sqrt(a*x**4 + b)*a*b*x**6 - 33*sqrt(a)*sqrt(a*x**4 + b)*b**2*x**2 + 48
*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**2*b*x**8 + 36*log((sqrt
(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a*b**2*x**4 + 16*a**3*x**12 - 48*a**
2*b*x**8 - 63*a*b**2*x**4 - 8*b**3)/(16*x**2*(4*sqrt(a*x**4 + b)*a*x**4 +
sqrt(a*x**4 + b)*b + 4*sqrt(a)*a*x**6 + 3*sqrt(a)*b*x**2))
```

3.530 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x \, dx = -\frac{b\sqrt{a+\frac{b}{x^4}}}{4x^2}+\frac{1}{2}a\sqrt{a+\frac{b}{x^4}}x^2-\frac{3}{4}a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2}\right)$$

output
```
-1/4*b*(a+b/x^4)^(1/2)/x^2+1/2*a*(a+b/x^4)^(1/2)*x^2-3/4*a*b^(1/2)*arctanh
(b^(1/2)/(a+b/x^4)^(1/2)/x^2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x \, dx = -\frac{\sqrt{a+\frac{b}{x^4}}\left((b-2ax^4)\sqrt{b+ax^4}+3a\sqrt{b}x^4\operatorname{arctanh}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)\right)}{4x^2\sqrt{b+ax^4}}$$

input
```
Integrate[(a + b/x^4)^(3/2)*x,x]
```

output
```
-1/4*(Sqrt[a + b/x^4]*((b - 2*a*x^4)*Sqrt[b + a*x^4] + 3*a*Sqrt[b]*x^4*Arc
Tanh[Sqrt[b + a*x^4]/Sqrt[b]]))/(x^2*Sqrt[b + a*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {858, 807, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^4}\right)^{3/2}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{3/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{\sqrt{a+\frac{b}{x^2}}x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{3/2}-3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)\right)$$

input
```
Int[(a + b/x^4)^(3/2)*x,x]
```

output
```
((a + b/x^2)^(3/2)*x - 3*b*(Sqrt[a + b/x^2]/(2*x^2) + (a*ArcTanh[Sqrt[b]/(
Sqrt[a + b/x^2]*x^2)])/(2*Sqrt[b])))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}} x^{2} \left(3 \ln\left(\frac{2 b+2 \sqrt{b}\, \sqrt{a\,x^{4}+b}}{x^{2}}\right) \sqrt{b}\, a\,x^{4}-2 a\,x^{4} \sqrt{a\,x^{4}+b}+b \sqrt{a\,x^{4}+b}\right)}{4 \left(a\,x^{4}+b \right)^{\frac{3}{2}}}$	85
risch	$-\frac{b \sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{4 x^{2}}+\frac{\left(\frac{\sqrt{a\,x^{4}+b}\, a}{2}-\frac{3 \ln\left(\frac{2 b+2 \sqrt{b}\, \sqrt{a\,x^{4}+b}}{x^{2}}\right) \sqrt{b}\, a}{4}\right) \sqrt{\frac{a\,x^{4}+b}{x^{4}}}\, x^{2}}{\sqrt{a\,x^{4}+b}}$	89

input
```
int((a+b/x^4)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/4*((a*x^4+b)/x^4)^(3/2)*x^2*(3*ln(2*(b^(1/2)*(a*x^4+b)^(1/2)+b)/x^2)*b^
(1/2)*a*x^4-2*a*x^4*(a*x^4+b)^(1/2)+b*(a*x^4+b)^(1/2))/(a*x^4+b)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.13

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx = \left[\frac{3\,a\sqrt{b}x^2\log\left(\frac{ax^4-2\,\sqrt{b}x^2\sqrt{\frac{ax^4+b}{x^4}}+2\,b}{x^4}\right)+2\,(2\,ax^4-b)\sqrt{\frac{ax^4+b}{x^4}}}{8\,x^2}, \frac{3\,a\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x^2\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right) ...}{4\,x^2}\right]$$

input
```
integrate((a+b/x^4)^(3/2)*x,x, algorithm="fricas")
```

output
```
[1/8*(3*a*sqrt(b)*x^2*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4) + 2
*b)/x^4) + 2*(2*a*x^4 - b)*sqrt((a*x^4 + b)/x^4))/x^2, 1/4*(3*a*sqrt(-b)*x
^2*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b)) + (2*a*x^4 - b)*
sqrt((a*x^4 + b)/x^4))/x^2]
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx = \frac{a^{\frac{3}{2}} x^2}{2\sqrt{1+\frac{b}{ax^4}}} + \frac{\sqrt{a}b}{4x^2\sqrt{1+\frac{b}{ax^4}}} - \frac{3a\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x^2}\right)}{4} - \frac{b^2}{4\sqrt{a}x^6\sqrt{1+\frac{b}{ax^4}}}$$

input
```
integrate((a+b/x**4)**(3/2)*x,x)
```

output
```
a**(3/2)*x**2/(2*sqrt(1 + b/(a*x**4))) + sqrt(a)*b/(4*x**2*sqrt(1 + b/(a*x
**4))) - 3*a*sqrt(b)*asinh(sqrt(b)/(sqrt(a)*x**2))/4 - b**2/(4*sqrt(a)*x**
6*sqrt(1 + b/(a*x**4)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx = \frac{1}{2}\sqrt{a+\frac{b}{x^4}}ax^2 - \frac{\sqrt{a+\frac{b}{x^4}}abx^2}{4\left(\left(a+\frac{b}{x^4}\right)x^4-b\right)} + \frac{3}{8}a\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2+\sqrt{b}}\right)$$

input
```
integrate((a+b/x^4)^(3/2)*x,x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^4)*a*x^2 - 1/4*sqrt(a + b/x^4)*a*b*x^2/((a + b/x^4)*x^4 -
 b) + 3/8*a*sqrt(b)*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x
^2 + sqrt(b)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx = \frac{1}{4}\left(\frac{3\,b\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\sqrt{ax^4+b} - \frac{\sqrt{ax^4+b}b}{ax^4}\right)a$$

input
```
integrate((a+b/x^4)^(3/2)*x,x, algorithm="giac")
```

output
```
1/4*(3*b*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(a*x^4 + b) - s
qrt(a*x^4 + b)*b/(a*x^4))*a
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx = \int x\left(a+\frac{b}{x^4}\right)^{3/2} dx$$

input
```
int(x*(a + b/x^4)^(3/2),x)
```

output
```
int(x*(a + b/x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 441, normalized size of antiderivative = 6.30

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x\,dx = \frac{8\sqrt{a}\,\sqrt{a\,x^4+b}\,a^2x^{10}+2\sqrt{a}\,\sqrt{a\,x^4+b}\,ab\,x^6-3\sqrt{a}\,\sqrt{a\,x^4+b}\,b^2x^2+12\sqrt{b}\,\sqrt{a\,x^4+b}\log\Big(}{}$$

input
```
int((a+b/x^4)^(3/2)*x,x)
```

output
```
(8*sqrt(a)*sqrt(a*x**4 + b)*a**2*x**10 + 2*sqrt(a)*sqrt(a*x**4 + b)*a*b*x*
*6 - 3*sqrt(a)*sqrt(a*x**4 + b)*b**2*x**2 + 12*sqrt(b)*sqrt(a*x**4 + b)*lo
g((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**2*x**8 + 3*sqrt(
b)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b
))*a*b*x**4 - 12*sqrt(b)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*
x**2 + sqrt(b))/sqrt(b))*a**2*x**8 - 3*sqrt(b)*sqrt(a*x**4 + b)*log((sqrt(
a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a*b*x**4 + 12*sqrt(b)*sqrt(
a)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**2*x**10 + 9
*sqrt(b)*sqrt(a)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*
a*b*x**6 - 12*sqrt(b)*sqrt(a)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(
b))/sqrt(b))*a**2*x**10 - 9*sqrt(b)*sqrt(a)*log((sqrt(a*x**4 + b) + sqrt(a
)*x**2 + sqrt(b))/sqrt(b))*a*b*x**6 + 8*a**3*x**12 + 6*a**2*b*x**8 - 3*a*b
**2*x**4 - b**3)/(4*x**4*(4*sqrt(a*x**4 + b)*a*x**4 + sqrt(a*x**4 + b)*b +
 4*sqrt(a)*a*x**6 + 3*sqrt(a)*b*x**2))
```

3.531 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x} dx=-\frac{1}{2}a\sqrt{a+\frac{b}{x^4}}-\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{3/2}+\frac{1}{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)$$

output
```
-1/2*a*(a+b/x^4)^(1/2)-1/6*(a+b/x^4)^(3/2)+1/2*a^(3/2)*arctanh((a+b/x^4)^(
1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x} dx=\frac{\sqrt{a+\frac{b}{x^4}}\left(-\sqrt{b+ax^4}(b+4ax^4)+3a^{3/2}x^6\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)\right)}{6x^4\sqrt{b+ax^4}}$$

input
```
Integrate[(a + b/x^4)^(3/2)/x,x]
```

output
```
(Sqrt[a + b/x^4]*(-(Sqrt[b + a*x^4]*(b + 4*a*x^4)) + 3*a^(3/2)*x^6*Log[Sqr
t[a]*x^2 + Sqrt[b + a*x^4]]))/(6*x^4*Sqrt[b + a*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \left(a+\frac{b}{x^4}\right)^{3/2}x^4d\frac{1}{x^4}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-a\int \sqrt{a+\frac{b}{x^4}}x^4d\frac{1}{x^4}-\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-a\left(a\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}+2\sqrt{a+\frac{b}{x^4}}\right)-\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-a\left(\frac{2a\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{b}+2\sqrt{a+\frac{b}{x^4}}\right)-\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-a\left(2\sqrt{a+\frac{b}{x^4}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)\right)-\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)$$

input
```
Int[(a + b/x^4)^(3/2)/x,x]
```

output
```
((-2*(a + b/x^4)^(3/2))/3 - a*(2*Sqrt[a + b/x^4] - 2*Sqrt[a]*ArcTanh[Sqrt[
a + b/x^4]/Sqrt[a]]))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

method	result	size
risch	$-\frac{\left(4 a \,x^{4}+b \right) \sqrt{\frac{a \,x^{4}+b}{x^{4}}}}{6 x^{4}}+\frac{a^{\frac{3}{2}} \ln \left(\sqrt{a}\, x^{2}+\sqrt{a \,x^{4}+b}\right) \sqrt{\frac{a \,x^{4}+b}{x^{4}}}\, x^{2}}{2 \sqrt{a \,x^{4}+b}}$	76
default	$-\frac{\left(\frac{a \,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}} \left(-3 a^{\frac{3}{2}} \ln \left(\sqrt{a}\, x^{2}+\sqrt{a \,x^{4}+b}\right) x^{6}+4 a \,x^{4} \sqrt{a \,x^{4}+b}+b \sqrt{a \,x^{4}+b}\right)}{6 \left(a \,x^{4}+b \right)^{\frac{3}{2}}}$	78

input
```
int((a+b/x^4)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(4*a*x^4+b)/x^4*((a*x^4+b)/x^4)^(1/2)+1/2*a^(3/2)*ln(a^(1/2)*x^2+(a*x
^4+b)^(1/2))*((a*x^4+b)/x^4)^(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx = \left[\frac{3\,a^{\frac{3}{2}}x^4\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-2\,(4\,ax^4+b)\sqrt{\frac{ax^4+b}{x^4}}}{12\,x^4}, -\frac{3\,\sqrt{-a}ax^4\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)+(4\,ax^4+b)\sqrt{\frac{ax^4+b}{x^4}}}{6\,x^4}\right]$$

input
```
integrate((a+b/x^4)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/12*(3*a^(3/2)*x^4*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) -
b) - 2*(4*a*x^4 + b)*sqrt((a*x^4 + b)/x^4))/x^4, -1/6*(3*sqrt(-a)*a*x^4*ar
ctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4)/a) + (4*a*x^4 + b)*sqrt((a*x^4 + b)/x^
4))/x^4]
```

Sympy [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx = -\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^4}}}{3}-\frac{a^{\frac{3}{2}}\log\left(\frac{b}{ax^4}\right)}{4}+\frac{a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{2}-\frac{\sqrt{a}b\sqrt{1+\frac{b}{ax^4}}}{6x^4}$$

input
```
integrate((a+b/x**4)**(3/2)/x,x)
```

output
```
-2*a**(3/2)*sqrt(1 + b/(a*x**4))/3 - a**(3/2)*log(b/(a*x**4))/4 + a**(3/2)
*log(sqrt(1 + b/(a*x**4)) + 1)/2 - sqrt(a)*b*sqrt(1 + b/(a*x**4))/(6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx = -\frac{1}{4}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)-\frac{1}{6}\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{a+\frac{b}{x^4}}\,a$$

input
```
integrate((a+b/x^4)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/4*a^(3/2)*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))
- 1/6*(a + b/x^4)^(3/2) - 1/2*sqrt(a + b/x^4)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(43) = 86.

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.07

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx = -\frac{1}{4}\,a^{\frac{3}{2}}\log\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2\right) + \frac{2\left(3\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^4a^{\frac{3}{2}}b-3\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2a^{\frac{3}{2}}b^2+2\,a^{\frac{3}{2}}b^3\right)}{3\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2-b\right)^3}$$

input
```
integrate((a+b/x^4)^(3/2)/x,x, algorithm="giac")
```

output
```
-1/4*a^(3/2)*log((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2) + 2/3*(3*(sqrt(a)*x^2
- sqrt(a*x^4 + b))^4*a^(3/2)*b - 3*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^2*a^(3/
2)*b^2 + 2*a^(3/2)*b^3)/((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2 - b)^3
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx=\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2}-\frac{a\,\sqrt{a+\frac{b}{x^4}}}{2}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{6}$$

input
```
int((a + b/x^4)^(3/2)/x,x)
```

output
```
(a^(3/2)*atanh((a + b/x^4)^(1/2)/a^(1/2)))/2 - (a*(a + b/x^4)^(1/2))/2 - (
a + b/x^4)^(3/2)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.56

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}\,dx=\frac{12\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a^2x^{10}+3\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)ab\,x^6-}{}$$

input
```
int((a+b/x^4)^(3/2)/x,x)
```

output
```
(12*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b)
)*a**2*x**10 + 3*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*
x**2)/sqrt(b))*a*b*x**6 - 16*sqrt(a)*sqrt(a*x**4 + b)*a**2*x**10 - 16*sqrt
(a)*sqrt(a*x**4 + b)*a*b*x**6 - 3*sqrt(a)*sqrt(a*x**4 + b)*b**2*x**2 + 12*
log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**3*x**12 + 9*log((sqrt(a*
x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**2*b*x**8 - 16*a**3*x**12 - 24*a**2*b
*x**8 - 9*a*b**2*x**4 - b**3)/(6*x**6*(4*sqrt(a*x**4 + b)*a*x**4 + sqrt(a*
x**4 + b)*b + 4*sqrt(a)*a*x**6 + 3*sqrt(a)*b*x**2))
```

3.532 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3} dx = -\frac{3a\sqrt{a+\frac{b}{x^4}}}{16x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{8x^2} - \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2}\right)}{16\sqrt{b}}$$

output
```
-3/16*a*(a+b/x^4)^(1/2)/x^2-1/8*(a+b/x^4)^(3/2)/x^2-3/16*a^2*arctanh(b^(1/
2)/(a+b/x^4)^(1/2)/x^2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3} dx = \frac{\sqrt{a+\frac{b}{x^4}}\left(-2b-5ax^4-\frac{3a^2x^8\operatorname{arctanh}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+ax^4}}\right)}{16x^6}$$

input
```
Integrate[(a + b/x^4)^(3/2)/x^3,x]
```

output
```
(Sqrt[a + b/x^4]*(-2*b - 5*a*x^4 - (3*a^2*x^8*ArcTanh[Sqrt[b + a*x^4]/Sqrt
[b]])/(Sqrt[b]*Sqrt[b + a*x^4])))/(16*x^6)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {858, 807, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{3/2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(-\frac{3}{4}a\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(-\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(-\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{b}{\sqrt{a+\frac{b}{x^2}}x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)$$

input
```
Int[(a + b/x^4)^(3/2)/x^3,x]
```

output
```
(-1/4*(a + b/x^2)^(3/2)/x^2 - (3*a*(Sqrt[a + b/x^2]/(2*x^2) + (a*ArcTanh[S
qrt[b]/(Sqrt[a + b/x^2]*x^2)])/(2*Sqrt[b])))/4)/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

method	result	size
risch	$-\frac{(5a\,x^4+2b)\sqrt{\frac{a\,x^4+b}{x^4}}}{16x^6}-\frac{3a^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^4+b}}{x^2}\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{16\sqrt{b}\,\sqrt{a\,x^4+b}}$	86
default	$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(3a^2\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^4+b}}{x^2}\right)x^8+5a\sqrt{a\,x^4+b}\,x^4\sqrt{b}+2b^{\frac{3}{2}}\sqrt{a\,x^4+b}\right)}{16x^2(a\,x^4+b)^{\frac{3}{2}}\sqrt{b}}$	93

input
```
int((a+b/x^4)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(5*a*x^4+2*b)/x^6*((a*x^4+b)/x^4)^(1/2)-3/16*a^2/b^(1/2)*ln((2*b+2*b
^(1/2)*(a*x^4+b)^(1/2))/x^2)*((a*x^4+b)/x^4)^(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.34

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,a^2\sqrt{b}x^6\log\left(\frac{ax^4-2\,\sqrt{b}x^2\sqrt{\frac{ax^4+b}{x^4}}+2\,b}{x^4}\right)-2\,(5\,abx^4+2\,b^2)\sqrt{\frac{ax^4+b}{x^4}}}{32\,bx^6}, \frac{3\,a^2\sqrt{-b}x^6\arctan\left(\right.}{}\right.$$

input
```
integrate((a+b/x^4)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/32*(3*a^2*sqrt(b)*x^6*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4)
+ 2*b)/x^4) - 2*(5*a*b*x^4 + 2*b^2)*sqrt((a*x^4 + b)/x^4))/(b*x^6), 1/16*(
3*a^2*sqrt(-b)*x^6*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b))
- (5*a*b*x^4 + 2*b^2)*sqrt((a*x^4 + b)/x^4))/(b*x^6)]
```

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx=-\frac{5a^{\frac{3}{2}}\sqrt{1+\frac{b}{ax^4}}}{16x^2}-\frac{\sqrt{a}b\sqrt{1+\frac{b}{ax^4}}}{8x^6}-\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x^2}\right)}{16\sqrt{b}}$$

input
```
integrate((a+b/x**4)**(3/2)/x**3,x)
```

output
```
-5*a**(3/2)*sqrt(1 + b/(a*x**4))/(16*x**2) - sqrt(a)*b*sqrt(1 + b/(a*x**4)
)/(8*x**6) - 3*a**2*asinh(sqrt(b)/(sqrt(a)*x**2))/(16*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx=\frac{3\,a^2\log\left(\frac{\sqrt{a+\frac{b}{x^4}}x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2+\sqrt{b}}\right)}{32\,\sqrt{b}}-\frac{5\,\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2x^6-3\,\sqrt{a+\frac{b}{x^4}}a^2bx^2}{16\,\left(\left(a+\frac{b}{x^4}\right)^2x^8-2\,\left(a+\frac{b}{x^4}\right)bx^4+b^2\right)}$$

input
```
integrate((a+b/x^4)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/32*a^2*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b
)))/sqrt(b) - 1/16*(5*(a + b/x^4)^(3/2)*a^2*x^6 - 3*sqrt(a + b/x^4)*a^2*b*
x^2)/((a + b/x^4)^2*x^8 - 2*(a + b/x^4)*b*x^4 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx = \frac{\frac{3\,a^3\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{5\,(ax^4+b)^{\frac{3}{2}}a^3 - 3\,\sqrt{ax^4+b}a^3b}{a^2x^8}}{16\,a}$$

input
```
integrate((a+b/x^4)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/16*(3*a^3*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) - (5*(a*x^4 + b)^(3/
2)*a^3 - 3*sqrt(a*x^4 + b)*a^3*b)/(a^2*x^8))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a + b/x^4)^(3/2)/x^3,x)
```

output
```
int((a + b/x^4)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 554, normalized size of antiderivative = 7.80

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^3}\,dx = \frac{24\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}\right)a^3x^{14} + 12\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}\right)}{}$$

input
```
int((a+b/x^4)^(3/2)/x^3,x)
```

output

```
(24*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b
))/sqrt(b))*a**3*x**14 + 12*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b)
 + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**2*b*x**10 - 24*sqrt(a)*sqrt(a*x**4
+ b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**3*x**14 -
 12*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b
))/sqrt(b))*a**2*b*x**10 - 40*sqrt(b)*sqrt(a*x**4 + b)*a**3*x**12 - 56*sqr
t(b)*sqrt(a*x**4 + b)*a**2*b*x**8 - 21*sqrt(b)*sqrt(a*x**4 + b)*a*b**2*x**
4 - 2*sqrt(b)*sqrt(a*x**4 + b)*b**3 - 40*sqrt(b)*sqrt(a)*a**3*x**14 - 76*s
qrt(b)*sqrt(a)*a**2*b*x**10 - 44*sqrt(b)*sqrt(a)*a*b**2*x**6 - 8*sqrt(b)*s
qrt(a)*b**3*x**2 + 24*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt
(b))*a**4*x**16 + 24*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(
b))*a**3*b*x**12 + 3*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(
b))*a**2*b**2*x**8 - 24*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sq
rt(b))*a**4*x**16 - 24*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqr
t(b))*a**3*b*x**12 - 3*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqr
t(b))*a**2*b**2*x**8)/(16*sqrt(b)*x**8*(8*sqrt(a)*sqrt(a*x**4 + b)*a*x**6
+ 4*sqrt(a)*sqrt(a*x**4 + b)*b*x**2 + 8*a**2*x**8 + 8*a*b*x**4 + b**2))
```

3.533 $\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2 \, dx = -\frac{b\sqrt{a+\frac{b}{x^4}}}{3x}+\frac{1}{3}a\sqrt{a+\frac{b}{x^4}}x^3 - \frac{2a^{3/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/3*b*(a+b/x^4)^(1/2)/x+1/3*a*(a+b/x^4)^(1/2)*x^3-2/3*a^(3/4)*b^(3/4)*((a
+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobi
AM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2 \, dx = -\frac{b\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{ax^4}{b}\right)}{3x\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(3/2)*x^2,x]
```

output
```
-1/3*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^4)/b)])/
(x*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 809, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{x^4}\right)^{3/2} dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{3/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{3/2}-2b\int \sqrt{a+\frac{b}{x^4}}d\frac{1}{x}$$

$$\Big\downarrow \text{748}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{3/2}-2b\left(\frac{2}{3}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{3/2}-2b\left(\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)$$

input
```
Int[(a + b/x^4)^(3/2)*x^2,x]
```

output
```
((a + b/x^4)^(3/2)*x^3)/3 - 2*b*(Sqrt[a + b/x^4]/(3*x) + (a^(3/4)*Sqrt[(a
+ b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*Ar
cTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(3*b^(1/4)*Sqrt[a + b/x^4]))
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{(a\,x^4-b)\sqrt{\frac{a\,x^4+b}{x^4}}}{3x}+\frac{4ab\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a\,x^4+b)}$	117
default	$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}} x^3\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^2x^8+4ab\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)x^3-\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,b^2\right)}{3(a\,x^4+b)^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$	138

input
```
int((a+b/x^4)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*x^4-b)/x*((a*x^4+b)/x^4)^(1/2)+4/3*a*b/(I*a^(1/2)/b^(1/2))^(1/2)*(1
-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)*El
lipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx=\int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2\,dx$$

input
```
integrate((a+b/x^4)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
integral((a*x^4 + b)*sqrt((a*x^4 + b)/x^4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx = -\frac{a^{\frac{3}{2}} x^3 \Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((a+b/x**4)**(3/2)*x**2,x)
```

output
```
-a**(3/2)*x**3*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), b*exp_polar(I*pi)/(
a*x**4))/(4*gamma(1/4))
```

Maxima [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2\,dx$$

input
```
integrate((a+b/x^4)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(3/2)*x^2, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2\,dx$$

input
```
integrate((a+b/x^4)^(3/2)*x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx=\int x^2\,\left(a+\frac{b}{x^4}\right)^{3/2}\,dx$$

input
```
int(x^2*(a + b/x^4)^(3/2),x)
```

output
```
int(x^2*(a + b/x^4)^(3/2), x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2\,dx=\frac{\sqrt{a\,x^4+b}\,a\,x^4-5\sqrt{a\,x^4+b}\,b-12\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^8+b\,x^4}dx\right)b^2x^3}{3x^3}$$

input
```
int((a+b/x^4)^(3/2)*x^2,x)
```

output
```
(sqrt(a*x**4 + b)*a*x**4 - 5*sqrt(a*x**4 + b)*b - 12*int(sqrt(a*x**4 + b)/
(a*x**8 + b*x**4),x)*b**2*x**3)/(3*x**3)
```

3.534 $\int \left(a+\frac{b}{x^4}\right)^{3/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 251

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = -\frac{b\sqrt{a+\frac{b}{x^4}}}{5x^3} - \frac{12a\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{5\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + a\sqrt{a+\frac{b}{x^4}}\,x + \frac{12a^{5/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{a+\frac{b}{x^4}}} - \frac{6a^{5/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5\sqrt{a+\frac{b}{x^4}}}$$

output

```
-1/5*b*(a+b/x^4)^(1/2)/x^3-12/5*a*b^(1/2)*(a+b/x^4)^(1/2)/(a^(1/2)+b^(1/2)
/x^2)/x+a*(a+b/x^4)^(1/2)*x+12/5*a^(5/4)*b^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/
2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*EllipticE(sin(2*arccot(a^(1/4)*x/b^
(1/4))),1/2*2^(1/2))/(a+b/x^4)^(1/2)-6/5*a^(5/4)*b^(1/4)*((a+b/x^4)/(a^(1/
2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^
(1/4)*x/b^(1/4)),1/2*2^(1/2))/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = -\frac{b\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{ax^4}{b}\right)}{5x^3\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(3/2),x]
```

output
```
-1/5*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((a*x^4)/b)])
/(x^3*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {773, 809, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{3/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$x\left(a+\frac{b}{x^4}\right)^{3/2} - 6b\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$x\left(a+\frac{b}{x^4}\right)^{3/2}-6b\left(\frac{2}{5}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)$$

$$\Big\downarrow \ 834$$

$$x\left(a+\frac{b}{x^4}\right)^{3/2}-6b\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)$$

$$\Big\downarrow \ 27$$

$$x\left(a+\frac{b}{x^4}\right)^{3/2}-6b\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)$$

$$\Big\downarrow \ 761$$

$$x\left(a+\frac{b}{x^4}\right)^{3/2}-$$
$$6b\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)$$

$$\Big\downarrow \ 1510$$

$$x\left(a+\frac{b}{x^4}\right)^{3/2}-$$
$$6b\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right.\right.}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}}{\sqrt{b}}\right.\right.$$

input
```
Int[(a + b/x^4)^(3/2),x]
```

output
```
(a + b/x^4)^(3/2)*x - 6*b*(Sqrt[a + b/x^4]/(5*x^3) + (2*a*(-((-(Sqrt[a + b
/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] +
Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4
)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^
4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b
^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3/4)*Sqrt[a + b/x^4])))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{(7a x^4+b)\sqrt{\frac{a x^4+b}{x^4}}}{5x^3}+\frac{12 i a^{\frac{3}{2}}\sqrt{b}\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right)\sqrt{\frac{a x^4+b}{x^4}}\,x^2}{5\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a x^4+b)}$
default	$-\frac{\left(\frac{a x^4+b}{x^4}\right)^{\frac{3}{2}} x\left(-12 i a^{\frac{3}{2}}\sqrt{b}\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x^5\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+12 i a^{\frac{3}{2}}\sqrt{b}\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x^5\,\mathrm{Elli}\right.}{5(a x^4+b)^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int((a+b/x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(7*a*x^4+b)/x^3*((a*x^4+b)/x^4)^(1/2)+12/5*I*a^(3/2)*b^(1/2)/(I*a^(1/
2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2
)^(1/2)/(a*x^4+b)*(EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-EllipticE(x*(I
*a^(1/2)/b^(1/2))^(1/2),I))*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/x^4)^(3/2),x, algorithm="fricas")
```

output
```
integral((a*x^4 + b)*sqrt((a*x^4 + b)/x^4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = -\frac{a^{\frac{3}{2}} x\Gamma\left(-\frac{1}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((a+b/x**4)**(3/2),x)
```

output
```
-a**(3/2)*x*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), b*exp_polar(I*pi)/(a*x
**4))/(4*gamma(3/4))
```

Maxima [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(3/2), x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.15

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = -\frac{x\left(a+\frac{b}{x^4}\right)^{3/2} {}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\ -\frac{1}{4};\ -\frac{a\,x^4}{b}\right)}{5\left(\frac{a\,x^4}{b}+1\right)^{3/2}}$$

input
```
int((a + b/x^4)^(3/2),x)
```

output
```
-(x*(a + b/x^4)^(3/2)*hypergeom([-3/2, -5/4], -1/4, -(a*x^4)/b))/(5*((a*x^
4)/b + 1)^(3/2))
```

Reduce [F]

$$\int \left(a+\frac{b}{x^4}\right)^{3/2} dx = \frac{\sqrt{a\,x^4+b}\,a\,x^4-\sqrt{a\,x^4+b}\,b-4\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{10}+b\,x^6}dx\right)b^2x^5}{x^5}$$

input
```
int((a+b/x^4)^(3/2),x)
```

output
```
(sqrt(a*x**4 + b)*a*x**4 - sqrt(a*x**4 + b)*b - 4*int(sqrt(a*x**4 + b)/(a*
x**10 + b*x**6),x)*b**2*x**5)/x**5
```

3.535 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2} dx = -\frac{2a\sqrt{a+\frac{b}{x^4}}}{7x} - \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x} - \frac{2a^{7/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-2/7*a*(a+b/x^4)^(1/2)/x-1/7*(a+b/x^4)^(3/2)/x-2/7*a^(7/4)*((a+b/x^4)/(a^(
1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(
a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/b^(1/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx = -\frac{b\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{ax^4}{b}\right)}{7x^5\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(3/2)/x^2,x]
```

output
```
-1/7*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((a*x^4)/b)])
/(x^5*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 748, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{3/2}d\frac{1}{x}$$

$$\Big\downarrow \text{748}$$

$$-\frac{6}{7}a\int \sqrt{a+\frac{b}{x^4}}d\frac{1}{x}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}$$

$$\Big\downarrow \text{748}$$

$$-\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}$$

$$\Big\downarrow \text{761}$$

$$-\frac{6}{7}a\left(\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}$$

input
```
Int[(a + b/x^4)^(3/2)/x^2,x]
```

output
```
-1/7*(a + b/x^4)^(3/2)/x - (6*a*(Sqrt[a + b/x^4]/(3*x) + (a^(3/4)*Sqrt[(a
+ b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*Ar
cTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(3*b^(1/4)*Sqrt[a + b/x^4])))/7
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(3a\,x^4+b)\sqrt{\frac{a\,x^4+b}{x^4}}}{7x^5}+\frac{4a^2\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a\,x^4+b)}$	117
default	$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(-4a^2\sqrt{\frac{-i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)x^7+3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^2x^8+4\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,ab\,x^4+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,b^2\right)}{7x(a\,x^4+b)^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$	154

input
```
int((a+b/x^4)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(3*a*x^4+b)/x^5*((a*x^4+b)/x^4)^(1/2)+4/7*a^2/(I*a^(1/2)/b^(1/2))^(1/
2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+
b)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx=-\frac{4\,a\sqrt{b}x^5\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\mid -1)+(3\,ax^4+b)\sqrt{\frac{ax^4+b}{x^4}}}{7\,x^5}$$

input
```
integrate((a+b/x^4)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-1/7*(4*a*sqrt(b)*x^5*(-a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4)), -1)
+ (3*a*x^4 + b)*sqrt((a*x^4 + b)/x^4))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx = -\frac{a^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((a+b/x**4)**(3/2)/x**2,x)
```

output
```
-a**(3/2)*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**4)
)/(4*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/x^4)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/x^4)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(3/2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx = -\frac{\left(a\,x^4+b\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\,\left(\frac{b}{a}+x^4\right)^{3/2}}$$

input
```
int((a + b/x^4)^(3/2)/x^2,x)
```

output
```
-((b + a*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, -b/(a*x^4)))/(x*(b/a + x^4
)^(3/2))
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}\,dx = \frac{-5\sqrt{a\,x^4+b}\,a\,x^4+\sqrt{a\,x^4+b}\,b+12\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{12}+b\,x^8}dx\right)b^2x^7}{5x^7}$$

input
```
int((a+b/x^4)^(3/2)/x^2,x)
```

output
```
( - 5*sqrt(a*x**4 + b)*a*x**4 + sqrt(a*x**4 + b)*b + 12*int(sqrt(a*x**4 +
b)/(a*x**12 + b*x**8),x)*b**2*x**7)/(5*x**7)
```

3.536 $\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 257

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx=-\frac{2a\sqrt{a+\frac{b}{x^4}}}{15x^3}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}-\frac{4a^2\sqrt{a+\frac{b}{x^4}}}{15\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\frac{4a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{2a^{9/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-2/15*a*(a+b/x^4)^(1/2)/x^3-1/9*(a+b/x^4)^(3/2)/x^3-4/15*a^2*(a+b/x^4)^(1/
2)/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)/x+4/15*a^(9/4)*((a+b/x^4)/(a^(1/2)+b^(1/2
)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*EllipticE(sin(2*arccot(a^(1/4)*x/b^(
1/4))),1/2*2^(1/2))/b^(3/4)/(a+b/x^4)^(1/2)-2/15*a^(9/4)*((a+b/x^4)/(a^(1/
2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^
(1/4)*x/b^(1/4)),1/2*2^(1/2))/b^(3/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.20

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx=-\frac{b\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{ax^4}{b}\right)}{9x^7\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(3/2)/x^4,x]
```

output
```
-1/9*(b*Sqrt[a + b/x^4]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^4)/b)])
/(x^7*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {858, 811, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$-\frac{2}{3}a\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}d\frac{1}{x}-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}$$

$$\Big\downarrow \text{811}$$

$$-\frac{2}{3}a\left(\frac{2}{5}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}$$

$\Big\downarrow$ 834

$$-\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}$$

$\Big\downarrow$ 27

$$-\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}$$

$\Big\downarrow$ 761

$$-\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}$$

$\Big\downarrow$ 1510

$$-\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)\right.}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}}{\sqrt{b}}\right.\right.$$

$$\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}$$

input
```
Int[(a + b/x^4)^(3/2)/x^4,x]
```

output
```
-1/9*(a + b/x^4)^(3/2)/x^3 - (2*a*(Sqrt[a + b/x^4]/(5*x^3) + (2*a*(-((-(Sq
rt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sq
rt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)
/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(
a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*
ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3/4)*Sqrt[a + b/x^4])))/5))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{(12a^2x^8+11ab\,x^4+5b^2)\sqrt{\frac{a\,x^4+b}{x^4}}}{45x^7b}+\frac{4ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right)\sqrt{\frac{a\,x^4+b}{x^4}}}{15\sqrt{b}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,(a\,x^4+b)}$
default	$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}\left(-12ia^{\frac{5}{2}}\sqrt{\frac{-i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x^9b\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+12ia^{\frac{5}{2}}\sqrt{\frac{-i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,x^9b\,\text{EllipticE}\left(\right.\right.}{45x^3(a\,x^4+b)^2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int((a+b/x^4)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/45*(12*a^2*x^8+11*a*b*x^4+5*b^2)/x^7/b*((a*x^4+b)/x^4)^(1/2)+4/15*I/b^(
1/2)*a^(5/2)/(I*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+
I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)*(EllipticF(x*(I*a^(1/2)/b^(1/2))^(1
/2),I)-EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/2),I))*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.42

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx =$$
$$-\frac{12\,a^2\sqrt{b}x^7\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|-1)-12\,a^2\sqrt{b}x^7\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|-1)+(12\,a^2x^8+}{45\,bx^7}$$

input
```
integrate((a+b/x^4)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/45*(12*a^2*sqrt(b)*x^7*(-a/b)^(3/4)*elliptic_e(arcsin(x*(-a/b)^(1/4)),
-1) - 12*a^2*sqrt(b)*x^7*(-a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4)), -
1) + (12*a^2*x^8 + 11*a*b*x^4 + 5*b^2)*sqrt((a*x^4 + b)/x^4))/(b*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx = -\frac{a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((a+b/x**4)**(3/2)/x**4,x)
```

output
```
-a**(3/2)*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4)
)/(4*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/x^4)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/x^4)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a + b/x^4)^(3/2)/x^4,x)
```

output
```
int((a + b/x^4)^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^4}\,dx=\frac{-7\sqrt{a\,x^4+b}\,a\,x^4-\sqrt{a\,x^4+b}\,b+12\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{14}+b\,x^{10}}dx\right)b^2x^9}{21x^9}$$

input
```
int((a+b/x^4)^(3/2)/x^4,x)
```

output
```
( - 7*sqrt(a*x**4 + b)*a*x**4 - sqrt(a*x**4 + b)*b + 12*int(sqrt(a*x**4 +
b)/(a*x**14 + b*x**10),x)*b**2*x**9)/(21*x**9)
```

3.537 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3 \, dx = -ab\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}b\left(a+\frac{b}{x^4}\right)^{3/2} + \frac{1}{4}a^2\sqrt{a+\frac{b}{x^4}}x^4 + \frac{5}{4}a^{3/2}b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)$$

output
```
-a*b*(a+b/x^4)^(1/2)-1/6*b*(a+b/x^4)^(3/2)+1/4*a^2*(a+b/x^4)^(1/2)*x^4+5/4
*a^(3/2)*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3 \, dx = \frac{\sqrt{a+\frac{b}{x^4}}\left(\sqrt{b+ax^4}(-2b^2-14abx^4+3a^2x^8)+15a^{3/2}bx^6\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)\right)}{12x^4\sqrt{b+ax^4}}$$

input
```
Integrate[(a + b/x^4)^(5/2)*x^3,x]
```

output
```
(Sqrt[a + b/x^4]*(Sqrt[b + a*x^4]*(-2*b^2 - 14*a*b*x^4 + 3*a^2*x^8) + 15*a
^(3/2)*b*x^6*Log[Sqrt[a]*x^2 + Sqrt[b + a*x^4]]))/(12*x^4*Sqrt[b + a*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^4}\right)^{5/2} dx$$

$$\Big\downarrow \text{ 798}$$

$$-\frac{1}{4} \int \left(a+\frac{b}{x^4}\right)^{5/2} x^8 d\frac{1}{x^4}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{4} \left(x^4 \left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5}{2} b \int \left(a+\frac{b}{x^4}\right)^{3/2} x^4 d\frac{1}{x^4}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{4} \left(x^4 \left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5}{2} b \left(a \int \sqrt{a+\frac{b}{x^4}} x^4 d\frac{1}{x^4}+\frac{2}{3} \left(a+\frac{b}{x^4}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{4} \left(x^4 \left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5}{2} b \left(a \left(a \int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}} d\frac{1}{x^4}+2 \sqrt{a+\frac{b}{x^4}}\right)+\frac{2}{3} \left(a+\frac{b}{x^4}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4} \left(x^4 \left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5}{2} b \left(a \left(\frac{2 a \int \frac{1}{\frac{1}{b x^8}-\frac{a}{b}} d\sqrt{a+\frac{b}{x^4}}}{b}+2 \sqrt{a+\frac{b}{x^4}}\right)+\frac{2}{3} \left(a+\frac{b}{x^4}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{4}\left(x^4\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{5}{2}b\left(a\left(2\sqrt{a+\frac{b}{x^4}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)\right)$$

input
```
Int[(a + b/x^4)^(5/2)*x^3,x]
```

output
```
((a + b/x^4)^(5/2)*x^4 - (5*b*((2*(a + b/x^4)^(3/2))/3 + a*(2*Sqrt[a + b/x
^4] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])))/2)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{(3a^2x^8-14ab\,x^4-2b^2)\sqrt{\frac{a\,x^4+b}{x^4}}}{12x^4}+\frac{5a^{\frac{3}{2}}b\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{4\sqrt{a\,x^4+b}}$	90
default	$\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}x^4\left(3a^2x^8\sqrt{a\,x^4+b}+15a^{\frac{3}{2}}b\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)x^6-14ab\sqrt{a\,x^4+b}\,x^4-2b^2\sqrt{a\,x^4+b}\right)}{12(a\,x^4+b)^{\frac{5}{2}}}$	103

input
```
int((a+b/x^4)^(5/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*a^2*x^8-14*a*b*x^4-2*b^2)/x^4*((a*x^4+b)/x^4)^(1/2)+5/4*a^(3/2)*b*
ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*((a*x^4+b)/x^4)^(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.99

$$\int\left(a+\frac{b}{x^4}\right)^{5/2}x^3\,dx=\left[\frac{15\,a^{\frac{3}{2}}bx^4\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+2\,(3\,a^2x^8-14\,abx^4-2\,b^2)\sqrt{\frac{ax^4+b}{x^4}}}{24\,x^4},\right.$$
$$\left.-\frac{15\,\sqrt{-a}abx^4\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)-(3\,a^2x^8-14\,abx^4-2\,b^2)\sqrt{\frac{ax^4+b}{x^4}}}{12\,x^4}\right]$$

input
```
integrate((a+b/x^4)^(5/2)*x^3,x, algorithm="fricas")
```

output
```
[1/24*(15*a^(3/2)*b*x^4*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4)
 - b) + 2*(3*a^2*x^8 - 14*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^4, -1/
12*(15*sqrt(-a)*a*b*x^4*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4)/a) - (3*a^2*
x^8 - 14*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^4]
```

Sympy [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx = \frac{a^{\frac{5}{2}}x^4\sqrt{1+\frac{b}{ax^4}}}{4} - \frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^4}}}{6} - \frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax^4}\right)}{8} + \frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{4} - \frac{\sqrt{a}b^2\sqrt{1+\frac{b}{ax^4}}}{6x^4}$$

input
```
integrate((a+b/x**4)**(5/2)*x**3,x)
```

output
```
a**(5/2)*x**4*sqrt(1 + b/(a*x**4))/4 - 7*a**(3/2)*b*sqrt(1 + b/(a*x**4))/6
 - 5*a**(3/2)*b*log(b/(a*x**4))/8 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x**4))
+ 1)/4 - sqrt(a)*b**2*sqrt(1 + b/(a*x**4))/(6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx = \frac{1}{4}\sqrt{a+\frac{b}{x^4}}a^2x^4 - \frac{5}{8}a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right) - \frac{1}{6}\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}b - \sqrt{a+\frac{b}{x^4}}ab$$

input
```
integrate((a+b/x^4)^(5/2)*x^3,x, algorithm="maxima")
```

output
```
1/4*sqrt(a + b/x^4)*a^2*x^4 - 5/8*a^(3/2)*b*log((sqrt(a + b/x^4) - sqrt(a)
)/(sqrt(a + b/x^4) + sqrt(a))) - 1/6*(a + b/x^4)^(3/2)*b - sqrt(a + b/x^4)
*a*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(63) = 126$.

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.75

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx = \frac{1}{4}\sqrt{ax^4+b}a^2x^2 - \frac{5}{8}a^{\frac{3}{2}}b\log\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2\right) + \frac{9\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^4a^{\frac{3}{2}}b^2 - 12\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2a^{\frac{3}{2}}b^3 + 7a^{\frac{3}{2}}b^4}{3\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2-b\right)^3}$$

input
```
integrate((a+b/x^4)^(5/2)*x^3,x, algorithm="giac")
```

output
```
1/4*sqrt(a*x^4 + b)*a^2*x^2 - 5/8*a^(3/2)*b*log((sqrt(a)*x^2 - sqrt(a*x^4
+ b))^2) + 1/3*(9*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^4*a^(3/2)*b^2 - 12*(sqrt
(a)*x^2 - sqrt(a*x^4 + b))^2*a^(3/2)*b^3 + 7*a^(3/2)*b^4)/((sqrt(a)*x^2 -
sqrt(a*x^4 + b))^2 - b)^3
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx = \frac{a^2\,x^4\,\sqrt{a+\frac{b}{x^4}}}{4} - \frac{b\,{\left(a+\frac{b}{x^4}\right)}^{3/2}}{6} - a\,b\,\sqrt{a+\frac{b}{x^4}} - \frac{a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^4}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{4}$$

input
```
int(x^3*(a + b/x^4)^(5/2),x)
```

output
```
(a^2*x^4*(a + b/x^4)^(1/2))/4 - (b*(a + b/x^4)^(3/2))/6 - (a^(3/2)*b*atan(
((a + b/x^4)^(1/2)*1i)/a^(1/2))*5i)/4 - a*b*(a + b/x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 5.49

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^3\,dx = \frac{480\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a^3b\,x^{14}+360\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a^2b^2x^{1}}{}$$

input
```
int((a+b/x^4)^(5/2)*x^3,x)
```

output
```
(480*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b
))*a**3*b*x**14 + 360*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqr
t(a)*x**2)/sqrt(b))*a**2*b**2*x**10 + 30*sqrt(a)*sqrt(a*x**4 + b)*log((sqr
t(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a*b**3*x**6 + 96*sqrt(a)*sqrt(a*x**
4 + b)*a**4*x**18 - 248*sqrt(a)*sqrt(a*x**4 + b)*a**3*b*x**14 - 534*sqrt(a
)*sqrt(a*x**4 + b)*a**2*b**2*x**10 - 215*sqrt(a)*sqrt(a*x**4 + b)*a*b**3*x
**6 - 20*sqrt(a)*sqrt(a*x**4 + b)*b**4*x**2 + 480*log((sqrt(a*x**4 + b) +
sqrt(a)*x**2)/sqrt(b))*a**4*b*x**16 + 600*log((sqrt(a*x**4 + b) + sqrt(a)*
x**2)/sqrt(b))*a**3*b**2*x**12 + 150*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)
/sqrt(b))*a**2*b**3*x**8 + 96*a**5*x**20 - 200*a**4*b*x**16 - 670*a**3*b**
2*x**12 - 445*a**2*b**3*x**8 - 80*a*b**4*x**4 - 4*b**5)/(24*x**6*(16*sqrt(
a*x**4 + b)*a**2*x**8 + 12*sqrt(a*x**4 + b)*a*b*x**4 + sqrt(a*x**4 + b)*b*
*2 + 16*sqrt(a)*a**2*x**10 + 20*sqrt(a)*a*b*x**6 + 5*sqrt(a)*b**2*x**2))
```

3.538 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 96

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = -\frac{b^2\sqrt{a+\frac{b}{x^4}}}{8x^6} - \frac{9ab\sqrt{a+\frac{b}{x^4}}}{16x^2} + \frac{1}{2}a^2\sqrt{a+\frac{b}{x^4}}x^2 - \frac{15}{16}a^2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2}\right)$$

output
```
-1/8*b^2*(a+b/x^4)^(1/2)/x^6-9/16*a*b*(a+b/x^4)^(1/2)/x^2+1/2*a^2*(a+b/x^4
)^(1/2)*x^2-15/16*a^2*b^(1/2)*arctanh(b^(1/2)/(a+b/x^4)^(1/2)/x^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = -\frac{\sqrt{a+\frac{b}{x^4}}\left(\sqrt{b+ax^4}(2b^2+9abx^4-8a^2x^8)+15a^2\sqrt{b}x^8\,\text{arctanh}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)\right)}{16x^6\sqrt{b+ax^4}}$$

input
```
Integrate[(a + b/x^4)^(5/2)*x,x]
```

output
```
-1/16*(Sqrt[a + b/x^4]*(Sqrt[b + a*x^4]*(2*b^2 + 9*a*b*x^4 - 8*a^2*x^8) +
15*a^2*Sqrt[b]*x^8*ArcTanh[Sqrt[b + a*x^4]/Sqrt[b]]))/(x^6*Sqrt[b + a*x^4]
)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {858, 807, 247, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^4}\right)^{5/2}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{5/2}x^3d\frac{1}{x}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{5/2}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\int \left(a+\frac{b}{x^2}\right)^{3/2}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x^2}+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)\right)$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{b}{\sqrt{a+\frac{b}{x^2}}x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(x\left(a+\frac{b}{x^2}\right)^{5/2}-5b\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)\right)$$

input
```
Int[(a + b/x^4)^(5/2)*x,x]
```

output
```
((a + b/x^2)^(5/2)*x - 5*b*((a + b/x^2)^(3/2)/(4*x^2) + (3*a*(Sqrt[a + b/x
^2]/(2*x^2) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x^2)])/(2*Sqrt[b])))/4))
/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

method	result	size
risch	$-\frac{b\left(9a\,x^{4}+2b\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{16x^{6}}+\frac{\left(\frac{\sqrt{a\,x^{4}+b}\,a^{2}}{2}-\frac{15\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right)\sqrt{b}\,a^{2}}{16}\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}{\sqrt{a\,x^{4}+b}}$	103
default	$\frac{\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{5}{2}}x^{2}\left(-15\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right)\sqrt{b}\,a^{2}x^{8}+8a^{2}x^{8}\sqrt{a\,x^{4}+b}-9ab\sqrt{a\,x^{4}+b}\,x^{4}-2b^{2}\sqrt{a\,x^{4}+b}\right)}{16\left(a\,x^{4}+b\right)^{\frac{5}{2}}}$	108

input
```
int((a+b/x^4)^(5/2)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/16*b*(9*a*x^4+2*b)/x^6*((a*x^4+b)/x^4)^(1/2)+(1/2*(a*x^4+b)^(1/2)*a^2-1
5/16*ln((2*b+2*b^(1/2)*(a*x^4+b)^(1/2))/x^2)*b^(1/2)*a^2)*((a*x^4+b)/x^4)^
(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.82

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = \left[\frac{15\,a^2\sqrt{b}x^6\log\left(\frac{ax^4-2\,\sqrt{b}x^2\sqrt{\frac{ax^4+b}{x^4}}+2\,b}{x^4}\right)+2\,(8\,a^2x^8-9\,abx^4-2\,b^2)\sqrt{\frac{ax^4+b}{x^4}}}{32\,x^6}, \frac{15\,a^2\sqrt{-b}x^6\,\mathrm{ar}\ldots}{}\right.$$

input
```
integrate((a+b/x^4)^(5/2)*x,x, algorithm="fricas")
```

output
```
[1/32*(15*a^2*sqrt(b)*x^6*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4)
 + 2*b)/x^4) + 2*(8*a^2*x^8 - 9*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^
6, 1/16*(15*a^2*sqrt(-b)*x^6*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/(a*
x^4 + b)) + (8*a^2*x^8 - 9*a*b*x^4 - 2*b^2)*sqrt((a*x^4 + b)/x^4))/x^6]
```

Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = \frac{a^{\frac{5}{2}}x^2}{2\sqrt{1+\frac{b}{ax^4}}} - \frac{a^{\frac{3}{2}}b}{16x^2\sqrt{1+\frac{b}{ax^4}}} - \frac{11\sqrt{a}b^2}{16x^6\sqrt{1+\frac{b}{ax^4}}} - \frac{15a^2\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x^2}\right)}{16} - \frac{b^3}{8\sqrt{a}x^{10}\sqrt{1+\frac{b}{ax^4}}}$$

input
```
integrate((a+b/x**4)**(5/2)*x,x)
```

output
```
a**(5/2)*x**2/(2*sqrt(1 + b/(a*x**4))) - a**(3/2)*b/(16*x**2*sqrt(1 + b/(a
*x**4))) - 11*sqrt(a)*b**2/(16*x**6*sqrt(1 + b/(a*x**4))) - 15*a**2*sqrt(b
)*asinh(sqrt(b)/(sqrt(a)*x**2))/16 - b**3/(8*sqrt(a)*x**10*sqrt(1 + b/(a*x
**4)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.45

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = \frac{1}{2}\sqrt{a+\frac{b}{x^4}}a^2x^2 + \frac{15}{32}a^2\sqrt{b}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2+\sqrt{b}}\right) - \frac{9\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2bx^6 - 7\sqrt{a+\frac{b}{x^4}}a^2b^2x^2}{16\left(\left(a+\frac{b}{x^4}\right)^2x^8 - 2\left(a+\frac{b}{x^4}\right)bx^4 + b^2\right)}$$

input
```
integrate((a+b/x^4)^(5/2)*x,x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^4)*a^2*x^2 + 15/32*a^2*sqrt(b)*log((sqrt(a + b/x^4)*x^2 -
 sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b))) - 1/16*(9*(a + b/x^4)^(3/2)*a^2
*b*x^6 - 7*sqrt(a + b/x^4)*a^2*b^2*x^2)/((a + b/x^4)^2*x^8 - 2*(a + b/x^4)
*b*x^4 + b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = \frac{\frac{15\,a^3b\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 8\sqrt{ax^4+b}a^3 - \frac{9\left(ax^4+b\right)^{\frac{3}{2}}a^3b - 7\sqrt{ax^4+b}a^3b^2}{a^2x^8}}{16\,a}$$

input
```
integrate((a+b/x^4)^(5/2)*x,x, algorithm="giac")
```

output
```
1/16*(15*a^3*b*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) + 8*sqrt(a*x^4 +
b)*a^3 - (9*(a*x^4 + b)^(3/2)*a^3*b - 7*sqrt(a*x^4 + b)*a^3*b^2)/(a^2*x^8)
)/a
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = \int x\left(a+\frac{b}{x^4}\right)^{5/2} dx$$

input
```
int(x*(a + b/x^4)^(5/2),x)
```

output
```
int(x*(a + b/x^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 713, normalized size of antiderivative = 7.43

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x\,dx = \text{Too large to display}$$

input
```
int((a+b/x^4)^(5/2)*x,x)
```

output

```
(128*sqrt(a)*sqrt(a*x**4 + b)*a**4*x**18 + 16*sqrt(a)*sqrt(a*x**4 + b)*a**
3*b*x**14 - 172*sqrt(a)*sqrt(a*x**4 + b)*a**2*b**2*x**10 - 85*sqrt(a)*sqrt
(a*x**4 + b)*a*b**3*x**6 - 10*sqrt(a)*sqrt(a*x**4 + b)*b**4*x**2 + 240*sqr
t(b)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt
(b))*a**4*x**16 + 180*sqrt(b)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqr
t(a)*x**2 - sqrt(b))/sqrt(b))*a**3*b*x**12 + 15*sqrt(b)*sqrt(a*x**4 + b)*l
og((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**2*b**2*x**8 - 2
40*sqrt(b)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b)
)/sqrt(b))*a**4*x**16 - 180*sqrt(b)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b)
 + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**3*b*x**12 - 15*sqrt(b)*sqrt(a*x**4
+ b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**2*b**2*x*
*8 + 240*sqrt(b)*sqrt(a)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/s
qrt(b))*a**4*x**18 + 300*sqrt(b)*sqrt(a)*log((sqrt(a*x**4 + b) + sqrt(a)*x
**2 - sqrt(b))/sqrt(b))*a**3*b*x**14 + 75*sqrt(b)*sqrt(a)*log((sqrt(a*x**4
 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**2*b**2*x**10 - 240*sqrt(b)*sqr
t(a)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**4*x**18 -
 300*sqrt(b)*sqrt(a)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(
b))*a**3*b*x**14 - 75*sqrt(b)*sqrt(a)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2
 + sqrt(b))/sqrt(b))*a**2*b**2*x**10 + 128*a**5*x**20 + 80*a**4*b*x**16 -
180*a**3*b**2*x**12 - 165*a**2*b**3*x**8 - 35*a*b**4*x**4 - 2*b**5)/(16...
```

3.539 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x} \, dx = -\frac{1}{2}a^2\sqrt{a+\frac{b}{x^4}} - \frac{1}{6}a\left(a+\frac{b}{x^4}\right)^{3/2} - \frac{1}{10}\left(a+\frac{b}{x^4}\right)^{5/2} + \frac{1}{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)$$

output
```
-1/2*a^2*(a+b/x^4)^(1/2)-1/6*a*(a+b/x^4)^(3/2)-1/10*(a+b/x^4)^(5/2)+1/2*a^
(5/2)*arctanh((a+b/x^4)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x} \, dx = \frac{\sqrt{a+\frac{b}{x^4}}\left(-3b^2-11abx^4-23a^2x^8+\frac{15a^{5/2}x^{10}\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{\sqrt{b+ax^4}}\right)}{30x^8}$$

input
```
Integrate[(a + b/x^4)^(5/2)/x,x]
```

output
```
(Sqrt[a + b/x^4]*(-3*b^2 - 11*a*b*x^4 - 23*a^2*x^8 + (15*a^(5/2)*x^10*Log[
Sqrt[a]*x^2 + Sqrt[b + a*x^4]])/Sqrt[b + a*x^4]))/(30*x^8)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \left(a+\frac{b}{x^4}\right)^{5/2}x^4d\frac{1}{x^4}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-a\int \left(a+\frac{b}{x^4}\right)^{3/2}x^4d\frac{1}{x^4}-\frac{2}{5}\left(a+\frac{b}{x^4}\right)^{5/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-a\left(a\int \sqrt{a+\frac{b}{x^4}}x^4d\frac{1}{x^4}+\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^4}\right)^{5/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-a\left(a\left(a\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}+2\sqrt{a+\frac{b}{x^4}}\right)+\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^4}\right)^{5/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-a\left(a\left(\frac{2a\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{b}+2\sqrt{a+\frac{b}{x^4}}\right)+\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^4}\right)^{5/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-a\left(a\left(2\sqrt{a+\frac{b}{x^4}}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+\frac{b}{x^4}\right)^{3/2}\right)-\frac{2}{5}\left(a+\frac{b}{x^4}\right)^{5/2}\right)$$

input
```
Int[(a + b/x^4)^(5/2)/x,x]
```

output
```
((-2*(a + b/x^4)^(5/2))/5 - a*((2*(a + b/x^4)^(3/2))/3 + a*(2*Sqrt[a + b/x
^4] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{(23a^2x^8+11ab\,x^4+3b^2)\sqrt{\frac{a\,x^4+b}{x^4}}}{30x^8}+\frac{a^{\frac{5}{2}}\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}{2\sqrt{a\,x^4+b}}$	89
default	$-\frac{\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}\left(-15a^{\frac{5}{2}}\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)x^{10}+23a^2x^8\sqrt{a\,x^4+b}+11ab\sqrt{a\,x^4+b}\,x^4+3b^2\sqrt{a\,x^4+b}\right)}{30(a\,x^4+b)^{\frac{5}{2}}}$	99

input
```
int((a+b/x^4)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(23*a^2*x^8+11*a*b*x^4+3*b^2)/x^8*((a*x^4+b)/x^4)^(1/2)+1/2*a^(5/2)*
ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*((a*x^4+b)/x^4)^(1/2)*x^2/(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.08

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx=\left[\frac{15\,a^{\frac{5}{2}}x^8\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-2\,(23\,a^2x^8+11\,abx^4+3\,b^2)\sqrt{\frac{ax^4+b}{x^4}}}{60\,x^8},\right.$$
$$\left.-\frac{15\,\sqrt{-a}a^2x^8\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)+(23\,a^2x^8+11\,abx^4+3\,b^2)\sqrt{\frac{ax^4+b}{x^4}}}{30\,x^8}\right]$$

input
```
integrate((a+b/x^4)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/60*(15*a^(5/2)*x^8*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) -
 b) - 2*(23*a^2*x^8 + 11*a*b*x^4 + 3*b^2)*sqrt((a*x^4 + b)/x^4))/x^8, -1/3
0*(15*sqrt(-a)*a^2*x^8*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4)/a) + (23*a^2*
x^8 + 11*a*b*x^4 + 3*b^2)*sqrt((a*x^4 + b)/x^4))/x^8]
```

Sympy [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx = -\frac{23a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^4}}}{30} - \frac{a^{\frac{5}{2}}\log\left(\frac{b}{ax^4}\right)}{4} + \frac{a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{2} - \frac{11a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^4}}}{30x^4} - \frac{\sqrt{a}b^2\sqrt{1+\frac{b}{ax^4}}}{10x^8}$$

input
```
integrate((a+b/x**4)**(5/2)/x,x)
```

output
```
-23*a**(5/2)*sqrt(1 + b/(a*x**4))/30 - a**(5/2)*log(b/(a*x**4))/4 + a**(5/
2)*log(sqrt(1 + b/(a*x**4)) + 1)/2 - 11*a**(3/2)*b*sqrt(1 + b/(a*x**4))/(3
0*x**4) - sqrt(a)*b**2*sqrt(1 + b/(a*x**4))/(10*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx = -\frac{1}{4}a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right) - \frac{1}{10}\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} - \frac{1}{6}\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a - \frac{1}{2}\sqrt{a+\frac{b}{x^4}}a^2$$

input
```
integrate((a+b/x^4)^(5/2)/x,x, algorithm="maxima")
```

output
```
-1/4*a^(5/2)*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))
- 1/10*(a + b/x^4)^(5/2) - 1/6*(a + b/x^4)^(3/2)*a - 1/2*sqrt(a + b/x^4)*a
^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(57) = 114$.

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.34

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx = -\frac{1}{4}a^{\frac{5}{2}}\log\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2\right) + \frac{45\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^8 a^{\frac{5}{2}}b - 90\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^6 a^{\frac{5}{2}}b^2 + 140\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^4 a^{\frac{5}{2}}b^3 - 70\left(\sqrt{a}x^2\right.}{15\left(\left(\sqrt{a}x^2-\sqrt{ax^4+b}\right)^2-b\right)^5}$$

input
```
integrate((a+b/x^4)^(5/2)/x,x, algorithm="giac")
```

output
```
-1/4*a^(5/2)*log((sqrt(a)*x^2 - sqrt(a*x^4 + b))^2) + 1/15*(45*(sqrt(a)*x^
2 - sqrt(a*x^4 + b))^8*a^(5/2)*b - 90*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^6*a^
(5/2)*b^2 + 140*(sqrt(a)*x^2 - sqrt(a*x^4 + b))^4*a^(5/2)*b^3 - 70*(sqrt(a
)*x^2 - sqrt(a*x^4 + b))^2*a^(5/2)*b^4 + 23*a^(5/2)*b^5)/((sqrt(a)*x^2 - s
qrt(a*x^4 + b))^2 - b)^5
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx = -\frac{a\left(a+\frac{b}{x^4}\right)^{3/2}}{6} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{10} - \frac{a^2\sqrt{a+\frac{b}{x^4}}}{2} - \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x^4}}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{2}$$

input
```
int((a + b/x^4)^(5/2)/x,x)
```

output
```
- (a^(5/2)*atan(((a + b/x^4)^(1/2)*1i)/a^(1/2))*1i)/2 - (a*(a + b/x^4)^(3/
2))/6 - (a + b/x^4)^(5/2)/10 - (a^2*(a + b/x^4)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.73

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}\,dx = \frac{240\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a^4x^{18}+180\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a^3b\,x}{}$$

input
```
int((a+b/x^4)^(5/2)/x,x)
```

output
```
(240*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b
))*a**4*x**18 + 180*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(
a)*x**2)/sqrt(b))*a**3*b*x**14 + 15*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x
**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**2*b**2*x**10 - 288*sqrt(a)*sqrt(a*x**
4 + b)*a**4*x**18 - 576*sqrt(a)*sqrt(a*x**4 + b)*a**3*b*x**14 - 378*sqrt(a
)*sqrt(a*x**4 + b)*a**2*b**2*x**10 - 115*sqrt(a)*sqrt(a*x**4 + b)*a*b**3*x
**6 - 15*sqrt(a)*sqrt(a*x**4 + b)*b**4*x**2 + 240*log((sqrt(a*x**4 + b) +
sqrt(a)*x**2)/sqrt(b))*a**5*x**20 + 300*log((sqrt(a*x**4 + b) + sqrt(a)*x*
*2)/sqrt(b))*a**4*b*x**16 + 75*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(
b))*a**3*b**2*x**12 - 288*a**5*x**20 - 720*a**4*b*x**16 - 630*a**3*b**2*x*
*12 - 250*a**2*b**3*x**8 - 50*a*b**4*x**4 - 3*b**5)/(30*x**10*(16*sqrt(a*x
**4 + b)*a**2*x**8 + 12*sqrt(a*x**4 + b)*a*b*x**4 + sqrt(a*x**4 + b)*b**2
+ 16*sqrt(a)*a**2*x**10 + 20*sqrt(a)*a*b*x**6 + 5*sqrt(a)*b**2*x**2))
```

3.540 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3} dx = -\frac{5a^2\sqrt{a+\frac{b}{x^4}}}{32x^2} - \frac{5a\left(a+\frac{b}{x^4}\right)^{3/2}}{48x^2} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{12x^2} - \frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2}\right)}{32\sqrt{b}}$$

output
```
-5/32*a^2*(a+b/x^4)^(1/2)/x^2-5/48*a*(a+b/x^4)^(3/2)/x^2-1/12*(a+b/x^4)^(5
/2)/x^2-5/32*a^3*arctanh(b^(1/2)/(a+b/x^4)^(1/2)/x^2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3} dx = \frac{\sqrt{a+\frac{b}{x^4}}\left(-8b^2-26abx^4-33a^2x^8-\frac{15a^3x^{12}\text{arctanh}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+ax^4}}\right)}{96x^{10}}$$

input
```
Integrate[(a + b/x^4)^(5/2)/x^3,x]
```

output
```
(Sqrt[a + b/x^4]*(-8*b^2 - 26*a*b*x^4 - 33*a^2*x^8 - (15*a^3*x^12*ArcTanh[
Sqrt[b + a*x^4]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + a*x^4])))/(96*x^10)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {858, 807, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^{5/2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\left(-\frac{5}{6}a\int \left(a+\frac{b}{x^2}\right)^{3/2}d\frac{1}{x^2}-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x^2}\right)$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\left(-\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{a+\frac{b}{x^2}}d\frac{1}{x^2}+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x^2}\right)$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\left(-\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x^2}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}\left(-\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{b}{\sqrt{a+\frac{b}{x^2}}x^2}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}+\frac{\sqrt{a+\frac{b}{x^2}}}{2x^2}\right)+\frac{\left(a+\frac{b}{x^2}\right)^{3/2}}{4x^2}\right)-\frac{\left(a+\frac{b}{x^2}\right)^{5/2}}{6x^2}\right)$$

input
```
Int[(a + b/x^4)^(5/2)/x^3,x]
```

output
```
(-1/6*(a + b/x^2)^(5/2)/x^2 - (5*a*((a + b/x^2)^(3/2)/(4*x^2) + (3*a*(Sqrt
[a + b/x^2]/(2*x^2) + (a*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x^2)])/(2*Sqrt[b
])))/4))/6)/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{\left(33 a^{2} x^{8}+26 a b\,x^{4}+8 b^{2}\right) \sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{96 x^{10}}-\frac{5 a^{3} \ln \left(\frac{2 b+2 \sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right) \sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}{32 \sqrt{b}\,\sqrt{a\,x^{4}+b}}$	97
default	$-\frac{\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{5}{2}} \left(15 a^{3} \ln \left(\frac{2 b+2 \sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right) x^{12}+33 a^{2} \sqrt{a\,x^{4}+b}\,\sqrt{b}\,x^{8}+26 b^{\frac{3}{2}} a \sqrt{a\,x^{4}+b}\,x^{4}+8 b^{\frac{5}{2}} \sqrt{a\,x^{4}+b}\right)}{96 x^{2} \left(a\,x^{4}+b\right)^{\frac{5}{2}} \sqrt{b}}$	113

input
```
int((a+b/x^4)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/96*(33*a^2*x^8+26*a*b*x^4+8*b^2)/x^10*((a*x^4+b)/x^4)^(1/2)-5/32*a^3/b^
(1/2)*ln((2*b+2*b^(1/2)*(a*x^4+b)^(1/2))/x^2)*((a*x^4+b)/x^4)^(1/2)*x^2/(a
*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.04

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx = \left[\frac{15\,a^3\sqrt{b}x^{10}\log\left(\frac{ax^4-2\,\sqrt{b}x^2\sqrt{\frac{ax^4+b}{x^4}}+2\,b}{x^4}\right)-2\,(33\,a^2bx^8+26\,ab^2x^4+8\,b^3)\sqrt{\frac{ax^4+b}{x^4}}}{192\,bx^{10}}, \frac{15\,a^3}{}\right.$$

input
```
integrate((a+b/x^4)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[1/192*(15*a^3*sqrt(b)*x^10*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^
4) + 2*b)/x^4) - 2*(33*a^2*b*x^8 + 26*a*b^2*x^4 + 8*b^3)*sqrt((a*x^4 + b)/
x^4))/(b*x^10), 1/96*(15*a^3*sqrt(-b)*x^10*arctan(sqrt(-b)*x^2*sqrt((a*x^4
 + b)/x^4)/(a*x^4 + b)) - (33*a^2*b*x^8 + 26*a*b^2*x^4 + 8*b^3)*sqrt((a*x^
4 + b)/x^4))/(b*x^10)]
```

Sympy [A] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx = -\frac{11a^{\frac{5}{2}}\sqrt{1+\frac{b}{ax^4}}}{32x^2} - \frac{13a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax^4}}}{48x^6} - \frac{\sqrt{a}b^2\sqrt{1+\frac{b}{ax^4}}}{12x^{10}} - \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x^2}\right)}{32\sqrt{b}}$$

input
```
integrate((a+b/x**4)**(5/2)/x**3,x)
```

output
```
-11*a**(5/2)*sqrt(1 + b/(a*x**4))/(32*x**2) - 13*a**(3/2)*b*sqrt(1 + b/(a*
x**4))/(48*x**6) - sqrt(a)*b**2*sqrt(1 + b/(a*x**4))/(12*x**10) - 5*a**3*a
sinh(sqrt(b)/(sqrt(a)*x**2))/(32*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(72) = 144.

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx = \frac{5\,a^3\log\left(\frac{\sqrt{a+\frac{b}{x^4}}x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2+\sqrt{b}}\right)}{64\,\sqrt{b}} - \frac{33\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}a^3x^{10} - 40\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^3bx^6 + 15\sqrt{a+\frac{b}{x^4}}a^3b^2x^2}{96\left(\left(a+\frac{b}{x^4}\right)^3x^{12} - 3\left(a+\frac{b}{x^4}\right)^2bx^8 + 3\left(a+\frac{b}{x^4}\right)b^2x^4 - b^3\right)}$$

input
```
integrate((a+b/x^4)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
5/64*a^3*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b
)))/sqrt(b) - 1/96*(33*(a + b/x^4)^(5/2)*a^3*x^10 - 40*(a + b/x^4)^(3/2)*a
^3*b*x^6 + 15*sqrt(a + b/x^4)*a^3*b^2*x^2)/((a + b/x^4)^3*x^12 - 3*(a + b/
x^4)^2*b*x^8 + 3*(a + b/x^4)*b^2*x^4 - b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx = \frac{1}{96}\,a^3\left(\frac{15\,\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{33\,(ax^4+b)^{\frac{5}{2}} - 40\,(ax^4+b)^{\frac{3}{2}}b + 15\,\sqrt{ax^4+b}b^2}{a^3x^{12}}\right)$$

input
```
integrate((a+b/x^4)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/96*a^3*(15*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b) - (33*(a*x^4 + b)^(
5/2) - 40*(a*x^4 + b)^(3/2)*b + 15*sqrt(a*x^4 + b)*b^2)/(a^3*x^12))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx$$

input
```
int((a + b/x^4)^(5/2)/x^3,x)
```

output
```
int((a + b/x^4)^(5/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 818, normalized size of antiderivative = 8.89

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b/x^4)^(5/2)/x^3,x)
```

output
```
(480*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(
b))/sqrt(b))*a**5*x**22 + 480*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 +
b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**4*b*x**18 + 90*sqrt(a)*sqrt(a*x**
4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**3*b**2*
x**14 - 480*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2
+ sqrt(b))/sqrt(b))*a**5*x**22 - 480*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*
x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**4*b*x**18 - 90*sqrt(a)*sqr
t(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**
3*b**2*x**14 - 1056*sqrt(b)*sqrt(a*x**4 + b)*a**5*x**20 - 2416*sqrt(b)*sqr
t(a*x**4 + b)*a**4*b*x**16 - 2098*sqrt(b)*sqrt(a*x**4 + b)*a**3*b**2*x**12
 - 885*sqrt(b)*sqrt(a*x**4 + b)*a**2*b**3*x**8 - 170*sqrt(b)*sqrt(a*x**4 +
 b)*a*b**4*x**4 - 8*sqrt(b)*sqrt(a*x**4 + b)*b**5 - 1056*sqrt(b)*sqrt(a)*a
**5*x**22 - 2944*sqrt(b)*sqrt(a)*a**4*b*x**18 - 3174*sqrt(b)*sqrt(a)*a**3*
b**2*x**14 - 1698*sqrt(b)*sqrt(a)*a**2*b**3*x**10 - 460*sqrt(b)*sqrt(a)*a*
b**4*x**6 - 48*sqrt(b)*sqrt(a)*b**5*x**2 + 480*log((sqrt(a*x**4 + b) + sqr
t(a)*x**2 - sqrt(b))/sqrt(b))*a**6*x**24 + 720*log((sqrt(a*x**4 + b) + sqr
t(a)*x**2 - sqrt(b))/sqrt(b))*a**5*b*x**20 + 270*log((sqrt(a*x**4 + b) + s
qrt(a)*x**2 - sqrt(b))/sqrt(b))*a**4*b**2*x**16 + 15*log((sqrt(a*x**4 + b)
 + sqrt(a)*x**2 - sqrt(b))/sqrt(b))*a**3*b**3*x**12 - 480*log((sqrt(a*x**4
 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))*a**6*x**24 - 720*log((sqrt(a*x...
```

3.541 $\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 151

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = -\frac{b^2\sqrt{a+\frac{b}{x^4}}}{7x^5}-\frac{16ab\sqrt{a+\frac{b}{x^4}}}{21x}+\frac{1}{3}a^2\sqrt{a+\frac{b}{x^4}}\,x^3 - \frac{20a^{7/4}b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{21\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/7*b^2*(a+b/x^4)^(1/2)/x^5-16/21*a*b*(a+b/x^4)^(1/2)/x+1/3*a^2*(a+b/x^4)
^(1/2)*x^3-20/21*a^(7/4)*b^(3/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)
*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(
1/2))/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = -\frac{b^2\sqrt{a+\frac{b}{x^4}}\ \text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{7}{4},-\frac{3}{4},-\frac{ax^4}{b}\right)}{7x^5\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(5/2)*x^2,x]
```

output
```
-1/7*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-5/2, -7/4, -3/4, -((a*x^4)/b)
])/(x^5*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 809, 748, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{x^4}\right)^{5/2}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{5/2} x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{5/2} - \frac{10}{3}b\int \left(a+\frac{b}{x^4}\right)^{3/2} d\frac{1}{x}$$

$$\Big\downarrow \text{748}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{10}{3}b\left(\frac{6}{7}a\int\sqrt{a+\frac{b}{x^4}}d\frac{1}{x}+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}\right)$$

$$\Big\downarrow \text{748}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{10}{3}b\left(\frac{6}{7}a\left(\frac{2}{3}a\int\frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^4}\right)^{5/2}-\frac{10}{3}b\left(\frac{6}{7}a\left(\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}\right)$$

input
```
Int[(a + b/x^4)^(5/2)*x^2,x]
```

output
```
((a + b/x^4)^(5/2)*x^3)/3 - (10*b*((a + b/x^4)^(3/2)/(7*x) + (6*a*(Sqrt[a
 + b/x^4]/(3*x) + (a^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqr
t[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(3*b^(1
/4)*Sqrt[a + b/x^4])))/7))/3
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
 + 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

method	result
risch	$\frac{\left(7a^{2}x^{8}-16ab\,x^{4}-3b^{2}\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{21x^{5}}+\frac{40a^{2}b\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}{21\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\left(a\,x^{4}+b\right)}$
default	$\frac{\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{5}{2}}x^{3}\left(7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^{3}x^{12}+40a^{2}b\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)x^{7}-9\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^{2}b\,x^{8}-19\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,b^{2}x^{4}-3\sqrt{}\right.}{21\left(a\,x^{4}+b\right)^{3}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int((a+b/x^4)^(5/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/21*(7*a^2*x^8-16*a*b*x^4-3*b^2)/x^5*((a*x^4+b)/x^4)^(1/2)+40/21*a^2*b/(I
*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/
2)*x^2)^(1/2)/(a*x^4+b)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*((a*x^4+b
)/x^4)^(1/2)*x^2
```

Fricas [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^2\,dx$$

input
```
integrate((a+b/x^4)^(5/2)*x^2,x, algorithm="fricas")
```

output
```
integral((a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt((a*x^4 + b)/x^4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = -\frac{a^{\frac{5}{2}}x^3\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((a+b/x**4)**(5/2)*x**2,x)
```

output
```
-a**(5/2)*x**3*gamma(-3/4)*hyper((-5/2, -3/4), (1/4,), b*exp_polar(I*pi)/(
a*x**4))/(4*gamma(1/4))
```

Maxima [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^2\,dx$$

input
```
integrate((a+b/x^4)^(5/2)*x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(5/2)*x^2, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^2\,dx$$

input
```
integrate((a+b/x^4)^(5/2)*x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(5/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = \int x^2\left(a+\frac{b}{x^4}\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b/x^4)^(5/2),x)
```

output
```
int(x^2*(a + b/x^4)^(5/2), x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2\,dx = \frac{\sqrt{a\,x^4+b}\,a^2x^8-8\sqrt{a\,x^4+b}\,ab\,x^4+3\sqrt{a\,x^4+b}\,b^2+24\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{12}+b\,x^8}dx\right)b^3x^7}{3x^7}$$

input
```
int((a+b/x^4)^(5/2)*x^2,x)
```

output
```
(sqrt(a*x**4 + b)*a**2*x**8 - 8*sqrt(a*x**4 + b)*a*b*x**4 + 3*sqrt(a*x**4
+ b)*b**2 + 24*int(sqrt(a*x**4 + b)/(a*x**12 + b*x**8),x)*b**3*x**7)/(3*x*
*7)
```

3.542 $\int \left(a+\frac{b}{x^4}\right)^{5/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 277

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = -\frac{b^2\sqrt{a+\frac{b}{x^4}}}{9x^7}-\frac{4ab\sqrt{a+\frac{b}{x^4}}}{9x^3}-\frac{8a^2\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{3\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+a^2\sqrt{a+\frac{b}{x^4}}x+\frac{8a^{9/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}-\frac{4a^{9/4}\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{3\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/9*b^2*(a+b/x^4)^(1/2)/x^7-4/9*a*b*(a+b/x^4)^(1/2)/x^3-8/3*a^2*b^(1/2)*(
a+b/x^4)^(1/2)/(a^(1/2)+b^(1/2)/x^2)/x+a^2*(a+b/x^4)^(1/2)*x+8/3*a^(9/4)*b
^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*Ell
ipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))/(a+b/x^4)^(1/2)-4/3*a
^(9/4)*b^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/
x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/(a+b/x^4)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = -\frac{b^2\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{9}{4},-\frac{5}{4},-\frac{ax^4}{b}\right)}{9x^7\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(5/2),x]
```

output
```
-1/9*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-5/2, -9/4, -5/4, -((a*x^4)/b)
])/(x^7*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {773, 809, 811, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx$$

$$\Big\downarrow \text{773}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{5/2} x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{809}$$

$$x\left(a+\frac{b}{x^4}\right)^{5/2} - 10b\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$x\left(a+\frac{b}{x^4}\right)^{5/2}-10b\left(\frac{2}{3}a\int\frac{\sqrt{a+\frac{b}{x^4}}}{x^2}d\frac{1}{x}+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)$$

$\downarrow$ 811

$$x\left(a+\frac{b}{x^4}\right)^{5/2}-10b\left(\frac{2}{3}a\left(\frac{2}{5}a\int\frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)$$

$\downarrow$ 834

$$x\left(a+\frac{b}{x^4}\right)^{5/2}-10b\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)$$

$\downarrow$ 27

$$x\left(a+\frac{b}{x^4}\right)^{5/2}-10b\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)$$

$\downarrow$ 761

$$x\left(a+\frac{b}{x^4}\right)^{5/2}-10b\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int\frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\frac{(}{}$$

$\downarrow$ 1510

$$x\left(a+\frac{b}{x^4}\right)^{5/2}-$$

$$10b\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\right.}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}}{\sqrt{b}}\right.\right.\right.$$

input
```
Int[(a + b/x^4)^(5/2),x]
```

output
```
(a + b/x^4)^(5/2)*x - 10*b*((a + b/x^4)^(3/2)/(9*x^3) + (2*a*(Sqrt[a + b/x
^4]/(5*x^3) + (2*a*(-((-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x)) + (a
^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)
*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x^4]))
/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a]
+ Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3/4)*S
qrt[a + b/x^4]))/5))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{\left(15a^{2}x^{8}+4ab\,x^{4}+b^{2}\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{9x^{7}}+\frac{8ia^{\frac{5}{2}}\sqrt{b}\,\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{3\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\left(a\,x^{4}+b\right)}$
default	$\frac{\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{5}{2}}x\left(24ia^{\frac{5}{2}}\sqrt{b}\,\sqrt{\frac{-i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,x^{9}\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-24ia^{\frac{5}{2}}\sqrt{b}\,\sqrt{\frac{-i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,x^{9}\operatorname{EllipticE}\right.}{9\left(a\,x^{4}+b\right)^{3}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int((a+b/x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(15*a^2*x^8+4*a*b*x^4+b^2)/x^7*((a*x^4+b)/x^4)^(1/2)+8/3*I*a^(5/2)*b^
(1/2)/(I*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/
2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)*(EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-
EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/2),I))*((a*x^4+b)/x^4)^(1/2)*x^2
```

Fricas [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b/x^4)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt((a*x^4 + b)/x^4)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = -\frac{a^{\frac{5}{2}} x \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((a+b/x**4)**(5/2),x)
```

output
```
-a**(5/2)*x*gamma(-1/4)*hyper((-5/2, -1/4), (3/4,), b*exp_polar(I*pi)/(a*x
**4))/(4*gamma(3/4))
```

Maxima [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b/x^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(5/2), x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = \int \left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b/x^4)^(5/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = -\frac{x\left(a+\frac{b}{x^4}\right)^{5/2} {}_2F_1\left(-\frac{5}{2},-\frac{9}{4};\ -\frac{5}{4};\ -\frac{a\,x^4}{b}\right)}{9\left(\frac{a\,x^4}{b}+1\right)^{5/2}}$$

input
```
int((a + b/x^4)^(5/2),x)
```

output
```
-(x*(a + b/x^4)^(5/2)*hypergeom([-5/2, -9/4], -5/4, -(a*x^4)/b))/(9*((a*x^
4)/b + 1)^(5/2))
```

Reduce [F]

$$\int \left(a+\frac{b}{x^4}\right)^{5/2} dx = \frac{21\sqrt{a\,x^4+b}\,a^2x^8-28\sqrt{a\,x^4+b}\,ab\,x^4+11\sqrt{a\,x^4+b}\,b^2+120\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{14}+b\,x^{10}}dx\right)b^3x^9}{21x^9}$$

input
```
int((a+b/x^4)^(5/2),x)
```

output
```
(21*sqrt(a*x**4 + b)*a**2*x**8 - 28*sqrt(a*x**4 + b)*a*b*x**4 + 11*sqrt(a*
x**4 + b)*b**2 + 120*int(sqrt(a*x**4 + b)/(a*x**14 + b*x**10),x)*b**3*x**9
)/(21*x**9)
```

3.543 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2} dx = -\frac{20a^2\sqrt{a+\frac{b}{x^4}}}{77x} - \frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{77x} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x} - \frac{20a^{11/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

output

```
-20/77*a^2*(a+b/x^4)^(1/2)/x-10/77*a*(a+b/x^4)^(3/2)/x-1/11*(a+b/x^4)^(5/2
)/x-20/77*a^(11/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1
/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/b^(1/4)/
(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx=-\frac{b^2\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{5}{2},-\frac{7}{4},-\frac{ax^4}{b}\right)}{11x^9\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(5/2)/x^2,x]
```

output
```
-1/11*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-11/4, -5/2, -7/4, -((a*x^4)/
b)])/(x^9*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {858, 748, 748, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx$$

$$\Big\downarrow \text{858}$$

$$-\int \left(a+\frac{b}{x^4}\right)^{5/2}d\frac{1}{x}$$

$$\Big\downarrow \text{748}$$

$$-\frac{10}{11}a\int \left(a+\frac{b}{x^4}\right)^{3/2}d\frac{1}{x}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x}$$

$$\Big\downarrow \text{748}$$

$$-\frac{10}{11}a\left(\frac{6}{7}a\int \sqrt{a+\frac{b}{x^4}}d\frac{1}{x}+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x}$$

$$\Big\downarrow \text{748}$$

$$-\frac{10}{11}a\left(\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x}$$

$$\Big\downarrow \text{761}$$

$$-\frac{10}{11}a\left(\frac{6}{7}a\left(\frac{a^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}}{3x}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{7x}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{11x}$$

input
```
Int[(a + b/x^4)^(5/2)/x^2,x]
```

output
```
-1/11*(a + b/x^4)^(5/2)/x - (10*a*((a + b/x^4)^(3/2)/(7*x) + (6*a*(Sqrt[a
+ b/x^4]/(3*x) + (a^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqr
t[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(3*b^(1
/4)*Sqrt[a + b/x^4])))/7))/11
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\left(37 a^{2} x^{8}+24 a b \,x^{4}+7 b^{2}\right) \sqrt{\frac{a \,x^{4}+b}{x^{4}}}}{77 x^{9}}+\frac{40 a^{3} \sqrt{1-\frac{i \sqrt{a}\, x^{2}}{\sqrt{b}}}\, \sqrt{1+\frac{i \sqrt{a}\, x^{2}}{\sqrt{b}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}, i\right) \sqrt{\frac{a \,x^{4}+b}{x^{4}}}\, x^{2}}{77 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, \left(a \,x^{4}+b \right)}$
default	$-\frac{\left(\frac{a \,x^{4}+b}{x^{4}}\right)^{\frac{5}{2}} \left(-40 a^{3} \sqrt{\frac{-i \sqrt{a}\, x^{2}+\sqrt{b}}{\sqrt{b}}}\, \sqrt{\frac{i \sqrt{a}\, x^{2}+\sqrt{b}}{\sqrt{b}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}, i\right) x^{11}+37 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, a^{3} x^{12}+61 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, a^{2} b \,x^{8}+31 \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}\, a \,b^{2} x^{4}-}{77 x \left(a \,x^{4}+b \right)^{3} \sqrt{\frac{i \sqrt{a}}{\sqrt{b}}}}$

input
```
int((a+b/x^4)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/77*(37*a^2*x^8+24*a*b*x^4+7*b^2)/x^9*((a*x^4+b)/x^4)^(1/2)+40/77*a^3/(I
*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/
2)*x^2)^(1/2)/(a*x^4+b)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*((a*x^4+b
)/x^4)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx = -\frac{40\,a^{2}\sqrt{b}x^{9}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\mid -1) + (37\,a^{2}x^{8} + 24\,abx^{4} + 7\,b^{2})\sqrt{\frac{ax^{4}+b}{x^{4}}}}{77\,x^{9}}$$

input
```
integrate((a+b/x^4)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
-1/77*(40*a^2*sqrt(b)*x^9*(-a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4)),
-1) + (37*a^2*x^8 + 24*a*b*x^4 + 7*b^2)*sqrt((a*x^4 + b)/x^4))/x^9
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.27

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx = -\frac{a^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((a+b/x**4)**(5/2)/x**2,x)
```

output
```
-a**(5/2)*gamma(1/4)*hyper((-5/2, 1/4), (5/4,), b*exp_polar(I*pi)/(a*x**4)
)/(4*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/x^4)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(5/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/x^4)^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(5/2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.27

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx=-\frac{\left(a\,x^4+b\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\,\left(\frac{b}{a}+x^4\right)^{5/2}}$$

input
```
int((a + b/x^4)^(5/2)/x^2,x)
```

output
```
-((b + a*x^4)^(5/2)*hypergeom([-5/2, 1/4], 5/4, -b/(a*x^4)))/(x*(b/a + x^4
)^(5/2))
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}\,dx=\frac{-3\sqrt{a\,x^4+b}\,a^2x^8-\sqrt{a\,x^4+b}\,b^2-8\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{16}+b\,x^{12}}dx\right)b^3x^{11}}{3x^{11}}$$

input
```
int((a+b/x^4)^(5/2)/x^2,x)
```

output
```
( - 3*sqrt(a*x**4 + b)*a**2*x**8 - sqrt(a*x**4 + b)*b**2 - 8*int(sqrt(a*x*
*4 + b)/(a*x**16 + b*x**12),x)*b**3*x**11)/(3*x**11)
```

3.544 $\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 278

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4} dx = -\frac{4a^2\sqrt{a+\frac{b}{x^4}}}{39x^3} - \frac{10a\left(a+\frac{b}{x^4}\right)^{3/2}}{117x^3} - \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3} - \frac{8a^3\sqrt{a+\frac{b}{x^4}}}{39\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{8a^{13/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{39b^{3/4}\sqrt{a+\frac{b}{x^4}}} - \frac{4a^{13/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{39b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

output

```
-4/39*a^2*(a+b/x^4)^(1/2)/x^3-10/117*a*(a+b/x^4)^(3/2)/x^3-1/13*(a+b/x^4)^
(5/2)/x^3-8/39*a^3*(a+b/x^4)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)/x+8/39*a^
(13/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*Ell
ipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))/b^(3/4)/(a+b/x^4)^(1/
2)-4/39*a^(13/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2
)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/b^(3/4)/(a
+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx=-\frac{b^2\sqrt{a+\frac{b}{x^4}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},-\frac{5}{2},-\frac{9}{4},-\frac{ax^4}{b}\right)}{13x^{11}\sqrt{1+\frac{ax^4}{b}}}$$

input
```
Integrate[(a + b/x^4)^(5/2)/x^4,x]
```

output
```
-1/13*(b^2*Sqrt[a + b/x^4]*Hypergeometric2F1[-13/4, -5/2, -9/4, -((a*x^4)/
b)])/(x^11*Sqrt[1 + (a*x^4)/b])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.533$, Rules used = {858, 811, 811, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{811}$$

$$-\frac{10}{13}a\int \frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{x^2}d\frac{1}{x}-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3}$$

$$\Big\downarrow \text{811}$$

$$-\frac{10}{13}a\left(\frac{2}{3}a\int \frac{\sqrt{a+\frac{b}{x^4}}}{x^2}d\frac{1}{x}+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3}$$

$$\Big\downarrow \text{811}$$

$$-\frac{10}{13}a\left(\frac{2}{3}a\left(\frac{2}{5}a\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3}$$

$$\Big\downarrow \text{834}$$

$$-\frac{10}{13}a\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10}{13}a\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\frac{\left(a+\frac{b}{x^4}\right)^{3/2}}{9x^3}\right)-\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{10}{13}a\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)+\frac{\sqrt{a+\frac{b}{x^4}}}{5x^3}\right)+\right.$$

$$\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3}$$

$$\Big\downarrow \text{1510}$$

$$-\frac{10}{13}a\left(\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\text{arc}\right.}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}}{\sqrt{b}}\right.\right.\right.$$

$$\frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{13x^3}$$

input
```
Int[(a + b/x^4)^(5/2)/x^4,x]
```

output
```
-1/13*(a + b/x^4)^(5/2)/x^3 - (10*a*((a + b/x^4)^(3/2)/(9*x^3) + (2*a*(Sqr
t[a + b/x^4]/(5*x^3) + (2*a*(-((-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)
*x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqr
t[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a +
 b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*
(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*
b^(3/4)*Sqrt[a + b/x^4])))/5))/3))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{\left(24a^{3}x^{12}+31a^{2}b\,x^{8}+28a\,b^{2}x^{4}+9b^{3}\right)\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}{117x^{11}b}+\frac{8ia^{\frac{7}{2}}\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right.}{39\sqrt{b}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\left(a\,x^{4}+b\right)}$
default	$-\frac{\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{5}{2}}\left(-24ia^{\frac{7}{2}}\sqrt{\frac{-i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,x^{13}b\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+24ia^{\frac{7}{2}}\sqrt{\frac{-i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,x^{13}b\,\text{Elliptic}\right.}{117x^{3}\left(a\,x^{4}+b\right)^{3}b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{}}{\sqrt{}}}}$

input
```
int((a+b/x^4)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/117*(24*a^3*x^12+31*a^2*b*x^8+28*a*b^2*x^4+9*b^3)/x^11/b*((a*x^4+b)/x^4
)^(1/2)+8/39*I/b^(1/2)*a^(7/2)/(I*a^(1/2)/b^(1/2))^(1/2)*(1-I*a^(1/2)/b^(1
/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)/(a*x^4+b)*(EllipticF(x*(I*a
^(1/2)/b^(1/2))^(1/2),I)-EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/2),I))*((a*x^4
+b)/x^4)^(1/2)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.43

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx = -\frac{24\,a^3\sqrt{b}x^{11}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|-1)-24\,a^3\sqrt{b}x^{11}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|-1)+(24\,a^3x^{12}}{117\,bx^{11}}$$

input
```
integrate((a+b/x^4)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
-1/117*(24*a^3*sqrt(b)*x^11*(-a/b)^(3/4)*elliptic_e(arcsin(x*(-a/b)^(1/4))
, -1) - 24*a^3*sqrt(b)*x^11*(-a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4))
, -1) + (24*a^3*x^12 + 31*a^2*b*x^8 + 28*a*b^2*x^4 + 9*b^3)*sqrt((a*x^4 +
b)/x^4))/(b*x^11)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx = -\frac{a^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((a+b/x**4)**(5/2)/x**4,x)
```

output
```
-a**(5/2)*gamma(3/4)*hyper((-5/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4)
)/(4*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/x^4)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(5/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/x^4)^(5/2)/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(5/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx$$

input
```
int((a + b/x^4)^(5/2)/x^4,x)
```

output
```
int((a + b/x^4)^(5/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^4}\right)^{5/2}}{x^4}\,dx = \frac{-77\sqrt{a\,x^4+b}\,a^2x^8-44\sqrt{a\,x^4+b}\,ab\,x^4-27\sqrt{a\,x^4+b}\,b^2-120\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^{18}+b\,x^{14}}dx\right)b^3x^{13}}{231x^{13}}$$

input
```
int((a+b/x^4)^(5/2)/x^4,x)
```

output
```
( - 77*sqrt(a*x**4 + b)*a**2*x**8 - 44*sqrt(a*x**4 + b)*a*b*x**4 - 27*sqrt
(a*x**4 + b)*b**2 - 120*int(sqrt(a*x**4 + b)/(a*x**18 + b*x**14),x)*b**3*x
**13)/(231*x**13)
```

3.545 $\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{\sqrt{a+\frac{b}{x^4}}x^4}{4a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
1/4*(a+b/x^4)^(1/2)*x^4/a-1/4*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{\sqrt{a}x^2(b+ax^4)-b\sqrt{b+ax^4}\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{4a^{3/2}\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[x^3/Sqrt[a + b/x^4],x]
```

output
```
(Sqrt[a]*x^2*(b + a*x^4) - b*Sqrt[b + a*x^4]*Log[Sqrt[a]*x^2 + Sqrt[b + a*
x^4]])/(4*a^(3/2)*Sqrt[a + b/x^4]*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \frac{x^8}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{b\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}}{2a}+\frac{x^4\sqrt{a+\frac{b}{x^4}}}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{a}+\frac{x^4\sqrt{a+\frac{b}{x^4}}}{a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{a+\frac{b}{x^4}}}{a}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[x^3/Sqrt[a + b/x^4],x]
```

output
```
((Sqrt[a + b/x^4]*x^4)/a - (b*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/a^(3/2))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{\sqrt{a\,x^{4}+b}\,\left(x^{2}\sqrt{a\,x^{4}+b}\,a^{\frac{3}{2}}-b\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)a\right)}{4\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}a^{\frac{5}{2}}}$	70
risch	$\frac{a\,x^{4}+b}{4a\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}-\frac{b\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)\sqrt{a\,x^{4}+b}}{4a^{\frac{3}{2}}\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}$	76

input
```
int(x^3/(a+b/x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*x^4+b)^(1/2)*(x^2*(a*x^4+b)^(1/2)*a^(3/2)-b*ln(a^(1/2)*x^2+(a*x^4+b
)^(1/2))*a)/((a*x^4+b)/x^4)^(1/2)/x^2/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.34

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx = \left[\frac{2\,ax^4\sqrt{\frac{ax^4+b}{x^4}}+\sqrt{a}b\log\left(-2\,ax^4+2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)}{8\,a^2}, \frac{ax^4\sqrt{\frac{ax^4+b}{x^4}}+\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)}{4\,a^2}\right]$$

input
```
integrate(x^3/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*a*x^4*sqrt((a*x^4 + b)/x^4) + sqrt(a)*b*log(-2*a*x^4 + 2*sqrt(a)*x
^4*sqrt((a*x^4 + b)/x^4) - b))/a^2, 1/4*(a*x^4*sqrt((a*x^4 + b)/x^4) + sqr
t(-a)*b*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4)/a))/a^2]
```

Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{b}x^2\sqrt{\frac{ax^4}{b}+1}}{4a} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate(x**3/(a+b/x**4)**(1/2),x)
```

output
```
sqrt(b)*x**2*sqrt(a*x**4/b + 1)/(4*a) - b*asinh(sqrt(a)*x**2/sqrt(b))/(4*a
**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{a+\frac{b}{x^4}}\,b}{4\left(\left(a+\frac{b}{x^4}\right)a-a^2\right)} + \frac{b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}$$

input
```
integrate(x^3/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(a + b/x^4)*b/((a + b/x^4)*a - a^2) + 1/8*b*log((sqrt(a + b/x^4) -
 sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{ax^4+b}\,x^2}{4\,a} + \frac{b\log\left(\left|-\sqrt{a}x^2+\sqrt{ax^4+b}\right|\right)}{4\,a^{\frac{3}{2}}}$$

input
```
integrate(x^3/(a+b/x^4)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a*x^4 + b)*x^2/a + 1/4*b*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))
/a^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{x^4\,\sqrt{a+\frac{b}{x^4}}}{4\,a} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\,a^{3/2}}$$

input
```
int(x^3/(a + b/x^4)^(1/2),x)
```

output
```
(x^4*(a + b/x^4)^(1/2))/(4*a) - (b*atanh((a + b/x^4)^(1/2)/a^(1/2)))/(4*a^
(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.44

$$\int \frac{x^3}{\sqrt{a+\frac{b}{x^4}}}\,dx$$
$$= \frac{-2\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)b\,x^2+2\sqrt{a}\,\sqrt{a\,x^4+b}\,a\,x^6+\sqrt{a}\,\sqrt{a\,x^4+b}\,b\,x^2-2\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right.}{4a\left(2\sqrt{a\,x^4+b}\,a\,x^2+2\sqrt{a}\,a\,x^4+\sqrt{a}\,b\right)}$$

input
```
int(x^3/(a+b/x^4)^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(
b))*b*x**2 + 2*sqrt(a)*sqrt(a*x**4 + b)*a*x**6 + sqrt(a)*sqrt(a*x**4 + b)*
b*x**2 - 2*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a*b*x**4 - log((
sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*b**2 + 2*a**2*x**8 + 2*a*b*x**4)
/(4*a*(2*sqrt(a*x**4 + b)*a*x**2 + 2*sqrt(a)*a*x**4 + sqrt(a)*b))
```

3.546 $\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{\sqrt{a+\frac{b}{x^4}}\,x^2}{2a}$$

output
```
1/2*(a+b/x^4)^(1/2)*x^2/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{\sqrt{a+\frac{b}{x^4}}\,x^2}{2a}$$

input
```
Integrate[x/Sqrt[a + b/x^4],x]
```

output
```
(Sqrt[a + b/x^4]*x^2)/(2*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{796}$$

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}}{2a}$$

input
```
Int[x/Sqrt[a + b/x^4],x]
```

output
```
(Sqrt[a + b/x^4]*x^2)/(2*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

method	result	size
orering	$\frac{a\,x^{4}+b}{2 a\,x^{2} \sqrt{a+\frac{b}{x^{4}}}}$	25
trager	$\frac{x^{2} \sqrt{-\frac{-a\,x^{4}-b}{x^{4}}}}{2 a}$	26
gosper	$\frac{a\,x^{4}+b}{2 a\,x^{2} \sqrt{\frac{a\,x^{4}+b}{x^{4}}}}$	29
default	$\frac{a\,x^{4}+b}{2 a\,x^{2} \sqrt{\frac{a\,x^{4}+b}{x^{4}}}}$	29
risch	$\frac{a\,x^{4}+b}{2 a\,x^{2} \sqrt{\frac{a\,x^{4}+b}{x^{4}}}}$	29

input
```
int(x/(a+b/x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^4+b)/a/x^2/(a+b/x^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{x^2\sqrt{\frac{ax^4+b}{x^4}}}{2\,a}$$

input
```
integrate(x/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*x^2*sqrt((a*x^4 + b)/x^4)/a
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{\sqrt{b}\sqrt{\frac{ax^4}{b}+1}}{2a}$$

input
```
integrate(x/(a+b/x**4)**(1/2),x)
```

output
```
sqrt(b)*sqrt(a*x**4/b + 1)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{\sqrt{a+\frac{b}{x^4}}x^2}{2\,a}$$

input
```
integrate(x/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^4)*x^2/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{\sqrt{ax^4+b}}{2\,a}$$

input
```
integrate(x/(a+b/x^4)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(a*x^4 + b)/a
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{x^2\,\sqrt{a+\frac{b}{x^4}}}{2\,a}$$

input
```
int(x/(a + b/x^4)^(1/2),x)
```

output
```
(x^2*(a + b/x^4)^(1/2))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{a}\,\sqrt{a\,x^4+b}\,x^2+a\,x^4+b}{2a\left(\sqrt{a\,x^4+b}+\sqrt{a}\,x^2\right)}$$

input
```
int(x/(a+b/x^4)^(1/2),x)
```

output
```
(sqrt(a)*sqrt(a*x**4 + b)*x**2 + a*x**4 + b)/(2*a*(sqrt(a*x**4 + b) + sqrt
(a)*x**2))
```

3.547 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
1/2*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x} \, dx = \frac{\sqrt{b+ax^4}\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{2\sqrt{a}\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[1/(Sqrt[a + b/x^4]*x),x]
```

output
```
(Sqrt[b + a*x^4]*Log[Sqrt[a]*x^2 + Sqrt[b + a*x^4]])/(2*Sqrt[a]*Sqrt[a + b
/x^4]*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b/x^4]*x),x]
```

output
```
ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

method	result	size
default	$\frac{\sqrt{a\,x^{4}+b}\,\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)}{2\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}\sqrt{a}}$	49

input
```
int(1/(a+b/x^4)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x^4+b)/x^4)^(1/2)/x^2*(a*x^4+b)^(1/2)*ln(a^(1/2)*x^2+(a*x^4+b)^(1/
2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx = \left[\frac{\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)}{4\,\sqrt{a}}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)}{2\,a}\right]$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*log(-2*a*x^4 - 2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b)/sqrt(a), -1/2
*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4)/a)/a]
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{2\sqrt{a}}$$

input
```
integrate(1/(a+b/x**4)**(1/2)/x,x)
```

output
```
asinh(sqrt(a)*x**2/sqrt(b))/(2*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{4\,\sqrt{a}}$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/4*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx = -\frac{\log\left(\left|-\sqrt{a}x^2+\sqrt{ax^4+b}\right|\right)}{2\sqrt{a}}$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
int(1/(x*(a + b/x^4)^(1/2)),x)
```

output
```
atanh((a + b/x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x}\,dx=\frac{\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)}{2\sqrt{a}}$$

input
```
int(1/(a+b/x^4)^(1/2)/x,x)
```

output
```
log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))/(2*sqrt(a))
```

3.548 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2}\right)}{2\sqrt{b}}$$

output
```
-1/2*arctanh(b^(1/2)/(a+b/x^4)^(1/2)/x^2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx=-\frac{\sqrt{b+ax^4}\operatorname{arctanh}\left(\frac{\sqrt{b+ax^4}}{\sqrt{b}}\right)}{2\sqrt{b}\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[1/(Sqrt[a + b/x^4]*x^3),x]
```

output
```
-1/2*(Sqrt[b + a*x^4]*ArcTanh[Sqrt[b + a*x^4]/Sqrt[b]])/(Sqrt[b]*Sqrt[a +
b/x^4]*x^2)
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x}d\frac{1}{x}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{a+\frac{b}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}\int \frac{1}{1-\frac{b}{\sqrt{a+\frac{b}{x^2}}x^2}}d\frac{1}{\sqrt{a+\frac{b}{x^2}}x^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{x^2\sqrt{a+\frac{b}{x^2}}}\right)}{2\sqrt{b}}$$

input
```
Int[1/(Sqrt[a + b/x^4]*x^3),x]
```

output
```
-1/2*ArcTanh[Sqrt[b]/(Sqrt[a + b/x^2]*x^2)]/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

method	result	size
default	$-\frac{\sqrt{a\,x^{4}+b}\,\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{a\,x^{4}+b}}{x^{2}}\right)}{2\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}\sqrt{b}}$	52

input
```
int(1/(a+b/x^4)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*x^4+b)/x^4)^(1/2)/x^2*(a*x^4+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(a*x^
4+b)^(1/2)+b)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx = \left[\frac{\log\left(\frac{ax^4-2\sqrt{b}x^2\sqrt{\frac{ax^4+b}{x^4}}+2b}{x^4}\right)}{4\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^2\sqrt{\frac{ax^4+b}{x^4}}}{ax^4+b}\right)}{2b}\right]$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*log((a*x^4 - 2*sqrt(b)*x^2*sqrt((a*x^4 + b)/x^4) + 2*b)/x^4)/sqrt(b),
 1/2*sqrt(-b)*arctan(sqrt(-b)*x^2*sqrt((a*x^4 + b)/x^4)/(a*x^4 + b))/b]
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{b}}{\sqrt{a}x^2}\right)}{2\sqrt{b}}$$

input
```
integrate(1/(a+b/x**4)**(1/2)/x**3,x)
```

output
```
-asinh(sqrt(b)/(sqrt(a)*x**2))/(2*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx = \frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}x^2-\sqrt{b}}{\sqrt{a+\frac{b}{x^4}}x^2+\sqrt{b}}\right)}{4\sqrt{b}}$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/4*log((sqrt(a + b/x^4)*x^2 - sqrt(b))/(sqrt(a + b/x^4)*x^2 + sqrt(b)))/s
qrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx=\frac{\arctan\left(\frac{\sqrt{ax^4+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}}$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(a*x^4 + b)/sqrt(-b))/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^3}\,dx=\int \frac{1}{x^3\sqrt{a+\frac{b}{x^4}}}\,dx$$

input
```
int(1/(x^3*(a + b/x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b/x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^3}\,dx=\frac{\sqrt{b}\left(\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}\right)-\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}\right)\right)}{2b}$$

input
```
int(1/(a+b/x^4)^(1/2)/x^3,x)
```

output
```
(sqrt(b)*(log((sqrt(a*x**4 + b) + sqrt(a)*x**2 - sqrt(b))/sqrt(b)) - log((
sqrt(a*x**4 + b) + sqrt(a)*x**2 + sqrt(b))/sqrt(b))))/(2*b)
```

3.549 $\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{a+\frac{b}{x^4}}\,x^3}{3a}+\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
1/3*(a+b/x^4)^(1/2)*x^3/a+1/6*b^(3/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^
(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/
2*2^(1/2))/a^(5/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{b+ax^4-b\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^4}{b}\right)}{3a\sqrt{a+\frac{b}{x^4}}\,x}$$

input
```
Integrate[x^2/Sqrt[a + b/x^4],x]
```

output
```
(b + a*x^4 - b*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x
^4)/b)])/(3*a*Sqrt[a + b/x^4]*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {858, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{3a}+\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}$$

$$\Big\downarrow \text{761}$$

$$\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+\frac{b}{x^4}}}+\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}$$

input
```
Int[x^2/Sqrt[a + b/x^4],x]
```

output
```
(Sqrt[a + b/x^4]*x^3)/(3*a) + (b^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]
/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)],
1/2])/(6*a^(5/4)*Sqrt[a + b/x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{a\,x^{4}+b}{3ax\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}-\frac{b\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)}{3a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}$	111
default	$\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a\,x^{5}-b\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,bx}{3\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$	124

input
```
int(x^2/(a+b/x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a/x*(a*x^4+b)/((a*x^4+b)/x^4)^(1/2)-1/3*b/a/(I*a^(1/2)/b^(1/2))^(1/2)*
(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)*EllipticF(
x*(I*a^(1/2)/b^(1/2))^(1/2),I)/((a*x^4+b)/x^4)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{x^3\sqrt{\frac{ax^4+b}{x^4}}-\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right)|-1\right)}{3\,a}$$

input
```
integrate(x^2/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^3*sqrt((a*x^4 + b)/x^4) - sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin((-
b/a)^(1/4)/x), -1))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.38

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx = -\frac{x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(x**2/(a+b/x**4)**(1/2),x)
```

output
```
-x**3*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*exp_polar(I*pi)/(a*x**4))/(
4*sqrt(a)*gamma(1/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx = \int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

input
```
integrate(x^2/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a + b/x^4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx = \int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

input
```
integrate(x^2/(a+b/x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a + b/x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx = \int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

input
```
int(x^2/(a + b/x^4)^(1/2),x)
```

output
```
int(x^2/(a + b/x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{a\,x^4+b}\,x-\left(\int \frac{\sqrt{a\,x^4+b}}{a\,x^4+b}dx\right)b}{3a}$$

input
```
int(x^2/(a+b/x^4)^(1/2),x)
```

output
```
(sqrt(a*x**4 + b)*x - int(sqrt(a*x**4 + b)/(a*x**4 + b),x)*b)/(3*a)
```

3.550 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 231

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx = -\frac{\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{a\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}+\frac{\sqrt{a+\frac{b}{x^4}}\,x}{a} + \frac{\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+\frac{b}{x^4}}} - \frac{\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-b^(1/2)*(a+b/x^4)^(1/2)/a/(a^(1/2)+b^(1/2)/x^2)/x+(a+b/x^4)^(1/2)*x/a+b^(
1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*Ellip
ticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))/a^(3/4)/(a+b/x^4)^(1/2)
-1/2*b^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^
2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/a^(3/4)/(a+b/x
^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{x\sqrt{1+\frac{ax^4}{b}}\ \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^4}{b}\right)}{3\sqrt{a+\frac{b}{x^4}}}$$

input
```
Integrate[1/Sqrt[a + b/x^4],x]
```

output
```
(x*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^4)/b)])/(3*
Sqrt[a + b/x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.545$, Rules used = {773, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}$$

$$\Big\downarrow \text{847}$$

$$\frac{x\sqrt{a+\frac{b}{x^4}}}{a}-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}}{a}$$

$$\Big\downarrow \text{834}$$

$$\frac{x\sqrt{a+\frac{b}{x^4}}}{a}-\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt{a+\frac{b}{x^4}}}{a}-\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{761}$$

$$\frac{x\sqrt{a+\frac{b}{x^4}}}{a}-\frac{b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int\frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{1510}$$

$$\frac{x\sqrt{a+\frac{b}{x^4}}}{a}-\frac{b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}}{\sqrt{b}}\right)}{a}$$

input
```
Int[1/Sqrt[a + b/x^4],x]
```

output
```
(Sqrt[a + b/x^4]*x)/a - (b*(-((-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*
x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt
[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a +
b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(
Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b
^(3/4)*Sqrt[a + b/x^4])))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{i\sqrt{b}\,\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right)}{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a}}$	113

input
```
int(1/(a+b/x^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/((a*x^4+b)/x^4)^(1/2)/x^2*b^(1/2)/(I*a^(1/2)/b^(1/2))^(1/2)*(-(I*a^(1/2)
*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)/a^(1/
2)*(EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-EllipticE(x*(I*a^(1/2)/b^(1/2
))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx = \frac{\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right) \mid -1) - \sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right) \mid -1) + x\sqrt{\frac{ax^4+b}{x^4}}}{a}$$

input
```
integrate(1/(a+b/x^4)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin((-b/a)^(1/4)/x), -1) - sqrt(a)*(-b
/a)^(3/4)*elliptic_f(arcsin((-b/a)^(1/4)/x), -1) + x*sqrt((a*x^4 + b)/x^4)
)/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx = -\frac{x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}\,\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(1/2),x)
```

output
```
-x*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*exp_polar(I*pi)/(a*x**4))/(4*s
qrt(a)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/x^4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a + b/x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx=\frac{x\,\sqrt{\frac{a\,x^4}{b}+1}\,\sqrt{x^4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a\,x^4}{b}\right)}{3\,\sqrt{a\,x^4+b}}$$

input
```
int(1/(a + b/x^4)^(1/2),x)
```

output
```
(x*((a*x^4)/b + 1)^(1/2)*(x^4)^(1/2)*hypergeom([1/2, 3/4], 7/4, -(a*x^4)/b
))/(3*(b + a*x^4)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}\,dx=\int \frac{\sqrt{a\,x^4+b}\,x^2}{a\,x^4+b}dx$$

input
```
int(1/(a+b/x^4)^(1/2),x)
```

output
```
int((sqrt(a*x**4 + b)*x**2)/(a*x**4 + b),x)
```

3.551 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx = -\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/2*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*Inver
seJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(1/4)/(a+b/x
^4)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx = -\frac{i\sqrt{1+\frac{ax^4}{b}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[1/(Sqrt[a + b/x^4]*x^2),x]
```

output
```
((-I)*Sqrt[1 + (a*x^4)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]*x]
, -1])/(Sqrt[(I*Sqrt[a])/Sqrt[b]]*Sqrt[a + b/x^4]*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {858, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}$$

$$\Big\downarrow \text{761}$$

$$-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[1/(Sqrt[a + b/x^4]*x^2),x]
```

output
```
-1/2*(Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*
EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a + b
/x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)}{\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$	86

input
```
int(1/(a+b/x^4)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/((a*x^4+b)/x^4)^(1/2)/x^2/(I*a^(1/2)/b^(1/2))^(1/2)*(-(I*a^(1/2)*x^2-b^(
1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(
I*a^(1/2)/b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx=-\frac{\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|-1\right)}{a}$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4)), -1)/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(1/2)/x**2,x)
```

output
```
-gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*exp_polar(I*pi)/(a*x**4))/(4*sqrt(
a)*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx = -\frac{\sqrt{\frac{b}{a}+x^4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\,\sqrt{a\,x^4+b}}$$

input
```
int(1/(x^2*(a + b/x^4)^(1/2)),x)
```

output
```
-((b/a + x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, -b/(a*x^4)))/(x*(b + a*x^4)
^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}\,dx = \int \frac{\sqrt{a\,x^4+b}}{a\,x^4+b}dx$$

input
```
int(1/(a+b/x^4)^(1/2)/x^2,x)
```

output
```
int(sqrt(a*x**4 + b)/(a*x**4 + b),x)
```

3.552 $\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 212

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^4}\,dx = -\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} + \frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+\frac{b}{x^4}}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-(a+b/x^4)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)/x+a^(1/4)*((a+b/x^4)/(a^(1/
2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*EllipticE(sin(2*arccot(a^(1
/4)*x/b^(1/4))),1/2*2^(1/2))/b^(3/4)/(a+b/x^4)^(1/2)-1/2*a^(1/4)*((a+b/x^4
)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*a
rccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/b^(3/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^4}\,dx = -\frac{\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{ax^4}{b}\right)}{\sqrt{a+\frac{b}{x^4}}x^3}$$

input
```
Integrate[1/(Sqrt[a + b/x^4]*x^4),x]
```

output
```
-((Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^4)/b)])/(S
qrt[a + b/x^4]*x^3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {858, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{x^4}}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{834}$$

$$\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}}{\sqrt{b}}-\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[1/(Sqrt[a + b/x^4]*x^4),x]
```

output
```
(-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x)) + (a^(1/4)*Sqrt[(a + b/x^4
)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcTan[b^
(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x^4]))/Sqrt[b] - (a^(1/4)*Sq
rt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*Elliptic
F[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3/4)*Sqrt[a + b/x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{a\,x^{4}+b}{b\,x^{3}\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}+\frac{i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)\right)}{\sqrt{b}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}$
default	$\frac{i\sqrt{a}\,\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,bx\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-i\sqrt{a}\,\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,bx\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)-\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{b}}{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{3}b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int(1/(a+b/x^4)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(a*x^4+b)/x^3/((a*x^4+b)/x^4)^(1/2)+I*a^(1/2)/b^(1/2)/(I*a^(1/2)/b^(1
/2))^(1/2)*(1-I*a^(1/2)/b^(1/2)*x^2)^(1/2)*(1+I*a^(1/2)/b^(1/2)*x^2)^(1/2)
*(EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-EllipticE(x*(I*a^(1/2)/b^(1/2))
^(1/2),I))/((a*x^4+b)/x^4)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^4}\,dx = -\frac{\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)|-1\right)-\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)|-1\right)+x\sqrt{\frac{ax^4+b}{x^4}}}{b}$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-(sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin(x*(-a/b)^(1/4)), -1) - sqrt(b)*(-
a/b)^(3/4)*elliptic_f(arcsin(x*(-a/b)^(1/4)), -1) + x*sqrt((a*x^4 + b)/x^4
))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^4}\,dx = -\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(1/2)/x**4,x)
```

output
```
-gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*exp_polar(I*pi)/(a*x**4))/(4*sqrt(
a)*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}x^4}}\,dx = \int \frac{1}{\sqrt{a + \frac{b}{x^4}x^4}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/x^4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}x^4}}\,dx = \int \frac{1}{\sqrt{a + \frac{b}{x^4}x^4}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/x^4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + \frac{b}{x^4}x^4}}\,dx = \int \frac{1}{x^4\sqrt{a + \frac{b}{x^4}}}\,dx$$

input
```
int(1/(x^4*(a + b/x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b/x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^4}}\,x^4}\,dx = \int \frac{\sqrt{a\,x^4+b}}{a\,x^6+b\,x^2}dx$$

input
```
int(1/(a+b/x^4)^(1/2)/x^4,x)
```

output
```
int(sqrt(a*x**4 + b)/(a*x**6 + b*x**2),x)
```

3.553 $\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx = \frac{3b}{4a^2\sqrt{a+\frac{b}{x^4}}} + \frac{x^4}{4a\sqrt{a+\frac{b}{x^4}}} - \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
3/4*b/a^2/(a+b/x^4)^(1/2)+1/4*x^4/a/(a+b/x^4)^(1/2)-3/4*b*arctanh((a+b/x^4
)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx = \frac{\sqrt{a}x^2(3b+ax^4) - 3b\sqrt{b+ax^4}\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{4a^{5/2}\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[x^3/(a + b/x^4)^(3/2),x]
```

output
```
(Sqrt[a]*x^2*(3*b + a*x^4) - 3*b*Sqrt[b + a*x^4]*Log[Sqrt[a]*x^2 + Sqrt[b
+ a*x^4]])/(4*a^(5/2)*Sqrt[a + b/x^4]*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \frac{x^8}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x^4}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{3b\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x^4}}{2a}+\frac{x^4}{a\sqrt{a+\frac{b}{x^4}}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\frac{3b\left(\frac{\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^4}}}\right)}{2a}+\frac{x^4}{a\sqrt{a+\frac{b}{x^4}}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{3b\left(\frac{2\int\frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^4}}}\right)}{2a}+\frac{x^4}{a\sqrt{a+\frac{b}{x^4}}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{x^4}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}+\frac{x^4}{a\sqrt{a+\frac{b}{x^4}}}\right)$$

input
```
Int[x^3/(a + b/x^4)^(3/2),x]
```

output
```
(x^4/(a*Sqrt[a + b/x^4]) + (3*b*(2/(a*Sqrt[a + b/x^4]) - (2*ArcTanh[Sqrt[a
 + b/x^4]/Sqrt[a]])/a^(3/2)))/(2*a))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{(a\,x^{4}+b)\left(x^{6} a^{\frac{7}{2}}+3 a^{\frac{5}{2}} b\,x^{2}-3 b \ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right) a^{2} \sqrt{a\,x^{4}+b}\right)}{4 \left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}} x^{6} a^{\frac{9}{2}}}$	79
risch	$\frac{a\,x^{4}+b}{4 a^{2} \sqrt{\frac{a\,x^{4}+b}{x^{4}}}}+\frac{\left(\frac{b\,x^{2}}{2 a^{2} \sqrt{a\,x^{4}+b}}-\frac{3 b \ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right)}{4 a^{\frac{5}{2}}}\right) \sqrt{a\,x^{4}+b}}{\sqrt{\frac{a\,x^{4}+b}{x^{4}}}\,x^{2}}$	96

input
```
int(x^3/(a+b/x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*x^4+b)*(x^6*a^(7/2)+3*a^(5/2)*b*x^2-3*b*ln(a^(1/2)*x^2+(a*x^4+b)^(1
/2))*a^2*(a*x^4+b)^(1/2))/((a*x^4+b)/x^4)^(3/2)/x^6/a^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.65

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \left[\frac{3\left(abx^4+b^2\right)\sqrt{a}\log\left(-2\,ax^4+2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+2\left(a^2x^8+3\,abx^4\right)\sqrt{\frac{ax^4+b}{x^4}}}{8\left(a^4x^4+a^3b\right)}, \frac{3\,(\ldots}{\ldots}\right.$$

input
```
integrate(x^3/(a+b/x^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a*b*x^4 + b^2)*sqrt(a)*log(-2*a*x^4 + 2*sqrt(a)*x^4*sqrt((a*x^4 +
 b)/x^4) - b) + 2*(a^2*x^8 + 3*a*b*x^4)*sqrt((a*x^4 + b)/x^4))/(a^4*x^4 +
a^3*b), 1/4*(3*(a*b*x^4 + b^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x
^4)/a) + (a^2*x^8 + 3*a*b*x^4)*sqrt((a*x^4 + b)/x^4))/(a^4*x^4 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{x^6}{4a\sqrt{b}\sqrt{\frac{ax^4}{b}+1}}+\frac{3\sqrt{b}x^2}{4a^2\sqrt{\frac{ax^4}{b}+1}}-\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}x^2}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}}$$

input
```
integrate(x**3/(a+b/x**4)**(3/2),x)
```

output
```
x**6/(4*a*sqrt(b)*sqrt(a*x**4/b + 1)) + 3*sqrt(b)*x**2/(4*a**2*sqrt(a*x**4
/b + 1)) - 3*b*asinh(sqrt(a)*x**2/sqrt(b))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{3\left(a+\frac{b}{x^4}\right)b-2\,ab}{4\left(\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x^4}}\,a^3\right)}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}$$

input

```
integrate(x^3/(a+b/x^4)^(3/2),x, algorithm="maxima")
```

output

```
1/4*(3*(a + b/x^4)*b - 2*a*b)/((a + b/x^4)^(3/2)*a^2 - sqrt(a + b/x^4)*a^3
) + 3/8*b*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(
5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{\left(\frac{x^4}{a}+\frac{3\,b}{a^2}\right)x^2}{4\sqrt{ax^4+b}}+\frac{3\,b\log\left(\left|-\sqrt{a}x^2+\sqrt{ax^4+b}\right|\right)}{4\,a^{\frac{5}{2}}}$$

input

```
integrate(x^3/(a+b/x^4)^(3/2),x, algorithm="giac")
```

output

```
1/4*(x^4/a + 3*b/a^2)*x^2/sqrt(a*x^4 + b) + 3/4*b*log(abs(-sqrt(a)*x^2 + s
qrt(a*x^4 + b)))/a^(5/2)
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{3\,b}{4\,a^2\,\sqrt{a+\frac{b}{x^4}}}+\frac{x^4}{4\,a\,\sqrt{a+\frac{b}{x^4}}}-\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

input
```
int(x^3/(a + b/x^4)^(3/2),x)
```

output
```
(3*b)/(4*a^2*(a + b/x^4)^(1/2)) + x^4/(4*a*(a + b/x^4)^(1/2)) - (3*b*atanh
((a + b/x^4)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.52

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{-48\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)ab\,x^6-36\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)b^2x^2}{}$$

input
```
int(x^3/(a+b/x^4)^(3/2),x)
```

output
```
( - 48*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt
(b))*a*b*x**6 - 36*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a
)*x**2)/sqrt(b))*b**2*x**2 + 16*sqrt(a)*sqrt(a*x**4 + b)*a**2*x**10 + 88*s
qrt(a)*sqrt(a*x**4 + b)*a*b*x**6 + 39*sqrt(a)*sqrt(a*x**4 + b)*b**2*x**2 -
 48*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**2*b*x**8 - 60*log((s
qrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a*b**2*x**4 - 12*log((sqrt(a*x**4
 + b) + sqrt(a)*x**2)/sqrt(b))*b**3 + 16*a**3*x**12 + 96*a**2*b*x**8 + 81*
a*b**2*x**4 + 9*b**3)/(16*a**2*(4*sqrt(a*x**4 + b)*a**2*x**6 + 3*sqrt(a*x*
*4 + b)*a*b*x**2 + 4*sqrt(a)*a**2*x**8 + 5*sqrt(a)*a*b*x**4 + sqrt(a)*b**2
))
```

3.554 $\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx = -\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}+\frac{\sqrt{a+\frac{b}{x^4}}x^2}{a^2}$$

output
```
-1/2*x^2/a/(a+b/x^4)^(1/2)+(a+b/x^4)^(1/2)*x^2/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx = \frac{2b+ax^4}{2a^2\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[x/(a + b/x^4)^(3/2),x]
```

output
```
(2*b + a*x^4)/(2*a^2*Sqrt[a + b/x^4]*x^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 805}$$

$$\frac{2\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}dx}{a}-\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x^2\sqrt{a+\frac{b}{x^4}}}{a^2}-\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[x/(a + b/x^4)^(3/2),x]
```

output
```
-1/2*x^2/(a*Sqrt[a + b/x^4]) + (Sqrt[a + b/x^4]*x^2)/a^2
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
orering	$\frac{(a\,x^4+2b)\,(a\,x^4+b)}{2a^2x^6\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}$	34
gosper	$\frac{(a\,x^4+b)\,(a\,x^4+2b)}{2a^2x^6\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}}$	38
default	$\frac{(a\,x^4+b)\,(a\,x^4+2b)}{2a^2x^6\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}}$	38
trager	$\frac{(a\,x^4+2b)x^2\sqrt{-\frac{-a\,x^4-b}{x^4}}}{2(a\,x^4+b)a^2}$	44
risch	$\frac{a\,x^4+b}{2a^2\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}+\frac{b}{2a^2\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}$	52

input
```
int(x/(a+b/x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^4+2*b)/a^2/x^6*(a*x^4+b)/(a+b/x^4)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\frac{(ax^6+2\,bx^2)\sqrt{\frac{ax^4+b}{x^4}}}{2\,(a^3x^4+a^2b)}$$

input
```
integrate(x/(a+b/x^4)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(a*x^6 + 2*b*x^2)*sqrt((a*x^4 + b)/x^4)/(a^3*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{x^4}{2a\sqrt{b}\sqrt{\frac{ax^4}{b}+1}} + \frac{\sqrt{b}}{a^2\sqrt{\frac{ax^4}{b}+1}}$$

input
```
integrate(x/(a+b/x**4)**(3/2),x)
```

output
```
x**4/(2*a*sqrt(b)*sqrt(a*x**4/b + 1)) + sqrt(b)/(a**2*sqrt(a*x**4/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{b}{x^4}}\,x^2}{2\,a^2} + \frac{b}{2\,\sqrt{a+\frac{b}{x^4}}\,a^2x^2}$$

input
```
integrate(x/(a+b/x^4)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^4)*x^2/a^2 + 1/2*b/(sqrt(a + b/x^4)*a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{\frac{\sqrt{ax^4+b}}{a} + \frac{b}{\sqrt{ax^4+b}a}}{2\,a}$$

input
```
integrate(x/(a+b/x^4)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(a*x^4 + b)/a + b/(sqrt(a*x^4 + b)*a))/a
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\frac{\frac{a\,x^4}{2}+b}{a^2\,x^2\,\sqrt{a+\frac{b}{x^4}}}$$

input
```
int(x/(a + b/x^4)^(3/2),x)
```

output
```
(b + (a*x^4)/2)/(a^2*x^2*(a + b/x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\frac{\sqrt{a\,x^4+b}\,a\,x^4+2\sqrt{a\,x^4+b}\,b+\sqrt{a}\,a\,x^6+2\sqrt{a}\,b\,x^2}{2a^2\left(\sqrt{a}\,\sqrt{a\,x^4+b}\,x^2+a\,x^4+b\right)}$$

input
```
int(x/(a+b/x^4)^(3/2),x)
```

output
```
(sqrt(a*x**4 + b)*a*x**4 + 2*sqrt(a*x**4 + b)*b + sqrt(a)*a*x**6 + 2*sqrt(
a)*b*x**2)/(2*a**2*(sqrt(a)*sqrt(a*x**4 + b)*x**2 + a*x**4 + b))
```

3.555 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x} \, dx = -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/2/a/(a+b/x^4)^(1/2)+1/2*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x} \, dx = \frac{-\sqrt{a}x^2+\sqrt{b+ax^4}\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{2a^{3/2}\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[1/((a + b/x^4)^(3/2)*x),x]
```

output
```
(-(Sqrt[a]*x^2) + Sqrt[b + a*x^4]*Log[Sqrt[a]*x^2 + Sqrt[b + a*x^4]])/(2*a
^(3/2)*Sqrt[a + b/x^4]*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^4}\right)^{3/2}}dx$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{4}\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x^4}$$

$$\Big\downarrow \ 61$$

$$\frac{1}{4}\left(-\frac{\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}}{a}-\frac{2}{a\sqrt{a+\frac{b}{x^4}}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(-\frac{2\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{ab}-\frac{2}{a\sqrt{a+\frac{b}{x^4}}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{a+\frac{b}{x^4}}}\right)$$

input
```
Int[1/((a + b/x^4)^(3/2)*x),x]
```

output
```
(-2/(a*Sqrt[a + b/x^4]) + (2*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/a^(3/2))/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{\left(a\,x^{4}+b\right)\left(-x^{2} a^{\frac{3}{2}}+\ln\left(\sqrt{a}\,x^{2}+\sqrt{a\,x^{4}+b}\right) a\sqrt{a\,x^{4}+b}\right)}{2\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}} x^{6} a^{\frac{5}{2}}}$	67

input
```
int(1/(a+b/x^4)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^4+b)*(-x^2*a^(3/2)+ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*a*(a*x^4+b)^(1
/2))/((a*x^4+b)/x^4)^(3/2)/x^6/a^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.35

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx = \left[-\frac{2\,ax^4\sqrt{\frac{ax^4+b}{x^4}} - (ax^4+b)\sqrt{a}\log\left(-2\,ax^4 - 2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}} - b\right)}{4\,(a^3x^4+a^2b)}, -\frac{ax^4\sqrt{\frac{ax^4+b}{x^4}} + (ax^4+b)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)}{2\,(a^3x^4+a^2b)}\right]$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*x^4*sqrt((a*x^4 + b)/x^4) - (a*x^4 + b)*sqrt(a)*log(-2*a*x^4 -
2*sqrt(a)*x^4*sqrt((a*x^4 + b)/x^4) - b))/(a^3*x^4 + a^2*b), -1/2*(a*x^4*s
qrt((a*x^4 + b)/x^4) + (a*x^4 + b)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x^4 +
b)/x^4)/a))/(a^3*x^4 + a^2*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(37) = 74$.

Time = 1.38 (sec) , antiderivative size = 187, normalized size of antiderivative = 4.07

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx = -\frac{2a^3x^4\sqrt{1+\frac{b}{ax^4}}}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b} - \frac{a^3x^4\log\left(\frac{b}{ax^4}\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b} + \frac{2a^3x^4\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b} - \frac{a^2b\log\left(\frac{b}{ax^4}\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b} + \frac{2a^2b\log\left(\sqrt{1+\frac{b}{ax^4}}+1\right)}{4a^{\frac{9}{2}}x^4+4a^{\frac{7}{2}}b}$$

input
```
integrate(1/(a+b/x**4)**(3/2)/x,x)
```

output
```
-2*a**3*x**4*sqrt(1 + b/(a*x**4))/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) - a**3*
x**4*log(b/(a*x**4))/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) + 2*a**3*x**4*log(sq
rt(1 + b/(a*x**4)) + 1)/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) - a**2*b*log(b/(a
*x**4))/(4*a**(9/2)*x**4 + 4*a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/(a*x**4
)) + 1)/(4*a**(9/2)*x**4 + 4*a**(7/2)*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}} - \frac{1}{2\,\sqrt{a+\frac{b}{x^4}}a}$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/4*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(3/2)
- 1/2/(sqrt(a + b/x^4)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx = -\frac{x^2}{2\,\sqrt{ax^4+b}a} - \frac{\log\left(\left|-\sqrt{a}x^2+\sqrt{ax^4+b}\right|\right)}{2\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x,x, algorithm="giac")
```

output
```
-1/2*x^2/(sqrt(a*x^4 + b)*a) - 1/2*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b))
)/a^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{1}{2\,a\,\sqrt{a+\frac{b}{x^4}}}$$

input
```
int(1/(x*(a + b/x^4)^(3/2)),x)
```

output
```
atanh((a + b/x^4)^(1/2)/a^(1/2))/(2*a^(3/2)) - 1/(2*a*(a + b/x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x}\,dx = \frac{\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)x^2 - 2\sqrt{a}\,\sqrt{a\,x^4+b}\,x^2 + \log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a\,x^4 +}{2a\left(\sqrt{a\,x^4+b}\,a\,x^2+\sqrt{a}\,a\,x^4+\sqrt{a}\,b\right)}$$

input
```
int(1/(a+b/x^4)^(3/2)/x,x)
```

output
```
(sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*x
**2 - 2*sqrt(a)*sqrt(a*x**4 + b)*x**2 + log((sqrt(a*x**4 + b) + sqrt(a)*x*
*2)/sqrt(b))*a*x**4 + log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*b - 2
*a*x**4 - b)/(2*a*(sqrt(a*x**4 + b)*a*x**2 + sqrt(a)*a*x**4 + sqrt(a)*b))
```

3.556 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3} \, dx = -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}x^2}$$

output
```
-1/2/a/(a+b/x^4)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3} \, dx = -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}x^2}$$

input
```
Integrate[1/((a + b/x^4)^(3/2)*x^3),x]
```

output
```
-1/2*1/(a*Sqrt[a + b/x^4]*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x^4}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{1}{2ax^2\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[1/((a + b/x^4)^(3/2)*x^3),x]
```

output
```
-1/2*1/(a*Sqrt[a + b/x^4]*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

method	result	size
orering	$-\frac{a\,x^{4}+b}{2 a\,x^{6} \left(a+\frac{b}{x^{4}}\right)^{\frac{3}{2}}}$	25
gosper	$-\frac{a\,x^{4}+b}{2 a\,x^{6} \left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}}}$	29
default	$-\frac{a\,x^{4}+b}{2 a\,x^{6} \left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}}}$	29
trager	$-\frac{x^{2} \sqrt{-\frac{-a\,x^{4}-b}{x^{4}}}}{2 a \left(a\,x^{4}+b\right)}$	35

input
```
int(1/(a+b/x^4)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x^4+b)/a/x^6/(a+b/x^4)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3}\,dx=-\frac{x^2\sqrt{\frac{ax^4+b}{x^4}}}{2\left(a^2x^4+ab\right)}$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*x^2*sqrt((a*x^4 + b)/x^4)/(a^2*x^4 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3}\,dx=-\frac{1}{2a\sqrt{b}\sqrt{\frac{ax^4}{b}+1}}$$

input
```
integrate(1/(a+b/x**4)**(3/2)/x**3,x)
```

output
```
-1/(2*a*sqrt(b)*sqrt(a*x**4/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^3}\,dx=-\frac{1}{2\sqrt{a+\frac{b}{x^4}}ax^2}$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2/(sqrt(a + b/x^4)*a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^3}\,dx=-\frac{1}{2\sqrt{ax^4+b}a}$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/2/(sqrt(a*x^4 + b)*a)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^3}\,dx=-\frac{x^2\sqrt{a+\frac{b}{x^4}}}{2\,a\,\left(a\,x^4+b\right)}$$

input
```
int(1/(x^3*(a + b/x^4)^(3/2)),x)
```

output
```
-(x^2*(a + b/x^4)^(1/2))/(2*a*(b + a*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^3}\,dx = \frac{-\sqrt{a\,x^4+b}-\sqrt{a}\,x^2}{2a\left(\sqrt{a}\,\sqrt{a\,x^4+b}\,x^2+a\,x^4+b\right)}$$

input
```
int(1/(a+b/x^4)^(3/2)/x^3,x)
```

output
```
( - (sqrt(a*x**4 + b) + sqrt(a)*x**2))/(2*a*(sqrt(a)*sqrt(a*x**4 + b)*x**2
 + a*x**4 + b))
```

3.557 $\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx = -\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}+\frac{5\sqrt{a+\frac{b}{x^4}}x^3}{6a^2} + \frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/2*x^3/a/(a+b/x^4)^(1/2)+5/6*(a+b/x^4)^(1/2)*x^3/a^2+5/12*b^(3/4)*((a+b/
x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(
2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/a^(9/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\frac{5b+2ax^4-5b\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^4}{b}\right)}{6a^2\sqrt{a+\frac{b}{x^4}}\,x}$$

input
```
Integrate[x^2/(a + b/x^4)^(3/2),x]
```

output
```
(5*b + 2*a*x^4 - 5*b*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4, 1/2, 5/4,
-((a*x^4)/b)])/(6*a^2*Sqrt[a + b/x^4]*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {858, 819, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$-\frac{5\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{2a}-\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{5\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{3a}-\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}\right)}{2a}-\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{5\left(-\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+\frac{b}{x^4}}}-\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}\right)}{2a}-\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[x^2/(a + b/x^4)^(3/2),x]
```

output
```
-1/2*x^3/(a*Sqrt[a + b/x^4]) - (5*(-1/3*(Sqrt[a + b/x^4]*x^3)/a - (b^(3/4)
*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*Ellip
ticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(6*a^(5/4)*Sqrt[a + b/x^4])))/(2
*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

method	result
default	$\frac{\left(a x^4+b\right)\left(2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a x^5-5b\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+5\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,b x\right)}{6\left(\frac{a x^4+b}{x^4}\right)^{\frac{3}{2}} x^6 a^2\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$
risch	$\frac{a x^4+b}{3a^2 x\sqrt{\frac{a x^4+b}{x^4}}}-\frac{b\left(b\left(\frac{x}{2b\sqrt{\left(x^4+\frac{b}{a}\right)a}}+\frac{\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)}{2b\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a x^4+b}}\right)+4a\left(-\frac{x}{2a\sqrt{\left(x^4+\frac{b}{a}\right)a}}+\frac{\sqrt{1-\frac{i\sqrt{a}\,x^2}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}}{\sqrt{}}}}{2a\sqrt{\frac{i\sqrt{}}{\sqrt{}}}}\right.\right.}{3a^2\sqrt{\frac{a x^4+b}{x^4}}\,x^2}$

input
```
int(x^2/(a+b/x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a*x^4+b)*(2*(I*a^(1/2)/b^(1/2))^(1/2)*a*x^5-5*b*(-(I*a^(1/2)*x^2-b^(1
/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I
*a^(1/2)/b^(1/2))^(1/2),I)+5*(I*a^(1/2)/b^(1/2))^(1/2)*b*x)/((a*x^4+b)/x^4
)^(3/2)/x^6/a^2/(I*a^(1/2)/b^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = -\frac{5\,(ax^4+b)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right) - (2\,ax^7+5\,bx^3)\sqrt{\frac{ax^4+b}{x^4}}}{6\,(a^3x^4+a^2b)}$$

input
```
integrate(x^2/(a+b/x^4)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(5*(a*x^4 + b)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin((-b/a)^(1/4)/x)
, -1) - (2*a*x^7 + 5*b*x^3)*sqrt((a*x^4 + b)/x^4))/(a^3*x^4 + a^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.32

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = -\frac{x^3\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(x**2/(a+b/x**4)**(3/2),x)
```

output
```
-x**3*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*exp_polar(I*pi)/(a*x**4))/(
4*a**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a + b/x^4)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a + b/x^4)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a + b/x^4)^(3/2),x)
```

output
```
int(x^2/(a + b/x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{\sqrt{a\,x^4+b}\,a\,x^5+5\sqrt{a\,x^4+b}\,bx-5\left(\int \frac{\sqrt{a\,x^4+b}}{a^2x^8+2ab\,x^4+b^2}dx\right)a\,b^2x^4-5\left(\int \frac{\sqrt{a\,x^4+b}}{a^2x^8+2ab\,x^4+b^2}dx\right)}{3a^2\,(a\,x^4+b)}$$

input
```
int(x^2/(a+b/x^4)^(3/2),x)
```

output
```
(sqrt(a*x**4 + b)*a*x**5 + 5*sqrt(a*x**4 + b)*b*x - 5*int(sqrt(a*x**4 + b)
/(a**2*x**8 + 2*a*b*x**4 + b**2),x)*a*b**2*x**4 - 5*int(sqrt(a*x**4 + b)/(
a**2*x**8 + 2*a*b*x**4 + b**2),x)*b**3)/(3*a**2*(a*x**4 + b))
```

3.558 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 258

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}} dx = -\frac{3\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{2a^2\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} - \frac{x}{2a\sqrt{a+\frac{b}{x^4}}} + \frac{3\sqrt{a+\frac{b}{x^4}}x}{2a^2} + \frac{3\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+\frac{b}{x^4}}} - \frac{3\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-3/2*b^(1/2)*(a+b/x^4)^(1/2)/a^2/(a^(1/2)+b^(1/2)/x^2)/x-1/2*x/a/(a+b/x^4)
^(1/2)+3/2*(a+b/x^4)^(1/2)*x/a^2+3/2*b^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x
^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4
))),1/2*2^(1/2))/a^(7/4)/(a+b/x^4)^(1/2)-3/4*b^(1/4)*((a+b/x^4)/(a^(1/2)+b
^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4
)*x/b^(1/4)),1/2*2^(1/2))/a^(7/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\frac{x-x\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{ax^4}{b}\right)}{a\sqrt{a+\frac{b}{x^4}}}$$

input
```
Integrate[(a + b/x^4)^(-3/2),x]
```

output
```
(x - x*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^4)/b)])
/(a*Sqrt[a + b/x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {773, 819, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$-\frac{3\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{2a}-\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{3\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}-\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{834}$$

$$-\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}-\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}-\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}-\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{1510}$$

$$-\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}}{\sqrt{b}}\right)}{a}-\cdots\right)}{2a}$$

$$\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[(a + b/x^4)^(-3/2),x]
```

output
```
-1/2*x/(a*Sqrt[a + b/x^4]) - (3*(-((Sqrt[a + b/x^4]*x)/a) + (b*(-((-(Sqrt[
a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[
a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)/(a
^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(a +
 b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*Arc
Tan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3/4)*Sqrt[a + b/x^4])))/a))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.73

method	result
default	$\frac{\left(a\,x^{4}+b\right)\left(-x^{3} a^{\frac{3}{2}} \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}+3 i\sqrt{b}\,\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,a\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}, i\right)-3 i\sqrt{b}\,\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,a\,\text{Ellipti}\right.}{2\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}} x^{6} a^{\frac{5}{2}} \sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int(1/(a+b/x^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^4+b)*(-x^3*a^(3/2)*(I*a^(1/2)/b^(1/2))^(1/2)+3*I*b^(1/2)*(-(I*a^(
1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*a
*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)-3*I*b^(1/2)*(-(I*a^(1/2)*x^2-b^(
1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*a*EllipticE(x
*(I*a^(1/2)/b^(1/2))^(1/2),I))/((a*x^4+b)/x^4)^(3/2)/x^6/a^(5/2)/(I*a^(1/2
)/b^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{3\left(ax^4+b\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) - 3\left(ax^4+b\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{b}{a}\right.}{x}\right.\right.}{2\left(a^3x^4+a^2b\right)}$$

input
```
integrate(1/(a+b/x^4)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(3*(a*x^4 + b)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin((-b/a)^(1/4)/x),
 -1) - 3*(a*x^4 + b)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin((-b/a)^(1/4)/x
), -1) + (2*a*x^5 + 3*b*x)*sqrt((a*x^4 + b)/x^4))/(a^3*x^4 + a^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = -\frac{x\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix}\right| \frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(3/2),x)
```

output
```
-x*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a
**(3/2)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx=\frac{x\left(\frac{a\,x^4}{b}+1\right)^{3/2}\sqrt{x^{12}}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a\,x^4}{b}\right)}{7\,\left(a\,x^4+b\right)^{3/2}}$$

input
```
int(1/(a + b/x^4)^(3/2),x)
```

output
```
(x*((a*x^4)/b + 1)^(3/2)*(x^12)^(1/2)*hypergeom([3/2, 7/4], 11/4, -(a*x^4)
/b))/(7*(b + a*x^4)^(3/2))
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx = \frac{\sqrt{a\,x^4+b}\,x^3-3\left(\int \frac{\sqrt{a\,x^4+b}\,x^2}{a^2x^8+2ab\,x^4+b^2}dx\right)ab\,x^4-3\left(\int \frac{\sqrt{a\,x^4+b}\,x^2}{a^2x^8+2ab\,x^4+b^2}dx\right)b^2}{a\left(a\,x^4+b\right)}$$

input
```
int(1/(a+b/x^4)^(3/2),x)
```

output
```
(sqrt(a*x**4 + b)*x**3 - 3*int((sqrt(a*x**4 + b)*x**2)/(a**2*x**8 + 2*a*b*
x**4 + b**2),x)*a*b*x**4 - 3*int((sqrt(a*x**4 + b)*x**2)/(a**2*x**8 + 2*a*
b*x**4 + b**2),x)*b**2)/(a*(a*x**4 + b))
```

3.559 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^2}\,dx=-\frac{1}{2a\sqrt{a+\frac{b}{x^4}}\,x}-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/2/a/(a+b/x^4)^(1/2)/x-1/4*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^
(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2)
)/a^(5/4)/b^(1/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2} \, dx = \frac{-1+\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^4}{b}\right)}{2a\sqrt{a+\frac{b}{x^4}}x}$$

input
```
Integrate[1/((a + b/x^4)^(3/2)*x^2),x]
```

output
```
(-1 + Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4, 1/2, 5/4, -((a*x^4)/b)])/
(2*a*Sqrt[a + b/x^4]*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {858, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{x^4}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}} d\frac{1}{x}$$

$$\Big\downarrow \text{ 749}$$

$$-\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{2a}-\frac{1}{2ax\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{1}{2ax\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[1/((a + b/x^4)^(3/2)*x^2),x]
```

output
```
-1/2*1/(a*Sqrt[a + b/x^4]*x) - (Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2
]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(
4*a^(5/4)*b^(1/4)*Sqrt[a + b/x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{\left(a\,x^{4}+b\right)\left(-\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\right)}{2\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{3}{2}}x^{6}a\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$	113

input
```
int(1/(a+b/x^4)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x^4+b)*(-(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+
b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)+x*(I*a^(1
/2)/b^(1/2))^(1/2))/((a*x^4+b)/x^4)^(3/2)/x^6/a/(I*a^(1/2)/b^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^2}\,dx = -\frac{ax^3\sqrt{\frac{ax^4+b}{x^4}}+(ax^4+b)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|-1\right)}{2\left(a^3x^4+a^2b\right)}$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(a*x^3*sqrt((a*x^4 + b)/x^4) + (a*x^4 + b)*sqrt(b)*(-a/b)^(3/4)*ellip
tic_f(arcsin(x*(-a/b)^(1/4)), -1))/(a^3*x^4 + a^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^2}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(3/2)/x**2,x)
```

output
```
-gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(3
/2)*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2}\,dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^4)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2}\,dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^4)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2}\, dx = -\frac{\left(\frac{b}{a}+x^4\right)^{3/2} {}_2F_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\,\left(a\,x^4+b\right)^{3/2}}$$

input
```
int(1/(x^2*(a + b/x^4)^(3/2)),x)
```

output
```
-((b/a + x^4)^(3/2)*hypergeom([1/4, 3/2], 5/4, -b/(a*x^4)))/(x*(b + a*x^4)
^(3/2))
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2}\, dx = \frac{-\sqrt{a\,x^4+b}\,x+\left(\int \frac{\sqrt{a\,x^4+b}}{a^2x^8+2ab\,x^4+b^2}dx\right)ab\,x^4+\left(\int \frac{\sqrt{a\,x^4+b}}{a^2x^8+2ab\,x^4+b^2}dx\right)b^2}{a\left(a\,x^4+b\right)}$$

input
```
int(1/(a+b/x^4)^(3/2)/x^2,x)
```

output
```
( - sqrt(a*x**4 + b)*x + int(sqrt(a*x**4 + b)/(a**2*x**8 + 2*a*b*x**4 + b*
*2),x)*a*b*x**4 + int(sqrt(a*x**4 + b)/(a**2*x**8 + 2*a*b*x**4 + b**2),x)*
b**2)/(a*(a*x**4 + b))
```

3.560 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 241

$$\begin{aligned}\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4} dx &= -\frac{1}{2a\sqrt{a+\frac{b}{x^4}}x^3}+\frac{\sqrt{a+\frac{b}{x^4}}}{2a\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} \\ &\quad -\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}} \\ &\quad +\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}\end{aligned}$$

output

```
-1/2/a/(a+b/x^4)^(1/2)/x^3+1/2*(a+b/x^4)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1/2)/
x^2)/x-1/2*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)
*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(
a+b/x^4)^(1/2)+1/4*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1
/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/a^(3/4)/
b^(3/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.22

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4} \, dx = \frac{x\sqrt{1+\frac{ax^4}{b}}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{ax^4}{b}\right)}{3b\sqrt{a+\frac{b}{x^4}}}$$

input
```
Integrate[1/((a + b/x^4)^(3/2)*x^4),x]
```

output
```
(x*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^4)/b)])/(3*
b*Sqrt[a + b/x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {858, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{x^4}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2} d\frac{1}{x}}{2a} - \frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \text{834}$$

$$\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}}{2a}-\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}}{2a}-\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}}{2a}-\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}}{\sqrt{b}}}{2a}-\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}$$

input
```
Int[1/((a + b/x^4)^(3/2)*x^4),x]
```

output
```
-1/2*1/(a*Sqrt[a + b/x^4]*x^3) + (-((-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]
/x^2)*x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a]
+ Sqrt[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqr
t[a + b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2
)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2]
)/(2*b^(3/4)*Sqrt[a + b/x^4]))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.78

method	result
default	$\frac{(a\,x^4+b)\left(x^3\sqrt{b}\,\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a}-i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,b\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)+i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,b\,\mathrm{EllipticE}\left(x\sqrt{\right.\right.}{2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{3}{2}}x^6 b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a}}$

input
```
int(1/(a+b/x^4)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^4+b)*(x^3*b^(1/2)*(I*a^(1/2)/b^(1/2))^(1/2)*a^(1/2)-I*(-(I*a^(1/2
)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*b*El
lipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)+I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2)
)^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*b*EllipticE(x*(I*a^(1/2)/b
^(1/2))^(1/2),I))/((a*x^4+b)/x^4)^(3/2)/x^6/b^(3/2)/(I*a^(1/2)/b^(1/2))^(1
/2)/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^4}\,dx = \frac{ax^5\sqrt{\frac{ax^4+b}{x^4}}+(ax^4+b)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\mid -1\right)-(ax^4+b)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F}{2\left(a^2bx^4+ab^2\right)}$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/2*(a*x^5*sqrt((a*x^4 + b)/x^4) + (a*x^4 + b)*sqrt(b)*(-a/b)^(3/4)*ellipt
ic_e(arcsin(x*(-a/b)^(1/4)), -1) - (a*x^4 + b)*sqrt(b)*(-a/b)^(3/4)*ellipt
ic_f(arcsin(x*(-a/b)^(1/4)), -1))/(a^2*b*x^4 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^4}\,dx = -\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(3/2)/x**4,x)
```

output
```
-gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(3
/2)*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^4}\,dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^4)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^4}\,dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/(a+b/x^4)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^4)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4}\,dx=\int \frac{1}{x^4\left(a+\frac{b}{x^4}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a + b/x^4)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b/x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2} x^4}\,dx=\int \frac{\sqrt{a\,x^4+b}\,x^2}{a^2x^8+2ab\,x^4+b^2}dx$$

input
```
int(1/(a+b/x^4)^(3/2)/x^4,x)
```

output
```
int((sqrt(a*x**4 + b)*x**2)/(a**2*x**8 + 2*a*b*x**4 + b**2),x)
```

3.561 $\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\frac{5b}{12a^2\left(a+\frac{b}{x^4}\right)^{3/2}}+\frac{5b}{4a^3\sqrt{a+\frac{b}{x^4}}}+\frac{x^4}{4a\left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{5b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
5/12*b/a^2/(a+b/x^4)^(3/2)+5/4*b/a^3/(a+b/x^4)^(1/2)+1/4*x^4/a/(a+b/x^4)^(
3/2)-5/4*b*arctanh((a+b/x^4)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\frac{\sqrt{a}x^2(15b^2+20abx^4+3a^2x^8)-15b(b+ax^4)^{3/2}\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{12a^{7/2}\sqrt{a+\frac{b}{x^4}}x^2\left(b+ax^4\right)}$$

input
```
Integrate[x^3/(a + b/x^4)^(5/2),x]
```

output
```
(Sqrt[a]*x^2*(15*b^2 + 20*a*b*x^4 + 3*a^2*x^8) - 15*b*(b + a*x^4)^(3/2)*Lo
g[Sqrt[a]*x^2 + Sqrt[b + a*x^4]])/(12*a^(7/2)*Sqrt[a + b/x^4]*x^2*(b + a*x
^4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}dx$$

$$\Big\downarrow \; 798$$

$$-\frac{1}{4}\int \frac{x^8}{\left(a+\frac{b}{x^4}\right)^{5/2}}d\frac{1}{x^4}$$

$$\Big\downarrow \; 52$$

$$\frac{1}{4}\left(\frac{5b\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{5/2}}d\frac{1}{x^4}}{2a}+\frac{x^4}{a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{4}\left(\frac{5b\left(\frac{\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x^4}}{a}+\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)}{2a}+\frac{x^4}{a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{4}\left(\frac{5b\left(\frac{\frac{\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^4}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)}{2a}+\frac{x^4}{a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^4}}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)}{2a}+\frac{x^4}{a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{5b\left(\frac{\frac{2}{a\sqrt{a+\frac{b}{x^4}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)}{2a}+\frac{x^4}{a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

input
```
Int[x^3/(a + b/x^4)^(5/2),x]
```

output
```
(x^4/(a*(a + b/x^4)^(3/2)) + (5*b*(2/(3*a*(a + b/x^4)^(3/2)) + (2/(a*Sqrt[
a + b/x^4]) - (2*ArcTanh[Sqrt[a + b/x^4]/Sqrt[a]])/a^(3/2))/a))/(2*a))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(68) = 136.

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

method	result
risch	$\frac{a x^4+b}{4a^3\sqrt{\frac{a x^4+b}{x^4}}}-\frac{b\left(\frac{5\ln\left(\sqrt{a}\, x^2+\sqrt{a x^4+b}\right)}{2\sqrt{a}}-\frac{3x^2}{\sqrt{a x^4+b}}+\frac{\sqrt{a x^4+b}\, x^2\left(2a x^4+3b\right)}{3a^2x^8+6ab x^4+3b^2}\right)\sqrt{a x^4+b}}{2a^3\sqrt{\frac{a x^4+b}{x^4}}\, x^2}$
default	$\frac{(a x^4+b)^{\frac{5}{2}}\left(3\sqrt{a x^4+b}\, a^{\frac{15}{2}} x^{10}+6a^{\frac{13}{2}} b\sqrt{a x^4+b}\, x^6+14a^{\frac{13}{2}}\sqrt{-\frac{\left(a x^2+\sqrt{-ab}\right)\left(-a x^2+\sqrt{-ab}\right)}{a}}\, b x^6-15\ln\left(\sqrt{a}\, x^2+\sqrt{a x^4+b}\right)a^7 b x^8+\right.}{12a^{\frac{13}{2}}\left(\frac{a x^4+b}{x^4}\right)^{\frac{5}{2}} x^{10}(-a}$

input
```
int(x^3/(a+b/x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^3*(a*x^4+b)/((a*x^4+b)/x^4)^(1/2)-1/2/a^3*b*(5/2*ln(a^(1/2)*x^2+(a*x
^4+b)^(1/2))/a^(1/2)-3*x^2/(a*x^4+b)^(1/2)+1/3*(a*x^4+b)^(1/2)*x^2*(2*a*x^
4+3*b)/(a^2*x^8+2*a*b*x^4+b^2))/((a*x^4+b)/x^4)^(1/2)/x^2*(a*x^4+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.85

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\left[\frac{15\left(a^2bx^8+2\,ab^2x^4+b^3\right)\sqrt{a}\log\left(-2\,ax^4+2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)+2\left(3\,a^3x^{12}+20\,a^2\right.}{24\left(a^6x^8+2\,a^5bx^4+a^4b^2\right)}\right.$$

input
```
integrate(x^3/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(15*(a^2*b*x^8 + 2*a*b^2*x^4 + b^3)*sqrt(a)*log(-2*a*x^4 + 2*sqrt(a)
*x^4*sqrt((a*x^4 + b)/x^4) - b) + 2*(3*a^3*x^12 + 20*a^2*b*x^8 + 15*a*b^2*
x^4)*sqrt((a*x^4 + b)/x^4))/(a^6*x^8 + 2*a^5*b*x^4 + a^4*b^2), 1/12*(15*(a
^2*b*x^8 + 2*a*b^2*x^4 + b^3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^
4)/a) + (3*a^3*x^12 + 20*a^2*b*x^8 + 15*a*b^2*x^4)*sqrt((a*x^4 + b)/x^4))/
(a^6*x^8 + 2*a^5*b*x^4 + a^4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(80) = 160$.

Time = 3.90 (sec) , antiderivative size = 819, normalized size of antiderivative = 9.31

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(a+b/x**4)**(5/2),x)
```

output
```
6*a**17*x**16*sqrt(1 + b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x*
*8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 46*a**16*b*x**12*sqrt(1
 + b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b*
*2*x**4 + 24*a**(33/2)*b**3) + 15*a**16*b*x**12*log(b/(a*x**4))/(24*a**(39
/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b*
*3) - 30*a**16*b*x**12*log(sqrt(1 + b/(a*x**4)) + 1)/(24*a**(39/2)*x**12 +
 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 70*a*
*15*b**2*x**8*sqrt(1 + b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x*
*8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 45*a**15*b**2*x**8*log(
b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*
x**4 + 24*a**(33/2)*b**3) - 90*a**15*b**2*x**8*log(sqrt(1 + b/(a*x**4)) +
1)/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24
*a**(33/2)*b**3) + 30*a**14*b**3*x**4*sqrt(1 + b/(a*x**4))/(24*a**(39/2)*x
**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) +
 45*a**14*b**3*x**4*log(b/(a*x**4))/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x
**8 + 72*a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) - 90*a**14*b**3*x**4*log
(sqrt(1 + b/(a*x**4)) + 1)/(24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*
a**(35/2)*b**2*x**4 + 24*a**(33/2)*b**3) + 15*a**13*b**4*log(b/(a*x**4))/(
24*a**(39/2)*x**12 + 72*a**(37/2)*b*x**8 + 72*a**(35/2)*b**2*x**4 + 24*a**
(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x**4)) + 1)/(24*a**(39/2...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{15\left(a+\frac{b}{x^4}\right)^2 b-10\left(a+\frac{b}{x^4}\right)ab-2\,a^2b}{12\left(\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^4\right)}+\frac{5\,b\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

input
```
integrate(x^3/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(15*(a + b/x^4)^2*b - 10*(a + b/x^4)*a*b - 2*a^2*b)/((a + b/x^4)^(5/2
)*a^3 - (a + b/x^4)^(3/2)*a^4) + 5/8*b*log((sqrt(a + b/x^4) - sqrt(a))/(sq
rt(a + b/x^4) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,x^4}{a}+\frac{20\,b}{a^2}\right)x^4+\frac{15\,b^2}{a^3}\right)x^2}{12\left(ax^4+b\right)^{\frac{3}{2}}}+\frac{5\,b\log\left(\left|-\sqrt{a}x^2+\sqrt{ax^4+b}\right|\right)}{4\,a^{\frac{7}{2}}}$$

input
```
integrate(x^3/(a+b/x^4)^(5/2),x, algorithm="giac")
```

output
```
1/12*((3*x^4/a + 20*b/a^2)*x^4 + 15*b^2/a^3)*x^2/(a*x^4 + b)^(3/2) + 5/4*b
*log(abs(-sqrt(a)*x^2 + sqrt(a*x^4 + b)))/a^(7/2)
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{5\,b}{3\,a^2\,\left(a+\frac{b}{x^4}\right)^{3/2}}+\frac{x^4}{4\,a\,\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{4\,a^{7/2}}+\frac{5\,b^2}{4\,a^3\,x^4\,\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
int(x^3/(a + b/x^4)^(5/2),x)
```

output
```
(5*b)/(3*a^2*(a + b/x^4)^(3/2)) + x^4/(4*a*(a + b/x^4)^(3/2)) - (5*b*atanh
((a + b/x^4)^(1/2)/a^(1/2)))/(4*a^(7/2)) + (5*b^2)/(4*a^3*x^4*(a + b/x^4)^
(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 588, normalized size of antiderivative = 6.68

$$\int \frac{x^3}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{-480\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a^3b\,x^{14}-1080\sqrt{a}\,\sqrt{a\,x^4+b}\,\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)}{}$$

input
```
int(x^3/(a+b/x^4)^(5/2),x)
```

output
```
( - 480*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqr
t(b))*a**3*b*x**14 - 1080*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) +
 sqrt(a)*x**2)/sqrt(b))*a**2*b**2*x**10 - 750*sqrt(a)*sqrt(a*x**4 + b)*log
((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a*b**3*x**6 - 150*sqrt(a)*sqrt
(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*b**4*x**2 + 96
*sqrt(a)*sqrt(a*x**4 + b)*a**4*x**18 + 632*sqrt(a)*sqrt(a*x**4 + b)*a**3*b
*x**14 + 786*sqrt(a)*sqrt(a*x**4 + b)*a**2*b**2*x**10 + 275*sqrt(a)*sqrt(a
*x**4 + b)*a*b**3*x**6 + 5*sqrt(a)*sqrt(a*x**4 + b)*b**4*x**2 - 480*log((s
qrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**4*b*x**16 - 1320*log((sqrt(a*x
**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**3*b**2*x**12 - 1230*log((sqrt(a*x**4
+ b) + sqrt(a)*x**2)/sqrt(b))*a**2*b**3*x**8 - 420*log((sqrt(a*x**4 + b) +
 sqrt(a)*x**2)/sqrt(b))*a*b**4*x**4 - 30*log((sqrt(a*x**4 + b) + sqrt(a)*x
**2)/sqrt(b))*b**5 + 96*a**5*x**20 + 680*a**4*b*x**16 + 1090*a**3*b**2*x**
12 + 595*a**2*b**3*x**8 + 80*a*b**4*x**4 - 5*b**5)/(24*a**3*(16*sqrt(a*x**
4 + b)*a**4*x**14 + 36*sqrt(a*x**4 + b)*a**3*b*x**10 + 25*sqrt(a*x**4 + b)
*a**2*b**2*x**6 + 5*sqrt(a*x**4 + b)*a*b**3*x**2 + 16*sqrt(a)*a**4*x**16 +
 44*sqrt(a)*a**3*b*x**12 + 41*sqrt(a)*a**2*b**2*x**8 + 14*sqrt(a)*a*b**3*x
**4 + sqrt(a)*b**4))
```

3.562 $\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}} dx = -\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{2x^2}{3a^2\sqrt{a+\frac{b}{x^4}}} + \frac{4\sqrt{a+\frac{b}{x^4}}x^2}{3a^3}$$

output
```
-1/6*x^2/a/(a+b/x^4)^(3/2)-2/3*x^2/a^2/(a+b/x^4)^(1/2)+4/3*(a+b/x^4)^(1/2)
*x^2/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}} dx = \frac{(b+ax^4)\left(8b^2+12abx^4+3a^2x^8\right)}{6a^3\left(a+\frac{b}{x^4}\right)^{5/2}x^{10}}$$

input
```
Integrate[x/(a + b/x^4)^(5/2),x]
```

output
```
((b + a*x^4)*(8*b^2 + 12*a*b*x^4 + 3*a^2*x^8))/(6*a^3*(a + b/x^4)^(5/2)*x^
10)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}dx$$

$$\Big\downarrow \text{805}$$

$$\frac{4\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{3/2}}dx}{3a}-\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \text{805}$$

$$\frac{4\left(\frac{2\int \frac{x}{\sqrt{a+\frac{b}{x^4}}}dx}{a}-\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{3a}-\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4\left(\frac{x^2\sqrt{a+\frac{b}{x^4}}}{a^2}-\frac{x^2}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{3a}-\frac{x^2}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
Int[x/(a + b/x^4)^(5/2),x]
```

output
```
-1/6*x^2/(a*(a + b/x^4)^(3/2)) + (4*(-1/2*x^2/(a*Sqrt[a + b/x^4]) + (Sqrt[
a + b/x^4]*x^2)/a^2))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

method	result	size
orering	$\frac{(3a^2x^8+12ab\,x^4+8b^2)(a\,x^4+b)}{6a^3x^{10}\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}$	46
gosper	$\frac{(a\,x^4+b)(3a^2x^8+12ab\,x^4+8b^2)}{6a^3x^{10}\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}}$	50
default	$\frac{(a\,x^4+b)(3a^2x^8+12ab\,x^4+8b^2)}{6a^3x^{10}\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}}$	50
trager	$\frac{x^2(3a^2x^8+12ab\,x^4+8b^2)\sqrt{-\frac{-a\,x^4-b}{x^4}}}{6a^3(a\,x^4+b)^2}$	56
risch	$\frac{a\,x^4+b}{2a^3\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}+\frac{(a\,x^4+b)(6a\,x^4+5b)b}{6a^3(a^2x^8+2ab\,x^4+b^2)\sqrt{\frac{a\,x^4+b}{x^4}}\,x^2}$	89

input
```
int(x/(a+b/x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*a^2*x^8+12*a*b*x^4+8*b^2)/a^3/x^10*(a*x^4+b)/(a+b/x^4)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{\left(3\,a^2x^{10}+12\,abx^6+8\,b^2x^2\right)\sqrt{\frac{ax^4+b}{x^4}}}{6\,\left(a^5x^8+2\,a^4bx^4+a^3b^2\right)}$$

input
```
integrate(x/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*a^2*x^10 + 12*a*b*x^6 + 8*b^2*x^2)*sqrt((a*x^4 + b)/x^4)/(a^5*x^8 +
 2*a^4*b*x^4 + a^3*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(56) = 112.

Time = 1.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.55

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{3a^2b^{\frac{9}{2}}x^8\sqrt{\frac{ax^4}{b}+1}}{6a^5b^4x^8+12a^4b^5x^4+6a^3b^6} + \frac{12ab^{\frac{11}{2}}x^4\sqrt{\frac{ax^4}{b}+1}}{6a^5b^4x^8+12a^4b^5x^4+6a^3b^6} + \frac{8b^{\frac{13}{2}}\sqrt{\frac{ax^4}{b}+1}}{6a^5b^4x^8+12a^4b^5x^4+6a^3b^6}$$

input
```
integrate(x/(a+b/x**4)**(5/2),x)
```

output
```
3*a**2*b**(9/2)*x**8*sqrt(a*x**4/b + 1)/(6*a**5*b**4*x**8 + 12*a**4*b**5*x
**4 + 6*a**3*b**6) + 12*a*b**(11/2)*x**4*sqrt(a*x**4/b + 1)/(6*a**5*b**4*x
**8 + 12*a**4*b**5*x**4 + 6*a**3*b**6) + 8*b**(13/2)*sqrt(a*x**4/b + 1)/(6
*a**5*b**4*x**8 + 12*a**4*b**5*x**4 + 6*a**3*b**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{\sqrt{a+\frac{b}{x^4}}x^2}{2\,a^3} + \frac{6\left(a+\frac{b}{x^4}\right)bx^4 - b^2}{6\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^3x^6}$$

input
```
integrate(x/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a + b/x^4)*x^2/a^3 + 1/6*(6*(a + b/x^4)*b*x^4 - b^2)/((a + b/x^4)
^(3/2)*a^3*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{\frac{3\sqrt{ax^4+b}}{a} + \frac{6\left(ax^4+b\right)b-b^2}{\left(ax^4+b\right)^{\frac{3}{2}}a}}{6\,a^2}$$

input
```
integrate(x/(a+b/x^4)^(5/2),x, algorithm="giac")
```

output
```
1/6*(3*sqrt(a*x^4 + b)/a + (6*(a*x^4 + b)*b - b^2)/((a*x^4 + b)^(3/2)*a))/
a^2
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{3\,a^2\,x^8 + 12\,a\,b\,x^4 + 8\,b^2}{6\,a^3\,x^6\,\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
int(x/(a + b/x^4)^(5/2),x)
```

output
```
(8*b^2 + 3*a^2*x^8 + 12*a*b*x^4)/(6*a^3*x^6*(a + b/x^4)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.06

$$\int \frac{x}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{12\sqrt{a\,x^4+b}\,a^3x^{12}+51\sqrt{a\,x^4+b}\,a^2b\,x^8+44\sqrt{a\,x^4+b}\,a\,b^2x^4+8\sqrt{a\,x^4+b}\,b^3+12\sqrt{a}}{6a^3\left(4\sqrt{a}\,\sqrt{a\,x^4+b}\,a^2x^{10}+7\sqrt{a}\,\sqrt{a\,x^4+b}\,ab\,x^6+3\sqrt{a}\,\sqrt{a\,x^4+b}\,b^2x^2}$$

input
```
int(x/(a+b/x^4)^(5/2),x)
```

output
```
(12*sqrt(a*x**4 + b)*a**3*x**12 + 51*sqrt(a*x**4 + b)*a**2*b*x**8 + 44*sqr
t(a*x**4 + b)*a*b**2*x**4 + 8*sqrt(a*x**4 + b)*b**3 + 12*sqrt(a)*a**3*x**1
4 + 57*sqrt(a)*a**2*b*x**10 + 68*sqrt(a)*a*b**2*x**6 + 24*sqrt(a)*b**3*x**
2)/(6*a**3*(4*sqrt(a)*sqrt(a*x**4 + b)*a**2*x**10 + 7*sqrt(a)*sqrt(a*x**4
+ b)*a*b*x**6 + 3*sqrt(a)*sqrt(a*x**4 + b)*b**2*x**2 + 4*a**3*x**12 + 9*a*
*2*b*x**8 + 6*a*b**2*x**4 + b**3))
```

3.563 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x} \, dx = -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{1}{2a^2\sqrt{a+\frac{b}{x^4}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/6/a/(a+b/x^4)^(3/2)-1/2/a^2/(a+b/x^4)^(1/2)+1/2*arctanh((a+b/x^4)^(1/2)
/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x} \, dx = \frac{-\sqrt{a}x^2(3b+4ax^4)+3(b+ax^4)^{3/2}\log\left(\sqrt{a}x^2+\sqrt{b+ax^4}\right)}{6a^{5/2}\sqrt{a+\frac{b}{x^4}}x^2\left(b+ax^4\right)}$$

input
```
Integrate[1/((a + b/x^4)^(5/2)*x),x]
```

output
```
(-(Sqrt[a]*x^2*(3*b + 4*a*x^4)) + 3*(b + a*x^4)^(3/2)*Log[Sqrt[a]*x^2 + Sq
rt[b + a*x^4]])/(6*a^(5/2)*Sqrt[a + b/x^4]*x^2*(b + a*x^4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{x^4}\right)^{5/2}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{4}\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{5/2}}d\frac{1}{x^4}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\frac{\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x^4}}{a}-\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\frac{\frac{\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x^4}}{a}+\frac{2}{a\sqrt{a+\frac{b}{x^4}}}}{a}-\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\frac{2\int \frac{1}{\frac{1}{bx^8}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^4}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{x^4}}}}{a}-\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{\frac{2}{a\sqrt{a+\frac{b}{x^4}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{a^{3/2}}}{a}-\frac{2}{3a\left(a+\frac{b}{x^4}\right)^{3/2}}\right)$$

input
```
Int[1/((a + b/x^4)^(5/2)*x),x]
```

output
```
(-2/(3*a*(a + b/x^4)^(3/2)) - (2/(a*Sqrt[a + b/x^4]) - (2*ArcTanh[Sqrt[a +
 b/x^4]/Sqrt[a]])/a^(3/2))/a)/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(48) = 96$.

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.45

method	result
default	$-\frac{(a\,x^4+b)^{\frac{5}{2}}\left(4a^{\frac{9}{2}}\sqrt{-\frac{\left(a\,x^2+\sqrt{-ab}\right)\left(-a\,x^2+\sqrt{-ab}\right)}{a}}\,x^6-3\ln\left(\sqrt{a}\,x^2+\sqrt{a\,x^4+b}\right)a^5x^8+3a^{\frac{7}{2}}b\sqrt{-\frac{\left(a\,x^2+\sqrt{-ab}\right)\left(-a\,x^2+\sqrt{-ab}\right)}{a}}\,x^2-6\right.}{6a^{\frac{7}{2}}\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}x^{10}\left(-a\,x^2+\sqrt{-ab}\right)^2\left(a\,x^2+\sqrt{-ab}\right)^2}$

input
```
int(1/(a+b/x^4)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(a*x^4+b)^(5/2)*(4*a^(9/2)*(-1/a*(a*x^2+(-a*b)^(1/2))*(-a*x^2+(-a*b)^
(1/2)))^(1/2)*x^6-3*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2))*a^5*x^8+3*a^(7/2)*b*(-
1/a*(a*x^2+(-a*b)^(1/2))*(-a*x^2+(-a*b)^(1/2)))^(1/2)*x^2-6*a^4*b*ln(a^(1/
2)*x^2+(a*x^4+b)^(1/2))*x^4-3*a^3*b^2*ln(a^(1/2)*x^2+(a*x^4+b)^(1/2)))/a^(
7/2)/((a*x^4+b)/x^4)^(5/2)/x^10/(-a*x^2+(-a*b)^(1/2))^2/(a*x^2+(-a*b)^(1/2
))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(48) = 96$.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.48

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x}\,dx = \left[\frac{3\left(a^2x^8+2\,abx^4+b^2\right)\sqrt{a}\log\left(-2\,ax^4-2\,\sqrt{a}x^4\sqrt{\frac{ax^4+b}{x^4}}-b\right)-2\left(4\,a^2x^8+3\,abx^4\right)}{12\left(a^5x^8+2\,a^4bx^4+a^3b^2\right)}\right.$$

$$\left. -\frac{3\left(a^2x^8+2\,abx^4+b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax^4+b}{x^4}}}{a}\right)+\left(4\,a^2x^8+3\,abx^4\right)\sqrt{\frac{ax^4+b}{x^4}}}{6\left(a^5x^8+2\,a^4bx^4+a^3b^2\right)}\right]$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(a)*log(-2*a*x^4 - 2*sqrt(a)*x^4*
sqrt((a*x^4 + b)/x^4) - b) - 2*(4*a^2*x^8 + 3*a*b*x^4)*sqrt((a*x^4 + b)/x^
4))/(a^5*x^8 + 2*a^4*b*x^4 + a^3*b^2), -1/6*(3*(a^2*x^8 + 2*a*b*x^4 + b^2)
*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x^4 + b)/x^4)/a) + (4*a^2*x^8 + 3*a*b*x^
4)*sqrt((a*x^4 + b)/x^4))/(a^5*x^8 + 2*a^4*b*x^4 + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(54) = 108$.

Time = 2.27 (sec) , antiderivative size = 743, normalized size of antiderivative = 11.61

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**4)**(5/2)/x,x)
```

output
```
-8*a**7*x**12*sqrt(1 + b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x*
*8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) - 3*a**7*x**12*log(b/(a*x
**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 +
 12*a**(13/2)*b**3) + 6*a**7*x**12*log(sqrt(1 + b/(a*x**4)) + 1)/(12*a**(1
9/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b
**3) - 14*a**6*b*x**8*sqrt(1 + b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(17
/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) - 9*a**6*b*x**8*l
og(b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b*
*2*x**4 + 12*a**(13/2)*b**3) + 18*a**6*b*x**8*log(sqrt(1 + b/(a*x**4)) + 1
)/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*
a**(13/2)*b**3) - 6*a**5*b**2*x**4*sqrt(1 + b/(a*x**4))/(12*a**(19/2)*x**1
2 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) - 9*
a**5*b**2*x**4*log(b/(a*x**4))/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 +
 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3) + 18*a**5*b**2*x**4*log(sqrt(
1 + b/(a*x**4)) + 1)/(12*a**(19/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15
/2)*b**2*x**4 + 12*a**(13/2)*b**3) - 3*a**4*b**3*log(b/(a*x**4))/(12*a**(1
9/2)*x**12 + 36*a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b
**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x**4)) + 1)/(12*a**(19/2)*x**12 + 36*
a**(17/2)*b*x**8 + 36*a**(15/2)*b**2*x**4 + 12*a**(13/2)*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^4}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^4}}+\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}} - \frac{4\,a+\frac{3\,b}{x^4}}{6\,\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x,x, algorithm="maxima")
```

output
```
-1/4*log((sqrt(a + b/x^4) - sqrt(a))/(sqrt(a + b/x^4) + sqrt(a)))/a^(5/2)
- 1/6*(4*a + 3*b/x^4)/((a + b/x^4)^(3/2)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x}\, dx = -\frac{\left(\frac{4\,x^4}{a}+\frac{3\,b}{a^2}\right)x^2}{6\,(ax^4+b)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{a}x^2+\sqrt{ax^4+b}\right|\right)}{2\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x,x, algorithm="giac")
```

output
```
-1/6*(4*x^4/a + 3*b/a^2)*x^2/(a*x^4 + b)^(3/2) - 1/2*log(abs(-sqrt(a)*x^2
+ sqrt(a*x^4 + b)))/a^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x}\, dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x^4}}}{\sqrt{a}}\right)}{2\,a^{5/2}} - \frac{\frac{a+\frac{b}{x^4}}{a^2}+\frac{1}{3\,a}}{2\,\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
int(1/(x*(a + b/x^4)^(5/2)),x)
```

output
```
atanh((a + b/x^4)^(1/2)/a^(1/2))/(2*a^(5/2)) - ((a + b/x^4)/a^2 + 1/(3*a))
/(2*(a + b/x^4)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.36

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x}\, dx = \frac{12\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)a^2x^{10}+21\sqrt{a}\,\sqrt{a\,x^4+b}\log\left(\frac{\sqrt{a\,x^4+b}+\sqrt{a}\,x^2}{\sqrt{b}}\right)ab\,x^6}{}$$

input
```
int(1/(a+b/x^4)^(5/2)/x,x)
```

output
```
(12*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b)
)*a**2*x**10 + 21*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b) + sqrt(a)
*x**2)/sqrt(b))*a*b*x**6 + 9*sqrt(a)*sqrt(a*x**4 + b)*log((sqrt(a*x**4 + b
) + sqrt(a)*x**2)/sqrt(b))*b**2*x**2 - 16*sqrt(a)*sqrt(a*x**4 + b)*a**2*x*
*10 - 16*sqrt(a)*sqrt(a*x**4 + b)*a*b*x**6 - 3*sqrt(a)*sqrt(a*x**4 + b)*b*
*2*x**2 + 12*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**3*x**12 + 2
7*log((sqrt(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a**2*b*x**8 + 18*log((sqr
t(a*x**4 + b) + sqrt(a)*x**2)/sqrt(b))*a*b**2*x**4 + 3*log((sqrt(a*x**4 +
b) + sqrt(a)*x**2)/sqrt(b))*b**3 - 16*a**3*x**12 - 24*a**2*b*x**8 - 9*a*b*
*2*x**4)/(6*a**2*(4*sqrt(a*x**4 + b)*a**3*x**10 + 7*sqrt(a*x**4 + b)*a**2*
b*x**6 + 3*sqrt(a*x**4 + b)*a*b**2*x**2 + 4*sqrt(a)*a**3*x**12 + 9*sqrt(a)
*a**2*b*x**8 + 6*sqrt(a)*a*b**2*x**4 + sqrt(a)*b**3))
```

3.564 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3} \, dx=-\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2} x^2}-\frac{1}{3a^2\sqrt{a+\frac{b}{x^4}}x^2}$$

output
```
-1/6/a/(a+b/x^4)^(3/2)/x^2-1/3/a^2/(a+b/x^4)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3} \, dx=\frac{(-2b-3ax^4)(b+ax^4)}{6a^2\left(a+\frac{b}{x^4}\right)^{5/2} x^{10}}$$

input
```
Integrate[1/((a + b/x^4)^(5/2)*x^3),x]
```

output
```
((-2*b - 3*a*x^4)*(b + a*x^4))/(6*a^2*(a + b/x^4)^(5/2)*x^10)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{x^4}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2b \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^7}dx}{a}-\frac{1}{2 a x^2 \left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{b}{3 a^2 x^6 \left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{1}{2 a x^2 \left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
Int[1/((a + b/x^4)^(5/2)*x^3),x]
```

output
```
-1/3*b/(a^2*(a + b/x^4)^(3/2)*x^6) - 1/(2*a*(a + b/x^4)^(3/2)*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
orering	$-\frac{(3a\,x^4+2b)\,(a\,x^4+b)}{6a^2x^{10}\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}$	35
gosper	$-\frac{(a\,x^4+b)\,(3a\,x^4+2b)}{6a^2x^{10}\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}}$	39
default	$-\frac{(a\,x^4+b)\,(3a\,x^4+2b)}{6a^2x^{10}\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}}$	39
trager	$-\frac{x^2(3a\,x^4+2b)\sqrt{-\frac{-a\,x^4-b}{x^4}}}{6a^2(a\,x^4+b)^2}$	45

input
```
int(1/(a+b/x^4)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*a*x^4+2*b)/a^2/x^10*(a*x^4+b)/(a+b/x^4)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x^3}\,dx = -\frac{(3\,ax^6+2\,bx^2)\sqrt{\frac{ax^4+b}{x^4}}}{6\,(a^4x^8+2\,a^3bx^4+a^2b^2)}$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
-1/6*(3*a*x^6 + 2*b*x^2)*sqrt((a*x^4 + b)/x^4)/(a^4*x^8 + 2*a^3*b*x^4 + a^
2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(39) = 78$.

Time = 1.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3}\, dx = -\frac{3ax^4}{6a^3\sqrt{b}x^4\sqrt{\frac{ax^4}{b}+1}+6a^2b^{\frac{3}{2}}\sqrt{\frac{ax^4}{b}+1}} - \frac{2b}{6a^3\sqrt{b}x^4\sqrt{\frac{ax^4}{b}+1}+6a^2b^{\frac{3}{2}}\sqrt{\frac{ax^4}{b}+1}}$$

input
```
integrate(1/(a+b/x**4)**(5/2)/x**3,x)
```

output
```
-3*a*x**4/(6*a**3*sqrt(b)*x**4*sqrt(a*x**4/b + 1) + 6*a**2*b**(3/2)*sqrt(a
*x**4/b + 1)) - 2*b/(6*a**3*sqrt(b)*x**4*sqrt(a*x**4/b + 1) + 6*a**2*b**(3
/2)*sqrt(a*x**4/b + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3}\, dx = -\frac{3\left(a+\frac{b}{x^4}\right)x^4-b}{6\left(a+\frac{b}{x^4}\right)^{\frac{3}{2}}a^2x^6}$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
-1/6*(3*(a + b/x^4)*x^4 - b)/((a + b/x^4)^(3/2)*a^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3} \, dx = -\frac{3\, ax^4+2\, b}{6\, (ax^4+b)^{\frac{3}{2}} a^2}$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^3,x, algorithm="giac")
```

output
```
-1/6*(3*a*x^4 + 2*b)/((a*x^4 + b)^(3/2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3} \, dx = -\frac{x^2\, \sqrt{a+\frac{b}{x^4}}\, (3\, a\, x^4+2\, b)}{6\, a^2\, (a\, x^4+b)^2}$$

input
```
int(1/(x^3*(a + b/x^4)^(5/2)),x)
```

output
```
-(x^2*(a + b/x^4)^(1/2)*(2*b + 3*a*x^4))/(6*a^2*(b + a*x^4)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.81

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^3} \, dx = \frac{-12\sqrt{a\, x^4+b}\, a^2x^8-11\sqrt{a\, x^4+b}\, ab\, x^4-2\sqrt{a\, x^4+b}\, b^2-12\sqrt{a}\, a^2x^{10}-17\sqrt{a}}{6a^2 \left(4\sqrt{a}\, \sqrt{a\, x^4+b}\, a^2x^{10}+7\sqrt{a}\, \sqrt{a\, x^4+b}\, ab\, x^6+3\sqrt{a}\, \sqrt{a\, x^4+b}\, b^2x^2+4a^3x^{12}+\right.}$$

input
```
int(1/(a+b/x^4)^(5/2)/x^3,x)
```

output
```
( - 12*sqrt(a*x**4 + b)*a**2*x**8 - 11*sqrt(a*x**4 + b)*a*b*x**4 - 2*sqrt(
a*x**4 + b)*b**2 - 12*sqrt(a)*a**2*x**10 - 17*sqrt(a)*a*b*x**6 - 6*sqrt(a)
*b**2*x**2)/(6*a**2*(4*sqrt(a)*sqrt(a*x**4 + b)*a**2*x**10 + 7*sqrt(a)*sqr
t(a*x**4 + b)*a*b*x**6 + 3*sqrt(a)*sqrt(a*x**4 + b)*b**2*x**2 + 4*a**3*x**
12 + 9*a**2*b*x**8 + 6*a*b**2*x**4 + b**3))
```

3.565 $\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 152

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = -\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}} - \frac{3x^3}{4a^2\sqrt{a+\frac{b}{x^4}}} + \frac{5\sqrt{a+\frac{b}{x^4}}\,x^3}{4a^3} + \frac{5b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{8a^{13/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/6*x^3/a/(a+b/x^4)^(3/2)-3/4*x^3/a^2/(a+b/x^4)^(1/2)+5/4*(a+b/x^4)^(1/2)
*x^3/a^3+5/8*b^(3/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^
(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/a^(13/
4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\frac{15b^2+21abx^4+4a^2x^8-15b(b+ax^4)\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^4}{b}\right)}{12a^3\sqrt{a+\frac{b}{x^4}}x\,(b+ax^4)}$$

input
```
Integrate[x^2/(a + b/x^4)^(5/2),x]
```

output
```
(15*b^2 + 21*a*b*x^4 + 4*a^2*x^8 - 15*b*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -((a*x^4)/b)])/(12*a^3*Sqrt[a + b/x^4]*x*(b
 + a*x^4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {858, 819, 819, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$-\frac{3\int \frac{x^4}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x}}{2a}-\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{3\left(\frac{5\int \frac{x^4}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{2a}+\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{2a}-\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 847

$$-\frac{3\left(\frac{5\left(-\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{3a}-\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}\right)}{2a}+\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{2a}-\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 761

$$-\frac{3\left(\frac{5\left(-\frac{b^{3/4}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+\frac{b}{x^4}}}-\frac{x^3\sqrt{a+\frac{b}{x^4}}}{3a}\right)}{2a}+\frac{x^3}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{2a}-\frac{x^3}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
Int[x^2/(a + b/x^4)^(5/2),x]
```

output
```
-1/6*x^3/(a*(a + b/x^4)^(3/2)) - (3*(x^3/(2*a*Sqrt[a + b/x^4]) + (5*(-1/3*
(Sqrt[a + b/x^4]*x^3)/a - (b^(3/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2
)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2]
)/(6*a^(5/4)*Sqrt[a + b/x^4])))/(2*a)))/(2*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.00

method	result
default	$\frac{4\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^{3}x^{13}-15\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)a^{2}b\,x^{8}+25\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^{2}b\,x^{9}-30\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\mathrm{Ell}}{12a^{3}\left(\frac{a\,x^{4}+b}{x^{4}}\right)^{\frac{5}{2}}x^{10}}$
risch	$\frac{a\,x^{4}+b}{3a^{3}x\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}-\frac{b\left(\frac{7\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a\,x^{4}+b}}-9b\left(\frac{x}{2b\sqrt{\left(x^{4}+\frac{b}{a}\right)a}}+\frac{\sqrt{1-\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\sqrt{1+\frac{i\sqrt{a}\,x^{2}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right.}{2b\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{a\,x^{4}+b}}\right.\right.}{3a^{3}\sqrt{\frac{a\,x^{4}+b}{x^{4}}}}$

input
```
int(x^2/(a+b/x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(4*(I*a^(1/2)/b^(1/2))^(1/2)*a^3*x^13-15*(-(I*a^(1/2)*x^2-b^(1/2))/b^
(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2
)/b^(1/2))^(1/2),I)*a^2*b*x^8+25*(I*a^(1/2)/b^(1/2))^(1/2)*a^2*b*x^9-30*(-
(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(
1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a*b^2*x^4+36*(I*a^(1/2)/b^(1
/2))^(1/2)*a*b^2*x^5-15*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/
2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*b^
3+15*(I*a^(1/2)/b^(1/2))^(1/2)*b^3*x)/a^3/((a*x^4+b)/x^4)^(5/2)/x^10/(I*a^
(1/2)/b^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = -\frac{15\left(a^2x^8+2\,abx^4+b^2\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-\left(4\,a^2x^{11}+21\,abx^7+15\,b^2x^3\right)\sqrt{\frac{ax^4+b}{x^4}}}{12\left(a^5x^8+2\,a^4bx^4+a^3b^2\right)}$$

input
```
integrate(x^2/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(15*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcs
in((-b/a)^(1/4)/x), -1) - (4*a^2*x^11 + 21*a*b*x^7 + 15*b^2*x^3)*sqrt((a*x
^4 + b)/x^4))/(a^5*x^8 + 2*a^4*b*x^4 + a^3*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = -\frac{x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(x**2/(a+b/x**4)**(5/2),x)
```

output
```
-x**3*gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*exp_polar(I*pi)/(a*x**4))/(
4*a**(5/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a + b/x^4)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/x^4)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a + b/x^4)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$$

input
```
int(x^2/(a + b/x^4)^(5/2),x)
```

output
```
int(x^2/(a + b/x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\frac{\sqrt{a\,x^4+b}\,a^2x^9+9\sqrt{a\,x^4+b}\,ab\,x^5+9\sqrt{a\,x^4+b}\,b^2x-9\left(\int \frac{\sqrt{a\,x^4+b}}{a^3x^{12}+3a^2b\,x^8+3a\,b^2x^4+b^3}dx\right)a}{3a^3\left(a^2x^8+2a\right.}$$

input
```
int(x^2/(a+b/x^4)^(5/2),x)
```

output
```
(sqrt(a*x**4 + b)*a**2*x**9 + 9*sqrt(a*x**4 + b)*a*b*x**5 + 9*sqrt(a*x**4
+ b)*b**2*x - 9*int(sqrt(a*x**4 + b)/(a**3*x**12 + 3*a**2*b*x**8 + 3*a*b**
2*x**4 + b**3),x)*a**2*b**3*x**8 - 18*int(sqrt(a*x**4 + b)/(a**3*x**12 + 3
*a**2*b*x**8 + 3*a*b**2*x**4 + b**3),x)*a*b**4*x**4 - 9*int(sqrt(a*x**4 +
b)/(a**3*x**12 + 3*a**2*b*x**8 + 3*a*b**2*x**4 + b**3),x)*b**5)/(3*a**3*(a
**2*x**8 + 2*a*b*x**4 + b**2))
```

3.566 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 277

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=-\frac{7\sqrt{b}\sqrt{a+\frac{b}{x^4}}}{4a^3\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}-\frac{7x}{12a^2\sqrt{a+\frac{b}{x^4}}}+\frac{7\sqrt{a+\frac{b}{x^4}}\,x}{4a^3}+\frac{7\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{11/4}\sqrt{a+\frac{b}{x^4}}}-\frac{7\sqrt[4]{b}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{8a^{11/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-7/4*b^(1/2)*(a+b/x^4)^(1/2)/a^3/(a^(1/2)+b^(1/2)/x^2)/x-1/6*x/a/(a+b/x^4)
^(3/2)-7/12*x/a^2/(a+b/x^4)^(1/2)+7/4*(a+b/x^4)^(1/2)*x/a^3+7/4*b^(1/4)*((
a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*EllipticE(si
n(2*arccot(a^(1/4)*x/b^(1/4))),1/2*2^(1/2))/a^(11/4)/(a+b/x^4)^(1/2)-7/8*b
^(1/4)*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*Inv
erseJacobiAM(2*arccot(a^(1/4)*x/b^(1/4)),1/2*2^(1/2))/a^(11/4)/(a+b/x^4)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.29

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\frac{7bx+3ax^5-7x(b+ax^4)\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{ax^4}{b}\right)}{3a^2\sqrt{a+\frac{b}{x^4}}\,(b+ax^4)}$$

input
```
Integrate[(a + b/x^4)^(-5/2),x]
```

output
```
(7*b*x + 3*a*x^5 - 7*x*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3
/4, 5/2, 7/4, -((a*x^4)/b)])/(3*a^2*Sqrt[a + b/x^4]*(b + a*x^4))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {773, 819, 819, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}dx$$

$$\Big\downarrow \text{773}$$

$$-\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{5/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{819}$$

$$-\frac{7\int \frac{x^2}{\left(a+\frac{b}{x^4}\right)^{3/2}}d\frac{1}{x}}{6a}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{7\left(\frac{3\int \frac{x^2}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{2a}+\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{6a}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 847

$$-\frac{7\left(\frac{3\left(\frac{b\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}+\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{6a}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 834

$$-\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}+\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{6a}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}+\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{6a}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 761

$$-\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}\right)}{a}-\frac{x\sqrt{a+\frac{b}{x^4}}}{a}\right)}{2a}+\frac{x}{2a\sqrt{a+\frac{b}{x^4}}}\right)}{6a}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$-\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a+\frac{b}{x^4}}}{x\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)}}{\sqrt{b}}\right)}{a}\right)}{2a}\right)}{6a}-\frac{x}{6a\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
Int[(a + b/x^4)^(-5/2),x]
```

output
```
-1/6*x/(a*(a + b/x^4)^(3/2)) - (7*(x/(2*a*Sqrt[a + b/x^4]) + (3*(-((Sqrt[a
 + b/x^4]*x)/a) + (b*(-((-(Sqrt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x)) +
(a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^
2)*EllipticE[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x^4]
))/Sqrt[b]) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a
] + Sqrt[b]/x^2)*EllipticF[2*ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3/4)
*Sqrt[a + b/x^4])))/a))/(2*a)))/(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.82

method	result
default	$\frac{-9a^{\frac{9}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,x^{11}+21i\sqrt{b}\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)a^{4}x^{8}-21i\sqrt{b}\sqrt{-\frac{i\sqrt{a}\,x^{2}-\sqrt{b}}{\sqrt{b}}}\sqrt{\frac{i\sqrt{a}\,x^{2}+\sqrt{b}}{\sqrt{b}}}\,\mathrm{EllipticE}\left(\right.}{}$

input
```
int(1/(a+b/x^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-9*a^(9/2)*(I*a^(1/2)/b^(1/2))^(1/2)*x^11+21*I*b^(1/2)*(-(I*a^(1/2)*
x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*Ellipt
icF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^4*x^8-21*I*b^(1/2)*(-(I*a^(1/2)*x^2-b
^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticE(x
*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^4*x^8-16*a^(7/2)*(I*a^(1/2)/b^(1/2))^(1/2)
*b*x^7+42*I*b^(3/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x
^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^3*x^
4-42*I*b^(3/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^
(1/2))/b^(1/2))^(1/2)*EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^3*x^4+21*
I*b^(5/2)*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2)
)/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^2-21*I*b^(5/2)
*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2)
)^(1/2)*EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^2-7*a^(5/2)*(I*a^(1/2)/
b^(1/2))^(1/2)*b^2*x^3)/a^(9/2)/((a*x^4+b)/x^4)^(5/2)/x^10/(I*a^(1/2)/b^(1
/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \frac{21\,(a^2x^8+2\,abx^4+b^2)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{b}{a}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-21\,(a^2x^8+2\,abx^4+b^2)\sqrt{}}{12\,(a^5x^8+2\,a^4bx^4}$$

input
```
integrate(1/(a+b/x^4)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(21*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsi
n((-b/a)^(1/4)/x), -1) - 21*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(a)*(-b/a)^(3/
4)*elliptic_f(arcsin((-b/a)^(1/4)/x), -1) + (12*a^2*x^9 + 35*a*b*x^5 + 21*
b^2*x)*sqrt((a*x^4 + b)/x^4))/(a^5*x^8 + 2*a^4*b*x^4 + a^3*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = -\frac{x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(5/2),x)
```

output
```
-x*gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a
**(5/2)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^4)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b/x^4)^(5/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^4)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.16

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\frac{x\left(\frac{a\,x^4}{b}+1\right)^{5/2}\sqrt{x^{20}}\;{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{11}{4};\ \frac{15}{4};\ -\frac{a\,x^4}{b}\right)}{11\,\left(a\,x^4+b\right)^{5/2}}$$

input
```
int(1/(a + b/x^4)^(5/2),x)
```

output
```
(x*((a*x^4)/b + 1)^(5/2)*(x^20)^(1/2)*hypergeom([5/2, 11/4], 15/4, -(a*x^4
)/b))/(11*(b + a*x^4)^(5/2))
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}}\,dx=\frac{3\sqrt{a\,x^4+b}\,a\,x^7+7\sqrt{a\,x^4+b}\,b\,x^3-21\left(\int \frac{\sqrt{a\,x^4+b}\,x^2}{a^3x^{12}+3a^2b\,x^8+3a\,b^2x^4+b^3}dx\right)a^2b^2x^8-42\left(\int \frac{}{a^3x}\right.}{3a^2\left(a^2x^8+2ab\,x^4+b\right.}$$

input
```
int(1/(a+b/x^4)^(5/2),x)
```

output
```
(3*sqrt(a*x**4 + b)*a*x**7 + 7*sqrt(a*x**4 + b)*b*x**3 - 21*int((sqrt(a*x*
*4 + b)*x**2)/(a**3*x**12 + 3*a**2*b*x**8 + 3*a*b**2*x**4 + b**3),x)*a**2*
b**2*x**8 - 42*int((sqrt(a*x**4 + b)*x**2)/(a**3*x**12 + 3*a**2*b*x**8 + 3
*a*b**2*x**4 + b**3),x)*a*b**3*x**4 - 21*int((sqrt(a*x**4 + b)*x**2)/(a**3
*x**12 + 3*a**2*b*x**8 + 3*a*b**2*x**4 + b**3),x)*b**4)/(3*a**2*(a**2*x**8
 + 2*a*b*x**4 + b**2))
```

3.567 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2} dx = -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2} x} - \frac{5}{12a^2\sqrt{a+\frac{b}{x^4}}\,x} - \frac{5\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{24a^{9/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/6/a/(a+b/x^4)^(3/2)/x-5/12/a^2/(a+b/x^4)^(1/2)/x-5/24*((a+b/x^4)/(a^(1/
2)+b^(1/2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^
(1/4)*x/b^(1/4)),1/2*2^(1/2))/a^(9/4)/b^(1/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2} \, dx = \frac{-5b-7ax^4+5(b+ax^4)\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^4}{b}\right)}{12a^2\sqrt{a+\frac{b}{x^4}}\,x\,(b+ax^4)}$$

input
```
Integrate[1/((a + b/x^4)^(5/2)*x^2),x]
```

output
```
(-5*b - 7*a*x^4 + 5*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[1/4,
 1/2, 5/4, -((a*x^4)/b)])/(12*a^2*Sqrt[a + b/x^4]*x*(b + a*x^4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {858, 749, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+\frac{b}{x^4}\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{858}$$

$$-\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}} d\frac{1}{x}$$

$$\Big\downarrow \text{749}$$

$$-\frac{5\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}} d\frac{1}{x}}{6a} - \frac{1}{6ax\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{5\left(\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{2a}+\frac{1}{2ax\sqrt{a+\frac{b}{x^4}}}\right)}{6a}-\frac{1}{6ax\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{5\left(\frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}+\frac{1}{2ax\sqrt{a+\frac{b}{x^4}}}\right)}{6a}-\frac{1}{6ax\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
Int[1/((a + b/x^4)^(5/2)*x^2),x]
```

output
```
-1/6*1/(a*(a + b/x^4)^(3/2)*x) - (5*(1/(2*a*Sqrt[a + b/x^4]*x) + (Sqrt[(a
+ b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*Ar
cTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(4*a^(5/4)*b^(1/4)*Sqrt[a + b/x^4])))/(6*
a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.13

method	result
default	$-\frac{-5\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)a^2x^8+7\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,a^2x^9-10\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},\right.}{12a^2\left(\frac{a\,x^4+b}{x^4}\right)^{\frac{5}{2}}x^{10}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}$

input
```
int(1/(a+b/x^4)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-5*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2)
)/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^2*x^8+7*(I*a^(
1/2)/b^(1/2))^(1/2)*a^2*x^9-10*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((
I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2
),I)*a*b*x^4+12*(I*a^(1/2)/b^(1/2))^(1/2)*a*b*x^5-5*(-(I*a^(1/2)*x^2-b^(1/
2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I*
a^(1/2)/b^(1/2))^(1/2),I)*b^2+5*(I*a^(1/2)/b^(1/2))^(1/2)*b^2*x)/a^2/((a*x
^4+b)/x^4)^(5/2)/x^10/(I*a^(1/2)/b^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x^2}\,dx =$$

$$-\frac{5\,(a^2x^8+2\,abx^4+b^2)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)\,|\,-1\right)+(7\,a^2x^7+5\,abx^3)\sqrt{\frac{ax^4+b}{x^4}}}{12\,(a^5x^8+2\,a^4bx^4+a^3b^2)}$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
-1/12*(5*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsi
n(x*(-a/b)^(1/4)), -1) + (7*a^2*x^7 + 5*a*b*x^3)*sqrt((a*x^4 + b)/x^4))/(a
^5*x^8 + 2*a^4*b*x^4 + a^3*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.28

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2}\, dx = -\frac{\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{5}{4} \end{matrix} \middle| \frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}} x \Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(5/2)/x**2,x)
```

output
```
-gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(5
/2)*x*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2}\, dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^2}\, dx$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^4)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2}\, dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^2}\, dx$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^4)^(5/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.30

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2}\,dx = -\frac{\left(\frac{b}{a}+x^4\right)^{5/2} {}_2\mathrm{F}_1\left(\frac{1}{4},\frac{5}{2};\ \frac{5}{4};\ -\frac{b}{a\,x^4}\right)}{x\,(a\,x^4+b)^{5/2}}$$

input
```
int(1/(x^2*(a + b/x^4)^(5/2)),x)
```

output
```
-((b/a + x^4)^(5/2)*hypergeom([1/4, 5/2], 5/4, -b/(a*x^4)))/(x*(b + a*x^4)
^(5/2))
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^2}\,dx = \frac{-\sqrt{a\,x^4+b}\,x+\left(\int \frac{\sqrt{a\,x^4+b}}{a^3x^{12}+3a^2b\,x^8+3a\,b^2x^4+b^3}dx\right)a\,b^2x^4+\left(\int \frac{\sqrt{a\,x^4+b}}{a^3x^{12}+3a^2b\,x^8+3a\,b^2x^4+b^3}dx\right)}{a^2\,(a\,x^4+b)}$$

input
```
int(1/(a+b/x^4)^(5/2)/x^2,x)
```

output
```
( - sqrt(a*x**4 + b)*x + int(sqrt(a*x**4 + b)/(a**3*x**12 + 3*a**2*b*x**8
+ 3*a*b**2*x**4 + b**3),x)*a*b**2*x**4 + int(sqrt(a*x**4 + b)/(a**3*x**12
+ 3*a**2*b*x**8 + 3*a*b**2*x**4 + b**3),x)*b**3)/(a**2*(a*x**4 + b))
```

3.568 $\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 262

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4} \, dx = -\frac{1}{6a\left(a+\frac{b}{x^4}\right)^{3/2} x^3} - \frac{1}{4a^2\sqrt{a+\frac{b}{x^4}}x^3} + \frac{\sqrt{a+\frac{b}{x^4}}}{4a^2\sqrt{b}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)x} - \frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}} + \frac{\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{8a^{7/4}b^{3/4}\sqrt{a+\frac{b}{x^4}}}$$

output
```
-1/6/a/(a+b/x^4)^(3/2)/x^3-1/4/a^2/(a+b/x^4)^(1/2)/x^3+1/4*(a+b/x^4)^(1/2)
/a^2/b^(1/2)/(a^(1/2)+b^(1/2)/x^2)/x-1/4*((a+b/x^4)/(a^(1/2)+b^(1/2)/x^2)^
2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*EllipticE(sin(2*arccot(a^(1/4)*x/b^(1/4))),
1/2*2^(1/2))/a^(7/4)/b^(3/4)/(a+b/x^4)^(1/2)+1/8*((a+b/x^4)/(a^(1/2)+b^(1/
2)/x^2)^2)^(1/2)*(a^(1/2)+b^(1/2)/x^2)*InverseJacobiAM(2*arccot(a^(1/4)*x/
b^(1/4)),1/2*2^(1/2))/a^(7/4)/b^(3/4)/(a+b/x^4)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.29

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = \frac{-bx+x(b+ax^4)\sqrt{1+\frac{ax^4}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{ax^4}{b}\right)}{3ab\sqrt{a+\frac{b}{x^4}}\,(b+ax^4)}$$

input
```
Integrate[1/((a + b/x^4)^(5/2)*x^4),x]
```

output
```
(-(b*x) + x*(b + a*x^4)*Sqrt[1 + (a*x^4)/b]*Hypergeometric2F1[3/4, 5/2, 7/
4, -((a*x^4)/b)])/(3*a*b*Sqrt[a + b/x^4]*(b + a*x^4))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {858, 819, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+\frac{b}{x^4}\right)^{5/2}}dx$$

$$\Big\downarrow \; 858$$

$$-\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2}x^2}d\frac{1}{x}$$

$$\Big\downarrow \; 819$$

$$-\frac{\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{3/2}x^2}d\frac{1}{x}}{2a}-\frac{1}{6ax^3\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{1}{\sqrt{a+\frac{b}{x^4}}x^2}d\frac{1}{x}}{2a}}{2a}-\frac{1}{6ax^3\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 834

$$-\frac{\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a}\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}}{2a}}{2a}-\frac{1}{6ax^3\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}}{2a}}{2a}-\frac{1}{6ax^3\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 761

$$-\frac{\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\int \frac{\sqrt{a}-\frac{\sqrt{b}}{x^2}}{\sqrt{a+\frac{b}{x^4}}}d\frac{1}{x}}{\sqrt{b}}}{2a}}{2a}-\frac{1}{6ax^3\left(a+\frac{b}{x^4}\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$-\frac{\frac{1}{2ax^3\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+\frac{b}{x^4}}}-\frac{\frac{\sqrt[4]{a}\sqrt{\frac{a+\frac{b}{x^4}}{\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)^2}}\left(\sqrt{a}+\frac{\sqrt{b}}{x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}x}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+\frac{b}{x^4}}}-\frac{\sqrt{a\cdots}}{x\left(\sqrt{a}\cdots\right.}}{\sqrt{b}}}{2a}}{2a}$$

$$\frac{1}{6ax^3\left(a+\frac{b}{x^4}\right)^{3/2}}$$

input
```
Int[1/((a + b/x^4)^(5/2)*x^4),x]
```

output
```
-1/6*1/(a*(a + b/x^4)^(3/2)*x^3) - (1/(2*a*Sqrt[a + b/x^4]*x^3) - (-((-(Sq
rt[a + b/x^4]/((Sqrt[a] + Sqrt[b]/x^2)*x)) + (a^(1/4)*Sqrt[(a + b/x^4)/(Sq
rt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticE[2*ArcTan[b^(1/4)
/(a^(1/4)*x)], 1/2])/(b^(1/4)*Sqrt[a + b/x^4]))/Sqrt[b]) + (a^(1/4)*Sqrt[(
a + b/x^4)/(Sqrt[a] + Sqrt[b]/x^2)^2]*(Sqrt[a] + Sqrt[b]/x^2)*EllipticF[2*
ArcTan[b^(1/4)/(a^(1/4)*x)], 1/2])/(2*b^(3/4)*Sqrt[a + b/x^4]))/(2*a))/(2*
a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.92

method	result
default	$-\frac{-3a^{\frac{7}{2}}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}\,\sqrt{b}\,x^{11}+3i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}},i\right)a^3b\,x^8-3i\sqrt{-\frac{i\sqrt{a}\,x^2-\sqrt{b}}{\sqrt{b}}}\,\sqrt{\frac{i\sqrt{a}\,x^2+\sqrt{b}}{\sqrt{b}}}\,\operatorname{EllipticE}\left(x\right.}{}$

input
```
int(1/(a+b/x^4)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*a^(7/2)*(I*a^(1/2)/b^(1/2))^(1/2)*b^(1/2)*x^11+3*I*(-(I*a^(1/2)*
x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*Ellipt
icF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^3*b*x^8-3*I*(-(I*a^(1/2)*x^2-b^(1/2))
/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticE(x*(I*a^(
1/2)/b^(1/2))^(1/2),I)*a^3*b*x^8-4*a^(5/2)*(I*a^(1/2)/b^(1/2))^(1/2)*b^(3/
2)*x^7+6*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2
))/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^2*b^2*x^4-6*I
*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2)
)^(1/2)*EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a^2*b^2*x^4-a^(3/2)*(I*a^
(1/2)/b^(1/2))^(1/2)*b^(5/2)*x^3+3*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1
/2)*((I*a^(1/2)*x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticF(x*(I*a^(1/2)/b^(1/2)
)^(1/2),I)*a*b^3-3*I*(-(I*a^(1/2)*x^2-b^(1/2))/b^(1/2))^(1/2)*((I*a^(1/2)*
x^2+b^(1/2))/b^(1/2))^(1/2)*EllipticE(x*(I*a^(1/2)/b^(1/2))^(1/2),I)*a*b^3
)/a^(5/2)/((a*x^4+b)/x^4)^(5/2)/x^10/b^(3/2)/(I*a^(1/2)/b^(1/2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = \frac{3\left(a^2x^8+2\,abx^4+b^2\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right) | -1\right) - 3\left(a^2x^8+2\,abx^4+b^2\right)}{12\left(a^4bx^8+2\,a^3b^2x^4+a\right.}$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
1/12*(3*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin
(x*(-a/b)^(1/4)), -1) - 3*(a^2*x^8 + 2*a*b*x^4 + b^2)*sqrt(b)*(-a/b)^(3/4)
*elliptic_f(arcsin(x*(-a/b)^(1/4)), -1) + (3*a^2*x^9 + a*b*x^5)*sqrt((a*x^
4 + b)/x^4))/(a^4*b*x^8 + 2*a^3*b^2*x^4 + a^2*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.15

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = -\frac{\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{be^{i\pi}}{ax^4}\right)}{4a^{\frac{5}{2}}x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/(a+b/x**4)**(5/2)/x**4,x)
```

output
```
-gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*exp_polar(I*pi)/(a*x**4))/(4*a**(5
/2)*x**3*gamma(7/4))
```

Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^4}\,dx$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(1/((a + b/x^4)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = \int \frac{1}{\left(a+\frac{b}{x^4}\right)^{\frac{5}{2}} x^4}\,dx$$

input
```
integrate(1/(a+b/x^4)^(5/2)/x^4,x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^4)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = \int \frac{1}{x^4 \left(a+\frac{b}{x^4}\right)^{5/2}}\,dx$$

input
```
int(1/(x^4*(a + b/x^4)^(5/2)),x)
```

output
```
int(1/(x^4*(a + b/x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+\frac{b}{x^4}\right)^{5/2} x^4}\,dx = \frac{-\sqrt{a\,x^4+b}\,x^3+3\left(\int \frac{\sqrt{a\,x^4+b}\,x^2}{a^3x^{12}+3a^2b\,x^8+3a\,b^2x^4+b^3}dx\right)a^2b\,x^8+6\left(\int \frac{\sqrt{a\,x^4+b}\,x^2}{a^3x^{12}+3a^2b\,x^8+3a\,b^2x^4+b^3}d\right.}{3a\left(a^2x^8+2ab\,x^4+b^2\right)}$$

input
```
int(1/(a+b/x^4)^(5/2)/x^4,x)
```

output
```
( - sqrt(a*x**4 + b)*x**3 + 3*int((sqrt(a*x**4 + b)*x**2)/(a**3*x**12 + 3*
a**2*b*x**8 + 3*a*b**2*x**4 + b**3),x)*a**2*b*x**8 + 6*int((sqrt(a*x**4 +
b)*x**2)/(a**3*x**12 + 3*a**2*b*x**8 + 3*a*b**2*x**4 + b**3),x)*a*b**2*x**
4 + 3*int((sqrt(a*x**4 + b)*x**2)/(a**3*x**12 + 3*a**2*b*x**8 + 3*a*b**2*x
**4 + b**3),x)*b**3)/(3*a*(a**2*x**8 + 2*a*b*x**4 + b**2))
```

3.569 $\int \frac{1}{a+\frac{b}{x^5}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 310

$$\begin{aligned}\int \frac{1}{a+\frac{b}{x^5}}\,dx = {} & \frac{x}{a} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\sqrt[5]{b}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\sqrt[5]{a}x}{\sqrt[5]{b}}\right)}{5a^{6/5}} \\
& + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\sqrt[5]{b}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sqrt[5]{a}x}{\sqrt[5]{b}}\right)}{5a^{6/5}} \\
& - \frac{\sqrt[5]{b}\log\left(\sqrt[5]{b}+\sqrt[5]{a}x\right)}{5a^{6/5}} \\
& + \frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}\log\left(b^{2/5}-\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+a^{2/5}x^2\right)}{20a^{6/5}} \\
& + \frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}\log\left(b^{2/5}-\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+a^{2/5}x^2\right)}{20a^{6/5}}\end{aligned}$$

output
```
x/a-1/10*(10+2*5^(1/2))^(1/2)*b^(1/5)*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*2
^(1/2)/(5+5^(1/2))^(1/2)*a^(1/5)*x/b^(1/5))/a^(6/5)-1/10*(10-2*5^(1/2))^(1
/2)*b^(1/5)*arctan(-1/5*(25+10*5^(1/2))^(1/2)+1/5*(50+10*5^(1/2))^(1/2)*a^
(1/5)*x/b^(1/5))/a^(6/5)-1/5*b^(1/5)*ln(b^(1/5)+a^(1/5)*x)/a^(6/5)+1/20*(-
5^(1/2)+1)*b^(1/5)*ln(b^(2/5)-1/2*(-5^(1/2)+1)*a^(1/5)*b^(1/5)*x+a^(2/5)*x
^2)/a^(6/5)+1/20*(5^(1/2)+1)*b^(1/5)*ln(b^(2/5)-1/2*(5^(1/2)+1)*a^(1/5)*b^
(1/5)*x+a^(2/5)*x^2)/a^(6/5)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx$$

$$= \frac{20\sqrt[5]{a}x-2\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{b}\arctan\left(\frac{\left(-1+\sqrt{5}\right)\sqrt[5]{b}+4\sqrt[5]{a}x}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{b}}\right)-2\sqrt{10-2\sqrt{5}}\sqrt[5]{b}\arctan\left(\frac{-\left(\left(1+\sqrt{5}\right)\sqrt[5]{b}\right)+4\sqrt[5]{}}{\sqrt{10-2\sqrt{5}}\sqrt[5]{b}}\right.}{}$$

input
```
Integrate[(a + b/x^5)^(-1),x]
```

output
```
(20*a^(1/5)*x - 2*Sqrt[2*(5 + Sqrt[5])]*b^(1/5)*ArcTan[((-1 + Sqrt[5])*b^(
1/5) + 4*a^(1/5)*x)/(Sqrt[2*(5 + Sqrt[5])]*b^(1/5))] - 2*Sqrt[10 - 2*Sqrt[
5]]*b^(1/5)*ArcTan[(-((1 + Sqrt[5])*b^(1/5)) + 4*a^(1/5)*x)/(Sqrt[10 - 2*S
qrt[5]]*b^(1/5))] - 4*b^(1/5)*Log[b^(1/5) + a^(1/5)*x] - (-1 + Sqrt[5])*b^
(1/5)*Log[b^(2/5) + ((-1 + Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 + a^(2/5)*x^2] +
(1 + Sqrt[5])*b^(1/5)*Log[b^(2/5) - ((1 + Sqrt[5])*a^(1/5)*b^(1/5)*x)/2 +
a^(2/5)*x^2])/(20*a^(6/5))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {772, 843, 751, 16, 27, 1142, 25, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{x^5}}dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{x^5}{ax^5+b}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{ax^5+b}dx}{a}$$

$$\Big\downarrow \text{751}$$

$$\frac{x}{a}-\frac{b\left(\frac{2\int \frac{4\sqrt[5]{b}-\left(1-\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{2\int \frac{4\sqrt[5]{b}-\left(1+\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{\int \frac{1}{\sqrt[5]{a}x+\sqrt[5]{b}}dx}{5b^{4/5}}\right)}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{x}{a}-\frac{b\left(\frac{2\int \frac{4\sqrt[5]{b}-\left(1-\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{2\int \frac{4\sqrt[5]{b}-\left(1+\sqrt{5}\right)\sqrt[5]{a}x}{2\left(2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}dx}{5b^{4/5}}+\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{4\sqrt[5]{b}-\left(1-\sqrt{5}\right)\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{5b^{4/5}}+\frac{\int \frac{4\sqrt[5]{b}-\left(1+\sqrt{5}\right)\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{5b^{4/5}}+\frac{\log\left(\sqrt[5]{a}x+\sqrt[5]{b}\right)}{5\sqrt[5]{a}b^{4/5}}\right)}{a}$$

$\downarrow$ 1142

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx-\frac{\left(1-\sqrt{5}\right)\int -\frac{\sqrt[5]{a}\left(\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x\right)}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{4\sqrt[5]{a}}}{5b^{4/5}}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{b}\int \frac{}{2a^{2/5}x^2-\left(1+\sqrt{5}\right.}}{}\right.}{a}$$

$\downarrow$ 25

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{\left(1-\sqrt{5}\right)\int \frac{\sqrt[5]{a}\left(\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x\right)}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{4\sqrt[5]{a}}}{5b^{4/5}}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{b}\int \frac{}{2a^{2/5}x^2-\left(1+\sqrt{5}\right.}}{}\right.}{a}$$

$\downarrow$ 27

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\left(5+\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx}{5b^{4/5}}+\frac{\frac{1}{2}\left(5-\sqrt{5}\right)\sqrt[5]{b}\int \frac{}{2a^{2/5}x^2-\left(1+\right.}}{}\right.}{a}$$

$\downarrow$ 1083

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx-\left(5+\sqrt{5}\right)\sqrt[5]{b}\int \frac{1}{-\left(4a^{2/5}x-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}\right)^2-2\left(5+\sqrt{5}\right)a^{2/5}b^{2/5}}d\left(4a^{2/5}x-\left(1-\sqrt{5}\right)\sqrt[5]{\right.}{5b^{4/5}}\right.}{}$$

$\downarrow$ 217

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{4}\left(1-\sqrt{5}\right)\int \frac{\left(1-\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{a}x}{2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}}dx+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4a^{2/5}x-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{a}\sqrt[5]{b}}\right)}{\sqrt[5]{a}}}{5b^{4/5}}+\frac{\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{\left(1+\sqrt{5}\right)\sqrt[5]{b}-4\sqrt[5]{\right.}{2a^{2/5}x^2-\left(1+\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{\right.}\right.}{a}$$

$\downarrow$ 1103

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4a^{2/5}x-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{a}\sqrt[5]{b}}\right)}{\sqrt[5]{a}}-\frac{\left(1-\sqrt{5}\right)\log\left(2a^{2/5}x^2-\left(1-\sqrt{5}\right)\sqrt[5]{a}\sqrt[5]{b}x+2b^{2/5}\right)}{4\sqrt[5]{a}}}{5b^{4/5}}+\frac{\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{4a^{2/5}x-\left(1+\sqrt{5}\right)\sqrt[5]{a}\right.}{\sqrt{2\left(5-\sqrt{5}\right)}\sqrt[5]{a}\sqrt[5]{\right.}\right)}{\sqrt[5]{a}}\right.}{a}$$

input
```
Int[(a + b/x^5)^(-1),x]
```

output
```
x/a - (b*(Log[b^(1/5) + a^(1/5)*x]/(5*a^(1/5)*b^(4/5)) + ((Sqrt[(5 + Sqrt[
5])/2]*ArcTan[(-((1 - Sqrt[5])*a^(1/5)*b^(1/5)) + 4*a^(2/5)*x)/(Sqrt[2*(5
+ Sqrt[5])]*a^(1/5)*b^(1/5))])/a^(1/5) - ((1 - Sqrt[5])*Log[2*b^(2/5) - (1
 - Sqrt[5])*a^(1/5)*b^(1/5)*x + 2*a^(2/5)*x^2])/(4*a^(1/5)))/(5*b^(4/5)) +
 ((Sqrt[(5 - Sqrt[5])/2]*ArcTan[(-((1 + Sqrt[5])*a^(1/5)*b^(1/5)) + 4*a^(2
/5)*x)/(Sqrt[2*(5 - Sqrt[5])]*a^(1/5)*b^(1/5))])/a^(1/5) - ((1 + Sqrt[5])*
Log[2*b^(2/5) - (1 + Sqrt[5])*a^(1/5)*b^(1/5)*x + 2*a^(2/5)*x^2])/(4*a^(1/
5)))/(5*b^(4/5))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 751
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k -
1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; r/(a*n)
  Int[1/(r + s*x), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 1)/2}], x]] /; Fre
eQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.11

method	result	size
risch	$\frac{x}{a}-\frac{b\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{5}a+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{4}}\right)}{5a^{2}}$	34
default	Expression too large to display	907

input
```
int(1/(a+b/x^5),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/5/a^2*b*sum(1/_R^4*ln(x-_R),_R=RootOf(_Z^5*a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 18648, normalized size of antiderivative = 60.15

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x^5),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.07

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx = \operatorname{RootSum}\left(3125t^5a^6+b,\left(t\mapsto t\log\left(-5ta+x\right)\right)\right)+\frac{x}{a}$$

input
```
integrate(1/(a+b/x**5),x)
```

output
```
RootSum(3125*_t**5*a**6 + b, Lambda(_t, _t*log(-5*_t*a + x))) + x/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx = -\frac{\frac{2\sqrt{5}b^{\frac{1}{5}}\left(\sqrt{5}+1\right)\arctan\left(\frac{4\,a^{\frac{2}{5}}\,x+a^{\frac{1}{5}}\,b^{\frac{1}{5}}\left(\sqrt{5}-1\right)}{a^{\frac{1}{5}}\,b^{\frac{1}{5}}\,\sqrt{2\,\sqrt{5}+10}}\right)}{a^{\frac{1}{5}}\,\sqrt{2\,\sqrt{5}+10}}+\frac{2\sqrt{5}b^{\frac{1}{5}}\left(\sqrt{5}-1\right)\arctan\left(\frac{4\,a^{\frac{2}{5}}\,x-a^{\frac{1}{5}}\,b^{\frac{1}{5}}\left(\sqrt{5}+1\right)}{a^{\frac{1}{5}}\,b^{\frac{1}{5}}\,\sqrt{-2\,\sqrt{5}+10}}\right)}{a^{\frac{1}{5}}\,\sqrt{-2\,\sqrt{5}+10}}-\frac{b^{\frac{1}{5}}\left(\sqrt{5}+3\right)\log\left(2\,a^{\frac{2}{5}}\,x^2-a^{\frac{1}{5}}\,b^{\frac{1}{5}}\,x\right.}{a^{\frac{1}{5}}\left(\sqrt{5}+1\right)}}{10\,a} + \frac{x}{a}$$

input
```
integrate(1/(a+b/x^5),x, algorithm="maxima")
```

output
```
-1/10*(2*sqrt(5)*b^(1/5)*(sqrt(5) + 1)*arctan((4*a^(2/5)*x + a^(1/5)*b^(1/
5)*(sqrt(5) - 1))/(a^(1/5)*b^(1/5)*sqrt(2*sqrt(5) + 10)))/(a^(1/5)*sqrt(2*
sqrt(5) + 10)) + 2*sqrt(5)*b^(1/5)*(sqrt(5) - 1)*arctan((4*a^(2/5)*x - a^(
1/5)*b^(1/5)*(sqrt(5) + 1))/(a^(1/5)*b^(1/5)*sqrt(-2*sqrt(5) + 10)))/(a^(1
/5)*sqrt(-2*sqrt(5) + 10)) - b^(1/5)*(sqrt(5) + 3)*log(2*a^(2/5)*x^2 - a^(
1/5)*b^(1/5)*x*(sqrt(5) + 1) + 2*b^(2/5))/(a^(1/5)*(sqrt(5) + 1)) - b^(1/5
)*(sqrt(5) - 3)*log(2*a^(2/5)*x^2 + a^(1/5)*b^(1/5)*x*(sqrt(5) - 1) + 2*b^
(2/5))/(a^(1/5)*(sqrt(5) - 1)) + 2*b^(1/5)*log(a^(1/5)*x + b^(1/5))/a^(1/5
))/a + x/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx = \frac{\left(-\frac{b}{a}\right)^{\frac{1}{5}}\log\left(\left|x-\left(-\frac{b}{a}\right)^{\frac{1}{5}}\right|\right)}{5\,a}+\frac{x}{a} - \frac{\left(-a^4b\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\left(\sqrt{5}-1\right)\left(-\frac{b}{a}\right)^{\frac{1}{5}}-4\,x}{\sqrt{2\sqrt{5}+10}\left(-\frac{b}{a}\right)^{\frac{1}{5}}}\right)}{10\,a^2} - \frac{\left(-a^4b\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\left(\sqrt{5}+1\right)\left(-\frac{b}{a}\right)^{\frac{1}{5}}+4\,x}{\sqrt{-2\sqrt{5}+10}\left(-\frac{b}{a}\right)^{\frac{1}{5}}}\right)}{10\,a^2} - \frac{\left(-a^4b\right)^{\frac{1}{5}}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{5}\left(-\frac{b}{a}\right)^{\frac{1}{5}}+\left(-\frac{b}{a}\right)^{\frac{1}{5}}\right)+\left(-\frac{b}{a}\right)^{\frac{2}{5}}\right)}{5\,a^2\left(\sqrt{5}-1\right)} + \frac{\left(-a^4b\right)^{\frac{1}{5}}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{5}\left(-\frac{b}{a}\right)^{\frac{1}{5}}-\left(-\frac{b}{a}\right)^{\frac{1}{5}}\right)+\left(-\frac{b}{a}\right)^{\frac{2}{5}}\right)}{5\,a^2\left(\sqrt{5}+1\right)}$$

input
```
integrate(1/(a+b/x^5),x, algorithm="giac")
```

output
```
1/5*(-b/a)^(1/5)*log(abs(x - (-b/a)^(1/5)))/a + x/a - 1/10*(-a^4*b)^(1/5)*
sqrt(2*sqrt(5) + 10)*arctan(-((sqrt(5) - 1)*(-b/a)^(1/5) - 4*x)/(sqrt(2*sq
rt(5) + 10)*(-b/a)^(1/5)))/a^2 - 1/10*(-a^4*b)^(1/5)*sqrt(-2*sqrt(5) + 10)
*arctan(((sqrt(5) + 1)*(-b/a)^(1/5) + 4*x)/(sqrt(-2*sqrt(5) + 10)*(-b/a)^(
1/5)))/a^2 - 1/5*(-a^4*b)^(1/5)*log(x^2 + 1/2*x*(sqrt(5)*(-b/a)^(1/5) + (-
b/a)^(1/5)) + (-b/a)^(2/5))/(a^2*(sqrt(5) - 1)) + 1/5*(-a^4*b)^(1/5)*log(x
^2 - 1/2*x*(sqrt(5)*(-b/a)^(1/5) - (-b/a)^(1/5)) + (-b/a)^(2/5))/(a^2*(sqr
t(5) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx = \frac{x}{a} + \frac{(-b)^{1/5}\ln\left((-b)^{16/5}+a^{1/5}\,b^3\,x\right)}{5\,a^{6/5}} - \frac{(-b)^{1/5}\ln\left(25\,a^{4/5}\,(-b)^{16/5}\left(\frac{\sqrt{-2\,\sqrt{5}-10}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-5\,a\,b^3\,x\right)\left(\frac{\sqrt{-2\,\sqrt{5}-10}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)}{a^{6/5}} + \frac{(-b)^{1/5}\ln\left(25\,a^{4/5}\,(-b)^{16/5}\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{-2\,\sqrt{5}-10}}{20}-\frac{1}{20}\right)+5\,a\,b^3\,x\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{-2\,\sqrt{5}-10}}{20}-\frac{1}{20}\right)}{a^{6/5}} - \frac{(-b)^{1/5}\ln\left(25\,a^{4/5}\,(-b)^{16/5}\left(\frac{\sqrt{5}}{20}-\frac{\sqrt{2\,\sqrt{5}-10}}{20}+\frac{1}{20}\right)-5\,a\,b^3\,x\right)\left(\frac{\sqrt{5}}{20}-\frac{\sqrt{2\,\sqrt{5}-10}}{20}+\frac{1}{20}\right)}{a^{6/5}} - \frac{(-b)^{1/5}\ln\left(25\,a^{4/5}\,(-b)^{16/5}\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2\,\sqrt{5}-10}}{20}+\frac{1}{20}\right)-5\,a\,b^3\,x\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2\,\sqrt{5}-10}}{20}+\frac{1}{20}\right)}{a^{6/5}}$$

input
```
int(1/(a + b/x^5),x)
```

output

```
x/a + ((-b)^(1/5)*log((-b)^(16/5) + a^(1/5)*b^3*x))/(5*a^(6/5)) - ((-b)^(1
/5)*log(25*a^(4/5)*(-b)^(16/5)*((- 2*5^(1/2) - 10)^(1/2)/20 - 5^(1/2)/20 +
 1/20) - 5*a*b^3*x)*((- 2*5^(1/2) - 10)^(1/2)/20 - 5^(1/2)/20 + 1/20))/a^(
6/5) + ((-b)^(1/5)*log(25*a^(4/5)*(-b)^(16/5)*(5^(1/2)/20 + (- 2*5^(1/2) -
 10)^(1/2)/20 - 1/20) + 5*a*b^3*x)*(5^(1/2)/20 + (- 2*5^(1/2) - 10)^(1/2)/
20 - 1/20))/a^(6/5) - ((-b)^(1/5)*log(25*a^(4/5)*(-b)^(16/5)*(5^(1/2)/20 -
 (2*5^(1/2) - 10)^(1/2)/20 + 1/20) - 5*a*b^3*x)*(5^(1/2)/20 - (2*5^(1/2) -
 10)^(1/2)/20 + 1/20))/a^(6/5) - ((-b)^(1/5)*log(25*a^(4/5)*(-b)^(16/5)*(5
^(1/2)/20 + (2*5^(1/2) - 10)^(1/2)/20 + 1/20) - 5*a*b^3*x)*(5^(1/2)/20 + (
2*5^(1/2) - 10)^(1/2)/20 + 1/20))/a^(6/5)
```

Reduce [F]

$$\int \frac{1}{a+\frac{b}{x^5}}\,dx=\frac{-\left(\int \frac{1}{a\,x^5+b}dx\right)b+x}{a}$$

input

```
int(1/(a+b/x^5),x)
```

output

```
( - int(1/(a*x**5 + b),x)*b + x)/a
```

3.570 $\int \frac{1}{\sqrt{a+\frac{b}{x^5}}x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}x}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

output
```
2/5*arctanh((a+b/x^5)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(27) = 54.

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}x}\,dx=\frac{2\sqrt{b+ax^5}\log\left(\sqrt{a}x^{5/2}+\sqrt{b+ax^5}\right)}{5\sqrt{a}\sqrt{a+\frac{b}{x^5}}x^{5/2}}$$

input
```
Integrate[1/(Sqrt[a + b/x^5]*x),x]
```

output
```
(2*Sqrt[b + a*x^5]*Log[Sqrt[a]*x^(5/2) + Sqrt[b + a*x^5]])/(5*Sqrt[a]*Sqrt
[a + b/x^5]*x^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{x^5}}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{5}\int \frac{x^5}{\sqrt{a+\frac{b}{x^5}}}d\frac{1}{x^5}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\frac{1}{bx^{10}}-\frac{a}{b}}d\sqrt{a+\frac{b}{x^5}}}{5b}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b/x^5]*x),x]
```

output
```
(2*ArcTanh[Sqrt[a + b/x^5]/Sqrt[a]])/(5*Sqrt[a])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\, x} dx$$

input
```
int(1/(a+b/x^5)^(1/2)/x,x)
```

output
```
int(1/(a+b/x^5)^(1/2)/x,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}x}\,dx = \left[\frac{\log\left(-8\,a^2x^{10}-8\,abx^5-b^2-4\,(2\,ax^{10}+bx^5)\sqrt{a}\sqrt{\frac{ax^5+b}{x^5}}\right)}{10\,\sqrt{a}},\ -\frac{\sqrt{-a}\arctan\left(\frac{2\,\sqrt{-a}x^5\sqrt{\frac{ax^5+b}{x^5}}}{2\,ax^5+b}\right)}{5\,a}\right]$$

input

```
integrate(1/(a+b/x^5)^(1/2)/x,x, algorithm="fricas")
```

output

```
[1/10*log(-8*a^2*x^10 - 8*a*b*x^5 - b^2 - 4*(2*a*x^10 + b*x^5)*sqrt(a)*sqr
t((a*x^5 + b)/x^5))/sqrt(a), -1/5*sqrt(-a)*arctan(2*sqrt(-a)*x^5*sqrt((a*x
^5 + b)/x^5)/(2*a*x^5 + b))/a]
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}x}\,dx = \frac{2\operatorname{asinh}\left(\frac{\sqrt{a}x^{\frac{5}{2}}}{\sqrt{b}}\right)}{5\sqrt{a}}$$

input

```
integrate(1/(a+b/x**5)**(1/2)/x,x)
```

output

```
2*asinh(sqrt(a)*x**(5/2)/sqrt(b))/(5*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx = -\frac{\log\left(\frac{\sqrt{a+\frac{b}{x^5}}-\sqrt{a}}{\sqrt{a+\frac{b}{x^5}}+\sqrt{a}}\right)}{5\sqrt{a}}$$

input
```
integrate(1/(a+b/x^5)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/5*log((sqrt(a + b/x^5) - sqrt(a))/(sqrt(a + b/x^5) + sqrt(a)))/sqrt(a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b/x^5)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

input
```
int(1/(x*(a + b/x^5)^(1/2)),x)
```

output
```
(2*atanh((a + b/x^5)^(1/2)/a^(1/2)))/(5*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{a+\frac{b}{x^5}}\,x}\,dx = \frac{\sqrt{a}\,\left(-\log\left(\sqrt{a\,x^5+b}-\sqrt{x}\,\sqrt{a}\,x^2\right)+\log\left(\sqrt{a\,x^5+b}+\sqrt{x}\,\sqrt{a}\,x^2\right)\right)}{5a}$$

input
```
int(1/(a+b/x^5)^(1/2)/x,x)
```

output
```
(sqrt(a)*( - log(sqrt(a*x**5 + b) - sqrt(x)*sqrt(a)*x**2) + log(sqrt(a*x**
5 + b) + sqrt(x)*sqrt(a)*x**2)))/(5*a)
```

3.571 $\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}x} \, dx = -\frac{2\arctan\left(\frac{\sqrt{-a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

output
```
-2/5*arctan((-a+b/x^5)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}x} \, dx = \frac{2\sqrt{-b+ax^5}\log\left(\sqrt{a}x^{5/2}+\sqrt{-b+ax^5}\right)}{5\sqrt{a}\sqrt{-a+\frac{b}{x^5}}x^{5/2}}$$

input
```
Integrate[1/(Sqrt[-a + b/x^5]*x),x]
```

output
```
(2*Sqrt[-b + a*x^5]*Log[Sqrt[a]*x^(5/2) + Sqrt[-b + a*x^5]])/(5*Sqrt[a]*Sq
rt[-a + b/x^5]*x^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\frac{b}{x^5}-a}}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{5}\int \frac{x^5}{\sqrt{\frac{b}{x^5}-a}}d\frac{1}{x^5}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2\int \frac{1}{\frac{a}{b}+\frac{1}{bx^{10}}}d\sqrt{\frac{b}{x^5}-a}}{5b}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2\arctan\left(\frac{\sqrt{\frac{b}{x^5}-a}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

input
```
Int[1/(Sqrt[-a + b/x^5]*x),x]
```

output
```
(-2*ArcTan[Sqrt[-a + b/x^5]/Sqrt[a]])/(5*Sqrt[a])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\, x}dx$$

input
```
int(1/(-a+b/x^5)^(1/2)/x,x)
```

output
```
int(1/(-a+b/x^5)^(1/2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.83

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}x}\,dx = \left[-\frac{\sqrt{-a}\log\left(-8\,a^2x^{10}+8\,abx^5-b^2+4\,(2\,ax^{10}-bx^5)\sqrt{-a}\sqrt{-\frac{ax^5-b}{x^5}}\right)}{10\,a},\ -\frac{\arctan\left(\frac{2\,\sqrt{a}x^5\sqrt{-\frac{ax^5-b}{x^5}}}{2\,ax^5-b}\right)}{5\,\sqrt{a}}\right]$$

input
```
integrate(1/(-a+b/x^5)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/10*sqrt(-a)*log(-8*a^2*x^10 + 8*a*b*x^5 - b^2 + 4*(2*a*x^10 - b*x^5)*s
qrt(-a)*sqrt(-(a*x^5 - b)/x^5))/a, -1/5*arctan(2*sqrt(a)*x^5*sqrt(-(a*x^5
- b)/x^5)/(2*a*x^5 - b))/sqrt(a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}x}\,dx = \begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{a}x^{\frac{5}{2}}}{\sqrt{b}}\right)}{5\sqrt{a}} & \text{for}\ \left|\frac{ax^5}{b}\right|>1 \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{a}x^{\frac{5}{2}}}{\sqrt{b}}\right)}{5\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-a+b/x**5)**(1/2)/x,x)
```

output
```
Piecewise((-2*I*acosh(sqrt(a)*x**(5/2)/sqrt(b))/(5*sqrt(a)), Abs(a*x**5/b)
 > 1), (2*asin(sqrt(a)*x**(5/2)/sqrt(b))/(5*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}x}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-a+\frac{b}{x^5}}}{\sqrt{a}}\right)}{5\sqrt{a}}$$

input
```
integrate(1/(-a+b/x^5)^(1/2)/x,x, algorithm="maxima")
```

output
```
-2/5*arctan(sqrt(-a + b/x^5)/sqrt(a))/sqrt(a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(-a+b/x^5)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\,x}\,dx=-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{\frac{b}{x^5}-a}}{\sqrt{a}}\right)}{5\,\sqrt{a}}$$

input
```
int(1/(x*(b/x^5 - a)^(1/2)),x)
```

output
```
-(2*atan((b/x^5 - a)^(1/2)/a^(1/2)))/(5*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{-a+\frac{b}{x^5}}\,x}\,dx=-\frac{\sqrt{a}\,atan\left(\frac{2\sqrt{a}\,\sqrt{-a\,x^5+b}\,a\,x^5-\sqrt{a}\,\sqrt{-a\,x^5+b}\,b}{2\sqrt{x}\,a^2x^7-2\sqrt{x}\,ab\,x^2}\right)}{5a}$$

input
```
int(1/(-a+b/x^5)^(1/2)/x,x)
```

output
```
( - sqrt(a)*atan((2*sqrt(a)*sqrt( - a*x**5 + b)*a*x**5 - sqrt(a)*sqrt( - a
*x**5 + b)*b)/(2*sqrt(x)*a**2*x**7 - 2*sqrt(x)*a*b*x**2)))/(5*a)
```

3.572 $\int \frac{1}{a+\frac{b}{x^6}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 155

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx=\frac{x}{a}-\frac{\sqrt[6]{b}\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3a^{7/6}}+\frac{\sqrt[6]{b}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{6a^{7/6}}-\frac{\sqrt[6]{b}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{6a^{7/6}}-\frac{\sqrt[6]{b}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x}{\sqrt[3]{b}+\sqrt[3]{a}x^2}\right)}{2\sqrt{3}a^{7/6}}$$

output
```
x/a-1/3*b^(1/6)*arctan(a^(1/6)*x/b^(1/6))/a^(7/6)-1/6*b^(1/6)*arctan(-3^(1
/2)+2*a^(1/6)*x/b^(1/6))/a^(7/6)-1/6*b^(1/6)*arctan(3^(1/2)+2*a^(1/6)*x/b^
(1/6))/a^(7/6)-1/6*b^(1/6)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x/(b^(1/3)+a^(1
/3)*x^2))*3^(1/2)/a^(7/6)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.17

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx = \frac{12\sqrt[6]{a}x-4\sqrt[6]{b}\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)+2\sqrt[6]{b}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)-2\sqrt[6]{b}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{a}x}{\sqrt[6]{b}}\right)+\sqrt{3}\sqrt[6]{b}\log}{12a^{7/6}}$$

input
```
Integrate[(a + b/x^6)^(-1),x]
```

output
```
(12*a^(1/6)*x - 4*b^(1/6)*ArcTan[(a^(1/6)*x)/b^(1/6)] + 2*b^(1/6)*ArcTan[S
qrt[3] - (2*a^(1/6)*x)/b^(1/6)] - 2*b^(1/6)*ArcTan[Sqrt[3] + (2*a^(1/6)*x)
/b^(1/6)] + Sqrt[3]*b^(1/6)*Log[b^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1
/3)*x^2] - Sqrt[3]*b^(1/6)*Log[b^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/
3)*x^2])/(12*a^(7/6))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.43, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.222$, Rules used = {772, 843, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{x^6}}dx$$

$$\Big\downarrow \text{772}$$

$$\int \frac{x^6}{ax^6+b}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{ax^6+b}dx}{a}$$

$$\Big\downarrow \text{753}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{1}{\sqrt[3]{a}x^2+\sqrt[3]{b}}dx}{3b^{2/3}}+\frac{\int \frac{2\sqrt[6]{b}-\sqrt{3}\sqrt[6]{a}x}{2\left(\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}\right)}dx}{3b^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{a}x+2\sqrt[6]{b}}{2\left(\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}\right)}dx}{3b^{5/6}}\right)}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{1}{\sqrt[3]{a}x^2+\sqrt[3]{b}}dx}{3b^{2/3}}+\frac{\int \frac{2\sqrt[6]{b}-\sqrt{3}\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{a}x+2\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}\right)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[6]{b}-\sqrt{3}\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{a}x+2\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}x}{\sqrt[6]{b}}\right)}{3\sqrt[6]{a}b^{5/6}}\right)}{a}$$

$$\Big\downarrow \text{1142}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\sqrt{3}\int -\frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x\right)}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\sqrt{3}\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}\cdots\right.}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{\cdots}}}{2\sqrt[6]{\cdots}}}{6b^{5/6}}\cdots\right.}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\sqrt{3}\int \frac{\sqrt[6]{a}\left(\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x\right)}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\sqrt{3}\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}x\cdots\right.}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{\cdots}}}{2\sqrt[6]{a}\cdots}}{6b^{5/6}}\cdots\right.}{a}$$

$$\Big\downarrow \; 27$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{b}\int \frac{1}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{a}\ldots}{\sqrt[3]{a}x^2+\sqrt{3}\ldots}}{6b^{5/6}}\ldots\right.}{a}$$

$$\Big\downarrow \; 1082$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)}{\sqrt{3}\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{a}x+\sqrt{3}\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\int \frac{1}{-\left(\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}+1\right)^2-\ldots}\ldots}{\sqrt{3}\sqrt[6]{\ldots}}}{6b^{5/6}}\ldots\right.}{a}$$

$$\Big\downarrow \; 217$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{b}-2\sqrt[6]{a}x}{\sqrt[3]{a}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)\right)}{\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{a}x+\sqrt{3}\sqrt[6]{b}}{\sqrt[3]{a}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}dx+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}+1\right)\right)}{\sqrt[6]{a}}}{6b^{5/6}}+\ldots\right.}{a}$$

$$\Big\downarrow \; 1103$$

$$\frac{x}{a}-\frac{b\left(\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}\right)\right)}{\sqrt[6]{a}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{2\sqrt[6]{a}}}{6b^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{a}x}{\sqrt{3}\sqrt[6]{b}}+1\right)\right)}{\sqrt[6]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}x^2+\sqrt[3]{b}\right)}{2\sqrt[6]{a}}}{6b^{5/6}}+\ldots\right.}{a}$$

input
```
Int[(a + b/x^6)^(-1),x]
```

output
```
x/a - (b*(ArcTan[(a^(1/6)*x)/b^(1/6)]/(3*a^(1/6)*b^(5/6)) + (-(ArcTan[Sqrt
[3]*(1 - (2*a^(1/6)*x)/(Sqrt[3]*b^(1/6)))]/a^(1/6)) - (Sqrt[3]*Log[b^(1/3)
 - Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/3)*x^2])/(2*a^(1/6)))/(6*b^(5/6)) + (A
rcTan[Sqrt[3]*(1 + (2*a^(1/6)*x)/(Sqrt[3]*b^(1/6)))]/a^(1/6) + (Sqrt[3]*Lo
g[b^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + a^(1/3)*x^2])/(2*a^(1/6)))/(6*b^(5
/6))))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.22

method	result
risch	$\frac{x}{a}-\frac{b\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}a+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{5}}\right)}{6a^{2}}$
default	$\frac{x}{a}-\frac{\left(\frac{\left(\frac{b}{a}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{3b}+\frac{\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}\ln\left(x^{2}+\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12b}+\frac{\left(\frac{b}{a}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6b}-\frac{\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}\ln\left(x^{2}-\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}x+\left(b\right.\right.}{12b}\right.}{a}$

input
```
int(1/(a+b/x^6),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/6/a^2*b*sum(1/_R^5*ln(x-_R),_R=RootOf(_Z^6*a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(105) = 210.

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.37

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx = -\frac{(\sqrt{-3}a+a)\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}(\sqrt{-3}a+a)\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}+x\right)-(\sqrt{-3}a+a)\left(-\frac{b}{a^7}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}(\sqrt{-3}a+a)(-\right.}{}$$

input
```
integrate(1/(a+b/x^6),x, algorithm="fricas")
```

output
```
-1/12*((sqrt(-3)*a + a)*(-b/a^7)^(1/6)*log(1/2*(sqrt(-3)*a + a)*(-b/a^7)^(
1/6) + x) - (sqrt(-3)*a + a)*(-b/a^7)^(1/6)*log(-1/2*(sqrt(-3)*a + a)*(-b/
a^7)^(1/6) + x) + (sqrt(-3)*a - a)*(-b/a^7)^(1/6)*log(1/2*(sqrt(-3)*a - a)
*(-b/a^7)^(1/6) + x) - (sqrt(-3)*a - a)*(-b/a^7)^(1/6)*log(-1/2*(sqrt(-3)*
a - a)*(-b/a^7)^(1/6) + x) + 2*a*(-b/a^7)^(1/6)*log(a*(-b/a^7)^(1/6) + x)
- 2*a*(-b/a^7)^(1/6)*log(-a*(-b/a^7)^(1/6) + x) - 12*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.14

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx = \operatorname{RootSum}\left(46656t^6a^7+b,\left(t\mapsto t\log\left(-6ta+x\right)\right)\right)+\frac{x}{a}$$

input
```
integrate(1/(a+b/x**6),x)
```

output
```
RootSum(46656*_t**6*a**7 + b, Lambda(_t, _t*log(-6*_t*a + x))) + x/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.25

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx =$$
$$-\frac{\frac{\sqrt{3}b^{\frac{1}{6}}\log\left(a^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+b^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}}-\frac{\sqrt{3}b^{\frac{1}{6}}\log\left(a^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+b^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}}+\frac{4\,b^{\frac{1}{3}}\arctan\left(\frac{a^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\,b^{\frac{1}{3}}\arctan\left(\frac{2\,a^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{12\,a}$$
$$+\frac{x}{a}$$

input
```
integrate(1/(a+b/x^6),x, algorithm="maxima")
```

output
```
-1/12*(sqrt(3)*b^(1/6)*log(a^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^(1/6)*x + b^(1/
3))/a^(1/6) - sqrt(3)*b^(1/6)*log(a^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x
+ b^(1/3))/a^(1/6) + 4*b^(1/3)*arctan(a^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/sqr
t(a^(1/3)*b^(1/3)) + 2*b^(1/3)*arctan((2*a^(1/3)*x + sqrt(3)*a^(1/6)*b^(1/
6))/sqrt(a^(1/3)*b^(1/3)))/sqrt(a^(1/3)*b^(1/3)) + 2*b^(1/3)*arctan((2*a^(
1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/sqrt(a^(1/3)*b^(1
/3)))/a + x/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.16

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx = \frac{x}{a}-\frac{\sqrt{3}(a^5b)^{\frac{1}{6}}\log\left(x^2+\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{6}}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12\,a^2}$$
$$+\frac{\sqrt{3}(a^5b)^{\frac{1}{6}}\log\left(x^2-\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{6}}+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12\,a^2}-\frac{(a^5b)^{\frac{1}{6}}\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{6\,a^2}$$
$$-\frac{(a^5b)^{\frac{1}{6}}\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{6}}}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{6\,a^2}-\frac{(a^5b)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{b}{a}\right)^{\frac{1}{6}}}\right)}{3\,a^2}$$

input
```
integrate(1/(a+b/x^6),x, algorithm="giac")
```

output
```
x/a - 1/12*sqrt(3)*(a^5*b)^(1/6)*log(x^2 + sqrt(3)*x*(b/a)^(1/6) + (b/a)^(
1/3))/a^2 + 1/12*sqrt(3)*(a^5*b)^(1/6)*log(x^2 - sqrt(3)*x*(b/a)^(1/6) + (
b/a)^(1/3))/a^2 - 1/6*(a^5*b)^(1/6)*arctan((2*x + sqrt(3)*(b/a)^(1/6))/(b/
a)^(1/6))/a^2 - 1/6*(a^5*b)^(1/6)*arctan((2*x - sqrt(3)*(b/a)^(1/6))/(b/a)
^(1/6))/a^2 - 1/3*(a^5*b)^(1/6)*arctan(x/(b/a)^(1/6))/a^2
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.46

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx = \frac{x}{a}+\frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{a^{1/6}\,x\,1\mathrm{i}}{(-b)^{1/6}}\right)\,1\mathrm{i}}{3\,a^{7/6}} + \frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{(-b)^{25/6}\,x\,1\mathrm{i}}{a^{1/6}\left(\frac{(-b)^{13/3}}{a^{1/3}}+\frac{\sqrt{3}\,(-b)^{13/3}\,1\mathrm{i}}{a^{1/3}}\right)}+\frac{\sqrt{3}\,(-b)^{25/6}\,x}{a^{1/6}\left(\frac{(-b)^{13/3}}{a^{1/3}}+\frac{\sqrt{3}\,(-b)^{13/3}\,1\mathrm{i}}{a^{1/3}}\right)}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3\,a^{7/6}} - \frac{(-b)^{1/6}\,\mathrm{atan}\left(\frac{(-b)^{25/6}\,x\,1\mathrm{i}}{a^{1/6}\left(\frac{(-b)^{13/3}}{a^{1/3}}-\frac{\sqrt{3}\,(-b)^{13/3}\,1\mathrm{i}}{a^{1/3}}\right)}-\frac{\sqrt{3}\,(-b)^{25/6}\,x}{a^{1/6}\left(\frac{(-b)^{13/3}}{a^{1/3}}-\frac{\sqrt{3}\,(-b)^{13/3}\,1\mathrm{i}}{a^{1/3}}\right)}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3\,a^{7/6}}$$

input
```
int(1/(a + b/x^6),x)
```

output
```
x/a + ((-b)^(1/6)*atan((a^(1/6)*x*1i)/(-b)^(1/6))*1i)/(3*a^(7/6)) + ((-b)^
(1/6)*atan(((-b)^(25/6)*x*1i)/(a^(1/6)*((-b)^(13/3)/a^(1/3) + (3^(1/2)*(-b
)^(13/3)*1i)/a^(1/3))) + (3^(1/2)*(-b)^(25/6)*x)/(a^(1/6)*((-b)^(13/3)/a^(
1/3) + (3^(1/2)*(-b)^(13/3)*1i)/a^(1/3))))*((3^(1/2)*1i)/2 - 1/2)*1i)/(3*a
^(7/6)) - ((-b)^(1/6)*atan(((-b)^(25/6)*x*1i)/(a^(1/6)*((-b)^(13/3)/a^(1/3
) - (3^(1/2)*(-b)^(13/3)*1i)/a^(1/3))) - (3^(1/2)*(-b)^(25/6)*x)/(a^(1/6)*
((-b)^(13/3)/a^(1/3) - (3^(1/2)*(-b)^(13/3)*1i)/a^(1/3))))*((3^(1/2)*1i)/2
 + 1/2)*1i)/(3*a^(7/6))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{1}{a+\frac{b}{x^6}}\,dx$$
$$= \frac{2b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2a^{\frac{1}{3}}x}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2a^{\frac{1}{3}}x}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-4b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{a^{\frac{1}{6}}x}{b^{\frac{1}{6}}}\right)+b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}\log\left(-b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}\,x\right.}{12a^{\frac{4}{3}}}$$

input
```
int(1/(a+b/x^6),x)
```

output
```
(2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*a**(1/3)*x)/(b**(
1/6)*a**(1/6))) - 2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*
a**(1/3)*x)/(b**(1/6)*a**(1/6))) - 4*b**(1/6)*a**(1/6)*atan((a**(1/3)*x)/(
b**(1/6)*a**(1/6))) + b**(1/6)*a**(1/6)*sqrt(3)*log( - b**(1/6)*a**(1/6)*s
qrt(3)*x + a**(1/3)*x**2 + b**(1/3)) - b**(1/6)*a**(1/6)*sqrt(3)*log(b**(1
/6)*a**(1/6)*sqrt(3)*x + a**(1/3)*x**2 + b**(1/3)) + 12*a**(1/3)*x)/(12*a*
*(1/3)*a)
```

3.573 $\int \frac{1}{a+\frac{b}{x^8}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 215

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx = \frac{x}{a}+\frac{\sqrt[8]{b}\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}}-\frac{\sqrt[8]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{4\sqrt{2}(-a)^{9/8}}+\frac{\sqrt[8]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{4\sqrt{2}(-a)^{9/8}}+\frac{\sqrt[8]{b}\operatorname{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{4(-a)^{9/8}}+\frac{\sqrt[8]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}x}{\sqrt[4]{b}+\sqrt[4]{-a}x^2}\right)}{4\sqrt{2}(-a)^{9/8}}$$

output
```
x/a+1/4*b^(1/8)*arctan((-a)^(1/8)*x/b^(1/8))/(-a)^(9/8)+1/8*b^(1/8)*arctan
(-1+2^(1/2)*(-a)^(1/8)*x/b^(1/8))*2^(1/2)/(-a)^(9/8)+1/8*b^(1/8)*arctan(1+
2^(1/2)*(-a)^(1/8)*x/b^(1/8))*2^(1/2)/(-a)^(9/8)+1/4*b^(1/8)*arctanh((-a)^
(1/8)*x/b^(1/8))/(-a)^(9/8)+1/8*b^(1/8)*arctanh(2^(1/2)*(-a)^(1/8)*b^(1/8)
*x/(b^(1/4)+(-a)^(1/4)*x^2))*2^(1/2)/(-a)^(9/8)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.71

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx$$
$$= \frac{8\sqrt[8]{a}x-2\sqrt[8]{b}\arctan\left(\frac{\sqrt[8]{a}x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{b}}-\tan\left(\frac{\pi}{8}\right)\right)\cos\left(\frac{\pi}{8}\right)-2\sqrt[8]{b}\arctan\left(\frac{\sqrt[8]{a}x\sec\left(\frac{\pi}{8}\right)}{\sqrt[8]{b}}+\tan\left(\frac{\pi}{8}\right)\right)\cos\left(\frac{\pi}{8}\right)+\sqrt[8]{}}{}$$

input
```
Integrate[(a + b/x^8)^(-1),x]
```

output
```
(8*a^(1/8)*x - 2*b^(1/8)*ArcTan[(a^(1/8)*x*Sec[Pi/8])/b^(1/8) - Tan[Pi/8]]
*Cos[Pi/8] - 2*b^(1/8)*ArcTan[(a^(1/8)*x*Sec[Pi/8])/b^(1/8) + Tan[Pi/8]]*C
os[Pi/8] + b^(1/8)*Cos[Pi/8]*Log[b^(1/4) + a^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)
*x*Cos[Pi/8]] - b^(1/8)*Cos[Pi/8]*Log[b^(1/4) + a^(1/4)*x^2 + 2*a^(1/8)*b^
(1/8)*x*Cos[Pi/8]] + 2*b^(1/8)*ArcTan[Cot[Pi/8] - (a^(1/8)*x*Csc[Pi/8])/b^
(1/8)]*Sin[Pi/8] - 2*b^(1/8)*ArcTan[Cot[Pi/8] + (a^(1/8)*x*Csc[Pi/8])/b^(1
/8)]*Sin[Pi/8] + b^(1/8)*Log[b^(1/4) + a^(1/4)*x^2 - 2*a^(1/8)*b^(1/8)*x*S
in[Pi/8]]*Sin[Pi/8] - b^(1/8)*Log[b^(1/4) + a^(1/4)*x^2 + 2*a^(1/8)*b^(1/8
)*x*Sin[Pi/8]]*Sin[Pi/8])/(8*a^(9/8))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.46, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.556, Rules used = {772, 843, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx$$
$$\Big\downarrow \; 772$$
$$\int \frac{x^8}{ax^8+b}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x}{a}-\frac{b\int \frac{1}{ax^8+b}dx}{a}$$

$$\Big\downarrow \text{758}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^4}dx}{2\sqrt{b}}+\frac{\int \frac{1}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{755}$$

$$\frac{x}{a}-\frac{b\left(\frac{\int \frac{1}{\sqrt{b}-\sqrt{-a}x^4}dx}{2\sqrt{b}}+\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{756}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int \frac{1}{\sqrt[4]{b}-\sqrt[4]{-a}x^2}dx}{2\sqrt[4]{b}}+\frac{\int \frac{1}{\sqrt[4]{-a}x^2+\sqrt[4]{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int \frac{1}{\sqrt[4]{b}-\sqrt[4]{-a}x^2}dx}{2\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}+\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\int \frac{\sqrt[4]{-a}x^2+\sqrt[4]{b}}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}\right)}{a}$$

$$\Big\downarrow \text{1476}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[4]{-a}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[4]{-a}}}{2\sqrt[4]{b}}+\frac{\int\frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}\right)}{a}$$

↓ 1082

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int\frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\text{ar}\cdots}{2\sqrt{\cdots}}\right)}{a}$$

↓ 217

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\int\frac{\sqrt[4]{b}-\sqrt[4]{-a}x^2}{\sqrt{-a}x^4+\sqrt{b}}dx}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}\right)}{a}$$

↓ 1479

$$\frac{x}{a}-\frac{b\left(\frac{\frac{-\frac{\int -\frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+\frac{\sqrt[8]{b}}{\sqrt[8]{-a}}\right)}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\cdots}{2\sqrt{b}}\cdots\right)}{a}$$

$\downarrow$ 25

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\frac{\int \frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+\frac{\sqrt[8]{b}}{\sqrt[8]{-a}}\right)}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctan}\cdots}{2\sqrt[8]{\cdots}}}{2\sqrt{b}}\cdots\right)}{a}$$

$\downarrow$ 27

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\frac{\int \frac{\frac{\sqrt{2}\sqrt[8]{b}}{\sqrt[8]{-a}}-2x}{x^2-\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}+\frac{\int \frac{\sqrt{2}x+\frac{\sqrt[8]{b}}{\sqrt[8]{-a}}}{x^2+\frac{\sqrt{2}\sqrt[8]{b}x}{\sqrt[8]{-a}}+\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}}dx}{2\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctan}\cdots}{2\sqrt[8]{\cdots}}}{2\sqrt{b}}\cdots\right)}{a}$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{a}-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{2\sqrt[8]{-a}b^{3/8}}}{2\sqrt{b}}+\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{-a}x}{\sqrt[8]{b}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}}{2\sqrt[4]{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}x+\sqrt[4]{-a}x^2+\sqrt[4]{b}\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{b}}-\cdots}{2\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{a}$$

input
```
Int[(a + b/x^8)^(-1),x]
```

output
```
x/a - (b*((ArcTan[((-a)^(1/8)*x)/b^(1/8)]/(2*(-a)^(1/8)*b^(3/8)) + ArcTanh
[((-a)^(1/8)*x)/b^(1/8)]/(2*(-a)^(1/8)*b^(3/8)))/(2*Sqrt[b]) + ((-(ArcTan[
1 - (Sqrt[2]*(-a)^(1/8)*x)/b^(1/8)]/(Sqrt[2]*(-a)^(1/8)*b^(1/8))) + ArcTan
[1 + (Sqrt[2]*(-a)^(1/8)*x)/b^(1/8)]/(Sqrt[2]*(-a)^(1/8)*b^(1/8)))/(2*b^(1
/4)) + (-1/2*Log[b^(1/4) - Sqrt[2]*(-a)^(1/8)*b^(1/8)*x + (-a)^(1/4)*x^2]/
(Sqrt[2]*(-a)^(1/8)*b^(1/8)) + Log[b^(1/4) + Sqrt[2]*(-a)^(1/8)*b^(1/8)*x
+ (-a)^(1/4)*x^2]/(2*Sqrt[2]*(-a)^(1/8)*b^(1/8)))/(2*b^(1/4)))/(2*Sqrt[b])
))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{x}{a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8a^{2}}$	34
risch	$\frac{x}{a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}+b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{7}}\right)}{8a^{2}}$	34

input
```
int(1/(a+b/x^8),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/8*b/a^2*sum(1/_R^7*ln(x-_R),_R=RootOf(_Z^8*a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx =$$

$$-\frac{(i+1)\ \sqrt{2}a\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}a\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}+x\right)-(i-1)\ \sqrt{2}a\left(-\frac{b}{a^9}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{2}a\left(-\frac{b}{a^9}\right.\right.}{}$$

input
```
integrate(1/(a+b/x^8),x, algorithm="fricas")
```

output
```
-1/16*((I + 1)*sqrt(2)*a*(-b/a^9)^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a*(-b/a^
9)^(1/8) + x) - (I - 1)*sqrt(2)*a*(-b/a^9)^(1/8)*log(-(1/2*I - 1/2)*sqrt(2
)*a*(-b/a^9)^(1/8) + x) + (I - 1)*sqrt(2)*a*(-b/a^9)^(1/8)*log((1/2*I - 1/
2)*sqrt(2)*a*(-b/a^9)^(1/8) + x) - (I + 1)*sqrt(2)*a*(-b/a^9)^(1/8)*log(-(
1/2*I + 1/2)*sqrt(2)*a*(-b/a^9)^(1/8) + x) + 2*a*(-b/a^9)^(1/8)*log(a*(-b/
a^9)^(1/8) + x) + 2*I*a*(-b/a^9)^(1/8)*log(I*a*(-b/a^9)^(1/8) + x) - 2*I*a
*(-b/a^9)^(1/8)*log(-I*a*(-b/a^9)^(1/8) + x) - 2*a*(-b/a^9)^(1/8)*log(-a*(
-b/a^9)^(1/8) + x) - 16*x)/a
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.10

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx = \operatorname{RootSum}\left(16777216t^8a^9+b,\left(t\mapsto t\log\left(-8ta+x\right)\right)\right)+\frac{x}{a}$$

input
```
integrate(1/(a+b/x**8),x)
```

output
```
RootSum(16777216*_t**8*a**9 + b, Lambda(_t, _t*log(-8*_t*a + x))) + x/a
```

Maxima [F]

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx=\int \frac{1}{a+\frac{b}{x^8}}\,dx$$

input
```
integrate(1/(a+b/x^8),x, algorithm="maxima")
```

output
```
-b*integrate(1/(a*x^8 + b), x)/a + x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(148) = 296$.

Time = 0.14 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.06

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx=\frac{x}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}{\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}}\right)}{4a\sqrt{2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2+x\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8a\sqrt{-2\sqrt{2}+4}}+\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2-x\sqrt{\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8a\sqrt{-2\sqrt{2}+4}}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2+x\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8a\sqrt{2\sqrt{2}+4}}+\frac{\left(\frac{b}{a}\right)^{\frac{1}{8}}\log\left(x^2-x\sqrt{-\sqrt{2}+2}\left(\frac{b}{a}\right)^{\frac{1}{8}}+\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)}{8a\sqrt{2\sqrt{2}+4}}$$

input
```
integrate(1/(a+b/x^8),x, algorithm="giac")
```

output
```
x/a - 1/4*(b/a)^(1/8)*arctan((2*x + sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(
sqrt(2) + 2)*(b/a)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) - 1/4*(b/a)^(1/8)*arct
an((2*x - sqrt(-sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(sqrt(2) + 2)*(b/a)^(1/8)))
/(a*sqrt(-2*sqrt(2) + 4)) - 1/4*(b/a)^(1/8)*arctan((2*x + sqrt(sqrt(2) + 2
)*(b/a)^(1/8))/(sqrt(-sqrt(2) + 2)*(b/a)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) -
 1/4*(b/a)^(1/8)*arctan((2*x - sqrt(sqrt(2) + 2)*(b/a)^(1/8))/(sqrt(-sqrt(
2) + 2)*(b/a)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/8*(b/a)^(1/8)*log(x^2 +
x*sqrt(sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) +
1/8*(b/a)^(1/8)*log(x^2 - x*sqrt(sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(
a*sqrt(-2*sqrt(2) + 4)) - 1/8*(b/a)^(1/8)*log(x^2 + x*sqrt(-sqrt(2) + 2)*(
b/a)^(1/8) + (b/a)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) + 1/8*(b/a)^(1/8)*log(x^
2 - x*sqrt(-sqrt(2) + 2)*(b/a)^(1/8) + (b/a)^(1/4))/(a*sqrt(2*sqrt(2) + 4)
)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.53

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx = \frac{x}{a}-\frac{(-b)^{1/8}\,\mathrm{atan}\left(\frac{a^{1/8}\,x}{(-b)^{1/8}}\right)}{4\,a^{9/8}}+\frac{(-b)^{1/8}\,\mathrm{atan}\left(\frac{a^{1/8}\,x\,1\mathrm{i}}{(-b)^{1/8}}\right)\,1\mathrm{i}}{4\,a^{9/8}} + \frac{\sqrt{2}\,(-b)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^{1/8}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-b)^{1/8}}\right)\,\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)}{a^{9/8}} + \frac{\sqrt{2}\,(-b)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^{1/8}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-b)^{1/8}}\right)\,\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)}{a^{9/8}}$$

input
```
int(1/(a + b/x^8),x)
```

output
```
x/a - ((-b)^(1/8)*atan((a^(1/8)*x)/(-b)^(1/8)))/(4*a^(9/8)) + ((-b)^(1/8)*
atan((a^(1/8)*x*1i)/(-b)^(1/8))*1i)/(4*a^(9/8)) - (2^(1/2)*(-b)^(1/8)*atan
((2^(1/2)*a^(1/8)*x*(1/2 - 1i/2))/(-b)^(1/8))*(1/8 + 1i/8))/a^(9/8) - (2^(
1/2)*(-b)^(1/8)*atan((2^(1/2)*a^(1/8)*x*(1/2 + 1i/2))/(-b)^(1/8))*(1/8 - 1
i/8))/a^(9/8)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.72

$$\int \frac{1}{a+\frac{b}{x^8}}\,dx = \frac{2b^{\frac{1}{8}}a^{\frac{7}{8}}\sqrt{\sqrt{2}+2}\,atan\left(\frac{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}-2a^{\frac{1}{4}}x}{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)-2b^{\frac{1}{8}}a^{\frac{7}{8}}\sqrt{\sqrt{2}+2}\,atan\left(\frac{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}+2a^{\frac{1}{4}}x}{b^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)+2b^{\frac{1}{8}}a^{\frac{7}{8}}\sqrt{-\sqrt{2}+}$$

input
```
int(1/(a+b/x^8),x)
```

output
```
(2*b**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt( - sqr
t(2) + 2) - 2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) - 2*b**(1
/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2
) + 2*a**(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) + 2*b**(1/8)*a**(
7/8)*sqrt( - sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*a*
*(1/4)*x)/(b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) - 2*b**(1/8)*a**(7/8)*
sqrt( - sqrt(2) + 2)*atan((b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*a**(1/4
)*x)/(b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) + b**(1/8)*a**(7/8)*sqrt( -
 sqrt(2) + 2)*log( - b**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*x + a**(1/4)*x
**2 + b**(1/4)) - b**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*log(b**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2)*x + a**(1/4)*x**2 + b**(1/4)) + b**(1/8)*a**(7/8
)*sqrt(sqrt(2) + 2)*log( - b**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)*x + a**(1/4
)*x**2 + b**(1/4)) - b**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*log(b**(1/8)*a**(
1/8)*sqrt(sqrt(2) + 2)*x + a**(1/4)*x**2 + b**(1/4)) + 16*a*x)/(16*a**2)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file