Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.2/47-1.1.3.2-e

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**701**]. This is test number [47].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (701)	0.00 (0)
Mathematica	100.00 (701)	0.00 (0)
Sympy	98.43 (690)	1.57 (11)
Fricas	84.02 (589)	15.98 (112)
Maple	78.74 (552)	21.26 (149)
Reduce	77.89 (546)	22.11 (155)
Maxima	77.46 (543)	22.54 (158)
Mupad	71.18 (499)	28.82 (202)
Giac	62.91 (441)	37.09 (260)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

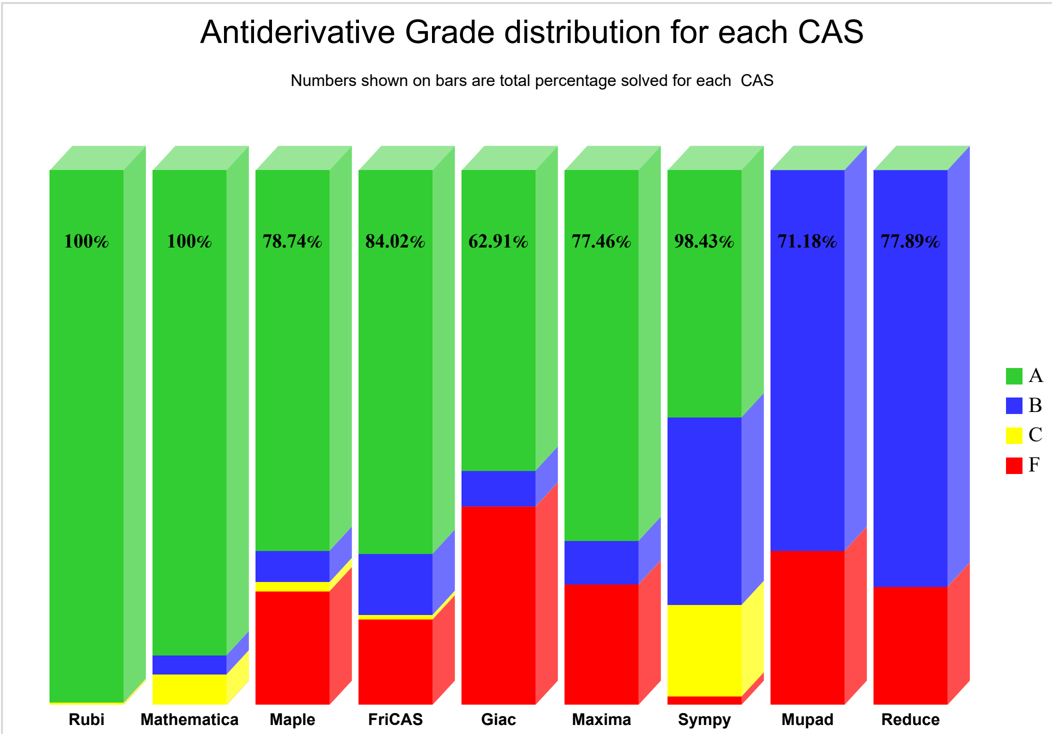
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

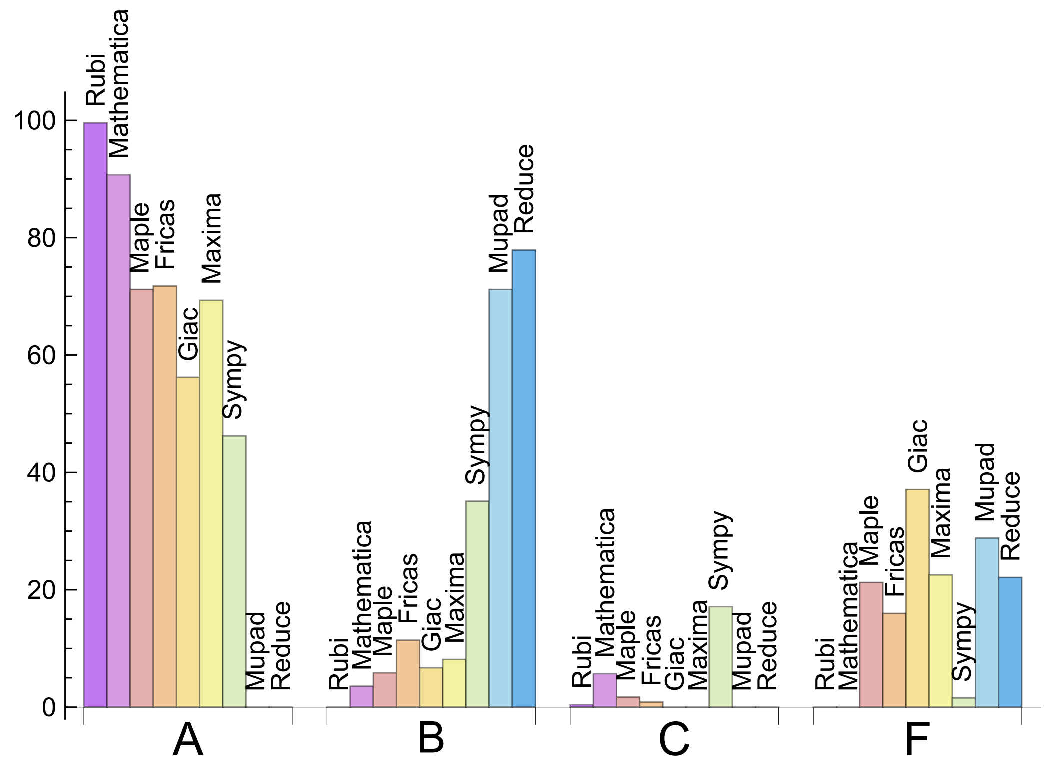
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.572	0.000	0.428	0.000
Mathematica	90.728	3.566	5.706	0.000
Fricas	71.755	11.412	0.856	15.977
Maple	71.184	5.849	1.712	21.255
Maxima	69.330	8.131	0.000	22.539
Giac	56.205	6.705	0.000	37.090
Sympy	46.220	35.093	17.118	1.569
Mupad	0.000	71.184	0.000	28.816
Reduce	0.000	77.889	0.000	22.111

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	11	0.00	90.91	9.09
Fricas	112	38.39	7.14	54.46
Maple	149	100.00	0.00	0.00
Reduce	155	100.00	0.00	0.00
Maxima	158	95.57	0.00	4.43
Mupad	202	0.00	100.00	0.00
Giac	260	98.85	0.00	1.15

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Fricas	0.09
Mathematica	0.13
Giac	0.13
Reduce	0.22
Rubi	0.34
Mupad	0.34
Sympy	3.44
Maple	3.72

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	66.82	1.09	51.00	0.96
Maple	68.89	1.12	56.00	0.89
Mupad	71.68	1.13	54.00	0.88
Rubi	75.03	1.06	57.00	1.00
Maxima	77.16	1.28	60.00	0.95
Giac	79.10	1.15	57.00	0.86
Reduce	79.66	1.26	57.50	0.94
Fricas	95.57	1.37	69.00	0.98
Sympy	257.05	3.61	87.00	1.32

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

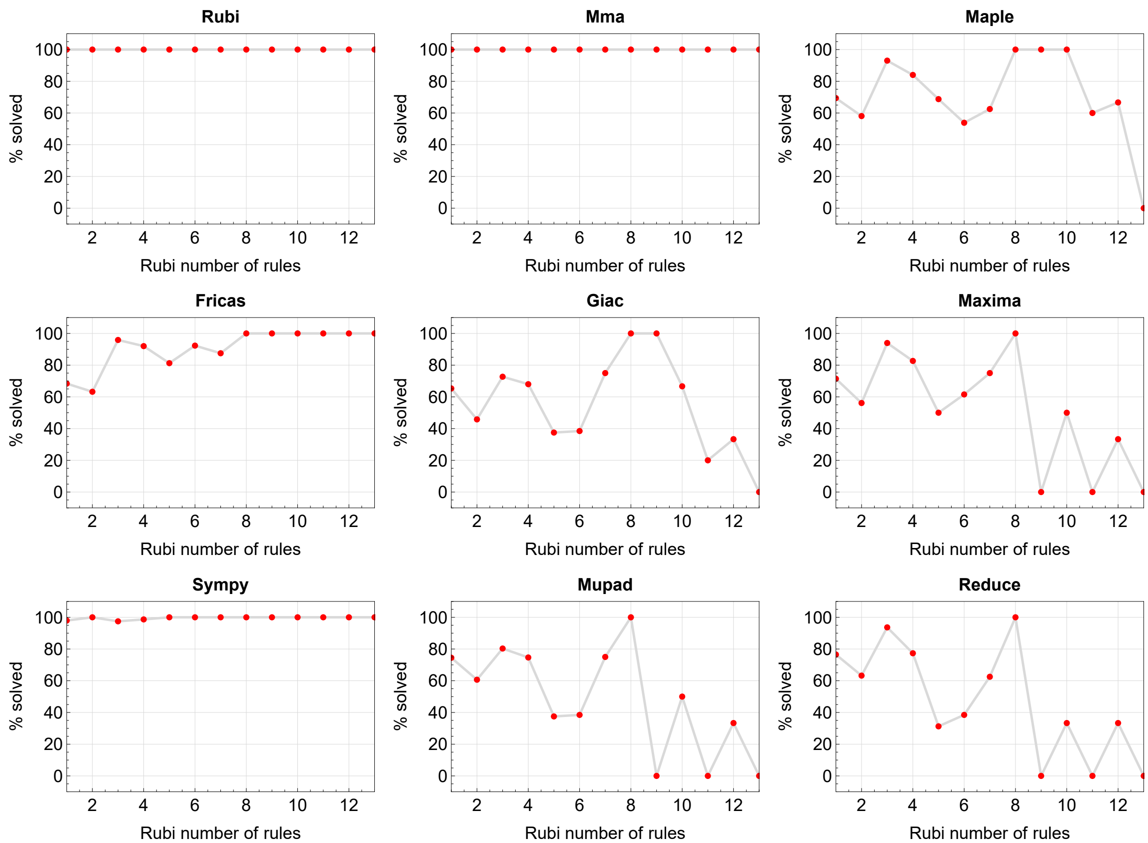


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

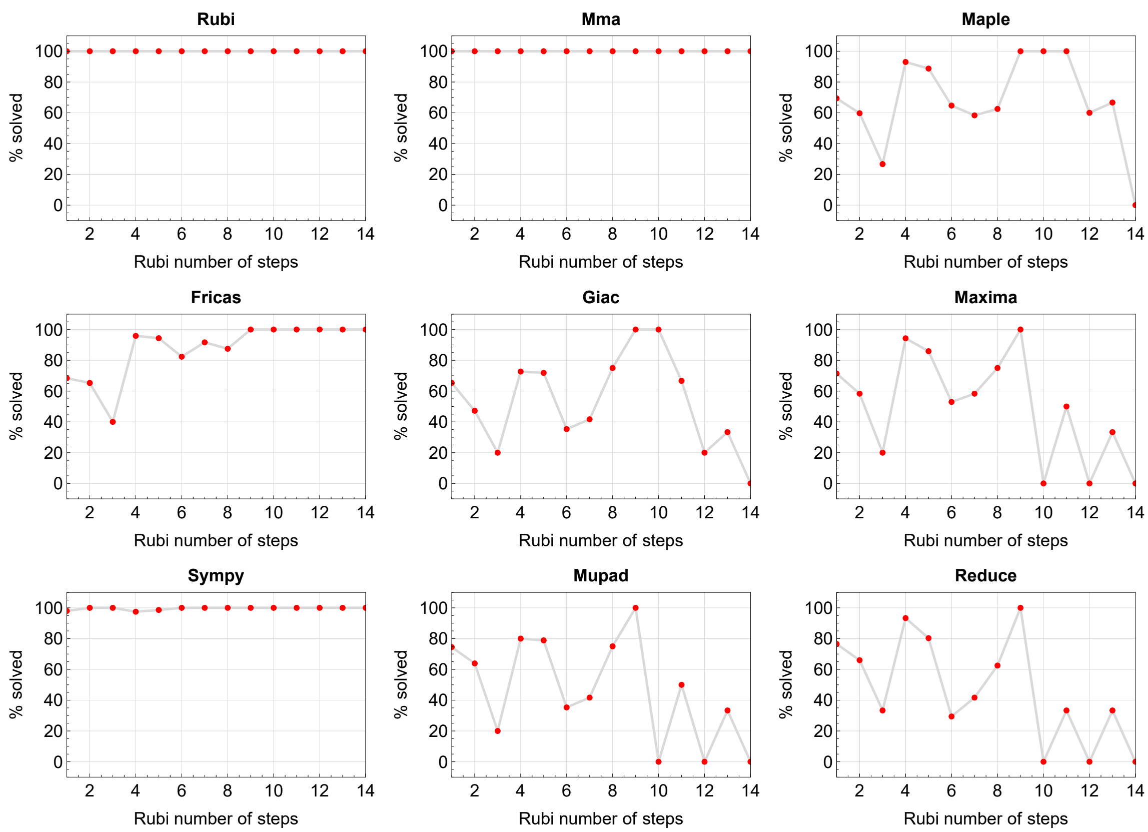


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

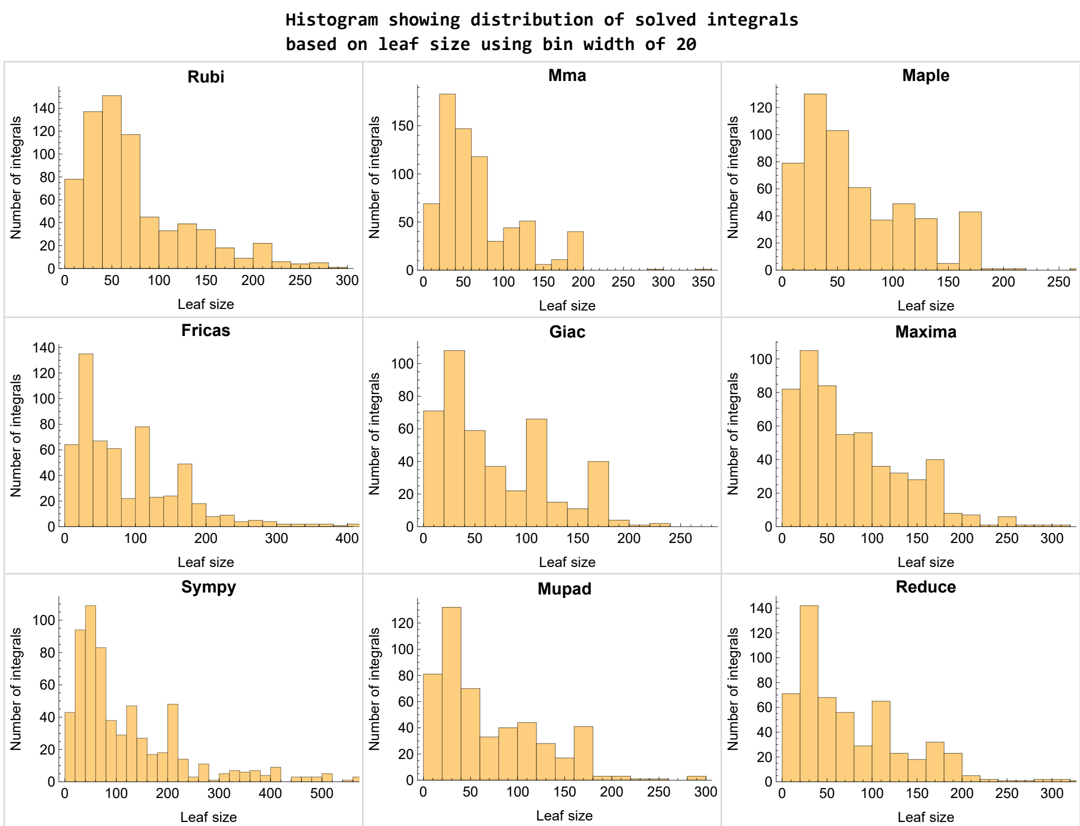


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

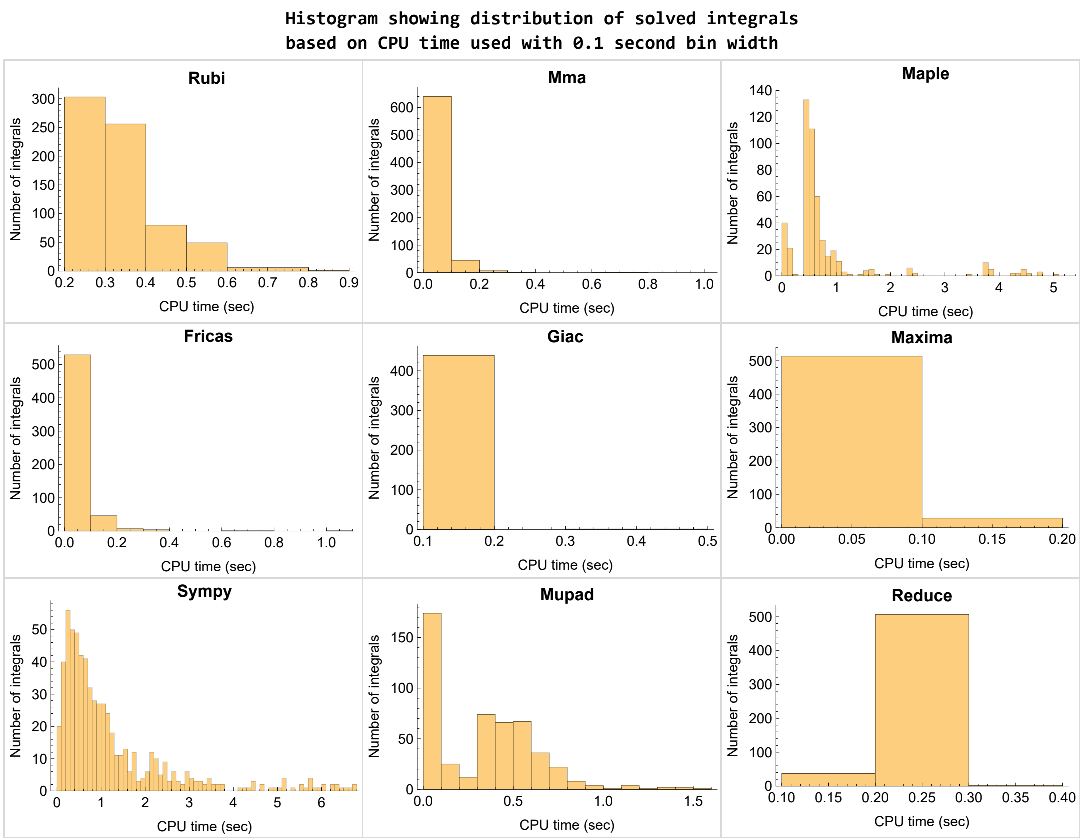


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

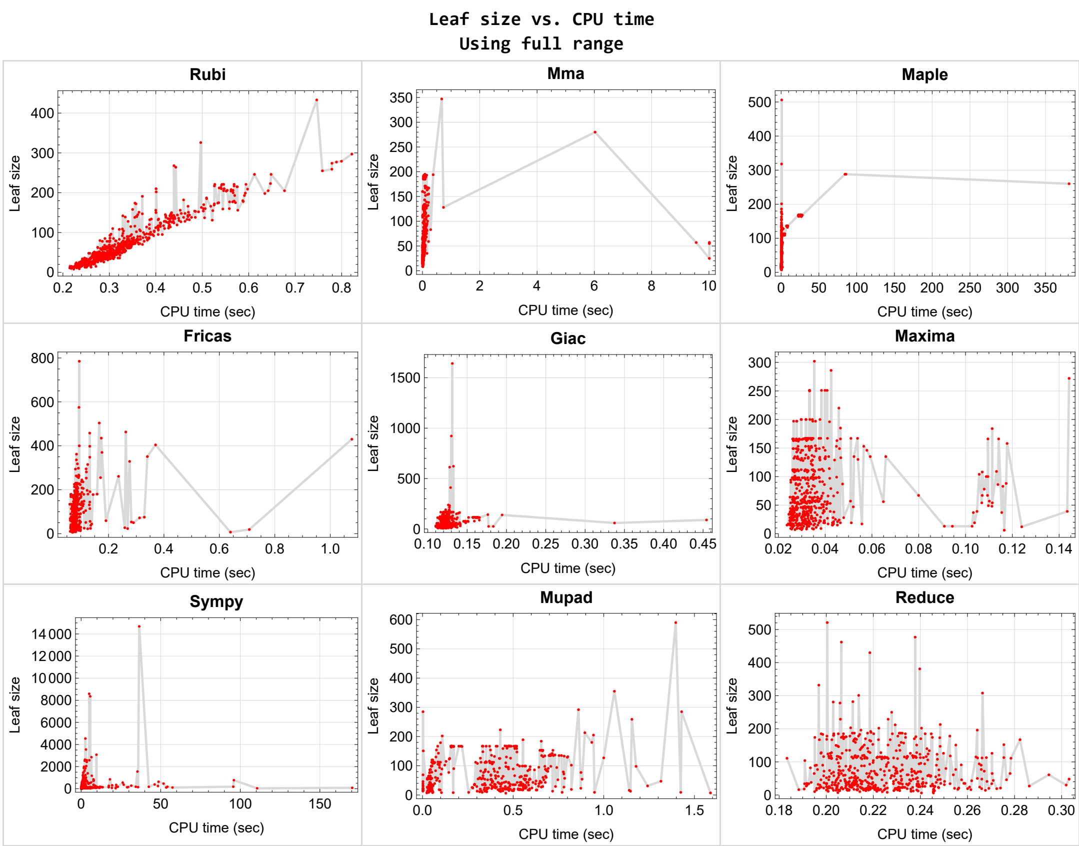


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{175, 280, 282, 283, 284\}$

Mathematica {}

Maple $\{282, 349, 356, 388, 399, 411, 431, 652\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

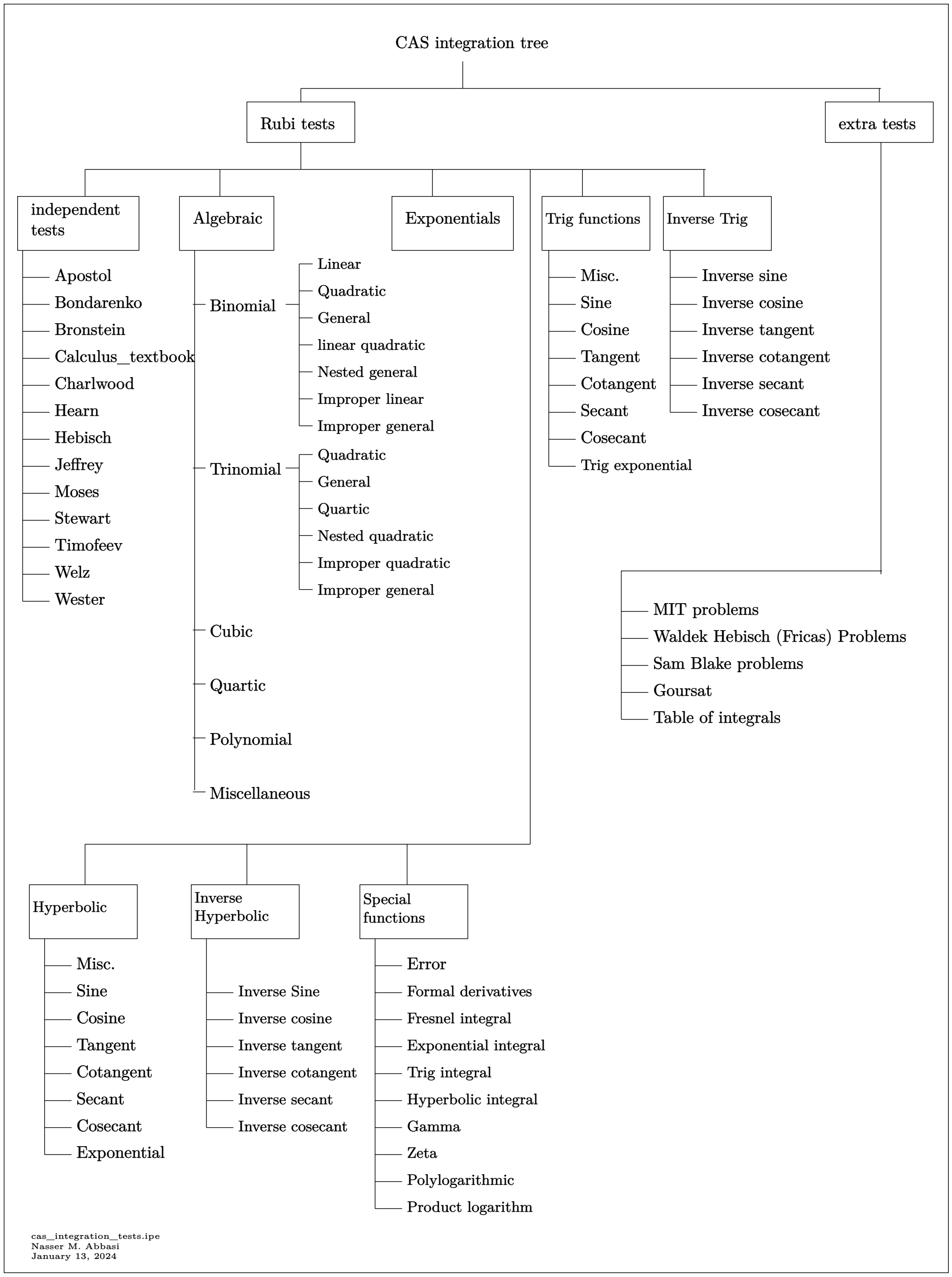
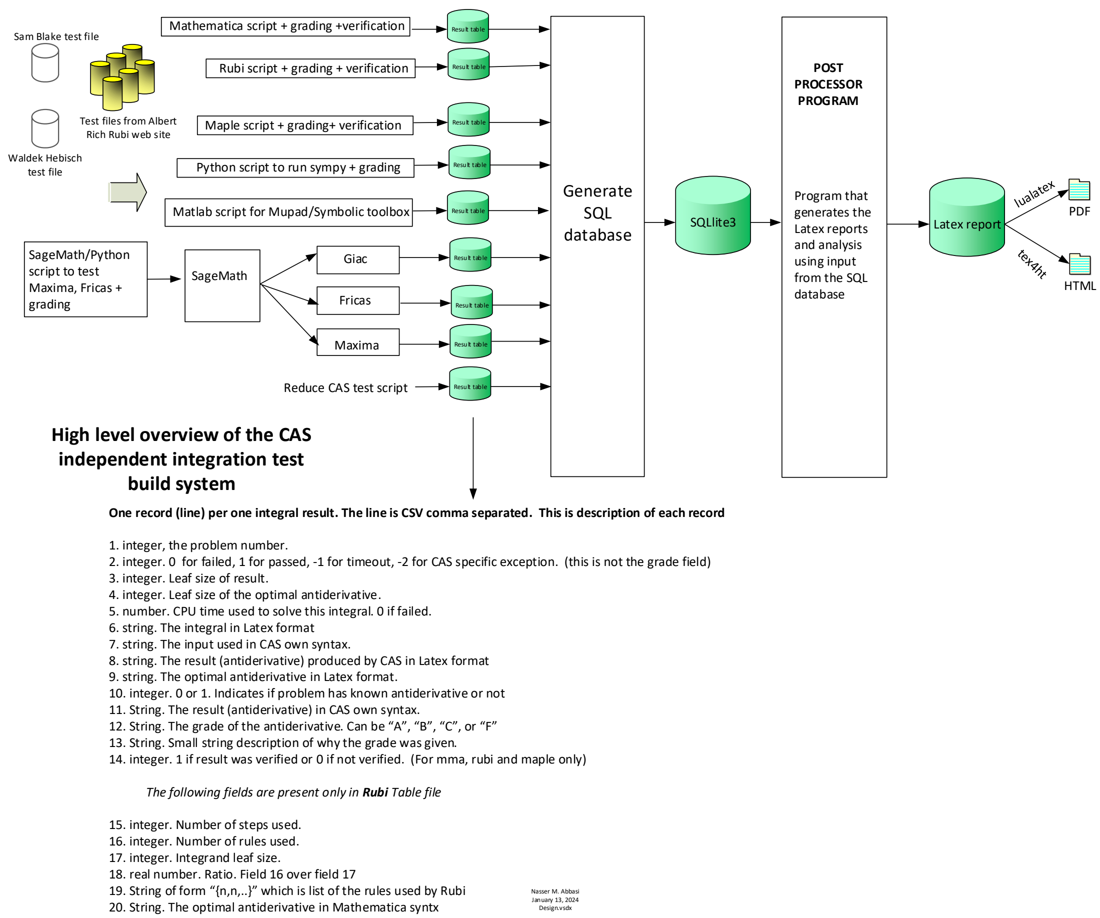


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 625, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701 }

B grade { }

C grade { 604, 624, 626 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 498, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 616, 617, 618, 619, 620, 621, 622, 623, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 661, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696 }

B grade { 39, 47, 54, 63, 64, 72, 73, 108, 160, 210, 219, 224, 235, 241, 284, 417, 429, 440, 441, 448, 449, 450, 628, 646, 648 }

C grade { 175, 281, 495, 496, 497, 499, 500, 501, 502, 503, 504, 564, 568, 569, 570, 584, 585, 604, 613, 614, 615, 624, 625, 626, 627, 629, 630, 659, 660, 662, 663, 664, 665, 666, 667, 697, 698, 699, 700, 701 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 160, 168, 169, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 361, 366, 371, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 438, 439, 442, 443, 444, 445, 446, 448, 449, 450, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 498, 499, 502, 507, 512, 517, 522, 527, 532, 535, 536, 537, 538, 539, 544, 545, 546, 547, 548, 553, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 567, 571, 572, 573, 574, 575, 580, 581, 582, 584, 585, 587, 588, 605, 606, 607, 620, 621, 624, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 647, 649, 650, 651, 653, 654, 655, 656, 657, 658, 676, 696 }

B grade { 27, 39, 47, 54, 63, 64, 72, 73, 100, 108, 116, 147, 148, 149, 150, 210, 219, 224, 235, 241, 387, 398, 403, 409, 410, 417, 428, 429, 430, 440, 441, 447, 451, 452, 554, 562, 583, 586, 645, 646, 648 }

C grade { 175, 282, 496, 497, 500, 501, 503, 504, 568, 569, 570, 652 }

F normal fail { 135, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 284, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 505, 506, 508, 509, 510, 511, 513, 514, 515, 516, 518, 519, 520, 521, 523, 524, 525, 526, 528, 529, 530, 531, 533, 534, 540, 541, 542, 543, 549, 550, 551, 552, 576, 577, 578, 579, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 622, 623, 625, 626, 627, 628, 629, 630, 631, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 697, 698, 699, 700, 701 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 149, 150, 156, 157, 160, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 325, 326, 327, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 356, 361, 366, 371, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 438, 439, 442, 443, 444, 445, 446, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 502, 503, 507, 512, 517, 522, 527, 532, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 571, 572, 575, 580, 581, 582, 583, 584, 585, 587, 588, 589, 605, 606, 607, 617, 618, 619, 620, 621, 625, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 649, 650, 651, 653, 654, 655, 656, 657, 658, 659, 661, 662, 663, 665, 666, 673, 676, 677, 678, 679, 686, 696, 697, 698, 699, 700, 701 }

B grade { 39, 47, 54, 63, 64, 72, 73, 100, 101, 102, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 143, 147, 148, 154, 155, 210, 219, 224, 235, 241, 260, 266, 267, 268, 269, 279, 323, 324, 328, 329, 330, 331, 332, 352, 353, 354, 355, 357, 358, 398, 409, 410, 417, 428, 429, 430, 440, 441, 447, 448, 449, 450, 451, 452, 564, 569, 570, 573, 574, 586, 590, 646, 648, 652 }

C grade { 497, 501, 504, 660, 664, 667 }

F normal fail { 151, 152, 158, 159, 171, 172, 173, 174, 175, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 576, 577, 578, 579, 631, 668, 669, 670, 671, 672, 674, 675, 685, 687, 688, 689, 690, 691, 692, 693, 694, 695 }

F(-1) timedout fail { 161, 162, 169, 170, 284, 335, 336, 337 }

F(-2) exception fail { 153, 505, 506, 508, 509, 510, 511, 513, 514, 515, 516, 518, 519, 520, 521, 523, 524, 525, 526, 528, 529, 530, 531, 533, 534, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 608, 609, 610, 611, 612, 613, 614, 615, 616, 622, 623, 624, 626, 627, 628, 629, 630, 680, 681, 682, 683, 684 }

Maxima

A grade { 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 180, 181, 182, 183, 184, 189, 190, 191, 192, 193, 198, 199, 200, 201, 202, 206, 207, 208, 209, 211, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 345, 346, 347, 348, 349, 352, 353, 354, 355, 356, 361, 366, 371, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 490, 491, 492, 493, 494, 498, 507, 512, 517, 522, 527, 532, 535, 536, 537, 538, 539, 544, 545, 546, 547, 548, 553, 554, 555, 556, 557, 558, 559, 560, 561, 565, 566, 567, 571, 572, 573, 574, 575, 580, 581, 582, 586, 587, 588, 589, 590, 604, 620, 621, 624, 625, 626, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 647, 649, 650, 652, 653, 654, 655, 656, 657, 658, 661, 673, 676, 686 }

B grade { 1, 2, 3, 4, 10, 11, 12, 13, 20, 21, 22, 31, 32, 39, 54, 72, 73, 100, 108, 116, 176, 177, 178, 179, 185, 186, 187, 188, 194, 195, 196, 197, 203, 204, 205, 210, 215, 224, 241, 281, 291, 292, 293, 301, 302, 310, 311, 403, 409, 417, 428, 429, 440, 441, 646, 648, 651 }

C grade { }

F normal fail { 151, 152, 153, 158, 159, 171, 172, 173, 174, 175, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 495, 496, 497, 499, 500, 501, 502, 503, 504, 505, 506, 508, 509, 510, 511, 513, 514, 515, 516, 518, 519, 520, 521, 523, 524, 525, 526, 528, 529, 530, 531, 533, 534, 540, 541, 542, 543, 549, 550, 551, 552, 562, 563, 564, 568, 569, 570, 576, 577, 578, 579, 583, 584, 585, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 622, 623, 627, 628, 629, 630, 631, 645, 659, 660, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701 }

F(-1) timedout fail { }

F(-2) exception fail { 343, 344, 350, 351, 357, 358, 486 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 166, 167, 168, 169, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 348, 377, 379, 380, 381, 382, 383, 387, 389, 390, 391, 392, 393, 394, 400, 401, 402, 403, 404, 405, 406, 412, 413, 414, 415, 416, 418, 419, 420, 421, 432, 433, 434, 435, 436, 437, 438, 439, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 457, 466, 475, 483, 491, 538, 547, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 580, 581, 582, 587, 588, 589, 590, 620, 621, 632, 633, 634, 635, 636, 637, 641, 650 }

B grade { 39, 47, 54, 63, 64, 72, 73, 108, 124, 125, 135, 139, 154, 155, 156, 157, 160, 210, 219, 224, 235, 241, 345, 346, 347, 352, 353, 354, 355, 398, 410, 417, 430, 440, 441, 447, 573, 574, 575, 586, 608, 646, 648, 652, 653, 654, 655 }

C grade { }

F normal fail { 151, 152, 153, 158, 159, 161, 162, 163, 164, 165, 171, 172, 173, 174, 175, 284, 342, 343, 344, 349, 350, 351, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 384, 385, 386, 388, 395, 396, 397, 399, 407, 408, 409, 411, 422, 423, 424, 425, 426, 427, 428, 429, 431, 453, 454, 455, 456, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 545, 546, 548, 549, 550, 551, 552, 553, 576, 577, 578, 579, 583, 584, 585, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 638, 639, 640, 642, 643, 644, 645, 647, 649, 651, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701 }

F(-1) timedout fail { }

F(-2) exception fail { 687, 688, 689 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 160, 162, 166, 167, 168, 169, 170, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 365, 366, 370, 371, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 457, 458, 466, 467, 475, 476, 483, 484, 490, 491, 492, 506, 511, 516, 521, 526, 531, 538, 547, 554, 555, 556, 557, 558, 559, 560, 561, 565, 566, 567, 571, 572, 573, 574, 575, 580, 581, 582, 586, 587, 588, 589, 590, 609, 617, 618, 619, 620, 621, 625, 632, 633, 634, 635, 636, 641, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 676, 681, 690 }

C grade { }

F normal fail { }

F(-1) timedout fail { 135, 144, 151, 152, 153, 158, 159, 161, 163, 164, 165, 171, 172, 173, 174, 175, 359, 362, 363, 364, 367, 368, 369, 372, 373, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 468, 469, 470, 471, 472, 473, 474, 477, 478, 479, 480, 481, 482, 485, 486, 487, 488, 489, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 527, 528, 529, 530, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 543, 544, 545, 546, 548, 549, 550, 551, 552, 553, 562, 563, 564, 568, 569, 570, 576, 577, 578, 579, 583, 584, 585, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 610, 611, 612, 613, 614, 615, 616, 622, 623, 624, 626, 627, 628, 629, 630, 631, 637, 638, 639, 640, 642, 643, 644, 645, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 677, 678, 679, 680, 682, 683, 684, 685, 686, 687, 688, 689, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 122, 127, 128, 132, 133, 134, 137, 138, 140, 141, 142, 143, 147, 148, 149, 150, 163, 166, 167, 168, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 249, 250, 252, 253, 254, 255, 273, 274, 275, 276, 277, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 334, 337, 341, 342, 349, 356, 374, 375, 376, 378, 379, 384, 385, 386, 388, 389, 390, 392, 393, 394, 395, 396, 399, 400, 401, 402, 405, 406, 407, 411, 412, 413, 414, 415, 416, 420, 421, 422, 423, 424, 425, 431, 432, 433, 434, 435, 436, 437, 438, 439, 445, 446, 453, 454, 455, 461, 462, 463, 464, 470, 471, 472, 473, 476, 477, 478, 479, 495, 499, 502, 512, 517, 522, 540, 541, 542, 543, 548, 549, 550, 551, 552, 559, 560, 561, 562, 565, 566, 567, 580, 581, 582, 605, 606, 607, 608, 609, 610, 638, 645, 647, 650, 661, 662, 665, 673, 677, 678, 679, 680, 681, 682, 686, 696 }

B grade { 27, 35, 39, 47, 54, 63, 64, 72, 73, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 129, 130, 131, 135, 136, 139, 144, 145, 156, 157, 160, 164, 165, 210, 219, 224, 235, 241, 251, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 278, 279, 281, 298, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 338, 339, 340, 343, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 357, 358, 361, 366, 371, 377, 380, 381, 382, 383, 387, 391, 397, 398, 403, 404, 408, 409, 410, 417, 418, 419, 426, 427, 428, 429, 430, 440, 441, 442, 443, 444, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 465, 466, 467, 468, 469, 474, 475, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 498, 507, 527, 532, 535, 536, 537, 538, 539, 544, 545, 546, 547, 554, 555, 556, 557, 558, 563, 564, 571, 572, 573, 574, 575, 583, 586, 587, 588, 589, 590, 611, 612, 620, 621, 632, 633, 634, 635, 636, 637, 639, 640, 641, 642, 643, 646, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 676, 683, 684, 697, 698, 699, 700, 701 }

C grade { 146, 151, 152, 153, 158, 159, 161, 162, 169, 170, 171, 172, 173, 174, 280, 282, 284, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 496, 497, 500, 501, 503, 504, 505, 506, 508, 509, 510, 511, 513, 514, 515, 516, 518, 519, 520, 521, 523, 524, 525, 526, 528, 529, 530, 531, 533, 534, 553, 568, 569, 570, 576, 577, 578, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 613, 614, 615, 616, 617, 618, 619, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 659, 660, 663, 664, 666, 667, 668, 669, 670, 671, 672, 674, 675, 685, 687, 688, 689, 690, 691, 692, 693, 694, 695 }

F normal fail { }

F(-1) timedout fail { 155, 247, 248, 256, 257, 336, 579, 584, 585, 644 }

F(-2) exception fail { 154 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 160, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 361, 366, 371, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 498, 535, 536, 537, 538, 544, 545, 546, 547, 554, 555, 556, 557, 558, 559, 560, 561, 565, 566, 567, 571, 572, 573, 574, 575, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 604, 609, 610, 617, 618, 619, 620, 621, 624, 625, 626, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 661, 673, 676, 681, 682, 686, 696, 697, 698, 699, 700, 701 }

C grade { }

F normal fail { 151, 152, 153, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 281, 284, 359, 360, 362, 363, 364, 365, 367, 368, 369, 370, 372, 373, 495, 496, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 539, 540, 541, 542, 543, 548, 549, 550, 551, 552, 553, 562, 563, 564, 568, 569, 570, 576, 577, 578, 579, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 611, 612, 613, 614, 615, 616, 622, 623, 627, 628, 629, 630, 631, 659, 660, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 677, 678, 679, 680, 683, 684, 685, 687, 688, 689, 690, 691, 692, 693, 694, 695 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	166	13	15	13	14	13
N.S.	1	1.00	1.00	0.74	8.74	0.68	0.79	0.68	0.74	0.68
time (sec)	N/A	0.241	0.012	0.107	0.031	0.068	0.556	0.119	0.209	0.030

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	132	13	15	13	14	13
N.S.	1	1.00	1.00	0.74	6.95	0.68	0.79	0.68	0.74	0.68
time (sec)	N/A	0.246	0.011	0.095	0.040	0.070	0.530	0.123	0.204	0.028

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	98	13	15	13	14	13
N.S.	1	1.00	1.00	0.74	5.16	0.68	0.79	0.68	0.74	0.68
time (sec)	N/A	0.243	0.011	0.095	0.029	0.078	0.547	0.116	0.203	0.028

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	64	13	15	13	14	13
N.S.	1	1.00	1.00	0.74	3.37	0.68	0.79	0.68	0.74	0.68
time (sec)	N/A	0.243	0.011	0.098	0.031	0.071	0.549	0.112	0.268	0.030

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	12	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.86	0.71
time (sec)	N/A	0.231	0.001	0.077	0.030	0.067	0.057	0.113	0.236	0.028

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	12	12	10	11
N.S.	1	1.00	1.00	0.92	0.85	1.08	0.92	0.92	0.77	0.85
time (sec)	N/A	0.247	0.013	0.107	0.032	0.071	0.089	0.120	0.224	0.315

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	16	14	13	13	12	13	18	13
N.S.	1	1.00	1.07	0.93	0.87	0.87	0.80	0.87	1.20	0.87
time (sec)	N/A	0.252	0.012	0.101	0.031	0.074	0.216	0.121	0.246	0.040

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	19	13
N.S.	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	1.00	0.68
time (sec)	N/A	0.261	0.016	0.105	0.029	0.076	0.220	0.117	0.230	0.030

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	19	15
N.S.	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	1.00	0.79
time (sec)	N/A	0.253	0.015	0.117	0.043	0.072	0.436	0.120	0.225	0.030

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	28	25	166	24	27	24	23	24
N.S.	1	1.06	0.88	0.78	5.19	0.75	0.84	0.75	0.72	0.75
time (sec)	N/A	0.301	0.019	0.570	0.028	0.072	0.261	0.110	0.207	0.044

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	28	25	132	24	27	24	23	24
N.S.	1	1.06	0.88	0.78	4.12	0.75	0.84	0.75	0.72	0.75
time (sec)	N/A	0.302	0.023	0.558	0.028	0.071	0.196	0.120	0.248	0.038

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	28	25	98	24	27	24	23	24
N.S.	1	1.06	0.88	0.78	3.06	0.75	0.84	0.75	0.72	0.75
time (sec)	N/A	0.293	0.018	0.563	0.031	0.066	0.154	0.120	0.201	0.277

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	28	25	64	24	27	24	23	24
N.S.	1	1.06	0.88	0.78	2.00	0.75	0.84	0.75	0.72	0.75
time (sec)	N/A	0.301	0.014	0.554	0.035	0.071	0.253	0.111	0.233	0.038

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	32	26	22	21	21	24	21	21	21
N.S.	1	1.19	0.96	0.81	0.78	0.78	0.89	0.78	0.78	0.78
time (sec)	N/A	0.280	0.002	0.566	0.031	0.082	0.084	0.118	0.220	0.035

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	30	21	20	19	22	20	20	18	22
N.S.	1	1.43	1.00	0.95	0.90	1.05	0.95	0.95	0.86	1.05
time (sec)	N/A	0.266	0.015	0.542	0.031	0.125	0.120	0.129	0.221	0.037

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	32	23	23	23	27	20	24	30	26
N.S.	1	1.33	0.96	0.96	0.96	1.12	0.83	1.00	1.25	1.08
time (sec)	N/A	0.276	0.022	0.630	0.025	0.067	0.279	0.117	0.221	0.274

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	28	25	24	24	26	24	30	24
N.S.	1	1.13	0.93	0.83	0.80	0.80	0.87	0.80	1.00	0.80
time (sec)	N/A	0.286	0.021	0.557	0.030	0.067	0.262	0.124	0.239	0.031

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	28	25	24	24	29	24	30	24
N.S.	1	1.06	0.88	0.78	0.75	0.75	0.91	0.75	0.94	0.75
time (sec)	N/A	0.281	0.018	0.572	0.031	0.073	0.281	0.121	0.197	0.029

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	28	25	24	24	29	24	30	24
N.S.	1	1.06	0.88	0.78	0.75	0.75	0.91	0.75	0.94	0.75
time (sec)	N/A	0.288	0.021	0.552	0.027	0.072	0.379	0.122	0.202	0.031

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	166	41	42	35	34	35
N.S.	1	1.04	0.87	0.77	3.53	0.87	0.89	0.74	0.72	0.74
time (sec)	N/A	0.315	0.021	0.641	0.029	0.065	0.559	0.119	0.241	0.047

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	43	36	132	41	44	35	34	35
N.S.	1	1.04	0.91	0.77	2.81	0.87	0.94	0.74	0.72	0.74
time (sec)	N/A	0.306	0.016	0.646	0.027	0.068	0.554	0.116	0.241	0.047

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	43	36	98	41	44	35	34	35
N.S.	1	1.04	0.91	0.77	2.09	0.87	0.94	0.74	0.72	0.74
time (sec)	N/A	0.309	0.016	0.636	0.035	0.076	0.504	0.116	0.193	0.046

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	49	43	35	64	40	41	34	34	34
N.S.	1	1.11	0.98	0.80	1.45	0.91	0.93	0.77	0.77	0.77
time (sec)	N/A	0.304	0.014	0.612	0.026	0.076	0.651	0.127	0.191	0.049

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	41	33	32	35	39	32	32	32
N.S.	1	1.05	1.08	0.87	0.84	0.92	1.03	0.84	0.84	0.84
time (sec)	N/A	0.282	0.003	0.613	0.026	0.076	0.386	0.110	0.197	0.049

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	45	37	32	31	34	37	32	30	34
N.S.	1	1.22	1.00	0.86	0.84	0.92	1.00	0.86	0.81	0.92
time (sec)	N/A	0.287	0.021	0.658	0.033	0.082	0.137	0.120	0.192	0.042

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	45	39	35	35	38	36	36	42	37
N.S.	1	1.18	1.03	0.92	0.92	1.00	0.95	0.95	1.11	0.97
time (sec)	N/A	0.297	0.029	0.701	0.026	0.089	0.227	0.122	0.208	0.287

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	41	36	33	32	39	33	41	33
N.S.	1	1.00	1.95	1.71	1.57	1.52	1.86	1.57	1.95	1.57
time (sec)	N/A	0.235	0.022	0.648	0.026	0.071	0.323	0.119	0.213	0.036

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	35	36	46	35	41	35
N.S.	1	1.04	0.87	0.77	0.74	0.77	0.98	0.74	0.87	0.74
time (sec)	N/A	0.301	0.021	0.643	0.026	0.070	0.400	0.120	0.215	0.286

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	41	36	35	36	42	35	41	35
N.S.	1	1.09	0.91	0.80	0.78	0.80	0.93	0.78	0.91	0.78
time (sec)	N/A	0.300	0.023	0.710	0.025	0.068	0.465	0.118	0.268	0.035

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	35	36	46	35	41	35
N.S.	1	1.04	0.87	0.77	0.74	0.77	0.98	0.74	0.87	0.74
time (sec)	N/A	0.302	0.022	0.725	0.031	0.071	0.450	0.118	0.210	0.288

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	67	58	166	63	73	57	58	57
N.S.	1	1.03	0.89	0.77	2.21	0.84	0.97	0.76	0.77	0.76
time (sec)	N/A	0.352	0.020	1.026	0.026	0.063	0.690	0.122	0.203	0.029

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	67	58	132	63	71	57	58	57
N.S.	1	1.01	0.92	0.79	1.81	0.86	0.97	0.78	0.79	0.78
time (sec)	N/A	0.349	0.021	0.947	0.031	0.069	0.648	0.121	0.199	0.027

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	77	67	57	98	62	70	56	58	56
N.S.	1	1.07	0.93	0.79	1.36	0.86	0.97	0.78	0.81	0.78
time (sec)	N/A	0.350	0.021	0.931	0.031	0.068	0.666	0.119	0.243	0.029

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	67	58	64	63	71	57	58	57
N.S.	1	1.02	0.84	0.72	0.80	0.79	0.89	0.71	0.72	0.71
time (sec)	N/A	0.352	0.020	0.918	0.037	0.070	0.527	0.121	0.228	0.028

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	65	55	54	58	66	54	56	54
N.S.	1	1.05	1.71	1.45	1.42	1.53	1.74	1.42	1.47	1.42
time (sec)	N/A	0.273	0.004	0.902	0.025	0.068	0.514	0.123	0.244	0.026

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	71	67	54	53	57	66	54	54	56
N.S.	1	1.09	1.03	0.83	0.82	0.88	1.02	0.83	0.83	0.86
time (sec)	N/A	0.324	0.023	1.006	0.032	0.077	0.170	0.124	0.218	0.033

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	62	55	55	61	61	56	67	57
N.S.	1	1.16	1.00	0.89	0.89	0.98	0.98	0.90	1.08	0.92
time (sec)	N/A	0.351	0.039	1.246	0.034	0.078	0.279	0.111	0.238	0.035

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	69	57	57	62	65	58	67	59
N.S.	1	1.08	1.05	0.86	0.86	0.94	0.98	0.88	1.02	0.89
time (sec)	N/A	0.351	0.040	0.961	0.027	0.084	0.286	0.121	0.211	0.316

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	65	58	55	56	66	55	65	55
N.S.	1	1.00	3.10	2.76	2.62	2.67	3.14	2.62	3.10	2.62
time (sec)	N/A	0.229	0.028	1.033	0.030	0.079	0.415	0.123	0.221	0.307

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	78	65	58	57	58	73	57	65	57
N.S.	1	1.11	0.93	0.83	0.81	0.83	1.04	0.81	0.93	0.81
time (sec)	N/A	0.285	0.032	0.977	0.026	0.077	0.443	0.114	0.218	0.318

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	65	58	57	58	75	57	65	57
N.S.	1	1.03	0.87	0.77	0.76	0.77	1.00	0.76	0.87	0.76
time (sec)	N/A	0.348	0.028	1.026	0.027	0.079	0.471	0.123	0.270	0.047

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	65	58	57	58	73	57	65	57
N.S.	1	1.03	0.89	0.79	0.78	0.79	1.00	0.78	0.89	0.78
time (sec)	N/A	0.331	0.034	0.979	0.035	0.076	0.803	0.129	0.229	0.050

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	144	126	113	166	118	139	112	117	112
N.S.	1	1.03	0.90	0.81	1.19	0.84	0.99	0.80	0.84	0.80
time (sec)	N/A	0.472	0.030	3.744	0.026	0.074	0.607	0.126	0.212	0.065

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	166	126	113	132	118	136	112	117	112
N.S.	1	1.02	0.78	0.70	0.81	0.73	0.84	0.69	0.72	0.69
time (sec)	N/A	0.469	0.029	3.757	0.038	0.071	0.471	0.117	0.205	0.062

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	126	113	98	118	139	112	117	112
N.S.	1	1.02	1.03	0.93	0.80	0.97	1.14	0.92	0.96	0.92
time (sec)	N/A	0.410	0.028	3.737	0.027	0.069	0.409	0.118	0.213	0.061

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	126	113	64	118	136	112	117	112
N.S.	1	1.02	1.58	1.41	0.80	1.48	1.70	1.40	1.46	1.40
time (sec)	N/A	0.359	0.025	3.747	0.028	0.085	0.314	0.122	0.201	0.060

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	124	110	30	113	133	109	115	109
N.S.	1	1.05	3.26	2.89	0.79	2.97	3.50	2.87	3.03	2.87
time (sec)	N/A	0.281	0.024	3.490	0.028	0.079	0.252	0.114	0.225	0.060

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	138	128	109	108	112	131	109	113	111
N.S.	1	1.08	1.00	0.85	0.84	0.88	1.02	0.85	0.88	0.87
time (sec)	N/A	0.422	0.033	3.767	0.032	0.098	0.325	0.130	0.226	0.065

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	135	123	110	110	117	124	111	128	112
N.S.	1	1.10	1.00	0.89	0.89	0.95	1.01	0.90	1.04	0.91
time (sec)	N/A	0.437	0.050	3.740	0.029	0.071	0.510	0.120	0.202	0.061

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	135	128	110	110	117	128	111	128	112
N.S.	1	1.06	1.01	0.87	0.87	0.92	1.01	0.87	1.01	0.88
time (sec)	N/A	0.431	0.056	3.805	0.027	0.076	0.590	0.125	0.238	0.051

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	135	124	110	110	117	128	111	128	112
N.S.	1	1.06	0.98	0.87	0.87	0.92	1.01	0.87	1.01	0.88
time (sec)	N/A	0.426	0.067	3.763	0.026	0.080	0.461	0.122	0.198	0.293

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	135	124	109	109	117	124	110	128	111
N.S.	1	1.11	1.02	0.89	0.89	0.96	1.02	0.90	1.05	0.91
time (sec)	N/A	0.441	0.082	3.757	0.036	0.080	0.404	0.121	0.219	0.058

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	140	128	111	111	117	131	112	128	114
N.S.	1	1.08	0.98	0.85	0.85	0.90	1.01	0.86	0.98	0.88
time (sec)	N/A	0.434	0.087	3.844	0.032	0.074	0.593	0.125	0.223	0.350

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	124	113	112	113	134	112	126	111
N.S.	1	1.04	2.70	2.46	2.43	2.46	2.91	2.43	2.74	2.41
time (sec)	N/A	0.258	0.045	3.778	0.026	0.073	0.648	0.122	0.250	0.345

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	110	124	113	112	113	138	112	126	112
N.S.	1	1.15	1.29	1.18	1.17	1.18	1.44	1.17	1.31	1.17
time (sec)	N/A	0.307	0.047	3.819	0.034	0.073	0.707	0.129	0.236	0.347

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	172	124	113	112	113	141	112	126	112
N.S.	1	1.18	0.85	0.77	0.77	0.77	0.97	0.77	0.86	0.77
time (sec)	N/A	0.357	0.057	3.877	0.030	0.066	0.871	0.131	0.207	0.353

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	124	113	112	113	138	112	126	112
N.S.	1	1.04	0.91	0.83	0.82	0.83	1.01	0.82	0.93	0.82
time (sec)	N/A	0.430	0.057	3.732	0.027	0.077	1.205	0.122	0.198	0.097

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	144	124	113	112	113	141	112	126	112
N.S.	1	1.03	0.89	0.81	0.80	0.81	1.01	0.80	0.90	0.80
time (sec)	N/A	0.431	0.055	3.851	0.032	0.069	1.177	0.124	0.202	0.352

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	246	187	168	200	173	214	167	178	167
N.S.	1	1.02	0.77	0.69	0.83	0.71	0.88	0.69	0.74	0.69
time (sec)	N/A	0.648	0.041	23.036	0.032	0.065	1.765	0.121	0.230	0.165

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	204	187	168	166	173	211	167	178	167
N.S.	1	1.01	0.93	0.83	0.82	0.86	1.04	0.83	0.88	0.83
time (sec)	N/A	0.552	0.038	23.044	0.031	0.073	1.728	0.116	0.253	0.157

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	163	187	168	132	173	209	167	178	167
N.S.	1	1.01	1.15	1.04	0.81	1.07	1.29	1.03	1.10	1.03
time (sec)	N/A	0.498	0.036	23.846	0.033	0.070	1.262	0.125	0.214	0.158

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	187	168	98	173	212	167	178	167
N.S.	1	1.02	1.53	1.38	0.80	1.42	1.74	1.37	1.46	1.37
time (sec)	N/A	0.439	0.038	22.957	0.030	0.067	1.196	0.124	0.210	0.157

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	187	168	64	173	204	167	178	167
N.S.	1	1.02	2.34	2.10	0.80	2.16	2.55	2.09	2.22	2.09
time (sec)	N/A	0.382	0.033	23.131	0.025	0.076	1.217	0.124	0.200	0.157

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	185	165	30	167	197	164	176	164
N.S.	1	1.05	4.87	4.34	0.79	4.39	5.18	4.32	4.63	4.32
time (sec)	N/A	0.285	0.031	23.049	0.027	0.076	0.987	0.125	0.208	0.158

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	213	189	164	163	167	211	164	174	166
N.S.	1	1.04	0.92	0.80	0.80	0.81	1.03	0.80	0.85	0.81
time (sec)	N/A	0.540	0.047	23.378	0.028	0.071	0.662	0.123	0.234	0.172

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	206	191	165	165	172	197	166	187	167
N.S.	1	1.07	0.99	0.86	0.86	0.90	1.03	0.86	0.97	0.87
time (sec)	N/A	0.554	0.070	23.839	0.030	0.077	0.825	0.132	0.232	0.412

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	204	189	165	163	172	196	164	187	167
N.S.	1	1.07	0.99	0.87	0.86	0.91	1.03	0.86	0.98	0.88
time (sec)	N/A	0.562	0.088	24.197	0.032	0.075	0.746	0.124	0.211	0.370

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	206	189	165	165	171	201	166	187	167
N.S.	1	1.05	0.96	0.84	0.84	0.87	1.03	0.85	0.95	0.85
time (sec)	N/A	0.562	0.084	23.966	0.028	0.076	0.689	0.125	0.232	0.334

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	206	189	165	165	172	199	166	187	167
N.S.	1	1.06	0.97	0.85	0.85	0.89	1.03	0.86	0.96	0.86
time (sec)	N/A	0.566	0.104	23.368	0.029	0.078	0.693	0.125	0.212	0.330

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	208	189	165	165	172	201	166	187	167
N.S.	1	1.06	0.96	0.84	0.84	0.88	1.03	0.85	0.95	0.85
time (sec)	N/A	0.562	0.117	23.076	0.033	0.088	0.805	0.124	0.218	0.331

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	209	189	167	167	172	202	168	187	168
N.S.	1	1.06	0.95	0.84	0.84	0.87	1.02	0.85	0.94	0.85
time (sec)	N/A	0.598	0.119	23.139	0.033	0.081	0.930	0.120	0.231	0.455

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	185	168	165	164	197	165	185	167
N.S.	1	1.00	8.81	8.00	7.86	7.81	9.38	7.86	8.81	7.95
time (sec)	N/A	0.236	0.069	23.100	0.032	0.080	0.876	0.123	0.234	0.210

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	78	185	168	167	168	209	167	185	167
N.S.	1	1.11	2.64	2.40	2.39	2.40	2.99	2.39	2.64	2.39
time (sec)	N/A	0.295	0.064	23.734	0.051	0.086	1.297	0.132	0.242	0.485

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	140	185	168	167	168	211	167	185	167
N.S.	1	1.17	1.54	1.40	1.39	1.40	1.76	1.39	1.54	1.39
time (sec)	N/A	0.359	0.082	23.078	0.032	0.080	1.330	0.120	0.220	0.200

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	202	185	168	167	168	214	167	185	167
N.S.	1	1.19	1.09	0.99	0.98	0.99	1.26	0.98	1.09	0.98
time (sec)	N/A	0.401	0.078	23.220	0.032	0.076	1.583	0.128	0.205	0.475

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	264	185	168	167	168	214	167	185	167
N.S.	1	1.20	0.84	0.76	0.76	0.76	0.97	0.76	0.84	0.76
time (sec)	N/A	0.443	0.081	23.905	0.032	0.073	1.922	0.121	0.228	0.465

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	326	185	168	167	168	212	167	185	167
N.S.	1	1.21	0.69	0.62	0.62	0.62	0.79	0.62	0.69	0.62
time (sec)	N/A	0.497	0.089	23.628	0.031	0.073	2.362	0.124	0.245	0.483

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	215	185	168	167	168	216	167	185	167
N.S.	1	1.02	0.88	0.80	0.79	0.80	1.02	0.79	0.88	0.79
time (sec)	N/A	0.575	0.080	23.615	0.034	0.071	2.606	0.123	0.203	0.214

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	217	185	168	167	168	216	167	185	167
N.S.	1	1.03	0.88	0.80	0.79	0.80	1.02	0.79	0.88	0.79
time (sec)	N/A	0.568	0.081	23.725	0.035	0.074	2.984	0.126	0.207	0.489

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	215	185	168	167	168	212	167	185	167
N.S.	1	1.04	0.89	0.81	0.81	0.81	1.02	0.81	0.89	0.81
time (sec)	N/A	0.575	0.080	23.814	0.033	0.071	3.130	0.127	0.198	0.476

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	110	100	88	129	88	109	89	89	87
N.S.	1	1.03	0.93	0.82	1.21	0.82	1.02	0.83	0.83	0.81
time (sec)	N/A	0.419	0.038	0.451	0.044	0.077	0.351	0.118	0.195	0.043

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	82	76	66	95	66	82	67	65	65
N.S.	1	1.04	0.96	0.84	1.20	0.84	1.04	0.85	0.82	0.82
time (sec)	N/A	0.363	0.031	0.451	0.028	0.071	0.337	0.118	0.202	0.034

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	54	52	44	61	43	54	45	41	43
N.S.	1	1.06	1.02	0.86	1.20	0.84	1.06	0.88	0.80	0.84
time (sec)	N/A	0.322	0.024	0.447	0.031	0.070	0.348	0.112	0.250	0.041

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	27	22	27	24	20	23
N.S.	1	1.04	1.00	0.89	1.00	0.81	1.00	0.89	0.74	0.85
time (sec)	N/A	0.282	0.003	0.447	0.033	0.077	0.277	0.123	0.218	0.043

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	30	21	20	20	37	22	18	17
N.S.	1	1.27	1.36	0.95	0.91	0.91	1.68	1.00	0.82	0.77
time (sec)	N/A	0.246	0.019	0.459	0.033	0.078	0.335	0.119	0.207	0.052

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	54	44	44	43	43	68	48	41	39
N.S.	1	1.15	0.94	0.94	0.91	0.91	1.45	1.02	0.87	0.83
time (sec)	N/A	0.310	0.031	0.455	0.034	0.079	0.445	0.123	0.239	0.067

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	82	69	66	64	69	99	69	66	60
N.S.	1	1.09	0.92	0.88	0.85	0.92	1.32	0.92	0.88	0.80
time (sec)	N/A	0.342	0.053	0.448	0.032	0.085	0.694	0.122	0.205	0.071

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	110	93	88	86	92	126	91	90	82
N.S.	1	1.07	0.90	0.85	0.83	0.89	1.22	0.88	0.87	0.80
time (sec)	N/A	0.386	0.066	0.471	0.029	0.084	1.500	0.119	0.198	0.321

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	114	119	95	129	128	272	100	112	99
N.S.	1	1.03	1.07	0.86	1.16	1.15	2.45	0.90	1.01	0.89
time (sec)	N/A	0.431	0.059	0.467	0.030	0.077	0.548	0.121	0.229	0.048

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	95	73	95	106	212	78	88	77
N.S.	1	1.04	1.14	0.88	1.14	1.28	2.55	0.94	1.06	0.93
time (sec)	N/A	0.384	0.045	0.460	0.028	0.070	0.407	0.116	0.215	0.039

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	58	67	51	60	82	134	52	62	54
N.S.	1	1.07	1.24	0.94	1.11	1.52	2.48	0.96	1.15	1.00
time (sec)	N/A	0.332	0.037	0.450	0.031	0.076	0.298	0.130	0.221	0.045

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	30	29	50	80	30	39	29
N.S.	1	1.00	0.88	0.91	0.88	1.52	2.42	0.91	1.18	0.88
time (sec)	N/A	0.286	0.006	0.448	0.038	0.075	0.310	0.120	0.213	0.299

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	43	37	35	34	67	151	36	53	32
N.S.	1	1.13	0.97	0.92	0.89	1.76	3.97	0.95	1.39	0.84
time (sec)	N/A	0.300	0.040	0.477	0.032	0.079	0.421	0.127	0.205	0.058

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	74	64	62	63	105	238	67	87	57
N.S.	1	1.10	0.96	0.93	0.94	1.57	3.55	1.00	1.30	0.85
time (sec)	N/A	0.356	0.095	0.479	0.031	0.081	0.545	0.124	0.210	0.084

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	102	91	84	88	133	333	90	118	81
N.S.	1	1.07	0.96	0.88	0.93	1.40	3.51	0.95	1.24	0.85
time (sec)	N/A	0.397	0.092	0.451	0.033	0.089	0.884	0.120	0.202	0.345

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	130	115	106	110	155	400	112	142	103
N.S.	1	1.06	0.93	0.86	0.89	1.26	3.25	0.91	1.15	0.84
time (sec)	N/A	0.450	0.113	0.467	0.033	0.083	1.265	0.115	0.215	0.092

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	119	119	100	128	157	413	101	134	108
N.S.	1	1.04	1.04	0.88	1.12	1.38	3.62	0.89	1.18	0.95
time (sec)	N/A	0.446	0.066	0.458	0.031	0.078	0.637	0.122	0.244	0.303

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	95	78	94	135	333	79	110	86
N.S.	1	1.08	1.08	0.89	1.07	1.53	3.78	0.90	1.25	0.98
time (sec)	N/A	0.389	0.058	0.454	0.025	0.094	0.509	0.121	0.202	0.040

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	66	57	60	101	233	53	83	66
N.S.	1	1.06	1.03	0.89	0.94	1.58	3.64	0.83	1.30	1.03
time (sec)	N/A	0.341	0.067	0.509	0.026	0.084	0.424	0.125	0.210	0.334

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	27	30	29	47	63	22	22	34
N.S.	1	1.00	1.69	1.88	1.81	2.94	3.94	1.38	1.38	2.12
time (sec)	N/A	0.231	0.003	0.710	0.027	0.082	0.408	0.121	0.232	0.347

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	61	44	48	54	115	364	48	97	52
N.S.	1	1.15	0.83	0.91	1.02	2.17	6.87	0.91	1.83	0.98
time (sec)	N/A	0.329	0.057	0.536	0.033	0.078	0.668	0.125	0.205	0.360

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	96	77	78	85	155	481	74	137	79
N.S.	1	1.13	0.91	0.92	1.00	1.82	5.66	0.87	1.61	0.93
time (sec)	N/A	0.385	0.101	0.467	0.028	0.092	1.615	0.123	0.214	0.363

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	119	104	100	110	182	612	101	168	102
N.S.	1	1.07	0.94	0.90	0.99	1.64	5.51	0.91	1.51	0.92
time (sec)	N/A	0.429	0.120	0.467	0.035	0.087	2.165	0.122	0.213	0.123

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	152	128	122	132	207	707	123	192	125
N.S.	1	1.09	0.92	0.88	0.95	1.49	5.09	0.88	1.38	0.90
time (sec)	N/A	0.492	0.132	0.467	0.030	0.090	2.981	0.115	0.219	0.424

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	159	143	135	163	245	966	121	213	148
N.S.	1	1.03	0.92	0.87	1.05	1.58	6.23	0.78	1.37	0.95
time (sec)	N/A	0.544	0.073	0.482	0.027	0.104	1.512	0.125	0.248	0.333

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	137	119	113	129	223	835	99	189	126
N.S.	1	1.05	0.91	0.86	0.98	1.70	6.37	0.76	1.44	0.96
time (sec)	N/A	0.505	0.066	0.497	0.038	0.098	1.111	0.121	0.219	0.319

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	108	93	92	95	191	704	73	163	106
N.S.	1	1.01	0.87	0.86	0.89	1.79	6.58	0.68	1.52	0.99
time (sec)	N/A	0.405	0.063	0.474	0.026	0.133	0.836	0.124	0.212	0.384

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	52	65	64	108	270	42	47	77
N.S.	1	1.00	2.48	3.10	3.05	5.14	12.86	2.00	2.24	3.67
time (sec)	N/A	0.230	0.040	0.469	0.027	0.078	1.131	0.123	0.208	0.145

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	30	31	30	96	136	22	53	57
N.S.	1	1.05	0.79	0.82	0.79	2.53	3.58	0.58	1.39	1.50
time (sec)	N/A	0.292	0.004	0.438	0.026	0.110	1.156	0.122	0.211	0.437

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	71	76	97	227	1059	69	203	94
N.S.	1	1.09	0.80	0.85	1.09	2.55	11.90	0.78	2.28	1.06
time (sec)	N/A	0.380	0.082	0.452	0.036	0.094	1.607	0.128	0.210	0.094

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	133	104	113	130	267	1241	101	250	123
N.S.	1	1.06	0.83	0.90	1.03	2.12	9.85	0.80	1.98	0.98
time (sec)	N/A	0.467	0.126	0.454	0.054	0.093	2.702	0.127	0.228	0.147

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	166	128	135	154	295	1391	119	282	147
N.S.	1	1.06	0.82	0.87	0.99	1.89	8.92	0.76	1.81	0.94
time (sec)	N/A	0.518	0.112	0.454	0.036	0.119	4.134	0.130	0.211	0.521

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	205	167	175	197	370	2048	143	332	202
N.S.	1	1.01	0.82	0.86	0.97	1.82	10.09	0.70	1.64	1.00
time (sec)	N/A	0.642	0.096	0.439	0.026	0.176	3.214	0.119	0.197	0.109

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	179	143	153	162	348	1839	119	308	179
N.S.	1	1.04	0.83	0.89	0.94	2.02	10.69	0.69	1.79	1.04
time (sec)	N/A	0.568	0.079	0.460	0.029	0.132	2.646	0.123	0.266	0.099

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	160	104	132	131	315	1629	95	281	159
N.S.	1	1.02	0.66	0.84	0.83	2.01	10.38	0.61	1.79	1.01
time (sec)	N/A	0.502	0.070	0.446	0.029	0.125	2.175	0.119	0.203	0.444

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	45	76	99	98	188	619	64	134	130
N.S.	1	1.05	1.77	2.30	2.28	4.37	14.40	1.49	3.12	3.02
time (sec)	N/A	0.264	0.052	0.529	0.027	0.104	2.457	0.119	0.230	0.463

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	52	65	64	177	410	42	110	110
N.S.	1	1.05	0.67	0.83	0.82	2.27	5.26	0.54	1.41	1.41
time (sec)	N/A	0.362	0.041	0.466	0.033	0.100	2.842	0.120	0.226	0.459

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	30	31	30	164	199	22	90	90
N.S.	1	1.05	0.79	0.82	0.79	4.32	5.24	0.58	2.37	2.37
time (sec)	N/A	0.295	0.023	0.480	0.026	0.099	2.247	0.125	0.196	0.172

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	151	106	118	163	398	2581	102	381	160
N.S.	1	1.06	0.74	0.83	1.14	2.78	18.05	0.71	2.66	1.12
time (sec)	N/A	0.470	0.117	0.458	0.045	0.132	3.496	0.134	0.240	0.466

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	199	139	163	196	435	2854	134	430	189
N.S.	1	1.08	0.76	0.89	1.07	2.36	15.51	0.73	2.34	1.03
time (sec)	N/A	0.593	0.137	0.444	0.042	0.174	6.456	0.121	0.219	0.554

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	223	163	186	220	463	3077	156	462	213
N.S.	1	1.03	0.75	0.86	1.01	2.13	14.18	0.72	2.13	0.98
time (sec)	N/A	0.647	0.163	0.479	0.046	0.262	9.619	0.139	0.206	0.896

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	27	19	16	15	15	19	17	13	10
N.S.	1	1.42	1.00	0.84	0.79	0.79	1.00	0.89	0.68	0.53
time (sec)	N/A	0.239	0.013	0.491	0.027	0.075	0.156	0.123	0.231	0.058

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	134	78	85	98	77	8588	191	76	98
N.S.	1	1.02	0.59	0.64	0.74	0.58	65.06	1.45	0.58	0.74
time (sec)	N/A	0.406	0.049	0.485	0.032	0.077	5.055	0.123	0.248	1.178

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	90	65	58	64	55	1987	134	52	64
N.S.	1	1.02	0.74	0.66	0.73	0.62	22.58	1.52	0.59	0.73
time (sec)	N/A	0.338	0.036	0.458	0.026	0.078	1.796	0.127	0.212	0.418

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	40	30	30	32	272	76	29	37
N.S.	1	1.05	0.95	0.71	0.71	0.76	6.48	1.81	0.69	0.88
time (sec)	N/A	0.282	0.023	0.464	0.027	0.092	0.810	0.125	0.204	0.397

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	45	43	32	49	112	75	43	42	31
N.S.	1	1.05	1.00	0.74	1.14	2.60	1.74	1.00	0.98	0.72
time (sec)	N/A	0.273	0.034	0.447	0.110	0.092	1.030	0.125	0.208	1.241

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	68	60	100	157	105	65	73	54
N.S.	1	1.03	0.88	0.78	1.30	2.04	1.36	0.84	0.95	0.70
time (sec)	N/A	0.301	0.093	0.544	0.110	0.090	2.525	0.124	0.206	0.880

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	145	90	88	166	208	170	97	112	91
N.S.	1	1.09	0.68	0.66	1.25	1.56	1.28	0.73	0.84	0.68
time (sec)	N/A	0.361	0.138	0.438	0.110	0.104	23.047	0.137	0.266	0.694

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	132	78	86	98	67	8356	85	65	98
N.S.	1	1.02	0.60	0.66	0.75	0.52	64.28	0.65	0.50	0.75
time (sec)	N/A	0.387	0.048	0.447	0.038	0.085	5.717	0.125	0.217	0.708

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	54	57	64	45	1872	57	41	64
N.S.	1	1.02	0.64	0.68	0.76	0.54	22.29	0.68	0.49	0.76
time (sec)	N/A	0.332	0.034	0.440	0.033	0.076	1.704	0.123	0.229	0.445

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	31	30	30	23	219	27	20	37
N.S.	1	1.05	0.78	0.75	0.75	0.58	5.48	0.68	0.50	0.92
time (sec)	N/A	0.278	0.020	0.439	0.031	0.078	0.761	0.121	0.204	0.434

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	37	91	24	23	34	19
N.S.	1	1.00	1.00	0.74	1.37	3.37	0.89	0.85	1.26	0.70
time (sec)	N/A	0.259	0.025	0.454	0.103	0.089	0.680	0.122	0.253	0.821

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	85	68	72	104	161	110	75	74	57
N.S.	1	1.06	0.85	0.90	1.30	2.01	1.38	0.94	0.92	0.71
time (sec)	N/A	0.309	0.086	0.443	0.106	0.095	3.068	0.123	0.207	0.784

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	90	124	166	208	173	109	112	94
N.S.	1	1.11	0.66	0.91	1.22	1.53	1.27	0.80	0.82	0.69
time (sec)	N/A	0.364	0.116	0.445	0.114	0.084	42.525	0.129	0.225	0.741

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	71	98	2628	231	86	0
N.S.	1	1.00	0.86	0.00	0.96	1.32	35.51	3.12	1.16	0.00
time (sec)	N/A	0.345	0.088	0.000	0.039	0.110	2.436	0.128	0.206	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	25	182	21	24	21
N.S.	1	1.00	1.00	0.96	0.91	1.09	7.91	0.91	1.04	0.91
time (sec)	N/A	0.240	0.035	0.449	0.032	0.112	0.414	0.123	0.231	0.556

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	6	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.67	0.78
time (sec)	N/A	0.235	0.001	0.082	0.027	0.069	0.095	0.118	0.240	0.024

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	20	10	9	12	17	14	13	13
N.S.	1	1.00	1.54	0.77	0.69	0.92	1.31	1.08	1.00	1.00
time (sec)	N/A	0.222	0.002	0.504	0.030	0.080	0.074	0.125	0.229	0.035

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	26	10	9	17	20	19	18	19
N.S.	1	1.00	2.00	0.77	0.69	1.31	1.54	1.46	1.38	1.46
time (sec)	N/A	0.224	0.011	0.583	0.038	0.068	0.086	0.122	0.218	0.035

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	19	16	22	15	17	15	13	15
N.S.	1	1.21	1.00	0.84	1.16	0.79	0.89	0.79	0.68	0.79
time (sec)	N/A	0.264	0.014	0.430	0.040	0.077	0.078	0.120	0.246	0.054

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	7	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.70	0.80
time (sec)	N/A	0.218	0.010	0.429	0.032	0.082	0.078	0.119	0.205	0.056

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	12	8	9	10	9
N.S.	1	1.00	1.00	0.91	0.82	1.09	0.73	0.82	0.91	0.82
time (sec)	N/A	0.230	0.010	0.437	0.029	0.074	0.208	0.113	0.195	0.034

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	20	12	9	11	9
N.S.	1	1.00	1.00	0.91	0.82	1.82	1.09	0.82	1.00	0.82
time (sec)	N/A	0.224	0.011	0.511	0.030	0.070	0.302	0.117	0.236	0.330

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	34	29	28	23	398	28	21	0
N.S.	1	1.04	0.74	0.63	0.61	0.50	8.65	0.61	0.46	0.00
time (sec)	N/A	0.289	0.019	0.441	0.048	0.072	0.935	0.120	0.222	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	9	31	9	11	9
N.S.	1	1.00	1.00	0.67	0.60	0.60	2.07	0.60	0.73	0.60
time (sec)	N/A	0.215	0.013	0.442	0.030	0.075	0.111	0.119	0.221	0.949

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	67	65	49	48	50	138	48	47	78
N.S.	1	1.16	1.12	0.84	0.83	0.86	2.38	0.83	0.81	1.34
time (sec)	N/A	0.319	0.051	0.458	0.111	0.079	0.831	0.127	0.248	0.052

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	78	506	83	260	150	106	278	292
N.S.	1	1.00	0.90	5.82	0.95	2.99	1.72	1.22	3.20	3.36
time (sec)	N/A	0.385	0.123	0.727	0.033	0.087	1.494	0.131	0.206	0.860

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	318	66	167	121	84	168	184
N.S.	1	1.00	0.90	4.54	0.94	2.39	1.73	1.20	2.40	2.63
time (sec)	N/A	0.341	0.098	0.599	0.032	0.085	1.124	0.122	0.200	0.655

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	178	45	89	78	58	86	103
N.S.	1	1.00	0.89	3.79	0.96	1.89	1.66	1.23	1.83	2.19
time (sec)	N/A	0.309	0.089	0.505	0.038	0.096	0.937	0.122	0.226	0.524

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	73	28	37	49	36	36	39
N.S.	1	1.00	1.00	2.43	0.93	1.23	1.63	1.20	1.20	1.30
time (sec)	N/A	0.280	0.063	0.230	0.030	0.088	0.734	0.118	0.242	0.425

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	0	0	0	82	0	158	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	2.22	0.00	4.27	0.00
time (sec)	N/A	0.274	0.067	0.000	0.000	0.000	0.753	0.000	0.213	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	0	0	0	478	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	12.92	0.00	0.00	0.00
time (sec)	N/A	0.270	0.073	0.000	0.000	0.000	1.198	0.000	0.269	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	63	65	0	0	0	46	0	0	0
N.S.	1	1.21	1.25	0.00	0.00	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.302	0.064	0.000	0.000	0.000	21.667	0.000	0.279	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	204	205	168	0	286	458	0	1642	521	590
N.S.	1	1.00	0.82	0.00	1.40	2.25	0.00	8.05	2.55	2.89
time (sec)	N/A	0.546	0.179	0.000	0.043	0.132	0.000	0.131	0.200	1.397

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	153	126	0	185	281	0	922	301	355
N.S.	1	1.01	0.83	0.00	1.22	1.85	0.00	6.07	1.98	2.34
time (sec)	N/A	0.485	0.133	0.000	0.047	0.120	0.000	0.130	0.214	1.059

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	95	0	104	148	14683	410	141	180
N.S.	1	1.01	0.95	0.00	1.04	1.48	146.83	4.10	1.41	1.80
time (sec)	N/A	0.389	0.093	0.000	0.035	0.112	36.496	0.129	0.218	0.934

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	49	42	0	45	56	445	94	46	41
N.S.	1	1.02	0.88	0.00	0.94	1.17	9.27	1.96	0.96	0.85
time (sec)	N/A	0.309	0.010	0.000	0.036	0.134	1.090	0.124	0.230	0.439

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	41	0	80	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.95	0.00	1.86	0.00
time (sec)	N/A	0.269	0.063	0.000	0.000	0.000	1.094	0.000	0.253	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	42	0	178	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.91	0.00	3.87	0.00
time (sec)	N/A	0.274	0.068	0.000	0.000	0.000	3.685	0.000	0.215	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	28	9	8	8	24	20	18	8
N.S.	1	1.00	2.33	0.75	0.67	0.67	2.00	1.67	1.50	0.67
time (sec)	N/A	0.223	0.016	0.486	0.035	0.074	0.131	0.122	0.236	0.105

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	150	194	0	184	0	41	0	15	0
N.S.	1	1.08	1.40	0.00	1.32	0.00	0.29	0.00	0.11	0.00
time (sec)	N/A	0.400	0.372	0.000	0.111	0.000	2.207	0.000	0.213	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	138	0	100	0	39	0	11	37
N.S.	1	1.05	1.68	0.00	1.22	0.00	0.48	0.00	0.13	0.45
time (sec)	N/A	0.301	0.211	0.000	0.109	0.000	0.626	0.000	0.246	0.484

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	35	0	35	27	76	0	15	0
N.S.	1	1.00	0.70	0.00	0.70	0.54	1.52	0.00	0.30	0.00
time (sec)	N/A	0.277	0.158	0.000	0.029	0.259	1.526	0.000	0.212	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	116	59	0	69	53	736	0	15	0
N.S.	1	1.12	0.57	0.00	0.66	0.51	7.08	0.00	0.14	0.00
time (sec)	N/A	0.355	0.211	0.000	0.030	0.283	7.976	0.000	0.284	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	182	83	0	103	75	1554	0	15	0
N.S.	1	1.15	0.53	0.00	0.65	0.47	9.84	0.00	0.09	0.00
time (sec)	N/A	0.460	0.288	0.000	0.035	0.329	35.389	0.000	0.211	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	80	0	98	72	155	89	15	98
N.S.	1	1.03	0.62	0.00	0.75	0.55	1.19	0.68	0.12	0.75
time (sec)	N/A	0.390	0.063	0.000	0.026	0.312	12.958	0.131	0.232	0.675

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	90	56	0	64	50	102	61	15	64
N.S.	1	1.05	0.65	0.00	0.74	0.58	1.19	0.71	0.17	0.74
time (sec)	N/A	0.342	0.054	0.000	0.026	0.290	2.300	0.131	0.196	0.679

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	31	30	30	23	49	30	15	29
N.S.	1	1.10	0.78	0.75	0.75	0.58	1.22	0.75	0.38	0.72
time (sec)	N/A	0.285	0.042	0.434	0.026	0.269	0.356	0.141	0.245	0.663

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	92	109	85	86	0	41	87	15	102
N.S.	1	1.08	1.28	1.00	1.01	0.00	0.48	1.02	0.18	1.20
time (sec)	N/A	0.346	0.141	0.441	0.114	0.000	0.747	0.165	0.227	0.821

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-1)	C	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	156	155	0	158	0	42	142	15	205
N.S.	1	1.05	1.05	0.00	1.07	0.00	0.28	0.96	0.10	1.39
time (sec)	N/A	0.400	0.219	0.000	0.118	0.000	3.401	0.176	0.230	0.943

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	41	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.72	0.00	0.26	0.00
time (sec)	N/A	0.306	10.013	0.000	0.000	0.000	1.467	0.000	0.238	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	41	0	13	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.72	0.00	0.23	0.00
time (sec)	N/A	0.308	9.555	0.000	0.000	0.000	0.539	0.000	0.222	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	42	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	0.27	0.00
time (sec)	N/A	0.304	10.013	0.000	0.000	0.000	0.928	0.000	0.221	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	48	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.84	0.00	0.26	0.00
time (sec)	N/A	0.305	10.016	0.000	0.000	0.000	4.390	0.000	0.242	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	433	25	18	0	0	34	0	61	0
N.S.	1	1.04	0.06	0.04	0.00	0.00	0.08	0.00	0.15	0.00
time (sec)	N/A	0.746	10.013	0.467	0.000	0.000	0.562	0.000	0.400	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	251	13	15	13	15	13
N.S.	1	1.00	1.00	0.74	13.21	0.68	0.79	0.68	0.79	0.68
time (sec)	N/A	0.246	0.013	0.099	0.040	0.070	0.530	0.120	0.225	0.031

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	200	13	15	13	15	13
N.S.	1	1.00	1.00	0.74	10.53	0.68	0.79	0.68	0.79	0.68
time (sec)	N/A	0.255	0.012	0.098	0.030	0.066	0.427	0.127	0.206	0.027

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	149	13	15	13	15	13
N.S.	1	1.00	1.00	0.74	7.84	0.68	0.79	0.68	0.79	0.68
time (sec)	N/A	0.252	0.012	0.102	0.028	0.064	0.379	0.120	0.230	0.029

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	98	13	15	13	15	13
N.S.	1	1.00	1.00	0.74	5.16	0.68	0.79	0.68	0.79	0.68
time (sec)	N/A	0.250	0.011	0.099	0.034	0.070	0.315	0.122	0.246	0.030

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	13	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.93	0.71
time (sec)	N/A	0.235	0.001	0.074	0.028	0.062	0.027	0.121	0.217	0.029

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	12	12	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.08	0.92	0.92	0.85	0.85
time (sec)	N/A	0.251	0.014	0.109	0.026	0.065	0.101	0.122	0.201	0.354

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	14	15	15	14	15	16	13
N.S.	1	1.00	1.12	0.82	0.88	0.88	0.82	0.88	0.94	0.76
time (sec)	N/A	0.254	0.017	0.102	0.032	0.075	0.277	0.119	0.221	0.032

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	16	15
N.S.	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	0.84	0.79
time (sec)	N/A	0.248	0.017	0.099	0.032	0.082	0.376	0.119	0.213	0.034

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	15	17	15	16	15
N.S.	1	1.00	1.00	0.74	0.79	0.79	0.89	0.79	0.84	0.79
time (sec)	N/A	0.246	0.015	0.099	0.029	0.091	0.383	0.117	0.210	0.036

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	32	25	250	24	31	24	26	24
N.S.	1	1.06	0.94	0.74	7.35	0.71	0.91	0.71	0.76	0.71
time (sec)	N/A	0.298	0.017	0.509	0.033	0.091	0.539	0.126	0.246	0.343

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	32	25	200	24	31	24	26	24
N.S.	1	1.06	0.94	0.74	5.88	0.71	0.91	0.71	0.76	0.71
time (sec)	N/A	0.298	0.018	0.516	0.030	0.085	0.485	0.116	0.213	0.041

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	32	25	148	24	31	24	26	24
N.S.	1	1.06	0.94	0.74	4.35	0.71	0.91	0.71	0.76	0.71
time (sec)	N/A	0.293	0.016	0.791	0.028	0.082	0.416	0.120	0.211	0.044

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	32	25	98	24	31	24	26	24
N.S.	1	1.06	0.94	0.74	2.88	0.71	0.91	0.71	0.76	0.71
time (sec)	N/A	0.290	0.015	0.490	0.026	0.135	0.397	0.116	0.210	0.040

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	30	22	21	21	27	21	24	21
N.S.	1	1.17	1.03	0.76	0.72	0.72	0.93	0.72	0.83	0.72
time (sec)	N/A	0.285	0.002	0.480	0.044	0.070	0.299	0.124	0.232	0.043

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	34	28	23	22	25	27	23	22	25
N.S.	1	1.21	1.00	0.82	0.79	0.89	0.96	0.82	0.79	0.89
time (sec)	N/A	0.275	0.017	0.513	0.025	0.076	0.102	0.121	0.215	0.038

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	29	25	24	24	26	24	26	24
N.S.	1	1.00	1.53	1.32	1.26	1.26	1.37	1.26	1.37	1.26
time (sec)	N/A	0.236	0.018	0.507	0.031	0.065	0.303	0.125	0.241	0.038

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	32	25	26	26	32	26	27	26
N.S.	1	1.06	0.94	0.74	0.76	0.76	0.94	0.76	0.79	0.76
time (sec)	N/A	0.284	0.021	0.494	0.026	0.071	0.261	0.121	0.220	0.031

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	32	25	26	26	32	26	27	26
N.S.	1	1.06	0.94	0.74	0.76	0.76	0.94	0.76	0.79	0.76
time (sec)	N/A	0.284	0.019	0.512	0.034	0.063	0.358	0.125	0.244	0.031

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	43	36	251	35	42	35	35	35
N.S.	1	1.04	0.91	0.77	5.34	0.74	0.89	0.74	0.74	0.74
time (sec)	N/A	0.316	0.017	0.606	0.033	0.076	0.608	0.114	0.215	0.049

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	43	36	200	35	42	35	35	35
N.S.	1	1.04	0.91	0.77	4.26	0.74	0.89	0.74	0.74	0.74
time (sec)	N/A	0.311	0.017	0.624	0.032	0.085	0.508	0.119	0.200	0.048

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	149	35	42	35	35	35
N.S.	1	1.04	0.87	0.77	3.17	0.74	0.89	0.74	0.74	0.74
time (sec)	N/A	0.305	0.020	0.583	0.033	0.068	0.457	0.135	0.192	0.048

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	43	36	98	35	42	35	35	35
N.S.	1	1.04	0.91	0.77	2.09	0.74	0.89	0.74	0.74	0.74
time (sec)	N/A	0.300	0.015	0.590	0.037	0.064	0.389	0.121	0.259	0.046

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	41	33	32	32	39	32	33	32
N.S.	1	1.12	0.98	0.79	0.76	0.76	0.93	0.76	0.79	0.76
time (sec)	N/A	0.289	0.003	0.555	0.031	0.067	0.378	0.121	0.219	0.046

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	45	36	31	30	33	36	31	30	33
N.S.	1	1.25	1.00	0.86	0.83	0.92	1.00	0.86	0.83	0.92
time (sec)	N/A	0.280	0.019	0.576	0.027	0.071	0.138	0.116	0.209	0.042

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	47	40	34	36	39	36	37	42	37
N.S.	1	1.21	1.03	0.87	0.92	1.00	0.92	0.95	1.08	0.95
time (sec)	N/A	0.284	0.033	0.608	0.039	0.069	0.261	0.128	0.212	0.043

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	41	36	35	35	41	35	38	35
N.S.	1	1.09	0.91	0.80	0.78	0.78	0.91	0.78	0.84	0.78
time (sec)	N/A	0.286	0.024	0.619	0.033	0.067	0.299	0.125	0.241	0.330

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	35	35	44	35	38	35
N.S.	1	1.04	0.87	0.77	0.74	0.74	0.94	0.74	0.81	0.74
time (sec)	N/A	0.290	0.022	0.582	0.026	0.065	0.345	0.123	0.209	0.339

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	69	58	251	69	75	57	57	57
N.S.	1	1.03	0.90	0.75	3.26	0.90	0.97	0.74	0.74	0.74
time (sec)	N/A	0.350	0.028	0.961	0.033	0.066	0.708	0.121	0.229	0.031

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	69	58	200	69	73	57	57	57
N.S.	1	1.05	0.92	0.77	2.67	0.92	0.97	0.76	0.76	0.76
time (sec)	N/A	0.343	0.021	0.947	0.038	0.064	0.571	0.127	0.244	0.029

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	69	58	149	69	75	57	57	57
N.S.	1	1.03	0.90	0.75	1.94	0.90	0.97	0.74	0.74	0.74
time (sec)	N/A	0.338	0.020	0.928	0.033	0.064	0.539	0.119	0.232	0.029

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	69	58	98	69	75	57	57	57
N.S.	1	1.03	0.90	0.75	1.27	0.90	0.97	0.74	0.74	0.74
time (sec)	N/A	0.340	0.020	0.947	0.036	0.062	0.535	0.127	0.218	0.031

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	67	55	54	61	68	54	55	54
N.S.	1	1.03	1.14	0.93	0.92	1.03	1.15	0.92	0.93	0.92
time (sec)	N/A	0.315	0.004	0.899	0.027	0.062	0.399	0.121	0.201	0.029

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	65	54	53	56	71	54	53	56
N.S.	1	1.12	1.00	0.83	0.82	0.86	1.09	0.83	0.82	0.86
time (sec)	N/A	0.319	0.026	1.145	0.030	0.066	0.692	0.122	0.219	0.034

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	76	69	57	59	62	66	60	64	59
N.S.	1	1.12	1.01	0.84	0.87	0.91	0.97	0.88	0.94	0.87
time (sec)	N/A	0.334	0.031	1.156	0.035	0.069	0.235	0.124	0.221	0.047

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	67	58	55	57	70	55	60	55
N.S.	1	1.00	3.19	2.76	2.62	2.71	3.33	2.62	2.86	2.62
time (sec)	N/A	0.228	0.028	1.012	0.028	0.096	0.281	0.123	0.244	0.326

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	67	58	57	59	73	57	60	57
N.S.	1	1.05	0.92	0.79	0.78	0.81	1.00	0.78	0.82	0.78
time (sec)	N/A	0.335	0.036	1.002	0.051	0.095	0.432	0.119	0.225	0.322

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	67	58	57	58	75	57	60	57
N.S.	1	1.03	0.89	0.77	0.76	0.77	1.00	0.76	0.80	0.76
time (sec)	N/A	0.336	0.031	0.990	0.033	0.087	0.485	0.124	0.248	0.050

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	67	58	57	59	76	57	60	57
N.S.	1	1.03	0.87	0.75	0.74	0.77	0.99	0.74	0.78	0.74
time (sec)	N/A	0.330	0.031	0.997	0.037	0.191	0.741	0.124	0.208	0.334

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	67	58	57	59	75	57	60	57
N.S.	1	1.05	0.89	0.77	0.76	0.79	1.00	0.76	0.80	0.76
time (sec)	N/A	0.339	0.032	1.457	0.033	0.082	0.856	0.123	0.205	0.052

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	148	130	113	251	124	144	112	112	112
N.S.	1	1.03	0.90	0.78	1.74	0.86	1.00	0.78	0.78	0.78
time (sec)	N/A	0.463	0.032	4.332	0.038	0.069	1.006	0.122	0.201	0.371

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	148	130	113	200	124	144	112	112	112
N.S.	1	1.03	0.90	0.78	1.39	0.86	1.00	0.78	0.78	0.78
time (sec)	N/A	0.455	0.028	4.333	0.033	0.076	1.064	0.123	0.264	0.063

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	185	130	113	149	124	141	112	112	112
N.S.	1	1.03	0.73	0.63	0.83	0.69	0.79	0.63	0.63	0.63
time (sec)	N/A	0.509	0.031	4.297	0.046	0.084	0.764	0.119	0.215	0.063

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	124	130	113	98	124	143	112	112	112
N.S.	1	1.03	1.08	0.94	0.82	1.03	1.19	0.93	0.93	0.93
time (sec)	N/A	0.409	0.029	4.438	0.026	0.069	0.695	0.126	0.225	0.063

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	128	110	47	117	136	109	110	109
N.S.	1	1.03	2.17	1.86	0.80	1.98	2.31	1.85	1.86	1.85
time (sec)	N/A	0.329	0.024	4.211	0.052	0.069	0.538	0.123	0.230	0.063

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	132	109	108	113	144	109	108	111
N.S.	1	1.04	0.97	0.80	0.79	0.83	1.06	0.80	0.79	0.82
time (sec)	N/A	0.422	0.034	4.716	0.027	0.077	1.199	0.124	0.247	0.067

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	135	125	110	110	116	131	111	119	112
N.S.	1	1.08	1.00	0.88	0.88	0.93	1.05	0.89	0.95	0.90
time (sec)	N/A	0.438	0.056	4.557	0.031	0.071	1.286	0.126	0.215	0.060

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	139	132	110	110	117	136	111	119	112
N.S.	1	1.06	1.01	0.84	0.84	0.89	1.04	0.85	0.91	0.85
time (sec)	N/A	0.440	0.046	4.444	0.035	0.073	1.499	0.129	0.212	0.323

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	136	132	112	112	118	133	113	119	113
N.S.	1	1.04	1.01	0.85	0.85	0.90	1.02	0.86	0.91	0.86
time (sec)	N/A	0.449	0.075	4.474	0.032	0.075	0.387	0.123	0.237	0.085

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	128	113	112	114	139	112	115	112
N.S.	1	1.04	2.78	2.46	2.43	2.48	3.02	2.43	2.50	2.43
time (sec)	N/A	0.255	0.047	4.414	0.037	0.071	0.682	0.126	0.233	0.386

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	142	128	113	112	114	143	112	115	112
N.S.	1	1.16	1.05	0.93	0.92	0.93	1.17	0.92	0.94	0.92
time (sec)	N/A	0.329	0.053	4.415	0.027	0.067	0.639	0.123	0.223	0.095

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	146	128	113	112	114	146	112	115	112
N.S.	1	1.01	0.89	0.78	0.78	0.79	1.01	0.78	0.80	0.78
time (sec)	N/A	0.432	0.058	4.707	0.038	0.070	0.868	0.125	0.216	0.379

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	148	128	113	112	114	146	112	115	112
N.S.	1	1.03	0.89	0.78	0.78	0.79	1.01	0.78	0.80	0.78
time (sec)	N/A	0.433	0.059	4.720	0.028	0.072	1.097	0.127	0.264	0.392

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	150	128	113	112	114	146	112	115	112
N.S.	1	1.04	0.89	0.78	0.78	0.79	1.01	0.78	0.80	0.78
time (sec)	N/A	0.434	0.060	4.522	0.027	0.133	1.540	0.138	0.211	0.099

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	144	128	113	112	114	144	112	115	112
N.S.	1	1.01	0.90	0.80	0.79	0.80	1.01	0.79	0.81	0.79
time (sec)	N/A	0.446	0.064	5.055	0.035	0.070	1.892	0.130	0.224	0.384

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	221	191	168	302	179	218	167	167	167
N.S.	1	1.02	0.88	0.77	1.39	0.82	1.00	0.77	0.77	0.77
time (sec)	N/A	0.594	0.040	25.442	0.035	0.093	1.808	0.125	0.222	0.172

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	221	191	168	251	179	218	167	167	167
N.S.	1	1.02	0.88	0.77	1.16	0.82	1.00	0.77	0.77	0.77
time (sec)	N/A	0.568	0.039	26.081	0.041	0.065	1.485	0.129	0.282	0.161

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	246	191	168	200	179	216	167	167	167
N.S.	1	1.01	0.78	0.69	0.82	0.73	0.89	0.68	0.68	0.68
time (sec)	N/A	0.612	0.038	25.822	0.035	0.064	1.327	0.120	0.202	0.161

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	187	191	168	149	179	214	167	167	167
N.S.	1	1.02	1.04	0.92	0.81	0.98	1.17	0.91	0.91	0.91
time (sec)	N/A	0.509	0.040	25.856	0.026	0.111	1.031	0.127	0.227	0.159

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	191	168	98	179	214	167	167	167
N.S.	1	1.02	1.57	1.38	0.80	1.47	1.75	1.37	1.37	1.37
time (sec)	N/A	0.427	0.037	26.086	0.028	0.065	0.848	0.131	0.197	0.160

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	189	165	47	172	207	164	165	164
N.S.	1	1.03	3.20	2.80	0.80	2.92	3.51	2.78	2.80	2.78
time (sec)	N/A	0.341	0.037	25.387	0.032	0.065	0.799	0.130	0.218	0.160

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	217	193	164	163	168	212	164	163	166
N.S.	1	1.04	0.92	0.78	0.78	0.80	1.01	0.78	0.78	0.79
time (sec)	N/A	0.526	0.047	26.740	0.026	0.065	0.450	0.128	0.219	0.172

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	212	195	165	167	173	204	168	174	167
N.S.	1	1.05	0.97	0.82	0.83	0.86	1.01	0.83	0.86	0.83
time (sec)	N/A	0.536	0.087	26.832	0.037	0.072	0.644	0.127	0.206	0.406

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	210	193	165	165	173	202	166	174	167
N.S.	1	1.05	0.96	0.82	0.82	0.86	1.01	0.83	0.87	0.84
time (sec)	N/A	0.547	0.083	26.849	0.032	0.072	0.516	0.125	0.245	0.084

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	210	193	165	165	173	202	166	174	167
N.S.	1	1.05	0.96	0.82	0.82	0.86	1.01	0.83	0.87	0.84
time (sec)	N/A	0.541	0.104	27.331	0.034	0.074	0.592	0.122	0.207	0.351

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	219	193	166	166	173	212	167	174	168
N.S.	1	1.04	0.91	0.79	0.79	0.82	1.00	0.79	0.82	0.80
time (sec)	N/A	0.541	0.148	26.766	0.047	0.073	0.806	0.127	0.195	0.477

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	189	168	167	169	209	167	170	166
N.S.	1	1.11	2.62	2.33	2.32	2.35	2.90	2.32	2.36	2.31
time (sec)	N/A	0.280	0.069	27.059	0.034	0.070	0.952	0.123	0.207	0.218

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	174	189	168	167	169	216	167	170	167
N.S.	1	1.18	1.28	1.14	1.13	1.14	1.46	1.13	1.15	1.13
time (sec)	N/A	0.355	0.084	26.330	0.041	0.101	1.218	0.122	0.217	0.514

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	268	189	168	167	169	216	167	170	167
N.S.	1	1.20	0.84	0.75	0.75	0.75	0.96	0.75	0.76	0.75
time (sec)	N/A	0.439	0.080	26.048	0.035	0.071	1.736	0.126	0.212	0.520

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	221	189	168	167	169	218	167	170	167
N.S.	1	1.03	0.88	0.78	0.78	0.79	1.01	0.78	0.79	0.78
time (sec)	N/A	0.527	0.080	26.188	0.054	0.070	1.873	0.124	0.212	0.511

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	221	189	168	167	169	219	167	170	167
N.S.	1	1.02	0.87	0.77	0.77	0.78	1.01	0.77	0.78	0.77
time (sec)	N/A	0.546	0.079	27.969	0.038	0.068	2.477	0.129	0.237	0.503

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	221	189	168	167	169	219	167	170	167
N.S.	1	1.02	0.87	0.77	0.77	0.78	1.01	0.77	0.78	0.77
time (sec)	N/A	0.542	0.085	26.796	0.032	0.071	3.042	0.129	0.240	0.513

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	170	152	132	197	133	0	133	131	130
N.S.	1	1.02	0.92	0.80	1.19	0.80	0.00	0.80	0.79	0.78
time (sec)	N/A	0.503	0.104	0.461	0.028	0.074	0.000	0.123	0.218	0.074

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	127	115	99	146	100	0	100	98	98
N.S.	1	1.02	0.93	0.80	1.18	0.81	0.00	0.81	0.79	0.79
time (sec)	N/A	0.429	0.066	0.466	0.040	0.069	0.000	0.115	0.220	0.050

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	78	66	95	66	80	67	65	64
N.S.	1	1.05	0.98	0.82	1.19	0.82	1.00	0.84	0.81	0.80
time (sec)	N/A	0.360	0.039	0.442	0.032	0.066	170.311	0.123	0.206	0.033

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	43	42	35	44	33	42	35	33	34
N.S.	1	1.02	1.00	0.83	1.05	0.79	1.00	0.83	0.79	0.81
time (sec)	N/A	0.289	0.007	0.444	0.031	0.070	0.184	0.120	0.234	0.047

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	30	21	20	20	37	22	20	17
N.S.	1	1.27	1.36	0.95	0.91	0.91	1.68	1.00	0.91	0.77
time (sec)	N/A	0.245	0.023	0.443	0.032	0.069	0.201	0.126	0.201	0.097

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	62	56	56	56	83	61	56	50
N.S.	1	1.11	0.98	0.89	0.89	0.89	1.32	0.97	0.89	0.79
time (sec)	N/A	0.336	0.034	0.445	0.031	0.082	0.553	0.119	0.212	0.077

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	95	87	86	93	129	91	91	83
N.S.	1	1.08	0.91	0.84	0.83	0.89	1.24	0.88	0.88	0.80
time (sec)	N/A	0.400	0.064	0.444	0.027	0.075	1.211	0.121	0.257	0.072

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	156	132	122	120	126	172	125	124	116
N.S.	1	1.05	0.89	0.82	0.81	0.85	1.15	0.84	0.83	0.78
time (sec)	N/A	0.452	0.084	0.447	0.035	0.108	2.258	0.126	0.243	0.369

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	27	25	16	15	17	22	17	17	10
N.S.	1	1.29	1.19	0.76	0.71	0.81	1.05	0.81	0.81	0.48
time (sec)	N/A	0.241	0.016	0.475	0.030	0.099	0.152	0.118	0.219	0.056

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	174	171	139	197	181	0	144	157	143
N.S.	1	1.02	1.00	0.81	1.15	1.06	0.00	0.84	0.92	0.84
time (sec)	N/A	0.533	0.091	0.677	0.036	0.084	0.000	0.127	0.233	0.088

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	128	134	106	146	147	0	111	124	110
N.S.	1	1.05	1.10	0.87	1.20	1.20	0.00	0.91	1.02	0.90
time (sec)	N/A	0.463	0.071	0.447	0.058	0.074	0.000	0.122	0.201	0.372

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	97	73	95	114	243	78	91	77
N.S.	1	1.04	1.14	0.86	1.12	1.34	2.86	0.92	1.07	0.91
time (sec)	N/A	0.379	0.050	0.441	0.027	0.069	48.287	0.123	0.232	0.337

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	47	56	41	44	68	109	41	57	43
N.S.	1	1.02	1.22	0.89	0.96	1.48	2.37	0.89	1.24	0.93
time (sec)	N/A	0.306	0.011	0.438	0.041	0.074	0.223	0.127	0.212	0.039

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	43	37	35	34	70	160	36	61	32
N.S.	1	1.13	0.97	0.92	0.89	1.84	4.21	0.95	1.61	0.84
time (sec)	N/A	0.301	0.047	0.450	0.033	0.072	0.538	0.121	0.222	0.058

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	87	77	73	73	122	272	77	104	67
N.S.	1	1.09	0.96	0.91	0.91	1.52	3.40	0.96	1.30	0.84
time (sec)	N/A	0.372	0.099	0.449	0.036	0.076	1.011	0.127	0.191	0.378

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	132	117	106	110	164	405	112	140	103
N.S.	1	1.06	0.94	0.85	0.88	1.31	3.24	0.90	1.12	0.82
time (sec)	N/A	0.472	0.115	0.473	0.028	0.082	2.465	0.112	0.243	0.093

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	172	154	139	143	196	505	145	173	136
N.S.	1	1.06	0.95	0.86	0.88	1.21	3.12	0.90	1.07	0.84
time (sec)	N/A	0.526	0.181	0.449	0.028	0.078	5.348	0.120	0.248	0.381

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	177	171	145	197	225	624	145	183	154
N.S.	1	1.04	1.00	0.85	1.15	1.32	3.65	0.85	1.07	0.90
time (sec)	N/A	0.565	0.085	0.451	0.028	0.082	1.194	0.127	0.199	0.077

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	138	134	112	146	192	493	112	150	120
N.S.	1	1.03	1.00	0.84	1.09	1.43	3.68	0.84	1.12	0.90
time (sec)	N/A	0.460	0.066	0.447	0.030	0.078	0.654	0.118	0.204	0.052

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	97	79	94	159	362	79	117	87
N.S.	1	1.06	1.08	0.88	1.04	1.77	4.02	0.88	1.30	0.97
time (sec)	N/A	0.393	0.057	0.458	0.033	0.075	0.333	0.119	0.237	0.347

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	47	47	46	113	228	44	81	53
N.S.	1	1.02	0.87	0.87	0.85	2.09	4.22	0.81	1.50	0.98
time (sec)	N/A	0.320	0.014	0.450	0.030	0.073	0.316	0.121	0.230	0.378

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	51	49	57	129	386	49	115	54
N.S.	1	1.09	0.91	0.88	1.02	2.30	6.89	0.88	2.05	0.96
time (sec)	N/A	0.331	0.067	0.451	0.034	0.078	0.619	0.123	0.212	0.379

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	111	93	90	97	191	561	90	159	89
N.S.	1	1.08	0.90	0.87	0.94	1.85	5.45	0.87	1.54	0.86
time (sec)	N/A	0.405	0.140	0.454	0.042	0.089	1.523	0.127	0.232	0.393

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	154	130	123	132	230	706	123	196	125
N.S.	1	1.05	0.89	0.84	0.90	1.58	4.84	0.84	1.34	0.86
time (sec)	N/A	0.484	0.142	0.477	0.028	0.082	3.621	0.123	0.264	0.151

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	193	167	156	165	263	847	156	229	159
N.S.	1	1.05	0.91	0.85	0.90	1.44	4.63	0.85	1.25	0.87
time (sec)	N/A	0.570	0.221	0.459	0.034	0.083	7.793	0.126	0.206	0.520

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	46	31	29	28	21	359	28	20	12
N.S.	1	1.10	0.74	0.69	0.67	0.50	8.55	0.67	0.48	0.29
time (sec)	N/A	0.265	0.017	0.545	0.025	0.076	0.859	0.123	0.203	0.431

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	25	18	19	23	20	19	21	18
N.S.	1	1.17	1.09	0.78	0.83	1.00	0.87	0.83	0.91	0.78
time (sec)	N/A	0.253	0.034	0.479	0.104	0.070	0.715	0.125	0.199	0.340

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	39	28	42	25	34	27	27	27
N.S.	1	1.05	1.00	0.72	1.08	0.64	0.87	0.69	0.69	0.69
time (sec)	N/A	0.283	0.018	0.463	0.027	0.072	0.088	0.115	0.245	0.026

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	14	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.88	0.75	0.75	0.75
time (sec)	N/A	0.234	0.014	0.569	0.124	0.079	0.092	0.125	0.237	0.032

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	8	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	0.67	0.67	0.67
time (sec)	N/A	0.219	0.012	0.429	0.026	0.073	0.094	0.123	0.226	0.169

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	0.75
time (sec)	N/A	0.223	0.013	0.504	0.117	0.070	0.111	0.120	0.246	0.460

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	24	7	12	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	2.18	0.64	1.09	0.64
time (sec)	N/A	0.215	0.022	0.463	0.030	0.640	0.098	0.123	0.239	1.589

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	19	49	9	18	9
N.S.	1	1.00	1.00	0.67	0.60	1.27	3.27	0.60	1.20	0.60
time (sec)	N/A	0.219	0.022	0.446	0.035	0.708	0.207	0.126	0.191	1.425

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	131	71	71	88	78	187	88	82	47
N.S.	1	1.49	0.81	0.81	1.00	0.89	2.12	1.00	0.93	0.53
time (sec)	N/A	0.521	0.087	0.437	0.117	0.074	0.865	0.123	0.193	0.094

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	205	126	124	272	147	462	139	17	223
N.S.	1	1.09	0.67	0.66	1.45	0.78	2.46	0.74	0.09	1.19
time (sec)	N/A	0.677	0.059	0.494	0.144	0.080	51.784	0.194	0.188	0.429

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	79	52	75	78	63	165	59	43	31
N.S.	1	1.18	0.78	1.12	1.16	0.94	2.46	0.88	0.64	0.46
time (sec)	N/A	0.313	0.092	0.475	0.108	0.078	2.128	0.337	0.262	0.482

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	56	45	38	62	55	60	42	32	27
N.S.	1	1.12	0.90	0.76	1.24	1.10	1.20	0.84	0.64	0.54
time (sec)	N/A	0.301	0.057	0.454	0.037	0.071	2.676	0.137	0.211	0.408

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	A	F(-1)	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	107	280	0	109	0	46	0	75	37
N.S.	1	1.13	2.95	0.00	1.15	0.00	0.48	0.00	0.79	0.39
time (sec)	N/A	0.365	6.025	0.000	0.113	0.000	1.926	0.000	0.912	0.555

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	14	15	13	15	13	16	13
N.S.	1	1.00	1.11	0.74	0.79	0.68	0.79	0.68	0.84	0.68
time (sec)	N/A	0.252	0.015	0.109	0.028	0.098	0.275	0.122	0.188	0.345

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	13	15	13	16	13
N.S.	1	1.00	1.00	0.74	0.79	0.68	0.79	0.68	0.84	0.68
time (sec)	N/A	0.248	0.002	0.102	0.037	0.075	0.182	0.118	0.261	0.031

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	13	15	13	16	13
N.S.	1	1.00	1.00	0.74	0.79	0.68	0.79	0.68	0.84	0.68
time (sec)	N/A	0.249	0.002	0.102	0.025	0.074	0.126	0.125	0.229	0.032

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	15	13	15	13	14	13
N.S.	1	1.00	1.00	0.74	0.79	0.68	0.79	0.68	0.74	0.68
time (sec)	N/A	0.245	0.003	0.106	0.025	0.074	0.094	0.124	0.210	0.032

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	12	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.86	0.71	0.71	0.71
time (sec)	N/A	0.233	0.001	0.086	0.030	0.078	0.021	0.116	0.214	0.032

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	19	12	12	15	14
N.S.	1	1.00	1.00	0.92	0.85	1.46	0.92	0.92	1.15	1.08
time (sec)	N/A	0.248	0.018	0.129	0.036	0.085	0.157	0.116	0.222	0.044

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	47	16	14	15	15	13
N.S.	1	1.00	1.00	0.82	2.76	0.94	0.82	0.88	0.88	0.76
time (sec)	N/A	0.248	0.016	0.108	0.033	0.076	0.220	0.137	0.213	0.325

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	14	98	16	17	15	15	13
N.S.	1	1.00	1.11	0.74	5.16	0.84	0.89	0.79	0.79	0.68
time (sec)	N/A	0.246	0.017	0.106	0.036	0.068	0.295	0.118	0.218	0.039

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	14	149	16	17	15	15	13
N.S.	1	1.00	1.11	0.74	7.84	0.84	0.89	0.79	0.79	0.68
time (sec)	N/A	0.245	0.017	0.114	0.026	0.073	0.394	0.115	0.223	0.346

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	25	26	24	31	24	27	24
N.S.	1	1.06	1.00	0.74	0.76	0.71	0.91	0.71	0.79	0.71
time (sec)	N/A	0.318	0.021	0.504	0.025	0.089	0.358	0.123	0.286	0.052

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	25	26	24	31	24	27	24
N.S.	1	1.06	1.00	0.74	0.76	0.71	0.91	0.71	0.79	0.71
time (sec)	N/A	0.322	0.019	0.502	0.031	0.067	0.271	0.121	0.210	0.365

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	25	26	24	31	24	27	24
N.S.	1	1.06	1.00	0.74	0.76	0.71	0.91	0.71	0.79	0.71
time (sec)	N/A	0.307	0.017	0.503	0.026	0.070	0.185	0.116	0.192	0.045

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	25	26	24	31	24	25	24
N.S.	1	1.06	1.00	0.74	0.76	0.71	0.91	0.71	0.74	0.71
time (sec)	N/A	0.299	0.016	0.497	0.026	0.066	0.122	0.122	0.200	0.045

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	16	25	14	21	21	24	21	21	21
N.S.	1	1.07	1.67	0.93	1.40	1.40	1.60	1.40	1.40	1.40
time (sec)	N/A	0.231	0.002	0.484	0.045	0.067	0.100	0.123	0.256	0.041

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	34	27	23	22	31	27	24	28	28
N.S.	1	1.21	0.96	0.82	0.79	1.11	0.96	0.86	1.00	1.00
time (sec)	N/A	0.298	0.031	0.502	0.038	0.071	0.193	0.130	0.225	0.043

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	36	34	25	47	27	29	26	26	24
N.S.	1	1.12	1.06	0.78	1.47	0.84	0.91	0.81	0.81	0.75
time (sec)	N/A	0.296	0.024	0.498	0.032	0.098	0.245	0.127	0.210	0.051

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	25	97	27	32	26	26	24
N.S.	1	1.06	1.00	0.74	2.85	0.79	0.94	0.76	0.76	0.71
time (sec)	N/A	0.294	0.022	0.502	0.027	0.074	0.319	0.121	0.201	0.403

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	34	25	149	27	32	26	26	24
N.S.	1	1.06	1.00	0.74	4.38	0.79	0.94	0.76	0.76	0.71
time (sec)	N/A	0.291	0.023	0.509	0.036	0.068	0.428	0.124	0.242	0.356

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	43	36	37	35	42	35	35	35
N.S.	1	1.04	0.91	0.77	0.79	0.74	0.89	0.74	0.74	0.74
time (sec)	N/A	0.327	0.018	0.569	0.031	0.065	0.412	0.115	0.231	0.054

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	37	35	42	35	35	35
N.S.	1	1.04	0.87	0.77	0.79	0.74	0.89	0.74	0.74	0.74
time (sec)	N/A	0.326	0.017	0.586	0.025	0.066	0.300	0.123	0.211	0.051

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	43	36	37	35	42	35	35	35
N.S.	1	1.04	0.91	0.77	0.79	0.74	0.89	0.74	0.74	0.74
time (sec)	N/A	0.320	0.014	0.584	0.027	0.077	0.207	0.112	0.218	0.050

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	41	33	37	32	39	32	33	32
N.S.	1	1.12	0.98	0.79	0.88	0.76	0.93	0.76	0.79	0.76
time (sec)	N/A	0.314	0.014	0.585	0.034	0.066	0.138	0.120	0.239	0.048

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	45	36	31	30	33	36	31	33	33
N.S.	1	1.25	1.00	0.86	0.83	0.92	1.00	0.86	0.92	0.92
time (sec)	N/A	0.302	0.005	0.546	0.030	0.064	0.114	0.121	0.212	0.046

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	47	40	34	33	39	36	37	37	37
N.S.	1	1.21	1.03	0.87	0.85	1.00	0.92	0.95	0.95	0.95
time (sec)	N/A	0.308	0.029	0.582	0.037	0.069	0.263	0.117	0.199	0.049

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	41	36	47	35	41	35	35	35
N.S.	1	1.09	0.91	0.80	1.04	0.78	0.91	0.78	0.78	0.78
time (sec)	N/A	0.316	0.021	0.599	0.034	0.068	0.336	0.125	0.235	0.041

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	98	35	44	35	35	35
N.S.	1	1.04	0.87	0.77	2.09	0.74	0.94	0.74	0.74	0.74
time (sec)	N/A	0.305	0.024	0.608	0.026	0.070	0.365	0.125	0.232	0.375

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	41	36	149	35	44	35	35	35
N.S.	1	1.04	0.87	0.77	3.17	0.74	0.94	0.74	0.74	0.74
time (sec)	N/A	0.316	0.024	0.890	0.037	0.068	0.489	0.123	0.220	0.046

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	140	126	110	122	111	143	111	109	108
N.S.	1	1.03	0.93	0.81	0.90	0.82	1.05	0.82	0.80	0.79
time (sec)	N/A	0.469	0.076	0.448	0.034	0.073	0.544	0.117	0.224	0.057

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	88	78	88	77	100	78	76	76
N.S.	1	1.03	0.94	0.83	0.94	0.82	1.06	0.83	0.81	0.81
time (sec)	N/A	0.387	0.034	0.444	0.027	0.066	0.312	0.124	0.232	0.039

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	51	44	54	43	58	45	43	42
N.S.	1	1.08	1.02	0.88	1.08	0.86	1.16	0.90	0.86	0.84
time (sec)	N/A	0.335	0.004	0.451	0.028	0.070	0.194	0.121	0.206	0.044

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	20	13	20	14	13	13
N.S.	1	1.00	1.00	0.93	1.33	0.87	1.33	0.93	0.87	0.87
time (sec)	N/A	0.232	0.012	0.451	0.033	0.079	0.244	0.120	0.206	0.040

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	56	48	44	44	47	73	49	49	41
N.S.	1	1.33	1.14	1.05	1.05	1.12	1.74	1.17	1.17	0.98
time (sec)	N/A	0.319	0.069	0.435	0.032	0.075	0.431	0.120	0.238	0.475

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	100	84	78	95	85	116	81	80	71
N.S.	1	1.20	1.01	0.94	1.14	1.02	1.40	0.98	0.96	0.86
time (sec)	N/A	0.373	0.070	0.440	0.035	0.077	0.989	0.123	0.251	0.102

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	142	121	109	146	118	158	113	113	105
N.S.	1	1.14	0.97	0.87	1.17	0.94	1.26	0.90	0.90	0.84
time (sec)	N/A	0.435	0.085	0.530	0.033	0.076	1.799	0.119	0.240	0.086

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	155	158	127	145	169	367	133	146	132
N.S.	1	1.03	1.05	0.85	0.97	1.13	2.45	0.89	0.97	0.88
time (sec)	N/A	0.523	0.077	0.571	0.036	0.079	0.951	0.118	0.205	0.397

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	121	95	112	137	277	100	113	99
N.S.	1	1.03	1.07	0.84	0.99	1.21	2.45	0.88	1.00	0.88
time (sec)	N/A	0.428	0.062	0.441	0.026	0.076	0.468	0.125	0.226	0.053

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	80	62	76	100	165	65	78	65
N.S.	1	1.07	1.19	0.93	1.13	1.49	2.46	0.97	1.16	0.97
time (sec)	N/A	0.353	0.013	0.442	0.032	0.075	0.287	0.117	0.236	0.043

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	30	34	56	99	30	44	29
N.S.	1	1.00	0.88	0.91	1.03	1.70	3.00	0.91	1.33	0.88
time (sec)	N/A	0.291	0.026	0.447	0.024	0.076	0.372	0.125	0.232	0.399

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	58	51	47	44	98	211	51	85	49
N.S.	1	1.26	1.11	1.02	0.96	2.13	4.59	1.11	1.85	1.07
time (sec)	N/A	0.327	0.094	0.445	0.025	0.077	0.860	0.117	0.229	0.384

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	104	95	84	95	148	340	90	118	81
N.S.	1	1.20	1.09	0.97	1.09	1.70	3.91	1.03	1.36	0.93
time (sec)	N/A	0.396	0.101	0.443	0.027	0.078	2.284	0.125	0.253	0.397

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	143	132	116	146	179	440	123	151	115
N.S.	1	1.16	1.07	0.94	1.19	1.46	3.58	1.00	1.23	0.93
time (sec)	N/A	0.469	0.189	0.447	0.030	0.078	4.618	0.126	0.216	0.130

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	177	171	145	167	225	624	145	183	154
N.S.	1	1.04	1.00	0.85	0.98	1.32	3.65	0.85	1.07	0.90
time (sec)	N/A	0.559	0.110	0.449	0.035	0.077	1.209	0.120	0.214	0.375

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	138	134	112	134	192	493	112	150	120
N.S.	1	1.03	1.00	0.84	1.00	1.43	3.68	0.84	1.12	0.90
time (sec)	N/A	0.485	0.095	0.449	0.033	0.079	0.550	0.129	0.231	0.056

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	97	79	101	159	362	79	117	87
N.S.	1	1.06	1.08	0.88	1.12	1.77	4.02	0.88	1.30	0.97
time (sec)	N/A	0.414	0.017	0.450	0.027	0.091	0.332	0.124	0.234	0.044

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	47	49	57	113	240	44	81	53
N.S.	1	1.02	0.87	0.91	1.06	2.09	4.44	0.81	1.50	0.98
time (sec)	N/A	0.334	0.043	0.465	0.031	0.073	0.716	0.127	0.212	0.377

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	51	49	46	129	406	49	115	54
N.S.	1	1.13	0.94	0.91	0.85	2.39	7.52	0.91	2.13	1.00
time (sec)	N/A	0.344	0.066	0.497	0.031	0.080	1.456	0.117	0.222	0.344

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	111	93	90	95	191	561	90	159	89
N.S.	1	1.19	1.00	0.97	1.02	2.05	6.03	0.97	1.71	0.96
time (sec)	N/A	0.427	0.152	0.502	0.027	0.078	4.203	0.121	0.212	0.107

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	154	130	123	146	230	707	123	196	125
N.S.	1	1.13	0.96	0.90	1.07	1.69	5.20	0.90	1.44	0.92
time (sec)	N/A	0.502	0.134	0.487	0.028	0.087	7.706	0.123	0.215	0.427

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	193	167	156	197	263	848	156	229	159
N.S.	1	1.12	0.97	0.90	1.14	1.52	4.90	0.90	1.32	0.92
time (sec)	N/A	0.591	0.213	0.500	0.035	0.093	18.105	0.124	0.226	0.509

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	35	28	40	27	34	28	27	27
N.S.	1	1.14	1.00	0.80	1.14	0.77	0.97	0.80	0.77	0.77
time (sec)	N/A	0.306	0.017	0.520	0.033	0.070	0.076	0.117	0.218	0.038

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	0	31	9	9	9
N.S.	1	1.00	1.00	0.67	0.60	0.00	2.07	0.60	0.60	0.60
time (sec)	N/A	0.239	0.027	0.534	0.028	0.000	0.281	0.130	0.245	0.770

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	0	0	13	48	13
N.S.	1	1.00	1.00	0.67	0.62	0.00	0.00	0.62	2.29	0.62
time (sec)	N/A	0.238	0.044	0.534	0.033	0.000	0.000	0.126	0.303	1.147

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	10	9	0	12	9	33	9
N.S.	1	1.00	1.00	0.67	0.60	0.00	0.80	0.60	2.20	0.60
time (sec)	N/A	0.227	0.028	0.463	0.025	0.000	0.166	0.118	0.213	0.255

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	19	28	51	29	24	20
N.S.	1	1.00	1.00	1.00	0.90	1.33	2.43	1.38	1.14	0.95
time (sec)	N/A	0.265	0.029	0.083	0.027	0.074	0.158	0.127	0.211	0.482

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	19	28	51	29	24	20
N.S.	1	1.00	1.00	1.00	0.90	1.33	2.43	1.38	1.14	0.95
time (sec)	N/A	0.266	0.024	0.069	0.030	0.074	0.199	0.125	0.265	0.467

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	19	28	51	29	24	20
N.S.	1	1.00	1.00	1.00	0.90	1.33	2.43	1.38	1.14	0.95
time (sec)	N/A	0.268	0.022	0.068	0.033	0.078	0.140	0.122	0.214	0.427

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	16	20	17	16	17	15
N.S.	1	1.00	1.00	1.00	1.00	1.25	1.06	1.00	1.06	0.94
time (sec)	N/A	0.248	0.005	0.042	0.030	0.074	0.041	0.118	0.212	0.345

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	18	14	18	15	17	0	15	13
N.S.	1	1.00	1.38	1.08	1.38	1.15	1.31	0.00	1.15	1.00
time (sec)	N/A	0.261	0.017	0.078	0.029	0.072	0.114	0.000	0.209	0.301

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	0	23	32	0	20	20
N.S.	1	1.00	1.00	0.95	0.00	1.05	1.45	0.00	0.91	0.91
time (sec)	N/A	0.267	0.020	0.056	0.000	0.077	0.207	0.000	0.269	0.319

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	21	0	23	60	0	24	20
N.S.	1	1.00	0.83	0.88	0.00	0.96	2.50	0.00	1.00	0.83
time (sec)	N/A	0.262	0.028	0.059	0.000	0.073	0.264	0.000	0.218	0.356

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	43	40	74	202	88	71	43
N.S.	1	1.00	0.86	0.98	0.91	1.68	4.59	2.00	1.61	0.98
time (sec)	N/A	0.302	0.078	0.480	0.027	0.082	0.373	0.125	0.230	0.609

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	44	41	78	211	91	72	43
N.S.	1	1.00	0.93	1.02	0.95	1.81	4.91	2.12	1.67	1.00
time (sec)	N/A	0.323	0.082	0.472	0.034	0.073	0.457	0.122	0.220	0.539

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	43	40	72	201	87	70	43
N.S.	1	1.00	0.84	0.98	0.91	1.64	4.57	1.98	1.59	0.98
time (sec)	N/A	0.305	0.068	0.489	0.039	0.071	0.305	0.124	0.240	0.522

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	37	38	65	182	73	66	36
N.S.	1	1.00	0.89	0.97	1.00	1.71	4.79	1.92	1.74	0.95
time (sec)	N/A	0.288	0.021	0.447	0.032	0.079	0.281	0.118	0.230	0.401

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	30	36	0	30	30
N.S.	1	1.00	1.00	0.97	1.06	0.94	1.12	0.00	0.94	0.94
time (sec)	N/A	0.278	0.027	0.474	0.026	0.075	0.129	0.000	0.211	0.385

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	43	0	68	190	0	71	43
N.S.	1	1.00	0.86	0.98	0.00	1.55	4.32	0.00	1.61	0.98
time (sec)	N/A	0.315	0.047	0.471	0.000	0.084	0.281	0.000	0.240	0.416

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	39	42	0	66	245	0	70	43
N.S.	1	1.00	0.78	0.84	0.00	1.32	4.90	0.00	1.40	0.86
time (sec)	N/A	0.315	0.053	0.468	0.000	0.080	0.287	0.000	0.239	0.384

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	65	61	145	507	188	151	66
N.S.	1	1.00	0.89	1.00	0.94	2.23	7.80	2.89	2.32	1.02
time (sec)	N/A	0.344	0.056	0.535	0.026	0.074	1.193	0.127	0.238	0.565

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	66	62	144	500	188	151	66
N.S.	1	1.00	0.86	1.00	0.94	2.18	7.58	2.85	2.29	1.00
time (sec)	N/A	0.341	0.051	0.528	0.027	0.075	0.620	0.127	0.226	0.574

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	65	61	145	500	188	151	66
N.S.	1	1.00	0.89	1.00	0.94	2.23	7.69	2.89	2.32	1.02
time (sec)	N/A	0.339	0.047	0.523	0.027	0.077	0.491	0.127	0.255	0.508

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	58	60	130	469	159	146	57
N.S.	1	1.00	0.90	0.97	1.00	2.17	7.82	2.65	2.43	0.95
time (sec)	N/A	0.322	0.014	0.492	0.027	0.072	0.442	0.119	0.215	0.404

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	46	44	48	44	53	0	44	46
N.S.	1	0.94	0.92	0.88	0.96	0.88	1.06	0.00	0.88	0.92
time (sec)	N/A	0.292	0.034	0.514	0.026	0.076	0.148	0.000	0.217	0.388

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	58	65	0	131	508	0	148	66
N.S.	1	1.00	0.88	0.98	0.00	1.98	7.70	0.00	2.24	1.00
time (sec)	N/A	0.347	0.060	0.526	0.000	0.087	0.411	0.000	0.205	0.404

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	65	0	134	627	0	152	66
N.S.	1	1.00	0.83	0.90	0.00	1.86	8.71	0.00	2.11	0.92
time (sec)	N/A	0.355	0.062	0.524	0.000	0.075	0.569	0.000	0.275	0.402

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	48	0	13	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.45	0.00	0.39	0.00
time (sec)	N/A	0.247	0.018	0.000	0.000	0.000	0.550	0.000	0.237	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	44	0	11	25
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.83	0.00	0.46	1.04
time (sec)	N/A	0.241	0.001	0.000	0.000	0.000	0.518	0.000	0.230	0.473

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	26	25	23	28	22	41	0	22	22
N.S.	1	1.13	1.09	1.00	1.22	0.96	1.78	0.00	0.96	0.96
time (sec)	N/A	0.257	0.028	0.549	0.034	0.083	0.473	0.000	0.202	0.382

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	0	0	0	49	0	18	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.44	0.00	0.53	0.00
time (sec)	N/A	0.259	0.024	0.000	0.000	0.000	0.620	0.000	0.217	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	0	0	0	54	0	18	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.50	0.00	0.50	0.00
time (sec)	N/A	0.260	0.022	0.000	0.000	0.000	0.641	0.000	0.202	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	335	0	26	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	10.15	0.00	0.79	0.00
time (sec)	N/A	0.257	0.016	0.000	0.000	0.000	0.755	0.000	0.205	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	318	0	24	25
N.S.	1	1.00	1.00	0.00	0.00	0.00	13.25	0.00	1.00	1.04
time (sec)	N/A	0.246	0.001	0.000	0.000	0.000	0.726	0.000	0.237	0.386

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	40	43	50	160	0	61	39
N.S.	1	1.00	1.13	1.03	1.10	1.28	4.10	0.00	1.56	1.00
time (sec)	N/A	0.309	0.043	0.534	0.027	0.080	0.775	0.000	0.235	0.343

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	0	0	0	367	0	34	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	10.79	0.00	1.00	0.00
time (sec)	N/A	0.268	0.025	0.000	0.000	0.000	0.939	0.000	0.217	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	0	0	0	400	0	34	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	11.11	0.00	0.94	0.00
time (sec)	N/A	0.259	0.025	0.000	0.000	0.000	1.003	0.000	0.207	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	1210	0	39	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	36.67	0.00	1.18	0.00
time (sec)	N/A	0.247	0.016	0.000	0.000	0.000	1.156	0.000	0.202	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	1226	0	37	25
N.S.	1	1.00	1.00	0.00	0.00	0.00	51.08	0.00	1.54	1.04
time (sec)	N/A	0.229	0.013	0.000	0.000	0.000	1.100	0.000	0.225	0.400

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	55	48	53	71	106	415	0	123	69
N.S.	1	0.95	0.83	0.91	1.22	1.83	7.16	0.00	2.12	1.19
time (sec)	N/A	0.316	0.073	0.551	0.031	0.076	1.436	0.000	0.247	0.377

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	0	0	0	1435	0	50	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	42.21	0.00	1.47	0.00
time (sec)	N/A	0.242	0.026	0.000	0.000	0.000	1.313	0.000	0.218	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	0	0	0	1479	0	50	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	41.08	0.00	1.39	0.00
time (sec)	N/A	0.247	0.027	0.000	0.000	0.000	1.464	0.000	0.249	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	23	22	36	0	20	20
N.S.	1	1.00	0.81	0.89	0.85	0.81	1.33	0.00	0.74	0.74
time (sec)	N/A	0.255	0.021	0.109	0.033	0.077	0.183	0.000	0.229	0.505

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	23	22	36	0	20	20
N.S.	1	1.00	0.81	0.89	0.85	0.81	1.33	0.00	0.74	0.74
time (sec)	N/A	0.249	0.022	0.102	0.036	0.075	0.219	0.000	0.197	0.484

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	23	22	36	0	20	20
N.S.	1	1.00	0.81	0.89	0.85	0.81	1.33	0.00	0.74	0.74
time (sec)	N/A	0.252	0.024	0.099	0.032	0.070	0.274	0.000	0.196	0.478

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	22	19	21	17	19	31	19	17	15
N.S.	1	1.16	1.00	1.11	0.89	1.00	1.63	1.00	0.89	0.79
time (sec)	N/A	0.248	0.014	0.091	0.025	0.073	0.193	0.125	0.256	0.477

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	18	14	18	15	17	0	15	13
N.S.	1	1.00	1.38	1.08	1.38	1.15	1.31	0.00	1.15	1.00
time (sec)	N/A	0.234	0.003	0.056	0.030	0.078	0.106	0.000	0.216	0.003

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	17	16	21	20	21	21	29
N.S.	1	1.00	1.31	1.06	1.00	1.31	1.25	1.31	1.31	1.81
time (sec)	N/A	0.241	0.026	0.093	0.025	0.078	0.552	0.118	0.226	0.734

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	24	25	20	39	20	22	20
N.S.	1	1.00	0.88	0.96	1.00	0.80	1.56	0.80	0.88	0.80
time (sec)	N/A	0.260	0.023	0.097	0.026	0.070	0.192	0.135	0.221	0.457

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	27	22	41	22	22	22
N.S.	1	1.00	0.81	0.89	1.00	0.81	1.52	0.81	0.81	0.81
time (sec)	N/A	0.260	0.024	0.101	0.026	0.069	0.196	0.129	0.248	0.448

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	27	22	41	22	22	22
N.S.	1	1.00	0.81	0.89	1.00	0.81	1.52	0.81	0.81	0.81
time (sec)	N/A	0.263	0.023	0.100	0.026	0.074	0.215	0.129	0.197	0.462

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	27	22	41	22	22	22
N.S.	1	1.00	0.81	0.89	1.00	0.81	1.52	0.81	0.81	0.81
time (sec)	N/A	0.265	0.023	0.106	0.035	0.076	0.215	0.126	0.220	0.441

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	40	35	40	39	35	63	0	33	33
N.S.	1	0.89	0.78	0.89	0.87	0.78	1.40	0.00	0.73	0.73
time (sec)	N/A	0.294	0.035	0.530	0.026	0.070	0.277	0.000	0.232	0.567

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	40	35	40	39	35	61	0	33	33
N.S.	1	0.89	0.78	0.89	0.87	0.78	1.36	0.00	0.73	0.73
time (sec)	N/A	0.300	0.027	0.544	0.025	0.075	0.320	0.000	0.245	0.557

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	40	35	40	39	35	63	0	33	33
N.S.	1	0.89	0.78	0.89	0.87	0.78	1.40	0.00	0.73	0.73
time (sec)	N/A	0.289	0.032	0.517	0.027	0.076	0.290	0.000	0.213	0.542

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	36	17	32	53	32	30	27
N.S.	1	1.00	1.00	1.89	0.89	1.68	2.79	1.68	1.58	1.42
time (sec)	N/A	0.235	0.018	0.519	0.026	0.077	0.272	0.122	0.211	0.557

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	34	30	36	0	30	30
N.S.	1	1.00	1.00	0.97	1.06	0.94	1.12	0.00	0.94	0.94
time (sec)	N/A	0.277	0.005	0.454	0.031	0.072	0.132	0.000	0.212	0.003

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	31	30	34	36	34	34	30
N.S.	1	1.00	1.03	1.03	1.00	1.13	1.20	1.13	1.13	1.00
time (sec)	N/A	0.286	0.035	0.484	0.030	0.076	0.767	0.125	0.229	0.640

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	33	34	38	39	38	38	34
N.S.	1	1.00	0.97	0.97	1.00	1.12	1.15	1.12	1.12	1.00
time (sec)	N/A	0.309	0.036	0.483	0.025	0.074	1.072	0.133	0.213	0.585

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	40	43	33	65	33	35	33
N.S.	1	1.00	1.46	1.67	1.79	1.38	2.71	1.38	1.46	1.38
time (sec)	N/A	0.247	0.028	0.516	0.026	0.071	0.274	0.135	0.211	0.534

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	40	35	40	45	35	70	35	35	35
N.S.	1	0.89	0.78	0.89	1.00	0.78	1.56	0.78	0.78	0.78
time (sec)	N/A	0.294	0.030	0.510	0.037	0.079	0.309	0.130	0.258	0.512

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	40	35	40	45	35	68	35	35	35
N.S.	1	0.89	0.78	0.89	1.00	0.78	1.51	0.78	0.78	0.78
time (sec)	N/A	0.293	0.029	0.510	0.026	0.071	0.299	0.130	0.218	0.498

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	40	35	40	45	35	70	35	35	35
N.S.	1	0.89	0.78	0.89	1.00	0.78	1.56	0.78	0.78	0.78
time (sec)	N/A	0.290	0.035	0.536	0.033	0.089	0.290	0.134	0.205	0.505

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	55	48	56	55	48	87	0	46	55
N.S.	1	0.87	0.76	0.89	0.87	0.76	1.38	0.00	0.73	0.87
time (sec)	N/A	0.315	0.030	0.677	0.034	0.079	0.418	0.000	0.201	0.591

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	55	48	56	55	48	88	0	46	55
N.S.	1	0.87	0.76	0.89	0.87	0.76	1.40	0.00	0.73	0.87
time (sec)	N/A	0.309	0.030	0.673	0.032	0.074	0.415	0.000	0.257	0.559

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	38	48	55	54	48	85	0	46	54
N.S.	1	0.95	1.20	1.38	1.35	1.20	2.12	0.00	1.15	1.35
time (sec)	N/A	0.295	0.029	0.703	0.032	0.075	0.488	0.000	0.235	0.569

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	52	17	45	76	45	43	51
N.S.	1	1.00	1.00	2.74	0.89	2.37	4.00	2.37	2.26	2.68
time (sec)	N/A	0.236	0.017	0.661	0.026	0.077	0.450	0.128	0.212	0.565

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	46	44	48	44	53	0	44	46
N.S.	1	0.94	0.92	0.88	0.96	0.88	1.06	0.00	0.88	0.92
time (sec)	N/A	0.294	0.001	0.642	0.031	0.073	0.145	0.000	0.204	0.003

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	46	47	48	47	48	53	48	48	47
N.S.	1	0.94	0.96	0.98	0.96	0.98	1.08	0.98	0.98	0.96
time (sec)	N/A	0.298	0.041	0.619	0.031	0.080	0.903	0.134	0.266	0.648

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	45	46	47	48	51	53	51	51	48
N.S.	1	0.94	0.96	0.98	1.00	1.06	1.10	1.06	1.06	1.00
time (sec)	N/A	0.304	0.048	0.724	0.035	0.118	1.178	0.136	0.210	0.626

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	49	48	49	52	51	56	51	51	52
N.S.	1	0.94	0.92	0.94	1.00	0.98	1.08	0.98	0.98	1.00
time (sec)	N/A	0.316	0.044	0.573	0.029	0.074	2.006	0.136	0.227	0.588

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	48	56	61	46	92	46	48	61
N.S.	1	1.00	2.00	2.33	2.54	1.92	3.83	1.92	2.00	2.54
time (sec)	N/A	0.244	0.031	0.783	0.035	0.072	0.445	0.135	0.213	0.537

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	48	56	63	48	94	48	48	63
N.S.	1	0.96	0.96	1.12	1.26	0.96	1.88	0.96	0.96	1.26
time (sec)	N/A	0.265	0.033	0.664	0.034	0.074	0.555	0.130	0.237	0.542

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	55	48	56	63	48	97	48	48	63
N.S.	1	0.87	0.76	0.89	1.00	0.76	1.54	0.76	0.76	1.00
time (sec)	N/A	0.301	0.032	0.653	0.037	0.073	0.448	0.138	0.205	0.546

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	55	48	56	63	48	95	48	48	63
N.S.	1	0.87	0.76	0.89	1.00	0.76	1.51	0.76	0.76	1.00
time (sec)	N/A	0.309	0.033	0.688	0.028	0.084	0.435	0.141	0.232	0.545

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	76	74	87	86	74	136	0	72	86
N.S.	1	0.90	0.88	1.04	1.02	0.88	1.62	0.00	0.86	1.02
time (sec)	N/A	0.351	0.041	1.644	0.040	0.086	0.973	0.000	0.232	0.618

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	57	74	88	87	74	138	0	72	87
N.S.	1	0.92	1.19	1.42	1.40	1.19	2.23	0.00	1.16	1.40
time (sec)	N/A	0.321	0.035	1.628	0.047	0.084	0.823	0.000	0.267	0.588

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	38	74	88	87	74	138	0	72	87
N.S.	1	0.95	1.85	2.20	2.18	1.85	3.45	0.00	1.80	2.18
time (sec)	N/A	0.286	0.033	1.651	0.029	0.085	0.823	0.000	0.219	0.592

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	84	17	71	124	71	69	83
N.S.	1	1.00	1.00	4.42	0.89	3.74	6.53	3.74	3.63	4.37
time (sec)	N/A	0.228	0.017	1.641	0.041	0.081	0.859	0.127	0.203	0.581

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	75	72	70	74	70	85	0	70	78
N.S.	1	0.89	0.86	0.83	0.88	0.83	1.01	0.00	0.83	0.93
time (sec)	N/A	0.331	0.039	0.781	0.032	0.077	0.262	0.000	0.236	0.414

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	74	73	80	79	75	85	75	75	79
N.S.	1	0.89	0.88	0.96	0.95	0.90	1.02	0.90	0.90	0.95
time (sec)	N/A	0.333	0.039	0.890	0.028	0.083	2.014	0.146	0.205	0.777

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	76	73	80	81	77	87	77	77	81
N.S.	1	0.89	0.86	0.94	0.95	0.91	1.02	0.91	0.91	0.95
time (sec)	N/A	0.339	0.050	0.895	0.030	0.082	2.472	0.145	0.196	0.691

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	76	77	80	83	77	87	77	77	83
N.S.	1	0.89	0.91	0.94	0.98	0.91	1.02	0.91	0.91	0.98
time (sec)	N/A	0.335	0.044	0.895	0.035	0.072	3.271	0.151	0.223	0.686

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	73	77	79	84	77	85	77	77	84
N.S.	1	0.89	0.94	0.96	1.02	0.94	1.04	0.94	0.94	1.02
time (sec)	N/A	0.336	0.064	0.951	0.035	0.076	4.459	0.159	0.255	0.684

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	77	74	81	88	77	88	77	77	88
N.S.	1	0.90	0.86	0.94	1.02	0.90	1.02	0.90	0.90	1.02
time (sec)	N/A	0.345	0.061	0.900	0.032	0.078	7.865	0.148	0.210	0.678

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	74	88	97	72	146	72	74	97
N.S.	1	1.00	3.08	3.67	4.04	3.00	6.08	3.00	3.08	4.04
time (sec)	N/A	0.235	0.043	1.542	0.034	0.080	0.907	0.147	0.210	0.594

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	74	88	99	74	150	74	74	99
N.S.	1	0.96	1.48	1.76	1.98	1.48	3.00	1.48	1.48	1.98
time (sec)	N/A	0.269	0.047	1.550	0.035	0.076	0.889	0.146	0.201	0.569

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	74	88	99	74	150	74	74	99
N.S.	1	1.01	0.96	1.14	1.29	0.96	1.95	0.96	0.96	1.29
time (sec)	N/A	0.300	0.046	1.615	0.045	0.089	0.929	0.151	0.227	0.564

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	83	74	88	99	74	148	74	74	99
N.S.	1	0.86	0.76	0.91	1.02	0.76	1.53	0.76	0.76	1.02
time (sec)	N/A	0.342	0.039	1.537	0.034	0.071	0.988	0.151	0.213	0.551

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	85	74	88	99	74	151	74	74	99
N.S.	1	0.86	0.75	0.89	1.00	0.75	1.53	0.75	0.75	1.00
time (sec)	N/A	0.348	0.044	1.524	0.037	0.078	0.834	0.133	0.230	0.535

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	128	113	136	135	113	212	0	111	135
N.S.	1	0.85	0.75	0.90	0.89	0.75	1.40	0.00	0.74	0.89
time (sec)	N/A	0.431	0.067	8.532	0.052	0.078	2.172	0.000	0.202	0.767

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	128	113	136	135	113	214	0	111	135
N.S.	1	0.85	0.75	0.90	0.89	0.75	1.42	0.00	0.74	0.89
time (sec)	N/A	0.418	0.064	8.919	0.066	0.076	2.114	0.000	0.278	0.722

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	133	113	135	134	113	212	0	111	134
N.S.	1	0.89	0.75	0.90	0.89	0.75	1.41	0.00	0.74	0.89
time (sec)	N/A	0.430	0.061	8.572	0.038	0.080	2.268	0.000	0.206	0.675

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	114	113	136	135	113	214	0	111	135
N.S.	1	0.89	0.88	1.06	1.05	0.88	1.67	0.00	0.87	1.05
time (sec)	N/A	0.403	0.043	8.628	0.040	0.081	2.165	0.000	0.216	0.701

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	95	113	136	135	113	212	0	111	135
N.S.	1	0.90	1.07	1.28	1.27	1.07	2.00	0.00	1.05	1.27
time (sec)	N/A	0.376	0.043	8.292	0.028	0.083	2.260	0.000	0.183	0.698

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	76	113	136	135	113	214	0	111	135
N.S.	1	0.90	1.35	1.62	1.61	1.35	2.55	0.00	1.32	1.61
time (sec)	N/A	0.348	0.041	8.451	0.035	0.073	2.247	0.000	0.234	0.705

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	57	113	136	135	113	211	0	111	135
N.S.	1	0.92	1.82	2.19	2.18	1.82	3.40	0.00	1.79	2.18
time (sec)	N/A	0.324	0.041	8.105	0.027	0.080	2.187	0.000	0.217	0.707

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	38	113	136	135	113	212	0	111	135
N.S.	1	0.95	2.82	3.40	3.38	2.82	5.30	0.00	2.78	3.38
time (sec)	N/A	0.288	0.044	7.959	0.042	0.070	2.121	0.000	0.211	0.657

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	132	17	110	189	110	108	131
N.S.	1	1.00	1.00	6.95	0.89	5.79	9.95	5.79	5.68	6.89
time (sec)	N/A	0.243	0.018	8.381	0.032	0.075	2.239	0.122	0.204	0.666

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	138	120	111	109	113	109	136	0	109	126
N.S.	1	0.87	0.80	0.79	0.82	0.79	0.99	0.00	0.79	0.91
time (sec)	N/A	0.386	0.060	1.928	0.039	0.074	0.309	0.000	0.215	0.486

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	117	112	128	127	114	134	114	114	127
N.S.	1	0.87	0.83	0.95	0.94	0.84	0.99	0.84	0.84	0.94
time (sec)	N/A	0.406	0.058	2.359	0.034	0.087	5.198	0.152	0.206	1.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	117	112	128	129	116	134	116	116	129
N.S.	1	0.87	0.83	0.95	0.96	0.86	0.99	0.86	0.86	0.96
time (sec)	N/A	0.417	0.058	2.385	0.035	0.074	6.739	0.158	0.235	0.799

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	115	116	128	131	116	133	116	116	131
N.S.	1	0.86	0.87	0.96	0.98	0.87	1.00	0.87	0.87	0.98
time (sec)	N/A	0.410	0.066	2.380	0.040	0.073	8.403	0.162	0.238	0.802

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	117	116	128	133	116	134	116	116	133
N.S.	1	0.87	0.86	0.95	0.99	0.86	0.99	0.86	0.86	0.99
time (sec)	N/A	0.405	0.064	2.416	0.047	0.103	10.523	0.164	0.242	0.781

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	115	116	128	135	116	133	116	116	135
N.S.	1	0.86	0.87	0.96	1.02	0.87	1.00	0.87	0.87	1.02
time (sec)	N/A	0.404	0.060	2.409	0.059	0.102	13.533	0.159	0.205	0.753

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	117	116	128	137	116	131	116	116	137
N.S.	1	0.87	0.86	0.95	1.01	0.86	0.97	0.86	0.86	1.01
time (sec)	N/A	0.409	0.059	2.398	0.034	0.077	17.318	0.161	0.199	0.731

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	116	116	127	138	116	134	116	116	138
N.S.	1	0.87	0.87	0.95	1.03	0.87	1.00	0.87	0.87	1.03
time (sec)	N/A	0.409	0.097	2.391	0.032	0.078	22.046	0.160	0.216	0.704

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	122	113	129	142	116	139	116	116	142
N.S.	1	0.87	0.81	0.92	1.01	0.83	0.99	0.83	0.83	1.01
time (sec)	N/A	0.410	0.072	2.370	0.029	0.073	35.774	0.157	0.238	0.715

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	113	136	151	111	221	111	113	151
N.S.	1	1.00	4.71	5.67	6.29	4.62	9.21	4.62	4.71	6.29
time (sec)	N/A	0.248	0.050	7.592	0.034	0.074	2.471	0.162	0.222	0.662

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	113	136	153	113	230	113	113	153
N.S.	1	0.96	2.26	2.72	3.06	2.26	4.60	2.26	2.26	3.06
time (sec)	N/A	0.274	0.049	7.453	0.034	0.074	2.548	0.161	0.222	0.648

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	113	136	153	113	228	113	113	153
N.S.	1	1.01	1.47	1.77	1.99	1.47	2.96	1.47	1.47	1.99
time (sec)	N/A	0.299	0.051	7.467	0.056	0.077	2.412	0.157	0.238	0.661

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	113	136	153	113	231	113	113	153
N.S.	1	1.04	1.09	1.31	1.47	1.09	2.22	1.09	1.09	1.47
time (sec)	N/A	0.321	0.050	7.299	0.028	0.078	2.362	0.166	0.217	0.664

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	138	113	136	153	113	230	113	113	153
N.S.	1	1.05	0.86	1.04	1.17	0.86	1.76	0.86	0.86	1.17
time (sec)	N/A	0.355	0.052	7.236	0.033	0.074	2.474	0.164	0.213	0.658

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	128	113	136	153	113	231	113	113	153
N.S.	1	0.85	0.75	0.90	1.01	0.75	1.53	0.75	0.75	1.01
time (sec)	N/A	0.437	0.056	7.308	0.032	0.078	2.355	0.158	0.268	0.656

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	128	113	136	153	113	230	113	113	153
N.S.	1	0.85	0.75	0.90	1.01	0.75	1.52	0.75	0.75	1.01
time (sec)	N/A	0.421	0.048	7.236	0.045	0.097	2.920	0.161	0.241	0.719

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	260	17	214	362	214	212	259
N.S.	1	1.00	1.00	13.68	0.89	11.26	19.05	11.26	11.16	13.63
time (sec)	N/A	0.235	0.018	381.253	0.032	0.077	26.736	0.128	0.229	1.155

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	135	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	8.44	0.88
time (sec)	N/A	0.235	0.004	0.600	0.026	0.060	0.053	0.126	0.210	0.443

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	135	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	8.44	0.88
time (sec)	N/A	0.233	0.003	0.987	0.028	0.069	0.059	0.123	0.209	0.431

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	135	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	8.44	0.88
time (sec)	N/A	0.230	0.003	1.760	0.027	0.065	0.056	0.117	0.231	0.424

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	288	25	194	345	25	202	285
N.S.	1	1.00	0.89	10.67	0.93	7.19	12.78	0.93	7.48	10.56
time (sec)	N/A	0.251	0.028	84.519	0.025	0.082	7.810	0.183	0.222	1.429

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	288	25	194	345	25	202	285
N.S.	1	1.00	0.89	10.67	0.93	7.19	12.78	0.93	7.48	10.56
time (sec)	N/A	0.251	0.001	85.907	0.028	0.079	7.817	0.177	0.240	0.003

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	71	65	79	72	65	90	0	65	0
N.S.	1	0.87	0.79	0.96	0.88	0.79	1.10	0.00	0.79	0.00
time (sec)	N/A	0.347	0.047	0.664	0.045	0.075	3.376	0.000	0.278	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	56	52	63	60	52	75	0	52	0
N.S.	1	0.88	0.81	0.98	0.94	0.81	1.17	0.00	0.81	0.00
time (sec)	N/A	0.331	0.041	0.563	0.035	0.078	2.036	0.000	0.213	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	38	47	45	38	60	0	38	0
N.S.	1	0.89	0.83	1.02	0.98	0.83	1.30	0.00	0.83	0.00
time (sec)	N/A	0.307	0.034	0.569	0.027	0.075	1.505	0.000	0.209	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	27	31	32	24	44	0	24	0
N.S.	1	0.93	0.96	1.11	1.14	0.86	1.57	0.00	0.86	0.00
time (sec)	N/A	0.288	0.031	0.664	0.029	0.077	1.082	0.000	0.198	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	31	16	15	15
N.S.	1	1.00	1.00	1.20	1.00	1.00	2.07	1.07	1.00	1.00
time (sec)	N/A	0.235	0.016	0.773	0.035	0.070	0.910	0.113	0.238	0.502

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	26	25	23	28	22	41	0	22	22
N.S.	1	1.13	1.09	1.00	1.22	0.96	1.78	0.00	0.96	0.96
time (sec)	N/A	0.250	0.005	0.750	0.028	0.074	0.370	0.000	0.231	0.003

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	41	42	37	76	0	38	0
N.S.	1	1.00	0.87	1.08	1.11	0.97	2.00	0.00	1.00	0.00
time (sec)	N/A	0.300	0.038	0.802	0.027	0.077	2.050	0.000	0.231	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	54	48	58	58	59	99	0	59	0
N.S.	1	0.95	0.84	1.02	1.02	1.04	1.74	0.00	1.04	0.00
time (sec)	N/A	0.318	0.064	0.612	0.036	0.073	3.786	0.000	0.207	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	70	63	75	71	72	114	0	72	0
N.S.	1	0.92	0.83	0.99	0.93	0.95	1.50	0.00	0.95	0.00
time (sec)	N/A	0.338	0.076	0.575	0.033	0.075	7.115	0.000	0.237	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	71	65	79	72	65	90	0	65	0
N.S.	1	0.87	0.79	0.96	0.88	0.79	1.10	0.00	0.79	0.00
time (sec)	N/A	0.344	0.002	0.546	0.034	0.077	3.575	0.000	0.195	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	56	52	63	60	52	75	0	52	0
N.S.	1	0.88	0.81	0.98	0.94	0.81	1.17	0.00	0.81	0.00
time (sec)	N/A	0.321	0.002	0.571	0.030	0.076	2.056	0.000	0.212	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	38	47	45	38	60	0	38	0
N.S.	1	0.89	0.83	1.02	0.98	0.83	1.30	0.00	0.83	0.00
time (sec)	N/A	0.299	0.006	0.548	0.028	0.079	1.528	0.000	0.220	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	27	31	32	24	44	0	24	0
N.S.	1	0.93	0.96	1.11	1.14	0.86	1.57	0.00	0.86	0.00
time (sec)	N/A	0.275	0.004	0.545	0.027	0.076	0.959	0.000	0.261	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	31	16	15	15
N.S.	1	1.00	1.00	1.20	1.00	1.00	2.07	1.07	1.00	1.00
time (sec)	N/A	0.229	0.001	0.552	0.045	0.071	0.840	0.121	0.196	0.002

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	26	25	23	28	22	41	0	22	22
N.S.	1	1.13	1.09	1.00	1.22	0.96	1.78	0.00	0.96	0.96
time (sec)	N/A	0.246	0.001	0.532	0.030	0.076	0.361	0.000	0.201	0.003

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	41	42	37	76	0	38	0
N.S.	1	1.00	0.87	1.08	1.11	0.97	2.00	0.00	1.00	0.00
time (sec)	N/A	0.281	0.001	0.576	0.028	0.081	1.737	0.000	0.218	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	54	48	58	58	59	99	0	59	0
N.S.	1	0.95	0.84	1.02	1.02	1.04	1.74	0.00	1.04	0.00
time (sec)	N/A	0.296	0.009	0.549	0.030	0.077	3.751	0.000	0.228	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	70	63	75	71	72	114	0	72	0
N.S.	1	0.92	0.83	0.99	0.93	0.95	1.50	0.00	0.95	0.00
time (sec)	N/A	0.312	0.002	0.589	0.032	0.081	6.977	0.000	0.225	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	60	54	70	63	55	82	0	55	0
N.S.	1	0.85	0.76	0.99	0.89	0.77	1.15	0.00	0.77	0.00
time (sec)	N/A	0.316	0.038	0.665	0.028	0.080	2.207	0.000	0.247	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	48	43	57	52	44	66	0	44	0
N.S.	1	0.86	0.77	1.02	0.93	0.79	1.18	0.00	0.79	0.00
time (sec)	N/A	0.304	0.034	0.621	0.028	0.140	1.451	0.000	0.271	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	38	33	44	42	34	56	0	34	0
N.S.	1	0.88	0.77	1.02	0.98	0.79	1.30	0.00	0.79	0.00
time (sec)	N/A	0.284	0.030	0.632	0.034	0.078	1.082	0.000	0.213	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	26	30	31	23	42	0	23	0
N.S.	1	0.93	0.96	1.11	1.15	0.85	1.56	0.00	0.85	0.00
time (sec)	N/A	0.269	0.027	0.668	0.034	0.079	0.795	0.000	0.205	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	31	16	15	15
N.S.	1	1.00	1.00	1.20	1.00	1.00	2.07	1.07	1.00	1.00
time (sec)	N/A	0.225	0.015	0.684	0.029	0.067	0.717	0.116	0.200	0.498

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	25	24	21	23	20	29	0	20	18
N.S.	1	1.14	1.09	0.95	1.05	0.91	1.32	0.00	0.91	0.82
time (sec)	N/A	0.235	0.024	0.631	0.028	0.073	0.337	0.000	0.255	0.416

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	33	34	34	54	0	34	0
N.S.	1	1.00	0.94	0.92	0.94	0.94	1.50	0.00	0.94	0.00
time (sec)	N/A	0.279	0.037	0.684	0.028	0.086	1.519	0.000	0.270	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	50	45	48	47	50	70	0	50	0
N.S.	1	0.94	0.85	0.91	0.89	0.94	1.32	0.00	0.94	0.00
time (sec)	N/A	0.289	0.053	0.708	0.039	0.081	2.652	0.000	0.264	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	62	56	61	58	61	82	0	61	0
N.S.	1	0.91	0.82	0.90	0.85	0.90	1.21	0.00	0.90	0.00
time (sec)	N/A	0.315	0.060	0.745	0.036	0.081	4.854	0.000	0.254	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	58	74	67	78	76	178	0	77	0
N.S.	1	0.88	1.12	1.02	1.18	1.15	2.70	0.00	1.17	0.00
time (sec)	N/A	0.328	0.050	0.600	0.034	0.082	11.847	0.000	0.266	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	43	55	51	61	59	133	0	61	0
N.S.	1	0.90	1.15	1.06	1.27	1.23	2.77	0.00	1.27	0.00
time (sec)	N/A	0.311	0.046	0.634	0.038	0.079	5.844	0.000	0.234	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	39	36	39	36	99	0	46	0
N.S.	1	0.94	1.18	1.09	1.18	1.09	3.00	0.00	1.39	0.00
time (sec)	N/A	0.297	0.038	0.619	0.032	0.090	6.644	0.000	0.277	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	17	65	17	19	21
N.S.	1	1.00	1.00	1.06	1.00	1.00	3.82	1.00	1.12	1.24
time (sec)	N/A	0.238	0.018	0.569	0.028	0.071	0.916	0.126	0.256	0.527

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	40	43	50	160	0	61	39
N.S.	1	1.00	1.13	1.03	1.10	1.28	4.10	0.00	1.56	1.00
time (sec)	N/A	0.310	0.010	0.532	0.034	0.078	0.725	0.000	0.295	0.002

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	54	49	60	62	82	262	0	93	0
N.S.	1	0.95	0.86	1.05	1.09	1.44	4.60	0.00	1.63	0.00
time (sec)	N/A	0.334	0.096	0.565	0.038	0.079	8.874	0.000	0.218	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	72	71	79	0	105	345	0	113	0
N.S.	1	0.92	0.91	1.01	0.00	1.35	4.42	0.00	1.45	0.00
time (sec)	N/A	0.365	0.099	0.644	0.000	0.118	22.005	0.000	0.237	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	85	83	95	94	116	384	0	126	0
N.S.	1	0.90	0.88	1.01	1.00	1.23	4.09	0.00	1.34	0.00
time (sec)	N/A	0.385	0.148	0.690	0.029	0.076	46.178	0.000	0.270	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	62	68	61	91	102	309	0	106	0
N.S.	1	0.89	0.97	0.87	1.30	1.46	4.41	0.00	1.51	0.00
time (sec)	N/A	0.346	0.071	0.784	0.039	0.078	19.466	0.000	0.243	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	51	47	47	66	76	260	0	90	0
N.S.	1	0.91	0.84	0.84	1.18	1.36	4.64	0.00	1.61	0.00
time (sec)	N/A	0.329	0.063	0.789	0.039	0.096	17.391	0.000	0.266	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	29	26	41	41	88	0	35	37
N.S.	1	1.00	1.21	1.08	1.71	1.71	3.67	0.00	1.46	1.54
time (sec)	N/A	0.246	0.038	0.671	0.039	0.078	1.758	0.000	0.249	0.543

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	31	136	17	30	33
N.S.	1	1.00	1.00	0.95	0.89	1.63	7.16	0.89	1.58	1.74
time (sec)	N/A	0.236	0.018	0.663	0.026	0.084	1.648	0.122	0.302	0.550

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	55	48	53	71	106	415	0	123	69
N.S.	1	0.95	0.83	0.91	1.22	1.83	7.16	0.00	2.12	1.19
time (sec)	N/A	0.329	0.001	0.538	0.027	0.071	1.172	0.000	0.240	0.003

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	71	68	70	91	139	547	0	153	0
N.S.	1	0.92	0.88	0.91	1.18	1.81	7.10	0.00	1.99	0.00
time (sec)	N/A	0.363	0.133	0.622	0.038	0.079	26.120	0.000	0.220	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	92	85	90	110	160	629	0	176	0
N.S.	1	0.91	0.84	0.89	1.09	1.58	6.23	0.00	1.74	0.00
time (sec)	N/A	0.395	0.119	0.740	0.039	0.086	48.621	0.000	0.238	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	32	79	0	131	41	0	21	0
N.S.	1	0.96	0.64	1.58	0.00	2.62	0.82	0.00	0.42	0.00
time (sec)	N/A	0.294	0.030	0.633	0.000	0.131	0.696	0.000	0.245	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	178	192	0	21	0
N.S.	1	1.00	0.21	0.34	0.00	1.11	1.20	0.00	0.13	0.00
time (sec)	N/A	0.558	0.030	0.747	0.000	0.144	1.150	0.000	0.265	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	259	34	54	0	255	267	0	21	0
N.S.	1	1.48	0.19	0.31	0.00	1.46	1.53	0.00	0.12	0.00
time (sec)	N/A	0.779	0.030	0.668	0.000	0.171	1.188	0.000	0.226	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	41	42	37	76	0	38	0
N.S.	1	1.00	0.87	1.08	1.11	0.97	2.00	0.00	1.00	0.00
time (sec)	N/A	0.311	0.010	0.607	0.027	0.078	1.627	0.000	0.247	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	32	79	0	131	41	0	21	0
N.S.	1	0.96	0.64	1.58	0.00	2.62	0.82	0.00	0.42	0.00
time (sec)	N/A	0.303	0.000	0.618	0.000	0.100	0.708	0.000	0.220	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	156	32	57	0	145	189	0	21	0
N.S.	1	0.99	0.20	0.36	0.00	0.92	1.20	0.00	0.13	0.00
time (sec)	N/A	0.576	0.032	0.665	0.000	0.105	1.125	0.000	0.213	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	255	32	56	0	197	265	0	21	0
N.S.	1	1.48	0.19	0.33	0.00	1.15	1.54	0.00	0.12	0.00
time (sec)	N/A	0.759	0.031	0.688	0.000	0.123	1.464	0.000	0.224	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	34	97	0	253	56	0	21	0
N.S.	1	1.03	0.50	1.43	0.00	3.72	0.82	0.00	0.31	0.00
time (sec)	N/A	0.336	0.030	0.671	0.000	0.120	1.090	0.000	0.268	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	178	34	73	0	180	230	0	21	0
N.S.	1	1.01	0.19	0.41	0.00	1.02	1.31	0.00	0.12	0.00
time (sec)	N/A	0.588	0.030	0.655	0.000	0.159	1.190	0.000	0.206	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	277	34	73	0	261	309	0	21	0
N.S.	1	1.45	0.18	0.38	0.00	1.37	1.62	0.00	0.11	0.00
time (sec)	N/A	0.789	0.046	0.659	0.000	0.236	1.308	0.000	0.216	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	57	0	0	0	51	0	95	0
N.S.	1	1.19	1.19	0.00	0.00	0.00	1.06	0.00	1.98	0.00
time (sec)	N/A	0.308	0.044	0.000	0.000	0.000	0.894	0.000	0.204	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	48	48	0	0	0	49	0	91	43
N.S.	1	1.23	1.23	0.00	0.00	0.00	1.26	0.00	2.33	1.10
time (sec)	N/A	0.282	0.006	0.000	0.000	0.000	0.779	0.000	0.257	0.655

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	43	42	36	54	88	76	0	41	0
N.S.	1	0.96	0.93	0.80	1.20	1.96	1.69	0.00	0.91	0.00
time (sec)	N/A	0.293	0.062	0.901	0.111	0.077	1.193	0.000	0.228	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	58	55	0	0	0	53	0	119	0
N.S.	1	1.18	1.12	0.00	0.00	0.00	1.08	0.00	2.43	0.00
time (sec)	N/A	0.304	0.051	0.000	0.000	0.000	1.044	0.000	0.258	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	57	0	0	0	54	0	123	0
N.S.	1	1.18	1.12	0.00	0.00	0.00	1.06	0.00	2.41	0.00
time (sec)	N/A	0.306	0.028	0.000	0.000	0.000	1.058	0.000	0.222	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	58	58	0	0	0	51	0	252	0
N.S.	1	1.21	1.21	0.00	0.00	0.00	1.06	0.00	5.25	0.00
time (sec)	N/A	0.289	0.023	0.000	0.000	0.000	1.245	0.000	0.227	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	49	49	0	0	0	49	0	241	43
N.S.	1	1.26	1.26	0.00	0.00	0.00	1.26	0.00	6.18	1.10
time (sec)	N/A	0.271	0.004	0.000	0.000	0.000	1.034	0.000	0.203	0.451

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	61	55	48	67	111	87	0	60	0
N.S.	1	0.95	0.86	0.75	1.05	1.73	1.36	0.00	0.94	0.00
time (sec)	N/A	0.295	0.055	0.871	0.109	0.084	1.734	0.000	0.212	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	56	0	0	0	53	0	297	0
N.S.	1	1.20	1.14	0.00	0.00	0.00	1.08	0.00	6.06	0.00
time (sec)	N/A	0.300	0.031	0.000	0.000	0.000	1.022	0.000	0.254	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	61	58	0	0	0	54	0	303	0
N.S.	1	1.20	1.14	0.00	0.00	0.00	1.06	0.00	5.94	0.00
time (sec)	N/A	0.298	0.030	0.000	0.000	0.000	1.043	0.000	0.236	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	60	0	0	0	51	0	488	0
N.S.	1	1.25	1.25	0.00	0.00	0.00	1.06	0.00	10.17	0.00
time (sec)	N/A	0.292	0.027	0.000	0.000	0.000	3.432	0.000	0.250	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	51	0	0	0	49	0	466	43
N.S.	1	1.31	1.31	0.00	0.00	0.00	1.26	0.00	11.95	1.10
time (sec)	N/A	0.277	0.018	0.000	0.000	0.000	2.389	0.000	0.215	0.476

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	79	69	62	83	141	117	0	81	0
N.S.	1	0.93	0.81	0.73	0.98	1.66	1.38	0.00	0.95	0.00
time (sec)	N/A	0.314	0.065	0.892	0.116	0.103	4.412	0.000	0.247	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	61	58	0	0	0	53	0	560	0
N.S.	1	1.24	1.18	0.00	0.00	0.00	1.08	0.00	11.43	0.00
time (sec)	N/A	0.292	0.034	0.000	0.000	0.000	2.723	0.000	0.225	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	60	0	0	0	54	0	568	0
N.S.	1	1.24	1.18	0.00	0.00	0.00	1.06	0.00	11.14	0.00
time (sec)	N/A	0.294	0.040	0.000	0.000	0.000	2.737	0.000	0.222	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	57	0	0	0	51	0	21	0
N.S.	1	1.19	1.19	0.00	0.00	0.00	1.06	0.00	0.44	0.00
time (sec)	N/A	0.283	0.026	0.000	0.000	0.000	0.733	0.000	0.227	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	48	48	0	0	0	49	0	20	43
N.S.	1	1.23	1.23	0.00	0.00	0.00	1.26	0.00	0.51	1.10
time (sec)	N/A	0.265	0.004	0.000	0.000	0.000	0.633	0.000	0.231	0.677

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	39	67	26	0	23	0
N.S.	1	1.00	1.00	0.82	1.39	2.39	0.93	0.00	0.82	0.00
time (sec)	N/A	0.249	0.033	0.881	0.143	0.080	0.684	0.000	0.210	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	58	55	0	0	0	51	0	27	0
N.S.	1	1.18	1.12	0.00	0.00	0.00	1.04	0.00	0.55	0.00
time (sec)	N/A	0.285	0.031	0.000	0.000	0.000	0.734	0.000	0.204	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	57	0	0	0	53	0	27	0
N.S.	1	1.18	1.12	0.00	0.00	0.00	1.04	0.00	0.53	0.00
time (sec)	N/A	0.289	0.032	0.000	0.000	0.000	0.818	0.000	0.217	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	60	0	0	0	51	0	34	0
N.S.	1	1.25	1.25	0.00	0.00	0.00	1.06	0.00	0.71	0.00
time (sec)	N/A	0.281	0.025	0.000	0.000	0.000	0.749	0.000	0.234	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	51	0	0	0	49	0	33	43
N.S.	1	1.31	1.31	0.00	0.00	0.00	1.26	0.00	0.85	1.10
time (sec)	N/A	0.268	0.005	0.000	0.000	0.000	0.730	0.000	0.237	0.544

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	46	48	39	57	141	184	0	37	0
N.S.	1	0.96	1.00	0.81	1.19	2.94	3.83	0.00	0.77	0.00
time (sec)	N/A	0.278	0.054	0.538	0.110	0.086	1.272	0.000	0.207	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	61	58	0	0	0	51	0	43	0
N.S.	1	1.24	1.18	0.00	0.00	0.00	1.04	0.00	0.88	0.00
time (sec)	N/A	0.291	0.035	0.000	0.000	0.000	1.081	0.000	0.243	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	60	0	0	0	53	0	43	0
N.S.	1	1.24	1.18	0.00	0.00	0.00	1.04	0.00	0.84	0.00
time (sec)	N/A	0.286	0.037	0.000	0.000	0.000	1.266	0.000	0.226	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	60	0	0	0	51	0	47	0
N.S.	1	1.25	1.25	0.00	0.00	0.00	1.06	0.00	0.98	0.00
time (sec)	N/A	0.288	0.025	0.000	0.000	0.000	1.175	0.000	0.233	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	51	51	0	0	0	49	0	46	43
N.S.	1	1.31	1.31	0.00	0.00	0.00	1.26	0.00	1.18	1.10
time (sec)	N/A	0.267	0.005	0.000	0.000	0.000	1.172	0.000	0.241	0.575

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	53	67	227	860	0	51	0
N.S.	1	1.00	0.88	0.77	0.97	3.29	12.46	0.00	0.74	0.00
time (sec)	N/A	0.300	0.091	0.541	0.107	0.089	2.861	0.000	0.271	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	61	58	0	0	0	51	0	59	0
N.S.	1	1.24	1.18	0.00	0.00	0.00	1.04	0.00	1.20	0.00
time (sec)	N/A	0.289	0.040	0.000	0.000	0.000	1.948	0.000	0.238	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	60	0	0	0	53	0	59	0
N.S.	1	1.24	1.18	0.00	0.00	0.00	1.04	0.00	1.16	0.00
time (sec)	N/A	0.293	0.039	0.000	0.000	0.000	2.654	0.000	0.230	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	84	57	67	66	66	2572	0	65	0
N.S.	1	0.91	0.62	0.73	0.72	0.72	27.96	0.00	0.71	0.00
time (sec)	N/A	0.339	0.045	0.539	0.044	0.080	5.158	0.000	0.238	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	63	44	54	53	53	1015	0	52	0
N.S.	1	0.93	0.65	0.79	0.78	0.78	14.93	0.00	0.76	0.00
time (sec)	N/A	0.301	0.043	0.539	0.042	0.072	3.076	0.000	0.200	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	42	31	41	39	39	338	0	38	0
N.S.	1	0.95	0.70	0.93	0.89	0.89	7.68	0.00	0.86	0.00
time (sec)	N/A	0.290	0.038	0.539	0.042	0.093	1.906	0.000	0.211	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	48	17	23	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	2.29	0.81	1.10	0.81
time (sec)	N/A	0.232	0.021	0.533	0.026	0.071	1.369	0.117	0.237	0.590

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	43	42	36	54	88	76	0	41	0
N.S.	1	0.96	0.93	0.80	1.20	1.96	1.69	0.00	0.91	0.00
time (sec)	N/A	0.277	0.004	0.549	0.107	0.081	0.934	0.000	0.247	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	49	48	0	0	116	49	0	50	0
N.S.	1	0.96	0.94	0.00	0.00	2.27	0.96	0.00	0.98	0.00
time (sec)	N/A	0.276	0.087	0.000	0.000	0.083	2.408	0.000	0.209	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	77	69	0	0	150	112	0	75	0
N.S.	1	0.92	0.82	0.00	0.00	1.79	1.33	0.00	0.89	0.00
time (sec)	N/A	0.301	0.114	0.000	0.000	0.086	6.532	0.000	0.200	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	109	81	0	0	180	143	0	96	0
N.S.	1	0.96	0.72	0.00	0.00	1.59	1.27	0.00	0.85	0.00
time (sec)	N/A	0.328	0.158	0.000	0.000	0.097	17.298	0.000	0.219	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	141	95	0	0	206	178	0	117	0
N.S.	1	0.99	0.67	0.00	0.00	1.45	1.25	0.00	0.82	0.00
time (sec)	N/A	0.358	0.159	0.000	0.000	0.109	54.177	0.000	0.236	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	80	57	54	66	53	2422	0	52	0
N.S.	1	0.91	0.65	0.61	0.75	0.60	27.52	0.00	0.59	0.00
time (sec)	N/A	0.343	0.045	0.552	0.041	0.099	3.547	0.000	0.239	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	61	44	41	53	40	916	0	39	0
N.S.	1	0.92	0.67	0.62	0.80	0.61	13.88	0.00	0.59	0.00
time (sec)	N/A	0.320	0.041	0.543	0.041	0.093	2.176	0.000	0.217	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	40	30	28	39	26	275	0	25	0
N.S.	1	0.95	0.71	0.67	0.93	0.62	6.55	0.00	0.60	0.00
time (sec)	N/A	0.287	0.038	0.544	0.040	0.088	1.432	0.000	0.239	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	44	17	16	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	2.32	0.89	0.84	0.89
time (sec)	N/A	0.236	0.021	0.556	0.025	0.093	0.993	0.119	0.209	0.553

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	39	67	26	0	23	0
N.S.	1	1.00	1.00	0.82	1.39	2.39	0.93	0.00	0.82	0.00
time (sec)	N/A	0.267	0.001	0.543	0.105	0.081	0.692	0.000	0.212	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	53	0	0	116	49	0	54	0
N.S.	1	0.96	1.00	0.00	0.00	2.19	0.92	0.00	1.02	0.00
time (sec)	N/A	0.286	0.076	0.000	0.000	0.076	3.322	0.000	0.216	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	83	68	0	0	154	117	0	75	0
N.S.	1	0.95	0.78	0.00	0.00	1.77	1.34	0.00	0.86	0.00
time (sec)	N/A	0.321	0.118	0.000	0.000	0.087	9.807	0.000	0.286	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	115	81	0	0	180	150	0	96	0
N.S.	1	0.99	0.70	0.00	0.00	1.55	1.29	0.00	0.83	0.00
time (sec)	N/A	0.346	0.120	0.000	0.000	0.126	29.553	0.000	0.219	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	147	94	0	0	206	182	0	117	0
N.S.	1	1.01	0.65	0.00	0.00	1.42	1.26	0.00	0.81	0.00
time (sec)	N/A	0.371	0.141	0.000	0.000	0.086	95.588	0.000	0.207	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	102	129	104	108	106	46	0	43	0
N.S.	1	0.96	1.22	0.98	1.02	1.00	0.43	0.00	0.41	0.00
time (sec)	N/A	0.358	0.096	0.878	0.107	0.092	0.791	0.000	0.243	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	36	17	32	53	32	30	27
N.S.	1	1.00	1.00	1.89	0.89	1.68	2.79	1.68	1.58	1.42
time (sec)	N/A	0.253	0.003	0.536	0.024	0.079	0.311	0.121	0.235	0.572

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	22	19	21	17	19	31	19	17	15
N.S.	1	1.16	1.00	1.11	0.89	1.00	1.63	1.00	0.89	0.79
time (sec)	N/A	0.267	0.002	0.072	0.026	0.079	0.203	0.113	0.213	0.495

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	15	31	16	15	15
N.S.	1	1.00	1.00	1.20	1.00	1.00	2.07	1.07	1.00	1.00
time (sec)	N/A	0.234	0.001	0.561	0.024	0.076	1.029	0.126	0.221	0.003

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	17	65	17	19	21
N.S.	1	1.00	1.00	1.06	1.00	1.00	3.82	1.00	1.12	1.24
time (sec)	N/A	0.235	0.003	0.598	0.033	0.069	0.862	0.122	0.272	0.002

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	31	136	17	30	33
N.S.	1	1.00	1.00	0.95	0.89	1.63	7.16	0.89	1.58	1.74
time (sec)	N/A	0.235	0.003	0.763	0.032	0.066	1.767	0.121	0.213	0.003

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	62	55	66	88	61	46	60
N.S.	1	1.00	0.75	0.93	0.82	0.99	1.31	0.91	0.69	0.90
time (sec)	N/A	0.332	0.044	0.753	0.031	0.073	0.436	0.148	0.198	0.682

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	37	44	39	48	65	43	33	43
N.S.	1	1.00	0.76	0.90	0.80	0.98	1.33	0.88	0.67	0.88
time (sec)	N/A	0.300	0.043	0.540	0.031	0.096	0.308	0.128	0.200	0.575

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	26	23	29	37	24	19	19
N.S.	1	1.00	0.79	0.90	0.79	1.00	1.28	0.83	0.66	0.66
time (sec)	N/A	0.269	0.023	0.092	0.025	0.095	0.203	0.125	0.266	0.538

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	54	0	98	29	23	20	0
N.S.	1	1.00	1.00	1.59	0.00	2.88	0.85	0.68	0.59	0.00
time (sec)	N/A	0.251	0.027	0.630	0.000	0.078	0.494	0.127	0.222	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	62	56	86	0	194	163	45	34	0
N.S.	1	1.09	0.98	1.51	0.00	3.40	2.86	0.79	0.60	0.00
time (sec)	N/A	0.275	0.057	0.693	0.000	0.082	1.082	0.136	0.220	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	32	101	0	318	683	62	48	0
N.S.	1	1.04	0.36	1.13	0.00	3.57	7.67	0.70	0.54	0.00
time (sec)	N/A	0.301	0.031	0.887	0.000	0.088	3.179	0.141	0.212	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	50	62	55	66	90	58	46	60
N.S.	1	1.00	0.72	0.90	0.80	0.96	1.30	0.84	0.67	0.87
time (sec)	N/A	0.321	0.041	0.721	0.026	0.076	0.409	0.135	0.224	0.664

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	37	44	39	48	65	40	33	43
N.S.	1	1.00	0.76	0.90	0.80	0.98	1.33	0.82	0.67	0.88
time (sec)	N/A	0.296	0.036	0.559	0.031	0.076	0.337	0.132	0.215	0.583

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	26	23	29	37	21	19	19
N.S.	1	1.00	0.79	0.90	0.79	1.00	1.28	0.72	0.66	0.66
time (sec)	N/A	0.268	0.023	0.095	0.036	0.100	0.235	0.127	0.234	0.528

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	138	32	31	0	400	160	131	20	0
N.S.	1	0.97	0.22	0.22	0.00	2.80	1.12	0.92	0.14	0.00
time (sec)	N/A	0.477	0.028	0.684	0.000	0.095	1.012	0.132	0.239	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	32	61	0	575	751	150	34	0
N.S.	1	1.00	0.19	0.36	0.00	3.40	4.44	0.89	0.20	0.00
time (sec)	N/A	0.525	0.029	0.698	0.000	0.094	1.487	0.136	0.226	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	32	76	0	785	2531	162	48	0
N.S.	1	1.00	0.16	0.38	0.00	3.92	12.66	0.81	0.24	0.00
time (sec)	N/A	0.566	0.034	0.910	0.000	0.095	2.241	0.140	0.208	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	21	19	18	46	19	18	19
N.S.	1	1.00	1.32	1.11	1.00	0.95	2.42	1.00	0.95	1.00
time (sec)	N/A	0.242	0.027	0.650	0.051	0.071	0.629	0.124	0.209	0.529

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	32	27	35	109	26	31	27
N.S.	1	1.00	1.00	1.19	1.00	1.30	4.04	0.96	1.15	1.00
time (sec)	N/A	0.253	0.032	1.098	0.041	0.081	10.589	0.129	0.250	0.780

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	78	75	362	3546	622	477	77
N.S.	1	1.00	0.89	1.04	1.00	4.83	47.28	8.29	6.36	1.03
time (sec)	N/A	0.362	0.063	0.729	0.029	0.085	3.025	0.133	0.238	0.874

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	51	151	988	238	177	52
N.S.	1	1.00	0.90	1.04	1.00	2.96	19.37	4.67	3.47	1.02
time (sec)	N/A	0.323	0.054	0.565	0.039	0.081	1.340	0.126	0.207	0.747

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	27	43	151	55	38	27
N.S.	1	1.00	1.00	1.04	1.00	1.59	5.59	2.04	1.41	1.00
time (sec)	N/A	0.271	0.028	0.098	0.036	0.079	0.611	0.120	0.204	0.656

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	0	0	133	0	15	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	3.32	0.00	0.38	0.00
time (sec)	N/A	0.265	0.028	0.000	0.000	0.000	0.706	0.000	0.254	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	0	0	1068	0	28	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	26.70	0.00	0.70	0.00
time (sec)	N/A	0.263	0.028	0.000	0.000	0.000	1.294	0.000	0.210	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	0	0	4539	0	41	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	113.48	0.00	1.02	0.00
time (sec)	N/A	0.263	0.029	0.000	0.000	0.000	2.676	0.000	0.219	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	0	0	0	0	0	132	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	3.30	0.00
time (sec)	N/A	0.263	0.055	0.000	0.000	0.000	0.000	0.000	112.611	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	57	70	67	58	94	58	59	72
N.S.	1	1.00	0.80	0.99	0.94	0.82	1.32	0.82	0.83	1.01
time (sec)	N/A	0.349	0.059	0.814	0.080	0.074	0.450	0.137	0.236	0.780

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	50	48	42	68	42	43	38
N.S.	1	1.00	0.81	0.96	0.92	0.81	1.31	0.81	0.83	0.73
time (sec)	N/A	0.317	0.043	0.595	0.028	0.078	0.296	0.129	0.236	0.623

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	29	28	25	39	25	26	23
N.S.	1	1.00	0.87	0.97	0.93	0.83	1.30	0.83	0.87	0.77
time (sec)	N/A	0.272	0.026	0.092	0.043	0.078	0.207	0.130	0.221	0.568

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	91	124	0	31	0
N.S.	1	1.00	1.00	1.85	0.00	2.76	3.76	0.00	0.94	0.00
time (sec)	N/A	0.267	0.035	0.664	0.000	0.099	6.323	0.000	0.204	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	53	95	0	207	0	0	94	0
N.S.	1	0.94	0.79	1.42	0.00	3.09	0.00	0.00	1.40	0.00
time (sec)	N/A	0.275	0.039	0.701	0.000	0.083	0.000	0.000	0.242	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	96	35	110	0	336	0	0	181	0
N.S.	1	0.99	0.36	1.13	0.00	3.46	0.00	0.00	1.87	0.00
time (sec)	N/A	0.303	0.034	0.834	0.000	0.084	0.000	0.000	0.203	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	38	17	37	92	71	36	17
N.S.	1	1.00	1.00	1.81	0.81	1.76	4.38	3.38	1.71	0.81
time (sec)	N/A	0.233	0.025	0.578	0.036	0.078	11.559	0.119	0.208	0.631

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	48	17	23	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	2.29	0.81	1.10	0.81
time (sec)	N/A	0.227	0.004	0.535	0.034	0.073	1.255	0.118	0.211	0.003

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	44	17	16	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	2.32	0.89	0.84	0.89
time (sec)	N/A	0.228	0.003	0.566	0.056	0.080	1.056	0.119	0.218	0.003

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	B	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	17	26	46	17	25	17
N.S.	1	1.00	1.00	0.00	0.89	1.37	2.42	0.89	1.32	0.89
time (sec)	N/A	0.232	0.022	0.000	0.030	0.071	2.955	0.120	0.210	0.582

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	B	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	0	17	40	70	17	38	17
N.S.	1	1.00	1.00	0.00	0.81	1.90	3.33	0.81	1.81	0.81
time (sec)	N/A	0.230	0.026	0.000	0.026	0.068	19.331	0.128	0.203	0.627

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	65	66	0	0	0	73	0	725	0
N.S.	1	1.18	1.20	0.00	0.00	0.00	1.33	0.00	13.18	0.00
time (sec)	N/A	0.301	0.039	0.000	0.000	0.000	9.000	0.000	0.241	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	65	0	0	0	73	0	176	0
N.S.	1	1.16	1.18	0.00	0.00	0.00	1.33	0.00	3.20	0.00
time (sec)	N/A	0.295	0.031	0.000	0.000	0.000	1.139	0.000	0.219	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	64	65	0	0	0	73	0	23	0
N.S.	1	1.16	1.18	0.00	0.00	0.00	1.33	0.00	0.42	0.00
time (sec)	N/A	0.294	0.033	0.000	0.000	0.000	0.995	0.000	0.200	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	67	68	0	0	0	73	0	36	0
N.S.	1	1.22	1.24	0.00	0.00	0.00	1.33	0.00	0.65	0.00
time (sec)	N/A	0.307	0.042	0.000	0.000	0.000	1.557	0.000	0.215	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	67	68	0	0	0	73	0	49	0
N.S.	1	1.22	1.24	0.00	0.00	0.00	1.33	0.00	0.89	0.00
time (sec)	N/A	0.306	0.042	0.000	0.000	0.000	9.505	0.000	0.257	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	70	68	0	0	0	60	0	316	0
N.S.	1	1.19	1.15	0.00	0.00	0.00	1.02	0.00	5.36	0.00
time (sec)	N/A	0.322	0.041	0.000	0.000	0.000	2.521	0.000	0.221	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	65	65	0	0	0	58	0	100	0
N.S.	1	1.16	1.16	0.00	0.00	0.00	1.04	0.00	1.79	0.00
time (sec)	N/A	0.310	0.037	0.000	0.000	0.000	1.758	0.000	0.239	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	65	64	0	0	0	66	0	156	0
N.S.	1	1.16	1.14	0.00	0.00	0.00	1.18	0.00	2.79	0.00
time (sec)	N/A	0.327	0.041	0.000	0.000	0.000	1.779	0.000	0.211	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	72	68	0	0	0	71	0	397	0
N.S.	1	1.18	1.11	0.00	0.00	0.00	1.16	0.00	6.51	0.00
time (sec)	N/A	0.331	0.042	0.000	0.000	0.000	2.929	0.000	0.243	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	75	77	0	0	0	82	0	1514	0
N.S.	1	1.14	1.17	0.00	0.00	0.00	1.24	0.00	22.94	0.00
time (sec)	N/A	0.327	0.042	0.000	0.000	0.000	1.024	0.000	0.240	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	69	69	0	0	0	80	0	279	0
N.S.	1	1.15	1.15	0.00	0.00	0.00	1.33	0.00	4.65	0.00
time (sec)	N/A	0.329	0.043	0.000	0.000	0.000	0.891	0.000	0.216	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	72	72	0	0	0	75	0	29	0
N.S.	1	1.14	1.14	0.00	0.00	0.00	1.19	0.00	0.46	0.00
time (sec)	N/A	0.320	0.043	0.000	0.000	0.000	0.917	0.000	0.229	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	75	77	0	0	0	80	0	31	0
N.S.	1	1.14	1.17	0.00	0.00	0.00	1.21	0.00	0.47	0.00
time (sec)	N/A	0.328	0.044	0.000	0.000	0.000	0.932	0.000	0.251	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	A	F(-2)	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	126	111	0	13	0	114	0	21	0
N.S.	1	8.40	7.40	0.00	0.87	0.00	7.60	0.00	1.40	0.00
time (sec)	N/A	0.435	0.202	0.000	0.103	0.000	2.053	0.000	0.217	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	191	100	98	0	180	148	0	28	0
N.S.	1	1.48	0.78	0.76	0.00	1.40	1.15	0.00	0.22	0.00
time (sec)	N/A	0.371	0.180	0.782	0.000	0.091	10.213	0.000	0.250	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	139	100	82	0	155	116	0	28	0
N.S.	1	1.42	1.02	0.84	0.00	1.58	1.18	0.00	0.29	0.00
time (sec)	N/A	0.334	0.070	0.767	0.000	0.088	3.157	0.000	0.258	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	85	81	64	0	119	49	0	28	0
N.S.	1	1.37	1.31	1.03	0.00	1.92	0.79	0.00	0.45	0.00
time (sec)	N/A	0.290	0.062	0.790	0.000	0.086	1.564	0.000	0.218	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	A	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	0	0	0	24	89	28	0
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.69	2.54	0.80	0.00
time (sec)	N/A	0.261	0.036	0.000	0.000	0.000	0.780	0.454	0.213	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	22	0	23	24
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.85	0.00	0.88	0.92
time (sec)	N/A	0.242	0.032	0.000	0.000	0.000	0.574	0.000	0.233	0.604

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	0	0	0	51	0	33	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.88	0.00	0.57	0.00
time (sec)	N/A	0.291	0.040	0.000	0.000	0.000	0.658	0.000	0.239	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	95	51	0	0	0	354	0	30	0
N.S.	1	1.07	0.57	0.00	0.00	0.00	3.98	0.00	0.34	0.00
time (sec)	N/A	0.348	0.040	0.000	0.000	0.000	0.823	0.000	0.226	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	132	64	0	0	0	605	0	30	0
N.S.	1	1.10	0.53	0.00	0.00	0.00	5.04	0.00	0.25	0.00
time (sec)	N/A	0.405	0.047	0.000	0.000	0.000	1.085	0.000	0.213	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	128	88	0	0	0	107	0	159	0
N.S.	1	0.94	0.65	0.00	0.00	0.00	0.79	0.00	1.17	0.00
time (sec)	N/A	0.340	0.083	0.000	0.000	0.000	53.464	0.000	0.291	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	98	86	0	0	0	107	0	130	0
N.S.	1	0.94	0.83	0.00	0.00	0.00	1.03	0.00	1.25	0.00
time (sec)	N/A	0.329	0.058	0.000	0.000	0.000	6.007	0.000	0.221	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	68	85	0	0	0	107	0	98	0
N.S.	1	0.94	1.18	0.00	0.00	0.00	1.49	0.00	1.36	0.00
time (sec)	N/A	0.290	0.047	0.000	0.000	0.000	1.208	0.000	0.208	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	66	0	0	0	107	0	33	0
N.S.	1	1.00	1.74	0.00	0.00	0.00	2.82	0.00	0.87	0.00
time (sec)	N/A	0.274	0.055	0.000	0.000	0.000	0.915	0.000	0.227	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	C	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	47	107	0	49	27
N.S.	1	1.00	1.00	0.00	0.00	1.62	3.69	0.00	1.69	0.93
time (sec)	N/A	0.252	0.046	0.000	0.000	0.077	1.567	0.000	0.241	0.557

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	C	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	0	0	93	107	0	98	39
N.S.	1	1.00	0.71	0.00	0.00	1.43	1.65	0.00	1.51	0.60
time (sec)	N/A	0.339	0.080	0.000	0.000	0.085	6.311	0.000	0.226	0.643

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	C	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	106	61	0	0	135	107	0	147	54
N.S.	1	1.09	0.63	0.00	0.00	1.39	1.10	0.00	1.52	0.56
time (sec)	N/A	0.405	0.085	0.000	0.000	0.109	57.401	0.000	0.248	0.594

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	48	16	24	18
N.S.	1	1.00	1.00	0.85	0.80	0.80	2.40	0.80	1.20	0.90
time (sec)	N/A	0.243	0.023	0.624	0.024	0.071	0.554	0.118	0.219	0.534

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	58	18	28	20
N.S.	1	1.00	1.00	0.86	0.82	0.82	2.64	0.82	1.27	0.91
time (sec)	N/A	0.242	0.026	0.598	0.025	0.076	0.493	0.121	0.208	0.538

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	81	75	0	0	0	110	0	32	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	1.38	0.00	0.40	0.00
time (sec)	N/A	0.358	0.143	0.000	0.000	0.000	1.364	0.000	0.207	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	79	0	0	0	128	0	22	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.58	0.00	0.27	0.00
time (sec)	N/A	0.382	0.125	0.000	0.000	0.000	1.280	0.000	0.206	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	A	F(-2)	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	160	128	40	37	0	194	0	25	0
N.S.	1	6.15	4.92	1.54	1.42	0.00	7.46	0.00	0.96	0.00
time (sec)	N/A	0.506	0.734	0.761	0.115	0.000	2.154	0.000	0.246	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	A	A	C	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	111	0	13	16	122	0	21	13
N.S.	1	1.00	7.40	0.00	0.87	1.07	8.13	0.00	1.40	0.87
time (sec)	N/A	0.271	0.018	0.000	0.091	0.077	6.047	0.000	0.274	0.718

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	A	F(-2)	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	126	111	0	13	0	114	0	21	0
N.S.	1	8.40	7.40	0.00	0.87	0.00	7.60	0.00	1.40	0.00
time (sec)	N/A	0.423	0.001	0.000	0.094	0.000	2.105	0.000	0.230	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	92	67	0	0	0	107	0	20	0
N.S.	1	0.95	0.69	0.00	0.00	0.00	1.10	0.00	0.21	0.00
time (sec)	N/A	0.328	0.087	0.000	0.000	0.000	0.955	0.000	0.222	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	143	347	0	0	0	131	0	96	0
N.S.	1	1.03	2.50	0.00	0.00	0.00	0.94	0.00	0.69	0.00
time (sec)	N/A	0.439	0.672	0.000	0.000	0.000	4.621	0.000	0.335	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	87	56	0	0	0	39	0	81	0
N.S.	1	0.98	0.63	0.00	0.00	0.00	0.44	0.00	0.91	0.00
time (sec)	N/A	0.352	0.039	0.000	0.000	0.000	0.772	0.000	0.237	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	58	0	0	0	46	0	22	0
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.40	0.00	0.19	0.00
time (sec)	N/A	0.380	0.041	0.000	0.000	0.000	4.449	0.000	0.224	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	63	0	0	0	70	0	173	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	1.13	0.00	2.79	0.00
time (sec)	N/A	0.327	0.039	0.000	0.000	0.000	5.634	0.000	0.271	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	23	27	78	23	28	23
N.S.	1	1.00	1.00	1.26	1.00	1.17	3.39	1.00	1.22	1.00
time (sec)	N/A	0.256	0.024	1.003	0.038	0.074	11.470	0.128	0.239	0.873

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	25	24	42	24	20	21
N.S.	1	1.00	1.00	1.00	1.19	1.14	2.00	1.14	0.95	1.00
time (sec)	N/A	0.246	0.003	0.099	0.044	0.075	1.186	0.120	0.215	0.590

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	32	22	19	18
N.S.	1	1.00	1.00	1.06	1.06	1.22	1.78	1.22	1.06	1.00
time (sec)	N/A	0.237	0.002	0.056	0.040	0.079	0.687	0.132	0.231	0.532

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	19	22	32	22	19	18
N.S.	1	1.00	1.00	1.06	1.06	1.22	1.78	1.22	1.06	1.00
time (sec)	N/A	0.238	0.002	0.049	0.043	0.087	0.607	0.115	0.219	0.544

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	29	20	17	16
N.S.	1	1.00	1.00	1.06	1.06	1.25	1.81	1.25	1.06	1.00
time (sec)	N/A	0.230	0.002	0.051	0.044	0.073	0.430	0.130	0.223	1.140

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	15	18	20	14	15	0
N.S.	1	1.00	1.00	1.07	1.07	1.29	1.43	1.00	1.07	0.00
time (sec)	N/A	0.237	0.001	0.061	0.043	0.072	0.134	0.127	0.205	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	27	0	19	0
N.S.	1	1.00	0.90	0.95	0.95	1.10	1.35	0.00	0.95	0.00
time (sec)	N/A	0.238	0.003	0.050	0.046	0.089	0.534	0.000	0.217	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	34	0	19	0
N.S.	1	1.00	0.90	0.95	0.95	1.10	1.70	0.00	0.95	0.00
time (sec)	N/A	0.238	0.002	0.056	0.037	0.076	0.655	0.000	0.216	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	19	22	34	0	19	0
N.S.	1	1.00	0.90	0.95	0.95	1.10	1.70	0.00	0.95	0.00
time (sec)	N/A	0.238	0.002	0.073	0.039	0.075	0.994	0.000	0.260	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	23	27	78	23	28	23
N.S.	1	1.00	1.00	1.26	1.00	1.17	3.39	1.00	1.22	1.00
time (sec)	N/A	0.247	0.000	1.099	0.026	0.079	11.580	0.127	0.205	0.003

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	40	61	51	62	274	0	58	0
N.S.	1	0.96	0.82	1.24	1.04	1.27	5.59	0.00	1.18	0.00
time (sec)	N/A	0.319	0.056	1.033	0.041	0.082	31.501	0.000	0.244	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	70	66	105	79	108	770	0	104	0
N.S.	1	0.93	0.88	1.40	1.05	1.44	10.27	0.00	1.39	0.00
time (sec)	N/A	0.371	0.076	1.148	0.042	0.083	95.968	0.000	0.220	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	95	78	171	114	158	0	0	167	0
N.S.	1	0.92	0.76	1.66	1.11	1.53	0.00	0.00	1.62	0.00
time (sec)	N/A	0.405	0.108	1.099	0.042	0.083	0.000	0.000	0.212	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	70	0	53	39	0	38	0
N.S.	1	1.00	1.00	2.19	0.00	1.66	1.22	0.00	1.19	0.00
time (sec)	N/A	0.270	0.067	0.879	0.000	0.081	3.472	0.000	0.194	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	113	136	151	111	221	111	113	151
N.S.	1	1.00	4.71	5.67	6.29	4.62	9.21	4.62	4.71	6.29
time (sec)	N/A	0.255	0.011	7.520	0.039	0.075	2.106	0.163	0.275	0.004

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	37	34	36	0	36	52
N.S.	1	1.00	0.97	0.97	1.23	1.13	1.20	0.00	1.20	1.73
time (sec)	N/A	0.266	0.031	0.985	0.036	0.081	110.567	0.000	0.226	0.669

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	108	91	90	90	97	90	92	92
N.S.	1	1.00	5.68	4.79	4.74	4.74	5.11	4.74	4.84	4.84
time (sec)	N/A	0.238	0.006	0.498	0.025	0.064	0.687	0.116	0.226	0.425

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	26	25	23	28	22	41	0	22	22
N.S.	1	1.13	1.09	1.00	1.22	0.96	1.78	0.00	0.96	0.96
time (sec)	N/A	0.256	0.005	0.562	0.034	0.069	0.391	0.000	0.208	0.002

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	23	18	15	22	45	18
N.S.	1	1.18	1.00	0.95	1.05	0.82	0.68	1.00	2.05	0.82
time (sec)	N/A	0.250	0.004	0.531	0.040	0.066	0.149	0.113	0.214	0.092

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	32	15	39	0	15	22
N.S.	1	1.00	1.00	1.07	2.13	1.00	2.60	0.00	1.00	1.47
time (sec)	N/A	0.260	0.017	0.634	0.033	0.079	0.465	0.000	0.209	0.477

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	70	64	47	201	71	172	1148	613	180	89
N.S.	1	0.91	0.67	2.87	1.01	2.46	16.40	8.76	2.57	1.27
time (sec)	N/A	0.347	0.029	0.585	0.036	0.088	1.673	0.128	0.203	0.500

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	57	56	87	337	135	94	95
N.S.	1	1.00	0.71	0.98	0.97	1.50	5.81	2.33	1.62	1.64
time (sec)	N/A	0.332	0.029	0.458	0.037	0.078	0.527	0.127	0.265	0.474

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	57	56	87	330	135	94	95
N.S.	1	1.00	0.71	0.98	0.97	1.50	5.69	2.33	1.62	1.64
time (sec)	N/A	0.328	0.042	0.460	0.065	0.087	0.372	0.125	0.239	0.470

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	39	57	56	87	323	135	88	95
N.S.	1	1.00	0.67	0.98	0.97	1.50	5.57	2.33	1.52	1.64
time (sec)	N/A	0.330	0.039	0.450	0.036	0.085	0.287	0.120	0.216	0.642

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	56	51	63	192	0	65	45
N.S.	1	1.00	0.69	1.08	0.98	1.21	3.69	0.00	1.25	0.87
time (sec)	N/A	0.320	0.057	0.054	0.039	0.082	0.314	0.000	0.212	0.460

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	64	60	56	81	388	0	90	89
N.S.	1	1.00	1.05	0.98	0.92	1.33	6.36	0.00	1.48	1.46
time (sec)	N/A	0.339	0.047	0.073	0.041	0.072	0.383	0.000	0.247	0.475

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	67	60	56	87	449	0	96	97
N.S.	1	1.00	1.06	0.95	0.89	1.38	7.13	0.00	1.52	1.54
time (sec)	N/A	0.338	0.049	0.080	0.040	0.072	0.494	0.000	0.240	0.460

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	180	39	0	0	329	233	0	32	0
N.S.	1	0.81	0.18	0.00	0.00	1.48	1.05	0.00	0.14	0.00
time (sec)	N/A	0.587	0.011	0.000	0.000	0.276	1.206	0.000	0.219	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	279	39	0	0	404	318	0	32	0
N.S.	1	1.18	0.16	0.00	0.00	1.70	1.34	0.00	0.14	0.00
time (sec)	N/A	0.800	0.011	0.000	0.000	0.370	1.371	0.000	0.219	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	51	43	0	63	59	36	0	46	0
N.S.	1	0.74	0.62	0.00	0.91	0.86	0.52	0.00	0.67	0.00
time (sec)	N/A	0.333	0.012	0.000	0.035	0.084	1.028	0.000	0.247	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	67	37	0	0	237	58	0	32	0
N.S.	1	0.91	0.50	0.00	0.00	3.20	0.78	0.00	0.43	0.00
time (sec)	N/A	0.320	0.010	0.000	0.000	0.103	0.798	0.000	0.209	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	175	37	0	0	255	223	0	32	0
N.S.	1	0.80	0.17	0.00	0.00	1.16	1.01	0.00	0.15	0.00
time (sec)	N/A	0.571	0.010	0.000	0.000	0.092	1.321	0.000	0.205	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	274	37	0	0	298	308	0	32	0
N.S.	1	1.17	0.16	0.00	0.00	1.27	1.32	0.00	0.14	0.00
time (sec)	N/A	0.779	0.010	0.000	0.000	0.091	1.506	0.000	0.257	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	90	39	0	0	504	87	0	32	0
N.S.	1	0.90	0.39	0.00	0.00	5.04	0.87	0.00	0.32	0.00
time (sec)	N/A	0.361	0.010	0.000	0.000	0.167	1.332	0.000	0.237	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	198	39	0	0	351	280	0	32	0
N.S.	1	0.80	0.16	0.00	0.00	1.43	1.14	0.00	0.13	0.00
time (sec)	N/A	0.635	0.011	0.000	0.000	0.340	1.252	0.000	0.228	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	297	39	0	0	430	371	0	32	0
N.S.	1	1.14	0.15	0.00	0.00	1.65	1.42	0.00	0.12	0.00
time (sec)	N/A	0.822	0.011	0.000	0.000	1.078	1.320	0.000	0.236	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	117	0	33	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.66	0.00	0.75	0.00
time (sec)	N/A	0.278	0.031	0.000	0.000	0.000	0.638	0.000	0.220	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	0	0	117	0	33	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	2.54	0.00	0.72	0.00
time (sec)	N/A	0.271	0.032	0.000	0.000	0.000	0.768	0.000	0.226	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	117	0	33	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.66	0.00	0.75	0.00
time (sec)	N/A	0.270	0.032	0.000	0.000	0.000	0.625	0.000	0.223	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	0	0	117	0	29	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	2.66	0.00	0.66	0.00
time (sec)	N/A	0.275	0.029	0.000	0.000	0.000	0.598	0.000	0.211	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	0	0	0	66	0	23	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	1.65	0.00	0.58	0.00
time (sec)	N/A	0.263	0.042	0.000	0.000	0.000	0.600	0.000	0.251	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	0	24	20	17	0	21	0
N.S.	1	1.00	1.04	0.00	0.86	0.71	0.61	0.00	0.75	0.00
time (sec)	N/A	0.256	0.005	0.000	0.046	0.079	0.541	0.000	0.206	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	0	0	0	63	0	37	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	1.26	0.00	0.74	0.00
time (sec)	N/A	0.274	0.039	0.000	0.000	0.000	0.604	0.000	0.203	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	0	0	0	66	0	40	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	1.27	0.00	0.77	0.00
time (sec)	N/A	0.279	0.030	0.000	0.000	0.000	0.653	0.000	0.225	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	31	25	22	22	73	0	25	29
N.S.	1	1.00	1.29	1.04	0.92	0.92	3.04	0.00	1.04	1.21
time (sec)	N/A	0.240	0.011	0.635	0.035	0.087	0.820	0.000	0.240	0.431

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	210	133	0	0	236	190	0	37	0
N.S.	1	1.18	0.75	0.00	0.00	1.33	1.07	0.00	0.21	0.00
time (sec)	N/A	0.400	0.115	0.000	0.000	0.108	9.437	0.000	0.213	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	158	121	0	0	197	150	0	37	0
N.S.	1	1.15	0.88	0.00	0.00	1.44	1.09	0.00	0.27	0.00
time (sec)	N/A	0.350	0.060	0.000	0.000	0.087	3.273	0.000	0.212	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	104	110	0	0	147	66	0	37	0
N.S.	1	1.14	1.21	0.00	0.00	1.62	0.73	0.00	0.41	0.00
time (sec)	N/A	0.330	0.044	0.000	0.000	0.090	1.645	0.000	0.204	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	78	0	0	0	31	0	37	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.57	0.00	0.69	0.00
time (sec)	N/A	0.316	0.045	0.000	0.000	0.000	0.922	0.000	0.241	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	0	31	0	33	29
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	1.06	0.94
time (sec)	N/A	0.242	0.010	0.000	0.000	0.000	0.745	0.000	0.210	0.457

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	65	41	0	0	0	71	0	43	0
N.S.	1	0.92	0.58	0.00	0.00	0.00	1.00	0.00	0.61	0.00
time (sec)	N/A	0.295	0.014	0.000	0.000	0.000	0.688	0.000	0.215	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	104	56	0	0	0	405	0	41	0
N.S.	1	0.93	0.50	0.00	0.00	0.00	3.62	0.00	0.37	0.00
time (sec)	N/A	0.367	0.017	0.000	0.000	0.000	0.978	0.000	0.208	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	145	69	0	0	0	677	0	41	0
N.S.	1	0.95	0.45	0.00	0.00	0.00	4.42	0.00	0.27	0.00
time (sec)	N/A	0.421	0.019	0.000	0.000	0.000	1.179	0.000	0.241	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	0	0	0	73	0	176	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.09	0.00	2.63	0.00
time (sec)	N/A	0.310	0.013	0.000	0.000	0.000	6.922	0.000	0.208	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	0	38	38	29	0	34	0
N.S.	1	1.00	0.94	0.00	1.06	1.06	0.81	0.00	0.94	0.00
time (sec)	N/A	0.277	0.014	0.000	0.044	0.136	5.925	0.000	0.200	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	62	0	0	0	63	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.323	0.031	0.000	0.000	0.000	5.165	0.000	0.214	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	62	0	0	0	63	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.323	0.032	0.000	0.000	0.000	5.539	0.000	0.272	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	0	0	0	60	0	509	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.97	0.00	8.21	0.00
time (sec)	N/A	0.318	0.026	0.000	0.000	0.000	5.139	0.000	0.223	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	46	0	96	47
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	2.09	1.02
time (sec)	N/A	0.282	0.003	0.000	0.000	0.000	1.122	0.000	0.208	1.315

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	59	0	0	0	70	0	238	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.09	0.00	3.72	0.00
time (sec)	N/A	0.323	0.031	0.000	0.000	0.000	4.975	0.000	0.272	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	0	0	0	76	0	266	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	1.15	0.00	4.03	0.00
time (sec)	N/A	0.335	0.040	0.000	0.000	0.000	5.720	0.000	0.215	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	0	0	0	76	0	266	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	1.15	0.00	4.03	0.00
time (sec)	N/A	0.341	0.031	0.000	0.000	0.000	5.505	0.000	0.213	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	78	68	0	0	0	66	0	83	0
N.S.	1	1.11	0.97	0.00	0.00	0.00	0.94	0.00	1.19	0.00
time (sec)	N/A	0.375	0.049	0.000	0.000	0.000	6.241	0.000	0.237	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	52	63	0	0	0	46	0	33	0
N.S.	1	0.83	1.00	0.00	0.00	0.00	0.73	0.00	0.52	0.00
time (sec)	N/A	0.332	0.031	0.000	0.000	0.000	2.929	0.000	0.238	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	74	0	75	49	0	50	0
N.S.	1	1.00	1.00	2.00	0.00	2.03	1.32	0.00	1.35	0.00
time (sec)	N/A	0.266	0.020	0.823	0.000	0.081	5.739	0.000	0.220	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	79	69	0	0	144	172	0	80	0
N.S.	1	0.95	0.83	0.00	0.00	1.73	2.07	0.00	0.96	0.00
time (sec)	N/A	0.358	0.072	0.000	0.000	0.132	5.713	0.000	0.230	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	127	69	0	0	215	566	0	120	0
N.S.	1	0.91	0.50	0.00	0.00	1.55	4.07	0.00	0.86	0.00
time (sec)	N/A	0.454	0.077	0.000	0.000	0.087	6.254	0.000	0.216	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	175	69	0	0	294	1168	0	175	0
N.S.	1	0.88	0.35	0.00	0.00	1.48	5.87	0.00	0.88	0.00
time (sec)	N/A	0.535	0.072	0.000	0.000	0.081	6.712	0.000	0.208	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	0	0	103	131	0	72	0
N.S.	1	1.00	0.84	0.00	0.00	1.24	1.58	0.00	0.87	0.00
time (sec)	N/A	0.367	0.074	0.000	0.000	0.079	28.921	0.000	0.199	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	90	75	0	0	154	172	0	87	0
N.S.	1	0.96	0.80	0.00	0.00	1.64	1.83	0.00	0.93	0.00
time (sec)	N/A	0.366	0.016	0.000	0.000	0.090	34.348	0.000	0.208	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [77] had the largest ratio of [.8000000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	13	0.154
2	A	2	2	1.00	13	0.154
3	A	2	2	1.00	13	0.154
4	A	2	2	1.00	11	0.182
5	A	1	1	1.00	9	0.111
6	A	2	2	1.00	13	0.154
7	A	2	2	1.00	13	0.154
8	A	2	2	1.00	13	0.154
9	A	2	2	1.00	13	0.154
10	A	4	3	1.06	15	0.200
11	A	4	3	1.06	15	0.200
12	A	4	3	1.06	15	0.200
13	A	4	3	1.06	13	0.231
14	A	4	3	1.19	11	0.273
15	A	4	3	1.43	15	0.200
16	A	4	3	1.33	15	0.200
17	A	4	3	1.13	15	0.200
18	A	4	3	1.06	15	0.200
19	A	4	3	1.06	15	0.200
20	A	4	3	1.04	15	0.200
21	A	4	3	1.04	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	3	1.04	15	0.200
23	A	4	3	1.11	13	0.231
24	A	4	3	1.05	11	0.273
25	A	4	3	1.22	15	0.200
26	A	4	3	1.18	15	0.200
27	A	1	1	1.00	15	0.067
28	A	4	3	1.04	15	0.200
29	A	4	3	1.09	15	0.200
30	A	4	3	1.04	15	0.200
31	A	4	3	1.03	15	0.200
32	A	4	3	1.01	15	0.200
33	A	4	3	1.07	15	0.200
34	A	4	3	1.02	13	0.231
35	A	4	3	1.05	11	0.273
36	A	4	3	1.09	15	0.200
37	A	4	3	1.16	15	0.200
38	A	4	3	1.08	15	0.200
39	A	1	1	1.00	15	0.067
40	A	5	4	1.11	15	0.267
41	A	4	3	1.03	15	0.200
42	A	4	3	1.03	15	0.200
43	A	4	3	1.03	15	0.200
44	A	4	3	1.02	15	0.200
45	A	4	3	1.02	15	0.200
46	A	4	3	1.02	13	0.231
47	A	4	3	1.05	11	0.273
48	A	4	3	1.08	15	0.200
49	A	4	3	1.10	15	0.200
50	A	4	3	1.06	15	0.200
51	A	4	3	1.06	15	0.200
52	A	4	3	1.11	15	0.200
53	A	4	3	1.08	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	1.04	15	0.200
55	A	6	5	1.15	15	0.333
56	A	8	7	1.18	15	0.467
57	A	4	3	1.04	15	0.200
58	A	4	3	1.03	15	0.200
59	A	4	3	1.02	15	0.200
60	A	4	3	1.01	15	0.200
61	A	4	3	1.01	15	0.200
62	A	4	3	1.02	15	0.200
63	A	4	3	1.02	13	0.231
64	A	4	3	1.05	11	0.273
65	A	4	3	1.04	15	0.200
66	A	4	3	1.07	15	0.200
67	A	4	3	1.07	15	0.200
68	A	4	3	1.05	15	0.200
69	A	4	3	1.06	15	0.200
70	A	4	3	1.06	15	0.200
71	A	4	3	1.06	15	0.200
72	A	1	1	1.00	15	0.067
73	A	5	4	1.11	15	0.267
74	A	7	6	1.17	15	0.400
75	A	9	8	1.19	15	0.533
76	A	11	10	1.20	15	0.667
77	A	13	12	1.21	15	0.800
78	A	4	3	1.02	15	0.200
79	A	4	3	1.03	15	0.200
80	A	4	3	1.04	15	0.200
81	A	4	3	1.03	15	0.200
82	A	4	3	1.04	15	0.200
83	A	4	3	1.06	13	0.231
84	A	4	3	1.04	11	0.273
85	A	5	4	1.27	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.15	15	0.200
87	A	4	3	1.09	15	0.200
88	A	4	3	1.07	15	0.200
89	A	4	3	1.03	15	0.200
90	A	4	3	1.04	15	0.200
91	A	4	3	1.07	13	0.231
92	A	4	3	1.00	11	0.273
93	A	4	3	1.13	15	0.200
94	A	4	3	1.10	15	0.200
95	A	4	3	1.07	15	0.200
96	A	4	3	1.06	15	0.200
97	A	4	3	1.04	15	0.200
98	A	4	3	1.08	15	0.200
99	A	4	3	1.06	13	0.231
100	A	1	1	1.00	11	0.091
101	A	4	3	1.15	15	0.200
102	A	4	3	1.13	15	0.200
103	A	4	3	1.07	15	0.200
104	A	4	3	1.09	15	0.200
105	A	4	3	1.03	15	0.200
106	A	4	3	1.05	15	0.200
107	A	4	3	1.01	15	0.200
108	A	1	1	1.00	13	0.077
109	A	4	3	1.05	11	0.273
110	A	4	3	1.09	15	0.200
111	A	4	3	1.06	15	0.200
112	A	4	3	1.06	15	0.200
113	A	4	3	1.01	15	0.200
114	A	4	3	1.04	15	0.200
115	A	4	3	1.02	15	0.200
116	A	4	3	1.05	15	0.200
117	A	4	3	1.05	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	3	1.05	11	0.273
119	A	4	3	1.06	15	0.200
120	A	4	3	1.08	15	0.200
121	A	4	3	1.03	15	0.200
122	A	5	4	1.42	15	0.267
123	A	4	3	1.02	17	0.176
124	A	4	3	1.02	15	0.200
125	A	4	3	1.05	13	0.231
126	A	5	4	1.05	17	0.235
127	A	6	5	1.03	17	0.294
128	A	8	7	1.09	17	0.412
129	A	4	3	1.02	17	0.176
130	A	4	3	1.02	15	0.200
131	A	4	3	1.05	13	0.231
132	A	4	3	1.00	17	0.176
133	A	6	5	1.06	17	0.294
134	A	8	7	1.11	17	0.412
135	A	4	3	1.00	17	0.176
136	A	1	1	1.00	17	0.059
137	A	2	2	1.00	13	0.154
138	A	1	1	1.00	15	0.067
139	A	1	1	1.00	15	0.067
140	A	4	3	1.21	15	0.200
141	A	1	1	1.00	15	0.067
142	A	1	1	1.00	15	0.067
143	A	1	1	1.00	15	0.067
144	A	4	3	1.04	17	0.176
145	A	1	1	1.00	17	0.059
146	A	8	7	1.16	15	0.467
147	A	2	2	1.00	15	0.133
148	A	2	2	1.00	15	0.133
149	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
150	A	2	2	1.00	13	0.154
151	A	3	2	1.00	15	0.133
152	A	3	2	1.00	15	0.133
153	A	4	3	1.21	15	0.200
154	A	4	3	1.00	15	0.200
155	A	4	3	1.01	15	0.200
156	A	4	3	1.01	13	0.231
157	A	4	3	1.02	11	0.273
158	A	3	2	1.00	15	0.133
159	A	3	2	1.00	15	0.133
160	A	1	1	1.00	15	0.067
161	A	5	4	1.08	17	0.235
162	A	3	2	1.05	13	0.154
163	A	2	2	1.00	17	0.118
164	A	4	4	1.12	17	0.235
165	A	6	6	1.15	17	0.353
166	A	4	3	1.03	17	0.176
167	A	4	3	1.05	17	0.176
168	A	4	3	1.10	17	0.176
169	A	6	5	1.08	17	0.294
170	A	8	7	1.05	17	0.412
171	A	4	3	1.00	17	0.176
172	A	4	3	1.00	15	0.200
173	A	4	3	1.00	17	0.176
174	A	4	3	1.00	17	0.176
175	A	6	5	1.04	17	0.294
176	A	2	2	1.00	13	0.154
177	A	2	2	1.00	13	0.154
178	A	2	2	1.00	13	0.154
179	A	2	2	1.00	11	0.182
180	A	1	1	1.00	9	0.111
181	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.00	13	0.154
183	A	2	2	1.00	13	0.154
184	A	2	2	1.00	13	0.154
185	A	4	3	1.06	15	0.200
186	A	4	3	1.06	15	0.200
187	A	4	3	1.06	15	0.200
188	A	4	3	1.06	13	0.231
189	A	4	3	1.17	11	0.273
190	A	4	3	1.21	15	0.200
191	A	1	1	1.00	15	0.067
192	A	4	3	1.06	15	0.200
193	A	4	3	1.06	15	0.200
194	A	4	3	1.04	15	0.200
195	A	4	3	1.04	15	0.200
196	A	4	3	1.04	15	0.200
197	A	4	3	1.04	13	0.231
198	A	4	3	1.12	11	0.273
199	A	4	3	1.25	15	0.200
200	A	4	3	1.21	15	0.200
201	A	4	3	1.09	15	0.200
202	A	4	3	1.04	15	0.200
203	A	4	3	1.03	15	0.200
204	A	4	3	1.05	15	0.200
205	A	4	3	1.03	15	0.200
206	A	4	3	1.03	13	0.231
207	A	4	3	1.03	11	0.273
208	A	4	3	1.12	15	0.200
209	A	4	3	1.12	15	0.200
210	A	1	1	1.00	15	0.067
211	A	4	3	1.05	15	0.200
212	A	4	3	1.03	15	0.200
213	A	4	3	1.03	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	4	3	1.05	15	0.200
215	A	4	3	1.03	15	0.200
216	A	4	3	1.03	15	0.200
217	A	4	3	1.03	15	0.200
218	A	4	3	1.03	13	0.231
219	A	4	3	1.03	11	0.273
220	A	4	3	1.04	15	0.200
221	A	4	3	1.08	15	0.200
222	A	4	3	1.06	15	0.200
223	A	4	3	1.04	15	0.200
224	A	4	3	1.04	15	0.200
225	A	7	6	1.16	15	0.400
226	A	4	3	1.01	15	0.200
227	A	4	3	1.03	15	0.200
228	A	4	3	1.04	15	0.200
229	A	4	3	1.01	15	0.200
230	A	4	3	1.02	15	0.200
231	A	4	3	1.02	15	0.200
232	A	4	3	1.01	15	0.200
233	A	4	3	1.02	15	0.200
234	A	4	3	1.02	13	0.231
235	A	4	3	1.03	11	0.273
236	A	4	3	1.04	15	0.200
237	A	4	3	1.05	15	0.200
238	A	4	3	1.05	15	0.200
239	A	4	3	1.05	15	0.200
240	A	4	3	1.04	15	0.200
241	A	5	4	1.11	15	0.267
242	A	8	7	1.18	15	0.467
243	A	11	10	1.20	15	0.667
244	A	4	3	1.03	15	0.200
245	A	4	3	1.02	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	4	3	1.02	15	0.200
247	A	4	3	1.02	15	0.200
248	A	4	3	1.02	15	0.200
249	A	4	3	1.05	13	0.231
250	A	4	3	1.02	11	0.273
251	A	5	4	1.27	15	0.267
252	A	4	3	1.11	15	0.200
253	A	4	3	1.08	15	0.200
254	A	4	3	1.05	15	0.200
255	A	5	4	1.29	15	0.267
256	A	4	3	1.02	15	0.200
257	A	4	3	1.05	15	0.200
258	A	4	3	1.04	13	0.231
259	A	4	3	1.02	11	0.273
260	A	4	3	1.13	15	0.200
261	A	4	3	1.09	15	0.200
262	A	4	3	1.06	15	0.200
263	A	4	3	1.06	15	0.200
264	A	4	3	1.04	15	0.200
265	A	4	3	1.03	15	0.200
266	A	4	3	1.06	13	0.231
267	A	4	3	1.02	11	0.273
268	A	4	3	1.09	15	0.200
269	A	4	3	1.08	15	0.200
270	A	4	3	1.05	15	0.200
271	A	4	3	1.05	15	0.200
272	A	4	3	1.10	11	0.273
273	A	6	5	1.17	15	0.333
274	A	4	3	1.05	15	0.200
275	A	4	3	1.00	9	0.333
276	A	1	1	1.00	15	0.067
277	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	1	1	1.00	17	0.059
279	A	1	1	1.00	17	0.059
280	A	13	12	1.49	15	0.800
281	A	11	10	1.09	15	0.667
282	A	7	6	1.18	13	0.462
283	A	7	6	1.12	11	0.545
284	A	5	4	1.13	13	0.308
285	A	2	2	1.00	13	0.154
286	A	2	2	1.00	13	0.154
287	A	2	2	1.00	13	0.154
288	A	2	2	1.00	11	0.182
289	A	1	1	1.00	9	0.111
290	A	2	2	1.00	13	0.154
291	A	2	2	1.00	13	0.154
292	A	2	2	1.00	13	0.154
293	A	2	2	1.00	13	0.154
294	A	5	4	1.06	15	0.267
295	A	5	4	1.06	15	0.267
296	A	5	4	1.06	15	0.267
297	A	5	4	1.06	13	0.308
298	A	1	1	1.07	11	0.091
299	A	5	4	1.21	15	0.267
300	A	5	4	1.12	15	0.267
301	A	5	4	1.06	15	0.267
302	A	5	4	1.06	15	0.267
303	A	5	4	1.04	15	0.267
304	A	5	4	1.04	15	0.267
305	A	5	4	1.04	15	0.267
306	A	5	4	1.12	13	0.308
307	A	5	4	1.25	11	0.364
308	A	5	4	1.21	15	0.267
309	A	5	4	1.09	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	5	4	1.04	15	0.267
311	A	5	4	1.04	15	0.267
312	A	5	4	1.03	15	0.267
313	A	5	4	1.03	13	0.308
314	A	5	4	1.08	11	0.364
315	A	2	2	1.00	15	0.133
316	A	5	4	1.33	15	0.267
317	A	5	4	1.20	15	0.267
318	A	5	4	1.14	15	0.267
319	A	5	4	1.03	15	0.267
320	A	5	4	1.03	13	0.308
321	A	5	4	1.07	11	0.364
322	A	5	4	1.00	15	0.267
323	A	5	4	1.26	15	0.267
324	A	5	4	1.20	15	0.267
325	A	5	4	1.16	15	0.267
326	A	5	4	1.04	15	0.267
327	A	5	4	1.03	13	0.308
328	A	5	4	1.06	11	0.364
329	A	5	4	1.02	15	0.267
330	A	5	4	1.13	15	0.267
331	A	5	4	1.19	15	0.267
332	A	5	4	1.13	15	0.267
333	A	5	4	1.12	15	0.267
334	A	5	4	1.14	11	0.364
335	A	1	1	1.00	17	0.059
336	A	1	1	1.00	23	0.043
337	A	1	1	1.00	17	0.059
338	A	2	2	1.00	11	0.182
339	A	2	2	1.00	11	0.182
340	A	2	2	1.00	9	0.222
341	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	2	2	1.00	11	0.182
343	A	2	2	1.00	11	0.182
344	A	2	2	1.00	11	0.182
345	A	2	2	1.00	13	0.154
346	A	2	2	1.00	13	0.154
347	A	2	2	1.00	11	0.182
348	A	2	2	1.00	9	0.222
349	A	4	3	1.00	13	0.231
350	A	2	2	1.00	13	0.154
351	A	2	2	1.00	13	0.154
352	A	2	2	1.00	13	0.154
353	A	2	2	1.00	13	0.154
354	A	2	2	1.00	11	0.182
355	A	2	2	1.00	9	0.222
356	A	4	3	0.94	13	0.231
357	A	2	2	1.00	13	0.154
358	A	2	2	1.00	13	0.154
359	A	1	1	1.00	11	0.091
360	A	1	1	1.00	9	0.111
361	A	5	4	1.13	13	0.308
362	A	1	1	1.00	13	0.077
363	A	1	1	1.00	13	0.077
364	A	1	1	1.00	11	0.091
365	A	1	1	1.00	9	0.111
366	A	4	3	1.00	13	0.231
367	A	1	1	1.00	13	0.077
368	A	1	1	1.00	13	0.077
369	A	1	1	1.00	11	0.091
370	A	1	1	1.00	9	0.111
371	A	4	3	0.95	13	0.231
372	A	1	1	1.00	13	0.077
373	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	2	2	1.00	15	0.133
375	A	2	2	1.00	15	0.133
376	A	2	2	1.00	15	0.133
377	A	2	2	1.16	13	0.154
378	A	2	2	1.00	11	0.182
379	A	2	2	1.00	15	0.133
380	A	2	2	1.00	15	0.133
381	A	2	2	1.00	15	0.133
382	A	2	2	1.00	15	0.133
383	A	2	2	1.00	15	0.133
384	A	4	3	0.89	17	0.176
385	A	4	3	0.89	17	0.176
386	A	4	3	0.89	17	0.176
387	A	1	1	1.00	15	0.067
388	A	4	3	1.00	13	0.231
389	A	4	3	1.00	17	0.176
390	A	4	3	1.00	17	0.176
391	A	1	1	1.00	17	0.059
392	A	4	3	0.89	17	0.176
393	A	4	3	0.89	17	0.176
394	A	4	3	0.89	17	0.176
395	A	4	3	0.87	17	0.176
396	A	4	3	0.87	17	0.176
397	A	4	3	0.95	17	0.176
398	A	1	1	1.00	15	0.067
399	A	4	3	0.94	13	0.231
400	A	4	3	0.94	17	0.176
401	A	4	3	0.94	17	0.176
402	A	4	3	0.94	17	0.176
403	A	1	1	1.00	17	0.059
404	A	4	3	0.96	17	0.176
405	A	4	3	0.87	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	4	3	0.87	17	0.176
407	A	4	3	0.90	17	0.176
408	A	4	3	0.92	17	0.176
409	A	4	3	0.95	17	0.176
410	A	1	1	1.00	15	0.067
411	A	4	3	0.89	13	0.231
412	A	4	3	0.89	17	0.176
413	A	4	3	0.89	17	0.176
414	A	4	3	0.89	17	0.176
415	A	4	3	0.89	17	0.176
416	A	4	3	0.90	17	0.176
417	A	1	1	1.00	17	0.059
418	A	4	3	0.96	17	0.176
419	A	5	4	1.01	17	0.235
420	A	4	3	0.86	17	0.176
421	A	4	3	0.86	17	0.176
422	A	4	3	0.85	17	0.176
423	A	4	3	0.85	17	0.176
424	A	4	3	0.89	17	0.176
425	A	4	3	0.89	17	0.176
426	A	4	3	0.90	17	0.176
427	A	4	3	0.90	17	0.176
428	A	4	3	0.92	17	0.176
429	A	4	3	0.95	17	0.176
430	A	1	1	1.00	15	0.067
431	A	4	3	0.87	13	0.231
432	A	4	3	0.87	17	0.176
433	A	4	3	0.87	17	0.176
434	A	4	3	0.86	17	0.176
435	A	4	3	0.87	17	0.176
436	A	4	3	0.86	17	0.176
437	A	4	3	0.87	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
438	A	4	3	0.87	17	0.176
439	A	4	3	0.87	17	0.176
440	A	1	1	1.00	17	0.059
441	A	4	3	0.96	17	0.176
442	A	5	4	1.01	17	0.235
443	A	6	5	1.04	17	0.294
444	A	7	6	1.05	17	0.353
445	A	4	3	0.85	17	0.176
446	A	4	3	0.85	17	0.176
447	A	1	1	1.00	15	0.067
448	A	1	1	1.00	13	0.077
449	A	1	1	1.00	13	0.077
450	A	1	1	1.00	13	0.077
451	A	1	1	1.00	19	0.053
452	A	1	1	1.00	23	0.043
453	A	4	3	0.87	17	0.176
454	A	4	3	0.88	17	0.176
455	A	4	3	0.89	17	0.176
456	A	4	3	0.93	17	0.176
457	A	1	1	1.00	15	0.067
458	A	5	4	1.13	13	0.308
459	A	4	3	1.00	17	0.176
460	A	4	3	0.95	17	0.176
461	A	4	3	0.92	17	0.176
462	A	4	3	0.87	19	0.158
463	A	4	3	0.88	19	0.158
464	A	4	3	0.89	19	0.158
465	A	4	3	0.93	19	0.158
466	A	1	1	1.00	15	0.067
467	A	5	4	1.13	13	0.308
468	A	4	3	1.00	17	0.176
469	A	4	3	0.95	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
470	A	4	3	0.92	19	0.158
471	A	4	3	0.85	17	0.176
472	A	4	3	0.86	17	0.176
473	A	4	3	0.88	17	0.176
474	A	4	3	0.93	17	0.176
475	A	1	1	1.00	15	0.067
476	A	5	4	1.14	13	0.308
477	A	4	3	1.00	17	0.176
478	A	4	3	0.94	17	0.176
479	A	4	3	0.91	17	0.176
480	A	4	3	0.88	17	0.176
481	A	4	3	0.90	17	0.176
482	A	4	3	0.94	17	0.176
483	A	1	1	1.00	15	0.067
484	A	4	3	1.00	13	0.231
485	A	4	3	0.95	17	0.176
486	A	4	3	0.92	17	0.176
487	A	4	3	0.90	17	0.176
488	A	4	3	0.89	17	0.176
489	A	4	3	0.91	17	0.176
490	A	1	1	1.00	17	0.059
491	A	1	1	1.00	15	0.067
492	A	4	3	0.95	13	0.231
493	A	4	3	0.92	17	0.176
494	A	4	3	0.91	17	0.176
495	A	5	4	0.96	19	0.211
496	A	11	10	1.00	19	0.526
497	A	11	10	1.48	19	0.526
498	A	4	3	1.00	17	0.176
499	A	5	4	0.96	19	0.211
500	A	12	11	0.99	19	0.579
501	A	12	11	1.48	19	0.579

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	6	5	1.03	19	0.263
503	A	13	12	1.01	19	0.632
504	A	13	12	1.45	19	0.632
505	A	2	2	1.19	13	0.154
506	A	2	2	1.23	11	0.182
507	A	5	4	0.96	15	0.267
508	A	2	2	1.18	15	0.133
509	A	2	2	1.18	15	0.133
510	A	2	2	1.21	13	0.154
511	A	2	2	1.26	11	0.182
512	A	6	5	0.95	15	0.333
513	A	2	2	1.20	15	0.133
514	A	2	2	1.20	15	0.133
515	A	2	2	1.25	13	0.154
516	A	2	2	1.31	11	0.182
517	A	7	6	0.93	15	0.400
518	A	2	2	1.24	15	0.133
519	A	2	2	1.24	15	0.133
520	A	2	2	1.19	13	0.154
521	A	2	2	1.23	11	0.182
522	A	4	3	1.00	15	0.200
523	A	2	2	1.18	15	0.133
524	A	2	2	1.18	15	0.133
525	A	2	2	1.25	13	0.154
526	A	2	2	1.31	11	0.182
527	A	5	4	0.96	15	0.267
528	A	2	2	1.24	15	0.133
529	A	2	2	1.24	15	0.133
530	A	2	2	1.25	13	0.154
531	A	2	2	1.31	11	0.182
532	A	6	5	1.00	15	0.333
533	A	2	2	1.24	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	2	2	1.24	15	0.133
535	A	4	3	0.91	19	0.158
536	A	4	3	0.93	19	0.158
537	A	4	3	0.95	19	0.158
538	A	1	1	1.00	17	0.059
539	A	5	4	0.96	15	0.267
540	A	5	4	0.96	19	0.211
541	A	6	5	0.92	19	0.263
542	A	7	6	0.96	19	0.316
543	A	8	7	0.99	19	0.368
544	A	4	3	0.91	19	0.158
545	A	4	3	0.92	19	0.158
546	A	4	3	0.95	19	0.158
547	A	1	1	1.00	17	0.059
548	A	4	3	1.00	15	0.200
549	A	5	4	0.96	19	0.211
550	A	6	5	0.95	19	0.263
551	A	7	6	0.99	19	0.316
552	A	8	7	1.01	19	0.368
553	A	7	6	0.96	15	0.400
554	A	1	1	1.00	15	0.067
555	A	2	2	1.16	13	0.154
556	A	1	1	1.00	15	0.067
557	A	1	1	1.00	15	0.067
558	A	1	1	1.00	15	0.067
559	A	2	2	1.00	19	0.105
560	A	2	2	1.00	19	0.105
561	A	2	2	1.00	17	0.118
562	A	3	2	1.00	19	0.105
563	A	4	3	1.09	19	0.158
564	A	5	4	1.04	19	0.211
565	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	2	2	1.00	19	0.105
567	A	2	2	1.00	17	0.118
568	A	10	9	0.97	19	0.474
569	A	11	10	1.00	19	0.526
570	A	12	11	1.00	19	0.579
571	A	1	1	1.00	15	0.067
572	A	1	1	1.00	15	0.067
573	A	2	2	1.00	13	0.154
574	A	2	2	1.00	13	0.154
575	A	2	2	1.00	11	0.182
576	A	1	1	1.00	13	0.077
577	A	1	1	1.00	13	0.077
578	A	1	1	1.00	13	0.077
579	A	1	1	1.00	13	0.077
580	A	2	2	1.00	17	0.118
581	A	2	2	1.00	17	0.118
582	A	2	2	1.00	15	0.133
583	A	3	2	1.00	17	0.118
584	A	4	3	0.94	17	0.176
585	A	5	4	0.99	17	0.235
586	A	1	1	1.00	17	0.059
587	A	1	1	1.00	17	0.059
588	A	1	1	1.00	17	0.059
589	A	1	1	1.00	17	0.059
590	A	1	1	1.00	17	0.059
591	A	2	2	1.18	15	0.133
592	A	2	2	1.16	15	0.133
593	A	2	2	1.16	15	0.133
594	A	2	2	1.22	15	0.133
595	A	2	2	1.22	15	0.133
596	A	2	2	1.19	19	0.105
597	A	2	2	1.16	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	2	2	1.16	19	0.105
599	A	2	2	1.18	19	0.105
600	A	2	2	1.14	19	0.105
601	A	2	2	1.15	17	0.118
602	A	2	2	1.14	19	0.105
603	A	2	2	1.14	19	0.105
604	C	1	1	8.40	42	0.024
605	A	6	5	1.48	21	0.238
606	A	5	4	1.42	21	0.190
607	A	4	3	1.37	21	0.143
608	A	4	3	1.00	21	0.143
609	A	1	1	1.00	21	0.048
610	A	2	2	1.00	21	0.095
611	A	3	3	1.07	21	0.143
612	A	4	4	1.10	21	0.190
613	A	7	6	0.94	19	0.316
614	A	6	5	0.94	19	0.263
615	A	5	4	0.94	19	0.211
616	A	4	3	1.00	19	0.158
617	A	1	1	1.00	19	0.053
618	A	2	2	1.00	19	0.105
619	A	3	3	1.09	19	0.158
620	A	1	1	1.00	15	0.067
621	A	1	1	1.00	17	0.059
622	A	2	2	1.01	17	0.118
623	A	2	2	1.00	19	0.105
624	C	1	1	6.15	45	0.022
625	A	2	2	1.00	37	0.054
626	C	1	1	8.40	42	0.024
627	A	3	2	0.95	19	0.105
628	A	4	3	1.03	21	0.143
629	A	3	2	0.98	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	4	3	1.00	21	0.143
631	A	2	2	1.00	13	0.154
632	A	1	1	1.00	15	0.067
633	A	2	2	1.00	11	0.182
634	A	2	2	1.00	11	0.182
635	A	2	2	1.00	9	0.222
636	A	2	2	1.00	7	0.286
637	A	3	2	1.00	11	0.182
638	A	2	2	1.00	11	0.182
639	A	2	2	1.00	11	0.182
640	A	2	2	1.00	11	0.182
641	A	1	1	1.00	15	0.067
642	A	4	3	0.96	17	0.176
643	A	4	3	0.93	17	0.176
644	A	4	3	0.92	17	0.176
645	A	1	1	1.00	21	0.048
646	A	1	1	1.00	17	0.059
647	A	1	1	1.00	17	0.059
648	A	1	1	1.00	13	0.077
649	A	5	4	1.13	13	0.308
650	A	5	4	1.18	13	0.308
651	A	2	2	1.00	15	0.133
652	A	2	2	0.91	15	0.133
653	A	2	2	1.00	15	0.133
654	A	2	2	1.00	15	0.133
655	A	2	2	1.00	13	0.154
656	A	2	2	1.00	15	0.133
657	A	2	2	1.00	15	0.133
658	A	2	2	1.00	15	0.133
659	A	12	11	0.81	21	0.524
660	A	12	11	1.18	21	0.524
661	A	5	4	0.74	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	6	5	0.91	21	0.238
663	A	13	12	0.80	21	0.571
664	A	13	12	1.17	21	0.571
665	A	7	6	0.90	21	0.286
666	A	14	13	0.80	21	0.619
667	A	14	13	1.14	21	0.619
668	A	1	1	1.00	17	0.059
669	A	1	1	1.00	17	0.059
670	A	1	1	1.00	17	0.059
671	A	1	1	1.00	17	0.059
672	A	1	1	1.00	15	0.067
673	A	2	2	1.00	17	0.118
674	A	1	1	1.00	17	0.059
675	A	1	1	1.00	17	0.059
676	A	1	1	1.00	17	0.059
677	A	7	6	1.18	23	0.261
678	A	6	5	1.15	23	0.217
679	A	5	4	1.14	23	0.174
680	A	5	4	1.00	23	0.174
681	A	1	1	1.00	23	0.043
682	A	2	2	0.92	23	0.087
683	A	3	3	0.93	23	0.130
684	A	4	4	0.95	23	0.174
685	A	2	2	1.00	15	0.133
686	A	2	2	1.00	17	0.118
687	A	2	2	1.00	17	0.118
688	A	2	2	1.00	17	0.118
689	A	2	2	1.00	15	0.133
690	A	2	2	1.00	9	0.222
691	A	2	2	1.00	17	0.118
692	A	2	2	1.00	17	0.118
693	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
694	A	4	3	1.11	21	0.143
695	A	4	3	0.83	20	0.150
696	A	1	1	1.00	23	0.043
697	A	2	2	0.95	23	0.087
698	A	3	3	0.91	23	0.130
699	A	4	4	0.88	23	0.174
700	A	2	2	1.00	22	0.091
701	A	2	2	0.96	24	0.083

CHAPTER 3

Listing of integrals

3.1 $\int \left(a+b\sqrt{x}\right) x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+b\sqrt{x}\right) x^4 \, dx = \frac{ax^5}{5} + \frac{2}{11} b x^{11/2}$$

output
```
1/5*a*x^5+2/11*b*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{x}\right) x^4 \, dx = \frac{1}{55}\left(11a+10b\sqrt{x}\right) x^5$$

input
```
Integrate[(a + b*Sqrt[x])*x^4,x]
```

output
```
((11*a + 10*b*Sqrt[x])*x^5)/55
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^4+bx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^5}{5}+\frac{2}{11}bx^{11/2}$$

input
```
Int[(a + b*Sqrt[x])*x^4,x]
```

output
```
(a*x^5)/5 + (2*b*x^(11/2))/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^{5}}{5}+\frac{2b\,x^{\frac{11}{2}}}{11}$	14
default	$\frac{a\,x^{5}}{5}+\frac{2b\,x^{\frac{11}{2}}}{11}$	14
trager	$\frac{a(x^{4}+x^{3}+x^{2}+x+1)(-1+x)}{5}+\frac{2b\,x^{\frac{11}{2}}}{11}$	26
orering	$\frac{19x^{5}(a+b\sqrt{x})}{55}-\frac{2x^{2}\left(\frac{b\,x^{\frac{7}{2}}}{2}+4(a+b\sqrt{x})x^{3}\right)}{55}$	38

input
```
int((a+b*x^(1/2))*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+2/11*b*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x^{4}\,dx=\frac{2}{11}\,bx^{\frac{11}{2}}+\frac{1}{5}\,ax^{5}$$

input
```
integrate((a+b*x^(1/2))*x^4,x, algorithm="fricas")
```

output
```
2/11*b*x^(11/2) + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)x^4\,dx=\frac{ax^5}{5}+\frac{2bx^{\frac{11}{2}}}{11}$$

input
```
integrate((a+b*x**(1/2))*x**4,x)
```

output
```
a*x**5/5 + 2*b*x**(11/2)/11
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 8.74

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)x^4\,dx=&\ \frac{2\left(b\sqrt{x}+a\right)^{11}}{11\,b^{10}}-\frac{9\left(b\sqrt{x}+a\right)^{10}a}{5\,b^{10}}+\frac{8\left(b\sqrt{x}+a\right)^{9}a^2}{b^{10}}\\&-\frac{21\left(b\sqrt{x}+a\right)^{8}a^3}{b^{10}}+\frac{36\left(b\sqrt{x}+a\right)^{7}a^4}{b^{10}}\\&-\frac{42\left(b\sqrt{x}+a\right)^{6}a^5}{b^{10}}+\frac{168\left(b\sqrt{x}+a\right)^{5}a^6}{5\,b^{10}}\\&-\frac{18\left(b\sqrt{x}+a\right)^{4}a^7}{b^{10}}+\frac{6\left(b\sqrt{x}+a\right)^{3}a^8}{b^{10}}-\frac{\left(b\sqrt{x}+a\right)^{2}a^9}{b^{10}}\end{aligned}$$

input
```
integrate((a+b*x^(1/2))*x^4,x, algorithm="maxima")
```

output
```
2/11*(b*sqrt(x) + a)^11/b^10 - 9/5*(b*sqrt(x) + a)^10*a/b^10 + 8*(b*sqrt(x
) + a)^9*a^2/b^10 - 21*(b*sqrt(x) + a)^8*a^3/b^10 + 36*(b*sqrt(x) + a)^7*a
^4/b^10 - 42*(b*sqrt(x) + a)^6*a^5/b^10 + 168/5*(b*sqrt(x) + a)^5*a^6/b^10
 - 18*(b*sqrt(x) + a)^4*a^7/b^10 + 6*(b*sqrt(x) + a)^3*a^8/b^10 - (b*sqrt(
x) + a)^2*a^9/b^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int (a+b\sqrt{x})\, x^4\, dx = \frac{2}{11}\, bx^{\frac{11}{2}} + \frac{1}{5}\, ax^5$$

input
```
integrate((a+b*x^(1/2))*x^4,x, algorithm="giac")
```

output
```
2/11*b*x^(11/2) + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int (a+b\sqrt{x})\, x^4\, dx = \frac{a\, x^5}{5} + \frac{2\, b\, x^{11/2}}{11}$$

input
```
int(x^4*(a + b*x^(1/2)),x)
```

output
```
(a*x^5)/5 + (2*b*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int (a+b\sqrt{x})\, x^4\, dx = \frac{x^5 \left(10\sqrt{x}\, b + 11a\right)}{55}$$

input
```
int((a+b*x^(1/2))*x^4,x)
```

output
```
(x**5*(10*sqrt(x)*b + 11*a))/55
```

3.2 $\int \left(a+b\sqrt{x}\right) x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+b\sqrt{x}\right) x^3 \, dx = \frac{ax^4}{4} + \frac{2}{9} b x^{9/2}$$

output
```
1/4*a*x^4+2/9*b*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{x}\right) x^3 \, dx = \frac{1}{36} \left(9a+8b\sqrt{x}\right) x^4$$

input
```
Integrate[(a + b*Sqrt[x])*x^3,x]
```

output
```
((9*a + 8*b*Sqrt[x])*x^4)/36
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^3+b x^{7/2}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^4}{4}+\frac{2}{9} b x^{9/2}$$

input
```
Int[(a + b*Sqrt[x])*x^3,x]
```

output
```
(a*x^4)/4 + (2*b*x^(9/2))/9
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^4}{4}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{a\,x^4}{4}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
trager	$\frac{a(x^3+x^2+x+1)(-1+x)}{4}+\frac{2b\,x^{\frac{9}{2}}}{9}$	23
orering	$\frac{5(a+b\sqrt{x})x^4}{12}-\frac{x^2\left(\frac{b\,x^{\frac{5}{2}}}{2}+3(a+b\sqrt{x})x^2\right)}{18}$	38

input
```
int((a+b*x^(1/2))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+2/9*b*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x^3\,dx=\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{1}{4}\,ax^4$$

input
```
integrate((a+b*x^(1/2))*x^3,x, algorithm="fricas")
```

output
```
2/9*b*x^(9/2) + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)x^3\,dx=\frac{ax^4}{4}+\frac{2bx^{\frac{9}{2}}}{9}$$

input
```
integrate((a+b*x**(1/2))*x**3,x)
```

output
```
a*x**4/4 + 2*b*x**(9/2)/9
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(13) = 26$.

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 6.95

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)x^3\,dx=&\frac{2\left(b\sqrt{x}+a\right)^9}{9\,b^8}-\frac{7\left(b\sqrt{x}+a\right)^8a}{4\,b^8}+\frac{6\left(b\sqrt{x}+a\right)^7a^2}{b^8}\\&-\frac{35\left(b\sqrt{x}+a\right)^6a^3}{3\,b^8}+\frac{14\left(b\sqrt{x}+a\right)^5a^4}{b^8}\\&-\frac{21\left(b\sqrt{x}+a\right)^4a^5}{2\,b^8}+\frac{14\left(b\sqrt{x}+a\right)^3a^6}{3\,b^8}-\frac{\left(b\sqrt{x}+a\right)^2a^7}{b^8}\end{aligned}$$

input
```
integrate((a+b*x^(1/2))*x^3,x, algorithm="maxima")
```

output
```
2/9*(b*sqrt(x) + a)^9/b^8 - 7/4*(b*sqrt(x) + a)^8*a/b^8 + 6*(b*sqrt(x) + a
)^7*a^2/b^8 - 35/3*(b*sqrt(x) + a)^6*a^3/b^8 + 14*(b*sqrt(x) + a)^5*a^4/b^
8 - 21/2*(b*sqrt(x) + a)^4*a^5/b^8 + 14/3*(b*sqrt(x) + a)^3*a^6/b^8 - (b*s
qrt(x) + a)^2*a^7/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x^3\,dx=\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{1}{4}\,ax^4$$

input
```
integrate((a+b*x^(1/2))*x^3,x, algorithm="giac")
```

output
```
2/9*b*x^(9/2) + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x^3\,dx=\frac{a\,x^4}{4}+\frac{2\,b\,x^{9/2}}{9}$$

input
```
int(x^3*(a + b*x^(1/2)),x)
```

output
```
(a*x^4)/4 + (2*b*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)x^3\,dx=\frac{x^4\left(8\sqrt{x}\,b+9a\right)}{36}$$

input
```
int((a+b*x^(1/2))*x^3,x)
```

output
```
(x**4*(8*sqrt(x)*b + 9*a))/36
```

3.3 $\int (a+b\sqrt{x}) x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int (a+b\sqrt{x}) x^2 \, dx = \frac{a x^3}{3} + \frac{2}{7} b x^{7/2}$$

output
```
1/3*a*x^3+2/7*b*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+b\sqrt{x}) x^2 \, dx = \frac{1}{21} (7a+6b\sqrt{x}) x^3$$

input
```
Integrate[(a + b*Sqrt[x])*x^2,x]
```

output
```
((7*a + 6*b*Sqrt[x])*x^3)/21
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^2+bx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{2}{7}bx^{7/2}$$

input
```
Int[(a + b*Sqrt[x])*x^2,x]
```

output
```
(a*x^3)/3 + (2*b*x^(7/2))/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^{3}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
default	$\frac{a\,x^{3}}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
trager	$\frac{a(x^{2}+x+1)(-1+x)}{3}+\frac{2b\,x^{\frac{7}{2}}}{7}$	20
orering	$\frac{11(a+b\sqrt{x})x^{3}}{21}-\frac{2x^{2}\left(\frac{b\,x^{\frac{3}{2}}}{2}+2(a+b\sqrt{x})x\right)}{21}$	36

input
```
int((a+b*x^(1/2))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+2/7*b*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x^{2}\,dx=\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{1}{3}\,ax^{3}$$

input
```
integrate((a+b*x^(1/2))*x^2,x, algorithm="fricas")
```

output
```
2/7*b*x^(7/2) + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)x^2\,dx=\frac{ax^3}{3}+\frac{2bx^{\frac{7}{2}}}{7}$$

input
```
integrate((a+b*x**(1/2))*x**2,x)
```

output
```
a*x**3/3 + 2*b*x**(7/2)/7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.16

$$\int \left(a+b\sqrt{x}\right)x^2\,dx=\frac{2\left(b\sqrt{x}+a\right)^7}{7\,b^6}-\frac{5\left(b\sqrt{x}+a\right)^6a}{3\,b^6}+\frac{4\left(b\sqrt{x}+a\right)^5a^2}{b^6}-\frac{5\left(b\sqrt{x}+a\right)^4a^3}{b^6}+\frac{10\left(b\sqrt{x}+a\right)^3a^4}{3\,b^6}-\frac{\left(b\sqrt{x}+a\right)^2a^5}{b^6}$$

input
```
integrate((a+b*x^(1/2))*x^2,x, algorithm="maxima")
```

output
```
2/7*(b*sqrt(x) + a)^7/b^6 - 5/3*(b*sqrt(x) + a)^6*a/b^6 + 4*(b*sqrt(x) + a
)^5*a^2/b^6 - 5*(b*sqrt(x) + a)^4*a^3/b^6 + 10/3*(b*sqrt(x) + a)^3*a^4/b^6
 - (b*sqrt(x) + a)^2*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right) x^2\, dx = \frac{2}{7}\, b x^{\frac{7}{2}} + \frac{1}{3}\, a x^3$$

input
```
integrate((a+b*x^(1/2))*x^2,x, algorithm="giac")
```

output
```
2/7*b*x^(7/2) + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right) x^2\, dx = \frac{a\, x^3}{3} + \frac{2\, b\, x^{7/2}}{7}$$

input
```
int(x^2*(a + b*x^(1/2)),x)
```

output
```
(a*x^3)/3 + (2*b*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right) x^2\, dx = \frac{x^3 \left(6\sqrt{x}\, b + 7a\right)}{21}$$

input
```
int((a+b*x^(1/2))*x^2,x)
```

output
```
(x**3*(6*sqrt(x)*b + 7*a))/21
```

3.4 $\int (a+b\sqrt{x})\, x\, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int (a+b\sqrt{x})\, x\, dx = \frac{ax^2}{2} + \frac{2}{5}bx^{5/2}$$

output
```
1/2*a*x^2+2/5*b*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+b\sqrt{x})\, x\, dx = \frac{1}{10}(5a+4b\sqrt{x})\, x^2$$

input
```
Integrate[(a + b*Sqrt[x])*x,x]
```

output
```
((5*a + 4*b*Sqrt[x])*x^2)/10
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax+bx^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}+\frac{2}{5}bx^{5/2}$$

input
```
Int[(a + b*Sqrt[x])*x,x]
```

output
```
(a*x^2)/2 + (2*b*x^(5/2))/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^{2}}{2}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
default	$\frac{a\,x^{2}}{2}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
trager	$\frac{(-1+x)a(1+x)}{2}+\frac{2b\,x^{\frac{5}{2}}}{5}$	17
orering	$\frac{7(a+b\sqrt{x})x^{2}}{10}-\frac{x^{2}\left(\frac{3b\sqrt{x}}{2}+a\right)}{5}$	27

input
```
int((a+b*x^(1/2))*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+2/5*b*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x\,dx=\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{1}{2}\,ax^{2}$$

input
```
integrate((a+b*x^(1/2))*x,x, algorithm="fricas")
```

output
```
2/5*b*x^(5/2) + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)x\,dx=\frac{ax^2}{2}+\frac{2bx^{\frac{5}{2}}}{5}$$

input
```
integrate((a+b*x**(1/2))*x,x)
```

output
```
a*x**2/2 + 2*b*x**(5/2)/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.37

$$\int \left(a+b\sqrt{x}\right)x\,dx=\frac{2\left(b\sqrt{x}+a\right)^5}{5\,b^4}-\frac{3\left(b\sqrt{x}+a\right)^4a}{2\,b^4}+\frac{2\left(b\sqrt{x}+a\right)^3a^2}{b^4}-\frac{\left(b\sqrt{x}+a\right)^2a^3}{b^4}$$

input
```
integrate((a+b*x^(1/2))*x,x, algorithm="maxima")
```

output
```
2/5*(b*sqrt(x) + a)^5/b^4 - 3/2*(b*sqrt(x) + a)^4*a/b^4 + 2*(b*sqrt(x) + a
)^3*a^2/b^4 - (b*sqrt(x) + a)^2*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x\,dx=\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{1}{2}\,ax^2$$

input
```
integrate((a+b*x^(1/2))*x,x, algorithm="giac")
```

output
```
2/5*b*x^(5/2) + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)x\,dx=\frac{a\,x^2}{2}+\frac{2\,b\,x^{5/2}}{5}$$

input
```
int(x*(a + b*x^(1/2)),x)
```

output
```
(a*x^2)/2 + (2*b*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)x\,dx=\frac{x^2\left(4\sqrt{x}\,b+5a\right)}{10}$$

input
```
int((a+b*x^(1/2))*x,x)
```

output
```
(x**2*(4*sqrt(x)*b + 5*a))/10
```

3.5 $\int (a+b\sqrt{x})\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int (a+b\sqrt{x})\,dx = ax + \frac{2}{3}bx^{3/2}$$

output
```
a*x+2/3*b*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (a+b\sqrt{x})\,dx = ax + \frac{2}{3}bx^{3/2}$$

input
```
Integrate[a + b*Sqrt[x],x]
```

output
```
a*x + (2*b*x^(3/2))/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{2}{3}bx^{3/2}$$

input
```
Int[a + b*Sqrt[x],x]
```

output
```
a*x + (2*b*x^(3/2))/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$ax+\frac{2bx^{\frac{3}{2}}}{3}$	11
default	$ax+\frac{2bx^{\frac{3}{2}}}{3}$	11
risch	$ax+\frac{2bx^{\frac{3}{2}}}{3}$	11
parts	$ax+\frac{2bx^{\frac{3}{2}}}{3}$	11
trager	$a(-1+x)+\frac{2bx^{\frac{3}{2}}}{3}$	13
orering	$\left(a+b\sqrt{x}\right)x-\frac{bx^{\frac{3}{2}}}{3}$	17

input
```
int(a+b*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*x+2/3*b*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right)\,dx=\frac{2}{3}\,bx^{\frac{3}{2}}+ax$$

input
```
integrate(a+b*x^(1/2),x, algorithm="fricas")
```

output
```
2/3*b*x^(3/2) + a*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt{x}\right) dx = ax + \frac{2bx^{\frac{3}{2}}}{3}$$

input
```
integrate(a+b*x**(1/2),x)
```

output
```
a*x + 2*b*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right) dx = \frac{2}{3}\,bx^{\frac{3}{2}} + ax$$

input
```
integrate(a+b*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*b*x^(3/2) + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right) dx = \frac{2}{3}\,bx^{\frac{3}{2}} + ax$$

input
```
integrate(a+b*x^(1/2),x, algorithm="giac")
```

output
```
2/3*b*x^(3/2) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right)\,dx=a\,x+\frac{2\,b\,x^{3/2}}{3}$$

input
```
int(a + b*x^(1/2),x)
```

output
```
a*x + (2*b*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt{x}\right)\,dx=\frac{x\left(2\sqrt{x}\,b+3a\right)}{3}$$

input
```
int(a+b*x^(1/2),x)
```

output
```
(x*(2*sqrt(x)*b + 3*a))/3
```

3.6 $\int \frac{a+b\sqrt{x}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{a+b\sqrt{x}}{x} \, dx = 2b\sqrt{x} + a\log(x)$$

output
```
2*b*x^(1/2)+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{x} \, dx = 2b\sqrt{x} + a\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])/x,x]
```

output
```
2*b*Sqrt[x] + a*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt{x}}{x}\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x}+\frac{b}{\sqrt{x}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a\log(x)+2b\sqrt{x}$$

input
```
Int[(a + b*Sqrt[x])/x,x]
```

output
```
2*b*Sqrt[x] + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$2b\sqrt{x}+a\ln\left(x\right)$	12
default	$2b\sqrt{x}+a\ln\left(x\right)$	12
trager	$2b\sqrt{x}+a\ln\left(x\right)$	12

input
```
int((a+b*x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
2*b*x^(1/2)+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sqrt{x}}{x}\,dx = 2\,a\log\left(\sqrt{x}\right)+2\,b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))/x,x, algorithm="fricas")
```

output
```
2*a*log(sqrt(x)) + 2*b*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sqrt{x}}{x}\,dx = a\log\left(x\right)+2b\sqrt{x}$$

input
```
integrate((a+b*x**(1/2))/x,x)
```

output
```
a*log(x) + 2*b*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sqrt{x}}{x}\,dx = a\log(x) + 2\,b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))/x,x, algorithm="maxima")
```

output
```
a*log(x) + 2*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sqrt{x}}{x}\,dx = a\log(|x|) + 2\,b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))/x,x, algorithm="giac")
```

output
```
a*log(abs(x)) + 2*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sqrt{x}}{x}\,dx = 2\,b\,\sqrt{x} + a\,\ln(x)$$

input
```
int((a + b*x^(1/2))/x,x)
```

output
```
2*b*x^(1/2) + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+b\sqrt{x}}{x}\,dx = 2\sqrt{x}\,b + \log(x)\,a$$

input
```
int((a+b*x^(1/2))/x,x)
```

output
```
2*sqrt(x)*b + log(x)*a
```

3.7 $\int \frac{a+b\sqrt{x}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx=-\frac{a}{x}-\frac{2b}{\sqrt{x}}$$

output
```
-a/x-2*b/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx=\frac{-a-2b\sqrt{x}}{x}$$

input
```
Integrate[(a + b*Sqrt[x])/x^2,x]
```

output
```
(-a - 2*b*Sqrt[x])/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^2}+\frac{b}{x^{3/2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{x}-\frac{2b}{\sqrt{x}}$$

input
```
Int[(a + b*Sqrt[x])/x^2,x]
```

output
```
-(a/x) - (2*b)/Sqrt[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{a}{x}-\frac{2b}{\sqrt{x}}$	14
default	$-\frac{a}{x}-\frac{2b}{\sqrt{x}}$	14
trager	$\frac{a(-1+x)}{x}-\frac{2b}{\sqrt{x}}$	16
orering	$-\frac{5(a+b\sqrt{x})}{x}-2\left(\frac{b}{2x^{\frac{5}{2}}}-\frac{2(a+b\sqrt{x})}{x^{3}}\right)x^{2}$	38

input
```
int((a+b*x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-2*b/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx = -\frac{2\,b\sqrt{x}+a}{x}$$

input
```
integrate((a+b*x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-(2*b*sqrt(x) + a)/x
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx = -\frac{a}{x}-\frac{2b}{\sqrt{x}}$$

input
```
integrate((a+b*x**(1/2))/x**2,x)
```

output
```
-a/x - 2*b/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx = -\frac{2\,b\sqrt{x}+a}{x}$$

input
```
integrate((a+b*x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-(2*b*sqrt(x) + a)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx = -\frac{2\,b\sqrt{x}+a}{x}$$

input
```
integrate((a+b*x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-(2*b*sqrt(x) + a)/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx = -\frac{a}{x}-\frac{2\,b}{\sqrt{x}}$$

input
```
int((a + b*x^(1/2))/x^2,x)
```

output
```
- a/x - (2*b)/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{a+b\sqrt{x}}{x^2}\,dx = \frac{-\sqrt{x}\,a-2bx}{\sqrt{x}\,x}$$

input
```
int((a+b*x^(1/2))/x^2,x)
```

output
```
( - sqrt(x)*a - 2*b*x)/(sqrt(x)*x)
```

3.8 $\int \frac{a+b\sqrt{x}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx=-\frac{a}{2x^2}-\frac{2b}{3x^{3/2}}$$

output
```
-1/2*a/x^2-2/3*b/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx=\frac{-3a-4b\sqrt{x}}{6x^2}$$

input
```
Integrate[(a + b*Sqrt[x])/x^3,x]
```

output
```
(-3*a - 4*b*Sqrt[x])/(6*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt{x}}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x^{5/2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}-\frac{2b}{3x^{3/2}}$$

input
```
Int[(a + b*Sqrt[x])/x^3,x]
```

output
```
-1/2*a/x^2 - (2*b)/(3*x^(3/2))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{a}{2x^2}-\frac{2b}{3x^{\frac{3}{2}}}$	14
default	$-\frac{a}{2x^2}-\frac{2b}{3x^{\frac{3}{2}}}$	14
trager	$\frac{(-1+x)a(1+x)}{2x^2}-\frac{2b}{3x^{\frac{3}{2}}}$	20
orering	$-\frac{3(a+b\sqrt{x})}{2x^2}-\frac{x^2\left(\frac{b}{2x^{\frac{7}{2}}}-\frac{3(a+b\sqrt{x})}{x^4}\right)}{3}$	38

input
```
int((a+b*x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-2/3*b/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx=-\frac{4\,b\sqrt{x}+3\,a}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/6*(4*b*sqrt(x) + 3*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx = -\frac{a}{2x^2} - \frac{2b}{3x^{\frac{3}{2}}}$$

input
```
integrate((a+b*x**(1/2))/x**3,x)
```

output
```
-a/(2*x**2) - 2*b/(3*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx = -\frac{4\,b\sqrt{x}+3\,a}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/6*(4*b*sqrt(x) + 3*a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx = -\frac{4\,b\sqrt{x}+3\,a}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))/x^3,x, algorithm="giac")
```

output
```
-1/6*(4*b*sqrt(x) + 3*a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx = -\frac{a}{2\,x^2} - \frac{2\,b}{3\,x^{3/2}}$$

input
```
int((a + b*x^(1/2))/x^3,x)
```

output
```
- a/(2*x^2) - (2*b)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{x^3}\,dx = \frac{-3\sqrt{x}\,a-4bx}{6\sqrt{x}\,x^2}$$

input
```
int((a+b*x^(1/2))/x^3,x)
```

output
```
( - 3*sqrt(x)*a - 4*b*x)/(6*sqrt(x)*x**2)
```

3.9 $\int \frac{a+b\sqrt{x}}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx = -\frac{a}{3x^3} - \frac{2b}{5x^{5/2}}$$

output
```
-1/3*a/x^3-2/5*b/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx = \frac{-5a-6b\sqrt{x}}{15x^3}$$

input
```
Integrate[(a + b*Sqrt[x])/x^4,x]
```

output
```
(-5*a - 6*b*Sqrt[x])/(15*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt{x}}{x^4} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{3x^3}-\frac{2b}{5x^{5/2}}$$

input
```
Int[(a + b*Sqrt[x])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b)/(5*x^(5/2))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{a}{3x^{3}}-\frac{2b}{5x^{\frac{5}{2}}}$	14
default	$-\frac{a}{3x^{3}}-\frac{2b}{5x^{\frac{5}{2}}}$	14
trager	$\frac{a(x^{2}+x+1)(-1+x)}{3x^{3}}-\frac{2b}{5x^{\frac{5}{2}}}$	23
orering	$-\frac{13(a+b\sqrt{x})}{15x^{3}}-\frac{2x^{2}\left(\frac{b}{2x^{\frac{9}{2}}}-\frac{4(a+b\sqrt{x})}{x^{5}}\right)}{15}$	38

input
```
int((a+b*x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-2/5*b/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx=-\frac{6\,b\sqrt{x}+5\,a}{15\,x^3}$$

input
```
integrate((a+b*x^(1/2))/x^4,x, algorithm="fricas")
```

output
```
-1/15*(6*b*sqrt(x) + 5*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx = -\frac{a}{3x^3} - \frac{2b}{5x^{\frac{5}{2}}}$$

input
```
integrate((a+b*x**(1/2))/x**4,x)
```

output
```
-a/(3*x**3) - 2*b/(5*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx = -\frac{6\,b\sqrt{x}+5\,a}{15\,x^3}$$

input
```
integrate((a+b*x^(1/2))/x^4,x, algorithm="maxima")
```

output
```
-1/15*(6*b*sqrt(x) + 5*a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx = -\frac{6\,b\sqrt{x}+5\,a}{15\,x^3}$$

input
```
integrate((a+b*x^(1/2))/x^4,x, algorithm="giac")
```

output
```
-1/15*(6*b*sqrt(x) + 5*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx = -\frac{5\,a+6\,b\,\sqrt{x}}{15\,x^3}$$

input
```
int((a + b*x^(1/2))/x^4,x)
```

output
```
-(5*a + 6*b*x^(1/2))/(15*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt{x}}{x^4}\,dx = \frac{-5\sqrt{x}\,a-6bx}{15\sqrt{x}\,x^3}$$

input
```
int((a+b*x^(1/2))/x^4,x)
```

output
```
( - 5*sqrt(x)*a - 6*b*x)/(15*sqrt(x)*x**3)
```

3.10 $\int \left(a+b\sqrt{x}\right)^2 x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \left(a+b\sqrt{x}\right)^2 x^4 \, dx = \frac{a^2x^5}{5} + \frac{4}{11}abx^{11/2} + \frac{b^2x^6}{6}$$

output
```
1/5*a^2*x^5+4/11*a*b*x^(11/2)+1/6*b^2*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^2 x^4 \, dx = \frac{1}{330}x^5\left(66a^2+120ab\sqrt{x}+55b^2x\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2*x^4,x]
```

output
```
(x^5*(66*a^2 + 120*a*b*Sqrt[x] + 55*b^2*x))/330
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\sqrt{x}\right)^2 \, dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^2 x^{9/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^2x^{11/2}+2abx^5+a^2x^{9/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2x^5}{10}+\frac{2}{11}abx^{11/2}+\frac{b^2x^6}{12}\right)$$

input
```
Int[(a + b*Sqrt[x])^2*x^4,x]
```

output
```
2*((a^2*x^5)/10 + (2*a*b*x^(11/2))/11 + (b^2*x^6)/12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{a^2x^5}{5}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{b^2x^6}{6}$	25
default	$\frac{a^2x^5}{5}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{b^2x^6}{6}$	25
trager	$\frac{(5b^2x^5+6a^2x^4+5b^2x^4+6a^2x^3+5b^2x^3+6a^2x^2+5b^2x^2+6a^2x+5b^2x+6a^2+5b^2)(-1+x)}{30}+\frac{4ab\,x^{\frac{11}{2}}}{11}$	93
orering	$\frac{x^5(-35b^2x+38a^2)(a+b\sqrt{x})^2}{-110b^2x+110a^2}-\frac{x^2(-5b^2x+6a^2)\left((a+b\sqrt{x})x^{\frac{7}{2}}b+4(a+b\sqrt{x})^2x^3\right)}{165(-b^2x+a^2)}$	96

input
```
int((a+b*x^(1/2))^2*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+4/11*a*b*x^(11/2)+1/6*b^2*x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2x^4\,dx=\frac{1}{6}\,b^2x^6+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{1}{5}\,a^2x^5$$

input
```
integrate((a+b*x^(1/2))^2*x^4,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 4/11*a*b*x^(11/2) + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^2 x^4\,dx=\frac{a^2x^5}{5}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{b^2x^6}{6}$$

input
```
integrate((a+b*x**(1/2))**2*x**4,x)
```

output
```
a**2*x**5/5 + 4*a*b*x**(11/2)/11 + b**2*x**6/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.19

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^2 x^4\,dx=&\frac{\left(b\sqrt{x}+a\right)^{12}}{6\,b^{10}}-\frac{18\left(b\sqrt{x}+a\right)^{11}a}{11\,b^{10}}+\frac{36\left(b\sqrt{x}+a\right)^{10}a^2}{5\,b^{10}}\\&-\frac{56\left(b\sqrt{x}+a\right)^{9}a^3}{3\,b^{10}}+\frac{63\left(b\sqrt{x}+a\right)^{8}a^4}{2\,b^{10}}\\&-\frac{36\left(b\sqrt{x}+a\right)^{7}a^5}{b^{10}}+\frac{28\left(b\sqrt{x}+a\right)^{6}a^6}{b^{10}}\\&-\frac{72\left(b\sqrt{x}+a\right)^{5}a^7}{5\,b^{10}}+\frac{9\left(b\sqrt{x}+a\right)^{4}a^8}{2\,b^{10}}-\frac{2\left(b\sqrt{x}+a\right)^{3}a^9}{3\,b^{10}}\end{aligned}$$

input
```
integrate((a+b*x^(1/2))^2*x^4,x, algorithm="maxima")
```

output
```
1/6*(b*sqrt(x) + a)^12/b^10 - 18/11*(b*sqrt(x) + a)^11*a/b^10 + 36/5*(b*sq
rt(x) + a)^10*a^2/b^10 - 56/3*(b*sqrt(x) + a)^9*a^3/b^10 + 63/2*(b*sqrt(x)
 + a)^8*a^4/b^10 - 36*(b*sqrt(x) + a)^7*a^5/b^10 + 28*(b*sqrt(x) + a)^6*a^
6/b^10 - 72/5*(b*sqrt(x) + a)^5*a^7/b^10 + 9/2*(b*sqrt(x) + a)^4*a^8/b^10
- 2/3*(b*sqrt(x) + a)^3*a^9/b^10
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^4\,dx = \frac{1}{6}\,b^2x^6 + \frac{4}{11}\,abx^{\frac{11}{2}} + \frac{1}{5}\,a^2x^5$$

input
```
integrate((a+b*x^(1/2))^2*x^4,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 4/11*a*b*x^(11/2) + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^4\,dx = \frac{a^2\,x^5}{5} + \frac{b^2\,x^6}{6} + \frac{4\,a\,b\,x^{11/2}}{11}$$

input
```
int(x^4*(a + b*x^(1/2))^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^6)/6 + (4*a*b*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^2 x^4\,dx = \frac{x^5\left(120\sqrt{x}\,ab + 66a^2 + 55b^2x\right)}{330}$$

input
```
int((a+b*x^(1/2))^2*x^4,x)
```

output
```
(x**5*(120*sqrt(x)*a*b + 66*a**2 + 55*b**2*x))/330
```

3.11 $\int \left(a+b\sqrt{x}\right)^2 x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \left(a+b\sqrt{x}\right)^2 x^3 \, dx = \frac{a^2x^4}{4} + \frac{4}{9}abx^{9/2} + \frac{b^2x^5}{5}$$

output
```
1/4*a^2*x^4+4/9*a*b*x^(9/2)+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^2 x^3 \, dx = \frac{1}{180}x^4\left(45a^2+80ab\sqrt{x}+36b^2x\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2*x^3,x]
```

output
```
(x^4*(45*a^2 + 80*a*b*Sqrt[x] + 36*b^2*x))/180
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\sqrt{x}\right)^2 \, dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^2 x^{7/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^2x^{9/2}+2abx^4+a^2x^{7/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2x^4}{8}+\frac{2}{9}abx^{9/2}+\frac{b^2x^5}{10}\right)$$

input
```
Int[(a + b*Sqrt[x])^2*x^3,x]
```

output
```
2*((a^2*x^4)/8 + (2*a*b*x^(9/2))/9 + (b^2*x^5)/10)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{a^{2} x^{4}}{4}+\frac{4 a b \,x^{\frac{9}{2}}}{9}+\frac{b^{2} x^{5}}{5}$	25
default	$\frac{a^{2} x^{4}}{4}+\frac{4 a b \,x^{\frac{9}{2}}}{9}+\frac{b^{2} x^{5}}{5}$	25
trager	$\frac{\left(4 b^{2} x^{4}+5 a^{2} x^{3}+4 b^{2} x^{3}+5 a^{2} x^{2}+4 b^{2} x^{2}+5 a^{2} x+4 b^{2} x+5 a^{2}+4 b^{2}\right) \left(-1+x\right)}{20}+\frac{4 a b \,x^{\frac{9}{2}}}{9}$	77
orering	$\frac{x^{4} \left(-68 b^{2} x+75 a^{2}\right) \left(a+b \sqrt{x}\right)^{2}}{-180 b^{2} x+180 a^{2}}-\frac{x^{2} \left(-4 b^{2} x+5 a^{2}\right) \left(\left(a+b \sqrt{x}\right) x^{\frac{5}{2}} b+3 \left(a+b \sqrt{x}\right)^{2} x^{2}\right)}{90 \left(-b^{2} x+a^{2}\right)}$	96

input
```
int((a+b*x^(1/2))^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+4/9*a*b*x^(9/2)+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^3\,dx=\frac{1}{5}\,b^{2}x^{5}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{1}{4}\,a^{2}x^{4}$$

input
```
integrate((a+b*x^(1/2))^2*x^3,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 4/9*a*b*x^(9/2) + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^2 x^3\,dx=\frac{a^2x^4}{4}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{b^2x^5}{5}$$

input
```
integrate((a+b*x**(1/2))**2*x**3,x)
```

output
```
a**2*x**4/4 + 4*a*b*x**(9/2)/9 + b**2*x**5/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.12

$$\int \left(a+b\sqrt{x}\right)^2 x^3\,dx=\frac{\left(b\sqrt{x}+a\right)^{10}}{5\,b^8}-\frac{14\left(b\sqrt{x}+a\right)^9a}{9\,b^8}+\frac{21\left(b\sqrt{x}+a\right)^8a^2}{4\,b^8}-\frac{10\left(b\sqrt{x}+a\right)^7a^3}{b^8}+\frac{35\left(b\sqrt{x}+a\right)^6a^4}{3\,b^8}-\frac{42\left(b\sqrt{x}+a\right)^5a^5}{5\,b^8}+\frac{7\left(b\sqrt{x}+a\right)^4a^6}{2\,b^8}-\frac{2\left(b\sqrt{x}+a\right)^3a^7}{3\,b^8}$$

input
```
integrate((a+b*x^(1/2))^2*x^3,x, algorithm="maxima")
```

output
```
1/5*(b*sqrt(x) + a)^10/b^8 - 14/9*(b*sqrt(x) + a)^9*a/b^8 + 21/4*(b*sqrt(x
) + a)^8*a^2/b^8 - 10*(b*sqrt(x) + a)^7*a^3/b^8 + 35/3*(b*sqrt(x) + a)^6*a
^4/b^8 - 42/5*(b*sqrt(x) + a)^5*a^5/b^8 + 7/2*(b*sqrt(x) + a)^4*a^6/b^8 -
2/3*(b*sqrt(x) + a)^3*a^7/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^3\,dx = \frac{1}{5}\,b^2x^5 + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{1}{4}\,a^2x^4$$

input
```
integrate((a+b*x^(1/2))^2*x^3,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 4/9*a*b*x^(9/2) + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^3\,dx = \frac{a^2\,x^4}{4} + \frac{b^2\,x^5}{5} + \frac{4\,a\,b\,x^{9/2}}{9}$$

input
```
int(x^3*(a + b*x^(1/2))^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^5)/5 + (4*a*b*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^2 x^3\,dx = \frac{x^4\left(80\sqrt{x}\,ab + 45a^2 + 36b^2x\right)}{180}$$

input
```
int((a+b*x^(1/2))^2*x^3,x)
```

output
```
(x**4*(80*sqrt(x)*a*b + 45*a**2 + 36*b**2*x))/180
```

3.12 $\int \left(a+b\sqrt{x}\right)^2 x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \left(a+b\sqrt{x}\right)^2 x^2 \, dx=\frac{a^2x^3}{3}+\frac{4}{7}abx^{7/2}+\frac{b^2x^4}{4}$$

output
```
1/3*a^2*x^3+4/7*a*b*x^(7/2)+1/4*b^2*x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^2 x^2 \, dx=\frac{1}{84}x^3\left(28a^2+48ab\sqrt{x}+21b^2x\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2*x^2,x]
```

output
```
(x^3*(28*a^2 + 48*a*b*Sqrt[x] + 21*b^2*x))/84
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{x}\right)^2 dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^2 x^{5/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^2x^{7/2}+2abx^3+a^2x^{5/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2x^3}{6}+\frac{2}{7}abx^{7/2}+\frac{b^2x^4}{8}\right)$$

input
```
Int[(a + b*Sqrt[x])^2*x^2,x]
```

output
```
2*((a^2*x^3)/6 + (2*a*b*x^(7/2))/7 + (b^2*x^4)/8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{a^{2} x^{3}}{3}+\frac{4 a b \,x^{\frac{7}{2}}}{7}+\frac{b^{2} x^{4}}{4}$	25
default	$\frac{a^{2} x^{3}}{3}+\frac{4 a b \,x^{\frac{7}{2}}}{7}+\frac{b^{2} x^{4}}{4}$	25
trager	$\frac{\left(3 b^{2} x^{3}+4 a^{2} x^{2}+3 b^{2} x^{2}+4 a^{2} x+3 b^{2} x+4 a^{2}+3 b^{2}\right) \left(-1+x \right)}{12}+\frac{4 a b \,x^{\frac{7}{2}}}{7}$	61
orering	$\frac{x^{3} \left(-39 b^{2} x+44 a^{2}\right) \left(a+b \sqrt{x}\right)^{2}}{-84 b^{2} x+84 a^{2}}-\frac{x^{2} \left(-3 b^{2} x+4 a^{2}\right) \left(\left(a+b \sqrt{x}\right) x^{\frac{3}{2}} b+2 \left(a+b \sqrt{x}\right)^{2} x\right)}{42 \left(-b^{2} x+a^{2}\right)}$	94

input
```
int((a+b*x^(1/2))^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+4/7*a*b*x^(7/2)+1/4*b^2*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^2\,dx=\frac{1}{4}\,b^{2}x^{4}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{1}{3}\,a^{2}x^{3}$$

input
```
integrate((a+b*x^(1/2))^2*x^2,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + 4/7*a*b*x^(7/2) + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^2 x^2\,dx = \frac{a^2x^3}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{b^2x^4}{4}$$

input
```
integrate((a+b*x**(1/2))**2*x**2,x)
```

output
```
a**2*x**3/3 + 4*a*b*x**(7/2)/7 + b**2*x**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.06

$$\int \left(a+b\sqrt{x}\right)^2 x^2\,dx = \frac{\left(b\sqrt{x}+a\right)^8}{4\,b^6}-\frac{10\,\left(b\sqrt{x}+a\right)^7a}{7\,b^6}+\frac{10\,\left(b\sqrt{x}+a\right)^6a^2}{3\,b^6}-\frac{4\,\left(b\sqrt{x}+a\right)^5a^3}{b^6}+\frac{5\,\left(b\sqrt{x}+a\right)^4a^4}{2\,b^6}-\frac{2\,\left(b\sqrt{x}+a\right)^3a^5}{3\,b^6}$$

input
```
integrate((a+b*x^(1/2))^2*x^2,x, algorithm="maxima")
```

output
```
1/4*(b*sqrt(x) + a)^8/b^6 - 10/7*(b*sqrt(x) + a)^7*a/b^6 + 10/3*(b*sqrt(x)
 + a)^6*a^2/b^6 - 4*(b*sqrt(x) + a)^5*a^3/b^6 + 5/2*(b*sqrt(x) + a)^4*a^4/
b^6 - 2/3*(b*sqrt(x) + a)^3*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^2\,dx=\frac{1}{4}\,b^2x^4+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{1}{3}\,a^2x^3$$

input
```
integrate((a+b*x^(1/2))^2*x^2,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + 4/7*a*b*x^(7/2) + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x^2\,dx=\frac{a^2\,x^3}{3}+\frac{b^2\,x^4}{4}+\frac{4\,a\,b\,x^{7/2}}{7}$$

input
```
int(x^2*(a + b*x^(1/2))^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^4)/4 + (4*a*b*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^2 x^2\,dx=\frac{x^3\left(48\sqrt{x}\,ab+28a^2+21b^2x\right)}{84}$$

input
```
int((a+b*x^(1/2))^2*x^2,x)
```

output
```
(x**3*(48*sqrt(x)*a*b + 28*a**2 + 21*b**2*x))/84
```

3.13 $\int \left(a+b\sqrt{x}\right)^2 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx=\frac{a^2x^2}{2}+\frac{4}{5}abx^{5/2}+\frac{b^2x^3}{3}$$

output
```
1/2*a^2*x^2+4/5*a*b*x^(5/2)+1/3*b^2*x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx=\frac{1}{30}x^2\left(15a^2+24ab\sqrt{x}+10b^2x\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2*x,x]
```

output
```
(x^2*(15*a^2 + 24*a*b*Sqrt[x] + 10*b^2*x))/30
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}\right)^2 dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^2 x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^2x^{5/2}+2abx^2+a^2x^{3/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2x^2}{4}+\frac{2}{5}abx^{5/2}+\frac{b^2x^3}{6}\right)$$

input
```
Int[(a + b*Sqrt[x])^2*x,x]
```

output
```
2*((a^2*x^2)/4 + (2*a*b*x^(5/2))/5 + (b^2*x^3)/6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{a^2x^2}{2}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{b^2x^3}{3}$	25
default	$\frac{a^2x^2}{2}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{b^2x^3}{3}$	25
trager	$\frac{(2b^2x^2+3a^2x+2b^2x+3a^2+2b^2)(-1+x)}{6}+\frac{4ab\,x^{\frac{5}{2}}}{5}$	45
orering	$\frac{x^2(-6b^2x+7a^2)(a+b\sqrt{x})^2}{-10b^2x+10a^2}-\frac{x^2(-2b^2x+3a^2)\left((a+b\sqrt{x})\sqrt{x}\,b+(a+b\sqrt{x})^2\right)}{15(-b^2x+a^2)}$	91

input
```
int((a+b*x^(1/2))^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+4/5*a*b*x^(5/2)+1/3*b^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx=\frac{1}{3}\,b^2x^3+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{1}{2}\,a^2x^2$$

input
```
integrate((a+b*x^(1/2))^2*x,x, algorithm="fricas")
```

output
```
1/3*b^2*x^3 + 4/5*a*b*x^(5/2) + 1/2*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx = \frac{a^2x^2}{2} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{b^2x^3}{3}$$

input
```
integrate((a+b*x**(1/2))**2*x,x)
```

output
```
a**2*x**2/2 + 4*a*b*x**(5/2)/5 + b**2*x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx = \frac{\left(b\sqrt{x}+a\right)^6}{3\,b^4} - \frac{6\left(b\sqrt{x}+a\right)^5 a}{5\,b^4} + \frac{3\left(b\sqrt{x}+a\right)^4 a^2}{2\,b^4} - \frac{2\left(b\sqrt{x}+a\right)^3 a^3}{3\,b^4}$$

input
```
integrate((a+b*x^(1/2))^2*x,x, algorithm="maxima")
```

output
```
1/3*(b*sqrt(x) + a)^6/b^4 - 6/5*(b*sqrt(x) + a)^5*a/b^4 + 3/2*(b*sqrt(x) +
 a)^4*a^2/b^4 - 2/3*(b*sqrt(x) + a)^3*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx = \frac{1}{3}\,b^2x^3 + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{1}{2}\,a^2x^2$$

input
```
integrate((a+b*x^(1/2))^2*x,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + 4/5*a*b*x^(5/2) + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx=\frac{a^2\,x^2}{2}+\frac{b^2\,x^3}{3}+\frac{4\,a\,b\,x^{5/2}}{5}$$

input
```
int(x*(a + b*x^(1/2))^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^3)/3 + (4*a*b*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^2 x\,dx=\frac{x^2\left(24\sqrt{x}\,ab+15a^2+10b^2x\right)}{30}$$

input
```
int((a+b*x^(1/2))^2*x,x)
```

output
```
(x**2*(24*sqrt(x)*a*b + 15*a**2 + 10*b**2*x))/30
```

3.14 $\int \left(a+b\sqrt{x}\right)^2 dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \left(a+b\sqrt{x}\right)^2 dx = a^2x+\frac{4}{3}abx^{3/2}+\frac{b^2x^2}{2}$$

output
```
a^2*x+4/3*a*b*x^(3/2)+1/2*b^2*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}\right)^2 dx = \frac{1}{6}x\left(6a^2+8ab\sqrt{x}+3b^2x\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2,x]
```

output
```
(x*(6*a^2 + 8*a*b*Sqrt[x] + 3*b^2*x))/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^2 dx$$

$$\Big\downarrow \text{774}$$

$$2\int \left(a+b\sqrt{x}\right)^2 \sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\sqrt{x}a^2+2bxa+b^2x^{3/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2x}{2}+\frac{2}{3}abx^{3/2}+\frac{b^2x^2}{4}\right)$$

input
```
Int[(a + b*Sqrt[x])^2,x]
```

output
```
2*((a^2*x)/2 + (2*a*b*x^(3/2))/3 + (b^2*x^2)/4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$a^2x+\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{b^2x^2}{2}$	22
default	$a^2x+\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{b^2x^2}{2}$	22
trager	$\frac{(-1+x)(b^2x+2a^2+b^2)}{2}+\frac{4ab\,x^{\frac{3}{2}}}{3}$	28
orering	$\frac{x(-5b^2x+6a^2)(a+b\sqrt{x})^2}{-6b^2x+6a^2}-\frac{x^{\frac{3}{2}}(-b^2x+2a^2)(a+b\sqrt{x})b}{3(-b^2x+a^2)}$	75

input
```
int((a+b*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+4/3*a*b*x^(3/2)+1/2*b^2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^2\,dx=\frac{1}{2}\,b^2x^2+\frac{4}{3}\,abx^{\frac{3}{2}}+a^2x$$

input
```
integrate((a+b*x^(1/2))^2,x, algorithm="fricas")
```

output
```
1/2*b^2*x^2 + 4/3*a*b*x^(3/2) + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt{x}\right)^2\, dx = a^2x + \frac{4abx^{\frac{3}{2}}}{3} + \frac{b^2x^2}{2}$$

input
```
integrate((a+b*x**(1/2))**2,x)
```

output
```
a**2*x + 4*a*b*x**(3/2)/3 + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^2\, dx = \frac{1}{2}\,b^2x^2 + \frac{4}{3}\,abx^{\frac{3}{2}} + a^2x$$

input
```
integrate((a+b*x^(1/2))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 + 4/3*a*b*x^(3/2) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^2\, dx = \frac{1}{2}\,b^2x^2 + \frac{4}{3}\,abx^{\frac{3}{2}} + a^2x$$

input
```
integrate((a+b*x^(1/2))^2,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + 4/3*a*b*x^(3/2) + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^2 \, dx = a^2 \, x + \frac{b^2 \, x^2}{2} + \frac{4 \, a \, b \, x^{3/2}}{3}$$

input
```
int((a + b*x^(1/2))^2,x)
```

output
```
a^2*x + (b^2*x^2)/2 + (4*a*b*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^2 \, dx = \frac{x\left(8\sqrt{x}\, ab + 6a^2 + 3b^2 x\right)}{6}$$

input
```
int((a+b*x^(1/2))^2,x)
```

output
```
(x*(8*sqrt(x)*a*b + 6*a**2 + 3*b**2*x))/6
```

3.15 $\int \frac{(a+b\sqrt{x})^2}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+b\sqrt{x})^2}{x} \, dx = 4ab\sqrt{x} + b^2x + a^2\log(x)$$

output
```
4*a*b*x^(1/2)+b^2*x+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sqrt{x})^2}{x} \, dx = 4ab\sqrt{x} + b^2x + a^2\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^2/x,x]
```

output
```
4*a*b*Sqrt[x] + b^2*x + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^2}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^2}{\sqrt{x}}+2ba+b^2\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(a^2\log\left(\sqrt{x}\right)+2ab\sqrt{x}+\frac{b^2x}{2}\right)$$

input
```
Int[(a + b*Sqrt[x])^2/x,x]
```

output
```
2*(2*a*b*Sqrt[x] + (b^2*x)/2 + a^2*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$4ab\sqrt{x}+b^2x+a^2\ln(x)$	20
default	$4ab\sqrt{x}+b^2x+a^2\ln(x)$	20
trager	$b^2(-1+x)+4ab\sqrt{x}+a^2\ln(x)$	22

input
```
int((a+b*x^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
4*a*b*x^(1/2)+b^2*x+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{(a+b\sqrt{x})^2}{x}\,dx = b^2x + 2\,a^2\log\left(\sqrt{x}\right) + 4\,ab\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^2/x,x, algorithm="fricas")
```

output
```
b^2*x + 2*a^2*log(sqrt(x)) + 4*a*b*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx = a^2\log\left(x\right)+4ab\sqrt{x}+b^2x$$

input
```
integrate((a+b*x**(1/2))**2/x,x)
```

output
```
a**2*log(x) + 4*a*b*sqrt(x) + b**2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx = b^2x+a^2\log\left(x\right)+4\,ab\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^2/x,x, algorithm="maxima")
```

output
```
b^2*x + a^2*log(x) + 4*a*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx = b^2x+a^2\log\left(|x|\right)+4\,ab\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^2/x,x, algorithm="giac")
```

output
```
b^2*x + a^2*log(abs(x)) + 4*a*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx = 2\,a^2\,\ln\left(\sqrt{x}\right)+b^2\,x+4\,a\,b\,\sqrt{x}$$

input
```
int((a + b*x^(1/2))^2/x,x)
```

output
```
2*a^2*log(x^(1/2)) + b^2*x + 4*a*b*x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x}\,dx = 4\sqrt{x}\,ab+\log(x)\,a^2+b^2x$$

input
```
int((a+b*x^(1/2))^2/x,x)
```

output
```
4*sqrt(x)*a*b + log(x)*a**2 + b**2*x
```

3.16 $\int \frac{(a+b\sqrt{x})^2}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{(a+b\sqrt{x})^2}{x^2} dx = -\frac{a^2}{x} - \frac{4ab}{\sqrt{x}} + b^2 \log(x)$$

output
```
-a^2/x-4*a*b/x^(1/2)+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sqrt{x})^2}{x^2} dx = -\frac{a(a+4b\sqrt{x})}{x} + b^2 \log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^2/x^2,x]
```

output
```
-((a*(a + 4*b*Sqrt[x]))/x) + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2} \, dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{\left(a+b\sqrt{x}\right)^2}{x^{3/2}}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$
$$2\int \left(\frac{a^2}{x^{3/2}}+\frac{2ba}{x}+\frac{b^2}{\sqrt{x}}\right)d\sqrt{x}$$
$$\Big\downarrow \text{2009}$$
$$2\left(-\frac{a^2}{2x}-\frac{2ab}{\sqrt{x}}+b^2\log\left(\sqrt{x}\right)\right)$$

input
```
Int[(a + b*Sqrt[x])^2/x^2,x]
```

output
```
2*(-1/2*a^2/x - (2*a*b)/Sqrt[x] + b^2*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{a^2}{x}-\frac{4ab}{\sqrt{x}}+b^2\ln\left(x\right)$	23
default	$-\frac{a^2}{x}-\frac{4ab}{\sqrt{x}}+b^2\ln\left(x\right)$	23
trager	$\frac{a^2(-1+x)}{x}-\frac{4ab}{\sqrt{x}}-b^2\ln\left(\frac{1}{x}\right)$	28

input
```
int((a+b*x^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x-4*a*b/x^(1/2)+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx=\frac{2\,b^2x\log\left(\sqrt{x}\right)-4\,ab\sqrt{x}-a^2}{x}$$

input
```
integrate((a+b*x^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
(2*b^2*x*log(sqrt(x)) - 4*a*b*sqrt(x) - a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx = -\frac{a^2}{x} - \frac{4ab}{\sqrt{x}} + b^2\log\left(x\right)$$

input
```
integrate((a+b*x**(1/2))**2/x**2,x)
```

output
```
-a**2/x - 4*a*b/sqrt(x) + b**2*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx = b^2\log\left(x\right) - \frac{4\,ab\sqrt{x}+a^2}{x}$$

input
```
integrate((a+b*x^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
b^2*log(x) - (4*a*b*sqrt(x) + a^2)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx = b^2\log\left(|x|\right) - \frac{4\,ab\sqrt{x}+a^2}{x}$$

input
```
integrate((a+b*x^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
b^2*log(abs(x)) - (4*a*b*sqrt(x) + a^2)/x
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx = 2\,b^2\,\ln\left(\sqrt{x}\right)-\frac{a^2+4\,a\,b\,\sqrt{x}}{x}$$

input
```
int((a + b*x^(1/2))^2/x^2,x)
```

output
```
2*b^2*log(x^(1/2)) - (a^2 + 4*a*b*x^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^2}\,dx = \frac{\sqrt{x}\log(x)\,b^2x-\sqrt{x}\,a^2-4abx}{\sqrt{x}\,x}$$

input
```
int((a+b*x^(1/2))^2/x^2,x)
```

output
```
(sqrt(x)*log(x)*b**2*x - sqrt(x)*a**2 - 4*a*b*x)/(sqrt(x)*x)
```

3.17 $\int \frac{(a+b\sqrt{x})^2}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{(a+b\sqrt{x})^2}{x^3} dx = -\frac{a^2}{2x^2} - \frac{4ab}{3x^{3/2}} - \frac{b^2}{x}$$

output
```
-1/2*a^2/x^2-4/3*a*b/x^(3/2)-b^2/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sqrt{x})^2}{x^3} dx = \frac{-3a^2 - 8ab\sqrt{x} - 6b^2x}{6x^2}$$

input
```
Integrate[(a + b*Sqrt[x])^2/x^3,x]
```

output
```
(-3*a^2 - 8*a*b*Sqrt[x] - 6*b^2*x)/(6*x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^2}{x^3}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{(a+b\sqrt{x})^2}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 53}$$

$$2\int \left(\frac{a^2}{x^{5/2}}+\frac{2ba}{x^2}+\frac{b^2}{x^{3/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{a^2}{4x^2}-\frac{2ab}{3x^{3/2}}-\frac{b^2}{2x}\right)$$

input
```
Int[(a + b*Sqrt[x])^2/x^3,x]
```

output
```
2*(-1/4*a^2/x^2 - (2*a*b)/(3*x^(3/2)) - b^2/(2*x))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{a^2}{2x^2}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{b^2}{x}$	25
default	$-\frac{a^2}{2x^2}-\frac{4ab}{3x^{\frac{3}{2}}}-\frac{b^2}{x}$	25
trager	$\frac{(-1+x)(a^2x+2b^2x+a^2)}{2x^2}-\frac{4ab}{3x^{\frac{3}{2}}}$	32
orering	$-\frac{(-14b^2x+9a^2)(a+b\sqrt{x})^2}{6x^2(-b^2x+a^2)}-\frac{(-2b^2x+a^2)x^2\left(\frac{(a+b\sqrt{x})b}{x^{\frac{7}{2}}}-\frac{3(a+b\sqrt{x})^2}{x^4}\right)}{3(-b^2x+a^2)}$	94

input
```
int((a+b*x^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-4/3*a*b/x^(3/2)-b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\sqrt{x})^2}{x^3}\,dx=-\frac{6\,b^2x+8\,ab\sqrt{x}+3\,a^2}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^2/x^3,x, algorithm="fricas")
```

output
```
-1/6*(6*b^2*x + 8*a*b*sqrt(x) + 3*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx = -\frac{a^2}{2x^2} - \frac{4ab}{3x^{\frac{3}{2}}} - \frac{b^2}{x}$$

input
```
integrate((a+b*x**(1/2))**2/x**3,x)
```

output
```
-a**2/(2*x**2) - 4*a*b/(3*x**(3/2)) - b**2/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx = -\frac{6\,b^2x + 8\,ab\sqrt{x} + 3\,a^2}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^2/x^3,x, algorithm="maxima")
```

output
```
-1/6*(6*b^2*x + 8*a*b*sqrt(x) + 3*a^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx = -\frac{6\,b^2x + 8\,ab\sqrt{x} + 3\,a^2}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^2/x^3,x, algorithm="giac")
```

output
```
-1/6*(6*b^2*x + 8*a*b*sqrt(x) + 3*a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx=-\frac{6\,b^2\,x+3\,a^2+8\,a\,b\,\sqrt{x}}{6\,x^2}$$

input
```
int((a + b*x^(1/2))^2/x^3,x)
```

output
```
-(6*b^2*x + 3*a^2 + 8*a*b*x^(1/2))/(6*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^3}\,dx=\frac{-3\sqrt{x}\,a^2-6\sqrt{x}\,b^2x-8abx}{6\sqrt{x}\,x^2}$$

input
```
int((a+b*x^(1/2))^2/x^3,x)
```

output
```
( - 3*sqrt(x)*a**2 - 6*sqrt(x)*b**2*x - 8*a*b*x)/(6*sqrt(x)*x**2)
```

3.18 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4} \, dx = -\frac{a^2}{3x^3} - \frac{4ab}{5x^{5/2}} - \frac{b^2}{2x^2}$$

output
```
-1/3*a^2/x^3-4/5*a*b/x^(5/2)-1/2*b^2/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4} \, dx = \frac{-10a^2 - 24ab\sqrt{x} - 15b^2x}{30x^3}$$

input
```
Integrate[(a + b*Sqrt[x])^2/x^4,x]
```

output
```
(-10*a^2 - 24*a*b*Sqrt[x] - 15*b^2*x)/(30*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^2}{x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^2}{x^{7/2}}+\frac{2ba}{x^3}+\frac{b^2}{x^{5/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^2}{6x^3}-\frac{2ab}{5x^{5/2}}-\frac{b^2}{4x^2}\right)$$

input
```
Int[(a + b*Sqrt[x])^2/x^4,x]
```

output
```
2*(-1/6*a^2/x^3 - (2*a*b)/(5*x^(5/2)) - b^2/(4*x^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{a^2}{3x^3}-\frac{4ab}{5x^{\frac{5}{2}}}-\frac{b^2}{2x^2}$	25
default	$-\frac{a^2}{3x^3}-\frac{4ab}{5x^{\frac{5}{2}}}-\frac{b^2}{2x^2}$	25
trager	$\frac{(-1+x)(2a^2x^2+3b^2x^2+2a^2x+3b^2x+2a^2)}{6x^3}-\frac{4ab}{5x^{\frac{5}{2}}}$	51
orering	$-\frac{(-33b^2x+26a^2)(a+b\sqrt{x})^2}{30x^3(-b^2x+a^2)}-\frac{(-3b^2x+2a^2)x^2\left(\frac{(a+b\sqrt{x})b}{x^{\frac{9}{2}}}-\frac{4(a+b\sqrt{x})^2}{x^5}\right)}{15(-b^2x+a^2)}$	96

input
```
int((a+b*x^(1/2))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3-4/5*a*b/x^(5/2)-1/2*b^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx=-\frac{15\,b^2x+24\,ab\sqrt{x}+10\,a^2}{30\,x^3}$$

input
```
integrate((a+b*x^(1/2))^2/x^4,x, algorithm="fricas")
```

output
```
-1/30*(15*b^2*x + 24*a*b*sqrt(x) + 10*a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx = -\frac{a^2}{3x^3} - \frac{4ab}{5x^{\frac{5}{2}}} - \frac{b^2}{2x^2}$$

input
```
integrate((a+b*x**(1/2))**2/x***4,x)
```

output
```
-a**2/(3*x***3) - 4*a*b/(5*x**(5/2)) - b**2/(2*x***2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx = -\frac{15\,b^2x + 24\,ab\sqrt{x} + 10\,a^2}{30\,x^3}$$

input
```
integrate((a+b*x^(1/2))^2/x^4,x, algorithm="maxima")
```

output
```
-1/30*(15*b^2*x + 24*a*b*sqrt(x) + 10*a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx = -\frac{15\,b^2x + 24\,ab\sqrt{x} + 10\,a^2}{30\,x^3}$$

input
```
integrate((a+b*x^(1/2))^2/x^4,x, algorithm="giac")
```

output
```
-1/30*(15*b^2*x + 24*a*b*sqrt(x) + 10*a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx = -\frac{15\,b^2\,x+10\,a^2+24\,a\,b\,\sqrt{x}}{30\,x^3}$$

input
```
int((a + b*x^(1/2))^2/x^4,x)
```

output
```
-(15*b^2*x + 10*a^2 + 24*a*b*x^(1/2))/(30*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^4}\,dx = \frac{-10\sqrt{x}\,a^2-15\sqrt{x}\,b^2x-24abx}{30\sqrt{x}\,x^3}$$

input
```
int((a+b*x^(1/2))^2/x^4,x)
```

output
```
( - 10*sqrt(x)*a**2 - 15*sqrt(x)*b**2*x - 24*a*b*x)/(30*sqrt(x)*x**3)
```

3.19 $\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5} \, dx = -\frac{a^2}{4x^4} - \frac{4ab}{7x^{7/2}} - \frac{b^2}{3x^3}$$

output
```
-1/4*a^2/x^4-4/7*a*b/x^(7/2)-1/3*b^2/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5} \, dx = \frac{-21a^2 - 48ab\sqrt{x} - 28b^2x}{84x^4}$$

input
```
Integrate[(a + b*Sqrt[x])^2/x^5,x]
```

output
```
(-21*a^2 - 48*a*b*Sqrt[x] - 28*b^2*x)/(84*x^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^2}{x^{9/2}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^2}{x^{9/2}}+\frac{2ba}{x^4}+\frac{b^2}{x^{7/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^2}{8x^4}-\frac{2ab}{7x^{7/2}}-\frac{b^2}{6x^3}\right)$$

input
```
Int[(a + b*Sqrt[x])^2/x^5,x]
```

output
```
2*(-1/8*a^2/x^4 - (2*a*b)/(7*x^(7/2)) - b^2/(6*x^3))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{a^2}{4x^4}-\frac{4ab}{7x^{\frac{7}{2}}}-\frac{b^2}{3x^3}$	25
default	$-\frac{a^2}{4x^4}-\frac{4ab}{7x^{\frac{7}{2}}}-\frac{b^2}{3x^3}$	25
trager	$\frac{(-1+x)(3a^2x^3+4b^2x^3+3a^2x^2+4b^2x^2+3a^2x+4b^2x+3a^2)}{12x^4}-\frac{4ab}{7x^{\frac{7}{2}}}$	67
orering	$-\frac{(-20b^2x+17a^2)(a+b\sqrt{x})^2}{28x^4(-b^2x+a^2)}-\frac{(-4b^2x+3a^2)x^2\left(\frac{(a+b\sqrt{x})b}{x^{\frac{11}{2}}}-\frac{5(a+b\sqrt{x})^2}{x^6}\right)}{42(-b^2x+a^2)}$	96

input
```
int((a+b*x^(1/2))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4-4/7*a*b/x^(7/2)-1/3*b^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\sqrt{x})^2}{x^5}\,dx=-\frac{28\,b^2x+48\,ab\sqrt{x}+21\,a^2}{84\,x^4}$$

input
```
integrate((a+b*x^(1/2))^2/x^5,x, algorithm="fricas")
```

output
```
-1/84*(28*b^2*x + 48*a*b*sqrt(x) + 21*a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx=-\frac{a^2}{4x^4}-\frac{4ab}{7x^{\frac{7}{2}}}-\frac{b^2}{3x^3}$$

input
```
integrate((a+b*x**(1/2))**2/x**5,x)
```

output
```
-a**2/(4*x**4) - 4*a*b/(7*x**(7/2)) - b**2/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx=-\frac{28\,b^2x+48\,ab\sqrt{x}+21\,a^2}{84\,x^4}$$

input
```
integrate((a+b*x^(1/2))^2/x^5,x, algorithm="maxima")
```

output
```
-1/84*(28*b^2*x + 48*a*b*sqrt(x) + 21*a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx=-\frac{28\,b^2x+48\,ab\sqrt{x}+21\,a^2}{84\,x^4}$$

input
```
integrate((a+b*x^(1/2))^2/x^5,x, algorithm="giac")
```

output
```
-1/84*(28*b^2*x + 48*a*b*sqrt(x) + 21*a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx=-\frac{28\,b^2\,x+21\,a^2+48\,a\,b\,\sqrt{x}}{84\,x^4}$$

input
```
int((a + b*x^(1/2))^2/x^5,x)
```

output
```
-(28*b^2*x + 21*a^2 + 48*a*b*x^(1/2))/(84*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt{x}\right)^2}{x^5}\,dx=\frac{-21\sqrt{x}\,a^2-28\sqrt{x}\,b^2x-48abx}{84\sqrt{x}\,x^4}$$

input
```
int((a+b*x^(1/2))^2/x^5,x)
```

output
```
( - 21*sqrt(x)*a**2 - 28*sqrt(x)*b**2*x - 48*a*b*x)/(84*sqrt(x)*x**4)
```

3.20 $\int \left(a+b\sqrt{x}\right)^3 x^4\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+b\sqrt{x}\right)^3 x^4\,dx=\frac{a^3x^5}{5}+\frac{6}{11}a^2bx^{11/2}+\frac{1}{2}ab^2x^6+\frac{2}{13}b^3x^{13/2}$$

output
```
1/5*a^3*x^5+6/11*a^2*b*x^(11/2)+1/2*a*b^2*x^6+2/13*b^3*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}\right)^3 x^4\,dx=\frac{x^5\left(286a^3+780a^2b\sqrt{x}+715ab^2x+220b^3x^{3/2}\right)}{1430}$$

input
```
Integrate[(a + b*Sqrt[x])^3*x^4,x]
```

output
```
(x^5*(286*a^3 + 780*a^2*b*Sqrt[x] + 715*a*b^2*x + 220*b^3*x^(3/2)))/1430
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sqrt{x}\right)^3 dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^3 x^{9/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^3x^6+3ab^2x^{11/2}+3a^2bx^5+a^3x^{9/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^3x^5}{10}+\frac{3}{11}a^2bx^{11/2}+\frac{1}{4}ab^2x^6+\frac{1}{13}b^3x^{13/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^3*x^4,x]
```

output
```
2*((a^3*x^5)/10 + (3*a^2*b*x^(11/2))/11 + (a*b^2*x^6)/4 + (b^3*x^(13/2))/1
3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^{3} x^{5}}{5}+\frac{6 a^{2} b \,x^{\frac{11}{2}}}{11}+\frac{a \,b^{2} x^{6}}{2}+\frac{2 b^{3} x^{\frac{13}{2}}}{13}$
default	$\frac{a^{3} x^{5}}{5}+\frac{6 a^{2} b \,x^{\frac{11}{2}}}{11}+\frac{a \,b^{2} x^{6}}{2}+\frac{2 b^{3} x^{\frac{13}{2}}}{13}$
trager	$\frac{a \left(5 b^{2} x^{5}+2 a^{2} x^{4}+5 b^{2} x^{4}+2 a^{2} x^{3}+5 b^{2} x^{3}+2 a^{2} x^{2}+5 b^{2} x^{2}+2 a^{2} x+5 b^{2} x+2 a^{2}+5 b^{2}\right) \left(-1+x \right)}{10}+\frac{2 b \,x^{\frac{11}{2}} \left(11 b^{2} x+39 a^{2}\right)}{143}$
orering	$\frac{x^{5} \left(1265 b^{4} x^{2}-2709 a^{2} b^{2} x+1482 a^{4}\right) \left(a+b \sqrt{x}\right)^{3}}{4290 \left(-b^{2} x+a^{2}\right)^{2}}-\frac{x^{2} \left(55 b^{4} x^{2}-129 a^{2} b^{2} x+78 a^{4}\right) \left(\frac{3 \left(a+b \sqrt{x}\right)^{2} x^{\frac{7}{2}} b}{2}+4 \left(a+b \sqrt{x}\right)^{3} x^{3}\right)}{2145 \left(-b^{2} x+a^{2}\right)^{2}}$

input
```
int((a+b*x^(1/2))^3*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3*x^5+6/11*a^2*b*x^(11/2)+1/2*a*b^2*x^6+2/13*b^3*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}\right)^3 x^4\,dx=\frac{1}{2}\,ab^2x^6+\frac{1}{5}\,a^3x^5+\frac{2}{143}\,\left(11\,b^3x^6+39\,a^2bx^5\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3*x^4,x, algorithm="fricas")
```

output
```
1/2*a*b^2*x^6 + 1/5*a^3*x^5 + 2/143*(11*b^3*x^6 + 39*a^2*b*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt{x}\right)^3 x^4\,dx = \frac{a^3x^5}{5}+\frac{6a^2bx^{\frac{11}{2}}}{11}+\frac{ab^2x^6}{2}+\frac{2b^3x^{\frac{13}{2}}}{13}$$

input
```
integrate((a+b*x**(1/2))**3*x**4,x)
```

output
```
a**3*x**5/5 + 6*a**2*b*x**(11/2)/11 + a*b**2*x**6/2 + 2*b**3*x**(13/2)/13
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.53

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^3 x^4\,dx &= \frac{2\left(b\sqrt{x}+a\right)^{13}}{13\,b^{10}}-\frac{3\left(b\sqrt{x}+a\right)^{12}a}{2\,b^{10}}+\frac{72\left(b\sqrt{x}+a\right)^{11}a^2}{11\,b^{10}}\\&\quad-\frac{84\left(b\sqrt{x}+a\right)^{10}a^3}{5\,b^{10}}+\frac{28\left(b\sqrt{x}+a\right)^{9}a^4}{b^{10}}\\&\quad-\frac{63\left(b\sqrt{x}+a\right)^{8}a^5}{2\,b^{10}}+\frac{24\left(b\sqrt{x}+a\right)^{7}a^6}{b^{10}}\\&\quad-\frac{12\left(b\sqrt{x}+a\right)^{6}a^7}{b^{10}}+\frac{18\left(b\sqrt{x}+a\right)^{5}a^8}{5\,b^{10}}-\frac{\left(b\sqrt{x}+a\right)^{4}a^9}{2\,b^{10}}\end{aligned}$$

input
```
integrate((a+b*x^(1/2))^3*x^4,x, algorithm="maxima")
```

output
```
2/13*(b*sqrt(x) + a)^13/b^10 - 3/2*(b*sqrt(x) + a)^12*a/b^10 + 72/11*(b*sq
rt(x) + a)^11*a^2/b^10 - 84/5*(b*sqrt(x) + a)^10*a^3/b^10 + 28*(b*sqrt(x)
+ a)^9*a^4/b^10 - 63/2*(b*sqrt(x) + a)^8*a^5/b^10 + 24*(b*sqrt(x) + a)^7*a
^6/b^10 - 12*(b*sqrt(x) + a)^6*a^7/b^10 + 18/5*(b*sqrt(x) + a)^5*a^8/b^10
- 1/2*(b*sqrt(x) + a)^4*a^9/b^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^3 x^4\,dx = \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{1}{2}\,ab^2x^6 + \frac{6}{11}\,a^2bx^{\frac{11}{2}} + \frac{1}{5}\,a^3x^5$$

input
```
integrate((a+b*x^(1/2))^3*x^4,x, algorithm="giac")
```

output
```
2/13*b^3*x^(13/2) + 1/2*a*b^2*x^6 + 6/11*a^2*b*x^(11/2) + 1/5*a^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^3 x^4\,dx = \frac{a^3\,x^5}{5} + \frac{2\,b^3\,x^{13/2}}{13} + \frac{a\,b^2\,x^6}{2} + \frac{6\,a^2\,b\,x^{11/2}}{11}$$

input
```
int(x^4*(a + b*x^(1/2))^3,x)
```

output
```
(a^3*x^5)/5 + (2*b^3*x^(13/2))/13 + (a*b^2*x^6)/2 + (6*a^2*b*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^3 x^4\,dx = \frac{x^5\left(780\sqrt{x}\,a^2b + 220\sqrt{x}\,b^3x + 286a^3 + 715a\,b^2x\right)}{1430}$$

input
```
int((a+b*x^(1/2))^3*x^4,x)
```

output
```
(x**5*(780*sqrt(x)*a**2*b + 220*sqrt(x)*b**3*x + 286*a**3 + 715*a*b**2*x))
/1430
```

3.21 $\int \left(a+b\sqrt{x}\right)^3 x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx=\frac{a^3x^4}{4}+\frac{2}{3}a^2bx^{9/2}+\frac{3}{5}ab^2x^5+\frac{2}{11}b^3x^{11/2}$$

output
```
1/4*a^3*x^4+2/3*a^2*b*x^(9/2)+3/5*a*b^2*x^5+2/11*b^3*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx=\frac{1}{660}\left(165a^3x^4+440a^2bx^{9/2}+396ab^2x^5+120b^3x^{11/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^3*x^3,x]
```

output
```
(165*a^3*x^4 + 440*a^2*b*x^(9/2) + 396*a*b^2*x^5 + 120*b^3*x^(11/2))/660
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{x}\right)^3 dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^3 x^{7/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^3x^5+3ab^2x^{9/2}+3a^2bx^4+a^3x^{7/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^3x^4}{8}+\frac{1}{3}a^2bx^{9/2}+\frac{3}{10}ab^2x^5+\frac{1}{11}b^3x^{11/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^3*x^3,x]
```

output
```
2*((a^3*x^4)/8 + (a^2*b*x^(9/2))/3 + (3*a*b^2*x^5)/10 + (b^3*x^(11/2))/11)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	siz
derivativedivides	$\frac{a^3x^4}{4}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{3a\,b^2x^5}{5}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
default	$\frac{a^3x^4}{4}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{3a\,b^2x^5}{5}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
trager	$\frac{a(12b^2x^4+5a^2x^3+12b^2x^3+5a^2x^2+12b^2x^2+5a^2x+12b^2x+5a^2+12b^2)(-1+x)}{20}+\frac{2b\,x^{\frac{9}{2}}(3b^2x+11a^2)}{33}$	89
orering	$\frac{x^4(228b^4x^2-493a^2b^2x+275a^4)(a+b\sqrt{x})^3}{660(-b^2x+a^2)^2}-\frac{x^2(36b^4x^2-87a^2b^2x+55a^4)\left(\frac{3(a+b\sqrt{x})^2x^{\frac{5}{2}}b}{2}+3(a+b\sqrt{x})^3x^2\right)}{990(-b^2x+a^2)^2}$	12

input
```
int((a+b*x^(1/2))^3*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*x^4+2/3*a^2*b*x^(9/2)+3/5*a*b^2*x^5+2/11*b^3*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx = \frac{3}{5}\,ab^2x^5+\frac{1}{4}\,a^3x^4+\frac{2}{33}\left(3\,b^3x^5+11\,a^2bx^4\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3*x^3,x, algorithm="fricas")
```

output
```
3/5*a*b^2*x^5 + 1/4*a^3*x^4 + 2/33*(3*b^3*x^5 + 11*a^2*b*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx = \frac{a^3x^4}{4} + \frac{2a^2bx^{\frac{9}{2}}}{3} + \frac{3ab^2x^5}{5} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

input
```
integrate((a+b*x**(1/2))**3*x**3,x)
```

output
```
a**3*x**4/4 + 2*a**2*b*x**(9/2)/3 + 3*a*b**2*x**5/5 + 2*b**3*x**(11/2)/11
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.81

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx = \frac{2\left(b\sqrt{x}+a\right)^{11}}{11\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^{10}a}{5\,b^8} + \frac{14\left(b\sqrt{x}+a\right)^9a^2}{3\,b^8} - \frac{35\left(b\sqrt{x}+a\right)^8a^3}{4\,b^8} + \frac{10\left(b\sqrt{x}+a\right)^7a^4}{b^8} - \frac{7\left(b\sqrt{x}+a\right)^6a^5}{b^8} + \frac{14\left(b\sqrt{x}+a\right)^5a^6}{5\,b^8} - \frac{\left(b\sqrt{x}+a\right)^4a^7}{2\,b^8}$$

input
```
integrate((a+b*x^(1/2))^3*x^3,x, algorithm="maxima")
```

output
```
2/11*(b*sqrt(x) + a)^11/b^8 - 7/5*(b*sqrt(x) + a)^10*a/b^8 + 14/3*(b*sqrt(
x) + a)^9*a^2/b^8 - 35/4*(b*sqrt(x) + a)^8*a^3/b^8 + 10*(b*sqrt(x) + a)^7*
a^4/b^8 - 7*(b*sqrt(x) + a)^6*a^5/b^8 + 14/5*(b*sqrt(x) + a)^5*a^6/b^8 - 1
/2*(b*sqrt(x) + a)^4*a^7/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx=\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{3}{5}\,ab^2x^5+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{1}{4}\,a^3x^4$$

input
```
integrate((a+b*x^(1/2))^3*x^3,x, algorithm="giac")
```

output
```
2/11*b^3*x^(11/2) + 3/5*a*b^2*x^5 + 2/3*a^2*b*x^(9/2) + 1/4*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx=\frac{a^3\,x^4}{4}+\frac{2\,b^3\,x^{11/2}}{11}+\frac{3\,a\,b^2\,x^5}{5}+\frac{2\,a^2\,b\,x^{9/2}}{3}$$

input
```
int(x^3*(a + b*x^(1/2))^3,x)
```

output
```
(a^3*x^4)/4 + (2*b^3*x^(11/2))/11 + (3*a*b^2*x^5)/5 + (2*a^2*b*x^(9/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^3 x^3\,dx=\frac{x^4\left(440\sqrt{x}\,a^2b+120\sqrt{x}\,b^3x+165a^3+396a\,b^2x\right)}{660}$$

input
```
int((a+b*x^(1/2))^3*x^3,x)
```

output
```
(x**4*(440*sqrt(x)*a**2*b + 120*sqrt(x)*b**3*x + 165*a**3 + 396*a*b**2*x))
/660
```

3.22 $\int \left(a+b\sqrt{x}\right)^3 x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+b\sqrt{x}\right)^3 x^2 \, dx=\frac{a^3x^3}{3}+\frac{6}{7}a^2bx^{7/2}+\frac{3}{4}ab^2x^4+\frac{2}{9}b^3x^{9/2}$$

output
```
1/3*a^3*x^3+6/7*a^2*b*x^(7/2)+3/4*a*b^2*x^4+2/9*b^3*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt{x}\right)^3 x^2 \, dx=\frac{1}{252}\left(84a^3x^3+216a^2bx^{7/2}+189ab^2x^4+56b^3x^{9/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^3*x^2,x]
```

output
```
(84*a^3*x^3 + 216*a^2*b*x^(7/2) + 189*a*b^2*x^4 + 56*b^3*x^(9/2))/252
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{x}\right)^3\,dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \left(a+b\sqrt{x}\right)^3 x^{5/2}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(b^3x^4+3ab^2x^{7/2}+3a^2bx^3+a^3x^{5/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{a^3x^3}{6}+\frac{3}{7}a^2bx^{7/2}+\frac{3}{8}ab^2x^4+\frac{1}{9}b^3x^{9/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^3*x^2,x]
```

output
```
2*((a^3*x^3)/6 + (3*a^2*b*x^(7/2))/7 + (3*a*b^2*x^4)/8 + (b^3*x^(9/2))/9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	siz
derivativedivides	$\frac{a^{3} x^{3}}{3}+\frac{6 a^{2} b \,x^{\frac{7}{2}}}{7}+\frac{3 a \,b^{2} x^{4}}{4}+\frac{2 b^{3} x^{\frac{9}{2}}}{9}$	36
default	$\frac{a^{3} x^{3}}{3}+\frac{6 a^{2} b \,x^{\frac{7}{2}}}{7}+\frac{3 a \,b^{2} x^{4}}{4}+\frac{2 b^{3} x^{\frac{9}{2}}}{9}$	36
trager	$\frac{a \left(9 b^{2} x^{3}+4 a^{2} x^{2}+9 b^{2} x^{2}+4 a^{2} x+9 b^{2} x+4 a^{2}+9 b^{2}\right) \left(-1+x\right)}{12}+\frac{2 b \,x^{\frac{7}{2}} \left(7 b^{2} x+27 a^{2}\right)}{63}$	73
orering	$\frac{x^{3} \left(315 b^{4} x^{2}-689 a^{2} b^{2} x+396 a^{4}\right) \left(a+b \sqrt{x}\right)^{3}}{756 \left(-b^{2} x+a^{2}\right)^{2}}-\frac{x^{2} \left(21 b^{4} x^{2}-53 a^{2} b^{2} x+36 a^{4}\right) \left(\frac{3 \left(a+b \sqrt{x}\right)^{2} x^{\frac{3}{2}} b}{2}+2 \left(a+b \sqrt{x}\right)^{3} x \right)}{378 \left(-b^{2} x+a^{2}\right)^{2}}$	119

input
```
int((a+b*x^(1/2))^3*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+6/7*a^2*b*x^(7/2)+3/4*a*b^2*x^4+2/9*b^3*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}\right)^3 x^2\,dx = \frac{3}{4}\,ab^2x^4 + \frac{1}{3}\,a^3x^3 + \frac{2}{63}\left(7\,b^3x^4 + 27\,a^2bx^3\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3*x^2,x, algorithm="fricas")
```

output
```
3/4*a*b^2*x^4 + 1/3*a^3*x^3 + 2/63*(7*b^3*x^4 + 27*a^2*b*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt{x}\right)^3 x^2\,dx = \frac{a^3x^3}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{3ab^2x^4}{4}+\frac{2b^3x^{\frac{9}{2}}}{9}$$

input
```
integrate((a+b*x**(1/2))**3*x**2,x)
```

output
```
a**3*x**3/3 + 6*a**2*b*x**(7/2)/7 + 3*a*b**2*x**4/4 + 2*b**3*x**(9/2)/9
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

$$\int \left(a+b\sqrt{x}\right)^3 x^2\,dx = \frac{2\left(b\sqrt{x}+a\right)^9}{9\,b^6}-\frac{5\left(b\sqrt{x}+a\right)^8 a}{4\,b^6}+\frac{20\left(b\sqrt{x}+a\right)^7 a^2}{7\,b^6} - \frac{10\left(b\sqrt{x}+a\right)^6 a^3}{3\,b^6}+\frac{2\left(b\sqrt{x}+a\right)^5 a^4}{b^6}-\frac{\left(b\sqrt{x}+a\right)^4 a^5}{2\,b^6}$$

input
```
integrate((a+b*x^(1/2))^3*x^2,x, algorithm="maxima")
```

output
```
2/9*(b*sqrt(x) + a)^9/b^6 - 5/4*(b*sqrt(x) + a)^8*a/b^6 + 20/7*(b*sqrt(x)
+ a)^7*a^2/b^6 - 10/3*(b*sqrt(x) + a)^6*a^3/b^6 + 2*(b*sqrt(x) + a)^5*a^4/
b^6 - 1/2*(b*sqrt(x) + a)^4*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^3 x^2\,dx = \frac{2}{9}\,b^3x^{\frac{9}{2}} + \frac{3}{4}\,ab^2x^4 + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{1}{3}\,a^3x^3$$

input
```
integrate((a+b*x^(1/2))^3*x^2,x, algorithm="giac")
```

output
```
2/9*b^3*x^(9/2) + 3/4*a*b^2*x^4 + 6/7*a^2*b*x^(7/2) + 1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^3 x^2\,dx = \frac{a^3\,x^3}{3} + \frac{2\,b^3\,x^{9/2}}{9} + \frac{3\,a\,b^2\,x^4}{4} + \frac{6\,a^2\,b\,x^{7/2}}{7}$$

input
```
int(x^2*(a + b*x^(1/2))^3,x)
```

output
```
(a^3*x^3)/3 + (2*b^3*x^(9/2))/9 + (3*a*b^2*x^4)/4 + (6*a^2*b*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^3 x^2\,dx = \frac{x^3\left(216\sqrt{x}\,a^2b + 56\sqrt{x}\,b^3x + 84a^3 + 189a\,b^2x\right)}{252}$$

input
```
int((a+b*x^(1/2))^3*x^2,x)
```

output
```
(x**3*(216*sqrt(x)*a**2*b + 56*sqrt(x)*b**3*x + 84*a**3 + 189*a*b**2*x))/2
52
```

3.23 $\int \left(a+b\sqrt{x}\right)^3 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx=\frac{a^3x^2}{2}+\frac{6}{5}a^2bx^{5/2}+ab^2x^3+\frac{2}{7}b^3x^{7/2}$$

output
```
1/2*a^3*x^2+6/5*a^2*b*x^(5/2)+a*b^2*x^3+2/7*b^3*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx=\frac{1}{70}\left(35a^3x^2+84a^2bx^{5/2}+70ab^2x^3+20b^3x^{7/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^3*x,x]
```

output
```
(35*a^3*x^2 + 84*a^2*b*x^(5/2) + 70*a*b^2*x^3 + 20*b^3*x^(7/2))/70
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}\right)^3\,dx$$

$$\Big\downarrow \; 798$$

$$2\int \left(a+b\sqrt{x}\right)^3 x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \; 49$$

$$2\int \left(x^{3/2}a^3+3bx^2a^2+3b^2x^{5/2}a+b^3x^3\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{a^3x^2}{4}+\frac{3}{5}a^2bx^{5/2}+\frac{1}{2}ab^2x^3+\frac{1}{7}b^3x^{7/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^3*x,x]
```

output
```
2*((a^3*x^2)/4 + (3*a^2*b*x^(5/2))/5 + (a*b^2*x^3)/2 + (b^3*x^(7/2))/7)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{a^{3} x^{2}}{2}+\frac{6 a^{2} b \,x^{\frac{5}{2}}}{5}+a \,b^{2} x^{3}+\frac{2 b^{3} x^{\frac{7}{2}}}{7}$	35
default	$\frac{a^{3} x^{2}}{2}+\frac{6 a^{2} b \,x^{\frac{5}{2}}}{5}+a \,b^{2} x^{3}+\frac{2 b^{3} x^{\frac{7}{2}}}{7}$	35
trager	$\frac{a \left(2 b^{2} x^{2}+a^{2} x+2 b^{2} x+a^{2}+2 b^{2}\right) \left(-1+x \right)}{2}+\frac{2 b \,x^{\frac{5}{2}} \left(5 b^{2} x+21 a^{2}\right)}{35}$	54
orering	$\frac{x^{2} \left(110 b^{4} x^{2}-243 a^{2} b^{2} x+147 a^{4}\right) \left(a+b \sqrt{x}\right)^{3}}{210 \left(-b^{2} x+a^{2}\right)^{2}}-\frac{x^{2} \left(10 b^{4} x^{2}-27 a^{2} b^{2} x+21 a^{4}\right) \left(\frac{3 \left(a+b \sqrt{x}\right)^{2} \sqrt{x}\, b}{2}+\left(a+b \sqrt{x}\right)^{3}\right)}{105 \left(-b^{2} x+a^{2}\right)^{2}}$	116

input
```
int((a+b*x^(1/2))^3*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+6/5*a^2*b*x^(5/2)+a*b^2*x^3+2/7*b^3*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx = ab^2x^3 + \frac{1}{2}\,a^3x^2 + \frac{2}{35}\,\left(5\,b^3x^3 + 21\,a^2bx^2\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3*x,x, algorithm="fricas")
```

output
```
a*b^2*x^3 + 1/2*a^3*x^2 + 2/35*(5*b^3*x^3 + 21*a^2*b*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx = \frac{a^3x^2}{2} + \frac{6a^2bx^{\frac{5}{2}}}{5} + ab^2x^3 + \frac{2b^3x^{\frac{7}{2}}}{7}$$

input
```
integrate((a+b*x**(1/2))**3*x,x)
```

output
```
a**3*x**2/2 + 6*a**2*b*x**(5/2)/5 + a*b**2*x**3 + 2*b**3*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx = \frac{2\left(b\sqrt{x}+a\right)^7}{7\,b^4} - \frac{\left(b\sqrt{x}+a\right)^6 a}{b^4} + \frac{6\left(b\sqrt{x}+a\right)^5 a^2}{5\,b^4} - \frac{\left(b\sqrt{x}+a\right)^4 a^3}{2\,b^4}$$

input
```
integrate((a+b*x^(1/2))^3*x,x, algorithm="maxima")
```

output
```
2/7*(b*sqrt(x) + a)^7/b^4 - (b*sqrt(x) + a)^6*a/b^4 + 6/5*(b*sqrt(x) + a)^
5*a^2/b^4 - 1/2*(b*sqrt(x) + a)^4*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx = \frac{2}{7}\,b^3x^{\frac{7}{2}} + ab^2x^3 + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + \frac{1}{2}\,a^3x^2$$

input
```
integrate((a+b*x^(1/2))^3*x,x, algorithm="giac")
```

output
```
2/7*b^3*x^(7/2) + a*b^2*x^3 + 6/5*a^2*b*x^(5/2) + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx=\frac{a^3\,x^2}{2}+\frac{2\,b^3\,x^{7/2}}{7}+a\,b^2\,x^3+\frac{6\,a^2\,b\,x^{5/2}}{5}$$

input
```
int(x*(a + b*x^(1/2))^3,x)
```

output
```
(a^3*x^2)/2 + (2*b^3*x^(7/2))/7 + a*b^2*x^3 + (6*a^2*b*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}\right)^3 x\,dx=\frac{x^2\left(84\sqrt{x}\,a^2b+20\sqrt{x}\,b^3x+35a^3+70a\,b^2x\right)}{70}$$

input
```
int((a+b*x^(1/2))^3*x,x)
```

output
```
(x**2*(84*sqrt(x)*a**2*b + 20*sqrt(x)*b**3*x + 35*a**3 + 70*a*b**2*x))/70
```

3.24 $\int \left(a+b\sqrt{x}\right)^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \left(a+b\sqrt{x}\right)^3 \, dx = -\frac{a\left(a+b\sqrt{x}\right)^4}{2b^2}+\frac{2\left(a+b\sqrt{x}\right)^5}{5b^2}$$

output
```
-1/2*a*(a+b*x^(1/2))^4/b^2+2/5*(a+b*x^(1/2))^5/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \left(a+b\sqrt{x}\right)^3 \, dx = \frac{1}{10}\left(10a^3x+20a^2bx^{3/2}+15ab^2x^2+4b^3x^{5/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^3,x]
```

output
```
(10*a^3*x + 20*a^2*b*x^(3/2) + 15*a*b^2*x^2 + 4*b^3*x^(5/2))/10
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^3 dx$$

$$\Big\downarrow \text{774}$$

$$2\int \left(a+b\sqrt{x}\right)^3 \sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^4}{b}-\frac{a\left(a+b\sqrt{x}\right)^3}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\left(a+b\sqrt{x}\right)^5}{5b^2}-\frac{a\left(a+b\sqrt{x}\right)^4}{4b^2}\right)$$

input
```
Int[(a + b*Sqrt[x])^3,x]
```

output
```
2*(-1/4*(a*(a + b*Sqrt[x])^4)/b^2 + (a + b*Sqrt[x])^5/(5*b^2))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{2b^3x^{\frac{5}{2}}}{5}+\frac{3a\,b^2x^2}{2}+2a^2b\,x^{\frac{3}{2}}+a^3x$	33
default	$\frac{2b^3x^{\frac{5}{2}}}{5}+\frac{3a\,b^2x^2}{2}+2a^2b\,x^{\frac{3}{2}}+a^3x$	33
trager	$\frac{(-1+x)(3b^2x+2a^2+3b^2)a}{2}+\frac{2b\,x^{\frac{3}{2}}\,(b^2x+5a^2)}{5}$	42
orering	$-\frac{(-7b^6x^3+15a^2b^4x^2+10a^6)\,(a+b\sqrt{x})^3}{10b^2(-b^2x+a^2)^2}+\frac{3(-b^6x^3+3a^2b^4x^2+10a^6)\sqrt{x}\,(a+b\sqrt{x})^2}{10b(-b^2x+a^2)^2}$	107

input
```
int((a+b*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*b^3*x^(5/2)+3/2*a*b^2*x^2+2*a^2*b*x^(3/2)+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt{x}\right)^3\,dx=\frac{3}{2}\,ab^2x^2+a^3x+\frac{2}{5}\,\left(b^3x^2+5\,a^2bx\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3,x, algorithm="fricas")
```

output
```
3/2*a*b^2*x^2 + a^3*x + 2/5*(b^3*x^2 + 5*a^2*b*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt{x}\right)^3\,dx=a^3x+2a^2bx^{\frac{3}{2}}+\frac{3ab^2x^2}{2}+\frac{2b^3x^{\frac{5}{2}}}{5}$$

input
```
integrate((a+b*x**(1/2))**3,x)
```

output
```
a**3*x + 2*a**2*b*x**(3/2) + 3*a*b**2*x**2/2 + 2*b**3*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^3\,dx=\frac{2}{5}\,b^3x^{\frac{5}{2}}+\frac{3}{2}\,ab^2x^2+2\,a^2bx^{\frac{3}{2}}+a^3x$$

input
```
integrate((a+b*x^(1/2))^3,x, algorithm="maxima")
```

output
```
2/5*b^3*x^(5/2) + 3/2*a*b^2*x^2 + 2*a^2*b*x^(3/2) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^3\,dx=\frac{2}{5}\,b^3x^{\frac{5}{2}}+\frac{3}{2}\,ab^2x^2+2\,a^2bx^{\frac{3}{2}}+a^3x$$

input
```
integrate((a+b*x^(1/2))^3,x, algorithm="giac")
```

output
```
2/5*b^3*x^(5/2) + 3/2*a*b^2*x^2 + 2*a^2*b*x^(3/2) + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^3 \, dx = a^3\,x + \frac{2\,b^3\,x^{5/2}}{5} + \frac{3\,a\,b^2\,x^2}{2} + 2\,a^2\,b\,x^{3/2}$$

input
```
int((a + b*x^(1/2))^3,x)
```

output
```
a^3*x + (2*b^3*x^(5/2))/5 + (3*a*b^2*x^2)/2 + 2*a^2*b*x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^3 \, dx = \frac{x\left(20\sqrt{x}\,a^2b + 4\sqrt{x}\,b^3x + 10a^3 + 15a\,b^2x\right)}{10}$$

input
```
int((a+b*x^(1/2))^3,x)
```

output
```
(x*(20*sqrt(x)*a**2*b + 4*sqrt(x)*b**3*x + 10*a**3 + 15*a*b**2*x))/10
```

3.25 $\int \frac{(a+b\sqrt{x})^3}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{(a+b\sqrt{x})^3}{x} \, dx = 6a^2b\sqrt{x} + 3ab^2x + \frac{2}{3}b^3x^{3/2} + a^3\log(x)$$

output
```
6*a^2*b*x^(1/2)+3*a*b^2*x+2/3*b^3*x^(3/2)+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sqrt{x})^3}{x} \, dx = 6a^2b\sqrt{x} + 3ab^2x + \frac{2}{3}b^3x^{3/2} + a^3\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^3/x,x]
```

output
```
6*a^2*b*Sqrt[x] + 3*a*b^2*x + (2*b^3*x^(3/2))/3 + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^3}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^3}{\sqrt{x}}+3ba^2+3b^2\sqrt{x}a+b^3x\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(a^3\log\left(\sqrt{x}\right)+3a^2b\sqrt{x}+\frac{3}{2}ab^2x+\frac{1}{3}b^3x^{3/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^3/x,x]
```

output
```
2*(3*a^2*b*Sqrt[x] + (3*a*b^2*x)/2 + (b^3*x^(3/2))/3 + a^3*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$6a^2b\sqrt{x}+3a\,b^2x+\frac{2b^3x^{\frac{3}{2}}}{3}+a^3\ln\left(x\right)$	32
default	$6a^2b\sqrt{x}+3a\,b^2x+\frac{2b^3x^{\frac{3}{2}}}{3}+a^3\ln\left(x\right)$	32
trager	$3a\,b^2(-1+x)+\frac{2(b^2x+9a^2)b\sqrt{x}}{3}+a^3\ln\left(x\right)$	34

input
```
int((a+b*x^(1/2))^3/x,x,method=_RETURNVERBOSE)
```

output
```
6*a^2*b*x^(1/2)+3*a*b^2*x+2/3*b^3*x^(3/2)+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x}\,dx = 3\,ab^2x+2\,a^3\log\left(\sqrt{x}\right)+\frac{2}{3}\left(b^3x+9\,a^2b\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3/x,x, algorithm="fricas")
```

output
```
3*a*b^2*x + 2*a^3*log(sqrt(x)) + 2/3*(b^3*x + 9*a^2*b)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x}\,dx = a^3\log\left(x\right)+6a^2b\sqrt{x}+3ab^2x+\frac{2b^3x^{\frac{3}{2}}}{3}$$

input
```
integrate((a+b*x**(1/2))**3/x,x)
```

output
```
a**3*log(x) + 6*a**2*b*sqrt(x) + 3*a*b**2*x + 2*b**3*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x}\,dx = \frac{2}{3}\,b^3x^{\frac{3}{2}}+3\,ab^2x+a^3\log\left(x\right)+6\,a^2b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3/x,x, algorithm="maxima")
```

output
```
2/3*b^3*x^(3/2) + 3*a*b^2*x + a^3*log(x) + 6*a^2*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x}\,dx = \frac{2}{3}\,b^3x^{\frac{3}{2}}+3\,ab^2x+a^3\log\left(|x|\right)+6\,a^2b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3/x,x, algorithm="giac")
```

output
```
2/3*b^3*x^(3/2) + 3*a*b^2*x + a^3*log(abs(x)) + 6*a^2*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sqrt{x})^3}{x}\,dx = 2\,a^3\,\ln\left(\sqrt{x}\right)+\frac{2\,b^3\,x^{3/2}}{3}+6\,a^2\,b\,\sqrt{x}+3\,a\,b^2\,x$$

input
```
int((a + b*x^(1/2))^3/x,x)
```

output
```
2*a^3*log(x^(1/2)) + (2*b^3*x^(3/2))/3 + 6*a^2*b*x^(1/2) + 3*a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\sqrt{x})^3}{x}\,dx = 6\sqrt{x}\,a^2b+\frac{2\sqrt{x}\,b^3x}{3}+\log(x)\,a^3+3a\,b^2x$$

input
```
int((a+b*x^(1/2))^3/x,x)
```

output
```
(18*sqrt(x)*a**2*b + 2*sqrt(x)*b**3*x + 3*log(x)*a**3 + 9*a*b**2*x)/3
```

3.26 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx = -\frac{a^3}{x} - \frac{6a^2b}{\sqrt{x}} + 2b^3\sqrt{x} + 3ab^2\log(x)$$

output
```
-a^3/x-6*a^2*b/x^(1/2)+2*b^3*x^(1/2)+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx = -\frac{a^3+6a^2b\sqrt{x}-2b^3x^{3/2}}{x} + 3ab^2\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^3/x^2,x]
```

output
```
-((a^3 + 6*a^2*b*Sqrt[x] - 2*b^3*x^(3/2))/x) + 3*a*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^3}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^3}{x^{3/2}}+\frac{3ba^2}{x}+\frac{3b^2a}{\sqrt{x}}+b^3\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^3}{2x}-\frac{3a^2b}{\sqrt{x}}+3ab^2\log\left(\sqrt{x}\right)+b^3\sqrt{x}\right)$$

input
```
Int[(a + b*Sqrt[x])^3/x^2,x]
```

output
```
2*(-1/2*a^3/x - (3*a^2*b)/Sqrt[x] + b^3*Sqrt[x] + 3*a*b^2*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{a^{3}}{x}-\frac{6a^{2}b}{\sqrt{x}}+2b^{3}\sqrt{x}+3a\,b^{2}\ln\left(x\right)$	35
default	$-\frac{a^{3}}{x}-\frac{6a^{2}b}{\sqrt{x}}+2b^{3}\sqrt{x}+3a\,b^{2}\ln\left(x\right)$	35
trager	$\frac{a^{3}\left(-1+x\right)}{x}-\frac{2\left(-b^{2}x+3a^{2}\right)b}{\sqrt{x}}-3a\,b^{2}\ln\left(\frac{1}{x}\right)$	40

input
```
int((a+b*x^(1/2))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x-6*a^2*b/x^(1/2)+2*b^3*x^(1/2)+3*a*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int\frac{\left(a+b\sqrt{x}\right)^{3}}{x^{2}}\,dx=\frac{6\,ab^{2}x\log\left(\sqrt{x}\right)-a^{3}+2\left(b^{3}x-3\,a^{2}b\right)\sqrt{x}}{x}$$

input
```
integrate((a+b*x^(1/2))^3/x^2,x, algorithm="fricas")
```

output
```
(6*a*b^2*x*log(sqrt(x)) - a^3 + 2*(b^3*x - 3*a^2*b)*sqrt(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\sqrt{x})^3}{x^2}\,dx = -\frac{a^3}{x} - \frac{6a^2b}{\sqrt{x}} + 3ab^2\log(x) + 2b^3\sqrt{x}$$

input
```
integrate((a+b*x**(1/2))**3/x**2,x)
```

output
```
-a**3/x - 6*a**2*b/sqrt(x) + 3*a*b**2*log(x) + 2*b**3*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\sqrt{x})^3}{x^2}\,dx = 3\,ab^2\log(x) + 2\,b^3\sqrt{x} - \frac{6\,a^2b\sqrt{x} + a^3}{x}$$

input
```
integrate((a+b*x^(1/2))^3/x^2,x, algorithm="maxima")
```

output
```
3*a*b^2*log(x) + 2*b^3*sqrt(x) - (6*a^2*b*sqrt(x) + a^3)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\sqrt{x})^3}{x^2}\,dx = 3\,ab^2\log(|x|) + 2\,b^3\sqrt{x} - \frac{6\,a^2b\sqrt{x} + a^3}{x}$$

input
```
integrate((a+b*x^(1/2))^3/x^2,x, algorithm="giac")
```

output
```
3*a*b^2*log(abs(x)) + 2*b^3*sqrt(x) - (6*a^2*b*sqrt(x) + a^3)/x
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx = 2\,b^3\,\sqrt{x}-\frac{a^3+6\,a^2\,b\,\sqrt{x}}{x}+6\,a\,b^2\,\ln\left(\sqrt{x}\right)$$

input
```
int((a + b*x^(1/2))^3/x^2,x)
```

output
```
2*b^3*x^(1/2) - (a^3 + 6*a^2*b*x^(1/2))/x + 6*a*b^2*log(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^2}\,dx = \frac{3\sqrt{x}\log(x)\,a\,b^2x-\sqrt{x}\,a^3-6a^2bx+2b^3x^2}{\sqrt{x}\,x}$$

input
```
int((a+b*x^(1/2))^3/x^2,x)
```

output
```
(3*sqrt(x)*log(x)*a*b**2*x - sqrt(x)*a**3 - 6*a**2*b*x + 2*b**3*x**2)/(sqr
t(x)*x)
```

3.27 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3} dx = -\frac{\left(a+b\sqrt{x}\right)^4}{2ax^2}$$

output
```
-1/2*(a+b*x^(1/2))^4/a/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3} dx = \frac{-a^3-4a^2b\sqrt{x}-6ab^2x-4b^3x^{3/2}}{2x^2}$$

input
```
Integrate[(a + b*Sqrt[x])^3/x^3,x]
```

output
```
(-a^3 - 4*a^2*b*Sqrt[x] - 6*a*b^2*x - 4*b^3*x^(3/2))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(a+b\sqrt{x}\right)^4}{2ax^2}$$

input
```
Int[(a + b*Sqrt[x])^3/x^3,x]
```

output
```
-1/2*(a + b*Sqrt[x])^4/(a*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

method	result	size
derivativedivides	$-\frac{3a\,b^{2}}{x}-\frac{2a^{2}b}{x^{\frac{3}{2}}}-\frac{a^{3}}{2x^{2}}-\frac{2b^{3}}{\sqrt{x}}$	36
default	$-\frac{3a\,b^{2}}{x}-\frac{2a^{2}b}{x^{\frac{3}{2}}}-\frac{a^{3}}{2x^{2}}-\frac{2b^{3}}{\sqrt{x}}$	36
trager	$\frac{(-1+x)(a^{2}x+6b^{2}x+a^{2})a}{2x^{2}}-\frac{2(b^{2}x+a^{2})b}{x^{\frac{3}{2}}}$	41
orering	$-\frac{(10b^{4}x^{2}-7a^{2}b^{2}x+3a^{4})(a+b\sqrt{x})^{3}}{2x^{2}(-b^{2}x+a^{2})^{2}}-\frac{(6b^{4}x^{2}-3a^{2}b^{2}x+a^{4})x^{2}\left(\frac{3(a+b\sqrt{x})^{2}b}{2x^{\frac{7}{2}}}-\frac{3(a+b\sqrt{x})^{3}}{x^{4}}\right)}{3(-b^{2}x+a^{2})^{2}}$	119

input
```
int((a+b*x^(1/2))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3*a*b^2/x-2*a^2*b/x^(3/2)-1/2*a^3/x^2-2*b^3/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \frac{(a+b\sqrt{x})^3}{x^3}\,dx=-\frac{6\,ab^2x+a^3+4\,(b^3x+a^2b)\sqrt{x}}{2\,x^2}$$

input
```
integrate((a+b*x^(1/2))^3/x^3,x, algorithm="fricas")
```

output
```
-1/2*(6*a*b^2*x + a^3 + 4*(b^3*x + a^2*b)*sqrt(x))/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{(a+b\sqrt{x})^3}{x^3}\,dx=-\frac{a^3}{2x^2}-\frac{2a^2b}{x^{\frac{3}{2}}}-\frac{3ab^2}{x}-\frac{2b^3}{\sqrt{x}}$$

input
```
integrate((a+b*x**(1/2))**3/x**3,x)
```

output
```
-a**3/(2*x**2) - 2*a**2*b/x**(3/2) - 3*a*b**2/x - 2*b**3/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}\,dx = -\frac{4\,b^3x^{\frac{3}{2}}+6\,ab^2x+4\,a^2b\sqrt{x}+a^3}{2\,x^2}$$

input
```
integrate((a+b*x^(1/2))^3/x^3,x, algorithm="maxima")
```

output
```
-1/2*(4*b^3*x^(3/2) + 6*a*b^2*x + 4*a^2*b*sqrt(x) + a^3)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}\,dx = -\frac{4\,b^3x^{\frac{3}{2}}+6\,ab^2x+4\,a^2b\sqrt{x}+a^3}{2\,x^2}$$

input
```
integrate((a+b*x^(1/2))^3/x^3,x, algorithm="giac")
```

output
```
-1/2*(4*b^3*x^(3/2) + 6*a*b^2*x + 4*a^2*b*sqrt(x) + a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}\,dx = -\frac{a^3+4\,b^3\,x^{3/2}+4\,a^2\,b\,\sqrt{x}+6\,a\,b^2\,x}{2\,x^2}$$

input
```
int((a + b*x^(1/2))^3/x^3,x)
```

output
```
-(a^3 + 4*b^3*x^(3/2) + 4*a^2*b*x^(1/2) + 6*a*b^2*x)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^3}\,dx = \frac{-\sqrt{x}\,a^3-6\sqrt{x}\,a\,b^2x-4a^2bx-4b^3x^2}{2\sqrt{x}\,x^2}$$

input
```
int((a+b*x^(1/2))^3/x^3,x)
```

output
```
( - sqrt(x)*a**3 - 6*sqrt(x)*a*b**2*x - 4*a**2*b*x - 4*b**3*x**2)/(2*sqrt(
x)*x**2)
```

3.28 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx = -\frac{a^3}{3x^3}-\frac{6a^2b}{5x^{5/2}}-\frac{3ab^2}{2x^2}-\frac{2b^3}{3x^{3/2}}$$

output
```
-1/3*a^3/x^3-6/5*a^2*b/x^(5/2)-3/2*a*b^2/x^2-2/3*b^3/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx = \frac{-10a^3-36a^2b\sqrt{x}-45ab^2x-20b^3x^{3/2}}{30x^3}$$

input
```
Integrate[(a + b*Sqrt[x])^3/x^4,x]
```

output
```
(-10*a^3 - 36*a^2*b*Sqrt[x] - 45*a*b^2*x - 20*b^3*x^(3/2))/(30*x^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^3}{x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^3}{x^{7/2}}+\frac{3ba^2}{x^3}+\frac{3b^2a}{x^{5/2}}+\frac{b^3}{x^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^3}{6x^3}-\frac{3a^2b}{5x^{5/2}}-\frac{3ab^2}{4x^2}-\frac{b^3}{3x^{3/2}}\right)$$

input
```
Int[(a + b*Sqrt[x])^3/x^4,x]
```

output
```
2*(-1/6*a^3/x^3 - (3*a^2*b)/(5*x^(5/2)) - (3*a*b^2)/(4*x^2) - b^3/(3*x^(3/
2)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{a^{3}}{3x^{3}}-\frac{6a^{2}b}{5x^{\frac{5}{2}}}-\frac{3a\,b^{2}}{2x^{2}}-\frac{2b^{3}}{3x^{\frac{3}{2}}}$	36
default	$-\frac{a^{3}}{3x^{3}}-\frac{6a^{2}b}{5x^{\frac{5}{2}}}-\frac{3a\,b^{2}}{2x^{2}}-\frac{2b^{3}}{3x^{\frac{3}{2}}}$	36
trager	$\frac{(-1+x)(2a^{2}x^{2}+9b^{2}x^{2}+2a^{2}x+9b^{2}x+2a^{2})a}{6x^{3}}-\frac{2(5b^{2}x+9a^{2})b}{15x^{\frac{5}{2}}}$	63
orering	$-\frac{(135b^{4}x^{2}-187a^{2}b^{2}x+78a^{4})(a+b\sqrt{x})^{3}}{90x^{3}(-b^{2}x+a^{2})^{2}}-\frac{(15b^{4}x^{2}-17a^{2}b^{2}x+6a^{4})x^{2}\left(\frac{3(a+b\sqrt{x})^{2}b}{2x^{\frac{9}{2}}}-\frac{4(a+b\sqrt{x})^{3}}{x^{5}}\right)}{45(-b^{2}x+a^{2})^{2}}$	121

input
```
int((a+b*x^(1/2))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3/x^3-6/5*a^2*b/x^(5/2)-3/2*a*b^2/x^2-2/3*b^3/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt{x}\right)^{3}}{x^{4}}\,dx=-\frac{45\,ab^{2}x+10\,a^{3}+4\,(5\,b^{3}x+9\,a^{2}b)\sqrt{x}}{30\,x^{3}}$$

input
```
integrate((a+b*x^(1/2))^3/x^4,x, algorithm="fricas")
```

output
```
-1/30*(45*a*b^2*x + 10*a^3 + 4*(5*b^3*x + 9*a^2*b)*sqrt(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx = -\frac{a^3}{3x^3} - \frac{6a^2b}{5x^{\frac{5}{2}}} - \frac{3ab^2}{2x^2} - \frac{2b^3}{3x^{\frac{3}{2}}}$$

input
```
integrate((a+b*x**(1/2))**3/x**4,x)
```

output
```
-a**3/(3*x**3) - 6*a**2*b/(5*x**(5/2)) - 3*a*b**2/(2*x**2) - 2*b**3/(3*x**
(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx = -\frac{20\,b^3x^{\frac{3}{2}} + 45\,ab^2x + 36\,a^2b\sqrt{x} + 10\,a^3}{30\,x^3}$$

input
```
integrate((a+b*x^(1/2))^3/x^4,x, algorithm="maxima")
```

output
```
-1/30*(20*b^3*x^(3/2) + 45*a*b^2*x + 36*a^2*b*sqrt(x) + 10*a^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx = -\frac{20\,b^3x^{\frac{3}{2}} + 45\,ab^2x + 36\,a^2b\sqrt{x} + 10\,a^3}{30\,x^3}$$

input
```
integrate((a+b*x^(1/2))^3/x^4,x, algorithm="giac")
```

output
```
-1/30*(20*b^3*x^(3/2) + 45*a*b^2*x + 36*a^2*b*sqrt(x) + 10*a^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx=-\frac{10\,a^3+20\,b^3\,x^{3/2}+36\,a^2\,b\,\sqrt{x}+45\,a\,b^2\,x}{30\,x^3}$$

input
```
int((a + b*x^(1/2))^3/x^4,x)
```

output
```
-(10*a^3 + 20*b^3*x^(3/2) + 36*a^2*b*x^(1/2) + 45*a*b^2*x)/(30*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^4}\,dx=\frac{-10\sqrt{x}\,a^3-45\sqrt{x}\,a\,b^2x-36a^2bx-20b^3x^2}{30\sqrt{x}\,x^3}$$

input
```
int((a+b*x^(1/2))^3/x^4,x)
```

output
```
( - 10*sqrt(x)*a**3 - 45*sqrt(x)*a*b**2*x - 36*a**2*b*x - 20*b**3*x**2)/(3
0*sqrt(x)*x**3)
```

3.29 $\int \frac{(a+b\sqrt{x})^3}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{(a+b\sqrt{x})^3}{x^5} dx = -\frac{a^3}{4x^4} - \frac{6a^2b}{7x^{7/2}} - \frac{ab^2}{x^3} - \frac{2b^3}{5x^{5/2}}$$

output
```
-1/4*a^3/x^4-6/7*a^2*b/x^(7/2)-a*b^2/x^3-2/5*b^3/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sqrt{x})^3}{x^5} dx = \frac{-35a^3 - 120a^2b\sqrt{x} - 140ab^2x - 56b^3x^{3/2}}{140x^4}$$

input
```
Integrate[(a + b*Sqrt[x])^3/x^5,x]
```

output
```
(-35*a^3 - 120*a^2*b*Sqrt[x] - 140*a*b^2*x - 56*b^3*x^(3/2))/(140*x^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^3}{x^{9/2}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^3}{x^{9/2}}+\frac{3ba^2}{x^4}+\frac{3b^2a}{x^{7/2}}+\frac{b^3}{x^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^3}{8x^4}-\frac{3a^2b}{7x^{7/2}}-\frac{ab^2}{2x^3}-\frac{b^3}{5x^{5/2}}\right)$$

input
```
Int[(a + b*Sqrt[x])^3/x^5,x]
```

output
```
2*(-1/8*a^3/x^4 - (3*a^2*b)/(7*x^(7/2)) - (a*b^2)/(2*x^3) - b^3/(5*x^(5/2)
))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{a^{3}}{4x^{4}}-\frac{6a^{2}b}{7x^{\frac{7}{2}}}-\frac{a\,b^{2}}{x^{3}}-\frac{2b^{3}}{5x^{\frac{5}{2}}}$	36
default	$-\frac{a^{3}}{4x^{4}}-\frac{6a^{2}b}{7x^{\frac{7}{2}}}-\frac{a\,b^{2}}{x^{3}}-\frac{2b^{3}}{5x^{\frac{5}{2}}}$	36
trager	$\frac{\left(-1+x\right)\left(a^{2}x^{3}+4b^{2}x^{3}+a^{2}x^{2}+4b^{2}x^{2}+a^{2}x+4b^{2}x+a^{2}\right)a}{4x^{4}}-\frac{2\left(7b^{2}x+15a^{2}\right)b}{35x^{\frac{7}{2}}}$	74
orering	$-\frac{\left(364b^{4}x^{2}-585a^{2}b^{2}x+255a^{4}\right)\left(a+b\sqrt{x}\right)^{3}}{420x^{4}\left(-b^{2}x+a^{2}\right)^{2}}-\frac{\left(28b^{4}x^{2}-39a^{2}b^{2}x+15a^{4}\right)x^{2}\left(\frac{3\left(a+b\sqrt{x}\right)^{2}b}{2x^{\frac{11}{2}}}-\frac{5\left(a+b\sqrt{x}\right)^{3}}{x^{6}}\right)}{210\left(-b^{2}x+a^{2}\right)^{2}}$	121

input
```
int((a+b*x^(1/2))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4-6/7*a^2*b/x^(7/2)-a*b^2/x^3-2/5*b^3/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{3}}{x^{5}}\,dx=-\frac{140\,ab^{2}x+35\,a^{3}+8\,\left(7\,b^{3}x+15\,a^{2}b\right)\sqrt{x}}{140\,x^{4}}$$

input
```
integrate((a+b*x^(1/2))^3/x^5,x, algorithm="fricas")
```

output
```
-1/140*(140*a*b^2*x + 35*a^3 + 8*(7*b^3*x + 15*a^2*b)*sqrt(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx = -\frac{a^3}{4x^4} - \frac{6a^2b}{7x^{\frac{7}{2}}} - \frac{ab^2}{x^3} - \frac{2b^3}{5x^{\frac{5}{2}}}$$

input
```
integrate((a+b*x**(1/2))**3/x**5,x)
```

output
```
-a**3/(4*x**4) - 6*a**2*b/(7*x**(7/2)) - a*b**2/x**3 - 2*b**3/(5*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx = -\frac{56\,b^3x^{\frac{3}{2}} + 140\,ab^2x + 120\,a^2b\sqrt{x} + 35\,a^3}{140\,x^4}$$

input
```
integrate((a+b*x^(1/2))^3/x^5,x, algorithm="maxima")
```

output
```
-1/140*(56*b^3*x^(3/2) + 140*a*b^2*x + 120*a^2*b*sqrt(x) + 35*a^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx = -\frac{56\,b^3x^{\frac{3}{2}} + 140\,ab^2x + 120\,a^2b\sqrt{x} + 35\,a^3}{140\,x^4}$$

input
```
integrate((a+b*x^(1/2))^3/x^5,x, algorithm="giac")
```

output
```
-1/140*(56*b^3*x^(3/2) + 140*a*b^2*x + 120*a^2*b*sqrt(x) + 35*a^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx = -\frac{35\,a^3+56\,b^3\,x^{3/2}+120\,a^2\,b\,\sqrt{x}+140\,a\,b^2\,x}{140\,x^4}$$

input
```
int((a + b*x^(1/2))^3/x^5,x)
```

output
```
-(35*a^3 + 56*b^3*x^(3/2) + 120*a^2*b*x^(1/2) + 140*a*b^2*x)/(140*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^5}\,dx = \frac{-35\sqrt{x}\,a^3-140\sqrt{x}\,a\,b^2x-120a^2bx-56b^3x^2}{140\sqrt{x}\,x^4}$$

input
```
int((a+b*x^(1/2))^3/x^5,x)
```

output
```
( - 35*sqrt(x)*a**3 - 140*sqrt(x)*a*b**2*x - 120*a**2*b*x - 56*b**3*x**2)/
(140*sqrt(x)*x**4)
```

3.30 $\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx=-\frac{a^3}{5x^5}-\frac{2a^2b}{3x^{9/2}}-\frac{3ab^2}{4x^4}-\frac{2b^3}{7x^{7/2}}$$

output
```
-1/5*a^3/x^5-2/3*a^2*b/x^(9/2)-3/4*a*b^2/x^4-2/7*b^3/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx=\frac{-84a^3-280a^2b\sqrt{x}-315ab^2x-120b^3x^{3/2}}{420x^5}$$

input
```
Integrate[(a + b*Sqrt[x])^3/x^6,x]
```

output
```
(-84*a^3 - 280*a^2*b*Sqrt[x] - 315*a*b^2*x - 120*b^3*x^(3/2))/(420*x^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}dx$$
$$\Big\downarrow \ 798$$
$$2\int \frac{\left(a+b\sqrt{x}\right)^3}{x^{11/2}}d\sqrt{x}$$
$$\Big\downarrow \ 53$$
$$2\int \left(\frac{a^3}{x^{11/2}}+\frac{3ba^2}{x^5}+\frac{3b^2a}{x^{9/2}}+\frac{b^3}{x^4}\right)d\sqrt{x}$$
$$\Big\downarrow \ 2009$$
$$2\left(-\frac{a^3}{10x^5}-\frac{a^2b}{3x^{9/2}}-\frac{3ab^2}{8x^4}-\frac{b^3}{7x^{7/2}}\right)$$

input
```
Int[(a + b*Sqrt[x])^3/x^6,x]
```

output
```
2*(-1/10*a^3/x^5 - (a^2*b)/(3*x^(9/2)) - (3*a*b^2)/(8*x^4) - b^3/(7*x^(7/2
)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{a^{3}}{5x^{5}}-\frac{2a^{2}b}{3x^{\frac{9}{2}}}-\frac{3a\,b^{2}}{4x^{4}}-\frac{2b^{3}}{7x^{\frac{7}{2}}}$	36
default	$-\frac{a^{3}}{5x^{5}}-\frac{2a^{2}b}{3x^{\frac{9}{2}}}-\frac{3a\,b^{2}}{4x^{4}}-\frac{2b^{3}}{7x^{\frac{7}{2}}}$	36
trager	$\frac{\left(-1+x\right)\left(4a^{2}x^{4}+15b^{2}x^{4}+4a^{2}x^{3}+15b^{2}x^{3}+4a^{2}x^{2}+15b^{2}x^{2}+4a^{2}x+15b^{2}x+4a^{2}\right)a}{20x^{5}}-\frac{2\left(3b^{2}x+7a^{2}\right)b}{21x^{\frac{9}{2}}}$	95
orering	$-\frac{\left(255b^{4}x^{2}-437a^{2}b^{2}x+196a^{4}\right)\left(a+b\sqrt{x}\right)^{3}}{420x^{5}\left(-b^{2}x+a^{2}\right)^{2}}-\frac{\left(45b^{4}x^{2}-69a^{2}b^{2}x+28a^{4}\right)x^{2}\left(\frac{3\left(a+b\sqrt{x}\right)^{2}b}{2x^{\frac{13}{2}}}-\frac{6\left(a+b\sqrt{x}\right)^{3}}{x^{7}}\right)}{630\left(-b^{2}x+a^{2}\right)^{2}}$	121

input
```
int((a+b*x^(1/2))^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^3/x^5-2/3*a^2*b/x^(9/2)-3/4*a*b^2/x^4-2/7*b^3/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt{x}\right)^{3}}{x^{6}}\,dx=-\frac{315\,ab^{2}x+84\,a^{3}+40\,\left(3\,b^{3}x+7\,a^{2}b\right)\sqrt{x}}{420\,x^{5}}$$

input
```
integrate((a+b*x^(1/2))^3/x^6,x, algorithm="fricas")
```

output
```
-1/420*(315*a*b^2*x + 84*a^3 + 40*(3*b^3*x + 7*a^2*b)*sqrt(x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx = -\frac{a^3}{5x^5}-\frac{2a^2b}{3x^{\frac{9}{2}}}-\frac{3ab^2}{4x^4}-\frac{2b^3}{7x^{\frac{7}{2}}}$$

input
```
integrate((a+b*x**(1/2))**3/x**6,x)
```

output
```
-a**3/(5*x**5) - 2*a**2*b/(3*x**(9/2)) - 3*a*b**2/(4*x**4) - 2*b**3/(7*x**
(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx = -\frac{120\,b^3x^{\frac{3}{2}}+315\,ab^2x+280\,a^2b\sqrt{x}+84\,a^3}{420\,x^5}$$

input
```
integrate((a+b*x^(1/2))^3/x^6,x, algorithm="maxima")
```

output
```
-1/420*(120*b^3*x^(3/2) + 315*a*b^2*x + 280*a^2*b*sqrt(x) + 84*a^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx = -\frac{120\,b^3x^{\frac{3}{2}}+315\,ab^2x+280\,a^2b\sqrt{x}+84\,a^3}{420\,x^5}$$

input
```
integrate((a+b*x^(1/2))^3/x^6,x, algorithm="giac")
```

output
```
-1/420*(120*b^3*x^(3/2) + 315*a*b^2*x + 280*a^2*b*sqrt(x) + 84*a^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx = -\frac{84\,a^3+120\,b^3\,x^{3/2}+280\,a^2\,b\,\sqrt{x}+315\,a\,b^2\,x}{420\,x^5}$$

input
```
int((a + b*x^(1/2))^3/x^6,x)
```

output
```
-(84*a^3 + 120*b^3*x^(3/2) + 280*a^2*b*x^(1/2) + 315*a*b^2*x)/(420*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^3}{x^6}\,dx = \frac{-84\sqrt{x}\,a^3-315\sqrt{x}\,a\,b^2x-280a^2bx-120b^3x^2}{420\sqrt{x}\,x^5}$$

input
```
int((a+b*x^(1/2))^3/x^6,x)
```

output
```
( - 84*sqrt(x)*a**3 - 315*sqrt(x)*a*b**2*x - 280*a**2*b*x - 120*b**3*x**2)
/(420*sqrt(x)*x**5)
```

3.31 $\int \left(a+b\sqrt{x}\right)^5 x^4\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \left(a+b\sqrt{x}\right)^5 x^4\,dx=\frac{a^5x^5}{5}+\frac{10}{11}a^4bx^{11/2}+\frac{5}{3}a^3b^2x^6+\frac{20}{13}a^2b^3x^{13/2}+\frac{5}{7}ab^4x^7+\frac{2}{15}b^5x^{15/2}$$

output
```
1/5*a^5*x^5+10/11*a^4*b*x^(11/2)+5/3*a^3*b^2*x^6+20/13*a^2*b^3*x^(13/2)+5/
7*a*b^4*x^7+2/15*b^5*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt{x}\right)^5 x^4\,dx =\frac{3003a^5x^5+13650a^4bx^{11/2}+25025a^3b^2x^6+23100a^2b^3x^{13/2}+10725ab^4x^7+2002b^5x^{15/2}}{15015}$$

input
```
Integrate[(a + b*Sqrt[x])^5*x^4,x]
```

output
```
(3003*a^5*x^5 + 13650*a^4*b*x^(11/2) + 25025*a^3*b^2*x^6 + 23100*a^2*b^3*x
^(13/2) + 10725*a*b^4*x^7 + 2002*b^5*x^(15/2))/15015
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sqrt{x}\right)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^5x^{9/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^5x^7+5ab^4x^{13/2}+10a^2b^3x^6+10a^3b^2x^{11/2}+5a^4bx^5+a^5x^{9/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^5x^5}{10}+\frac{5}{11}a^4bx^{11/2}+\frac{5}{6}a^3b^2x^6+\frac{10}{13}a^2b^3x^{13/2}+\frac{5}{14}ab^4x^7+\frac{1}{15}b^5x^{15/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^5*x^4,x]
```

output
```
2*((a^5*x^5)/10 + (5*a^4*b*x^(11/2))/11 + (5*a^3*b^2*x^6)/6 + (10*a^2*b^3*
x^(13/2))/13 + (5*a*b^4*x^7)/14 + (b^5*x^(15/2))/15)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^5x^5}{5}+\frac{10a^4b\,x^{\frac{11}{2}}}{11}+\frac{5a^3b^2x^6}{3}+\frac{20a^2b^3x^{\frac{13}{2}}}{13}+\frac{5a\,b^4x^7}{7}+\frac{2b^5x^{\frac{15}{2}}}{15}$
default	$\frac{a^5x^5}{5}+\frac{10a^4b\,x^{\frac{11}{2}}}{11}+\frac{5a^3b^2x^6}{3}+\frac{20a^2b^3x^{\frac{13}{2}}}{13}+\frac{5a\,b^4x^7}{7}+\frac{2b^5x^{\frac{15}{2}}}{15}$
orering	$\frac{x^5(19305b^8x^4-81125a^2b^6x^3+129398a^4b^4x^2-93345a^6b^2x+25935a^8)(a+b\sqrt{x})^5}{75075(-b^2x+a^2)^4}-\frac{2x^2(715b^8x^4-3245a^2b^6x^3+562}{}$
trager	$\frac{a(75b^4x^6+175a^2b^2x^5+75b^4x^5+21a^4x^4+175a^2b^2x^4+75b^4x^4+21a^4x^3+175a^2b^2x^3+75b^4x^3+21a^4x^2+175a^2b^2x^2+75}{105}$

input
```
int((a+b*x^(1/2))^5*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*x^5+10/11*a^4*b*x^(11/2)+5/3*a^3*b^2*x^6+20/13*a^2*b^3*x^(13/2)+5/
7*a*b^4*x^7+2/15*b^5*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^5 x^4\,dx = \frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^3b^2x^6+\frac{1}{5}\,a^5x^5 + \frac{2}{2145}\left(143\,b^5x^7+1650\,a^2b^3x^6+975\,a^4bx^5\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5*x^4,x, algorithm="fricas")
```

output
```
5/7*a*b^4*x^7 + 5/3*a^3*b^2*x^6 + 1/5*a^5*x^5 + 2/2145*(143*b^5*x^7 + 1650
*a^2*b^3*x^6 + 975*a^4*b*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{x}\right)^5 x^4\,dx = \frac{a^5x^5}{5} + \frac{10a^4bx^{\frac{11}{2}}}{11} + \frac{5a^3b^2x^6}{3} + \frac{20a^2b^3x^{\frac{13}{2}}}{13} + \frac{5ab^4x^7}{7} + \frac{2b^5x^{\frac{15}{2}}}{15}$$

input
```
integrate((a+b*x**(1/2))**5*x**4,x)
```

output
```
a**5*x**5/5 + 10*a**4*b*x**(11/2)/11 + 5*a**3*b**2*x**6/3 + 20*a**2*b**3*x
**(13/2)/13 + 5*a*b**4*x**7/7 + 2*b**5*x**(15/2)/15
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(57) = 114.

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^5 x^4\,dx =& \frac{2\left(b\sqrt{x}+a\right)^{15}}{15\,b^{10}} - \frac{9\left(b\sqrt{x}+a\right)^{14}a}{7\,b^{10}} + \frac{72\left(b\sqrt{x}+a\right)^{13}a^2}{13\,b^{10}} \\ &- \frac{14\left(b\sqrt{x}+a\right)^{12}a^3}{b^{10}} + \frac{252\left(b\sqrt{x}+a\right)^{11}a^4}{11\,b^{10}} \\ &- \frac{126\left(b\sqrt{x}+a\right)^{10}a^5}{5\,b^{10}} + \frac{56\left(b\sqrt{x}+a\right)^{9}a^6}{3\,b^{10}} \\ &- \frac{9\left(b\sqrt{x}+a\right)^{8}a^7}{b^{10}} + \frac{18\left(b\sqrt{x}+a\right)^{7}a^8}{7\,b^{10}} - \frac{\left(b\sqrt{x}+a\right)^{6}a^9}{3\,b^{10}}\end{aligned}$$

input
```
integrate((a+b*x^(1/2))^5*x^4,x, algorithm="maxima")
```

output
```
2/15*(b*sqrt(x) + a)^15/b^10 - 9/7*(b*sqrt(x) + a)^14*a/b^10 + 72/13*(b*sq
rt(x) + a)^13*a^2/b^10 - 14*(b*sqrt(x) + a)^12*a^3/b^10 + 252/11*(b*sqrt(x
) + a)^11*a^4/b^10 - 126/5*(b*sqrt(x) + a)^10*a^5/b^10 + 56/3*(b*sqrt(x) +
 a)^9*a^6/b^10 - 9*(b*sqrt(x) + a)^8*a^7/b^10 + 18/7*(b*sqrt(x) + a)^7*a^8
/b^10 - 1/3*(b*sqrt(x) + a)^6*a^9/b^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt{x}\right)^5 x^4\,dx = \frac{2}{15}\,b^5x^{\frac{15}{2}} + \frac{5}{7}\,ab^4x^7 + \frac{20}{13}\,a^2b^3x^{\frac{13}{2}} + \frac{5}{3}\,a^3b^2x^6 + \frac{10}{11}\,a^4bx^{\frac{11}{2}} + \frac{1}{5}\,a^5x^5$$

input
```
integrate((a+b*x^(1/2))^5*x^4,x, algorithm="giac")
```

output
```
2/15*b^5*x^(15/2) + 5/7*a*b^4*x^7 + 20/13*a^2*b^3*x^(13/2) + 5/3*a^3*b^2*x
^6 + 10/11*a^4*b*x^(11/2) + 1/5*a^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt{x}\right)^5 x^4\,dx = \frac{a^5\,x^5}{5} + \frac{2\,b^5\,x^{15/2}}{15} + \frac{5\,a\,b^4\,x^7}{7} + \frac{10\,a^4\,b\,x^{11/2}}{11} + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{20\,a^2\,b^3\,x^{13/2}}{13}$$

input
```
int(x^4*(a + b*x^(1/2))^5,x)
```

output
```
(a^5*x^5)/5 + (2*b^5*x^(15/2))/15 + (5*a*b^4*x^7)/7 + (10*a^4*b*x^(11/2))/
11 + (5*a^3*b^2*x^6)/3 + (20*a^2*b^3*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}\right)^5 x^4\,dx$$
$$=\frac{x^5\left(13650\sqrt{x}\,a^4b+23100\sqrt{x}\,a^2b^3x+2002\sqrt{x}\,b^5x^2+3003a^5+25025a^3b^2x+10725a\,b^4x^2\right)}{15015}$$

input
```
int((a+b*x^(1/2))^5*x^4,x)
```

output
```
(x**5*(13650*sqrt(x)*a**4*b + 23100*sqrt(x)*a**2*b**3*x + 2002*sqrt(x)*b**
5*x**2 + 3003*a**5 + 25025*a**3*b**2*x + 10725*a*b**4*x**2))/15015
```

3.32 $\int \left(a+b\sqrt{x}\right)^5 x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \left(a+b\sqrt{x}\right)^5 x^3 \, dx = \frac{a^5x^4}{4}+\frac{10}{9}a^4bx^{9/2}+2a^3b^2x^5+\frac{20}{11}a^2b^3x^{11/2}+\frac{5}{6}ab^4x^6+\frac{2}{13}b^5x^{13/2}$$

output
```
1/4*a^5*x^4+10/9*a^4*b*x^(9/2)+2*a^3*b^2*x^5+20/11*a^2*b^3*x^(11/2)+5/6*a*
b^4*x^6+2/13*b^5*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt{x}\right)^5 x^3 \, dx = \frac{1287a^5x^4+5720a^4bx^{9/2}+10296a^3b^2x^5+9360a^2b^3x^{11/2}+4290ab^4x^6+792b^5x^{13/2}}{5148}$$

input
```
Integrate[(a + b*Sqrt[x])^5*x^3,x]
```

output
```
(1287*a^5*x^4 + 5720*a^4*b*x^(9/2) + 10296*a^3*b^2*x^5 + 9360*a^2*b^3*x^(1
1/2) + 4290*a*b^4*x^6 + 792*b^5*x^(13/2))/5148
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{x}\right)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^5 x^{7/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(b^5x^6+5ab^4x^{11/2}+10a^2b^3x^5+10a^3b^2x^{9/2}+5a^4bx^4+a^5x^{7/2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^5x^4}{8}+\frac{5}{9}a^4bx^{9/2}+a^3b^2x^5+\frac{10}{11}a^2b^3x^{11/2}+\frac{5}{12}ab^4x^6+\frac{1}{13}b^5x^{13/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^5*x^3,x]
```

output
```
2*((a^5*x^4)/8 + (5*a^4*b*x^(9/2))/9 + a^3*b^2*x^5 + (10*a^2*b^3*x^(11/2))
/11 + (5*a*b^4*x^6)/12 + (b^5*x^(13/2))/13)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a^5x^4}{4}+\frac{10a^4b\,x^{\frac{9}{2}}}{9}+2a^3b^2x^5+\frac{20a^2b^3x^{\frac{11}{2}}}{11}+\frac{5a\,b^4x^6}{6}+\frac{2b^5x^{\frac{13}{2}}}{13}$
default	$\frac{a^5x^4}{4}+\frac{10a^4b\,x^{\frac{9}{2}}}{9}+2a^3b^2x^5+\frac{20a^2b^3x^{\frac{11}{2}}}{11}+\frac{5a\,b^4x^6}{6}+\frac{2b^5x^{\frac{13}{2}}}{13}$
trager	$\frac{a(10b^4x^5+24a^2b^2x^4+10b^4x^4+3a^4x^3+24a^2b^2x^3+10b^4x^3+3a^4x^2+24a^2b^2x^2+10b^4x^2+3a^4x+24a^2b^2x+10b^4x+3a^4+}{12}$
orering	$\frac{x^4(7590b^8x^4-32004a^2b^6x^3+51395a^4b^4x^2-37570a^6b^2x+10725a^8)(a+b\sqrt{x})^5}{25740(-b^2x+a^2)^4}-\frac{x^2(330b^8x^4-1524a^2b^6x^3+2705a^4}{}$

input
```
int((a+b*x^(1/2))^5*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*x^4+10/9*a^4*b*x^(9/2)+2*a^3*b^2*x^5+20/11*a^2*b^3*x^(11/2)+5/6*a*
b^4*x^6+2/13*b^5*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int (a+b\sqrt{x})^5\,x^3\,dx=\frac{5}{6}\,ab^4x^6+2\,a^3b^2x^5+\frac{1}{4}\,a^5x^4+\frac{2}{1287}\,(99\,b^5x^6+1170\,a^2b^3x^5+715\,a^4bx^4)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5*x^3,x, algorithm="fricas")
```

output
```
5/6*a*b^4*x^6 + 2*a^3*b^2*x^5 + 1/4*a^5*x^4 + 2/1287*(99*b^5*x^6 + 1170*a^
2*b^3*x^5 + 715*a^4*b*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{x}\right)^5 x^3\,dx = \frac{a^5x^4}{4} + \frac{10a^4bx^{\frac{9}{2}}}{9} + 2a^3b^2x^5 + \frac{20a^2b^3x^{\frac{11}{2}}}{11} + \frac{5ab^4x^6}{6} + \frac{2b^5x^{\frac{13}{2}}}{13}$$

input
```
integrate((a+b*x**(1/2))**5*x**3,x)
```

output
```
a**5*x**4/4 + 10*a**4*b*x**(9/2)/9 + 2*a**3*b**2*x**5 + 20*a**2*b**3*x**(1
1/2)/11 + 5*a*b**4*x**6/6 + 2*b**5*x**(13/2)/13
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.81

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^5 x^3\,dx = &\frac{2\left(b\sqrt{x}+a\right)^{13}}{13\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^{12}a}{6\,b^8} + \frac{42\left(b\sqrt{x}+a\right)^{11}a^2}{11\,b^8} \\ &- \frac{7\left(b\sqrt{x}+a\right)^{10}a^3}{b^8} + \frac{70\left(b\sqrt{x}+a\right)^9a^4}{9\,b^8} \\ &- \frac{21\left(b\sqrt{x}+a\right)^8a^5}{4\,b^8} + \frac{2\left(b\sqrt{x}+a\right)^7a^6}{b^8} - \frac{\left(b\sqrt{x}+a\right)^6a^7}{3\,b^8}\end{aligned}$$

input
```
integrate((a+b*x^(1/2))^5*x^3,x, algorithm="maxima")
```

output
```
2/13*(b*sqrt(x) + a)^13/b^8 - 7/6*(b*sqrt(x) + a)^12*a/b^8 + 42/11*(b*sqrt
(x) + a)^11*a^2/b^8 - 7*(b*sqrt(x) + a)^10*a^3/b^8 + 70/9*(b*sqrt(x) + a)^
9*a^4/b^8 - 21/4*(b*sqrt(x) + a)^8*a^5/b^8 + 2*(b*sqrt(x) + a)^7*a^6/b^8 -
 1/3*(b*sqrt(x) + a)^6*a^7/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^5 x^3\,dx = \frac{2}{13}\,b^5x^{\frac{13}{2}} + \frac{5}{6}\,ab^4x^6 + \frac{20}{11}\,a^2b^3x^{\frac{11}{2}} + 2\,a^3b^2x^5 + \frac{10}{9}\,a^4bx^{\frac{9}{2}} + \frac{1}{4}\,a^5x^4$$

input
```
integrate((a+b*x^(1/2))^5*x^3,x, algorithm="giac")
```

output
```
2/13*b^5*x^(13/2) + 5/6*a*b^4*x^6 + 20/11*a^2*b^3*x^(11/2) + 2*a^3*b^2*x^5
 + 10/9*a^4*b*x^(9/2) + 1/4*a^5*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^5 x^3\,dx = \frac{a^5\,x^4}{4} + \frac{2\,b^5\,x^{13/2}}{13} + \frac{5\,a\,b^4\,x^6}{6} + \frac{10\,a^4\,b\,x^{9/2}}{9} + 2\,a^3\,b^2\,x^5 + \frac{20\,a^2\,b^3\,x^{11/2}}{11}$$

input
```
int(x^3*(a + b*x^(1/2))^5,x)
```

output
```
(a^5*x^4)/4 + (2*b^5*x^(13/2))/13 + (5*a*b^4*x^6)/6 + (10*a^4*b*x^(9/2))/9
 + 2*a^3*b^2*x^5 + (20*a^2*b^3*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)^5 x^3\,dx = \frac{x^4\left(5720\sqrt{x}\,a^4b + 9360\sqrt{x}\,a^2b^3x + 792\sqrt{x}\,b^5x^2 + 1287a^5 + 10296a^3b^2x + 4290a\,b^4x^2\right)}{5148}$$

input
```
int((a+b*x^(1/2))^5*x^3,x)
```

output
```
(x**4*(5720*sqrt(x)*a**4*b + 9360*sqrt(x)*a**2*b**3*x + 792*sqrt(x)*b**5*x
**2 + 1287*a**5 + 10296*a**3*b**2*x + 4290*a*b**4*x**2))/5148
```

3.33 $\int \left(a+b\sqrt{x}\right)^5 x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \left(a+b\sqrt{x}\right)^5 x^2 \, dx = \frac{a^5x^3}{3}+\frac{10}{7}a^4bx^{7/2}+\frac{5}{2}a^3b^2x^4+\frac{20}{9}a^2b^3x^{9/2}+ab^4x^5+\frac{2}{11}b^5x^{11/2}$$

output
```
1/3*a^5*x^3+10/7*a^4*b*x^(7/2)+5/2*a^3*b^2*x^4+20/9*a^2*b^3*x^(9/2)+a*b^4*
x^5+2/11*b^5*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt{x}\right)^5 x^2 \, dx = \frac{462a^5x^3+1980a^4bx^{7/2}+3465a^3b^2x^4+3080a^2b^3x^{9/2}+1386ab^4x^5+252b^5x^{11/2}}{1386}$$

input
```
Integrate[(a + b*Sqrt[x])^5*x^2,x]
```

output
```
(462*a^5*x^3 + 1980*a^4*b*x^(7/2) + 3465*a^3*b^2*x^4 + 3080*a^2*b^3*x^(9/2
) + 1386*a*b^4*x^5 + 252*b^5*x^(11/2))/1386
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{x}\right)^5 dx$$

$$\Big\downarrow \ 798$$

$$2\int \left(a+b\sqrt{x}\right)^5 x^{5/2}d\sqrt{x}$$

$$\Big\downarrow \ 49$$

$$2\int \left(x^{5/2}a^5+5bx^3a^4+10b^2x^{7/2}a^3+10b^3x^4a^2+5b^4x^{9/2}a+b^5x^5\right)d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{a^5x^3}{6}+\frac{5}{7}a^4bx^{7/2}+\frac{5}{4}a^3b^2x^4+\frac{10}{9}a^2b^3x^{9/2}+\frac{1}{2}ab^4x^5+\frac{1}{11}b^5x^{11/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^5*x^2,x]
```

output
```
2*((a^5*x^3)/6 + (5*a^4*b*x^(7/2))/7 + (5*a^3*b^2*x^4)/4 + (10*a^2*b^3*x^(
9/2))/9 + (a*b^4*x^5)/2 + (b^5*x^(11/2))/11)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{a^{5} x^{3}}{3}+\frac{10 a^{4} b \,x^{\frac{7}{2}}}{7}+\frac{5 a^{3} b^{2} x^{4}}{2}+\frac{20 a^{2} b^{3} x^{\frac{9}{2}}}{9}+a \,b^{4} x^{5}+\frac{2 b^{5} x^{\frac{11}{2}}}{11}$
default	$\frac{a^{5} x^{3}}{3}+\frac{10 a^{4} b \,x^{\frac{7}{2}}}{7}+\frac{5 a^{3} b^{2} x^{4}}{2}+\frac{20 a^{2} b^{3} x^{\frac{9}{2}}}{9}+a \,b^{4} x^{5}+\frac{2 b^{5} x^{\frac{11}{2}}}{11}$
trager	$\frac{a \left(6 b^{4} x^{4}+15 a^{2} b^{2} x^{3}+6 b^{4} x^{3}+2 a^{4} x^{2}+15 a^{2} b^{2} x^{2}+6 b^{4} x^{2}+2 a^{4} x+15 a^{2} b^{2} x+6 b^{4} x+2 a^{4}+15 a^{2} b^{2}+6 b^{4}\right) \left(-1+x \right)}{6}+\frac{2 b \,x^{\frac{7}{2}} (}{}$
orering	$\frac{x^{3} \left(2394 b^{8} x^{4}-10115 a^{2} b^{6} x^{3}+16350 a^{4} b^{4} x^{2}-12155 a^{6} b^{2} x +3630 a^{8}\right) \left(a +b \sqrt{x}\right)^{5}}{6930 \left(-b^{2} x +a^{2}\right)^{4}}-\frac{x^{2} \left(126 b^{8} x^{4}-595 a^{2} b^{6} x^{3}+1090 a^{4} b^{4}}{}$

input
```
int((a+b*x^(1/2))^5*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^5*x^3+10/7*a^4*b*x^(7/2)+5/2*a^3*b^2*x^4+20/9*a^2*b^3*x^(9/2)+a*b^4*
x^5+2/11*b^5*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt{x}\right)^5 x^2\,dx = ab^{4} x^{5} + \frac{5}{2} \, a^{3} b^{2} x^{4} + \frac{1}{3} \, a^{5} x^{3} + \frac{2}{693} \, {\left(63 \, b^{5} x^{5} + 770 \, a^{2} b^{3} x^{4} + 495 \, a^{4} b x^{3}\right)} \sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5*x^2,x, algorithm="fricas")
```

output
```
a*b^4*x^5 + 5/2*a^3*b^2*x^4 + 1/3*a^5*x^3 + 2/693*(63*b^5*x^5 + 770*a^2*b^
3*x^4 + 495*a^4*b*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{x}\right)^5 x^2\,dx=\frac{a^5x^3}{3}+\frac{10a^4bx^{\frac{7}{2}}}{7}+\frac{5a^3b^2x^4}{2}+\frac{20a^2b^3x^{\frac{9}{2}}}{9}+ab^4x^5+\frac{2b^5x^{\frac{11}{2}}}{11}$$

input
```
integrate((a+b*x**(1/2))**5*x**2,x)
```

output
```
a**5*x**3/3 + 10*a**4*b*x**(7/2)/7 + 5*a**3*b**2*x**4/2 + 20*a**2*b**3*x**
(9/2)/9 + a*b**4*x**5 + 2*b**5*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int \left(a+b\sqrt{x}\right)^5 x^2\,dx=\frac{2\left(b\sqrt{x}+a\right)^{11}}{11\,b^6}-\frac{\left(b\sqrt{x}+a\right)^{10}a}{b^6}+\frac{20\left(b\sqrt{x}+a\right)^{9}a^2}{9\,b^6}-\frac{5\left(b\sqrt{x}+a\right)^{8}a^3}{2\,b^6}+\frac{10\left(b\sqrt{x}+a\right)^{7}a^4}{7\,b^6}-\frac{\left(b\sqrt{x}+a\right)^{6}a^5}{3\,b^6}$$

input
```
integrate((a+b*x^(1/2))^5*x^2,x, algorithm="maxima")
```

output
```
2/11*(b*sqrt(x) + a)^11/b^6 - (b*sqrt(x) + a)^10*a/b^6 + 20/9*(b*sqrt(x) +
 a)^9*a^2/b^6 - 5/2*(b*sqrt(x) + a)^8*a^3/b^6 + 10/7*(b*sqrt(x) + a)^7*a^4
/b^6 - 1/3*(b*sqrt(x) + a)^6*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^5 x^2\,dx = \frac{2}{11}\,b^5x^{\frac{11}{2}} + ab^4x^5 + \frac{20}{9}\,a^2b^3x^{\frac{9}{2}} + \frac{5}{2}\,a^3b^2x^4 + \frac{10}{7}\,a^4bx^{\frac{7}{2}} + \frac{1}{3}\,a^5x^3$$

input
```
integrate((a+b*x^(1/2))^5*x^2,x, algorithm="giac")
```

output
```
2/11*b^5*x^(11/2) + a*b^4*x^5 + 20/9*a^2*b^3*x^(9/2) + 5/2*a^3*b^2*x^4 + 1
0/7*a^4*b*x^(7/2) + 1/3*a^5*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^5 x^2\,dx = \frac{a^5\,x^3}{3} + \frac{2\,b^5\,x^{11/2}}{11} + a\,b^4\,x^5 + \frac{10\,a^4\,b\,x^{7/2}}{7} + \frac{5\,a^3\,b^2\,x^4}{2} + \frac{20\,a^2\,b^3\,x^{9/2}}{9}$$

input
```
int(x^2*(a + b*x^(1/2))^5,x)
```

output
```
(a^5*x^3)/3 + (2*b^5*x^(11/2))/11 + a*b^4*x^5 + (10*a^4*b*x^(7/2))/7 + (5*
a^3*b^2*x^4)/2 + (20*a^2*b^3*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \left(a+b\sqrt{x}\right)^5 x^2\,dx = \frac{x^3\left(1980\sqrt{x}\,a^4b + 3080\sqrt{x}\,a^2b^3x + 252\sqrt{x}\,b^5x^2 + 462a^5 + 3465a^3b^2x + 1386a\,b^4x^2\right)}{1386}$$

input
```
int((a+b*x^(1/2))^5*x^2,x)
```

output
```
(x**3*(1980*sqrt(x)*a**4*b + 3080*sqrt(x)*a**2*b**3*x + 252*sqrt(x)*b**5*x
**2 + 462*a**5 + 3465*a**3*b**2*x + 1386*a*b**4*x**2))/1386
```

3.34 $\int \left(a+b\sqrt{x}\right)^5 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \left(a+b\sqrt{x}\right)^5 x\,dx=-\frac{a^3\left(a+b\sqrt{x}\right)^6}{3b^4}+\frac{6a^2\left(a+b\sqrt{x}\right)^7}{7b^4}-\frac{3a\left(a+b\sqrt{x}\right)^8}{4b^4}+\frac{2\left(a+b\sqrt{x}\right)^9}{9b^4}$$

output
```
-1/3*a^3*(a+b*x^(1/2))^6/b^4+6/7*a^2*(a+b*x^(1/2))^7/b^4-3/4*a*(a+b*x^(1/2
))^8/b^4+2/9*(a+b*x^(1/2))^9/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^5 x\,dx=\frac{1}{252}\left(126a^5x^2+504a^4bx^{5/2}+840a^3b^2x^3+720a^2b^3x^{7/2}+315ab^4x^4+56b^5x^{9/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^5*x,x]
```

output
```
(126*a^5*x^2 + 504*a^4*b*x^(5/2) + 840*a^3*b^2*x^3 + 720*a^2*b^3*x^(7/2) +
 315*a*b^4*x^4 + 56*b^5*x^(9/2))/252
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}\right)^5 dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^5 x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^8}{b^3}-\frac{3a\left(a+b\sqrt{x}\right)^7}{b^3}+\frac{3a^2\left(a+b\sqrt{x}\right)^6}{b^3}-\frac{a^3\left(a+b\sqrt{x}\right)^5}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^3\left(a+b\sqrt{x}\right)^6}{6b^4}+\frac{3a^2\left(a+b\sqrt{x}\right)^7}{7b^4}+\frac{\left(a+b\sqrt{x}\right)^9}{9b^4}-\frac{3a\left(a+b\sqrt{x}\right)^8}{8b^4}\right)$$

input
```
Int[(a + b*Sqrt[x])^5*x,x]
```

output
```
2*(-1/6*(a^3*(a + b*Sqrt[x])^6)/b^4 + (3*a^2*(a + b*Sqrt[x])^7)/(7*b^4) -
(3*a*(a + b*Sqrt[x])^8)/(8*b^4) + (a + b*Sqrt[x])^9/(9*b^4))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{2b^5x^{\frac{9}{2}}}{9}+\frac{5a\,b^4x^4}{4}+\frac{20a^2b^3x^{\frac{7}{2}}}{7}+\frac{10a^3b^2x^3}{3}+2a^4b\,x^{\frac{5}{2}}+\frac{a^5x^2}{2}$
default	$\frac{2b^5x^{\frac{9}{2}}}{9}+\frac{5a\,b^4x^4}{4}+\frac{20a^2b^3x^{\frac{7}{2}}}{7}+\frac{10a^3b^2x^3}{3}+2a^4b\,x^{\frac{5}{2}}+\frac{a^5x^2}{2}$
trager	$\frac{a(15b^4x^3+40a^2b^2x^2+15b^4x^2+6a^4x+40a^2b^2x+15b^4x+6a^4+40a^2b^2+15b^4)(-1+x)}{12}+\frac{2b\,x^{\frac{5}{2}}(7b^4x^2+90a^2b^2x+63a^4}{63}$
orering	$-\frac{(-105b^{12}x^6+442a^2b^{10}x^5-715a^4x^4b^8+540a^6b^6x^3+1260a^{10}x\,b^2+378a^{12})(a+b\sqrt{x})^5}{252b^4(-b^2x+a^2)^4}+\frac{(-7b^{12}x^6+34a^2b^{10}x^5-6}{}$

input
```
int((a+b*x^(1/2))^5*x,x,method=_RETURNVERBOSE)
```

output
```
2/9*b^5*x^(9/2)+5/4*a*b^4*x^4+20/7*a^2*b^3*x^(7/2)+10/3*a^3*b^2*x^3+2*a^4*
b*x^(5/2)+1/2*a^5*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int\left(a+b\sqrt{x}\right)^5x\,dx=\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^3b^2x^3+\frac{1}{2}\,a^5x^2+\frac{2}{63}\left(7\,b^5x^4+90\,a^2b^3x^3+63\,a^4bx^2\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5*x,x, algorithm="fricas")
```

output
```
5/4*a*b^4*x^4 + 10/3*a^3*b^2*x^3 + 1/2*a^5*x^2 + 2/63*(7*b^5*x^4 + 90*a^2*
b^3*x^3 + 63*a^4*b*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt{x}\right)^5 x\,dx = \frac{a^5x^2}{2} + 2a^4bx^{\frac{5}{2}} + \frac{10a^3b^2x^3}{3} + \frac{20a^2b^3x^{\frac{7}{2}}}{7} + \frac{5ab^4x^4}{4} + \frac{2b^5x^{\frac{9}{2}}}{9}$$

input
```
integrate((a+b*x**(1/2))**5*x,x)
```

output
```
a**5*x**2/2 + 2*a**4*b*x**(5/2) + 10*a**3*b**2*x**3/3 + 20*a**2*b**3*x**(7
/2)/7 + 5*a*b**4*x**4/4 + 2*b**5*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^5 x\,dx = \frac{2\left(b\sqrt{x}+a\right)^9}{9\,b^4} - \frac{3\left(b\sqrt{x}+a\right)^8 a}{4\,b^4} + \frac{6\left(b\sqrt{x}+a\right)^7 a^2}{7\,b^4} - \frac{\left(b\sqrt{x}+a\right)^6 a^3}{3\,b^4}$$

input
```
integrate((a+b*x^(1/2))^5*x,x, algorithm="maxima")
```

output
```
2/9*(b*sqrt(x) + a)^9/b^4 - 3/4*(b*sqrt(x) + a)^8*a/b^4 + 6/7*(b*sqrt(x) +
 a)^7*a^2/b^4 - 1/3*(b*sqrt(x) + a)^6*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right)^5 x\,dx = \frac{2}{9}\,b^5x^{\frac{9}{2}} + \frac{5}{4}\,ab^4x^4 + \frac{20}{7}\,a^2b^3x^{\frac{7}{2}} + \frac{10}{3}\,a^3b^2x^3 + 2\,a^4bx^{\frac{5}{2}} + \frac{1}{2}\,a^5x^2$$

input
```
integrate((a+b*x^(1/2))^5*x,x, algorithm="giac")
```

output
```
2/9*b^5*x^(9/2) + 5/4*a*b^4*x^4 + 20/7*a^2*b^3*x^(7/2) + 10/3*a^3*b^2*x^3
+ 2*a^4*b*x^(5/2) + 1/2*a^5*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right)^5 x\,dx = \frac{a^5\,x^2}{2} + \frac{2\,b^5\,x^{9/2}}{9} + \frac{5\,a\,b^4\,x^4}{4} + 2\,a^4\,b\,x^{5/2} + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{20\,a^2\,b^3\,x^{7/2}}{7}$$

input
```
int(x*(a + b*x^(1/2))^5,x)
```

output
```
(a^5*x^2)/2 + (2*b^5*x^(9/2))/9 + (5*a*b^4*x^4)/4 + 2*a^4*b*x^(5/2) + (10*
a^3*b^2*x^3)/3 + (20*a^2*b^3*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^5 x\,dx = \frac{x^2\left(504\sqrt{x}\,a^4b + 720\sqrt{x}\,a^2b^3x + 56\sqrt{x}\,b^5x^2 + 126a^5 + 840a^3b^2x + 315a\,b^4x^2\right)}{252}$$

input
```
int((a+b*x^(1/2))^5*x,x)
```

output
```
(x**2*(504*sqrt(x)*a**4*b + 720*sqrt(x)*a**2*b**3*x + 56*sqrt(x)*b**5*x**2
 + 126*a**5 + 840*a**3*b**2*x + 315*a*b**4*x**2))/252
```

3.35 $\int \left(a+b\sqrt{x}\right)^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \left(a+b\sqrt{x}\right)^5 \, dx = -\frac{a\left(a+b\sqrt{x}\right)^6}{3b^2}+\frac{2\left(a+b\sqrt{x}\right)^7}{7b^2}$$

output
```
-1/3*a*(a+b*x^(1/2))^6/b^2+2/7*(a+b*x^(1/2))^7/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \left(a+b\sqrt{x}\right)^5 \, dx = \frac{1}{21}\left(21a^5x+70a^4bx^{3/2}+105a^3b^2x^2+84a^2b^3x^{5/2}+35ab^4x^3+6b^5x^{7/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^5,x]
```

output
```
(21*a^5*x + 70*a^4*b*x^(3/2) + 105*a^3*b^2*x^2 + 84*a^2*b^3*x^(5/2) + 35*a
*b^4*x^3 + 6*b^5*x^(7/2))/21
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^5 dx$$

$$\Big\downarrow \text{774}$$

$$2\int \left(a+b\sqrt{x}\right)^5 \sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^6}{b}-\frac{a\left(a+b\sqrt{x}\right)^5}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\left(a+b\sqrt{x}\right)^7}{7b^2}-\frac{a\left(a+b\sqrt{x}\right)^6}{6b^2}\right)$$

input
```
Int[(a + b*Sqrt[x])^5,x]
```

output
```
2*(-1/6*(a*(a + b*Sqrt[x])^6)/b^2 + (a + b*Sqrt[x])^7/(7*b^2))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{2b^5x^{\frac{7}{2}}}{7}+\frac{5a\,b^4x^3}{3}+4a^2b^3x^{\frac{5}{2}}+5a^3b^2x^2+\frac{10a^4b\,x^{\frac{3}{2}}}{3}+a^5x$
default	$\frac{2b^5x^{\frac{7}{2}}}{7}+\frac{5a\,b^4x^3}{3}+4a^2b^3x^{\frac{5}{2}}+5a^3b^2x^2+\frac{10a^4b\,x^{\frac{3}{2}}}{3}+a^5x$
trager	$\frac{a(5b^4x^2+15a^2b^2x+5b^4x+3a^4+15a^2b^2+5b^4)(-1+x)}{3}+\frac{2b\,x^{\frac{3}{2}}(3b^4x^2+42a^2b^2x+35a^4)}{21}$
orering	$\frac{(11b^{10}x^5-45a^2b^8x^4+70a^4b^6x^3+315a^8b^2x+49a^{10})(a+b\sqrt{x})^5}{21b^2(-b^2x+a^2)^4}-\frac{5(b^{10}x^5-5a^2b^8x^4+10a^4b^6x^3+105a^8b^2x+49a^{10})\sqrt{}}{21b(-b^2x+a^2)^4}$

input
```
int((a+b*x^(1/2))^5,x,method=_RETURNVERBOSE)
```

output
```
2/7*b^5*x^(7/2)+5/3*a*b^4*x^3+4*a^2*b^3*x^(5/2)+5*a^3*b^2*x^2+10/3*a^4*b*x
^(3/2)+a^5*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int\left(a+b\sqrt{x}\right)^5\,dx=\frac{5}{3}\,ab^4x^3+5\,a^3b^2x^2+a^5x+\frac{2}{21}\left(3\,b^5x^3+42\,a^2b^3x^2+35\,a^4bx\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5,x, algorithm="fricas")
```

output
```
5/3*a*b^4*x^3 + 5*a^3*b^2*x^2 + a^5*x + 2/21*(3*b^5*x^3 + 42*a^2*b^3*x^2 +
 35*a^4*b*x)*sqrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \left(a+b\sqrt{x}\right)^5 \, dx = a^5x + \frac{10a^4bx^{\frac{3}{2}}}{3} + 5a^3b^2x^2 + 4a^2b^3x^{\frac{5}{2}} + \frac{5ab^4x^3}{3} + \frac{2b^5x^{\frac{7}{2}}}{7}$$

input
```
integrate((a+b*x**(1/2))**5,x)
```

output
```
a**5*x + 10*a**4*b*x**(3/2)/3 + 5*a**3*b**2*x**2 + 4*a**2*b**3*x**(5/2) +
5*a*b**4*x**3/3 + 2*b**5*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \left(a+b\sqrt{x}\right)^5 \, dx = \frac{2}{7}\,b^5x^{\frac{7}{2}} + \frac{5}{3}\,ab^4x^3 + 4\,a^2b^3x^{\frac{5}{2}} + 5\,a^3b^2x^2 + \frac{10}{3}\,a^4bx^{\frac{3}{2}} + a^5x$$

input
```
integrate((a+b*x^(1/2))^5,x, algorithm="maxima")
```

output
```
2/7*b^5*x^(7/2) + 5/3*a*b^4*x^3 + 4*a^2*b^3*x^(5/2) + 5*a^3*b^2*x^2 + 10/3
*a^4*b*x^(3/2) + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \left(a+b\sqrt{x}\right)^5\,dx=\frac{2}{7}\,b^5x^{\frac{7}{2}}+\frac{5}{3}\,ab^4x^3+4\,a^2b^3x^{\frac{5}{2}}+5\,a^3b^2x^2+\frac{10}{3}\,a^4bx^{\frac{3}{2}}+a^5x$$

input
```
integrate((a+b*x^(1/2))^5,x, algorithm="giac")
```

output
```
2/7*b^5*x^(7/2) + 5/3*a*b^4*x^3 + 4*a^2*b^3*x^(5/2) + 5*a^3*b^2*x^2 + 10/3
*a^4*b*x^(3/2) + a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \left(a+b\sqrt{x}\right)^5\,dx=a^5\,x+\frac{2\,b^5\,x^{7/2}}{7}+\frac{5\,a\,b^4\,x^3}{3}+\frac{10\,a^4\,b\,x^{3/2}}{3}+5\,a^3\,b^2\,x^2+4\,a^2\,b^3\,x^{5/2}$$

input
```
int((a + b*x^(1/2))^5,x)
```

output
```
a^5*x + (2*b^5*x^(7/2))/7 + (5*a*b^4*x^3)/3 + (10*a^4*b*x^(3/2))/3 + 5*a^3
*b^2*x^2 + 4*a^2*b^3*x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \left(a+b\sqrt{x}\right)^5\,dx=\frac{x\left(70\sqrt{x}\,a^4b+84\sqrt{x}\,a^2b^3x+6\sqrt{x}\,b^5x^2+21a^5+105a^3b^2x+35a\,b^4x^2\right)}{21}$$

input
```
int((a+b*x^(1/2))^5,x)
```

output
```
(x*(70*sqrt(x)*a**4*b + 84*sqrt(x)*a**2*b**3*x + 6*sqrt(x)*b**5*x**2 + 21*
a**5 + 105*a**3*b**2*x + 35*a*b**4*x**2))/21
```

3.36 $\int \frac{(a+b\sqrt{x})^5}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{(a+b\sqrt{x})^5}{x} \, dx = 10a^4b\sqrt{x} + 10a^3b^2x + \frac{20}{3}a^2b^3x^{3/2} + \frac{5}{2}ab^4x^2 + \frac{2}{5}b^5x^{5/2} + a^5\log(x)$$

output
```
10*a^4*b*x^(1/2)+10*a^3*b^2*x+20/3*a^2*b^3*x^(3/2)+5/2*a*b^4*x^2+2/5*b^5*x
^(5/2)+a^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\sqrt{x})^5}{x} \, dx = \frac{1}{30}b\sqrt{x}\left(300a^4 + 300a^3b\sqrt{x} + 200a^2b^2x + 75ab^3x^{3/2} + 12b^4x^2\right) + 2a^5\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^5/x,x]
```

output
```
(b*Sqrt[x]*(300*a^4 + 300*a^3*b*Sqrt[x] + 200*a^2*b^2*x + 75*a*b^3*x^(3/2)
 + 12*b^4*x^2))/30 + 2*a^5*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^5}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(\frac{a^5}{\sqrt{x}}+5ba^4+10b^2\sqrt{x}a^3+10b^3xa^2+5b^4x^{3/2}a+b^5x^2\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(a^5\log\left(\sqrt{x}\right)+5a^4b\sqrt{x}+5a^3b^2x+\frac{10}{3}a^2b^3x^{3/2}+\frac{5}{4}ab^4x^2+\frac{1}{5}b^5x^{5/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^5/x,x]
```

output
```
2*(5*a^4*b*Sqrt[x] + 5*a^3*b^2*x + (10*a^2*b^3*x^(3/2))/3 + (5*a*b^4*x^2)/
4 + (b^5*x^(5/2))/5 + a^5*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$10a^4b\sqrt{x}+10a^3b^2x+\frac{20a^2b^3x^{\frac{3}{2}}}{3}+\frac{5b^4x^2a}{2}+\frac{2b^5x^{\frac{5}{2}}}{5}+a^5\ln\left(x\right)$	54
default	$10a^4b\sqrt{x}+10a^3b^2x+\frac{20a^2b^3x^{\frac{3}{2}}}{3}+\frac{5b^4x^2a}{2}+\frac{2b^5x^{\frac{5}{2}}}{5}+a^5\ln\left(x\right)$	54
trager	$\frac{5(-1+x)\left(b^2x+4a^2+b^2\right)a\,b^2}{2}+\frac{2b\left(3b^4x^2+50a^2b^2x+75a^4\right)\sqrt{x}}{15}+a^5\ln\left(x\right)$	60

input
```
int((a+b*x^(1/2))^5/x,x,method=_RETURNVERBOSE)
```

output
```
10*a^4*b*x^(1/2)+10*a^3*b^2*x+20/3*a^2*b^3*x^(3/2)+5/2*b^4*x^2*a+2/5*b^5*x
^(5/2)+a^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int\frac{\left(a+b\sqrt{x}\right)^5}{x}\,dx=\frac{5}{2}\,ab^4x^2+10\,a^3b^2x+2\,a^5\log\left(\sqrt{x}\right)+\frac{2}{15}\left(3\,b^5x^2+50\,a^2b^3x+75\,a^4b\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5/x,x, algorithm="fricas")
```

output
```
5/2*a*b^4*x^2 + 10*a^3*b^2*x + 2*a^5*log(sqrt(x)) + 2/15*(3*b^5*x^2 + 50*a
^2*b^3*x + 75*a^4*b)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x}\,dx = a^5\log\left(x\right)+10a^4b\sqrt{x}+10a^3b^2x+\frac{20a^2b^3x^{\frac{3}{2}}}{3}+\frac{5ab^4x^2}{2}+\frac{2b^5x^{\frac{5}{2}}}{5}$$

input
```
integrate((a+b*x**(1/2))**5/x,x)
```

output
```
a**5*log(x) + 10*a**4*b*sqrt(x) + 10*a**3*b**2*x + 20*a**2*b**3*x**(3/2)/3
 + 5*a*b**4*x**2/2 + 2*b**5*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x}\,dx = \frac{2}{5}\,b^5x^{\frac{5}{2}}+\frac{5}{2}\,ab^4x^2+\frac{20}{3}\,a^2b^3x^{\frac{3}{2}}+10\,a^3b^2x+a^5\log\left(x\right)+10\,a^4b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5/x,x, algorithm="maxima")
```

output
```
2/5*b^5*x^(5/2) + 5/2*a*b^4*x^2 + 20/3*a^2*b^3*x^(3/2) + 10*a^3*b^2*x + a^
5*log(x) + 10*a^4*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\sqrt{x})^5}{x}\,dx = \frac{2}{5}b^5x^{\frac{5}{2}} + \frac{5}{2}ab^4x^2 + \frac{20}{3}a^2b^3x^{\frac{3}{2}} + 10\,a^3b^2x + a^5\log(|x|) + 10\,a^4b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^5/x,x, algorithm="giac")
```

output
```
2/5*b^5*x^(5/2) + 5/2*a*b^4*x^2 + 20/3*a^2*b^3*x^(3/2) + 10*a^3*b^2*x + a^
5*log(abs(x)) + 10*a^4*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\sqrt{x})^5}{x}\,dx = 2\,a^5\,\ln\left(\sqrt{x}\right) + \frac{2\,b^5\,x^{5/2}}{5} + 10\,a^3\,b^2\,x + \frac{5\,a\,b^4\,x^2}{2} + 10\,a^4\,b\,\sqrt{x} + \frac{20\,a^2\,b^3\,x^{3/2}}{3}$$

input
```
int((a + b*x^(1/2))^5/x,x)
```

output
```
2*a^5*log(x^(1/2)) + (2*b^5*x^(5/2))/5 + 10*a^3*b^2*x + (5*a*b^4*x^2)/2 +
10*a^4*b*x^(1/2) + (20*a^2*b^3*x^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\sqrt{x})^5}{x}\,dx = 10\sqrt{x}\,a^4b + \frac{20\sqrt{x}\,a^2b^3x}{3} + \frac{2\sqrt{x}\,b^5x^2}{5} + \log(x)\,a^5 + 10a^3b^2x + \frac{5a\,b^4x^2}{2}$$

input
```
int((a+b*x^(1/2))^5/x,x)
```

output
```
(300*sqrt(x)*a**4*b + 200*sqrt(x)*a**2*b**3*x + 12*sqrt(x)*b**5*x**2 + 30*
log(x)*a**5 + 300*a**3*b**2*x + 75*a*b**4*x**2)/30
```

3.37 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2} \, dx = -\frac{a^5}{x} - \frac{10a^4b}{\sqrt{x}} + 20a^2b^3\sqrt{x} + 5ab^4x + \frac{2}{3}b^5x^{3/2} + 10a^3b^2\log(x)$$

output
```
-a^5/x-10*a^4*b/x^(1/2)+20*a^2*b^3*x^(1/2)+5*a*b^4*x+2/3*b^5*x^(3/2)+10*a^
3*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2} \, dx = -\frac{a^5}{x} - \frac{10a^4b}{\sqrt{x}} + 20a^2b^3\sqrt{x} + 5ab^4x + \frac{2}{3}b^5x^{3/2} + 10a^3b^2\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^5/x^2,x]
```

output
```
-(a^5/x) - (10*a^4*b)/Sqrt[x] + 20*a^2*b^3*Sqrt[x] + 5*a*b^4*x + (2*b^5*x^
(3/2))/3 + 10*a^3*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx$$

$$\Big\downarrow \; 798$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^5}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \; 49$$

$$2\int \left(\frac{a^5}{x^{3/2}}+\frac{5ba^4}{x}+\frac{10b^2a^3}{\sqrt{x}}+10b^3a^2+5b^4\sqrt{x}a+b^5x\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(-\frac{a^5}{2x}-\frac{5a^4b}{\sqrt{x}}+10a^3b^2\log\left(\sqrt{x}\right)+10a^2b^3\sqrt{x}+\frac{5}{2}ab^4x+\frac{1}{3}b^5x^{3/2}\right)$$

input
```
Int[(a + b*Sqrt[x])^5/x^2,x]
```

output
```
2*(-1/2*a^5/x - (5*a^4*b)/Sqrt[x] + 10*a^2*b^3*Sqrt[x] + (5*a*b^4*x)/2 + (
b^5*x^(3/2))/3 + 10*a^3*b^2*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{a^5}{x}-\frac{10a^4b}{\sqrt{x}}+20a^2b^3\sqrt{x}+5b^4xa+\frac{2b^5x^{\frac{3}{2}}}{3}+10a^3b^2\ln\left(x\right)$	55
default	$-\frac{a^5}{x}-\frac{10a^4b}{\sqrt{x}}+20a^2b^3\sqrt{x}+5b^4xa+\frac{2b^5x^{\frac{3}{2}}}{3}+10a^3b^2\ln\left(x\right)$	55
trager	$\frac{(-1+x)(5b^4x+a^4)a}{x}-\frac{2(-b^4x^2-30a^2b^2x+15a^4)b}{3\sqrt{x}}-10a^3b^2\ln\left(\frac{1}{x}\right)$	61

input
```
int((a+b*x^(1/2))^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^5/x-10*a^4*b/x^(1/2)+20*a^2*b^3*x^(1/2)+5*b^4*x*a+2/3*b^5*x^(3/2)+10*a^
3*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int\frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx=\frac{15\,ab^4x^2+60\,a^3b^2x\log\left(\sqrt{x}\right)-3\,a^5+2\,(b^5x^2+30\,a^2b^3x-15\,a^4b)\sqrt{x}}{3\,x}$$

input
```
integrate((a+b*x^(1/2))^5/x^2,x, algorithm="fricas")
```

output
```
1/3*(15*a*b^4*x^2 + 60*a^3*b^2*x*log(sqrt(x)) - 3*a^5 + 2*(b^5*x^2 + 30*a^
2*b^3*x - 15*a^4*b)*sqrt(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sqrt{x})^5}{x^2}\,dx = -\frac{a^5}{x} - \frac{10a^4b}{\sqrt{x}} + 10a^3b^2\log(x) + 20a^2b^3\sqrt{x} + 5ab^4x + \frac{2b^5x^{\frac{3}{2}}}{3}$$

input
```
integrate((a+b*x**(1/2))**5/x**2,x)
```

output
```
-a**5/x - 10*a**4*b/sqrt(x) + 10*a**3*b**2*log(x) + 20*a**2*b**3*sqrt(x) +
 5*a*b**4*x + 2*b**5*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\sqrt{x})^5}{x^2}\,dx = \frac{2}{3}\,b^5x^{\frac{3}{2}} + 5\,ab^4x + 10\,a^3b^2\log(x) + 20\,a^2b^3\sqrt{x} - \frac{10\,a^4b\sqrt{x} + a^5}{x}$$

input
```
integrate((a+b*x^(1/2))^5/x^2,x, algorithm="maxima")
```

output
```
2/3*b^5*x^(3/2) + 5*a*b^4*x + 10*a^3*b^2*log(x) + 20*a^2*b^3*sqrt(x) - (10
*a^4*b*sqrt(x) + a^5)/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx = \frac{2}{3}\,b^5x^{\frac{3}{2}} + 5\,ab^4x + 10\,a^3b^2\log\left(|x|\right) + 20\,a^2b^3\sqrt{x} - \frac{10\,a^4b\sqrt{x}+a^5}{x}$$

input
```
integrate((a+b*x^(1/2))^5/x^2,x, algorithm="giac")
```

output
```
2/3*b^5*x^(3/2) + 5*a*b^4*x + 10*a^3*b^2*log(abs(x)) + 20*a^2*b^3*sqrt(x)
- (10*a^4*b*sqrt(x) + a^5)/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx = \frac{2\,b^5\,x^{3/2}}{3} - \frac{a^5+10\,a^4\,b\,\sqrt{x}}{x} + 20\,a^3\,b^2\,\ln\left(\sqrt{x}\right) + 20\,a^2\,b^3\,\sqrt{x} + 5\,a\,b^4\,x$$

input
```
int((a + b*x^(1/2))^5/x^2,x)
```

output
```
(2*b^5*x^(3/2))/3 - (a^5 + 10*a^4*b*x^(1/2))/x + 20*a^3*b^2*log(x^(1/2)) +
 20*a^2*b^3*x^(1/2) + 5*a*b^4*x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^2}\,dx = \frac{30\sqrt{x}\log(x)\,a^3b^2x - 3\sqrt{x}\,a^5 + 15\sqrt{x}\,a\,b^4x^2 - 30a^4bx + 60a^2b^3x^2 + 2b^5x^3}{3\sqrt{x}\,x}$$

input
```
int((a+b*x^(1/2))^5/x^2,x)
```

output
```
(30*sqrt(x)*log(x)*a**3*b**2*x - 3*sqrt(x)*a**5 + 15*sqrt(x)*a*b**4*x**2 -
 30*a**4*b*x + 60*a**2*b**3*x**2 + 2*b**5*x**3)/(3*sqrt(x)*x)
```

3.38 $\int \frac{(a+b\sqrt{x})^5}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx=-\frac{a^5}{2x^2}-\frac{10a^4b}{3x^{3/2}}-\frac{10a^3b^2}{x}-\frac{20a^2b^3}{\sqrt{x}}+2b^5\sqrt{x}+5ab^4\log(x)$$

output
```
-1/2*a^5/x^2-10/3*a^4*b/x^(3/2)-10*a^3*b^2/x-20*a^2*b^3/x^(1/2)+2*b^5*x^(1
/2)+5*a*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx=\frac{-3a^5-20a^4b\sqrt{x}-60a^3b^2x-120a^2b^3x^{3/2}+12b^5x^{5/2}}{6x^2}+10ab^4\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^5/x^3,x]
```

output
```
(-3*a^5 - 20*a^4*b*Sqrt[x] - 60*a^3*b^2*x - 120*a^2*b^3*x^(3/2) + 12*b^5*x
^(5/2))/(6*x^2) + 10*a*b^4*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^5}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^5}{x^{5/2}}+\frac{5ba^4}{x^2}+\frac{10b^2a^3}{x^{3/2}}+\frac{10b^3a^2}{x}+\frac{5b^4a}{\sqrt{x}}+b^5\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^5}{4x^2}-\frac{5a^4b}{3x^{3/2}}-\frac{5a^3b^2}{x}-\frac{10a^2b^3}{\sqrt{x}}+5ab^4\log\left(\sqrt{x}\right)+b^5\sqrt{x}\right)$$

input
```
Int[(a + b*Sqrt[x])^5/x^3,x]
```

output
```
2*(-1/4*a^5/x^2 - (5*a^4*b)/(3*x^(3/2)) - (5*a^3*b^2)/x - (10*a^2*b^3)/Sqr
t[x] + b^5*Sqrt[x] + 5*a*b^4*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{a^{5}}{2x^{2}}-\frac{10a^{4}b}{3x^{\frac{3}{2}}}-\frac{10a^{3}b^{2}}{x}-\frac{20a^{2}b^{3}}{\sqrt{x}}+2b^{5}\sqrt{x}+5a\,b^{4}\ln\left(x\right)$	57
default	$-\frac{a^{5}}{2x^{2}}-\frac{10a^{4}b}{3x^{\frac{3}{2}}}-\frac{10a^{3}b^{2}}{x}-\frac{20a^{2}b^{3}}{\sqrt{x}}+2b^{5}\sqrt{x}+5a\,b^{4}\ln\left(x\right)$	57
trager	$\frac{(-1+x)(a^{2}x+20b^{2}x+a^{2})a^{3}}{2x^{2}}-\frac{2(-3b^{4}x^{2}+30a^{2}b^{2}x+5a^{4})b}{3x^{\frac{3}{2}}}-5a\,b^{4}\ln\left(\frac{1}{x}\right)$	67

input
```
int((a+b*x^(1/2))^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^5/x^2-10/3*a^4*b/x^(3/2)-10*a^3*b^2/x-20*a^2*b^3/x^(1/2)+2*b^5*x^(1
/2)+5*a*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int\frac{\left(a+b\sqrt{x}\right)^{5}}{x^{3}}\,dx=\frac{60\,ab^{4}x^{2}\log\left(\sqrt{x}\right)-60\,a^{3}b^{2}x-3\,a^{5}+4\,(3\,b^{5}x^{2}-30\,a^{2}b^{3}x-5\,a^{4}b)\sqrt{x}}{6\,x^{2}}$$

input
```
integrate((a+b*x^(1/2))^5/x^3,x, algorithm="fricas")
```

output
```
1/6*(60*a*b^4*x^2*log(sqrt(x)) - 60*a^3*b^2*x - 3*a^5 + 4*(3*b^5*x^2 - 30*
a^2*b^3*x - 5*a^4*b)*sqrt(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx = -\frac{a^5}{2x^2} - \frac{10a^4b}{3x^{\frac{3}{2}}} - \frac{10a^3b^2}{x} - \frac{20a^2b^3}{\sqrt{x}} + 5ab^4\log(x) + 2b^5\sqrt{x}$$

input
```
integrate((a+b*x**(1/2))**5/x**3,x)
```

output
```
-a**5/(2*x**2) - 10*a**4*b/(3*x**(3/2)) - 10*a**3*b**2/x - 20*a**2*b**3/sq
rt(x) + 5*a*b**4*log(x) + 2*b**5*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx = 5\,ab^4\log(x) + 2\,b^5\sqrt{x} - \frac{120\,a^2b^3x^{\frac{3}{2}} + 60\,a^3b^2x + 20\,a^4b\sqrt{x} + 3\,a^5}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^5/x^3,x, algorithm="maxima")
```

output
```
5*a*b^4*log(x) + 2*b^5*sqrt(x) - 1/6*(120*a^2*b^3*x^(3/2) + 60*a^3*b^2*x +
 20*a^4*b*sqrt(x) + 3*a^5)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx = 5\,ab^4\log\left(|x|\right) + 2\,b^5\sqrt{x} - \frac{120\,a^2b^3x^{\frac{3}{2}} + 60\,a^3b^2x + 20\,a^4b\sqrt{x} + 3\,a^5}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^5/x^3,x, algorithm="giac")
```

output
```
5*a*b^4*log(abs(x)) + 2*b^5*sqrt(x) - 1/6*(120*a^2*b^3*x^(3/2) + 60*a^3*b^
2*x + 20*a^4*b*sqrt(x) + 3*a^5)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx = 2\,b^5\,\sqrt{x} - \frac{\frac{a^5}{2} + 10\,a^3\,b^2\,x + \frac{10\,a^4\,b\,\sqrt{x}}{3} + 20\,a^2\,b^3\,x^{3/2}}{x^2} + 10\,a\,b^4\,\ln\left(\sqrt{x}\right)$$

input
```
int((a + b*x^(1/2))^5/x^3,x)
```

output
```
2*b^5*x^(1/2) - (a^5/2 + 10*a^3*b^2*x + (10*a^4*b*x^(1/2))/3 + 20*a^2*b^3*
x^(3/2))/x^2 + 10*a*b^4*log(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^3}\,dx = \frac{30\sqrt{x}\,\log(x)\,a\,b^4x^2 - 3\sqrt{x}\,a^5 - 60\sqrt{x}\,a^3b^2x - 20a^4bx - 120a^2b^3x^2 + 12b^5x^3}{6\sqrt{x}\,x^2}$$

input
```
int((a+b*x^(1/2))^5/x^3,x)
```

output
```
(30*sqrt(x)*log(x)*a*b**4*x**2 - 3*sqrt(x)*a**5 - 60*sqrt(x)*a**3*b**2*x -
 20*a**4*b*x - 120*a**2*b**3*x**2 + 12*b**5*x**3)/(6*sqrt(x)*x**2)
```

3.39 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\,dx=-\frac{\left(a+b\sqrt{x}\right)^6}{3ax^3}$$

output
```
-1/3*(a+b*x^(1/2))^6/a/x^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(21) = 42.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.10

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\,dx=\frac{-a^5-6a^4b\sqrt{x}-15a^3b^2x-20a^2b^3x^{3/2}-15ab^4x^2-6b^5x^{5/2}}{3x^3}$$

input
```
Integrate[(a + b*Sqrt[x])^5/x^4,x]
```

output
```
(-a^5 - 6*a^4*b*Sqrt[x] - 15*a^3*b^2*x - 20*a^2*b^3*x^(3/2) - 15*a*b^4*x^2
 - 6*b^5*x^(5/2))/(3*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(a+b\sqrt{x}\right)^6}{3ax^3}$$

input
```
Int[(a + b*Sqrt[x])^5/x^4,x]
```

output
```
-1/3*(a + b*Sqrt[x])^6/(a*x^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 1.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

method	result
derivativedivides	$-\frac{2b^5}{\sqrt{x}}-\frac{5a\,b^4}{x}-\frac{2a^4b}{x^{\frac{5}{2}}}-\frac{a^5}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{20a^2b^3}{3x^{\frac{3}{2}}}$
default	$-\frac{2b^5}{\sqrt{x}}-\frac{5a\,b^4}{x}-\frac{2a^4b}{x^{\frac{5}{2}}}-\frac{a^5}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{20a^2b^3}{3x^{\frac{3}{2}}}$
trager	$\frac{(-1+x)(a^4x^2+15a^2b^2x^2+15b^4x^2+a^4x+15a^2b^2x+a^4)a}{3x^3}-\frac{2(3b^4x^2+10a^2b^2x+3a^4)b}{3x^{\frac{5}{2}}}$
orering	$-\frac{(75b^8x^4-35a^2b^6x^3+90a^4b^4x^2-55a^6b^2x+13a^8)(a+b\sqrt{x})^5}{15x^3(-b^2x+a^2)^4}-\frac{2(15b^8x^4-5a^2b^6x^3+10a^4b^4x^2-5a^6b^2x+a^8)x^2\left(\frac{5}{}\right.}{15(-b^2x+a^2)^4}$

input
```
int((a+b*x^(1/2))^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2*b^5/x^(1/2)-5*a*b^4/x-2*a^4*b/x^(5/2)-1/3*a^5/x^3-5*a^3*b^2/x^2-20/3*a^
2*b^3/x^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.67

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}\,dx = -\frac{15\,ab^4x^2+15\,a^3b^2x+a^5+2\left(3\,b^5x^2+10\,a^2b^3x+3\,a^4b\right)\sqrt{x}}{3\,x^3}$$

input
```
integrate((a+b*x^(1/2))^5/x^4,x, algorithm="fricas")
```

output
```
-1/3*(15*a*b^4*x^2 + 15*a^3*b^2*x + a^5 + 2*(3*b^5*x^2 + 10*a^2*b^3*x + 3*
a^4*b)*sqrt(x))/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(17) = 34$.

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \frac{(a+b\sqrt{x})^5}{x^4}\,dx = -\frac{a^5}{3x^3} - \frac{2a^4b}{x^{\frac{5}{2}}} - \frac{5a^3b^2}{x^2} - \frac{20a^2b^3}{3x^{\frac{3}{2}}} - \frac{5ab^4}{x} - \frac{2b^5}{\sqrt{x}}$$

input
```
integrate((a+b*x**(1/2))**5/x**4,x)
```

output
```
-a**5/(3*x**3) - 2*a**4*b/x**(5/2) - 5*a**3*b**2/x**2 - 20*a**2*b**3/(3*x*
*(3/2)) - 5*a*b**4/x - 2*b**5/sqrt(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{(a+b\sqrt{x})^5}{x^4}\,dx = -\frac{6\,b^5x^{\frac{5}{2}} + 15\,ab^4x^2 + 20\,a^2b^3x^{\frac{3}{2}} + 15\,a^3b^2x + 6\,a^4b\sqrt{x} + a^5}{3\,x^3}$$

input
```
integrate((a+b*x^(1/2))^5/x^4,x, algorithm="maxima")
```

output
```
-1/3*(6*b^5*x^(5/2) + 15*a*b^4*x^2 + 20*a^2*b^3*x^(3/2) + 15*a^3*b^2*x + 6
*a^4*b*sqrt(x) + a^5)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{(a+b\sqrt{x})^5}{x^4}\,dx = -\frac{6\,b^5x^{\frac{5}{2}} + 15\,ab^4x^2 + 20\,a^2b^3x^{\frac{3}{2}} + 15\,a^3b^2x + 6\,a^4b\sqrt{x} + a^5}{3\,x^3}$$

input
```
integrate((a+b*x^(1/2))^5/x^4,x, algorithm="giac")
```

output
```
-1/3*(6*b^5*x^(5/2) + 15*a*b^4*x^2 + 20*a^2*b^3*x^(3/2) + 15*a^3*b^2*x + 6
*a^4*b*sqrt(x) + a^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{(a+b\sqrt{x})^5}{x^4}\,dx = -\frac{a^5 + 6\,b^5\,x^{5/2} + 15\,a^3\,b^2\,x + 15\,a\,b^4\,x^2 + 6\,a^4\,b\,\sqrt{x} + 20\,a^2\,b^3\,x^{3/2}}{3\,x^3}$$

input
```
int((a + b*x^(1/2))^5/x^4,x)
```

output
```
-(a^5 + 6*b^5*x^(5/2) + 15*a^3*b^2*x + 15*a*b^4*x^2 + 6*a^4*b*x^(1/2) + 20
*a^2*b^3*x^(3/2))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.10

$$\int \frac{(a+b\sqrt{x})^5}{x^4}\,dx = \frac{-\sqrt{x}\,a^5 - 15\sqrt{x}\,a^3b^2x - 15\sqrt{x}\,a\,b^4x^2 - 6a^4bx - 20a^2b^3x^2 - 6b^5x^3}{3\sqrt{x}\,x^3}$$

input
```
int((a+b*x^(1/2))^5/x^4,x)
```

output
```
( - sqrt(x)*a**5 - 15*sqrt(x)*a**3*b**2*x - 15*sqrt(x)*a*b**4*x**2 - 6*a**
4*b*x - 20*a**2*b**3*x**2 - 6*b**5*x**3)/(3*sqrt(x)*x**3)
```

3.40 $\int \frac{(a+b\sqrt{x})^5}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{(a+b\sqrt{x})^5}{x^5}\,dx = -\frac{(a+b\sqrt{x})^6}{4ax^4} + \frac{b(a+b\sqrt{x})^6}{14a^2x^{7/2}} - \frac{b^2(a+b\sqrt{x})^6}{84a^3x^3}$$

output
```
-1/4*(a+b*x^(1/2))^6/a/x^4+1/14*b*(a+b*x^(1/2))^6/a^2/x^(7/2)-1/84*b^2*(a+
b*x^(1/2))^6/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\sqrt{x})^5}{x^5}\,dx = \frac{-21a^5-120a^4b\sqrt{x}-280a^3b^2x-336a^2b^3x^{3/2}-210ab^4x^2-56b^5x^{5/2}}{84x^4}$$

input
```
Integrate[(a + b*Sqrt[x])^5/x^5,x]
```

output
```
(-21*a^5 - 120*a^4*b*Sqrt[x] - 280*a^3*b^2*x - 336*a^2*b^3*x^(3/2) - 210*a
*b^4*x^2 - 56*b^5*x^(5/2))/(84*x^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^5}{x^{9/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 55}$$

$$2\left(-\frac{b\int \frac{\left(a+b\sqrt{x}\right)^5}{x^4}d\sqrt{x}}{4a}-\frac{\left(a+b\sqrt{x}\right)^6}{8ax^4}\right)$$

$$\Big\downarrow \text{ 55}$$

$$2\left(-\frac{b\left(-\frac{b\int \frac{\left(a+b\sqrt{x}\right)^5}{x^{7/2}}d\sqrt{x}}{7a}-\frac{\left(a+b\sqrt{x}\right)^6}{7ax^{7/2}}\right)}{4a}-\frac{\left(a+b\sqrt{x}\right)^6}{8ax^4}\right)$$

$$\Big\downarrow \text{ 48}$$

$$2\left(-\frac{b\left(\frac{b\left(a+b\sqrt{x}\right)^6}{42a^2x^3}-\frac{\left(a+b\sqrt{x}\right)^6}{7ax^{7/2}}\right)}{4a}-\frac{\left(a+b\sqrt{x}\right)^6}{8ax^4}\right)$$

input
```
Int[(a + b*Sqrt[x])^5/x^5,x]
```

output
```
2*(-1/4*(b*(-1/7*(a + b*Sqrt[x])^6/(a*x^(7/2)) + (b*(a + b*Sqrt[x])^6)/(42
*a^2*x^3)))/a - (a + b*Sqrt[x])^6/(8*a*x^4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{2b^5}{3x^{\frac{3}{2}}}-\frac{10a^4b}{7x^{\frac{7}{2}}}-\frac{5a\,b^4}{2x^2}-\frac{10a^3b^2}{3x^3}-\frac{a^5}{4x^4}-\frac{4a^2b^3}{x^{\frac{5}{2}}}$
default	$-\frac{2b^5}{3x^{\frac{3}{2}}}-\frac{10a^4b}{7x^{\frac{7}{2}}}-\frac{5a\,b^4}{2x^2}-\frac{10a^3b^2}{3x^3}-\frac{a^5}{4x^4}-\frac{4a^2b^3}{x^{\frac{5}{2}}}$
trager	$\frac{(-1+x)\left(3a^4x^3+40a^2b^2x^3+30b^4x^3+3a^4x^2+40a^2b^2x^2+30b^4x^2+3a^4x+40a^2b^2x+3a^4\right)a}{12x^4}-\frac{2\left(7b^4x^2+42a^2b^2x+15a^4\right.}{21x^{\frac{7}{2}}}$
orering	$-\frac{\left(126b^8x^4-308a^2b^6x^3+377a^4b^4x^2-222a^6b^2x+51a^8\right)\left(a+b\sqrt{x}\right)^5}{84x^4\left(-b^2x+a^2\right)^4}-\frac{\left(70b^8x^4-140a^2b^6x^3+145a^4b^4x^2-74a^6b^2x+1\right.}{210(-b^2x+a^2}$

input
```
int((a+b*x^(1/2))^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/3*b^5/x^(3/2)-10/7*a^4*b/x^(7/2)-5/2*a*b^4/x^2-10/3*a^3*b^2/x^3-1/4*a^5
/x^4-4*a^2*b^3/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx = -\frac{210\,ab^4x^2+280\,a^3b^2x+21\,a^5+8\,(7\,b^5x^2+42\,a^2b^3x+15\,a^4b)\sqrt{x}}{84\,x^4}$$

input
```
integrate((a+b*x^(1/2))^5/x^5,x, algorithm="fricas")
```

output
```
-1/84*(210*a*b^4*x^2 + 280*a^3*b^2*x + 21*a^5 + 8*(7*b^5*x^2 + 42*a^2*b^3*
x + 15*a^4*b)*sqrt(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx = -\frac{a^5}{4x^4}-\frac{10a^4b}{7x^{\frac{7}{2}}}-\frac{10a^3b^2}{3x^3}-\frac{4a^2b^3}{x^{\frac{5}{2}}}-\frac{5ab^4}{2x^2}-\frac{2b^5}{3x^{\frac{3}{2}}}$$

input
```
integrate((a+b*x**(1/2))**5/x**5,x)
```

output
```
-a**5/(4*x**4) - 10*a**4*b/(7*x**(7/2)) - 10*a**3*b**2/(3*x**3) - 4*a**2*b
**3/x**(5/2) - 5*a*b**4/(2*x**2) - 2*b**5/(3*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx = -\frac{56\,b^5x^{\frac{5}{2}}+210\,ab^4x^2+336\,a^2b^3x^{\frac{3}{2}}+280\,a^3b^2x+120\,a^4b\sqrt{x}+21\,a^5}{84\,x^4}$$

input
```
integrate((a+b*x^(1/2))^5/x^5,x, algorithm="maxima")
```

output
```
-1/84*(56*b^5*x^(5/2) + 210*a*b^4*x^2 + 336*a^2*b^3*x^(3/2) + 280*a^3*b^2*
x + 120*a^4*b*sqrt(x) + 21*a^5)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx = -\frac{56\,b^5x^{\frac{5}{2}}+210\,ab^4x^2+336\,a^2b^3x^{\frac{3}{2}}+280\,a^3b^2x+120\,a^4b\sqrt{x}+21\,a^5}{84\,x^4}$$

input
```
integrate((a+b*x^(1/2))^5/x^5,x, algorithm="giac")
```

output
```
-1/84*(56*b^5*x^(5/2) + 210*a*b^4*x^2 + 336*a^2*b^3*x^(3/2) + 280*a^3*b^2*
x + 120*a^4*b*sqrt(x) + 21*a^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx = -\frac{21\,a^5+56\,b^5\,x^{5/2}+280\,a^3\,b^2\,x+210\,a\,b^4\,x^2+120\,a^4\,b\,\sqrt{x}+336\,a^2\,b^3\,x^{3/2}}{84\,x^4}$$

input
```
int((a + b*x^(1/2))^5/x^5,x)
```

output
```
-(21*a^5 + 56*b^5*x^(5/2) + 280*a^3*b^2*x + 210*a*b^4*x^2 + 120*a^4*b*x^(1
/2) + 336*a^2*b^3*x^(3/2))/(84*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^5}\,dx = \frac{-21\sqrt{x}\,a^5-280\sqrt{x}\,a^3b^2x-210\sqrt{x}\,a\,b^4x^2-120a^4bx-336a^2b^3x^2-56b^5x^3}{84\sqrt{x}\,x^4}$$

input
```
int((a+b*x^(1/2))^5/x^5,x)
```

output
```
( - 21*sqrt(x)*a**5 - 280*sqrt(x)*a**3*b**2*x - 210*sqrt(x)*a*b**4*x**2 -
120*a**4*b*x - 336*a**2*b**3*x**2 - 56*b**5*x**3)/(84*sqrt(x)*x**4)
```

3.41 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx=-\frac{a^5}{5x^5}-\frac{10a^4b}{9x^{9/2}}-\frac{5a^3b^2}{2x^4}-\frac{20a^2b^3}{7x^{7/2}}-\frac{5ab^4}{3x^3}-\frac{2b^5}{5x^{5/2}}$$

output
```
-1/5*a^5/x^5-10/9*a^4*b/x^(9/2)-5/2*a^3*b^2/x^4-20/7*a^2*b^3/x^(7/2)-5/3*a
*b^4/x^3-2/5*b^5/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx =\frac{-126a^5-700a^4b\sqrt{x}-1575a^3b^2x-1800a^2b^3x^{3/2}-1050ab^4x^2-252b^5x^{5/2}}{630x^5}$$

input
```
Integrate[(a + b*Sqrt[x])^5/x^6,x]
```

output
```
(-126*a^5 - 700*a^4*b*Sqrt[x] - 1575*a^3*b^2*x - 1800*a^2*b^3*x^(3/2) - 10
50*a*b^4*x^2 - 252*b^5*x^(5/2))/(630*x^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^5}{x^{11/2}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^5}{x^{11/2}}+\frac{5ba^4}{x^5}+\frac{10b^2a^3}{x^{9/2}}+\frac{10b^3a^2}{x^4}+\frac{5b^4a}{x^{7/2}}+\frac{b^5}{x^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^5}{10x^5}-\frac{5a^4b}{9x^{9/2}}-\frac{5a^3b^2}{4x^4}-\frac{10a^2b^3}{7x^{7/2}}-\frac{5ab^4}{6x^3}-\frac{b^5}{5x^{5/2}}\right)$$

input
```
Int[(a + b*Sqrt[x])^5/x^6,x]
```

output
```
2*(-1/10*a^5/x^5 - (5*a^4*b)/(9*x^(9/2)) - (5*a^3*b^2)/(4*x^4) - (10*a^2*b
^3)/(7*x^(7/2)) - (5*a*b^4)/(6*x^3) - b^5/(5*x^(5/2)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{a^5}{5x^5}-\frac{10a^4b}{9x^{\frac{9}{2}}}-\frac{5a^3b^2}{2x^4}-\frac{20a^2b^3}{7x^{\frac{7}{2}}}-\frac{5a\,b^4}{3x^3}-\frac{2b^5}{5x^{\frac{5}{2}}}$
default	$-\frac{a^5}{5x^5}-\frac{10a^4b}{9x^{\frac{9}{2}}}-\frac{5a^3b^2}{2x^4}-\frac{20a^2b^3}{7x^{\frac{7}{2}}}-\frac{5a\,b^4}{3x^3}-\frac{2b^5}{5x^{\frac{5}{2}}}$
trager	$\frac{(-1+x)(6a^4x^4+75a^2b^2x^4+50b^4x^4+6a^4x^3+75a^2b^2x^3+50b^4x^3+6a^4x^2+75a^2b^2x^2+50b^4x^2+6a^4x+75a^2b^2x+6a^4)a}{30x^5}$
orering	$-\frac{(2730b^8x^4-8325a^2b^6x^3+10642a^4b^4x^2-6365a^6b^2x+1470a^8)(a+b\sqrt{x})^5}{3150x^5(-b^2x+a^2)^4}-\frac{(210b^8x^4-555a^2b^6x^3+626a^4b^4x^2-33}{1575}$

input
```
int((a+b*x^(1/2))^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^5/x^5-10/9*a^4*b/x^(9/2)-5/2*a^3*b^2/x^4-20/7*a^2*b^3/x^(7/2)-5/3*a
*b^4/x^3-2/5*b^5/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx = -\frac{1050\,ab^4x^2+1575\,a^3b^2x+126\,a^5+4\,(63\,b^5x^2+450\,a^2b^3x+175\,a^4b)\sqrt{x}}{630\,x^5}$$

input
```
integrate((a+b*x^(1/2))^5/x^6,x, algorithm="fricas")
```

output
```
-1/630*(1050*a*b^4*x^2 + 1575*a^3*b^2*x + 126*a^5 + 4*(63*b^5*x^2 + 450*a^
2*b^3*x + 175*a^4*b)*sqrt(x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx = -\frac{a^5}{5x^5} - \frac{10a^4b}{9x^{\frac{9}{2}}} - \frac{5a^3b^2}{2x^4} - \frac{20a^2b^3}{7x^{\frac{7}{2}}} - \frac{5ab^4}{3x^3} - \frac{2b^5}{5x^{\frac{5}{2}}}$$

input
```
integrate((a+b*x**(1/2))**5/x**6,x)
```

output
```
-a**5/(5*x**5) - 10*a**4*b/(9*x**(9/2)) - 5*a**3*b**2/(2*x**4) - 20*a**2*b
**3/(7*x**(7/2)) - 5*a*b**4/(3*x**3) - 2*b**5/(5*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx = -\frac{252\,b^5x^{\frac{5}{2}} + 1050\,ab^4x^2 + 1800\,a^2b^3x^{\frac{3}{2}} + 1575\,a^3b^2x + 700\,a^4b\sqrt{x} + 126\,a^5}{630\,x^5}$$

input
```
integrate((a+b*x^(1/2))^5/x^6,x, algorithm="maxima")
```

output
```
-1/630*(252*b^5*x^(5/2) + 1050*a*b^4*x^2 + 1800*a^2*b^3*x^(3/2) + 1575*a^3
*b^2*x + 700*a^4*b*sqrt(x) + 126*a^5)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx = -\frac{252\,b^5x^{\frac{5}{2}} + 1050\,ab^4x^2 + 1800\,a^2b^3x^{\frac{3}{2}} + 1575\,a^3b^2x + 700\,a^4b\sqrt{x} + 126\,a^5}{630\,x^5}$$

input
```
integrate((a+b*x^(1/2))^5/x^6,x, algorithm="giac")
```

output
```
-1/630*(252*b^5*x^(5/2) + 1050*a*b^4*x^2 + 1800*a^2*b^3*x^(3/2) + 1575*a^3
*b^2*x + 700*a^4*b*sqrt(x) + 126*a^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx = -\frac{126\,a^5 + 252\,b^5\,x^{5/2} + 1575\,a^3\,b^2\,x + 1050\,a\,b^4\,x^2 + 700\,a^4\,b\,\sqrt{x} + 1800\,a^2\,b^3\,x^{3/2}}{630\,x^5}$$

input
```
int((a + b*x^(1/2))^5/x^6,x)
```

output
```
-(126*a^5 + 252*b^5*x^(5/2) + 1575*a^3*b^2*x + 1050*a*b^4*x^2 + 700*a^4*b*
x^(1/2) + 1800*a^2*b^3*x^(3/2))/(630*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^6}\,dx = \frac{-126\sqrt{x}\,a^5-1575\sqrt{x}\,a^3b^2x-1050\sqrt{x}\,a\,b^4x^2-700a^4bx-1800a^2b^3x^2-252b^5x^3}{630\sqrt{x}\,x^5}$$

input
```
int((a+b*x^(1/2))^5/x^6,x)
```

output
```
( - 126*sqrt(x)*a**5 - 1575*sqrt(x)*a**3*b**2*x - 1050*sqrt(x)*a*b**4*x**2
 - 700*a**4*b*x - 1800*a**2*b**3*x**2 - 252*b**5*x**3)/(630*sqrt(x)*x**5)
```

3.42 $\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx=-\frac{a^5}{6x^6}-\frac{10a^4b}{11x^{11/2}}-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{9/2}}-\frac{5ab^4}{4x^4}-\frac{2b^5}{7x^{7/2}}$$

output
```
-1/6*a^5/x^6-10/11*a^4*b/x^(11/2)-2*a^3*b^2/x^5-20/9*a^2*b^3/x^(9/2)-5/4*a
*b^4/x^4-2/7*b^5/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx=\frac{-462a^5-2520a^4b\sqrt{x}-5544a^3b^2x-6160a^2b^3x^{3/2}-3465ab^4x^2-792b^5x^{5/2}}{2772x^6}$$

input
```
Integrate[(a + b*Sqrt[x])^5/x^7,x]
```

output
```
(-462*a^5 - 2520*a^4*b*Sqrt[x] - 5544*a^3*b^2*x - 6160*a^2*b^3*x^(3/2) - 3
465*a*b^4*x^2 - 792*b^5*x^(5/2))/(2772*x^6)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^5}{x^{13/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 53}$$

$$2\int \left(\frac{a^5}{x^{13/2}}+\frac{5ba^4}{x^6}+\frac{10b^2a^3}{x^{11/2}}+\frac{10b^3a^2}{x^5}+\frac{5b^4a}{x^{9/2}}+\frac{b^5}{x^4}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{a^5}{12x^6}-\frac{5a^4b}{11x^{11/2}}-\frac{a^3b^2}{x^5}-\frac{10a^2b^3}{9x^{9/2}}-\frac{5ab^4}{8x^4}-\frac{b^5}{7x^{7/2}}\right)$$

input
```
Int[(a + b*Sqrt[x])^5/x^7,x]
```

output
```
2*(-1/12*a^5/x^6 - (5*a^4*b)/(11*x^(11/2)) - (a^3*b^2)/x^5 - (10*a^2*b^3)/
(9*x^(9/2)) - (5*a*b^4)/(8*x^4) - b^5/(7*x^(7/2)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{a^5}{6x^6}-\frac{10a^4b}{11x^{\frac{11}{2}}}-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{\frac{9}{2}}}-\frac{5a\,b^4}{4x^4}-\frac{2b^5}{7x^{\frac{7}{2}}}$
default	$-\frac{a^5}{6x^6}-\frac{10a^4b}{11x^{\frac{11}{2}}}-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{\frac{9}{2}}}-\frac{5a\,b^4}{4x^4}-\frac{2b^5}{7x^{\frac{7}{2}}}$
orering	$-\frac{(8415b^8x^4-28006a^2b^6x^3+37065a^4b^4x^2-22540a^6b^2x+5250a^8)\left(a+b\sqrt{x}\right)^5}{13860x^6(-b^2x+a^2)^4}-\frac{(495b^8x^4-1474a^2b^6x^3+1765a^4b^4x$
trager	$\frac{(-1+x)(2a^4x^5+24a^2b^2x^5+15b^4x^5+2a^4x^4+24a^2b^2x^4+15b^4x^4+2a^4x^3+24a^2b^2x^3+15b^4x^3+2a^4x^2+24a^2b^2x^2+15b^4}{12x^6}$

input
```
int((a+b*x^(1/2))^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^5/x^6-10/11*a^4*b/x^(11/2)-2*a^3*b^2/x^5-20/9*a^2*b^3/x^(9/2)-5/4*a
*b^4/x^4-2/7*b^5/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx = -\frac{3465\,ab^4x^2+5544\,a^3b^2x+462\,a^5+8\,(99\,b^5x^2+770\,a^2b^3x+315\,a^4b)\sqrt{x}}{2772\,x^6}$$

input
```
integrate((a+b*x^(1/2))^5/x^7,x, algorithm="fricas")
```

output
```
-1/2772*(3465*a*b^4*x^2 + 5544*a^3*b^2*x + 462*a^5 + 8*(99*b^5*x^2 + 770*a
^2*b^3*x + 315*a^4*b)*sqrt(x))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx = -\frac{a^5}{6x^6}-\frac{10a^4b}{11x^{\frac{11}{2}}}-\frac{2a^3b^2}{x^5}-\frac{20a^2b^3}{9x^{\frac{9}{2}}}-\frac{5ab^4}{4x^4}-\frac{2b^5}{7x^{\frac{7}{2}}}$$

input
```
integrate((a+b*x**(1/2))**5/x**7,x)
```

output
```
-a**5/(6*x**6) - 10*a**4*b/(11*x**(11/2)) - 2*a**3*b**2/x**5 - 20*a**2*b**
3/(9*x**(9/2)) - 5*a*b**4/(4*x**4) - 2*b**5/(7*x**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx = -\frac{792\,b^5x^{\frac{5}{2}}+3465\,ab^4x^2+6160\,a^2b^3x^{\frac{3}{2}}+5544\,a^3b^2x+2520\,a^4b\sqrt{x}+462\,a^5}{2772\,x^6}$$

input
```
integrate((a+b*x^(1/2))^5/x^7,x, algorithm="maxima")
```

output
```
-1/2772*(792*b^5*x^(5/2) + 3465*a*b^4*x^2 + 6160*a^2*b^3*x^(3/2) + 5544*a^
3*b^2*x + 2520*a^4*b*sqrt(x) + 462*a^5)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx = -\frac{792\,b^5x^{\frac{5}{2}}+3465\,ab^4x^2+6160\,a^2b^3x^{\frac{3}{2}}+5544\,a^3b^2x+2520\,a^4b\sqrt{x}+462\,a^5}{2772\,x^6}$$

input
```
integrate((a+b*x^(1/2))^5/x^7,x, algorithm="giac")
```

output
```
-1/2772*(792*b^5*x^(5/2) + 3465*a*b^4*x^2 + 6160*a^2*b^3*x^(3/2) + 5544*a^
3*b^2*x + 2520*a^4*b*sqrt(x) + 462*a^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx = -\frac{\frac{a^5}{6}+\frac{2\,b^5\,x^{5/2}}{7}+2\,a^3\,b^2\,x+\frac{5\,a\,b^4\,x^2}{4}+\frac{10\,a^4\,b\,\sqrt{x}}{11}+\frac{20\,a^2\,b^3\,x^{3/2}}{9}}{x^6}$$

input
```
int((a + b*x^(1/2))^5/x^7,x)
```

output
```
-(a^5/6 + (2*b^5*x^(5/2))/7 + 2*a^3*b^2*x + (5*a*b^4*x^2)/4 + (10*a^4*b*x^
(1/2))/11 + (20*a^2*b^3*x^(3/2))/9)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt{x}\right)^5}{x^7}\,dx = \frac{-462\sqrt{x}\,a^5-5544\sqrt{x}\,a^3b^2x-3465\sqrt{x}\,a\,b^4x^2-2520a^4bx-6160a^2b^3x^2-792b^5x^3}{2772\sqrt{x}\,x^6}$$

input
```
int((a+b*x^(1/2))^5/x^7,x)
```

output
```
( - 462*sqrt(x)*a**5 - 5544*sqrt(x)*a**3*b**2*x - 3465*sqrt(x)*a*b**4*x**2
 - 2520*a**4*b*x - 6160*a**2*b**3*x**2 - 792*b**5*x**3)/(2772*sqrt(x)*x**6
)
```

3.43 $\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx = \frac{a^{10}x^5}{5}+\frac{20}{11}a^9bx^{11/2}+\frac{15}{2}a^8b^2x^6+\frac{240}{13}a^7b^3x^{13/2}+30a^6b^4x^7+\frac{168}{5}a^5b^5x^{15/2}+\frac{105}{4}a^4b^6x^8+\frac{240}{17}a^3b^7x^{17/2}+5a^2b^8x^9+\frac{20}{19}ab^9x^{19/2}+\frac{b^{10}x^{10}}{10}$$

output
```
1/5*a^10*x^5+20/11*a^9*b*x^(11/2)+15/2*a^8*b^2*x^6+240/13*a^7*b^3*x^(13/2)
+30*a^6*b^4*x^7+168/5*a^5*b^5*x^(15/2)+105/4*a^4*b^6*x^8+240/17*a^3*b^7*x^
(17/2)+5*a^2*b^8*x^9+20/19*a*b^9*x^(19/2)+1/10*b^10*x^10
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx = \frac{184756a^{10}x^5+1679600a^9bx^{11/2}+6928350a^8b^2x^6+17054400a^7b^3x^{13/2}+27713400a^6b^4x^7+31039008a^5\cdots}{923780\cdots}$$

input
```
Integrate[(a + b*Sqrt[x])^10*x^4,x]
```

output
```
(184756*a^10*x^5 + 1679600*a^9*b*x^(11/2) + 6928350*a^8*b^2*x^6 + 17054400
*a^7*b^3*x^(13/2) + 27713400*a^6*b^4*x^7 + 31039008*a^5*b^5*x^(15/2) + 242
49225*a^4*b^6*x^8 + 13041600*a^3*b^7*x^(17/2) + 4618900*a^2*b^8*x^9 + 9724
00*a*b^9*x^(19/2) + 92378*b^10*x^10)/923780
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\sqrt{x}\right)^{10} dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^{10} x^{9/2} d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(x^{9/2}a^{10}+10bx^5a^9+45b^2x^{11/2}a^8+120b^3x^6a^7+210b^4x^{13/2}a^6+252b^5x^7a^5+210b^6x^{15/2}a^4+120b^7x^8a^3+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^{10}x^5}{10}+\frac{10}{11}a^9bx^{11/2}+\frac{15}{4}a^8b^2x^6+\frac{120}{13}a^7b^3x^{13/2}+15a^6b^4x^7+\frac{84}{5}a^5b^5x^{15/2}+\frac{105}{8}a^4b^6x^8+\frac{120}{17}a^3b^7x^{17/2}+\right.$$

input
```
Int[(a + b*Sqrt[x])^10*x^4,x]
```

output
```
2*((a^10*x^5)/10 + (10*a^9*b*x^(11/2))/11 + (15*a^8*b^2*x^6)/4 + (120*a^7*
b^3*x^(13/2))/13 + 15*a^6*b^4*x^7 + (84*a^5*b^5*x^(15/2))/5 + (105*a^4*b^6
*x^8)/8 + (120*a^3*b^7*x^(17/2))/17 + (5*a^2*b^8*x^9)/2 + (10*a*b^9*x^(19/
2))/19 + (b^10*x^10)/20)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a^{10}x^5}{5}+\frac{20a^9b\,x^{\frac{11}{2}}}{11}+\frac{15a^8b^2x^6}{2}+\frac{240a^7b^3x^{\frac{13}{2}}}{13}+30a^6b^4x^7+\frac{168a^5b^5x^{\frac{15}{2}}}{5}+\frac{105a^4b^6x^8}{4}+\frac{240a^3b^7x^{\frac{17}{2}}}{17}$
default	$\frac{a^{10}x^5}{5}+\frac{20a^9b\,x^{\frac{11}{2}}}{11}+\frac{15a^8b^2x^6}{2}+\frac{240a^7b^3x^{\frac{13}{2}}}{13}+30a^6b^4x^7+\frac{168a^5b^5x^{\frac{15}{2}}}{5}+\frac{105a^4b^6x^8}{4}+\frac{240a^3b^7x^{\frac{17}{2}}}{17}$
orering	$\frac{x^5(-899470b^{18}x^9+8308300a^2b^{16}x^8-34245783a^4b^{14}x^7+82760700a^6b^{12}x^6-129416850a^8b^{10}x^5+136083780a^{10}b^8x^4-}{4618900(-b^2x+a^2)^9}$
trager	$(2b^{10}x^9+100a^2b^8x^8+2b^{10}x^8+525a^4b^6x^7+100a^2b^8x^7+2b^{10}x^7+600a^6b^4x^6+525a^4b^6x^6+100a^2b^8x^6+2b^{10}x^6+150a^8b$

input
```
int((a+b*x^(1/2))^10*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^10*x^5+20/11*a^9*b*x^(11/2)+15/2*a^8*b^2*x^6+240/13*a^7*b^3*x^(13/2)
+30*a^6*b^4*x^7+168/5*a^5*b^5*x^(15/2)+105/4*a^4*b^6*x^8+240/17*a^3*b^7*x^
(17/2)+5*a^2*b^8*x^9+20/19*a*b^9*x^(19/2)+1/10*b^10*x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx = \frac{1}{10}\,b^{10}x^{10}+5\,a^2b^8x^9+\frac{105}{4}\,a^4b^6x^8+30\,a^6b^4x^7+\frac{15}{2}\,a^8b^2x^6+\frac{1}{5}\,a^{10}x^5 + \frac{4}{230945}\left(60775\,ab^9x^9+815100\,a^3b^7x^8+1939938\,a^5b^5x^7+1065900\,a^7b^3x^6+104975\,a^9bx^5\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^10*x^4,x, algorithm="fricas")
```

output
```
1/10*b^10*x^10 + 5*a^2*b^8*x^9 + 105/4*a^4*b^6*x^8 + 30*a^6*b^4*x^7 + 15/2
*a^8*b^2*x^6 + 1/5*a^10*x^5 + 4/230945*(60775*a*b^9*x^9 + 815100*a^3*b^7*x
^8 + 1939938*a^5*b^5*x^7 + 1065900*a^7*b^3*x^6 + 104975*a^9*b*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx = \frac{a^{10}x^5}{5}+\frac{20a^9bx^{\frac{11}{2}}}{11}+\frac{15a^8b^2x^6}{2}+\frac{240a^7b^3x^{\frac{13}{2}}}{13} + 30a^6b^4x^7+\frac{168a^5b^5x^{\frac{15}{2}}}{5}+\frac{105a^4b^6x^8}{4} + \frac{240a^3b^7x^{\frac{17}{2}}}{17}+5a^2b^8x^9+\frac{20ab^9x^{\frac{19}{2}}}{19}+\frac{b^{10}x^{10}}{10}$$

input
```
integrate((a+b*x**(1/2))**10*x**4,x)
```

output
```
a**10*x**5/5 + 20*a**9*b*x**(11/2)/11 + 15*a**8*b**2*x**6/2 + 240*a**7*b**
3*x**(13/2)/13 + 30*a**6*b**4*x**7 + 168*a**5*b**5*x**(15/2)/5 + 105*a**4*
b**6*x**8/4 + 240*a**3*b**7*x**(17/2)/17 + 5*a**2*b**8*x**9 + 20*a*b**9*x*
*(19/2)/19 + b**10*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \left(a+b\sqrt{x}\right)^{10}x^4\,dx=\frac{\left(b\sqrt{x}+a\right)^{20}}{10\,b^{10}}-\frac{18\left(b\sqrt{x}+a\right)^{19}a}{19\,b^{10}}+\frac{4\left(b\sqrt{x}+a\right)^{18}a^2}{b^{10}}-\frac{168\left(b\sqrt{x}+a\right)^{17}a^3}{17\,b^{10}}+\frac{63\left(b\sqrt{x}+a\right)^{16}a^4}{4\,b^{10}}-\frac{84\left(b\sqrt{x}+a\right)^{15}a^5}{5\,b^{10}}+\frac{12\left(b\sqrt{x}+a\right)^{14}a^6}{b^{10}}-\frac{72\left(b\sqrt{x}+a\right)^{13}a^7}{13\,b^{10}}+\frac{3\left(b\sqrt{x}+a\right)^{12}a^8}{2\,b^{10}}-\frac{2\left(b\sqrt{x}+a\right)^{11}a^9}{11\,b^{10}}$$

input
```
integrate((a+b*x^(1/2))^10*x^4,x, algorithm="maxima")
```

output
```
1/10*(b*sqrt(x) + a)^20/b^10 - 18/19*(b*sqrt(x) + a)^19*a/b^10 + 4*(b*sqrt
(x) + a)^18*a^2/b^10 - 168/17*(b*sqrt(x) + a)^17*a^3/b^10 + 63/4*(b*sqrt(x
) + a)^16*a^4/b^10 - 84/5*(b*sqrt(x) + a)^15*a^5/b^10 + 12*(b*sqrt(x) + a)
^14*a^6/b^10 - 72/13*(b*sqrt(x) + a)^13*a^7/b^10 + 3/2*(b*sqrt(x) + a)^12*
a^8/b^10 - 2/11*(b*sqrt(x) + a)^11*a^9/b^10
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^{10}x^4\,dx=\frac{1}{10}\,b^{10}x^{10}+\frac{20}{19}\,ab^9x^{\frac{19}{2}}+5\,a^2b^8x^9+\frac{240}{17}\,a^3b^7x^{\frac{17}{2}}+\frac{105}{4}\,a^4b^6x^8+\frac{168}{5}\,a^5b^5x^{\frac{15}{2}}+30\,a^6b^4x^7+\frac{240}{13}\,a^7b^3x^{\frac{13}{2}}+\frac{15}{2}\,a^8b^2x^6+\frac{20}{11}\,a^9bx^{\frac{11}{2}}+\frac{1}{5}\,a^{10}x^5$$

input
```
integrate((a+b*x^(1/2))^10*x^4,x, algorithm="giac")
```

output
```
1/10*b^10*x^10 + 20/19*a*b^9*x^(19/2) + 5*a^2*b^8*x^9 + 240/17*a^3*b^7*x^(
17/2) + 105/4*a^4*b^6*x^8 + 168/5*a^5*b^5*x^(15/2) + 30*a^6*b^4*x^7 + 240/
13*a^7*b^3*x^(13/2) + 15/2*a^8*b^2*x^6 + 20/11*a^9*b*x^(11/2) + 1/5*a^10*x
^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx = \frac{a^{10}\,x^5}{5}+\frac{b^{10}\,x^{10}}{10}+\frac{20\,a^9\,b\,x^{11/2}}{11}+\frac{20\,a\,b^9\,x^{19/2}}{19}+\frac{15\,a^8\,b^2\,x^6}{2}+30\,a^6\,b^4\,x^7+\frac{105\,a^4\,b^6\,x^8}{4}+5\,a^2\,b^8\,x^9+\frac{240\,a^7\,b^3\,x^{13/2}}{13}+\frac{168\,a^5\,b^5\,x^{15/2}}{5}+\frac{240\,a^3\,b^7\,x^{17/2}}{17}$$

input
```
int(x^4*(a + b*x^(1/2))^10,x)
```

output
```
(a^10*x^5)/5 + (b^10*x^10)/10 + (20*a^9*b*x^(11/2))/11 + (20*a*b^9*x^(19/2
))/19 + (15*a^8*b^2*x^6)/2 + 30*a^6*b^4*x^7 + (105*a^4*b^6*x^8)/4 + 5*a^2*
b^8*x^9 + (240*a^7*b^3*x^(13/2))/13 + (168*a^5*b^5*x^(15/2))/5 + (240*a^3*
b^7*x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^{10} x^4\,dx = \frac{x^5\left(1679600\sqrt{x}\,a^9b+17054400\sqrt{x}\,a^7b^3x+31039008\sqrt{x}\,a^5b^5x^2+13041600\sqrt{x}\,a^3b^7x^3+972400\sqrt{x}\,a\,b^9x\right.}{9237}$$

input
```
int((a+b*x^(1/2))^10*x^4,x)
```

output
```
(x**5*(1679600*sqrt(x)*a**9*b + 17054400*sqrt(x)*a**7*b**3*x + 31039008*sq
rt(x)*a**5*b**5*x**2 + 13041600*sqrt(x)*a**3*b**7*x**3 + 972400*sqrt(x)*a*
b**9*x**4 + 184756*a**10 + 6928350*a**8*b**2*x + 27713400*a**6*b**4*x**2 +
 24249225*a**4*b**6*x**3 + 4618900*a**2*b**8*x**4 + 92378*b**10*x**5))/923
780
```

3.44 $\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 162

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx = &-\frac{2a^7\left(a+b\sqrt{x}\right)^{11}}{11b^8}+\frac{7a^6\left(a+b\sqrt{x}\right)^{12}}{6b^8}-\frac{42a^5\left(a+b\sqrt{x}\right)^{13}}{13b^8}\\&+\frac{5a^4\left(a+b\sqrt{x}\right)^{14}}{b^8}-\frac{14a^3\left(a+b\sqrt{x}\right)^{15}}{3b^8}\\&+\frac{21a^2\left(a+b\sqrt{x}\right)^{16}}{8b^8}-\frac{14a\left(a+b\sqrt{x}\right)^{17}}{17b^8}+\frac{\left(a+b\sqrt{x}\right)^{18}}{9b^8}\end{aligned}$$

output
```
-2/11*a^7*(a+b*x^(1/2))^11/b^8+7/6*a^6*(a+b*x^(1/2))^12/b^8-42/13*a^5*(a+b
*x^(1/2))^13/b^8+5*a^4*(a+b*x^(1/2))^14/b^8-14/3*a^3*(a+b*x^(1/2))^15/b^8+
21/8*a^2*(a+b*x^(1/2))^16/b^8-14/17*a*(a+b*x^(1/2))^17/b^8+1/9*(a+b*x^(1/2
))^18/b^8
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx$$
$$=\frac{43758a^{10}x^4+388960a^9bx^{9/2}+1575288a^8b^2x^5+3818880a^7b^3x^{11/2}+6126120a^6b^4x^6+6785856a^5b^5x^{13/2}}{175032}$$

input
```
Integrate[(a + b*Sqrt[x])^10*x^3,x]
```

output
```
(43758*a^10*x^4 + 388960*a^9*b*x^(9/2) + 1575288*a^8*b^2*x^5 + 3818880*a^7
*b^3*x^(11/2) + 6126120*a^6*b^4*x^6 + 6785856*a^5*b^5*x^(13/2) + 5250960*a
^4*b^6*x^7 + 2800512*a^3*b^7*x^(15/2) + 984555*a^2*b^8*x^8 + 205920*a*b^9*
x^(17/2) + 19448*b^10*x^9)/175032
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{x}\right)^{10}\,dx$$
$$\Big\downarrow \text{798}$$
$$2\int \left(a+b\sqrt{x}\right)^{10}x^{7/2}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$
$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{17}}{b^7}-\frac{7a\left(a+b\sqrt{x}\right)^{16}}{b^7}+\frac{21a^2\left(a+b\sqrt{x}\right)^{15}}{b^7}-\frac{35a^3\left(a+b\sqrt{x}\right)^{14}}{b^7}+\frac{35a^4\left(a+b\sqrt{x}\right)^{13}}{b^7}-\frac{21a^5\left(a+\right.}{b^7}\right.$$
$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^7\left(a+b\sqrt{x}\right)^{11}}{11b^8}+\frac{7a^6\left(a+b\sqrt{x}\right)^{12}}{12b^8}-\frac{21a^5\left(a+b\sqrt{x}\right)^{13}}{13b^8}+\frac{5a^4\left(a+b\sqrt{x}\right)^{14}}{2b^8}-\frac{7a^3\left(a+b\sqrt{x}\right)^{15}}{3b^8}+\frac{21a^2\left(a+b\right.}{16b^8}\right.$$

input
```
Int[(a + b*Sqrt[x])^10*x^3,x]
```

output
```
2*(-1/11*(a^7*(a + b*Sqrt[x])^11)/b^8 + (7*a^6*(a + b*Sqrt[x])^12)/(12*b^8
) - (21*a^5*(a + b*Sqrt[x])^13)/(13*b^8) + (5*a^4*(a + b*Sqrt[x])^14)/(2*b
^8) - (7*a^3*(a + b*Sqrt[x])^15)/(3*b^8) + (21*a^2*(a + b*Sqrt[x])^16)/(16
*b^8) - (7*a*(a + b*Sqrt[x])^17)/(17*b^8) + (a + b*Sqrt[x])^18/(18*b^8))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{b^{10}x^9}{9}+\frac{20a\,b^9x^{\frac{17}{2}}}{17}+\frac{45a^2b^8x^8}{8}+16a^3b^7x^{\frac{15}{2}}+30a^4b^6x^7+\frac{504a^5b^5x^{\frac{13}{2}}}{13}+35a^6b^4x^6+\frac{240a^7b^3x^{\frac{1}{}}}{11}$
default	$\frac{b^{10}x^9}{9}+\frac{20a\,b^9x^{\frac{17}{2}}}{17}+\frac{45a^2b^8x^8}{8}+16a^3b^7x^{\frac{15}{2}}+30a^4b^6x^7+\frac{504a^5b^5x^{\frac{13}{2}}}{13}+35a^6b^4x^6+\frac{240a^7b^3x^{\frac{1}{}}}{11}$
orering	$-\frac{(188760b^{26}x^{13}-1742169a^2b^{24}x^{12}+7177500a^4b^{22}x^{11}-17346150a^6b^{20}x^{10}+27149500a^8b^{18}x^9-28616715a^{10}b^{16}x^8+}{}$
trager	$\frac{(8b^{10}x^8+405a^2b^8x^7+8b^{10}x^7+2160a^4b^6x^6+405a^2b^8x^6+8b^{10}x^6+2520a^6b^4x^5+2160a^4b^6x^5+405a^2b^8x^5+8b^{10}x^5+648}{}$

input
```
int((a+b*x^(1/2))^10*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^10*x^9+20/17*a*b^9*x^(17/2)+45/8*a^2*b^8*x^8+16*a^3*b^7*x^(15/2)+30*
a^4*b^6*x^7+504/13*a^5*b^5*x^(13/2)+35*a^6*b^4*x^6+240/11*a^7*b^3*x^(11/2)
+9*a^8*b^2*x^5+20/9*a^9*b*x^(9/2)+1/4*a^10*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt{x}\right)^{10}x^3\,dx=\frac{1}{9}\,b^{10}x^9+\frac{45}{8}\,a^2b^8x^8+30\,a^4b^6x^7+35\,a^6b^4x^6+9\,a^8b^2x^5+\frac{1}{4}\,a^{10}x^4+\frac{4}{21879}\left(6435\,ab^9x^8+87516\,a^3b^7x^7+212058\,a^5b^5x^6+119340\,a^7b^3x^5+12155\,a^9bx^4\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^10*x^3,x, algorithm="fricas")
```

output
```
1/9*b^10*x^9 + 45/8*a^2*b^8*x^8 + 30*a^4*b^6*x^7 + 35*a^6*b^4*x^6 + 9*a^8*
b^2*x^5 + 1/4*a^10*x^4 + 4/21879*(6435*a*b^9*x^8 + 87516*a^3*b^7*x^7 + 212
058*a^5*b^5*x^6 + 119340*a^7*b^3*x^5 + 12155*a^9*b*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx = \frac{a^{10}x^4}{4} + \frac{20a^9bx^{\frac{9}{2}}}{9} + 9a^8b^2x^5 + \frac{240a^7b^3x^{\frac{11}{2}}}{11} + 35a^6b^4x^6 + \frac{504a^5b^5x^{\frac{13}{2}}}{13} + 30a^4b^6x^7 + 16a^3b^7x^{\frac{15}{2}} + \frac{45a^2b^8x^8}{8} + \frac{20ab^9x^{\frac{17}{2}}}{17} + \frac{b^{10}x^9}{9}$$

input
```
integrate((a+b*x**(1/2))**10*x**3,x)
```

output
```
a**10*x**4/4 + 20*a**9*b*x**(9/2)/9 + 9*a**8*b**2*x**5 + 240*a**7*b**3*x**
(11/2)/11 + 35*a**6*b**4*x**6 + 504*a**5*b**5*x**(13/2)/13 + 30*a**4*b**6*
x**7 + 16*a**3*b**7*x**(15/2) + 45*a**2*b**8*x**8/8 + 20*a*b**9*x**(17/2)/
17 + b**10*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx = \frac{\left(b\sqrt{x}+a\right)^{18}}{9\,b^8} - \frac{14\left(b\sqrt{x}+a\right)^{17}a}{17\,b^8} + \frac{21\left(b\sqrt{x}+a\right)^{16}a^2}{8\,b^8} - \frac{14\left(b\sqrt{x}+a\right)^{15}a^3}{3\,b^8} + \frac{5\left(b\sqrt{x}+a\right)^{14}a^4}{b^8} - \frac{42\left(b\sqrt{x}+a\right)^{13}a^5}{13\,b^8} + \frac{7\left(b\sqrt{x}+a\right)^{12}a^6}{6\,b^8} - \frac{2\left(b\sqrt{x}+a\right)^{11}a^7}{11\,b^8}$$

input
```
integrate((a+b*x^(1/2))^10*x^3,x, algorithm="maxima")
```

output
```
1/9*(b*sqrt(x) + a)^18/b^8 - 14/17*(b*sqrt(x) + a)^17*a/b^8 + 21/8*(b*sqrt
(x) + a)^16*a^2/b^8 - 14/3*(b*sqrt(x) + a)^15*a^3/b^8 + 5*(b*sqrt(x) + a)^
14*a^4/b^8 - 42/13*(b*sqrt(x) + a)^13*a^5/b^8 + 7/6*(b*sqrt(x) + a)^12*a^6
/b^8 - 2/11*(b*sqrt(x) + a)^11*a^7/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.69

$$\int (a+b\sqrt{x})^{10} x^3\,dx = \frac{1}{9}\,b^{10}x^9 + \frac{20}{17}\,ab^9x^{\frac{17}{2}} + \frac{45}{8}\,a^2b^8x^8 + 16\,a^3b^7x^{\frac{15}{2}} + 30\,a^4b^6x^7 + \frac{504}{13}\,a^5b^5x^{\frac{13}{2}} + 35\,a^6b^4x^6 + \frac{240}{11}\,a^7b^3x^{\frac{11}{2}} + 9\,a^8b^2x^5 + \frac{20}{9}\,a^9bx^{\frac{9}{2}} + \frac{1}{4}\,a^{10}x^4$$

input
```
integrate((a+b*x^(1/2))^10*x^3,x, algorithm="giac")
```

output
```
1/9*b^10*x^9 + 20/17*a*b^9*x^(17/2) + 45/8*a^2*b^8*x^8 + 16*a^3*b^7*x^(15/
2) + 30*a^4*b^6*x^7 + 504/13*a^5*b^5*x^(13/2) + 35*a^6*b^4*x^6 + 240/11*a^
7*b^3*x^(11/2) + 9*a^8*b^2*x^5 + 20/9*a^9*b*x^(9/2) + 1/4*a^10*x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.69

$$\int (a+b\sqrt{x})^{10} x^3\,dx = \frac{a^{10}\,x^4}{4} + \frac{b^{10}\,x^9}{9} + \frac{20\,a^9\,b\,x^{9/2}}{9} + \frac{20\,a\,b^9\,x^{17/2}}{17} + 9\,a^8\,b^2\,x^5 + 35\,a^6\,b^4\,x^6 + 30\,a^4\,b^6\,x^7 + \frac{45\,a^2\,b^8\,x^8}{8} + \frac{240\,a^7\,b^3\,x^{11/2}}{11} + \frac{504\,a^5\,b^5\,x^{13/2}}{13} + 16\,a^3\,b^7\,x^{15/2}$$

input
```
int(x^3*(a + b*x^(1/2))^10,x)
```

output
```
(a^10*x^4)/4 + (b^10*x^9)/9 + (20*a^9*b*x^(9/2))/9 + (20*a*b^9*x^(17/2))/1
7 + 9*a^8*b^2*x^5 + 35*a^6*b^4*x^6 + 30*a^4*b^6*x^7 + (45*a^2*b^8*x^8)/8 +
 (240*a^7*b^3*x^(11/2))/11 + (504*a^5*b^5*x^(13/2))/13 + 16*a^3*b^7*x^(15/
2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^{10} x^3\,dx$$
$$= \frac{x^4\left(388960\sqrt{x}\,a^9b+3818880\sqrt{x}\,a^7b^3x+6785856\sqrt{x}\,a^5b^5x^2+2800512\sqrt{x}\,a^3b^7x^3+205920\sqrt{x}\,a\,b^9x^4+4\right.}{175032}$$

input
```
int((a+b*x^(1/2))^10*x^3,x)
```

output
```
(x**4*(388960*sqrt(x)*a**9*b + 3818880*sqrt(x)*a**7*b**3*x + 6785856*sqrt(
x)*a**5*b**5*x**2 + 2800512*sqrt(x)*a**3*b**7*x**3 + 205920*sqrt(x)*a*b**9
*x**4 + 43758*a**10 + 1575288*a**8*b**2*x + 6126120*a**6*b**4*x**2 + 52509
60*a**4*b**6*x**3 + 984555*a**2*b**8*x**4 + 19448*b**10*x**5))/175032
```

3.45 $\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx = -\frac{2a^5\left(a+b\sqrt{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt{x}\right)^{12}}{6b^6}-\frac{20a^3\left(a+b\sqrt{x}\right)^{13}}{13b^6}+\frac{10a^2\left(a+b\sqrt{x}\right)^{14}}{7b^6}-\frac{2a\left(a+b\sqrt{x}\right)^{15}}{3b^6}+\frac{\left(a+b\sqrt{x}\right)^{16}}{8b^6}$$

output
```
-2/11*a^5*(a+b*x^(1/2))^11/b^6+5/6*a^4*(a+b*x^(1/2))^12/b^6-20/13*a^3*(a+b
*x^(1/2))^13/b^6+10/7*a^2*(a+b*x^(1/2))^14/b^6-2/3*a*(a+b*x^(1/2))^15/b^6+
1/8*(a+b*x^(1/2))^16/b^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx = \frac{8008a^{10}x^3+68640a^9bx^{7/2}+270270a^8b^2x^4+640640a^7b^3x^{9/2}+1009008a^6b^4x^5+1100736a^5b^5x^{11/2}+84}{24024}$$

input
```
Integrate[(a + b*Sqrt[x])^10*x^2,x]
```

output
```
(8008*a^10*x^3 + 68640*a^9*b*x^(7/2) + 270270*a^8*b^2*x^4 + 640640*a^7*b^3
*x^(9/2) + 1009008*a^6*b^4*x^5 + 1100736*a^5*b^5*x^(11/2) + 840840*a^4*b^6
*x^6 + 443520*a^3*b^7*x^(13/2) + 154440*a^2*b^8*x^7 + 32032*a*b^9*x^(15/2)
 + 3003*b^10*x^8)/24024
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{x}\right)^{10}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^{10}x^{5/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{15}}{b^5}-\frac{5a\left(a+b\sqrt{x}\right)^{14}}{b^5}+\frac{10a^2\left(a+b\sqrt{x}\right)^{13}}{b^5}-\frac{10a^3\left(a+b\sqrt{x}\right)^{12}}{b^5}+\frac{5a^4\left(a+b\sqrt{x}\right)^{11}}{b^5}-\frac{a^5\left(a+b\sqrt{x}\right)}{b^5}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^5\left(a+b\sqrt{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt{x}\right)^{12}}{12b^6}-\frac{10a^3\left(a+b\sqrt{x}\right)^{13}}{13b^6}+\frac{5a^2\left(a+b\sqrt{x}\right)^{14}}{7b^6}+\frac{\left(a+b\sqrt{x}\right)^{16}}{16b^6}-\frac{a\left(a+b\sqrt{x}\right)^{15}}{3b^6}\right.$$

input
```
Int[(a + b*Sqrt[x])^10*x^2,x]
```

output
```
2*(-1/11*(a^5*(a + b*Sqrt[x])^11)/b^6 + (5*a^4*(a + b*Sqrt[x])^12)/(12*b^6
) - (10*a^3*(a + b*Sqrt[x])^13)/(13*b^6) + (5*a^2*(a + b*Sqrt[x])^14)/(7*b
^6) - (a*(a + b*Sqrt[x])^15)/(3*b^6) + (a + b*Sqrt[x])^16/(16*b^6))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{b^{10}x^8}{8}+\frac{4a\,b^9x^{\frac{15}{2}}}{3}+\frac{45a^2b^8x^7}{7}+\frac{240a^3b^7x^{\frac{13}{2}}}{13}+35a^4b^6x^6+\frac{504a^5b^5x^{\frac{11}{2}}}{11}+42a^6b^4x^5+\frac{80a^7b^3x^{\frac{9}{2}}}{3}+$
default	$\frac{b^{10}x^8}{8}+\frac{4a\,b^9x^{\frac{15}{2}}}{3}+\frac{45a^2b^8x^7}{7}+\frac{240a^3b^7x^{\frac{13}{2}}}{13}+35a^4b^6x^6+\frac{504a^5b^5x^{\frac{11}{2}}}{11}+42a^6b^4x^5+\frac{80a^7b^3x^{\frac{9}{2}}}{3}+$
orering	$\frac{(-29029b^{24}x^{12}+267300a^2b^{22}x^{11}-1098750a^4b^{20}x^{10}+2650060a^6b^{18}x^9-4141935a^8b^{16}x^8+4365288a^{10}b^{14}x^7-312018}{120120b^6(-b^2x}$
trager	$\frac{(21b^{10}x^7+1080a^2b^8x^6+21b^{10}x^6+5880a^4b^6x^5+1080a^2b^8x^5+21b^{10}x^5+7056a^6b^4x^4+5880a^4b^6x^4+1080a^2b^8x^4+21b^{10}}{}$

input
```
int((a+b*x^(1/2))^10*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^10*x^8+4/3*a*b^9*x^(15/2)+45/7*a^2*b^8*x^7+240/13*a^3*b^7*x^(13/2)+3
5*a^4*b^6*x^6+504/11*a^5*b^5*x^(11/2)+42*a^6*b^4*x^5+80/3*a^7*b^3*x^(9/2)+
45/4*a^8*b^2*x^4+20/7*a^9*b*x^(7/2)+1/3*a^10*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx = \frac{1}{8}\,b^{10}x^8 + \frac{45}{7}\,a^2b^8x^7 + 35\,a^4b^6x^6 + 42\,a^6b^4x^5 + \frac{45}{4}\,a^8b^2x^4 + \frac{1}{3}\,a^{10}x^3 + \frac{4}{3003}\left(1001\,ab^9x^7 + 13860\,a^3b^7x^6 + 34398\,a^5b^5x^5 + 20020\,a^7b^3x^4 + 2145\,a^9bx^3\right)\sqrt{x}$$

input

```
integrate((a+b*x^(1/2))^10*x^2,x, algorithm="fricas")
```

output

```
1/8*b^10*x^8 + 45/7*a^2*b^8*x^7 + 35*a^4*b^6*x^6 + 42*a^6*b^4*x^5 + 45/4*a
^8*b^2*x^4 + 1/3*a^10*x^3 + 4/3003*(1001*a*b^9*x^7 + 13860*a^3*b^7*x^6 + 3
4398*a^5*b^5*x^5 + 20020*a^7*b^3*x^4 + 2145*a^9*b*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx = \frac{a^{10}x^3}{3} + \frac{20a^9bx^{\frac{7}{2}}}{7} + \frac{45a^8b^2x^4}{4} + \frac{80a^7b^3x^{\frac{9}{2}}}{3} + 42a^6b^4x^5 + \frac{504a^5b^5x^{\frac{11}{2}}}{11} + 35a^4b^6x^6 + \frac{240a^3b^7x^{\frac{13}{2}}}{13} + \frac{45a^2b^8x^7}{7} + \frac{4ab^9x^{\frac{15}{2}}}{3} + \frac{b^{10}x^8}{8}$$

input

```
integrate((a+b*x**(1/2))**10*x**2,x)
```

output

```
a**10*x**3/3 + 20*a**9*b*x**(7/2)/7 + 45*a**8*b**2*x**4/4 + 80*a**7*b**3*x
**(9/2)/3 + 42*a**6*b**4*x**5 + 504*a**5*b**5*x**(11/2)/11 + 35*a**4*b**6*
x**6 + 240*a**3*b**7*x**(13/2)/13 + 45*a**2*b**8*x**7/7 + 4*a*b**9*x**(15/
2)/3 + b**10*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^{10}x^2\,dx=\frac{\left(b\sqrt{x}+a\right)^{16}}{8\,b^6}-\frac{2\left(b\sqrt{x}+a\right)^{15}a}{3\,b^6}+\frac{10\left(b\sqrt{x}+a\right)^{14}a^2}{7\,b^6}-\frac{20\left(b\sqrt{x}+a\right)^{13}a^3}{13\,b^6}+\frac{5\left(b\sqrt{x}+a\right)^{12}a^4}{6\,b^6}-\frac{2\left(b\sqrt{x}+a\right)^{11}a^5}{11\,b^6}$$

input
```
integrate((a+b*x^(1/2))^10*x^2,x, algorithm="maxima")
```

output
```
1/8*(b*sqrt(x) + a)^16/b^6 - 2/3*(b*sqrt(x) + a)^15*a/b^6 + 10/7*(b*sqrt(x
) + a)^14*a^2/b^6 - 20/13*(b*sqrt(x) + a)^13*a^3/b^6 + 5/6*(b*sqrt(x) + a)
^12*a^4/b^6 - 2/11*(b*sqrt(x) + a)^11*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt{x}\right)^{10}x^2\,dx=\frac{1}{8}\,b^{10}x^8+\frac{4}{3}\,ab^9x^{\frac{15}{2}}+\frac{45}{7}\,a^2b^8x^7+\frac{240}{13}\,a^3b^7x^{\frac{13}{2}}+35\,a^4b^6x^6+\frac{504}{11}\,a^5b^5x^{\frac{11}{2}}+42\,a^6b^4x^5+\frac{80}{3}\,a^7b^3x^{\frac{9}{2}}+\frac{45}{4}\,a^8b^2x^4+\frac{20}{7}\,a^9bx^{\frac{7}{2}}+\frac{1}{3}\,a^{10}x^3$$

input
```
integrate((a+b*x^(1/2))^10*x^2,x, algorithm="giac")
```

output
```
1/8*b^10*x^8 + 4/3*a*b^9*x^(15/2) + 45/7*a^2*b^8*x^7 + 240/13*a^3*b^7*x^(1
3/2) + 35*a^4*b^6*x^6 + 504/11*a^5*b^5*x^(11/2) + 42*a^6*b^4*x^5 + 80/3*a^
7*b^3*x^(9/2) + 45/4*a^8*b^2*x^4 + 20/7*a^9*b*x^(7/2) + 1/3*a^10*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx = \frac{a^{10}\,x^3}{3}+\frac{b^{10}\,x^8}{8}+\frac{20\,a^9\,b\,x^{7/2}}{7}+\frac{4\,a\,b^9\,x^{15/2}}{3}+\frac{45\,a^8\,b^2\,x^4}{4}+42\,a^6\,b^4\,x^5+35\,a^4\,b^6\,x^6+\frac{45\,a^2\,b^8\,x^7}{7}+\frac{80\,a^7\,b^3\,x^{9/2}}{3}+\frac{504\,a^5\,b^5\,x^{11/2}}{11}+\frac{240\,a^3\,b^7\,x^{13/2}}{13}$$

input
```
int(x^2*(a + b*x^(1/2))^10,x)
```

output
```
(a^10*x^3)/3 + (b^10*x^8)/8 + (20*a^9*b*x^(7/2))/7 + (4*a*b^9*x^(15/2))/3
+ (45*a^8*b^2*x^4)/4 + 42*a^6*b^4*x^5 + 35*a^4*b^6*x^6 + (45*a^2*b^8*x^7)/
7 + (80*a^7*b^3*x^(9/2))/3 + (504*a^5*b^5*x^(11/2))/11 + (240*a^3*b^7*x^(1
3/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}\right)^{10} x^2\,dx = \frac{x^3\left(68640\sqrt{x}\,a^9b+640640\sqrt{x}\,a^7b^3x+1100736\sqrt{x}\,a^5b^5x^2+443520\sqrt{x}\,a^3b^7x^3+32032\sqrt{x}\,a\,b^9x^4+8008a\right.}{24024}$$

input
```
int((a+b*x^(1/2))^10*x^2,x)
```

output
```
(x**3*(68640*sqrt(x)*a**9*b + 640640*sqrt(x)*a**7*b**3*x + 1100736*sqrt(x)
*a**5*b**5*x**2 + 443520*sqrt(x)*a**3*b**7*x**3 + 32032*sqrt(x)*a*b**9*x**
4 + 8008*a**10 + 270270*a**8*b**2*x + 1009008*a**6*b**4*x**2 + 840840*a**4
*b**6*x**3 + 154440*a**2*b**8*x**4 + 3003*b**10*x**5))/24024
```

3.46 $\int \left(a+b\sqrt{x}\right)^{10} x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \left(a+b\sqrt{x}\right)^{10} x\,dx=-\frac{2a^3\left(a+b\sqrt{x}\right)^{11}}{11b^4}+\frac{a^2\left(a+b\sqrt{x}\right)^{12}}{2b^4}-\frac{6a\left(a+b\sqrt{x}\right)^{13}}{13b^4}+\frac{\left(a+b\sqrt{x}\right)^{14}}{7b^4}$$

output
```
-2/11*a^3*(a+b*x^(1/2))^11/b^4+1/2*a^2*(a+b*x^(1/2))^12/b^4-6/13*a*(a+b*x^
(1/2))^13/b^4+1/7*(a+b*x^(1/2))^14/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \left(a+b\sqrt{x}\right)^{10} x\,dx$$
$$=\frac{1001a^{10}x^2+8008a^9bx^{5/2}+30030a^8b^2x^3+68640a^7b^3x^{7/2}+105105a^6b^4x^4+112112a^5b^5x^{9/2}+84084a^4b}{2002}$$

input
```
Integrate[(a + b*Sqrt[x])^10*x,x]
```

output
```
(1001*a^10*x^2 + 8008*a^9*b*x^(5/2) + 30030*a^8*b^2*x^3 + 68640*a^7*b^3*x^
(7/2) + 105105*a^6*b^4*x^4 + 112112*a^5*b^5*x^(9/2) + 84084*a^4*b^6*x^5 +
43680*a^3*b^7*x^(11/2) + 15015*a^2*b^8*x^6 + 3080*a*b^9*x^(13/2) + 286*b^1
0*x^7)/2002
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}\right)^{10}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^{10}x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{13}}{b^3}-\frac{3a\left(a+b\sqrt{x}\right)^{12}}{b^3}+\frac{3a^2\left(a+b\sqrt{x}\right)^{11}}{b^3}-\frac{a^3\left(a+b\sqrt{x}\right)^{10}}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^3\left(a+b\sqrt{x}\right)^{11}}{11b^4}+\frac{a^2\left(a+b\sqrt{x}\right)^{12}}{4b^4}+\frac{\left(a+b\sqrt{x}\right)^{14}}{14b^4}-\frac{3a\left(a+b\sqrt{x}\right)^{13}}{13b^4}\right)$$

input
```
Int[(a + b*Sqrt[x])^10*x,x]
```

output
```
2*(-1/11*(a^3*(a + b*Sqrt[x])^11)/b^4 + (a^2*(a + b*Sqrt[x])^12)/(4*b^4) -
 (3*a*(a + b*Sqrt[x])^13)/(13*b^4) + (a + b*Sqrt[x])^14/(14*b^4))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{b^{10}x^7}{7}+\frac{20a\,b^9x^{\frac{13}{2}}}{13}+\frac{15a^2b^8x^6}{2}+\frac{240a^3b^7x^{\frac{11}{2}}}{11}+42a^4b^6x^5+56a^5b^5x^{\frac{9}{2}}+\frac{105a^6b^4x^4}{2}+\frac{240a^7b^3x^{\frac{7}{2}}}{7}$
default	$\frac{b^{10}x^7}{7}+\frac{20a\,b^9x^{\frac{13}{2}}}{13}+\frac{15a^2b^8x^6}{2}+\frac{240a^3b^7x^{\frac{11}{2}}}{11}+42a^4b^6x^5+56a^5b^5x^{\frac{9}{2}}+\frac{105a^6b^4x^4}{2}+\frac{240a^7b^3x^{\frac{7}{2}}}{7}$
orering	$-\frac{(550b^{22}x^{11}-5037a^2b^{20}x^{10}+20580a^4b^{18}x^9-49305a^6b^{16}x^8+76500a^8b^{14}x^7-80010a^{10}b^{12}x^6+56784a^{12}b^{10}x^5+2702}{2002b^4(-b^2x+a^2)^9}$
trager	$(2b^{10}x^6+105a^2b^8x^5+2b^{10}x^5+588a^4b^6x^4+105a^2b^8x^4+2b^{10}x^4+735a^6b^4x^3+588a^4b^6x^3+105a^2b^8x^3+2b^{10}x^3+210a^8b$

input
```
int((a+b*x^(1/2))^10*x,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^10*x^7+20/13*a*b^9*x^(13/2)+15/2*a^2*b^8*x^6+240/11*a^3*b^7*x^(11/2)
+42*a^4*b^6*x^5+56*a^5*b^5*x^(9/2)+105/2*a^6*b^4*x^4+240/7*a^7*b^3*x^(7/2)
+15*a^8*b^2*x^3+4*a^9*b*x^(5/2)+1/2*a^10*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \left(a+b\sqrt{x}\right)^{10} x\,dx = \frac{1}{7}\,b^{10}x^7 + \frac{15}{2}\,a^2b^8x^6 + 42\,a^4b^6x^5 + \frac{105}{2}\,a^6b^4x^4 + 15\,a^8b^2x^3 + \frac{1}{2}\,a^{10}x^2 + \frac{4}{1001}\left(385\,ab^9x^6 + 5460\,a^3b^7x^5 + 14014\,a^5b^5x^4 + 8580\,a^7b^3x^3 + 1001\,a^9bx^2\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^10*x,x, algorithm="fricas")
```

output
```
1/7*b^10*x^7 + 15/2*a^2*b^8*x^6 + 42*a^4*b^6*x^5 + 105/2*a^6*b^4*x^4 + 15*
a^8*b^2*x^3 + 1/2*a^10*x^2 + 4/1001*(385*a*b^9*x^6 + 5460*a^3*b^7*x^5 + 14
014*a^5*b^5*x^4 + 8580*a^7*b^3*x^3 + 1001*a^9*b*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70

$$\int \left(a+b\sqrt{x}\right)^{10} x\,dx = \frac{a^{10}x^2}{2} + 4a^9bx^{\frac{5}{2}} + 15a^8b^2x^3 + \frac{240a^7b^3x^{\frac{7}{2}}}{7} + \frac{105a^6b^4x^4}{2} + 56a^5b^5x^{\frac{9}{2}} + 42a^4b^6x^5 + \frac{240a^3b^7x^{\frac{11}{2}}}{11} + \frac{15a^2b^8x^6}{2} + \frac{20ab^9x^{\frac{13}{2}}}{13} + \frac{b^{10}x^7}{7}$$

input
```
integrate((a+b*x**(1/2))**10*x,x)
```

output
```
a**10*x**2/2 + 4*a**9*b*x**(5/2) + 15*a**8*b**2*x**3 + 240*a**7*b**3*x**(7
/2)/7 + 105*a**6*b**4*x**4/2 + 56*a**5*b**5*x**(9/2) + 42*a**4*b**6*x**5 +
 240*a**3*b**7*x**(11/2)/11 + 15*a**2*b**8*x**6/2 + 20*a*b**9*x**(13/2)/13
 + b**10*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^{10} x\,dx = \frac{\left(b\sqrt{x}+a\right)^{14}}{7\,b^4} - \frac{6\left(b\sqrt{x}+a\right)^{13}a}{13\,b^4} + \frac{\left(b\sqrt{x}+a\right)^{12}a^2}{2\,b^4} - \frac{2\left(b\sqrt{x}+a\right)^{11}a^3}{11\,b^4}$$

input
```
integrate((a+b*x^(1/2))^10*x,x, algorithm="maxima")
```

output
```
1/7*(b*sqrt(x) + a)^14/b^4 - 6/13*(b*sqrt(x) + a)^13*a/b^4 + 1/2*(b*sqrt(x
) + a)^12*a^2/b^4 - 2/11*(b*sqrt(x) + a)^11*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{10} x\,dx = {} & \frac{1}{7}\,b^{10}x^7 + \frac{20}{13}\,ab^9x^{\frac{13}{2}} + \frac{15}{2}\,a^2b^8x^6 + \frac{240}{11}\,a^3b^7x^{\frac{11}{2}} \\ & + 42\,a^4b^6x^5 + 56\,a^5b^5x^{\frac{9}{2}} + \frac{105}{2}\,a^6b^4x^4 \\ & + \frac{240}{7}\,a^7b^3x^{\frac{7}{2}} + 15\,a^8b^2x^3 + 4\,a^9bx^{\frac{5}{2}} + \frac{1}{2}\,a^{10}x^2\end{aligned}$$

input
```
integrate((a+b*x^(1/2))^10*x,x, algorithm="giac")
```

output
```
1/7*b^10*x^7 + 20/13*a*b^9*x^(13/2) + 15/2*a^2*b^8*x^6 + 240/11*a^3*b^7*x^
(11/2) + 42*a^4*b^6*x^5 + 56*a^5*b^5*x^(9/2) + 105/2*a^6*b^4*x^4 + 240/7*a
^7*b^3*x^(7/2) + 15*a^8*b^2*x^3 + 4*a^9*b*x^(5/2) + 1/2*a^10*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\int \left(a+b\sqrt{x}\right)^{10} x\,dx = \frac{a^{10}\,x^2}{2}+\frac{b^{10}\,x^7}{7}+4\,a^9\,b\,x^{5/2}+\frac{20\,a\,b^9\,x^{13/2}}{13}+15\,a^8\,b^2\,x^3+\frac{105\,a^6\,b^4\,x^4}{2}+42\,a^4\,b^6\,x^5+\frac{15\,a^2\,b^8\,x^6}{2}+\frac{240\,a^7\,b^3\,x^{7/2}}{7}+56\,a^5\,b^5\,x^{9/2}+\frac{240\,a^3\,b^7\,x^{11/2}}{11}$$

input
```
int(x*(a + b*x^(1/2))^10,x)
```

output
```
(a^10*x^2)/2 + (b^10*x^7)/7 + 4*a^9*b*x^(5/2) + (20*a*b^9*x^(13/2))/13 + 1
5*a^8*b^2*x^3 + (105*a^6*b^4*x^4)/2 + 42*a^4*b^6*x^5 + (15*a^2*b^8*x^6)/2
+ (240*a^7*b^3*x^(7/2))/7 + 56*a^5*b^5*x^(9/2) + (240*a^3*b^7*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int \left(a+b\sqrt{x}\right)^{10} x\,dx = \frac{x^2\big(8008\sqrt{x}\,a^9b+68640\sqrt{x}\,a^7b^3x+112112\sqrt{x}\,a^5b^5x^2+43680\sqrt{x}\,a^3b^7x^3+3080\sqrt{x}\,a\,b^9x^4+1001a^{10}+3}{2002}$$

input
```
int((a+b*x^(1/2))^10*x,x)
```

output
```
(x**2*(8008*sqrt(x)*a**9*b + 68640*sqrt(x)*a**7*b**3*x + 112112*sqrt(x)*a*
*5*b**5*x**2 + 43680*sqrt(x)*a**3*b**7*x**3 + 3080*sqrt(x)*a*b**9*x**4 + 1
001*a**10 + 30030*a**8*b**2*x + 105105*a**6*b**4*x**2 + 84084*a**4*b**6*x*
*3 + 15015*a**2*b**8*x**4 + 286*b**10*x**5))/2002
```

3.47 $\int \left(a+b\sqrt{x}\right)^{10} dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \left(a+b\sqrt{x}\right)^{10} dx = -\frac{2a\left(a+b\sqrt{x}\right)^{11}}{11b^2}+\frac{\left(a+b\sqrt{x}\right)^{12}}{6b^2}$$

output
```
-2/11*a*(a+b*x^(1/2))^11/b^2+1/6*(a+b*x^(1/2))^12/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(38) = 76.

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.26

$$\int \left(a+b\sqrt{x}\right)^{10} dx = \frac{1}{66}\left(66a^{10}x+440a^9bx^{3/2}+1485a^8b^2x^2+3168a^7b^3x^{5/2}+4620a^6b^4x^3+4752a^5b^5x^{7/2}+3465a^4b^6x^4+1760a^3b^7x^{9/2}+594a^2b^8x^5+120ab^9x^{11/2}+11b^{10}x^6\right)$$

input
```
Integrate[(a + b*Sqrt[x])^10,x]
```

output
```
(66*a^10*x + 440*a^9*b*x^(3/2) + 1485*a^8*b^2*x^2 + 3168*a^7*b^3*x^(5/2) +
 4620*a^6*b^4*x^3 + 4752*a^5*b^5*x^(7/2) + 3465*a^4*b^6*x^4 + 1760*a^3*b^7
*x^(9/2) + 594*a^2*b^8*x^5 + 120*a*b^9*x^(11/2) + 11*b^10*x^6)/66
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^{10} dx$$

$$\Big\downarrow \text{774}$$

$$2\int \left(a+b\sqrt{x}\right)^{10}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{11}}{b}-\frac{a\left(a+b\sqrt{x}\right)^{10}}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\left(a+b\sqrt{x}\right)^{12}}{12b^2}-\frac{a\left(a+b\sqrt{x}\right)^{11}}{11b^2}\right)$$

input
```
Int[(a + b*Sqrt[x])^10,x]
```

output
```
2*(-1/11*(a*(a + b*Sqrt[x])^11)/b^2 + (a + b*Sqrt[x])^12/(12*b^2))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(30) = 60$.

Time = 3.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.89

method	result
derivativedivides	$\frac{b^{10}x^6}{6}+\frac{20a\,b^9x^{\frac{11}{2}}}{11}+9a^2b^8x^5+\frac{80a^3b^7x^{\frac{9}{2}}}{3}+\frac{105a^4b^6x^4}{2}+72a^5b^5x^{\frac{7}{2}}+70a^6b^4x^3+48a^7b^3x^{\frac{5}{2}}+$
default	$\frac{b^{10}x^6}{6}+\frac{20a\,b^9x^{\frac{11}{2}}}{11}+9a^2b^8x^5+\frac{80a^3b^7x^{\frac{9}{2}}}{3}+\frac{105a^4b^6x^4}{2}+72a^5b^5x^{\frac{7}{2}}+70a^6b^4x^3+48a^7b^3x^{\frac{5}{2}}+$
trager	$\frac{(b^{10}x^5+54a^2b^8x^4+b^{10}x^4+315a^4b^6x^3+54a^2b^8x^3+b^{10}x^3+420a^6b^4x^2+315a^4b^6x^2+54a^2b^8x^2+b^{10}x^2+135a^8b^2x+420}{6}$
orering	$\frac{(-21b^{20}x^{10}+190a^2b^{18}x^9-765a^4b^{16}x^8+1800a^6b^{14}x^7-2730a^8b^{12}x^6+2772a^{10}b^{10}x^5+18480a^{14}b^6x^3+26235a^{16}b^4x^2+}{66b^2(-b^2x+a^2)^9}$

input
```
int((a+b*x^(1/2))^10,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^10*x^6+20/11*a*b^9*x^(11/2)+9*a^2*b^8*x^5+80/3*a^3*b^7*x^(9/2)+105/2
*a^4*b^6*x^4+72*a^5*b^5*x^(7/2)+70*a^6*b^4*x^3+48*a^7*b^3*x^(5/2)+45/2*a^8
*b^2*x^2+20/3*a^9*b*x^(3/2)+a^10*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.97

$$\int \left(a+b\sqrt{x}\right)^{10} dx = \frac{1}{6}b^{10}x^6 + 9a^2b^8x^5 + \frac{105}{2}a^4b^6x^4 + 70a^6b^4x^3 + \frac{45}{2}a^8b^2x^2 + a^{10}x + \frac{4}{33}\left(15ab^9x^5 + 220a^3b^7x^4 + 594a^5b^5x^3 + 396a^7b^3x^2 + 55a^9bx\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^10,x, algorithm="fricas")
```

output
```
1/6*b^10*x^6 + 9*a^2*b^8*x^5 + 105/2*a^4*b^6*x^4 + 70*a^6*b^4*x^3 + 45/2*a
^8*b^2*x^2 + a^10*x + 4/33*(15*a*b^9*x^5 + 220*a^3*b^7*x^4 + 594*a^5*b^5*x
^3 + 396*a^7*b^3*x^2 + 55*a^9*b*x)*sqrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(32) = 64$.

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.50

$$\int \left(a+b\sqrt{x}\right)^{10} dx = a^{10}x + \frac{20a^9bx^{\frac{3}{2}}}{3} + \frac{45a^8b^2x^2}{2} + 48a^7b^3x^{\frac{5}{2}} + 70a^6b^4x^3 + 72a^5b^5x^{\frac{7}{2}} + \frac{105a^4b^6x^4}{2} + \frac{80a^3b^7x^{\frac{9}{2}}}{3} + 9a^2b^8x^5 + \frac{20ab^9x^{\frac{11}{2}}}{11} + \frac{b^{10}x^6}{6}$$

input
```
integrate((a+b*x**(1/2))**10,x)
```

output
```
a**10*x + 20*a**9*b*x**(3/2)/3 + 45*a**8*b**2*x**2/2 + 48*a**7*b**3*x**(5/
2) + 70*a**6*b**4*x**3 + 72*a**5*b**5*x**(7/2) + 105*a**4*b**6*x**4/2 + 80
*a**3*b**7*x**(9/2)/3 + 9*a**2*b**8*x**5 + 20*a*b**9*x**(11/2)/11 + b**10*
x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)^{10}\,dx=\frac{\left(b\sqrt{x}+a\right)^{12}}{6\,b^2}-\frac{2\,\left(b\sqrt{x}+a\right)^{11}a}{11\,b^2}$$

input
```
integrate((a+b*x^(1/2))^10,x, algorithm="maxima")
```

output
```
1/6*(b*sqrt(x) + a)^12/b^2 - 2/11*(b*sqrt(x) + a)^11*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(30) = 60.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int \left(a+b\sqrt{x}\right)^{10}\,dx=\frac{1}{6}\,b^{10}x^6+\frac{20}{11}\,ab^9x^{\frac{11}{2}}+9\,a^2b^8x^5+\frac{80}{3}\,a^3b^7x^{\frac{9}{2}}+\frac{105}{2}\,a^4b^6x^4+72\,a^5b^5x^{\frac{7}{2}}+70\,a^6b^4x^3+48\,a^7b^3x^{\frac{5}{2}}+\frac{45}{2}\,a^8b^2x^2+\frac{20}{3}\,a^9bx^{\frac{3}{2}}+a^{10}x$$

input
```
integrate((a+b*x^(1/2))^10,x, algorithm="giac")
```

output
```
1/6*b^10*x^6 + 20/11*a*b^9*x^(11/2) + 9*a^2*b^8*x^5 + 80/3*a^3*b^7*x^(9/2)
 + 105/2*a^4*b^6*x^4 + 72*a^5*b^5*x^(7/2) + 70*a^6*b^4*x^3 + 48*a^7*b^3*x^
(5/2) + 45/2*a^8*b^2*x^2 + 20/3*a^9*b*x^(3/2) + a^10*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int \left(a+b\sqrt{x}\right)^{10}\,dx=a^{10}\,x+\frac{b^{10}\,x^6}{6}+\frac{20\,a^9\,b\,x^{3/2}}{3}+\frac{20\,a\,b^9\,x^{11/2}}{11}+\frac{45\,a^8\,b^2\,x^2}{2}+70\,a^6\,b^4\,x^3+\frac{105\,a^4\,b^6\,x^4}{2}+9\,a^2\,b^8\,x^5+48\,a^7\,b^3\,x^{5/2}+72\,a^5\,b^5\,x^{7/2}+\frac{80\,a^3\,b^7\,x^{9/2}}{3}$$

input
```
int((a + b*x^(1/2))^10,x)
```

output
```
a^10*x + (b^10*x^6)/6 + (20*a^9*b*x^(3/2))/3 + (20*a*b^9*x^(11/2))/11 + (4
5*a^8*b^2*x^2)/2 + 70*a^6*b^4*x^3 + (105*a^4*b^6*x^4)/2 + 9*a^2*b^8*x^5 +
48*a^7*b^3*x^(5/2) + 72*a^5*b^5*x^(7/2) + (80*a^3*b^7*x^(9/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.03

$$\int \left(a+b\sqrt{x}\right)^{10}\,dx$$
$$=\frac{x\left(440\sqrt{x}\,a^9b+3168\sqrt{x}\,a^7b^3x+4752\sqrt{x}\,a^5b^5x^2+1760\sqrt{x}\,a^3b^7x^3+120\sqrt{x}\,a\,b^9x^4+66a^{10}+1485a^8b^2x\right.}{66}$$

input
```
int((a+b*x^(1/2))^10,x)
```

output
```
(x*(440*sqrt(x)*a**9*b + 3168*sqrt(x)*a**7*b**3*x + 4752*sqrt(x)*a**5*b**5
*x**2 + 1760*sqrt(x)*a**3*b**7*x**3 + 120*sqrt(x)*a*b**9*x**4 + 66*a**10 +
 1485*a**8*b**2*x + 4620*a**6*b**4*x**2 + 3465*a**4*b**6*x**3 + 594*a**2*b
**8*x**4 + 11*b**10*x**5))/66
```

3.48 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x} \, dx = 20a^9b\sqrt{x}+45a^8b^2x+80a^7b^3x^{3/2}+105a^6b^4x^2 +\frac{504}{5}a^5b^5x^{5/2}+70a^4b^6x^3+\frac{240}{7}a^3b^7x^{7/2} +\frac{45}{4}a^2b^8x^4+\frac{20}{9}ab^9x^{9/2}+\frac{b^{10}x^5}{5}+a^{10}\log(x)$$

output
```
20*a^9*b*x^(1/2)+45*a^8*b^2*x+80*a^7*b^3*x^(3/2)+105*a^6*b^4*x^2+504/5*a^5
*b^5*x^(5/2)+70*a^4*b^6*x^3+240/7*a^3*b^7*x^(7/2)+45/4*a^2*b^8*x^4+20/9*a*
b^9*x^(9/2)+1/5*b^10*x^5+a^10*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x} \, dx = \frac{25200a^9b\sqrt{x}+56700a^8b^2x+100800a^7b^3x^{3/2}+132300a^6b^4x^2+127008a^5b^5x^{5/2}+88200a^4b^6x^3+43200}{1260} + 2a^{10}\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^10/x,x]
```

output
```
(25200*a^9*b*Sqrt[x] + 56700*a^8*b^2*x + 100800*a^7*b^3*x^(3/2) + 132300*a
^6*b^4*x^2 + 127008*a^5*b^5*x^(5/2) + 88200*a^4*b^6*x^3 + 43200*a^3*b^7*x^
(7/2) + 14175*a^2*b^8*x^4 + 2800*a*b^9*x^(9/2) + 252*b^10*x^5)/1260 + 2*a^
10*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx$$

$$\Big\downarrow \; 798$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{\sqrt{x}}\,d\sqrt{x}$$

$$\Big\downarrow \; 49$$

$$2\int \left(\frac{a^{10}}{\sqrt{x}}+10ba^9+45b^2\sqrt{x}a^8+120b^3xa^7+210b^4x^{3/2}a^6+252b^5x^2a^5+210b^6x^{5/2}a^4+120b^7x^3a^3+45b^8x^{7/2}a^2$$

$$\Big\downarrow \; 2009$$

$$2\Big(a^{10}\log\left(\sqrt{x}\right)+10a^9b\sqrt{x}+\frac{45}{2}a^8b^2x+40a^7b^3x^{3/2}+\frac{105}{2}a^6b^4x^2+\frac{252}{5}a^5b^5x^{5/2}+35a^4b^6x^3+\frac{120}{7}a^3b^7x^{7/2}+$$

input
```
Int[(a + b*Sqrt[x])^10/x,x]
```

output
```
2*(10*a^9*b*Sqrt[x] + (45*a^8*b^2*x)/2 + 40*a^7*b^3*x^(3/2) + (105*a^6*b^4
*x^2)/2 + (252*a^5*b^5*x^(5/2))/5 + 35*a^4*b^6*x^3 + (120*a^3*b^7*x^(7/2))
/7 + (45*a^2*b^8*x^4)/8 + (10*a*b^9*x^(9/2))/9 + (b^10*x^5)/10 + a^10*Log[
Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

method	result
derivativedivides	$20a^9b\sqrt{x}+45a^8b^2x+80a^7b^3x^{\frac{3}{2}}+105a^6b^4x^2+\frac{504a^5b^5x^{\frac{5}{2}}}{5}+70a^4b^6x^3+\frac{240a^3b^7x^{\frac{7}{2}}}{7}+\frac{45a$
default	$20a^9b\sqrt{x}+45a^8b^2x+80a^7b^3x^{\frac{3}{2}}+105a^6b^4x^2+\frac{504a^5b^5x^{\frac{5}{2}}}{5}+70a^4b^6x^3+\frac{240a^3b^7x^{\frac{7}{2}}}{7}+\frac{45a$
trager	$\frac{b^2(4b^8x^4+225a^2b^6x^3+4b^8x^3+1400a^4b^4x^2+225a^2b^6x^2+4b^8x^2+2100a^6b^2x+1400a^4b^4x+225a^2b^6x+4b^8x+900a^8+21}{20}$

input
```
int((a+b*x^(1/2))^10/x,x,method=_RETURNVERBOSE)
```

output
```
20*a^9*b*x^(1/2)+45*a^8*b^2*x+80*a^7*b^3*x^(3/2)+105*a^6*b^4*x^2+504/5*a^5
*b^5*x^(5/2)+70*a^4*b^6*x^3+240/7*a^3*b^7*x^(7/2)+45/4*a^2*b^8*x^4+20/9*a*
b^9*x^(9/2)+1/5*b^10*x^5+a^10*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx = \frac{1}{5}\,b^{10}x^5 + \frac{45}{4}\,a^2b^8x^4 + 70\,a^4b^6x^3 + 105\,a^6b^4x^2 + 45\,a^8b^2x + 2\,a^{10}\log\left(\sqrt{x}\right) + \frac{4}{315}\left(175\,ab^9x^4 + 2700\,a^3b^7x^3 + 7938\,a^5b^5x^2 + 6300\,a^7b^3x + 1575\,a^9b\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^10/x,x, algorithm="fricas")
```

output
```
1/5*b^10*x^5 + 45/4*a^2*b^8*x^4 + 70*a^4*b^6*x^3 + 105*a^6*b^4*x^2 + 45*a^
8*b^2*x + 2*a^10*log(sqrt(x)) + 4/315*(175*a*b^9*x^4 + 2700*a^3*b^7*x^3 +
7938*a^5*b^5*x^2 + 6300*a^7*b^3*x + 1575*a^9*b)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx = a^{10}\log\left(x\right) + 20a^9b\sqrt{x} + 45a^8b^2x + 80a^7b^3x^{\frac{3}{2}} + 105a^6b^4x^2 + \frac{504a^5b^5x^{\frac{5}{2}}}{5} + 70a^4b^6x^3 + \frac{240a^3b^7x^{\frac{7}{2}}}{7} + \frac{45a^2b^8x^4}{4} + \frac{20ab^9x^{\frac{9}{2}}}{9} + \frac{b^{10}x^5}{5}$$

input
```
integrate((a+b*x**(1/2))**10/x,x)
```

output
```
a**10*log(x) + 20*a**9*b*sqrt(x) + 45*a**8*b**2*x + 80*a**7*b**3*x**(3/2)
+ 105*a**6*b**4*x**2 + 504*a**5*b**5*x**(5/2)/5 + 70*a**4*b**6*x**3 + 240*
a**3*b**7*x**(7/2)/7 + 45*a**2*b**8*x**4/4 + 20*a*b**9*x**(9/2)/9 + b**10*
x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx = \frac{1}{5}\,b^{10}x^5 + \frac{20}{9}\,ab^9x^{\frac{9}{2}} + \frac{45}{4}\,a^2b^8x^4 + \frac{240}{7}\,a^3b^7x^{\frac{7}{2}} + 70\,a^4b^6x^3 + \frac{504}{5}\,a^5b^5x^{\frac{5}{2}} + 105\,a^6b^4x^2 + 80\,a^7b^3x^{\frac{3}{2}} + 45\,a^8b^2x + a^{10}\log\left(x\right) + 20\,a^9b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^10/x,x, algorithm="maxima")
```

output
```
1/5*b^10*x^5 + 20/9*a*b^9*x^(9/2) + 45/4*a^2*b^8*x^4 + 240/7*a^3*b^7*x^(7/
2) + 70*a^4*b^6*x^3 + 504/5*a^5*b^5*x^(5/2) + 105*a^6*b^4*x^2 + 80*a^7*b^3
*x^(3/2) + 45*a^8*b^2*x + a^10*log(x) + 20*a^9*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x}\,dx = \frac{1}{5}\,b^{10}x^5 + \frac{20}{9}\,ab^9x^{\frac{9}{2}} + \frac{45}{4}\,a^2b^8x^4 + \frac{240}{7}\,a^3b^7x^{\frac{7}{2}} + 70\,a^4b^6x^3 + \frac{504}{5}\,a^5b^5x^{\frac{5}{2}} + 105\,a^6b^4x^2 + 80\,a^7b^3x^{\frac{3}{2}} + 45\,a^8b^2x + a^{10}\log\left(|x|\right) + 20\,a^9b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^10/x,x, algorithm="giac")
```

output
```
1/5*b^10*x^5 + 20/9*a*b^9*x^(9/2) + 45/4*a^2*b^8*x^4 + 240/7*a^3*b^7*x^(7/
2) + 70*a^4*b^6*x^3 + 504/5*a^5*b^5*x^(5/2) + 105*a^6*b^4*x^2 + 80*a^7*b^3
*x^(3/2) + 45*a^8*b^2*x + a^10*log(abs(x)) + 20*a^9*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{(a+b\sqrt{x})^{10}}{x}\,dx = 2\,a^{10}\,\ln\left(\sqrt{x}\right) + \frac{b^{10}\,x^5}{5} + 45\,a^8\,b^2\,x + 20\,a^9\,b\,\sqrt{x} + \frac{20\,a\,b^9\,x^{9/2}}{9} + 105\,a^6\,b^4\,x^2 + 70\,a^4\,b^6\,x^3 + \frac{45\,a^2\,b^8\,x^4}{4} + 80\,a^7\,b^3\,x^{3/2} + \frac{504\,a^5\,b^5\,x^{5/2}}{5} + \frac{240\,a^3\,b^7\,x^{7/2}}{7}$$

input
```
int((a + b*x^(1/2))^10/x,x)
```

output
```
2*a^10*log(x^(1/2)) + (b^10*x^5)/5 + 45*a^8*b^2*x + 20*a^9*b*x^(1/2) + (20
*a*b^9*x^(9/2))/9 + 105*a^6*b^4*x^2 + 70*a^4*b^6*x^3 + (45*a^2*b^8*x^4)/4
+ 80*a^7*b^3*x^(3/2) + (504*a^5*b^5*x^(5/2))/5 + (240*a^3*b^7*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\sqrt{x})^{10}}{x}\,dx = 20\sqrt{x}\,a^9b + 80\sqrt{x}\,a^7b^3x + \frac{504\sqrt{x}\,a^5b^5x^2}{5} + \frac{240\sqrt{x}\,a^3b^7x^3}{7} + \frac{20\sqrt{x}\,a\,b^9x^4}{9} + \log(x)\,a^{10} + 45a^8b^2x + 105a^6b^4x^2 + 70a^4b^6x^3 + \frac{45a^2b^8x^4}{4} + \frac{b^{10}x^5}{5}$$

input
```
int((a+b*x^(1/2))^10/x,x)
```

output
```
(25200*sqrt(x)*a**9*b + 100800*sqrt(x)*a**7*b**3*x + 127008*sqrt(x)*a**5*b
**5*x**2 + 43200*sqrt(x)*a**3*b**7*x**3 + 2800*sqrt(x)*a*b**9*x**4 + 1260*
log(x)*a**10 + 56700*a**8*b**2*x + 132300*a**6*b**4*x**2 + 88200*a**4*b**6
*x**3 + 14175*a**2*b**8*x**4 + 252*b**10*x**5)/1260
```

3.49 $\int \frac{(a+b\sqrt{x})^{10}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{(a+b\sqrt{x})^{10}}{x^2} dx = -\frac{a^{10}}{x} - \frac{20a^9b}{\sqrt{x}} + 240a^7b^3\sqrt{x} + 210a^6b^4x + 168a^5b^5x^{3/2} + 105a^4b^6x^2 + 48a^3b^7x^{5/2} + 15a^2b^8x^3 + \frac{20}{7}ab^9x^{7/2} + \frac{b^{10}x^4}{4} + 45a^8b^2\log(x)$$

output
```
-a^10/x-20*a^9*b/x^(1/2)+240*a^7*b^3*x^(1/2)+210*a^6*b^4*x+168*a^5*b^5*x^(
3/2)+105*a^4*b^6*x^2+48*a^3*b^7*x^(5/2)+15*a^2*b^8*x^3+20/7*a*b^9*x^(7/2)+
1/4*b^10*x^4+45*a^8*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sqrt{x})^{10}}{x^2} dx = -\frac{a^{10}}{x} - \frac{20a^9b}{\sqrt{x}} + 240a^7b^3\sqrt{x} + 210a^6b^4x + 168a^5b^5x^{3/2} + 105a^4b^6x^2 + 48a^3b^7x^{5/2} + 15a^2b^8x^3 + \frac{20}{7}ab^9x^{7/2} + \frac{b^{10}x^4}{4} + 45a^8b^2\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^2,x]
```

output
```
-(a^10/x) - (20*a^9*b)/Sqrt[x] + 240*a^7*b^3*Sqrt[x] + 210*a^6*b^4*x + 168
*a^5*b^5*x^(3/2) + 105*a^4*b^6*x^2 + 48*a^3*b^7*x^(5/2) + 15*a^2*b^8*x^3 +
 (20*a*b^9*x^(7/2))/7 + (b^10*x^4)/4 + 45*a^8*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^{10}}{x^{3/2}}+\frac{10ba^9}{x}+\frac{45b^2a^8}{\sqrt{x}}+120b^3a^7+210b^4\sqrt{x}a^6+252b^5xa^5+210b^6x^{3/2}a^4+120b^7x^2a^3+45b^8x^{5/2}a^2+1$$

$$\Big\downarrow \text{2009}$$

$$2\Bigg(-\frac{a^{10}}{2x}-\frac{10a^9b}{\sqrt{x}}+45a^8b^2\log\left(\sqrt{x}\right)+120a^7b^3\sqrt{x}+105a^6b^4x+84a^5b^5x^{3/2}+\frac{105}{2}a^4b^6x^2+24a^3b^7x^{5/2}+\frac{15}{2}a^2b$$

input
```
Int[(a + b*Sqrt[x])^10/x^2,x]
```

output
```
2*(-1/2*a^10/x - (10*a^9*b)/Sqrt[x] + 120*a^7*b^3*Sqrt[x] + 105*a^6*b^4*x
+ 84*a^5*b^5*x^(3/2) + (105*a^4*b^6*x^2)/2 + 24*a^3*b^7*x^(5/2) + (15*a^2*
b^8*x^3)/2 + (10*a*b^9*x^(7/2))/7 + (b^10*x^4)/8 + 45*a^8*b^2*Log[Sqrt[x]]
)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{a^{10}}{x}-\frac{20a^9b}{\sqrt{x}}+240a^7b^3\sqrt{x}+210a^6b^4x+168a^5b^5x^{\frac{3}{2}}+105a^4b^6x^2+48a^3b^7x^{\frac{5}{2}}+15a^2b$
default	$-\frac{a^{10}}{x}-\frac{20a^9b}{\sqrt{x}}+240a^7b^3\sqrt{x}+210a^6b^4x+168a^5b^5x^{\frac{3}{2}}+105a^4b^6x^2+48a^3b^7x^{\frac{5}{2}}+15a^2b$
trager	$\frac{(-1+x)(b^{10}x^4+60a^2b^8x^3+b^{10}x^3+420a^4b^6x^2+60a^2b^8x^2+b^{10}x^2+840a^6b^4x+420a^4b^6x+60a^2b^8x+b^{10}x+4a^{10})}{4x}-\frac{4}{}$

input
```
int((a+b*x^(1/2))^10/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^10/x-20*a^9*b/x^(1/2)+240*a^7*b^3*x^(1/2)+210*a^6*b^4*x+168*a^5*b^5*x^(
3/2)+105*a^4*b^6*x^2+48*a^3*b^7*x^(5/2)+15*a^2*b^8*x^3+20/7*a*b^9*x^(7/2)+
1/4*b^10*x^4+45*a^8*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx$$
$$=\frac{7\,b^{10}x^5+420\,a^2b^8x^4+2940\,a^4b^6x^3+5880\,a^6b^4x^2+2520\,a^8b^2x\log\left(\sqrt{x}\right)-28\,a^{10}+16\,(5\,ab^9x^4+84\,a^3b^7}{28\,x}$$

input
```
integrate((a+b*x^(1/2))^10/x^2,x, algorithm="fricas")
```

output
```
1/28*(7*b^10*x^5 + 420*a^2*b^8*x^4 + 2940*a^4*b^6*x^3 + 5880*a^6*b^4*x^2 +
 2520*a^8*b^2*x*log(sqrt(x)) - 28*a^10 + 16*(5*a*b^9*x^4 + 84*a^3*b^7*x^3
+ 294*a^5*b^5*x^2 + 420*a^7*b^3*x - 35*a^9*b)*sqrt(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx=-\frac{a^{10}}{x}-\frac{20a^9b}{\sqrt{x}}+45a^8b^2\log\left(x\right)+240a^7b^3\sqrt{x}+210a^6b^4x$$
$$+168a^5b^5x^{\frac{3}{2}}+105a^4b^6x^2+48a^3b^7x^{\frac{5}{2}}+15a^2b^8x^3+\frac{20ab^9x^{\frac{7}{2}}}{7}+\frac{b^{10}x^4}{4}$$

input
```
integrate((a+b*x**(1/2))**10/x**2,x)
```

output
```
-a**10/x - 20*a**9*b/sqrt(x) + 45*a**8*b**2*log(x) + 240*a**7*b**3*sqrt(x)
 + 210*a**6*b**4*x + 168*a**5*b**5*x**(3/2) + 105*a**4*b**6*x**2 + 48*a**3
*b**7*x**(5/2) + 15*a**2*b**8*x**3 + 20*a*b**9*x**(7/2)/7 + b**10*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx = \frac{1}{4}\,b^{10}x^4+\frac{20}{7}\,ab^9x^{\frac{7}{2}}+15\,a^2b^8x^3+48\,a^3b^7x^{\frac{5}{2}}+105\,a^4b^6x^2+168\,a^5b^5x^{\frac{3}{2}} + 210\,a^6b^4x+45\,a^8b^2\log\left(x\right)+240\,a^7b^3\sqrt{x}-\frac{20\,a^9b\sqrt{x}+a^{10}}{x}$$

input
```
integrate((a+b*x^(1/2))^10/x^2,x, algorithm="maxima")
```

output
```
1/4*b^10*x^4 + 20/7*a*b^9*x^(7/2) + 15*a^2*b^8*x^3 + 48*a^3*b^7*x^(5/2) +
105*a^4*b^6*x^2 + 168*a^5*b^5*x^(3/2) + 210*a^6*b^4*x + 45*a^8*b^2*log(x)
+ 240*a^7*b^3*sqrt(x) - (20*a^9*b*sqrt(x) + a^10)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx = \frac{1}{4}\,b^{10}x^4+\frac{20}{7}\,ab^9x^{\frac{7}{2}}+15\,a^2b^8x^3+48\,a^3b^7x^{\frac{5}{2}}+105\,a^4b^6x^2+168\,a^5b^5x^{\frac{3}{2}} + 210\,a^6b^4x+45\,a^8b^2\log\left(|x|\right)+240\,a^7b^3\sqrt{x}-\frac{20\,a^9b\sqrt{x}+a^{10}}{x}$$

input
```
integrate((a+b*x^(1/2))^10/x^2,x, algorithm="giac")
```

output
```
1/4*b^10*x^4 + 20/7*a*b^9*x^(7/2) + 15*a^2*b^8*x^3 + 48*a^3*b^7*x^(5/2) +
105*a^4*b^6*x^2 + 168*a^5*b^5*x^(3/2) + 210*a^6*b^4*x + 45*a^8*b^2*log(abs
(x)) + 240*a^7*b^3*sqrt(x) - (20*a^9*b*sqrt(x) + a^10)/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx = \frac{b^{10}\,x^4}{4}-\frac{a^{10}+20\,a^9\,b\,\sqrt{x}}{x}+90\,a^8\,b^2\,\ln\left(\sqrt{x}\right)+210\,a^6\,b^4\,x+\frac{20\,a\,b^9\,x^{7/2}}{7}+105\,a^4\,b^6\,x^2+15\,a^2\,b^8\,x^3+240\,a^7\,b^3\,\sqrt{x}+168\,a^5\,b^5\,x^{3/2}+48\,a^3\,b^7\,x^{5/2}$$

input
```
int((a + b*x^(1/2))^10/x^2,x)
```

output
```
(b^10*x^4)/4 - (a^10 + 20*a^9*b*x^(1/2))/x + 90*a^8*b^2*log(x^(1/2)) + 210
*a^6*b^4*x + (20*a*b^9*x^(7/2))/7 + 105*a^4*b^6*x^2 + 15*a^2*b^8*x^3 + 240
*a^7*b^3*x^(1/2) + 168*a^5*b^5*x^(3/2) + 48*a^3*b^7*x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^2}\,dx = \frac{1260\sqrt{x}\,\log(x)\,a^8b^2x-28\sqrt{x}\,a^{10}+5880\sqrt{x}\,a^6b^4x^2+2940\sqrt{x}\,a^4b^6x^3+420\sqrt{x}\,a^2b^8x^4+7\sqrt{x}\,b^{10}x^5-56}{28\sqrt{x}\,x}$$

input
```
int((a+b*x^(1/2))^10/x^2,x)
```

output
```
(1260*sqrt(x)*log(x)*a**8*b**2*x - 28*sqrt(x)*a**10 + 5880*sqrt(x)*a**6*b*
*4*x**2 + 2940*sqrt(x)*a**4*b**6*x**3 + 420*sqrt(x)*a**2*b**8*x**4 + 7*sqr
t(x)*b**10*x**5 - 560*a**9*b*x + 6720*a**7*b**3*x**2 + 4704*a**5*b**5*x**3
 + 1344*a**3*b**7*x**4 + 80*a*b**9*x**5)/(28*sqrt(x)*x)
```

3.50 $\int \frac{(a+b\sqrt{x})^{10}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{(a+b\sqrt{x})^{10}}{x^3}\,dx = -\frac{a^{10}}{2x^2}-\frac{20a^9b}{3x^{3/2}}-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+504a^5b^5\sqrt{x}+210a^4b^6x+80a^3b^7x^{3/2}+\frac{45}{2}a^2b^8x^2+4ab^9x^{5/2}+\frac{b^{10}x^3}{3}+210a^6b^4\log(x)$$

output
```
-1/2*a^10/x^2-20/3*a^9*b/x^(3/2)-45*a^8*b^2/x-240*a^7*b^3/x^(1/2)+504*a^5*
b^5*x^(1/2)+210*a^4*b^6*x+80*a^3*b^7*x^(3/2)+45/2*a^2*b^8*x^2+4*a*b^9*x^(5
/2)+1/3*b^10*x^3+210*a^6*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\sqrt{x})^{10}}{x^3}\,dx = \frac{-3a^{10}-40a^9b\sqrt{x}-270a^8b^2x-1440a^7b^3x^{3/2}+3024a^5b^5x^{5/2}+1260a^4b^6x^3+480a^3b^7x^{7/2}+135a^2b^8x^4}{6x^2}+420a^6b^4\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^3,x]
```

output
```
(-3*a^10 - 40*a^9*b*Sqrt[x] - 270*a^8*b^2*x - 1440*a^7*b^3*x^(3/2) + 3024*
a^5*b^5*x^(5/2) + 1260*a^4*b^6*x^3 + 480*a^3*b^7*x^(7/2) + 135*a^2*b^8*x^4
 + 24*a*b^9*x^(9/2) + 2*b^10*x^5)/(6*x^2) + 420*a^6*b^4*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^{10}}{x^{5/2}}+\frac{10ba^9}{x^2}+\frac{45b^2a^8}{x^{3/2}}+\frac{120b^3a^7}{x}+\frac{210b^4a^6}{\sqrt{x}}+252b^5a^5+210b^6\sqrt{x}a^4+120b^7xa^3+45b^8x^{3/2}a^2+10b^9x^2$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{10}}{4x^2}-\frac{10a^9b}{3x^{3/2}}-\frac{45a^8b^2}{2x}-\frac{120a^7b^3}{\sqrt{x}}+210a^6b^4\log\left(\sqrt{x}\right)+252a^5b^5\sqrt{x}+105a^4b^6x+40a^3b^7x^{3/2}+\frac{45}{4}a^2b^8x^2\right.$$

input
```
Int[(a + b*Sqrt[x])^10/x^3,x]
```

output
```
2*(-1/4*a^10/x^2 - (10*a^9*b)/(3*x^(3/2)) - (45*a^8*b^2)/(2*x) - (120*a^7*
b^3)/Sqrt[x] + 252*a^5*b^5*Sqrt[x] + 105*a^4*b^6*x + 40*a^3*b^7*x^(3/2) +
(45*a^2*b^8*x^2)/4 + 2*a*b^9*x^(5/2) + (b^10*x^3)/6 + 210*a^6*b^4*Log[Sqrt
[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{a^{10}}{2x^2}-\frac{20a^9b}{3x^{\frac{3}{2}}}-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+504a^5b^5\sqrt{x}+210a^4b^6x+80a^3b^7x^{\frac{3}{2}}+\frac{45a^2b^8x^2}{2}+4a\,b$
default	$-\frac{a^{10}}{2x^2}-\frac{20a^9b}{3x^{\frac{3}{2}}}-\frac{45a^8b^2}{x}-\frac{240a^7b^3}{\sqrt{x}}+504a^5b^5\sqrt{x}+210a^4b^6x+80a^3b^7x^{\frac{3}{2}}+\frac{45a^2b^8x^2}{2}+4a\,b$
trager	$\frac{(-1+x)(2b^{10}x^4+135a^2b^8x^3+2b^{10}x^3+1260a^4b^6x^2+135a^2b^8x^2+2b^{10}x^2+3a^{10}x+270a^8b^2x+3a^{10})}{6x^2}-\frac{4(-3b^8x^4-60$

input
```
int((a+b*x^(1/2))^10/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^10/x^2-20/3*a^9*b/x^(3/2)-45*a^8*b^2/x-240*a^7*b^3/x^(1/2)+504*a^5*
b^5*x^(1/2)+210*a^4*b^6*x+80*a^3*b^7*x^(3/2)+45/2*a^2*b^8*x^2+4*a*b^9*x^(5
/2)+1/3*b^10*x^3+210*a^6*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx$$
$$= \frac{2\,b^{10}x^5 + 135\,a^2b^8x^4 + 1260\,a^4b^6x^3 + 2520\,a^6b^4x^2\log\left(\sqrt{x}\right) - 270\,a^8b^2x - 3\,a^{10} + 8\,(3\,ab^9x^4 + 60\,a^3b^7x^3}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^10/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^10*x^5 + 135*a^2*b^8*x^4 + 1260*a^4*b^6*x^3 + 2520*a^6*b^4*x^2*lo
g(sqrt(x)) - 270*a^8*b^2*x - 3*a^10 + 8*(3*a*b^9*x^4 + 60*a^3*b^7*x^3 + 37
8*a^5*b^5*x^2 - 180*a^7*b^3*x - 5*a^9*b)*sqrt(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx = -\frac{a^{10}}{2x^2} - \frac{20a^9b}{3x^{\frac{3}{2}}} - \frac{45a^8b^2}{x} - \frac{240a^7b^3}{\sqrt{x}} + 210a^6b^4\log(x) + 504a^5b^5\sqrt{x} + 210a^4b^6x + 80a^3b^7x^{\frac{3}{2}} + \frac{45a^2b^8x^2}{2} + 4ab^9x^{\frac{5}{2}} + \frac{b^{10}x^3}{3}$$

input
```
integrate((a+b*x**(1/2))**10/x**3,x)
```

output
```
-a**10/(2*x**2) - 20*a**9*b/(3*x**(3/2)) - 45*a**8*b**2/x - 240*a**7*b**3/
sqrt(x) + 210*a**6*b**4*log(x) + 504*a**5*b**5*sqrt(x) + 210*a**4*b**6*x +
 80*a**3*b**7*x**(3/2) + 45*a**2*b**8*x**2/2 + 4*a*b**9*x**(5/2) + b**10*x
**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx = \frac{1}{3}\,b^{10}x^3 + 4\,ab^9x^{\frac{5}{2}} + \frac{45}{2}\,a^2b^8x^2 + 80\,a^3b^7x^{\frac{3}{2}} + 210\,a^4b^6x + 210\,a^6b^4\log\left(x\right) + 504\,a^5b^5\sqrt{x} - \frac{1440\,a^7b^3x^{\frac{3}{2}} + 270\,a^8b^2x + 40\,a^9b\sqrt{x} + 3\,a^{10}}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^10/x^3,x, algorithm="maxima")
```

output
```
1/3*b^10*x^3 + 4*a*b^9*x^(5/2) + 45/2*a^2*b^8*x^2 + 80*a^3*b^7*x^(3/2) + 2
10*a^4*b^6*x + 210*a^6*b^4*log(x) + 504*a^5*b^5*sqrt(x) - 1/6*(1440*a^7*b^
3*x^(3/2) + 270*a^8*b^2*x + 40*a^9*b*sqrt(x) + 3*a^10)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx = \frac{1}{3}\,b^{10}x^3 + 4\,ab^9x^{\frac{5}{2}} + \frac{45}{2}\,a^2b^8x^2 + 80\,a^3b^7x^{\frac{3}{2}} + 210\,a^4b^6x + 210\,a^6b^4\log\left(|x|\right) + 504\,a^5b^5\sqrt{x} - \frac{1440\,a^7b^3x^{\frac{3}{2}} + 270\,a^8b^2x + 40\,a^9b\sqrt{x} + 3\,a^{10}}{6\,x^2}$$

input
```
integrate((a+b*x^(1/2))^10/x^3,x, algorithm="giac")
```

output
```
1/3*b^10*x^3 + 4*a*b^9*x^(5/2) + 45/2*a^2*b^8*x^2 + 80*a^3*b^7*x^(3/2) + 2
10*a^4*b^6*x + 210*a^6*b^4*log(abs(x)) + 504*a^5*b^5*sqrt(x) - 1/6*(1440*a
^7*b^3*x^(3/2) + 270*a^8*b^2*x + 40*a^9*b*sqrt(x) + 3*a^10)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx = \frac{b^{10}\,x^3}{3}-\frac{\frac{a^{10}}{2}+45\,a^8\,b^2\,x+\frac{20\,a^9\,b\,\sqrt{x}}{3}+240\,a^7\,b^3\,x^{3/2}}{x^2}+420\,a^6\,b^4\,\ln\left(\sqrt{x}\right)+210\,a^4\,b^6\,x+4\,a\,b^9\,x^{5/2}+\frac{45\,a^2\,b^8\,x^2}{2}+504\,a^5\,b^5\,\sqrt{x}+80\,a^3\,b^7\,x^{3/2}$$

input
```
int((a + b*x^(1/2))^10/x^3,x)
```

output
```
(b^10*x^3)/3 - (a^10/2 + 45*a^8*b^2*x + (20*a^9*b*x^(1/2))/3 + 240*a^7*b^3
*x^(3/2))/x^2 + 420*a^6*b^4*log(x^(1/2)) + 210*a^4*b^6*x + 4*a*b^9*x^(5/2)
 + (45*a^2*b^8*x^2)/2 + 504*a^5*b^5*x^(1/2) + 80*a^3*b^7*x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^3}\,dx = \frac{1260\sqrt{x}\,\log(x)\,a^6b^4x^2-3\sqrt{x}\,a^{10}-270\sqrt{x}\,a^8b^2x+1260\sqrt{x}\,a^4b^6x^3+135\sqrt{x}\,a^2b^8x^4+2\sqrt{x}\,b^{10}x^5-40a^9b\cdots}{6\sqrt{x}\,x^2}$$

input
```
int((a+b*x^(1/2))^10/x^3,x)
```

output
```
(1260*sqrt(x)*log(x)*a**6*b**4*x**2 - 3*sqrt(x)*a**10 - 270*sqrt(x)*a**8*b
**2*x + 1260*sqrt(x)*a**4*b**6*x**3 + 135*sqrt(x)*a**2*b**8*x**4 + 2*sqrt(
x)*b**10*x**5 - 40*a**9*b*x - 1440*a**7*b**3*x**2 + 3024*a**5*b**5*x**3 +
480*a**3*b**7*x**4 + 24*a*b**9*x**5)/(6*sqrt(x)*x**2)
```

3.51 $\int \frac{(a+b\sqrt{x})^{10}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{(a+b\sqrt{x})^{10}}{x^4}\,dx = -\frac{a^{10}}{3x^3}-\frac{4a^9b}{x^{5/2}}-\frac{45a^8b^2}{2x^2}-\frac{80a^7b^3}{x^{3/2}}-\frac{210a^6b^4}{x}-\frac{504a^5b^5}{\sqrt{x}}+240a^3b^7\sqrt{x}+45a^2b^8x+\frac{20}{3}ab^9x^{3/2}+\frac{b^{10}x^2}{2}+210a^4b^6\log(x)$$

output
```
-1/3*a^10/x^3-4*a^9*b/x^(5/2)-45/2*a^8*b^2/x^2-80*a^7*b^3/x^(3/2)-210*a^6*
b^4/x-504*a^5*b^5/x^(1/2)+240*a^3*b^7*x^(1/2)+45*a^2*b^8*x+20/3*a*b^9*x^(3
/2)+1/2*b^10*x^2+210*a^4*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sqrt{x})^{10}}{x^4}\,dx = -\frac{2a^{10}+24a^9b\sqrt{x}+135a^8b^2x+480a^7b^3x^{3/2}+1260a^6b^4x^2+3024a^5b^5x^{5/2}-1440a^3b^7x^{7/2}-270a^2b^8x^4}{6x^3}+210a^4b^6\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^4,x]
```

output
```
-1/6*(2*a^10 + 24*a^9*b*Sqrt[x] + 135*a^8*b^2*x + 480*a^7*b^3*x^(3/2) + 12
60*a^6*b^4*x^2 + 3024*a^5*b^5*x^(5/2) - 1440*a^3*b^7*x^(7/2) - 270*a^2*b^8
*x^4 - 40*a*b^9*x^(9/2) - 3*b^10*x^5)/x^3 + 210*a^4*b^6*Log[x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^{10}}{x^{7/2}}+\frac{10ba^9}{x^3}+\frac{45b^2a^8}{x^{5/2}}+\frac{120b^3a^7}{x^2}+\frac{210b^4a^6}{x^{3/2}}+\frac{252b^5a^5}{x}+\frac{210b^6a^4}{\sqrt{x}}+120b^7a^3+45b^8\sqrt{x}a^2+10b^9xa+b$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{10}}{6x^3}-\frac{2a^9b}{x^{5/2}}-\frac{45a^8b^2}{4x^2}-\frac{40a^7b^3}{x^{3/2}}-\frac{105a^6b^4}{x}-\frac{252a^5b^5}{\sqrt{x}}+210a^4b^6\log\left(\sqrt{x}\right)+120a^3b^7\sqrt{x}+\frac{45}{2}a^2b^8x+\frac{10}{3}ab$$

input
```
Int[(a + b*Sqrt[x])^10/x^4,x]
```

output

```
2*(-1/6*a^10/x^3 - (2*a^9*b)/x^(5/2) - (45*a^8*b^2)/(4*x^2) - (40*a^7*b^3)
/x^(3/2) - (105*a^6*b^4)/x - (252*a^5*b^5)/Sqrt[x] + 120*a^3*b^7*Sqrt[x] +
 (45*a^2*b^8*x)/2 + (10*a*b^9*x^(3/2))/3 + (b^10*x^2)/4 + 210*a^4*b^6*Log[
Sqrt[x]])
```

Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{a^{10}}{3x^{3}}-\frac{4a^{9}b}{x^{\frac{5}{2}}}-\frac{45a^{8}b^{2}}{2x^{2}}-\frac{80a^{7}b^{3}}{x^{\frac{3}{2}}}-\frac{210a^{6}b^{4}}{x}-\frac{504a^{5}b^{5}}{\sqrt{x}}+240a^{3}b^{7}\sqrt{x}+45a^{2}b^{8}x+\frac{20a\,b^{9}x^{\frac{3}{2}}}{3}+$
default	$-\frac{a^{10}}{3x^{3}}-\frac{4a^{9}b}{x^{\frac{5}{2}}}-\frac{45a^{8}b^{2}}{2x^{2}}-\frac{80a^{7}b^{3}}{x^{\frac{3}{2}}}-\frac{210a^{6}b^{4}}{x}-\frac{504a^{5}b^{5}}{\sqrt{x}}+240a^{3}b^{7}\sqrt{x}+45a^{2}b^{8}x+\frac{20a\,b^{9}x^{\frac{3}{2}}}{3}+$
trager	$\frac{(-1+x)(3b^{10}x^{4}+270a^{2}b^{8}x^{3}+3b^{10}x^{3}+2a^{10}x^{2}+135a^{8}b^{2}x^{2}+1260a^{6}b^{4}x^{2}+2a^{10}x+135a^{8}b^{2}x+2a^{10})}{6x^{3}}-\frac{4(-5b^{8}x^{4}-18$

input

```
int((a+b*x^(1/2))^10/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*a^10/x^3-4*a^9*b/x^(5/2)-45/2*a^8*b^2/x^2-80*a^7*b^3/x^(3/2)-210*a^6*
b^4/x-504*a^5*b^5/x^(1/2)+240*a^3*b^7*x^(1/2)+45*a^2*b^8*x+20/3*a*b^9*x^(3
/2)+1/2*b^10*x^2+210*a^4*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx = \frac{3\,b^{10}x^5+270\,a^2b^8x^4+2520\,a^4b^6x^3\log\left(\sqrt{x}\right)-1260\,a^6b^4x^2-135\,a^8b^2x-2\,a^{10}+8\,(5\,ab^9x^4+180\,a^3b^7x^3}{6\,x^3}$$

input
```
integrate((a+b*x^(1/2))^10/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*b^10*x^5 + 270*a^2*b^8*x^4 + 2520*a^4*b^6*x^3*log(sqrt(x)) - 1260*a
^6*b^4*x^2 - 135*a^8*b^2*x - 2*a^10 + 8*(5*a*b^9*x^4 + 180*a^3*b^7*x^3 - 3
78*a^5*b^5*x^2 - 60*a^7*b^3*x - 3*a^9*b)*sqrt(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx = -\frac{a^{10}}{3x^3}-\frac{4a^9b}{x^{\frac{5}{2}}}-\frac{45a^8b^2}{2x^2}-\frac{80a^7b^3}{x^{\frac{3}{2}}}-\frac{210a^6b^4}{x}-\frac{504a^5b^5}{\sqrt{x}}+210a^4b^6\log{(x)}+240a^3b^7\sqrt{x}+45a^2b^8x+\frac{20ab^9x^{\frac{3}{2}}}{3}+\frac{b^{10}x^2}{2}$$

input
```
integrate((a+b*x**(1/2))**10/x**4,x)
```

output
```
-a**10/(3*x**3) - 4*a**9*b/x**(5/2) - 45*a**8*b**2/(2*x**2) - 80*a**7*b**3
/x**(3/2) - 210*a**6*b**4/x - 504*a**5*b**5/sqrt(x) + 210*a**4*b**6*log(x)
 + 240*a**3*b**7*sqrt(x) + 45*a**2*b**8*x + 20*a*b**9*x**(3/2)/3 + b**10*x
**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx$$
$$= \frac{1}{2}\,b^{10}x^2 + \frac{20}{3}\,ab^9x^{\frac{3}{2}} + 45\,a^2b^8x + 210\,a^4b^6\log\left(x\right) + 240\,a^3b^7\sqrt{x} - \frac{3024\,a^5b^5x^{\frac{5}{2}} + 1260\,a^6b^4x^2 + 480\,a^7b^3x^{\frac{3}{2}} + 135\,a^8b^2x + 24\,a^9b\sqrt{x} + 2\,a^{10}}{6\,x^3}$$

input
```
integrate((a+b*x^(1/2))^10/x^4,x, algorithm="maxima")
```

output
```
1/2*b^10*x^2 + 20/3*a*b^9*x^(3/2) + 45*a^2*b^8*x + 210*a^4*b^6*log(x) + 24
0*a^3*b^7*sqrt(x) - 1/6*(3024*a^5*b^5*x^(5/2) + 1260*a^6*b^4*x^2 + 480*a^7
*b^3*x^(3/2) + 135*a^8*b^2*x + 24*a^9*b*sqrt(x) + 2*a^10)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx$$
$$= \frac{1}{2}\,b^{10}x^2 + \frac{20}{3}\,ab^9x^{\frac{3}{2}} + 45\,a^2b^8x + 210\,a^4b^6\log\left(|x|\right) + 240\,a^3b^7\sqrt{x} - \frac{3024\,a^5b^5x^{\frac{5}{2}} + 1260\,a^6b^4x^2 + 480\,a^7b^3x^{\frac{3}{2}} + 135\,a^8b^2x + 24\,a^9b\sqrt{x} + 2\,a^{10}}{6\,x^3}$$

input
```
integrate((a+b*x^(1/2))^10/x^4,x, algorithm="giac")
```

output
```
1/2*b^10*x^2 + 20/3*a*b^9*x^(3/2) + 45*a^2*b^8*x + 210*a^4*b^6*log(abs(x))
 + 240*a^3*b^7*sqrt(x) - 1/6*(3024*a^5*b^5*x^(5/2) + 1260*a^6*b^4*x^2 + 48
0*a^7*b^3*x^(3/2) + 135*a^8*b^2*x + 24*a^9*b*sqrt(x) + 2*a^10)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx$$
$$= \frac{b^{10}\,x^2}{2} - \frac{\frac{a^{10}}{3} + \frac{45\,a^8\,b^2\,x}{2} + 4\,a^9\,b\,\sqrt{x} + 210\,a^6\,b^4\,x^2 + 80\,a^7\,b^3\,x^{3/2} + 504\,a^5\,b^5\,x^{5/2}}{x^3}$$
$$+ 420\,a^4\,b^6\,\ln\left(\sqrt{x}\right) + 45\,a^2\,b^8\,x + \frac{20\,a\,b^9\,x^{3/2}}{3} + 240\,a^3\,b^7\,\sqrt{x}$$

input
```
int((a + b*x^(1/2))^10/x^4,x)
```

output
```
(b^10*x^2)/2 - (a^10/3 + (45*a^8*b^2*x)/2 + 4*a^9*b*x^(1/2) + 210*a^6*b^4*
x^2 + 80*a^7*b^3*x^(3/2) + 504*a^5*b^5*x^(5/2))/x^3 + 420*a^4*b^6*log(x^(1
/2)) + 45*a^2*b^8*x + (20*a*b^9*x^(3/2))/3 + 240*a^3*b^7*x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^4}\,dx$$
$$= \frac{1260\sqrt{x}\log(x)\,a^4b^6x^3 - 2\sqrt{x}\,a^{10} - 135\sqrt{x}\,a^8b^2x - 1260\sqrt{x}\,a^6b^4x^2 + 270\sqrt{x}\,a^2b^8x^4 + 3\sqrt{x}\,b^{10}x^5 - 24a^9b}{6\sqrt{x}\,x^3}$$

input
```
int((a+b*x^(1/2))^10/x^4,x)
```

output
```
(1260*sqrt(x)*log(x)*a**4*b**6*x**3 - 2*sqrt(x)*a**10 - 135*sqrt(x)*a**8*b
**2*x - 1260*sqrt(x)*a**6*b**4*x**2 + 270*sqrt(x)*a**2*b**8*x**4 + 3*sqrt(
x)*b**10*x**5 - 24*a**9*b*x - 480*a**7*b**3*x**2 - 3024*a**5*b**5*x**3 + 1
440*a**3*b**7*x**4 + 40*a*b**9*x**5)/(6*sqrt(x)*x**3)
```

3.52 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = -\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{7/2}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{5/2}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{3/2}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+20ab^9\sqrt{x}+b^{10}x+45a^2b^8\log(x)$$

output
```
-1/4*a^10/x^4-20/7*a^9*b/x^(7/2)-15*a^8*b^2/x^3-48*a^7*b^3/x^(5/2)-105*a^6
*b^4/x^2-168*a^5*b^5/x^(3/2)-210*a^4*b^6/x-240*a^3*b^7/x^(1/2)+20*a*b^9*x^
(1/2)+b^10*x+45*a^2*b^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = -\frac{7a^{10}+80a^9b\sqrt{x}+420a^8b^2x+1344a^7b^3x^{3/2}+2940a^6b^4x^2+4704a^5b^5x^{5/2}+5880a^4b^6x^3+6720a^3b^7x}{28x^4}+45a^2b^8\log(x)$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^5,x]
```

output
```
-1/28*(7*a^10 + 80*a^9*b*Sqrt[x] + 420*a^8*b^2*x + 1344*a^7*b^3*x^(3/2) +
2940*a^6*b^4*x^2 + 4704*a^5*b^5*x^(5/2) + 5880*a^4*b^6*x^3 + 6720*a^3*b^7*
x^(7/2) - 560*a*b^9*x^(9/2) - 28*b^10*x^5)/x^4 + 45*a^2*b^8*Log[x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{9/2}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^{10}}{x^{9/2}}+\frac{10ba^9}{x^4}+\frac{45b^2a^8}{x^{7/2}}+\frac{120b^3a^7}{x^3}+\frac{210b^4a^6}{x^{5/2}}+\frac{252b^5a^5}{x^2}+\frac{210b^6a^4}{x^{3/2}}+\frac{120b^7a^3}{x}+\frac{45b^8a^2}{\sqrt{x}}+10b^9a+b^{10}\sqrt{\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{10}}{8x^4}-\frac{10a^9b}{7x^{7/2}}-\frac{15a^8b^2}{2x^3}-\frac{24a^7b^3}{x^{5/2}}-\frac{105a^6b^4}{2x^2}-\frac{84a^5b^5}{x^{3/2}}-\frac{105a^4b^6}{x}-\frac{120a^3b^7}{\sqrt{x}}+45a^2b^8\log\left(\sqrt{x}\right)+10ab^9\sqrt{x}\right.$$

input
```
Int[(a + b*Sqrt[x])^10/x^5,x]
```

output
```
2*(-1/8*a^10/x^4 - (10*a^9*b)/(7*x^(7/2)) - (15*a^8*b^2)/(2*x^3) - (24*a^7
*b^3)/x^(5/2) - (105*a^6*b^4)/(2*x^2) - (84*a^5*b^5)/x^(3/2) - (105*a^4*b^
6)/x - (120*a^3*b^7)/Sqrt[x] + 10*a*b^9*Sqrt[x] + (b^10*x)/2 + 45*a^2*b^8*
Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{\frac{7}{2}}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{\frac{5}{2}}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{\frac{3}{2}}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+20a\,b^9\sqrt{x}+b^{10}$
default	$-\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{\frac{7}{2}}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{\frac{5}{2}}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{\frac{3}{2}}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+20a\,b^9\sqrt{x}+b^{10}$
trager	$\frac{(-1+x)(4b^{10}x^4+a^{10}x^3+60a^8b^2x^3+420a^6b^4x^3+840a^4b^6x^3+a^{10}x^2+60a^8b^2x^2+420a^6b^4x^2+a^{10}x+60a^8b^2x+a^{10})}{4x^4}-$

input
```
int((a+b*x^(1/2))^10/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^10/x^4-20/7*a^9*b/x^(7/2)-15*a^8*b^2/x^3-48*a^7*b^3/x^(5/2)-105*a^6
*b^4/x^2-168*a^5*b^5/x^(3/2)-210*a^4*b^6/x-240*a^3*b^7/x^(1/2)+20*a*b^9*x^
(1/2)+b^10*x+45*a^2*b^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = \frac{28\,b^{10}x^5+2520\,a^2b^8x^4\log\left(\sqrt{x}\right)-5880\,a^4b^6x^3-2940\,a^6b^4x^2-420\,a^8b^2x-7\,a^{10}+16\,(35\,ab^9x^4-420\,a}{28\,x^4}$$

input
```
integrate((a+b*x^(1/2))^10/x^5,x, algorithm="fricas")
```

output
```
1/28*(28*b^10*x^5 + 2520*a^2*b^8*x^4*log(sqrt(x)) - 5880*a^4*b^6*x^3 - 294
0*a^6*b^4*x^2 - 420*a^8*b^2*x - 7*a^10 + 16*(35*a*b^9*x^4 - 420*a^3*b^7*x^
3 - 294*a^5*b^5*x^2 - 84*a^7*b^3*x - 5*a^9*b)*sqrt(x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = -\frac{a^{10}}{4x^4}-\frac{20a^9b}{7x^{\frac{7}{2}}}-\frac{15a^8b^2}{x^3}-\frac{48a^7b^3}{x^{\frac{5}{2}}}-\frac{105a^6b^4}{x^2}-\frac{168a^5b^5}{x^{\frac{3}{2}}}-\frac{210a^4b^6}{x}-\frac{240a^3b^7}{\sqrt{x}}+45a^2b^8\log\left(x\right)+20ab^9\sqrt{x}+b^{10}x$$

input
```
integrate((a+b*x**(1/2))**10/x**5,x)
```

output
```
-a**10/(4*x**4) - 20*a**9*b/(7*x**(7/2)) - 15*a**8*b**2/x**3 - 48*a**7*b**
3/x**(5/2) - 105*a**6*b**4/x**2 - 168*a**5*b**5/x**(3/2) - 210*a**4*b**6/x
 - 240*a**3*b**7/sqrt(x) + 45*a**2*b**8*log(x) + 20*a*b**9*sqrt(x) + b**10
*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = b^{10}x + 45\,a^2b^8\log\left(x\right) + 20\,ab^9\sqrt{x} - \frac{6720\,a^3b^7x^{\frac{7}{2}} + 5880\,a^4b^6x^3 + 4704\,a^5b^5x^{\frac{5}{2}} + 2940\,a^6b^4x^2 + 1344\,a^7b^3x^{\frac{3}{2}} + 420\,a^8b^2x + 80\,a^9b\sqrt{x} + 7\,a}{28\,x^4}$$

input
```
integrate((a+b*x^(1/2))^10/x^5,x, algorithm="maxima")
```

output
```
b^10*x + 45*a^2*b^8*log(x) + 20*a*b^9*sqrt(x) - 1/28*(6720*a^3*b^7*x^(7/2)
 + 5880*a^4*b^6*x^3 + 4704*a^5*b^5*x^(5/2) + 2940*a^6*b^4*x^2 + 1344*a^7*b
^3*x^(3/2) + 420*a^8*b^2*x + 80*a^9*b*sqrt(x) + 7*a^10)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = b^{10}x + 45\,a^2b^8\log\left(|x|\right) + 20\,ab^9\sqrt{x} - \frac{6720\,a^3b^7x^{\frac{7}{2}} + 5880\,a^4b^6x^3 + 4704\,a^5b^5x^{\frac{5}{2}} + 2940\,a^6b^4x^2 + 1344\,a^7b^3x^{\frac{3}{2}} + 420\,a^8b^2x + 80\,a^9b\sqrt{x} + 7\,a}{28\,x^4}$$

input
```
integrate((a+b*x^(1/2))^10/x^5,x, algorithm="giac")
```

output
```
b^10*x + 45*a^2*b^8*log(abs(x)) + 20*a*b^9*sqrt(x) - 1/28*(6720*a^3*b^7*x^
(7/2) + 5880*a^4*b^6*x^3 + 4704*a^5*b^5*x^(5/2) + 2940*a^6*b^4*x^2 + 1344*
a^7*b^3*x^(3/2) + 420*a^8*b^2*x + 80*a^9*b*sqrt(x) + 7*a^10)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = b^{10}\,x - \frac{\frac{a^{10}}{4}+15\,a^8\,b^2\,x+\frac{20\,a^9\,b\,\sqrt{x}}{7}+105\,a^6\,b^4\,x^2+210\,a^4\,b^6\,x^3+48\,a^7\,b^3\,x^{3/2}+168\,a^5\,b^5\,x^{5/2}+240\,a^3\,b^7\,x^{7/2}}{x^4} + 90\,a^2\,b^8\,\ln\left(\sqrt{x}\right)+20\,a\,b^9\,\sqrt{x}$$

input
```
int((a + b*x^(1/2))^10/x^5,x)
```

output
```
b^10*x - (a^10/4 + 15*a^8*b^2*x + (20*a^9*b*x^(1/2))/7 + 105*a^6*b^4*x^2 +
 210*a^4*b^6*x^3 + 48*a^7*b^3*x^(3/2) + 168*a^5*b^5*x^(5/2) + 240*a^3*b^7*
x^(7/2))/x^4 + 90*a^2*b^8*log(x^(1/2)) + 20*a*b^9*x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^5}\,dx = \frac{1260\sqrt{x}\log(x)\,a^2b^8x^4-7\sqrt{x}\,a^{10}-420\sqrt{x}\,a^8b^2x-2940\sqrt{x}\,a^6b^4x^2-5880\sqrt{x}\,a^4b^6x^3+28\sqrt{x}\,b^{10}x^5-80}{28\sqrt{x}\,x^4}$$

input
```
int((a+b*x^(1/2))^10/x^5,x)
```

output
```
(1260*sqrt(x)*log(x)*a**2*b**8*x**4 - 7*sqrt(x)*a**10 - 420*sqrt(x)*a**8*b
**2*x - 2940*sqrt(x)*a**6*b**4*x**2 - 5880*sqrt(x)*a**4*b**6*x**3 + 28*sqr
t(x)*b**10*x**5 - 80*a**9*b*x - 1344*a**7*b**3*x**2 - 4704*a**5*b**5*x**3
- 6720*a**3*b**7*x**4 + 560*a*b**9*x**5)/(28*sqrt(x)*x**4)
```

3.53 $\int \frac{(a+b\sqrt{x})^{10}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 130

$$\int \frac{(a+b\sqrt{x})^{10}}{x^6}\,dx = -\frac{a^{10}}{5x^5}-\frac{20a^9b}{9x^{9/2}}-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{7/2}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{5/2}} -\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{3/2}}-\frac{45a^2b^8}{x}-\frac{20ab^9}{\sqrt{x}}+b^{10}\log(x)$$

output
```
-1/5*a^10/x^5-20/9*a^9*b/x^(9/2)-45/4*a^8*b^2/x^4-240/7*a^7*b^3/x^(7/2)-70
*a^6*b^4/x^3-504/5*a^5*b^5/x^(5/2)-105*a^4*b^6/x^2-80*a^3*b^7/x^(3/2)-45*a
^2*b^8/x-20*a*b^9/x^(1/2)+b^10*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sqrt{x})^{10}}{x^6}\,dx = \frac{-252a^{10}-2800a^9b\sqrt{x}-14175a^8b^2x-43200a^7b^3x^{3/2}-88200a^6b^4x^2-127008a^5b^5x^{5/2}-132300a^4b^6x^3}{1260x^5} + 2b^{10}\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^6,x]
```

output
```
(-252*a^10 - 2800*a^9*b*Sqrt[x] - 14175*a^8*b^2*x - 43200*a^7*b^3*x^(3/2)
- 88200*a^6*b^4*x^2 - 127008*a^5*b^5*x^(5/2) - 132300*a^4*b^6*x^3 - 100800
*a^3*b^7*x^(7/2) - 56700*a^2*b^8*x^4 - 25200*a*b^9*x^(9/2))/(1260*x^5) + 2
*b^10*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11/2}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^{10}}{x^{11/2}}+\frac{10ba^9}{x^5}+\frac{45b^2a^8}{x^{9/2}}+\frac{120b^3a^7}{x^4}+\frac{210b^4a^6}{x^{7/2}}+\frac{252b^5a^5}{x^3}+\frac{210b^6a^4}{x^{5/2}}+\frac{120b^7a^3}{x^2}+\frac{45b^8a^2}{x^{3/2}}+\frac{10b^9a}{x}+\frac{b^{10}}{\sqrt{x}}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{10}}{10x^5}-\frac{10a^9b}{9x^{9/2}}-\frac{45a^8b^2}{8x^4}-\frac{120a^7b^3}{7x^{7/2}}-\frac{35a^6b^4}{x^3}-\frac{252a^5b^5}{5x^{5/2}}-\frac{105a^4b^6}{2x^2}-\frac{40a^3b^7}{x^{3/2}}-\frac{45a^2b^8}{2x}-\frac{10ab^9}{\sqrt{x}}+b^{10}\log(\right.$$

input
```
Int[(a + b*Sqrt[x])^10/x^6,x]
```

output
```
2*(-1/10*a^10/x^5 - (10*a^9*b)/(9*x^(9/2)) - (45*a^8*b^2)/(8*x^4) - (120*a
^7*b^3)/(7*x^(7/2)) - (35*a^6*b^4)/x^3 - (252*a^5*b^5)/(5*x^(5/2)) - (105*
a^4*b^6)/(2*x^2) - (40*a^3*b^7)/x^(3/2) - (45*a^2*b^8)/(2*x) - (10*a*b^9)/
Sqrt[x] + b^10*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{a^{10}}{5x^5}-\frac{20a^9b}{9x^{\frac{9}{2}}}-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{\frac{7}{2}}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{\frac{5}{2}}}-\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{\frac{3}{2}}}-\frac{45a^2b^8}{x}-\frac{20a\,b^9}{\sqrt{x}}+$
default	$-\frac{a^{10}}{5x^5}-\frac{20a^9b}{9x^{\frac{9}{2}}}-\frac{45a^8b^2}{4x^4}-\frac{240a^7b^3}{7x^{\frac{7}{2}}}-\frac{70a^6b^4}{x^3}-\frac{504a^5b^5}{5x^{\frac{5}{2}}}-\frac{105a^4b^6}{x^2}-\frac{80a^3b^7}{x^{\frac{3}{2}}}-\frac{45a^2b^8}{x}-\frac{20a\,b^9}{\sqrt{x}}+$
trager	$\frac{(-1+x)(4a^8x^4+225a^6b^2x^4+1400a^4x^4b^4+2100a^2b^6x^4+900b^8x^4+4a^8x^3+225a^6b^2x^3+1400a^4b^4x^3+2100a^2b^6x^3+4a^8}{20x^5}$

input
```
int((a+b*x^(1/2))^10/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^10/x^5-20/9*a^9*b/x^(9/2)-45/4*a^8*b^2/x^4-240/7*a^7*b^3/x^(7/2)-70
*a^6*b^4/x^3-504/5*a^5*b^5/x^(5/2)-105*a^4*b^6/x^2-80*a^3*b^7/x^(3/2)-45*a
^2*b^8/x-20*a*b^9/x^(1/2)+b^10*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx$$
$$= \frac{2520\,b^{10}x^5\log\left(\sqrt{x}\right) - 56700\,a^2b^8x^4 - 132300\,a^4b^6x^3 - 88200\,a^6b^4x^2 - 14175\,a^8b^2x - 252\,a^{10} - 16\,(1575}{1260\,x^5}$$

input
```
integrate((a+b*x^(1/2))^10/x^6,x, algorithm="fricas")
```

output
```
1/1260*(2520*b^10*x^5*log(sqrt(x)) - 56700*a^2*b^8*x^4 - 132300*a^4*b^6*x^
3 - 88200*a^6*b^4*x^2 - 14175*a^8*b^2*x - 252*a^10 - 16*(1575*a*b^9*x^4 +
6300*a^3*b^7*x^3 + 7938*a^5*b^5*x^2 + 2700*a^7*b^3*x + 175*a^9*b)*sqrt(x))
/x^5
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx = -\frac{a^{10}}{5x^5} - \frac{20a^9b}{9x^{\frac{9}{2}}} - \frac{45a^8b^2}{4x^4} - \frac{240a^7b^3}{7x^{\frac{7}{2}}} - \frac{70a^6b^4}{x^3} - \frac{504a^5b^5}{5x^{\frac{5}{2}}} - \frac{105a^4b^6}{x^2} - \frac{80a^3b^7}{x^{\frac{3}{2}}} - \frac{45a^2b^8}{x} - \frac{20ab^9}{\sqrt{x}} + b^{10}\log(x)$$

input
```
integrate((a+b*x**(1/2))**10/x**6,x)
```

output
```
-a**10/(5*x**5) - 20*a**9*b/(9*x**(9/2)) - 45*a**8*b**2/(4*x**4) - 240*a**
7*b**3/(7*x**(7/2)) - 70*a**6*b**4/x**3 - 504*a**5*b**5/(5*x**(5/2)) - 105
*a**4*b**6/x**2 - 80*a**3*b**7/x**(3/2) - 45*a**2*b**8/x - 20*a*b**9/sqrt(
x) + b**10*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx = b^{10}\log\left(x\right) - \frac{25200\,ab^9x^{\frac{9}{2}} + 56700\,a^2b^8x^4 + 100800\,a^3b^7x^{\frac{7}{2}} + 132300\,a^4b^6x^3 + 127008\,a^5b^5x^{\frac{5}{2}} + 88200\,a^6b^4x^2 + 432}{1260\,x^5}$$

input
```
integrate((a+b*x^(1/2))^10/x^6,x, algorithm="maxima")
```

output
```
b^10*log(x) - 1/1260*(25200*a*b^9*x^(9/2) + 56700*a^2*b^8*x^4 + 100800*a^3
*b^7*x^(7/2) + 132300*a^4*b^6*x^3 + 127008*a^5*b^5*x^(5/2) + 88200*a^6*b^4
*x^2 + 43200*a^7*b^3*x^(3/2) + 14175*a^8*b^2*x + 2800*a^9*b*sqrt(x) + 252*
a^10)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx = b^{10}\log\left(|x|\right) - \frac{25200\,ab^9x^{\frac{9}{2}} + 56700\,a^2b^8x^4 + 100800\,a^3b^7x^{\frac{7}{2}} + 132300\,a^4b^6x^3 + 127008\,a^5b^5x^{\frac{5}{2}} + 88200\,a^6b^4x^2 + 432}{1260\,x^5}$$

input
```
integrate((a+b*x^(1/2))^10/x^6,x, algorithm="giac")
```

output
```
b^10*log(abs(x)) - 1/1260*(25200*a*b^9*x^(9/2) + 56700*a^2*b^8*x^4 + 10080
0*a^3*b^7*x^(7/2) + 132300*a^4*b^6*x^3 + 127008*a^5*b^5*x^(5/2) + 88200*a^
6*b^4*x^2 + 43200*a^7*b^3*x^(3/2) + 14175*a^8*b^2*x + 2800*a^9*b*sqrt(x) +
 252*a^10)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx = 2\,b^{10}\,\ln\left(\sqrt{x}\right) - \frac{\frac{a^{10}}{5} + \frac{45\,a^8\,b^2\,x}{4} + \frac{20\,a^9\,b\,\sqrt{x}}{9} + 20\,a\,b^9\,x^{9/2} + 70\,a^6\,b^4\,x^2 + 105\,a^4\,b^6\,x^3 + 45\,a^2\,b^8\,x^4 + \frac{240\,a^7\,b^3\,x^{3/2}}{7} + \frac{504\,a^5\,b^5}{5}}{x^5}$$

input
```
int((a + b*x^(1/2))^10/x^6,x)
```

output
```
2*b^10*log(x^(1/2)) - (a^10/5 + (45*a^8*b^2*x)/4 + (20*a^9*b*x^(1/2))/9 +
20*a*b^9*x^(9/2) + 70*a^6*b^4*x^2 + 105*a^4*b^6*x^3 + 45*a^2*b^8*x^4 + (24
0*a^7*b^3*x^(3/2))/7 + (504*a^5*b^5*x^(5/2))/5 + 80*a^3*b^7*x^(7/2))/x^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}\,dx = \frac{1260\sqrt{x}\log(x)\,b^{10}x^5 - 252\sqrt{x}\,a^{10} - 14175\sqrt{x}\,a^8b^2x - 88200\sqrt{x}\,a^6b^4x^2 - 132300\sqrt{x}\,a^4b^6x^3 - 56700\sqrt{x}}{1260\sqrt{x}\,x^5}$$

input
```
int((a+b*x^(1/2))^10/x^6,x)
```

output
```
(1260*sqrt(x)*log(x)*b**10*x**5 - 252*sqrt(x)*a**10 - 14175*sqrt(x)*a**8*b
**2*x - 88200*sqrt(x)*a**6*b**4*x**2 - 132300*sqrt(x)*a**4*b**6*x**3 - 567
00*sqrt(x)*a**2*b**8*x**4 - 2800*a**9*b*x - 43200*a**7*b**3*x**2 - 127008*
a**5*b**5*x**3 - 100800*a**3*b**7*x**4 - 25200*a*b**9*x**5)/(1260*sqrt(x)*
x**5)
```

3.54 $\int \frac{(a+b\sqrt{x})^{10}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{(a+b\sqrt{x})^{10}}{x^7}\,dx = -\frac{(a+b\sqrt{x})^{11}}{6ax^6} + \frac{b(a+b\sqrt{x})^{11}}{66a^2x^{11/2}}$$

output
```
-1/6*(a+b*x^(1/2))^11/a/x^6+1/66*b*(a+b*x^(1/2))^11/a^2/x^(11/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(46) = 92.

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.70

$$\int \frac{(a+b\sqrt{x})^{10}}{x^7}\,dx$$
$$= \frac{-11a^{10} - 120a^9b\sqrt{x} - 594a^8b^2x - 1760a^7b^3x^{3/2} - 3465a^6b^4x^2 - 4752a^5b^5x^{5/2} - 4620a^4b^6x^3 - 3168a^3b^7}{66x^6}$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^7,x]
```

output
```
(-11*a^10 - 120*a^9*b*Sqrt[x] - 594*a^8*b^2*x - 1760*a^7*b^3*x^(3/2) - 346
5*a^6*b^4*x^2 - 4752*a^5*b^5*x^(5/2) - 4620*a^4*b^6*x^3 - 3168*a^3*b^7*x^(
7/2) - 1485*a^2*b^8*x^4 - 440*a*b^9*x^(9/2) - 66*b^10*x^5)/(66*x^6)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{13/2}}d\sqrt{x}$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{b\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}d\sqrt{x}}{12a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{12ax^6}\right)$$

$$\Big\downarrow \text{48}$$

$$2\left(\frac{b\left(a+b\sqrt{x}\right)^{11}}{132a^2x^{11/2}}-\frac{\left(a+b\sqrt{x}\right)^{11}}{12ax^6}\right)$$

input
```
Int[(a + b*Sqrt[x])^10/x^7,x]
```

output
```
2*(-1/12*(a + b*Sqrt[x])^11/(a*x^6) + (b*(a + b*Sqrt[x])^11)/(132*a^2*x^(1
1/2)))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 3.78 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.46

method	result
derivativedivides	$-\frac{70a^4b^6}{x^3}-\frac{72a^5b^5}{x^{\frac{7}{2}}}-\frac{a^{10}}{6x^6}-\frac{105a^6b^4}{2x^4}-\frac{20a\,b^9}{3x^{\frac{3}{2}}}-\frac{9a^8b^2}{x^5}-\frac{b^{10}}{x}-\frac{80a^7b^3}{3x^{\frac{9}{2}}}-\frac{48a^3b^7}{x^{\frac{5}{2}}}-\frac{45a^2b^8}{2x^2}-\frac{20a^9b}{11x^{\frac{11}{2}}}$
default	$-\frac{70a^4b^6}{x^3}-\frac{72a^5b^5}{x^{\frac{7}{2}}}-\frac{a^{10}}{6x^6}-\frac{105a^6b^4}{2x^4}-\frac{20a\,b^9}{3x^{\frac{3}{2}}}-\frac{9a^8b^2}{x^5}-\frac{b^{10}}{x}-\frac{80a^7b^3}{3x^{\frac{9}{2}}}-\frac{48a^3b^7}{x^{\frac{5}{2}}}-\frac{45a^2b^8}{2x^2}-\frac{20a^9b}{11x^{\frac{11}{2}}}$
trager	$\frac{(-1+x)(a^{10}x^5+54a^8b^2x^5+315a^6b^4x^5+420a^4b^6x^5+135a^2b^8x^5+6b^{10}x^5+a^{10}x^4+54a^8b^2x^4+315a^6b^4x^4+420a^4b^6x^4+}{6x^6}$
orering	$-\frac{(-770b^{18}x^9+891a^2b^{16}x^8-7260a^4b^{14}x^7+13585a^6b^{12}x^6-18900a^8b^{10}x^5+17850a^{10}b^8x^4-11400a^{12}b^6x^3+4725a^{14}b}{330x^6(-b^2x+a^2)^9}$

input
```
int((a+b*x^(1/2))^10/x^7,x,method=_RETURNVERBOSE)
```

output
```
-70*a^4*b^6/x^3-72*a^5*b^5/x^(7/2)-1/6*a^10/x^6-105/2*a^6*b^4/x^4-20/3*a*b
^9/x^(3/2)-9*a^8*b^2/x^5-b^10/x-80/3*a^7*b^3/x^(9/2)-48*a^3*b^7/x^(5/2)-45
/2*a^2*b^8/x^2-20/11*a^9*b/x^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(36) = 72$.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.46

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx = -\frac{66\,b^{10}x^5+1485\,a^2b^8x^4+4620\,a^4b^6x^3+3465\,a^6b^4x^2+594\,a^8b^2x+11\,a^{10}+8\,(55\,ab^9x^4+396\,a^3b^7x^3+}{66\,x^6}$$

input
```
integrate((a+b*x^(1/2))^10/x^7,x, algorithm="fricas")
```

output
```
-1/66*(66*b^10*x^5 + 1485*a^2*b^8*x^4 + 4620*a^4*b^6*x^3 + 3465*a^6*b^4*x^
2 + 594*a^8*b^2*x + 11*a^10 + 8*(55*a*b^9*x^4 + 396*a^3*b^7*x^3 + 594*a^5*
b^5*x^2 + 220*a^7*b^3*x + 15*a^9*b)*sqrt(x))/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(37) = 74$.

Time = 0.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.91

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx = -\frac{a^{10}}{6x^6}-\frac{20a^9b}{11x^{\frac{11}{2}}}-\frac{9a^8b^2}{x^5}-\frac{80a^7b^3}{3x^{\frac{9}{2}}}-\frac{105a^6b^4}{2x^4} -\frac{72a^5b^5}{x^{\frac{7}{2}}}-\frac{70a^4b^6}{x^3}-\frac{48a^3b^7}{x^{\frac{5}{2}}}-\frac{45a^2b^8}{2x^2}-\frac{20ab^9}{3x^{\frac{3}{2}}}-\frac{b^{10}}{x}$$

input
```
integrate((a+b*x**(1/2))**10/x**7,x)
```

output
```
-a**10/(6*x**6) - 20*a**9*b/(11*x**(11/2)) - 9*a**8*b**2/x**5 - 80*a**7*b*
*3/(3*x**(9/2)) - 105*a**6*b**4/(2*x**4) - 72*a**5*b**5/x**(7/2) - 70*a**4
*b**6/x**3 - 48*a**3*b**7/x**(5/2) - 45*a**2*b**8/(2*x**2) - 20*a*b**9/(3*
x**(3/2)) - b**10/x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx = -\frac{66\,b^{10}x^5 + 440\,ab^9x^{\frac{9}{2}} + 1485\,a^2b^8x^4 + 3168\,a^3b^7x^{\frac{7}{2}} + 4620\,a^4b^6x^3 + 4752\,a^5b^5x^{\frac{5}{2}} + 3465\,a^6b^4x^2 + 1760}{66\,x^6}$$

input
```
integrate((a+b*x^(1/2))^10/x^7,x, algorithm="maxima")
```

output
```
-1/66*(66*b^10*x^5 + 440*a*b^9*x^(9/2) + 1485*a^2*b^8*x^4 + 3168*a^3*b^7*x
^(7/2) + 4620*a^4*b^6*x^3 + 4752*a^5*b^5*x^(5/2) + 3465*a^6*b^4*x^2 + 1760
*a^7*b^3*x^(3/2) + 594*a^8*b^2*x + 120*a^9*b*sqrt(x) + 11*a^10)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx = -\frac{66\,b^{10}x^5 + 440\,ab^9x^{\frac{9}{2}} + 1485\,a^2b^8x^4 + 3168\,a^3b^7x^{\frac{7}{2}} + 4620\,a^4b^6x^3 + 4752\,a^5b^5x^{\frac{5}{2}} + 3465\,a^6b^4x^2 + 1760}{66\,x^6}$$

input
```
integrate((a+b*x^(1/2))^10/x^7,x, algorithm="giac")
```

output
```
-1/66*(66*b^10*x^5 + 440*a*b^9*x^(9/2) + 1485*a^2*b^8*x^4 + 3168*a^3*b^7*x
^(7/2) + 4620*a^4*b^6*x^3 + 4752*a^5*b^5*x^(5/2) + 3465*a^6*b^4*x^2 + 1760
*a^7*b^3*x^(3/2) + 594*a^8*b^2*x + 120*a^9*b*sqrt(x) + 11*a^10)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.41

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx = -\frac{\frac{a^{10}}{6}+b^{10}\,x^5+9\,a^8\,b^2\,x+\frac{20\,a^9\,b\,\sqrt{x}}{11}+\frac{20\,a\,b^9\,x^{9/2}}{3}+\frac{105\,a^6\,b^4\,x^2}{2}+70\,a^4\,b^6\,x^3+\frac{45\,a^2\,b^8\,x^4}{2}+\frac{80\,a^7\,b^3\,x^{3/2}}{3}+72\,a^5}{x^6}$$

input
```
int((a + b*x^(1/2))^10/x^7,x)
```

output
```
-(a^10/6 + b^10*x^5 + 9*a^8*b^2*x + (20*a^9*b*x^(1/2))/11 + (20*a*b^9*x^(9
/2))/3 + (105*a^6*b^4*x^2)/2 + 70*a^4*b^6*x^3 + (45*a^2*b^8*x^4)/2 + (80*a
^7*b^3*x^(3/2))/3 + 72*a^5*b^5*x^(5/2) + 48*a^3*b^7*x^(7/2))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}\,dx = \frac{-11\sqrt{x}\,a^{10}-594\sqrt{x}\,a^8b^2x-3465\sqrt{x}\,a^6b^4x^2-4620\sqrt{x}\,a^4b^6x^3-1485\sqrt{x}\,a^2b^8x^4-66\sqrt{x}\,b^{10}x^5-120a^9}{66\sqrt{x}\,x^6}$$

input
```
int((a+b*x^(1/2))^10/x^7,x)
```

output
```
( - 11*sqrt(x)*a**10 - 594*sqrt(x)*a**8*b**2*x - 3465*sqrt(x)*a**6*b**4*x**
*2 - 4620*sqrt(x)*a**4*b**6*x**3 - 1485*sqrt(x)*a**2*b**8*x**4 - 66*sqrt(x
)*b**10*x**5 - 120*a**9*b*x - 1760*a**7*b**3*x**2 - 4752*a**5*b**5*x**3 -
3168*a**3*b**7*x**4 - 440*a*b**9*x**5)/(66*sqrt(x)*x**6)
```

3.55 $\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx=-\frac{\left(a+b\sqrt{x}\right)^{11}}{7ax^7}+\frac{3b\left(a+b\sqrt{x}\right)^{11}}{91a^2x^{13/2}}-\frac{b^2\left(a+b\sqrt{x}\right)^{11}}{182a^3x^6}+\frac{b^3\left(a+b\sqrt{x}\right)^{11}}{2002a^4x^{11/2}}$$

output
```
-1/7*(a+b*x^(1/2))^11/a/x^7+3/91*b*(a+b*x^(1/2))^11/a^2/x^(13/2)-1/182*b^2
*(a+b*x^(1/2))^11/a^3/x^6+1/2002*b^3*(a+b*x^(1/2))^11/a^4/x^(11/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx$$
$$=\frac{-286a^{10}-3080a^9b\sqrt{x}-15015a^8b^2x-43680a^7b^3x^{3/2}-84084a^6b^4x^2-112112a^5b^5x^{5/2}-105105a^4b^6x^3}{2002x^7}$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^8,x]
```

output
```
(-286*a^10 - 3080*a^9*b*Sqrt[x] - 15015*a^8*b^2*x - 43680*a^7*b^3*x^(3/2)
- 84084*a^6*b^4*x^2 - 112112*a^5*b^5*x^(5/2) - 105105*a^4*b^6*x^3 - 68640*
a^3*b^7*x^(7/2) - 30030*a^2*b^8*x^4 - 8008*a*b^9*x^(9/2) - 1001*b^10*x^5)/
(2002*x^7)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{15/2}}d\sqrt{x}$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{3b\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}d\sqrt{x}}{14a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{14ax^7}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{3b\left(-\frac{2b\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{13/2}}d\sqrt{x}}{13a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{13ax^{13/2}}\right)}{14a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{14ax^7}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{(a+b\sqrt{x})^{10}}{x^6}d\sqrt{x}}{12a}-\frac{(a+b\sqrt{x})^{11}}{12ax^6}\right)}{13a}-\frac{(a+b\sqrt{x})^{11}}{13ax^{13/2}}\right)}{14a}-\frac{(a+b\sqrt{x})^{11}}{14ax^7}\right)$$

$$\Big\downarrow \text{48}$$

$$2\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+b\sqrt{x})^{11}}{132a^2x^{11/2}}-\frac{(a+b\sqrt{x})^{11}}{12ax^6}\right)}{13a}-\frac{(a+b\sqrt{x})^{11}}{13ax^{13/2}}\right)}{14a}-\frac{(a+b\sqrt{x})^{11}}{14ax^7}\right)$$

input
```
Int[(a + b*Sqrt[x])^10/x^8,x]
```

output
```
2*((-3*b*((-2*b*(-1/12*(a + b*Sqrt[x])^11/(a*x^6) + (b*(a + b*Sqrt[x])^11)
/(132*a^2*x^(11/2))))/(13*a) - (a + b*Sqrt[x])^11/(13*a*x^(13/2))))/(14*a)
 - (a + b*Sqrt[x])^11/(14*a*x^7))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

method	result
derivativedivides	$-\frac{b^{10}}{2x^2}-\frac{20a^9b}{13x^{\frac{13}{2}}}-\frac{15a^8b^2}{2x^6}-\frac{105a^4b^6}{2x^4}-\frac{4a\,b^9}{x^{\frac{5}{2}}}-\frac{42a^6b^4}{x^5}-\frac{56a^5b^5}{x^{\frac{9}{2}}}-\frac{a^{10}}{7x^7}-\frac{15a^2b^8}{x^3}-\frac{240a^3b^7}{7x^{\frac{7}{2}}}-\frac{240a}{11x}$
default	$-\frac{b^{10}}{2x^2}-\frac{20a^9b}{13x^{\frac{13}{2}}}-\frac{15a^8b^2}{2x^6}-\frac{105a^4b^6}{2x^4}-\frac{4a\,b^9}{x^{\frac{5}{2}}}-\frac{42a^6b^4}{x^5}-\frac{56a^5b^5}{x^{\frac{9}{2}}}-\frac{a^{10}}{7x^7}-\frac{15a^2b^8}{x^3}-\frac{240a^3b^7}{7x^{\frac{7}{2}}}-\frac{240a}{11x}$
orering	$-\frac{(-11011b^{18}x^9+55770a^2b^{16}x^8-182325a^4b^{14}x^7+371280a^6b^{12}x^6-506730a^8b^{10}x^5+472500a^{10}b^8x^4-298425a^{12}b^6x^3}{10010x^7(-b^2x+a^2)^9}$
trager	$\frac{(-1+x)(2a^{10}x^6+105a^8b^2x^6+588a^6b^4x^6+735a^4b^6x^6+210a^2b^8x^6+7b^{10}x^6+2a^{10}x^5+105a^8b^2x^5+588a^6b^4x^5+735a^4b}{}$

input
```
int((a+b*x^(1/2))^10/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^10/x^2-20/13*a^9*b/x^(13/2)-15/2*a^8*b^2/x^6-105/2*a^4*b^6/x^4-4*a*
b^9/x^(5/2)-42*a^6*b^4/x^5-56*a^5*b^5/x^(9/2)-1/7*a^10/x^7-15*a^2*b^8/x^3-
240/7*a^3*b^7/x^(7/2)-240/11*a^7*b^3/x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx =$$
$$-\frac{1001\,b^{10}x^5+30030\,a^2b^8x^4+105105\,a^4b^6x^3+84084\,a^6b^4x^2+15015\,a^8b^2x+286\,a^{10}+8\,(1001\,ab^9x^4+}{2002\,x^7}$$

input
```
integrate((a+b*x^(1/2))^10/x^8,x, algorithm="fricas")
```

output
```
-1/2002*(1001*b^10*x^5 + 30030*a^2*b^8*x^4 + 105105*a^4*b^6*x^3 + 84084*a^
6*b^4*x^2 + 15015*a^8*b^2*x + 286*a^10 + 8*(1001*a*b^9*x^4 + 8580*a^3*b^7*
x^3 + 14014*a^5*b^5*x^2 + 5460*a^7*b^3*x + 385*a^9*b)*sqrt(x))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx = -\frac{a^{10}}{7x^7} - \frac{20a^9b}{13x^{\frac{13}{2}}} - \frac{15a^8b^2}{2x^6} - \frac{240a^7b^3}{11x^{\frac{11}{2}}} - \frac{42a^6b^4}{x^5} - \frac{56a^5b^5}{x^{\frac{9}{2}}} - \frac{105a^4b^6}{2x^4} - \frac{240a^3b^7}{7x^{\frac{7}{2}}} - \frac{15a^2b^8}{x^3} - \frac{4ab^9}{x^{\frac{5}{2}}} - \frac{b^{10}}{2x^2}$$

input
```
integrate((a+b*x**(1/2))**10/x**8,x)
```

output
```
-a**10/(7*x**7) - 20*a**9*b/(13*x**(13/2)) - 15*a**8*b**2/(2*x**6) - 240*a
**7*b**3/(11*x**(11/2)) - 42*a**6*b**4/x**5 - 56*a**5*b**5/x**(9/2) - 105*
a**4*b**6/(2*x**4) - 240*a**3*b**7/(7*x**(7/2)) - 15*a**2*b**8/x**3 - 4*a*
b**9/x**(5/2) - b**10/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx = -\frac{1001\,b^{10}x^5 + 8008\,ab^9x^{\frac{9}{2}} + 30030\,a^2b^8x^4 + 68640\,a^3b^7x^{\frac{7}{2}} + 105105\,a^4b^6x^3 + 112112\,a^5b^5x^{\frac{5}{2}} + 84084\,a^6}{2002\,x^7}$$

input
```
integrate((a+b*x^(1/2))^10/x^8,x, algorithm="maxima")
```

output
```
-1/2002*(1001*b^10*x^5 + 8008*a*b^9*x^(9/2) + 30030*a^2*b^8*x^4 + 68640*a^
3*b^7*x^(7/2) + 105105*a^4*b^6*x^3 + 112112*a^5*b^5*x^(5/2) + 84084*a^6*b^
4*x^2 + 43680*a^7*b^3*x^(3/2) + 15015*a^8*b^2*x + 3080*a^9*b*sqrt(x) + 286
*a^10)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx = -\frac{1001\,b^{10}x^5 + 8008\,ab^9x^{\frac{9}{2}} + 30030\,a^2b^8x^4 + 68640\,a^3b^7x^{\frac{7}{2}} + 105105\,a^4b^6x^3 + 112112\,a^5b^5x^{\frac{5}{2}} + 84084\,a^6}{2002\,x^7}$$

input
```
integrate((a+b*x^(1/2))^10/x^8,x, algorithm="giac")
```

output
```
-1/2002*(1001*b^10*x^5 + 8008*a*b^9*x^(9/2) + 30030*a^2*b^8*x^4 + 68640*a^
3*b^7*x^(7/2) + 105105*a^4*b^6*x^3 + 112112*a^5*b^5*x^(5/2) + 84084*a^6*b^
4*x^2 + 43680*a^7*b^3*x^(3/2) + 15015*a^8*b^2*x + 3080*a^9*b*sqrt(x) + 286
*a^10)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx = -\frac{\frac{a^{10}}{7} + \frac{b^{10}\,x^5}{2} + \frac{15\,a^8\,b^2\,x}{2} + \frac{20\,a^9\,b\,\sqrt{x}}{13} + 4\,a\,b^9\,x^{9/2} + 42\,a^6\,b^4\,x^2 + \frac{105\,a^4\,b^6\,x^3}{2} + 15\,a^2\,b^8\,x^4 + \frac{240\,a^7\,b^3\,x^{3/2}}{11} + 56}{x^7}$$

input
```
int((a + b*x^(1/2))^10/x^8,x)
```

output
```
-(a^10/7 + (b^10*x^5)/2 + (15*a^8*b^2*x)/2 + (20*a^9*b*x^(1/2))/13 + 4*a*b
^9*x^(9/2) + 42*a^6*b^4*x^2 + (105*a^4*b^6*x^3)/2 + 15*a^2*b^8*x^4 + (240*
a^7*b^3*x^(3/2))/11 + 56*a^5*b^5*x^(5/2) + (240*a^3*b^7*x^(7/2))/7)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}\,dx$$
$$= \frac{-286\sqrt{x}\,a^{10}-15015\sqrt{x}\,a^8b^2x-84084\sqrt{x}\,a^6b^4x^2-105105\sqrt{x}\,a^4b^6x^3-30030\sqrt{x}\,a^2b^8x^4-1001\sqrt{x}\,b^{10}x}{2002\sqrt{x}\,x^7}$$

input
```
int((a+b*x^(1/2))^10/x^8,x)
```

output
```
( - 286*sqrt(x)*a**10 - 15015*sqrt(x)*a**8*b**2*x - 84084*sqrt(x)*a**6*b**
4*x**2 - 105105*sqrt(x)*a**4*b**6*x**3 - 30030*sqrt(x)*a**2*b**8*x**4 - 10
01*sqrt(x)*b**10*x**5 - 3080*a**9*b*x - 43680*a**7*b**3*x**2 - 112112*a**5
*b**5*x**3 - 68640*a**3*b**7*x**4 - 8008*a*b**9*x**5)/(2002*sqrt(x)*x**7)
```

3.56 $\int \frac{(a+b\sqrt{x})^{10}}{x^9} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{(a+b\sqrt{x})^{10}}{x^9} \, dx = -\frac{(a+b\sqrt{x})^{11}}{8ax^8} + \frac{b(a+b\sqrt{x})^{11}}{24a^2x^{15/2}} - \frac{b^2(a+b\sqrt{x})^{11}}{84a^3x^7} + \frac{b^3(a+b\sqrt{x})^{11}}{364a^4x^{13/2}} - \frac{b^4(a+b\sqrt{x})^{11}}{2184a^5x^6} + \frac{b^5(a+b\sqrt{x})^{11}}{24024a^6x^{11/2}}$$

output
```
-1/8*(a+b*x^(1/2))^11/a/x^8+1/24*b*(a+b*x^(1/2))^11/a^2/x^(15/2)-1/84*b^2*
(a+b*x^(1/2))^11/a^3/x^7+1/364*b^3*(a+b*x^(1/2))^11/a^4/x^(13/2)-1/2184*b^
4*(a+b*x^(1/2))^11/a^5/x^6+1/24024*b^5*(a+b*x^(1/2))^11/a^6/x^(11/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\sqrt{x})^{10}}{x^9} \, dx = \frac{-3003a^{10} - 32032a^9b\sqrt{x} - 154440a^8b^2x - 443520a^7b^3x^{3/2} - 840840a^6b^4x^2 - 1100736a^5b^5x^{5/2} - 10090}{24024x^8}$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^9,x]
```

output
```
(-3003*a^10 - 32032*a^9*b*Sqrt[x] - 154440*a^8*b^2*x - 443520*a^7*b^3*x^(3
/2) - 840840*a^6*b^4*x^2 - 1100736*a^5*b^5*x^(5/2) - 1009008*a^4*b^6*x^3 -
 640640*a^3*b^7*x^(7/2) - 270270*a^2*b^8*x^4 - 68640*a*b^9*x^(9/2) - 8008*
b^10*x^5)/(24024*x^8)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{17/2}}d\sqrt{x}$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{5b\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^8}d\sqrt{x}}{16a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{16ax^8}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{5b\left(-\frac{4b\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{15/2}}d\sqrt{x}}{15a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{15ax^{15/2}}\right)}{16a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{16ax^8}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\int\frac{\left(a+b\sqrt{x}\right)^{10}}{x^7}d\sqrt{x}}{14a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{14ax^7}\right)}{15a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{15ax^{15/2}}\right)}{16a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{16ax^8}\right)$$

$$\Big\downarrow \; 55$$

$$2\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\int\frac{\left(a+b\sqrt{x}\right)^{10}}{x^{13/2}}d\sqrt{x}}{13a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{13ax^{13/2}}\right)}{14a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{14ax^7}\right)}{15a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{15ax^{15/2}}\right)}{16a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{16ax^8}\right)$$

$$\Big\downarrow \; 55$$

$$2\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{\left(a+b\sqrt{x}\right)^{10}}{x^6}d\sqrt{x}}{12a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{12ax^6}\right)}{13a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{13ax^{13/2}}\right)}{14a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{14ax^7}\right)}{15a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{15ax^{15/2}}\right)}{16a}-\frac{\left(a+b\sqrt{x}\right)^{11}}{16ax^8}\right)$$

$$\Big\downarrow \; 48$$

$$2\left(-\frac{5b\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(\frac{b(a+b\sqrt{x})^{11}}{132a^2x^{11/2}}-\frac{(a+b\sqrt{x})^{11}}{12ax^6}\right)}{13a}-\frac{(a+b\sqrt{x})^{11}}{13ax^{13/2}}\right)}{14a}-\frac{(a+b\sqrt{x})^{11}}{14ax^7}\right)}{15a}-\frac{(a+b\sqrt{x})^{11}}{15ax^{15/2}}\right)}{16a}-\frac{(a+b\sqrt{x})^{11}}{16ax^8}\right)$$

input
```
Int[(a + b*Sqrt[x])^10/x^9,x]
```

output
```
2*((-5*b*((-4*b*((-3*b*((-2*b*(-1/12*(a + b*Sqrt[x])^11/(a*x^6) + (b*(a +
b*Sqrt[x])^11)/(132*a^2*x^(11/2))))/(13*a) - (a + b*Sqrt[x])^11/(13*a*x^(1
3/2))))/(14*a) - (a + b*Sqrt[x])^11/(14*a*x^7)))/(15*a) - (a + b*Sqrt[x])^
11/(15*a*x^(15/2))))/(16*a) - (a + b*Sqrt[x])^11/(16*a*x^8))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{504a^5b^5}{11x^{\frac{11}{2}}}-\frac{45a^8b^2}{7x^7}-\frac{b^{10}}{3x^3}-\frac{a^{10}}{8x^8}-\frac{240a^7b^3}{13x^{\frac{13}{2}}}-\frac{4a^9b}{3x^{\frac{15}{2}}}-\frac{35a^6b^4}{x^6}-\frac{42a^4b^6}{x^5}-\frac{45a^2b^8}{4x^4}-\frac{80a^3b^7}{3x^{\frac{9}{2}}}-\frac{20a}{7x}$
default	$-\frac{504a^5b^5}{11x^{\frac{11}{2}}}-\frac{45a^8b^2}{7x^7}-\frac{b^{10}}{3x^3}-\frac{a^{10}}{8x^8}-\frac{240a^7b^3}{13x^{\frac{13}{2}}}-\frac{4a^9b}{3x^{\frac{15}{2}}}-\frac{35a^6b^4}{x^6}-\frac{42a^4b^6}{x^5}-\frac{45a^2b^8}{4x^4}-\frac{80a^3b^7}{3x^{\frac{9}{2}}}-\frac{20a}{7x}$
orering	$-\frac{(-85800b^{18}x^9+559130a^2b^{16}x^8-1867320a^4b^{14}x^7+3854340a^6b^{12}x^6-5284296a^8b^{10}x^5+4930875a^{10}b^8x^4-3110940}{120120x^8(-b^2x+a^2)^9}$
trager	$\frac{(-1+x)(21a^{10}x^7+1080a^8b^2x^7+5880a^6b^4x^7+7056a^4b^6x^7+1890a^2b^8x^7+56b^{10}x^7+21a^{10}x^6+1080a^8b^2x^6+5880a^6b^4}{}$

input
```
int((a+b*x^(1/2))^10/x^9,x,method=_RETURNVERBOSE)
```

output
```
-504/11*a^5*b^5/x^(11/2)-45/7*a^8*b^2/x^7-1/3*b^10/x^3-1/8*a^10/x^8-240/13
*a^7*b^3/x^(13/2)-4/3*a^9*b/x^(15/2)-35*a^6*b^4/x^6-42*a^4*b^6/x^5-45/4*a^
2*b^8/x^4-80/3*a^3*b^7/x^(9/2)-20/7*a*b^9/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx = -\frac{8008\,b^{10}x^5+270270\,a^2b^8x^4+1009008\,a^4b^6x^3+840840\,a^6b^4x^2+154440\,a^8b^2x+3003\,a^{10}+32\,(2145\,a}{24024\,x^8}$$

input
```
integrate((a+b*x^(1/2))^10/x^9,x, algorithm="fricas")
```

output
```
-1/24024*(8008*b^10*x^5 + 270270*a^2*b^8*x^4 + 1009008*a^4*b^6*x^3 + 84084
0*a^6*b^4*x^2 + 154440*a^8*b^2*x + 3003*a^10 + 32*(2145*a*b^9*x^4 + 20020*
a^3*b^7*x^3 + 34398*a^5*b^5*x^2 + 13860*a^7*b^3*x + 1001*a^9*b)*sqrt(x))/x
^8
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx = -\frac{a^{10}}{8x^8}-\frac{4a^9b}{3x^{\frac{15}{2}}}-\frac{45a^8b^2}{7x^7}-\frac{240a^7b^3}{13x^{\frac{13}{2}}}-\frac{35a^6b^4}{x^6}-\frac{504a^5b^5}{11x^{\frac{11}{2}}}-\frac{42a^4b^6}{x^5}-\frac{80a^3b^7}{3x^{\frac{9}{2}}}-\frac{45a^2b^8}{4x^4}-\frac{20ab^9}{7x^{\frac{7}{2}}}-\frac{b^{10}}{3x^3}$$

input
```
integrate((a+b*x**(1/2))**10/x**9,x)
```

output
```
-a**10/(8*x**8) - 4*a**9*b/(3*x**(15/2)) - 45*a**8*b**2/(7*x**7) - 240*a**
7*b**3/(13*x**(13/2)) - 35*a**6*b**4/x**6 - 504*a**5*b**5/(11*x**(11/2)) -
 42*a**4*b**6/x**5 - 80*a**3*b**7/(3*x**(9/2)) - 45*a**2*b**8/(4*x**4) - 2
0*a*b**9/(7*x**(7/2)) - b**10/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx = -\frac{8008\,b^{10}x^5+68640\,ab^9x^{\frac{9}{2}}+270270\,a^2b^8x^4+640640\,a^3b^7x^{\frac{7}{2}}+1009008\,a^4b^6x^3+1100736\,a^5b^5x^{\frac{5}{2}}+84}{24024\,x^8}$$

input
```
integrate((a+b*x^(1/2))^10/x^9,x, algorithm="maxima")
```

output
```
-1/24024*(8008*b^10*x^5 + 68640*a*b^9*x^(9/2) + 270270*a^2*b^8*x^4 + 64064
0*a^3*b^7*x^(7/2) + 1009008*a^4*b^6*x^3 + 1100736*a^5*b^5*x^(5/2) + 840840
*a^6*b^4*x^2 + 443520*a^7*b^3*x^(3/2) + 154440*a^8*b^2*x + 32032*a^9*b*sqr
t(x) + 3003*a^10)/x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx = -\frac{8008\,b^{10}x^5+68640\,ab^9x^{\frac{9}{2}}+270270\,a^2b^8x^4+640640\,a^3b^7x^{\frac{7}{2}}+1009008\,a^4b^6x^3+1100736\,a^5b^5x^{\frac{5}{2}}+84}{24024\,x^8}$$

input
```
integrate((a+b*x^(1/2))^10/x^9,x, algorithm="giac")
```

output
```
-1/24024*(8008*b^10*x^5 + 68640*a*b^9*x^(9/2) + 270270*a^2*b^8*x^4 + 64064
0*a^3*b^7*x^(7/2) + 1009008*a^4*b^6*x^3 + 1100736*a^5*b^5*x^(5/2) + 840840
*a^6*b^4*x^2 + 443520*a^7*b^3*x^(3/2) + 154440*a^8*b^2*x + 32032*a^9*b*sqr
t(x) + 3003*a^10)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx = -\frac{\frac{a^{10}}{8}+\frac{b^{10}\,x^5}{3}+\frac{45\,a^8\,b^2\,x}{7}+\frac{4\,a^9\,b\,\sqrt{x}}{3}+\frac{20\,a\,b^9\,x^{9/2}}{7}+35\,a^6\,b^4\,x^2+42\,a^4\,b^6\,x^3+\frac{45\,a^2\,b^8\,x^4}{4}+\frac{240\,a^7\,b^3\,x^{3/2}}{13}+\frac{504\,a^5}{1}}{x^8}$$

input
```
int((a + b*x^(1/2))^10/x^9,x)
```

output
```
-(a^10/8 + (b^10*x^5)/3 + (45*a^8*b^2*x)/7 + (4*a^9*b*x^(1/2))/3 + (20*a*b
^9*x^(9/2))/7 + 35*a^6*b^4*x^2 + 42*a^4*b^6*x^3 + (45*a^2*b^8*x^4)/4 + (24
0*a^7*b^3*x^(3/2))/13 + (504*a^5*b^5*x^(5/2))/11 + (80*a^3*b^7*x^(7/2))/3)
/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^9}\,dx = \frac{-3003\sqrt{x}\,a^{10}-154440\sqrt{x}\,a^8b^2x-840840\sqrt{x}\,a^6b^4x^2-1009008\sqrt{x}\,a^4b^6x^3-270270\sqrt{x}\,a^2b^8x^4-8008\sqrt{}}{24024\sqrt{x}\,x^8}$$

input
```
int((a+b*x^(1/2))^10/x^9,x)
```

output
```
( - 3003*sqrt(x)*a**10 - 154440*sqrt(x)*a**8*b**2*x - 840840*sqrt(x)*a**6*
b**4*x**2 - 1009008*sqrt(x)*a**4*b**6*x**3 - 270270*sqrt(x)*a**2*b**8*x**4
 - 8008*sqrt(x)*b**10*x**5 - 32032*a**9*b*x - 443520*a**7*b**3*x**2 - 1100
736*a**5*b**5*x**3 - 640640*a**3*b**7*x**4 - 68640*a*b**9*x**5)/(24024*sqr
t(x)*x**8)
```

3.57 $\int \frac{(a+b\sqrt{x})^{10}}{x^{10}} dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{(a+b\sqrt{x})^{10}}{x^{10}} dx = -\frac{a^{10}}{9x^9} - \frac{20a^9b}{17x^{17/2}} - \frac{45a^8b^2}{8x^8} - \frac{16a^7b^3}{x^{15/2}} - \frac{30a^6b^4}{x^7} - \frac{504a^5b^5}{13x^{13/2}} - \frac{35a^4b^6}{x^6} - \frac{240a^3b^7}{11x^{11/2}} - \frac{9a^2b^8}{x^5} - \frac{20ab^9}{9x^{9/2}} - \frac{b^{10}}{4x^4}$$

output
```
-1/9*a^10/x^9-20/17*a^9*b/x^(17/2)-45/8*a^8*b^2/x^8-16*a^7*b^3/x^(15/2)-30
*a^6*b^4/x^7-504/13*a^5*b^5/x^(13/2)-35*a^4*b^6/x^6-240/11*a^3*b^7/x^(11/2
)-9*a^2*b^8/x^5-20/9*a*b^9/x^(9/2)-1/4*b^10/x^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\sqrt{x})^{10}}{x^{10}} dx = \frac{-19448a^{10} - 205920a^9b\sqrt{x} - 984555a^8b^2x - 2800512a^7b^3x^{3/2} - 5250960a^6b^4x^2 - 6785856a^5b^5x^{5/2} - 6}{175032x^9}$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^10,x]
```

output
```
(-19448*a^10 - 205920*a^9*b*Sqrt[x] - 984555*a^8*b^2*x - 2800512*a^7*b^3*x
^(3/2) - 5250960*a^6*b^4*x^2 - 6785856*a^5*b^5*x^(5/2) - 6126120*a^4*b^6*x
^3 - 3818880*a^3*b^7*x^(7/2) - 1575288*a^2*b^8*x^4 - 388960*a*b^9*x^(9/2)
- 43758*b^10*x^5)/(175032*x^9)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{19/2}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^{10}}{x^{19/2}}+\frac{10ba^9}{x^9}+\frac{45b^2a^8}{x^{17/2}}+\frac{120b^3a^7}{x^8}+\frac{210b^4a^6}{x^{15/2}}+\frac{252b^5a^5}{x^7}+\frac{210b^6a^4}{x^{13/2}}+\frac{120b^7a^3}{x^6}+\frac{45b^8a^2}{x^{11/2}}+\frac{10b^9a}{x^5}+\frac{b^{10}}{x^{9/}}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{10}}{18x^9}-\frac{10a^9b}{17x^{17/2}}-\frac{45a^8b^2}{16x^8}-\frac{8a^7b^3}{x^{15/2}}-\frac{15a^6b^4}{x^7}-\frac{252a^5b^5}{13x^{13/2}}-\frac{35a^4b^6}{2x^6}-\frac{120a^3b^7}{11x^{11/2}}-\frac{9a^2b^8}{2x^5}-\frac{10ab^9}{9x^{9/2}}-\frac{b^{10}}{8x^4}\right)$$

input
```
Int[(a + b*Sqrt[x])^10/x^10,x]
```

output
```
2*(-1/18*a^10/x^9 - (10*a^9*b)/(17*x^(17/2)) - (45*a^8*b^2)/(16*x^8) - (8*
a^7*b^3)/x^(15/2) - (15*a^6*b^4)/x^7 - (252*a^5*b^5)/(13*x^(13/2)) - (35*a
^4*b^6)/(2*x^6) - (120*a^3*b^7)/(11*x^(11/2)) - (9*a^2*b^8)/(2*x^5) - (10*
a*b^9)/(9*x^(9/2)) - b^10/(8*x^4))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{a^{10}}{9x^9}-\frac{20a^9b}{17x^{\frac{17}{2}}}-\frac{45a^8b^2}{8x^8}-\frac{16a^7b^3}{x^{\frac{15}{2}}}-\frac{30a^6b^4}{x^7}-\frac{504a^5b^5}{13x^{\frac{13}{2}}}-\frac{35a^4b^6}{x^6}-\frac{240a^3b^7}{11x^{\frac{11}{2}}}-\frac{9a^2b^8}{x^5}-\frac{20a\,b^9}{9x^{\frac{9}{2}}}-$
default	$-\frac{a^{10}}{9x^9}-\frac{20a^9b}{17x^{\frac{17}{2}}}-\frac{45a^8b^2}{8x^8}-\frac{16a^7b^3}{x^{\frac{15}{2}}}-\frac{30a^6b^4}{x^7}-\frac{504a^5b^5}{13x^{\frac{13}{2}}}-\frac{35a^4b^6}{x^6}-\frac{240a^3b^7}{11x^{\frac{11}{2}}}-\frac{9a^2b^8}{x^5}-\frac{20a\,b^9}{9x^{\frac{9}{2}}}-$
orering	$-\frac{(-92378b^{18}x^9+668304a^2b^{16}x^8-2313156a^4b^{14}x^7+4857240a^6b^{12}x^6-6718707a^8b^{10}x^5+6298684a^{10}b^8x^4-3983190}{175032x^9(-b^2x+a^2)^9}$
trager	$(-1+x)(8a^{10}x^8+405a^8b^2x^8+2160a^6b^4x^8+2520a^4b^6x^8+648a^2b^8x^8+18b^{10}x^8+8a^{10}x^7+405a^8b^2x^7+2160a^6b^4x^7+25$

input
```
int((a+b*x^(1/2))^10/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^10/x^9-20/17*a^9*b/x^(17/2)-45/8*a^8*b^2/x^8-16*a^7*b^3/x^(15/2)-30
*a^6*b^4/x^7-504/13*a^5*b^5/x^(13/2)-35*a^4*b^6/x^6-240/11*a^3*b^7/x^(11/2
)-9*a^2*b^8/x^5-20/9*a*b^9/x^(9/2)-1/4*b^10/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx = -\frac{43758\,b^{10}x^5 + 1575288\,a^2b^8x^4 + 6126120\,a^4b^6x^3 + 5250960\,a^6b^4x^2 + 984555\,a^8b^2x + 19448\,a^{10} + 32\,(1}{175032\,x^9}$$

input
```
integrate((a+b*x^(1/2))^10/x^10,x, algorithm="fricas")
```

output
```
-1/175032*(43758*b^10*x^5 + 1575288*a^2*b^8*x^4 + 6126120*a^4*b^6*x^3 + 52
50960*a^6*b^4*x^2 + 984555*a^8*b^2*x + 19448*a^10 + 32*(12155*a*b^9*x^4 +
119340*a^3*b^7*x^3 + 212058*a^5*b^5*x^2 + 87516*a^7*b^3*x + 6435*a^9*b)*sq
rt(x))/x^9
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx = -\frac{a^{10}}{9x^9} - \frac{20a^9b}{17x^{\frac{17}{2}}} - \frac{45a^8b^2}{8x^8} - \frac{16a^7b^3}{x^{\frac{15}{2}}} - \frac{30a^6b^4}{x^7} - \frac{504a^5b^5}{13x^{\frac{13}{2}}} - \frac{35a^4b^6}{x^6} - \frac{240a^3b^7}{11x^{\frac{11}{2}}} - \frac{9a^2b^8}{x^5} - \frac{20ab^9}{9x^{\frac{9}{2}}} - \frac{b^{10}}{4x^4}$$

input
```
integrate((a+b*x**(1/2))**10/x**10,x)
```

output
```
-a**10/(9*x**9) - 20*a**9*b/(17*x**(17/2)) - 45*a**8*b**2/(8*x**8) - 16*a*
*7*b**3/x**(15/2) - 30*a**6*b**4/x**7 - 504*a**5*b**5/(13*x**(13/2)) - 35*
a**4*b**6/x**6 - 240*a**3*b**7/(11*x**(11/2)) - 9*a**2*b**8/x**5 - 20*a*b*
*9/(9*x**(9/2)) - b**10/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx = -\frac{43758\,b^{10}x^5+388960\,ab^9x^{\frac{9}{2}}+1575288\,a^2b^8x^4+3818880\,a^3b^7x^{\frac{7}{2}}+6126120\,a^4b^6x^3+6785856\,a^5b^5x^{\frac{5}{2}}}{175032\,x^9}$$

input
```
integrate((a+b*x^(1/2))^10/x^10,x, algorithm="maxima")
```

output
```
-1/175032*(43758*b^10*x^5 + 388960*a*b^9*x^(9/2) + 1575288*a^2*b^8*x^4 + 3
818880*a^3*b^7*x^(7/2) + 6126120*a^4*b^6*x^3 + 6785856*a^5*b^5*x^(5/2) + 5
250960*a^6*b^4*x^2 + 2800512*a^7*b^3*x^(3/2) + 984555*a^8*b^2*x + 205920*a
^9*b*sqrt(x) + 19448*a^10)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx = -\frac{43758\,b^{10}x^5+388960\,ab^9x^{\frac{9}{2}}+1575288\,a^2b^8x^4+3818880\,a^3b^7x^{\frac{7}{2}}+6126120\,a^4b^6x^3+6785856\,a^5b^5x^{\frac{5}{2}}}{175032\,x^9}$$

input
```
integrate((a+b*x^(1/2))^10/x^10,x, algorithm="giac")
```

output
```
-1/175032*(43758*b^10*x^5 + 388960*a*b^9*x^(9/2) + 1575288*a^2*b^8*x^4 + 3
818880*a^3*b^7*x^(7/2) + 6126120*a^4*b^6*x^3 + 6785856*a^5*b^5*x^(5/2) + 5
250960*a^6*b^4*x^2 + 2800512*a^7*b^3*x^(3/2) + 984555*a^8*b^2*x + 205920*a
^9*b*sqrt(x) + 19448*a^10)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx = -\frac{\frac{a^{10}}{9}+\frac{b^{10}\,x^5}{4}+\frac{45\,a^8\,b^2\,x}{8}+\frac{20\,a^9\,b\,\sqrt{x}}{17}+\frac{20\,a\,b^9\,x^{9/2}}{9}+30\,a^6\,b^4\,x^2+35\,a^4\,b^6\,x^3+9\,a^2\,b^8\,x^4+16\,a^7\,b^3\,x^{3/2}+\frac{504}{}}{x^9}$$

input
```
int((a + b*x^(1/2))^10/x^10,x)
```

output
```
-(a^10/9 + (b^10*x^5)/4 + (45*a^8*b^2*x)/8 + (20*a^9*b*x^(1/2))/17 + (20*a
*b^9*x^(9/2))/9 + 30*a^6*b^4*x^2 + 35*a^4*b^6*x^3 + 9*a^2*b^8*x^4 + 16*a^7
*b^3*x^(3/2) + (504*a^5*b^5*x^(5/2))/13 + (240*a^3*b^7*x^(7/2))/11)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{10}}\,dx = \frac{-19448\sqrt{x}\,a^{10}-984555\sqrt{x}\,a^8b^2x-5250960\sqrt{x}\,a^6b^4x^2-6126120\sqrt{x}\,a^4b^6x^3-1575288\sqrt{x}\,a^2b^8x^4-43}{175032\sqrt{x}}$$

input
```
int((a+b*x^(1/2))^10/x^10,x)
```

output
```
( - 19448*sqrt(x)*a**10 - 984555*sqrt(x)*a**8*b**2*x - 5250960*sqrt(x)*a**
6*b**4*x**2 - 6126120*sqrt(x)*a**4*b**6*x**3 - 1575288*sqrt(x)*a**2*b**8*x
**4 - 43758*sqrt(x)*b**10*x**5 - 205920*a**9*b*x - 2800512*a**7*b**3*x**2
- 6785856*a**5*b**5*x**3 - 3818880*a**3*b**7*x**4 - 388960*a*b**9*x**5)/(1
75032*sqrt(x)*x**9)
```

3.58 $\int \frac{(a+b\sqrt{x})^{10}}{x^{11}}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int \frac{(a+b\sqrt{x})^{10}}{x^{11}}\, dx = -\frac{a^{10}}{10x^{10}} - \frac{20a^9b}{19x^{19/2}} - \frac{5a^8b^2}{x^9} - \frac{240a^7b^3}{17x^{17/2}} - \frac{105a^6b^4}{4x^8} - \frac{168a^5b^5}{5x^{15/2}} - \frac{30a^4b^6}{x^7} - \frac{240a^3b^7}{13x^{13/2}} - \frac{15a^2b^8}{2x^6} - \frac{20ab^9}{11x^{11/2}} - \frac{b^{10}}{5x^5}$$

output
```
-1/10*a^10/x^10-20/19*a^9*b/x^(19/2)-5*a^8*b^2/x^9-240/17*a^7*b^3/x^(17/2)
-105/4*a^6*b^4/x^8-168/5*a^5*b^5/x^(15/2)-30*a^4*b^6/x^7-240/13*a^3*b^7/x^
(13/2)-15/2*a^2*b^8/x^6-20/11*a*b^9/x^(11/2)-1/5*b^10/x^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\sqrt{x})^{10}}{x^{11}}\, dx = \frac{-92378a^{10} - 972400a^9b\sqrt{x} - 4618900a^8b^2x - 13041600a^7b^3x^{3/2} - 24249225a^6b^4x^2 - 31039008a^5b^5x^{5/2}}{923780x^{10}}$$

input
```
Integrate[(a + b*Sqrt[x])^10/x^11,x]
```

output
```
(-92378*a^10 - 972400*a^9*b*Sqrt[x] - 4618900*a^8*b^2*x - 13041600*a^7*b^3
*x^(3/2) - 24249225*a^6*b^4*x^2 - 31039008*a^5*b^5*x^(5/2) - 27713400*a^4*
b^6*x^3 - 17054400*a^3*b^7*x^(7/2) - 6928350*a^2*b^8*x^4 - 1679600*a*b^9*x
^(9/2) - 184756*b^10*x^5)/(923780*x^10)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{21/2}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^{10}}{x^{21/2}}+\frac{10ba^9}{x^{10}}+\frac{45b^2a^8}{x^{19/2}}+\frac{120b^3a^7}{x^9}+\frac{210b^4a^6}{x^{17/2}}+\frac{252b^5a^5}{x^8}+\frac{210b^6a^4}{x^{15/2}}+\frac{120b^7a^3}{x^7}+\frac{45b^8a^2}{x^{13/2}}+\frac{10b^9a}{x^6}+\frac{b^{10}}{x^{11/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{10}}{20x^{10}}-\frac{10a^9b}{19x^{19/2}}-\frac{5a^8b^2}{2x^9}-\frac{120a^7b^3}{17x^{17/2}}-\frac{105a^6b^4}{8x^8}-\frac{84a^5b^5}{5x^{15/2}}-\frac{15a^4b^6}{x^7}-\frac{120a^3b^7}{13x^{13/2}}-\frac{15a^2b^8}{4x^6}-\frac{10ab^9}{11x^{11/2}}-\frac{b^{10}}{10x^5}\right.$$

input
```
Int[(a + b*Sqrt[x])^10/x^11,x]
```

output
```
2*(-1/20*a^10/x^10 - (10*a^9*b)/(19*x^(19/2)) - (5*a^8*b^2)/(2*x^9) - (120
*a^7*b^3)/(17*x^(17/2)) - (105*a^6*b^4)/(8*x^8) - (84*a^5*b^5)/(5*x^(15/2)
) - (15*a^4*b^6)/x^7 - (120*a^3*b^7)/(13*x^(13/2)) - (15*a^2*b^8)/(4*x^6)
- (10*a*b^9)/(11*x^(11/2)) - b^10/(10*x^5))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 3.85 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{a^{10}}{10x^{10}}-\frac{20a^9b}{19x^{\frac{19}{2}}}-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{\frac{17}{2}}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{\frac{15}{2}}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{\frac{13}{2}}}-\frac{15a^2b^8}{2x^6}-\frac{20a\,b^9}{11x^{\frac{11}{2}}}$
default	$-\frac{a^{10}}{10x^{10}}-\frac{20a^9b}{19x^{\frac{19}{2}}}-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{\frac{17}{2}}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{\frac{15}{2}}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{\frac{13}{2}}}-\frac{15a^2b^8}{2x^6}-\frac{20a\,b^9}{11x^{\frac{11}{2}}}$
orering	$-\frac{(-1931540b^{18}x^9+14777250a^2b^{16}x^8-52535304a^4b^{14}x^7+111965955a^6b^{12}x^6-156244340a^8b^{10}x^5+147267450a^{10}b^8}{4618900x^{10}(-b^2x+a^2)^9}$
trager	$(-1+x)(2a^{10}x^9+100a^8b^2x^9+525a^6b^4x^9+600a^4b^6x^9+150a^2b^8x^9+4b^{10}x^9+2a^{10}x^8+100a^8b^2x^8+525a^6b^4x^8+600a^4b$

input
```
int((a+b*x^(1/2))^10/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^10/x^10-20/19*a^9*b/x^(19/2)-5*a^8*b^2/x^9-240/17*a^7*b^3/x^(17/2)
-105/4*a^6*b^4/x^8-168/5*a^5*b^5/x^(15/2)-30*a^4*b^6/x^7-240/13*a^3*b^7/x^
(13/2)-15/2*a^2*b^8/x^6-20/11*a*b^9/x^(11/2)-1/5*b^10/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx = -\frac{184756\,b^{10}x^5+6928350\,a^2b^8x^4+27713400\,a^4b^6x^3+24249225\,a^6b^4x^2+4618900\,a^8b^2x+92378\,a^{10}+}{923780\,x^{10}}$$

input
```
integrate((a+b*x^(1/2))^10/x^11,x, algorithm="fricas")
```

output
```
-1/923780*(184756*b^10*x^5 + 6928350*a^2*b^8*x^4 + 27713400*a^4*b^6*x^3 +
24249225*a^6*b^4*x^2 + 4618900*a^8*b^2*x + 92378*a^10 + 16*(104975*a*b^9*x
^4 + 1065900*a^3*b^7*x^3 + 1939938*a^5*b^5*x^2 + 815100*a^7*b^3*x + 60775*
a^9*b)*sqrt(x))/x^10
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx = -\frac{a^{10}}{10x^{10}}-\frac{20a^9b}{19x^{\frac{19}{2}}}-\frac{5a^8b^2}{x^9}-\frac{240a^7b^3}{17x^{\frac{17}{2}}}-\frac{105a^6b^4}{4x^8}-\frac{168a^5b^5}{5x^{\frac{15}{2}}}-\frac{30a^4b^6}{x^7}-\frac{240a^3b^7}{13x^{\frac{13}{2}}}-\frac{15a^2b^8}{2x^6}-\frac{20ab^9}{11x^{\frac{11}{2}}}-\frac{b^{10}}{5x^5}$$

input
```
integrate((a+b*x**(1/2))**10/x**11,x)
```

output
```
-a**10/(10*x**10) - 20*a**9*b/(19*x**(19/2)) - 5*a**8*b**2/x**9 - 240*a**7
*b**3/(17*x**(17/2)) - 105*a**6*b**4/(4*x**8) - 168*a**5*b**5/(5*x**(15/2)
) - 30*a**4*b**6/x**7 - 240*a**3*b**7/(13*x**(13/2)) - 15*a**2*b**8/(2*x**
6) - 20*a*b**9/(11*x**(11/2)) - b**10/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx = -\frac{184756\,b^{10}x^5 + 1679600\,ab^9x^{\frac{9}{2}} + 6928350\,a^2b^8x^4 + 17054400\,a^3b^7x^{\frac{7}{2}} + 27713400\,a^4b^6x^3 + 31039008\,a^5}{923780\,x^{10}}$$

input
```
integrate((a+b*x^(1/2))^10/x^11,x, algorithm="maxima")
```

output
```
-1/923780*(184756*b^10*x^5 + 1679600*a*b^9*x^(9/2) + 6928350*a^2*b^8*x^4 +
 17054400*a^3*b^7*x^(7/2) + 27713400*a^4*b^6*x^3 + 31039008*a^5*b^5*x^(5/2
) + 24249225*a^6*b^4*x^2 + 13041600*a^7*b^3*x^(3/2) + 4618900*a^8*b^2*x +
972400*a^9*b*sqrt(x) + 92378*a^10)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx = -\frac{184756\,b^{10}x^5 + 1679600\,ab^9x^{\frac{9}{2}} + 6928350\,a^2b^8x^4 + 17054400\,a^3b^7x^{\frac{7}{2}} + 27713400\,a^4b^6x^3 + 31039008\,a^5}{923780\,x^{10}}$$

input
```
integrate((a+b*x^(1/2))^10/x^11,x, algorithm="giac")
```

output
```
-1/923780*(184756*b^10*x^5 + 1679600*a*b^9*x^(9/2) + 6928350*a^2*b^8*x^4 +
 17054400*a^3*b^7*x^(7/2) + 27713400*a^4*b^6*x^3 + 31039008*a^5*b^5*x^(5/2
) + 24249225*a^6*b^4*x^2 + 13041600*a^7*b^3*x^(3/2) + 4618900*a^8*b^2*x +
972400*a^9*b*sqrt(x) + 92378*a^10)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx = -\frac{\frac{a^{10}}{10}+\frac{b^{10}\,x^5}{5}+5\,a^8\,b^2\,x+\frac{20\,a^9\,b\,\sqrt{x}}{19}+\frac{20\,a\,b^9\,x^{9/2}}{11}+\frac{105\,a^6\,b^4\,x^2}{4}+30\,a^4\,b^6\,x^3+\frac{15\,a^2\,b^8\,x^4}{2}+\frac{240\,a^7\,b^3\,x^{3/2}}{17}+\frac{168\,a^5}{}}{x^{10}}$$

input
```
int((a + b*x^(1/2))^10/x^11,x)
```

output
```
-(a^10/10 + (b^10*x^5)/5 + 5*a^8*b^2*x + (20*a^9*b*x^(1/2))/19 + (20*a*b^9
*x^(9/2))/11 + (105*a^6*b^4*x^2)/4 + 30*a^4*b^6*x^3 + (15*a^2*b^8*x^4)/2 +
 (240*a^7*b^3*x^(3/2))/17 + (168*a^5*b^5*x^(5/2))/5 + (240*a^3*b^7*x^(7/2)
)/13)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt{x}\right)^{10}}{x^{11}}\,dx = \frac{-92378\sqrt{x}\,a^{10}-4618900\sqrt{x}\,a^8b^2x-24249225\sqrt{x}\,a^6b^4x^2-27713400\sqrt{x}\,a^4b^6x^3-6928350\sqrt{x}\,a^2b^8x^4-}{92378}$$

input
```
int((a+b*x^(1/2))^10/x^11,x)
```

output
```
( - 92378*sqrt(x)*a**10 - 4618900*sqrt(x)*a**8*b**2*x - 24249225*sqrt(x)*a
**6*b**4*x**2 - 27713400*sqrt(x)*a**4*b**6*x**3 - 6928350*sqrt(x)*a**2*b**
8*x**4 - 184756*sqrt(x)*b**10*x**5 - 972400*a**9*b*x - 13041600*a**7*b**3*
x**2 - 31039008*a**5*b**5*x**3 - 17054400*a**3*b**7*x**4 - 1679600*a*b**9*
x**5)/(923780*sqrt(x)*x**10)
```

3.59 $\int \left(a+b\sqrt{x}\right)^{15} x^5 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 242

$$\int \left(a+b\sqrt{x}\right)^{15} x^5 \, dx = -\frac{a^{11}\left(a+b\sqrt{x}\right)^{16}}{8b^{12}}+\frac{22a^{10}\left(a+b\sqrt{x}\right)^{17}}{17b^{12}}-\frac{55a^9\left(a+b\sqrt{x}\right)^{18}}{9b^{12}}+\frac{330a^8\left(a+b\sqrt{x}\right)^{19}}{19b^{12}}-\frac{33a^7\left(a+b\sqrt{x}\right)^{20}}{b^{12}}+\frac{44a^6\left(a+b\sqrt{x}\right)^{21}}{b^{12}}-\frac{42a^5\left(a+b\sqrt{x}\right)^{22}}{b^{12}}+\frac{660a^4\left(a+b\sqrt{x}\right)^{23}}{23b^{12}}-\frac{55a^3\left(a+b\sqrt{x}\right)^{24}}{4b^{12}}+\frac{22a^2\left(a+b\sqrt{x}\right)^{25}}{5b^{12}}-\frac{11a\left(a+b\sqrt{x}\right)^{26}}{13b^{12}}+\frac{2\left(a+b\sqrt{x}\right)^{27}}{27b^{12}}$$

output
```
-1/8*a^11*(a+b*x^(1/2))^16/b^12+22/17*a^10*(a+b*x^(1/2))^17/b^12-55/9*a^9*
(a+b*x^(1/2))^18/b^12+330/19*a^8*(a+b*x^(1/2))^19/b^12-33*a^7*(a+b*x^(1/2)
)^20/b^12+44*a^6*(a+b*x^(1/2))^21/b^12-42*a^5*(a+b*x^(1/2))^22/b^12+660/23
*a^4*(a+b*x^(1/2))^23/b^12-55/4*a^3*(a+b*x^(1/2))^24/b^12+22/5*a^2*(a+b*x^
(1/2))^25/b^12-11/13*a*(a+b*x^(1/2))^26/b^12+2/27*(a+b*x^(1/2))^27/b^12
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}\right)^{15} x^5\,dx$$
$$= \frac{17383860a^{15}x^6 + 240699600a^{14}bx^{13/2} + 1564547400a^{13}b^2x^7 + 6327725040a^{12}b^3x^{15/2} + 17796726675a^{11}b}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15*x^5,x]
```

output
```
(17383860*a^15*x^6 + 240699600*a^14*b*x^(13/2) + 1564547400*a^13*b^2*x^7 +
 6327725040*a^12*b^3*x^(15/2) + 17796726675*a^11*b^4*x^8 + 36849692880*a^1
0*b^5*x^(17/2) + 58004146200*a^9*b^6*x^9 + 70651666800*a^8*b^7*x^(19/2) +
67119083460*a^7*b^8*x^10 + 49717839600*a^6*b^9*x^(21/2) + 28474762680*a^5*
b^10*x^11 + 12380331600*a^4*b^11*x^(23/2) + 3954828150*a^3*b^12*x^12 + 876
146544*a^2*b^13*x^(25/2) + 120349800*a*b^14*x^13 + 7726160*b^15*x^(27/2))/
104303160
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\sqrt{x}\right)^{15}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^{15} x^{11/2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{26}}{b^{11}}-\frac{11a\left(a+b\sqrt{x}\right)^{25}}{b^{11}}+\frac{55a^2\left(a+b\sqrt{x}\right)^{24}}{b^{11}}-\frac{165a^3\left(a+b\sqrt{x}\right)^{23}}{b^{11}}+\frac{330a^4\left(a+b\sqrt{x}\right)^{22}}{b^{11}}-\frac{462a^5\left(\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{11}\left(a+b\sqrt{x}\right)^{16}}{16b^{12}}+\frac{11a^{10}\left(a+b\sqrt{x}\right)^{17}}{17b^{12}}-\frac{55a^9\left(a+b\sqrt{x}\right)^{18}}{18b^{12}}+\frac{165a^8\left(a+b\sqrt{x}\right)^{19}}{19b^{12}}-\frac{33a^7\left(a+b\sqrt{x}\right)^{20}}{2b^{12}}+\frac{22a}{}\right.$$

input
```
Int[(a + b*Sqrt[x])^15*x^5,x]
```

output
```
2*(-1/16*(a^11*(a + b*Sqrt[x])^16)/b^12 + (11*a^10*(a + b*Sqrt[x])^17)/(17
*b^12) - (55*a^9*(a + b*Sqrt[x])^18)/(18*b^12) + (165*a^8*(a + b*Sqrt[x])^
19)/(19*b^12) - (33*a^7*(a + b*Sqrt[x])^20)/(2*b^12) + (22*a^6*(a + b*Sqrt
[x])^21)/b^12 - (21*a^5*(a + b*Sqrt[x])^22)/b^12 + (330*a^4*(a + b*Sqrt[x]
)^23)/(23*b^12) - (55*a^3*(a + b*Sqrt[x])^24)/(8*b^12) + (11*a^2*(a + b*Sq
rt[x])^25)/(5*b^12) - (11*a*(a + b*Sqrt[x])^26)/(26*b^12) + (a + b*Sqrt[x]
)^27/(27*b^12))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2b^{15}x^{\frac{27}{2}}}{27}+\frac{15a\,b^{14}x^{13}}{13}+\frac{42a^2b^{13}x^{\frac{25}{2}}}{5}+\frac{455a^3b^{12}x^{12}}{12}+\frac{2730a^4b^{11}x^{\frac{23}{2}}}{23}+273a^5b^{10}x^{11}+\frac{1430a^6b^9x^{\frac{21}{2}}}{3}$
default	$\frac{2b^{15}x^{\frac{27}{2}}}{27}+\frac{15a\,b^{14}x^{13}}{13}+\frac{42a^2b^{13}x^{\frac{25}{2}}}{5}+\frac{455a^3b^{12}x^{12}}{12}+\frac{2730a^4b^{11}x^{\frac{23}{2}}}{23}+273a^5b^{10}x^{11}+\frac{1430a^6b^9x^{\frac{21}{2}}}{3}$
orering	$\frac{(45465480b^{40}x^{20}-647229338a^2b^{38}x^{19}+4285323042a^4b^{36}x^{18}-17496401598a^6b^{34}x^{17}+49231260806a^8b^{32}x^{16}-10103}{}$
trager	$\frac{a(1080b^{14}x^{12}+35490a^2b^{12}x^{11}+1080b^{14}x^{11}+255528a^4b^{10}x^{10}+35490x^{10}b^{12}a^2+1080b^{14}x^{10}+602316a^6b^8x^9+255528}{}$

input
```
int((a+b*x^(1/2))^15*x^5,x,method=_RETURNVERBOSE)
```

output
```
2/27*b^15*x^(27/2)+15/13*a*b^14*x^13+42/5*a^2*b^13*x^(25/2)+455/12*a^3*b^1
2*x^12+2730/23*a^4*b^11*x^(23/2)+273*a^5*b^10*x^11+1430/3*a^6*b^9*x^(21/2)
+1287/2*a^7*b^8*x^10+12870/19*a^8*b^7*x^(19/2)+5005/9*a^9*b^6*x^9+6006/17*
a^10*b^5*x^(17/2)+1365/8*a^11*b^4*x^8+182/3*a^12*b^3*x^(15/2)+15*a^13*b^2*
x^7+30/13*a^14*b*x^(13/2)+1/6*a^15*x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right)^{15}x^5\,dx=\frac{15}{13}\,ab^{14}x^{13}+\frac{455}{12}\,a^3b^{12}x^{12}+273\,a^5b^{10}x^{11}+\frac{1287}{2}\,a^7b^8x^{10}+\frac{5005}{9}\,a^9b^6x^9+\frac{1365}{8}\,a^{11}b^4x^8+15\,a^{13}b^2x^7+\frac{1}{6}\,a^{15}x^6+\frac{2}{13037895}\,(482885\,b^{15}x^{13}+54759159\,a^2b^{13}x^{12}+773770725\,a^4b^{11}x^{11}+3107364975\,a^6b^9x^{10}+4415729$$

input
```
integrate((a+b*x^(1/2))^15*x^5,x, algorithm="fricas")
```

output
```
15/13*a*b^14*x^13 + 455/12*a^3*b^12*x^12 + 273*a^5*b^10*x^11 + 1287/2*a^7*
b^8*x^10 + 5005/9*a^9*b^6*x^9 + 1365/8*a^11*b^4*x^8 + 15*a^13*b^2*x^7 + 1/
6*a^15*x^6 + 2/13037895*(482885*b^15*x^13 + 54759159*a^2*b^13*x^12 + 77377
0725*a^4*b^11*x^11 + 3107364975*a^6*b^9*x^10 + 4415729175*a^8*b^7*x^9 + 23
03105805*a^10*b^5*x^8 + 395482815*a^12*b^3*x^7 + 15043725*a^14*b*x^6)*sqrt
(x)
```

Sympy [A] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^{15} x^5\,dx = \frac{a^{15}x^6}{6} + \frac{30a^{14}bx^{\frac{13}{2}}}{13} + 15a^{13}b^2x^7 + \frac{182a^{12}b^3x^{\frac{15}{2}}}{3} + \frac{1365a^{11}b^4x^8}{8} + \frac{6006a^{10}b^5x^{\frac{17}{2}}}{17} + \frac{5005a^9b^6x^9}{9} + \frac{12870a^8b^7x^{\frac{19}{2}}}{19} + \frac{1287a^7b^8x^{10}}{2} + \frac{1430a^6b^9x^{\frac{21}{2}}}{3} + 273a^5b^{10}x^{11} + \frac{2730a^4b^{11}x^{\frac{23}{2}}}{23} + \frac{455a^3b^{12}x^{12}}{12} + \frac{42a^2b^{13}x^{\frac{25}{2}}}{5} + \frac{15ab^{14}x^{13}}{13} + \frac{2b^{15}x^{\frac{27}{2}}}{27}$$

input
```
integrate((a+b*x**(1/2))**15*x**5,x)
```

output
```
a**15*x**6/6 + 30*a**14*b*x**(13/2)/13 + 15*a**13*b**2*x**7 + 182*a**12*b*
*3*x**(15/2)/3 + 1365*a**11*b**4*x**8/8 + 6006*a**10*b**5*x**(17/2)/17 + 5
005*a**9*b**6*x**9/9 + 12870*a**8*b**7*x**(19/2)/19 + 1287*a**7*b**8*x**10
/2 + 1430*a**6*b**9*x**(21/2)/3 + 273*a**5*b**10*x**11 + 2730*a**4*b**11*x
**(23/2)/23 + 455*a**3*b**12*x**12/12 + 42*a**2*b**13*x**(25/2)/5 + 15*a*b
**14*x**13/13 + 2*b**15*x**(27/2)/27
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt{x}\right)^{15}x^5\,dx=\frac{2\left(b\sqrt{x}+a\right)^{27}}{27\,b^{12}}-\frac{11\left(b\sqrt{x}+a\right)^{26}a}{13\,b^{12}}+\frac{22\left(b\sqrt{x}+a\right)^{25}a^2}{5\,b^{12}}-\frac{55\left(b\sqrt{x}+a\right)^{24}a^3}{4\,b^{12}}+\frac{660\left(b\sqrt{x}+a\right)^{23}a^4}{23\,b^{12}}-\frac{42\left(b\sqrt{x}+a\right)^{22}a^5}{b^{12}}+\frac{44\left(b\sqrt{x}+a\right)^{21}a^6}{b^{12}}-\frac{33\left(b\sqrt{x}+a\right)^{20}a^7}{b^{12}}+\frac{330\left(b\sqrt{x}+a\right)^{19}a^8}{19\,b^{12}}-\frac{55\left(b\sqrt{x}+a\right)^{18}a^9}{9\,b^{12}}+\frac{22\left(b\sqrt{x}+a\right)^{17}a^{10}}{17\,b^{12}}-\frac{\left(b\sqrt{x}+a\right)^{16}a^{11}}{8\,b^{12}}$$

input
```
integrate((a+b*x^(1/2))^15*x^5,x, algorithm="maxima")
```

output
```
2/27*(b*sqrt(x) + a)^27/b^12 - 11/13*(b*sqrt(x) + a)^26*a/b^12 + 22/5*(b*s
qrt(x) + a)^25*a^2/b^12 - 55/4*(b*sqrt(x) + a)^24*a^3/b^12 + 660/23*(b*sqr
t(x) + a)^23*a^4/b^12 - 42*(b*sqrt(x) + a)^22*a^5/b^12 + 44*(b*sqrt(x) + a
)^21*a^6/b^12 - 33*(b*sqrt(x) + a)^20*a^7/b^12 + 330/19*(b*sqrt(x) + a)^19
*a^8/b^12 - 55/9*(b*sqrt(x) + a)^18*a^9/b^12 + 22/17*(b*sqrt(x) + a)^17*a^
10/b^12 - 1/8*(b*sqrt(x) + a)^16*a^11/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.69

$$\int \left(a+b\sqrt{x}\right)^{15}x^5\,dx=\frac{2}{27}\,b^{15}x^{\frac{27}{2}}+\frac{15}{13}\,ab^{14}x^{13}+\frac{42}{5}\,a^2b^{13}x^{\frac{25}{2}}+\frac{455}{12}\,a^3b^{12}x^{12}+\frac{2730}{23}\,a^4b^{11}x^{\frac{23}{2}}+273\,a^5b^{10}x^{11}+\frac{1430}{3}\,a^6b^9x^{\frac{21}{2}}+\frac{1287}{2}\,a^7b^8x^{10}+\frac{12870}{19}\,a^8b^7x^{\frac{19}{2}}+\frac{5005}{9}\,a^9b^6x^9+\frac{6006}{17}\,a^{10}b^5x^{\frac{17}{2}}+\frac{1365}{8}\,a^{11}b^4x^8+\frac{182}{3}\,a^{12}b^3x^{\frac{15}{2}}+15\,a^{13}b^2x^7+\frac{30}{13}\,a^{14}bx^{\frac{13}{2}}+\frac{1}{6}\,a^{15}x^6$$

input
```
integrate((a+b*x^(1/2))^15*x^5,x, algorithm="giac")
```

output
```
2/27*b^15*x^(27/2) + 15/13*a*b^14*x^13 + 42/5*a^2*b^13*x^(25/2) + 455/12*a
^3*b^12*x^12 + 2730/23*a^4*b^11*x^(23/2) + 273*a^5*b^10*x^11 + 1430/3*a^6*
b^9*x^(21/2) + 1287/2*a^7*b^8*x^10 + 12870/19*a^8*b^7*x^(19/2) + 5005/9*a^
9*b^6*x^9 + 6006/17*a^10*b^5*x^(17/2) + 1365/8*a^11*b^4*x^8 + 182/3*a^12*b
^3*x^(15/2) + 15*a^13*b^2*x^7 + 30/13*a^14*b*x^(13/2) + 1/6*a^15*x^6
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.69

$$\int \left(a+b\sqrt{x}\right)^{15} x^5\,dx = \frac{a^{15}\,x^6}{6}+\frac{2\,b^{15}\,x^{27/2}}{27}+\frac{15\,a\,b^{14}\,x^{13}}{13}+\frac{30\,a^{14}\,b\,x^{13/2}}{13}+15\,a^{13}\,b^2\,x^7+\frac{1365\,a^{11}\,b^4\,x^8}{8}+\frac{5005\,a^9\,b^6\,x^9}{9}+\frac{1287\,a^7\,b^8\,x^{10}}{2}+273\,a^5\,b^{10}\,x^{11}+\frac{455\,a^3\,b^{12}\,x^{12}}{12}+\frac{182\,a^{12}\,b^3\,x^{15/2}}{3}+\frac{6006\,a^{10}\,b^5\,x^{17/2}}{17}+\frac{12870\,a^8\,b^7\,x^{19/2}}{19}+\frac{1430\,a^6\,b^9\,x^{21/2}}{3}+\frac{2730\,a^4\,b^{11}\,x^{23/2}}{23}+\frac{42\,a^2\,b^{13}\,x^{25/2}}{5}$$

input
```
int(x^5*(a + b*x^(1/2))^15,x)
```

output
```
(a^15*x^6)/6 + (2*b^15*x^(27/2))/27 + (15*a*b^14*x^13)/13 + (30*a^14*b*x^(
13/2))/13 + 15*a^13*b^2*x^7 + (1365*a^11*b^4*x^8)/8 + (5005*a^9*b^6*x^9)/9
 + (1287*a^7*b^8*x^10)/2 + 273*a^5*b^10*x^11 + (455*a^3*b^12*x^12)/12 + (1
82*a^12*b^3*x^(15/2))/3 + (6006*a^10*b^5*x^(17/2))/17 + (12870*a^8*b^7*x^(
19/2))/19 + (1430*a^6*b^9*x^(21/2))/3 + (2730*a^4*b^11*x^(23/2))/23 + (42*
a^2*b^13*x^(25/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^{15} x^5\,dx$$
$$= \frac{x^6\left(240699600\sqrt{x}\,a^{14}b+6327725040\sqrt{x}\,a^{12}b^3x+36849692880\sqrt{x}\,a^{10}b^5x^2+70651666800\sqrt{x}\,a^8b^7x^3+4\right.}{}$$

input
```
int((a+b*x^(1/2))^15*x^5,x)
```

output
```
(x**6*(240699600*sqrt(x)*a**14*b + 6327725040*sqrt(x)*a**12*b**3*x + 36849
692880*sqrt(x)*a**10*b**5*x**2 + 70651666800*sqrt(x)*a**8*b**7*x**3 + 4971
7839600*sqrt(x)*a**6*b**9*x**4 + 12380331600*sqrt(x)*a**4*b**11*x**5 + 876
146544*sqrt(x)*a**2*b**13*x**6 + 7726160*sqrt(x)*b**15*x**7 + 17383860*a**
15 + 1564547400*a**13*b**2*x + 17796726675*a**11*b**4*x**2 + 58004146200*a
**9*b**6*x**3 + 67119083460*a**7*b**8*x**4 + 28474762680*a**5*b**10*x**5 +
 3954828150*a**3*b**12*x**6 + 120349800*a*b**14*x**7))/104303160
```

3.60 $\int \left(a+b\sqrt{x}\right)^{15} x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 202

$$\int \left(a+b\sqrt{x}\right)^{15} x^4 \, dx = -\frac{a^9\left(a+b\sqrt{x}\right)^{16}}{8b^{10}}+\frac{18a^8\left(a+b\sqrt{x}\right)^{17}}{17b^{10}}-\frac{4a^7\left(a+b\sqrt{x}\right)^{18}}{b^{10}}+\frac{168a^6\left(a+b\sqrt{x}\right)^{19}}{19b^{10}}-\frac{63a^5\left(a+b\sqrt{x}\right)^{20}}{5b^{10}}+\frac{12a^4\left(a+b\sqrt{x}\right)^{21}}{b^{10}}-\frac{84a^3\left(a+b\sqrt{x}\right)^{22}}{11b^{10}}+\frac{72a^2\left(a+b\sqrt{x}\right)^{23}}{23b^{10}}-\frac{3a\left(a+b\sqrt{x}\right)^{24}}{4b^{10}}+\frac{2\left(a+b\sqrt{x}\right)^{25}}{25b^{10}}$$

output
```
-1/8*a^9*(a+b*x^(1/2))^16/b^10+18/17*a^8*(a+b*x^(1/2))^17/b^10-4*a^7*(a+b*
x^(1/2))^18/b^10+168/19*a^6*(a+b*x^(1/2))^19/b^10-63/5*a^5*(a+b*x^(1/2))^2
0/b^10+12*a^4*(a+b*x^(1/2))^21/b^10-84/11*a^3*(a+b*x^(1/2))^22/b^10+72/23*
a^2*(a+b*x^(1/2))^23/b^10-3/4*a*(a+b*x^(1/2))^24/b^10+2/25*(a+b*x^(1/2))^2
5/b^10
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt{x}\right)^{15} x^4\,dx$$
$$= \frac{3268760a^{15}x^5 + 44574000a^{14}bx^{11/2} + 286016500a^{13}b^2x^6 + 1144066000a^{12}b^3x^{13/2} + 3187041000a^{11}b^4x^7 -}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15*x^4,x]
```

output
```
(3268760*a^15*x^5 + 44574000*a^14*b*x^(11/2) + 286016500*a^13*b^2*x^6 + 11
44066000*a^12*b^3*x^(13/2) + 3187041000*a^11*b^4*x^7 + 6544057520*a^10*b^5
*x^(15/2) + 10225089875*a^9*b^6*x^8 + 12373218000*a^8*b^7*x^(17/2) + 11685
817000*a^7*b^8*x^9 + 8610602000*a^6*b^9*x^(19/2) + 4908043140*a^5*b^10*x^1
0 + 2124694000*a^4*b^11*x^(21/2) + 676039000*a^3*b^12*x^11 + 149226000*a^2
*b^13*x^(23/2) + 20429750*a*b^14*x^12 + 1307504*b^15*x^(25/2))/16343800
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sqrt{x}\right)^{15}dx$$
$$\Big\downarrow \text{798}$$
$$2\int \left(a+b\sqrt{x}\right)^{15} x^{9/2}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$
$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{24}}{b^9} - \frac{9a\left(a+b\sqrt{x}\right)^{23}}{b^9} + \frac{36a^2\left(a+b\sqrt{x}\right)^{22}}{b^9} - \frac{84a^3\left(a+b\sqrt{x}\right)^{21}}{b^9} + \frac{126a^4\left(a+b\sqrt{x}\right)^{20}}{b^9} - \frac{126a^5\left(a\right.}{b}\right.$$

$$\downarrow \text{2009}$$

$$2\left(-\frac{a^9\left(a+b\sqrt{x}\right)^{16}}{16b^{10}}+\frac{9a^8\left(a+b\sqrt{x}\right)^{17}}{17b^{10}}-\frac{2a^7\left(a+b\sqrt{x}\right)^{18}}{b^{10}}+\frac{84a^6\left(a+b\sqrt{x}\right)^{19}}{19b^{10}}-\frac{63a^5\left(a+b\sqrt{x}\right)^{20}}{10b^{10}}+\frac{6a^4\left(a+b\right.}{b^{10}}\right.$$

input
```
Int[(a + b*Sqrt[x])^15*x^4,x]
```

output
```
2*(-1/16*(a^9*(a + b*Sqrt[x])^16)/b^10 + (9*a^8*(a + b*Sqrt[x])^17)/(17*b^
10) - (2*a^7*(a + b*Sqrt[x])^18)/b^10 + (84*a^6*(a + b*Sqrt[x])^19)/(19*b^
10) - (63*a^5*(a + b*Sqrt[x])^20)/(10*b^10) + (6*a^4*(a + b*Sqrt[x])^21)/b
^10 - (42*a^3*(a + b*Sqrt[x])^22)/(11*b^10) + (36*a^2*(a + b*Sqrt[x])^23)/
(23*b^10) - (3*a*(a + b*Sqrt[x])^24)/(8*b^10) + (a + b*Sqrt[x])^25/(25*b^1
0))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2b^{15}x^{\frac{25}{2}}}{25}+\frac{5a\,b^{14}x^{12}}{4}+\frac{210a^2b^{13}x^{\frac{23}{2}}}{23}+\frac{455a^3b^{12}x^{11}}{11}+130a^4b^{11}x^{\frac{21}{2}}+\frac{3003a^5b^{10}x^{10}}{10}+\frac{10010a^6b^9x^{\frac{19}{2}}}{19}$
default	$\frac{2b^{15}x^{\frac{25}{2}}}{25}+\frac{5a\,b^{14}x^{12}}{4}+\frac{210a^2b^{13}x^{\frac{23}{2}}}{23}+\frac{455a^3b^{12}x^{11}}{11}+130a^4b^{11}x^{\frac{21}{2}}+\frac{3003a^5b^{10}x^{10}}{10}+\frac{10010a^6b^9x^{\frac{19}{2}}}{19}$
orering	$-\frac{(-38407930b^{38}x^{19}+546370650a^2b^{36}x^{18}-3615112254a^4b^{34}x^{17}+14751217390a^6b^{32}x^{16}-41486940495a^8b^{30}x^{15}+85}{}$
trager	$\frac{a(550b^{14}x^{11}+18200x^{10}b^{12}a^2+550b^{14}x^{10}+132132b^{10}x^9a^4+18200x^9b^{12}a^2+550b^{14}x^9+314600a^6b^8x^8+132132b^{10}x^8a}{}$

input
```
int((a+b*x^(1/2))^15*x^4,x,method=_RETURNVERBOSE)
```

output
```
2/25*b^15*x^(25/2)+5/4*a*b^14*x^12+210/23*a^2*b^13*x^(23/2)+455/11*a^3*b^1
2*x^11+130*a^4*b^11*x^(21/2)+3003/10*a^5*b^10*x^10+10010/19*a^6*b^9*x^(19/
2)+715*a^7*b^8*x^9+12870/17*a^8*b^7*x^(17/2)+5005/8*a^9*b^6*x^8+2002/5*a^1
0*b^5*x^(15/2)+195*a^11*b^4*x^7+70*a^12*b^3*x^(13/2)+35/2*a^13*b^2*x^6+30/
11*a^14*b*x^(11/2)+1/5*a^15*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt{x}\right)^{15}x^4\,dx=\frac{5}{4}\,ab^{14}x^{12}+\frac{455}{11}\,a^3b^{12}x^{11}+\frac{3003}{10}\,a^5b^{10}x^{10}+715\,a^7b^8x^9+\frac{5005}{8}\,a^9b^6x^8+195\,a^{11}b^4x^7+\frac{35}{2}\,a^{13}b^2x^6+\frac{1}{5}\,a^{15}x^5+\frac{2}{2042975}\,(81719\,b^{15}x^{12}+9326625\,a^2b^{13}x^{11}+132793375\,a^4b^{11}x^{10}+538162625\,a^6b^9x^9+773326125\,a^8b$$

input
```
integrate((a+b*x^(1/2))^15*x^4,x, algorithm="fricas")
```

output
```
5/4*a*b^14*x^12 + 455/11*a^3*b^12*x^11 + 3003/10*a^5*b^10*x^10 + 715*a^7*b
^8*x^9 + 5005/8*a^9*b^6*x^8 + 195*a^11*b^4*x^7 + 35/2*a^13*b^2*x^6 + 1/5*a
^15*x^5 + 2/2042975*(81719*b^15*x^12 + 9326625*a^2*b^13*x^11 + 132793375*a
^4*b^11*x^10 + 538162625*a^6*b^9*x^9 + 773326125*a^8*b^7*x^8 + 409003595*a
^10*b^5*x^7 + 71504125*a^12*b^3*x^6 + 2785875*a^14*b*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04

$$\int \left(a+b\sqrt{x}\right)^{15} x^4\,dx = \frac{a^{15}x^5}{5} + \frac{30a^{14}bx^{\frac{11}{2}}}{11} + \frac{35a^{13}b^2x^6}{2} + 70a^{12}b^3x^{\frac{13}{2}} + 195a^{11}b^4x^7 + \frac{2002a^{10}b^5x^{\frac{15}{2}}}{5} + \frac{5005a^9b^6x^8}{8} + \frac{12870a^8b^7x^{\frac{17}{2}}}{17} + 715a^7b^8x^9 + \frac{10010a^6b^9x^{\frac{19}{2}}}{19} + \frac{3003a^5b^{10}x^{10}}{10} + 130a^4b^{11}x^{\frac{21}{2}} + \frac{455a^3b^{12}x^{11}}{11} + \frac{210a^2b^{13}x^{\frac{23}{2}}}{23} + \frac{5ab^{14}x^{12}}{4} + \frac{2b^{15}x^{\frac{25}{2}}}{25}$$

input
```
integrate((a+b*x**(1/2))**15*x**4,x)
```

output
```
a**15*x**5/5 + 30*a**14*b*x**(11/2)/11 + 35*a**13*b**2*x**6/2 + 70*a**12*b
**3*x**(13/2) + 195*a**11*b**4*x**7 + 2002*a**10*b**5*x**(15/2)/5 + 5005*a
**9*b**6*x**8/8 + 12870*a**8*b**7*x**(17/2)/17 + 715*a**7*b**8*x**9 + 1001
0*a**6*b**9*x**(19/2)/19 + 3003*a**5*b**10*x**10/10 + 130*a**4*b**11*x**(2
1/2) + 455*a**3*b**12*x**11/11 + 210*a**2*b**13*x**(23/2)/23 + 5*a*b**14*x
**12/4 + 2*b**15*x**(25/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt{x}\right)^{15}x^4\,dx = \frac{2\left(b\sqrt{x}+a\right)^{25}}{25\,b^{10}} - \frac{3\left(b\sqrt{x}+a\right)^{24}a}{4\,b^{10}} + \frac{72\left(b\sqrt{x}+a\right)^{23}a^2}{23\,b^{10}} - \frac{84\left(b\sqrt{x}+a\right)^{22}a^3}{11\,b^{10}} + \frac{12\left(b\sqrt{x}+a\right)^{21}a^4}{b^{10}} - \frac{63\left(b\sqrt{x}+a\right)^{20}a^5}{5\,b^{10}} + \frac{168\left(b\sqrt{x}+a\right)^{19}a^6}{19\,b^{10}} - \frac{4\left(b\sqrt{x}+a\right)^{18}a^7}{b^{10}} + \frac{18\left(b\sqrt{x}+a\right)^{17}a^8}{17\,b^{10}} - \frac{\left(b\sqrt{x}+a\right)^{16}a^9}{8\,b^{10}}$$

input
```
integrate((a+b*x^(1/2))^15*x^4,x, algorithm="maxima")
```

output
```
2/25*(b*sqrt(x) + a)^25/b^10 - 3/4*(b*sqrt(x) + a)^24*a/b^10 + 72/23*(b*sq
rt(x) + a)^23*a^2/b^10 - 84/11*(b*sqrt(x) + a)^22*a^3/b^10 + 12*(b*sqrt(x)
 + a)^21*a^4/b^10 - 63/5*(b*sqrt(x) + a)^20*a^5/b^10 + 168/19*(b*sqrt(x) +
 a)^19*a^6/b^10 - 4*(b*sqrt(x) + a)^18*a^7/b^10 + 18/17*(b*sqrt(x) + a)^17
*a^8/b^10 - 1/8*(b*sqrt(x) + a)^16*a^9/b^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt{x}\right)^{15}x^4\,dx = \frac{2}{25}\,b^{15}x^{\frac{25}{2}} + \frac{5}{4}\,ab^{14}x^{12} + \frac{210}{23}\,a^2b^{13}x^{\frac{23}{2}} + \frac{455}{11}\,a^3b^{12}x^{11} + 130\,a^4b^{11}x^{\frac{21}{2}} + \frac{3003}{10}\,a^5b^{10}x^{10} + \frac{10010}{19}\,a^6b^9x^{\frac{19}{2}} + 715\,a^7b^8x^9 + \frac{12870}{17}\,a^8b^7x^{\frac{17}{2}} + \frac{5005}{8}\,a^9b^6x^8 + \frac{2002}{5}\,a^{10}b^5x^{\frac{15}{2}} + 195\,a^{11}b^4x^7 + 70\,a^{12}b^3x^{\frac{13}{2}} + \frac{35}{2}\,a^{13}b^2x^6 + \frac{30}{11}\,a^{14}bx^{\frac{11}{2}} + \frac{1}{5}\,a^{15}x^5$$

input
```
integrate((a+b*x^(1/2))^15*x^4,x, algorithm="giac")
```

output
```
2/25*b^15*x^(25/2) + 5/4*a*b^14*x^12 + 210/23*a^2*b^13*x^(23/2) + 455/11*a
^3*b^12*x^11 + 130*a^4*b^11*x^(21/2) + 3003/10*a^5*b^10*x^10 + 10010/19*a^
6*b^9*x^(19/2) + 715*a^7*b^8*x^9 + 12870/17*a^8*b^7*x^(17/2) + 5005/8*a^9*
b^6*x^8 + 2002/5*a^10*b^5*x^(15/2) + 195*a^11*b^4*x^7 + 70*a^12*b^3*x^(13/
2) + 35/2*a^13*b^2*x^6 + 30/11*a^14*b*x^(11/2) + 1/5*a^15*x^5
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt{x}\right)^{15} x^4\,dx = \frac{a^{15}\,x^5}{5}+\frac{2\,b^{15}\,x^{25/2}}{25}+\frac{5\,a\,b^{14}\,x^{12}}{4}+\frac{30\,a^{14}\,b\,x^{11/2}}{11}+\frac{35\,a^{13}\,b^2\,x^6}{2}+195\,a^{11}\,b^4\,x^7+\frac{5005\,a^9\,b^6\,x^8}{8}+715\,a^7\,b^8\,x^9+\frac{3003\,a^5\,b^{10}\,x^{10}}{10}+\frac{455\,a^3\,b^{12}\,x^{11}}{11}+70\,a^{12}\,b^3\,x^{13/2}+\frac{2002\,a^{10}\,b^5\,x^{15/2}}{5}+\frac{12870\,a^8\,b^7\,x^{17/2}}{17}+\frac{10010\,a^6\,b^9\,x^{19/2}}{19}+130\,a^4\,b^{11}\,x^{21/2}+\frac{210\,a^2\,b^{13}\,x^{23/2}}{23}$$

input
```
int(x^4*(a + b*x^(1/2))^15,x)
```

output
```
(a^15*x^5)/5 + (2*b^15*x^(25/2))/25 + (5*a*b^14*x^12)/4 + (30*a^14*b*x^(11
/2))/11 + (35*a^13*b^2*x^6)/2 + 195*a^11*b^4*x^7 + (5005*a^9*b^6*x^8)/8 +
715*a^7*b^8*x^9 + (3003*a^5*b^10*x^10)/10 + (455*a^3*b^12*x^11)/11 + 70*a^
12*b^3*x^(13/2) + (2002*a^10*b^5*x^(15/2))/5 + (12870*a^8*b^7*x^(17/2))/17
 + (10010*a^6*b^9*x^(19/2))/19 + 130*a^4*b^11*x^(21/2) + (210*a^2*b^13*x^(
23/2))/23
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^{15} x^4\,dx$$
$$= \frac{x^5\left(44574000\sqrt{x}\,a^{14}b+1144066000\sqrt{x}\,a^{12}b^3x+6544057520\sqrt{x}\,a^{10}b^5x^2+12373218000\sqrt{x}\,a^8b^7x^3+861\right.}{}$$

input
```
int((a+b*x^(1/2))^15*x^4,x)
```

output
```
(x**5*(44574000*sqrt(x)*a**14*b + 1144066000*sqrt(x)*a**12*b**3*x + 654405
7520*sqrt(x)*a**10*b**5*x**2 + 12373218000*sqrt(x)*a**8*b**7*x**3 + 861060
2000*sqrt(x)*a**6*b**9*x**4 + 2124694000*sqrt(x)*a**4*b**11*x**5 + 1492260
00*sqrt(x)*a**2*b**13*x**6 + 1307504*sqrt(x)*b**15*x**7 + 3268760*a**15 +
286016500*a**13*b**2*x + 3187041000*a**11*b**4*x**2 + 10225089875*a**9*b**
6*x**3 + 11685817000*a**7*b**8*x**4 + 4908043140*a**5*b**10*x**5 + 6760390
00*a**3*b**12*x**6 + 20429750*a*b**14*x**7))/16343800
```

3.61 $\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 162

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx = -\frac{a^7\left(a+b\sqrt{x}\right)^{16}}{8b^8}+\frac{14a^6\left(a+b\sqrt{x}\right)^{17}}{17b^8}-\frac{7a^5\left(a+b\sqrt{x}\right)^{18}}{3b^8}+\frac{70a^4\left(a+b\sqrt{x}\right)^{19}}{19b^8}-\frac{7a^3\left(a+b\sqrt{x}\right)^{20}}{2b^8}+\frac{2a^2\left(a+b\sqrt{x}\right)^{21}}{b^8}-\frac{7a\left(a+b\sqrt{x}\right)^{22}}{11b^8}+\frac{2\left(a+b\sqrt{x}\right)^{23}}{23b^8}$$

output
```
-1/8*a^7*(a+b*x^(1/2))^16/b^8+14/17*a^6*(a+b*x^(1/2))^17/b^8-7/3*a^5*(a+b*
x^(1/2))^18/b^8+70/19*a^4*(a+b*x^(1/2))^19/b^8-7/2*a^3*(a+b*x^(1/2))^20/b^
8+2*a^2*(a+b*x^(1/2))^21/b^8-7/11*a*(a+b*x^(1/2))^22/b^8+2/23*(a+b*x^(1/2)
)^23/b^8
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx$$
$$= \frac{490314a^{15}x^4 + 6537520a^{14}bx^{9/2} + 41186376a^{13}b^2x^5 + 162249360a^{12}b^3x^{11/2} + 446185740a^{11}b^4x^6 + 90610}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15*x^3,x]
```

output
```
(490314*a^15*x^4 + 6537520*a^14*b*x^(9/2) + 41186376*a^13*b^2*x^5 + 162249
360*a^12*b^3*x^(11/2) + 446185740*a^11*b^4*x^6 + 906100272*a^10*b^5*x^(13/
2) + 1402298040*a^9*b^6*x^7 + 1682757648*a^8*b^7*x^(15/2) + 1577585295*a^7
*b^8*x^8 + 1154833680*a^6*b^9*x^(17/2) + 654405752*a^5*b^10*x^9 + 28180152
0*a^4*b^11*x^(19/2) + 89237148*a^3*b^12*x^10 + 19612560*a^2*b^13*x^(21/2)
+ 2674440*a*b^14*x^11 + 170544*b^15*x^(23/2))/1961256
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{x}\right)^{15}\,dx$$
$$\Big\downarrow \text{798}$$
$$2\int \left(a+b\sqrt{x}\right)^{15} x^{7/2}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$
$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{22}}{b^7} - \frac{7a\left(a+b\sqrt{x}\right)^{21}}{b^7} + \frac{21a^2\left(a+b\sqrt{x}\right)^{20}}{b^7} - \frac{35a^3\left(a+b\sqrt{x}\right)^{19}}{b^7} + \frac{35a^4\left(a+b\sqrt{x}\right)^{18}}{b^7} - \frac{21a^5(a+}{b^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^7\left(a+b\sqrt{x}\right)^{16}}{16b^8}+\frac{7a^6\left(a+b\sqrt{x}\right)^{17}}{17b^8}-\frac{7a^5\left(a+b\sqrt{x}\right)^{18}}{6b^8}+\frac{35a^4\left(a+b\sqrt{x}\right)^{19}}{19b^8}-\frac{7a^3\left(a+b\sqrt{x}\right)^{20}}{4b^8}+\frac{a^2\left(a+b\sqrt{x}\right)^{21}}{b^8}\right.$$

input
```
Int[(a + b*Sqrt[x])^15*x^3,x]
```

output
```
2*(-1/16*(a^7*(a + b*Sqrt[x])^16)/b^8 + (7*a^6*(a + b*Sqrt[x])^17)/(17*b^8
) - (7*a^5*(a + b*Sqrt[x])^18)/(6*b^8) + (35*a^4*(a + b*Sqrt[x])^19)/(19*b
^8) - (7*a^3*(a + b*Sqrt[x])^20)/(4*b^8) + (a^2*(a + b*Sqrt[x])^21)/b^8 -
(7*a*(a + b*Sqrt[x])^22)/(22*b^8) + (a + b*Sqrt[x])^23/(23*b^8))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.85 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{2b^{15}x^{\frac{23}{2}}}{23}+\frac{15b^{14}x^{11}a}{11}+10a^2b^{13}x^{\frac{21}{2}}+\frac{91a^3b^{12}x^{10}}{2}+\frac{2730a^4b^{11}x^{\frac{19}{2}}}{19}+\frac{1001b^{10}x^9a^5}{3}+\frac{10010a^6b^9x^{\frac{17}{2}}}{17}+$
default	$\frac{2b^{15}x^{\frac{23}{2}}}{23}+\frac{15b^{14}x^{11}a}{11}+10a^2b^{13}x^{\frac{21}{2}}+\frac{91a^3b^{12}x^{10}}{2}+\frac{2730a^4b^{11}x^{\frac{19}{2}}}{19}+\frac{1001b^{10}x^9a^5}{3}+\frac{10010a^6b^9x^{\frac{17}{2}}}{17}+$
orering	$\frac{(1666680b^{36}x^{18}-23678484a^2b^{34}x^{17}+156463580a^4b^{32}x^{16}-637591955a^6b^{30}x^{15}+1790864075a^8b^{28}x^{14}-3669868895a}{}$
trager	$\frac{a(360b^{14}x^{10}+12012x^9b^{12}a^2+360b^{14}x^9+88088b^{10}x^8a^4+12012b^{12}x^8a^2+360b^{14}x^8+212355x^7b^8a^6+88088b^{10}x^7a^4+1}{}$

input
```
int((a+b*x^(1/2))^15*x^3,x,method=_RETURNVERBOSE)
```

output
```
2/23*b^15*x^(23/2)+15/11*b^14*x^11*a+10*a^2*b^13*x^(21/2)+91/2*a^3*b^12*x^
10+2730/19*a^4*b^11*x^(19/2)+1001/3*b^10*x^9*a^5+10010/17*a^6*b^9*x^(17/2)
+6435/8*b^8*x^8*a^7+858*a^8*b^7*x^(15/2)+715*a^9*b^6*x^7+462*a^10*b^5*x^(1
3/2)+455/2*a^11*b^4*x^6+910/11*a^12*b^3*x^(11/2)+21*a^13*b^2*x^5+10/3*a^14
*b*x^(9/2)+1/4*a^15*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.07

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx = \frac{15}{11}\,ab^{14}x^{11}+\frac{91}{2}\,a^3b^{12}x^{10}+\frac{1001}{3}\,a^5b^{10}x^9 + \frac{6435}{8}\,a^7b^8x^8+715\,a^9b^6x^7+\frac{455}{2}\,a^{11}b^4x^6+21\,a^{13}b^2x^5+\frac{1}{4}\,a^{15}x^4 + \frac{2}{245157}\left(10659\,b^{15}x^{11}+1225785\,a^2b^{13}x^{10}+17612595\,a^4b^{11}x^9+72177105\,a^6b^9x^8+105172353\,a^8b^7x^7\right.$$

input
```
integrate((a+b*x^(1/2))^15*x^3,x, algorithm="fricas")
```

output
```
15/11*a*b^14*x^11 + 91/2*a^3*b^12*x^10 + 1001/3*a^5*b^10*x^9 + 6435/8*a^7*
b^8*x^8 + 715*a^9*b^6*x^7 + 455/2*a^11*b^4*x^6 + 21*a^13*b^2*x^5 + 1/4*a^1
5*x^4 + 2/245157*(10659*b^15*x^11 + 1225785*a^2*b^13*x^10 + 17612595*a^4*b
^11*x^9 + 72177105*a^6*b^9*x^8 + 105172353*a^8*b^7*x^7 + 56631267*a^10*b^5
*x^6 + 10140585*a^12*b^3*x^5 + 408595*a^14*b*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.29

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx = \frac{a^{15}x^4}{4} + \frac{10a^{14}bx^{\frac{9}{2}}}{3} + 21a^{13}b^2x^5 + \frac{910a^{12}b^3x^{\frac{11}{2}}}{11} + \frac{455a^{11}b^4x^6}{2} + 462a^{10}b^5x^{\frac{13}{2}} + 715a^9b^6x^7 + 858a^8b^7x^{\frac{15}{2}} + \frac{6435a^7b^8x^8}{8} + \frac{10010a^6b^9x^{\frac{17}{2}}}{17} + \frac{1001a^5b^{10}x^9}{3} + \frac{2730a^4b^{11}x^{\frac{19}{2}}}{19} + \frac{91a^3b^{12}x^{10}}{2} + 10a^2b^{13}x^{\frac{21}{2}} + \frac{15ab^{14}x^{11}}{11} + \frac{2b^{15}x^{\frac{23}{2}}}{23}$$

input
```
integrate((a+b*x**(1/2))**15*x**3,x)
```

output
```
a**15*x**4/4 + 10*a**14*b*x**(9/2)/3 + 21*a**13*b**2*x**5 + 910*a**12*b**3
*x**(11/2)/11 + 455*a**11*b**4*x**6/2 + 462*a**10*b**5*x**(13/2) + 715*a**
9*b**6*x**7 + 858*a**8*b**7*x**(15/2) + 6435*a**7*b**8*x**8/8 + 10010*a**6
*b**9*x**(17/2)/17 + 1001*a**5*b**10*x**9/3 + 2730*a**4*b**11*x**(19/2)/19
 + 91*a**3*b**12*x**10/2 + 10*a**2*b**13*x**(21/2) + 15*a*b**14*x**11/11 +
 2*b**15*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx = \frac{2\left(b\sqrt{x}+a\right)^{23}}{23\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^{22}a}{11\,b^8} + \frac{2\left(b\sqrt{x}+a\right)^{21}a^2}{b^8} - \frac{7\left(b\sqrt{x}+a\right)^{20}a^3}{2\,b^8} + \frac{70\left(b\sqrt{x}+a\right)^{19}a^4}{19\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^{18}a^5}{3\,b^8} + \frac{14\left(b\sqrt{x}+a\right)^{17}a^6}{17\,b^8} - \frac{\left(b\sqrt{x}+a\right)^{16}a^7}{8\,b^8}$$

input
```
integrate((a+b*x^(1/2))^15*x^3,x, algorithm="maxima")
```

output
```
2/23*(b*sqrt(x) + a)^23/b^8 - 7/11*(b*sqrt(x) + a)^22*a/b^8 + 2*(b*sqrt(x)
 + a)^21*a^2/b^8 - 7/2*(b*sqrt(x) + a)^20*a^3/b^8 + 70/19*(b*sqrt(x) + a)^
19*a^4/b^8 - 7/3*(b*sqrt(x) + a)^18*a^5/b^8 + 14/17*(b*sqrt(x) + a)^17*a^6
/b^8 - 1/8*(b*sqrt(x) + a)^16*a^7/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx = \frac{2}{23}\,b^{15}x^{\frac{23}{2}} + \frac{15}{11}\,ab^{14}x^{11} + 10\,a^2b^{13}x^{\frac{21}{2}} + \frac{91}{2}\,a^3b^{12}x^{10} + \frac{2730}{19}\,a^4b^{11}x^{\frac{19}{2}} + \frac{1001}{3}\,a^5b^{10}x^9 + \frac{10010}{17}\,a^6b^9x^{\frac{17}{2}} + \frac{6435}{8}\,a^7b^8x^8 + 858\,a^8b^7x^{\frac{15}{2}} + 715\,a^9b^6x^7 + 462\,a^{10}b^5x^{\frac{13}{2}} + \frac{455}{2}\,a^{11}b^4x^6 + \frac{910}{11}\,a^{12}b^3x^{\frac{11}{2}} + 21\,a^{13}b^2x^5 + \frac{10}{3}\,a^{14}bx^{\frac{9}{2}} + \frac{1}{4}\,a^{15}x^4$$

input
```
integrate((a+b*x^(1/2))^15*x^3,x, algorithm="giac")
```

output
```
2/23*b^15*x^(23/2) + 15/11*a*b^14*x^11 + 10*a^2*b^13*x^(21/2) + 91/2*a^3*b
^12*x^10 + 2730/19*a^4*b^11*x^(19/2) + 1001/3*a^5*b^10*x^9 + 10010/17*a^6*
b^9*x^(17/2) + 6435/8*a^7*b^8*x^8 + 858*a^8*b^7*x^(15/2) + 715*a^9*b^6*x^7
 + 462*a^10*b^5*x^(13/2) + 455/2*a^11*b^4*x^6 + 910/11*a^12*b^3*x^(11/2) +
 21*a^13*b^2*x^5 + 10/3*a^14*b*x^(9/2) + 1/4*a^15*x^4
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx = \frac{a^{15}\,x^4}{4}+\frac{2\,b^{15}\,x^{23/2}}{23}+\frac{15\,a\,b^{14}\,x^{11}}{11}+\frac{10\,a^{14}\,b\,x^{9/2}}{3}+21\,a^{13}\,b^2\,x^5+\frac{455\,a^{11}\,b^4\,x^6}{2}+715\,a^9\,b^6\,x^7+\frac{6435\,a^7\,b^8\,x^8}{8}+\frac{1001\,a^5\,b^{10}\,x^9}{3}+\frac{91\,a^3\,b^{12}\,x^{10}}{2}+\frac{910\,a^{12}\,b^3\,x^{11/2}}{11}+462\,a^{10}\,b^5\,x^{13/2}+858\,a^8\,b^7\,x^{15/2}+\frac{10010\,a^6\,b^9\,x^{17/2}}{17}+\frac{2730\,a^4\,b^{11}\,x^{19/2}}{19}+10\,a^2\,b^{13}\,x^{21/2}$$

input
```
int(x^3*(a + b*x^(1/2))^15,x)
```

output
```
(a^15*x^4)/4 + (2*b^15*x^(23/2))/23 + (15*a*b^14*x^11)/11 + (10*a^14*b*x^(
9/2))/3 + 21*a^13*b^2*x^5 + (455*a^11*b^4*x^6)/2 + 715*a^9*b^6*x^7 + (6435
*a^7*b^8*x^8)/8 + (1001*a^5*b^10*x^9)/3 + (91*a^3*b^12*x^10)/2 + (910*a^12
*b^3*x^(11/2))/11 + 462*a^10*b^5*x^(13/2) + 858*a^8*b^7*x^(15/2) + (10010*
a^6*b^9*x^(17/2))/17 + (2730*a^4*b^11*x^(19/2))/19 + 10*a^2*b^13*x^(21/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10

$$\int \left(a+b\sqrt{x}\right)^{15} x^3\,dx$$
$$= \frac{x^4\left(6537520\sqrt{x}\,a^{14}b+162249360\sqrt{x}\,a^{12}b^3x+906100272\sqrt{x}\,a^{10}b^5x^2+1682757648\sqrt{x}\,a^8b^7x^3+1154833\right.}{}$$

input
```
int((a+b*x^(1/2))^15*x^3,x)
```

output
```
(x**4*(6537520*sqrt(x)*a**14*b + 162249360*sqrt(x)*a**12*b**3*x + 90610027
2*sqrt(x)*a**10*b**5*x**2 + 1682757648*sqrt(x)*a**8*b**7*x**3 + 1154833680
*sqrt(x)*a**6*b**9*x**4 + 281801520*sqrt(x)*a**4*b**11*x**5 + 19612560*sqr
t(x)*a**2*b**13*x**6 + 170544*sqrt(x)*b**15*x**7 + 490314*a**15 + 41186376
*a**13*b**2*x + 446185740*a**11*b**4*x**2 + 1402298040*a**9*b**6*x**3 + 15
77585295*a**7*b**8*x**4 + 654405752*a**5*b**10*x**5 + 89237148*a**3*b**12*
x**6 + 2674440*a*b**14*x**7))/1961256
```

3.62 $\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx = -\frac{a^5\left(a+b\sqrt{x}\right)^{16}}{8b^6}+\frac{10a^4\left(a+b\sqrt{x}\right)^{17}}{17b^6}-\frac{10a^3\left(a+b\sqrt{x}\right)^{18}}{9b^6}+\frac{20a^2\left(a+b\sqrt{x}\right)^{19}}{19b^6}-\frac{a\left(a+b\sqrt{x}\right)^{20}}{2b^6}+\frac{2\left(a+b\sqrt{x}\right)^{21}}{21b^6}$$

output
```
-1/8*a^5*(a+b*x^(1/2))^16/b^6+10/17*a^4*(a+b*x^(1/2))^17/b^6-10/9*a^3*(a+b
*x^(1/2))^18/b^6+20/19*a^2*(a+b*x^(1/2))^19/b^6-1/2*a*(a+b*x^(1/2))^20/b^6
+2/21*(a+b*x^(1/2))^21/b^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.53

$$\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx = \frac{54264a^{15}x^3+697680a^{14}bx^{7/2}+4273290a^{13}b^2x^4+16460080a^{12}b^3x^{9/2}+44442216a^{11}b^4x^5+88884432a^{10}\cdots}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15*x^2,x]
```

output
```
(54264*a^15*x^3 + 697680*a^14*b*x^(7/2) + 4273290*a^13*b^2*x^4 + 16460080*
a^12*b^3*x^(9/2) + 44442216*a^11*b^4*x^5 + 88884432*a^10*b^5*x^(11/2) + 13
5795660*a^9*b^6*x^6 + 161164080*a^8*b^7*x^(13/2) + 149652360*a^7*b^8*x^7 +
 108636528*a^6*b^9*x^(15/2) + 61108047*a^5*b^10*x^8 + 26142480*a^4*b^11*x^
(17/2) + 8230040*a^3*b^12*x^9 + 1799280*a^2*b^13*x^(19/2) + 244188*a*b^14*
x^10 + 15504*b^15*x^(21/2))/162792
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\sqrt{x}\right)^{15} dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^{15} x^{5/2} d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{20}}{b^5}-\frac{5a\left(a+b\sqrt{x}\right)^{19}}{b^5}+\frac{10a^2\left(a+b\sqrt{x}\right)^{18}}{b^5}-\frac{10a^3\left(a+b\sqrt{x}\right)^{17}}{b^5}+\frac{5a^4\left(a+b\sqrt{x}\right)^{16}}{b^5}-\frac{a^5\left(a+b\sqrt{x}\right.}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^5\left(a+b\sqrt{x}\right)^{16}}{16b^6}+\frac{5a^4\left(a+b\sqrt{x}\right)^{17}}{17b^6}-\frac{5a^3\left(a+b\sqrt{x}\right)^{18}}{9b^6}+\frac{10a^2\left(a+b\sqrt{x}\right)^{19}}{19b^6}+\frac{\left(a+b\sqrt{x}\right)^{21}}{21b^6}-\frac{a\left(a+b\sqrt{x}\right)^{20}}{4b^6}\right.$$

input
```
Int[(a + b*Sqrt[x])^15*x^2,x]
```

output
```
2*(-1/16*(a^5*(a + b*Sqrt[x])^16)/b^6 + (5*a^4*(a + b*Sqrt[x])^17)/(17*b^6
) - (5*a^3*(a + b*Sqrt[x])^18)/(9*b^6) + (10*a^2*(a + b*Sqrt[x])^19)/(19*b
^6) - (a*(a + b*Sqrt[x])^20)/(4*b^6) + (a + b*Sqrt[x])^21/(21*b^6))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 22.96 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{2b^{15}x^{\frac{21}{2}}}{21}+\frac{3a\,b^{14}x^{10}}{2}+\frac{210a^{2}b^{13}x^{\frac{19}{2}}}{19}+\frac{455a^{3}b^{12}x^{9}}{9}+\frac{2730a^{4}b^{11}x^{\frac{17}{2}}}{17}+\frac{3003a^{5}b^{10}x^{8}}{8}+\frac{2002a^{6}b^{9}x^{\frac{15}{2}}}{3}+$
default	$\frac{2b^{15}x^{\frac{21}{2}}}{21}+\frac{3a\,b^{14}x^{10}}{2}+\frac{210a^{2}b^{13}x^{\frac{19}{2}}}{19}+\frac{455a^{3}b^{12}x^{9}}{9}+\frac{2730a^{4}b^{11}x^{\frac{17}{2}}}{17}+\frac{3003a^{5}b^{10}x^{8}}{8}+\frac{2002a^{6}b^{9}x^{\frac{15}{2}}}{3}+$
orering	$-\frac{(-453492b^{34}x^{17}+6428380a^{2}b^{32}x^{16}-42376425a^{4}b^{30}x^{15}+172245645a^{6}b^{28}x^{14}-482496245a^{8}b^{26}x^{13}+985927761a^{10}}{}$
trager	$\frac{a(756b^{14}x^{9}+25480b^{12}x^{8}a^{2}+756b^{14}x^{8}+189189b^{10}x^{7}a^{4}+25480a^{2}b^{12}x^{7}+756x^{7}b^{14}+463320b^{8}x^{6}a^{6}+189189a^{4}b^{10}x^{6}+}{}$

input
```
int((a+b*x^(1/2))^15*x^2,x,method=_RETURNVERBOSE)
```

output
```
2/21*b^15*x^(21/2)+3/2*a*b^14*x^10+210/19*a^2*b^13*x^(19/2)+455/9*a^3*b^12
*x^9+2730/17*a^4*b^11*x^(17/2)+3003/8*a^5*b^10*x^8+2002/3*a^6*b^9*x^(15/2)
+6435/7*x^7*b^8*a^7+990*a^8*b^7*x^(13/2)+5005/6*a^9*b^6*x^6+546*a^10*b^5*x
^(11/2)+273*a^11*b^4*x^5+910/9*a^12*b^3*x^(9/2)+105/4*a^13*b^2*x^4+30/7*a^
14*b*x^(7/2)+1/3*a^15*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.42

$$\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx = \frac{3}{2}\,ab^{14}x^{10} + \frac{455}{9}\,a^3b^{12}x^9 + \frac{3003}{8}\,a^5b^{10}x^8 + \frac{6435}{7}\,a^7b^8x^7 + \frac{5005}{6}\,a^9b^6x^6 + 273\,a^{11}b^4x^5 + \frac{105}{4}\,a^{13}b^2x^4 + \frac{1}{3}\,a^{15}x^3 + \frac{2}{20349}\,\left(969\,b^{15}x^{10} + 112455\,a^2b^{13}x^9 + 1633905\,a^4b^{11}x^8 + 6789783\,a^6b^9x^7 + 10072755\,a^8b^7x^6 + 555527$$

input
```
integrate((a+b*x^(1/2))^15*x^2,x, algorithm="fricas")
```

output
```
3/2*a*b^14*x^10 + 455/9*a^3*b^12*x^9 + 3003/8*a^5*b^10*x^8 + 6435/7*a^7*b^
8*x^7 + 5005/6*a^9*b^6*x^6 + 273*a^11*b^4*x^5 + 105/4*a^13*b^2*x^4 + 1/3*a
^15*x^3 + 2/20349*(969*b^15*x^10 + 112455*a^2*b^13*x^9 + 1633905*a^4*b^11*
x^8 + 6789783*a^6*b^9*x^7 + 10072755*a^8*b^7*x^6 + 5555277*a^10*b^5*x^5 +
1028755*a^12*b^3*x^4 + 43605*a^14*b*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.74

$$\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx = \frac{a^{15}x^3}{3} + \frac{30a^{14}bx^{\frac{7}{2}}}{7} + \frac{105a^{13}b^2x^4}{4} + \frac{910a^{12}b^3x^{\frac{9}{2}}}{9} + 273a^{11}b^4x^5 + 546a^{10}b^5x^{\frac{11}{2}} + \frac{5005a^9b^6x^6}{6} + 990a^8b^7x^{\frac{13}{2}} + \frac{6435a^7b^8x^7}{7} + \frac{2002a^6b^9x^{\frac{15}{2}}}{3} + \frac{3003a^5b^{10}x^8}{8} + \frac{2730a^4b^{11}x^{\frac{17}{2}}}{17} + \frac{455a^3b^{12}x^9}{9} + \frac{210a^2b^{13}x^{\frac{19}{2}}}{19} + \frac{3ab^{14}x^{10}}{2} + \frac{2b^{15}x^{\frac{21}{2}}}{21}$$

input
```
integrate((a+b*x**(1/2))**15*x**2,x)
```

output
```
a**15*x**3/3 + 30*a**14*b*x**(7/2)/7 + 105*a**13*b**2*x**4/4 + 910*a**12*b
**3*x**(9/2)/9 + 273*a**11*b**4*x**5 + 546*a**10*b**5*x**(11/2) + 5005*a**
9*b**6*x**6/6 + 990*a**8*b**7*x**(13/2) + 6435*a**7*b**8*x**7/7 + 2002*a**
6*b**9*x**(15/2)/3 + 3003*a**5*b**10*x**8/8 + 2730*a**4*b**11*x**(17/2)/17
 + 455*a**3*b**12*x**9/9 + 210*a**2*b**13*x**(19/2)/19 + 3*a*b**14*x**10/2
 + 2*b**15*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^{15}x^2\,dx = \frac{2\left(b\sqrt{x}+a\right)^{21}}{21\,b^6} - \frac{\left(b\sqrt{x}+a\right)^{20}a}{2\,b^6} + \frac{20\left(b\sqrt{x}+a\right)^{19}a^2}{19\,b^6} - \frac{10\left(b\sqrt{x}+a\right)^{18}a^3}{9\,b^6} + \frac{10\left(b\sqrt{x}+a\right)^{17}a^4}{17\,b^6} - \frac{\left(b\sqrt{x}+a\right)^{16}a^5}{8\,b^6}$$

input
```
integrate((a+b*x^(1/2))^15*x^2,x, algorithm="maxima")
```

output
```
2/21*(b*sqrt(x) + a)^21/b^6 - 1/2*(b*sqrt(x) + a)^20*a/b^6 + 20/19*(b*sqrt
(x) + a)^19*a^2/b^6 - 10/9*(b*sqrt(x) + a)^18*a^3/b^6 + 10/17*(b*sqrt(x) +
 a)^17*a^4/b^6 - 1/8*(b*sqrt(x) + a)^16*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\int \left(a+b\sqrt{x}\right)^{15}x^2\,dx = \frac{2}{21}\,b^{15}x^{\frac{21}{2}} + \frac{3}{2}\,ab^{14}x^{10} + \frac{210}{19}\,a^2b^{13}x^{\frac{19}{2}} + \frac{455}{9}\,a^3b^{12}x^9 + \frac{2730}{17}\,a^4b^{11}x^{\frac{17}{2}} + \frac{3003}{8}\,a^5b^{10}x^8 + \frac{2002}{3}\,a^6b^9x^{\frac{15}{2}} + \frac{6435}{7}\,a^7b^8x^7 + 990\,a^8b^7x^{\frac{13}{2}} + \frac{5005}{6}\,a^9b^6x^6 + 546\,a^{10}b^5x^{\frac{11}{2}} + 273\,a^{11}b^4x^5 + \frac{910}{9}\,a^{12}b^3x^{\frac{9}{2}} + \frac{105}{4}\,a^{13}b^2x^4 + \frac{30}{7}\,a^{14}bx^{\frac{7}{2}} + \frac{1}{3}\,a^{15}x^3$$

input
```
integrate((a+b*x^(1/2))^15*x^2,x, algorithm="giac")
```

output
```
2/21*b^15*x^(21/2) + 3/2*a*b^14*x^10 + 210/19*a^2*b^13*x^(19/2) + 455/9*a^
3*b^12*x^9 + 2730/17*a^4*b^11*x^(17/2) + 3003/8*a^5*b^10*x^8 + 2002/3*a^6*
b^9*x^(15/2) + 6435/7*a^7*b^8*x^7 + 990*a^8*b^7*x^(13/2) + 5005/6*a^9*b^6*
x^6 + 546*a^10*b^5*x^(11/2) + 273*a^11*b^4*x^5 + 910/9*a^12*b^3*x^(9/2) +
105/4*a^13*b^2*x^4 + 30/7*a^14*b*x^(7/2) + 1/3*a^15*x^3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx = \frac{a^{15}\,x^3}{3}+\frac{2\,b^{15}\,x^{21/2}}{21}+\frac{30\,a^{14}\,b\,x^{7/2}}{7}+\frac{3\,a\,b^{14}\,x^{10}}{2}+\frac{105\,a^{13}\,b^2\,x^4}{4} + 273\,a^{11}\,b^4\,x^5+\frac{5005\,a^9\,b^6\,x^6}{6}+\frac{6435\,a^7\,b^8\,x^7}{7}+\frac{3003\,a^5\,b^{10}\,x^8}{8} + \frac{455\,a^3\,b^{12}\,x^9}{9}+\frac{910\,a^{12}\,b^3\,x^{9/2}}{9}+546\,a^{10}\,b^5\,x^{11/2}+990\,a^8\,b^7\,x^{13/2} + \frac{2002\,a^6\,b^9\,x^{15/2}}{3}+\frac{2730\,a^4\,b^{11}\,x^{17/2}}{17}+\frac{210\,a^2\,b^{13}\,x^{19/2}}{19}$$

input
```
int(x^2*(a + b*x^(1/2))^15,x)
```

output
```
(a^15*x^3)/3 + (2*b^15*x^(21/2))/21 + (30*a^14*b*x^(7/2))/7 + (3*a*b^14*x^
10)/2 + (105*a^13*b^2*x^4)/4 + 273*a^11*b^4*x^5 + (5005*a^9*b^6*x^6)/6 + (
6435*a^7*b^8*x^7)/7 + (3003*a^5*b^10*x^8)/8 + (455*a^3*b^12*x^9)/9 + (910*
a^12*b^3*x^(9/2))/9 + 546*a^10*b^5*x^(11/2) + 990*a^8*b^7*x^(13/2) + (2002
*a^6*b^9*x^(15/2))/3 + (2730*a^4*b^11*x^(17/2))/17 + (210*a^2*b^13*x^(19/2
))/19
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.46

$$\int \left(a+b\sqrt{x}\right)^{15} x^2\,dx$$
$$= \frac{x^3\left(697680\sqrt{x}\,a^{14}b+16460080\sqrt{x}\,a^{12}b^3x+88884432\sqrt{x}\,a^{10}b^5x^2+161164080\sqrt{x}\,a^8b^7x^3+108636528\sqrt{\right.}{}$$

input
```
int((a+b*x^(1/2))^15*x^2,x)
```

output
```
(x**3*(697680*sqrt(x)*a**14*b + 16460080*sqrt(x)*a**12*b**3*x + 88884432*s
qrt(x)*a**10*b**5*x**2 + 161164080*sqrt(x)*a**8*b**7*x**3 + 108636528*sqrt
(x)*a**6*b**9*x**4 + 26142480*sqrt(x)*a**4*b**11*x**5 + 1799280*sqrt(x)*a*
*2*b**13*x**6 + 15504*sqrt(x)*b**15*x**7 + 54264*a**15 + 4273290*a**13*b**
2*x + 44442216*a**11*b**4*x**2 + 135795660*a**9*b**6*x**3 + 149652360*a**7
*b**8*x**4 + 61108047*a**5*b**10*x**5 + 8230040*a**3*b**12*x**6 + 244188*a
*b**14*x**7))/162792
```

3.63 $\int \left(a+b\sqrt{x}\right)^{15} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \left(a+b\sqrt{x}\right)^{15} x \, dx = -\frac{a^3\left(a+b\sqrt{x}\right)^{16}}{8b^4}+\frac{6a^2\left(a+b\sqrt{x}\right)^{17}}{17b^4} -\frac{a\left(a+b\sqrt{x}\right)^{18}}{3b^4}+\frac{2\left(a+b\sqrt{x}\right)^{19}}{19b^4}$$

output
```
-1/8*a^3*(a+b*x^(1/2))^16/b^4+6/17*a^2*(a+b*x^(1/2))^17/b^4-1/3*a*(a+b*x^(
1/2))^18/b^4+2/19*(a+b*x^(1/2))^19/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 187 vs. 2(80) = 160.

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.34

$$\int \left(a+b\sqrt{x}\right)^{15} x \, dx = \frac{3876a^{15}x^2+46512a^{14}bx^{5/2}+271320a^{13}b^2x^3+1007760a^{12}b^3x^{7/2}+2645370a^{11}b^4x^4+5173168a^{10}b^5x^{9/2}}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15*x,x]
```

output
```
(3876*a^15*x^2 + 46512*a^14*b*x^(5/2) + 271320*a^13*b^2*x^3 + 1007760*a^12
*b^3*x^(7/2) + 2645370*a^11*b^4*x^4 + 5173168*a^10*b^5*x^(9/2) + 7759752*a
^9*b^6*x^5 + 9069840*a^8*b^7*x^(11/2) + 8314020*a^7*b^8*x^6 + 5969040*a^6*
b^9*x^(13/2) + 3325608*a^5*b^10*x^7 + 1410864*a^4*b^11*x^(15/2) + 440895*a
^3*b^12*x^8 + 95760*a^2*b^13*x^(17/2) + 12920*a*b^14*x^9 + 816*b^15*x^(19/
2))/7752
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}\right)^{15}dx$$

$$\Big\downarrow \; 798$$

$$2\int \left(a+b\sqrt{x}\right)^{15}x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \; 49$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{18}}{b^3}-\frac{3a\left(a+b\sqrt{x}\right)^{17}}{b^3}+\frac{3a^2\left(a+b\sqrt{x}\right)^{16}}{b^3}-\frac{a^3\left(a+b\sqrt{x}\right)^{15}}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(-\frac{a^3\left(a+b\sqrt{x}\right)^{16}}{16b^4}+\frac{3a^2\left(a+b\sqrt{x}\right)^{17}}{17b^4}+\frac{\left(a+b\sqrt{x}\right)^{19}}{19b^4}-\frac{a\left(a+b\sqrt{x}\right)^{18}}{6b^4}\right)$$

input
```
Int[(a + b*Sqrt[x])^15*x,x]
```

output
```
2*(-1/16*(a^3*(a + b*Sqrt[x])^16)/b^4 + (3*a^2*(a + b*Sqrt[x])^17)/(17*b^4
) - (a*(a + b*Sqrt[x])^18)/(6*b^4) + (a + b*Sqrt[x])^19/(19*b^4))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(64) = 128$.

Time = 23.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.10

method	result
derivativedivides	$\frac{2b^{15}x^{\frac{19}{2}}}{19}+\frac{5a\,b^{14}x^{9}}{3}+\frac{210a^{2}b^{13}x^{\frac{17}{2}}}{17}+\frac{455a^{3}b^{12}x^{8}}{8}+182a^{4}b^{11}x^{\frac{15}{2}}+429a^{5}b^{10}x^{7}+770a^{6}b^{9}x^{\frac{13}{2}}+$
default	$\frac{2b^{15}x^{\frac{19}{2}}}{19}+\frac{5a\,b^{14}x^{9}}{3}+\frac{210a^{2}b^{13}x^{\frac{17}{2}}}{17}+\frac{455a^{3}b^{12}x^{8}}{8}+182a^{4}b^{11}x^{\frac{15}{2}}+429a^{5}b^{10}x^{7}+770a^{6}b^{9}x^{\frac{13}{2}}+$
orering	$(4760x^{16}b^{32}-67221x^{15}b^{30}a^{2}+441285x^{14}b^{28}a^{4}-1785385x^{13}b^{26}a^{6}+4975425x^{12}b^{24}a^{8}-10107825x^{11}b^{22}a^{10}+1544843$
trager	$a(40b^{14}x^{8}+1365a^{2}b^{12}x^{7}+40x^{7}b^{14}+10296a^{4}b^{10}x^{6}+1365a^{2}b^{12}x^{6}+40b^{14}x^{6}+25740a^{6}b^{8}x^{5}+10296a^{4}b^{10}x^{5}+1365a^{2}b^{1}$

input
```
int((a+b*x^(1/2))^15*x,x,method=_RETURNVERBOSE)
```

output
```
2/19*b^15*x^(19/2)+5/3*a*b^14*x^9+210/17*a^2*b^13*x^(17/2)+455/8*a^3*b^12*
x^8+182*a^4*b^11*x^(15/2)+429*a^5*b^10*x^7+770*a^6*b^9*x^(13/2)+2145/2*a^7
*b^8*x^6+1170*a^8*b^7*x^(11/2)+1001*a^9*b^6*x^5+2002/3*a^10*b^5*x^(9/2)+13
65/4*a^11*b^4*x^4+130*a^12*b^3*x^(7/2)+35*a^13*b^2*x^3+6*a^14*b*x^(5/2)+1/
2*a^15*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \left(a+b\sqrt{x}\right)^{15} x\,dx = \frac{5}{3}\,ab^{14}x^9 + \frac{455}{8}\,a^3b^{12}x^8 + 429\,a^5b^{10}x^7 + \frac{2145}{2}\,a^7b^8x^6 + 1001\,a^9b^6x^5 + \frac{1365}{4}\,a^{11}b^4x^4 + 35\,a^{13}b^2x^3 + \frac{1}{2}\,a^{15}x^2 + \frac{2}{969}\left(51\,b^{15}x^9 + 5985\,a^2b^{13}x^8 + 88179\,a^4b^{11}x^7 + 373065\,a^6b^9x^6 + 566865\,a^8b^7x^5 + 323323\,a^{10}b^5x^4 + 62$$

input
```
integrate((a+b*x^(1/2))^15*x,x, algorithm="fricas")
```

output
```
5/3*a*b^14*x^9 + 455/8*a^3*b^12*x^8 + 429*a^5*b^10*x^7 + 2145/2*a^7*b^8*x^
6 + 1001*a^9*b^6*x^5 + 1365/4*a^11*b^4*x^4 + 35*a^13*b^2*x^3 + 1/2*a^15*x^
2 + 2/969*(51*b^15*x^9 + 5985*a^2*b^13*x^8 + 88179*a^4*b^11*x^7 + 373065*a
^6*b^9*x^6 + 566865*a^8*b^7*x^5 + 323323*a^10*b^5*x^4 + 62985*a^12*b^3*x^3
 + 2907*a^14*b*x^2)*sqrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(71) = 142$.

Time = 1.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.55

$$\int \left(a+b\sqrt{x}\right)^{15} x\,dx = \frac{a^{15}x^2}{2} + 6a^{14}bx^{\frac{5}{2}} + 35a^{13}b^2x^3 + 130a^{12}b^3x^{\frac{7}{2}} + \frac{1365a^{11}b^4x^4}{4} + \frac{2002a^{10}b^5x^{\frac{9}{2}}}{3} + 1001a^9b^6x^5 + 1170a^8b^7x^{\frac{11}{2}} + \frac{2145a^7b^8x^6}{2} + 770a^6b^9x^{\frac{13}{2}} + 429a^5b^{10}x^7 + 182a^4b^{11}x^{\frac{15}{2}} + \frac{455a^3b^{12}x^8}{8} + \frac{210a^2b^{13}x^{\frac{17}{2}}}{17} + \frac{5ab^{14}x^9}{3} + \frac{2b^{15}x^{\frac{19}{2}}}{19}$$

input
```
integrate((a+b*x**(1/2))**15*x,x)
```

output
```
a**15*x**2/2 + 6*a**14*b*x**(5/2) + 35*a**13*b**2*x**3 + 130*a**12*b**3*x*
*(7/2) + 1365*a**11*b**4*x**4/4 + 2002*a**10*b**5*x**(9/2)/3 + 1001*a**9*b
**6*x**5 + 1170*a**8*b**7*x**(11/2) + 2145*a**7*b**8*x**6/2 + 770*a**6*b**
9*x**(13/2) + 429*a**5*b**10*x**7 + 182*a**4*b**11*x**(15/2) + 455*a**3*b*
*12*x**8/8 + 210*a**2*b**13*x**(17/2)/17 + 5*a*b**14*x**9/3 + 2*b**15*x**(
19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^{15} x\,dx = \frac{2\left(b\sqrt{x}+a\right)^{19}}{19\,b^4} - \frac{\left(b\sqrt{x}+a\right)^{18}a}{3\,b^4} + \frac{6\left(b\sqrt{x}+a\right)^{17}a^2}{17\,b^4} - \frac{\left(b\sqrt{x}+a\right)^{16}a^3}{8\,b^4}$$

input
```
integrate((a+b*x^(1/2))^15*x,x, algorithm="maxima")
```

output
```
2/19*(b*sqrt(x) + a)^19/b^4 - 1/3*(b*sqrt(x) + a)^18*a/b^4 + 6/17*(b*sqrt(
x) + a)^17*a^2/b^4 - 1/8*(b*sqrt(x) + a)^16*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(64) = 128.

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.09

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15} x\,dx = {} & \frac{2}{19}\,b^{15}x^{\frac{19}{2}} + \frac{5}{3}\,ab^{14}x^9 + \frac{210}{17}\,a^2b^{13}x^{\frac{17}{2}} + \frac{455}{8}\,a^3b^{12}x^8 \\ & + 182\,a^4b^{11}x^{\frac{15}{2}} + 429\,a^5b^{10}x^7 + 770\,a^6b^9x^{\frac{13}{2}} + \frac{2145}{2}\,a^7b^8x^6 \\ & + 1170\,a^8b^7x^{\frac{11}{2}} + 1001\,a^9b^6x^5 + \frac{2002}{3}\,a^{10}b^5x^{\frac{9}{2}} + \frac{1365}{4}\,a^{11}b^4x^4 \\ & + 130\,a^{12}b^3x^{\frac{7}{2}} + 35\,a^{13}b^2x^3 + 6\,a^{14}bx^{\frac{5}{2}} + \frac{1}{2}\,a^{15}x^2\end{aligned}$$

input
```
integrate((a+b*x^(1/2))^15*x,x, algorithm="giac")
```

output
```
2/19*b^15*x^(19/2) + 5/3*a*b^14*x^9 + 210/17*a^2*b^13*x^(17/2) + 455/8*a^3
*b^12*x^8 + 182*a^4*b^11*x^(15/2) + 429*a^5*b^10*x^7 + 770*a^6*b^9*x^(13/2
) + 2145/2*a^7*b^8*x^6 + 1170*a^8*b^7*x^(11/2) + 1001*a^9*b^6*x^5 + 2002/3
*a^10*b^5*x^(9/2) + 1365/4*a^11*b^4*x^4 + 130*a^12*b^3*x^(7/2) + 35*a^13*b
^2*x^3 + 6*a^14*b*x^(5/2) + 1/2*a^15*x^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.09

$$\int \left(a+b\sqrt{x}\right)^{15} x\,dx = \frac{a^{15}\,x^2}{2} + \frac{2\,b^{15}\,x^{19/2}}{19} + 6\,a^{14}\,b\,x^{5/2} + \frac{5\,a\,b^{14}\,x^9}{3} + 35\,a^{13}\,b^2\,x^3 + \frac{1365\,a^{11}\,b^4\,x^4}{4} + 1001\,a^9\,b^6\,x^5 + \frac{2145\,a^7\,b^8\,x^6}{2} + 429\,a^5\,b^{10}\,x^7 + \frac{455\,a^3\,b^{12}\,x^8}{8} + 130\,a^{12}\,b^3\,x^{7/2} + \frac{2002\,a^{10}\,b^5\,x^{9/2}}{3} + 1170\,a^8\,b^7\,x^{11/2} + 770\,a^6\,b^9\,x^{13/2} + 182\,a^4\,b^{11}\,x^{15/2} + \frac{210\,a^2\,b^{13}\,x^{17/2}}{17}$$

input
```
int(x*(a + b*x^(1/2))^15,x)
```

output
```
(a^15*x^2)/2 + (2*b^15*x^(19/2))/19 + 6*a^14*b*x^(5/2) + (5*a*b^14*x^9)/3
+ 35*a^13*b^2*x^3 + (1365*a^11*b^4*x^4)/4 + 1001*a^9*b^6*x^5 + (2145*a^7*b
^8*x^6)/2 + 429*a^5*b^10*x^7 + (455*a^3*b^12*x^8)/8 + 130*a^12*b^3*x^(7/2)
 + (2002*a^10*b^5*x^(9/2))/3 + 1170*a^8*b^7*x^(11/2) + 770*a^6*b^9*x^(13/2
) + 182*a^4*b^11*x^(15/2) + (210*a^2*b^13*x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int \left(a+b\sqrt{x}\right)^{15} x\,dx = \frac{x^2\left(46512\sqrt{x}\,a^{14}b + 1007760\sqrt{x}\,a^{12}b^3x + 5173168\sqrt{x}\,a^{10}b^5x^2 + 9069840\sqrt{x}\,a^8b^7x^3 + 5969040\sqrt{x}\,a^6b^9x^4\right.}{}$$

input
```
int((a+b*x^(1/2))^15*x,x)
```

output

```
(x**2*(46512*sqrt(x)*a**14*b + 1007760*sqrt(x)*a**12*b**3*x + 5173168*sqrt
(x)*a**10*b**5*x**2 + 9069840*sqrt(x)*a**8*b**7*x**3 + 5969040*sqrt(x)*a**
6*b**9*x**4 + 1410864*sqrt(x)*a**4*b**11*x**5 + 95760*sqrt(x)*a**2*b**13*x
**6 + 816*sqrt(x)*b**15*x**7 + 3876*a**15 + 271320*a**13*b**2*x + 2645370*
a**11*b**4*x**2 + 7759752*a**9*b**6*x**3 + 8314020*a**7*b**8*x**4 + 332560
8*a**5*b**10*x**5 + 440895*a**3*b**12*x**6 + 12920*a*b**14*x**7))/7752
```

3.64 $\int \left(a+b\sqrt{x}\right)^{15}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \left(a+b\sqrt{x}\right)^{15}\,dx=-\frac{a\left(a+b\sqrt{x}\right)^{16}}{8b^2}+\frac{2\left(a+b\sqrt{x}\right)^{17}}{17b^2}$$

output
```
-1/8*a*(a+b*x^(1/2))^16/b^2+2/17*(a+b*x^(1/2))^17/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 185 vs. 2(38) = 76.

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.87

$$\int \left(a+b\sqrt{x}\right)^{15}\,dx=\frac{1}{136}\left(136a^{15}x+1360a^{14}bx^{3/2}+7140a^{13}b^2x^2+24752a^{12}b^3x^{5/2}+61880a^{11}b^4x^3+116688a^{10}b^5x^{7/2}+170170a^9b^6x^4+194480a^8b^7x^{9/2}+175032a^7b^8x^5+123760a^6b^9x^{11/2}+68068a^5b^{10}x^6+28560a^4b^{11}x^{13/2}+8840a^3b^{12}x^7+1904a^2b^{13}x^{15/2}+255ab^{14}x^8+16b^{15}x^{17/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^15,x]
```

output
```
(136*a^15*x + 1360*a^14*b*x^(3/2) + 7140*a^13*b^2*x^2 + 24752*a^12*b^3*x^(
5/2) + 61880*a^11*b^4*x^3 + 116688*a^10*b^5*x^(7/2) + 170170*a^9*b^6*x^4 +
 194480*a^8*b^7*x^(9/2) + 175032*a^7*b^8*x^5 + 123760*a^6*b^9*x^(11/2) + 6
8068*a^5*b^10*x^6 + 28560*a^4*b^11*x^(13/2) + 8840*a^3*b^12*x^7 + 1904*a^2
*b^13*x^(15/2) + 255*a*b^14*x^8 + 16*b^15*x^(17/2))/136
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^{15} dx$$

$$\Big\downarrow \ 774$$

$$2\int \left(a+b\sqrt{x}\right)^{15} \sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \ 49$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{16}}{b}-\frac{a\left(a+b\sqrt{x}\right)^{15}}{b}\right) d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{\left(a+b\sqrt{x}\right)^{17}}{17b^2}-\frac{a\left(a+b\sqrt{x}\right)^{16}}{16b^2}\right)$$

input
```
Int[(a + b*Sqrt[x])^15,x]
```

output
```
2*(-1/16*(a*(a + b*Sqrt[x])^16)/b^2 + (a + b*Sqrt[x])^17/(17*b^2))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(30) = 60$.

Time = 23.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 4.34

method	result
derivativedivides	$\frac{2b^{15}x^{\frac{17}{2}}}{17}+\frac{15a\,b^{14}x^{8}}{8}+14a^{2}b^{13}x^{\frac{15}{2}}+65a^{3}b^{12}x^{7}+210a^{4}b^{11}x^{\frac{13}{2}}+\frac{1001a^{5}b^{10}x^{6}}{2}+910a^{6}b^{9}x^{\frac{11}{2}}$
default	$\frac{2b^{15}x^{\frac{17}{2}}}{17}+\frac{15a\,b^{14}x^{8}}{8}+14a^{2}b^{13}x^{\frac{15}{2}}+65a^{3}b^{12}x^{7}+210a^{4}b^{11}x^{\frac{13}{2}}+\frac{1001a^{5}b^{10}x^{6}}{2}+910a^{6}b^{9}x^{\frac{11}{2}}$
orering	$-\frac{(-31b^{30}x^{15}+435a^{2}b^{28}x^{14}-2835a^{4}b^{26}x^{13}+11375a^{6}b^{24}x^{12}-31395a^{8}b^{22}x^{11}+63063a^{10}b^{20}x^{10}-95095a^{12}b^{18}x^{9}+10}{}$
trager	$\frac{a(15x^{7}b^{14}+520a^{2}b^{12}x^{6}+15b^{14}x^{6}+4004a^{4}b^{10}x^{5}+520a^{2}b^{12}x^{5}+15b^{14}x^{5}+10296a^{6}b^{8}x^{4}+4004a^{4}b^{10}x^{4}+520b^{12}x^{4}a^{2}+1}{}$

input
```
int((a+b*x^(1/2))^15,x,method=_RETURNVERBOSE)
```

output
```
2/17*b^15*x^(17/2)+15/8*a*b^14*x^8+14*a^2*b^13*x^(15/2)+65*a^3*b^12*x^7+21
0*a^4*b^11*x^(13/2)+1001/2*a^5*b^10*x^6+910*a^6*b^9*x^(11/2)+1287*a^7*b^8*
x^5+1430*a^8*b^7*x^(9/2)+5005/4*a^9*b^6*x^4+858*a^10*b^5*x^(7/2)+455*a^11*
b^4*x^3+182*a^12*b^3*x^(5/2)+105/2*a^13*b^2*x^2+10*a^14*b*x^(3/2)+a^15*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 4.39

$$\int \left(a+b\sqrt{x}\right)^{15} dx = \frac{15}{8}ab^{14}x^8 + 65a^3b^{12}x^7 + \frac{1001}{2}a^5b^{10}x^6 + 1287a^7b^8x^5 + \frac{5005}{4}a^9b^6x^4 + 455a^{11}b^4x^3 + \frac{105}{2}a^{13}b^2x^2 + a^{15}x + \frac{2}{17}\left(b^{15}x^8 + 119a^2b^{13}x^7 + 1785a^4b^{11}x^6 + 7735a^6b^9x^5 + 12155a^8b^7x^4 + 7293a^{10}b^5x^3 + 1547a^{12}b^3x^2 +$$

input
```
integrate((a+b*x^(1/2))^15,x, algorithm="fricas")
```

output
```
15/8*a*b^14*x^8 + 65*a^3*b^12*x^7 + 1001/2*a^5*b^10*x^6 + 1287*a^7*b^8*x^5
 + 5005/4*a^9*b^6*x^4 + 455*a^11*b^4*x^3 + 105/2*a^13*b^2*x^2 + a^15*x + 2
/17*(b^15*x^8 + 119*a^2*b^13*x^7 + 1785*a^4*b^11*x^6 + 7735*a^6*b^9*x^5 +
12155*a^8*b^7*x^4 + 7293*a^10*b^5*x^3 + 1547*a^12*b^3*x^2 + 85*a^14*b*x)*s
qrt(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(32) = 64$.

Time = 0.99 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.18

$$\int \left(a+b\sqrt{x}\right)^{15} dx = a^{15}x + 10a^{14}bx^{\frac{3}{2}} + \frac{105a^{13}b^2x^2}{2} + 182a^{12}b^3x^{\frac{5}{2}} + 455a^{11}b^4x^3 + 858a^{10}b^5x^{\frac{7}{2}} + \frac{5005a^9b^6x^4}{4} + 1430a^8b^7x^{\frac{9}{2}} + 1287a^7b^8x^5 + 910a^6b^9x^{\frac{11}{2}} + \frac{1001a^5b^{10}x^6}{2} + 210a^4b^{11}x^{\frac{13}{2}} + 65a^3b^{12}x^7 + 14a^2b^{13}x^{\frac{15}{2}} + \frac{15ab^{14}x^8}{8} + \frac{2b^{15}x^{\frac{17}{2}}}{17}$$

input
```
integrate((a+b*x**(1/2))**15,x)
```

output
```
a**15*x + 10*a**14*b*x**(3/2) + 105*a**13*b**2*x**2/2 + 182*a**12*b**3*x**
(5/2) + 455*a**11*b**4*x**3 + 858*a**10*b**5*x**(7/2) + 5005*a**9*b**6*x**
4/4 + 1430*a**8*b**7*x**(9/2) + 1287*a**7*b**8*x**5 + 910*a**6*b**9*x**(11
/2) + 1001*a**5*b**10*x**6/2 + 210*a**4*b**11*x**(13/2) + 65*a**3*b**12*x*
*7 + 14*a**2*b**13*x**(15/2) + 15*a*b**14*x**8/8 + 2*b**15*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)^{15}\,dx=\frac{2\left(b\sqrt{x}+a\right)^{17}}{17\,b^2}-\frac{\left(b\sqrt{x}+a\right)^{16}a}{8\,b^2}$$

input
```
integrate((a+b*x^(1/2))^15,x, algorithm="maxima")
```

output
```
2/17*(b*sqrt(x) + a)^17/b^2 - 1/8*(b*sqrt(x) + a)^16*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(30) = 60.

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

$$\begin{aligned}\int \left(a+b\sqrt{x}\right)^{15}\,dx=&\ \frac{2}{17}\,b^{15}x^{\frac{17}{2}}+\frac{15}{8}\,ab^{14}x^8+14\,a^2b^{13}x^{\frac{15}{2}}+65\,a^3b^{12}x^7\\&+210\,a^4b^{11}x^{\frac{13}{2}}+\frac{1001}{2}\,a^5b^{10}x^6+910\,a^6b^9x^{\frac{11}{2}}+1287\,a^7b^8x^5\\&+1430\,a^8b^7x^{\frac{9}{2}}+\frac{5005}{4}\,a^9b^6x^4+858\,a^{10}b^5x^{\frac{7}{2}}+455\,a^{11}b^4x^3\\&+182\,a^{12}b^3x^{\frac{5}{2}}+\frac{105}{2}\,a^{13}b^2x^2+10\,a^{14}bx^{\frac{3}{2}}+a^{15}x\end{aligned}$$

input
```
integrate((a+b*x^(1/2))^15,x, algorithm="giac")
```

output
```
2/17*b^15*x^(17/2) + 15/8*a*b^14*x^8 + 14*a^2*b^13*x^(15/2) + 65*a^3*b^12*
x^7 + 210*a^4*b^11*x^(13/2) + 1001/2*a^5*b^10*x^6 + 910*a^6*b^9*x^(11/2) +
 1287*a^7*b^8*x^5 + 1430*a^8*b^7*x^(9/2) + 5005/4*a^9*b^6*x^4 + 858*a^10*b
^5*x^(7/2) + 455*a^11*b^4*x^3 + 182*a^12*b^3*x^(5/2) + 105/2*a^13*b^2*x^2
+ 10*a^14*b*x^(3/2) + a^15*x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

$$\int \left(a+b\sqrt{x}\right)^{15}\,dx = a^{15}\,x+\frac{2\,b^{15}\,x^{17/2}}{17}+10\,a^{14}\,b\,x^{3/2}+\frac{15\,a\,b^{14}\,x^8}{8}+\frac{105\,a^{13}\,b^2\,x^2}{2}+455\,a^{11}\,b^4\,x^3+\frac{5005\,a^9\,b^6\,x^4}{4}+1287\,a^7\,b^8\,x^5+\frac{1001\,a^5\,b^{10}\,x^6}{2}+65\,a^3\,b^{12}\,x^7+182\,a^{12}\,b^3\,x^{5/2}+858\,a^{10}\,b^5\,x^{7/2}+1430\,a^8\,b^7\,x^{9/2}+910\,a^6\,b^9\,x^{11/2}+210\,a^4\,b^{11}\,x^{13/2}+14\,a^2\,b^{13}\,x^{15/2}$$

input
```
int((a + b*x^(1/2))^15,x)
```

output
```
a^15*x + (2*b^15*x^(17/2))/17 + 10*a^14*b*x^(3/2) + (15*a*b^14*x^8)/8 + (1
05*a^13*b^2*x^2)/2 + 455*a^11*b^4*x^3 + (5005*a^9*b^6*x^4)/4 + 1287*a^7*b^
8*x^5 + (1001*a^5*b^10*x^6)/2 + 65*a^3*b^12*x^7 + 182*a^12*b^3*x^(5/2) + 8
58*a^10*b^5*x^(7/2) + 1430*a^8*b^7*x^(9/2) + 910*a^6*b^9*x^(11/2) + 210*a^
4*b^11*x^(13/2) + 14*a^2*b^13*x^(15/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 4.63

$$\int \left(a+b\sqrt{x}\right)^{15}\,dx = \frac{x\left(1360\sqrt{x}\,a^{14}b+24752\sqrt{x}\,a^{12}b^3x+116688\sqrt{x}\,a^{10}b^5x^2+194480\sqrt{x}\,a^8b^7x^3+123760\sqrt{x}\,a^6b^9x^4+28560\right.}{}$$

input
```
int((a+b*x^(1/2))^15,x)
```

output
```
(x*(1360*sqrt(x)*a**14*b + 24752*sqrt(x)*a**12*b**3*x + 116688*sqrt(x)*a**
10*b**5*x**2 + 194480*sqrt(x)*a**8*b**7*x**3 + 123760*sqrt(x)*a**6*b**9*x*
*4 + 28560*sqrt(x)*a**4*b**11*x**5 + 1904*sqrt(x)*a**2*b**13*x**6 + 16*sqr
t(x)*b**15*x**7 + 136*a**15 + 7140*a**13*b**2*x + 61880*a**11*b**4*x**2 +
170170*a**9*b**6*x**3 + 175032*a**7*b**8*x**4 + 68068*a**5*b**10*x**5 + 88
40*a**3*b**12*x**6 + 255*a*b**14*x**7))/136
```

3.65 $\int \frac{(a+b\sqrt{x})^{15}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 205

$$\begin{aligned}\int \frac{(a+b\sqrt{x})^{15}}{x} dx = {} & 30a^{14}b\sqrt{x} + 105a^{13}b^2x + \frac{910}{3}a^{12}b^3x^{3/2} + \frac{1365}{2}a^{11}b^4x^2 \\ & + \frac{6006}{5}a^{10}b^5x^{5/2} + \frac{5005}{3}a^9b^6x^3 + \frac{12870}{7}a^8b^7x^{7/2} + \frac{6435}{4}a^7b^8x^4 \\ & + \frac{10010}{9}a^6b^9x^{9/2} + \frac{3003}{5}a^5b^{10}x^5 + \frac{2730}{11}a^4b^{11}x^{11/2} + \frac{455}{6}a^3b^{12}x^6 \\ & + \frac{210}{13}a^2b^{13}x^{13/2} + \frac{15}{7}ab^{14}x^7 + \frac{2}{15}b^{15}x^{15/2} + a^{15}\log(x)\end{aligned}$$

output
```
30*a^14*b*x^(1/2)+105*a^13*b^2*x+910/3*a^12*b^3*x^(3/2)+1365/2*a^11*b^4*x^
2+6006/5*a^10*b^5*x^(5/2)+5005/3*a^9*b^6*x^3+12870/7*a^8*b^7*x^(7/2)+6435/
4*a^7*b^8*x^4+10010/9*a^6*b^9*x^(9/2)+3003/5*a^5*b^10*x^5+2730/11*a^4*b^11
*x^(11/2)+455/6*a^3*b^12*x^6+210/13*a^2*b^13*x^(13/2)+15/7*a*b^14*x^7+2/15
*b^15*x^(15/2)+a^15*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx$$
$$= \frac{5405400a^{14}b\sqrt{x} + 18918900a^{13}b^2x + 54654600a^{12}b^3x^{3/2} + 122972850a^{11}b^4x^2 + 216432216a^{10}b^5x^{5/2} + 30}{}$$
$$+ 2a^{15}\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^15/x,x]
```

output
```
(5405400*a^14*b*Sqrt[x] + 18918900*a^13*b^2*x + 54654600*a^12*b^3*x^(3/2)
+ 122972850*a^11*b^4*x^2 + 216432216*a^10*b^5*x^(5/2) + 300600300*a^9*b^6*
x^3 + 331273800*a^8*b^7*x^(7/2) + 289864575*a^7*b^8*x^4 + 200400200*a^6*b^
9*x^(9/2) + 108216108*a^5*b^10*x^5 + 44717400*a^4*b^11*x^(11/2) + 13663650
*a^3*b^12*x^6 + 2910600*a^2*b^13*x^(13/2) + 386100*a*b^14*x^7 + 24024*b^15
*x^(15/2))/180180 + 2*a^15*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx$$
$$\Big\downarrow \ 798$$
$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{\sqrt{x}}d\sqrt{x}$$
$$\Big\downarrow \ 49$$

$$2\int\left(\frac{a^{15}}{\sqrt{x}}+15ba^{14}+105b^2\sqrt{x}a^{13}+455b^3xa^{12}+1365b^4x^{3/2}a^{11}+3003b^5x^2a^{10}+5005b^6x^{5/2}a^9+6435b^7x^3a^8+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(a^{15}\log\left(\sqrt{x}\right)+15a^{14}b\sqrt{x}+\frac{105}{2}a^{13}b^2x+\frac{455}{3}a^{12}b^3x^{3/2}+\frac{1365}{4}a^{11}b^4x^2+\frac{3003}{5}a^{10}b^5x^{5/2}+\frac{5005}{6}a^9b^6x^3+\frac{643}{7}\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x,x]
```

output
```
2*(15*a^14*b*Sqrt[x] + (105*a^13*b^2*x)/2 + (455*a^12*b^3*x^(3/2))/3 + (13
65*a^11*b^4*x^2)/4 + (3003*a^10*b^5*x^(5/2))/5 + (5005*a^9*b^6*x^3)/6 + (6
435*a^8*b^7*x^(7/2))/7 + (6435*a^7*b^8*x^4)/8 + (5005*a^6*b^9*x^(9/2))/9 +
 (3003*a^5*b^10*x^5)/10 + (1365*a^4*b^11*x^(11/2))/11 + (455*a^3*b^12*x^6)
/12 + (105*a^2*b^13*x^(13/2))/13 + (15*a*b^14*x^7)/14 + (b^15*x^(15/2))/15
 + a^15*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

method	result
derivativedivides	$30a^{14}b\sqrt{x}+105a^{13}b^{2}x+\frac{910a^{12}b^{3}x^{\frac{3}{2}}}{3}+\frac{1365a^{11}b^{4}x^{2}}{2}+\frac{6006a^{10}b^{5}x^{\frac{5}{2}}}{5}+\frac{5005a^{9}b^{6}x^{3}}{3}+\frac{12870a^{8}b^{7}x^{\frac{7}{2}}}{7}$
default	$30a^{14}b\sqrt{x}+105a^{13}b^{2}x+\frac{910a^{12}b^{3}x^{\frac{3}{2}}}{3}+\frac{1365a^{11}b^{4}x^{2}}{2}+\frac{6006a^{10}b^{5}x^{\frac{5}{2}}}{5}+\frac{5005a^{9}b^{6}x^{3}}{3}+\frac{12870a^{8}b^{7}x^{\frac{7}{2}}}{7}$
trager	$a\,b^{2}(900x^{6}b^{12}+31850x^{5}b^{10}a^{2}+900x^{5}b^{12}+252252x^{4}b^{8}a^{4}+31850b^{10}x^{4}a^{2}+900b^{12}x^{4}+675675x^{3}b^{6}a^{6}+252252b^{8}x^{3}a^{4}+$

input
```
int((a+b*x^(1/2))^15/x,x,method=_RETURNVERBOSE)
```

output
```
30*a^14*b*x^(1/2)+105*a^13*b^2*x+910/3*a^12*b^3*x^(3/2)+1365/2*a^11*b^4*x^
2+6006/5*a^10*b^5*x^(5/2)+5005/3*a^9*b^6*x^3+12870/7*a^8*b^7*x^(7/2)+6435/
4*a^7*b^8*x^4+10010/9*a^6*b^9*x^(9/2)+3003/5*a^5*b^10*x^5+2730/11*a^4*b^11
*x^(11/2)+455/6*a^3*b^12*x^6+210/13*a^2*b^13*x^(13/2)+15/7*a*b^14*x^7+2/15
*b^15*x^(15/2)+a^15*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx = \frac{15}{7}\,ab^{14}x^{7}+\frac{455}{6}\,a^{3}b^{12}x^{6}+\frac{3003}{5}\,a^{5}b^{10}x^{5}+\frac{6435}{4}\,a^{7}b^{8}x^{4} + \frac{5005}{3}\,a^{9}b^{6}x^{3}+\frac{1365}{2}\,a^{11}b^{4}x^{2}+105\,a^{13}b^{2}x+2\,a^{15}\log\left(\sqrt{x}\right) + \frac{2}{45045}\left(3003\,b^{15}x^{7}+363825\,a^{2}b^{13}x^{6}+5589675\,a^{4}b^{11}x^{5}+25050025\,a^{6}b^{9}x^{4}+41409225\,a^{8}b^{7}x^{3}+27054\right.$$

input
```
integrate((a+b*x^(1/2))^15/x,x, algorithm="fricas")
```

output
```
15/7*a*b^14*x^7 + 455/6*a^3*b^12*x^6 + 3003/5*a^5*b^10*x^5 + 6435/4*a^7*b^
8*x^4 + 5005/3*a^9*b^6*x^3 + 1365/2*a^11*b^4*x^2 + 105*a^13*b^2*x + 2*a^15
*log(sqrt(x)) + 2/45045*(3003*b^15*x^7 + 363825*a^2*b^13*x^6 + 5589675*a^4
*b^11*x^5 + 25050025*a^6*b^9*x^4 + 41409225*a^8*b^7*x^3 + 27054027*a^10*b^
5*x^2 + 6831825*a^12*b^3*x + 675675*a^14*b)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx = a^{15}\log{(x)} + 30a^{14}b\sqrt{x} + 105a^{13}b^2x + \frac{910a^{12}b^3x^{\frac{3}{2}}}{3} + \frac{1365a^{11}b^4x^2}{2} + \frac{6006a^{10}b^5x^{\frac{5}{2}}}{5} + \frac{5005a^9b^6x^3}{3} + \frac{12870a^8b^7x^{\frac{7}{2}}}{7} + \frac{6435a^7b^8x^4}{4} + \frac{10010a^6b^9x^{\frac{9}{2}}}{9} + \frac{3003a^5b^{10}x^5}{5} + \frac{2730a^4b^{11}x^{\frac{11}{2}}}{11} + \frac{455a^3b^{12}x^6}{6} + \frac{210a^2b^{13}x^{\frac{13}{2}}}{13} + \frac{15ab^{14}x^7}{7} + \frac{2b^{15}x^{\frac{15}{2}}}{15}$$

input
```
integrate((a+b*x**(1/2))**15/x,x)
```

output
```
a**15*log(x) + 30*a**14*b*sqrt(x) + 105*a**13*b**2*x + 910*a**12*b**3*x**(
3/2)/3 + 1365*a**11*b**4*x**2/2 + 6006*a**10*b**5*x**(5/2)/5 + 5005*a**9*b
**6*x**3/3 + 12870*a**8*b**7*x**(7/2)/7 + 6435*a**7*b**8*x**4/4 + 10010*a*
*6*b**9*x**(9/2)/9 + 3003*a**5*b**10*x**5/5 + 2730*a**4*b**11*x**(11/2)/11
 + 455*a**3*b**12*x**6/6 + 210*a**2*b**13*x**(13/2)/13 + 15*a*b**14*x**7/7
 + 2*b**15*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx = \frac{2}{15}\,b^{15}x^{\frac{15}{2}} + \frac{15}{7}\,ab^{14}x^7 + \frac{210}{13}\,a^2b^{13}x^{\frac{13}{2}} + \frac{455}{6}\,a^3b^{12}x^6 + \frac{2730}{11}\,a^4b^{11}x^{\frac{11}{2}} + \frac{3003}{5}\,a^5b^{10}x^5 + \frac{10010}{9}\,a^6b^9x^{\frac{9}{2}} + \frac{6435}{4}\,a^7b^8x^4 + \frac{12870}{7}\,a^8b^7x^{\frac{7}{2}} + \frac{5005}{3}\,a^9b^6x^3 + \frac{6006}{5}\,a^{10}b^5x^{\frac{5}{2}} + \frac{1365}{2}\,a^{11}b^4x^2 + \frac{910}{3}\,a^{12}b^3x^{\frac{3}{2}} + 105\,a^{13}b^2x + a^{15}\log{(x)} + 30\,a^{14}b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^15/x,x, algorithm="maxima")
```

output
```
2/15*b^15*x^(15/2) + 15/7*a*b^14*x^7 + 210/13*a^2*b^13*x^(13/2) + 455/6*a^
3*b^12*x^6 + 2730/11*a^4*b^11*x^(11/2) + 3003/5*a^5*b^10*x^5 + 10010/9*a^6
*b^9*x^(9/2) + 6435/4*a^7*b^8*x^4 + 12870/7*a^8*b^7*x^(7/2) + 5005/3*a^9*b
^6*x^3 + 6006/5*a^10*b^5*x^(5/2) + 1365/2*a^11*b^4*x^2 + 910/3*a^12*b^3*x^
(3/2) + 105*a^13*b^2*x + a^15*log(x) + 30*a^14*b*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx = \frac{2}{15}\,b^{15}x^{\frac{15}{2}} + \frac{15}{7}\,ab^{14}x^{7} + \frac{210}{13}\,a^{2}b^{13}x^{\frac{13}{2}} + \frac{455}{6}\,a^{3}b^{12}x^{6} + \frac{2730}{11}\,a^{4}b^{11}x^{\frac{11}{2}} + \frac{3003}{5}\,a^{5}b^{10}x^{5} + \frac{10010}{9}\,a^{6}b^{9}x^{\frac{9}{2}} + \frac{6435}{4}\,a^{7}b^{8}x^{4} + \frac{12870}{7}\,a^{8}b^{7}x^{\frac{7}{2}} + \frac{5005}{3}\,a^{9}b^{6}x^{3} + \frac{6006}{5}\,a^{10}b^{5}x^{\frac{5}{2}} + \frac{1365}{2}\,a^{11}b^{4}x^{2} + \frac{910}{3}\,a^{12}b^{3}x^{\frac{3}{2}} + 105\,a^{13}b^{2}x + a^{15}\log\left(|x|\right) + 30\,a^{14}b\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^15/x,x, algorithm="giac")
```

output
```
2/15*b^15*x^(15/2) + 15/7*a*b^14*x^7 + 210/13*a^2*b^13*x^(13/2) + 455/6*a^
3*b^12*x^6 + 2730/11*a^4*b^11*x^(11/2) + 3003/5*a^5*b^10*x^5 + 10010/9*a^6
*b^9*x^(9/2) + 6435/4*a^7*b^8*x^4 + 12870/7*a^8*b^7*x^(7/2) + 5005/3*a^9*b
^6*x^3 + 6006/5*a^10*b^5*x^(5/2) + 1365/2*a^11*b^4*x^2 + 910/3*a^12*b^3*x^
(3/2) + 105*a^13*b^2*x + a^15*log(abs(x)) + 30*a^14*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x}\,dx = 2\,a^{15}\,\ln\left(\sqrt{x}\right) + \frac{2\,b^{15}\,x^{15/2}}{15} + 105\,a^{13}\,b^2\,x + 30\,a^{14}\,b\,\sqrt{x} + \frac{15\,a\,b^{14}\,x^7}{7} + \frac{1365\,a^{11}\,b^4\,x^2}{2} + \frac{5005\,a^9\,b^6\,x^3}{3} + \frac{6435\,a^7\,b^8\,x^4}{4} + \frac{3003\,a^5\,b^{10}\,x^5}{5} + \frac{910\,a^{12}\,b^3\,x^{3/2}}{3} + \frac{455\,a^3\,b^{12}\,x^6}{6} + \frac{6006\,a^{10}\,b^5\,x^{5/2}}{5} + \frac{12870\,a^8\,b^7\,x^{7/2}}{7} + \frac{10010\,a^6\,b^9\,x^{9/2}}{9} + \frac{2730\,a^4\,b^{11}\,x^{11/2}}{11} + \frac{210\,a^2\,b^{13}\,x^{13/2}}{13}$$

input
```
int((a + b*x^(1/2))^15/x,x)
```

output
```
2*a^15*log(x^(1/2)) + (2*b^15*x^(15/2))/15 + 105*a^13*b^2*x + 30*a^14*b*x^
(1/2) + (15*a*b^14*x^7)/7 + (1365*a^11*b^4*x^2)/2 + (5005*a^9*b^6*x^3)/3 +
 (6435*a^7*b^8*x^4)/4 + (3003*a^5*b^10*x^5)/5 + (910*a^12*b^3*x^(3/2))/3 +
 (455*a^3*b^12*x^6)/6 + (6006*a^10*b^5*x^(5/2))/5 + (12870*a^8*b^7*x^(7/2)
)/7 + (10010*a^6*b^9*x^(9/2))/9 + (2730*a^4*b^11*x^(11/2))/11 + (210*a^2*b
^13*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\sqrt{x})^{15}}{x}\,dx = 30\sqrt{x}\,a^{14}b + \frac{910\sqrt{x}\,a^{12}b^3x}{3} + \frac{6006\sqrt{x}\,a^{10}b^5x^2}{5} + \frac{12870\sqrt{x}\,a^8b^7x^3}{7} + \frac{10010\sqrt{x}\,a^6b^9x^4}{9} + \frac{2730\sqrt{x}\,a^4b^{11}x^5}{11} + \frac{210\sqrt{x}\,a^2b^{13}x^6}{13} + \frac{2\sqrt{x}\,b^{15}x^7}{15} + \log(x)\,a^{15} + 105a^{13}b^2x + \frac{1365a^{11}b^4x^2}{2} + \frac{5005a^9b^6x^3}{3} + \frac{6435a^7b^8x^4}{4} + \frac{3003a^5b^{10}x^5}{5} + \frac{455a^3b^{12}x^6}{6} + \frac{15a\,b^{14}x^7}{7}$$

input
```
int((a+b*x^(1/2))^15/x,x)
```

output
```
(5405400*sqrt(x)*a**14*b + 54654600*sqrt(x)*a**12*b**3*x + 216432216*sqrt(
x)*a**10*b**5*x**2 + 331273800*sqrt(x)*a**8*b**7*x**3 + 200400200*sqrt(x)*
a**6*b**9*x**4 + 44717400*sqrt(x)*a**4*b**11*x**5 + 2910600*sqrt(x)*a**2*b
**13*x**6 + 24024*sqrt(x)*b**15*x**7 + 180180*log(x)*a**15 + 18918900*a**1
3*b**2*x + 122972850*a**11*b**4*x**2 + 300600300*a**9*b**6*x**3 + 28986457
5*a**7*b**8*x**4 + 108216108*a**5*b**10*x**5 + 13663650*a**3*b**12*x**6 +
386100*a*b**14*x**7)/180180
```

3.66 $\int \frac{(a+b\sqrt{x})^{15}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 192

$$\int \frac{(a+b\sqrt{x})^{15}}{x^2} dx = -\frac{a^{15}}{x} - \frac{30a^{14}b}{\sqrt{x}} + 910a^{12}b^3\sqrt{x} + 1365a^{11}b^4x + 2002a^{10}b^5x^{3/2} + \frac{5005}{2}a^9b^6x^2 + 2574a^8b^7x^{5/2} + 2145a^7b^8x^3 + 1430a^6b^9x^{7/2} + \frac{3003}{4}a^5b^{10}x^4 + \frac{910}{3}a^4b^{11}x^{9/2} + 91a^3b^{12}x^5 + \frac{210}{11}a^2b^{13}x^{11/2} + \frac{5}{2}ab^{14}x^6 + \frac{2}{13}b^{15}x^{13/2} + 105a^{13}b^2\log(x)$$

output

```
-a^15/x-30*a^14*b/x^(1/2)+910*a^12*b^3*x^(1/2)+1365*a^11*b^4*x+2002*a^10*b
^5*x^(3/2)+5005/2*a^9*b^6*x^2+2574*a^8*b^7*x^(5/2)+2145*a^7*b^8*x^3+1430*a
^6*b^9*x^(7/2)+3003/4*a^5*b^10*x^4+910/3*a^4*b^11*x^(9/2)+91*a^3*b^12*x^5+
210/11*a^2*b^13*x^(11/2)+5/2*a*b^14*x^6+2/13*b^15*x^(13/2)+105*a^13*b^2*ln
(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\sqrt{x})^{15}}{x^2} \, dx$$
$$= \frac{-1716a^{15} - 51480a^{14}b\sqrt{x} + 1561560a^{12}b^3x^{3/2} + 2342340a^{11}b^4x^2 + 3435432a^{10}b^5x^{5/2} + 4294290a^9b^6x^3 -}{}$$
$$+ 210a^{13}b^2\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^2,x]
```

output
```
(-1716*a^15 - 51480*a^14*b*Sqrt[x] + 1561560*a^12*b^3*x^(3/2) + 2342340*a^
11*b^4*x^2 + 3435432*a^10*b^5*x^(5/2) + 4294290*a^9*b^6*x^3 + 4416984*a^8*
b^7*x^(7/2) + 3680820*a^7*b^8*x^4 + 2453880*a^6*b^9*x^(9/2) + 1288287*a^5*
b^10*x^5 + 520520*a^4*b^11*x^(11/2) + 156156*a^3*b^12*x^6 + 32760*a^2*b^13
*x^(13/2) + 4290*a*b^14*x^7 + 264*b^15*x^(15/2))/(1716*x) + 210*a^13*b^2*L
og[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^{15}}{x^2} \, dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{(a+b\sqrt{x})^{15}}{x^{3/2}} d\sqrt{x}$$
$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^{15}}{x^{3/2}}+\frac{15ba^{14}}{x}+\frac{105b^2a^{13}}{\sqrt{x}}+455b^3a^{12}+1365b^4\sqrt{x}a^{11}+3003b^5xa^{10}+5005b^6x^{3/2}a^9+6435b^7x^2a^8+6435\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{15}}{2x}-\frac{15a^{14}b}{\sqrt{x}}+105a^{13}b^2\log\left(\sqrt{x}\right)+455a^{12}b^3\sqrt{x}+\frac{1365}{2}a^{11}b^4x+1001a^{10}b^5x^{3/2}+\frac{5005}{4}a^9b^6x^2+1287a^8b^7\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^2,x]
```

output
```
2*(-1/2*a^15/x - (15*a^14*b)/Sqrt[x] + 455*a^12*b^3*Sqrt[x] + (1365*a^11*b
^4*x)/2 + 1001*a^10*b^5*x^(3/2) + (5005*a^9*b^6*x^2)/4 + 1287*a^8*b^7*x^(5
/2) + (2145*a^7*b^8*x^3)/2 + 715*a^6*b^9*x^(7/2) + (3003*a^5*b^10*x^4)/8 +
 (455*a^4*b^11*x^(9/2))/3 + (91*a^3*b^12*x^5)/2 + (105*a^2*b^13*x^(11/2))/
11 + (5*a*b^14*x^6)/4 + (b^15*x^(13/2))/13 + 105*a^13*b^2*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.84 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{a^{15}}{x}-\frac{30a^{14}b}{\sqrt{x}}+910a^{12}b^3\sqrt{x}+1365a^{11}b^4x+2002a^{10}b^5x^{\frac{3}{2}}+\frac{5005a^9b^6x^2}{2}+2574a^8b^7x^{\frac{5}{2}}+$
default	$-\frac{a^{15}}{x}-\frac{30a^{14}b}{\sqrt{x}}+910a^{12}b^3\sqrt{x}+1365a^{11}b^4x+2002a^{10}b^5x^{\frac{3}{2}}+\frac{5005a^9b^6x^2}{2}+2574a^8b^7x^{\frac{5}{2}}+$
trager	$\frac{(-1+x)(10b^{14}x^6+364a^2b^{12}x^5+10b^{14}x^5+3003a^4b^{10}x^4+364b^{12}x^4a^2+10b^{14}x^4+8580a^6b^8x^3+3003b^{10}x^3a^4+364b^{12}x^3}{}$

input
```
int((a+b*x^(1/2))^15/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^15/x-30*a^14*b/x^(1/2)+910*a^12*b^3*x^(1/2)+1365*a^11*b^4*x+2002*a^10*b
^5*x^(3/2)+5005/2*a^9*b^6*x^2+2574*a^8*b^7*x^(5/2)+2145*a^7*b^8*x^3+1430*a
^6*b^9*x^(7/2)+3003/4*a^5*b^10*x^4+910/3*a^4*b^11*x^(9/2)+91*a^3*b^12*x^5+
210/11*a^2*b^13*x^(11/2)+5/2*a*b^14*x^6+2/13*b^15*x^(13/2)+105*a^13*b^2*ln
(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx = \frac{4290\,ab^{14}x^7+156156\,a^3b^{12}x^6+1288287\,a^5b^{10}x^5+3680820\,a^7b^8x^4+4294290\,a^9b^6x^3+2342340\,a^{11}b^4x^2}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^2,x, algorithm="fricas")
```

output
```
1/1716*(4290*a*b^14*x^7 + 156156*a^3*b^12*x^6 + 1288287*a^5*b^10*x^5 + 368
0820*a^7*b^8*x^4 + 4294290*a^9*b^6*x^3 + 2342340*a^11*b^4*x^2 + 360360*a^1
3*b^2*x*log(sqrt(x)) - 1716*a^15 + 8*(33*b^15*x^7 + 4095*a^2*b^13*x^6 + 65
065*a^4*b^11*x^5 + 306735*a^6*b^9*x^4 + 552123*a^8*b^7*x^3 + 429429*a^10*b
^5*x^2 + 195195*a^12*b^3*x - 6435*a^14*b)*sqrt(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx = -\frac{a^{15}}{x}-\frac{30a^{14}b}{\sqrt{x}}+105a^{13}b^2\log{(x)}+910a^{12}b^3\sqrt{x} +1365a^{11}b^4x+2002a^{10}b^5x^{\frac{3}{2}}+\frac{5005a^9b^6x^2}{2}+2574a^8b^7x^{\frac{5}{2}} +2145a^7b^8x^3+1430a^6b^9x^{\frac{7}{2}}+\frac{3003a^5b^{10}x^4}{4}+\frac{910a^4b^{11}x^{\frac{9}{2}}}{3} +91a^3b^{12}x^5+\frac{210a^2b^{13}x^{\frac{11}{2}}}{11}+\frac{5ab^{14}x^6}{2}+\frac{2b^{15}x^{\frac{13}{2}}}{13}$$

input
```
integrate((a+b*x**(1/2))**15/x**2,x)
```

output
```
-a**15/x - 30*a**14*b/sqrt(x) + 105*a**13*b**2*log(x) + 910*a**12*b**3*sqr
t(x) + 1365*a**11*b**4*x + 2002*a**10*b**5*x**(3/2) + 5005*a**9*b**6*x**2/
2 + 2574*a**8*b**7*x**(5/2) + 2145*a**7*b**8*x**3 + 1430*a**6*b**9*x**(7/2
) + 3003*a**5*b**10*x**4/4 + 910*a**4*b**11*x**(9/2)/3 + 91*a**3*b**12*x**
5 + 210*a**2*b**13*x**(11/2)/11 + 5*a*b**14*x**6/2 + 2*b**15*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx = \frac{2}{13}\,b^{15}x^{\frac{13}{2}}+\frac{5}{2}\,ab^{14}x^6+\frac{210}{11}\,a^2b^{13}x^{\frac{11}{2}}+91\,a^3b^{12}x^5 +\frac{910}{3}\,a^4b^{11}x^{\frac{9}{2}}+\frac{3003}{4}\,a^5b^{10}x^4+1430\,a^6b^9x^{\frac{7}{2}}+2145\,a^7b^8x^3 +2574\,a^8b^7x^{\frac{5}{2}}+\frac{5005}{2}\,a^9b^6x^2+2002\,a^{10}b^5x^{\frac{3}{2}}+1365\,a^{11}b^4x +105\,a^{13}b^2\log{(x)}+910\,a^{12}b^3\sqrt{x}-\frac{30\,a^{14}b\sqrt{x}+a^{15}}{x}$$

input
```
integrate((a+b*x^(1/2))^15/x^2,x, algorithm="maxima")
```

output
```
2/13*b^15*x^(13/2) + 5/2*a*b^14*x^6 + 210/11*a^2*b^13*x^(11/2) + 91*a^3*b^
12*x^5 + 910/3*a^4*b^11*x^(9/2) + 3003/4*a^5*b^10*x^4 + 1430*a^6*b^9*x^(7/
2) + 2145*a^7*b^8*x^3 + 2574*a^8*b^7*x^(5/2) + 5005/2*a^9*b^6*x^2 + 2002*a
^10*b^5*x^(3/2) + 1365*a^11*b^4*x + 105*a^13*b^2*log(x) + 910*a^12*b^3*sqr
t(x) - (30*a^14*b*sqrt(x) + a^15)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx = \frac{2}{13}\,b^{15}x^{\frac{13}{2}} + \frac{5}{2}\,ab^{14}x^6 + \frac{210}{11}\,a^2b^{13}x^{\frac{11}{2}} + 91\,a^3b^{12}x^5 + \frac{910}{3}\,a^4b^{11}x^{\frac{9}{2}} + \frac{3003}{4}\,a^5b^{10}x^4 + 1430\,a^6b^9x^{\frac{7}{2}} + 2145\,a^7b^8x^3 + 2574\,a^8b^7x^{\frac{5}{2}} + \frac{5005}{2}\,a^9b^6x^2 + 2002\,a^{10}b^5x^{\frac{3}{2}} + 1365\,a^{11}b^4x + 105\,a^{13}b^2\log\left(|x|\right) + 910\,a^{12}b^3\sqrt{x} - \frac{30\,a^{14}b\sqrt{x} + a^{15}}{x}$$

input
```
integrate((a+b*x^(1/2))^15/x^2,x, algorithm="giac")
```

output
```
2/13*b^15*x^(13/2) + 5/2*a*b^14*x^6 + 210/11*a^2*b^13*x^(11/2) + 91*a^3*b^
12*x^5 + 910/3*a^4*b^11*x^(9/2) + 3003/4*a^5*b^10*x^4 + 1430*a^6*b^9*x^(7/
2) + 2145*a^7*b^8*x^3 + 2574*a^8*b^7*x^(5/2) + 5005/2*a^9*b^6*x^2 + 2002*a
^10*b^5*x^(3/2) + 1365*a^11*b^4*x + 105*a^13*b^2*log(abs(x)) + 910*a^12*b^
3*sqrt(x) - (30*a^14*b*sqrt(x) + a^15)/x
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx = \frac{2\,b^{15}\,x^{13/2}}{13} - \frac{a^{15} + 30\,a^{14}\,b\,\sqrt{x}}{x} + 210\,a^{13}\,b^2\,\ln\left(\sqrt{x}\right) + 1365\,a^{11}\,b^4\,x + \frac{5\,a\,b^{14}\,x^6}{2} + \frac{5005\,a^9\,b^6\,x^2}{2} + 2145\,a^7\,b^8\,x^3 + 910\,a^{12}\,b^3\,\sqrt{x} + \frac{3003\,a^5\,b^{10}\,x^4}{4} + 91\,a^3\,b^{12}\,x^5 + 2002\,a^{10}\,b^5\,x^{3/2} + 2574\,a^8\,b^7\,x^{5/2} + 1430\,a^6\,b^9\,x^{7/2} + \frac{910\,a^4\,b^{11}\,x^{9/2}}{3} + \frac{210\,a^2\,b^{13}\,x^{11/2}}{11}$$

input
```
int((a + b*x^(1/2))^15/x^2,x)
```

output
```
(2*b^15*x^(13/2))/13 - (a^15 + 30*a^14*b*x^(1/2))/x + 210*a^13*b^2*log(x^(
1/2)) + 1365*a^11*b^4*x + (5*a*b^14*x^6)/2 + (5005*a^9*b^6*x^2)/2 + 2145*a
^7*b^8*x^3 + 910*a^12*b^3*x^(1/2) + (3003*a^5*b^10*x^4)/4 + 91*a^3*b^12*x^
5 + 2002*a^10*b^5*x^(3/2) + 2574*a^8*b^7*x^(5/2) + 1430*a^6*b^9*x^(7/2) +
(910*a^4*b^11*x^(9/2))/3 + (210*a^2*b^13*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^2}\,dx = \frac{180180\sqrt{x}\log(x)\,a^{13}b^2x-1716\sqrt{x}\,a^{15}+2342340\sqrt{x}\,a^{11}b^4x^2+4294290\sqrt{x}\,a^9b^6x^3+3680820\sqrt{x}\,a^7b^8x^4}{}$$

input
```
int((a+b*x^(1/2))^15/x^2,x)
```

output
```
(180180*sqrt(x)*log(x)*a**13*b**2*x - 1716*sqrt(x)*a**15 + 2342340*sqrt(x)
*a**11*b**4*x**2 + 4294290*sqrt(x)*a**9*b**6*x**3 + 3680820*sqrt(x)*a**7*b
**8*x**4 + 1288287*sqrt(x)*a**5*b**10*x**5 + 156156*sqrt(x)*a**3*b**12*x**
6 + 4290*sqrt(x)*a*b**14*x**7 - 51480*a**14*b*x + 1561560*a**12*b**3*x**2
+ 3435432*a**10*b**5*x**3 + 4416984*a**8*b**7*x**4 + 2453880*a**6*b**9*x**
5 + 520520*a**4*b**11*x**6 + 32760*a**2*b**13*x**7 + 264*b**15*x**8)/(1716
*sqrt(x)*x)
```

3.67 $\int \frac{(a+b\sqrt{x})^{15}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 190

$$\int \frac{(a+b\sqrt{x})^{15}}{x^3}\,dx = -\frac{a^{15}}{2x^2} - \frac{10a^{14}b}{x^{3/2}} - \frac{105a^{13}b^2}{x} - \frac{910a^{12}b^3}{\sqrt{x}} + 6006a^{10}b^5\sqrt{x} + 5005a^9b^6x + 4290a^8b^7x^{3/2} + \frac{6435}{2}a^7b^8x^2 + 2002a^6b^9x^{5/2} + 1001a^5b^{10}x^3 + 390a^4b^{11}x^{7/2} + \frac{455}{4}a^3b^{12}x^4 + \frac{70}{3}a^2b^{13}x^{9/2} + 3ab^{14}x^5 + \frac{2}{11}b^{15}x^{11/2} + 1365a^{11}b^4\log(x)$$

output
```
-1/2*a^15/x^2-10*a^14*b/x^(3/2)-105*a^13*b^2/x-910*a^12*b^3/x^(1/2)+6006*a
^10*b^5*x^(1/2)+5005*a^9*b^6*x+4290*a^8*b^7*x^(3/2)+6435/2*a^7*b^8*x^2+200
2*a^6*b^9*x^(5/2)+1001*a^5*b^10*x^3+390*a^4*b^11*x^(7/2)+455/4*a^3*b^12*x^
4+70/3*a^2*b^13*x^(9/2)+3*a*b^14*x^5+2/11*b^15*x^(11/2)+1365*a^11*b^4*ln(x
)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx$$
$$= \frac{-66a^{15} - 1320a^{14}b\sqrt{x} - 13860a^{13}b^2x - 120120a^{12}b^3x^{3/2} + 792792a^{10}b^5x^{5/2} + 660660a^9b^6x^3 + 566280a^{8}\cdots}{\cdots}$$
$$+ 2730a^{11}b^4\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^3,x]
```

output
```
(-66*a^15 - 1320*a^14*b*Sqrt[x] - 13860*a^13*b^2*x - 120120*a^12*b^3*x^(3/
2) + 792792*a^10*b^5*x^(5/2) + 660660*a^9*b^6*x^3 + 566280*a^8*b^7*x^(7/2)
 + 424710*a^7*b^8*x^4 + 264264*a^6*b^9*x^(9/2) + 132132*a^5*b^10*x^5 + 514
80*a^4*b^11*x^(11/2) + 15015*a^3*b^12*x^6 + 3080*a^2*b^13*x^(13/2) + 396*a
*b^14*x^7 + 24*b^15*x^(15/2))/(132*x^2) + 2730*a^11*b^4*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{5/2}}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$
$$2\int \left(\frac{a^{15}}{x^{5/2}} + \frac{15ba^{14}}{x^2} + \frac{105b^2a^{13}}{x^{3/2}} + \frac{455b^3a^{12}}{x} + \frac{1365b^4a^{11}}{\sqrt{x}} + 3003b^5a^{10} + 5005b^6\sqrt{x}a^9 + 6435b^7xa^8 + 6435b^8x^{3/}\cdots$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{15}}{4x^2}-\frac{5a^{14}b}{x^{3/2}}-\frac{105a^{13}b^2}{2x}-\frac{455a^{12}b^3}{\sqrt{x}}+1365a^{11}b^4\log\left(\sqrt{x}\right)+3003a^{10}b^5\sqrt{x}+\frac{5005}{2}a^9b^6x+2145a^8b^7x^{3/2}+\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^3,x]
```

output
```
2*(-1/4*a^15/x^2 - (5*a^14*b)/x^(3/2) - (105*a^13*b^2)/(2*x) - (455*a^12*b
^3)/Sqrt[x] + 3003*a^10*b^5*Sqrt[x] + (5005*a^9*b^6*x)/2 + 2145*a^8*b^7*x^
(3/2) + (6435*a^7*b^8*x^2)/4 + 1001*a^6*b^9*x^(5/2) + (1001*a^5*b^10*x^3)/
2 + 195*a^4*b^11*x^(7/2) + (455*a^3*b^12*x^4)/8 + (35*a^2*b^13*x^(9/2))/3
+ (3*a*b^14*x^5)/2 + (b^15*x^(11/2))/11 + 1365*a^11*b^4*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 24.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{a^{15}}{2x^2}-\frac{10a^{14}b}{x^{\frac{3}{2}}}-\frac{105a^{13}b^2}{x}-\frac{910a^{12}b^3}{\sqrt{x}}+6006a^{10}b^5\sqrt{x}+5005a^9b^6x+4290a^8b^7x^{\frac{3}{2}}+\frac{6435a^7}{2}$
default	$-\frac{a^{15}}{2x^2}-\frac{10a^{14}b}{x^{\frac{3}{2}}}-\frac{105a^{13}b^2}{x}-\frac{910a^{12}b^3}{\sqrt{x}}+6006a^{10}b^5\sqrt{x}+5005a^9b^6x+4290a^8b^7x^{\frac{3}{2}}+\frac{6435a^7}{2}$
trager	$\frac{(-1+x)(12b^{14}x^6+455a^2b^{12}x^5+12b^{14}x^5+4004a^4b^{10}x^4+455b^{12}x^4a^2+12b^{14}x^4+12870a^6b^8x^3+4004b^{10}x^3a^4+455b^{12}a}{4x^2}$

input
```
int((a+b*x^(1/2))^15/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^15/x^2-10*a^14*b/x^(3/2)-105*a^13*b^2/x-910*a^12*b^3/x^(1/2)+6006*a
^10*b^5*x^(1/2)+5005*a^9*b^6*x+4290*a^8*b^7*x^(3/2)+6435/2*a^7*b^8*x^2+200
2*a^6*b^9*x^(5/2)+1001*a^5*b^10*x^3+390*a^4*b^11*x^(7/2)+455/4*a^3*b^12*x^
4+70/3*a^2*b^13*x^(9/2)+3*a*b^14*x^5+2/11*b^15*x^(11/2)+1365*a^11*b^4*ln(x
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx = \frac{396\,ab^{14}x^7+15015\,a^3b^{12}x^6+132132\,a^5b^{10}x^5+424710\,a^7b^8x^4+660660\,a^9b^6x^3+360360\,a^{11}b^4x^2\log\left(\sqrt{x}\right.}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^3,x, algorithm="fricas")
```

output
```
1/132*(396*a*b^14*x^7 + 15015*a^3*b^12*x^6 + 132132*a^5*b^10*x^5 + 424710*
a^7*b^8*x^4 + 660660*a^9*b^6*x^3 + 360360*a^11*b^4*x^2*log(sqrt(x)) - 1386
0*a^13*b^2*x - 66*a^15 + 8*(3*b^15*x^7 + 385*a^2*b^13*x^6 + 6435*a^4*b^11*
x^5 + 33033*a^6*b^9*x^4 + 70785*a^8*b^7*x^3 + 99099*a^10*b^5*x^2 - 15015*a
^12*b^3*x - 165*a^14*b)*sqrt(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx = -\frac{a^{15}}{2x^2} - \frac{10a^{14}b}{x^{\frac{3}{2}}} - \frac{105a^{13}b^2}{x} - \frac{910a^{12}b^3}{\sqrt{x}} + 1365a^{11}b^4\log(x) + 6006a^{10}b^5\sqrt{x} + 5005a^9b^6x + 4290a^8b^7x^{\frac{3}{2}} + \frac{6435a^7b^8x^2}{2} + 2002a^6b^9x^{\frac{5}{2}} + 1001a^5b^{10}x^3 + 390a^4b^{11}x^{\frac{7}{2}} + \frac{455a^3b^{12}x^4}{4} + \frac{70a^2b^{13}x^{\frac{9}{2}}}{3} + 3ab^{14}x^5 + \frac{2b^{15}x^{\frac{11}{2}}}{11}$$

input
```
integrate((a+b*x**(1/2))**15/x**3,x)
```

output
```
-a**15/(2*x**2) - 10*a**14*b/x**(3/2) - 105*a**13*b**2/x - 910*a**12*b**3/
sqrt(x) + 1365*a**11*b**4*log(x) + 6006*a**10*b**5*sqrt(x) + 5005*a**9*b**
6*x + 4290*a**8*b**7*x**(3/2) + 6435*a**7*b**8*x**2/2 + 2002*a**6*b**9*x**
(5/2) + 1001*a**5*b**10*x**3 + 390*a**4*b**11*x**(7/2) + 455*a**3*b**12*x*
*4/4 + 70*a**2*b**13*x**(9/2)/3 + 3*a*b**14*x**5 + 2*b**15*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx = \frac{2}{11}\,b^{15}x^{\frac{11}{2}} + 3\,ab^{14}x^5 + \frac{70}{3}\,a^2b^{13}x^{\frac{9}{2}} + \frac{455}{4}\,a^3b^{12}x^4 + 390\,a^4b^{11}x^{\frac{7}{2}} + 1001\,a^5b^{10}x^3 + 2002\,a^6b^9x^{\frac{5}{2}} + \frac{6435}{2}\,a^7b^8x^2 + 4290\,a^8b^7x^{\frac{3}{2}} + 5005\,a^9b^6x + 1365\,a^{11}b^4\log(x) + 6006\,a^{10}b^5\sqrt{x} - \frac{1820\,a^{12}b^3x^{\frac{3}{2}} + 210\,a^{13}b^2x + 20\,a^{14}b\sqrt{x} + a^{15}}{2\,x^2}$$

input
```
integrate((a+b*x^(1/2))^15/x^3,x, algorithm="maxima")
```

output
```
2/11*b^15*x^(11/2) + 3*a*b^14*x^5 + 70/3*a^2*b^13*x^(9/2) + 455/4*a^3*b^12
*x^4 + 390*a^4*b^11*x^(7/2) + 1001*a^5*b^10*x^3 + 2002*a^6*b^9*x^(5/2) + 6
435/2*a^7*b^8*x^2 + 4290*a^8*b^7*x^(3/2) + 5005*a^9*b^6*x + 1365*a^11*b^4*
log(x) + 6006*a^10*b^5*sqrt(x) - 1/2*(1820*a^12*b^3*x^(3/2) + 210*a^13*b^2
*x + 20*a^14*b*sqrt(x) + a^15)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx = \frac{2}{11}\,b^{15}x^{\frac{11}{2}} + 3\,ab^{14}x^5 + \frac{70}{3}\,a^2b^{13}x^{\frac{9}{2}} + \frac{455}{4}\,a^3b^{12}x^4 + 390\,a^4b^{11}x^{\frac{7}{2}} + 1001\,a^5b^{10}x^3 + 2002\,a^6b^9x^{\frac{5}{2}} + \frac{6435}{2}\,a^7b^8x^2 + 4290\,a^8b^7x^{\frac{3}{2}} + 5005\,a^9b^6x + 1365\,a^{11}b^4\log\left(|x|\right) + 6006\,a^{10}b^5\sqrt{x} - \frac{1820\,a^{12}b^3x^{\frac{3}{2}} + 210\,a^{13}b^2x + 20\,a^{14}b\sqrt{x} + a^{15}}{2\,x^2}$$

input
```
integrate((a+b*x^(1/2))^15/x^3,x, algorithm="giac")
```

output
```
2/11*b^15*x^(11/2) + 3*a*b^14*x^5 + 70/3*a^2*b^13*x^(9/2) + 455/4*a^3*b^12
*x^4 + 390*a^4*b^11*x^(7/2) + 1001*a^5*b^10*x^3 + 2002*a^6*b^9*x^(5/2) + 6
435/2*a^7*b^8*x^2 + 4290*a^8*b^7*x^(3/2) + 5005*a^9*b^6*x + 1365*a^11*b^4*
log(abs(x)) + 6006*a^10*b^5*sqrt(x) - 1/2*(1820*a^12*b^3*x^(3/2) + 210*a^1
3*b^2*x + 20*a^14*b*sqrt(x) + a^15)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx = \frac{2\,b^{15}\,x^{11/2}}{11} - \frac{\frac{a^{15}}{2} + 105\,a^{13}\,b^2\,x + 10\,a^{14}\,b\,\sqrt{x} + 910\,a^{12}\,b^3\,x^{3/2}}{x^2} + 2730\,a^{11}\,b^4\,\ln\left(\sqrt{x}\right) + 5005\,a^9\,b^6\,x + 3\,a\,b^{14}\,x^5 + \frac{6435\,a^7\,b^8\,x^2}{2} + 1001\,a^5\,b^{10}\,x^3 + 6006\,a^{10}\,b^5\,\sqrt{x} + \frac{455\,a^3\,b^{12}\,x^4}{4} + 4290\,a^8\,b^7\,x^{3/2} + 2002\,a^6\,b^9\,x^{5/2} + 390\,a^4\,b^{11}\,x^{7/2} + \frac{70\,a^2\,b^{13}\,x^{9/2}}{3}$$

input
```
int((a + b*x^(1/2))^15/x^3,x)
```

output
```
(2*b^15*x^(11/2))/11 - (a^15/2 + 105*a^13*b^2*x + 10*a^14*b*x^(1/2) + 910*
a^12*b^3*x^(3/2))/x^2 + 2730*a^11*b^4*log(x^(1/2)) + 5005*a^9*b^6*x + 3*a*
b^14*x^5 + (6435*a^7*b^8*x^2)/2 + 1001*a^5*b^10*x^3 + 6006*a^10*b^5*x^(1/2
) + (455*a^3*b^12*x^4)/4 + 4290*a^8*b^7*x^(3/2) + 2002*a^6*b^9*x^(5/2) + 3
90*a^4*b^11*x^(7/2) + (70*a^2*b^13*x^(9/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^3}\,dx$$
$$= \frac{180180\sqrt{x}\log(x)\,a^{11}b^4x^2 - 66\sqrt{x}\,a^{15} - 13860\sqrt{x}\,a^{13}b^2x + 660660\sqrt{x}\,a^9b^6x^3 + 424710\sqrt{x}\,a^7b^8x^4 + 1321}{}$$

input
```
int((a+b*x^(1/2))^15/x^3,x)
```

output
```
(180180*sqrt(x)*log(x)*a**11*b**4*x**2 - 66*sqrt(x)*a**15 - 13860*sqrt(x)*
a**13*b**2*x + 660660*sqrt(x)*a**9*b**6*x**3 + 424710*sqrt(x)*a**7*b**8*x*
*4 + 132132*sqrt(x)*a**5*b**10*x**5 + 15015*sqrt(x)*a**3*b**12*x**6 + 396*
sqrt(x)*a*b**14*x**7 - 1320*a**14*b*x - 120120*a**12*b**3*x**2 + 792792*a*
*10*b**5*x**3 + 566280*a**8*b**7*x**4 + 264264*a**6*b**9*x**5 + 51480*a**4
*b**11*x**6 + 3080*a**2*b**13*x**7 + 24*b**15*x**8)/(132*sqrt(x)*x**2)
```

3.68 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 196

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx = -\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{5/2}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{3/2}}-\frac{1365a^{11}b^4}{x} -\frac{6006a^{10}b^5}{\sqrt{x}}+12870a^8b^7\sqrt{x}+6435a^7b^8x+\frac{10010}{3}a^6b^9x^{3/2} +\frac{3003}{2}a^5b^{10}x^2+546a^4b^{11}x^{5/2}+\frac{455}{3}a^3b^{12}x^3 +30a^2b^{13}x^{7/2}+\frac{15}{4}ab^{14}x^4+\frac{2}{9}b^{15}x^{9/2}+5005a^9b^6\log(x)$$

output
```
-1/3*a^15/x^3-6*a^14*b/x^(5/2)-105/2*a^13*b^2/x^2-910/3*a^12*b^3/x^(3/2)-1
365*a^11*b^4/x-6006*a^10*b^5/x^(1/2)+12870*a^8*b^7*x^(1/2)+6435*a^7*b^8*x+
10010/3*a^6*b^9*x^(3/2)+3003/2*a^5*b^10*x^2+546*a^4*b^11*x^(5/2)+455/3*a^3
*b^12*x^3+30*a^2*b^13*x^(7/2)+15/4*a*b^14*x^4+2/9*b^15*x^(9/2)+5005*a^9*b^
6*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sqrt{x})^{15}}{x^4} dx$$
$$= \frac{-12a^{15} - 216a^{14}b\sqrt{x} - 1890a^{13}b^2x - 10920a^{12}b^3x^{3/2} - 49140a^{11}b^4x^2 - 216216a^{10}b^5x^{5/2} + 463320a^8b^7x$$
$$+ 10010a^9b^6\log(\sqrt{x})$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^4,x]
```

output
```
(-12*a^15 - 216*a^14*b*Sqrt[x] - 1890*a^13*b^2*x - 10920*a^12*b^3*x^(3/2)
- 49140*a^11*b^4*x^2 - 216216*a^10*b^5*x^(5/2) + 463320*a^8*b^7*x^(7/2) +
231660*a^7*b^8*x^4 + 120120*a^6*b^9*x^(9/2) + 54054*a^5*b^10*x^5 + 19656*a
^4*b^11*x^(11/2) + 5460*a^3*b^12*x^6 + 1080*a^2*b^13*x^(13/2) + 135*a*b^14
*x^7 + 8*b^15*x^(15/2))/(36*x^3) + 10010*a^9*b^6*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^{15}}{x^4} dx$$
$$\Big\downarrow \; 798$$
$$2\int \frac{(a+b\sqrt{x})^{15}}{x^{7/2}} d\sqrt{x}$$
$$\Big\downarrow \; 49$$
$$2\int \left(\frac{a^{15}}{x^{7/2}} + \frac{15ba^{14}}{x^3} + \frac{105b^2a^{13}}{x^{5/2}} + \frac{455b^3a^{12}}{x^2} + \frac{1365b^4a^{11}}{x^{3/2}} + \frac{3003b^5a^{10}}{x} + \frac{5005b^6a^9}{\sqrt{x}} + 6435b^7a^8 + 6435b^8\sqrt{x}a^7 \right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{15}}{6x^3}-\frac{3a^{14}b}{x^{5/2}}-\frac{105a^{13}b^2}{4x^2}-\frac{455a^{12}b^3}{3x^{3/2}}-\frac{1365a^{11}b^4}{2x}-\frac{3003a^{10}b^5}{\sqrt{x}}+5005a^9b^6\log\left(\sqrt{x}\right)+6435a^8b^7\sqrt{x}+\frac{6435}{2}\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^4,x]
```

output
```
2*(-1/6*a^15/x^3 - (3*a^14*b)/x^(5/2) - (105*a^13*b^2)/(4*x^2) - (455*a^12
*b^3)/(3*x^(3/2)) - (1365*a^11*b^4)/(2*x) - (3003*a^10*b^5)/Sqrt[x] + 6435
*a^8*b^7*Sqrt[x] + (6435*a^7*b^8*x)/2 + (5005*a^6*b^9*x^(3/2))/3 + (3003*a
^5*b^10*x^2)/4 + 273*a^4*b^11*x^(5/2) + (455*a^3*b^12*x^3)/6 + 15*a^2*b^13
*x^(7/2) + (15*a*b^14*x^4)/8 + (b^15*x^(9/2))/9 + 5005*a^9*b^6*Log[Sqrt[x]
])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.97 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{\frac{5}{2}}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{\frac{3}{2}}}-\frac{1365a^{11}b^4}{x}-\frac{6006a^{10}b^5}{\sqrt{x}}+12870a^8b^7\sqrt{x}+6435a^7b^8x$ -
default	$-\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{\frac{5}{2}}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{\frac{3}{2}}}-\frac{1365a^{11}b^4}{x}-\frac{6006a^{10}b^5}{\sqrt{x}}+12870a^8b^7\sqrt{x}+6435a^7b^8x$ -
trager	$\frac{(-1+x)(45b^{14}x^6+1820a^2b^{12}x^5+45b^{14}x^5+18018a^4b^{10}x^4+1820b^{12}x^4a^2+45b^{14}x^4+77220a^6b^8x^3+18018b^{10}x^3a^4+182}{12x^3}$

input
```
int((a+b*x^(1/2))^15/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^15/x^3-6*a^14*b/x^(5/2)-105/2*a^13*b^2/x^2-910/3*a^12*b^3/x^(3/2)-1
365*a^11*b^4/x-6006*a^10*b^5/x^(1/2)+12870*a^8*b^7*x^(1/2)+6435*a^7*b^8*x+
10010/3*a^6*b^9*x^(3/2)+3003/2*a^5*b^10*x^2+546*a^4*b^11*x^(5/2)+455/3*a^3
*b^12*x^3+30*a^2*b^13*x^(7/2)+15/4*a*b^14*x^4+2/9*b^15*x^(9/2)+5005*a^9*b^
6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx$$
$$=\frac{135\,ab^{14}x^7+5460\,a^3b^{12}x^6+54054\,a^5b^{10}x^5+231660\,a^7b^8x^4+360360\,a^9b^6x^3\log\left(\sqrt{x}\right)-49140\,a^{11}b^4x^2-}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^4,x, algorithm="fricas")
```

output
```
1/36*(135*a*b^14*x^7 + 5460*a^3*b^12*x^6 + 54054*a^5*b^10*x^5 + 231660*a^7
*b^8*x^4 + 360360*a^9*b^6*x^3*log(sqrt(x)) - 49140*a^11*b^4*x^2 - 1890*a^1
3*b^2*x - 12*a^15 + 8*(b^15*x^7 + 135*a^2*b^13*x^6 + 2457*a^4*b^11*x^5 + 1
5015*a^6*b^9*x^4 + 57915*a^8*b^7*x^3 - 27027*a^10*b^5*x^2 - 1365*a^12*b^3*
x - 27*a^14*b)*sqrt(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx = -\frac{a^{15}}{3x^3}-\frac{6a^{14}b}{x^{\frac{5}{2}}}-\frac{105a^{13}b^2}{2x^2}-\frac{910a^{12}b^3}{3x^{\frac{3}{2}}}-\frac{1365a^{11}b^4}{x}-\frac{6006a^{10}b^5}{\sqrt{x}}+5005a^9b^6\log(x)+12870a^8b^7\sqrt{x}+6435a^7b^8x+\frac{10010a^6b^9x^{\frac{3}{2}}}{3}+\frac{3003a^5b^{10}x^2}{2}+546a^4b^{11}x^{\frac{5}{2}}+\frac{455a^3b^{12}x^3}{3}+30a^2b^{13}x^{\frac{7}{2}}+\frac{15ab^{14}x^4}{4}+\frac{2b^{15}x^{\frac{9}{2}}}{9}$$

input
```
integrate((a+b*x**(1/2))**15/x**4,x)
```

output
```
-a**15/(3*x**3) - 6*a**14*b/x**(5/2) - 105*a**13*b**2/(2*x**2) - 910*a**12
*b**3/(3*x**(3/2)) - 1365*a**11*b**4/x - 6006*a**10*b**5/sqrt(x) + 5005*a*
*9*b**6*log(x) + 12870*a**8*b**7*sqrt(x) + 6435*a**7*b**8*x + 10010*a**6*b
**9*x**(3/2)/3 + 3003*a**5*b**10*x**2/2 + 546*a**4*b**11*x**(5/2) + 455*a*
*3*b**12*x**3/3 + 30*a**2*b**13*x**(7/2) + 15*a*b**14*x**4/4 + 2*b**15*x**
(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx = \frac{2}{9}\,b^{15}x^{\frac{9}{2}}+\frac{15}{4}\,ab^{14}x^4+30\,a^2b^{13}x^{\frac{7}{2}}+\frac{455}{3}\,a^3b^{12}x^3+546\,a^4b^{11}x^{\frac{5}{2}}+\frac{3003}{2}\,a^5b^{10}x^2+\frac{10010}{3}\,a^6b^9x^{\frac{3}{2}}+6435\,a^7b^8x+5005\,a^9b^6\log(x)+12870\,a^8b^7\sqrt{x}-\frac{36036\,a^{10}b^5x^{\frac{5}{2}}+8190\,a^{11}b^4x^2+1820\,a^{12}b^3x^{\frac{3}{2}}+315\,a^{13}b^2x+36\,a^{14}b\sqrt{x}+2\,a^{15}}{6\,x^3}$$

input
```
integrate((a+b*x^(1/2))^15/x^4,x, algorithm="maxima")
```

output
```
2/9*b^15*x^(9/2) + 15/4*a*b^14*x^4 + 30*a^2*b^13*x^(7/2) + 455/3*a^3*b^12*
x^3 + 546*a^4*b^11*x^(5/2) + 3003/2*a^5*b^10*x^2 + 10010/3*a^6*b^9*x^(3/2)
 + 6435*a^7*b^8*x + 5005*a^9*b^6*log(x) + 12870*a^8*b^7*sqrt(x) - 1/6*(360
36*a^10*b^5*x^(5/2) + 8190*a^11*b^4*x^2 + 1820*a^12*b^3*x^(3/2) + 315*a^13
*b^2*x + 36*a^14*b*sqrt(x) + 2*a^15)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx = \frac{2}{9}\,b^{15}x^{\frac{9}{2}} + \frac{15}{4}\,ab^{14}x^4 + 30\,a^2b^{13}x^{\frac{7}{2}} + \frac{455}{3}\,a^3b^{12}x^3 + 546\,a^4b^{11}x^{\frac{5}{2}} + \frac{3003}{2}\,a^5b^{10}x^2 + \frac{10010}{3}\,a^6b^9x^{\frac{3}{2}} + 6435\,a^7b^8x + 5005\,a^9b^6\log\left(|x|\right) + 12870\,a^8b^7\sqrt{x} - \frac{36036\,a^{10}b^5x^{\frac{5}{2}} + 8190\,a^{11}b^4x^2 + 1820\,a^{12}b^3x^{\frac{3}{2}} + 315\,a^{13}b^2x + 36\,a^{14}b\sqrt{x} + 2\,a^{15}}{6\,x^3}$$

input
```
integrate((a+b*x^(1/2))^15/x^4,x, algorithm="giac")
```

output
```
2/9*b^15*x^(9/2) + 15/4*a*b^14*x^4 + 30*a^2*b^13*x^(7/2) + 455/3*a^3*b^12*
x^3 + 546*a^4*b^11*x^(5/2) + 3003/2*a^5*b^10*x^2 + 10010/3*a^6*b^9*x^(3/2)
 + 6435*a^7*b^8*x + 5005*a^9*b^6*log(abs(x)) + 12870*a^8*b^7*sqrt(x) - 1/6
*(36036*a^10*b^5*x^(5/2) + 8190*a^11*b^4*x^2 + 1820*a^12*b^3*x^(3/2) + 315
*a^13*b^2*x + 36*a^14*b*sqrt(x) + 2*a^15)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx = \frac{2\,b^{15}\,x^{9/2}}{9} - \frac{\frac{a^{15}}{3} + \frac{105\,a^{13}\,b^2\,x}{2} + 6\,a^{14}\,b\,\sqrt{x} + 1365\,a^{11}\,b^4\,x^2 + \frac{910\,a^{12}\,b^3\,x^{3/2}}{3} + 6006\,a^{10}\,b^5\,x^{5/2}}{x^3} + 10010\,a^9\,b^6\,\ln\left(\sqrt{x}\right) + 6435\,a^7\,b^8\,x + \frac{15\,a\,b^{14}\,x^4}{4} + \frac{3003\,a^5\,b^{10}\,x^2}{2} + \frac{455\,a^3\,b^{12}\,x^3}{3} + 12870\,a^8\,b^7\,\sqrt{x} + \frac{10010\,a}{}$$

input
```
int((a + b*x^(1/2))^15/x^4,x)
```

output
```
(2*b^15*x^(9/2))/9 - (a^15/3 + (105*a^13*b^2*x)/2 + 6*a^14*b*x^(1/2) + 136
5*a^11*b^4*x^2 + (910*a^12*b^3*x^(3/2))/3 + 6006*a^10*b^5*x^(5/2))/x^3 + 1
0010*a^9*b^6*log(x^(1/2)) + 6435*a^7*b^8*x + (15*a*b^14*x^4)/4 + (3003*a^5
*b^10*x^2)/2 + (455*a^3*b^12*x^3)/3 + 12870*a^8*b^7*x^(1/2) + (10010*a^6*b
^9*x^(3/2))/3 + 546*a^4*b^11*x^(5/2) + 30*a^2*b^13*x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^4}\,dx = \frac{180180\sqrt{x}\log(x)\,a^9b^6x^3 - 12\sqrt{x}\,a^{15} - 1890\sqrt{x}\,a^{13}b^2x - 49140\sqrt{x}\,a^{11}b^4x^2 + 231660\sqrt{x}\,a^7b^8x^4 + 54054\sqrt{}}{}$$

input
```
int((a+b*x^(1/2))^15/x^4,x)
```

output
```
(180180*sqrt(x)*log(x)*a**9*b**6*x**3 - 12*sqrt(x)*a**15 - 1890*sqrt(x)*a*
*13*b**2*x - 49140*sqrt(x)*a**11*b**4*x**2 + 231660*sqrt(x)*a**7*b**8*x**4
 + 54054*sqrt(x)*a**5*b**10*x**5 + 5460*sqrt(x)*a**3*b**12*x**6 + 135*sqrt
(x)*a*b**14*x**7 - 216*a**14*b*x - 10920*a**12*b**3*x**2 - 216216*a**10*b*
*5*x**3 + 463320*a**8*b**7*x**4 + 120120*a**6*b**9*x**5 + 19656*a**4*b**11
*x**6 + 1080*a**2*b**13*x**7 + 8*b**15*x**8)/(36*sqrt(x)*x**3)
```

3.69 $\int \frac{(a+b\sqrt{x})^{15}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 194

$$\int \frac{(a+b\sqrt{x})^{15}}{x^6}\,dx = -\frac{a^{15}}{5x^5} - \frac{10a^{14}b}{3x^{9/2}} - \frac{105a^{13}b^2}{4x^4} - \frac{130a^{12}b^3}{x^{7/2}} - \frac{455a^{11}b^4}{x^3} - \frac{6006a^{10}b^5}{5x^{5/2}} - \frac{5005a^9b^6}{2x^2} - \frac{4290a^8b^7}{x^{3/2}} - \frac{6435a^7b^8}{x} - \frac{10010a^6b^9}{\sqrt{x}} + 2730a^4b^{11}\sqrt{x} + 455a^3b^{12}x + 70a^2b^{13}x^{3/2} + \frac{15}{2}ab^{14}x^2 + \frac{2}{5}b^{15}x^{5/2} + 3003a^5b^{10}\log(x)$$

output
```
-1/5*a^15/x^5-10/3*a^14*b/x^(9/2)-105/4*a^13*b^2/x^4-130*a^12*b^3/x^(7/2)-
455*a^11*b^4/x^3-6006/5*a^10*b^5/x^(5/2)-5005/2*a^9*b^6/x^2-4290*a^8*b^7/x
^(3/2)-6435*a^7*b^8/x-10010*a^6*b^9/x^(1/2)+2730*a^4*b^11*x^(1/2)+455*a^3*
b^12*x+70*a^2*b^13*x^(3/2)+15/2*a*b^14*x^2+2/5*b^15*x^(5/2)+3003*a^5*b^10*
ln(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx$$
$$= \frac{-12a^{15}-200a^{14}b\sqrt{x}-1575a^{13}b^2x-7800a^{12}b^3x^{3/2}-27300a^{11}b^4x^2-72072a^{10}b^5x^{5/2}-150150a^9b^6x^3 -}{}$$
$$+6006a^5b^{10}\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^6,x]
```

output
```
(-12*a^15 - 200*a^14*b*Sqrt[x] - 1575*a^13*b^2*x - 7800*a^12*b^3*x^(3/2) -
 27300*a^11*b^4*x^2 - 72072*a^10*b^5*x^(5/2) - 150150*a^9*b^6*x^3 - 257400
*a^8*b^7*x^(7/2) - 386100*a^7*b^8*x^4 - 600600*a^6*b^9*x^(9/2) + 163800*a^
4*b^11*x^(11/2) + 27300*a^3*b^12*x^6 + 4200*a^2*b^13*x^(13/2) + 450*a*b^14
*x^7 + 24*b^15*x^(15/2))/(60*x^5) + 6006*a^5*b^10*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11/2}}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$
$$2\int \left(\frac{a^{15}}{x^{11/2}}+\frac{15ba^{14}}{x^5}+\frac{105b^2a^{13}}{x^{9/2}}+\frac{455b^3a^{12}}{x^4}+\frac{1365b^4a^{11}}{x^{7/2}}+\frac{3003b^5a^{10}}{x^3}+\frac{5005b^6a^9}{x^{5/2}}+\frac{6435b^7a^8}{x^2}+\frac{6435b^8a^7}{x^{3/2}}+\right.$$

$$\downarrow \text{2009}$$

$$2\left(-\frac{a^{15}}{10x^5}-\frac{5a^{14}b}{3x^{9/2}}-\frac{105a^{13}b^2}{8x^4}-\frac{65a^{12}b^3}{x^{7/2}}-\frac{455a^{11}b^4}{2x^3}-\frac{3003a^{10}b^5}{5x^{5/2}}-\frac{5005a^9b^6}{4x^2}-\frac{2145a^8b^7}{x^{3/2}}-\frac{6435a^7b^8}{2x}-\frac{5005}{\sqrt{}}\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^6,x]
```

output
```
2*(-1/10*a^15/x^5 - (5*a^14*b)/(3*x^(9/2)) - (105*a^13*b^2)/(8*x^4) - (65*
a^12*b^3)/x^(7/2) - (455*a^11*b^4)/(2*x^3) - (3003*a^10*b^5)/(5*x^(5/2)) -
 (5005*a^9*b^6)/(4*x^2) - (2145*a^8*b^7)/x^(3/2) - (6435*a^7*b^8)/(2*x) -
(5005*a^6*b^9)/Sqrt[x] + 1365*a^4*b^11*Sqrt[x] + (455*a^3*b^12*x)/2 + 35*a
^2*b^13*x^(3/2) + (15*a*b^14*x^2)/4 + (b^15*x^(5/2))/5 + 3003*a^5*b^10*Log
[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{a^{15}}{5x^5}-\frac{10a^{14}b}{3x^{\frac{9}{2}}}-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{\frac{7}{2}}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{\frac{5}{2}}}-\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{\frac{3}{2}}}-\frac{6435a^7b^8}{x}$
default	$-\frac{a^{15}}{5x^5}-\frac{10a^{14}b}{3x^{\frac{9}{2}}}-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{\frac{7}{2}}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{\frac{5}{2}}}-\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{\frac{3}{2}}}-\frac{6435a^7b^8}{x}$
trager	$\frac{(-1+x)(150b^{14}x^6+9100a^2b^{12}x^5+150b^{14}x^5+4a^{14}x^4+525a^{12}b^2x^4+9100b^4x^4a^{10}+50050x^4b^6a^8+128700a^6b^8x^4+4a^{1}}{20x^5}$

input
```
int((a+b*x^(1/2))^15/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^15/x^5-10/3*a^14*b/x^(9/2)-105/4*a^13*b^2/x^4-130*a^12*b^3/x^(7/2)-
455*a^11*b^4/x^3-6006/5*a^10*b^5/x^(5/2)-5005/2*a^9*b^6/x^2-4290*a^8*b^7/x
^(3/2)-6435*a^7*b^8/x-10010*a^6*b^9/x^(1/2)+2730*a^4*b^11*x^(1/2)+455*a^3*
b^12*x+70*a^2*b^13*x^(3/2)+15/2*a*b^14*x^2+2/5*b^15*x^(5/2)+3003*a^5*b^10*
ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx = \frac{450\,ab^{14}x^7+27300\,a^3b^{12}x^6+360360\,a^5b^{10}x^5\log\left(\sqrt{x}\right)-386100\,a^7b^8x^4-150150\,a^9b^6x^3-27300\,a^{11}b^4x^{2}}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^6,x, algorithm="fricas")
```

output
```
1/60*(450*a*b^14*x^7 + 27300*a^3*b^12*x^6 + 360360*a^5*b^10*x^5*log(sqrt(x
)) - 386100*a^7*b^8*x^4 - 150150*a^9*b^6*x^3 - 27300*a^11*b^4*x^2 - 1575*a
^13*b^2*x - 12*a^15 + 8*(3*b^15*x^7 + 525*a^2*b^13*x^6 + 20475*a^4*b^11*x^
5 - 75075*a^6*b^9*x^4 - 32175*a^8*b^7*x^3 - 9009*a^10*b^5*x^2 - 975*a^12*b
^3*x - 25*a^14*b)*sqrt(x))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx = -\frac{a^{15}}{5x^5}-\frac{10a^{14}b}{3x^{\frac{9}{2}}}-\frac{105a^{13}b^2}{4x^4}-\frac{130a^{12}b^3}{x^{\frac{7}{2}}}-\frac{455a^{11}b^4}{x^3}-\frac{6006a^{10}b^5}{5x^{\frac{5}{2}}} -\frac{5005a^9b^6}{2x^2}-\frac{4290a^8b^7}{x^{\frac{3}{2}}}-\frac{6435a^7b^8}{x}-\frac{10010a^6b^9}{\sqrt{x}}+3003a^5b^{10}\log(x) + 2730a^4b^{11}\sqrt{x}+455a^3b^{12}x+70a^2b^{13}x^{\frac{3}{2}}+\frac{15ab^{14}x^2}{2}+\frac{2b^{15}x^{\frac{5}{2}}}{5}$$

input
```
integrate((a+b*x**(1/2))**15/x**6,x)
```

output
```
-a**15/(5*x**5) - 10*a**14*b/(3*x**(9/2)) - 105*a**13*b**2/(4*x**4) - 130*
a**12*b**3/x**(7/2) - 455*a**11*b**4/x**3 - 6006*a**10*b**5/(5*x**(5/2)) -
 5005*a**9*b**6/(2*x**2) - 4290*a**8*b**7/x**(3/2) - 6435*a**7*b**8/x - 10
010*a**6*b**9/sqrt(x) + 3003*a**5*b**10*log(x) + 2730*a**4*b**11*sqrt(x) +
 455*a**3*b**12*x + 70*a**2*b**13*x**(3/2) + 15*a*b**14*x**2/2 + 2*b**15*x
**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx = \frac{2}{5}\,b^{15}x^{\frac{5}{2}}+\frac{15}{2}\,ab^{14}x^2+70\,a^2b^{13}x^{\frac{3}{2}}+455\,a^3b^{12}x+3003\,a^5b^{10}\log(x)+2730\,a^4b^{11}\sqrt{x} - \frac{600600\,a^6b^9x^{\frac{9}{2}}+386100\,a^7b^8x^4+257400\,a^8b^7x^{\frac{7}{2}}+150150\,a^9b^6x^3+72072\,a^{10}b^5x^{\frac{5}{2}}+27300\,a^{11}b^4x^2+}{60\,x^5}$$

input
```
integrate((a+b*x^(1/2))^15/x^6,x, algorithm="maxima")
```

output
```
2/5*b^15*x^(5/2) + 15/2*a*b^14*x^2 + 70*a^2*b^13*x^(3/2) + 455*a^3*b^12*x
+ 3003*a^5*b^10*log(x) + 2730*a^4*b^11*sqrt(x) - 1/60*(600600*a^6*b^9*x^(9
/2) + 386100*a^7*b^8*x^4 + 257400*a^8*b^7*x^(7/2) + 150150*a^9*b^6*x^3 + 7
2072*a^10*b^5*x^(5/2) + 27300*a^11*b^4*x^2 + 7800*a^12*b^3*x^(3/2) + 1575*
a^13*b^2*x + 200*a^14*b*sqrt(x) + 12*a^15)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx$$
$$= \frac{2}{5}\,b^{15}x^{\frac{5}{2}} + \frac{15}{2}\,ab^{14}x^2 + 70\,a^2b^{13}x^{\frac{3}{2}} + 455\,a^3b^{12}x + 3003\,a^5b^{10}\log\left(|x|\right) + 2730\,a^4b^{11}\sqrt{x}$$
$$- \frac{600600\,a^6b^9x^{\frac{9}{2}} + 386100\,a^7b^8x^4 + 257400\,a^8b^7x^{\frac{7}{2}} + 150150\,a^9b^6x^3 + 72072\,a^{10}b^5x^{\frac{5}{2}} + 27300\,a^{11}b^4x^2 +}{60\,x^5}$$

input
```
integrate((a+b*x^(1/2))^15/x^6,x, algorithm="giac")
```

output
```
2/5*b^15*x^(5/2) + 15/2*a*b^14*x^2 + 70*a^2*b^13*x^(3/2) + 455*a^3*b^12*x
+ 3003*a^5*b^10*log(abs(x)) + 2730*a^4*b^11*sqrt(x) - 1/60*(600600*a^6*b^9
*x^(9/2) + 386100*a^7*b^8*x^4 + 257400*a^8*b^7*x^(7/2) + 150150*a^9*b^6*x^
3 + 72072*a^10*b^5*x^(5/2) + 27300*a^11*b^4*x^2 + 7800*a^12*b^3*x^(3/2) +
1575*a^13*b^2*x + 200*a^14*b*sqrt(x) + 12*a^15)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx = \frac{2\,b^{15}\,x^{5/2}}{5}$$
$$- \frac{\frac{a^{15}}{5} + \frac{105\,a^{13}\,b^2\,x}{4} + \frac{10\,a^{14}\,b\,\sqrt{x}}{3} + 455\,a^{11}\,b^4\,x^2 + \frac{5005\,a^9\,b^6\,x^3}{2} + 6435\,a^7\,b^8\,x^4 + 130\,a^{12}\,b^3\,x^{3/2} + \frac{6006\,a^{10}\,b^5\,x^{5/2}}{5}}{x^5}$$
$$+ 6006\,a^5\,b^{10}\,\ln\left(\sqrt{x}\right) + 455\,a^3\,b^{12}\,x + \frac{15\,a\,b^{14}\,x^2}{2} + 2730\,a^4\,b^{11}\,\sqrt{x} + 70\,a^2\,b^{13}\,x^{3/2}$$

input
```
int((a + b*x^(1/2))^15/x^6,x)
```

output
```
(2*b^15*x^(5/2))/5 - (a^15/5 + (105*a^13*b^2*x)/4 + (10*a^14*b*x^(1/2))/3
+ 455*a^11*b^4*x^2 + (5005*a^9*b^6*x^3)/2 + 6435*a^7*b^8*x^4 + 130*a^12*b^
3*x^(3/2) + (6006*a^10*b^5*x^(5/2))/5 + 4290*a^8*b^7*x^(7/2) + 10010*a^6*b
^9*x^(9/2))/x^5 + 6006*a^5*b^10*log(x^(1/2)) + 455*a^3*b^12*x + (15*a*b^14
*x^2)/2 + 2730*a^4*b^11*x^(1/2) + 70*a^2*b^13*x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^6}\,dx$$
$$= \frac{180180\sqrt{x}\log(x)\,a^5b^{10}x^5 - 12\sqrt{x}\,a^{15} - 1575\sqrt{x}\,a^{13}b^2x - 27300\sqrt{x}\,a^{11}b^4x^2 - 150150\sqrt{x}\,a^9b^6x^3 - 38610}{}$$

input
```
int((a+b*x^(1/2))^15/x^6,x)
```

output
```
(180180*sqrt(x)*log(x)*a**5*b**10*x**5 - 12*sqrt(x)*a**15 - 1575*sqrt(x)*a
**13*b**2*x - 27300*sqrt(x)*a**11*b**4*x**2 - 150150*sqrt(x)*a**9*b**6*x**
3 - 386100*sqrt(x)*a**7*b**8*x**4 + 27300*sqrt(x)*a**3*b**12*x**6 + 450*sq
rt(x)*a*b**14*x**7 - 200*a**14*b*x - 7800*a**12*b**3*x**2 - 72072*a**10*b*
*5*x**3 - 257400*a**8*b**7*x**4 - 600600*a**6*b**9*x**5 + 163800*a**4*b**1
1*x**6 + 4200*a**2*b**13*x**7 + 24*b**15*x**8)/(60*sqrt(x)*x**5)
```

3.70 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 196

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx = -\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{11/2}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{9/2}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{7/2}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{5/2}}-\frac{6435a^7b^8}{2x^2}-\frac{10010a^6b^9}{3x^{3/2}}-\frac{3003a^5b^{10}}{x}-\frac{2730a^4b^{11}}{\sqrt{x}}+210a^2b^{13}\sqrt{x}+15ab^{14}x+\frac{2}{3}b^{15}x^{3/2}+455a^3b^{12}\log(x)$$

output

```
-1/6*a^15/x^6-30/11*a^14*b/x^(11/2)-21*a^13*b^2/x^5-910/9*a^12*b^3/x^(9/2)
-1365/4*a^11*b^4/x^4-858*a^10*b^5/x^(7/2)-5005/3*a^9*b^6/x^3-2574*a^8*b^7/
x^(5/2)-6435/2*a^7*b^8/x^2-10010/3*a^6*b^9/x^(3/2)-3003*a^5*b^10/x-2730*a^
4*b^11/x^(1/2)+210*a^2*b^13*x^(1/2)+15*a*b^14*x+2/3*b^15*x^(3/2)+455*a^3*b
^12*ln(x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx$$
$$= \frac{-66a^{15} - 1080a^{14}b\sqrt{x} - 8316a^{13}b^2x - 40040a^{12}b^3x^{3/2} - 135135a^{11}b^4x^2 - 339768a^{10}b^5x^{5/2} - 660660a^9b}{}$$
$$+ 910a^3b^{12}\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^7,x]
```

output
```
(-66*a^15 - 1080*a^14*b*Sqrt[x] - 8316*a^13*b^2*x - 40040*a^12*b^3*x^(3/2)
 - 135135*a^11*b^4*x^2 - 339768*a^10*b^5*x^(5/2) - 660660*a^9*b^6*x^3 - 10
19304*a^8*b^7*x^(7/2) - 1274130*a^7*b^8*x^4 - 1321320*a^6*b^9*x^(9/2) - 11
89188*a^5*b^10*x^5 - 1081080*a^4*b^11*x^(11/2) + 83160*a^2*b^13*x^(13/2) +
 5940*a*b^14*x^7 + 264*b^15*x^(15/2))/(396*x^6) + 910*a^3*b^12*Log[Sqrt[x]
]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(\frac{a^{15}}{x^{13/2}}+\frac{15ba^{14}}{x^6}+\frac{105b^2a^{13}}{x^{11/2}}+\frac{455b^3a^{12}}{x^5}+\frac{1365b^4a^{11}}{x^{9/2}}+\frac{3003b^5a^{10}}{x^4}+\frac{5005b^6a^9}{x^{7/2}}+\frac{6435b^7a^8}{x^3}+\frac{6435b^8a^7}{x^{5/2}}+\right.$$

$$\Big\downarrow \; 2009$$

$$2\left(-\frac{a^{15}}{12x^6}-\frac{15a^{14}b}{11x^{11/2}}-\frac{21a^{13}b^2}{2x^5}-\frac{455a^{12}b^3}{9x^{9/2}}-\frac{1365a^{11}b^4}{8x^4}-\frac{429a^{10}b^5}{x^{7/2}}-\frac{5005a^9b^6}{6x^3}-\frac{1287a^8b^7}{x^{5/2}}-\frac{6435a^7b^8}{4x^2}-\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^7,x]
```

output
```
2*(-1/12*a^15/x^6 - (15*a^14*b)/(11*x^(11/2)) - (21*a^13*b^2)/(2*x^5) - (4
55*a^12*b^3)/(9*x^(9/2)) - (1365*a^11*b^4)/(8*x^4) - (429*a^10*b^5)/x^(7/2
) - (5005*a^9*b^6)/(6*x^3) - (1287*a^8*b^7)/x^(5/2) - (6435*a^7*b^8)/(4*x^
2) - (5005*a^6*b^9)/(3*x^(3/2)) - (3003*a^5*b^10)/(2*x) - (1365*a^4*b^11)/
Sqrt[x] + 105*a^2*b^13*Sqrt[x] + (15*a*b^14*x)/2 + (b^15*x^(3/2))/3 + 455*
a^3*b^12*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{\frac{11}{2}}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{\frac{9}{2}}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{\frac{7}{2}}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{\frac{5}{2}}}-\frac{6435a^7b^8}{2x^2}$
default	$-\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{\frac{11}{2}}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{\frac{9}{2}}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{\frac{7}{2}}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{\frac{5}{2}}}-\frac{6435a^7b^8}{2x^2}$
trager	$(-1+x)(180b^{14}x^6+2a^{14}x^5+252a^{12}b^2x^5+4095a^{10}b^4x^5+20020a^8b^6x^5+38610a^6b^8x^5+36036a^4b^{10}x^5+2a^{14}x^4+252a^1$

input
```
int((a+b*x^(1/2))^15/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^15/x^6-30/11*a^14*b/x^(11/2)-21*a^13*b^2/x^5-910/9*a^12*b^3/x^(9/2)
-1365/4*a^11*b^4/x^4-858*a^10*b^5/x^(7/2)-5005/3*a^9*b^6/x^3-2574*a^8*b^7/
x^(5/2)-6435/2*a^7*b^8/x^2-10010/3*a^6*b^9/x^(3/2)-3003*a^5*b^10/x-2730*a^
4*b^11/x^(1/2)+210*a^2*b^13*x^(1/2)+15*a*b^14*x+2/3*b^15*x^(3/2)+455*a^3*b
^12*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx = \frac{5940\,ab^{14}x^7+360360\,a^3b^{12}x^6\log\left(\sqrt{x}\right)-1189188\,a^5b^{10}x^5-1274130\,a^7b^8x^4-660660\,a^9b^6x^3-135135\,a}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^7,x, algorithm="fricas")
```

output
```
1/396*(5940*a*b^14*x^7 + 360360*a^3*b^12*x^6*log(sqrt(x)) - 1189188*a^5*b^
10*x^5 - 1274130*a^7*b^8*x^4 - 660660*a^9*b^6*x^3 - 135135*a^11*b^4*x^2 -
8316*a^13*b^2*x - 66*a^15 + 8*(33*b^15*x^7 + 10395*a^2*b^13*x^6 - 135135*a
^4*b^11*x^5 - 165165*a^6*b^9*x^4 - 127413*a^8*b^7*x^3 - 42471*a^10*b^5*x^2
 - 5005*a^12*b^3*x - 135*a^14*b)*sqrt(x))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx = -\frac{a^{15}}{6x^6}-\frac{30a^{14}b}{11x^{\frac{11}{2}}}-\frac{21a^{13}b^2}{x^5}-\frac{910a^{12}b^3}{9x^{\frac{9}{2}}}-\frac{1365a^{11}b^4}{4x^4}-\frac{858a^{10}b^5}{x^{\frac{7}{2}}}-\frac{5005a^9b^6}{3x^3}-\frac{2574a^8b^7}{x^{\frac{5}{2}}}-\frac{6435a^7b^8}{2x^2}-\frac{10010a^6b^9}{3x^{\frac{3}{2}}}-\frac{3003a^5b^{10}}{x}-\frac{2730a^4b^{11}}{\sqrt{x}}+455a^3b^{12}\log(x)+210a^2b^{13}\sqrt{x}+15ab^{14}x+\frac{2b^{15}x^{\frac{3}{2}}}{3}$$

input
```
integrate((a+b*x**(1/2))**15/x**7,x)
```

output
```
-a**15/(6*x**6) - 30*a**14*b/(11*x**(11/2)) - 21*a**13*b**2/x**5 - 910*a**
12*b**3/(9*x**(9/2)) - 1365*a**11*b**4/(4*x**4) - 858*a**10*b**5/x**(7/2)
- 5005*a**9*b**6/(3*x**3) - 2574*a**8*b**7/x**(5/2) - 6435*a**7*b**8/(2*x*
*2) - 10010*a**6*b**9/(3*x**(3/2)) - 3003*a**5*b**10/x - 2730*a**4*b**11/s
qrt(x) + 455*a**3*b**12*log(x) + 210*a**2*b**13*sqrt(x) + 15*a*b**14*x + 2
*b**15*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx = \frac{2}{3}\,b^{15}x^{\frac{3}{2}}+15\,ab^{14}x+455\,a^3b^{12}\log(x)+210\,a^2b^{13}\sqrt{x}-\frac{1081080\,a^4b^{11}x^{\frac{11}{2}}+1189188\,a^5b^{10}x^5+1321320\,a^6b^9x^{\frac{9}{2}}+1274130\,a^7b^8x^4+1019304\,a^8b^7x^{\frac{7}{2}}+660660}{396\,x}$$

input
```
integrate((a+b*x^(1/2))^15/x^7,x, algorithm="maxima")
```

output
```
2/3*b^15*x^(3/2) + 15*a*b^14*x + 455*a^3*b^12*log(x) + 210*a^2*b^13*sqrt(x
) - 1/396*(1081080*a^4*b^11*x^(11/2) + 1189188*a^5*b^10*x^5 + 1321320*a^6*
b^9*x^(9/2) + 1274130*a^7*b^8*x^4 + 1019304*a^8*b^7*x^(7/2) + 660660*a^9*b
^6*x^3 + 339768*a^10*b^5*x^(5/2) + 135135*a^11*b^4*x^2 + 40040*a^12*b^3*x^
(3/2) + 8316*a^13*b^2*x + 1080*a^14*b*sqrt(x) + 66*a^15)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx = \frac{2}{3}\,b^{15}x^{\frac{3}{2}} + 15\,ab^{14}x + 455\,a^3b^{12}\log\left(|x|\right) + 210\,a^2b^{13}\sqrt{x} - \frac{1081080\,a^4b^{11}x^{\frac{11}{2}} + 1189188\,a^5b^{10}x^5 + 1321320\,a^6b^9x^{\frac{9}{2}} + 1274130\,a^7b^8x^4 + 1019304\,a^8b^7x^{\frac{7}{2}} + 660660 \ldots}{396\,x \ldots}$$

input
```
integrate((a+b*x^(1/2))^15/x^7,x, algorithm="giac")
```

output
```
2/3*b^15*x^(3/2) + 15*a*b^14*x + 455*a^3*b^12*log(abs(x)) + 210*a^2*b^13*s
qrt(x) - 1/396*(1081080*a^4*b^11*x^(11/2) + 1189188*a^5*b^10*x^5 + 1321320
*a^6*b^9*x^(9/2) + 1274130*a^7*b^8*x^4 + 1019304*a^8*b^7*x^(7/2) + 660660*
a^9*b^6*x^3 + 339768*a^10*b^5*x^(5/2) + 135135*a^11*b^4*x^2 + 40040*a^12*b
^3*x^(3/2) + 8316*a^13*b^2*x + 1080*a^14*b*sqrt(x) + 66*a^15)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx = \frac{2\,b^{15}\,x^{3/2}}{3} - \frac{\frac{a^{15}}{6} + 21\,a^{13}\,b^2\,x + \frac{30\,a^{14}\,b\,\sqrt{x}}{11} + \frac{1365\,a^{11}\,b^4\,x^2}{4} + \frac{5005\,a^9\,b^6\,x^3}{3} + \frac{6435\,a^7\,b^8\,x^4}{2} + 3003\,a^5\,b^{10}\,x^5 + \frac{910\,a^{12}\,b^3\,x^{3/2}}{9} + 85 \ldots}{x^6} + 910\,a^3\,b^{12}\,\ln\left(\sqrt{x}\right) + 210\,a^2\,b^{13}\,\sqrt{x} + 15\,a\,b^{14}\,x$$

input
```
int((a + b*x^(1/2))^15/x^7,x)
```

output
```
(2*b^15*x^(3/2))/3 - (a^15/6 + 21*a^13*b^2*x + (30*a^14*b*x^(1/2))/11 + (1
365*a^11*b^4*x^2)/4 + (5005*a^9*b^6*x^3)/3 + (6435*a^7*b^8*x^4)/2 + 3003*a
^5*b^10*x^5 + (910*a^12*b^3*x^(3/2))/9 + 858*a^10*b^5*x^(5/2) + 2574*a^8*b
^7*x^(7/2) + (10010*a^6*b^9*x^(9/2))/3 + 2730*a^4*b^11*x^(11/2))/x^6 + 910
*a^3*b^12*log(x^(1/2)) + 210*a^2*b^13*x^(1/2) + 15*a*b^14*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^7}\,dx$$
$$= \frac{180180\sqrt{x}\log(x)\,a^3b^{12}x^6 - 66\sqrt{x}\,a^{15} - 8316\sqrt{x}\,a^{13}b^2x - 135135\sqrt{x}\,a^{11}b^4x^2 - 660660\sqrt{x}\,a^9b^6x^3 - 1274}{}$$

input
```
int((a+b*x^(1/2))^15/x^7,x)
```

output
```
(180180*sqrt(x)*log(x)*a**3*b**12*x**6 - 66*sqrt(x)*a**15 - 8316*sqrt(x)*a
**13*b**2*x - 135135*sqrt(x)*a**11*b**4*x**2 - 660660*sqrt(x)*a**9*b**6*x*
*3 - 1274130*sqrt(x)*a**7*b**8*x**4 - 1189188*sqrt(x)*a**5*b**10*x**5 + 59
40*sqrt(x)*a*b**14*x**7 - 1080*a**14*b*x - 40040*a**12*b**3*x**2 - 339768*
a**10*b**5*x**3 - 1019304*a**8*b**7*x**4 - 1321320*a**6*b**9*x**5 - 108108
0*a**4*b**11*x**6 + 83160*a**2*b**13*x**7 + 264*b**15*x**8)/(396*sqrt(x)*x
**6)
```

3.71 $\int \frac{(a+b\sqrt{x})^{15}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 198

$$\int \frac{(a+b\sqrt{x})^{15}}{x^8} dx = -\frac{a^{15}}{7x^7} - \frac{30a^{14}b}{13x^{13/2}} - \frac{35a^{13}b^2}{2x^6} - \frac{910a^{12}b^3}{11x^{11/2}} - \frac{273a^{11}b^4}{x^5} - \frac{2002a^{10}b^5}{3x^{9/2}} - \frac{5005a^9b^6}{4x^4} - \frac{12870a^8b^7}{7x^{7/2}} - \frac{2145a^7b^8}{x^3} - \frac{2002a^6b^9}{x^{5/2}} - \frac{3003a^5b^{10}}{2x^2} - \frac{910a^4b^{11}}{x^{3/2}} - \frac{455a^3b^{12}}{x} - \frac{210a^2b^{13}}{\sqrt{x}} + 2b^{15}\sqrt{x} + 15ab^{14}\log(x)$$

output
```
-1/7*a^15/x^7-30/13*a^14*b/x^(13/2)-35/2*a^13*b^2/x^6-910/11*a^12*b^3/x^(1
1/2)-273*a^11*b^4/x^5-2002/3*a^10*b^5/x^(9/2)-5005/4*a^9*b^6/x^4-12870/7*a
^8*b^7/x^(7/2)-2145*a^7*b^8/x^3-2002*a^6*b^9/x^(5/2)-3003/2*a^5*b^10/x^2-9
10*a^4*b^11/x^(3/2)-455*a^3*b^12/x-210*a^2*b^13/x^(1/2)+2*b^15*x^(1/2)+15*
a*b^14*ln(x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\sqrt{x})^{15}}{x^8}\,dx$$
$$= \frac{-1716a^{15} - 27720a^{14}b\sqrt{x} - 210210a^{13}b^2x - 993720a^{12}b^3x^{3/2} - 3279276a^{11}b^4x^2 - 8016008a^{10}b^5x^{5/2} - 1$$
$$+ 30ab^{14}\log\left(\sqrt{x}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^8,x]
```

output
```
(-1716*a^15 - 27720*a^14*b*Sqrt[x] - 210210*a^13*b^2*x - 993720*a^12*b^3*x
^(3/2) - 3279276*a^11*b^4*x^2 - 8016008*a^10*b^5*x^(5/2) - 15030015*a^9*b^
6*x^3 - 22084920*a^8*b^7*x^(7/2) - 25765740*a^7*b^8*x^4 - 24048024*a^6*b^9
*x^(9/2) - 18036018*a^5*b^10*x^5 - 10930920*a^4*b^11*x^(11/2) - 5465460*a^
3*b^12*x^6 - 2522520*a^2*b^13*x^(13/2) + 24024*b^15*x^(15/2))/(12012*x^7)
+ 30*a*b^14*Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^{15}}{x^8}\,dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{(a+b\sqrt{x})^{15}}{x^{15/2}}\,d\sqrt{x}$$
$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{a^{15}}{x^{15/2}}+\frac{15ba^{14}}{x^7}+\frac{105b^2a^{13}}{x^{13/2}}+\frac{455b^3a^{12}}{x^6}+\frac{1365b^4a^{11}}{x^{11/2}}+\frac{3003b^5a^{10}}{x^5}+\frac{5005b^6a^9}{x^{9/2}}+\frac{6435b^7a^8}{x^4}+\frac{6435b^8a^7}{x^{7/2}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{15}}{14x^7}-\frac{15a^{14}b}{13x^{13/2}}-\frac{35a^{13}b^2}{4x^6}-\frac{455a^{12}b^3}{11x^{11/2}}-\frac{273a^{11}b^4}{2x^5}-\frac{1001a^{10}b^5}{3x^{9/2}}-\frac{5005a^9b^6}{8x^4}-\frac{6435a^8b^7}{7x^{7/2}}-\frac{2145a^7b^8}{2x^3}-\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^8,x]
```

output
```
2*(-1/14*a^15/x^7 - (15*a^14*b)/(13*x^(13/2)) - (35*a^13*b^2)/(4*x^6) - (4
55*a^12*b^3)/(11*x^(11/2)) - (273*a^11*b^4)/(2*x^5) - (1001*a^10*b^5)/(3*x
^(9/2)) - (5005*a^9*b^6)/(8*x^4) - (6435*a^8*b^7)/(7*x^(7/2)) - (2145*a^7*
b^8)/(2*x^3) - (1001*a^6*b^9)/x^(5/2) - (3003*a^5*b^10)/(4*x^2) - (455*a^4
*b^11)/x^(3/2) - (455*a^3*b^12)/(2*x) - (105*a^2*b^13)/Sqrt[x] + b^15*Sqrt
[x] + 15*a*b^14*Log[Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{a^{15}}{7x^7}-\frac{30a^{14}b}{13x^{\frac{13}{2}}}-\frac{35a^{13}b^2}{2x^6}-\frac{910a^{12}b^3}{11x^{\frac{11}{2}}}-\frac{273a^{11}b^4}{x^5}-\frac{2002a^{10}b^5}{3x^{\frac{9}{2}}}-\frac{5005a^9b^6}{4x^4}-\frac{12870a^8b^7}{7x^{\frac{7}{2}}}-\frac{2145a^7b^8}{x^3}$
default	$-\frac{a^{15}}{7x^7}-\frac{30a^{14}b}{13x^{\frac{13}{2}}}-\frac{35a^{13}b^2}{2x^6}-\frac{910a^{12}b^3}{11x^{\frac{11}{2}}}-\frac{273a^{11}b^4}{x^5}-\frac{2002a^{10}b^5}{3x^{\frac{9}{2}}}-\frac{5005a^9b^6}{4x^4}-\frac{12870a^8b^7}{7x^{\frac{7}{2}}}-\frac{2145a^7b^8}{x^3}$
trager	$(-1+x)(4a^{12}x^6+490a^{10}b^2x^6+7644a^8b^4x^6+35035a^6x^6b^6+60060b^8x^6a^4+42042a^2b^{10}x^6+12740x^6b^{12}+4a^{12}x^5+490a$

input
```
int((a+b*x^(1/2))^15/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^15/x^7-30/13*a^14*b/x^(13/2)-35/2*a^13*b^2/x^6-910/11*a^12*b^3/x^(1
1/2)-273*a^11*b^4/x^5-2002/3*a^10*b^5/x^(9/2)-5005/4*a^9*b^6/x^4-12870/7*a
^8*b^7/x^(7/2)-2145*a^7*b^8/x^3-2002*a^6*b^9/x^(5/2)-3003/2*a^5*b^10/x^2-9
10*a^4*b^11/x^(3/2)-455*a^3*b^12/x-210*a^2*b^13/x^(1/2)+2*b^15*x^(1/2)+15*
a*b^14*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx = \frac{360360\,ab^{14}x^7\log\left(\sqrt{x}\right)-5465460\,a^3b^{12}x^6-18036018\,a^5b^{10}x^5-25765740\,a^7b^8x^4-15030015\,a^9b^6x^3-}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^8,x, algorithm="fricas")
```

output
```
1/12012*(360360*a*b^14*x^7*log(sqrt(x)) - 5465460*a^3*b^12*x^6 - 18036018*
a^5*b^10*x^5 - 25765740*a^7*b^8*x^4 - 15030015*a^9*b^6*x^3 - 3279276*a^11*
b^4*x^2 - 210210*a^13*b^2*x - 1716*a^15 + 8*(3003*b^15*x^7 - 315315*a^2*b^
13*x^6 - 1366365*a^4*b^11*x^5 - 3006003*a^6*b^9*x^4 - 2760615*a^8*b^7*x^3
- 1002001*a^10*b^5*x^2 - 124215*a^12*b^3*x - 3465*a^14*b)*sqrt(x))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx = -\frac{a^{15}}{7x^7} - \frac{30a^{14}b}{13x^{\frac{13}{2}}} - \frac{35a^{13}b^2}{2x^6} - \frac{910a^{12}b^3}{11x^{\frac{11}{2}}} - \frac{273a^{11}b^4}{x^5} - \frac{2002a^{10}b^5}{3x^{\frac{9}{2}}} - \frac{5005a^9b^6}{4x^4} - \frac{12870a^8b^7}{7x^{\frac{7}{2}}} - \frac{2145a^7b^8}{x^3} - \frac{2002a^6b^9}{x^{\frac{5}{2}}} - \frac{3003a^5b^{10}}{2x^2} - \frac{910a^4b^{11}}{x^{\frac{3}{2}}} - \frac{455a^3b^{12}}{x} - \frac{210a^2b^{13}}{\sqrt{x}} + 15ab^{14}\log(x) + 2b^{15}\sqrt{x}$$

input
```
integrate((a+b*x**(1/2))**15/x**8,x)
```

output
```
-a**15/(7*x**7) - 30*a**14*b/(13*x**(13/2)) - 35*a**13*b**2/(2*x**6) - 910
*a**12*b**3/(11*x**(11/2)) - 273*a**11*b**4/x**5 - 2002*a**10*b**5/(3*x**(
9/2)) - 5005*a**9*b**6/(4*x**4) - 12870*a**8*b**7/(7*x**(7/2)) - 2145*a**7
*b**8/x**3 - 2002*a**6*b**9/x**(5/2) - 3003*a**5*b**10/(2*x**2) - 910*a**4
*b**11/x**(3/2) - 455*a**3*b**12/x - 210*a**2*b**13/sqrt(x) + 15*a*b**14*l
og(x) + 2*b**15*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx = 15\,ab^{14}\log(x) + 2\,b^{15}\sqrt{x} - \frac{2522520\,a^2b^{13}x^{\frac{13}{2}} + 5465460\,a^3b^{12}x^6 + 10930920\,a^4b^{11}x^{\frac{11}{2}} + 18036018\,a^5b^{10}x^5 + 24048024\,a^6b^9x^{\frac{9}{2}} + 25}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^8,x, algorithm="maxima")
```

output
```
15*a*b^14*log(x) + 2*b^15*sqrt(x) - 1/12012*(2522520*a^2*b^13*x^(13/2) + 5
465460*a^3*b^12*x^6 + 10930920*a^4*b^11*x^(11/2) + 18036018*a^5*b^10*x^5 +
 24048024*a^6*b^9*x^(9/2) + 25765740*a^7*b^8*x^4 + 22084920*a^8*b^7*x^(7/2
) + 15030015*a^9*b^6*x^3 + 8016008*a^10*b^5*x^(5/2) + 3279276*a^11*b^4*x^2
 + 993720*a^12*b^3*x^(3/2) + 210210*a^13*b^2*x + 27720*a^14*b*sqrt(x) + 17
16*a^15)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx = 15\,ab^{14}\log\left(|x|\right) + 2\,b^{15}\sqrt{x}$$
$$-\frac{2522520\,a^2b^{13}x^{\frac{13}{2}} + 5465460\,a^3b^{12}x^6 + 10930920\,a^4b^{11}x^{\frac{11}{2}} + 18036018\,a^5b^{10}x^5 + 24048024\,a^6b^9x^{\frac{9}{2}} + 25}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^8,x, algorithm="giac")
```

output
```
15*a*b^14*log(abs(x)) + 2*b^15*sqrt(x) - 1/12012*(2522520*a^2*b^13*x^(13/2
) + 5465460*a^3*b^12*x^6 + 10930920*a^4*b^11*x^(11/2) + 18036018*a^5*b^10*
x^5 + 24048024*a^6*b^9*x^(9/2) + 25765740*a^7*b^8*x^4 + 22084920*a^8*b^7*x
^(7/2) + 15030015*a^9*b^6*x^3 + 8016008*a^10*b^5*x^(5/2) + 3279276*a^11*b^
4*x^2 + 993720*a^12*b^3*x^(3/2) + 210210*a^13*b^2*x + 27720*a^14*b*sqrt(x)
 + 1716*a^15)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^8}\,dx = 2\,b^{15}\,\sqrt{x} - \frac{a^{15}}{7\,x^7} - \frac{30\,a^{14}\,b}{13\,x^{13/2}} - \frac{455\,a^3\,b^{12}}{x} - \frac{3003\,a^5\,b^{10}}{2\,x^2}$$
$$- \frac{210\,a^2\,b^{13}}{\sqrt{x}} - \frac{2145\,a^7\,b^8}{x^3} - \frac{5005\,a^9\,b^6}{4\,x^4} - \frac{910\,a^4\,b^{11}}{x^{3/2}}$$
$$- \frac{273\,a^{11}\,b^4}{x^5} - \frac{35\,a^{13}\,b^2}{2\,x^6} - \frac{2002\,a^6\,b^9}{x^{5/2}} - \frac{12870\,a^8\,b^7}{7\,x^{7/2}}$$
$$- \frac{2002\,a^{10}\,b^5}{3\,x^{9/2}} - \frac{910\,a^{12}\,b^3}{11\,x^{11/2}} + 30\,a\,b^{14}\,\ln\left(\sqrt{x}\right)$$

input
```
int((a + b*x^(1/2))^15/x^8,x)
```

output
```
2*b^15*x^(1/2) - a^15/(7*x^7) - (30*a^14*b)/(13*x^(13/2)) - (455*a^3*b^12)
/x - (3003*a^5*b^10)/(2*x^2) - (210*a^2*b^13)/x^(1/2) - (2145*a^7*b^8)/x^3
 - (5005*a^9*b^6)/(4*x^4) - (910*a^4*b^11)/x^(3/2) - (273*a^11*b^4)/x^5 -
(35*a^13*b^2)/(2*x^6) - (2002*a^6*b^9)/x^(5/2) - (12870*a^8*b^7)/(7*x^(7/2
)) - (2002*a^10*b^5)/(3*x^(9/2)) - (910*a^12*b^3)/(11*x^(11/2)) + 30*a*b^1
4*log(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\sqrt{x})^{15}}{x^8}\,dx = \frac{180180\sqrt{x}\log(x)\,a\,b^{14}x^7 - 1716\sqrt{x}\,a^{15} - 210210\sqrt{x}\,a^{13}b^2x - 3279276\sqrt{x}\,a^{11}b^4x^2 - 15030015\sqrt{x}\,a^9b^6x^3}{}$$

input
```
int((a+b*x^(1/2))^15/x^8,x)
```

output
```
(180180*sqrt(x)*log(x)*a*b**14*x**7 - 1716*sqrt(x)*a**15 - 210210*sqrt(x)*
a**13*b**2*x - 3279276*sqrt(x)*a**11*b**4*x**2 - 15030015*sqrt(x)*a**9*b**
6*x**3 - 25765740*sqrt(x)*a**7*b**8*x**4 - 18036018*sqrt(x)*a**5*b**10*x**
5 - 5465460*sqrt(x)*a**3*b**12*x**6 - 27720*a**14*b*x - 993720*a**12*b**3*
x**2 - 8016008*a**10*b**5*x**3 - 22084920*a**8*b**7*x**4 - 24048024*a**6*b
**9*x**5 - 10930920*a**4*b**11*x**6 - 2522520*a**2*b**13*x**7 + 24024*b**1
5*x**8)/(12012*sqrt(x)*x**7)
```

3.72 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx=-\frac{\left(a+b\sqrt{x}\right)^{16}}{8ax^8}$$

output
```
-1/8*(a+b*x^(1/2))^16/a/x^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 185 vs. 2(21) = 42.

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx$$
$$=\frac{-a^{15}-16a^{14}b\sqrt{x}-120a^{13}b^2x-560a^{12}b^3x^{3/2}-1820a^{11}b^4x^2-4368a^{10}b^5x^{5/2}-8008a^9b^6x^3-11440a^8b}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^9,x]
```

output
```
(-a^15 - 16*a^14*b*Sqrt[x] - 120*a^13*b^2*x - 560*a^12*b^3*x^(3/2) - 1820*
a^11*b^4*x^2 - 4368*a^10*b^5*x^(5/2) - 8008*a^9*b^6*x^3 - 11440*a^8*b^7*x^
(7/2) - 12870*a^7*b^8*x^4 - 11440*a^6*b^9*x^(9/2) - 8008*a^5*b^10*x^5 - 43
68*a^4*b^11*x^(11/2) - 1820*a^3*b^12*x^6 - 560*a^2*b^13*x^(13/2) - 120*a*b
^14*x^7 - 16*b^15*x^(15/2))/(8*x^8)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^{15}}{x^9}\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+b\sqrt{x})^{16}}{8ax^8}$$

input
```
Int[(a + b*Sqrt[x])^15/x^9,x]
```

output
```
-1/8*(a + b*Sqrt[x])^16/(a*x^8)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(17) = 34$.

Time = 23.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 8.00

method	result
derivativedivides	$-\frac{546a^{10}b^5}{x^{\frac{11}{2}}} - \frac{a^{15}}{8x^8} - \frac{2a^{14}b}{x^{\frac{15}{2}}} - \frac{6435a^7b^8}{4x^4} - \frac{546a^4b^{11}}{x^{\frac{5}{2}}} - \frac{455a^3b^{12}}{2x^2} - \frac{15a^{13}b^2}{x^7} - \frac{2b^{15}}{\sqrt{x}} - \frac{15a\,b^{14}}{x} - \frac{1430a^6}{x^{\frac{7}{2}}}$
default	$-\frac{546a^{10}b^5}{x^{\frac{11}{2}}} - \frac{a^{15}}{8x^8} - \frac{2a^{14}b}{x^{\frac{15}{2}}} - \frac{6435a^7b^8}{4x^4} - \frac{546a^4b^{11}}{x^{\frac{5}{2}}} - \frac{455a^3b^{12}}{2x^2} - \frac{15a^{13}b^2}{x^7} - \frac{2b^{15}}{\sqrt{x}} - \frac{15a\,b^{14}}{x} - \frac{1430a^6}{x^{\frac{7}{2}}}$
orering	$-\frac{(200b^{28}x^{14}+2940a^2b^{26}x^{13}+16380a^4b^{24}x^{12}+18590a^6b^{22}x^{11}+26026a^8b^{20}x^{10}-22750a^{10}b^{18}x^9+36550a^{12}b^{16}x^8-40}{40x^8}$
trager	$\underline{(-1+x)(a^{14}x^7+120a^{12}b^2x^7+1820a^{10}b^4x^7+8008a^8b^6x^7+12870a^6b^8x^7+8008b^{10}x^7a^4+1820a^2b^{12}x^7+120x^7b^{14}+a^{14}}$

input
```
int((a+b*x^(1/2))^15/x^9,x,method=_RETURNVERBOSE)
```

output
```
-546*a^10*b^5/x^(11/2)-1/8*a^15/x^8-2*a^14*b/x^(15/2)-6435/4*a^7*b^8/x^4-5
46*a^4*b^11/x^(5/2)-455/2*a^3*b^12/x^2-15*a^13*b^2/x^7-2*b^15/x^(1/2)-15*a
*b^14/x-1430*a^6*b^9/x^(7/2)-1001*a^9*b^6/x^5-1430*a^8*b^7/x^(9/2)-70*a^2*
b^13/x^(3/2)-455/2*a^11*b^4/x^6-1001*a^5*b^10/x^3-70*a^12*b^3/x^(13/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 7.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx = -\frac{120\,ab^{14}x^7 + 1820\,a^3b^{12}x^6 + 8008\,a^5b^{10}x^5 + 12870\,a^7b^8x^4 + 8008\,a^9b^6x^3 + 1820\,a^{11}b^4x^2 + 120\,a^{13}b^2x +}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^9,x, algorithm="fricas")
```

output
```
-1/8*(120*a*b^14*x^7 + 1820*a^3*b^12*x^6 + 8008*a^5*b^10*x^5 + 12870*a^7*b
^8*x^4 + 8008*a^9*b^6*x^3 + 1820*a^11*b^4*x^2 + 120*a^13*b^2*x + a^15 + 16
*(b^15*x^7 + 35*a^2*b^13*x^6 + 273*a^4*b^11*x^5 + 715*a^6*b^9*x^4 + 715*a^
8*b^7*x^3 + 273*a^10*b^5*x^2 + 35*a^12*b^3*x + a^14*b)*sqrt(x))/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(17) = 34$.

Time = 0.88 (sec) , antiderivative size = 197, normalized size of antiderivative = 9.38

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx = -\frac{a^{15}}{8x^8} - \frac{2a^{14}b}{x^{\frac{15}{2}}} - \frac{15a^{13}b^2}{x^7} - \frac{70a^{12}b^3}{x^{\frac{13}{2}}} - \frac{455a^{11}b^4}{2x^6} - \frac{546a^{10}b^5}{x^{\frac{11}{2}}} - \frac{1001a^9b^6}{x^5} - \frac{1430a^8b^7}{x^{\frac{9}{2}}} - \frac{6435a^7b^8}{4x^4} - \frac{1430a^6b^9}{x^{\frac{7}{2}}} - \frac{1001a^5b^{10}}{x^3} - \frac{546a^4b^{11}}{x^{\frac{5}{2}}} - \frac{455a^3b^{12}}{2x^2} - \frac{70a^2b^{13}}{x^{\frac{3}{2}}} - \frac{15ab^{14}}{x} - \frac{2b^{15}}{\sqrt{x}}$$

input
```
integrate((a+b*x**(1/2))**15/x**9,x)
```

output
```
-a**15/(8*x**8) - 2*a**14*b/x**(15/2) - 15*a**13*b**2/x**7 - 70*a**12*b**3
/x**(13/2) - 455*a**11*b**4/(2*x**6) - 546*a**10*b**5/x**(11/2) - 1001*a**
9*b**6/x**5 - 1430*a**8*b**7/x**(9/2) - 6435*a**7*b**8/(4*x**4) - 1430*a**
6*b**9/x**(7/2) - 1001*a**5*b**10/x**3 - 546*a**4*b**11/x**(5/2) - 455*a**
3*b**12/(2*x**2) - 70*a**2*b**13/x**(3/2) - 15*a*b**14/x - 2*b**15/sqrt(x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 7.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx = -\frac{16\,b^{15}x^{\frac{15}{2}} + 120\,ab^{14}x^7 + 560\,a^2b^{13}x^{\frac{13}{2}} + 1820\,a^3b^{12}x^6 + 4368\,a^4b^{11}x^{\frac{11}{2}} + 8008\,a^5b^{10}x^5 + 11440\,a^6b^9x^{\frac{9}{2}}}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^9,x, algorithm="maxima")
```

output
```
-1/8*(16*b^15*x^(15/2) + 120*a*b^14*x^7 + 560*a^2*b^13*x^(13/2) + 1820*a^3
*b^12*x^6 + 4368*a^4*b^11*x^(11/2) + 8008*a^5*b^10*x^5 + 11440*a^6*b^9*x^(
9/2) + 12870*a^7*b^8*x^4 + 11440*a^8*b^7*x^(7/2) + 8008*a^9*b^6*x^3 + 4368
*a^10*b^5*x^(5/2) + 1820*a^11*b^4*x^2 + 560*a^12*b^3*x^(3/2) + 120*a^13*b^
2*x + 16*a^14*b*sqrt(x) + a^15)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 7.86

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx = -\frac{16\,b^{15}x^{\frac{15}{2}} + 120\,ab^{14}x^7 + 560\,a^2b^{13}x^{\frac{13}{2}} + 1820\,a^3b^{12}x^6 + 4368\,a^4b^{11}x^{\frac{11}{2}} + 8008\,a^5b^{10}x^5 + 11440\,a^6b^9x^{\frac{9}{2}}}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^9,x, algorithm="giac")
```

output
```
-1/8*(16*b^15*x^(15/2) + 120*a*b^14*x^7 + 560*a^2*b^13*x^(13/2) + 1820*a^3
*b^12*x^6 + 4368*a^4*b^11*x^(11/2) + 8008*a^5*b^10*x^5 + 11440*a^6*b^9*x^(
9/2) + 12870*a^7*b^8*x^4 + 11440*a^8*b^7*x^(7/2) + 8008*a^9*b^6*x^3 + 4368
*a^10*b^5*x^(5/2) + 1820*a^11*b^4*x^2 + 560*a^12*b^3*x^(3/2) + 120*a^13*b^
2*x + 16*a^14*b*sqrt(x) + a^15)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 7.95

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx = -\frac{\frac{a^{15}}{8} + 2\,b^{15}\,x^{15/2} + 15\,a^{13}\,b^2\,x + 2\,a^{14}\,b\,\sqrt{x} + 15\,a\,b^{14}\,x^7 + \frac{455\,a^{11}\,b^4\,x^2}{2} + 1001\,a^9\,b^6\,x^3 + \frac{6435\,a^7\,b^8\,x^4}{4} + 100}{}$$

input
```
int((a + b*x^(1/2))^15/x^9,x)
```

output
```
-(a^15/8 + 2*b^15*x^(15/2) + 15*a^13*b^2*x + 2*a^14*b*x^(1/2) + 15*a*b^14*
x^7 + (455*a^11*b^4*x^2)/2 + 1001*a^9*b^6*x^3 + (6435*a^7*b^8*x^4)/4 + 100
1*a^5*b^10*x^5 + 70*a^12*b^3*x^(3/2) + (455*a^3*b^12*x^6)/2 + 546*a^10*b^5
*x^(5/2) + 1430*a^8*b^7*x^(7/2) + 1430*a^6*b^9*x^(9/2) + 546*a^4*b^11*x^(1
1/2) + 70*a^2*b^13*x^(13/2))/x^8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}\,dx = \frac{-\sqrt{x}\,a^{15}-120\sqrt{x}\,a^{13}b^2x-1820\sqrt{x}\,a^{11}b^4x^2-8008\sqrt{x}\,a^9b^6x^3-12870\sqrt{x}\,a^7b^8x^4-8008\sqrt{x}\,a^5b^{10}x^5-1}{}$$

input
```
int((a+b*x^(1/2))^15/x^9,x)
```

output
```
( - sqrt(x)*a**15 - 120*sqrt(x)*a**13*b**2*x - 1820*sqrt(x)*a**11*b**4*x**
2 - 8008*sqrt(x)*a**9*b**6*x**3 - 12870*sqrt(x)*a**7*b**8*x**4 - 8008*sqrt
(x)*a**5*b**10*x**5 - 1820*sqrt(x)*a**3*b**12*x**6 - 120*sqrt(x)*a*b**14*x
**7 - 16*a**14*b*x - 560*a**12*b**3*x**2 - 4368*a**10*b**5*x**3 - 11440*a*
*8*b**7*x**4 - 11440*a**6*b**9*x**5 - 4368*a**4*b**11*x**6 - 560*a**2*b**1
3*x**7 - 16*b**15*x**8)/(8*sqrt(x)*x**8)
```

3.73 $\int \frac{(a+b\sqrt{x})^{15}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{10}}\,dx = -\frac{(a+b\sqrt{x})^{16}}{9ax^9}+\frac{2b(a+b\sqrt{x})^{16}}{153a^2x^{17/2}}-\frac{b^2(a+b\sqrt{x})^{16}}{1224a^3x^8}$$

output
```
-1/9*(a+b*x^(1/2))^16/a/x^9+2/153*b*(a+b*x^(1/2))^16/a^2/x^(17/2)-1/1224*b
^2*(a+b*x^(1/2))^16/a^3/x^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 185 vs. 2(70) = 140.

Time = 0.06 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.64

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{10}}\,dx$$
$$= \frac{-136a^{15}-2160a^{14}b\sqrt{x}-16065a^{13}b^2x-74256a^{12}b^3x^{3/2}-238680a^{11}b^4x^2-565488a^{10}b^5x^{5/2}-102102\ldots}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^10,x]
```

output
```
(-136*a^15 - 2160*a^14*b*Sqrt[x] - 16065*a^13*b^2*x - 74256*a^12*b^3*x^(3/
2) - 238680*a^11*b^4*x^2 - 565488*a^10*b^5*x^(5/2) - 1021020*a^9*b^6*x^3 -
 1432080*a^8*b^7*x^(7/2) - 1575288*a^7*b^8*x^4 - 1361360*a^6*b^9*x^(9/2) -
 918918*a^5*b^10*x^5 - 477360*a^4*b^11*x^(11/2) - 185640*a^3*b^12*x^6 - 51
408*a^2*b^13*x^(13/2) - 9180*a*b^14*x^7 - 816*b^15*x^(15/2))/(1224*x^9)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{19/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 55}$$

$$2\left(-\frac{b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}d\sqrt{x}}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)$$

$$\Big\downarrow \text{ 55}$$

$$2\left(-\frac{b\left(-\frac{b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17/2}}d\sqrt{x}}{17a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)$$

$$\Big\downarrow \text{ 48}$$

$$2\left(-\frac{b\left(\frac{b\left(a+b\sqrt{x}\right)^{16}}{272a^2x^8}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)$$

input
```
Int[(a + b*Sqrt[x])^15/x^10,x]
```

output
```
2*(-1/9*(b*(-1/17*(a + b*Sqrt[x])^16/(a*x^(17/2)) + (b*(a + b*Sqrt[x])^16)
/(272*a^2*x^8)))/a - (a + b*Sqrt[x])^16/(18*a*x^9))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(56) = 112.

Time = 23.73 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

method	result
derivativedivides	$-\frac{2b^{15}}{3x^{\frac{3}{2}}}-\frac{a^{15}}{9x^9}-\frac{1170a^8b^7}{x^{\frac{11}{2}}}-\frac{455a^3b^{12}}{3x^3}-\frac{5005a^9b^6}{6x^6}-\frac{10010a^6b^9}{9x^{\frac{9}{2}}}-\frac{462a^{10}b^5}{x^{\frac{13}{2}}}-\frac{15a\,b^{14}}{2x^2}-\frac{182a^{12}b^3}{3x^{\frac{15}{2}}}-$
default	$-\frac{2b^{15}}{3x^{\frac{3}{2}}}-\frac{a^{15}}{9x^9}-\frac{1170a^8b^7}{x^{\frac{11}{2}}}-\frac{455a^3b^{12}}{3x^3}-\frac{5005a^9b^6}{6x^6}-\frac{10010a^6b^9}{9x^{\frac{9}{2}}}-\frac{462a^{10}b^5}{x^{\frac{13}{2}}}-\frac{15a\,b^{14}}{2x^2}-\frac{182a^{12}b^3}{3x^{\frac{15}{2}}}-$
orering	$-\frac{(5508b^{28}x^{14}-15708a^2b^{26}x^{13}+155142a^4b^{24}x^{12}-473382a^6b^{22}x^{11}+1209754a^8b^{20}x^{10}-2257770a^{10}b^{18}x^9+3216213a}{}$
trager	$\frac{(-1+x)(8a^{14}x^8+945a^{12}b^2x^8+14040a^{10}b^4x^8+60060a^8b^6x^8+92664a^6b^8x^8+54054a^4b^{10}x^8+10920a^2b^{12}x^8+540b^{14}x^8}{}$

input
```
int((a+b*x^(1/2))^15/x^10,x,method=_RETURNVERBOSE)
```

output
```
-2/3*b^15/x^(3/2)-1/9*a^15/x^9-1170*a^8*b^7/x^(11/2)-455/3*a^3*b^12/x^3-50
05/6*a^9*b^6/x^6-10010/9*a^6*b^9/x^(9/2)-462*a^10*b^5/x^(13/2)-15/2*a*b^14
/x^2-182/3*a^12*b^3/x^(15/2)-3003/4*a^5*b^10/x^4-195*a^11*b^4/x^7-105/8*a^
13*b^2/x^8-390*a^4*b^11/x^(7/2)-30/17*a^14*b/x^(17/2)-42*a^2*b^13/x^(5/2)-
1287*a^7*b^8/x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(56) = 112.

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{10}}\,dx =$$
$$-\frac{9180\,ab^{14}x^7+185640\,a^3b^{12}x^6+918918\,a^5b^{10}x^5+1575288\,a^7b^8x^4+1021020\,a^9b^6x^3+238680\,a^{11}b^4x^2}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^10,x, algorithm="fricas")
```

output
```
-1/1224*(9180*a*b^14*x^7 + 185640*a^3*b^12*x^6 + 918918*a^5*b^10*x^5 + 157
5288*a^7*b^8*x^4 + 1021020*a^9*b^6*x^3 + 238680*a^11*b^4*x^2 + 16065*a^13*
b^2*x + 136*a^15 + 16*(51*b^15*x^7 + 3213*a^2*b^13*x^6 + 29835*a^4*b^11*x^
5 + 85085*a^6*b^9*x^4 + 89505*a^8*b^7*x^3 + 35343*a^10*b^5*x^2 + 4641*a^12
*b^3*x + 135*a^14*b)*sqrt(x))/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(61) = 122$.

Time = 1.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.99

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx = -\frac{a^{15}}{9x^9} - \frac{30a^{14}b}{17x^{\frac{17}{2}}} - \frac{105a^{13}b^2}{8x^8} - \frac{182a^{12}b^3}{3x^{\frac{15}{2}}} - \frac{195a^{11}b^4}{x^7} - \frac{462a^{10}b^5}{x^{\frac{13}{2}}} - \frac{5005a^9b^6}{6x^6} - \frac{1170a^8b^7}{x^{\frac{11}{2}}} - \frac{1287a^7b^8}{x^5} - \frac{10010a^6b^9}{9x^{\frac{9}{2}}} - \frac{3003a^5b^{10}}{4x^4} - \frac{390a^4b^{11}}{x^{\frac{7}{2}}} - \frac{455a^3b^{12}}{3x^3} - \frac{42a^2b^{13}}{x^{\frac{5}{2}}} - \frac{15ab^{14}}{2x^2} - \frac{2b^{15}}{3x^{\frac{3}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**10,x)
```

output
```
-a**15/(9*x**9) - 30*a**14*b/(17*x**(17/2)) - 105*a**13*b**2/(8*x**8) - 18
2*a**12*b**3/(3*x**(15/2)) - 195*a**11*b**4/x**7 - 462*a**10*b**5/x**(13/2
) - 5005*a**9*b**6/(6*x**6) - 1170*a**8*b**7/x**(11/2) - 1287*a**7*b**8/x*
*5 - 10010*a**6*b**9/(9*x**(9/2)) - 3003*a**5*b**10/(4*x**4) - 390*a**4*b*
*11/x**(7/2) - 455*a**3*b**12/(3*x**3) - 42*a**2*b**13/x**(5/2) - 15*a*b**
14/(2*x**2) - 2*b**15/(3*x**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(56) = 112$.

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.39

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx = -\frac{816\,b^{15}x^{\frac{15}{2}} + 9180\,ab^{14}x^7 + 51408\,a^2b^{13}x^{\frac{13}{2}} + 185640\,a^3b^{12}x^6 + 477360\,a^4b^{11}x^{\frac{11}{2}} + 918918\,a^5b^{10}x^5 + 13}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^10,x, algorithm="maxima")
```

output
```
-1/1224*(816*b^15*x^(15/2) + 9180*a*b^14*x^7 + 51408*a^2*b^13*x^(13/2) + 1
85640*a^3*b^12*x^6 + 477360*a^4*b^11*x^(11/2) + 918918*a^5*b^10*x^5 + 1361
360*a^6*b^9*x^(9/2) + 1575288*a^7*b^8*x^4 + 1432080*a^8*b^7*x^(7/2) + 1021
020*a^9*b^6*x^3 + 565488*a^10*b^5*x^(5/2) + 238680*a^11*b^4*x^2 + 74256*a^
12*b^3*x^(3/2) + 16065*a^13*b^2*x + 2160*a^14*b*sqrt(x) + 136*a^15)/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.39

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx = -\frac{816\,b^{15}x^{\frac{15}{2}} + 9180\,ab^{14}x^{7} + 51408\,a^{2}b^{13}x^{\frac{13}{2}} + 185640\,a^{3}b^{12}x^{6} + 477360\,a^{4}b^{11}x^{\frac{11}{2}} + 918918\,a^{5}b^{10}x^{5} + 13}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^10,x, algorithm="giac")
```

output
```
-1/1224*(816*b^15*x^(15/2) + 9180*a*b^14*x^7 + 51408*a^2*b^13*x^(13/2) + 1
85640*a^3*b^12*x^6 + 477360*a^4*b^11*x^(11/2) + 918918*a^5*b^10*x^5 + 1361
360*a^6*b^9*x^(9/2) + 1575288*a^7*b^8*x^4 + 1432080*a^8*b^7*x^(7/2) + 1021
020*a^9*b^6*x^3 + 565488*a^10*b^5*x^(5/2) + 238680*a^11*b^4*x^2 + 74256*a^
12*b^3*x^(3/2) + 16065*a^13*b^2*x + 2160*a^14*b*sqrt(x) + 136*a^15)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.39

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx = -\frac{\frac{a^{15}}{9} + \frac{2\,b^{15}\,x^{15/2}}{3} + \frac{105\,a^{13}\,b^2\,x}{8} + \frac{30\,a^{14}\,b\,\sqrt{x}}{17} + \frac{15\,a\,b^{14}\,x^7}{2} + 195\,a^{11}\,b^4\,x^2 + \frac{5005\,a^9\,b^6\,x^3}{6} + 1287\,a^7\,b^8\,x^4 + \frac{3003\,a^5\,b}{4}}{}$$

input
```
int((a + b*x^(1/2))^15/x^10,x)
```

output
```
-(a^15/9 + (2*b^15*x^(15/2))/3 + (105*a^13*b^2*x)/8 + (30*a^14*b*x^(1/2))/
17 + (15*a*b^14*x^7)/2 + 195*a^11*b^4*x^2 + (5005*a^9*b^6*x^3)/6 + 1287*a^
7*b^8*x^4 + (3003*a^5*b^10*x^5)/4 + (182*a^12*b^3*x^(3/2))/3 + (455*a^3*b^
12*x^6)/3 + 462*a^10*b^5*x^(5/2) + 1170*a^8*b^7*x^(7/2) + (10010*a^6*b^9*x
^(9/2))/9 + 390*a^4*b^11*x^(11/2) + 42*a^2*b^13*x^(13/2))/x^9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.64

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}\,dx$$
$$= \frac{-136\sqrt{x}\,a^{15}-16065\sqrt{x}\,a^{13}b^2x-238680\sqrt{x}\,a^{11}b^4x^2-1021020\sqrt{x}\,a^9b^6x^3-1575288\sqrt{x}\,a^7b^8x^4-91891}{}$$

input
```
int((a+b*x^(1/2))^15/x^10,x)
```

output
```
( - 136*sqrt(x)*a**15 - 16065*sqrt(x)*a**13*b**2*x - 238680*sqrt(x)*a**11*
b**4*x**2 - 1021020*sqrt(x)*a**9*b**6*x**3 - 1575288*sqrt(x)*a**7*b**8*x**
4 - 918918*sqrt(x)*a**5*b**10*x**5 - 185640*sqrt(x)*a**3*b**12*x**6 - 9180
*sqrt(x)*a*b**14*x**7 - 2160*a**14*b*x - 74256*a**12*b**3*x**2 - 565488*a*
*10*b**5*x**3 - 1432080*a**8*b**7*x**4 - 1361360*a**6*b**9*x**5 - 477360*a
**4*b**11*x**6 - 51408*a**2*b**13*x**7 - 816*b**15*x**8)/(1224*sqrt(x)*x**
9)
```

3.74 $\int \frac{(a+b\sqrt{x})^{15}}{x^{11}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{11}} \, dx = -\frac{(a+b\sqrt{x})^{16}}{10ax^{10}} + \frac{2b(a+b\sqrt{x})^{16}}{95a^2x^{19/2}} - \frac{b^2(a+b\sqrt{x})^{16}}{285a^3x^9} + \frac{2b^3(a+b\sqrt{x})^{16}}{4845a^4x^{17/2}} - \frac{b^4(a+b\sqrt{x})^{16}}{38760a^5x^8}$$

output
```
-1/10*(a+b*x^(1/2))^16/a/x^10+2/95*b*(a+b*x^(1/2))^16/a^2/x^(19/2)-1/285*b
^2*(a+b*x^(1/2))^16/a^3/x^9+2/4845*b^3*(a+b*x^(1/2))^16/a^4/x^(17/2)-1/387
60*b^4*(a+b*x^(1/2))^16/a^5/x^8
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{11}} \, dx = \frac{-3876a^{15} - 61200a^{14}b\sqrt{x} - 452200a^{13}b^2x - 2074800a^{12}b^3x^{3/2} - 6613425a^{11}b^4x^2 - 15519504a^{10}b^5x^{5/2} - \cdots}{\cdots}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^11,x]
```

output
```
(-3876*a^15 - 61200*a^14*b*Sqrt[x] - 452200*a^13*b^2*x - 2074800*a^12*b^3*
x^(3/2) - 6613425*a^11*b^4*x^2 - 15519504*a^10*b^5*x^(5/2) - 27713400*a^9*
b^6*x^3 - 38372400*a^8*b^7*x^(7/2) - 41570100*a^7*b^8*x^4 - 35271600*a^6*b
^9*x^(9/2) - 23279256*a^5*b^10*x^5 - 11757200*a^4*b^11*x^(11/2) - 4408950*
a^3*b^12*x^6 - 1162800*a^2*b^13*x^(13/2) - 193800*a*b^14*x^7 - 15504*b^15*
x^(15/2))/(38760*x^10)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{21/2}}d\sqrt{x}$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}d\sqrt{x}}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{b\left(-\frac{3b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{19/2}}d\sqrt{x}}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{b\left(-\frac{3b\left(-\frac{b\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}d\sqrt{x}}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(-\frac{b\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17/2}}d\sqrt{x}}{17a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)$$

$$\Big\downarrow \text{48}$$

$$2\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(\frac{b\left(a+b\sqrt{x}\right)^{16}}{272a^2x^8}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)$$

input
```
Int[(a + b*Sqrt[x])^15/x^11,x]
```

output
```
2*(-1/5*(b*((-3*b*(-1/9*(b*(-1/17*(a + b*Sqrt[x])^16/(a*x^(17/2)) + (b*(a
+ b*Sqrt[x])^16)/(272*a^2*x^8)))/a - (a + b*Sqrt[x])^16/(18*a*x^9)))/(19*a
) - (a + b*Sqrt[x])^16/(19*a*x^(19/2))))/a - (a + b*Sqrt[x])^16/(20*a*x^10
))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 23.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.40

method	result
derivativedivides	$-\frac{2002a^{10}b^5}{5x^{\frac{15}{2}}}-\frac{2145a^7b^8}{2x^6}-\frac{990a^8b^7}{x^{\frac{13}{2}}}-\frac{3003a^5b^{10}}{5x^5}-\frac{5a\,b^{14}}{x^3}-\frac{910a^6b^9}{x^{\frac{11}{2}}}-\frac{910a^4b^{11}}{3x^{\frac{9}{2}}}-\frac{30a^2b^{13}}{x^{\frac{7}{2}}}-\frac{715a^9b^6}{x^7}$
default	$-\frac{2002a^{10}b^5}{5x^{\frac{15}{2}}}-\frac{2145a^7b^8}{2x^6}-\frac{990a^8b^7}{x^{\frac{13}{2}}}-\frac{3003a^5b^{10}}{5x^5}-\frac{5a\,b^{14}}{x^3}-\frac{910a^6b^9}{x^{\frac{11}{2}}}-\frac{910a^4b^{11}}{3x^{\frac{9}{2}}}-\frac{30a^2b^{13}}{x^{\frac{7}{2}}}-\frac{715a^9b^6}{x^7}$
orering	$-\frac{(503880b^{28}x^{14}-3924450a^2b^{26}x^{13}+20986602a^4b^{24}x^{12}-72600710a^6b^{22}x^{11}+180495630a^8b^{20}x^{10}-334639305a^{10}b^{18}}{}$
trager	$\frac{(-1+x)(12a^{14}x^9+1400a^{12}b^2x^9+20475a^{10}b^4x^9+85800a^8b^6x^9+128700a^6b^8x^9+72072a^4b^{10}x^9+13650a^2b^{12}x^9+600b^{}}{}$

input
```
int((a+b*x^(1/2))^15/x^11,x,method=_RETURNVERBOSE)
```

output
```
-2002/5*a^10*b^5/x^(15/2)-2145/2*a^7*b^8/x^6-990*a^8*b^7/x^(13/2)-3003/5*a
^5*b^10/x^5-5*a*b^14/x^3-910*a^6*b^9/x^(11/2)-910/3*a^4*b^11/x^(9/2)-30*a^
2*b^13/x^(7/2)-715*a^9*b^6/x^7-910/17*a^12*b^3/x^(17/2)-30/19*a^14*b/x^(19
/2)-455/4*a^3*b^12/x^4-35/3*a^13*b^2/x^9-2/5*b^15/x^(5/2)-1/10*a^15/x^10-1
365/8*a^11*b^4/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx = -\frac{193800\,ab^{14}x^7+4408950\,a^3b^{12}x^6+23279256\,a^5b^{10}x^5+41570100\,a^7b^8x^4+27713400\,a^9b^6x^3+661342}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^11,x, algorithm="fricas")
```

output
```
-1/38760*(193800*a*b^14*x^7 + 4408950*a^3*b^12*x^6 + 23279256*a^5*b^10*x^5
 + 41570100*a^7*b^8*x^4 + 27713400*a^9*b^6*x^3 + 6613425*a^11*b^4*x^2 + 45
2200*a^13*b^2*x + 3876*a^15 + 16*(969*b^15*x^7 + 72675*a^2*b^13*x^6 + 7348
25*a^4*b^11*x^5 + 2204475*a^6*b^9*x^4 + 2398275*a^8*b^7*x^3 + 969969*a^10*
b^5*x^2 + 129675*a^12*b^3*x + 3825*a^14*b)*sqrt(x))/x^10
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.76

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx = -\frac{a^{15}}{10x^{10}} - \frac{30a^{14}b}{19x^{\frac{19}{2}}} - \frac{35a^{13}b^2}{3x^9} - \frac{910a^{12}b^3}{17x^{\frac{17}{2}}} - \frac{1365a^{11}b^4}{8x^8} - \frac{2002a^{10}b^5}{5x^{\frac{15}{2}}} - \frac{715a^9b^6}{x^7} - \frac{990a^8b^7}{x^{\frac{13}{2}}} - \frac{2145a^7b^8}{2x^6} - \frac{910a^6b^9}{x^{\frac{11}{2}}} - \frac{3003a^5b^{10}}{5x^5} - \frac{910a^4b^{11}}{3x^{\frac{9}{2}}} - \frac{455a^3b^{12}}{4x^4} - \frac{30a^2b^{13}}{x^{\frac{7}{2}}} - \frac{5ab^{14}}{x^3} - \frac{2b^{15}}{5x^{\frac{5}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**11,x)
```

output
```
-a**15/(10*x**10) - 30*a**14*b/(19*x**(19/2)) - 35*a**13*b**2/(3*x**9) - 9
10*a**12*b**3/(17*x**(17/2)) - 1365*a**11*b**4/(8*x**8) - 2002*a**10*b**5/
(5*x**(15/2)) - 715*a**9*b**6/x**7 - 990*a**8*b**7/x**(13/2) - 2145*a**7*b
**8/(2*x**6) - 910*a**6*b**9/x**(11/2) - 3003*a**5*b**10/(5*x**5) - 910*a*
*4*b**11/(3*x**(9/2)) - 455*a**3*b**12/(4*x**4) - 30*a**2*b**13/x**(7/2) -
 5*a*b**14/x**3 - 2*b**15/(5*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx = -\frac{15504\,b^{15}x^{\frac{15}{2}} + 193800\,ab^{14}x^7 + 1162800\,a^2b^{13}x^{\frac{13}{2}} + 4408950\,a^3b^{12}x^6 + 11757200\,a^4b^{11}x^{\frac{11}{2}} + 23279256}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^11,x, algorithm="maxima")
```

output
```
-1/38760*(15504*b^15*x^(15/2) + 193800*a*b^14*x^7 + 1162800*a^2*b^13*x^(13
/2) + 4408950*a^3*b^12*x^6 + 11757200*a^4*b^11*x^(11/2) + 23279256*a^5*b^1
0*x^5 + 35271600*a^6*b^9*x^(9/2) + 41570100*a^7*b^8*x^4 + 38372400*a^8*b^7
*x^(7/2) + 27713400*a^9*b^6*x^3 + 15519504*a^10*b^5*x^(5/2) + 6613425*a^11
*b^4*x^2 + 2074800*a^12*b^3*x^(3/2) + 452200*a^13*b^2*x + 61200*a^14*b*sqr
t(x) + 3876*a^15)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx =$$
$$-\frac{15504\,b^{15}x^{\frac{15}{2}}+193800\,ab^{14}x^{7}+1162800\,a^{2}b^{13}x^{\frac{13}{2}}+4408950\,a^{3}b^{12}x^{6}+11757200\,a^{4}b^{11}x^{\frac{11}{2}}+23279256}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^11,x, algorithm="giac")
```

output

```
-1/38760*(15504*b^15*x^(15/2) + 193800*a*b^14*x^7 + 1162800*a^2*b^13*x^(13
/2) + 4408950*a^3*b^12*x^6 + 11757200*a^4*b^11*x^(11/2) + 23279256*a^5*b^1
0*x^5 + 35271600*a^6*b^9*x^(9/2) + 41570100*a^7*b^8*x^4 + 38372400*a^8*b^7
*x^(7/2) + 27713400*a^9*b^6*x^3 + 15519504*a^10*b^5*x^(5/2) + 6613425*a^11
*b^4*x^2 + 2074800*a^12*b^3*x^(3/2) + 452200*a^13*b^2*x + 61200*a^14*b*sqr
t(x) + 3876*a^15)/x^10
```


Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}\,dx =$$
$$-\frac{\frac{a^{15}}{10}+\frac{2\,b^{15}\,x^{15/2}}{5}+\frac{35\,a^{13}\,b^{2}\,x}{3}+\frac{30\,a^{14}\,b\,\sqrt{x}}{19}+5\,a\,b^{14}\,x^{7}+\frac{1365\,a^{11}\,b^{4}\,x^{2}}{8}+715\,a^{9}\,b^{6}\,x^{3}+\frac{2145\,a^{7}\,b^{8}\,x^{4}}{2}+\frac{3003\,a^{5}\,b^{10}\,x}{5}}{}$$

input
```
int((a + b*x^(1/2))^15/x^11,x)
```

output
```
-(a^15/10 + (2*b^15*x^(15/2))/5 + (35*a^13*b^2*x)/3 + (30*a^14*b*x^(1/2))/
19 + 5*a*b^14*x^7 + (1365*a^11*b^4*x^2)/8 + 715*a^9*b^6*x^3 + (2145*a^7*b^
8*x^4)/2 + (3003*a^5*b^10*x^5)/5 + (910*a^12*b^3*x^(3/2))/17 + (455*a^3*b^
12*x^6)/4 + (2002*a^10*b^5*x^(5/2))/5 + 990*a^8*b^7*x^(7/2) + 910*a^6*b^9*
x^(9/2) + (910*a^4*b^11*x^(11/2))/3 + 30*a^2*b^13*x^(13/2))/x^10
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{11}}\,dx$$
$$= \frac{-3876\sqrt{x}\,a^{15} - 452200\sqrt{x}\,a^{13}b^2x - 6613425\sqrt{x}\,a^{11}b^4x^2 - 27713400\sqrt{x}\,a^9b^6x^3 - 41570100\sqrt{x}\,a^7b^8x^4 -$$

input
```
int((a+b*x^(1/2))^15/x^11,x)
```

output
```
( - 3876*sqrt(x)*a**15 - 452200*sqrt(x)*a**13*b**2*x - 6613425*sqrt(x)*a**
11*b**4*x**2 - 27713400*sqrt(x)*a**9*b**6*x**3 - 41570100*sqrt(x)*a**7*b**
8*x**4 - 23279256*sqrt(x)*a**5*b**10*x**5 - 4408950*sqrt(x)*a**3*b**12*x**
6 - 193800*sqrt(x)*a*b**14*x**7 - 61200*a**14*b*x - 2074800*a**12*b**3*x**
2 - 15519504*a**10*b**5*x**3 - 38372400*a**8*b**7*x**4 - 35271600*a**6*b**
9*x**5 - 11757200*a**4*b**11*x**6 - 1162800*a**2*b**13*x**7 - 15504*b**15*
x**8)/(38760*sqrt(x)*x**10)
```

3.75 $\int \frac{(a+b\sqrt{x})^{15}}{x^{12}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 170

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{12}}\,dx = -\frac{(a+b\sqrt{x})^{16}}{11ax^{11}}+\frac{2b(a+b\sqrt{x})^{16}}{77a^2x^{21/2}}-\frac{b^2(a+b\sqrt{x})^{16}}{154a^3x^{10}}+\frac{2b^3(a+b\sqrt{x})^{16}}{1463a^4x^{19/2}}-\frac{b^4(a+b\sqrt{x})^{16}}{4389a^5x^9}+\frac{2b^5(a+b\sqrt{x})^{16}}{74613a^6x^{17/2}}-\frac{b^6(a+b\sqrt{x})^{16}}{596904a^7x^8}$$

output
```
-1/11*(a+b*x^(1/2))^16/a/x^11+2/77*b*(a+b*x^(1/2))^16/a^2/x^(21/2)-1/154*b
^2*(a+b*x^(1/2))^16/a^3/x^10+2/1463*b^3*(a+b*x^(1/2))^16/a^4/x^(19/2)-1/43
89*b^4*(a+b*x^(1/2))^16/a^5/x^9+2/74613*b^5*(a+b*x^(1/2))^16/a^6/x^(17/2)-
1/596904*b^6*(a+b*x^(1/2))^16/a^7/x^8
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx = \frac{-54264a^{15}-852720a^{14}b\sqrt{x}-6267492a^{13}b^2x-28588560a^{12}b^3x^{3/2}-90530440a^{11}b^4x^2-210882672a^{10}}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^12,x]
```

output
```
(-54264*a^15 - 852720*a^14*b*Sqrt[x] - 6267492*a^13*b^2*x - 28588560*a^12*
b^3*x^(3/2) - 90530440*a^11*b^4*x^2 - 210882672*a^10*b^5*x^(5/2) - 3734380
65*a^9*b^6*x^3 - 512143632*a^8*b^7*x^(7/2) - 548725320*a^7*b^8*x^4 - 45961
6080*a^6*b^9*x^(9/2) - 298750452*a^5*b^10*x^5 - 148140720*a^4*b^11*x^(11/2
) - 54318264*a^3*b^12*x^6 - 13927760*a^2*b^13*x^(13/2) - 2238390*a*b^14*x^
7 - 170544*b^15*x^(15/2))/(596904*x^11)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {798, 55, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx$$

$$\Big\downarrow \ 798$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{23/2}}d\sqrt{x}$$

$$\Big\downarrow \ 55$$

$$2\left(-\frac{3b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}d\sqrt{x}}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right)$$

$$\downarrow 55$$

$$2\left(-\frac{3b\left(-\frac{5b\int \frac{(a+b\sqrt{x})^{15}}{x^{21/2}}d\sqrt{x}}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)$$

$$\downarrow 55$$

$$2\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\int \frac{(a+b\sqrt{x})^{15}}{x^{10}}d\sqrt{x}}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)$$

$$\downarrow 55$$

$$2\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\int \frac{(a+b\sqrt{x})^{15}}{x^{19/2}}d\sqrt{x}}{19a}-\frac{(a+b\sqrt{x})^{16}}{19ax^{19/2}}\right)}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)$$

$$\downarrow 55$$

$$2\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\int\frac{(a+b\sqrt{x})^{15}}{x^9}d\sqrt{x}}{9a}-\frac{(a+b\sqrt{x})^{16}}{18ax^9}\right)}{19a}-\frac{(a+b\sqrt{x})^{16}}{19ax^{19/2}}\right)}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(-\frac{b\int\frac{(a+b\sqrt{x})^{15}}{x^{17/2}}d\sqrt{x}}{17a}-\frac{(a+b\sqrt{x})^{16}}{17ax^{17/2}}\right)}{9a}-\frac{(a+b\sqrt{x})^{16}}{18ax^{9}}\right)}{19a}-\frac{(a+b\sqrt{x})^{16}}{19ax^{19/2}}\right)}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b}{22a}\right.$$

↓ 48

$$2\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(\frac{b\left(a+b\sqrt{x}\right)^{16}}{272a^2x^8}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^12,x]
```

output
```
2*((-3*b*((-5*b*(-1/5*(b*((-3*b*(-1/9*(b*(-1/17*(a + b*Sqrt[x])^16/(a*x^(1
7/2)) + (b*(a + b*Sqrt[x])^16)/(272*a^2*x^8)))/a - (a + b*Sqrt[x])^16/(18*
a*x^9)))/(19*a) - (a + b*Sqrt[x])^16/(19*a*x^(19/2))))/a - (a + b*Sqrt[x])
^16/(20*a*x^10)))/(21*a) - (a + b*Sqrt[x])^16/(21*a*x^(21/2))))/(11*a) - (
a + b*Sqrt[x])^16/(22*a*x^11))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 23.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{858a^8b^7}{x^{\frac{15}{2}}}-\frac{910a^{12}b^3}{19x^{\frac{19}{2}}}-\frac{770a^6b^9}{x^{\frac{13}{2}}}-\frac{455a^{11}b^4}{3x^9}-\frac{2b^{15}}{7x^{\frac{7}{2}}}-\frac{6006a^{10}b^5}{17x^{\frac{17}{2}}}-\frac{1001a^5b^{10}}{2x^6}-\frac{5005a^9b^6}{8x^8}-\frac{70a^2b^{13}}{3x^{\frac{9}{2}}}$
default	$-\frac{858a^8b^7}{x^{\frac{15}{2}}}-\frac{910a^{12}b^3}{19x^{\frac{19}{2}}}-\frac{770a^6b^9}{x^{\frac{13}{2}}}-\frac{455a^{11}b^4}{3x^9}-\frac{2b^{15}}{7x^{\frac{7}{2}}}-\frac{6006a^{10}b^5}{17x^{\frac{17}{2}}}-\frac{1001a^5b^{10}}{2x^6}-\frac{5005a^9b^6}{8x^8}-\frac{70a^2b^{13}}{3x^{\frac{9}{2}}}$
orering	$-\frac{(1812030b^{28}x^{14}-17956862a^2b^{26}x^{13}+96702970a^4b^{24}x^{12}-339728170a^6b^{22}x^{11}+848722875a^8b^{20}x^{10}-1575629055a}{}$
trager	$\frac{(-1+x)(168a^{14}x^{10}+19404a^{12}b^2x^{10}+280280a^{10}b^4x^{10}+1156155a^8b^6x^{10}+1698840a^6b^8x^{10}+924924a^4b^{10}x^{10}+168168}{}$

input
```
int((a+b*x^(1/2))^15/x^12,x,method=_RETURNVERBOSE)
```

output
```
-858*a^8*b^7/x^(15/2)-910/19*a^12*b^3/x^(19/2)-770*a^6*b^9/x^(13/2)-455/3*
a^11*b^4/x^9-2/7*b^15/x^(7/2)-6006/17*a^10*b^5/x^(17/2)-1001/2*a^5*b^10/x^
6-5005/8*a^9*b^6/x^8-70/3*a^2*b^13/x^(9/2)-1/11*a^15/x^11-21/2*a^13*b^2/x^
10-10/7*a^14*b/x^(21/2)-2730/11*a^4*b^11/x^(11/2)-6435/7*a^7*b^8/x^7-15/4*
a*b^14/x^4-91*a^3*b^12/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx = -\frac{2238390\,ab^{14}x^7+54318264\,a^3b^{12}x^6+298750452\,a^5b^{10}x^5+548725320\,a^7b^8x^4+373438065\,a^9b^6x^3+9}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^12,x, algorithm="fricas")
```

output
```
-1/596904*(2238390*a*b^14*x^7 + 54318264*a^3*b^12*x^6 + 298750452*a^5*b^10
*x^5 + 548725320*a^7*b^8*x^4 + 373438065*a^9*b^6*x^3 + 90530440*a^11*b^4*x
^2 + 6267492*a^13*b^2*x + 54264*a^15 + 16*(10659*b^15*x^7 + 870485*a^2*b^1
3*x^6 + 9258795*a^4*b^11*x^5 + 28726005*a^6*b^9*x^4 + 32008977*a^8*b^7*x^3
 + 13180167*a^10*b^5*x^2 + 1786785*a^12*b^3*x + 53295*a^14*b)*sqrt(x))/x^1
1
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx = -\frac{a^{15}}{11x^{11}} - \frac{10a^{14}b}{7x^{\frac{21}{2}}} - \frac{21a^{13}b^2}{2x^{10}} - \frac{910a^{12}b^3}{19x^{\frac{19}{2}}} - \frac{455a^{11}b^4}{3x^9} - \frac{6006a^{10}b^5}{17x^{\frac{17}{2}}} - \frac{5005a^9b^6}{8x^8} - \frac{858a^8b^7}{x^{\frac{15}{2}}} - \frac{6435a^7b^8}{7x^7} - \frac{770a^6b^9}{x^{\frac{13}{2}}} - \frac{1001a^5b^{10}}{2x^6} - \frac{2730a^4b^{11}}{11x^{\frac{11}{2}}} - \frac{91a^3b^{12}}{x^5} - \frac{70a^2b^{13}}{3x^{\frac{9}{2}}} - \frac{15ab^{14}}{4x^4} - \frac{2b^{15}}{7x^{\frac{7}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**12,x)
```

output
```
-a**15/(11*x**11) - 10*a**14*b/(7*x**(21/2)) - 21*a**13*b**2/(2*x**10) - 9
10*a**12*b**3/(19*x**(19/2)) - 455*a**11*b**4/(3*x**9) - 6006*a**10*b**5/(
17*x**(17/2)) - 5005*a**9*b**6/(8*x**8) - 858*a**8*b**7/x**(15/2) - 6435*a
**7*b**8/(7*x**7) - 770*a**6*b**9/x**(13/2) - 1001*a**5*b**10/(2*x**6) - 2
730*a**4*b**11/(11*x**(11/2)) - 91*a**3*b**12/x**5 - 70*a**2*b**13/(3*x**(
9/2)) - 15*a*b**14/(4*x**4) - 2*b**15/(7*x**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx =$$

$$-\frac{170544\,b^{15}x^{\frac{15}{2}} + 2238390\,ab^{14}x^7 + 13927760\,a^2b^{13}x^{\frac{13}{2}} + 54318264\,a^3b^{12}x^6 + 148140720\,a^4b^{11}x^{\frac{11}{2}} + 298\cdots}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^12,x, algorithm="maxima")
```

output
```
-1/596904*(170544*b^15*x^(15/2) + 2238390*a*b^14*x^7 + 13927760*a^2*b^13*x
^(13/2) + 54318264*a^3*b^12*x^6 + 148140720*a^4*b^11*x^(11/2) + 298750452*
a^5*b^10*x^5 + 459616080*a^6*b^9*x^(9/2) + 548725320*a^7*b^8*x^4 + 5121436
32*a^8*b^7*x^(7/2) + 373438065*a^9*b^6*x^3 + 210882672*a^10*b^5*x^(5/2) +
90530440*a^11*b^4*x^2 + 28588560*a^12*b^3*x^(3/2) + 6267492*a^13*b^2*x + 8
52720*a^14*b*sqrt(x) + 54264*a^15)/x^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{12}}\,dx = -\frac{170544\,b^{15}x^{\frac{15}{2}} + 2238390\,ab^{14}x^7 + 13927760\,a^2b^{13}x^{\frac{13}{2}} + 54318264\,a^3b^{12}x^6 + 148140720\,a^4b^{11}x^{\frac{11}{2}} + 298}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^12,x, algorithm="giac")
```

output
```
-1/596904*(170544*b^15*x^(15/2) + 2238390*a*b^14*x^7 + 13927760*a^2*b^13*x
^(13/2) + 54318264*a^3*b^12*x^6 + 148140720*a^4*b^11*x^(11/2) + 298750452*
a^5*b^10*x^5 + 459616080*a^6*b^9*x^(9/2) + 548725320*a^7*b^8*x^4 + 5121436
32*a^8*b^7*x^(7/2) + 373438065*a^9*b^6*x^3 + 210882672*a^10*b^5*x^(5/2) +
90530440*a^11*b^4*x^2 + 28588560*a^12*b^3*x^(3/2) + 6267492*a^13*b^2*x + 8
52720*a^14*b*sqrt(x) + 54264*a^15)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{12}}\,dx = -\frac{\frac{a^{15}}{11} + \frac{2\,b^{15}\,x^{15/2}}{7} + \frac{21\,a^{13}\,b^2\,x}{2} + \frac{10\,a^{14}\,b\,\sqrt{x}}{7} + \frac{15\,a\,b^{14}\,x^7}{4} + \frac{455\,a^{11}\,b^4\,x^2}{3} + \frac{5005\,a^9\,b^6\,x^3}{8} + \frac{6435\,a^7\,b^8\,x^4}{7} + \frac{1001\,a^5\,b^{10}\,x^5}{2} + }{x}$$

input
```
int((a + b*x^(1/2))^15/x^12,x)
```

output
```
-(a^15/11 + (2*b^15*x^(15/2))/7 + (21*a^13*b^2*x)/2 + (10*a^14*b*x^(1/2))/
7 + (15*a*b^14*x^7)/4 + (455*a^11*b^4*x^2)/3 + (5005*a^9*b^6*x^3)/8 + (643
5*a^7*b^8*x^4)/7 + (1001*a^5*b^10*x^5)/2 + (910*a^12*b^3*x^(3/2))/19 + 91*
a^3*b^12*x^6 + (6006*a^10*b^5*x^(5/2))/17 + 858*a^8*b^7*x^(7/2) + 770*a^6*
b^9*x^(9/2) + (2730*a^4*b^11*x^(11/2))/11 + (70*a^2*b^13*x^(13/2))/3)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}\,dx = \frac{-54264\sqrt{x}\,a^{15}-6267492\sqrt{x}\,a^{13}b^2x-90530440\sqrt{x}\,a^{11}b^4x^2-373438065\sqrt{x}\,a^9b^6x^3-548725320\sqrt{x}\,a^7b$$

input
```
int((a+b*x^(1/2))^15/x^12,x)
```

output
```
( - 54264*sqrt(x)*a**15 - 6267492*sqrt(x)*a**13*b**2*x - 90530440*sqrt(x)*
a**11*b**4*x**2 - 373438065*sqrt(x)*a**9*b**6*x**3 - 548725320*sqrt(x)*a**
7*b**8*x**4 - 298750452*sqrt(x)*a**5*b**10*x**5 - 54318264*sqrt(x)*a**3*b*
*12*x**6 - 2238390*sqrt(x)*a*b**14*x**7 - 852720*a**14*b*x - 28588560*a**1
2*b**3*x**2 - 210882672*a**10*b**5*x**3 - 512143632*a**8*b**7*x**4 - 45961
6080*a**6*b**9*x**5 - 148140720*a**4*b**11*x**6 - 13927760*a**2*b**13*x**7
 - 170544*b**15*x**8)/(596904*sqrt(x)*x**11)
```

3.76 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 220

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx = & -\frac{\left(a+b\sqrt{x}\right)^{16}}{12ax^{12}}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{69a^2x^{23/2}}-\frac{7b^2\left(a+b\sqrt{x}\right)^{16}}{759a^3x^{11}} \\ & +\frac{2b^3\left(a+b\sqrt{x}\right)^{16}}{759a^4x^{21/2}}-\frac{b^4\left(a+b\sqrt{x}\right)^{16}}{1518a^5x^{10}}+\frac{2b^5\left(a+b\sqrt{x}\right)^{16}}{14421a^6x^{19/2}} \\ & -\frac{b^6\left(a+b\sqrt{x}\right)^{16}}{43263a^7x^9}+\frac{2b^7\left(a+b\sqrt{x}\right)^{16}}{735471a^8x^{17/2}}-\frac{b^8\left(a+b\sqrt{x}\right)^{16}}{5883768a^9x^8}\end{aligned}$$

output
```
-1/12*(a+b*x^(1/2))^16/a/x^12+2/69*b*(a+b*x^(1/2))^16/a^2/x^(23/2)-7/759*b
^2*(a+b*x^(1/2))^16/a^3/x^11+2/759*b^3*(a+b*x^(1/2))^16/a^4/x^(21/2)-1/151
8*b^4*(a+b*x^(1/2))^16/a^5/x^10+2/14421*b^5*(a+b*x^(1/2))^16/a^6/x^(19/2)-
1/43263*b^6*(a+b*x^(1/2))^16/a^7/x^9+2/735471*b^7*(a+b*x^(1/2))^16/a^8/x^(
17/2)-1/5883768*b^8*(a+b*x^(1/2))^16/a^9/x^8
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx$$
$$= \frac{-490314a^{15} - 7674480a^{14}b\sqrt{x} - 56163240a^{13}b^2x - 254963280a^{12}b^3x^{3/2} - 803134332a^{11}b^4x^2 - 1859890}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^13,x]
```

output
```
(-490314*a^15 - 7674480*a^14*b*Sqrt[x] - 56163240*a^13*b^2*x - 254963280*a
^12*b^3*x^(3/2) - 803134332*a^11*b^4*x^2 - 1859890032*a^10*b^5*x^(5/2) - 3
272028760*a^9*b^6*x^3 - 4454358480*a^8*b^7*x^(7/2) - 4732755885*a^7*b^8*x^
4 - 3926434512*a^6*b^9*x^(9/2) - 2524136472*a^5*b^10*x^5 - 1235591280*a^4*
b^11*x^(11/2) - 446185740*a^3*b^12*x^6 - 112326480*a^2*b^13*x^(13/2) - 176
51304*a*b^14*x^7 - 1307504*b^15*x^(15/2))/(5883768*x^12)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {798, 55, 55, 55, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx$$
$$\Big\downarrow \text{ 798}$$
$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{25/2}}d\sqrt{x}$$
$$\Big\downarrow \text{ 55}$$
$$2\left(-\frac{b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{12}}d\sqrt{x}}{3a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{24ax^{12}}\right)$$

$$\Big\downarrow \ 55$$

$$2\left(-\frac{b\left(-\frac{7b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{23/2}}d\sqrt{x}}{23a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{23ax^{23/2}}\right)}{3a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{24ax^{12}}\right)$$

$$\Big\downarrow \ 55$$

$$2\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{11}}d\sqrt{x}}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right)}{23a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{23ax^{23/2}}\right)}{3a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{24ax^{12}}\right)$$

$$\Big\downarrow \ 55$$

$$2\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{21/2}}d\sqrt{x}}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right)}{23a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{23ax^{23/2}}\right)}{3a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{24ax^{12}}\right)$$

$$\Big\downarrow \ 55$$

$$2\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^{10}}d\sqrt{x}}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right)}{23a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{23ax^{23/2}}\right)}{3a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{24ax^{12}}\right)$$

$\Big\downarrow$ 55

$$2\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^{19/2}}d\sqrt{x}}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right)}{23a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{23ax^{23/2}}\right)}{3a}-\frac{(a+b}{24a}$$

↓ 55

$$\Bigg(b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\int\frac{(a+b\sqrt{x})^{15}}{x^9}d\sqrt{x}}{9a}-\frac{(a+b\sqrt{x})^{16}}{18ax^9}\right)}{19a}-\frac{(a+b\sqrt{x})^{16}}{19ax^{19/2}}\right)}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)}{23a}-\frac{(a+b\sqrt{\dots}}{23ax^{23\dots}}$$

↓ 55

$$\left(\left(7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(-\frac{b\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17/2}}\,d\sqrt{x}}{17a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^{9}}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{\left(a+b\sqrt{\right.}}{22a}\right.\right.\right.$$

$\downarrow$ 48

$$\Bigg(\Bigg(b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(\frac{b\left(a+b\sqrt{x}\right)^{16}}{272a^2x^8}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right)}{23a} -$$

input
```
Int[(a + b*Sqrt[x])^15/x^13,x]
```

output
```
2*(-1/3*(b*((-7*b*((-3*b*((-5*b*(-1/5*(b*((-3*b*(-1/9*(b*(-1/17*(a + b*Sqr
t[x])^16/(a*x^(17/2)) + (b*(a + b*Sqrt[x])^16)/(272*a^2*x^8)))/a - (a + b*
Sqrt[x])^16/(18*a*x^9)))/(19*a) - (a + b*Sqrt[x])^16/(19*a*x^(19/2))))/a -
 (a + b*Sqrt[x])^16/(20*a*x^10)))/(21*a) - (a + b*Sqrt[x])^16/(21*a*x^(21/
2))))/(11*a) - (a + b*Sqrt[x])^16/(22*a*x^11)))/(23*a) - (a + b*Sqrt[x])^1
6/(23*a*x^(23/2))))/a - (a + b*Sqrt[x])^16/(24*a*x^12))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 23.90 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{30a^{14}b}{23x^{\frac{23}{2}}}-\frac{130a^{12}b^{3}}{3x^{\frac{21}{2}}}-\frac{2002a^{6}b^{9}}{3x^{\frac{15}{2}}}-\frac{2b^{15}}{9x^{\frac{9}{2}}}-\frac{273a^{11}b^{4}}{2x^{10}}-\frac{5005a^{9}b^{6}}{9x^{9}}-\frac{a^{15}}{12x^{12}}-\frac{3a\,b^{14}}{x^{5}}-\frac{105a^{13}b^{2}}{11x^{11}}-210$
default	$-\frac{30a^{14}b}{23x^{\frac{23}{2}}}-\frac{130a^{12}b^{3}}{3x^{\frac{21}{2}}}-\frac{2002a^{6}b^{9}}{3x^{\frac{15}{2}}}-\frac{2b^{15}}{9x^{\frac{9}{2}}}-\frac{273a^{11}b^{4}}{2x^{10}}-\frac{5005a^{9}b^{6}}{9x^{9}}-\frac{a^{15}}{12x^{12}}-\frac{3a\,b^{14}}{x^{5}}-\frac{105a^{13}b^{2}}{11x^{11}}-210$
orering	$-\frac{(41186376b^{28}x^{14}-454506220a^{2}b^{26}x^{13}+2529574500a^{4}b^{24}x^{12}-9035261235a^{6}b^{22}x^{11}+22770783035a^{8}b^{20}x^{10}-4247}{}$
trager	$\frac{(-1+x)(66a^{14}x^{11}+7560a^{12}b^{2}x^{11}+108108a^{10}b^{4}x^{11}+440440a^{8}b^{6}x^{11}+637065a^{6}b^{8}x^{11}+339768a^{4}b^{10}x^{11}+60060a^{2}b^{12}}{}$

input
```
int((a+b*x^(1/2))^15/x^13,x,method=_RETURNVERBOSE)
```

output
```
-30/23*a^14*b/x^(23/2)-130/3*a^12*b^3/x^(21/2)-2002/3*a^6*b^9/x^(15/2)-2/9
*b^15/x^(9/2)-273/2*a^11*b^4/x^10-5005/9*a^9*b^6/x^9-1/12*a^15/x^12-3*a*b^
14/x^5-105/11*a^13*b^2/x^11-210*a^4*b^11/x^(13/2)-210/11*a^2*b^13/x^(11/2)
-455/6*a^3*b^12/x^6-429*a^5*b^10/x^7-12870/17*a^8*b^7/x^(17/2)-6006/19*a^1
0*b^5/x^(19/2)-6435/8*a^7*b^8/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx =$$
$$-\frac{17651304\,ab^{14}x^{7}+446185740\,a^{3}b^{12}x^{6}+2524136472\,a^{5}b^{10}x^{5}+4732755885\,a^{7}b^{8}x^{4}+3272028760\,a^{9}b^{6}}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^13,x, algorithm="fricas")
```

output
```
-1/5883768*(17651304*a*b^14*x^7 + 446185740*a^3*b^12*x^6 + 2524136472*a^5*
b^10*x^5 + 4732755885*a^7*b^8*x^4 + 3272028760*a^9*b^6*x^3 + 803134332*a^1
1*b^4*x^2 + 56163240*a^13*b^2*x + 490314*a^15 + 16*(81719*b^15*x^7 + 70204
05*a^2*b^13*x^6 + 77224455*a^4*b^11*x^5 + 245402157*a^6*b^9*x^4 + 27839740
5*a^8*b^7*x^3 + 116243127*a^10*b^5*x^2 + 15935205*a^12*b^3*x + 479655*a^14
*b)*sqrt(x))/x^12
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx = -\frac{a^{15}}{12x^{12}} - \frac{30a^{14}b}{23x^{\frac{23}{2}}} - \frac{105a^{13}b^2}{11x^{11}} - \frac{130a^{12}b^3}{3x^{\frac{21}{2}}} - \frac{273a^{11}b^4}{2x^{10}} - \frac{6006a^{10}b^5}{19x^{\frac{19}{2}}} - \frac{5005a^9b^6}{9x^9} - \frac{12870a^8b^7}{17x^{\frac{17}{2}}} - \frac{6435a^7b^8}{8x^8} - \frac{2002a^6b^9}{3x^{\frac{15}{2}}} - \frac{429a^5b^{10}}{x^7} - \frac{210a^4b^{11}}{x^{\frac{13}{2}}} - \frac{455a^3b^{12}}{6x^6} - \frac{210a^2b^{13}}{11x^{\frac{11}{2}}} - \frac{3ab^{14}}{x^5} - \frac{2b^{15}}{9x^{\frac{9}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**13,x)
```

output
```
-a**15/(12*x**12) - 30*a**14*b/(23*x**(23/2)) - 105*a**13*b**2/(11*x**11)
- 130*a**12*b**3/(3*x**(21/2)) - 273*a**11*b**4/(2*x**10) - 6006*a**10*b**
5/(19*x**(19/2)) - 5005*a**9*b**6/(9*x**9) - 12870*a**8*b**7/(17*x**(17/2)
) - 6435*a**7*b**8/(8*x**8) - 2002*a**6*b**9/(3*x**(15/2)) - 429*a**5*b**1
0/x**7 - 210*a**4*b**11/x**(13/2) - 455*a**3*b**12/(6*x**6) - 210*a**2*b**
13/(11*x**(11/2)) - 3*a*b**14/x**5 - 2*b**15/(9*x**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx = -\frac{1307504\,b^{15}x^{\frac{15}{2}} + 17651304\,ab^{14}x^7 + 112326480\,a^2b^{13}x^{\frac{13}{2}} + 446185740\,a^3b^{12}x^6 + 1235591280\,a^4b^{11}x^{\frac{11}{2}}}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^13,x, algorithm="maxima")
```

output
```
-1/5883768*(1307504*b^15*x^(15/2) + 17651304*a*b^14*x^7 + 112326480*a^2*b^
13*x^(13/2) + 446185740*a^3*b^12*x^6 + 1235591280*a^4*b^11*x^(11/2) + 2524
136472*a^5*b^10*x^5 + 3926434512*a^6*b^9*x^(9/2) + 4732755885*a^7*b^8*x^4
+ 4454358480*a^8*b^7*x^(7/2) + 3272028760*a^9*b^6*x^3 + 1859890032*a^10*b^
5*x^(5/2) + 803134332*a^11*b^4*x^2 + 254963280*a^12*b^3*x^(3/2) + 56163240
*a^13*b^2*x + 7674480*a^14*b*sqrt(x) + 490314*a^15)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{13}}\,dx = -\frac{1307504\,b^{15}x^{\frac{15}{2}} + 17651304\,ab^{14}x^7 + 112326480\,a^2b^{13}x^{\frac{13}{2}} + 446185740\,a^3b^{12}x^6 + 1235591280\,a^4b^{11}x^{\frac{11}{2}}$$

input
```
integrate((a+b*x^(1/2))^15/x^13,x, algorithm="giac")
```

output
```
-1/5883768*(1307504*b^15*x^(15/2) + 17651304*a*b^14*x^7 + 112326480*a^2*b^
13*x^(13/2) + 446185740*a^3*b^12*x^6 + 1235591280*a^4*b^11*x^(11/2) + 2524
136472*a^5*b^10*x^5 + 3926434512*a^6*b^9*x^(9/2) + 4732755885*a^7*b^8*x^4
+ 4454358480*a^8*b^7*x^(7/2) + 3272028760*a^9*b^6*x^3 + 1859890032*a^10*b^
5*x^(5/2) + 803134332*a^11*b^4*x^2 + 254963280*a^12*b^3*x^(3/2) + 56163240
*a^13*b^2*x + 7674480*a^14*b*sqrt(x) + 490314*a^15)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{13}}\,dx = -\frac{\frac{a^{15}}{12} + \frac{2\,b^{15}\,x^{15/2}}{9} + \frac{105\,a^{13}\,b^2\,x}{11} + \frac{30\,a^{14}\,b\,\sqrt{x}}{23} + 3\,a\,b^{14}\,x^7 + \frac{273\,a^{11}\,b^4\,x^2}{2} + \frac{5005\,a^9\,b^6\,x^3}{9} + \frac{6435\,a^7\,b^8\,x^4}{8} + 429\,a^5\,b^{10}\,x}{}$$

input
```
int((a + b*x^(1/2))^15/x^13,x)
```

output
```
-(a^15/12 + (2*b^15*x^(15/2))/9 + (105*a^13*b^2*x)/11 + (30*a^14*b*x^(1/2)
)/23 + 3*a*b^14*x^7 + (273*a^11*b^4*x^2)/2 + (5005*a^9*b^6*x^3)/9 + (6435*
a^7*b^8*x^4)/8 + 429*a^5*b^10*x^5 + (130*a^12*b^3*x^(3/2))/3 + (455*a^3*b^
12*x^6)/6 + (6006*a^10*b^5*x^(5/2))/19 + (12870*a^8*b^7*x^(7/2))/17 + (200
2*a^6*b^9*x^(9/2))/3 + 210*a^4*b^11*x^(11/2) + (210*a^2*b^13*x^(13/2))/11)
/x^12
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}\,dx$$
$$= \frac{-490314\sqrt{x}\,a^{15}-56163240\sqrt{x}\,a^{13}b^{2}x-803134332\sqrt{x}\,a^{11}b^{4}x^{2}-3272028760\sqrt{x}\,a^{9}b^{6}x^{3}-4732755885\sqrt{}}{}$$

input
```
int((a+b*x^(1/2))^15/x^13,x)
```

output
```
( - 490314*sqrt(x)*a**15 - 56163240*sqrt(x)*a**13*b**2*x - 803134332*sqrt(
x)*a**11*b**4*x**2 - 3272028760*sqrt(x)*a**9*b**6*x**3 - 4732755885*sqrt(x
)*a**7*b**8*x**4 - 2524136472*sqrt(x)*a**5*b**10*x**5 - 446185740*sqrt(x)*
a**3*b**12*x**6 - 17651304*sqrt(x)*a*b**14*x**7 - 7674480*a**14*b*x - 2549
63280*a**12*b**3*x**2 - 1859890032*a**10*b**5*x**3 - 4454358480*a**8*b**7*
x**4 - 3926434512*a**6*b**9*x**5 - 1235591280*a**4*b**11*x**6 - 112326480*
a**2*b**13*x**7 - 1307504*b**15*x**8)/(5883768*sqrt(x)*x**12)
```

3.77 $\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\begin{aligned}\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx =& -\frac{\left(a+b\sqrt{x}\right)^{16}}{13ax^{13}}+\frac{2b\left(a+b\sqrt{x}\right)^{16}}{65a^2x^{25/2}}-\frac{3b^2\left(a+b\sqrt{x}\right)^{16}}{260a^3x^{12}}\\ &+\frac{6b^3\left(a+b\sqrt{x}\right)^{16}}{1495a^4x^{23/2}}-\frac{21b^4\left(a+b\sqrt{x}\right)^{16}}{16445a^5x^{11}}\\ &+\frac{6b^5\left(a+b\sqrt{x}\right)^{16}}{16445a^6x^{21/2}}-\frac{3b^6\left(a+b\sqrt{x}\right)^{16}}{32890a^7x^{10}}+\frac{6b^7\left(a+b\sqrt{x}\right)^{16}}{312455a^8x^{19/2}}\\ &-\frac{b^8\left(a+b\sqrt{x}\right)^{16}}{312455a^9x^9}+\frac{2b^9\left(a+b\sqrt{x}\right)^{16}}{5311735a^{10}x^{17/2}}-\frac{b^{10}\left(a+b\sqrt{x}\right)^{16}}{42493880a^{11}x^8}\end{aligned}$$

output
```
-1/13*(a+b*x^(1/2))^16/a/x^13+2/65*b*(a+b*x^(1/2))^16/a^2/x^(25/2)-3/260*b
^2*(a+b*x^(1/2))^16/a^3/x^12+6/1495*b^3*(a+b*x^(1/2))^16/a^4/x^(23/2)-21/1
6445*b^4*(a+b*x^(1/2))^16/a^5/x^11+6/16445*b^5*(a+b*x^(1/2))^16/a^6/x^(21/
2)-3/32890*b^6*(a+b*x^(1/2))^16/a^7/x^10+6/312455*b^7*(a+b*x^(1/2))^16/a^8
/x^(19/2)-1/312455*b^8*(a+b*x^(1/2))^16/a^9/x^9+2/5311735*b^9*(a+b*x^(1/2)
)^16/a^10/x^(17/2)-1/42493880*b^10*(a+b*x^(1/2))^16/a^11/x^8
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx$$
$$= \frac{-3268760a^{15}-50992656a^{14}b\sqrt{x}-371821450a^{13}b^2x-1681279600a^{12}b^3x^{3/2}-5273104200a^{11}b^4x^2-12}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^14,x]
```

output
```
(-3268760*a^15 - 50992656*a^14*b*Sqrt[x] - 371821450*a^13*b^2*x - 16812796
00*a^12*b^3*x^(3/2) - 5273104200*a^11*b^4*x^2 - 12153249680*a^10*b^5*x^(5/
2) - 21268186940*a^9*b^6*x^3 - 28784012400*a^8*b^7*x^(7/2) - 30383124200*a
^7*b^8*x^4 - 25021396400*a^6*b^9*x^(9/2) - 15951140205*a^5*b^10*x^5 - 7733
886160*a^4*b^11*x^(11/2) - 2762102200*a^3*b^12*x^6 - 686439600*a^2*b^13*x^
(13/2) - 106234700*a*b^14*x^7 - 7726160*b^15*x^(15/2))/(42493880*x^13)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {798, 55, 55, 55, 55, 55, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx$$

$$\Big\downarrow \ 798$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{27/2}}d\sqrt{x}$$

$$\Big\downarrow \ 55$$

$$2\left(-\frac{5b\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{13}}d\sqrt{x}}{13a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{26ax^{13}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{5b\left(-\frac{9b\int \frac{(a+b\sqrt{x})^{15}}{x^{25/2}}d\sqrt{x}}{25a}-\frac{(a+b\sqrt{x})^{16}}{25ax^{25/2}}\right)}{13a}-\frac{(a+b\sqrt{x})^{16}}{26ax^{13}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{5b\left(-\frac{9b\left(-\frac{b\int \frac{(a+b\sqrt{x})^{15}}{x^{12}}d\sqrt{x}}{3a}-\frac{(a+b\sqrt{x})^{16}}{24ax^{12}}\right)}{25a}-\frac{(a+b\sqrt{x})^{16}}{25ax^{25/2}}\right)}{13a}-\frac{(a+b\sqrt{x})^{16}}{26ax^{13}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{5b\left(-\frac{9b\left(-\frac{b\left(-\frac{7b\int \frac{(a+b\sqrt{x})^{15}}{x^{23/2}}d\sqrt{x}}{23a}-\frac{(a+b\sqrt{x})^{16}}{23ax^{23/2}}\right)}{3a}-\frac{(a+b\sqrt{x})^{16}}{24ax^{12}}\right)}{25a}-\frac{(a+b\sqrt{x})^{16}}{25ax^{25/2}}\right)}{13a}-\frac{(a+b\sqrt{x})^{16}}{26ax^{13}}\right)$$

$$\Big\downarrow \text{55}$$

$$2\left(-\frac{5b\left(-\frac{9b\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\int\frac{(a+b\sqrt{x})^{15}}{x^{11}}d\sqrt{x}}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)}{23a}-\frac{(a+b\sqrt{x})^{16}}{23ax^{23/2}}\right)}{3a}-\frac{(a+b\sqrt{x})^{16}}{24ax^{12}}\right)}{25a}-\frac{(a+b\sqrt{x})^{16}}{25ax^{25/2}}\right)}{13a}-\frac{(a+b\sqrt{x})^{16}}{26ax^{13}}\right)$$

$$\Big\downarrow \; 55$$

$$2\left(-\frac{5b\left(-\frac{9b\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\int\frac{(a+b\sqrt{x})^{15}}{x^{21/2}}d\sqrt{x}}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)}{23a}-\frac{(a+b\sqrt{x})^{16}}{23ax^{23/2}}\right)}{3a}-\frac{(a+b\sqrt{x})^{16}}{24ax^{12}}\right)}{25a}-\frac{(a+b\sqrt{x})^{16}}{25ax^{25/2}}\right)}{13a}-\frac{(a+}{26}\right.$$

$\downarrow$ 55

$$\Bigg(\Bigg(5b\left(-\frac{9b\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\int\frac{(a+b\sqrt{x})^{15}}{x^{10}}d\sqrt{x}}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)}{23a}-\frac{(a+b\sqrt{x})^{16}}{23ax^{23/2}}\right)}{3a}-\frac{(a+b\sqrt{x})^{16}}{24ax^{12}}\right)}{25a}-\frac{(a+b}{25ax}$$

$\downarrow$ 55

$$9b\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\int\frac{(a+b\sqrt{x})^{15}}{x^{19/2}}d\sqrt{x}}{19a}-\frac{(a+b\sqrt{x})^{16}}{19ax^{19/2}}\right)}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+b\sqrt{x})^{16}}{22ax^{11}}\right)}{23a}-\frac{(a+b\sqrt{x})^{16}}{23ax^{23/2}}\right)}{3a}-\frac{(a+}{2}$$

$$\Big\downarrow 55$$

$$\left(\left(\left(b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\int\frac{\left(a+b\sqrt{x}\right)^{15}}{x^9}d\sqrt{x}}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{22ax^{11}}\right)}{23a}-\right.\right.\right.\right.$$

$\downarrow$ 55

$$7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(-\frac{b\int\frac{(a+b\sqrt{x})^{15}}{x^{17/2}}d\sqrt{x}}{17a}-\frac{(a+b\sqrt{x})^{16}}{17ax^{17/2}}\right)}{9a}-\frac{(a+b\sqrt{x})^{16}}{18ax^{9}}\right)}{19a}-\frac{(a+b\sqrt{x})^{16}}{19ax^{19/2}}\right)}{5a}-\frac{(a+b\sqrt{x})^{16}}{20ax^{10}}\right)}{21a}-\frac{(a+b\sqrt{x})^{1}}{21ax^{21/2}}\right)}{11a}\right.$$

↓ 48

$$\Bigg(\Bigg(\Bigg(\Bigg(b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(\frac{b\left(a+b\sqrt{x}\right)^{16}}{272a^2x^8}-\frac{\left(a+b\sqrt{x}\right)^{16}}{17ax^{17/2}}\right)}{9a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{18ax^9}\right)}{19a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{19ax^{19/2}}\right)}{5a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{20ax^{10}}\right)}{21a}-\frac{\left(a+b\sqrt{x}\right)^{16}}{21ax^{21/2}}\right)}{11a}-\frac{(a+}{2}\cdots\right)}{23a}\cdots$$

input
```
Int[(a + b*Sqrt[x])^15/x^14,x]
```

output
```
2*((-5*b*((-9*b*(-1/3*(b*((-7*b*((-3*b*((-5*b*(-1/5*(b*((-3*b*(-1/9*(b*(-1
/17*(a + b*Sqrt[x])^16/(a*x^(17/2)) + (b*(a + b*Sqrt[x])^16)/(272*a^2*x^8)
))/a - (a + b*Sqrt[x])^16/(18*a*x^9)))/(19*a) - (a + b*Sqrt[x])^16/(19*a*x
^(19/2))))/a - (a + b*Sqrt[x])^16/(20*a*x^10)))/(21*a) - (a + b*Sqrt[x])^1
6/(21*a*x^(21/2))))/(11*a) - (a + b*Sqrt[x])^16/(22*a*x^11)))/(23*a) - (a
+ b*Sqrt[x])^16/(23*a*x^(23/2))))/a - (a + b*Sqrt[x])^16/(24*a*x^12)))/(25
*a) - (a + b*Sqrt[x])^16/(25*a*x^(25/2))))/(13*a) - (a + b*Sqrt[x])^16/(26
*a*x^13))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 23.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.62

method	result
derivativedivides	$-\frac{12870a^8b^7}{19x^{\frac{19}{2}}}-\frac{3003a^5b^{10}}{8x^8}-\frac{35a^{13}b^2}{4x^{12}}-\frac{2b^{15}}{11x^{\frac{11}{2}}}-\frac{1001a^9b^6}{2x^{10}}-\frac{715a^7b^8}{x^9}-\frac{1365a^{11}b^4}{11x^{11}}-\frac{10010a^6b^9}{17x^{\frac{17}{2}}}-\frac{65a}{}$
default	$-\frac{12870a^8b^7}{19x^{\frac{19}{2}}}-\frac{3003a^5b^{10}}{8x^8}-\frac{35a^{13}b^2}{4x^{12}}-\frac{2b^{15}}{11x^{\frac{11}{2}}}-\frac{1001a^9b^6}{2x^{10}}-\frac{715a^7b^8}{x^9}-\frac{1365a^{11}b^4}{11x^{11}}-\frac{10010a^6b^9}{17x^{\frac{17}{2}}}-\frac{65a}{}$
orering	$-\frac{(241442500b^{28}x^{14}-2828220300a^2b^{26}x^{13}+16174233075a^4b^{24}x^{12}-58649365775a^6b^{22}x^{11}+149144490495a^8b^{20}x^{10}}{}$
trager	$\frac{(-1+x)(88a^{14}x^{12}+10010a^{12}b^2x^{12}+141960a^{10}b^4x^{12}+572572a^8b^6x^{12}+817960a^6b^8x^{12}+429429a^4b^{10}x^{12}+74360a^2b^{1}}{}$

input
```
int((a+b*x^(1/2))^15/x^14,x,method=_RETURNVERBOSE)
```

output
```
-12870/19*a^8*b^7/x^(19/2)-3003/8*a^5*b^10/x^8-35/4*a^13*b^2/x^12-2/11*b^1
5/x^(11/2)-1001/2*a^9*b^6/x^10-715*a^7*b^8/x^9-1365/11*a^11*b^4/x^11-10010
/17*a^6*b^9/x^(17/2)-65*a^3*b^12/x^7-6/5*a^14*b/x^(25/2)-5/2*a*b^14/x^6-91
0/23*a^12*b^3/x^(23/2)-182*a^4*b^11/x^(15/2)-1/13*a^15/x^13-286*a^10*b^5/x
^(21/2)-210/13*a^2*b^13/x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.62

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx = -\frac{106234700\,ab^{14}x^7+2762102200\,a^3b^{12}x^6+15951140205\,a^5b^{10}x^5+30383124200\,a^7b^8x^4+21268186940}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^14,x, algorithm="fricas")
```

output
```
-1/42493880*(106234700*a*b^14*x^7 + 2762102200*a^3*b^12*x^6 + 15951140205*
a^5*b^10*x^5 + 30383124200*a^7*b^8*x^4 + 21268186940*a^9*b^6*x^3 + 5273104
200*a^11*b^4*x^2 + 371821450*a^13*b^2*x + 3268760*a^15 + 16*(482885*b^15*x
^7 + 42902475*a^2*b^13*x^6 + 483367885*a^4*b^11*x^5 + 1563837275*a^6*b^9*x
^4 + 1799000775*a^8*b^7*x^3 + 759578105*a^10*b^5*x^2 + 105079975*a^12*b^3*
x + 3187041*a^14*b)*sqrt(x))/x^13
```

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx = -\frac{a^{15}}{13x^{13}} - \frac{6a^{14}b}{5x^{\frac{25}{2}}} - \frac{35a^{13}b^2}{4x^{12}} - \frac{910a^{12}b^3}{23x^{\frac{23}{2}}} - \frac{1365a^{11}b^4}{11x^{11}} - \frac{286a^{10}b^5}{x^{\frac{21}{2}}} - \frac{1001a^9b^6}{2x^{10}} - \frac{12870a^8b^7}{19x^{\frac{19}{2}}} - \frac{715a^7b^8}{x^9} - \frac{10010a^6b^9}{17x^{\frac{17}{2}}} - \frac{3003a^5b^{10}}{8x^8} - \frac{182a^4b^{11}}{x^{\frac{15}{2}}} - \frac{65a^3b^{12}}{x^7} - \frac{210a^2b^{13}}{13x^{\frac{13}{2}}} - \frac{5ab^{14}}{2x^6} - \frac{2b^{15}}{11x^{\frac{11}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**14,x)
```

output
```
-a**15/(13*x**13) - 6*a**14*b/(5*x**(25/2)) - 35*a**13*b**2/(4*x**12) - 91
0*a**12*b**3/(23*x**(23/2)) - 1365*a**11*b**4/(11*x**11) - 286*a**10*b**5/
x**(21/2) - 1001*a**9*b**6/(2*x**10) - 12870*a**8*b**7/(19*x**(19/2)) - 71
5*a**7*b**8/x**9 - 10010*a**6*b**9/(17*x**(17/2)) - 3003*a**5*b**10/(8*x**
8) - 182*a**4*b**11/x**(15/2) - 65*a**3*b**12/x**7 - 210*a**2*b**13/(13*x*
*(13/2)) - 5*a*b**14/(2*x**6) - 2*b**15/(11*x**(11/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.62

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx =$$

$$-\frac{7726160\,b^{15}x^{\frac{15}{2}} + 106234700\,ab^{14}x^7 + 686439600\,a^2b^{13}x^{\frac{13}{2}} + 2762102200\,a^3b^{12}x^6 + 7733886160\,a^4b^{11}x}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^14,x, algorithm="maxima")
```

output
```
-1/42493880*(7726160*b^15*x^(15/2) + 106234700*a*b^14*x^7 + 686439600*a^2*
b^13*x^(13/2) + 2762102200*a^3*b^12*x^6 + 7733886160*a^4*b^11*x^(11/2) + 1
5951140205*a^5*b^10*x^5 + 25021396400*a^6*b^9*x^(9/2) + 30383124200*a^7*b^
8*x^4 + 28784012400*a^8*b^7*x^(7/2) + 21268186940*a^9*b^6*x^3 + 1215324968
0*a^10*b^5*x^(5/2) + 5273104200*a^11*b^4*x^2 + 1681279600*a^12*b^3*x^(3/2)
 + 371821450*a^13*b^2*x + 50992656*a^14*b*sqrt(x) + 3268760*a^15)/x^13
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{14}}\,dx = -\frac{7726160\,b^{15}x^{\frac{15}{2}} + 106234700\,ab^{14}x^7 + 686439600\,a^2b^{13}x^{\frac{13}{2}} + 2762102200\,a^3b^{12}x^6 + 7733886160\,a^4b^{11}x}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^14,x, algorithm="giac")
```

output
```
-1/42493880*(7726160*b^15*x^(15/2) + 106234700*a*b^14*x^7 + 686439600*a^2*
b^13*x^(13/2) + 2762102200*a^3*b^12*x^6 + 7733886160*a^4*b^11*x^(11/2) + 1
5951140205*a^5*b^10*x^5 + 25021396400*a^6*b^9*x^(9/2) + 30383124200*a^7*b^
8*x^4 + 28784012400*a^8*b^7*x^(7/2) + 21268186940*a^9*b^6*x^3 + 1215324968
0*a^10*b^5*x^(5/2) + 5273104200*a^11*b^4*x^2 + 1681279600*a^12*b^3*x^(3/2)
 + 371821450*a^13*b^2*x + 50992656*a^14*b*sqrt(x) + 3268760*a^15)/x^13
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{14}}\,dx = -\frac{\frac{a^{15}}{13} + \frac{2\,b^{15}\,x^{15/2}}{11} + \frac{35\,a^{13}\,b^2\,x}{4} + \frac{6\,a^{14}\,b\,\sqrt{x}}{5} + \frac{5\,a\,b^{14}\,x^7}{2} + \frac{1365\,a^{11}\,b^4\,x^2}{11} + \frac{1001\,a^9\,b^6\,x^3}{2} + 715\,a^7\,b^8\,x^4 + \frac{3003\,a^5\,b^{10}\,x^5}{8}}{}$$

input
```
int((a + b*x^(1/2))^15/x^14,x)
```

output
```
-(a^15/13 + (2*b^15*x^(15/2))/11 + (35*a^13*b^2*x)/4 + (6*a^14*b*x^(1/2))/
5 + (5*a*b^14*x^7)/2 + (1365*a^11*b^4*x^2)/11 + (1001*a^9*b^6*x^3)/2 + 715
*a^7*b^8*x^4 + (3003*a^5*b^10*x^5)/8 + (910*a^12*b^3*x^(3/2))/23 + 65*a^3*
b^12*x^6 + 286*a^10*b^5*x^(5/2) + (12870*a^8*b^7*x^(7/2))/19 + (10010*a^6*
b^9*x^(9/2))/17 + 182*a^4*b^11*x^(11/2) + (210*a^2*b^13*x^(13/2))/13)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.69

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{14}}\,dx = \frac{-3268760\sqrt{x}\,a^{15}-371821450\sqrt{x}\,a^{13}b^2x-5273104200\sqrt{x}\,a^{11}b^4x^2-21268186940\sqrt{x}\,a^9b^6x^3-3038312}{}$$

input
```
int((a+b*x^(1/2))^15/x^14,x)
```

output
```
( - 3268760*sqrt(x)*a**15 - 371821450*sqrt(x)*a**13*b**2*x - 5273104200*sq
rt(x)*a**11*b**4*x**2 - 21268186940*sqrt(x)*a**9*b**6*x**3 - 30383124200*s
qrt(x)*a**7*b**8*x**4 - 15951140205*sqrt(x)*a**5*b**10*x**5 - 2762102200*s
qrt(x)*a**3*b**12*x**6 - 106234700*sqrt(x)*a*b**14*x**7 - 50992656*a**14*b
*x - 1681279600*a**12*b**3*x**2 - 12153249680*a**10*b**5*x**3 - 2878401240
0*a**8*b**7*x**4 - 25021396400*a**6*b**9*x**5 - 7733886160*a**4*b**11*x**6
 - 686439600*a**2*b**13*x**7 - 7726160*b**15*x**8)/(42493880*sqrt(x)*x**13
)
```

3.78 $\int \frac{(a+b\sqrt{x})^{15}}{x^{15}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 211

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{15}}\,dx = -\frac{a^{15}}{14x^{14}} - \frac{10a^{14}b}{9x^{27/2}} - \frac{105a^{13}b^2}{13x^{13}} - \frac{182a^{12}b^3}{5x^{25/2}} - \frac{455a^{11}b^4}{4x^{12}} - \frac{6006a^{10}b^5}{23x^{23/2}} - \frac{455a^9b^6}{x^{11}} - \frac{4290a^8b^7}{7x^{21/2}} - \frac{1287a^7b^8}{2x^{10}} - \frac{10010a^6b^9}{19x^{19/2}} - \frac{1001a^5b^{10}}{3x^9} - \frac{2730a^4b^{11}}{17x^{17/2}} - \frac{455a^3b^{12}}{8x^8} - \frac{14a^2b^{13}}{x^{15/2}} - \frac{15ab^{14}}{7x^7} - \frac{2b^{15}}{13x^{13/2}}$$

output

```
-1/14*a^15/x^14-10/9*a^14*b/x^(27/2)-105/13*a^13*b^2/x^13-182/5*a^12*b^3/x
^(25/2)-455/4*a^11*b^4/x^12-6006/23*a^10*b^5/x^(23/2)-455*a^9*b^6/x^11-429
0/7*a^8*b^7/x^(21/2)-1287/2*a^7*b^8/x^10-10010/19*a^6*b^9/x^(19/2)-1001/3*
a^5*b^10/x^9-2730/17*a^4*b^11/x^(17/2)-455/8*a^3*b^12/x^8-14*a^2*b^13/x^(1
5/2)-15/7*a*b^14/x^7-2/13*b^15/x^(13/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{15}}\,dx = \frac{-17383860a^{15}-270415600a^{14}b\sqrt{x}-1965713400a^{13}b^2x-8858815056a^{12}b^3x^{3/2}-27683797050a^{11}b^4x^2}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^15,x]
```

output
```
(-17383860*a^15 - 270415600*a^14*b*Sqrt[x] - 1965713400*a^13*b^2*x - 88588
15056*a^12*b^3*x^(3/2) - 27683797050*a^11*b^4*x^2 - 63552368880*a^10*b^5*x
^(5/2) - 110735188200*a^9*b^6*x^3 - 149153518800*a^8*b^7*x^(7/2) - 1566111
94740*a^7*b^8*x^4 - 128219691600*a^6*b^9*x^(9/2) - 81205804680*a^5*b^10*x^
5 - 39083007600*a^4*b^11*x^(11/2) - 13841898525*a^3*b^12*x^6 - 3407236560*
a^2*b^13*x^(13/2) - 521515800*a*b^14*x^7 - 37442160*b^15*x^(15/2))/(243374
040*x^14)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{15}}\,dx$$

$$\Big\downarrow \; 798$$

$$2\int \frac{(a+b\sqrt{x})^{15}}{x^{29/2}}d\sqrt{x}$$

$$\Big\downarrow \; 53$$

$$2\int\left(\frac{a^{15}}{x^{29/2}}+\frac{15ba^{14}}{x^{14}}+\frac{105b^2a^{13}}{x^{27/2}}+\frac{455b^3a^{12}}{x^{13}}+\frac{1365b^4a^{11}}{x^{25/2}}+\frac{3003b^5a^{10}}{x^{12}}+\frac{5005b^6a^9}{x^{23/2}}+\frac{6435b^7a^8}{x^{11}}+\frac{6435b^8a^7}{x^{21/2}}+\right.$$

$\downarrow$ 2009

$$2\left(-\frac{a^{15}}{28x^{14}}-\frac{5a^{14}b}{9x^{27/2}}-\frac{105a^{13}b^2}{26x^{13}}-\frac{91a^{12}b^3}{5x^{25/2}}-\frac{455a^{11}b^4}{8x^{12}}-\frac{3003a^{10}b^5}{23x^{23/2}}-\frac{455a^9b^6}{2x^{11}}-\frac{2145a^8b^7}{7x^{21/2}}-\frac{1287a^7b^8}{4x^{10}}-\frac{5005}{19x}\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^15,x]
```

output
```
2*(-1/28*a^15/x^14 - (5*a^14*b)/(9*x^(27/2)) - (105*a^13*b^2)/(26*x^13) -
(91*a^12*b^3)/(5*x^(25/2)) - (455*a^11*b^4)/(8*x^12) - (3003*a^10*b^5)/(23
*x^(23/2)) - (455*a^9*b^6)/(2*x^11) - (2145*a^8*b^7)/(7*x^(21/2)) - (1287*
a^7*b^8)/(4*x^10) - (5005*a^6*b^9)/(19*x^(19/2)) - (1001*a^5*b^10)/(6*x^9)
 - (1365*a^4*b^11)/(17*x^(17/2)) - (455*a^3*b^12)/(16*x^8) - (7*a^2*b^13)/
x^(15/2) - (15*a*b^14)/(14*x^7) - b^15/(13*x^(13/2)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.62 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{a^{15}}{14x^{14}}-\frac{10a^{14}b}{9x^{\frac{27}{2}}}-\frac{105a^{13}b^{2}}{13x^{13}}-\frac{182a^{12}b^{3}}{5x^{\frac{25}{2}}}-\frac{455a^{11}b^{4}}{4x^{12}}-\frac{6006a^{10}b^{5}}{23x^{\frac{23}{2}}}-\frac{455a^{9}b^{6}}{x^{11}}-\frac{4290a^{8}b^{7}}{7x^{\frac{21}{2}}}-\frac{1287a^{7}b^{8}}{2x^{10}}$
default	$-\frac{a^{15}}{14x^{14}}-\frac{10a^{14}b}{9x^{\frac{27}{2}}}-\frac{105a^{13}b^{2}}{13x^{13}}-\frac{182a^{12}b^{3}}{5x^{\frac{25}{2}}}-\frac{455a^{11}b^{4}}{4x^{12}}-\frac{6006a^{10}b^{5}}{23x^{\frac{23}{2}}}-\frac{455a^{9}b^{6}}{x^{11}}-\frac{4290a^{8}b^{7}}{7x^{\frac{21}{2}}}-\frac{1287a^{7}b^{8}}{2x^{10}}$
orering	$-\frac{(77558760b^{28}x^{14}-943074405a^{2}b^{26}x^{13}+5505144645a^{4}b^{24}x^{12}-20214204465a^{6}b^{22}x^{11}+51822798105a^{8}b^{20}x^{10}-977}{}$
trager	$\frac{(-1+x)(156a^{14}x^{13}+17640a^{12}b^{2}x^{13}+248430a^{10}b^{4}x^{13}+993720a^{8}b^{6}x^{13}+1405404a^{6}b^{8}x^{13}+728728a^{4}b^{10}x^{13}+124215a}{}$

input
```
int((a+b*x^(1/2))^15/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/14*a^15/x^14-10/9*a^14*b/x^(27/2)-105/13*a^13*b^2/x^13-182/5*a^12*b^3/x
^(25/2)-455/4*a^11*b^4/x^12-6006/23*a^10*b^5/x^(23/2)-455*a^9*b^6/x^11-429
0/7*a^8*b^7/x^(21/2)-1287/2*a^7*b^8/x^10-10010/19*a^6*b^9/x^(19/2)-1001/3*
a^5*b^10/x^9-2730/17*a^4*b^11/x^(17/2)-455/8*a^3*b^12/x^8-14*a^2*b^13/x^(1
5/2)-15/7*a*b^14/x^7-2/13*b^15/x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx = -\frac{521515800\,ab^{14}x^{7}+13841898525\,a^{3}b^{12}x^{6}+81205804680\,a^{5}b^{10}x^{5}+156611194740\,a^{7}b^{8}x^{4}+11073518}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^15,x, algorithm="fricas")
```

output
```
-1/243374040*(521515800*a*b^14*x^7 + 13841898525*a^3*b^12*x^6 + 8120580468
0*a^5*b^10*x^5 + 156611194740*a^7*b^8*x^4 + 110735188200*a^9*b^6*x^3 + 276
83797050*a^11*b^4*x^2 + 1965713400*a^13*b^2*x + 17383860*a^15 + 16*(234013
5*b^15*x^7 + 212952285*a^2*b^13*x^6 + 2442687975*a^4*b^11*x^5 + 8013730725
*a^6*b^9*x^4 + 9322094925*a^8*b^7*x^3 + 3972023055*a^10*b^5*x^2 + 55367594
1*a^12*b^3*x + 16900975*a^14*b)*sqrt(x))/x^14
```

Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx = -\frac{a^{15}}{14x^{14}} - \frac{10a^{14}b}{9x^{\frac{27}{2}}} - \frac{105a^{13}b^2}{13x^{13}} - \frac{182a^{12}b^3}{5x^{\frac{25}{2}}} - \frac{455a^{11}b^4}{4x^{12}} - \frac{6006a^{10}b^5}{23x^{\frac{23}{2}}} - \frac{455a^9b^6}{x^{11}} - \frac{4290a^8b^7}{7x^{\frac{21}{2}}} - \frac{1287a^7b^8}{2x^{10}} - \frac{10010a^6b^9}{19x^{\frac{19}{2}}} - \frac{1001a^5b^{10}}{3x^9} - \frac{2730a^4b^{11}}{17x^{\frac{17}{2}}} - \frac{455a^3b^{12}}{8x^8} - \frac{14a^2b^{13}}{x^{\frac{15}{2}}} - \frac{15ab^{14}}{7x^7} - \frac{2b^{15}}{13x^{\frac{13}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**15,x)
```

output
```
-a**15/(14*x**14) - 10*a**14*b/(9*x**(27/2)) - 105*a**13*b**2/(13*x**13) -
 182*a**12*b**3/(5*x**(25/2)) - 455*a**11*b**4/(4*x**12) - 6006*a**10*b**5
/(23*x**(23/2)) - 455*a**9*b**6/x**11 - 4290*a**8*b**7/(7*x**(21/2)) - 128
7*a**7*b**8/(2*x**10) - 10010*a**6*b**9/(19*x**(19/2)) - 1001*a**5*b**10/(
3*x**9) - 2730*a**4*b**11/(17*x**(17/2)) - 455*a**3*b**12/(8*x**8) - 14*a*
*2*b**13/x**(15/2) - 15*a*b**14/(7*x**7) - 2*b**15/(13*x**(13/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx =$$

$$-\frac{37442160\,b^{15}x^{\frac{15}{2}} + 521515800\,ab^{14}x^7 + 3407236560\,a^2b^{13}x^{\frac{13}{2}} + 13841898525\,a^3b^{12}x^6 + 39083007600\,a}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^15,x, algorithm="maxima")
```

output
```
-1/243374040*(37442160*b^15*x^(15/2) + 521515800*a*b^14*x^7 + 3407236560*a
^2*b^13*x^(13/2) + 13841898525*a^3*b^12*x^6 + 39083007600*a^4*b^11*x^(11/2
) + 81205804680*a^5*b^10*x^5 + 128219691600*a^6*b^9*x^(9/2) + 156611194740
*a^7*b^8*x^4 + 149153518800*a^8*b^7*x^(7/2) + 110735188200*a^9*b^6*x^3 + 6
3552368880*a^10*b^5*x^(5/2) + 27683797050*a^11*b^4*x^2 + 8858815056*a^12*b
^3*x^(3/2) + 1965713400*a^13*b^2*x + 270415600*a^14*b*sqrt(x) + 17383860*a
^15)/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx =$$
$$-\frac{37442160\,b^{15}x^{\frac{15}{2}}+521515800\,ab^{14}x^{7}+3407236560\,a^{2}b^{13}x^{\frac{13}{2}}+13841898525\,a^{3}b^{12}x^{6}+39083007600\,a$$

input
```
integrate((a+b*x^(1/2))^15/x^15,x, algorithm="giac")
```

output
```
-1/243374040*(37442160*b^15*x^(15/2) + 521515800*a*b^14*x^7 + 3407236560*a
^2*b^13*x^(13/2) + 13841898525*a^3*b^12*x^6 + 39083007600*a^4*b^11*x^(11/2
) + 81205804680*a^5*b^10*x^5 + 128219691600*a^6*b^9*x^(9/2) + 156611194740
*a^7*b^8*x^4 + 149153518800*a^8*b^7*x^(7/2) + 110735188200*a^9*b^6*x^3 + 6
3552368880*a^10*b^5*x^(5/2) + 27683797050*a^11*b^4*x^2 + 8858815056*a^12*b
^3*x^(3/2) + 1965713400*a^13*b^2*x + 270415600*a^14*b*sqrt(x) + 17383860*a
^15)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx = -\frac{\frac{a^{15}}{14}+\frac{2\,b^{15}\,x^{15/2}}{13}+\frac{105\,a^{13}\,b^2\,x}{13}+\frac{10\,a^{14}\,b\,\sqrt{x}}{9}+\frac{15\,a\,b^{14}\,x^7}{7}+\frac{455\,a^{11}\,b^4\,x^2}{4}+455\,a^9\,b^6\,x^3+\frac{1287\,a^7\,b^8\,x^4}{2}+\frac{1001\,a^5\,b^{10}\,x^5}{3}+\dots}{\dots}$$

input
```
int((a + b*x^(1/2))^15/x^15,x)
```

output
```
-(a^15/14 + (2*b^15*x^(15/2))/13 + (105*a^13*b^2*x)/13 + (10*a^14*b*x^(1/2
))/9 + (15*a*b^14*x^7)/7 + (455*a^11*b^4*x^2)/4 + 455*a^9*b^6*x^3 + (1287*
a^7*b^8*x^4)/2 + (1001*a^5*b^10*x^5)/3 + (182*a^12*b^3*x^(3/2))/5 + (455*a
^3*b^12*x^6)/8 + (6006*a^10*b^5*x^(5/2))/23 + (4290*a^8*b^7*x^(7/2))/7 + (
10010*a^6*b^9*x^(9/2))/19 + (2730*a^4*b^11*x^(11/2))/17 + 14*a^2*b^13*x^(1
3/2))/x^14
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{15}}\,dx = \frac{-17383860\sqrt{x}\,a^{15}-1965713400\sqrt{x}\,a^{13}b^2x-27683797050\sqrt{x}\,a^{11}b^4x^2-110735188200\sqrt{x}\,a^9b^6x^3-156\dots}{\dots}$$

input
```
int((a+b*x^(1/2))^15/x^15,x)
```

output
```
( - 17383860*sqrt(x)*a**15 - 1965713400*sqrt(x)*a**13*b**2*x - 27683797050
*sqrt(x)*a**11*b**4*x**2 - 110735188200*sqrt(x)*a**9*b**6*x**3 - 156611194
740*sqrt(x)*a**7*b**8*x**4 - 81205804680*sqrt(x)*a**5*b**10*x**5 - 1384189
8525*sqrt(x)*a**3*b**12*x**6 - 521515800*sqrt(x)*a*b**14*x**7 - 270415600*
a**14*b*x - 8858815056*a**12*b**3*x**2 - 63552368880*a**10*b**5*x**3 - 149
153518800*a**8*b**7*x**4 - 128219691600*a**6*b**9*x**5 - 39083007600*a**4*
b**11*x**6 - 3407236560*a**2*b**13*x**7 - 37442160*b**15*x**8)/(243374040*
sqrt(x)*x**14)
```

3.79 $\int \frac{(a+b\sqrt{x})^{15}}{x^{16}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 211

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{16}}\,dx = -\frac{a^{15}}{15x^{15}} - \frac{30a^{14}b}{29x^{29/2}} - \frac{15a^{13}b^2}{2x^{14}} - \frac{910a^{12}b^3}{27x^{27/2}} - \frac{105a^{11}b^4}{x^{13}} - \frac{6006a^{10}b^5}{25x^{25/2}} - \frac{5005a^9b^6}{12x^{12}} - \frac{12870a^8b^7}{23x^{23/2}} - \frac{585a^7b^8}{x^{11}} - \frac{1430a^6b^9}{3x^{21/2}} - \frac{3003a^5b^{10}}{10x^{10}} - \frac{2730a^4b^{11}}{19x^{19/2}} - \frac{455a^3b^{12}}{9x^9} - \frac{210a^2b^{13}}{17x^{17/2}} - \frac{15ab^{14}}{8x^8} - \frac{2b^{15}}{15x^{15/2}}$$

output
```
-1/15*a^15/x^15-30/29*a^14*b/x^(29/2)-15/2*a^13*b^2/x^14-910/27*a^12*b^3/x
^(27/2)-105*a^11*b^4/x^13-6006/25*a^10*b^5/x^(25/2)-5005/12*a^9*b^6/x^12-1
2870/23*a^8*b^7/x^(23/2)-585*a^7*b^8/x^11-1430/3*a^6*b^9/x^(21/2)-3003/10*
a^5*b^10/x^10-2730/19*a^4*b^11/x^(19/2)-455/9*a^3*b^12/x^9-210/17*a^2*b^13
/x^(17/2)-15/8*a*b^14/x^8-2/15*b^15/x^(15/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx = \frac{-77558760a^{15}-1203498000a^{14}b\sqrt{x}-8725360500a^{13}b^2x-39210262000a^{12}b^3x^{3/2}-122155047000a^{11}b}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^16,x]
```

output
```
(-77558760*a^15 - 1203498000*a^14*b*Sqrt[x] - 8725360500*a^13*b^2*x - 3921
0262000*a^12*b^3*x^(3/2) - 122155047000*a^11*b^4*x^2 - 279490747536*a^10*b
^5*x^(5/2) - 485226992250*a^9*b^6*x^3 - 650987766000*a^8*b^7*x^(7/2) - 680
578119000*a^7*b^8*x^4 - 554545134000*a^6*b^9*x^(9/2) - 349363434420*a^5*b^
10*x^5 - 167159538000*a^4*b^11*x^(11/2) - 58815393000*a^3*b^12*x^6 - 14371
182000*a^2*b^13*x^(13/2) - 2181340125*a*b^14*x^7 - 155117520*b^15*x^(15/2)
)/(1163381400*x^15)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx$$

$$\Big\downarrow \ 798$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{31/2}}d\sqrt{x}$$

$$\Big\downarrow \ 53$$

$$2\int\left(\frac{a^{15}}{x^{31/2}}+\frac{15ba^{14}}{x^{15}}+\frac{105b^2a^{13}}{x^{29/2}}+\frac{455b^3a^{12}}{x^{14}}+\frac{1365b^4a^{11}}{x^{27/2}}+\frac{3003b^5a^{10}}{x^{13}}+\frac{5005b^6a^9}{x^{25/2}}+\frac{6435b^7a^8}{x^{12}}+\frac{6435b^8a^7}{x^{23/2}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{15}}{30x^{15}}-\frac{15a^{14}b}{29x^{29/2}}-\frac{15a^{13}b^2}{4x^{14}}-\frac{455a^{12}b^3}{27x^{27/2}}-\frac{105a^{11}b^4}{2x^{13}}-\frac{3003a^{10}b^5}{25x^{25/2}}-\frac{5005a^9b^6}{24x^{12}}-\frac{6435a^8b^7}{23x^{23/2}}-\frac{585a^7b^8}{2x^{11}}-\frac{715}{3x}\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^16,x]
```

output
```
2*(-1/30*a^15/x^15 - (15*a^14*b)/(29*x^(29/2)) - (15*a^13*b^2)/(4*x^14) -
(455*a^12*b^3)/(27*x^(27/2)) - (105*a^11*b^4)/(2*x^13) - (3003*a^10*b^5)/(
25*x^(25/2)) - (5005*a^9*b^6)/(24*x^12) - (6435*a^8*b^7)/(23*x^(23/2)) - (
585*a^7*b^8)/(2*x^11) - (715*a^6*b^9)/(3*x^(21/2)) - (3003*a^5*b^10)/(20*x
^10) - (1365*a^4*b^11)/(19*x^(19/2)) - (455*a^3*b^12)/(18*x^9) - (105*a^2*
b^13)/(17*x^(17/2)) - (15*a*b^14)/(16*x^8) - b^15/(15*x^(15/2)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.72 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{a^{15}}{15x^{15}}-\frac{30a^{14}b}{29x^{\frac{29}{2}}}-\frac{15a^{13}b^{2}}{2x^{14}}-\frac{910a^{12}b^{3}}{27x^{\frac{27}{2}}}-\frac{105a^{11}b^{4}}{x^{13}}-\frac{6006a^{10}b^{5}}{25x^{\frac{25}{2}}}-\frac{5005a^{9}b^{6}}{12x^{12}}-\frac{12870a^{8}b^{7}}{23x^{\frac{23}{2}}}-\frac{585a^{7}b^{8}}{x^{11}}$
default	$-\frac{a^{15}}{15x^{15}}-\frac{30a^{14}b}{29x^{\frac{29}{2}}}-\frac{15a^{13}b^{2}}{2x^{14}}-\frac{910a^{12}b^{3}}{27x^{\frac{27}{2}}}-\frac{105a^{11}b^{4}}{x^{13}}-\frac{6006a^{10}b^{5}}{25x^{\frac{25}{2}}}-\frac{5005a^{9}b^{6}}{12x^{12}}-\frac{12870a^{8}b^{7}}{23x^{\frac{23}{2}}}-\frac{585a^{7}b^{8}}{x^{11}}$
orering	$-\frac{(4798948275b^{28}x^{14}-59846658375a^{2}b^{26}x^{13}+354823433055a^{4}b^{24}x^{12}-1316246744475a^{6}b^{22}x^{11}+3398132958675a^{8}}{}$
trager	Expression too large to display

input
```
int((a+b*x^(1/2))^15/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^15/x^15-30/29*a^14*b/x^(29/2)-15/2*a^13*b^2/x^14-910/27*a^12*b^3/x
^(27/2)-105*a^11*b^4/x^13-6006/25*a^10*b^5/x^(25/2)-5005/12*a^9*b^6/x^12-1
2870/23*a^8*b^7/x^(23/2)-585*a^7*b^8/x^11-1430/3*a^6*b^9/x^(21/2)-3003/10*
a^5*b^10/x^10-2730/19*a^4*b^11/x^(19/2)-455/9*a^3*b^12/x^9-210/17*a^2*b^13
/x^(17/2)-15/8*a*b^14/x^8-2/15*b^15/x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx = -\frac{2181340125\,ab^{14}x^{7}+58815393000\,a^{3}b^{12}x^{6}+349363434420\,a^{5}b^{10}x^{5}+680578119000\,a^{7}b^{8}x^{4}+4852269}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^16,x, algorithm="fricas")
```

output
```
-1/1163381400*(2181340125*a*b^14*x^7 + 58815393000*a^3*b^12*x^6 + 34936343
4420*a^5*b^10*x^5 + 680578119000*a^7*b^8*x^4 + 485226992250*a^9*b^6*x^3 +
122155047000*a^11*b^4*x^2 + 8725360500*a^13*b^2*x + 77558760*a^15 + 16*(96
94845*b^15*x^7 + 898198875*a^2*b^13*x^6 + 10447471125*a^4*b^11*x^5 + 34659
070875*a^6*b^9*x^4 + 40686735375*a^8*b^7*x^3 + 17468171721*a^10*b^5*x^2 +
2450641375*a^12*b^3*x + 75218625*a^14*b)*sqrt(x))/x^15
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{16}}\,dx = -\frac{a^{15}}{15x^{15}} - \frac{30a^{14}b}{29x^{\frac{29}{2}}} - \frac{15a^{13}b^2}{2x^{14}} - \frac{910a^{12}b^3}{27x^{\frac{27}{2}}} - \frac{105a^{11}b^4}{x^{13}} - \frac{6006a^{10}b^5}{25x^{\frac{25}{2}}} - \frac{5005a^9b^6}{12x^{12}} - \frac{12870a^8b^7}{23x^{\frac{23}{2}}} - \frac{585a^7b^8}{x^{11}} - \frac{1430a^6b^9}{3x^{\frac{21}{2}}} - \frac{3003a^5b^{10}}{10x^{10}} - \frac{2730a^4b^{11}}{19x^{\frac{19}{2}}} - \frac{455a^3b^{12}}{9x^9} - \frac{210a^2b^{13}}{17x^{\frac{17}{2}}} - \frac{15ab^{14}}{8x^8} - \frac{2b^{15}}{15x^{\frac{15}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**16,x)
```

output
```
-a**15/(15*x**15) - 30*a**14*b/(29*x**(29/2)) - 15*a**13*b**2/(2*x**14) -
910*a**12*b**3/(27*x**(27/2)) - 105*a**11*b**4/x**13 - 6006*a**10*b**5/(25
*x**(25/2)) - 5005*a**9*b**6/(12*x**12) - 12870*a**8*b**7/(23*x**(23/2)) -
 585*a**7*b**8/x**11 - 1430*a**6*b**9/(3*x**(21/2)) - 3003*a**5*b**10/(10*
x**10) - 2730*a**4*b**11/(19*x**(19/2)) - 455*a**3*b**12/(9*x**9) - 210*a*
*2*b**13/(17*x**(17/2)) - 15*a*b**14/(8*x**8) - 2*b**15/(15*x**(15/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{16}}\,dx = -\frac{155117520\,b^{15}x^{\frac{15}{2}} + 2181340125\,ab^{14}x^7 + 14371182000\,a^2b^{13}x^{\frac{13}{2}} + 58815393000\,a^3b^{12}x^6 + 1671595380}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^16,x, algorithm="maxima")
```

output
```
-1/1163381400*(155117520*b^15*x^(15/2) + 2181340125*a*b^14*x^7 + 143711820
00*a^2*b^13*x^(13/2) + 58815393000*a^3*b^12*x^6 + 167159538000*a^4*b^11*x^
(11/2) + 349363434420*a^5*b^10*x^5 + 554545134000*a^6*b^9*x^(9/2) + 680578
119000*a^7*b^8*x^4 + 650987766000*a^8*b^7*x^(7/2) + 485226992250*a^9*b^6*x
^3 + 279490747536*a^10*b^5*x^(5/2) + 122155047000*a^11*b^4*x^2 + 392102620
00*a^12*b^3*x^(3/2) + 8725360500*a^13*b^2*x + 1203498000*a^14*b*sqrt(x) +
77558760*a^15)/x^15
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx =$$

$$-\frac{155117520\,b^{15}x^{\frac{15}{2}} + 2181340125\,ab^{14}x^{7} + 14371182000\,a^{2}b^{13}x^{\frac{13}{2}} + 58815393000\,a^{3}b^{12}x^{6} + 1671595380}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^16,x, algorithm="giac")
```

output
```
-1/1163381400*(155117520*b^15*x^(15/2) + 2181340125*a*b^14*x^7 + 143711820
00*a^2*b^13*x^(13/2) + 58815393000*a^3*b^12*x^6 + 167159538000*a^4*b^11*x^
(11/2) + 349363434420*a^5*b^10*x^5 + 554545134000*a^6*b^9*x^(9/2) + 680578
119000*a^7*b^8*x^4 + 650987766000*a^8*b^7*x^(7/2) + 485226992250*a^9*b^6*x
^3 + 279490747536*a^10*b^5*x^(5/2) + 122155047000*a^11*b^4*x^2 + 392102620
00*a^12*b^3*x^(3/2) + 8725360500*a^13*b^2*x + 1203498000*a^14*b*sqrt(x) +
77558760*a^15)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx = -\frac{\frac{a^{15}}{15}+\frac{2\,b^{15}\,x^{15/2}}{15}+\frac{15\,a^{13}\,b^2\,x}{2}+\frac{30\,a^{14}\,b\,\sqrt{x}}{29}+\frac{15\,a\,b^{14}\,x^7}{8}+105\,a^{11}\,b^4\,x^2+\frac{5005\,a^9\,b^6\,x^3}{12}+585\,a^7\,b^8\,x^4+\frac{3003\,a^5\,b^{10}\,x^5}{10}}{}$$

input
```
int((a + b*x^(1/2))^15/x^16,x)
```

output
```
-(a^15/15 + (2*b^15*x^(15/2))/15 + (15*a^13*b^2*x)/2 + (30*a^14*b*x^(1/2))
/29 + (15*a*b^14*x^7)/8 + 105*a^11*b^4*x^2 + (5005*a^9*b^6*x^3)/12 + 585*a
^7*b^8*x^4 + (3003*a^5*b^10*x^5)/10 + (910*a^12*b^3*x^(3/2))/27 + (455*a^3
*b^12*x^6)/9 + (6006*a^10*b^5*x^(5/2))/25 + (12870*a^8*b^7*x^(7/2))/23 + (
1430*a^6*b^9*x^(9/2))/3 + (2730*a^4*b^11*x^(11/2))/19 + (210*a^2*b^13*x^(1
3/2))/17)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{16}}\,dx = \frac{-77558760\sqrt{x}\,a^{15}-8725360500\sqrt{x}\,a^{13}b^2x-122155047000\sqrt{x}\,a^{11}b^4x^2-485226992250\sqrt{x}\,a^9b^6x^3-68}{}$$

input
```
int((a+b*x^(1/2))^15/x^16,x)
```

output
```
( - 77558760*sqrt(x)*a**15 - 8725360500*sqrt(x)*a**13*b**2*x - 12215504700
0*sqrt(x)*a**11*b**4*x**2 - 485226992250*sqrt(x)*a**9*b**6*x**3 - 68057811
9000*sqrt(x)*a**7*b**8*x**4 - 349363434420*sqrt(x)*a**5*b**10*x**5 - 58815
393000*sqrt(x)*a**3*b**12*x**6 - 2181340125*sqrt(x)*a*b**14*x**7 - 1203498
000*a**14*b*x - 39210262000*a**12*b**3*x**2 - 279490747536*a**10*b**5*x**3
 - 650987766000*a**8*b**7*x**4 - 554545134000*a**6*b**9*x**5 - 16715953800
0*a**4*b**11*x**6 - 14371182000*a**2*b**13*x**7 - 155117520*b**15*x**8)/(1
163381400*sqrt(x)*x**15)
```

3.80 $\int \frac{(a+b\sqrt{x})^{15}}{x^{17}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 207

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{17}}\,dx = -\frac{a^{15}}{16x^{16}}-\frac{30a^{14}b}{31x^{31/2}}-\frac{7a^{13}b^2}{x^{15}}-\frac{910a^{12}b^3}{29x^{29/2}}-\frac{195a^{11}b^4}{2x^{14}}-\frac{2002a^{10}b^5}{9x^{27/2}}-\frac{385a^9b^6}{x^{13}}-\frac{2574a^8b^7}{5x^{25/2}}-\frac{2145a^7b^8}{4x^{12}}-\frac{10010a^6b^9}{23x^{23/2}}-\frac{273a^5b^{10}}{x^{11}}-\frac{130a^4b^{11}}{x^{21/2}}-\frac{91a^3b^{12}}{2x^{10}}-\frac{210a^2b^{13}}{19x^{19/2}}-\frac{5ab^{14}}{3x^9}-\frac{2b^{15}}{17x^{17/2}}$$

output

```
-1/16*a^15/x^16-30/31*a^14*b/x^(31/2)-7*a^13*b^2/x^15-910/29*a^12*b^3/x^(2
9/2)-195/2*a^11*b^4/x^14-2002/9*a^10*b^5/x^(27/2)-385*a^9*b^6/x^13-2574/5*
a^8*b^7/x^(25/2)-2145/4*a^7*b^8/x^12-10010/23*a^6*b^9/x^(23/2)-273*a^5*b^1
0/x^11-130*a^4*b^11/x^(21/2)-91/2*a^3*b^12/x^10-210/19*a^2*b^13/x^(19/2)-5
/3*a*b^14/x^9-2/17*b^15/x^(17/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{17}}\,dx = \frac{-300540195a^{15} - 4653525600a^{14}b\sqrt{x} - 33660501840a^{13}b^2x - 150891904800a^{12}b^3x^{3/2} - 468842704200a}{}$$

input
```
Integrate[(a + b*Sqrt[x])^15/x^17,x]
```

output
```
(-300540195*a^15 - 4653525600*a^14*b*Sqrt[x] - 33660501840*a^13*b^2*x - 15
0891904800*a^12*b^3*x^(3/2) - 468842704200*a^11*b^4*x^2 - 1069655947360*a^
10*b^5*x^(5/2) - 1851327601200*a^9*b^6*x^3 - 2475489478176*a^8*b^7*x^(7/2)
 - 2578634873100*a^7*b^8*x^4 - 2092805114400*a^6*b^9*x^(9/2) - 13127595717
60*a^5*b^10*x^5 - 625123605600*a^4*b^11*x^(11/2) - 218793261960*a^3*b^12*x
^6 - 53148160800*a^2*b^13*x^(13/2) - 8014405200*a*b^14*x^7 - 565722720*b^1
5*x^(15/2))/(4808643120*x^16)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^{15}}{x^{17}}\,dx$$

$$\Big\downarrow \; 798$$

$$2\int \frac{(a+b\sqrt{x})^{15}}{x^{33/2}}d\sqrt{x}$$

$$\Big\downarrow \; 53$$

$$2\int\left(\frac{a^{15}}{x^{33/2}}+\frac{15ba^{14}}{x^{16}}+\frac{105b^2a^{13}}{x^{31/2}}+\frac{455b^3a^{12}}{x^{15}}+\frac{1365b^4a^{11}}{x^{29/2}}+\frac{3003b^5a^{10}}{x^{14}}+\frac{5005b^6a^9}{x^{27/2}}+\frac{6435b^7a^8}{x^{13}}+\frac{6435b^8a^7}{x^{25/2}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^{15}}{32x^{16}}-\frac{15a^{14}b}{31x^{31/2}}-\frac{7a^{13}b^2}{2x^{15}}-\frac{455a^{12}b^3}{29x^{29/2}}-\frac{195a^{11}b^4}{4x^{14}}-\frac{1001a^{10}b^5}{9x^{27/2}}-\frac{385a^9b^6}{2x^{13}}-\frac{1287a^8b^7}{5x^{25/2}}-\frac{2145a^7b^8}{8x^{12}}-\frac{5005}{23x}\right.$$

input
```
Int[(a + b*Sqrt[x])^15/x^17,x]
```

output
```
2*(-1/32*a^15/x^16 - (15*a^14*b)/(31*x^(31/2)) - (7*a^13*b^2)/(2*x^15) - (
455*a^12*b^3)/(29*x^(29/2)) - (195*a^11*b^4)/(4*x^14) - (1001*a^10*b^5)/(9
*x^(27/2)) - (385*a^9*b^6)/(2*x^13) - (1287*a^8*b^7)/(5*x^(25/2)) - (2145*
a^7*b^8)/(8*x^12) - (5005*a^6*b^9)/(23*x^(23/2)) - (273*a^5*b^10)/(2*x^11)
 - (65*a^4*b^11)/x^(21/2) - (91*a^3*b^12)/(4*x^10) - (105*a^2*b^13)/(19*x^
(19/2)) - (5*a*b^14)/(6*x^9) - b^15/(17*x^(17/2)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 23.81 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{a^{15}}{16x^{16}}-\frac{30a^{14}b}{31x^{\frac{31}{2}}}-\frac{7a^{13}b^{2}}{x^{15}}-\frac{910a^{12}b^{3}}{29x^{\frac{29}{2}}}-\frac{195a^{11}b^{4}}{2x^{14}}-\frac{2002a^{10}b^{5}}{9x^{\frac{27}{2}}}-\frac{385a^{9}b^{6}}{x^{13}}-\frac{2574a^{8}b^{7}}{5x^{\frac{25}{2}}}-\frac{2145a^{7}b^{8}}{4x^{12}}-$
default	$-\frac{a^{15}}{16x^{16}}-\frac{30a^{14}b}{31x^{\frac{31}{2}}}-\frac{7a^{13}b^{2}}{x^{15}}-\frac{910a^{12}b^{3}}{29x^{\frac{29}{2}}}-\frac{195a^{11}b^{4}}{2x^{14}}-\frac{2002a^{10}b^{5}}{9x^{\frac{27}{2}}}-\frac{385a^{9}b^{6}}{x^{13}}-\frac{2574a^{8}b^{7}}{5x^{\frac{25}{2}}}-\frac{2145a^{7}b^{8}}{4x^{12}}-$
orering	$-\frac{(17443117200b^{28}x^{14}-221502776040a^{2}b^{26}x^{13}+1329113424600a^{4}b^{24}x^{12}-4971688367100a^{6}b^{22}x^{11}+129120975963}{}$
trager	Expression too large to display

input
```
int((a+b*x^(1/2))^15/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*a^15/x^16-30/31*a^14*b/x^(31/2)-7*a^13*b^2/x^15-910/29*a^12*b^3/x^(2
9/2)-195/2*a^11*b^4/x^14-2002/9*a^10*b^5/x^(27/2)-385*a^9*b^6/x^13-2574/5*
a^8*b^7/x^(25/2)-2145/4*a^7*b^8/x^12-10010/23*a^6*b^9/x^(23/2)-273*a^5*b^1
0/x^11-130*a^4*b^11/x^(21/2)-91/2*a^3*b^12/x^10-210/19*a^2*b^13/x^(19/2)-5
/3*a*b^14/x^9-2/17*b^15/x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx = -\frac{8014405200\,ab^{14}x^{7}+218793261960\,a^{3}b^{12}x^{6}+1312759571760\,a^{5}b^{10}x^{5}+2578634873100\,a^{7}b^{8}x^{4}+185}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^17,x, algorithm="fricas")
```

output
```
-1/4808643120*(8014405200*a*b^14*x^7 + 218793261960*a^3*b^12*x^6 + 1312759
571760*a^5*b^10*x^5 + 2578634873100*a^7*b^8*x^4 + 1851327601200*a^9*b^6*x^
3 + 468842704200*a^11*b^4*x^2 + 33660501840*a^13*b^2*x + 300540195*a^15 +
32*(17678835*b^15*x^7 + 1660880025*a^2*b^13*x^6 + 19535112675*a^4*b^11*x^5
 + 65400159825*a^6*b^9*x^4 + 77359046193*a^8*b^7*x^3 + 33426748355*a^10*b^
5*x^2 + 4715372025*a^12*b^3*x + 145422675*a^14*b)*sqrt(x))/x^16
```

Sympy [A] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx = -\frac{a^{15}}{16x^{16}} - \frac{30a^{14}b}{31x^{\frac{31}{2}}} - \frac{7a^{13}b^2}{x^{15}} - \frac{910a^{12}b^3}{29x^{\frac{29}{2}}} - \frac{195a^{11}b^4}{2x^{14}} - \frac{2002a^{10}b^5}{9x^{\frac{27}{2}}} - \frac{385a^9b^6}{x^{13}} - \frac{2574a^8b^7}{5x^{\frac{25}{2}}} - \frac{2145a^7b^8}{4x^{12}} - \frac{10010a^6b^9}{23x^{\frac{23}{2}}} - \frac{273a^5b^{10}}{x^{11}} - \frac{130a^4b^{11}}{x^{\frac{21}{2}}} - \frac{91a^3b^{12}}{2x^{10}} - \frac{210a^2b^{13}}{19x^{\frac{19}{2}}} - \frac{5ab^{14}}{3x^9} - \frac{2b^{15}}{17x^{\frac{17}{2}}}$$

input
```
integrate((a+b*x**(1/2))**15/x**17,x)
```

output
```
-a**15/(16*x**16) - 30*a**14*b/(31*x**(31/2)) - 7*a**13*b**2/x**15 - 910*a
**12*b**3/(29*x**(29/2)) - 195*a**11*b**4/(2*x**14) - 2002*a**10*b**5/(9*x
**(27/2)) - 385*a**9*b**6/x**13 - 2574*a**8*b**7/(5*x**(25/2)) - 2145*a**7
*b**8/(4*x**12) - 10010*a**6*b**9/(23*x**(23/2)) - 273*a**5*b**10/x**11 -
130*a**4*b**11/x**(21/2) - 91*a**3*b**12/(2*x**10) - 210*a**2*b**13/(19*x*
*(19/2)) - 5*a*b**14/(3*x**9) - 2*b**15/(17*x**(17/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx = -\frac{565722720\,b^{15}x^{\frac{15}{2}} + 8014405200\,ab^{14}x^7 + 53148160800\,a^2b^{13}x^{\frac{13}{2}} + 218793261960\,a^3b^{12}x^6 + 625123605}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^17,x, algorithm="maxima")
```

output
```
-1/4808643120*(565722720*b^15*x^(15/2) + 8014405200*a*b^14*x^7 + 531481608
00*a^2*b^13*x^(13/2) + 218793261960*a^3*b^12*x^6 + 625123605600*a^4*b^11*x
^(11/2) + 1312759571760*a^5*b^10*x^5 + 2092805114400*a^6*b^9*x^(9/2) + 257
8634873100*a^7*b^8*x^4 + 2475489478176*a^8*b^7*x^(7/2) + 1851327601200*a^9
*b^6*x^3 + 1069655947360*a^10*b^5*x^(5/2) + 468842704200*a^11*b^4*x^2 + 15
0891904800*a^12*b^3*x^(3/2) + 33660501840*a^13*b^2*x + 4653525600*a^14*b*s
qrt(x) + 300540195*a^15)/x^16
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx =$$
$$-\frac{565722720\,b^{15}x^{\frac{15}{2}} + 8014405200\,ab^{14}x^{7} + 53148160800\,a^{2}b^{13}x^{\frac{13}{2}} + 218793261960\,a^{3}b^{12}x^{6} + 62512360\cdots}{}$$

input
```
integrate((a+b*x^(1/2))^15/x^17,x, algorithm="giac")
```

output
```
-1/4808643120*(565722720*b^15*x^(15/2) + 8014405200*a*b^14*x^7 + 531481608
00*a^2*b^13*x^(13/2) + 218793261960*a^3*b^12*x^6 + 625123605600*a^4*b^11*x
^(11/2) + 1312759571760*a^5*b^10*x^5 + 2092805114400*a^6*b^9*x^(9/2) + 257
8634873100*a^7*b^8*x^4 + 2475489478176*a^8*b^7*x^(7/2) + 1851327601200*a^9
*b^6*x^3 + 1069655947360*a^10*b^5*x^(5/2) + 468842704200*a^11*b^4*x^2 + 15
0891904800*a^12*b^3*x^(3/2) + 33660501840*a^13*b^2*x + 4653525600*a^14*b*s
qrt(x) + 300540195*a^15)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx =$$
$$-\frac{\frac{a^{15}}{16}+\frac{2\,b^{15}\,x^{15/2}}{17}+7\,a^{13}\,b^2\,x+\frac{30\,a^{14}\,b\,\sqrt{x}}{31}+\frac{5\,a\,b^{14}\,x^7}{3}+\frac{195\,a^{11}\,b^4\,x^2}{2}+385\,a^9\,b^6\,x^3+\frac{2145\,a^7\,b^8\,x^4}{4}+273\,a^5\,b^{10}\,x}{}$$

input
```
int((a + b*x^(1/2))^15/x^17,x)
```

output
```
-(a^15/16 + (2*b^15*x^(15/2))/17 + 7*a^13*b^2*x + (30*a^14*b*x^(1/2))/31 +
 (5*a*b^14*x^7)/3 + (195*a^11*b^4*x^2)/2 + 385*a^9*b^6*x^3 + (2145*a^7*b^8
*x^4)/4 + 273*a^5*b^10*x^5 + (910*a^12*b^3*x^(3/2))/29 + (91*a^3*b^12*x^6)
/2 + (2002*a^10*b^5*x^(5/2))/9 + (2574*a^8*b^7*x^(7/2))/5 + (10010*a^6*b^9
*x^(9/2))/23 + 130*a^4*b^11*x^(11/2) + (210*a^2*b^13*x^(13/2))/19)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt{x}\right)^{15}}{x^{17}}\,dx$$
$$=\frac{-300540195\sqrt{x}\,a^{15}-33660501840\sqrt{x}\,a^{13}b^2x-468842704200\sqrt{x}\,a^{11}b^4x^2-1851327601200\sqrt{x}\,a^9b^6x^3-}{}$$

input
```
int((a+b*x^(1/2))^15/x^17,x)
```

output
```
( - 300540195*sqrt(x)*a**15 - 33660501840*sqrt(x)*a**13*b**2*x - 468842704
200*sqrt(x)*a**11*b**4*x**2 - 1851327601200*sqrt(x)*a**9*b**6*x**3 - 25786
34873100*sqrt(x)*a**7*b**8*x**4 - 1312759571760*sqrt(x)*a**5*b**10*x**5 -
218793261960*sqrt(x)*a**3*b**12*x**6 - 8014405200*sqrt(x)*a*b**14*x**7 - 4
653525600*a**14*b*x - 150891904800*a**12*b**3*x**2 - 1069655947360*a**10*b
**5*x**3 - 2475489478176*a**8*b**7*x**4 - 2092805114400*a**6*b**9*x**5 - 6
25123605600*a**4*b**11*x**6 - 53148160800*a**2*b**13*x**7 - 565722720*b**1
5*x**8)/(4808643120*sqrt(x)*x**16)
```

3.81 $\int \frac{x^3}{a+b\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx=\frac{2a^6\sqrt{x}}{b^7}-\frac{a^5x}{b^6}+\frac{2a^4x^{3/2}}{3b^5}-\frac{a^3x^2}{2b^4}+\frac{2a^2x^{5/2}}{5b^3}-\frac{ax^3}{3b^2}+\frac{2x^{7/2}}{7b}-\frac{2a^7\log\left(a+b\sqrt{x}\right)}{b^8}$$

output
```
2*a^6*x^(1/2)/b^7-a^5*x/b^6+2/3*a^4*x^(3/2)/b^5-1/2*a^3*x^2/b^4+2/5*a^2*x^
(5/2)/b^3-1/3*a*x^3/b^2+2/7*x^(7/2)/b-2*a^7*ln(a+b*x^(1/2))/b^8
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx=\frac{\sqrt{x}\left(420a^6-210a^5b\sqrt{x}+140a^4b^2x-105a^3b^3x^{3/2}+84a^2b^4x^2-70ab^5x^{5/2}+60b^6x^3\right)}{210b^7}-\frac{2a^7\log\left(a+b\sqrt{x}\right)}{b^8}$$

input
```
Integrate[x^3/(a + b*Sqrt[x]),x]
```

output
```
(Sqrt[x]*(420*a^6 - 210*a^5*b*Sqrt[x] + 140*a^4*b^2*x - 105*a^3*b^3*x^(3/2
) + 84*a^2*b^4*x^2 - 70*a*b^5*x^(5/2) + 60*b^6*x^3))/(210*b^7) - (2*a^7*Lo
g[a + b*Sqrt[x]])/b^8
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{7/2}}{a+b\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^7}{b^7\left(a+b\sqrt{x}\right)}+\frac{a^6}{b^7}-\frac{\sqrt{x}a^5}{b^6}+\frac{xa^4}{b^5}-\frac{x^{3/2}a^3}{b^4}+\frac{x^2a^2}{b^3}-\frac{x^{5/2}a}{b^2}+\frac{x^3}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^7\log\left(a+b\sqrt{x}\right)}{b^8}+\frac{a^6\sqrt{x}}{b^7}-\frac{a^5x}{2b^6}+\frac{a^4x^{3/2}}{3b^5}-\frac{a^3x^2}{4b^4}+\frac{a^2x^{5/2}}{5b^3}-\frac{ax^3}{6b^2}+\frac{x^{7/2}}{7b}\right)$$

input
```
Int[x^3/(a + b*Sqrt[x]),x]
```

output
```
2*((a^6*Sqrt[x])/b^7 - (a^5*x)/(2*b^6) + (a^4*x^(3/2))/(3*b^5) - (a^3*x^2)
/(4*b^4) + (a^2*x^(5/2))/(5*b^3) - (a*x^3)/(6*b^2) + x^(7/2)/(7*b) - (a^7*
Log[a + b*Sqrt[x]])/b^8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2x^{\frac{7}{2}}b^6}{7}-\frac{a\,b^5x^3}{3}+\frac{2x^{\frac{5}{2}}a^2b^4}{5}-\frac{a^3b^3x^2}{2}+\frac{2x^{\frac{3}{2}}a^4b^2}{3}-a^5xb+2\sqrt{x}\,a^6}{b^7}-\frac{2a^7\ln\left(a+b\sqrt{x}\right)}{b^8}$	88
default	$\frac{\frac{2x^{\frac{7}{2}}b^6}{7}-\frac{a\,b^5x^3}{3}+\frac{2x^{\frac{5}{2}}a^2b^4}{5}-\frac{a^3b^3x^2}{2}+\frac{2x^{\frac{3}{2}}a^4b^2}{3}-a^5xb+2\sqrt{x}\,a^6}{b^7}-\frac{2a^7\ln\left(a+b\sqrt{x}\right)}{b^8}$	88

input
```
int(x^3/(a+b*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/b^7*(1/7*x^(7/2)*b^6-1/6*a*b^5*x^3+1/5*x^(5/2)*a^2*b^4-1/4*a^3*b^3*x^2+1
/3*x^(3/2)*a^4*b^2-1/2*a^5*x*b+x^(1/2)*a^6)-2*a^7*ln(a+b*x^(1/2))/b^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx = -\frac{70\,ab^6x^3 + 105\,a^3b^4x^2 + 210\,a^5b^2x + 420\,a^7\log\left(b\sqrt{x}+a\right) - 4\,(15\,b^7x^3 + 21\,a^2b^5x^2 + 35\,a^4b^3x + 105\,a}{210\,b^8}$$

input
```
integrate(x^3/(a+b*x^(1/2)),x, algorithm="fricas")
```

output
```
-1/210*(70*a*b^6*x^3 + 105*a^3*b^4*x^2 + 210*a^5*b^2*x + 420*a^7*log(b*sqr
t(x) + a) - 4*(15*b^7*x^3 + 21*a^2*b^5*x^2 + 35*a^4*b^3*x + 105*a^6*b)*sqr
t(x))/b^8
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx = \begin{cases} -\frac{2a^7\log\left(\frac{a}{b}+\sqrt{x}\right)}{b^8} + \frac{2a^6\sqrt{x}}{b^7} - \frac{a^5x}{b^6} + \frac{2a^4x^{\frac{3}{2}}}{3b^5} - \frac{a^3x^2}{2b^4} + \frac{2a^2x^{\frac{5}{2}}}{5b^3} - \frac{ax^3}{3b^2} + \frac{2x^{\frac{7}{2}}}{7b} & \text{for } b \neq 0 \\ \frac{x^4}{4a} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(a+b*x**(1/2)),x)
```

output
```
Piecewise((-2*a**7*log(a/b + sqrt(x))/b**8 + 2*a**6*sqrt(x)/b**7 - a**5*x/
b**6 + 2*a**4*x**(3/2)/(3*b**5) - a**3*x**2/(2*b**4) + 2*a**2*x**(5/2)/(5*
b**3) - a*x**3/(3*b**2) + 2*x**(7/2)/(7*b), Ne(b, 0)), (x**4/(4*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx = -\frac{2\,a^7\log\left(b\sqrt{x}+a\right)}{b^8} + \frac{2\left(b\sqrt{x}+a\right)^7}{7\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^6 a}{3\,b^8} + \frac{42\left(b\sqrt{x}+a\right)^5 a^2}{5\,b^8} - \frac{35\left(b\sqrt{x}+a\right)^4 a^3}{2\,b^8} + \frac{70\left(b\sqrt{x}+a\right)^3 a^4}{3\,b^8} - \frac{21\left(b\sqrt{x}+a\right)^2 a^5}{b^8} + \frac{14\left(b\sqrt{x}+a\right) a^6}{b^8}$$

input
```
integrate(x^3/(a+b*x^(1/2)),x, algorithm="maxima")
```

output
```
-2*a^7*log(b*sqrt(x) + a)/b^8 + 2/7*(b*sqrt(x) + a)^7/b^8 - 7/3*(b*sqrt(x)
 + a)^6*a/b^8 + 42/5*(b*sqrt(x) + a)^5*a^2/b^8 - 35/2*(b*sqrt(x) + a)^4*a^
3/b^8 + 70/3*(b*sqrt(x) + a)^3*a^4/b^8 - 21*(b*sqrt(x) + a)^2*a^5/b^8 + 14
*(b*sqrt(x) + a)*a^6/b^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx = -\frac{2\,a^7\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8} + \frac{60\,b^6x^{\frac{7}{2}} - 70\,ab^5x^3 + 84\,a^2b^4x^{\frac{5}{2}} - 105\,a^3b^3x^2 + 140\,a^4b^2x^{\frac{3}{2}} - 210\,a^5bx + 420\,a^6\sqrt{x}}{210\,b^7}$$

input
```
integrate(x^3/(a+b*x^(1/2)),x, algorithm="giac")
```

output
```
-2*a^7*log(abs(b*sqrt(x) + a))/b^8 + 1/210*(60*b^6*x^(7/2) - 70*a*b^5*x^3
+ 84*a^2*b^4*x^(5/2) - 105*a^3*b^3*x^2 + 140*a^4*b^2*x^(3/2) - 210*a^5*b*x
 + 420*a^6*sqrt(x))/b^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx = \frac{2\,x^{7/2}}{7\,b}-\frac{a\,x^3}{3\,b^2}-\frac{a^5\,x}{b^6}-\frac{2\,a^7\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}-\frac{a^3\,x^2}{2\,b^4}+\frac{2\,a^2\,x^{5/2}}{5\,b^3}+\frac{2\,a^4\,x^{3/2}}{3\,b^5}+\frac{2\,a^6\,\sqrt{x}}{b^7}$$

input
```
int(x^3/(a + b*x^(1/2)),x)
```

output
```
(2*x^(7/2))/(7*b) - (a*x^3)/(3*b^2) - (a^5*x)/b^6 - (2*a^7*log(a + b*x^(1/
2)))/b^8 - (a^3*x^2)/(2*b^4) + (2*a^2*x^(5/2))/(5*b^3) + (2*a^4*x^(3/2))/(
3*b^5) + (2*a^6*x^(1/2))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{a+b\sqrt{x}}\,dx = \frac{420\sqrt{x}\,a^6b+140\sqrt{x}\,a^4b^3x+84\sqrt{x}\,a^2b^5x^2+60\sqrt{x}\,b^7x^3-420\log\left(\sqrt{x}\,b+a\right)a^7-210a^5b^2x-105a^3b^4x^2}{210b^8}$$

input
```
int(x^3/(a+b*x^(1/2)),x)
```

output
```
(420*sqrt(x)*a**6*b + 140*sqrt(x)*a**4*b**3*x + 84*sqrt(x)*a**2*b**5*x**2
+ 60*sqrt(x)*b**7*x**3 - 420*log(sqrt(x)*b + a)*a**7 - 210*a**5*b**2*x - 1
05*a**3*b**4*x**2 - 70*a*b**6*x**3)/(210*b**8)
```

3.82 $\int \frac{x^2}{a+b\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx=\frac{2a^4\sqrt{x}}{b^5}-\frac{a^3x}{b^4}+\frac{2a^2x^{3/2}}{3b^3}-\frac{ax^2}{2b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^5\log\left(a+b\sqrt{x}\right)}{b^6}$$

output
```
2*a^4*x^(1/2)/b^5-a^3*x/b^4+2/3*a^2*x^(3/2)/b^3-1/2*a*x^2/b^2+2/5*x^(5/2)/
b-2*a^5*ln(a+b*x^(1/2))/b^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx=\frac{\sqrt{x}\left(60a^4-30a^3b\sqrt{x}+20a^2b^2x-15ab^3x^{3/2}+12b^4x^2\right)}{30b^5}-\frac{2a^5\log\left(a+b\sqrt{x}\right)}{b^6}$$

input
```
Integrate[x^2/(a + b*Sqrt[x]),x]
```

output
```
(Sqrt[x]*(60*a^4 - 30*a^3*b*Sqrt[x] + 20*a^2*b^2*x - 15*a*b^3*x^(3/2) + 12
*b^4*x^2))/(30*b^5) - (2*a^5*Log[a + b*Sqrt[x]])/b^6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sqrt{x}}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{x^{5/2}}{a+b\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(-\frac{a^5}{b^5\left(a+b\sqrt{x}\right)}+\frac{a^4}{b^5}-\frac{\sqrt{x}a^3}{b^4}+\frac{xa^2}{b^3}-\frac{x^{3/2}a}{b^2}+\frac{x^2}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{a^5\log\left(a+b\sqrt{x}\right)}{b^6}+\frac{a^4\sqrt{x}}{b^5}-\frac{a^3x}{2b^4}+\frac{a^2x^{3/2}}{3b^3}-\frac{ax^2}{4b^2}+\frac{x^{5/2}}{5b}\right)$$

input
```
Int[x^2/(a + b*Sqrt[x]),x]
```

output
```
2*((a^4*Sqrt[x])/b^5 - (a^3*x)/(2*b^4) + (a^2*x^(3/2))/(3*b^3) - (a*x^2)/(
4*b^2) + x^(5/2)/(5*b) - (a^5*Log[a + b*Sqrt[x]])/b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{2x^{\frac{5}{2}}b^4}{5}-\frac{a\,b^3x^2}{2}+\frac{2x^{\frac{3}{2}}a^2b^2}{3}-a^3bx+2\sqrt{x}\,a^4}{b^5}-\frac{2a^5\ln\left(a+b\sqrt{x}\right)}{b^6}$	66
default	$\frac{\frac{2x^{\frac{5}{2}}b^4}{5}-\frac{a\,b^3x^2}{2}+\frac{2x^{\frac{3}{2}}a^2b^2}{3}-a^3bx+2\sqrt{x}\,a^4}{b^5}-\frac{2a^5\ln\left(a+b\sqrt{x}\right)}{b^6}$	66

input
```
int(x^2/(a+b*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/5*x^(5/2)*b^4-1/4*a*b^3*x^2+1/3*x^(3/2)*a^2*b^2-1/2*a^3*b*x+x^(1/
2)*a^4)-2*a^5*ln(a+b*x^(1/2))/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx = -\frac{15\,ab^4x^2+30\,a^3b^2x+60\,a^5\log\left(b\sqrt{x}+a\right)-4\left(3\,b^5x^2+5\,a^2b^3x+15\,a^4b\right)\sqrt{x}}{30\,b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2)),x, algorithm="fricas")
```

output
```
-1/30*(15*a*b^4*x^2 + 30*a^3*b^2*x + 60*a^5*log(b*sqrt(x) + a) - 4*(3*b^5*
x^2 + 5*a^2*b^3*x + 15*a^4*b)*sqrt(x))/b^6
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx = \begin{cases} -\frac{2a^5\log\left(\frac{a}{b}+\sqrt{x}\right)}{b^6} + \frac{2a^4\sqrt{x}}{b^5} - \frac{a^3x}{b^4} + \frac{2a^2x^{\frac{3}{2}}}{3b^3} - \frac{ax^2}{2b^2} + \frac{2x^{\frac{5}{2}}}{5b} & \text{for } b \neq 0 \\ \frac{x^3}{3a} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(a+b*x**(1/2)),x)
```

output
```
Piecewise((-2*a**5*log(a/b + sqrt(x))/b**6 + 2*a**4*sqrt(x)/b**5 - a**3*x/
b**4 + 2*a**2*x**(3/2)/(3*b**3) - a*x**2/(2*b**2) + 2*x**(5/2)/(5*b), Ne(b
, 0)), (x**3/(3*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx = -\frac{2\,a^5\log\left(b\sqrt{x}+a\right)}{b^6} + \frac{2\,\left(b\sqrt{x}+a\right)^5}{5\,b^6} - \frac{5\,\left(b\sqrt{x}+a\right)^4 a}{2\,b^6} + \frac{20\,\left(b\sqrt{x}+a\right)^3 a^2}{3\,b^6} - \frac{10\,\left(b\sqrt{x}+a\right)^2 a^3}{b^6} + \frac{10\,\left(b\sqrt{x}+a\right) a^4}{b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2)),x, algorithm="maxima")
```

output
```
-2*a^5*log(b*sqrt(x) + a)/b^6 + 2/5*(b*sqrt(x) + a)^5/b^6 - 5/2*(b*sqrt(x)
 + a)^4*a/b^6 + 20/3*(b*sqrt(x) + a)^3*a^2/b^6 - 10*(b*sqrt(x) + a)^2*a^3/
b^6 + 10*(b*sqrt(x) + a)*a^4/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx = -\frac{2\,a^5\log\left(\left|b\sqrt{x}+a\right|\right)}{b^6} + \frac{12\,b^4x^{\frac{5}{2}} - 15\,ab^3x^2 + 20\,a^2b^2x^{\frac{3}{2}} - 30\,a^3bx + 60\,a^4\sqrt{x}}{30\,b^5}$$

input
```
integrate(x^2/(a+b*x^(1/2)),x, algorithm="giac")
```

output
```
-2*a^5*log(abs(b*sqrt(x) + a))/b^6 + 1/30*(12*b^4*x^(5/2) - 15*a*b^3*x^2 +
 20*a^2*b^2*x^(3/2) - 30*a^3*b*x + 60*a^4*sqrt(x))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx = \frac{2\,x^{5/2}}{5\,b} - \frac{a\,x^2}{2\,b^2} - \frac{a^3\,x}{b^4} - \frac{2\,a^5\,\ln\left(a+b\,\sqrt{x}\right)}{b^6} + \frac{2\,a^2\,x^{3/2}}{3\,b^3} + \frac{2\,a^4\,\sqrt{x}}{b^5}$$

input
```
int(x^2/(a + b*x^(1/2)),x)
```

output
```
(2*x^(5/2))/(5*b) - (a*x^2)/(2*b^2) - (a^3*x)/b^4 - (2*a^5*log(a + b*x^(1/
2)))/b^6 + (2*a^2*x^(3/2))/(3*b^3) + (2*a^4*x^(1/2))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{a+b\sqrt{x}}\,dx = \frac{60\sqrt{x}\,a^4b + 20\sqrt{x}\,a^2b^3x + 12\sqrt{x}\,b^5x^2 - 60\log\left(\sqrt{x}\,b+a\right)a^5 - 30a^3b^2x - 15a\,b^4x^2}{30b^6}$$

input
```
int(x^2/(a+b*x^(1/2)),x)
```

output
```
(60*sqrt(x)*a**4*b + 20*sqrt(x)*a**2*b**3*x + 12*sqrt(x)*b**5*x**2 - 60*lo
g(sqrt(x)*b + a)*a**5 - 30*a**3*b**2*x - 15*a*b**4*x**2)/(30*b**6)
```

3.83 $\int \frac{x}{a+b\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x}{a+b\sqrt{x}}\,dx=\frac{2a^2\sqrt{x}}{b^3}-\frac{ax}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^3\log\left(a+b\sqrt{x}\right)}{b^4}$$

output
```
2*a^2*x^(1/2)/b^3-a*x/b^2+2/3*x^(3/2)/b-2*a^3*ln(a+b*x^(1/2))/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{x}{a+b\sqrt{x}}\,dx=\frac{\sqrt{x}\left(6a^2-3ab\sqrt{x}+2b^2x\right)}{3b^3}-\frac{2a^3\log\left(a+b\sqrt{x}\right)}{b^4}$$

input
```
Integrate[x/(a + b*Sqrt[x]),x]
```

output
```
(Sqrt[x]*(6*a^2 - 3*a*b*Sqrt[x] + 2*b^2*x))/(3*b^3) - (2*a^3*Log[a + b*Sqr
t[x]])/b^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sqrt{x}}dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{x^{3/2}}{a+b\sqrt{x}}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$
$$2\int \left(-\frac{a^3}{b^3\left(a+b\sqrt{x}\right)}+\frac{a^2}{b^3}-\frac{\sqrt{x}a}{b^2}+\frac{x}{b}\right)d\sqrt{x}$$
$$\Big\downarrow \text{2009}$$
$$2\left(-\frac{a^3\log\left(a+b\sqrt{x}\right)}{b^4}+\frac{a^2\sqrt{x}}{b^3}-\frac{ax}{2b^2}+\frac{x^{3/2}}{3b}\right)$$

input
```
Int[x/(a + b*Sqrt[x]),x]
```

output
```
2*((a^2*Sqrt[x])/b^3 - (a*x)/(2*b^2) + x^(3/2)/(3*b) - (a^3*Log[a + b*Sqrt
[x]])/b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{2b^2x^{\frac{3}{2}}}{3}-abx+2a^2\sqrt{x}}{b^3}-\frac{2a^3\ln\left(a+b\sqrt{x}\right)}{b^4}$	44
default	$\frac{\frac{2b^2x^{\frac{3}{2}}}{3}-abx+2a^2\sqrt{x}}{b^3}-\frac{2a^3\ln\left(a+b\sqrt{x}\right)}{b^4}$	44

input
```
int(x/(a+b*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/b^3*(1/3*b^2*x^(3/2)-1/2*a*b*x+a^2*x^(1/2))-2*a^3*ln(a+b*x^(1/2))/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{x}{a+b\sqrt{x}}\,dx = -\frac{3\,ab^2x+6\,a^3\log\left(b\sqrt{x}+a\right)-2\,(b^3x+3\,a^2b)\sqrt{x}}{3\,b^4}$$

input
```
integrate(x/(a+b*x^(1/2)),x, algorithm="fricas")
```

output
```
-1/3*(3*a*b^2*x + 6*a^3*log(b*sqrt(x) + a) - 2*(b^3*x + 3*a^2*b)*sqrt(x))/
b^4
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{x}{a+b\sqrt{x}}\,dx = \begin{cases} -\frac{2a^3\log\left(\frac{a}{b}+\sqrt{x}\right)}{b^4} + \frac{2a^2\sqrt{x}}{b^3} - \frac{ax}{b^2} + \frac{2x^{\frac{3}{2}}}{3b} & \text{for } b \neq 0 \\ \frac{x^2}{2a} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b*x**(1/2)),x)
```

output
```
Piecewise((-2*a**3*log(a/b + sqrt(x))/b**4 + 2*a**2*sqrt(x)/b**3 - a*x/b**
2 + 2*x**(3/2)/(3*b), Ne(b, 0)), (x**2/(2*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{x}{a+b\sqrt{x}}\,dx = -\frac{2\,a^3\log\left(b\sqrt{x}+a\right)}{b^4} + \frac{2\left(b\sqrt{x}+a\right)^3}{3\,b^4} - \frac{3\left(b\sqrt{x}+a\right)^2 a}{b^4} + \frac{6\left(b\sqrt{x}+a\right)a^2}{b^4}$$

input
```
integrate(x/(a+b*x^(1/2)),x, algorithm="maxima")
```

output
```
-2*a^3*log(b*sqrt(x) + a)/b^4 + 2/3*(b*sqrt(x) + a)^3/b^4 - 3*(b*sqrt(x) +
 a)^2*a/b^4 + 6*(b*sqrt(x) + a)*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x}{a+b\sqrt{x}}\,dx = -\frac{2\,a^3\log\left(\left|b\sqrt{x}+a\right|\right)}{b^4} + \frac{2\,b^2x^{\frac{3}{2}} - 3\,abx + 6\,a^2\sqrt{x}}{3\,b^3}$$

input
```
integrate(x/(a+b*x^(1/2)),x, algorithm="giac")
```

output
```
-2*a^3*log(abs(b*sqrt(x) + a))/b^4 + 1/3*(2*b^2*x^(3/2) - 3*a*b*x + 6*a^2*
sqrt(x))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{x}{a+b\sqrt{x}}\,dx = \frac{2\,x^{3/2}}{3\,b} - \frac{2\,a^3\,\ln\left(a+b\,\sqrt{x}\right)}{b^4} + \frac{2\,a^2\,\sqrt{x}}{b^3} - \frac{a\,x}{b^2}$$

input
```
int(x/(a + b*x^(1/2)),x)
```

output
```
(2*x^(3/2))/(3*b) - (2*a^3*log(a + b*x^(1/2)))/b^4 + (2*a^2*x^(1/2))/b^3 -
 (a*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{x}{a+b\sqrt{x}}\,dx = \frac{6\sqrt{x}\,a^2b+2\sqrt{x}\,b^3x-6\log\left(\sqrt{x}\,b+a\right)a^3-3a\,b^2x}{3b^4}$$

input
```
int(x/(a+b*x^(1/2)),x)
```

output
```
(6*sqrt(x)*a**2*b + 2*sqrt(x)*b**3*x - 6*log(sqrt(x)*b + a)*a**3 - 3*a*b**
2*x)/(3*b**4)
```

3.84 $\int \frac{1}{a+b\sqrt{x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{a+b\sqrt{x}}\,dx=\frac{2\sqrt{x}}{b}-\frac{2a\log\left(a+b\sqrt{x}\right)}{b^2}$$

output
```
2*x^(1/2)/b-2*a*ln(a+b*x^(1/2))/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sqrt{x}}\,dx=\frac{2\sqrt{x}}{b}-\frac{2a\log\left(a+b\sqrt{x}\right)}{b^2}$$

input
```
Integrate[(a + b*Sqrt[x])^(-1),x]
```

output
```
(2*Sqrt[x])/b - (2*a*Log[a + b*Sqrt[x]])/b^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sqrt{x}}dx$$

$$\Big\downarrow \text{774}$$

$$2\int \frac{\sqrt{x}}{a+b\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\frac{1}{b}-\frac{a}{b\left(a+b\sqrt{x}\right)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\sqrt{x}}{b}-\frac{a\log\left(a+b\sqrt{x}\right)}{b^2}\right)$$

input
```
Int[(a + b*Sqrt[x])^(-1),x]
```

output
```
2*(Sqrt[x]/b - (a*Log[a + b*Sqrt[x]])/b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{2a\ln\left(a+b\sqrt{x}\right)}{b^{2}}$	24
default	$\frac{2\sqrt{x}}{b}+\frac{a\ln\left(b\sqrt{x}-a\right)}{b^{2}}-\frac{a\ln\left(a+b\sqrt{x}\right)}{b^{2}}-\frac{a\ln\left(b^{2}x-a^{2}\right)}{b^{2}}$	57

input
```
int(1/(a+b*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b-2*a*ln(a+b*x^(1/2))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+b\sqrt{x}}\,dx=-\frac{2\left(a\log\left(b\sqrt{x}+a\right)-b\sqrt{x}\right)}{b^{2}}$$

input
```
integrate(1/(a+b*x^(1/2)),x, algorithm="fricas")
```

output
```
-2*(a*log(b*sqrt(x) + a) - b*sqrt(x))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sqrt{x}}\,dx = \begin{cases} -\frac{2a\log\left(\frac{a}{b}+\sqrt{x}\right)}{b^2}+\frac{2\sqrt{x}}{b} & \text{for } b\neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2)),x)
```

output
```
Piecewise((-2*a*log(a/b + sqrt(x))/b**2 + 2*sqrt(x)/b, Ne(b, 0)), (x/a, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sqrt{x}}\,dx = -\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\left(b\sqrt{x}+a\right)}{b^2}$$

input
```
integrate(1/(a+b*x^(1/2)),x, algorithm="maxima")
```

output
```
-2*a*log(b*sqrt(x) + a)/b^2 + 2*(b*sqrt(x) + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+b\sqrt{x}}\,dx = -\frac{2\,a\log\left(\left|b\sqrt{x}+a\right|\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

input
```
integrate(1/(a+b*x^(1/2)),x, algorithm="giac")
```

output
```
-2*a*log(abs(b*sqrt(x) + a))/b^2 + 2*sqrt(x)/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\sqrt{x}}\,dx=\frac{2\,\sqrt{x}}{b}-\frac{2\,a\,\ln\left(a+b\,\sqrt{x}\right)}{b^2}$$

input
```
int(1/(a + b*x^(1/2)),x)
```

output
```
(2*x^(1/2))/b - (2*a*log(a + b*x^(1/2)))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{a+b\sqrt{x}}\,dx=\frac{2\sqrt{x}\,b-2\log\left(\sqrt{x}\,b+a\right)a}{b^2}$$

input
```
int(1/(a+b*x^(1/2)),x)
```

output
```
(2*(sqrt(x)*b - log(sqrt(x)*b + a)*a))/b**2
```

3.85 $\int \frac{1}{(a+b\sqrt{x})x} dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx=-\frac{2\log\left(a+b\sqrt{x}\right)}{a}+\frac{\log(x)}{a}$$

output
```
-2*ln(a+b*x^(1/2))/a+ln(x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx=-\frac{2\log\left(a^2+ab\sqrt{x}\right)}{a}+\frac{2\log\left(\sqrt{x}\right)}{a}$$

input
```
Integrate[1/((a + b*Sqrt[x])*x),x]
```

output
```
(-2*Log[a^2 + a*b*Sqrt[x]])/a + (2*Log[Sqrt[x]])/a
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{x}\right)}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{47}$$

$$2\left(\frac{\int \frac{1}{\sqrt{x}}d\sqrt{x}}{a}-\frac{b\int \frac{1}{a+b\sqrt{x}}d\sqrt{x}}{a}\right)$$

$$\Big\downarrow \text{14}$$

$$2\left(\frac{\log\left(\sqrt{x}\right)}{a}-\frac{b\int \frac{1}{a+b\sqrt{x}}d\sqrt{x}}{a}\right)$$

$$\Big\downarrow \text{16}$$

$$2\left(\frac{\log\left(\sqrt{x}\right)}{a}-\frac{\log\left(a+b\sqrt{x}\right)}{a}\right)$$

input
```
Int[1/((a + b*Sqrt[x])*x),x]
```

output
```
2*(-(Log[a + b*Sqrt[x]]/a) + Log[Sqrt[x]]/a)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2\ln\left(a+b\sqrt{x}\right)}{a}+\frac{\ln\left(x\right)}{a}$	21
default	$-\frac{2\ln\left(a+b\sqrt{x}\right)}{a}+\frac{\ln\left(x\right)}{a}$	21

input
```
int(1/(a+b*x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-2*ln(a+b*x^(1/2))/a+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx = -\frac{2\left(\log\left(b\sqrt{x}+a\right)-\log\left(\sqrt{x}\right)\right)}{a}$$

input
```
integrate(1/(a+b*x^(1/2))/x,x, algorithm="fricas")
```

output
```
-2*(log(b*sqrt(x) + a) - log(sqrt(x)))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\ a=0\wedge b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\ a=0 \\ \frac{\log\left(x\right)}{a} & \text{for}\ b=0 \\ \frac{\log\left(x\right)}{a}-\frac{2\log\left(\frac{a}{b}+\sqrt{x}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))/x,x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)),
(log(x)/a, Eq(b, 0)), (log(x)/a - 2*log(a/b + sqrt(x))/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx=-\frac{2\,\log\left(b\sqrt{x}+a\right)}{a}+\frac{\log\left(x\right)}{a}$$

input
```
integrate(1/(a+b*x^(1/2))/x,x, algorithm="maxima")
```

output
```
-2*log(b*sqrt(x) + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx=-\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a}+\frac{\log\left(\left|x\right|\right)}{a}$$

input
```
integrate(1/(a+b*x^(1/2))/x,x, algorithm="giac")
```

output
```
-2*log(abs(b*sqrt(x) + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx=-\frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a}$$

input
```
int(1/(x*(a + b*x^(1/2))),x)
```

output
```
-(4*atanh((2*b*x^(1/2))/a + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x}\,dx = \frac{-2\log\left(\sqrt{x}\,b+a\right)+2\log\left(\sqrt{x}\right)}{a}$$

input
```
int(1/(a+b*x^(1/2))/x,x)
```

output
```
(2*( - log(sqrt(x)*b + a) + log(sqrt(x))))/a
```

3.86 $\int \frac{1}{(a+b\sqrt{x})x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx = -\frac{1}{ax}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{b^2\log(x)}{a^3}$$

output
```
-1/a/x+2*b/a^2/x^(1/2)-2*b^2*ln(a+b*x^(1/2))/a^3+b^2*ln(x)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx = \frac{-a\left(a-2b\sqrt{x}\right)-2b^2x\log\left(a+b\sqrt{x}\right)+b^2x\log(x)}{a^3x}$$

input
```
Integrate[1/((a + b*Sqrt[x])*x^2),x]
```

output
```
(-(a*(a - 2*b*Sqrt[x])) - 2*b^2*x*Log[a + b*Sqrt[x]] + b^2*x*Log[x])/(a^3*
x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt{x}\right)} \, dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right) x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{b^3}{a^3 \left(a+b\sqrt{x}\right)}+\frac{b^2}{a^3\sqrt{x}}-\frac{b}{a^2x}+\frac{1}{ax^{3/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{b^2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{b^2\log\left(\sqrt{x}\right)}{a^3}+\frac{b}{a^2\sqrt{x}}-\frac{1}{2ax}\right)$$

input
```
Int[1/((a + b*Sqrt[x])*x^2),x]
```

output
```
2*(-1/2*1/(a*x) + b/(a^2*Sqrt[x]) - (b^2*Log[a + b*Sqrt[x]])/a^3 + (b^2*Lo
g[Sqrt[x]])/a^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{1}{ax}+\frac{2b}{a^{2}\sqrt{x}}-\frac{2b^{2}\ln\left(a+b\sqrt{x}\right)}{a^{3}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}$	44
default	$-\frac{1}{ax}+\frac{2b}{a^{2}\sqrt{x}}-\frac{2b^{2}\ln\left(a+b\sqrt{x}\right)}{a^{3}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}$	44

input
```
int(1/(a+b*x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a/x+2*b/a^2/x^(1/2)-2*b^2*ln(a+b*x^(1/2))/a^3+b^2*ln(x)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^{2}}\,dx=-\frac{2\,b^{2}x\log\left(b\sqrt{x}+a\right)-2\,b^{2}x\log\left(\sqrt{x}\right)-2\,ab\sqrt{x}+a^{2}}{a^{3}x}$$

input
```
integrate(1/(a+b*x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-(2*b^2*x*log(b*sqrt(x) + a) - 2*b^2*x*log(sqrt(x)) - 2*a*b*sqrt(x) + a^2)
/(a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a=0 \\ -\frac{1}{ax} & \text{for } b=0 \\ -\frac{1}{ax}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\log(x)}{a^3}-\frac{2b^2\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))/x**2,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0
)), (-1/(a*x), Eq(b, 0)), (-1/(a*x) + 2*b/(a**2*sqrt(x)) + b**2*log(x)/a**
3 - 2*b**2*log(a/b + sqrt(x))/a**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx = -\frac{2\,b^2\log\left(b\sqrt{x}+a\right)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,b\sqrt{x}-a}{a^2x}$$

input
```
integrate(1/(a+b*x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-2*b^2*log(b*sqrt(x) + a)/a^3 + b^2*log(x)/a^3 + (2*b*sqrt(x) - a)/(a^2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx = -\frac{2\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,ab\sqrt{x}-a^2}{a^3x}$$

input
```
integrate(1/(a+b*x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-2*b^2*log(abs(b*sqrt(x) + a))/a^3 + b^2*log(abs(x))/a^3 + (2*a*b*sqrt(x)
- a^2)/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx = -\frac{\frac{1}{a}-\frac{2\,b\,\sqrt{x}}{a^2}}{x}-\frac{4\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^3}$$

input
```
int(1/(x^2*(a + b*x^(1/2))),x)
```

output
```
- (1/a - (2*b*x^(1/2))/a^2)/x - (4*b^2*atanh((2*b*x^(1/2))/a + 1))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^2}\,dx = \frac{2\sqrt{x}\,ab-2\log\left(\sqrt{x}\,b+a\right)b^2x+2\log\left(\sqrt{x}\right)b^2x-a^2}{a^3x}$$

input
```
int(1/(a+b*x^(1/2))/x^2,x)
```

output
```
(2*sqrt(x)*a*b - 2*log(sqrt(x)*b + a)*b**2*x + 2*log(sqrt(x))*b**2*x - a**
2)/(a**3*x)
```

3.87 $\int \frac{1}{(a+b\sqrt{x})x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}\,dx = -\frac{1}{2ax^2}+\frac{2b}{3a^2x^{3/2}}-\frac{b^2}{a^3x}+\frac{2b^3}{a^4\sqrt{x}}-\frac{2b^4\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{b^4\log(x)}{a^5}$$

output
```
-1/2/a/x^2+2/3*b/a^2/x^(3/2)-b^2/a^3/x+2*b^3/a^4/x^(1/2)-2*b^4*ln(a+b*x^(1
/2))/a^5+b^4*ln(x)/a^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}\,dx = \frac{\frac{a\left(-3a^3+4a^2b\sqrt{x}-6ab^2x+12b^3x^{3/2}\right)}{x^2}-12b^4\log\left(a+b\sqrt{x}\right)+6b^4\log(x)}{6a^5}$$

input
```
Integrate[1/((a + b*Sqrt[x])*x^3),x]
```

output
```
((a*(-3*a^3 + 4*a^2*b*Sqrt[x] - 6*a*b^2*x + 12*b^3*x^(3/2)))/x^2 - 12*b^4*
Log[a + b*Sqrt[x]] + 6*b^4*Log[x])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\sqrt{x}\right)}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 54}$$

$$2\int \left(-\frac{b^5}{a^5\left(a+b\sqrt{x}\right)}+\frac{b^4}{a^5\sqrt{x}}-\frac{b^3}{a^4x}+\frac{b^2}{a^3x^{3/2}}-\frac{b}{a^2x^2}+\frac{1}{ax^{5/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{b^4\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{b^4\log\left(\sqrt{x}\right)}{a^5}+\frac{b^3}{a^4\sqrt{x}}-\frac{b^2}{2a^3x}+\frac{b}{3a^2x^{3/2}}-\frac{1}{4ax^2}\right)$$

input
```
Int[1/((a + b*Sqrt[x])*x^3),x]
```

output
```
2*(-1/4*1/(a*x^2) + b/(3*a^2*x^(3/2)) - b^2/(2*a^3*x) + b^3/(a^4*Sqrt[x])
- (b^4*Log[a + b*Sqrt[x]])/a^5 + (b^4*Log[Sqrt[x]])/a^5)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{1}{2a\,x^{2}}+\frac{2b}{3a^{2}x^{\frac{3}{2}}}-\frac{b^{2}}{a^{3}x}+\frac{2b^{3}}{a^{4}\sqrt{x}}-\frac{2b^{4}\ln\left(a+b\sqrt{x}\right)}{a^{5}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}$	66
default	$-\frac{1}{2a\,x^{2}}+\frac{2b}{3a^{2}x^{\frac{3}{2}}}-\frac{b^{2}}{a^{3}x}+\frac{2b^{3}}{a^{4}\sqrt{x}}-\frac{2b^{4}\ln\left(a+b\sqrt{x}\right)}{a^{5}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}$	66

input
```
int(1/(a+b*x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+2/3*b/a^2/x^(3/2)-b^2/a^3/x+2*b^3/a^4/x^(1/2)-2*b^4*ln(a+b*x^(1
/2))/a^5+b^4*ln(x)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^{3}}\,dx = -\frac{12\,b^{4}x^{2}\log\left(b\sqrt{x}+a\right)-12\,b^{4}x^{2}\log\left(\sqrt{x}\right)+6\,a^{2}b^{2}x+3\,a^{4}-4\left(3\,ab^{3}x+a^{3}b\right)\sqrt{x}}{6\,a^{5}x^{2}}$$

input
```
integrate(1/(a+b*x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/6*(12*b^4*x^2*log(b*sqrt(x) + a) - 12*b^4*x^2*log(sqrt(x)) + 6*a^2*b^2*
x + 3*a^4 - 4*(3*a*b^3*x + a^3*b)*sqrt(x))/(a^5*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0 \\ -\frac{1}{2ax^2} & \text{for } b=0 \\ -\frac{1}{2ax^2}+\frac{2b}{3a^2x^{\frac{3}{2}}}-\frac{b^2}{a^3x}+\frac{2b^3}{a^4\sqrt{x}}+\frac{b^4\log(x)}{a^5}-\frac{2b^4\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))/x**3,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0
)), (-1/(2*a*x**2), Eq(b, 0)), (-1/(2*a*x**2) + 2*b/(3*a**2*x**(3/2)) - b*
*2/(a**3*x) + 2*b**3/(a**4*sqrt(x)) + b**4*log(x)/a**5 - 2*b**4*log(a/b +
sqrt(x))/a**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^3}\,dx = -\frac{2\,b^4\log\left(b\sqrt{x}+a\right)}{a^5}+\frac{b^4\log(x)}{a^5}+\frac{12\,b^3x^{\frac{3}{2}}-6\,ab^2x+4\,a^2b\sqrt{x}-3\,a^3}{6\,a^4x^2}$$

input
```
integrate(1/(a+b*x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-2*b^4*log(b*sqrt(x) + a)/a^5 + b^4*log(x)/a^5 + 1/6*(12*b^3*x^(3/2) - 6*a
*b^2*x + 4*a^2*b*sqrt(x) - 3*a^3)/(a^4*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+b\sqrt{x})\,x^3}\,dx = -\frac{2\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^5} + \frac{b^4\log\left(|x|\right)}{a^5} + \frac{12\,ab^3x^{\frac{3}{2}} - 6\,a^2b^2x + 4\,a^3b\sqrt{x} - 3\,a^4}{6\,a^5x^2}$$

input
```
integrate(1/(a+b*x^(1/2))/x^3,x, algorithm="giac")
```

output
```
-2*b^4*log(abs(b*sqrt(x) + a))/a^5 + b^4*log(abs(x))/a^5 + 1/6*(12*a*b^3*x
^(3/2) - 6*a^2*b^2*x + 4*a^3*b*sqrt(x) - 3*a^4)/(a^5*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+b\sqrt{x})\,x^3}\,dx = -\frac{\frac{1}{2\,a} - \frac{2\,b\,\sqrt{x}}{3\,a^2} + \frac{b^2\,x}{a^3} - \frac{2\,b^3\,x^{3/2}}{a^4}}{x^2} - \frac{4\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^5}$$

input
```
int(1/(x^3*(a + b*x^(1/2))),x)
```

output
```
- (1/(2*a) - (2*b*x^(1/2))/(3*a^2) + (b^2*x)/a^3 - (2*b^3*x^(3/2))/a^4)/x^
2 - (4*b^4*atanh((2*b*x^(1/2))/a + 1))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+b\sqrt{x})\,x^3}\,dx = \frac{4\sqrt{x}\,a^3b + 12\sqrt{x}\,a\,b^3x - 12\log\left(\sqrt{x}\,b+a\right)b^4x^2 + 12\log\left(\sqrt{x}\right)b^4x^2 - 3a^4 - 6a^2b^2x}{6a^5x^2}$$

input
```
int(1/(a+b*x^(1/2))/x^3,x)
```

output
```
(4*sqrt(x)*a**3*b + 12*sqrt(x)*a*b**3*x - 12*log(sqrt(x)*b + a)*b**4*x**2
+ 12*log(sqrt(x))*b**4*x**2 - 3*a**4 - 6*a**2*b**2*x)/(6*a**5*x**2)
```

3.88 $\int \frac{1}{(a+b\sqrt{x})x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx=-\frac{1}{3ax^3}+\frac{2b}{5a^2x^{5/2}}-\frac{b^2}{2a^3x^2}+\frac{2b^3}{3a^4x^{3/2}}-\frac{b^4}{a^5x}+\frac{2b^5}{a^6\sqrt{x}}-\frac{2b^6\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}$$

output
```
-1/3/a/x^3+2/5*b/a^2/x^(5/2)-1/2*b^2/a^3/x^2+2/3*b^3/a^4/x^(3/2)-b^4/a^5/x
+2*b^5/a^6/x^(1/2)-2*b^6*ln(a+b*x^(1/2))/a^7+b^6*ln(x)/a^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx=\frac{\frac{a\left(-10a^5+12a^4b\sqrt{x}-15a^3b^2x+20a^2b^3x^{3/2}-30ab^4x^2+60b^5x^{5/2}\right)}{x^3}-60b^6\log\left(a+b\sqrt{x}\right)+30b^6\log(x)}{30a^7}$$

input
```
Integrate[1/((a + b*Sqrt[x])*x^4),x]
```

output
```
((a*(-10*a^5 + 12*a^4*b*Sqrt[x] - 15*a^3*b^2*x + 20*a^2*b^3*x^(3/2) - 30*a
*b^4*x^2 + 60*b^5*x^(5/2)))/x^3 - 60*b^6*Log[a + b*Sqrt[x]] + 30*b^6*Log[x
])/(30*a^7)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\sqrt{x}\right)}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{b^7}{a^7\left(a+b\sqrt{x}\right)}+\frac{b^6}{a^7\sqrt{x}}-\frac{b^5}{a^6x}+\frac{b^4}{a^5x^{3/2}}-\frac{b^3}{a^4x^2}+\frac{b^2}{a^3x^{5/2}}-\frac{b}{a^2x^3}+\frac{1}{ax^{7/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{b^6\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{b^6\log\left(\sqrt{x}\right)}{a^7}+\frac{b^5}{a^6\sqrt{x}}-\frac{b^4}{2a^5x}+\frac{b^3}{3a^4x^{3/2}}-\frac{b^2}{4a^3x^2}+\frac{b}{5a^2x^{5/2}}-\frac{1}{6ax^3}\right)$$

input
```
Int[1/((a + b*Sqrt[x])*x^4),x]
```

output
```
2*(-1/6*1/(a*x^3) + b/(5*a^2*x^(5/2)) - b^2/(4*a^3*x^2) + b^3/(3*a^4*x^(3/
2)) - b^4/(2*a^5*x) + b^5/(a^6*Sqrt[x]) - (b^6*Log[a + b*Sqrt[x]])/a^7 + (
b^6*Log[Sqrt[x]])/a^7)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{1}{3 a \,x^{3}}+\frac{2 b}{5 a^{2} x^{\frac{5}{2}}}-\frac{b^{2}}{2 a^{3} x^{2}}+\frac{2 b^{3}}{3 a^{4} x^{\frac{3}{2}}}-\frac{b^{4}}{a^{5} x}+\frac{2 b^{5}}{a^{6} \sqrt{x}}-\frac{2 b^{6} \ln \left(a +b \sqrt{x}\right)}{a^{7}}+\frac{b^{6} \ln \left(x \right)}{a^{7}}$	88
default	$-\frac{1}{3 a \,x^{3}}+\frac{2 b}{5 a^{2} x^{\frac{5}{2}}}-\frac{b^{2}}{2 a^{3} x^{2}}+\frac{2 b^{3}}{3 a^{4} x^{\frac{3}{2}}}-\frac{b^{4}}{a^{5} x}+\frac{2 b^{5}}{a^{6} \sqrt{x}}-\frac{2 b^{6} \ln \left(a +b \sqrt{x}\right)}{a^{7}}+\frac{b^{6} \ln \left(x \right)}{a^{7}}$	88

input
```
int(1/(a+b*x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+2/5*b/a^2/x^(5/2)-1/2*b^2/a^3/x^2+2/3*b^3/a^4/x^(3/2)-b^4/a^5/x
+2*b^5/a^6/x^(1/2)-2*b^6*ln(a+b*x^(1/2))/a^7+b^6*ln(x)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx = -\frac{60\,b^6x^3\log\left(b\sqrt{x}+a\right)-60\,b^6x^3\log\left(\sqrt{x}\right)+30\,a^2b^4x^2+15\,a^4b^2x+10\,a^6-4\left(15\,ab^5x^2+5\,a^3b^3x+3\,a\right.}{30\,a^7x^3}$$

input
```
integrate(1/(a+b*x^(1/2))/x^4,x, algorithm="fricas")
```

output
```
-1/30*(60*b^6*x^3*log(b*sqrt(x) + a) - 60*b^6*x^3*log(sqrt(x)) + 30*a^2*b^
4*x^2 + 15*a^4*b^2*x + 10*a^6 - 4*(15*a*b^5*x^2 + 5*a^3*b^3*x + 3*a^5*b)*s
qrt(x))/(a^7*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ -\frac{1}{3ax^3} & \text{for } b = 0 \\ -\frac{1}{3ax^3} + \frac{2b}{5a^2x^{\frac{5}{2}}} - \frac{b^2}{2a^3x^2} + \frac{2b^3}{3a^4x^{\frac{3}{2}}} - \frac{b^4}{a^5x} + \frac{2b^5}{a^6\sqrt{x}} + \frac{b^6\log(x)}{a^7} - \frac{2b^6\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^7} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))/x**4,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0
)), (-1/(3*a*x**3), Eq(b, 0)), (-1/(3*a*x**3) + 2*b/(5*a**2*x**(5/2)) - b*
*2/(2*a**3*x**2) + 2*b**3/(3*a**4*x**(3/2)) - b**4/(a**5*x) + 2*b**5/(a**6
*sqrt(x)) + b**6*log(x)/a**7 - 2*b**6*log(a/b + sqrt(x))/a**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx = -\frac{2\,b^6\log\left(b\sqrt{x}+a\right)}{a^7} + \frac{b^6\log(x)}{a^7} + \frac{60\,b^5x^{\frac{5}{2}} - 30\,ab^4x^2 + 20\,a^2b^3x^{\frac{3}{2}} - 15\,a^3b^2x + 12\,a^4b\sqrt{x} - 10\,a^5}{30\,a^6x^3}$$

input
```
integrate(1/(a+b*x^(1/2))/x^4,x, algorithm="maxima")
```

output
```
-2*b^6*log(b*sqrt(x) + a)/a^7 + b^6*log(x)/a^7 + 1/30*(60*b^5*x^(5/2) - 30
*a*b^4*x^2 + 20*a^2*b^3*x^(3/2) - 15*a^3*b^2*x + 12*a^4*b*sqrt(x) - 10*a^5
)/(a^6*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx = -\frac{2\,b^6\log\left(\left|b\sqrt{x}+a\right|\right)}{a^7}+\frac{b^6\log\left(\left|x\right|\right)}{a^7} + \frac{60\,ab^5x^{\frac{5}{2}}-30\,a^2b^4x^2+20\,a^3b^3x^{\frac{3}{2}}-15\,a^4b^2x+12\,a^5b\sqrt{x}-10\,a^6}{30\,a^7x^3}$$

input
```
integrate(1/(a+b*x^(1/2))/x^4,x, algorithm="giac")
```

output
```
-2*b^6*log(abs(b*sqrt(x) + a))/a^7 + b^6*log(abs(x))/a^7 + 1/30*(60*a*b^5*
x^(5/2) - 30*a^2*b^4*x^2 + 20*a^3*b^3*x^(3/2) - 15*a^4*b^2*x + 12*a^5*b*sq
rt(x) - 10*a^6)/(a^7*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+b\sqrt{x}\right)x^4}\,dx = -\frac{\frac{1}{3\,a}-\frac{2\,b\,\sqrt{x}}{5\,a^2}+\frac{b^2\,x}{2\,a^3}+\frac{b^4\,x^2}{a^5}-\frac{2\,b^3\,x^{3/2}}{3\,a^4}-\frac{2\,b^5\,x^{5/2}}{a^6}}{x^3} - \frac{4\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^7}$$

input
```
int(1/(x^4*(a + b*x^(1/2))),x)
```

output
```
- (1/(3*a) - (2*b*x^(1/2))/(5*a^2) + (b^2*x)/(2*a^3) + (b^4*x^2)/a^5 - (2*
b^3*x^(3/2))/(3*a^4) - (2*b^5*x^(5/2))/a^6)/x^3 - (4*b^6*atanh((2*b*x^(1/2
))/a + 1))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+b\sqrt{x})\,x^4}\,dx$$
$$= \frac{12\sqrt{x}\,a^5b+20\sqrt{x}\,a^3b^3x+60\sqrt{x}\,a\,b^5x^2-60\log\left(\sqrt{x}\,b+a\right)b^6x^3+60\log\left(\sqrt{x}\right)b^6x^3-10a^6-15a^4b^2x-}{30a^7x^3}$$

input
```
int(1/(a+b*x^(1/2))/x^4,x)
```

output
```
(12*sqrt(x)*a**5*b + 20*sqrt(x)*a**3*b**3*x + 60*sqrt(x)*a*b**5*x**2 - 60*
log(sqrt(x)*b + a)*b**6*x**3 + 60*log(sqrt(x))*b**6*x**3 - 10*a**6 - 15*a*
*4*b**2*x - 30*a**2*b**4*x**2)/(30*a**7*x**3)
```

3.89 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 111

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{2a^7}{b^8\left(a+b\sqrt{x}\right)} - \frac{12a^5\sqrt{x}}{b^7} + \frac{5a^4x}{b^6} - \frac{8a^3x^{3/2}}{3b^5} + \frac{3a^2x^2}{2b^4} - \frac{4ax^{5/2}}{5b^3} + \frac{x^3}{3b^2} + \frac{14a^6\log\left(a+b\sqrt{x}\right)}{b^8}$$

output
```
2*a^7/b^8/(a+b*x^(1/2))-12*a^5*x^(1/2)/b^7+5*a^4*x/b^6-8/3*a^3*x^(3/2)/b^5
+3/2*a^2*x^2/b^4-4/5*a*x^(5/2)/b^3+1/3*x^3/b^2+14*a^6*ln(a+b*x^(1/2))/b^8
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{60a^7 - 360a^6b\sqrt{x} - 210a^5b^2x + 70a^4b^3x^{3/2} - 35a^3b^4x^2 + 21a^2b^5x^{5/2} - 14ab^6x^3 + 10b^7x^{7/2}}{30b^8\left(a+b\sqrt{x}\right)} + \frac{14a^6\log\left(a+b\sqrt{x}\right)}{b^8}$$

input
```
Integrate[x^3/(a + b*Sqrt[x])^2,x]
```

output
```
(60*a^7 - 360*a^6*b*Sqrt[x] - 210*a^5*b^2*x + 70*a^4*b^3*x^(3/2) - 35*a^3*
b^4*x^2 + 21*a^2*b^5*x^(5/2) - 14*a*b^6*x^3 + 10*b^7*x^(7/2))/(30*b^8*(a +
 b*Sqrt[x])) + (14*a^6*Log[a + b*Sqrt[x]])/b^8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2} \, dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{7/2}}{\left(a+b\sqrt{x}\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^7}{b^7\left(a+b\sqrt{x}\right)^2}+\frac{7a^6}{b^7\left(a+b\sqrt{x}\right)}-\frac{6a^5}{b^7}+\frac{5\sqrt{x}a^4}{b^6}-\frac{4xa^3}{b^5}+\frac{3x^{3/2}a^2}{b^4}-\frac{2x^2a}{b^3}+\frac{x^{5/2}}{b^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^7}{b^8\left(a+b\sqrt{x}\right)}+\frac{7a^6\log\left(a+b\sqrt{x}\right)}{b^8}-\frac{6a^5\sqrt{x}}{b^7}+\frac{5a^4x}{2b^6}-\frac{4a^3x^{3/2}}{3b^5}+\frac{3a^2x^2}{4b^4}-\frac{2ax^{5/2}}{5b^3}+\frac{x^3}{6b^2}\right)$$

input
```
Int[x^3/(a + b*Sqrt[x])^2,x]
```

output
```
2*(a^7/(b^8*(a + b*Sqrt[x])) - (6*a^5*Sqrt[x])/b^7 + (5*a^4*x)/(2*b^6) - (
4*a^3*x^(3/2))/(3*b^5) + (3*a^2*x^2)/(4*b^4) - (2*a*x^(5/2))/(5*b^3) + x^3
/(6*b^2) + (7*a^6*Log[a + b*Sqrt[x]])/b^8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2\left(-\frac{x^{3} b^{5}}{6}+\frac{2 a \,x^{\frac{5}{2}} b^{4}}{5}-\frac{3 a^{2} b^{3} x^{2}}{4}+\frac{4 a^{3} x^{\frac{3}{2}} b^{2}}{3}-\frac{5 a^{4} b x}{2}+6 a^{5} \sqrt{x}\right)}{b^{7}}+\frac{2 a^{7}}{b^{8} \left(a +b \sqrt{x}\right)}+\frac{14 a^{6} \ln \left(a +b \sqrt{x}\right)}{b^{8}}$	95
default	$-\frac{2\left(-\frac{x^{3} b^{5}}{6}+\frac{2 a \,x^{\frac{5}{2}} b^{4}}{5}-\frac{3 a^{2} b^{3} x^{2}}{4}+\frac{4 a^{3} x^{\frac{3}{2}} b^{2}}{3}-\frac{5 a^{4} b x}{2}+6 a^{5} \sqrt{x}\right)}{b^{7}}+\frac{2 a^{7}}{b^{8} \left(a +b \sqrt{x}\right)}+\frac{14 a^{6} \ln \left(a +b \sqrt{x}\right)}{b^{8}}$	95

input
```
int(x^3/(a+b*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^7*(-1/6*x^3*b^5+2/5*a*x^(5/2)*b^4-3/4*a^2*b^3*x^2+4/3*a^3*x^(3/2)*b^2
-5/2*a^4*b*x+6*a^5*x^(1/2))+2*a^7/b^8/(a+b*x^(1/2))+14*a^6*ln(a+b*x^(1/2))
/b^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{10\,b^8x^4+35\,a^2b^6x^3+105\,a^4b^4x^2-150\,a^6b^2x-60\,a^8+420\,(a^6b^2x-a^8)\log\left(b\sqrt{x}+a\right)-4\,(6\,ab^7x^3+14}{30\,(b^{10}x-a^2b^8)}$$

input
```
integrate(x^3/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

output
```
1/30*(10*b^8*x^4 + 35*a^2*b^6*x^3 + 105*a^4*b^4*x^2 - 150*a^6*b^2*x - 60*a
^8 + 420*(a^6*b^2*x - a^8)*log(b*sqrt(x) + a) - 4*(6*a*b^7*x^3 + 14*a^3*b^
5*x^2 + 70*a^5*b^3*x - 105*a^7*b)*sqrt(x))/(b^10*x - a^2*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(109) = 218.

Time = 0.55 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.45

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \begin{cases} \frac{420a^7\log\left(\frac{a}{b}+\sqrt{x}\right)}{30ab^8+30b^9\sqrt{x}}+\frac{420a^7}{30ab^8+30b^9\sqrt{x}}+\frac{420a^6b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{30ab^8+30b^9\sqrt{x}}-\frac{210a^5b^2x}{30ab^8+30b^9\sqrt{x}}+\frac{70a^4b^3x^{\frac{3}{2}}}{30ab^8+30b^9\sqrt{x}}-\frac{35a^3b^4x^2}{30ab^8+30b^9\sqrt{x}}+\frac{21a^2b}{30ab^8+3} \\ \frac{x^4}{4a^2} \end{cases}$$

input
```
integrate(x**3/(a+b*x**(1/2))**2,x)
```

output
```
Piecewise((420*a**7*log(a/b + sqrt(x))/(30*a*b**8 + 30*b**9*sqrt(x)) + 420
*a**7/(30*a*b**8 + 30*b**9*sqrt(x)) + 420*a**6*b*sqrt(x)*log(a/b + sqrt(x)
)/(30*a*b**8 + 30*b**9*sqrt(x)) - 210*a**5*b**2*x/(30*a*b**8 + 30*b**9*sqr
t(x)) + 70*a**4*b**3*x**(3/2)/(30*a*b**8 + 30*b**9*sqrt(x)) - 35*a**3*b**4
*x**2/(30*a*b**8 + 30*b**9*sqrt(x)) + 21*a**2*b**5*x**(5/2)/(30*a*b**8 + 3
0*b**9*sqrt(x)) - 14*a*b**6*x**3/(30*a*b**8 + 30*b**9*sqrt(x)) + 10*b**7*x
**(7/2)/(30*a*b**8 + 30*b**9*sqrt(x)), Ne(b, 0)), (x**4/(4*a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{14\,a^6\log\left(b\sqrt{x}+a\right)}{b^8} + \frac{\left(b\sqrt{x}+a\right)^6}{3\,b^8} - \frac{14\left(b\sqrt{x}+a\right)^5a}{5\,b^8} + \frac{21\left(b\sqrt{x}+a\right)^4a^2}{2\,b^8} - \frac{70\left(b\sqrt{x}+a\right)^3a^3}{3\,b^8} + \frac{35\left(b\sqrt{x}+a\right)^2a^4}{b^8} - \frac{42\left(b\sqrt{x}+a\right)a^5}{b^8} + \frac{2\,a^7}{\left(b\sqrt{x}+a\right)b^8}$$

input
```
integrate(x^3/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

output
```
14*a^6*log(b*sqrt(x) + a)/b^8 + 1/3*(b*sqrt(x) + a)^6/b^8 - 14/5*(b*sqrt(x
) + a)^5*a/b^8 + 21/2*(b*sqrt(x) + a)^4*a^2/b^8 - 70/3*(b*sqrt(x) + a)^3*a
^3/b^8 + 35*(b*sqrt(x) + a)^2*a^4/b^8 - 42*(b*sqrt(x) + a)*a^5/b^8 + 2*a^7
/((b*sqrt(x) + a)*b^8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{14\,a^6\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8} + \frac{2\,a^7}{\left(b\sqrt{x}+a\right)b^8} + \frac{10\,b^{10}x^3 - 24\,ab^9x^{\frac{5}{2}} + 45\,a^2b^8x^2 - 80\,a^3b^7x^{\frac{3}{2}} + 150\,a^4b^6x - 360\,a^5b^5\sqrt{x}}{30\,b^{12}}$$

input
```
integrate(x^3/(a+b*x^(1/2))^2,x, algorithm="giac")
```

output
```
14*a^6*log(abs(b*sqrt(x) + a))/b^8 + 2*a^7/((b*sqrt(x) + a)*b^8) + 1/30*(1
0*b^10*x^3 - 24*a*b^9*x^(5/2) + 45*a^2*b^8*x^2 - 80*a^3*b^7*x^(3/2) + 150*
a^4*b^6*x - 360*a^5*b^5*sqrt(x))/b^12
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{x^3}{3\,b^2}+\frac{2\,a^7}{b\,\left(a\,b^7+b^8\,\sqrt{x}\right)}-\frac{4\,a\,x^{5/2}}{5\,b^3}+\frac{5\,a^4\,x}{b^6}+\frac{14\,a^6\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}+\frac{3\,a^2\,x^2}{2\,b^4}-\frac{8\,a^3\,x^{3/2}}{3\,b^5}-\frac{12\,a^5\,\sqrt{x}}{b^7}$$

input
```
int(x^3/(a + b*x^(1/2))^2,x)
```

output
```
x^3/(3*b^2) + (2*a^7)/(b*(a*b^7 + b^8*x^(1/2))) - (4*a*x^(5/2))/(5*b^3) +
(5*a^4*x)/b^6 + (14*a^6*log(a + b*x^(1/2)))/b^8 + (3*a^2*x^2)/(2*b^4) - (8
*a^3*x^(3/2))/(3*b^5) - (12*a^5*x^(1/2))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{420\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^6b-420\sqrt{x}\,a^6b+70\sqrt{x}\,a^4b^3x+21\sqrt{x}\,a^2b^5x^2+10\sqrt{x}\,b^7x^3+420\log\left(\sqrt{x}\,b+a\right)a}{30b^8\left(\sqrt{x}\,b+a\right)}$$

input
```
int(x^3/(a+b*x^(1/2))^2,x)
```

output
```
(420*sqrt(x)*log(sqrt(x)*b + a)*a**6*b - 420*sqrt(x)*a**6*b + 70*sqrt(x)*a
**4*b**3*x + 21*sqrt(x)*a**2*b**5*x**2 + 10*sqrt(x)*b**7*x**3 + 420*log(sq
rt(x)*b + a)*a**7 - 210*a**5*b**2*x - 35*a**3*b**4*x**2 - 14*a*b**6*x**3)/
(30*b**8*(sqrt(x)*b + a))
```

3.90 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx=\frac{2a^5}{b^6\left(a+b\sqrt{x}\right)}-\frac{8a^3\sqrt{x}}{b^5}+\frac{3a^2x}{b^4}-\frac{4ax^{3/2}}{3b^3}+\frac{x^2}{2b^2}+\frac{10a^4\log\left(a+b\sqrt{x}\right)}{b^6}$$

output
```
2*a^5/b^6/(a+b*x^(1/2))-8*a^3*x^(1/2)/b^5+3*a^2*x/b^4-4/3*a*x^(3/2)/b^3+1/
2*x^2/b^2+10*a^4*ln(a+b*x^(1/2))/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx=\frac{12a^5-48a^4b\sqrt{x}-30a^3b^2x+10a^2b^3x^{3/2}-5ab^4x^2+3b^5x^{5/2}}{6b^6\left(a+b\sqrt{x}\right)}+\frac{10a^4\log\left(a+b\sqrt{x}\right)}{b^6}$$

input
```
Integrate[x^2/(a + b*Sqrt[x])^2,x]
```

output
```
(12*a^5 - 48*a^4*b*Sqrt[x] - 30*a^3*b^2*x + 10*a^2*b^3*x^(3/2) - 5*a*b^4*x
^2 + 3*b^5*x^(5/2))/(6*b^6*(a + b*Sqrt[x])) + (10*a^4*Log[a + b*Sqrt[x]])/
b^6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{5/2}}{\left(a+b\sqrt{x}\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^5}{b^5\left(a+b\sqrt{x}\right)^2}+\frac{5a^4}{b^5\left(a+b\sqrt{x}\right)}-\frac{4a^3}{b^5}+\frac{3\sqrt{x}a^2}{b^4}-\frac{2xa}{b^3}+\frac{x^{3/2}}{b^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^5}{b^6\left(a+b\sqrt{x}\right)}+\frac{5a^4\log\left(a+b\sqrt{x}\right)}{b^6}-\frac{4a^3\sqrt{x}}{b^5}+\frac{3a^2x}{2b^4}-\frac{2ax^{3/2}}{3b^3}+\frac{x^2}{4b^2}\right)$$

input
```
Int[x^2/(a + b*Sqrt[x])^2,x]
```

output
```
2*(a^5/(b^6*(a + b*Sqrt[x])) - (4*a^3*Sqrt[x])/b^5 + (3*a^2*x)/(2*b^4) - (
2*a*x^(3/2))/(3*b^3) + x^2/(4*b^2) + (5*a^4*Log[a + b*Sqrt[x]])/b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{2\left(-\frac{b^{3}x^{2}}{4}+\frac{2a\,b^{2}x^{\frac{3}{2}}}{3}-\frac{3a^{2}bx}{2}+4a^{3}\sqrt{x}\right)}{b^{5}}+\frac{2a^{5}}{b^{6}\left(a+b\sqrt{x}\right)}+\frac{10a^{4}\ln\left(a+b\sqrt{x}\right)}{b^{6}}$	73
default	$-\frac{2\left(-\frac{b^{3}x^{2}}{4}+\frac{2a\,b^{2}x^{\frac{3}{2}}}{3}-\frac{3a^{2}bx}{2}+4a^{3}\sqrt{x}\right)}{b^{5}}+\frac{2a^{5}}{b^{6}\left(a+b\sqrt{x}\right)}+\frac{10a^{4}\ln\left(a+b\sqrt{x}\right)}{b^{6}}$	73

input
```
int(x^2/(a+b*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^5*(-1/4*b^3*x^2+2/3*a*b^2*x^(3/2)-3/2*a^2*b*x+4*a^3*x^(1/2))+2*a^5/b^
6/(a+b*x^(1/2))+10*a^4*ln(a+b*x^(1/2))/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{3\,b^6x^3 + 15\,a^2b^4x^2 - 18\,a^4b^2x - 12\,a^6 + 60\,(a^4b^2x - a^6)\log\left(b\sqrt{x}+a\right) - 4\,(2\,ab^5x^2 + 10\,a^3b^3x - 15\,a^5b)}{6\,(b^8x - a^2b^6)}$$

input
```
integrate(x^2/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^6*x^3 + 15*a^2*b^4*x^2 - 18*a^4*b^2*x - 12*a^6 + 60*(a^4*b^2*x -
a^6)*log(b*sqrt(x) + a) - 4*(2*a*b^5*x^2 + 10*a^3*b^3*x - 15*a^5*b)*sqrt(x
))/(b^8*x - a^2*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(80) = 160.

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.55

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx = \begin{cases} \frac{60a^5\log\left(\frac{a}{b}+\sqrt{x}\right)}{6ab^6+6b^7\sqrt{x}} + \frac{60a^5}{6ab^6+6b^7\sqrt{x}} + \frac{60a^4b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{6ab^6+6b^7\sqrt{x}} - \frac{30a^3b^2x}{6ab^6+6b^7\sqrt{x}} + \frac{10a^2b^3x^{\frac{3}{2}}}{6ab^6+6b^7\sqrt{x}} - \frac{5ab^4x^2}{6ab^6+6b^7\sqrt{x}} + \frac{3b^5x^{\frac{5}{2}}}{6ab^6+6b^7\sqrt{x}} & \text{fo} \\ \frac{x^3}{3a^2} & \text{oth} \end{cases}$$

input
```
integrate(x**2/(a+b*x**(1/2))**2,x)
```

output
```
Piecewise((60*a**5*log(a/b + sqrt(x))/(6*a*b**6 + 6*b**7*sqrt(x)) + 60*a**
5/(6*a*b**6 + 6*b**7*sqrt(x)) + 60*a**4*b*sqrt(x)*log(a/b + sqrt(x))/(6*a*
b**6 + 6*b**7*sqrt(x)) - 30*a**3*b**2*x/(6*a*b**6 + 6*b**7*sqrt(x)) + 10*a
**2*b**3*x**(3/2)/(6*a*b**6 + 6*b**7*sqrt(x)) - 5*a*b**4*x**2/(6*a*b**6 +
6*b**7*sqrt(x)) + 3*b**5*x**(5/2)/(6*a*b**6 + 6*b**7*sqrt(x)), Ne(b, 0)),
(x**3/(3*a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{10\,a^4\log\left(b\sqrt{x}+a\right)}{b^6} + \frac{\left(b\sqrt{x}+a\right)^4}{2\,b^6} - \frac{10\left(b\sqrt{x}+a\right)^3 a}{3\,b^6} + \frac{10\left(b\sqrt{x}+a\right)^2 a^2}{b^6} - \frac{20\left(b\sqrt{x}+a\right)a^3}{b^6} + \frac{2\,a^5}{\left(b\sqrt{x}+a\right)b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

output
```
10*a^4*log(b*sqrt(x) + a)/b^6 + 1/2*(b*sqrt(x) + a)^4/b^6 - 10/3*(b*sqrt(x
) + a)^3*a/b^6 + 10*(b*sqrt(x) + a)^2*a^2/b^6 - 20*(b*sqrt(x) + a)*a^3/b^6
 + 2*a^5/((b*sqrt(x) + a)*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{10\,a^4\log\left(\left|b\sqrt{x}+a\right|\right)}{b^6} + \frac{2\,a^5}{\left(b\sqrt{x}+a\right)b^6} + \frac{3\,b^6x^2 - 8\,ab^5x^{\frac{3}{2}} + 18\,a^2b^4x - 48\,a^3b^3\sqrt{x}}{6\,b^8}$$

input
```
integrate(x^2/(a+b*x^(1/2))^2,x, algorithm="giac")
```

output
```
10*a^4*log(abs(b*sqrt(x) + a))/b^6 + 2*a^5/((b*sqrt(x) + a)*b^6) + 1/6*(3*
b^6*x^2 - 8*a*b^5*x^(3/2) + 18*a^2*b^4*x - 48*a^3*b^3*sqrt(x))/b^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{x^2}{2\,b^2}+\frac{2\,a^5}{b\,\left(a\,b^5+b^6\,\sqrt{x}\right)}+\frac{3\,a^2\,x}{b^4}-\frac{4\,a\,x^{3/2}}{3\,b^3}+\frac{10\,a^4\,\ln\left(a+b\,\sqrt{x}\right)}{b^6}-\frac{8\,a^3\,\sqrt{x}}{b^5}$$

input
```
int(x^2/(a + b*x^(1/2))^2,x)
```

output
```
x^2/(2*b^2) + (2*a^5)/(b*(a*b^5 + b^6*x^(1/2))) + (3*a^2*x)/b^4 - (4*a*x^(
3/2))/(3*b^3) + (10*a^4*log(a + b*x^(1/2)))/b^6 - (8*a^3*x^(1/2))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{60\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^4b-60\sqrt{x}\,a^4b+10\sqrt{x}\,a^2b^3x+3\sqrt{x}\,b^5x^2+60\log\left(\sqrt{x}\,b+a\right)a^5-30a^3b^2x-5a\,b^4}{6b^6\left(\sqrt{x}\,b+a\right)}$$

input
```
int(x^2/(a+b*x^(1/2))^2,x)
```

output
```
(60*sqrt(x)*log(sqrt(x)*b + a)*a**4*b - 60*sqrt(x)*a**4*b + 10*sqrt(x)*a**
2*b**3*x + 3*sqrt(x)*b**5*x**2 + 60*log(sqrt(x)*b + a)*a**5 - 30*a**3*b**2
*x - 5*a*b**4*x**2)/(6*b**6*(sqrt(x)*b + a))
```

3.91 $\int \frac{x}{\left(a+b\sqrt{x}\right)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2} \, dx = \frac{2a^3}{b^4\left(a+b\sqrt{x}\right)} - \frac{4a\sqrt{x}}{b^3} + \frac{x}{b^2} + \frac{6a^2\log\left(a+b\sqrt{x}\right)}{b^4}$$

output
```
2*a^3/b^4/(a+b*x^(1/2))-4*a*x^(1/2)/b^3+x/b^2+6*a^2*ln(a+b*x^(1/2))/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2} \, dx = \frac{2a^3-4a^2b\sqrt{x}-3ab^2x+b^3x^{3/2}}{b^4\left(a+b\sqrt{x}\right)} + \frac{6a^2\log\left(a+b\sqrt{x}\right)}{b^4}$$

input
```
Integrate[x/(a + b*Sqrt[x])^2,x]
```

output
```
(2*a^3 - 4*a^2*b*Sqrt[x] - 3*a*b^2*x + b^3*x^(3/2))/(b^4*(a + b*Sqrt[x]))
+ (6*a^2*Log[a + b*Sqrt[x]])/b^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{x^{3/2}}{\left(a+b\sqrt{x}\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(-\frac{a^3}{b^3\left(a+b\sqrt{x}\right)^2}+\frac{3a^2}{b^3\left(a+b\sqrt{x}\right)}-\frac{2a}{b^3}+\frac{\sqrt{x}}{b^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{a^3}{b^4\left(a+b\sqrt{x}\right)}+\frac{3a^2\log\left(a+b\sqrt{x}\right)}{b^4}-\frac{2a\sqrt{x}}{b^3}+\frac{x}{2b^2}\right)$$

input
```
Int[x/(a + b*Sqrt[x])^2,x]
```

output
```
2*(a^3/(b^4*(a + b*Sqrt[x])) - (2*a*Sqrt[x])/b^3 + x/(2*b^2) + (3*a^2*Log[
a + b*Sqrt[x]])/b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{2\left(-\frac{bx}{2}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{3}}{b^{4}\left(a+b\sqrt{x}\right)}+\frac{6a^{2}\ln\left(a+b\sqrt{x}\right)}{b^{4}}$	51
default	$-\frac{2\left(-\frac{bx}{2}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{3}}{b^{4}\left(a+b\sqrt{x}\right)}+\frac{6a^{2}\ln\left(a+b\sqrt{x}\right)}{b^{4}}$	51

input
```
int(x/(a+b*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^3*(-1/2*b*x+2*a*x^(1/2))+2*a^3/b^4/(a+b*x^(1/2))+6*a^2*ln(a+b*x^(1/2)
)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{b^4x^2-a^2b^2x-2\,a^4+6\,(a^2b^2x-a^4)\log\left(b\sqrt{x}+a\right)-2\,(2\,ab^3x-3\,a^3b)\sqrt{x}}{b^6x-a^2b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

output
```
(b^4*x^2 - a^2*b^2*x - 2*a^4 + 6*(a^2*b^2*x - a^4)*log(b*sqrt(x) + a) - 2*
(2*a*b^3*x - 3*a^3*b)*sqrt(x))/(b^6*x - a^2*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(51) = 102.

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.48

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx = \begin{cases} \frac{6a^3\log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^4+b^5\sqrt{x}}+\frac{6a^3}{ab^4+b^5\sqrt{x}}+\frac{6a^2b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^4+b^5\sqrt{x}}-\frac{3ab^2x}{ab^4+b^5\sqrt{x}}+\frac{b^3x^{\frac{3}{2}}}{ab^4+b^5\sqrt{x}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b*x**(1/2))**2,x)
```

output
```
Piecewise((6*a**3*log(a/b + sqrt(x))/(a*b**4 + b**5*sqrt(x)) + 6*a**3/(a*b
**4 + b**5*sqrt(x)) + 6*a**2*b*sqrt(x)*log(a/b + sqrt(x))/(a*b**4 + b**5*s
qrt(x)) - 3*a*b**2*x/(a*b**4 + b**5*sqrt(x)) + b**3*x**(3/2)/(a*b**4 + b**
5*sqrt(x)), Ne(b, 0)), (x**2/(2*a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{6\,a^2\log\left(b\sqrt{x}+a\right)}{b^4} + \frac{\left(b\sqrt{x}+a\right)^2}{b^4} - \frac{6\left(b\sqrt{x}+a\right)a}{b^4} + \frac{2\,a^3}{\left(b\sqrt{x}+a\right)b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

output
```
6*a^2*log(b*sqrt(x) + a)/b^4 + (b*sqrt(x) + a)^2/b^4 - 6*(b*sqrt(x) + a)*a
/b^4 + 2*a^3/((b*sqrt(x) + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{6\,a^2\log\left(\left|b\sqrt{x}+a\right|\right)}{b^4} + \frac{2\,a^3}{\left(b\sqrt{x}+a\right)b^4} + \frac{b^2x-4\,ab\sqrt{x}}{b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^2,x, algorithm="giac")
```

output
```
6*a^2*log(abs(b*sqrt(x) + a))/b^4 + 2*a^3/((b*sqrt(x) + a)*b^4) + (b^2*x -
 4*a*b*sqrt(x))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{x}{b^2} + \frac{2\,a^3}{b\,\left(a\,b^3+b^4\,\sqrt{x}\right)} - \frac{4\,a\,\sqrt{x}}{b^3} + \frac{6\,a^2\,\ln\left(a+b\,\sqrt{x}\right)}{b^4}$$

input
```
int(x/(a + b*x^(1/2))^2,x)
```

output
```
x/b^2 + (2*a^3)/(b*(a*b^3 + b^4*x^(1/2))) - (4*a*x^(1/2))/b^3 + (6*a^2*log
(a + b*x^(1/2)))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{6\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^2b-6\sqrt{x}\,a^2b+\sqrt{x}\,b^3x+6\log\left(\sqrt{x}\,b+a\right)a^3-3a\,b^2x}{b^4\left(\sqrt{x}\,b+a\right)}$$

input
```
int(x/(a+b*x^(1/2))^2,x)
```

output
```
(6*sqrt(x)*log(sqrt(x)*b + a)*a**2*b - 6*sqrt(x)*a**2*b + sqrt(x)*b**3*x +
 6*log(sqrt(x)*b + a)*a**3 - 3*a*b**2*x)/(b**4*(sqrt(x)*b + a))
```

3.92 $\int \frac{1}{(a+b\sqrt{x})^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{1}{(a+b\sqrt{x})^2} \, dx = \frac{2a}{b^2 (a+b\sqrt{x})} + \frac{2\log(a+b\sqrt{x})}{b^2}$$

output
```
2*a/b^2/(a+b*x^(1/2))+2*ln(a+b*x^(1/2))/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+b\sqrt{x})^2} \, dx = \frac{2\left(\frac{a}{a+b\sqrt{x}} + \log(a+b\sqrt{x})\right)}{b^2}$$

input
```
Integrate[(a + b*Sqrt[x])^(-2),x]
```

output
```
(2*(a/(a + b*Sqrt[x]) + Log[a + b*Sqrt[x]]))/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2}dx$$

$$\Big\downarrow \text{ 774}$$

$$2\int \frac{\sqrt{x}}{\left(a+b\sqrt{x}\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(\frac{1}{b\left(a+b\sqrt{x}\right)}-\frac{a}{b\left(a+b\sqrt{x}\right)^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{a}{b^2\left(a+b\sqrt{x}\right)}+\frac{\log\left(a+b\sqrt{x}\right)}{b^2}\right)$$

input
```
Int[(a + b*Sqrt[x])^(-2),x]
```

output
```
2*(a/(b^2*(a + b*Sqrt[x])) + Log[a + b*Sqrt[x]]/b^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	s
derivativedivides	$\frac{2a}{b^2(a+b\sqrt{x})}+\frac{2\ln(a+b\sqrt{x})}{b^2}$	3
default	$\frac{a^2}{(-b^2x+a^2)b^2}+\frac{\ln(b^2x-a^2)}{b^2}-\frac{a^2}{b^2(b^2x-a^2)}+\frac{a}{b^2(b\sqrt{x}-a)}-\frac{\ln(b\sqrt{x}-a)}{b^2}+\frac{a}{b^2(a+b\sqrt{x})}+\frac{\ln(a+b\sqrt{x})}{b^2}$	1

input
```
int(1/(a+b*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*a/b^2/(a+b*x^(1/2))+2*ln(a+b*x^(1/2))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx=\frac{2\left(ab\sqrt{x}-a^2+(b^2x-a^2)\log\left(b\sqrt{x}+a\right)\right)}{b^4x-a^2b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

output
```
2*(a*b*sqrt(x) - a^2 + (b^2*x - a^2)*log(b*sqrt(x) + a))/(b^4*x - a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.42

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx = \begin{cases} \frac{2a\log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^2+b^3\sqrt{x}} + \frac{2a}{ab^2+b^3\sqrt{x}} + \frac{2b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{ab^2+b^3\sqrt{x}} & \text{for } b \neq 0 \\ \frac{x}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**2,x)
```

output
```
Piecewise((2*a*log(a/b + sqrt(x))/(a*b**2 + b**3*sqrt(x)) + 2*a/(a*b**2 +
b**3*sqrt(x)) + 2*b*sqrt(x)*log(a/b + sqrt(x))/(a*b**2 + b**3*sqrt(x)), Ne
(b, 0)), (x/a**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{2\,\log\left(b\sqrt{x}+a\right)}{b^2} + \frac{2\,a}{\left(b\sqrt{x}+a\right)b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

output
```
2*log(b*sqrt(x) + a)/b^2 + 2*a/((b*sqrt(x) + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx = \frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{b^2} + \frac{2\,a}{\left(b\sqrt{x}+a\right)b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^2,x, algorithm="giac")
```

output
```
2*log(abs(b*sqrt(x) + a))/b^2 + 2*a/((b*sqrt(x) + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx=\frac{2\,\ln\left(a+b\,\sqrt{x}\right)}{b^2}+\frac{2\,a}{b^2\,\left(a+b\,\sqrt{x}\right)}$$

input
```
int(1/(a + b*x^(1/2))^2,x)
```

output
```
(2*log(a + b*x^(1/2)))/b^2 + (2*a)/(b^2*(a + b*x^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx=\frac{2\sqrt{x}\log\left(\sqrt{x}\,b+a\right)b-2\sqrt{x}\,b+2\log\left(\sqrt{x}\,b+a\right)a}{b^2\left(\sqrt{x}\,b+a\right)}$$

input
```
int(1/(a+b*x^(1/2))^2,x)
```

output
```
(2*(sqrt(x)*log(sqrt(x)*b + a)*b - sqrt(x)*b + log(sqrt(x)*b + a)*a))/(b**
2*(sqrt(x)*b + a))
```

3.93 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = \frac{2}{a\left(a+b\sqrt{x}\right)} - \frac{2\log\left(a+b\sqrt{x}\right)}{a^2} + \frac{\log(x)}{a^2}$$

output
```
2/a/(a+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^2+ln(x)/a^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = \frac{2\left(\frac{a}{a+b\sqrt{x}} - \log\left(a+b\sqrt{x}\right) + \frac{\log(x)}{2}\right)}{a^2}$$

input
```
Integrate[1/((a + b*Sqrt[x])^2*x),x]
```

output
```
(2*(a/(a + b*Sqrt[x]) - Log[a + b*Sqrt[x]] + Log[x]/2))/a^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{x}\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^2\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{b}{a^2\left(a+b\sqrt{x}\right)}-\frac{b}{a\left(a+b\sqrt{x}\right)^2}+\frac{1}{a^2\sqrt{x}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\log\left(a+b\sqrt{x}\right)}{a^2}+\frac{\log\left(\sqrt{x}\right)}{a^2}+\frac{1}{a\left(a+b\sqrt{x}\right)}\right)$$

input
```
Int[1/((a + b*Sqrt[x])^2*x),x]
```

output
```
2*(1/(a*(a + b*Sqrt[x])) - Log[a + b*Sqrt[x]]/a^2 + Log[Sqrt[x]]/a^2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{2}{a(a+b\sqrt{x})}-\frac{2\ln(a+b\sqrt{x})}{a^2}+\frac{\ln(x)}{a^2}$	35
default	$\frac{2}{a(a+b\sqrt{x})}-\frac{2\ln(a+b\sqrt{x})}{a^2}+\frac{\ln(x)}{a^2}$	35

input
```
int(1/(a+b*x^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
2/a/(a+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^2+ln(x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = \frac{2\left(ab\sqrt{x}-a^2-(b^2x-a^2)\log\left(b\sqrt{x}+a\right)+(b^2x-a^2)\log\left(\sqrt{x}\right)\right)}{a^2b^2x-a^4}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x,x, algorithm="fricas")
```

output
```
2*(a*b*sqrt(x) - a^2 - (b^2*x - a^2)*log(b*sqrt(x) + a) + (b^2*x - a^2)*lo
g(sqrt(x)))/(a^2*b^2*x - a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(32) = 64$.

Time = 0.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.97

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = \begin{cases} \frac{\tilde{\infty}}{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{\log(x)}{a^2} & \text{for } b = 0 \\ -\frac{1}{b^2 x} & \text{for } a = 0 \\ \frac{a\sqrt{x}\log(x)}{a^3\sqrt{x}+a^2bx} - \frac{2a\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^3\sqrt{x}+a^2bx} + \frac{2a\sqrt{x}}{a^3\sqrt{x}+a^2bx} + \frac{bx\log(x)}{a^3\sqrt{x}+a^2bx} - \frac{2bx\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^3\sqrt{x}+a^2bx} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**2/x,x)
```

output
```
Piecewise((zoo/x, Eq(a, 0) & Eq(b, 0)), (log(x)/a**2, Eq(b, 0)), (-1/(b**2
*x), Eq(a, 0)), (a*sqrt(x)*log(x)/(a**3*sqrt(x) + a**2*b*x) - 2*a*sqrt(x)*
log(a/b + sqrt(x))/(a**3*sqrt(x) + a**2*b*x) + 2*a*sqrt(x)/(a**3*sqrt(x) +
 a**2*b*x) + b*x*log(x)/(a**3*sqrt(x) + a**2*b*x) - 2*b*x*log(a/b + sqrt(x
))/(a**3*sqrt(x) + a**2*b*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = \frac{2}{ab\sqrt{x}+a^2} - \frac{2\log\left(b\sqrt{x}+a\right)}{a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x,x, algorithm="maxima")
```

output
```
2/(a*b*sqrt(x) + a^2) - 2*log(b*sqrt(x) + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = -\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{2}{\left(b\sqrt{x}+a\right)a}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x,x, algorithm="giac")
```

output
```
-2*log(abs(b*sqrt(x) + a))/a^2 + log(abs(x))/a^2 + 2/((b*sqrt(x) + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = \frac{2}{a\,\left(a+b\,\sqrt{x}\right)} - \frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^2}$$

input
```
int(1/(x*(a + b*x^(1/2))^2),x)
```

output
```
2/(a*(a + b*x^(1/2))) - (4*atanh((2*b*x^(1/2))/a + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x}\,dx = \frac{-2\sqrt{x}\log\left(\sqrt{x}\,b+a\right)b+2\sqrt{x}\log\left(\sqrt{x}\right)b-2\sqrt{x}\,b-2\log\left(\sqrt{x}\,b+a\right)a+2\log\left(\sqrt{x}\right)a}{a^2\left(\sqrt{x}\,b+a\right)}$$

input
```
int(1/(a+b*x^(1/2))^2/x,x)
```

output
```
(2*( - sqrt(x)*log(sqrt(x)*b + a)*b + sqrt(x)*log(sqrt(x))*b - sqrt(x)*b -
 log(sqrt(x)*b + a)*a + log(sqrt(x))*a))/(a**2*(sqrt(x)*b + a))
```

3.94 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2} \, dx = \frac{2b^2}{a^3\left(a+b\sqrt{x}\right)} - \frac{1}{a^2x} + \frac{4b}{a^3\sqrt{x}} - \frac{6b^2\log\left(a+b\sqrt{x}\right)}{a^4} + \frac{3b^2\log(x)}{a^4}$$

output
```
2*b^2/a^3/(a+b*x^(1/2))-1/a^2/x+4*b/a^3/x^(1/2)-6*b^2*ln(a+b*x^(1/2))/a^4+
3*b^2*ln(x)/a^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2} \, dx = \frac{\frac{a\left(-a^2+3ab\sqrt{x}+6b^2x\right)}{\left(a+b\sqrt{x}\right)x} - 6b^2\log\left(a+b\sqrt{x}\right) + 3b^2\log(x)}{a^4}$$

input
```
Integrate[1/((a + b*Sqrt[x])^2*x^2),x]
```

output
```
((a*(-a^2 + 3*a*b*Sqrt[x] + 6*b^2*x))/((a + b*Sqrt[x])*x) - 6*b^2*Log[a +
b*Sqrt[x]] + 3*b^2*Log[x])/a^4
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt{x}\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{3b^3}{a^4\left(a+b\sqrt{x}\right)}-\frac{b^3}{a^3\left(a+b\sqrt{x}\right)^2}+\frac{3b^2}{a^4\sqrt{x}}-\frac{2b}{a^3x}+\frac{1}{a^2x^{3/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{3b^2\log\left(a+b\sqrt{x}\right)}{a^4}+\frac{3b^2\log\left(\sqrt{x}\right)}{a^4}+\frac{b^2}{a^3\left(a+b\sqrt{x}\right)}+\frac{2b}{a^3\sqrt{x}}-\frac{1}{2a^2x}\right)$$

input
```
Int[1/((a + b*Sqrt[x])^2*x^2),x]
```

output
```
2*(b^2/(a^3*(a + b*Sqrt[x])) - 1/(2*a^2*x) + (2*b)/(a^3*Sqrt[x]) - (3*b^2*
Log[a + b*Sqrt[x]])/a^4 + (3*b^2*Log[Sqrt[x]])/a^4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2b^2}{a^3(a+b\sqrt{x})}-\frac{1}{a^2x}+\frac{4b}{a^3\sqrt{x}}-\frac{6b^2\ln(a+b\sqrt{x})}{a^4}+\frac{3b^2\ln(x)}{a^4}$	62
default	$\frac{2b^2}{a^3(a+b\sqrt{x})}-\frac{1}{a^2x}+\frac{4b}{a^3\sqrt{x}}-\frac{6b^2\ln(a+b\sqrt{x})}{a^4}+\frac{3b^2\ln(x)}{a^4}$	62

input
```
int(1/(a+b*x^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*b^2/a^3/(a+b*x^(1/2))-1/a^2/x+4*b/a^3/x^(1/2)-6*b^2*ln(a+b*x^(1/2))/a^4+
3*b^2*ln(x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2x^2}\,dx = -\frac{3\,a^2b^2x-a^4+6\,(b^4x^2-a^2b^2x)\log\left(b\sqrt{x}+a\right)-6\,(b^4x^2-a^2b^2x)\log\left(\sqrt{x}\right)-2\,(3\,ab^3x-2\,a^3b)\sqrt{x}}{a^4b^2x^2-a^6x}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
-(3*a^2*b^2*x - a^4 + 6*(b^4*x^2 - a^2*b^2*x)*log(b*sqrt(x) + a) - 6*(b^4*
x^2 - a^2*b^2*x)*log(sqrt(x)) - 2*(3*a*b^3*x - 2*a^3*b)*sqrt(x))/(a^4*b^2*
x^2 - a^6*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(65) = 130$.

Time = 0.54 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.55

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^2} \\ -\frac{1}{a^2x} \\ -\frac{1}{2b^2x^2} \\ -\frac{a^3\sqrt{x}}{a^5x^{\frac{3}{2}}+a^4bx^2} + \frac{3a^2bx}{a^5x^{\frac{3}{2}}+a^4bx^2} + \frac{3ab^2x^{\frac{3}{2}}\log(x)}{a^5x^{\frac{3}{2}}+a^4bx^2} - \frac{6ab^2x^{\frac{3}{2}}\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5x^{\frac{3}{2}}+a^4bx^2} + \frac{6ab^2x^{\frac{3}{2}}}{a^5x^{\frac{3}{2}}+a^4bx^2} + \frac{3b^3x^2\log(x)}{a^5x^{\frac{3}{2}}+a^4bx^2} - \frac{6b^3x^2\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5x^{\frac{3}{2}}+a^4bx^2} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**2/x**2,x)
```

output
```
Piecewise((zoo/x**2, Eq(a, 0) & Eq(b, 0)), (-1/(a**2*x), Eq(b, 0)), (-1/(2
*b**2*x**2), Eq(a, 0)), (-a**3*sqrt(x)/(a**5*x**(3/2) + a**4*b*x**2) + 3*a
**2*b*x/(a**5*x**(3/2) + a**4*b*x**2) + 3*a*b**2*x**(3/2)*log(x)/(a**5*x**
(3/2) + a**4*b*x**2) - 6*a*b**2*x**(3/2)*log(a/b + sqrt(x))/(a**5*x**(3/2)
 + a**4*b*x**2) + 6*a*b**2*x**(3/2)/(a**5*x**(3/2) + a**4*b*x**2) + 3*b**3
*x**2*log(x)/(a**5*x**(3/2) + a**4*b*x**2) - 6*b**3*x**2*log(a/b + sqrt(x)
)/(a**5*x**(3/2) + a**4*b*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx = \frac{6\,b^2x+3\,ab\sqrt{x}-a^2}{a^3bx^{\frac{3}{2}}+a^4x} - \frac{6\,b^2\log\left(b\sqrt{x}+a\right)}{a^4} + \frac{3\,b^2\log\left(x\right)}{a^4}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
(6*b^2*x + 3*a*b*sqrt(x) - a^2)/(a^3*b*x^(3/2) + a^4*x) - 6*b^2*log(b*sqrt
(x) + a)/a^4 + 3*b^2*log(x)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx = -\frac{6\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^4} + \frac{3\,b^2\log\left(\left|x\right|\right)}{a^4} + \frac{6\,ab^2x+3\,a^2b\sqrt{x}-a^3}{\left(b\sqrt{x}+a\right)a^4x}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
-6*b^2*log(abs(b*sqrt(x) + a))/a^4 + 3*b^2*log(abs(x))/a^4 + (6*a*b^2*x +
3*a^2*b*sqrt(x) - a^3)/((b*sqrt(x) + a)*a^4*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx = \frac{\frac{3\,b\,\sqrt{x}}{a^2}-\frac{1}{a}+\frac{6\,b^2\,x}{a^3}}{a\,x+b\,x^{3/2}} - \frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^4}$$

input
```
int(1/(x^2*(a + b*x^(1/2))^2),x)
```

output
```
((3*b*x^(1/2))/a^2 - 1/a + (6*b^2*x)/a^3)/(a*x + b*x^(3/2)) - (12*b^2*atan
h((2*b*x^(1/2))/a + 1))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^2}\,dx$$
$$= \frac{-6\sqrt{x}\log\left(\sqrt{x}\,b+a\right)b^3x+6\sqrt{x}\log\left(\sqrt{x}\right)b^3x+3\sqrt{x}\,a^2b-6\sqrt{x}\,b^3x-6\log\left(\sqrt{x}\,b+a\right)a\,b^2x+6\log\left(\sqrt{x}\right)}{a^4x\left(\sqrt{x}\,b+a\right)}$$

input
```
int(1/(a+b*x^(1/2))^2/x^2,x)
```

output
```
( - 6*sqrt(x)*log(sqrt(x)*b + a)*b**3*x + 6*sqrt(x)*log(sqrt(x))*b**3*x +
3*sqrt(x)*a**2*b - 6*sqrt(x)*b**3*x - 6*log(sqrt(x)*b + a)*a*b**2*x + 6*lo
g(sqrt(x))*a*b**2*x - a**3)/(a**4*x*(sqrt(x)*b + a))
```

3.95 $\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3} \, dx = \frac{2b^4}{a^5\left(a+b\sqrt{x}\right)} - \frac{1}{2a^2x^2} + \frac{4b}{3a^3x^{3/2}} - \frac{3b^2}{a^4x} + \frac{8b^3}{a^5\sqrt{x}} - \frac{10b^4\log\left(a+b\sqrt{x}\right)}{a^6} + \frac{5b^4\log(x)}{a^6}$$

output
```
2*b^4/a^5/(a+b*x^(1/2))-1/2/a^2/x^2+4/3*b/a^3/x^(3/2)-3*b^2/a^4/x+8*b^3/a^
5/x^(1/2)-10*b^4*ln(a+b*x^(1/2))/a^6+5*b^4*ln(x)/a^6
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3} \, dx = \frac{\frac{a\left(-3a^4+5a^3b\sqrt{x}-10a^2b^2x+30ab^3x^{3/2}+60b^4x^2\right)}{\left(a+b\sqrt{x}\right)x^2} - 60b^4\log\left(a+b\sqrt{x}\right) + 30b^4\log(x)}{6a^6}$$

input
```
Integrate[1/((a + b*Sqrt[x])^2*x^3),x]
```

output
```
((a*(-3*a^4 + 5*a^3*b*Sqrt[x] - 10*a^2*b^2*x + 30*a*b^3*x^(3/2) + 60*b^4*x
^2))/((a + b*Sqrt[x])*x^2) - 60*b^4*Log[a + b*Sqrt[x]] + 30*b^4*Log[x])/(6
*a^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sqrt{x}\right)^2} \, dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{5b^5}{a^6\left(a+b\sqrt{x}\right)}-\frac{b^5}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{5b^4}{a^6\sqrt{x}}-\frac{4b^3}{a^5x}+\frac{3b^2}{a^4x^{3/2}}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^{5/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{5b^4\log\left(a+b\sqrt{x}\right)}{a^6}+\frac{5b^4\log\left(\sqrt{x}\right)}{a^6}+\frac{b^4}{a^5\left(a+b\sqrt{x}\right)}+\frac{4b^3}{a^5\sqrt{x}}-\frac{3b^2}{2a^4x}+\frac{2b}{3a^3x^{3/2}}-\frac{1}{4a^2x^2}\right)$$

input
```
Int[1/((a + b*Sqrt[x])^2*x^3),x]
```

output
```
2*(b^4/(a^5*(a + b*Sqrt[x])) - 1/(4*a^2*x^2) + (2*b)/(3*a^3*x^(3/2)) - (3*
b^2)/(2*a^4*x) + (4*b^3)/(a^5*Sqrt[x]) - (5*b^4*Log[a + b*Sqrt[x]])/a^6 +
(5*b^4*Log[Sqrt[x]])/a^6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2b^4}{a^5(a+b\sqrt{x})}-\frac{1}{2a^2x^2}+\frac{4b}{3a^3x^{\frac{3}{2}}}-\frac{3b^2}{a^4x}+\frac{8b^3}{a^5\sqrt{x}}-\frac{10b^4\ln(a+b\sqrt{x})}{a^6}+\frac{5b^4\ln(x)}{a^6}$	84
default	$\frac{2b^4}{a^5(a+b\sqrt{x})}-\frac{1}{2a^2x^2}+\frac{4b}{3a^3x^{\frac{3}{2}}}-\frac{3b^2}{a^4x}+\frac{8b^3}{a^5\sqrt{x}}-\frac{10b^4\ln(a+b\sqrt{x})}{a^6}+\frac{5b^4\ln(x)}{a^6}$	84

input
```
int(1/(a+b*x^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*b^4/a^5/(a+b*x^(1/2))-1/2/a^2/x^2+4/3*b/a^3/x^(3/2)-3*b^2/a^4/x+8*b^3/a^
5/x^(1/2)-10*b^4*ln(a+b*x^(1/2))/a^6+5*b^4*ln(x)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx = -\frac{30\,a^2b^4x^2-15\,a^4b^2x-3\,a^6+60\,(b^6x^3-a^2b^4x^2)\log\left(b\sqrt{x}+a\right)-60\,(b^6x^3-a^2b^4x^2)\log\left(\sqrt{x}\right)-4\,(15}{6\,(a^6b^2x^3-a^8x^2)}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^3,x, algorithm="fricas")
```

output
```
-1/6*(30*a^2*b^4*x^2 - 15*a^4*b^2*x - 3*a^6 + 60*(b^6*x^3 - a^2*b^4*x^2)*l
og(b*sqrt(x) + a) - 60*(b^6*x^3 - a^2*b^4*x^2)*log(sqrt(x)) - 4*(15*a*b^5*
x^2 - 10*a^3*b^3*x - 2*a^5*b)*sqrt(x))/(a^6*b^2*x^3 - a^8*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(94) = 188$.

Time = 0.88 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.51

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^3} \\ -\frac{1}{2a^2x^2} \\ -\frac{1}{3b^2x^3} \\ -\frac{3a^5\sqrt{x}}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{5a^4bx}{6a^7x^{\frac{5}{2}}+6a^6bx^3} - \frac{10a^3b^2x^{\frac{3}{2}}}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{30a^2b^3x^2}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{30ab^4x^{\frac{5}{2}}\log(x)}{6a^7x^{\frac{5}{2}}+6a^6bx^3} - \frac{60ab^4x^{\frac{5}{2}}\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^7x^{\frac{5}{2}}+6a^6bx^3} + \frac{60ab^4\cdots}{6a^7x^{\frac{5}{2}}+\cdots} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**2/x**3,x)
```

output
```
Piecewise((zoo/x**3, Eq(a, 0) & Eq(b, 0)), (-1/(2*a**2*x**2), Eq(b, 0)), (
-1/(3*b**2*x**3), Eq(a, 0)), (-3*a**5*sqrt(x)/(6*a**7*x**(5/2) + 6*a**6*b*
x**3) + 5*a**4*b*x/(6*a**7*x**(5/2) + 6*a**6*b*x**3) - 10*a**3*b**2*x**(3/
2)/(6*a**7*x**(5/2) + 6*a**6*b*x**3) + 30*a**2*b**3*x**2/(6*a**7*x**(5/2)
+ 6*a**6*b*x**3) + 30*a*b**4*x**(5/2)*log(x)/(6*a**7*x**(5/2) + 6*a**6*b*x
**3) - 60*a*b**4*x**(5/2)*log(a/b + sqrt(x))/(6*a**7*x**(5/2) + 6*a**6*b*x
**3) + 60*a*b**4*x**(5/2)/(6*a**7*x**(5/2) + 6*a**6*b*x**3) + 30*b**5*x**3
*log(x)/(6*a**7*x**(5/2) + 6*a**6*b*x**3) - 60*b**5*x**3*log(a/b + sqrt(x)
)/(6*a**7*x**(5/2) + 6*a**6*b*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx = \frac{60\,b^4x^2+30\,ab^3x^{\frac{3}{2}}-10\,a^2b^2x+5\,a^3b\sqrt{x}-3\,a^4}{6\left(a^5bx^{\frac{5}{2}}+a^6x^2\right)} - \frac{10\,b^4\log\left(b\sqrt{x}+a\right)}{a^6} + \frac{5\,b^4\log\left(x\right)}{a^6}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^3,x, algorithm="maxima")
```

output
```
1/6*(60*b^4*x^2 + 30*a*b^3*x^(3/2) - 10*a^2*b^2*x + 5*a^3*b*sqrt(x) - 3*a^
4)/(a^5*b*x^(5/2) + a^6*x^2) - 10*b^4*log(b*sqrt(x) + a)/a^6 + 5*b^4*log(x
)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx = -\frac{10\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^6} + \frac{5\,b^4\log\left(\left|x\right|\right)}{a^6} + \frac{60\,ab^4x^2+30\,a^2b^3x^{\frac{3}{2}}-10\,a^3b^2x+5\,a^4b\sqrt{x}-3\,a^5}{6\left(b\sqrt{x}+a\right)a^6x^2}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^3,x, algorithm="giac")
```

output
```
-10*b^4*log(abs(b*sqrt(x) + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/6*(60*a*b^
4*x^2 + 30*a^2*b^3*x^(3/2) - 10*a^3*b^2*x + 5*a^4*b*sqrt(x) - 3*a^5)/((b*s
qrt(x) + a)*a^6*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx = \frac{\frac{5\,b\,\sqrt{x}}{6\,a^2}-\frac{1}{2\,a}-\frac{5\,b^2\,x}{3\,a^3}+\frac{10\,b^4\,x^2}{a^5}+\frac{5\,b^3\,x^{3/2}}{a^4}}{a\,x^2+b\,x^{5/2}}-\frac{20\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^6}$$

input
```
int(1/(x^3*(a + b*x^(1/2))^2),x)
```

output
```
((5*b*x^(1/2))/(6*a^2) - 1/(2*a) - (5*b^2*x)/(3*a^3) + (10*b^4*x^2)/a^5 +
(5*b^3*x^(3/2))/a^4)/(a*x^2 + b*x^(5/2)) - (20*b^4*atanh((2*b*x^(1/2))/a +
 1))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^3}\,dx$$
$$= \frac{-60\sqrt{x}\log\left(\sqrt{x}\,b+a\right)b^5x^2+60\sqrt{x}\log\left(\sqrt{x}\right)b^5x^2+5\sqrt{x}\,a^4b+30\sqrt{x}\,a^2b^3x-60\sqrt{x}\,b^5x^2-60\log\left(\sqrt{x}\,b\right.}{6a^6x^2\left(\sqrt{x}\,b+a\right)}$$

input
```
int(1/(a+b*x^(1/2))^2/x^3,x)
```

output
```
( - 60*sqrt(x)*log(sqrt(x)*b + a)*b**5*x**2 + 60*sqrt(x)*log(sqrt(x))*b**5
*x**2 + 5*sqrt(x)*a**4*b + 30*sqrt(x)*a**2*b**3*x - 60*sqrt(x)*b**5*x**2 -
 60*log(sqrt(x)*b + a)*a*b**4*x**2 + 60*log(sqrt(x))*a*b**4*x**2 - 3*a**5
- 10*a**3*b**2*x)/(6*a**6*x**2*(sqrt(x)*b + a))
```

3.96 $\int \frac{1}{(a+b\sqrt{x})^2 x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4} dx = \frac{2b^6}{a^7\left(a+b\sqrt{x}\right)} - \frac{1}{3a^2x^3} + \frac{4b}{5a^3x^{5/2}} - \frac{3b^2}{2a^4x^2} + \frac{8b^3}{3a^5x^{3/2}} - \frac{5b^4}{a^6x} + \frac{12b^5}{a^7\sqrt{x}} - \frac{14b^6\log\left(a+b\sqrt{x}\right)}{a^8} + \frac{7b^6\log(x)}{a^8}$$

output
```
2*b^6/a^7/(a+b*x^(1/2))-1/3/a^2/x^3+4/5*b/a^3/x^(5/2)-3/2*b^2/a^4/x^2+8/3*
b^3/a^5/x^(3/2)-5*b^4/a^6/x+12*b^5/a^7/x^(1/2)-14*b^6*ln(a+b*x^(1/2))/a^8+
7*b^6*ln(x)/a^8
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4} dx = \frac{\frac{a\left(-10a^6+14a^5b\sqrt{x}-21a^4b^2x+35a^3b^3x^{3/2}-70a^2b^4x^2+210ab^5x^{5/2}+420b^6x^3\right)}{\left(a+b\sqrt{x}\right)x^3} - 420b^6\log\left(a+b\sqrt{x}\right) + 210b^6\log(x)}{30a^8}$$

input
```
Integrate[1/((a + b*Sqrt[x])^2*x^4),x]
```

output
```
((a*(-10*a^6 + 14*a^5*b*Sqrt[x] - 21*a^4*b^2*x + 35*a^3*b^3*x^(3/2) - 70*a
^2*b^4*x^2 + 210*a*b^5*x^(5/2) + 420*b^6*x^3))/((a + b*Sqrt[x])*x^3) - 420
*b^6*Log[a + b*Sqrt[x]] + 210*b^6*Log[x])/(30*a^8)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\sqrt{x}\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{7b^7}{a^8\left(a+b\sqrt{x}\right)}-\frac{b^7}{a^7\left(a+b\sqrt{x}\right)^2}+\frac{7b^6}{a^8\sqrt{x}}-\frac{6b^5}{a^7x}+\frac{5b^4}{a^6x^{3/2}}-\frac{4b^3}{a^5x^2}+\frac{3b^2}{a^4x^{5/2}}-\frac{2b}{a^3x^3}+\frac{1}{a^2x^{7/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{7b^6\log\left(a+b\sqrt{x}\right)}{a^8}+\frac{7b^6\log\left(\sqrt{x}\right)}{a^8}+\frac{b^6}{a^7\left(a+b\sqrt{x}\right)}+\frac{6b^5}{a^7\sqrt{x}}-\frac{5b^4}{2a^6x}+\frac{4b^3}{3a^5x^{3/2}}-\frac{3b^2}{4a^4x^2}+\frac{2b}{5a^3x^{5/2}}-\frac{1}{6a^2x^3}\right.$$

input
```
Int[1/((a + b*Sqrt[x])^2*x^4),x]
```

output
```
2*(b^6/(a^7*(a + b*Sqrt[x])) - 1/(6*a^2*x^3) + (2*b)/(5*a^3*x^(5/2)) - (3*
b^2)/(4*a^4*x^2) + (4*b^3)/(3*a^5*x^(3/2)) - (5*b^4)/(2*a^6*x) + (6*b^5)/(
a^7*Sqrt[x]) - (7*b^6*Log[a + b*Sqrt[x]])/a^8 + (7*b^6*Log[Sqrt[x]])/a^8)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

method	result	siz
derivativedivides	$\frac{2b^6}{a^7(a+b\sqrt{x})}-\frac{1}{3a^2x^3}+\frac{4b}{5a^3x^{\frac{5}{2}}}-\frac{3b^2}{2a^4x^2}+\frac{8b^3}{3a^5x^{\frac{3}{2}}}-\frac{5b^4}{a^6x}+\frac{12b^5}{a^7\sqrt{x}}-\frac{14b^6\ln(a+b\sqrt{x})}{a^8}+\frac{7b^6\ln(x)}{a^8}$	10
default	$\frac{2b^6}{a^7(a+b\sqrt{x})}-\frac{1}{3a^2x^3}+\frac{4b}{5a^3x^{\frac{5}{2}}}-\frac{3b^2}{2a^4x^2}+\frac{8b^3}{3a^5x^{\frac{3}{2}}}-\frac{5b^4}{a^6x}+\frac{12b^5}{a^7\sqrt{x}}-\frac{14b^6\ln(a+b\sqrt{x})}{a^8}+\frac{7b^6\ln(x)}{a^8}$	10

input
```
int(1/(a+b*x^(1/2))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
2*b^6/a^7/(a+b*x^(1/2))-1/3/a^2/x^3+4/5*b/a^3/x^(5/2)-3/2*b^2/a^4/x^2+8/3*
b^3/a^5/x^(3/2)-5*b^4/a^6/x+12*b^5/a^7/x^(1/2)-14*b^6*ln(a+b*x^(1/2))/a^8+
7*b^6*ln(x)/a^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx = -\frac{210\,a^2b^6x^3 - 105\,a^4b^4x^2 - 35\,a^6b^2x - 10\,a^8 + 420\,(b^8x^4 - a^2b^6x^3)\log\left(b\sqrt{x}+a\right) - 420\,(b^8x^4 - a^2b^6x^3}{30\,(a^8b^2x^4 - a^{10}x^3)}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^4,x, algorithm="fricas")
```

output
```
-1/30*(210*a^2*b^6*x^3 - 105*a^4*b^4*x^2 - 35*a^6*b^2*x - 10*a^8 + 420*(b^
8*x^4 - a^2*b^6*x^3)*log(b*sqrt(x) + a) - 420*(b^8*x^4 - a^2*b^6*x^3)*log(
sqrt(x)) - 4*(105*a*b^7*x^3 - 70*a^3*b^5*x^2 - 14*a^5*b^3*x - 6*a^7*b)*sqr
t(x))/(a^8*b^2*x^4 - a^10*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(122) = 244.

Time = 1.26 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.25

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^4} \\ -\frac{1}{3a^2x^3} \\ -\frac{1}{4b^2x^4} \\ -\frac{10a^7\sqrt{x}}{30a^9x^{\frac{7}{2}}+30a^8bx^4} + \frac{14a^6bx}{30a^9x^{\frac{7}{2}}+30a^8bx^4} - \frac{21a^5b^2x^{\frac{3}{2}}}{30a^9x^{\frac{7}{2}}+30a^8bx^4} + \frac{35a^4b^3x^2}{30a^9x^{\frac{7}{2}}+30a^8bx^4} - \frac{70a^3b^4x^{\frac{5}{2}}}{30a^9x^{\frac{7}{2}}+30a^8bx^4} + \frac{210a^2b^5x^3}{30a^9x^{\frac{7}{2}}+30a^8bx^4} + \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**2/x**4,x)
```

output
```
Piecewise((zoo/x**4, Eq(a, 0) & Eq(b, 0)), (-1/(3*a**2*x**3), Eq(b, 0)), (
-1/(4*b**2*x**4), Eq(a, 0)), (-10*a**7*sqrt(x)/(30*a**9*x**(7/2) + 30*a**8
*b*x**4) + 14*a**6*b*x/(30*a**9*x**(7/2) + 30*a**8*b*x**4) - 21*a**5*b**2*
x**(3/2)/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 35*a**4*b**3*x**2/(30*a**9*
x**(7/2) + 30*a**8*b*x**4) - 70*a**3*b**4*x**(5/2)/(30*a**9*x**(7/2) + 30*
a**8*b*x**4) + 210*a**2*b**5*x**3/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 21
0*a*b**6*x**(7/2)*log(x)/(30*a**9*x**(7/2) + 30*a**8*b*x**4) - 420*a*b**6*
x**(7/2)*log(a/b + sqrt(x))/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 420*a*b*
*6*x**(7/2)/(30*a**9*x**(7/2) + 30*a**8*b*x**4) + 210*b**7*x**4*log(x)/(30
*a**9*x**(7/2) + 30*a**8*b*x**4) - 420*b**7*x**4*log(a/b + sqrt(x))/(30*a*
*9*x**(7/2) + 30*a**8*b*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx = \frac{420\,b^6x^3+210\,ab^5x^{\frac{5}{2}}-70\,a^2b^4x^2+35\,a^3b^3x^{\frac{3}{2}}-21\,a^4b^2x+14\,a^5b\sqrt{x}-10\,a^6}{30\left(a^7bx^{\frac{7}{2}}+a^8x^3\right)} - \frac{14\,b^6\log\left(b\sqrt{x}+a\right)}{a^8}+\frac{7\,b^6\log\left(x\right)}{a^8}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^4,x, algorithm="maxima")
```

output
```
1/30*(420*b^6*x^3 + 210*a*b^5*x^(5/2) - 70*a^2*b^4*x^2 + 35*a^3*b^3*x^(3/2
) - 21*a^4*b^2*x + 14*a^5*b*sqrt(x) - 10*a^6)/(a^7*b*x^(7/2) + a^8*x^3) -
14*b^6*log(b*sqrt(x) + a)/a^8 + 7*b^6*log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx = -\frac{14\,b^6\log\left(\left|b\sqrt{x}+a\right|\right)}{a^8} + \frac{7\,b^6\log\left(|x|\right)}{a^8} + \frac{420\,ab^6x^3 + 210\,a^2b^5x^{\frac{5}{2}} - 70\,a^3b^4x^2 + 35\,a^4b^3x^{\frac{3}{2}} - 21\,a^5b^2x + 14\,a^6b\sqrt{x} - 10\,a^7}{30\left(b\sqrt{x}+a\right)a^8x^3}$$

input
```
integrate(1/(a+b*x^(1/2))^2/x^4,x, algorithm="giac")
```

output
```
-14*b^6*log(abs(b*sqrt(x) + a))/a^8 + 7*b^6*log(abs(x))/a^8 + 1/30*(420*a*
b^6*x^3 + 210*a^2*b^5*x^(5/2) - 70*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) - 21*a
^5*b^2*x + 14*a^6*b*sqrt(x) - 10*a^7)/((b*sqrt(x) + a)*a^8*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx = \frac{\frac{7\,b\,\sqrt{x}}{15\,a^2} - \frac{1}{3\,a} - \frac{7\,b^2\,x}{10\,a^3} - \frac{7\,b^4\,x^2}{3\,a^5} + \frac{7\,b^3\,x^{3/2}}{6\,a^4} + \frac{14\,b^6\,x^3}{a^7} + \frac{7\,b^5\,x^{5/2}}{a^6}}{a\,x^3 + b\,x^{7/2}} - \frac{28\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^8}$$

input
```
int(1/(x^4*(a + b*x^(1/2))^2),x)
```

output
```
((7*b*x^(1/2))/(15*a^2) - 1/(3*a) - (7*b^2*x)/(10*a^3) - (7*b^4*x^2)/(3*a^
5) + (7*b^3*x^(3/2))/(6*a^4) + (14*b^6*x^3)/a^7 + (7*b^5*x^(5/2))/a^6)/(a*
x^3 + b*x^(7/2)) - (28*b^6*atanh((2*b*x^(1/2))/a + 1))/a^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^2 x^4}\,dx$$
$$= \frac{-420\sqrt{x}\log\left(\sqrt{x}\,b+a\right)b^7x^3+420\sqrt{x}\log\left(\sqrt{x}\right)b^7x^3+14\sqrt{x}\,a^6b+35\sqrt{x}\,a^4b^3x+210\sqrt{x}\,a^2b^5x^2-420\sqrt{}}{30a^8x^3\left(\sqrt{x}\,b+a\right)}$$

input
```
int(1/(a+b*x^(1/2))^2/x^4,x)
```

output
```
( - 420*sqrt(x)*log(sqrt(x)*b + a)*b**7*x**3 + 420*sqrt(x)*log(sqrt(x))*b*
*7*x**3 + 14*sqrt(x)*a**6*b + 35*sqrt(x)*a**4*b**3*x + 210*sqrt(x)*a**2*b*
*5*x**2 - 420*sqrt(x)*b**7*x**3 - 420*log(sqrt(x)*b + a)*a*b**6*x**3 + 420
*log(sqrt(x))*a*b**6*x**3 - 10*a**7 - 21*a**5*b**2*x - 70*a**3*b**4*x**2)/
(30*a**8*x**3*(sqrt(x)*b + a))
```

3.97 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 114

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{a^7}{b^8\left(a+b\sqrt{x}\right)^2} - \frac{14a^6}{b^8\left(a+b\sqrt{x}\right)} + \frac{30a^4\sqrt{x}}{b^7} - \frac{10a^3x}{b^6} + \frac{4a^2x^{3/2}}{b^5} - \frac{3ax^2}{2b^4} + \frac{2x^{5/2}}{5b^3} - \frac{42a^5\log\left(a+b\sqrt{x}\right)}{b^8}$$

output
```
a^7/b^8/(a+b*x^(1/2))^2-14*a^6/b^8/(a+b*x^(1/2))+30*a^4*x^(1/2)/b^7-10*a^3
*x/b^6+4*a^2*x^(3/2)/b^5-3/2*a*x^2/b^4+2/5*x^(5/2)/b^3-42*a^5*ln(a+b*x^(1/
2))/b^8
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{-130a^7+160a^6b\sqrt{x}+500a^5b^2x+140a^4b^3x^{3/2}-35a^3b^4x^2+14a^2b^5x^{5/2}-7ab^6x^3+4b^7x^{7/2}}{10b^8\left(a+b\sqrt{x}\right)^2} - \frac{42a^5\log\left(a+b\sqrt{x}\right)}{b^8}$$

input
```
Integrate[x^3/(a + b*Sqrt[x])^3,x]
```

output
```
(-130*a^7 + 160*a^6*b*Sqrt[x] + 500*a^5*b^2*x + 140*a^4*b^3*x^(3/2) - 35*a
^3*b^4*x^2 + 14*a^2*b^5*x^(5/2) - 7*a*b^6*x^3 + 4*b^7*x^(7/2))/(10*b^8*(a
+ b*Sqrt[x])^2) - (42*a^5*Log[a + b*Sqrt[x]])/b^8
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{7/2}}{\left(a+b\sqrt{x}\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^7}{b^7\left(a+b\sqrt{x}\right)^3}+\frac{7a^6}{b^7\left(a+b\sqrt{x}\right)^2}-\frac{21a^5}{b^7\left(a+b\sqrt{x}\right)}+\frac{15a^4}{b^7}-\frac{10\sqrt{x}a^3}{b^6}+\frac{6xa^2}{b^5}-\frac{3x^{3/2}a}{b^4}+\frac{x^2}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^7}{2b^8\left(a+b\sqrt{x}\right)^2}-\frac{7a^6}{b^8\left(a+b\sqrt{x}\right)}-\frac{21a^5\log\left(a+b\sqrt{x}\right)}{b^8}+\frac{15a^4\sqrt{x}}{b^7}-\frac{5a^3x}{b^6}+\frac{2a^2x^{3/2}}{b^5}-\frac{3ax^2}{4b^4}+\frac{x^{5/2}}{5b^3}\right)$$

input
```
Int[x^3/(a + b*Sqrt[x])^3,x]
```

output
```
2*(a^7/(2*b^8*(a + b*Sqrt[x])^2) - (7*a^6)/(b^8*(a + b*Sqrt[x])) + (15*a^4
*Sqrt[x])/b^7 - (5*a^3*x)/b^6 + (2*a^2*x^(3/2))/b^5 - (3*a*x^2)/(4*b^4) +
x^(5/2)/(5*b^3) - (21*a^5*Log[a + b*Sqrt[x]])/b^8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{2x^{\frac{5}{2}}b^4}{5}-\frac{3a\,b^3x^2}{2}+4x^{\frac{3}{2}}a^2b^2-10a^3bx+30\sqrt{x}\,a^4}{b^7}+\frac{a^7}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{42a^5\ln\left(a+b\sqrt{x}\right)}{b^8}-\frac{14a^6}{b^8\left(a+b\sqrt{x}\right)}$	100
default	$\frac{\frac{2x^{\frac{5}{2}}b^4}{5}-\frac{3a\,b^3x^2}{2}+4x^{\frac{3}{2}}a^2b^2-10a^3bx+30\sqrt{x}\,a^4}{b^7}+\frac{a^7}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{42a^5\ln\left(a+b\sqrt{x}\right)}{b^8}-\frac{14a^6}{b^8\left(a+b\sqrt{x}\right)}$	100

input
```
int(x^3/(a+b*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/b^7*(1/5*x^(5/2)*b^4-3/4*a*b^3*x^2+2*x^(3/2)*a^2*b^2-5*a^3*b*x+15*x^(1/2
)*a^4)+a^7/b^8/(a+b*x^(1/2))^2-42*a^5*ln(a+b*x^(1/2))/b^8-14*a^6/b^8/(a+b*
x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{15\,ab^8x^4+70\,a^3b^6x^3-185\,a^5b^4x^2-50\,a^7b^2x+130\,a^9+420\left(a^5b^4x^2-2\,a^7b^2x+a^9\right)\log\left(b\sqrt{x}+a\right)-}{10\left(b^{12}x^2-2\,a^2b^{10}x+a^4b^8\right)}$$

input
```
integrate(x^3/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/10*(15*a*b^8*x^4 + 70*a^3*b^6*x^3 - 185*a^5*b^4*x^2 - 50*a^7*b^2*x + 13
0*a^9 + 420*(a^5*b^4*x^2 - 2*a^7*b^2*x + a^9)*log(b*sqrt(x) + a) - 4*(b^9*
x^4 + 8*a^2*b^7*x^3 + 56*a^4*b^5*x^2 - 175*a^6*b^3*x + 105*a^8*b)*sqrt(x))
/(b^12*x^2 - 2*a^2*b^10*x + a^4*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(112) = 224$.

Time = 0.64 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.62

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx = \begin{cases} -\frac{420a^7\log\left(\frac{a}{b}+\sqrt{x}\right)}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x} - \frac{630a^7}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x} - \frac{840a^6b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x} - \frac{840a^6b\sqrt{x}}{10a^2b^8+20ab^9\sqrt{x}+10b^{10}x} - \frac{420a^5b^2x\,\mathrm{l}}{10a^2b^8+20a} \\ \frac{x^4}{4a^3} \end{cases}$$

input
```
integrate(x**3/(a+b*x**(1/2))**3,x)
```

output
```
Piecewise((-420*a**7*log(a/b + sqrt(x))/(10*a**2*b**8 + 20*a*b**9*sqrt(x)
+ 10*b**10*x) - 630*a**7/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10*x) -
 840*a**6*b*sqrt(x)*log(a/b + sqrt(x))/(10*a**2*b**8 + 20*a*b**9*sqrt(x) +
 10*b**10*x) - 840*a**6*b*sqrt(x)/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b
**10*x) - 420*a**5*b**2*x*log(a/b + sqrt(x))/(10*a**2*b**8 + 20*a*b**9*sqr
t(x) + 10*b**10*x) + 140*a**4*b**3*x**(3/2)/(10*a**2*b**8 + 20*a*b**9*sqrt
(x) + 10*b**10*x) - 35*a**3*b**4*x**2/(10*a**2*b**8 + 20*a*b**9*sqrt(x) +
10*b**10*x) + 14*a**2*b**5*x**(5/2)/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10
*b**10*x) - 7*a*b**6*x**3/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10*x)
+ 4*b**7*x**(7/2)/(10*a**2*b**8 + 20*a*b**9*sqrt(x) + 10*b**10*x), Ne(b, 0
)), (x**4/(4*a**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{42\,a^5\log\left(b\sqrt{x}+a\right)}{b^8} + \frac{2\left(b\sqrt{x}+a\right)^5}{5\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^4 a}{2\,b^8} + \frac{14\left(b\sqrt{x}+a\right)^3 a^2}{b^8} - \frac{35\left(b\sqrt{x}+a\right)^2 a^3}{b^8} + \frac{70\left(b\sqrt{x}+a\right)a^4}{b^8} - \frac{14\,a^6}{\left(b\sqrt{x}+a\right)b^8} + \frac{a^7}{\left(b\sqrt{x}+a\right)^2 b^8}$$

input
```
integrate(x^3/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

output
```
-42*a^5*log(b*sqrt(x) + a)/b^8 + 2/5*(b*sqrt(x) + a)^5/b^8 - 7/2*(b*sqrt(x
) + a)^4*a/b^8 + 14*(b*sqrt(x) + a)^3*a^2/b^8 - 35*(b*sqrt(x) + a)^2*a^3/b
^8 + 70*(b*sqrt(x) + a)*a^4/b^8 - 14*a^6/((b*sqrt(x) + a)*b^8) + a^7/((b*s
qrt(x) + a)^2*b^8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{42\,a^5\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8} - \frac{14\,a^6b\sqrt{x}+13\,a^7}{\left(b\sqrt{x}+a\right)^2b^8} + \frac{4\,b^{12}x^{\frac{5}{2}}-15\,ab^{11}x^2+40\,a^2b^{10}x^{\frac{3}{2}}-100\,a^3b^9x+300\,a^4b^8\sqrt{x}}{10\,b^{15}}$$

input
```
integrate(x^3/(a+b*x^(1/2))^3,x, algorithm="giac")
```

output
```
-42*a^5*log(abs(b*sqrt(x) + a))/b^8 - (14*a^6*b*sqrt(x) + 13*a^7)/((b*sqrt
(x) + a)^2*b^8) + 1/10*(4*b^12*x^(5/2) - 15*a*b^11*x^2 + 40*a^2*b^10*x^(3/
2) - 100*a^3*b^9*x + 300*a^4*b^8*sqrt(x))/b^15
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{2\,x^{5/2}}{5\,b^3} - \frac{\frac{13\,a^7}{b}+14\,a^6\,\sqrt{x}}{b^9\,x+a^2\,b^7+2\,a\,b^8\,\sqrt{x}} - \frac{3\,a\,x^2}{2\,b^4} - \frac{10\,a^3\,x}{b^6} - \frac{42\,a^5\,\ln\left(a+b\,\sqrt{x}\right)}{b^8} + \frac{4\,a^2\,x^{3/2}}{b^5} + \frac{30\,a^4\,\sqrt{x}}{b^7}$$

input
```
int(x^3/(a + b*x^(1/2))^3,x)
```

output
```
(2*x^(5/2))/(5*b^3) - ((13*a^7)/b + 14*a^6*x^(1/2))/(b^9*x + a^2*b^7 + 2*a
*b^8*x^(1/2)) - (3*a*x^2)/(2*b^4) - (10*a^3*x)/b^6 - (42*a^5*log(a + b*x^(
1/2)))/b^8 + (4*a^2*x^(3/2))/b^5 + (30*a^4*x^(1/2))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^3}\,dx$$
$$= \frac{-840\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^6b+140\sqrt{x}\,a^4b^3x+14\sqrt{x}\,a^2b^5x^2+4\sqrt{x}\,b^7x^3-420\log\left(\sqrt{x}\,b+a\right)a^7-420\log}{10b^8\left(2\sqrt{x}\,ab+a^2+b^2x\right)}$$

input
```
int(x^3/(a+b*x^(1/2))^3,x)
```

output
```
( - 840*sqrt(x)*log(sqrt(x)*b + a)*a**6*b + 140*sqrt(x)*a**4*b**3*x + 14*s
qrt(x)*a**2*b**5*x**2 + 4*sqrt(x)*b**7*x**3 - 420*log(sqrt(x)*b + a)*a**7
- 420*log(sqrt(x)*b + a)*a**5*b**2*x - 210*a**7 + 420*a**5*b**2*x - 35*a**
3*b**4*x**2 - 7*a*b**6*x**3)/(10*b**8*(2*sqrt(x)*a*b + a**2 + b**2*x))
```

3.98 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3} \, dx = \frac{a^5}{b^6\left(a+b\sqrt{x}\right)^2} - \frac{10a^4}{b^6\left(a+b\sqrt{x}\right)} + \frac{12a^2\sqrt{x}}{b^5} - \frac{3ax}{b^4} + \frac{2x^{3/2}}{3b^3} - \frac{20a^3\log\left(a+b\sqrt{x}\right)}{b^6}$$

output
```
a^5/b^6/(a+b*x^(1/2))^2-10*a^4/b^6/(a+b*x^(1/2))+12*a^2*x^(1/2)/b^5-3*a*x/
b^4+2/3*x^(3/2)/b^3-20*a^3*ln(a+b*x^(1/2))/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3} \, dx = \frac{-27a^5+6a^4b\sqrt{x}+63a^3b^2x+20a^2b^3x^{3/2}-5ab^4x^2+2b^5x^{5/2}}{3b^6\left(a+b\sqrt{x}\right)^2} - \frac{20a^3\log\left(a+b\sqrt{x}\right)}{b^6}$$

input
```
Integrate[x^2/(a + b*Sqrt[x])^3,x]
```

output
```
(-27*a^5 + 6*a^4*b*Sqrt[x] + 63*a^3*b^2*x + 20*a^2*b^3*x^(3/2) - 5*a*b^4*x
^2 + 2*b^5*x^(5/2))/(3*b^6*(a + b*Sqrt[x])^2) - (20*a^3*Log[a + b*Sqrt[x]]
)/b^6
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{5/2}}{\left(a+b\sqrt{x}\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^5}{b^5\left(a+b\sqrt{x}\right)^3}+\frac{5a^4}{b^5\left(a+b\sqrt{x}\right)^2}-\frac{10a^3}{b^5\left(a+b\sqrt{x}\right)}+\frac{6a^2}{b^5}-\frac{3\sqrt{x}a}{b^4}+\frac{x}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^5}{2b^6\left(a+b\sqrt{x}\right)^2}-\frac{5a^4}{b^6\left(a+b\sqrt{x}\right)}-\frac{10a^3\log\left(a+b\sqrt{x}\right)}{b^6}+\frac{6a^2\sqrt{x}}{b^5}-\frac{3ax}{2b^4}+\frac{x^{3/2}}{3b^3}\right)$$

input
```
Int[x^2/(a + b*Sqrt[x])^3,x]
```

output
```
2*(a^5/(2*b^6*(a + b*Sqrt[x])^2) - (5*a^4)/(b^6*(a + b*Sqrt[x])) + (6*a^2*
Sqrt[x])/b^5 - (3*a*x)/(2*b^4) + x^(3/2)/(3*b^3) - (10*a^3*Log[a + b*Sqrt[
x]])/b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{2b^2x^{\frac{3}{2}}}{3}-3abx+12a^2\sqrt{x}}{b^5}-\frac{20a^3\ln\left(a+b\sqrt{x}\right)}{b^6}+\frac{a^5}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{10a^4}{b^6\left(a+b\sqrt{x}\right)}$	78
default	$\frac{\frac{2b^2x^{\frac{3}{2}}}{3}-3abx+12a^2\sqrt{x}}{b^5}-\frac{20a^3\ln\left(a+b\sqrt{x}\right)}{b^6}+\frac{a^5}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{10a^4}{b^6\left(a+b\sqrt{x}\right)}$	78

input
```
int(x^2/(a+b*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/3*b^2*x^(3/2)-3/2*a*b*x+6*a^2*x^(1/2))-20*a^3*ln(a+b*x^(1/2))/b^6
+a^5/b^6/(a+b*x^(1/2))^2-10*a^4/b^6/(a+b*x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx =$$
$$-\frac{9\,ab^6x^3-18\,a^3b^4x^2-24\,a^5b^2x+27\,a^7+60\left(a^3b^4x^2-2\,a^5b^2x+a^7\right)\log\left(b\sqrt{x}+a\right)-2\left(b^7x^3+16\,a^2b^5\right.}{3\left(b^{10}x^2-2\,a^2b^8x+a^4b^6\right)}$$

input
```
integrate(x^2/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/3*(9*a*b^6*x^3 - 18*a^3*b^4*x^2 - 24*a^5*b^2*x + 27*a^7 + 60*(a^3*b^4*x
^2 - 2*a^5*b^2*x + a^7)*log(b*sqrt(x) + a) - 2*(b^7*x^3 + 16*a^2*b^5*x^2 -
 50*a^4*b^3*x + 30*a^6*b)*sqrt(x))/(b^10*x^2 - 2*a^2*b^8*x + a^4*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(85) = 170$.

Time = 0.51 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.78

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx$$
$$=\begin{cases} -\frac{60a^5\log\left(\frac{a}{b}+\sqrt{x}\right)}{3a^2b^6+6ab^7\sqrt{x}+3b^8x}-\frac{90a^5}{3a^2b^6+6ab^7\sqrt{x}+3b^8x}-\frac{120a^4b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{3a^2b^6+6ab^7\sqrt{x}+3b^8x}-\frac{120a^4b\sqrt{x}}{3a^2b^6+6ab^7\sqrt{x}+3b^8x}-\frac{60a^3b^2x\log\left(\frac{a}{b}+\sqrt{x}\right)}{3a^2b^6+6ab^7\sqrt{x}+3b^8x}+\ldots \\ \frac{x^3}{3a^3} \end{cases}$$

input
```
integrate(x**2/(a+b*x**(1/2))**3,x)
```

output
```
Piecewise((-60*a**5*log(a/b + sqrt(x))/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3
*b**8*x) - 90*a**5/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) - 120*a**4*
b*sqrt(x)*log(a/b + sqrt(x))/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) -
 120*a**4*b*sqrt(x)/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) - 60*a**3*
b**2*x*log(a/b + sqrt(x))/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) + 20
*a**2*b**3*x**(3/2)/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) - 5*a*b**4
*x**2/(3*a**2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x) + 2*b**5*x**(5/2)/(3*a**
2*b**6 + 6*a*b**7*sqrt(x) + 3*b**8*x), Ne(b, 0)), (x**3/(3*a**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{20\,a^3\log\left(b\sqrt{x}+a\right)}{b^6}+\frac{2\left(b\sqrt{x}+a\right)^3}{3\,b^6}-\frac{5\left(b\sqrt{x}+a\right)^2a}{b^6}+\frac{20\left(b\sqrt{x}+a\right)a^2}{b^6}-\frac{10\,a^4}{\left(b\sqrt{x}+a\right)b^6}+\frac{a^5}{\left(b\sqrt{x}+a\right)^2b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

output
```
-20*a^3*log(b*sqrt(x) + a)/b^6 + 2/3*(b*sqrt(x) + a)^3/b^6 - 5*(b*sqrt(x)
+ a)^2*a/b^6 + 20*(b*sqrt(x) + a)*a^2/b^6 - 10*a^4/((b*sqrt(x) + a)*b^6) +
 a^5/((b*sqrt(x) + a)^2*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{20\,a^3\log\left(\left|b\sqrt{x}+a\right|\right)}{b^6}-\frac{10\,a^4b\sqrt{x}+9\,a^5}{\left(b\sqrt{x}+a\right)^2b^6}+\frac{2\,b^6x^{\frac{3}{2}}-9\,ab^5x+36\,a^2b^4\sqrt{x}}{3\,b^9}$$

input
```
integrate(x^2/(a+b*x^(1/2))^3,x, algorithm="giac")
```

output
```
-20*a^3*log(abs(b*sqrt(x) + a))/b^6 - (10*a^4*b*sqrt(x) + 9*a^5)/((b*sqrt(
x) + a)^2*b^6) + 1/3*(2*b^6*x^(3/2) - 9*a*b^5*x + 36*a^2*b^4*sqrt(x))/b^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{2\,x^{3/2}}{3\,b^3} - \frac{\frac{9\,a^5}{b}+10\,a^4\,\sqrt{x}}{b^7\,x+a^2\,b^5+2\,a\,b^6\,\sqrt{x}} - \frac{20\,a^3\,\ln\left(a+b\,\sqrt{x}\right)}{b^6} + \frac{12\,a^2\,\sqrt{x}}{b^5} - \frac{3\,a\,x}{b^4}$$

input
```
int(x^2/(a + b*x^(1/2))^3,x)
```

output
```
(2*x^(3/2))/(3*b^3) - ((9*a^5)/b + 10*a^4*x^(1/2))/(b^7*x + a^2*b^5 + 2*a*
b^6*x^(1/2)) - (20*a^3*log(a + b*x^(1/2)))/b^6 + (12*a^2*x^(1/2))/b^5 - (3
*a*x)/b^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{-120\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^4b+20\sqrt{x}\,a^2b^3x+2\sqrt{x}\,b^5x^2-60\log\left(\sqrt{x}\,b+a\right)a^5-60\log\left(\sqrt{x}\,b+a\right)a^3b^2x-}{3b^6\left(2\sqrt{x}\,ab+a^2+b^2x\right)}$$

input
```
int(x^2/(a+b*x^(1/2))^3,x)
```

output
```
( - 120*sqrt(x)*log(sqrt(x)*b + a)*a**4*b + 20*sqrt(x)*a**2*b**3*x + 2*sqr
t(x)*b**5*x**2 - 60*log(sqrt(x)*b + a)*a**5 - 60*log(sqrt(x)*b + a)*a**3*b
**2*x - 30*a**5 + 60*a**3*b**2*x - 5*a*b**4*x**2)/(3*b**6*(2*sqrt(x)*a*b +
 a**2 + b**2*x))
```

3.99 $\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx=\frac{a^3}{b^4\left(a+b\sqrt{x}\right)^2}-\frac{6a^2}{b^4\left(a+b\sqrt{x}\right)}+\frac{2\sqrt{x}}{b^3}-\frac{6a\log\left(a+b\sqrt{x}\right)}{b^4}$$

output
```
a^3/b^4/(a+b*x^(1/2))^2-6*a^2/b^4/(a+b*x^(1/2))+2*x^(1/2)/b^3-6*a*ln(a+b*x
^(1/2))/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx=\frac{-5a^3-4a^2b\sqrt{x}+4ab^2x+2b^3x^{3/2}}{b^4\left(a+b\sqrt{x}\right)^2}-\frac{6a\log\left(a+b\sqrt{x}\right)}{b^4}$$

input
```
Integrate[x/(a + b*Sqrt[x])^3,x]
```

output
```
(-5*a^3 - 4*a^2*b*Sqrt[x] + 4*a*b^2*x + 2*b^3*x^(3/2))/(b^4*(a + b*Sqrt[x]
)^2) - (6*a*Log[a + b*Sqrt[x]])/b^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{3/2}}{\left(a+b\sqrt{x}\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^3}{b^3\left(a+b\sqrt{x}\right)^3}+\frac{3a^2}{b^3\left(a+b\sqrt{x}\right)^2}-\frac{3a}{b^3\left(a+b\sqrt{x}\right)}+\frac{1}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^3}{2b^4\left(a+b\sqrt{x}\right)^2}-\frac{3a^2}{b^4\left(a+b\sqrt{x}\right)}-\frac{3a\log\left(a+b\sqrt{x}\right)}{b^4}+\frac{\sqrt{x}}{b^3}\right)$$

input
```
Int[x/(a + b*Sqrt[x])^3,x]
```

output
```
2*(a^3/(2*b^4*(a + b*Sqrt[x])^2) - (3*a^2)/(b^4*(a + b*Sqrt[x])) + Sqrt[x]
/b^3 - (3*a*Log[a + b*Sqrt[x]])/b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{a^3}{b^4\left(a+b\sqrt{x}\right)^2}-\frac{6a^2}{b^4\left(a+b\sqrt{x}\right)}+\frac{2\sqrt{x}}{b^3}-\frac{6a\ln\left(a+b\sqrt{x}\right)}{b^4}$	57
default	$\frac{a^3}{b^4\left(a+b\sqrt{x}\right)^2}-\frac{6a^2}{b^4\left(a+b\sqrt{x}\right)}+\frac{2\sqrt{x}}{b^3}-\frac{6a\ln\left(a+b\sqrt{x}\right)}{b^4}$	57

input
```
int(x/(a+b*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/b^4/(a+b*x^(1/2))^2-6*a^2/b^4/(a+b*x^(1/2))+2*x^(1/2)/b^3-6*a*ln(a+b*x
^(1/2))/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.58

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{7\,a^3b^2x-5\,a^5-6\,(ab^4x^2-2\,a^3b^2x+a^5)\log\left(b\sqrt{x}+a\right)+2\,(b^5x^2-5\,a^2b^3x+3\,a^4b)\sqrt{x}}{b^8x^2-2\,a^2b^6x+a^4b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

output
```
(7*a^3*b^2*x - 5*a^5 - 6*(a*b^4*x^2 - 2*a^3*b^2*x + a^5)*log(b*sqrt(x) + a
) + 2*(b^5*x^2 - 5*a^2*b^3*x + 3*a^4*b)*sqrt(x))/(b^8*x^2 - 2*a^2*b^6*x +
a^4*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(60) = 120$.

Time = 0.42 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.64

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx = \begin{cases} -\frac{6a^3\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{9a^3}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{12a^2b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{12a^2b\sqrt{x}}{a^2b^4+2ab^5\sqrt{x}+b^6x} - \frac{6ab^2x\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^2b^4+2ab^5\sqrt{x}+b^6x} + \frac{2b^3x^{\frac{3}{2}}}{a^2b^4+2ab^5\sqrt{}} \\ \frac{x^2}{2a^3} \end{cases}$$

input
```
integrate(x/(a+b*x**(1/2))**3,x)
```

output
```
Piecewise((-6*a**3*log(a/b + sqrt(x))/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6
*x) - 9*a**3/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) - 12*a**2*b*sqrt(x)*l
og(a/b + sqrt(x))/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) - 12*a**2*b*sqrt
(x)/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) - 6*a*b**2*x*log(a/b + sqrt(x)
)/(a**2*b**4 + 2*a*b**5*sqrt(x) + b**6*x) + 2*b**3*x**(3/2)/(a**2*b**4 + 2
*a*b**5*sqrt(x) + b**6*x), Ne(b, 0)), (x**2/(2*a**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{6\,a\log\left(b\sqrt{x}+a\right)}{b^4}+\frac{2\left(b\sqrt{x}+a\right)}{b^4}-\frac{6\,a^2}{\left(b\sqrt{x}+a\right)b^4}+\frac{a^3}{\left(b\sqrt{x}+a\right)^2b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

output
```
-6*a*log(b*sqrt(x) + a)/b^4 + 2*(b*sqrt(x) + a)/b^4 - 6*a^2/((b*sqrt(x) +
a)*b^4) + a^3/((b*sqrt(x) + a)^2*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{6\,a\log\left(\left|b\sqrt{x}+a\right|\right)}{b^4}+\frac{2\sqrt{x}}{b^3}-\frac{6\,a^2b\sqrt{x}+5\,a^3}{\left(b\sqrt{x}+a\right)^2b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^3,x, algorithm="giac")
```

output
```
-6*a*log(abs(b*sqrt(x) + a))/b^4 + 2*sqrt(x)/b^3 - (6*a^2*b*sqrt(x) + 5*a^
3)/((b*sqrt(x) + a)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{2\sqrt{x}}{b^3}-\frac{\frac{5\,a^3}{b}+6\,a^2\sqrt{x}}{b^5\,x+a^2\,b^3+2\,a\,b^4\sqrt{x}}-\frac{6\,a\,\ln\left(a+b\sqrt{x}\right)}{b^4}$$

input
```
int(x/(a + b*x^(1/2))^3,x)
```

output
```
(2*x^(1/2))/b^3 - ((5*a^3)/b + 6*a^2*x^(1/2))/(b^5*x + a^2*b^3 + 2*a*b^4*x
^(1/2)) - (6*a*log(a + b*x^(1/2)))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{-12\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^2b+2\sqrt{x}\,b^3x-6\log\left(\sqrt{x}\,b+a\right)a^3-6\log\left(\sqrt{x}\,b+a\right)a\,b^2x-3a^3+6a\,b^2x}{b^4\left(2\sqrt{x}\,ab+a^2+b^2x\right)}$$

input
```
int(x/(a+b*x^(1/2))^3,x)
```

output
```
( - 12*sqrt(x)*log(sqrt(x)*b + a)*a**2*b + 2*sqrt(x)*b**3*x - 6*log(sqrt(x
)*b + a)*a**3 - 6*log(sqrt(x)*b + a)*a*b**2*x - 3*a**3 + 6*a*b**2*x)/(b**4
*(2*sqrt(x)*a*b + a**2 + b**2*x))
```

3.100 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3} \, dx = \frac{x}{a\left(a+b\sqrt{x}\right)^2}$$

output
```
x/a/(a+b*x^(1/2))^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3} \, dx = \frac{-a-2b\sqrt{x}}{b^2\left(a+b\sqrt{x}\right)^2}$$

input
```
Integrate[(a + b*Sqrt[x])^(-3),x]
```

output
```
(-a - 2*b*Sqrt[x])/(b^2*(a + b*Sqrt[x])^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}dx$$
$$\Big\downarrow \text{ 746}$$
$$\frac{x}{a\left(a+b\sqrt{x}\right)^2}$$

input
```
Int[(a + b*Sqrt[x])^(-3),x]
```

output
```
x/(a*(a + b*Sqrt[x])^2)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(14) = 28.

Time = 0.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{a}{b^2(a+b\sqrt{x})^2}-\frac{2}{b^2(a+b\sqrt{x})}$
trager	$\frac{(-1+x)(a^2b^2x-3b^4x+a^4+a^2b^2)a}{(-b^2x+a^2)^2(a^4-2a^2b^2+b^4)}-\frac{2b\,x^{\frac{3}{2}}}{(-b^2x+a^2)^2}$
default	$\frac{a^3}{2(-b^2x+a^2)^2b^2}-\frac{1}{b^2(b\sqrt{x}-a)}-\frac{a}{2b^2(b\sqrt{x}-a)^2}-\frac{1}{b^2(a+b\sqrt{x})}+\frac{a}{2b^2(a+b\sqrt{x})^2}+3a\,b^2\Big(\frac{a^2}{2b^4(b^2x-a^2)^2}+$

input
```
int(1/(a+b*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
a/b^2/(a+b*x^(1/2))^2-2/b^2/(a+b*x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.94

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{2\,b^3x^{\frac{3}{2}}-3\,ab^2x+a^3}{b^6x^2-2\,a^2b^4x+a^4b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^3,x, algorithm="fricas")
```

output
```
-(2*b^3*x^(3/2) - 3*a*b^2*x + a^3)/(b^6*x^2 - 2*a^2*b^4*x + a^4*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(12) = 24$.

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.94

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx = \begin{cases} -\frac{a}{a^2b^2+2ab^3\sqrt{x}+b^4x}-\frac{2b\sqrt{x}}{a^2b^2+2ab^3\sqrt{x}+b^4x} & \text{for } b\neq 0 \\ \frac{x}{a^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**3,x)
```

output
```
Piecewise((-a/(a**2*b**2 + 2*a*b**3*sqrt(x) + b**4*x) - 2*b*sqrt(x)/(a**2*
b**2 + 2*a*b**3*sqrt(x) + b**4*x), Ne(b, 0)), (x/a**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{2}{\left(b\sqrt{x}+a\right)b^2} + \frac{a}{\left(b\sqrt{x}+a\right)^2 b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^3,x, algorithm="maxima")
```

output
```
-2/((b*sqrt(x) + a)*b^2) + a/((b*sqrt(x) + a)^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{2\,b\sqrt{x}+a}{\left(b\sqrt{x}+a\right)^2 b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^3,x, algorithm="giac")
```

output
```
-(2*b*sqrt(x) + a)/((b*sqrt(x) + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx = -\frac{\frac{a}{b^2}+\frac{2\sqrt{x}}{b}}{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}$$

input
```
int(1/(a + b*x^(1/2))^3,x)
```

output
```
-(a/b^2 + (2*x^(1/2))/b)/(b^2*x + a^2 + 2*a*b*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3}\,dx = \frac{x}{a\left(2\sqrt{x}\,ab+a^2+b^2x\right)}$$

input
```
int(1/(a+b*x^(1/2))^3,x)
```

output
```
x/(a*(2*sqrt(x)*a*b + a**2 + b**2*x))
```

3.101 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx = \frac{1}{a\left(a+b\sqrt{x}\right)^2}+\frac{2}{a^2\left(a+b\sqrt{x}\right)}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{\log(x)}{a^3}$$

output
```
1/a/(a+b*x^(1/2))^2+2/a^2/(a+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^3+ln(x)/a^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx = \frac{\frac{a\left(3a+2b\sqrt{x}\right)}{\left(a+b\sqrt{x}\right)^2}-2\log\left(a+b\sqrt{x}\right)+\log(x)}{a^3}$$

input
```
Integrate[1/((a + b*Sqrt[x])^3*x),x]
```

output
```
((a*(3*a + 2*b*Sqrt[x]))/(a + b*Sqrt[x])^2 - 2*Log[a + b*Sqrt[x]] + Log[x]
)/a^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^3\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{b}{a^3\left(a+b\sqrt{x}\right)}-\frac{b}{a^2\left(a+b\sqrt{x}\right)^2}-\frac{b}{a\left(a+b\sqrt{x}\right)^3}+\frac{1}{a^3\sqrt{x}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\log\left(a+b\sqrt{x}\right)}{a^3}+\frac{\log\left(\sqrt{x}\right)}{a^3}+\frac{1}{a^2\left(a+b\sqrt{x}\right)}+\frac{1}{2a\left(a+b\sqrt{x}\right)^2}\right)$$

input
```
Int[1/((a + b*Sqrt[x])^3*x),x]
```

output
```
2*(1/(2*a*(a + b*Sqrt[x])^2) + 1/(a^2*(a + b*Sqrt[x])) - Log[a + b*Sqrt[x]
]/a^3 + Log[Sqrt[x]]/a^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{1}{a\left(a+b\sqrt{x}\right)^{2}}+\frac{2}{a^{2}\left(a+b\sqrt{x}\right)}-\frac{2\ln\left(a+b\sqrt{x}\right)}{a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$	48
default	$\frac{1}{a\left(a+b\sqrt{x}\right)^{2}}+\frac{2}{a^{2}\left(a+b\sqrt{x}\right)}-\frac{2\ln\left(a+b\sqrt{x}\right)}{a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$	48

input
```
int(1/(a+b*x^(1/2))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/a/(a+b*x^(1/2))^2+2/a^2/(a+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^3+ln(x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.17

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^{3}x}\,dx = -\frac{a^{2}b^{2}x-3\,a^{4}+2\left(b^{4}x^{2}-2\,a^{2}b^{2}x+a^{4}\right)\log\left(b\sqrt{x}+a\right)-2\left(b^{4}x^{2}-2\,a^{2}b^{2}x+a^{4}\right)\log\left(\sqrt{x}\right)-2\left(ab^{3}x-2\right.}{a^{3}b^{4}x^{2}-2\,a^{5}b^{2}x+a^{7}}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x,x, algorithm="fricas")
```

output
```
-(a^2*b^2*x - 3*a^4 + 2*(b^4*x^2 - 2*a^2*b^2*x + a^4)*log(b*sqrt(x) + a) -
 2*(b^4*x^2 - 2*a^2*b^2*x + a^4)*log(sqrt(x)) - 2*(a*b^3*x - 2*a^3*b)*sqrt
(x))/(a^3*b^4*x^2 - 2*a^5*b^2*x + a^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(48) = 96$.

Time = 0.67 (sec) , antiderivative size = 364, normalized size of antiderivative = 6.87

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{\log(x)}{a^3} \\ -\frac{2}{3b^3x^{\frac{3}{2}}} \\ \frac{a^2\sqrt{x}\log(x)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} - \frac{2a^2\sqrt{x}\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} + \frac{3a^2\sqrt{x}}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} + \frac{2abx\log(x)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} - \frac{4abx\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^5\sqrt{x}+2a^4bx+a^3b^2x^{\frac{3}{2}}} + \cdots \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**3/x,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (log(x)/a**3, Eq(b, 0)), (-
2/(3*b**3*x**(3/2)), Eq(a, 0)), (a**2*sqrt(x)*log(x)/(a**5*sqrt(x) + 2*a**
4*b*x + a**3*b**2*x**(3/2)) - 2*a**2*sqrt(x)*log(a/b + sqrt(x))/(a**5*sqrt
(x) + 2*a**4*b*x + a**3*b**2*x**(3/2)) + 3*a**2*sqrt(x)/(a**5*sqrt(x) + 2*
a**4*b*x + a**3*b**2*x**(3/2)) + 2*a*b*x*log(x)/(a**5*sqrt(x) + 2*a**4*b*x
 + a**3*b**2*x**(3/2)) - 4*a*b*x*log(a/b + sqrt(x))/(a**5*sqrt(x) + 2*a**4
*b*x + a**3*b**2*x**(3/2)) + 2*a*b*x/(a**5*sqrt(x) + 2*a**4*b*x + a**3*b**
2*x**(3/2)) + b**2*x**(3/2)*log(x)/(a**5*sqrt(x) + 2*a**4*b*x + a**3*b**2*
x**(3/2)) - 2*b**2*x**(3/2)*log(a/b + sqrt(x))/(a**5*sqrt(x) + 2*a**4*b*x
+ a**3*b**2*x**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx = \frac{2\,b\sqrt{x}+3\,a}{a^2b^2x+2\,a^3b\sqrt{x}+a^4} - \frac{2\,\log\left(b\sqrt{x}+a\right)}{a^3} + \frac{\log\left(x\right)}{a^3}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x,x, algorithm="maxima")
```

output
```
(2*b*sqrt(x) + 3*a)/(a^2*b^2*x + 2*a^3*b*sqrt(x) + a^4) - 2*log(b*sqrt(x)
+ a)/a^3 + log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx = -\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a^3} + \frac{\log\left(\left|x\right|\right)}{a^3} + \frac{2\,ab\sqrt{x}+3\,a^2}{\left(b\sqrt{x}+a\right)^2 a^3}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x,x, algorithm="giac")
```

output
```
-2*log(abs(b*sqrt(x) + a))/a^3 + log(abs(x))/a^3 + (2*a*b*sqrt(x) + 3*a^2)
/((b*sqrt(x) + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx = \frac{\frac{3}{a}+\frac{2\,b\,\sqrt{x}}{a^2}}{b^2\,x+a^2+2\,a\,b\,\sqrt{x}} - \frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^3}$$

input
```
int(1/(x*(a + b*x^(1/2))^3),x)
```

output
```
(3/a + (2*b*x^(1/2))/a^2)/(b^2*x + a^2 + 2*a*b*x^(1/2)) - (4*atanh((2*b*x^
(1/2))/a + 1))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x}\,dx$$
$$= \frac{-4\sqrt{x}\log\left(\sqrt{x}\,b+a\right)ab+4\sqrt{x}\log\left(\sqrt{x}\right)ab-2\log\left(\sqrt{x}\,b+a\right)a^2-2\log\left(\sqrt{x}\,b+a\right)b^2x+2\log\left(\sqrt{x}\right)a^2+}{a^3\left(2\sqrt{x}\,ab+a^2+b^2x\right)}$$

input
```
int(1/(a+b*x^(1/2))^3/x,x)
```

output
```
( - 4*sqrt(x)*log(sqrt(x)*b + a)*a*b + 4*sqrt(x)*log(sqrt(x))*a*b - 2*log(
sqrt(x)*b + a)*a**2 - 2*log(sqrt(x)*b + a)*b**2*x + 2*log(sqrt(x))*a**2 +
2*log(sqrt(x))*b**2*x + 2*a**2 - b**2*x)/(a**3*(2*sqrt(x)*a*b + a**2 + b**
2*x))
```

3.102 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2} \, dx = \frac{b^2}{a^3\left(a+b\sqrt{x}\right)^2} + \frac{6b^2}{a^4\left(a+b\sqrt{x}\right)} - \frac{1}{a^3 x} + \frac{6b}{a^4\sqrt{x}} - \frac{12b^2\log\left(a+b\sqrt{x}\right)}{a^5} + \frac{6b^2\log(x)}{a^5}$$

output
```
b^2/a^3/(a+b*x^(1/2))^2+6*b^2/a^4/(a+b*x^(1/2))-1/a^3/x+6*b/a^4/x^(1/2)-12
*b^2*ln(a+b*x^(1/2))/a^5+6*b^2*ln(x)/a^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2} \, dx = \frac{\frac{a\left(-a^3+4a^2b\sqrt{x}+18ab^2x+12b^3x^{3/2}\right)}{\left(a+b\sqrt{x}\right)^2 x} - 12b^2\log\left(a+b\sqrt{x}\right) + 6b^2\log(x)}{a^5}$$

input
```
Integrate[1/((a + b*Sqrt[x])^3*x^2),x]
```

output
```
((a*(-a^3 + 4*a^2*b*Sqrt[x] + 18*a*b^2*x + 12*b^3*x^(3/2)))/((a + b*Sqrt[x
])^2*x) - 12*b^2*Log[a + b*Sqrt[x]] + 6*b^2*Log[x])/a^5
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt{x}\right)^3} \, dx$$

$$\Big\downarrow \ 798$$

$$2 \int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^{3/2}} d\sqrt{x}$$

$$\Big\downarrow \ 54$$

$$2 \int \left(-\frac{6b^3}{a^5 \left(a+b\sqrt{x}\right)} - \frac{3b^3}{a^4 \left(a+b\sqrt{x}\right)^2} - \frac{b^3}{a^3 \left(a+b\sqrt{x}\right)^3} + \frac{6b^2}{a^5\sqrt{x}} - \frac{3b}{a^4 x} + \frac{1}{a^3 x^{3/2}}\right) d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2 \left(-\frac{6b^2 \log\left(a+b\sqrt{x}\right)}{a^5} + \frac{6b^2 \log\left(\sqrt{x}\right)}{a^5} + \frac{3b^2}{a^4 \left(a+b\sqrt{x}\right)} + \frac{3b}{a^4\sqrt{x}} + \frac{b^2}{2a^3 \left(a+b\sqrt{x}\right)^2} - \frac{1}{2a^3 x}\right)$$

input
```
Int[1/((a + b*Sqrt[x])^3*x^2),x]
```

output
```
2*(b^2/(2*a^3*(a + b*Sqrt[x])^2) + (3*b^2)/(a^4*(a + b*Sqrt[x])) - 1/(2*a^
3*x) + (3*b)/(a^4*Sqrt[x]) - (6*b^2*Log[a + b*Sqrt[x]])/a^5 + (6*b^2*Log[S
qrt[x]])/a^5)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{b^2}{a^3(a+b\sqrt{x})^2}+\frac{6b^2}{a^4(a+b\sqrt{x})}-\frac{1}{a^3x}+\frac{6b}{a^4\sqrt{x}}-\frac{12b^2\ln(a+b\sqrt{x})}{a^5}+\frac{6b^2\ln(x)}{a^5}$	78
default	$\frac{b^2}{a^3(a+b\sqrt{x})^2}+\frac{6b^2}{a^4(a+b\sqrt{x})}-\frac{1}{a^3x}+\frac{6b}{a^4\sqrt{x}}-\frac{12b^2\ln(a+b\sqrt{x})}{a^5}+\frac{6b^2\ln(x)}{a^5}$	78

input
```
int(1/(a+b*x^(1/2))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/a^3/(a+b*x^(1/2))^2+6*b^2/a^4/(a+b*x^(1/2))-1/a^3/x+6*b/a^4/x^(1/2)-12
*b^2*ln(a+b*x^(1/2))/a^5+6*b^2*ln(x)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(77) = 154$.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.82

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx = -\frac{6\,a^2b^4x^2 - 9\,a^4b^2x + a^6 + 12\,(b^6x^3 - 2\,a^2b^4x^2 + a^4b^2x)\log\left(b\sqrt{x}+a\right) - 12\,(b^6x^3 - 2\,a^2b^4x^2 + a^4b^2x)\,\mathrm{l}}{a^5b^4x^3 - 2\,a^7b^2x^2 + a^9x}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^2,x, algorithm="fricas")
```

output
```
-(6*a^2*b^4*x^2 - 9*a^4*b^2*x + a^6 + 12*(b^6*x^3 - 2*a^2*b^4*x^2 + a^4*b^
2*x)*log(b*sqrt(x) + a) - 12*(b^6*x^3 - 2*a^2*b^4*x^2 + a^4*b^2*x)*log(sqr
t(x)) - 2*(6*a*b^5*x^2 - 10*a^3*b^3*x + 3*a^5*b)*sqrt(x))/(a^5*b^4*x^3 - 2
*a^7*b^2*x^2 + a^9*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(82) = 164$.

Time = 1.61 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.66

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{1}{a^3x} \\ -\frac{2}{5b^3x^{\frac{5}{2}}} \\ -\frac{a^4\sqrt{x}}{a^7x^{\frac{3}{2}}+2a^6bx^2+a^5b^2x^{\frac{5}{2}}} + \frac{4a^3bx}{a^7x^{\frac{3}{2}}+2a^6bx^2+a^5b^2x^{\frac{5}{2}}} + \frac{6a^2b^2x^{\frac{3}{2}}\log(x)}{a^7x^{\frac{3}{2}}+2a^6bx^2+a^5b^2x^{\frac{5}{2}}} - \frac{12a^2b^2x^{\frac{3}{2}}\log\left(\frac{a}{b}+\sqrt{x}\right)}{a^7x^{\frac{3}{2}}+2a^6bx^2+a^5b^2x^{\frac{5}{2}}} + \frac{18a^2b^2x^{\frac{3}{2}}}{a^7x^{\frac{3}{2}}+2a^6bx^2+a^5b^2x} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**3/x**2,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-1/(a**3*x), Eq(b, 0)), (-
2/(5*b**3*x**(5/2)), Eq(a, 0)), (-a**4*sqrt(x)/(a**7*x**(3/2) + 2*a**6*b*x
**2 + a**5*b**2*x**(5/2)) + 4*a**3*b*x/(a**7*x**(3/2) + 2*a**6*b*x**2 + a*
*5*b**2*x**(5/2)) + 6*a**2*b**2*x**(3/2)*log(x)/(a**7*x**(3/2) + 2*a**6*b*
x**2 + a**5*b**2*x**(5/2)) - 12*a**2*b**2*x**(3/2)*log(a/b + sqrt(x))/(a**
7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 18*a**2*b**2*x**(3/2)/(
a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 12*a*b**3*x**2*log(x
)/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) - 24*a*b**3*x**2*lo
g(a/b + sqrt(x))/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 12
*a*b**3*x**2/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) + 6*b**4
*x**(5/2)*log(x)/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b**2*x**(5/2)) - 12
*b**4*x**(5/2)*log(a/b + sqrt(x))/(a**7*x**(3/2) + 2*a**6*b*x**2 + a**5*b*
*2*x**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx = \frac{12\,b^3 x^{\frac{3}{2}} + 18\,ab^2 x + 4\,a^2 b\sqrt{x} - a^3}{a^4 b^2 x^2 + 2\,a^5 b x^{\frac{3}{2}} + a^6 x} - \frac{12\,b^2 \log\left(b\sqrt{x} + a\right)}{a^5} + \frac{6\,b^2 \log\left(x\right)}{a^5}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^2,x, algorithm="maxima")
```

output
```
(12*b^3*x^(3/2) + 18*a*b^2*x + 4*a^2*b*sqrt(x) - a^3)/(a^4*b^2*x^2 + 2*a^5
*b*x^(3/2) + a^6*x) - 12*b^2*log(b*sqrt(x) + a)/a^5 + 6*b^2*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx = -\frac{12\,b^2 \log\left(\left|b\sqrt{x} + a\right|\right)}{a^5} + \frac{6\,b^2 \log\left(\left|x\right|\right)}{a^5} + \frac{12\,b^3 x^{\frac{3}{2}} + 18\,ab^2 x + 4\,a^2 b\sqrt{x} - a^3}{\left(bx + a\sqrt{x}\right)^2 a^4}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^2,x, algorithm="giac")
```

output
```
-12*b^2*log(abs(b*sqrt(x) + a))/a^5 + 6*b^2*log(abs(x))/a^5 + (12*b^3*x^(3
/2) + 18*a*b^2*x + 4*a^2*b*sqrt(x) - a^3)/((b*x + a*sqrt(x))^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx=\frac{\frac{4\,b\,\sqrt{x}}{a^2}-\frac{1}{a}+\frac{18\,b^2\,x}{a^3}+\frac{12\,b^3\,x^{3/2}}{a^4}}{a^2\,x+b^2\,x^2+2\,a\,b\,x^{3/2}}-\frac{24\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^5}$$

input
```
int(1/(x^2*(a + b*x^(1/2))^3),x)
```

output
```
((4*b*x^(1/2))/a^2 - 1/a + (18*b^2*x)/a^3 + (12*b^3*x^(3/2))/a^4)/(a^2*x +
 b^2*x^2 + 2*a*b*x^(3/2)) - (24*b^2*atanh((2*b*x^(1/2))/a + 1))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^2}\,dx$$
$$=\frac{-24\sqrt{x}\log(\sqrt{x}\,b+a)\,a\,b^3x+24\sqrt{x}\log(\sqrt{x})\,a\,b^3x+4\sqrt{x}\,a^3b-12\log(\sqrt{x}\,b+a)\,a^2b^2x-12\log(\sqrt{x}\,b+}{a^5x\left(2\sqrt{x}\,ab+a^2+b^2x\right)}$$

input
```
int(1/(a+b*x^(1/2))^3/x^2,x)
```

output
```
( - 24*sqrt(x)*log(sqrt(x)*b + a)*a*b**3*x + 24*sqrt(x)*log(sqrt(x))*a*b**
3*x + 4*sqrt(x)*a**3*b - 12*log(sqrt(x)*b + a)*a**2*b**2*x - 12*log(sqrt(x
)*b + a)*b**4*x**2 + 12*log(sqrt(x))*a**2*b**2*x + 12*log(sqrt(x))*b**4*x*
*2 - a**4 + 12*a**2*b**2*x - 6*b**4*x**2)/(a**5*x*(2*sqrt(x)*a*b + a**2 +
b**2*x))
```

3.103 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 111

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3} \, dx = \frac{b^4}{a^5\left(a+b\sqrt{x}\right)^2} + \frac{10b^4}{a^6\left(a+b\sqrt{x}\right)} - \frac{1}{2a^3x^2} + \frac{2b}{a^4x^{3/2}} - \frac{6b^2}{a^5x} + \frac{20b^3}{a^6\sqrt{x}} - \frac{30b^4\log\left(a+b\sqrt{x}\right)}{a^7} + \frac{15b^4\log(x)}{a^7}$$

output
```
b^4/a^5/(a+b*x^(1/2))^2+10*b^4/a^6/(a+b*x^(1/2))-1/2/a^3/x^2+2*b/a^4/x^(3/
2)-6*b^2/a^5/x+20*b^3/a^6/x^(1/2)-30*b^4*ln(a+b*x^(1/2))/a^7+15*b^4*ln(x)/
a^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3} \, dx = \frac{\frac{a\left(-a^5+2a^4b\sqrt{x}-5a^3b^2x+20a^2b^3x^{3/2}+90ab^4x^2+60b^5x^{5/2}\right)}{\left(a+b\sqrt{x}\right)^2x^2} - 60b^4\log\left(a+b\sqrt{x}\right) + 30b^4\log(x)}{2a^7}$$

input
```
Integrate[1/((a + b*Sqrt[x])^3*x^3),x]
```

output
```
((a*(-a^5 + 2*a^4*b*Sqrt[x] - 5*a^3*b^2*x + 20*a^2*b^3*x^(3/2) + 90*a*b^4*
x^2 + 60*b^5*x^(5/2)))/((a + b*Sqrt[x])^2*x^2) - 60*b^4*Log[a + b*Sqrt[x]]
 + 30*b^4*Log[x])/(2*a^7)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sqrt{x}\right)^3} \, dx$$

$$\Big\downarrow \ 798$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \ 54$$

$$2\int \left(-\frac{15b^5}{a^7\left(a+b\sqrt{x}\right)}-\frac{5b^5}{a^6\left(a+b\sqrt{x}\right)^2}-\frac{b^5}{a^5\left(a+b\sqrt{x}\right)^3}+\frac{15b^4}{a^7\sqrt{x}}-\frac{10b^3}{a^6x}+\frac{6b^2}{a^5x^{3/2}}-\frac{3b}{a^4x^2}+\frac{1}{a^3x^{5/2}}\right)d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2\left(-\frac{15b^4\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^4\log\left(\sqrt{x}\right)}{a^7}+\frac{5b^4}{a^6\left(a+b\sqrt{x}\right)}+\frac{10b^3}{a^6\sqrt{x}}+\frac{b^4}{2a^5\left(a+b\sqrt{x}\right)^2}-\frac{3b^2}{a^5x}+\frac{b}{a^4x^{3/2}}-\frac{1}{4a^3x^2}\right)$$

input
```
Int[1/((a + b*Sqrt[x])^3*x^3),x]
```

output
```
2*(b^4/(2*a^5*(a + b*Sqrt[x])^2) + (5*b^4)/(a^6*(a + b*Sqrt[x])) - 1/(4*a^
3*x^2) + b/(a^4*x^(3/2)) - (3*b^2)/(a^5*x) + (10*b^3)/(a^6*Sqrt[x]) - (15*
b^4*Log[a + b*Sqrt[x]])/a^7 + (15*b^4*Log[Sqrt[x]])/a^7)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{b^4}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{10b^4}{a^6\left(a+b\sqrt{x}\right)}-\frac{1}{2a^3x^2}+\frac{2b}{a^4x^{\frac{3}{2}}}-\frac{6b^2}{a^5x}+\frac{20b^3}{a^6\sqrt{x}}-\frac{30b^4\ln\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^4\ln\left(x\right)}{a^7}$	100
default	$\frac{b^4}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{10b^4}{a^6\left(a+b\sqrt{x}\right)}-\frac{1}{2a^3x^2}+\frac{2b}{a^4x^{\frac{3}{2}}}-\frac{6b^2}{a^5x}+\frac{20b^3}{a^6\sqrt{x}}-\frac{30b^4\ln\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^4\ln\left(x\right)}{a^7}$	100

input
```
int(1/(a+b*x^(1/2))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
b^4/a^5/(a+b*x^(1/2))^2+10*b^4/a^6/(a+b*x^(1/2))-1/2/a^3/x^2+2*b/a^4/x^(3/
2)-6*b^2/a^5/x+20*b^3/a^6/x^(1/2)-30*b^4*ln(a+b*x^(1/2))/a^7+15*b^4*ln(x)/
a^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+b\sqrt{x})^3 x^3} dx = -\frac{30 a^2 b^6 x^3 - 45 a^4 b^4 x^2 + 10 a^6 b^2 x + a^8 + 60 (b^8 x^4 - 2 a^2 b^6 x^3 + a^4 b^4 x^2) \log(b\sqrt{x} + a) - 60 (b^8 x^4 - 2 a}{2 (a^7 b^4 x^4 - 2 a^9 b^2 x^3 + a^{11} x^2)}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^3,x, algorithm="fricas")
```

output
```
-1/2*(30*a^2*b^6*x^3 - 45*a^4*b^4*x^2 + 10*a^6*b^2*x + a^8 + 60*(b^8*x^4 -
 2*a^2*b^6*x^3 + a^4*b^4*x^2)*log(b*sqrt(x) + a) - 60*(b^8*x^4 - 2*a^2*b^6
*x^3 + a^4*b^4*x^2)*log(sqrt(x)) - 4*(15*a*b^7*x^3 - 25*a^3*b^5*x^2 + 8*a^
5*b^3*x + a^7*b)*sqrt(x))/(a^7*b^4*x^4 - 2*a^9*b^2*x^3 + a^11*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(109) = 218.

Time = 2.16 (sec) , antiderivative size = 612, normalized size of antiderivative = 5.51

$$\int \frac{1}{(a+b\sqrt{x})^3 x^3} dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{1}{2a^3x^2} \\ -\frac{2}{7b^3x^{\frac{7}{2}}} \\ -\frac{a^6\sqrt{x}}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}} + \frac{2a^5bx}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}} - \frac{5a^4b^2x^{\frac{3}{2}}}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}} + \frac{20a^3b^3x^2}{2a^9x^{\frac{5}{2}}+4a^8bx^3+2a^7b^2x^{\frac{7}{2}}} + \frac{30a^2b^4x}{2a^9x^{\frac{5}{2}}+4a^8b} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**3/x**3,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-1/(2*a**3*x**2), Eq(b, 0)
), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-a**6*sqrt(x)/(2*a**9*x**(5/2) + 4*a
**8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 2*a**5*b*x/(2*a**9*x**(5/2) + 4*a**8*
b*x**3 + 2*a**7*b**2*x**(7/2)) - 5*a**4*b**2*x**(3/2)/(2*a**9*x**(5/2) + 4
*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 20*a**3*b**3*x**2/(2*a**9*x**(5/2)
+ 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 30*a**2*b**4*x**(5/2)*log(x)/(2*
a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) - 60*a**2*b**4*x**(5
/2)*log(a/b + sqrt(x))/(2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(
7/2)) + 90*a**2*b**4*x**(5/2)/(2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b*
*2*x**(7/2)) + 60*a*b**5*x**3*log(x)/(2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*
a**7*b**2*x**(7/2)) - 120*a*b**5*x**3*log(a/b + sqrt(x))/(2*a**9*x**(5/2)
+ 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 60*a*b**5*x**3/(2*a**9*x**(5/2)
+ 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) + 30*b**6*x**(7/2)*log(x)/(2*a**9*
x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)) - 60*b**6*x**(7/2)*log(a/
b + sqrt(x))/(2*a**9*x**(5/2) + 4*a**8*b*x**3 + 2*a**7*b**2*x**(7/2)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3}\,dx = \frac{60\,b^5x^{\frac{5}{2}} + 90\,ab^4x^2 + 20\,a^2b^3x^{\frac{3}{2}} - 5\,a^3b^2x + 2\,a^4b\sqrt{x} - a^5}{2\left(a^6b^2x^3 + 2\,a^7bx^{\frac{5}{2}} + a^8x^2\right)} - \frac{30\,b^4\log\left(b\sqrt{x}+a\right)}{a^7} + \frac{15\,b^4\log\left(x\right)}{a^7}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^3,x, algorithm="maxima")
```

output
```
1/2*(60*b^5*x^(5/2) + 90*a*b^4*x^2 + 20*a^2*b^3*x^(3/2) - 5*a^3*b^2*x + 2*
a^4*b*sqrt(x) - a^5)/(a^6*b^2*x^3 + 2*a^7*b*x^(5/2) + a^8*x^2) - 30*b^4*lo
g(b*sqrt(x) + a)/a^7 + 15*b^4*log(x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3}\,dx = -\frac{30\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^7}+\frac{15\,b^4\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^{\frac{5}{2}}+90\,a^2b^4x^2+20\,a^3b^3x^{\frac{3}{2}}-5\,a^4b^2x+2\,a^5b\sqrt{x}-a^6}{2\left(b\sqrt{x}+a\right)^2a^7x^2}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^3,x, algorithm="giac")
```

output
```
-30*b^4*log(abs(b*sqrt(x) + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/2*(60*a*b
^5*x^(5/2) + 90*a^2*b^4*x^2 + 20*a^3*b^3*x^(3/2) - 5*a^4*b^2*x + 2*a^5*b*s
qrt(x) - a^6)/((b*sqrt(x) + a)^2*a^7*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3}\,dx = \frac{\frac{b\sqrt{x}}{a^2}-\frac{1}{2\,a}-\frac{5\,b^2\,x}{2\,a^3}+\frac{45\,b^4\,x^2}{a^5}+\frac{10\,b^3\,x^{3/2}}{a^4}+\frac{30\,b^5\,x^{5/2}}{a^6}}{a^2\,x^2+b^2\,x^3+2\,a\,b\,x^{5/2}}-\frac{60\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^7}$$

input
```
int(1/(x^3*(a + b*x^(1/2))^3),x)
```

output
```
((b*x^(1/2))/a^2 - 1/(2*a) - (5*b^2*x)/(2*a^3) + (45*b^4*x^2)/a^5 + (10*b^
3*x^(3/2))/a^4 + (30*b^5*x^(5/2))/a^6)/(a^2*x^2 + b^2*x^3 + 2*a*b*x^(5/2))
 - (60*b^4*atanh((2*b*x^(1/2))/a + 1))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^3}\,dx$$
$$= \frac{-120\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a\,b^5x^2+120\sqrt{x}\log\left(\sqrt{x}\right)a\,b^5x^2+2\sqrt{x}\,a^5b+20\sqrt{x}\,a^3b^3x-60\log\left(\sqrt{x}\,b+a\right)a^2b}{2a^7x^2\left(2\sqrt{x}\,a\right.}$$

input
```
int(1/(a+b*x^(1/2))^3/x^3,x)
```

output
```
( - 120*sqrt(x)*log(sqrt(x)*b + a)*a*b**5*x**2 + 120*sqrt(x)*log(sqrt(x))*
a*b**5*x**2 + 2*sqrt(x)*a**5*b + 20*sqrt(x)*a**3*b**3*x - 60*log(sqrt(x)*b
 + a)*a**2*b**4*x**2 - 60*log(sqrt(x)*b + a)*b**6*x**3 + 60*log(sqrt(x))*a
**2*b**4*x**2 + 60*log(sqrt(x))*b**6*x**3 - a**6 - 5*a**4*b**2*x + 60*a**2
*b**4*x**2 - 30*b**6*x**3)/(2*a**7*x**2*(2*sqrt(x)*a*b + a**2 + b**2*x))
```

3.104 $\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 139

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4} \, dx = \frac{b^6}{a^7\left(a+b\sqrt{x}\right)^2} + \frac{14b^6}{a^8\left(a+b\sqrt{x}\right)} - \frac{1}{3a^3x^3} + \frac{6b}{5a^4x^{5/2}} - \frac{3b^2}{a^5x^2} + \frac{20b^3}{3a^6x^{3/2}} - \frac{15b^4}{a^7x} + \frac{42b^5}{a^8\sqrt{x}} - \frac{56b^6\log\left(a+b\sqrt{x}\right)}{a^9} + \frac{28b^6\log(x)}{a^9}$$

output
```
b^6/a^7/(a+b*x^(1/2))^2+14*b^6/a^8/(a+b*x^(1/2))-1/3/a^3/x^3+6/5*b/a^4/x^(
5/2)-3*b^2/a^5/x^2+20/3*b^3/a^6/x^(3/2)-15*b^4/a^7/x+42*b^5/a^8/x^(1/2)-56
*b^6*ln(a+b*x^(1/2))/a^9+28*b^6*ln(x)/a^9
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4} \, dx = \frac{\frac{a\left(-5a^7+8a^6b\sqrt{x}-14a^5b^2x+28a^4b^3x^{3/2}-70a^3b^4x^2+280a^2b^5x^{5/2}+1260ab^6x^3+840b^7x^{7/2}\right)}{\left(a+b\sqrt{x}\right)^2x^3} - 840b^6\log\left(a+b\sqrt{x}\right) + 420b^6\log(x)}{15a^9}$$

input
```
Integrate[1/((a + b*Sqrt[x])^3*x^4),x]
```

output
```
((a*(-5*a^7 + 8*a^6*b*Sqrt[x] - 14*a^5*b^2*x + 28*a^4*b^3*x^(3/2) - 70*a^3
*b^4*x^2 + 280*a^2*b^5*x^(5/2) + 1260*a*b^6*x^3 + 840*b^7*x^(7/2)))/((a +
b*Sqrt[x])^2*x^3) - 840*b^6*Log[a + b*Sqrt[x]] + 420*b^6*Log[x])/(15*a^9)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\sqrt{x}\right)^3} \, dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{28b^7}{a^9\left(a+b\sqrt{x}\right)}-\frac{7b^7}{a^8\left(a+b\sqrt{x}\right)^2}-\frac{b^7}{a^7\left(a+b\sqrt{x}\right)^3}+\frac{28b^6}{a^9\sqrt{x}}-\frac{21b^5}{a^8x}+\frac{15b^4}{a^7x^{3/2}}-\frac{10b^3}{a^6x^2}+\frac{6b^2}{a^5x^{5/2}}-\frac{3b}{a^4x^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{28b^6\log\left(a+b\sqrt{x}\right)}{a^9}+\frac{28b^6\log\left(\sqrt{x}\right)}{a^9}+\frac{7b^6}{a^8\left(a+b\sqrt{x}\right)}+\frac{21b^5}{a^8\sqrt{x}}+\frac{b^6}{2a^7\left(a+b\sqrt{x}\right)^2}-\frac{15b^4}{2a^7x}+\frac{10b^3}{3a^6x^{3/2}}-\frac{3b^2}{2a^5x}\right.$$

input
```
Int[1/((a + b*Sqrt[x])^3*x^4),x]
```

output
```
2*(b^6/(2*a^7*(a + b*Sqrt[x])^2) + (7*b^6)/(a^8*(a + b*Sqrt[x])) - 1/(6*a^
3*x^3) + (3*b)/(5*a^4*x^(5/2)) - (3*b^2)/(2*a^5*x^2) + (10*b^3)/(3*a^6*x^(
3/2)) - (15*b^4)/(2*a^7*x) + (21*b^5)/(a^8*Sqrt[x]) - (28*b^6*Log[a + b*Sq
rt[x]])/a^9 + (28*b^6*Log[Sqrt[x]])/a^9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{b^6}{a^7(a+b\sqrt{x})^2}+\frac{14b^6}{a^8(a+b\sqrt{x})}-\frac{1}{3a^3x^3}+\frac{6b}{5a^4x^{\frac{5}{2}}}-\frac{3b^2}{a^5x^2}+\frac{20b^3}{3a^6x^{\frac{3}{2}}}-\frac{15b^4}{a^7x}+\frac{42b^5}{a^8\sqrt{x}}-\frac{56b^6\ln(a+b\sqrt{x})}{a^9}+$
default	$\frac{b^6}{a^7(a+b\sqrt{x})^2}+\frac{14b^6}{a^8(a+b\sqrt{x})}-\frac{1}{3a^3x^3}+\frac{6b}{5a^4x^{\frac{5}{2}}}-\frac{3b^2}{a^5x^2}+\frac{20b^3}{3a^6x^{\frac{3}{2}}}-\frac{15b^4}{a^7x}+\frac{42b^5}{a^8\sqrt{x}}-\frac{56b^6\ln(a+b\sqrt{x})}{a^9}+$

input
```
int(1/(a+b*x^(1/2))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
b^6/a^7/(a+b*x^(1/2))^2+14*b^6/a^8/(a+b*x^(1/2))-1/3/a^3/x^3+6/5*b/a^4/x^(
5/2)-3*b^2/a^5/x^2+20/3*b^3/a^6/x^(3/2)-15*b^4/a^7/x+42*b^5/a^8/x^(1/2)-56
*b^6*ln(a+b*x^(1/2))/a^9+28*b^6*ln(x)/a^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx = -\frac{420\,a^2b^8x^4 - 630\,a^4b^6x^3 + 140\,a^6b^4x^2 + 35\,a^8b^2x + 5\,a^{10} + 840\,(b^{10}x^5 - 2\,a^2b^8x^4 + a^4b^6x^3)\log\left(b\sqrt{x}+\right.}{15\,(a^9b^4x^5}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^4,x, algorithm="fricas")
```

output
```
-1/15*(420*a^2*b^8*x^4 - 630*a^4*b^6*x^3 + 140*a^6*b^4*x^2 + 35*a^8*b^2*x
+ 5*a^10 + 840*(b^10*x^5 - 2*a^2*b^8*x^4 + a^4*b^6*x^3)*log(b*sqrt(x) + a)
 - 840*(b^10*x^5 - 2*a^2*b^8*x^4 + a^4*b^6*x^3)*log(sqrt(x)) - 2*(420*a*b^
9*x^4 - 700*a^3*b^7*x^3 + 224*a^5*b^5*x^2 + 32*a^7*b^3*x + 9*a^9*b)*sqrt(x
))/(a^9*b^4*x^5 - 2*a^11*b^2*x^4 + a^13*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(138) = 276.

Time = 2.98 (sec) , antiderivative size = 707, normalized size of antiderivative = 5.09

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2))**3/x**4,x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-1/(3*a**3*x**3), Eq(b, 0)
), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (-5*a**8*sqrt(x)/(15*a**11*x**(7/2) +
 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 8*a**7*b*x/(15*a**11*x**(7/2)
+ 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) - 14*a**6*b**2*x**(3/2)/(15*a**
11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 28*a**5*b**3*x**2
/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) - 70*a**4*b
**4*x**(5/2)/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2))
 + 280*a**3*b**5*x**3/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*
x**(9/2)) + 420*a**2*b**6*x**(7/2)*log(x)/(15*a**11*x**(7/2) + 30*a**10*b*
x**4 + 15*a**9*b**2*x**(9/2)) - 840*a**2*b**6*x**(7/2)*log(a/b + sqrt(x))/
(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 1260*a**2*
b**6*x**(7/2)/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)
) + 840*a*b**7*x**4*log(x)/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*
b**2*x**(9/2)) - 1680*a*b**7*x**4*log(a/b + sqrt(x))/(15*a**11*x**(7/2) +
30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 840*a*b**7*x**4/(15*a**11*x**(7
/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) + 420*b**8*x**(9/2)*log(x)/
(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*b**2*x**(9/2)) - 840*b**8*x
**(9/2)*log(a/b + sqrt(x))/(15*a**11*x**(7/2) + 30*a**10*b*x**4 + 15*a**9*
b**2*x**(9/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx = \frac{840\,b^7x^{\frac{7}{2}} + 1260\,ab^6x^3 + 280\,a^2b^5x^{\frac{5}{2}} - 70\,a^3b^4x^2 + 28\,a^4b^3x^{\frac{3}{2}} - 14\,a^5b^2x + 8\,a^6b\sqrt{x} - 5\,a^7}{15\left(a^8b^2x^4 + 2\,a^9bx^{\frac{7}{2}} + a^{10}x^3\right)} - \frac{56\,b^6\log\left(b\sqrt{x}+a\right)}{a^9} + \frac{28\,b^6\log\left(x\right)}{a^9}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^4,x, algorithm="maxima")
```

output
```
1/15*(840*b^7*x^(7/2) + 1260*a*b^6*x^3 + 280*a^2*b^5*x^(5/2) - 70*a^3*b^4*
x^2 + 28*a^4*b^3*x^(3/2) - 14*a^5*b^2*x + 8*a^6*b*sqrt(x) - 5*a^7)/(a^8*b^
2*x^4 + 2*a^9*b*x^(7/2) + a^10*x^3) - 56*b^6*log(b*sqrt(x) + a)/a^9 + 28*b
^6*log(x)/a^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx = -\frac{56\,b^6\log\left(\left|b\sqrt{x}+a\right|\right)}{a^9} + \frac{28\,b^6\log\left(\left|x\right|\right)}{a^9} + \frac{840\,ab^7x^{\frac{7}{2}} + 1260\,a^2b^6x^3 + 280\,a^3b^5x^{\frac{5}{2}} - 70\,a^4b^4x^2 + 28\,a^5b^3x^{\frac{3}{2}} - 14\,a^6b^2x + 8\,a^7b\sqrt{x} - 5\,a^8}{15\,\left(b\sqrt{x}+a\right)^2a^9x^3}$$

input
```
integrate(1/(a+b*x^(1/2))^3/x^4,x, algorithm="giac")
```

output
```
-56*b^6*log(abs(b*sqrt(x) + a))/a^9 + 28*b^6*log(abs(x))/a^9 + 1/15*(840*a
*b^7*x^(7/2) + 1260*a^2*b^6*x^3 + 280*a^3*b^5*x^(5/2) - 70*a^4*b^4*x^2 + 2
8*a^5*b^3*x^(3/2) - 14*a^6*b^2*x + 8*a^7*b*sqrt(x) - 5*a^8)/((b*sqrt(x) +
a)^2*a^9*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx = \frac{\frac{8\,b\,\sqrt{x}}{15\,a^2} - \frac{1}{3\,a} - \frac{14\,b^2\,x}{15\,a^3} - \frac{14\,b^4\,x^2}{3\,a^5} + \frac{28\,b^3\,x^{3/2}}{15\,a^4} + \frac{84\,b^6\,x^3}{a^7} + \frac{56\,b^5\,x^{5/2}}{3\,a^6} + \frac{56\,b^7\,x^{7/2}}{a^8}}{a^2\,x^3 + b^2\,x^4 + 2\,a\,b\,x^{7/2}} - \frac{112\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a} + 1\right)}{a^9}$$

input
```
int(1/(x^4*(a + b*x^(1/2))^3),x)
```

output
```
((8*b*x^(1/2))/(15*a^2) - 1/(3*a) - (14*b^2*x)/(15*a^3) - (14*b^4*x^2)/(3*
a^5) + (28*b^3*x^(3/2))/(15*a^4) + (84*b^6*x^3)/a^7 + (56*b^5*x^(5/2))/(3*
a^6) + (56*b^7*x^(7/2))/a^8)/(a^2*x^3 + b^2*x^4 + 2*a*b*x^(7/2)) - (112*b^
6*atanh((2*b*x^(1/2))/a + 1))/a^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^3 x^4}\,dx$$
$$= \frac{-1680\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a\,b^7x^3+1680\sqrt{x}\log\left(\sqrt{x}\right)a\,b^7x^3+8\sqrt{x}\,a^7b+28\sqrt{x}\,a^5b^3x+280\sqrt{x}\,a^3b^5x^2-8}{}$$

input
```
int(1/(a+b*x^(1/2))^3/x^4,x)
```

output
```
( - 1680*sqrt(x)*log(sqrt(x)*b + a)*a*b**7*x**3 + 1680*sqrt(x)*log(sqrt(x)
)*a*b**7*x**3 + 8*sqrt(x)*a**7*b + 28*sqrt(x)*a**5*b**3*x + 280*sqrt(x)*a*
*3*b**5*x**2 - 840*log(sqrt(x)*b + a)*a**2*b**6*x**3 - 840*log(sqrt(x)*b +
 a)*b**8*x**4 + 840*log(sqrt(x))*a**2*b**6*x**3 + 840*log(sqrt(x))*b**8*x*
*4 - 5*a**8 - 14*a**6*b**2*x - 70*a**4*b**4*x**2 + 840*a**2*b**6*x**3 - 42
0*b**8*x**4)/(15*a**9*x**3*(2*sqrt(x)*a*b + a**2 + b**2*x))
```

3.105 $\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 155

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{a^9}{2b^{10}\left(a+b\sqrt{x}\right)^4} - \frac{6a^8}{b^{10}\left(a+b\sqrt{x}\right)^3} + \frac{36a^7}{b^{10}\left(a+b\sqrt{x}\right)^2} - \frac{168a^6}{b^{10}\left(a+b\sqrt{x}\right)} + \frac{140a^4\sqrt{x}}{b^9} - \frac{35a^3x}{b^8} + \frac{10a^2x^{3/2}}{b^7} - \frac{5ax^2}{2b^6} + \frac{2x^{5/2}}{5b^5} - \frac{252a^5\log\left(a+b\sqrt{x}\right)}{b^{10}}$$

output
```
1/2*a^9/b^10/(a+b*x^(1/2))^4-6*a^8/b^10/(a+b*x^(1/2))^3+36*a^7/b^10/(a+b*x
^(1/2))^2-168*a^6/b^10/(a+b*x^(1/2))+140*a^4*x^(1/2)/b^9-35*a^3*x/b^8+10*a
^2*x^(3/2)/b^7-5/2*a*x^2/b^6+2/5*x^(5/2)/b^5-252*a^5*ln(a+b*x^(1/2))/b^10
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx$$
$$=\frac{-1375a^9-2980a^8b\sqrt{x}+570a^7b^2x+5420a^6b^3x^{3/2}+3875a^5b^4x^2+504a^4b^5x^{5/2}-84a^3b^6x^3+24a^2b^7x^{7/}}{10b^{10}\left(a+b\sqrt{x}\right)^4}$$
$$-\frac{252a^5\log\left(a+b\sqrt{x}\right)}{b^{10}}$$

input
```
Integrate[x^4/(a + b*Sqrt[x])^5,x]
```

output
```
(-1375*a^9 - 2980*a^8*b*Sqrt[x] + 570*a^7*b^2*x + 5420*a^6*b^3*x^(3/2) + 3
875*a^5*b^4*x^2 + 504*a^4*b^5*x^(5/2) - 84*a^3*b^6*x^3 + 24*a^2*b^7*x^(7/2
) - 9*a*b^8*x^4 + 4*b^9*x^(9/2))/(10*b^10*(a + b*Sqrt[x])^4) - (252*a^5*Lo
g[a + b*Sqrt[x]])/b^10
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{x^{9/2}}{\left(a+b\sqrt{x}\right)^5}d\sqrt{x}$$
$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^9}{b^9\left(a+b\sqrt{x}\right)^5}+\frac{9a^8}{b^9\left(a+b\sqrt{x}\right)^4}-\frac{36a^7}{b^9\left(a+b\sqrt{x}\right)^3}+\frac{84a^6}{b^9\left(a+b\sqrt{x}\right)^2}-\frac{126a^5}{b^9\left(a+b\sqrt{x}\right)}+\frac{70a^4}{b^9}-\frac{35\sqrt{x}a^3}{b^8}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^9}{4b^{10}\left(a+b\sqrt{x}\right)^4}-\frac{3a^8}{b^{10}\left(a+b\sqrt{x}\right)^3}+\frac{18a^7}{b^{10}\left(a+b\sqrt{x}\right)^2}-\frac{84a^6}{b^{10}\left(a+b\sqrt{x}\right)}-\frac{126a^5\log\left(a+b\sqrt{x}\right)}{b^{10}}+\frac{70a^4\sqrt{x}}{b^9}-\right.$$

input
```
Int[x^4/(a + b*Sqrt[x])^5,x]
```

output
```
2*(a^9/(4*b^10*(a + b*Sqrt[x])^4) - (3*a^8)/(b^10*(a + b*Sqrt[x])^3) + (18
*a^7)/(b^10*(a + b*Sqrt[x])^2) - (84*a^6)/(b^10*(a + b*Sqrt[x])) + (70*a^4
*Sqrt[x])/b^9 - (35*a^3*x)/(2*b^8) + (5*a^2*x^(3/2))/b^7 - (5*a*x^2)/(4*b^
6) + x^(5/2)/(5*b^5) - (126*a^5*Log[a + b*Sqrt[x]])/b^10)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{2x^{\frac{5}{2}}b^4}{5}-\frac{5ab^3x^2}{2}+10x^{\frac{3}{2}}a^2b^2-35a^3bx+140\sqrt{x}\,a^4}{b^9}-\frac{6a^8}{b^{10}(a+b\sqrt{x})^3}+\frac{a^9}{2b^{10}(a+b\sqrt{x})^4}-\frac{252a^5\ln(a+b\sqrt{x})}{b^{10}}+\frac{}{b^{10}}$
default	$\frac{\frac{2x^{\frac{5}{2}}b^4}{5}-\frac{5ab^3x^2}{2}+10x^{\frac{3}{2}}a^2b^2-35a^3bx+140\sqrt{x}\,a^4}{b^9}-\frac{6a^8}{b^{10}(a+b\sqrt{x})^3}+\frac{a^9}{2b^{10}(a+b\sqrt{x})^4}-\frac{252a^5\ln(a+b\sqrt{x})}{b^{10}}+\frac{}{b^{10}}$

input
```
int(x^4/(a+b*x^(1/2))^5,x,method=_RETURNVERBOSE)
```

output
```
2/b^9*(1/5*x^(5/2)*b^4-5/4*a*b^3*x^2+5*x^(3/2)*a^2*b^2-35/2*a^3*b*x+70*x^(
1/2)*a^4)-6*a^8/b^10/(a+b*x^(1/2))^3+1/2*a^9/b^10/(a+b*x^(1/2))^4-252*a^5*
ln(a+b*x^(1/2))/b^10+36*a^7/b^10/(a+b*x^(1/2))^2-168*a^6/b^10/(a+b*x^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{(a+b\sqrt{x})^5}\,dx = -\frac{25\,ab^{12}x^6+250\,a^3b^{10}x^5-1250\,a^5b^8x^4-40\,a^7b^6x^3+3840\,a^9b^4x^2-4240\,a^{11}b^2x+1375\,a^{13}+2520\,(a^5}{}$$

input
```
integrate(x^4/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

output
```
-1/10*(25*a*b^12*x^6 + 250*a^3*b^10*x^5 - 1250*a^5*b^8*x^4 - 40*a^7*b^6*x^
3 + 3840*a^9*b^4*x^2 - 4240*a^11*b^2*x + 1375*a^13 + 2520*(a^5*b^8*x^4 - 4
*a^7*b^6*x^3 + 6*a^9*b^4*x^2 - 4*a^11*b^2*x + a^13)*log(b*sqrt(x) + a) - 4
*(b^13*x^6 + 21*a^2*b^11*x^5 + 256*a^4*b^9*x^4 - 1674*a^6*b^7*x^3 + 3066*a
^8*b^5*x^2 - 2310*a^10*b^3*x + 630*a^12*b)*sqrt(x))/(b^18*x^4 - 4*a^2*b^16
*x^3 + 6*a^4*b^14*x^2 - 4*a^6*b^12*x + a^8*b^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. $2(151) = 302$.

Time = 1.51 (sec) , antiderivative size = 966, normalized size of antiderivative = 6.23

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(a+b*x**(1/2))**5,x)
```

output
```
Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (x**5/(5*a**5), Eq(b, 0)),
(zoo*x**5, Eq(a, -b*sqrt(x))), (-2520*a**9*log(a/b + sqrt(x))/(10*a**4*b**
10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b*
*14*x**2) - 5250*a**9/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**
12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) - 10080*a**8*b*sqrt(x)*log(a/b
 + sqrt(x))/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*
a*b**13*x**(3/2) + 10*b**14*x**2) - 18480*a**8*b*sqrt(x)/(10*a**4*b**10 +
40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x
**2) - 15120*a**7*b**2*x*log(a/b + sqrt(x))/(10*a**4*b**10 + 40*a**3*b**11
*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) - 22680*
a**7*b**2*x/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*
a*b**13*x**(3/2) + 10*b**14*x**2) - 10080*a**6*b**3*x**(3/2)*log(a/b + sqr
t(x))/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**1
3*x**(3/2) + 10*b**14*x**2) - 10080*a**6*b**3*x**(3/2)/(10*a**4*b**10 + 40
*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**
2) - 2520*a**5*b**4*x**2*log(a/b + sqrt(x))/(10*a**4*b**10 + 40*a**3*b**11
*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b**14*x**2) + 504*a*
*4*b**5*x**(5/2)/(10*a**4*b**10 + 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x
+ 40*a*b**13*x**(3/2) + 10*b**14*x**2) - 84*a**3*b**6*x**3/(10*a**4*b**10
+ 40*a**3*b**11*sqrt(x) + 60*a**2*b**12*x + 40*a*b**13*x**(3/2) + 10*b*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{252\,a^5\log\left(b\sqrt{x}+a\right)}{b^{10}} + \frac{2\left(b\sqrt{x}+a\right)^5}{5\,b^{10}} - \frac{9\left(b\sqrt{x}+a\right)^4 a}{2\,b^{10}} + \frac{24\left(b\sqrt{x}+a\right)^3 a^2}{b^{10}} - \frac{84\left(b\sqrt{x}+a\right)^2 a^3}{b^{10}} + \frac{252\left(b\sqrt{x}+a\right)a^4}{b^{10}} - \frac{168\,a^6}{\left(b\sqrt{x}+a\right)b^{10}} + \frac{36\,a^7}{\left(b\sqrt{x}+a\right)^2 b^{10}} - \frac{6\,a^8}{\left(b\sqrt{x}+a\right)^3 b^{10}} + \frac{a^9}{2\left(b\sqrt{x}+a\right)^4 b^{10}}$$

input
```
integrate(x^4/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

output
```
-252*a^5*log(b*sqrt(x) + a)/b^10 + 2/5*(b*sqrt(x) + a)^5/b^10 - 9/2*(b*sqr
t(x) + a)^4*a/b^10 + 24*(b*sqrt(x) + a)^3*a^2/b^10 - 84*(b*sqrt(x) + a)^2*
a^3/b^10 + 252*(b*sqrt(x) + a)*a^4/b^10 - 168*a^6/((b*sqrt(x) + a)*b^10) +
 36*a^7/((b*sqrt(x) + a)^2*b^10) - 6*a^8/((b*sqrt(x) + a)^3*b^10) + 1/2*a^
9/((b*sqrt(x) + a)^4*b^10)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{252\,a^5\log\left(\left|b\sqrt{x}+a\right|\right)}{b^{10}} - \frac{336\,a^6b^3x^{\frac{3}{2}} + 936\,a^7b^2x + 876\,a^8b\sqrt{x} + 275\,a^9}{2\left(b\sqrt{x}+a\right)^4 b^{10}} + \frac{4\,b^{20}x^{\frac{5}{2}} - 25\,ab^{19}x^2 + 100\,a^2b^{18}x^{\frac{3}{2}} - 350\,a^3b^{17}x + 1400\,a^4b^{16}\sqrt{x}}{10\,b^{25}}$$

input
```
integrate(x^4/(a+b*x^(1/2))^5,x, algorithm="giac")
```

output
```
-252*a^5*log(abs(b*sqrt(x) + a))/b^10 - 1/2*(336*a^6*b^3*x^(3/2) + 936*a^7
*b^2*x + 876*a^8*b*sqrt(x) + 275*a^9)/((b*sqrt(x) + a)^4*b^10) + 1/10*(4*b
^20*x^(5/2) - 25*a*b^19*x^2 + 100*a^2*b^18*x^(3/2) - 350*a^3*b^17*x + 1400
*a^4*b^16*sqrt(x))/b^25
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{2\,x^{5/2}}{5\,b^5} - \frac{\frac{275\,a^9}{2\,b} + 438\,a^8\,\sqrt{x} + 168\,a^6\,b^2\,x^{3/2} + 468\,a^7\,b\,x}{a^4\,b^9 + b^{13}\,x^2 + 6\,a^2\,b^{11}\,x + 4\,a\,b^{12}\,x^{3/2} + 4\,a^3\,b^{10}\,\sqrt{x}} - \frac{5\,a\,x^2}{2\,b^6} - \frac{35\,a^3\,x}{b^8} - \frac{252\,a^5\,\ln\left(a+b\,\sqrt{x}\right)}{b^{10}} + \frac{10\,a^2\,x^{3/2}}{b^7} + \frac{140\,a^4\,\sqrt{x}}{b^9}$$

input
```
int(x^4/(a + b*x^(1/2))^5,x)
```

output
```
(2*x^(5/2))/(5*b^5) - ((275*a^9)/(2*b) + 438*a^8*x^(1/2) + 168*a^6*b^2*x^(
3/2) + 468*a^7*b*x)/(a^4*b^9 + b^13*x^2 + 6*a^2*b^11*x + 4*a*b^12*x^(3/2)
+ 4*a^3*b^10*x^(1/2)) - (5*a*x^2)/(2*b^6) - (35*a^3*x)/b^8 - (252*a^5*log(
a + b*x^(1/2)))/b^10 + (10*a^2*x^(3/2))/b^7 + (140*a^4*x^(1/2))/b^9
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.37

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{-10080\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^8b - 10080\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^6b^3x - 8400\sqrt{x}\,a^8b + 504\sqrt{x}\,a^4b^5x^2 + 24\sqrt{x}\,a}{}$$

input
```
int(x^4/(a+b*x^(1/2))^5,x)
```

output
```
( - 10080*sqrt(x)*log(sqrt(x)*b + a)*a**8*b - 10080*sqrt(x)*log(sqrt(x)*b
+ a)*a**6*b**3*x - 8400*sqrt(x)*a**8*b + 504*sqrt(x)*a**4*b**5*x**2 + 24*s
qrt(x)*a**2*b**7*x**3 + 4*sqrt(x)*b**9*x**4 - 2520*log(sqrt(x)*b + a)*a**9
 - 15120*log(sqrt(x)*b + a)*a**7*b**2*x - 2520*log(sqrt(x)*b + a)*a**5*b**
4*x**2 - 2730*a**9 - 7560*a**7*b**2*x + 2520*a**5*b**4*x**2 - 84*a**3*b**6
*x**3 - 9*a*b**8*x**4)/(10*b**10*(4*sqrt(x)*a**3*b + 4*sqrt(x)*a*b**3*x +
a**4 + 6*a**2*b**2*x + b**4*x**2))
```

3.106 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{a^7}{2b^8\left(a+b\sqrt{x}\right)^4} - \frac{14a^6}{3b^8\left(a+b\sqrt{x}\right)^3} + \frac{21a^5}{b^8\left(a+b\sqrt{x}\right)^2} - \frac{70a^4}{b^8\left(a+b\sqrt{x}\right)} + \frac{30a^2\sqrt{x}}{b^7} - \frac{5ax}{b^6} + \frac{2x^{3/2}}{3b^5} - \frac{70a^3\log\left(a+b\sqrt{x}\right)}{b^8}$$

output
```
1/2*a^7/b^8/(a+b*x^(1/2))^4-14/3*a^6/b^8/(a+b*x^(1/2))^3+21*a^5/b^8/(a+b*x
^(1/2))^2-70*a^4/b^8/(a+b*x^(1/2))+30*a^2*x^(1/2)/b^7-5*a*x/b^6+2/3*x^(3/2
)/b^5-70*a^3*ln(a+b*x^(1/2))/b^8
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{-319a^7-856a^6b\sqrt{x}-444a^5b^2x+544a^4b^3x^{3/2}+556a^3b^4x^2+84a^2b^5x^{5/2}-14ab^6x^3+4b^7x^{7/2}}{6b^8\left(a+b\sqrt{x}\right)^4} - \frac{70a^3\log\left(a+b\sqrt{x}\right)}{b^8}$$

input
```
Integrate[x^3/(a + b*Sqrt[x])^5,x]
```

output
```
(-319*a^7 - 856*a^6*b*Sqrt[x] - 444*a^5*b^2*x + 544*a^4*b^3*x^(3/2) + 556*
a^3*b^4*x^2 + 84*a^2*b^5*x^(5/2) - 14*a*b^6*x^3 + 4*b^7*x^(7/2))/(6*b^8*(a
 + b*Sqrt[x])^4) - (70*a^3*Log[a + b*Sqrt[x]])/b^8
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{7/2}}{\left(a+b\sqrt{x}\right)^5}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^7}{b^7\left(a+b\sqrt{x}\right)^5}+\frac{7a^6}{b^7\left(a+b\sqrt{x}\right)^4}-\frac{21a^5}{b^7\left(a+b\sqrt{x}\right)^3}+\frac{35a^4}{b^7\left(a+b\sqrt{x}\right)^2}-\frac{35a^3}{b^7\left(a+b\sqrt{x}\right)}+\frac{15a^2}{b^7}-\frac{5\sqrt{x}a}{b^6}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^7}{4b^8\left(a+b\sqrt{x}\right)^4}-\frac{7a^6}{3b^8\left(a+b\sqrt{x}\right)^3}+\frac{21a^5}{2b^8\left(a+b\sqrt{x}\right)^2}-\frac{35a^4}{b^8\left(a+b\sqrt{x}\right)}-\frac{35a^3\log\left(a+b\sqrt{x}\right)}{b^8}+\frac{15a^2\sqrt{x}}{b^7}-\frac{5a}{2b^6}\right.$$

input
```
Int[x^3/(a + b*Sqrt[x])^5,x]
```

output
```
2*(a^7/(4*b^8*(a + b*Sqrt[x])^4) - (7*a^6)/(3*b^8*(a + b*Sqrt[x])^3) + (21
*a^5)/(2*b^8*(a + b*Sqrt[x])^2) - (35*a^4)/(b^8*(a + b*Sqrt[x])) + (15*a^2
*Sqrt[x])/b^7 - (5*a*x)/(2*b^6) + x^(3/2)/(3*b^5) - (35*a^3*Log[a + b*Sqrt
[x]])/b^8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{2b^2x^{\frac{3}{2}}}{3}-5abx+30a^2\sqrt{x}}{b^7}-\frac{14a^6}{3b^8\left(a+b\sqrt{x}\right)^3}+\frac{21a^5}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{70a^3\ln\left(a+b\sqrt{x}\right)}{b^8}-\frac{70a^4}{b^8\left(a+b\sqrt{x}\right)}+\frac{a^7}{2b^8\left(a+b\sqrt{x}\right)^4}$
default	$\frac{\frac{2b^2x^{\frac{3}{2}}}{3}-5abx+30a^2\sqrt{x}}{b^7}-\frac{14a^6}{3b^8\left(a+b\sqrt{x}\right)^3}+\frac{21a^5}{b^8\left(a+b\sqrt{x}\right)^2}-\frac{70a^3\ln\left(a+b\sqrt{x}\right)}{b^8}-\frac{70a^4}{b^8\left(a+b\sqrt{x}\right)}+\frac{a^7}{2b^8\left(a+b\sqrt{x}\right)^4}$

input
```
int(x^3/(a+b*x^(1/2))^5,x,method=_RETURNVERBOSE)
```

output
```
2/b^7*(1/3*b^2*x^(3/2)-5/2*a*b*x+15*a^2*x^(1/2))-14/3*a^6/b^8/(a+b*x^(1/2)
)^3+21*a^5/b^8/(a+b*x^(1/2))^2-70*a^3*ln(a+b*x^(1/2))/b^8-70*a^4/b^8/(a+b*
x^(1/2))+1/2*a^7/b^8/(a+b*x^(1/2))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(111) = 222$.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.70

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{30\,ab^{10}x^5 - 120\,a^3b^8x^4 - 366\,a^5b^6x^3 + 1179\,a^7b^4x^2 - 1066\,a^9b^2x + 319\,a^{11} + 420\,(a^3b^8x^4 - 4\,a^5b^6x^3 +}{6\,(b^{16}x^4 - 4\,a^2b^{14}}$$

input
```
integrate(x^3/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

output
```
-1/6*(30*a*b^10*x^5 - 120*a^3*b^8*x^4 - 366*a^5*b^6*x^3 + 1179*a^7*b^4*x^2
 - 1066*a^9*b^2*x + 319*a^11 + 420*(a^3*b^8*x^4 - 4*a^5*b^6*x^3 + 6*a^7*b^
4*x^2 - 4*a^9*b^2*x + a^11)*log(b*sqrt(x) + a) - 4*(b^11*x^5 + 41*a^2*b^9*
x^4 - 279*a^4*b^7*x^3 + 511*a^6*b^5*x^2 - 385*a^8*b^3*x + 105*a^10*b)*sqrt
(x))/(b^16*x^4 - 4*a^2*b^14*x^3 + 6*a^4*b^12*x^2 - 4*a^6*b^10*x + a^8*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(126) = 252$.

Time = 1.11 (sec) , antiderivative size = 835, normalized size of antiderivative = 6.37

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(a+b*x**(1/2))**5,x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (x**4/(4*a**5), Eq(b, 0)),
(zoo*x**4, Eq(a, -b*sqrt(x))), (-420*a**7*log(a/b + sqrt(x))/(6*a**4*b**8
+ 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x
**2) - 875*a**7/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24
*a*b**11*x**(3/2) + 6*b**12*x**2) - 1680*a**6*b*sqrt(x)*log(a/b + sqrt(x))
/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/
2) + 6*b**12*x**2) - 3080*a**6*b*sqrt(x)/(6*a**4*b**8 + 24*a**3*b**9*sqrt(
x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 2520*a**5*b**
2*x*log(a/b + sqrt(x))/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10
*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 3780*a**5*b**2*x/(6*a**4*b**8 +
 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x*
*2) - 1680*a**4*b**3*x**(3/2)*log(a/b + sqrt(x))/(6*a**4*b**8 + 24*a**3*b*
*9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 1680*
a**4*b**3*x**(3/2)/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x +
 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 420*a**3*b**4*x**2*log(a/b + sqrt(x
))/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(
3/2) + 6*b**12*x**2) + 84*a**2*b**5*x**(5/2)/(6*a**4*b**8 + 24*a**3*b**9*s
qrt(x) + 36*a**2*b**10*x + 24*a*b**11*x**(3/2) + 6*b**12*x**2) - 14*a*b**6
*x**3/(6*a**4*b**8 + 24*a**3*b**9*sqrt(x) + 36*a**2*b**10*x + 24*a*b**11*x
**(3/2) + 6*b**12*x**2) + 4*b**7*x**(7/2)/(6*a**4*b**8 + 24*a**3*b**9*s...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{70\,a^3\log\left(b\sqrt{x}+a\right)}{b^8}+\frac{2\left(b\sqrt{x}+a\right)^3}{3\,b^8} - \frac{7\left(b\sqrt{x}+a\right)^2a}{b^8}+\frac{42\left(b\sqrt{x}+a\right)a^2}{b^8}-\frac{70\,a^4}{\left(b\sqrt{x}+a\right)b^8} + \frac{21\,a^5}{\left(b\sqrt{x}+a\right)^2b^8}-\frac{14\,a^6}{3\left(b\sqrt{x}+a\right)^3b^8}+\frac{a^7}{2\left(b\sqrt{x}+a\right)^4b^8}$$

input
```
integrate(x^3/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

output
```
-70*a^3*log(b*sqrt(x) + a)/b^8 + 2/3*(b*sqrt(x) + a)^3/b^8 - 7*(b*sqrt(x)
+ a)^2*a/b^8 + 42*(b*sqrt(x) + a)*a^2/b^8 - 70*a^4/((b*sqrt(x) + a)*b^8) +
 21*a^5/((b*sqrt(x) + a)^2*b^8) - 14/3*a^6/((b*sqrt(x) + a)^3*b^8) + 1/2*a
^7/((b*sqrt(x) + a)^4*b^8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{70\,a^3\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8} - \frac{420\,a^4b^3x^{\frac{3}{2}}+1134\,a^5b^2x+1036\,a^6b\sqrt{x}+319\,a^7}{6\left(b\sqrt{x}+a\right)^4b^8} + \frac{2\,b^{10}x^{\frac{3}{2}}-15\,ab^9x+90\,a^2b^8\sqrt{x}}{3\,b^{15}}$$

input
```
integrate(x^3/(a+b*x^(1/2))^5,x, algorithm="giac")
```

output
```
-70*a^3*log(abs(b*sqrt(x) + a))/b^8 - 1/6*(420*a^4*b^3*x^(3/2) + 1134*a^5*
b^2*x + 1036*a^6*b*sqrt(x) + 319*a^7)/((b*sqrt(x) + a)^4*b^8) + 1/3*(2*b^1
0*x^(3/2) - 15*a*b^9*x + 90*a^2*b^8*sqrt(x))/b^15
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{2\,x^{3/2}}{3\,b^5} - \frac{\frac{319\,a^7}{6\,b}+\frac{518\,a^6\,\sqrt{x}}{3}+70\,a^4\,b^2\,x^{3/2}+189\,a^5\,b\,x}{a^4\,b^7+b^{11}\,x^2+6\,a^2\,b^9\,x+4\,a\,b^{10}\,x^{3/2}+4\,a^3\,b^8\,\sqrt{x}} - \frac{70\,a^3\,\ln\left(a+b\,\sqrt{x}\right)}{b^8} + \frac{30\,a^2\,\sqrt{x}}{b^7} - \frac{5\,a\,x}{b^6}$$

input
```
int(x^3/(a + b*x^(1/2))^5,x)
```

output
```
(2*x^(3/2))/(3*b^5) - ((319*a^7)/(6*b) + (518*a^6*x^(1/2))/3 + 70*a^4*b^2*
x^(3/2) + 189*a^5*b*x)/(a^4*b^7 + b^11*x^2 + 6*a^2*b^9*x + 4*a*b^10*x^(3/2
) + 4*a^3*b^8*x^(1/2)) - (70*a^3*log(a + b*x^(1/2)))/b^8 + (30*a^2*x^(1/2)
)/b^7 - (5*a*x)/b^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{-1680\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^6b-1680\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^4b^3x-1400\sqrt{x}\,a^6b+84\sqrt{x}\,a^2b^5x^2+4\sqrt{x}\,b^7x^3-}{6b^8\left(4\sqrt{x}\,a^3b-\right.}$$

input
```
int(x^3/(a+b*x^(1/2))^5,x)
```

output
```
( - 1680*sqrt(x)*log(sqrt(x)*b + a)*a**6*b - 1680*sqrt(x)*log(sqrt(x)*b +
a)*a**4*b**3*x - 1400*sqrt(x)*a**6*b + 84*sqrt(x)*a**2*b**5*x**2 + 4*sqrt(
x)*b**7*x**3 - 420*log(sqrt(x)*b + a)*a**7 - 2520*log(sqrt(x)*b + a)*a**5*
b**2*x - 420*log(sqrt(x)*b + a)*a**3*b**4*x**2 - 455*a**7 - 1260*a**5*b**2
*x + 420*a**3*b**4*x**2 - 14*a*b**6*x**3)/(6*b**8*(4*sqrt(x)*a**3*b + 4*sq
rt(x)*a*b**3*x + a**4 + 6*a**2*b**2*x + b**4*x**2))
```

3.107 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx=\frac{a^5}{2b^6\left(a+b\sqrt{x}\right)^4}-\frac{10a^4}{3b^6\left(a+b\sqrt{x}\right)^3}+\frac{10a^3}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{20a^2}{b^6\left(a+b\sqrt{x}\right)}+\frac{2\sqrt{x}}{b^5}-\frac{10a\log\left(a+b\sqrt{x}\right)}{b^6}$$

output
```
1/2*a^5/b^6/(a+b*x^(1/2))^4-10/3*a^4/b^6/(a+b*x^(1/2))^3+10*a^3/b^6/(a+b*x
^(1/2))^2-20*a^2/b^6/(a+b*x^(1/2))+2*x^(1/2)/b^5-10*a*ln(a+b*x^(1/2))/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx=\frac{-77a^5-248a^4b\sqrt{x}-252a^3b^2x-48a^2b^3x^{3/2}+48ab^4x^2+12b^5x^{5/2}}{6b^6\left(a+b\sqrt{x}\right)^4}-\frac{10a\log\left(a+b\sqrt{x}\right)}{b^6}$$

input
```
Integrate[x^2/(a + b*Sqrt[x])^5,x]
```

output
```
(-77*a^5 - 248*a^4*b*Sqrt[x] - 252*a^3*b^2*x - 48*a^2*b^3*x^(3/2) + 48*a*b
^4*x^2 + 12*b^5*x^(5/2))/(6*b^6*(a + b*Sqrt[x])^4) - (10*a*Log[a + b*Sqrt[
x]])/b^6
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{5/2}}{\left(a+b\sqrt{x}\right)^5}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^5}{b^5\left(a+b\sqrt{x}\right)^5}+\frac{5a^4}{b^5\left(a+b\sqrt{x}\right)^4}-\frac{10a^3}{b^5\left(a+b\sqrt{x}\right)^3}+\frac{10a^2}{b^5\left(a+b\sqrt{x}\right)^2}-\frac{5a}{b^5\left(a+b\sqrt{x}\right)}+\frac{1}{b^5}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^5}{4b^6\left(a+b\sqrt{x}\right)^4}-\frac{5a^4}{3b^6\left(a+b\sqrt{x}\right)^3}+\frac{5a^3}{b^6\left(a+b\sqrt{x}\right)^2}-\frac{10a^2}{b^6\left(a+b\sqrt{x}\right)}-\frac{5a\log\left(a+b\sqrt{x}\right)}{b^6}+\frac{\sqrt{x}}{b^5}\right)$$

input
```
Int[x^2/(a + b*Sqrt[x])^5,x]
```

output
```
2*(a^5/(4*b^6*(a + b*Sqrt[x])^4) - (5*a^4)/(3*b^6*(a + b*Sqrt[x])^3) + (5*
a^3)/(b^6*(a + b*Sqrt[x])^2) - (10*a^2)/(b^6*(a + b*Sqrt[x])) + Sqrt[x]/b^
5 - (5*a*Log[a + b*Sqrt[x]])/b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{a^5}{2b^6(a+b\sqrt{x})^4}-\frac{10a^4}{3b^6(a+b\sqrt{x})^3}+\frac{10a^3}{b^6(a+b\sqrt{x})^2}-\frac{20a^2}{b^6(a+b\sqrt{x})}+\frac{2\sqrt{x}}{b^5}-\frac{10a\ln(a+b\sqrt{x})}{b^6}$	92
default	$\frac{a^5}{2b^6(a+b\sqrt{x})^4}-\frac{10a^4}{3b^6(a+b\sqrt{x})^3}+\frac{10a^3}{b^6(a+b\sqrt{x})^2}-\frac{20a^2}{b^6(a+b\sqrt{x})}+\frac{2\sqrt{x}}{b^5}-\frac{10a\ln(a+b\sqrt{x})}{b^6}$	92

input
```
int(x^2/(a+b*x^(1/2))^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^5/b^6/(a+b*x^(1/2))^4-10/3*a^4/b^6/(a+b*x^(1/2))^3+10*a^3/b^6/(a+b*x
^(1/2))^2-20*a^2/b^6/(a+b*x^(1/2))+2*x^(1/2)/b^5-10*a*ln(a+b*x^(1/2))/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(91) = 182.

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx$$
$$=\frac{180\,a^3b^6x^3-357\,a^5b^4x^2+278\,a^7b^2x-77\,a^9-60\left(ab^8x^4-4\,a^3b^6x^3+6\,a^5b^4x^2-4\,a^7b^2x+a^9\right)\log\left(b\sqrt{x}\right.}{6\left(b^{14}x^4-4\,a^2b^{12}x^3+6\,a^4b^{10}x^2-4\,a^6b^8x+\right.}$$

input
```
integrate(x^2/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

output
```
1/6*(180*a^3*b^6*x^3 - 357*a^5*b^4*x^2 + 278*a^7*b^2*x - 77*a^9 - 60*(a*b^
8*x^4 - 4*a^3*b^6*x^3 + 6*a^5*b^4*x^2 - 4*a^7*b^2*x + a^9)*log(b*sqrt(x) +
 a) + 4*(3*b^9*x^4 - 42*a^2*b^7*x^3 + 73*a^4*b^5*x^2 - 55*a^6*b^3*x + 15*a
^8*b)*sqrt(x))/(b^14*x^4 - 4*a^2*b^12*x^3 + 6*a^4*b^10*x^2 - 4*a^6*b^8*x +
 a^8*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(100) = 200.

Time = 0.84 (sec) , antiderivative size = 704, normalized size of antiderivative = 6.58

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx$$
$$=\begin{cases}\tilde{\infty}\sqrt{x}\\ \frac{x^3}{3a^5}\\ \tilde{\infty}x^3\\ -\frac{60a^5\log\left(\frac{a}{b}+\sqrt{x}\right)}{6a^4b^6+24a^3b^7\sqrt{x}+36a^2b^8x+24ab^9x^{\frac{3}{2}}+6b^{10}x^2}-\frac{125a^5}{6a^4b^6+24a^3b^7\sqrt{x}+36a^2b^8x+24ab^9x^{\frac{3}{2}}+6b^{10}x^2}-\frac{240a^4b\sqrt{x}\log\left(\frac{a}{b}+\sqrt{\right.}}{6a^4b^6+24a^3b^7\sqrt{x}+36a^2b^8x+24a}\end{cases}$$

input
```
integrate(x**2/(a+b*x**(1/2))**5,x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (x**3/(3*a**5), Eq(b, 0)), (
zoo*x**3, Eq(a, -b*sqrt(x))), (-60*a**5*log(a/b + sqrt(x))/(6*a**4*b**6 +
24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2)
 - 125*a**5/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b*
*9*x**(3/2) + 6*b**10*x**2) - 240*a**4*b*sqrt(x)*log(a/b + sqrt(x))/(6*a**
4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b*
*10*x**2) - 440*a**4*b*sqrt(x)/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a*
*2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) - 360*a**3*b**2*x*log(a/b +
 sqrt(x))/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9
*x**(3/2) + 6*b**10*x**2) - 540*a**3*b**2*x/(6*a**4*b**6 + 24*a**3*b**7*sq
rt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) - 240*a**2*b**
3*x**(3/2)*log(a/b + sqrt(x))/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**
2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) - 240*a**2*b**3*x**(3/2)/(6*
a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6
*b**10*x**2) - 60*a*b**4*x**2*log(a/b + sqrt(x))/(6*a**4*b**6 + 24*a**3*b*
*7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9*x**(3/2) + 6*b**10*x**2) + 12*b**5
*x**(5/2)/(6*a**4*b**6 + 24*a**3*b**7*sqrt(x) + 36*a**2*b**8*x + 24*a*b**9
*x**(3/2) + 6*b**10*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{10\,a\log\left(b\sqrt{x}+a\right)}{b^6} + \frac{2\left(b\sqrt{x}+a\right)}{b^6} - \frac{20\,a^2}{(b\sqrt{x}+a)b^6} + \frac{10\,a^3}{\left(b\sqrt{x}+a\right)^2 b^6} - \frac{10\,a^4}{3\left(b\sqrt{x}+a\right)^3 b^6} + \frac{a^5}{2\left(b\sqrt{x}+a\right)^4 b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

output
```
-10*a*log(b*sqrt(x) + a)/b^6 + 2*(b*sqrt(x) + a)/b^6 - 20*a^2/((b*sqrt(x)
+ a)*b^6) + 10*a^3/((b*sqrt(x) + a)^2*b^6) - 10/3*a^4/((b*sqrt(x) + a)^3*b
^6) + 1/2*a^5/((b*sqrt(x) + a)^4*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{10\,a\log\left(\left|b\sqrt{x}+a\right|\right)}{b^6}+\frac{2\sqrt{x}}{b^5} - \frac{120\,a^2b^3x^{\frac{3}{2}}+300\,a^3b^2x+260\,a^4b\sqrt{x}+77\,a^5}{6\left(b\sqrt{x}+a\right)^4b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^5,x, algorithm="giac")
```

output
```
-10*a*log(abs(b*sqrt(x) + a))/b^6 + 2*sqrt(x)/b^5 - 1/6*(120*a^2*b^3*x^(3/
2) + 300*a^3*b^2*x + 260*a^4*b*sqrt(x) + 77*a^5)/((b*sqrt(x) + a)^4*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{2\sqrt{x}}{b^5} - \frac{\frac{77\,a^5}{6\,b}+\frac{130\,a^4\sqrt{x}}{3}+20\,a^2\,b^2\,x^{3/2}+50\,a^3\,b\,x}{a^4\,b^5+b^9\,x^2+6\,a^2\,b^7\,x+4\,a\,b^8\,x^{3/2}+4\,a^3\,b^6\,\sqrt{x}} - \frac{10\,a\,\ln\left(a+b\,\sqrt{x}\right)}{b^6}$$

input
```
int(x^2/(a + b*x^(1/2))^5,x)
```

output
```
(2*x^(1/2))/b^5 - ((77*a^5)/(6*b) + (130*a^4*x^(1/2))/3 + 20*a^2*b^2*x^(3/
2) + 50*a^3*b*x)/(a^4*b^5 + b^9*x^2 + 6*a^2*b^7*x + 4*a*b^8*x^(3/2) + 4*a^
3*b^6*x^(1/2)) - (10*a*log(a + b*x^(1/2)))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^5}\,dx$$
$$= \frac{-240\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^4b-240\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^2b^3x-200\sqrt{x}\,a^4b+12\sqrt{x}\,b^5x^2-60\log\left(\sqrt{x}\,b+a\right)}{6b^6\left(4\sqrt{x}\,a^3b+4\sqrt{x}\,a\,b^3x+a^4+\right.}$$

input
```
int(x^2/(a+b*x^(1/2))^5,x)
```

output
```
( - 240*sqrt(x)*log(sqrt(x)*b + a)*a**4*b - 240*sqrt(x)*log(sqrt(x)*b + a)
*a**2*b**3*x - 200*sqrt(x)*a**4*b + 12*sqrt(x)*b**5*x**2 - 60*log(sqrt(x)*
b + a)*a**5 - 360*log(sqrt(x)*b + a)*a**3*b**2*x - 60*log(sqrt(x)*b + a)*a
*b**4*x**2 - 65*a**5 - 180*a**3*b**2*x + 60*a*b**4*x**2)/(6*b**6*(4*sqrt(x
)*a**3*b + 4*sqrt(x)*a*b**3*x + a**4 + 6*a**2*b**2*x + b**4*x**2))
```

3.108 $\int \frac{x}{\left(a+b\sqrt{x}\right)^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5} \, dx = \frac{x^2}{2a\left(a+b\sqrt{x}\right)^4}$$

output
```
1/2*x^2/a/(a+b*x^(1/2))^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5} \, dx = \frac{-a^3-4a^2b\sqrt{x}-6ab^2x-4b^3x^{3/2}}{2b^4\left(a+b\sqrt{x}\right)^4}$$

input
```
Integrate[x/(a + b*Sqrt[x])^5,x]
```

output
```
(-a^3 - 4*a^2*b*Sqrt[x] - 6*a*b^2*x - 4*b^3*x^(3/2))/(2*b^4*(a + b*Sqrt[x]
)^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}dx$$

$$\Big\downarrow \text{796}$$

$$\frac{x^2}{2a\left(a+b\sqrt{x}\right)^4}$$

input
```
Int[x/(a + b*Sqrt[x])^5,x]
```

output
```
x^2/(2*a*(a + b*Sqrt[x])^4)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(17) = 34$.

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.10

method	result
derivativedivides	$\frac{3a}{b^4(a+b\sqrt{x})^2}-\frac{2}{b^4(a+b\sqrt{x})}-\frac{2a^2}{b^4(a+b\sqrt{x})^3}+\frac{a^3}{2b^4(a+b\sqrt{x})^4}$
default	$\frac{3a}{b^4(a+b\sqrt{x})^2}-\frac{2}{b^4(a+b\sqrt{x})}-\frac{2a^2}{b^4(a+b\sqrt{x})^3}+\frac{a^3}{2b^4(a+b\sqrt{x})^4}$
trager	$\frac{(-1+x)(a^6b^4x^3-4a^4b^6x^3+5a^2b^8x^3-10b^{10}x^3+6a^8b^2x^2-23a^6b^4x^2+36a^4b^6x^2+5a^2b^8x^2+a^{10}x+2a^8b^2x-23a^6b^4x-4}{2(-b^2x+a^2)^4(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)}$

input
```
int(x/(a+b*x^(1/2))^5,x,method=_RETURNVERBOSE)
```

output
```
3/b^4*a/(a+b*x^(1/2))^2-2/b^4/(a+b*x^(1/2))-2*a^2/b^4/(a+b*x^(1/2))^3+1/2*
a^3/b^4/(a+b*x^(1/2))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 5.14

$$\int \frac{x}{(a+b\sqrt{x})^5}\,dx = \frac{10\,ab^6x^3-5\,a^3b^4x^2+4\,a^5b^2x-a^7-4\,(b^7x^3+a^2b^5x^2)\sqrt{x}}{2\,(b^{12}x^4-4\,a^2b^{10}x^3+6\,a^4b^8x^2-4\,a^6b^6x+a^8b^4)}$$

input
```
integrate(x/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

output
```
1/2*(10*a*b^6*x^3 - 5*a^3*b^4*x^2 + 4*a^5*b^2*x - a^7 - 4*(b^7*x^3 + a^2*b
^5*x^2)*sqrt(x))/(b^12*x^4 - 4*a^2*b^10*x^3 + 6*a^4*b^8*x^2 - 4*a^6*b^6*x
+ a^8*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(15) = 30$.

Time = 1.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 12.86

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \tilde{\infty}x^2 \\ \frac{x^2}{2a^5} \\ -\frac{a^3}{2a^4b^4+8a^3b^5\sqrt{x}+12a^2b^6x+8ab^7x^{\frac{3}{2}}+2b^8x^2} - \frac{4a^2b\sqrt{x}}{2a^4b^4+8a^3b^5\sqrt{x}+12a^2b^6x+8ab^7x^{\frac{3}{2}}+2b^8x^2} - \frac{6ab^2x}{2a^4b^4+8a^3b^5\sqrt{x}+12a^2b^6x+8ab^7x^{\frac{3}{2}}+2b^8x^2} \end{cases}$$

input
```
integrate(x/(a+b*x**(1/2))**5,x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (zoo*x**2, Eq(a, -b*sqrt(x))
), (x**2/(2*a**5), Eq(b, 0)), (-a**3/(2*a**4*b**4 + 8*a**3*b**5*sqrt(x) +
12*a**2*b**6*x + 8*a*b**7*x**(3/2) + 2*b**8*x**2) - 4*a**2*b*sqrt(x)/(2*a*
*4*b**4 + 8*a**3*b**5*sqrt(x) + 12*a**2*b**6*x + 8*a*b**7*x**(3/2) + 2*b**
8*x**2) - 6*a*b**2*x/(2*a**4*b**4 + 8*a**3*b**5*sqrt(x) + 12*a**2*b**6*x +
 8*a*b**7*x**(3/2) + 2*b**8*x**2) - 4*b**3*x**(3/2)/(2*a**4*b**4 + 8*a**3*
b**5*sqrt(x) + 12*a**2*b**6*x + 8*a*b**7*x**(3/2) + 2*b**8*x**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.05

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{2}{\left(b\sqrt{x}+a\right)b^4} + \frac{3\,a}{\left(b\sqrt{x}+a\right)^2b^4} - \frac{2\,a^2}{\left(b\sqrt{x}+a\right)^3b^4} + \frac{a^3}{2\,\left(b\sqrt{x}+a\right)^4b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

output
```
-2/((b*sqrt(x) + a)*b^4) + 3*a/((b*sqrt(x) + a)^2*b^4) - 2*a^2/((b*sqrt(x)
 + a)^3*b^4) + 1/2*a^3/((b*sqrt(x) + a)^4*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{4\,b^3x^{\frac{3}{2}}+6\,ab^2x+4\,a^2b\sqrt{x}+a^3}{2\left(b\sqrt{x}+a\right)^4b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^5,x, algorithm="giac")
```

output
```
-1/2*(4*b^3*x^(3/2) + 6*a*b^2*x + 4*a^2*b*sqrt(x) + a^3)/((b*sqrt(x) + a)^
4*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.67

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{\frac{a^3}{2\,b^4}+\frac{2\,x^{3/2}}{b}+\frac{2\,a^2\,\sqrt{x}}{b^3}+\frac{3\,a\,x}{b^2}}{a^4+b^4\,x^2+6\,a^2\,b^2\,x+4\,a^3\,b\,\sqrt{x}+4\,a\,b^3\,x^{3/2}}$$

input
```
int(x/(a + b*x^(1/2))^5,x)
```

output
```
-(a^3/(2*b^4) + (2*x^(3/2))/b + (2*a^2*x^(1/2))/b^3 + (3*a*x)/b^2)/(a^4 +
b^4*x^2 + 6*a^2*b^2*x + 4*a^3*b*x^(1/2) + 4*a*b^3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^5}\,dx=\frac{x^2}{2a\left(4\sqrt{x}\,a^3b+4\sqrt{x}\,a\,b^3x+a^4+6a^2b^2x+b^4x^2\right)}$$

input
```
int(x/(a+b*x^(1/2))^5,x)
```

output
```
x**2/(2*a*(4*sqrt(x)*a**3*b + 4*sqrt(x)*a*b**3*x + a**4 + 6*a**2*b**2*x +
b**4*x**2))
```

3.109 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx=\frac{a}{2b^2\left(a+b\sqrt{x}\right)^4}-\frac{2}{3b^2\left(a+b\sqrt{x}\right)^3}$$

output
```
1/2*a/b^2/(a+b*x^(1/2))^4-2/3/b^2/(a+b*x^(1/2))^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx=\frac{-a-4b\sqrt{x}}{6b^2\left(a+b\sqrt{x}\right)^4}$$

input
```
Integrate[(a + b*Sqrt[x])^(-5),x]
```

output
```
(-a - 4*b*Sqrt[x])/(6*b^2*(a + b*Sqrt[x])^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}dx$$

$$\Big\downarrow \text{774}$$

$$2\int \frac{\sqrt{x}}{\left(a+b\sqrt{x}\right)^5}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{1}{b\left(a+b\sqrt{x}\right)^4}-\frac{a}{b\left(a+b\sqrt{x}\right)^5}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a}{4b^2\left(a+b\sqrt{x}\right)^4}-\frac{1}{3b^2\left(a+b\sqrt{x}\right)^3}\right)$$

input
```
Int[(a + b*Sqrt[x])^(-5),x]
```

output
```
2*(a/(4*b^2*(a + b*Sqrt[x])^4) - 1/(3*b^2*(a + b*Sqrt[x])^3))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a}{2b^2(a+b\sqrt{x})^4}-\frac{2}{3b^2(a+b\sqrt{x})^3}$
default	$\frac{a^5}{4(-b^2x+a^2)^4b^2}-\frac{1}{3b^2(b\sqrt{x}-a)^3}-\frac{a}{4b^2(b\sqrt{x}-a)^4}-\frac{1}{3b^2(a+b\sqrt{x})^3}+\frac{a}{4b^2(a+b\sqrt{x})^4}+10a^3b^2\Big(\frac{1}{3b^4(b^2x-a}$
trager	$\frac{(-1+x)(a^4b^6x^3-10a^2b^8x^3-15b^{10}x^3-4a^6b^4x^2+41a^4b^6x^2+50a^2b^8x^2-15b^{10}x^2+21a^8b^2x-124a^6b^4x+41a^4b^6x-10a^2}{6(-b^2x+a^2)^4(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)}$

input
```
int(1/(a+b*x^(1/2))^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/b^2/(a+b*x^(1/2))^4-2/3/b^2/(a+b*x^(1/2))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(30) = 60.

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+b\sqrt{x})^5}\,dx=\frac{15\,ab^4x^2+10\,a^3b^2x-a^5-4\,(b^5x^2+5\,a^2b^3x)\sqrt{x}}{6\,(b^{10}x^4-4\,a^2b^8x^3+6\,a^4b^6x^2-4\,a^6b^4x+a^8b^2)}$$

input
```
integrate(1/(a+b*x^(1/2))^5,x, algorithm="fricas")
```

output
```
1/6*(15*a*b^4*x^2 + 10*a^3*b^2*x - a^5 - 4*(b^5*x^2 + 5*a^2*b^3*x)*sqrt(x)
)/(b^10*x^4 - 4*a^2*b^8*x^3 + 6*a^4*b^6*x^2 - 4*a^6*b^4*x + a^8*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(32) = 64$.

Time = 1.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.58

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0 \wedge b=0 \\ \tilde{\infty}x & \text{for } a=-b\sqrt{x} \\ \frac{x}{a^5} & \text{for } b=0 \\ -\frac{a}{6a^4b^2+24a^3b^3\sqrt{x}+36a^2b^4x+24ab^5x^{\frac{3}{2}}+6b^6x^2} - \frac{4b\sqrt{x}}{6a^4b^2+24a^3b^3\sqrt{x}+36a^2b^4x+24ab^5x^{\frac{3}{2}}+6b^6x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**5,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (zoo*x, Eq(a, -b*sqrt(x))),
 (x/a**5, Eq(b, 0)), (-a/(6*a**4*b**2 + 24*a**3*b**3*sqrt(x) + 36*a**2*b**
4*x + 24*a*b**5*x**(3/2) + 6*b**6*x**2) - 4*b*sqrt(x)/(6*a**4*b**2 + 24*a*
*3*b**3*sqrt(x) + 36*a**2*b**4*x + 24*a*b**5*x**(3/2) + 6*b**6*x**2), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{2}{3\left(b\sqrt{x}+a\right)^3b^2} + \frac{a}{2\left(b\sqrt{x}+a\right)^4b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^5,x, algorithm="maxima")
```

output
```
-2/3/((b*sqrt(x) + a)^3*b^2) + 1/2*a/((b*sqrt(x) + a)^4*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{4\,b\sqrt{x}+a}{6\left(b\sqrt{x}+a\right)^4 b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^5,x, algorithm="giac")
```

output
```
-1/6*(4*b*sqrt(x) + a)/((b*sqrt(x) + a)^4*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx = -\frac{\frac{a}{6\,b^2}+\frac{2\,\sqrt{x}}{3\,b}}{a^4+b^4\,x^2+6\,a^2\,b^2\,x+4\,a^3\,b\,\sqrt{x}+4\,a\,b^3\,x^{3/2}}$$

input
```
int(1/(a + b*x^(1/2))^5,x)
```

output
```
-(a/(6*b^2) + (2*x^(1/2))/(3*b))/(a^4 + b^4*x^2 + 6*a^2*b^2*x + 4*a^3*b*x^
(1/2) + 4*a*b^3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5}\,dx = \frac{-4\sqrt{x}\,b-a}{6b^2\left(4\sqrt{x}\,a^3b+4\sqrt{x}\,a\,b^3x+a^4+6a^2b^2x+b^4x^2\right)}$$

input
```
int(1/(a+b*x^(1/2))^5,x)
```

output
```
( - 4*sqrt(x)*b - a)/(6*b**2*(4*sqrt(x)*a**3*b + 4*sqrt(x)*a*b**3*x + a**4
 + 6*a**2*b**2*x + b**4*x**2))
```

3.110 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx=\frac{1}{2a\left(a+b\sqrt{x}\right)^4}+\frac{2}{3a^2\left(a+b\sqrt{x}\right)^3}+\frac{1}{a^3\left(a+b\sqrt{x}\right)^2}+\frac{2}{a^4\left(a+b\sqrt{x}\right)}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{\log(x)}{a^5}$$

output
```
1/2/a/(a+b*x^(1/2))^4+2/3/a^2/(a+b*x^(1/2))^3+1/a^3/(a+b*x^(1/2))^2+2/a^4/
(a+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^5+ln(x)/a^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx=\frac{\frac{a\left(25a^3+52a^2b\sqrt{x}+42ab^2x+12b^3x^{3/2}\right)}{\left(a+b\sqrt{x}\right)^4}-12\log\left(a+b\sqrt{x}\right)+6\log(x)}{6a^5}$$

input
```
Integrate[1/((a + b*Sqrt[x])^5*x),x]
```

output
```
((a*(25*a^3 + 52*a^2*b*Sqrt[x] + 42*a*b^2*x + 12*b^3*x^(3/2)))/(a + b*Sqrt
[x])^4 - 12*Log[a + b*Sqrt[x]] + 6*Log[x])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{x}\right)^5}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^5\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{b}{a^5\left(a+b\sqrt{x}\right)}-\frac{b}{a^4\left(a+b\sqrt{x}\right)^2}-\frac{b}{a^3\left(a+b\sqrt{x}\right)^3}-\frac{b}{a^2\left(a+b\sqrt{x}\right)^4}-\frac{b}{a\left(a+b\sqrt{x}\right)^5}+\frac{1}{a^5\sqrt{x}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\log\left(a+b\sqrt{x}\right)}{a^5}+\frac{\log\left(\sqrt{x}\right)}{a^5}+\frac{1}{a^4\left(a+b\sqrt{x}\right)}+\frac{1}{2a^3\left(a+b\sqrt{x}\right)^2}+\frac{1}{3a^2\left(a+b\sqrt{x}\right)^3}+\frac{1}{4a\left(a+b\sqrt{x}\right)^4}\right)$$

input
```
Int[1/((a + b*Sqrt[x])^5*x),x]
```

output
```
2*(1/(4*a*(a + b*Sqrt[x])^4) + 1/(3*a^2*(a + b*Sqrt[x])^3) + 1/(2*a^3*(a +
 b*Sqrt[x])^2) + 1/(a^4*(a + b*Sqrt[x])) - Log[a + b*Sqrt[x]]/a^5 + Log[Sq
rt[x]]/a^5)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{1}{2a\left(a+b\sqrt{x}\right)^{4}}+\frac{2}{3a^{2}\left(a+b\sqrt{x}\right)^{3}}+\frac{1}{a^{3}\left(a+b\sqrt{x}\right)^{2}}+\frac{2}{a^{4}\left(a+b\sqrt{x}\right)}-\frac{2\ln\left(a+b\sqrt{x}\right)}{a^{5}}+\frac{\ln\left(x\right)}{a^{5}}$	76
default	$\frac{1}{2a\left(a+b\sqrt{x}\right)^{4}}+\frac{2}{3a^{2}\left(a+b\sqrt{x}\right)^{3}}+\frac{1}{a^{3}\left(a+b\sqrt{x}\right)^{2}}+\frac{2}{a^{4}\left(a+b\sqrt{x}\right)}-\frac{2\ln\left(a+b\sqrt{x}\right)}{a^{5}}+\frac{\ln\left(x\right)}{a^{5}}$	76

input
```
int(1/(a+b*x^(1/2))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(a+b*x^(1/2))^4+2/3/a^2/(a+b*x^(1/2))^3+1/a^3/(a+b*x^(1/2))^2+2/a^4/
(a+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^5+ln(x)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(75) = 150$.

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.55

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx =$$
$$-\frac{6\,a^2 b^6 x^3 - 21\,a^4 b^4 x^2 + 16\,a^6 b^2 x - 25\,a^8 + 12\,(b^8 x^4 - 4\,a^2 b^6 x^3 + 6\,a^4 b^4 x^2 - 4\,a^6 b^2 x + a^8)\log\left(b\sqrt{x} + a\right.}{6\,(a^5 b^8 x^4 - 4\,a^7 b^6 x^3}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x,x, algorithm="fricas")
```

output
```
-1/6*(6*a^2*b^6*x^3 - 21*a^4*b^4*x^2 + 16*a^6*b^2*x - 25*a^8 + 12*(b^8*x^4
 - 4*a^2*b^6*x^3 + 6*a^4*b^4*x^2 - 4*a^6*b^2*x + a^8)*log(b*sqrt(x) + a) -
 12*(b^8*x^4 - 4*a^2*b^6*x^3 + 6*a^4*b^4*x^2 - 4*a^6*b^2*x + a^8)*log(sqrt
(x)) - 4*(3*a*b^7*x^3 - 11*a^3*b^5*x^2 + 14*a^5*b^3*x - 12*a^7*b)*sqrt(x))
/(a^5*b^8*x^4 - 4*a^7*b^6*x^3 + 6*a^9*b^4*x^2 - 4*a^11*b^2*x + a^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(82) = 164$.

Time = 1.61 (sec) , antiderivative size = 1059, normalized size of antiderivative = 11.90

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2))**5/x,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (log(x)/a**5, Eq(b, 0)), (-
2/(5*b**5*x**(5/2)), Eq(a, 0)), (zoo*log(x), Eq(a, -b*sqrt(x))), (6*a**4*s
qrt(x)*log(x)/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a
**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) - 12*a**4*sqrt(x)*log(a/b + sqrt(x))
/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2
 + 6*a**5*b**4*x**(5/2)) + 25*a**4*sqrt(x)/(6*a**9*sqrt(x) + 24*a**8*b*x +
 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) + 24*a*
*3*b*x*log(x)/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a
**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) - 48*a**3*b*x*log(a/b + sqrt(x))/(6*
a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6
*a**5*b**4*x**(5/2)) + 52*a**3*b*x/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7
*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) + 36*a**2*b**2*
x**(3/2)*log(x)/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24
*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) - 72*a**2*b**2*x**(3/2)*log(a/b +
sqrt(x))/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b
**3*x**2 + 6*a**5*b**4*x**(5/2)) + 42*a**2*b**2*x**(3/2)/(6*a**9*sqrt(x) +
 24*a**8*b*x + 36*a**7*b**2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**
(5/2)) + 24*a*b**3*x**2*log(x)/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**
2*x**(3/2) + 24*a**6*b**3*x**2 + 6*a**5*b**4*x**(5/2)) - 48*a*b**3*x**2*lo
g(a/b + sqrt(x))/(6*a**9*sqrt(x) + 24*a**8*b*x + 36*a**7*b**2*x**(3/2) ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx = \frac{12\,b^3x^{\frac{3}{2}}+42\,ab^2x+52\,a^2b\sqrt{x}+25\,a^3}{6\left(a^4b^4x^2+4\,a^5b^3x^{\frac{3}{2}}+6\,a^6b^2x+4\,a^7b\sqrt{x}+a^8\right)} - \frac{2\,\log\left(b\sqrt{x}+a\right)}{a^5} + \frac{\log\left(x\right)}{a^5}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x,x, algorithm="maxima")
```

output
```
1/6*(12*b^3*x^(3/2) + 42*a*b^2*x + 52*a^2*b*sqrt(x) + 25*a^3)/(a^4*b^4*x^2
 + 4*a^5*b^3*x^(3/2) + 6*a^6*b^2*x + 4*a^7*b*sqrt(x) + a^8) - 2*log(b*sqrt
(x) + a)/a^5 + log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx = -\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a^5} + \frac{\log\left(|x|\right)}{a^5} + \frac{12\,ab^3x^{\frac{3}{2}} + 42\,a^2b^2x + 52\,a^3b\sqrt{x} + 25\,a^4}{6\left(b\sqrt{x}+a\right)^4a^5}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x,x, algorithm="giac")
```

output
```
-2*log(abs(b*sqrt(x) + a))/a^5 + log(abs(x))/a^5 + 1/6*(12*a*b^3*x^(3/2) +
 42*a^2*b^2*x + 52*a^3*b*sqrt(x) + 25*a^4)/((b*sqrt(x) + a)^4*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx = \frac{\frac{25}{6\,a} + \frac{26\,b\,\sqrt{x}}{3\,a^2} + \frac{7\,b^2\,x}{a^3} + \frac{2\,b^3\,x^{3/2}}{a^4}}{a^4 + b^4\,x^2 + 6\,a^2\,b^2\,x + 4\,a^3\,b\,\sqrt{x} + 4\,a\,b^3\,x^{3/2}} - \frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^5}$$

input
```
int(1/(x*(a + b*x^(1/2))^5),x)
```

output
```
(25/(6*a) + (26*b*x^(1/2))/(3*a^2) + (7*b^2*x)/a^3 + (2*b^3*x^(3/2))/a^4)/
(a^4 + b^4*x^2 + 6*a^2*b^2*x + 4*a^3*b*x^(1/2) + 4*a*b^3*x^(3/2)) - (4*ata
nh((2*b*x^(1/2))/a + 1))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x}\,dx$$
$$= \frac{-48\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^3b-48\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a\,b^3x+48\sqrt{x}\log\left(\sqrt{x}\right)a^3b+48\sqrt{x}\log\left(\sqrt{x}\right)a\,b^3x+40}{}$$

input
```
int(1/(a+b*x^(1/2))^5/x,x)
```

output
```
( - 48*sqrt(x)*log(sqrt(x)*b + a)*a**3*b - 48*sqrt(x)*log(sqrt(x)*b + a)*a
*b**3*x + 48*sqrt(x)*log(sqrt(x))*a**3*b + 48*sqrt(x)*log(sqrt(x))*a*b**3*
x + 40*sqrt(x)*a**3*b - 12*log(sqrt(x)*b + a)*a**4 - 72*log(sqrt(x)*b + a)
*a**2*b**2*x - 12*log(sqrt(x)*b + a)*b**4*x**2 + 12*log(sqrt(x))*a**4 + 72
*log(sqrt(x))*a**2*b**2*x + 12*log(sqrt(x))*b**4*x**2 + 22*a**4 + 24*a**2*
b**2*x - 3*b**4*x**2)/(6*a**5*(4*sqrt(x)*a**3*b + 4*sqrt(x)*a*b**3*x + a**
4 + 6*a**2*b**2*x + b**4*x**2))
```

3.111 $\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = \frac{b^2}{2a^3\left(a+b\sqrt{x}\right)^4}+\frac{2b^2}{a^4\left(a+b\sqrt{x}\right)^3} + \frac{6b^2}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{20b^2}{a^6\left(a+b\sqrt{x}\right)}-\frac{1}{a^5x} + \frac{10b}{a^6\sqrt{x}}-\frac{30b^2\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^2\log(x)}{a^7}$$

output
```
1/2*b^2/a^3/(a+b*x^(1/2))^4+2*b^2/a^4/(a+b*x^(1/2))^3+6*b^2/a^5/(a+b*x^(1/
2))^2+20*b^2/a^6/(a+b*x^(1/2))-1/a^5/x+10*b/a^6/x^(1/2)-30*b^2*ln(a+b*x^(1
/2))/a^7+15*b^2*ln(x)/a^7
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = \frac{\frac{a\left(-2a^5+12a^4b\sqrt{x}+125a^3b^2x+260a^2b^3x^{3/2}+210ab^4x^2+60b^5x^{5/2}\right)}{\left(a+b\sqrt{x}\right)^4x}-60b^2\log\left(a+b\sqrt{x}\right)+30b^2\log(x)}{2a^7}$$

input
```
Integrate[1/((a + b*Sqrt[x])^5*x^2),x]
```

output
```
((a*(-2*a^5 + 12*a^4*b*Sqrt[x] + 125*a^3*b^2*x + 260*a^2*b^3*x^(3/2) + 210
*a*b^4*x^2 + 60*b^5*x^(5/2)))/((a + b*Sqrt[x])^4*x) - 60*b^2*Log[a + b*Sqr
t[x]] + 30*b^2*Log[x])/(2*a^7)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\sqrt{x}\right)^5}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{15b^3}{a^7\left(a+b\sqrt{x}\right)}-\frac{10b^3}{a^6\left(a+b\sqrt{x}\right)^2}-\frac{6b^3}{a^5\left(a+b\sqrt{x}\right)^3}-\frac{3b^3}{a^4\left(a+b\sqrt{x}\right)^4}-\frac{b^3}{a^3\left(a+b\sqrt{x}\right)^5}+\frac{15b^2}{a^7\sqrt{x}}-\frac{5b}{a^6x}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{15b^2\log\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^2\log\left(\sqrt{x}\right)}{a^7}+\frac{10b^2}{a^6\left(a+b\sqrt{x}\right)}+\frac{5b}{a^6\sqrt{x}}+\frac{3b^2}{a^5\left(a+b\sqrt{x}\right)^2}-\frac{1}{2a^5x}+\frac{b^2}{a^4\left(a+b\sqrt{x}\right)^3}+\right.$$

input
```
Int[1/((a + b*Sqrt[x])^5*x^2),x]
```

output
```
2*(b^2/(4*a^3*(a + b*Sqrt[x])^4) + b^2/(a^4*(a + b*Sqrt[x])^3) + (3*b^2)/(
a^5*(a + b*Sqrt[x])^2) + (10*b^2)/(a^6*(a + b*Sqrt[x])) - 1/(2*a^5*x) + (5
*b)/(a^6*Sqrt[x]) - (15*b^2*Log[a + b*Sqrt[x]])/a^7 + (15*b^2*Log[Sqrt[x]]
)/a^7)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{b^2}{2a^3\left(a+b\sqrt{x}\right)^4}+\frac{2b^2}{a^4\left(a+b\sqrt{x}\right)^3}+\frac{6b^2}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{20b^2}{a^6\left(a+b\sqrt{x}\right)}-\frac{1}{a^5x}+\frac{10b}{a^6\sqrt{x}}-\frac{30b^2\ln\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^2\ln}{a^7}$
default	$\frac{b^2}{2a^3\left(a+b\sqrt{x}\right)^4}+\frac{2b^2}{a^4\left(a+b\sqrt{x}\right)^3}+\frac{6b^2}{a^5\left(a+b\sqrt{x}\right)^2}+\frac{20b^2}{a^6\left(a+b\sqrt{x}\right)}-\frac{1}{a^5x}+\frac{10b}{a^6\sqrt{x}}-\frac{30b^2\ln\left(a+b\sqrt{x}\right)}{a^7}+\frac{15b^2\ln}{a^7}$

input
```
int(1/(a+b*x^(1/2))^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/a^3/(a+b*x^(1/2))^4+2*b^2/a^4/(a+b*x^(1/2))^3+6*b^2/a^5/(a+b*x^(1/
2))^2+20*b^2/a^6/(a+b*x^(1/2))-1/a^5/x+10*b/a^6/x^(1/2)-30*b^2*ln(a+b*x^(1
/2))/a^7+15*b^2*ln(x)/a^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(112) = 224.

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.12

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = -\frac{30\,a^2b^8x^4-105\,a^4b^6x^3+130\,a^6b^4x^2-65\,a^8b^2x+2\,a^{10}+60\,(b^{10}x^5-4\,a^2b^8x^4+6\,a^4b^6x^3-4\,a^6b^4x^2+}{2\,(a}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x^2,x, algorithm="fricas")
```

output
```
-1/2*(30*a^2*b^8*x^4 - 105*a^4*b^6*x^3 + 130*a^6*b^4*x^2 - 65*a^8*b^2*x +
2*a^10 + 60*(b^10*x^5 - 4*a^2*b^8*x^4 + 6*a^4*b^6*x^3 - 4*a^6*b^4*x^2 + a^
8*b^2*x)*log(b*sqrt(x) + a) - 60*(b^10*x^5 - 4*a^2*b^8*x^4 + 6*a^4*b^6*x^3
 - 4*a^6*b^4*x^2 + a^8*b^2*x)*log(sqrt(x)) - 4*(15*a*b^9*x^4 - 55*a^3*b^7*
x^3 + 73*a^5*b^5*x^2 - 40*a^7*b^3*x + 5*a^9*b)*sqrt(x))/(a^7*b^8*x^5 - 4*a
^9*b^6*x^4 + 6*a^11*b^4*x^3 - 4*a^13*b^2*x^2 + a^15*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. 2(121) = 242.

Time = 2.70 (sec) , antiderivative size = 1241, normalized size of antiderivative = 9.85

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2))**5/x**2,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-1/(a**5*x), Eq(b, 0)), (-
2/(7*b**5*x**(7/2)), Eq(a, 0)), (zoo/x, Eq(a, -b*sqrt(x))), (-2*a**6*sqrt(
x)/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**
3*x**3 + 2*a**7*b**4*x**(7/2)) + 12*a**5*b*x/(2*a**11*x**(3/2) + 8*a**10*b
*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) +
 30*a**4*b**2*x**(3/2)*log(x)/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9
*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) - 60*a**4*b**2*x
**(3/2)*log(a/b + sqrt(x))/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b*
*2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 125*a**4*b**2*x**
(3/2)/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*
b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 120*a**3*b**3*x**2*log(x)/(2*a**11*x**
(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7
*b**4*x**(7/2)) - 240*a**3*b**3*x**2*log(a/b + sqrt(x))/(2*a**11*x**(3/2)
+ 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*
x**(7/2)) + 260*a**3*b**3*x**2/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**
9*b**2*x**(5/2) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 180*a**2*b**4
*x**(5/2)*log(x)/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2
) + 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) - 360*a**2*b**4*x**(5/2)*log(
a/b + sqrt(x))/(2*a**11*x**(3/2) + 8*a**10*b*x**2 + 12*a**9*b**2*x**(5/2)
+ 8*a**8*b**3*x**3 + 2*a**7*b**4*x**(7/2)) + 210*a**2*b**4*x**(5/2)/(2*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = \frac{60\,b^5x^{\frac{5}{2}} + 210\,ab^4x^2 + 260\,a^2b^3x^{\frac{3}{2}} + 125\,a^3b^2x + 12\,a^4b\sqrt{x} - 2\,a^5}{2\left(a^6b^4x^3 + 4\,a^7b^3x^{\frac{5}{2}} + 6\,a^8b^2x^2 + 4\,a^9bx^{\frac{3}{2}} + a^{10}x\right)} - \frac{30\,b^2\log\left(b\sqrt{x}+a\right)}{a^7} + \frac{15\,b^2\log\left(x\right)}{a^7}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x^2,x, algorithm="maxima")
```

output
```
1/2*(60*b^5*x^(5/2) + 210*a*b^4*x^2 + 260*a^2*b^3*x^(3/2) + 125*a^3*b^2*x
+ 12*a^4*b*sqrt(x) - 2*a^5)/(a^6*b^4*x^3 + 4*a^7*b^3*x^(5/2) + 6*a^8*b^2*x
^2 + 4*a^9*b*x^(3/2) + a^10*x) - 30*b^2*log(b*sqrt(x) + a)/a^7 + 15*b^2*lo
g(x)/a^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = -\frac{30\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^7} + \frac{15\,b^2\log\left(\left|x\right|\right)}{a^7} + \frac{60\,ab^5x^{\frac{5}{2}} + 210\,a^2b^4x^2 + 260\,a^3b^3x^{\frac{3}{2}} + 125\,a^4b^2x + 12\,a^5b\sqrt{x} - 2\,a^6}{2\left(b\sqrt{x}+a\right)^4a^7x}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x^2,x, algorithm="giac")
```

output
```
-30*b^2*log(abs(b*sqrt(x) + a))/a^7 + 15*b^2*log(abs(x))/a^7 + 1/2*(60*a*b
^5*x^(5/2) + 210*a^2*b^4*x^2 + 260*a^3*b^3*x^(3/2) + 125*a^4*b^2*x + 12*a^
5*b*sqrt(x) - 2*a^6)/((b*sqrt(x) + a)^4*a^7*x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx = \frac{\frac{6\,b\,\sqrt{x}}{a^2} - \frac{1}{a} + \frac{125\,b^2\,x}{2\,a^3} + \frac{105\,b^4\,x^2}{a^5} + \frac{130\,b^3\,x^{3/2}}{a^4} + \frac{30\,b^5\,x^{5/2}}{a^6}}{a^4\,x + b^4\,x^3 + 4\,a^3\,b\,x^{3/2} + 4\,a\,b^3\,x^{5/2} + 6\,a^2\,b^2\,x^2} - \frac{60\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^7}$$

input
```
int(1/(x^2*(a + b*x^(1/2))^5),x)
```

output
```
((6*b*x^(1/2))/a^2 - 1/a + (125*b^2*x)/(2*a^3) + (105*b^4*x^2)/a^5 + (130*
b^3*x^(3/2))/a^4 + (30*b^5*x^(5/2))/a^6)/(a^4*x + b^4*x^3 + 4*a^3*b*x^(3/2
) + 4*a*b^3*x^(5/2) + 6*a^2*b^2*x^2) - (60*b^2*atanh((2*b*x^(1/2))/a + 1))
/a^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.98

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^2}\,dx$$
$$= \frac{-240\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^3b^3x-240\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a\,b^5x^2+240\sqrt{x}\log\left(\sqrt{x}\right)a^3b^3x+240\sqrt{x}\log\left(\sqrt{x}\right)}{}$$

input
```
int(1/(a+b*x^(1/2))^5/x^2,x)
```

output
```
( - 240*sqrt(x)*log(sqrt(x)*b + a)*a**3*b**3*x - 240*sqrt(x)*log(sqrt(x)*b
 + a)*a*b**5*x**2 + 240*sqrt(x)*log(sqrt(x))*a**3*b**3*x + 240*sqrt(x)*log
(sqrt(x))*a*b**5*x**2 + 12*sqrt(x)*a**5*b + 200*sqrt(x)*a**3*b**3*x - 60*l
og(sqrt(x)*b + a)*a**4*b**2*x - 360*log(sqrt(x)*b + a)*a**2*b**4*x**2 - 60
*log(sqrt(x)*b + a)*b**6*x**3 + 60*log(sqrt(x))*a**4*b**2*x + 360*log(sqrt
(x))*a**2*b**4*x**2 + 60*log(sqrt(x))*b**6*x**3 - 2*a**6 + 110*a**4*b**2*x
 + 120*a**2*b**4*x**2 - 15*b**6*x**3)/(2*a**7*x*(4*sqrt(x)*a**3*b + 4*sqrt
(x)*a*b**3*x + a**4 + 6*a**2*b**2*x + b**4*x**2))
```

3.112 $\int \frac{1}{(a+b\sqrt{x})^5 x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 156

$$\int \frac{1}{(a+b\sqrt{x})^5 x^3} \, dx = \frac{b^4}{2a^5(a+b\sqrt{x})^4} + \frac{10b^4}{3a^6(a+b\sqrt{x})^3} + \frac{15b^4}{a^7(a+b\sqrt{x})^2} + \frac{70b^4}{a^8(a+b\sqrt{x})} - \frac{1}{2a^5x^2} + \frac{10b}{3a^6x^{3/2}} - \frac{15b^2}{a^7x} + \frac{70b^3}{a^8\sqrt{x}} - \frac{140b^4\log(a+b\sqrt{x})}{a^9} + \frac{70b^4\log(x)}{a^9}$$

output
```
1/2*b^4/a^5/(a+b*x^(1/2))^4+10/3*b^4/a^6/(a+b*x^(1/2))^3+15*b^4/a^7/(a+b*x
^(1/2))^2+70*b^4/a^8/(a+b*x^(1/2))-1/2/a^5/x^2+10/3*b/a^6/x^(3/2)-15*b^2/a
^7/x+70*b^3/a^8/x^(1/2)-140*b^4*ln(a+b*x^(1/2))/a^9+70*b^4*ln(x)/a^9
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+b\sqrt{x})^5 x^3} \, dx = \frac{\frac{a(-3a^7+8a^6b\sqrt{x}-28a^5b^2x+168a^4b^3x^{3/2}+1750a^3b^4x^2+3640a^2b^5x^{5/2}+2940ab^6x^3+840b^7x^{7/2})}{(a+b\sqrt{x})^4x^2} - 840b^4\log(a+b\sqrt{x}) + 420b^4 \mathrm{lo}}{6a^9}$$

input
```
Integrate[1/((a + b*Sqrt[x])^5*x^3),x]
```

output
```
((a*(-3*a^7 + 8*a^6*b*Sqrt[x] - 28*a^5*b^2*x + 168*a^4*b^3*x^(3/2) + 1750*
a^3*b^4*x^2 + 3640*a^2*b^5*x^(5/2) + 2940*a*b^6*x^3 + 840*b^7*x^(7/2)))/((
a + b*Sqrt[x])^4*x^2) - 840*b^4*Log[a + b*Sqrt[x]] + 420*b^4*Log[x])/(6*a^
9)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sqrt{x}\right)^5} \, dx$$

$$\Big\downarrow \ 798$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \ 54$$

$$2\int \left(-\frac{70b^5}{a^9\left(a+b\sqrt{x}\right)}-\frac{35b^5}{a^8\left(a+b\sqrt{x}\right)^2}-\frac{15b^5}{a^7\left(a+b\sqrt{x}\right)^3}-\frac{5b^5}{a^6\left(a+b\sqrt{x}\right)^4}-\frac{b^5}{a^5\left(a+b\sqrt{x}\right)^5}+\frac{70b^4}{a^9\sqrt{x}}-\frac{35b^3}{a^8x}+\right.$$

$$\Big\downarrow \ 2009$$

$$2\left(-\frac{70b^4\log\left(a+b\sqrt{x}\right)}{a^9}+\frac{70b^4\log\left(\sqrt{x}\right)}{a^9}+\frac{35b^4}{a^8\left(a+b\sqrt{x}\right)}+\frac{35b^3}{a^8\sqrt{x}}+\frac{15b^4}{2a^7\left(a+b\sqrt{x}\right)^2}-\frac{15b^2}{2a^7x}+\frac{5b^4}{3a^6\left(a+b\sqrt{x}\right)^3}\right.$$

input
```
Int[1/((a + b*Sqrt[x])^5*x^3),x]
```

output
```
2*(b^4/(4*a^5*(a + b*Sqrt[x])^4) + (5*b^4)/(3*a^6*(a + b*Sqrt[x])^3) + (15
*b^4)/(2*a^7*(a + b*Sqrt[x])^2) + (35*b^4)/(a^8*(a + b*Sqrt[x])) - 1/(4*a^
5*x^2) + (5*b)/(3*a^6*x^(3/2)) - (15*b^2)/(2*a^7*x) + (35*b^3)/(a^8*Sqrt[x
]) - (70*b^4*Log[a + b*Sqrt[x]])/a^9 + (70*b^4*Log[Sqrt[x]])/a^9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{b^4}{2a^5\left(a+b\sqrt{x}\right)^4}+\frac{10b^4}{3a^6\left(a+b\sqrt{x}\right)^3}+\frac{15b^4}{a^7\left(a+b\sqrt{x}\right)^2}+\frac{70b^4}{a^8\left(a+b\sqrt{x}\right)}-\frac{1}{2a^5x^2}+\frac{10b}{3a^6x^{\frac{3}{2}}}-\frac{15b^2}{a^7x}+\frac{70b^3}{a^8\sqrt{x}}-140$
default	$\frac{b^4}{2a^5\left(a+b\sqrt{x}\right)^4}+\frac{10b^4}{3a^6\left(a+b\sqrt{x}\right)^3}+\frac{15b^4}{a^7\left(a+b\sqrt{x}\right)^2}+\frac{70b^4}{a^8\left(a+b\sqrt{x}\right)}-\frac{1}{2a^5x^2}+\frac{10b}{3a^6x^{\frac{3}{2}}}-\frac{15b^2}{a^7x}+\frac{70b^3}{a^8\sqrt{x}}-140$

input
```
int(1/(a+b*x^(1/2))^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^4/a^5/(a+b*x^(1/2))^4+10/3*b^4/a^6/(a+b*x^(1/2))^3+15*b^4/a^7/(a+b*x
^(1/2))^2+70*b^4/a^8/(a+b*x^(1/2))-1/2/a^5/x^2+10/3*b/a^6/x^(3/2)-15*b^2/a
^7/x+70*b^3/a^8/x^(1/2)-140*b^4*ln(a+b*x^(1/2))/a^9+70*b^4*ln(x)/a^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(134) = 268.

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx = -\frac{420\,a^2b^{10}x^5 - 1470\,a^4b^8x^4 + 1820\,a^6b^6x^3 - 875\,a^8b^4x^2 + 78\,a^{10}b^2x + 3\,a^{12} + 840\,(b^{12}x^6 - 4\,a^2b^{10}x^5 + \cdots}{}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x^3,x, algorithm="fricas")
```

output
```
-1/6*(420*a^2*b^10*x^5 - 1470*a^4*b^8*x^4 + 1820*a^6*b^6*x^3 - 875*a^8*b^4
*x^2 + 78*a^10*b^2*x + 3*a^12 + 840*(b^12*x^6 - 4*a^2*b^10*x^5 + 6*a^4*b^8
*x^4 - 4*a^6*b^6*x^3 + a^8*b^4*x^2)*log(b*sqrt(x) + a) - 840*(b^12*x^6 - 4
*a^2*b^10*x^5 + 6*a^4*b^8*x^4 - 4*a^6*b^6*x^3 + a^8*b^4*x^2)*log(sqrt(x))
- 4*(210*a*b^11*x^5 - 770*a^3*b^9*x^4 + 1022*a^5*b^7*x^3 - 558*a^7*b^5*x^2
 + 85*a^9*b^3*x + 5*a^11*b)*sqrt(x))/(a^9*b^8*x^6 - 4*a^11*b^6*x^5 + 6*a^1
3*b^4*x^4 - 4*a^15*b^2*x^3 + a^17*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1391 vs. 2(151) = 302.

Time = 4.13 (sec) , antiderivative size = 1391, normalized size of antiderivative = 8.92

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2))**5/x**3,x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-1/(2*a**5*x**2), Eq(b, 0)
), (-2/(9*b**5*x**(9/2)), Eq(a, 0)), (zoo/x**2, Eq(a, -b*sqrt(x))), (-3*a*
*8*sqrt(x)/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) +
24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 8*a**7*b*x/(6*a**13*x**(5/2)
+ 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b
**4*x**(9/2)) - 28*a**6*b**2*x**(3/2)/(6*a**13*x**(5/2) + 24*a**12*b*x**3
+ 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 16
8*a**5*b**3*x**2/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7
/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 420*a**4*b**4*x**(5/2)*
log(x)/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a
**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) - 840*a**4*b**4*x**(5/2)*log(a/b +
sqrt(x))/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24
*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 1750*a**4*b**4*x**(5/2)/(6*a**1
3*x**(5/2) + 24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4
 + 6*a**9*b**4*x**(9/2)) + 1680*a**3*b**5*x**3*log(x)/(6*a**13*x**(5/2) +
24*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**
4*x**(9/2)) - 3360*a**3*b**5*x**3*log(a/b + sqrt(x))/(6*a**13*x**(5/2) + 2
4*a**12*b*x**3 + 36*a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4
*x**(9/2)) + 3640*a**3*b**5*x**3/(6*a**13*x**(5/2) + 24*a**12*b*x**3 + 36*
a**11*b**2*x**(7/2) + 24*a**10*b**3*x**4 + 6*a**9*b**4*x**(9/2)) + 2520...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx = \frac{840\,b^7x^{\frac{7}{2}} + 2940\,ab^6x^3 + 3640\,a^2b^5x^{\frac{5}{2}} + 1750\,a^3b^4x^2 + 168\,a^4b^3x^{\frac{3}{2}} - 28\,a^5b^2x + 8\,a^6b\sqrt{x} - 3\,a^7}{6\left(a^8b^4x^4 + 4\,a^9b^3x^{\frac{7}{2}} + 6\,a^{10}b^2x^3 + 4\,a^{11}bx^{\frac{5}{2}} + a^{12}x^2\right)} - \frac{140\,b^4\log\left(b\sqrt{x}+a\right)}{a^9} + \frac{70\,b^4\log\left(x\right)}{a^9}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x^3,x, algorithm="maxima")
```

output
```
1/6*(840*b^7*x^(7/2) + 2940*a*b^6*x^3 + 3640*a^2*b^5*x^(5/2) + 1750*a^3*b^
4*x^2 + 168*a^4*b^3*x^(3/2) - 28*a^5*b^2*x + 8*a^6*b*sqrt(x) - 3*a^7)/(a^8
*b^4*x^4 + 4*a^9*b^3*x^(7/2) + 6*a^10*b^2*x^3 + 4*a^11*b*x^(5/2) + a^12*x^
2) - 140*b^4*log(b*sqrt(x) + a)/a^9 + 70*b^4*log(x)/a^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx = -\frac{140\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^9} + \frac{70\,b^4\log\left(|x|\right)}{a^9} + \frac{840\,b^7x^{\frac{7}{2}} + 2940\,ab^6x^3 + 3640\,a^2b^5x^{\frac{5}{2}} + 1750\,a^3b^4x^2 + 168\,a^4b^3x^{\frac{3}{2}} - 28\,a^5b^2x + 8\,a^6b\sqrt{x} - 3\,a^7}{6\left(bx+a\sqrt{x}\right)^4a^8}$$

input
```
integrate(1/(a+b*x^(1/2))^5/x^3,x, algorithm="giac")
```

output
```
-140*b^4*log(abs(b*sqrt(x) + a))/a^9 + 70*b^4*log(abs(x))/a^9 + 1/6*(840*b
^7*x^(7/2) + 2940*a*b^6*x^3 + 3640*a^2*b^5*x^(5/2) + 1750*a^3*b^4*x^2 + 16
8*a^4*b^3*x^(3/2) - 28*a^5*b^2*x + 8*a^6*b*sqrt(x) - 3*a^7)/((b*x + a*sqrt
(x))^4*a^8)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx = \frac{\frac{4\,b\,\sqrt{x}}{3\,a^2} - \frac{1}{2\,a} - \frac{14\,b^2\,x}{3\,a^3} + \frac{875\,b^4\,x^2}{3\,a^5} + \frac{28\,b^3\,x^{3/2}}{a^4} + \frac{490\,b^6\,x^3}{a^7} + \frac{1820\,b^5\,x^{5/2}}{3\,a^6} + \frac{140\,b^7\,x^{7/2}}{a^8}}{a^4\,x^2 + b^4\,x^4 + 4\,a^3\,b\,x^{5/2} + 4\,a\,b^3\,x^{7/2} + 6\,a^2\,b^2\,x^3} - \frac{280\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^9}$$

input
```
int(1/(x^3*(a + b*x^(1/2))^5),x)
```

output
```
((4*b*x^(1/2))/(3*a^2) - 1/(2*a) - (14*b^2*x)/(3*a^3) + (875*b^4*x^2)/(3*a
^5) + (28*b^3*x^(3/2))/a^4 + (490*b^6*x^3)/a^7 + (1820*b^5*x^(5/2))/(3*a^6
) + (140*b^7*x^(7/2))/a^8)/(a^4*x^2 + b^4*x^4 + 4*a^3*b*x^(5/2) + 4*a*b^3*
x^(7/2) + 6*a^2*b^2*x^3) - (280*b^4*atanh((2*b*x^(1/2))/a + 1))/a^9
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^5 x^3}\,dx$$
$$= \frac{-3360\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^3b^5x^2 - 3360\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a\,b^7x^3 + 3360\sqrt{x}\log\left(\sqrt{x}\right)a^3b^5x^2 + 3360\sqrt{x}\log}{}$$

input
```
int(1/(a+b*x^(1/2))^5/x^3,x)
```

output
```
( - 3360*sqrt(x)*log(sqrt(x)*b + a)*a**3*b**5*x**2 - 3360*sqrt(x)*log(sqrt
(x)*b + a)*a*b**7*x**3 + 3360*sqrt(x)*log(sqrt(x))*a**3*b**5*x**2 + 3360*s
qrt(x)*log(sqrt(x))*a*b**7*x**3 + 8*sqrt(x)*a**7*b + 168*sqrt(x)*a**5*b**3
*x + 2800*sqrt(x)*a**3*b**5*x**2 - 840*log(sqrt(x)*b + a)*a**4*b**4*x**2 -
 5040*log(sqrt(x)*b + a)*a**2*b**6*x**3 - 840*log(sqrt(x)*b + a)*b**8*x**4
 + 840*log(sqrt(x))*a**4*b**4*x**2 + 5040*log(sqrt(x))*a**2*b**6*x**3 + 84
0*log(sqrt(x))*b**8*x**4 - 3*a**8 - 28*a**6*b**2*x + 1540*a**4*b**4*x**2 +
 1680*a**2*b**6*x**3 - 210*b**8*x**4)/(6*a**9*x**2*(4*sqrt(x)*a**3*b + 4*s
qrt(x)*a*b**3*x + a**4 + 6*a**2*b**2*x + b**4*x**2))
```

3.113 $\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 203

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{2a^{11}}{7b^{12}\left(a+b\sqrt{x}\right)^7} - \frac{11a^{10}}{3b^{12}\left(a+b\sqrt{x}\right)^6} + \frac{22a^9}{b^{12}\left(a+b\sqrt{x}\right)^5} - \frac{165a^8}{2b^{12}\left(a+b\sqrt{x}\right)^4} + \frac{220a^7}{b^{12}\left(a+b\sqrt{x}\right)^3} - \frac{462a^6}{b^{12}\left(a+b\sqrt{x}\right)^2} + \frac{924a^5}{b^{12}\left(a+b\sqrt{x}\right)} - \frac{240a^3\sqrt{x}}{b^{11}} + \frac{36a^2x}{b^{10}} - \frac{16ax^{3/2}}{3b^9} + \frac{x^2}{2b^8} + \frac{660a^4\log\left(a+b\sqrt{x}\right)}{b^{12}}$$

output
```
2/7*a^11/b^12/(a+b*x^(1/2))^7-11/3*a^10/b^12/(a+b*x^(1/2))^6+22*a^9/b^12/(
a+b*x^(1/2))^5-165/2*a^8/b^12/(a+b*x^(1/2))^4+220*a^7/b^12/(a+b*x^(1/2))^3
-462*a^6/b^12/(a+b*x^(1/2))^2+924*a^5/b^12/(a+b*x^(1/2))-240*a^3*x^(1/2)/b
^11+36*a^2*x/b^10-16/3*a*x^(3/2)/b^9+1/2*x^2/b^8+660*a^4*ln(a+b*x^(1/2))/b
^12
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx$$
$$= \frac{25961a^{11}+154007a^{10}b\sqrt{x}+365001a^9b^2x+414295a^8b^3x^{3/2}+171745a^7b^4x^2-90993a^6b^5x^{5/2}-127351}{42b^{12}\left(a+b\sqrt{x}\right)^7}$$
$$+\frac{660a^4\log\left(a+b\sqrt{x}\right)}{b^{12}}$$

input
```
Integrate[x^5/(a + b*Sqrt[x])^8,x]
```

output
```
(25961*a^11 + 154007*a^10*b*Sqrt[x] + 365001*a^9*b^2*x + 414295*a^8*b^3*x^
(3/2) + 171745*a^7*b^4*x^2 - 90993*a^6*b^5*x^(5/2) - 127351*a^5*b^6*x^3 -
45913*a^4*b^7*x^(7/2) - 3465*a^3*b^8*x^4 + 385*a^2*b^9*x^(9/2) - 77*a*b^10
*x^5 + 21*b^11*x^(11/2))/(42*b^12*(a + b*Sqrt[x])^7) + (660*a^4*Log[a + b*
Sqrt[x]])/b^12
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx$$
$$\Big\downarrow \; 798$$
$$2\int \frac{x^{11/2}}{\left(a+b\sqrt{x}\right)^8}\,d\sqrt{x}$$
$$\Big\downarrow \; 49$$

$$2\int\left(-\frac{a^{11}}{b^{11}\left(a+b\sqrt{x}\right)^8}+\frac{11a^{10}}{b^{11}\left(a+b\sqrt{x}\right)^7}-\frac{55a^9}{b^{11}\left(a+b\sqrt{x}\right)^6}+\frac{165a^8}{b^{11}\left(a+b\sqrt{x}\right)^5}-\frac{330a^7}{b^{11}\left(a+b\sqrt{x}\right)^4}+\frac{462a^6}{b^{11}\left(a+b\sqrt{x}\right.}\cdots\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^{11}}{7b^{12}\left(a+b\sqrt{x}\right)^7}-\frac{11a^{10}}{6b^{12}\left(a+b\sqrt{x}\right)^6}+\frac{11a^9}{b^{12}\left(a+b\sqrt{x}\right)^5}-\frac{165a^8}{4b^{12}\left(a+b\sqrt{x}\right)^4}+\frac{110a^7}{b^{12}\left(a+b\sqrt{x}\right)^3}-\frac{231a^6}{b^{12}\left(a+b\sqrt{x}\right)}\cdots\right.$$

input
```
Int[x^5/(a + b*Sqrt[x])^8,x]
```

output
```
2*(a^11/(7*b^12*(a + b*Sqrt[x])^7) - (11*a^10)/(6*b^12*(a + b*Sqrt[x])^6)
+ (11*a^9)/(b^12*(a + b*Sqrt[x])^5) - (165*a^8)/(4*b^12*(a + b*Sqrt[x])^4)
 + (110*a^7)/(b^12*(a + b*Sqrt[x])^3) - (231*a^6)/(b^12*(a + b*Sqrt[x])^2)
 + (462*a^5)/(b^12*(a + b*Sqrt[x])) - (120*a^3*Sqrt[x])/b^11 + (18*a^2*x)/
b^10 - (8*a*x^(3/2))/(3*b^9) + x^2/(4*b^8) + (330*a^4*Log[a + b*Sqrt[x]])/
b^12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{2\left(-\frac{b^3x^2}{4}+\frac{8a\,b^2x^{\frac{3}{2}}}{3}-18a^2bx+120a^3\sqrt{x}\right)}{b^{11}}-\frac{11a^{10}}{3b^{12}\left(a+b\sqrt{x}\right)^6}+\frac{220a^7}{b^{12}\left(a+b\sqrt{x}\right)^3}+\frac{2a^{11}}{7b^{12}\left(a+b\sqrt{x}\right)^7}-\frac{462a^6}{b^{12}(a+b\sqrt{}}$
default	$-\frac{2\left(-\frac{b^3x^2}{4}+\frac{8a\,b^2x^{\frac{3}{2}}}{3}-18a^2bx+120a^3\sqrt{x}\right)}{b^{11}}-\frac{11a^{10}}{3b^{12}\left(a+b\sqrt{x}\right)^6}+\frac{220a^7}{b^{12}\left(a+b\sqrt{x}\right)^3}+\frac{2a^{11}}{7b^{12}\left(a+b\sqrt{x}\right)^7}-\frac{462a^6}{b^{12}(a+b\sqrt{}}$

input
```
int(x^5/(a+b*x^(1/2))^8,x,method=_RETURNVERBOSE)
```

output
```
-2/b^11*(-1/4*b^3*x^2+8/3*a*b^2*x^(3/2)-18*a^2*b*x+120*a^3*x^(1/2))-11/3*a
^10/b^12/(a+b*x^(1/2))^6+220*a^7/b^12/(a+b*x^(1/2))^3+2/7*a^11/b^12/(a+b*x
^(1/2))^7-462*a^6/b^12/(a+b*x^(1/2))^2+22*a^9/b^12/(a+b*x^(1/2))^5+660*a^4
*ln(a+b*x^(1/2))/b^12+924*a^5/b^12/(a+b*x^(1/2))-165/2*a^8/b^12/(a+b*x^(1/
2))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(173) = 346.

Time = 0.18 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.82

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx$$
$$=\frac{21\,b^{18}x^9+1365\,a^2b^{16}x^8-10143\,a^4b^{14}x^7-27195\,a^6b^{12}x^6+227094\,a^8b^{10}x^5-540190\,a^{10}b^8x^4+661465\,a}{}$$

input
```
integrate(x^5/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

output
```
1/42*(21*b^18*x^9 + 1365*a^2*b^16*x^8 - 10143*a^4*b^14*x^7 - 27195*a^6*b^1
2*x^6 + 227094*a^8*b^10*x^5 - 540190*a^10*b^8*x^4 + 661465*a^12*b^6*x^3 -
455091*a^14*b^4*x^2 + 167867*a^16*b^2*x - 25961*a^18 + 27720*(a^4*b^14*x^7
 - 7*a^6*b^12*x^6 + 21*a^8*b^10*x^5 - 35*a^10*b^8*x^4 + 35*a^12*b^6*x^3 -
21*a^14*b^4*x^2 + 7*a^16*b^2*x - a^18)*log(b*sqrt(x) + a) - 8*(28*a*b^17*x
^8 + 1064*a^3*b^15*x^7 - 13083*a^5*b^13*x^6 + 48580*a^7*b^11*x^5 - 92323*a
^9*b^9*x^4 + 101376*a^11*b^7*x^3 - 65373*a^13*b^5*x^2 + 23100*a^15*b^3*x -
 3465*a^17*b)*sqrt(x))/(b^26*x^7 - 7*a^2*b^24*x^6 + 21*a^4*b^22*x^5 - 35*a
^6*b^20*x^4 + 35*a^8*b^18*x^3 - 21*a^10*b^16*x^2 + 7*a^12*b^14*x - a^14*b^
12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. $2(197) = 394$.

Time = 3.21 (sec) , antiderivative size = 2048, normalized size of antiderivative = 10.09

$$\int \frac{x^5}{\left(a + b\sqrt{x}\right)^8} \, dx = \text{Too large to display}$$

input
```
integrate(x**5/(a+b*x**(1/2))**8,x)
```

output
```
Piecewise((27720*a**11*log(a/b + sqrt(x))/(42*a**7*b**12 + 294*a**6*b**13*
sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x*
*2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 718
74*a**11/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470
*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 29
4*a*b**18*x**3 + 42*b**19*x**(7/2)) + 194040*a**10*b*sqrt(x)*log(a/b + sqr
t(x))/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a*
*4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a
*b**18*x**3 + 42*b**19*x**(7/2)) + 475398*a**10*b*sqrt(x)/(42*a**7*b**12 +
 294*a**6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 14
70*a**3*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19
*x**(7/2)) + 582120*a**9*b**2*x*log(a/b + sqrt(x))/(42*a**7*b**12 + 294*a*
*6*b**13*sqrt(x) + 882*a**5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3
*b**16*x**2 + 882*a**2*b**17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/
2)) + 1329174*a**9*b**2*x/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a*
*5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b*
*17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 970200*a**8*b**3*x*
*(3/2)*log(a/b + sqrt(x))/(42*a**7*b**12 + 294*a**6*b**13*sqrt(x) + 882*a*
*5*b**14*x + 1470*a**4*b**15*x**(3/2) + 1470*a**3*b**16*x**2 + 882*a**2*b*
*17*x**(5/2) + 294*a*b**18*x**3 + 42*b**19*x**(7/2)) + 2021250*a**8*b**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{660\,a^4\log\left(b\sqrt{x}+a\right)}{b^{12}} + \frac{\left(b\sqrt{x}+a\right)^4}{2\,b^{12}} - \frac{22\left(b\sqrt{x}+a\right)^3 a}{3\,b^{12}} + \frac{55\left(b\sqrt{x}+a\right)^2 a^2}{b^{12}} - \frac{330\left(b\sqrt{x}+a\right)a^3}{b^{12}} + \frac{924\,a^5}{\left(b\sqrt{x}+a\right)b^{12}} - \frac{462\,a^6}{\left(b\sqrt{x}+a\right)^2 b^{12}} + \frac{220\,a^7}{\left(b\sqrt{x}+a\right)^3 b^{12}} - \frac{165\,a^8}{2\left(b\sqrt{x}+a\right)^4 b^{12}} + \frac{22\,a^9}{\left(b\sqrt{x}+a\right)^5 b^{12}} - \frac{11\,a^{10}}{3\left(b\sqrt{x}+a\right)^6 b^{12}} + \frac{2\,a^{11}}{7\left(b\sqrt{x}+a\right)^7 b^{12}}$$

input
```
integrate(x^5/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

output
```
660*a^4*log(b*sqrt(x) + a)/b^12 + 1/2*(b*sqrt(x) + a)^4/b^12 - 22/3*(b*sqr
t(x) + a)^3*a/b^12 + 55*(b*sqrt(x) + a)^2*a^2/b^12 - 330*(b*sqrt(x) + a)*a
^3/b^12 + 924*a^5/((b*sqrt(x) + a)*b^12) - 462*a^6/((b*sqrt(x) + a)^2*b^12
) + 220*a^7/((b*sqrt(x) + a)^3*b^12) - 165/2*a^8/((b*sqrt(x) + a)^4*b^12)
+ 22*a^9/((b*sqrt(x) + a)^5*b^12) - 11/3*a^10/((b*sqrt(x) + a)^6*b^12) + 2
/7*a^11/((b*sqrt(x) + a)^7*b^12)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{660\,a^4\log\left(\left|b\sqrt{x}+a\right|\right)}{b^{12}} + \frac{38808\,a^5b^6x^3+213444\,a^6b^5x^{\frac{5}{2}}+494340\,a^7b^4x^2+615615\,a^8b^3x^{\frac{3}{2}}+434049\,a^9b^2x+164087\,a^{10}b\sqrt{x}+2}{42\left(b\sqrt{x}+a\right)^7b^{12}} + \frac{3\,b^{24}x^2-32\,ab^{23}x^{\frac{3}{2}}+216\,a^2b^{22}x-1440\,a^3b^{21}\sqrt{x}}{6\,b^{32}}$$

input
```
integrate(x^5/(a+b*x^(1/2))^8,x, algorithm="giac")
```

output
```
660*a^4*log(abs(b*sqrt(x) + a))/b^12 + 1/42*(38808*a^5*b^6*x^3 + 213444*a^
6*b^5*x^(5/2) + 494340*a^7*b^4*x^2 + 615615*a^8*b^3*x^(3/2) + 434049*a^9*b
^2*x + 164087*a^10*b*sqrt(x) + 25961*a^11)/((b*sqrt(x) + a)^7*b^12) + 1/6*
(3*b^24*x^2 - 32*a*b^23*x^(3/2) + 216*a^2*b^22*x - 1440*a^3*b^21*sqrt(x))/
b^32
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{\frac{25961\,a^{11}}{42\,b}+\frac{23441\,a^{10}\,\sqrt{x}}{6}+11770\,a^7\,b^3\,x^2+924\,a^5\,b^5\,x^3+\frac{29315\,a^8\,b^2\,x^{3/2}}{2}+5082\,a^6\,b^4\,x^{5/2}+\frac{20669\,a^9\,b\,x}{2}}{a^7\,b^{11}+b^{18}\,x^{7/2}+21\,a^5\,b^{13}\,x+7\,a\,b^{17}\,x^3+35\,a^3\,b^{15}\,x^2+7\,a^6\,b^{12}\,\sqrt{x}+35\,a^4\,b^{14}\,x^{3/2}+21\,a^2\,b^{16}\,x^{5/2}} + \frac{x^2}{2\,b^8}+\frac{36\,a^2\,x}{b^{10}}-\frac{16\,a\,x^{3/2}}{3\,b^9}+\frac{660\,a^4\,\ln\left(a+b\,\sqrt{x}\right)}{b^{12}}-\frac{240\,a^3\,\sqrt{x}}{b^{11}}$$

input
```
int(x^5/(a + b*x^(1/2))^8,x)
```

output
```
((25961*a^11)/(42*b) + (23441*a^10*x^(1/2))/6 + 11770*a^7*b^3*x^2 + 924*a^
5*b^5*x^3 + (29315*a^8*b^2*x^(3/2))/2 + 5082*a^6*b^4*x^(5/2) + (20669*a^9*
b*x)/2)/(a^7*b^11 + b^18*x^(7/2) + 21*a^5*b^13*x + 7*a*b^17*x^3 + 35*a^3*b
^15*x^2 + 7*a^6*b^12*x^(1/2) + 35*a^4*b^14*x^(3/2) + 21*a^2*b^16*x^(5/2))
+ x^2/(2*b^8) + (36*a^2*x)/b^10 - (16*a*x^(3/2))/(3*b^9) + (660*a^4*log(a
+ b*x^(1/2)))/b^12 - (240*a^3*x^(1/2))/b^11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.64

$$\int \frac{x^5}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{194040\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^{10}b+970200\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^8b^3x+582120\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^6b^5x^2+27}{}$$

input
```
int(x^5/(a+b*x^(1/2))^8,x)
```

output
```
(194040*sqrt(x)*log(sqrt(x)*b + a)*a**10*b + 970200*sqrt(x)*log(sqrt(x)*b
+ a)*a**8*b**3*x + 582120*sqrt(x)*log(sqrt(x)*b + a)*a**6*b**5*x**2 + 2772
0*sqrt(x)*log(sqrt(x)*b + a)*a**4*b**7*x**3 + 281358*sqrt(x)*a**10*b + 105
1050*sqrt(x)*a**8*b**3*x + 291060*sqrt(x)*a**6*b**5*x**2 - 27720*sqrt(x)*a
**4*b**7*x**3 + 385*sqrt(x)*a**2*b**9*x**4 + 21*sqrt(x)*b**11*x**5 + 27720
*log(sqrt(x)*b + a)*a**11 + 582120*log(sqrt(x)*b + a)*a**9*b**2*x + 970200
*log(sqrt(x)*b + a)*a**7*b**4*x**2 + 194040*log(sqrt(x)*b + a)*a**5*b**6*x
**3 + 44154*a**11 + 747054*a**9*b**2*x + 808500*a**7*b**4*x**2 - 3465*a**3
*b**8*x**4 - 77*a*b**10*x**5)/(42*b**12*(7*sqrt(x)*a**6*b + 35*sqrt(x)*a**
4*b**3*x + 21*sqrt(x)*a**2*b**5*x**2 + sqrt(x)*b**7*x**3 + a**7 + 21*a**5*
b**2*x + 35*a**3*b**4*x**2 + 7*a*b**6*x**3))
```

3.114 $\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 172

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{2a^9}{7b^{10}\left(a+b\sqrt{x}\right)^7} - \frac{3a^8}{b^{10}\left(a+b\sqrt{x}\right)^6} + \frac{72a^7}{5b^{10}\left(a+b\sqrt{x}\right)^5} - \frac{42a^6}{b^{10}\left(a+b\sqrt{x}\right)^4} + \frac{84a^5}{b^{10}\left(a+b\sqrt{x}\right)^3} - \frac{126a^4}{b^{10}\left(a+b\sqrt{x}\right)^2} + \frac{168a^3}{b^{10}\left(a+b\sqrt{x}\right)} - \frac{16a\sqrt{x}}{b^9} + \frac{x}{b^8} + \frac{72a^2\log\left(a+b\sqrt{x}\right)}{b^{10}}$$

output
```
2/7*a^9/b^10/(a+b*x^(1/2))^7-3*a^8/b^10/(a+b*x^(1/2))^6+72/5*a^7/b^10/(a+b
*x^(1/2))^5-42*a^6/b^10/(a+b*x^(1/2))^4+84*a^5/b^10/(a+b*x^(1/2))^3-126*a^
4/b^10/(a+b*x^(1/2))^2+168*a^3/b^10/(a+b*x^(1/2))-16*a*x^(1/2)/b^9+x/b^8+7
2*a^2*ln(a+b*x^(1/2))/b^10
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx$$
$$=\frac{3349a^9+20923a^8b\sqrt{x}+53949a^7b^2x+72275a^6b^3x^{3/2}+50225a^5b^4x^2+12495a^4b^5x^{5/2}-4655a^3b^6x^3-3}{35b^{10}\left(a+b\sqrt{x}\right)^7}$$
$$+\frac{72a^2\log\left(a+b\sqrt{x}\right)}{b^{10}}$$

input
```
Integrate[x^4/(a + b*Sqrt[x])^8,x]
```

output
```
(3349*a^9 + 20923*a^8*b*Sqrt[x] + 53949*a^7*b^2*x + 72275*a^6*b^3*x^(3/2)
+ 50225*a^5*b^4*x^2 + 12495*a^4*b^5*x^(5/2) - 4655*a^3*b^6*x^3 - 3185*a^2*
b^7*x^(7/2) - 315*a*b^8*x^4 + 35*b^9*x^(9/2))/(35*b^10*(a + b*Sqrt[x])^7)
+ (72*a^2*Log[a + b*Sqrt[x]])/b^10
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}dx$$
$$\Big\downarrow \text{ 798}$$
$$2\int \frac{x^{9/2}}{\left(a+b\sqrt{x}\right)^8}d\sqrt{x}$$
$$\Big\downarrow \text{ 49}$$

$$2\int \left(-\frac{a^9}{b^9 \left(a+b\sqrt{x}\right)^8}+\frac{9a^8}{b^9 \left(a+b\sqrt{x}\right)^7}-\frac{36a^7}{b^9 \left(a+b\sqrt{x}\right)^6}+\frac{84a^6}{b^9 \left(a+b\sqrt{x}\right)^5}-\frac{126a^5}{b^9 \left(a+b\sqrt{x}\right)^4}+\frac{126a^4}{b^9 \left(a+b\sqrt{x}\right)^3}-\right.$$

$\big\downarrow$ 2009

$$2\left(\frac{a^9}{7b^{10} \left(a+b\sqrt{x}\right)^7}-\frac{3a^8}{2b^{10} \left(a+b\sqrt{x}\right)^6}+\frac{36a^7}{5b^{10} \left(a+b\sqrt{x}\right)^5}-\frac{21a^6}{b^{10} \left(a+b\sqrt{x}\right)^4}+\frac{42a^5}{b^{10} \left(a+b\sqrt{x}\right)^3}-\frac{63a^4}{b^{10} \left(a+b\sqrt{x}\right)\right.$$

input
```
Int[x^4/(a + b*Sqrt[x])^8,x]
```

output
```
2*(a^9/(7*b^10*(a + b*Sqrt[x])^7) - (3*a^8)/(2*b^10*(a + b*Sqrt[x])^6) + (
36*a^7)/(5*b^10*(a + b*Sqrt[x])^5) - (21*a^6)/(b^10*(a + b*Sqrt[x])^4) + (
42*a^5)/(b^10*(a + b*Sqrt[x])^3) - (63*a^4)/(b^10*(a + b*Sqrt[x])^2) + (84
*a^3)/(b^10*(a + b*Sqrt[x])) - (8*a*Sqrt[x])/b^9 + x/(2*b^8) + (36*a^2*Log
[a + b*Sqrt[x]])/b^10)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{2\left(-\frac{bx}{2}+8a\sqrt{x}\right)}{b^9}+\frac{72a^7}{5b^{10}\left(a+b\sqrt{x}\right)^5}-\frac{126a^4}{b^{10}\left(a+b\sqrt{x}\right)^2}-\frac{3a^8}{b^{10}\left(a+b\sqrt{x}\right)^6}-\frac{42a^6}{b^{10}\left(a+b\sqrt{x}\right)^4}+\frac{84a^5}{b^{10}\left(a+b\sqrt{x}\right)^3}+$
default	$-\frac{2\left(-\frac{bx}{2}+8a\sqrt{x}\right)}{b^9}+\frac{72a^7}{5b^{10}\left(a+b\sqrt{x}\right)^5}-\frac{126a^4}{b^{10}\left(a+b\sqrt{x}\right)^2}-\frac{3a^8}{b^{10}\left(a+b\sqrt{x}\right)^6}-\frac{42a^6}{b^{10}\left(a+b\sqrt{x}\right)^4}+\frac{84a^5}{b^{10}\left(a+b\sqrt{x}\right)^3}+$

input
```
int(x^4/(a+b*x^(1/2))^8,x,method=_RETURNVERBOSE)
```

output
```
-2/b^9*(-1/2*b*x+8*a*x^(1/2))+72/5*a^7/b^10/(a+b*x^(1/2))^5-126*a^4/b^10/(
a+b*x^(1/2))^2-3*a^8/b^10/(a+b*x^(1/2))^6-42*a^6/b^10/(a+b*x^(1/2))^4+84*a
^5/b^10/(a+b*x^(1/2))^3+2/7*a^9/b^10/(a+b*x^(1/2))^7+168*a^3/b^10/(a+b*x^(
1/2))+72*a^2*ln(a+b*x^(1/2))/b^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(150) = 300.

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.02

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx$$
$$= \frac{35\,b^{16}x^8-245\,a^2b^{14}x^7-9555\,a^4b^{12}x^6+41405\,a^6b^{10}x^5-83720\,a^8b^8x^4+94745\,a^{10}b^6x^3-62139\,a^{12}b^4x^2}{}$$

input
```
integrate(x^4/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

output
```
1/35*(35*b^16*x^8 - 245*a^2*b^14*x^7 - 9555*a^4*b^12*x^6 + 41405*a^6*b^10*
x^5 - 83720*a^8*b^8*x^4 + 94745*a^10*b^6*x^3 - 62139*a^12*b^4*x^2 + 22183*
a^14*b^2*x - 3349*a^16 + 2520*(a^2*b^14*x^7 - 7*a^4*b^12*x^6 + 21*a^6*b^10
*x^5 - 35*a^8*b^8*x^4 + 35*a^10*b^6*x^3 - 21*a^12*b^4*x^2 + 7*a^14*b^2*x -
 a^16)*log(b*sqrt(x) + a) - 8*(70*a*b^15*x^7 - 1225*a^3*b^13*x^6 + 4410*a^
5*b^11*x^5 - 8393*a^7*b^9*x^4 + 9216*a^9*b^7*x^3 - 5943*a^11*b^5*x^2 + 210
0*a^13*b^3*x - 315*a^15*b)*sqrt(x))/(b^24*x^7 - 7*a^2*b^22*x^6 + 21*a^4*b^
20*x^5 - 35*a^6*b^18*x^4 + 35*a^8*b^16*x^3 - 21*a^10*b^14*x^2 + 7*a^12*b^1
2*x - a^14*b^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. $2(167) = 334$.

Time = 2.65 (sec) , antiderivative size = 1839, normalized size of antiderivative = 10.69

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx = \text{Too large to display}$$

input
```
integrate(x**4/(a+b*x**(1/2))**8,x)
```

output
```
Piecewise((2520*a**9*log(a/b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*sq
rt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2
 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 6534*
a**9/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**
4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*
b**16*x**3 + 35*b**17*x**(7/2)) + 17640*a**8*b*sqrt(x)*log(a/b + sqrt(x))/
(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**
13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16
*x**3 + 35*b**17*x**(7/2)) + 43218*a**8*b*sqrt(x)/(35*a**7*b**10 + 245*a**
6*b**11*sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*
b**14*x**2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2
)) + 52920*a**7*b**2*x*log(a/b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*
sqrt(x) + 735*a**5*b**12*x + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x*
*2 + 735*a**2*b**15*x**(5/2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 120
834*a**7*b**2*x/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x
 + 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/
2) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 88200*a**6*b**3*x**(3/2)*log(
a/b + sqrt(x))/(35*a**7*b**10 + 245*a**6*b**11*sqrt(x) + 735*a**5*b**12*x
+ 1225*a**4*b**13*x**(3/2) + 1225*a**3*b**14*x**2 + 735*a**2*b**15*x**(5/2
) + 245*a*b**16*x**3 + 35*b**17*x**(7/2)) + 183750*a**6*b**3*x**(3/2)/(...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{72\,a^2\log\left(b\sqrt{x}+a\right)}{b^{10}} + \frac{\left(b\sqrt{x}+a\right)^2}{b^{10}} - \frac{18\,\left(b\sqrt{x}+a\right)a}{b^{10}} + \frac{168\,a^3}{\left(b\sqrt{x}+a\right)b^{10}} - \frac{126\,a^4}{\left(b\sqrt{x}+a\right)^2b^{10}} + \frac{84\,a^5}{\left(b\sqrt{x}+a\right)^3b^{10}} - \frac{42\,a^6}{\left(b\sqrt{x}+a\right)^4b^{10}} + \frac{72\,a^7}{5\,\left(b\sqrt{x}+a\right)^5b^{10}} - \frac{3\,a^8}{\left(b\sqrt{x}+a\right)^6b^{10}} + \frac{2\,a^9}{7\,\left(b\sqrt{x}+a\right)^7b^{10}}$$

input
```
integrate(x^4/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

output
```
72*a^2*log(b*sqrt(x) + a)/b^10 + (b*sqrt(x) + a)^2/b^10 - 18*(b*sqrt(x) +
a)*a/b^10 + 168*a^3/((b*sqrt(x) + a)*b^10) - 126*a^4/((b*sqrt(x) + a)^2*b^
10) + 84*a^5/((b*sqrt(x) + a)^3*b^10) - 42*a^6/((b*sqrt(x) + a)^4*b^10) +
72/5*a^7/((b*sqrt(x) + a)^5*b^10) - 3*a^8/((b*sqrt(x) + a)^6*b^10) + 2/7*a
^9/((b*sqrt(x) + a)^7*b^10)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{72\,a^2\log\left(\left|b\sqrt{x}+a\right|\right)}{b^{10}} + \frac{b^8x-16\,ab^7\sqrt{x}}{b^{16}} + \frac{5880\,a^3b^6x^3+30870\,a^4b^5x^{\frac{5}{2}}+69090\,a^5b^4x^2+83790\,a^6b^3x^{\frac{3}{2}}+57834\,a^7b^2x+21483\,a^8b\sqrt{x}+3349\,a^9}{35\left(b\sqrt{x}+a\right)^7b^{10}}$$

input
```
integrate(x^4/(a+b*x^(1/2))^8,x, algorithm="giac")
```

output
```
72*a^2*log(abs(b*sqrt(x) + a))/b^10 + (b^8*x - 16*a*b^7*sqrt(x))/b^16 + 1/
35*(5880*a^3*b^6*x^3 + 30870*a^4*b^5*x^(5/2) + 69090*a^5*b^4*x^2 + 83790*a
^6*b^3*x^(3/2) + 57834*a^7*b^2*x + 21483*a^8*b*sqrt(x) + 3349*a^9)/((b*sqr
t(x) + a)^7*b^10)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{x}{b^8} + \frac{\frac{3349\,a^9}{35\,b}+\frac{3069\,a^8\,\sqrt{x}}{5}+1974\,a^5\,b^3\,x^2+168\,a^3\,b^5\,x^3+2394\,a^6\,b^2\,x^{3/2}+882\,a^4\,b^4\,x^{5/2}+\frac{8262\,a^7\,b\,x}{5}}{a^7\,b^9+b^{16}\,x^{7/2}+21\,a^5\,b^{11}\,x+7\,a\,b^{15}\,x^3+35\,a^3\,b^{13}\,x^2+7\,a^6\,b^{10}\,\sqrt{x}+35\,a^4\,b^{12}\,x^{3/2}+21\,a^2\,b^{14}\,x^{5/2}} - \frac{16\,a\,\sqrt{x}}{b^9} + \frac{72\,a^2\,\ln\left(a+b\,\sqrt{x}\right)}{b^{10}}$$

input
```
int(x^4/(a + b*x^(1/2))^8,x)
```

output
```
x/b^8 + ((3349*a^9)/(35*b) + (3069*a^8*x^(1/2))/5 + 1974*a^5*b^3*x^2 + 168
*a^3*b^5*x^3 + 2394*a^6*b^2*x^(3/2) + 882*a^4*b^4*x^(5/2) + (8262*a^7*b*x)
/5)/(a^7*b^9 + b^16*x^(7/2) + 21*a^5*b^11*x + 7*a*b^15*x^3 + 35*a^3*b^13*x
^2 + 7*a^6*b^10*x^(1/2) + 35*a^4*b^12*x^(3/2) + 21*a^2*b^14*x^(5/2)) - (16
*a*x^(1/2))/b^9 + (72*a^2*log(a + b*x^(1/2)))/b^10
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.79

$$\int \frac{x^4}{(a+b\sqrt{x})^8}\,dx$$
$$= \frac{17640\sqrt{x}\log(\sqrt{x}\,b+a)\,a^8b + 88200\sqrt{x}\log(\sqrt{x}\,b+a)\,a^6b^3x + 52920\sqrt{x}\log(\sqrt{x}\,b+a)\,a^4b^5x^2 + 2520\sqrt{}}{}$$

input
```
int(x^4/(a+b*x^(1/2))^8,x)
```

output
```
(17640*sqrt(x)*log(sqrt(x)*b + a)*a**8*b + 88200*sqrt(x)*log(sqrt(x)*b + a
)*a**6*b**3*x + 52920*sqrt(x)*log(sqrt(x)*b + a)*a**4*b**5*x**2 + 2520*sqr
t(x)*log(sqrt(x)*b + a)*a**2*b**7*x**3 + 25578*sqrt(x)*a**8*b + 95550*sqrt
(x)*a**6*b**3*x + 26460*sqrt(x)*a**4*b**5*x**2 - 2520*sqrt(x)*a**2*b**7*x*
*3 + 35*sqrt(x)*b**9*x**4 + 2520*log(sqrt(x)*b + a)*a**9 + 52920*log(sqrt(
x)*b + a)*a**7*b**2*x + 88200*log(sqrt(x)*b + a)*a**5*b**4*x**2 + 17640*lo
g(sqrt(x)*b + a)*a**3*b**6*x**3 + 4014*a**9 + 67914*a**7*b**2*x + 73500*a*
*5*b**4*x**2 - 315*a*b**8*x**4)/(35*b**10*(7*sqrt(x)*a**6*b + 35*sqrt(x)*a
**4*b**3*x + 21*sqrt(x)*a**2*b**5*x**2 + sqrt(x)*b**7*x**3 + a**7 + 21*a**
5*b**2*x + 35*a**3*b**4*x**2 + 7*a*b**6*x**3))
```

3.115 $\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 157

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{2a^7}{7b^8\left(a+b\sqrt{x}\right)^7} - \frac{7a^6}{3b^8\left(a+b\sqrt{x}\right)^6} + \frac{42a^5}{5b^8\left(a+b\sqrt{x}\right)^5} - \frac{35a^4}{2b^8\left(a+b\sqrt{x}\right)^4} + \frac{70a^3}{3b^8\left(a+b\sqrt{x}\right)^3} - \frac{21a^2}{b^8\left(a+b\sqrt{x}\right)^2} + \frac{14a}{b^8\left(a+b\sqrt{x}\right)} + \frac{2\log\left(a+b\sqrt{x}\right)}{b^8}$$

output
```
2/7*a^7/b^8/(a+b*x^(1/2))^7-7/3*a^6/b^8/(a+b*x^(1/2))^6+42/5*a^5/b^8/(a+b*
x^(1/2))^5-35/2*a^4/b^8/(a+b*x^(1/2))^4+70/3*a^3/b^8/(a+b*x^(1/2))^3-21*a^
2/b^8/(a+b*x^(1/2))^2+14*a/b^8/(a+b*x^(1/2))+2*ln(a+b*x^(1/2))/b^8
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{a\left(1089a^6+7203a^5b\sqrt{x}+20139a^4b^2x+30625a^3b^3x^{3/2}+26950a^2b^4x^2+13230ab^5x^{5/2}+2940b^6x^3\right)}{210b^8\left(a+b\sqrt{x}\right)^7} + \frac{2\log\left(a+b\sqrt{x}\right)}{b^8}$$

input
```
Integrate[x^3/(a + b*Sqrt[x])^8,x]
```

output
```
(a*(1089*a^6 + 7203*a^5*b*Sqrt[x] + 20139*a^4*b^2*x + 30625*a^3*b^3*x^(3/2
) + 26950*a^2*b^4*x^2 + 13230*a*b^5*x^(5/2) + 2940*b^6*x^3))/(210*b^8*(a +
 b*Sqrt[x])^7) + (2*Log[a + b*Sqrt[x]])/b^8
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{7/2}}{\left(a+b\sqrt{x}\right)^8}\,d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(-\frac{a^7}{b^7\left(a+b\sqrt{x}\right)^8}+\frac{7a^6}{b^7\left(a+b\sqrt{x}\right)^7}-\frac{21a^5}{b^7\left(a+b\sqrt{x}\right)^6}+\frac{35a^4}{b^7\left(a+b\sqrt{x}\right)^5}-\frac{35a^3}{b^7\left(a+b\sqrt{x}\right)^4}+\frac{21a^2}{b^7\left(a+b\sqrt{x}\right)^3}-\right.$$

$$\downarrow \text{2009}$$

$$2\left(\frac{a^7}{7b^8\left(a+b\sqrt{x}\right)^7}-\frac{7a^6}{6b^8\left(a+b\sqrt{x}\right)^6}+\frac{21a^5}{5b^8\left(a+b\sqrt{x}\right)^5}-\frac{35a^4}{4b^8\left(a+b\sqrt{x}\right)^4}+\frac{35a^3}{3b^8\left(a+b\sqrt{x}\right)^3}-\frac{21a^2}{2b^8\left(a+b\sqrt{x}\right)^2}\right.$$

input
```
Int[x^3/(a + b*Sqrt[x])^8,x]
```

output
```
2*(a^7/(7*b^8*(a + b*Sqrt[x])^7) - (7*a^6)/(6*b^8*(a + b*Sqrt[x])^6) + (21
*a^5)/(5*b^8*(a + b*Sqrt[x])^5) - (35*a^4)/(4*b^8*(a + b*Sqrt[x])^4) + (35
*a^3)/(3*b^8*(a + b*Sqrt[x])^3) - (21*a^2)/(2*b^8*(a + b*Sqrt[x])^2) + (7*
a)/(b^8*(a + b*Sqrt[x])) + Log[a + b*Sqrt[x]]/b^8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2a^7}{7b^8(a+b\sqrt{x})^7}-\frac{7a^6}{3b^8(a+b\sqrt{x})^6}+\frac{42a^5}{5b^8(a+b\sqrt{x})^5}-\frac{35a^4}{2b^8(a+b\sqrt{x})^4}+\frac{70a^3}{3b^8(a+b\sqrt{x})^3}-\frac{21a^2}{b^8(a+b\sqrt{x})^2}+\frac{14}{b^8(a+}$
default	$\frac{2a^7}{7b^8(a+b\sqrt{x})^7}-\frac{7a^6}{3b^8(a+b\sqrt{x})^6}+\frac{42a^5}{5b^8(a+b\sqrt{x})^5}-\frac{35a^4}{2b^8(a+b\sqrt{x})^4}+\frac{70a^3}{3b^8(a+b\sqrt{x})^3}-\frac{21a^2}{b^8(a+b\sqrt{x})^2}+\frac{14}{b^8(a+}$

input
```
int(x^3/(a+b*x^(1/2))^8,x,method=_RETURNVERBOSE)
```

output
```
2/7*a^7/b^8/(a+b*x^(1/2))^7-7/3*a^6/b^8/(a+b*x^(1/2))^6+42/5*a^5/b^8/(a+b*
x^(1/2))^5-35/2*a^4/b^8/(a+b*x^(1/2))^4+70/3*a^3/b^8/(a+b*x^(1/2))^3-21*a^
2/b^8/(a+b*x^(1/2))^2+14*a/b^8/(a+b*x^(1/2))+2*ln(a+b*x^(1/2))/b^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(131) = 262$.

Time = 0.12 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.01

$$\int \frac{x^3}{(a+b\sqrt{x})^8}\,dx =$$
$$-\frac{7350\,a^2b^{12}x^6 - 16905\,a^4b^{10}x^5 + 32585\,a^6b^8x^4 - 34370\,a^8b^6x^3 + 21504\,a^{10}b^4x^2 - 7413\,a^{12}b^2x + 1089\,a^{1}}{}$$

input
```
integrate(x^3/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

output
```
-1/210*(7350*a^2*b^12*x^6 - 16905*a^4*b^10*x^5 + 32585*a^6*b^8*x^4 - 34370
*a^8*b^6*x^3 + 21504*a^10*b^4*x^2 - 7413*a^12*b^2*x + 1089*a^14 - 420*(b^1
4*x^7 - 7*a^2*b^12*x^6 + 21*a^4*b^10*x^5 - 35*a^6*b^8*x^4 + 35*a^8*b^6*x^3
 - 21*a^10*b^4*x^2 + 7*a^12*b^2*x - a^14)*log(b*sqrt(x) + a) - 4*(735*a*b^
13*x^6 - 980*a^3*b^11*x^5 + 2891*a^5*b^9*x^4 - 3072*a^7*b^7*x^3 + 1981*a^9
*b^5*x^2 - 700*a^11*b^3*x + 105*a^13*b)*sqrt(x))/(b^22*x^7 - 7*a^2*b^20*x^
6 + 21*a^4*b^18*x^5 - 35*a^6*b^16*x^4 + 35*a^8*b^14*x^3 - 21*a^10*b^12*x^2
 + 7*a^12*b^10*x - a^14*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1629 vs. $2(150) = 300$.

Time = 2.18 (sec) , antiderivative size = 1629, normalized size of antiderivative = 10.38

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx = \text{Too large to display}$$

input
```
integrate(x**3/(a+b*x**(1/2))**8,x)
```

output
```
Piecewise((420*a**7*log(a/b + sqrt(x))/(210*a**7*b**8 + 1470*a**6*b**9*sqr
t(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2
 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 10
89*a**7/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350
*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1
470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 2940*a**6*b*sqrt(x)*log(a/b + sqr
t(x))/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a
**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 147
0*a*b**14*x**3 + 210*b**15*x**(7/2)) + 7203*a**6*b*sqrt(x)/(210*a**7*b**8
+ 1470*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) +
7350*a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*
b**15*x**(7/2)) + 8820*a**5*b**2*x*log(a/b + sqrt(x))/(210*a**7*b**8 + 147
0*a**6*b**9*sqrt(x) + 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*
a**3*b**12*x**2 + 4410*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15
*x**(7/2)) + 20139*a**5*b**2*x/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x) + 4
410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 4410*
a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 14700*a**4
*b**3*x**(3/2)*log(a/b + sqrt(x))/(210*a**7*b**8 + 1470*a**6*b**9*sqrt(x)
+ 4410*a**5*b**10*x + 7350*a**4*b**11*x**(3/2) + 7350*a**3*b**12*x**2 + 44
10*a**2*b**13*x**(5/2) + 1470*a*b**14*x**3 + 210*b**15*x**(7/2)) + 3062...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{2\,\log\left(b\sqrt{x}+a\right)}{b^8} + \frac{14\,a}{\left(b\sqrt{x}+a\right)b^8} - \frac{21\,a^2}{\left(b\sqrt{x}+a\right)^2 b^8} + \frac{70\,a^3}{3\,\left(b\sqrt{x}+a\right)^3 b^8} - \frac{35\,a^4}{2\,\left(b\sqrt{x}+a\right)^4 b^8} + \frac{42\,a^5}{5\,\left(b\sqrt{x}+a\right)^5 b^8} - \frac{7\,a^6}{3\,\left(b\sqrt{x}+a\right)^6 b^8} + \frac{2\,a^7}{7\,\left(b\sqrt{x}+a\right)^7 b^8}$$

input
```
integrate(x^3/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

output
```
2*log(b*sqrt(x) + a)/b^8 + 14*a/((b*sqrt(x) + a)*b^8) - 21*a^2/((b*sqrt(x)
 + a)^2*b^8) + 70/3*a^3/((b*sqrt(x) + a)^3*b^8) - 35/2*a^4/((b*sqrt(x) + a
)^4*b^8) + 42/5*a^5/((b*sqrt(x) + a)^5*b^8) - 7/3*a^6/((b*sqrt(x) + a)^6*b
^8) + 2/7*a^7/((b*sqrt(x) + a)^7*b^8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{b^8} + \frac{2940\,ab^5x^3 + 13230\,a^2b^4x^{\frac{5}{2}} + 26950\,a^3b^3x^2 + 30625\,a^4b^2x^{\frac{3}{2}} + 20139\,a^5bx + 7203\,a^6\sqrt{x} + \frac{1089\,a^7}{b}}{210\,\left(b\sqrt{x}+a\right)^7 b^7}$$

input
```
integrate(x^3/(a+b*x^(1/2))^8,x, algorithm="giac")
```

output
```
2*log(abs(b*sqrt(x) + a))/b^8 + 1/210*(2940*a*b^5*x^3 + 13230*a^2*b^4*x^(5
/2) + 26950*a^3*b^3*x^2 + 30625*a^4*b^2*x^(3/2) + 20139*a^5*b*x + 7203*a^6
*sqrt(x) + 1089*a^7/b)/((b*sqrt(x) + a)^7*b^7)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{\frac{363\,a^7}{70\,b^8}+\frac{14\,a\,x^3}{b^2}+\frac{959\,a^5\,x}{10\,b^6}+\frac{385\,a^3\,x^2}{3\,b^4}+\frac{63\,a^2\,x^{5/2}}{b^3}+\frac{875\,a^4\,x^{3/2}}{6\,b^5}+\frac{343\,a^6\,\sqrt{x}}{10\,b^7}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}} + \frac{2\,\ln\left(a+b\,\sqrt{x}\right)}{b^8}$$

input
```
int(x^3/(a + b*x^(1/2))^8,x)
```

output
```
((363*a^7)/(70*b^8) + (14*a*x^3)/b^2 + (959*a^5*x)/(10*b^6) + (385*a^3*x^2
)/(3*b^4) + (63*a^2*x^(5/2))/b^3 + (875*a^4*x^(3/2))/(6*b^5) + (343*a^6*x^
(1/2))/(10*b^7))/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*x^3 + 7*a^6*b
*x^(1/2) + 35*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x^(5/2)) + (2*
log(a + b*x^(1/2)))/b^8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.79

$$\int \frac{x^3}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{2940\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^6b+14700\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^4b^3x+8820\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^2b^5x^2+420\sqrt{x}\log}{}$$

input
```
int(x^3/(a+b*x^(1/2))^8,x)
```

output
```
(2940*sqrt(x)*log(sqrt(x)*b + a)*a**6*b + 14700*sqrt(x)*log(sqrt(x)*b + a)
*a**4*b**3*x + 8820*sqrt(x)*log(sqrt(x)*b + a)*a**2*b**5*x**2 + 420*sqrt(x
)*log(sqrt(x)*b + a)*b**7*x**3 + 4263*sqrt(x)*a**6*b + 15925*sqrt(x)*a**4*
b**3*x + 4410*sqrt(x)*a**2*b**5*x**2 - 420*sqrt(x)*b**7*x**3 + 420*log(sqr
t(x)*b + a)*a**7 + 8820*log(sqrt(x)*b + a)*a**5*b**2*x + 14700*log(sqrt(x)
*b + a)*a**3*b**4*x**2 + 2940*log(sqrt(x)*b + a)*a*b**6*x**3 + 669*a**7 +
11319*a**5*b**2*x + 12250*a**3*b**4*x**2)/(210*b**8*(7*sqrt(x)*a**6*b + 35
*sqrt(x)*a**4*b**3*x + 21*sqrt(x)*a**2*b**5*x**2 + sqrt(x)*b**7*x**3 + a**
7 + 21*a**5*b**2*x + 35*a**3*b**4*x**2 + 7*a*b**6*x**3))
```

3.116 $\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx=\frac{2x^3}{7a\left(a+b\sqrt{x}\right)^7}+\frac{x^3}{21a^2\left(a+b\sqrt{x}\right)^6}$$

output
```
2/7*x^3/a/(a+b*x^(1/2))^7+1/21*x^3/a^2/(a+b*x^(1/2))^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx=\frac{-a^5-7a^4b\sqrt{x}-21a^3b^2x-35a^2b^3x^{3/2}-35ab^4x^2-21b^5x^{5/2}}{21b^6\left(a+b\sqrt{x}\right)^7}$$

input
```
Integrate[x^2/(a + b*Sqrt[x])^8,x]
```

output
```
(-a^5 - 7*a^4*b*Sqrt[x] - 21*a^3*b^2*x - 35*a^2*b^3*x^(3/2) - 35*a*b^4*x^2
 - 21*b^5*x^(5/2))/(21*b^6*(a + b*Sqrt[x])^7)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{5/2}}{\left(a+b\sqrt{x}\right)^8}d\sqrt{x}$$

$$\Big\downarrow \text{55}$$

$$2\left(\frac{\int \frac{x^{5/2}}{\left(a+b\sqrt{x}\right)^7}d\sqrt{x}}{7a}+\frac{x^3}{7a\left(a+b\sqrt{x}\right)^7}\right)$$

$$\Big\downarrow \text{48}$$

$$2\left(\frac{x^3}{42a^2\left(a+b\sqrt{x}\right)^6}+\frac{x^3}{7a\left(a+b\sqrt{x}\right)^7}\right)$$

input
```
Int[x^2/(a + b*Sqrt[x])^8,x]
```

output
```
2*(x^3/(7*a*(a + b*Sqrt[x])^7) + x^3/(42*a^2*(a + b*Sqrt[x])^6))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(35) = 70$.

Time = 0.53 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

method	result
derivativedivides	$-\frac{5a^2}{b^6(a+b\sqrt{x})^4}-\frac{1}{b^6(a+b\sqrt{x})^2}+\frac{2a^5}{7b^6(a+b\sqrt{x})^7}-\frac{5a^4}{3b^6(a+b\sqrt{x})^6}+\frac{4a^3}{b^6(a+b\sqrt{x})^5}+\frac{10a}{3b^6(a+b\sqrt{x})^3}$
default	$-\frac{5a^2}{b^6(a+b\sqrt{x})^4}-\frac{1}{b^6(a+b\sqrt{x})^2}+\frac{2a^5}{7b^6(a+b\sqrt{x})^7}-\frac{5a^4}{3b^6(a+b\sqrt{x})^6}+\frac{4a^3}{b^6(a+b\sqrt{x})^5}+\frac{10a}{3b^6(a+b\sqrt{x})^3}$
trager	$(-1+x)(-a^{12}b^8x^6+7a^{10}b^{10}x^6-21a^8b^{12}x^6+42a^6b^{14}x^6+105a^4b^{16}x^6+231a^2b^{18}x^6+21b^{20}x^6+28a^{14}b^6x^5-197a^{12}b^8x$

input
```
int(x^2/(a+b*x^(1/2))^8,x,method=_RETURNVERBOSE)
```

output
```
-5/b^6*a^2/(a+b*x^(1/2))^4-1/b^6/(a+b*x^(1/2))^2+2/7*a^5/b^6/(a+b*x^(1/2))
^7-5/3*a^4/b^6/(a+b*x^(1/2))^6+4/b^6*a^3/(a+b*x^(1/2))^5+10/3*a/b^6/(a+b*x
^(1/2))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(35) = 70$.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 4.37

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{21\,b^{12}x^6+231\,a^2b^{10}x^5+105\,a^4b^8x^4+42\,a^6b^6x^3-21\,a^8b^4x^2+7\,a^{10}b^2x-a^{12}-16\left(7\,ab^{11}x^5+14\,a^3b^9\right.}{21\left(b^{20}x^7-7\,a^2b^{18}x^6+21\,a^4b^{16}x^5-35\,a^6b^{14}x^4+35\,a^8b^{12}x^3-21\,a^{10}b^{10}x^2+7\,a^{12}b^8x-a\right.}$$

input
```
integrate(x^2/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

output
```
-1/21*(21*b^12*x^6 + 231*a^2*b^10*x^5 + 105*a^4*b^8*x^4 + 42*a^6*b^6*x^3 -
 21*a^8*b^4*x^2 + 7*a^10*b^2*x - a^12 - 16*(7*a*b^11*x^5 + 14*a^3*b^9*x^4
+ 3*a^5*b^7*x^3)*sqrt(x))/(b^20*x^7 - 7*a^2*b^18*x^6 + 21*a^4*b^16*x^5 - 3
5*a^6*b^14*x^4 + 35*a^8*b^12*x^3 - 21*a^10*b^10*x^2 + 7*a^12*b^8*x - a^14*
b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(36) = 72$.

Time = 2.46 (sec) , antiderivative size = 619, normalized size of antiderivative = 14.40

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx = \begin{cases} -\frac{a^5}{21a^7b^6+147a^6b^7\sqrt{x}+441a^5b^8x+735a^4b^9x^{\frac{3}{2}}+735a^3b^{10}x^2+441a^2b^{11}x^{\frac{5}{2}}+147ab^{12}x^3+21b^{13}x^{\frac{7}{2}}} - \frac{}{21a^7b^6+147a^6b^7\sqrt{x}+441a^5b^8x+735a} \\ \frac{x^3}{3a^8} \end{cases}$$

input
```
integrate(x**2/(a+b*x**(1/2))**8,x)
```

output
```
Piecewise((-a**5/(21*a**7*b**6 + 147*a**6*b**7*sqrt(x) + 441*a**5*b**8*x +
 735*a**4*b**9*x**(3/2) + 735*a**3*b**10*x**2 + 441*a**2*b**11*x**(5/2) +
147*a*b**12*x**3 + 21*b**13*x**(7/2)) - 7*a**4*b*sqrt(x)/(21*a**7*b**6 + 1
47*a**6*b**7*sqrt(x) + 441*a**5*b**8*x + 735*a**4*b**9*x**(3/2) + 735*a**3
*b**10*x**2 + 441*a**2*b**11*x**(5/2) + 147*a*b**12*x**3 + 21*b**13*x**(7/
2)) - 21*a**3*b**2*x/(21*a**7*b**6 + 147*a**6*b**7*sqrt(x) + 441*a**5*b**8
*x + 735*a**4*b**9*x**(3/2) + 735*a**3*b**10*x**2 + 441*a**2*b**11*x**(5/2
) + 147*a*b**12*x**3 + 21*b**13*x**(7/2)) - 35*a**2*b**3*x**(3/2)/(21*a**7
*b**6 + 147*a**6*b**7*sqrt(x) + 441*a**5*b**8*x + 735*a**4*b**9*x**(3/2) +
 735*a**3*b**10*x**2 + 441*a**2*b**11*x**(5/2) + 147*a*b**12*x**3 + 21*b**
13*x**(7/2)) - 35*a*b**4*x**2/(21*a**7*b**6 + 147*a**6*b**7*sqrt(x) + 441*
a**5*b**8*x + 735*a**4*b**9*x**(3/2) + 735*a**3*b**10*x**2 + 441*a**2*b**1
1*x**(5/2) + 147*a*b**12*x**3 + 21*b**13*x**(7/2)) - 21*b**5*x**(5/2)/(21*
a**7*b**6 + 147*a**6*b**7*sqrt(x) + 441*a**5*b**8*x + 735*a**4*b**9*x**(3/
2) + 735*a**3*b**10*x**2 + 441*a**2*b**11*x**(5/2) + 147*a*b**12*x**3 + 21
*b**13*x**(7/2)), Ne(b, 0)), (x**3/(3*a**8), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{1}{\left(b\sqrt{x}+a\right)^2b^6} + \frac{10\,a}{3\left(b\sqrt{x}+a\right)^3b^6} - \frac{5\,a^2}{\left(b\sqrt{x}+a\right)^4b^6} + \frac{4\,a^3}{\left(b\sqrt{x}+a\right)^5b^6} - \frac{5\,a^4}{3\left(b\sqrt{x}+a\right)^6b^6} + \frac{2\,a^5}{7\left(b\sqrt{x}+a\right)^7b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

output
```
-1/((b*sqrt(x) + a)^2*b^6) + 10/3*a/((b*sqrt(x) + a)^3*b^6) - 5*a^2/((b*sq
rt(x) + a)^4*b^6) + 4*a^3/((b*sqrt(x) + a)^5*b^6) - 5/3*a^4/((b*sqrt(x) +
a)^6*b^6) + 2/7*a^5/((b*sqrt(x) + a)^7*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{21\,b^5x^{\frac{5}{2}} + 35\,ab^4x^2 + 35\,a^2b^3x^{\frac{3}{2}} + 21\,a^3b^2x + 7\,a^4b\sqrt{x} + a^5}{21\,\left(b\sqrt{x}+a\right)^7b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^8,x, algorithm="giac")
```

output
```
-1/21*(21*b^5*x^(5/2) + 35*a*b^4*x^2 + 35*a^2*b^3*x^(3/2) + 21*a^3*b^2*x +
 7*a^4*b*sqrt(x) + a^5)/((b*sqrt(x) + a)^7*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.02

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{\frac{a^5}{21\,b^6}+\frac{x^{5/2}}{b}+\frac{5\,a\,x^2}{3\,b^2}+\frac{a^3\,x}{b^4}+\frac{5\,a^2\,x^{3/2}}{3\,b^3}+\frac{a^4\,\sqrt{x}}{3\,b^5}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}$$

input
```
int(x^2/(a + b*x^(1/2))^8,x)
```

output
```
-(a^5/(21*b^6) + x^(5/2)/b + (5*a*x^2)/(3*b^2) + (a^3*x)/b^4 + (5*a^2*x^(3
/2))/(3*b^3) + (a^4*x^(1/2))/(3*b^5))/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x +
7*a*b^6*x^3 + 7*a^6*b*x^(1/2) + 35*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a
^2*b^5*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.12

$$\int \frac{x^2}{\left(a+b\sqrt{x}\right)^8}\,dx$$
$$=\frac{-7\sqrt{x}\,a^4b-35\sqrt{x}\,a^2b^3x-21\sqrt{x}\,b^5x^2-a^5-21a^3b^2x-35a\,b^4x^2}{21b^6\left(7\sqrt{x}\,a^6b+35\sqrt{x}\,a^4b^3x+21\sqrt{x}\,a^2b^5x^2+\sqrt{x}\,b^7x^3+a^7+21a^5b^2x+35a^3b^4x^2+7a\,b^6x^3\right)}$$

input
```
int(x^2/(a+b*x^(1/2))^8,x)
```

output
```
( - 7*sqrt(x)*a**4*b - 35*sqrt(x)*a**2*b**3*x - 21*sqrt(x)*b**5*x**2 - a**
5 - 21*a**3*b**2*x - 35*a*b**4*x**2)/(21*b**6*(7*sqrt(x)*a**6*b + 35*sqrt(
x)*a**4*b**3*x + 21*sqrt(x)*a**2*b**5*x**2 + sqrt(x)*b**7*x**3 + a**7 + 21
*a**5*b**2*x + 35*a**3*b**4*x**2 + 7*a*b**6*x**3))
```

3.117 $\int \frac{x}{\left(a+b\sqrt{x}\right)^8} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8} \, dx = \frac{2a^3}{7b^4\left(a+b\sqrt{x}\right)^7} - \frac{a^2}{b^4\left(a+b\sqrt{x}\right)^6} + \frac{6a}{5b^4\left(a+b\sqrt{x}\right)^5} - \frac{1}{2b^4\left(a+b\sqrt{x}\right)^4}$$

output
```
2/7*a^3/b^4/(a+b*x^(1/2))^7-a^2/b^4/(a+b*x^(1/2))^6+6/5*a/b^4/(a+b*x^(1/2)
)^5-1/2/b^4/(a+b*x^(1/2))^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8} \, dx = \frac{-a^3-7a^2b\sqrt{x}-21ab^2x-35b^3x^{3/2}}{70b^4\left(a+b\sqrt{x}\right)^7}$$

input
```
Integrate[x/(a + b*Sqrt[x])^8,x]
```

output
```
(-a^3 - 7*a^2*b*Sqrt[x] - 21*a*b^2*x - 35*b^3*x^(3/2))/(70*b^4*(a + b*Sqrt
[x])^7)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{3/2}}{\left(a+b\sqrt{x}\right)^8}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{a^3}{b^3\left(a+b\sqrt{x}\right)^8}+\frac{3a^2}{b^3\left(a+b\sqrt{x}\right)^7}-\frac{3a}{b^3\left(a+b\sqrt{x}\right)^6}+\frac{1}{b^3\left(a+b\sqrt{x}\right)^5}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^3}{7b^4\left(a+b\sqrt{x}\right)^7}-\frac{a^2}{2b^4\left(a+b\sqrt{x}\right)^6}+\frac{3a}{5b^4\left(a+b\sqrt{x}\right)^5}-\frac{1}{4b^4\left(a+b\sqrt{x}\right)^4}\right)$$

input
```
Int[x/(a + b*Sqrt[x])^8,x]
```

output
```
2*(a^3/(7*b^4*(a + b*Sqrt[x])^7) - a^2/(2*b^4*(a + b*Sqrt[x])^6) + (3*a)/(
5*b^4*(a + b*Sqrt[x])^5) - 1/(4*b^4*(a + b*Sqrt[x])^4))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2a^3}{7b^4(a+b\sqrt{x})^7}-\frac{a^2}{b^4(a+b\sqrt{x})^6}+\frac{6a}{5b^4(a+b\sqrt{x})^5}-\frac{1}{2b^4(a+b\sqrt{x})^4}$
default	$\frac{2a^3}{7b^4(a+b\sqrt{x})^7}-\frac{a^2}{b^4(a+b\sqrt{x})^6}+\frac{6a}{5b^4(a+b\sqrt{x})^5}-\frac{1}{2b^4(a+b\sqrt{x})^4}$
trager	$\frac{(-1+x)(-a^{10}b^{10}x^6+7a^8b^{12}x^6+14a^6b^{14}x^6+630a^4b^{16}x^6+595a^2b^{18}x^6+35b^{20}x^6+7a^{12}b^8x^5-50a^{10}b^{10}x^5-91a^8b^{12}x^5}{}$

input
```
int(x/(a+b*x^(1/2))^8,x,method=_RETURNVERBOSE)
```

output
```
2/7*a^3/b^4/(a+b*x^(1/2))^7-a^2/b^4/(a+b*x^(1/2))^6+6/5*a/b^4/(a+b*x^(1/2)
)^5-1/2/b^4/(a+b*x^(1/2))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.27

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{35\,b^{10}x^5 + 595\,a^2b^8x^4 + 630\,a^4b^6x^3 + 14\,a^6b^4x^2 + 7\,a^8b^2x - a^{10} - 32\,(7\,ab^9x^4 + 26\,a^3b^7x^3 + 7\,a^5b^5x^2)\sqrt{}}{70\,(b^{18}x^7 - 7\,a^2b^{16}x^6 + 21\,a^4b^{14}x^5 - 35\,a^6b^{12}x^4 + 35\,a^8b^{10}x^3 - 21\,a^{10}b^8x^2 + 7\,a^{12}b^6x - a^{14}b^4)}$$

input
```
integrate(x/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

output
```
-1/70*(35*b^10*x^5 + 595*a^2*b^8*x^4 + 630*a^4*b^6*x^3 + 14*a^6*b^4*x^2 +
7*a^8*b^2*x - a^10 - 32*(7*a*b^9*x^4 + 26*a^3*b^7*x^3 + 7*a^5*b^5*x^2)*sqr
t(x))/(b^18*x^7 - 7*a^2*b^16*x^6 + 21*a^4*b^14*x^5 - 35*a^6*b^12*x^4 + 35*
a^8*b^10*x^3 - 21*a^10*b^8*x^2 + 7*a^12*b^6*x - a^14*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(71) = 142$.

Time = 2.84 (sec) , antiderivative size = 410, normalized size of antiderivative = 5.26

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx = \begin{cases} -\frac{a^3}{70a^7b^4+490a^6b^5\sqrt{x}+1470a^5b^6x+2450a^4b^7x^{\frac{3}{2}}+2450a^3b^8x^2+1470a^2b^9x^{\frac{5}{2}}+490ab^{10}x^3+70b^{11}x^{\frac{7}{2}}} - \frac{}{70a^7b^4+490a^6b^5\sqrt{x}+1470a^5b^6x+2} \\ \frac{x^2}{2a^8} \end{cases}$$

input
```
integrate(x/(a+b*x**(1/2))**8,x)
```

output
```
Piecewise((-a**3/(70*a**7*b**4 + 490*a**6*b**5*sqrt(x) + 1470*a**5*b**6*x
+ 2450*a**4*b**7*x**(3/2) + 2450*a**3*b**8*x**2 + 1470*a**2*b**9*x**(5/2)
+ 490*a*b**10*x**3 + 70*b**11*x**(7/2)) - 7*a**2*b*sqrt(x)/(70*a**7*b**4 +
 490*a**6*b**5*sqrt(x) + 1470*a**5*b**6*x + 2450*a**4*b**7*x**(3/2) + 2450
*a**3*b**8*x**2 + 1470*a**2*b**9*x**(5/2) + 490*a*b**10*x**3 + 70*b**11*x*
*(7/2)) - 21*a*b**2*x/(70*a**7*b**4 + 490*a**6*b**5*sqrt(x) + 1470*a**5*b*
*6*x + 2450*a**4*b**7*x**(3/2) + 2450*a**3*b**8*x**2 + 1470*a**2*b**9*x**(
5/2) + 490*a*b**10*x**3 + 70*b**11*x**(7/2)) - 35*b**3*x**(3/2)/(70*a**7*b
**4 + 490*a**6*b**5*sqrt(x) + 1470*a**5*b**6*x + 2450*a**4*b**7*x**(3/2) +
 2450*a**3*b**8*x**2 + 1470*a**2*b**9*x**(5/2) + 490*a*b**10*x**3 + 70*b**
11*x**(7/2)), Ne(b, 0)), (x**2/(2*a**8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{1}{2\left(b\sqrt{x}+a\right)^4b^4}+\frac{6\,a}{5\left(b\sqrt{x}+a\right)^5b^4}-\frac{a^2}{\left(b\sqrt{x}+a\right)^6b^4}+\frac{2\,a^3}{7\left(b\sqrt{x}+a\right)^7b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

output
```
-1/2/((b*sqrt(x) + a)^4*b^4) + 6/5*a/((b*sqrt(x) + a)^5*b^4) - a^2/((b*sqr
t(x) + a)^6*b^4) + 2/7*a^3/((b*sqrt(x) + a)^7*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{35\,b^3x^{\frac{3}{2}}+21\,ab^2x+7\,a^2b\sqrt{x}+a^3}{70\left(b\sqrt{x}+a\right)^7b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^8,x, algorithm="giac")
```

output
```
-1/70*(35*b^3*x^(3/2) + 21*a*b^2*x + 7*a^2*b*sqrt(x) + a^3)/((b*sqrt(x) +
a)^7*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{\frac{a^3}{70\,b^4}+\frac{x^{3/2}}{2\,b}+\frac{a^2\,\sqrt{x}}{10\,b^3}+\frac{3\,a\,x}{10\,b^2}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}$$

input
```
int(x/(a + b*x^(1/2))^8,x)
```

output
```
-(a^3/(70*b^4) + x^(3/2)/(2*b) + (a^2*x^(1/2))/(10*b^3) + (3*a*x)/(10*b^2)
)/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*x^3 + 7*a^6*b*x^(1/2) + 35*a
^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.41

$$\int \frac{x}{\left(a+b\sqrt{x}\right)^8}\,dx = \frac{-7\sqrt{x}\,a^2b-35\sqrt{x}\,b^3x-a^3-21a\,b^2x}{70b^4\left(7\sqrt{x}\,a^6b+35\sqrt{x}\,a^4b^3x+21\sqrt{x}\,a^2b^5x^2+\sqrt{x}\,b^7x^3+a^7+21a^5b^2x+35a^3b^4x^2+7a\,b^6x^3\right)}$$

input
```
int(x/(a+b*x^(1/2))^8,x)
```

output
```
( - 7*sqrt(x)*a**2*b - 35*sqrt(x)*b**3*x - a**3 - 21*a*b**2*x)/(70*b**4*(7
*sqrt(x)*a**6*b + 35*sqrt(x)*a**4*b**3*x + 21*sqrt(x)*a**2*b**5*x**2 + sqr
t(x)*b**7*x**3 + a**7 + 21*a**5*b**2*x + 35*a**3*b**4*x**2 + 7*a*b**6*x**3
))
```

3.118 $\int \frac{1}{(a+b\sqrt{x})^8} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{1}{(a+b\sqrt{x})^8} \, dx = \frac{2a}{7b^2(a+b\sqrt{x})^7} - \frac{1}{3b^2(a+b\sqrt{x})^6}$$

output
```
2/7*a/b^2/(a+b*x^(1/2))^7-1/3/b^2/(a+b*x^(1/2))^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+b\sqrt{x})^8} \, dx = \frac{-a-7b\sqrt{x}}{21b^2(a+b\sqrt{x})^7}$$

input
```
Integrate[(a + b*Sqrt[x])^(-8),x]
```

output
```
(-a - 7*b*Sqrt[x])/(21*b^2*(a + b*Sqrt[x])^7)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8}dx$$

$$\Big\downarrow \text{774}$$

$$2\int \frac{\sqrt{x}}{\left(a+b\sqrt{x}\right)^8}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{1}{b\left(a+b\sqrt{x}\right)^7}-\frac{a}{b\left(a+b\sqrt{x}\right)^8}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a}{7b^2\left(a+b\sqrt{x}\right)^7}-\frac{1}{6b^2\left(a+b\sqrt{x}\right)^6}\right)$$

input
```
Int[(a + b*Sqrt[x])^(-8),x]
```

output
```
2*(a/(7*b^2*(a + b*Sqrt[x])^7) - 1/(6*b^2*(a + b*Sqrt[x])^6))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{2a}{7b^2(a+b\sqrt{x})^7}-\frac{1}{3b^2(a+b\sqrt{x})^6}$
default	$28a^2b^6\left(-\frac{3a^2}{5b^8(b^2x-a^2)^5}-\frac{a^4}{2b^8(b^2x-a^2)^6}-\frac{a^6}{7b^8(b^2x-a^2)^7}-\frac{1}{4(b^2x-a^2)^4b^8}\right)+\frac{a^8}{7(-b^2x+a^2)^7b^2}+\frac{}{6b^2(b}$
trager	$\frac{(-1+x)(-a^8b^{12}x^6+28a^6b^{14}x^6+210a^4b^{16}x^6+140a^2b^{18}x^6+7b^{20}x^6+7a^{10}b^{10}x^5-197a^8b^{12}x^5-1442a^6b^{14}x^5-770a^4b^{1}}{}$

input
```
int(1/(a+b*x^(1/2))^8,x,method=_RETURNVERBOSE)
```

output
```
2/7*a/b^2/(a+b*x^(1/2))^7-1/3/b^2/(a+b*x^(1/2))^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(30) = 60.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{7\,b^8x^4+140\,a^2b^6x^3+210\,a^4b^4x^2+28\,a^6b^2x-a^8-16\left(3\,ab^7x^3+14\,a^3b^5x^2+7\,a^5b^3x\right)\sqrt{x}}{21\left(b^{16}x^7-7\,a^2b^{14}x^6+21\,a^4b^{12}x^5-35\,a^6b^{10}x^4+35\,a^8b^8x^3-21\,a^{10}b^6x^2+7\,a^{12}b^4x-a^{14}b^2\right)}$$

input
```
integrate(1/(a+b*x^(1/2))^8,x, algorithm="fricas")
```

output
```
-1/21*(7*b^8*x^4 + 140*a^2*b^6*x^3 + 210*a^4*b^4*x^2 + 28*a^6*b^2*x - a^8
- 16*(3*a*b^7*x^3 + 14*a^3*b^5*x^2 + 7*a^5*b^3*x)*sqrt(x))/(b^16*x^7 - 7*a
^2*b^14*x^6 + 21*a^4*b^12*x^5 - 35*a^6*b^10*x^4 + 35*a^8*b^8*x^3 - 21*a^10
*b^6*x^2 + 7*a^12*b^4*x - a^14*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(34) = 68$.

Time = 2.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 5.24

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx = \begin{cases} -\frac{a}{21a^7b^2+147a^6b^3\sqrt{x}+441a^5b^4x+735a^4b^5x^{\frac{3}{2}}+735a^3b^6x^2+441a^2b^7x^{\frac{5}{2}}+147ab^8x^3+21b^9x^{\frac{7}{2}}} - \frac{}{21a^7b^2+147a^6b^3\sqrt{x}+441a^5b^4x+735a^4b^5 \cdots} \\ \frac{x}{a^8} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/2))**8,x)
```

output
```
Piecewise((-a/(21*a**7*b**2 + 147*a**6*b**3*sqrt(x) + 441*a**5*b**4*x + 73
5*a**4*b**5*x**(3/2) + 735*a**3*b**6*x**2 + 441*a**2*b**7*x**(5/2) + 147*a
*b**8*x**3 + 21*b**9*x**(7/2)) - 7*b*sqrt(x)/(21*a**7*b**2 + 147*a**6*b**3
*sqrt(x) + 441*a**5*b**4*x + 735*a**4*b**5*x**(3/2) + 735*a**3*b**6*x**2 +
 441*a**2*b**7*x**(5/2) + 147*a*b**8*x**3 + 21*b**9*x**(7/2)), Ne(b, 0)),
(x/a**8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx = -\frac{1}{3\left(b\sqrt{x}+a\right)^6b^2} + \frac{2\,a}{7\left(b\sqrt{x}+a\right)^7b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^8,x, algorithm="maxima")
```

output
```
-1/3/((b*sqrt(x) + a)^6*b^2) + 2/7*a/((b*sqrt(x) + a)^7*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx=-\frac{7\,b\sqrt{x}+a}{21\,\left(b\sqrt{x}+a\right)^7b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^8,x, algorithm="giac")
```

output
```
-1/21*(7*b*sqrt(x) + a)/((b*sqrt(x) + a)^7*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.37

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx=$$

$$-\frac{\frac{a}{21\,b^2}+\frac{\sqrt{x}}{3\,b}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}}$$

input
```
int(1/(a + b*x^(1/2))^8,x)
```

output
```
-(a/(21*b^2) + x^(1/2)/(3*b))/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*
x^3 + 7*a^6*b*x^(1/2) + 35*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x
^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.37

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8}\,dx$$
$$= \frac{-7\sqrt{x}\,b-a}{21b^2\left(7\sqrt{x}\,a^6b+35\sqrt{x}\,a^4b^3x+21\sqrt{x}\,a^2b^5x^2+\sqrt{x}\,b^7x^3+a^7+21a^5b^2x+35a^3b^4x^2+7a\,b^6x^3\right)}$$

input
```
int(1/(a+b*x^(1/2))^8,x)
```

output
```
( - 7*sqrt(x)*b - a)/(21*b**2*(7*sqrt(x)*a**6*b + 35*sqrt(x)*a**4*b**3*x +
 21*sqrt(x)*a**2*b**5*x**2 + sqrt(x)*b**7*x**3 + a**7 + 21*a**5*b**2*x + 3
5*a**3*b**4*x**2 + 7*a*b**6*x**3))
```

3.119 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 143

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx = \frac{2}{7a\left(a+b\sqrt{x}\right)^7}+\frac{1}{3a^2\left(a+b\sqrt{x}\right)^6}+\frac{2}{5a^3\left(a+b\sqrt{x}\right)^5}+\frac{1}{2a^4\left(a+b\sqrt{x}\right)^4}+\frac{2}{3a^5\left(a+b\sqrt{x}\right)^3}+\frac{1}{a^6\left(a+b\sqrt{x}\right)^2}+\frac{2}{a^7\left(a+b\sqrt{x}\right)}-\frac{2\log\left(a+b\sqrt{x}\right)}{a^8}+\frac{\log(x)}{a^8}$$

output
```
2/7/a/(a+b*x^(1/2))^7+1/3/a^2/(a+b*x^(1/2))^6+2/5/a^3/(a+b*x^(1/2))^5+1/2/
a^4/(a+b*x^(1/2))^4+2/3/a^5/(a+b*x^(1/2))^3+1/a^6/(a+b*x^(1/2))^2+2/a^7/(a
+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^8+ln(x)/a^8
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx = \frac{\frac{a\left(1089a^6+4683a^5b\sqrt{x}+9639a^4b^2x+11165a^3b^3x^{3/2}+7490a^2b^4x^2+2730ab^5x^{5/2}+420b^6x^3\right)}{\left(a+b\sqrt{x}\right)^7}-420\log\left(a+b\sqrt{x}\right)+210\log(x)}{210a^8}$$

input
```
Integrate[1/((a + b*Sqrt[x])^8*x),x]
```

output
```
((a*(1089*a^6 + 4683*a^5*b*Sqrt[x] + 9639*a^4*b^2*x + 11165*a^3*b^3*x^(3/2
) + 7490*a^2*b^4*x^2 + 2730*a*b^5*x^(5/2) + 420*b^6*x^3))/(a + b*Sqrt[x])^
7 - 420*Log[a + b*Sqrt[x]] + 210*Log[x])/(210*a^8)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{x}\right)^8}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^8\sqrt{x}}\,d\sqrt{x}$$

$$\Big\downarrow \text{54}$$

$$2\int \left(-\frac{b}{a^8\left(a+b\sqrt{x}\right)}-\frac{b}{a^7\left(a+b\sqrt{x}\right)^2}-\frac{b}{a^6\left(a+b\sqrt{x}\right)^3}-\frac{b}{a^5\left(a+b\sqrt{x}\right)^4}-\frac{b}{a^4\left(a+b\sqrt{x}\right)^5}-\frac{b}{a^3\left(a+b\sqrt{x}\right)^6}-\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{\log\left(a+b\sqrt{x}\right)}{a^8}+\frac{\log\left(\sqrt{x}\right)}{a^8}+\frac{1}{a^7\left(a+b\sqrt{x}\right)}+\frac{1}{2a^6\left(a+b\sqrt{x}\right)^2}+\frac{1}{3a^5\left(a+b\sqrt{x}\right)^3}+\frac{1}{4a^4\left(a+b\sqrt{x}\right)^4}+\frac{}{5a^3}\right.$$

input
```
Int[1/((a + b*Sqrt[x])^8*x),x]
```

output
```
2*(1/(7*a*(a + b*Sqrt[x])^7) + 1/(6*a^2*(a + b*Sqrt[x])^6) + 1/(5*a^3*(a +
 b*Sqrt[x])^5) + 1/(4*a^4*(a + b*Sqrt[x])^4) + 1/(3*a^5*(a + b*Sqrt[x])^3)
 + 1/(2*a^6*(a + b*Sqrt[x])^2) + 1/(a^7*(a + b*Sqrt[x])) - Log[a + b*Sqrt[
x]]/a^8 + Log[Sqrt[x]]/a^8)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2}{7a\left(a+b\sqrt{x}\right)^7}+\frac{1}{3a^2\left(a+b\sqrt{x}\right)^6}+\frac{2}{5a^3\left(a+b\sqrt{x}\right)^5}+\frac{1}{2a^4\left(a+b\sqrt{x}\right)^4}+\frac{2}{3a^5\left(a+b\sqrt{x}\right)^3}+\frac{1}{a^6\left(a+b\sqrt{x}\right)^2}+\frac{}{a^7(a-}$
default	$\frac{2}{7a\left(a+b\sqrt{x}\right)^7}+\frac{1}{3a^2\left(a+b\sqrt{x}\right)^6}+\frac{2}{5a^3\left(a+b\sqrt{x}\right)^5}+\frac{1}{2a^4\left(a+b\sqrt{x}\right)^4}+\frac{2}{3a^5\left(a+b\sqrt{x}\right)^3}+\frac{1}{a^6\left(a+b\sqrt{x}\right)^2}+\frac{}{a^7(a-}$

input
```
int(1/(a+b*x^(1/2))^8/x,x,method=_RETURNVERBOSE)
```

output
```
2/7/a/(a+b*x^(1/2))^7+1/3/a^2/(a+b*x^(1/2))^6+2/5/a^3/(a+b*x^(1/2))^5+1/2/
a^4/(a+b*x^(1/2))^4+2/3/a^5/(a+b*x^(1/2))^3+1/a^6/(a+b*x^(1/2))^2+2/a^7/(a
+b*x^(1/2))-2*ln(a+b*x^(1/2))/a^8+ln(x)/a^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.78

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx = -\frac{210\,a^2b^{12}x^6 - 1365\,a^4b^{10}x^5 + 3745\,a^6b^8x^4 - 5530\,a^8b^6x^3 + 5964\,a^{10}b^4x^2 - 273\,a^{12}b^2x + 1089\,a^{14} + 420}{}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x,x, algorithm="fricas")
```

output
```
-1/210*(210*a^2*b^12*x^6 - 1365*a^4*b^10*x^5 + 3745*a^6*b^8*x^4 - 5530*a^8
*b^6*x^3 + 5964*a^10*b^4*x^2 - 273*a^12*b^2*x + 1089*a^14 + 420*(b^14*x^7
- 7*a^2*b^12*x^6 + 21*a^4*b^10*x^5 - 35*a^6*b^8*x^4 + 35*a^8*b^6*x^3 - 21*
a^10*b^4*x^2 + 7*a^12*b^2*x - a^14)*log(b*sqrt(x) + a) - 420*(b^14*x^7 - 7
*a^2*b^12*x^6 + 21*a^4*b^10*x^5 - 35*a^6*b^8*x^4 + 35*a^8*b^6*x^3 - 21*a^1
0*b^4*x^2 + 7*a^12*b^2*x - a^14)*log(sqrt(x)) - 4*(105*a*b^13*x^6 - 700*a^
3*b^11*x^5 + 1981*a^5*b^9*x^4 - 3072*a^7*b^7*x^3 + 2891*a^9*b^5*x^2 - 980*
a^11*b^3*x + 735*a^13*b)*sqrt(x))/(a^8*b^14*x^7 - 7*a^10*b^12*x^6 + 21*a^1
2*b^10*x^5 - 35*a^14*b^8*x^4 + 35*a^16*b^6*x^3 - 21*a^18*b^4*x^2 + 7*a^20*
b^2*x - a^22)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2581 vs. 2(133) = 266.

Time = 3.50 (sec) , antiderivative size = 2581, normalized size of antiderivative = 18.05

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2))**8/x,x)
```

output
```
Piecewise((zoo/x**4, Eq(a, 0) & Eq(b, 0)), (log(x)/a**8, Eq(b, 0)), (-1/(4
*b**8*x**4), Eq(a, 0)), (210*a**7*sqrt(x)*log(x)/(210*a**15*sqrt(x) + 1470
*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*
b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*
b**7*x**4) - 420*a**7*sqrt(x)*log(a/b + sqrt(x))/(210*a**15*sqrt(x) + 1470
*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*
b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*
b**7*x**4) + 1089*a**7*sqrt(x)/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*
a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 44
10*a**10*b**5*x**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) + 1470*
a**6*b*x*log(x)/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(
3/2) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x
**3 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) - 2940*a**6*b*x*log(a/
b + sqrt(x))/(210*a**15*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2
) + 7350*a**12*b**3*x**2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3
 + 1470*a**9*b**6*x**(7/2) + 210*a**8*b**7*x**4) + 4683*a**6*b*x/(210*a**1
5*sqrt(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x*
*2 + 7350*a**11*b**4*x**(5/2) + 4410*a**10*b**5*x**3 + 1470*a**9*b**6*x**(
7/2) + 210*a**8*b**7*x**4) + 4410*a**5*b**2*x**(3/2)*log(x)/(210*a**15*sqr
t(x) + 1470*a**14*b*x + 4410*a**13*b**2*x**(3/2) + 7350*a**12*b**3*x**2...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx = \frac{420\,b^6x^3+2730\,ab^5x^{\frac{5}{2}}+7490\,a^2b^4x^2+11165\,a^3b^3x^{\frac{3}{2}}+9639\,a^4b^2x+4683\,a^5b\sqrt{x}+1089\,a^6}{210\,\left(a^7b^7x^{\frac{7}{2}}+7\,a^8b^6x^3+21\,a^9b^5x^{\frac{5}{2}}+35\,a^{10}b^4x^2+35\,a^{11}b^3x^{\frac{3}{2}}+21\,a^{12}b^2x+7\,a^{13}b\sqrt{x}+a^{14}\right)} - \frac{2\,\log\left(b\sqrt{x}+a\right)}{a^8} + \frac{\log\left(x\right)}{a^8}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x,x, algorithm="maxima")
```

output
```
1/210*(420*b^6*x^3 + 2730*a*b^5*x^(5/2) + 7490*a^2*b^4*x^2 + 11165*a^3*b^3
*x^(3/2) + 9639*a^4*b^2*x + 4683*a^5*b*sqrt(x) + 1089*a^6)/(a^7*b^7*x^(7/2
) + 7*a^8*b^6*x^3 + 21*a^9*b^5*x^(5/2) + 35*a^10*b^4*x^2 + 35*a^11*b^3*x^(
3/2) + 21*a^12*b^2*x + 7*a^13*b*sqrt(x) + a^14) - 2*log(b*sqrt(x) + a)/a^8
 + log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\,dx = -\frac{2\,\log\left(\left|b\sqrt{x}+a\right|\right)}{a^8} + \frac{\log\left(\left|x\right|\right)}{a^8} + \frac{420\,ab^6x^3+2730\,a^2b^5x^{\frac{5}{2}}+7490\,a^3b^4x^2+11165\,a^4b^3x^{\frac{3}{2}}+9639\,a^5b^2x+4683\,a^6b\sqrt{x}+1089\,a^7}{210\,\left(b\sqrt{x}+a\right)^7a^8}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x,x, algorithm="giac")
```

output
```
-2*log(abs(b*sqrt(x) + a))/a^8 + log(abs(x))/a^8 + 1/210*(420*a*b^6*x^3 +
2730*a^2*b^5*x^(5/2) + 7490*a^3*b^4*x^2 + 11165*a^4*b^3*x^(3/2) + 9639*a^5
*b^2*x + 4683*a^6*b*sqrt(x) + 1089*a^7)/((b*sqrt(x) + a)^7*a^8)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\, dx = \frac{\frac{363}{70\,a}+\frac{223\,b\,\sqrt{x}}{10\,a^2}+\frac{459\,b^2\,x}{10\,a^3}+\frac{107\,b^4\,x^2}{3\,a^5}+\frac{319\,b^3\,x^{3/2}}{6\,a^4}+\frac{2\,b^6\,x^3}{a^7}+\frac{13\,b^5\,x^{5/2}}{a^6}}{a^7+b^7\,x^{7/2}+21\,a^5\,b^2\,x+7\,a\,b^6\,x^3+7\,a^6\,b\,\sqrt{x}+35\,a^3\,b^4\,x^2+35\,a^4\,b^3\,x^{3/2}+21\,a^2\,b^5\,x^{5/2}} - \frac{4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^8}$$

input
```
int(1/(x*(a + b*x^(1/2))^8),x)
```

output
```
(363/(70*a) + (223*b*x^(1/2))/(10*a^2) + (459*b^2*x)/(10*a^3) + (107*b^4*x
^2)/(3*a^5) + (319*b^3*x^(3/2))/(6*a^4) + (2*b^6*x^3)/a^7 + (13*b^5*x^(5/2
))/a^6)/(a^7 + b^7*x^(7/2) + 21*a^5*b^2*x + 7*a*b^6*x^3 + 7*a^6*b*x^(1/2)
+ 35*a^3*b^4*x^2 + 35*a^4*b^3*x^(3/2) + 21*a^2*b^5*x^(5/2)) - (4*atanh((2*
b*x^(1/2))/a + 1))/a^8
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.66

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x}\, dx = \frac{-2940\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^6b-14700\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^4b^3x-8820\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^2b^5x^2-420\sqrt{x}}{}$$

input
```
int(1/(a+b*x^(1/2))^8/x,x)
```

output

```
( - 2940*sqrt(x)*log(sqrt(x)*b + a)*a**6*b - 14700*sqrt(x)*log(sqrt(x)*b +
 a)*a**4*b**3*x - 8820*sqrt(x)*log(sqrt(x)*b + a)*a**2*b**5*x**2 - 420*sqr
t(x)*log(sqrt(x)*b + a)*b**7*x**3 + 2940*sqrt(x)*log(sqrt(x))*a**6*b + 147
00*sqrt(x)*log(sqrt(x))*a**4*b**3*x + 8820*sqrt(x)*log(sqrt(x))*a**2*b**5*
x**2 + 420*sqrt(x)*log(sqrt(x))*b**7*x**3 + 4263*sqrt(x)*a**6*b + 9065*sqr
t(x)*a**4*b**3*x + 1470*sqrt(x)*a**2*b**5*x**2 - 60*sqrt(x)*b**7*x**3 - 42
0*log(sqrt(x)*b + a)*a**7 - 8820*log(sqrt(x)*b + a)*a**5*b**2*x - 14700*lo
g(sqrt(x)*b + a)*a**3*b**4*x**2 - 2940*log(sqrt(x)*b + a)*a*b**6*x**3 + 42
0*log(sqrt(x))*a**7 + 8820*log(sqrt(x))*a**5*b**2*x + 14700*log(sqrt(x))*a
**3*b**4*x**2 + 2940*log(sqrt(x))*a*b**6*x**3 + 1029*a**7 + 8379*a**5*b**2
*x + 5390*a**3*b**4*x**2)/(210*a**8*(7*sqrt(x)*a**6*b + 35*sqrt(x)*a**4*b*
*3*x + 21*sqrt(x)*a**2*b**5*x**2 + sqrt(x)*b**7*x**3 + a**7 + 21*a**5*b**2
*x + 35*a**3*b**4*x**2 + 7*a*b**6*x**3))
```

3.120 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 184

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2} \, dx = {} & \frac{2b^2}{7a^3\left(a+b\sqrt{x}\right)^7}+\frac{b^2}{a^4\left(a+b\sqrt{x}\right)^6}+\frac{12b^2}{5a^5\left(a+b\sqrt{x}\right)^5} \\ & +\frac{5b^2}{a^6\left(a+b\sqrt{x}\right)^4}+\frac{10b^2}{a^7\left(a+b\sqrt{x}\right)^3} \\ & +\frac{21b^2}{a^8\left(a+b\sqrt{x}\right)^2}+\frac{56b^2}{a^9\left(a+b\sqrt{x}\right)}-\frac{1}{a^8x} \\ & +\frac{16b}{a^9\sqrt{x}}-\frac{72b^2\log\left(a+b\sqrt{x}\right)}{a^{10}}+\frac{36b^2\log(x)}{a^{10}}\end{aligned}$$

output
```
2/7*b^2/a^3/(a+b*x^(1/2))^7+b^2/a^4/(a+b*x^(1/2))^6+12/5*b^2/a^5/(a+b*x^(1
/2))^5+5*b^2/a^6/(a+b*x^(1/2))^4+10*b^2/a^7/(a+b*x^(1/2))^3+21*b^2/a^8/(a+
b*x^(1/2))^2+56*b^2/a^9/(a+b*x^(1/2))-1/a^8/x+16*b/a^9/x^(1/2)-72*b^2*ln(a
+b*x^(1/2))/a^10+36*b^2*ln(x)/a^10
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx$$
$$= \frac{\frac{a\left(-35a^8+315a^7b\sqrt{x}+6534a^6b^2x+28098a^5b^3x^{3/2}+57834a^4b^4x^2+66990a^3b^5x^{5/2}+44940a^2b^6x^3+16380ab^7x^{7/2}+2520b^8x^4\right)}{\left(a+b\sqrt{x}\right)^7 x} - 2520b^2 \log}{35a^{10}}$$

input
```
Integrate[1/((a + b*Sqrt[x])^8*x^2),x]
```

output
```
((a*(-35*a^8 + 315*a^7*b*Sqrt[x] + 6534*a^6*b^2*x + 28098*a^5*b^3*x^(3/2)
+ 57834*a^4*b^4*x^2 + 66990*a^3*b^5*x^(5/2) + 44940*a^2*b^6*x^3 + 16380*a*
b^7*x^(7/2) + 2520*b^8*x^4))/((a + b*Sqrt[x])^7*x) - 2520*b^2*Log[a + b*Sq
rt[x]] + 1260*b^2*Log[x])/(35*a^10)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\sqrt{x}\right)^8}dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^{3/2}}d\sqrt{x}$$
$$\Big\downarrow \text{54}$$
$$2\int \left(-\frac{36b^3}{a^{10}\left(a+b\sqrt{x}\right)}-\frac{28b^3}{a^9\left(a+b\sqrt{x}\right)^2}-\frac{21b^3}{a^8\left(a+b\sqrt{x}\right)^3}-\frac{15b^3}{a^7\left(a+b\sqrt{x}\right)^4}-\frac{10b^3}{a^6\left(a+b\sqrt{x}\right)^5}-\frac{6b^3}{a^5\left(a+b\sqrt{x}\right)^6}-\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{36b^2\log\left(a+b\sqrt{x}\right)}{a^{10}}+\frac{36b^2\log\left(\sqrt{x}\right)}{a^{10}}+\frac{28b^2}{a^9\left(a+b\sqrt{x}\right)}+\frac{8b}{a^9\sqrt{x}}+\frac{21b^2}{2a^8\left(a+b\sqrt{x}\right)^2}-\frac{1}{2a^8x}+\frac{5b^2}{a^7\left(a+b\sqrt{x}\right)^3}\right.$$

input
```
Int[1/((a + b*Sqrt[x])^8*x^2),x]
```

output
```
2*(b^2/(7*a^3*(a + b*Sqrt[x])^7) + b^2/(2*a^4*(a + b*Sqrt[x])^6) + (6*b^2)
/(5*a^5*(a + b*Sqrt[x])^5) + (5*b^2)/(2*a^6*(a + b*Sqrt[x])^4) + (5*b^2)/(
a^7*(a + b*Sqrt[x])^3) + (21*b^2)/(2*a^8*(a + b*Sqrt[x])^2) + (28*b^2)/(a^
9*(a + b*Sqrt[x])) - 1/(2*a^8*x) + (8*b)/(a^9*Sqrt[x]) - (36*b^2*Log[a + b
*Sqrt[x]])/a^10 + (36*b^2*Log[Sqrt[x]])/a^10)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^2}{7a^3\left(a+b\sqrt{x}\right)^7}+\frac{b^2}{a^4\left(a+b\sqrt{x}\right)^6}+\frac{12b^2}{5a^5\left(a+b\sqrt{x}\right)^5}+\frac{5b^2}{a^6\left(a+b\sqrt{x}\right)^4}+\frac{10b^2}{a^7\left(a+b\sqrt{x}\right)^3}+\frac{21b^2}{a^8\left(a+b\sqrt{x}\right)^2}+\frac{56b^2}{a^9\left(a+b\right.}$
default	$\frac{2b^2}{7a^3\left(a+b\sqrt{x}\right)^7}+\frac{b^2}{a^4\left(a+b\sqrt{x}\right)^6}+\frac{12b^2}{5a^5\left(a+b\sqrt{x}\right)^5}+\frac{5b^2}{a^6\left(a+b\sqrt{x}\right)^4}+\frac{10b^2}{a^7\left(a+b\sqrt{x}\right)^3}+\frac{21b^2}{a^8\left(a+b\sqrt{x}\right)^2}+\frac{56b^2}{a^9\left(a+b\right.}$

input
```
int(1/(a+b*x^(1/2))^8/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*b^2/a^3/(a+b*x^(1/2))^7+b^2/a^4/(a+b*x^(1/2))^6+12/5*b^2/a^5/(a+b*x^(1
/2))^5+5*b^2/a^6/(a+b*x^(1/2))^4+10*b^2/a^7/(a+b*x^(1/2))^3+21*b^2/a^8/(a+
b*x^(1/2))^2+56*b^2/a^9/(a+b*x^(1/2))-1/a^8/x+16*b/a^9/x^(1/2)-72*b^2*ln(a
+b*x^(1/2))/a^10+36*b^2*ln(x)/a^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(162) = 324$.

Time = 0.17 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.36

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx = -\frac{1260\,a^2b^{14}x^7 - 8190\,a^4b^{12}x^6 + 22470\,a^6b^{10}x^5 - 33495\,a^8b^8x^4 + 28924\,a^{10}b^6x^3 - 13888\,a^{12}b^4x^2 + 3594}{}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x^2,x, algorithm="fricas")
```

output
```
-1/35*(1260*a^2*b^14*x^7 - 8190*a^4*b^12*x^6 + 22470*a^6*b^10*x^5 - 33495*
a^8*b^8*x^4 + 28924*a^10*b^6*x^3 - 13888*a^12*b^4*x^2 + 3594*a^14*b^2*x -
35*a^16 + 2520*(b^16*x^8 - 7*a^2*b^14*x^7 + 21*a^4*b^12*x^6 - 35*a^6*b^10*
x^5 + 35*a^8*b^8*x^4 - 21*a^10*b^6*x^3 + 7*a^12*b^4*x^2 - a^14*b^2*x)*log(
b*sqrt(x) + a) - 2520*(b^16*x^8 - 7*a^2*b^14*x^7 + 21*a^4*b^12*x^6 - 35*a^
6*b^10*x^5 + 35*a^8*b^8*x^4 - 21*a^10*b^6*x^3 + 7*a^12*b^4*x^2 - a^14*b^2*
x)*log(sqrt(x)) - 8*(315*a*b^15*x^7 - 2100*a^3*b^13*x^6 + 5943*a^5*b^11*x^
5 - 9216*a^7*b^9*x^4 + 8393*a^9*b^7*x^3 - 4410*a^11*b^5*x^2 + 1225*a^13*b^
3*x - 70*a^15*b)*sqrt(x))/(a^10*b^14*x^8 - 7*a^12*b^12*x^7 + 21*a^14*b^10*
x^6 - 35*a^16*b^8*x^5 + 35*a^18*b^6*x^4 - 21*a^20*b^4*x^3 + 7*a^22*b^2*x^2
 - a^24*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2854 vs. $2(178) = 356$.

Time = 6.46 (sec) , antiderivative size = 2854, normalized size of antiderivative = 15.51

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2))**8/x**2,x)
```

output
```
Piecewise((zoo/x**5, Eq(a, 0) & Eq(b, 0)), (-1/(a**8*x), Eq(b, 0)), (-1/(5
*b**8*x**5), Eq(a, 0)), (-35*a**9*sqrt(x)/(35*a**17*x**(3/2) + 245*a**16*b
*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x
**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x*
*5) + 315*a**8*b*x/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*
x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**
5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 1260*a**7*b**2*x*
*(3/2)*log(x)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5
/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**
4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) - 2520*a**7*b**2*x**(3/2
)*log(a/b + sqrt(x))/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**
2*x**(5/2) + 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b
**5*x**4 + 245*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 6534*a**7*b**2*
x**(3/2)/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) +
 1225*a**14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 2
45*a**11*b**6*x**(9/2) + 35*a**10*b**7*x**5) + 8820*a**6*b**3*x**2*log(x)/
(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1225*a**
14*b**3*x**3 + 1225*a**13*b**4*x**(7/2) + 735*a**12*b**5*x**4 + 245*a**11*
b**6*x**(9/2) + 35*a**10*b**7*x**5) - 17640*a**6*b**3*x**2*log(a/b + sqrt(
x))/(35*a**17*x**(3/2) + 245*a**16*b*x**2 + 735*a**15*b**2*x**(5/2) + 1...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx = \frac{2520\,b^8x^4+16380\,ab^7x^{\frac{7}{2}}+44940\,a^2b^6x^3+66990\,a^3b^5x^{\frac{5}{2}}+57834\,a^4b^4x^2+28098\,a^5b^3x^{\frac{3}{2}}+6534\,a^6b^2x+\cdots}{35\left(a^9b^7x^{\frac{9}{2}}+7\,a^{10}b^6x^4+21\,a^{11}b^5x^{\frac{7}{2}}+35\,a^{12}b^4x^3+35\,a^{13}b^3x^{\frac{5}{2}}+21\,a^{14}b^2x^2+7\,a^{15}bx^{\frac{3}{2}}+\cdots\right.} - \frac{72\,b^2\log\left(b\sqrt{x}+a\right)}{a^{10}} + \frac{36\,b^2\log\left(x\right)}{a^{10}}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x^2,x, algorithm="maxima")
```

output
```
1/35*(2520*b^8*x^4 + 16380*a*b^7*x^(7/2) + 44940*a^2*b^6*x^3 + 66990*a^3*b
^5*x^(5/2) + 57834*a^4*b^4*x^2 + 28098*a^5*b^3*x^(3/2) + 6534*a^6*b^2*x +
315*a^7*b*sqrt(x) - 35*a^8)/(a^9*b^7*x^(9/2) + 7*a^10*b^6*x^4 + 21*a^11*b^
5*x^(7/2) + 35*a^12*b^4*x^3 + 35*a^13*b^3*x^(5/2) + 21*a^14*b^2*x^2 + 7*a^
15*b*x^(3/2) + a^16*x) - 72*b^2*log(b*sqrt(x) + a)/a^10 + 36*b^2*log(x)/a^
10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx = -\frac{72\,b^2\log\left(\left|b\sqrt{x}+a\right|\right)}{a^{10}} + \frac{36\,b^2\log\left(\left|x\right|\right)}{a^{10}} + \frac{2520\,ab^8x^4+16380\,a^2b^7x^{\frac{7}{2}}+44940\,a^3b^6x^3+66990\,a^4b^5x^{\frac{5}{2}}+57834\,a^5b^4x^2+28098\,a^6b^3x^{\frac{3}{2}}+6534\,a^7b\cdots}{35\left(b\sqrt{x}+a\right)^7a^{10}x}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x^2,x, algorithm="giac")
```

output
```
-72*b^2*log(abs(b*sqrt(x) + a))/a^10 + 36*b^2*log(abs(x))/a^10 + 1/35*(252
0*a*b^8*x^4 + 16380*a^2*b^7*x^(7/2) + 44940*a^3*b^6*x^3 + 66990*a^4*b^5*x^
(5/2) + 57834*a^5*b^4*x^2 + 28098*a^6*b^3*x^(3/2) + 6534*a^7*b^2*x + 315*a
^8*b*sqrt(x) - 35*a^9)/((b*sqrt(x) + a)^7*a^10*x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx$$
$$= \frac{\frac{9\,b\,\sqrt{x}}{a^2}-\frac{1}{a}+\frac{6534\,b^2\,x}{35\,a^3}+\frac{8262\,b^4\,x^2}{5\,a^5}+\frac{4014\,b^3\,x^{3/2}}{5\,a^4}+\frac{1284\,b^6\,x^3}{a^7}+\frac{1914\,b^5\,x^{5/2}}{a^6}+\frac{72\,b^8\,x^4}{a^9}+\frac{468\,b^7\,x^{7/2}}{a^8}}{a^7\,x+b^7\,x^{9/2}+7\,a\,b^6\,x^4+7\,a^6\,b\,x^{3/2}+21\,a^5\,b^2\,x^2+35\,a^3\,b^4\,x^3+35\,a^4\,b^3\,x^{5/2}+21\,a^2\,b^5\,x^{7/2}} - \frac{144\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^{10}}$$

input
```
int(1/(x^2*(a + b*x^(1/2)))^8),x)
```

output
```
((9*b*x^(1/2))/a^2 - 1/a + (6534*b^2*x)/(35*a^3) + (8262*b^4*x^2)/(5*a^5)
+ (4014*b^3*x^(3/2))/(5*a^4) + (1284*b^6*x^3)/a^7 + (1914*b^5*x^(5/2))/a^6
 + (72*b^8*x^4)/a^9 + (468*b^7*x^(7/2))/a^8)/(a^7*x + b^7*x^(9/2) + 7*a*b^
6*x^4 + 7*a^6*b*x^(3/2) + 21*a^5*b^2*x^2 + 35*a^3*b^4*x^3 + 35*a^4*b^3*x^(
5/2) + 21*a^2*b^5*x^(7/2)) - (144*b^2*atanh((2*b*x^(1/2))/a + 1))/a^10
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.34

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^2}\,dx$$
$$= \frac{-88200\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^4b^5x^2-52920\sqrt{x}\log\left(\sqrt{x}\,b+a\right)a^2b^7x^3-35a^9+25578\sqrt{x}\,a^6b^3x-88200\log}{}$$

input
```
int(1/(a+b*x^(1/2))^8/x^2,x)
```

output
```
( - 17640*sqrt(x)*log(sqrt(x)*b + a)*a**6*b**3*x - 88200*sqrt(x)*log(sqrt(
x)*b + a)*a**4*b**5*x**2 - 52920*sqrt(x)*log(sqrt(x)*b + a)*a**2*b**7*x**3
 - 2520*sqrt(x)*log(sqrt(x)*b + a)*b**9*x**4 + 17640*sqrt(x)*log(sqrt(x))*
a**6*b**3*x + 88200*sqrt(x)*log(sqrt(x))*a**4*b**5*x**2 + 52920*sqrt(x)*lo
g(sqrt(x))*a**2*b**7*x**3 + 2520*sqrt(x)*log(sqrt(x))*b**9*x**4 + 315*sqrt
(x)*a**8*b + 25578*sqrt(x)*a**6*b**3*x + 54390*sqrt(x)*a**4*b**5*x**2 + 88
20*sqrt(x)*a**2*b**7*x**3 - 360*sqrt(x)*b**9*x**4 - 2520*log(sqrt(x)*b + a
)*a**7*b**2*x - 52920*log(sqrt(x)*b + a)*a**5*b**4*x**2 - 88200*log(sqrt(x
)*b + a)*a**3*b**6*x**3 - 17640*log(sqrt(x)*b + a)*a*b**8*x**4 + 2520*log(
sqrt(x))*a**7*b**2*x + 52920*log(sqrt(x))*a**5*b**4*x**2 + 88200*log(sqrt(
x))*a**3*b**6*x**3 + 17640*log(sqrt(x))*a*b**8*x**4 - 35*a**9 + 6174*a**7*
b**2*x + 50274*a**5*b**4*x**2 + 32340*a**3*b**6*x**3)/(35*a**10*x*(7*sqrt(
x)*a**6*b + 35*sqrt(x)*a**4*b**3*x + 21*sqrt(x)*a**2*b**5*x**2 + sqrt(x)*b
**7*x**3 + a**7 + 21*a**5*b**2*x + 35*a**3*b**4*x**2 + 7*a*b**6*x**3))
```

3.121 $\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx &= \frac{2b^4}{7a^5\left(a+b\sqrt{x}\right)^7}+\frac{5b^4}{3a^6\left(a+b\sqrt{x}\right)^6}+\frac{6b^4}{a^7\left(a+b\sqrt{x}\right)^5} \\ &\quad+\frac{35b^4}{2a^8\left(a+b\sqrt{x}\right)^4}+\frac{140b^4}{3a^9\left(a+b\sqrt{x}\right)^3}+\frac{126b^4}{a^{10}\left(a+b\sqrt{x}\right)^2} \\ &\quad+\frac{420b^4}{a^{11}\left(a+b\sqrt{x}\right)}-\frac{1}{2a^8x^2}+\frac{16b}{3a^9x^{3/2}}-\frac{36b^2}{a^{10}x} \\ &\quad+\frac{240b^3}{a^{11}\sqrt{x}}-\frac{660b^4\log\left(a+b\sqrt{x}\right)}{a^{12}}+\frac{330b^4\log(x)}{a^{12}}\end{aligned}$$

output
```
2/7*b^4/a^5/(a+b*x^(1/2))^7+5/3*b^4/a^6/(a+b*x^(1/2))^6+6*b^4/a^7/(a+b*x^(
1/2))^5+35/2*b^4/a^8/(a+b*x^(1/2))^4+140/3*b^4/a^9/(a+b*x^(1/2))^3+126*b^4
/a^10/(a+b*x^(1/2))^2+420*b^4/a^11/(a+b*x^(1/2))-1/2/a^8/x^2+16/3*b/a^9/x^
(3/2)-36*b^2/a^10/x+240*b^3/a^11/x^(1/2)-660*b^4*ln(a+b*x^(1/2))/a^12+330*
b^4*ln(x)/a^12
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx = \frac{\frac{a\left(-21a^{10}+77a^9b\sqrt{x}-385a^8b^2x+3465a^7b^3x^{3/2}+71874a^6b^4x^2+309078a^5b^5x^{5/2}+636174a^4b^6x^3+736890a^3b^7x^{7/2}+494340a^2b^8x^4+180180ab^9\right.}{\left(a+b\sqrt{x}\right)^7x^2}}{42a^{12}}$$

input
```
Integrate[1/((a + b*Sqrt[x])^8*x^3),x]
```

output
```
((a*(-21*a^10 + 77*a^9*b*Sqrt[x] - 385*a^8*b^2*x + 3465*a^7*b^3*x^(3/2) +
71874*a^6*b^4*x^2 + 309078*a^5*b^5*x^(5/2) + 636174*a^4*b^6*x^3 + 736890*a
^3*b^7*x^(7/2) + 494340*a^2*b^8*x^4 + 180180*a*b^9*x^(9/2) + 27720*b^10*x^
5))/((a + b*Sqrt[x])^7*x^2) - 27720*b^4*Log[a + b*Sqrt[x]] + 13860*b^4*Log
[x])/(42*a^12)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\sqrt{x}\right)^8}\,dx$$

$$\Big\downarrow \; 798$$

$$2\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \; 54$$

$$2\int \left(-\frac{330b^5}{a^{12}\left(a+b\sqrt{x}\right)}-\frac{210b^5}{a^{11}\left(a+b\sqrt{x}\right)^2}-\frac{126b^5}{a^{10}\left(a+b\sqrt{x}\right)^3}-\frac{70b^5}{a^9\left(a+b\sqrt{x}\right)^4}-\frac{35b^5}{a^8\left(a+b\sqrt{x}\right)^5}-\frac{15b^5}{a^7\left(a+b\sqrt{x}\right)^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{330b^4\log\left(a+b\sqrt{x}\right)}{a^{12}}+\frac{330b^4\log\left(\sqrt{x}\right)}{a^{12}}+\frac{210b^4}{a^{11}\left(a+b\sqrt{x}\right)}+\frac{120b^3}{a^{11}\sqrt{x}}+\frac{63b^4}{a^{10}\left(a+b\sqrt{x}\right)^2}-\frac{18b^2}{a^{10}x}+\frac{70b^4}{3a^9\left(a+b\sqrt{\right.}}\right.$$

input
```
Int[1/((a + b*Sqrt[x])^8*x^3),x]
```

output
```
2*(b^4/(7*a^5*(a + b*Sqrt[x])^7) + (5*b^4)/(6*a^6*(a + b*Sqrt[x])^6) + (3*
b^4)/(a^7*(a + b*Sqrt[x])^5) + (35*b^4)/(4*a^8*(a + b*Sqrt[x])^4) + (70*b^
4)/(3*a^9*(a + b*Sqrt[x])^3) + (63*b^4)/(a^10*(a + b*Sqrt[x])^2) + (210*b^
4)/(a^11*(a + b*Sqrt[x])) - 1/(4*a^8*x^2) + (8*b)/(3*a^9*x^(3/2)) - (18*b^
2)/(a^10*x) + (120*b^3)/(a^11*Sqrt[x]) - (330*b^4*Log[a + b*Sqrt[x]])/a^12
 + (330*b^4*Log[Sqrt[x]])/a^12)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{2b^4}{7a^5\left(a+b\sqrt{x}\right)^7}+\frac{5b^4}{3a^6\left(a+b\sqrt{x}\right)^6}+\frac{6b^4}{a^7\left(a+b\sqrt{x}\right)^5}+\frac{35b^4}{2a^8\left(a+b\sqrt{x}\right)^4}+\frac{140b^4}{3a^9\left(a+b\sqrt{x}\right)^3}+\frac{126b^4}{a^{10}\left(a+b\sqrt{x}\right)^2}+\frac{4}{a^{11}(}$
default	$\frac{2b^4}{7a^5\left(a+b\sqrt{x}\right)^7}+\frac{5b^4}{3a^6\left(a+b\sqrt{x}\right)^6}+\frac{6b^4}{a^7\left(a+b\sqrt{x}\right)^5}+\frac{35b^4}{2a^8\left(a+b\sqrt{x}\right)^4}+\frac{140b^4}{3a^9\left(a+b\sqrt{x}\right)^3}+\frac{126b^4}{a^{10}\left(a+b\sqrt{x}\right)^2}+\frac{4}{a^{11}(}$

input
```
int(1/(a+b*x^(1/2))^8/x^3,x,method=_RETURNVERBOSE)
```

output
```
2/7*b^4/a^5/(a+b*x^(1/2))^7+5/3*b^4/a^6/(a+b*x^(1/2))^6+6*b^4/a^7/(a+b*x^(
1/2))^5+35/2*b^4/a^8/(a+b*x^(1/2))^4+140/3*b^4/a^9/(a+b*x^(1/2))^3+126*b^4
/a^10/(a+b*x^(1/2))^2+420*b^4/a^11/(a+b*x^(1/2))-1/2/a^8/x^2+16/3*b/a^9/x^
(3/2)-36*b^2/a^10/x+240*b^3/a^11/x^(1/2)-660*b^4*ln(a+b*x^(1/2))/a^12+330*
b^4*ln(x)/a^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(185) = 370.

Time = 0.26 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.13

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx =$$
$$-\frac{13860\,a^2b^{16}x^8-90090\,a^4b^{14}x^7+247170\,a^6b^{12}x^6-368445\,a^8b^{10}x^5+318087\,a^{10}b^8x^4-154532\,a^{12}b^6x^3}{}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x^3,x, algorithm="fricas")
```

output
```
-1/42*(13860*a^2*b^16*x^8 - 90090*a^4*b^14*x^7 + 247170*a^6*b^12*x^6 - 368
445*a^8*b^10*x^5 + 318087*a^10*b^8*x^4 - 154532*a^12*b^6*x^3 + 36104*a^14*
b^4*x^2 - 1365*a^16*b^2*x - 21*a^18 + 27720*(b^18*x^9 - 7*a^2*b^16*x^8 + 2
1*a^4*b^14*x^7 - 35*a^6*b^12*x^6 + 35*a^8*b^10*x^5 - 21*a^10*b^8*x^4 + 7*a
^12*b^6*x^3 - a^14*b^4*x^2)*log(b*sqrt(x) + a) - 27720*(b^18*x^9 - 7*a^2*b
^16*x^8 + 21*a^4*b^14*x^7 - 35*a^6*b^12*x^6 + 35*a^8*b^10*x^5 - 21*a^10*b^
8*x^4 + 7*a^12*b^6*x^3 - a^14*b^4*x^2)*log(sqrt(x)) - 8*(3465*a*b^17*x^8 -
 23100*a^3*b^15*x^7 + 65373*a^5*b^13*x^6 - 101376*a^7*b^11*x^5 + 92323*a^9
*b^9*x^4 - 48580*a^11*b^7*x^3 + 13083*a^13*b^5*x^2 - 1064*a^15*b^3*x - 28*
a^17*b)*sqrt(x))/(a^12*b^14*x^9 - 7*a^14*b^12*x^8 + 21*a^16*b^10*x^7 - 35*
a^18*b^8*x^6 + 35*a^20*b^6*x^5 - 21*a^22*b^4*x^4 + 7*a^24*b^2*x^3 - a^26*x
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3077 vs. $2(212) = 424$.

Time = 9.62 (sec) , antiderivative size = 3077, normalized size of antiderivative = 14.18

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/2))**8/x**3,x)
```

output
```
Piecewise((zoo/x**6, Eq(a, 0) & Eq(b, 0)), (-1/(2*a**8*x**2), Eq(b, 0)), (
-1/(6*b**8*x**6), Eq(a, 0)), (-21*a**11*sqrt(x)/(42*a**19*x**(5/2) + 294*a
**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*
b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*
b**7*x**6) + 77*a**10*b*x/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**1
7*b**2*x**(7/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a*
*14*b**5*x**5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) - 385*a**9*
b**2*x**(3/2)/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7
/2) + 1470*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**
5 + 294*a**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 3465*a**8*b**3*x**2/(
42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**1
6*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b
**6*x**(11/2) + 42*a**12*b**7*x**6) + 13860*a**7*b**4*x**(5/2)*log(x)/(42*
a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16*b
**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a**13*b**6
*x**(11/2) + 42*a**12*b**7*x**6) - 27720*a**7*b**4*x**(5/2)*log(a/b + sqrt
(x))/(42*a**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 147
0*a**16*b**3*x**4 + 1470*a**15*b**4*x**(9/2) + 882*a**14*b**5*x**5 + 294*a
**13*b**6*x**(11/2) + 42*a**12*b**7*x**6) + 71874*a**7*b**4*x**(5/2)/(42*a
**19*x**(5/2) + 294*a**18*b*x**3 + 882*a**17*b**2*x**(7/2) + 1470*a**16...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx$$
$$= \frac{27720\,b^{10}x^5 + 180180\,ab^9x^{\frac{9}{2}} + 494340\,a^2b^8x^4 + 736890\,a^3b^7x^{\frac{7}{2}} + 636174\,a^4b^6x^3 + 309078\,a^5b^5x^{\frac{5}{2}} + 7187}{42\left(a^{11}b^7x^{\frac{11}{2}} + 7\,a^{12}b^6x^5 + 21\,a^{13}b^5x^{\frac{9}{2}} + 35\,a^{14}b^4x^4 + 35\,a^{15}b^3x^{\frac{7}{2}} + 21\,a^{16}\right.}$$
$$- \frac{660\,b^4\log\left(b\sqrt{x}+a\right)}{a^{12}} + \frac{330\,b^4\log\left(x\right)}{a^{12}}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x^3,x, algorithm="maxima")
```

output
```
1/42*(27720*b^10*x^5 + 180180*a*b^9*x^(9/2) + 494340*a^2*b^8*x^4 + 736890*
a^3*b^7*x^(7/2) + 636174*a^4*b^6*x^3 + 309078*a^5*b^5*x^(5/2) + 71874*a^6*
b^4*x^2 + 3465*a^7*b^3*x^(3/2) - 385*a^8*b^2*x + 77*a^9*b*sqrt(x) - 21*a^1
0)/(a^11*b^7*x^(11/2) + 7*a^12*b^6*x^5 + 21*a^13*b^5*x^(9/2) + 35*a^14*b^4
*x^4 + 35*a^15*b^3*x^(7/2) + 21*a^16*b^2*x^3 + 7*a^17*b*x^(5/2) + a^18*x^2
) - 660*b^4*log(b*sqrt(x) + a)/a^12 + 330*b^4*log(x)/a^12
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx = -\frac{660\,b^4\log\left(\left|b\sqrt{x}+a\right|\right)}{a^{12}} + \frac{330\,b^4\log\left(|x|\right)}{a^{12}} + \frac{27720\,ab^{10}x^5 + 180180\,a^2b^9x^{\frac{9}{2}} + 494340\,a^3b^8x^4 + 736890\,a^4b^7x^{\frac{7}{2}} + 636174\,a^5b^6x^3 + 309078\,a^6b^5x^{\frac{5}{2}} + \cdots}{42\left(b\sqrt{x}+a\right)^7a^{12}x^2}$$

input
```
integrate(1/(a+b*x^(1/2))^8/x^3,x, algorithm="giac")
```

output
```
-660*b^4*log(abs(b*sqrt(x) + a))/a^12 + 330*b^4*log(abs(x))/a^12 + 1/42*(2
7720*a*b^10*x^5 + 180180*a^2*b^9*x^(9/2) + 494340*a^3*b^8*x^4 + 736890*a^4
*b^7*x^(7/2) + 636174*a^5*b^6*x^3 + 309078*a^6*b^5*x^(5/2) + 71874*a^7*b^4
*x^2 + 3465*a^8*b^3*x^(3/2) - 385*a^9*b^2*x + 77*a^10*b*sqrt(x) - 21*a^11)
/((b*sqrt(x) + a)^7*a^12*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx = \frac{\frac{11\,b\,\sqrt{x}}{6\,a^2} - \frac{1}{2\,a} - \frac{55\,b^2\,x}{6\,a^3} + \frac{11979\,b^4\,x^2}{7\,a^5} + \frac{165\,b^3\,x^{3/2}}{2\,a^4} + \frac{15147\,b^6\,x^3}{a^7} + \frac{7359\,b^5\,x^{5/2}}{a^6} + \frac{11770\,b^8\,x^4}{a^9} + \frac{17545\,b^7\,x^{7/2}}{a^8} + \frac{660\,b^{10}\,x^5}{a^{11}} + \cdots}{a^7\,x^2 + b^7\,x^{11/2} + 7\,a\,b^6\,x^5 + 7\,a^6\,b\,x^{5/2} + 21\,a^5\,b^2\,x^3 + 35\,a^3\,b^4\,x^4 + 35\,a^4\,b^3\,x^{7/2} + 21\,a^2\,b^5\,x^{9/2} + \cdots} - \frac{1320\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^{12}}$$

input
```
int(1/(x^3*(a + b*x^(1/2))^8),x)
```

output
```
((11*b*x^(1/2))/(6*a^2) - 1/(2*a) - (55*b^2*x)/(6*a^3) + (11979*b^4*x^2)/(
7*a^5) + (165*b^3*x^(3/2))/(2*a^4) + (15147*b^6*x^3)/a^7 + (7359*b^5*x^(5/
2))/a^6 + (11770*b^8*x^4)/a^9 + (17545*b^7*x^(7/2))/a^8 + (660*b^10*x^5)/a
^11 + (4290*b^9*x^(9/2))/a^10)/(a^7*x^2 + b^7*x^(11/2) + 7*a*b^6*x^5 + 7*a
^6*b*x^(5/2) + 21*a^5*b^2*x^3 + 35*a^3*b^4*x^4 + 35*a^4*b^3*x^(7/2) + 21*a
^2*b^5*x^(9/2)) - (1320*b^4*atanh((2*b*x^(1/2))/a + 1))/a^12
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.13

$$\int \frac{1}{\left(a+b\sqrt{x}\right)^8 x^3}\,dx$$
$$= \frac{-27720\sqrt{x}\log\left(\sqrt{x}\,b+a\right)b^{11}x^5+27720\sqrt{x}\log\left(\sqrt{x}\right)b^{11}x^5-970200\log\left(\sqrt{x}\,b+a\right)a^3b^8x^4-194040\log(}{}$$

input
```
int(1/(a+b*x^(1/2))^8/x^3,x)
```

output
```
( - 194040*sqrt(x)*log(sqrt(x)*b + a)*a**6*b**5*x**2 - 970200*sqrt(x)*log(
sqrt(x)*b + a)*a**4*b**7*x**3 - 582120*sqrt(x)*log(sqrt(x)*b + a)*a**2*b**
9*x**4 - 27720*sqrt(x)*log(sqrt(x)*b + a)*b**11*x**5 + 194040*sqrt(x)*log(
sqrt(x))*a**6*b**5*x**2 + 970200*sqrt(x)*log(sqrt(x))*a**4*b**7*x**3 + 582
120*sqrt(x)*log(sqrt(x))*a**2*b**9*x**4 + 27720*sqrt(x)*log(sqrt(x))*b**11
*x**5 + 77*sqrt(x)*a**10*b + 3465*sqrt(x)*a**8*b**3*x + 281358*sqrt(x)*a**
6*b**5*x**2 + 598290*sqrt(x)*a**4*b**7*x**3 + 97020*sqrt(x)*a**2*b**9*x**4
 - 3960*sqrt(x)*b**11*x**5 - 27720*log(sqrt(x)*b + a)*a**7*b**4*x**2 - 582
120*log(sqrt(x)*b + a)*a**5*b**6*x**3 - 970200*log(sqrt(x)*b + a)*a**3*b**
8*x**4 - 194040*log(sqrt(x)*b + a)*a*b**10*x**5 + 27720*log(sqrt(x))*a**7*
b**4*x**2 + 582120*log(sqrt(x))*a**5*b**6*x**3 + 970200*log(sqrt(x))*a**3*
b**8*x**4 + 194040*log(sqrt(x))*a*b**10*x**5 - 21*a**11 - 385*a**9*b**2*x
+ 67914*a**7*b**4*x**2 + 553014*a**5*b**6*x**3 + 355740*a**3*b**8*x**4)/(4
2*a**12*x**2*(7*sqrt(x)*a**6*b + 35*sqrt(x)*a**4*b**3*x + 21*sqrt(x)*a**2*
b**5*x**2 + sqrt(x)*b**7*x**3 + a**7 + 21*a**5*b**2*x + 35*a**3*b**4*x**2
+ 7*a*b**6*x**3))
```

3.122 $\int \frac{1}{(2+b\sqrt{x})x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx = -\log\left(2+b\sqrt{x}\right)+\frac{\log(x)}{2}$$

output
```
-ln(2+b*x^(1/2))+1/2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx = -\log\left(2+b\sqrt{x}\right)+\log\left(\sqrt{x}\right)$$

input
```
Integrate[1/((2 + b*Sqrt[x])*x),x]
```

output
```
-Log[2 + b*Sqrt[x]] + Log[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(b\sqrt{x}+2\right)}dx$$

$$\Big\downarrow \ 798$$

$$2\int \frac{1}{\left(\sqrt{x}b+2\right)\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \ 47$$

$$2\left(\frac{1}{2}\int \frac{1}{\sqrt{x}}d\sqrt{x}-\frac{1}{2}b\int \frac{1}{\sqrt{x}b+2}d\sqrt{x}\right)$$

$$\Big\downarrow \ 14$$

$$2\left(\frac{\log\left(\sqrt{x}\right)}{2}-\frac{1}{2}b\int \frac{1}{\sqrt{x}b+2}d\sqrt{x}\right)$$

$$\Big\downarrow \ 16$$

$$2\left(\frac{\log\left(\sqrt{x}\right)}{2}-\frac{1}{2}\log\left(b\sqrt{x}+2\right)\right)$$

input
```
Int[1/((2 + b*Sqrt[x])*x),x]
```

output
```
2*(-1/2*Log[2 + b*Sqrt[x]] + Log[Sqrt[x]]/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\ln\left(2+b\sqrt{x}\right)+\frac{\ln(x)}{2}$	16
default	$-\ln\left(2+b\sqrt{x}\right)+\frac{\ln(x)}{2}$	16
meijerg	$\frac{\ln(x)}{2}-\ln\left(2\right)+\ln\left(b\right)-\ln\left(1+\frac{b\sqrt{x}}{2}\right)$	23

input
```
int(1/(2+b*x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(2+b*x^(1/2))+1/2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx=-\log\left(b\sqrt{x}+2\right)+\log\left(\sqrt{x}\right)$$

input
```
integrate(1/(2+b*x^(1/2))/x,x, algorithm="fricas")
```

output
```
-log(b*sqrt(x) + 2) + log(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx=\begin{cases}\frac{\log(x)}{2}-\log\left(\sqrt{x}+\frac{2}{b}\right) & \text{for } b\neq 0\\ \frac{\log(x)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(2+b*x**(1/2))/x,x)
```

output
```
Piecewise((log(x)/2 - log(sqrt(x) + 2/b), Ne(b, 0)), (log(x)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(2+b\sqrt{x}\right)x}\,dx=-\log\left(b\sqrt{x}+2\right)+\frac{1}{2}\log(x)$$

input
```
integrate(1/(2+b*x^(1/2))/x,x, algorithm="maxima")
```

output
```
-log(b*sqrt(x) + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{(2+b\sqrt{x})\,x}\,dx = -\log\left(\left|b\sqrt{x}+2\right|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

input
```
integrate(1/(2+b*x^(1/2))/x,x, algorithm="giac")
```

output
```
-log(abs(b*sqrt(x) + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int \frac{1}{(2+b\sqrt{x})\,x}\,dx = -2\,\mathrm{atanh}\left(b\,\sqrt{x}+1\right)$$

input
```
int(1/(x*(b*x^(1/2) + 2)),x)
```

output
```
-2*atanh(b*x^(1/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{(2+b\sqrt{x})\,x}\,dx = -\log\left(\sqrt{x}\,b+2\right)+\log\left(\sqrt{x}\right)$$

input
```
int(1/(2+b*x^(1/2))/x,x)
```

output
```
 - log(sqrt(x)*b + 2) + log(sqrt(x))
```

3.123 $\int \sqrt{a+b\sqrt{x}}x^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 132

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx = -\frac{4a^5\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}+\frac{4a^4\left(a+b\sqrt{x}\right)^{5/2}}{b^6}-\frac{40a^3\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}+\frac{40a^2\left(a+b\sqrt{x}\right)^{9/2}}{9b^6}-\frac{20a\left(a+b\sqrt{x}\right)^{11/2}}{11b^6}+\frac{4\left(a+b\sqrt{x}\right)^{13/2}}{13b^6}$$

output
```
-4/3*a^5*(a+b*x^(1/2))^(3/2)/b^6+4*a^4*(a+b*x^(1/2))^(5/2)/b^6-40/7*a^3*(a
+b*x^(1/2))^(7/2)/b^6+40/9*a^2*(a+b*x^(1/2))^(9/2)/b^6-20/11*a*(a+b*x^(1/2
))^(11/2)/b^6+4/13*(a+b*x^(1/2))^(13/2)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx = \frac{4\left(a+b\sqrt{x}\right)^{3/2}\left(-256a^5+384a^4b\sqrt{x}-480a^3b^2x+560a^2b^3x^{3/2}-630ab^4x^2+693b^5x^{5/2}\right)}{9009b^6}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x]]*x^2,x]
```

output
```
(4*(a + b*Sqrt[x])^(3/2)*(-256*a^5 + 384*a^4*b*Sqrt[x] - 480*a^3*b^2*x + 5
60*a^2*b^3*x^(3/2) - 630*a*b^4*x^2 + 693*b^5*x^(5/2)))/(9009*b^6)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+b\sqrt{x}}\,dx$$

$$\Big\downarrow \; 798$$

$$2\int \sqrt{a+b\sqrt{x}}x^{5/2}d\sqrt{x}$$

$$\Big\downarrow \; 53$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{11/2}}{b^5}-\frac{5a\left(a+b\sqrt{x}\right)^{9/2}}{b^5}+\frac{10a^2\left(a+b\sqrt{x}\right)^{7/2}}{b^5}-\frac{10a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^5}+\frac{5a^4\left(a+b\sqrt{x}\right)^{3/2}}{b^5}-\frac{a^5\sqrt{a}\right.$$

$$\Big\downarrow \; 2009$$

$$2\left(-\frac{2a^5\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}+\frac{2a^4\left(a+b\sqrt{x}\right)^{5/2}}{b^6}-\frac{20a^3\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}+\frac{20a^2\left(a+b\sqrt{x}\right)^{9/2}}{9b^6}+\frac{2\left(a+b\sqrt{x}\right)^{13/2}}{13b^6}-\frac{10a(\right.$$

input
```
Int[Sqrt[a + b*Sqrt[x]]*x^2,x]
```

output
```
2*((-2*a^5*(a + b*Sqrt[x])^(3/2))/(3*b^6) + (2*a^4*(a + b*Sqrt[x])^(5/2))/
b^6 - (20*a^3*(a + b*Sqrt[x])^(7/2))/(7*b^6) + (20*a^2*(a + b*Sqrt[x])^(9/
2))/(9*b^6) - (10*a*(a + b*Sqrt[x])^(11/2))/(11*b^6) + (2*(a + b*Sqrt[x])^
(13/2))/(13*b^6))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{\frac{4(a+b\sqrt{x})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{x})^{\frac{11}{2}}}{11}+\frac{40a^2(a+b\sqrt{x})^{\frac{9}{2}}}{9}-\frac{40a^3(a+b\sqrt{x})^{\frac{7}{2}}}{7}+4a^4(a+b\sqrt{x})^{\frac{5}{2}}-\frac{4a^5(a+b\sqrt{x})^{\frac{3}{2}}}{3}}{b^6}$	85
default	$\frac{\frac{4(a+b\sqrt{x})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{x})^{\frac{11}{2}}}{11}+\frac{40a^2(a+b\sqrt{x})^{\frac{9}{2}}}{9}-\frac{40a^3(a+b\sqrt{x})^{\frac{7}{2}}}{7}+4a^4(a+b\sqrt{x})^{\frac{5}{2}}-\frac{4a^5(a+b\sqrt{x})^{\frac{3}{2}}}{3}}{b^6}$	85

input
```
int((a+b*x^(1/2))^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
4/b^6*(1/13*(a+b*x^(1/2))^(13/2)-5/11*a*(a+b*x^(1/2))^(11/2)+10/9*a^2*(a+b
*x^(1/2))^(9/2)-10/7*a^3*(a+b*x^(1/2))^(7/2)+a^4*(a+b*x^(1/2))^(5/2)-1/3*a
^5*(a+b*x^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.58

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx$$
$$= \frac{4\left(693\,b^6x^3-70\,a^2b^4x^2-96\,a^4b^2x-256\,a^6+\left(63\,ab^5x^2+80\,a^3b^3x+128\,a^5b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{9009\,b^6}$$

input
```
integrate((a+b*x^(1/2))^(1/2)*x^2,x, algorithm="fricas")
```

output
```
4/9009*(693*b^6*x^3 - 70*a^2*b^4*x^2 - 96*a^4*b^2*x - 256*a^6 + (63*a*b^5*
x^2 + 80*a^3*b^3*x + 128*a^5*b)*sqrt(x))*sqrt(b*sqrt(x) + a)/b^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8588 vs. 2(124) = 248.

Time = 5.06 (sec) , antiderivative size = 8588, normalized size of antiderivative = 65.06

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**(1/2))**(1/2)*x**2,x)
```

output
```
-1024*a**(153/2)*x**18*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 1351
35*a**69*b**7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**
(39/2) + 12297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 450
90045*a**64*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*
b**14*x**22 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23
+ 12297285*a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**5
7*b**19*x**(49/2) + 135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2))
 + 1024*a**(153/2)*x**18/(9009*a**70*b**6*x**18 + 135135*a**69*b**7*x**(37
/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12297285*a*
*66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64*b**12*x*
*21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22 + 450900
45*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*a**59*b**
17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*x**(49/2) +
135135*a**56*b**20*x**25 + 9009*a**55*b**21*x**(51/2)) - 14848*a**(151/2)*
b*x**(37/2)*sqrt(1 + b*sqrt(x)/a)/(9009*a**70*b**6*x**18 + 135135*a**69*b*
*7*x**(37/2) + 945945*a**68*b**8*x**19 + 4099095*a**67*b**9*x**(39/2) + 12
297285*a**66*b**10*x**20 + 27054027*a**65*b**11*x**(41/2) + 45090045*a**64
*b**12*x**21 + 57972915*a**63*b**13*x**(43/2) + 57972915*a**62*b**14*x**22
 + 45090045*a**61*b**15*x**(45/2) + 27054027*a**60*b**16*x**23 + 12297285*
a**59*b**17*x**(47/2) + 4099095*a**58*b**18*x**24 + 945945*a**57*b**19*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx = \frac{4\left(b\sqrt{x}+a\right)^{\frac{13}{2}}}{13\,b^6} - \frac{20\left(b\sqrt{x}+a\right)^{\frac{11}{2}}a}{11\,b^6} + \frac{40\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a^2}{9\,b^6} - \frac{40\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^3}{7\,b^6} + \frac{4\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^4}{b^6} - \frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^5}{3\,b^6}$$

input
```
integrate((a+b*x^(1/2))^(1/2)*x^2,x, algorithm="maxima")
```

output
```
4/13*(b*sqrt(x) + a)^(13/2)/b^6 - 20/11*(b*sqrt(x) + a)^(11/2)*a/b^6 + 40/
9*(b*sqrt(x) + a)^(9/2)*a^2/b^6 - 40/7*(b*sqrt(x) + a)^(7/2)*a^3/b^6 + 4*(
b*sqrt(x) + a)^(5/2)*a^4/b^6 - 4/3*(b*sqrt(x) + a)^(3/2)*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx = \frac{4\left(\frac{13\left(63\,(b\sqrt{x}+a)^{\frac{11}{2}}-385\,(b\sqrt{x}+a)^{\frac{9}{2}}a+990\,(b\sqrt{x}+a)^{\frac{7}{2}}a^2-1386\,(b\sqrt{x}+a)^{\frac{5}{2}}a^3+1155\,(b\sqrt{x}+a)^{\frac{3}{2}}a^4-693\,\sqrt{b\sqrt{x}+a}\,a^5\right)a}{b^5}+\frac{3\left(231\,(b\sqrt{x}\right.}{}\right.}{9009\,b}$$

input
```
integrate((a+b*x^(1/2))^(1/2)*x^2,x, algorithm="giac")
```

output
```
4/9009*(13*(63*(b*sqrt(x) + a)^(11/2) - 385*(b*sqrt(x) + a)^(9/2)*a + 990*
(b*sqrt(x) + a)^(7/2)*a^2 - 1386*(b*sqrt(x) + a)^(5/2)*a^3 + 1155*(b*sqrt(
x) + a)^(3/2)*a^4 - 693*sqrt(b*sqrt(x) + a)*a^5)*a/b^5 + 3*(231*(b*sqrt(x)
 + a)^(13/2) - 1638*(b*sqrt(x) + a)^(11/2)*a + 5005*(b*sqrt(x) + a)^(9/2)*
a^2 - 8580*(b*sqrt(x) + a)^(7/2)*a^3 + 9009*(b*sqrt(x) + a)^(5/2)*a^4 - 60
06*(b*sqrt(x) + a)^(3/2)*a^5 + 3003*sqrt(b*sqrt(x) + a)*a^6)/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx = \frac{4\,{\left(a+b\,\sqrt{x}\right)}^{13/2}}{13\,b^6}-\frac{20\,a\,{\left(a+b\,\sqrt{x}\right)}^{11/2}}{11\,b^6}-\frac{4\,a^5\,{\left(a+b\,\sqrt{x}\right)}^{3/2}}{3\,b^6}+\frac{4\,a^4\,{\left(a+b\,\sqrt{x}\right)}^{5/2}}{b^6}-\frac{40\,a^3\,{\left(a+b\,\sqrt{x}\right)}^{7/2}}{7\,b^6}+\frac{40\,a^2\,{\left(a+b\,\sqrt{x}\right)}^{9/2}}{9\,b^6}$$

input
```
int(x^2*(a + b*x^(1/2))^(1/2),x)
```

output
```
(4*(a + b*x^(1/2))^(13/2))/(13*b^6) - (20*a*(a + b*x^(1/2))^(11/2))/(11*b^
6) - (4*a^5*(a + b*x^(1/2))^(3/2))/(3*b^6) + (4*a^4*(a + b*x^(1/2))^(5/2))
/b^6 - (40*a^3*(a + b*x^(1/2))^(7/2))/(7*b^6) + (40*a^2*(a + b*x^(1/2))^(9
/2))/(9*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \sqrt{a+b\sqrt{x}}x^2\,dx$$
$$=\frac{4\sqrt{\sqrt{x}\,b+a}\left(128\sqrt{x}\,a^5b+80\sqrt{x}\,a^3b^3x+63\sqrt{x}\,a\,b^5x^2-256a^6-96a^4b^2x-70a^2b^4x^2+693b^6x^3\right)}{9009b^6}$$

input
```
int((a+b*x^(1/2))^(1/2)*x^2,x)
```

output
```
(4*sqrt(sqrt(x)*b + a)*(128*sqrt(x)*a**5*b + 80*sqrt(x)*a**3*b**3*x + 63*s
qrt(x)*a*b**5*x**2 - 256*a**6 - 96*a**4*b**2*x - 70*a**2*b**4*x**2 + 693*b
**6*x**3))/(9009*b**6)
```

3.124 $\int \sqrt{a+b\sqrt{x}}x\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \sqrt{a+b\sqrt{x}}x\,dx = -\frac{4a^3\left(a+b\sqrt{x}\right)^{3/2}}{3b^4}+\frac{12a^2\left(a+b\sqrt{x}\right)^{5/2}}{5b^4} - \frac{12a\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}+\frac{4\left(a+b\sqrt{x}\right)^{9/2}}{9b^4}$$

output
```
-4/3*a^3*(a+b*x^(1/2))^(3/2)/b^4+12/5*a^2*(a+b*x^(1/2))^(5/2)/b^4-12/7*a*(
a+b*x^(1/2))^(7/2)/b^4+4/9*(a+b*x^(1/2))^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \sqrt{a+b\sqrt{x}}x\,dx = \frac{4\sqrt{a+b\sqrt{x}}\left(-16a^4+8a^3b\sqrt{x}-6a^2b^2x+5ab^3x^{3/2}+35b^4x^2\right)}{315b^4}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x]]*x,x]
```

output
```
(4*Sqrt[a + b*Sqrt[x]]*(-16*a^4 + 8*a^3*b*Sqrt[x] - 6*a^2*b^2*x + 5*a*b^3*
x^(3/2) + 35*b^4*x^2))/(315*b^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\sqrt{x}}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \sqrt{a+b\sqrt{x}}x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{7/2}}{b^3}-\frac{3a\left(a+b\sqrt{x}\right)^{5/2}}{b^3}+\frac{3a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^3}-\frac{a^3\sqrt{a+b\sqrt{x}}}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{2a^3\left(a+b\sqrt{x}\right)^{3/2}}{3b^4}+\frac{6a^2\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}+\frac{2\left(a+b\sqrt{x}\right)^{9/2}}{9b^4}-\frac{6a\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x]]*x,x]
```

output
```
2*((-2*a^3*(a + b*Sqrt[x])^(3/2))/(3*b^4) + (6*a^2*(a + b*Sqrt[x])^(5/2))/
(5*b^4) - (6*a*(a + b*Sqrt[x])^(7/2))/(7*b^4) + (2*(a + b*Sqrt[x])^(9/2))/
(9*b^4))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{\frac{4(a+b\sqrt{x})^{\frac{9}{2}}}{9}-\frac{12a(a+b\sqrt{x})^{\frac{7}{2}}}{7}+\frac{12a^2(a+b\sqrt{x})^{\frac{5}{2}}}{5}-\frac{4a^3(a+b\sqrt{x})^{\frac{3}{2}}}{3}}{b^4}$	58
default	$\frac{\frac{4(a+b\sqrt{x})^{\frac{9}{2}}}{9}-\frac{12a(a+b\sqrt{x})^{\frac{7}{2}}}{7}+\frac{12a^2(a+b\sqrt{x})^{\frac{5}{2}}}{5}-\frac{4a^3(a+b\sqrt{x})^{\frac{3}{2}}}{3}}{b^4}$	58

input
```
int((a+b*x^(1/2))^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
4/b^4*(1/9*(a+b*x^(1/2))^(9/2)-3/7*a*(a+b*x^(1/2))^(7/2)+3/5*a^2*(a+b*x^(1
/2))^(5/2)-1/3*a^3*(a+b*x^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \sqrt{a+b\sqrt{x}}x\,dx = \frac{4\left(35\,b^4x^2-6\,a^2b^2x-16\,a^4+\left(5\,ab^3x+8\,a^3b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{315\,b^4}$$

input
```
integrate((a+b*x^(1/2))^(1/2)*x,x, algorithm="fricas")
```

output
```
4/315*(35*b^4*x^2 - 6*a^2*b^2*x - 16*a^4 + (5*a*b^3*x + 8*a^3*b)*sqrt(x))*
sqrt(b*sqrt(x) + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. 2(82) = 164.

Time = 1.80 (sec) , antiderivative size = 1987, normalized size of antiderivative = 22.58

$$\int \sqrt{a+b\sqrt{x}}x\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**(1/2))**(1/2)*x,x)
```

output
```
-64*a**(49/2)*x**8*sqrt(1 + b*sqrt(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19
*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*
a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a**14*b**10*x**11) + 64
*a**(49/2)*x**8/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a*
*18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a
**15*b**9*x**(21/2) + 315*a**14*b**10*x**11) - 352*a**(47/2)*b*x**(17/2)*s
qrt(1 + b*sqrt(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 47
25*a**18*b**6*x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1
890*a**15*b**9*x**(21/2) + 315*a**14*b**10*x**11) + 384*a**(47/2)*b*x**(17
/2)/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**
9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x*
*(21/2) + 315*a**14*b**10*x**11) - 792*a**(45/2)*b**2*x**9*sqrt(1 + b*sqrt
(x)/a)/(315*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*
x**9 + 6300*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9
*x**(21/2) + 315*a**14*b**10*x**11) + 960*a**(45/2)*b**2*x**9/(315*a**20*b
**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300*a**17*b
**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/2) + 315*a*
*14*b**10*x**11) - 924*a**(43/2)*b**3*x**(19/2)*sqrt(1 + b*sqrt(x)/a)/(315
*a**20*b**4*x**8 + 1890*a**19*b**5*x**(17/2) + 4725*a**18*b**6*x**9 + 6300
*a**17*b**7*x**(19/2) + 4725*a**16*b**8*x**10 + 1890*a**15*b**9*x**(21/...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \sqrt{a+b\sqrt{x}}x\,dx = \frac{4\left(b\sqrt{x}+a\right)^{\frac{9}{2}}}{9\,b^4} - \frac{12\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a}{7\,b^4} + \frac{12\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^2}{5\,b^4} - \frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^3}{3\,b^4}$$

input
```
integrate((a+b*x^(1/2))^(1/2)*x,x, algorithm="maxima")
```

output
```
4/9*(b*sqrt(x) + a)^(9/2)/b^4 - 12/7*(b*sqrt(x) + a)^(7/2)*a/b^4 + 12/5*(b
*sqrt(x) + a)^(5/2)*a^2/b^4 - 4/3*(b*sqrt(x) + a)^(3/2)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \sqrt{a+b\sqrt{x}}x\,dx$$
$$= \frac{4\left(\frac{9\left(5\left(b\sqrt{x}+a\right)^{\frac{7}{2}}-21\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a+35\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2-35\sqrt{b\sqrt{x}+a}a^3\right)a}{b^3}+\frac{35\left(b\sqrt{x}+a\right)^{\frac{9}{2}}-180\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a+378\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^2-420\left(b\sqrt{\cdots}\right.}{b^3}\right.}{315\,b}$$

input
```
integrate((a+b*x^(1/2))^(1/2)*x,x, algorithm="giac")
```

output
```
4/315*(9*(5*(b*sqrt(x) + a)^(7/2) - 21*(b*sqrt(x) + a)^(5/2)*a + 35*(b*sqr
t(x) + a)^(3/2)*a^2 - 35*sqrt(b*sqrt(x) + a)*a^3)*a/b^3 + (35*(b*sqrt(x) +
 a)^(9/2) - 180*(b*sqrt(x) + a)^(7/2)*a + 378*(b*sqrt(x) + a)^(5/2)*a^2 -
420*(b*sqrt(x) + a)^(3/2)*a^3 + 315*sqrt(b*sqrt(x) + a)*a^4)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73

$$\int \sqrt{a+b\sqrt{x}}x\,dx = \frac{4\left(a+b\sqrt{x}\right)^{9/2}}{9\,b^4} - \frac{12\,a\left(a+b\sqrt{x}\right)^{7/2}}{7\,b^4} - \frac{4\,a^3\left(a+b\sqrt{x}\right)^{3/2}}{3\,b^4} + \frac{12\,a^2\left(a+b\sqrt{x}\right)^{5/2}}{5\,b^4}$$

input
```
int(x*(a + b*x^(1/2))^(1/2),x)
```

output
```
(4*(a + b*x^(1/2))^(9/2))/(9*b^4) - (12*a*(a + b*x^(1/2))^(7/2))/(7*b^4) -
 (4*a^3*(a + b*x^(1/2))^(3/2))/(3*b^4) + (12*a^2*(a + b*x^(1/2))^(5/2))/(5
*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \sqrt{a+b\sqrt{x}}x\,dx = \frac{4\sqrt{\sqrt{x}\,b+a}\,\left(8\sqrt{x}\,a^3b+5\sqrt{x}\,a\,b^3x-16a^4-6a^2b^2x+35b^4x^2\right)}{315b^4}$$

input
```
int((a+b*x^(1/2))^(1/2)*x,x)
```

output
```
(4*sqrt(sqrt(x)*b + a)*(8*sqrt(x)*a**3*b + 5*sqrt(x)*a*b**3*x - 16*a**4 -
6*a**2*b**2*x + 35*b**4*x**2))/(315*b**4)
```

3.125 $\int \sqrt{a+b\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \sqrt{a+b\sqrt{x}}\,dx=-\frac{4a\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}+\frac{4\left(a+b\sqrt{x}\right)^{5/2}}{5b^2}$$

output
```
-4/3*a*(a+b*x^(1/2))^(3/2)/b^2+4/5*(a+b*x^(1/2))^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \sqrt{a+b\sqrt{x}}\,dx=\frac{4\sqrt{a+b\sqrt{x}}\left(-2a^2+ab\sqrt{x}+3b^2x\right)}{15b^2}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[x]]*(-2*a^2 + a*b*Sqrt[x] + 3*b^2*x))/(15*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 774}$$

$$2\int \sqrt{a+b\sqrt{x}}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{ 53}$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{3/2}}{b}-\frac{a\sqrt{a+b\sqrt{x}}}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{2\left(a+b\sqrt{x}\right)^{5/2}}{5b^2}-\frac{2a\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x]],x]
```

output
```
2*((-2*a*(a + b*Sqrt[x])^(3/2))/(3*b^2) + (2*(a + b*Sqrt[x])^(5/2))/(5*b^2
))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{x}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{x}\right)^{\frac{3}{2}}}{3}}{b^{2}}$	30
default	$\frac{\frac{4\left(a+b\sqrt{x}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{x}\right)^{\frac{3}{2}}}{3}}{b^{2}}$	30

input
```
int((a+b*x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/b^2*(1/5*(a+b*x^(1/2))^(5/2)-1/3*a*(a+b*x^(1/2))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \sqrt{a+b\sqrt{x}}\,dx = \frac{4\left(3\,b^{2}x+ab\sqrt{x}-2\,a^{2}\right)\sqrt{b\sqrt{x}+a}}{15\,b^{2}}$$

input
```
integrate((a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/15*(3*b^2*x + a*b*sqrt(x) - 2*a^2)*sqrt(b*sqrt(x) + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(37) = 74$.

Time = 0.81 (sec) , antiderivative size = 272, normalized size of antiderivative = 6.48

$$\int \sqrt{a+b\sqrt{x}}\,dx = -\frac{8a^{\frac{9}{2}}x^2\sqrt{1+\frac{b\sqrt{x}}{a}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}} + \frac{8a^{\frac{9}{2}}x^2}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}} - \frac{4a^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{1+\frac{b\sqrt{x}}{a}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}} + \frac{8a^{\frac{7}{2}}bx^{\frac{5}{2}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}} + \frac{16a^{\frac{5}{2}}b^2x^3\sqrt{1+\frac{b\sqrt{x}}{a}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}} + \frac{12a^{\frac{3}{2}}b^3x^{\frac{7}{2}}\sqrt{1+\frac{b\sqrt{x}}{a}}}{15a^2b^2x^2+15ab^3x^{\frac{5}{2}}}$$

input
```
integrate((a+b*x**(1/2))**(1/2),x)
```

output
```
-8*a**(9/2)*x**2*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x**2 + 15*a*b**3*x**(
5/2)) + 8*a**(9/2)*x**2/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5/2)) - 4*a**(7
/2)*b*x**(5/2)*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5/
2)) + 8*a**(7/2)*b*x**(5/2)/(15*a**2*b**2*x**2 + 15*a*b**3*x**(5/2)) + 16*
a**(5/2)*b**2*x**3*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x**2 + 15*a*b**3*x*
*(5/2)) + 12*a**(3/2)*b**3*x**(7/2)*sqrt(1 + b*sqrt(x)/a)/(15*a**2*b**2*x*
*2 + 15*a*b**3*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \sqrt{a+b\sqrt{x}}\,dx = \frac{4\left(b\sqrt{x}+a\right)^{\frac{5}{2}}}{5\,b^2} - \frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a}{3\,b^2}$$

input
```
integrate((a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/5*(b*sqrt(x) + a)^(5/2)/b^2 - 4/3*(b*sqrt(x) + a)^(3/2)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.81

$$\int \sqrt{a+b\sqrt{x}}\,dx = \frac{4\left(\frac{5\left(\left(b\sqrt{x}+a\right)^{\frac{3}{2}}-3\sqrt{b\sqrt{x}+a}a\right)a}{b}+\frac{3\left(b\sqrt{x}+a\right)^{\frac{5}{2}}-10\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a+15\sqrt{b\sqrt{x}+a}a^2}{b}\right)}{15\,b}$$

input
```
integrate((a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/15*(5*((b*sqrt(x) + a)^(3/2) - 3*sqrt(b*sqrt(x) + a)*a)*a/b + (3*(b*sqrt
(x) + a)^(5/2) - 10*(b*sqrt(x) + a)^(3/2)*a + 15*sqrt(b*sqrt(x) + a)*a^2)/
b)/b
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \sqrt{a+b\sqrt{x}}\,dx = \frac{x\sqrt{a+b\sqrt{x}}\,{}_2F_1\left(-\frac{1}{2},2;\ 3;\ -\frac{b\sqrt{x}}{a}\right)}{\sqrt{\frac{b\sqrt{x}}{a}+1}}$$

input
```
int((a + b*x^(1/2))^(1/2),x)
```

output
```
(x*(a + b*x^(1/2))^(1/2)*hypergeom([-1/2, 2], 3, -(b*x^(1/2))/a))/((b*x^(1
/2))/a + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

$$\int \sqrt{a+b\sqrt{x}}\,dx=\frac{4\sqrt{\sqrt{x}\,b+a}\,\left(\sqrt{x}\,ab-2a^2+3b^2x\right)}{15b^2}$$

input
```
int((a+b*x^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*b + a)*(sqrt(x)*a*b - 2*a**2 + 3*b**2*x))/(15*b**2)
```

3.126 $\int \frac{\sqrt{a+b\sqrt{x}}}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x} \, dx = 4\sqrt{a+b\sqrt{x}} - 4\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$$

output
```
4*(a+b*x^(1/2))^(1/2)-4*a^(1/2)*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x} \, dx = 4\sqrt{a+b\sqrt{x}} - 4\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[x]]/x,x]
```

output
```
4*Sqrt[a + b*Sqrt[x]] - 4*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x} \, dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{\sqrt{a+b\sqrt{x}}}{\sqrt{x}}d\sqrt{x}$$
$$\Big\downarrow \text{60}$$
$$2\left(a\int \frac{1}{\sqrt{a+b\sqrt{x}}\sqrt{x}}d\sqrt{x}+2\sqrt{a+b\sqrt{x}}\right)$$
$$\Big\downarrow \text{73}$$
$$2\left(\frac{2a\int \frac{1}{\frac{x}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{x}}}{b}+2\sqrt{a+b\sqrt{x}}\right)$$
$$\Big\downarrow \text{221}$$
$$2\left(2\sqrt{a+b\sqrt{x}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x]]/x,x]
```

output
```
2*(2*Sqrt[a + b*Sqrt[x]] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$4\sqrt{a+b\sqrt{x}}-4\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$	32
default	$4\sqrt{a+b\sqrt{x}}-4\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$	32

input
```
int((a+b*x^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
4*(a+b*x^(1/2))^(1/2)-4*a^(1/2)*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx = \left[2\sqrt{a}\log\left(\frac{bx-2\sqrt{b\sqrt{x}+a}\sqrt{a}\sqrt{x}+2a\sqrt{x}}{x}\right)+4\sqrt{b\sqrt{x}+a},\,4\sqrt{-a}\arctan\left(\frac{\left(\sqrt{-a}b\sqrt{x}-\sqrt{-a}a\right)\sqrt{b\sqrt{x}+a}}{b^2x-a^2}\right)+4\sqrt{b\sqrt{x}+a}\right]$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[2*sqrt(a)*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2*a*sqrt(x))
/x) + 4*sqrt(b*sqrt(x) + a), 4*sqrt(-a)*arctan((sqrt(-a)*b*sqrt(x) - sqrt(
-a)*a)*sqrt(b*sqrt(x) + a)/(b^2*x - a^2)) + 4*sqrt(b*sqrt(x) + a)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.

Time = 1.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx = -4\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)+\frac{4a}{\sqrt{b}\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{4\sqrt{b}\sqrt[4]{x}}{\sqrt{\frac{a}{b\sqrt{x}}+1}}$$

input
```
integrate((a+b*x**(1/2))**(1/2)/x,x)
```

output
```
-4*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(1/4))) + 4*a/(sqrt(b)*x**(1/4)*sqrt(
a/(b*sqrt(x)) + 1)) + 4*sqrt(b)*x**(1/4)/sqrt(a/(b*sqrt(x)) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx = 2\sqrt{a}\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)+4\sqrt{b\sqrt{x}+a}$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
2*sqrt(a)*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(
a))) + 4*sqrt(b*sqrt(x) + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx = 4\,b\left(\frac{a\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b}+\frac{\sqrt{b\sqrt{x}+a}}{b}\right)$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
4*b*(a*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*b) + sqrt(b*sqrt(x)
+ a)/b)
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx = 4\sqrt{a+b\sqrt{x}}-4\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)$$

input
```
int((a + b*x^(1/2))^(1/2)/x,x)
```

output
```
4*(a + b*x^(1/2))^(1/2) - 4*a^(1/2)*atanh((a + b*x^(1/2))^(1/2)/a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x}\,dx = 4\sqrt{\sqrt{x}\,b+a}+2\sqrt{a}\log\left(\sqrt{\sqrt{x}\,b+a}-\sqrt{a}\right) - 2\sqrt{a}\log\left(\sqrt{\sqrt{x}\,b+a}+\sqrt{a}\right)$$

input
```
int((a+b*x^(1/2))^(1/2)/x,x)
```

output
```
2*(2*sqrt(sqrt(x)*b + a) + sqrt(a)*log(sqrt(sqrt(x)*b + a) - sqrt(a)) - sq
rt(a)*log(sqrt(sqrt(x)*b + a) + sqrt(a)))
```

3.127 $\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx = -\frac{\sqrt{a+b\sqrt{x}}}{x} - \frac{b\sqrt{a+b\sqrt{x}}}{2a\sqrt{x}} + \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-(a+b*x^(1/2))^(1/2)/x-1/2*b*(a+b*x^(1/2))^(1/2)/a/x^(1/2)+1/2*b^2*arctanh
((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx = \frac{\left(-2a-b\sqrt{x}\right)\sqrt{a+b\sqrt{x}}}{2ax} + \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x]]/x^2,x]
```

output
```
((-2*a - b*Sqrt[x])*Sqrt[a + b*Sqrt[x]])/(2*a*x) + (b^2*ArcTanh[Sqrt[a + b
*Sqrt[x]]/Sqrt[a]])/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\sqrt{a+b\sqrt{x}}}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{51}$$

$$2\left(\frac{1}{4}b\int \frac{1}{\sqrt{a+b\sqrt{x}}x}d\sqrt{x}-\frac{\sqrt{a+b\sqrt{x}}}{2x}\right)$$

$$\Big\downarrow \text{52}$$

$$2\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{a+b\sqrt{x}}\sqrt{x}}d\sqrt{x}}{2a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{2x}\right)$$

$$\Big\downarrow \text{73}$$

$$2\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{x}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{x}}}{a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{2x}\right)$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{2x}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x]]/x^2,x]
```

output
```
2*(-1/2*Sqrt[a + b*Sqrt[x]]/x + (b*(-(Sqrt[a + b*Sqrt[x]]/(a*Sqrt[x])) + (
b*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/a^(3/2)))/4)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$4b^{2}\left(-\frac{\frac{(a+b\sqrt{x})^{\frac{3}{2}}}{8a}+\frac{\sqrt{a+b\sqrt{x}}}{8}}{b^{2}x}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	60
default	$4b^{2}\left(-\frac{\frac{(a+b\sqrt{x})^{\frac{3}{2}}}{8a}+\frac{\sqrt{a+b\sqrt{x}}}{8}}{b^{2}x}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	60

input
```
int((a+b*x^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
4*b^2*(-(1/8/a*(a+b*x^(1/2))^(3/2)+1/8*(a+b*x^(1/2))^(1/2))/b^2/x+1/8/a^(3
/2)*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx = \left[\frac{\sqrt{a}b^{2}x\log\left(\frac{bx+2\sqrt{b\sqrt{x}+a}\sqrt{a}\sqrt{x}+2\,a\sqrt{x}}{x}\right)-2\left(ab\sqrt{x}+2\,a^{2}\right)\sqrt{b\sqrt{x}+a}}{4\,a^{2}x}, -\frac{\sqrt{-a}b^{2}x\arctan\left(\frac{\left(\sqrt{-a}b\sqrt{x}-\sqrt{-a}a\right)\sqrt{b\sqrt{x}+a}}{b^{2}x-a^{2}}\right)+\left(ab\sqrt{x}+2\,a^{2}\right)\sqrt{b\sqrt{x}+a}}{2\,a^{2}x}\right]$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b^2*x*log((b*x + 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2*a
*sqrt(x))/x) - 2*(a*b*sqrt(x) + 2*a^2)*sqrt(b*sqrt(x) + a))/(a^2*x), -1/2*
(sqrt(-a)*b^2*x*arctan((sqrt(-a)*b*sqrt(x) - sqrt(-a)*a)*sqrt(b*sqrt(x) +
a)/(b^2*x - a^2)) + (a*b*sqrt(x) + 2*a^2)*sqrt(b*sqrt(x) + a))/(a^2*x)]
```

Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx = -\frac{a}{\sqrt{b}x^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} - \frac{3\sqrt{b}}{2x^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} - \frac{b^{\frac{3}{2}}}{2a\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}} + \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((a+b*x**(1/2))**(1/2)/x**2,x)
```

output
```
-a/(sqrt(b)*x**(5/4)*sqrt(a/(b*sqrt(x)) + 1)) - 3*sqrt(b)/(2*x**(3/4)*sqrt
(a/(b*sqrt(x)) + 1)) - b**(3/2)/(2*a*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1)) + b
**2*asinh(sqrt(a)/(sqrt(b)*x**(1/4)))/(2*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx = -\frac{b^2\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}} - \frac{\left(b\sqrt{x}+a\right)^{\frac{3}{2}}b^2+\sqrt{b\sqrt{x}+a}\,ab^2}{2\left(\left(b\sqrt{x}+a\right)^2a-2\left(b\sqrt{x}+a\right)a^2+a^3\right)}$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a
)))/a^(3/2) - 1/2*((b*sqrt(x) + a)^(3/2)*b^2 + sqrt(b*sqrt(x) + a)*a*b^2)/
((b*sqrt(x) + a)^2*a - 2*(b*sqrt(x) + a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx = -\frac{1}{2}\,b^3\left(\frac{\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab} + \frac{\left(b\sqrt{x}+a\right)^{\frac{3}{2}} + \sqrt{b\sqrt{x}+a}\,a}{ab^3x}\right)$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/2*b^3*(arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a*b) + ((b*sqrt(x
) + a)^(3/2) + sqrt(b*sqrt(x) + a)*a)/(a*b^3*x))
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,\sqrt{x}}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{\sqrt{a+b\,\sqrt{x}}}{2\,x} - \frac{\left(a+b\,\sqrt{x}\right)^{3/2}}{2\,a\,x}$$

input
```
int((a + b*x^(1/2))^(1/2)/x^2,x)
```

output
```
(b^2*atanh((a + b*x^(1/2))^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^(1/2))^(
1/2)/(2*x) - (a + b*x^(1/2))^(3/2)/(2*a*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^2}\,dx$$
$$= \frac{-2\sqrt{x}\,\sqrt{\sqrt{x}\,b+a}\,ab - 4\sqrt{\sqrt{x}\,b+a}\,a^2 - \sqrt{a}\log\left(\sqrt{\sqrt{x}\,b+a}-\sqrt{a}\right)b^2x + \sqrt{a}\log\left(\sqrt{\sqrt{x}\,b+a}+\sqrt{a}\right)b}{4a^2x}$$

input
```
int((a+b*x^(1/2))^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(x)*sqrt(sqrt(x)*b + a)*a*b - 4*sqrt(sqrt(x)*b + a)*a**2 - sqrt(
a)*log(sqrt(sqrt(x)*b + a) - sqrt(a))*b**2*x + sqrt(a)*log(sqrt(sqrt(x)*b
+ a) + sqrt(a))*b**2*x)/(4*a**2*x)
```

3.128 $\int \frac{\sqrt{a+b\sqrt{x}}}{x^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3} \, dx = -\frac{\sqrt{a+b\sqrt{x}}}{2x^2} - \frac{b\sqrt{a+b\sqrt{x}}}{12ax^{3/2}} + \frac{5b^2\sqrt{a+b\sqrt{x}}}{48a^2x} - \frac{5b^3\sqrt{a+b\sqrt{x}}}{32a^3\sqrt{x}} + \frac{5b^4 \text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{7/2}}$$

output
```
-1/2*(a+b*x^(1/2))^(1/2)/x^2-1/12*b*(a+b*x^(1/2))^(1/2)/a/x^(3/2)+5/48*b^2
*(a+b*x^(1/2))^(1/2)/a^2/x-5/32*b^3*(a+b*x^(1/2))^(1/2)/a^3/x^(1/2)+5/32*b
^4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3} \, dx = \frac{\sqrt{a+b\sqrt{x}}\left(-48a^3 - 8a^2b\sqrt{x} + 10ab^2x - 15b^3x^{3/2}\right)}{96a^3x^2} + \frac{5b^4 \text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{7/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[x]]/x^3,x]
```

output
```
(Sqrt[a + b*Sqrt[x]]*(-48*a^3 - 8*a^2*b*Sqrt[x] + 10*a*b^2*x - 15*b^3*x^(3
/2)))/(96*a^3*x^2) + (5*b^4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/(32*a^(7
/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {798, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3} \, dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\sqrt{a+b\sqrt{x}}}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{51}$$

$$2\left(\frac{1}{8}b\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2}d\sqrt{x}-\frac{\sqrt{a+b\sqrt{x}}}{4x^2}\right)$$

$$\Big\downarrow \text{52}$$

$$2\left(\frac{1}{8}b\left(-\frac{5b\int \frac{1}{\sqrt{a+b\sqrt{x}}x^{3/2}}d\sqrt{x}}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{4x^2}\right)$$

$$\Big\downarrow \text{52}$$

$$2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{\sqrt{a+b\sqrt{x}}x}d\sqrt{x}}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{4x^2}\right)$$

$$\Big\downarrow \text{52}$$

$$2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{a+b\sqrt{x}}\sqrt{x}}d\sqrt{x}}{2a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{4x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{x}}}{a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{4x^2}\right)$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)-\frac{\sqrt{a+b\sqrt{x}}}{4x^2}\right)$$

input
```
Int[Sqrt[a + b*Sqrt[x]]/x^3,x]
```

output
```
2*(-1/4*Sqrt[a + b*Sqrt[x]]/x^2 + (b*(-1/3*Sqrt[a + b*Sqrt[x]]/(a*x^(3/2))
 - (5*b*(-1/2*Sqrt[a + b*Sqrt[x]]/(a*x) - (3*b*(-(Sqrt[a + b*Sqrt[x]]/(a*S
qrt[x])) + (b*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a
)))/8)
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$4b^4\left(-\frac{\frac{5(a+b\sqrt{x})^{\frac{7}{2}}}{128a^3}-\frac{55(a+b\sqrt{x})^{\frac{5}{2}}}{384a^2}+\frac{73(a+b\sqrt{x})^{\frac{3}{2}}}{384a}+\frac{5\sqrt{a+b\sqrt{x}}}{128}}{b^4x^2}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{128a^{\frac{7}{2}}}\right)$	88
default	$4b^4\left(-\frac{\frac{5(a+b\sqrt{x})^{\frac{7}{2}}}{128a^3}-\frac{55(a+b\sqrt{x})^{\frac{5}{2}}}{384a^2}+\frac{73(a+b\sqrt{x})^{\frac{3}{2}}}{384a}+\frac{5\sqrt{a+b\sqrt{x}}}{128}}{b^4x^2}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{128a^{\frac{7}{2}}}\right)$	88

input
```
int((a+b*x^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
4*b^4*(-(5/128/a^3*(a+b*x^(1/2))^(7/2)-55/384/a^2*(a+b*x^(1/2))^(5/2)+73/3
84/a*(a+b*x^(1/2))^(3/2)+5/128*(a+b*x^(1/2))^(1/2))/b^4/x^2+5/128/a^(7/2)*
arctanh((a+b*x^(1/2))^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx = \left[\frac{15\,\sqrt{a}b^4x^2\log\left(\frac{bx+2\,\sqrt{b\sqrt{x}+a}\sqrt{a}\sqrt{x}+2\,a\sqrt{x}}{x}\right)+2\,\left(10\,a^2b^2x-48\,a^4-\left(15\,ab^3x+8\,a^3b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{192\,a^4x^2},\ -\frac{15\,\sqrt{-a}b^4x^2\arctan\left(\frac{\left(\sqrt{-a}b\sqrt{x}-\sqrt{-a}a\right)\sqrt{b\sqrt{x}+a}}{b^2x-a^2}\right)-\left(10\,a^2b^2x-48\,a^4-\left(15\,ab^3x+8\,a^3b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{96\,a^4x^2}\right]$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/192*(15*sqrt(a)*b^4*x^2*log((b*x + 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x
) + 2*a*sqrt(x))/x) + 2*(10*a^2*b^2*x - 48*a^4 - (15*a*b^3*x + 8*a^3*b)*sq
rt(x))*sqrt(b*sqrt(x) + a))/(a^4*x^2), -1/96*(15*sqrt(-a)*b^4*x^2*arctan((
sqrt(-a)*b*sqrt(x) - sqrt(-a)*a)*sqrt(b*sqrt(x) + a)/(b^2*x - a^2)) - (10*
a^2*b^2*x - 48*a^4 - (15*a*b^3*x + 8*a^3*b)*sqrt(x))*sqrt(b*sqrt(x) + a))/
(a^4*x^2)]
```

Sympy [A] (verification not implemented)

Time = 23.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx = -\frac{a}{2\sqrt{b}x^{\frac{9}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} - \frac{7\sqrt{b}}{12x^{\frac{7}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} + \frac{b^{\frac{3}{2}}}{48ax^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} - \frac{5b^{\frac{5}{2}}}{96a^2x^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} - \frac{5b^{\frac{7}{2}}}{32a^3\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}} + \frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)}{32a^{\frac{7}{2}}}$$

input
```
integrate((a+b*x**(1/2))**(1/2)/x**3,x)
```

output
```
-a/(2*sqrt(b)*x**(9/4)*sqrt(a/(b*sqrt(x)) + 1)) - 7*sqrt(b)/(12*x**(7/4)*s
qrt(a/(b*sqrt(x)) + 1)) + b**(3/2)/(48*a*x**(5/4)*sqrt(a/(b*sqrt(x)) + 1))
 - 5*b**(5/2)/(96*a**2*x**(3/4)*sqrt(a/(b*sqrt(x)) + 1)) - 5*b**(7/2)/(32*
a**3*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*x**
(1/4)))/(32*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx = -\frac{5\,b^4\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{64\,a^{\frac{7}{2}}} - \frac{15\left(b\sqrt{x}+a\right)^{\frac{7}{2}}b^4 - 55\left(b\sqrt{x}+a\right)^{\frac{5}{2}}ab^4 + 73\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2b^4 + 15\sqrt{b\sqrt{x}+a}a^3b^4}{96\left(\left(b\sqrt{x}+a\right)^4a^3 - 4\left(b\sqrt{x}+a\right)^3a^4 + 6\left(b\sqrt{x}+a\right)^2a^5 - 4\left(b\sqrt{x}+a\right)a^6 + a^7\right)}$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-5/64*b^4*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(
a)))/a^(7/2) - 1/96*(15*(b*sqrt(x) + a)^(7/2)*b^4 - 55*(b*sqrt(x) + a)^(5/
2)*a*b^4 + 73*(b*sqrt(x) + a)^(3/2)*a^2*b^4 + 15*sqrt(b*sqrt(x) + a)*a^3*b
^4)/((b*sqrt(x) + a)^4*a^3 - 4*(b*sqrt(x) + a)^3*a^4 + 6*(b*sqrt(x) + a)^2
*a^5 - 4*(b*sqrt(x) + a)*a^6 + a^7)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx =$$
$$-\frac{1}{96}\,b^5\left(\frac{15\,\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}+\frac{15\,\left(b\sqrt{x}+a\right)^{\frac{7}{2}}-55\,\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a+73\,\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2+15\,\sqrt{b\sqrt{x}+a}\right.}{a^3b^5x^2}$$

input
```
integrate((a+b*x^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/96*b^5*(15*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a^3*b) + (15*
(b*sqrt(x) + a)^(7/2) - 55*(b*sqrt(x) + a)^(5/2)*a + 73*(b*sqrt(x) + a)^(3
/2)*a^2 + 15*sqrt(b*sqrt(x) + a)*a^3)/(a^3*b^5*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx = \frac{55\,\left(a+b\,\sqrt{x}\right)^{5/2}}{96\,a^2\,x^2}-\frac{73\,\left(a+b\,\sqrt{x}\right)^{3/2}}{96\,a\,x^2}-\frac{5\,\sqrt{a+b\,\sqrt{x}}}{32\,x^2}-\frac{5\,\left(a+b\,\sqrt{x}\right)^{7/2}}{32\,a^3\,x^2}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,\sqrt{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{32\,a^{7/2}}$$

input
```
int((a + b*x^(1/2))^(1/2)/x^3,x)
```

output
```
(55*(a + b*x^(1/2))^(5/2))/(96*a^2*x^2) - (b^4*atan(((a + b*x^(1/2))^(1/2)
*1i)/a^(1/2))*5i)/(32*a^(7/2)) - (73*(a + b*x^(1/2))^(3/2))/(96*a*x^2) - (
5*(a + b*x^(1/2))^(1/2))/(32*x^2) - (5*(a + b*x^(1/2))^(7/2))/(32*a^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\sqrt{x}}}{x^3}\,dx$$
$$= \frac{-16\sqrt{x}\,\sqrt{\sqrt{x}\,b+a}\,a^3b-30\sqrt{x}\,\sqrt{\sqrt{x}\,b+a}\,a\,b^3x-96\sqrt{\sqrt{x}\,b+a}\,a^4+20\sqrt{\sqrt{x}\,b+a}\,a^2b^2x-15\sqrt{a}\log\Big(}{192a^4x^2}$$

input
```
int((a+b*x^(1/2))^(1/2)/x^3,x)
```

output
```
( - 16*sqrt(x)*sqrt(sqrt(x)*b + a)*a**3*b - 30*sqrt(x)*sqrt(sqrt(x)*b + a)
*a*b**3*x - 96*sqrt(sqrt(x)*b + a)*a**4 + 20*sqrt(sqrt(x)*b + a)*a**2*b**2
*x - 15*sqrt(a)*log(sqrt(sqrt(x)*b + a) - sqrt(a))*b**4*x**2 + 15*sqrt(a)*
log(sqrt(sqrt(x)*b + a) + sqrt(a))*b**4*x**2)/(192*a**4*x**2)
```

3.129 $\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = -\frac{4a^5\sqrt{a+b\sqrt{x}}}{b^6}+\frac{20a^4\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}-\frac{8a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^6}+\frac{40a^2\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}-\frac{20a\left(a+b\sqrt{x}\right)^{9/2}}{9b^6}+\frac{4\left(a+b\sqrt{x}\right)^{11/2}}{11b^6}$$

output
```
-4*a^5*(a+b*x^(1/2))^(1/2)/b^6+20/3*a^4*(a+b*x^(1/2))^(3/2)/b^6-8*a^3*(a+b
*x^(1/2))^(5/2)/b^6+40/7*a^2*(a+b*x^(1/2))^(7/2)/b^6-20/9*a*(a+b*x^(1/2))^
(9/2)/b^6+4/11*(a+b*x^(1/2))^(11/2)/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\sqrt{a+b\sqrt{x}}\left(-256a^5+128a^4b\sqrt{x}-96a^3b^2x+80a^2b^3x^{3/2}-70ab^4x^2+63b^5x^{5/2}\right)}{693b^6}$$

input
```
Integrate[x^2/Sqrt[a + b*Sqrt[x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[x]]*(-256*a^5 + 128*a^4*b*Sqrt[x] - 96*a^3*b^2*x + 80*a
^2*b^3*x^(3/2) - 70*a*b^4*x^2 + 63*b^5*x^(5/2)))/(693*b^6)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{5/2}}{\sqrt{a+b\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{a^5}{b^5\sqrt{a+b\sqrt{x}}}+\frac{5\sqrt{a+b\sqrt{x}}a^4}{b^5}-\frac{10\left(a+b\sqrt{x}\right)^{3/2}a^3}{b^5}+\frac{10\left(a+b\sqrt{x}\right)^{5/2}a^2}{b^5}-\frac{5\left(a+b\sqrt{x}\right)^{7/2}a}{b^5}+\frac{(a+b\sqrt{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{2a^5\sqrt{a+b\sqrt{x}}}{b^6}+\frac{10a^4\left(a+b\sqrt{x}\right)^{3/2}}{3b^6}-\frac{4a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^6}+\frac{20a^2\left(a+b\sqrt{x}\right)^{7/2}}{7b^6}+\frac{2\left(a+b\sqrt{x}\right)^{11/2}}{11b^6}-\frac{10a(a\right.$$

input
```
Int[x^2/Sqrt[a + b*Sqrt[x]],x]
```

output
```
2*((-2*a^5*Sqrt[a + b*Sqrt[x]])/b^6 + (10*a^4*(a + b*Sqrt[x])^(3/2))/(3*b^
6) - (4*a^3*(a + b*Sqrt[x])^(5/2))/b^6 + (20*a^2*(a + b*Sqrt[x])^(7/2))/(7
*b^6) - (10*a*(a + b*Sqrt[x])^(9/2))/(9*b^6) + (2*(a + b*Sqrt[x])^(11/2))/
(11*b^6))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{\frac{4(a+b\sqrt{x})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{x})^{\frac{9}{2}}}{9}+\frac{40a^2(a+b\sqrt{x})^{\frac{7}{2}}}{7}-8a^3(a+b\sqrt{x})^{\frac{5}{2}}+\frac{20a^4(a+b\sqrt{x})^{\frac{3}{2}}}{3}-4a^5\sqrt{a+b\sqrt{x}}}{b^6}$	86
default	$\frac{\frac{4(a+b\sqrt{x})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{x})^{\frac{9}{2}}}{9}+\frac{40a^2(a+b\sqrt{x})^{\frac{7}{2}}}{7}-8a^3(a+b\sqrt{x})^{\frac{5}{2}}+\frac{20a^4(a+b\sqrt{x})^{\frac{3}{2}}}{3}-4a^5\sqrt{a+b\sqrt{x}}}{b^6}$	86

input
```
int(x^2/(a+b*x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/b^6*(1/11*(a+b*x^(1/2))^(11/2)-5/9*a*(a+b*x^(1/2))^(9/2)+10/7*a^2*(a+b*x
^(1/2))^(7/2)-2*a^3*(a+b*x^(1/2))^(5/2)+5/3*a^4*(a+b*x^(1/2))^(3/2)-a^5*(a
+b*x^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = -\frac{4\left(70\,ab^4x^2+96\,a^3b^2x+256\,a^5-\left(63\,b^5x^2+80\,a^2b^3x+128\,a^4b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{693\,b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/693*(70*a*b^4*x^2 + 96*a^3*b^2*x + 256*a^5 - (63*b^5*x^2 + 80*a^2*b^3*x
 + 128*a^4*b)*sqrt(x))*sqrt(b*sqrt(x) + a)/b^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8356 vs. 2(122) = 244.

Time = 5.72 (sec) , antiderivative size = 8356, normalized size of antiderivative = 64.28

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = \text{Too large to display}$$

input
```
integrate(x**2/(a+b*x**(1/2))**(1/2),x)
```

output
```
-1024*a**(151/2)*x**18*sqrt(1 + b*sqrt(x)/a)/(693*a**70*b**6*x**18 + 10395
*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19 + 315315*a**67*b**9*x**(39/
2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b**11*x**(41/2) + 3468465*a*
*64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2) + 4459455*a**62*b**14*x**2
2 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**60*b**16*x**23 + 945945*a**
59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 + 72765*a**57*b**19*x**(49/2
) + 10395*a**56*b**20*x**25 + 693*a**55*b**21*x**(51/2)) + 1024*a**(151/2)
*x**18/(693*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b*
*8*x**19 + 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 208107
9*a**65*b**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*
x**(43/2) + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 20
81079*a**60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**1
8*x**24 + 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**5
5*b**21*x**(51/2)) - 14848*a**(149/2)*b*x**(37/2)*sqrt(1 + b*sqrt(x)/a)/(6
93*a**70*b**6*x**18 + 10395*a**69*b**7*x**(37/2) + 72765*a**68*b**8*x**19
+ 315315*a**67*b**9*x**(39/2) + 945945*a**66*b**10*x**20 + 2081079*a**65*b
**11*x**(41/2) + 3468465*a**64*b**12*x**21 + 4459455*a**63*b**13*x**(43/2)
 + 4459455*a**62*b**14*x**22 + 3468465*a**61*b**15*x**(45/2) + 2081079*a**
60*b**16*x**23 + 945945*a**59*b**17*x**(47/2) + 315315*a**58*b**18*x**24 +
 72765*a**57*b**19*x**(49/2) + 10395*a**56*b**20*x**25 + 693*a**55*b**2...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(b\sqrt{x}+a\right)^{\frac{11}{2}}}{11\,b^6} - \frac{20\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a}{9\,b^6} + \frac{40\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^2}{7\,b^6} - \frac{8\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^3}{b^6} + \frac{20\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^4}{3\,b^6} - \frac{4\sqrt{b\sqrt{x}+a}a^5}{b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/11*(b*sqrt(x) + a)^(11/2)/b^6 - 20/9*(b*sqrt(x) + a)^(9/2)*a/b^6 + 40/7*
(b*sqrt(x) + a)^(7/2)*a^2/b^6 - 8*(b*sqrt(x) + a)^(5/2)*a^3/b^6 + 20/3*(b*
sqrt(x) + a)^(3/2)*a^4/b^6 - 4*sqrt(b*sqrt(x) + a)*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(63\left(b\sqrt{x}+a\right)^{\frac{11}{2}} - 385\left(b\sqrt{x}+a\right)^{\frac{9}{2}}a + 990\left(b\sqrt{x}+a\right)^{\frac{7}{2}}a^2 - 1386\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a^3 + 1155\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a\right.}{693\,b^6}$$

input
```
integrate(x^2/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/693*(63*(b*sqrt(x) + a)^(11/2) - 385*(b*sqrt(x) + a)^(9/2)*a + 990*(b*sq
rt(x) + a)^(7/2)*a^2 - 1386*(b*sqrt(x) + a)^(5/2)*a^3 + 1155*(b*sqrt(x) +
a)^(3/2)*a^4 - 693*sqrt(b*sqrt(x) + a)*a^5)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(a+b\sqrt{x}\right)^{11/2}}{11\,b^6} - \frac{20\,a\left(a+b\sqrt{x}\right)^{9/2}}{9\,b^6} - \frac{4\,a^5\sqrt{a+b\sqrt{x}}}{b^6} + \frac{20\,a^4\left(a+b\sqrt{x}\right)^{3/2}}{3\,b^6} - \frac{8\,a^3\left(a+b\sqrt{x}\right)^{5/2}}{b^6} + \frac{40\,a^2\left(a+b\sqrt{x}\right)^{7/2}}{7\,b^6}$$

input
```
int(x^2/(a + b*x^(1/2))^(1/2),x)
```

output
```
(4*(a + b*x^(1/2))^(11/2))/(11*b^6) - (20*a*(a + b*x^(1/2))^(9/2))/(9*b^6)
 - (4*a^5*(a + b*x^(1/2))^(1/2))/b^6 + (20*a^4*(a + b*x^(1/2))^(3/2))/(3*b
^6) - (8*a^3*(a + b*x^(1/2))^(5/2))/b^6 + (40*a^2*(a + b*x^(1/2))^(7/2))/(
7*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{\sqrt{a+b\sqrt{x}}}\,dx$$
$$= \frac{4\sqrt{\sqrt{x}\,b+a}\,\left(128\sqrt{x}\,a^4b+80\sqrt{x}\,a^2b^3x+63\sqrt{x}\,b^5x^2-256a^5-96a^3b^2x-70a\,b^4x^2\right)}{693b^6}$$

input
```
int(x^2/(a+b*x^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*b + a)*(128*sqrt(x)*a**4*b + 80*sqrt(x)*a**2*b**3*x + 63*s
qrt(x)*b**5*x**2 - 256*a**5 - 96*a**3*b**2*x - 70*a*b**4*x**2))/(693*b**6)
```

3.130 $\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = -\frac{4a^3\sqrt{a+b\sqrt{x}}}{b^4}+\frac{4a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^4}-\frac{12a\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}+\frac{4\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}$$

output
```
-4*a^3*(a+b*x^(1/2))^(1/2)/b^4+4*a^2*(a+b*x^(1/2))^(3/2)/b^4-12/5*a*(a+b*x
^(1/2))^(5/2)/b^4+4/7*(a+b*x^(1/2))^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\sqrt{a+b\sqrt{x}}\left(-16a^3+8a^2b\sqrt{x}-6ab^2x+5b^3x^{3/2}\right)}{35b^4}$$

input
```
Integrate[x/Sqrt[a + b*Sqrt[x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[x]]*(-16*a^3 + 8*a^2*b*Sqrt[x] - 6*a*b^2*x + 5*b^3*x^(3
/2)))/(35*b^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{x^{3/2}}{\sqrt{a+b\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{a^3}{b^3\sqrt{a+b\sqrt{x}}}+\frac{3\sqrt{a+b\sqrt{x}}a^2}{b^3}-\frac{3\left(a+b\sqrt{x}\right)^{3/2}a}{b^3}+\frac{\left(a+b\sqrt{x}\right)^{5/2}}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{2a^3\sqrt{a+b\sqrt{x}}}{b^4}+\frac{2a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^4}+\frac{2\left(a+b\sqrt{x}\right)^{7/2}}{7b^4}-\frac{6a\left(a+b\sqrt{x}\right)^{5/2}}{5b^4}\right)$$

input
```
Int[x/Sqrt[a + b*Sqrt[x]],x]
```

output
```
2*((-2*a^3*Sqrt[a + b*Sqrt[x]])/b^4 + (2*a^2*(a + b*Sqrt[x])^(3/2))/b^4 -
(6*a*(a + b*Sqrt[x])^(5/2))/(5*b^4) + (2*(a + b*Sqrt[x])^(7/2))/(7*b^4))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{x}\right)^{\frac{7}{2}}}{7}-\frac{12a\left(a+b\sqrt{x}\right)^{\frac{5}{2}}}{5}+4a^{2}\left(a+b\sqrt{x}\right)^{\frac{3}{2}}-4a^{3}\sqrt{a+b\sqrt{x}}}{b^{4}}$	57
default	$\frac{\frac{4\left(a+b\sqrt{x}\right)^{\frac{7}{2}}}{7}-\frac{12a\left(a+b\sqrt{x}\right)^{\frac{5}{2}}}{5}+4a^{2}\left(a+b\sqrt{x}\right)^{\frac{3}{2}}-4a^{3}\sqrt{a+b\sqrt{x}}}{b^{4}}$	57

input
```
int(x/(a+b*x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/b^4*(1/7*(a+b*x^(1/2))^(7/2)-3/5*a*(a+b*x^(1/2))^(5/2)+a^2*(a+b*x^(1/2))
^(3/2)-a^3*(a+b*x^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = -\frac{4\left(6\,ab^{2}x+16\,a^{3}-\left(5\,b^{3}x+8\,a^{2}b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{35\,b^{4}}$$

input
```
integrate(x/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/35*(6*a*b^2*x + 16*a^3 - (5*b^3*x + 8*a^2*b)*sqrt(x))*sqrt(b*sqrt(x) +
a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1872 vs. 2(78) = 156.

Time = 1.70 (sec) , antiderivative size = 1872, normalized size of antiderivative = 22.29

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*x**(1/2))**(1/2),x)
```

output
```
-64*a**(47/2)*x**8*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b
**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16
*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 64*a**(47
/2)*x**8/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x
**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**
(21/2) + 35*a**14*b**10*x**11) - 352*a**(45/2)*b*x**(17/2)*sqrt(1 + b*sqrt
(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**
9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(2
1/2) + 35*a**14*b**10*x**11) + 384*a**(45/2)*b*x**(17/2)/(35*a**20*b**4*x*
*8 + 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(1
9/2) + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x*
*11) - 792*a**(43/2)*b**2*x**9*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 +
 210*a**19*b**5*x**(17/2) + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2)
 + 525*a**16*b**8*x**10 + 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11)
 + 960*a**(43/2)*b**2*x**9/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2)
+ 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 +
210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) - 924*a**(41/2)*b**3*x**(
19/2)*sqrt(1 + b*sqrt(x)/a)/(35*a**20*b**4*x**8 + 210*a**19*b**5*x**(17/2)
 + 525*a**18*b**6*x**9 + 700*a**17*b**7*x**(19/2) + 525*a**16*b**8*x**10 +
 210*a**15*b**9*x**(21/2) + 35*a**14*b**10*x**11) + 1280*a**(41/2)*b**3...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(b\sqrt{x}+a\right)^{\frac{7}{2}}}{7\,b^4} - \frac{12\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a}{5\,b^4} + \frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2}{b^4} - \frac{4\,\sqrt{b\sqrt{x}+a}a^3}{b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/7*(b*sqrt(x) + a)^(7/2)/b^4 - 12/5*(b*sqrt(x) + a)^(5/2)*a/b^4 + 4*(b*sq
rt(x) + a)^(3/2)*a^2/b^4 - 4*sqrt(b*sqrt(x) + a)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(5\left(b\sqrt{x}+a\right)^{\frac{7}{2}}-21\left(b\sqrt{x}+a\right)^{\frac{5}{2}}a+35\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2-35\sqrt{b\sqrt{x}+a}a^3\right)}{35\,b^4}$$

input
```
integrate(x/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/35*(5*(b*sqrt(x) + a)^(7/2) - 21*(b*sqrt(x) + a)^(5/2)*a + 35*(b*sqrt(x)
 + a)^(3/2)*a^2 - 35*sqrt(b*sqrt(x) + a)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(a+b\sqrt{x}\right)^{7/2}}{7\,b^4}-\frac{12\,a\left(a+b\sqrt{x}\right)^{5/2}}{5\,b^4}-\frac{4\,a^3\sqrt{a+b\sqrt{x}}}{b^4}+\frac{4\,a^2\left(a+b\sqrt{x}\right)^{3/2}}{b^4}$$

input
```
int(x/(a + b*x^(1/2))^(1/2),x)
```

output
```
(4*(a + b*x^(1/2))^(7/2))/(7*b^4) - (12*a*(a + b*x^(1/2))^(5/2))/(5*b^4) -
 (4*a^3*(a + b*x^(1/2))^(1/2))/b^4 + (4*a^2*(a + b*x^(1/2))^(3/2))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.49

$$\int \frac{x}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\sqrt{\sqrt{x}\,b+a}\,\left(8\sqrt{x}\,a^2b+5\sqrt{x}\,b^3x-16a^3-6a\,b^2x\right)}{35b^4}$$

input
```
int(x/(a+b*x^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*b + a)*(8*sqrt(x)*a**2*b + 5*sqrt(x)*b**3*x - 16*a**3 - 6*
a*b**2*x))/(35*b**4)
```

3.131 $\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx=-\frac{4a\sqrt{a+b\sqrt{x}}}{b^2}+\frac{4\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}$$

output
```
-4*a*(a+b*x^(1/2))^(1/2)/b^2+4/3*(a+b*x^(1/2))^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx=\frac{4\left(-2a+b\sqrt{x}\right)\sqrt{a+b\sqrt{x}}}{3b^2}$$

input
```
Integrate[1/Sqrt[a + b*Sqrt[x]],x]
```

output
```
(4*(-2*a + b*Sqrt[x])*Sqrt[a + b*Sqrt[x]])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{774}$$

$$2\int \frac{\sqrt{x}}{\sqrt{a+b\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{\sqrt{a+b\sqrt{x}}}{b}-\frac{a}{b\sqrt{a+b\sqrt{x}}}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{2\left(a+b\sqrt{x}\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a+b\sqrt{x}}}{b^2}\right)$$

input
```
Int[1/Sqrt[a + b*Sqrt[x]],x]
```

output
```
2*((-2*a*Sqrt[a + b*Sqrt[x]])/b^2 + (2*(a + b*Sqrt[x])^(3/2))/(3*b^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{x}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{x}}}{b^{2}}$	30
default	$\frac{\frac{4\left(a+b\sqrt{x}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{x}}}{b^{2}}$	30

input
```
int(1/(a+b*x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/b^2*(1/3*(a+b*x^(1/2))^(3/2)-a*(a+b*x^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx=\frac{4\,\sqrt{b\sqrt{x}+a}\left(b\sqrt{x}-2\,a\right)}{3\,b^{2}}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/3*sqrt(b*sqrt(x) + a)*(b*sqrt(x) - 2*a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(36) = 72$.

Time = 0.76 (sec) , antiderivative size = 219, normalized size of antiderivative = 5.48

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx = -\frac{8a^{\frac{7}{2}}x^2\sqrt{1+\frac{b\sqrt{x}}{a}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}} + \frac{8a^{\frac{7}{2}}x^2}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}} - \frac{4a^{\frac{5}{2}}bx^{\frac{5}{2}}\sqrt{1+\frac{b\sqrt{x}}{a}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}} + \frac{8a^{\frac{5}{2}}bx^{\frac{5}{2}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}} + \frac{4a^{\frac{3}{2}}b^2x^3\sqrt{1+\frac{b\sqrt{x}}{a}}}{3a^2b^2x^2+3ab^3x^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b*x**(1/2))**(1/2),x)
```

output
```
-8*a**(7/2)*x**2*sqrt(1 + b*sqrt(x)/a)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/
2)) + 8*a**(7/2)*x**2/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2)) - 4*a**(5/2)*
b*x**(5/2)*sqrt(1 + b*sqrt(x)/a)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2)) +
8*a**(5/2)*b*x**(5/2)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2)) + 4*a**(3/2)*
b**2*x**3*sqrt(1 + b*sqrt(x)/a)/(3*a**2*b**2*x**2 + 3*a*b**3*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(b\sqrt{x}+a\right)^{\frac{3}{2}}}{3\,b^2} - \frac{4\,\sqrt{b\sqrt{x}+a}a}{b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/3*(b*sqrt(x) + a)^(3/2)/b^2 - 4*sqrt(b*sqrt(x) + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\left(\left(b\sqrt{x}+a\right)^{\frac{3}{2}} - 3\sqrt{b\sqrt{x}+a}a\right)}{3\,b^2}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/3*((b*sqrt(x) + a)^(3/2) - 3*sqrt(b*sqrt(x) + a)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{x\sqrt{\frac{b\sqrt{x}}{a}+1}\,{}_2F_1\left(\frac{1}{2},2;\ 3;\ -\frac{b\sqrt{x}}{a}\right)}{\sqrt{a+b\sqrt{x}}}$$

input
```
int(1/(a + b*x^(1/2))^(1/2),x)
```

output
```
(x*((b*x^(1/2))/a + 1)^(1/2)*hypergeom([1/2, 2], 3, -(b*x^(1/2))/a))/(a +
b*x^(1/2))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx = \frac{4\sqrt{\sqrt{x}\,b+a}\left(\sqrt{x}\,b-2a\right)}{3b^2}$$

input
```
int(1/(a+b*x^(1/2))^(1/2),x)
```

output
```
(4*sqrt(sqrt(x)*b + a)*(sqrt(x)*b - 2*a))/(3*b**2)
```

3.132 $\int \frac{1}{\sqrt{a+b\sqrt{x}x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1}{\sqrt{a+b\sqrt{x}x}}\,dx=-\frac{4\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\sqrt{x}x}}\,dx=-\frac{4\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[x]]*x),x]
```

output
```
(-4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\sqrt{x}}}dx$$
$$\Big\downarrow \text{798}$$
$$2\int \frac{1}{\sqrt{a+b\sqrt{x}}\sqrt{x}}d\sqrt{x}$$
$$\Big\downarrow \text{73}$$
$$\frac{4\int \frac{1}{\frac{x}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{x}}}{b}$$
$$\Big\downarrow \text{221}$$
$$-\frac{4\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[a + b*Sqrt[x]]*x),x]
```

output
```
(-4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{4 \operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$	20
default	$-\frac{4 \operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$	20

input
```
int(1/(a+b*x^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(19) = 38$.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.37

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x}\,dx = \left[\frac{2\log\left(\frac{bx-2\sqrt{b\sqrt{x}+a}\sqrt{a}\sqrt{x}+2a\sqrt{x}}{x}\right)}{\sqrt{a}}, \frac{4\sqrt{-a}\arctan\left(\frac{(\sqrt{-a}b\sqrt{x}-\sqrt{-a}a)\sqrt{b\sqrt{x}+a}}{b^2x-a^2}\right)}{a}\right]$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
[2*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2*a*sqrt(x))/x)/sqrt
(a), 4*sqrt(-a)*arctan((sqrt(-a)*b*sqrt(x) - sqrt(-a)*a)*sqrt(b*sqrt(x) +
a)/(b^2*x - a^2))/a]
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{a+b\sqrt{x}x}}\,dx = -\frac{4\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)}{\sqrt{a}}$$

input
```
integrate(1/(a+b*x**(1/2))**(1/2)/x,x)
```

output
```
-4*asinh(sqrt(a)/(sqrt(b)*x**(1/4)))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{a+b\sqrt{x}x}}\,dx = \frac{2\,\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
2*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a)))/sqr
t(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x}\,dx = \frac{4\ \arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
4*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x}\,dx = -\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,\sqrt{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^(1/2))^(1/2)),x)
```

output
```
-(4*atanh((a + b*x^(1/2))^(1/2)/a^(1/2)))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x}\,dx = \frac{2\sqrt{a}\left(\log\left(\sqrt{\sqrt{x}\,b+a}-\sqrt{a}\right)-\log\left(\sqrt{\sqrt{x}\,b+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int(1/(a+b*x^(1/2))^(1/2)/x,x)
```

output
```
(2*sqrt(a)*(log(sqrt(sqrt(x)*b + a) - sqrt(a)) - log(sqrt(sqrt(x)*b + a) +
 sqrt(a))))/a
```

3.133 $\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2} dx = -\frac{\sqrt{a+b\sqrt{x}}}{ax} + \frac{3b\sqrt{a+b\sqrt{x}}}{2a^2\sqrt{x}} - \frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-(a+b*x^(1/2))^(1/2)/a/x+3/2*b*(a+b*x^(1/2))^(1/2)/a^2/x^(1/2)-3/2*b^2*arc
tanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2} dx = \frac{\sqrt{a+b\sqrt{x}}(-2a+3b\sqrt{x})}{2a^2x} - \frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[x]]*x^2),x]
```

output
```
(Sqrt[a + b*Sqrt[x]]*(-2*a + 3*b*Sqrt[x]))/(2*a^2*x) - (3*b^2*ArcTanh[Sqrt
[a + b*Sqrt[x]]/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\sqrt{a+b\sqrt{x}}x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{52}$$

$$2\left(-\frac{3b\int \frac{1}{\sqrt{a+b\sqrt{x}}x}d\sqrt{x}}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)$$

$$\Big\downarrow \text{52}$$

$$2\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{a+b\sqrt{x}}\sqrt{x}}d\sqrt{x}}{2a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)$$

$$\Big\downarrow \text{73}$$

$$2\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{x}}}{a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)$$

$$\Big\downarrow \text{221}$$

$$2\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)$$

input
```
Int[1/(Sqrt[a + b*Sqrt[x]]*x^2),x]
```

output
```
2*(-1/2*Sqrt[a + b*Sqrt[x]]/(a*x) - (3*b*(-(Sqrt[a + b*Sqrt[x]]/(a*Sqrt[x]
)) + (b*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/a^(3/2)))/(4*a))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$4b^{2}\left(-\frac{\sqrt{a+b\sqrt{x}}}{4a\,b^{2}x}-\frac{3\left(-\frac{\sqrt{a+b\sqrt{x}}}{2ab\sqrt{x}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	72
default	$4b^{2}\left(-\frac{\sqrt{a+b\sqrt{x}}}{4a\,b^{2}x}-\frac{3\left(-\frac{\sqrt{a+b\sqrt{x}}}{2ab\sqrt{x}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	72

input
```
int(1/(a+b*x^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
4*b^2*(-1/4*(a+b*x^(1/2))^(1/2)/a/b^2/x-3/4/a*(-1/2*(a+b*x^(1/2))^(1/2)/a/
b/x^(1/2)+1/2/a^(3/2)*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.01

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^{2}}\,dx$$
$$=\left[\frac{3\sqrt{a}b^{2}x\log\left(\frac{bx-2\sqrt{b\sqrt{x}+a}\sqrt{a}\sqrt{x}+2\,a\sqrt{x}}{x}\right)+2\left(3\,ab\sqrt{x}-2\,a^{2}\right)\sqrt{b\sqrt{x}+a}}{4\,a^{3}x},\frac{3\sqrt{-a}b^{2}x\arctan\left(\frac{(\sqrt{-a}b\sqrt{x}-\sqrt{-a}\right.}{b^{2}x-a}\right.$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(3*sqrt(a)*b^2*x*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(x) + 2
*a*sqrt(x))/x) + 2*(3*a*b*sqrt(x) - 2*a^2)*sqrt(b*sqrt(x) + a))/(a^3*x), 1
/2*(3*sqrt(-a)*b^2*x*arctan((sqrt(-a)*b*sqrt(x) - sqrt(-a)*a)*sqrt(b*sqrt(
x) + a)/(b^2*x - a^2)) + (3*a*b*sqrt(x) - 2*a^2)*sqrt(b*sqrt(x) + a))/(a^3
*x)]
```

Sympy [A] (verification not implemented)

Time = 3.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x^2}\,dx = -\frac{1}{\sqrt{b}x^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} + \frac{\sqrt{b}}{2ax^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}} + \frac{3b^{\frac{3}{2}}}{2a^2\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}} - \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)}{2a^{\frac{5}{2}}}$$

input
```
integrate(1/(a+b*x**(1/2))**(1/2)/x**2,x)
```

output
```
-1/(sqrt(b)*x**(5/4)*sqrt(a/(b*sqrt(x)) + 1)) + sqrt(b)/(2*a*x**(3/4)*sqrt
(a/(b*sqrt(x)) + 1)) + 3*b**(3/2)/(2*a**2*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1)
) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*x**(1/4)))/(2*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}\,x^2}\,dx = \frac{3\,b^2\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}} + \frac{3\left(b\sqrt{x}+a\right)^{\frac{3}{2}}b^2 - 5\sqrt{b\sqrt{x}+a}\,ab^2}{2\left(\left(b\sqrt{x}+a\right)^2a^2 - 2\left(b\sqrt{x}+a\right)a^3 + a^4\right)}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
3/4*b^2*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(a)
))/a^(5/2) + 1/2*(3*(b*sqrt(x) + a)^(3/2)*b^2 - 5*sqrt(b*sqrt(x) + a)*a*b^
2)/((b*sqrt(x) + a)^2*a^2 - 2*(b*sqrt(x) + a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(b\sqrt{x}+a)^{\frac{3}{2}}b^3-5\,\sqrt{b\sqrt{x}+a}ab^3}{a^2b^2x}}{2\,b}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*(3*b^3*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*sqr
t(x) + a)^(3/2)*b^3 - 5*sqrt(b*sqrt(x) + a)*a*b^3)/(a^2*b^2*x))/b
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2}\,dx = \frac{3\,\left(a+b\,\sqrt{x}\right)^{3/2}}{2\,a^2\,x} - \frac{5\,\sqrt{a+b\,\sqrt{x}}}{2\,a\,x} - \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,\sqrt{x}}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int(1/(x^2*(a + b*x^(1/2))^(1/2)),x)
```

output
```
(3*(a + b*x^(1/2))^(3/2))/(2*a^2*x) - (5*(a + b*x^(1/2))^(1/2))/(2*a*x) -
(3*b^2*atanh((a + b*x^(1/2))^(1/2)/a^(1/2)))/(2*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2}\,dx = \frac{6\sqrt{x}\,\sqrt{\sqrt{x}\,b+a}\,ab-4\sqrt{\sqrt{x}\,b+a}\,a^2+3\sqrt{a}\log\left(\sqrt{\sqrt{x}\,b+a}-\sqrt{a}\right)b^2x-3\sqrt{a}\log\left(\sqrt{\sqrt{x}\,b+a}+\sqrt{a}\right)}{4a^3x}$$

input
```
int(1/(a+b*x^(1/2))^(1/2)/x^2,x)
```

output
```
(6*sqrt(x)*sqrt(sqrt(x)*b + a)*a*b - 4*sqrt(sqrt(x)*b + a)*a**2 + 3*sqrt(a
)*log(sqrt(sqrt(x)*b + a) - sqrt(a))*b**2*x - 3*sqrt(a)*log(sqrt(sqrt(x)*b
 + a) + sqrt(a))*b**2*x)/(4*a**3*x)
```

3.134 $\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 136

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3} dx = -\frac{\sqrt{a+b\sqrt{x}}}{2ax^2} + \frac{7b\sqrt{a+b\sqrt{x}}}{12a^2x^{3/2}} - \frac{35b^2\sqrt{a+b\sqrt{x}}}{48a^3x} + \frac{35b^3\sqrt{a+b\sqrt{x}}}{32a^4\sqrt{x}} - \frac{35b^4\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{9/2}}$$

output
```
-1/2*(a+b*x^(1/2))^(1/2)/a/x^2+7/12*b*(a+b*x^(1/2))^(1/2)/a^2/x^(3/2)-35/4
8*b^2*(a+b*x^(1/2))^(1/2)/a^3/x+35/32*b^3*(a+b*x^(1/2))^(1/2)/a^4/x^(1/2)-
35/32*b^4*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3} dx = \frac{\sqrt{a+b\sqrt{x}}\left(-48a^3+56a^2b\sqrt{x}-70ab^2x+105b^3x^{3/2}\right)}{96a^4x^2} - \frac{35b^4\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{32a^{9/2}}$$

input
```
Integrate[1/(Sqrt[a + b*Sqrt[x]]*x^3),x]
```

output
```
(Sqrt[a + b*Sqrt[x]]*(-48*a^3 + 56*a^2*b*Sqrt[x] - 70*a*b^2*x + 105*b^3*x^
(3/2)))/(96*a^4*x^2) - (35*b^4*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/(32*a
^(9/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {798, 52, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{x}}}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{1}{\sqrt{a+b\sqrt{x}}x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{52}$$

$$2\left(-\frac{7b\int \frac{1}{\sqrt{a+b\sqrt{x}}x^2}d\sqrt{x}}{8a}-\frac{\sqrt{a+b\sqrt{x}}}{4ax^2}\right)$$

$$\Big\downarrow \text{52}$$

$$2\left(-\frac{7b\left(-\frac{5b\int \frac{1}{\sqrt{a+b\sqrt{x}}x^{3/2}}d\sqrt{x}}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{x}}}{4ax^2}\right)$$

$$\Big\downarrow \text{52}$$

$$2\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{\sqrt{a+b\sqrt{x}}x}d\sqrt{x}}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{x}}}{4ax^2}\right)$$

$$\Big\downarrow \ 52$$

$$2\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{a+b\sqrt{x}}\sqrt{x}}d\sqrt{x}}{2a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{x}}}{4ax^2}\right)$$

$$\Big\downarrow \ 73$$

$$2\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x}{b}-\frac{a}{b}}d\sqrt{a+b\sqrt{x}}}{a}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{x}}}{4ax^2}\right)$$

$$\Big\downarrow \ 221$$

$$2\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\sqrt{x}}}{a\sqrt{x}}\right)}{4a}-\frac{\sqrt{a+b\sqrt{x}}}{2ax}\right)}{6a}-\frac{\sqrt{a+b\sqrt{x}}}{3ax^{3/2}}\right)}{8a}-\frac{\sqrt{a+b\sqrt{x}}}{4ax^2}\right)$$

input
```
Int[1/(Sqrt[a + b*Sqrt[x]]*x^3),x]
```

output
```
2*(-1/4*Sqrt[a + b*Sqrt[x]]/(a*x^2) - (7*b*(-1/3*Sqrt[a + b*Sqrt[x]]/(a*x^
(3/2)) - (5*b*(-1/2*Sqrt[a + b*Sqrt[x]]/(a*x) - (3*b*(-(Sqrt[a + b*Sqrt[x]
]/(a*Sqrt[x])) + (b*ArcTanh[Sqrt[a + b*Sqrt[x]]/Sqrt[a]])/a^(3/2)))/(4*a))
)/(6*a)))/(8*a))
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$4b^4\left(-\frac{\sqrt{a+b\sqrt{x}}}{8a\,b^4x^2}-\frac{7\left(-\frac{\sqrt{a+b\sqrt{x}}}{6a\,b^3x^{\frac{3}{2}}}+\frac{5\sqrt{a+b\sqrt{x}}}{24a\,b^2x}+\frac{5\left(-\frac{3\sqrt{a+b\sqrt{x}}}{8ab\sqrt{x}}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}\right)}{8a}\right)$	124
default	$4b^4\left(-\frac{\sqrt{a+b\sqrt{x}}}{8a\,b^4x^2}-\frac{7\left(-\frac{\sqrt{a+b\sqrt{x}}}{6a\,b^3x^{\frac{3}{2}}}+\frac{5\sqrt{a+b\sqrt{x}}}{24a\,b^2x}+\frac{5\left(-\frac{3\sqrt{a+b\sqrt{x}}}{8ab\sqrt{x}}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{x}}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}\right)}{8a}\right)$	124

input
```
int(1/(a+b*x^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
4*b^4*(-1/8*(a+b*x^(1/2))^(1/2)/a/b^4/x^2-7/8/a*(-1/6*(a+b*x^(1/2))^(1/2)/
a/b^3/x^(3/2)+5/6/a*(1/4*(a+b*x^(1/2))^(1/2)/a/b^2/x+3/4/a*(-1/2*(a+b*x^(1
/2))^(1/2)/a/b/x^(1/2)+1/2/a^(3/2)*arctanh((a+b*x^(1/2))^(1/2)/a^(1/2)))))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3}\,dx$$
$$= \left[\frac{105\,\sqrt{a}b^4x^2\log\left(\frac{bx-2\,\sqrt{b\sqrt{x}+a}\sqrt{a}\sqrt{x}+2\,a\sqrt{x}}{x}\right)-2\left(70\,a^2b^2x+48\,a^4-7\left(15\,ab^3x+8\,a^3b\right)\sqrt{x}\right)\sqrt{b\sqrt{x}+a}}{192\,a^5x^2}, -\right.$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/192*(105*sqrt(a)*b^4*x^2*log((b*x - 2*sqrt(b*sqrt(x) + a)*sqrt(a)*sqrt(
x) + 2*a*sqrt(x))/x) - 2*(70*a^2*b^2*x + 48*a^4 - 7*(15*a*b^3*x + 8*a^3*b)
*sqrt(x))*sqrt(b*sqrt(x) + a))/(a^5*x^2), 1/96*(105*sqrt(-a)*b^4*x^2*arcta
n((sqrt(-a)*b*sqrt(x) - sqrt(-a)*a)*sqrt(b*sqrt(x) + a)/(b^2*x - a^2)) - (
70*a^2*b^2*x + 48*a^4 - 7*(15*a*b^3*x + 8*a^3*b)*sqrt(x))*sqrt(b*sqrt(x) +
 a))/(a^5*x^2)]
```

Sympy [A] (verification not implemented)

Time = 42.52 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3}\,dx = -\frac{1}{2\sqrt{b}x^{\frac{9}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{\sqrt{b}}{12ax^{\frac{7}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{7b^{\frac{3}{2}}}{48a^2x^{\frac{5}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{35b^{\frac{5}{2}}}{96a^3x^{\frac{3}{4}}\sqrt{\frac{a}{b\sqrt{x}}+1}}+\frac{35b^{\frac{7}{2}}}{32a^4\sqrt[4]{x}\sqrt{\frac{a}{b\sqrt{x}}+1}}-\frac{35b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt[4]{x}}\right)}{32a^{\frac{9}{2}}}$$

input
```
integrate(1/(a+b*x**(1/2))**(1/2)/x**3,x)
```

output
```
-1/(2*sqrt(b)*x**(9/4)*sqrt(a/(b*sqrt(x)) + 1)) + sqrt(b)/(12*a*x**(7/4)*s
qrt(a/(b*sqrt(x)) + 1)) - 7*b**(3/2)/(48*a**2*x**(5/4)*sqrt(a/(b*sqrt(x))
+ 1)) + 35*b**(5/2)/(96*a**3*x**(3/4)*sqrt(a/(b*sqrt(x)) + 1)) + 35*b**(7/
2)/(32*a**4*x**(1/4)*sqrt(a/(b*sqrt(x)) + 1)) - 35*b**4*asinh(sqrt(a)/(sqr
t(b)*x**(1/4)))/(32*a**(9/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3}\,dx = \frac{35\,b^4\log\left(\frac{\sqrt{b\sqrt{x}+a}-\sqrt{a}}{\sqrt{b\sqrt{x}+a}+\sqrt{a}}\right)}{64\,a^{\frac{9}{2}}} + \frac{105\left(b\sqrt{x}+a\right)^{\frac{7}{2}}b^4 - 385\left(b\sqrt{x}+a\right)^{\frac{5}{2}}ab^4 + 511\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2b^4 - 279\sqrt{b\sqrt{x}+a}\,a^3b^4}{96\left(\left(b\sqrt{x}+a\right)^4a^4 - 4\left(b\sqrt{x}+a\right)^3a^5 + 6\left(b\sqrt{x}+a\right)^2a^6 - 4\left(b\sqrt{x}+a\right)a^7 + a^8\right)}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
35/64*b^4*log((sqrt(b*sqrt(x) + a) - sqrt(a))/(sqrt(b*sqrt(x) + a) + sqrt(
a)))/a^(9/2) + 1/96*(105*(b*sqrt(x) + a)^(7/2)*b^4 - 385*(b*sqrt(x) + a)^(
5/2)*a*b^4 + 511*(b*sqrt(x) + a)^(3/2)*a^2*b^4 - 279*sqrt(b*sqrt(x) + a)*a
^3*b^4)/((b*sqrt(x) + a)^4*a^4 - 4*(b*sqrt(x) + a)^3*a^5 + 6*(b*sqrt(x) +
a)^2*a^6 - 4*(b*sqrt(x) + a)*a^7 + a^8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3}\,dx = \frac{\frac{105\,b^5\arctan\left(\frac{\sqrt{b\sqrt{x}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{105\left(b\sqrt{x}+a\right)^{\frac{7}{2}}b^5 - 385\left(b\sqrt{x}+a\right)^{\frac{5}{2}}ab^5 + 511\left(b\sqrt{x}+a\right)^{\frac{3}{2}}a^2b^5 - 279\sqrt{b\sqrt{x}+a}\,a^3b^5}{a^4b^4x^2}}{96\,b}$$

input
```
integrate(1/(a+b*x^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/96*(105*b^5*arctan(sqrt(b*sqrt(x) + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(
b*sqrt(x) + a)^(7/2)*b^5 - 385*(b*sqrt(x) + a)^(5/2)*a*b^5 + 511*(b*sqrt(x
) + a)^(3/2)*a^2*b^5 - 279*sqrt(b*sqrt(x) + a)*a^3*b^5)/(a^4*b^4*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3}\,dx = \frac{511\,\left(a+b\,\sqrt{x}\right)^{3/2}}{96\,a^2\,x^2} - \frac{93\,\sqrt{a+b\,\sqrt{x}}}{32\,a\,x^2} - \frac{385\,\left(a+b\,\sqrt{x}\right)^{5/2}}{96\,a^3\,x^2} + \frac{35\,\left(a+b\,\sqrt{x}\right)^{7/2}}{32\,a^4\,x^2} + \frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,\sqrt{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,35\mathrm{i}}{32\,a^{9/2}}$$

input
```
int(1/(x^3*(a + b*x^(1/2))^(1/2)),x)
```

output
```
(b^4*atan(((a + b*x^(1/2))^(1/2)*1i)/a^(1/2))*35i)/(32*a^(9/2)) - (93*(a +
 b*x^(1/2))^(1/2))/(32*a*x^2) + (511*(a + b*x^(1/2))^(3/2))/(96*a^2*x^2) -
 (385*(a + b*x^(1/2))^(5/2))/(96*a^3*x^2) + (35*(a + b*x^(1/2))^(7/2))/(32
*a^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+b\sqrt{x}}x^3}\,dx = \frac{112\sqrt{x}\,\sqrt{\sqrt{x}\,b+a}\,a^3b + 210\sqrt{x}\,\sqrt{\sqrt{x}\,b+a}\,a\,b^3x - 96\sqrt{\sqrt{x}\,b+a}\,a^4 - 140\sqrt{\sqrt{x}\,b+a}\,a^2b^2x + 105\sqrt{a}\,\mathrm{lo}}{192a^5x^2}$$

input
```
int(1/(a+b*x^(1/2))^(1/2)/x^3,x)
```

output
```
(112*sqrt(x)*sqrt(sqrt(x)*b + a)*a**3*b + 210*sqrt(x)*sqrt(sqrt(x)*b + a)*
a*b**3*x - 96*sqrt(sqrt(x)*b + a)*a**4 - 140*sqrt(sqrt(x)*b + a)*a**2*b**2
*x + 105*sqrt(a)*log(sqrt(sqrt(x)*b + a) - sqrt(a))*b**4*x**2 - 105*sqrt(a
)*log(sqrt(sqrt(x)*b + a) + sqrt(a))*b**4*x**2)/(192*a**5*x**2)
```

3.135 $\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\,dx=\frac{2a^2\left(a+b\sqrt{x}\right)^{1+n}}{b^3(1+n)}-\frac{4a\left(a+b\sqrt{x}\right)^{2+n}}{b^3(2+n)}+\frac{2\left(a+b\sqrt{x}\right)^{3+n}}{b^3(3+n)}$$

output
```
2*a^2*(a+b*x^(1/2))^(1+n)/b^3/(1+n)-4*a*(a+b*x^(1/2))^(2+n)/b^3/(2+n)+2*(a
+b*x^(1/2))^(3+n)/b^3/(3+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\,dx=\frac{2\left(a+b\sqrt{x}\right)^{1+n}\left(2a^2-2ab(1+n)\sqrt{x}+b^2(2+3n+n^2)\,x\right)}{b^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[(a + b*Sqrt[x])^n*Sqrt[x],x]
```

output
```
(2*(a + b*Sqrt[x])^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*Sqrt[x] + b^2*(2 + 3*n +
 n^2)*x))/(b^3*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+b\sqrt{x}\right)^n dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^n x d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\frac{a^2\left(a+b\sqrt{x}\right)^n}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)^{n+1}}{b^2}+\frac{\left(a+b\sqrt{x}\right)^{n+2}}{b^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^2\left(a+b\sqrt{x}\right)^{n+1}}{b^3(n+1)}-\frac{2a\left(a+b\sqrt{x}\right)^{n+2}}{b^3(n+2)}+\frac{\left(a+b\sqrt{x}\right)^{n+3}}{b^3(n+3)}\right)$$

input
```
Int[(a + b*Sqrt[x])^n*Sqrt[x],x]
```

output
```
2*((a^2*(a + b*Sqrt[x])^(1 + n))/(b^3*(1 + n)) - (2*a*(a + b*Sqrt[x])^(2 +
 n))/(b^3*(2 + n)) + (a + b*Sqrt[x])^(3 + n)/(b^3*(3 + n)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^{n} \sqrt{x}dx$$

input
```
int((a+b*x^(1/2))^n*x^(1/2),x)
```

output
```
int((a+b*x^(1/2))^n*x^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \left(a+b\sqrt{x}\right)^{n} \sqrt{x}\,dx = \frac{2\left(2a^{3}+(ab^{2}n^{2}+ab^{2}n)x-(2a^{2}bn-(b^{3}n^{2}+3b^{3}n+2b^{3})x)\sqrt{x}\right)\left(b\sqrt{x}+a\right)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}}$$

input
```
integrate((a+b*x^(1/2))^n*x^(1/2),x, algorithm="fricas")
```

output
```
2*(2*a^3 + (a*b^2*n^2 + a*b^2*n)*x - (2*a^2*b*n - (b^3*n^2 + 3*b^3*n + 2*b
^3)*x)*sqrt(x))*(b*sqrt(x) + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. 2(65) = 130.

Time = 2.44 (sec) , antiderivative size = 2628, normalized size of antiderivative = 35.51

$$\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**(1/2))**n*x**(1/2),x)
```

output
```
4*a**3*a**(n + 3)*x**(9/2)*(1 + b*sqrt(x)/a)**(n + 3)/(a**3*b**3*n**3*x**(
9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x
**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*
x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11
/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**
6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) - 4*a**3*a**(n + 3)*x**(9/2)/(
a**3*b**3*n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9
/2) + 6*a**3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**
5 + 33*a**2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18
*a*b**5*n**2*x**(11/2) + 33*a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**
6*n**3*x**6 + 6*b**6*n**2*x**6 + 11*b**6*n*x**6 + 6*b**6*x**6) - 4*a**2*a*
*(n + 3)*b*n*x**5*(1 + b*sqrt(x)/a)**(n + 3)/(a**3*b**3*n**3*x**(9/2) + 6*
a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a**3*b**3*x**(9/2) +
 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**2*b**4*n*x**5 + 18
*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5*n**2*x**(11/2) + 33*
a*b**5*n*x**(11/2) + 18*a*b**5*x**(11/2) + b**6*n**3*x**6 + 6*b**6*n**2*x*
*6 + 11*b**6*n*x**6 + 6*b**6*x**6) - 12*a**2*a**(n + 3)*b*x**5/(a**3*b**3*
n**3*x**(9/2) + 6*a**3*b**3*n**2*x**(9/2) + 11*a**3*b**3*n*x**(9/2) + 6*a*
*3*b**3*x**(9/2) + 3*a**2*b**4*n**3*x**5 + 18*a**2*b**4*n**2*x**5 + 33*a**
2*b**4*n*x**5 + 18*a**2*b**4*x**5 + 3*a*b**5*n**3*x**(11/2) + 18*a*b**5...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\,dx = \frac{2\left((n^2+3n+2)b^3x^{\frac{3}{2}}+(n^2+n)ab^2x-2a^2bn\sqrt{x}+2a^3\right)\left(b\sqrt{x}+a\right)^n}{(n^3+6n^2+11n+6)b^3}$$

input
```
integrate((a+b*x^(1/2))^n*x^(1/2),x, algorithm="maxima")
```

output
```
2*((n^2 + 3*n + 2)*b^3*x^(3/2) + (n^2 + n)*a*b^2*x - 2*a^2*b*n*sqrt(x) + 2
*a^3)*(b*sqrt(x) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(68) = 136.

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.12

$$\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)^3\left(b\sqrt{x}+a\right)^n n^2-2\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^n an^2+\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)^n a^2n^2+3\left(b\sqrt{x}+\right.\right.}{}$$

input
```
integrate((a+b*x^(1/2))^n*x^(1/2),x, algorithm="giac")
```

output
```
2*((b*sqrt(x) + a)^3*(b*sqrt(x) + a)^n*n^2 - 2*(b*sqrt(x) + a)^2*(b*sqrt(x
) + a)^n*a*n^2 + (b*sqrt(x) + a)*(b*sqrt(x) + a)^n*a^2*n^2 + 3*(b*sqrt(x)
+ a)^3*(b*sqrt(x) + a)^n*n - 8*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^n*a*n + 5
*(b*sqrt(x) + a)*(b*sqrt(x) + a)^n*a^2*n + 2*(b*sqrt(x) + a)^3*(b*sqrt(x)
+ a)^n - 6*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^n*a + 6*(b*sqrt(x) + a)*(b*sq
rt(x) + a)^n*a^2)/((b^2*n^3 + 6*b^2*n^2 + 11*b^2*n + 6*b^2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\, dx = \int \sqrt{x}\, \left(a+b\,\sqrt{x}\right)^n\, dx$$

input
```
int(x^(1/2)*(a + b*x^(1/2))^n,x)
```

output
```
int(x^(1/2)*(a + b*x^(1/2))^n, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \left(a+b\sqrt{x}\right)^n \sqrt{x}\, dx = \frac{2\left(\sqrt{x}\,b+a\right)^n \left(-2\sqrt{x}\,a^2bn+\sqrt{x}\,b^3n^2x+3\sqrt{x}\,b^3nx+2\sqrt{x}\,b^3x+2a^3+a\,b^2n^2x+a\,b^2nx\right)}{b^3\left(n^3+6n^2+11n+6\right)}$$

input
```
int((a+b*x^(1/2))^n*x^(1/2),x)
```

output
```
(2*(sqrt(x)*b + a)**n*( - 2*sqrt(x)*a**2*b*n + sqrt(x)*b**3*n**2*x + 3*sqr
t(x)*b**3*n*x + 2*sqrt(x)*b**3*x + 2*a**3 + a*b**2*n**2*x + a*b**2*n*x))/(
b**3*(n**3 + 6*n**2 + 11*n + 6))
```

3.136 $\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx=\frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b(1+n)}$$

output
```
2*(a+b*x^(1/2))^(1+n)/b/(1+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx=\frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b(1+n)}$$

input
```
Integrate[(a + b*Sqrt[x])^n/Sqrt[x],x]
```

output
```
(2*(a + b*Sqrt[x])^(1 + n))/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{2\left(a+b\sqrt{x}\right)^{n+1}}{b(n+1)}$$

input
```
Int[(a + b*Sqrt[x])^n/Sqrt[x],x]
```

output
```
(2*(a + b*Sqrt[x])^(1 + n))/(b*(1 + n))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b(1+n)}$	22
default	$\frac{2\left(a+b\sqrt{x}\right)^{1+n}}{b(1+n)}$	22

input
```
int((a+b*x^(1/2))^n/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a+b*x^(1/2))^(1+n)/b/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx = \frac{2\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)^n}{bn+b}$$

input
```
integrate((a+b*x^(1/2))^n/x^(1/2),x, algorithm="fricas")
```

output
```
2*(b*sqrt(x) + a)*(b*sqrt(x) + a)^n/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(17) = 34$.

Time = 0.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 7.91

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \wedge n = -1 \\ 2\cdot 0^n\sqrt{x} & \text{for } a = -b\sqrt{x} \\ 2a^n\sqrt{x} & \text{for } b = 0 \\ \frac{2\log\left(\frac{a}{b}+\sqrt{x}\right)}{b} & \text{for } n = -1 \\ \frac{2a^2\left(a+b\sqrt{x}\right)^n}{abn+ab+b^2n\sqrt{x}+b^2\sqrt{x}} + \frac{4ab\sqrt{x}\left(a+b\sqrt{x}\right)^n}{abn+ab+b^2n\sqrt{x}+b^2\sqrt{x}} + \frac{2b^2x\left(a+b\sqrt{x}\right)^n}{abn+ab+b^2n\sqrt{x}+b^2\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**(1/2))**n/x**(1/2),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0) & Eq(n, -1)), (2*0**n*sqrt(x),
 Eq(a, -b*sqrt(x))), (2*a**n*sqrt(x), Eq(b, 0)), (2*log(a/b + sqrt(x))/b,
Eq(n, -1)), (2*a**2*(a + b*sqrt(x))**n/(a*b*n + a*b + b**2*n*sqrt(x) + b**
2*sqrt(x)) + 4*a*b*sqrt(x)*(a + b*sqrt(x))**n/(a*b*n + a*b + b**2*n*sqrt(x
) + b**2*sqrt(x)) + 2*b**2*x*(a + b*sqrt(x))**n/(a*b*n + a*b + b**2*n*sqrt
(x) + b**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx=\frac{2\left(b\sqrt{x}+a\right)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*x^(1/2))^n/x^(1/2),x, algorithm="maxima")
```

output
```
2*(b*sqrt(x) + a)^(n + 1)/(b*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx=\frac{2\left(b\sqrt{x}+a\right)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*x^(1/2))^n/x^(1/2),x, algorithm="giac")
```

output
```
2*(b*sqrt(x) + a)^(n + 1)/(b*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx=\frac{2\,{\left(a+b\,\sqrt{x}\right)}^{n+1}}{b\,\left(n+1\right)}$$

input
```
int((a + b*x^(1/2))^n/x^(1/2),x)
```

output
```
(2*(a + b*x^(1/2))^(n + 1))/(b*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b\sqrt{x}\right)^n}{\sqrt{x}}\,dx=\frac{2\left(\sqrt{x}\,b+a\right)^n\left(\sqrt{x}\,b+a\right)}{b\left(n+1\right)}$$

input
```
int((a+b*x^(1/2))^n/x^(1/2),x)
```

output
```
(2*(sqrt(x)*b + a)**n*(sqrt(x)*b + a))/(b*(n + 1))
```

3.137 $\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx=\left(1+\sqrt{x}\right)^2$$

output
```
(1+x^(1/2))^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx=2\sqrt{x}+x$$

input
```
Integrate[(1 + Sqrt[x])/Sqrt[x],x]
```

output
```
2*Sqrt[x] + x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}+1}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{1}{\sqrt{x}}+1\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x+2\sqrt{x}$$

input
```
Int[(1 + Sqrt[x])/Sqrt[x],x]
```

output
```
2*Sqrt[x] + x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$x+2\sqrt{x}$	8
default	$x+2\sqrt{x}$	8
trager	$-1+x+2\sqrt{x}$	9
orering	$\sqrt{x}\,(1+\sqrt{x})-2x^{2}\left(\frac{1}{2x}-\frac{1+\sqrt{x}}{2x^{\frac{3}{2}}}\right)$	32

input
```
int((1+x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x+2*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = x+2\,\sqrt{x}$$

input
```
integrate((1+x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
x + 2*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = 2\sqrt{x}+x$$

input
```
integrate((1+x**(1/2))/x**(1/2),x)
```

output
```
2*sqrt(x) + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = \left(\sqrt{x}+1\right)^2$$

input
```
integrate((1+x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
(sqrt(x) + 1)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = x+2\sqrt{x}$$

input
```
integrate((1+x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
x + 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = x+2\sqrt{x}$$

input
```
int((x^(1/2) + 1)/x^(1/2),x)
```

output
```
x + 2*x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = 2\sqrt{x}+x$$

input
```
int((1+x^(1/2))/x^(1/2),x)
```

output
```
2*sqrt(x) + x
```

3.138 $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx=\frac{2}{3}(1+\sqrt{x})^3$$

output
```
2/3*(1+x^(1/2))^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx=2\sqrt{x}+2x+\frac{2x^{3/2}}{3}$$

input
```
Integrate[(1 + Sqrt[x])^2/Sqrt[x],x]
```

output
```
2*Sqrt[x] + 2*x + (2*x^(3/2))/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\sqrt{x}+1)^2}{\sqrt{x}} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{2}{3}(\sqrt{x}+1)^3$$

input
```
Int[(1 + Sqrt[x])^2/Sqrt[x],x]
```

output
```
(2*(1 + Sqrt[x])^3)/3
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2(1+\sqrt{x})^3}{3}$	10
default	$\frac{2x^{\frac{3}{2}}}{3}+2x+2\sqrt{x}$	15
trager	$-2+2x+\left(2+\frac{2x}{3}\right)\sqrt{x}$	15
orering	$\frac{(1+x)(1+\sqrt{x})^2}{\sqrt{x}}-\frac{2(x^2+3)x\left(\frac{1+\sqrt{x}}{x}-\frac{(1+\sqrt{x})^2}{2x^{\frac{3}{2}}}\right)}{3(-1+x)}$	51

input
```
int((1+x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x^(1/2))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx=\frac{2}{3}\,(x+3)\sqrt{x}+2\,x$$

input
```
integrate((1+x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x + 3)*sqrt(x) + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx = \frac{2x^{\frac{3}{2}}}{3} + 2\sqrt{x} + 2x$$

input
```
integrate((1+x**(1/2))**2/x**(1/2),x)
```

output
```
2*x**(3/2)/3 + 2*sqrt(x) + 2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx = \frac{2}{3}\left(\sqrt{x}+1\right)^3$$

input
```
integrate((1+x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/3*(sqrt(x) + 1)^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx = \frac{2}{3}\,x^{\frac{3}{2}} + 2\,x + 2\,\sqrt{x}$$

input
```
integrate((1+x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

output
```
2/3*x^(3/2) + 2*x + 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}\,dx = \frac{2\,\sqrt{x}\,\left(x+3\,\sqrt{x}+3\right)}{3}$$

input
```
int((x^(1/2) + 1)^2/x^(1/2),x)
```

output
```
(2*x^(1/2)*(x + 3*x^(1/2) + 3))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,x}{3}+2\sqrt{x}+2x$$

input
```
int((1+x^(1/2))^2/x^(1/2),x)
```

output
```
(2*(sqrt(x)*x + 3*sqrt(x) + 3*x))/3
```

3.139 $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx = \frac{1}{2}(1+\sqrt{x})^4$$

output
```
1/2*(1+x^(1/2))^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\left(4+6\sqrt{x}+4x+x^{3/2}\right)$$

input
```
Integrate[(1 + Sqrt[x])^3/Sqrt[x],x]
```

output
```
(Sqrt[x]*(4 + 6*Sqrt[x] + 4*x + x^(3/2)))/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{x}+1\right)^3}{\sqrt{x}} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4$$

input
```
Int[(1 + Sqrt[x])^3/Sqrt[x],x]
```

output
```
(1 + Sqrt[x])^4/2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\left(1+\sqrt{x}\right)^{4}}{2}$	10
trager	$\frac{(-1+x)(7+x)}{2}+(2+2x)\sqrt{x}$	19
default	$\frac{x^{2}}{2}+2x^{\frac{3}{2}}+3x+2\sqrt{x}$	20
orering	$\frac{\sqrt{x}\,(5x^{2}-9x+6)\left(1+\sqrt{x}\right)^{3}}{6(-1+x)^{2}}-\frac{x^{2}(x^{2}-3x+6)\left(\frac{3\left(1+\sqrt{x}\right)^{2}}{2x}-\frac{\left(1+\sqrt{x}\right)^{3}}{2x^{\frac{3}{2}}}\right)}{3(-1+x)^{2}}$	72

input
```
int((1+x^(1/2))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1+x^(1/2))^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\left(1+\sqrt{x}\right)^{3}}{\sqrt{x}}\,dx=\frac{1}{2}\,x^{2}+2\,(x+1)\sqrt{x}+3\,x$$

input
```
integrate((1+x^(1/2))^3/x^(1/2),x, algorithm="fricas")
```

output
```
1/2*x^2 + 2*(x + 1)*sqrt(x) + 3*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx = 2x^{\frac{3}{2}} + 2\sqrt{x} + \frac{x^2}{2} + 3x$$

input

```
integrate((1+x**(1/2))**3/x**(1/2),x)
```

output

```
2*x**(3/2) + 2*sqrt(x) + x**2/2 + 3*x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx = \frac{1}{2}\left(\sqrt{x}+1\right)^4$$

input

```
integrate((1+x^(1/2))^3/x^(1/2),x, algorithm="maxima")
```

output

```
1/2*(sqrt(x) + 1)^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}\,dx = \frac{1}{2}\,x^2 + 2\,x^{\frac{3}{2}} + 3\,x + 2\,\sqrt{x}$$

input

```
integrate((1+x^(1/2))^3/x^(1/2),x, algorithm="giac")
```

output

```
1/2*x^2 + 2*x^(3/2) + 3*x + 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}\,dx = 3\,x+\frac{x^2}{2}+2\,\sqrt{x}+2\,x^{3/2}$$

input
```
int((x^(1/2) + 1)^3/x^(1/2),x)
```

output
```
3*x + x^2/2 + 2*x^(1/2) + 2*x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}\,dx = 2\sqrt{x}\,x+2\sqrt{x}+\frac{x^2}{2}+3x$$

input
```
int((1+x^(1/2))^3/x^(1/2),x)
```

output
```
(4*sqrt(x)*x + 4*sqrt(x) + x**2 + 6*x)/2
```

3.140 $\int \frac{\sqrt{x}}{1+\sqrt{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\sqrt{x}}{1+\sqrt{x}} \, dx = -2\sqrt{x}+x+2\log \left(1+\sqrt{x}\right)$$

output
```
-2*x^(1/2)+x+2*ln(1+x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+\sqrt{x}} \, dx = -2\sqrt{x}+x+2\log \left(1+\sqrt{x}\right)$$

input
```
Integrate[Sqrt[x]/(1 + Sqrt[x]),x]
```

output
```
-2*Sqrt[x] + x + 2*Log[1 + Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x}+1}dx$$
$$\Big\downarrow \text{ 798}$$
$$2\int \frac{x}{\sqrt{x}+1}d\sqrt{x}$$
$$\Big\downarrow \text{ 49}$$
$$2\int \left(\sqrt{x}+\frac{1}{\sqrt{x}+1}-1\right)d\sqrt{x}$$
$$\Big\downarrow \text{ 2009}$$
$$2\left(\frac{x}{2}-\sqrt{x}+\log\left(\sqrt{x}+1\right)\right)$$

input
```
Int[Sqrt[x]/(1 + Sqrt[x]),x]
```

output
```
2*(-Sqrt[x] + x/2 + Log[1 + Sqrt[x]])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-2\sqrt{x}+x+2\ln\left(1+\sqrt{x}\right)$	16
default	$-2\sqrt{x}+x+2\ln\left(1+\sqrt{x}\right)$	16
trager	$x-2-2\sqrt{x}+\ln\left(2\sqrt{x}+x+1\right)$	18
meijerg	$-\frac{\sqrt{x}\,(-3\sqrt{x}+6)}{3}+2\ln\left(1+\sqrt{x}\right)$	22

input
```
int(x^(1/2)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*x^(1/2)+x+2*ln(1+x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = x - 2\,\sqrt{x} + 2\,\log\left(\sqrt{x}+1\right)$$

input
```
integrate(x^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
```

output
```
x - 2*sqrt(x) + 2*log(sqrt(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = -2\sqrt{x} + x + 2\log\left(\sqrt{x}+1\right)$$

input
```
integrate(x**(1/2)/(1+x**(1/2)),x)
```

output
```
-2*sqrt(x) + x + 2*log(sqrt(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = \left(\sqrt{x}+1\right)^2 - 4\sqrt{x} + 2\log\left(\sqrt{x}+1\right) - 4$$

input
```
integrate(x^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
```

output
```
(sqrt(x) + 1)^2 - 4*sqrt(x) + 2*log(sqrt(x) + 1) - 4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = x - 2\sqrt{x} + 2\log\left(\sqrt{x}+1\right)$$

input
```
integrate(x^(1/2)/(1+x^(1/2)),x, algorithm="giac")
```

output
```
x - 2*sqrt(x) + 2*log(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = x + 2\,\ln\left(\sqrt{x}+1\right) - 2\,\sqrt{x}$$

input
```
int(x^(1/2)/(x^(1/2) + 1),x)
```

output
```
x + 2*log(x^(1/2) + 1) - 2*x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx = -2\sqrt{x} + 2\log\left(\sqrt{x}+1\right) + x$$

input
```
int(x^(1/2)/(1+x^(1/2)),x)
```

output
```
- 2*sqrt(x) + 2*log(sqrt(x) + 1) + x
```

3.141 $\int \frac{1}{(1+\sqrt{x})\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx=2\log\left(1+\sqrt{x}\right)$$

output
```
2*ln(1+x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx=2\log\left(1+\sqrt{x}\right)$$

input
```
Integrate[1/((1 + Sqrt[x])*Sqrt[x]),x]
```

output
```
2*Log[1 + Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sqrt{x}+1)\sqrt{x}}dx$$
$$\Big\downarrow \text{ 792}$$
$$2\log\left(\sqrt{x}+1\right)$$

input
```
Int[1/((1 + Sqrt[x])*Sqrt[x]),x]
```

output
```
2*Log[1 + Sqrt[x]]
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$2\ln\left(1+\sqrt{x}\right)$	9
default	$2\ln\left(1+\sqrt{x}\right)$	9
meijerg	$2\ln\left(1+\sqrt{x}\right)$	9
trager	$\ln\left(2\sqrt{x}+x+1\right)$	10

input
```
int(1/x^(1/2)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*ln(1+x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx = 2\,\log\left(\sqrt{x}+1\right)$$

input
```
integrate(1/(1+x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*log(sqrt(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx = 2\log\left(\sqrt{x}+1\right)$$

input
```
integrate(1/(1+x**(1/2))/x**(1/2),x)
```

output
```
2*log(sqrt(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx = 2\,\log\left(\sqrt{x}+1\right)$$

input
```
integrate(1/(1+x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*log(sqrt(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx = 2\,\log\left(\sqrt{x}+1\right)$$

input
```
integrate(1/(1+x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*log(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx = 2\,\ln\left(\sqrt{x}+1\right)$$

input
```
int(1/(x^(1/2)*(x^(1/2) + 1)),x)
```

output
```
2*log(x^(1/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(1+\sqrt{x}\right)\sqrt{x}}\,dx = 2\log\left(\sqrt{x}+1\right)$$

input
```
int(1/(1+x^(1/2))/x^(1/2),x)
```

output
```
2*log(sqrt(x) + 1)
```

3.142 $\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx=-\frac{2}{1+\sqrt{x}}$$

output
```
-2/(1+x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx=-\frac{2}{1+\sqrt{x}}$$

input
```
Integrate[1/((1 + Sqrt[x])^2*Sqrt[x]),x]
```

output
```
-2/(1 + Sqrt[x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x}+1\right)^2\sqrt{x}}dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{2}{\sqrt{x}+1}$$

input
```
Int[1/((1 + Sqrt[x])^2*Sqrt[x]),x]
```

output
```
-2/(1 + Sqrt[x])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{2}{1+\sqrt{x}}$	10
default	$-\frac{2}{1+\sqrt{x}}$	10
meijerg	$\frac{2\sqrt{x}}{1+\sqrt{x}}$	13
trager	$-\frac{2(x-2)}{-1+x}-\frac{2\sqrt{x}}{-1+x}$	22

input
```
int(1/(1+x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(1+x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx=-\frac{2\left(\sqrt{x}-1\right)}{x-1}$$

input
```
integrate(1/(1+x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(x) - 1)/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx=-\frac{2}{\sqrt{x}+1}$$

input
```
integrate(1/(1+x**(1/2))**2/x**(1/2),x)
```

output
```
-2/(sqrt(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx=-\frac{2}{\sqrt{x}+1}$$

input
```
integrate(1/(1+x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx=-\frac{2}{\sqrt{x}+1}$$

input
```
integrate(1/(1+x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

output
```
-2/(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2\sqrt{x}}\,dx=-\frac{2}{\sqrt{x}+1}$$

input
```
int(1/(x^(1/2)*(x^(1/2) + 1)^2),x)
```

output
```
-2/(x^(1/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(1+\sqrt{x}\right)^2 \sqrt{x}}\,dx = \frac{2\sqrt{x}}{\sqrt{x}+1}$$

input
```
int(1/(1+x^(1/2))^2/x^(1/2),x)
```

output
```
(2*sqrt(x))/(sqrt(x) + 1)
```

3.143 $\int \frac{1}{\left(1+\sqrt{x}\right)^3\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1}{\left(1+\sqrt{x}\right)^3\sqrt{x}}\,dx = -\frac{1}{\left(1+\sqrt{x}\right)^2}$$

output
```
-1/(1+x^(1/2))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+\sqrt{x}\right)^3\sqrt{x}}\,dx = -\frac{1}{\left(1+\sqrt{x}\right)^2}$$

input
```
Integrate[1/((1 + Sqrt[x])^3*Sqrt[x]),x]
```

output
```
-(1 + Sqrt[x])^(-2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x}+1\right)^3 \sqrt{x}} \, dx$$

$$\Big\downarrow \; 793$$

$$-\frac{1}{\left(\sqrt{x}+1\right)^2}$$

input
```
Int[1/((1 + Sqrt[x])^3*Sqrt[x]),x]
```

output
```
-(1 + Sqrt[x])^(-2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{1}{(1+\sqrt{x})^2}$	10
default	$-\frac{1}{(1+\sqrt{x})^2}$	10
meijerg	$\frac{\sqrt{x}\,(\sqrt{x}+2)}{(1+\sqrt{x})^2}$	17
trager	$\frac{(x-2)(-1+3x)}{(-1+x)^2}+\frac{2\sqrt{x}}{(-1+x)^2}$	26

input
```
int(1/(1+x^(1/2))^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+x^(1/2))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{1}{(1+\sqrt{x})^3\sqrt{x}}\,dx = -\frac{x-2\sqrt{x}+1}{x^2-2\,x+1}$$

input
```
integrate(1/(1+x^(1/2))^3/x^(1/2),x, algorithm="fricas")
```

output
```
-(x - 2*sqrt(x) + 1)/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{(1+\sqrt{x})^3\sqrt{x}}\,dx = -\frac{1}{2\sqrt{x}+x+1}$$

input
```
integrate(1/(1+x**(1/2))**3/x**(1/2),x)
```

output
```
-1/(2*sqrt(x) + x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1+\sqrt{x}\right)^3 \sqrt{x}}\, dx = -\frac{1}{\left(\sqrt{x}+1\right)^2}$$

input
```
integrate(1/(1+x^(1/2))^3/x^(1/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(x) + 1)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1+\sqrt{x}\right)^3 \sqrt{x}}\, dx = -\frac{1}{\left(\sqrt{x}+1\right)^2}$$

input
```
integrate(1/(1+x^(1/2))^3/x^(1/2),x, algorithm="giac")
```

output
```
-1/(sqrt(x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(1+\sqrt{x}\right)^3 \sqrt{x}}\, dx = -\frac{1}{\left(\sqrt{x}+1\right)^2}$$

input
```
int(1/(x^(1/2)*(x^(1/2) + 1)^3),x)
```

output
```
-1/(x^(1/2) + 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+\sqrt{x}\right)^3 \sqrt{x}}\, dx = -\frac{1}{2\sqrt{x}+x+1}$$

input
```
int(1/(1+x^(1/2))^3/x^(1/2),x)
```

output
```
( - 1)/(2*sqrt(x) + x + 1)
```

3.144 $\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx=\frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}-\frac{8}{5}\left(1+\sqrt{x}\right)^{5/2}+\frac{4}{7}\left(1+\sqrt{x}\right)^{7/2}$$

output
```
4/3*(1+x^(1/2))^(3/2)-8/5*(1+x^(1/2))^(5/2)+4/7*(1+x^(1/2))^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx=\frac{4}{105}\sqrt{1+\sqrt{x}}\left(8-4\sqrt{x}+3x+15x^{3/2}\right)$$

input
```
Integrate[Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

output
```
(4*Sqrt[1 + Sqrt[x]]*(8 - 4*Sqrt[x] + 3*x + 15*x^(3/2)))/105
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x}+1}\sqrt{x}\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \sqrt{\sqrt{x}+1}xd\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(\left(\sqrt{x}+1\right)^{5/2}-2\left(\sqrt{x}+1\right)^{3/2}+\sqrt{\sqrt{x}+1}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{2}{7}\left(\sqrt{x}+1\right)^{7/2}-\frac{4}{5}\left(\sqrt{x}+1\right)^{5/2}+\frac{2}{3}\left(\sqrt{x}+1\right)^{3/2}\right)$$

input
```
Int[Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

output
```
2*((2*(1 + Sqrt[x])^(3/2))/3 - (4*(1 + Sqrt[x])^(5/2))/5 + (2*(1 + Sqrt[x]
)^(7/2))/7)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{4(1+\sqrt{x})^{\frac{3}{2}}}{3}-\frac{8(1+\sqrt{x})^{\frac{5}{2}}}{5}+\frac{4(1+\sqrt{x})^{\frac{7}{2}}}{7}$	29
default	$\frac{4(1+\sqrt{x})^{\frac{3}{2}}}{3}-\frac{8(1+\sqrt{x})^{\frac{5}{2}}}{5}+\frac{4(1+\sqrt{x})^{\frac{7}{2}}}{7}$	29
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\,(1+\sqrt{x})^{\frac{3}{2}}(15x-12\sqrt{x}+8)}{105}}{\sqrt{\pi}}$	34

input
```
int((1+x^(1/2))^(1/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*(1+x^(1/2))^(3/2)-8/5*(1+x^(1/2))^(5/2)+4/7*(1+x^(1/2))^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx=\frac{4}{105}\left((15\,x-4)\sqrt{x}+3\,x+8\right)\sqrt{\sqrt{x}+1}$$

input
```
integrate((1+x^(1/2))^(1/2)*x^(1/2),x, algorithm="fricas")
```

output
```
4/105*((15*x - 4)*sqrt(x) + 3*x + 8)*sqrt(sqrt(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(39) = 78.

Time = 0.94 (sec) , antiderivative size = 398, normalized size of antiderivative = 8.65

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{60x^{\frac{15}{2}}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} + \frac{200x^{\frac{13}{2}}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} + \frac{60x^{\frac{11}{2}}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} - \frac{96x^{\frac{11}{2}}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} + \frac{32x^{\frac{9}{2}}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} - \frac{32x^{\frac{9}{2}}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} + \frac{192x^{7}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} + \frac{80x^{6}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} - \frac{32x^{6}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} + \frac{80x^{5}\sqrt{\sqrt{x}+1}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}} - \frac{96x^{5}}{315x^{\frac{11}{2}}+105x^{\frac{9}{2}}+105x^{6}+315x^{5}}$$

input
```
integrate((1+x**(1/2))**(1/2)*x**(1/2),x)
```

output
```
60*x**(15/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 105*x**(9/2) + 105*x**6 +
315*x**5) + 200*x**(13/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 105*x**(9/2)
+ 105*x**6 + 315*x**5) + 60*x**(11/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2) + 1
05*x**(9/2) + 105*x**6 + 315*x**5) - 96*x**(11/2)/(315*x**(11/2) + 105*x**
(9/2) + 105*x**6 + 315*x**5) + 32*x**(9/2)*sqrt(sqrt(x) + 1)/(315*x**(11/2
) + 105*x**(9/2) + 105*x**6 + 315*x**5) - 32*x**(9/2)/(315*x**(11/2) + 105
*x**(9/2) + 105*x**6 + 315*x**5) + 192*x**7*sqrt(sqrt(x) + 1)/(315*x**(11/
2) + 105*x**(9/2) + 105*x**6 + 315*x**5) + 80*x**6*sqrt(sqrt(x) + 1)/(315*
x**(11/2) + 105*x**(9/2) + 105*x**6 + 315*x**5) - 32*x**6/(315*x**(11/2) +
 105*x**(9/2) + 105*x**6 + 315*x**5) + 80*x**5*sqrt(sqrt(x) + 1)/(315*x**(
11/2) + 105*x**(9/2) + 105*x**6 + 315*x**5) - 96*x**5/(315*x**(11/2) + 105
*x**(9/2) + 105*x**6 + 315*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{4}{7}\left(\sqrt{x}+1\right)^{\frac{7}{2}} - \frac{8}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}} + \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+x^(1/2))^(1/2)*x^(1/2),x, algorithm="maxima")
```

output
```
4/7*(sqrt(x) + 1)^(7/2) - 8/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x) + 1)^(3/2
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{4}{7}\left(\sqrt{x}+1\right)^{\frac{7}{2}} - \frac{8}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}} + \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+x^(1/2))^(1/2)*x^(1/2),x, algorithm="giac")
```

output
```
4/7*(sqrt(x) + 1)^(7/2) - 8/5*(sqrt(x) + 1)^(5/2) + 4/3*(sqrt(x) + 1)^(3/2
)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \int \sqrt{x}\,\sqrt{\sqrt{x}+1}\,dx$$

input
```
int(x^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

output
```
int(x^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.46

$$\int \sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{4\sqrt{\sqrt{x}+1}\,\left(15\sqrt{x}\,x-4\sqrt{x}+3x+8\right)}{105}$$

input
```
int((1+x^(1/2))^(1/2)*x^(1/2),x)
```

output
```
(4*sqrt(sqrt(x) + 1)*(15*sqrt(x)*x - 4*sqrt(x) + 3*x + 8))/105
```

3.145 $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx = \frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}$$

output
```
4/3*(1+x^(1/2))^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx = \frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}$$

input
```
Integrate[Sqrt[1 + Sqrt[x]]/Sqrt[x],x]
```

output
```
(4*(1 + Sqrt[x])^(3/2))/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{4}{3} \left(\sqrt{x}+1\right)^{3/2}$$

input
```
Int[Sqrt[1 + Sqrt[x]]/Sqrt[x],x]
```

output
```
(4*(1 + Sqrt[x])^(3/2))/3
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{4\left(1+\sqrt{x}\right)^{\frac{3}{2}}}{3}$	10
default	$\frac{4\left(1+\sqrt{x}\right)^{\frac{3}{2}}}{3}$	10
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(2+2\sqrt{x}\right)\sqrt{1+\sqrt{x}}}{3}}{\sqrt{\pi}}$	31

input
```
int((1+x^(1/2))^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*(1+x^(1/2))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
4/3*(sqrt(x) + 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \frac{4\sqrt{x}\sqrt{\sqrt{x}+1}}{3} + \frac{4\sqrt{\sqrt{x}+1}}{3}$$

input
```
integrate((1+x**(1/2))**(1/2)/x**(1/2),x)
```

output
```
4*sqrt(x)*sqrt(sqrt(x) + 1)/3 + 4*sqrt(sqrt(x) + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
4/3*(sqrt(x) + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
4/3*(sqrt(x) + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \frac{4\left(\sqrt{x}+1\right)^{3/2}}{3}$$

input
```
int((x^(1/2) + 1)^(1/2)/x^(1/2),x)
```

output
```
(4*(x^(1/2) + 1)^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \frac{4\sqrt{\sqrt{x}+1}\,(\sqrt{x}+1)}{3}$$

input
```
int((1+x^(1/2))^(1/2)/x^(1/2),x)
```

output
```
(4*sqrt(sqrt(x) + 1)*(sqrt(x) + 1))/3
```

3.146 $\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx=-3\sqrt[3]{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-3\log\left(1+\sqrt[6]{x}\right)+\log\left(1+\sqrt{x}\right)$$

output
```
-3*x^(1/3)+6/5*x^(5/6)-2*3^(1/2)*arctan(1/3*(1-2*x^(1/6))*3^(1/2))-3*ln(1+
x^(1/6))+ln(1+x^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx=-3\sqrt[3]{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-2\log\left(1+\sqrt[6]{x}\right)+\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)$$

input
```
Integrate[x^(1/3)/(1 + Sqrt[x]),x]
```

output
```
-3*x^(1/3) + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/Sqrt[3]] - 2
*Log[1 + x^(1/6)] + Log[1 - x^(1/6) + x^(1/3)]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {864, 60, 60, 68, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}}{\sqrt{x}+1} \, dx$$

$$\Big\downarrow \text{864}$$

$$2 \int \frac{x^{5/6}}{\sqrt{x}+1} d\sqrt{x}$$

$$\Big\downarrow \text{60}$$

$$2 \left(\frac{3x^{5/6}}{5} - \int \frac{\sqrt[3]{x}}{\sqrt{x}+1} d\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$2 \left(\int \frac{1}{\left(\sqrt{x}+1\right) \sqrt[6]{x}} d\sqrt{x} + \frac{3x^{5/6}}{5} - \frac{3\sqrt[3]{x}}{2}\right)$$

$$\Big\downarrow \text{68}$$

$$2 \left(-\frac{3}{2} \int \frac{1}{\sqrt[6]{x}+1} d\sqrt[6]{x} + \frac{3}{2} \int \frac{1}{x-\sqrt[6]{x}+1} d\sqrt[6]{x} + \frac{3x^{5/6}}{5} - \frac{3\sqrt[3]{x}}{2} + \frac{1}{2} \log\left(\sqrt{x}+1\right)\right)$$

$$\Big\downarrow \text{16}$$

$$2 \left(\frac{3}{2} \int \frac{1}{x-\sqrt[6]{x}+1} d\sqrt[6]{x} + \frac{3x^{5/6}}{5} - \frac{3\sqrt[3]{x}}{2} - \frac{3}{2} \log\left(\sqrt[6]{x}+1\right) + \frac{1}{2} \log\left(\sqrt{x}+1\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$2 \left(-3 \int \frac{1}{-x-3} d\left(2\sqrt[6]{x}-1\right) + \frac{3x^{5/6}}{5} - \frac{3\sqrt[3]{x}}{2} - \frac{3}{2} \log\left(\sqrt[6]{x}+1\right) + \frac{1}{2} \log\left(\sqrt{x}+1\right)\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\sqrt{3}\arctan\left(\frac{2\sqrt[6]{x}-1}{\sqrt{3}}\right)+\frac{3x^{5/6}}{5}-\frac{3\sqrt[3]{x}}{2}-\frac{3}{2}\log\left(\sqrt[6]{x}+1\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)\right)$$

input
```
Int[x^(1/3)/(1 + Sqrt[x]),x]
```

output
```
2*((-3*x^(1/3))/2 + (3*x^(5/6))/5 + Sqrt[3]*ArcTan[(-1 + 2*x^(1/6))/Sqrt[3
]] - (3*Log[1 + x^(1/6)])/2 + Log[1 + Sqrt[x]]/2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 68
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5}-3x^{\frac{1}{3}}-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+2\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$	49
default	$\frac{6x^{\frac{5}{6}}}{5}-3x^{\frac{1}{3}}-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+2\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$	49
meijerg	$-\frac{3x^{\frac{1}{3}}\left(-8\sqrt{x}+20\right)}{20}+2x^{\frac{1}{3}}\left(-\frac{\ln\left(1+x^{\frac{1}{6}}\right)}{x^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)}{2x^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{6}}}{2-x^{\frac{1}{6}}}\right)}{x^{\frac{1}{3}}}\right)$	72

input
```
int(x^(1/3)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
6/5*x^(5/6)-3*x^(1/3)-2*ln(1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1)+2*3^(1/2)*arct
an(1/3*(2*x^(1/6)-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right) + \frac{6}{5}x^{\frac{5}{6}} - 3x^{\frac{1}{3}} + \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 2\log\left(x^{\frac{1}{6}} + 1\right)$$

input
```
integrate(x^(1/3)/(1+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 3*x^(1
/3) + log(x^(1/3) - x^(1/6) + 1) - 2*log(x^(1/6) + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx = \frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)} - \frac{8\sqrt[3]{x}\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)} - \frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{i\pi}{3}} + 1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16\log\left(-\sqrt[6]{x}e^{i\pi} + 1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{5i\pi}{3}} + 1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**(1/3)/(1+x**(1/2)),x)
```

output
```
16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
 - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_polar(I*pi/3) + 1)*gamma(8/3)/(3*ga
mma(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi) + 1)*gamma(8/3)/(3*gamma(11/
3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_polar(5*I*pi/3) + 1)*gamma(8/3)/(
3*gamma(11/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}} - 3x^{\frac{1}{3}}+\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-2\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(x^(1/3)/(1+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) - 3*x^(1/3) +
log(x^(1/3) - x^(1/6) + 1) - 2*log(x^(1/6) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}} - 3x^{\frac{1}{3}}+\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-2\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(x^(1/3)/(1+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) - 3*x^(1/3) +
log(x^(1/3) - x^(1/6) + 1) - 2*log(x^(1/6) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx = \frac{6\,x^{5/6}}{5} - \ln\left(9\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2 + 36\,x^{1/6}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right) + \ln\left(9\left(1+\sqrt{3}\,1\mathrm{i}\right)^2 + 36\,x^{1/6}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right) - 3\,x^{1/3} - 2\,\ln\left(36\,x^{1/6}+36\right)$$

input
```
int(x^(1/3)/(x^(1/2) + 1),x)
```

output
```
log(9*(3^(1/2)*1i + 1)^2 + 36*x^(1/6))*(3^(1/2)*1i + 1) - log(9*(3^(1/2)*1
i - 1)^2 + 36*x^(1/6))*(3^(1/2)*1i - 1) - 2*log(36*x^(1/6) + 36) - 3*x^(1/
3) + (6*x^(5/6))/5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\,atan\left(\frac{2x^{\frac{1}{6}}-1}{\sqrt{3}}\right) + \frac{6x^{\frac{5}{6}}}{5} - 3x^{\frac{1}{3}} - 2\log\left(x^{\frac{1}{6}}+1\right) + \log\left(-x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)$$

input
```
int(x^(1/3)/(1+x^(1/2)),x)
```

output
```
(10*sqrt(3)*atan((2*x**(1/6) - 1)/sqrt(3)) + 6*x**(5/6) - 15*x**(1/3) - 10
*log(x**(1/6) + 1) + 5*log( - x**(1/6) + x**(1/3) + 1))/5
```

3.147 $\int \left(a+b\sqrt{x}\right)^4 x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \left(a+b\sqrt{x}\right)^4 x^m \, dx = \frac{a^4x^{1+m}}{1+m}+\frac{8a^3bx^{\frac{3}{2}+m}}{3+2m}+\frac{6a^2b^2x^{2+m}}{2+m}+\frac{8ab^3x^{\frac{5}{2}+m}}{5+2m}+\frac{b^4x^{3+m}}{3+m}$$

output
```
a^4*x^(1+m)/(1+m)+8*a^3*b*x^(3/2+m)/(3+2*m)+6*a^2*b^2*x^(2+m)/(2+m)+8*a*b^
3*x^(5/2+m)/(5+2*m)+b^4*x^(3+m)/(3+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}\right)^4 x^m \, dx = x^{1+m}\left(\frac{a^4}{1+m}+\frac{8a^3b\sqrt{x}}{3+2m}+\frac{6a^2b^2x}{2+m}+\frac{8ab^3x^{3/2}}{5+2m}+\frac{b^4x^2}{3+m}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^4*x^m,x]
```

output
```
x^(1 + m)*(a^4/(1 + m) + (8*a^3*b*Sqrt[x])/(3 + 2*m) + (6*a^2*b^2*x)/(2 +
m) + (8*a*b^3*x^(3/2))/(5 + 2*m) + (b^4*x^2)/(3 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+b\sqrt{x}\right)^4 dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^4x^m+4a^3bx^{m+\frac{1}{2}}+6a^2b^2x^{m+1}+4ab^3x^{m+\frac{3}{2}}+b^4x^{m+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4x^{m+1}}{m+1}+\frac{8a^3bx^{m+\frac{3}{2}}}{2m+3}+\frac{6a^2b^2x^{m+2}}{m+2}+\frac{8ab^3x^{m+\frac{5}{2}}}{2m+5}+\frac{b^4x^{m+3}}{m+3}$$

input
```
Int[(a + b*Sqrt[x])^4*x^m,x]
```

output
```
(a^4*x^(1 + m))/(1 + m) + (8*a^3*b*x^(3/2 + m))/(3 + 2*m) + (6*a^2*b^2*x^(
2 + m))/(2 + m) + (8*a*b^3*x^(5/2 + m))/(5 + 2*m) + (b^4*x^(3 + m))/(3 + m
)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(83) = 166.

Time = 0.73 (sec) , antiderivative size = 506, normalized size of antiderivative = 5.82

method	result
orering	$\frac{(-8b^6m^4x^3+24a^2b^4m^4x^2-54b^6m^3x^3-24a^4b^2m^4x+174a^2b^4m^3x^2-133b^6m^2x^3+8a^6m^4-186a^4b^2m^3x+435a^2b^4m^2x^2-141x^3b^6}{(2m^2+5m+3)(2m^2+9m}$

input
```
int((a+b*x^(1/2))^4*x^m,x,method=_RETURNVERBOSE)
```

output
```
(-8*b^6*m^4*x^3+24*a^2*b^4*m^4*x^2-54*b^6*m^3*x^3-24*a^4*b^2*m^4*x+174*a^2
*b^4*m^3*x^2-133*b^6*m^2*x^3+8*a^6*m^4-186*a^4*b^2*m^3*x+435*a^2*b^4*m^2*x
^2-141*b^6*m*x^3+66*a^6*m^3-495*a^4*b^2*m^2*x+453*a^2*b^4*m*x^2-54*b^6*x^3
+193*a^6*m^2-519*a^4*b^2*m*x+168*a^2*b^4*x^2+231*a^6*m-180*a^4*b^2*x+90*a^
6)*x/(2*m^2+5*m+3)/(2*m^2+9*m+10)/(3+m)/(-b^2*x+a^2)^3*(a+b*x^(1/2))^4*x^m
-2*(-2*b^6*m^3*x^3+6*a^2*b^4*m^3*x^2-9*b^6*m^2*x^3-6*a^4*b^2*m^3*x+33*a^2*
b^4*m^2*x^2-13*b^6*m*x^3+2*a^6*m^3-39*a^4*b^2*m^2*x+51*a^2*b^4*m*x^2-6*b^6
*x^3+15*a^6*m^2-75*a^4*b^2*m*x+24*a^2*b^4*x^2+37*a^6*m-36*a^4*b^2*x+30*a^6
)/(2*m^2+5*m+3)/(2*m^2+9*m+10)/(3+m)/(-b^2*x+a^2)^3*x^2*(2*(a+b*x^(1/2))^3
*x^m*b/x^(1/2)+(a+b*x^(1/2))^4*x^m*m/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(83) = 166.

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.99

$$\int \left(a+b\sqrt{x}\right)^4 x^m\,dx$$
$$= \frac{((4\,b^4m^4 + 28\,b^4m^3 + 71\,b^4m^2 + 77\,b^4m + 30\,b^4)x^3 + 6\,(4\,a^2b^2m^4 + 32\,a^2b^2m^3 + 91\,a^2b^2m^2 + 108\,a^2b^2m}{}$$

input
```
integrate((a+b*x^(1/2))^4*x^m,x, algorithm="fricas")
```

output
```
((4*b^4*m^4 + 28*b^4*m^3 + 71*b^4*m^2 + 77*b^4*m + 30*b^4)*x^3 + 6*(4*a^2*
b^2*m^4 + 32*a^2*b^2*m^3 + 91*a^2*b^2*m^2 + 108*a^2*b^2*m + 45*a^2*b^2)*x^
2 + (4*a^4*m^4 + 36*a^4*m^3 + 119*a^4*m^2 + 171*a^4*m + 90*a^4)*x + 8*((2*
a*b^3*m^4 + 15*a*b^3*m^3 + 40*a*b^3*m^2 + 45*a*b^3*m + 18*a*b^3)*x^2 + (2*
a^3*b*m^4 + 17*a^3*b*m^3 + 52*a^3*b*m^2 + 67*a^3*b*m + 30*a^3*b)*x)*sqrt(x
))*x^m/(4*m^5 + 40*m^4 + 155*m^3 + 290*m^2 + 261*m + 90)
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.72

$$\int \left(a+b\sqrt{x}\right)^4 x^m\,dx = a^4\left(\begin{cases}\frac{x^{m+1}}{m+1} & \text{for}\ m\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right) + 8a^3b\left(\begin{cases}\frac{x^{\frac{3}{2}}x^m}{2m+3} & \text{for}\ m\neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^m\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right) + 6a^2b^2\left(\begin{cases}\frac{x^2x^m}{m+2} & \text{for}\ m\neq -2\\ x^2x^m\log(x) & \text{otherwise}\end{cases}\right) + 8ab^3\left(\begin{cases}\frac{x^{\frac{5}{2}}x^m}{2m+5} & \text{for}\ m\neq -\frac{5}{2}\\ x^{\frac{5}{2}}x^m\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right) + b^4\left(\begin{cases}\frac{x^3x^m}{m+3} & \text{for}\ m\neq -3\\ x^3x^m\log(x) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**(1/2))**4*x**m,x)
```

output
```
a**4*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 8*a**3*b
*Piecewise((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (x**(3/2)*x**m*log(sqrt
(x)), True)) + 6*a**2*b**2*Piecewise((x**2*x**m/(m + 2), Ne(m, -2)), (x**2
*x**m*log(x), True)) + 8*a*b**3*Piecewise((x**(5/2)*x**m/(2*m + 5), Ne(m,
-5/2)), (x**(5/2)*x**m*log(sqrt(x)), True)) + b**4*Piecewise((x**3*x**m/(m
 + 3), Ne(m, -3)), (x**3*x**m*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt{x}\right)^4 x^m\,dx = \frac{b^4x^{m+3}}{m+3}+\frac{8\,ab^3x^{m+\frac{5}{2}}}{2\,m+5}+\frac{6\,a^2b^2x^{m+2}}{m+2}+\frac{8\,a^3bx^{m+\frac{3}{2}}}{2\,m+3}+\frac{a^4x^{m+1}}{m+1}$$

input
```
integrate((a+b*x^(1/2))^4*x^m,x, algorithm="maxima")
```

output
```
b^4*x^(m + 3)/(m + 3) + 8*a*b^3*x^(m + 5/2)/(2*m + 5) + 6*a^2*b^2*x^(m + 2
)/(m + 2) + 8*a^3*b*x^(m + 3/2)/(2*m + 3) + a^4*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int \left(a+b\sqrt{x}\right)^4 x^m\,dx = \frac{b^4x^3\sqrt{x}^{2\,m}}{m+3}+\frac{8\,ab^3x^{\frac{5}{2}}\sqrt{x}^{2\,m}}{2\,m+5}+\frac{6\,a^2b^2x^2\sqrt{x}^{2\,m}}{m+2}+\frac{8\,a^3bx^{\frac{3}{2}}\sqrt{x}^{2\,m}}{2\,m+3}+\frac{a^4x\sqrt{x}^{2\,m}}{m+1}$$

input
```
integrate((a+b*x^(1/2))^4*x^m,x, algorithm="giac")
```

output
```
b^4*x^3*sqrt(x)^(2*m)/(m + 3) + 8*a*b^3*x^(5/2)*sqrt(x)^(2*m)/(2*m + 5) +
6*a^2*b^2*x^2*sqrt(x)^(2*m)/(m + 2) + 8*a^3*b*x^(3/2)*sqrt(x)^(2*m)/(2*m +
 3) + a^4*x*sqrt(x)^(2*m)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.36

$$\int \left(a+b\sqrt{x}\right)^4 x^m\,dx = \frac{b^4\,x^m\,x^3\,\left(4\,m^4+28\,m^3+71\,m^2+77\,m+30\right)}{4\,m^5+40\,m^4+155\,m^3+290\,m^2+261\,m+90} + \frac{a^4\,x\,x^m\,\left(4\,m^4+36\,m^3+119\,m^2+171\,m+90\right)}{4\,m^5+40\,m^4+155\,m^3+290\,m^2+261\,m+90} + \frac{8\,a\,b^3\,x^m\,x^{5/2}\,\left(2\,m^4+15\,m^3+40\,m^2+45\,m+18\right)}{4\,m^5+40\,m^4+155\,m^3+290\,m^2+261\,m+90} + \frac{8\,a^3\,b\,x^m\,x^{3/2}\,\left(2\,m^4+17\,m^3+52\,m^2+67\,m+30\right)}{4\,m^5+40\,m^4+155\,m^3+290\,m^2+261\,m+90} + \frac{6\,a^2\,b^2\,x^m\,x^2\,\left(4\,m^4+32\,m^3+91\,m^2+108\,m+45\right)}{4\,m^5+40\,m^4+155\,m^3+290\,m^2+261\,m+90}$$

input
```
int(x^m*(a + b*x^(1/2))^4,x)
```

output
```
(b^4*x^m*x^3*(77*m + 71*m^2 + 28*m^3 + 4*m^4 + 30))/(261*m + 290*m^2 + 155
*m^3 + 40*m^4 + 4*m^5 + 90) + (a^4*x*x^m*(171*m + 119*m^2 + 36*m^3 + 4*m^4
 + 90))/(261*m + 290*m^2 + 155*m^3 + 40*m^4 + 4*m^5 + 90) + (8*a*b^3*x^m*x
^(5/2)*(45*m + 40*m^2 + 15*m^3 + 2*m^4 + 18))/(261*m + 290*m^2 + 155*m^3 +
 40*m^4 + 4*m^5 + 90) + (8*a^3*b*x^m*x^(3/2)*(67*m + 52*m^2 + 17*m^3 + 2*m
^4 + 30))/(261*m + 290*m^2 + 155*m^3 + 40*m^4 + 4*m^5 + 90) + (6*a^2*b^2*x
^m*x^2*(108*m + 91*m^2 + 32*m^3 + 4*m^4 + 45))/(261*m + 290*m^2 + 155*m^3
+ 40*m^4 + 4*m^5 + 90)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.20

$$\int \left(a+b\sqrt{x}\right)^4 x^m\,dx$$

$$= \frac{x^m x\left(16\sqrt{x}\,a^3 b\,m^4+136\sqrt{x}\,a^3 b\,m^3+416\sqrt{x}\,a^3 b\,m^2+536\sqrt{x}\,a^3 bm+4b^4m^4x^2+28b^4m^3x^2+71b^4m^2x^2\right.}{}$$

input
```
int((a+b*x^(1/2))^4*x^m,x)
```

output
```
(x**m*x*(16*sqrt(x)*a**3*b*m**4 + 136*sqrt(x)*a**3*b*m**3 + 416*sqrt(x)*a*
*3*b*m**2 + 536*sqrt(x)*a**3*b*m + 240*sqrt(x)*a**3*b + 16*sqrt(x)*a*b**3*
m**4*x + 120*sqrt(x)*a*b**3*m**3*x + 320*sqrt(x)*a*b**3*m**2*x + 360*sqrt(
x)*a*b**3*m*x + 144*sqrt(x)*a*b**3*x + 4*a**4*m**4 + 36*a**4*m**3 + 119*a*
*4*m**2 + 171*a**4*m + 90*a**4 + 24*a**2*b**2*m**4*x + 192*a**2*b**2*m**3*
x + 546*a**2*b**2*m**2*x + 648*a**2*b**2*m*x + 270*a**2*b**2*x + 4*b**4*m*
*4*x**2 + 28*b**4*m**3*x**2 + 71*b**4*m**2*x**2 + 77*b**4*m*x**2 + 30*b**4
*x**2))/(4*m**5 + 40*m**4 + 155*m**3 + 290*m**2 + 261*m + 90)
```

3.148 $\int \left(a+b\sqrt{x}\right)^3 x^m\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx=\frac{a^3x^{1+m}}{1+m}+\frac{6a^2bx^{\frac{3}{2}+m}}{3+2m}+\frac{3ab^2x^{2+m}}{2+m}+\frac{2b^3x^{\frac{5}{2}+m}}{5+2m}$$

output
```
a^3*x^(1+m)/(1+m)+6*a^2*b*x^(3/2+m)/(3+2*m)+3*a*b^2*x^(2+m)/(2+m)+2*b^3*x^
(5/2+m)/(5+2*m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx=x^{1+m}\left(\frac{a^3}{1+m}+\frac{6a^2b\sqrt{x}}{3+2m}+\frac{3ab^2x}{2+m}+\frac{2b^3x^{3/2}}{5+2m}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^3*x^m,x]
```

output
```
x^(1 + m)*(a^3/(1 + m) + (6*a^2*b*Sqrt[x])/(3 + 2*m) + (3*a*b^2*x)/(2 + m)
 + (2*b^3*x^(3/2))/(5 + 2*m))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+b\sqrt{x}\right)^3 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^m+3a^2bx^{m+\frac{1}{2}}+3ab^2x^{m+1}+b^3x^{m+\frac{3}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^{m+1}}{m+1}+\frac{6a^2bx^{m+\frac{3}{2}}}{2m+3}+\frac{3ab^2x^{m+2}}{m+2}+\frac{2b^3x^{m+\frac{5}{2}}}{2m+5}$$

input
```
Int[(a + b*Sqrt[x])^3*x^m,x]
```

output
```
(a^3*x^(1 + m))/(1 + m) + (6*a^2*b*x^(3/2 + m))/(3 + 2*m) + (3*a*b^2*x^(2
+ m))/(2 + m) + (2*b^3*x^(5/2 + m))/(5 + 2*m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(66) = 132$.

Time = 0.60 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.54

method	result
orering	$\frac{(8b^4m^3x^2-16a^2b^2m^3x+34b^4m^2x^2+8a^4m^3-76a^2b^2m^2x+47b^4m\,x^2+42a^4m^2-106a^2b^2mx+21b^4x^2+67a^4m-45a^2b^2x+30a^4)x(}{(2m^2+5m+3)(2m^2+9m+10)(-b^2x+a^2)^2}$

input
```
int((a+b*x^(1/2))^3*x^m,x,method=_RETURNVERBOSE)
```

output
```
(8*b^4*m^3*x^2-16*a^2*b^2*m^3*x+34*b^4*m^2*x^2+8*a^4*m^3-76*a^2*b^2*m^2*x+
47*b^4*m*x^2+42*a^4*m^2-106*a^2*b^2*m*x+21*b^4*x^2+67*a^4*m-45*a^2*b^2*x+3
0*a^4)*x/(2*m^2+5*m+3)/(2*m^2+9*m+10)/(-b^2*x+a^2)^2*(a+b*x^(1/2))^3*x^m-2
*(2*b^4*m^2*x^2-4*a^2*b^2*m^2*x+5*b^4*m*x^2+2*a^4*m^2-14*a^2*b^2*m*x+3*b^4
*x^2+9*a^4*m-9*a^2*b^2*x+10*a^4)/(2*m^2+5*m+3)/(2*m^2+9*m+10)/(-b^2*x+a^2)
^2*x^2*(3/2*(a+b*x^(1/2))^2*x^m*b/x^(1/2)+(a+b*x^(1/2))^3*x^m*m/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(66) = 132$.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.39

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx$$
$$= \frac{(3\,(4\,ab^2m^3 + 20\,ab^2m^2 + 31\,ab^2m + 15\,ab^2)x^2 + (4\,a^3m^3 + 24\,a^3m^2 + 47\,a^3m + 30\,a^3)x + 2\,((2\,b^3m^3 +}{4\,m^4 + 28\,m^3 + 71\,m^2 + 77\,m}$$

input
```
integrate((a+b*x^(1/2))^3*x^m,x, algorithm="fricas")
```

output
```
(3*(4*a*b^2*m^3 + 20*a*b^2*m^2 + 31*a*b^2*m + 15*a*b^2)*x^2 + (4*a^3*m^3 +
 24*a^3*m^2 + 47*a^3*m + 30*a^3)*x + 2*((2*b^3*m^3 + 9*b^3*m^2 + 13*b^3*m
+ 6*b^3)*x^2 + 3*(2*a^2*b*m^3 + 11*a^2*b*m^2 + 19*a^2*b*m + 10*a^2*b)*x)*s
qrt(x))*x^m/(4*m^4 + 28*m^3 + 71*m^2 + 77*m + 30)
```

Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.73

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx = a^3\left(\begin{cases}\frac{x^{m+1}}{m+1} & \text{for}\ m\neq -1\\ \log\left(x\right) & \text{otherwise}\end{cases}\right) + 6a^2b\left(\begin{cases}\frac{x^{\frac{3}{2}}x^m}{2m+3} & \text{for}\ m\neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^m\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right) + 3ab^2\left(\begin{cases}\frac{x^2x^m}{m+2} & \text{for}\ m\neq -2\\ x^2x^m\log\left(x\right) & \text{otherwise}\end{cases}\right) + 2b^3\left(\begin{cases}\frac{x^{\frac{5}{2}}x^m}{2m+5} & \text{for}\ m\neq -\frac{5}{2}\\ x^{\frac{5}{2}}x^m\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**(1/2))**3*x**m,x)
```

output
```
a**3*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 6*a**2*b
*Piecewise((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (x**(3/2)*x**m*log(sqrt
(x)), True)) + 3*a*b**2*Piecewise((x**2*x**m/(m + 2), Ne(m, -2)), (x**2*x*
*m*log(x), True)) + 2*b**3*Piecewise((x**(5/2)*x**m/(2*m + 5), Ne(m, -5/2)
), (x**(5/2)*x**m*log(sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx = \frac{2\,b^3x^{m+\frac{5}{2}}}{2\,m+5} + \frac{3\,ab^2x^{m+2}}{m+2} + \frac{6\,a^2bx^{m+\frac{3}{2}}}{2\,m+3} + \frac{a^3x^{m+1}}{m+1}$$

input
```
integrate((a+b*x^(1/2))^3*x^m,x, algorithm="maxima")
```

output
```
2*b^3*x^(m + 5/2)/(2*m + 5) + 3*a*b^2*x^(m + 2)/(m + 2) + 6*a^2*b*x^(m + 3
/2)/(2*m + 3) + a^3*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx = \frac{2\,b^3x^{\frac{5}{2}}\sqrt{x}^{2\,m}}{2\,m+5} + \frac{3\,ab^2x^2\sqrt{x}^{2\,m}}{m+2} + \frac{6\,a^2bx^{\frac{3}{2}}\sqrt{x}^{2\,m}}{2\,m+3} + \frac{a^3x\sqrt{x}^{2\,m}}{m+1}$$

input
```
integrate((a+b*x^(1/2))^3*x^m,x, algorithm="giac")
```

output
```
2*b^3*x^(5/2)*sqrt(x)^(2*m)/(2*m + 5) + 3*a*b^2*x^2*sqrt(x)^(2*m)/(m + 2)
+ 6*a^2*b*x^(3/2)*sqrt(x)^(2*m)/(2*m + 3) + a^3*x*sqrt(x)^(2*m)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx = x^m\left(\frac{a^3\,x\,(4\,m^3+24\,m^2+47\,m+30)}{4\,m^4+28\,m^3+71\,m^2+77\,m+30} + \frac{2\,b^3\,x^{5/2}\,(2\,m^3+9\,m^2+13\,m+6)}{4\,m^4+28\,m^3+71\,m^2+77\,m+30} + \frac{6\,a^2\,b\,x^{3/2}\,(2\,m^3+11\,m^2+19\,m+10)}{4\,m^4+28\,m^3+71\,m^2+77\,m+30} + \frac{3\,a\,b^2\,x^2\,(4\,m^3+20\,m^2+31\,m+15)}{4\,m^4+28\,m^3+71\,m^2+77\,m+30}\right)$$

input
```
int(x^m*(a + b*x^(1/2))^3,x)
```

output
```
x^m*((a^3*x*(47*m + 24*m^2 + 4*m^3 + 30))/(77*m + 71*m^2 + 28*m^3 + 4*m^4
+ 30) + (2*b^3*x^(5/2)*(13*m + 9*m^2 + 2*m^3 + 6))/(77*m + 71*m^2 + 28*m^3
 + 4*m^4 + 30) + (6*a^2*b*x^(3/2)*(19*m + 11*m^2 + 2*m^3 + 10))/(77*m + 71
*m^2 + 28*m^3 + 4*m^4 + 30) + (3*a*b^2*x^2*(31*m + 20*m^2 + 4*m^3 + 15))/(
77*m + 71*m^2 + 28*m^3 + 4*m^4 + 30))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

$$\int \left(a+b\sqrt{x}\right)^3 x^m\,dx$$
$$= \frac{x^m x\left(12\sqrt{x}\,a^2b\,m^3+66\sqrt{x}\,a^2b\,m^2+114\sqrt{x}\,a^2bm+60\sqrt{x}\,a^2b+4\sqrt{x}\,b^3m^3x+18\sqrt{x}\,b^3m^2x+26\sqrt{x}\,b^3m\right.}{4m^4+28m^3+71}$$

input
```
int((a+b*x^(1/2))^3*x^m,x)
```

output
```
(x**m*x*(12*sqrt(x)*a**2*b*m**3 + 66*sqrt(x)*a**2*b*m**2 + 114*sqrt(x)*a**
2*b*m + 60*sqrt(x)*a**2*b + 4*sqrt(x)*b**3*m**3*x + 18*sqrt(x)*b**3*m**2*x
 + 26*sqrt(x)*b**3*m*x + 12*sqrt(x)*b**3*x + 4*a**3*m**3 + 24*a**3*m**2 +
47*a**3*m + 30*a**3 + 12*a*b**2*m**3*x + 60*a*b**2*m**2*x + 93*a*b**2*m*x
+ 45*a*b**2*x))/(4*m**4 + 28*m**3 + 71*m**2 + 77*m + 30)
```

3.149 $\int \left(a+b\sqrt{x}\right)^2 x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+b\sqrt{x}\right)^2 x^m \, dx = \frac{a^2 x^{1+m}}{1+m} + \frac{4abx^{\frac{3}{2}+m}}{3+2m} + \frac{b^2 x^{2+m}}{2+m}$$

output
```
a^2*x^(1+m)/(1+m)+4*a*b*x^(3/2+m)/(3+2*m)+b^2*x^(2+m)/(2+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt{x}\right)^2 x^m \, dx = x^{1+m}\left(\frac{a^2}{1+m} + \frac{4ab\sqrt{x}}{3+2m} + \frac{b^2 x}{2+m}\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2*x^m,x]
```

output
```
x^(1 + m)*(a^2/(1 + m) + (4*a*b*Sqrt[x])/(3 + 2*m) + (b^2*x)/(2 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+b\sqrt{x}\right)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2 x^m+2 a b x^{m+\frac{1}{2}}+b^2 x^{m+1}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 x^{m+1}}{m+1}+\frac{4 a b x^{m+\frac{3}{2}}}{2 m+3}+\frac{b^2 x^{m+2}}{m+2}$$

input
```
Int[(a + b*Sqrt[x])^2*x^m,x]
```

output
```
(a^2*x^(1 + m))/(1 + m) + (4*a*b*x^(3/2 + m))/(3 + 2*m) + (b^2*x^(2 + m))/
(2 + m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(45) = 90$.

Time = 0.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.79

method	result
orering	$\frac{(-4b^2m^2x+4a^2m^2-9b^2mx+11a^2m-5b^2x+6a^2)x(a+b\sqrt{x})^2x^m}{(2m^2+5m+3)(2+m)(-b^2x+a^2)} - \frac{2(-b^2mx+a^2m-b^2x+2a^2)x^2\left(\frac{(a+b\sqrt{x})x^m b}{\sqrt{x}}+\frac{(a+b\sqrt{x})^2x^m m}{x}\right)}{(2m^2+5m+3)(2+m)(-b^2x+a^2)}$

input
```
int((a+b*x^(1/2))^2*x^m,x,method=_RETURNVERBOSE)
```

output
```
(-4*b^2*m^2*x+4*a^2*m^2-9*b^2*m*x+11*a^2*m-5*b^2*x+6*a^2)*x/(2*m^2+5*m+3)/
(2+m)/(-b^2*x+a^2)*(a+b*x^(1/2))^2*x^m-2*(-b^2*m*x+a^2*m-b^2*x+2*a^2)/(2*m
^2+5*m+3)/(2+m)/(-b^2*x+a^2)*x^2*((a+b*x^(1/2))*x^m*b/x^(1/2)+(a+b*x^(1/2)
)^2*x^m*m/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.89

$$\int \left(a+b\sqrt{x}\right)^2 x^m\,dx = \frac{\left((2\,b^2m^2+5\,b^2m+3\,b^2)x^2+4\,(abm^2+3\,abm+2\,ab)x^{\frac{3}{2}}+(2\,a^2m^2+7\,a^2m+6\,a^2)x\right)x^m}{2\,m^3+9\,m^2+13\,m+6}$$

input
```
integrate((a+b*x^(1/2))^2*x^m,x, algorithm="fricas")
```

output
```
((2*b^2*m^2 + 5*b^2*m + 3*b^2)*x^2 + 4*(a*b*m^2 + 3*a*b*m + 2*a*b)*x^(3/2)
 + (2*a^2*m^2 + 7*a^2*m + 6*a^2)*x)*x^m/(2*m^3 + 9*m^2 + 13*m + 6)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int \left(a+b\sqrt{x}\right)^2 x^m\,dx = a^2\left(\begin{cases}\frac{x^{m+1}}{m+1} & \text{for } m\neq -1\\ \log(x) & \text{otherwise}\end{cases}\right) + 4ab\left(\begin{cases}\frac{x^{\frac{3}{2}}x^m}{2m+3} & \text{for } m\neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^m\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right) + b^2\left(\begin{cases}\frac{x^2x^m}{m+2} & \text{for } m\neq -2\\ x^2x^m\log(x) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**(1/2))**2*x**m,x)
```

output
```
a**2*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 4*a*b*Pi
ecewise((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (x**(3/2)*x**m*log(sqrt(x)
), True)) + b**2*Piecewise((x**2*x**m/(m + 2), Ne(m, -2)), (x**2*x**m*log(
x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}\right)^2 x^m\,dx = \frac{b^2x^{m+2}}{m+2} + \frac{4\,abx^{m+\frac{3}{2}}}{2\,m+3} + \frac{a^2x^{m+1}}{m+1}$$

input
```
integrate((a+b*x^(1/2))^2*x^m,x, algorithm="maxima")
```

output
```
b^2*x^(m + 2)/(m + 2) + 4*a*b*x^(m + 3/2)/(2*m + 3) + a^2*x^(m + 1)/(m + 1
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \left(a+b\sqrt{x}\right)^2 x^m\,dx = \frac{b^2x^2\sqrt{x}^{2\,m}}{m+2} + \frac{4\,abx^{\frac{3}{2}}\sqrt{x}^{2\,m}}{2\,m+3} + \frac{a^2x\sqrt{x}^{2\,m}}{m+1}$$

input
```
integrate((a+b*x^(1/2))^2*x^m,x, algorithm="giac")
```

output
```
b^2*x^2*sqrt(x)^(2*m)/(m + 2) + 4*a*b*x^(3/2)*sqrt(x)^(2*m)/(2*m + 3) + a^
2*x*sqrt(x)^(2*m)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.19

$$\int \left(a+b\sqrt{x}\right)^2 x^m\,dx = x^m\left(\frac{b^2\,x^2\,(2\,m^2+5\,m+3)}{2\,m^3+9\,m^2+13\,m+6} + \frac{a^2\,x\,(2\,m^2+7\,m+6)}{2\,m^3+9\,m^2+13\,m+6} + \frac{4\,a\,b\,x^{3/2}\,(m^2+3\,m+2)}{2\,m^3+9\,m^2+13\,m+6}\right)$$

input
```
int(x^m*(a + b*x^(1/2))^2,x)
```

output
```
x^m*((b^2*x^2*(5*m + 2*m^2 + 3))/(13*m + 9*m^2 + 2*m^3 + 6) + (a^2*x*(7*m
+ 2*m^2 + 6))/(13*m + 9*m^2 + 2*m^3 + 6) + (4*a*b*x^(3/2)*(3*m + m^2 + 2))
/(13*m + 9*m^2 + 2*m^3 + 6))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.83

$$\int \left(a+b\sqrt{x}\right)^2 x^m\,dx = \frac{x^m x\left(4\sqrt{x}\,ab\,m^2+12\sqrt{x}\,abm+8\sqrt{x}\,ab+2a^2m^2+7a^2m+6a^2+2b^2m^2x+5b^2mx+3b^2x\right)}{2m^3+9m^2+13m+6}$$

input
```
int((a+b*x^(1/2))^2*x^m,x)
```

output
```
(x**m*x*(4*sqrt(x)*a*b*m**2 + 12*sqrt(x)*a*b*m + 8*sqrt(x)*a*b + 2*a**2*m*
*2 + 7*a**2*m + 6*a**2 + 2*b**2*m**2*x + 5*b**2*m*x + 3*b**2*x))/(2*m**3 +
 9*m**2 + 13*m + 6)
```

3.150 $\int \left(a+b\sqrt{x}\right) x^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(a+b\sqrt{x}\right) x^m \, dx = \frac{a x^{1+m}}{1+m} + \frac{2 b x^{\frac{3}{2}+m}}{3+2m}$$

output
```
a*x^(1+m)/(1+m)+2*b*x^(3/2+m)/(3+2*m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{x}\right) x^m \, dx = \frac{a x^{1+m}}{1+m} + \frac{2 b x^{\frac{3}{2}+m}}{3+2m}$$

input
```
Integrate[(a + b*Sqrt[x])*x^m,x]
```

output
```
(a*x^(1 + m))/(1 + m) + (2*b*x^(3/2 + m))/(3 + 2*m)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+b\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(a x^m+b x^{m+\frac{1}{2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a x^{m+1}}{m+1}+\frac{2 b x^{m+\frac{3}{2}}}{2 m+3}$$

input
```
Int[(a + b*Sqrt[x])*x^m,x]
```

output
```
(a*x^(1 + m))/(1 + m) + (2*b*x^(3/2 + m))/(3 + 2*m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

method	result	size
orering	$\frac{x(4m+3)(a+b\sqrt{x})x^m}{2m^2+5m+3} - \frac{2x^2\left(\frac{b\,x^m}{2\sqrt{x}}+\frac{(a+b\sqrt{x})x^m m}{x}\right)}{2m^2+5m+3}$	73

input
```
int((a+b*x^(1/2))*x^m,x,method=_RETURNVERBOSE)
```

output
```
x*(4*m+3)/(2*m^2+5*m+3)*(a+b*x^(1/2))*x^m-2/(2*m^2+5*m+3)*x^2*(1/2*b/x^(1/
2)*x^m+(a+b*x^(1/2))*x^m*m/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\sqrt{x}\right)x^m\,dx = \frac{\left(2\,(bm+b)x^{\frac{3}{2}}+(2\,am+3\,a)x\right)x^m}{2\,m^2+5\,m+3}$$

input
```
integrate((a+b*x^(1/2))*x^m,x, algorithm="fricas")
```

output
```
(2*(b*m + b)*x^(3/2) + (2*a*m + 3*a)*x)*x^m/(2*m^2 + 5*m + 3)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \left(a+b\sqrt{x}\right)x^m\,dx$$
$$= a\left(\begin{cases}\frac{x^{m+1}}{m+1} & \text{for } m \neq -1\\ \log(x) & \text{otherwise}\end{cases}\right) + 2b\left(\begin{cases}\frac{x^{\frac{3}{2}}x^m}{2m+3} & \text{for } m \neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^m\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**(1/2))*x**m,x)
```

output
```
a*Piecewise((x**(m + 1)/(m + 1), Ne(m, -1)), (log(x), True)) + 2*b*Piecewi
se((x**(3/2)*x**m/(2*m + 3), Ne(m, -3/2)), (x**(3/2)*x**m*log(sqrt(x)), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt{x}\right)x^m\,dx=\frac{2\,bx^{m+\frac{3}{2}}}{2\,m+3}+\frac{ax^{m+1}}{m+1}$$

input
```
integrate((a+b*x^(1/2))*x^m,x, algorithm="maxima")
```

output
```
2*b*x^(m + 3/2)/(2*m + 3) + a*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \left(a+b\sqrt{x}\right)x^m\,dx=\frac{2\,bx^{\frac{3}{2}}\sqrt{x}^{2\,m}}{2\,m+3}+\frac{ax\sqrt{x}^{2\,m}}{m+1}$$

input
```
integrate((a+b*x^(1/2))*x^m,x, algorithm="giac")
```

output
```
2*b*x^(3/2)*sqrt(x)^(2*m)/(2*m + 3) + a*x*sqrt(x)^(2*m)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \left(a+b\sqrt{x}\right) x^m\,dx=\frac{x^{m+1}\,\left(3\,a+2\,a\,m+2\,b\,\sqrt{x}+2\,b\,m\,\sqrt{x}\right)}{2\,m^2+5\,m+3}$$

input
```
int(x^m*(a + b*x^(1/2)),x)
```

output
```
(x^(m + 1)*(3*a + 2*a*m + 2*b*x^(1/2) + 2*b*m*x^(1/2)))/(5*m + 2*m^2 + 3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \left(a+b\sqrt{x}\right) x^m\,dx=\frac{x^m x\left(2\sqrt{x}\,bm+2\sqrt{x}\,b+2am+3a\right)}{2m^2+5m+3}$$

input
```
int((a+b*x^(1/2))*x^m,x)
```

output
```
(x**m*x*(2*sqrt(x)*b*m + 2*sqrt(x)*b + 2*a*m + 3*a))/(2*m**2 + 5*m + 3)
```

3.151 $\int \frac{x^m}{a+b\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,2(1+m),3+2m,-\frac{b\sqrt{x}}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*hypergeom([1, 2+2*m],[3+2*m],-b*x^(1/2)/a)/a/(1+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,2+2m,3+2m,-\frac{b\sqrt{x}}{a}\right)}{a+am}$$

input
```
Integrate[x^m/(a + b*Sqrt[x]),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, 2 + 2*m, 3 + 2*m, -((b*Sqrt[x])/a)])/(a +
a*m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {864, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+b\sqrt{x}}dx$$
$$\Big\downarrow \; 864$$
$$2\int \frac{x^{\frac{1}{2}(2m+1)}}{a+b\sqrt{x}}d\sqrt{x}$$
$$\Big\downarrow \; 74$$
$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(1,2(m+1),2m+3,-\frac{b\sqrt{x}}{a}\right)}{a(m+1)}$$

input
```
Int[x^m/(a + b*Sqrt[x]),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((b*Sqrt[x])/a)])/(a*
(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{x^m}{a+b\sqrt{x}}dx$$

input
```
int(x^m/(a+b*x^(1/2)),x)
```

output
```
int(x^m/(a+b*x^(1/2)),x)
```

Fricas [F]

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx = \int \frac{x^m}{b\sqrt{x}+a}\,dx$$

input
```
integrate(x^m/(a+b*x^(1/2)),x, algorithm="fricas")
```

output
```
integral((b*sqrt(x)*x^m - a*x^m)/(b^2*x - a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.22

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx = \frac{4mx^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a\Gamma(2m+3)} + \frac{4x^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a\Gamma(2m+3)}$$

input
```
integrate(x**m/(a+b*x**(1/2)),x)
```

output
```
4*m*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m
 + 2)/(a*gamma(2*m + 3)) + 4*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)
/a, 1, 2*m + 2)*gamma(2*m + 2)/(a*gamma(2*m + 3))
```

Maxima [F]

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx = \int \frac{x^m}{b\sqrt{x}+a}\,dx$$

input
```
integrate(x^m/(a+b*x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*sqrt(x) + a), x)
```

Giac [F]

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx = \int \frac{x^m}{b\sqrt{x}+a}\,dx$$

input
```
integrate(x^m/(a+b*x^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^m/(b*sqrt(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx = \int \frac{x^m}{a+b\,\sqrt{x}}\,dx$$

input
```
int(x^m/(a + b*x^(1/2)),x)
```

output
```
int(x^m/(a + b*x^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^m}{a+b\sqrt{x}}\,dx$$
$$= \frac{2x^{m+\frac{1}{2}}bm - 2x^m am - x^m a - 2\left(\int \frac{x^{m+\frac{1}{2}}}{-b^2x^2+a^2x}dx\right)a^2b\,m^2 - \left(\int \frac{x^{m+\frac{1}{2}}}{-b^2x^2+a^2x}dx\right)a^2bm + 2\left(\int \frac{x^m}{-b^2x^2+a^2x}dx\right)a^3r}{b^2m\,(2m+1)}$$

input
```
int(x^m/(a+b*x^(1/2)),x)
```

output
```
(2*x**((2*m + 1)/2)*b*m - 2*x**m*a*m - x**m*a - 2*int(x**((2*m + 1)/2)/(a*
*2*x - b**2*x**2),x)*a**2*b*m**2 - int(x**((2*m + 1)/2)/(a**2*x - b**2*x**
2),x)*a**2*b*m + 2*int(x**m/(a**2*x - b**2*x**2),x)*a**3*m**2 + int(x**m/(
a**2*x - b**2*x**2),x)*a**3*m)/(b**2*m*(2*m + 1))
```

3.152 $\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, 2(1+m), 3+2m, -\frac{b\sqrt{x}}{a}\right)}{a^2(1+m)}$$

output
```
x^(1+m)*hypergeom([2, 2+2*m],[3+2*m],-b*x^(1/2)/a)/a^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, 2(1+m), 1+2(1+m), -\frac{b\sqrt{x}}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[x^m/(a + b*Sqrt[x])^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, 2*(1 + m), 1 + 2*(1 + m), -((b*Sqrt[x])/a)
])/(a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {864, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx$$
$$\Big\downarrow \text{864}$$
$$2\int \frac{x^{\frac{1}{2}(2m+1)}}{\left(a+b\sqrt{x}\right)^2}d\sqrt{x}$$
$$\Big\downarrow \text{74}$$
$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(2,2(m+1),2m+3,-\frac{b\sqrt{x}}{a}\right)}{a^2(m+1)}$$

input
```
Int[x^m/(a + b*Sqrt[x])^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, 2*(1 + m), 3 + 2*m, -((b*Sqrt[x])/a)])/(a^
2*(1 + m))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}dx$$

input
```
int(x^m/(a+b*x^(1/2))^2,x)
```

output
```
int(x^m/(a+b*x^(1/2))^2,x)
```

Fricas [F]

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx = \int \frac{x^m}{\left(b\sqrt{x}+a\right)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^(1/2))^2,x, algorithm="fricas")
```

output
```
integral(-(2*a*b*sqrt(x)*x^m - (b^2*x + a^2)*x^m)/(b^4*x^2 - 2*a^2*b^2*x +
 a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 478, normalized size of antiderivative = 12.92

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx = -\frac{8am^2x^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)} - \frac{12amx^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)} + \frac{4amx^{m+1}\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)} - \frac{4ax^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)} + \frac{4ax^{m+1}\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)} - \frac{8bm^2\sqrt{x}x^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)} - \frac{12bm\sqrt{x}x^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)} - \frac{4b\sqrt{x}x^{m+1}\Phi\left(\frac{b\sqrt{x}e^{i\pi}}{a},1,2m+2\right)\Gamma(2m+2)}{a^3\Gamma\left(2m+3\right)+a^2b\sqrt{x}\Gamma\left(2m+3\right)}$$

input
```
integrate(x**m/(a+b*x**(1/2))**2,x)
```

output
```
-8*a*m**2*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gam
ma(2*m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 12*a*m
*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m +
2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) + 4*a*m*x**(m + 1
)*gamma(2*m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 4
*a*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*gamma(2*m
+ 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) + 4*a*x**(m + 1
)*gamma(2*m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 8
*b*m**2*sqrt(x)*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m +
2)*gamma(2*m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) -
12*b*m*sqrt(x)*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2
)*gamma(2*m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3)) - 4
*b*sqrt(x)*x**(m + 1)*lerchphi(b*sqrt(x)*exp_polar(I*pi)/a, 1, 2*m + 2)*ga
mma(2*m + 2)/(a**3*gamma(2*m + 3) + a**2*b*sqrt(x)*gamma(2*m + 3))
```

Maxima [F]

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx=\int \frac{x^m}{\left(b\sqrt{x}+a\right)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^(1/2))^2,x, algorithm="maxima")
```

output
```
-(2*m + 1)*integrate(x^m/(a*b*sqrt(x) + a^2), x) + 2*x*x^m/(a*b*sqrt(x) +
a^2)
```

Giac [F]

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx=\int \frac{x^m}{\left(b\sqrt{x}+a\right)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^(1/2))^2,x, algorithm="giac")
```

output
```
integrate(x^m/(b*sqrt(x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx = \int \frac{x^m}{\left(a+b\,\sqrt{x}\right)^2}\,dx$$

input
```
int(x^m/(a + b*x^(1/2))^2,x)
```

output
```
int(x^m/(a + b*x^(1/2))^2, x)
```

Reduce [F]

$$\int \frac{x^m}{\left(a+b\sqrt{x}\right)^2}\,dx = \text{too large to display}$$

input
```
int(x^m/(a+b*x^(1/2))^2,x)
```

output

```
( - 4*x**((2*m + 1)/2)*a**2*b*m + x**((2*m + 1)/2)*a**2*b - 2*x**((2*m + 1
)/2)*b**3*m**2*x + 3*x**((2*m + 1)/2)*b**3*m*x - x**((2*m + 1)/2)*b**3*x -
 8*sqrt(x)*int(x**((2*m + 1)/2)/(2*a**4*m*x - a**4*x - 4*a**2*b**2*m*x**2
+ 2*a**2*b**2*x**2 + 2*b**4*m*x**3 - b**4*x**3),x)*a**5*b**2*m**4 + 8*sqrt
(x)*int(x**((2*m + 1)/2)/(2*a**4*m*x - a**4*x - 4*a**2*b**2*m*x**2 + 2*a**
2*b**2*x**2 + 2*b**4*m*x**3 - b**4*x**3),x)*a**5*b**2*m**3 + 2*sqrt(x)*int
(x**((2*m + 1)/2)/(2*a**4*m*x - a**4*x - 4*a**2*b**2*m*x**2 + 2*a**2*b**2*
x**2 + 2*b**4*m*x**3 - b**4*x**3),x)*a**5*b**2*m**2 - 2*sqrt(x)*int(x**((2
*m + 1)/2)/(2*a**4*m*x - a**4*x - 4*a**2*b**2*m*x**2 + 2*a**2*b**2*x**2 +
2*b**4*m*x**3 - b**4*x**3),x)*a**5*b**2*m + 8*sqrt(x)*int(x**((2*m + 1)/2)
/(2*a**4*m*x - a**4*x - 4*a**2*b**2*m*x**2 + 2*a**2*b**2*x**2 + 2*b**4*m*x
**3 - b**4*x**3),x)*a**3*b**4*m**4*x - 8*sqrt(x)*int(x**((2*m + 1)/2)/(2*a
**4*m*x - a**4*x - 4*a**2*b**2*m*x**2 + 2*a**2*b**2*x**2 + 2*b**4*m*x**3 -
 b**4*x**3),x)*a**3*b**4*m**3*x - 2*sqrt(x)*int(x**((2*m + 1)/2)/(2*a**4*m
*x - a**4*x - 4*a**2*b**2*m*x**2 + 2*a**2*b**2*x**2 + 2*b**4*m*x**3 - b**4
*x**3),x)*a**3*b**4*m**2*x + 2*sqrt(x)*int(x**((2*m + 1)/2)/(2*a**4*m*x -
a**4*x - 4*a**2*b**2*m*x**2 + 2*a**2*b**2*x**2 + 2*b**4*m*x**3 - b**4*x**3
),x)*a**3*b**4*m*x + 4*sqrt(x)*int(x**m/(2*a**4*m**2*x - 3*a**4*m*x + a**4
*x - 4*a**2*b**2*m**2*x**2 + 6*a**2*b**2*m*x**2 - 2*a**2*b**2*x**2 + 2*b**
4*m**2*x**3 - 3*b**4*m*x**3 + b**4*x**3),x)*a**6*b*m**5 - 4*sqrt(x)*int...
```

3.153 $\int \left(a+b\sqrt{x}\right)^p x^m\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx = -\frac{2\left(a+b\sqrt{x}\right)^{1+p} x^{1+m}\,\text{Hypergeometric2F1}\left(1,3+2m+p,2+p,\frac{a+b\sqrt{x}}{a}\right)}{a(1+p)}$$

output
```
-2*(a+b*x^(1/2))^(p+1)*x^(1+m)*hypergeom([1, 3+2*m+p],[2+p],(a+b*x^(1/2))/
a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx = \frac{\left(a+b\sqrt{x}\right)^p \left(1+\frac{b\sqrt{x}}{a}\right)^{-p} x^{1+m}\,\text{Hypergeometric2F1}\left(2(1+m),-p,1+2(1+m),-\frac{b\sqrt{x}}{a}\right)}{1+m}$$

input
```
Integrate[(a + b*Sqrt[x])^p*x^m,x]
```

output
```
((a + b*Sqrt[x])^p*x^(1 + m)*Hypergeometric2F1[2*(1 + m), -p, 1 + 2*(1 + m
), -((b*Sqrt[x])/a)])/((1 + m)*(1 + (b*Sqrt[x])/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {864, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+b\sqrt{x}\right)^p \, dx$$

$$\Big\downarrow \text{864}$$

$$2\int \left(a+b\sqrt{x}\right)^p x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{76}$$

$$2\left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p}\int \left(\frac{\sqrt{x}b}{a}+1\right)^p x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{m+1}\left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(2(m+1),-p,2m+3,-\frac{b\sqrt{x}}{a}\right)}{m+1}$$

input
```
Int[(a + b*Sqrt[x])^p*x^m,x]
```

output
```
((a + b*Sqrt[x])^p*x^(1 + m)*Hypergeometric2F1[2*(1 + m), -p, 3 + 2*m, -((
b*Sqrt[x])/a)])/((1 + m)*(1 + (b*Sqrt[x])/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^p x^m dx$$

input
```
int((a+b*x^(1/2))^p*x^m,x)
```

output
```
int((a+b*x^(1/2))^p*x^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/2))^p*x^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.67 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx = \frac{2a^p x^{m+1}\Gamma(2m+2)\,{}_2F_1\left(\begin{matrix} -p, 2m+2 \\ 2m+3 \end{matrix}\middle| \frac{b\sqrt{x}e^{i\pi}}{a}\right)}{\Gamma(2m+3)}$$

input
```
integrate((a+b*x**(1/2))**p*x**m,x)
```

output
```
2*a**p*x**(m + 1)*gamma(2*m + 2)*hyper((-p, 2*m + 2), (2*m + 3,), b*sqrt(x
)*exp_polar(I*pi)/a)/gamma(2*m + 3)
```

Maxima [F]

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx = \int \left(b\sqrt{x}+a\right)^p x^m\,dx$$

input
```
integrate((a+b*x^(1/2))^p*x^m,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(x) + a)^p*x^m, x)
```

Giac [F]

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx=\int \left(b\sqrt{x}+a\right)^p x^m\,dx$$

input
```
integrate((a+b*x^(1/2))^p*x^m,x, algorithm="giac")
```

output
```
integrate((b*sqrt(x) + a)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx=\int x^m \left(a+b\,\sqrt{x}\right)^p\,dx$$

input
```
int(x^m*(a + b*x^(1/2))^p,x)
```

output
```
int(x^m*(a + b*x^(1/2))^p, x)
```

Reduce [F]

$$\int \left(a+b\sqrt{x}\right)^p x^m\,dx=\text{too large to display}$$

input
```
int((a+b*x^(1/2))^p*x^m,x)
```

output
```
(2*(2*x**((2*m + 1)/2)*(sqrt(x)*b + a)**p*a*b*m*p + x**((2*m + 1)/2)*(sqrt
(x)*b + a)**p*a*b*p**2 - 2*x**m*(sqrt(x)*b + a)**p*a**2*m*p - x**m*(sqrt(x
)*b + a)**p*a**2*p + 4*x**m*(sqrt(x)*b + a)**p*b**2*m**2*x + 4*x**m*(sqrt(
x)*b + a)**p*b**2*m*p*x + 2*x**m*(sqrt(x)*b + a)**p*b**2*m*x + x**m*(sqrt(
x)*b + a)**p*b**2*p**2*x + x**m*(sqrt(x)*b + a)**p*b**2*p*x + 16*int((x**m
*(sqrt(x)*b + a)**p)/(8*a**2*m**3*x + 12*a**2*m**2*p*x + 12*a**2*m**2*x +
6*a**2*m*p**2*x + 12*a**2*m*p*x + 4*a**2*m*x + a**2*p**3*x + 3*a**2*p**2*x
 + 2*a**2*p*x - 8*b**2*m**3*x**2 - 12*b**2*m**2*p*x**2 - 12*b**2*m**2*x**2
 - 6*b**2*m*p**2*x**2 - 12*b**2*m*p*x**2 - 4*b**2*m*x**2 - b**2*p**3*x**2
- 3*b**2*p**2*x**2 - 2*b**2*p*x**2),x)*a**4*m**5*p + 24*int((x**m*(sqrt(x)
*b + a)**p)/(8*a**2*m**3*x + 12*a**2*m**2*p*x + 12*a**2*m**2*x + 6*a**2*m*
p**2*x + 12*a**2*m*p*x + 4*a**2*m*x + a**2*p**3*x + 3*a**2*p**2*x + 2*a**2
*p*x - 8*b**2*m**3*x**2 - 12*b**2*m**2*p*x**2 - 12*b**2*m**2*x**2 - 6*b**2
*m*p**2*x**2 - 12*b**2*m*p*x**2 - 4*b**2*m*x**2 - b**2*p**3*x**2 - 3*b**2*
p**2*x**2 - 2*b**2*p*x**2),x)*a**4*m**4*p**2 + 32*int((x**m*(sqrt(x)*b + a
)**p)/(8*a**2*m**3*x + 12*a**2*m**2*p*x + 12*a**2*m**2*x + 6*a**2*m*p**2*x
 + 12*a**2*m*p*x + 4*a**2*m*x + a**2*p**3*x + 3*a**2*p**2*x + 2*a**2*p*x -
 8*b**2*m**3*x**2 - 12*b**2*m**2*p*x**2 - 12*b**2*m**2*x**2 - 6*b**2*m*p**
2*x**2 - 12*b**2*m*p*x**2 - 4*b**2*m*x**2 - b**2*p**3*x**2 - 3*b**2*p**2*x
**2 - 2*b**2*p*x**2),x)*a**4*m**4*p + 12*int((x**m*(sqrt(x)*b + a)**p)/...
```

3.154 $\int \left(a+b\sqrt{x}\right)^p x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 204

$$\int \left(a+b\sqrt{x}\right)^p x^3\,dx = -\frac{2a^7\left(a+b\sqrt{x}\right)^{1+p}}{b^8(1+p)}+\frac{14a^6\left(a+b\sqrt{x}\right)^{2+p}}{b^8(2+p)}-\frac{42a^5\left(a+b\sqrt{x}\right)^{3+p}}{b^8(3+p)}+\frac{70a^4\left(a+b\sqrt{x}\right)^{4+p}}{b^8(4+p)}-\frac{70a^3\left(a+b\sqrt{x}\right)^{5+p}}{b^8(5+p)}+\frac{42a^2\left(a+b\sqrt{x}\right)^{6+p}}{b^8(6+p)}-\frac{14a\left(a+b\sqrt{x}\right)^{7+p}}{b^8(7+p)}+\frac{2\left(a+b\sqrt{x}\right)^{8+p}}{b^8(8+p)}$$

output
```
-2*a^7*(a+b*x^(1/2))^(p+1)/b^8/(p+1)+14*a^6*(a+b*x^(1/2))^(2+p)/b^8/(2+p)-
42*a^5*(a+b*x^(1/2))^(3+p)/b^8/(3+p)+70*a^4*(a+b*x^(1/2))^(4+p)/b^8/(4+p)-
70*a^3*(a+b*x^(1/2))^(5+p)/b^8/(5+p)+42*a^2*(a+b*x^(1/2))^(6+p)/b^8/(6+p)-
14*a*(a+b*x^(1/2))^(7+p)/b^8/(7+p)+2*(a+b*x^(1/2))^(8+p)/b^8/(8+p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt{x}\right)^p x^3\,dx = \frac{2\left(-\frac{a^7}{1+p}+\frac{7a^6\left(a+b\sqrt{x}\right)}{2+p}-\frac{21a^5\left(a+b\sqrt{x}\right)^2}{3+p}+\frac{35a^4\left(a+b\sqrt{x}\right)^3}{4+p}-\frac{35a^3\left(a+b\sqrt{x}\right)^4}{5+p}+\frac{21a^2\left(a+b\sqrt{x}\right)^5}{6+p}-\frac{7a\left(a+b\sqrt{x}\right)^6}{7+p}+\frac{\left(a+b\sqrt{x}\right)^7}{8+p}\right.}{b^8}$$

input
```
Integrate[(a + b*Sqrt[x])^p*x^3,x]
```

output
```
(2*(-(a^7/(1 + p)) + (7*a^6*(a + b*Sqrt[x]))/(2 + p) - (21*a^5*(a + b*Sqrt
[x])^2)/(3 + p) + (35*a^4*(a + b*Sqrt[x])^3)/(4 + p) - (35*a^3*(a + b*Sqrt
[x])^4)/(5 + p) + (21*a^2*(a + b*Sqrt[x])^5)/(6 + p) - (7*a*(a + b*Sqrt[x]
)^6)/(7 + p) + (a + b*Sqrt[x])^7/(8 + p))*(a + b*Sqrt[x])^(1 + p))/b^8
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{x}\right)^p\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^p x^{7/2}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{a^7\left(a+b\sqrt{x}\right)^p}{b^7}+\frac{7a^6\left(a+b\sqrt{x}\right)^{p+1}}{b^7}-\frac{21a^5\left(a+b\sqrt{x}\right)^{p+2}}{b^7}+\frac{35a^4\left(a+b\sqrt{x}\right)^{p+3}}{b^7}-\frac{35a^3\left(a+b\sqrt{x}\right)^{p+4}}{b^7}+\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^7\left(a+b\sqrt{x}\right)^{p+1}}{b^8(p+1)}+\frac{7a^6\left(a+b\sqrt{x}\right)^{p+2}}{b^8(p+2)}-\frac{21a^5\left(a+b\sqrt{x}\right)^{p+3}}{b^8(p+3)}+\frac{35a^4\left(a+b\sqrt{x}\right)^{p+4}}{b^8(p+4)}-\frac{35a^3\left(a+b\sqrt{x}\right)^{p+5}}{b^8(p+5)}+\right.$$

input
```
Int[(a + b*Sqrt[x])^p*x^3,x]
```

output
```
2*(-((a^7*(a + b*Sqrt[x])^(1 + p))/(b^8*(1 + p))) + (7*a^6*(a + b*Sqrt[x])
^(2 + p))/(b^8*(2 + p)) - (21*a^5*(a + b*Sqrt[x])^(3 + p))/(b^8*(3 + p)) +
 (35*a^4*(a + b*Sqrt[x])^(4 + p))/(b^8*(4 + p)) - (35*a^3*(a + b*Sqrt[x])^
(5 + p))/(b^8*(5 + p)) + (21*a^2*(a + b*Sqrt[x])^(6 + p))/(b^8*(6 + p)) -
(7*a*(a + b*Sqrt[x])^(7 + p))/(b^8*(7 + p)) + (a + b*Sqrt[x])^(8 + p)/(b^8
*(8 + p)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^p x^3 dx$$

input
```
int((a+b*x^(1/2))^p*x^3,x)
```

output
```
int((a+b*x^(1/2))^p*x^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(188) = 376$.

Time = 0.13 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.25

$$\int \left(a+b\sqrt{x}\right)^p x^3 \, dx = -\frac{2\left(5040\,a^8-\left(b^8p^7+28\,b^8p^6+322\,b^8p^5+1960\,b^8p^4+6769\,b^8p^3+13132\,b^8p^2+13068\,b^8p+5040\,b^8\right)x^4\right.}{}$$

input
```
integrate((a+b*x^(1/2))^p*x^3,x, algorithm="fricas")
```

output
```
-2*(5040*a^8 - (b^8*p^7 + 28*b^8*p^6 + 322*b^8*p^5 + 1960*b^8*p^4 + 6769*b
^8*p^3 + 13132*b^8*p^2 + 13068*b^8*p + 5040*b^8)*x^4 + 7*(a^2*b^6*p^6 + 15
*a^2*b^6*p^5 + 85*a^2*b^6*p^4 + 225*a^2*b^6*p^3 + 274*a^2*b^6*p^2 + 120*a^
2*b^6*p)*x^3 + 210*(a^4*b^4*p^4 + 6*a^4*b^4*p^3 + 11*a^4*b^4*p^2 + 6*a^4*b
^4*p)*x^2 + 2520*(a^6*b^2*p^2 + a^6*b^2*p)*x - (5040*a^7*b*p + (a*b^7*p^7
+ 21*a*b^7*p^6 + 175*a*b^7*p^5 + 735*a*b^7*p^4 + 1624*a*b^7*p^3 + 1764*a*b
^7*p^2 + 720*a*b^7*p)*x^3 + 42*(a^3*b^5*p^5 + 10*a^3*b^5*p^4 + 35*a^3*b^5*
p^3 + 50*a^3*b^5*p^2 + 24*a^3*b^5*p)*x^2 + 840*(a^5*b^3*p^3 + 3*a^5*b^3*p^
2 + 2*a^5*b^3*p)*x)*sqrt(x))*(b*sqrt(x) + a)^p/(b^8*p^8 + 36*b^8*p^7 + 546
*b^8*p^6 + 4536*b^8*p^5 + 22449*b^8*p^4 + 67284*b^8*p^3 + 118124*b^8*p^2 +
 109584*b^8*p + 40320*b^8)
```

Sympy [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt{x}\right)^p x^3\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**(1/2))**p*x**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.40

$$\int \left(a+b\sqrt{x}\right)^p x^3\,dx = \frac{2\left((p^7+28\,p^6+322\,p^5+1960\,p^4+6769\,p^3+13132\,p^2+13068\,p+5040)b^8x^4+(p^7+21\,p^6+175\,p^5+\right.}{}$$

input
```
integrate((a+b*x^(1/2))^p*x^3,x, algorithm="maxima")
```

output
```
2*((p^7 + 28*p^6 + 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5
040)*b^8*x^4 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 7
20*p)*a*b^7*x^(7/2) - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p
)*a^2*b^6*x^3 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*a^3*b^5*x^(5/2)
 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*a^4*b^4*x^2 + 840*(p^3 + 3*p^2 + 2*p)*
a^5*b^3*x^(3/2) - 2520*(p^2 + p)*a^6*b^2*x + 5040*a^7*b*p*sqrt(x) - 5040*a
^8)*(b*sqrt(x) + a)^p/((p^8 + 36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67
284*p^3 + 118124*p^2 + 109584*p + 40320)*b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1642 vs. $2(188) = 376$.

Time = 0.13 (sec) , antiderivative size = 1642, normalized size of antiderivative = 8.05

$$\int \left(a+b\sqrt{x}\right)^p x^3\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x^(1/2))^p*x^3,x, algorithm="giac")
```

output
```
2*((b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p^7 - 7*(b*sqrt(x) + a)^7*(b*sqrt(x
) + a)^p*a*p^7 + 21*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^7 - 35*(b*sq
rt(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p^7 + 35*(b*sqrt(x) + a)^4*(b*sqrt(x) +
 a)^p*a^4*p^7 - 21*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^5*p^7 + 7*(b*sqrt
(x) + a)^2*(b*sqrt(x) + a)^p*a^6*p^7 - (b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a
^7*p^7 + 28*(b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p^6 - 203*(b*sqrt(x) + a)^
7*(b*sqrt(x) + a)^p*a*p^6 + 630*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^
6 - 1085*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p^6 + 1120*(b*sqrt(x) + a
)^4*(b*sqrt(x) + a)^p*a^4*p^6 - 693*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^
5*p^6 + 238*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^6*p^6 - 35*(b*sqrt(x) +
a)*(b*sqrt(x) + a)^p*a^7*p^6 + 322*(b*sqrt(x) + a)^8*(b*sqrt(x) + a)^p*p^5
 - 2401*(b*sqrt(x) + a)^7*(b*sqrt(x) + a)^p*a*p^5 + 7686*(b*sqrt(x) + a)^6
*(b*sqrt(x) + a)^p*a^2*p^5 - 13685*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a^3
*p^5 + 14630*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^4*p^5 - 9387*(b*sqrt(x)
 + a)^3*(b*sqrt(x) + a)^p*a^5*p^5 + 3346*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)
^p*a^6*p^5 - 511*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^7*p^5 + 1960*(b*sqrt(
x) + a)^8*(b*sqrt(x) + a)^p*p^4 - 14945*(b*sqrt(x) + a)^7*(b*sqrt(x) + a)^
p*a*p^4 + 49140*(b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*a^2*p^4 - 90335*(b*sqr
t(x) + a)^5*(b*sqrt(x) + a)^p*a^3*p^4 + 100240*(b*sqrt(x) + a)^4*(b*sqrt(x
) + a)^p*a^4*p^4 - 67095*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a^5*p^4 + ...
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.89

$$\int \left(a+b\sqrt{x}\right)^p x^3\,dx$$
$$= \left(a+b\sqrt{x}\right)^p \left(\frac{2x^4\left(p^7+28p^6+322p^5+1960p^4+6769p^3+13132p^2+13068p+5040\right)}{p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320}\right.$$
$$-\frac{10080a^8}{b^8\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}$$
$$+\frac{10080a^7p\sqrt{x}}{b^7\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}$$
$$-\frac{5040a^6px\left(p+1\right)}{b^6\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}$$
$$-\frac{14a^2px^3\left(p^5+15p^4+85p^3+225p^2+274p+120\right)}{b^2\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}$$
$$+\frac{1680a^5px^{3/2}\left(p^2+3p+2\right)}{b^5\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}$$
$$-\frac{420a^4px^2\left(p^3+6p^2+11p+6\right)}{b^4\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}$$
$$+\frac{2apx^{7/2}\left(p^6+21p^5+175p^4+735p^3+1624p^2+1764p+720\right)}{b\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}$$
$$\left.+\frac{84a^3px^{5/2}\left(p^4+10p^3+35p^2+50p+24\right)}{b^3\left(p^8+36p^7+546p^6+4536p^5+22449p^4+67284p^3+118124p^2+109584p+40320\right)}\right)$$

input
```
int(x^3*(a + b*x^(1/2))^p,x)
```

output
```
(a + b*x^(1/2))^p*((2*x^4*(13068*p + 13132*p^2 + 6769*p^3 + 1960*p^4 + 322
*p^5 + 28*p^6 + p^7 + 5040))/(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^
4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320) - (10080*a^8)/(b^8*(109584*
p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8
 + 40320)) + (10080*a^7*p*x^(1/2))/(b^7*(109584*p + 118124*p^2 + 67284*p^3
 + 22449*p^4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)) - (5040*a^6*p*x
*(p + 1))/(b^6*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p^5 +
 546*p^6 + 36*p^7 + p^8 + 40320)) - (14*a^2*p*x^3*(274*p + 225*p^2 + 85*p^
3 + 15*p^4 + p^5 + 120))/(b^2*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p
^4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)) + (1680*a^5*p*x^(3/2)*(3*
p + p^2 + 2))/(b^5*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p
^5 + 546*p^6 + 36*p^7 + p^8 + 40320)) - (420*a^4*p*x^2*(11*p + 6*p^2 + p^3
 + 6))/(b^4*(109584*p + 118124*p^2 + 67284*p^3 + 22449*p^4 + 4536*p^5 + 54
6*p^6 + 36*p^7 + p^8 + 40320)) + (2*a*p*x^(7/2)*(1764*p + 1624*p^2 + 735*p
^3 + 175*p^4 + 21*p^5 + p^6 + 720))/(b*(109584*p + 118124*p^2 + 67284*p^3
+ 22449*p^4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)) + (84*a^3*p*x^(5
/2)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))/(b^3*(109584*p + 118124*p^2 + 672
84*p^3 + 22449*p^4 + 4536*p^5 + 546*p^6 + 36*p^7 + p^8 + 40320)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.55

$$\int \left(a+b\sqrt{x}\right)^p x^3\,dx$$

$$= \frac{2\left(\sqrt{x}\,b+a\right)^p \left(b^8p^7x^4+5040\sqrt{x}\,a^7bp-2520a^6b^2p^2x-2520a^6b^2px-210a^4b^4p^4x^2-1260a^4b^4p^3x^2-231\right.}{}$$

input
```
int((a+b*x^(1/2))^p*x^3,x)
```

output

```
(2*(sqrt(x)*b + a)**p*(5040*sqrt(x)*a**7*b*p + 840*sqrt(x)*a**5*b**3*p**3*
x + 2520*sqrt(x)*a**5*b**3*p**2*x + 1680*sqrt(x)*a**5*b**3*p*x + 42*sqrt(x
)*a**3*b**5*p**5*x**2 + 420*sqrt(x)*a**3*b**5*p**4*x**2 + 1470*sqrt(x)*a**
3*b**5*p**3*x**2 + 2100*sqrt(x)*a**3*b**5*p**2*x**2 + 1008*sqrt(x)*a**3*b*
*5*p*x**2 + sqrt(x)*a*b**7*p**7*x**3 + 21*sqrt(x)*a*b**7*p**6*x**3 + 175*s
qrt(x)*a*b**7*p**5*x**3 + 735*sqrt(x)*a*b**7*p**4*x**3 + 1624*sqrt(x)*a*b*
*7*p**3*x**3 + 1764*sqrt(x)*a*b**7*p**2*x**3 + 720*sqrt(x)*a*b**7*p*x**3 -
 5040*a**8 - 2520*a**6*b**2*p**2*x - 2520*a**6*b**2*p*x - 210*a**4*b**4*p*
*4*x**2 - 1260*a**4*b**4*p**3*x**2 - 2310*a**4*b**4*p**2*x**2 - 1260*a**4*
b**4*p*x**2 - 7*a**2*b**6*p**6*x**3 - 105*a**2*b**6*p**5*x**3 - 595*a**2*b
**6*p**4*x**3 - 1575*a**2*b**6*p**3*x**3 - 1918*a**2*b**6*p**2*x**3 - 840*
a**2*b**6*p*x**3 + b**8*p**7*x**4 + 28*b**8*p**6*x**4 + 322*b**8*p**5*x**4
 + 1960*b**8*p**4*x**4 + 6769*b**8*p**3*x**4 + 13132*b**8*p**2*x**4 + 1306
8*b**8*p*x**4 + 5040*b**8*x**4))/(b**8*(p**8 + 36*p**7 + 546*p**6 + 4536*p
**5 + 22449*p**4 + 67284*p**3 + 118124*p**2 + 109584*p + 40320))
```

3.155 $\int \left(a+b\sqrt{x}\right)^p x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 152

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx=-\frac{2a^5\left(a+b\sqrt{x}\right)^{1+p}}{b^6(1+p)}+\frac{10a^4\left(a+b\sqrt{x}\right)^{2+p}}{b^6(2+p)}-\frac{20a^3\left(a+b\sqrt{x}\right)^{3+p}}{b^6(3+p)}+\frac{20a^2\left(a+b\sqrt{x}\right)^{4+p}}{b^6(4+p)}-\frac{10a\left(a+b\sqrt{x}\right)^{5+p}}{b^6(5+p)}+\frac{2\left(a+b\sqrt{x}\right)^{6+p}}{b^6(6+p)}$$

output
```
-2*a^5*(a+b*x^(1/2))^(p+1)/b^6/(p+1)+10*a^4*(a+b*x^(1/2))^(2+p)/b^6/(2+p)-
20*a^3*(a+b*x^(1/2))^(3+p)/b^6/(3+p)+20*a^2*(a+b*x^(1/2))^(4+p)/b^6/(4+p)-
10*a*(a+b*x^(1/2))^(5+p)/b^6/(5+p)+2*(a+b*x^(1/2))^(6+p)/b^6/(6+p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx=\frac{2\left(-\frac{a^5}{1+p}+\frac{5a^4\left(a+b\sqrt{x}\right)}{2+p}-\frac{10a^3\left(a+b\sqrt{x}\right)^2}{3+p}+\frac{10a^2\left(a+b\sqrt{x}\right)^3}{4+p}-\frac{5a\left(a+b\sqrt{x}\right)^4}{5+p}+\frac{\left(a+b\sqrt{x}\right)^5}{6+p}\right)\left(a+b\sqrt{x}\right)^{1+p}}{b^6}$$

input
```
Integrate[(a + b*Sqrt[x])^p*x^2,x]
```

output
```
(2*(-(a^5/(1 + p)) + (5*a^4*(a + b*Sqrt[x]))/(2 + p) - (10*a^3*(a + b*Sqrt
[x])^2)/(3 + p) + (10*a^2*(a + b*Sqrt[x])^3)/(4 + p) - (5*a*(a + b*Sqrt[x]
)^4)/(5 + p) + (a + b*Sqrt[x])^5/(6 + p))*(a + b*Sqrt[x])^(1 + p))/b^6
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{x}\right)^p\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^p x^{5/2}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{a^5\left(a+b\sqrt{x}\right)^p}{b^5}+\frac{5a^4\left(a+b\sqrt{x}\right)^{p+1}}{b^5}-\frac{10a^3\left(a+b\sqrt{x}\right)^{p+2}}{b^5}+\frac{10a^2\left(a+b\sqrt{x}\right)^{p+3}}{b^5}-\frac{5a\left(a+b\sqrt{x}\right)^{p+4}}{b^5}+\frac{(a}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^5\left(a+b\sqrt{x}\right)^{p+1}}{b^6(p+1)}+\frac{5a^4\left(a+b\sqrt{x}\right)^{p+2}}{b^6(p+2)}-\frac{10a^3\left(a+b\sqrt{x}\right)^{p+3}}{b^6(p+3)}+\frac{10a^2\left(a+b\sqrt{x}\right)^{p+4}}{b^6(p+4)}-\frac{5a\left(a+b\sqrt{x}\right)^{p+5}}{b^6(p+5)}+\frac{(a}{b}\right.$$

input
```
Int[(a + b*Sqrt[x])^p*x^2,x]
```

output
```
2*(-((a^5*(a + b*Sqrt[x])^(1 + p))/(b^6*(1 + p))) + (5*a^4*(a + b*Sqrt[x])
^(2 + p))/(b^6*(2 + p)) - (10*a^3*(a + b*Sqrt[x])^(3 + p))/(b^6*(3 + p)) +
 (10*a^2*(a + b*Sqrt[x])^(4 + p))/(b^6*(4 + p)) - (5*a*(a + b*Sqrt[x])^(5
+ p))/(b^6*(5 + p)) + (a + b*Sqrt[x])^(6 + p)/(b^6*(6 + p)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^p x^2 dx$$

input
```
int((a+b*x^(1/2))^p*x^2,x)
```

output
```
int((a+b*x^(1/2))^p*x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(140) = 280.

Time = 0.12 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.85

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx = -\frac{2\left(120\,a^6-(b^6p^5+15\,b^6p^4+85\,b^6p^3+225\,b^6p^2+274\,b^6p+120\,b^6)x^3+5\,(a^2b^4p^4+6\,a^2b^4p^3+11\,a^2b^4\right.}{b^6p^6}$$

input
```
integrate((a+b*x^(1/2))^p*x^2,x, algorithm="fricas")
```

output
```
-2*(120*a^6 - (b^6*p^5 + 15*b^6*p^4 + 85*b^6*p^3 + 225*b^6*p^2 + 274*b^6*p
 + 120*b^6)*x^3 + 5*(a^2*b^4*p^4 + 6*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 6*a^2*
b^4*p)*x^2 + 60*(a^4*b^2*p^2 + a^4*b^2*p)*x - (120*a^5*b*p + (a*b^5*p^5 +
10*a*b^5*p^4 + 35*a*b^5*p^3 + 50*a*b^5*p^2 + 24*a*b^5*p)*x^2 + 20*(a^3*b^3
*p^3 + 3*a^3*b^3*p^2 + 2*a^3*b^3*p)*x)*sqrt(x))*(b*sqrt(x) + a)^p/(b^6*p^6
 + 21*b^6*p^5 + 175*b^6*p^4 + 735*b^6*p^3 + 1624*b^6*p^2 + 1764*b^6*p + 72
0*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**(1/2))**p*x**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.22

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx = \frac{2\left(\left(p^5+15\,p^4+85\,p^3+225\,p^2+274\,p+120\right)b^6x^3+\left(p^5+10\,p^4+35\,p^3+50\,p^2+24\,p\right)ab^5x^{\frac{5}{2}}-5\left(p^4+\cdots\right.\right.}{\left(p^6+21\,p^5+175\,p^4+735\cdots\right.}$$

input
```
integrate((a+b*x^(1/2))^p*x^2,x, algorithm="maxima")
```

output
```
2*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*b^6*x^3 + (p^5 + 10*p^4
 + 35*p^3 + 50*p^2 + 24*p)*a*b^5*x^(5/2) - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*
a^2*b^4*x^2 + 20*(p^3 + 3*p^2 + 2*p)*a^3*b^3*x^(3/2) - 60*(p^2 + p)*a^4*b^
2*x + 120*a^5*b*p*sqrt(x) - 120*a^6)*(b*sqrt(x) + a)^p/((p^6 + 21*p^5 + 17
5*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(140) = 280$.

Time = 0.13 (sec) , antiderivative size = 922, normalized size of antiderivative = 6.07

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x^(1/2))^p*x^2,x, algorithm="giac")
```

output
```
2*((b*sqrt(x) + a)^6*(b*sqrt(x) + a)^p*p^5 - 5*(b*sqrt(x) + a)^5*(b*sqrt(x
) + a)^p*a*p^5 + 10*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2*p^5 - 10*(b*sq
rt(x) + a)^3*(b*sqrt(x) + a)^p*a^3*p^5 + 5*(b*sqrt(x) + a)^2*(b*sqrt(x) +
a)^p*a^4*p^5 - (b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^5*p^5 + 15*(b*sqrt(x) +
 a)^6*(b*sqrt(x) + a)^p*p^4 - 80*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p^4
 + 170*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2*p^4 - 180*(b*sqrt(x) + a)^3
*(b*sqrt(x) + a)^p*a^3*p^4 + 95*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p^
4 - 20*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^5*p^4 + 85*(b*sqrt(x) + a)^6*(b
*sqrt(x) + a)^p*p^3 - 475*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p^3 + 1070
*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2*p^3 - 1210*(b*sqrt(x) + a)^3*(b*s
qrt(x) + a)^p*a^3*p^3 + 685*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p^3 -
155*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^5*p^3 + 225*(b*sqrt(x) + a)^6*(b*s
qrt(x) + a)^p*p^2 - 1300*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p^2 + 3070*
(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2*p^2 - 3720*(b*sqrt(x) + a)^3*(b*sq
rt(x) + a)^p*a^3*p^2 + 2305*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p^2 -
580*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^5*p^2 + 274*(b*sqrt(x) + a)^6*(b*s
qrt(x) + a)^p*p - 1620*(b*sqrt(x) + a)^5*(b*sqrt(x) + a)^p*a*p + 3960*(b*s
qrt(x) + a)^4*(b*sqrt(x) + a)^p*a^2*p - 5080*(b*sqrt(x) + a)^3*(b*sqrt(x)
+ a)^p*a^3*p + 3510*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^4*p - 1044*(b*sq
rt(x) + a)*(b*sqrt(x) + a)^p*a^5*p + 120*(b*sqrt(x) + a)^6*(b*sqrt(x) +...
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.34

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx$$
$$= \left(a+b\sqrt{x}\right)^p \left(\frac{2\,x^3\,(p^5+15\,p^4+85\,p^3+225\,p^2+274\,p+120)}{p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720} - \frac{240\,a^6}{b^6\,(p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720)} + \frac{240\,a^5\,p\,\sqrt{x}}{b^5\,(p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720)} + \frac{2\,a\,p\,x^{5/2}\,(p^4+10\,p^3+35\,p^2+50\,p+24)}{b\,(p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720)} + \frac{40\,a^3\,p\,x^{3/2}\,(p^2+3\,p+2)}{b^3\,(p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720)} - \frac{10\,a^2\,p\,x^2\,(p^3+6\,p^2+11\,p+6)}{b^2\,(p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720)} - \frac{120\,a^4\,p\,x\,(p+1)}{b^4\,(p^6+21\,p^5+175\,p^4+735\,p^3+1624\,p^2+1764\,p+720)}\right)$$

input
```
int(x^2*(a + b*x^(1/2))^p,x)
```

output
```
(a + b*x^(1/2))^p*((2*x^3*(274*p + 225*p^2 + 85*p^3 + 15*p^4 + p^5 + 120))
/(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720) - (240*a^6)/
(b^6*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) + (240*
a^5*p*x^(1/2))/(b^5*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6
+ 720)) + (2*a*p*x^(5/2)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24))/(b*(1764*p +
 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) + (40*a^3*p*x^(3/2)*(
3*p + p^2 + 2))/(b^3*(1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6
 + 720)) - (10*a^2*p*x^2*(11*p + 6*p^2 + p^3 + 6))/(b^2*(1764*p + 1624*p^2
 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)) - (120*a^4*p*x*(p + 1))/(b^4*(
1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6 + 720)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.98

$$\int \left(a+b\sqrt{x}\right)^p x^2\,dx$$
$$= \frac{2\left(\sqrt{x}\,b+a\right)^p \left(120\sqrt{x}\,a^5bp+20\sqrt{x}\,a^3b^3p^3x+60\sqrt{x}\,a^3b^3p^2x+40\sqrt{x}\,a^3b^3px+\sqrt{x}\,a\,b^5p^5x^2+10\sqrt{x}\,a\,b^5p\right.}{}$$

input
```
int((a+b*x^(1/2))^p*x^2,x)
```

output
```
(2*(sqrt(x)*b + a)**p*(120*sqrt(x)*a**5*b*p + 20*sqrt(x)*a**3*b**3*p**3*x
+ 60*sqrt(x)*a**3*b**3*p**2*x + 40*sqrt(x)*a**3*b**3*p*x + sqrt(x)*a*b**5*
p**5*x**2 + 10*sqrt(x)*a*b**5*p**4*x**2 + 35*sqrt(x)*a*b**5*p**3*x**2 + 50
*sqrt(x)*a*b**5*p**2*x**2 + 24*sqrt(x)*a*b**5*p*x**2 - 120*a**6 - 60*a**4*
b**2*p**2*x - 60*a**4*b**2*p*x - 5*a**2*b**4*p**4*x**2 - 30*a**2*b**4*p**3
*x**2 - 55*a**2*b**4*p**2*x**2 - 30*a**2*b**4*p*x**2 + b**6*p**5*x**3 + 15
*b**6*p**4*x**3 + 85*b**6*p**3*x**3 + 225*b**6*p**2*x**3 + 274*b**6*p*x**3
 + 120*b**6*x**3))/(b**6*(p**6 + 21*p**5 + 175*p**4 + 735*p**3 + 1624*p**2
 + 1764*p + 720))
```

3.156 $\int \left(a+b\sqrt{x}\right)^p x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \left(a+b\sqrt{x}\right)^p x\,dx=-\frac{2a^3\left(a+b\sqrt{x}\right)^{1+p}}{b^4(1+p)}+\frac{6a^2\left(a+b\sqrt{x}\right)^{2+p}}{b^4(2+p)}-\frac{6a\left(a+b\sqrt{x}\right)^{3+p}}{b^4(3+p)}+\frac{2\left(a+b\sqrt{x}\right)^{4+p}}{b^4(4+p)}$$

output
```
-2*a^3*(a+b*x^(1/2))^(p+1)/b^4/(p+1)+6*a^2*(a+b*x^(1/2))^(2+p)/b^4/(2+p)-6
*a*(a+b*x^(1/2))^(3+p)/b^4/(3+p)+2*(a+b*x^(1/2))^(4+p)/b^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt{x}\right)^p x\,dx =\frac{2\left(a+b\sqrt{x}\right)^{1+p}\left(-6a^3+6a^2b(1+p)\sqrt{x}-3ab^2(2+3p+p^2)\,x+b^3(6+11p+6p^2+p^3)\,x^{3/2}\right)}{b^4(1+p)(2+p)(3+p)(4+p)}$$

input
```
Integrate[(a + b*Sqrt[x])^p*x,x]
```

output
```
(2*(a + b*Sqrt[x])^(1 + p)*(-6*a^3 + 6*a^2*b*(1 + p)*Sqrt[x] - 3*a*b^2*(2
+ 3*p + p^2)*x + b^3*(6 + 11*p + 6*p^2 + p^3)*x^(3/2)))/(b^4*(1 + p)*(2 +
p)*(3 + p)*(4 + p))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{x}\right)^p\,dx$$

$$\Big\downarrow \text{798}$$

$$2\int \left(a+b\sqrt{x}\right)^p x^{3/2}d\sqrt{x}$$

$$\Big\downarrow \text{53}$$

$$2\int \left(-\frac{a^3\left(a+b\sqrt{x}\right)^p}{b^3}+\frac{3a^2\left(a+b\sqrt{x}\right)^{p+1}}{b^3}-\frac{3a\left(a+b\sqrt{x}\right)^{p+2}}{b^3}+\frac{\left(a+b\sqrt{x}\right)^{p+3}}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{a^3\left(a+b\sqrt{x}\right)^{p+1}}{b^4(p+1)}+\frac{3a^2\left(a+b\sqrt{x}\right)^{p+2}}{b^4(p+2)}-\frac{3a\left(a+b\sqrt{x}\right)^{p+3}}{b^4(p+3)}+\frac{\left(a+b\sqrt{x}\right)^{p+4}}{b^4(p+4)}\right)$$

input
```
Int[(a + b*Sqrt[x])^p*x,x]
```

output
```
2*(-((a^3*(a + b*Sqrt[x])^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*Sqrt[x])
^(2 + p))/(b^4*(2 + p)) - (3*a*(a + b*Sqrt[x])^(3 + p))/(b^4*(3 + p)) + (a
 + b*Sqrt[x])^(4 + p)/(b^4*(4 + p)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^p x dx$$

input
```
int((a+b*x^(1/2))^p*x,x)
```

output
```
int((a+b*x^(1/2))^p*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48

$$\int \left(a+b\sqrt{x}\right)^p x\,dx = -\frac{2\,(6\,a^4-(b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4)x^2+3\,(a^2b^2p^2+a^2b^2p)x-(6\,a^3bp+(ab^3p^3+3\,ab^3p^2+2\,ab^3p}{b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4}$$

input
```
integrate((a+b*x^(1/2))^p*x,x, algorithm="fricas")
```

output
```
-2*(6*a^4 - (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^2 + 3*(a^2*b^2*p^2
+ a^2*b^2*p)*x - (6*a^3*b*p + (a*b^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x)*sqr
t(x))*(b*sqrt(x) + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24
*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14683 vs. $2(88) = 176$.

Time = 36.50 (sec) , antiderivative size = 14683, normalized size of antiderivative = 146.83

$$\int \left(a+b\sqrt{x}\right)^p x\,dx = \text{Too large to display}$$

input
```
integrate((a+b*x**(1/2))**p*x,x)
```

output
```
-12*a**6*a**(p + 4)*x**8*(1 + b*sqrt(x)/a)**(p + 4)/(a**6*b**4*p**4*x**8 +
 10*a**6*b**4*p**3*x**8 + 35*a**6*b**4*p**2*x**8 + 50*a**6*b**4*p*x**8 + 2
4*a**6*b**4*x**8 + 6*a**5*b**5*p**4*x**(17/2) + 60*a**5*b**5*p**3*x**(17/2
) + 210*a**5*b**5*p**2*x**(17/2) + 300*a**5*b**5*p*x**(17/2) + 144*a**5*b*
*5*x**(17/2) + 15*a**4*b**6*p**4*x**9 + 150*a**4*b**6*p**3*x**9 + 525*a**4
*b**6*p**2*x**9 + 750*a**4*b**6*p*x**9 + 360*a**4*b**6*x**9 + 20*a**3*b**7
*p**4*x**(19/2) + 200*a**3*b**7*p**3*x**(19/2) + 700*a**3*b**7*p**2*x**(19
/2) + 1000*a**3*b**7*p*x**(19/2) + 480*a**3*b**7*x**(19/2) + 15*a**2*b**8*
p**4*x**10 + 150*a**2*b**8*p**3*x**10 + 525*a**2*b**8*p**2*x**10 + 750*a**
2*b**8*p*x**10 + 360*a**2*b**8*x**10 + 6*a*b**9*p**4*x**(21/2) + 60*a*b**9
*p**3*x**(21/2) + 210*a*b**9*p**2*x**(21/2) + 300*a*b**9*p*x**(21/2) + 144
*a*b**9*x**(21/2) + b**10*p**4*x**11 + 10*b**10*p**3*x**11 + 35*b**10*p**2
*x**11 + 50*b**10*p*x**11 + 24*b**10*x**11) + 12*a**6*a**(p + 4)*x**8/(a**
6*b**4*p**4*x**8 + 10*a**6*b**4*p**3*x**8 + 35*a**6*b**4*p**2*x**8 + 50*a*
*6*b**4*p*x**8 + 24*a**6*b**4*x**8 + 6*a**5*b**5*p**4*x**(17/2) + 60*a**5*
b**5*p**3*x**(17/2) + 210*a**5*b**5*p**2*x**(17/2) + 300*a**5*b**5*p*x**(1
7/2) + 144*a**5*b**5*x**(17/2) + 15*a**4*b**6*p**4*x**9 + 150*a**4*b**6*p*
*3*x**9 + 525*a**4*b**6*p**2*x**9 + 750*a**4*b**6*p*x**9 + 360*a**4*b**6*x
**9 + 20*a**3*b**7*p**4*x**(19/2) + 200*a**3*b**7*p**3*x**(19/2) + 700*a**
3*b**7*p**2*x**(19/2) + 1000*a**3*b**7*p*x**(19/2) + 480*a**3*b**7*x**(...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \left(a+b\sqrt{x}\right)^p x\,dx = \frac{2\left((p^3+6\,p^2+11\,p+6)b^4x^2+(p^3+3\,p^2+2\,p)ab^3x^{\frac{3}{2}}-3\,(p^2+p)a^2b^2x+6\,a^3bp\sqrt{x}-6\,a^4\right)\left(b\sqrt{x}+a\right)^p}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

input
```
integrate((a+b*x^(1/2))^p*x,x, algorithm="maxima")
```

output
```
2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^2 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^(3/2) -
3*(p^2 + p)*a^2*b^2*x + 6*a^3*b*p*sqrt(x) - 6*a^4)*(b*sqrt(x) + a)^p/((p^4
 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(92) = 184.

Time = 0.13 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.10

$$\int \left(a+b\sqrt{x}\right)^p x\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)^4\left(b\sqrt{x}+a\right)^pp^3-3\left(b\sqrt{x}+a\right)^3\left(b\sqrt{x}+a\right)^pap^3+3\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^pa^2p^3-\left(b\sqrt{x}+\right.\right.}{}$$

input
```
integrate((a+b*x^(1/2))^p*x,x, algorithm="giac")
```

output
```
2*((b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*p^3 - 3*(b*sqrt(x) + a)^3*(b*sqrt(x
) + a)^p*a*p^3 + 3*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^2*p^3 - (b*sqrt(x
) + a)*(b*sqrt(x) + a)^p*a^3*p^3 + 6*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*p
^2 - 21*(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a*p^2 + 24*(b*sqrt(x) + a)^2*(
b*sqrt(x) + a)^p*a^2*p^2 - 9*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^3*p^2 + 1
1*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p*p - 42*(b*sqrt(x) + a)^3*(b*sqrt(x)
+ a)^p*a*p + 57*(b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*a^2*p - 26*(b*sqrt(x)
+ a)*(b*sqrt(x) + a)^p*a^3*p + 6*(b*sqrt(x) + a)^4*(b*sqrt(x) + a)^p - 24*
(b*sqrt(x) + a)^3*(b*sqrt(x) + a)^p*a + 36*(b*sqrt(x) + a)^2*(b*sqrt(x) +
a)^p*a^2 - 24*(b*sqrt(x) + a)*(b*sqrt(x) + a)^p*a^3)/((b^3*p^4 + 10*b^3*p^
3 + 35*b^3*p^2 + 50*b^3*p + 24*b^3)*b)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.80

$$\int \left(a+b\sqrt{x}\right)^p x\,dx = \left(a+b\,\sqrt{x}\right)^p \left(\frac{2\,x^2\,(p^3+6\,p^2+11\,p+6)}{p^4+10\,p^3+35\,p^2+50\,p+24} - \frac{12\,a^4}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{12\,a^3\,p\,\sqrt{x}}{b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} - \frac{6\,a^2\,p\,x\,(p+1)}{b^2\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{2\,a\,p\,x^{3/2}\,(p^2+3\,p+2)}{b\,(p^4+10\,p^3+35\,p^2+50\,p+24)}\right)$$

input
```
int(x*(a + b*x^(1/2))^p,x)
```

output
```
(a + b*x^(1/2))^p*((2*x^2*(11*p + 6*p^2 + p^3 + 6))/(50*p + 35*p^2 + 10*p^
3 + p^4 + 24) - (12*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (12*a
^3*p*x^(1/2))/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (6*a^2*p*x*(p +
1))/(b^2*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (2*a*p*x^(3/2)*(3*p + p^2
+ 2))/(b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \left(a+b\sqrt{x}\right)^p x\,dx$$
$$= \frac{2\left(\sqrt{x}\,b+a\right)^p \left(6\sqrt{x}\,a^3bp+\sqrt{x}\,a\,b^3p^3x+3\sqrt{x}\,a\,b^3p^2x+2\sqrt{x}\,a\,b^3px-6a^4-3a^2b^2p^2x-3a^2b^2px+b^4p^3x\right.}{b^4\left(p^4+10p^3+35p^2+50p+24\right)}$$

input
```
int((a+b*x^(1/2))^p*x,x)
```

output
```
(2*(sqrt(x)*b + a)**p*(6*sqrt(x)*a**3*b*p + sqrt(x)*a*b**3*p**3*x + 3*sqrt
(x)*a*b**3*p**2*x + 2*sqrt(x)*a*b**3*p*x - 6*a**4 - 3*a**2*b**2*p**2*x - 3
*a**2*b**2*p*x + b**4*p**3*x**2 + 6*b**4*p**2*x**2 + 11*b**4*p*x**2 + 6*b*
*4*x**2))/(b**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.157 $\int \left(a+b\sqrt{x}\right)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \left(a+b\sqrt{x}\right)^p\,dx=-\frac{2a\left(a+b\sqrt{x}\right)^{1+p}}{b^2(1+p)}+\frac{2\left(a+b\sqrt{x}\right)^{2+p}}{b^2(2+p)}$$

output
```
-2*a*(a+b*x^(1/2))^(p+1)/b^2/(p+1)+2*(a+b*x^(1/2))^(2+p)/b^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^p\,dx=\frac{2\left(a+b\sqrt{x}\right)^{1+p}\left(-a+b(1+p)\sqrt{x}\right)}{b^2(1+p)(2+p)}$$

input
```
Integrate[(a + b*Sqrt[x])^p,x]
```

output
```
(2*(a + b*Sqrt[x])^(1 + p)*(-a + b*(1 + p)*Sqrt[x]))/(b^2*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^p\,dx$$

$$\Big\downarrow \ 774$$

$$2\int \left(a+b\sqrt{x}\right)^p\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \ 53$$

$$2\int \left(\frac{\left(a+b\sqrt{x}\right)^{p+1}}{b}-\frac{a\left(a+b\sqrt{x}\right)^p}{b}\right)d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{\left(a+b\sqrt{x}\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+b\sqrt{x}\right)^{p+1}}{b^2(p+1)}\right)$$

input
```
Int[(a + b*Sqrt[x])^p,x]
```

output
```
2*(-((a*(a + b*Sqrt[x])^(1 + p))/(b^2*(1 + p))) + (a + b*Sqrt[x])^(2 + p)/
(b^2*(2 + p)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^{p} dx$$

input
```
int((a+b*x^(1/2))^p,x)
```

output
```
int((a+b*x^(1/2))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \left(a+b\sqrt{x}\right)^p\,dx=\frac{2\left(abp\sqrt{x}-a^2+(b^2p+b^2)x\right)\left(b\sqrt{x}+a\right)^p}{b^2p^2+3\,b^2p+2\,b^2}$$

input
```
integrate((a+b*x^(1/2))^p,x, algorithm="fricas")
```

output
```
2*(a*b*p*sqrt(x) - a^2 + (b^2*p + b^2)*x)*(b*sqrt(x) + a)^p/(b^2*p^2 + 3*b
^2*p + 2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(41) = 82$.

Time = 1.09 (sec) , antiderivative size = 445, normalized size of antiderivative = 9.27

$$\int \left(a+b\sqrt{x}\right)^p\,dx = -\frac{2aa^{p+2}x^2\left(1+\frac{b\sqrt{x}}{a}\right)^{p+2}}{ab^2p^2x^2+3ab^2px^2+2ab^2x^2+b^3p^2x^{\frac{5}{2}}+3b^3px^{\frac{5}{2}}+2b^3x^{\frac{5}{2}}} + \frac{2aa^{p+2}x^2}{ab^2p^2x^2+3ab^2px^2+2ab^2x^2+b^3p^2x^{\frac{5}{2}}+3b^3px^{\frac{5}{2}}+2b^3x^{\frac{5}{2}}} + \frac{2a^{p+2}bpx^{\frac{5}{2}}\left(1+\frac{b\sqrt{x}}{a}\right)^{p+2}}{ab^2p^2x^2+3ab^2px^2+2ab^2x^2+b^3p^2x^{\frac{5}{2}}+3b^3px^{\frac{5}{2}}+2b^3x^{\frac{5}{2}}} + \frac{2a^{p+2}bx^{\frac{5}{2}}\left(1+\frac{b\sqrt{x}}{a}\right)^{p+2}}{ab^2p^2x^2+3ab^2px^2+2ab^2x^2+b^3p^2x^{\frac{5}{2}}+3b^3px^{\frac{5}{2}}+2b^3x^{\frac{5}{2}}} + \frac{2a^{p+2}bx^{\frac{5}{2}}}{ab^2p^2x^2+3ab^2px^2+2ab^2x^2+b^3p^2x^{\frac{5}{2}}+3b^3px^{\frac{5}{2}}+2b^3x^{\frac{5}{2}}}$$

input
```
integrate((a+b*x**(1/2))**p,x)
```

output
```
-2*a*a**(p + 2)*x**2*(1 + b*sqrt(x)/a)**(p + 2)/(a*b**2*p**2*x**2 + 3*a*b*
*2*p*x**2 + 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**
3*x**(5/2)) + 2*a*a**(p + 2)*x**2/(a*b**2*p**2*x**2 + 3*a*b**2*p*x**2 + 2*
a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) + 2*b**3*x**(5/2)) +
2*a**(p + 2)*b*p*x**(5/2)*(1 + b*sqrt(x)/a)**(p + 2)/(a*b**2*p**2*x**2 + 3
*a*b**2*p*x**2 + 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(5/2) +
2*b**3*x**(5/2)) + 2*a**(p + 2)*b*x**(5/2)*(1 + b*sqrt(x)/a)**(p + 2)/(a*b
**2*p**2*x**2 + 3*a*b**2*p*x**2 + 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b
**3*p*x**(5/2) + 2*b**3*x**(5/2)) + 2*a**(p + 2)*b*x**(5/2)/(a*b**2*p**2*x
**2 + 3*a*b**2*p*x**2 + 2*a*b**2*x**2 + b**3*p**2*x**(5/2) + 3*b**3*p*x**(
5/2) + 2*b**3*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt{x}\right)^p\,dx = \frac{2\left(b^2(p+1)x+abp\sqrt{x}-a^2\right)\left(b\sqrt{x}+a\right)^p}{(p^2+3\,p+2)b^2}$$

input
```
integrate((a+b*x^(1/2))^p,x, algorithm="maxima")
```

output
```
2*(b^2*(p + 1)*x + a*b*p*sqrt(x) - a^2)*(b*sqrt(x) + a)^p/((p^2 + 3*p + 2)
*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(44) = 88.

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \left(a+b\sqrt{x}\right)^p\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^p p-\left(b\sqrt{x}+a\right)\left(b\sqrt{x}+a\right)^p ap+\left(b\sqrt{x}+a\right)^2\left(b\sqrt{x}+a\right)^p-2\left(b\sqrt{x}+a\right)\left(b\sqrt{x}-\right.\right.}{(p^2+3\,p+2)b^2}$$

input
```
integrate((a+b*x^(1/2))^p,x, algorithm="giac")
```

output
```
2*((b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p*p - (b*sqrt(x) + a)*(b*sqrt(x) + a)
^p*a*p + (b*sqrt(x) + a)^2*(b*sqrt(x) + a)^p - 2*(b*sqrt(x) + a)*(b*sqrt(x
) + a)^p*a)/((p^2 + 3*p + 2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{x}\right)^p \, dx = \frac{x\left(a+b\sqrt{x}\right)^p {}_2\mathrm{F}_1\left(2,-p;\ 3;\ -\frac{b\sqrt{x}}{a}\right)}{\left(\frac{b\sqrt{x}}{a}+1\right)^p}$$

input
```
int((a + b*x^(1/2))^p,x)
```

output
```
(x*(a + b*x^(1/2))^p*hypergeom([2, -p], 3, -(b*x^(1/2))/a))/((b*x^(1/2))/a
 + 1)^p
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}\right)^p \, dx = \frac{2\left(\sqrt{x}\,b+a\right)^p \left(\sqrt{x}\,abp-a^2+b^2px+b^2x\right)}{b^2\left(p^2+3p+2\right)}$$

input
```
int((a+b*x^(1/2))^p,x)
```

output
```
(2*(sqrt(x)*b + a)**p*(sqrt(x)*a*b*p - a**2 + b**2*p*x + b**2*x))/(b**2*(p
**2 + 3*p + 2))
```

3.158 $\int \frac{(a+b\sqrt{x})^p}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{(a+b\sqrt{x})^p}{x} \, dx = -\frac{2(a+b\sqrt{x})^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sqrt{x}}{a}\right)}{a(1+p)}$$

output
```
-2*(a+b*x^(1/2))^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^(1/2)/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sqrt{x})^p}{x} \, dx = -\frac{2(a+b\sqrt{x})^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{b\sqrt{x}}{a}\right)}{a(1+p)}$$

input
```
Integrate[(a + b*Sqrt[x])^p/x,x]
```

output
```
(-2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sqrt
[x])/a])/(a*(1 + p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\left(a+b\sqrt{x}\right)^p}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2\left(a+b\sqrt{x}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{\sqrt{x}b}{a}+1\right)}{a(p+1)}$$

input
```
Int[(a + b*Sqrt[x])^p/x,x]
```

output
```
(-2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sqrt
[x])/a])/(a*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x}\,dx$$

input
```
int((a+b*x^(1/2))^p/x,x)
```

output
```
int((a+b*x^(1/2))^p/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x}\,dx = \int \frac{\left(b\sqrt{x}+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/2))^p/x,x, algorithm="fricas")
```

output
```
integral((b*sqrt(x) + a)^p/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x}\,dx = -\frac{2b^p x^{\frac{p}{2}}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p,-p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{b\sqrt{x}}\right)}{\Gamma\left(1-p\right)}$$

input
```
integrate((a+b*x**(1/2))**p/x,x)
```

output
```
-2*b**p*x**(p/2)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*
sqrt(x)))/gamma(1 - p)
```

Maxima [F]

$$\int \frac{(a+b\sqrt{x})^p}{x}\,dx = \int \frac{(b\sqrt{x}+a)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/2))^p/x,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(x) + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+b\sqrt{x})^p}{x}\,dx = \int \frac{(b\sqrt{x}+a)^p}{x}\,dx$$

input
```
integrate((a+b*x^(1/2))^p/x,x, algorithm="giac")
```

output
```
integrate((b*sqrt(x) + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sqrt{x})^p}{x}\,dx = \int \frac{(a+b\,\sqrt{x})^p}{x}\,dx$$

input
```
int((a + b*x^(1/2))^p/x,x)
```

output
```
int((a + b*x^(1/2))^p/x, x)
```

Reduce [F]

$$\int \frac{(a+b\sqrt{x})^p}{x}\,dx = \frac{2(\sqrt{x}\,b+a)^p + \left(\int \frac{(\sqrt{x}\,b+a)^p}{-b^2x^2+a^2x}dx\right)a^2p - \left(\int \frac{(\sqrt{x}\,b+a)^p}{\sqrt{x}\,a^2-\sqrt{x}\,b^2x}dx\right)abp}{p}$$

input
```
int((a+b*x^(1/2))^p/x,x)
```

output
```
(2*(sqrt(x)*b + a)**p + int((sqrt(x)*b + a)**p/(a**2*x - b**2*x**2),x)*a**
2*p - int((sqrt(x)*b + a)**p/(sqrt(x)*a**2 - sqrt(x)*b**2*x),x)*a*b*p)/p
```

3.159 $\int \frac{(a+b\sqrt{x})^p}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{(a+b\sqrt{x})^p}{x^2} dx = -\frac{2b^2(a+b\sqrt{x})^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b\sqrt{x}}{a}\right)}{a^3(1+p)}$$

output
```
-2*b^2*(a+b*x^(1/2))^(p+1)*hypergeom([3, p+1],[2+p],1+b*x^(1/2)/a)/a^3/(p+
1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\sqrt{x})^p}{x^2} dx = -\frac{2b^2(a+b\sqrt{x})^{1+p}\,\text{Hypergeometric2F1}\left(3,1+p,2+p,1+\frac{b\sqrt{x}}{a}\right)}{a^3(1+p)}$$

input
```
Integrate[(a + b*Sqrt[x])^p/x^2,x]
```

output
```
(-2*b^2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*
Sqrt[x])/a])/(a^3*(1 + p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt{x})^p}{x^2} dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{(a+b\sqrt{x})^p}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{75}$$

$$-\frac{2b^2(a+b\sqrt{x})^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{\sqrt{x}b}{a}+1\right)}{a^3(p+1)}$$

input
```
Int[(a + b*Sqrt[x])^p/x^2,x]
```

output
```
(-2*b^2*(a + b*Sqrt[x])^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, 1 + (b*
Sqrt[x])/a])/(a^3*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}dx$$

input
```
int((a+b*x^(1/2))^p/x^2,x)
```

output
```
int((a+b*x^(1/2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx=\int \frac{\left(b\sqrt{x}+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/2))^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*sqrt(x) + a)^p/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx=-\frac{2b^px^{\frac{p}{2}-1}\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}-p,2-p\\3-p\end{matrix}\middle|\frac{ae^{i\pi}}{b\sqrt{x}}\right)}{\Gamma\left(3-p\right)}$$

input
```
integrate((a+b*x**(1/2))**p/x**2,x)
```

output
```
-2*b**p*x**(p/2 - 1)*gamma(2 - p)*hyper((-p, 2 - p), (3 - p,), a*exp_polar
(I*pi)/(b*sqrt(x)))/gamma(3 - p)
```

Maxima [F]

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx = \int \frac{\left(b\sqrt{x}+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/2))^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(x) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx = \int \frac{\left(b\sqrt{x}+a\right)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^(1/2))^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*sqrt(x) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx = \int \frac{\left(a+b\,\sqrt{x}\right)^p}{x^2}\,dx$$

input
```
int((a + b*x^(1/2))^p/x^2,x)
```

output
```
int((a + b*x^(1/2))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\sqrt{x}\right)^p}{x^2}\,dx$$
$$=\frac{-2\sqrt{x}\left(\sqrt{x}\,b+a\right)^p bp-2\left(\sqrt{x}\,b+a\right)^p a+\left(\int \frac{\left(\sqrt{x}\,b+a\right)^p}{-b^2x^2+a^2x}dx\right)a\,b^2p^2x-\left(\int \frac{\left(\sqrt{x}\,b+a\right)^p}{-b^2x^2+a^2x}dx\right)a\,b^2px-\left(\int \frac{\sqrt{x}\,(\sqrt{}}{-b^2x}\right.}{2ax}$$

input
```
int((a+b*x^(1/2))^p/x^2,x)
```

output
```
( - 2*sqrt(x)*(sqrt(x)*b + a)**p*b*p - 2*(sqrt(x)*b + a)**p*a + int((sqrt(
x)*b + a)**p/(a**2*x - b**2*x**2),x)*a*b**2*p**2*x - int((sqrt(x)*b + a)**
p/(a**2*x - b**2*x**2),x)*a*b**2*p*x - int((sqrt(x)*(sqrt(x)*b + a)**p)/(a
**2*x - b**2*x**2),x)*b**3*p**2*x + int((sqrt(x)*(sqrt(x)*b + a)**p)/(a**2
*x - b**2*x**2),x)*b**3*p*x)/(2*a*x)
```

3.160 $\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2}{3}\log\left(1+x^{3/2}\right)$$

output
```
2/3*ln(1+x^(3/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2}{3}\log\left(1+\sqrt{x}\right)+\frac{2}{3}\log\left(1-\sqrt{x}+x\right)$$

input
```
Integrate[Sqrt[x]/(1 + x^(3/2)),x]
```

output
```
(2*Log[1 + Sqrt[x]])/3 + (2*Log[1 - Sqrt[x] + x])/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x^{3/2}+1} \, dx$$
$$\Big\downarrow \; 792$$
$$\frac{2}{3} \log \left(x^{3/2}+1\right)$$

input
```
Int[Sqrt[x]/(1 + x^(3/2)),x]
```

output
```
(2*Log[1 + x^(3/2)])/3
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2\ln\left(1+x^{\frac{3}{2}}\right)}{3}$	9
default	$\frac{2\ln\left(1+x^{\frac{3}{2}}\right)}{3}$	9
meijerg	$\frac{2\ln\left(1+x^{\frac{3}{2}}\right)}{3}$	9
trager	$-\frac{\ln\left(-\frac{-x^{3}+2x^{\frac{3}{2}}-1}{\left(x^{2}+x+1\right)^{2}(-1+x)^{2}}\right)}{3}$	31

input
```
int(x^(1/2)/(1+x^(3/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(1+x^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2}{3}\,\log\left(x^{\frac{3}{2}}+1\right)$$

input
```
integrate(x^(1/2)/(1+x^(3/2)),x, algorithm="fricas")
```

output
```
2/3*log(x^(3/2) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2\log\left(\sqrt{x}+1\right)}{3} + \frac{2\log\left(-\sqrt{x}+x+1\right)}{3}$$

input
```
integrate(x**(1/2)/(1+x**(3/2)),x)
```

output
```
2*log(sqrt(x) + 1)/3 + 2*log(-sqrt(x) + x + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2}{3}\log\left(x^{\frac{3}{2}}+1\right)$$

input
```
integrate(x^(1/2)/(1+x^(3/2)),x, algorithm="maxima")
```

output
```
2/3*log(x^(3/2) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2}{3}\log\left(x-\sqrt{x}+1\right) + \frac{2}{3}\log\left(\sqrt{x}+1\right)$$

input
```
integrate(x^(1/2)/(1+x^(3/2)),x, algorithm="giac")
```

output
```
2/3*log(x - sqrt(x) + 1) + 2/3*log(sqrt(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2\ \ln\left(x^{3/2}+1\right)}{3}$$

input
```
int(x^(1/2)/(x^(3/2) + 1),x)
```

output
```
(2*log(x^(3/2) + 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{x}}{1+x^{3/2}}\,dx = \frac{2\log\left(-\sqrt{x}+x+1\right)}{3}+\frac{2\log\left(\sqrt{x}+1\right)}{3}$$

input
```
int(x^(1/2)/(1+x^(3/2)),x)
```

output
```
(2*(log( - sqrt(x) + x + 1) + log(sqrt(x) + 1)))/3
```

3.161 $\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{x^3}{(a+bx^{3/2})^{2/3}}\,dx = -\frac{5ax\sqrt[3]{a+bx^{3/2}}}{9b^2} + \frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{3b} - \frac{10a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}} - \frac{5a^2\log\left(\sqrt[3]{b}\sqrt{x}-\sqrt[3]{a+bx^{3/2}}\right)}{9b^{8/3}}$$

output

```
-5/9*a*x*(a+b*x^(3/2))^(1/3)/b^2+1/3*x^(5/2)*(a+b*x^(3/2))^(1/3)/b-10/27*a
^2*arctan(1/3*(1+2*b^(1/3)*x^(1/2)/(a+b*x^(3/2))^(1/3))*3^(1/2))*3^(1/2)/b
^(8/3)-5/9*a^2*ln(b^(1/3)*x^(1/2)-(a+b*x^(3/2))^(1/3))/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.40

$$\int \frac{x^3}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^{3/2}}\left(-5ax+3bx^{5/2}\right)}{9b^2} - \frac{10a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt{x}}{\sqrt[3]{b}\sqrt{x}+2\sqrt[3]{a+bx^{3/2}}}\right)}{9\sqrt{3}b^{8/3}} - \frac{10a^2\log\left(-\sqrt[3]{b}\sqrt{x}+\sqrt[3]{a+bx^{3/2}}\right)}{27b^{8/3}} + \frac{5a^2\log\left(b^{2/3}x+\sqrt[3]{b}\sqrt{x}\sqrt[3]{a+bx^{3/2}}+\left(a+bx^{3/2}\right)^{2/3}\right)}{27b^{8/3}}$$

input
```
Integrate[x^3/(a + b*x^(3/2))^(2/3),x]
```

output
```
((a + b*x^(3/2))^(1/3)*(-5*a*x + 3*b*x^(5/2)))/(9*b^2) - (10*a^2*ArcTan[(S
qrt[3]*b^(1/3)*Sqrt[x])/(b^(1/3)*Sqrt[x] + 2*(a + b*x^(3/2))^(1/3))])/(9*S
qrt[3]*b^(8/3)) - (10*a^2*Log[-(b^(1/3)*Sqrt[x]) + (a + b*x^(3/2))^(1/3)])
/(27*b^(8/3)) + (5*a^2*Log[b^(2/3)*x + b^(1/3)*Sqrt[x]*(a + b*x^(3/2))^(1/
3) + (a + b*x^(3/2))^(2/3)])/(27*b^(8/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {864, 843, 843, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^{3/2})^{2/3}}\,dx$$

$$\Big\downarrow \ 864$$

$$2\int \frac{x^{7/2}}{(bx^{3/2}+a)^{2/3}}d\sqrt{x}$$

$$\Big\downarrow \text{ 843}$$

$$2\left(\frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{6b}-\frac{5a\int \frac{x^2}{\left(bx^{3/2}+a\right)^{2/3}}d\sqrt{x}}{6b}\right)$$

$$\Big\downarrow \text{ 843}$$

$$2\left(\frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{6b}-\frac{5a\left(\frac{x\sqrt[3]{a+bx^{3/2}}}{3b}-\frac{2a\int \frac{\sqrt{x}}{\left(bx^{3/2}+a\right)^{2/3}}d\sqrt{x}}{3b}\right)}{6b}\right)$$

$$\Big\downarrow \text{ 853}$$

$$2\left(\frac{x^{5/2}\sqrt[3]{a+bx^{3/2}}}{6b}-\frac{5a\left(\frac{x\sqrt[3]{a+bx^{3/2}}}{3b}-\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}\sqrt{x}-\sqrt[3]{a+bx^{3/2}}\right)}{2b^{2/3}}\right)}{3b}\right)}{6b}\right)$$

input
```
Int[x^3/(a + b*x^(3/2))^(2/3),x]
```

output
```
2*((x^(5/2)*(a + b*x^(3/2))^(1/3))/(6*b) - (5*a*((x*(a + b*x^(3/2))^(1/3))
/(3*b) - (2*a*(-(ArcTan[(1 + (2*b^(1/3)*Sqrt[x])/(a + b*x^(3/2))^(1/3))/Sq
rt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*Sqrt[x] - (a + b*x^(3/2))^(1/3)]/(
2*b^(2/3))))/(3*b)))/(6*b))
```

Defintions of rubi rules used

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(x^3/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x^3/(a+b*x^(3/2))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^{3/2})^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.29

$$\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{2x^4\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**3/(a+b*x**(3/2))**(2/3),x)
```

output
```
2*x**4*gamma(8/3)*hyper((2/3, 8/3), (11/3,), b*x**(3/2)*exp_polar(I*pi)/a)
/(3*a**(2/3)*gamma(11/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.32

$$\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{8}{3}}} + \frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{\sqrt{x}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}{x}\right)}{27\,b^{\frac{8}{3}}} - \frac{10\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{27\,b^{\frac{8}{3}}} + \frac{\frac{8\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^2b}{\sqrt{x}}-\frac{5\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^2}{x^2}}{9\left(b^4-\frac{2\left(bx^{\frac{3}{2}}+a\right)b^3}{x^{\frac{3}{2}}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^2b^2}{x^3}\right)}$$

input
```
integrate(x^3/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
10/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^(3/2) + a)^(1/3)/sq
rt(x))/b^(1/3))/b^(8/3) + 5/27*a^2*log(b^(2/3) + (b*x^(3/2) + a)^(1/3)*b^(
1/3)/sqrt(x) + (b*x^(3/2) + a)^(2/3)/x)/b^(8/3) - 10/27*a^2*log(-b^(1/3) +
 (b*x^(3/2) + a)^(1/3)/sqrt(x))/b^(8/3) + 1/9*(8*(b*x^(3/2) + a)^(1/3)*a^2
*b/sqrt(x) - 5*(b*x^(3/2) + a)^(4/3)*a^2/x^2)/(b^4 - 2*(b*x^(3/2) + a)*b^3
/x^(3/2) + (b*x^(3/2) + a)^2*b^2/x^3)
```

Giac [F]

$$\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx=\int \frac{x^3}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(x^3/(b*x^(3/2) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx=\int \frac{x^3}{\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

input
```
int(x^3/(a + b*x^(3/2))^(2/3),x)
```

output
```
int(x^3/(a + b*x^(3/2))^(2/3), x)
```

Reduce [F]

$$\int \frac{x^3}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{x^3}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(x^3/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x**3/(sqrt(x)*b*x + a)**(2/3),x)
```

3.162 $\int \frac{1}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{1}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{\log\left(\sqrt[3]{b}\sqrt{x}-\sqrt[3]{a+bx^{3/2}}\right)}{b^{2/3}}$$

output
```
-2/3*arctan(1/3*(1+2*b^(1/3)*x^(1/2)/(a+b*x^(3/2))^(1/3))*3^(1/2))*3^(1/2)
/b^(2/3)-ln(b^(1/3)*x^(1/2)-(a+b*x^(3/2))^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt{x}}{\sqrt[3]{b}\sqrt{x}+2\sqrt[3]{a+bx^{3/2}}}\right) - 2\log\left(-\sqrt[3]{b}\sqrt{x}+\sqrt[3]{a+bx^{3/2}}\right) + \log\left(b^{2/3}x\right.}{3b^{2/3}}$$

input
```
Integrate[(a + b*x^(3/2))^(-2/3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*Sqrt[x])/(b^(1/3)*Sqrt[x] + 2*(a + b*x
^(3/2))^(1/3))] - 2*Log[-(b^(1/3)*Sqrt[x]) + (a + b*x^(3/2))^(1/3)] + Log[
b^(2/3)*x + b^(1/3)*Sqrt[x]*(a + b*x^(3/2))^(1/3) + (a + b*x^(3/2))^(2/3)]
)/(3*b^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {774, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^{3/2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 774}$$

$$2\int \frac{\sqrt{x}}{\left(bx^{3/2}+a\right)^{2/3}}d\sqrt{x}$$

$$\Big\downarrow \text{ 853}$$

$$2\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt{x}}{\sqrt[3]{a+bx^{3/2}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}\sqrt{x}-\sqrt[3]{a+bx^{3/2}}\right)}{2b^{2/3}}\right)$$

input
```
Int[(a + b*x^(3/2))^(-2/3),x]
```

output
```
2*(-(ArcTan[(1 + (2*b^(1/3)*Sqrt[x])/(a + b*x^(3/2))^(1/3))/Sqrt[3]]/(Sqrt
[3]*b^(2/3))) - Log[b^(1/3)*Sqrt[x] - (a + b*x^(3/2))^(1/3)]/(2*b^(2/3)))
```

Defintions of rubi rules used

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/(a+b*x^(3/2))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^{3/2})^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+bx^{3/2})^{2/3}}\,dx = \frac{2x\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/(a+b*x**(3/2))**(2/3),x)
```

output
```
2*x*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**(3/2)*exp_polar(I*pi)/a)/(3*
a**(2/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^{3/2})^{2/3}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{3b^{\frac{1}{3}}}\right)}{3b^{\frac{2}{3}}} + \frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{\sqrt{x}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}{x}\right)}{3b^{\frac{2}{3}}} - \frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{\sqrt{x}}\right)}{3b^{\frac{2}{3}}}$$

input
```
integrate(1/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^(3/2) + a)^(1/3)/sqrt(x))
/b^(1/3))/b^(2/3) + 1/3*log(b^(2/3) + (b*x^(3/2) + a)^(1/3)*b^(1/3)/sqrt(x
) + (b*x^(3/2) + a)^(2/3)/x)/b^(2/3) - 2/3*log(-b^(1/3) + (b*x^(3/2) + a)^
(1/3)/sqrt(x))/b^(2/3)
```

Giac [F]

$$\int \frac{1}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^(3/2) + a)^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a+bx^{3/2})^{2/3}}\,dx = \frac{x\left(\frac{b\,x^{3/2}}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{b\,x^{3/2}}{a}\right)}{\left(a+b\,x^{3/2}\right)^{2/3}}$$

input
```
int(1/(a + b*x^(3/2))^(2/3),x)
```

output
```
(x*((b*x^(3/2))/a + 1)^(2/3)*hypergeom([2/3, 2/3], 5/3, -(b*x^(3/2))/a))/(
a + b*x^(3/2))^(2/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{1}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/(sqrt(x)*b*x + a)**(2/3),x)
```

3.163 $\int \frac{1}{x^3\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \frac{1}{x^3\left(a+bx^{3/2}\right)^{2/3}}\,dx=-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}+\frac{3b\sqrt[3]{a+bx^{3/2}}}{2a^2\sqrt{x}}$$

output
```
-1/2*(a+b*x^(3/2))^(1/3)/a/x^2+3/2*b*(a+b*x^(3/2))^(1/3)/a^2/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3\left(a+bx^{3/2}\right)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^{3/2}}\left(-a+3bx^{3/2}\right)}{2a^2x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^(3/2))^(2/3)),x]
```

output
```
((a + b*x^(3/2))^(1/3)*(-a + 3*b*x^(3/2)))/(2*a^2*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^{3/2}\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b \int \frac{1}{x^{3/2}\left(bx^{3/2}+a\right)^{2/3}}dx}{4a}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{3b\sqrt[3]{a+bx^{3/2}}}{2a^2\sqrt{x}}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}$$

input
```
Int[1/(x^3*(a + b*x^(3/2))^(2/3)),x]
```

output
```
-1/2*(a + b*x^(3/2))^(1/3)/(a*x^2) + (3*b*(a + b*x^(3/2))^(1/3))/(2*a^2*Sq
rt[x])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{1}{x^3 \left(a + b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/x^3/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/x^3/(a+b*x^(3/2))^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^3 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{\left(3\,bx^{\frac{3}{2}} - a\right)\left(bx^{\frac{3}{2}} + a\right)^{\frac{1}{3}}}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
1/2*(3*b*x^(3/2) - a)*(b*x^(3/2) + a)^(1/3)/(a^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^3 \left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^{\frac{3}{2}}}+1}\Gamma\left(-\frac{4}{3}\right)}{9ax^{\frac{3}{2}}\Gamma\left(\frac{2}{3}\right)} + \frac{2b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^{\frac{3}{2}}}+1}\Gamma\left(-\frac{4}{3}\right)}{3a^2\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**3/(a+b*x**(3/2))**(2/3),x)
```

output
```
-2*b**(1/3)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-4/3)/(9*a*x**(3/2)*gamma(2/
3)) + 2*b**(4/3)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-4/3)/(3*a**2*gamma(2/3
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{\frac{4\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b}{\sqrt{x}} - \frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}}{x^2}}{2\,a^2}$$

input
```
integrate(1/x^3/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/2*(4*(b*x^(3/2) + a)^(1/3)*b/sqrt(x) - (b*x^(3/2) + a)^(4/3)/x^2)/a^2
```

Giac [F]

$$\int \frac{1}{x^3 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(3/2) + a)^(2/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{x^3\,\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

input
```
int(1/(x^3*(a + b*x^(3/2))^(2/3)),x)
```

output
```
int(1/(x^3*(a + b*x^(3/2))^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}} x^3}dx$$

input
```
int(1/x^3/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/((sqrt(x)*b*x + a)**(2/3)*x**3),x)
```

3.164 $\int \frac{1}{x^6\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \frac{1}{x^6\left(a+bx^{3/2}\right)^{2/3}}\,dx=-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}+\frac{9b\sqrt[3]{a+bx^{3/2}}}{35a^2x^{7/2}}-\frac{27b^2\sqrt[3]{a+bx^{3/2}}}{70a^3x^2}+\frac{81b^3\sqrt[3]{a+bx^{3/2}}}{70a^4\sqrt{x}}$$

output
```
-1/5*(a+b*x^(3/2))^(1/3)/a/x^5+9/35*b*(a+b*x^(3/2))^(1/3)/a^2/x^(7/2)-27/7
0*b^2*(a+b*x^(3/2))^(1/3)/a^3/x^2+81/70*b^3*(a+b*x^(3/2))^(1/3)/a^4/x^(1/2
)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^6\left(a+bx^{3/2}\right)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^{3/2}}\left(-14a^3+18a^2bx^{3/2}-27ab^2x^3+81b^3x^{9/2}\right)}{70a^4x^5}$$

input
```
Integrate[1/(x^6*(a + b*x^(3/2))^(2/3)),x]
```

output
```
((a + b*x^(3/2))^(1/3)*(-14*a^3 + 18*a^2*b*x^(3/2) - 27*a*b^2*x^3 + 81*b^3
*x^(9/2)))/(70*a^4*x^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+bx^{3/2}\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{9b\int \frac{1}{x^{9/2}\left(bx^{3/2}+a\right)^{2/3}}dx}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{9b\left(-\frac{6b\int \frac{1}{x^3\left(bx^{3/2}+a\right)^{2/3}}dx}{7a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{7ax^{7/2}}\right)}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{9b\left(-\frac{6b\left(-\frac{3b\int \frac{1}{x^{3/2}\left(bx^{3/2}+a\right)^{2/3}}dx}{4a}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}\right)}{7a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{7ax^{7/2}}\right)}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{9b\left(-\frac{6b\left(\frac{3b\sqrt[3]{a+bx^{3/2}}}{2a^2\sqrt{x}}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}\right)}{7a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{7ax^{7/2}}\right)}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}$$

input
```
Int[1/(x^6*(a + b*x^(3/2))^(2/3)),x]
```

output
```
-1/5*(a + b*x^(3/2))^(1/3)/(a*x^5) - (9*b*((-2*(a + b*x^(3/2))^(1/3))/(7*a
*x^(7/2)) - (6*b*(-1/2*(a + b*x^(3/2))^(1/3)/(a*x^2) + (3*b*(a + b*x^(3/2)
)^(1/3))/(2*a^2*Sqrt[x])))/(7*a)))/(10*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(a +b \,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}d x$$

input
```
int(1/x^6/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/x^6/(a+b*x^(3/2))^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^6 \left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{\left(27\,ab^2x^3+14\,a^3-9\,\left(9\,b^3x^4+2\,a^2bx\right)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{70\,a^4x^5}$$

input
```
integrate(1/x^6/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
-1/70*(27*a*b^2*x^3 + 14*a^3 - 9*(9*b^3*x^4 + 2*a^2*b*x)*sqrt(x))*(b*x^(3/
2) + a)^(1/3)/(a^4*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(95) = 190.

Time = 7.98 (sec) , antiderivative size = 736, normalized size of antiderivative = 7.08

$$\int \frac{1}{x^6 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**6/(a+b*x**(3/2))**(2/3),x)
```

output
```
-56*a**6*b**(28/3)*x**9*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*
b**9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**
11*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) - 96*a**5*b**(31
/3)*x**(21/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(2
7/2)*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11*x**(33/
2)*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) - 60*a**4*b**(34/3)*x**12*
(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(27/2)*gamma(2/3
) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11*x**(33/2)*gamma(2/3)
+ 81*a**4*b**12*x**18*gamma(2/3)) + 160*a**3*b**(37/3)*x**(27/2)*(a/(b*x**
(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(27/2)*gamma(2/3) + 243*a
**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11*x**(33/2)*gamma(2/3) + 81*a**4
*b**12*x**18*gamma(2/3)) + 720*a**2*b**(40/3)*x**15*(a/(b*x**(3/2)) + 1)**
(1/3)*gamma(-10/3)/(81*a**7*b**9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*x**
15*gamma(2/3) + 243*a**5*b**11*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**18*
gamma(2/3)) + 864*a*b**(43/3)*x**(33/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(
-10/3)/(81*a**7*b**9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3
) + 243*a**5*b**11*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3))
+ 324*b**(46/3)*x**18*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b*
*9*x**(27/2)*gamma(2/3) + 243*a**6*b**10*x**15*gamma(2/3) + 243*a**5*b**11
*x**(33/2)*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^6 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{\frac{140 \left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}} b^3}{\sqrt{x}} - \frac{105 \left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}} b^2}{x^2} + \frac{60 \left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}} b}{x^{\frac{7}{2}}} - \frac{14 \left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}}{x^5}}{70\,a^4}$$

input
```
integrate(1/x^6/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/70*(140*(b*x^(3/2) + a)^(1/3)*b^3/sqrt(x) - 105*(b*x^(3/2) + a)^(4/3)*b^
2/x^2 + 60*(b*x^(3/2) + a)^(7/3)*b/x^(7/2) - 14*(b*x^(3/2) + a)^(10/3)/x^5
)/a^4
```

Giac [F]

$$\int \frac{1}{x^6 \left(a+bx^{3/2}\right)^{2/3}} \, dx = \int \frac{1}{\left(bx^{\frac{3}{2}} + a\right)^{\frac{2}{3}} x^6} \, dx$$

input
```
integrate(1/x^6/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(3/2) + a)^(2/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(a+bx^{3/2}\right)^{2/3}} \, dx = \int \frac{1}{x^6 \left(a+b\,x^{3/2}\right)^{2/3}} \, dx$$

input
```
int(1/(x^6*(a + b*x^(3/2))^(2/3)),x)
```

output
```
int(1/(x^6*(a + b*x^(3/2))^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(a+bx^{3/2}\right)^{2/3}} \, dx = \int \frac{1}{\left(\sqrt{x}\,bx + a\right)^{\frac{2}{3}} x^6} dx$$

input
```
int(1/x^6/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/((sqrt(x)*b*x + a)**(2/3)*x**6),x)
```

3.165 $\int \frac{1}{x^9\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int \frac{1}{x^9\left(a+bx^{3/2}\right)^{2/3}}\,dx=-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}+\frac{15b\sqrt[3]{a+bx^{3/2}}}{104a^2x^{13/2}}-\frac{9b^2\sqrt[3]{a+bx^{3/2}}}{52a^3x^5}+\frac{81b^3\sqrt[3]{a+bx^{3/2}}}{364a^4x^{7/2}}-\frac{243b^4\sqrt[3]{a+bx^{3/2}}}{728a^5x^2}+\frac{729b^5\sqrt[3]{a+bx^{3/2}}}{728a^6\sqrt{x}}$$

output
```
-1/8*(a+b*x^(3/2))^(1/3)/a/x^8+15/104*b*(a+b*x^(3/2))^(1/3)/a^2/x^(13/2)-9
/52*b^2*(a+b*x^(3/2))^(1/3)/a^3/x^5+81/364*b^3*(a+b*x^(3/2))^(1/3)/a^4/x^(
7/2)-243/728*b^4*(a+b*x^(3/2))^(1/3)/a^5/x^2+729/728*b^5*(a+b*x^(3/2))^(1/
3)/a^6/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^9\left(a+bx^{3/2}\right)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^{3/2}}\left(-91a^5+105a^4bx^{3/2}-126a^3b^2x^3+162a^2b^3x^{9/2}-243ab^4x^6+729b^5\right.}{728a^6x^8}$$

input
```
Integrate[1/(x^9*(a + b*x^(3/2))^(2/3)),x]
```

output
```
((a + b*x^(3/2))^(1/3)*(-91*a^5 + 105*a^4*b*x^(3/2) - 126*a^3*b^2*x^3 + 16
2*a^2*b^3*x^(9/2) - 243*a*b^4*x^6 + 729*b^5*x^(15/2)))/(728*a^6*x^8)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {803, 803, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+bx^{3/2}\right)^{2/3}} \, dx$$

$$\Big\downarrow \; 803$$

$$-\frac{15b\int \frac{1}{x^{15/2}\left(bx^{3/2}+a\right)^{2/3}}dx}{16a}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

$$\Big\downarrow \; 803$$

$$-\frac{15b\left(-\frac{12b\int \frac{1}{x^{6}\left(bx^{3/2}+a\right)^{2/3}}dx}{13a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{13ax^{13/2}}\right)}{16a}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

$$\Big\downarrow \; 803$$

$$-\frac{15b\left(-\frac{12b\left(-\frac{9b\int \frac{1}{x^{9/2}\left(bx^{3/2}+a\right)^{2/3}}dx}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}\right)}{13a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{13ax^{13/2}}\right)}{16a}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

$$\Big\downarrow \; 803$$

$$-\frac{15b\left(-\frac{12b\left(-\frac{9b\left(-\frac{6b\int \frac{1}{x^3\left(bx^{3/2}+a\right)^{2/3}}dx}{7a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{7ax^{7/2}}\right)}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}\right)}{13a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{13ax^{13/2}}\right)}{16a}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

$\downarrow$ 803

$$-\frac{15b\left(-\frac{12b\left(-\frac{9b\left(-\frac{6b\left(-\frac{3b\int \frac{1}{x^{3/2}\left(bx^{3/2}+a\right)^{2/3}}dx}{4a}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}\right)}{7a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{7ax^{7/2}}\right)}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}\right)}{13a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{13ax^{13/2}}\right)}{16a}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

$\downarrow$ 796

$$-\frac{15b\left(-\frac{12b\left(-\frac{9b\left(-\frac{6b\left(\frac{3b\sqrt[3]{a+bx^{3/2}}}{2a^2\sqrt{x}}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^2}\right)}{7a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{7ax^{7/2}}\right)}{10a}-\frac{\sqrt[3]{a+bx^{3/2}}}{5ax^5}\right)}{13a}-\frac{2\sqrt[3]{a+bx^{3/2}}}{13ax^{13/2}}\right)}{16a}-\frac{\sqrt[3]{a+bx^{3/2}}}{8ax^8}$$

input
```
Int[1/(x^9*(a + b*x^(3/2))^(2/3)),x]
```

output
```
-1/8*(a + b*x^(3/2))^(1/3)/(a*x^8) - (15*b*((-2*(a + b*x^(3/2))^(1/3))/(13
*a*x^(13/2)) - (12*b*(-1/5*(a + b*x^(3/2))^(1/3)/(a*x^5) - (9*b*((-2*(a +
b*x^(3/2))^(1/3))/(7*a*x^(7/2)) - (6*b*(-1/2*(a + b*x^(3/2))^(1/3)/(a*x^2)
 + (3*b*(a + b*x^(3/2))^(1/3))/(2*a^2*Sqrt[x])))/(7*a)))/(10*a)))/(13*a)))
/(16*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{1}{x^9 \left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/x^9/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/x^9/(a+b*x^(3/2))^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^9 (a+bx^{3/2})^{2/3}}\,dx = -\frac{\left(243\,ab^4x^6+126\,a^3b^2x^3+91\,a^5-3\left(243\,b^5x^7+54\,a^2b^3x^4+35\,a^4bx\right)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{728\,a^6x^8}$$

input
```
integrate(1/x^9/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
-1/728*(243*a*b^4*x^6 + 126*a^3*b^2*x^3 + 91*a^5 - 3*(243*b^5*x^7 + 54*a^2
*b^3*x^4 + 35*a^4*b*x)*sqrt(x))*(b*x^(3/2) + a)^(1/3)/(a^6*x^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1554 vs. $2(148) = 296$.

Time = 35.39 (sec) , antiderivative size = 1554, normalized size of antiderivative = 9.84

$$\int \frac{1}{x^9 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**9/(a+b*x**(3/2))**(2/3),x)
```

output
```
-7280*a**10*b**(76/3)*x**30*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*
a**11*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 729
0*a**9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 364
5*a**7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 280
00*a**9*b**(79/3)*x**(63/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*
a**11*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 729
0*a**9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 364
5*a**7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 408
80*a**8*b**(82/3)*x**33*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**1
1*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a*
*9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a*
*7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 26240*a
**7*b**(85/3)*x**(69/2)*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**1
1*b**25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a*
*9*b**27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a*
*7*b**29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) - 7840*a*
*6*b**(88/3)*x**36*(a/(b*x**(3/2)) + 1)**(1/3)*gamma(-16/3)/(729*a**11*b**
25*x**(75/2)*gamma(2/3) + 3645*a**10*b**26*x**39*gamma(2/3) + 7290*a**9*b*
*27*x**(81/2)*gamma(2/3) + 7290*a**8*b**28*x**42*gamma(2/3) + 3645*a**7*b*
*29*x**(87/2)*gamma(2/3) + 729*a**6*b**30*x**45*gamma(2/3)) + 24640*a**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^9 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{\frac{1456\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}b^5}{\sqrt{x}} - \frac{1820\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}b^4}{x^2} + \frac{2080\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}b^3}{x^{\frac{7}{2}}} - \frac{1456\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}b^2}{x^5} + \frac{560\left(bx^{\frac{3}{2}}+a\right)}{x^{\frac{13}{2}}}}{728\,a^6}$$

input
```
integrate(1/x^9/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/728*(1456*(b*x^(3/2) + a)^(1/3)*b^5/sqrt(x) - 1820*(b*x^(3/2) + a)^(4/3)
*b^4/x^2 + 2080*(b*x^(3/2) + a)^(7/3)*b^3/x^(7/2) - 1456*(b*x^(3/2) + a)^(
10/3)*b^2/x^5 + 560*(b*x^(3/2) + a)^(13/3)*b/x^(13/2) - 91*(b*x^(3/2) + a)
^(16/3)/x^8)/a^6
```

Giac [F]

$$\int \frac{1}{x^9 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^9}\,dx$$

input
```
integrate(1/x^9/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(3/2) + a)^(2/3)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{x^9 \left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

input
```
int(1/(x^9*(a + b*x^(3/2))^(2/3)),x)
```

output
```
int(1/(x^9*(a + b*x^(3/2))^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^9 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}} x^9}dx$$

input
```
int(1/x^9/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/((sqrt(x)*b*x + a)**(2/3)*x**9),x)
```

3.166 $\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2a^5\sqrt[3]{a+bx^{3/2}}}{b^6}+\frac{5a^4\left(a+bx^{3/2}\right)^{4/3}}{2b^6} -\frac{20a^3\left(a+bx^{3/2}\right)^{7/3}}{7b^6}+\frac{2a^2\left(a+bx^{3/2}\right)^{10/3}}{b^6}-\frac{10a\left(a+bx^{3/2}\right)^{13/3}}{13b^6}+\frac{\left(a+bx^{3/2}\right)^{16/3}}{8b^6}$$

output
```
-2*a^5*(a+b*x^(3/2))^(1/3)/b^6+5/2*a^4*(a+b*x^(3/2))^(4/3)/b^6-20/7*a^3*(a
+b*x^(3/2))^(7/3)/b^6+2*a^2*(a+b*x^(3/2))^(10/3)/b^6-10/13*a*(a+b*x^(3/2))
^(13/3)/b^6+1/8*(a+b*x^(3/2))^(16/3)/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \frac{x^8}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^{3/2}}\left(-729a^5+243a^4bx^{3/2}-162a^3b^2x^3+126a^2b^3x^{9/2}-105ab^4x^6+91b^5x^{15}\right.}{728b^6}$$

input
```
Integrate[x^8/(a + b*x^(3/2))^(2/3),x]
```

output
```
((a + b*x^(3/2))^(1/3)*(-729*a^5 + 243*a^4*b*x^(3/2) - 162*a^3*b^2*x^3 + 1
26*a^2*b^3*x^(9/2) - 105*a*b^4*x^6 + 91*b^5*x^(15/2)))/(728*b^6)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{2}{3}\int \frac{x^{15/2}}{\left(bx^{3/2}+a\right)^{2/3}}dx^{3/2}$$

$$\Big\downarrow \text{53}$$

$$\frac{2}{3}\int \left(-\frac{a^5}{b^5\left(bx^{3/2}+a\right)^{2/3}}+\frac{5\sqrt[3]{bx^{3/2}+a}a^4}{b^5}-\frac{10\left(bx^{3/2}+a\right)^{4/3}a^3}{b^5}+\frac{10\left(bx^{3/2}+a\right)^{7/3}a^2}{b^5}-\frac{5\left(bx^{3/2}+a\right)^{10/3}a}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\left(-\frac{3a^5\sqrt[3]{a+bx^{3/2}}}{b^6}+\frac{15a^4\left(a+bx^{3/2}\right)^{4/3}}{4b^6}-\frac{30a^3\left(a+bx^{3/2}\right)^{7/3}}{7b^6}+\frac{3a^2\left(a+bx^{3/2}\right)^{10/3}}{b^6}+\frac{3\left(a+bx^{3/2}\right)^{16/3}}{16b^6}-\right.$$

input
```
Int[x^8/(a + b*x^(3/2))^(2/3),x]
```

output
```
(2*((-3*a^5*(a + b*x^(3/2))^(1/3))/b^6 + (15*a^4*(a + b*x^(3/2))^(4/3))/(4
*b^6) - (30*a^3*(a + b*x^(3/2))^(7/3))/(7*b^6) + (3*a^2*(a + b*x^(3/2))^(1
0/3))/b^6 - (15*a*(a + b*x^(3/2))^(13/3))/(13*b^6) + (3*(a + b*x^(3/2))^(1
6/3))/(16*b^6)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^8}{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(x^8/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x^8/(a+b*x^(3/2))^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int \frac{x^8}{(a+bx^{3/2})^{2/3}}\,dx = -\frac{\left(105\,ab^4x^6+162\,a^3b^2x^3+729\,a^5-(91\,b^5x^7+126\,a^2b^3x^4+243\,a^4bx)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{728\,b^6}$$

input
```
integrate(x^8/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
-1/728*(105*a*b^4*x^6 + 162*a^3*b^2*x^3 + 729*a^5 - (91*b^5*x^7 + 126*a^2*
b^3*x^4 + 243*a^4*b*x)*sqrt(x))*(b*x^(3/2) + a)^(1/3)/b^6
```

Sympy [A] (verification not implemented)

Time = 12.96 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19

$$\int \frac{x^8}{(a+bx^{3/2})^{2/3}}\,dx = \begin{cases} -\frac{729a^5\sqrt[3]{a+bx^{\frac{3}{2}}}}{728b^6} + \frac{243a^4x^{\frac{3}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{728b^5} - \frac{81a^3x^3\sqrt[3]{a+bx^{\frac{3}{2}}}}{364b^4} + \frac{9a^2x^{\frac{9}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{52b^3} - \frac{15a \\ \frac{x^9}{9a^{\frac{2}{3}}} \end{cases}$$

input
```
integrate(x**8/(a+b*x**(3/2))**(2/3),x)
```

output
```
Piecewise((-729*a**5*(a + b*x**(3/2))**(1/3)/(728*b**6) + 243*a**4*x**(3/2
)*(a + b*x**(3/2))**(1/3)/(728*b**5) - 81*a**3*x**3*(a + b*x**(3/2))**(1/3
)/(364*b**4) + 9*a**2*x**(9/2)*(a + b*x**(3/2))**(1/3)/(52*b**3) - 15*a*x*
*6*(a + b*x**(3/2))**(1/3)/(104*b**2) + x**(15/2)*(a + b*x**(3/2))**(1/3)/
(8*b), Ne(b, 0)), (x**9/(9*a**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{16}{3}}}{8\,b^6} - \frac{10\left(bx^{\frac{3}{2}}+a\right)^{\frac{13}{3}}a}{13\,b^6} + \frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}a^2}{b^6} - \frac{20\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a^3}{7\,b^6} + \frac{5\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^4}{2\,b^6} - \frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^5}{b^6}$$

input
```
integrate(x^8/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/8*(b*x^(3/2) + a)^(16/3)/b^6 - 10/13*(b*x^(3/2) + a)^(13/3)*a/b^6 + 2*(b
*x^(3/2) + a)^(10/3)*a^2/b^6 - 20/7*(b*x^(3/2) + a)^(7/3)*a^3/b^6 + 5/2*(b
*x^(3/2) + a)^(4/3)*a^4/b^6 - 2*(b*x^(3/2) + a)^(1/3)*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^5}{b^6} + \frac{91\left(bx^{\frac{3}{2}}+a\right)^{\frac{16}{3}} - 560\left(bx^{\frac{3}{2}}+a\right)^{\frac{13}{3}}a + 1456\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}a^2 - 2080\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a^3 + 1820\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^4}{728\,b^6}$$

input
```
integrate(x^8/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
-2*(b*x^(3/2) + a)^(1/3)*a^5/b^6 + 1/728*(91*(b*x^(3/2) + a)^(16/3) - 560*
(b*x^(3/2) + a)^(13/3)*a + 1456*(b*x^(3/2) + a)^(10/3)*a^2 - 2080*(b*x^(3/
2) + a)^(7/3)*a^3 + 1820*(b*x^(3/2) + a)^(4/3)*a^4)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{\left(a+b\,x^{3/2}\right)^{16/3}}{8\,b^6} - \frac{10\,a\,\left(a+b\,x^{3/2}\right)^{13/3}}{13\,b^6} - \frac{2\,a^5\,\left(a+b\,x^{3/2}\right)^{1/3}}{b^6} + \frac{5\,a^4\,\left(a+b\,x^{3/2}\right)^{4/3}}{2\,b^6} - \frac{20\,a^3\,\left(a+b\,x^{3/2}\right)^{7/3}}{7\,b^6} + \frac{2\,a^2\,\left(a+b\,x^{3/2}\right)^{10/3}}{b^6}$$

input
```
int(x^8/(a + b*x^(3/2))^(2/3),x)
```

output
```
(a + b*x^(3/2))^(16/3)/(8*b^6) - (10*a*(a + b*x^(3/2))^(13/3))/(13*b^6) -
(2*a^5*(a + b*x^(3/2))^(1/3))/b^6 + (5*a^4*(a + b*x^(3/2))^(4/3))/(2*b^6)
- (20*a^3*(a + b*x^(3/2))^(7/3))/(7*b^6) + (2*a^2*(a + b*x^(3/2))^(10/3))/
b^6
```

Reduce [F]

$$\int \frac{x^8}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{x^8}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(x^8/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x**8/(sqrt(x)*b*x + a)**(2/3),x)
```

3.167 $\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2a^3\sqrt[3]{a+bx^{3/2}}}{b^4} + \frac{3a^2\left(a+bx^{3/2}\right)^{4/3}}{2b^4} - \frac{6a\left(a+bx^{3/2}\right)^{7/3}}{7b^4} + \frac{\left(a+bx^{3/2}\right)^{10/3}}{5b^4}$$

output
```
-2*a^3*(a+b*x^(3/2))^(1/3)/b^4+3/2*a^2*(a+b*x^(3/2))^(4/3)/b^4-6/7*a*(a+b*
x^(3/2))^(7/3)/b^4+1/5*(a+b*x^(3/2))^(10/3)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^{3/2}}\left(-81a^3+27a^2bx^{3/2}-18ab^2x^3+14b^3x^{9/2}\right)}{70b^4}$$

input
```
Integrate[x^5/(a + b*x^(3/2))^(2/3),x]
```

output
```
((a + b*x^(3/2))^(1/3)*(-81*a^3 + 27*a^2*b*x^(3/2) - 18*a*b^2*x^3 + 14*b^3
*x^(9/2)))/(70*b^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{2}{3}\int \frac{x^{9/2}}{\left(bx^{3/2}+a\right)^{2/3}}dx^{3/2}$$

$$\Big\downarrow \text{53}$$

$$\frac{2}{3}\int \left(-\frac{a^3}{b^3\left(bx^{3/2}+a\right)^{2/3}}+\frac{3\sqrt[3]{bx^{3/2}+a}a^2}{b^3}-\frac{3\left(bx^{3/2}+a\right)^{4/3}a}{b^3}+\frac{\left(bx^{3/2}+a\right)^{7/3}}{b^3}\right)dx^{3/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\left(-\frac{3a^3\sqrt[3]{a+bx^{3/2}}}{b^4}+\frac{9a^2\left(a+bx^{3/2}\right)^{4/3}}{4b^4}+\frac{3\left(a+bx^{3/2}\right)^{10/3}}{10b^4}-\frac{9a\left(a+bx^{3/2}\right)^{7/3}}{7b^4}\right)$$

input
```
Int[x^5/(a + b*x^(3/2))^(2/3),x]
```

output
```
(2*((-3*a^3*(a + b*x^(3/2))^(1/3))/b^4 + (9*a^2*(a + b*x^(3/2))^(4/3))/(4*
b^4) - (9*a*(a + b*x^(3/2))^(7/3))/(7*b^4) + (3*(a + b*x^(3/2))^(10/3))/(1
0*b^4)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^5}{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(x^5/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x^5/(a+b*x^(3/2))^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

$$\int \frac{x^5}{(a+bx^{3/2})^{2/3}}\,dx = -\frac{\left(18\,ab^2x^3+81\,a^3-\left(14\,b^3x^4+27\,a^2bx\right)\sqrt{x}\right)\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}}{70\,b^4}$$

input
```
integrate(x^5/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
-1/70*(18*a*b^2*x^3 + 81*a^3 - (14*b^3*x^4 + 27*a^2*b*x)*sqrt(x))*(b*x^(3/
2) + a)^(1/3)/b^4
```

Sympy [A] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(a+bx^{3/2})^{2/3}}\,dx = \begin{cases} -\frac{81a^3\sqrt[3]{a+bx^{\frac{3}{2}}}}{70b^4} + \frac{27a^2x^{\frac{3}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{70b^3} - \frac{9ax^3\sqrt[3]{a+bx^{\frac{3}{2}}}}{35b^2} + \frac{x^{\frac{9}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{5b} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(a+b*x**(3/2))**(2/3),x)
```

output
```
Piecewise((-81*a**3*(a + b*x**(3/2))**(1/3)/(70*b**4) + 27*a**2*x**(3/2)*(
a + b*x**(3/2))**(1/3)/(70*b**3) - 9*a*x**3*(a + b*x**(3/2))**(1/3)/(35*b*
*2) + x**(9/2)*(a + b*x**(3/2))**(1/3)/(5*b), Ne(b, 0)), (x**6/(6*a**(2/3)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}}}{5\,b^4} - \frac{6\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a}{7\,b^4} + \frac{3\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^2}{2\,b^4} - \frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^3}{b^4}$$

input
```
integrate(x^5/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/5*(b*x^(3/2) + a)^(10/3)/b^4 - 6/7*(b*x^(3/2) + a)^(7/3)*a/b^4 + 3/2*(b*
x^(3/2) + a)^(4/3)*a^2/b^4 - 2*(b*x^(3/2) + a)^(1/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{(a+bx^{3/2})^{2/3}}\,dx = -\frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^3}{b^4} + \frac{14\left(bx^{\frac{3}{2}}+a\right)^{\frac{10}{3}} - 60\left(bx^{\frac{3}{2}}+a\right)^{\frac{7}{3}}a + 105\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}a^2}{70\,b^4}$$

input
```
integrate(x^5/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
-2*(b*x^(3/2) + a)^(1/3)*a^3/b^4 + 1/70*(14*(b*x^(3/2) + a)^(10/3) - 60*(b
*x^(3/2) + a)^(7/3)*a + 105*(b*x^(3/2) + a)^(4/3)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\left(a+b\,x^{3/2}\right)^{10/3}}{5\,b^4} - \frac{6\,a\,\left(a+b\,x^{3/2}\right)^{7/3}}{7\,b^4} - \frac{2\,a^3\,\left(a+b\,x^{3/2}\right)^{1/3}}{b^4} + \frac{3\,a^2\,\left(a+b\,x^{3/2}\right)^{4/3}}{2\,b^4}$$

input
```
int(x^5/(a + b*x^(3/2))^(2/3),x)
```

output
```
(a + b*x^(3/2))^(10/3)/(5*b^4) - (6*a*(a + b*x^(3/2))^(7/3))/(7*b^4) - (2*
a^3*(a + b*x^(3/2))^(1/3))/b^4 + (3*a^2*(a + b*x^(3/2))^(4/3))/(2*b^4)
```

Reduce [F]

$$\int \frac{x^5}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{x^5}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(x^5/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x**5/(sqrt(x)*b*x + a)**(2/3),x)
```

3.168 $\int \frac{x^2}{\left(a+bx^{3/2}\right)^{2/3}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{x^2}{\left(a+bx^{3/2}\right)^{2/3}} \, dx=-\frac{2a\sqrt[3]{a+bx^{3/2}}}{b^2}+\frac{\left(a+bx^{3/2}\right)^{4/3}}{2b^2}$$

output
```
-2*a*(a+b*x^(3/2))^(1/3)/b^2+1/2*(a+b*x^(3/2))^(4/3)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\left(a+bx^{3/2}\right)^{2/3}} \, dx=\frac{\left(-3a+bx^{3/2}\right)\sqrt[3]{a+bx^{3/2}}}{2b^2}$$

input
```
Integrate[x^2/(a + b*x^(3/2))^(2/3),x]
```

output
```
((-3*a + b*x^(3/2))*(a + b*x^(3/2))^(1/3))/(2*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+bx^{3/2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{2}{3}\int \frac{x^{3/2}}{\left(bx^{3/2}+a\right)^{2/3}}dx^{3/2}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{2}{3}\int \left(\frac{\sqrt[3]{bx^{3/2}+a}}{b}-\frac{a}{b\left(bx^{3/2}+a\right)^{2/3}}\right)dx^{3/2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}\left(\frac{3\left(a+bx^{3/2}\right)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx^{3/2}}}{b^2}\right)$$

input
```
Int[x^2/(a + b*x^(3/2))^(2/3),x]
```

output
```
(2*((-3*a*(a + b*x^(3/2))^(1/3))/b^2 + (3*(a + b*x^(3/2))^(4/3))/(4*b^2)))
/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{4}{3}}}{2}-2a\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}}{b^{2}}$	30
default	$\frac{\frac{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{4}{3}}}{2}-2a\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}}{b^{2}}$	30

input
```
int(x^2/(a+b*x^(3/2))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*(1/4*(a+b*x^(3/2))^(4/3)-a*(a+b*x^(3/2))^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}\left(bx^{\frac{3}{2}}-3\,a\right)}{2\,b^2}$$

input
```
integrate(x^2/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
1/2*(b*x^(3/2) + a)^(1/3)*(b*x^(3/2) - 3*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a+bx^{3/2})^{2/3}}\,dx = \begin{cases} -\frac{3a\sqrt[3]{a+bx^{\frac{3}{2}}}}{2b^2} + \frac{x^{\frac{3}{2}}\sqrt[3]{a+bx^{\frac{3}{2}}}}{2b} & \text{for}\ b \neq 0 \\ \frac{x^3}{3a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(a+b*x**(3/2))**(2/3),x)
```

output
```
Piecewise((-3*a*(a + b*x**(3/2))**(1/3)/(2*b**2) + x**(3/2)*(a + b*x**(3/2
))**(1/3)/(2*b), Ne(b, 0)), (x**3/(3*a**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}}{2\,b^2} - \frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a}{b^2}$$

input
```
integrate(x^2/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
1/2*(b*x^(3/2) + a)^(4/3)/b^2 - 2*(b*x^(3/2) + a)^(1/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{(a+bx^{3/2})^{2/3}}\,dx = \frac{\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}}{2\,b^2} - \frac{2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a}{b^2}$$

input
```
integrate(x^2/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
1/2*(b*x^(3/2) + a)^(4/3)/b^2 - 2*(b*x^(3/2) + a)^(1/3)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(a+bx^{3/2})^{2/3}}\,dx = -\frac{4\,a\left(a+b\,x^{3/2}\right)^{1/3}-\left(a+b\,x^{3/2}\right)^{4/3}}{2\,b^2}$$

input
```
int(x^2/(a + b*x^(3/2))^(2/3),x)
```

output
```
-(4*a*(a + b*x^(3/2))^(1/3) - (a + b*x^(3/2))^(4/3))/(2*b^2)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x^2}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(x^2/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x**2/(sqrt(x)*b*x + a)**(2/3),x)
```

3.169 $\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^{3/2}}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{a^{2/3}}$$

output
```
-2/3*arctan(1/3*(a^(1/3)+2*(a+b*x^(3/2))^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a
^(2/3)-1/2*ln(x)/a^(2/3)+ln(a^(1/3)-(a+b*x^(3/2))^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^{3/2}}\right)+\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^{3/2}}+\left(a+bx^{3/2}\right)^{2/3}\right)}{3a^{2/3}}$$

input
```
Integrate[1/(x*(a + b*x^(3/2))^(2/3)),x]
```

output
```
-1/3*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^(3/2))^(1/3))/a^(1/3))/Sqrt[3]] -
2*Log[-a^(1/3) + (a + b*x^(3/2))^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^(
3/2))^(1/3) + (a + b*x^(3/2))^(2/3)])/a^(2/3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {798, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{2}{3}\int \frac{1}{x^{3/2}\left(bx^{3/2}+a\right)^{2/3}}dx^{3/2}$$

$$\Big\downarrow \text{69}$$

$$\frac{2}{3}\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^{3/2}+a}}d\sqrt[3]{bx^{3/2}+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^3+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^{3/2}+a}}d\sqrt[3]{bx^{3/2}+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2}{3}\left(-\frac{3\int \frac{1}{x^3+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^{3/2}+a}}d\sqrt[3]{bx^{3/2}+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{2a^{2/3}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2}{3}\left(\frac{3\int \frac{1}{-x^3-3}d\left(\frac{2\sqrt[3]{bx^{3/2}+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{2a^{2/3}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{2a^{2/3}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)$$

input
```
Int[1/(x*(a + b*x^(3/2))^(2/3)),x]
```

output
```
(2*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^(3/2))^(1/3))/a^(1/3))/Sqrt[3]])/a^
(2/3)) - Log[x^(3/2)]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^(3/2))^(1/3)
])/(2*a^(2/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2\ln\left(\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left(\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}$
default	$\frac{2\ln\left(\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left(\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}$

input
```
int(1/x/(a+b*x^(3/2))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
2/3/a^(2/3)*ln((a+b*x^(3/2))^(1/3)-a^(1/3))-1/3/a^(2/3)*ln((a+b*x^(3/2))^(
2/3)+a^(1/3)*(a+b*x^(3/2))^(1/3)+a^(2/3))-2/3/a^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/a^(1/3)*(a+b*x^(3/2))^(1/3)+1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^{\frac{3}{2}}}\right)}{3b^{\frac{2}{3}}x\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(a+b*x**(3/2))**(2/3),x)
```

output
```
-2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**(3/2)))/(3
*b**(2/3)*x*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{2}{3}}} - \frac{\log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}+\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{2}{3}}} + \frac{2\log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^(3/2) + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) - 1/3*log((b*x^(3/2) + a)^(2/3) + (b*x^(3/2) + a)^(1/3)*a^(1/3)
 + a^(2/3))/a^(2/3) + 2/3*log((b*x^(3/2) + a)^(1/3) - a^(1/3))/a^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}} - \frac{\log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}+\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{2}{3}}} + \frac{2\,\log\left(\left|\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^(3/2) + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) - 1/3*log((b*x^(3/2) + a)^(2/3) + (b*x^(3/2) + a)^(1/3)*a^(1/3)
 + a^(2/3))/a^(2/3) + 2/3*log(abs((b*x^(3/2) + a)^(1/3) - a^(1/3)))/a^(2/3
)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{2\,\ln\left(6\,\left(a+b\,x^{3/2}\right)^{1/3}-6\,a^{1/3}\right)}{3\,a^{2/3}} + \frac{\ln\left(3\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)-6\,\left(a+b\,x^{3/2}\right)^{1/3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3\,a^{2/3}} - \frac{\ln\left(3\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)+6\,\left(a+b\,x^{3/2}\right)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3\,a^{2/3}}$$

input
```
int(1/(x*(a + b*x^(3/2))^(2/3)),x)
```

output
```
(2*log(6*(a + b*x^(3/2))^(1/3) - 6*a^(1/3)))/(3*a^(2/3)) + (log(3*a^(1/3)*
(3^(1/2)*1i - 1) - 6*(a + b*x^(3/2))^(1/3))*(3^(1/2)*1i - 1))/(3*a^(2/3))
- (log(3*a^(1/3)*(3^(1/2)*1i + 1) + 6*(a + b*x^(3/2))^(1/3))*(3^(1/2)*1i +
 1))/(3*a^(2/3))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^{3/2}\right)^{2/3}}\,dx=\int \frac{1}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}x}dx$$

input
```
int(1/x/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/((sqrt(x)*b*x + a)**(2/3)*x),x)
```

3.170 $\int \frac{1}{x^4\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 148

$$\int \frac{1}{x^4 \left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^{3/2}}}{3ax^3}+\frac{5b\sqrt[3]{a+bx^{3/2}}}{9a^2x^{3/2}} -\frac{10b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^{3/2}}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{9a^{8/3}}$$

output
```
-1/3*(a+b*x^(3/2))^(1/3)/a/x^3+5/9*b*(a+b*x^(3/2))^(1/3)/a^2/x^(3/2)-10/27
*b^2*arctan(1/3*(a^(1/3)+2*(a+b*x^(3/2))^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a
^(8/3)-5/18*b^2*ln(x)/a^(8/3)+5/9*b^2*ln(a^(1/3)-(a+b*x^(3/2))^(1/3))/a^(8
/3)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{\frac{3a^{2/3}\sqrt[3]{a+bx^{3/2}}\left(-3a+5bx^{3/2}\right)}{x^3} - 10\sqrt{3}b^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 10b^2\log\left(-\sqrt[3]{\vphantom{a}}\right.}{27a^{8/3}}$$

input
```
Integrate[1/(x^4*(a + b*x^(3/2))^(2/3)),x]
```

output
```
((3*a^(2/3)*(a + b*x^(3/2))^(1/3)*(-3*a + 5*b*x^(3/2)))/x^3 - 10*Sqrt[3]*b
^2*ArcTan[(1 + (2*(a + b*x^(3/2))^(1/3))/a^(1/3))/Sqrt[3]] + 10*b^2*Log[-a
^(1/3) + (a + b*x^(3/2))^(1/3)] - 5*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^(3/
2))^(1/3) + (a + b*x^(3/2))^(2/3)])/(27*a^(8/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {798, 52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+bx^{3/2}\right)^{2/3}}dx$$

$$\Big\downarrow \; 798$$

$$\frac{2}{3}\int \frac{1}{x^{9/2}\left(bx^{3/2}+a\right)^{2/3}}dx^{3/2}$$

$$\Big\downarrow \; 52$$

$$\frac{2}{3}\left(-\frac{5b\int \frac{1}{x^3\left(bx^{3/2}+a\right)^{2/3}}dx^{3/2}}{6a}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^3}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{2}{3}\left(-\frac{5b\left(-\frac{2b\int \frac{1}{x^{3/2}\left(bx^{3/2}+a\right)^{2/3}}dx^{3/2}}{3a}-\frac{\sqrt[3]{a+bx^{3/2}}}{ax^{3/2}}\right)}{6a}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^3}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{2}{3}\left(-\frac{5b\left(-\frac{2b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^{3/2}+a}}d\sqrt[3]{bx^{3/2}+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^3+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^{3/2}+a}}d\sqrt[3]{bx^{3/2}+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+b}}{ax^{3/}}\right.}{6a}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{2}{3}\left(-\frac{5b\left(-\frac{2b\left(-\frac{3\int \frac{1}{x^3+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^{3/2}+a}}d\sqrt[3]{bx^{3/2}+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{2a^{2/3}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^{3/2}}}{ax^{3/2}}\right)}{6a}-\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}\left(-\frac{5b\left(-\frac{2b\left(\frac{3\int \frac{1}{-x^3-3}d\left(\frac{2\sqrt[3]{bx^{3/2}+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{2a^{2/3}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^{3/2}}}{ax^{3/2}}\right)}{6a}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{3}\left(-\frac{5b\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^{3/2}}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^{3/2}}\right)}{2a^{2/3}}-\frac{\log\left(x^{3/2}\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^{3/2}}}{ax^{3/2}}\right)}{6a}-\frac{\sqrt[3]{a+bx^{3/2}}}{2ax^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^(3/2))^(2/3)),x]
```

output
```
(2*(-1/2*(a + b*x^(3/2))^(1/3)/(a*x^3) - (5*b*(-((a + b*x^(3/2))^(1/3)/(a*
x^(3/2))) - (2*b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^(3/2))^(1/3))/a^(1/3)
)/Sqrt[3]])/a^(2/3)) - Log[x^(3/2)]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*
x^(3/2))^(1/3)])/(2*a^(2/3))))/(3*a)))/(6*a)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{1}{x^4 \left(a + b\, x^{\frac{3}{2}}\right)^{\frac{2}{3}}} dx$$

input
```
int(1/x^4/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/x^4/(a+b*x^(3/2))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + bx^{3/2}\right)^{2/3}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^4 \left(a + bx^{3/2}\right)^{2/3}}\, dx = -\frac{2\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^{\frac{3}{2}}}\right)}{3b^{\frac{2}{3}} x^4 \Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(1/x**4/(a+b*x**(3/2))**(2/3),x)
```

output
```
-2*gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**(3/2)))/(
3*b**(2/3)*x**4*gamma(11/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^4 \left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{10\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{8}{3}}} - \frac{5\,b^2 \log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}} + \left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{27\,a^{\frac{8}{3}}} + \frac{10\,b^2 \log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{27\,a^{\frac{8}{3}}} + \frac{5\left(bx^{\frac{3}{2}}+a\right)^{\frac{4}{3}}b^2 - 8\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}ab^2}{9\left(\left(bx^{\frac{3}{2}}+a\right)^2 a^2 - 2\left(bx^{\frac{3}{2}}+a\right)a^3 + a^4\right)}$$

input
```
integrate(1/x^4/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
-10/27*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^(3/2) + a)^(1/3) + a^(1/3))/
a^(1/3))/a^(8/3) - 5/27*b^2*log((b*x^(3/2) + a)^(2/3) + (b*x^(3/2) + a)^(1
/3)*a^(1/3) + a^(2/3))/a^(8/3) + 10/27*b^2*log((b*x^(3/2) + a)^(1/3) - a^(
1/3))/a^(8/3) + 1/9*(5*(b*x^(3/2) + a)^(4/3)*b^2 - 8*(b*x^(3/2) + a)^(1/3)
*a*b^2)/((b*x^(3/2) + a)^2*a^2 - 2*(b*x^(3/2) + a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4 \left(a+bx^{3/2}\right)^{2/3}}\,dx = -\frac{\frac{10\sqrt{3}b^3 \arctan\left(\frac{\sqrt{3}\left(2\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}} + \frac{5\,b^3 \log\left(\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}+\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}} - \frac{10\,b^3 \log\left(\left|\left(bx^{\frac{3}{2}}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}} - \frac{3\left(5\left(bx^{\frac{3}{2}} \cdots\right.\right.}{\cdots}}{27\,b}$$

input
```
integrate(1/x^4/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
-1/27*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^(3/2) + a)^(1/3) + a^(1/3
))/a^(1/3))/a^(8/3) + 5*b^3*log((b*x^(3/2) + a)^(2/3) + (b*x^(3/2) + a)^(1
/3)*a^(1/3) + a^(2/3))/a^(8/3) - 10*b^3*log(abs((b*x^(3/2) + a)^(1/3) - a^
(1/3)))/a^(8/3) - 3*(5*(b*x^(3/2) + a)^(4/3)*b^3 - 8*(b*x^(3/2) + a)^(1/3)
*a*b^3)/(a^2*b^2*x^3))/b
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^4 (a+bx^{3/2})^{2/3}}\,dx = \frac{10\,b^2\,\ln\left(\left(a+b\,x^{3/2}\right)^{1/3}-a^{1/3}\right)}{27\,a^{8/3}} - \frac{\frac{8\,b^2\,\left(a+b\,x^{3/2}\right)^{1/3}}{9\,a}-\frac{5\,b^2\,\left(a+b\,x^{3/2}\right)^{4/3}}{9\,a^2}}{\left(a+b\,x^{3/2}\right)^2-2\,a\,\left(a+b\,x^{3/2}\right)+a^2} + \frac{\ln\left(\frac{-5\,b^2+\sqrt{3}\,b^2\,5{}\mathrm{i}}{3\,a^{5/3}}-\frac{10\,b^2\,\left(a+b\,x^{3/2}\right)^{1/3}}{3\,a^2}\right)\,\left(-5\,b^2+\sqrt{3}\,b^2\,5{}\mathrm{i}\right)}{27\,a^{8/3}} - \frac{\ln\left(\frac{5\,b^2+\sqrt{3}\,b^2\,5{}\mathrm{i}}{3\,a^{5/3}}+\frac{10\,b^2\,\left(a+b\,x^{3/2}\right)^{1/3}}{3\,a^2}\right)\,\left(5\,b^2+\sqrt{3}\,b^2\,5{}\mathrm{i}\right)}{27\,a^{8/3}}$$

input
```
int(1/(x^4*(a + b*x^(3/2))^(2/3)),x)
```

output
```
(10*b^2*log((a + b*x^(3/2))^(1/3) - a^(1/3)))/(27*a^(8/3)) - ((8*b^2*(a +
b*x^(3/2))^(1/3))/(9*a) - (5*b^2*(a + b*x^(3/2))^(4/3))/(9*a^2))/((a + b*x
^(3/2))^2 - 2*a*(a + b*x^(3/2)) + a^2) + (log((3^(1/2)*b^2*5i - 5*b^2)/(3*
a^(5/3)) - (10*b^2*(a + b*x^(3/2))^(1/3))/(3*a^2))*(3^(1/2)*b^2*5i - 5*b^2
))/(27*a^(8/3)) - (log((3^(1/2)*b^2*5i + 5*b^2)/(3*a^(5/3)) + (10*b^2*(a +
 b*x^(3/2))^(1/3))/(3*a^2))*(3^(1/2)*b^2*5i + 5*b^2))/(27*a^(8/3))
```

Reduce [F]

$$\int \frac{1}{x^4 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}} x^4}dx$$

input
```
int(1/x^4/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/((sqrt(x)*b*x + a)**(2/3)*x**4),x)
```

3.171 $\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx=\frac{x^5\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{10}{3},\frac{13}{3},-\frac{bx^{3/2}}{a}\right)}{5\left(a+bx^{3/2}\right)^{2/3}}$$

output
```
1/5*x^5*(1+b*x^(3/2)/a)^(2/3)*hypergeom([2/3, 10/3],[13/3],-b*x^(3/2)/a)/(
a+b*x^(3/2))^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx=\frac{x^5\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{10}{3},\frac{13}{3},-\frac{bx^{3/2}}{a}\right)}{5\left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Integrate[x^4/(a + b*x^(3/2))^(2/3),x]
```

output
```
(x^5*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 10/3, 13/3, -((b*x^(
3/2))/a)])/(5*(a + b*x^(3/2))^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{x^{9/2}}{\left(bx^{3/2}+a\right)^{2/3}}d\sqrt{x}$$

$$\Big\downarrow \text{889}$$

$$\frac{2\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\int \frac{x^{9/2}}{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}}d\sqrt{x}}{\left(a+bx^{3/2}\right)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^5\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{10}{3},\frac{13}{3},-\frac{bx^{3/2}}{a}\right)}{5\left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Int[x^4/(a + b*x^(3/2))^(2/3),x]
```

output

```
(x^5*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 10/3, 13/3, -((b*x^(
3/2))/a)])/(5*(a + b*x^(3/2))^(2/3))
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(x^4/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x^4/(a+b*x^(3/2))^(2/3),x)
```

Fricas [F]

$$\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx=\int \frac{x^4}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^(11/2) - a*x^4)*(b*x^(3/2) + a)^(1/3)/(b^2*x^3 - a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{2x^5\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(x**4/(a+b*x**(3/2))**(2/3),x)
```

output
```
2*x**5*gamma(10/3)*hyper((2/3, 10/3), (13/3,), b*x**(3/2)*exp_polar(I*pi)/
a)/(3*a**(2/3)*gamma(13/3))
```

Maxima [F]

$$\int \frac{x^4}{\left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{x^4}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^(3/2) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x^4}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^(3/2) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x^4}{(a+b\,x^{3/2})^{2/3}}\,dx$$

input
```
int(x^4/(a + b*x^(3/2))^(2/3),x)
```

output
```
int(x^4/(a + b*x^(3/2))^(2/3), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x^4}{(\sqrt{x}\,bx+a)^{\frac{2}{3}}}\,dx$$

input
```
int(x^4/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x**4/(sqrt(x)*b*x + a)**(2/3),x)
```

3.172 $\int \frac{x}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{x}{\left(a+bx^{3/2}\right)^{2/3}}\,dx=\frac{x^2\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^{3/2}}{a}\right)}{2\left(a+bx^{3/2}\right)^{2/3}}$$

output
```
1/2*x^2*(1+b*x^(3/2)/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^(3/2)/a)/(a+
b*x^(3/2))^(2/3)
```

Mathematica [A] (verified)

Time = 9.56 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+bx^{3/2}\right)^{2/3}}\,dx=\frac{x^2\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^{3/2}}{a}\right)}{2\left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Integrate[x/(a + b*x^(3/2))^(2/3),x]
```

output
```
(x^2*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^(3/
2))/a)])/(2*(a + b*x^(3/2))^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+bx^{3/2}\right)^{2/3}}\,dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{x^{3/2}}{\left(bx^{3/2}+a\right)^{2/3}}d\sqrt{x}$$

$$\Big\downarrow \text{889}$$

$$\frac{2\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\int \frac{x^{3/2}}{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}}d\sqrt{x}}{\left(a+bx^{3/2}\right)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^{3/2}}{a}\right)}{2\left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Int[x/(a + b*x^(3/2))^(2/3),x]
```

output
```
(x^2*(1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^(3/
2))/a)])/(2*(a + b*x^(3/2))^(2/3))
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\left(a+b\,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}dx$$

input
```
int(x/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x/(a+b*x^(3/2))^(2/3),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^{3/2})^{2/3}}\,dx=\int \frac{x}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^(5/2) - a*x)*(b*x^(3/2) + a)^(1/3)/(b^2*x^3 - a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+bx^{3/2})^{2/3}}\,dx = \frac{2x^2\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x/(a+b*x**(3/2))**(2/3),x)
```

output
```
2*x**2*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**(3/2)*exp_polar(I*pi)/a)/
(3*a**(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^(3/2) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(x/(b*x^(3/2) + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x}{(a+b\,x^{3/2})^{2/3}}\,dx$$

input
```
int(x/(a + b*x^(3/2))^(2/3),x)
```

output
```
int(x/(a + b*x^(3/2))^(2/3), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^{3/2})^{2/3}}\,dx = \int \frac{x}{(\sqrt{x}\,bx+a)^{\frac{2}{3}}}dx$$

input
```
int(x/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(x/(sqrt(x)*b*x + a)**(2/3),x)
```

3.173 $\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx=-\frac{\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^{3/2}}{a}\right)}{x\left(a+bx^{3/2}\right)^{2/3}}$$

output
```
-(1+b*x^(3/2)/a)^(2/3)*hypergeom([-2/3, 2/3],[1/3],-b*x^(3/2)/a)/x/(a+b*x^
(3/2))^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx=-\frac{\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^{3/2}}{a}\right)}{x\left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Integrate[1/(x^2*(a + b*x^(3/2))^(2/3)),x]
```

output
```
-(((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^(3/2
))/a)])/(x*(a + b*x^(3/2))^(2/3)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^{3/2}\right)^{2/3}}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{1}{x^{3/2} \left(bx^{3/2}+a\right)^{2/3}}d\sqrt{x}$$

$$\Big\downarrow \text{889}$$

$$\frac{2\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\int \frac{1}{x^{3/2}\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}}d\sqrt{x}}{\left(a+bx^{3/2}\right)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^{3/2}}{a}\right)}{x\left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Int[1/(x^2*(a + b*x^(3/2))^(2/3)),x]
```

output

```
-(((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^(3/2
))/a)])/(x*(a + b*x^(3/2))^(2/3)))
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(a +b \,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}d x$$

input
```
int(1/x^2/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/x^2/(a+b*x^(3/2))^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^{3/2}\right)^{2/3}} \, dx=\int { \frac{1}{{\left(b x^{\frac{3}{2}} + a\right)}^{\frac{2}{3}} x^{2}} \,d x }$$

input
```
integrate(1/x^2/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^(3/2) + a)^(1/3)*(b*x^(3/2) - a)/(b^2*x^5 - a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{2\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^{\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**2/(a+b*x**(3/2))**(2/3),x)
```

output
```
2*gamma(-2/3)*hyper((-2/3, 2/3), (1/3,), b*x**(3/2)*exp_polar(I*pi)/a)/(3*
a**(2/3)*x*gamma(1/3))
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^(3/2) + a)^(2/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(3/2) + a)^(2/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{x^2\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

input
```
int(1/(x^2*(a + b*x^(3/2))^(2/3)),x)
```

output
```
int(1/(x^2*(a + b*x^(3/2))^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}x^2}dx$$

input
```
int(1/x^2/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/((sqrt(x)*b*x + a)**(2/3)*x**2),x)
```

3.174 $\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx=-\frac{\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{8}{3},\frac{2}{3},-\frac{5}{3},-\frac{bx^{3/2}}{a}\right)}{4x^4\left(a+bx^{3/2}\right)^{2/3}}$$

output
```
-1/4*(1+b*x^(3/2)/a)^(2/3)*hypergeom([-8/3, 2/3],[-5/3],-b*x^(3/2)/a)/x^4/
(a+b*x^(3/2))^(2/3)
```

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx=-\frac{\left(1+\frac{bx^{3/2}}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{8}{3},\frac{2}{3},-\frac{5}{3},-\frac{bx^{3/2}}{a}\right)}{4x^4\left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Integrate[1/(x^5*(a + b*x^(3/2))^(2/3)),x]
```

output
```
-1/4*((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-8/3, 2/3, -5/3, -((b*x^
(3/2))/a)])/(x^4*(a + b*x^(3/2))^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {864, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+bx^{3/2}\right)^{2/3}} \, dx$$

$$\Big\downarrow \ 864$$

$$2\int \frac{1}{x^{9/2} \left(bx^{3/2}+a\right)^{2/3}} d\sqrt{x}$$

$$\Big\downarrow \ 889$$

$$\frac{2\left(\frac{bx^{3/2}}{a}+1\right)^{2/3} \int \frac{1}{x^{9/2}\left(\frac{bx^{3/2}}{a}+1\right)^{2/3}} d\sqrt{x}}{\left(a+bx^{3/2}\right)^{2/3}}$$

$$\Big\downarrow \ 888$$

$$-\frac{\left(\frac{bx^{3/2}}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{8}{3},\frac{2}{3},-\frac{5}{3},-\frac{bx^{3/2}}{a}\right)}{4x^4 \left(a+bx^{3/2}\right)^{2/3}}$$

input
```
Int[1/(x^5*(a + b*x^(3/2))^(2/3)),x]
```

output

```
-1/4*((1 + (b*x^(3/2))/a)^(2/3)*Hypergeometric2F1[-8/3, 2/3, -5/3, -((b*x^
(3/2))/a)])/(x^4*(a + b*x^(3/2))^(2/3))
```

Defintions of rubi rules used

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(a +b \,x^{\frac{3}{2}}\right)^{\frac{2}{3}}}d x$$

input
```
int(1/x^5/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/x^5/(a+b*x^(3/2))^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{x^5 \left(a+b x^{3/2}\right)^{2/3}}\, dx = \int \frac{1}{{\left(b x^{\frac{3}{2}} + a\right)}^{\frac{2}{3}} x^{5}}\, dx$$

input
```
integrate(1/x^5/(a+b*x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^(3/2) + a)^(1/3)*(b*x^(3/2) - a)/(b^2*x^8 - a^2*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \frac{2\Gamma\left(-\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{8}{3}, \frac{2}{3} \\ -\frac{5}{3} \end{matrix}\middle| \frac{bx^{\frac{3}{2}} e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}} x^4 \Gamma\left(-\frac{5}{3}\right)}$$

input
```
integrate(1/x**5/(a+b*x**(3/2))**(2/3),x)
```

output
```
2*gamma(-8/3)*hyper((-8/3, 2/3), (-5/3,), b*x**(3/2)*exp_polar(I*pi)/a)/(3
*a**(2/3)*x**4*gamma(-5/3))
```

Maxima [F]

$$\int \frac{1}{x^5 \left(a+bx^{3/2}\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}} x^5}\,dx$$

input
```
integrate(1/x^5/(a+b*x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^(3/2) + a)^(2/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^{\frac{3}{2}}+a\right)^{\frac{2}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(a+b*x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(3/2) + a)^(2/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx=\int \frac{1}{x^5\left(a+b\,x^{3/2}\right)^{2/3}}\,dx$$

input
```
int(1/(x^5*(a + b*x^(3/2))^(2/3)),x)
```

output
```
int(1/(x^5*(a + b*x^(3/2))^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\left(a+bx^{3/2}\right)^{2/3}}\,dx=\int \frac{1}{\left(\sqrt{x}\,bx+a\right)^{\frac{2}{3}}x^5}dx$$

input
```
int(1/x^5/(a+b*x^(3/2))^(2/3),x)
```

output
```
int(1/((sqrt(x)*b*x + a)**(2/3)*x**5),x)
```

3.175 $\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 415

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx = \frac{4\sqrt{1+bx^{9/2}}}{3b^{2/3}\left(1+\sqrt{3}+\sqrt[3]{b}x^{3/2}\right)} - \frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{b}x^{3/2}\right)\sqrt{\frac{1-\sqrt[3]{b}x^{3/2}+b^{2/3}x^3}{\left(1+\sqrt{3}+\sqrt[3]{b}x^{3/2}\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+\sqrt[3]{b}x^{3/2}}{1+\sqrt{3}+\sqrt[3]{b}x^{3/2}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{\frac{1+\sqrt[3]{b}x^{3/2}}{\left(1+\sqrt{3}+\sqrt[3]{b}x^{3/2}\right)^2}}\sqrt{1+bx^{9/2}}} + \frac{4\sqrt{2}\left(1+\sqrt[3]{b}x^{3/2}\right)\sqrt{\frac{1-\sqrt[3]{b}x^{3/2}+b^{2/3}x^3}{\left(1+\sqrt{3}+\sqrt[3]{b}x^{3/2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+\sqrt[3]{b}x^{3/2}}{1+\sqrt{3}+\sqrt[3]{b}x^{3/2}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{1+\sqrt[3]{b}x^{3/2}}{\left(1+\sqrt{3}+\sqrt[3]{b}x^{3/2}\right)^2}}\sqrt{1+bx^{9/2}}}$$

output
```
4/3*(1+b*x^(9/2))^(1/2)/b^(2/3)/(1+3^(1/2)+b^(1/3)*x^(3/2))-2/3*(1/2*6^(1/
2)-1/2*2^(1/2))*(1+b^(1/3)*x^(3/2))*((1-b^(1/3)*x^(3/2)+b^(2/3)*x^3)/(1+3^
(1/2)+b^(1/3)*x^(3/2))^2)^(1/2)*EllipticE((1-3^(1/2)+b^(1/3)*x^(3/2))/(1+3
^(1/2)+b^(1/3)*x^(3/2)),I*3^(1/2)+2*I)*3^(1/4)/b^(2/3)/((1+b^(1/3)*x^(3/2)
)/(1+3^(1/2)+b^(1/3)*x^(3/2))^2)^(1/2)/(1+b*x^(9/2))^(1/2)+4/9*2^(1/2)*(1+
b^(1/3)*x^(3/2))*((1-b^(1/3)*x^(3/2)+b^(2/3)*x^3)/(1+3^(1/2)+b^(1/3)*x^(3/
2))^2)^(1/2)*EllipticF((1-3^(1/2)+b^(1/3)*x^(3/2))/(1+3^(1/2)+b^(1/3)*x^(3
/2)),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/((1+b^(1/3)*x^(3/2))/(1+3^(1/2)+b^(1/3
)*x^(3/2))^2)^(1/2)/(1+b*x^(9/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx = \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-bx^{9/2}\right)$$

input
```
Integrate[x^2/Sqrt[1 + b*x^(9/2)],x]
```

output
```
(x^3*Hypergeometric2F1[1/2, 2/3, 5/3, -(b*x^(9/2))])/3
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {864, 807, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{bx^{9/2}+1}}dx$$

$$\Big\downarrow \text{864}$$

$$2\int \frac{x^{5/2}}{\sqrt{bx^{9/2}+1}}d\sqrt{x}$$

$\Big\downarrow$ 807

$$\frac{2}{3}\int \frac{x^{3/2}}{\sqrt{bx^{3/2}+1}}dx^{3/2}$$

$\Big\downarrow$ 832

$$\frac{2}{3}\left(\frac{\int \frac{\sqrt[3]{b}x^{3/2}-\sqrt{3}+1}{\sqrt{bx^{3/2}+1}}dx^{3/2}}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{bx^{3/2}+1}}dx^{3/2}}{\sqrt[3]{b}}\right)$$

$\Big\downarrow$ 759

$$\frac{2}{3}\left(\frac{\int \frac{\sqrt[3]{b}x^{3/2}-\sqrt{3}+1}{\sqrt{bx^{3/2}+1}}dx^{3/2}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}x^{3/2}+1\right)\sqrt{\frac{b^{2/3}x-\sqrt[3]{b}x^{3/2}+1}{\left(\sqrt[3]{b}x^{3/2}+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^{3/2}-\sqrt{}}{\sqrt[3]{b}x^{3/2}+\sqrt{}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^{3/2}+1}{\left(\sqrt[3]{b}x^{3/2}+\sqrt{3}+1\right)^2}}\sqrt{bx^{3/2}+1}}\right.$$

$\Big\downarrow$ 2416

$$\frac{2}{3}\left(\frac{\frac{2\sqrt{bx^{3/2}+1}}{\sqrt[3]{b}\left(\sqrt[3]{b}x^{3/2}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}x^{3/2}+1\right)\sqrt{\frac{b^{2/3}x-\sqrt[3]{b}x^{3/2}+1}{\left(\sqrt[3]{b}x^{3/2}+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x^{3/2}-\sqrt{3}+1}{\sqrt[3]{b}x^{3/2}+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{b}x^{3/2}+1}{\left(\sqrt[3]{b}x^{3/2}+\sqrt{3}+1\right)^2}}\sqrt{bx^{3/2}+1}}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)}{}\right.$$

input
```
Int[x^2/Sqrt[1 + b*x^(9/2)],x]
```

output
```
(2*(((2*Sqrt[1 + b*x^(3/2)])/(b^(1/3)*(1 + Sqrt[3] + b^(1/3)*x^(3/2))) - (
3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + b^(1/3)*x^(3/2))*Sqrt[(1 + b^(2/3)*x - b^(1
/3)*x^(3/2))/(1 + Sqrt[3] + b^(1/3)*x^(3/2))^2]*EllipticE[ArcSin[(1 - Sqrt
[3] + b^(1/3)*x^(3/2))/(1 + Sqrt[3] + b^(1/3)*x^(3/2))], -7 - 4*Sqrt[3]])/
(b^(1/3)*Sqrt[(1 + b^(1/3)*x^(3/2))/(1 + Sqrt[3] + b^(1/3)*x^(3/2))^2]*Sqr
t[1 + b*x^(3/2)]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + b^(1/
3)*x^(3/2))*Sqrt[(1 + b^(2/3)*x - b^(1/3)*x^(3/2))/(1 + Sqrt[3] + b^(1/3)*
x^(3/2))^2]*EllipticF[ArcSin[(1 - Sqrt[3] + b^(1/3)*x^(3/2))/(1 + Sqrt[3]
+ b^(1/3)*x^(3/2))], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(1 + b^(1/3)*x
^(3/2))/(1 + Sqrt[3] + b^(1/3)*x^(3/2))^2]*Sqrt[1 + b*x^(3/2)])))/3
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.04

method	result	size
meijerg	$\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{5}{3}\right], -b\, x^{\frac{9}{2}}\right)}{3}$	18

input
```
int(x^2/(1+b*x^(9/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([1/2,2/3],[5/3],-b*x^(9/2))
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\, dx = \int \frac{x^2}{\sqrt{bx^{\frac{9}{2}}+1}}\, dx$$

input
```
integrate(x^2/(1+b*x^(9/2))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^(13/2) - x^2)*sqrt(b*x^(9/2) + 1)/(b^2*x^9 - 1), x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.08

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx = \frac{2x^3\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|bx^{\frac{9}{2}}e^{i\pi}\right)}{9\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x**2/(1+b*x**(9/2))**(1/2),x)
```

output
```
2*x**3*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**(9/2)*exp_polar(I*pi))/(9
*gamma(5/3))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx = \int \frac{x^2}{\sqrt{bx^{\frac{9}{2}}+1}}\,dx$$

input
```
integrate(x^2/(1+b*x^(9/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^(9/2) + 1), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx = \int \frac{x^2}{\sqrt{bx^{\frac{9}{2}}+1}}\,dx$$

input
```
integrate(x^2/(1+b*x^(9/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^(9/2) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx=\int \frac{x^2}{\sqrt{b\,x^{9/2}+1}}\,dx$$

input
```
int(x^2/(b*x^(9/2) + 1)^(1/2),x)
```

output
```
int(x^2/(b*x^(9/2) + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1+bx^{9/2}}}\,dx=-\left(\int \frac{\sqrt{\sqrt{x}\,b\,x^4+1}\,x^2}{b^2x^9-1}dx\right)+\left(\int \frac{\sqrt{x}\,\sqrt{\sqrt{x}\,b\,x^4+1}\,x^6}{b^2x^9-1}dx\right)b$$

input
```
int(x^2/(1+b*x^(9/2))^(1/2),x)
```

output
```
- int((sqrt(sqrt(x)*b*x**4 + 1)*x**2)/(b**2*x**9 - 1),x) + int((sqrt(x)*s
qrt(sqrt(x)*b*x**4 + 1)*x**6)/(b**2*x**9 - 1),x)*b
```

3.176 $\int \left(a+b\sqrt[3]{x}\right) x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+b\sqrt[3]{x}\right) x^4 \, dx = \frac{ax^5}{5}+\frac{3}{16}bx^{16/3}$$

output
```
1/5*a*x^5+3/16*b*x^(16/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right) x^4 \, dx = \frac{1}{80}\left(16a+15b\sqrt[3]{x}\right) x^5$$

input
```
Integrate[(a + b*x^(1/3))*x^4,x]
```

output
```
((16*a + 15*b*x^(1/3))*x^5)/80
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a + b\sqrt[3]{x}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^4 + bx^{13/3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^5}{5} + \frac{3}{16}bx^{16/3}$$

input
```
Int[(a + b*x^(1/3))*x^4,x]
```

output
```
(a*x^5)/5 + (3*b*x^(16/3))/16
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^{5}}{5}+\frac{3 b\,x^{\frac{16}{3}}}{16}$	14
default	$\frac{a\,x^{5}}{5}+\frac{3 b\,x^{\frac{16}{3}}}{16}$	14
trager	$\frac{a\left(x^{4}+x^{3}+x^{2}+x+1\right)\left(-1+x\right)}{5}+\frac{3 b\,x^{\frac{16}{3}}}{16}$	26
orering	$\frac{7 x^{5}\left(a+b\,x^{\frac{1}{3}}\right)}{20}-\frac{3 x^{2}\left(\frac{b\,x^{\frac{10}{3}}}{3}+4\left(a+b\,x^{\frac{1}{3}}\right)x^{3}\right)}{80}$	38

input
```
int((a+b*x^(1/3))*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+3/16*b*x^(16/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)x^4\,dx=\frac{3}{16}\,bx^{\frac{16}{3}}+\frac{1}{5}\,ax^5$$

input
```
integrate((a+b*x^(1/3))*x^4,x, algorithm="fricas")
```

output
```
3/16*b*x^(16/3) + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int (a+b\sqrt[3]{x})\,x^4\,dx=\frac{ax^5}{5}+\frac{3bx^{\frac{16}{3}}}{16}$$

input
```
integrate((a+b*x**(1/3))*x**4,x)
```

output
```
a*x**5/5 + 3*b*x**(16/3)/16
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(13) = 26$.

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 13.21

$$\int (a+b\sqrt[3]{x})\,x^4\,dx=\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}}{16\,b^{15}}-\frac{14\left(bx^{\frac{1}{3}}+a\right)^{15}a}{5\,b^{15}}+\frac{39\left(bx^{\frac{1}{3}}+a\right)^{14}a^2}{2\,b^{15}}-\frac{84\left(bx^{\frac{1}{3}}+a\right)^{13}a^3}{b^{15}}+\frac{1001\left(bx^{\frac{1}{3}}+a\right)^{12}a^4}{4\,b^{15}}-\frac{546\left(bx^{\frac{1}{3}}+a\right)^{11}a^5}{b^{15}}+\frac{9009\left(bx^{\frac{1}{3}}+a\right)^{10}a^6}{10\,b^{15}}-\frac{1144\left(bx^{\frac{1}{3}}+a\right)^{9}a^7}{b^{15}}+\frac{9009\left(bx^{\frac{1}{3}}+a\right)^{8}a^8}{8\,b^{15}}-\frac{858\left(bx^{\frac{1}{3}}+a\right)^{7}a^9}{b^{15}}+\frac{1001\left(bx^{\frac{1}{3}}+a\right)^{6}a^{10}}{2\,b^{15}}-\frac{1092\left(bx^{\frac{1}{3}}+a\right)^{5}a^{11}}{5\,b^{15}}+\frac{273\left(bx^{\frac{1}{3}}+a\right)^{4}a^{12}}{4\,b^{15}}-\frac{14\left(bx^{\frac{1}{3}}+a\right)^{3}a^{13}}{b^{15}}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{2}a^{14}}{2\,b^{15}}$$

input
```
integrate((a+b*x^(1/3))*x^4,x, algorithm="maxima")
```

output
```
3/16*(b*x^(1/3) + a)^16/b^15 - 14/5*(b*x^(1/3) + a)^15*a/b^15 + 39/2*(b*x^
(1/3) + a)^14*a^2/b^15 - 84*(b*x^(1/3) + a)^13*a^3/b^15 + 1001/4*(b*x^(1/3
) + a)^12*a^4/b^15 - 546*(b*x^(1/3) + a)^11*a^5/b^15 + 9009/10*(b*x^(1/3)
+ a)^10*a^6/b^15 - 1144*(b*x^(1/3) + a)^9*a^7/b^15 + 9009/8*(b*x^(1/3) + a
)^8*a^8/b^15 - 858*(b*x^(1/3) + a)^7*a^9/b^15 + 1001/2*(b*x^(1/3) + a)^6*a
^10/b^15 - 1092/5*(b*x^(1/3) + a)^5*a^11/b^15 + 273/4*(b*x^(1/3) + a)^4*a^
12/b^15 - 14*(b*x^(1/3) + a)^3*a^13/b^15 + 3/2*(b*x^(1/3) + a)^2*a^14/b^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)x^4\,dx=\frac{3}{16}\,bx^{\frac{16}{3}}+\frac{1}{5}\,ax^5$$

input
```
integrate((a+b*x^(1/3))*x^4,x, algorithm="giac")
```

output
```
3/16*b*x^(16/3) + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)x^4\,dx=\frac{a\,x^5}{5}+\frac{3\,b\,x^{16/3}}{16}$$

input
```
int(x^4*(a + b*x^(1/3)),x)
```

output
```
(a*x^5)/5 + (3*b*x^(16/3))/16
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right)x^4\,dx=\frac{x^5\left(15x^{\frac{1}{3}}b+16a\right)}{80}$$

input
```
int((a+b*x^(1/3))*x^4,x)
```

output
```
(x**5*(15*x**(1/3)*b + 16*a))/80
```

3.177 $\int \left(a+b\sqrt[3]{x}\right) x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+b\sqrt[3]{x}\right) x^3 \, dx = \frac{ax^4}{4}+\frac{3}{13}bx^{13/3}$$

output
```
1/4*a*x^4+3/13*b*x^(13/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right) x^3 \, dx = \frac{1}{52}\left(13a+12b\sqrt[3]{x}\right) x^4$$

input
```
Integrate[(a + b*x^(1/3))*x^3,x]
```

output
```
((13*a + 12*b*x^(1/3))*x^4)/52
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\sqrt[3]{x}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^3+b x^{10/3}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^4}{4}+\frac{3}{13} b x^{13/3}$$

input
```
Int[(a + b*x^(1/3))*x^3,x]
```

output
```
(a*x^4)/4 + (3*b*x^(13/3))/13
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^{4}}{4}+\frac{3b\,x^{\frac{13}{3}}}{13}$	14
default	$\frac{a\,x^{4}}{4}+\frac{3b\,x^{\frac{13}{3}}}{13}$	14
trager	$\frac{a(x^{3}+x^{2}+x+1)(-1+x)}{4}+\frac{3b\,x^{\frac{13}{3}}}{13}$	23
orering	$\frac{11\left(a+b\,x^{\frac{1}{3}}\right)x^{4}}{26}-\frac{3x^{2}\left(\frac{b\,x^{\frac{7}{3}}}{3}+3\left(a+b\,x^{\frac{1}{3}}\right)x^{2}\right)}{52}$	38

input
```
int((a+b*x^(1/3))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+3/13*b*x^(13/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)x^{3}\,dx=\frac{3}{13}\,bx^{\frac{13}{3}}+\frac{1}{4}\,ax^{4}$$

input
```
integrate((a+b*x^(1/3))*x^3,x, algorithm="fricas")
```

output
```
3/13*b*x^(13/3) + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right)x^3\,dx=\frac{ax^4}{4}+\frac{3bx^{\frac{13}{3}}}{13}$$

input
```
integrate((a+b*x**(1/3))*x**3,x)
```

output
```
a*x**4/4 + 3*b*x**(13/3)/13
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 10.53

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)x^3\,dx=&\frac{3\left(bx^{\frac{1}{3}}+a\right)^{13}}{13\,b^{12}}-\frac{11\left(bx^{\frac{1}{3}}+a\right)^{12}a}{4\,b^{12}}+\frac{15\left(bx^{\frac{1}{3}}+a\right)^{11}a^2}{b^{12}}\\&-\frac{99\left(bx^{\frac{1}{3}}+a\right)^{10}a^3}{2\,b^{12}}+\frac{110\left(bx^{\frac{1}{3}}+a\right)^{9}a^4}{b^{12}}-\frac{693\left(bx^{\frac{1}{3}}+a\right)^{8}a^5}{4\,b^{12}}\\&+\frac{198\left(bx^{\frac{1}{3}}+a\right)^{7}a^6}{b^{12}}-\frac{165\left(bx^{\frac{1}{3}}+a\right)^{6}a^7}{b^{12}}+\frac{99\left(bx^{\frac{1}{3}}+a\right)^{5}a^8}{b^{12}}\\&-\frac{165\left(bx^{\frac{1}{3}}+a\right)^{4}a^9}{4\,b^{12}}+\frac{11\left(bx^{\frac{1}{3}}+a\right)^{3}a^{10}}{b^{12}}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^{2}a^{11}}{2\,b^{12}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))*x^3,x, algorithm="maxima")
```

output
```
3/13*(b*x^(1/3) + a)^13/b^12 - 11/4*(b*x^(1/3) + a)^12*a/b^12 + 15*(b*x^(1
/3) + a)^11*a^2/b^12 - 99/2*(b*x^(1/3) + a)^10*a^3/b^12 + 110*(b*x^(1/3) +
 a)^9*a^4/b^12 - 693/4*(b*x^(1/3) + a)^8*a^5/b^12 + 198*(b*x^(1/3) + a)^7*
a^6/b^12 - 165*(b*x^(1/3) + a)^6*a^7/b^12 + 99*(b*x^(1/3) + a)^5*a^8/b^12
- 165/4*(b*x^(1/3) + a)^4*a^9/b^12 + 11*(b*x^(1/3) + a)^3*a^10/b^12 - 3/2*
(b*x^(1/3) + a)^2*a^11/b^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right) x^3 \, dx = \frac{3}{13} b x^{\frac{13}{3}} + \frac{1}{4} a x^4$$

input
```
integrate((a+b*x^(1/3))*x^3,x, algorithm="giac")
```

output
```
3/13*b*x^(13/3) + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right) x^3 \, dx = \frac{a\,x^4}{4} + \frac{3\,b\,x^{13/3}}{13}$$

input
```
int(x^3*(a + b*x^(1/3)),x)
```

output
```
(a*x^4)/4 + (3*b*x^(13/3))/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right) x^3 \, dx = \frac{x^4 \left(12 x^{\frac{1}{3}} b + 13 a\right)}{52}$$

input
```
int((a+b*x^(1/3))*x^3,x)
```

output
```
(x**4*(12*x**(1/3)*b + 13*a))/52
```

3.178 $\int \left(a+b\sqrt[3]{x}\right) x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+b\sqrt[3]{x}\right) x^2 \, dx = \frac{ax^3}{3} + \frac{3}{10} b x^{10/3}$$

output
```
1/3*a*x^3+3/10*b*x^(10/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right) x^2 \, dx = \frac{1}{30}\left(10a+9b\sqrt[3]{x}\right) x^3$$

input
```
Integrate[(a + b*x^(1/3))*x^2,x]
```

output
```
((10*a + 9*b*x^(1/3))*x^3)/30
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt[3]{x}\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^2+bx^{7/3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{3}{10}bx^{10/3}$$

input
```
Int[(a + b*x^(1/3))*x^2,x]
```

output
```
(a*x^3)/3 + (3*b*x^(10/3))/10
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^{3}}{3}+\frac{3 b\,x^{\frac{10}{3}}}{10}$	14
default	$\frac{a\,x^{3}}{3}+\frac{3 b\,x^{\frac{10}{3}}}{10}$	14
trager	$\frac{a\left(x^{2}+x+1\right)\left(-1+x\right)}{3}+\frac{3 b\,x^{\frac{10}{3}}}{10}$	20
orering	$\frac{8\left(a+b\,x^{\frac{1}{3}}\right)x^{3}}{15}-\frac{x^{2}\left(\frac{b\,x^{\frac{4}{3}}}{3}+2\left(a+b\,x^{\frac{1}{3}}\right)x\right)}{10}$	36

input
```
int((a+b*x^(1/3))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+3/10*b*x^(10/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)x^2\,dx=\frac{3}{10}\,bx^{\frac{10}{3}}+\frac{1}{3}\,ax^3$$

input
```
integrate((a+b*x^(1/3))*x^2,x, algorithm="fricas")
```

output
```
3/10*b*x^(10/3) + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right)x^2\,dx=\frac{ax^3}{3}+\frac{3bx^{\frac{10}{3}}}{10}$$

input
```
integrate((a+b*x**(1/3))*x**2,x)
```

output
```
a*x**3/3 + 3*b*x**(10/3)/10
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 7.84

$$\int \left(a+b\sqrt[3]{x}\right)x^2\,dx=\frac{3\left(bx^{\frac{1}{3}}+a\right)^{10}}{10\,b^9}-\frac{8\left(bx^{\frac{1}{3}}+a\right)^{9}a}{3\,b^9}+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{8}a^2}{2\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{7}a^3}{b^9}+\frac{35\left(bx^{\frac{1}{3}}+a\right)^{6}a^4}{b^9}-\frac{168\left(bx^{\frac{1}{3}}+a\right)^{5}a^5}{5\,b^9}+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{4}a^6}{b^9}-\frac{8\left(bx^{\frac{1}{3}}+a\right)^{3}a^7}{b^9}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{2}a^8}{2\,b^9}$$

input
```
integrate((a+b*x^(1/3))*x^2,x, algorithm="maxima")
```

output
```
3/10*(b*x^(1/3) + a)^10/b^9 - 8/3*(b*x^(1/3) + a)^9*a/b^9 + 21/2*(b*x^(1/3
) + a)^8*a^2/b^9 - 24*(b*x^(1/3) + a)^7*a^3/b^9 + 35*(b*x^(1/3) + a)^6*a^4
/b^9 - 168/5*(b*x^(1/3) + a)^5*a^5/b^9 + 21*(b*x^(1/3) + a)^4*a^6/b^9 - 8*
(b*x^(1/3) + a)^3*a^7/b^9 + 3/2*(b*x^(1/3) + a)^2*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)x^2\,dx = \frac{3}{10}\,bx^{\frac{10}{3}} + \frac{1}{3}\,ax^3$$

input
```
integrate((a+b*x^(1/3))*x^2,x, algorithm="giac")
```

output
```
3/10*b*x^(10/3) + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)x^2\,dx = \frac{a\,x^3}{3} + \frac{3\,b\,x^{10/3}}{10}$$

input
```
int(x^2*(a + b*x^(1/3)),x)
```

output
```
(a*x^3)/3 + (3*b*x^(10/3))/10
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right)x^2\,dx = \frac{x^3\left(9x^{\frac{1}{3}}b+10a\right)}{30}$$

input
```
int((a+b*x^(1/3))*x^2,x)
```

output
```
(x**3*(9*x**(1/3)*b + 10*a))/30
```

3.179 $\int \left(a+b\sqrt[3]{x}\right) x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \left(a+b\sqrt[3]{x}\right) x\,dx=\frac{ax^2}{2}+\frac{3}{7}bx^{7/3}$$

output
```
1/2*a*x^2+3/7*b*x^(7/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right) x\,dx=\frac{1}{14}\left(7a+6b\sqrt[3]{x}\right)x^2$$

input
```
Integrate[(a + b*x^(1/3))*x,x]
```

output
```
((7*a + 6*b*x^(1/3))*x^2)/14
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt[3]{x}\right)\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(ax+bx^{4/3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^2}{2}+\frac{3}{7}bx^{7/3}$$

input
```
Int[(a + b*x^(1/3))*x,x]
```

output
```
(a*x^2)/2 + (3*b*x^(7/3))/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a\,x^{2}}{2}+\frac{3 b\,x^{\frac{7}{3}}}{7}$	14
default	$\frac{a\,x^{2}}{2}+\frac{3 b\,x^{\frac{7}{3}}}{7}$	14
trager	$\frac{(-1+x) a (1+x)}{2}+\frac{3 b\,x^{\frac{7}{3}}}{7}$	17
orering	$\frac{5\left(a+b\,x^{\frac{1}{3}}\right) x^{2}}{7}-\frac{3 x^{2} \left(\frac{4 b\,x^{\frac{1}{3}}}{3}+a\right)}{14}$	27

input
```
int((a+b*x^(1/3))*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+3/7*b*x^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right) x\,dx=\frac{3}{7}\,bx^{\frac{7}{3}}+\frac{1}{2}\,ax^{2}$$

input
```
integrate((a+b*x^(1/3))*x,x, algorithm="fricas")
```

output
```
3/7*b*x^(7/3) + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right)x\,dx=\frac{ax^2}{2}+\frac{3bx^{\frac{7}{3}}}{7}$$

input
```
integrate((a+b*x**(1/3))*x,x)
```

output
```
a*x**2/2 + 3*b*x**(7/3)/7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.16

$$\int \left(a+b\sqrt[3]{x}\right)x\,dx=\frac{3\left(bx^{\frac{1}{3}}+a\right)^7}{7\,b^6}-\frac{5\left(bx^{\frac{1}{3}}+a\right)^6a}{2\,b^6}+\frac{6\left(bx^{\frac{1}{3}}+a\right)^5a^2}{b^6}-\frac{15\left(bx^{\frac{1}{3}}+a\right)^4a^3}{2\,b^6}+\frac{5\left(bx^{\frac{1}{3}}+a\right)^3a^4}{b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^2a^5}{2\,b^6}$$

input
```
integrate((a+b*x^(1/3))*x,x, algorithm="maxima")
```

output
```
3/7*(b*x^(1/3) + a)^7/b^6 - 5/2*(b*x^(1/3) + a)^6*a/b^6 + 6*(b*x^(1/3) + a
)^5*a^2/b^6 - 15/2*(b*x^(1/3) + a)^4*a^3/b^6 + 5*(b*x^(1/3) + a)^3*a^4/b^6
 - 3/2*(b*x^(1/3) + a)^2*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right) x \, dx = \frac{3}{7} b x^{\frac{7}{3}} + \frac{1}{2} a x^2$$

input
```
integrate((a+b*x^(1/3))*x,x, algorithm="giac")
```

output
```
3/7*b*x^(7/3) + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right) x \, dx = \frac{a\,x^2}{2} + \frac{3\,b\,x^{7/3}}{7}$$

input
```
int(x*(a + b*x^(1/3)),x)
```

output
```
(a*x^2)/2 + (3*b*x^(7/3))/7
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right) x \, dx = \frac{x^2 \left(6x^{\frac{1}{3}} b + 7a\right)}{14}$$

input
```
int((a+b*x^(1/3))*x,x)
```

output
```
(x**2*(6*x**(1/3)*b + 7*a))/14
```

3.180 $\int \left(a+b\sqrt[3]{x}\right) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \left(a+b\sqrt[3]{x}\right) \, dx=a x+\frac{3}{4} b x^{4/3}$$

output
```
a*x+3/4*b*x^(4/3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right) \, dx=a x+\frac{3}{4} b x^{4/3}$$

input
```
Integrate[a + b*x^(1/3),x]
```

output
```
a*x + (3*b*x^(4/3))/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{3}{4}bx^{4/3}$$

input
```
Int[a + b*x^(1/3),x]
```

output
```
a*x + (3*b*x^(4/3))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$ax+\frac{3b\,x^{\frac{4}{3}}}{4}$	11
default	$ax+\frac{3b\,x^{\frac{4}{3}}}{4}$	11
risch	$ax+\frac{3b\,x^{\frac{4}{3}}}{4}$	11
parts	$ax+\frac{3b\,x^{\frac{4}{3}}}{4}$	11
trager	$a(-1+x)+\frac{3b\,x^{\frac{4}{3}}}{4}$	13
orering	$\left(a+b\,x^{\frac{1}{3}}\right)x-\frac{b\,x^{\frac{4}{3}}}{4}$	17

input
```
int(a+b*x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
a*x+3/4*b*x^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)\,dx=\frac{3}{4}\,bx^{\frac{4}{3}}+ax$$

input
```
integrate(a+b*x^(1/3),x, algorithm="fricas")
```

output
```
3/4*b*x^(4/3) + a*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt[3]{x}\right) dx = ax + \frac{3bx^{\frac{4}{3}}}{4}$$

input
```
integrate(a+b*x**(1/3),x)
```

output
```
a*x + 3*b*x**(4/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right) dx = \frac{3}{4}bx^{\frac{4}{3}} + ax$$

input
```
integrate(a+b*x^(1/3),x, algorithm="maxima")
```

output
```
3/4*b*x^(4/3) + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right) dx = \frac{3}{4}bx^{\frac{4}{3}} + ax$$

input
```
integrate(a+b*x^(1/3),x, algorithm="giac")
```

output
```
3/4*b*x^(4/3) + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)\,dx=a\,x+\frac{3\,b\,x^{4/3}}{4}$$

input
```
int(a + b*x^(1/3),x)
```

output
```
a*x + (3*b*x^(4/3))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}\right)\,dx=\frac{x\left(3x^{\frac{1}{3}}b+4a\right)}{4}$$

input
```
int(a+b*x^(1/3),x)
```

output
```
(x*(3*x**(1/3)*b + 4*a))/4
```

3.181 $\int \frac{a+b\sqrt[3]{x}}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = 3b\sqrt[3]{x}+a\log(x)$$

output
```
3*b*x^(1/3)+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = 3b\sqrt[3]{x}+a\log(x)$$

input
```
Integrate[(a + b*x^(1/3))/x,x]
```

output
```
3*b*x^(1/3) + a*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt[3]{x}}{x} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x}+\frac{b}{x^{2/3}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)+3b\sqrt[3]{x}$$

input
```
Int[(a + b*x^(1/3))/x,x]
```

output
```
3*b*x^(1/3) + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$3b\,x^{\frac{1}{3}}+a\ln\left(x\right)$	12
default	$3b\,x^{\frac{1}{3}}+a\ln\left(x\right)$	12
trager	$3b\,x^{\frac{1}{3}}+a\ln\left(x\right)$	12

input
```
int((a+b*x^(1/3))/x,x,method=_RETURNVERBOSE)
```

output
```
3*b*x^(1/3)+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = 3\,a\log\left(x^{\frac{1}{3}}\right)+3\,bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))/x,x, algorithm="fricas")
```

output
```
3*a*log(x^(1/3)) + 3*b*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = a\log\left(x\right)+3b\sqrt[3]{x}$$

input
```
integrate((a+b*x**(1/3))/x,x)
```

output
```
a*log(x) + 3*b*x**(1/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = a\log\left(x\right)+3\,bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))/x,x, algorithm="maxima")
```

output
```
a*log(x) + 3*b*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = a\log\left(|x|\right)+3\,bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))/x,x, algorithm="giac")
```

output
```
a*log(abs(x)) + 3*b*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = 3\,b\,x^{1/3}+a\,\ln\left(x\right)$$

input
```
int((a + b*x^(1/3))/x,x)
```

output
```
3*b*x^(1/3) + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+b\sqrt[3]{x}}{x}\,dx = 3x^{\frac{1}{3}}b+\log(x)\,a$$

input
```
int((a+b*x^(1/3))/x,x)
```

output
```
3*x**(1/3)*b + log(x)*a
```

3.182 $\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx=-\frac{a}{x}-\frac{3b}{2x^{2/3}}$$

output
```
-a/x-3/2*b/x^(2/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx=\frac{-2a-3b\sqrt[3]{x}}{2x}$$

input
```
Integrate[(a + b*x^(1/3))/x^2,x]
```

output
```
(-2*a - 3*b*x^(1/3))/(2*x)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt[3]{x}}{x^2} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^2}+\frac{b}{x^{5/3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{x}-\frac{3b}{2x^{2/3}}$$

input
```
Int[(a + b*x^(1/3))/x^2,x]
```

output
```
-(a/x) - (3*b)/(2*x^(2/3))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{a}{x}-\frac{3b}{2x^{\frac{2}{3}}}$	14
default	$-\frac{a}{x}-\frac{3b}{2x^{\frac{2}{3}}}$	14
trager	$\frac{a(-1+x)}{x}-\frac{3b}{2x^{\frac{2}{3}}}$	16
orering	$-\frac{4\left(a+b\,x^{\frac{1}{3}}\right)}{x}-\frac{3\left(\frac{b}{3x^{\frac{8}{3}}}-\frac{2\left(a+b\,x^{\frac{1}{3}}\right)}{x^{3}}\right)x^{2}}{2}$	38

input
```
int((a+b*x^(1/3))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-3/2*b/x^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx=-\frac{3\,bx^{\frac{1}{3}}+2\,a}{2\,x}$$

input
```
integrate((a+b*x^(1/3))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(3*b*x^(1/3) + 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx = -\frac{a}{x} - \frac{3b}{2x^{\frac{2}{3}}}$$

input
```
integrate((a+b*x**(1/3))/x**2,x)
```

output
```
-a/x - 3*b/(2*x**(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx = -\frac{3\,bx^{\frac{1}{3}}+2\,a}{2\,x}$$

input
```
integrate((a+b*x^(1/3))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*x^(1/3) + 2*a)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx = -\frac{3\,bx^{\frac{1}{3}}+2\,a}{2\,x}$$

input
```
integrate((a+b*x^(1/3))/x^2,x, algorithm="giac")
```

output
```
-1/2*(3*b*x^(1/3) + 2*a)/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx=-\frac{a}{x}-\frac{3\,b}{2\,x^{2/3}}$$

input
```
int((a + b*x^(1/3))/x^2,x)
```

output
```
- a/x - (3*b)/(2*x^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{a+b\sqrt[3]{x}}{x^2}\,dx=\frac{-2x^{\frac{2}{3}}a-3bx}{2x^{\frac{5}{3}}}$$

input
```
int((a+b*x^(1/3))/x^2,x)
```

output
```
( - 2*x**(2/3)*a - 3*b*x)/(2*x**(2/3)*x)
```

3.183 $\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx = -\frac{a}{2x^2}-\frac{3b}{5x^{5/3}}$$

output
```
-1/2*a/x^2-3/5*b/x^(5/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx = \frac{-5a-6b\sqrt[3]{x}}{10x^2}$$

input
```
Integrate[(a + b*x^(1/3))/x^3,x]
```

output
```
(-5*a - 6*b*x^(1/3))/(10*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x^{8/3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}-\frac{3b}{5x^{5/3}}$$

input
```
Int[(a + b*x^(1/3))/x^3,x]
```

output
```
-1/2*a/x^2 - (3*b)/(5*x^(5/3))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{a}{2x^{2}}-\frac{3b}{5x^{\frac{5}{3}}}$	14
default	$-\frac{a}{2x^{2}}-\frac{3b}{5x^{\frac{5}{3}}}$	14
trager	$\frac{(-1+x)a(1+x)}{2x^{2}}-\frac{3b}{5x^{\frac{5}{3}}}$	20
orering	$-\frac{7\left(a+b\,x^{\frac{1}{3}}\right)}{5x^{2}}-\frac{3x^{2}\left(\frac{b}{3x^{\frac{11}{3}}}-\frac{3\left(a+b\,x^{\frac{1}{3}}\right)}{x^{4}}\right)}{10}$	38

input
```
int((a+b*x^(1/3))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-3/5*b/x^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx=-\frac{6\,bx^{\frac{1}{3}}+5\,a}{10\,x^{2}}$$

input
```
integrate((a+b*x^(1/3))/x^3,x, algorithm="fricas")
```

output
```
-1/10*(6*b*x^(1/3) + 5*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a + b\sqrt[3]{x}}{x^3}\,dx = -\frac{a}{2x^2} - \frac{3b}{5x^{\frac{5}{3}}}$$

input
```
integrate((a+b*x**(1/3))/x**3,x)
```

output
```
-a/(2*x**2) - 3*b/(5*x**(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a + b\sqrt[3]{x}}{x^3}\,dx = -\frac{6\,bx^{\frac{1}{3}} + 5\,a}{10\,x^2}$$

input
```
integrate((a+b*x^(1/3))/x^3,x, algorithm="maxima")
```

output
```
-1/10*(6*b*x^(1/3) + 5*a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a + b\sqrt[3]{x}}{x^3}\,dx = -\frac{6\,bx^{\frac{1}{3}} + 5\,a}{10\,x^2}$$

input
```
integrate((a+b*x^(1/3))/x^3,x, algorithm="giac")
```

output
```
-1/10*(6*b*x^(1/3) + 5*a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx = -\frac{5\,a+6\,b\,x^{1/3}}{10\,x^2}$$

input
```
int((a + b*x^(1/3))/x^3,x)
```

output
```
-(5*a + 6*b*x^(1/3))/(10*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{a+b\sqrt[3]{x}}{x^3}\,dx = \frac{-5x^{\frac{2}{3}}a-6bx}{10x^{\frac{8}{3}}}$$

input
```
int((a+b*x^(1/3))/x^3,x)
```

output
```
( - 5*x**(2/3)*a - 6*b*x)/(10*x**(2/3)*x**2)
```

3.184 $\int \frac{a+b\sqrt[3]{x}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+b\sqrt[3]{x}}{x^4} dx = -\frac{a}{3x^3} - \frac{3b}{8x^{8/3}}$$

output
```
-1/3*a/x^3-3/8*b/x^(8/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sqrt[3]{x}}{x^4} dx = \frac{-8a-9b\sqrt[3]{x}}{24x^3}$$

input
```
Integrate[(a + b*x^(1/3))/x^4,x]
```

output
```
(-8*a - 9*b*x^(1/3))/(24*x^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sqrt[3]{x}}{x^4} \, dx$$

$$\Big\downarrow \ 802$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^{11/3}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a}{3x^3}-\frac{3b}{8x^{8/3}}$$

input
```
Int[(a + b*x^(1/3))/x^4,x]
```

output
```
-1/3*a/x^3 - (3*b)/(8*x^(8/3))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{a}{3x^{3}}-\frac{3b}{8x^{\frac{8}{3}}}$	14
default	$-\frac{a}{3x^{3}}-\frac{3b}{8x^{\frac{8}{3}}}$	14
trager	$\frac{a\left(x^{2}+x+1\right)\left(-1+x\right)}{3x^{3}}-\frac{3b}{8x^{\frac{8}{3}}}$	23
orering	$-\frac{5\left(a+b\,x^{\frac{1}{3}}\right)}{6x^{3}}-\frac{x^{2}\left(\frac{b}{3x^{\frac{14}{3}}}-\frac{4\left(a+b\,x^{\frac{1}{3}}\right)}{x^{5}}\right)}{8}$	38

input
```
int((a+b*x^(1/3))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-3/8*b/x^(8/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt[3]{x}}{x^{4}}\,dx=-\frac{9\,bx^{\frac{1}{3}}+8\,a}{24\,x^{3}}$$

input
```
integrate((a+b*x^(1/3))/x^4,x, algorithm="fricas")
```

output
```
-1/24*(9*b*x^(1/3) + 8*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sqrt[3]{x}}{x^4}\,dx = -\frac{a}{3x^3} - \frac{3b}{8x^{\frac{8}{3}}}$$

input
```
integrate((a+b*x**(1/3))/x**4,x)
```

output
```
-a/(3*x**3) - 3*b/(8*x**(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt[3]{x}}{x^4}\,dx = -\frac{9\,bx^{\frac{1}{3}} + 8\,a}{24\,x^3}$$

input
```
integrate((a+b*x^(1/3))/x^4,x, algorithm="maxima")
```

output
```
-1/24*(9*b*x^(1/3) + 8*a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sqrt[3]{x}}{x^4}\,dx = -\frac{9\,bx^{\frac{1}{3}} + 8\,a}{24\,x^3}$$

input
```
integrate((a+b*x^(1/3))/x^4,x, algorithm="giac")
```

output
```
-1/24*(9*b*x^(1/3) + 8*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a + b\sqrt[3]{x}}{x^4}\,dx = -\frac{8\,a + 9\,b\,x^{1/3}}{24\,x^3}$$

input
```
int((a + b*x^(1/3))/x^4,x)
```

output
```
-(8*a + 9*b*x^(1/3))/(24*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{a + b\sqrt[3]{x}}{x^4}\,dx = \frac{-8x^{\frac{2}{3}}a - 9bx}{24x^{\frac{11}{3}}}$$

input
```
int((a+b*x^(1/3))/x^4,x)
```

output
```
( - 8*x**(2/3)*a - 9*b*x)/(24*x**(2/3)*x**3)
```

3.185 $\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx=\frac{a^2x^5}{5}+\frac{3}{8}abx^{16/3}+\frac{3}{17}b^2x^{17/3}$$

output
```
1/5*a^2*x^5+3/8*a*b*x^(16/3)+3/17*b^2*x^(17/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx=\frac{1}{680}\left(136a^2x^5+255abx^{16/3}+120b^2x^{17/3}\right)$$

input
```
Integrate[(a + b*x^(1/3))^2*x^4,x]
```

output
```
(136*a^2*x^5 + 255*a*b*x^(16/3) + 120*b^2*x^(17/3))/680
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\sqrt[3]{x}\right)^2 \, dx$$

$$\Big\downarrow \ 798$$

$$3\int \left(a+b\sqrt[3]{x}\right)^2 x^{14/3} d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(b^2x^{16/3}+2abx^5+a^2x^{14/3}\right) d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(\frac{a^2x^5}{15}+\frac{1}{8}abx^{16/3}+\frac{1}{17}b^2x^{17/3}\right)$$

input
```
Int[(a + b*x^(1/3))^2*x^4,x]
```

output
```
3*((a^2*x^5)/15 + (a*b*x^(16/3))/8 + (b^2*x^(17/3))/17)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^2x^5}{5}+\frac{3ab\,x^{\frac{16}{3}}}{8}+\frac{3b^2x^{\frac{17}{3}}}{17}$
default	$\frac{a^2x^5}{5}+\frac{3ab\,x^{\frac{16}{3}}}{8}+\frac{3b^2x^{\frac{17}{3}}}{17}$
trager	$\frac{a^2(x^4+x^3+x^2+x+1)(-1+x)}{5}+\frac{3ab\,x^{\frac{16}{3}}}{8}+\frac{3b^2x^{\frac{17}{3}}}{17}$
orering	$\frac{79x^5\left(a+b\,x^{\frac{1}{3}}\right)^2}{170}-\frac{117x^2\left(\frac{2\left(a+b\,x^{\frac{1}{3}}\right)x^{\frac{10}{3}}b}{3}+4\left(a+b\,x^{\frac{1}{3}}\right)^2x^3\right)}{1360}+\frac{9x^3\left(\frac{2b^2x^{\frac{8}{3}}}{9}+\frac{44\left(a+b\,x^{\frac{1}{3}}\right)x^{\frac{7}{3}}b}{9}+12\left(a+b\,x^{\frac{1}{3}}\right)^2 x \ldots\right.}{1360}$

input
```
int((a+b*x^(1/3))^2*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+3/8*a*b*x^(16/3)+3/17*b^2*x^(17/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2x^4\,dx=\frac{3}{17}\,b^2x^{\frac{17}{3}}+\frac{3}{8}\,abx^{\frac{16}{3}}+\frac{1}{5}\,a^2x^5$$

input
```
integrate((a+b*x^(1/3))^2*x^4,x, algorithm="fricas")
```

output
```
3/17*b^2*x^(17/3) + 3/8*a*b*x^(16/3) + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx = \frac{a^2x^5}{5} + \frac{3abx^{\frac{16}{3}}}{8} + \frac{3b^2x^{\frac{17}{3}}}{17}$$

input
```
integrate((a+b*x**(1/3))**2*x**4,x)
```

output
```
a**2*x**5/5 + 3*a*b*x**(16/3)/8 + 3*b**2*x**(17/3)/17
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 250, normalized size of antiderivative = 7.35

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx = {} & \frac{3\left(bx^{\frac{1}{3}}+a\right)^{17}}{17\,b^{15}} - \frac{21\left(bx^{\frac{1}{3}}+a\right)^{16}a}{8\,b^{15}} + \frac{91\left(bx^{\frac{1}{3}}+a\right)^{15}a^2}{5\,b^{15}} \\ & - \frac{78\left(bx^{\frac{1}{3}}+a\right)^{14}a^3}{b^{15}} + \frac{231\left(bx^{\frac{1}{3}}+a\right)^{13}a^4}{b^{15}} - \frac{1001\left(bx^{\frac{1}{3}}+a\right)^{12}a^5}{2\,b^{15}} \\ & + \frac{819\left(bx^{\frac{1}{3}}+a\right)^{11}a^6}{b^{15}} - \frac{5148\left(bx^{\frac{1}{3}}+a\right)^{10}a^7}{5\,b^{15}} + \frac{1001\left(bx^{\frac{1}{3}}+a\right)^{9}a^8}{b^{15}} \\ & - \frac{3003\left(bx^{\frac{1}{3}}+a\right)^{8}a^9}{4\,b^{15}} + \frac{429\left(bx^{\frac{1}{3}}+a\right)^{7}a^{10}}{b^{15}} - \frac{182\left(bx^{\frac{1}{3}}+a\right)^{6}a^{11}}{b^{15}} \\ & + \frac{273\left(bx^{\frac{1}{3}}+a\right)^{5}a^{12}}{5\,b^{15}} - \frac{21\left(bx^{\frac{1}{3}}+a\right)^{4}a^{13}}{2\,b^{15}} + \frac{\left(bx^{\frac{1}{3}}+a\right)^{3}a^{14}}{b^{15}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^2*x^4,x, algorithm="maxima")
```

output
```
3/17*(b*x^(1/3) + a)^17/b^15 - 21/8*(b*x^(1/3) + a)^16*a/b^15 + 91/5*(b*x^
(1/3) + a)^15*a^2/b^15 - 78*(b*x^(1/3) + a)^14*a^3/b^15 + 231*(b*x^(1/3) +
 a)^13*a^4/b^15 - 1001/2*(b*x^(1/3) + a)^12*a^5/b^15 + 819*(b*x^(1/3) + a)
^11*a^6/b^15 - 5148/5*(b*x^(1/3) + a)^10*a^7/b^15 + 1001*(b*x^(1/3) + a)^9
*a^8/b^15 - 3003/4*(b*x^(1/3) + a)^8*a^9/b^15 + 429*(b*x^(1/3) + a)^7*a^10
/b^15 - 182*(b*x^(1/3) + a)^6*a^11/b^15 + 273/5*(b*x^(1/3) + a)^5*a^12/b^1
5 - 21/2*(b*x^(1/3) + a)^4*a^13/b^15 + (b*x^(1/3) + a)^3*a^14/b^15
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx = \frac{3}{17}\,b^2x^{\frac{17}{3}} + \frac{3}{8}\,abx^{\frac{16}{3}} + \frac{1}{5}\,a^2x^5$$

input
```
integrate((a+b*x^(1/3))^2*x^4,x, algorithm="giac")
```

output
```
3/17*b^2*x^(17/3) + 3/8*a*b*x^(16/3) + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx = \frac{a^2\,x^5}{5} + \frac{3\,b^2\,x^{17/3}}{17} + \frac{3\,a\,b\,x^{16/3}}{8}$$

input
```
int(x^4*(a + b*x^(1/3))^2,x)
```

output
```
(a^2*x^5)/5 + (3*b^2*x^(17/3))/17 + (3*a*b*x^(16/3))/8
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^4\,dx=\frac{x^5\left(120x^{\frac{2}{3}}b^2+255x^{\frac{1}{3}}ab+136a^2\right)}{680}$$

input
```
int((a+b*x^(1/3))^2*x^4,x)
```

output
```
(x**5*(120*x**(2/3)*b**2 + 255*x**(1/3)*a*b + 136*a**2))/680
```

3.186 $\int \left(a+b\sqrt[3]{x}\right)^2 x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^3 \, dx = \frac{a^2x^4}{4}+\frac{6}{13}abx^{13/3}+\frac{3}{14}b^2x^{14/3}$$

output
```
1/4*a^2*x^4+6/13*a*b*x^(13/3)+3/14*b^2*x^(14/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^3 \, dx = \frac{1}{364}\left(91a^2+168ab\sqrt[3]{x}+78b^2x^{2/3}\right)x^4$$

input
```
Integrate[(a + b*x^(1/3))^2*x^3,x]
```

output
```
((91*a^2 + 168*a*b*x^(1/3) + 78*b^2*x^(2/3))*x^4)/364
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt[3]{x}\right)^2\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^2 x^{11/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(b^2x^{13/3}+2abx^4+a^2x^{11/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2x^4}{12}+\frac{2}{13}abx^{13/3}+\frac{1}{14}b^2x^{14/3}\right)$$

input
```
Int[(a + b*x^(1/3))^2*x^3,x]
```

output
```
3*((a^2*x^4)/12 + (2*a*b*x^(13/3))/13 + (b^2*x^(14/3))/14)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^{2} x^{4}}{4}+\frac{6 a b \,x^{\frac{13}{3}}}{13}+\frac{3 b^{2} x^{\frac{14}{3}}}{14}$
default	$\frac{a^{2} x^{4}}{4}+\frac{6 a b \,x^{\frac{13}{3}}}{13}+\frac{3 b^{2} x^{\frac{14}{3}}}{14}$
trager	$\frac{a^{2} \left(x^{3}+x^{2}+x +1\right) \left(-1+x \right)}{4}+\frac{6 a b \,x^{\frac{13}{3}}}{13}+\frac{3 b^{2} x^{\frac{14}{3}}}{14}$
orering	$\frac{199 \left(a +b \,x^{\frac{1}{3}}\right)^{2} x^{4}}{364}-\frac{45 x^{2} \left(\frac{2 \left(a +b \,x^{\frac{1}{3}}\right) x^{\frac{7}{3}} b}{3}+3 \left(a +b \,x^{\frac{1}{3}}\right)^{2} x^{2}\right)}{364}+\frac{9 x^{3} \left(\frac{2 b^{2} x^{\frac{5}{3}}}{9}+\frac{32 \left(a +b \,x^{\frac{1}{3}}\right) x^{\frac{4}{3}} b}{9}+6 \left(a +b \,x^{\frac{1}{3}}\right)^{2} x\right)}{728}$

input
```
int((a+b*x^(1/3))^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+6/13*a*b*x^(13/3)+3/14*b^2*x^(14/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^3\,dx=\frac{3}{14}\,b^{2}x^{\frac{14}{3}}+\frac{6}{13}\,abx^{\frac{13}{3}}+\frac{1}{4}\,a^{2}x^{4}$$

input
```
integrate((a+b*x^(1/3))^2*x^3,x, algorithm="fricas")
```

output
```
3/14*b^2*x^(14/3) + 6/13*a*b*x^(13/3) + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^3\,dx=\frac{a^2x^4}{4}+\frac{6abx^{\frac{13}{3}}}{13}+\frac{3b^2x^{\frac{14}{3}}}{14}$$

input
```
integrate((a+b*x**(1/3))**2*x**3,x)
```

output
```
a**2*x**4/4 + 6*a*b*x**(13/3)/13 + 3*b**2*x**(14/3)/14
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.88

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^2 x^3\,dx=&\frac{3\left(bx^{\frac{1}{3}}+a\right)^{14}}{14\,b^{12}}-\frac{33\left(bx^{\frac{1}{3}}+a\right)^{13}a}{13\,b^{12}}+\frac{55\left(bx^{\frac{1}{3}}+a\right)^{12}a^2}{4\,b^{12}}\\&-\frac{45\left(bx^{\frac{1}{3}}+a\right)^{11}a^3}{b^{12}}+\frac{99\left(bx^{\frac{1}{3}}+a\right)^{10}a^4}{b^{12}}-\frac{154\left(bx^{\frac{1}{3}}+a\right)^{9}a^5}{b^{12}}\\&+\frac{693\left(bx^{\frac{1}{3}}+a\right)^{8}a^6}{4\,b^{12}}-\frac{990\left(bx^{\frac{1}{3}}+a\right)^{7}a^7}{7\,b^{12}}+\frac{165\left(bx^{\frac{1}{3}}+a\right)^{6}a^8}{2\,b^{12}}\\&-\frac{33\left(bx^{\frac{1}{3}}+a\right)^{5}a^9}{b^{12}}+\frac{33\left(bx^{\frac{1}{3}}+a\right)^{4}a^{10}}{4\,b^{12}}-\frac{\left(bx^{\frac{1}{3}}+a\right)^{3}a^{11}}{b^{12}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^2*x^3,x, algorithm="maxima")
```

output
```
3/14*(b*x^(1/3) + a)^14/b^12 - 33/13*(b*x^(1/3) + a)^13*a/b^12 + 55/4*(b*x
^(1/3) + a)^12*a^2/b^12 - 45*(b*x^(1/3) + a)^11*a^3/b^12 + 99*(b*x^(1/3) +
 a)^10*a^4/b^12 - 154*(b*x^(1/3) + a)^9*a^5/b^12 + 693/4*(b*x^(1/3) + a)^8
*a^6/b^12 - 990/7*(b*x^(1/3) + a)^7*a^7/b^12 + 165/2*(b*x^(1/3) + a)^6*a^8
/b^12 - 33*(b*x^(1/3) + a)^5*a^9/b^12 + 33/4*(b*x^(1/3) + a)^4*a^10/b^12 -
 (b*x^(1/3) + a)^3*a^11/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^3\,dx=\frac{3}{14}\,b^2x^{\frac{14}{3}}+\frac{6}{13}\,abx^{\frac{13}{3}}+\frac{1}{4}\,a^2x^4$$

input
```
integrate((a+b*x^(1/3))^2*x^3,x, algorithm="giac")
```

output
```
3/14*b^2*x^(14/3) + 6/13*a*b*x^(13/3) + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^3\,dx=\frac{a^2\,x^4}{4}+\frac{3\,b^2\,x^{14/3}}{14}+\frac{6\,a\,b\,x^{13/3}}{13}$$

input
```
int(x^3*(a + b*x^(1/3))^2,x)
```

output
```
(a^2*x^4)/4 + (3*b^2*x^(14/3))/14 + (6*a*b*x^(13/3))/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^3\,dx=\frac{x^4\left(78x^{\frac{2}{3}}b^2+168x^{\frac{1}{3}}ab+91a^2\right)}{364}$$

input
```
int((a+b*x^(1/3))^2*x^3,x)
```

output
```
(x**4*(78*x**(2/3)*b**2 + 168*x**(1/3)*a*b + 91*a**2))/364
```

3.187 $\int \left(a+b\sqrt[3]{x}\right)^2 x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^2 \, dx=\frac{a^2x^3}{3}+\frac{3}{5}abx^{10/3}+\frac{3}{11}b^2x^{11/3}$$

output
```
1/3*a^2*x^3+3/5*a*b*x^(10/3)+3/11*b^2*x^(11/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^2 \, dx=\frac{1}{165}\left(55a^2+99ab\sqrt[3]{x}+45b^2x^{2/3}\right)x^3$$

input
```
Integrate[(a + b*x^(1/3))^2*x^2,x]
```

output
```
((55*a^2 + 99*a*b*x^(1/3) + 45*b^2*x^(2/3))*x^3)/165
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\sqrt[3]{x}\right)^2 dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^2 x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(b^2x^{10/3}+2abx^3+a^2x^{8/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2x^3}{9}+\frac{1}{5}abx^{10/3}+\frac{1}{11}b^2x^{11/3}\right)$$

input
```
Int[(a + b*x^(1/3))^2*x^2,x]
```

output
```
3*((a^2*x^3)/9 + (a*b*x^(10/3))/5 + (b^2*x^(11/3))/11)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^{2} x^{3}}{3}+\frac{3 a b \,x^{\frac{10}{3}}}{5}+\frac{3 b^{2} x^{\frac{11}{3}}}{11}$
default	$\frac{a^{2} x^{3}}{3}+\frac{3 a b \,x^{\frac{10}{3}}}{5}+\frac{3 b^{2} x^{\frac{11}{3}}}{11}$
trager	$\frac{a^{2} \left(x^{2}+x+1\right) \left(-1+x\right)}{3}+\frac{3 a b \,x^{\frac{10}{3}}}{5}+\frac{3 b^{2} x^{\frac{11}{3}}}{11}$
orering	$\frac{109 \left(a+b \,x^{\frac{1}{3}}\right)^{2} x^{3}}{165}-\frac{21 x^{2} \left(\frac{2 \left(a+b \,x^{\frac{1}{3}}\right) x^{\frac{4}{3}} b}{3}+2 \left(a+b \,x^{\frac{1}{3}}\right)^{2} x\right)}{110}+\frac{3 x^{3} \left(\frac{2 b^{2} x^{\frac{2}{3}}}{9}+\frac{20 \left(a+b \,x^{\frac{1}{3}}\right) x^{\frac{1}{3}} b}{9}+2 \left(a+b \,x^{\frac{1}{3}}\right)^{2}\right)}{110}$

input
```
int((a+b*x^(1/3))^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+3/5*a*b*x^(10/3)+3/11*b^2*x^(11/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^2\,dx=\frac{3}{11}\,b^{2}x^{\frac{11}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{1}{3}\,a^{2}x^{3}$$

input
```
integrate((a+b*x^(1/3))^2*x^2,x, algorithm="fricas")
```

output
```
3/11*b^2*x^(11/3) + 3/5*a*b*x^(10/3) + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^2\,dx=\frac{a^2x^3}{3}+\frac{3abx^{\frac{10}{3}}}{5}+\frac{3b^2x^{\frac{11}{3}}}{11}$$

input
```
integrate((a+b*x**(1/3))**2*x**2,x)
```

output
```
a**2*x**3/3 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(11/3)/11
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.35

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^2 x^2\,dx=&\frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}}{11\,b^9}-\frac{12\left(bx^{\frac{1}{3}}+a\right)^{10}a}{5\,b^9}+\frac{28\left(bx^{\frac{1}{3}}+a\right)^{9}a^2}{3\,b^9}\\&-\frac{21\left(bx^{\frac{1}{3}}+a\right)^{8}a^3}{b^9}+\frac{30\left(bx^{\frac{1}{3}}+a\right)^{7}a^4}{b^9}-\frac{28\left(bx^{\frac{1}{3}}+a\right)^{6}a^5}{b^9}\\&+\frac{84\left(bx^{\frac{1}{3}}+a\right)^{5}a^6}{5\,b^9}-\frac{6\left(bx^{\frac{1}{3}}+a\right)^{4}a^7}{b^9}+\frac{\left(bx^{\frac{1}{3}}+a\right)^{3}a^8}{b^9}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^2*x^2,x, algorithm="maxima")
```

output
```
3/11*(b*x^(1/3) + a)^11/b^9 - 12/5*(b*x^(1/3) + a)^10*a/b^9 + 28/3*(b*x^(1
/3) + a)^9*a^2/b^9 - 21*(b*x^(1/3) + a)^8*a^3/b^9 + 30*(b*x^(1/3) + a)^7*a
^4/b^9 - 28*(b*x^(1/3) + a)^6*a^5/b^9 + 84/5*(b*x^(1/3) + a)^5*a^6/b^9 - 6
*(b*x^(1/3) + a)^4*a^7/b^9 + (b*x^(1/3) + a)^3*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^2\,dx = \frac{3}{11}\,b^2x^{\frac{11}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{1}{3}\,a^2x^3$$

input
```
integrate((a+b*x^(1/3))^2*x^2,x, algorithm="giac")
```

output
```
3/11*b^2*x^(11/3) + 3/5*a*b*x^(10/3) + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^2\,dx = \frac{a^2\,x^3}{3}+\frac{3\,b^2\,x^{11/3}}{11}+\frac{3\,a\,b\,x^{10/3}}{5}$$

input
```
int(x^2*(a + b*x^(1/3))^2,x)
```

output
```
(a^2*x^3)/3 + (3*b^2*x^(11/3))/11 + (3*a*b*x^(10/3))/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^2 x^2\,dx = \frac{x^3\left(45x^{\frac{2}{3}}b^2+99x^{\frac{1}{3}}ab+55a^2\right)}{165}$$

input
```
int((a+b*x^(1/3))^2*x^2,x)
```

output
```
(x**3*(45*x**(2/3)*b**2 + 99*x**(1/3)*a*b + 55*a**2))/165
```

3.188 $\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx=\frac{a^2x^2}{2}+\frac{6}{7}abx^{7/3}+\frac{3}{8}b^2x^{8/3}$$

output
```
1/2*a^2*x^2+6/7*a*b*x^(7/3)+3/8*b^2*x^(8/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx=\frac{1}{56}\left(28a^2+48ab\sqrt[3]{x}+21b^2x^{2/3}\right)x^2$$

input
```
Integrate[(a + b*x^(1/3))^2*x,x]
```

output
```
((28*a^2 + 48*a*b*x^(1/3) + 21*b^2*x^(2/3))*x^2)/56
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt[3]{x}\right)^2 dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^2 x^{5/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(b^2x^{7/3}+2abx^2+a^2x^{5/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2x^2}{6}+\frac{2}{7}abx^{7/3}+\frac{1}{8}b^2x^{8/3}\right)$$

input
```
Int[(a + b*x^(1/3))^2*x,x]
```

output
```
3*((a^2*x^2)/6 + (2*a*b*x^(7/3))/7 + (b^2*x^(8/3))/8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{a^{2} x^{2}}{2}+\frac{6 a b \,x^{\frac{7}{3}}}{7}+\frac{3 b^{2} x^{\frac{8}{3}}}{8}$	25
default	$\frac{a^{2} x^{2}}{2}+\frac{6 a b \,x^{\frac{7}{3}}}{7}+\frac{3 b^{2} x^{\frac{8}{3}}}{8}$	25
trager	$\frac{\left(-1+x \right) a^{2} \left(1+x \right)}{2}+\frac{6 a b \,x^{\frac{7}{3}}}{7}+\frac{3 b^{2} x^{\frac{8}{3}}}{8}$	28
orering	$\frac{23 \left(a +b \,x^{\frac{1}{3}}\right)^{2} x^{2}}{28}-\frac{9 x^{2} \left(\frac{2 \left(a +b \,x^{\frac{1}{3}}\right) x^{\frac{1}{3}} b}{3}+\left(a +b \,x^{\frac{1}{3}}\right)^{2}\right)}{28}+\frac{9 x^{3} \left(\frac{2 b^{2}}{9 x^{\frac{1}{3}}}+\frac{8 \left(a +b \,x^{\frac{1}{3}}\right) b}{9 x^{\frac{2}{3}}}\right)}{112}$	71

input
```
int((a+b*x^(1/3))^2*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+6/7*a*b*x^(7/3)+3/8*b^2*x^(8/3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx=\frac{3}{8}\,b^{2}x^{\frac{8}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{1}{2}\,a^{2}x^{2}$$

input
```
integrate((a+b*x^(1/3))^2*x,x, algorithm="fricas")
```

output
```
3/8*b^2*x^(8/3) + 6/7*a*b*x^(7/3) + 1/2*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx = \frac{a^2x^2}{2} + \frac{6abx^{\frac{7}{3}}}{7} + \frac{3b^2x^{\frac{8}{3}}}{8}$$

input
```
integrate((a+b*x**(1/3))**2*x,x)
```

output
```
a**2*x**2/2 + 6*a*b*x**(7/3)/7 + 3*b**2*x**(8/3)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(24) = 48.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.88

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx = \frac{3\left(bx^{\frac{1}{3}}+a\right)^8}{8\,b^6} - \frac{15\left(bx^{\frac{1}{3}}+a\right)^7 a}{7\,b^6} + \frac{5\left(bx^{\frac{1}{3}}+a\right)^6 a^2}{b^6} - \frac{6\left(bx^{\frac{1}{3}}+a\right)^5 a^3}{b^6} + \frac{15\left(bx^{\frac{1}{3}}+a\right)^4 a^4}{4\,b^6} - \frac{\left(bx^{\frac{1}{3}}+a\right)^3 a^5}{b^6}$$

input
```
integrate((a+b*x^(1/3))^2*x,x, algorithm="maxima")
```

output
```
3/8*(b*x^(1/3) + a)^8/b^6 - 15/7*(b*x^(1/3) + a)^7*a/b^6 + 5*(b*x^(1/3) +
a)^6*a^2/b^6 - 6*(b*x^(1/3) + a)^5*a^3/b^6 + 15/4*(b*x^(1/3) + a)^4*a^4/b^
6 - (b*x^(1/3) + a)^3*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx = \frac{3}{8}b^2x^{\frac{8}{3}} + \frac{6}{7}abx^{\frac{7}{3}} + \frac{1}{2}a^2x^2$$

input
```
integrate((a+b*x^(1/3))^2*x,x, algorithm="giac")
```

output
```
3/8*b^2*x^(8/3) + 6/7*a*b*x^(7/3) + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx = \frac{a^2\,x^2}{2} + \frac{3\,b^2\,x^{8/3}}{8} + \frac{6\,a\,b\,x^{7/3}}{7}$$

input
```
int(x*(a + b*x^(1/3))^2,x)
```

output
```
(a^2*x^2)/2 + (3*b^2*x^(8/3))/8 + (6*a*b*x^(7/3))/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^2 x\,dx = \frac{x^2\left(21x^{\frac{2}{3}}b^2 + 48x^{\frac{1}{3}}ab + 28a^2\right)}{56}$$

input
```
int((a+b*x^(1/3))^2*x,x)
```

output
```
(x**2*(21*x**(2/3)*b**2 + 48*x**(1/3)*a*b + 28*a**2))/56
```

3.189 $\int \left(a+b\sqrt[3]{x}\right)^2 dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \left(a+b\sqrt[3]{x}\right)^2 dx=a^2x+\frac{3}{2}abx^{4/3}+\frac{3}{5}b^2x^{5/3}$$

output
```
a^2*x+3/2*a*b*x^(4/3)+3/5*b^2*x^(5/3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt[3]{x}\right)^2 dx=\frac{1}{10}\left(10a^2x+15abx^{4/3}+6b^2x^{5/3}\right)$$

input
```
Integrate[(a + b*x^(1/3))^2,x]
```

output
```
(10*a^2*x + 15*a*b*x^(4/3) + 6*b^2*x^(5/3))/10
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}\right)^2 dx$$

$$\Big\downarrow \text{774}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^2 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x^{2/3}a^2+2bxa+b^2x^{4/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2x}{3}+\frac{1}{2}abx^{4/3}+\frac{1}{5}b^2x^{5/3}\right)$$

input
```
Int[(a + b*x^(1/3))^2,x]
```

output
```
3*((a^2*x)/3 + (a*b*x^(4/3))/2 + (b^2*x^(5/3))/5)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$a^{2}x+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	22
default	$a^{2}x+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	22
trager	$a^{2}(-1+x)+\frac{3ab\,x^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{5}{3}}}{5}$	24
orering	$\left(a+b\,x^{\frac{1}{3}}\right)^{2}x-\frac{3\left(a+b\,x^{\frac{1}{3}}\right)x^{\frac{4}{3}}b}{10}+\frac{9x^{3}\left(\frac{2b^{2}}{9x^{\frac{4}{3}}}-\frac{4\left(a+b\,x^{\frac{1}{3}}\right)b}{9x^{\frac{5}{3}}}\right)}{20}$	53

input
```
int((a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+3/2*a*b*x^(4/3)+3/5*b^2*x^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt[3]{x}\right)^{2}\,dx=\frac{3}{5}\,b^{2}x^{\frac{5}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+a^{2}x$$

input
```
integrate((a+b*x^(1/3))^2,x, algorithm="fricas")
```

output
```
3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}\right)^2 dx=a^2x+\frac{3abx^{\frac{4}{3}}}{2}+\frac{3b^2x^{\frac{5}{3}}}{5}$$

input
```
integrate((a+b*x**(1/3))**2,x)
```

output
```
a**2*x + 3*a*b*x**(4/3)/2 + 3*b**2*x**(5/3)/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt[3]{x}\right)^2 dx=\frac{3}{5}b^2x^{\frac{5}{3}}+\frac{3}{2}abx^{\frac{4}{3}}+a^2x$$

input
```
integrate((a+b*x^(1/3))^2,x, algorithm="maxima")
```

output
```
3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt[3]{x}\right)^2 dx=\frac{3}{5}b^2x^{\frac{5}{3}}+\frac{3}{2}abx^{\frac{4}{3}}+a^2x$$

input
```
integrate((a+b*x^(1/3))^2,x, algorithm="giac")
```

output
```
3/5*b^2*x^(5/3) + 3/2*a*b*x^(4/3) + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt[3]{x}\right)^2\,dx=a^2\,x+\frac{3\,b^2\,x^{5/3}}{5}+\frac{3\,a\,b\,x^{4/3}}{2}$$

input
```
int((a + b*x^(1/3))^2,x)
```

output
```
a^2*x + (3*b^2*x^(5/3))/5 + (3*a*b*x^(4/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt[3]{x}\right)^2\,dx=\frac{x\left(6x^{\frac{2}{3}}b^2+15x^{\frac{1}{3}}ab+10a^2\right)}{10}$$

input
```
int((a+b*x^(1/3))^2,x)
```

output
```
(x*(6*x**(2/3)*b**2 + 15*x**(1/3)*a*b + 10*a**2))/10
```

3.190 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx=6ab\sqrt[3]{x}+\frac{3}{2}b^2x^{2/3}+a^2\log(x)$$

output
```
6*a*b*x^(1/3)+3/2*b^2*x^(2/3)+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx=\frac{3}{2}b\left(4a+b\sqrt[3]{x}\right)\sqrt[3]{x}+a^2\log(x)$$

input
```
Integrate[(a + b*x^(1/3))^2/x,x]
```

output
```
(3*b*(4*a + b*x^(1/3))*x^(1/3))/2 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx$$

$$\Big\downarrow \ 798$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(\frac{a^2}{\sqrt[3]{x}}+2ba+b^2\sqrt[3]{x}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(a^2\log\left(\sqrt[3]{x}\right)+2ab\sqrt[3]{x}+\frac{1}{2}b^2x^{2/3}\right)$$

input
```
Int[(a + b*x^(1/3))^2/x,x]
```

output
```
3*(2*a*b*x^(1/3) + (b^2*x^(2/3))/2 + a^2*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$6ab\,x^{\frac{1}{3}}+\frac{3b^{2}x^{\frac{2}{3}}}{2}+a^{2}\ln\left(x\right)$	23
default	$6ab\,x^{\frac{1}{3}}+\frac{3b^{2}x^{\frac{2}{3}}}{2}+a^{2}\ln\left(x\right)$	23
trager	$6ab\,x^{\frac{1}{3}}+\frac{3b^{2}x^{\frac{2}{3}}}{2}+a^{2}\ln\left(x\right)$	23

input
```
int((a+b*x^(1/3))^2/x,x,method=_RETURNVERBOSE)
```

output
```
6*a*b*x^(1/3)+3/2*b^2*x^(2/3)+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int\frac{\left(a+b\sqrt[3]{x}\right)^{2}}{x}\,dx=3\,a^{2}\log\left(x^{\frac{1}{3}}\right)+\frac{3}{2}\,b^{2}x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^2/x,x, algorithm="fricas")
```

output
```
3*a^2*log(x^(1/3)) + 3/2*b^2*x^(2/3) + 6*a*b*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx = a^2\log\left(x\right)+6ab\sqrt[3]{x}+\frac{3b^2x^{\frac{2}{3}}}{2}$$

input
```
integrate((a+b*x**(1/3))**2/x,x)
```

output
```
a**2*log(x) + 6*a*b*x**(1/3) + 3*b**2*x**(2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx = a^2\log\left(x\right)+\frac{3}{2}\,b^2x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 3/2*b^2*x^(2/3) + 6*a*b*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx = a^2\log\left(|x|\right)+\frac{3}{2}\,b^2x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^2/x,x, algorithm="giac")
```

output
```
a^2*log(abs(x)) + 3/2*b^2*x^(2/3) + 6*a*b*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx = 3\,a^2\,\ln\left(x^{1/3}\right)+\frac{3\,b^2\,x^{2/3}}{2}+6\,a\,b\,x^{1/3}$$

input
```
int((a + b*x^(1/3))^2/x,x)
```

output
```
3*a^2*log(x^(1/3)) + (3*b^2*x^(2/3))/2 + 6*a*b*x^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x}\,dx = \frac{3x^{\frac{2}{3}}b^2}{2}+6x^{\frac{1}{3}}ab+\log(x)\,a^2$$

input
```
int((a+b*x^(1/3))^2/x,x)
```

output
```
(3*x**(2/3)*b**2 + 12*x**(1/3)*a*b + 2*log(x)*a**2)/2
```

3.191 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx=-\frac{\left(a+b\sqrt[3]{x}\right)^3}{ax}$$

output
```
-(a+b*x^(1/3))^3/a/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx=\frac{-a^2-3ab\sqrt[3]{x}-3b^2x^{2/3}}{x}$$

input
```
Integrate[(a + b*x^(1/3))^2/x^2,x]
```

output
```
(-a^2 - 3*a*b*x^(1/3) - 3*b^2*x^(2/3))/x
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2} \, dx$$

$$\Big\downarrow \; 796$$

$$-\frac{\left(a+b\sqrt[3]{x}\right)^3}{ax}$$

input
```
Int[(a + b*x^(1/3))^2/x^2,x]
```

output
```
-((a + b*x^(1/3))^3/(a*x))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$-\frac{3b^2}{x^{\frac{1}{3}}}-\frac{3ab}{x^{\frac{2}{3}}}-\frac{a^2}{x}$	25
default	$-\frac{3b^2}{x^{\frac{1}{3}}}-\frac{3ab}{x^{\frac{2}{3}}}-\frac{a^2}{x}$	25
trager	$\frac{a^2(-1+x)}{x}-\frac{3ab}{x^{\frac{2}{3}}}-\frac{3b^2}{x^{\frac{1}{3}}}$	27
orering	$-\frac{19\left(a+b\,x^{\frac{1}{3}}\right)^2}{x}-\frac{45\left(\frac{2\left(a+b\,x^{\frac{1}{3}}\right)b}{3x^{\frac{8}{3}}}-\frac{2\left(a+b\,x^{\frac{1}{3}}\right)^2}{x^3}\right)x^2}{2}-\frac{9x^3\left(\frac{2b^2}{9x^{\frac{10}{3}}}-\frac{28\left(a+b\,x^{\frac{1}{3}}\right)b}{9x^{\frac{11}{3}}}+\frac{6\left(a+b\,x^{\frac{1}{3}}\right)^2}{x^4}\right)}{2}$	90

input
```
int((a+b*x^(1/3))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3*b^2/x^(1/3)-3*a*b/x^(2/3)-a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx=-\frac{3\,b^2x^{\frac{2}{3}}+3\,abx^{\frac{1}{3}}+a^2}{x}$$

input
```
integrate((a+b*x^(1/3))^2/x^2,x, algorithm="fricas")
```

output
```
-(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx=-\frac{a^2}{x}-\frac{3ab}{x^{\frac{2}{3}}}-\frac{3b^2}{\sqrt[3]{x}}$$

input
```
integrate((a+b*x**(1/3))**2/x**2,x)
```

output
```
-a**2/x - 3*a*b/x**(2/3) - 3*b**2/x**(1/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\sqrt[3]{x})^2}{x^2}\,dx = -\frac{3\,b^2x^{\frac{2}{3}}+3\,abx^{\frac{1}{3}}+a^2}{x}$$

input
```
integrate((a+b*x^(1/3))^2/x^2,x, algorithm="maxima")
```

output
```
-(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\sqrt[3]{x})^2}{x^2}\,dx = -\frac{3\,b^2x^{\frac{2}{3}}+3\,abx^{\frac{1}{3}}+a^2}{x}$$

input
```
integrate((a+b*x^(1/3))^2/x^2,x, algorithm="giac")
```

output
```
-(3*b^2*x^(2/3) + 3*a*b*x^(1/3) + a^2)/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\sqrt[3]{x})^2}{x^2}\,dx = -\frac{a^2}{x}-\frac{3\,b^2}{x^{1/3}}-\frac{3\,a\,b}{x^{2/3}}$$

input
```
int((a + b*x^(1/3))^2/x^2,x)
```

output
```
- a^2/x - (3*b^2)/x^(1/3) - (3*a*b)/x^(2/3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^2}\,dx=\frac{-x^{\frac{2}{3}}a^2-3x^{\frac{4}{3}}b^2-3abx}{x^{\frac{5}{3}}}$$

input
```
int((a+b*x^(1/3))^2/x^2,x)
```

output
```
( - x**(2/3)*a**2 - 3*x**(1/3)*b**2*x - 3*a*b*x)/(x**(2/3)*x)
```

3.192 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3} \, dx=-\frac{a^2}{2x^2}-\frac{6ab}{5x^{5/3}}-\frac{3b^2}{4x^{4/3}}$$

output
```
-1/2*a^2/x^2-6/5*a*b/x^(5/3)-3/4*b^2/x^(4/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3} \, dx=\frac{-10a^2-24ab\sqrt[3]{x}-15b^2x^{2/3}}{20x^2}$$

input
```
Integrate[(a + b*x^(1/3))^2/x^3,x]
```

output
```
(-10*a^2 - 24*a*b*x^(1/3) - 15*b^2*x^(2/3))/(20*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx$$
$$\Big\downarrow \text{ 798}$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^{7/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 53}$$
$$3\int \left(\frac{a^2}{x^{7/3}}+\frac{2ba}{x^2}+\frac{b^2}{x^{5/3}}\right)d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 2009}$$
$$3\left(-\frac{a^2}{6x^2}-\frac{2ab}{5x^{5/3}}-\frac{b^2}{4x^{4/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^2/x^3,x]
```

output
```
3*(-1/6*a^2/x^2 - (2*a*b)/(5*x^(5/3)) - b^2/(4*x^(4/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{a^2}{2x^2}-\frac{6ab}{5x^{\frac{5}{3}}}-\frac{3b^2}{4x^{\frac{4}{3}}}$	25
default	$-\frac{a^2}{2x^2}-\frac{6ab}{5x^{\frac{5}{3}}}-\frac{3b^2}{4x^{\frac{4}{3}}}$	25
trager	$\frac{(-1+x)a^2(1+x)}{2x^2}-\frac{6ab}{5x^{\frac{5}{3}}}-\frac{3b^2}{4x^{\frac{4}{3}}}$	31
orering	$-\frac{16\left(a+b\,x^{\frac{1}{3}}\right)^2}{5x^2}-\frac{9x^2\left(\frac{2\left(a+b\,x^{\frac{1}{3}}\right)b}{3x^{\frac{11}{3}}}-\frac{3\left(a+b\,x^{\frac{1}{3}}\right)^2}{x^4}\right)}{5}-\frac{9x^3\left(\frac{2b^2}{9x^{\frac{13}{3}}}-\frac{40\left(a+b\,x^{\frac{1}{3}}\right)b}{9x^{\frac{14}{3}}}+\frac{12\left(a+b\,x^{\frac{1}{3}}\right)^2}{x^5}\right)}{40}$	90

input
```
int((a+b*x^(1/3))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-6/5*a*b/x^(5/3)-3/4*b^2/x^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx=-\frac{15\,b^2x^{\frac{2}{3}}+24\,abx^{\frac{1}{3}}+10\,a^2}{20\,x^2}$$

input
```
integrate((a+b*x^(1/3))^2/x^3,x, algorithm="fricas")
```

output
```
-1/20*(15*b^2*x^(2/3) + 24*a*b*x^(1/3) + 10*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx=-\frac{a^2}{2x^2}-\frac{6ab}{5x^{\frac{5}{3}}}-\frac{3b^2}{4x^{\frac{4}{3}}}$$

input
```
integrate((a+b*x**(1/3))**2/x**3,x)
```

output
```
-a**2/(2*x**2) - 6*a*b/(5*x**(5/3)) - 3*b**2/(4*x**(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx=-\frac{15\,b^2x^{\frac{2}{3}}+24\,abx^{\frac{1}{3}}+10\,a^2}{20\,x^2}$$

input
```
integrate((a+b*x^(1/3))^2/x^3,x, algorithm="maxima")
```

output
```
-1/20*(15*b^2*x^(2/3) + 24*a*b*x^(1/3) + 10*a^2)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx=-\frac{15\,b^2x^{\frac{2}{3}}+24\,abx^{\frac{1}{3}}+10\,a^2}{20\,x^2}$$

input
```
integrate((a+b*x^(1/3))^2/x^3,x, algorithm="giac")
```

output
```
-1/20*(15*b^2*x^(2/3) + 24*a*b*x^(1/3) + 10*a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx=-\frac{10\,a^2+15\,b^2\,x^{2/3}+24\,a\,b\,x^{1/3}}{20\,x^2}$$

input
```
int((a + b*x^(1/3))^2/x^3,x)
```

output
```
-(10*a^2 + 15*b^2*x^(2/3) + 24*a*b*x^(1/3))/(20*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^3}\,dx=\frac{-10x^{\frac{2}{3}}a^2-15x^{\frac{4}{3}}b^2-24abx}{20x^{\frac{8}{3}}}$$

input
```
int((a+b*x^(1/3))^2/x^3,x)
```

output
```
( - 10*x**(2/3)*a**2 - 15*x**(1/3)*b**2*x - 24*a*b*x)/(20*x**(2/3)*x**2)
```

3.193 $\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx=-\frac{a^2}{3x^3}-\frac{3ab}{4x^{8/3}}-\frac{3b^2}{7x^{7/3}}$$

output
```
-1/3*a^2/x^3-3/4*a*b/x^(8/3)-3/7*b^2/x^(7/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx=\frac{-28a^2-63ab\sqrt[3]{x}-36b^2x^{2/3}}{84x^3}$$

input
```
Integrate[(a + b*x^(1/3))^2/x^4,x]
```

output
```
(-28*a^2 - 63*a*b*x^(1/3) - 36*b^2*x^(2/3))/(84*x^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx$$

$$\Big\downarrow \; 798$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 53$$

$$3\int \left(\frac{a^2}{x^{10/3}}+\frac{2ba}{x^3}+\frac{b^2}{x^{8/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \; 2009$$

$$3\left(-\frac{a^2}{9x^3}-\frac{ab}{4x^{8/3}}-\frac{b^2}{7x^{7/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^2/x^4,x]
```

output
```
3*(-1/9*a^2/x^3 - (a*b)/(4*x^(8/3)) - b^2/(7*x^(7/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{a^{2}}{3x^{3}}-\frac{3ab}{4x^{\frac{8}{3}}}-\frac{3b^{2}}{7x^{\frac{7}{3}}}$	25
default	$-\frac{a^{2}}{3x^{3}}-\frac{3ab}{4x^{\frac{8}{3}}}-\frac{3b^{2}}{7x^{\frac{7}{3}}}$	25
trager	$\frac{a^{2}\left(x^{2}+x+1\right)\left(-1+x\right)}{3x^{3}}-\frac{3ab}{4x^{\frac{8}{3}}}-\frac{3b^{2}}{7x^{\frac{7}{3}}}$	34
orering	$-\frac{34\left(a+b\,x^{\frac{1}{3}}\right)^{2}}{21x^{3}}-\frac{33x^{2}\left(\frac{2\left(a+b\,x^{\frac{1}{3}}\right)b}{3x^{\frac{14}{3}}}-\frac{4\left(a+b\,x^{\frac{1}{3}}\right)^{2}}{x^{5}}\right)}{56}-\frac{3x^{3}\left(\frac{2b^{2}}{9x^{\frac{16}{3}}}-\frac{52\left(a+b\,x^{\frac{1}{3}}\right)b}{9x^{\frac{17}{3}}}+\frac{20\left(a+b\,x^{\frac{1}{3}}\right)^{2}}{x^{6}}\right)}{56}$	90

input
```
int((a+b*x^(1/3))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3-3/4*a*b/x^(8/3)-3/7*b^2/x^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx=-\frac{36\,b^{2}x^{\frac{2}{3}}+63\,abx^{\frac{1}{3}}+28\,a^{2}}{84\,x^{3}}$$

input
```
integrate((a+b*x^(1/3))^2/x^4,x, algorithm="fricas")
```

output
```
-1/84*(36*b^2*x^(2/3) + 63*a*b*x^(1/3) + 28*a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx = -\frac{a^2}{3x^3} - \frac{3ab}{4x^{\frac{8}{3}}} - \frac{3b^2}{7x^{\frac{7}{3}}}$$

input
```
integrate((a+b*x**(1/3))**2/x**4,x)
```

output
```
-a**2/(3*x**3) - 3*a*b/(4*x**(8/3)) - 3*b**2/(7*x**(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx = -\frac{36\,b^2x^{\frac{2}{3}} + 63\,abx^{\frac{1}{3}} + 28\,a^2}{84\,x^3}$$

input
```
integrate((a+b*x^(1/3))^2/x^4,x, algorithm="maxima")
```

output
```
-1/84*(36*b^2*x^(2/3) + 63*a*b*x^(1/3) + 28*a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx = -\frac{36\,b^2x^{\frac{2}{3}} + 63\,abx^{\frac{1}{3}} + 28\,a^2}{84\,x^3}$$

input
```
integrate((a+b*x^(1/3))^2/x^4,x, algorithm="giac")
```

output
```
-1/84*(36*b^2*x^(2/3) + 63*a*b*x^(1/3) + 28*a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx=-\frac{28\,a^2+36\,b^2\,x^{2/3}+63\,a\,b\,x^{1/3}}{84\,x^3}$$

input
```
int((a + b*x^(1/3))^2/x^4,x)
```

output
```
-(28*a^2 + 36*b^2*x^(2/3) + 63*a*b*x^(1/3))/(84*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^2}{x^4}\,dx=\frac{-28x^{\frac{2}{3}}a^2-36x^{\frac{4}{3}}b^2-63abx}{84x^{\frac{11}{3}}}$$

input
```
int((a+b*x^(1/3))^2/x^4,x)
```

output
```
( - 28*x**(2/3)*a**2 - 36*x**(1/3)*b**2*x - 63*a*b*x)/(84*x**(2/3)*x**3)
```

3.194 $\int \left(a+b\sqrt[3]{x}\right)^3 x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^4 \, dx=\frac{a^3x^5}{5}+\frac{9}{16}a^2bx^{16/3}+\frac{9}{17}ab^2x^{17/3}+\frac{b^3x^6}{6}$$

output
```
1/5*a^3*x^5+9/16*a^2*b*x^(16/3)+9/17*a*b^2*x^(17/3)+1/6*b^3*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^4 \, dx=\frac{816a^3x^5+2295a^2bx^{16/3}+2160ab^2x^{17/3}+680b^3x^6}{4080}$$

input
```
Integrate[(a + b*x^(1/3))^3*x^4,x]
```

output
```
(816*a^3*x^5 + 2295*a^2*b*x^(16/3) + 2160*a*b^2*x^(17/3) + 680*b^3*x^6)/40
80
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b \sqrt[3]{x}\right)^3 dx$$

$$\Big\downarrow \text{798}$$

$$3 \int \left(a+b \sqrt[3]{x}\right)^3 x^{14/3} d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3 \int \left(b^3 x^{17/3}+3 a b^2 x^{16/3}+3 a^2 b x^5+a^3 x^{14/3}\right) d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3 \left(\frac{a^3 x^5}{15}+\frac{3}{16} a^2 b x^{16/3}+\frac{3}{17} a b^2 x^{17/3}+\frac{b^3 x^6}{18}\right)$$

input
```
Int[(a + b*x^(1/3))^3*x^4,x]
```

output
```
3*((a^3*x^5)/15 + (3*a^2*b*x^(16/3))/16 + (3*a*b^2*x^(17/3))/17 + (b^3*x^6
)/18)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^3x^5}{5}+\frac{9a^2b\,x^{\frac{16}{3}}}{16}+\frac{9a\,b^2x^{\frac{17}{3}}}{17}+\frac{b^3x^6}{6}$
default	$\frac{a^3x^5}{5}+\frac{9a^2b\,x^{\frac{16}{3}}}{16}+\frac{9a\,b^2x^{\frac{17}{3}}}{17}+\frac{b^3x^6}{6}$
trager	$\frac{(5b^3x^5+6a^3x^4+5b^3x^4+6a^3x^3+5b^3x^3+6a^3x^2+5b^3x^2+6a^3x+5b^3x+6a^3+5b^3)(-1+x)}{30}+\frac{9a^2b\,x^{\frac{16}{3}}}{16}+\frac{9a\,b^2x^{\frac{17}{3}}}{17}$
orering	$\frac{x^5(1805b^3x+1896a^3)\left(a+b\,x^{\frac{1}{3}}\right)^3}{4080b^3x+4080a^3}-\frac{3x^2(35b^3x+39a^3)\left(\left(a+b\,x^{\frac{1}{3}}\right)^2x^{\frac{10}{3}}b+4\left(a+b\,x^{\frac{1}{3}}\right)^3x^3\right)}{1360(b^3x+a^3)}+\frac{3x^3(5b^3x+6a^3)\left(\ldots\right.}{\ldots}$

input
```
int((a+b*x^(1/3))^3*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3*x^5+9/16*a^2*b*x^(16/3)+9/17*a*b^2*x^(17/3)+1/6*b^3*x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^4\,dx=\frac{1}{6}\,b^3x^6+\frac{9}{17}\,ab^2x^{\frac{17}{3}}+\frac{9}{16}\,a^2bx^{\frac{16}{3}}+\frac{1}{5}\,a^3x^5$$

input
```
integrate((a+b*x^(1/3))^3*x^4,x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 9/17*a*b^2*x^(17/3) + 9/16*a^2*b*x^(16/3) + 1/5*a^3*x^5
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^4\,dx=\frac{a^3x^5}{5}+\frac{9a^2bx^{\frac{16}{3}}}{16}+\frac{9ab^2x^{\frac{17}{3}}}{17}+\frac{b^3x^6}{6}$$

input
```
integrate((a+b*x**(1/3))**3*x**4,x)
```

output
```
a**3*x**5/5 + 9*a**2*b*x**(16/3)/16 + 9*a*b**2*x**(17/3)/17 + b**3*x**6/6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 5.34

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^3 x^4\,dx=&\frac{\left(bx^{\frac{1}{3}}+a\right)^{18}}{6\,b^{15}}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^{17}a}{17\,b^{15}}+\frac{273\left(bx^{\frac{1}{3}}+a\right)^{16}a^2}{16\,b^{15}}\\&-\frac{364\left(bx^{\frac{1}{3}}+a\right)^{15}a^3}{5\,b^{15}}+\frac{429\left(bx^{\frac{1}{3}}+a\right)^{14}a^4}{2\,b^{15}}-\frac{462\left(bx^{\frac{1}{3}}+a\right)^{13}a^5}{b^{15}}\\&+\frac{3003\left(bx^{\frac{1}{3}}+a\right)^{12}a^6}{4\,b^{15}}-\frac{936\left(bx^{\frac{1}{3}}+a\right)^{11}a^7}{b^{15}}\\&+\frac{9009\left(bx^{\frac{1}{3}}+a\right)^{10}a^8}{10\,b^{15}}-\frac{2002\left(bx^{\frac{1}{3}}+a\right)^{9}a^9}{3\,b^{15}}\\&+\frac{3003\left(bx^{\frac{1}{3}}+a\right)^{8}a^{10}}{8\,b^{15}}-\frac{156\left(bx^{\frac{1}{3}}+a\right)^{7}a^{11}}{b^{15}}\\&+\frac{91\left(bx^{\frac{1}{3}}+a\right)^{6}a^{12}}{2\,b^{15}}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^{5}a^{13}}{5\,b^{15}}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{4}a^{14}}{4\,b^{15}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^3*x^4,x, algorithm="maxima")
```

output
```
1/6*(b*x^(1/3) + a)^18/b^15 - 42/17*(b*x^(1/3) + a)^17*a/b^15 + 273/16*(b*
x^(1/3) + a)^16*a^2/b^15 - 364/5*(b*x^(1/3) + a)^15*a^3/b^15 + 429/2*(b*x^
(1/3) + a)^14*a^4/b^15 - 462*(b*x^(1/3) + a)^13*a^5/b^15 + 3003/4*(b*x^(1/
3) + a)^12*a^6/b^15 - 936*(b*x^(1/3) + a)^11*a^7/b^15 + 9009/10*(b*x^(1/3)
 + a)^10*a^8/b^15 - 2002/3*(b*x^(1/3) + a)^9*a^9/b^15 + 3003/8*(b*x^(1/3)
+ a)^8*a^10/b^15 - 156*(b*x^(1/3) + a)^7*a^11/b^15 + 91/2*(b*x^(1/3) + a)^
6*a^12/b^15 - 42/5*(b*x^(1/3) + a)^5*a^13/b^15 + 3/4*(b*x^(1/3) + a)^4*a^1
4/b^15
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^4\,dx=\frac{1}{6}\,b^3x^6+\frac{9}{17}\,ab^2x^{\frac{17}{3}}+\frac{9}{16}\,a^2bx^{\frac{16}{3}}+\frac{1}{5}\,a^3x^5$$

input
```
integrate((a+b*x^(1/3))^3*x^4,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 + 9/17*a*b^2*x^(17/3) + 9/16*a^2*b*x^(16/3) + 1/5*a^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^4\,dx=\frac{a^3\,x^5}{5}+\frac{b^3\,x^6}{6}+\frac{9\,a^2\,b\,x^{16/3}}{16}+\frac{9\,a\,b^2\,x^{17/3}}{17}$$

input
```
int(x^4*(a + b*x^(1/3))^3,x)
```

output
```
(a^3*x^5)/5 + (b^3*x^6)/6 + (9*a^2*b*x^(16/3))/16 + (9*a*b^2*x^(17/3))/17
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^4\,dx = \frac{x^5\left(2160x^{\frac{2}{3}}a\,b^2+2295x^{\frac{1}{3}}a^2b+816a^3+680b^3x\right)}{4080}$$

input
```
int((a+b*x^(1/3))^3*x^4,x)
```

output
```
(x**5*(2160*x**(2/3)*a*b**2 + 2295*x**(1/3)*a**2*b + 816*a**3 + 680*b**3*x
))/4080
```

3.195 $\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx=\frac{a^3x^4}{4}+\frac{9}{13}a^2bx^{13/3}+\frac{9}{14}ab^2x^{14/3}+\frac{b^3x^5}{5}$$

output
```
1/4*a^3*x^4+9/13*a^2*b*x^(13/3)+9/14*a*b^2*x^(14/3)+1/5*b^3*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx=\frac{455a^3x^4+1260a^2bx^{13/3}+1170ab^2x^{14/3}+364b^3x^5}{1820}$$

input
```
Integrate[(a + b*x^(1/3))^3*x^3,x]
```

output
```
(455*a^3*x^4 + 1260*a^2*b*x^(13/3) + 1170*a*b^2*x^(14/3) + 364*b^3*x^5)/18
20
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt[3]{x}\right)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^3 x^{11/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(b^3x^{14/3}+3ab^2x^{13/3}+3a^2bx^4+a^3x^{11/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^3x^4}{12}+\frac{3}{13}a^2bx^{13/3}+\frac{3}{14}ab^2x^{14/3}+\frac{b^3x^5}{15}\right)$$

input
```
Int[(a + b*x^(1/3))^3*x^3,x]
```

output
```
3*((a^3*x^4)/12 + (3*a^2*b*x^(13/3))/13 + (3*a*b^2*x^(14/3))/14 + (b^3*x^5
)/15)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^3x^4}{4}+\frac{9a^2b\,x^{\frac{13}{3}}}{13}+\frac{9a\,b^2x^{\frac{14}{3}}}{14}+\frac{b^3x^5}{5}$
default	$\frac{a^3x^4}{4}+\frac{9a^2b\,x^{\frac{13}{3}}}{13}+\frac{9a\,b^2x^{\frac{14}{3}}}{14}+\frac{b^3x^5}{5}$
trager	$\frac{(4b^3x^4+5a^3x^3+4b^3x^3+5a^3x^2+4b^3x^2+5a^3x+4b^3x+5a^3+4b^3)(-1+x)}{20}+\frac{9a^2b\,x^{\frac{13}{3}}}{13}+\frac{9a\,b^2x^{\frac{14}{3}}}{14}$
orering	$\frac{x^4(188b^3x+199a^3)\left(a+b\,x^{\frac{1}{3}}\right)^3}{364b^3x+364a^3}-\frac{9x^2(22b^3x+25a^3)\left(\left(a+b\,x^{\frac{1}{3}}\right)^2x^{\frac{7}{3}}b+3\left(a+b\,x^{\frac{1}{3}}\right)^3x^2\right)}{1820(b^3x+a^3)}+\frac{9x^3(4b^3x+5a^3)\left(\frac{2\left(a\right.}{}\right.}{}$

input
```
int((a+b*x^(1/3))^3*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*x^4+9/13*a^2*b*x^(13/3)+9/14*a*b^2*x^(14/3)+1/5*b^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int\left(a+b\sqrt[3]{x}\right)^3x^3\,dx=\frac{1}{5}\,b^3x^5+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{13}\,a^2bx^{\frac{13}{3}}+\frac{1}{4}\,a^3x^4$$

input
```
integrate((a+b*x^(1/3))^3*x^3,x, algorithm="fricas")
```

output
```
1/5*b^3*x^5 + 9/14*a*b^2*x^(14/3) + 9/13*a^2*b*x^(13/3) + 1/4*a^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx=\frac{a^3x^4}{4}+\frac{9a^2bx^{\frac{13}{3}}}{13}+\frac{9ab^2x^{\frac{14}{3}}}{14}+\frac{b^3x^5}{5}$$

input
```
integrate((a+b*x**(1/3))**3*x**3,x)
```

output
```
a**3*x**4/4 + 9*a**2*b*x**(13/3)/13 + 9*a*b**2*x**(14/3)/14 + b**3*x**5/5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.26

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx=&\frac{\left(bx^{\frac{1}{3}}+a\right)^{15}}{5\,b^{12}}-\frac{33\left(bx^{\frac{1}{3}}+a\right)^{14}a}{14\,b^{12}}+\frac{165\left(bx^{\frac{1}{3}}+a\right)^{13}a^2}{13\,b^{12}}\\&-\frac{165\left(bx^{\frac{1}{3}}+a\right)^{12}a^3}{4\,b^{12}}+\frac{90\left(bx^{\frac{1}{3}}+a\right)^{11}a^4}{b^{12}}-\frac{693\left(bx^{\frac{1}{3}}+a\right)^{10}a^5}{5\,b^{12}}\\&+\frac{154\left(bx^{\frac{1}{3}}+a\right)^{9}a^6}{b^{12}}-\frac{495\left(bx^{\frac{1}{3}}+a\right)^{8}a^7}{4\,b^{12}}+\frac{495\left(bx^{\frac{1}{3}}+a\right)^{7}a^8}{7\,b^{12}}\\&-\frac{55\left(bx^{\frac{1}{3}}+a\right)^{6}a^9}{2\,b^{12}}+\frac{33\left(bx^{\frac{1}{3}}+a\right)^{5}a^{10}}{5\,b^{12}}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^{4}a^{11}}{4\,b^{12}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^3*x^3,x, algorithm="maxima")
```

output
```
1/5*(b*x^(1/3) + a)^15/b^12 - 33/14*(b*x^(1/3) + a)^14*a/b^12 + 165/13*(b*
x^(1/3) + a)^13*a^2/b^12 - 165/4*(b*x^(1/3) + a)^12*a^3/b^12 + 90*(b*x^(1/
3) + a)^11*a^4/b^12 - 693/5*(b*x^(1/3) + a)^10*a^5/b^12 + 154*(b*x^(1/3) +
 a)^9*a^6/b^12 - 495/4*(b*x^(1/3) + a)^8*a^7/b^12 + 495/7*(b*x^(1/3) + a)^
7*a^8/b^12 - 55/2*(b*x^(1/3) + a)^6*a^9/b^12 + 33/5*(b*x^(1/3) + a)^5*a^10
/b^12 - 3/4*(b*x^(1/3) + a)^4*a^11/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx = \frac{1}{5}\,b^3x^5 + \frac{9}{14}\,ab^2x^{\frac{14}{3}} + \frac{9}{13}\,a^2bx^{\frac{13}{3}} + \frac{1}{4}\,a^3x^4$$

input
```
integrate((a+b*x^(1/3))^3*x^3,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 + 9/14*a*b^2*x^(14/3) + 9/13*a^2*b*x^(13/3) + 1/4*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx = \frac{a^3\,x^4}{4} + \frac{b^3\,x^5}{5} + \frac{9\,a^2\,b\,x^{13/3}}{13} + \frac{9\,a\,b^2\,x^{14/3}}{14}$$

input
```
int(x^3*(a + b*x^(1/3))^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^5)/5 + (9*a^2*b*x^(13/3))/13 + (9*a*b^2*x^(14/3))/14
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^3\,dx = \frac{x^4\left(1170x^{\frac{2}{3}}a\,b^2 + 1260x^{\frac{1}{3}}a^2b + 455a^3 + 364b^3x\right)}{1820}$$

input
```
int((a+b*x^(1/3))^3*x^3,x)
```

output
```
(x**4*(1170*x**(2/3)*a*b**2 + 1260*x**(1/3)*a**2*b + 455*a**3 + 364*b**3*x
))/1820
```

3.196 $\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx=\frac{a^3x^3}{3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{11}ab^2x^{11/3}+\frac{b^3x^4}{4}$$

output
```
1/3*a^3*x^3+9/10*a^2*b*x^(10/3)+9/11*a*b^2*x^(11/3)+1/4*b^3*x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx=\frac{1}{660}x^3\left(220a^3+594a^2b\sqrt[3]{x}+540ab^2x^{2/3}+165b^3x\right)$$

input
```
Integrate[(a + b*x^(1/3))^3*x^2,x]
```

output
```
(x^3*(220*a^3 + 594*a^2*b*x^(1/3) + 540*a*b^2*x^(2/3) + 165*b^3*x))/660
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\sqrt[3]{x}\right)^3 \, dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^3 x^{8/3} d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(b^3x^{11/3}+3ab^2x^{10/3}+3a^2bx^3+a^3x^{8/3}\right) d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^3x^3}{9}+\frac{3}{10}a^2bx^{10/3}+\frac{3}{11}ab^2x^{11/3}+\frac{b^3x^4}{12}\right)$$

input
```
Int[(a + b*x^(1/3))^3*x^2,x]
```

output
```
3*((a^3*x^3)/9 + (3*a^2*b*x^(10/3))/10 + (3*a*b^2*x^(11/3))/11 + (b^3*x^4)
/12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^3x^3}{3}+\frac{9a^2b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^2x^{\frac{11}{3}}}{11}+\frac{b^3x^4}{4}$
default	$\frac{a^3x^3}{3}+\frac{9a^2b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^2x^{\frac{11}{3}}}{11}+\frac{b^3x^4}{4}$
trager	$\frac{(3b^3x^3+4a^3x^2+3b^3x^2+4a^3x+3b^3x+4a^3+3b^3)(-1+x)}{12}+\frac{9a^2b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^2x^{\frac{11}{3}}}{11}$
orering	$\frac{x^3(102b^3x+109a^3)\left(a+b\,x^{\frac{1}{3}}\right)^3}{165b^3x+165a^3}-\frac{3x^2(6b^3x+7a^3)\left(\left(a+b\,x^{\frac{1}{3}}\right)^2x^{\frac{4}{3}}b+2\left(a+b\,x^{\frac{1}{3}}\right)^3x\right)}{110(b^3x+a^3)}+\frac{3x^3(3b^3x+4a^3)\left(\frac{2\left(a+b\,x\right.}{}\right.}{}$

input
```
int((a+b*x^(1/3))^3*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+9/10*a^2*b*x^(10/3)+9/11*a*b^2*x^(11/3)+1/4*b^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3x^2\,dx=\frac{1}{4}\,b^3x^4+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{10}\,a^2bx^{\frac{10}{3}}+\frac{1}{3}\,a^3x^3$$

input
```
integrate((a+b*x^(1/3))^3*x^2,x, algorithm="fricas")
```

output
```
1/4*b^3*x^4 + 9/11*a*b^2*x^(11/3) + 9/10*a^2*b*x^(10/3) + 1/3*a^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx=\frac{a^3x^3}{3}+\frac{9a^2bx^{\frac{10}{3}}}{10}+\frac{9ab^2x^{\frac{11}{3}}}{11}+\frac{b^3x^4}{4}$$

input
```
integrate((a+b*x**(1/3))**3*x**2,x)
```

output
```
a**3*x**3/3 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(11/3)/11 + b**3*x**4/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.17

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx={}&\frac{\left(bx^{\frac{1}{3}}+a\right)^{12}}{4\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{11}a}{11\,b^9}+\frac{42\left(bx^{\frac{1}{3}}+a\right)^{10}a^2}{5\,b^9}\\&-\frac{56\left(bx^{\frac{1}{3}}+a\right)^{9}a^3}{3\,b^9}+\frac{105\left(bx^{\frac{1}{3}}+a\right)^{8}a^4}{4\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{7}a^5}{b^9}\\&+\frac{14\left(bx^{\frac{1}{3}}+a\right)^{6}a^6}{b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{5}a^7}{5\,b^9}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{4}a^8}{4\,b^9}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^3*x^2,x, algorithm="maxima")
```

output
```
1/4*(b*x^(1/3) + a)^12/b^9 - 24/11*(b*x^(1/3) + a)^11*a/b^9 + 42/5*(b*x^(1
/3) + a)^10*a^2/b^9 - 56/3*(b*x^(1/3) + a)^9*a^3/b^9 + 105/4*(b*x^(1/3) +
a)^8*a^4/b^9 - 24*(b*x^(1/3) + a)^7*a^5/b^9 + 14*(b*x^(1/3) + a)^6*a^6/b^9
 - 24/5*(b*x^(1/3) + a)^5*a^7/b^9 + 3/4*(b*x^(1/3) + a)^4*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx=\frac{1}{4}\,b^3x^4+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{10}\,a^2bx^{\frac{10}{3}}+\frac{1}{3}\,a^3x^3$$

input
```
integrate((a+b*x^(1/3))^3*x^2,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 + 9/11*a*b^2*x^(11/3) + 9/10*a^2*b*x^(10/3) + 1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx=\frac{a^3\,x^3}{3}+\frac{b^3\,x^4}{4}+\frac{9\,a^2\,b\,x^{10/3}}{10}+\frac{9\,a\,b^2\,x^{11/3}}{11}$$

input
```
int(x^2*(a + b*x^(1/3))^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^4)/4 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(11/3))/11
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x^2\,dx=\frac{x^3\left(540x^{\frac{2}{3}}a\,b^2+594x^{\frac{1}{3}}a^2b+220a^3+165b^3x\right)}{660}$$

input
```
int((a+b*x^(1/3))^3*x^2,x)
```

output
```
(x**3*(540*x**(2/3)*a*b**2 + 594*x**(1/3)*a**2*b + 220*a**3 + 165*b**3*x))
/660
```

3.197 $\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx=\frac{a^3x^2}{2}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{8}ab^2x^{8/3}+\frac{b^3x^3}{3}$$

output
```
1/2*a^3*x^2+9/7*a^2*b*x^(7/3)+9/8*a*b^2*x^(8/3)+1/3*b^3*x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx=\frac{1}{168}\left(84a^3x^2+216a^2bx^{7/3}+189ab^2x^{8/3}+56b^3x^3\right)$$

input
```
Integrate[(a + b*x^(1/3))^3*x,x]
```

output
```
(84*a^3*x^2 + 216*a^2*b*x^(7/3) + 189*a*b^2*x^(8/3) + 56*b^3*x^3)/168
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt[3]{x}\right)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^3 x^{5/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x^{5/3}a^3+3bx^2a^2+3b^2x^{7/3}a+b^3x^{8/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^3x^2}{6}+\frac{3}{7}a^2bx^{7/3}+\frac{3}{8}ab^2x^{8/3}+\frac{b^3x^3}{9}\right)$$

input
```
Int[(a + b*x^(1/3))^3*x,x]
```

output
```
3*((a^3*x^2)/6 + (3*a^2*b*x^(7/3))/7 + (3*a*b^2*x^(8/3))/8 + (b^3*x^3)/9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^3x^2}{2}+\frac{9a^2b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^2x^{\frac{8}{3}}}{8}+\frac{b^3x^3}{3}$
default	$\frac{a^3x^2}{2}+\frac{9a^2b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^2x^{\frac{8}{3}}}{8}+\frac{b^3x^3}{3}$
trager	$\frac{(2b^3x^2+3a^3x+2b^3x+3a^3+2b^3)(-1+x)}{6}+\frac{9a^2b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^2x^{\frac{8}{3}}}{8}$
orering	$\frac{x^2(64b^3x+69a^3)\left(a+b\,x^{\frac{1}{3}}\right)^3}{84b^3x+84a^3}-\frac{3x^2(5b^3x+6a^3)\left(\left(a+b\,x^{\frac{1}{3}}\right)^2x^{\frac{1}{3}}b+\left(a+b\,x^{\frac{1}{3}}\right)^3\right)}{56(b^3x+a^3)}+\frac{3x^3(2b^3x+3a^3)\left(\frac{2\left(a+b\,x^{\frac{1}{3}}\right)b}{3x^{\frac{1}{3}}}\right.}{112(b^3x+a^3}$

input
```
int((a+b*x^(1/3))^3*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+9/7*a^2*b*x^(7/3)+9/8*a*b^2*x^(8/3)+1/3*b^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3x\,dx=\frac{1}{3}\,b^3x^3+\frac{9}{8}\,ab^2x^{\frac{8}{3}}+\frac{9}{7}\,a^2bx^{\frac{7}{3}}+\frac{1}{2}\,a^3x^2$$

input
```
integrate((a+b*x^(1/3))^3*x,x, algorithm="fricas")
```

output
```
1/3*b^3*x^3 + 9/8*a*b^2*x^(8/3) + 9/7*a^2*b*x^(7/3) + 1/2*a^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx=\frac{a^3x^2}{2}+\frac{9a^2bx^{\frac{7}{3}}}{7}+\frac{9ab^2x^{\frac{8}{3}}}{8}+\frac{b^3x^3}{3}$$

input
```
integrate((a+b*x**(1/3))**3*x,x)
```

output
```
a**3*x**2/2 + 9*a**2*b*x**(7/3)/7 + 9*a*b**2*x**(8/3)/8 + b**3*x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(35) = 70.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

$$\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx=\frac{\left(bx^{\frac{1}{3}}+a\right)^9}{3\,b^6}-\frac{15\left(bx^{\frac{1}{3}}+a\right)^8a}{8\,b^6}+\frac{30\left(bx^{\frac{1}{3}}+a\right)^7a^2}{7\,b^6}-\frac{5\left(bx^{\frac{1}{3}}+a\right)^6a^3}{b^6}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^5a^4}{b^6}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^4a^5}{4\,b^6}$$

input
```
integrate((a+b*x^(1/3))^3*x,x, algorithm="maxima")
```

output
```
1/3*(b*x^(1/3) + a)^9/b^6 - 15/8*(b*x^(1/3) + a)^8*a/b^6 + 30/7*(b*x^(1/3)
 + a)^7*a^2/b^6 - 5*(b*x^(1/3) + a)^6*a^3/b^6 + 3*(b*x^(1/3) + a)^5*a^4/b^
6 - 3/4*(b*x^(1/3) + a)^4*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx = \frac{1}{3}\,b^3x^3 + \frac{9}{8}\,ab^2x^{\frac{8}{3}} + \frac{9}{7}\,a^2bx^{\frac{7}{3}} + \frac{1}{2}\,a^3x^2$$

input
```
integrate((a+b*x^(1/3))^3*x,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 + 9/8*a*b^2*x^(8/3) + 9/7*a^2*b*x^(7/3) + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx = \frac{a^3\,x^2}{2} + \frac{b^3\,x^3}{3} + \frac{9\,a^2\,b\,x^{7/3}}{7} + \frac{9\,a\,b^2\,x^{8/3}}{8}$$

input
```
int(x*(a + b*x^(1/3))^3,x)
```

output
```
(a^3*x^2)/2 + (b^3*x^3)/3 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(8/3))/8
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^3 x\,dx = \frac{x^2\left(189x^{\frac{2}{3}}a\,b^2 + 216x^{\frac{1}{3}}a^2b + 84a^3 + 56b^3x\right)}{168}$$

input
```
int((a+b*x^(1/3))^3*x,x)
```

output
```
(x**2*(189*x**(2/3)*a*b**2 + 216*x**(1/3)*a**2*b + 84*a**3 + 56*b**3*x))/1
68
```

3.198 $\int \left(a+b\sqrt[3]{x}\right)^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \left(a+b\sqrt[3]{x}\right)^3 \, dx = a^3x+\frac{9}{4}a^2bx^{4/3}+\frac{9}{5}ab^2x^{5/3}+\frac{b^3x^2}{2}$$

output
```
a^3*x+9/4*a^2*b*x^(4/3)+9/5*a*b^2*x^(5/3)+1/2*b^3*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \left(a+b\sqrt[3]{x}\right)^3 \, dx = \frac{1}{20}\left(20a^3x+45a^2bx^{4/3}+36ab^2x^{5/3}+10b^3x^2\right)$$

input
```
Integrate[(a + b*x^(1/3))^3,x]
```

output
```
(20*a^3*x + 45*a^2*b*x^(4/3) + 36*a*b^2*x^(5/3) + 10*b^3*x^2)/20
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}\right)^3 \, dx$$

$$\Big\downarrow \text{774}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^3 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x^{2/3}a^3+3bxa^2+3b^2x^{4/3}a+b^3x^{5/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^3x}{3}+\frac{3}{4}a^2bx^{4/3}+\frac{3}{5}ab^2x^{5/3}+\frac{b^3x^2}{6}\right)$$

input
```
Int[(a + b*x^(1/3))^3,x]
```

output
```
3*((a^3*x)/3 + (3*a^2*b*x^(4/3))/4 + (3*a*b^2*x^(5/3))/5 + (b^3*x^2)/6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$a^{3}x+\frac{9a^{2}b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^{2}x^{\frac{5}{3}}}{5}+\frac{b^{3}x^{2}}{2}$	33
default	$a^{3}x+\frac{9a^{2}b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^{2}x^{\frac{5}{3}}}{5}+\frac{b^{3}x^{2}}{2}$	33
trager	$\frac{(-1+x)(b^{3}x+2a^{3}+b^{3})}{2}+\frac{9a^{2}b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^{2}x^{\frac{5}{3}}}{5}$	39
orering	$\frac{x(19b^{3}x+20a^{3})\left(a+b\,x^{\frac{1}{3}}\right)^{3}}{20b^{3}x+20a^{3}}-\frac{9\left(a+b\,x^{\frac{1}{3}}\right)^{2}x^{\frac{4}{3}}b}{20}+\frac{9x^{3}(b^{3}x+2a^{3})\left(\frac{2\left(a+b\,x^{\frac{1}{3}}\right)b^{2}}{3x^{\frac{4}{3}}}-\frac{2\left(a+b\,x^{\frac{1}{3}}\right)^{2}b}{3x^{\frac{5}{3}}}\right)}{40(b^{3}x+a^{3})}$	110

input
```
int((a+b*x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+9/4*a^2*b*x^(4/3)+9/5*a*b^2*x^(5/3)+1/2*b^3*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^{3}\,dx=\frac{1}{2}\,b^{3}x^{2}+\frac{9}{5}\,ab^{2}x^{\frac{5}{3}}+\frac{9}{4}\,a^{2}bx^{\frac{4}{3}}+a^{3}x$$

input
```
integrate((a+b*x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}\right)^3\,dx=a^3x+\frac{9a^2bx^{\frac{4}{3}}}{4}+\frac{9ab^2x^{\frac{5}{3}}}{5}+\frac{b^3x^2}{2}$$

input
```
integrate((a+b*x**(1/3))**3,x)
```

output
```
a**3*x + 9*a**2*b*x**(4/3)/4 + 9*a*b**2*x**(5/3)/5 + b**3*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^3\,dx=\frac{1}{2}\,b^3x^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

input
```
integrate((a+b*x^(1/3))^3,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^3\,dx=\frac{1}{2}\,b^3x^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

input
```
integrate((a+b*x^(1/3))^3,x, algorithm="giac")
```

output
```
1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^3\,dx=a^3\,x+\frac{b^3\,x^2}{2}+\frac{9\,a^2\,b\,x^{4/3}}{4}+\frac{9\,a\,b^2\,x^{5/3}}{5}$$

input
```
int((a + b*x^(1/3))^3,x)
```

output
```
a^3*x + (b^3*x^2)/2 + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(5/3))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right)^3\,dx=\frac{x\left(36x^{\frac{2}{3}}a\,b^2+45x^{\frac{1}{3}}a^2b+20a^3+10b^3x\right)}{20}$$

input
```
int((a+b*x^(1/3))^3,x)
```

output
```
(x*(36*x**(2/3)*a*b**2 + 45*x**(1/3)*a**2*b + 20*a**3 + 10*b**3*x))/20
```

3.199 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x} dx = 9a^2b\sqrt[3]{x}+\frac{9}{2}ab^2x^{2/3}+b^3x+a^3\log(x)$$

output
```
9*a^2*b*x^(1/3)+9/2*a*b^2*x^(2/3)+b^3*x+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x} dx = 9a^2b\sqrt[3]{x}+\frac{9}{2}ab^2x^{2/3}+b^3x+a^3\log(x)$$

input
```
Integrate[(a + b*x^(1/3))^3/x,x]
```

output
```
9*a^2*b*x^(1/3) + (9*a*b^2*x^(2/3))/2 + b^3*x + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^3}{\sqrt[3]{x}}+3ba^2+3b^2\sqrt[3]{x}a+b^3x^{2/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(a^3\log\left(\sqrt[3]{x}\right)+3a^2b\sqrt[3]{x}+\frac{3}{2}ab^2x^{2/3}+\frac{b^3x}{3}\right)$$

input
```
Int[(a + b*x^(1/3))^3/x,x]
```

output
```
3*(3*a^2*b*x^(1/3) + (3*a*b^2*x^(2/3))/2 + (b^3*x)/3 + a^3*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$9a^2 b\, x^{\frac{1}{3}} + \frac{9 a\, b^2 x^{\frac{2}{3}}}{2} + b^3 x + a^3 \ln\left(x\right)$	31
default	$9a^2 b\, x^{\frac{1}{3}} + \frac{9 a\, b^2 x^{\frac{2}{3}}}{2} + b^3 x + a^3 \ln\left(x\right)$	31
trager	$b^3(-1+x) + 9a^2 b\, x^{\frac{1}{3}} + \frac{9 a\, b^2 x^{\frac{2}{3}}}{2} + a^3 \ln\left(x\right)$	33

input
```
int((a+b*x^(1/3))^3/x,x,method=_RETURNVERBOSE)
```

output
```
9*a^2*b*x^(1/3)+9/2*a*b^2*x^(2/3)+b^3*x+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\, dx = b^3 x + 3\, a^3 \log\left(x^{\frac{1}{3}}\right) + \frac{9}{2}\, a b^2 x^{\frac{2}{3}} + 9\, a^2 b x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^3/x,x, algorithm="fricas")
```

output
```
b^3*x + 3*a^3*log(x^(1/3)) + 9/2*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx = a^3\log\left(x\right)+9a^2b\sqrt[3]{x}+\frac{9ab^2x^{\frac{2}{3}}}{2}+b^3x$$

input
```
integrate((a+b*x**(1/3))**3/x,x)
```

output
```
a**3*log(x) + 9*a**2*b*x**(1/3) + 9*a*b**2*x**(2/3)/2 + b**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx = b^3x+a^3\log\left(x\right)+\frac{9}{2}\,ab^2x^{\frac{2}{3}}+9\,a^2bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^3/x,x, algorithm="maxima")
```

output
```
b^3*x + a^3*log(x) + 9/2*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx = b^3x+a^3\log\left(|x|\right)+\frac{9}{2}\,ab^2x^{\frac{2}{3}}+9\,a^2bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^3/x,x, algorithm="giac")
```

output
```
b^3*x + a^3*log(abs(x)) + 9/2*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx = 3\,a^3\,\ln\left(x^{1/3}\right)+b^3\,x+9\,a^2\,b\,x^{1/3}+\frac{9\,a\,b^2\,x^{2/3}}{2}$$

input
```
int((a + b*x^(1/3))^3/x,x)
```

output
```
3*a^3*log(x^(1/3)) + b^3*x + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(2/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x}\,dx = \frac{9x^{\frac{2}{3}}a\,b^2}{2}+9x^{\frac{1}{3}}a^2b+\log(x)\,a^3+b^3x$$

input
```
int((a+b*x^(1/3))^3/x,x)
```

output
```
(9*x**(2/3)*a*b**2 + 18*x**(1/3)*a**2*b + 2*log(x)*a**3 + 2*b**3*x)/2
```

3.200 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx=-\frac{a^3}{x}-\frac{9a^2b}{2x^{2/3}}-\frac{9ab^2}{\sqrt[3]{x}}+b^3\log(x)$$

output
```
-a^3/x-9/2*a^2*b/x^(2/3)-9*a*b^2/x^(1/3)+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx=-\frac{a\left(2a^2+9ab\sqrt[3]{x}+18b^2x^{2/3}\right)}{2x}+b^3\log(x)$$

input
```
Integrate[(a + b*x^(1/3))^3/x^2,x]
```

output
```
-1/2*(a*(2*a^2 + 9*a*b*x^(1/3) + 18*b^2*x^(2/3)))/x + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^3}{x^{4/3}}+\frac{3ba^2}{x}+\frac{3b^2a}{x^{2/3}}+\frac{b^3}{\sqrt[3]{x}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^3}{3x}-\frac{3a^2b}{2x^{2/3}}-\frac{3ab^2}{\sqrt[3]{x}}+b^3\log\left(\sqrt[3]{x}\right)\right)$$

input
```
Int[(a + b*x^(1/3))^3/x^2,x]
```

output
```
3*(-1/3*a^3/x - (3*a^2*b)/(2*x^(2/3)) - (3*a*b^2)/x^(1/3) + b^3*Log[x^(1/3
)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{a^3}{x}-\frac{9a^2b}{2x^{\frac{2}{3}}}-\frac{9a\,b^2}{x^{\frac{1}{3}}}+b^3\ln\left(x\right)$	34
default	$-\frac{a^3}{x}-\frac{9a^2b}{2x^{\frac{2}{3}}}-\frac{9a\,b^2}{x^{\frac{1}{3}}}+b^3\ln\left(x\right)$	34
trager	$\frac{a^3(-1+x)}{x}-\frac{9a^2b}{2x^{\frac{2}{3}}}-\frac{9a\,b^2}{x^{\frac{1}{3}}}+b^3\ln\left(x\right)$	36

input
```
int((a+b*x^(1/3))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x-9/2*a^2*b/x^(2/3)-9*a*b^2/x^(1/3)+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx=\frac{6\,b^3x\log\left(x^{\frac{1}{3}}\right)-18\,ab^2x^{\frac{2}{3}}-9\,a^2bx^{\frac{1}{3}}-2\,a^3}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^3/x^2,x, algorithm="fricas")
```

output
```
1/2*(6*b^3*x*log(x^(1/3)) - 18*a*b^2*x^(2/3) - 9*a^2*b*x^(1/3) - 2*a^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx=-\frac{a^3}{x}-\frac{9a^2b}{2x^{\frac{2}{3}}}-\frac{9ab^2}{\sqrt[3]{x}}+b^3\log\left(x\right)$$

input
```
integrate((a+b*x**(1/3))**3/x**2,x)
```

output
```
-a**3/x - 9*a**2*b/(2*x**(2/3)) - 9*a*b**2/x**(1/3) + b**3*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx=b^3\log\left(x\right)-\frac{18\,ab^2x^{\frac{2}{3}}+9\,a^2bx^{\frac{1}{3}}+2\,a^3}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^3/x^2,x, algorithm="maxima")
```

output
```
b^3*log(x) - 1/2*(18*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3) + 2*a^3)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx=b^3\log\left(|x|\right)-\frac{18\,ab^2x^{\frac{2}{3}}+9\,a^2bx^{\frac{1}{3}}+2\,a^3}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^3/x^2,x, algorithm="giac")
```

output
```
b^3*log(abs(x)) - 1/2*(18*a*b^2*x^(2/3) + 9*a^2*b*x^(1/3) + 2*a^3)/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx = 3\,b^3\,\ln\left(x^{1/3}\right)-\frac{a^3+\frac{9\,a^2\,b\,x^{1/3}}{2}+9\,a\,b^2\,x^{2/3}}{x}$$

input
```
int((a + b*x^(1/3))^3/x^2,x)
```

output
```
3*b^3*log(x^(1/3)) - (a^3 + (9*a^2*b*x^(1/3))/2 + 9*a*b^2*x^(2/3))/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^2}\,dx = \frac{6x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)b^3-2x^{\frac{2}{3}}a^3-18x^{\frac{4}{3}}a\,b^2-9a^2bx}{2x^{\frac{5}{3}}}$$

input
```
int((a+b*x^(1/3))^3/x^2,x)
```

output
```
(6*x**(2/3)*log(x**(1/3))*b**3*x - 2*x**(2/3)*a**3 - 18*x**(1/3)*a*b**2*x
- 9*a**2*b*x)/(2*x**(2/3)*x)
```

3.201 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx=-\frac{a^3}{2x^2}-\frac{9a^2b}{5x^{5/3}}-\frac{9ab^2}{4x^{4/3}}-\frac{b^3}{x}$$

output
```
-1/2*a^3/x^2-9/5*a^2*b/x^(5/3)-9/4*a*b^2/x^(4/3)-b^3/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx=\frac{-10a^3-36a^2b\sqrt[3]{x}-45ab^2x^{2/3}-20b^3x}{20x^2}$$

input
```
Integrate[(a + b*x^(1/3))^3/x^3,x]
```

output
```
(-10*a^3 - 36*a^2*b*x^(1/3) - 45*a*b^2*x^(2/3) - 20*b^3*x)/(20*x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}dx$$

$$\Big\downarrow \text{ 798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 53}$$

$$3\int \left(\frac{a^3}{x^{7/3}}+\frac{3ba^2}{x^2}+\frac{3b^2a}{x^{5/3}}+\frac{b^3}{x^{4/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(-\frac{a^3}{6x^2}-\frac{3a^2b}{5x^{5/3}}-\frac{3ab^2}{4x^{4/3}}-\frac{b^3}{3x}\right)$$

input
```
Int[(a + b*x^(1/3))^3/x^3,x]
```

output
```
3*(-1/6*a^3/x^2 - (3*a^2*b)/(5*x^(5/3)) - (3*a*b^2)/(4*x^(4/3)) - b^3/(3*x
))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{a^{3}}{2x^{2}}-\frac{9a^{2}b}{5x^{\frac{5}{3}}}-\frac{9a\,b^{2}}{4x^{\frac{4}{3}}}-\frac{b^{3}}{x}$
default	$-\frac{a^{3}}{2x^{2}}-\frac{9a^{2}b}{5x^{\frac{5}{3}}}-\frac{9a\,b^{2}}{4x^{\frac{4}{3}}}-\frac{b^{3}}{x}$
trager	$\frac{(-1+x)(a^{3}x+2b^{3}x+a^{3})}{2x^{2}}-\frac{9a^{2}b}{5x^{\frac{5}{3}}}-\frac{9a\,b^{2}}{4x^{\frac{4}{3}}}$
orering	$-\frac{(23b^{3}x+16a^{3})\left(a+b\,x^{\frac{1}{3}}\right)^{3}}{5x^{2}(b^{3}x+a^{3})}-\frac{9x^{2}(7b^{3}x+4a^{3})\left(\frac{\left(a+b\,x^{\frac{1}{3}}\right)^{2}b}{x^{\frac{11}{3}}}-\frac{3\left(a+b\,x^{\frac{1}{3}}\right)^{3}}{x^{4}}\right)}{20(b^{3}x+a^{3})}-\frac{9(2b^{3}x+a^{3})x^{3}\left(\frac{2\left(a+b\,x^{\frac{1}{3}}\right)b^{2}}{3x^{\frac{13}{3}}}-\cdots\right.}{40(b^{3}\cdots}$

input
```
int((a+b*x^(1/3))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2-9/5*a^2*b/x^(5/3)-9/4*a*b^2/x^(4/3)-b^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx=-\frac{20\,b^3x+45\,ab^2x^{\frac{2}{3}}+36\,a^2bx^{\frac{1}{3}}+10\,a^3}{20\,x^2}$$

input
```
integrate((a+b*x^(1/3))^3/x^3,x, algorithm="fricas")
```

output
```
-1/20*(20*b^3*x + 45*a*b^2*x^(2/3) + 36*a^2*b*x^(1/3) + 10*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx = -\frac{a^3}{2x^2} - \frac{9a^2b}{5x^{\frac{5}{3}}} - \frac{9ab^2}{4x^{\frac{4}{3}}} - \frac{b^3}{x}$$

input
```
integrate((a+b*x**(1/3))**3/x**3,x)
```

output
```
-a**3/(2*x**2) - 9*a**2*b/(5*x**(5/3)) - 9*a*b**2/(4*x**(4/3)) - b**3/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx = -\frac{20\,b^3x + 45\,ab^2x^{\frac{2}{3}} + 36\,a^2bx^{\frac{1}{3}} + 10\,a^3}{20\,x^2}$$

input
```
integrate((a+b*x^(1/3))^3/x^3,x, algorithm="maxima")
```

output
```
-1/20*(20*b^3*x + 45*a*b^2*x^(2/3) + 36*a^2*b*x^(1/3) + 10*a^3)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx = -\frac{20\,b^3x + 45\,ab^2x^{\frac{2}{3}} + 36\,a^2bx^{\frac{1}{3}} + 10\,a^3}{20\,x^2}$$

input
```
integrate((a+b*x^(1/3))^3/x^3,x, algorithm="giac")
```

output
```
-1/20*(20*b^3*x + 45*a*b^2*x^(2/3) + 36*a^2*b*x^(1/3) + 10*a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx=-\frac{20\,b^3\,x+10\,a^3+36\,a^2\,b\,x^{1/3}+45\,a\,b^2\,x^{2/3}}{20\,x^2}$$

input
```
int((a + b*x^(1/3))^3/x^3,x)
```

output
```
-(20*b^3*x + 10*a^3 + 36*a^2*b*x^(1/3) + 45*a*b^2*x^(2/3))/(20*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^3}\,dx=\frac{-10x^{\frac{2}{3}}a^3-20x^{\frac{5}{3}}b^3-45x^{\frac{4}{3}}a\,b^2-36a^2bx}{20x^{\frac{8}{3}}}$$

input
```
int((a+b*x^(1/3))^3/x^3,x)
```

output
```
( - 10*x**(2/3)*a**3 - 20*x**(2/3)*b**3*x - 45*x**(1/3)*a*b**2*x - 36*a**2
*b*x)/(20*x**(2/3)*x**2)
```

3.202 $\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4} dx = -\frac{a^3}{3x^3} - \frac{9a^2b}{8x^{8/3}} - \frac{9ab^2}{7x^{7/3}} - \frac{b^3}{2x^2}$$

output
```
-1/3*a^3/x^3-9/8*a^2*b/x^(8/3)-9/7*a*b^2/x^(7/3)-1/2*b^3/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4} dx = \frac{-56a^3 - 189a^2b\sqrt[3]{x} - 216ab^2x^{2/3} - 84b^3x}{168x^3}$$

input
```
Integrate[(a + b*x^(1/3))^3/x^4,x]
```

output
```
(-56*a^3 - 189*a^2*b*x^(1/3) - 216*a*b^2*x^(2/3) - 84*b^3*x)/(168*x^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}dx$$

$$\Big\downarrow \ 798$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 53$$

$$3\int \left(\frac{a^3}{x^{10/3}}+\frac{3ba^2}{x^3}+\frac{3b^2a}{x^{8/3}}+\frac{b^3}{x^{7/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(-\frac{a^3}{9x^3}-\frac{3a^2b}{8x^{8/3}}-\frac{3ab^2}{7x^{7/3}}-\frac{b^3}{6x^2}\right)$$

input
```
Int[(a + b*x^(1/3))^3/x^4,x]
```

output
```
3*(-1/9*a^3/x^3 - (3*a^2*b)/(8*x^(8/3)) - (3*a*b^2)/(7*x^(7/3)) - b^3/(6*x
^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{a^3}{3x^3}-\frac{9a^2b}{8x^{\frac{8}{3}}}-\frac{9a\,b^2}{7x^{\frac{7}{3}}}-\frac{b^3}{2x^2}$
default	$-\frac{a^3}{3x^3}-\frac{9a^2b}{8x^{\frac{8}{3}}}-\frac{9a\,b^2}{7x^{\frac{7}{3}}}-\frac{b^3}{2x^2}$
trager	$\frac{(-1+x)(2a^3x^2+3b^3x^2+2a^3x+3b^3x+2a^3)}{6x^3}-\frac{9a^2b}{8x^{\frac{8}{3}}}-\frac{9a\,b^2}{7x^{\frac{7}{3}}}$
orering	$-\frac{(327b^3x+272a^3)\left(a+b\,x^{\frac{1}{3}}\right)^3}{168x^3(b^3x+a^3)}-\frac{3x^2(15b^3x+11a^3)\left(\frac{\left(a+b\,x^{\frac{1}{3}}\right)^2 b}{x^{\frac{14}{3}}}-\frac{4\left(a+b\,x^{\frac{1}{3}}\right)^3}{x^5}\right)}{56(b^3x+a^3)}-$ $3(3b^3x+2a^3)x^3$ (2 (a+b x

input
```
int((a+b*x^(1/3))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3/x^3-9/8*a^2*b/x^(8/3)-9/7*a*b^2/x^(7/3)-1/2*b^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx=-\frac{84\,b^3x+216\,ab^2x^{\frac{2}{3}}+189\,a^2bx^{\frac{1}{3}}+56\,a^3}{168\,x^3}$$

input
```
integrate((a+b*x^(1/3))^3/x^4,x, algorithm="fricas")
```

output
```
-1/168*(84*b^3*x + 216*a*b^2*x^(2/3) + 189*a^2*b*x^(1/3) + 56*a^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx=-\frac{a^3}{3x^3}-\frac{9a^2b}{8x^{\frac{8}{3}}}-\frac{9ab^2}{7x^{\frac{7}{3}}}-\frac{b^3}{2x^2}$$

input
```
integrate((a+b*x**(1/3))**3/x***4,x)
```

output
```
-a**3/(3*x**3) - 9*a**2*b/(8*x**(8/3)) - 9*a*b**2/(7*x**(7/3)) - b**3/(2*x
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx=-\frac{84\,b^3x+216\,ab^2x^{\frac{2}{3}}+189\,a^2bx^{\frac{1}{3}}+56\,a^3}{168\,x^3}$$

input
```
integrate((a+b*x^(1/3))^3/x^4,x, algorithm="maxima")
```

output
```
-1/168*(84*b^3*x + 216*a*b^2*x^(2/3) + 189*a^2*b*x^(1/3) + 56*a^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx=-\frac{84\,b^3x+216\,ab^2x^{\frac{2}{3}}+189\,a^2bx^{\frac{1}{3}}+56\,a^3}{168\,x^3}$$

input
```
integrate((a+b*x^(1/3))^3/x^4,x, algorithm="giac")
```

output
```
-1/168*(84*b^3*x + 216*a*b^2*x^(2/3) + 189*a^2*b*x^(1/3) + 56*a^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx=-\frac{84\,b^3\,x+56\,a^3+189\,a^2\,b\,x^{1/3}+216\,a\,b^2\,x^{2/3}}{168\,x^3}$$

input
```
int((a + b*x^(1/3))^3/x^4,x)
```

output
```
-(84*b^3*x + 56*a^3 + 189*a^2*b*x^(1/3) + 216*a*b^2*x^(2/3))/(168*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^3}{x^4}\,dx=\frac{-56x^{\frac{2}{3}}a^3-84x^{\frac{5}{3}}b^3-216x^{\frac{4}{3}}a\,b^2-189a^2bx}{168x^{\frac{11}{3}}}$$

input
```
int((a+b*x^(1/3))^3/x^4,x)
```

output
```
( - 56*x**(2/3)*a**3 - 84*x**(2/3)*b**3*x - 216*x**(1/3)*a*b**2*x - 189*a*
*2*b*x)/(168*x**(2/3)*x**3)
```

3.203 $\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx=\frac{a^5x^5}{5}+\frac{15}{16}a^4bx^{16/3}+\frac{30}{17}a^3b^2x^{17/3}+\frac{5}{3}a^2b^3x^6+\frac{15}{19}ab^4x^{19/3}+\frac{3}{20}b^5x^{20/3}$$

output
```
1/5*a^5*x^5+15/16*a^4*b*x^(16/3)+30/17*a^3*b^2*x^(17/3)+5/3*a^2*b^3*x^6+15
/19*a*b^4*x^(19/3)+3/20*b^5*x^(20/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx =\frac{15504a^5x^5+72675a^4bx^{16/3}+136800a^3b^2x^{17/3}+129200a^2b^3x^6+61200ab^4x^{19/3}+11628b^5x^{20/3}}{77520}$$

input
```
Integrate[(a + b*x^(1/3))^5*x^4,x]
```

output
```
(15504*a^5*x^5 + 72675*a^4*b*x^(16/3) + 136800*a^3*b^2*x^(17/3) + 129200*a
^2*b^3*x^6 + 61200*a*b^4*x^(19/3) + 11628*b^5*x^(20/3))/77520
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sqrt[3]{x}\right)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^5x^{14/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(b^5x^{19/3}+5ab^4x^6+10a^2b^3x^{17/3}+10a^3b^2x^{16/3}+5a^4bx^5+a^5x^{14/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^5x^5}{15}+\frac{5}{16}a^4bx^{16/3}+\frac{10}{17}a^3b^2x^{17/3}+\frac{5}{9}a^2b^3x^6+\frac{5}{19}ab^4x^{19/3}+\frac{1}{20}b^5x^{20/3}\right)$$

input
```
Int[(a + b*x^(1/3))^5*x^4,x]
```

output
```
3*((a^5*x^5)/15 + (5*a^4*b*x^(16/3))/16 + (10*a^3*b^2*x^(17/3))/17 + (5*a^
2*b^3*x^6)/9 + (5*a*b^4*x^(19/3))/19 + (b^5*x^(20/3))/20)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a^5x^5}{5}+\frac{15a^4b\,x^{\frac{16}{3}}}{16}+\frac{30a^3b^2x^{\frac{17}{3}}}{17}+\frac{5a^2b^3x^6}{3}+\frac{15a\,b^4x^{\frac{19}{3}}}{19}+\frac{3b^5x^{\frac{20}{3}}}{20}$
default	$\frac{a^5x^5}{5}+\frac{15a^4b\,x^{\frac{16}{3}}}{16}+\frac{30a^3b^2x^{\frac{17}{3}}}{17}+\frac{5a^2b^3x^6}{3}+\frac{15a\,b^4x^{\frac{19}{3}}}{19}+\frac{3b^5x^{\frac{20}{3}}}{20}$
trager	$\frac{a^2(25b^3x^5+3a^3x^4+25b^3x^4+3a^3x^3+25b^3x^3+3a^3x^2+25b^3x^2+3a^3x+25b^3x+3a^3+25b^3)(-1+x)}{15}+\frac{15ab\,x^{\frac{16}{3}}(16b^3x\cdots}{304}$
orering	$\frac{x^5(156400b^9x^3+485423a^3b^6x^2+507721a^6b^3x+180120a^9)\left(a+b\,x^{\frac{1}{3}}\right)^5}{387600(b^3x+a^3)^3}-\frac{3x^2(5440b^9x^3+17670a^3b^6x^2+19523a^6b^3x\cdots}{25840\cdots}$

input
```
int((a+b*x^(1/3))^5*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*x^5+15/16*a^4*b*x^(16/3)+30/17*a^3*b^2*x^(17/3)+5/3*a^2*b^3*x^6+15
/19*a*b^4*x^(19/3)+3/20*b^5*x^(20/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx = \frac{5}{3}\,a^2b^3x^6+\frac{1}{5}\,a^5x^5+\frac{3}{340}\,\left(17\,b^5x^6+200\,a^3b^2x^5\right)x^{\frac{2}{3}} + \frac{15}{304}\,\left(16\,ab^4x^6+19\,a^4bx^5\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5*x^4,x, algorithm="fricas")
```

output
```
5/3*a^2*b^3*x^6 + 1/5*a^5*x^5 + 3/340*(17*b^5*x^6 + 200*a^3*b^2*x^5)*x^(2/
3) + 15/304*(16*a*b^4*x^6 + 19*a^4*b*x^5)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx=\frac{a^5x^5}{5}+\frac{15a^4bx^{\frac{16}{3}}}{16}+\frac{30a^3b^2x^{\frac{17}{3}}}{17}+\frac{5a^2b^3x^6}{3}+\frac{15ab^4x^{\frac{19}{3}}}{19}+\frac{3b^5x^{\frac{20}{3}}}{20}$$

input
```
integrate((a+b*x**(1/3))**5*x**4,x)
```

output
```
a**5*x**5/5 + 15*a**4*b*x**(16/3)/16 + 30*a**3*b**2*x**(17/3)/17 + 5*a**2*
b**3*x**6/3 + 15*a*b**4*x**(19/3)/19 + 3*b**5*x**(20/3)/20
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(57) = 114.

Time = 0.03 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.26

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx=&\frac{3\left(bx^{\frac{1}{3}}+a\right)^{20}}{20\,b^{15}}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^{19}a}{19\,b^{15}}+\frac{91\left(bx^{\frac{1}{3}}+a\right)^{18}a^2}{6\,b^{15}}\\&-\frac{1092\left(bx^{\frac{1}{3}}+a\right)^{17}a^3}{17\,b^{15}}+\frac{3003\left(bx^{\frac{1}{3}}+a\right)^{16}a^4}{16\,b^{15}}\\&-\frac{2002\left(bx^{\frac{1}{3}}+a\right)^{15}a^5}{5\,b^{15}}+\frac{1287\left(bx^{\frac{1}{3}}+a\right)^{14}a^6}{2\,b^{15}}\\&-\frac{792\left(bx^{\frac{1}{3}}+a\right)^{13}a^7}{b^{15}}+\frac{3003\left(bx^{\frac{1}{3}}+a\right)^{12}a^8}{4\,b^{15}}-\frac{546\left(bx^{\frac{1}{3}}+a\right)^{11}a^9}{b^{15}}\\&+\frac{3003\left(bx^{\frac{1}{3}}+a\right)^{10}a^{10}}{10\,b^{15}}-\frac{364\left(bx^{\frac{1}{3}}+a\right)^{9}a^{11}}{3\,b^{15}}\\&+\frac{273\left(bx^{\frac{1}{3}}+a\right)^{8}a^{12}}{8\,b^{15}}-\frac{6\left(bx^{\frac{1}{3}}+a\right)^{7}a^{13}}{b^{15}}+\frac{\left(bx^{\frac{1}{3}}+a\right)^{6}a^{14}}{2\,b^{15}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^5*x^4,x, algorithm="maxima")
```

output
```
3/20*(b*x^(1/3) + a)^20/b^15 - 42/19*(b*x^(1/3) + a)^19*a/b^15 + 91/6*(b*x
^(1/3) + a)^18*a^2/b^15 - 1092/17*(b*x^(1/3) + a)^17*a^3/b^15 + 3003/16*(b
*x^(1/3) + a)^16*a^4/b^15 - 2002/5*(b*x^(1/3) + a)^15*a^5/b^15 + 1287/2*(b
*x^(1/3) + a)^14*a^6/b^15 - 792*(b*x^(1/3) + a)^13*a^7/b^15 + 3003/4*(b*x^
(1/3) + a)^12*a^8/b^15 - 546*(b*x^(1/3) + a)^11*a^9/b^15 + 3003/10*(b*x^(1
/3) + a)^10*a^10/b^15 - 364/3*(b*x^(1/3) + a)^9*a^11/b^15 + 273/8*(b*x^(1/
3) + a)^8*a^12/b^15 - 6*(b*x^(1/3) + a)^7*a^13/b^15 + 1/2*(b*x^(1/3) + a)^
6*a^14/b^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^4 \, dx = \frac{3}{20} \, b^5 x^{\frac{20}{3}} + \frac{15}{19} \, a b^4 x^{\frac{19}{3}} + \frac{5}{3} \, a^2 b^3 x^6 + \frac{30}{17} \, a^3 b^2 x^{\frac{17}{3}} + \frac{15}{16} \, a^4 b x^{\frac{16}{3}} + \frac{1}{5} \, a^5 x^5$$

input
```
integrate((a+b*x^(1/3))^5*x^4,x, algorithm="giac")
```

output
```
3/20*b^5*x^(20/3) + 15/19*a*b^4*x^(19/3) + 5/3*a^2*b^3*x^6 + 30/17*a^3*b^2
*x^(17/3) + 15/16*a^4*b*x^(16/3) + 1/5*a^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^4 \, dx = \frac{a^5\,x^5}{5} + \frac{3\,b^5\,x^{20/3}}{20} + \frac{15\,a^4\,b\,x^{16/3}}{16} + \frac{15\,a\,b^4\,x^{19/3}}{19} + \frac{5\,a^2\,b^3\,x^6}{3} + \frac{30\,a^3\,b^2\,x^{17/3}}{17}$$

input
```
int(x^4*(a + b*x^(1/3))^5,x)
```

output
```
(a^5*x^5)/5 + (3*b^5*x^(20/3))/20 + (15*a^4*b*x^(16/3))/16 + (15*a*b^4*x^(
19/3))/19 + (5*a^2*b^3*x^6)/3 + (30*a^3*b^2*x^(17/3))/17
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^4\,dx = \frac{x^5\left(136800x^{\frac{2}{3}}a^3b^2+11628x^{\frac{5}{3}}b^5+72675x^{\frac{1}{3}}a^4b+61200x^{\frac{4}{3}}a\,b^4+15504a^5+129200a^2b^3x\right)}{77520}$$

input
```
int((a+b*x^(1/3))^5*x^4,x)
```

output
```
(x**5*(136800*x**(2/3)*a**3*b**2 + 11628*x**(2/3)*b**5*x + 72675*x**(1/3)*
a**4*b + 61200*x**(1/3)*a*b**4*x + 15504*a**5 + 129200*a**2*b**3*x))/77520
```

3.204 $\int \left(a+b\sqrt[3]{x}\right)^5 x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^3 \, dx=\frac{a^5x^4}{4}+\frac{15}{13}a^4bx^{13/3}+\frac{15}{7}a^3b^2x^{14/3}+2a^2b^3x^5+\frac{15}{16}ab^4x^{16/3}+\frac{3}{17}b^5x^{17/3}$$

output
```
1/4*a^5*x^4+15/13*a^4*b*x^(13/3)+15/7*a^3*b^2*x^(14/3)+2*a^2*b^3*x^5+15/16
*a*b^4*x^(16/3)+3/17*b^5*x^(17/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^3 \, dx = \frac{6188a^5x^4+28560a^4bx^{13/3}+53040a^3b^2x^{14/3}+49504a^2b^3x^5+23205ab^4x^{16/3}+4368b^5x^{17/3}}{24752}$$

input
```
Integrate[(a + b*x^(1/3))^5*x^3,x]
```

output
```
(6188*a^5*x^4 + 28560*a^4*b*x^(13/3) + 53040*a^3*b^2*x^(14/3) + 49504*a^2*
b^3*x^5 + 23205*a*b^4*x^(16/3) + 4368*b^5*x^(17/3))/24752
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt[3]{x}\right)^5 dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^5 x^{11/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(b^5x^{16/3}+5ab^4x^5+10a^2b^3x^{14/3}+10a^3b^2x^{13/3}+5a^4bx^4+a^5x^{11/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^5x^4}{12}+\frac{5}{13}a^4bx^{13/3}+\frac{5}{7}a^3b^2x^{14/3}+\frac{2}{3}a^2b^3x^5+\frac{5}{16}ab^4x^{16/3}+\frac{1}{17}b^5x^{17/3}\right)$$

input
```
Int[(a + b*x^(1/3))^5*x^3,x]
```

output
```
3*((a^5*x^4)/12 + (5*a^4*b*x^(13/3))/13 + (5*a^3*b^2*x^(14/3))/7 + (2*a^2*
b^3*x^5)/3 + (5*a*b^4*x^(16/3))/16 + (b^5*x^(17/3))/17)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^{5} x^{4}}{4}+\frac{15 a^{4} b\, x^{\frac{13}{3}}}{13}+\frac{15 a^{3} b^{2} x^{\frac{14}{3}}}{7}+2 a^{2} b^{3} x^{5}+\frac{15 a\, b^{4} x^{\frac{16}{3}}}{16}+\frac{3 b^{5} x^{\frac{17}{3}}}{17}$
default	$\frac{a^{5} x^{4}}{4}+\frac{15 a^{4} b\, x^{\frac{13}{3}}}{13}+\frac{15 a^{3} b^{2} x^{\frac{14}{3}}}{7}+2 a^{2} b^{3} x^{5}+\frac{15 a\, b^{4} x^{\frac{16}{3}}}{16}+\frac{3 b^{5} x^{\frac{17}{3}}}{17}$
trager	$\frac{a^{2} \left(8 b^{3} x^{4}+a^{3} x^{3}+8 b^{3} x^{3}+a^{3} x^{2}+8 b^{3} x^{2}+a^{3} x+8 b^{3} x+a^{3}+8 b^{3}\right) \left(-1+x\right)}{4}+\frac{15 a b\, x^{\frac{13}{3}} \left(13 b^{3} x+16 a^{3}\right)}{208}+\frac{3 b^{2} x^{\frac{14}{3}} \left(7 b^{3} x+\right.}{119}$
orering	$\frac{x^{4} \left(115024 b^{9} x^{3}+357745 a^{3} b^{6} x^{2}+376250 a^{6} b^{3} x+135320 a^{9}\right) \left(a+b\, x^{\frac{1}{3}}\right)^{5}}{247520 \left(b^{3} x+a^{3}\right)^{3}}-\frac{9 x^{2} \left(1183 b^{9} x^{3}+3873 a^{3} b^{6} x^{2}+4345 a^{6} b^{3} x+\right.}{123760}$

input
```
int((a+b*x^(1/3))^5*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*x^4+15/13*a^4*b*x^(13/3)+15/7*a^3*b^2*x^(14/3)+2*a^2*b^3*x^5+15/16
*a*b^4*x^(16/3)+3/17*b^5*x^(17/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^3\,dx = 2\,a^2b^3x^5+\frac{1}{4}\,a^5x^4+\frac{3}{119}\left(7\,b^5x^5+85\,a^3b^2x^4\right)x^{\frac{2}{3}}+\frac{15}{208}\left(13\,ab^4x^5+16\,a^4bx^4\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5*x^3,x, algorithm="fricas")
```

output
```
2*a^2*b^3*x^5 + 1/4*a^5*x^4 + 3/119*(7*b^5*x^5 + 85*a^3*b^2*x^4)*x^(2/3) +
 15/208*(13*a*b^4*x^5 + 16*a^4*b*x^4)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^3\,dx=\frac{a^5x^4}{4}+\frac{15a^4bx^{\frac{13}{3}}}{13}+\frac{15a^3b^2x^{\frac{14}{3}}}{7}+2a^2b^3x^5+\frac{15ab^4x^{\frac{16}{3}}}{16}+\frac{3b^5x^{\frac{17}{3}}}{17}$$

input
```
integrate((a+b*x**(1/3))**5*x**3,x)
```

output
```
a**5*x**4/4 + 15*a**4*b*x**(13/3)/13 + 15*a**3*b**2*x**(14/3)/7 + 2*a**2*b
**3*x**5 + 15*a*b**4*x**(16/3)/16 + 3*b**5*x**(17/3)/17
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(57) = 114.

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.67

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^5 x^3\,dx=&\frac{3\left(bx^{\frac{1}{3}}+a\right)^{17}}{17\,b^{12}}-\frac{33\left(bx^{\frac{1}{3}}+a\right)^{16}a}{16\,b^{12}}+\frac{11\left(bx^{\frac{1}{3}}+a\right)^{15}a^2}{b^{12}}\\&-\frac{495\left(bx^{\frac{1}{3}}+a\right)^{14}a^3}{14\,b^{12}}+\frac{990\left(bx^{\frac{1}{3}}+a\right)^{13}a^4}{13\,b^{12}}-\frac{231\left(bx^{\frac{1}{3}}+a\right)^{12}a^5}{2\,b^{12}}\\&+\frac{126\left(bx^{\frac{1}{3}}+a\right)^{11}a^6}{b^{12}}-\frac{99\left(bx^{\frac{1}{3}}+a\right)^{10}a^7}{b^{12}}+\frac{55\left(bx^{\frac{1}{3}}+a\right)^{9}a^8}{b^{12}}\\&-\frac{165\left(bx^{\frac{1}{3}}+a\right)^{8}a^9}{8\,b^{12}}+\frac{33\left(bx^{\frac{1}{3}}+a\right)^{7}a^{10}}{7\,b^{12}}-\frac{\left(bx^{\frac{1}{3}}+a\right)^{6}a^{11}}{2\,b^{12}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^5*x^3,x, algorithm="maxima")
```

output
```
3/17*(b*x^(1/3) + a)^17/b^12 - 33/16*(b*x^(1/3) + a)^16*a/b^12 + 11*(b*x^(
1/3) + a)^15*a^2/b^12 - 495/14*(b*x^(1/3) + a)^14*a^3/b^12 + 990/13*(b*x^(
1/3) + a)^13*a^4/b^12 - 231/2*(b*x^(1/3) + a)^12*a^5/b^12 + 126*(b*x^(1/3)
 + a)^11*a^6/b^12 - 99*(b*x^(1/3) + a)^10*a^7/b^12 + 55*(b*x^(1/3) + a)^9*
a^8/b^12 - 165/8*(b*x^(1/3) + a)^8*a^9/b^12 + 33/7*(b*x^(1/3) + a)^7*a^10/
b^12 - 1/2*(b*x^(1/3) + a)^6*a^11/b^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^3\,dx = \frac{3}{17}\,b^5x^{\frac{17}{3}} + \frac{15}{16}\,ab^4x^{\frac{16}{3}} + 2\,a^2b^3x^5 + \frac{15}{7}\,a^3b^2x^{\frac{14}{3}} + \frac{15}{13}\,a^4bx^{\frac{13}{3}} + \frac{1}{4}\,a^5x^4$$

input
```
integrate((a+b*x^(1/3))^5*x^3,x, algorithm="giac")
```

output
```
3/17*b^5*x^(17/3) + 15/16*a*b^4*x^(16/3) + 2*a^2*b^3*x^5 + 15/7*a^3*b^2*x^
(14/3) + 15/13*a^4*b*x^(13/3) + 1/4*a^5*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^3\,dx = \frac{a^5\,x^4}{4} + \frac{3\,b^5\,x^{17/3}}{17} + \frac{15\,a^4\,b\,x^{13/3}}{13} + \frac{15\,a\,b^4\,x^{16/3}}{16} + 2\,a^2\,b^3\,x^5 + \frac{15\,a^3\,b^2\,x^{14/3}}{7}$$

input
```
int(x^3*(a + b*x^(1/3))^5,x)
```

output
```
(a^5*x^4)/4 + (3*b^5*x^(17/3))/17 + (15*a^4*b*x^(13/3))/13 + (15*a*b^4*x^(
16/3))/16 + 2*a^2*b^3*x^5 + (15*a^3*b^2*x^(14/3))/7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^3\,dx = \frac{x^4\left(53040x^{\frac{2}{3}}a^3b^2+4368x^{\frac{5}{3}}b^5+28560x^{\frac{1}{3}}a^4b+23205x^{\frac{4}{3}}a\,b^4+6188a^5+49504a^2b^3x\right)}{24752}$$

input
```
int((a+b*x^(1/3))^5*x^3,x)
```

output
```
(x**4*(53040*x**(2/3)*a**3*b**2 + 4368*x**(2/3)*b**5*x + 28560*x**(1/3)*a*
*4*b + 23205*x**(1/3)*a*b**4*x + 6188*a**5 + 49504*a**2*b**3*x))/24752
```

3.205 $\int \left(a+b\sqrt[3]{x}\right)^5 x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^2 \, dx=\frac{a^5x^3}{3}+\frac{3}{2}a^4bx^{10/3}+\frac{30}{11}a^3b^2x^{11/3}+\frac{5}{2}a^2b^3x^4+\frac{15}{13}ab^4x^{13/3}+\frac{3}{14}b^5x^{14/3}$$

output
```
1/3*a^5*x^3+3/2*a^4*b*x^(10/3)+30/11*a^3*b^2*x^(11/3)+5/2*a^2*b^3*x^4+15/1
3*a*b^4*x^(13/3)+3/14*b^5*x^(14/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^2 \, dx =\frac{2002a^5x^3+9009a^4bx^{10/3}+16380a^3b^2x^{11/3}+15015a^2b^3x^4+6930ab^4x^{13/3}+1287b^5x^{14/3}}{6006}$$

input
```
Integrate[(a + b*x^(1/3))^5*x^2,x]
```

output
```
(2002*a^5*x^3 + 9009*a^4*b*x^(10/3) + 16380*a^3*b^2*x^(11/3) + 15015*a^2*b
^3*x^4 + 6930*a*b^4*x^(13/3) + 1287*b^5*x^(14/3))/6006
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt[3]{x}\right)^5dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^5x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x^{8/3}a^5+5bx^3a^4+10b^2x^{10/3}a^3+10b^3x^{11/3}a^2+5b^4x^4a+b^5x^{13/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^5x^3}{9}+\frac{1}{2}a^4bx^{10/3}+\frac{10}{11}a^3b^2x^{11/3}+\frac{5}{6}a^2b^3x^4+\frac{5}{13}ab^4x^{13/3}+\frac{1}{14}b^5x^{14/3}\right)$$

input
```
Int[(a + b*x^(1/3))^5*x^2,x]
```

output
```
3*((a^5*x^3)/9 + (a^4*b*x^(10/3))/2 + (10*a^3*b^2*x^(11/3))/11 + (5*a^2*b^
3*x^4)/6 + (5*a*b^4*x^(13/3))/13 + (b^5*x^(14/3))/14)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a^5x^3}{3}+\frac{3a^4b\,x^{\frac{10}{3}}}{2}+\frac{30a^3b^2x^{\frac{11}{3}}}{11}+\frac{5a^2b^3x^4}{2}+\frac{15a\,b^4x^{\frac{13}{3}}}{13}+\frac{3b^5x^{\frac{14}{3}}}{14}$
default	$\frac{a^5x^3}{3}+\frac{3a^4b\,x^{\frac{10}{3}}}{2}+\frac{30a^3b^2x^{\frac{11}{3}}}{11}+\frac{5a^2b^3x^4}{2}+\frac{15a\,b^4x^{\frac{13}{3}}}{13}+\frac{3b^5x^{\frac{14}{3}}}{14}$
trager	$\frac{a^2(15b^3x^3+2a^3x^2+15b^3x^2+2a^3x+15b^3x+2a^3+15b^3)(-1+x)}{6}+\frac{3ab\,x^{\frac{10}{3}}(10b^3x+13a^3)}{26}+\frac{3b^2x^{\frac{11}{3}}(11b^3x+140a^3}{154}$
orering	$\frac{x^3(32835b^9x^3+102178a^3b^6x^2+108038a^6b^3x+39676a^9)\left(a+b\,x^{\frac{1}{3}}\right)^5}{60060(b^3x+a^3)^3}-\frac{3x^2(825b^9x^3+2718a^3b^6x^2+3104a^6b^3x+127}{20020(b^3x}$

input
```
int((a+b*x^(1/3))^5*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^5*x^3+3/2*a^4*b*x^(10/3)+30/11*a^3*b^2*x^(11/3)+5/2*a^2*b^3*x^4+15/1
3*a*b^4*x^(13/3)+3/14*b^5*x^(14/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^2\,dx = \frac{5}{2}\,a^2b^3x^4+\frac{1}{3}\,a^5x^3+\frac{3}{154}\,\left(11\,b^5x^4+140\,a^3b^2x^3\right)x^{\frac{2}{3}} + \frac{3}{26}\,\left(10\,ab^4x^4+13\,a^4bx^3\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5*x^2,x, algorithm="fricas")
```

output
```
5/2*a^2*b^3*x^4 + 1/3*a^5*x^3 + 3/154*(11*b^5*x^4 + 140*a^3*b^2*x^3)*x^(2/
3) + 3/26*(10*a*b^4*x^4 + 13*a^4*b*x^3)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^2\,dx = \frac{a^5x^3}{3}+\frac{3a^4bx^{\frac{10}{3}}}{2}+\frac{30a^3b^2x^{\frac{11}{3}}}{11}+\frac{5a^2b^3x^4}{2}+\frac{15ab^4x^{\frac{13}{3}}}{13}+\frac{3b^5x^{\frac{14}{3}}}{14}$$

input
```
integrate((a+b*x**(1/3))**5*x**2,x)
```

output
```
a**5*x**3/3 + 3*a**4*b*x**(10/3)/2 + 30*a**3*b**2*x**(11/3)/11 + 5*a**2*b*
*3*x**4/2 + 15*a*b**4*x**(13/3)/13 + 3*b**5*x**(14/3)/14
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(57) = 114.

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^5 x^2\,dx &= \frac{3\left(bx^{\frac{1}{3}}+a\right)^{14}}{14\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{13}a}{13\,b^9}+\frac{7\left(bx^{\frac{1}{3}}+a\right)^{12}a^2}{b^9}\\&\quad-\frac{168\left(bx^{\frac{1}{3}}+a\right)^{11}a^3}{11\,b^9}+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{10}a^4}{b^9}-\frac{56\left(bx^{\frac{1}{3}}+a\right)^{9}a^5}{3\,b^9}\\&\quad+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{8}a^6}{2\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{7}a^7}{7\,b^9}+\frac{\left(bx^{\frac{1}{3}}+a\right)^{6}a^8}{2\,b^9}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^5*x^2,x, algorithm="maxima")
```

output
```
3/14*(b*x^(1/3) + a)^14/b^9 - 24/13*(b*x^(1/3) + a)^13*a/b^9 + 7*(b*x^(1/3
) + a)^12*a^2/b^9 - 168/11*(b*x^(1/3) + a)^11*a^3/b^9 + 21*(b*x^(1/3) + a)
^10*a^4/b^9 - 56/3*(b*x^(1/3) + a)^9*a^5/b^9 + 21/2*(b*x^(1/3) + a)^8*a^6/
b^9 - 24/7*(b*x^(1/3) + a)^7*a^7/b^9 + 1/2*(b*x^(1/3) + a)^6*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^2\,dx=\frac{3}{14}\,b^5x^{\frac{14}{3}}+\frac{15}{13}\,ab^4x^{\frac{13}{3}}+\frac{5}{2}\,a^2b^3x^4+\frac{30}{11}\,a^3b^2x^{\frac{11}{3}}+\frac{3}{2}\,a^4bx^{\frac{10}{3}}+\frac{1}{3}\,a^5x^3$$

input
```
integrate((a+b*x^(1/3))^5*x^2,x, algorithm="giac")
```

output
```
3/14*b^5*x^(14/3) + 15/13*a*b^4*x^(13/3) + 5/2*a^2*b^3*x^4 + 30/11*a^3*b^2
*x^(11/3) + 3/2*a^4*b*x^(10/3) + 1/3*a^5*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^2\,dx=\frac{a^5\,x^3}{3}+\frac{3\,b^5\,x^{14/3}}{14}+\frac{3\,a^4\,b\,x^{10/3}}{2}+\frac{15\,a\,b^4\,x^{13/3}}{13}+\frac{5\,a^2\,b^3\,x^4}{2}+\frac{30\,a^3\,b^2\,x^{11/3}}{11}$$

input
```
int(x^2*(a + b*x^(1/3))^5,x)
```

output
```
(a^5*x^3)/3 + (3*b^5*x^(14/3))/14 + (3*a^4*b*x^(10/3))/2 + (15*a*b^4*x^(13
/3))/13 + (5*a^2*b^3*x^4)/2 + (30*a^3*b^2*x^(11/3))/11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x^2\,dx=\frac{x^3\left(16380x^{\frac{2}{3}}a^3b^2+1287x^{\frac{5}{3}}b^5+9009x^{\frac{1}{3}}a^4b+6930x^{\frac{4}{3}}a\,b^4+2002a^5+15015a^2b^3x\right)}{6006}$$

input
```
int((a+b*x^(1/3))^5*x^2,x)
```

output
```
(x**3*(16380*x**(2/3)*a**3*b**2 + 1287*x**(2/3)*b**5*x + 9009*x**(1/3)*a**
4*b + 6930*x**(1/3)*a*b**4*x + 2002*a**5 + 15015*a**2*b**3*x))/6006
```

3.206 $\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx=\frac{a^5x^2}{2}+\frac{15}{7}a^4bx^{7/3}+\frac{15}{4}a^3b^2x^{8/3}+\frac{10}{3}a^2b^3x^3+\frac{3}{2}ab^4x^{10/3}+\frac{3}{11}b^5x^{11/3}$$

output
```
1/2*a^5*x^2+15/7*a^4*b*x^(7/3)+15/4*a^3*b^2*x^(8/3)+10/3*a^2*b^3*x^3+3/2*a
*b^4*x^(10/3)+3/11*b^5*x^(11/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx=\frac{1}{924}\left(462a^5x^2+1980a^4bx^{7/3}+3465a^3b^2x^{8/3}+3080a^2b^3x^3+1386ab^4x^{10/3}+252b^5x^{11/3}\right)$$

input
```
Integrate[(a + b*x^(1/3))^5*x,x]
```

output
```
(462*a^5*x^2 + 1980*a^4*b*x^(7/3) + 3465*a^3*b^2*x^(8/3) + 3080*a^2*b^3*x^
3 + 1386*a*b^4*x^(10/3) + 252*b^5*x^(11/3))/924
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt[3]{x}\right)^5 dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^5 x^{5/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x^{5/3}a^5+5bx^2a^4+10b^2x^{7/3}a^3+10b^3x^{8/3}a^2+5b^4x^3a+b^5x^{10/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^5x^2}{6}+\frac{5}{7}a^4bx^{7/3}+\frac{5}{4}a^3b^2x^{8/3}+\frac{10}{9}a^2b^3x^3+\frac{1}{2}ab^4x^{10/3}+\frac{1}{11}b^5x^{11/3}\right)$$

input
```
Int[(a + b*x^(1/3))^5*x,x]
```

output
```
3*((a^5*x^2)/6 + (5*a^4*b*x^(7/3))/7 + (5*a^3*b^2*x^(8/3))/4 + (10*a^2*b^3
*x^3)/9 + (a*b^4*x^(10/3))/2 + (b^5*x^(11/3))/11)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a^5x^2}{2}+\frac{15a^4b\,x^{\frac{7}{3}}}{7}+\frac{15a^3b^2x^{\frac{8}{3}}}{4}+\frac{10a^2b^3x^3}{3}+\frac{3a\,b^4x^{\frac{10}{3}}}{2}+\frac{3b^5x^{\frac{11}{3}}}{11}$
default	$\frac{a^5x^2}{2}+\frac{15a^4b\,x^{\frac{7}{3}}}{7}+\frac{15a^3b^2x^{\frac{8}{3}}}{4}+\frac{10a^2b^3x^3}{3}+\frac{3a\,b^4x^{\frac{10}{3}}}{2}+\frac{3b^5x^{\frac{11}{3}}}{11}$
trager	$\frac{a^2(20b^3x^2+3a^3x+20b^3x+3a^3+20b^3)(-1+x)}{6}+\frac{3ab\,x^{\frac{7}{3}}(7b^3x+10a^3)}{14}+\frac{3b^2x^{\frac{8}{3}}(4b^3x+55a^3)}{44}$
orering	$-\frac{(-3052b^{15}x^5-9449a^3b^{12}x^4-9985a^6b^9x^3-1485a^9b^6x^2-23595a^{12}b^3x+8250a^{15})\left(a+b\,x^{\frac{1}{3}}\right)^5}{4620b^6(b^3x+a^3)^3}+\frac{3(-98b^{15}x^5-321}{}$

input
```
int((a+b*x^(1/3))^5*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^5*x^2+15/7*a^4*b*x^(7/3)+15/4*a^3*b^2*x^(8/3)+10/3*a^2*b^3*x^3+3/2*a
*b^4*x^(10/3)+3/11*b^5*x^(11/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx=\frac{10}{3}\,a^2b^3x^3+\frac{1}{2}\,a^5x^2+\frac{3}{44}\,\left(4\,b^5x^3+55\,a^3b^2x^2\right)x^{\frac{2}{3}}+\frac{3}{14}\,\left(7\,ab^4x^3+10\,a^4bx^2\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5*x,x, algorithm="fricas")
```

output
```
10/3*a^2*b^3*x^3 + 1/2*a^5*x^2 + 3/44*(4*b^5*x^3 + 55*a^3*b^2*x^2)*x^(2/3)
 + 3/14*(7*a*b^4*x^3 + 10*a^4*b*x^2)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx = \frac{a^5x^2}{2} + \frac{15a^4bx^{\frac{7}{3}}}{7} + \frac{15a^3b^2x^{\frac{8}{3}}}{4} + \frac{10a^2b^3x^3}{3} + \frac{3ab^4x^{\frac{10}{3}}}{2} + \frac{3b^5x^{\frac{11}{3}}}{11}$$

input
```
integrate((a+b*x**(1/3))**5*x,x)
```

output
```
a**5*x**2/2 + 15*a**4*b*x**(7/3)/7 + 15*a**3*b**2*x**(8/3)/4 + 10*a**2*b**
3*x**3/3 + 3*a*b**4*x**(10/3)/2 + 3*b**5*x**(11/3)/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx = \frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}}{11\,b^6} - \frac{3\left(bx^{\frac{1}{3}}+a\right)^{10}a}{2\,b^6} + \frac{10\left(bx^{\frac{1}{3}}+a\right)^{9}a^2}{3\,b^6} - \frac{15\left(bx^{\frac{1}{3}}+a\right)^{8}a^3}{4\,b^6} + \frac{15\left(bx^{\frac{1}{3}}+a\right)^{7}a^4}{7\,b^6} - \frac{\left(bx^{\frac{1}{3}}+a\right)^{6}a^5}{2\,b^6}$$

input
```
integrate((a+b*x^(1/3))^5*x,x, algorithm="maxima")
```

output
```
3/11*(b*x^(1/3) + a)^11/b^6 - 3/2*(b*x^(1/3) + a)^10*a/b^6 + 10/3*(b*x^(1/
3) + a)^9*a^2/b^6 - 15/4*(b*x^(1/3) + a)^8*a^3/b^6 + 15/7*(b*x^(1/3) + a)^
7*a^4/b^6 - 1/2*(b*x^(1/3) + a)^6*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx = \frac{3}{11}\,b^5x^{\frac{11}{3}} + \frac{3}{2}\,ab^4x^{\frac{10}{3}} + \frac{10}{3}\,a^2b^3x^3 + \frac{15}{4}\,a^3b^2x^{\frac{8}{3}} + \frac{15}{7}\,a^4bx^{\frac{7}{3}} + \frac{1}{2}\,a^5x^2$$

input
```
integrate((a+b*x^(1/3))^5*x,x, algorithm="giac")
```

output
```
3/11*b^5*x^(11/3) + 3/2*a*b^4*x^(10/3) + 10/3*a^2*b^3*x^3 + 15/4*a^3*b^2*x
^(8/3) + 15/7*a^4*b*x^(7/3) + 1/2*a^5*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx = \frac{a^5\,x^2}{2} + \frac{3\,b^5\,x^{11/3}}{11} + \frac{15\,a^4\,b\,x^{7/3}}{7} + \frac{3\,a\,b^4\,x^{10/3}}{2} + \frac{10\,a^2\,b^3\,x^3}{3} + \frac{15\,a^3\,b^2\,x^{8/3}}{4}$$

input
```
int(x*(a + b*x^(1/3))^5,x)
```

output
```
(a^5*x^2)/2 + (3*b^5*x^(11/3))/11 + (15*a^4*b*x^(7/3))/7 + (3*a*b^4*x^(10/
3))/2 + (10*a^2*b^3*x^3)/3 + (15*a^3*b^2*x^(8/3))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt[3]{x}\right)^5 x\,dx = \frac{x^2\left(3465x^{\frac{2}{3}}a^3b^2 + 252x^{\frac{5}{3}}b^5 + 1980x^{\frac{1}{3}}a^4b + 1386x^{\frac{4}{3}}a\,b^4 + 462a^5 + 3080a^2b^3x\right)}{924}$$

input
```
int((a+b*x^(1/3))^5*x,x)
```

output
```
(x**2*(3465*x**(2/3)*a**3*b**2 + 252*x**(2/3)*b**5*x + 1980*x**(1/3)*a**4*
b + 1386*x**(1/3)*a*b**4*x + 462*a**5 + 3080*a**2*b**3*x))/924
```

3.207 $\int \left(a+b\sqrt[3]{x}\right)^5 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \left(a+b\sqrt[3]{x}\right)^5 \, dx=\frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{2b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^7}{7b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^8}{8b^3}$$

output
```
1/2*a^2*(a+b*x^(1/3))^6/b^3-6/7*a*(a+b*x^(1/3))^7/b^3+3/8*(a+b*x^(1/3))^8/
b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \left(a+b\sqrt[3]{x}\right)^5 \, dx=\frac{1}{56}\left(56a^5x+210a^4bx^{4/3}+336a^3b^2x^{5/3}+280a^2b^3x^2+120ab^4x^{7/3}+21b^5x^{8/3}\right)$$

input
```
Integrate[(a + b*x^(1/3))^5,x]
```

output
```
(56*a^5*x + 210*a^4*b*x^(4/3) + 336*a^3*b^2*x^(5/3) + 280*a^2*b^3*x^2 + 12
0*a*b^4*x^(7/3) + 21*b^5*x^(8/3))/56
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}\right)^5 dx$$

$$\Big\downarrow \text{ 774}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^5 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^7}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^6}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^5}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^6}{6b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^8}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^7}{7b^3}\right)$$

input
```
Int[(a + b*x^(1/3))^5,x]
```

output
```
3*((a^2*(a + b*x^(1/3))^6)/(6*b^3) - (2*a*(a + b*x^(1/3))^7)/(7*b^3) + (a
+ b*x^(1/3))^8/(8*b^3))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{3b^5x^{\frac{8}{3}}}{8}+\frac{15a\,b^4x^{\frac{7}{3}}}{7}+5a^2b^3x^2+6a^3b^2x^{\frac{5}{3}}+\frac{15a^4b\,x^{\frac{4}{3}}}{4}+a^5x$
default	$\frac{3b^5x^{\frac{8}{3}}}{8}+\frac{15a\,b^4x^{\frac{7}{3}}}{7}+5a^2b^3x^2+6a^3b^2x^{\frac{5}{3}}+\frac{15a^4b\,x^{\frac{4}{3}}}{4}+a^5x$
trager	$(-1+x)\left(5b^3x+a^3+5b^3\right)a^2+\frac{15ab\,x^{\frac{4}{3}}\left(4b^3x+7a^3\right)}{28}+\frac{3b^2x^{\frac{5}{3}}\left(b^3x+16a^3\right)}{8}$
orering	$-\frac{\left(-46x^4b^{12}-139a^3b^9x^3-81x^2b^6a^6-469x\,b^3a^9+59a^{12}\right)\left(a+b\,x^{\frac{1}{3}}\right)^5}{56b^3\left(b^3x+a^3\right)^3}-\frac{15\left(b^9x^3+3a^3b^6x^2+25a^9\right)x^{\frac{4}{3}}\left(a+b\,x^{\frac{1}{3}}\right)^4b}{28\left(b^3x+a^3\right)^3}$

input
```
int((a+b*x^(1/3))^5,x,method=_RETURNVERBOSE)
```

output
```
3/8*b^5*x^(8/3)+15/7*a*b^4*x^(7/3)+5*a^2*b^3*x^2+6*a^3*b^2*x^(5/3)+15/4*a^
4*b*x^(4/3)+a^5*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt[3]{x}\right)^5\,dx=5\,a^2b^3x^2+a^5x+\frac{3}{8}\left(b^5x^2+16\,a^3b^2x\right)x^{\frac{2}{3}}+\frac{15}{28}\left(4\,ab^4x^2+7\,a^4bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5,x, algorithm="fricas")
```

output
```
5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*
b^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \left(a+b\sqrt[3]{x}\right)^5\,dx=a^5x+\frac{15a^4bx^{\frac{4}{3}}}{4}+6a^3b^2x^{\frac{5}{3}}+5a^2b^3x^2+\frac{15ab^4x^{\frac{7}{3}}}{7}+\frac{3b^5x^{\frac{8}{3}}}{8}$$

input
```
integrate((a+b*x**(1/3))**5,x)
```

output
```
a**5*x + 15*a**4*b*x**(4/3)/4 + 6*a**3*b**2*x**(5/3) + 5*a**2*b**3*x**2 +
15*a*b**4*x**(7/3)/7 + 3*b**5*x**(8/3)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt[3]{x}\right)^5\,dx=\frac{3}{8}\,b^5x^{\frac{8}{3}}+\frac{15}{7}\,ab^4x^{\frac{7}{3}}+5\,a^2b^3x^2+6\,a^3b^2x^{\frac{5}{3}}+\frac{15}{4}\,a^4bx^{\frac{4}{3}}+a^5x$$

input
```
integrate((a+b*x^(1/3))^5,x, algorithm="maxima")
```

output
```
3/8*b^5*x^(8/3) + 15/7*a*b^4*x^(7/3) + 5*a^2*b^3*x^2 + 6*a^3*b^2*x^(5/3) +
 15/4*a^4*b*x^(4/3) + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt[3]{x}\right)^5 \, dx = \frac{3}{8}\,b^5x^{\frac{8}{3}} + \frac{15}{7}\,ab^4x^{\frac{7}{3}} + 5\,a^2b^3x^2 + 6\,a^3b^2x^{\frac{5}{3}} + \frac{15}{4}\,a^4bx^{\frac{4}{3}} + a^5x$$

input
```
integrate((a+b*x^(1/3))^5,x, algorithm="giac")
```

output
```
3/8*b^5*x^(8/3) + 15/7*a*b^4*x^(7/3) + 5*a^2*b^3*x^2 + 6*a^3*b^2*x^(5/3) +
 15/4*a^4*b*x^(4/3) + a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt[3]{x}\right)^5 \, dx = a^5\,x + \frac{3\,b^5\,x^{8/3}}{8} + \frac{15\,a^4\,b\,x^{4/3}}{4} + \frac{15\,a\,b^4\,x^{7/3}}{7} + 5\,a^2\,b^3\,x^2 + 6\,a^3\,b^2\,x^{5/3}$$

input
```
int((a + b*x^(1/3))^5,x)
```

output
```
a^5*x + (3*b^5*x^(8/3))/8 + (15*a^4*b*x^(4/3))/4 + (15*a*b^4*x^(7/3))/7 +
5*a^2*b^3*x^2 + 6*a^3*b^2*x^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}\right)^5 \, dx = \frac{x\left(336x^{\frac{2}{3}}a^3b^2 + 21x^{\frac{5}{3}}b^5 + 210x^{\frac{1}{3}}a^4b + 120x^{\frac{4}{3}}a\,b^4 + 56a^5 + 280a^2b^3x\right)}{56}$$

input
```
int((a+b*x^(1/3))^5,x)
```

output
```
(x*(336*x**(2/3)*a**3*b**2 + 21*x**(2/3)*b**5*x + 210*x**(1/3)*a**4*b + 12
0*x**(1/3)*a*b**4*x + 56*a**5 + 280*a**2*b**3*x))/56
```

3.208 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx = 15a^4b\sqrt[3]{x}+15a^3b^2x^{2/3}+10a^2b^3x+\frac{15}{4}ab^4x^{4/3}+\frac{3}{5}b^5x^{5/3}+a^5\log(x)$$

output
```
15*a^4*b*x^(1/3)+15*a^3*b^2*x^(2/3)+10*a^2*b^3*x+15/4*a*b^4*x^(4/3)+3/5*b^
5*x^(5/3)+a^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx = 15a^4b\sqrt[3]{x}+15a^3b^2x^{2/3}+10a^2b^3x+\frac{15}{4}ab^4x^{4/3}+\frac{3}{5}b^5x^{5/3}+a^5\log(x)$$

input
```
Integrate[(a + b*x^(1/3))^5/x,x]
```

output
```
15*a^4*b*x^(1/3) + 15*a^3*b^2*x^(2/3) + 10*a^2*b^3*x + (15*a*b^4*x^(4/3))/
4 + (3*b^5*x^(5/3))/5 + a^5*Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx$$

$$\Big\downarrow \ 798$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(\frac{a^5}{\sqrt[3]{x}}+5ba^4+10b^2\sqrt[3]{x}a^3+10b^3x^{2/3}a^2+5b^4xa+b^5x^{4/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(a^5\log\left(\sqrt[3]{x}\right)+5a^4b\sqrt[3]{x}+5a^3b^2x^{2/3}+\frac{10}{3}a^2b^3x+\frac{5}{4}ab^4x^{4/3}+\frac{1}{5}b^5x^{5/3}\right)$$

input
```
Int[(a + b*x^(1/3))^5/x,x]
```

output
```
3*(5*a^4*b*x^(1/3) + 5*a^3*b^2*x^(2/3) + (10*a^2*b^3*x)/3 + (5*a*b^4*x^(4/
3))/4 + (b^5*x^(5/3))/5 + a^5*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$15a^4b\,x^{\frac{1}{3}}+15a^3b^2x^{\frac{2}{3}}+10a^2b^3x+\frac{15a\,b^4x^{\frac{4}{3}}}{4}+\frac{3b^5x^{\frac{5}{3}}}{5}+a^5\ln\left(x\right)$	54
default	$15a^4b\,x^{\frac{1}{3}}+15a^3b^2x^{\frac{2}{3}}+10a^2b^3x+\frac{15a\,b^4x^{\frac{4}{3}}}{4}+\frac{3b^5x^{\frac{5}{3}}}{5}+a^5\ln\left(x\right)$	54
trager	$10a^2b^3(-1+x)+\frac{15(b^3x+4a^3)ab\,x^{\frac{1}{3}}}{4}+\frac{3(b^3x+25a^3)b^2x^{\frac{2}{3}}}{5}+a^5\ln\left(x\right)$	56

input
```
int((a+b*x^(1/3))^5/x,x,method=_RETURNVERBOSE)
```

output
```
15*a^4*b*x^(1/3)+15*a^3*b^2*x^(2/3)+10*a^2*b^3*x+15/4*a*b^4*x^(4/3)+3/5*b^
5*x^(5/3)+a^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int\frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx=10\,a^2b^3x+3\,a^5\log\left(x^{\frac{1}{3}}\right)+\frac{3}{5}\left(b^5x+25\,a^3b^2\right)x^{\frac{2}{3}}+\frac{15}{4}\left(ab^4x+4\,a^4b\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5/x,x, algorithm="fricas")
```

output
```
10*a^2*b^3*x + 3*a^5*log(x^(1/3)) + 3/5*(b^5*x + 25*a^3*b^2)*x^(2/3) + 15/
4*(a*b^4*x + 4*a^4*b)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx = 3a^5\log\left(\sqrt[3]{x}\right)+15a^4b\sqrt[3]{x}+15a^3b^2x^{\frac{2}{3}}+10a^2b^3x+\frac{15ab^4x^{\frac{4}{3}}}{4}+\frac{3b^5x^{\frac{5}{3}}}{5}$$

input
```
integrate((a+b*x**(1/3))**5/x,x)
```

output
```
3*a**5*log(x**(1/3)) + 15*a**4*b*x**(1/3) + 15*a**3*b**2*x**(2/3) + 10*a**
2*b**3*x + 15*a*b**4*x**(4/3)/4 + 3*b**5*x**(5/3)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx = \frac{3}{5}\,b^5x^{\frac{5}{3}}+\frac{15}{4}\,ab^4x^{\frac{4}{3}}+10\,a^2b^3x+a^5\log\left(x\right)+15\,a^3b^2x^{\frac{2}{3}}+15\,a^4bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5/x,x, algorithm="maxima")
```

output
```
3/5*b^5*x^(5/3) + 15/4*a*b^4*x^(4/3) + 10*a^2*b^3*x + a^5*log(x) + 15*a^3*
b^2*x^(2/3) + 15*a^4*b*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx = \frac{3}{5}\,b^5x^{\frac{5}{3}}+\frac{15}{4}\,ab^4x^{\frac{4}{3}}+10\,a^2b^3x+a^5\log\left(|x|\right)+15\,a^3b^2x^{\frac{2}{3}}+15\,a^4bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^5/x,x, algorithm="giac")
```

output
```
3/5*b^5*x^(5/3) + 15/4*a*b^4*x^(4/3) + 10*a^2*b^3*x + a^5*log(abs(x)) + 15
*a^3*b^2*x^(2/3) + 15*a^4*b*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx = 3\,a^5\,\ln\left(x^{1/3}\right)+\frac{3\,b^5\,x^{5/3}}{5}+10\,a^2\,b^3\,x+15\,a^4\,b\,x^{1/3}+\frac{15\,a\,b^4\,x^{4/3}}{4}+15\,a^3\,b^2\,x^{2/3}$$

input
```
int((a + b*x^(1/3))^5/x,x)
```

output
```
3*a^5*log(x^(1/3)) + (3*b^5*x^(5/3))/5 + 10*a^2*b^3*x + 15*a^4*b*x^(1/3) +
 (15*a*b^4*x^(4/3))/4 + 15*a^3*b^2*x^(2/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x}\,dx = 15x^{\frac{2}{3}}a^3b^2+\frac{3x^{\frac{5}{3}}b^5}{5}+15x^{\frac{1}{3}}a^4b+\frac{15x^{\frac{4}{3}}a\,b^4}{4}+\log(x)\,a^5+10a^2b^3x$$

input
```
int((a+b*x^(1/3))^5/x,x)
```

output
```
(300*x**(2/3)*a**3*b**2 + 12*x**(2/3)*b**5*x + 300*x**(1/3)*a**4*b + 75*x*
*(1/3)*a*b**4*x + 20*log(x)*a**5 + 200*a**2*b**3*x)/20
```

3.209 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx=-\frac{a^5}{x}-\frac{15a^4b}{2x^{2/3}}-\frac{30a^3b^2}{\sqrt[3]{x}}+15ab^4\sqrt[3]{x}+\frac{3}{2}b^5x^{2/3}+10a^2b^3\log(x)$$

output
```
-a^5/x-15/2*a^4*b/x^(2/3)-30*a^3*b^2/x^(1/3)+15*a*b^4*x^(1/3)+3/2*b^5*x^(2
/3)+10*a^2*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx =\frac{-2a^5-15a^4b\sqrt[3]{x}-60a^3b^2x^{2/3}+30ab^4x^{4/3}+3b^5x^{5/3}+20a^2b^3x\log(x)}{2x}$$

input
```
Integrate[(a + b*x^(1/3))^5/x^2,x]
```

output
```
(-2*a^5 - 15*a^4*b*x^(1/3) - 60*a^3*b^2*x^(2/3) + 30*a*b^4*x^(4/3) + 3*b^5
*x^(5/3) + 20*a^2*b^3*x*Log[x])/(2*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 49}$$

$$3\int \left(\frac{a^5}{x^{4/3}}+\frac{5ba^4}{x}+\frac{10b^2a^3}{x^{2/3}}+\frac{10b^3a^2}{\sqrt[3]{x}}+5b^4a+b^5\sqrt[3]{x}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(-\frac{a^5}{3x}-\frac{5a^4b}{2x^{2/3}}-\frac{10a^3b^2}{\sqrt[3]{x}}+10a^2b^3\log\left(\sqrt[3]{x}\right)+5ab^4\sqrt[3]{x}+\frac{1}{2}b^5x^{2/3}\right)$$

input
```
Int[(a + b*x^(1/3))^5/x^2,x]
```

output
```
3*(-1/3*a^5/x - (5*a^4*b)/(2*x^(2/3)) - (10*a^3*b^2)/x^(1/3) + 5*a*b^4*x^(
1/3) + (b^5*x^(2/3))/2 + 10*a^2*b^3*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{a^{5}}{x}-\frac{15 a^{4} b}{2 x^{\frac{2}{3}}}-\frac{30 a^{3} b^{2}}{x^{\frac{1}{3}}}+15 a\, b^{4} x^{\frac{1}{3}}+\frac{3 b^{5} x^{\frac{2}{3}}}{2}+10 a^{2} b^{3} \ln \left(x \right)$	57
default	$-\frac{a^{5}}{x}-\frac{15 a^{4} b}{2 x^{\frac{2}{3}}}-\frac{30 a^{3} b^{2}}{x^{\frac{1}{3}}}+15 a\, b^{4} x^{\frac{1}{3}}+\frac{3 b^{5} x^{\frac{2}{3}}}{2}+10 a^{2} b^{3} \ln \left(x \right)$	57
trager	$\frac{a^{5} \left(-1+x \right)}{x}-\frac{15 \left(-2 b^{3} x +a^{3}\right) a b}{2 x^{\frac{2}{3}}}-\frac{3 \left(-b^{3} x +20 a^{3}\right) b^{2}}{2 x^{\frac{1}{3}}}-10 a^{2} b^{3} \ln \left(\frac{1}{x}\right)$	61

input
```
int((a+b*x^(1/3))^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^5/x-15/2*a^4*b/x^(2/3)-30*a^3*b^2/x^(1/3)+15*a*b^4*x^(1/3)+3/2*b^5*x^(2
/3)+10*a^2*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx = \frac{60\,a^2b^3x\log\left(x^{\frac{1}{3}}\right)-2\,a^5+3\,(b^5x-20\,a^3b^2)x^{\frac{2}{3}}+15\,(2\,ab^4x-a^4b)x^{\frac{1}{3}}}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^5/x^2,x, algorithm="fricas")
```

output
```
1/2*(60*a^2*b^3*x*log(x^(1/3)) - 2*a^5 + 3*(b^5*x - 20*a^3*b^2)*x^(2/3) +
15*(2*a*b^4*x - a^4*b)*x^(1/3))/x
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx = -\frac{a^5}{x}-\frac{15a^4b}{2x^{\frac{2}{3}}}-\frac{30a^3b^2}{\sqrt[3]{x}}+10a^2b^3\log(x)+15ab^4\sqrt[3]{x}+\frac{3b^5x^{\frac{2}{3}}}{2}$$

input
```
integrate((a+b*x**(1/3))**5/x**2,x)
```

output
```
-a**5/x - 15*a**4*b/(2*x**(2/3)) - 30*a**3*b**2/x**(1/3) + 10*a**2*b**3*lo
g(x) + 15*a*b**4*x**(1/3) + 3*b**5*x**(2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx = 10\,a^2b^3\log(x)+\frac{3}{2}\,b^5x^{\frac{2}{3}}+15\,ab^4x^{\frac{1}{3}}-\frac{60\,a^3b^2x^{\frac{2}{3}}+15\,a^4bx^{\frac{1}{3}}+2\,a^5}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^5/x^2,x, algorithm="maxima")
```

output
```
10*a^2*b^3*log(x) + 3/2*b^5*x^(2/3) + 15*a*b^4*x^(1/3) - 1/2*(60*a^3*b^2*x
^(2/3) + 15*a^4*b*x^(1/3) + 2*a^5)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx = 10\,a^2b^3\log\left(|x|\right)+\frac{3}{2}\,b^5x^{\frac{2}{3}}+15\,ab^4x^{\frac{1}{3}}-\frac{60\,a^3b^2x^{\frac{2}{3}}+15\,a^4bx^{\frac{1}{3}}+2\,a^5}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^5/x^2,x, algorithm="giac")
```

output
```
10*a^2*b^3*log(abs(x)) + 3/2*b^5*x^(2/3) + 15*a*b^4*x^(1/3) - 1/2*(60*a^3*
b^2*x^(2/3) + 15*a^4*b*x^(1/3) + 2*a^5)/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx = \frac{3\,b^5\,x^{2/3}}{2}-\frac{a^5+\frac{15\,a^4\,b\,x^{1/3}}{2}+30\,a^3\,b^2\,x^{2/3}}{x}+30\,a^2\,b^3\,\ln\left(x^{1/3}\right)+15\,a\,b^4\,x^{1/3}$$

input
```
int((a + b*x^(1/3))^5/x^2,x)
```

output
```
(3*b^5*x^(2/3))/2 - (a^5 + (15*a^4*b*x^(1/3))/2 + 30*a^3*b^2*x^(2/3))/x +
30*a^2*b^3*log(x^(1/3)) + 15*a*b^4*x^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^2}\,dx = \frac{60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)a^2b^3-2x^{\frac{2}{3}}a^5-60x^{\frac{4}{3}}a^3b^2+3x^{\frac{7}{3}}b^5-15a^4bx+30a\,b^4x^2}{2x^{\frac{5}{3}}}$$

input
```
int((a+b*x^(1/3))^5/x^2,x)
```

output
```
(60*x**(2/3)*log(x**(1/3))*a**2*b**3*x - 2*x**(2/3)*a**5 - 60*x**(1/3)*a**
3*b**2*x + 3*x**(1/3)*b**5*x**2 - 15*a**4*b*x + 30*a*b**4*x**2)/(2*x**(2/3
)*x)
```

3.210 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = -\frac{\left(a+b\sqrt[3]{x}\right)^6}{2ax^2}$$

output
```
-1/2*(a+b*x^(1/3))^6/a/x^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(21) = 42.

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.19

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = \frac{-a^5-6a^4b\sqrt[3]{x}-15a^3b^2x^{2/3}-20a^2b^3x-15ab^4x^{4/3}-6b^5x^{5/3}}{2x^2}$$

input
```
Integrate[(a + b*x^(1/3))^5/x^3,x]
```

output
```
(-a^5 - 6*a^4*b*x^(1/3) - 15*a^3*b^2*x^(2/3) - 20*a^2*b^3*x - 15*a*b^4*x^(
4/3) - 6*b^5*x^(5/3))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{\left(a+b\sqrt[3]{x}\right)^6}{2ax^2}$$

input
```
Int[(a + b*x^(1/3))^5/x^3,x]
```

output
```
-1/2*(a + b*x^(1/3))^6/(a*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 1.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

method	result
derivativedivides	$-\frac{15a\,b^4}{2x^{\frac{2}{3}}}-\frac{3a^4b}{x^{\frac{5}{3}}}-\frac{a^5}{2x^2}-\frac{3b^5}{x^{\frac{1}{3}}}-\frac{15a^3b^2}{2x^{\frac{4}{3}}}-\frac{10a^2b^3}{x}$
default	$-\frac{15a\,b^4}{2x^{\frac{2}{3}}}-\frac{3a^4b}{x^{\frac{5}{3}}}-\frac{a^5}{2x^2}-\frac{3b^5}{x^{\frac{1}{3}}}-\frac{15a^3b^2}{2x^{\frac{4}{3}}}-\frac{10a^2b^3}{x}$
trager	$\frac{(-1+x)(a^3x+20b^3x+a^3)a^2}{2x^2}-\frac{3(5b^3x+2a^3)ab}{2x^{\frac{5}{3}}}-\frac{3(2b^3x+5a^3)b^2}{2x^{\frac{4}{3}}}$
orering	$-\frac{(190b^9x^3-29a^3b^6x^2+101a^6b^3x+32a^9)\left(a+b\,x^{\frac{1}{3}}\right)^5}{10x^2(b^3x+a^3)^3}-\frac{9x^2(25b^9x^3-6a^3b^6x^2+7a^6b^3x+2a^9)\left(\frac{5\left(a+b\,x^{\frac{1}{3}}\right)^4b}{3x^{\frac{11}{3}}}-\frac{3\left(a \right.}{}\right.}{10(b^3x+a^3)^3}$

input
```
int((a+b*x^(1/3))^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
-15/2*a*b^4/x^(2/3)-3*a^4*b/x^(5/3)-1/2*a^5/x^2-3*b^5/x^(1/3)-15/2*a^3*b^2
/x^(4/3)-10/x*a^2*b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.71

$$\int\frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx=-\frac{20\,a^2b^3x+a^5+3\,(2\,b^5x+5\,a^3b^2)x^{\frac{2}{3}}+3\,(5\,ab^4x+2\,a^4b)x^{\frac{1}{3}}}{2\,x^2}$$

input
```
integrate((a+b*x^(1/3))^5/x^3,x, algorithm="fricas")
```

output
```
-1/2*(20*a^2*b^3*x + a^5 + 3*(2*b^5*x + 5*a^3*b^2)*x^(2/3) + 3*(5*a*b^4*x
+ 2*a^4*b)*x^(1/3))/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = -\frac{a^5}{2x^2}-\frac{3a^4b}{x^{\frac{5}{3}}}-\frac{15a^3b^2}{2x^{\frac{4}{3}}}-\frac{10a^2b^3}{x}-\frac{15ab^4}{2x^{\frac{2}{3}}}-\frac{3b^5}{\sqrt[3]{x}}$$

input
```
integrate((a+b*x**(1/3))**5/x**3,x)
```

output
```
-a**5/(2*x**2) - 3*a**4*b/x**(5/3) - 15*a**3*b**2/(2*x**(4/3)) - 10*a**2*b
**3/x - 15*a*b**4/(2*x**(2/3)) - 3*b**5/x**(1/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = -\frac{6\,b^5x^{\frac{5}{3}}+15\,ab^4x^{\frac{4}{3}}+20\,a^2b^3x+15\,a^3b^2x^{\frac{2}{3}}+6\,a^4bx^{\frac{1}{3}}+a^5}{2\,x^2}$$

input
```
integrate((a+b*x^(1/3))^5/x^3,x, algorithm="maxima")
```

output
```
-1/2*(6*b^5*x^(5/3) + 15*a*b^4*x^(4/3) + 20*a^2*b^3*x + 15*a^3*b^2*x^(2/3)
 + 6*a^4*b*x^(1/3) + a^5)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = -\frac{6\,b^5x^{\frac{5}{3}} + 15\,ab^4x^{\frac{4}{3}} + 20\,a^2b^3x + 15\,a^3b^2x^{\frac{2}{3}} + 6\,a^4bx^{\frac{1}{3}} + a^5}{2\,x^2}$$

input
```
integrate((a+b*x^(1/3))^5/x^3,x, algorithm="giac")
```

output
```
-1/2*(6*b^5*x^(5/3) + 15*a*b^4*x^(4/3) + 20*a^2*b^3*x + 15*a^3*b^2*x^(2/3)
 + 6*a^4*b*x^(1/3) + a^5)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = -\frac{a^5 + 6\,b^5\,x^{5/3} + 20\,a^2\,b^3\,x + 6\,a^4\,b\,x^{1/3} + 15\,a\,b^4\,x^{4/3} + 15\,a^3\,b^2\,x^{2/3}}{2\,x^2}$$

input
```
int((a + b*x^(1/3))^5/x^3,x)
```

output
```
-(a^5 + 6*b^5*x^(5/3) + 20*a^2*b^3*x + 6*a^4*b*x^(1/3) + 15*a*b^4*x^(4/3)
+ 15*a^3*b^2*x^(2/3))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^3}\,dx = \frac{-x^{\frac{2}{3}}a^5 - 20x^{\frac{5}{3}}a^2b^3 - 15x^{\frac{4}{3}}a^3b^2 - 6x^{\frac{7}{3}}b^5 - 6a^4bx - 15a\,b^4x^2}{2x^{\frac{8}{3}}}$$

input
```
int((a+b*x^(1/3))^5/x^3,x)
```

output
```
( - x**(2/3)*a**5 - 20*x**(2/3)*a**2*b**3*x - 15*x**(1/3)*a**3*b**2*x - 6*
x**(1/3)*b**5*x**2 - 6*a**4*b*x - 15*a*b**4*x**2)/(2*x**(2/3)*x**2)
```

3.211 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx = -\frac{a^5}{3x^3}-\frac{15a^4b}{8x^{8/3}}-\frac{30a^3b^2}{7x^{7/3}}-\frac{5a^2b^3}{x^2}-\frac{3ab^4}{x^{5/3}}-\frac{3b^5}{4x^{4/3}}$$

output
```
-1/3*a^5/x^3-15/8*a^4*b/x^(8/3)-30/7*a^3*b^2/x^(7/3)-5*a^2*b^3/x^2-3*a*b^4
/x^(5/3)-3/4*b^5/x^(4/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx = \frac{-56a^5-315a^4b\sqrt[3]{x}-720a^3b^2x^{2/3}-840a^2b^3x-504ab^4x^{4/3}-126b^5x^{5/3}}{168x^3}$$

input
```
Integrate[(a + b*x^(1/3))^5/x^4,x]
```

output
```
(-56*a^5 - 315*a^4*b*x^(1/3) - 720*a^3*b^2*x^(2/3) - 840*a^2*b^3*x - 504*a
*b^4*x^(4/3) - 126*b^5*x^(5/3))/(168*x^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^5}{x^{10/3}}+\frac{5ba^4}{x^3}+\frac{10b^2a^3}{x^{8/3}}+\frac{10b^3a^2}{x^{7/3}}+\frac{5b^4a}{x^2}+\frac{b^5}{x^{5/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^5}{9x^3}-\frac{5a^4b}{8x^{8/3}}-\frac{10a^3b^2}{7x^{7/3}}-\frac{5a^2b^3}{3x^2}-\frac{ab^4}{x^{5/3}}-\frac{b^5}{4x^{4/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^5/x^4,x]
```

output
```
3*(-1/9*a^5/x^3 - (5*a^4*b)/(8*x^(8/3)) - (10*a^3*b^2)/(7*x^(7/3)) - (5*a^
2*b^3)/(3*x^2) - (a*b^4)/x^(5/3) - b^5/(4*x^(4/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{a^{5}}{3x^{3}}-\frac{15a^{4}b}{8x^{\frac{8}{3}}}-\frac{30a^{3}b^{2}}{7x^{\frac{7}{3}}}-\frac{5a^{2}b^{3}}{x^{2}}-\frac{3a\,b^{4}}{x^{\frac{5}{3}}}-\frac{3b^{5}}{4x^{\frac{4}{3}}}$
default	$-\frac{a^{5}}{3x^{3}}-\frac{15a^{4}b}{8x^{\frac{8}{3}}}-\frac{30a^{3}b^{2}}{7x^{\frac{7}{3}}}-\frac{5a^{2}b^{3}}{x^{2}}-\frac{3a\,b^{4}}{x^{\frac{5}{3}}}-\frac{3b^{5}}{4x^{\frac{4}{3}}}$
trager	$\frac{(-1+x)(a^{3}x^{2}+15b^{3}x^{2}+a^{3}x+15b^{3}x+a^{3})a^{2}}{3x^{3}}-\frac{3(8b^{3}x+5a^{3})ab}{8x^{\frac{8}{3}}}-\frac{3(7b^{3}x+40a^{3})b^{2}}{28x^{\frac{7}{3}}}$
orering	$-\frac{(2688b^{9}x^{3}+5161a^{3}b^{6}x^{2}+4445a^{6}b^{3}x+1360a^{9})\left(a+b\,x^{\frac{1}{3}}\right)^{5}}{840x^{3}(b^{3}x+a^{3})^{3}}-\frac{3x^{2}(336b^{9}x^{3}+522a^{3}b^{6}x^{2}+395a^{6}b^{3}x+110a^{9})\left(\frac{5(a}{\ldots}\right.}{560(b^{3}x+a^{3})^{3}}$

input
```
int((a+b*x^(1/3))^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^5/x^3-15/8*a^4*b/x^(8/3)-30/7*a^3*b^2/x^(7/3)-5*a^2*b^3/x^2-3*a*b^4
/x^(5/3)-3/4*b^5/x^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx=-\frac{840\,a^2b^3x+56\,a^5+18\,(7\,b^5x+40\,a^3b^2)x^{\frac{2}{3}}+63\,(8\,ab^4x+5\,a^4b)x^{\frac{1}{3}}}{168\,x^3}$$

input
```
integrate((a+b*x^(1/3))^5/x^4,x, algorithm="fricas")
```

output
```
-1/168*(840*a^2*b^3*x + 56*a^5 + 18*(7*b^5*x + 40*a^3*b^2)*x^(2/3) + 63*(8
*a*b^4*x + 5*a^4*b)*x^(1/3))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx=-\frac{a^5}{3x^3}-\frac{15a^4b}{8x^{\frac{8}{3}}}-\frac{30a^3b^2}{7x^{\frac{7}{3}}}-\frac{5a^2b^3}{x^2}-\frac{3ab^4}{x^{\frac{5}{3}}}-\frac{3b^5}{4x^{\frac{4}{3}}}$$

input
```
integrate((a+b*x**(1/3))**5/x**4,x)
```

output
```
-a**5/(3*x**3) - 15*a**4*b/(8*x**(8/3)) - 30*a**3*b**2/(7*x**(7/3)) - 5*a*
*2*b**3/x**2 - 3*a*b**4/x**(5/3) - 3*b**5/(4*x**(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx =-\frac{126\,b^5x^{\frac{5}{3}}+504\,ab^4x^{\frac{4}{3}}+840\,a^2b^3x+720\,a^3b^2x^{\frac{2}{3}}+315\,a^4bx^{\frac{1}{3}}+56\,a^5}{168\,x^3}$$

input
```
integrate((a+b*x^(1/3))^5/x^4,x, algorithm="maxima")
```

output
```
-1/168*(126*b^5*x^(5/3) + 504*a*b^4*x^(4/3) + 840*a^2*b^3*x + 720*a^3*b^2*
x^(2/3) + 315*a^4*b*x^(1/3) + 56*a^5)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx = -\frac{126\,b^5x^{\frac{5}{3}}+504\,ab^4x^{\frac{4}{3}}+840\,a^2b^3x+720\,a^3b^2x^{\frac{2}{3}}+315\,a^4bx^{\frac{1}{3}}+56\,a^5}{168\,x^3}$$

input
```
integrate((a+b*x^(1/3))^5/x^4,x, algorithm="giac")
```

output
```
-1/168*(126*b^5*x^(5/3) + 504*a*b^4*x^(4/3) + 840*a^2*b^3*x + 720*a^3*b^2*
x^(2/3) + 315*a^4*b*x^(1/3) + 56*a^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx = -\frac{\frac{a^5}{3}+\frac{3\,b^5\,x^{5/3}}{4}+5\,a^2\,b^3\,x+\frac{15\,a^4\,b\,x^{1/3}}{8}+3\,a\,b^4\,x^{4/3}+\frac{30\,a^3\,b^2\,x^{2/3}}{7}}{x^3}$$

input
```
int((a + b*x^(1/3))^5/x^4,x)
```

output
```
-(a^5/3 + (3*b^5*x^(5/3))/4 + 5*a^2*b^3*x + (15*a^4*b*x^(1/3))/8 + 3*a*b^4
*x^(4/3) + (30*a^3*b^2*x^(2/3))/7)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^4}\,dx = \frac{-56x^{\frac{2}{3}}a^5-840x^{\frac{5}{3}}a^2b^3-720x^{\frac{4}{3}}a^3b^2-126x^{\frac{7}{3}}b^5-315a^4bx-504a\,b^4x^2}{168x^{\frac{11}{3}}}$$

input
```
int((a+b*x^(1/3))^5/x^4,x)
```

output
```
( - 56*x**(2/3)*a**5 - 840*x**(2/3)*a**2*b**3*x - 720*x**(1/3)*a**3*b**2*x
 - 126*x**(1/3)*b**5*x**2 - 315*a**4*b*x - 504*a*b**4*x**2)/(168*x**(2/3)*
x**3)
```

3.212 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx=-\frac{a^5}{4x^4}-\frac{15a^4b}{11x^{11/3}}-\frac{3a^3b^2}{x^{10/3}}-\frac{10a^2b^3}{3x^3}-\frac{15ab^4}{8x^{8/3}}-\frac{3b^5}{7x^{7/3}}$$

output
```
-1/4*a^5/x^4-15/11*a^4*b/x^(11/3)-3*a^3*b^2/x^(10/3)-10/3*a^2*b^3/x^3-15/8
*a*b^4/x^(8/3)-3/7*b^5/x^(7/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx =\frac{-462a^5-2520a^4b\sqrt[3]{x}-5544a^3b^2x^{2/3}-6160a^2b^3x-3465ab^4x^{4/3}-792b^5x^{5/3}}{1848x^4}$$

input
```
Integrate[(a + b*x^(1/3))^5/x^5,x]
```

output
```
(-462*a^5 - 2520*a^4*b*x^(1/3) - 5544*a^3*b^2*x^(2/3) - 6160*a^2*b^3*x - 3
465*a*b^4*x^(4/3) - 792*b^5*x^(5/3))/(1848*x^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}dx$$
$$\Big\downarrow \text{798}$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^{13/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{53}$$
$$3\int \left(\frac{a^5}{x^{13/3}}+\frac{5ba^4}{x^4}+\frac{10b^2a^3}{x^{11/3}}+\frac{10b^3a^2}{x^{10/3}}+\frac{5b^4a}{x^3}+\frac{b^5}{x^{8/3}}\right)d\sqrt[3]{x}$$
$$\Big\downarrow \text{2009}$$
$$3\left(-\frac{a^5}{12x^4}-\frac{5a^4b}{11x^{11/3}}-\frac{a^3b^2}{x^{10/3}}-\frac{10a^2b^3}{9x^3}-\frac{5ab^4}{8x^{8/3}}-\frac{b^5}{7x^{7/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^5/x^5,x]
```

output
```
3*(-1/12*a^5/x^4 - (5*a^4*b)/(11*x^(11/3)) - (a^3*b^2)/x^(10/3) - (10*a^2*
b^3)/(9*x^3) - (5*a*b^4)/(8*x^(8/3)) - b^5/(7*x^(7/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{a^{5}}{4 x^{4}}-\frac{15 a^{4} b}{11 x^{\frac{11}{3}}}-\frac{3 a^{3} b^{2}}{x^{\frac{10}{3}}}-\frac{10 a^{2} b^{3}}{3 x^{3}}-\frac{15 a\, b^{4}}{8 x^{\frac{8}{3}}}-\frac{3 b^{5}}{7 x^{\frac{7}{3}}}$
default	$-\frac{a^{5}}{4 x^{4}}-\frac{15 a^{4} b}{11 x^{\frac{11}{3}}}-\frac{3 a^{3} b^{2}}{x^{\frac{10}{3}}}-\frac{10 a^{2} b^{3}}{3 x^{3}}-\frac{15 a\, b^{4}}{8 x^{\frac{8}{3}}}-\frac{3 b^{5}}{7 x^{\frac{7}{3}}}$
trager	$\frac{\left(-1+x \right) \left(3 a^{3} x^{3}+40 b^{3} x^{3}+3 a^{3} x^{2}+40 b^{3} x^{2}+3 a^{3} x +40 b^{3} x +3 a^{3}\right) a^{2}}{12 x^{4}}-\frac{15 \left(11 b^{3} x +8 a^{3}\right) a b}{88 x^{\frac{11}{3}}}-\frac{3 \left(b^{3} x +7 a^{3}\right) b^{2}}{7 x^{\frac{10}{3}}}$
orering	$-\frac{\left(5984 b^{9} x^{3}+14395 a^{3} b^{6} x^{2}+12782 a^{6} b^{3} x +3948 a^{9}\right) \left(a +b \,x^{\frac{1}{3}}\right)^{5}}{3696 x^{4} \left(b^{3} x +a^{3}\right)^{3}}-\frac{3 x^{2} \left(605 b^{9} x^{3}+1275 a^{3} b^{6} x^{2}+1027 a^{6} b^{3} x +294 a^{9}\right)}{3080 \left(b^{3} x +a^{3}\right)^{3}}$

input
```
int((a+b*x^(1/3))^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^5/x^4-15/11*a^4*b/x^(11/3)-3*a^3*b^2/x^(10/3)-10/3*a^2*b^3/x^3-15/8
*a*b^4/x^(8/3)-3/7*b^5/x^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx = -\frac{6160\,a^2b^3x+462\,a^5+792\,(b^5x+7\,a^3b^2)x^{\frac{2}{3}}+315\,(11\,ab^4x+8\,a^4b)x^{\frac{1}{3}}}{1848\,x^4}$$

input
```
integrate((a+b*x^(1/3))^5/x^5,x, algorithm="fricas")
```

output
```
-1/1848*(6160*a^2*b^3*x + 462*a^5 + 792*(b^5*x + 7*a^3*b^2)*x^(2/3) + 315*
(11*a*b^4*x + 8*a^4*b)*x^(1/3))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx = -\frac{a^5}{4x^4}-\frac{15a^4b}{11x^{\frac{11}{3}}}-\frac{3a^3b^2}{x^{\frac{10}{3}}}-\frac{10a^2b^3}{3x^3}-\frac{15ab^4}{8x^{\frac{8}{3}}}-\frac{3b^5}{7x^{\frac{7}{3}}}$$

input
```
integrate((a+b*x**(1/3))**5/x**5,x)
```

output
```
-a**5/(4*x**4) - 15*a**4*b/(11*x**(11/3)) - 3*a**3*b**2/x**(10/3) - 10*a**
2*b**3/(3*x**3) - 15*a*b**4/(8*x**(8/3)) - 3*b**5/(7*x**(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx = -\frac{792\,b^5x^{\frac{5}{3}}+3465\,ab^4x^{\frac{4}{3}}+6160\,a^2b^3x+5544\,a^3b^2x^{\frac{2}{3}}+2520\,a^4bx^{\frac{1}{3}}+462\,a^5}{1848\,x^4}$$

input
```
integrate((a+b*x^(1/3))^5/x^5,x, algorithm="maxima")
```

output
```
-1/1848*(792*b^5*x^(5/3) + 3465*a*b^4*x^(4/3) + 6160*a^2*b^3*x + 5544*a^3*
b^2*x^(2/3) + 2520*a^4*b*x^(1/3) + 462*a^5)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx = -\frac{792\,b^5x^{\frac{5}{3}}+3465\,ab^4x^{\frac{4}{3}}+6160\,a^2b^3x+5544\,a^3b^2x^{\frac{2}{3}}+2520\,a^4bx^{\frac{1}{3}}+462\,a^5}{1848\,x^4}$$

input
```
integrate((a+b*x^(1/3))^5/x^5,x, algorithm="giac")
```

output
```
-1/1848*(792*b^5*x^(5/3) + 3465*a*b^4*x^(4/3) + 6160*a^2*b^3*x + 5544*a^3*
b^2*x^(2/3) + 2520*a^4*b*x^(1/3) + 462*a^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx = -\frac{\frac{a^5}{4}+\frac{3\,b^5\,x^{5/3}}{7}+\frac{10\,a^2\,b^3\,x}{3}+\frac{15\,a^4\,b\,x^{1/3}}{11}+\frac{15\,a\,b^4\,x^{4/3}}{8}+3\,a^3\,b^2\,x^{2/3}}{x^4}$$

input
```
int((a + b*x^(1/3))^5/x^5,x)
```

output
```
-(a^5/4 + (3*b^5*x^(5/3))/7 + (10*a^2*b^3*x)/3 + (15*a^4*b*x^(1/3))/11 + (
15*a*b^4*x^(4/3))/8 + 3*a^3*b^2*x^(2/3))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^5}\,dx$$
$$= \frac{-462x^{\frac{2}{3}}a^5-6160x^{\frac{5}{3}}a^2b^3-5544x^{\frac{4}{3}}a^3b^2-792x^{\frac{7}{3}}b^5-2520a^4bx-3465a\,b^4x^2}{1848x^{\frac{14}{3}}}$$

input
```
int((a+b*x^(1/3))^5/x^5,x)
```

output
```
( - 462*x**(2/3)*a**5 - 6160*x**(2/3)*a**2*b**3*x - 5544*x**(1/3)*a**3*b**
2*x - 792*x**(1/3)*b**5*x**2 - 2520*a**4*b*x - 3465*a*b**4*x**2)/(1848*x**
(2/3)*x**4)
```

3.213 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx=-\frac{a^5}{5x^5}-\frac{15a^4b}{14x^{14/3}}-\frac{30a^3b^2}{13x^{13/3}}-\frac{5a^2b^3}{2x^4}-\frac{15ab^4}{11x^{11/3}}-\frac{3b^5}{10x^{10/3}}$$

output
```
-1/5*a^5/x^5-15/14*a^4*b/x^(14/3)-30/13*a^3*b^2/x^(13/3)-5/2*a^2*b^3/x^4-1
5/11*a*b^4/x^(11/3)-3/10*b^5/x^(10/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx =\frac{-2002a^5-10725a^4b\sqrt[3]{x}-23100a^3b^2x^{2/3}-25025a^2b^3x-13650ab^4x^{4/3}-3003b^5x^{5/3}}{10010x^5}$$

input
```
Integrate[(a + b*x^(1/3))^5/x^6,x]
```

output
```
(-2002*a^5 - 10725*a^4*b*x^(1/3) - 23100*a^3*b^2*x^(2/3) - 25025*a^2*b^3*x
 - 13650*a*b^4*x^(4/3) - 3003*b^5*x^(5/3))/(10010*x^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^{16/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^5}{x^{16/3}}+\frac{5ba^4}{x^5}+\frac{10b^2a^3}{x^{14/3}}+\frac{10b^3a^2}{x^{13/3}}+\frac{5b^4a}{x^4}+\frac{b^5}{x^{11/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^5}{15x^5}-\frac{5a^4b}{14x^{14/3}}-\frac{10a^3b^2}{13x^{13/3}}-\frac{5a^2b^3}{6x^4}-\frac{5ab^4}{11x^{11/3}}-\frac{b^5}{10x^{10/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^5/x^6,x]
```

output
```
3*(-1/15*a^5/x^5 - (5*a^4*b)/(14*x^(14/3)) - (10*a^3*b^2)/(13*x^(13/3)) -
(5*a^2*b^3)/(6*x^4) - (5*a*b^4)/(11*x^(11/3)) - b^5/(10*x^(10/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{a^5}{5x^5}-\frac{15a^4b}{14x^{\frac{14}{3}}}-\frac{30a^3b^2}{13x^{\frac{13}{3}}}-\frac{5a^2b^3}{2x^4}-\frac{15a\,b^4}{11x^{\frac{11}{3}}}-\frac{3b^5}{10x^{\frac{10}{3}}}$
default	$-\frac{a^5}{5x^5}-\frac{15a^4b}{14x^{\frac{14}{3}}}-\frac{30a^3b^2}{13x^{\frac{13}{3}}}-\frac{5a^2b^3}{2x^4}-\frac{15a\,b^4}{11x^{\frac{11}{3}}}-\frac{3b^5}{10x^{\frac{10}{3}}}$
trager	$\frac{(-1+x)(2a^3x^4+25b^3x^4+2a^3x^3+25b^3x^3+2a^3x^2+25b^3x^2+2a^3x+25b^3x+2a^3)a^2}{10x^5}-\frac{15(14b^3x+11a^3)ab}{154x^{\frac{14}{3}}}-\frac{3(13b^3x}{13}$
orering	$-\frac{(106925b^9x^3+278294a^3b^6x^2+254038a^6b^3x+79420a^9)\left(a+b\,x^{\frac{1}{3}}\right)^5}{100100x^5(b^3x+a^3)^3}-\frac{9x^2(3185b^9x^3+7530a^3b^6x^2+6368a^6b^3x+1}{100100(b^3x}$

input
```
int((a+b*x^(1/3))^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^5/x^5-15/14*a^4*b/x^(14/3)-30/13*a^3*b^2/x^(13/3)-5/2*a^2*b^3/x^4-1
5/11*a*b^4/x^(11/3)-3/10*b^5/x^(10/3)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx = -\frac{25025\,a^2b^3x+2002\,a^5+231\,(13\,b^5x+100\,a^3b^2)x^{\frac{2}{3}}+975\,(14\,ab^4x+11\,a^4b)x^{\frac{1}{3}}}{10010\,x^5}$$

input
```
integrate((a+b*x^(1/3))^5/x^6,x, algorithm="fricas")
```

output
```
-1/10010*(25025*a^2*b^3*x + 2002*a^5 + 231*(13*b^5*x + 100*a^3*b^2)*x^(2/3
) + 975*(14*a*b^4*x + 11*a^4*b)*x^(1/3))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx = -\frac{a^5}{5x^5}-\frac{15a^4b}{14x^{\frac{14}{3}}}-\frac{30a^3b^2}{13x^{\frac{13}{3}}}-\frac{5a^2b^3}{2x^4}-\frac{15ab^4}{11x^{\frac{11}{3}}}-\frac{3b^5}{10x^{\frac{10}{3}}}$$

input
```
integrate((a+b*x**(1/3))**5/x**6,x)
```

output
```
-a**5/(5*x**5) - 15*a**4*b/(14*x**(14/3)) - 30*a**3*b**2/(13*x**(13/3)) -
5*a**2*b**3/(2*x**4) - 15*a*b**4/(11*x**(11/3)) - 3*b**5/(10*x**(10/3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx = -\frac{3003\,b^5x^{\frac{5}{3}}+13650\,ab^4x^{\frac{4}{3}}+25025\,a^2b^3x+23100\,a^3b^2x^{\frac{2}{3}}+10725\,a^4bx^{\frac{1}{3}}+2002\,a^5}{10010\,x^5}$$

input
```
integrate((a+b*x^(1/3))^5/x^6,x, algorithm="maxima")
```

output
```
-1/10010*(3003*b^5*x^(5/3) + 13650*a*b^4*x^(4/3) + 25025*a^2*b^3*x + 23100
*a^3*b^2*x^(2/3) + 10725*a^4*b*x^(1/3) + 2002*a^5)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx = -\frac{3003\,b^5x^{\frac{5}{3}}+13650\,ab^4x^{\frac{4}{3}}+25025\,a^2b^3x+23100\,a^3b^2x^{\frac{2}{3}}+10725\,a^4bx^{\frac{1}{3}}+2002\,a^5}{10010\,x^5}$$

input
```
integrate((a+b*x^(1/3))^5/x^6,x, algorithm="giac")
```

output
```
-1/10010*(3003*b^5*x^(5/3) + 13650*a*b^4*x^(4/3) + 25025*a^2*b^3*x + 23100
*a^3*b^2*x^(2/3) + 10725*a^4*b*x^(1/3) + 2002*a^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx = -\frac{\frac{a^5}{5}+\frac{3\,b^5\,x^{5/3}}{10}+\frac{5\,a^2\,b^3\,x}{2}+\frac{15\,a^4\,b\,x^{1/3}}{14}+\frac{15\,a\,b^4\,x^{4/3}}{11}+\frac{30\,a^3\,b^2\,x^{2/3}}{13}}{x^5}$$

input
```
int((a + b*x^(1/3))^5/x^6,x)
```

output
```
-(a^5/5 + (3*b^5*x^(5/3))/10 + (5*a^2*b^3*x)/2 + (15*a^4*b*x^(1/3))/14 + (
15*a*b^4*x^(4/3))/11 + (30*a^3*b^2*x^(2/3))/13)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^6}\,dx$$
$$= \frac{-2002x^{\frac{2}{3}}a^5-25025x^{\frac{5}{3}}a^2b^3-23100x^{\frac{4}{3}}a^3b^2-3003x^{\frac{7}{3}}b^5-10725a^4bx-13650a\,b^4x^2}{10010x^{\frac{17}{3}}}$$

input
```
int((a+b*x^(1/3))^5/x^6,x)
```

output
```
( - 2002*x**(2/3)*a**5 - 25025*x**(2/3)*a**2*b**3*x - 23100*x**(1/3)*a**3*
b**2*x - 3003*x**(1/3)*b**5*x**2 - 10725*a**4*b*x - 13650*a*b**4*x**2)/(10
010*x**(2/3)*x**5)
```

3.214 $\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx=-\frac{a^5}{6x^6}-\frac{15a^4b}{17x^{17/3}}-\frac{15a^3b^2}{8x^{16/3}}-\frac{2a^2b^3}{x^5}-\frac{15ab^4}{14x^{14/3}}-\frac{3b^5}{13x^{13/3}}$$

output
```
-1/6*a^5/x^6-15/17*a^4*b/x^(17/3)-15/8*a^3*b^2/x^(16/3)-2*a^2*b^3/x^5-15/1
4*a*b^4/x^(14/3)-3/13*b^5/x^(13/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx =\frac{-6188a^5-32760a^4b\sqrt[3]{x}-69615a^3b^2x^{2/3}-74256a^2b^3x-39780ab^4x^{4/3}-8568b^5x^{5/3}}{37128x^6}$$

input
```
Integrate[(a + b*x^(1/3))^5/x^7,x]
```

output
```
(-6188*a^5 - 32760*a^4*b*x^(1/3) - 69615*a^3*b^2*x^(2/3) - 74256*a^2*b^3*x
 - 39780*a*b^4*x^(4/3) - 8568*b^5*x^(5/3))/(37128*x^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^{19/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^5}{x^{19/3}}+\frac{5ba^4}{x^6}+\frac{10b^2a^3}{x^{17/3}}+\frac{10b^3a^2}{x^{16/3}}+\frac{5b^4a}{x^5}+\frac{b^5}{x^{14/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^5}{18x^6}-\frac{5a^4b}{17x^{17/3}}-\frac{5a^3b^2}{8x^{16/3}}-\frac{2a^2b^3}{3x^5}-\frac{5ab^4}{14x^{14/3}}-\frac{b^5}{13x^{13/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^5/x^7,x]
```

output
```
3*(-1/18*a^5/x^6 - (5*a^4*b)/(17*x^(17/3)) - (5*a^3*b^2)/(8*x^(16/3)) - (2
*a^2*b^3)/(3*x^5) - (5*a*b^4)/(14*x^(14/3)) - b^5/(13*x^(13/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{a^5}{6x^6}-\frac{15a^4b}{17x^{\frac{17}{3}}}-\frac{15a^3b^2}{8x^{\frac{16}{3}}}-\frac{2a^2b^3}{x^5}-\frac{15a\,b^4}{14x^{\frac{14}{3}}}-\frac{3b^5}{13x^{\frac{13}{3}}}$
default	$-\frac{a^5}{6x^6}-\frac{15a^4b}{17x^{\frac{17}{3}}}-\frac{15a^3b^2}{8x^{\frac{16}{3}}}-\frac{2a^2b^3}{x^5}-\frac{15a\,b^4}{14x^{\frac{14}{3}}}-\frac{3b^5}{13x^{\frac{13}{3}}}$
trager	$\frac{(-1+x)(a^3x^5+12b^3x^5+a^3x^4+12b^3x^4+a^3x^3+12b^3x^3+a^3x^2+12b^3x^2+a^3x+12b^3x+a^3)a^2}{6x^6}-\frac{15(17b^3x+14a^3)ab}{238x^{\frac{17}{3}}}-$
orering	$-\frac{(147288b^9x^3+398515a^3b^6x^2+370475a^6b^3x+116935a^9)\left(a+b\,x^{\frac{1}{3}}\right)^5}{185640x^6(b^3x+a^3)^3}-\frac{3x^2(3468b^9x^3+8703a^3b^6x^2+7600a^6b^3x+}{61880(b^3}$

input
```
int((a+b*x^(1/3))^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^5/x^6-15/17*a^4*b/x^(17/3)-15/8*a^3*b^2/x^(16/3)-2*a^2*b^3/x^5-15/1
4*a*b^4/x^(14/3)-3/13*b^5/x^(13/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx = -\frac{74256\,a^2b^3x+6188\,a^5+1071\,(8\,b^5x+65\,a^3b^2)x^{\frac{2}{3}}+2340\,(17\,ab^4x+14\,a^4b)x^{\frac{1}{3}}}{37128\,x^6}$$

input
```
integrate((a+b*x^(1/3))^5/x^7,x, algorithm="fricas")
```

output
```
-1/37128*(74256*a^2*b^3*x + 6188*a^5 + 1071*(8*b^5*x + 65*a^3*b^2)*x^(2/3)
 + 2340*(17*a*b^4*x + 14*a^4*b)*x^(1/3))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx = -\frac{a^5}{6x^6}-\frac{15a^4b}{17x^{\frac{17}{3}}}-\frac{15a^3b^2}{8x^{\frac{16}{3}}}-\frac{2a^2b^3}{x^5}-\frac{15ab^4}{14x^{\frac{14}{3}}}-\frac{3b^5}{13x^{\frac{13}{3}}}$$

input
```
integrate((a+b*x**(1/3))**5/x**7,x)
```

output
```
-a**5/(6*x**6) - 15*a**4*b/(17*x**(17/3)) - 15*a**3*b**2/(8*x**(16/3)) - 2
*a**2*b**3/x**5 - 15*a*b**4/(14*x**(14/3)) - 3*b**5/(13*x**(13/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx = -\frac{8568\,b^5x^{\frac{5}{3}}+39780\,ab^4x^{\frac{4}{3}}+74256\,a^2b^3x+69615\,a^3b^2x^{\frac{2}{3}}+32760\,a^4bx^{\frac{1}{3}}+6188\,a^5}{37128\,x^6}$$

input
```
integrate((a+b*x^(1/3))^5/x^7,x, algorithm="maxima")
```

output
```
-1/37128*(8568*b^5*x^(5/3) + 39780*a*b^4*x^(4/3) + 74256*a^2*b^3*x + 69615
*a^3*b^2*x^(2/3) + 32760*a^4*b*x^(1/3) + 6188*a^5)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx = -\frac{8568\,b^5x^{\frac{5}{3}}+39780\,ab^4x^{\frac{4}{3}}+74256\,a^2b^3x+69615\,a^3b^2x^{\frac{2}{3}}+32760\,a^4bx^{\frac{1}{3}}+6188\,a^5}{37128\,x^6}$$

input
```
integrate((a+b*x^(1/3))^5/x^7,x, algorithm="giac")
```

output
```
-1/37128*(8568*b^5*x^(5/3) + 39780*a*b^4*x^(4/3) + 74256*a^2*b^3*x + 69615
*a^3*b^2*x^(2/3) + 32760*a^4*b*x^(1/3) + 6188*a^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx = -\frac{\frac{a^5}{6}+\frac{3\,b^5\,x^{5/3}}{13}+2\,a^2\,b^3\,x+\frac{15\,a^4\,b\,x^{1/3}}{17}+\frac{15\,a\,b^4\,x^{4/3}}{14}+\frac{15\,a^3\,b^2\,x^{2/3}}{8}}{x^6}$$

input
```
int((a + b*x^(1/3))^5/x^7,x)
```

output
```
-(a^5/6 + (3*b^5*x^(5/3))/13 + 2*a^2*b^3*x + (15*a^4*b*x^(1/3))/17 + (15*a
*b^4*x^(4/3))/14 + (15*a^3*b^2*x^(2/3))/8)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^5}{x^7}\,dx$$
$$= \frac{-6188x^{\frac{2}{3}}a^5 - 74256x^{\frac{5}{3}}a^2b^3 - 69615x^{\frac{4}{3}}a^3b^2 - 8568x^{\frac{7}{3}}b^5 - 32760a^4bx - 39780a\,b^4x^2}{37128x^{\frac{20}{3}}}$$

input
```
int((a+b*x^(1/3))^5/x^7,x)
```

output
```
( - 6188*x**(2/3)*a**5 - 74256*x**(2/3)*a**2*b**3*x - 69615*x**(1/3)*a**3*
b**2*x - 8568*x**(1/3)*b**5*x**2 - 32760*a**4*b*x - 39780*a*b**4*x**2)/(37
128*x**(2/3)*x**6)
```

3.215 $\int \left(a+b\sqrt[3]{x}\right)^{10} x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{10} x^4 \, dx = {} & \frac{a^{10}x^5}{5}+\frac{15}{8}a^9bx^{16/3}+\frac{135}{17}a^8b^2x^{17/3}+20a^7b^3x^6 \\ & +\frac{630}{19}a^6b^4x^{19/3}+\frac{189}{5}a^5b^5x^{20/3}+30a^4b^6x^7 \\ & +\frac{180}{11}a^3b^7x^{22/3}+\frac{135}{23}a^2b^8x^{23/3}+\frac{5}{4}ab^9x^8+\frac{3}{25}b^{10}x^{25/3}\end{aligned}$$

output
```
1/5*a^10*x^5+15/8*a^9*b*x^(16/3)+135/17*a^8*b^2*x^(17/3)+20*a^7*b^3*x^6+63
0/19*a^6*b^4*x^(19/3)+189/5*a^5*b^5*x^(20/3)+30*a^4*b^6*x^7+180/11*a^3*b^7
*x^(22/3)+135/23*a^2*b^8*x^(23/3)+5/4*a*b^9*x^8+3/25*b^10*x^(25/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^4 \, dx$$

$$= \frac{3268760a^{10}x^5+30644625a^9bx^{16/3}+129789000a^8b^2x^{17/3}+326876000a^7b^3x^6+541926000a^6b^4x^{19/3}+6}{}$$

input
```
Integrate[(a + b*x^(1/3))^10*x^4,x]
```

output
```
(3268760*a^10*x^5 + 30644625*a^9*b*x^(16/3) + 129789000*a^8*b^2*x^(17/3) +
 326876000*a^7*b^3*x^6 + 541926000*a^6*b^4*x^(19/3) + 617795640*a^5*b^5*x^
(20/3) + 490314000*a^4*b^6*x^7 + 267444000*a^3*b^7*x^(22/3) + 95931000*a^2
*b^8*x^(23/3) + 20429750*a*b^9*x^8 + 1961256*b^10*x^(25/3))/16343800
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\sqrt[3]{x}\right)^{10} dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{10} x^{14/3} d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \Big(x^{14/3}a^{10}+10bx^5a^9+45b^2x^{16/3}a^8+120b^3x^{17/3}a^7+210b^4x^6a^6+252b^5x^{19/3}a^5+210b^6x^{20/3}a^4+120b^7x^7a$$

$$\Big\downarrow \text{2009}$$

$$3\bigg(\frac{a^{10}x^5}{15}+\frac{5}{8}a^9bx^{16/3}+\frac{45}{17}a^8b^2x^{17/3}+\frac{20}{3}a^7b^3x^6+\frac{210}{19}a^6b^4x^{19/3}+\frac{63}{5}a^5b^5x^{20/3}+10a^4b^6x^7+\frac{60}{11}a^3b^7x^{22/3}+\frac{4}{2}$$

input
```
Int[(a + b*x^(1/3))^10*x^4,x]
```

output
```
3*((a^10*x^5)/15 + (5*a^9*b*x^(16/3))/8 + (45*a^8*b^2*x^(17/3))/17 + (20*a
^7*b^3*x^6)/3 + (210*a^6*b^4*x^(19/3))/19 + (63*a^5*b^5*x^(20/3))/5 + 10*a
^4*b^6*x^7 + (60*a^3*b^7*x^(22/3))/11 + (45*a^2*b^8*x^(23/3))/23 + (5*a*b^
9*x^8)/12 + (b^10*x^(25/3))/25)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^{10}x^5}{5}+\frac{15a^9b\,x^{\frac{16}{3}}}{8}+\frac{135a^8b^2x^{\frac{17}{3}}}{17}+20a^7b^3x^6+\frac{630a^6b^4x^{\frac{19}{3}}}{19}+\frac{189a^5b^5x^{\frac{20}{3}}}{5}+30a^4b^6x^7+\frac{180a^3b^7}{11}$
default	$\frac{a^{10}x^5}{5}+\frac{15a^9b\,x^{\frac{16}{3}}}{8}+\frac{135a^8b^2x^{\frac{17}{3}}}{17}+20a^7b^3x^6+\frac{630a^6b^4x^{\frac{19}{3}}}{19}+\frac{189a^5b^5x^{\frac{20}{3}}}{5}+30a^4b^6x^7+\frac{180a^3b^7}{11}$
trager	$a\left(25b^9x^7+600x^6a^3b^6+25b^9x^6+400a^6b^3x^5+600a^3b^6x^5+25b^9x^5+4a^9x^4+400a^6b^3x^4+600a^3b^6x^4+25b^9x^4+4a^9x^3+4\right.$
orering	$(27002800b^{39}x^{13}+219460412a^3b^{36}x^{12}+782727824a^6b^{33}x^{11}+1601733910a^9b^{30}x^{10}+2059980272a^{12}b^{27}x^9+17090423$

input
```
int((a+b*x^(1/3))^10*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^10*x^5+15/8*a^9*b*x^(16/3)+135/17*a^8*b^2*x^(17/3)+20*a^7*b^3*x^6+63
0/19*a^6*b^4*x^(19/3)+189/5*a^5*b^5*x^(20/3)+30*a^4*b^6*x^7+180/11*a^3*b^7
*x^(22/3)+135/23*a^2*b^8*x^(23/3)+5/4*a*b^9*x^8+3/25*b^10*x^(25/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^4\,dx$$
$$=\frac{5}{4}\,ab^9x^8+30\,a^4b^6x^7+20\,a^7b^3x^6+\frac{1}{5}\,a^{10}x^5+\frac{27}{1955}\,\left(425\,a^2b^8x^7+2737\,a^5b^5x^6+575\,a^8b^2x^5\right)x^{\frac{2}{3}}+\frac{3}{41800}\,\left(1672\,b^{10}x^8+228000\,a^3b^7x^7+462000\,a^6b^4x^6+26125\,a^9bx^5\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^10*x^4,x, algorithm="fricas")
```

output
```
5/4*a*b^9*x^8 + 30*a^4*b^6*x^7 + 20*a^7*b^3*x^6 + 1/5*a^10*x^5 + 27/1955*(
425*a^2*b^8*x^7 + 2737*a^5*b^5*x^6 + 575*a^8*b^2*x^5)*x^(2/3) + 3/41800*(1
672*b^10*x^8 + 228000*a^3*b^7*x^7 + 462000*a^6*b^4*x^6 + 26125*a^9*b*x^5)*
x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^4\,dx=\frac{a^{10}x^5}{5}+\frac{15a^9bx^{\frac{16}{3}}}{8}+\frac{135a^8b^2x^{\frac{17}{3}}}{17}+20a^7b^3x^6+\frac{630a^6b^4x^{\frac{19}{3}}}{19}+\frac{189a^5b^5x^{\frac{20}{3}}}{5}+30a^4b^6x^7+\frac{180a^3b^7x^{\frac{22}{3}}}{11}+\frac{135a^2b^8x^{\frac{23}{3}}}{23}+\frac{5ab^9x^8}{4}+\frac{3b^{10}x^{\frac{25}{3}}}{25}$$

input
```
integrate((a+b*x**(1/3))**10*x**4,x)
```

output
```
a**10*x**5/5 + 15*a**9*b*x**(16/3)/8 + 135*a**8*b**2*x**(17/3)/17 + 20*a**
7*b**3*x**6 + 630*a**6*b**4*x**(19/3)/19 + 189*a**5*b**5*x**(20/3)/5 + 30*
a**4*b**6*x**7 + 180*a**3*b**7*x**(22/3)/11 + 135*a**2*b**8*x**(23/3)/23 +
 5*a*b**9*x**8/4 + 3*b**10*x**(25/3)/25
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(112) = 224$.

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.74

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^4\,dx = \frac{3\left(bx^{\frac{1}{3}}+a\right)^{25}}{25\,b^{15}} - \frac{7\left(bx^{\frac{1}{3}}+a\right)^{24}a}{4\,b^{15}} + \frac{273\left(bx^{\frac{1}{3}}+a\right)^{23}a^2}{23\,b^{15}} - \frac{546\left(bx^{\frac{1}{3}}+a\right)^{22}a^3}{11\,b^{15}} + \frac{143\left(bx^{\frac{1}{3}}+a\right)^{21}a^4}{b^{15}} - \frac{3003\left(bx^{\frac{1}{3}}+a\right)^{20}a^5}{10\,b^{15}} + \frac{9009\left(bx^{\frac{1}{3}}+a\right)^{19}a^6}{19\,b^{15}} - \frac{572\left(bx^{\frac{1}{3}}+a\right)^{18}a^7}{b^{15}} + \frac{9009\left(bx^{\frac{1}{3}}+a\right)^{17}a^8}{17\,b^{15}} - \frac{3003\left(bx^{\frac{1}{3}}+a\right)^{16}a^9}{8\,b^{15}} + \frac{1001\left(bx^{\frac{1}{3}}+a\right)^{15}a^{10}}{5\,b^{15}} - \frac{78\left(bx^{\frac{1}{3}}+a\right)^{14}a^{11}}{b^{15}} + \frac{21\left(bx^{\frac{1}{3}}+a\right)^{13}a^{12}}{b^{15}} - \frac{7\left(bx^{\frac{1}{3}}+a\right)^{12}a^{13}}{2\,b^{15}} + \frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}a^{14}}{11\,b^{15}}$$

input
```
integrate((a+b*x^(1/3))^10*x^4,x, algorithm="maxima")
```

output
```
3/25*(b*x^(1/3) + a)^25/b^15 - 7/4*(b*x^(1/3) + a)^24*a/b^15 + 273/23*(b*x
^(1/3) + a)^23*a^2/b^15 - 546/11*(b*x^(1/3) + a)^22*a^3/b^15 + 143*(b*x^(1
/3) + a)^21*a^4/b^15 - 3003/10*(b*x^(1/3) + a)^20*a^5/b^15 + 9009/19*(b*x^
(1/3) + a)^19*a^6/b^15 - 572*(b*x^(1/3) + a)^18*a^7/b^15 + 9009/17*(b*x^(1
/3) + a)^17*a^8/b^15 - 3003/8*(b*x^(1/3) + a)^16*a^9/b^15 + 1001/5*(b*x^(1
/3) + a)^15*a^10/b^15 - 78*(b*x^(1/3) + a)^14*a^11/b^15 + 21*(b*x^(1/3) +
a)^13*a^12/b^15 - 7/2*(b*x^(1/3) + a)^12*a^13/b^15 + 3/11*(b*x^(1/3) + a)^
11*a^14/b^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^4\,dx = \frac{3}{25}\,b^{10}x^{\frac{25}{3}}+\frac{5}{4}\,ab^9x^8+\frac{135}{23}\,a^2b^8x^{\frac{23}{3}}+\frac{180}{11}\,a^3b^7x^{\frac{22}{3}} + 30\,a^4b^6x^7+\frac{189}{5}\,a^5b^5x^{\frac{20}{3}}+\frac{630}{19}\,a^6b^4x^{\frac{19}{3}} + 20\,a^7b^3x^6+\frac{135}{17}\,a^8b^2x^{\frac{17}{3}}+\frac{15}{8}\,a^9bx^{\frac{16}{3}}+\frac{1}{5}\,a^{10}x^5$$

input
```
integrate((a+b*x^(1/3))^10*x^4,x, algorithm="giac")
```

output
```
3/25*b^10*x^(25/3) + 5/4*a*b^9*x^8 + 135/23*a^2*b^8*x^(23/3) + 180/11*a^3*
b^7*x^(22/3) + 30*a^4*b^6*x^7 + 189/5*a^5*b^5*x^(20/3) + 630/19*a^6*b^4*x^
(19/3) + 20*a^7*b^3*x^6 + 135/17*a^8*b^2*x^(17/3) + 15/8*a^9*b*x^(16/3) +
1/5*a^10*x^5
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^4\,dx = \frac{a^{10}\,x^5}{5}+\frac{3\,b^{10}\,x^{25/3}}{25}+\frac{5\,a\,b^9\,x^8}{4}+\frac{15\,a^9\,b\,x^{16/3}}{8}+20\,a^7\,b^3\,x^6 + 30\,a^4\,b^6\,x^7+\frac{135\,a^8\,b^2\,x^{17/3}}{17}+\frac{630\,a^6\,b^4\,x^{19/3}}{19} + \frac{189\,a^5\,b^5\,x^{20/3}}{5}+\frac{180\,a^3\,b^7\,x^{22/3}}{11}+\frac{135\,a^2\,b^8\,x^{23/3}}{23}$$

input
```
int(x^4*(a + b*x^(1/3))^10,x)
```

output
```
(a^10*x^5)/5 + (3*b^10*x^(25/3))/25 + (5*a*b^9*x^8)/4 + (15*a^9*b*x^(16/3)
)/8 + 20*a^7*b^3*x^6 + 30*a^4*b^6*x^7 + (135*a^8*b^2*x^(17/3))/17 + (630*a
^6*b^4*x^(19/3))/19 + (189*a^5*b^5*x^(20/3))/5 + (180*a^3*b^7*x^(22/3))/11
 + (135*a^2*b^8*x^(23/3))/23
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^4\,dx$$
$$= \frac{x^5\left(129789000x^{\frac{2}{3}}a^8b^2+617795640x^{\frac{5}{3}}a^5b^5+95931000x^{\frac{8}{3}}a^2b^8+30644625x^{\frac{1}{3}}a^9b+541926000x^{\frac{4}{3}}a^6b^4+2\cdots\right.}{163\cdots}$$

input
```
int((a+b*x^(1/3))^10*x^4,x)
```

output
```
(x**5*(129789000*x**(2/3)*a**8*b**2 + 617795640*x**(2/3)*a**5*b**5*x + 959
31000*x**(2/3)*a**2*b**8*x**2 + 30644625*x**(1/3)*a**9*b + 541926000*x**(1
/3)*a**6*b**4*x + 267444000*x**(1/3)*a**3*b**7*x**2 + 1961256*x**(1/3)*b**
10*x**3 + 3268760*a**10 + 326876000*a**7*b**3*x + 490314000*a**4*b**6*x**2
 + 20429750*a*b**9*x**3))/16343800
```

3.216 $\int \left(a+b\sqrt[3]{x}\right)^{10} x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^3\,dx = \frac{a^{10}x^4}{4}+\frac{30}{13}a^9bx^{13/3}+\frac{135}{14}a^8b^2x^{14/3}+24a^7b^3x^5+\frac{315}{8}a^6b^4x^{16/3}+\frac{756}{17}a^5b^5x^{17/3}+35a^4b^6x^6+\frac{360}{19}a^3b^7x^{19/3}+\frac{27}{4}a^2b^8x^{20/3}+\frac{10}{7}ab^9x^7+\frac{3}{22}b^{10}x^{22/3}$$

output
```
1/4*a^10*x^4+30/13*a^9*b*x^(13/3)+135/14*a^8*b^2*x^(14/3)+24*a^7*b^3*x^5+3
15/8*a^6*b^4*x^(16/3)+756/17*a^5*b^5*x^(17/3)+35*a^4*b^6*x^6+360/19*a^3*b^
7*x^(19/3)+27/4*a^2*b^8*x^(20/3)+10/7*a*b^9*x^7+3/22*b^10*x^(22/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^3\,dx = \frac{646646a^{10}x^4+5969040a^9bx^{13/3}+24942060a^8b^2x^{14/3}+62078016a^7b^3x^5+101846745a^6b^4x^{16/3}+115020\cdots}{2\cdots}$$

input
```
Integrate[(a + b*x^(1/3))^10*x^3,x]
```

output
```
(646646*a^10*x^4 + 5969040*a^9*b*x^(13/3) + 24942060*a^8*b^2*x^(14/3) + 62
078016*a^7*b^3*x^5 + 101846745*a^6*b^4*x^(16/3) + 115026912*a^5*b^5*x^(17/
3) + 90530440*a^4*b^6*x^6 + 49008960*a^3*b^7*x^(19/3) + 17459442*a^2*b^8*x
^(20/3) + 3695120*a*b^9*x^7 + 352716*b^10*x^(22/3))/2586584
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt[3]{x}\right)^{10}\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{10}x^{11/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x^{11/3}a^{10}+10bx^4a^9+45b^2x^{13/3}a^8+120b^3x^{14/3}a^7+210b^4x^5a^6+252b^5x^{16/3}a^5+210b^6x^{17/3}a^4+120b^7x^6a^3\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^{10}x^4}{12}+\frac{10}{13}a^9bx^{13/3}+\frac{45}{14}a^8b^2x^{14/3}+8a^7b^3x^5+\frac{105}{8}a^6b^4x^{16/3}+\frac{252}{17}a^5b^5x^{17/3}+\frac{35}{3}a^4b^6x^6+\frac{120}{19}a^3b^7x^{19/3}+\right.$$

input
```
Int[(a + b*x^(1/3))^10*x^3,x]
```

output
```
3*((a^10*x^4)/12 + (10*a^9*b*x^(13/3))/13 + (45*a^8*b^2*x^(14/3))/14 + 8*a
^7*b^3*x^5 + (105*a^6*b^4*x^(16/3))/8 + (252*a^5*b^5*x^(17/3))/17 + (35*a^
4*b^6*x^6)/3 + (120*a^3*b^7*x^(19/3))/19 + (9*a^2*b^8*x^(20/3))/4 + (10*a*
b^9*x^7)/21 + (b^10*x^(22/3))/22)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{a^{10}x^4}{4}+\frac{30a^9b\,x^{\frac{13}{3}}}{13}+\frac{135a^8b^2x^{\frac{14}{3}}}{14}+24a^7b^3x^5+\frac{315a^6b^4x^{\frac{16}{3}}}{8}+\frac{756a^5b^5x^{\frac{17}{3}}}{17}+35a^4b^6x^6+\frac{360a^3b^7}{19}$
default	$\frac{a^{10}x^4}{4}+\frac{30a^9b\,x^{\frac{13}{3}}}{13}+\frac{135a^8b^2x^{\frac{14}{3}}}{14}+24a^7b^3x^5+\frac{315a^6b^4x^{\frac{16}{3}}}{8}+\frac{756a^5b^5x^{\frac{17}{3}}}{17}+35a^4b^6x^6+\frac{360a^3b^7}{19}$
trager	$\frac{a(40b^9x^6+980a^3b^6x^5+40b^9x^5+672a^6b^3x^4+980a^3b^6x^4+40b^9x^4+7a^9x^3+672a^6b^3x^3+980a^3b^6x^3+40b^9x^3+7a^9x^2+6}{28}$
orering	$\frac{(4795258b^{36}x^{12}+38923300a^3b^{33}x^{11}+138659729a^6b^{30}x^{10}+283479367a^9b^{27}x^9+364445277a^{12}b^{24}x^8+302627820a^{15}b}{}$

input
```
int((a+b*x^(1/3))^10*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^10*x^4+30/13*a^9*b*x^(13/3)+135/14*a^8*b^2*x^(14/3)+24*a^7*b^3*x^5+3
15/8*a^6*b^4*x^(16/3)+756/17*a^5*b^5*x^(17/3)+35*a^4*b^6*x^6+360/19*a^3*b^
7*x^(19/3)+27/4*a^2*b^8*x^(20/3)+10/7*a*b^9*x^7+3/22*b^10*x^(22/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

$$\int \left(a + b\sqrt[3]{x}\right)^{10} x^3 \, dx = \frac{10}{7} \, a b^{9} x^{7} + 35 \, a^{4} b^{6} x^{6} + 24 \, a^{7} b^{3} x^{5} + \frac{1}{4} \, a^{10} x^{4} + \frac{27}{476} \, {\left(119 \, a^{2} b^{8} x^{6} + 784 \, a^{5} b^{5} x^{5} + 170 \, a^{8} b^{2} x^{4}\right)} x^{\frac{2}{3}} + \frac{3}{21736} \, {\left(988 \, b^{10} x^{7} + 137280 \, a^{3} b^{7} x^{6} + 285285 \, a^{6} b^{4} x^{5} + 16720 \, a^{9} b x^{4}\right)} x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^10*x^3,x, algorithm="fricas")
```

output
```
10/7*a*b^9*x^7 + 35*a^4*b^6*x^6 + 24*a^7*b^3*x^5 + 1/4*a^10*x^4 + 27/476*(
119*a^2*b^8*x^6 + 784*a^5*b^5*x^5 + 170*a^8*b^2*x^4)*x^(2/3) + 3/21736*(98
8*b^10*x^7 + 137280*a^3*b^7*x^6 + 285285*a^6*b^4*x^5 + 16720*a^9*b*x^4)*x^
(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

$$\int \left(a + b\sqrt[3]{x}\right)^{10} x^3 \, dx = \frac{a^{10}x^{4}}{4} + \frac{30a^{9}bx^{\frac{13}{3}}}{13} + \frac{135a^{8}b^{2}x^{\frac{14}{3}}}{14} + 24a^{7}b^{3}x^{5} + \frac{315a^{6}b^{4}x^{\frac{16}{3}}}{8} + \frac{756a^{5}b^{5}x^{\frac{17}{3}}}{17} + 35a^{4}b^{6}x^{6} + \frac{360a^{3}b^{7}x^{\frac{19}{3}}}{19} + \frac{27a^{2}b^{8}x^{\frac{20}{3}}}{4} + \frac{10ab^{9}x^{7}}{7} + \frac{3b^{10}x^{\frac{22}{3}}}{22}$$

input
```
integrate((a+b*x**(1/3))**10*x**3,x)
```

output
```
a**10*x**4/4 + 30*a**9*b*x**(13/3)/13 + 135*a**8*b**2*x**(14/3)/14 + 24*a*
*7*b**3*x**5 + 315*a**6*b**4*x**(16/3)/8 + 756*a**5*b**5*x**(17/3)/17 + 35
*a**4*b**6*x**6 + 360*a**3*b**7*x**(19/3)/19 + 27*a**2*b**8*x**(20/3)/4 +
10*a*b**9*x**7/7 + 3*b**10*x**(22/3)/22
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.39

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^3\,dx = \frac{3\left(bx^{\frac{1}{3}}+a\right)^{22}}{22\,b^{12}} - \frac{11\left(bx^{\frac{1}{3}}+a\right)^{21}a}{7\,b^{12}} + \frac{33\left(bx^{\frac{1}{3}}+a\right)^{20}a^2}{4\,b^{12}} - \frac{495\left(bx^{\frac{1}{3}}+a\right)^{19}a^3}{19\,b^{12}} + \frac{55\left(bx^{\frac{1}{3}}+a\right)^{18}a^4}{b^{12}} - \frac{1386\left(bx^{\frac{1}{3}}+a\right)^{17}a^5}{17\,b^{12}} + \frac{693\left(bx^{\frac{1}{3}}+a\right)^{16}a^6}{8\,b^{12}} - \frac{66\left(bx^{\frac{1}{3}}+a\right)^{15}a^7}{b^{12}} + \frac{495\left(bx^{\frac{1}{3}}+a\right)^{14}a^8}{14\,b^{12}} - \frac{165\left(bx^{\frac{1}{3}}+a\right)^{13}a^9}{13\,b^{12}} + \frac{11\left(bx^{\frac{1}{3}}+a\right)^{12}a^{10}}{4\,b^{12}} - \frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}a^{11}}{11\,b^{12}}$$

input
```
integrate((a+b*x^(1/3))^10*x^3,x, algorithm="maxima")
```

output
```
3/22*(b*x^(1/3) + a)^22/b^12 - 11/7*(b*x^(1/3) + a)^21*a/b^12 + 33/4*(b*x^
(1/3) + a)^20*a^2/b^12 - 495/19*(b*x^(1/3) + a)^19*a^3/b^12 + 55*(b*x^(1/3
) + a)^18*a^4/b^12 - 1386/17*(b*x^(1/3) + a)^17*a^5/b^12 + 693/8*(b*x^(1/3
) + a)^16*a^6/b^12 - 66*(b*x^(1/3) + a)^15*a^7/b^12 + 495/14*(b*x^(1/3) +
a)^14*a^8/b^12 - 165/13*(b*x^(1/3) + a)^13*a^9/b^12 + 11/4*(b*x^(1/3) + a)
^12*a^10/b^12 - 3/11*(b*x^(1/3) + a)^11*a^11/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^3\,dx = \frac{3}{22}\,b^{10}x^{\frac{22}{3}} + \frac{10}{7}\,ab^9x^7 + \frac{27}{4}\,a^2b^8x^{\frac{20}{3}} + \frac{360}{19}\,a^3b^7x^{\frac{19}{3}} + 35\,a^4b^6x^6 + \frac{756}{17}\,a^5b^5x^{\frac{17}{3}} + \frac{315}{8}\,a^6b^4x^{\frac{16}{3}} + 24\,a^7b^3x^5 + \frac{135}{14}\,a^8b^2x^{\frac{14}{3}} + \frac{30}{13}\,a^9bx^{\frac{13}{3}} + \frac{1}{4}\,a^{10}x^4$$

input
```
integrate((a+b*x^(1/3))^10*x^3,x, algorithm="giac")
```

output
```
3/22*b^10*x^(22/3) + 10/7*a*b^9*x^7 + 27/4*a^2*b^8*x^(20/3) + 360/19*a^3*b
^7*x^(19/3) + 35*a^4*b^6*x^6 + 756/17*a^5*b^5*x^(17/3) + 315/8*a^6*b^4*x^(
16/3) + 24*a^7*b^3*x^5 + 135/14*a^8*b^2*x^(14/3) + 30/13*a^9*b*x^(13/3) +
1/4*a^10*x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^3\,dx = \frac{a^{10}\,x^4}{4}+\frac{3\,b^{10}\,x^{22/3}}{22}+\frac{10\,a\,b^9\,x^7}{7}+\frac{30\,a^9\,b\,x^{13/3}}{13}+24\,a^7\,b^3\,x^5+35\,a^4\,b^6\,x^6+\frac{135\,a^8\,b^2\,x^{14/3}}{14}+\frac{315\,a^6\,b^4\,x^{16/3}}{8}+\frac{756\,a^5\,b^5\,x^{17/3}}{17}+\frac{360\,a^3\,b^7\,x^{19/3}}{19}+\frac{27\,a^2\,b^8\,x^{20/3}}{4}$$

input
```
int(x^3*(a + b*x^(1/3))^10,x)
```

output
```
(a^10*x^4)/4 + (3*b^10*x^(22/3))/22 + (10*a*b^9*x^7)/7 + (30*a^9*b*x^(13/3
))/13 + 24*a^7*b^3*x^5 + 35*a^4*b^6*x^6 + (135*a^8*b^2*x^(14/3))/14 + (315
*a^6*b^4*x^(16/3))/8 + (756*a^5*b^5*x^(17/3))/17 + (360*a^3*b^7*x^(19/3))/
19 + (27*a^2*b^8*x^(20/3))/4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^3\,dx = \frac{x^4\left(24942060x^{\frac{2}{3}}a^8b^2+115026912x^{\frac{5}{3}}a^5b^5+17459442x^{\frac{8}{3}}a^2b^8+5969040x^{\frac{1}{3}}a^9b+101846745x^{\frac{4}{3}}a^6b^4+490\right.}{258658}$$

input
```
int((a+b*x^(1/3))^10*x^3,x)
```

output
```
(x**4*(24942060*x**(2/3)*a**8*b**2 + 115026912*x**(2/3)*a**5*b**5*x + 1745
9442*x**(2/3)*a**2*b**8*x**2 + 5969040*x**(1/3)*a**9*b + 101846745*x**(1/3
)*a**6*b**4*x + 49008960*x**(1/3)*a**3*b**7*x**2 + 352716*x**(1/3)*b**10*x
**3 + 646646*a**10 + 62078016*a**7*b**3*x + 90530440*a**4*b**6*x**2 + 3695
120*a*b**9*x**3))/2586584
```

3.217 $\int \left(a+b\sqrt[3]{x}\right)^{10} x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 179

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{10} x^2\,dx = {} & \frac{3a^8\left(a+b\sqrt[3]{x}\right)^{11}}{11b^9}-\frac{2a^7\left(a+b\sqrt[3]{x}\right)^{12}}{b^9}+\frac{84a^6\left(a+b\sqrt[3]{x}\right)^{13}}{13b^9}\\ & -\frac{12a^5\left(a+b\sqrt[3]{x}\right)^{14}}{b^9}+\frac{14a^4\left(a+b\sqrt[3]{x}\right)^{15}}{b^9}-\frac{21a^3\left(a+b\sqrt[3]{x}\right)^{16}}{2b^9}\\ & +\frac{84a^2\left(a+b\sqrt[3]{x}\right)^{17}}{17b^9}-\frac{4a\left(a+b\sqrt[3]{x}\right)^{18}}{3b^9}+\frac{3\left(a+b\sqrt[3]{x}\right)^{19}}{19b^9}\end{aligned}$$

output
```
3/11*a^8*(a+b*x^(1/3))^11/b^9-2*a^7*(a+b*x^(1/3))^12/b^9+84/13*a^6*(a+b*x^
(1/3))^13/b^9-12*a^5*(a+b*x^(1/3))^14/b^9+14*a^4*(a+b*x^(1/3))^15/b^9-21/2
*a^3*(a+b*x^(1/3))^16/b^9+84/17*a^2*(a+b*x^(1/3))^17/b^9-4/3*a*(a+b*x^(1/3
))^18/b^9+3/19*(a+b*x^(1/3))^19/b^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^2\,dx$$
$$=\frac{92378a^{10}x^3+831402a^9bx^{10/3}+3401190a^8b^2x^{11/3}+8314020a^7b^3x^4+13430340a^6b^4x^{13/3}+14965236a^5b\cdots}{277134\cdots}$$

input
```
Integrate[(a + b*x^(1/3))^10*x^2,x]
```

output
```
(92378*a^10*x^3 + 831402*a^9*b*x^(10/3) + 3401190*a^8*b^2*x^(11/3) + 83140
20*a^7*b^3*x^4 + 13430340*a^6*b^4*x^(13/3) + 14965236*a^5*b^5*x^(14/3) + 1
1639628*a^4*b^6*x^5 + 6235515*a^3*b^7*x^(16/3) + 2200770*a^2*b^8*x^(17/3)
+ 461890*a*b^9*x^6 + 43758*b^10*x^(19/3))/277134
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt[3]{x}\right)^{10}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{10}x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^{18}}{b^8}-\frac{8a\left(a+b\sqrt[3]{x}\right)^{17}}{b^8}+\frac{28a^2\left(a+b\sqrt[3]{x}\right)^{16}}{b^8}-\frac{56a^3\left(a+b\sqrt[3]{x}\right)^{15}}{b^8}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^{14}}{b^8}-\frac{56a^5\left(a+\cdots\right.}{b^8\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^8(a+b\sqrt[3]{x})^{11}}{11b^9}-\frac{2a^7(a+b\sqrt[3]{x})^{12}}{3b^9}+\frac{28a^6(a+b\sqrt[3]{x})^{13}}{13b^9}-\frac{4a^5(a+b\sqrt[3]{x})^{14}}{b^9}+\frac{14a^4(a+b\sqrt[3]{x})^{15}}{3b^9}-\frac{7a^3(a+b\right.$$

input
```
Int[(a + b*x^(1/3))^10*x^2,x]
```

output
```
3*((a^8*(a + b*x^(1/3))^11)/(11*b^9) - (2*a^7*(a + b*x^(1/3))^12)/(3*b^9)
+ (28*a^6*(a + b*x^(1/3))^13)/(13*b^9) - (4*a^5*(a + b*x^(1/3))^14)/b^9 +
(14*a^4*(a + b*x^(1/3))^15)/(3*b^9) - (7*a^3*(a + b*x^(1/3))^16)/(2*b^9) +
 (28*a^2*(a + b*x^(1/3))^17)/(17*b^9) - (4*a*(a + b*x^(1/3))^18)/(9*b^9) +
 (a + b*x^(1/3))^19/(19*b^9))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{3b^{10}x^{\frac{19}{3}}}{19}+\frac{5a\,b^9x^6}{3}+\frac{135a^2b^8x^{\frac{17}{3}}}{17}+\frac{45a^3b^7x^{\frac{16}{3}}}{2}+42a^4b^6x^5+54a^5b^5x^{\frac{14}{3}}+\frac{630a^6b^4x^{\frac{13}{3}}}{13}+30a^7b^3$
default	$\frac{3b^{10}x^{\frac{19}{3}}}{19}+\frac{5a\,b^9x^6}{3}+\frac{135a^2b^8x^{\frac{17}{3}}}{17}+\frac{45a^3b^7x^{\frac{16}{3}}}{2}+42a^4b^6x^5+54a^5b^5x^{\frac{14}{3}}+\frac{630a^6b^4x^{\frac{13}{3}}}{13}+30a^7b^3$
trager	$\frac{a(5b^9x^5+126a^3b^6x^4+5b^9x^4+90a^6b^3x^3+126a^3b^6x^3+5b^9x^3+a^9x^2+90a^6b^3x^2+126a^3b^6x^2+5b^9x^2+a^9x+90a^6b^3x+12}{3}$
orering	$\frac{(584870b^{33}x^{11}+4735159a^3b^{30}x^{10}+16820744a^6b^{27}x^9+34284768a^9b^{24}x^8+43943340a^{12}b^{21}x^7+36398922a^{15}b^{18}x^6+76}{1385}$

input
```
int((a+b*x^(1/3))^10*x^2,x,method=_RETURNVERBOSE)
```

output
```
3/19*b^10*x^(19/3)+5/3*a*b^9*x^6+135/17*a^2*b^8*x^(17/3)+45/2*a^3*b^7*x^(1
6/3)+42*a^4*b^6*x^5+54*a^5*b^5*x^(14/3)+630/13*a^6*b^4*x^(13/3)+30*a^7*b^3
*x^4+135/11*a^8*b^2*x^(11/3)+3*a^9*b*x^(10/3)+1/3*a^10*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{10}x^2\,dx &= \frac{5}{3}\,ab^9x^6+42\,a^4b^6x^5+30\,a^7b^3x^4+\frac{1}{3}\,a^{10}x^3\\ &\quad+\frac{27}{187}\left(55\,a^2b^8x^5+374\,a^5b^5x^4+85\,a^8b^2x^3\right)x^{\frac{2}{3}}\\ &\quad+\frac{3}{494}\left(26\,b^{10}x^6+3705\,a^3b^7x^5+7980\,a^6b^4x^4+494\,a^9bx^3\right)x^{\frac{1}{3}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^10*x^2,x, algorithm="fricas")
```

output
```
5/3*a*b^9*x^6 + 42*a^4*b^6*x^5 + 30*a^7*b^3*x^4 + 1/3*a^10*x^3 + 27/187*(5
5*a^2*b^8*x^5 + 374*a^5*b^5*x^4 + 85*a^8*b^2*x^3)*x^(2/3) + 3/494*(26*b^10
*x^6 + 3705*a^3*b^7*x^5 + 7980*a^6*b^4*x^4 + 494*a^9*b*x^3)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^2\,dx=\frac{a^{10}x^3}{3}+3a^9bx^{\frac{10}{3}}+\frac{135a^8b^2x^{\frac{11}{3}}}{11}+30a^7b^3x^4+\frac{630a^6b^4x^{\frac{13}{3}}}{13}+54a^5b^5x^{\frac{14}{3}}+42a^4b^6x^5+\frac{45a^3b^7x^{\frac{16}{3}}}{2}+\frac{135a^2b^8x^{\frac{17}{3}}}{17}+\frac{5ab^9x^6}{3}+\frac{3b^{10}x^{\frac{19}{3}}}{19}$$

input
```
integrate((a+b*x**(1/3))**10*x**2,x)
```

output
```
a**10*x**3/3 + 3*a**9*b*x**(10/3) + 135*a**8*b**2*x**(11/3)/11 + 30*a**7*b
**3*x**4 + 630*a**6*b**4*x**(13/3)/13 + 54*a**5*b**5*x**(14/3) + 42*a**4*b
**6*x**5 + 45*a**3*b**7*x**(16/3)/2 + 135*a**2*b**8*x**(17/3)/17 + 5*a*b**
9*x**6/3 + 3*b**10*x**(19/3)/19
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x^2\,dx=\frac{3\left(bx^{\frac{1}{3}}+a\right)^{19}}{19\,b^9}-\frac{4\left(bx^{\frac{1}{3}}+a\right)^{18}a}{3\,b^9}+\frac{84\left(bx^{\frac{1}{3}}+a\right)^{17}a^2}{17\,b^9}-\frac{21\left(bx^{\frac{1}{3}}+a\right)^{16}a^3}{2\,b^9}+\frac{14\left(bx^{\frac{1}{3}}+a\right)^{15}a^4}{b^9}-\frac{12\left(bx^{\frac{1}{3}}+a\right)^{14}a^5}{b^9}+\frac{84\left(bx^{\frac{1}{3}}+a\right)^{13}a^6}{13\,b^9}-\frac{2\left(bx^{\frac{1}{3}}+a\right)^{12}a^7}{b^9}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}a^8}{11\,b^9}$$

input
```
integrate((a+b*x^(1/3))^10*x^2,x, algorithm="maxima")
```

output
```
3/19*(b*x^(1/3) + a)^19/b^9 - 4/3*(b*x^(1/3) + a)^18*a/b^9 + 84/17*(b*x^(1
/3) + a)^17*a^2/b^9 - 21/2*(b*x^(1/3) + a)^16*a^3/b^9 + 14*(b*x^(1/3) + a)
^15*a^4/b^9 - 12*(b*x^(1/3) + a)^14*a^5/b^9 + 84/13*(b*x^(1/3) + a)^13*a^6
/b^9 - 2*(b*x^(1/3) + a)^12*a^7/b^9 + 3/11*(b*x^(1/3) + a)^11*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^2\,dx = \frac{3}{19}\,b^{10}x^{\frac{19}{3}} + \frac{5}{3}\,ab^9x^6 + \frac{135}{17}\,a^2b^8x^{\frac{17}{3}} + \frac{45}{2}\,a^3b^7x^{\frac{16}{3}} + 42\,a^4b^6x^5 + 54\,a^5b^5x^{\frac{14}{3}} + \frac{630}{13}\,a^6b^4x^{\frac{13}{3}} + 30\,a^7b^3x^4 + \frac{135}{11}\,a^8b^2x^{\frac{11}{3}} + 3\,a^9bx^{\frac{10}{3}} + \frac{1}{3}\,a^{10}x^3$$

input
```
integrate((a+b*x^(1/3))^10*x^2,x, algorithm="giac")
```

output
```
3/19*b^10*x^(19/3) + 5/3*a*b^9*x^6 + 135/17*a^2*b^8*x^(17/3) + 45/2*a^3*b^
7*x^(16/3) + 42*a^4*b^6*x^5 + 54*a^5*b^5*x^(14/3) + 630/13*a^6*b^4*x^(13/3
) + 30*a^7*b^3*x^4 + 135/11*a^8*b^2*x^(11/3) + 3*a^9*b*x^(10/3) + 1/3*a^10
*x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^2\,dx = \frac{a^{10}\,x^3}{3} + \frac{3\,b^{10}\,x^{19/3}}{19} + \frac{5\,a\,b^9\,x^6}{3} + 3\,a^9\,b\,x^{10/3} + 30\,a^7\,b^3\,x^4 + 42\,a^4\,b^6\,x^5 + \frac{135\,a^8\,b^2\,x^{11/3}}{11} + \frac{630\,a^6\,b^4\,x^{13/3}}{13} + 54\,a^5\,b^5\,x^{14/3} + \frac{45\,a^3\,b^7\,x^{16/3}}{2} + \frac{135\,a^2\,b^8\,x^{17/3}}{17}$$

input
```
int(x^2*(a + b*x^(1/3))^10,x)
```

output
```
(a^10*x^3)/3 + (3*b^10*x^(19/3))/19 + (5*a*b^9*x^6)/3 + 3*a^9*b*x^(10/3) +
 30*a^7*b^3*x^4 + 42*a^4*b^6*x^5 + (135*a^8*b^2*x^(11/3))/11 + (630*a^6*b^
4*x^(13/3))/13 + 54*a^5*b^5*x^(14/3) + (45*a^3*b^7*x^(16/3))/2 + (135*a^2*
b^8*x^(17/3))/17
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x^2\,dx = \frac{x^3\Big(3401190x^{\frac{2}{3}}a^8b^2+14965236x^{\frac{5}{3}}a^5b^5+2200770x^{\frac{8}{3}}a^2b^8+831402x^{\frac{1}{3}}a^9b+13430340x^{\frac{4}{3}}a^6b^4+6235515x}{277134}$$

input
```
int((a+b*x^(1/3))^10*x^2,x)
```

output
```
(x**3*(3401190*x**(2/3)*a**8*b**2 + 14965236*x**(2/3)*a**5*b**5*x + 220077
0*x**(2/3)*a**2*b**8*x**2 + 831402*x**(1/3)*a**9*b + 13430340*x**(1/3)*a**
6*b**4*x + 6235515*x**(1/3)*a**3*b**7*x**2 + 43758*x**(1/3)*b**10*x**3 + 9
2378*a**10 + 8314020*a**7*b**3*x + 11639628*a**4*b**6*x**2 + 461890*a*b**9
*x**3))/277134
```

3.218 $\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 120

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx=-\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{12}}{4b^6}-\frac{30a^3\left(a+b\sqrt[3]{x}\right)^{13}}{13b^6}+\frac{15a^2\left(a+b\sqrt[3]{x}\right)^{14}}{7b^6}-\frac{a\left(a+b\sqrt[3]{x}\right)^{15}}{b^6}+\frac{3\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}$$

output
```
-3/11*a^5*(a+b*x^(1/3))^11/b^6+5/4*a^4*(a+b*x^(1/3))^12/b^6-30/13*a^3*(a+b
*x^(1/3))^13/b^6+15/7*a^2*(a+b*x^(1/3))^14/b^6-a*(a+b*x^(1/3))^15/b^6+3/16
*(a+b*x^(1/3))^16/b^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx$$
$$=\frac{8008a^{10}x^2+68640a^9bx^{7/3}+270270a^8b^2x^{8/3}+640640a^7b^3x^3+1009008a^6b^4x^{10/3}+1100736a^5b^5x^{11/3}+}{16016}$$

input
```
Integrate[(a + b*x^(1/3))^10*x,x]
```

output
```
(8008*a^10*x^2 + 68640*a^9*b*x^(7/3) + 270270*a^8*b^2*x^(8/3) + 640640*a^7
*b^3*x^3 + 1009008*a^6*b^4*x^(10/3) + 1100736*a^5*b^5*x^(11/3) + 840840*a^
4*b^6*x^4 + 443520*a^3*b^7*x^(13/3) + 154440*a^2*b^8*x^(14/3) + 32032*a*b^
9*x^5 + 3003*b^10*x^(16/3))/16016
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt[3]{x}\right)^{10}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{10}x^{5/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^{15}}{b^5}-\frac{5a\left(a+b\sqrt[3]{x}\right)^{14}}{b^5}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^{13}}{b^5}-\frac{10a^3\left(a+b\sqrt[3]{x}\right)^{12}}{b^5}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{11}}{b^5}-\frac{a^5\left(a+b\sqrt[3]{x}\right.}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^5\left(a+b\sqrt[3]{x}\right)^{11}}{11b^6}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{12}}{12b^6}-\frac{10a^3\left(a+b\sqrt[3]{x}\right)^{13}}{13b^6}+\frac{5a^2\left(a+b\sqrt[3]{x}\right)^{14}}{7b^6}+\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}-\frac{a\left(a+b\sqrt[3]{x}\right)^{1}}{3b^6}\right.$$

input
```
Int[(a + b*x^(1/3))^10*x,x]
```

output
```
3*(-1/11*(a^5*(a + b*x^(1/3))^11)/b^6 + (5*a^4*(a + b*x^(1/3))^12)/(12*b^6
) - (10*a^3*(a + b*x^(1/3))^13)/(13*b^6) + (5*a^2*(a + b*x^(1/3))^14)/(7*b
^6) - (a*(a + b*x^(1/3))^15)/(3*b^6) + (a + b*x^(1/3))^16/(16*b^6))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{3b^{10}x^{\frac{16}{3}}}{16}+2a\,b^{9}x^{5}+\frac{135a^{2}b^{8}x^{\frac{14}{3}}}{14}+\frac{360a^{3}b^{7}x^{\frac{13}{3}}}{13}+\frac{105a^{4}b^{6}x^{4}}{2}+\frac{756a^{5}b^{5}x^{\frac{11}{3}}}{11}+63a^{6}b^{4}x^{\frac{10}{3}}+40a^{7}$
default	$\frac{3b^{10}x^{\frac{16}{3}}}{16}+2a\,b^{9}x^{5}+\frac{135a^{2}b^{8}x^{\frac{14}{3}}}{14}+\frac{360a^{3}b^{7}x^{\frac{13}{3}}}{13}+\frac{105a^{4}b^{6}x^{4}}{2}+\frac{756a^{5}b^{5}x^{\frac{11}{3}}}{11}+63a^{6}b^{4}x^{\frac{10}{3}}+40a^{7}$
trager	$\frac{a(4b^{9}x^{4}+105a^{3}b^{6}x^{3}+4b^{9}x^{3}+80a^{6}b^{3}x^{2}+105a^{3}b^{6}x^{2}+4b^{9}x^{2}+a^{9}x+80a^{6}b^{3}x+105a^{3}b^{6}x+4b^{9}x+a^{9}+80a^{6}b^{3}+105a^{3}b^{6}+}{2}$
orering	$\frac{(39182b^{30}x^{10}+315739a^{3}b^{27}x^{9}+1115310a^{6}b^{24}x^{8}+2257716a^{9}b^{21}x^{7}+2869188a^{12}b^{18}x^{6}+1107960a^{15}b^{15}x^{5}+45943380}{80080b^{6}(b^{3}x+a^{3})^{8}}$

input
```
int((a+b*x^(1/3))^10*x,x,method=_RETURNVERBOSE)
```

output
```
3/16*b^10*x^(16/3)+2*a*b^9*x^5+135/14*a^2*b^8*x^(14/3)+360/13*a^3*b^7*x^(1
3/3)+105/2*a^4*b^6*x^4+756/11*a^5*b^5*x^(11/3)+63*a^6*b^4*x^(10/3)+40*a^7*
b^3*x^3+135/8*a^8*b^2*x^(8/3)+30/7*a^9*b*x^(7/3)+1/2*a^10*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x\,dx = 2\,ab^{9}x^{5} + \frac{105}{2}\,a^{4}b^{6}x^{4} + 40\,a^{7}b^{3}x^{3} + \frac{1}{2}\,a^{10}x^{2} + \frac{27}{616}\left(220\,a^{2}b^{8}x^{4} + 1568\,a^{5}b^{5}x^{3} + 385\,a^{8}b^{2}x^{2}\right)x^{\frac{2}{3}} + \frac{3}{1456}\left(91\,b^{10}x^{5} + 13440\,a^{3}b^{7}x^{4} + 30576\,a^{6}b^{4}x^{3} + 2080\,a^{9}bx^{2}\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^10*x,x, algorithm="fricas")
```

output
```
2*a*b^9*x^5 + 105/2*a^4*b^6*x^4 + 40*a^7*b^3*x^3 + 1/2*a^10*x^2 + 27/616*(
220*a^2*b^8*x^4 + 1568*a^5*b^5*x^3 + 385*a^8*b^2*x^2)*x^(2/3) + 3/1456*(91
*b^10*x^5 + 13440*a^3*b^7*x^4 + 30576*a^6*b^4*x^3 + 2080*a^9*b*x^2)*x^(1/3
)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.19

$$\int \left(a+b\sqrt[3]{x}\right)^{10}x\,dx = \frac{a^{10}x^{2}}{2} + \frac{30a^{9}bx^{\frac{7}{3}}}{7} + \frac{135a^{8}b^{2}x^{\frac{8}{3}}}{8} + 40a^{7}b^{3}x^{3} + 63a^{6}b^{4}x^{\frac{10}{3}} + \frac{756a^{5}b^{5}x^{\frac{11}{3}}}{11} + \frac{105a^{4}b^{6}x^{4}}{2} + \frac{360a^{3}b^{7}x^{\frac{13}{3}}}{13} + \frac{135a^{2}b^{8}x^{\frac{14}{3}}}{14} + 2ab^{9}x^{5} + \frac{3b^{10}x^{\frac{16}{3}}}{16}$$

input
```
integrate((a+b*x**(1/3))**10*x,x)
```

output
```
a**10*x**2/2 + 30*a**9*b*x**(7/3)/7 + 135*a**8*b**2*x**(8/3)/8 + 40*a**7*b
**3*x**3 + 63*a**6*b**4*x**(10/3) + 756*a**5*b**5*x**(11/3)/11 + 105*a**4*
b**6*x**4/2 + 360*a**3*b**7*x**(13/3)/13 + 135*a**2*b**8*x**(14/3)/14 + 2*
a*b**9*x**5 + 3*b**10*x**(16/3)/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx = \frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}}{16\,b^6} - \frac{\left(bx^{\frac{1}{3}}+a\right)^{15}a}{b^6} + \frac{15\left(bx^{\frac{1}{3}}+a\right)^{14}a^2}{7\,b^6} - \frac{30\left(bx^{\frac{1}{3}}+a\right)^{13}a^3}{13\,b^6} + \frac{5\left(bx^{\frac{1}{3}}+a\right)^{12}a^4}{4\,b^6} - \frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}a^5}{11\,b^6}$$

input
```
integrate((a+b*x^(1/3))^10*x,x, algorithm="maxima")
```

output
```
3/16*(b*x^(1/3) + a)^16/b^6 - (b*x^(1/3) + a)^15*a/b^6 + 15/7*(b*x^(1/3) +
 a)^14*a^2/b^6 - 30/13*(b*x^(1/3) + a)^13*a^3/b^6 + 5/4*(b*x^(1/3) + a)^12
*a^4/b^6 - 3/11*(b*x^(1/3) + a)^11*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx = \frac{3}{16}\,b^{10}x^{\frac{16}{3}} + 2\,ab^9x^5 + \frac{135}{14}\,a^2b^8x^{\frac{14}{3}} + \frac{360}{13}\,a^3b^7x^{\frac{13}{3}} + \frac{105}{2}\,a^4b^6x^4 + \frac{756}{11}\,a^5b^5x^{\frac{11}{3}} + 63\,a^6b^4x^{\frac{10}{3}} + 40\,a^7b^3x^3 + \frac{135}{8}\,a^8b^2x^{\frac{8}{3}} + \frac{30}{7}\,a^9bx^{\frac{7}{3}} + \frac{1}{2}\,a^{10}x^2$$

input
```
integrate((a+b*x^(1/3))^10*x,x, algorithm="giac")
```

output
```
3/16*b^10*x^(16/3) + 2*a*b^9*x^5 + 135/14*a^2*b^8*x^(14/3) + 360/13*a^3*b^
7*x^(13/3) + 105/2*a^4*b^6*x^4 + 756/11*a^5*b^5*x^(11/3) + 63*a^6*b^4*x^(1
0/3) + 40*a^7*b^3*x^3 + 135/8*a^8*b^2*x^(8/3) + 30/7*a^9*b*x^(7/3) + 1/2*a
^10*x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx = \frac{a^{10}\,x^2}{2}+\frac{3\,b^{10}\,x^{16/3}}{16}+2\,a\,b^9\,x^5+\frac{30\,a^9\,b\,x^{7/3}}{7}+40\,a^7\,b^3\,x^3+\frac{105\,a^4\,b^6\,x^4}{2}+\frac{135\,a^8\,b^2\,x^{8/3}}{8}+63\,a^6\,b^4\,x^{10/3}+\frac{756\,a^5\,b^5\,x^{11/3}}{11}+\frac{360\,a^3\,b^7\,x^{13/3}}{13}+\frac{135\,a^2\,b^8\,x^{14/3}}{14}$$

input
```
int(x*(a + b*x^(1/3))^10,x)
```

output
```
(a^10*x^2)/2 + (3*b^10*x^(16/3))/16 + 2*a*b^9*x^5 + (30*a^9*b*x^(7/3))/7 +
 40*a^7*b^3*x^3 + (105*a^4*b^6*x^4)/2 + (135*a^8*b^2*x^(8/3))/8 + 63*a^6*b
^4*x^(10/3) + (756*a^5*b^5*x^(11/3))/11 + (360*a^3*b^7*x^(13/3))/13 + (135
*a^2*b^8*x^(14/3))/14
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt[3]{x}\right)^{10} x\,dx = \frac{x^2\Big(270270x^{\frac{2}{3}}a^8b^2+1100736x^{\frac{5}{3}}a^5b^5+154440x^{\frac{8}{3}}a^2b^8+68640x^{\frac{1}{3}}a^9b+1009008x^{\frac{4}{3}}a^6b^4+443520x^{\frac{7}{3}}a^3b^7+}{16016}$$

input
```
int((a+b*x^(1/3))^10*x,x)
```

output
```
(x**2*(270270*x**(2/3)*a**8*b**2 + 1100736*x**(2/3)*a**5*b**5*x + 154440*x
**(2/3)*a**2*b**8*x**2 + 68640*x**(1/3)*a**9*b + 1009008*x**(1/3)*a**6*b**
4*x + 443520*x**(1/3)*a**3*b**7*x**2 + 3003*x**(1/3)*b**10*x**3 + 8008*a**
10 + 640640*a**7*b**3*x + 840840*a**4*b**6*x**2 + 32032*a*b**9*x**3))/1601
6
```

3.219 $\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^{11}}{11b^3}-\frac{a\left(a+b\sqrt[3]{x}\right)^{12}}{2b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^{13}}{13b^3}$$

output
```
3/11*a^2*(a+b*x^(1/3))^11/b^3-1/2*a*(a+b*x^(1/3))^12/b^3+3/13*(a+b*x^(1/3)
)^13/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(59) = 118.

Time = 0.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.17

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx=\frac{1}{286}\left(286a^{10}x+2145a^9bx^{4/3}+7722a^8b^2x^{5/3}+17160a^7b^3x^2+25740a^6b^4x^{7/3}+27027a^5b^5x^{8/3}+20020a^4b^6x^3+10296a^3b^7x^{10/3}+3510a^2b^8x^{11/3}+715ab^9x^4+66b^{10}x^{13/3}\right)$$

input
```
Integrate[(a + b*x^(1/3))^10,x]
```

output
```
(286*a^10*x + 2145*a^9*b*x^(4/3) + 7722*a^8*b^2*x^(5/3) + 17160*a^7*b^3*x^
2 + 25740*a^6*b^4*x^(7/3) + 27027*a^5*b^5*x^(8/3) + 20020*a^4*b^6*x^3 + 10
296*a^3*b^7*x^(10/3) + 3510*a^2*b^8*x^(11/3) + 715*a*b^9*x^4 + 66*b^10*x^(
13/3))/286
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}\right)^{10}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{10}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^{12}}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^{11}}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^{10}}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^{11}}{11b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^{13}}{13b^3}-\frac{a\left(a+b\sqrt[3]{x}\right)^{12}}{6b^3}\right)$$

input
```
Int[(a + b*x^(1/3))^10,x]
```

output
```
3*((a^2*(a + b*x^(1/3))^11)/(11*b^3) - (a*(a + b*x^(1/3))^12)/(6*b^3) + (a
 + b*x^(1/3))^13/(13*b^3))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(47) = 94$.

Time = 4.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{3b^{10}x^{\frac{13}{3}}}{13}+\frac{5a\,b^{9}x^{4}}{2}+\frac{135a^{2}b^{8}x^{\frac{11}{3}}}{11}+36a^{3}b^{7}x^{\frac{10}{3}}+70a^{4}b^{6}x^{3}+\frac{189a^{5}b^{5}x^{\frac{8}{3}}}{2}+90a^{6}b^{4}x^{\frac{7}{3}}+60a^{7}b^{3}$
default	$\frac{3b^{10}x^{\frac{13}{3}}}{13}+\frac{5a\,b^{9}x^{4}}{2}+\frac{135a^{2}b^{8}x^{\frac{11}{3}}}{11}+36a^{3}b^{7}x^{\frac{10}{3}}+70a^{4}b^{6}x^{3}+\frac{189a^{5}b^{5}x^{\frac{8}{3}}}{2}+90a^{6}b^{4}x^{\frac{7}{3}}+60a^{7}b^{3}$
trager	$\frac{a\left(5b^{9}x^{3}+140a^{3}b^{6}x^{2}+5b^{9}x^{2}+120a^{6}b^{3}x+140a^{3}b^{6}x+5b^{9}x+2a^{9}+120a^{6}b^{3}+140a^{3}b^{6}+5b^{9}\right)\left(-1+x\right)}{2}+\frac{3b\,x^{\frac{4}{3}}\left(2b^{9}x^{3}+3\right.}{}$
orering	$\frac{\left(166b^{27}x^{9}+1329a^{3}b^{24}x^{8}+4656a^{6}b^{21}x^{7}+9324a^{9}b^{18}x^{6}+6636a^{12}b^{15}x^{5}+176946a^{15}b^{12}x^{4}-635376a^{18}b^{9}x^{3}+495276a^{21}\right.}{286b^{3}\left(b^{3}x+a^{3}\right)^{8}}$

input
```
int((a+b*x^(1/3))^10,x,method=_RETURNVERBOSE)
```

output
```
3/13*b^10*x^(13/3)+5/2*a*b^9*x^4+135/11*a^2*b^8*x^(11/3)+36*a^3*b^7*x^(10/
3)+70*a^4*b^6*x^3+189/2*a^5*b^5*x^(8/3)+90*a^6*b^4*x^(7/3)+60*a^7*b^3*x^2+
27*a^8*b^2*x^(5/3)+15/2*a^9*b*x^(4/3)+a^10*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(47) = 94$.

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.98

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx = \frac{5}{2}\,ab^9x^4 + 70\,a^4b^6x^3 + 60\,a^7b^3x^2 + a^{10}x + \frac{27}{22}\left(10\,a^2b^8x^3 + 77\,a^5b^5x^2 + 22\,a^8b^2x\right)x^{\frac{2}{3}} + \frac{3}{26}\left(2\,b^{10}x^4 + 312\,a^3b^7x^3 + 780\,a^6b^4x^2 + 65\,a^9bx\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^10,x, algorithm="fricas")
```

output
```
5/2*a*b^9*x^4 + 70*a^4*b^6*x^3 + 60*a^7*b^3*x^2 + a^10*x + 27/22*(10*a^2*b
^8*x^3 + 77*a^5*b^5*x^2 + 22*a^8*b^2*x)*x^(2/3) + 3/26*(2*b^10*x^4 + 312*a
^3*b^7*x^3 + 780*a^6*b^4*x^2 + 65*a^9*b*x)*x^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(53) = 106$.

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.31

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx = a^{10}x + \frac{15a^9bx^{\frac{4}{3}}}{2} + 27a^8b^2x^{\frac{5}{3}} + 60a^7b^3x^2 + 90a^6b^4x^{\frac{7}{3}} + \frac{189a^5b^5x^{\frac{8}{3}}}{2} + 70a^4b^6x^3 + 36a^3b^7x^{\frac{10}{3}} + \frac{135a^2b^8x^{\frac{11}{3}}}{11} + \frac{5ab^9x^4}{2} + \frac{3b^{10}x^{\frac{13}{3}}}{13}$$

input
```
integrate((a+b*x**(1/3))**10,x)
```

output
```
a**10*x + 15*a**9*b*x**(4/3)/2 + 27*a**8*b**2*x**(5/3) + 60*a**7*b**3*x**2
 + 90*a**6*b**4*x**(7/3) + 189*a**5*b**5*x**(8/3)/2 + 70*a**4*b**6*x**3 +
36*a**3*b**7*x**(10/3) + 135*a**2*b**8*x**(11/3)/11 + 5*a*b**9*x**4/2 + 3*
b**10*x**(13/3)/13
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx=\frac{3\left(bx^{\frac{1}{3}}+a\right)^{13}}{13\,b^3}-\frac{\left(bx^{\frac{1}{3}}+a\right)^{12}a}{2\,b^3}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}a^2}{11\,b^3}$$

input
```
integrate((a+b*x^(1/3))^10,x, algorithm="maxima")
```

output
```
3/13*(b*x^(1/3) + a)^13/b^3 - 1/2*(b*x^(1/3) + a)^12*a/b^3 + 3/11*(b*x^(1/
3) + a)^11*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(47) = 94.

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx=\frac{3}{13}\,b^{10}x^{\frac{13}{3}}+\frac{5}{2}\,ab^9x^4+\frac{135}{11}\,a^2b^8x^{\frac{11}{3}}+36\,a^3b^7x^{\frac{10}{3}}+70\,a^4b^6x^3+\frac{189}{2}\,a^5b^5x^{\frac{8}{3}}+90\,a^6b^4x^{\frac{7}{3}}+60\,a^7b^3x^2+27\,a^8b^2x^{\frac{5}{3}}+\frac{15}{2}\,a^9bx^{\frac{4}{3}}+a^{10}x$$

input
```
integrate((a+b*x^(1/3))^10,x, algorithm="giac")
```

output
```
3/13*b^10*x^(13/3) + 5/2*a*b^9*x^4 + 135/11*a^2*b^8*x^(11/3) + 36*a^3*b^7*
x^(10/3) + 70*a^4*b^6*x^3 + 189/2*a^5*b^5*x^(8/3) + 90*a^6*b^4*x^(7/3) + 6
0*a^7*b^3*x^2 + 27*a^8*b^2*x^(5/3) + 15/2*a^9*b*x^(4/3) + a^10*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx = a^{10}\,x+\frac{3\,b^{10}\,x^{13/3}}{13}+\frac{5\,a\,b^9\,x^4}{2}+\frac{15\,a^9\,b\,x^{4/3}}{2}+60\,a^7\,b^3\,x^2+70\,a^4\,b^6\,x^3+27\,a^8\,b^2\,x^{5/3}+90\,a^6\,b^4\,x^{7/3}+\frac{189\,a^5\,b^5\,x^{8/3}}{2}+36\,a^3\,b^7\,x^{10/3}+\frac{135\,a^2\,b^8\,x^{11/3}}{11}$$

input
```
int((a + b*x^(1/3))^10,x)
```

output
```
a^10*x + (3*b^10*x^(13/3))/13 + (5*a*b^9*x^4)/2 + (15*a^9*b*x^(4/3))/2 + 6
0*a^7*b^3*x^2 + 70*a^4*b^6*x^3 + 27*a^8*b^2*x^(5/3) + 90*a^6*b^4*x^(7/3) +
 (189*a^5*b^5*x^(8/3))/2 + 36*a^3*b^7*x^(10/3) + (135*a^2*b^8*x^(11/3))/11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.86

$$\int \left(a+b\sqrt[3]{x}\right)^{10}\,dx = \frac{x\Big(7722x^{\frac{2}{3}}a^8b^2+27027x^{\frac{5}{3}}a^5b^5+3510x^{\frac{8}{3}}a^2b^8+2145x^{\frac{1}{3}}a^9b+25740x^{\frac{4}{3}}a^6b^4+10296x^{\frac{7}{3}}a^3b^7+66x^{\frac{10}{3}}b^{10}+}{286}$$

input
```
int((a+b*x^(1/3))^10,x)
```

output
```
(x*(7722*x**(2/3)*a**8*b**2 + 27027*x**(2/3)*a**5*b**5*x + 3510*x**(2/3)*a
**2*b**8*x**2 + 2145*x**(1/3)*a**9*b + 25740*x**(1/3)*a**6*b**4*x + 10296*
x**(1/3)*a**3*b**7*x**2 + 66*x**(1/3)*b**10*x**3 + 286*a**10 + 17160*a**7*
b**3*x + 20020*a**4*b**6*x**2 + 715*a*b**9*x**3))/286
```

3.220 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = 30a^9b\sqrt[3]{x}+\frac{135}{2}a^8b^2x^{2/3}+120a^7b^3x+\frac{315}{2}a^6b^4x^{4/3}+\frac{756}{5}a^5b^5x^{5/3}+105a^4b^6x^2+\frac{360}{7}a^3b^7x^{7/3}+\frac{135}{8}a^2b^8x^{8/3}+\frac{10}{3}ab^9x^3+\frac{3}{10}b^{10}x^{10/3}+a^{10}\log(x)$$

output
```
30*a^9*b*x^(1/3)+135/2*a^8*b^2*x^(2/3)+120*a^7*b^3*x+315/2*a^6*b^4*x^(4/3)
+756/5*a^5*b^5*x^(5/3)+105*a^4*b^6*x^2+360/7*a^3*b^7*x^(7/3)+135/8*a^2*b^8
*x^(8/3)+10/3*a*b^9*x^3+3/10*b^10*x^(10/3)+a^10*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = \frac{1}{840}\left(25200a^9b\sqrt[3]{x}+56700a^8b^2x^{2/3}+100800a^7b^3x+132300a^6b^4x^{4/3}+127008a^5b^5x^{5/3}+88200a^4b^6x^2+43200a^3b^7x^{7/3}+14175a^2b^8x^{8/3}+2800ab^9x^3+252b^{10}x^{10/3}\right)+3a^{10}\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^10/x,x]
```

output
```
(25200*a^9*b*x^(1/3) + 56700*a^8*b^2*x^(2/3) + 100800*a^7*b^3*x + 132300*a
^6*b^4*x^(4/3) + 127008*a^5*b^5*x^(5/3) + 88200*a^4*b^6*x^2 + 43200*a^3*b^
7*x^(7/3) + 14175*a^2*b^8*x^(8/3) + 2800*a*b^9*x^3 + 252*b^10*x^(10/3))/84
0 + 3*a^10*Log[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^{10}}{\sqrt[3]{x}}+10ba^9+45b^2\sqrt[3]{x}a^8+120b^3x^{2/3}a^7+210b^4xa^6+252b^5x^{4/3}a^5+210b^6x^{5/3}a^4+120b^7x^2a^3+45b^8x^{7/3}$$

$$\Big\downarrow \text{2009}$$

$$3\left(a^{10}\log\left(\sqrt[3]{x}\right)+10a^9b\sqrt[3]{x}+\frac{45}{2}a^8b^2x^{2/3}+40a^7b^3x+\frac{105}{2}a^6b^4x^{4/3}+\frac{252}{5}a^5b^5x^{5/3}+35a^4b^6x^2+\frac{120}{7}a^3b^7x^{7/3}+$$

input
```
Int[(a + b*x^(1/3))^10/x,x]
```

output
```
3*(10*a^9*b*x^(1/3) + (45*a^8*b^2*x^(2/3))/2 + 40*a^7*b^3*x + (105*a^6*b^4
*x^(4/3))/2 + (252*a^5*b^5*x^(5/3))/5 + 35*a^4*b^6*x^2 + (120*a^3*b^7*x^(7
/3))/7 + (45*a^2*b^8*x^(8/3))/8 + (10*a*b^9*x^3)/9 + (b^10*x^(10/3))/10 +
a^10*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

method	result
derivativedivides	$30a^9b\,x^{\frac{1}{3}}+\frac{135a^8b^2x^{\frac{2}{3}}}{2}+120a^7b^3x+\frac{315a^6b^4x^{\frac{4}{3}}}{2}+\frac{756a^5b^5x^{\frac{5}{3}}}{5}+105a^4b^6x^2+\frac{360a^3b^7x^{\frac{7}{3}}}{7}+135$
default	$30a^9b\,x^{\frac{1}{3}}+\frac{135a^8b^2x^{\frac{2}{3}}}{2}+120a^7b^3x+\frac{315a^6b^4x^{\frac{4}{3}}}{2}+\frac{756a^5b^5x^{\frac{5}{3}}}{5}+105a^4b^6x^2+\frac{360a^3b^7x^{\frac{7}{3}}}{7}+135$
trager	$\frac{5a\,b^3(2b^6x^2+63a^3b^3x+2b^6x+72a^6+63a^3b^3+2b^6)(-1+x)}{3}+\frac{3b(7b^9x^3+1200a^3b^6x^2+3675a^6b^3x+700a^9)x^{\frac{1}{3}}}{70}+27$

input
```
int((a+b*x^(1/3))^10/x,x,method=_RETURNVERBOSE)
```

output
```
30*a^9*b*x^(1/3)+135/2*a^8*b^2*x^(2/3)+120*a^7*b^3*x+315/2*a^6*b^4*x^(4/3)
+756/5*a^5*b^5*x^(5/3)+105*a^4*b^6*x^2+360/7*a^3*b^7*x^(7/3)+135/8*a^2*b^8
*x^(8/3)+10/3*a*b^9*x^3+3/10*b^10*x^(10/3)+a^10*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = \frac{10}{3}\,ab^9x^3+105\,a^4b^6x^2+120\,a^7b^3x+3\,a^{10}\log\left(x^{\frac{1}{3}}\right) + \frac{27}{40}\left(25\,a^2b^8x^2+224\,a^5b^5x+100\,a^8b^2\right)x^{\frac{2}{3}} + \frac{3}{70}\left(7\,b^{10}x^3+1200\,a^3b^7x^2+3675\,a^6b^4x+700\,a^9b\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^10/x,x, algorithm="fricas")
```

output
```
10/3*a*b^9*x^3 + 105*a^4*b^6*x^2 + 120*a^7*b^3*x + 3*a^10*log(x^(1/3)) + 2
7/40*(25*a^2*b^8*x^2 + 224*a^5*b^5*x + 100*a^8*b^2)*x^(2/3) + 3/70*(7*b^10
*x^3 + 1200*a^3*b^7*x^2 + 3675*a^6*b^4*x + 700*a^9*b)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = 3a^{10}\log\left(\sqrt[3]{x}\right)+30a^9b\sqrt[3]{x}+\frac{135a^8b^2x^{\frac{2}{3}}}{2}+120a^7b^3x + \frac{315a^6b^4x^{\frac{4}{3}}}{2}+\frac{756a^5b^5x^{\frac{5}{3}}}{5}+105a^4b^6x^2 + \frac{360a^3b^7x^{\frac{7}{3}}}{7}+\frac{135a^2b^8x^{\frac{8}{3}}}{8}+\frac{10ab^9x^3}{3}+\frac{3b^{10}x^{\frac{10}{3}}}{10}$$

input
```
integrate((a+b*x**(1/3))**10/x,x)
```

output
```
3*a**10*log(x**(1/3)) + 30*a**9*b*x**(1/3) + 135*a**8*b**2*x**(2/3)/2 + 12
0*a**7*b**3*x + 315*a**6*b**4*x**(4/3)/2 + 756*a**5*b**5*x**(5/3)/5 + 105*
a**4*b**6*x**2 + 360*a**3*b**7*x**(7/3)/7 + 135*a**2*b**8*x**(8/3)/8 + 10*
a*b**9*x**3/3 + 3*b**10*x**(10/3)/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = \frac{3}{10}\,b^{10}x^{\frac{10}{3}} + \frac{10}{3}\,ab^{9}x^{3} + \frac{135}{8}\,a^{2}b^{8}x^{\frac{8}{3}} + \frac{360}{7}\,a^{3}b^{7}x^{\frac{7}{3}} + 105\,a^{4}b^{6}x^{2} + \frac{756}{5}\,a^{5}b^{5}x^{\frac{5}{3}} + \frac{315}{2}\,a^{6}b^{4}x^{\frac{4}{3}} + 120\,a^{7}b^{3}x + a^{10}\log\left(x\right) + \frac{135}{2}\,a^{8}b^{2}x^{\frac{2}{3}} + 30\,a^{9}bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^10/x,x, algorithm="maxima")
```

output
```
3/10*b^10*x^(10/3) + 10/3*a*b^9*x^3 + 135/8*a^2*b^8*x^(8/3) + 360/7*a^3*b^
7*x^(7/3) + 105*a^4*b^6*x^2 + 756/5*a^5*b^5*x^(5/3) + 315/2*a^6*b^4*x^(4/3
) + 120*a^7*b^3*x + a^10*log(x) + 135/2*a^8*b^2*x^(2/3) + 30*a^9*b*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = \frac{3}{10}\,b^{10}x^{\frac{10}{3}} + \frac{10}{3}\,ab^{9}x^{3} + \frac{135}{8}\,a^{2}b^{8}x^{\frac{8}{3}} + \frac{360}{7}\,a^{3}b^{7}x^{\frac{7}{3}} + 105\,a^{4}b^{6}x^{2} + \frac{756}{5}\,a^{5}b^{5}x^{\frac{5}{3}} + \frac{315}{2}\,a^{6}b^{4}x^{\frac{4}{3}} + 120\,a^{7}b^{3}x + a^{10}\log\left(|x|\right) + \frac{135}{2}\,a^{8}b^{2}x^{\frac{2}{3}} + 30\,a^{9}bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^10/x,x, algorithm="giac")
```

output
```
3/10*b^10*x^(10/3) + 10/3*a*b^9*x^3 + 135/8*a^2*b^8*x^(8/3) + 360/7*a^3*b^
7*x^(7/3) + 105*a^4*b^6*x^2 + 756/5*a^5*b^5*x^(5/3) + 315/2*a^6*b^4*x^(4/3
) + 120*a^7*b^3*x + a^10*log(abs(x)) + 135/2*a^8*b^2*x^(2/3) + 30*a^9*b*x^
(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = 3\,a^{10}\,\ln\left(x^{1/3}\right)+\frac{3\,b^{10}\,x^{10/3}}{10}+120\,a^7\,b^3\,x+\frac{10\,a\,b^9\,x^3}{3}+30\,a^9\,b\,x^{1/3}+105\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{2}+\frac{315\,a^6\,b^4\,x^{4/3}}{2}+\frac{756\,a^5\,b^5\,x^{5/3}}{5}+\frac{360\,a^3\,b^7\,x^{7/3}}{7}+\frac{135\,a^2\,b^8\,x^{8/3}}{8}$$

input
```
int((a + b*x^(1/3))^10/x,x)
```

output
```
3*a^10*log(x^(1/3)) + (3*b^10*x^(10/3))/10 + 120*a^7*b^3*x + (10*a*b^9*x^3
)/3 + 30*a^9*b*x^(1/3) + 105*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/2 + (315*
a^6*b^4*x^(4/3))/2 + (756*a^5*b^5*x^(5/3))/5 + (360*a^3*b^7*x^(7/3))/7 + (
135*a^2*b^8*x^(8/3))/8
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x}\,dx = \frac{135x^{\frac{2}{3}}a^8b^2}{2}+\frac{756x^{\frac{5}{3}}a^5b^5}{5}+\frac{135x^{\frac{8}{3}}a^2b^8}{8}+30x^{\frac{1}{3}}a^9b+\frac{315x^{\frac{4}{3}}a^6b^4}{2}+\frac{360x^{\frac{7}{3}}a^3b^7}{7}+\frac{3x^{\frac{10}{3}}b^{10}}{10}+\log(x)\,a^{10}+120a^7b^3x+105a^4b^6x^2+\frac{10a\,b^9x^3}{3}$$

input
```
int((a+b*x^(1/3))^10/x,x)
```

output
```
(56700*x**(2/3)*a**8*b**2 + 127008*x**(2/3)*a**5*b**5*x + 14175*x**(2/3)*a
**2*b**8*x**2 + 25200*x**(1/3)*a**9*b + 132300*x**(1/3)*a**6*b**4*x + 4320
0*x**(1/3)*a**3*b**7*x**2 + 252*x**(1/3)*b**10*x**3 + 840*log(x)*a**10 + 1
00800*a**7*b**3*x + 88200*a**4*b**6*x**2 + 2800*a*b**9*x**3)/840
```

3.221 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2} \, dx = -\frac{a^{10}}{x} - \frac{15a^9b}{x^{2/3}} - \frac{135a^8b^2}{\sqrt[3]{x}} + 630a^6b^4\sqrt[3]{x} + 378a^5b^5x^{2/3} + 210a^4b^6x + 90a^3b^7x^{4/3} + 27a^2b^8x^{5/3} + 5ab^9x^2 + \frac{3}{7}b^{10}x^{7/3} + 120a^7b^3\log(x)$$

output
```
-a^10/x-15*a^9*b/x^(2/3)-135*a^8*b^2/x^(1/3)+630*a^6*b^4*x^(1/3)+378*a^5*b
^5*x^(2/3)+210*a^4*b^6*x+90*a^3*b^7*x^(4/3)+27*a^2*b^8*x^(5/3)+5*a*b^9*x^2
+3/7*b^10*x^(7/3)+120*a^7*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2} \, dx = -\frac{a^{10}}{x} - \frac{15a^9b}{x^{2/3}} - \frac{135a^8b^2}{\sqrt[3]{x}} + 630a^6b^4\sqrt[3]{x} + 378a^5b^5x^{2/3} + 210a^4b^6x + 90a^3b^7x^{4/3} + 27a^2b^8x^{5/3} + 5ab^9x^2 + \frac{3}{7}b^{10}x^{7/3} + 120a^7b^3\log(x)$$

input
```
Integrate[(a + b*x^(1/3))^10/x^2,x]
```

output
```
-(a^10/x) - (15*a^9*b)/x^(2/3) - (135*a^8*b^2)/x^(1/3) + 630*a^6*b^4*x^(1/
3) + 378*a^5*b^5*x^(2/3) + 210*a^4*b^6*x + 90*a^3*b^7*x^(4/3) + 27*a^2*b^8
*x^(5/3) + 5*a*b^9*x^2 + (3*b^10*x^(7/3))/7 + 120*a^7*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^{10}}{x^{4/3}}+\frac{10ba^9}{x}+\frac{45b^2a^8}{x^{2/3}}+\frac{120b^3a^7}{\sqrt[3]{x}}+210b^4a^6+252b^5\sqrt[3]{x}a^5+210b^6x^{2/3}a^4+120b^7xa^3+45b^8x^{4/3}a^2+10b$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{10}}{3x}-\frac{5a^9b}{x^{2/3}}-\frac{45a^8b^2}{\sqrt[3]{x}}+120a^7b^3\log\left(\sqrt[3]{x}\right)+210a^6b^4\sqrt[3]{x}+126a^5b^5x^{2/3}+70a^4b^6x+30a^3b^7x^{4/3}+9a^2b^8x^{5/3}$$

input
```
Int[(a + b*x^(1/3))^10/x^2,x]
```

output
```
3*(-1/3*a^10/x - (5*a^9*b)/x^(2/3) - (45*a^8*b^2)/x^(1/3) + 210*a^6*b^4*x^
(1/3) + 126*a^5*b^5*x^(2/3) + 70*a^4*b^6*x + 30*a^3*b^7*x^(4/3) + 9*a^2*b^
8*x^(5/3) + (5*a*b^9*x^2)/3 + (b^10*x^(7/3))/7 + 120*a^7*b^3*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{a^{10}}{x}-\frac{15a^9b}{x^{\frac{2}{3}}}-\frac{135a^8b^2}{x^{\frac{1}{3}}}+630a^6b^4x^{\frac{1}{3}}+378a^5b^5x^{\frac{2}{3}}+210a^4b^6x+90a^3b^7x^{\frac{4}{3}}+27a^2b^8x^{\frac{5}{3}}$
default	$-\frac{a^{10}}{x}-\frac{15a^9b}{x^{\frac{2}{3}}}-\frac{135a^8b^2}{x^{\frac{1}{3}}}+630a^6b^4x^{\frac{1}{3}}+378a^5b^5x^{\frac{2}{3}}+210a^4b^6x+90a^3b^7x^{\frac{4}{3}}+27a^2b^8x^{\frac{5}{3}}$
trager	$\frac{(-1+x)(5b^9x^2+210a^3b^6x+5b^9x+a^9)a}{x}-\frac{3(-b^9x^3-210a^3b^6x^2-1470a^6b^3x+35a^9)b}{7x^{\frac{2}{3}}}-\frac{27(-b^6x^2-14a^3b^3x+5a^6)}{x^{\frac{1}{3}}}$

input
```
int((a+b*x^(1/3))^10/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^10/x-15*a^9*b/x^(2/3)-135*a^8*b^2/x^(1/3)+630*a^6*b^4*x^(1/3)+378*a^5*b
^5*x^(2/3)+210*a^4*b^6*x+90*a^3*b^7*x^(4/3)+27*a^2*b^8*x^(5/3)+5*a*b^9*x^2
+3/7*b^10*x^(7/3)+120*a^7*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx$$
$$= \frac{35\,ab^9x^3+1470\,a^4b^6x^2+2520\,a^7b^3x\log\left(x^{\frac{1}{3}}\right)-7\,a^{10}+189\,(a^2b^8x^2+14\,a^5b^5x-5\,a^8b^2)x^{\frac{2}{3}}+3\,(b^{10}x^3+}{7\,x}$$

input
```
integrate((a+b*x^(1/3))^10/x^2,x, algorithm="fricas")
```

output
```
1/7*(35*a*b^9*x^3 + 1470*a^4*b^6*x^2 + 2520*a^7*b^3*x*log(x^(1/3)) - 7*a^1
0 + 189*(a^2*b^8*x^2 + 14*a^5*b^5*x - 5*a^8*b^2)*x^(2/3) + 3*(b^10*x^3 + 2
10*a^3*b^7*x^2 + 1470*a^6*b^4*x - 35*a^9*b)*x^(1/3))/x
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx = -\frac{a^{10}}{x}-\frac{15a^9b}{x^{\frac{2}{3}}}-\frac{135a^8b^2}{\sqrt[3]{x}}+360a^7b^3\log\left(\sqrt[3]{x}\right)+630a^6b^4\sqrt[3]{x}$$
$$+378a^5b^5x^{\frac{2}{3}}+210a^4b^6x+90a^3b^7x^{\frac{4}{3}}+27a^2b^8x^{\frac{5}{3}}+5ab^9x^2+\frac{3b^{10}x^{\frac{7}{3}}}{7}$$

input
```
integrate((a+b*x**(1/3))**10/x**2,x)
```

output
```
-a**10/x - 15*a**9*b/x**(2/3) - 135*a**8*b**2/x**(1/3) + 360*a**7*b**3*log
(x**(1/3)) + 630*a**6*b**4*x**(1/3) + 378*a**5*b**5*x**(2/3) + 210*a**4*b*
*6*x + 90*a**3*b**7*x**(4/3) + 27*a**2*b**8*x**(5/3) + 5*a*b**9*x**2 + 3*b
**10*x**(7/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\sqrt[3]{x})^{10}}{x^2}\,dx = \frac{3}{7}\,b^{10}x^{\frac{7}{3}} + 5\,ab^9x^2 + 27\,a^2b^8x^{\frac{5}{3}} + 90\,a^3b^7x^{\frac{4}{3}} + 210\,a^4b^6x + 120\,a^7b^3\log(x) + 378\,a^5b^5x^{\frac{2}{3}} + 630\,a^6b^4x^{\frac{1}{3}} - \frac{135\,a^8b^2x^{\frac{2}{3}} + 15\,a^9bx^{\frac{1}{3}} + a^{10}}{x}$$

input
```
integrate((a+b*x^(1/3))^10/x^2,x, algorithm="maxima")
```

output
```
3/7*b^10*x^(7/3) + 5*a*b^9*x^2 + 27*a^2*b^8*x^(5/3) + 90*a^3*b^7*x^(4/3) +
 210*a^4*b^6*x + 120*a^7*b^3*log(x) + 378*a^5*b^5*x^(2/3) + 630*a^6*b^4*x^
(1/3) - (135*a^8*b^2*x^(2/3) + 15*a^9*b*x^(1/3) + a^10)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\sqrt[3]{x})^{10}}{x^2}\,dx = \frac{3}{7}\,b^{10}x^{\frac{7}{3}} + 5\,ab^9x^2 + 27\,a^2b^8x^{\frac{5}{3}} + 90\,a^3b^7x^{\frac{4}{3}} + 210\,a^4b^6x + 120\,a^7b^3\log(|x|) + 378\,a^5b^5x^{\frac{2}{3}} + 630\,a^6b^4x^{\frac{1}{3}} - \frac{135\,a^8b^2x^{\frac{2}{3}} + 15\,a^9bx^{\frac{1}{3}} + a^{10}}{x}$$

input
```
integrate((a+b*x^(1/3))^10/x^2,x, algorithm="giac")
```

output
```
3/7*b^10*x^(7/3) + 5*a*b^9*x^2 + 27*a^2*b^8*x^(5/3) + 90*a^3*b^7*x^(4/3) +
 210*a^4*b^6*x + 120*a^7*b^3*log(abs(x)) + 378*a^5*b^5*x^(2/3) + 630*a^6*b
^4*x^(1/3) - (135*a^8*b^2*x^(2/3) + 15*a^9*b*x^(1/3) + a^10)/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx = \frac{3\,b^{10}\,x^{7/3}}{7} - \frac{a^{10}+15\,a^9\,b\,x^{1/3}+135\,a^8\,b^2\,x^{2/3}}{x} +360\,a^7\,b^3\,\ln\left(x^{1/3}\right)+210\,a^4\,b^6\,x+5\,a\,b^9\,x^2+630\,a^6\,b^4\,x^{1/3}+378\,a^5\,b^5\,x^{2/3}+90\,a^3\,b^7\,x^{4/3}+27\,a^2\,b^8\,x^{5/3}$$

input
```
int((a + b*x^(1/3))^10/x^2,x)
```

output
```
(3*b^10*x^(7/3))/7 - (a^10 + 15*a^9*b*x^(1/3) + 135*a^8*b^2*x^(2/3))/x + 3
60*a^7*b^3*log(x^(1/3)) + 210*a^4*b^6*x + 5*a*b^9*x^2 + 630*a^6*b^4*x^(1/3
) + 378*a^5*b^5*x^(2/3) + 90*a^3*b^7*x^(4/3) + 27*a^2*b^8*x^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^2}\,dx = \frac{2520x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)a^7b^3-7x^{\frac{2}{3}}a^{10}+1470x^{\frac{8}{3}}a^4b^6+35x^{\frac{11}{3}}a\,b^9-945x^{\frac{4}{3}}a^8b^2+2646x^{\frac{7}{3}}a^5b^5+189x^{\frac{10}{3}}a^2b^8-10}{7x^{\frac{5}{3}}}$$

input
```
int((a+b*x^(1/3))^10/x^2,x)
```

output
```
(2520*x**(2/3)*log(x**(1/3))*a**7*b**3*x - 7*x**(2/3)*a**10 + 1470*x**(2/3
)*a**4*b**6*x**2 + 35*x**(2/3)*a*b**9*x**3 - 945*x**(1/3)*a**8*b**2*x + 26
46*x**(1/3)*a**5*b**5*x**2 + 189*x**(1/3)*a**2*b**8*x**3 - 105*a**9*b*x +
4410*a**6*b**4*x**2 + 630*a**3*b**7*x**3 + 3*b**10*x**4)/(7*x**(2/3)*x)
```

3.222 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx = -\frac{a^{10}}{2x^2}-\frac{6a^9b}{x^{5/3}}-\frac{135a^8b^2}{4x^{4/3}}-\frac{120a^7b^3}{x}-\frac{315a^6b^4}{x^{2/3}}-\frac{756a^5b^5}{\sqrt[3]{x}}+360a^3b^7\sqrt[3]{x}+\frac{135}{2}a^2b^8x^{2/3}+10ab^9x+\frac{3}{4}b^{10}x^{4/3}+210a^4b^6\log(x)$$

output
```
-1/2*a^10/x^2-6*a^9*b/x^(5/3)-135/4*a^8*b^2/x^(4/3)-120*a^7*b^3/x-315*a^6*
b^4/x^(2/3)-756*a^5*b^5/x^(1/3)+360*a^3*b^7*x^(1/3)+135/2*a^2*b^8*x^(2/3)+
10*a*b^9*x+3/4*b^10*x^(4/3)+210*a^4*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx = \frac{-2a^{10}-24a^9b\sqrt[3]{x}-135a^8b^2x^{2/3}-480a^7b^3x-1260a^6b^4x^{4/3}-3024a^5b^5x^{5/3}+1440a^3b^7x^{7/3}+270a^2b^8x}{4x^2}+630a^4b^6\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^10/x^3,x]
```

output
```
(-2*a^10 - 24*a^9*b*x^(1/3) - 135*a^8*b^2*x^(2/3) - 480*a^7*b^3*x - 1260*a
^6*b^4*x^(4/3) - 3024*a^5*b^5*x^(5/3) + 1440*a^3*b^7*x^(7/3) + 270*a^2*b^8
*x^(8/3) + 40*a*b^9*x^3 + 3*b^10*x^(10/3))/(4*x^2) + 630*a^4*b^6*Log[x^(1/
3)]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^{10}}{x^{7/3}}+\frac{10ba^9}{x^2}+\frac{45b^2a^8}{x^{5/3}}+\frac{120b^3a^7}{x^{4/3}}+\frac{210b^4a^6}{x}+\frac{252b^5a^5}{x^{2/3}}+\frac{210b^6a^4}{\sqrt[3]{x}}+120b^7a^3+45b^8\sqrt[3]{x}a^2+10b^9x^{2/3}a\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{10}}{6x^2}-\frac{2a^9b}{x^{5/3}}-\frac{45a^8b^2}{4x^{4/3}}-\frac{40a^7b^3}{x}-\frac{105a^6b^4}{x^{2/3}}-\frac{252a^5b^5}{\sqrt[3]{x}}+210a^4b^6\log\left(\sqrt[3]{x}\right)+120a^3b^7\sqrt[3]{x}+\frac{45}{2}a^2b^8x^{2/3}+\right.$$

input
```
Int[(a + b*x^(1/3))^10/x^3,x]
```

output
```
3*(-1/6*a^10/x^2 - (2*a^9*b)/x^(5/3) - (45*a^8*b^2)/(4*x^(4/3)) - (40*a^7*
b^3)/x - (105*a^6*b^4)/x^(2/3) - (252*a^5*b^5)/x^(1/3) + 120*a^3*b^7*x^(1/
3) + (45*a^2*b^8*x^(2/3))/2 + (10*a*b^9*x)/3 + (b^10*x^(4/3))/4 + 210*a^4*
b^6*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{a^{10}}{2x^{2}}-\frac{6a^{9}b}{x^{\frac{5}{3}}}-\frac{135a^{8}b^{2}}{4x^{\frac{4}{3}}}-\frac{120a^{7}b^{3}}{x}-\frac{315a^{6}b^{4}}{x^{\frac{2}{3}}}-\frac{756a^{5}b^{5}}{x^{\frac{1}{3}}}+360a^{3}b^{7}x^{\frac{1}{3}}+\frac{135a^{2}b^{8}x^{\frac{2}{3}}}{2}+10a\,b^{9}x$
default	$-\frac{a^{10}}{2x^{2}}-\frac{6a^{9}b}{x^{\frac{5}{3}}}-\frac{135a^{8}b^{2}}{4x^{\frac{4}{3}}}-\frac{120a^{7}b^{3}}{x}-\frac{315a^{6}b^{4}}{x^{\frac{2}{3}}}-\frac{756a^{5}b^{5}}{x^{\frac{1}{3}}}+360a^{3}b^{7}x^{\frac{1}{3}}+\frac{135a^{2}b^{8}x^{\frac{2}{3}}}{2}+10a\,b^{9}x$
trager	$\frac{(20b^{9}x^{2}+a^{9}x+240a^{6}b^{3}x+a^{9})a(-1+x)}{2x^{2}}-\frac{3(-b^{9}x^{3}-480a^{3}b^{6}x^{2}+420a^{6}b^{3}x+8a^{9})b}{4x^{\frac{5}{3}}}-\frac{27(-10b^{6}x^{2}+112a^{3}b^{3}x+5a}{4x^{\frac{4}{3}}}$

input
```
int((a+b*x^(1/3))^10/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^10/x^2-6*a^9*b/x^(5/3)-135/4*a^8*b^2/x^(4/3)-120*a^7*b^3/x-315*a^6*
b^4/x^(2/3)-756*a^5*b^5/x^(1/3)+360*a^3*b^7*x^(1/3)+135/2*a^2*b^8*x^(2/3)+
10*a*b^9*x+3/4*b^10*x^(4/3)+210*a^4*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx = \frac{40\,ab^9x^3+2520\,a^4b^6x^2\log\left(x^{\frac{1}{3}}\right)-480\,a^7b^3x-2\,a^{10}+27\,(10\,a^2b^8x^2-112\,a^5b^5x-5\,a^8b^2)x^{\frac{2}{3}}+3\,(b^{10}x^3}{4\,x^2}$$

input
```
integrate((a+b*x^(1/3))^10/x^3,x, algorithm="fricas")
```

output
```
1/4*(40*a*b^9*x^3 + 2520*a^4*b^6*x^2*log(x^(1/3)) - 480*a^7*b^3*x - 2*a^10
 + 27*(10*a^2*b^8*x^2 - 112*a^5*b^5*x - 5*a^8*b^2)*x^(2/3) + 3*(b^10*x^3 +
 480*a^3*b^7*x^2 - 420*a^6*b^4*x - 8*a^9*b)*x^(1/3))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx = -\frac{a^{10}}{2x^2}-\frac{6a^9b}{x^{\frac{5}{3}}}-\frac{135a^8b^2}{4x^{\frac{4}{3}}}-\frac{120a^7b^3}{x}-\frac{315a^6b^4}{x^{\frac{2}{3}}}-\frac{756a^5b^5}{\sqrt[3]{x}} + 630a^4b^6\log\left(\sqrt[3]{x}\right)+360a^3b^7\sqrt[3]{x}+\frac{135a^2b^8x^{\frac{2}{3}}}{2}+10ab^9x+\frac{3b^{10}x^{\frac{4}{3}}}{4}$$

input
```
integrate((a+b*x**(1/3))**10/x**3,x)
```

output
```
-a**10/(2*x**2) - 6*a**9*b/x**(5/3) - 135*a**8*b**2/(4*x**(4/3)) - 120*a**
7*b**3/x - 315*a**6*b**4/x**(2/3) - 756*a**5*b**5/x**(1/3) + 630*a**4*b**6
*log(x**(1/3)) + 360*a**3*b**7*x**(1/3) + 135*a**2*b**8*x**(2/3)/2 + 10*a*
b**9*x + 3*b**10*x**(4/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx = \frac{3}{4}\,b^{10}x^{\frac{4}{3}} + 10\,ab^9x + 210\,a^4b^6\log\left(x\right) + \frac{135}{2}\,a^2b^8x^{\frac{2}{3}} + 360\,a^3b^7x^{\frac{1}{3}} - \frac{3024\,a^5b^5x^{\frac{5}{3}} + 1260\,a^6b^4x^{\frac{4}{3}} + 480\,a^7b^3x + 135\,a^8b^2x^{\frac{2}{3}} + 24\,a^9bx^{\frac{1}{3}} + 2\,a^{10}}{4\,x^2}$$

input
```
integrate((a+b*x^(1/3))^10/x^3,x, algorithm="maxima")
```

output
```
3/4*b^10*x^(4/3) + 10*a*b^9*x + 210*a^4*b^6*log(x) + 135/2*a^2*b^8*x^(2/3)
 + 360*a^3*b^7*x^(1/3) - 1/4*(3024*a^5*b^5*x^(5/3) + 1260*a^6*b^4*x^(4/3)
+ 480*a^7*b^3*x + 135*a^8*b^2*x^(2/3) + 24*a^9*b*x^(1/3) + 2*a^10)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx = \frac{3}{4}\,b^{10}x^{\frac{4}{3}} + 10\,ab^9x + 210\,a^4b^6\log\left(|x|\right) + \frac{135}{2}\,a^2b^8x^{\frac{2}{3}} + 360\,a^3b^7x^{\frac{1}{3}} - \frac{3024\,a^5b^5x^{\frac{5}{3}} + 1260\,a^6b^4x^{\frac{4}{3}} + 480\,a^7b^3x + 135\,a^8b^2x^{\frac{2}{3}} + 24\,a^9bx^{\frac{1}{3}} + 2\,a^{10}}{4\,x^2}$$

input
```
integrate((a+b*x^(1/3))^10/x^3,x, algorithm="giac")
```

output
```
3/4*b^10*x^(4/3) + 10*a*b^9*x + 210*a^4*b^6*log(abs(x)) + 135/2*a^2*b^8*x^
(2/3) + 360*a^3*b^7*x^(1/3) - 1/4*(3024*a^5*b^5*x^(5/3) + 1260*a^6*b^4*x^(
4/3) + 480*a^7*b^3*x + 135*a^8*b^2*x^(2/3) + 24*a^9*b*x^(1/3) + 2*a^10)/x^
2
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx$$
$$= \frac{3\,b^{10}\,x^{4/3}}{4} - \frac{\frac{a^{10}}{2}+120\,a^7\,b^3\,x+6\,a^9\,b\,x^{1/3}+\frac{135\,a^8\,b^2\,x^{2/3}}{4}+315\,a^6\,b^4\,x^{4/3}+756\,a^5\,b^5\,x^{5/3}}{x^2} + 630\,a^4\,b^6\,\ln\left(x^{1/3}\right)+360\,a^3\,b^7\,x^{1/3}+\frac{135\,a^2\,b^8\,x^{2/3}}{2}+10\,a\,b^9\,x$$

input
```
int((a + b*x^(1/3))^10/x^3,x)
```

output
```
(3*b^10*x^(4/3))/4 - (a^10/2 + 120*a^7*b^3*x + 6*a^9*b*x^(1/3) + (135*a^8*
b^2*x^(2/3))/4 + 315*a^6*b^4*x^(4/3) + 756*a^5*b^5*x^(5/3))/x^2 + 630*a^4*
b^6*log(x^(1/3)) + 360*a^3*b^7*x^(1/3) + (135*a^2*b^8*x^(2/3))/2 + 10*a*b^
9*x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^3}\,dx$$
$$= \frac{2520x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}\right)a^4b^6-2x^{\frac{2}{3}}a^{10}-480x^{\frac{5}{3}}a^7b^3+40x^{\frac{11}{3}}a\,b^9-135x^{\frac{4}{3}}a^8b^2-3024x^{\frac{7}{3}}a^5b^5+270x^{\frac{10}{3}}a^2b^8-24a}{4x^{\frac{8}{3}}}$$

input
```
int((a+b*x^(1/3))^10/x^3,x)
```

output
```
(2520*x**(2/3)*log(x**(1/3))*a**4*b**6*x**2 - 2*x**(2/3)*a**10 - 480*x**(2
/3)*a**7*b**3*x + 40*x**(2/3)*a*b**9*x**3 - 135*x**(1/3)*a**8*b**2*x - 302
4*x**(1/3)*a**5*b**5*x**2 + 270*x**(1/3)*a**2*b**8*x**3 - 24*a**9*b*x - 12
60*a**6*b**4*x**2 + 1440*a**3*b**7*x**3 + 3*b**10*x**4)/(4*x**(2/3)*x**2)
```

3.223 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx = -\frac{a^{10}}{3x^3}-\frac{15a^9b}{4x^{8/3}}-\frac{135a^8b^2}{7x^{7/3}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{5/3}}-\frac{189a^5b^5}{x^{4/3}}-\frac{210a^4b^6}{x}-\frac{180a^3b^7}{x^{2/3}}-\frac{135a^2b^8}{\sqrt[3]{x}}+3b^{10}\sqrt[3]{x}+10ab^9\log(x)$$

output
```
-1/3*a^10/x^3-15/4*a^9*b/x^(8/3)-135/7*a^8*b^2/x^(7/3)-60*a^7*b^3/x^2-126*
a^6*b^4/x^(5/3)-189*a^5*b^5/x^(4/3)-210*a^4*b^6/x-180*a^3*b^7/x^(2/3)-135*
a^2*b^8/x^(1/3)+3*b^10*x^(1/3)+10*a*b^9*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx = \frac{-28a^{10}-315a^9b\sqrt[3]{x}-1620a^8b^2x^{2/3}-5040a^7b^3x-10584a^6b^4x^{4/3}-15876a^5b^5x^{5/3}-17640a^4b^6x^2-15}{84x^3} + 30ab^9\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^10/x^4,x]
```

output
```
(-28*a^10 - 315*a^9*b*x^(1/3) - 1620*a^8*b^2*x^(2/3) - 5040*a^7*b^3*x - 10
584*a^6*b^4*x^(4/3) - 15876*a^5*b^5*x^(5/3) - 17640*a^4*b^6*x^2 - 15120*a^
3*b^7*x^(7/3) - 11340*a^2*b^8*x^(8/3) + 252*b^10*x^(10/3))/(84*x^3) + 30*a
*b^9*Log[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^{10}}{x^{10/3}}+\frac{10ba^9}{x^3}+\frac{45b^2a^8}{x^{8/3}}+\frac{120b^3a^7}{x^{7/3}}+\frac{210b^4a^6}{x^2}+\frac{252b^5a^5}{x^{5/3}}+\frac{210b^6a^4}{x^{4/3}}+\frac{120b^7a^3}{x}+\frac{45b^8a^2}{x^{2/3}}+\frac{10b^9a}{\sqrt[3]{x}}+b^{10}\right)$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{10}}{9x^3}-\frac{5a^9b}{4x^{8/3}}-\frac{45a^8b^2}{7x^{7/3}}-\frac{20a^7b^3}{x^2}-\frac{42a^6b^4}{x^{5/3}}-\frac{63a^5b^5}{x^{4/3}}-\frac{70a^4b^6}{x}-\frac{60a^3b^7}{x^{2/3}}-\frac{45a^2b^8}{\sqrt[3]{x}}+10ab^9\log\left(\sqrt[3]{x}\right)+b^{10}\right.$$

input
```
Int[(a + b*x^(1/3))^10/x^4,x]
```

output
```
3*(-1/9*a^10/x^3 - (5*a^9*b)/(4*x^(8/3)) - (45*a^8*b^2)/(7*x^(7/3)) - (20*
a^7*b^3)/x^2 - (42*a^6*b^4)/x^(5/3) - (63*a^5*b^5)/x^(4/3) - (70*a^4*b^6)/
x - (60*a^3*b^7)/x^(2/3) - (45*a^2*b^8)/x^(1/3) + b^10*x^(1/3) + 10*a*b^9*
Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{a^{10}}{3x^3}-\frac{15a^9b}{4x^{\frac{8}{3}}}-\frac{135a^8b^2}{7x^{\frac{7}{3}}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{\frac{5}{3}}}-\frac{189a^5b^5}{x^{\frac{4}{3}}}-\frac{210a^4b^6}{x}-\frac{180a^3b^7}{x^{\frac{2}{3}}}-\frac{135a^2b^8}{x^{\frac{1}{3}}}+3b^{10}x$
default	$-\frac{a^{10}}{3x^3}-\frac{15a^9b}{4x^{\frac{8}{3}}}-\frac{135a^8b^2}{7x^{\frac{7}{3}}}-\frac{60a^7b^3}{x^2}-\frac{126a^6b^4}{x^{\frac{5}{3}}}-\frac{189a^5b^5}{x^{\frac{4}{3}}}-\frac{210a^4b^6}{x}-\frac{180a^3b^7}{x^{\frac{2}{3}}}-\frac{135a^2b^8}{x^{\frac{1}{3}}}+3b^{10}x$
trager	$\frac{(-1+x)(a^6x^2+180a^3b^3x^2+630b^6x^2+a^6x+180a^3b^3x+a^6)a^4}{3x^3}-\frac{3(-4b^9x^3+240a^3b^6x^2+168a^6b^3x+5a^9)b}{4x^{\frac{8}{3}}}-\frac{27(35$

input
```
int((a+b*x^(1/3))^10/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^10/x^3-15/4*a^9*b/x^(8/3)-135/7*a^8*b^2/x^(7/3)-60*a^7*b^3/x^2-126*
a^6*b^4/x^(5/3)-189*a^5*b^5/x^(4/3)-210*a^4*b^6/x-180*a^3*b^7/x^(2/3)-135*
a^2*b^8/x^(1/3)+3*b^10*x^(1/3)+10*a*b^9*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx = \frac{2520\,ab^9x^3\log\left(x^{\frac{1}{3}}\right) - 17640\,a^4b^6x^2 - 5040\,a^7b^3x - 28\,a^{10} - 324\,(35\,a^2b^8x^2 + 49\,a^5b^5x + 5\,a^8b^2)x^{\frac{2}{3}} + 63}{84\,x^3}$$

input
```
integrate((a+b*x^(1/3))^10/x^4,x, algorithm="fricas")
```

output
```
1/84*(2520*a*b^9*x^3*log(x^(1/3)) - 17640*a^4*b^6*x^2 - 5040*a^7*b^3*x - 2
8*a^10 - 324*(35*a^2*b^8*x^2 + 49*a^5*b^5*x + 5*a^8*b^2)*x^(2/3) + 63*(4*b
^10*x^3 - 240*a^3*b^7*x^2 - 168*a^6*b^4*x - 5*a^9*b)*x^(1/3))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx = -\frac{a^{10}}{3x^3} - \frac{15a^9b}{4x^{\frac{8}{3}}} - \frac{135a^8b^2}{7x^{\frac{7}{3}}} - \frac{60a^7b^3}{x^2} - \frac{126a^6b^4}{x^{\frac{5}{3}}} - \frac{189a^5b^5}{x^{\frac{4}{3}}} - \frac{210a^4b^6}{x} - \frac{180a^3b^7}{x^{\frac{2}{3}}} - \frac{135a^2b^8}{\sqrt[3]{x}} + 10ab^9\log(x) + 3b^{10}\sqrt[3]{x}$$

input
```
integrate((a+b*x**(1/3))**10/x**4,x)
```

output
```
-a**10/(3*x**3) - 15*a**9*b/(4*x**(8/3)) - 135*a**8*b**2/(7*x**(7/3)) - 60
*a**7*b**3/x**2 - 126*a**6*b**4/x**(5/3) - 189*a**5*b**5/x**(4/3) - 210*a*
*4*b**6/x - 180*a**3*b**7/x**(2/3) - 135*a**2*b**8/x**(1/3) + 10*a*b**9*lo
g(x) + 3*b**10*x**(1/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx = 10\,ab^9\log\left(x\right)+3\,b^{10}x^{\frac{1}{3}} - \frac{11340\,a^2b^8x^{\frac{8}{3}}+15120\,a^3b^7x^{\frac{7}{3}}+17640\,a^4b^6x^2+15876\,a^5b^5x^{\frac{5}{3}}+10584\,a^6b^4x^{\frac{4}{3}}+5040\,a^7b^3x+1620\,a^8b}{84\,x^3}$$

input
```
integrate((a+b*x^(1/3))^10/x^4,x, algorithm="maxima")
```

output
```
10*a*b^9*log(x) + 3*b^10*x^(1/3) - 1/84*(11340*a^2*b^8*x^(8/3) + 15120*a^3
*b^7*x^(7/3) + 17640*a^4*b^6*x^2 + 15876*a^5*b^5*x^(5/3) + 10584*a^6*b^4*x
^(4/3) + 5040*a^7*b^3*x + 1620*a^8*b^2*x^(2/3) + 315*a^9*b*x^(1/3) + 28*a^
10)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx = 10\,ab^9\log\left(|x|\right)+3\,b^{10}x^{\frac{1}{3}} - \frac{11340\,a^2b^8x^{\frac{8}{3}}+15120\,a^3b^7x^{\frac{7}{3}}+17640\,a^4b^6x^2+15876\,a^5b^5x^{\frac{5}{3}}+10584\,a^6b^4x^{\frac{4}{3}}+5040\,a^7b^3x+1620\,a^8b}{84\,x^3}$$

input
```
integrate((a+b*x^(1/3))^10/x^4,x, algorithm="giac")
```

output
```
10*a*b^9*log(abs(x)) + 3*b^10*x^(1/3) - 1/84*(11340*a^2*b^8*x^(8/3) + 1512
0*a^3*b^7*x^(7/3) + 17640*a^4*b^6*x^2 + 15876*a^5*b^5*x^(5/3) + 10584*a^6*
b^4*x^(4/3) + 5040*a^7*b^3*x + 1620*a^8*b^2*x^(2/3) + 315*a^9*b*x^(1/3) +
28*a^10)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx = 3\,b^{10}\,x^{1/3}-\frac{a^{10}}{3\,x^3}-\frac{15\,a^9\,b}{4\,x^{8/3}}-\frac{210\,a^4\,b^6}{x}-\frac{60\,a^7\,b^3}{x^2}-\frac{135\,a^2\,b^8}{x^{1/3}}-\frac{180\,a^3\,b^7}{x^{2/3}}-\frac{189\,a^5\,b^5}{x^{4/3}}-\frac{126\,a^6\,b^4}{x^{5/3}}-\frac{135\,a^8\,b^2}{7\,x^{7/3}}+30\,a\,b^9\,\ln\left(x^{1/3}\right)$$

input
```
int((a + b*x^(1/3))^10/x^4,x)
```

output
```
3*b^10*x^(1/3) - a^10/(3*x^3) - (15*a^9*b)/(4*x^(8/3)) - (210*a^4*b^6)/x -
 (60*a^7*b^3)/x^2 - (135*a^2*b^8)/x^(1/3) - (180*a^3*b^7)/x^(2/3) - (189*a
^5*b^5)/x^(4/3) - (126*a^6*b^4)/x^(5/3) - (135*a^8*b^2)/(7*x^(7/3)) + 30*a
*b^9*log(x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}\,dx$$
$$=\frac{2520x^{\frac{11}{3}}\log\left(x^{\frac{1}{3}}\right)a\,b^9-28x^{\frac{2}{3}}a^{10}-5040x^{\frac{5}{3}}a^7b^3-17640x^{\frac{8}{3}}a^4b^6-1620x^{\frac{4}{3}}a^8b^2-15876x^{\frac{7}{3}}a^5b^5-11340x^{\frac{10}{3}}}{84x^{\frac{11}{3}}}$$

input
```
int((a+b*x^(1/3))^10/x^4,x)
```

output
```
(2520*x**(2/3)*log(x**(1/3))*a*b**9*x**3 - 28*x**(2/3)*a**10 - 5040*x**(2/
3)*a**7*b**3*x - 17640*x**(2/3)*a**4*b**6*x**2 - 1620*x**(1/3)*a**8*b**2*x
 - 15876*x**(1/3)*a**5*b**5*x**2 - 11340*x**(1/3)*a**2*b**8*x**3 - 315*a**
9*b*x - 10584*a**6*b**4*x**2 - 15120*a**3*b**7*x**3 + 252*b**10*x**4)/(84*
x**(2/3)*x**3)
```

3.224 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5} dx = -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{4ax^4} + \frac{b\left(a+b\sqrt[3]{x}\right)^{11}}{44a^2x^{11/3}}$$

output
```
-1/4*(a+b*x^(1/3))^11/a/x^4+1/44*b*(a+b*x^(1/3))^11/a^2/x^(11/3)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(46) = 92.

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5} dx = \frac{-11a^{10} - 120a^9b\sqrt[3]{x} - 594a^8b^2x^{2/3} - 1760a^7b^3x - 3465a^6b^4x^{4/3} - 4752a^5b^5x^{5/3} - 4620a^4b^6x^2 - 3168a^3}{44x^4}$$

input
```
Integrate[(a + b*x^(1/3))^10/x^5,x]
```

output
```
(-11*a^10 - 120*a^9*b*x^(1/3) - 594*a^8*b^2*x^(2/3) - 1760*a^7*b^3*x - 346
5*a^6*b^4*x^(4/3) - 4752*a^5*b^5*x^(5/3) - 4620*a^4*b^6*x^2 - 3168*a^3*b^7
*x^(7/3) - 1485*a^2*b^8*x^(8/3) - 440*a*b^9*x^3 - 66*b^10*x^(10/3))/(44*x^
4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{13/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{b\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}d\sqrt[3]{x}}{12a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{12ax^4}\right)$$

$$\Big\downarrow \text{48}$$

$$3\left(\frac{b\left(a+b\sqrt[3]{x}\right)^{11}}{132a^2x^{11/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{12ax^4}\right)$$

input
```
Int[(a + b*x^(1/3))^10/x^5,x]
```

output
```
3*(-1/12*(a + b*x^(1/3))^11/(a*x^4) + (b*(a + b*x^(1/3))^11)/(132*a^2*x^(1
1/3)))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(36) = 72$.

Time = 4.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.46

method	result
derivativedivides	$-\frac{27a^8b^2}{2x^{\frac{10}{3}}}-\frac{3b^{10}}{2x^{\frac{2}{3}}}-\frac{10a\,b^9}{x}-\frac{105a^4b^6}{x^2}-\frac{a^{10}}{4x^4}-\frac{72a^3b^7}{x^{\frac{5}{3}}}-\frac{135a^2b^8}{4x^{\frac{4}{3}}}-\frac{108a^5b^5}{x^{\frac{7}{3}}}-\frac{315a^6b^4}{4x^{\frac{8}{3}}}-\frac{30a^9b}{11x^{\frac{11}{3}}}-$
default	$-\frac{27a^8b^2}{2x^{\frac{10}{3}}}-\frac{3b^{10}}{2x^{\frac{2}{3}}}-\frac{10a\,b^9}{x}-\frac{105a^4b^6}{x^2}-\frac{a^{10}}{4x^4}-\frac{72a^3b^7}{x^{\frac{5}{3}}}-\frac{135a^2b^8}{4x^{\frac{4}{3}}}-\frac{108a^5b^5}{x^{\frac{7}{3}}}-\frac{315a^6b^4}{4x^{\frac{8}{3}}}-\frac{30a^9b}{11x^{\frac{11}{3}}}-$
trager	$\frac{(-1+x)\left(a^9x^3+160a^6b^3x^3+420a^3b^6x^3+40b^9x^3+a^9x^2+160a^6b^3x^2+420a^3b^6x^2+a^9x+160a^6b^3x+a^9\right)a}{4x^4}-\frac{3\left(22b^9x^3\right.}{}$
orering	$-\frac{\left(682b^{24}x^8-3388a^3b^{21}x^7+10406a^6b^{18}x^6+2884a^9b^{15}x^5+7154a^{12}b^{12}x^4+5460a^{15}b^9x^3+2688a^{18}b^6x^2+759a^{21}b^3x+\right.}{88x^4(b^3x+a^3)^8}$

input
```
int((a+b*x^(1/3))^10/x^5,x,method=_RETURNVERBOSE)
```

output
```
-27/2*a^8*b^2/x^(10/3)-3/2*b^10/x^(2/3)-10*a*b^9/x-105*a^4*b^6/x^2-1/4*a^1
0/x^4-72*a^3*b^7/x^(5/3)-135/4*a^2*b^8/x^(4/3)-108*a^5*b^5/x^(7/3)-315/4*a
^6*b^4/x^(8/3)-30/11*a^9*b/x^(11/3)-40/x^3*a^7*b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(36) = 72.

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx = -\frac{440\,ab^9x^3+4620\,a^4b^6x^2+1760\,a^7b^3x+11\,a^{10}+297\,(5\,a^2b^8x^2+16\,a^5b^5x+2\,a^8b^2)x^{\frac{2}{3}}+3\,(22\,b^{10}x^3+}{44\,x^4}$$

input
```
integrate((a+b*x^(1/3))^10/x^5,x, algorithm="fricas")
```

output
```
-1/44*(440*a*b^9*x^3 + 4620*a^4*b^6*x^2 + 1760*a^7*b^3*x + 11*a^10 + 297*(
5*a^2*b^8*x^2 + 16*a^5*b^5*x + 2*a^8*b^2)*x^(2/3) + 3*(22*b^10*x^3 + 1056*
a^3*b^7*x^2 + 1155*a^6*b^4*x + 40*a^9*b)*x^(1/3))/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(37) = 74.

Time = 0.68 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.02

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx = -\frac{a^{10}}{4x^4}-\frac{30a^9b}{11x^{\frac{11}{3}}}-\frac{27a^8b^2}{2x^{\frac{10}{3}}}-\frac{40a^7b^3}{x^3}-\frac{315a^6b^4}{4x^{\frac{8}{3}}}-\frac{108a^5b^5}{x^{\frac{7}{3}}} -\frac{105a^4b^6}{x^2}-\frac{72a^3b^7}{x^{\frac{5}{3}}}-\frac{135a^2b^8}{4x^{\frac{4}{3}}}-\frac{10ab^9}{x}-\frac{3b^{10}}{2x^{\frac{2}{3}}}$$

input
```
integrate((a+b*x**(1/3))**10/x**5,x)
```

output
```
-a**10/(4*x**4) - 30*a**9*b/(11*x**(11/3)) - 27*a**8*b**2/(2*x**(10/3)) -
40*a**7*b**3/x**3 - 315*a**6*b**4/(4*x**(8/3)) - 108*a**5*b**5/x**(7/3) -
105*a**4*b**6/x**2 - 72*a**3*b**7/x**(5/3) - 135*a**2*b**8/(4*x**(4/3)) -
10*a*b**9/x - 3*b**10/(2*x**(2/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx = -\frac{66\,b^{10}x^{\frac{10}{3}}+440\,ab^9x^3+1485\,a^2b^8x^{\frac{8}{3}}+3168\,a^3b^7x^{\frac{7}{3}}+4620\,a^4b^6x^2+4752\,a^5b^5x^{\frac{5}{3}}+3465\,a^6b^4x^{\frac{4}{3}}+176}{44\,x^4}$$

input
```
integrate((a+b*x^(1/3))^10/x^5,x, algorithm="maxima")
```

output
```
-1/44*(66*b^10*x^(10/3) + 440*a*b^9*x^3 + 1485*a^2*b^8*x^(8/3) + 3168*a^3*
b^7*x^(7/3) + 4620*a^4*b^6*x^2 + 4752*a^5*b^5*x^(5/3) + 3465*a^6*b^4*x^(4/
3) + 1760*a^7*b^3*x + 594*a^8*b^2*x^(2/3) + 120*a^9*b*x^(1/3) + 11*a^10)/x
^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx = -\frac{66\,b^{10}x^{\frac{10}{3}}+440\,ab^9x^3+1485\,a^2b^8x^{\frac{8}{3}}+3168\,a^3b^7x^{\frac{7}{3}}+4620\,a^4b^6x^2+4752\,a^5b^5x^{\frac{5}{3}}+3465\,a^6b^4x^{\frac{4}{3}}+176}{44\,x^4}$$

input
```
integrate((a+b*x^(1/3))^10/x^5,x, algorithm="giac")
```

output
```
-1/44*(66*b^10*x^(10/3) + 440*a*b^9*x^3 + 1485*a^2*b^8*x^(8/3) + 3168*a^3*
b^7*x^(7/3) + 4620*a^4*b^6*x^2 + 4752*a^5*b^5*x^(5/3) + 3465*a^6*b^4*x^(4/
3) + 1760*a^7*b^3*x + 594*a^8*b^2*x^(2/3) + 120*a^9*b*x^(1/3) + 11*a^10)/x
^4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx = -\frac{\frac{a^{10}}{4}+\frac{3\,b^{10}\,x^{10/3}}{2}+40\,a^7\,b^3\,x+10\,a\,b^9\,x^3+\frac{30\,a^9\,b\,x^{1/3}}{11}+105\,a^4\,b^6\,x^2+\frac{27\,a^8\,b^2\,x^{2/3}}{2}+\frac{315\,a^6\,b^4\,x^{4/3}}{4}+108\,a^5\,b^5}{x^4}$$

input
```
int((a + b*x^(1/3))^10/x^5,x)
```

output
```
-(a^10/4 + (3*b^10*x^(10/3))/2 + 40*a^7*b^3*x + 10*a*b^9*x^3 + (30*a^9*b*x
^(1/3))/11 + 105*a^4*b^6*x^2 + (27*a^8*b^2*x^(2/3))/2 + (315*a^6*b^4*x^(4/
3))/4 + 108*a^5*b^5*x^(5/3) + 72*a^3*b^7*x^(7/3) + (135*a^2*b^8*x^(8/3))/4
)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}\,dx = \frac{-11x^{\frac{2}{3}}a^{10}-1760x^{\frac{5}{3}}a^7b^3-4620x^{\frac{8}{3}}a^4b^6-440x^{\frac{11}{3}}a\,b^9-594x^{\frac{4}{3}}a^8b^2-4752x^{\frac{7}{3}}a^5b^5-1485x^{\frac{10}{3}}a^2b^8-120a^9}{44x^{\frac{14}{3}}}$$

input
```
int((a+b*x^(1/3))^10/x^5,x)
```

output
```
( - 11*x**(2/3)*a**10 - 1760*x**(2/3)*a**7*b**3*x - 4620*x**(2/3)*a**4*b**
6*x**2 - 440*x**(2/3)*a*b**9*x**3 - 594*x**(1/3)*a**8*b**2*x - 4752*x**(1/
3)*a**5*b**5*x**2 - 1485*x**(1/3)*a**2*b**8*x**3 - 120*a**9*b*x - 3465*a**
6*b**4*x**2 - 3168*a**3*b**7*x**3 - 66*b**10*x**4)/(44*x**(2/3)*x**4)
```

3.225 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = -\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{5ax^5} + \frac{2b\left(a+b\sqrt[3]{x}\right)^{11}}{35a^2x^{14/3}} - \frac{6b^2\left(a+b\sqrt[3]{x}\right)^{11}}{455a^3x^{13/3}} + \frac{b^3\left(a+b\sqrt[3]{x}\right)^{11}}{455a^4x^4} - \frac{b^4\left(a+b\sqrt[3]{x}\right)^{11}}{5005a^5x^{11/3}}$$

output
```
-1/5*(a+b*x^(1/3))^11/a/x^5+2/35*b*(a+b*x^(1/3))^11/a^2/x^(14/3)-6/455*b^2
*(a+b*x^(1/3))^11/a^3/x^(13/3)+1/455*b^3*(a+b*x^(1/3))^11/a^4/x^4-1/5005*b
^4*(a+b*x^(1/3))^11/a^5/x^(11/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = \frac{-1001a^{10} - 10725a^9b\sqrt[3]{x} - 51975a^8b^2x^{2/3} - 150150a^7b^3x - 286650a^6b^4x^{4/3} - 378378a^5b^5x^{5/3} - 350350}{5005x^5}$$

input
```
Integrate[(a + b*x^(1/3))^10/x^6,x]
```

output
```
(-1001*a^10 - 10725*a^9*b*x^(1/3) - 51975*a^8*b^2*x^(2/3) - 150150*a^7*b^3
*x - 286650*a^6*b^4*x^(4/3) - 378378*a^5*b^5*x^(5/3) - 350350*a^4*b^6*x^2
- 225225*a^3*b^7*x^(7/3) - 96525*a^2*b^8*x^(8/3) - 25025*a*b^9*x^3 - 3003*
b^10*x^(10/3))/(5005*x^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{16/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{4b\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^5}d\sqrt[3]{x}}{15a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{15ax^5}\right)$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{4b\left(-\frac{3b\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{14/3}}d\sqrt[3]{x}}{14a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{14ax^{14/3}}\right)}{15a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{15ax^5}\right)$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\int\frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{13/3}}d\sqrt[3]{x}}{13a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{13ax^{13/3}}\right)}{14a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{14ax^{14/3}}\right)}{15a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{15ax^5}\right)$$

$$\Big\downarrow \; 55$$

$$3\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(-\frac{b\int\frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^4}d\sqrt[3]{x}}{12a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{12ax^4}\right)}{13a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{13ax^{13/3}}\right)}{14a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{14ax^{14/3}}\right)}{15a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{15ax^5}\right)$$

$$\Big\downarrow \color{blue}{48}$$

$$3\left(-\frac{4b\left(-\frac{3b\left(-\frac{2b\left(\frac{b\left(a+b\sqrt[3]{x}\right)^{11}}{132a^2x^{11/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{12ax^4}\right)}{13a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{13ax^{13/3}}\right)}{14a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{14ax^{14/3}}\right)}{15a}-\frac{\left(a+b\sqrt[3]{x}\right)^{11}}{15ax^5}\right)$$

input
```
Int[(a + b*x^(1/3))^10/x^6,x]
```

output
```
3*((-4*b*((-3*b*((-2*b*(-1/12*(a + b*x^(1/3))^11/(a*x^4) + (b*(a + b*x^(1/
3))^11)/(132*a^2*x^(11/3))))/(13*a) - (a + b*x^(1/3))^11/(13*a*x^(13/3))))
/(14*a) - (a + b*x^(1/3))^11/(14*a*x^(14/3))))/(15*a) - (a + b*x^(1/3))^11
/(15*a*x^5))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

method	result
derivativedivides	$-\frac{5a\,b^9}{x^2}-\frac{135a^8b^2}{13x^{\frac{13}{3}}}-\frac{378a^5b^5}{5x^{\frac{10}{3}}}-\frac{630a^6b^4}{11x^{\frac{11}{3}}}-\frac{15a^9b}{7x^{\frac{14}{3}}}-\frac{70a^4b^6}{x^3}-\frac{45a^3b^7}{x^{\frac{8}{3}}}-\frac{3b^{10}}{5x^{\frac{5}{3}}}-\frac{30a^7b^3}{x^4}-\frac{135a^2b^8}{7x^{\frac{7}{3}}}-$
default	$-\frac{5a\,b^9}{x^2}-\frac{135a^8b^2}{13x^{\frac{13}{3}}}-\frac{378a^5b^5}{5x^{\frac{10}{3}}}-\frac{630a^6b^4}{11x^{\frac{11}{3}}}-\frac{15a^9b}{7x^{\frac{14}{3}}}-\frac{70a^4b^6}{x^3}-\frac{45a^3b^7}{x^{\frac{8}{3}}}-\frac{3b^{10}}{5x^{\frac{5}{3}}}-\frac{30a^7b^3}{x^4}-\frac{135a^2b^8}{7x^{\frac{7}{3}}}-$
trager	$\frac{(-1+x)(a^9x^4+150a^6b^3x^4+350a^3b^6x^4+25b^9x^4+a^9x^3+150a^6b^3x^3+350a^3b^6x^3+25b^9x^3+a^9x^2+150a^6b^3x^2+350a^3b^6}{5x^5}$
orering	$-\frac{(60775b^{24}x^8+207493a^3b^{21}x^7+726236a^6b^{18}x^6+1311400a^9b^{15}x^5+1550682a^{12}b^{12}x^4+1192536a^{15}b^9x^3+579036a^{18}}{25025x^5(b^3x+a^3)^8}$

input
```
int((a+b*x^(1/3))^10/x^6,x,method=_RETURNVERBOSE)
```

output
```
-5/x^2*a*b^9-135/13*a^8*b^2/x^(13/3)-378/5*a^5*b^5/x^(10/3)-630/11*a^6*b^4
/x^(11/3)-15/7*a^9*b/x^(14/3)-70*a^4*b^6/x^3-45*a^3*b^7/x^(8/3)-3/5*b^10/x
^(5/3)-30/x^4*a^7*b^3-135/7*a^2*b^8/x^(7/3)-1/5*a^10/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = -\frac{25025\,ab^9x^3+350350\,a^4b^6x^2+150150\,a^7b^3x+1001\,a^{10}+297\,(325\,a^2b^8x^2+1274\,a^5b^5x+175\,a^8b^2)x^{\frac{2}{3}}}{5005\,x^5}$$

input
```
integrate((a+b*x^(1/3))^10/x^6,x, algorithm="fricas")
```

output
```
-1/5005*(25025*a*b^9*x^3 + 350350*a^4*b^6*x^2 + 150150*a^7*b^3*x + 1001*a^
10 + 297*(325*a^2*b^8*x^2 + 1274*a^5*b^5*x + 175*a^8*b^2)*x^(2/3) + 39*(77
*b^10*x^3 + 5775*a^3*b^7*x^2 + 7350*a^6*b^4*x + 275*a^9*b)*x^(1/3))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = -\frac{a^{10}}{5x^5}-\frac{15a^9b}{7x^{\frac{14}{3}}}-\frac{135a^8b^2}{13x^{\frac{13}{3}}}-\frac{30a^7b^3}{x^4}-\frac{630a^6b^4}{11x^{\frac{11}{3}}}-\frac{378a^5b^5}{5x^{\frac{10}{3}}}-\frac{70a^4b^6}{x^3}-\frac{45a^3b^7}{x^{\frac{8}{3}}}-\frac{135a^2b^8}{7x^{\frac{7}{3}}}-\frac{5ab^9}{x^2}-\frac{3b^{10}}{5x^{\frac{5}{3}}}$$

input
```
integrate((a+b*x**(1/3))**10/x**6,x)
```

output
```
-a**10/(5*x**5) - 15*a**9*b/(7*x**(14/3)) - 135*a**8*b**2/(13*x**(13/3)) -
 30*a**7*b**3/x**4 - 630*a**6*b**4/(11*x**(11/3)) - 378*a**5*b**5/(5*x**(1
0/3)) - 70*a**4*b**6/x**3 - 45*a**3*b**7/x**(8/3) - 135*a**2*b**8/(7*x**(7
/3)) - 5*a*b**9/x**2 - 3*b**10/(5*x**(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = -\frac{3003\,b^{10}x^{\frac{10}{3}} + 25025\,ab^9x^3 + 96525\,a^2b^8x^{\frac{8}{3}} + 225225\,a^3b^7x^{\frac{7}{3}} + 350350\,a^4b^6x^2 + 378378\,a^5b^5x^{\frac{5}{3}} + 2866}{5005\,x^5}$$

input
```
integrate((a+b*x^(1/3))^10/x^6,x, algorithm="maxima")
```

output
```
-1/5005*(3003*b^10*x^(10/3) + 25025*a*b^9*x^3 + 96525*a^2*b^8*x^(8/3) + 22
5225*a^3*b^7*x^(7/3) + 350350*a^4*b^6*x^2 + 378378*a^5*b^5*x^(5/3) + 28665
0*a^6*b^4*x^(4/3) + 150150*a^7*b^3*x + 51975*a^8*b^2*x^(2/3) + 10725*a^9*b
*x^(1/3) + 1001*a^10)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = -\frac{3003\,b^{10}x^{\frac{10}{3}} + 25025\,ab^9x^3 + 96525\,a^2b^8x^{\frac{8}{3}} + 225225\,a^3b^7x^{\frac{7}{3}} + 350350\,a^4b^6x^2 + 378378\,a^5b^5x^{\frac{5}{3}} + 2866}{5005\,x^5}$$

input
```
integrate((a+b*x^(1/3))^10/x^6,x, algorithm="giac")
```

output
```
-1/5005*(3003*b^10*x^(10/3) + 25025*a*b^9*x^3 + 96525*a^2*b^8*x^(8/3) + 22
5225*a^3*b^7*x^(7/3) + 350350*a^4*b^6*x^2 + 378378*a^5*b^5*x^(5/3) + 28665
0*a^6*b^4*x^(4/3) + 150150*a^7*b^3*x + 51975*a^8*b^2*x^(2/3) + 10725*a^9*b
*x^(1/3) + 1001*a^10)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = -\frac{\frac{a^{10}}{5}+\frac{3\,b^{10}\,x^{10/3}}{5}+30\,a^7\,b^3\,x+5\,a\,b^9\,x^3+\frac{15\,a^9\,b\,x^{1/3}}{7}+70\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{13}+\frac{630\,a^6\,b^4\,x^{4/3}}{11}+\frac{378\,a^5\,b^5\,x^{5/3}}{5}}{x^5}$$

input
```
int((a + b*x^(1/3))^10/x^6,x)
```

output
```
-(a^10/5 + (3*b^10*x^(10/3))/5 + 30*a^7*b^3*x + 5*a*b^9*x^3 + (15*a^9*b*x^
(1/3))/7 + 70*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/13 + (630*a^6*b^4*x^(4/3
))/11 + (378*a^5*b^5*x^(5/3))/5 + 45*a^3*b^7*x^(7/3) + (135*a^2*b^8*x^(8/3
))/7)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^6}\,dx = \frac{-1001x^{\frac{2}{3}}a^{10}-150150x^{\frac{5}{3}}a^7b^3-350350x^{\frac{8}{3}}a^4b^6-25025x^{\frac{11}{3}}a\,b^9-51975x^{\frac{4}{3}}a^8b^2-378378x^{\frac{7}{3}}a^5b^5-96525}{5005x^{\frac{17}{3}}}$$

input
```
int((a+b*x^(1/3))^10/x^6,x)
```

output
```
( - 1001*x**(2/3)*a**10 - 150150*x**(2/3)*a**7*b**3*x - 350350*x**(2/3)*a*
*4*b**6*x**2 - 25025*x**(2/3)*a*b**9*x**3 - 51975*x**(1/3)*a**8*b**2*x - 3
78378*x**(1/3)*a**5*b**5*x**2 - 96525*x**(1/3)*a**2*b**8*x**3 - 10725*a**9
*b*x - 286650*a**6*b**4*x**2 - 225225*a**3*b**7*x**3 - 3003*b**10*x**4)/(5
005*x**(2/3)*x**5)
```

3.226 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx = -\frac{a^{10}}{6x^6}-\frac{30a^9b}{17x^{17/3}}-\frac{135a^8b^2}{16x^{16/3}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{14/3}}-\frac{756a^5b^5}{13x^{13/3}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{11/3}}-\frac{27a^2b^8}{2x^{10/3}}-\frac{10ab^9}{3x^3}-\frac{3b^{10}}{8x^{8/3}}$$

output
```
-1/6*a^10/x^6-30/17*a^9*b/x^(17/3)-135/16*a^8*b^2/x^(16/3)-24*a^7*b^3/x^5-
45*a^6*b^4/x^(14/3)-756/13*a^5*b^5/x^(13/3)-105/2*a^4*b^6/x^4-360/11*a^3*b
^7/x^(11/3)-27/2*a^2*b^8/x^(10/3)-10/3*a*b^9/x^3-3/8*b^10/x^(8/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx = \frac{-19448a^{10}-205920a^9b\sqrt[3]{x}-984555a^8b^2x^{2/3}-2800512a^7b^3x-5250960a^6b^4x^{4/3}-6785856a^5b^5x^{5/3}-}{116688x^6}$$

input
```
Integrate[(a + b*x^(1/3))^10/x^7,x]
```

output
```
(-19448*a^10 - 205920*a^9*b*x^(1/3) - 984555*a^8*b^2*x^(2/3) - 2800512*a^7
*b^3*x - 5250960*a^6*b^4*x^(4/3) - 6785856*a^5*b^5*x^(5/3) - 6126120*a^4*b
^6*x^2 - 3818880*a^3*b^7*x^(7/3) - 1575288*a^2*b^8*x^(8/3) - 388960*a*b^9*
x^3 - 43758*b^10*x^(10/3))/(116688*x^6)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{19/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^{10}}{x^{19/3}}+\frac{10ba^9}{x^6}+\frac{45b^2a^8}{x^{17/3}}+\frac{120b^3a^7}{x^{16/3}}+\frac{210b^4a^6}{x^5}+\frac{252b^5a^5}{x^{14/3}}+\frac{210b^6a^4}{x^{13/3}}+\frac{120b^7a^3}{x^4}+\frac{45b^8a^2}{x^{11/3}}+\frac{10b^9a}{x^{10/3}}+\frac{b^{10}}{x^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{10}}{18x^6}-\frac{10a^9b}{17x^{17/3}}-\frac{45a^8b^2}{16x^{16/3}}-\frac{8a^7b^3}{x^5}-\frac{15a^6b^4}{x^{14/3}}-\frac{252a^5b^5}{13x^{13/3}}-\frac{35a^4b^6}{2x^4}-\frac{120a^3b^7}{11x^{11/3}}-\frac{9a^2b^8}{2x^{10/3}}-\frac{10ab^9}{9x^3}-\frac{b^{10}}{8x^{8/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^10/x^7,x]
```

output
```
3*(-1/18*a^10/x^6 - (10*a^9*b)/(17*x^(17/3)) - (45*a^8*b^2)/(16*x^(16/3))
- (8*a^7*b^3)/x^5 - (15*a^6*b^4)/x^(14/3) - (252*a^5*b^5)/(13*x^(13/3)) -
(35*a^4*b^6)/(2*x^4) - (120*a^3*b^7)/(11*x^(11/3)) - (9*a^2*b^8)/(2*x^(10/
3)) - (10*a*b^9)/(9*x^3) - b^10/(8*x^(8/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{a^{10}}{6x^6}-\frac{30a^9b}{17x^{\frac{17}{3}}}-\frac{135a^8b^2}{16x^{\frac{16}{3}}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{\frac{14}{3}}}-\frac{756a^5b^5}{13x^{\frac{13}{3}}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{\frac{11}{3}}}-\frac{27a^2b^8}{2x^{\frac{10}{3}}}-\frac{10a\,b^9}{3x^3}-$
default	$-\frac{a^{10}}{6x^6}-\frac{30a^9b}{17x^{\frac{17}{3}}}-\frac{135a^8b^2}{16x^{\frac{16}{3}}}-\frac{24a^7b^3}{x^5}-\frac{45a^6b^4}{x^{\frac{14}{3}}}-\frac{756a^5b^5}{13x^{\frac{13}{3}}}-\frac{105a^4b^6}{2x^4}-\frac{360a^3b^7}{11x^{\frac{11}{3}}}-\frac{27a^2b^8}{2x^{\frac{10}{3}}}-\frac{10a\,b^9}{3x^3}-$
trager	$\frac{(-1+x)(a^9x^5+144a^6b^3x^5+315a^3b^6x^5+20b^9x^5+a^9x^4+144a^6b^3x^4+315a^3b^6x^4+20b^9x^4+a^9x^3+144a^6b^3x^3+315a^3b^6}{6x^6}$
orering	$-\frac{(1614184b^{24}x^8+8998100a^3b^{21}x^7+27273508a^6b^{18}x^6+49821324a^9b^{15}x^5+58538784a^{12}b^{12}x^4+44761983a^{15}b^9x^3+2}{1166880x^6(b^3x+a^3)^8}$

input
```
int((a+b*x^(1/3))^10/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^10/x^6-30/17*a^9*b/x^(17/3)-135/16*a^8*b^2/x^(16/3)-24*a^7*b^3/x^5-
45*a^6*b^4/x^(14/3)-756/13*a^5*b^5/x^(13/3)-105/2*a^4*b^6/x^4-360/11*a^3*b
^7/x^(11/3)-27/2*a^2*b^8/x^(10/3)-10/3*a*b^9/x^3-3/8*b^10/x^(8/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx = -\frac{388960\,ab^9x^3 + 6126120\,a^4b^6x^2 + 2800512\,a^7b^3x + 19448\,a^{10} + 15147\,(104\,a^2b^8x^2 + 448\,a^5b^5x + 65\,a^8}{116688\,x^6}$$

input
```
integrate((a+b*x^(1/3))^10/x^7,x, algorithm="fricas")
```

output
```
-1/116688*(388960*a*b^9*x^3 + 6126120*a^4*b^6*x^2 + 2800512*a^7*b^3*x + 19
448*a^10 + 15147*(104*a^2*b^8*x^2 + 448*a^5*b^5*x + 65*a^8*b^2)*x^(2/3) +
234*(187*b^10*x^3 + 16320*a^3*b^7*x^2 + 22440*a^6*b^4*x + 880*a^9*b)*x^(1/
3))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx = -\frac{a^{10}}{6x^6} - \frac{30a^9b}{17x^{\frac{17}{3}}} - \frac{135a^8b^2}{16x^{\frac{16}{3}}} - \frac{24a^7b^3}{x^5} - \frac{45a^6b^4}{x^{\frac{14}{3}}} - \frac{756a^5b^5}{13x^{\frac{13}{3}}} - \frac{105a^4b^6}{2x^4} - \frac{360a^3b^7}{11x^{\frac{11}{3}}} - \frac{27a^2b^8}{2x^{\frac{10}{3}}} - \frac{10ab^9}{3x^3} - \frac{3b^{10}}{8x^{\frac{8}{3}}}$$

input
```
integrate((a+b*x**(1/3))**10/x**7,x)
```

output
```
-a**10/(6*x**6) - 30*a**9*b/(17*x**(17/3)) - 135*a**8*b**2/(16*x**(16/3))
- 24*a**7*b**3/x**5 - 45*a**6*b**4/x**(14/3) - 756*a**5*b**5/(13*x**(13/3)
) - 105*a**4*b**6/(2*x**4) - 360*a**3*b**7/(11*x**(11/3)) - 27*a**2*b**8/(
2*x**(10/3)) - 10*a*b**9/(3*x**3) - 3*b**10/(8*x**(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx =$$
$$-\frac{43758\,b^{10}x^{\frac{10}{3}}+388960\,ab^{9}x^{3}+1575288\,a^{2}b^{8}x^{\frac{8}{3}}+3818880\,a^{3}b^{7}x^{\frac{7}{3}}+6126120\,a^{4}b^{6}x^{2}+6785856\,a^{5}b^{5}x^{\frac{5}{3}}}{116688\,x^{6}}$$

input
```
integrate((a+b*x^(1/3))^10/x^7,x, algorithm="maxima")
```

output
```
-1/116688*(43758*b^10*x^(10/3) + 388960*a*b^9*x^3 + 1575288*a^2*b^8*x^(8/3
) + 3818880*a^3*b^7*x^(7/3) + 6126120*a^4*b^6*x^2 + 6785856*a^5*b^5*x^(5/3
) + 5250960*a^6*b^4*x^(4/3) + 2800512*a^7*b^3*x + 984555*a^8*b^2*x^(2/3) +
 205920*a^9*b*x^(1/3) + 19448*a^10)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx =$$
$$-\frac{43758\,b^{10}x^{\frac{10}{3}}+388960\,ab^{9}x^{3}+1575288\,a^{2}b^{8}x^{\frac{8}{3}}+3818880\,a^{3}b^{7}x^{\frac{7}{3}}+6126120\,a^{4}b^{6}x^{2}+6785856\,a^{5}b^{5}x^{\frac{5}{3}}}{116688\,x^{6}}$$

input
```
integrate((a+b*x^(1/3))^10/x^7,x, algorithm="giac")
```

output
```
-1/116688*(43758*b^10*x^(10/3) + 388960*a*b^9*x^3 + 1575288*a^2*b^8*x^(8/3
) + 3818880*a^3*b^7*x^(7/3) + 6126120*a^4*b^6*x^2 + 6785856*a^5*b^5*x^(5/3
) + 5250960*a^6*b^4*x^(4/3) + 2800512*a^7*b^3*x + 984555*a^8*b^2*x^(2/3) +
 205920*a^9*b*x^(1/3) + 19448*a^10)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx = -\frac{\frac{a^{10}}{6}+\frac{3\,b^{10}\,x^{10/3}}{8}+24\,a^7\,b^3\,x+\frac{10\,a\,b^9\,x^3}{3}+\frac{30\,a^9\,b\,x^{1/3}}{17}+\frac{105\,a^4\,b^6\,x^2}{2}+\frac{135\,a^8\,b^2\,x^{2/3}}{16}+45\,a^6\,b^4\,x^{4/3}+\frac{756\,a^5\,b^5\,x^{5/3}}{13}}{x^6}$$

input
```
int((a + b*x^(1/3))^10/x^7,x)
```

output
```
-(a^10/6 + (3*b^10*x^(10/3))/8 + 24*a^7*b^3*x + (10*a*b^9*x^3)/3 + (30*a^9
*b*x^(1/3))/17 + (105*a^4*b^6*x^2)/2 + (135*a^8*b^2*x^(2/3))/16 + 45*a^6*b
^4*x^(4/3) + (756*a^5*b^5*x^(5/3))/13 + (360*a^3*b^7*x^(7/3))/11 + (27*a^2
*b^8*x^(8/3))/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^7}\,dx = \frac{-19448x^{\frac{2}{3}}a^{10}-2800512x^{\frac{5}{3}}a^7b^3-6126120x^{\frac{8}{3}}a^4b^6-388960x^{\frac{11}{3}}a\,b^9-984555x^{\frac{4}{3}}a^8b^2-6785856x^{\frac{7}{3}}a^5b^5-}{116688x^{\frac{20}{3}}}$$

input
```
int((a+b*x^(1/3))^10/x^7,x)
```

output
```
( - 19448*x**(2/3)*a**10 - 2800512*x**(2/3)*a**7*b**3*x - 6126120*x**(2/3)
*a**4*b**6*x**2 - 388960*x**(2/3)*a*b**9*x**3 - 984555*x**(1/3)*a**8*b**2*
x - 6785856*x**(1/3)*a**5*b**5*x**2 - 1575288*x**(1/3)*a**2*b**8*x**3 - 20
5920*a**9*b*x - 5250960*a**6*b**4*x**2 - 3818880*a**3*b**7*x**3 - 43758*b*
*10*x**4)/(116688*x**(2/3)*x**6)
```

3.227 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = -\frac{a^{10}}{7x^7}-\frac{3a^9b}{2x^{20/3}}-\frac{135a^8b^2}{19x^{19/3}}-\frac{20a^7b^3}{x^6}-\frac{630a^6b^4}{17x^{17/3}}-\frac{189a^5b^5}{4x^{16/3}}-\frac{42a^4b^6}{x^5}-\frac{180a^3b^7}{7x^{14/3}}-\frac{135a^2b^8}{13x^{13/3}}-\frac{5ab^9}{2x^4}-\frac{3b^{10}}{11x^{11/3}}$$

output
```
-1/7*a^10/x^7-3/2*a^9*b/x^(20/3)-135/19*a^8*b^2/x^(19/3)-20*a^7*b^3/x^6-63
0/17*a^6*b^4/x^(17/3)-189/4*a^5*b^5/x^(16/3)-42*a^4*b^6/x^5-180/7*a^3*b^7/
x^(14/3)-135/13*a^2*b^8/x^(13/3)-5/2*a*b^9/x^4-3/11*b^10/x^(11/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = \frac{-184756a^{10}-1939938a^9b\sqrt[3]{x}-9189180a^8b^2x^{2/3}-25865840a^7b^3x-47927880a^6b^4x^{4/3}-61108047a^5b^5 \ldots}{1293292 \ldots}$$

input
```
Integrate[(a + b*x^(1/3))^10/x^8,x]
```

output
```
(-184756*a^10 - 1939938*a^9*b*x^(1/3) - 9189180*a^8*b^2*x^(2/3) - 25865840
*a^7*b^3*x - 47927880*a^6*b^4*x^(4/3) - 61108047*a^5*b^5*x^(5/3) - 5431826
4*a^4*b^6*x^2 - 33256080*a^3*b^7*x^(7/3) - 13430340*a^2*b^8*x^(8/3) - 3233
230*a*b^9*x^3 - 352716*b^10*x^(10/3))/(1293292*x^7)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{22/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^{10}}{x^{22/3}}+\frac{10ba^9}{x^7}+\frac{45b^2a^8}{x^{20/3}}+\frac{120b^3a^7}{x^{19/3}}+\frac{210b^4a^6}{x^6}+\frac{252b^5a^5}{x^{17/3}}+\frac{210b^6a^4}{x^{16/3}}+\frac{120b^7a^3}{x^5}+\frac{45b^8a^2}{x^{14/3}}+\frac{10b^9a}{x^{13/3}}+\frac{b^{10}}{x^4}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{10}}{21x^7}-\frac{a^9b}{2x^{20/3}}-\frac{45a^8b^2}{19x^{19/3}}-\frac{20a^7b^3}{3x^6}-\frac{210a^6b^4}{17x^{17/3}}-\frac{63a^5b^5}{4x^{16/3}}-\frac{14a^4b^6}{x^5}-\frac{60a^3b^7}{7x^{14/3}}-\frac{45a^2b^8}{13x^{13/3}}-\frac{5ab^9}{6x^4}-\frac{b^{10}}{11x^{11/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^10/x^8,x]
```

output
```
3*(-1/21*a^10/x^7 - (a^9*b)/(2*x^(20/3)) - (45*a^8*b^2)/(19*x^(19/3)) - (2
0*a^7*b^3)/(3*x^6) - (210*a^6*b^4)/(17*x^(17/3)) - (63*a^5*b^5)/(4*x^(16/3
)) - (14*a^4*b^6)/x^5 - (60*a^3*b^7)/(7*x^(14/3)) - (45*a^2*b^8)/(13*x^(13
/3)) - (5*a*b^9)/(6*x^4) - b^10/(11*x^(11/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{a^{10}}{7x^{7}}-\frac{3a^{9}b}{2x^{\frac{20}{3}}}-\frac{135a^{8}b^{2}}{19x^{\frac{19}{3}}}-\frac{20a^{7}b^{3}}{x^{6}}-\frac{630a^{6}b^{4}}{17x^{\frac{17}{3}}}-\frac{189a^{5}b^{5}}{4x^{\frac{16}{3}}}-\frac{42a^{4}b^{6}}{x^{5}}-\frac{180a^{3}b^{7}}{7x^{\frac{14}{3}}}-\frac{135a^{2}b^{8}}{13x^{\frac{13}{3}}}-\frac{5a\,b^{9}}{2x^{4}}-$
default	$-\frac{a^{10}}{7x^{7}}-\frac{3a^{9}b}{2x^{\frac{20}{3}}}-\frac{135a^{8}b^{2}}{19x^{\frac{19}{3}}}-\frac{20a^{7}b^{3}}{x^{6}}-\frac{630a^{6}b^{4}}{17x^{\frac{17}{3}}}-\frac{189a^{5}b^{5}}{4x^{\frac{16}{3}}}-\frac{42a^{4}b^{6}}{x^{5}}-\frac{180a^{3}b^{7}}{7x^{\frac{14}{3}}}-\frac{135a^{2}b^{8}}{13x^{\frac{13}{3}}}-\frac{5a\,b^{9}}{2x^{4}}-$
trager	$\frac{(-1+x)(2a^{9}x^{6}+280a^{6}b^{3}x^{6}+588x^{6}a^{3}b^{6}+35b^{9}x^{6}+2a^{9}x^{5}+280a^{6}b^{3}x^{5}+588a^{3}b^{6}x^{5}+35b^{9}x^{5}+2a^{9}x^{4}+280a^{6}b^{3}x^{4}+588a}{14x}$
orering	$-\frac{(6195140b^{24}x^{8}+39951547a^{3}b^{21}x^{7}+123848592a^{6}b^{18}x^{6}+228405744a^{9}b^{15}x^{5}+269271000a^{12}b^{12}x^{4}+206050922a^{15}b^{9}}{6466460x^{7}(b^{3}x+a^{3})^{8}}$

input
```
int((a+b*x^(1/3))^10/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^10/x^7-3/2*a^9*b/x^(20/3)-135/19*a^8*b^2/x^(19/3)-20*a^7*b^3/x^6-63
0/17*a^6*b^4/x^(17/3)-189/4*a^5*b^5/x^(16/3)-42*a^4*b^6/x^5-180/7*a^3*b^7/
x^(14/3)-135/13*a^2*b^8/x^(13/3)-5/2*a*b^9/x^4-3/11*b^10/x^(11/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = -\frac{3233230\,ab^9x^3 + 54318264\,a^4b^6x^2 + 25865840\,a^7b^3x + 184756\,a^{10} + 35343\,(380\,a^2b^8x^2 + 1729\,a^5b^5x +}{1293292\,x^7}$$

input
```
integrate((a+b*x^(1/3))^10/x^8,x, algorithm="fricas")
```

output
```
-1/1293292*(3233230*a*b^9*x^3 + 54318264*a^4*b^6*x^2 + 25865840*a^7*b^3*x
+ 184756*a^10 + 35343*(380*a^2*b^8*x^2 + 1729*a^5*b^5*x + 260*a^8*b^2)*x^(
2/3) + 1482*(238*b^10*x^3 + 22440*a^3*b^7*x^2 + 32340*a^6*b^4*x + 1309*a^9
*b)*x^(1/3))/x^7
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = -\frac{a^{10}}{7x^7} - \frac{3a^9b}{2x^{\frac{20}{3}}} - \frac{135a^8b^2}{19x^{\frac{19}{3}}} - \frac{20a^7b^3}{x^6} - \frac{630a^6b^4}{17x^{\frac{17}{3}}} - \frac{189a^5b^5}{4x^{\frac{16}{3}}} - \frac{42a^4b^6}{x^5} - \frac{180a^3b^7}{7x^{\frac{14}{3}}} - \frac{135a^2b^8}{13x^{\frac{13}{3}}} - \frac{5ab^9}{2x^4} - \frac{3b^{10}}{11x^{\frac{11}{3}}}$$

input
```
integrate((a+b*x**(1/3))**10/x**8,x)
```

output
```
-a**10/(7*x**7) - 3*a**9*b/(2*x**(20/3)) - 135*a**8*b**2/(19*x**(19/3)) -
20*a**7*b**3/x**6 - 630*a**6*b**4/(17*x**(17/3)) - 189*a**5*b**5/(4*x**(16
/3)) - 42*a**4*b**6/x**5 - 180*a**3*b**7/(7*x**(14/3)) - 135*a**2*b**8/(13
*x**(13/3)) - 5*a*b**9/(2*x**4) - 3*b**10/(11*x**(11/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = -\frac{352716\,b^{10}x^{\frac{10}{3}} + 3233230\,ab^9x^3 + 13430340\,a^2b^8x^{\frac{8}{3}} + 33256080\,a^3b^7x^{\frac{7}{3}} + 54318264\,a^4b^6x^2 + 61108047}{1293292}$$

input
```
integrate((a+b*x^(1/3))^10/x^8,x, algorithm="maxima")
```

output
```
-1/1293292*(352716*b^10*x^(10/3) + 3233230*a*b^9*x^3 + 13430340*a^2*b^8*x^
(8/3) + 33256080*a^3*b^7*x^(7/3) + 54318264*a^4*b^6*x^2 + 61108047*a^5*b^5
*x^(5/3) + 47927880*a^6*b^4*x^(4/3) + 25865840*a^7*b^3*x + 9189180*a^8*b^2
*x^(2/3) + 1939938*a^9*b*x^(1/3) + 184756*a^10)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = -\frac{352716\,b^{10}x^{\frac{10}{3}} + 3233230\,ab^9x^3 + 13430340\,a^2b^8x^{\frac{8}{3}} + 33256080\,a^3b^7x^{\frac{7}{3}} + 54318264\,a^4b^6x^2 + 61108047}{1293292}$$

input
```
integrate((a+b*x^(1/3))^10/x^8,x, algorithm="giac")
```

output
```
-1/1293292*(352716*b^10*x^(10/3) + 3233230*a*b^9*x^3 + 13430340*a^2*b^8*x^
(8/3) + 33256080*a^3*b^7*x^(7/3) + 54318264*a^4*b^6*x^2 + 61108047*a^5*b^5
*x^(5/3) + 47927880*a^6*b^4*x^(4/3) + 25865840*a^7*b^3*x + 9189180*a^8*b^2
*x^(2/3) + 1939938*a^9*b*x^(1/3) + 184756*a^10)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = -\frac{\frac{a^{10}}{7}+\frac{3\,b^{10}\,x^{10/3}}{11}+20\,a^7\,b^3\,x+\frac{5\,a\,b^9\,x^3}{2}+\frac{3\,a^9\,b\,x^{1/3}}{2}+42\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{19}+\frac{630\,a^6\,b^4\,x^{4/3}}{17}+\frac{189\,a^5\,b^5\,x^{5/3}}{4}+}{x^7}$$

input
```
int((a + b*x^(1/3))^10/x^8,x)
```

output
```
-(a^10/7 + (3*b^10*x^(10/3))/11 + 20*a^7*b^3*x + (5*a*b^9*x^3)/2 + (3*a^9*
b*x^(1/3))/2 + 42*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/19 + (630*a^6*b^4*x^
(4/3))/17 + (189*a^5*b^5*x^(5/3))/4 + (180*a^3*b^7*x^(7/3))/7 + (135*a^2*b
^8*x^(8/3))/13)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^8}\,dx = \frac{-184756x^{\frac{2}{3}}a^{10}-25865840x^{\frac{5}{3}}a^7b^3-54318264x^{\frac{8}{3}}a^4b^6-3233230x^{\frac{11}{3}}a\,b^9-9189180x^{\frac{4}{3}}a^8b^2-61108047x}{1293292x^{\frac{2}{3}}}$$

input
```
int((a+b*x^(1/3))^10/x^8,x)
```

output
```
( - 184756*x**(2/3)*a**10 - 25865840*x**(2/3)*a**7*b**3*x - 54318264*x**(2
/3)*a**4*b**6*x**2 - 3233230*x**(2/3)*a*b**9*x**3 - 9189180*x**(1/3)*a**8*
b**2*x - 61108047*x**(1/3)*a**5*b**5*x**2 - 13430340*x**(1/3)*a**2*b**8*x*
*3 - 1939938*a**9*b*x - 47927880*a**6*b**4*x**2 - 33256080*a**3*b**7*x**3
- 352716*b**10*x**4)/(1293292*x**(2/3)*x**7)
```

3.228 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 144

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = -\frac{a^{10}}{8x^8}-\frac{30a^9b}{23x^{23/3}}-\frac{135a^8b^2}{22x^{22/3}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{20/3}}-\frac{756a^5b^5}{19x^{19/3}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{17/3}}-\frac{135a^2b^8}{16x^{16/3}}-\frac{2ab^9}{x^5}-\frac{3b^{10}}{14x^{14/3}}$$

output
```
-1/8*a^10/x^8-30/23*a^9*b/x^(23/3)-135/22*a^8*b^2/x^(22/3)-120/7*a^7*b^3/x
^7-63/2*a^6*b^4/x^(20/3)-756/19*a^5*b^5/x^(19/3)-35*a^4*b^6/x^6-360/17*a^3
*b^7/x^(17/3)-135/16*a^2*b^8/x^(16/3)-2*a*b^9/x^5-3/14*b^10/x^(14/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = \frac{-1144066a^{10}-11938080a^9b\sqrt[3]{x}-56163240a^8b^2x^{2/3}-156900480a^7b^3x-288304632a^6b^4x^{4/3}-364174\cdots}{91\cdots}$$

input
```
Integrate[(a + b*x^(1/3))^10/x^9,x]
```

output
```
(-1144066*a^10 - 11938080*a^9*b*x^(1/3) - 56163240*a^8*b^2*x^(2/3) - 15690
0480*a^7*b^3*x - 288304632*a^6*b^4*x^(4/3) - 364174272*a^5*b^5*x^(5/3) - 3
20338480*a^4*b^6*x^2 - 193818240*a^3*b^7*x^(7/3) - 77224455*a^2*b^8*x^(8/3
) - 18305056*a*b^9*x^3 - 1961256*b^10*x^(10/3))/(9152528*x^8)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{25/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^{10}}{x^{25/3}}+\frac{10ba^9}{x^8}+\frac{45b^2a^8}{x^{23/3}}+\frac{120b^3a^7}{x^{22/3}}+\frac{210b^4a^6}{x^7}+\frac{252b^5a^5}{x^{20/3}}+\frac{210b^6a^4}{x^{19/3}}+\frac{120b^7a^3}{x^6}+\frac{45b^8a^2}{x^{17/3}}+\frac{10b^9a}{x^{16/3}}+\frac{b^{10}}{x^5}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{10}}{24x^8}-\frac{10a^9b}{23x^{23/3}}-\frac{45a^8b^2}{22x^{22/3}}-\frac{40a^7b^3}{7x^7}-\frac{21a^6b^4}{2x^{20/3}}-\frac{252a^5b^5}{19x^{19/3}}-\frac{35a^4b^6}{3x^6}-\frac{120a^3b^7}{17x^{17/3}}-\frac{45a^2b^8}{16x^{16/3}}-\frac{2ab^9}{3x^5}-\frac{b^{10}}{14x^{14/}}\right.$$

input
```
Int[(a + b*x^(1/3))^10/x^9,x]
```

output
```
3*(-1/24*a^10/x^8 - (10*a^9*b)/(23*x^(23/3)) - (45*a^8*b^2)/(22*x^(22/3))
- (40*a^7*b^3)/(7*x^7) - (21*a^6*b^4)/(2*x^(20/3)) - (252*a^5*b^5)/(19*x^(
19/3)) - (35*a^4*b^6)/(3*x^6) - (120*a^3*b^7)/(17*x^(17/3)) - (45*a^2*b^8)
/(16*x^(16/3)) - (2*a*b^9)/(3*x^5) - b^10/(14*x^(14/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{a^{10}}{8x^8}-\frac{30a^9b}{23x^{\frac{23}{3}}}-\frac{135a^8b^2}{22x^{\frac{22}{3}}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{\frac{20}{3}}}-\frac{756a^5b^5}{19x^{\frac{19}{3}}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{\frac{17}{3}}}-\frac{135a^2b^8}{16x^{\frac{16}{3}}}-\frac{2a\,b^9}{x^5}-$
default	$-\frac{a^{10}}{8x^8}-\frac{30a^9b}{23x^{\frac{23}{3}}}-\frac{135a^8b^2}{22x^{\frac{22}{3}}}-\frac{120a^7b^3}{7x^7}-\frac{63a^6b^4}{2x^{\frac{20}{3}}}-\frac{756a^5b^5}{19x^{\frac{19}{3}}}-\frac{35a^4b^6}{x^6}-\frac{360a^3b^7}{17x^{\frac{17}{3}}}-\frac{135a^2b^8}{16x^{\frac{16}{3}}}-\frac{2a\,b^9}{x^5}-$
trager	$\frac{(-1+x)\left(7a^9x^7+960a^6b^3x^7+1960a^3b^6x^7+112b^9x^7+7a^9x^6+960a^6b^3x^6+1960x^6a^3b^6+112b^9x^6+7a^9x^5+960a^6b^3x^5+\right.}{}$
orering	$-\frac{\left(66846142b^{24}x^8+460892124a^3b^{21}x^7+1463394780a^6b^{18}x^6+2729238987a^9b^{15}x^5+3235741676a^{12}b^{12}x^4+2483170\right.}{91525280x^8\left(b^3x+a^3\right)^8}$

input
```
int((a+b*x^(1/3))^10/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^10/x^8-30/23*a^9*b/x^(23/3)-135/22*a^8*b^2/x^(22/3)-120/7*a^7*b^3/x
^7-63/2*a^6*b^4/x^(20/3)-756/19*a^5*b^5/x^(19/3)-35*a^4*b^6/x^6-360/17*a^3
*b^7/x^(17/3)-135/16*a^2*b^8/x^(16/3)-2*a*b^9/x^5-3/14*b^10/x^(14/3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = -\frac{18305056\,ab^9x^3 + 320338480\,a^4b^6x^2 + 156900480\,a^7b^3x + 1144066\,a^{10} + 73899\,(1045\,a^2b^8x^2 + 4928\,a^5}{9152528\,x^8}$$

input
```
integrate((a+b*x^(1/3))^10/x^9,x, algorithm="fricas")
```

output
```
-1/9152528*(18305056*a*b^9*x^3 + 320338480*a^4*b^6*x^2 + 156900480*a^7*b^3
*x + 1144066*a^10 + 73899*(1045*a^2*b^8*x^2 + 4928*a^5*b^5*x + 760*a^8*b^2
)*x^(2/3) + 5016*(391*b^10*x^3 + 38640*a^3*b^7*x^2 + 57477*a^6*b^4*x + 238
0*a^9*b)*x^(1/3))/x^8
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = -\frac{a^{10}}{8x^8} - \frac{30a^9b}{23x^{\frac{23}{3}}} - \frac{135a^8b^2}{22x^{\frac{22}{3}}} - \frac{120a^7b^3}{7x^7} - \frac{63a^6b^4}{2x^{\frac{20}{3}}} - \frac{756a^5b^5}{19x^{\frac{19}{3}}} - \frac{35a^4b^6}{x^6} - \frac{360a^3b^7}{17x^{\frac{17}{3}}} - \frac{135a^2b^8}{16x^{\frac{16}{3}}} - \frac{2ab^9}{x^5} - \frac{3b^{10}}{14x^{\frac{14}{3}}}$$

input
```
integrate((a+b*x**(1/3))**10/x**9,x)
```

output
```
-a**10/(8*x**8) - 30*a**9*b/(23*x**(23/3)) - 135*a**8*b**2/(22*x**(22/3))
- 120*a**7*b**3/(7*x**7) - 63*a**6*b**4/(2*x**(20/3)) - 756*a**5*b**5/(19*
x**(19/3)) - 35*a**4*b**6/x**6 - 360*a**3*b**7/(17*x**(17/3)) - 135*a**2*b
**8/(16*x**(16/3)) - 2*a*b**9/x**5 - 3*b**10/(14*x**(14/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = -\frac{1961256\,b^{10}x^{\frac{10}{3}} + 18305056\,ab^{9}x^{3} + 77224455\,a^{2}b^{8}x^{\frac{8}{3}} + 193818240\,a^{3}b^{7}x^{\frac{7}{3}} + 320338480\,a^{4}b^{6}x^{2} + 3641}{915}$$

input
```
integrate((a+b*x^(1/3))^10/x^9,x, algorithm="maxima")
```

output
```
-1/9152528*(1961256*b^10*x^(10/3) + 18305056*a*b^9*x^3 + 77224455*a^2*b^8*
x^(8/3) + 193818240*a^3*b^7*x^(7/3) + 320338480*a^4*b^6*x^2 + 364174272*a^
5*b^5*x^(5/3) + 288304632*a^6*b^4*x^(4/3) + 156900480*a^7*b^3*x + 56163240
*a^8*b^2*x^(2/3) + 11938080*a^9*b*x^(1/3) + 1144066*a^10)/x^8
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = -\frac{1961256\,b^{10}x^{\frac{10}{3}} + 18305056\,ab^{9}x^{3} + 77224455\,a^{2}b^{8}x^{\frac{8}{3}} + 193818240\,a^{3}b^{7}x^{\frac{7}{3}} + 320338480\,a^{4}b^{6}x^{2} + 3641}{915}$$

input
```
integrate((a+b*x^(1/3))^10/x^9,x, algorithm="giac")
```

output
```
-1/9152528*(1961256*b^10*x^(10/3) + 18305056*a*b^9*x^3 + 77224455*a^2*b^8*
x^(8/3) + 193818240*a^3*b^7*x^(7/3) + 320338480*a^4*b^6*x^2 + 364174272*a^
5*b^5*x^(5/3) + 288304632*a^6*b^4*x^(4/3) + 156900480*a^7*b^3*x + 56163240
*a^8*b^2*x^(2/3) + 11938080*a^9*b*x^(1/3) + 1144066*a^10)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = -\frac{\frac{a^{10}}{8}+\frac{3\,b^{10}\,x^{10/3}}{14}+\frac{120\,a^7\,b^3\,x}{7}+2\,a\,b^9\,x^3+\frac{30\,a^9\,b\,x^{1/3}}{23}+35\,a^4\,b^6\,x^2+\frac{135\,a^8\,b^2\,x^{2/3}}{22}+\frac{63\,a^6\,b^4\,x^{4/3}}{2}+\frac{756\,a^5\,b^5\,x^{5/3}}{19}}{x^8}$$

input
```
int((a + b*x^(1/3))^10/x^9,x)
```

output
```
-(a^10/8 + (3*b^10*x^(10/3))/14 + (120*a^7*b^3*x)/7 + 2*a*b^9*x^3 + (30*a^
9*b*x^(1/3))/23 + 35*a^4*b^6*x^2 + (135*a^8*b^2*x^(2/3))/22 + (63*a^6*b^4*
x^(4/3))/2 + (756*a^5*b^5*x^(5/3))/19 + (360*a^3*b^7*x^(7/3))/17 + (135*a^
2*b^8*x^(8/3))/16)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^9}\,dx = \frac{-1144066x^{\frac{2}{3}}a^{10}-156900480x^{\frac{5}{3}}a^7b^3-320338480x^{\frac{8}{3}}a^4b^6-18305056x^{\frac{11}{3}}a\,b^9-56163240x^{\frac{4}{3}}a^8b^2-36417}{91525}$$

input
```
int((a+b*x^(1/3))^10/x^9,x)
```

output
```
( - 1144066*x**(2/3)*a**10 - 156900480*x**(2/3)*a**7*b**3*x - 320338480*x*
*(2/3)*a**4*b**6*x**2 - 18305056*x**(2/3)*a*b**9*x**3 - 56163240*x**(1/3)*
a**8*b**2*x - 364174272*x**(1/3)*a**5*b**5*x**2 - 77224455*x**(1/3)*a**2*b
**8*x**3 - 11938080*a**9*b*x - 288304632*a**6*b**4*x**2 - 193818240*a**3*b
**7*x**3 - 1961256*b**10*x**4)/(9152528*x**(2/3)*x**8)
```

3.229 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 142

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx = -\frac{a^{10}}{9x^9}-\frac{15a^9b}{13x^{26/3}}-\frac{27a^8b^2}{5x^{25/3}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{23/3}}-\frac{378a^5b^5}{11x^{22/3}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{20/3}}-\frac{135a^2b^8}{19x^{19/3}}-\frac{5ab^9}{3x^6}-\frac{3b^{10}}{17x^{17/3}}$$

output
```
-1/9*a^10/x^9-15/13*a^9*b/x^(26/3)-27/5*a^8*b^2/x^(25/3)-15*a^7*b^3/x^8-63
0/23*a^6*b^4/x^(23/3)-378/11*a^5*b^5/x^(22/3)-30*a^4*b^6/x^7-18*a^3*b^7/x^
(20/3)-135/19*a^2*b^8/x^(19/3)-5/3*a*b^9/x^6-3/17*b^10/x^(17/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx = \frac{-5311735a^{10}-55160325a^9b\sqrt[3]{x}-258150321a^8b^2x^{2/3}-717084225a^7b^3x-1309458150a^6b^4x^{4/3}-1642}{}$$

input
```
Integrate[(a + b*x^(1/3))^10/x^10,x]
```

output
```
(-5311735*a^10 - 55160325*a^9*b*x^(1/3) - 258150321*a^8*b^2*x^(2/3) - 7170
84225*a^7*b^3*x - 1309458150*a^6*b^4*x^(4/3) - 1642774770*a^5*b^5*x^(5/3)
- 1434168450*a^4*b^6*x^2 - 860501070*a^3*b^7*x^(7/3) - 339671475*a^2*b^8*x
^(8/3) - 79676025*a*b^9*x^3 - 8436285*b^10*x^(10/3))/(47805615*x^9)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}dx$$

$$\Big\downarrow \ 798$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{28/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 53$$

$$3\int \left(\frac{a^{10}}{x^{28/3}}+\frac{10ba^9}{x^9}+\frac{45b^2a^8}{x^{26/3}}+\frac{120b^3a^7}{x^{25/3}}+\frac{210b^4a^6}{x^8}+\frac{252b^5a^5}{x^{23/3}}+\frac{210b^6a^4}{x^{22/3}}+\frac{120b^7a^3}{x^7}+\frac{45b^8a^2}{x^{20/3}}+\frac{10b^9a}{x^{19/3}}+\frac{b^{10}}{x^6}$$

$$\Big\downarrow \ 2009$$

$$3\left(-\frac{a^{10}}{27x^9}-\frac{5a^9b}{13x^{26/3}}-\frac{9a^8b^2}{5x^{25/3}}-\frac{5a^7b^3}{x^8}-\frac{210a^6b^4}{23x^{23/3}}-\frac{126a^5b^5}{11x^{22/3}}-\frac{10a^4b^6}{x^7}-\frac{6a^3b^7}{x^{20/3}}-\frac{45a^2b^8}{19x^{19/3}}-\frac{5ab^9}{9x^6}-\frac{b^{10}}{17x^{17/3}}\right)$$

input
```
Int[(a + b*x^(1/3))^10/x^10,x]
```

output
```
3*(-1/27*a^10/x^9 - (5*a^9*b)/(13*x^(26/3)) - (9*a^8*b^2)/(5*x^(25/3)) - (
5*a^7*b^3)/x^8 - (210*a^6*b^4)/(23*x^(23/3)) - (126*a^5*b^5)/(11*x^(22/3))
 - (10*a^4*b^6)/x^7 - (6*a^3*b^7)/x^(20/3) - (45*a^2*b^8)/(19*x^(19/3)) -
(5*a*b^9)/(9*x^6) - b^10/(17*x^(17/3)))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{a^{10}}{9x^9}-\frac{15a^9b}{13x^{\frac{26}{3}}}-\frac{27a^8b^2}{5x^{\frac{25}{3}}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{\frac{23}{3}}}-\frac{378a^5b^5}{11x^{\frac{22}{3}}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{\frac{20}{3}}}-\frac{135a^2b^8}{19x^{\frac{19}{3}}}-\frac{5a\,b^9}{3x^6}-$
default	$-\frac{a^{10}}{9x^9}-\frac{15a^9b}{13x^{\frac{26}{3}}}-\frac{27a^8b^2}{5x^{\frac{25}{3}}}-\frac{15a^7b^3}{x^8}-\frac{630a^6b^4}{23x^{\frac{23}{3}}}-\frac{378a^5b^5}{11x^{\frac{22}{3}}}-\frac{30a^4b^6}{x^7}-\frac{18a^3b^7}{x^{\frac{20}{3}}}-\frac{135a^2b^8}{19x^{\frac{19}{3}}}-\frac{5a\,b^9}{3x^6}-$
trager	$(-1+x)(a^9x^8+135a^6b^3x^8+270a^3b^6x^8+15b^9x^8+a^9x^7+135a^6b^3x^7+270a^3b^6x^7+15b^9x^7+a^9x^6+135a^6b^3x^6+270x^6a^3$
orering	$-\frac{(281702858b^{24}x^8+201600750a^3b^{21}x^7+651592968a^6b^{18}x^6+1227009280a^9b^{15}x^5+1462848590a^{12}b^{12}x^4+112634050}{47805615x^9(b^3x+a^3)^8}$

input
```
int((a+b*x^(1/3))^10/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^10/x^9-15/13*a^9*b/x^(26/3)-27/5*a^8*b^2/x^(25/3)-15*a^7*b^3/x^8-63
0/23*a^6*b^4/x^(23/3)-378/11*a^5*b^5/x^(22/3)-30*a^4*b^6/x^7-18*a^3*b^7/x^
(20/3)-135/19*a^2*b^8/x^(19/3)-5/3*a*b^9/x^6-3/17*b^10/x^(17/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx = -\frac{79676025\,ab^9x^3 + 1434168450\,a^4b^6x^2 + 717084225\,a^7b^3x + 5311735\,a^{10} + 1235169\,(275\,a^2b^8x^2 + 1330}{47805615\,x^9}$$

input
```
integrate((a+b*x^(1/3))^10/x^10,x, algorithm="fricas")
```

output
```
-1/47805615*(79676025*a*b^9*x^3 + 1434168450*a^4*b^6*x^2 + 717084225*a^7*b
^3*x + 5311735*a^10 + 1235169*(275*a^2*b^8*x^2 + 1330*a^5*b^5*x + 209*a^8*
b^2)*x^(2/3) + 28215*(299*b^10*x^3 + 30498*a^3*b^7*x^2 + 46410*a^6*b^4*x +
 1955*a^9*b)*x^(1/3))/x^9
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx = -\frac{a^{10}}{9x^9} - \frac{15a^9b}{13x^{\frac{26}{3}}} - \frac{27a^8b^2}{5x^{\frac{25}{3}}} - \frac{15a^7b^3}{x^8} - \frac{630a^6b^4}{23x^{\frac{23}{3}}} - \frac{378a^5b^5}{11x^{\frac{22}{3}}} - \frac{30a^4b^6}{x^7} - \frac{18a^3b^7}{x^{\frac{20}{3}}} - \frac{135a^2b^8}{19x^{\frac{19}{3}}} - \frac{5ab^9}{3x^6} - \frac{3b^{10}}{17x^{\frac{17}{3}}}$$

input
```
integrate((a+b*x**(1/3))**10/x**10,x)
```

output
```
-a**10/(9*x**9) - 15*a**9*b/(13*x**(26/3)) - 27*a**8*b**2/(5*x**(25/3)) -
15*a**7*b**3/x**8 - 630*a**6*b**4/(23*x**(23/3)) - 378*a**5*b**5/(11*x**(2
2/3)) - 30*a**4*b**6/x**7 - 18*a**3*b**7/x**(20/3) - 135*a**2*b**8/(19*x**
(19/3)) - 5*a*b**9/(3*x**6) - 3*b**10/(17*x**(17/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx =$$

$$-\frac{8436285\,b^{10}x^{\frac{10}{3}}+79676025\,ab^9x^3+339671475\,a^2b^8x^{\frac{8}{3}}+860501070\,a^3b^7x^{\frac{7}{3}}+1434168450\,a^4b^6x^2+16\cdots}{4\cdots}$$

input
```
integrate((a+b*x^(1/3))^10/x^10,x, algorithm="maxima")
```

output
```
-1/47805615*(8436285*b^10*x^(10/3) + 79676025*a*b^9*x^3 + 339671475*a^2*b^
8*x^(8/3) + 860501070*a^3*b^7*x^(7/3) + 1434168450*a^4*b^6*x^2 + 164277477
0*a^5*b^5*x^(5/3) + 1309458150*a^6*b^4*x^(4/3) + 717084225*a^7*b^3*x + 258
150321*a^8*b^2*x^(2/3) + 55160325*a^9*b*x^(1/3) + 5311735*a^10)/x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx =$$

$$-\frac{8436285\,b^{10}x^{\frac{10}{3}}+79676025\,ab^9x^3+339671475\,a^2b^8x^{\frac{8}{3}}+860501070\,a^3b^7x^{\frac{7}{3}}+1434168450\,a^4b^6x^2+16\cdots}{4\cdots}$$

input
```
integrate((a+b*x^(1/3))^10/x^10,x, algorithm="giac")
```

output
```
-1/47805615*(8436285*b^10*x^(10/3) + 79676025*a*b^9*x^3 + 339671475*a^2*b^
8*x^(8/3) + 860501070*a^3*b^7*x^(7/3) + 1434168450*a^4*b^6*x^2 + 164277477
0*a^5*b^5*x^(5/3) + 1309458150*a^6*b^4*x^(4/3) + 717084225*a^7*b^3*x + 258
150321*a^8*b^2*x^(2/3) + 55160325*a^9*b*x^(1/3) + 5311735*a^10)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx = -\frac{\frac{a^{10}}{9}+\frac{3\,b^{10}\,x^{10/3}}{17}+15\,a^7\,b^3\,x+\frac{5\,a\,b^9\,x^3}{3}+\frac{15\,a^9\,b\,x^{1/3}}{13}+30\,a^4\,b^6\,x^2+\frac{27\,a^8\,b^2\,x^{2/3}}{5}+\frac{630\,a^6\,b^4\,x^{4/3}}{23}+\frac{378\,a^5\,b^5\,x^{5/3}}{11}+\cdots}{x^9}$$

input
```
int((a + b*x^(1/3))^10/x^10,x)
```

output
```
-(a^10/9 + (3*b^10*x^(10/3))/17 + 15*a^7*b^3*x + (5*a*b^9*x^3)/3 + (15*a^9
*b*x^(1/3))/13 + 30*a^4*b^6*x^2 + (27*a^8*b^2*x^(2/3))/5 + (630*a^6*b^4*x^
(4/3))/23 + (378*a^5*b^5*x^(5/3))/11 + 18*a^3*b^7*x^(7/3) + (135*a^2*b^8*x
^(8/3))/19)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{10}}{x^{10}}\,dx = \frac{-5311735x^{\frac{2}{3}}a^{10}-717084225x^{\frac{5}{3}}a^7b^3-1434168450x^{\frac{8}{3}}a^4b^6-79676025x^{\frac{11}{3}}a\,b^9-258150321x^{\frac{4}{3}}a^8b^2-164\cdots}{478\cdots}$$

input
```
int((a+b*x^(1/3))^10/x^10,x)
```

output
```
( - 5311735*x**(2/3)*a**10 - 717084225*x**(2/3)*a**7*b**3*x - 1434168450*x
**(2/3)*a**4*b**6*x**2 - 79676025*x**(2/3)*a*b**9*x**3 - 258150321*x**(1/3
)*a**8*b**2*x - 1642774770*x**(1/3)*a**5*b**5*x**2 - 339671475*x**(1/3)*a*
*2*b**8*x**3 - 55160325*a**9*b*x - 1309458150*a**6*b**4*x**2 - 860501070*a
**3*b**7*x**3 - 8436285*b**10*x**4)/(47805615*x**(2/3)*x**9)
```

3.230 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^5\,dx$

Optimal result

Integrand size = 15, antiderivative size = 217

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^5\,dx = \frac{a^{15}x^6}{6}+\frac{45}{19}a^{14}bx^{19/3}+\frac{63}{4}a^{13}b^2x^{20/3}+65a^{12}b^3x^7+\frac{4095}{22}a^{11}b^4x^{22/3} +\frac{9009}{23}a^{10}b^5x^{23/3}+\frac{5005}{8}a^9b^6x^8+\frac{3861}{5}a^8b^7x^{25/3}+\frac{1485}{2}a^7b^8x^{26/3} +\frac{5005}{9}a^6b^9x^9+\frac{1287}{4}a^5b^{10}x^{28/3}+\frac{4095}{29}a^4b^{11}x^{29/3} +\frac{91}{2}a^3b^{12}x^{10}+\frac{315}{31}a^2b^{13}x^{31/3}+\frac{45}{32}ab^{14}x^{32/3}+\frac{b^{15}x^{11}}{11}$$

output
```
1/6*a^15*x^6+45/19*a^14*b*x^(19/3)+63/4*a^13*b^2*x^(20/3)+65*a^12*b^3*x^7+
4095/22*a^11*b^4*x^(22/3)+9009/23*a^10*b^5*x^(23/3)+5005/8*a^9*b^6*x^8+386
1/5*a^8*b^7*x^(25/3)+1485/2*a^7*b^8*x^(26/3)+5005/9*a^6*b^9*x^9+1287/4*a^5
*b^10*x^(28/3)+4095/29*a^4*b^11*x^(29/3)+91/2*a^3*b^12*x^10+315/31*a^2*b^1
3*x^(31/3)+45/32*a*b^14*x^(32/3)+1/11*b^15*x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^5\,dx$$
$$=\frac{1037158320a^{15}x^6+14738565600a^{14}bx^{19/3}+98011461240a^{13}b^2x^{20/3}+404491744800a^{12}b^3x^7+1158317\ldots}{}$$

input
```
Integrate[(a + b*x^(1/3))^15*x^5,x]
```

output
```
(1037158320*a^15*x^6 + 14738565600*a^14*b*x^(19/3) + 98011461240*a^13*b^2*
x^(20/3) + 404491744800*a^12*b^3*x^7 + 1158317269200*a^11*b^4*x^(22/3) + 2
437502427360*a^10*b^5*x^(23/3) + 3893233043700*a^9*b^6*x^8 + 4805361928224
*a^8*b^7*x^(25/3) + 4620540315600*a^7*b^8*x^(26/3) + 3460651594400*a^6*b^9
*x^9 + 2002234136760*a^5*b^10*x^(28/3) + 878723445600*a^4*b^11*x^(29/3) +
283144221360*a^3*b^12*x^10 + 63233200800*a^2*b^13*x^(31/3) + 8751023325*a*
b^14*x^(32/3) + 565722720*b^15*x^11)/6222949920
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\sqrt[3]{x}\right)^{15}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{15}x^{17/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \Big(x^{17/3}a^{15}+15bx^6a^{14}+105b^2x^{19/3}a^{13}+455b^3x^{20/3}a^{12}+1365b^4x^7a^{11}+3003b^5x^{22/3}a^{10}+5005b^6x^{23/3}a^9+\ldots$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^{15}x^6}{18}+\frac{15}{19}a^{14}bx^{19/3}+\frac{21}{4}a^{13}b^2x^{20/3}+\frac{65}{3}a^{12}b^3x^7+\frac{1365}{22}a^{11}b^4x^{22/3}+\frac{3003}{23}a^{10}b^5x^{23/3}+\frac{5005}{24}a^9b^6x^8+\frac{1287}{5}\right.$$

input
```
Int[(a + b*x^(1/3))^15*x^5,x]
```

output
```
3*((a^15*x^6)/18 + (15*a^14*b*x^(19/3))/19 + (21*a^13*b^2*x^(20/3))/4 + (6
5*a^12*b^3*x^7)/3 + (1365*a^11*b^4*x^(22/3))/22 + (3003*a^10*b^5*x^(23/3))
/23 + (5005*a^9*b^6*x^8)/24 + (1287*a^8*b^7*x^(25/3))/5 + (495*a^7*b^8*x^(
26/3))/2 + (5005*a^6*b^9*x^9)/27 + (429*a^5*b^10*x^(28/3))/4 + (1365*a^4*b
^11*x^(29/3))/29 + (91*a^3*b^12*x^10)/6 + (105*a^2*b^13*x^(31/3))/31 + (15
*a*b^14*x^(32/3))/32 + (b^15*x^11)/33)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 25.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^{15}x^6}{6}+\frac{45a^{14}b\,x^{\frac{19}{3}}}{19}+\frac{63a^{13}b^2x^{\frac{20}{3}}}{4}+65a^{12}b^3x^7+\frac{4095a^{11}b^4x^{\frac{22}{3}}}{22}+\frac{9009a^{10}b^5x^{\frac{23}{3}}}{23}+\frac{5005a^9b^6x^8}{8}+$
default	$\frac{a^{15}x^6}{6}+\frac{45a^{14}b\,x^{\frac{19}{3}}}{19}+\frac{63a^{13}b^2x^{\frac{20}{3}}}{4}+65a^{12}b^3x^7+\frac{4095a^{11}b^4x^{\frac{22}{3}}}{22}+\frac{9009a^{10}b^5x^{\frac{23}{3}}}{23}+\frac{5005a^9b^6x^8}{8}+$
trager	$(72b^{15}x^{10}+36036a^3b^{12}x^9+72x^9b^{15}+440440a^6b^9x^8+36036x^8b^{12}a^3+72x^8b^{15}+495495a^9b^6x^7+440440b^9x^7a^6+36036$
orering	$-(-55728250200b^{57}x^{19}-731345381325a^3b^{54}x^{18}-4435050755905a^6b^{51}x^{17}-16459729168610a^9b^{48}x^{16}-4172358203$

input
```
int((a+b*x^(1/3))^15*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^15*x^6+45/19*a^14*b*x^(19/3)+63/4*a^13*b^2*x^(20/3)+65*a^12*b^3*x^7+
4095/22*a^11*b^4*x^(22/3)+9009/23*a^10*b^5*x^(23/3)+5005/8*a^9*b^6*x^8+386
1/5*a^8*b^7*x^(25/3)+1485/2*a^7*b^8*x^(26/3)+5005/9*a^6*b^9*x^9+1287/4*a^5
*b^10*x^(28/3)+4095/29*a^4*b^11*x^(29/3)+91/2*a^3*b^12*x^10+315/31*a^2*b^1
3*x^(31/3)+45/32*a*b^14*x^(32/3)+1/11*b^15*x^11
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^5\,dx$$
$$=\frac{1}{11}\,b^{15}x^{11}+\frac{91}{2}\,a^3b^{12}x^{10}+\frac{5005}{9}\,a^6b^9x^9+\frac{5005}{8}\,a^9b^6x^8+65\,a^{12}b^3x^7+\frac{1}{6}\,a^{15}x^6$$
$$+\frac{9}{21344}\left(3335\,ab^{14}x^{10}+334880\,a^4b^{11}x^9+1760880\,a^7b^8x^8+928928\,a^{10}b^5x^7+37352\,a^{13}b^2x^6\right)x^{\frac{2}{3}}$$
$$+\frac{9}{129580}\left(146300\,a^2b^{13}x^{10}+4632485\,a^5b^{10}x^9+11117964\,a^8b^7x^8+2679950\,a^{11}b^4x^7+34100\,a^{14}bx^6\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^15*x^5,x, algorithm="fricas")
```

output
```
1/11*b^15*x^11 + 91/2*a^3*b^12*x^10 + 5005/9*a^6*b^9*x^9 + 5005/8*a^9*b^6*
x^8 + 65*a^12*b^3*x^7 + 1/6*a^15*x^6 + 9/21344*(3335*a*b^14*x^10 + 334880*
a^4*b^11*x^9 + 1760880*a^7*b^8*x^8 + 928928*a^10*b^5*x^7 + 37352*a^13*b^2*
x^6)*x^(2/3) + 9/129580*(146300*a^2*b^13*x^10 + 4632485*a^5*b^10*x^9 + 111
17964*a^8*b^7*x^8 + 2679950*a^11*b^4*x^7 + 34100*a^14*b*x^6)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^5\,dx = \frac{a^{15}x^6}{6}+\frac{45a^{14}bx^{\frac{19}{3}}}{19}+\frac{63a^{13}b^2x^{\frac{20}{3}}}{4}+65a^{12}b^3x^7+\frac{4095a^{11}b^4x^{\frac{22}{3}}}{22}+\frac{9009a^{10}b^5x^{\frac{23}{3}}}{23}+\frac{5005a^9b^6x^8}{8}+\frac{3861a^8b^7x^{\frac{25}{3}}}{5}+\frac{1485a^7b^8x^{\frac{26}{3}}}{2}+\frac{5005a^6b^9x^9}{9}+\frac{1287a^5b^{10}x^{\frac{28}{3}}}{4}+\frac{4095a^4b^{11}x^{\frac{29}{3}}}{29}+\frac{91a^3b^{12}x^{10}}{2}+\frac{315a^2b^{13}x^{\frac{31}{3}}}{31}+\frac{45ab^{14}x^{\frac{32}{3}}}{32}+\frac{b^{15}x^{11}}{11}$$

input
```
integrate((a+b*x**(1/3))**15*x**5,x)
```

output
```
a**15*x**6/6 + 45*a**14*b*x**(19/3)/19 + 63*a**13*b**2*x**(20/3)/4 + 65*a*
*12*b**3*x**7 + 4095*a**11*b**4*x**(22/3)/22 + 9009*a**10*b**5*x**(23/3)/2
3 + 5005*a**9*b**6*x**8/8 + 3861*a**8*b**7*x**(25/3)/5 + 1485*a**7*b**8*x*
*(26/3)/2 + 5005*a**6*b**9*x**9/9 + 1287*a**5*b**10*x**(28/3)/4 + 4095*a**
4*b**11*x**(29/3)/29 + 91*a**3*b**12*x**10/2 + 315*a**2*b**13*x**(31/3)/31
 + 45*a*b**14*x**(32/3)/32 + b**15*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.39

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^5\,dx=\frac{\left(bx^{\frac{1}{3}}+a\right)^{33}}{11\,b^{18}}-\frac{51\left(bx^{\frac{1}{3}}+a\right)^{32}a}{32\,b^{18}}+\frac{408\left(bx^{\frac{1}{3}}+a\right)^{31}a^2}{31\,b^{18}}-\frac{68\left(bx^{\frac{1}{3}}+a\right)^{30}a^3}{b^{18}}+\frac{7140\left(bx^{\frac{1}{3}}+a\right)^{29}a^4}{29\,b^{18}}-\frac{663\left(bx^{\frac{1}{3}}+a\right)^{28}a^5}{b^{18}}+\frac{12376\left(bx^{\frac{1}{3}}+a\right)^{27}a^6}{9\,b^{18}}-\frac{2244\left(bx^{\frac{1}{3}}+a\right)^{26}a^7}{b^{18}}+\frac{14586\left(bx^{\frac{1}{3}}+a\right)^{25}a^8}{5\,b^{18}}-\frac{12155\left(bx^{\frac{1}{3}}+a\right)^{24}a^9}{4\,b^{18}}+\frac{58344\left(bx^{\frac{1}{3}}+a\right)^{23}a^{10}}{23\,b^{18}}-\frac{18564\left(bx^{\frac{1}{3}}+a\right)^{22}a^{11}}{11\,b^{18}}+\frac{884\left(bx^{\frac{1}{3}}+a\right)^{21}a^{12}}{b^{18}}-\frac{357\left(bx^{\frac{1}{3}}+a\right)^{20}a^{13}}{b^{18}}+\frac{2040\left(bx^{\frac{1}{3}}+a\right)^{19}a^{14}}{19\,b^{18}}-\frac{68\left(bx^{\frac{1}{3}}+a\right)^{18}a^{15}}{3\,b^{18}}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{17}a^{16}}{b^{18}}-\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^{17}}{16\,b^{18}}$$

input
```
integrate((a+b*x^(1/3))^15*x^5,x, algorithm="maxima")
```

output
```
1/11*(b*x^(1/3) + a)^33/b^18 - 51/32*(b*x^(1/3) + a)^32*a/b^18 + 408/31*(b
*x^(1/3) + a)^31*a^2/b^18 - 68*(b*x^(1/3) + a)^30*a^3/b^18 + 7140/29*(b*x^
(1/3) + a)^29*a^4/b^18 - 663*(b*x^(1/3) + a)^28*a^5/b^18 + 12376/9*(b*x^(1
/3) + a)^27*a^6/b^18 - 2244*(b*x^(1/3) + a)^26*a^7/b^18 + 14586/5*(b*x^(1/
3) + a)^25*a^8/b^18 - 12155/4*(b*x^(1/3) + a)^24*a^9/b^18 + 58344/23*(b*x^
(1/3) + a)^23*a^10/b^18 - 18564/11*(b*x^(1/3) + a)^22*a^11/b^18 + 884*(b*x
^(1/3) + a)^21*a^12/b^18 - 357*(b*x^(1/3) + a)^20*a^13/b^18 + 2040/19*(b*x
^(1/3) + a)^19*a^14/b^18 - 68/3*(b*x^(1/3) + a)^18*a^15/b^18 + 3*(b*x^(1/3
) + a)^17*a^16/b^18 - 3/16*(b*x^(1/3) + a)^16*a^17/b^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^5\,dx = \frac{1}{11}\,b^{15}x^{11} + \frac{45}{32}\,ab^{14}x^{\frac{32}{3}} + \frac{315}{31}\,a^2b^{13}x^{\frac{31}{3}} + \frac{91}{2}\,a^3b^{12}x^{10} + \frac{4095}{29}\,a^4b^{11}x^{\frac{29}{3}} + \frac{1287}{4}\,a^5b^{10}x^{\frac{28}{3}} + \frac{5005}{9}\,a^6b^9x^9 + \frac{1485}{2}\,a^7b^8x^{\frac{26}{3}} + \frac{3861}{5}\,a^8b^7x^{\frac{25}{3}} + \frac{5005}{8}\,a^9b^6x^8 + \frac{9009}{23}\,a^{10}b^5x^{\frac{23}{3}} + \frac{4095}{22}\,a^{11}b^4x^{\frac{22}{3}} + 65\,a^{12}b^3x^7 + \frac{63}{4}\,a^{13}b^2x^{\frac{20}{3}} + \frac{45}{19}\,a^{14}bx^{\frac{19}{3}} + \frac{1}{6}\,a^{15}x^6$$

input
```
integrate((a+b*x^(1/3))^15*x^5,x, algorithm="giac")
```

output
```
1/11*b^15*x^11 + 45/32*a*b^14*x^(32/3) + 315/31*a^2*b^13*x^(31/3) + 91/2*a
^3*b^12*x^10 + 4095/29*a^4*b^11*x^(29/3) + 1287/4*a^5*b^10*x^(28/3) + 5005
/9*a^6*b^9*x^9 + 1485/2*a^7*b^8*x^(26/3) + 3861/5*a^8*b^7*x^(25/3) + 5005/
8*a^9*b^6*x^8 + 9009/23*a^10*b^5*x^(23/3) + 4095/22*a^11*b^4*x^(22/3) + 65
*a^12*b^3*x^7 + 63/4*a^13*b^2*x^(20/3) + 45/19*a^14*b*x^(19/3) + 1/6*a^15*
x^6
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^5\,dx = \frac{a^{15}\,x^6}{6} + \frac{b^{15}\,x^{11}}{11} + \frac{45\,a^{14}\,b\,x^{19/3}}{19} + \frac{45\,a\,b^{14}\,x^{32/3}}{32} + 65\,a^{12}\,b^3\,x^7 + \frac{5005\,a^9\,b^6\,x^8}{8} + \frac{5005\,a^6\,b^9\,x^9}{9} + \frac{91\,a^3\,b^{12}\,x^{10}}{2} + \frac{63\,a^{13}\,b^2\,x^{20/3}}{4} + \frac{4095\,a^{11}\,b^4\,x^{22/3}}{22} + \frac{9009\,a^{10}\,b^5\,x^{23/3}}{23} + \frac{3861\,a^8\,b^7\,x^{25/3}}{5} + \frac{1485\,a^7\,b^8\,x^{26/3}}{2} + \frac{1287\,a^5\,b^{10}\,x^{28/3}}{4} + \frac{4095\,a^4\,b^{11}\,x^{29/3}}{29} + \frac{315\,a^2\,b^{13}\,x^{31/3}}{31}$$

input
```
int(x^5*(a + b*x^(1/3))^15,x)
```

output
```
(a^15*x^6)/6 + (b^15*x^11)/11 + (45*a^14*b*x^(19/3))/19 + (45*a*b^14*x^(32
/3))/32 + 65*a^12*b^3*x^7 + (5005*a^9*b^6*x^8)/8 + (5005*a^6*b^9*x^9)/9 +
(91*a^3*b^12*x^10)/2 + (63*a^13*b^2*x^(20/3))/4 + (4095*a^11*b^4*x^(22/3))
/22 + (9009*a^10*b^5*x^(23/3))/23 + (3861*a^8*b^7*x^(25/3))/5 + (1485*a^7*
b^8*x^(26/3))/2 + (1287*a^5*b^10*x^(28/3))/4 + (4095*a^4*b^11*x^(29/3))/29
 + (315*a^2*b^13*x^(31/3))/31
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^5\,dx$$

$$=\frac{x^6\Big(98011461240x^{\frac{2}{3}}a^{13}b^2+2437502427360x^{\frac{5}{3}}a^{10}b^5+4620540315600x^{\frac{8}{3}}a^7b^8+878723445600x^{\frac{11}{3}}a^4b^{11}+}{}$$

input
```
int((a+b*x^(1/3))^15*x^5,x)
```

output
```
(x**6*(98011461240*x**(2/3)*a**13*b**2 + 2437502427360*x**(2/3)*a**10*b**5
*x + 4620540315600*x**(2/3)*a**7*b**8*x**2 + 878723445600*x**(2/3)*a**4*b*
*11*x**3 + 8751023325*x**(2/3)*a*b**14*x**4 + 14738565600*x**(1/3)*a**14*b
 + 1158317269200*x**(1/3)*a**11*b**4*x + 4805361928224*x**(1/3)*a**8*b**7*
x**2 + 2002234136760*x**(1/3)*a**5*b**10*x**3 + 63233200800*x**(1/3)*a**2*
b**13*x**4 + 1037158320*a**15 + 404491744800*a**12*b**3*x + 3893233043700*
a**9*b**6*x**2 + 3460651594400*a**6*b**9*x**3 + 283144221360*a**3*b**12*x*
*4 + 565722720*b**15*x**5))/6222949920
```

3.231 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 217

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^4 \, dx = \frac{a^{15}x^5}{5} + \frac{45}{16}a^{14}bx^{16/3} + \frac{315}{17}a^{13}b^2x^{17/3} + \frac{455}{6}a^{12}b^3x^6 + \frac{4095}{19}a^{11}b^4x^{19/3} + \frac{9009}{20}a^{10}b^5x^{20/3} + 715a^9b^6x^7 + \frac{1755}{2}a^8b^7x^{22/3} + \frac{19305}{23}a^7b^8x^{23/3} + \frac{5005}{8}a^6b^9x^8 + \frac{9009}{25}a^5b^{10}x^{25/3} + \frac{315}{2}a^4b^{11}x^{26/3} + \frac{455}{9}a^3b^{12}x^9 + \frac{45}{4}a^2b^{13}x^{28/3} + \frac{45}{29}ab^{14}x^{29/3} + \frac{b^{15}x^{10}}{10}$$

output

```
1/5*a^15*x^5+45/16*a^14*b*x^(16/3)+315/17*a^13*b^2*x^(17/3)+455/6*a^12*b^3
*x^6+4095/19*a^11*b^4*x^(19/3)+9009/20*a^10*b^5*x^(20/3)+715*a^9*b^6*x^7+1
755/2*a^8*b^7*x^(22/3)+19305/23*a^7*b^8*x^(23/3)+5005/8*a^6*b^9*x^8+9009/2
5*a^5*b^10*x^(25/3)+315/2*a^4*b^11*x^(26/3)+455/9*a^3*b^12*x^9+45/4*a^2*b^
13*x^(28/3)+45/29*a*b^14*x^(29/3)+1/10*b^15*x^10
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^4\,dx = \frac{155117520a^{15}x^5+2181340125a^{14}bx^{16/3}+14371182000a^{13}b^2x^{17/3}+58815393000a^{12}b^3x^6+16715953800}{}$$

input
```
Integrate[(a + b*x^(1/3))^15*x^4,x]
```

output
```
(155117520*a^15*x^5 + 2181340125*a^14*b*x^(16/3) + 14371182000*a^13*b^2*x^
(17/3) + 58815393000*a^12*b^3*x^6 + 167159538000*a^11*b^4*x^(19/3) + 34936
3434420*a^10*b^5*x^(20/3) + 554545134000*a^9*b^6*x^7 + 680578119000*a^8*b^
7*x^(22/3) + 650987766000*a^7*b^8*x^(23/3) + 485226992250*a^6*b^9*x^8 + 27
9490747536*a^5*b^10*x^(25/3) + 122155047000*a^4*b^11*x^(26/3) + 3921026200
0*a^3*b^12*x^9 + 8725360500*a^2*b^13*x^(28/3) + 1203498000*a*b^14*x^(29/3)
 + 77558760*b^15*x^10)/775587600
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\sqrt[3]{x}\right)^{15}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{15} x^{14/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(x^{14/3}a^{15}+15bx^5a^{14}+105b^2x^{16/3}a^{13}+455b^3x^{17/3}a^{12}+1365b^4x^6a^{11}+3003b^5x^{19/3}a^{10}+5005b^6x^{20/3}a^9+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^{15}x^5}{15}+\frac{15}{16}a^{14}bx^{16/3}+\frac{105}{17}a^{13}b^2x^{17/3}+\frac{455}{18}a^{12}b^3x^6+\frac{1365}{19}a^{11}b^4x^{19/3}+\frac{3003}{20}a^{10}b^5x^{20/3}+\frac{715}{3}a^9b^6x^7+\frac{585}{2}\right.$$

input
```
Int[(a + b*x^(1/3))^15*x^4,x]
```

output
```
3*((a^15*x^5)/15 + (15*a^14*b*x^(16/3))/16 + (105*a^13*b^2*x^(17/3))/17 +
(455*a^12*b^3*x^6)/18 + (1365*a^11*b^4*x^(19/3))/19 + (3003*a^10*b^5*x^(20
/3))/20 + (715*a^9*b^6*x^7)/3 + (585*a^8*b^7*x^(22/3))/2 + (6435*a^7*b^8*x
^(23/3))/23 + (5005*a^6*b^9*x^8)/24 + (3003*a^5*b^10*x^(25/3))/25 + (105*a
^4*b^11*x^(26/3))/2 + (455*a^3*b^12*x^9)/27 + (15*a^2*b^13*x^(28/3))/4 + (
15*a*b^14*x^(29/3))/29 + (b^15*x^10)/30)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a^{15}x^5}{5}+\frac{45a^{14}b\,x^{\frac{16}{3}}}{16}+\frac{315a^{13}b^2x^{\frac{17}{3}}}{17}+\frac{455a^{12}b^3x^6}{6}+\frac{4095a^{11}b^4x^{\frac{19}{3}}}{19}+\frac{9009a^{10}b^5x^{\frac{20}{3}}}{20}+715a^9b^6x^7+$
default	$\frac{a^{15}x^5}{5}+\frac{45a^{14}b\,x^{\frac{16}{3}}}{16}+\frac{315a^{13}b^2x^{\frac{17}{3}}}{17}+\frac{455a^{12}b^3x^6}{6}+\frac{4095a^{11}b^4x^{\frac{19}{3}}}{19}+\frac{9009a^{10}b^5x^{\frac{20}{3}}}{20}+715a^9b^6x^7+$
trager	$\frac{(36x^9b^{15}+18200x^8b^{12}a^3+36x^8b^{15}+225225b^9x^7a^6+18200x^7b^{12}a^3+36x^7b^{15}+257400b^6x^6a^9+225225x^6b^9a^6+18200a}{}$
orering	$-\frac{(-37943617500b^{54}x^{18}-497508540760a^3b^{51}x^{17}-3014219523770a^6b^{48}x^{16}-11175963403150a^9b^{45}x^{15}-2830254821}{}$

input
```
int((a+b*x^(1/3))^15*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^15*x^5+45/16*a^14*b*x^(16/3)+315/17*a^13*b^2*x^(17/3)+455/6*a^12*b^3
*x^6+4095/19*a^11*b^4*x^(19/3)+9009/20*a^10*b^5*x^(20/3)+715*a^9*b^6*x^7+1
755/2*a^8*b^7*x^(22/3)+19305/23*a^7*b^8*x^(23/3)+5005/8*a^6*b^9*x^8+9009/2
5*a^5*b^10*x^(25/3)+315/2*a^4*b^11*x^(26/3)+455/9*a^3*b^12*x^9+45/4*a^2*b^
13*x^(28/3)+45/29*a*b^14*x^(29/3)+1/10*b^15*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^4\,dx$$
$$=\frac{1}{10}\,b^{15}x^{10}+\frac{455}{9}\,a^3b^{12}x^9+\frac{5005}{8}\,a^6b^9x^8+715\,a^9b^6x^7+\frac{455}{6}\,a^{12}b^3x^6+\frac{1}{5}\,a^{15}x^5$$
$$+\frac{9}{226780}\left(39100\,ab^{14}x^9+3968650\,a^4b^{11}x^8+21149700\,a^7b^8x^7+11350339\,a^{10}b^5x^6+466900\,a^{13}b^2x^5\right)x^{\frac{2}{3}}$$
$$+\frac{9}{7600}\left(9500\,a^2b^{13}x^9+304304\,a^5b^{10}x^8+741000\,a^8b^7x^7+182000\,a^{11}b^4x^6+2375\,a^{14}bx^5\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^15*x^4,x, algorithm="fricas")
```

output
```
1/10*b^15*x^10 + 455/9*a^3*b^12*x^9 + 5005/8*a^6*b^9*x^8 + 715*a^9*b^6*x^7
 + 455/6*a^12*b^3*x^6 + 1/5*a^15*x^5 + 9/226780*(39100*a*b^14*x^9 + 396865
0*a^4*b^11*x^8 + 21149700*a^7*b^8*x^7 + 11350339*a^10*b^5*x^6 + 466900*a^1
3*b^2*x^5)*x^(2/3) + 9/7600*(9500*a^2*b^13*x^9 + 304304*a^5*b^10*x^8 + 741
000*a^8*b^7*x^7 + 182000*a^11*b^4*x^6 + 2375*a^14*b*x^5)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^4\,dx = \frac{a^{15}x^5}{5}+\frac{45a^{14}bx^{\frac{16}{3}}}{16}+\frac{315a^{13}b^2x^{\frac{17}{3}}}{17}+\frac{455a^{12}b^3x^6}{6}+\frac{4095a^{11}b^4x^{\frac{19}{3}}}{19}+\frac{9009a^{10}b^5x^{\frac{20}{3}}}{20}+715a^9b^6x^7+\frac{1755a^8b^7x^{\frac{22}{3}}}{2}+\frac{19305a^7b^8x^{\frac{23}{3}}}{23}+\frac{5005a^6b^9x^8}{8}+\frac{9009a^5b^{10}x^{\frac{25}{3}}}{25}+\frac{315a^4b^{11}x^{\frac{26}{3}}}{2}+\frac{455a^3b^{12}x^9}{9}+\frac{45a^2b^{13}x^{\frac{28}{3}}}{4}+\frac{45ab^{14}x^{\frac{29}{3}}}{29}+\frac{b^{15}x^{10}}{10}$$

input
```
integrate((a+b*x**(1/3))**15*x**4,x)
```

output
```
a**15*x**5/5 + 45*a**14*b*x**(16/3)/16 + 315*a**13*b**2*x**(17/3)/17 + 455
*a**12*b**3*x**6/6 + 4095*a**11*b**4*x**(19/3)/19 + 9009*a**10*b**5*x**(20
/3)/20 + 715*a**9*b**6*x**7 + 1755*a**8*b**7*x**(22/3)/2 + 19305*a**7*b**8
*x**(23/3)/23 + 5005*a**6*b**9*x**8/8 + 9009*a**5*b**10*x**(25/3)/25 + 315
*a**4*b**11*x**(26/3)/2 + 455*a**3*b**12*x**9/9 + 45*a**2*b**13*x**(28/3)/
4 + 45*a*b**14*x**(29/3)/29 + b**15*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.16

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^4\,dx = \frac{\left(bx^{\frac{1}{3}}+a\right)^{30}}{10\,b^{15}} - \frac{42\left(bx^{\frac{1}{3}}+a\right)^{29}a}{29\,b^{15}} + \frac{39\left(bx^{\frac{1}{3}}+a\right)^{28}a^2}{4\,b^{15}} - \frac{364\left(bx^{\frac{1}{3}}+a\right)^{27}a^3}{9\,b^{15}} + \frac{231\left(bx^{\frac{1}{3}}+a\right)^{26}a^4}{2\,b^{15}} - \frac{6006\left(bx^{\frac{1}{3}}+a\right)^{25}a^5}{25\,b^{15}} + \frac{3003\left(bx^{\frac{1}{3}}+a\right)^{24}a^6}{8\,b^{15}} - \frac{10296\left(bx^{\frac{1}{3}}+a\right)^{23}a^7}{23\,b^{15}} + \frac{819\left(bx^{\frac{1}{3}}+a\right)^{22}a^8}{2\,b^{15}} - \frac{286\left(bx^{\frac{1}{3}}+a\right)^{21}a^9}{b^{15}} + \frac{3003\left(bx^{\frac{1}{3}}+a\right)^{20}a^{10}}{20\,b^{15}} - \frac{1092\left(bx^{\frac{1}{3}}+a\right)^{19}a^{11}}{19\,b^{15}} + \frac{91\left(bx^{\frac{1}{3}}+a\right)^{18}a^{12}}{6\,b^{15}} - \frac{42\left(bx^{\frac{1}{3}}+a\right)^{17}a^{13}}{17\,b^{15}} + \frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^{14}}{16\,b^{15}}$$

input
```
integrate((a+b*x^(1/3))^15*x^4,x, algorithm="maxima")
```

output
```
1/10*(b*x^(1/3) + a)^30/b^15 - 42/29*(b*x^(1/3) + a)^29*a/b^15 + 39/4*(b*x
^(1/3) + a)^28*a^2/b^15 - 364/9*(b*x^(1/3) + a)^27*a^3/b^15 + 231/2*(b*x^(
1/3) + a)^26*a^4/b^15 - 6006/25*(b*x^(1/3) + a)^25*a^5/b^15 + 3003/8*(b*x^
(1/3) + a)^24*a^6/b^15 - 10296/23*(b*x^(1/3) + a)^23*a^7/b^15 + 819/2*(b*x
^(1/3) + a)^22*a^8/b^15 - 286*(b*x^(1/3) + a)^21*a^9/b^15 + 3003/20*(b*x^(
1/3) + a)^20*a^10/b^15 - 1092/19*(b*x^(1/3) + a)^19*a^11/b^15 + 91/6*(b*x^
(1/3) + a)^18*a^12/b^15 - 42/17*(b*x^(1/3) + a)^17*a^13/b^15 + 3/16*(b*x^(
1/3) + a)^16*a^14/b^15
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^4\,dx = \frac{1}{10}\,b^{15}x^{10} + \frac{45}{29}\,ab^{14}x^{\frac{29}{3}} + \frac{45}{4}\,a^2b^{13}x^{\frac{28}{3}} + \frac{455}{9}\,a^3b^{12}x^9 + \frac{315}{2}\,a^4b^{11}x^{\frac{26}{3}} + \frac{9009}{25}\,a^5b^{10}x^{\frac{25}{3}} + \frac{5005}{8}\,a^6b^9x^8 + \frac{19305}{23}\,a^7b^8x^{\frac{23}{3}} + \frac{1755}{2}\,a^8b^7x^{\frac{22}{3}} + 715\,a^9b^6x^7 + \frac{9009}{20}\,a^{10}b^5x^{\frac{20}{3}} + \frac{4095}{19}\,a^{11}b^4x^{\frac{19}{3}} + \frac{455}{6}\,a^{12}b^3x^6 + \frac{315}{17}\,a^{13}b^2x^{\frac{17}{3}} + \frac{45}{16}\,a^{14}bx^{\frac{16}{3}} + \frac{1}{5}\,a^{15}x^5$$

input
```
integrate((a+b*x^(1/3))^15*x^4,x, algorithm="giac")
```

output
```
1/10*b^15*x^10 + 45/29*a*b^14*x^(29/3) + 45/4*a^2*b^13*x^(28/3) + 455/9*a^
3*b^12*x^9 + 315/2*a^4*b^11*x^(26/3) + 9009/25*a^5*b^10*x^(25/3) + 5005/8*
a^6*b^9*x^8 + 19305/23*a^7*b^8*x^(23/3) + 1755/2*a^8*b^7*x^(22/3) + 715*a^
9*b^6*x^7 + 9009/20*a^10*b^5*x^(20/3) + 4095/19*a^11*b^4*x^(19/3) + 455/6*
a^12*b^3*x^6 + 315/17*a^13*b^2*x^(17/3) + 45/16*a^14*b*x^(16/3) + 1/5*a^15
*x^5
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^4\,dx = \frac{a^{15}\,x^5}{5} + \frac{b^{15}\,x^{10}}{10} + \frac{45\,a^{14}\,b\,x^{16/3}}{16} + \frac{45\,a\,b^{14}\,x^{29/3}}{29} + \frac{455\,a^{12}\,b^3\,x^6}{6} + 715\,a^9\,b^6\,x^7 + \frac{5005\,a^6\,b^9\,x^8}{8} + \frac{455\,a^3\,b^{12}\,x^9}{9} + \frac{315\,a^{13}\,b^2\,x^{17/3}}{17} + \frac{4095\,a^{11}\,b^4\,x^{19/3}}{19} + \frac{9009\,a^{10}\,b^5\,x^{20/3}}{20} + \frac{1755\,a^8\,b^7\,x^{22/3}}{2} + \frac{19305\,a^7\,b^8\,x^{23/3}}{23} + \frac{9009\,a^5\,b^{10}\,x^{25/3}}{25} + \frac{315\,a^4\,b^{11}\,x^{26/3}}{2} + \frac{45\,a^2\,b^{13}\,x^{28/3}}{4}$$

input
```
int(x^4*(a + b*x^(1/3))^15,x)
```

output

```
(a^15*x^5)/5 + (b^15*x^10)/10 + (45*a^14*b*x^(16/3))/16 + (45*a*b^14*x^(29
/3))/29 + (455*a^12*b^3*x^6)/6 + 715*a^9*b^6*x^7 + (5005*a^6*b^9*x^8)/8 +
(455*a^3*b^12*x^9)/9 + (315*a^13*b^2*x^(17/3))/17 + (4095*a^11*b^4*x^(19/3
))/19 + (9009*a^10*b^5*x^(20/3))/20 + (1755*a^8*b^7*x^(22/3))/2 + (19305*a
^7*b^8*x^(23/3))/23 + (9009*a^5*b^10*x^(25/3))/25 + (315*a^4*b^11*x^(26/3)
)/2 + (45*a^2*b^13*x^(28/3))/4
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^4 \, dx$$

$$= \frac{x^5 \left(14371182000 x^{\frac{2}{3}} a^{13} b^2 + 349363434420 x^{\frac{5}{3}} a^{10} b^5 + 650987766000 x^{\frac{8}{3}} a^7 b^8 + 122155047000 x^{\frac{11}{3}} a^4 b^{11} + 12\right.}{}$$

input

```
int((a+b*x^(1/3))^15*x^4,x)
```

output

```
(x**5*(14371182000*x**(2/3)*a**13*b**2 + 349363434420*x**(2/3)*a**10*b**5*
x + 650987766000*x**(2/3)*a**7*b**8*x**2 + 122155047000*x**(2/3)*a**4*b**1
1*x**3 + 1203498000*x**(2/3)*a*b**14*x**4 + 2181340125*x**(1/3)*a**14*b +
167159538000*x**(1/3)*a**11*b**4*x + 680578119000*x**(1/3)*a**8*b**7*x**2
+ 279490747536*x**(1/3)*a**5*b**10*x**3 + 8725360500*x**(1/3)*a**2*b**13*x
**4 + 155117520*a**15 + 58815393000*a**12*b**3*x + 554545134000*a**9*b**6*
x**2 + 485226992250*a**6*b**9*x**3 + 39210262000*a**3*b**12*x**4 + 7755876
0*b**15*x**5))/775587600
```

3.232 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 244

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx = -\frac{3a^{11}\left(a+b\sqrt[3]{x}\right)^{16}}{16b^{12}}+\frac{33a^{10}\left(a+b\sqrt[3]{x}\right)^{17}}{17b^{12}}-\frac{55a^9\left(a+b\sqrt[3]{x}\right)^{18}}{6b^{12}}+\frac{495a^8\left(a+b\sqrt[3]{x}\right)^{19}}{19b^{12}}-\frac{99a^7\left(a+b\sqrt[3]{x}\right)^{20}}{2b^{12}}+\frac{66a^6\left(a+b\sqrt[3]{x}\right)^{21}}{b^{12}}-\frac{63a^5\left(a+b\sqrt[3]{x}\right)^{22}}{b^{12}}+\frac{990a^4\left(a+b\sqrt[3]{x}\right)^{23}}{23b^{12}}-\frac{165a^3\left(a+b\sqrt[3]{x}\right)^{24}}{8b^{12}}+\frac{33a^2\left(a+b\sqrt[3]{x}\right)^{25}}{5b^{12}}-\frac{33a\left(a+b\sqrt[3]{x}\right)^{26}}{26b^{12}}+\frac{\left(a+b\sqrt[3]{x}\right)^{27}}{9b^{12}}$$

output
```
-3/16*a^11*(a+b*x^(1/3))^16/b^12+33/17*a^10*(a+b*x^(1/3))^17/b^12-55/6*a^9
*(a+b*x^(1/3))^18/b^12+495/19*a^8*(a+b*x^(1/3))^19/b^12-99/2*a^7*(a+b*x^(1
/3))^20/b^12+66*a^6*(a+b*x^(1/3))^21/b^12-63*a^5*(a+b*x^(1/3))^22/b^12+990
/23*a^4*(a+b*x^(1/3))^23/b^12-165/8*a^3*(a+b*x^(1/3))^24/b^12+33/5*a^2*(a+
b*x^(1/3))^25/b^12-33/26*a*(a+b*x^(1/3))^26/b^12+1/9*(a+b*x^(1/3))^27/b^12
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^3 \, dx$$
$$= \frac{17383860a^{15}x^4 + 240699600a^{14}bx^{13/3} + 1564547400a^{13}b^2x^{14/3} + 6327725040a^{12}b^3x^5 + 17796726675a^{11}b$$

input
```
Integrate[(a + b*x^(1/3))^15*x^3,x]
```

output
```
(17383860*a^15*x^4 + 240699600*a^14*b*x^(13/3) + 1564547400*a^13*b^2*x^(14
/3) + 6327725040*a^12*b^3*x^5 + 17796726675*a^11*b^4*x^(16/3) + 3684969288
0*a^10*b^5*x^(17/3) + 58004146200*a^9*b^6*x^6 + 70651666800*a^8*b^7*x^(19/
3) + 67119083460*a^7*b^8*x^(20/3) + 49717839600*a^6*b^9*x^7 + 28474762680*
a^5*b^10*x^(22/3) + 12380331600*a^4*b^11*x^(23/3) + 3954828150*a^3*b^12*x^
8 + 876146544*a^2*b^13*x^(25/3) + 120349800*a*b^14*x^(26/3) + 7726160*b^15
*x^9)/69535440
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\sqrt[3]{x}\right)^{15} dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{15} x^{11/3} d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^{26}}{b^{11}}-\frac{11a\left(a+b\sqrt[3]{x}\right)^{25}}{b^{11}}+\frac{55a^2\left(a+b\sqrt[3]{x}\right)^{24}}{b^{11}}-\frac{165a^3\left(a+b\sqrt[3]{x}\right)^{23}}{b^{11}}+\frac{330a^4\left(a+b\sqrt[3]{x}\right)^{22}}{b^{11}}-\frac{462a^5}{\right.$$

$\downarrow$ 2009

$$3\left(-\frac{a^{11}\left(a+b\sqrt[3]{x}\right)^{16}}{16b^{12}}+\frac{11a^{10}\left(a+b\sqrt[3]{x}\right)^{17}}{17b^{12}}-\frac{55a^9\left(a+b\sqrt[3]{x}\right)^{18}}{18b^{12}}+\frac{165a^8\left(a+b\sqrt[3]{x}\right)^{19}}{19b^{12}}-\frac{33a^7\left(a+b\sqrt[3]{x}\right)^{20}}{2b^{12}}+\frac{22a\right.$$

input
```
Int[(a + b*x^(1/3))^15*x^3,x]
```

output
```
3*(-1/16*(a^11*(a + b*x^(1/3))^16)/b^12 + (11*a^10*(a + b*x^(1/3))^17)/(17
*b^12) - (55*a^9*(a + b*x^(1/3))^18)/(18*b^12) + (165*a^8*(a + b*x^(1/3))^
19)/(19*b^12) - (33*a^7*(a + b*x^(1/3))^20)/(2*b^12) + (22*a^6*(a + b*x^(1
/3))^21)/b^12 - (21*a^5*(a + b*x^(1/3))^22)/b^12 + (330*a^4*(a + b*x^(1/3)
)^23)/(23*b^12) - (55*a^3*(a + b*x^(1/3))^24)/(8*b^12) + (11*a^2*(a + b*x^
(1/3))^25)/(5*b^12) - (11*a*(a + b*x^(1/3))^26)/(26*b^12) + (a + b*x^(1/3)
)^27/(27*b^12))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 25.82 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{x^9 b^{15}}{9}+\frac{45 a\, b^{14} x^{\frac{26}{3}}}{26}+\frac{63 a^2 b^{13} x^{\frac{25}{3}}}{5}+\frac{455 x^8 b^{12} a^3}{8}+\frac{4095 a^4 b^{11} x^{\frac{23}{3}}}{23}+\frac{819 a^5 b^{10} x^{\frac{22}{3}}}{2}+715 b^9 x^7 a^6+38$
default	$\frac{x^9 b^{15}}{9}+\frac{45 a\, b^{14} x^{\frac{26}{3}}}{26}+\frac{63 a^2 b^{13} x^{\frac{25}{3}}}{5}+\frac{455 x^8 b^{12} a^3}{8}+\frac{4095 a^4 b^{11} x^{\frac{23}{3}}}{23}+\frac{819 a^5 b^{10} x^{\frac{22}{3}}}{2}+715 b^9 x^7 a^6+38$
trager	$(8x^8b^{15}+4095x^7b^{12}a^3+8x^7b^{15}+51480x^6b^9a^6+4095a^3b^{12}x^6+8b^{15}x^6+60060x^5b^6a^9+51480x^5b^9a^6+4095a^3b^{12}x^5+8b$
orering	$-\frac{(-3748376240b^{51}x^{17}-49081901050a^3b^{48}x^{16}-296935418780a^6b^{45}x^{15}-1099219638790a^9b^{42}x^{14}-2778952406615a}{}$

input
```
int((a+b*x^(1/3))^15*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9*b^15+45/26*a*b^14*x^(26/3)+63/5*a^2*b^13*x^(25/3)+455/8*x^8*b^12*a
^3+4095/23*a^4*b^11*x^(23/3)+819/2*a^5*b^10*x^(22/3)+715*b^9*x^7*a^6+3861/
4*a^7*b^8*x^(20/3)+19305/19*a^8*b^7*x^(19/3)+5005/6*b^6*x^6*a^9+9009/17*a^
10*b^5*x^(17/3)+4095/16*a^11*b^4*x^(16/3)+91*b^3*x^5*a^12+45/2*a^13*b^2*x^
(14/3)+45/13*a^14*b*x^(13/3)+1/4*x^4*a^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.73

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx$$
$$=\frac{1}{9}\,b^{15}x^9+\frac{455}{8}\,a^3b^{12}x^8+715\,a^6b^9x^7+\frac{5005}{6}\,a^9b^6x^6+91\,a^{12}b^3x^5+\frac{1}{4}\,a^{15}x^4$$
$$+\frac{9}{20332}\left(3910\,ab^{14}x^8+402220\,a^4b^{11}x^7+2180607\,a^7b^8x^6+1197196\,a^{10}b^5x^5+50830\,a^{13}b^2x^4\right)x^{\frac{2}{3}}$$
$$+\frac{9}{19760}\left(27664\,a^2b^{13}x^8+899080\,a^5b^{10}x^7+2230800\,a^8b^7x^6+561925\,a^{11}b^4x^5+7600\,a^{14}bx^4\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^15*x^3,x, algorithm="fricas")
```

output
```
1/9*b^15*x^9 + 455/8*a^3*b^12*x^8 + 715*a^6*b^9*x^7 + 5005/6*a^9*b^6*x^6 +
 91*a^12*b^3*x^5 + 1/4*a^15*x^4 + 9/20332*(3910*a*b^14*x^8 + 402220*a^4*b^
11*x^7 + 2180607*a^7*b^8*x^6 + 1197196*a^10*b^5*x^5 + 50830*a^13*b^2*x^4)*
x^(2/3) + 9/19760*(27664*a^2*b^13*x^8 + 899080*a^5*b^10*x^7 + 2230800*a^8*
b^7*x^6 + 561925*a^11*b^4*x^5 + 7600*a^14*b*x^4)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^3\,dx = \frac{a^{15}x^4}{4}+\frac{45a^{14}bx^{\frac{13}{3}}}{13}+\frac{45a^{13}b^2x^{\frac{14}{3}}}{2}+91a^{12}b^3x^5+\frac{4095a^{11}b^4x^{\frac{16}{3}}}{16}+\frac{9009a^{10}b^5x^{\frac{17}{3}}}{17}+\frac{5005a^9b^6x^6}{6}+\frac{19305a^8b^7x^{\frac{19}{3}}}{19}+\frac{3861a^7b^8x^{\frac{20}{3}}}{4}+715a^6b^9x^7+\frac{819a^5b^{10}x^{\frac{22}{3}}}{2}+\frac{4095a^4b^{11}x^{\frac{23}{3}}}{23}+\frac{455a^3b^{12}x^8}{8}+\frac{63a^2b^{13}x^{\frac{25}{3}}}{5}+\frac{45ab^{14}x^{\frac{26}{3}}}{26}+\frac{b^{15}x^9}{9}$$

input
```
integrate((a+b*x**(1/3))**15*x**3,x)
```

output
```
a**15*x**4/4 + 45*a**14*b*x**(13/3)/13 + 45*a**13*b**2*x**(14/3)/2 + 91*a*
*12*b**3*x**5 + 4095*a**11*b**4*x**(16/3)/16 + 9009*a**10*b**5*x**(17/3)/1
7 + 5005*a**9*b**6*x**6/6 + 19305*a**8*b**7*x**(19/3)/19 + 3861*a**7*b**8*
x**(20/3)/4 + 715*a**6*b**9*x**7 + 819*a**5*b**10*x**(22/3)/2 + 4095*a**4*
b**11*x**(23/3)/23 + 455*a**3*b**12*x**8/8 + 63*a**2*b**13*x**(25/3)/5 + 4
5*a*b**14*x**(26/3)/26 + b**15*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx = \frac{\left(bx^{\frac{1}{3}}+a\right)^{27}}{9\,b^{12}} - \frac{33\left(bx^{\frac{1}{3}}+a\right)^{26}a}{26\,b^{12}} + \frac{33\left(bx^{\frac{1}{3}}+a\right)^{25}a^2}{5\,b^{12}} - \frac{165\left(bx^{\frac{1}{3}}+a\right)^{24}a^3}{8\,b^{12}} + \frac{990\left(bx^{\frac{1}{3}}+a\right)^{23}a^4}{23\,b^{12}} - \frac{63\left(bx^{\frac{1}{3}}+a\right)^{22}a^5}{b^{12}} + \frac{66\left(bx^{\frac{1}{3}}+a\right)^{21}a^6}{b^{12}} - \frac{99\left(bx^{\frac{1}{3}}+a\right)^{20}a^7}{2\,b^{12}} + \frac{495\left(bx^{\frac{1}{3}}+a\right)^{19}a^8}{19\,b^{12}} - \frac{55\left(bx^{\frac{1}{3}}+a\right)^{18}a^9}{6\,b^{12}} + \frac{33\left(bx^{\frac{1}{3}}+a\right)^{17}a^{10}}{17\,b^{12}} - \frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^{11}}{16\,b^{12}}$$

input
```
integrate((a+b*x^(1/3))^15*x^3,x, algorithm="maxima")
```

output
```
1/9*(b*x^(1/3) + a)^27/b^12 - 33/26*(b*x^(1/3) + a)^26*a/b^12 + 33/5*(b*x^
(1/3) + a)^25*a^2/b^12 - 165/8*(b*x^(1/3) + a)^24*a^3/b^12 + 990/23*(b*x^(
1/3) + a)^23*a^4/b^12 - 63*(b*x^(1/3) + a)^22*a^5/b^12 + 66*(b*x^(1/3) + a
)^21*a^6/b^12 - 99/2*(b*x^(1/3) + a)^20*a^7/b^12 + 495/19*(b*x^(1/3) + a)^
19*a^8/b^12 - 55/6*(b*x^(1/3) + a)^18*a^9/b^12 + 33/17*(b*x^(1/3) + a)^17*
a^10/b^12 - 3/16*(b*x^(1/3) + a)^16*a^11/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx = \frac{1}{9}\,b^{15}x^9 + \frac{45}{26}\,ab^{14}x^{\frac{26}{3}} + \frac{63}{5}\,a^2b^{13}x^{\frac{25}{3}} + \frac{455}{8}\,a^3b^{12}x^8 + \frac{4095}{23}\,a^4b^{11}x^{\frac{23}{3}} + \frac{819}{2}\,a^5b^{10}x^{\frac{22}{3}} + 715\,a^6b^9x^7 + \frac{3861}{4}\,a^7b^8x^{\frac{20}{3}} + \frac{19305}{19}\,a^8b^7x^{\frac{19}{3}} + \frac{5005}{6}\,a^9b^6x^6 + \frac{9009}{17}\,a^{10}b^5x^{\frac{17}{3}} + \frac{4095}{16}\,a^{11}b^4x^{\frac{16}{3}} + 91\,a^{12}b^3x^5 + \frac{45}{2}\,a^{13}b^2x^{\frac{14}{3}} + \frac{45}{13}\,a^{14}bx^{\frac{13}{3}} + \frac{1}{4}\,a^{15}x^4$$

input
```
integrate((a+b*x^(1/3))^15*x^3,x, algorithm="giac")
```

output
```
1/9*b^15*x^9 + 45/26*a*b^14*x^(26/3) + 63/5*a^2*b^13*x^(25/3) + 455/8*a^3*
b^12*x^8 + 4095/23*a^4*b^11*x^(23/3) + 819/2*a^5*b^10*x^(22/3) + 715*a^6*b
^9*x^7 + 3861/4*a^7*b^8*x^(20/3) + 19305/19*a^8*b^7*x^(19/3) + 5005/6*a^9*
b^6*x^6 + 9009/17*a^10*b^5*x^(17/3) + 4095/16*a^11*b^4*x^(16/3) + 91*a^12*
b^3*x^5 + 45/2*a^13*b^2*x^(14/3) + 45/13*a^14*b*x^(13/3) + 1/4*a^15*x^4
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx = \frac{a^{15}\,x^4}{4}+\frac{b^{15}\,x^9}{9}+\frac{45\,a^{14}\,b\,x^{13/3}}{13}+\frac{45\,a\,b^{14}\,x^{26/3}}{26}+91\,a^{12}\,b^3\,x^5+\frac{5005\,a^9\,b^6\,x^6}{6}+715\,a^6\,b^9\,x^7+\frac{455\,a^3\,b^{12}\,x^8}{8}+\frac{45\,a^{13}\,b^2\,x^{14/3}}{2}+\frac{4095\,a^{11}\,b^4\,x^{16/3}}{16}+\frac{9009\,a^{10}\,b^5\,x^{17/3}}{17}+\frac{19305\,a^8\,b^7\,x^{19/3}}{19}+\frac{3861\,a^7\,b^8\,x^{20/3}}{4}+\frac{819\,a^5\,b^{10}\,x^{22/3}}{2}+\frac{4095\,a^4\,b^{11}\,x^{23/3}}{23}+\frac{63\,a^2\,b^{13}\,x^{25/3}}{5}$$

input
```
int(x^3*(a + b*x^(1/3))^15,x)
```

output
```
(a^15*x^4)/4 + (b^15*x^9)/9 + (45*a^14*b*x^(13/3))/13 + (45*a*b^14*x^(26/3
))/26 + 91*a^12*b^3*x^5 + (5005*a^9*b^6*x^6)/6 + 715*a^6*b^9*x^7 + (455*a^
3*b^12*x^8)/8 + (45*a^13*b^2*x^(14/3))/2 + (4095*a^11*b^4*x^(16/3))/16 + (
9009*a^10*b^5*x^(17/3))/17 + (19305*a^8*b^7*x^(19/3))/19 + (3861*a^7*b^8*x
^(20/3))/4 + (819*a^5*b^10*x^(22/3))/2 + (4095*a^4*b^11*x^(23/3))/23 + (63
*a^2*b^13*x^(25/3))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^3\,dx$$
$$= \frac{x^4\left(1564547400x^{\frac{2}{3}}a^{13}b^2+36849692880x^{\frac{5}{3}}a^{10}b^5+67119083460x^{\frac{8}{3}}a^7b^8+12380331600x^{\frac{11}{3}}a^4b^{11}+120349\right.}{}$$

input
```
int((a+b*x^(1/3))^15*x^3,x)
```

output
```
(x**4*(1564547400*x**(2/3)*a**13*b**2 + 36849692880*x**(2/3)*a**10*b**5*x
+ 67119083460*x**(2/3)*a**7*b**8*x**2 + 12380331600*x**(2/3)*a**4*b**11*x*
*3 + 120349800*x**(2/3)*a*b**14*x**4 + 240699600*x**(1/3)*a**14*b + 177967
26675*x**(1/3)*a**11*b**4*x + 70651666800*x**(1/3)*a**8*b**7*x**2 + 284747
62680*x**(1/3)*a**5*b**10*x**3 + 876146544*x**(1/3)*a**2*b**13*x**4 + 1738
3860*a**15 + 6327725040*a**12*b**3*x + 58004146200*a**9*b**6*x**2 + 497178
39600*a**6*b**9*x**3 + 3954828150*a**3*b**12*x**4 + 7726160*b**15*x**5))/6
9535440
```

3.233 $\int \left(a+b\sqrt[3]{x}\right)^{15} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 183

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^2 \, dx = \frac{3a^8\left(a+b\sqrt[3]{x}\right)^{16}}{16b^9} - \frac{24a^7\left(a+b\sqrt[3]{x}\right)^{17}}{17b^9} + \frac{14a^6\left(a+b\sqrt[3]{x}\right)^{18}}{3b^9} - \frac{168a^5\left(a+b\sqrt[3]{x}\right)^{19}}{19b^9} + \frac{21a^4\left(a+b\sqrt[3]{x}\right)^{20}}{2b^9} - \frac{8a^3\left(a+b\sqrt[3]{x}\right)^{21}}{b^9} + \frac{42a^2\left(a+b\sqrt[3]{x}\right)^{22}}{11b^9} - \frac{24a\left(a+b\sqrt[3]{x}\right)^{23}}{23b^9} + \frac{\left(a+b\sqrt[3]{x}\right)^{24}}{8b^9}$$

output
```
3/16*a^8*(a+b*x^(1/3))^16/b^9-24/17*a^7*(a+b*x^(1/3))^17/b^9+14/3*a^6*(a+b
*x^(1/3))^18/b^9-168/19*a^5*(a+b*x^(1/3))^19/b^9+21/2*a^4*(a+b*x^(1/3))^20
/b^9-8*a^3*(a+b*x^(1/3))^21/b^9+42/11*a^2*(a+b*x^(1/3))^22/b^9-24/23*a*(a+
b*x^(1/3))^23/b^9+1/8*(a+b*x^(1/3))^24/b^9
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.04

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^2\,dx = \frac{1307504a^{15}x^3+17651304a^{14}bx^{10/3}+112326480a^{13}b^2x^{11/3}+446185740a^{12}b^3x^4+1235591280a^{11}b^4x^{13/3}}{}$$

input
```
Integrate[(a + b*x^(1/3))^15*x^2,x]
```

output
```
(1307504*a^15*x^3 + 17651304*a^14*b*x^(10/3) + 112326480*a^13*b^2*x^(11/3)
 + 446185740*a^12*b^3*x^4 + 1235591280*a^11*b^4*x^(13/3) + 2524136472*a^10
*b^5*x^(14/3) + 3926434512*a^9*b^6*x^5 + 4732755885*a^8*b^7*x^(16/3) + 445
4358480*a^7*b^8*x^(17/3) + 3272028760*a^6*b^9*x^6 + 1859890032*a^5*b^10*x^
(19/3) + 803134332*a^4*b^11*x^(20/3) + 254963280*a^3*b^12*x^7 + 56163240*a
^2*b^13*x^(22/3) + 7674480*a*b^14*x^(23/3) + 490314*b^15*x^8)/3922512
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt[3]{x}\right)^{15}\,dx$$

$$\Big\downarrow \ 798$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{15} x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^{23}}{b^8}-\frac{8a\left(a+b\sqrt[3]{x}\right)^{22}}{b^8}+\frac{28a^2\left(a+b\sqrt[3]{x}\right)^{21}}{b^8}-\frac{56a^3\left(a+b\sqrt[3]{x}\right)^{20}}{b^8}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^{19}}{b^8}-\frac{56a^5\left(a+\right.}{b^8}\right.$$

$$\downarrow \text{2009}$$

$$3\left(\frac{a^8\left(a+b\sqrt[3]{x}\right)^{16}}{16b^9}-\frac{8a^7\left(a+b\sqrt[3]{x}\right)^{17}}{17b^9}+\frac{14a^6\left(a+b\sqrt[3]{x}\right)^{18}}{9b^9}-\frac{56a^5\left(a+b\sqrt[3]{x}\right)^{19}}{19b^9}+\frac{7a^4\left(a+b\sqrt[3]{x}\right)^{20}}{2b^9}-\frac{8a^3\left(a+b\right.}{3b^9}\right.$$

input
```
Int[(a + b*x^(1/3))^15*x^2,x]
```

output
```
3*((a^8*(a + b*x^(1/3))^16)/(16*b^9) - (8*a^7*(a + b*x^(1/3))^17)/(17*b^9)
 + (14*a^6*(a + b*x^(1/3))^18)/(9*b^9) - (56*a^5*(a + b*x^(1/3))^19)/(19*b
^9) + (7*a^4*(a + b*x^(1/3))^20)/(2*b^9) - (8*a^3*(a + b*x^(1/3))^21)/(3*b
^9) + (14*a^2*(a + b*x^(1/3))^22)/(11*b^9) - (8*a*(a + b*x^(1/3))^23)/(23*
b^9) + (a + b*x^(1/3))^24/(24*b^9))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 25.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{x^8 b^{15}}{8}+\frac{45a\,b^{14}x^{\frac{23}{3}}}{23}+\frac{315a^2b^{13}x^{\frac{22}{3}}}{22}+65x^7b^{12}a^3+\frac{819a^4b^{11}x^{\frac{20}{3}}}{4}+\frac{9009a^5b^{10}x^{\frac{19}{3}}}{19}+\frac{5005x^6b^9a^6}{6}+$ 19
default	$\frac{x^8 b^{15}}{8}+\frac{45a\,b^{14}x^{\frac{23}{3}}}{23}+\frac{315a^2b^{13}x^{\frac{22}{3}}}{22}+65x^7b^{12}a^3+\frac{819a^4b^{11}x^{\frac{20}{3}}}{4}+\frac{9009a^5b^{10}x^{\frac{19}{3}}}{19}+\frac{5005x^6b^9a^6}{6}+$ 19
trager	$(3x^7b^{15}+1560a^3b^{12}x^6+3b^{15}x^6+20020x^5b^9a^6+1560a^3b^{12}x^5+3b^{15}x^5+24024x^4b^6a^9+20020b^9x^4a^6+1560a^3b^{12}x^4+3b$
orering	$-\frac{(-33624300b^{48}x^{16}-439429770a^3b^{45}x^{15}-2652694720a^6b^{42}x^{14}-9795979490a^9b^{39}x^{13}-24696833200a^{12}b^{36}x^{12}-44}{}$

input
```
int((a+b*x^(1/3))^15*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*b^15+45/23*a*b^14*x^(23/3)+315/22*a^2*b^13*x^(22/3)+65*x^7*b^12*a^
3+819/4*a^4*b^11*x^(20/3)+9009/19*a^5*b^10*x^(19/3)+5005/6*x^6*b^9*a^6+193
05/17*a^7*b^8*x^(17/3)+19305/16*a^8*b^7*x^(16/3)+1001*x^5*b^6*a^9+1287/2*a
^10*b^5*x^(14/3)+315*a^11*b^4*x^(13/3)+455/4*b^3*x^4*a^12+315/11*a^13*b^2*
x^(11/3)+9/2*a^14*b*x^(10/3)+1/3*a^15*x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^2\,dx$$
$$=\frac{1}{8}b^{15}x^8+65\,a^3b^{12}x^7+\frac{5005}{6}\,a^6b^9x^6+1001\,a^9b^6x^5+\frac{455}{4}\,a^{12}b^3x^4+\frac{1}{3}\,a^{15}x^3$$
$$+\frac{9}{17204}\left(3740\,ab^{14}x^7+391391\,a^4b^{11}x^6+2170740\,a^7b^8x^5+1230086\,a^{10}b^5x^4+54740\,a^{13}b^2x^3\right)x^{\frac{2}{3}}$$
$$+\frac{9}{3344}\left(5320\,a^2b^{13}x^7+176176\,a^5b^{10}x^6+448305\,a^8b^7x^5+117040\,a^{11}b^4x^4+1672\,a^{14}bx^3\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^15*x^2,x, algorithm="fricas")
```

output
```
1/8*b^15*x^8 + 65*a^3*b^12*x^7 + 5005/6*a^6*b^9*x^6 + 1001*a^9*b^6*x^5 + 4
55/4*a^12*b^3*x^4 + 1/3*a^15*x^3 + 9/17204*(3740*a*b^14*x^7 + 391391*a^4*b
^11*x^6 + 2170740*a^7*b^8*x^5 + 1230086*a^10*b^5*x^4 + 54740*a^13*b^2*x^3)
*x^(2/3) + 9/3344*(5320*a^2*b^13*x^7 + 176176*a^5*b^10*x^6 + 448305*a^8*b^
7*x^5 + 117040*a^11*b^4*x^4 + 1672*a^14*b*x^3)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.17

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^2\,dx = \frac{a^{15}x^3}{3}+\frac{9a^{14}bx^{\frac{10}{3}}}{2}+\frac{315a^{13}b^2x^{\frac{11}{3}}}{11}+\frac{455a^{12}b^3x^4}{4}+315a^{11}b^4x^{\frac{13}{3}}+\frac{1287a^{10}b^5x^{\frac{14}{3}}}{2}+1001a^9b^6x^5+\frac{19305a^8b^7x^{\frac{16}{3}}}{16}+\frac{19305a^7b^8x^{\frac{17}{3}}}{17}+\frac{5005a^6b^9x^6}{6}+\frac{9009a^5b^{10}x^{\frac{19}{3}}}{19}+\frac{819a^4b^{11}x^{\frac{20}{3}}}{4}+65a^3b^{12}x^7+\frac{315a^2b^{13}x^{\frac{22}{3}}}{22}+\frac{45ab^{14}x^{\frac{23}{3}}}{23}+\frac{b^{15}x^8}{8}$$

input
```
integrate((a+b*x**(1/3))**15*x**2,x)
```

output
```
a**15*x**3/3 + 9*a**14*b*x**(10/3)/2 + 315*a**13*b**2*x**(11/3)/11 + 455*a
**12*b**3*x**4/4 + 315*a**11*b**4*x**(13/3) + 1287*a**10*b**5*x**(14/3)/2
+ 1001*a**9*b**6*x**5 + 19305*a**8*b**7*x**(16/3)/16 + 19305*a**7*b**8*x**
(17/3)/17 + 5005*a**6*b**9*x**6/6 + 9009*a**5*b**10*x**(19/3)/19 + 819*a**
4*b**11*x**(20/3)/4 + 65*a**3*b**12*x**7 + 315*a**2*b**13*x**(22/3)/22 + 4
5*a*b**14*x**(23/3)/23 + b**15*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^2\,dx=\frac{\left(bx^{\frac{1}{3}}+a\right)^{24}}{8\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{23}a}{23\,b^9}+\frac{42\left(bx^{\frac{1}{3}}+a\right)^{22}a^2}{11\,b^9}-\frac{8\left(bx^{\frac{1}{3}}+a\right)^{21}a^3}{b^9}+\frac{21\left(bx^{\frac{1}{3}}+a\right)^{20}a^4}{2\,b^9}-\frac{168\left(bx^{\frac{1}{3}}+a\right)^{19}a^5}{19\,b^9}+\frac{14\left(bx^{\frac{1}{3}}+a\right)^{18}a^6}{3\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^{17}a^7}{17\,b^9}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^8}{16\,b^9}$$

input
```
integrate((a+b*x^(1/3))^15*x^2,x, algorithm="maxima")
```

output
```
1/8*(b*x^(1/3) + a)^24/b^9 - 24/23*(b*x^(1/3) + a)^23*a/b^9 + 42/11*(b*x^(
1/3) + a)^22*a^2/b^9 - 8*(b*x^(1/3) + a)^21*a^3/b^9 + 21/2*(b*x^(1/3) + a)
^20*a^4/b^9 - 168/19*(b*x^(1/3) + a)^19*a^5/b^9 + 14/3*(b*x^(1/3) + a)^18*
a^6/b^9 - 24/17*(b*x^(1/3) + a)^17*a^7/b^9 + 3/16*(b*x^(1/3) + a)^16*a^8/b
^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^{15}x^2\,dx=\frac{1}{8}\,b^{15}x^8+\frac{45}{23}\,ab^{14}x^{\frac{23}{3}}+\frac{315}{22}\,a^2b^{13}x^{\frac{22}{3}}+65\,a^3b^{12}x^7+\frac{819}{4}\,a^4b^{11}x^{\frac{20}{3}}+\frac{9009}{19}\,a^5b^{10}x^{\frac{19}{3}}+\frac{5005}{6}\,a^6b^9x^6+\frac{19305}{17}\,a^7b^8x^{\frac{17}{3}}+\frac{19305}{16}\,a^8b^7x^{\frac{16}{3}}+1001\,a^9b^6x^5+\frac{1287}{2}\,a^{10}b^5x^{\frac{14}{3}}+315\,a^{11}b^4x^{\frac{13}{3}}+\frac{455}{4}\,a^{12}b^3x^4+\frac{315}{11}\,a^{13}b^2x^{\frac{11}{3}}+\frac{9}{2}\,a^{14}bx^{\frac{10}{3}}+\frac{1}{3}\,a^{15}x^3$$

input
```
integrate((a+b*x^(1/3))^15*x^2,x, algorithm="giac")
```

output
```
1/8*b^15*x^8 + 45/23*a*b^14*x^(23/3) + 315/22*a^2*b^13*x^(22/3) + 65*a^3*b
^12*x^7 + 819/4*a^4*b^11*x^(20/3) + 9009/19*a^5*b^10*x^(19/3) + 5005/6*a^6
*b^9*x^6 + 19305/17*a^7*b^8*x^(17/3) + 19305/16*a^8*b^7*x^(16/3) + 1001*a^
9*b^6*x^5 + 1287/2*a^10*b^5*x^(14/3) + 315*a^11*b^4*x^(13/3) + 455/4*a^12*
b^3*x^4 + 315/11*a^13*b^2*x^(11/3) + 9/2*a^14*b*x^(10/3) + 1/3*a^15*x^3
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^2\,dx = \frac{a^{15}\,x^3}{3}+\frac{b^{15}\,x^8}{8}+\frac{9\,a^{14}\,b\,x^{10/3}}{2}+\frac{45\,a\,b^{14}\,x^{23/3}}{23}+\frac{455\,a^{12}\,b^3\,x^4}{4}+1001\,a^9\,b^6\,x^5+\frac{5005\,a^6\,b^9\,x^6}{6}+65\,a^3\,b^{12}\,x^7+\frac{315\,a^{13}\,b^2\,x^{11/3}}{11}+315\,a^{11}\,b^4\,x^{13/3}+\frac{1287\,a^{10}\,b^5\,x^{14/3}}{2}+\frac{19305\,a^8\,b^7\,x^{16/3}}{16}+\frac{19305\,a^7\,b^8\,x^{17/3}}{17}+\frac{9009\,a^5\,b^{10}\,x^{19/3}}{19}+\frac{819\,a^4\,b^{11}\,x^{20/3}}{4}+\frac{315\,a^2\,b^{13}\,x^{22/3}}{22}$$

input
```
int(x^2*(a + b*x^(1/3))^15,x)
```

output
```
(a^15*x^3)/3 + (b^15*x^8)/8 + (9*a^14*b*x^(10/3))/2 + (45*a*b^14*x^(23/3))
/23 + (455*a^12*b^3*x^4)/4 + 1001*a^9*b^6*x^5 + (5005*a^6*b^9*x^6)/6 + 65*
a^3*b^12*x^7 + (315*a^13*b^2*x^(11/3))/11 + 315*a^11*b^4*x^(13/3) + (1287*
a^10*b^5*x^(14/3))/2 + (19305*a^8*b^7*x^(16/3))/16 + (19305*a^7*b^8*x^(17/
3))/17 + (9009*a^5*b^10*x^(19/3))/19 + (819*a^4*b^11*x^(20/3))/4 + (315*a^
2*b^13*x^(22/3))/22
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x^2\,dx$$
$$= \frac{x^3\Big(112326480x^{\frac{2}{3}}a^{13}b^2+2524136472x^{\frac{5}{3}}a^{10}b^5+4454358480x^{\frac{8}{3}}a^7b^8+803134332x^{\frac{11}{3}}a^4b^{11}+7674480x^{\frac{14}{3}}a}{}$$

input
```
int((a+b*x^(1/3))^15*x^2,x)
```

output
```
(x**3*(112326480*x**(2/3)*a**13*b**2 + 2524136472*x**(2/3)*a**10*b**5*x +
4454358480*x**(2/3)*a**7*b**8*x**2 + 803134332*x**(2/3)*a**4*b**11*x**3 +
7674480*x**(2/3)*a*b**14*x**4 + 17651304*x**(1/3)*a**14*b + 1235591280*x**
(1/3)*a**11*b**4*x + 4732755885*x**(1/3)*a**8*b**7*x**2 + 1859890032*x**(1
/3)*a**5*b**10*x**3 + 56163240*x**(1/3)*a**2*b**13*x**4 + 1307504*a**15 +
446185740*a**12*b**3*x + 3926434512*a**9*b**6*x**2 + 3272028760*a**6*b**9*
x**3 + 254963280*a**3*b**12*x**4 + 490314*b**15*x**5))/3922512
```

3.234 $\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 122

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx = -\frac{3a^5\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}+\frac{15a^4\left(a+b\sqrt[3]{x}\right)^{17}}{17b^6}-\frac{5a^3\left(a+b\sqrt[3]{x}\right)^{18}}{3b^6}+\frac{30a^2\left(a+b\sqrt[3]{x}\right)^{19}}{19b^6}-\frac{3a\left(a+b\sqrt[3]{x}\right)^{20}}{4b^6}+\frac{\left(a+b\sqrt[3]{x}\right)^{21}}{7b^6}$$

output
```
-3/16*a^5*(a+b*x^(1/3))^16/b^6+15/17*a^4*(a+b*x^(1/3))^17/b^6-5/3*a^3*(a+b
*x^(1/3))^18/b^6+30/19*a^2*(a+b*x^(1/3))^19/b^6-3/4*a*(a+b*x^(1/3))^20/b^6
+1/7*(a+b*x^(1/3))^21/b^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.57

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx = \frac{54264a^{15}x^2+697680a^{14}bx^{7/3}+4273290a^{13}b^2x^{8/3}+16460080a^{12}b^3x^3+44442216a^{11}b^4x^{10/3}+88884432}{}$$

input
```
Integrate[(a + b*x^(1/3))^15*x,x]
```

output
```
(54264*a^15*x^2 + 697680*a^14*b*x^(7/3) + 4273290*a^13*b^2*x^(8/3) + 16460
080*a^12*b^3*x^3 + 44442216*a^11*b^4*x^(10/3) + 88884432*a^10*b^5*x^(11/3)
 + 135795660*a^9*b^6*x^4 + 161164080*a^8*b^7*x^(13/3) + 149652360*a^7*b^8*
x^(14/3) + 108636528*a^6*b^9*x^5 + 61108047*a^5*b^10*x^(16/3) + 26142480*a
^4*b^11*x^(17/3) + 8230040*a^3*b^12*x^6 + 1799280*a^2*b^13*x^(19/3) + 2441
88*a*b^14*x^(20/3) + 15504*b^15*x^7)/108528
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt[3]{x}\right)^{15}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{15}x^{5/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^{20}}{b^5}-\frac{5a\left(a+b\sqrt[3]{x}\right)^{19}}{b^5}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^{18}}{b^5}-\frac{10a^3\left(a+b\sqrt[3]{x}\right)^{17}}{b^5}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{16}}{b^5}-\frac{a^5\left(a+b\sqrt[3]{\ }\right.}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^5\left(a+b\sqrt[3]{x}\right)^{16}}{16b^6}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)^{17}}{17b^6}-\frac{5a^3\left(a+b\sqrt[3]{x}\right)^{18}}{9b^6}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^{19}}{19b^6}+\frac{\left(a+b\sqrt[3]{x}\right)^{21}}{21b^6}-\frac{a\left(a+b\sqrt[3]{x}\right)^{2}}{4b^6}\right.$$

input
```
Int[(a + b*x^(1/3))^15*x,x]
```

output
```
3*(-1/16*(a^5*(a + b*x^(1/3))^16)/b^6 + (5*a^4*(a + b*x^(1/3))^17)/(17*b^6
) - (5*a^3*(a + b*x^(1/3))^18)/(9*b^6) + (10*a^2*(a + b*x^(1/3))^19)/(19*b
^6) - (a*(a + b*x^(1/3))^20)/(4*b^6) + (a + b*x^(1/3))^21/(21*b^6))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{x^7 b^{15}}{7}+\frac{9 a\, b^{14} x^{\frac{20}{3}}}{4}+\frac{315 a^2 b^{13} x^{\frac{19}{3}}}{19}+\frac{455 a^3 b^{12} x^6}{6}+\frac{4095 a^4 b^{11} x^{\frac{17}{3}}}{17}+\frac{9009 a^5 b^{10} x^{\frac{16}{3}}}{16}+1001 x^5 b^9 a^6+$
default	$\frac{x^7 b^{15}}{7}+\frac{9 a\, b^{14} x^{\frac{20}{3}}}{4}+\frac{315 a^2 b^{13} x^{\frac{19}{3}}}{19}+\frac{455 a^3 b^{12} x^6}{6}+\frac{4095 a^4 b^{11} x^{\frac{17}{3}}}{17}+\frac{9009 a^5 b^{10} x^{\frac{16}{3}}}{16}+1001 x^5 b^9 a^6+$
trager	$\frac{(12 b^{15} x^6+6370 a^3 b^{12} x^5+12 b^{15} x^5+84084 b^9 x^4 a^6+6370 a^3 b^{12} x^4+12 b^{15} x^4+105105 a^9 b^6 x^3+84084 a^6 b^9 x^3+6370 a^3 b^{12} x}{}$
orering	$-\frac{(-7339920 b^{45} x^{15}-95678765 a^3 b^{42} x^{14}-575889475 a^6 b^{39} x^{13}-2119459160 a^9 b^{36} x^{12}-5322210530 a^{12} b^{33} x^{11}-963173}{}$

input
```
int((a+b*x^(1/3))^15*x,x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*b^15+9/4*a*b^14*x^(20/3)+315/19*a^2*b^13*x^(19/3)+455/6*a^3*b^12*x
^6+4095/17*a^4*b^11*x^(17/3)+9009/16*a^5*b^10*x^(16/3)+1001*x^5*b^9*a^6+19
305/14*a^7*b^8*x^(14/3)+1485*a^8*b^7*x^(13/3)+5005/4*x^4*b^6*a^9+819*a^10*
b^5*x^(11/3)+819/2*a^11*b^4*x^(10/3)+455/3*a^12*b^3*x^3+315/8*a^13*b^2*x^(
8/3)+45/7*a^14*b*x^(7/3)+1/2*a^15*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.47

$$\begin{aligned}
&\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx \\
&= \frac{1}{7}\,b^{15}x^7 + \frac{455}{6}\,a^3b^{12}x^6 + 1001\,a^6b^9x^5 + \frac{5005}{4}\,a^9b^6x^4 + \frac{455}{3}\,a^{12}b^3x^3 + \frac{1}{2}\,a^{15}x^2 \\
&\quad + \frac{9}{952}\left(238\,ab^{14}x^6 + 25480\,a^4b^{11}x^5 + 145860\,a^7b^8x^4 + 86632\,a^{10}b^5x^3 + 4165\,a^{13}b^2x^2\right)x^{\frac{2}{3}} \\
&\quad + \frac{9}{2128}\left(3920\,a^2b^{13}x^6 + 133133\,a^5b^{10}x^5 + 351120\,a^8b^7x^4 + 96824\,a^{11}b^4x^3 + 1520\,a^{14}bx^2\right)x^{\frac{1}{3}}
\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^15*x,x, algorithm="fricas")
```

output
```
1/7*b^15*x^7 + 455/6*a^3*b^12*x^6 + 1001*a^6*b^9*x^5 + 5005/4*a^9*b^6*x^4
+ 455/3*a^12*b^3*x^3 + 1/2*a^15*x^2 + 9/952*(238*a*b^14*x^6 + 25480*a^4*b^
11*x^5 + 145860*a^7*b^8*x^4 + 86632*a^10*b^5*x^3 + 4165*a^13*b^2*x^2)*x^(2
/3) + 9/2128*(3920*a^2*b^13*x^6 + 133133*a^5*b^10*x^5 + 351120*a^8*b^7*x^4
 + 96824*a^11*b^4*x^3 + 1520*a^14*b*x^2)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.75

$$\begin{aligned}
\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx &= \frac{a^{15}x^2}{2} + \frac{45a^{14}bx^{\frac{7}{3}}}{7} + \frac{315a^{13}b^2x^{\frac{8}{3}}}{8} + \frac{455a^{12}b^3x^3}{3} \\
&\quad + \frac{819a^{11}b^4x^{\frac{10}{3}}}{2} + 819a^{10}b^5x^{\frac{11}{3}} + \frac{5005a^9b^6x^4}{4} + 1485a^8b^7x^{\frac{13}{3}} \\
&\quad + \frac{19305a^7b^8x^{\frac{14}{3}}}{14} + 1001a^6b^9x^5 + \frac{9009a^5b^{10}x^{\frac{16}{3}}}{16} + \frac{4095a^4b^{11}x^{\frac{17}{3}}}{17} \\
&\quad + \frac{455a^3b^{12}x^6}{6} + \frac{315a^2b^{13}x^{\frac{19}{3}}}{19} + \frac{9ab^{14}x^{\frac{20}{3}}}{4} + \frac{b^{15}x^7}{7}
\end{aligned}$$

input
```
integrate((a+b*x**(1/3))**15*x,x)
```

output
```
a**15*x**2/2 + 45*a**14*b*x**(7/3)/7 + 315*a**13*b**2*x**(8/3)/8 + 455*a**
12*b**3*x**3/3 + 819*a**11*b**4*x**(10/3)/2 + 819*a**10*b**5*x**(11/3) + 5
005*a**9*b**6*x**4/4 + 1485*a**8*b**7*x**(13/3) + 19305*a**7*b**8*x**(14/3
)/14 + 1001*a**6*b**9*x**5 + 9009*a**5*b**10*x**(16/3)/16 + 4095*a**4*b**1
1*x**(17/3)/17 + 455*a**3*b**12*x**6/6 + 315*a**2*b**13*x**(19/3)/19 + 9*a
*b**14*x**(20/3)/4 + b**15*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx = \frac{\left(bx^{\frac{1}{3}}+a\right)^{21}}{7\,b^6} - \frac{3\left(bx^{\frac{1}{3}}+a\right)^{20}a}{4\,b^6} + \frac{30\left(bx^{\frac{1}{3}}+a\right)^{19}a^2}{19\,b^6} - \frac{5\left(bx^{\frac{1}{3}}+a\right)^{18}a^3}{3\,b^6} + \frac{15\left(bx^{\frac{1}{3}}+a\right)^{17}a^4}{17\,b^6} - \frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^5}{16\,b^6}$$

input
```
integrate((a+b*x^(1/3))^15*x,x, algorithm="maxima")
```

output
```
1/7*(b*x^(1/3) + a)^21/b^6 - 3/4*(b*x^(1/3) + a)^20*a/b^6 + 30/19*(b*x^(1/
3) + a)^19*a^2/b^6 - 5/3*(b*x^(1/3) + a)^18*a^3/b^6 + 15/17*(b*x^(1/3) + a
)^17*a^4/b^6 - 3/16*(b*x^(1/3) + a)^16*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx = \frac{1}{7}\,b^{15}x^7 + \frac{9}{4}\,ab^{14}x^{\frac{20}{3}} + \frac{315}{19}\,a^2b^{13}x^{\frac{19}{3}} + \frac{455}{6}\,a^3b^{12}x^6 + \frac{4095}{17}\,a^4b^{11}x^{\frac{17}{3}} + \frac{9009}{16}\,a^5b^{10}x^{\frac{16}{3}} + 1001\,a^6b^9x^5 + \frac{19305}{14}\,a^7b^8x^{\frac{14}{3}} + 1485\,a^8b^7x^{\frac{13}{3}} + \frac{5005}{4}\,a^9b^6x^4 + 819\,a^{10}b^5x^{\frac{11}{3}} + \frac{819}{2}\,a^{11}b^4x^{\frac{10}{3}} + \frac{455}{3}\,a^{12}b^3x^3 + \frac{315}{8}\,a^{13}b^2x^{\frac{8}{3}} + \frac{45}{7}\,a^{14}bx^{\frac{7}{3}} + \frac{1}{2}\,a^{15}x^2$$

input
```
integrate((a+b*x^(1/3))^15*x,x, algorithm="giac")
```

output
```
1/7*b^15*x^7 + 9/4*a*b^14*x^(20/3) + 315/19*a^2*b^13*x^(19/3) + 455/6*a^3*
b^12*x^6 + 4095/17*a^4*b^11*x^(17/3) + 9009/16*a^5*b^10*x^(16/3) + 1001*a^
6*b^9*x^5 + 19305/14*a^7*b^8*x^(14/3) + 1485*a^8*b^7*x^(13/3) + 5005/4*a^9
*b^6*x^4 + 819*a^10*b^5*x^(11/3) + 819/2*a^11*b^4*x^(10/3) + 455/3*a^12*b^
3*x^3 + 315/8*a^13*b^2*x^(8/3) + 45/7*a^14*b*x^(7/3) + 1/2*a^15*x^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx = \frac{a^{15}\,x^2}{2}+\frac{b^{15}\,x^7}{7}+\frac{45\,a^{14}\,b\,x^{7/3}}{7}+\frac{9\,a\,b^{14}\,x^{20/3}}{4}+\frac{455\,a^{12}\,b^3\,x^3}{3}+\frac{5005\,a^9\,b^6\,x^4}{4}+1001\,a^6\,b^9\,x^5+\frac{455\,a^3\,b^{12}\,x^6}{6}+\frac{315\,a^{13}\,b^2\,x^{8/3}}{8}+\frac{819\,a^{11}\,b^4\,x^{10/3}}{2}+819\,a^{10}\,b^5\,x^{11/3}+1485\,a^8\,b^7\,x^{13/3}+\frac{19305\,a^7\,b^8\,x^{14/3}}{14}+\frac{9009\,a^5\,b^{10}\,x^{16/3}}{16}+\frac{4095\,a^4\,b^{11}\,x^{17/3}}{17}+\frac{315\,a^2\,b^{13}\,x^{19/3}}{19}$$

input
```
int(x*(a + b*x^(1/3))^15,x)
```

output
```
(a^15*x^2)/2 + (b^15*x^7)/7 + (45*a^14*b*x^(7/3))/7 + (9*a*b^14*x^(20/3))/
4 + (455*a^12*b^3*x^3)/3 + (5005*a^9*b^6*x^4)/4 + 1001*a^6*b^9*x^5 + (455*
a^3*b^12*x^6)/6 + (315*a^13*b^2*x^(8/3))/8 + (819*a^11*b^4*x^(10/3))/2 + 8
19*a^10*b^5*x^(11/3) + 1485*a^8*b^7*x^(13/3) + (19305*a^7*b^8*x^(14/3))/14
 + (9009*a^5*b^10*x^(16/3))/16 + (4095*a^4*b^11*x^(17/3))/17 + (315*a^2*b^
13*x^(19/3))/19
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.37

$$\int \left(a+b\sqrt[3]{x}\right)^{15} x\,dx$$
$$= \frac{x^2\left(4273290x^{\frac{2}{3}}a^{13}b^2+88884432x^{\frac{5}{3}}a^{10}b^5+149652360x^{\frac{8}{3}}a^7b^8+26142480x^{\frac{11}{3}}a^4b^{11}+244188x^{\frac{14}{3}}a\,b^{14}+69\right.}{}$$

input
```
int((a+b*x^(1/3))^15*x,x)
```

output
```
(x**2*(4273290*x**(2/3)*a**13*b**2 + 88884432*x**(2/3)*a**10*b**5*x + 1496
52360*x**(2/3)*a**7*b**8*x**2 + 26142480*x**(2/3)*a**4*b**11*x**3 + 244188
*x**(2/3)*a*b**14*x**4 + 697680*x**(1/3)*a**14*b + 44442216*x**(1/3)*a**11
*b**4*x + 161164080*x**(1/3)*a**8*b**7*x**2 + 61108047*x**(1/3)*a**5*b**10
*x**3 + 1799280*x**(1/3)*a**2*b**13*x**4 + 54264*a**15 + 16460080*a**12*b*
*3*x + 135795660*a**9*b**6*x**2 + 108636528*a**6*b**9*x**3 + 8230040*a**3*
b**12*x**4 + 15504*b**15*x**5))/108528
```

3.235 $\int \left(a+b\sqrt[3]{x}\right)^{15} dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \left(a+b\sqrt[3]{x}\right)^{15} dx=\frac{3a^2\left(a+b\sqrt[3]{x}\right)^{16}}{16b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^{17}}{17b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^{18}}{6b^3}$$

output
```
3/16*a^2*(a+b*x^(1/3))^16/b^3-6/17*a*(a+b*x^(1/3))^17/b^3+1/6*(a+b*x^(1/3)
)^18/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(59) = 118.

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.20

$$\int \left(a+b\sqrt[3]{x}\right)^{15} dx=\frac{1}{816}\left(816a^{15}x+9180a^{14}bx^{4/3}+51408a^{13}b^2x^{5/3}+185640a^{12}b^3x^2+477360a^{11}b^4x^{7/3}+918918a^{10}b^5x^{8/3}+1361360a^9b^6x^3+1575288a^8b^7x^{10/3}+1432080a^7b^8x^{11/3}+1021020a^6b^9x^4+565488a^5b^{10}x^{13/3}+238680a^4b^{11}x^{14/3}+74256a^3b^{12}x^5+16065a^2b^{13}x^{16/3}+2160ab^{14}x^{17/3}+136b^{15}x^6\right)$$

input
```
Integrate[(a + b*x^(1/3))^15,x]
```

output
```
(816*a^15*x + 9180*a^14*b*x^(4/3) + 51408*a^13*b^2*x^(5/3) + 185640*a^12*b
^3*x^2 + 477360*a^11*b^4*x^(7/3) + 918918*a^10*b^5*x^(8/3) + 1361360*a^9*b
^6*x^3 + 1575288*a^8*b^7*x^(10/3) + 1432080*a^7*b^8*x^(11/3) + 1021020*a^6
*b^9*x^4 + 565488*a^5*b^10*x^(13/3) + 238680*a^4*b^11*x^(14/3) + 74256*a^3
*b^12*x^5 + 16065*a^2*b^13*x^(16/3) + 2160*a*b^14*x^(17/3) + 136*b^15*x^6)
/816
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt[3]{x}\right)^{15}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \left(a+b\sqrt[3]{x}\right)^{15}x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{\left(a+b\sqrt[3]{x}\right)^{17}}{b^2}-\frac{2a\left(a+b\sqrt[3]{x}\right)^{16}}{b^2}+\frac{a^2\left(a+b\sqrt[3]{x}\right)^{15}}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2\left(a+b\sqrt[3]{x}\right)^{16}}{16b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^{18}}{18b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^{17}}{17b^3}\right)$$

input
```
Int[(a + b*x^(1/3))^15,x]
```

output
```
3*((a^2*(a + b*x^(1/3))^16)/(16*b^3) - (2*a*(a + b*x^(1/3))^17)/(17*b^3) +
 (a + b*x^(1/3))^18/(18*b^3))
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(47) = 94$.

Time = 25.39 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.80

method	result
derivativedivides	$\frac{b^{15}x^6}{6}+\frac{45a\,b^{14}x^{\frac{17}{3}}}{17}+\frac{315a^2b^{13}x^{\frac{16}{3}}}{16}+91a^3b^{12}x^5+\frac{585a^4b^{11}x^{\frac{14}{3}}}{2}+693a^5b^{10}x^{\frac{13}{3}}+\frac{5005b^9x^4a^6}{4}+1$
default	$\frac{b^{15}x^6}{6}+\frac{45a\,b^{14}x^{\frac{17}{3}}}{17}+\frac{315a^2b^{13}x^{\frac{16}{3}}}{16}+91a^3b^{12}x^5+\frac{585a^4b^{11}x^{\frac{14}{3}}}{2}+693a^5b^{10}x^{\frac{13}{3}}+\frac{5005b^9x^4a^6}{4}+1$
trager	$(2b^{15}x^5+1092a^3b^{12}x^4+2b^{15}x^4+15015a^6b^9x^3+1092a^3b^{12}x^3+2b^{15}x^3+20020a^9b^6x^2+15015a^6b^9x^2+1092a^3b^{12}x^2+2b$
orering	$-\frac{(-361b^{42}x^{14}-4694a^3b^{39}x^{13}-28171a^6b^{36}x^{12}-103324a^9b^{33}x^{11}-258401a^{12}b^{30}x^{10}-465322a^{15}b^{27}x^9-347490a^{18}b^2}{}$

input
```
int((a+b*x^(1/3))^15,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^15*x^6+45/17*a*b^14*x^(17/3)+315/16*a^2*b^13*x^(16/3)+91*a^3*b^12*x^
5+585/2*a^4*b^11*x^(14/3)+693*a^5*b^10*x^(13/3)+5005/4*b^9*x^4*a^6+1755*a^
7*b^8*x^(11/3)+3861/2*a^8*b^7*x^(10/3)+5005/3*a^9*b^6*x^3+9009/8*a^10*b^5*
x^(8/3)+585*a^11*b^4*x^(7/3)+455/2*a^12*b^3*x^2+63*a^13*b^2*x^(5/3)+45/4*a
^14*b*x^(4/3)+a^15*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(47) = 94$.

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.92

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{15}\,dx &= \frac{1}{6}\,b^{15}x^6+91\,a^3b^{12}x^5+\frac{5005}{4}\,a^6b^9x^4+\frac{5005}{3}\,a^9b^6x^3+\frac{455}{2}\,a^{12}b^3x^2+a^{15}x\\&\quad+\frac{9}{136}\left(40\,ab^{14}x^5+4420\,a^4b^{11}x^4+26520\,a^7b^8x^3+17017\,a^{10}b^5x^2+952\,a^{13}b^2x\right)x^{\frac{2}{3}}\\&\quad+\frac{9}{16}\left(35\,a^2b^{13}x^5+1232\,a^5b^{10}x^4+3432\,a^8b^7x^3+1040\,a^{11}b^4x^2+20\,a^{14}bx\right)x^{\frac{1}{3}}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^15,x, algorithm="fricas")
```

output
```
1/6*b^15*x^6 + 91*a^3*b^12*x^5 + 5005/4*a^6*b^9*x^4 + 5005/3*a^9*b^6*x^3 +
 455/2*a^12*b^3*x^2 + a^15*x + 9/136*(40*a*b^14*x^5 + 4420*a^4*b^11*x^4 +
26520*a^7*b^8*x^3 + 17017*a^10*b^5*x^2 + 952*a^13*b^2*x)*x^(2/3) + 9/16*(3
5*a^2*b^13*x^5 + 1232*a^5*b^10*x^4 + 3432*a^8*b^7*x^3 + 1040*a^11*b^4*x^2
+ 20*a^14*b*x)*x^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(53) = 106$.

Time = 0.80 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.51

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{15}\,dx &= a^{15}x+\frac{45a^{14}bx^{\frac{4}{3}}}{4}+63a^{13}b^2x^{\frac{5}{3}}+\frac{455a^{12}b^3x^2}{2}+585a^{11}b^4x^{\frac{7}{3}}\\&\quad+\frac{9009a^{10}b^5x^{\frac{8}{3}}}{8}+\frac{5005a^9b^6x^3}{3}+\frac{3861a^8b^7x^{\frac{10}{3}}}{2}\\&\quad+1755a^7b^8x^{\frac{11}{3}}+\frac{5005a^6b^9x^4}{4}+693a^5b^{10}x^{\frac{13}{3}}+\frac{585a^4b^{11}x^{\frac{14}{3}}}{2}\\&\quad+91a^3b^{12}x^5+\frac{315a^2b^{13}x^{\frac{16}{3}}}{16}+\frac{45ab^{14}x^{\frac{17}{3}}}{17}+\frac{b^{15}x^6}{6}\end{aligned}$$

input
```
integrate((a+b*x**(1/3))**15,x)
```

output
```
a**15*x + 45*a**14*b*x**(4/3)/4 + 63*a**13*b**2*x**(5/3) + 455*a**12*b**3*
x**2/2 + 585*a**11*b**4*x**(7/3) + 9009*a**10*b**5*x**(8/3)/8 + 5005*a**9*
b**6*x**3/3 + 3861*a**8*b**7*x**(10/3)/2 + 1755*a**7*b**8*x**(11/3) + 5005
*a**6*b**9*x**4/4 + 693*a**5*b**10*x**(13/3) + 585*a**4*b**11*x**(14/3)/2
+ 91*a**3*b**12*x**5 + 315*a**2*b**13*x**(16/3)/16 + 45*a*b**14*x**(17/3)/
17 + b**15*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt[3]{x}\right)^{15}\,dx=\frac{\left(bx^{\frac{1}{3}}+a\right)^{18}}{6\,b^3}-\frac{6\left(bx^{\frac{1}{3}}+a\right)^{17}a}{17\,b^3}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^{16}a^2}{16\,b^3}$$

input
```
integrate((a+b*x^(1/3))^15,x, algorithm="maxima")
```

output
```
1/6*(b*x^(1/3) + a)^18/b^3 - 6/17*(b*x^(1/3) + a)^17*a/b^3 + 3/16*(b*x^(1/
3) + a)^16*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(47) = 94.

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.78

$$\begin{aligned}\int \left(a+b\sqrt[3]{x}\right)^{15}\,dx=&\ \frac{1}{6}\,b^{15}x^6+\frac{45}{17}\,ab^{14}x^{\frac{17}{3}}+\frac{315}{16}\,a^2b^{13}x^{\frac{16}{3}}+91\,a^3b^{12}x^5\\&+\frac{585}{2}\,a^4b^{11}x^{\frac{14}{3}}+693\,a^5b^{10}x^{\frac{13}{3}}+\frac{5005}{4}\,a^6b^9x^4+1755\,a^7b^8x^{\frac{11}{3}}\\&+\frac{3861}{2}\,a^8b^7x^{\frac{10}{3}}+\frac{5005}{3}\,a^9b^6x^3+\frac{9009}{8}\,a^{10}b^5x^{\frac{8}{3}}+585\,a^{11}b^4x^{\frac{7}{3}}\\&+\frac{455}{2}\,a^{12}b^3x^2+63\,a^{13}b^2x^{\frac{5}{3}}+\frac{45}{4}\,a^{14}bx^{\frac{4}{3}}+a^{15}x\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^15,x, algorithm="giac")
```

output
```
1/6*b^15*x^6 + 45/17*a*b^14*x^(17/3) + 315/16*a^2*b^13*x^(16/3) + 91*a^3*b
^12*x^5 + 585/2*a^4*b^11*x^(14/3) + 693*a^5*b^10*x^(13/3) + 5005/4*a^6*b^9
*x^4 + 1755*a^7*b^8*x^(11/3) + 3861/2*a^8*b^7*x^(10/3) + 5005/3*a^9*b^6*x^
3 + 9009/8*a^10*b^5*x^(8/3) + 585*a^11*b^4*x^(7/3) + 455/2*a^12*b^3*x^2 +
63*a^13*b^2*x^(5/3) + 45/4*a^14*b*x^(4/3) + a^15*x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.78

$$\int \left(a+b\sqrt[3]{x}\right)^{15}\,dx = a^{15}\,x+\frac{b^{15}\,x^6}{6}+\frac{45\,a^{14}\,b\,x^{4/3}}{4}+\frac{45\,a\,b^{14}\,x^{17/3}}{17}+\frac{455\,a^{12}\,b^3\,x^2}{2}+\frac{5005\,a^9\,b^6\,x^3}{3}+\frac{5005\,a^6\,b^9\,x^4}{4}+91\,a^3\,b^{12}\,x^5+63\,a^{13}\,b^2\,x^{5/3}+585\,a^{11}\,b^4\,x^{7/3}+\frac{9009\,a^{10}\,b^5\,x^{8/3}}{8}+\frac{3861\,a^8\,b^7\,x^{10/3}}{2}+1755\,a^7\,b^8\,x^{11/3}+693\,a^5\,b^{10}\,x^{13/3}+\frac{585\,a^4\,b^{11}\,x^{14/3}}{2}+\frac{315\,a^2\,b^{13}\,x^{16/3}}{16}$$

input
```
int((a + b*x^(1/3))^15,x)
```

output
```
a^15*x + (b^15*x^6)/6 + (45*a^14*b*x^(4/3))/4 + (45*a*b^14*x^(17/3))/17 +
(455*a^12*b^3*x^2)/2 + (5005*a^9*b^6*x^3)/3 + (5005*a^6*b^9*x^4)/4 + 91*a^
3*b^12*x^5 + 63*a^13*b^2*x^(5/3) + 585*a^11*b^4*x^(7/3) + (9009*a^10*b^5*x
^(8/3))/8 + (3861*a^8*b^7*x^(10/3))/2 + 1755*a^7*b^8*x^(11/3) + 693*a^5*b^
10*x^(13/3) + (585*a^4*b^11*x^(14/3))/2 + (315*a^2*b^13*x^(16/3))/16
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.80

$$\int \left(a+b\sqrt[3]{x}\right)^{15}\,dx = \frac{x\left(51408x^{\frac{2}{3}}a^{13}b^2+918918x^{\frac{5}{3}}a^{10}b^5+1432080x^{\frac{8}{3}}a^7b^8+238680x^{\frac{11}{3}}a^4b^{11}+2160x^{\frac{14}{3}}a\,b^{14}+9180x^{\frac{1}{3}}a^{14}b+\cdots\right.}{\cdots}$$

input
```
int((a+b*x^(1/3))^15,x)
```

output
```
(x*(51408*x**(2/3)*a**13*b**2 + 918918*x**(2/3)*a**10*b**5*x + 1432080*x**
(2/3)*a**7*b**8*x**2 + 238680*x**(2/3)*a**4*b**11*x**3 + 2160*x**(2/3)*a*b
**14*x**4 + 9180*x**(1/3)*a**14*b + 477360*x**(1/3)*a**11*b**4*x + 1575288
*x**(1/3)*a**8*b**7*x**2 + 565488*x**(1/3)*a**5*b**10*x**3 + 16065*x**(1/3
)*a**2*b**13*x**4 + 816*a**15 + 185640*a**12*b**3*x + 1361360*a**9*b**6*x*
*2 + 1021020*a**6*b**9*x**3 + 74256*a**3*b**12*x**4 + 136*b**15*x**5))/816
```

3.236 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 209

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx = 45a^{14}b\sqrt[3]{x} + \frac{315}{2}a^{13}b^2x^{2/3} + 455a^{12}b^3x + \frac{4095}{4}a^{11}b^4x^{4/3} + \frac{9009}{5}a^{10}b^5x^{5/3} + \frac{5005}{2}a^9b^6x^2 + \frac{19305}{7}a^8b^7x^{7/3} + \frac{19305}{8}a^7b^8x^{8/3} + \frac{5005}{3}a^6b^9x^3 + \frac{9009}{10}a^5b^{10}x^{10/3} + \frac{4095}{11}a^4b^{11}x^{11/3} + \frac{455}{4}a^3b^{12}x^4 + \frac{315}{13}a^2b^{13}x^{13/3} + \frac{45}{14}ab^{14}x^{14/3} + \frac{b^{15}x^5}{5} + a^{15}\log(x)$$

output
```
45*a^14*b*x^(1/3)+315/2*a^13*b^2*x^(2/3)+455*a^12*b^3*x+4095/4*a^11*b^4*x^
(4/3)+9009/5*a^10*b^5*x^(5/3)+5005/2*a^9*b^6*x^2+19305/7*a^8*b^7*x^(7/3)+1
9305/8*a^7*b^8*x^(8/3)+5005/3*a^6*b^9*x^3+9009/10*a^5*b^10*x^(10/3)+4095/1
1*a^4*b^11*x^(11/3)+455/4*a^3*b^12*x^4+315/13*a^2*b^13*x^(13/3)+45/14*a*b^
14*x^(14/3)+1/5*b^15*x^5+a^15*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx$$
$$= \frac{5405400a^{14}b\sqrt[3]{x}+18918900a^{13}b^2x^{2/3}+54654600a^{12}b^3x+122972850a^{11}b^4x^{4/3}+216432216a^{10}b^5x^{5/3}+\cdots}{}$$
$$+3a^{15}\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^15/x,x]
```

output
```
(5405400*a^14*b*x^(1/3) + 18918900*a^13*b^2*x^(2/3) + 54654600*a^12*b^3*x
+ 122972850*a^11*b^4*x^(4/3) + 216432216*a^10*b^5*x^(5/3) + 300600300*a^9*
b^6*x^2 + 331273800*a^8*b^7*x^(7/3) + 289864575*a^7*b^8*x^(8/3) + 20040020
0*a^6*b^9*x^3 + 108216108*a^5*b^10*x^(10/3) + 44717400*a^4*b^11*x^(11/3) +
 13663650*a^3*b^12*x^4 + 2910600*a^2*b^13*x^(13/3) + 386100*a*b^14*x^(14/3
) + 24024*b^15*x^5)/120120 + 3*a^15*Log[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx$$
$$\Big\downarrow \text{ 798}$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{\sqrt[3]{x}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 49}$$

$$3\int \left(\frac{a^{15}}{\sqrt[3]{x}}+15ba^{14}+105b^2\sqrt[3]{x}a^{13}+455b^3x^{2/3}a^{12}+1365b^4xa^{11}+3003b^5x^{4/3}a^{10}+5005b^6x^{5/3}a^9+6435b^7x^2a^8$$

$$\big\downarrow 2009$$

$$3\left(a^{15}\log\left(\sqrt[3]{x}\right)+15a^{14}b\sqrt[3]{x}+\frac{105}{2}a^{13}b^2x^{2/3}+\frac{455}{3}a^{12}b^3x+\frac{1365}{4}a^{11}b^4x^{4/3}+\frac{3003}{5}a^{10}b^5x^{5/3}+\frac{5005}{6}a^9b^6x^2+\frac{6}{-}\right.$$

input
```
Int[(a + b*x^(1/3))^15/x,x]
```

output
```
3*(15*a^14*b*x^(1/3) + (105*a^13*b^2*x^(2/3))/2 + (455*a^12*b^3*x)/3 + (13
65*a^11*b^4*x^(4/3))/4 + (3003*a^10*b^5*x^(5/3))/5 + (5005*a^9*b^6*x^2)/6
+ (6435*a^8*b^7*x^(7/3))/7 + (6435*a^7*b^8*x^(8/3))/8 + (5005*a^6*b^9*x^3)
/9 + (3003*a^5*b^10*x^(10/3))/10 + (1365*a^4*b^11*x^(11/3))/11 + (455*a^3*
b^12*x^4)/12 + (105*a^2*b^13*x^(13/3))/13 + (15*a*b^14*x^(14/3))/14 + (b^1
5*x^5)/15 + a^15*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.74 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

method	result
derivativedivides	$45a^{14}b\,x^{\frac{1}{3}}+\frac{315a^{13}b^{2}x^{\frac{2}{3}}}{2}+455a^{12}b^{3}x+\frac{4095a^{11}b^{4}x^{\frac{4}{3}}}{4}+\frac{9009a^{10}b^{5}x^{\frac{5}{3}}}{5}+\frac{5005a^{9}b^{6}x^{2}}{2}+\frac{19305a^{8}b^{7}x^{\frac{7}{3}}}{7}$
default	$45a^{14}b\,x^{\frac{1}{3}}+\frac{315a^{13}b^{2}x^{\frac{2}{3}}}{2}+455a^{12}b^{3}x+\frac{4095a^{11}b^{4}x^{\frac{4}{3}}}{4}+\frac{9009a^{10}b^{5}x^{\frac{5}{3}}}{5}+\frac{5005a^{9}b^{6}x^{2}}{2}+\frac{19305a^{8}b^{7}x^{\frac{7}{3}}}{7}$
trager	$\frac{b^{3}(12x^{4}b^{12}+6825x^{3}b^{9}a^{3}+12x^{3}b^{12}+100100x^{2}b^{6}a^{6}+6825x^{2}b^{9}a^{3}+12b^{12}x^{2}+150150x\,b^{3}a^{9}+100100x\,b^{6}a^{6}+6825x\,b^{9}a^{3}}{60}$

input
```
int((a+b*x^(1/3))^15/x,x,method=_RETURNVERBOSE)
```

output
```
45*a^14*b*x^(1/3)+315/2*a^13*b^2*x^(2/3)+455*a^12*b^3*x+4095/4*a^11*b^4*x^
(4/3)+9009/5*a^10*b^5*x^(5/3)+5005/2*a^9*b^6*x^2+19305/7*a^8*b^7*x^(7/3)+1
9305/8*a^7*b^8*x^(8/3)+5005/3*a^6*b^9*x^3+9009/10*a^5*b^10*x^(10/3)+4095/1
1*a^4*b^11*x^(11/3)+455/4*a^3*b^12*x^4+315/13*a^2*b^13*x^(13/3)+45/14*a*b^
14*x^(14/3)+1/5*b^15*x^5+a^15*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx$$
$$=\frac{1}{5}\,b^{15}x^{5}+\frac{455}{4}\,a^{3}b^{12}x^{4}+\frac{5005}{3}\,a^{6}b^{9}x^{3}+\frac{5005}{2}\,a^{9}b^{6}x^{2}+455\,a^{12}b^{3}x+3\,a^{15}\log\left(x^{\frac{1}{3}}\right)$$
$$+\frac{9}{3080}\left(1100\,ab^{14}x^{4}+127400\,a^{4}b^{11}x^{3}+825825\,a^{7}b^{8}x^{2}+616616\,a^{10}b^{5}x+53900\,a^{13}b^{2}\right)x^{\frac{2}{3}}$$
$$+\frac{9}{1820}\left(4900\,a^{2}b^{13}x^{4}+182182\,a^{5}b^{10}x^{3}+557700\,a^{8}b^{7}x^{2}+207025\,a^{11}b^{4}x+9100\,a^{14}b\right)x^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^15/x,x, algorithm="fricas")
```

output
```
1/5*b^15*x^5 + 455/4*a^3*b^12*x^4 + 5005/3*a^6*b^9*x^3 + 5005/2*a^9*b^6*x^
2 + 455*a^12*b^3*x + 3*a^15*log(x^(1/3)) + 9/3080*(1100*a*b^14*x^4 + 12740
0*a^4*b^11*x^3 + 825825*a^7*b^8*x^2 + 616616*a^10*b^5*x + 53900*a^13*b^2)*
x^(2/3) + 9/1820*(4900*a^2*b^13*x^4 + 182182*a^5*b^10*x^3 + 557700*a^8*b^7
*x^2 + 207025*a^11*b^4*x + 9100*a^14*b)*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx = a^{15}\log\left(x\right)+45a^{14}b\sqrt[3]{x}+\frac{315a^{13}b^2x^{\frac{2}{3}}}{2}+455a^{12}b^3x+\frac{4095a^{11}b^4x^{\frac{4}{3}}}{4}+\frac{9009a^{10}b^5x^{\frac{5}{3}}}{5}+\frac{5005a^9b^6x^2}{2}+\frac{19305a^8b^7x^{\frac{7}{3}}}{7}+\frac{19305a^7b^8x^{\frac{8}{3}}}{8}+\frac{5005a^6b^9x^3}{3}+\frac{9009a^5b^{10}x^{\frac{10}{3}}}{10}+\frac{4095a^4b^{11}x^{\frac{11}{3}}}{11}+\frac{455a^3b^{12}x^4}{4}+\frac{315a^2b^{13}x^{\frac{13}{3}}}{13}+\frac{45ab^{14}x^{\frac{14}{3}}}{14}+\frac{b^{15}x^5}{5}$$

input
```
integrate((a+b*x**(1/3))**15/x,x)
```

output
```
a**15*log(x) + 45*a**14*b*x**(1/3) + 315*a**13*b**2*x**(2/3)/2 + 455*a**12
*b**3*x + 4095*a**11*b**4*x**(4/3)/4 + 9009*a**10*b**5*x**(5/3)/5 + 5005*a
**9*b**6*x**2/2 + 19305*a**8*b**7*x**(7/3)/7 + 19305*a**7*b**8*x**(8/3)/8
+ 5005*a**6*b**9*x**3/3 + 9009*a**5*b**10*x**(10/3)/10 + 4095*a**4*b**11*x
**(11/3)/11 + 455*a**3*b**12*x**4/4 + 315*a**2*b**13*x**(13/3)/13 + 45*a*b
**14*x**(14/3)/14 + b**15*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx = \frac{1}{5}\,b^{15}x^5+\frac{45}{14}\,ab^{14}x^{\frac{14}{3}}+\frac{315}{13}\,a^2b^{13}x^{\frac{13}{3}}+\frac{455}{4}\,a^3b^{12}x^4+\frac{4095}{11}\,a^4b^{11}x^{\frac{11}{3}}+\frac{9009}{10}\,a^5b^{10}x^{\frac{10}{3}}+\frac{5005}{3}\,a^6b^9x^3+\frac{19305}{8}\,a^7b^8x^{\frac{8}{3}}+\frac{19305}{7}\,a^8b^7x^{\frac{7}{3}}+\frac{5005}{2}\,a^9b^6x^2+\frac{9009}{5}\,a^{10}b^5x^{\frac{5}{3}}+\frac{4095}{4}\,a^{11}b^4x^{\frac{4}{3}}+455\,a^{12}b^3x+a^{15}\log\left(x\right)+\frac{315}{2}\,a^{13}b^2x^{\frac{2}{3}}+45\,a^{14}bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^15/x,x, algorithm="maxima")
```

output
```
1/5*b^15*x^5 + 45/14*a*b^14*x^(14/3) + 315/13*a^2*b^13*x^(13/3) + 455/4*a^
3*b^12*x^4 + 4095/11*a^4*b^11*x^(11/3) + 9009/10*a^5*b^10*x^(10/3) + 5005/
3*a^6*b^9*x^3 + 19305/8*a^7*b^8*x^(8/3) + 19305/7*a^8*b^7*x^(7/3) + 5005/2
*a^9*b^6*x^2 + 9009/5*a^10*b^5*x^(5/3) + 4095/4*a^11*b^4*x^(4/3) + 455*a^1
2*b^3*x + a^15*log(x) + 315/2*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx = \frac{1}{5}\,b^{15}x^5+\frac{45}{14}\,ab^{14}x^{\frac{14}{3}}+\frac{315}{13}\,a^2b^{13}x^{\frac{13}{3}}+\frac{455}{4}\,a^3b^{12}x^4+\frac{4095}{11}\,a^4b^{11}x^{\frac{11}{3}}+\frac{9009}{10}\,a^5b^{10}x^{\frac{10}{3}}+\frac{5005}{3}\,a^6b^9x^3+\frac{19305}{8}\,a^7b^8x^{\frac{8}{3}}+\frac{19305}{7}\,a^8b^7x^{\frac{7}{3}}+\frac{5005}{2}\,a^9b^6x^2+\frac{9009}{5}\,a^{10}b^5x^{\frac{5}{3}}+\frac{4095}{4}\,a^{11}b^4x^{\frac{4}{3}}+455\,a^{12}b^3x+a^{15}\log\left(|x|\right)+\frac{315}{2}\,a^{13}b^2x^{\frac{2}{3}}+45\,a^{14}bx^{\frac{1}{3}}$$

input
```
integrate((a+b*x^(1/3))^15/x,x, algorithm="giac")
```

output
```
1/5*b^15*x^5 + 45/14*a*b^14*x^(14/3) + 315/13*a^2*b^13*x^(13/3) + 455/4*a^
3*b^12*x^4 + 4095/11*a^4*b^11*x^(11/3) + 9009/10*a^5*b^10*x^(10/3) + 5005/
3*a^6*b^9*x^3 + 19305/8*a^7*b^8*x^(8/3) + 19305/7*a^8*b^7*x^(7/3) + 5005/2
*a^9*b^6*x^2 + 9009/5*a^10*b^5*x^(5/3) + 4095/4*a^11*b^4*x^(4/3) + 455*a^1
2*b^3*x + a^15*log(abs(x)) + 315/2*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx = 3\,a^{15}\,\ln\left(x^{1/3}\right)+\frac{b^{15}\,x^5}{5}+455\,a^{12}\,b^3\,x+45\,a^{14}\,b\,x^{1/3} + \frac{45\,a\,b^{14}\,x^{14/3}}{14}+\frac{5005\,a^9\,b^6\,x^2}{2}+\frac{5005\,a^6\,b^9\,x^3}{3} + \frac{455\,a^3\,b^{12}\,x^4}{4}+\frac{315\,a^{13}\,b^2\,x^{2/3}}{2}+\frac{4095\,a^{11}\,b^4\,x^{4/3}}{4} + \frac{9009\,a^{10}\,b^5\,x^{5/3}}{5}+\frac{19305\,a^8\,b^7\,x^{7/3}}{7}+\frac{19305\,a^7\,b^8\,x^{8/3}}{8} + \frac{9009\,a^5\,b^{10}\,x^{10/3}}{10}+\frac{4095\,a^4\,b^{11}\,x^{11/3}}{11}+\frac{315\,a^2\,b^{13}\,x^{13/3}}{13}$$

input
```
int((a + b*x^(1/3))^15/x,x)
```

output
```
3*a^15*log(x^(1/3)) + (b^15*x^5)/5 + 455*a^12*b^3*x + 45*a^14*b*x^(1/3) +
(45*a*b^14*x^(14/3))/14 + (5005*a^9*b^6*x^2)/2 + (5005*a^6*b^9*x^3)/3 + (4
55*a^3*b^12*x^4)/4 + (315*a^13*b^2*x^(2/3))/2 + (4095*a^11*b^4*x^(4/3))/4
+ (9009*a^10*b^5*x^(5/3))/5 + (19305*a^8*b^7*x^(7/3))/7 + (19305*a^7*b^8*x
^(8/3))/8 + (9009*a^5*b^10*x^(10/3))/10 + (4095*a^4*b^11*x^(11/3))/11 + (3
15*a^2*b^13*x^(13/3))/13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x}\,dx = \frac{315x^{\frac{2}{3}}a^{13}b^{2}}{2}+\frac{9009x^{\frac{5}{3}}a^{10}b^{5}}{5}+\frac{19305x^{\frac{8}{3}}a^{7}b^{8}}{8}+\frac{4095x^{\frac{11}{3}}a^{4}b^{11}}{11} + \frac{45x^{\frac{14}{3}}a\,b^{14}}{14}+45x^{\frac{1}{3}}a^{14}b+\frac{4095x^{\frac{4}{3}}a^{11}b^{4}}{4}+\frac{19305x^{\frac{7}{3}}a^{8}b^{7}}{7} + \frac{9009x^{\frac{10}{3}}a^{5}b^{10}}{10}+\frac{315x^{\frac{13}{3}}a^{2}b^{13}}{13}+\log(x)\,a^{15}+455a^{12}b^{3}x + \frac{5005a^{9}b^{6}x^{2}}{2}+\frac{5005a^{6}b^{9}x^{3}}{3}+\frac{455a^{3}b^{12}x^{4}}{4}+\frac{b^{15}x^{5}}{5}$$

input
```
int((a+b*x^(1/3))^15/x,x)
```

output
```
(18918900*x**(2/3)*a**13*b**2 + 216432216*x**(2/3)*a**10*b**5*x + 28986457
5*x**(2/3)*a**7*b**8*x**2 + 44717400*x**(2/3)*a**4*b**11*x**3 + 386100*x**
(2/3)*a*b**14*x**4 + 5405400*x**(1/3)*a**14*b + 122972850*x**(1/3)*a**11*b
**4*x + 331273800*x**(1/3)*a**8*b**7*x**2 + 108216108*x**(1/3)*a**5*b**10*
x**3 + 2910600*x**(1/3)*a**2*b**13*x**4 + 120120*log(x)*a**15 + 54654600*a
**12*b**3*x + 300600300*a**9*b**6*x**2 + 200400200*a**6*b**9*x**3 + 136636
50*a**3*b**12*x**4 + 24024*b**15*x**5)/120120
```

3.237 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 202

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx = -\frac{a^{15}}{x}-\frac{45a^{14}b}{2x^{2/3}}-\frac{315a^{13}b^2}{\sqrt[3]{x}}+4095a^{11}b^4\sqrt[3]{x}+\frac{9009}{2}a^{10}b^5x^{2/3} +5005a^9b^6x+\frac{19305}{4}a^8b^7x^{4/3}+3861a^7b^8x^{5/3}+\frac{5005}{2}a^6b^9x^2 +1287a^5b^{10}x^{7/3}+\frac{4095}{8}a^4b^{11}x^{8/3}+\frac{455}{3}a^3b^{12}x^3 +\frac{63}{2}a^2b^{13}x^{10/3}+\frac{45}{11}ab^{14}x^{11/3}+\frac{b^{15}x^4}{4}+455a^{12}b^3\log(x)$$

output
```
-a^15/x-45/2*a^14*b/x^(2/3)-315*a^13*b^2/x^(1/3)+4095*a^11*b^4*x^(1/3)+900
9/2*a^10*b^5*x^(2/3)+5005*a^9*b^6*x+19305/4*a^8*b^7*x^(4/3)+3861*a^7*b^8*x
^(5/3)+5005/2*a^6*b^9*x^2+1287*a^5*b^10*x^(7/3)+4095/8*a^4*b^11*x^(8/3)+45
5/3*a^3*b^12*x^3+63/2*a^2*b^13*x^(10/3)+45/11*a*b^14*x^(11/3)+1/4*b^15*x^4
+455*a^12*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx$$
$$= \frac{-264a^{15}-5940a^{14}b\sqrt[3]{x}-83160a^{13}b^2x^{2/3}+1081080a^{11}b^4x^{4/3}+1189188a^{10}b^5x^{5/3}+1321320a^9b^6x^2+1}{}$$
$$+1365a^{12}b^3\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^15/x^2,x]
```

output
```
(-264*a^15 - 5940*a^14*b*x^(1/3) - 83160*a^13*b^2*x^(2/3) + 1081080*a^11*b
^4*x^(4/3) + 1189188*a^10*b^5*x^(5/3) + 1321320*a^9*b^6*x^2 + 1274130*a^8*
b^7*x^(7/3) + 1019304*a^7*b^8*x^(8/3) + 660660*a^6*b^9*x^3 + 339768*a^5*b^
10*x^(10/3) + 135135*a^4*b^11*x^(11/3) + 40040*a^3*b^12*x^4 + 8316*a^2*b^1
3*x^(13/3) + 1080*a*b^14*x^(14/3) + 66*b^15*x^5)/(264*x) + 1365*a^12*b^3*L
og[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx$$

$$\Big\downarrow \; 798$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 49$$

$$3\int \left(\frac{a^{15}}{x^{4/3}}+\frac{15ba^{14}}{x}+\frac{105b^2a^{13}}{x^{2/3}}+\frac{455b^3a^{12}}{\sqrt[3]{x}}+1365b^4a^{11}+3003b^5\sqrt[3]{x}a^{10}+5005b^6x^{2/3}a^9+6435b^7xa^8+6435b^8\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{15}}{3x}-\frac{15a^{14}b}{2x^{2/3}}-\frac{105a^{13}b^2}{\sqrt[3]{x}}+455a^{12}b^3\log\left(\sqrt[3]{x}\right)+1365a^{11}b^4\sqrt[3]{x}+\frac{3003}{2}a^{10}b^5x^{2/3}+\frac{5005}{3}a^9b^6x+\frac{6435}{4}a^8b^7x\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^2,x]
```

output
```
3*(-1/3*a^15/x - (15*a^14*b)/(2*x^(2/3)) - (105*a^13*b^2)/x^(1/3) + 1365*a
^11*b^4*x^(1/3) + (3003*a^10*b^5*x^(2/3))/2 + (5005*a^9*b^6*x)/3 + (6435*a
^8*b^7*x^(4/3))/4 + 1287*a^7*b^8*x^(5/3) + (5005*a^6*b^9*x^2)/6 + 429*a^5*
b^10*x^(7/3) + (1365*a^4*b^11*x^(8/3))/8 + (455*a^3*b^12*x^3)/9 + (21*a^2*
b^13*x^(10/3))/2 + (15*a*b^14*x^(11/3))/11 + (b^15*x^4)/12 + 455*a^12*b^3*
Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.83 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{a^{15}}{x}-\frac{45a^{14}b}{2x^{\frac{2}{3}}}-\frac{315a^{13}b^{2}}{x^{\frac{1}{3}}}+4095a^{11}b^{4}x^{\frac{1}{3}}+\frac{9009a^{10}b^{5}x^{\frac{2}{3}}}{2}+5005a^{9}b^{6}x+\frac{19305a^{8}b^{7}x^{\frac{4}{3}}}{4}+3861$
default	$-\frac{a^{15}}{x}-\frac{45a^{14}b}{2x^{\frac{2}{3}}}-\frac{315a^{13}b^{2}}{x^{\frac{1}{3}}}+4095a^{11}b^{4}x^{\frac{1}{3}}+\frac{9009a^{10}b^{5}x^{\frac{2}{3}}}{2}+5005a^{9}b^{6}x+\frac{19305a^{8}b^{7}x^{\frac{4}{3}}}{4}+3861$
trager	$\frac{(-1+x)(3b^{15}x^{4}+1820a^{3}b^{12}x^{3}+3b^{15}x^{3}+30030a^{6}b^{9}x^{2}+1820a^{3}b^{12}x^{2}+3b^{15}x^{2}+60060a^{9}b^{6}x+30030a^{6}b^{9}x+1820a^{3}b^{12}}{12x}$

input
```
int((a+b*x^(1/3))^15/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^15/x-45/2*a^14*b/x^(2/3)-315*a^13*b^2/x^(1/3)+4095*a^11*b^4*x^(1/3)+900
9/2*a^10*b^5*x^(2/3)+5005*a^9*b^6*x+19305/4*a^8*b^7*x^(4/3)+3861*a^7*b^8*x
^(5/3)+5005/2*a^6*b^9*x^2+1287*a^5*b^10*x^(7/3)+4095/8*a^4*b^11*x^(8/3)+45
5/3*a^3*b^12*x^3+63/2*a^2*b^13*x^(10/3)+45/11*a*b^14*x^(11/3)+1/4*b^15*x^4
+455*a^12*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{2}}\,dx$$
$$=\frac{66\,b^{15}x^{5}+40040\,a^{3}b^{12}x^{4}+660660\,a^{6}b^{9}x^{3}+1321320\,a^{9}b^{6}x^{2}+360360\,a^{12}b^{3}x\log\left(x^{\frac{1}{3}}\right)-264\,a^{15}+27\,(40}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^2,x, algorithm="fricas")
```

output
```
1/264*(66*b^15*x^5 + 40040*a^3*b^12*x^4 + 660660*a^6*b^9*x^3 + 1321320*a^9
*b^6*x^2 + 360360*a^12*b^3*x*log(x^(1/3)) - 264*a^15 + 27*(40*a*b^14*x^4 +
 5005*a^4*b^11*x^3 + 37752*a^7*b^8*x^2 + 44044*a^10*b^5*x - 3080*a^13*b^2)
*x^(2/3) + 594*(14*a^2*b^13*x^4 + 572*a^5*b^10*x^3 + 2145*a^8*b^7*x^2 + 18
20*a^11*b^4*x - 10*a^14*b)*x^(1/3))/x
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx = -\frac{a^{15}}{x} - \frac{45a^{14}b}{2x^{\frac{2}{3}}} - \frac{315a^{13}b^2}{\sqrt[3]{x}} + 455a^{12}b^3\log(x) + 4095a^{11}b^4\sqrt[3]{x} + \frac{9009a^{10}b^5x^{\frac{2}{3}}}{2} + 5005a^9b^6x + \frac{19305a^8b^7x^{\frac{4}{3}}}{4} + 3861a^7b^8x^{\frac{5}{3}} + \frac{5005a^6b^9x^2}{2} + 1287a^5b^{10}x^{\frac{7}{3}} + \frac{4095a^4b^{11}x^{\frac{8}{3}}}{8} + \frac{455a^3b^{12}x^3}{3} + \frac{63a^2b^{13}x^{\frac{10}{3}}}{2} + \frac{45ab^{14}x^{\frac{11}{3}}}{11} + \frac{b^{15}x^4}{4}$$

input
```
integrate((a+b*x**(1/3))**15/x**2,x)
```

output
```
-a**15/x - 45*a**14*b/(2*x**(2/3)) - 315*a**13*b**2/x**(1/3) + 455*a**12*b
**3*log(x) + 4095*a**11*b**4*x**(1/3) + 9009*a**10*b**5*x**(2/3)/2 + 5005*
a**9*b**6*x + 19305*a**8*b**7*x**(4/3)/4 + 3861*a**7*b**8*x**(5/3) + 5005*
a**6*b**9*x**2/2 + 1287*a**5*b**10*x**(7/3) + 4095*a**4*b**11*x**(8/3)/8 +
 455*a**3*b**12*x**3/3 + 63*a**2*b**13*x**(10/3)/2 + 45*a*b**14*x**(11/3)/
11 + b**15*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx = \frac{1}{4}\,b^{15}x^4 + \frac{45}{11}\,ab^{14}x^{\frac{11}{3}} + \frac{63}{2}\,a^2b^{13}x^{\frac{10}{3}} + \frac{455}{3}\,a^3b^{12}x^3 + \frac{4095}{8}\,a^4b^{11}x^{\frac{8}{3}} + 1287\,a^5b^{10}x^{\frac{7}{3}} + \frac{5005}{2}\,a^6b^9x^2 + 3861\,a^7b^8x^{\frac{5}{3}} + \frac{19305}{4}\,a^8b^7x^{\frac{4}{3}} + 5005\,a^9b^6x + 455\,a^{12}b^3\log(x) + \frac{9009}{2}\,a^{10}b^5x^{\frac{2}{3}} + 4095\,a^{11}b^4x^{\frac{1}{3}} - \frac{630\,a^{13}b^2x^{\frac{2}{3}} + 45\,a^{14}bx^{\frac{1}{3}} + 2\,a^{15}}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^15/x^2,x, algorithm="maxima")
```

output
```
1/4*b^15*x^4 + 45/11*a*b^14*x^(11/3) + 63/2*a^2*b^13*x^(10/3) + 455/3*a^3*
b^12*x^3 + 4095/8*a^4*b^11*x^(8/3) + 1287*a^5*b^10*x^(7/3) + 5005/2*a^6*b^
9*x^2 + 3861*a^7*b^8*x^(5/3) + 19305/4*a^8*b^7*x^(4/3) + 5005*a^9*b^6*x +
455*a^12*b^3*log(x) + 9009/2*a^10*b^5*x^(2/3) + 4095*a^11*b^4*x^(1/3) - 1/
2*(630*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3) + 2*a^15)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx = \frac{1}{4}\,b^{15}x^4 + \frac{45}{11}\,ab^{14}x^{\frac{11}{3}} + \frac{63}{2}\,a^2b^{13}x^{\frac{10}{3}} + \frac{455}{3}\,a^3b^{12}x^3 + \frac{4095}{8}\,a^4b^{11}x^{\frac{8}{3}} + 1287\,a^5b^{10}x^{\frac{7}{3}} + \frac{5005}{2}\,a^6b^9x^2 + 3861\,a^7b^8x^{\frac{5}{3}} + \frac{19305}{4}\,a^8b^7x^{\frac{4}{3}} + 5005\,a^9b^6x + 455\,a^{12}b^3\log\left(|x|\right) + \frac{9009}{2}\,a^{10}b^5x^{\frac{2}{3}} + 4095\,a^{11}b^4x^{\frac{1}{3}} - \frac{630\,a^{13}b^2x^{\frac{2}{3}} + 45\,a^{14}bx^{\frac{1}{3}} + 2\,a^{15}}{2\,x}$$

input
```
integrate((a+b*x^(1/3))^15/x^2,x, algorithm="giac")
```

output
```
1/4*b^15*x^4 + 45/11*a*b^14*x^(11/3) + 63/2*a^2*b^13*x^(10/3) + 455/3*a^3*
b^12*x^3 + 4095/8*a^4*b^11*x^(8/3) + 1287*a^5*b^10*x^(7/3) + 5005/2*a^6*b^
9*x^2 + 3861*a^7*b^8*x^(5/3) + 19305/4*a^8*b^7*x^(4/3) + 5005*a^9*b^6*x +
455*a^12*b^3*log(abs(x)) + 9009/2*a^10*b^5*x^(2/3) + 4095*a^11*b^4*x^(1/3)
 - 1/2*(630*a^13*b^2*x^(2/3) + 45*a^14*b*x^(1/3) + 2*a^15)/x
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx = \frac{b^{15}\,x^4}{4} - \frac{a^{15} + \frac{45\,a^{14}\,b\,x^{1/3}}{2} + 315\,a^{13}\,b^2\,x^{2/3}}{x} + 1365\,a^{12}\,b^3\,\ln\left(x^{1/3}\right) + 5005\,a^9\,b^6\,x + \frac{45\,a\,b^{14}\,x^{11/3}}{11} + \frac{5005\,a^6\,b^9\,x^2}{2} + \frac{455\,a^3\,b^{12}\,x^3}{3} + 4095\,a^{11}\,b^4\,x^{1/3} + \frac{9009}{}$$

input
```
int((a + b*x^(1/3))^15/x^2,x)
```

output
```
(b^15*x^4)/4 - (a^15 + (45*a^14*b*x^(1/3))/2 + 315*a^13*b^2*x^(2/3))/x + 1
365*a^12*b^3*log(x^(1/3)) + 5005*a^9*b^6*x + (45*a*b^14*x^(11/3))/11 + (50
05*a^6*b^9*x^2)/2 + (455*a^3*b^12*x^3)/3 + 4095*a^11*b^4*x^(1/3) + (9009*a
^10*b^5*x^(2/3))/2 + (19305*a^8*b^7*x^(4/3))/4 + 3861*a^7*b^8*x^(5/3) + 12
87*a^5*b^10*x^(7/3) + (4095*a^4*b^11*x^(8/3))/8 + (63*a^2*b^13*x^(10/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^2}\,dx = \frac{360360x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)a^{12}b^3-264x^{\frac{2}{3}}a^{15}+1321320x^{\frac{8}{3}}a^9b^6+660660x^{\frac{11}{3}}a^6b^9+40040x^{\frac{14}{3}}a^3b^{12}+66x^{\frac{17}{3}}b^{15}-8}{}$$

input
```
int((a+b*x^(1/3))^15/x^2,x)
```

output
```
(360360*x**(2/3)*log(x**(1/3))*a**12*b**3*x - 264*x**(2/3)*a**15 + 1321320
*x**(2/3)*a**9*b**6*x**2 + 660660*x**(2/3)*a**6*b**9*x**3 + 40040*x**(2/3)
*a**3*b**12*x**4 + 66*x**(2/3)*b**15*x**5 - 83160*x**(1/3)*a**13*b**2*x +
1189188*x**(1/3)*a**10*b**5*x**2 + 1019304*x**(1/3)*a**7*b**8*x**3 + 13513
5*x**(1/3)*a**4*b**11*x**4 + 1080*x**(1/3)*a*b**14*x**5 - 5940*a**14*b*x +
 1081080*a**11*b**4*x**2 + 1274130*a**8*b**7*x**3 + 339768*a**5*b**10*x**4
 + 8316*a**2*b**13*x**5)/(264*x**(2/3)*x)
```

3.238 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 200

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx = & -\frac{a^{15}}{2x^2}-\frac{9a^{14}b}{x^{5/3}}-\frac{315a^{13}b^2}{4x^{4/3}}-\frac{455a^{12}b^3}{x}-\frac{4095a^{11}b^4}{2x^{2/3}}\\ & -\frac{9009a^{10}b^5}{\sqrt[3]{x}}+19305a^8b^7\sqrt[3]{x}+\frac{19305}{2}a^7b^8x^{2/3}\\ & +5005a^6b^9x+\frac{9009}{4}a^5b^{10}x^{4/3}+819a^4b^{11}x^{5/3}+\frac{455}{2}a^3b^{12}x^2\\ & +45a^2b^{13}x^{7/3}+\frac{45}{8}ab^{14}x^{8/3}+\frac{b^{15}x^3}{3}+5005a^9b^6\log(x)\end{aligned}$$

output
```
-1/2*a^15/x^2-9*a^14*b/x^(5/3)-315/4*a^13*b^2/x^(4/3)-455*a^12*b^3/x-4095/
2*a^11*b^4/x^(2/3)-9009*a^10*b^5/x^(1/3)+19305*a^8*b^7*x^(1/3)+19305/2*a^7
*b^8*x^(2/3)+5005*a^6*b^9*x+9009/4*a^5*b^10*x^(4/3)+819*a^4*b^11*x^(5/3)+4
55/2*a^3*b^12*x^2+45*a^2*b^13*x^(7/3)+45/8*a*b^14*x^(8/3)+1/3*b^15*x^3+500
5*a^9*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sqrt[3]{x})^{15}}{x^3}\,dx = \frac{-12a^{15}-216a^{14}b\sqrt[3]{x}-1890a^{13}b^2x^{2/3}-10920a^{12}b^3x-49140a^{11}b^4x^{4/3}-216216a^{10}b^5x^{5/3}+463320a^8b^7\ldots}{\ldots} + 15015a^9b^6\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^15/x^3,x]
```

output
```
(-12*a^15 - 216*a^14*b*x^(1/3) - 1890*a^13*b^2*x^(2/3) - 10920*a^12*b^3*x
- 49140*a^11*b^4*x^(4/3) - 216216*a^10*b^5*x^(5/3) + 463320*a^8*b^7*x^(7/3
) + 231660*a^7*b^8*x^(8/3) + 120120*a^6*b^9*x^3 + 54054*a^5*b^10*x^(10/3)
+ 19656*a^4*b^11*x^(11/3) + 5460*a^3*b^12*x^4 + 1080*a^2*b^13*x^(13/3) + 1
35*a*b^14*x^(14/3) + 8*b^15*x^5)/(24*x^2) + 15015*a^9*b^6*Log[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sqrt[3]{x})^{15}}{x^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{(a+b\sqrt[3]{x})^{15}}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^{15}}{x^{7/3}}+\frac{15ba^{14}}{x^2}+\frac{105b^2a^{13}}{x^{5/3}}+\frac{455b^3a^{12}}{x^{4/3}}+\frac{1365b^4a^{11}}{x}+\frac{3003b^5a^{10}}{x^{2/3}}+\frac{5005b^6a^9}{\sqrt[3]{x}}+6435b^7a^8+6435b^8\sqrt[3]{x}a^7\ldots\right.$$

$$\Big\downarrow 2009$$

$$3\left(-\frac{a^{15}}{6x^2}-\frac{3a^{14}b}{x^{5/3}}-\frac{105a^{13}b^2}{4x^{4/3}}-\frac{455a^{12}b^3}{3x}-\frac{1365a^{11}b^4}{2x^{2/3}}-\frac{3003a^{10}b^5}{\sqrt[3]{x}}+5005a^9b^6\log\left(\sqrt[3]{x}\right)+6435a^8b^7\sqrt[3]{x}+\frac{6435}{2}\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^3,x]
```

output
```
3*(-1/6*a^15/x^2 - (3*a^14*b)/x^(5/3) - (105*a^13*b^2)/(4*x^(4/3)) - (455*
a^12*b^3)/(3*x) - (1365*a^11*b^4)/(2*x^(2/3)) - (3003*a^10*b^5)/x^(1/3) +
6435*a^8*b^7*x^(1/3) + (6435*a^7*b^8*x^(2/3))/2 + (5005*a^6*b^9*x)/3 + (30
03*a^5*b^10*x^(4/3))/4 + 273*a^4*b^11*x^(5/3) + (455*a^3*b^12*x^2)/6 + 15*
a^2*b^13*x^(7/3) + (15*a*b^14*x^(8/3))/8 + (b^15*x^3)/9 + 5005*a^9*b^6*Log
[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{a^{15}}{2x^2}-\frac{9a^{14}b}{x^{\frac{5}{3}}}-\frac{315a^{13}b^2}{4x^{\frac{4}{3}}}-\frac{455a^{12}b^3}{x}-\frac{4095a^{11}b^4}{2x^{\frac{2}{3}}}-\frac{9009a^{10}b^5}{x^{\frac{1}{3}}}+19305a^8b^7x^{\frac{1}{3}}+\frac{19305a^7b^8x^{\frac{2}{3}}}{2}+$
default	$-\frac{a^{15}}{2x^2}-\frac{9a^{14}b}{x^{\frac{5}{3}}}-\frac{315a^{13}b^2}{4x^{\frac{4}{3}}}-\frac{455a^{12}b^3}{x}-\frac{4095a^{11}b^4}{2x^{\frac{2}{3}}}-\frac{9009a^{10}b^5}{x^{\frac{1}{3}}}+19305a^8b^7x^{\frac{1}{3}}+\frac{19305a^7b^8x^{\frac{2}{3}}}{2}+$
trager	$\frac{(-1+x)(2b^{15}x^4+1365a^3b^{12}x^3+2b^{15}x^3+30030a^6b^9x^2+1365a^3b^{12}x^2+2b^{15}x^2+3a^{15}x+2730a^{12}b^3x+3a^{15})}{6x^2}-\frac{9(-20}{}$

input
```
int((a+b*x^(1/3))^15/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^15/x^2-9*a^14*b/x^(5/3)-315/4*a^13*b^2/x^(4/3)-455*a^12*b^3/x-4095/
2*a^11*b^4/x^(2/3)-9009*a^10*b^5/x^(1/3)+19305*a^8*b^7*x^(1/3)+19305/2*a^7
*b^8*x^(2/3)+5005*a^6*b^9*x+9009/4*a^5*b^10*x^(4/3)+819*a^4*b^11*x^(5/3)+4
55/2*a^3*b^12*x^2+45*a^2*b^13*x^(7/3)+45/8*a*b^14*x^(8/3)+1/3*b^15*x^3+500
5*a^9*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx = \frac{8\,b^{15}x^5+5460\,a^3b^{12}x^4+120120\,a^6b^9x^3+360360\,a^9b^6x^2\log\left(x^{\frac{1}{3}}\right)-10920\,a^{12}b^3x-12\,a^{15}+27\,(5\,ab^{14}x^4}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^3,x, algorithm="fricas")
```

output
```
1/24*(8*b^15*x^5 + 5460*a^3*b^12*x^4 + 120120*a^6*b^9*x^3 + 360360*a^9*b^6
*x^2*log(x^(1/3)) - 10920*a^12*b^3*x - 12*a^15 + 27*(5*a*b^14*x^4 + 728*a^
4*b^11*x^3 + 8580*a^7*b^8*x^2 - 8008*a^10*b^5*x - 70*a^13*b^2)*x^(2/3) + 5
4*(20*a^2*b^13*x^4 + 1001*a^5*b^10*x^3 + 8580*a^8*b^7*x^2 - 910*a^11*b^4*x
 - 4*a^14*b)*x^(1/3))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx = -\frac{a^{15}}{2x^2} - \frac{9a^{14}b}{x^{\frac{5}{3}}} - \frac{315a^{13}b^2}{4x^{\frac{4}{3}}} - \frac{455a^{12}b^3}{x} - \frac{4095a^{11}b^4}{2x^{\frac{2}{3}}} - \frac{9009a^{10}b^5}{\sqrt[3]{x}} + 5005a^9b^6\log(x) + 19305a^8b^7\sqrt[3]{x} + \frac{19305a^7b^8x^{\frac{2}{3}}}{2} + 5005a^6b^9x + \frac{9009a^5b^{10}x^{\frac{4}{3}}}{4} + 819a^4b^{11}x^{\frac{5}{3}} + \frac{455a^3b^{12}x^2}{2} + 45a^2b^{13}x^{\frac{7}{3}} + \frac{45ab^{14}x^{\frac{8}{3}}}{8} + \frac{b^{15}x^3}{3}$$

input
```
integrate((a+b*x**(1/3))**15/x**3,x)
```

output
```
-a**15/(2*x**2) - 9*a**14*b/x**(5/3) - 315*a**13*b**2/(4*x**(4/3)) - 455*a
**12*b**3/x - 4095*a**11*b**4/(2*x**(2/3)) - 9009*a**10*b**5/x**(1/3) + 50
05*a**9*b**6*log(x) + 19305*a**8*b**7*x**(1/3) + 19305*a**7*b**8*x**(2/3)/
2 + 5005*a**6*b**9*x + 9009*a**5*b**10*x**(4/3)/4 + 819*a**4*b**11*x**(5/3
) + 455*a**3*b**12*x**2/2 + 45*a**2*b**13*x**(7/3) + 45*a*b**14*x**(8/3)/8
 + b**15*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx = \frac{1}{3}\,b^{15}x^3 + \frac{45}{8}\,ab^{14}x^{\frac{8}{3}} + 45\,a^2b^{13}x^{\frac{7}{3}} + \frac{455}{2}\,a^3b^{12}x^2 + 819\,a^4b^{11}x^{\frac{5}{3}} + \frac{9009}{4}\,a^5b^{10}x^{\frac{4}{3}} + 5005\,a^6b^9x + 5005\,a^9b^6\log(x) + \frac{19305}{2}\,a^7b^8x^{\frac{2}{3}} + 19305\,a^8b^7x^{\frac{1}{3}} - \frac{36036\,a^{10}b^5x^{\frac{5}{3}} + 8190\,a^{11}b^4x^{\frac{4}{3}} + 1820\,a^{12}b^3x + 315\,a^{13}b^2x^{\frac{2}{3}} + 36\,a^{14}bx^{\frac{1}{3}} + 2\,a^{15}}{4\,x^2}$$

input
```
integrate((a+b*x^(1/3))^15/x^3,x, algorithm="maxima")
```

output
```
1/3*b^15*x^3 + 45/8*a*b^14*x^(8/3) + 45*a^2*b^13*x^(7/3) + 455/2*a^3*b^12*
x^2 + 819*a^4*b^11*x^(5/3) + 9009/4*a^5*b^10*x^(4/3) + 5005*a^6*b^9*x + 50
05*a^9*b^6*log(x) + 19305/2*a^7*b^8*x^(2/3) + 19305*a^8*b^7*x^(1/3) - 1/4*
(36036*a^10*b^5*x^(5/3) + 8190*a^11*b^4*x^(4/3) + 1820*a^12*b^3*x + 315*a^
13*b^2*x^(2/3) + 36*a^14*b*x^(1/3) + 2*a^15)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx &= \frac{1}{3}\,b^{15}x^3+\frac{45}{8}\,ab^{14}x^{\frac{8}{3}}+45\,a^2b^{13}x^{\frac{7}{3}}+\frac{455}{2}\,a^3b^{12}x^2+819\,a^4b^{11}x^{\frac{5}{3}}+\frac{9009}{4}\,a^5b^{10}x^{\frac{4}{3}}\\ &\quad+5005\,a^6b^9x+5005\,a^9b^6\log\left(|x|\right)+\frac{19305}{2}\,a^7b^8x^{\frac{2}{3}}+19305\,a^8b^7x^{\frac{1}{3}}\\ &\quad-\frac{36036\,a^{10}b^5x^{\frac{5}{3}}+8190\,a^{11}b^4x^{\frac{4}{3}}+1820\,a^{12}b^3x+315\,a^{13}b^2x^{\frac{2}{3}}+36\,a^{14}bx^{\frac{1}{3}}+2\,a^{15}}{4\,x^2}\end{aligned}$$

input
```
integrate((a+b*x^(1/3))^15/x^3,x, algorithm="giac")
```

output
```
1/3*b^15*x^3 + 45/8*a*b^14*x^(8/3) + 45*a^2*b^13*x^(7/3) + 455/2*a^3*b^12*
x^2 + 819*a^4*b^11*x^(5/3) + 9009/4*a^5*b^10*x^(4/3) + 5005*a^6*b^9*x + 50
05*a^9*b^6*log(abs(x)) + 19305/2*a^7*b^8*x^(2/3) + 19305*a^8*b^7*x^(1/3) -
 1/4*(36036*a^10*b^5*x^(5/3) + 8190*a^11*b^4*x^(4/3) + 1820*a^12*b^3*x + 3
15*a^13*b^2*x^(2/3) + 36*a^14*b*x^(1/3) + 2*a^15)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx &= \frac{b^{15}\,x^3}{3}\\ &\quad-\frac{\frac{a^{15}}{2}+455\,a^{12}\,b^3\,x+9\,a^{14}\,b\,x^{1/3}+\frac{315\,a^{13}\,b^2\,x^{2/3}}{4}+\frac{4095\,a^{11}\,b^4\,x^{4/3}}{2}+9009\,a^{10}\,b^5\,x^{5/3}}{x^2}\\ &\quad+15015\,a^9\,b^6\,\ln\left(x^{1/3}\right)+5005\,a^6\,b^9\,x+\frac{45\,a\,b^{14}\,x^{8/3}}{8}+\frac{455\,a^3\,b^{12}\,x^2}{2}+19305\,a^8\,b^7\,x^{1/3}+\frac{19305\,a^7\,b^8\,x^{2/3}}{2}+9\dots\end{aligned}$$

input
```
int((a + b*x^(1/3))^15/x^3,x)
```

output
```
(b^15*x^3)/3 - (a^15/2 + 455*a^12*b^3*x + 9*a^14*b*x^(1/3) + (315*a^13*b^2
*x^(2/3))/4 + (4095*a^11*b^4*x^(4/3))/2 + 9009*a^10*b^5*x^(5/3))/x^2 + 150
15*a^9*b^6*log(x^(1/3)) + 5005*a^6*b^9*x + (45*a*b^14*x^(8/3))/8 + (455*a^
3*b^12*x^2)/2 + 19305*a^8*b^7*x^(1/3) + (19305*a^7*b^8*x^(2/3))/2 + (9009*
a^5*b^10*x^(4/3))/4 + 819*a^4*b^11*x^(5/3) + 45*a^2*b^13*x^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^3}\,dx$$
$$= \frac{360360x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}\right)a^9b^6 - 12x^{\frac{2}{3}}a^{15} - 10920x^{\frac{5}{3}}a^{12}b^3 + 120120x^{\frac{11}{3}}a^6b^9 + 5460x^{\frac{14}{3}}a^3b^{12} + 8x^{\frac{17}{3}}b^{15} - 1890x}{}$$

input
```
int((a+b*x^(1/3))^15/x^3,x)
```

output
```
(360360*x**(2/3)*log(x**(1/3))*a**9*b**6*x**2 - 12*x**(2/3)*a**15 - 10920*
x**(2/3)*a**12*b**3*x + 120120*x**(2/3)*a**6*b**9*x**3 + 5460*x**(2/3)*a**
3*b**12*x**4 + 8*x**(2/3)*b**15*x**5 - 1890*x**(1/3)*a**13*b**2*x - 216216
*x**(1/3)*a**10*b**5*x**2 + 231660*x**(1/3)*a**7*b**8*x**3 + 19656*x**(1/3
)*a**4*b**11*x**4 + 135*x**(1/3)*a*b**14*x**5 - 216*a**14*b*x - 49140*a**1
1*b**4*x**2 + 463320*a**8*b**7*x**3 + 54054*a**5*b**10*x**4 + 1080*a**2*b*
*13*x**5)/(24*x**(2/3)*x**2)
```

3.239 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 200

$$\begin{aligned}\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4} dx = &-\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{8/3}}-\frac{45a^{13}b^2}{x^{7/3}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{5/3}}\\ &-\frac{9009a^{10}b^5}{4x^{4/3}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{2/3}}-\frac{19305a^7b^8}{\sqrt[3]{x}}\\ &+9009a^5b^{10}\sqrt[3]{x}+\frac{4095}{2}a^4b^{11}x^{2/3}+455a^3b^{12}x\\ &+\frac{315}{4}a^2b^{13}x^{4/3}+9ab^{14}x^{5/3}+\frac{b^{15}x^2}{2}+5005a^6b^9\log(x)\end{aligned}$$

output
```
-1/3*a^15/x^3-45/8*a^14*b/x^(8/3)-45*a^13*b^2/x^(7/3)-455/2*a^12*b^3/x^2-8
19*a^11*b^4/x^(5/3)-9009/4*a^10*b^5/x^(4/3)-5005*a^9*b^6/x-19305/2*a^8*b^7
/x^(2/3)-19305*a^7*b^8/x^(1/3)+9009*a^5*b^10*x^(1/3)+4095/2*a^4*b^11*x^(2/
3)+455*a^3*b^12*x+315/4*a^2*b^13*x^(4/3)+9*a*b^14*x^(5/3)+1/2*b^15*x^2+500
5*a^6*b^9*ln(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx$$
$$= \frac{-8a^{15}-135a^{14}b\sqrt[3]{x}-1080a^{13}b^2x^{2/3}-5460a^{12}b^3x-19656a^{11}b^4x^{4/3}-54054a^{10}b^5x^{5/3}-120120a^9b^6x^2}{}$$
$$+15015a^6b^9\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^15/x^4,x]
```

output
```
(-8*a^15 - 135*a^14*b*x^(1/3) - 1080*a^13*b^2*x^(2/3) - 5460*a^12*b^3*x -
19656*a^11*b^4*x^(4/3) - 54054*a^10*b^5*x^(5/3) - 120120*a^9*b^6*x^2 - 231
660*a^8*b^7*x^(7/3) - 463320*a^7*b^8*x^(8/3) + 216216*a^5*b^10*x^(10/3) +
49140*a^4*b^11*x^(11/3) + 10920*a^3*b^12*x^4 + 1890*a^2*b^13*x^(13/3) + 21
6*a*b^14*x^(14/3) + 12*b^15*x^5)/(24*x^3) + 15015*a^6*b^9*Log[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx$$
$$\Big\downarrow \text{798}$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{49}$$
$$3\int \left(\frac{a^{15}}{x^{10/3}}+\frac{15ba^{14}}{x^3}+\frac{105b^2a^{13}}{x^{8/3}}+\frac{455b^3a^{12}}{x^{7/3}}+\frac{1365b^4a^{11}}{x^2}+\frac{3003b^5a^{10}}{x^{5/3}}+\frac{5005b^6a^9}{x^{4/3}}+\frac{6435b^7a^8}{x}+\frac{6435b^8a^7}{x^{2/3}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{15}}{9x^3}-\frac{15a^{14}b}{8x^{8/3}}-\frac{15a^{13}b^2}{x^{7/3}}-\frac{455a^{12}b^3}{6x^2}-\frac{273a^{11}b^4}{x^{5/3}}-\frac{3003a^{10}b^5}{4x^{4/3}}-\frac{5005a^9b^6}{3x}-\frac{6435a^8b^7}{2x^{2/3}}-\frac{6435a^7b^8}{\sqrt[3]{x}}+5005\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^4,x]
```

output
```
3*(-1/9*a^15/x^3 - (15*a^14*b)/(8*x^(8/3)) - (15*a^13*b^2)/x^(7/3) - (455*
a^12*b^3)/(6*x^2) - (273*a^11*b^4)/x^(5/3) - (3003*a^10*b^5)/(4*x^(4/3)) -
 (5005*a^9*b^6)/(3*x) - (6435*a^8*b^7)/(2*x^(2/3)) - (6435*a^7*b^8)/x^(1/3
) + 3003*a^5*b^10*x^(1/3) + (1365*a^4*b^11*x^(2/3))/2 + (455*a^3*b^12*x)/3
 + (105*a^2*b^13*x^(4/3))/4 + 3*a*b^14*x^(5/3) + (b^15*x^2)/6 + 5005*a^6*b
^9*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 27.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{\frac{8}{3}}}-\frac{45a^{13}b^2}{x^{\frac{7}{3}}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{\frac{5}{3}}}-\frac{9009a^{10}b^5}{4x^{\frac{4}{3}}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{\frac{2}{3}}}-\frac{19305a^7b^8}{x^{\frac{1}{3}}}$
default	$-\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{\frac{8}{3}}}-\frac{45a^{13}b^2}{x^{\frac{7}{3}}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{\frac{5}{3}}}-\frac{9009a^{10}b^5}{4x^{\frac{4}{3}}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{\frac{2}{3}}}-\frac{19305a^7b^8}{x^{\frac{1}{3}}}$
trager	$\frac{(-1+x)(3b^{15}x^4+2730a^3b^{12}x^3+3b^{15}x^3+2a^{15}x^2+1365a^{12}b^3x^2+30030a^9b^6x^2+2a^{15}x+1365a^{12}b^3x+2a^{15})}{6x^3}-\frac{9(-70}{}$

input
```
int((a+b*x^(1/3))^15/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^15/x^3-45/8*a^14*b/x^(8/3)-45*a^13*b^2/x^(7/3)-455/2*a^12*b^3/x^2-8
19*a^11*b^4/x^(5/3)-9009/4*a^10*b^5/x^(4/3)-5005*a^9*b^6/x-19305/2*a^8*b^7
/x^(2/3)-19305*a^7*b^8/x^(1/3)+9009*a^5*b^10*x^(1/3)+4095/2*a^4*b^11*x^(2/
3)+455*a^3*b^12*x+315/4*a^2*b^13*x^(4/3)+9*a*b^14*x^(5/3)+1/2*b^15*x^2+500
5*a^6*b^9*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx = \frac{12\,b^{15}x^5+10920\,a^3b^{12}x^4+360360\,a^6b^9x^3\log\left(x^{\frac{1}{3}}\right)-120120\,a^9b^6x^2-5460\,a^{12}b^3x-8\,a^{15}+54\,(4\,ab^{14}x^4}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^4,x, algorithm="fricas")
```

output
```
1/24*(12*b^15*x^5 + 10920*a^3*b^12*x^4 + 360360*a^6*b^9*x^3*log(x^(1/3)) -
 120120*a^9*b^6*x^2 - 5460*a^12*b^3*x - 8*a^15 + 54*(4*a*b^14*x^4 + 910*a^
4*b^11*x^3 - 8580*a^7*b^8*x^2 - 1001*a^10*b^5*x - 20*a^13*b^2)*x^(2/3) + 2
7*(70*a^2*b^13*x^4 + 8008*a^5*b^10*x^3 - 8580*a^8*b^7*x^2 - 728*a^11*b^4*x
 - 5*a^14*b)*x^(1/3))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx = -\frac{a^{15}}{3x^3}-\frac{45a^{14}b}{8x^{\frac{8}{3}}}-\frac{45a^{13}b^2}{x^{\frac{7}{3}}}-\frac{455a^{12}b^3}{2x^2}-\frac{819a^{11}b^4}{x^{\frac{5}{3}}}-\frac{9009a^{10}b^5}{4x^{\frac{4}{3}}}-\frac{5005a^9b^6}{x}-\frac{19305a^8b^7}{2x^{\frac{2}{3}}}-\frac{19305a^7b^8}{\sqrt[3]{x}}+5005a^6b^9\log(x)+9009a^5b^{10}\sqrt[3]{x}+\frac{4095a^4b^{11}x^{\frac{2}{3}}}{2}+455a^3b^{12}x+\frac{315a^2b^{13}x^{\frac{4}{3}}}{4}+9ab^{14}x^{\frac{5}{3}}+\frac{b^{15}x^2}{2}$$

input
```
integrate((a+b*x**(1/3))**15/x**4,x)
```

output
```
-a**15/(3*x**3) - 45*a**14*b/(8*x**(8/3)) - 45*a**13*b**2/x**(7/3) - 455*a
**12*b**3/(2*x**2) - 819*a**11*b**4/x**(5/3) - 9009*a**10*b**5/(4*x**(4/3)
) - 5005*a**9*b**6/x - 19305*a**8*b**7/(2*x**(2/3)) - 19305*a**7*b**8/x**(
1/3) + 5005*a**6*b**9*log(x) + 9009*a**5*b**10*x**(1/3) + 4095*a**4*b**11*
x**(2/3)/2 + 455*a**3*b**12*x + 315*a**2*b**13*x**(4/3)/4 + 9*a*b**14*x**(
5/3) + b**15*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx = \frac{1}{2}\,b^{15}x^2+9\,ab^{14}x^{\frac{5}{3}}+\frac{315}{4}\,a^2b^{13}x^{\frac{4}{3}}+455\,a^3b^{12}x+5005\,a^6b^9\log(x)+\frac{4095}{2}\,a^4b^{11}x^{\frac{2}{3}}+9009\,a^5b^{10}x^{\frac{1}{3}}-\frac{463320\,a^7b^8x^{\frac{8}{3}}+231660\,a^8b^7x^{\frac{7}{3}}+120120\,a^9b^6x^2+54054\,a^{10}b^5x^{\frac{5}{3}}+19656\,a^{11}b^4x^{\frac{4}{3}}+5460\,a^{12}b^3x+10}{24\,x^3}$$

input
```
integrate((a+b*x^(1/3))^15/x^4,x, algorithm="maxima")
```

output
```
1/2*b^15*x^2 + 9*a*b^14*x^(5/3) + 315/4*a^2*b^13*x^(4/3) + 455*a^3*b^12*x
+ 5005*a^6*b^9*log(x) + 4095/2*a^4*b^11*x^(2/3) + 9009*a^5*b^10*x^(1/3) -
1/24*(463320*a^7*b^8*x^(8/3) + 231660*a^8*b^7*x^(7/3) + 120120*a^9*b^6*x^2
 + 54054*a^10*b^5*x^(5/3) + 19656*a^11*b^4*x^(4/3) + 5460*a^12*b^3*x + 108
0*a^13*b^2*x^(2/3) + 135*a^14*b*x^(1/3) + 8*a^15)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.83

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx = \frac{1}{2}\,b^{15}x^2 + 9\,ab^{14}x^{\frac{5}{3}} + \frac{315}{4}\,a^2b^{13}x^{\frac{4}{3}} + 455\,a^3b^{12}x + 5005\,a^6b^9\log\left(|x|\right) + \frac{4095}{2}\,a^4b^{11}x^{\frac{2}{3}} + 9009\,a^5b^{10}x^{\frac{1}{3}} - \frac{463320\,a^7b^8x^{\frac{8}{3}} + 231660\,a^8b^7x^{\frac{7}{3}} + 120120\,a^9b^6x^2 + 54054\,a^{10}b^5x^{\frac{5}{3}} + 19656\,a^{11}b^4x^{\frac{4}{3}} + 5460\,a^{12}b^3x + 10}{24\,x^3}$$

input
```
integrate((a+b*x^(1/3))^15/x^4,x, algorithm="giac")
```

output
```
1/2*b^15*x^2 + 9*a*b^14*x^(5/3) + 315/4*a^2*b^13*x^(4/3) + 455*a^3*b^12*x
+ 5005*a^6*b^9*log(abs(x)) + 4095/2*a^4*b^11*x^(2/3) + 9009*a^5*b^10*x^(1/
3) - 1/24*(463320*a^7*b^8*x^(8/3) + 231660*a^8*b^7*x^(7/3) + 120120*a^9*b^
6*x^2 + 54054*a^10*b^5*x^(5/3) + 19656*a^11*b^4*x^(4/3) + 5460*a^12*b^3*x
+ 1080*a^13*b^2*x^(2/3) + 135*a^14*b*x^(1/3) + 8*a^15)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^4}\,dx = \frac{b^{15}\,x^2}{2} - \frac{\frac{a^{15}}{3} + \frac{455\,a^{12}\,b^3\,x}{2} + \frac{45\,a^{14}\,b\,x^{1/3}}{8} + 5005\,a^9\,b^6\,x^2 + 45\,a^{13}\,b^2\,x^{2/3} + 819\,a^{11}\,b^4\,x^{4/3} + \frac{9009\,a^{10}\,b^5\,x^{5/3}}{4} + \frac{19305\,a^8\,b^7}{2}}{x^3} + 15015\,a^6\,b^9\,\ln\left(x^{1/3}\right) + 455\,a^3\,b^{12}\,x + 9\,a\,b^{14}\,x^{5/3} + 9009\,a^5\,b^{10}\,x^{1/3} + \frac{4095\,a^4\,b^{11}\,x^{2/3}}{2} + \frac{315\,a^2\,b^{13}\,x^{4/3}}{4}$$

input
```
int((a + b*x^(1/3))^15/x^4,x)
```

output
```
(b^15*x^2)/2 - (a^15/3 + (455*a^12*b^3*x)/2 + (45*a^14*b*x^(1/3))/8 + 5005
*a^9*b^6*x^2 + 45*a^13*b^2*x^(2/3) + 819*a^11*b^4*x^(4/3) + (9009*a^10*b^5
*x^(5/3))/4 + (19305*a^8*b^7*x^(7/3))/2 + 19305*a^7*b^8*x^(8/3))/x^3 + 150
15*a^6*b^9*log(x^(1/3)) + 455*a^3*b^12*x + 9*a*b^14*x^(5/3) + 9009*a^5*b^1
0*x^(1/3) + (4095*a^4*b^11*x^(2/3))/2 + (315*a^2*b^13*x^(4/3))/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \frac{(a+b\sqrt[3]{x})^{15}}{x^4}\,dx$$
$$= \frac{360360x^{\frac{11}{3}}\log\left(x^{\frac{1}{3}}\right)a^6b^9 - 8x^{\frac{2}{3}}a^{15} - 5460x^{\frac{5}{3}}a^{12}b^3 - 120120x^{\frac{8}{3}}a^9b^6 + 10920x^{\frac{14}{3}}a^3b^{12} + 12x^{\frac{17}{3}}b^{15} - 1080x}{}$$

input
```
int((a+b*x^(1/3))^15/x^4,x)
```

output
```
(360360*x**(2/3)*log(x**(1/3))*a**6*b**9*x**3 - 8*x**(2/3)*a**15 - 5460*x*
*(2/3)*a**12*b**3*x - 120120*x**(2/3)*a**9*b**6*x**2 + 10920*x**(2/3)*a**3
*b**12*x**4 + 12*x**(2/3)*b**15*x**5 - 1080*x**(1/3)*a**13*b**2*x - 54054*
x**(1/3)*a**10*b**5*x**2 - 463320*x**(1/3)*a**7*b**8*x**3 + 49140*x**(1/3)
*a**4*b**11*x**4 + 216*x**(1/3)*a*b**14*x**5 - 135*a**14*b*x - 19656*a**11
*b**4*x**2 - 231660*a**8*b**7*x**3 + 216216*a**5*b**10*x**4 + 1890*a**2*b*
*13*x**5)/(24*x**(2/3)*x**3)
```

3.240 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 211

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx = -\frac{a^{15}}{5x^5}-\frac{45a^{14}b}{14x^{14/3}}-\frac{315a^{13}b^2}{13x^{13/3}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{11/3}}-\frac{9009a^{10}b^5}{10x^{10/3}}-\frac{5005a^9b^6}{3x^3}-\frac{19305a^8b^7}{8x^{8/3}}-\frac{19305a^7b^8}{7x^{7/3}}-\frac{5005a^6b^9}{2x^2}-\frac{9009a^5b^{10}}{5x^{5/3}}-\frac{4095a^4b^{11}}{4x^{4/3}}-\frac{455a^3b^{12}}{x}-\frac{315a^2b^{13}}{2x^{2/3}}-\frac{45ab^{14}}{\sqrt[3]{x}}+b^{15}\log(x)$$

output

```
-1/5*a^15/x^5-45/14*a^14*b/x^(14/3)-315/13*a^13*b^2/x^(13/3)-455/4*a^12*b^
3/x^4-4095/11*a^11*b^4/x^(11/3)-9009/10*a^10*b^5/x^(10/3)-5005/3*a^9*b^6/x
^3-19305/8*a^8*b^7/x^(8/3)-19305/7*a^7*b^8/x^(7/3)-5005/2*a^6*b^9/x^2-9009
/5*a^5*b^10/x^(5/3)-4095/4*a^4*b^11/x^(4/3)-455*a^3*b^12/x-315/2*a^2*b^13/
x^(2/3)-45*a*b^14/x^(1/3)+b^15*ln(x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx$$
$$= \frac{-24024a^{15} - 386100a^{14}b\sqrt[3]{x} - 2910600a^{13}b^2x^{2/3} - 13663650a^{12}b^3x - 44717400a^{11}b^4x^{4/3} - 108216108a}{}$$
$$+ 3b^{15}\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(a + b*x^(1/3))^15/x^6,x]
```

output
```
(-24024*a^15 - 386100*a^14*b*x^(1/3) - 2910600*a^13*b^2*x^(2/3) - 13663650
*a^12*b^3*x - 44717400*a^11*b^4*x^(4/3) - 108216108*a^10*b^5*x^(5/3) - 200
400200*a^9*b^6*x^2 - 289864575*a^8*b^7*x^(7/3) - 331273800*a^7*b^8*x^(8/3)
 - 300600300*a^6*b^9*x^3 - 216432216*a^5*b^10*x^(10/3) - 122972850*a^4*b^1
1*x^(11/3) - 54654600*a^3*b^12*x^4 - 18918900*a^2*b^13*x^(13/3) - 5405400*
a*b^14*x^(14/3))/(120120*x^5) + 3*b^15*Log[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx$$
$$\Big\downarrow \ 798$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{16/3}}\,d\sqrt[3]{x}$$
$$\Big\downarrow \ 49$$

$$3\int \left(\frac{a^{15}}{x^{16/3}}+\frac{15ba^{14}}{x^5}+\frac{105b^2a^{13}}{x^{14/3}}+\frac{455b^3a^{12}}{x^{13/3}}+\frac{1365b^4a^{11}}{x^4}+\frac{3003b^5a^{10}}{x^{11/3}}+\frac{5005b^6a^9}{x^{10/3}}+\frac{6435b^7a^8}{x^3}+\frac{6435b^8a^7}{x^{8/3}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{15}}{15x^5}-\frac{15a^{14}b}{14x^{14/3}}-\frac{105a^{13}b^2}{13x^{13/3}}-\frac{455a^{12}b^3}{12x^4}-\frac{1365a^{11}b^4}{11x^{11/3}}-\frac{3003a^{10}b^5}{10x^{10/3}}-\frac{5005a^9b^6}{9x^3}-\frac{6435a^8b^7}{8x^{8/3}}-\frac{6435a^7b^8}{7x^{7/3}}-\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^6,x]
```

output
```
3*(-1/15*a^15/x^5 - (15*a^14*b)/(14*x^(14/3)) - (105*a^13*b^2)/(13*x^(13/3
)) - (455*a^12*b^3)/(12*x^4) - (1365*a^11*b^4)/(11*x^(11/3)) - (3003*a^10*
b^5)/(10*x^(10/3)) - (5005*a^9*b^6)/(9*x^3) - (6435*a^8*b^7)/(8*x^(8/3)) -
 (6435*a^7*b^8)/(7*x^(7/3)) - (5005*a^6*b^9)/(6*x^2) - (3003*a^5*b^10)/(5*
x^(5/3)) - (1365*a^4*b^11)/(4*x^(4/3)) - (455*a^3*b^12)/(3*x) - (105*a^2*b
^13)/(2*x^(2/3)) - (15*a*b^14)/x^(1/3) + b^15*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.77 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{a^{15}}{5x^5}-\frac{45a^{14}b}{14x^{\frac{14}{3}}}-\frac{315a^{13}b^2}{13x^{\frac{13}{3}}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{\frac{11}{3}}}-\frac{9009a^{10}b^5}{10x^{\frac{10}{3}}}-\frac{5005a^9b^6}{3x^3}-\frac{19305a^8b^7}{8x^{\frac{8}{3}}}-\frac{19305a^7}{7x^{\frac{7}{3}}}$
default	$-\frac{a^{15}}{5x^5}-\frac{45a^{14}b}{14x^{\frac{14}{3}}}-\frac{315a^{13}b^2}{13x^{\frac{13}{3}}}-\frac{455a^{12}b^3}{4x^4}-\frac{4095a^{11}b^4}{11x^{\frac{11}{3}}}-\frac{9009a^{10}b^5}{10x^{\frac{10}{3}}}-\frac{5005a^9b^6}{3x^3}-\frac{19305a^8b^7}{8x^{\frac{8}{3}}}-\frac{19305a^7}{7x^{\frac{7}{3}}}$
trager	$\frac{(-1+x)(12x^4a^{12}+6825x^4b^3a^9+100100x^4b^6a^6+150150a^3b^9x^4+27300x^4b^{12}+12a^{12}x^3+6825a^9x^3b^3+100100b^6x^3a^6+}{60x^5}$

input
```
int((a+b*x^(1/3))^15/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^15/x^5-45/14*a^14*b/x^(14/3)-315/13*a^13*b^2/x^(13/3)-455/4*a^12*b^
3/x^4-4095/11*a^11*b^4/x^(11/3)-9009/10*a^10*b^5/x^(10/3)-5005/3*a^9*b^6/x
^3-19305/8*a^8*b^7/x^(8/3)-19305/7*a^7*b^8/x^(7/3)-5005/2*a^6*b^9/x^2-9009
/5*a^5*b^10/x^(5/3)-4095/4*a^4*b^11/x^(4/3)-455*a^3*b^12/x-315/2*a^2*b^13/
x^(2/3)-45*a*b^14/x^(1/3)+b^15*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx$$
$$=\frac{360360\,b^{15}x^5\log\left(x^{\frac{1}{3}}\right)-54654600\,a^3b^{12}x^4-300600300\,a^6b^9x^3-200400200\,a^9b^6x^2-13663650\,a^{12}b^3x-}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^6,x, algorithm="fricas")
```

output
```
1/120120*(360360*b^15*x^5*log(x^(1/3)) - 54654600*a^3*b^12*x^4 - 300600300
*a^6*b^9*x^3 - 200400200*a^9*b^6*x^2 - 13663650*a^12*b^3*x - 24024*a^15 -
594*(9100*a*b^14*x^4 + 207025*a^4*b^11*x^3 + 557700*a^7*b^8*x^2 + 182182*a
^10*b^5*x + 4900*a^13*b^2)*x^(2/3) - 351*(53900*a^2*b^13*x^4 + 616616*a^5*
b^10*x^3 + 825825*a^8*b^7*x^2 + 127400*a^11*b^4*x + 1100*a^14*b)*x^(1/3))/
x^5
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx = -\frac{a^{15}}{5x^5} - \frac{45a^{14}b}{14x^{\frac{14}{3}}} - \frac{315a^{13}b^2}{13x^{\frac{13}{3}}} - \frac{455a^{12}b^3}{4x^4} - \frac{4095a^{11}b^4}{11x^{\frac{11}{3}}} - \frac{9009a^{10}b^5}{10x^{\frac{10}{3}}} - \frac{5005a^9b^6}{3x^3} - \frac{19305a^8b^7}{8x^{\frac{8}{3}}} - \frac{19305a^7b^8}{7x^{\frac{7}{3}}} - \frac{5005a^6b^9}{2x^2} - \frac{9009a^5b^{10}}{5x^{\frac{5}{3}}} - \frac{4095a^4b^{11}}{4x^{\frac{4}{3}}} - \frac{455a^3b^{12}}{x} - \frac{315a^2b^{13}}{2x^{\frac{2}{3}}} - \frac{45ab^{14}}{\sqrt[3]{x}} + b^{15}\log(x)$$

input
```
integrate((a+b*x**(1/3))**15/x**6,x)
```

output
```
-a**15/(5*x**5) - 45*a**14*b/(14*x**(14/3)) - 315*a**13*b**2/(13*x**(13/3)
) - 455*a**12*b**3/(4*x**4) - 4095*a**11*b**4/(11*x**(11/3)) - 9009*a**10*
b**5/(10*x**(10/3)) - 5005*a**9*b**6/(3*x**3) - 19305*a**8*b**7/(8*x**(8/3
)) - 19305*a**7*b**8/(7*x**(7/3)) - 5005*a**6*b**9/(2*x**2) - 9009*a**5*b*
*10/(5*x**(5/3)) - 4095*a**4*b**11/(4*x**(4/3)) - 455*a**3*b**12/x - 315*a
**2*b**13/(2*x**(2/3)) - 45*a*b**14/x**(1/3) + b**15*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx = b^{15}\log(x) - \frac{5405400\,ab^{14}x^{\frac{14}{3}} + 18918900\,a^2b^{13}x^{\frac{13}{3}} + 54654600\,a^3b^{12}x^4 + 122972850\,a^4b^{11}x^{\frac{11}{3}} + 216432216\,a^5b^{10}x^{\frac{10}{3}}}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^6,x, algorithm="maxima")
```

output
```
b^15*log(x) - 1/120120*(5405400*a*b^14*x^(14/3) + 18918900*a^2*b^13*x^(13/
3) + 54654600*a^3*b^12*x^4 + 122972850*a^4*b^11*x^(11/3) + 216432216*a^5*b
^10*x^(10/3) + 300600300*a^6*b^9*x^3 + 331273800*a^7*b^8*x^(8/3) + 2898645
75*a^8*b^7*x^(7/3) + 200400200*a^9*b^6*x^2 + 108216108*a^10*b^5*x^(5/3) +
44717400*a^11*b^4*x^(4/3) + 13663650*a^12*b^3*x + 2910600*a^13*b^2*x^(2/3)
 + 386100*a^14*b*x^(1/3) + 24024*a^15)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx = b^{15}\log\left(|x|\right)$$
$$-\frac{5405400\,ab^{14}x^{\frac{14}{3}} + 18918900\,a^2b^{13}x^{\frac{13}{3}} + 54654600\,a^3b^{12}x^4 + 122972850\,a^4b^{11}x^{\frac{11}{3}} + 216432216\,a^5b^{10}x^{\frac{10}{3}}}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^6,x, algorithm="giac")
```

output
```
b^15*log(abs(x)) - 1/120120*(5405400*a*b^14*x^(14/3) + 18918900*a^2*b^13*x
^(13/3) + 54654600*a^3*b^12*x^4 + 122972850*a^4*b^11*x^(11/3) + 216432216*
a^5*b^10*x^(10/3) + 300600300*a^6*b^9*x^3 + 331273800*a^7*b^8*x^(8/3) + 28
9864575*a^8*b^7*x^(7/3) + 200400200*a^9*b^6*x^2 + 108216108*a^10*b^5*x^(5/
3) + 44717400*a^11*b^4*x^(4/3) + 13663650*a^12*b^3*x + 2910600*a^13*b^2*x^
(2/3) + 386100*a^14*b*x^(1/3) + 24024*a^15)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx = 3\,b^{15}\,\ln\left(x^{1/3}\right) - \frac{a^{15}}{5\,x^5} - \frac{45\,a\,b^{14}}{x^{1/3}} - \frac{45\,a^{14}\,b}{14\,x^{14/3}} - \frac{455\,a^3\,b^{12}}{x} - \frac{5005\,a^6\,b^9}{2\,x^2} - \frac{5005\,a^9\,b^6}{3\,x^3} - \frac{455\,a^{12}\,b^3}{4\,x^4} - \frac{315\,a^2\,b^{13}}{2\,x^{2/3}} - \frac{4095\,a^4\,b^{11}}{4\,x^{4/3}} - \frac{9009\,a^5\,b^{10}}{5\,x^{5/3}} - \frac{19305\,a^7\,b^8}{7\,x^{7/3}} - \frac{19305\,a^8\,b^7}{8\,x^{8/3}} - \frac{9009\,a^{10}\,b^5}{10\,x^{10/3}} - \frac{4095\,a^{11}\,b^4}{11\,x^{11/3}} - \frac{315\,a^{13}\,b^2}{13\,x^{13/3}}$$

input
```
int((a + b*x^(1/3))^15/x^6,x)
```

output
```
3*b^15*log(x^(1/3)) - a^15/(5*x^5) - (45*a*b^14)/x^(1/3) - (45*a^14*b)/(14
*x^(14/3)) - (455*a^3*b^12)/x - (5005*a^6*b^9)/(2*x^2) - (5005*a^9*b^6)/(3
*x^3) - (455*a^12*b^3)/(4*x^4) - (315*a^2*b^13)/(2*x^(2/3)) - (4095*a^4*b^
11)/(4*x^(4/3)) - (9009*a^5*b^10)/(5*x^(5/3)) - (19305*a^7*b^8)/(7*x^(7/3)
) - (19305*a^8*b^7)/(8*x^(8/3)) - (9009*a^10*b^5)/(10*x^(10/3)) - (4095*a^
11*b^4)/(11*x^(11/3)) - (315*a^13*b^2)/(13*x^(13/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,dx$$
$$= \frac{360360x^{\frac{17}{3}}\log\left(x^{\frac{1}{3}}\right)b^{15} - 24024x^{\frac{2}{3}}a^{15} - 13663650x^{\frac{5}{3}}a^{12}b^3 - 200400200x^{\frac{8}{3}}a^9b^6 - 300600300x^{\frac{11}{3}}a^6b^9 - 54}{}$$

input
```
int((a+b*x^(1/3))^15/x^6,x)
```

output
```
(360360*x**(2/3)*log(x**(1/3))*b**15*x**5 - 24024*x**(2/3)*a**15 - 1366365
0*x**(2/3)*a**12*b**3*x - 200400200*x**(2/3)*a**9*b**6*x**2 - 300600300*x*
*(2/3)*a**6*b**9*x**3 - 54654600*x**(2/3)*a**3*b**12*x**4 - 2910600*x**(1/
3)*a**13*b**2*x - 108216108*x**(1/3)*a**10*b**5*x**2 - 331273800*x**(1/3)*
a**7*b**8*x**3 - 122972850*x**(1/3)*a**4*b**11*x**4 - 5405400*x**(1/3)*a*b
**14*x**5 - 386100*a**14*b*x - 44717400*a**11*b**4*x**2 - 289864575*a**8*b
**7*x**3 - 216432216*a**5*b**10*x**4 - 18918900*a**2*b**13*x**5)/(120120*x
**(2/3)*x**5)
```

3.241 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx=-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{6ax^6}+\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{51a^2x^{17/3}}-\frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{816a^3x^{16/3}}$$

output
```
-1/6*(a+b*x^(1/3))^16/a/x^6+1/51*b*(a+b*x^(1/3))^16/a^2/x^(17/3)-1/816*b^2
*(a+b*x^(1/3))^16/a^3/x^(16/3)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(72) = 144.

Time = 0.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.62

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx$$
$$=\frac{-136a^{15}-2160a^{14}b\sqrt[3]{x}-16065a^{13}b^2x^{2/3}-74256a^{12}b^3x-238680a^{11}b^4x^{4/3}-565488a^{10}b^5x^{5/3}-10210}{}$$

input
```
Integrate[(a + b*x^(1/3))^15/x^7,x]
```

output
```
(-136*a^15 - 2160*a^14*b*x^(1/3) - 16065*a^13*b^2*x^(2/3) - 74256*a^12*b^3
*x - 238680*a^11*b^4*x^(4/3) - 565488*a^10*b^5*x^(5/3) - 1021020*a^9*b^6*x
^2 - 1432080*a^8*b^7*x^(7/3) - 1575288*a^7*b^8*x^(8/3) - 1361360*a^6*b^9*x
^3 - 918918*a^5*b^10*x^(10/3) - 477360*a^4*b^11*x^(11/3) - 185640*a^3*b^12
*x^4 - 51408*a^2*b^13*x^(13/3) - 9180*a*b^14*x^(14/3) - 816*b^15*x^5)/(816
*x^6)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{19/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}d\sqrt[3]{x}}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^6}\right)$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{b\left(-\frac{b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{17/3}}d\sqrt[3]{x}}{17a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{17ax^{17/3}}\right)}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^6}\right)$$

$$\Big\downarrow \text{48}$$

$$3\left(-\frac{b\left(\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{272a^2x^{16/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{17ax^{17/3}}\right)}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^6}\right)$$

input
```
Int[(a + b*x^(1/3))^15/x^7,x]
```

output
```
3*(-1/9*(b*(-1/17*(a + b*x^(1/3))^16/(a*x^(17/3)) + (b*(a + b*x^(1/3))^16)
/(272*a^2*x^(16/3))))/a - (a + b*x^(1/3))^16/(18*a*x^6))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(56) = 112$.

Time = 27.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.33

method	result
derivativedivides	$-\frac{5005a^6b^9}{3x^3}-\frac{45a\,b^{14}}{4x^{\frac{4}{3}}}-\frac{a^{15}}{6x^6}-\frac{455a^3b^{12}}{2x^2}-\frac{91a^{12}b^3}{x^5}-\frac{585a^{11}b^4}{2x^{\frac{14}{3}}}-\frac{693a^{10}b^5}{x^{\frac{13}{3}}}-\frac{585a^4b^{11}}{x^{\frac{7}{3}}}-\frac{5005a^9b^6}{4x^4}-$
default	$-\frac{5005a^6b^9}{3x^3}-\frac{45a\,b^{14}}{4x^{\frac{4}{3}}}-\frac{a^{15}}{6x^6}-\frac{455a^3b^{12}}{2x^2}-\frac{91a^{12}b^3}{x^5}-\frac{585a^{11}b^4}{2x^{\frac{14}{3}}}-\frac{693a^{10}b^5}{x^{\frac{13}{3}}}-\frac{585a^4b^{11}}{x^{\frac{7}{3}}}-\frac{5005a^9b^6}{4x^4}-$
trager	$\frac{(-1+x)\left(2a^{15}x^5+1092a^{12}b^3x^5+15015x^5b^6a^9+20020x^5b^9a^6+2730a^3b^{12}x^5+12b^{15}x^5+2x^4a^{15}+1092b^3x^4a^{12}+15015x\right.}{}$
orering	$-\frac{(18768b^{39}x^{13}-218790a^3b^{36}x^{12}+1679158a^6b^{33}x^{11}-2021776a^9b^{30}x^{10}+4206124a^{12}b^{27}x^9+2925637a^{15}b^{24}x^8+4705}{}$

input

```
int((a+b*x^(1/3))^15/x^7,x,method=_RETURNVERBOSE)
```

output

```
-5005/3*a^6*b^9/x^3-45/4*a*b^14/x^(4/3)-1/6*a^15/x^6-455/2*a^3*b^12/x^2-91
*a^12*b^3/x^5-585/2*a^11*b^4/x^(14/3)-693*a^10*b^5/x^(13/3)-585*a^4*b^11/x
^(7/3)-5005/4*a^9*b^6/x^4-3861/2*a^7*b^8/x^(10/3)-b^15/x-315/16*a^13*b^2/x
^(16/3)-63*a^2*b^13/x^(5/3)-45/17*a^14*b/x^(17/3)-1755*a^8*b^7/x^(11/3)-90
09/8*a^5*b^10/x^(8/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(56) = 112$.

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.35

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx =$$
$$-\frac{816\,b^{15}x^5+185640\,a^3b^{12}x^4+1361360\,a^6b^9x^3+1021020\,a^9b^6x^2+74256\,a^{12}b^3x+136\,a^{15}+459\,(20\,ab}{}$$

input

```
integrate((a+b*x^(1/3))^15/x^7,x, algorithm="fricas")
```

output
```
-1/816*(816*b^15*x^5 + 185640*a^3*b^12*x^4 + 1361360*a^6*b^9*x^3 + 1021020
*a^9*b^6*x^2 + 74256*a^12*b^3*x + 136*a^15 + 459*(20*a*b^14*x^4 + 1040*a^4
*b^11*x^3 + 3432*a^7*b^8*x^2 + 1232*a^10*b^5*x + 35*a^13*b^2)*x^(2/3) + 54
*(952*a^2*b^13*x^4 + 17017*a^5*b^10*x^3 + 26520*a^8*b^7*x^2 + 4420*a^11*b^
4*x + 40*a^14*b)*x^(1/3))/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(61) = 122$.

Time = 0.95 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.90

$$\int \frac{(a+b\sqrt[3]{x})^{15}}{x^7}\,dx = -\frac{a^{15}}{6x^6} - \frac{45a^{14}b}{17x^{\frac{17}{3}}} - \frac{315a^{13}b^2}{16x^{\frac{16}{3}}} - \frac{91a^{12}b^3}{x^5} - \frac{585a^{11}b^4}{2x^{\frac{14}{3}}} - \frac{693a^{10}b^5}{x^{\frac{13}{3}}} - \frac{5005a^9b^6}{4x^4} - \frac{1755a^8b^7}{x^{\frac{11}{3}}} - \frac{3861a^7b^8}{2x^{\frac{10}{3}}} - \frac{5005a^6b^9}{3x^3} - \frac{9009a^5b^{10}}{8x^{\frac{8}{3}}} - \frac{585a^4b^{11}}{x^{\frac{7}{3}}} - \frac{455a^3b^{12}}{2x^2} - \frac{63a^2b^{13}}{x^{\frac{5}{3}}} - \frac{45ab^{14}}{4x^{\frac{4}{3}}} - \frac{b^{15}}{x}$$

input
```
integrate((a+b*x**(1/3))**15/x**7,x)
```

output
```
-a**15/(6*x**6) - 45*a**14*b/(17*x**(17/3)) - 315*a**13*b**2/(16*x**(16/3)
) - 91*a**12*b**3/x**5 - 585*a**11*b**4/(2*x**(14/3)) - 693*a**10*b**5/x**
(13/3) - 5005*a**9*b**6/(4*x**4) - 1755*a**8*b**7/x**(11/3) - 3861*a**7*b*
*8/(2*x**(10/3)) - 5005*a**6*b**9/(3*x**3) - 9009*a**5*b**10/(8*x**(8/3))
- 585*a**4*b**11/x**(7/3) - 455*a**3*b**12/(2*x**2) - 63*a**2*b**13/x**(5/
3) - 45*a*b**14/(4*x**(4/3)) - b**15/x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(56) = 112$.

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.32

$$\int \frac{(a+b\sqrt[3]{x})^{15}}{x^7}\,dx = -\frac{816\,b^{15}x^5 + 9180\,ab^{14}x^{\frac{14}{3}} + 51408\,a^2b^{13}x^{\frac{13}{3}} + 185640\,a^3b^{12}x^4 + 477360\,a^4b^{11}x^{\frac{11}{3}} + 918918\,a^5b^{10}x^{\frac{10}{3}} + 1}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^7,x, algorithm="maxima")
```

output
```
-1/816*(816*b^15*x^5 + 9180*a*b^14*x^(14/3) + 51408*a^2*b^13*x^(13/3) + 18
5640*a^3*b^12*x^4 + 477360*a^4*b^11*x^(11/3) + 918918*a^5*b^10*x^(10/3) +
1361360*a^6*b^9*x^3 + 1575288*a^7*b^8*x^(8/3) + 1432080*a^8*b^7*x^(7/3) +
1021020*a^9*b^6*x^2 + 565488*a^10*b^5*x^(5/3) + 238680*a^11*b^4*x^(4/3) +
74256*a^12*b^3*x + 16065*a^13*b^2*x^(2/3) + 2160*a^14*b*x^(1/3) + 136*a^15
)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.32

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx = -\frac{816\,b^{15}x^5 + 9180\,ab^{14}x^{\frac{14}{3}} + 51408\,a^2b^{13}x^{\frac{13}{3}} + 185640\,a^3b^{12}x^4 + 477360\,a^4b^{11}x^{\frac{11}{3}} + 918918\,a^5b^{10}x^{\frac{10}{3}} + 1}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^7,x, algorithm="giac")
```

output
```
-1/816*(816*b^15*x^5 + 9180*a*b^14*x^(14/3) + 51408*a^2*b^13*x^(13/3) + 18
5640*a^3*b^12*x^4 + 477360*a^4*b^11*x^(11/3) + 918918*a^5*b^10*x^(10/3) +
1361360*a^6*b^9*x^3 + 1575288*a^7*b^8*x^(8/3) + 1432080*a^8*b^7*x^(7/3) +
1021020*a^9*b^6*x^2 + 565488*a^10*b^5*x^(5/3) + 238680*a^11*b^4*x^(4/3) +
74256*a^12*b^3*x + 16065*a^13*b^2*x^(2/3) + 2160*a^14*b*x^(1/3) + 136*a^15
)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.31

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx = -\frac{\frac{a^{15}}{6}+b^{15}\,x^5+91\,a^{12}\,b^3\,x+\frac{45\,a^{14}\,b\,x^{1/3}}{17}+\frac{45\,a\,b^{14}\,x^{14/3}}{4}+\frac{5005\,a^9\,b^6\,x^2}{4}+\frac{5005\,a^6\,b^9\,x^3}{3}+\frac{455\,a^3\,b^{12}\,x^4}{2}+\frac{315\,a^{13}\,b^2\,x^{2/}}{16}$$

input
```
int((a + b*x^(1/3))^15/x^7,x)
```

output
```
-(a^15/6 + b^15*x^5 + 91*a^12*b^3*x + (45*a^14*b*x^(1/3))/17 + (45*a*b^14*
x^(14/3))/4 + (5005*a^9*b^6*x^2)/4 + (5005*a^6*b^9*x^3)/3 + (455*a^3*b^12*
x^4)/2 + (315*a^13*b^2*x^(2/3))/16 + (585*a^11*b^4*x^(4/3))/2 + 693*a^10*b
^5*x^(5/3) + 1755*a^8*b^7*x^(7/3) + (3861*a^7*b^8*x^(8/3))/2 + (9009*a^5*b
^10*x^(10/3))/8 + 585*a^4*b^11*x^(11/3) + 63*a^2*b^13*x^(13/3))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.36

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}\,dx = \frac{-136x^{\frac{2}{3}}a^{15}-74256x^{\frac{5}{3}}a^{12}b^3-1021020x^{\frac{8}{3}}a^9b^6-1361360x^{\frac{11}{3}}a^6b^9-185640x^{\frac{14}{3}}a^3b^{12}-816x^{\frac{17}{3}}b^{15}-1606}{}$$

input
```
int((a+b*x^(1/3))^15/x^7,x)
```

output
```
( - 136*x**(2/3)*a**15 - 74256*x**(2/3)*a**12*b**3*x - 1021020*x**(2/3)*a*
*9*b**6*x**2 - 1361360*x**(2/3)*a**6*b**9*x**3 - 185640*x**(2/3)*a**3*b**1
2*x**4 - 816*x**(2/3)*b**15*x**5 - 16065*x**(1/3)*a**13*b**2*x - 565488*x*
*(1/3)*a**10*b**5*x**2 - 1575288*x**(1/3)*a**7*b**8*x**3 - 477360*x**(1/3)
*a**4*b**11*x**4 - 9180*x**(1/3)*a*b**14*x**5 - 2160*a**14*b*x - 238680*a*
*11*b**4*x**2 - 1432080*a**8*b**7*x**3 - 918918*a**5*b**10*x**4 - 51408*a*
*2*b**13*x**5)/(816*x**(2/3)*x**6)
```

3.242 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx=-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{7ax^7}+\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{28a^2x^{20/3}}-\frac{b^2\left(a+b\sqrt[3]{x}\right)^{16}}{133a^3x^{19/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{798a^4x^6}-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{6783a^5x^{17/3}}+\frac{b^5\left(a+b\sqrt[3]{x}\right)^{16}}{108528a^6x^{16/3}}$$

output
```
-1/7*(a+b*x^(1/3))^16/a/x^7+1/28*b*(a+b*x^(1/3))^16/a^2/x^(20/3)-1/133*b^2
*(a+b*x^(1/3))^16/a^3/x^(19/3)+1/798*b^3*(a+b*x^(1/3))^16/a^4/x^6-1/6783*b
^4*(a+b*x^(1/3))^16/a^5/x^(17/3)+1/108528*b^5*(a+b*x^(1/3))^16/a^6/x^(16/3
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.28

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx$$
$$=\frac{-15504a^{15}-244188a^{14}b\sqrt[3]{x}-1799280a^{13}b^2x^{2/3}-8230040a^{12}b^3x-26142480a^{11}b^4x^{4/3}-61108047a^{10}b}{}$$

input
```
Integrate[(a + b*x^(1/3))^15/x^8,x]
```

output
```
(-15504*a^15 - 244188*a^14*b*x^(1/3) - 1799280*a^13*b^2*x^(2/3) - 8230040*
a^12*b^3*x - 26142480*a^11*b^4*x^(4/3) - 61108047*a^10*b^5*x^(5/3) - 10863
6528*a^9*b^6*x^2 - 149652360*a^8*b^7*x^(7/3) - 161164080*a^7*b^8*x^(8/3) -
 135795660*a^6*b^9*x^3 - 88884432*a^5*b^10*x^(10/3) - 44442216*a^4*b^11*x^
(11/3) - 16460080*a^3*b^12*x^4 - 4273290*a^2*b^13*x^(13/3) - 697680*a*b^14
*x^(14/3) - 54264*b^15*x^5)/(108528*x^7)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{22/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{5b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}d\sqrt[3]{x}}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^7}\right)$$

$$\Big\downarrow \text{55}$$

$$3\left(-\frac{5b\left(-\frac{b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{20/3}}d\sqrt[3]{x}}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^7}\right)$$

$$\Big\downarrow \; 55$$

$$3\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{19/3}}d\sqrt[3]{x}}{19a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}}\right)}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^{7}}\right)$$

$$\Big\downarrow \; 55$$

$$3\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\int\frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}d\sqrt[3]{x}}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^6}\right)}{19a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}}\right)}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^7}\right)$$

$$\Big\downarrow \; 55$$

$$3\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(-\frac{b\int\frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{17/3}}d\sqrt[3]{x}}{17a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{17ax^{17/3}}\right)}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^{6}}\right)}{19a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}}\right)}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{(a+b\sqrt[3]{x})}{21ax^{7}}\right.$$

$$\Big\downarrow \; 48$$

$$3\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{272a^2x^{16/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{17ax^{17/3}}\right)}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^6}\right)}{19a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}}\right)}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^7}\right)$$

input
```
Int[(a + b*x^(1/3))^15/x^8,x]
```

output
```
3*((-5*b*(-1/5*(b*((-3*b*(-1/9*(b*(-1/17*(a + b*x^(1/3))^16/(a*x^(17/3)) +
 (b*(a + b*x^(1/3))^16)/(272*a^2*x^(16/3))))/a - (a + b*x^(1/3))^16/(18*a*
x^6)))/(19*a) - (a + b*x^(1/3))^16/(19*a*x^(19/3))))/a - (a + b*x^(1/3))^1
6/(20*a*x^(20/3))))/(21*a) - (a + b*x^(1/3))^16/(21*a*x^7))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 26.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.14

method	result
derivativedivides	$-\frac{315a^2b^{13}}{8x^{\frac{8}{3}}}-\frac{315a^{13}b^2}{19x^{\frac{19}{3}}}-\frac{9009a^{10}b^5}{16x^{\frac{16}{3}}}-\frac{45a\,b^{14}}{7x^{\frac{7}{3}}}-\frac{819a^4b^{11}}{2x^{\frac{10}{3}}}-\frac{1485a^7b^8}{x^{\frac{13}{3}}}-\frac{455a^3b^{12}}{3x^3}-\frac{9a^{14}b}{4x^{\frac{20}{3}}}-\frac{455a^{12}b}{6x^6}$
default	$-\frac{315a^2b^{13}}{8x^{\frac{8}{3}}}-\frac{315a^{13}b^2}{19x^{\frac{19}{3}}}-\frac{9009a^{10}b^5}{16x^{\frac{16}{3}}}-\frac{45a\,b^{14}}{7x^{\frac{7}{3}}}-\frac{819a^4b^{11}}{2x^{\frac{10}{3}}}-\frac{1485a^7b^8}{x^{\frac{13}{3}}}-\frac{455a^3b^{12}}{3x^3}-\frac{9a^{14}b}{4x^{\frac{20}{3}}}-\frac{455a^{12}b}{6x^6}$
trager	$\frac{(-1+x)(12a^{15}x^6+6370a^{12}b^3x^6+84084x^6b^6a^9+105105x^6b^9a^6+12740a^3b^{12}x^6+42b^{15}x^6+12a^{15}x^5+6370a^{12}b^3x^5+84}{}$
orering	$-\frac{(7393470b^{39}x^{13}+25513124a^3b^{36}x^{12}+239723494a^6b^{33}x^{11}+685802663a^9b^{30}x^{10}+1677560885a^{12}b^{27}x^9+289062974}{}$

input
```
int((a+b*x^(1/3))^15/x^8,x,method=_RETURNVERBOSE)
```

output
```
-315/8*a^2*b^13/x^(8/3)-315/19*a^13*b^2/x^(19/3)-9009/16*a^10*b^5/x^(16/3)
-45/7*a*b^14/x^(7/3)-819/2*a^4*b^11/x^(10/3)-1485*a^7*b^8/x^(13/3)-455/3*a
^3*b^12/x^3-9/4*a^14*b/x^(20/3)-455/6*a^12*b^3/x^6-4095/17*a^11*b^4/x^(17/
3)-819*a^5*b^10/x^(11/3)-19305/14*a^8*b^7/x^(14/3)-5005/4*a^6*b^9/x^4-1/7*
a^15/x^7-1/2*b^15/x^2-1001*a^9*b^6/x^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx =$$
$$-\frac{54264\,b^{15}x^5+16460080\,a^3b^{12}x^4+135795660\,a^6b^9x^3+108636528\,a^9b^6x^2+8230040\,a^{12}b^3x+15504\,a^1}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^8,x, algorithm="fricas")
```

output
```
-1/108528*(54264*b^15*x^5 + 16460080*a^3*b^12*x^4 + 135795660*a^6*b^9*x^3
+ 108636528*a^9*b^6*x^2 + 8230040*a^12*b^3*x + 15504*a^15 + 459*(1520*a*b^
14*x^4 + 96824*a^4*b^11*x^3 + 351120*a^7*b^8*x^2 + 133133*a^10*b^5*x + 392
0*a^13*b^2)*x^(2/3) + 1026*(4165*a^2*b^13*x^4 + 86632*a^5*b^10*x^3 + 14586
0*a^8*b^7*x^2 + 25480*a^11*b^4*x + 238*a^14*b)*x^(1/3))/x^7
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx = -\frac{a^{15}}{7x^7} - \frac{9a^{14}b}{4x^{\frac{20}{3}}} - \frac{315a^{13}b^2}{19x^{\frac{19}{3}}} - \frac{455a^{12}b^3}{6x^6} - \frac{4095a^{11}b^4}{17x^{\frac{17}{3}}} - \frac{9009a^{10}b^5}{16x^{\frac{16}{3}}} - \frac{1001a^9b^6}{x^5} - \frac{19305a^8b^7}{14x^{\frac{14}{3}}} - \frac{1485a^7b^8}{x^{\frac{13}{3}}} - \frac{5005a^6b^9}{4x^4} - \frac{819a^5b^{10}}{x^{\frac{11}{3}}} - \frac{819a^4b^{11}}{2x^{\frac{10}{3}}} - \frac{455a^3b^{12}}{3x^3} - \frac{315a^2b^{13}}{8x^{\frac{8}{3}}} - \frac{45ab^{14}}{7x^{\frac{7}{3}}} - \frac{b^{15}}{2x^2}$$

input
```
integrate((a+b*x**(1/3))**15/x**8,x)
```

output
```
-a**15/(7*x**7) - 9*a**14*b/(4*x**(20/3)) - 315*a**13*b**2/(19*x**(19/3))
- 455*a**12*b**3/(6*x**6) - 4095*a**11*b**4/(17*x**(17/3)) - 9009*a**10*b*
*5/(16*x**(16/3)) - 1001*a**9*b**6/x**5 - 19305*a**8*b**7/(14*x**(14/3)) -
 1485*a**7*b**8/x**(13/3) - 5005*a**6*b**9/(4*x**4) - 819*a**5*b**10/x**(1
1/3) - 819*a**4*b**11/(2*x**(10/3)) - 455*a**3*b**12/(3*x**3) - 315*a**2*b
**13/(8*x**(8/3)) - 45*a*b**14/(7*x**(7/3)) - b**15/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx = -\frac{54264\,b^{15}x^5 + 697680\,ab^{14}x^{\frac{14}{3}} + 4273290\,a^2b^{13}x^{\frac{13}{3}} + 16460080\,a^3b^{12}x^4 + 44442216\,a^4b^{11}x^{\frac{11}{3}} + 888844}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^8,x, algorithm="maxima")
```

output
```
-1/108528*(54264*b^15*x^5 + 697680*a*b^14*x^(14/3) + 4273290*a^2*b^13*x^(1
3/3) + 16460080*a^3*b^12*x^4 + 44442216*a^4*b^11*x^(11/3) + 88884432*a^5*b
^10*x^(10/3) + 135795660*a^6*b^9*x^3 + 161164080*a^7*b^8*x^(8/3) + 1496523
60*a^8*b^7*x^(7/3) + 108636528*a^9*b^6*x^2 + 61108047*a^10*b^5*x^(5/3) + 2
6142480*a^11*b^4*x^(4/3) + 8230040*a^12*b^3*x + 1799280*a^13*b^2*x^(2/3) +
 244188*a^14*b*x^(1/3) + 15504*a^15)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\sqrt[3]{x})^{15}}{x^8}\,dx = -\frac{54264\,b^{15}x^5+697680\,ab^{14}x^{\frac{14}{3}}+4273290\,a^2b^{13}x^{\frac{13}{3}}+16460080\,a^3b^{12}x^4+44442216\,a^4b^{11}x^{\frac{11}{3}}+888844}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^8,x, algorithm="giac")
```

output
```
-1/108528*(54264*b^15*x^5 + 697680*a*b^14*x^(14/3) + 4273290*a^2*b^13*x^(1
3/3) + 16460080*a^3*b^12*x^4 + 44442216*a^4*b^11*x^(11/3) + 88884432*a^5*b
^10*x^(10/3) + 135795660*a^6*b^9*x^3 + 161164080*a^7*b^8*x^(8/3) + 1496523
60*a^8*b^7*x^(7/3) + 108636528*a^9*b^6*x^2 + 61108047*a^10*b^5*x^(5/3) + 2
6142480*a^11*b^4*x^(4/3) + 8230040*a^12*b^3*x + 1799280*a^13*b^2*x^(2/3) +
 244188*a^14*b*x^(1/3) + 15504*a^15)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\sqrt[3]{x})^{15}}{x^8}\,dx = -\frac{\frac{a^{15}}{7}+\frac{b^{15}\,x^5}{2}+\frac{455\,a^{12}\,b^3\,x}{6}+\frac{9\,a^{14}\,b\,x^{1/3}}{4}+\frac{45\,a\,b^{14}\,x^{14/3}}{7}+1001\,a^9\,b^6\,x^2+\frac{5005\,a^6\,b^9\,x^3}{4}+\frac{455\,a^3\,b^{12}\,x^4}{3}+\frac{315\,a^{13}\,b^2\,x^{2/}}{19}}{}$$

input
```
int((a + b*x^(1/3))^15/x^8,x)
```

output
```
-(a^15/7 + (b^15*x^5)/2 + (455*a^12*b^3*x)/6 + (9*a^14*b*x^(1/3))/4 + (45*
a*b^14*x^(14/3))/7 + 1001*a^9*b^6*x^2 + (5005*a^6*b^9*x^3)/4 + (455*a^3*b^
12*x^4)/3 + (315*a^13*b^2*x^(2/3))/19 + (4095*a^11*b^4*x^(4/3))/17 + (9009
*a^10*b^5*x^(5/3))/16 + (19305*a^8*b^7*x^(7/3))/14 + 1485*a^7*b^8*x^(8/3)
+ 819*a^5*b^10*x^(10/3) + (819*a^4*b^11*x^(11/3))/2 + (315*a^2*b^13*x^(13/
3))/8)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}\,dx$$
$$= \frac{-15504x^{\frac{2}{3}}a^{15}-8230040x^{\frac{5}{3}}a^{12}b^3-108636528x^{\frac{8}{3}}a^9b^6-135795660x^{\frac{11}{3}}a^6b^9-16460080x^{\frac{14}{3}}a^3b^{12}-54264}{}$$

input
```
int((a+b*x^(1/3))^15/x^8,x)
```

output
```
( - 15504*x**(2/3)*a**15 - 8230040*x**(2/3)*a**12*b**3*x - 108636528*x**(2
/3)*a**9*b**6*x**2 - 135795660*x**(2/3)*a**6*b**9*x**3 - 16460080*x**(2/3)
*a**3*b**12*x**4 - 54264*x**(2/3)*b**15*x**5 - 1799280*x**(1/3)*a**13*b**2
*x - 61108047*x**(1/3)*a**10*b**5*x**2 - 161164080*x**(1/3)*a**7*b**8*x**3
 - 44442216*x**(1/3)*a**4*b**11*x**4 - 697680*x**(1/3)*a*b**14*x**5 - 2441
88*a**14*b*x - 26142480*a**11*b**4*x**2 - 149652360*a**8*b**7*x**3 - 88884
432*a**5*b**10*x**4 - 4273290*a**2*b**13*x**5)/(108528*x**(2/3)*x**7)
```

3.243 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 224

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx=-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{8ax^8}+\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{23a^2x^{23/3}}-\frac{7b^2\left(a+b\sqrt[3]{x}\right)^{16}}{506a^3x^{22/3}}+\frac{b^3\left(a+b\sqrt[3]{x}\right)^{16}}{253a^4x^7}-\frac{b^4\left(a+b\sqrt[3]{x}\right)^{16}}{1012a^5x^{20/3}}+\frac{b^5\left(a+b\sqrt[3]{x}\right)^{16}}{4807a^6x^{19/3}}-\frac{b^6\left(a+b\sqrt[3]{x}\right)^{16}}{28842a^7x^6}+\frac{b^7\left(a+b\sqrt[3]{x}\right)^{16}}{245157a^8x^{17/3}}-\frac{b^8\left(a+b\sqrt[3]{x}\right)^{16}}{3922512a^9x^{16/3}}$$

output
```
-1/8*(a+b*x^(1/3))^16/a/x^8+1/23*b*(a+b*x^(1/3))^16/a^2/x^(23/3)-7/506*b^2
*(a+b*x^(1/3))^16/a^3/x^(22/3)+1/253*b^3*(a+b*x^(1/3))^16/a^4/x^7-1/1012*b
^4*(a+b*x^(1/3))^16/a^5/x^(20/3)+1/4807*b^5*(a+b*x^(1/3))^16/a^6/x^(19/3)-
1/28842*b^6*(a+b*x^(1/3))^16/a^7/x^6+1/245157*b^7*(a+b*x^(1/3))^16/a^8/x^(
17/3)-1/3922512*b^8*(a+b*x^(1/3))^16/a^9/x^(16/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx$$
$$= \frac{-490314a^{15}-7674480a^{14}b\sqrt[3]{x}-56163240a^{13}b^2x^{2/3}-254963280a^{12}b^3x-803134332a^{11}b^4x^{4/3}-18598}{}$$

input
```
Integrate[(a + b*x^(1/3))^15/x^9,x]
```

output
```
(-490314*a^15 - 7674480*a^14*b*x^(1/3) - 56163240*a^13*b^2*x^(2/3) - 25496
3280*a^12*b^3*x - 803134332*a^11*b^4*x^(4/3) - 1859890032*a^10*b^5*x^(5/3)
 - 3272028760*a^9*b^6*x^2 - 4454358480*a^8*b^7*x^(7/3) - 4732755885*a^7*b^
8*x^(8/3) - 3926434512*a^6*b^9*x^3 - 2524136472*a^5*b^10*x^(10/3) - 123559
1280*a^4*b^11*x^(11/3) - 446185740*a^3*b^12*x^4 - 112326480*a^2*b^13*x^(13
/3) - 17651304*a*b^14*x^(14/3) - 1307504*b^15*x^5)/(3922512*x^8)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {798, 55, 55, 55, 55, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx$$
$$\Big\downarrow \; 798$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{25/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \; 55$$

$$3\left(-\frac{b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^8}d\sqrt[3]{x}}{3a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{24ax^8}\right)$$

$\Big\downarrow$ 55

$$3\left(-\frac{b\left(-\frac{7b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{23/3}}d\sqrt[3]{x}}{23a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{23ax^{23/3}}\right)}{3a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{24ax^8}\right)$$

$\Big\downarrow$ 55

$$3\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{22/3}}d\sqrt[3]{x}}{11a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{22ax^{22/3}}\right)}{23a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{23ax^{23/3}}\right)}{3a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{24ax^8}\right)$$

$\Big\downarrow$ 55

$$3\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\int\frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^7}d\sqrt[3]{x}}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^7}\right)}{11a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{22ax^{22/3}}\right)}{23a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{23ax^{23/3}}\right)}{3a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{24ax^8}\right)$$

$$\downarrow 55$$

$$3\left(-\frac{b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\int\frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{20/3}}d\sqrt[3]{x}}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^{7}}\right)}{11a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{22ax^{22/3}}\right)}{23a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{23ax^{23/3}}\right)}{3a}-\frac{(a+b\sqrt[3]{x}}{24ax^{8}}$$

$\downarrow$ 55

$$\left(b\left(-\frac{7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\int\frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{19/3}}d\sqrt[3]{x}}{19a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}}\right)}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^{7}}\right)}{11a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{22ax^{22/3}}\right)}{23a}-\frac{\left(a+b\sqrt[3]{x}\right.}{23ax^{23/}}\right.\right.$$

$\downarrow$ 55

$$\Bigg(\Bigg(\Bigg(7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\int\frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^6}\,d\sqrt[3]{x}}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^6}\right)}{19a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}}\right)}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^7}\right)}{11a}-\frac{\left(a+b\sqrt[3]{3}\right.}{22ax^{22}}\right.$$

$\Big\downarrow$ 55

$$\Bigg(\Bigg(\Bigg(\Bigg(3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(-\frac{b\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{17/3}}\,d\sqrt[3]{x}}{17a} - \frac{\left(a+b\sqrt[3]{x}\right)^{16}}{17ax^{17/3}} \right)}{9a} - \frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^{6}} \right)}{19a} - \frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}} \right)}{5a} - \frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}} \right)}{21a} - \frac{\left(a+ \cdots\right.}{\cdots} \cdots$$

$\downarrow$ 48

$$\Bigg(\Bigg(7b\left(-\frac{3b\left(-\frac{5b\left(-\frac{b\left(-\frac{3b\left(-\frac{b\left(\frac{b\left(a+b\sqrt[3]{x}\right)^{16}}{272a^2x^{16/3}}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{17ax^{17/3}}\right)}{9a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{18ax^6}\right)}{19a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{19ax^{19/3}}\right)}{5a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{20ax^{20/3}}\right)}{21a}-\frac{\left(a+b\sqrt[3]{x}\right)^{16}}{21ax^7}\right)}{11a}\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^9,x]
```

output
```
3*(-1/3*(b*((-7*b*((-3*b*((-5*b*(-1/5*(b*((-3*b*(-1/9*(b*(-1/17*(a + b*x^(
1/3))^16/(a*x^(17/3)) + (b*(a + b*x^(1/3))^16)/(272*a^2*x^(16/3)))))/a - (a
 + b*x^(1/3))^16/(18*a*x^6)))/(19*a) - (a + b*x^(1/3))^16/(19*a*x^(19/3)))
)/a - (a + b*x^(1/3))^16/(20*a*x^(20/3))))/(21*a) - (a + b*x^(1/3))^16/(21
*a*x^7)))/(11*a) - (a + b*x^(1/3))^16/(22*a*x^(22/3))))/(23*a) - (a + b*x^
(1/3))^16/(23*a*x^(23/3))))/a - (a + b*x^(1/3))^16/(24*a*x^8))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 26.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{9a\,b^{14}}{2x^{\frac{10}{3}}}-\frac{819a^{11}b^{4}}{4x^{\frac{20}{3}}}-\frac{65a^{12}b^{3}}{x^{7}}-\frac{315a^{2}b^{13}}{11x^{\frac{11}{3}}}-\frac{a^{15}}{8x^{8}}-\frac{455a^{3}b^{12}}{4x^{4}}-\frac{19305a^{8}b^{7}}{17x^{\frac{17}{3}}}-\frac{5005a^{9}b^{6}}{6x^{6}}-\frac{19305a^{7}b^{8}}{16x^{\frac{16}{3}}}$
default	$-\frac{9a\,b^{14}}{2x^{\frac{10}{3}}}-\frac{819a^{11}b^{4}}{4x^{\frac{20}{3}}}-\frac{65a^{12}b^{3}}{x^{7}}-\frac{315a^{2}b^{13}}{11x^{\frac{11}{3}}}-\frac{a^{15}}{8x^{8}}-\frac{455a^{3}b^{12}}{4x^{4}}-\frac{19305a^{8}b^{7}}{17x^{\frac{17}{3}}}-\frac{5005a^{9}b^{6}}{6x^{6}}-\frac{19305a^{7}b^{8}}{16x^{\frac{16}{3}}}$
trager	$\frac{(-1+x)(3a^{15}x^{7}+1560a^{12}b^{3}x^{7}+20020a^{9}b^{6}x^{7}+24024x^{7}b^{9}a^{6}+2730x^{7}b^{12}a^{3}+8x^{7}b^{15}+3a^{15}x^{6}+1560a^{12}b^{3}x^{6}+20020x^{6}}{}$
orering	$-\frac{(1655775552b^{39}x^{13}+1304205524a^{3}b^{36}x^{12}+6913095280a^{6}b^{33}x^{11}+22914492601a^{9}b^{30}x^{10}+53679593513a^{12}b^{27}x^{9}+9}{}$

input
```
int((a+b*x^(1/3))^15/x^9,x,method=_RETURNVERBOSE)
```

output
```
-9/2*a*b^14/x^(10/3)-819/4*a^11*b^4/x^(20/3)-65*a^12*b^3/x^7-315/11*a^2*b^
13/x^(11/3)-1/8*a^15/x^8-455/4*a^3*b^12/x^4-19305/17*a^8*b^7/x^(17/3)-5005
/6*a^9*b^6/x^6-19305/16*a^7*b^8/x^(16/3)-45/23*a^14*b/x^(23/3)-9009/19*a^1
0*b^5/x^(19/3)-1001*a^6*b^9/x^5-1/3*b^15/x^3-315/22*a^13*b^2/x^(22/3)-315*
a^4*b^11/x^(13/3)-1287/2*a^5*b^10/x^(14/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{9}}\,dx =$$
$$-\frac{1307504\,b^{15}x^{5}+446185740\,a^{3}b^{12}x^{4}+3926434512\,a^{6}b^{9}x^{3}+3272028760\,a^{9}b^{6}x^{2}+254963280\,a^{12}b^{3}x+}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^9,x, algorithm="fricas")
```

output
```
-1/3922512*(1307504*b^15*x^5 + 446185740*a^3*b^12*x^4 + 3926434512*a^6*b^9
*x^3 + 3272028760*a^9*b^6*x^2 + 254963280*a^12*b^3*x + 490314*a^15 + 10557
*(1672*a*b^14*x^4 + 117040*a^4*b^11*x^3 + 448305*a^7*b^8*x^2 + 176176*a^10
*b^5*x + 5320*a^13*b^2)*x^(2/3) + 2052*(54740*a^2*b^13*x^4 + 1230086*a^5*b
^10*x^3 + 2170740*a^8*b^7*x^2 + 391391*a^11*b^4*x + 3740*a^14*b)*x^(1/3))/
x^8
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx = -\frac{a^{15}}{8x^8} - \frac{45a^{14}b}{23x^{\frac{23}{3}}} - \frac{315a^{13}b^2}{22x^{\frac{22}{3}}} - \frac{65a^{12}b^3}{x^7} - \frac{819a^{11}b^4}{4x^{\frac{20}{3}}} - \frac{9009a^{10}b^5}{19x^{\frac{19}{3}}} - \frac{5005a^9b^6}{6x^6} - \frac{19305a^8b^7}{17x^{\frac{17}{3}}} - \frac{19305a^7b^8}{16x^{\frac{16}{3}}} - \frac{1001a^6b^9}{x^5} - \frac{1287a^5b^{10}}{2x^{\frac{14}{3}}} - \frac{315a^4b^{11}}{x^{\frac{13}{3}}} - \frac{455a^3b^{12}}{4x^4} - \frac{315a^2b^{13}}{11x^{\frac{11}{3}}} - \frac{9ab^{14}}{2x^{\frac{10}{3}}} - \frac{b^{15}}{3x^3}$$

input
```
integrate((a+b*x**(1/3))**15/x**9,x)
```

output
```
-a**15/(8*x**8) - 45*a**14*b/(23*x**(23/3)) - 315*a**13*b**2/(22*x**(22/3)
) - 65*a**12*b**3/x**7 - 819*a**11*b**4/(4*x**(20/3)) - 9009*a**10*b**5/(1
9*x**(19/3)) - 5005*a**9*b**6/(6*x**6) - 19305*a**8*b**7/(17*x**(17/3)) -
19305*a**7*b**8/(16*x**(16/3)) - 1001*a**6*b**9/x**5 - 1287*a**5*b**10/(2*
x**(14/3)) - 315*a**4*b**11/x**(13/3) - 455*a**3*b**12/(4*x**4) - 315*a**2
*b**13/(11*x**(11/3)) - 9*a*b**14/(2*x**(10/3)) - b**15/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx = -\frac{1307504\,b^{15}x^5 + 17651304\,ab^{14}x^{\frac{14}{3}} + 112326480\,a^2b^{13}x^{\frac{13}{3}} + 446185740\,a^3b^{12}x^4 + 1235591280\,a^4b^{11}x^{\frac{11}{3}}}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^9,x, algorithm="maxima")
```

output
```
-1/3922512*(1307504*b^15*x^5 + 17651304*a*b^14*x^(14/3) + 112326480*a^2*b^
13*x^(13/3) + 446185740*a^3*b^12*x^4 + 1235591280*a^4*b^11*x^(11/3) + 2524
136472*a^5*b^10*x^(10/3) + 3926434512*a^6*b^9*x^3 + 4732755885*a^7*b^8*x^(
8/3) + 4454358480*a^8*b^7*x^(7/3) + 3272028760*a^9*b^6*x^2 + 1859890032*a^
10*b^5*x^(5/3) + 803134332*a^11*b^4*x^(4/3) + 254963280*a^12*b^3*x + 56163
240*a^13*b^2*x^(2/3) + 7674480*a^14*b*x^(1/3) + 490314*a^15)/x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx = -\frac{1307504\,b^{15}x^5 + 17651304\,ab^{14}x^{\frac{14}{3}} + 112326480\,a^2b^{13}x^{\frac{13}{3}} + 446185740\,a^3b^{12}x^4 + 1235591280\,a^4b^{11}x^{\frac{11}{3}}}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^9,x, algorithm="giac")
```

output
```
-1/3922512*(1307504*b^15*x^5 + 17651304*a*b^14*x^(14/3) + 112326480*a^2*b^
13*x^(13/3) + 446185740*a^3*b^12*x^4 + 1235591280*a^4*b^11*x^(11/3) + 2524
136472*a^5*b^10*x^(10/3) + 3926434512*a^6*b^9*x^3 + 4732755885*a^7*b^8*x^(
8/3) + 4454358480*a^8*b^7*x^(7/3) + 3272028760*a^9*b^6*x^2 + 1859890032*a^
10*b^5*x^(5/3) + 803134332*a^11*b^4*x^(4/3) + 254963280*a^12*b^3*x + 56163
240*a^13*b^2*x^(2/3) + 7674480*a^14*b*x^(1/3) + 490314*a^15)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx = -\frac{\frac{a^{15}}{8} + \frac{b^{15}\,x^5}{3} + 65\,a^{12}\,b^3\,x + \frac{45\,a^{14}\,b\,x^{1/3}}{23} + \frac{9\,a\,b^{14}\,x^{14/3}}{2} + \frac{5005\,a^9\,b^6\,x^2}{6} + 1001\,a^6\,b^9\,x^3 + \frac{455\,a^3\,b^{12}\,x^4}{4} + \frac{315\,a^{13}\,b^2\,x^{2}}{22}}{}$$

input
```
int((a + b*x^(1/3))^15/x^9,x)
```

output
```
-(a^15/8 + (b^15*x^5)/3 + 65*a^12*b^3*x + (45*a^14*b*x^(1/3))/23 + (9*a*b^
14*x^(14/3))/2 + (5005*a^9*b^6*x^2)/6 + 1001*a^6*b^9*x^3 + (455*a^3*b^12*x
^4)/4 + (315*a^13*b^2*x^(2/3))/22 + (819*a^11*b^4*x^(4/3))/4 + (9009*a^10*
b^5*x^(5/3))/19 + (19305*a^8*b^7*x^(7/3))/17 + (19305*a^7*b^8*x^(8/3))/16
+ (1287*a^5*b^10*x^(10/3))/2 + 315*a^4*b^11*x^(11/3) + (315*a^2*b^13*x^(13
/3))/11)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^9}\,dx$$
$$= \frac{-490314x^{\frac{2}{3}}a^{15} - 254963280x^{\frac{5}{3}}a^{12}b^3 - 3272028760x^{\frac{8}{3}}a^9b^6 - 3926434512x^{\frac{11}{3}}a^6b^9 - 446185740x^{\frac{14}{3}}a^3b^{12} -}{}$$

input
```
int((a+b*x^(1/3))^15/x^9,x)
```

output
```
( - 490314*x**(2/3)*a**15 - 254963280*x**(2/3)*a**12*b**3*x - 3272028760*x
**(2/3)*a**9*b**6*x**2 - 3926434512*x**(2/3)*a**6*b**9*x**3 - 446185740*x*
*(2/3)*a**3*b**12*x**4 - 1307504*x**(2/3)*b**15*x**5 - 56163240*x**(1/3)*a
**13*b**2*x - 1859890032*x**(1/3)*a**10*b**5*x**2 - 4732755885*x**(1/3)*a*
*7*b**8*x**3 - 1235591280*x**(1/3)*a**4*b**11*x**4 - 17651304*x**(1/3)*a*b
**14*x**5 - 7674480*a**14*b*x - 803134332*a**11*b**4*x**2 - 4454358480*a**
8*b**7*x**3 - 2524136472*a**5*b**10*x**4 - 112326480*a**2*b**13*x**5)/(392
2512*x**(2/3)*x**8)
```

3.244 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 215

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx = -\frac{a^{15}}{9x^9}-\frac{45a^{14}b}{26x^{26/3}}-\frac{63a^{13}b^2}{5x^{25/3}}-\frac{455a^{12}b^3}{8x^8}-\frac{4095a^{11}b^4}{23x^{23/3}} -\frac{819a^{10}b^5}{2x^{22/3}}-\frac{715a^9b^6}{x^7}-\frac{3861a^8b^7}{4x^{20/3}}-\frac{19305a^7b^8}{19x^{19/3}}-\frac{5005a^6b^9}{6x^6} -\frac{9009a^5b^{10}}{17x^{17/3}}-\frac{4095a^4b^{11}}{16x^{16/3}}-\frac{91a^3b^{12}}{x^5}-\frac{45a^2b^{13}}{2x^{14/3}}-\frac{45ab^{14}}{13x^{13/3}}-\frac{b^{15}}{4x^4}$$

output
```
-1/9*a^15/x^9-45/26*a^14*b/x^(26/3)-63/5*a^13*b^2/x^(25/3)-455/8*a^12*b^3/
x^8-4095/23*a^11*b^4/x^(23/3)-819/2*a^10*b^5/x^(22/3)-715*a^9*b^6/x^7-3861
/4*a^8*b^7/x^(20/3)-19305/19*a^7*b^8/x^(19/3)-5005/6*a^6*b^9/x^6-9009/17*a
^5*b^10/x^(17/3)-4095/16*a^4*b^11/x^(16/3)-91*a^3*b^12/x^5-45/2*a^2*b^13/x
^(14/3)-45/13*a*b^14/x^(13/3)-1/4*b^15/x^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx = \frac{-7726160a^{15}-120349800a^{14}b\sqrt[3]{x}-876146544a^{13}b^2x^{2/3}-3954828150a^{12}b^3x-12380331600a^{11}b^4x^{4/3}}{}$$

input
```
Integrate[(a + b*x^(1/3))^15/x^10,x]
```

output
```
(-7726160*a^15 - 120349800*a^14*b*x^(1/3) - 876146544*a^13*b^2*x^(2/3) - 3
954828150*a^12*b^3*x - 12380331600*a^11*b^4*x^(4/3) - 28474762680*a^10*b^5
*x^(5/3) - 49717839600*a^9*b^6*x^2 - 67119083460*a^8*b^7*x^(7/3) - 7065166
6800*a^7*b^8*x^(8/3) - 58004146200*a^6*b^9*x^3 - 36849692880*a^5*b^10*x^(1
0/3) - 17796726675*a^4*b^11*x^(11/3) - 6327725040*a^3*b^12*x^4 - 156454740
0*a^2*b^13*x^(13/3) - 240699600*a*b^14*x^(14/3) - 17383860*b^15*x^5)/(6953
5440*x^9)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx$$

$$\Big\downarrow \; 798$$

$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{28/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 53$$

$$3\int\left(\frac{a^{15}}{x^{28/3}}+\frac{15ba^{14}}{x^9}+\frac{105b^2a^{13}}{x^{26/3}}+\frac{455b^3a^{12}}{x^{25/3}}+\frac{1365b^4a^{11}}{x^8}+\frac{3003b^5a^{10}}{x^{23/3}}+\frac{5005b^6a^9}{x^{22/3}}+\frac{6435b^7a^8}{x^7}+\frac{6435b^8a^7}{x^{20/3}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{15}}{27x^9}-\frac{15a^{14}b}{26x^{26/3}}-\frac{21a^{13}b^2}{5x^{25/3}}-\frac{455a^{12}b^3}{24x^8}-\frac{1365a^{11}b^4}{23x^{23/3}}-\frac{273a^{10}b^5}{2x^{22/3}}-\frac{715a^9b^6}{3x^7}-\frac{1287a^8b^7}{4x^{20/3}}-\frac{6435a^7b^8}{19x^{19/3}}-\frac{500}{18}\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^10,x]
```

output
```
3*(-1/27*a^15/x^9 - (15*a^14*b)/(26*x^(26/3)) - (21*a^13*b^2)/(5*x^(25/3))
 - (455*a^12*b^3)/(24*x^8) - (1365*a^11*b^4)/(23*x^(23/3)) - (273*a^10*b^5
)/(2*x^(22/3)) - (715*a^9*b^6)/(3*x^7) - (1287*a^8*b^7)/(4*x^(20/3)) - (64
35*a^7*b^8)/(19*x^(19/3)) - (5005*a^6*b^9)/(18*x^6) - (3003*a^5*b^10)/(17*
x^(17/3)) - (1365*a^4*b^11)/(16*x^(16/3)) - (91*a^3*b^12)/(3*x^5) - (15*a^
2*b^13)/(2*x^(14/3)) - (15*a*b^14)/(13*x^(13/3)) - b^15/(12*x^4))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{a^{15}}{9x^{9}}-\frac{45a^{14}b}{26x^{\frac{26}{3}}}-\frac{63a^{13}b^{2}}{5x^{\frac{25}{3}}}-\frac{455a^{12}b^{3}}{8x^{8}}-\frac{4095a^{11}b^{4}}{23x^{\frac{23}{3}}}-\frac{819a^{10}b^{5}}{2x^{\frac{22}{3}}}-\frac{715a^{9}b^{6}}{x^{7}}-\frac{3861a^{8}b^{7}}{4x^{\frac{20}{3}}}-\frac{19305a^{7}b^{8}}{19x^{\frac{19}{3}}}-$
default	$-\frac{a^{15}}{9x^{9}}-\frac{45a^{14}b}{26x^{\frac{26}{3}}}-\frac{63a^{13}b^{2}}{5x^{\frac{25}{3}}}-\frac{455a^{12}b^{3}}{8x^{8}}-\frac{4095a^{11}b^{4}}{23x^{\frac{23}{3}}}-\frac{819a^{10}b^{5}}{2x^{\frac{22}{3}}}-\frac{715a^{9}b^{6}}{x^{7}}-\frac{3861a^{8}b^{7}}{4x^{\frac{20}{3}}}-\frac{19305a^{7}b^{8}}{19x^{\frac{19}{3}}}-$
trager	$\frac{(-1+x)(8a^{15}x^{8}+4095a^{12}b^{3}x^{8}+51480a^{9}b^{6}x^{8}+60060a^{6}b^{9}x^{8}+6552x^{8}b^{12}a^{3}+18x^{8}b^{15}+8a^{15}x^{7}+4095a^{12}b^{3}x^{7}+51480a}{}$
orering	$-\frac{(4225615520b^{39}x^{13}+4117674015a^{3}b^{36}x^{12}+21508117995a^{6}b^{33}x^{11}+71972062890a^{9}b^{30}x^{10}+168623489874a^{12}b^{27}x^{9}}{}$

input
```
int((a+b*x^(1/3))^15/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^15/x^9-45/26*a^14*b/x^(26/3)-63/5*a^13*b^2/x^(25/3)-455/8*a^12*b^3/
x^8-4095/23*a^11*b^4/x^(23/3)-819/2*a^10*b^5/x^(22/3)-715*a^9*b^6/x^7-3861
/4*a^8*b^7/x^(20/3)-19305/19*a^7*b^8/x^(19/3)-5005/6*a^6*b^9/x^6-9009/17*a
^5*b^10/x^(17/3)-4095/16*a^4*b^11/x^(16/3)-91*a^3*b^12/x^5-45/2*a^2*b^13/x
^(14/3)-45/13*a*b^14/x^(13/3)-1/4*b^15/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx = -\frac{17383860\,b^{15}x^{5}+6327725040\,a^{3}b^{12}x^{4}+58004146200\,a^{6}b^{9}x^{3}+49717839600\,a^{9}b^{6}x^{2}+3954828150\,a^{12}}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^10,x, algorithm="fricas")
```

output
```
-1/69535440*(17383860*b^15*x^5 + 6327725040*a^3*b^12*x^4 + 58004146200*a^6
*b^9*x^3 + 49717839600*a^9*b^6*x^2 + 3954828150*a^12*b^3*x + 7726160*a^15
+ 31671*(7600*a*b^14*x^4 + 561925*a^4*b^11*x^3 + 2230800*a^7*b^8*x^2 + 899
080*a^10*b^5*x + 27664*a^13*b^2)*x^(2/3) + 30780*(50830*a^2*b^13*x^4 + 119
7196*a^5*b^10*x^3 + 2180607*a^8*b^7*x^2 + 402220*a^11*b^4*x + 3910*a^14*b)
*x^(1/3))/x^9
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx = -\frac{a^{15}}{9x^9}-\frac{45a^{14}b}{26x^{\frac{26}{3}}}-\frac{63a^{13}b^2}{5x^{\frac{25}{3}}}-\frac{455a^{12}b^3}{8x^8}-\frac{4095a^{11}b^4}{23x^{\frac{23}{3}}}-\frac{819a^{10}b^5}{2x^{\frac{22}{3}}}-\frac{715a^9b^6}{x^7}-\frac{3861a^8b^7}{4x^{\frac{20}{3}}}-\frac{19305a^7b^8}{19x^{\frac{19}{3}}}-\frac{5005a^6b^9}{6x^6}-\frac{9009a^5b^{10}}{17x^{\frac{17}{3}}}-\frac{4095a^4b^{11}}{16x^{\frac{16}{3}}}-\frac{91a^3b^{12}}{x^5}-\frac{45a^2b^{13}}{2x^{\frac{14}{3}}}-\frac{45ab^{14}}{13x^{\frac{13}{3}}}-\frac{b^{15}}{4x^4}$$

input
```
integrate((a+b*x**(1/3))**15/x**10,x)
```

output
```
-a**15/(9*x**9) - 45*a**14*b/(26*x**(26/3)) - 63*a**13*b**2/(5*x**(25/3))
- 455*a**12*b**3/(8*x**8) - 4095*a**11*b**4/(23*x**(23/3)) - 819*a**10*b**
5/(2*x**(22/3)) - 715*a**9*b**6/x**7 - 3861*a**8*b**7/(4*x**(20/3)) - 1930
5*a**7*b**8/(19*x**(19/3)) - 5005*a**6*b**9/(6*x**6) - 9009*a**5*b**10/(17
*x**(17/3)) - 4095*a**4*b**11/(16*x**(16/3)) - 91*a**3*b**12/x**5 - 45*a**
2*b**13/(2*x**(14/3)) - 45*a*b**14/(13*x**(13/3)) - b**15/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx = -\frac{17383860\,b^{15}x^5+240699600\,ab^{14}x^{\frac{14}{3}}+1564547400\,a^2b^{13}x^{\frac{13}{3}}+6327725040\,a^3b^{12}x^4+17796726675\,a^4b}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^10,x, algorithm="maxima")
```

output
```
-1/69535440*(17383860*b^15*x^5 + 240699600*a*b^14*x^(14/3) + 1564547400*a^
2*b^13*x^(13/3) + 6327725040*a^3*b^12*x^4 + 17796726675*a^4*b^11*x^(11/3)
+ 36849692880*a^5*b^10*x^(10/3) + 58004146200*a^6*b^9*x^3 + 70651666800*a^
7*b^8*x^(8/3) + 67119083460*a^8*b^7*x^(7/3) + 49717839600*a^9*b^6*x^2 + 28
474762680*a^10*b^5*x^(5/3) + 12380331600*a^11*b^4*x^(4/3) + 3954828150*a^1
2*b^3*x + 876146544*a^13*b^2*x^(2/3) + 120349800*a^14*b*x^(1/3) + 7726160*
a^15)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx =$$

$$-\frac{17383860\,b^{15}x^5+240699600\,ab^{14}x^{\frac{14}{3}}+1564547400\,a^2b^{13}x^{\frac{13}{3}}+6327725040\,a^3b^{12}x^4+17796726675\,a^4b^{11}\cdots}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^10,x, algorithm="giac")
```

output
```
-1/69535440*(17383860*b^15*x^5 + 240699600*a*b^14*x^(14/3) + 1564547400*a^
2*b^13*x^(13/3) + 6327725040*a^3*b^12*x^4 + 17796726675*a^4*b^11*x^(11/3)
+ 36849692880*a^5*b^10*x^(10/3) + 58004146200*a^6*b^9*x^3 + 70651666800*a^
7*b^8*x^(8/3) + 67119083460*a^8*b^7*x^(7/3) + 49717839600*a^9*b^6*x^2 + 28
474762680*a^10*b^5*x^(5/3) + 12380331600*a^11*b^4*x^(4/3) + 3954828150*a^1
2*b^3*x + 876146544*a^13*b^2*x^(2/3) + 120349800*a^14*b*x^(1/3) + 7726160*
a^15)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx = -\frac{\frac{a^{15}}{9}+\frac{b^{15}\,x^5}{4}+\frac{455\,a^{12}\,b^3\,x}{8}+\frac{45\,a^{14}\,b\,x^{1/3}}{26}+\frac{45\,a\,b^{14}\,x^{14/3}}{13}+715\,a^9\,b^6\,x^2+\frac{5005\,a^6\,b^9\,x^3}{6}+91\,a^3\,b^{12}\,x^4+\frac{63\,a^{13}\,b^2\,x^{2/}}{5}}{}$$

input
```
int((a + b*x^(1/3))^15/x^10,x)
```

output
```
-(a^15/9 + (b^15*x^5)/4 + (455*a^12*b^3*x)/8 + (45*a^14*b*x^(1/3))/26 + (4
5*a*b^14*x^(14/3))/13 + 715*a^9*b^6*x^2 + (5005*a^6*b^9*x^3)/6 + 91*a^3*b^
12*x^4 + (63*a^13*b^2*x^(2/3))/5 + (4095*a^11*b^4*x^(4/3))/23 + (819*a^10*
b^5*x^(5/3))/2 + (3861*a^8*b^7*x^(7/3))/4 + (19305*a^7*b^8*x^(8/3))/19 + (
9009*a^5*b^10*x^(10/3))/17 + (4095*a^4*b^11*x^(11/3))/16 + (45*a^2*b^13*x^
(13/3))/2)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{10}}\,dx = \frac{-7726160x^{\frac{2}{3}}a^{15}-3954828150x^{\frac{5}{3}}a^{12}b^3-49717839600x^{\frac{8}{3}}a^9b^6-58004146200x^{\frac{11}{3}}a^6b^9-6327725040x^{\frac{14}{3}}a}{}$$

input
```
int((a+b*x^(1/3))^15/x^10,x)
```

output
```
( - 7726160*x**(2/3)*a**15 - 3954828150*x**(2/3)*a**12*b**3*x - 4971783960
0*x**(2/3)*a**9*b**6*x**2 - 58004146200*x**(2/3)*a**6*b**9*x**3 - 63277250
40*x**(2/3)*a**3*b**12*x**4 - 17383860*x**(2/3)*b**15*x**5 - 876146544*x**
(1/3)*a**13*b**2*x - 28474762680*x**(1/3)*a**10*b**5*x**2 - 70651666800*x*
*(1/3)*a**7*b**8*x**3 - 17796726675*x**(1/3)*a**4*b**11*x**4 - 240699600*x
**(1/3)*a*b**14*x**5 - 120349800*a**14*b*x - 12380331600*a**11*b**4*x**2 -
 67119083460*a**8*b**7*x**3 - 36849692880*a**5*b**10*x**4 - 1564547400*a**
2*b**13*x**5)/(69535440*x**(2/3)*x**9)
```

3.245 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 217

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx = -\frac{a^{15}}{10x^{10}}-\frac{45a^{14}b}{29x^{29/3}}-\frac{45a^{13}b^2}{4x^{28/3}}-\frac{455a^{12}b^3}{9x^9}-\frac{315a^{11}b^4}{2x^{26/3}}-\frac{9009a^{10}b^5}{25x^{25/3}}-\frac{5005a^9b^6}{8x^8}-\frac{19305a^8b^7}{23x^{23/3}}-\frac{1755a^7b^8}{2x^{22/3}}-\frac{715a^6b^9}{x^7}-\frac{9009a^5b^{10}}{20x^{20/3}}-\frac{4095a^4b^{11}}{19x^{19/3}}-\frac{455a^3b^{12}}{6x^6}-\frac{315a^2b^{13}}{17x^{17/3}}-\frac{45ab^{14}}{16x^{16/3}}-\frac{b^{15}}{5x^5}$$

output
```
-1/10*a^15/x^10-45/29*a^14*b/x^(29/3)-45/4*a^13*b^2/x^(28/3)-455/9*a^12*b^
3/x^9-315/2*a^11*b^4/x^(26/3)-9009/25*a^10*b^5/x^(25/3)-5005/8*a^9*b^6/x^8
-19305/23*a^8*b^7/x^(23/3)-1755/2*a^7*b^8/x^(22/3)-715*a^6*b^9/x^7-9009/20
*a^5*b^10/x^(20/3)-4095/19*a^4*b^11/x^(19/3)-455/6*a^3*b^12/x^6-315/17*a^2
*b^13/x^(17/3)-45/16*a*b^14/x^(16/3)-1/5*b^15/x^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx$$
$$= \frac{-77558760a^{15} - 1203498000a^{14}b\sqrt[3]{x} - 8725360500a^{13}b^2x^{2/3} - 39210262000a^{12}b^3x - 122155047000a^{11}b}{}$$

input
```
Integrate[(a + b*x^(1/3))^15/x^11,x]
```

output
```
(-77558760*a^15 - 1203498000*a^14*b*x^(1/3) - 8725360500*a^13*b^2*x^(2/3)
- 39210262000*a^12*b^3*x - 122155047000*a^11*b^4*x^(4/3) - 279490747536*a^
10*b^5*x^(5/3) - 485226992250*a^9*b^6*x^2 - 650987766000*a^8*b^7*x^(7/3) -
 680578119000*a^7*b^8*x^(8/3) - 554545134000*a^6*b^9*x^3 - 349363434420*a^
5*b^10*x^(10/3) - 167159538000*a^4*b^11*x^(11/3) - 58815393000*a^3*b^12*x^
4 - 14371182000*a^2*b^13*x^(13/3) - 2181340125*a*b^14*x^(14/3) - 155117520
*b^15*x^5)/(775587600*x^10)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx$$
$$\Big\downarrow \text{ 798}$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{31/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{ 53}$$

$$3\int \left(\frac{a^{15}}{x^{31/3}}+\frac{15ba^{14}}{x^{10}}+\frac{105b^2a^{13}}{x^{29/3}}+\frac{455b^3a^{12}}{x^{28/3}}+\frac{1365b^4a^{11}}{x^9}+\frac{3003b^5a^{10}}{x^{26/3}}+\frac{5005b^6a^9}{x^{25/3}}+\frac{6435b^7a^8}{x^8}+\frac{6435b^8a^7}{x^{23/3}}+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{15}}{30x^{10}}-\frac{15a^{14}b}{29x^{29/3}}-\frac{15a^{13}b^2}{4x^{28/3}}-\frac{455a^{12}b^3}{27x^9}-\frac{105a^{11}b^4}{2x^{26/3}}-\frac{3003a^{10}b^5}{25x^{25/3}}-\frac{5005a^9b^6}{24x^8}-\frac{6435a^8b^7}{23x^{23/3}}-\frac{585a^7b^8}{2x^{22/3}}-\frac{715}{3}\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^11,x]
```

output
```
3*(-1/30*a^15/x^10 - (15*a^14*b)/(29*x^(29/3)) - (15*a^13*b^2)/(4*x^(28/3)
) - (455*a^12*b^3)/(27*x^9) - (105*a^11*b^4)/(2*x^(26/3)) - (3003*a^10*b^5
)/(25*x^(25/3)) - (5005*a^9*b^6)/(24*x^8) - (6435*a^8*b^7)/(23*x^(23/3)) -
 (585*a^7*b^8)/(2*x^(22/3)) - (715*a^6*b^9)/(3*x^7) - (3003*a^5*b^10)/(20*
x^(20/3)) - (1365*a^4*b^11)/(19*x^(19/3)) - (455*a^3*b^12)/(18*x^6) - (105
*a^2*b^13)/(17*x^(17/3)) - (15*a*b^14)/(16*x^(16/3)) - b^15/(15*x^5))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 27.97 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{a^{15}}{10x^{10}}-\frac{45a^{14}b}{29x^{\frac{29}{3}}}-\frac{45a^{13}b^{2}}{4x^{\frac{28}{3}}}-\frac{455a^{12}b^{3}}{9x^{9}}-\frac{315a^{11}b^{4}}{2x^{\frac{26}{3}}}-\frac{9009a^{10}b^{5}}{25x^{\frac{25}{3}}}-\frac{5005a^{9}b^{6}}{8x^{8}}-\frac{19305a^{8}b^{7}}{23x^{\frac{23}{3}}}-\frac{1755a^{7}b}{2x^{\frac{22}{3}}}$
default	$-\frac{a^{15}}{10x^{10}}-\frac{45a^{14}b}{29x^{\frac{29}{3}}}-\frac{45a^{13}b^{2}}{4x^{\frac{28}{3}}}-\frac{455a^{12}b^{3}}{9x^{9}}-\frac{315a^{11}b^{4}}{2x^{\frac{26}{3}}}-\frac{9009a^{10}b^{5}}{25x^{\frac{25}{3}}}-\frac{5005a^{9}b^{6}}{8x^{8}}-\frac{19305a^{8}b^{7}}{23x^{\frac{23}{3}}}-\frac{1755a^{7}b}{2x^{\frac{22}{3}}}$
trager	$(-1+x)(36a^{15}x^{9}+18200a^{12}b^{3}x^{9}+225225a^{9}b^{6}x^{9}+257400a^{6}b^{9}x^{9}+27300a^{3}b^{12}x^{9}+72x^{9}b^{15}+36a^{15}x^{8}+18200a^{12}b^{3}x^{8}$
orering	$-\frac{(18363177000b^{39}x^{13}+196657369545a^{3}b^{36}x^{12}+1049795347965a^{6}b^{33}x^{11}+3548747254710a^{9}b^{30}x^{10}+835208082308}{}$

input
```
int((a+b*x^(1/3))^15/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^15/x^10-45/29*a^14*b/x^(29/3)-45/4*a^13*b^2/x^(28/3)-455/9*a^12*b^
3/x^9-315/2*a^11*b^4/x^(26/3)-9009/25*a^10*b^5/x^(25/3)-5005/8*a^9*b^6/x^8
-19305/23*a^8*b^7/x^(23/3)-1755/2*a^7*b^8/x^(22/3)-715*a^6*b^9/x^7-9009/20
*a^5*b^10/x^(20/3)-4095/19*a^4*b^11/x^(19/3)-455/6*a^3*b^12/x^6-315/17*a^2
*b^13/x^(17/3)-45/16*a*b^14/x^(16/3)-1/5*b^15/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx =$$
$$-\frac{155117520\,b^{15}x^{5}+58815393000\,a^{3}b^{12}x^{4}+554545134000\,a^{6}b^{9}x^{3}+485226992250\,a^{9}b^{6}x^{2}+392102620}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^11,x, algorithm="fricas")
```

output
```
-1/775587600*(155117520*b^15*x^5 + 58815393000*a^3*b^12*x^4 + 554545134000
*a^6*b^9*x^3 + 485226992250*a^9*b^6*x^2 + 39210262000*a^12*b^3*x + 7755876
0*a^15 + 918459*(2375*a*b^14*x^4 + 182000*a^4*b^11*x^3 + 741000*a^7*b^8*x^
2 + 304304*a^10*b^5*x + 9500*a^13*b^2)*x^(2/3) + 30780*(466900*a^2*b^13*x^
4 + 11350339*a^5*b^10*x^3 + 21149700*a^8*b^7*x^2 + 3968650*a^11*b^4*x + 39
100*a^14*b)*x^(1/3))/x^10
```

Sympy [A] (verification not implemented)

Time = 2.48 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx = -\frac{a^{15}}{10x^{10}} - \frac{45a^{14}b}{29x^{\frac{29}{3}}} - \frac{45a^{13}b^2}{4x^{\frac{28}{3}}} - \frac{455a^{12}b^3}{9x^9} - \frac{315a^{11}b^4}{2x^{\frac{26}{3}}} - \frac{9009a^{10}b^5}{25x^{\frac{25}{3}}} - \frac{5005a^9b^6}{8x^8} - \frac{19305a^8b^7}{23x^{\frac{23}{3}}} - \frac{1755a^7b^8}{2x^{\frac{22}{3}}} - \frac{715a^6b^9}{x^7} - \frac{9009a^5b^{10}}{20x^{\frac{20}{3}}} - \frac{4095a^4b^{11}}{19x^{\frac{19}{3}}} - \frac{455a^3b^{12}}{6x^6} - \frac{315a^2b^{13}}{17x^{\frac{17}{3}}} - \frac{45ab^{14}}{16x^{\frac{16}{3}}} - \frac{b^{15}}{5x^5}$$

input
```
integrate((a+b*x**(1/3))**15/x**11,x)
```

output
```
-a**15/(10*x**10) - 45*a**14*b/(29*x**(29/3)) - 45*a**13*b**2/(4*x**(28/3)
) - 455*a**12*b**3/(9*x**9) - 315*a**11*b**4/(2*x**(26/3)) - 9009*a**10*b*
*5/(25*x**(25/3)) - 5005*a**9*b**6/(8*x**8) - 19305*a**8*b**7/(23*x**(23/3
)) - 1755*a**7*b**8/(2*x**(22/3)) - 715*a**6*b**9/x**7 - 9009*a**5*b**10/(
20*x**(20/3)) - 4095*a**4*b**11/(19*x**(19/3)) - 455*a**3*b**12/(6*x**6) -
 315*a**2*b**13/(17*x**(17/3)) - 45*a*b**14/(16*x**(16/3)) - b**15/(5*x**5
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx = -\frac{155117520\,b^{15}x^5 + 2181340125\,ab^{14}x^{\frac{14}{3}} + 14371182000\,a^2b^{13}x^{\frac{13}{3}} + 58815393000\,a^3b^{12}x^4 + 1671595380}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^11,x, algorithm="maxima")
```

output
```
-1/775587600*(155117520*b^15*x^5 + 2181340125*a*b^14*x^(14/3) + 1437118200
0*a^2*b^13*x^(13/3) + 58815393000*a^3*b^12*x^4 + 167159538000*a^4*b^11*x^(
11/3) + 349363434420*a^5*b^10*x^(10/3) + 554545134000*a^6*b^9*x^3 + 680578
119000*a^7*b^8*x^(8/3) + 650987766000*a^8*b^7*x^(7/3) + 485226992250*a^9*b
^6*x^2 + 279490747536*a^10*b^5*x^(5/3) + 122155047000*a^11*b^4*x^(4/3) + 3
9210262000*a^12*b^3*x + 8725360500*a^13*b^2*x^(2/3) + 1203498000*a^14*b*x^
(1/3) + 77558760*a^15)/x^10
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx =$$

$$-\frac{155117520\,b^{15}x^5 + 2181340125\,ab^{14}x^{\frac{14}{3}} + 14371182000\,a^2b^{13}x^{\frac{13}{3}} + 58815393000\,a^3b^{12}x^4 + 16715953800}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^11,x, algorithm="giac")
```

output
```
-1/775587600*(155117520*b^15*x^5 + 2181340125*a*b^14*x^(14/3) + 1437118200
0*a^2*b^13*x^(13/3) + 58815393000*a^3*b^12*x^4 + 167159538000*a^4*b^11*x^(
11/3) + 349363434420*a^5*b^10*x^(10/3) + 554545134000*a^6*b^9*x^3 + 680578
119000*a^7*b^8*x^(8/3) + 650987766000*a^8*b^7*x^(7/3) + 485226992250*a^9*b
^6*x^2 + 279490747536*a^10*b^5*x^(5/3) + 122155047000*a^11*b^4*x^(4/3) + 3
9210262000*a^12*b^3*x + 8725360500*a^13*b^2*x^(2/3) + 1203498000*a^14*b*x^
(1/3) + 77558760*a^15)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx = -\frac{\frac{a^{15}}{10}+\frac{b^{15}\,x^5}{5}+\frac{455\,a^{12}\,b^3\,x}{9}+\frac{45\,a^{14}\,b\,x^{1/3}}{29}+\frac{45\,a\,b^{14}\,x^{14/3}}{16}+\frac{5005\,a^9\,b^6\,x^2}{8}+715\,a^6\,b^9\,x^3+\frac{455\,a^3\,b^{12}\,x^4}{6}+\frac{45\,a^{13}\,b^2\,x^{2/3}}{4}$$

input
```
int((a + b*x^(1/3))^15/x^11,x)
```

output
```
-(a^15/10 + (b^15*x^5)/5 + (455*a^12*b^3*x)/9 + (45*a^14*b*x^(1/3))/29 + (
45*a*b^14*x^(14/3))/16 + (5005*a^9*b^6*x^2)/8 + 715*a^6*b^9*x^3 + (455*a^3
*b^12*x^4)/6 + (45*a^13*b^2*x^(2/3))/4 + (315*a^11*b^4*x^(4/3))/2 + (9009*
a^10*b^5*x^(5/3))/25 + (19305*a^8*b^7*x^(7/3))/23 + (1755*a^7*b^8*x^(8/3))
/2 + (9009*a^5*b^10*x^(10/3))/20 + (4095*a^4*b^11*x^(11/3))/19 + (315*a^2*
b^13*x^(13/3))/17)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{11}}\,dx = \frac{-77558760x^{\frac{2}{3}}a^{15}-39210262000x^{\frac{5}{3}}a^{12}b^3-485226992250x^{\frac{8}{3}}a^9b^6-554545134000x^{\frac{11}{3}}a^6b^9-5881539300$$

input
```
int((a+b*x^(1/3))^15/x^11,x)
```

output
```
( - 77558760*x**(2/3)*a**15 - 39210262000*x**(2/3)*a**12*b**3*x - 48522699
2250*x**(2/3)*a**9*b**6*x**2 - 554545134000*x**(2/3)*a**6*b**9*x**3 - 5881
5393000*x**(2/3)*a**3*b**12*x**4 - 155117520*x**(2/3)*b**15*x**5 - 8725360
500*x**(1/3)*a**13*b**2*x - 279490747536*x**(1/3)*a**10*b**5*x**2 - 680578
119000*x**(1/3)*a**7*b**8*x**3 - 167159538000*x**(1/3)*a**4*b**11*x**4 - 2
181340125*x**(1/3)*a*b**14*x**5 - 1203498000*a**14*b*x - 122155047000*a**1
1*b**4*x**2 - 650987766000*a**8*b**7*x**3 - 349363434420*a**5*b**10*x**4 -
 14371182000*a**2*b**13*x**5)/(775587600*x**(2/3)*x**10)
```

3.246 $\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 217

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx = -\frac{a^{15}}{11x^{11}}-\frac{45a^{14}b}{32x^{32/3}}-\frac{315a^{13}b^2}{31x^{31/3}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{29/3}}-\frac{1287a^{10}b^5}{4x^{28/3}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a^8b^7}{2x^{26/3}}-\frac{3861a^7b^8}{5x^{25/3}}-\frac{5005a^6b^9}{8x^8}-\frac{9009a^5b^{10}}{23x^{23/3}}-\frac{4095a^4b^{11}}{22x^{22/3}}-\frac{65a^3b^{12}}{x^7}-\frac{63a^2b^{13}}{4x^{20/3}}-\frac{45ab^{14}}{19x^{19/3}}-\frac{b^{15}}{6x^6}$$

output

```
-1/11*a^15/x^11-45/32*a^14*b/x^(32/3)-315/31*a^13*b^2/x^(31/3)-91/2*a^12*b
^3/x^10-4095/29*a^11*b^4/x^(29/3)-1287/4*a^10*b^5/x^(28/3)-5005/9*a^9*b^6/
x^9-1485/2*a^8*b^7/x^(26/3)-3861/5*a^7*b^8/x^(25/3)-5005/8*a^6*b^9/x^8-900
9/23*a^5*b^10/x^(23/3)-4095/22*a^4*b^11/x^(22/3)-65*a^3*b^12/x^7-63/4*a^2*
b^13/x^(20/3)-45/19*a*b^14/x^(19/3)-1/6*b^15/x^6
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx$$
$$= \frac{-565722720a^{15} - 8751023325a^{14}b\sqrt[3]{x} - 63233200800a^{13}b^2x^{2/3} - 283144221360a^{12}b^3x - 878723445600a}{}$$

input
```
Integrate[(a + b*x^(1/3))^15/x^12,x]
```

output
```
(-565722720*a^15 - 8751023325*a^14*b*x^(1/3) - 63233200800*a^13*b^2*x^(2/3
) - 283144221360*a^12*b^3*x - 878723445600*a^11*b^4*x^(4/3) - 200223413676
0*a^10*b^5*x^(5/3) - 3460651594400*a^9*b^6*x^2 - 4620540315600*a^8*b^7*x^(
7/3) - 4805361928224*a^7*b^8*x^(8/3) - 3893233043700*a^6*b^9*x^3 - 2437502
427360*a^5*b^10*x^(10/3) - 1158317269200*a^4*b^11*x^(11/3) - 404491744800*
a^3*b^12*x^4 - 98011461240*a^2*b^13*x^(13/3) - 14738565600*a*b^14*x^(14/3)
 - 1037158320*b^15*x^5)/(6222949920*x^11)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx$$
$$\Big\downarrow \; 798$$
$$3\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{34/3}}d\sqrt[3]{x}$$
$$\Big\downarrow \; 53$$

$$3\int\left(\frac{a^{15}}{x^{34/3}}+\frac{15ba^{14}}{x^{11}}+\frac{105b^2a^{13}}{x^{32/3}}+\frac{455b^3a^{12}}{x^{31/3}}+\frac{1365b^4a^{11}}{x^{10}}+\frac{3003b^5a^{10}}{x^{29/3}}+\frac{5005b^6a^9}{x^{28/3}}+\frac{6435b^7a^8}{x^9}+\frac{6435b^8a^7}{x^{26/3}}+\right.$$

$$\downarrow \text{2009}$$

$$3\left(-\frac{a^{15}}{33x^{11}}-\frac{15a^{14}b}{32x^{32/3}}-\frac{105a^{13}b^2}{31x^{31/3}}-\frac{91a^{12}b^3}{6x^{10}}-\frac{1365a^{11}b^4}{29x^{29/3}}-\frac{429a^{10}b^5}{4x^{28/3}}-\frac{5005a^9b^6}{27x^9}-\frac{495a^8b^7}{2x^{26/3}}-\frac{1287a^7b^8}{5x^{25/3}}-\frac{500}{2}\right.$$

input
```
Int[(a + b*x^(1/3))^15/x^12,x]
```

output
```
3*(-1/33*a^15/x^11 - (15*a^14*b)/(32*x^(32/3)) - (105*a^13*b^2)/(31*x^(31/
3)) - (91*a^12*b^3)/(6*x^10) - (1365*a^11*b^4)/(29*x^(29/3)) - (429*a^10*b
^5)/(4*x^(28/3)) - (5005*a^9*b^6)/(27*x^9) - (495*a^8*b^7)/(2*x^(26/3)) -
(1287*a^7*b^8)/(5*x^(25/3)) - (5005*a^6*b^9)/(24*x^8) - (3003*a^5*b^10)/(2
3*x^(23/3)) - (1365*a^4*b^11)/(22*x^(22/3)) - (65*a^3*b^12)/(3*x^7) - (21*
a^2*b^13)/(4*x^(20/3)) - (15*a*b^14)/(19*x^(19/3)) - b^15/(18*x^6))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 26.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{a^{15}}{11x^{11}}-\frac{45a^{14}b}{32x^{\frac{32}{3}}}-\frac{315a^{13}b^2}{31x^{\frac{31}{3}}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{\frac{29}{3}}}-\frac{1287a^{10}b^5}{4x^{\frac{28}{3}}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a^8b^7}{2x^{\frac{26}{3}}}-\frac{3861a^7b}{5x^{\frac{25}{3}}}$
default	$-\frac{a^{15}}{11x^{11}}-\frac{45a^{14}b}{32x^{\frac{32}{3}}}-\frac{315a^{13}b^2}{31x^{\frac{31}{3}}}-\frac{91a^{12}b^3}{2x^{10}}-\frac{4095a^{11}b^4}{29x^{\frac{29}{3}}}-\frac{1287a^{10}b^5}{4x^{\frac{28}{3}}}-\frac{5005a^9b^6}{9x^9}-\frac{1485a^8b^7}{2x^{\frac{26}{3}}}-\frac{3861a^7b}{5x^{\frac{25}{3}}}$
orering	$-\frac{(17222286840b^{39}x^{13}+194195866800a^3b^{36}x^{12}+1057995417516a^6b^{33}x^{11}+3613484571268a^9b^{30}x^{10}+855211100186}{}$
trager	$\frac{(-1+x)(72a^{15}x^{10}+36036a^{12}b^3x^{10}+440440a^9b^6x^{10}+495495a^6b^9x^{10}+51480a^3b^{12}x^{10}+132b^{15}x^{10}+72a^{15}x^9+36036a}{}$

input
```
int((a+b*x^(1/3))^15/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^15/x^11-45/32*a^14*b/x^(32/3)-315/31*a^13*b^2/x^(31/3)-91/2*a^12*b
^3/x^10-4095/29*a^11*b^4/x^(29/3)-1287/4*a^10*b^5/x^(28/3)-5005/9*a^9*b^6/
x^9-1485/2*a^8*b^7/x^(26/3)-3861/5*a^7*b^8/x^(25/3)-5005/8*a^6*b^9/x^8-900
9/23*a^5*b^10/x^(23/3)-4095/22*a^4*b^11/x^(22/3)-65*a^3*b^12/x^7-63/4*a^2*
b^13/x^(20/3)-45/19*a*b^14/x^(19/3)-1/6*b^15/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx =$$
$$-\frac{1037158320\,b^{15}x^5+404491744800\,a^3b^{12}x^4+3893233043700\,a^6b^9x^3+3460651594400\,a^9b^6x^2+28314}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^12,x, algorithm="fricas")
```

output
```
-1/6222949920*(1037158320*b^15*x^5 + 404491744800*a^3*b^12*x^4 + 389323304
3700*a^6*b^9*x^3 + 3460651594400*a^9*b^6*x^2 + 283144221360*a^12*b^3*x + 5
65722720*a^15 + 432216*(34100*a*b^14*x^4 + 2679950*a^4*b^11*x^3 + 11117964
*a^7*b^8*x^2 + 4632485*a^10*b^5*x + 146300*a^13*b^2)*x^(2/3) + 2623995*(37
352*a^2*b^13*x^4 + 928928*a^5*b^10*x^3 + 1760880*a^8*b^7*x^2 + 334880*a^11
*b^4*x + 3335*a^14*b)*x^(1/3))/x^11
```

Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx = -\frac{a^{15}}{11x^{11}} - \frac{45a^{14}b}{32x^{\frac{32}{3}}} - \frac{315a^{13}b^2}{31x^{\frac{31}{3}}} - \frac{91a^{12}b^3}{2x^{10}} - \frac{4095a^{11}b^4}{29x^{\frac{29}{3}}} - \frac{1287a^{10}b^5}{4x^{\frac{28}{3}}} - \frac{5005a^9b^6}{9x^9} - \frac{1485a^8b^7}{2x^{\frac{26}{3}}} - \frac{3861a^7b^8}{5x^{\frac{25}{3}}} - \frac{5005a^6b^9}{8x^8} - \frac{9009a^5b^{10}}{23x^{\frac{23}{3}}} - \frac{4095a^4b^{11}}{22x^{\frac{22}{3}}} - \frac{65a^3b^{12}}{x^7} - \frac{63a^2b^{13}}{4x^{\frac{20}{3}}} - \frac{45ab^{14}}{19x^{\frac{19}{3}}} - \frac{b^{15}}{6x^6}$$

input
```
integrate((a+b*x**(1/3))**15/x**12,x)
```

output
```
-a**15/(11*x**11) - 45*a**14*b/(32*x**(32/3)) - 315*a**13*b**2/(31*x**(31/
3)) - 91*a**12*b**3/(2*x**10) - 4095*a**11*b**4/(29*x**(29/3)) - 1287*a**1
0*b**5/(4*x**(28/3)) - 5005*a**9*b**6/(9*x**9) - 1485*a**8*b**7/(2*x**(26/
3)) - 3861*a**7*b**8/(5*x**(25/3)) - 5005*a**6*b**9/(8*x**8) - 9009*a**5*b
**10/(23*x**(23/3)) - 4095*a**4*b**11/(22*x**(22/3)) - 65*a**3*b**12/x**7
- 63*a**2*b**13/(4*x**(20/3)) - 45*a*b**14/(19*x**(19/3)) - b**15/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx = -\frac{1037158320\,b^{15}x^5 + 14738565600\,ab^{14}x^{\frac{14}{3}} + 98011461240\,a^2b^{13}x^{\frac{13}{3}} + 404491744800\,a^3b^{12}x^4 + 1158317}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^12,x, algorithm="maxima")
```

output
```
-1/6222949920*(1037158320*b^15*x^5 + 14738565600*a*b^14*x^(14/3) + 9801146
1240*a^2*b^13*x^(13/3) + 404491744800*a^3*b^12*x^4 + 1158317269200*a^4*b^1
1*x^(11/3) + 2437502427360*a^5*b^10*x^(10/3) + 3893233043700*a^6*b^9*x^3 +
 4805361928224*a^7*b^8*x^(8/3) + 4620540315600*a^8*b^7*x^(7/3) + 346065159
4400*a^9*b^6*x^2 + 2002234136760*a^10*b^5*x^(5/3) + 878723445600*a^11*b^4*
x^(4/3) + 283144221360*a^12*b^3*x + 63233200800*a^13*b^2*x^(2/3) + 8751023
325*a^14*b*x^(1/3) + 565722720*a^15)/x^11
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx = -\frac{1037158320\,b^{15}x^5+14738565600\,ab^{14}x^{\frac{14}{3}}+98011461240\,a^2b^{13}x^{\frac{13}{3}}+404491744800\,a^3b^{12}x^4+1158317\cdots}{}$$

input
```
integrate((a+b*x^(1/3))^15/x^12,x, algorithm="giac")
```

output
```
-1/6222949920*(1037158320*b^15*x^5 + 14738565600*a*b^14*x^(14/3) + 9801146
1240*a^2*b^13*x^(13/3) + 404491744800*a^3*b^12*x^4 + 1158317269200*a^4*b^1
1*x^(11/3) + 2437502427360*a^5*b^10*x^(10/3) + 3893233043700*a^6*b^9*x^3 +
 4805361928224*a^7*b^8*x^(8/3) + 4620540315600*a^8*b^7*x^(7/3) + 346065159
4400*a^9*b^6*x^2 + 2002234136760*a^10*b^5*x^(5/3) + 878723445600*a^11*b^4*
x^(4/3) + 283144221360*a^12*b^3*x + 63233200800*a^13*b^2*x^(2/3) + 8751023
325*a^14*b*x^(1/3) + 565722720*a^15)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx = -\frac{\frac{a^{15}}{11}+\frac{b^{15}\,x^5}{6}+\frac{91\,a^{12}\,b^3\,x}{2}+\frac{45\,a^{14}\,b\,x^{1/3}}{32}+\frac{45\,a\,b^{14}\,x^{14/3}}{19}+\frac{5005\,a^9\,b^6\,x^2}{9}+\frac{5005\,a^6\,b^9\,x^3}{8}+65\,a^3\,b^{12}\,x^4+\frac{315\,a^{13}\,b^2\,x^{2/3}}{31}}{}$$

input

```
int((a + b*x^(1/3))^15/x^12,x)
```

output

```
-(a^15/11 + (b^15*x^5)/6 + (91*a^12*b^3*x)/2 + (45*a^14*b*x^(1/3))/32 + (4
5*a*b^14*x^(14/3))/19 + (5005*a^9*b^6*x^2)/9 + (5005*a^6*b^9*x^3)/8 + 65*a
^3*b^12*x^4 + (315*a^13*b^2*x^(2/3))/31 + (4095*a^11*b^4*x^(4/3))/29 + (12
87*a^10*b^5*x^(5/3))/4 + (1485*a^8*b^7*x^(7/3))/2 + (3861*a^7*b^8*x^(8/3))
/5 + (9009*a^5*b^10*x^(10/3))/23 + (4095*a^4*b^11*x^(11/3))/22 + (63*a^2*b
^13*x^(13/3))/4)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+b\sqrt[3]{x}\right)^{15}}{x^{12}}\,dx = \frac{-565722720x^{\frac{2}{3}}a^{15}-283144221360x^{\frac{5}{3}}a^{12}b^3-3460651594400x^{\frac{8}{3}}a^9b^6-3893233043700x^{\frac{11}{3}}a^6b^9-404491}{}$$

input

```
int((a+b*x^(1/3))^15/x^12,x)
```

output

```
( - 565722720*x**(2/3)*a**15 - 283144221360*x**(2/3)*a**12*b**3*x - 346065
1594400*x**(2/3)*a**9*b**6*x**2 - 3893233043700*x**(2/3)*a**6*b**9*x**3 -
404491744800*x**(2/3)*a**3*b**12*x**4 - 1037158320*x**(2/3)*b**15*x**5 - 6
3233200800*x**(1/3)*a**13*b**2*x - 2002234136760*x**(1/3)*a**10*b**5*x**2
- 4805361928224*x**(1/3)*a**7*b**8*x**3 - 1158317269200*x**(1/3)*a**4*b**1
1*x**4 - 14738565600*x**(1/3)*a*b**14*x**5 - 8751023325*a**14*b*x - 878723
445600*a**11*b**4*x**2 - 4620540315600*a**8*b**7*x**3 - 2437502427360*a**5
*b**10*x**4 - 98011461240*a**2*b**13*x**5)/(6222949920*x**(2/3)*x**11)
```

3.247 $\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 166

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx=\frac{3a^{10}\sqrt[3]{x}}{b^{11}}-\frac{3a^9x^{2/3}}{2b^{10}}+\frac{a^8x}{b^9}-\frac{3a^7x^{4/3}}{4b^8}+\frac{3a^6x^{5/3}}{5b^7}-\frac{a^5x^2}{2b^6}+\frac{3a^4x^{7/3}}{7b^5}-\frac{3a^3x^{8/3}}{8b^4}+\frac{a^2x^3}{3b^3}-\frac{3ax^{10/3}}{10b^2}+\frac{3x^{11/3}}{11b}-\frac{3a^{11}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}$$

output
```
3*a^10*x^(1/3)/b^11-3/2*a^9*x^(2/3)/b^10+a^8*x/b^9-3/4*a^7*x^(4/3)/b^8+3/5
*a^6*x^(5/3)/b^7-1/2*a^5*x^2/b^6+3/7*a^4*x^(7/3)/b^5-3/8*a^3*x^(8/3)/b^4+1
/3*a^2*x^3/b^3-3/10*a*x^(10/3)/b^2+3/11*x^(11/3)/b-3*a^11*ln(a+b*x^(1/3))/
b^12
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx =\frac{27720a^{10}\sqrt[3]{x}-13860a^9bx^{2/3}+9240a^8b^2x-6930a^7b^3x^{4/3}+5544a^6b^4x^{5/3}-4620a^5b^5x^2+3960a^4b^6x^{7/3}}{9240b^{11}}-\frac{3a^{11}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}$$

input
```
Integrate[x^3/(a + b*x^(1/3)),x]
```

output
```
(27720*a^10*x^(1/3) - 13860*a^9*b*x^(2/3) + 9240*a^8*b^2*x - 6930*a^7*b^3*
x^(4/3) + 5544*a^6*b^4*x^(5/3) - 4620*a^5*b^5*x^2 + 3960*a^4*b^6*x^(7/3) -
 3465*a^3*b^7*x^(8/3) + 3080*a^2*b^8*x^3 - 2772*a*b^9*x^(10/3) + 2520*b^10
*x^(11/3))/(9240*b^11) - (3*a^11*Log[a + b*x^(1/3)])/b^12
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{11/3}}{a+b\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{a^{11}}{b^{11}\left(a+b\sqrt[3]{x}\right)}+\frac{a^{10}}{b^{11}}-\frac{\sqrt[3]{x}a^9}{b^{10}}+\frac{x^{2/3}a^8}{b^9}-\frac{xa^7}{b^8}+\frac{x^{4/3}a^6}{b^7}-\frac{x^{5/3}a^5}{b^6}+\frac{x^2a^4}{b^5}-\frac{x^{7/3}a^3}{b^4}+\frac{x^{8/3}a^2}{b^3}-\frac{x^3a}{b^2}+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^{11}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}+\frac{a^{10}\sqrt[3]{x}}{b^{11}}-\frac{a^9x^{2/3}}{2b^{10}}+\frac{a^8x}{3b^9}-\frac{a^7x^{4/3}}{4b^8}+\frac{a^6x^{5/3}}{5b^7}-\frac{a^5x^2}{6b^6}+\frac{a^4x^{7/3}}{7b^5}-\frac{a^3x^{8/3}}{8b^4}+\frac{a^2x^3}{9b^3}-\frac{ax^{10}}{10}\right.$$

input
```
Int[x^3/(a + b*x^(1/3)),x]
```

output
```
3*((a^10*x^(1/3))/b^11 - (a^9*x^(2/3))/(2*b^10) + (a^8*x)/(3*b^9) - (a^7*x
^(4/3))/(4*b^8) + (a^6*x^(5/3))/(5*b^7) - (a^5*x^2)/(6*b^6) + (a^4*x^(7/3)
)/(7*b^5) - (a^3*x^(8/3))/(8*b^4) + (a^2*x^3)/(9*b^3) - (a*x^(10/3))/(10*b
^2) + x^(11/3)/(11*b) - (a^11*Log[a + b*x^(1/3)])/b^12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{3x^{\frac{11}{3}}b^{10}}{11}-\frac{3a\,x^{\frac{10}{3}}b^{9}}{10}+\frac{a^{2}b^{8}x^{3}}{3}-\frac{3a^{3}x^{\frac{8}{3}}b^{7}}{8}+\frac{3a^{4}x^{\frac{7}{3}}b^{6}}{7}-\frac{a^{5}b^{5}x^{2}}{2}+\frac{3a^{6}x^{\frac{5}{3}}b^{4}}{5}-\frac{3a^{7}x^{\frac{4}{3}}b^{3}}{4}+a^{8}b^{2}x-\frac{3b\,a^{9}x^{\frac{2}{3}}}{2}+3a^{10}x^{\frac{1}{3}}}{b^{11}}-3a^{1}\ldots$
default	$\frac{\frac{3x^{\frac{11}{3}}b^{10}}{11}-\frac{3a\,x^{\frac{10}{3}}b^{9}}{10}+\frac{a^{2}b^{8}x^{3}}{3}-\frac{3a^{3}x^{\frac{8}{3}}b^{7}}{8}+\frac{3a^{4}x^{\frac{7}{3}}b^{6}}{7}-\frac{a^{5}b^{5}x^{2}}{2}+\frac{3a^{6}x^{\frac{5}{3}}b^{4}}{5}-\frac{3a^{7}x^{\frac{4}{3}}b^{3}}{4}+a^{8}b^{2}x-\frac{3b\,a^{9}x^{\frac{2}{3}}}{2}+3a^{10}x^{\frac{1}{3}}}{b^{11}}-3a^{1}\ldots$

input
```
int(x^3/(a+b*x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/b^11*(1/11*x^(11/3)*b^10-1/10*a*x^(10/3)*b^9+1/9*a^2*b^8*x^3-1/8*a^3*x^(
8/3)*b^7+1/7*a^4*x^(7/3)*b^6-1/6*a^5*b^5*x^2+1/5*a^6*x^(5/3)*b^4-1/4*a^7*x
^(4/3)*b^3+1/3*a^8*b^2*x-1/2*b*a^9*x^(2/3)+a^10*x^(1/3))-3*a^11*ln(a+b*x^(
1/3))/b^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx = \frac{3080\,a^2b^9x^3 - 4620\,a^5b^6x^2 + 9240\,a^8b^3x - 27720\,a^{11}\log\left(bx^{\frac{1}{3}} + a\right) + 63\,(40\,b^{11}x^3 - 55\,a^3b^8x^2 + 88\,a^6b^5x}{9240\,b^{12}}$$

input
```
integrate(x^3/(a+b*x^(1/3)),x, algorithm="fricas")
```

output
```
1/9240*(3080*a^2*b^9*x^3 - 4620*a^5*b^6*x^2 + 9240*a^8*b^3*x - 27720*a^11*
log(b*x^(1/3) + a) + 63*(40*b^11*x^3 - 55*a^3*b^8*x^2 + 88*a^6*b^5*x - 220
*a^9*b^2)*x^(2/3) - 198*(14*a*b^10*x^3 - 20*a^4*b^7*x^2 + 35*a^7*b^4*x - 1
40*a^10*b)*x^(1/3))/b^12
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a+b*x**(1/3)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx = -\frac{3\,a^{11}\log\left(bx^{\frac{1}{3}}+a\right)}{b^{12}} + \frac{3\left(bx^{\frac{1}{3}}+a\right)^{11}}{11\,b^{12}} - \frac{33\left(bx^{\frac{1}{3}}+a\right)^{10}a}{10\,b^{12}} + \frac{55\left(bx^{\frac{1}{3}}+a\right)^{9}a^2}{3\,b^{12}} - \frac{495\left(bx^{\frac{1}{3}}+a\right)^{8}a^3}{8\,b^{12}} + \frac{990\left(bx^{\frac{1}{3}}+a\right)^{7}a^4}{7\,b^{12}} - \frac{231\left(bx^{\frac{1}{3}}+a\right)^{6}a^5}{b^{12}} + \frac{1386\left(bx^{\frac{1}{3}}+a\right)^{5}a^6}{5\,b^{12}} - \frac{495\left(bx^{\frac{1}{3}}+a\right)^{4}a^7}{2\,b^{12}} + \frac{165\left(bx^{\frac{1}{3}}+a\right)^{3}a^8}{b^{12}} - \frac{165\left(bx^{\frac{1}{3}}+a\right)^{2}a^9}{2\,b^{12}} + \frac{33\left(bx^{\frac{1}{3}}+a\right)a^{10}}{b^{12}}$$

input
```
integrate(x^3/(a+b*x^(1/3)),x, algorithm="maxima")
```

output
```
-3*a^11*log(b*x^(1/3) + a)/b^12 + 3/11*(b*x^(1/3) + a)^11/b^12 - 33/10*(b*
x^(1/3) + a)^10*a/b^12 + 55/3*(b*x^(1/3) + a)^9*a^2/b^12 - 495/8*(b*x^(1/3
) + a)^8*a^3/b^12 + 990/7*(b*x^(1/3) + a)^7*a^4/b^12 - 231*(b*x^(1/3) + a)
^6*a^5/b^12 + 1386/5*(b*x^(1/3) + a)^5*a^6/b^12 - 495/2*(b*x^(1/3) + a)^4*
a^7/b^12 + 165*(b*x^(1/3) + a)^3*a^8/b^12 - 165/2*(b*x^(1/3) + a)^2*a^9/b^
12 + 33*(b*x^(1/3) + a)*a^10/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx = -\frac{3\,a^{11}\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^{12}} + \frac{2520\,b^{10}x^{\frac{11}{3}} - 2772\,ab^9x^{\frac{10}{3}} + 3080\,a^2b^8x^3 - 3465\,a^3b^7x^{\frac{8}{3}} + 3960\,a^4b^6x^{\frac{7}{3}} - 4620\,a^5b^5x^2 + 5544\,a^6b^4x^{\frac{5}{3}} - }{9240\,b^{11}}$$

input
```
integrate(x^3/(a+b*x^(1/3)),x, algorithm="giac")
```

output
```
-3*a^11*log(abs(b*x^(1/3) + a))/b^12 + 1/9240*(2520*b^10*x^(11/3) - 2772*a
*b^9*x^(10/3) + 3080*a^2*b^8*x^3 - 3465*a^3*b^7*x^(8/3) + 3960*a^4*b^6*x^(
7/3) - 4620*a^5*b^5*x^2 + 5544*a^6*b^4*x^(5/3) - 6930*a^7*b^3*x^(4/3) + 92
40*a^8*b^2*x - 13860*a^9*b*x^(2/3) + 27720*a^10*x^(1/3))/b^11
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx = \frac{3\,x^{11/3}}{11\,b} - \frac{3\,a\,x^{10/3}}{10\,b^2} + \frac{a^8\,x}{b^9} - \frac{3\,a^{11}\,\ln\left(a+b\,x^{1/3}\right)}{b^{12}} + \frac{a^2\,x^3}{3\,b^3} - \frac{a^5\,x^2}{2\,b^6} - \frac{3\,a^3\,x^{8/3}}{8\,b^4} + \frac{3\,a^4\,x^{7/3}}{7\,b^5} + \frac{3\,a^6\,x^{5/3}}{5\,b^7} - \frac{3\,a^7\,x^{4/3}}{4\,b^8} - \frac{3\,a^9\,x^{2/3}}{2\,b^{10}} + \frac{3\,a^{10}\,x^{1/3}}{b^{11}}$$

input
```
int(x^3/(a + b*x^(1/3)),x)
```

output
```
(3*x^(11/3))/(11*b) - (3*a*x^(10/3))/(10*b^2) + (a^8*x)/b^9 - (3*a^11*log(
a + b*x^(1/3)))/b^12 + (a^2*x^3)/(3*b^3) - (a^5*x^2)/(2*b^6) - (3*a^3*x^(8
/3))/(8*b^4) + (3*a^4*x^(7/3))/(7*b^5) + (3*a^6*x^(5/3))/(5*b^7) - (3*a^7*
x^(4/3))/(4*b^8) - (3*a^9*x^(2/3))/(2*b^10) + (3*a^10*x^(1/3))/b^11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{a+b\sqrt[3]{x}}\,dx = \frac{-13860x^{\frac{2}{3}}a^9b^2 + 5544x^{\frac{5}{3}}a^6b^5 - 3465x^{\frac{8}{3}}a^3b^8 + 2520x^{\frac{11}{3}}b^{11} + 27720x^{\frac{1}{3}}a^{10}b - 6930x^{\frac{4}{3}}a^7b^4 + 3960x^{\frac{7}{3}}a^4b^7 -}{9240b^{12}}$$

input
```
int(x^3/(a+b*x^(1/3)),x)
```

output
```
( - 13860*x**(2/3)*a**9*b**2 + 5544*x**(2/3)*a**6*b**5*x - 3465*x**(2/3)*a
**3*b**8*x**2 + 2520*x**(2/3)*b**11*x**3 + 27720*x**(1/3)*a**10*b - 6930*x
**(1/3)*a**7*b**4*x + 3960*x**(1/3)*a**4*b**7*x**2 - 2772*x**(1/3)*a*b**10
*x**3 - 27720*log(x**(1/3)*b + a)*a**11 + 9240*a**8*b**3*x - 4620*a**5*b**
6*x**2 + 3080*a**2*b**9*x**3)/(9240*b**12)
```

3.248 $\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx=-\frac{3a^7\sqrt[3]{x}}{b^8}+\frac{3a^6x^{2/3}}{2b^7}-\frac{a^5x}{b^6}+\frac{3a^4x^{4/3}}{4b^5}-\frac{3a^3x^{5/3}}{5b^4}+\frac{a^2x^2}{2b^3}-\frac{3ax^{7/3}}{7b^2}+\frac{3x^{8/3}}{8b}+\frac{3a^8\log\left(a+b\sqrt[3]{x}\right)}{b^9}$$

output
```
-3*a^7*x^(1/3)/b^8+3/2*a^6*x^(2/3)/b^7-a^5*x/b^6+3/4*a^4*x^(4/3)/b^5-3/5*a
^3*x^(5/3)/b^4+1/2*a^2*x^2/b^3-3/7*a*x^(7/3)/b^2+3/8*x^(8/3)/b+3*a^8*ln(a+
b*x^(1/3))/b^9
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx$$
$$=\frac{\sqrt[3]{x}(-840a^7+420a^6b\sqrt[3]{x}-280a^5b^2x^{2/3}+210a^4b^3x-168a^3b^4x^{4/3}+140a^2b^5x^{5/3}-120ab^6x^2+105b^7x^7}{280b^8}$$
$$+\frac{3a^8\log\left(a+b\sqrt[3]{x}\right)}{b^9}$$

input
```
Integrate[x^2/(a + b*x^(1/3)),x]
```

output
```
(x^(1/3)*(-840*a^7 + 420*a^6*b*x^(1/3) - 280*a^5*b^2*x^(2/3) + 210*a^4*b^3
*x - 168*a^3*b^4*x^(4/3) + 140*a^2*b^5*x^(5/3) - 120*a*b^6*x^2 + 105*b^7*x
^(7/3)))/(280*b^8) + (3*a^8*Log[a + b*x^(1/3)])/b^9
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{8/3}}{a+b\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^8}{b^8\left(a+b\sqrt[3]{x}\right)}-\frac{a^7}{b^8}+\frac{\sqrt[3]{x}a^6}{b^7}-\frac{x^{2/3}a^5}{b^6}+\frac{xa^4}{b^5}-\frac{x^{4/3}a^3}{b^4}+\frac{x^{5/3}a^2}{b^3}-\frac{x^2a}{b^2}+\frac{x^{7/3}}{b}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^8\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{a^7\sqrt[3]{x}}{b^8}+\frac{a^6x^{2/3}}{2b^7}-\frac{a^5x}{3b^6}+\frac{a^4x^{4/3}}{4b^5}-\frac{a^3x^{5/3}}{5b^4}+\frac{a^2x^2}{6b^3}-\frac{ax^{7/3}}{7b^2}+\frac{x^{8/3}}{8b}\right)$$

input
```
Int[x^2/(a + b*x^(1/3)),x]
```

output
```
3*(-((a^7*x^(1/3))/b^8) + (a^6*x^(2/3))/(2*b^7) - (a^5*x)/(3*b^6) + (a^4*x
^(4/3))/(4*b^5) - (a^3*x^(5/3))/(5*b^4) + (a^2*x^2)/(6*b^3) - (a*x^(7/3))/
(7*b^2) + x^(8/3)/(8*b) + (a^8*Log[a + b*x^(1/3)])/b^9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{3\left(-\frac{x^{\frac{8}{3}} b^{7}}{8}+\frac{a x^{\frac{7}{3}} b^{6}}{7}-\frac{a^{2} x^{2} b^{5}}{6}+\frac{a^{3} x^{\frac{5}{3}} b^{4}}{5}-\frac{a^{4} x^{\frac{4}{3}} b^{3}}{4}+\frac{a^{5} x b^{2}}{3}-\frac{b a^{6} x^{\frac{2}{3}}}{2}+a^{7} x^{\frac{1}{3}}\right)}{b^{8}}+\frac{3 a^{8} \ln\left(a+b x^{\frac{1}{3}}\right)}{b^{9}}$	99
default	$-\frac{3\left(-\frac{x^{\frac{8}{3}} b^{7}}{8}+\frac{a x^{\frac{7}{3}} b^{6}}{7}-\frac{a^{2} x^{2} b^{5}}{6}+\frac{a^{3} x^{\frac{5}{3}} b^{4}}{5}-\frac{a^{4} x^{\frac{4}{3}} b^{3}}{4}+\frac{a^{5} x b^{2}}{3}-\frac{b a^{6} x^{\frac{2}{3}}}{2}+a^{7} x^{\frac{1}{3}}\right)}{b^{8}}+\frac{3 a^{8} \ln\left(a+b x^{\frac{1}{3}}\right)}{b^{9}}$	99

input
```
int(x^2/(a+b*x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-3/b^8*(-1/8*x^(8/3)*b^7+1/7*a*x^(7/3)*b^6-1/6*a^2*x^2*b^5+1/5*a^3*x^(5/3)
*b^4-1/4*a^4*x^(4/3)*b^3+1/3*a^5*x*b^2-1/2*b*a^6*x^(2/3)+a^7*x^(1/3))+3*a^
8*ln(a+b*x^(1/3))/b^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx = \frac{140\,a^2b^6x^2-280\,a^5b^3x+840\,a^8\log\left(bx^{\frac{1}{3}}+a\right)+21\,(5\,b^8x^2-8\,a^3b^5x+20\,a^6b^2)x^{\frac{2}{3}}-30\,(4\,ab^7x^2-7\,a^4b}{280\,b^9}$$

input

```
integrate(x^2/(a+b*x^(1/3)),x, algorithm="fricas")
```

output

```
1/280*(140*a^2*b^6*x^2 - 280*a^5*b^3*x + 840*a^8*log(b*x^(1/3) + a) + 21*(
5*b^8*x^2 - 8*a^3*b^5*x + 20*a^6*b^2)*x^(2/3) - 30*(4*a*b^7*x^2 - 7*a^4*b^
4*x + 28*a^7*b)*x^(1/3))/b^9
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx = \text{Timed out}$$

input

```
integrate(x**2/(a+b*x**(1/3)),x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx = {} & \frac{3\,a^8\log\left(bx^{\frac{1}{3}}+a\right)}{b^9}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^8}{8\,b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)^7a}{7\,b^9}\\ & +\frac{14\left(bx^{\frac{1}{3}}+a\right)^6a^2}{b^9}-\frac{168\left(bx^{\frac{1}{3}}+a\right)^5a^3}{5\,b^9}+\frac{105\left(bx^{\frac{1}{3}}+a\right)^4a^4}{2\,b^9}\\ & -\frac{56\left(bx^{\frac{1}{3}}+a\right)^3a^5}{b^9}+\frac{42\left(bx^{\frac{1}{3}}+a\right)^2a^6}{b^9}-\frac{24\left(bx^{\frac{1}{3}}+a\right)a^7}{b^9}\end{aligned}$$

input
```
integrate(x^2/(a+b*x^(1/3)),x, algorithm="maxima")
```

output
```
3*a^8*log(b*x^(1/3) + a)/b^9 + 3/8*(b*x^(1/3) + a)^8/b^9 - 24/7*(b*x^(1/3)
 + a)^7*a/b^9 + 14*(b*x^(1/3) + a)^6*a^2/b^9 - 168/5*(b*x^(1/3) + a)^5*a^3
/b^9 + 105/2*(b*x^(1/3) + a)^4*a^4/b^9 - 56*(b*x^(1/3) + a)^3*a^5/b^9 + 42
*(b*x^(1/3) + a)^2*a^6/b^9 - 24*(b*x^(1/3) + a)*a^7/b^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx = \frac{3\,a^8\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^9} + \frac{105\,b^7x^{\frac{8}{3}} - 120\,ab^6x^{\frac{7}{3}} + 140\,a^2b^5x^2 - 168\,a^3b^4x^{\frac{5}{3}} + 210\,a^4b^3x^{\frac{4}{3}} - 280\,a^5b^2x + 420\,a^6bx^{\frac{2}{3}} - 840\,a^7x^{\frac{1}{3}}}{280\,b^8}$$

input
```
integrate(x^2/(a+b*x^(1/3)),x, algorithm="giac")
```

output
```
3*a^8*log(abs(b*x^(1/3) + a))/b^9 + 1/280*(105*b^7*x^(8/3) - 120*a*b^6*x^(
7/3) + 140*a^2*b^5*x^2 - 168*a^3*b^4*x^(5/3) + 210*a^4*b^3*x^(4/3) - 280*a
^5*b^2*x + 420*a^6*b*x^(2/3) - 840*a^7*x^(1/3))/b^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx = \frac{3\,x^{8/3}}{8\,b} - \frac{a^5\,x}{b^6} - \frac{3\,a\,x^{7/3}}{7\,b^2} + \frac{3\,a^8\,\ln\left(a+b\,x^{1/3}\right)}{b^9} + \frac{a^2\,x^2}{2\,b^3} - \frac{3\,a^3\,x^{5/3}}{5\,b^4} + \frac{3\,a^4\,x^{4/3}}{4\,b^5} + \frac{3\,a^6\,x^{2/3}}{2\,b^7} - \frac{3\,a^7\,x^{1/3}}{b^8}$$

input
```
int(x^2/(a + b*x^(1/3)),x)
```

output
```
(3*x^(8/3))/(8*b) - (a^5*x)/b^6 - (3*a*x^(7/3))/(7*b^2) + (3*a^8*log(a + b
*x^(1/3)))/b^9 + (a^2*x^2)/(2*b^3) - (3*a^3*x^(5/3))/(5*b^4) + (3*a^4*x^(4
/3))/(4*b^5) + (3*a^6*x^(2/3))/(2*b^7) - (3*a^7*x^(1/3))/b^8
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{a+b\sqrt[3]{x}}\,dx = \frac{420x^{\frac{2}{3}}a^6b^2 - 168x^{\frac{5}{3}}a^3b^5 + 105x^{\frac{8}{3}}b^8 - 840x^{\frac{1}{3}}a^7b + 210x^{\frac{4}{3}}a^4b^4 - 120x^{\frac{7}{3}}a\,b^7 + 840\log\left(x^{\frac{1}{3}}b+a\right)a^8 - 280a}{280b^9}$$

input
```
int(x^2/(a+b*x^(1/3)),x)
```

output
```
(420*x**(2/3)*a**6*b**2 - 168*x**(2/3)*a**3*b**5*x + 105*x**(2/3)*b**8*x**
2 - 840*x**(1/3)*a**7*b + 210*x**(1/3)*a**4*b**4*x - 120*x**(1/3)*a*b**7*x
**2 + 840*log(x**(1/3)*b + a)*a**8 - 280*a**5*b**3*x + 140*a**2*b**6*x**2)
/(280*b**9)
```

3.249 $\int \frac{x}{a+b\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx=\frac{3a^4\sqrt[3]{x}}{b^5}-\frac{3a^3x^{2/3}}{2b^4}+\frac{a^2x}{b^3}-\frac{3ax^{4/3}}{4b^2}+\frac{3x^{5/3}}{5b}-\frac{3a^5\log\left(a+b\sqrt[3]{x}\right)}{b^6}$$

output
```
3*a^4*x^(1/3)/b^5-3/2*a^3*x^(2/3)/b^4+a^2*x/b^3-3/4*a*x^(4/3)/b^2+3/5*x^(5
/3)/b-3*a^5*ln(a+b*x^(1/3))/b^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx=\frac{\sqrt[3]{x}\left(60a^4-30a^3b\sqrt[3]{x}+20a^2b^2x^{2/3}-15ab^3x+12b^4x^{4/3}\right)}{20b^5}-\frac{3a^5\log\left(a+b\sqrt[3]{x}\right)}{b^6}$$

input
```
Integrate[x/(a + b*x^(1/3)),x]
```

output
```
(x^(1/3)*(60*a^4 - 30*a^3*b*x^(1/3) + 20*a^2*b^2*x^(2/3) - 15*a*b^3*x + 12
*b^4*x^(4/3)))/(20*b^5) - (3*a^5*Log[a + b*x^(1/3)])/b^6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{5/3}}{a+b\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{a^5}{b^5\left(a+b\sqrt[3]{x}\right)}+\frac{a^4}{b^5}-\frac{\sqrt[3]{x}a^3}{b^4}+\frac{x^{2/3}a^2}{b^3}-\frac{xa}{b^2}+\frac{x^{4/3}}{b}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^5\log\left(a+b\sqrt[3]{x}\right)}{b^6}+\frac{a^4\sqrt[3]{x}}{b^5}-\frac{a^3x^{2/3}}{2b^4}+\frac{a^2x}{3b^3}-\frac{ax^{4/3}}{4b^2}+\frac{x^{5/3}}{5b}\right)$$

input
```
Int[x/(a + b*x^(1/3)),x]
```

output
```
3*((a^4*x^(1/3))/b^5 - (a^3*x^(2/3))/(2*b^4) + (a^2*x)/(3*b^3) - (a*x^(4/3
))/(4*b^2) + x^(5/3)/(5*b) - (a^5*Log[a + b*x^(1/3)])/b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{3x^{\frac{5}{3}}b^4}{5}-\frac{3a\,x^{\frac{4}{3}}b^3}{4}+a^2b^2x-\frac{3b\,a^3x^{\frac{2}{3}}}{2}+3a^4x^{\frac{1}{3}}}{b^5}-\frac{3a^5\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^6}$	66
default	$\frac{\frac{3x^{\frac{5}{3}}b^4}{5}-\frac{3a\,x^{\frac{4}{3}}b^3}{4}+a^2b^2x-\frac{3b\,a^3x^{\frac{2}{3}}}{2}+3a^4x^{\frac{1}{3}}}{b^5}-\frac{3a^5\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^6}$	66

input
```
int(x/(a+b*x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/b^5*(1/5*x^(5/3)*b^4-1/4*a*x^(4/3)*b^3+1/3*a^2*b^2*x-1/2*b*a^3*x^(2/3)+a
^4*x^(1/3))-3*a^5*ln(a+b*x^(1/3))/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx = \frac{20\,a^2b^3x-60\,a^5\log\left(bx^{\frac{1}{3}}+a\right)+6\,(2\,b^5x-5\,a^3b^2)x^{\frac{2}{3}}-15\,(ab^4x-4\,a^4b)x^{\frac{1}{3}}}{20\,b^6}$$

input
```
integrate(x/(a+b*x^(1/3)),x, algorithm="fricas")
```

output
```
1/20*(20*a^2*b^3*x - 60*a^5*log(b*x^(1/3) + a) + 6*(2*b^5*x - 5*a^3*b^2)*x
^(2/3) - 15*(a*b^4*x - 4*a^4*b)*x^(1/3))/b^6
```

Sympy [A] (verification not implemented)

Time = 170.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx = -\frac{3a^5\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{b^6} + \frac{3a^4\sqrt[3]{x}}{b^5} - \frac{3a^3x^{\frac{2}{3}}}{2b^4} + \frac{a^2x}{b^3} - \frac{3ax^{\frac{4}{3}}}{4b^2} + \frac{3x^{\frac{5}{3}}}{5b}$$

input
```
integrate(x/(a+b*x**(1/3)),x)
```

output
```
-3*a**5*log(1 + b*x**(1/3)/a)/b**6 + 3*a**4*x**(1/3)/b**5 - 3*a**3*x**(2/3
)/(2*b**4) + a**2*x/b**3 - 3*a*x**(4/3)/(4*b**2) + 3*x**(5/3)/(5*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx = -\frac{3\,a^5\log\left(bx^{\frac{1}{3}}+a\right)}{b^6} + \frac{3\left(bx^{\frac{1}{3}}+a\right)^5}{5\,b^6} - \frac{15\left(bx^{\frac{1}{3}}+a\right)^4a}{4\,b^6} + \frac{10\left(bx^{\frac{1}{3}}+a\right)^3a^2}{b^6} - \frac{15\left(bx^{\frac{1}{3}}+a\right)^2a^3}{b^6} + \frac{15\left(bx^{\frac{1}{3}}+a\right)a^4}{b^6}$$

input
```
integrate(x/(a+b*x^(1/3)),x, algorithm="maxima")
```

output
```
-3*a^5*log(b*x^(1/3) + a)/b^6 + 3/5*(b*x^(1/3) + a)^5/b^6 - 15/4*(b*x^(1/3
) + a)^4*a/b^6 + 10*(b*x^(1/3) + a)^3*a^2/b^6 - 15*(b*x^(1/3) + a)^2*a^3/b
^6 + 15*(b*x^(1/3) + a)*a^4/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx = -\frac{3\,a^5\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^6} + \frac{12\,b^4x^{\frac{5}{3}} - 15\,ab^3x^{\frac{4}{3}} + 20\,a^2b^2x - 30\,a^3bx^{\frac{2}{3}} + 60\,a^4x^{\frac{1}{3}}}{20\,b^5}$$

input
```
integrate(x/(a+b*x^(1/3)),x, algorithm="giac")
```

output
```
-3*a^5*log(abs(b*x^(1/3) + a))/b^6 + 1/20*(12*b^4*x^(5/3) - 15*a*b^3*x^(4/
3) + 20*a^2*b^2*x - 30*a^3*b*x^(2/3) + 60*a^4*x^(1/3))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx = \frac{3\,x^{5/3}}{5\,b} + \frac{a^2\,x}{b^3} - \frac{3\,a\,x^{4/3}}{4\,b^2} - \frac{3\,a^5\,\ln\left(a+b\,x^{1/3}\right)}{b^6} - \frac{3\,a^3\,x^{2/3}}{2\,b^4} + \frac{3\,a^4\,x^{1/3}}{b^5}$$

input
```
int(x/(a + b*x^(1/3)),x)
```

output
```
(3*x^(5/3))/(5*b) + (a^2*x)/b^3 - (3*a*x^(4/3))/(4*b^2) - (3*a^5*log(a + b
*x^(1/3)))/b^6 - (3*a^3*x^(2/3))/(2*b^4) + (3*a^4*x^(1/3))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+b\sqrt[3]{x}}\,dx = \frac{-30x^{\frac{2}{3}}a^3b^2 + 12x^{\frac{5}{3}}b^5 + 60x^{\frac{1}{3}}a^4b - 15x^{\frac{4}{3}}a\,b^4 - 60\log\left(x^{\frac{1}{3}}b+a\right)a^5 + 20a^2b^3x}{20b^6}$$

input
```
int(x/(a+b*x^(1/3)),x)
```

output
```
( - 30*x**(2/3)*a**3*b**2 + 12*x**(2/3)*b**5*x + 60*x**(1/3)*a**4*b - 15*x
**(1/3)*a*b**4*x - 60*log(x**(1/3)*b + a)*a**5 + 20*a**2*b**3*x)/(20*b**6)
```

3.250 $\int \frac{1}{a+b\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx = -\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{2/3}}{2b}+\frac{3a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

output
```
-3*a*x^(1/3)/b^2+3/2*x^(2/3)/b+3*a^2*ln(a+b*x^(1/3))/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx = \frac{3\left(-2a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{2b^2}+\frac{3a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

input
```
Integrate[(a + b*x^(1/3))^(-1),x]
```

output
```
(3*(-2*a + b*x^(1/3))*x^(1/3))/(2*b^2) + (3*a^2*Log[a + b*x^(1/3)])/b^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{x^{2/3}}{a+b\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)}-\frac{a}{b^2}+\frac{\sqrt[3]{x}}{b}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2\log\left(a+b\sqrt[3]{x}\right)}{b^3}-\frac{a\sqrt[3]{x}}{b^2}+\frac{x^{2/3}}{2b}\right)$$

input
```
Int[(a + b*x^(1/3))^(-1),x]
```

output
```
3*(-((a*x^(1/3))/b^2) + x^(2/3)/(2*b) + (a^2*Log[a + b*x^(1/3)])/b^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{3\left(-\frac{b\,x^{\frac{2}{3}}}{2}+a\,x^{\frac{1}{3}}\right)}{b^{2}}+\frac{3a^{2}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^{3}}$	35
default	$\frac{a^{2}\ln\left(b^{3}x+a^{3}\right)}{b^{3}}+\frac{3x^{\frac{2}{3}}}{2b}-\frac{a^{2}\ln\left(b^{2}x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^{2}\right)}{b^{3}}+\frac{2a^{2}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^{3}}-\frac{3a\,x^{\frac{1}{3}}}{b^{2}}$	79

input
```
int(1/(a+b*x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-3/b^2*(-1/2*b*x^(2/3)+a*x^(1/3))+3*a^2*ln(a+b*x^(1/3))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx=\frac{3\left(2\,a^{2}\log\left(bx^{\frac{1}{3}}+a\right)+b^{2}x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^{3}}$$

input
```
integrate(1/(a+b*x^(1/3)),x, algorithm="fricas")
```

output
```
3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx = \begin{cases} \frac{3a^2\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{b^3} - \frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{\frac{2}{3}}}{2b} & \text{for } b \neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3)),x)
```

output
```
Piecewise((3*a**2*log(a/b + x**(1/3))/b**3 - 3*a*x**(1/3)/b**2 + 3*x**(2/3
)/(2*b), Ne(b, 0)), (x/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx = \frac{3\,a^2\log\left(bx^{\frac{1}{3}}+a\right)}{b^3} + \frac{3\left(bx^{\frac{1}{3}}+a\right)^2}{2\,b^3} - \frac{6\left(bx^{\frac{1}{3}}+a\right)a}{b^3}$$

input
```
integrate(1/(a+b*x^(1/3)),x, algorithm="maxima")
```

output
```
3*a^2*log(b*x^(1/3) + a)/b^3 + 3/2*(b*x^(1/3) + a)^2/b^3 - 6*(b*x^(1/3) +
a)*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx = \frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3} + \frac{3\left(bx^{\frac{2}{3}}-2\,ax^{\frac{1}{3}}\right)}{2\,b^2}$$

input
```
integrate(1/(a+b*x^(1/3)),x, algorithm="giac")
```

output
```
3*a^2*log(abs(b*x^(1/3) + a))/b^3 + 3/2*(b*x^(2/3) - 2*a*x^(1/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx = \frac{6\,a^2\,\ln\left(a+b\,x^{1/3}\right)+3\,b^2\,x^{2/3}-6\,a\,b\,x^{1/3}}{2\,b^3}$$

input
```
int(1/(a + b*x^(1/3)),x)
```

output
```
(6*a^2*log(a + b*x^(1/3)) + 3*b^2*x^(2/3) - 6*a*b*x^(1/3))/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{1}{a+b\sqrt[3]{x}}\,dx = \frac{\frac{3x^{\frac{2}{3}}b^2}{2}-3x^{\frac{1}{3}}ab+3\log\left(x^{\frac{1}{3}}b+a\right)a^2}{b^3}$$

input
```
int(1/(a+b*x^(1/3)),x)
```

output
```
(3*(x**(2/3)*b**2 - 2*x**(1/3)*a*b + 2*log(x**(1/3)*b + a)*a**2))/(2*b**3)
```

3.251 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx=-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a}+\frac{\log(x)}{a}$$

output
```
-3*ln(a+b*x^(1/3))/a+ln(x)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx=-\frac{3\log\left(a^2+ab\sqrt[3]{x}\right)}{a}+\frac{3\log\left(\sqrt[3]{x}\right)}{a}$$

input
```
Integrate[1/((a + b*x^(1/3))*x),x]
```

output
```
(-3*Log[a^2 + a*b*x^(1/3)])/a + (3*Log[x^(1/3)])/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt[3]{x}\right)}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{47}$$

$$3\left(\frac{\int \frac{1}{\sqrt[3]{x}}d\sqrt[3]{x}}{a}-\frac{b\int \frac{1}{a+b\sqrt[3]{x}}d\sqrt[3]{x}}{a}\right)$$

$$\Big\downarrow \text{14}$$

$$3\left(\frac{\log\left(\sqrt[3]{x}\right)}{a}-\frac{b\int \frac{1}{a+b\sqrt[3]{x}}d\sqrt[3]{x}}{a}\right)$$

$$\Big\downarrow \text{16}$$

$$3\left(\frac{\log\left(\sqrt[3]{x}\right)}{a}-\frac{\log\left(a+b\sqrt[3]{x}\right)}{a}\right)$$

input
```
Int[1/((a + b*x^(1/3))*x),x]
```

output
```
3*(-(Log[a + b*x^(1/3)]/a) + Log[x^(1/3)]/a)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a}+\frac{\ln\left(x\right)}{a}$	21
default	$-\frac{3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a}+\frac{\ln\left(x\right)}{a}$	21

input
```
int(1/(a+b*x^(1/3))/x,x,method=_RETURNVERBOSE)
```

output
```
-3*ln(a+b*x^(1/3))/a+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx = -\frac{3\left(\log\left(bx^{\frac{1}{3}}+a\right)-\log\left(x^{\frac{1}{3}}\right)\right)}{a}$$

input
```
integrate(1/(a+b*x^(1/3))/x,x, algorithm="fricas")
```

output
```
-3*(log(b*x^(1/3) + a) - log(x^(1/3)))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a=0 \wedge b=0 \\ -\frac{3}{b\sqrt[3]{x}} & \text{for } a=0 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ \frac{\log(x)}{a}-\frac{3\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))/x,x)
```

output
```
Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (-3/(b*x**(1/3)), Eq(a, 0))
, (log(x)/a, Eq(b, 0)), (log(x)/a - 3*log(a/b + x**(1/3))/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx = -\frac{3\,\log\left(bx^{\frac{1}{3}}+a\right)}{a}+\frac{\log\left(x\right)}{a}$$

input
```
integrate(1/(a+b*x^(1/3))/x,x, algorithm="maxima")
```

output
```
-3*log(b*x^(1/3) + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx = -\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a}+\frac{\log\left(|x|\right)}{a}$$

input
```
integrate(1/(a+b*x^(1/3))/x,x, algorithm="giac")
```

output
```
-3*log(abs(b*x^(1/3) + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx = -\frac{6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a}$$

input
```
int(1/(x*(a + b*x^(1/3))),x)
```

output
```
-(6*atanh((2*b*x^(1/3))/a + 1))/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x}\,dx = \frac{3\log\left(x^{\frac{1}{3}}\right)-3\log\left(x^{\frac{1}{3}}b+a\right)}{a}$$

input
```
int(1/(a+b*x^(1/3))/x,x)
```

output
```
(3*(log(x**(1/3)) - log(x**(1/3)*b + a)))/a
```

3.252 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx = -\frac{1}{ax}+\frac{3b}{2a^2x^{2/3}}-\frac{3b^2}{a^3\sqrt[3]{x}}+\frac{3b^3\log\left(a+b\sqrt[3]{x}\right)}{a^4}-\frac{b^3\log(x)}{a^4}$$

output
```
-1/a/x+3/2*b/a^2/x^(2/3)-3*b^2/a^3/x^(1/3)+3*b^3*ln(a+b*x^(1/3))/a^4-b^3*l
n(x)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx = -\frac{2a^3-3a^2b\sqrt[3]{x}+6ab^2x^{2/3}-6b^3x\log\left(a+b\sqrt[3]{x}\right)+2b^3x\log(x)}{2a^4x}$$

input
```
Integrate[1/((a + b*x^(1/3))*x^2),x]
```

output
```
-1/2*(2*a^3 - 3*a^2*b*x^(1/3) + 6*a*b^2*x^(2/3) - 6*b^3*x*Log[a + b*x^(1/3
)] + 2*b^3*x*Log[x])/(a^4*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt[3]{x}\right)} \, dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{b^4}{a^4\left(a+b\sqrt[3]{x}\right)}-\frac{b^3}{a^4\sqrt[3]{x}}+\frac{b^2}{a^3x^{2/3}}-\frac{b}{a^2x}+\frac{1}{ax^{4/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{b^3\log\left(a+b\sqrt[3]{x}\right)}{a^4}-\frac{b^3\log\left(\sqrt[3]{x}\right)}{a^4}-\frac{b^2}{a^3\sqrt[3]{x}}+\frac{b}{2a^2x^{2/3}}-\frac{1}{3ax}\right)$$

input
```
Int[1/((a + b*x^(1/3))*x^2),x]
```

output
```
3*(-1/3*1/(a*x) + b/(2*a^2*x^(2/3)) - b^2/(a^3*x^(1/3)) + (b^3*Log[a + b*x
^(1/3)])/a^4 - (b^3*Log[x^(1/3)])/a^4)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{1}{ax}+\frac{3b}{2a^2x^{\frac{2}{3}}}-\frac{3b^2}{a^3x^{\frac{1}{3}}}+\frac{3b^3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^4}-\frac{b^3\ln(x)}{a^4}$	56
default	$-\frac{1}{ax}+\frac{3b}{2a^2x^{\frac{2}{3}}}-\frac{3b^2}{a^3x^{\frac{1}{3}}}+\frac{3b^3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^4}-\frac{b^3\ln(x)}{a^4}$	56

input
```
int(1/(a+b*x^(1/3))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a/x+3/2*b/a^2/x^(2/3)-3*b^2/a^3/x^(1/3)+3*b^3*ln(a+b*x^(1/3))/a^4-b^3*l
n(x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx=\frac{6\,b^3x\log\left(bx^{\frac{1}{3}}+a\right)-6\,b^3x\log\left(x^{\frac{1}{3}}\right)-6\,ab^2x^{\frac{2}{3}}+3\,a^2bx^{\frac{1}{3}}-2\,a^3}{2\,a^4x}$$

input
```
integrate(1/(a+b*x^(1/3))/x^2,x, algorithm="fricas")
```

output
```
1/2*(6*b^3*x*log(b*x^(1/3) + a) - 6*b^3*x*log(x^(1/3)) - 6*a*b^2*x^(2/3) +
 3*a^2*b*x^(1/3) - 2*a^3)/(a^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for}\ a=0 \wedge b=0 \\ -\frac{3}{4bx^{\frac{4}{3}}} & \text{for}\ a=0 \\ -\frac{1}{ax} & \text{for}\ b=0 \\ -\frac{1}{ax}+\frac{3b}{2a^2x^{\frac{2}{3}}}-\frac{3b^2}{a^3\sqrt[3]{x}}-\frac{b^3\log(x)}{a^4}+\frac{3b^3\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))/x**2,x)
```

output
```
Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b*x**(4/3)), Eq(a, 0
)), (-1/(a*x), Eq(b, 0)), (-1/(a*x) + 3*b/(2*a**2*x**(2/3)) - 3*b**2/(a**3
*x**(1/3)) - b**3*log(x)/a**4 + 3*b**3*log(a/b + x**(1/3))/a**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx = \frac{3\,b^3\log\left(bx^{\frac{1}{3}}+a\right)}{a^4}-\frac{b^3\log(x)}{a^4}-\frac{6\,b^2x^{\frac{2}{3}}-3\,abx^{\frac{1}{3}}+2\,a^2}{2\,a^3x}$$

input
```
integrate(1/(a+b*x^(1/3))/x^2,x, algorithm="maxima")
```

output
```
3*b^3*log(b*x^(1/3) + a)/a^4 - b^3*log(x)/a^4 - 1/2*(6*b^2*x^(2/3) - 3*a*b
*x^(1/3) + 2*a^2)/(a^3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx = \frac{3\,b^3\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^4} - \frac{b^3\log\left(|x|\right)}{a^4} - \frac{6\,ab^2x^{\frac{2}{3}} - 3\,a^2bx^{\frac{1}{3}} + 2\,a^3}{2\,a^4x}$$

input
```
integrate(1/(a+b*x^(1/3))/x^2,x, algorithm="giac")
```

output
```
3*b^3*log(abs(b*x^(1/3) + a))/a^4 - b^3*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^(
2/3) - 3*a^2*b*x^(1/3) + 2*a^3)/(a^4*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx = \frac{6\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^4} - \frac{\frac{1}{a} - \frac{3\,b\,x^{1/3}}{2\,a^2} + \frac{3\,b^2\,x^{2/3}}{a^3}}{x}$$

input
```
int(1/(x^2*(a + b*x^(1/3))),x)
```

output
```
(6*b^3*atanh((2*b*x^(1/3))/a + 1))/a^4 - (1/a - (3*b*x^(1/3))/(2*a^2) + (3
*b^2*x^(2/3))/a^3)/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^2}\,dx = \frac{-6x^{\frac{2}{3}}a\,b^2 + 3x^{\frac{1}{3}}a^2b - 6\log\left(x^{\frac{1}{3}}\right)b^3x + 6\log\left(x^{\frac{1}{3}}b+a\right)b^3x - 2a^3}{2a^4x}$$

input
```
int(1/(a+b*x^(1/3))/x^2,x)
```

output
```
( - 6*x**(2/3)*a*b**2 + 3*x**(1/3)*a**2*b - 6*log(x**(1/3))*b**3*x + 6*log
(x**(1/3)*b + a)*b**3*x - 2*a**3)/(2*a**4*x)
```

3.253 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx=-\frac{1}{2ax^2}+\frac{3b}{5a^2x^{5/3}}-\frac{3b^2}{4a^3x^{4/3}}+\frac{b^3}{a^4x}-\frac{3b^4}{2a^5x^{2/3}}+\frac{3b^5}{a^6\sqrt[3]{x}}-\frac{3b^6\log\left(a+b\sqrt[3]{x}\right)}{a^7}+\frac{b^6\log(x)}{a^7}$$

output
```
-1/2/a/x^2+3/5*b/a^2/x^(5/3)-3/4*b^2/a^3/x^(4/3)+b^3/a^4/x-3/2*b^4/a^5/x^(
2/3)+3*b^5/a^6/x^(1/3)-3*b^6*ln(a+b*x^(1/3))/a^7+b^6*ln(x)/a^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx=\frac{\frac{a\left(-10a^5+12a^4b\sqrt[3]{x}-15a^3b^2x^{2/3}+20a^2b^3x-30ab^4x^{4/3}+60b^5x^{5/3}\right)}{x^2}-60b^6\log\left(a+b\sqrt[3]{x}\right)+20b^6\log(x)}{20a^7}$$

input
```
Integrate[1/((a + b*x^(1/3))*x^3),x]
```

output
```
((a*(-10*a^5 + 12*a^4*b*x^(1/3) - 15*a^3*b^2*x^(2/3) + 20*a^2*b^3*x - 30*a
*b^4*x^(4/3) + 60*b^5*x^(5/3)))/x^2 - 60*b^6*Log[a + b*x^(1/3)] + 20*b^6*L
og[x])/(20*a^7)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\sqrt[3]{x}\right)}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(-\frac{b^7}{a^7\left(a+b\sqrt[3]{x}\right)}+\frac{b^6}{a^7\sqrt[3]{x}}-\frac{b^5}{a^6x^{2/3}}+\frac{b^4}{a^5x}-\frac{b^3}{a^4x^{4/3}}+\frac{b^2}{a^3x^{5/3}}-\frac{b}{a^2x^2}+\frac{1}{ax^{7/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{b^6\log\left(a+b\sqrt[3]{x}\right)}{a^7}+\frac{b^6\log\left(\sqrt[3]{x}\right)}{a^7}+\frac{b^5}{a^6\sqrt[3]{x}}-\frac{b^4}{2a^5x^{2/3}}+\frac{b^3}{3a^4x}-\frac{b^2}{4a^3x^{4/3}}+\frac{b}{5a^2x^{5/3}}-\frac{1}{6ax^2}\right)$$

input
```
Int[1/((a + b*x^(1/3))*x^3),x]
```

output
```
3*(-1/6*1/(a*x^2) + b/(5*a^2*x^(5/3)) - b^2/(4*a^3*x^(4/3)) + b^3/(3*a^4*x
) - b^4/(2*a^5*x^(2/3)) + b^5/(a^6*x^(1/3)) - (b^6*Log[a + b*x^(1/3)])/a^7
 + (b^6*Log[x^(1/3)])/a^7)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{1}{2a\,x^{2}}+\frac{3b}{5a^{2}x^{\frac{5}{3}}}-\frac{3b^{2}}{4a^{3}x^{\frac{4}{3}}}+\frac{b^{3}}{a^{4}x}-\frac{3b^{4}}{2a^{5}x^{\frac{2}{3}}}+\frac{3b^{5}}{a^{6}x^{\frac{1}{3}}}-\frac{3b^{6}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{7}}+\frac{b^{6}\ln\left(x\right)}{a^{7}}$	87
default	$-\frac{1}{2a\,x^{2}}+\frac{3b}{5a^{2}x^{\frac{5}{3}}}-\frac{3b^{2}}{4a^{3}x^{\frac{4}{3}}}+\frac{b^{3}}{a^{4}x}-\frac{3b^{4}}{2a^{5}x^{\frac{2}{3}}}+\frac{3b^{5}}{a^{6}x^{\frac{1}{3}}}-\frac{3b^{6}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{7}}+\frac{b^{6}\ln\left(x\right)}{a^{7}}$	87

input
```
int(1/(a+b*x^(1/3))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+3/5*b/a^2/x^(5/3)-3/4*b^2/a^3/x^(4/3)+b^3/a^4/x-3/2*b^4/a^5/x^(
2/3)+3*b^5/a^6/x^(1/3)-3*b^6*ln(a+b*x^(1/3))/a^7+b^6*ln(x)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx =$$
$$-\frac{60\,b^6x^2\log\left(bx^{\frac{1}{3}}+a\right)-60\,b^6x^2\log\left(x^{\frac{1}{3}}\right)-20\,a^3b^3x+10\,a^6-15\,(4\,ab^5x-a^4b^2)x^{\frac{2}{3}}+6\,(5\,a^2b^4x-2\,a}{20\,a^7x^2}$$

input
```
integrate(1/(a+b*x^(1/3))/x^3,x, algorithm="fricas")
```

output
```
-1/20*(60*b^6*x^2*log(b*x^(1/3) + a) - 60*b^6*x^2*log(x^(1/3)) - 20*a^3*b^
3*x + 10*a^6 - 15*(4*a*b^5*x - a^4*b^2)*x^(2/3) + 6*(5*a^2*b^4*x - 2*a^5*b
)*x^(1/3))/(a^7*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx$$
$$=\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{3}}} & \text{for } a=0\wedge b=0 \\ -\frac{3}{7bx^{\frac{7}{3}}} & \text{for } a=0 \\ -\frac{1}{2ax^2} & \text{for } b=0 \\ -\frac{1}{2ax^2}+\frac{3b}{5a^2x^{\frac{5}{3}}}-\frac{3b^2}{4a^3x^{\frac{4}{3}}}+\frac{b^3}{a^4x}-\frac{3b^4}{2a^5x^{\frac{2}{3}}}+\frac{3b^5}{a^6\sqrt[3]{x}}+\frac{b^6\log(x)}{a^7}-\frac{3b^6\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^7} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))/x**3,x)
```

output
```
Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (-3/(7*b*x**(7/3)), Eq(a, 0
)), (-1/(2*a*x**2), Eq(b, 0)), (-1/(2*a*x**2) + 3*b/(5*a**2*x**(5/3)) - 3*
b**2/(4*a**3*x**(4/3)) + b**3/(a**4*x) - 3*b**4/(2*a**5*x**(2/3)) + 3*b**5
/(a**6*x**(1/3)) + b**6*log(x)/a**7 - 3*b**6*log(a/b + x**(1/3))/a**7, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx = -\frac{3\,b^6\log\left(bx^{\frac{1}{3}}+a\right)}{a^7} + \frac{b^6\log\left(x\right)}{a^7} + \frac{60\,b^5x^{\frac{5}{3}} - 30\,ab^4x^{\frac{4}{3}} + 20\,a^2b^3x - 15\,a^3b^2x^{\frac{2}{3}} + 12\,a^4bx^{\frac{1}{3}} - 10\,a^5}{20\,a^6x^2}$$

input
```
integrate(1/(a+b*x^(1/3))/x^3,x, algorithm="maxima")
```

output
```
-3*b^6*log(b*x^(1/3) + a)/a^7 + b^6*log(x)/a^7 + 1/20*(60*b^5*x^(5/3) - 30
*a*b^4*x^(4/3) + 20*a^2*b^3*x - 15*a^3*b^2*x^(2/3) + 12*a^4*b*x^(1/3) - 10
*a^5)/(a^6*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx = -\frac{3\,b^6\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^7} + \frac{b^6\log\left(|x|\right)}{a^7} + \frac{60\,ab^5x^{\frac{5}{3}} - 30\,a^2b^4x^{\frac{4}{3}} + 20\,a^3b^3x - 15\,a^4b^2x^{\frac{2}{3}} + 12\,a^5bx^{\frac{1}{3}} - 10\,a^6}{20\,a^7x^2}$$

input
```
integrate(1/(a+b*x^(1/3))/x^3,x, algorithm="giac")
```

output
```
-3*b^6*log(abs(b*x^(1/3) + a))/a^7 + b^6*log(abs(x))/a^7 + 1/20*(60*a*b^5*
x^(5/3) - 30*a^2*b^4*x^(4/3) + 20*a^3*b^3*x - 15*a^4*b^2*x^(2/3) + 12*a^5*
b*x^(1/3) - 10*a^6)/(a^7*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx = -\frac{\frac{1}{2\,a}-\frac{3\,b\,x^{1/3}}{5\,a^2}-\frac{b^3\,x}{a^4}+\frac{3\,b^2\,x^{2/3}}{4\,a^3}+\frac{3\,b^4\,x^{4/3}}{2\,a^5}-\frac{3\,b^5\,x^{5/3}}{a^6}}{x^2} - \frac{6\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^7}$$

input
```
int(1/(x^3*(a + b*x^(1/3))),x)
```

output
```
- (1/(2*a) - (3*b*x^(1/3))/(5*a^2) - (b^3*x)/a^4 + (3*b^2*x^(2/3))/(4*a^3)
 + (3*b^4*x^(4/3))/(2*a^5) - (3*b^5*x^(5/3))/a^6)/x^2 - (6*b^6*atanh((2*b*
x^(1/3))/a + 1))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^3}\,dx = \frac{-15x^{\frac{2}{3}}a^4b^2+60x^{\frac{5}{3}}a\,b^5+12x^{\frac{1}{3}}a^5b-30x^{\frac{4}{3}}a^2b^4+60\log\left(x^{\frac{1}{3}}\right)b^6x^2-60\log\left(x^{\frac{1}{3}}b+a\right)b^6x^2-10a^6+20a^3}{20a^7x^2}$$

input
```
int(1/(a+b*x^(1/3))/x^3,x)
```

output
```
( - 15*x**(2/3)*a**4*b**2 + 60*x**(2/3)*a*b**5*x + 12*x**(1/3)*a**5*b - 30
*x**(1/3)*a**2*b**4*x + 60*log(x**(1/3))*b**6*x**2 - 60*log(x**(1/3)*b + a
)*b**6*x**2 - 10*a**6 + 20*a**3*b**3*x)/(20*a**7*x**2)
```

3.254 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = -\frac{1}{3ax^3}+\frac{3b}{8a^2x^{8/3}}-\frac{3b^2}{7a^3x^{7/3}}+\frac{b^3}{2a^4x^2}-\frac{3b^4}{5a^5x^{5/3}}+\frac{3b^5}{4a^6x^{4/3}}-\frac{b^6}{a^7x}+\frac{3b^7}{2a^8x^{2/3}}-\frac{3b^8}{a^9\sqrt[3]{x}}+\frac{3b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}$$

output
```
-1/3/a/x^3+3/8*b/a^2/x^(8/3)-3/7*b^2/a^3/x^(7/3)+1/2*b^3/a^4/x^2-3/5*b^4/a
^5/x^(5/3)+3/4*b^5/a^6/x^(4/3)-b^6/a^7/x+3/2*b^7/a^8/x^(2/3)-3*b^8/a^9/x^(
1/3)+3*b^9*ln(a+b*x^(1/3))/a^10-b^9*ln(x)/a^10
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = \frac{\frac{a\left(-280a^8+315a^7b\sqrt[3]{x}-360a^6b^2x^{2/3}+420a^5b^3x-504a^4b^4x^{4/3}+630a^3b^5x^{5/3}-840a^2b^6x^2+1260ab^7x^{7/3}-2520b^8x^{8/3}\right)}{x^3}+2520b^9\log\left(a\right.}{840a^{10}}$$

input
```
Integrate[1/((a + b*x^(1/3))*x^4),x]
```

output
```
((a*(-280*a^8 + 315*a^7*b*x^(1/3) - 360*a^6*b^2*x^(2/3) + 420*a^5*b^3*x -
504*a^4*b^4*x^(4/3) + 630*a^3*b^5*x^(5/3) - 840*a^2*b^6*x^2 + 1260*a*b^7*x
^(7/3) - 2520*b^8*x^(8/3)))/x^3 + 2520*b^9*Log[a + b*x^(1/3)] - 840*b^9*Lo
g[x])/(840*a^10)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\sqrt[3]{x}\right)} \, dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right) x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{b^{10}}{a^{10}\left(a+b\sqrt[3]{x}\right)}-\frac{b^9}{a^{10}\sqrt[3]{x}}+\frac{b^8}{a^9x^{2/3}}-\frac{b^7}{a^8x}+\frac{b^6}{a^7x^{4/3}}-\frac{b^5}{a^6x^{5/3}}+\frac{b^4}{a^5x^2}-\frac{b^3}{a^4x^{7/3}}+\frac{b^2}{a^3x^{8/3}}-\frac{b}{a^2x^3}+\frac{1}{ax^{10/3}}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{10}}-\frac{b^9\log\left(\sqrt[3]{x}\right)}{a^{10}}-\frac{b^8}{a^9\sqrt[3]{x}}+\frac{b^7}{2a^8x^{2/3}}-\frac{b^6}{3a^7x}+\frac{b^5}{4a^6x^{4/3}}-\frac{b^4}{5a^5x^{5/3}}+\frac{b^3}{6a^4x^2}-\frac{b^2}{7a^3x^{7/3}}+\frac{}{8a^2}\right.$$

input
```
Int[1/((a + b*x^(1/3))*x^4),x]
```

output
```
3*(-1/9*1/(a*x^3) + b/(8*a^2*x^(8/3)) - b^2/(7*a^3*x^(7/3)) + b^3/(6*a^4*x
^2) - b^4/(5*a^5*x^(5/3)) + b^5/(4*a^6*x^(4/3)) - b^6/(3*a^7*x) + b^7/(2*a
^8*x^(2/3)) - b^8/(a^9*x^(1/3)) + (b^9*Log[a + b*x^(1/3)])/a^10 - (b^9*Log
[x^(1/3)])/a^10)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{1}{3a\,x^{3}}+\frac{3b}{8a^{2}x^{\frac{8}{3}}}-\frac{3b^{2}}{7a^{3}x^{\frac{7}{3}}}+\frac{b^{3}}{2a^{4}x^{2}}-\frac{3b^{4}}{5a^{5}x^{\frac{5}{3}}}+\frac{3b^{5}}{4a^{6}x^{\frac{4}{3}}}-\frac{b^{6}}{a^{7}x}+\frac{3b^{7}}{2a^{8}x^{\frac{2}{3}}}-\frac{3b^{8}}{a^{9}x^{\frac{1}{3}}}+\frac{3b^{9}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{10}}$
default	$-\frac{1}{3a\,x^{3}}+\frac{3b}{8a^{2}x^{\frac{8}{3}}}-\frac{3b^{2}}{7a^{3}x^{\frac{7}{3}}}+\frac{b^{3}}{2a^{4}x^{2}}-\frac{3b^{4}}{5a^{5}x^{\frac{5}{3}}}+\frac{3b^{5}}{4a^{6}x^{\frac{4}{3}}}-\frac{b^{6}}{a^{7}x}+\frac{3b^{7}}{2a^{8}x^{\frac{2}{3}}}-\frac{3b^{8}}{a^{9}x^{\frac{1}{3}}}+\frac{3b^{9}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{10}}$

input
```
int(1/(a+b*x^(1/3))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+3/8*b/a^2/x^(8/3)-3/7*b^2/a^3/x^(7/3)+1/2*b^3/a^4/x^2-3/5*b^4/a
^5/x^(5/3)+3/4*b^5/a^6/x^(4/3)-b^6/a^7/x+3/2*b^7/a^8/x^(2/3)-3*b^8/a^9/x^(
1/3)+3*b^9*ln(a+b*x^(1/3))/a^10-b^9*ln(x)/a^10
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = \frac{2520\,b^9x^3\log\left(bx^{\frac{1}{3}}+a\right)-2520\,b^9x^3\log\left(x^{\frac{1}{3}}\right)-840\,a^3b^6x^2+420\,a^6b^3x-280\,a^9-90\,(28\,ab^8x^2-7\,a^4b^5}{840\,a^{10}x^3}$$

input
```
integrate(1/(a+b*x^(1/3))/x^4,x, algorithm="fricas")
```

output
```
1/840*(2520*b^9*x^3*log(b*x^(1/3) + a) - 2520*b^9*x^3*log(x^(1/3)) - 840*a
^3*b^6*x^2 + 420*a^6*b^3*x - 280*a^9 - 90*(28*a*b^8*x^2 - 7*a^4*b^5*x + 4*
a^7*b^2)*x^(2/3) + 63*(20*a^2*b^7*x^2 - 8*a^5*b^4*x + 5*a^8*b)*x^(1/3))/(a
^10*x^3)
```

Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{10}{3}}} & \text{for} \\ -\frac{3}{10bx^{\frac{10}{3}}} & \text{for} \\ -\frac{1}{3ax^3} & \text{for} \\ -\frac{1}{3ax^3}+\frac{3b}{8a^2x^{\frac{8}{3}}}-\frac{3b^2}{7a^3x^{\frac{7}{3}}}+\frac{b^3}{2a^4x^2}-\frac{3b^4}{5a^5x^{\frac{5}{3}}}+\frac{3b^5}{4a^6x^{\frac{4}{3}}}-\frac{b^6}{a^7x}+\frac{3b^7}{2a^8x^{\frac{2}{3}}}-\frac{3b^8}{a^9\sqrt[3]{x}}-\frac{b^9\log(x)}{a^{10}}+\frac{3b^9\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^{10}} & \text{ot} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))/x**4,x)
```

output
```
Piecewise((zoo/x**(10/3), Eq(a, 0) & Eq(b, 0)), (-3/(10*b*x**(10/3)), Eq(a
, 0)), (-1/(3*a*x**3), Eq(b, 0)), (-1/(3*a*x**3) + 3*b/(8*a**2*x**(8/3)) -
 3*b**2/(7*a**3*x**(7/3)) + b**3/(2*a**4*x**2) - 3*b**4/(5*a**5*x**(5/3))
+ 3*b**5/(4*a**6*x**(4/3)) - b**6/(a**7*x) + 3*b**7/(2*a**8*x**(2/3)) - 3*
b**8/(a**9*x**(1/3)) - b**9*log(x)/a**10 + 3*b**9*log(a/b + x**(1/3))/a**1
0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = \frac{3\,b^9\log\left(bx^{\frac{1}{3}}+a\right)}{a^{10}} - \frac{b^9\log\left(x\right)}{a^{10}} - \frac{2520\,b^8x^{\frac{8}{3}} - 1260\,ab^7x^{\frac{7}{3}} + 840\,a^2b^6x^2 - 630\,a^3b^5x^{\frac{5}{3}} + 504\,a^4b^4x^{\frac{4}{3}} - 420\,a^5b^3x + 360\,a^6b^2x^{\frac{2}{3}} - 315\,a^7bx}{840\,a^9x^3}$$

input
```
integrate(1/(a+b*x^(1/3))/x^4,x, algorithm="maxima")
```

output
```
3*b^9*log(b*x^(1/3) + a)/a^10 - b^9*log(x)/a^10 - 1/840*(2520*b^8*x^(8/3)
- 1260*a*b^7*x^(7/3) + 840*a^2*b^6*x^2 - 630*a^3*b^5*x^(5/3) + 504*a^4*b^4
*x^(4/3) - 420*a^5*b^3*x + 360*a^6*b^2*x^(2/3) - 315*a^7*b*x^(1/3) + 280*a
^8)/(a^9*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = \frac{3\,b^9\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^{10}} - \frac{b^9\log\left(|x|\right)}{a^{10}} - \frac{2520\,ab^8x^{\frac{8}{3}} - 1260\,a^2b^7x^{\frac{7}{3}} + 840\,a^3b^6x^2 - 630\,a^4b^5x^{\frac{5}{3}} + 504\,a^5b^4x^{\frac{4}{3}} - 420\,a^6b^3x + 360\,a^7b^2x^{\frac{2}{3}} - 315\,a^8}{840\,a^{10}x^3}$$

input
```
integrate(1/(a+b*x^(1/3))/x^4,x, algorithm="giac")
```

output
```
3*b^9*log(abs(b*x^(1/3) + a))/a^10 - b^9*log(abs(x))/a^10 - 1/840*(2520*a*
b^8*x^(8/3) - 1260*a^2*b^7*x^(7/3) + 840*a^3*b^6*x^2 - 630*a^4*b^5*x^(5/3)
 + 504*a^5*b^4*x^(4/3) - 420*a^6*b^3*x + 360*a^7*b^2*x^(2/3) - 315*a^8*b*x
^(1/3) + 280*a^9)/(a^10*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = -\frac{280\,a^9-5040\,b^9\,x^3\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)-420\,a^6\,b^3\,x-315\,a^8\,b\,x^{1/3}+2520\,a\,b^8\,x^{8/3}+840\,a^3\,b^6\,x^2+36}{840\,a^{10}\,x^3}$$

input
```
int(1/(x^4*(a + b*x^(1/3))),x)
```

output
```
-(280*a^9 - 5040*b^9*x^3*atanh((2*b*x^(1/3))/a + 1) - 420*a^6*b^3*x - 315*
a^8*b*x^(1/3) + 2520*a*b^8*x^(8/3) + 840*a^3*b^6*x^2 + 360*a^7*b^2*x^(2/3)
 + 504*a^5*b^4*x^(4/3) - 630*a^4*b^5*x^(5/3) - 1260*a^2*b^7*x^(7/3))/(840*
a^10*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)x^4}\,dx = \frac{-360x^{\frac{2}{3}}a^7b^2+630x^{\frac{5}{3}}a^4b^5-2520x^{\frac{8}{3}}a\,b^8+315x^{\frac{1}{3}}a^8b-504x^{\frac{4}{3}}a^5b^4+1260x^{\frac{7}{3}}a^2b^7-2520\log\left(x^{\frac{1}{3}}\right)b^9x^3+}{840a^{10}x^3}$$

input
```
int(1/(a+b*x^(1/3))/x^4,x)
```

output
```
( - 360*x**(2/3)*a**7*b**2 + 630*x**(2/3)*a**4*b**5*x - 2520*x**(2/3)*a*b*
*8*x**2 + 315*x**(1/3)*a**8*b - 504*x**(1/3)*a**5*b**4*x + 1260*x**(1/3)*a
**2*b**7*x**2 - 2520*log(x**(1/3))*b**9*x**3 + 2520*log(x**(1/3)*b + a)*b*
*9*x**3 - 280*a**9 + 420*a**6*b**3*x - 840*a**3*b**6*x**2)/(840*a**10*x**3
)
```

3.255 $\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx=-\frac{3}{2}\log\left(2+b\sqrt[3]{x}\right)+\frac{\log(x)}{2}$$

output
```
-3/2*ln(2+b*x^(1/3))+1/2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx=-\frac{3}{2}\log\left(2+b\sqrt[3]{x}\right)+\frac{3}{2}\log\left(\sqrt[3]{x}\right)$$

input
```
Integrate[1/((2 + b*x^(1/3))*x),x]
```

output
```
(-3*Log[2 + b*x^(1/3)])/2 + (3*Log[x^(1/3)])/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(b\sqrt[3]{x}+2\right)}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}b+2\right)\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{47}$$

$$3\left(\frac{1}{2}\int \frac{1}{\sqrt[3]{x}}d\sqrt[3]{x}-\frac{1}{2}b\int \frac{1}{\sqrt[3]{x}b+2}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{14}$$

$$3\left(\frac{1}{2}\log\left(\sqrt[3]{x}\right)-\frac{1}{2}b\int \frac{1}{\sqrt[3]{x}b+2}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{16}$$

$$3\left(\frac{1}{2}\log\left(\sqrt[3]{x}\right)-\frac{1}{2}\log\left(b\sqrt[3]{x}+2\right)\right)$$

input
```
Int[1/((2 + b*x^(1/3))*x),x]
```

output
```
3*(-1/2*Log[2 + b*x^(1/3)] + Log[x^(1/3)]/2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{3\ln\left(2+b\,x^{\frac{1}{3}}\right)}{2}+\frac{\ln\left(x\right)}{2}$	16
default	$-\frac{3\ln\left(2+b\,x^{\frac{1}{3}}\right)}{2}+\frac{\ln\left(x\right)}{2}$	16
meijerg	$\frac{\ln\left(x\right)}{2}-\frac{3\ln\left(2\right)}{2}+\frac{3\ln\left(b\right)}{2}-\frac{3\ln\left(1+\frac{b\,x^{\frac{1}{3}}}{2}\right)}{2}$	25

input
```
int(1/(2+b*x^(1/3))/x,x,method=_RETURNVERBOSE)
```

output
```
-3/2*ln(2+b*x^(1/3))+1/2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx = -\frac{3}{2}\,\log\left(bx^{\frac{1}{3}}+2\right)+\frac{3}{2}\,\log\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(1/(2+b*x^(1/3))/x,x, algorithm="fricas")
```

output
```
-3/2*log(b*x^(1/3) + 2) + 3/2*log(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx = \begin{cases} \frac{\log(x)}{2} - \frac{3\log\left(\sqrt[3]{x}+\frac{2}{b}\right)}{2} & \text{for } b \neq 0 \\ \frac{\log(x)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2+b*x**(1/3))/x,x)
```

output
```
Piecewise((log(x)/2 - 3*log(x**(1/3) + 2/b)/2, Ne(b, 0)), (log(x)/2, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx = -\frac{3}{2}\,\log\left(bx^{\frac{1}{3}}+2\right)+\frac{1}{2}\,\log\left(x\right)$$

input
```
integrate(1/(2+b*x^(1/3))/x,x, algorithm="maxima")
```

output
```
-3/2*log(b*x^(1/3) + 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx=-\frac{3}{2}\,\log\left(\left|bx^{\frac{1}{3}}+2\right|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

input
```
integrate(1/(2+b*x^(1/3))/x,x, algorithm="giac")
```

output
```
-3/2*log(abs(b*x^(1/3) + 2)) + 1/2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.48

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx=-3\,\mathrm{atanh}\left(b\,x^{1/3}+1\right)$$

input
```
int(1/(x*(b*x^(1/3) + 2)),x)
```

output
```
-3*atanh(b*x^(1/3) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(2+b\sqrt[3]{x}\right)x}\,dx=\frac{3\log\left(x^{\frac{1}{3}}\right)}{2}-\frac{3\log\left(x^{\frac{1}{3}}b+2\right)}{2}$$

input
```
int(1/(2+b*x^(1/3))/x,x)
```

output
```
(3*(log(x**(1/3)) - log(x**(1/3)*b + 2)))/2
```

3.256 $\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 171

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx=\frac{3a^{11}}{b^{12}\left(a+b\sqrt[3]{x}\right)}-\frac{30a^9\sqrt[3]{x}}{b^{11}}+\frac{27a^8x^{2/3}}{2b^{10}}-\frac{8a^7x}{b^9}+\frac{21a^6x^{4/3}}{4b^8}-\frac{18a^5x^{5/3}}{5b^7}+\frac{5a^4x^2}{2b^6}-\frac{12a^3x^{7/3}}{7b^5}+\frac{9a^2x^{8/3}}{8b^4}-\frac{2ax^3}{3b^3}+\frac{3x^{10/3}}{10b^2}+\frac{33a^{10}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}$$

output
```
3*a^11/b^12/(a+b*x^(1/3))-30*a^9*x^(1/3)/b^11+27/2*a^8*x^(2/3)/b^10-8*a^7*
x/b^9+21/4*a^6*x^(4/3)/b^8-18/5*a^5*x^(5/3)/b^7+5/2*a^4*x^2/b^6-12/7*a^3*x
^(7/3)/b^5+9/8*a^2*x^(8/3)/b^4-2/3*a*x^3/b^3+3/10*x^(10/3)/b^2+33*a^10*ln(
a+b*x^(1/3))/b^12
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx=\frac{2520a^{11}-25200a^{10}b\sqrt[3]{x}-13860a^9b^2x^{2/3}+4620a^8b^3x-2310a^7b^4x^{4/3}+1386a^6b^5x^{5/3}-924a^5b^6x^2+66}{840b^{12}\left(a+b\sqrt[3]{x}\right)}+\frac{33a^{10}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}$$

input
```
Integrate[x^3/(a + b*x^(1/3))^2,x]
```

output
```
(2520*a^11 - 25200*a^10*b*x^(1/3) - 13860*a^9*b^2*x^(2/3) + 4620*a^8*b^3*x
 - 2310*a^7*b^4*x^(4/3) + 1386*a^6*b^5*x^(5/3) - 924*a^5*b^6*x^2 + 660*a^4
*b^7*x^(7/3) - 495*a^3*b^8*x^(8/3) + 385*a^2*b^9*x^3 - 308*a*b^10*x^(10/3)
 + 252*b^11*x^(11/3))/(840*b^12*(a + b*x^(1/3))) + (33*a^10*Log[a + b*x^(1
/3)])/b^12
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{11/3}}{\left(a+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{a^{11}}{b^{11}\left(a+b\sqrt[3]{x}\right)^2}+\frac{11a^{10}}{b^{11}\left(a+b\sqrt[3]{x}\right)}-\frac{10a^9}{b^{11}}+\frac{9\sqrt[3]{x}a^8}{b^{10}}-\frac{8x^{2/3}a^7}{b^9}+\frac{7xa^6}{b^8}-\frac{6x^{4/3}a^5}{b^7}+\frac{5x^{5/3}a^4}{b^6}-\frac{4x^2a^3}{b^5}+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^{11}}{b^{12}\left(a+b\sqrt[3]{x}\right)}+\frac{11a^{10}\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}-\frac{10a^9\sqrt[3]{x}}{b^{11}}+\frac{9a^8x^{2/3}}{2b^{10}}-\frac{8a^7x}{3b^9}+\frac{7a^6x^{4/3}}{4b^8}-\frac{6a^5x^{5/3}}{5b^7}+\frac{5a^4x^2}{6b^6}-\frac{4a^3x^{7/3}}{7b^5}\right.$$

input
```
Int[x^3/(a + b*x^(1/3))^2,x]
```

output
```
3*(a^11/(b^12*(a + b*x^(1/3))) - (10*a^9*x^(1/3))/b^11 + (9*a^8*x^(2/3))/(
2*b^10) - (8*a^7*x)/(3*b^9) + (7*a^6*x^(4/3))/(4*b^8) - (6*a^5*x^(5/3))/(5
*b^7) + (5*a^4*x^2)/(6*b^6) - (4*a^3*x^(7/3))/(7*b^5) + (3*a^2*x^(8/3))/(8
*b^4) - (2*a*x^3)/(9*b^3) + x^(10/3)/(10*b^2) + (11*a^10*Log[a + b*x^(1/3)
])/b^12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{3\left(-\frac{x^{\frac{10}{3}} b^{9}}{10}+\frac{2 a \,x^{3} b^{8}}{9}-\frac{3 a^{2} x^{\frac{8}{3}} b^{7}}{8}+\frac{4 a^{3} x^{\frac{7}{3}} b^{6}}{7}-\frac{5 a^{4} x^{2} b^{5}}{6}+\frac{6 a^{5} x^{\frac{5}{3}} b^{4}}{5}-\frac{7 a^{6} x^{\frac{4}{3}} b^{3}}{4}+\frac{8 a^{7} x \,b^{2}}{3}-\frac{9 a^{8} x^{\frac{2}{3}} b}{2}+10 a^{9} x^{\frac{1}{3}}\right)}{b^{11}}+\frac{}{b^{12}(}$
default	$-\frac{3\left(-\frac{x^{\frac{10}{3}} b^{9}}{10}+\frac{2 a \,x^{3} b^{8}}{9}-\frac{3 a^{2} x^{\frac{8}{3}} b^{7}}{8}+\frac{4 a^{3} x^{\frac{7}{3}} b^{6}}{7}-\frac{5 a^{4} x^{2} b^{5}}{6}+\frac{6 a^{5} x^{\frac{5}{3}} b^{4}}{5}-\frac{7 a^{6} x^{\frac{4}{3}} b^{3}}{4}+\frac{8 a^{7} x \,b^{2}}{3}-\frac{9 a^{8} x^{\frac{2}{3}} b}{2}+10 a^{9} x^{\frac{1}{3}}\right)}{b^{11}}+\frac{}{b^{12}(}$

input
```
int(x^3/(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
-3/b^11*(-1/10*x^(10/3)*b^9+2/9*a*x^3*b^8-3/8*a^2*x^(8/3)*b^7+4/7*a^3*x^(7
/3)*b^6-5/6*a^4*x^2*b^5+6/5*a^5*x^(5/3)*b^4-7/4*a^6*x^(4/3)*b^3+8/3*a^7*x*
b^2-9/2*a^8*x^(2/3)*b+10*a^9*x^(1/3))+3*a^11/b^12/(a+b*x^(1/3))+33*a^10*ln
(a+b*x^(1/3))/b^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{560\,ab^{12}x^4 - 1540\,a^4b^9x^3 + 4620\,a^7b^6x^2 + 6720\,a^{10}b^3x - 2520\,a^{13} - 27720\,\left(a^{10}b^3x + a^{13}\right)\log\left(bx^{\frac{1}{3}} + a\right)}{}$$

input
```
integrate(x^3/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

output
```
-1/840*(560*a*b^12*x^4 - 1540*a^4*b^9*x^3 + 4620*a^7*b^6*x^2 + 6720*a^10*b
^3*x - 2520*a^13 - 27720*(a^10*b^3*x + a^13)*log(b*x^(1/3) + a) - 63*(15*a
^2*b^11*x^3 - 33*a^5*b^8*x^2 + 132*a^8*b^5*x + 220*a^11*b^2)*x^(2/3) - 18*
(14*b^13*x^4 - 66*a^3*b^10*x^3 + 165*a^6*b^7*x^2 - 1155*a^9*b^4*x - 1540*a
^12*b)*x^(1/3))/(b^15*x + a^3*b^12)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a+b*x**(1/3))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{33\,a^{10}\log\left(bx^{\frac{1}{3}}+a\right)}{b^{12}} + \frac{3\left(bx^{\frac{1}{3}}+a\right)^{10}}{10\,b^{12}} - \frac{11\left(bx^{\frac{1}{3}}+a\right)^{9}a}{3\,b^{12}} + \frac{165\left(bx^{\frac{1}{3}}+a\right)^{8}a^2}{8\,b^{12}} - \frac{495\left(bx^{\frac{1}{3}}+a\right)^{7}a^3}{7\,b^{12}} + \frac{165\left(bx^{\frac{1}{3}}+a\right)^{6}a^4}{b^{12}} - \frac{1386\left(bx^{\frac{1}{3}}+a\right)^{5}a^5}{5\,b^{12}} + \frac{693\left(bx^{\frac{1}{3}}+a\right)^{4}a^6}{2\,b^{12}} - \frac{330\left(bx^{\frac{1}{3}}+a\right)^{3}a^7}{b^{12}} + \frac{495\left(bx^{\frac{1}{3}}+a\right)^{2}a^8}{2\,b^{12}} - \frac{165\left(bx^{\frac{1}{3}}+a\right)a^9}{b^{12}} + \frac{3\,a^{11}}{\left(bx^{\frac{1}{3}}+a\right)b^{12}}$$

input
```
integrate(x^3/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

output
```
33*a^10*log(b*x^(1/3) + a)/b^12 + 3/10*(b*x^(1/3) + a)^10/b^12 - 11/3*(b*x
^(1/3) + a)^9*a/b^12 + 165/8*(b*x^(1/3) + a)^8*a^2/b^12 - 495/7*(b*x^(1/3)
 + a)^7*a^3/b^12 + 165*(b*x^(1/3) + a)^6*a^4/b^12 - 1386/5*(b*x^(1/3) + a)
^5*a^5/b^12 + 693/2*(b*x^(1/3) + a)^4*a^6/b^12 - 330*(b*x^(1/3) + a)^3*a^7
/b^12 + 495/2*(b*x^(1/3) + a)^2*a^8/b^12 - 165*(b*x^(1/3) + a)*a^9/b^12 +
3*a^11/((b*x^(1/3) + a)*b^12)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{33\,a^{10}\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^{12}} + \frac{3\,a^{11}}{\left(bx^{\frac{1}{3}}+a\right)b^{12}} + \frac{252\,b^{18}x^{\frac{10}{3}} - 560\,ab^{17}x^3 + 945\,a^2b^{16}x^{\frac{8}{3}} - 1440\,a^3b^{15}x^{\frac{7}{3}} + 2100\,a^4b^{14}x^2 - 3024\,a^5b^{13}x^{\frac{5}{3}} + 4410\,a^6b^{12}x^{\frac{4}{3}}}{840\,b^{20}}$$

input
```
integrate(x^3/(a+b*x^(1/3))^2,x, algorithm="giac")
```

output
```
33*a^10*log(abs(b*x^(1/3) + a))/b^12 + 3*a^11/((b*x^(1/3) + a)*b^12) + 1/8
40*(252*b^18*x^(10/3) - 560*a*b^17*x^3 + 945*a^2*b^16*x^(8/3) - 1440*a^3*b
^15*x^(7/3) + 2100*a^4*b^14*x^2 - 3024*a^5*b^13*x^(5/3) + 4410*a^6*b^12*x^
(4/3) - 6720*a^7*b^11*x + 11340*a^8*b^10*x^(2/3) - 25200*a^9*b^9*x^(1/3))/
b^20
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{3\,x^{10/3}}{10\,b^2}+\frac{3\,a^{11}}{b\,\left(a\,b^{11}+b^{12}\,x^{1/3}\right)}-\frac{2\,a\,x^3}{3\,b^3}-\frac{8\,a^7\,x}{b^9}+\frac{33\,a^{10}\,\ln\left(a+b\,x^{1/3}\right)}{b^{12}}+\frac{5\,a^4\,x^2}{2\,b^6}+\frac{9\,a^2\,x^{8/3}}{8\,b^4}-\frac{12\,a^3\,x^{7/3}}{7\,b^5}-\frac{18\,a^5\,x^{5/3}}{5\,b^7}+\frac{21\,a^6\,x^{4/3}}{4\,b^8}+\frac{27\,a^8\,x^{2/3}}{2\,b^{10}}-\frac{30\,a^9\,x^{1/3}}{b^{11}}$$

input
```
int(x^3/(a + b*x^(1/3))^2,x)
```

output
```
(3*x^(10/3))/(10*b^2) + (3*a^11)/(b*(a*b^11 + b^12*x^(1/3))) - (2*a*x^3)/(
3*b^3) - (8*a^7*x)/b^9 + (33*a^10*log(a + b*x^(1/3)))/b^12 + (5*a^4*x^2)/(
2*b^6) + (9*a^2*x^(8/3))/(8*b^4) - (12*a^3*x^(7/3))/(7*b^5) - (18*a^5*x^(5
/3))/(5*b^7) + (21*a^6*x^(4/3))/(4*b^8) + (27*a^8*x^(2/3))/(2*b^10) - (30*
a^9*x^(1/3))/b^11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{-13860x^{\frac{2}{3}}a^9b^2+1386x^{\frac{5}{3}}a^6b^5-495x^{\frac{8}{3}}a^3b^8+252x^{\frac{11}{3}}b^{11}+27720x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^{10}b-27720x^{\frac{1}{3}}a^{10}b-}{840b^{12}\left(\right.}$$

input
```
int(x^3/(a+b*x^(1/3))^2,x)
```

output
```
( - 13860*x**(2/3)*a**9*b**2 + 1386*x**(2/3)*a**6*b**5*x - 495*x**(2/3)*a*
*3*b**8*x**2 + 252*x**(2/3)*b**11*x**3 + 27720*x**(1/3)*log(x**(1/3)*b + a
)*a**10*b - 27720*x**(1/3)*a**10*b - 2310*x**(1/3)*a**7*b**4*x + 660*x**(1
/3)*a**4*b**7*x**2 - 308*x**(1/3)*a*b**10*x**3 + 27720*log(x**(1/3)*b + a)
*a**11 + 4620*a**8*b**3*x - 924*a**5*b**6*x**2 + 385*a**2*b**9*x**3)/(840*
b**12*(x**(1/3)*b + a))
```

3.257 $\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2} \, dx = -\frac{3a^8}{b^9\left(a+b\sqrt[3]{x}\right)}+\frac{21a^6\sqrt[3]{x}}{b^8}-\frac{9a^5x^{2/3}}{b^7}+\frac{5a^4x}{b^6}-\frac{3a^3x^{4/3}}{b^5}+\frac{9a^2x^{5/3}}{5b^4}-\frac{ax^2}{b^3}+\frac{3x^{7/3}}{7b^2}-\frac{24a^7\log\left(a+b\sqrt[3]{x}\right)}{b^9}$$

output
```
-3*a^8/b^9/(a+b*x^(1/3))+21*a^6*x^(1/3)/b^8-9*a^5*x^(2/3)/b^7+5*a^4*x/b^6-
3*a^3*x^(4/3)/b^5+9/5*a^2*x^(5/3)/b^4-a*x^2/b^3+3/7*x^(7/3)/b^2-24*a^7*ln(
a+b*x^(1/3))/b^9
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2} \, dx = \frac{-105a^8+735a^7b\sqrt[3]{x}+420a^6b^2x^{2/3}-140a^5b^3x+70a^4b^4x^{4/3}-42a^3b^5x^{5/3}+28a^2b^6x^2-20ab^7x^{7/3}+15}{35b^9\left(a+b\sqrt[3]{x}\right)} - \frac{24a^7\log\left(a+b\sqrt[3]{x}\right)}{b^9}$$

input
```
Integrate[x^2/(a + b*x^(1/3))^2,x]
```

output
```
(-105*a^8 + 735*a^7*b*x^(1/3) + 420*a^6*b^2*x^(2/3) - 140*a^5*b^3*x + 70*a
^4*b^4*x^(4/3) - 42*a^3*b^5*x^(5/3) + 28*a^2*b^6*x^2 - 20*a*b^7*x^(7/3) +
15*b^8*x^(8/3))/(35*b^9*(a + b*x^(1/3))) - (24*a^7*Log[a + b*x^(1/3)])/b^9
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{8/3}}{\left(a+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^8}{b^8\left(a+b\sqrt[3]{x}\right)^2}-\frac{8a^7}{b^8\left(a+b\sqrt[3]{x}\right)}+\frac{7a^6}{b^8}-\frac{6\sqrt[3]{x}a^5}{b^7}+\frac{5x^{2/3}a^4}{b^6}-\frac{4xa^3}{b^5}+\frac{3x^{4/3}a^2}{b^4}-\frac{2x^{5/3}a}{b^3}+\frac{x^2}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^8}{b^9\left(a+b\sqrt[3]{x}\right)}-\frac{8a^7\log\left(a+b\sqrt[3]{x}\right)}{b^9}+\frac{7a^6\sqrt[3]{x}}{b^8}-\frac{3a^5x^{2/3}}{b^7}+\frac{5a^4x}{3b^6}-\frac{a^3x^{4/3}}{b^5}+\frac{3a^2x^{5/3}}{5b^4}-\frac{ax^2}{3b^3}+\frac{x^{7/3}}{7b^2}\right)$$

input
```
Int[x^2/(a + b*x^(1/3))^2,x]
```

output
```
3*(-(a^8/(b^9*(a + b*x^(1/3)))) + (7*a^6*x^(1/3))/b^8 - (3*a^5*x^(2/3))/b^
7 + (5*a^4*x)/(3*b^6) - (a^3*x^(4/3))/b^5 + (3*a^2*x^(5/3))/(5*b^4) - (a*x
^2)/(3*b^3) + x^(7/3)/(7*b^2) - (8*a^7*Log[a + b*x^(1/3)])/b^9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{3x^{\frac{7}{3}}b^{6}}{7}-a\,b^{5}x^{2}+\frac{9a^{2}x^{\frac{5}{3}}b^{4}}{5}-3a^{3}x^{\frac{4}{3}}b^{3}+5a^{4}b^{2}x-9a^{5}x^{\frac{2}{3}}b+21a^{6}x^{\frac{1}{3}}}{b^{8}}-\frac{24a^{7}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^{9}}-\frac{3a^{8}}{b^{9}\left(a+b\,x^{\frac{1}{3}}\right)}$	106
default	$\frac{\frac{3x^{\frac{7}{3}}b^{6}}{7}-a\,b^{5}x^{2}+\frac{9a^{2}x^{\frac{5}{3}}b^{4}}{5}-3a^{3}x^{\frac{4}{3}}b^{3}+5a^{4}b^{2}x-9a^{5}x^{\frac{2}{3}}b+21a^{6}x^{\frac{1}{3}}}{b^{8}}-\frac{24a^{7}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^{9}}-\frac{3a^{8}}{b^{9}\left(a+b\,x^{\frac{1}{3}}\right)}$	106

input
```
int(x^2/(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
3/b^8*(1/7*x^(7/3)*b^6-1/3*a*b^5*x^2+3/5*a^2*x^(5/3)*b^4-a^3*x^(4/3)*b^3+5
/3*a^4*b^2*x-3*a^5*x^(2/3)*b+7*a^6*x^(1/3))-24*a^7*ln(a+b*x^(1/3))/b^9-3*a
^8/b^9/(a+b*x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{35\,ab^9x^3 - 140\,a^4b^6x^2 - 175\,a^7b^3x + 105\,a^{10} + 840\,(a^7b^3x + a^{10})\log\left(bx^{\frac{1}{3}} + a\right) - 21\,(3\,a^2b^8x^2 - 12\,a^5}{35\,(b^{12}x + a^3b^9)}$$

input
```
integrate(x^2/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

output
```
-1/35*(35*a*b^9*x^3 - 140*a^4*b^6*x^2 - 175*a^7*b^3*x + 105*a^10 + 840*(a^
7*b^3*x + a^10)*log(b*x^(1/3) + a) - 21*(3*a^2*b^8*x^2 - 12*a^5*b^5*x - 20
*a^8*b^2)*x^(2/3) - 15*(b^10*x^3 - 6*a^3*b^7*x^2 + 42*a^6*b^4*x + 56*a^9*b
)*x^(1/3))/(b^12*x + a^3*b^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(a+b*x**(1/3))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{24\,a^7\log\left(bx^{\frac{1}{3}}+a\right)}{b^9}+\frac{3\left(bx^{\frac{1}{3}}+a\right)^7}{7\,b^9}-\frac{4\left(bx^{\frac{1}{3}}+a\right)^6 a}{b^9}+\frac{84\left(bx^{\frac{1}{3}}+a\right)^5 a^2}{5\,b^9}-\frac{42\left(bx^{\frac{1}{3}}+a\right)^4 a^3}{b^9}+\frac{70\left(bx^{\frac{1}{3}}+a\right)^3 a^4}{b^9}-\frac{84\left(bx^{\frac{1}{3}}+a\right)^2 a^5}{b^9}+\frac{84\left(bx^{\frac{1}{3}}+a\right)a^6}{b^9}-\frac{3\,a^8}{\left(bx^{\frac{1}{3}}+a\right)b^9}$$

input
```
integrate(x^2/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

output
```
-24*a^7*log(b*x^(1/3) + a)/b^9 + 3/7*(b*x^(1/3) + a)^7/b^9 - 4*(b*x^(1/3)
+ a)^6*a/b^9 + 84/5*(b*x^(1/3) + a)^5*a^2/b^9 - 42*(b*x^(1/3) + a)^4*a^3/b
^9 + 70*(b*x^(1/3) + a)^3*a^4/b^9 - 84*(b*x^(1/3) + a)^2*a^5/b^9 + 84*(b*x
^(1/3) + a)*a^6/b^9 - 3*a^8/((b*x^(1/3) + a)*b^9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{24\,a^7\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^9}-\frac{3\,a^8}{\left(bx^{\frac{1}{3}}+a\right)b^9}+\frac{15\,b^{12}x^{\frac{7}{3}}-35\,ab^{11}x^2+63\,a^2b^{10}x^{\frac{5}{3}}-105\,a^3b^9x^{\frac{4}{3}}+175\,a^4b^8x-315\,a^5b^7x^{\frac{2}{3}}+735\,a^6b^6x^{\frac{1}{3}}}{35\,b^{14}}$$

input
```
integrate(x^2/(a+b*x^(1/3))^2,x, algorithm="giac")
```

output
```
-24*a^7*log(abs(b*x^(1/3) + a))/b^9 - 3*a^8/((b*x^(1/3) + a)*b^9) + 1/35*(
15*b^12*x^(7/3) - 35*a*b^11*x^2 + 63*a^2*b^10*x^(5/3) - 105*a^3*b^9*x^(4/3
) + 175*a^4*b^8*x - 315*a^5*b^7*x^(2/3) + 735*a^6*b^6*x^(1/3))/b^14
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{3\,x^{7/3}}{7\,b^2} - \frac{3\,a^8}{b\,\left(a\,b^8+b^9\,x^{1/3}\right)} - \frac{a\,x^2}{b^3} + \frac{5\,a^4\,x}{b^6} - \frac{24\,a^7\,\ln\left(a+b\,x^{1/3}\right)}{b^9} + \frac{9\,a^2\,x^{5/3}}{5\,b^4} - \frac{3\,a^3\,x^{4/3}}{b^5} - \frac{9\,a^5\,x^{2/3}}{b^7} + \frac{21\,a^6\,x^{1/3}}{b^8}$$

input
```
int(x^2/(a + b*x^(1/3))^2,x)
```

output
```
(3*x^(7/3))/(7*b^2) - (3*a^8)/(b*(a*b^8 + b^9*x^(1/3))) - (a*x^2)/b^3 + (5
*a^4*x)/b^6 - (24*a^7*log(a + b*x^(1/3)))/b^9 + (9*a^2*x^(5/3))/(5*b^4) -
(3*a^3*x^(4/3))/b^5 - (9*a^5*x^(2/3))/b^7 + (21*a^6*x^(1/3))/b^8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{420x^{\frac{2}{3}}a^6b^2 - 42x^{\frac{5}{3}}a^3b^5 + 15x^{\frac{8}{3}}b^8 - 840x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^7b + 840x^{\frac{1}{3}}a^7b + 70x^{\frac{4}{3}}a^4b^4 - 20x^{\frac{7}{3}}a\,b^7 - 840\,\mathrm{lo}}{35b^9\left(x^{\frac{1}{3}}b+a\right)}$$

input
```
int(x^2/(a+b*x^(1/3))^2,x)
```

output
```
(420*x**(2/3)*a**6*b**2 - 42*x**(2/3)*a**3*b**5*x + 15*x**(2/3)*b**8*x**2
- 840*x**(1/3)*log(x**(1/3)*b + a)*a**7*b + 840*x**(1/3)*a**7*b + 70*x**(1
/3)*a**4*b**4*x - 20*x**(1/3)*a*b**7*x**2 - 840*log(x**(1/3)*b + a)*a**8 -
 140*a**5*b**3*x + 28*a**2*b**6*x**2)/(35*b**9*(x**(1/3)*b + a))
```

3.258 $\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{3a^5}{b^6\left(a+b\sqrt[3]{x}\right)} - \frac{12a^3\sqrt[3]{x}}{b^5} + \frac{9a^2x^{2/3}}{2b^4} - \frac{2ax}{b^3} + \frac{3x^{4/3}}{4b^2} + \frac{15a^4\log\left(a+b\sqrt[3]{x}\right)}{b^6}$$

output
```
3*a^5/b^6/(a+b*x^(1/3))-12*a^3*x^(1/3)/b^5+9/2*a^2*x^(2/3)/b^4-2*a*x/b^3+3
/4*x^(4/3)/b^2+15*a^4*ln(a+b*x^(1/3))/b^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{12a^5-48a^4b\sqrt[3]{x}-30a^3b^2x^{2/3}+10a^2b^3x-5ab^4x^{4/3}+3b^5x^{5/3}}{4b^6\left(a+b\sqrt[3]{x}\right)} + \frac{15a^4\log\left(a+b\sqrt[3]{x}\right)}{b^6}$$

input
```
Integrate[x/(a + b*x^(1/3))^2,x]
```

output
```
(12*a^5 - 48*a^4*b*x^(1/3) - 30*a^3*b^2*x^(2/3) + 10*a^2*b^3*x - 5*a*b^4*x
^(4/3) + 3*b^5*x^(5/3))/(4*b^6*(a + b*x^(1/3))) + (15*a^4*Log[a + b*x^(1/3
)])/b^6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{5/3}}{\left(a+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{a^5}{b^5\left(a+b\sqrt[3]{x}\right)^2}+\frac{5a^4}{b^5\left(a+b\sqrt[3]{x}\right)}-\frac{4a^3}{b^5}+\frac{3\sqrt[3]{x}a^2}{b^4}-\frac{2x^{2/3}a}{b^3}+\frac{x}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^5}{b^6\left(a+b\sqrt[3]{x}\right)}+\frac{5a^4\log\left(a+b\sqrt[3]{x}\right)}{b^6}-\frac{4a^3\sqrt[3]{x}}{b^5}+\frac{3a^2x^{2/3}}{2b^4}-\frac{2ax}{3b^3}+\frac{x^{4/3}}{4b^2}\right)$$

input
```
Int[x/(a + b*x^(1/3))^2,x]
```

output
```
3*(a^5/(b^6*(a + b*x^(1/3))) - (4*a^3*x^(1/3))/b^5 + (3*a^2*x^(2/3))/(2*b^
4) - (2*a*x)/(3*b^3) + x^(4/3)/(4*b^2) + (5*a^4*Log[a + b*x^(1/3)])/b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{3\left(-\frac{x^{\frac{4}{3}} b^{3}}{4}+\frac{2 a \,b^{2} x}{3}-\frac{3 a^{2} b \,x^{\frac{2}{3}}}{2}+4 a^{3} x^{\frac{1}{3}}\right)}{b^{5}}+\frac{3 a^{5}}{b^{6} \left(a +b \,x^{\frac{1}{3}}\right)}+\frac{15 a^{4} \ln \left(a +b \,x^{\frac{1}{3}}\right)}{b^{6}}$	73
default	$-\frac{3\left(-\frac{x^{\frac{4}{3}} b^{3}}{4}+\frac{2 a \,b^{2} x}{3}-\frac{3 a^{2} b \,x^{\frac{2}{3}}}{2}+4 a^{3} x^{\frac{1}{3}}\right)}{b^{5}}+\frac{3 a^{5}}{b^{6} \left(a +b \,x^{\frac{1}{3}}\right)}+\frac{15 a^{4} \ln \left(a +b \,x^{\frac{1}{3}}\right)}{b^{6}}$	73

input
```
int(x/(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
-3/b^5*(-1/4*x^(4/3)*b^3+2/3*a*b^2*x-3/2*a^2*b*x^(2/3)+4*a^3*x^(1/3))+3*a^
5/b^6/(a+b*x^(1/3))+15*a^4*ln(a+b*x^(1/3))/b^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{8\,ab^6x^2+8\,a^4b^3x-12\,a^7-60\,(a^4b^3x+a^7)\log\left(bx^{\frac{1}{3}}+a\right)-6\,(3\,a^2b^5x+5\,a^5b^2)x^{\frac{2}{3}}-3\,(b^7x^2-15\,a^3b}{4\,(b^9x+a^3b^6)}$$

input
```
integrate(x/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

output
```
-1/4*(8*a*b^6*x^2 + 8*a^4*b^3*x - 12*a^7 - 60*(a^4*b^3*x + a^7)*log(b*x^(1
/3) + a) - 6*(3*a^2*b^5*x + 5*a^5*b^2)*x^(2/3) - 3*(b^7*x^2 - 15*a^3*b^4*x
 - 20*a^6*b)*x^(1/3))/(b^9*x + a^3*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(83) = 166.

Time = 48.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.86

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{60a^5x^{\frac{80}{3}}\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}} + \frac{60a^4bx^{27}\log\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}} - \frac{60a^4bx^{27}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}} - \frac{30a^3b^2x^{\frac{82}{3}}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}} + \frac{10a^2b^3x^{\frac{83}{3}}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}} - \frac{5ab^4x^{28}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}} + \frac{3b^5x^{\frac{85}{3}}}{4ab^6x^{\frac{80}{3}}+4b^7x^{27}}$$

input
```
integrate(x/(a+b*x**(1/3))**2,x)
```

output
```
60*a**5*x**(80/3)*log(1 + b*x**(1/3)/a)/(4*a*b**6*x**(80/3) + 4*b**7*x**27
) + 60*a**4*b*x**27*log(1 + b*x**(1/3)/a)/(4*a*b**6*x**(80/3) + 4*b**7*x**
27) - 60*a**4*b*x**27/(4*a*b**6*x**(80/3) + 4*b**7*x**27) - 30*a**3*b**2*x
**(82/3)/(4*a*b**6*x**(80/3) + 4*b**7*x**27) + 10*a**2*b**3*x**(83/3)/(4*a
*b**6*x**(80/3) + 4*b**7*x**27) - 5*a*b**4*x**28/(4*a*b**6*x**(80/3) + 4*b
**7*x**27) + 3*b**5*x**(85/3)/(4*a*b**6*x**(80/3) + 4*b**7*x**27)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{15\,a^4\log\left(bx^{\frac{1}{3}}+a\right)}{b^6} + \frac{3\left(bx^{\frac{1}{3}}+a\right)^4}{4\,b^6} - \frac{5\left(bx^{\frac{1}{3}}+a\right)^3 a}{b^6} + \frac{15\left(bx^{\frac{1}{3}}+a\right)^2 a^2}{b^6} - \frac{30\left(bx^{\frac{1}{3}}+a\right)a^3}{b^6} + \frac{3\,a^5}{\left(bx^{\frac{1}{3}}+a\right)b^6}$$

input
```
integrate(x/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

output
```
15*a^4*log(b*x^(1/3) + a)/b^6 + 3/4*(b*x^(1/3) + a)^4/b^6 - 5*(b*x^(1/3) +
 a)^3*a/b^6 + 15*(b*x^(1/3) + a)^2*a^2/b^6 - 30*(b*x^(1/3) + a)*a^3/b^6 +
3*a^5/((b*x^(1/3) + a)*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{15\,a^4\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^6} + \frac{3\,a^5}{\left(bx^{\frac{1}{3}}+a\right)b^6} + \frac{3\,b^6x^{\frac{4}{3}} - 8\,ab^5x + 18\,a^2b^4x^{\frac{2}{3}} - 48\,a^3b^3x^{\frac{1}{3}}}{4\,b^8}$$

input
```
integrate(x/(a+b*x^(1/3))^2,x, algorithm="giac")
```

output
```
15*a^4*log(abs(b*x^(1/3) + a))/b^6 + 3*a^5/((b*x^(1/3) + a)*b^6) + 1/4*(3*
b^6*x^(4/3) - 8*a*b^5*x + 18*a^2*b^4*x^(2/3) - 48*a^3*b^3*x^(1/3))/b^8
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{3\,x^{4/3}}{4\,b^2}+\frac{3\,a^5}{b\,\left(a\,b^5+b^6\,x^{1/3}\right)}+\frac{15\,a^4\,\ln\left(a+b\,x^{1/3}\right)}{b^6}+\frac{9\,a^2\,x^{2/3}}{2\,b^4}-\frac{12\,a^3\,x^{1/3}}{b^5}-\frac{2\,a\,x}{b^3}$$

input
```
int(x/(a + b*x^(1/3))^2,x)
```

output
```
(3*x^(4/3))/(4*b^2) + (3*a^5)/(b*(a*b^5 + b^6*x^(1/3))) + (15*a^4*log(a +
b*x^(1/3)))/b^6 + (9*a^2*x^(2/3))/(2*b^4) - (12*a^3*x^(1/3))/b^5 - (2*a*x)
/b^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{-30x^{\frac{2}{3}}a^3b^2+3x^{\frac{5}{3}}b^5+60x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^4b-60x^{\frac{1}{3}}a^4b-5x^{\frac{4}{3}}a\,b^4+60\log\left(x^{\frac{1}{3}}b+a\right)a^5+10a^2b^3x}{4b^6\left(x^{\frac{1}{3}}b+a\right)}$$

input
```
int(x/(a+b*x^(1/3))^2,x)
```

output
```
( - 30*x**(2/3)*a**3*b**2 + 3*x**(2/3)*b**5*x + 60*x**(1/3)*log(x**(1/3)*b
 + a)*a**4*b - 60*x**(1/3)*a**4*b - 5*x**(1/3)*a*b**4*x + 60*log(x**(1/3)*
b + a)*a**5 + 10*a**2*b**3*x)/(4*b**6*(x**(1/3)*b + a))
```

3.259 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx=-\frac{3a^2}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{3\sqrt[3]{x}}{b^2}-\frac{6a\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

output
```
-3*a^2/b^3/(a+b*x^(1/3))+3*x^(1/3)/b^2-6*a*ln(a+b*x^(1/3))/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx=\frac{3\left(-a^2+ab\sqrt[3]{x}+b^2x^{2/3}\right)}{b^3\left(a+b\sqrt[3]{x}\right)}-\frac{6a\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

input
```
Integrate[(a + b*x^(1/3))^(-2),x]
```

output
```
(3*(-a^2 + a*b*x^(1/3) + b^2*x^(2/3)))/(b^3*(a + b*x^(1/3))) - (6*a*Log[a
+ b*x^(1/3)])/b^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^2}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)}+\frac{1}{b^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^2}{b^3\left(a+b\sqrt[3]{x}\right)}-\frac{2a\log\left(a+b\sqrt[3]{x}\right)}{b^3}+\frac{\sqrt[3]{x}}{b^2}\right)$$

input
```
Int[(a + b*x^(1/3))^(-2),x]
```

output
```
3*(-(a^2/(b^3*(a + b*x^(1/3)))) + x^(1/3)/b^2 - (2*a*Log[a + b*x^(1/3)])/b
^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{3a^2}{b^3\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{3x^{\frac{1}{3}}}{b^2}-\frac{6a\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^3}$
default	$-\frac{3a^4}{(b^3x+a^3)b^3}+b^4\left(\frac{3x^{\frac{1}{3}}}{b^6}+\frac{a\left(\frac{a\,x^{\frac{1}{3}}+\frac{a^2}{b}}{b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2}+\frac{2\ln\left(b^2x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^2\right)}{b}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{3}}b^2-ab\right)\sqrt{3}}{3ab}\right)}{b}\right)}{3b^6}-\right.$

input
```
int(1/(a+b*x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
-3*a^2/b^3/(a+b*x^(1/3))+3*x^(1/3)/b^2-6*a*ln(a+b*x^(1/3))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{3\left(a^2b^2x^{\frac{2}{3}} + a^4 + 2\,(ab^3x + a^4)\log\left(bx^{\frac{1}{3}} + a\right) - (b^4x + 2\,a^3b)x^{\frac{1}{3}}\right)}{b^6x + a^3b^3}$$

input
```
integrate(1/(a+b*x^(1/3))^2,x, algorithm="fricas")
```

output
```
-3*(a^2*b^2*x^(2/3) + a^4 + 2*(a*b^3*x + a^4)*log(b*x^(1/3) + a) - (b^4*x
+ 2*a^3*b)*x^(1/3))/(b^6*x + a^3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(42) = 84.

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.37

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \begin{cases} -\frac{6a^2\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{ab^3+b^4\sqrt[3]{x}} - \frac{6a^2}{ab^3+b^4\sqrt[3]{x}} - \frac{6ab\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{ab^3+b^4\sqrt[3]{x}} + \frac{3b^2x^{\frac{2}{3}}}{ab^3+b^4\sqrt[3]{x}} & \text{for } b \neq 0 \\ \frac{x}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**2,x)
```

output
```
Piecewise((-6*a**2*log(a/b + x**(1/3))/(a*b**3 + b**4*x**(1/3)) - 6*a**2/(
a*b**3 + b**4*x**(1/3)) - 6*a*b*x**(1/3)*log(a/b + x**(1/3))/(a*b**3 + b**
4*x**(1/3)) + 3*b**2*x**(2/3)/(a*b**3 + b**4*x**(1/3)), Ne(b, 0)), (x/a**2
, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{6\,a\log\left(bx^{\frac{1}{3}}+a\right)}{b^3}+\frac{3\left(bx^{\frac{1}{3}}+a\right)}{b^3}-\frac{3\,a^2}{\left(bx^{\frac{1}{3}}+a\right)b^3}$$

input
```
integrate(1/(a+b*x^(1/3))^2,x, algorithm="maxima")
```

output
```
-6*a*log(b*x^(1/3) + a)/b^3 + 3*(b*x^(1/3) + a)/b^3 - 3*a^2/((b*x^(1/3) +
a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = -\frac{6\,a\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3}+\frac{3\,x^{\frac{1}{3}}}{b^2}-\frac{3\,a^2}{\left(bx^{\frac{1}{3}}+a\right)b^3}$$

input
```
integrate(1/(a+b*x^(1/3))^2,x, algorithm="giac")
```

output
```
-6*a*log(abs(b*x^(1/3) + a))/b^3 + 3*x^(1/3)/b^2 - 3*a^2/((b*x^(1/3) + a)*
b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{3\,x^{1/3}}{b^2}-\frac{3\,a^2}{a\,b^3+b^4\,x^{1/3}}-\frac{6\,a\,\ln\left(a+b\,x^{1/3}\right)}{b^3}$$

input
```
int(1/(a + b*x^(1/3))^2,x)
```

output
```
(3*x^(1/3))/b^2 - (3*a^2)/(a*b^3 + b^4*x^(1/3)) - (6*a*log(a + b*x^(1/3)))
/b^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2}\,dx = \frac{3x^{\frac{2}{3}}b^2-6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)ab+6x^{\frac{1}{3}}ab-6\log\left(x^{\frac{1}{3}}b+a\right)a^2}{b^3\left(x^{\frac{1}{3}}b+a\right)}$$

input
```
int(1/(a+b*x^(1/3))^2,x)
```

output
```
(3*(x**(2/3)*b**2 - 2*x**(1/3)*log(x**(1/3)*b + a)*a*b + 2*x**(1/3)*a*b -
2*log(x**(1/3)*b + a)*a**2))/(b**3*(x**(1/3)*b + a))
```

3.260 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x} \, dx = \frac{3}{a\left(a+b\sqrt[3]{x}\right)} - \frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^2} + \frac{\log(x)}{a^2}$$

output
```
3/a/(a+b*x^(1/3))-3*ln(a+b*x^(1/3))/a^2+ln(x)/a^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x} \, dx = \frac{3\left(\frac{a}{a+b\sqrt[3]{x}} - \log\left(a+b\sqrt[3]{x}\right) + \frac{\log(x)}{3}\right)}{a^2}$$

input
```
Integrate[1/((a + b*x^(1/3))^2*x),x]
```

output
```
(3*(a/(a + b*x^(1/3)) - Log[a + b*x^(1/3)] + Log[x]/3))/a^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+b \sqrt[3]{x}\right)^2} \, dx$$

$$\Big\downarrow \text{798}$$

$$3 \int \frac{1}{\left(a+b \sqrt[3]{x}\right)^2 \sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3 \int \left(-\frac{b}{a^2 \left(a+b \sqrt[3]{x}\right)}-\frac{b}{a \left(a+b \sqrt[3]{x}\right)^2}+\frac{1}{a^2 \sqrt[3]{x}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3 \left(-\frac{\log \left(a+b \sqrt[3]{x}\right)}{a^2}+\frac{\log \left(\sqrt[3]{x}\right)}{a^2}+\frac{1}{a \left(a+b \sqrt[3]{x}\right)}\right)$$

input
```
Int[1/((a + b*x^(1/3))^2*x),x]
```

output
```
3*(1/(a*(a + b*x^(1/3))) - Log[a + b*x^(1/3)]/a^2 + Log[x^(1/3)]/a^2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{3}{a\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{2}}+\frac{\ln(x)}{a^{2}}$	35
default	$\frac{3}{a\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{2}}+\frac{\ln(x)}{a^{2}}$	35

input
```
int(1/(a+b*x^(1/3))^2/x,x,method=_RETURNVERBOSE)
```

output
```
3/a/(a+b*x^(1/3))-3*ln(a+b*x^(1/3))/a^2+ln(x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx$$
$$=\frac{3\left(ab^2x^{\frac{2}{3}}-a^2bx^{\frac{1}{3}}+a^3-(b^3x+a^3)\log\left(bx^{\frac{1}{3}}+a\right)+(b^3x+a^3)\log\left(x^{\frac{1}{3}}\right)\right)}{a^2b^3x+a^5}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x,x, algorithm="fricas")
```

output
```
3*(a*b^2*x^(2/3) - a^2*b*x^(1/3) + a^3 - (b^3*x + a^3)*log(b*x^(1/3) + a)
+ (b^3*x + a^3)*log(x^(1/3)))/(a^2*b^3*x + a^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(32) = 64$.

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.21

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for } a=0 \wedge b=0 \\ \frac{\log(x)}{a^2} & \text{for } b=0 \\ -\frac{3}{2b^2x^{\frac{2}{3}}} & \text{for } a=0 \\ \frac{ax^{\frac{2}{3}}\log(x)}{a^3x^{\frac{2}{3}}+a^2bx} - \frac{3ax^{\frac{2}{3}}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^3x^{\frac{2}{3}}+a^2bx} + \frac{3ax^{\frac{2}{3}}}{a^3x^{\frac{2}{3}}+a^2bx} + \frac{bx\log(x)}{a^3x^{\frac{2}{3}}+a^2bx} - \frac{3bx\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^3x^{\frac{2}{3}}+a^2bx} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**2/x,x)
```

output
```
Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (log(x)/a**2, Eq(b, 0)), (-
3/(2*b**2*x**(2/3)), Eq(a, 0)), (a*x**(2/3)*log(x)/(a**3*x**(2/3) + a**2*b
*x) - 3*a*x**(2/3)*log(a/b + x**(1/3))/(a**3*x**(2/3) + a**2*b*x) + 3*a*x*
*(2/3)/(a**3*x**(2/3) + a**2*b*x) + b*x*log(x)/(a**3*x**(2/3) + a**2*b*x)
- 3*b*x*log(a/b + x**(1/3))/(a**3*x**(2/3) + a**2*b*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx = \frac{3}{abx^{\frac{1}{3}}+a^2} - \frac{3\log\left(bx^{\frac{1}{3}}+a\right)}{a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x,x, algorithm="maxima")
```

output
```
3/(a*b*x^(1/3) + a^2) - 3*log(b*x^(1/3) + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx = -\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{3}{\left(bx^{\frac{1}{3}}+a\right)a}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x,x, algorithm="giac")
```

output
```
-3*log(abs(b*x^(1/3) + a))/a^2 + log(abs(x))/a^2 + 3/((b*x^(1/3) + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx = \frac{3}{a\,\left(a+b\,x^{1/3}\right)}-\frac{6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^2}$$

input
```
int(1/(x*(a + b*x^(1/3))^2),x)
```

output
```
3/(a*(a + b*x^(1/3))) - (6*atanh((2*b*x^(1/3))/a + 1))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x}\,dx$$
$$= \frac{3x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}\right)b-3x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b-3x^{\frac{1}{3}}b+3\log\left(x^{\frac{1}{3}}\right)a-3\log\left(x^{\frac{1}{3}}b+a\right)a}{a^2\left(x^{\frac{1}{3}}b+a\right)}$$

input
```
int(1/(a+b*x^(1/3))^2/x,x)
```

output
```
(3*(x**(1/3)*log(x**(1/3))*b - x**(1/3)*log(x**(1/3)*b + a)*b - x**(1/3)*b
 + log(x**(1/3))*a - log(x**(1/3)*b + a)*a))/(a**2*(x**(1/3)*b + a))
```

3.261 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx = -\frac{3b^3}{a^4\left(a+b\sqrt[3]{x}\right)} - \frac{1}{a^2x} + \frac{3b}{a^3x^{2/3}} - \frac{9b^2}{a^4\sqrt[3]{x}} + \frac{12b^3\log\left(a+b\sqrt[3]{x}\right)}{a^5} - \frac{4b^3\log(x)}{a^5}$$

output
```
-3*b^3/a^4/(a+b*x^(1/3))-1/a^2/x+3*b/a^3/x^(2/3)-9*b^2/a^4/x^(1/3)+12*b^3*
ln(a+b*x^(1/3))/a^5-4*b^3*ln(x)/a^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx = \frac{-\frac{a^4-2a^3b\sqrt[3]{x}+6a^2b^2x^{2/3}+12ab^3x}{ax+bx^{4/3}} + 12b^3\log\left(a+b\sqrt[3]{x}\right) - 4b^3\log(x)}{a^5}$$

input
```
Integrate[1/((a + b*x^(1/3))^2*x^2),x]
```

output
```
(-((a^4 - 2*a^3*b*x^(1/3) + 6*a^2*b^2*x^(2/3) + 12*a*b^3*x)/(a*x + b*x^(4/
3))) + 12*b^3*Log[a + b*x^(1/3)] - 4*b^3*Log[x])/a^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt[3]{x}\right)^2} dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{4b^4}{a^5\left(a+b\sqrt[3]{x}\right)}+\frac{b^4}{a^4\left(a+b\sqrt[3]{x}\right)^2}-\frac{4b^3}{a^5\sqrt[3]{x}}+\frac{3b^2}{a^4x^{2/3}}-\frac{2b}{a^3x}+\frac{1}{a^2x^{4/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{4b^3\log\left(a+b\sqrt[3]{x}\right)}{a^5}-\frac{4b^3\log\left(\sqrt[3]{x}\right)}{a^5}-\frac{b^3}{a^4\left(a+b\sqrt[3]{x}\right)}-\frac{3b^2}{a^4\sqrt[3]{x}}+\frac{b}{a^3x^{2/3}}-\frac{1}{3a^2x}\right)$$

input
```
Int[1/((a + b*x^(1/3))^2*x^2),x]
```

output
```
3*(-(b^3/(a^4*(a + b*x^(1/3)))) - 1/(3*a^2*x) + b/(a^3*x^(2/3)) - (3*b^2)/
(a^4*x^(1/3)) + (4*b^3*Log[a + b*x^(1/3)])/a^5 - (4*b^3*Log[x^(1/3)])/a^5)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{3b^{3}}{a^{4}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{a^{2}x}+\frac{3b}{a^{3}x^{\frac{2}{3}}}-\frac{9b^{2}}{a^{4}x^{\frac{1}{3}}}+\frac{12b^{3}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{5}}-\frac{4b^{3}\ln\left(x\right)}{a^{5}}$	73
default	$-\frac{3b^{3}}{a^{4}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{a^{2}x}+\frac{3b}{a^{3}x^{\frac{2}{3}}}-\frac{9b^{2}}{a^{4}x^{\frac{1}{3}}}+\frac{12b^{3}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{5}}-\frac{4b^{3}\ln\left(x\right)}{a^{5}}$	73

input
```
int(1/(a+b*x^(1/3))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3*b^3/a^4/(a+b*x^(1/3))-1/a^2/x+3*b/a^3/x^(2/3)-9*b^2/a^4/x^(1/3)+12*b^3*
ln(a+b*x^(1/3))/a^5-4*b^3*ln(x)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx = -\frac{4\,a^3b^3x + a^6 - 12\,(b^6x^2 + a^3b^3x)\log\left(bx^{\frac{1}{3}} + a\right) + 12\,(b^6x^2 + a^3b^3x)\log\left(x^{\frac{1}{3}}\right) + 3\,(4\,ab^5x + 3\,a^4b^2)x^{\frac{2}{3}}}{a^5b^3x^2 + a^8x}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^2,x, algorithm="fricas")
```

output
```
-(4*a^3*b^3*x + a^6 - 12*(b^6*x^2 + a^3*b^3*x)*log(b*x^(1/3) + a) + 12*(b^
6*x^2 + a^3*b^3*x)*log(x^(1/3)) + 3*(4*a*b^5*x + 3*a^4*b^2)*x^(2/3) - 3*(2
*a^2*b^4*x + a^5*b)*x^(1/3))/(a^5*b^3*x^2 + a^8*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(78) = 156.

Time = 1.01 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.40

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} \\ -\frac{1}{a^2x} \\ -\frac{3}{5b^2x^{\frac{5}{3}}} \\ -\frac{a^4x^{\frac{2}{3}}}{a^6x^{\frac{5}{3}}+a^5bx^2} + \frac{2a^3bx}{a^6x^{\frac{5}{3}}+a^5bx^2} - \frac{6a^2b^2x^{\frac{4}{3}}}{a^6x^{\frac{5}{3}}+a^5bx^2} - \frac{4ab^3x^{\frac{5}{3}}\log(x)}{a^6x^{\frac{5}{3}}+a^5bx^2} + \frac{12ab^3x^{\frac{5}{3}}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{a^6x^{\frac{5}{3}}+a^5bx^2} - \frac{12ab^3x^{\frac{5}{3}}}{a^6x^{\frac{5}{3}}+a^5bx^2} - \frac{4b^4x^2\log(x)}{a^6x^{\frac{5}{3}}+a^5bx^2} + \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**2/x**2,x)
```

output
```
Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-1/(a**2*x), Eq(b, 0)), (-
3/(5*b**2*x**(5/3)), Eq(a, 0)), (-a**4*x**(2/3)/(a**6*x**(5/3) + a**5*b*x*
*2) + 2*a**3*b*x/(a**6*x**(5/3) + a**5*b*x**2) - 6*a**2*b**2*x**(4/3)/(a**
6*x**(5/3) + a**5*b*x**2) - 4*a*b**3*x**(5/3)*log(x)/(a**6*x**(5/3) + a**5
*b*x**2) + 12*a*b**3*x**(5/3)*log(a/b + x**(1/3))/(a**6*x**(5/3) + a**5*b*
x**2) - 12*a*b**3*x**(5/3)/(a**6*x**(5/3) + a**5*b*x**2) - 4*b**4*x**2*log
(x)/(a**6*x**(5/3) + a**5*b*x**2) + 12*b**4*x**2*log(a/b + x**(1/3))/(a**6
*x**(5/3) + a**5*b*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx = -\frac{12\,b^3x + 6\,ab^2x^{\frac{2}{3}} - 2\,a^2bx^{\frac{1}{3}} + a^3}{a^4bx^{\frac{4}{3}} + a^5x} + \frac{12\,b^3\log\left(bx^{\frac{1}{3}} + a\right)}{a^5} - \frac{4\,b^3\log\left(x\right)}{a^5}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^2,x, algorithm="maxima")
```

output
```
-(12*b^3*x + 6*a*b^2*x^(2/3) - 2*a^2*b*x^(1/3) + a^3)/(a^4*b*x^(4/3) + a^5
*x) + 12*b^3*log(b*x^(1/3) + a)/a^5 - 4*b^3*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx = \frac{12\,b^3\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{a^5} - \frac{4\,b^3\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x + 6\,a^2b^2x^{\frac{2}{3}} - 2\,a^3bx^{\frac{1}{3}} + a^4}{\left(bx^{\frac{1}{3}} + a\right)a^5x}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^2,x, algorithm="giac")
```

output
```
12*b^3*log(abs(b*x^(1/3) + a))/a^5 - 4*b^3*log(abs(x))/a^5 - (12*a*b^3*x +
 6*a^2*b^2*x^(2/3) - 2*a^3*b*x^(1/3) + a^4)/((b*x^(1/3) + a)*a^5*x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx=\frac{24\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^5}-\frac{\frac{1}{a}-\frac{2\,b\,x^{1/3}}{a^2}+\frac{12\,b^3\,x}{a^4}+\frac{6\,b^2\,x^{2/3}}{a^3}}{a\,x+b\,x^{4/3}}$$

input
```
int(1/(x^2*(a + b*x^(1/3))^2),x)
```

output
```
(24*b^3*atanh((2*b*x^(1/3))/a + 1))/a^5 - (1/a - (2*b*x^(1/3))/a^2 + (12*b
^3*x)/a^4 + (6*b^2*x^(2/3))/a^3)/(a*x + b*x^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^2}\,dx$$
$$=\frac{-6x^{\frac{2}{3}}a^2b^2-12x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}\right)b^4+12x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^4+2x^{\frac{1}{3}}a^3b+12x^{\frac{4}{3}}b^4-12\log\left(x^{\frac{1}{3}}\right)a\,b^3x+12\log\left(x\right.}{a^5x\left(x^{\frac{1}{3}}b+a\right)}$$

input
```
int(1/(a+b*x^(1/3))^2/x^2,x)
```

output
```
( - 6*x**(2/3)*a**2*b**2 - 12*x**(1/3)*log(x**(1/3))*b**4*x + 12*x**(1/3)*
log(x**(1/3)*b + a)*b**4*x + 2*x**(1/3)*a**3*b + 12*x**(1/3)*b**4*x - 12*l
og(x**(1/3))*a*b**3*x + 12*log(x**(1/3)*b + a)*a*b**3*x - a**4)/(a**5*x*(x
**(1/3)*b + a))
```

3.262 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3} dx = \frac{3b^6}{a^7\left(a+b\sqrt[3]{x}\right)} - \frac{1}{2a^2x^2} + \frac{6b}{5a^3x^{5/3}} - \frac{9b^2}{4a^4x^{4/3}} + \frac{4b^3}{a^5x} - \frac{15b^4}{2a^6x^{2/3}} + \frac{18b^5}{a^7\sqrt[3]{x}} - \frac{21b^6\log\left(a+b\sqrt[3]{x}\right)}{a^8} + \frac{7b^6\log(x)}{a^8}$$

output
```
3*b^6/a^7/(a+b*x^(1/3))-1/2/a^2/x^2+6/5*b/a^3/x^(5/3)-9/4*b^2/a^4/x^(4/3)+
4*b^3/a^5/x-15/2*b^4/a^6/x^(2/3)+18*b^5/a^7/x^(1/3)-21*b^6*ln(a+b*x^(1/3))
/a^8+7*b^6*ln(x)/a^8
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3} dx = \frac{\frac{a\left(-10a^6+14a^5b\sqrt[3]{x}-21a^4b^2x^{2/3}+35a^3b^3x-70a^2b^4x^{4/3}+210ab^5x^{5/3}+420b^6x^2\right)}{\left(a+b\sqrt[3]{x}\right)x^2} - 420b^6\log\left(a+b\sqrt[3]{x}\right) + 140b^6\log(x)}{20a^8}$$

input
```
Integrate[1/((a + b*x^(1/3))^2*x^3),x]
```

output
```
((a*(-10*a^6 + 14*a^5*b*x^(1/3) - 21*a^4*b^2*x^(2/3) + 35*a^3*b^3*x - 70*a
^2*b^4*x^(4/3) + 210*a*b^5*x^(5/3) + 420*b^6*x^2))/((a + b*x^(1/3))*x^2) -
 420*b^6*Log[a + b*x^(1/3)] + 140*b^6*Log[x])/(20*a^8)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sqrt[3]{x}\right)^2} dx$$

$$\Big\downarrow \; 798$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 54$$

$$3\int \left(-\frac{7b^7}{a^8\left(a+b\sqrt[3]{x}\right)}-\frac{b^7}{a^7\left(a+b\sqrt[3]{x}\right)^2}+\frac{7b^6}{a^8\sqrt[3]{x}}-\frac{6b^5}{a^7x^{2/3}}+\frac{5b^4}{a^6x}-\frac{4b^3}{a^5x^{4/3}}+\frac{3b^2}{a^4x^{5/3}}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^{7/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \; 2009$$

$$3\left(-\frac{7b^6\log\left(a+b\sqrt[3]{x}\right)}{a^8}+\frac{7b^6\log\left(\sqrt[3]{x}\right)}{a^8}+\frac{b^6}{a^7\left(a+b\sqrt[3]{x}\right)}+\frac{6b^5}{a^7\sqrt[3]{x}}-\frac{5b^4}{2a^6x^{2/3}}+\frac{4b^3}{3a^5x}-\frac{3b^2}{4a^4x^{4/3}}+\frac{2b}{5a^3x^{5/3}}-\frac{1}{6a\cdots}\right.$$

input
```
Int[1/((a + b*x^(1/3))^2*x^3),x]
```

output
```
3*(b^6/(a^7*(a + b*x^(1/3))) - 1/(6*a^2*x^2) + (2*b)/(5*a^3*x^(5/3)) - (3*
b^2)/(4*a^4*x^(4/3)) + (4*b^3)/(3*a^5*x) - (5*b^4)/(2*a^6*x^(2/3)) + (6*b^
5)/(a^7*x^(1/3)) - (7*b^6*Log[a + b*x^(1/3)])/a^8 + (7*b^6*Log[x^(1/3)])/a
^8)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

method	result	s
derivativedivides	$\frac{3b^6}{a^7\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{2a^2x^2}+\frac{6b}{5a^3x^{\frac{5}{3}}}-\frac{9b^2}{4a^4x^{\frac{4}{3}}}+\frac{4b^3}{a^5x}-\frac{15b^4}{2a^6x^{\frac{2}{3}}}+\frac{18b^5}{a^7x^{\frac{1}{3}}}-\frac{21b^6\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^8}+\frac{7b^6\ln(x)}{a^8}$	1
default	$\frac{3b^6}{a^7\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{2a^2x^2}+\frac{6b}{5a^3x^{\frac{5}{3}}}-\frac{9b^2}{4a^4x^{\frac{4}{3}}}+\frac{4b^3}{a^5x}-\frac{15b^4}{2a^6x^{\frac{2}{3}}}+\frac{18b^5}{a^7x^{\frac{1}{3}}}-\frac{21b^6\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^8}+\frac{7b^6\ln(x)}{a^8}$	1

input
```
int(1/(a+b*x^(1/3))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
3*b^6/a^7/(a+b*x^(1/3))-1/2/a^2/x^2+6/5*b/a^3/x^(5/3)-9/4*b^2/a^4/x^(4/3)+
4*b^3/a^5/x-15/2*b^4/a^6/x^(2/3)+18*b^5/a^7/x^(1/3)-21*b^6*ln(a+b*x^(1/3))
/a^8+7*b^6*ln(x)/a^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx$$
$$= \frac{140\,a^3b^6x^2+70\,a^6b^3x-10\,a^9-420\,(b^9x^3+a^3b^6x^2)\log\left(bx^{\frac{1}{3}}+a\right)+420\,(b^9x^3+a^3b^6x^2)\log\left(x^{\frac{1}{3}}\right)+15}{20\,(a^8b^3x^3+a^{11}x^2)}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^3,x, algorithm="fricas")
```

output
```
1/20*(140*a^3*b^6*x^2 + 70*a^6*b^3*x - 10*a^9 - 420*(b^9*x^3 + a^3*b^6*x^2
)*log(b*x^(1/3) + a) + 420*(b^9*x^3 + a^3*b^6*x^2)*log(x^(1/3)) + 15*(28*a
*b^8*x^2 + 21*a^4*b^5*x - 3*a^7*b^2)*x^(2/3) - 6*(35*a^2*b^7*x^2 + 21*a^5*
b^4*x - 4*a^8*b)*x^(1/3))/(a^8*b^3*x^3 + a^11*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(124) = 248.

Time = 2.47 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.24

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{8}{3}}} \\ -\frac{1}{2a^2x^2} \\ -\frac{3}{8b^2x^{\frac{8}{3}}} \\ -\frac{10a^7x^{\frac{2}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+\frac{14a^6bx}{20a^9x^{\frac{8}{3}}+20a^8bx^3}-\frac{21a^5b^2x^{\frac{4}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+\frac{35a^4b^3x^{\frac{5}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}-\frac{70a^3b^4x^2}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+\frac{210a^2b^5x^{\frac{7}{3}}}{20a^9x^{\frac{8}{3}}+20a^8bx^3}+ \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**2/x**3,x)
```

output
```
Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (-1/(2*a**2*x**2), Eq(b, 0)
), (-3/(8*b**2*x**(8/3)), Eq(a, 0)), (-10*a**7*x**(2/3)/(20*a**9*x**(8/3)
+ 20*a**8*b*x**3) + 14*a**6*b*x/(20*a**9*x**(8/3) + 20*a**8*b*x**3) - 21*a
**5*b**2*x**(4/3)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) + 35*a**4*b**3*x**(5
/3)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) - 70*a**3*b**4*x**2/(20*a**9*x**(8
/3) + 20*a**8*b*x**3) + 210*a**2*b**5*x**(7/3)/(20*a**9*x**(8/3) + 20*a**8
*b*x**3) + 140*a*b**6*x**(8/3)*log(x)/(20*a**9*x**(8/3) + 20*a**8*b*x**3)
- 420*a*b**6*x**(8/3)*log(a/b + x**(1/3))/(20*a**9*x**(8/3) + 20*a**8*b*x*
*3) + 420*a*b**6*x**(8/3)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) + 140*b**7*x
**3*log(x)/(20*a**9*x**(8/3) + 20*a**8*b*x**3) - 420*b**7*x**3*log(a/b + x
**(1/3))/(20*a**9*x**(8/3) + 20*a**8*b*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx = \frac{420\,b^6x^2+210\,ab^5x^{\frac{5}{3}}-70\,a^2b^4x^{\frac{4}{3}}+35\,a^3b^3x-21\,a^4b^2x^{\frac{2}{3}}+14\,a^5bx^{\frac{1}{3}}-10\,a^6}{20\left(a^7bx^{\frac{7}{3}}+a^8x^2\right)} - \frac{21\,b^6\log\left(bx^{\frac{1}{3}}+a\right)}{a^8} + \frac{7\,b^6\log\left(x\right)}{a^8}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^3,x, algorithm="maxima")
```

output
```
1/20*(420*b^6*x^2 + 210*a*b^5*x^(5/3) - 70*a^2*b^4*x^(4/3) + 35*a^3*b^3*x
- 21*a^4*b^2*x^(2/3) + 14*a^5*b*x^(1/3) - 10*a^6)/(a^7*b*x^(7/3) + a^8*x^2
) - 21*b^6*log(b*x^(1/3) + a)/a^8 + 7*b^6*log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx$$
$$= -\frac{21\,b^6\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^8} + \frac{7\,b^6\log\left(|x|\right)}{a^8}$$
$$+ \frac{420\,ab^6x^2 + 210\,a^2b^5x^{\frac{5}{3}} - 70\,a^3b^4x^{\frac{4}{3}} + 35\,a^4b^3x - 21\,a^5b^2x^{\frac{2}{3}} + 14\,a^6bx^{\frac{1}{3}} - 10\,a^7}{20\left(bx^{\frac{1}{3}}+a\right)a^8x^2}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^3,x, algorithm="giac")
```

output
```
-21*b^6*log(abs(b*x^(1/3) + a))/a^8 + 7*b^6*log(abs(x))/a^8 + 1/20*(420*a*
b^6*x^2 + 210*a^2*b^5*x^(5/3) - 70*a^3*b^4*x^(4/3) + 35*a^4*b^3*x - 21*a^5
*b^2*x^(2/3) + 14*a^6*b*x^(1/3) - 10*a^7)/((b*x^(1/3) + a)*a^8*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx = \frac{\frac{7\,b\,x^{1/3}}{10\,a^2} - \frac{1}{2\,a} + \frac{7\,b^3\,x}{4\,a^4} - \frac{21\,b^2\,x^{2/3}}{20\,a^3} + \frac{21\,b^6\,x^2}{a^7} - \frac{7\,b^4\,x^{4/3}}{2\,a^5} + \frac{21\,b^5\,x^{5/3}}{2\,a^6}}{a\,x^2 + b\,x^{7/3}}$$
$$- \frac{42\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^8}$$

input
```
int(1/(x^3*(a + b*x^(1/3))^2),x)
```

output
```
((7*b*x^(1/3))/(10*a^2) - 1/(2*a) + (7*b^3*x)/(4*a^4) - (21*b^2*x^(2/3))/(
20*a^3) + (21*b^6*x^2)/a^7 - (7*b^4*x^(4/3))/(2*a^5) + (21*b^5*x^(5/3))/(2
*a^6))/(a*x^2 + b*x^(7/3)) - (42*b^6*atanh((2*b*x^(1/3))/a + 1))/a^8
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^3}\,dx$$
$$= \frac{-21x^{\frac{2}{3}}a^5b^2 + 210x^{\frac{5}{3}}a^2b^5 + 420x^{\frac{7}{3}}\log\left(x^{\frac{1}{3}}\right)b^7 - 420x^{\frac{7}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^7 + 14x^{\frac{1}{3}}a^6b - 70x^{\frac{4}{3}}a^3b^4 - 420x^{\frac{7}{3}}b^7}{20a^8x^2\left(x^{\frac{1}{3}}b+a\right)}$$

input
```
int(1/(a+b*x^(1/3))^2/x^3,x)
```

output
```
( - 21*x**(2/3)*a**5*b**2 + 210*x**(2/3)*a**2*b**5*x + 420*x**(1/3)*log(x*
*(1/3))*b**7*x**2 - 420*x**(1/3)*log(x**(1/3)*b + a)*b**7*x**2 + 14*x**(1/
3)*a**6*b - 70*x**(1/3)*a**3*b**4*x - 420*x**(1/3)*b**7*x**2 + 420*log(x**
(1/3))*a*b**6*x**2 - 420*log(x**(1/3)*b + a)*a*b**6*x**2 - 10*a**7 + 35*a*
*4*b**3*x)/(20*a**8*x**2*(x**(1/3)*b + a))
```

3.263 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 162

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx = &-\frac{3b^9}{a^{10}\left(a+b\sqrt[3]{x}\right)}-\frac{1}{3a^2x^3}+\frac{3b}{4a^3x^{8/3}}-\frac{9b^2}{7a^4x^{7/3}}\\ &+\frac{2b^3}{a^5x^2}-\frac{3b^4}{a^6x^{5/3}}+\frac{9b^5}{2a^7x^{4/3}}-\frac{7b^6}{a^8x}+\frac{12b^7}{a^9x^{2/3}}\\ &-\frac{27b^8}{a^{10}\sqrt[3]{x}}+\frac{30b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{11}}-\frac{10b^9\log(x)}{a^{11}}\end{aligned}$$

output
```
-3*b^9/a^10/(a+b*x^(1/3))-1/3/a^2/x^3+3/4*b/a^3/x^(8/3)-9/7*b^2/a^4/x^(7/3
)+2*b^3/a^5/x^2-3*b^4/a^6/x^(5/3)+9/2*b^5/a^7/x^(4/3)-7*b^6/a^8/x+12*b^7/a
^9/x^(2/3)-27*b^8/a^10/x^(1/3)+30*b^9*ln(a+b*x^(1/3))/a^11-10*b^9*ln(x)/a^
11
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx = -\frac{\frac{a\left(28a^9-35a^8b\sqrt[3]{x}+45a^7b^2x^{2/3}-60a^6b^3x+84a^5b^4x^{4/3}-126a^4b^5x^{5/3}+210a^3b^6x^2-420a^2b^7x^{7/3}+1260ab^8x^{8/3}+2520b^9x^3\right)}{\left(a+b\sqrt[3]{x}\right)x^3}-2520b^9}{84a^{11}}$$

input
```
Integrate[1/((a + b*x^(1/3))^2*x^4),x]
```

output
```
-1/84*((a*(28*a^9 - 35*a^8*b*x^(1/3) + 45*a^7*b^2*x^(2/3) - 60*a^6*b^3*x +
 84*a^5*b^4*x^(4/3) - 126*a^4*b^5*x^(5/3) + 210*a^3*b^6*x^2 - 420*a^2*b^7*
x^(7/3) + 1260*a*b^8*x^(8/3) + 2520*b^9*x^3))/((a + b*x^(1/3))*x^3) - 2520
*b^9*Log[a + b*x^(1/3)] + 840*b^9*Log[x])/a^11
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+b\sqrt[3]{x}\right)^2}\,dx$$

$$\Big\downarrow \ 798$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^{10/3}}\,d\sqrt[3]{x}$$

$$\Big\downarrow \ 54$$

$$3\int \left(\frac{10b^{10}}{a^{11}\left(a+b\sqrt[3]{x}\right)}+\frac{b^{10}}{a^{10}\left(a+b\sqrt[3]{x}\right)^2}-\frac{10b^9}{a^{11}\sqrt[3]{x}}+\frac{9b^8}{a^{10}x^{2/3}}-\frac{8b^7}{a^9x}+\frac{7b^6}{a^8x^{4/3}}-\frac{6b^5}{a^7x^{5/3}}+\frac{5b^4}{a^6x^2}-\frac{4b^3}{a^5x^{7/3}}+\frac{3b}{a^4x}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{10b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{11}}-\frac{10b^9\log\left(\sqrt[3]{x}\right)}{a^{11}}-\frac{b^9}{a^{10}\left(a+b\sqrt[3]{x}\right)}-\frac{9b^8}{a^{10}\sqrt[3]{x}}+\frac{4b^7}{a^9x^{2/3}}-\frac{7b^6}{3a^8x}+\frac{3b^5}{2a^7x^{4/3}}-\frac{b^4}{a^6x^{5/3}}+\frac{2b^3}{3a^5}\right.$$

input
```
Int[1/((a + b*x^(1/3))^2*x^4),x]
```

output
```
3*(-(b^9/(a^10*(a + b*x^(1/3)))) - 1/(9*a^2*x^3) + b/(4*a^3*x^(8/3)) - (3*
b^2)/(7*a^4*x^(7/3)) + (2*b^3)/(3*a^5*x^2) - b^4/(a^6*x^(5/3)) + (3*b^5)/(
2*a^7*x^(4/3)) - (7*b^6)/(3*a^8*x) + (4*b^7)/(a^9*x^(2/3)) - (9*b^8)/(a^10
*x^(1/3)) + (10*b^9*Log[a + b*x^(1/3)])/a^11 - (10*b^9*Log[x^(1/3)])/a^11)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{3b^9}{a^{10}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{3a^2x^3}+\frac{3b}{4a^3x^{\frac{8}{3}}}-\frac{9b^2}{7a^4x^{\frac{7}{3}}}+\frac{2b^3}{a^5x^2}-\frac{3b^4}{a^6x^{\frac{5}{3}}}+\frac{9b^5}{2a^7x^{\frac{4}{3}}}-\frac{7b^6}{a^8x}+\frac{12b^7}{a^9x^{\frac{2}{3}}}-\frac{27b^8}{a^{10}x^{\frac{1}{3}}}+$
default	$-\frac{3b^9}{a^{10}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{3a^2x^3}+\frac{3b}{4a^3x^{\frac{8}{3}}}-\frac{9b^2}{7a^4x^{\frac{7}{3}}}+\frac{2b^3}{a^5x^2}-\frac{3b^4}{a^6x^{\frac{5}{3}}}+\frac{9b^5}{2a^7x^{\frac{4}{3}}}-\frac{7b^6}{a^8x}+\frac{12b^7}{a^9x^{\frac{2}{3}}}-\frac{27b^8}{a^{10}x^{\frac{1}{3}}}+$

input
```
int(1/(a+b*x^(1/3))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-3*b^9/a^10/(a+b*x^(1/3))-1/3/a^2/x^3+3/4*b/a^3/x^(8/3)-9/7*b^2/a^4/x^(7/3
)+2*b^3/a^5/x^2-3*b^4/a^6/x^(5/3)+9/2*b^5/a^7/x^(4/3)-7*b^6/a^8/x+12*b^7/a
^9/x^(2/3)-27*b^8/a^10/x^(1/3)+30*b^9*ln(a+b*x^(1/3))/a^11-10*b^9*ln(x)/a^
11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx =$$
$$-\frac{840\,a^3b^9x^3 + 420\,a^6b^6x^2 - 140\,a^9b^3x + 28\,a^{12} - 2520\,(b^{12}x^4 + a^3b^9x^3)\log\left(bx^{\frac{1}{3}} + a\right) + 2520\,(b^{12}x^4 + a}{8}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^4,x, algorithm="fricas")
```

output
```
-1/84*(840*a^3*b^9*x^3 + 420*a^6*b^6*x^2 - 140*a^9*b^3*x + 28*a^12 - 2520*
(b^12*x^4 + a^3*b^9*x^3)*log(b*x^(1/3) + a) + 2520*(b^12*x^4 + a^3*b^9*x^3
)*log(x^(1/3)) + 18*(140*a*b^11*x^3 + 105*a^4*b^8*x^2 - 15*a^7*b^5*x + 6*a
^10*b^2)*x^(2/3) - 63*(20*a^2*b^10*x^3 + 12*a^5*b^7*x^2 - 3*a^8*b^4*x + a^
11*b)*x^(1/3))/(a^11*b^3*x^4 + a^14*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(163) = 326$.

Time = 5.35 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.12

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{3}}} \\ -\frac{1}{3a^2x^3} \\ -\frac{3}{11b^2x^{\frac{11}{3}}} \\ -\frac{28a^{10}x^{\frac{2}{3}}}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4} + \frac{35a^9bx}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4} - \frac{45a^8b^2x^{\frac{4}{3}}}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4} + \frac{60a^7b^3x^{\frac{5}{3}}}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4} - \frac{84a^6b^4x^2}{84a^{12}x^{\frac{11}{3}}+84a^{11}bx^4} + \frac{126a}{84a^{12}x^{\frac{1}{3}}} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**2/x**4,x)
```

output
```
Piecewise((zoo/x**(11/3), Eq(a, 0) & Eq(b, 0)), (-1/(3*a**2*x**3), Eq(b, 0
)), (-3/(11*b**2*x**(11/3)), Eq(a, 0)), (-28*a**10*x**(2/3)/(84*a**12*x**(
11/3) + 84*a**11*b*x**4) + 35*a**9*b*x/(84*a**12*x**(11/3) + 84*a**11*b*x*
*4) - 45*a**8*b**2*x**(4/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) + 60*a*
*7*b**3*x**(5/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) - 84*a**6*b**4*x**
2/(84*a**12*x**(11/3) + 84*a**11*b*x**4) + 126*a**5*b**5*x**(7/3)/(84*a**1
2*x**(11/3) + 84*a**11*b*x**4) - 210*a**4*b**6*x**(8/3)/(84*a**12*x**(11/3
) + 84*a**11*b*x**4) + 420*a**3*b**7*x**3/(84*a**12*x**(11/3) + 84*a**11*b
*x**4) - 1260*a**2*b**8*x**(10/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) -
 840*a*b**9*x**(11/3)*log(x)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) + 2520
*a*b**9*x**(11/3)*log(a/b + x**(1/3))/(84*a**12*x**(11/3) + 84*a**11*b*x**
4) - 2520*a*b**9*x**(11/3)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) - 840*b*
*10*x**4*log(x)/(84*a**12*x**(11/3) + 84*a**11*b*x**4) + 2520*b**10*x**4*l
og(a/b + x**(1/3))/(84*a**12*x**(11/3) + 84*a**11*b*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx = -\frac{2520\,b^9x^3 + 1260\,ab^8x^{\frac{8}{3}} - 420\,a^2b^7x^{\frac{7}{3}} + 210\,a^3b^6x^2 - 126\,a^4b^5x^{\frac{5}{3}} + 84\,a^5b^4x^{\frac{4}{3}} - 60\,a^6b^3x + 45\,a^7b^2x^{\frac{2}{3}} +}{84\left(a^{10}bx^{\frac{10}{3}} + a^{11}x^3\right)} + \frac{30\,b^9\log\left(bx^{\frac{1}{3}} + a\right)}{a^{11}} - \frac{10\,b^9\log\left(x\right)}{a^{11}}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^4,x, algorithm="maxima")
```

output
```
-1/84*(2520*b^9*x^3 + 1260*a*b^8*x^(8/3) - 420*a^2*b^7*x^(7/3) + 210*a^3*b
^6*x^2 - 126*a^4*b^5*x^(5/3) + 84*a^5*b^4*x^(4/3) - 60*a^6*b^3*x + 45*a^7*
b^2*x^(2/3) - 35*a^8*b*x^(1/3) + 28*a^9)/(a^10*b*x^(10/3) + a^11*x^3) + 30
*b^9*log(b*x^(1/3) + a)/a^11 - 10*b^9*log(x)/a^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx = \frac{30\,b^9\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{a^{11}} - \frac{10\,b^9\log\left(|x|\right)}{a^{11}} - \frac{2520\,ab^9x^3 + 1260\,a^2b^8x^{\frac{8}{3}} - 420\,a^3b^7x^{\frac{7}{3}} + 210\,a^4b^6x^2 - 126\,a^5b^5x^{\frac{5}{3}} + 84\,a^6b^4x^{\frac{4}{3}} - 60\,a^7b^3x + 45\,a^8b^2x}{84\left(bx^{\frac{1}{3}} + a\right)a^{11}x^3}$$

input
```
integrate(1/(a+b*x^(1/3))^2/x^4,x, algorithm="giac")
```

output
```
30*b^9*log(abs(b*x^(1/3) + a))/a^11 - 10*b^9*log(abs(x))/a^11 - 1/84*(2520
*a*b^9*x^3 + 1260*a^2*b^8*x^(8/3) - 420*a^3*b^7*x^(7/3) + 210*a^4*b^6*x^2
- 126*a^5*b^5*x^(5/3) + 84*a^6*b^4*x^(4/3) - 60*a^7*b^3*x + 45*a^8*b^2*x^(
2/3) - 35*a^9*b*x^(1/3) + 28*a^10)/((b*x^(1/3) + a)*a^11*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx = \frac{60\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^{11}} - \frac{\frac{1}{3\,a}-\frac{5\,b\,x^{1/3}}{12\,a^2}-\frac{5\,b^3\,x}{7\,a^4}+\frac{15\,b^2\,x^{2/3}}{28\,a^3}+\frac{5\,b^6\,x^2}{2\,a^7}+\frac{b^4\,x^{4/3}}{a^5}-\frac{3\,b^5\,x^{5/3}}{2\,a^6}+\frac{30\,b^9\,x^3}{a^{10}}-\frac{5\,b^7\,x^{7/3}}{a^8}+\frac{15\,b^8\,x^{8/3}}{a^9}}{a\,x^3+b\,x^{10/3}}$$

input
```
int(1/(x^4*(a + b*x^(1/3))^2),x)
```

output
```
(60*b^9*atanh((2*b*x^(1/3))/a + 1))/a^11 - (1/(3*a) - (5*b*x^(1/3))/(12*a^
2) - (5*b^3*x)/(7*a^4) + (15*b^2*x^(2/3))/(28*a^3) + (5*b^6*x^2)/(2*a^7) +
 (b^4*x^(4/3))/a^5 - (3*b^5*x^(5/3))/(2*a^6) + (30*b^9*x^3)/a^10 - (5*b^7*
x^(7/3))/a^8 + (15*b^8*x^(8/3))/a^9)/(a*x^3 + b*x^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^2 x^4}\,dx = \frac{-45x^{\frac{2}{3}}a^8b^2+126x^{\frac{5}{3}}a^5b^5-1260x^{\frac{8}{3}}a^2b^8-2520x^{\frac{10}{3}}\log\left(x^{\frac{1}{3}}\right)b^{10}+2520x^{\frac{10}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^{10}+35x^{\frac{1}{3}}a^9b-}{8}$$

input
```
int(1/(a+b*x^(1/3))^2/x^4,x)
```

output
```
( - 45*x**(2/3)*a**8*b**2 + 126*x**(2/3)*a**5*b**5*x - 1260*x**(2/3)*a**2*
b**8*x**2 - 2520*x**(1/3)*log(x**(1/3))*b**10*x**3 + 2520*x**(1/3)*log(x**
(1/3)*b + a)*b**10*x**3 + 35*x**(1/3)*a**9*b - 84*x**(1/3)*a**6*b**4*x + 4
20*x**(1/3)*a**3*b**7*x**2 + 2520*x**(1/3)*b**10*x**3 - 2520*log(x**(1/3))
*a*b**9*x**3 + 2520*log(x**(1/3)*b + a)*a*b**9*x**3 - 28*a**10 + 60*a**7*b
**3*x - 210*a**4*b**6*x**2)/(84*a**11*x**3*(x**(1/3)*b + a))
```

3.264 $\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 171

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{3a^{11}}{2b^{12}\left(a+b\sqrt[3]{x}\right)^2} - \frac{33a^{10}}{b^{12}\left(a+b\sqrt[3]{x}\right)} + \frac{135a^8\sqrt[3]{x}}{b^{11}} - \frac{54a^7x^{2/3}}{b^{10}} + \frac{28a^6x}{b^9} - \frac{63a^5x^{4/3}}{4b^8} + \frac{9a^4x^{5/3}}{b^7} - \frac{5a^3x^2}{b^6} + \frac{18a^2x^{7/3}}{7b^5} - \frac{9ax^{8/3}}{8b^4} + \frac{x^3}{3b^3} - \frac{165a^9\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}$$

output
```
3/2*a^11/b^12/(a+b*x^(1/3))^2-33*a^10/b^12/(a+b*x^(1/3))+135*a^8*x^(1/3)/b
^11-54*a^7*x^(2/3)/b^10+28*a^6*x/b^9-63/4*a^5*x^(4/3)/b^8+9*a^4*x^(5/3)/b^
7-5*a^3*x^2/b^6+18/7*a^2*x^(7/3)/b^5-9/8*a*x^(8/3)/b^4+1/3*x^3/b^3-165*a^9
*ln(a+b*x^(1/3))/b^12
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$= \frac{-5292a^{11}+17136a^{10}b\sqrt[3]{x}+36288a^9b^2x^{2/3}+9240a^8b^3x-2310a^7b^4x^{4/3}+924a^6b^5x^{5/3}-462a^5b^6x^2+2}{168b^{12}\left(a+b\sqrt[3]{x}\right)^2}$$
$$-\frac{165a^9\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}$$

input
```
Integrate[x^3/(a + b*x^(1/3))^3,x]
```

output
```
(-5292*a^11 + 17136*a^10*b*x^(1/3) + 36288*a^9*b^2*x^(2/3) + 9240*a^8*b^3*
x - 2310*a^7*b^4*x^(4/3) + 924*a^6*b^5*x^(5/3) - 462*a^5*b^6*x^2 + 264*a^4
*b^7*x^(7/3) - 165*a^3*b^8*x^(8/3) + 110*a^2*b^9*x^3 - 77*a*b^10*x^(10/3)
+ 56*b^11*x^(11/3))/(168*b^12*(a + b*x^(1/3))^2) - (165*a^9*Log[a + b*x^(1
/3)])/b^12
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$\Big\downarrow \text{798}$$
$$3\int \frac{x^{11/3}}{\left(a+b\sqrt[3]{x}\right)^3}\,d\sqrt[3]{x}$$
$$\Big\downarrow \text{49}$$

$$3\int\left(-\frac{a^{11}}{b^{11}\left(a+b\sqrt[3]{x}\right)^3}+\frac{11a^{10}}{b^{11}\left(a+b\sqrt[3]{x}\right)^2}-\frac{55a^9}{b^{11}\left(a+b\sqrt[3]{x}\right)}+\frac{45a^8}{b^{11}}-\frac{36\sqrt[3]{x}a^7}{b^{10}}+\frac{28x^{2/3}a^6}{b^9}-\frac{21xa^5}{b^8}+\frac{15x^{4/3}a^4}{b^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^{11}}{2b^{12}\left(a+b\sqrt[3]{x}\right)^2}-\frac{11a^{10}}{b^{12}\left(a+b\sqrt[3]{x}\right)}-\frac{55a^9\log\left(a+b\sqrt[3]{x}\right)}{b^{12}}+\frac{45a^8\sqrt[3]{x}}{b^{11}}-\frac{18a^7x^{2/3}}{b^{10}}+\frac{28a^6x}{3b^9}-\frac{21a^5x^{4/3}}{4b^8}+\frac{3a^4x}{b^7}\right.$$

input
```
Int[x^3/(a + b*x^(1/3))^3,x]
```

output
```
3*(a^11/(2*b^12*(a + b*x^(1/3))^2) - (11*a^10)/(b^12*(a + b*x^(1/3))) + (4
5*a^8*x^(1/3))/b^11 - (18*a^7*x^(2/3))/b^10 + (28*a^6*x)/(3*b^9) - (21*a^5
*x^(4/3))/(4*b^8) + (3*a^4*x^(5/3))/b^7 - (5*a^3*x^2)/(3*b^6) + (6*a^2*x^(
7/3))/(7*b^5) - (3*a*x^(8/3))/(8*b^4) + x^3/(9*b^3) - (55*a^9*Log[a + b*x^
(1/3)])/b^12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{b^8x^3}{3}-\frac{9x^{\frac{8}{3}}a\,b^7}{8}+\frac{18x^{\frac{7}{3}}a^2b^6}{7}-5a^3b^5x^2+9x^{\frac{5}{3}}a^4b^4-\frac{63x^{\frac{4}{3}}a^5b^3}{4}+28a^6b^2x-54x^{\frac{2}{3}}a^7b+135x^{\frac{1}{3}}a^8}{b^{11}}-\frac{33a^{10}}{b^{12}\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{}{2b}$
default	$\frac{\frac{b^8x^3}{3}-\frac{9x^{\frac{8}{3}}a\,b^7}{8}+\frac{18x^{\frac{7}{3}}a^2b^6}{7}-5a^3b^5x^2+9x^{\frac{5}{3}}a^4b^4-\frac{63x^{\frac{4}{3}}a^5b^3}{4}+28a^6b^2x-54x^{\frac{2}{3}}a^7b+135x^{\frac{1}{3}}a^8}{b^{11}}-\frac{33a^{10}}{b^{12}\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{}{2b}$

input
```
int(x^3/(a+b*x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
3/b^11*(1/9*b^8*x^3-3/8*x^(8/3)*a*b^7+6/7*x^(7/3)*a^2*b^6-5/3*a^3*b^5*x^2+
3*x^(5/3)*a^4*b^4-21/4*x^(4/3)*a^5*b^3+28/3*a^6*b^2*x-18*x^(2/3)*a^7*b+45*
x^(1/3)*a^8)-33*a^10/b^12/(a+b*x^(1/3))+3/2*a^11/b^12/(a+b*x^(1/3))^2-165*
a^9*ln(a+b*x^(1/3))/b^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.32

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{56\,b^{15}x^5-728\,a^3b^{12}x^4+3080\,a^6b^9x^3+8568\,a^9b^6x^2-1344\,a^{12}b^3x-5292\,a^{15}-27720\left(a^9b^6x^2+2\,a^{12}b^3x\right.}{}$$

input
```
integrate(x^3/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/168*(56*b^15*x^5 - 728*a^3*b^12*x^4 + 3080*a^6*b^9*x^3 + 8568*a^9*b^6*x^
2 - 1344*a^12*b^3*x - 5292*a^15 - 27720*(a^9*b^6*x^2 + 2*a^12*b^3*x + a^15
)*log(b*x^(1/3) + a) - 63*(3*a*b^14*x^4 - 18*a^4*b^11*x^3 + 99*a^7*b^8*x^2
 + 352*a^10*b^5*x + 220*a^13*b^2)*x^(2/3) + 18*(24*a^2*b^13*x^4 - 99*a^5*b
^10*x^3 + 990*a^8*b^7*x^2 + 2695*a^11*b^4*x + 1540*a^14*b)*x^(1/3))/(b^18*
x^2 + 2*a^3*b^15*x + a^6*b^12)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(170) = 340$.

Time = 1.19 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.65

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$= \begin{cases} -\frac{27720a^{11}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{168a^2b^{12}+336ab^{13}\sqrt[3]{x}+168b^{14}x^{\frac{2}{3}}} - \frac{41580a^{11}}{168a^2b^{12}+336ab^{13}\sqrt[3]{x}+168b^{14}x^{\frac{2}{3}}} - \frac{55440a^{10}b\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{168a^2b^{12}+336ab^{13}\sqrt[3]{x}+168b^{14}x^{\frac{2}{3}}} - \frac{55440a}{168a^2b^{12}+336ab} \\ \frac{x^4}{4a^3} \end{cases}$$

input
```
integrate(x**3/(a+b*x**(1/3))**3,x)
```

output
```
Piecewise((-27720*a**11*log(a/b + x**(1/3))/(168*a**2*b**12 + 336*a*b**13*
x**(1/3) + 168*b**14*x**(2/3)) - 41580*a**11/(168*a**2*b**12 + 336*a*b**13
*x**(1/3) + 168*b**14*x**(2/3)) - 55440*a**10*b*x**(1/3)*log(a/b + x**(1/3
))/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) - 55440*a*
*10*b*x**(1/3)/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)
) - 27720*a**9*b**2*x**(2/3)*log(a/b + x**(1/3))/(168*a**2*b**12 + 336*a*b
**13*x**(1/3) + 168*b**14*x**(2/3)) + 9240*a**8*b**3*x/(168*a**2*b**12 + 3
36*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) - 2310*a**7*b**4*x**(4/3)/(168*a
**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) + 924*a**6*b**5*x**
(5/3)/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) - 462*a
**5*b**6*x**2/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3))
 + 264*a**4*b**7*x**(7/3)/(168*a**2*b**12 + 336*a*b**13*x**(1/3) + 168*b**
14*x**(2/3)) - 165*a**3*b**8*x**(8/3)/(168*a**2*b**12 + 336*a*b**13*x**(1/
3) + 168*b**14*x**(2/3)) + 110*a**2*b**9*x**3/(168*a**2*b**12 + 336*a*b**1
3*x**(1/3) + 168*b**14*x**(2/3)) - 77*a*b**10*x**(10/3)/(168*a**2*b**12 +
336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)) + 56*b**11*x**(11/3)/(168*a**2*
b**12 + 336*a*b**13*x**(1/3) + 168*b**14*x**(2/3)), Ne(b, 0)), (x**4/(4*a*
*3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = -\frac{165\,a^9\log\left(bx^{\frac{1}{3}}+a\right)}{b^{12}} + \frac{\left(bx^{\frac{1}{3}}+a\right)^9}{3\,b^{12}} - \frac{33\left(bx^{\frac{1}{3}}+a\right)^8 a}{8\,b^{12}} + \frac{165\left(bx^{\frac{1}{3}}+a\right)^7 a^2}{7\,b^{12}} - \frac{165\left(bx^{\frac{1}{3}}+a\right)^6 a^3}{2\,b^{12}} + \frac{198\left(bx^{\frac{1}{3}}+a\right)^5 a^4}{b^{12}} - \frac{693\left(bx^{\frac{1}{3}}+a\right)^4 a^5}{2\,b^{12}} + \frac{462\left(bx^{\frac{1}{3}}+a\right)^3 a^6}{b^{12}} - \frac{495\left(bx^{\frac{1}{3}}+a\right)^2 a^7}{b^{12}} + \frac{495\left(bx^{\frac{1}{3}}+a\right)a^8}{b^{12}} - \frac{33\,a^{10}}{\left(bx^{\frac{1}{3}}+a\right)b^{12}} + \frac{3\,a^{11}}{2\left(bx^{\frac{1}{3}}+a\right)^2 b^{12}}$$

input
```
integrate(x^3/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

output
```
-165*a^9*log(b*x^(1/3) + a)/b^12 + 1/3*(b*x^(1/3) + a)^9/b^12 - 33/8*(b*x^
(1/3) + a)^8*a/b^12 + 165/7*(b*x^(1/3) + a)^7*a^2/b^12 - 165/2*(b*x^(1/3)
+ a)^6*a^3/b^12 + 198*(b*x^(1/3) + a)^5*a^4/b^12 - 693/2*(b*x^(1/3) + a)^4
*a^5/b^12 + 462*(b*x^(1/3) + a)^3*a^6/b^12 - 495*(b*x^(1/3) + a)^2*a^7/b^1
2 + 495*(b*x^(1/3) + a)*a^8/b^12 - 33*a^10/((b*x^(1/3) + a)*b^12) + 3/2*a^
11/((b*x^(1/3) + a)^2*b^12)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = -\frac{165\,a^9\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^{12}} - \frac{3\left(22\,a^{10}bx^{\frac{1}{3}}+21\,a^{11}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2 b^{12}} + \frac{56\,b^{24}x^3 - 189\,ab^{23}x^{\frac{8}{3}} + 432\,a^2b^{22}x^{\frac{7}{3}} - 840\,a^3b^{21}x^2 + 1512\,a^4b^{20}x^{\frac{5}{3}} - 2646\,a^5b^{19}x^{\frac{4}{3}} + 4704\,a^6b^{18}x - 90}{168\,b^{27}}$$

input
```
integrate(x^3/(a+b*x^(1/3))^3,x, algorithm="giac")
```

output
```
-165*a^9*log(abs(b*x^(1/3) + a))/b^12 - 3/2*(22*a^10*b*x^(1/3) + 21*a^11)/
((b*x^(1/3) + a)^2*b^12) + 1/168*(56*b^24*x^3 - 189*a*b^23*x^(8/3) + 432*a
^2*b^22*x^(7/3) - 840*a^3*b^21*x^2 + 1512*a^4*b^20*x^(5/3) - 2646*a^5*b^19
*x^(4/3) + 4704*a^6*b^18*x - 9072*a^7*b^17*x^(2/3) + 22680*a^8*b^16*x^(1/3
))/b^27
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx=\frac{x^3}{3\,b^3}-\frac{\frac{63\,a^{11}}{2\,b}+33\,a^{10}\,x^{1/3}}{a^2\,b^{11}+b^{13}\,x^{2/3}+2\,a\,b^{12}\,x^{1/3}}-\frac{9\,a\,x^{8/3}}{8\,b^4}+\frac{28\,a^6\,x}{b^9}-\frac{165\,a^9\,\ln\left(a+b\,x^{1/3}\right)}{b^{12}}-\frac{5\,a^3\,x^2}{b^6}+\frac{18\,a^2\,x^{7/3}}{7\,b^5}+\frac{9\,a^4\,x^{5/3}}{b^7}-\frac{63\,a^5\,x^{4/3}}{4\,b^8}-\frac{54\,a^7\,x^{2/3}}{b^{10}}+\frac{135\,a^8\,x^{1/3}}{b^{11}}$$

input
```
int(x^3/(a + b*x^(1/3))^3,x)
```

output
```
x^3/(3*b^3) - ((63*a^11)/(2*b) + 33*a^10*x^(1/3))/(a^2*b^11 + b^13*x^(2/3)
 + 2*a*b^12*x^(1/3)) - (9*a*x^(8/3))/(8*b^4) + (28*a^6*x)/b^9 - (165*a^9*l
og(a + b*x^(1/3)))/b^12 - (5*a^3*x^2)/b^6 + (18*a^2*x^(7/3))/(7*b^5) + (9*
a^4*x^(5/3))/b^7 - (63*a^5*x^(4/3))/(4*b^8) - (54*a^7*x^(2/3))/b^10 + (135
*a^8*x^(1/3))/b^11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\left(a+b\sqrt[3]{x}\right)^3}\,dx=\frac{-27720x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^9b^2+27720x^{\frac{2}{3}}a^9b^2+924x^{\frac{5}{3}}a^6b^5-165x^{\frac{8}{3}}a^3b^8+56x^{\frac{11}{3}}b^{11}-55440x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+\right.}{16}$$

input
```
int(x^3/(a+b*x^(1/3))^3,x)
```

output
```
( - 27720*x**(2/3)*log(x**(1/3)*b + a)*a**9*b**2 + 27720*x**(2/3)*a**9*b**
2 + 924*x**(2/3)*a**6*b**5*x - 165*x**(2/3)*a**3*b**8*x**2 + 56*x**(2/3)*b
**11*x**3 - 55440*x**(1/3)*log(x**(1/3)*b + a)*a**10*b - 2310*x**(1/3)*a**
7*b**4*x + 264*x**(1/3)*a**4*b**7*x**2 - 77*x**(1/3)*a*b**10*x**3 - 27720*
log(x**(1/3)*b + a)*a**11 - 13860*a**11 + 9240*a**8*b**3*x - 462*a**5*b**6
*x**2 + 110*a**2*b**9*x**3)/(168*b**12*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a
**2))
```

3.265 $\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx=-\frac{3a^8}{2b^9\left(a+b\sqrt[3]{x}\right)^2}+\frac{24a^7}{b^9\left(a+b\sqrt[3]{x}\right)}-\frac{63a^5\sqrt[3]{x}}{b^8}+\frac{45a^4x^{2/3}}{2b^7}-\frac{10a^3x}{b^6}+\frac{9a^2x^{4/3}}{2b^5}-\frac{9ax^{5/3}}{5b^4}+\frac{x^2}{2b^3}+\frac{84a^6\log\left(a+b\sqrt[3]{x}\right)}{b^9}$$

output
```
-3/2*a^8/b^9/(a+b*x^(1/3))^2+24*a^7/b^9/(a+b*x^(1/3))-63*a^5*x^(1/3)/b^8+4
5/2*a^4*x^(2/3)/b^7-10*a^3*x/b^6+9/2*a^2*x^(4/3)/b^5-9/5*a*x^(5/3)/b^4+1/2
*x^2/b^3+84*a^6*ln(a+b*x^(1/3))/b^9
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$=\frac{225a^8-390a^7b\sqrt[3]{x}-1035a^6b^2x^{2/3}-280a^5b^3x+70a^4b^4x^{4/3}-28a^3b^5x^{5/3}+14a^2b^6x^2-8ab^7x^{7/3}+5b^8x}{10b^9\left(a+b\sqrt[3]{x}\right)^2}$$
$$+\frac{84a^6\log\left(a+b\sqrt[3]{x}\right)}{b^9}$$

input
```
Integrate[x^2/(a + b*x^(1/3))^3,x]
```

output
```
(225*a^8 - 390*a^7*b*x^(1/3) - 1035*a^6*b^2*x^(2/3) - 280*a^5*b^3*x + 70*a
^4*b^4*x^(4/3) - 28*a^3*b^5*x^(5/3) + 14*a^2*b^6*x^2 - 8*a*b^7*x^(7/3) + 5
*b^8*x^(8/3))/(10*b^9*(a + b*x^(1/3))^2) + (84*a^6*Log[a + b*x^(1/3)])/b^9
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{8/3}}{\left(a+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^8}{b^8\left(a+b\sqrt[3]{x}\right)^3}-\frac{8a^7}{b^8\left(a+b\sqrt[3]{x}\right)^2}+\frac{28a^6}{b^8\left(a+b\sqrt[3]{x}\right)}-\frac{21a^5}{b^8}+\frac{15\sqrt[3]{x}a^4}{b^7}-\frac{10x^{2/3}a^3}{b^6}+\frac{6xa^2}{b^5}-\frac{3x^{4/3}a}{b^4}+\frac{x^{5/3}}{b^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^8}{2b^9\left(a+b\sqrt[3]{x}\right)^2}+\frac{8a^7}{b^9\left(a+b\sqrt[3]{x}\right)}+\frac{28a^6\log\left(a+b\sqrt[3]{x}\right)}{b^9}-\frac{21a^5\sqrt[3]{x}}{b^8}+\frac{15a^4x^{2/3}}{2b^7}-\frac{10a^3x}{3b^6}+\frac{3a^2x^{4/3}}{2b^5}-\frac{3ax^{5/}}{5b^4}\right.$$

input
```
Int[x^2/(a + b*x^(1/3))^3,x]
```

output
```
3*(-1/2*a^8/(b^9*(a + b*x^(1/3))^2) + (8*a^7)/(b^9*(a + b*x^(1/3))) - (21*
a^5*x^(1/3))/b^8 + (15*a^4*x^(2/3))/(2*b^7) - (10*a^3*x)/(3*b^6) + (3*a^2*
x^(4/3))/(2*b^5) - (3*a*x^(5/3))/(5*b^4) + x^2/(6*b^3) + (28*a^6*Log[a + b
*x^(1/3)])/b^9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{3\left(-\frac{b^{5}x^{2}}{6}+\frac{3x^{\frac{5}{3}}a\,b^{4}}{5}-\frac{3x^{\frac{4}{3}}a^{2}b^{3}}{2}+\frac{10a^{3}b^{2}x}{3}-\frac{15x^{\frac{2}{3}}a^{4}b}{2}+21x^{\frac{1}{3}}a^{5}\right)}{b^{8}}+\frac{84a^{6}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^{9}}+\frac{24a^{7}}{b^{9}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{3}{2b^{9}\left(a+\right.}$
default	$-\frac{3\left(-\frac{b^{5}x^{2}}{6}+\frac{3x^{\frac{5}{3}}a\,b^{4}}{5}-\frac{3x^{\frac{4}{3}}a^{2}b^{3}}{2}+\frac{10a^{3}b^{2}x}{3}-\frac{15x^{\frac{2}{3}}a^{4}b}{2}+21x^{\frac{1}{3}}a^{5}\right)}{b^{8}}+\frac{84a^{6}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^{9}}+\frac{24a^{7}}{b^{9}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{3}{2b^{9}\left(a+\right.}$

input
```
int(x^2/(a+b*x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
-3/b^8*(-1/6*b^5*x^2+3/5*x^(5/3)*a*b^4-3/2*x^(4/3)*a^2*b^3+10/3*a^3*b^2*x-
15/2*x^(2/3)*a^4*b+21*x^(1/3)*a^5)+84*a^6*ln(a+b*x^(1/3))/b^9+24*a^7/b^9/(
a+b*x^(1/3))-3/2*a^8/b^9/(a+b*x^(1/3))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$= \frac{5\,b^{12}x^4 - 90\,a^3b^9x^3 - 195\,a^6b^6x^2 + 170\,a^9b^3x + 225\,a^{12} + 840\,(a^6b^6x^2 + 2\,a^9b^3x + a^{12})\log\left(bx^{\frac{1}{3}} + a\right) - 3}{10\,(b^{15}x^2 + 2\,a}$$

input
```
integrate(x^2/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/10*(5*b^12*x^4 - 90*a^3*b^9*x^3 - 195*a^6*b^6*x^2 + 170*a^9*b^3*x + 225*
a^12 + 840*(a^6*b^6*x^2 + 2*a^9*b^3*x + a^12)*log(b*x^(1/3) + a) - 3*(6*a*
b^11*x^3 - 63*a^4*b^8*x^2 - 224*a^7*b^5*x - 140*a^10*b^2)*x^(2/3) + 15*(3*
a^2*b^10*x^3 - 36*a^5*b^7*x^2 - 98*a^8*b^4*x - 56*a^11*b)*x^(1/3))/(b^15*x
^2 + 2*a^3*b^12*x + a^6*b^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(131) = 262.

Time = 0.65 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.68

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$= \begin{cases} \frac{840a^8\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}} + \frac{1260a^8}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}} + \frac{1680a^7b\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}} + \frac{1680a^7b\sqrt[3]{x}}{10a^2b^9+20ab^{10}\sqrt[3]{x}+10b^{11}x^{\frac{2}{3}}} + \\ \frac{x^3}{3a^3} \end{cases}$$

input
```
integrate(x**2/(a+b*x**(1/3))**3,x)
```

output
```
Piecewise((840*a**8*log(a/b + x**(1/3))/(10*a**2*b**9 + 20*a*b**10*x**(1/3
) + 10*b**11*x**(2/3)) + 1260*a**8/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 1
0*b**11*x**(2/3)) + 1680*a**7*b*x**(1/3)*log(a/b + x**(1/3))/(10*a**2*b**9
 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2/3)) + 1680*a**7*b*x**(1/3)/(10*a**
2*b**9 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2/3)) + 840*a**6*b**2*x**(2/3)
*log(a/b + x**(1/3))/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2/
3)) - 280*a**5*b**3*x/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**11*x**(2
/3)) + 70*a**4*b**4*x**(4/3)/(10*a**2*b**9 + 20*a*b**10*x**(1/3) + 10*b**1
1*x**(2/3)) - 28*a**3*b**5*x**(5/3)/(10*a**2*b**9 + 20*a*b**10*x**(1/3) +
10*b**11*x**(2/3)) + 14*a**2*b**6*x**2/(10*a**2*b**9 + 20*a*b**10*x**(1/3)
 + 10*b**11*x**(2/3)) - 8*a*b**7*x**(7/3)/(10*a**2*b**9 + 20*a*b**10*x**(1
/3) + 10*b**11*x**(2/3)) + 5*b**8*x**(8/3)/(10*a**2*b**9 + 20*a*b**10*x**(
1/3) + 10*b**11*x**(2/3)), Ne(b, 0)), (x**3/(3*a**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{84\,a^6\log\left(bx^{\frac{1}{3}}+a\right)}{b^9} + \frac{\left(bx^{\frac{1}{3}}+a\right)^6}{2\,b^9} - \frac{24\left(bx^{\frac{1}{3}}+a\right)^5 a}{5\,b^9} + \frac{21\left(bx^{\frac{1}{3}}+a\right)^4 a^2}{b^9} - \frac{56\left(bx^{\frac{1}{3}}+a\right)^3 a^3}{b^9} + \frac{105\left(bx^{\frac{1}{3}}+a\right)^2 a^4}{b^9} - \frac{168\left(bx^{\frac{1}{3}}+a\right)a^5}{b^9} + \frac{24\,a^7}{\left(bx^{\frac{1}{3}}+a\right)b^9} - \frac{3\,a^8}{2\left(bx^{\frac{1}{3}}+a\right)^2 b^9}$$

input
```
integrate(x^2/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

output
```
84*a^6*log(b*x^(1/3) + a)/b^9 + 1/2*(b*x^(1/3) + a)^6/b^9 - 24/5*(b*x^(1/3
) + a)^5*a/b^9 + 21*(b*x^(1/3) + a)^4*a^2/b^9 - 56*(b*x^(1/3) + a)^3*a^3/b
^9 + 105*(b*x^(1/3) + a)^2*a^4/b^9 - 168*(b*x^(1/3) + a)*a^5/b^9 + 24*a^7/
((b*x^(1/3) + a)*b^9) - 3/2*a^8/((b*x^(1/3) + a)^2*b^9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{84\,a^6\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^9}+\frac{3\left(16\,a^7bx^{\frac{1}{3}}+15\,a^8\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^9} + \frac{5\,b^{15}x^2-18\,ab^{14}x^{\frac{5}{3}}+45\,a^2b^{13}x^{\frac{4}{3}}-100\,a^3b^{12}x+225\,a^4b^{11}x^{\frac{2}{3}}-630\,a^5b^{10}x^{\frac{1}{3}}}{10\,b^{18}}$$

input
```
integrate(x^2/(a+b*x^(1/3))^3,x, algorithm="giac")
```

output
```
84*a^6*log(abs(b*x^(1/3) + a))/b^9 + 3/2*(16*a^7*b*x^(1/3) + 15*a^8)/((b*x
^(1/3) + a)^2*b^9) + 1/10*(5*b^15*x^2 - 18*a*b^14*x^(5/3) + 45*a^2*b^13*x^
(4/3) - 100*a^3*b^12*x + 225*a^4*b^11*x^(2/3) - 630*a^5*b^10*x^(1/3))/b^18
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{\frac{45\,a^8}{2\,b}+24\,a^7\,x^{1/3}}{a^2\,b^8+b^{10}\,x^{2/3}+2\,a\,b^9\,x^{1/3}}+\frac{x^2}{2\,b^3}-\frac{10\,a^3\,x}{b^6}-\frac{9\,a\,x^{5/3}}{5\,b^4} + \frac{84\,a^6\,\ln\left(a+b\,x^{1/3}\right)}{b^9}+\frac{9\,a^2\,x^{4/3}}{2\,b^5}+\frac{45\,a^4\,x^{2/3}}{2\,b^7}-\frac{63\,a^5\,x^{1/3}}{b^8}$$

input
```
int(x^2/(a + b*x^(1/3))^3,x)
```

output
```
((45*a^8)/(2*b) + 24*a^7*x^(1/3))/(a^2*b^8 + b^10*x^(2/3) + 2*a*b^9*x^(1/3
)) + x^2/(2*b^3) - (10*a^3*x)/b^6 - (9*a*x^(5/3))/(5*b^4) + (84*a^6*log(a
+ b*x^(1/3)))/b^9 + (9*a^2*x^(4/3))/(2*b^5) + (45*a^4*x^(2/3))/(2*b^7) - (
63*a^5*x^(1/3))/b^8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$= \frac{840x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^6b^2 - 840x^{\frac{2}{3}}a^6b^2 - 28x^{\frac{5}{3}}a^3b^5 + 5x^{\frac{8}{3}}b^8 + 1680x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^7b + 70x^{\frac{4}{3}}a^4b^4 - 8x}{10b^9\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)}$$

input
```
int(x^2/(a+b*x^(1/3))^3,x)
```

output
```
(840*x**(2/3)*log(x**(1/3)*b + a)*a**6*b**2 - 840*x**(2/3)*a**6*b**2 - 28*
x**(2/3)*a**3*b**5*x + 5*x**(2/3)*b**8*x**2 + 1680*x**(1/3)*log(x**(1/3)*b
 + a)*a**7*b + 70*x**(1/3)*a**4*b**4*x - 8*x**(1/3)*a*b**7*x**2 + 840*log(
x**(1/3)*b + a)*a**8 + 420*a**8 - 280*a**5*b**3*x + 14*a**2*b**6*x**2)/(10
*b**9*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2))
```

3.266 $\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx=\frac{3a^5}{2b^6\left(a+b\sqrt[3]{x}\right)^2}-\frac{15a^4}{b^6\left(a+b\sqrt[3]{x}\right)}+\frac{18a^2\sqrt[3]{x}}{b^5}-\frac{9ax^{2/3}}{2b^4}+\frac{x}{b^3}-\frac{30a^3\log\left(a+b\sqrt[3]{x}\right)}{b^6}$$

output
```
3/2*a^5/b^6/(a+b*x^(1/3))^2-15*a^4/b^6/(a+b*x^(1/3))+18*a^2*x^(1/3)/b^5-9/
2*a*x^(2/3)/b^4+x/b^3-30*a^3*ln(a+b*x^(1/3))/b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx=\frac{-27a^5+6a^4b\sqrt[3]{x}+63a^3b^2x^{2/3}+20a^2b^3x-5ab^4x^{4/3}+2b^5x^{5/3}}{2b^6\left(a+b\sqrt[3]{x}\right)^2}-\frac{30a^3\log\left(a+b\sqrt[3]{x}\right)}{b^6}$$

input
```
Integrate[x/(a + b*x^(1/3))^3,x]
```

output
```
(-27*a^5 + 6*a^4*b*x^(1/3) + 63*a^3*b^2*x^(2/3) + 20*a^2*b^3*x - 5*a*b^4*x
^(4/3) + 2*b^5*x^(5/3))/(2*b^6*(a + b*x^(1/3))^2) - (30*a^3*Log[a + b*x^(1
/3)])/b^6
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{5/3}}{\left(a+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{a^5}{b^5\left(a+b\sqrt[3]{x}\right)^3}+\frac{5a^4}{b^5\left(a+b\sqrt[3]{x}\right)^2}-\frac{10a^3}{b^5\left(a+b\sqrt[3]{x}\right)}+\frac{6a^2}{b^5}-\frac{3\sqrt[3]{x}a}{b^4}+\frac{x^{2/3}}{b^3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^5}{2b^6\left(a+b\sqrt[3]{x}\right)^2}-\frac{5a^4}{b^6\left(a+b\sqrt[3]{x}\right)}-\frac{10a^3\log\left(a+b\sqrt[3]{x}\right)}{b^6}+\frac{6a^2\sqrt[3]{x}}{b^5}-\frac{3ax^{2/3}}{2b^4}+\frac{x}{3b^3}\right)$$

input
```
Int[x/(a + b*x^(1/3))^3,x]
```

output
```
3*(a^5/(2*b^6*(a + b*x^(1/3))^2) - (5*a^4)/(b^6*(a + b*x^(1/3))) + (6*a^2*
x^(1/3))/b^5 - (3*a*x^(2/3))/(2*b^4) + x/(3*b^3) - (10*a^3*Log[a + b*x^(1/
3)])/b^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{b^2x-\frac{9x^{\frac{2}{3}}ab}{2}+18x^{\frac{1}{3}}a^2}{b^5}-\frac{15a^4}{b^6\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{3a^5}{2b^6\left(a+b\,x^{\frac{1}{3}}\right)^2}-\frac{30a^3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^6}$	79
default	$\frac{b^2x-\frac{9x^{\frac{2}{3}}ab}{2}+18x^{\frac{1}{3}}a^2}{b^5}-\frac{15a^4}{b^6\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{3a^5}{2b^6\left(a+b\,x^{\frac{1}{3}}\right)^2}-\frac{30a^3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^6}$	79

input
```
int(x/(a+b*x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
3/b^5*(1/3*b^2*x-3/2*x^(2/3)*a*b+6*x^(1/3)*a^2)-15*a^4/b^6/(a+b*x^(1/3))+3
/2*a^5/b^6/(a+b*x^(1/3))^2-30*a^3*ln(a+b*x^(1/3))/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(76) = 152$.

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{2\,b^9x^3+4\,a^3b^6x^2-34\,a^6b^3x-27\,a^9-60\,(a^3b^6x^2+2\,a^6b^3x+a^9)\log\left(bx^{\frac{1}{3}}+a\right)-3\,(3\,ab^8x^2+16\,a^4b^5x}{2\,(b^{12}x^2+2\,a^3b^9x+a^6b^6)}$$

input
```
integrate(x/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^9*x^3 + 4*a^3*b^6*x^2 - 34*a^6*b^3*x - 27*a^9 - 60*(a^3*b^6*x^2 +
 2*a^6*b^3*x + a^9)*log(b*x^(1/3) + a) - 3*(3*a*b^8*x^2 + 16*a^4*b^5*x + 1
0*a^7*b^2)*x^(2/3) + 3*(12*a^2*b^7*x^2 + 35*a^5*b^4*x + 20*a^8*b)*x^(1/3))
/(b^12*x^2 + 2*a^3*b^9*x + a^6*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(87) = 174$.

Time = 0.33 (sec) , antiderivative size = 362, normalized size of antiderivative = 4.02

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \begin{cases} -\frac{60a^5\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}} - \frac{90a^5}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}} - \frac{120a^4b\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}} - \frac{120a^4b\sqrt[3]{x}}{2a^2b^6+4ab^7\sqrt[3]{x}+2b^8x^{\frac{2}{3}}} - \frac{60a^3b^2x^{\frac{2}{3}}\log(}{2a^2b^6+4ab^7\sqrt[}} \\ \frac{x^2}{2a^3} \end{cases}$$

input
```
integrate(x/(a+b*x**(1/3))**3,x)
```

output
```
Piecewise((-60*a**5*log(a/b + x**(1/3))/(2*a**2*b**6 + 4*a*b**7*x**(1/3) +
 2*b**8*x**(2/3)) - 90*a**5/(2*a**2*b**6 + 4*a*b**7*x**(1/3) + 2*b**8*x**(
2/3)) - 120*a**4*b*x**(1/3)*log(a/b + x**(1/3))/(2*a**2*b**6 + 4*a*b**7*x*
*(1/3) + 2*b**8*x**(2/3)) - 120*a**4*b*x**(1/3)/(2*a**2*b**6 + 4*a*b**7*x*
*(1/3) + 2*b**8*x**(2/3)) - 60*a**3*b**2*x**(2/3)*log(a/b + x**(1/3))/(2*a
**2*b**6 + 4*a*b**7*x**(1/3) + 2*b**8*x**(2/3)) + 20*a**2*b**3*x/(2*a**2*b
**6 + 4*a*b**7*x**(1/3) + 2*b**8*x**(2/3)) - 5*a*b**4*x**(4/3)/(2*a**2*b**
6 + 4*a*b**7*x**(1/3) + 2*b**8*x**(2/3)) + 2*b**5*x**(5/3)/(2*a**2*b**6 +
4*a*b**7*x**(1/3) + 2*b**8*x**(2/3)), Ne(b, 0)), (x**2/(2*a**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = -\frac{30\,a^3\log\left(bx^{\frac{1}{3}}+a\right)}{b^6}+\frac{\left(bx^{\frac{1}{3}}+a\right)^3}{b^6}-\frac{15\left(bx^{\frac{1}{3}}+a\right)^2a}{2\,b^6}+\frac{30\left(bx^{\frac{1}{3}}+a\right)a^2}{b^6}-\frac{15\,a^4}{\left(bx^{\frac{1}{3}}+a\right)b^6}+\frac{3\,a^5}{2\left(bx^{\frac{1}{3}}+a\right)^2b^6}$$

input
```
integrate(x/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

output
```
-30*a^3*log(b*x^(1/3) + a)/b^6 + (b*x^(1/3) + a)^3/b^6 - 15/2*(b*x^(1/3) +
 a)^2*a/b^6 + 30*(b*x^(1/3) + a)*a^2/b^6 - 15*a^4/((b*x^(1/3) + a)*b^6) +
3/2*a^5/((b*x^(1/3) + a)^2*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = -\frac{30\,a^3\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^6}-\frac{3\left(10\,a^4bx^{\frac{1}{3}}+9\,a^5\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2b^6}+\frac{2\,b^6x-9\,ab^5x^{\frac{2}{3}}+36\,a^2b^4x^{\frac{1}{3}}}{2\,b^9}$$

input
```
integrate(x/(a+b*x^(1/3))^3,x, algorithm="giac")
```

output
```
-30*a^3*log(abs(b*x^(1/3) + a))/b^6 - 3/2*(10*a^4*b*x^(1/3) + 9*a^5)/((b*x
^(1/3) + a)^2*b^6) + 1/2*(2*b^6*x - 9*a*b^5*x^(2/3) + 36*a^2*b^4*x^(1/3))/
b^9
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx=\frac{x}{b^3}-\frac{\frac{27\,a^5}{2\,b}+15\,a^4\,x^{1/3}}{a^2\,b^5+b^7\,x^{2/3}+2\,a\,b^6\,x^{1/3}}-\frac{9\,a\,x^{2/3}}{2\,b^4}-\frac{30\,a^3\,\ln\left(a+b\,x^{1/3}\right)}{b^6}+\frac{18\,a^2\,x^{1/3}}{b^5}$$

input
```
int(x/(a + b*x^(1/3))^3,x)
```

output
```
x/b^3 - ((27*a^5)/(2*b) + 15*a^4*x^(1/3))/(a^2*b^5 + b^7*x^(2/3) + 2*a*b^6
*x^(1/3)) - (9*a*x^(2/3))/(2*b^4) - (30*a^3*log(a + b*x^(1/3)))/b^6 + (18*
a^2*x^(1/3))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

$$\int \frac{x}{\left(a+b\sqrt[3]{x}\right)^3}\,dx$$
$$=\frac{-60x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^3b^2+60x^{\frac{2}{3}}a^3b^2+2x^{\frac{5}{3}}b^5-120x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)a^4b-5x^{\frac{4}{3}}a\,b^4-60\log\left(x^{\frac{1}{3}}b+a\right)a}{2b^6\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)}$$

input
```
int(x/(a+b*x^(1/3))^3,x)
```

output
```
( - 60*x**(2/3)*log(x**(1/3)*b + a)*a**3*b**2 + 60*x**(2/3)*a**3*b**2 + 2*
x**(2/3)*b**5*x - 120*x**(1/3)*log(x**(1/3)*b + a)*a**4*b - 5*x**(1/3)*a*b
**4*x - 60*log(x**(1/3)*b + a)*a**5 - 30*a**5 + 20*a**2*b**3*x)/(2*b**6*(x
**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2))
```

3.267 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3} dx = -\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{6a}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{3\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

output
```
-3/2*a^2/b^3/(a+b*x^(1/3))^2+6*a/b^3/(a+b*x^(1/3))+3*ln(a+b*x^(1/3))/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3} dx = \frac{3a\left(3a+4b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{3\log\left(a+b\sqrt[3]{x}\right)}{b^3}$$

input
```
Integrate[(a + b*x^(1/3))^(-3),x]
```

output
```
(3*a*(3*a + 4*b*x^(1/3)))/(2*b^3*(a + b*x^(1/3))^2) + (3*Log[a + b*x^(1/3)
])/b^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{x^{2/3}}{\left(a+b\sqrt[3]{x}\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^2}{b^2\left(a+b\sqrt[3]{x}\right)^3}-\frac{2a}{b^2\left(a+b\sqrt[3]{x}\right)^2}+\frac{1}{b^2\left(a+b\sqrt[3]{x}\right)}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^2}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)}+\frac{\log\left(a+b\sqrt[3]{x}\right)}{b^3}\right)$$

input
```
Int[(a + b*x^(1/3))^(-3),x]
```

output
```
3*(-1/2*a^2/(b^3*(a + b*x^(1/3))^2) + (2*a)/(b^3*(a + b*x^(1/3))) + Log[a
+ b*x^(1/3)]/b^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{3a^2}{2b^3\left(a+b\,x^{\frac{1}{3}}\right)^2}+\frac{6a}{b^3\left(a+b\,x^{\frac{1}{3}}\right)}+\frac{3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{b^3}$
default	$-\frac{a^6}{(b^3x+a^3)^2b^3}+\frac{\ln(b^3x+a^3)}{b^3}+\frac{2a^3}{b^3(b^3x+a^3)}-7a^3b^3\left(\frac{a^3}{2b^6(b^3x+a^3)^2}-\frac{1}{(b^3x+a^3)b^6}\right)+6a^2b^4\left(-\frac{3abx}{(b\right.$

input
```
int(1/(a+b*x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
-3/2*a^2/b^3/(a+b*x^(1/3))^2+6*a/b^3/(a+b*x^(1/3))+3*ln(a+b*x^(1/3))/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(46) = 92.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.09

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{3\left(6\,a^3b^3x+3\,a^6+2\,(b^6x^2+2\,a^3b^3x+a^6)\log\left(bx^{\frac{1}{3}}+a\right)+(4\,ab^5x+a^4b^2)x^{\frac{2}{3}}-(5\,a^2b^4x+2\,a^5b)x^{\frac{1}{3}}\right)}{2\,(b^9x^2+2\,a^3b^6x+a^6b^3)}$$

input
```
integrate(1/(a+b*x^(1/3))^3,x, algorithm="fricas")
```

output
```
3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
 a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^
9*x^2 + 2*a^3*b^6*x + a^6*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(49) = 98.

Time = 0.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.22

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \begin{cases} \frac{6a^2\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}} + \frac{9a^2}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}} + \frac{12ab\sqrt[3]{x}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}} + \frac{12ab\sqrt[3]{x}}{2a^2b^3+4ab^4\sqrt[3]{x}+2b^5x^{\frac{2}{3}}} + \frac{6b^2x^{\frac{2}{3}}\log\left(\frac{a}{b}+\sqrt[3]{\right.}}{2a^2b^3+4ab^4\sqrt[3]{x}+\right.} \\ \frac{x}{a^3} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**3,x)
```

output
```
Piecewise((6*a**2*log(a/b + x**(1/3))/(2*a**2*b**3 + 4*a*b**4*x**(1/3) + 2
*b**5*x**(2/3)) + 9*a**2/(2*a**2*b**3 + 4*a*b**4*x**(1/3) + 2*b**5*x**(2/3
)) + 12*a*b*x**(1/3)*log(a/b + x**(1/3))/(2*a**2*b**3 + 4*a*b**4*x**(1/3)
+ 2*b**5*x**(2/3)) + 12*a*b*x**(1/3)/(2*a**2*b**3 + 4*a*b**4*x**(1/3) + 2*
b**5*x**(2/3)) + 6*b**2*x**(2/3)*log(a/b + x**(1/3))/(2*a**2*b**3 + 4*a*b*
*4*x**(1/3) + 2*b**5*x**(2/3)), Ne(b, 0)), (x/a**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{3\,\log\left(bx^{\frac{1}{3}}+a\right)}{b^3} + \frac{6\,a}{\left(bx^{\frac{1}{3}}+a\right)b^3} - \frac{3\,a^2}{2\left(bx^{\frac{1}{3}}+a\right)^2 b^3}$$

input
```
integrate(1/(a+b*x^(1/3))^3,x, algorithm="maxima")
```

output
```
3*log(b*x^(1/3) + a)/b^3 + 6*a/((b*x^(1/3) + a)*b^3) - 3/2*a^2/((b*x^(1/3)
 + a)^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3} + \frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^2}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2 b^2}$$

input
```
integrate(1/(a+b*x^(1/3))^3,x, algorithm="giac")
```

output
```
3*log(abs(b*x^(1/3) + a))/b^3 + 3/2*(4*a*x^(1/3) + 3*a^2/b)/((b*x^(1/3) +
a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx=\frac{\frac{9\,a^2}{2\,b^3}+\frac{6\,a\,x^{1/3}}{b^2}}{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}+\frac{3\,\ln\left(a+b\,x^{1/3}\right)}{b^3}$$

input
```
int(1/(a + b*x^(1/3))^3,x)
```

output
```
((9*a^2)/(2*b^3) + (6*a*x^(1/3))/b^2)/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))
+ (3*log(a + b*x^(1/3)))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3}\,dx = \frac{3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^2-3x^{\frac{2}{3}}b^2+6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)ab+3\log\left(x^{\frac{1}{3}}b+a\right)a^2+\frac{3a^2}{2}}{b^3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)}$$

input
```
int(1/(a+b*x^(1/3))^3,x)
```

output
```
(3*(2*x**(2/3)*log(x**(1/3)*b + a)*b**2 - 2*x**(2/3)*b**2 + 4*x**(1/3)*log
(x**(1/3)*b + a)*a*b + 2*log(x**(1/3)*b + a)*a**2 + a**2))/(2*b**3*(x**(2/
3)*b**2 + 2*x**(1/3)*a*b + a**2))
```

3.268 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x} \, dx=\frac{3}{2a\left(a+b\sqrt[3]{x}\right)^2}+\frac{3}{a^2\left(a+b\sqrt[3]{x}\right)}-\frac{3\log\left(a+b\sqrt[3]{x}\right)}{a^3}+\frac{\log(x)}{a^3}$$

output
```
3/2/a/(a+b*x^(1/3))^2+3/a^2/(a+b*x^(1/3))-3*ln(a+b*x^(1/3))/a^3+ln(x)/a^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x} \, dx=\frac{3\left(\frac{a\left(3a+2b\sqrt[3]{x}\right)}{\left(a+b\sqrt[3]{x}\right)^2}-2\log\left(a+b\sqrt[3]{x}\right)+\frac{2\log(x)}{3}\right)}{2a^3}$$

input
```
Integrate[1/((a + b*x^(1/3))^3*x),x]
```

output
```
(3*((a*(3*a + 2*b*x^(1/3)))/(a + b*x^(1/3))^2 - 2*Log[a + b*x^(1/3)] + (2*
Log[x])/3))/(2*a^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt[3]{x}\right)^3}dx$$

$$\Big\downarrow \text{ 798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 54}$$

$$3\int \left(-\frac{b}{a^3\left(a+b\sqrt[3]{x}\right)}-\frac{b}{a^2\left(a+b\sqrt[3]{x}\right)^2}-\frac{b}{a\left(a+b\sqrt[3]{x}\right)^3}+\frac{1}{a^3\sqrt[3]{x}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(-\frac{\log\left(a+b\sqrt[3]{x}\right)}{a^3}+\frac{\log\left(\sqrt[3]{x}\right)}{a^3}+\frac{1}{a^2\left(a+b\sqrt[3]{x}\right)}+\frac{1}{2a\left(a+b\sqrt[3]{x}\right)^2}\right)$$

input
```
Int[1/((a + b*x^(1/3))^3*x),x]
```

output
```
3*(1/(2*a*(a + b*x^(1/3))^2) + 1/(a^2*(a + b*x^(1/3))) - Log[a + b*x^(1/3)
]/a^3 + Log[x^(1/3)]/a^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{3}{2a\left(a+b\,x^{\frac{1}{3}}\right)^{2}}+\frac{3}{a^{2}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$	49
default	$\frac{3}{2a\left(a+b\,x^{\frac{1}{3}}\right)^{2}}+\frac{3}{a^{2}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{3\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$	49

input
```
int(1/(a+b*x^(1/3))^3/x,x,method=_RETURNVERBOSE)
```

output
```
3/2/a/(a+b*x^(1/3))^2+3/a^2/(a+b*x^(1/3))-3*ln(a+b*x^(1/3))/a^3+ln(x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(48) = 96$.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.30

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx = \frac{3\left(3\,a^6-2\,(b^6x^2+2\,a^3b^3x+a^6)\log\left(bx^{\frac{1}{3}}+a\right)+2\,(b^6x^2+2\,a^3b^3x+a^6)\log\left(x^{\frac{1}{3}}\right)+(2\,ab^5x+5\,a^4b^2)x^{\frac{2}{3}}\right.}{2\,(a^3b^6x^2+2\,a^6b^3x+a^9)}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x,x, algorithm="fricas")
```

output
```
3/2*(3*a^6 - 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) + a) + 2*(b^6*x
^2 + 2*a^3*b^3*x + a^6)*log(x^(1/3)) + (2*a*b^5*x + 5*a^4*b^2)*x^(2/3) - (
a^2*b^4*x + 4*a^5*b)*x^(1/3))/(a^3*b^6*x^2 + 2*a^6*b^3*x + a^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(49) = 98$.

Time = 0.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 6.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx = \begin{cases} \frac{\tilde{\infty}}{x} \\ \frac{\log(x)}{a^3} \\ -\frac{1}{b^3x} \\ \frac{2a^2x^{\frac{2}{3}}\log(x)}{2a^5x^{\frac{2}{3}}+4a^4bx+2a^3b^2x^{\frac{4}{3}}} - \frac{6a^2x^{\frac{2}{3}}\log\left(\frac{a}{b}+\sqrt[3]{x}\right)}{2a^5x^{\frac{2}{3}}+4a^4bx+2a^3b^2x^{\frac{4}{3}}} + \frac{9a^2x^{\frac{2}{3}}}{2a^5x^{\frac{2}{3}}+4a^4bx+2a^3b^2x^{\frac{4}{3}}} + \frac{4abx\log(x)}{2a^5x^{\frac{2}{3}}+4a^4bx+2a^3b^2x^{\frac{4}{3}}} - \frac{12abx\log\left(\frac{a}{b}+\sqrt[3]{x}\right.}{2a^5x^{\frac{2}{3}}+4a^4bx+2a^3b} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**3/x,x)
```

output
```
Piecewise((zoo/x, Eq(a, 0) & Eq(b, 0)), (log(x)/a**3, Eq(b, 0)), (-1/(b**3
*x), Eq(a, 0)), (2*a**2*x**(2/3)*log(x)/(2*a**5*x**(2/3) + 4*a**4*b*x + 2*
a**3*b**2*x**(4/3)) - 6*a**2*x**(2/3)*log(a/b + x**(1/3))/(2*a**5*x**(2/3)
 + 4*a**4*b*x + 2*a**3*b**2*x**(4/3)) + 9*a**2*x**(2/3)/(2*a**5*x**(2/3) +
 4*a**4*b*x + 2*a**3*b**2*x**(4/3)) + 4*a*b*x*log(x)/(2*a**5*x**(2/3) + 4*
a**4*b*x + 2*a**3*b**2*x**(4/3)) - 12*a*b*x*log(a/b + x**(1/3))/(2*a**5*x*
*(2/3) + 4*a**4*b*x + 2*a**3*b**2*x**(4/3)) + 6*a*b*x/(2*a**5*x**(2/3) + 4
*a**4*b*x + 2*a**3*b**2*x**(4/3)) + 2*b**2*x**(4/3)*log(x)/(2*a**5*x**(2/3
) + 4*a**4*b*x + 2*a**3*b**2*x**(4/3)) - 6*b**2*x**(4/3)*log(a/b + x**(1/3
))/(2*a**5*x**(2/3) + 4*a**4*b*x + 2*a**3*b**2*x**(4/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx = \frac{3\left(2\,bx^{\frac{1}{3}}+3\,a\right)}{2\left(a^2b^2x^{\frac{2}{3}}+2\,a^3bx^{\frac{1}{3}}+a^4\right)}-\frac{3\,\log\left(bx^{\frac{1}{3}}+a\right)}{a^3}+\frac{\log\left(x\right)}{a^3}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x,x, algorithm="maxima")
```

output
```
3/2*(2*b*x^(1/3) + 3*a)/(a^2*b^2*x^(2/3) + 2*a^3*b*x^(1/3) + a^4) - 3*log(
b*x^(1/3) + a)/a^3 + log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx = -\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^3}+\frac{\log\left(\left|x\right|\right)}{a^3}+\frac{3\left(2\,abx^{\frac{1}{3}}+3\,a^2\right)}{2\left(bx^{\frac{1}{3}}+a\right)^2a^3}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x,x, algorithm="giac")
```

output
```
-3*log(abs(b*x^(1/3) + a))/a^3 + log(abs(x))/a^3 + 3/2*(2*a*b*x^(1/3) + 3*
a^2)/((b*x^(1/3) + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx = \frac{\frac{9}{2\,a}+\frac{3\,b\,x^{1/3}}{a^2}}{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}} - \frac{6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^3}$$

input
```
int(1/(x*(a + b*x^(1/3))^3),x)
```

output
```
(9/(2*a) + (3*b*x^(1/3))/a^2)/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3)) - (6*ata
nh((2*b*x^(1/3))/a + 1))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.05

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x}\,dx = \frac{3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)b^2 - 3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^2 - \frac{3x^{\frac{2}{3}}b^2}{2} + 6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}\right)ab - 6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}b+a\right)ab + 3\log\left(x^{\frac{1}{3}}\right)a^2}{a^3\left(x^{\frac{2}{3}}b^2+2x^{\frac{1}{3}}ab+a^2\right)}$$

input
```
int(1/(a+b*x^(1/3))^3/x,x)
```

output
```
(3*(2*x**(2/3)*log(x**(1/3))*b**2 - 2*x**(2/3)*log(x**(1/3)*b + a)*b**2 -
x**(2/3)*b**2 + 4*x**(1/3)*log(x**(1/3))*a*b - 4*x**(1/3)*log(x**(1/3)*b +
 a)*a*b + 2*log(x**(1/3))*a**2 - 2*log(x**(1/3)*b + a)*a**2 + 2*a**2))/(2*
a**3*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2))
```

3.269 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx = -\frac{3b^3}{2a^4\left(a+b\sqrt[3]{x}\right)^2}-\frac{12b^3}{a^5\left(a+b\sqrt[3]{x}\right)}-\frac{1}{a^3x}+\frac{9b}{2a^4x^{2/3}} -\frac{18b^2}{a^5\sqrt[3]{x}}+\frac{30b^3\log\left(a+b\sqrt[3]{x}\right)}{a^6}-\frac{10b^3\log(x)}{a^6}$$

output
```
-3/2*b^3/a^4/(a+b*x^(1/3))^2-12*b^3/a^5/(a+b*x^(1/3))-1/a^3/x+9/2*b/a^4/x^
(2/3)-18*b^2/a^5/x^(1/3)+30*b^3*ln(a+b*x^(1/3))/a^6-10*b^3*ln(x)/a^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx = -\frac{\frac{a\left(2a^4-5a^3b\sqrt[3]{x}+20a^2b^2x^{2/3}+90ab^3x+60b^4x^{4/3}\right)}{\left(a+b\sqrt[3]{x}\right)^2 x}-60b^3\log\left(a+b\sqrt[3]{x}\right)+20b^3\log(x)}{2a^6}$$

input
```
Integrate[1/((a + b*x^(1/3))^3*x^2),x]
```

output
```
-1/2*((a*(2*a^4 - 5*a^3*b*x^(1/3) + 20*a^2*b^2*x^(2/3) + 90*a*b^3*x + 60*b
^4*x^(4/3)))/((a + b*x^(1/3))^2*x) - 60*b^3*Log[a + b*x^(1/3)] + 20*b^3*Lo
g[x])/a^6
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt[3]{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{10b^4}{a^6\left(a+b\sqrt[3]{x}\right)}+\frac{4b^4}{a^5\left(a+b\sqrt[3]{x}\right)^2}+\frac{b^4}{a^4\left(a+b\sqrt[3]{x}\right)^3}-\frac{10b^3}{a^6\sqrt[3]{x}}+\frac{6b^2}{a^5x^{2/3}}-\frac{3b}{a^4x}+\frac{1}{a^3x^{4/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{10b^3\log\left(a+b\sqrt[3]{x}\right)}{a^6}-\frac{10b^3\log\left(\sqrt[3]{x}\right)}{a^6}-\frac{4b^3}{a^5\left(a+b\sqrt[3]{x}\right)}-\frac{6b^2}{a^5\sqrt[3]{x}}-\frac{b^3}{2a^4\left(a+b\sqrt[3]{x}\right)^2}+\frac{3b}{2a^4x^{2/3}}-\frac{1}{3a^3x}\right)$$

input
```
Int[1/((a + b*x^(1/3))^3*x^2),x]
```

output
```
3*(-1/2*b^3/(a^4*(a + b*x^(1/3))^2) - (4*b^3)/(a^5*(a + b*x^(1/3))) - 1/(3
*a^3*x) + (3*b)/(2*a^4*x^(2/3)) - (6*b^2)/(a^5*x^(1/3)) + (10*b^3*Log[a +
b*x^(1/3)])/a^6 - (10*b^3*Log[x^(1/3)])/a^6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{3b^{3}}{2a^{4}\left(a+b\,x^{\frac{1}{3}}\right)^{2}}-\frac{12b^{3}}{a^{5}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{a^{3}x}+\frac{9b}{2a^{4}x^{\frac{2}{3}}}-\frac{18b^{2}}{a^{5}x^{\frac{1}{3}}}+\frac{30b^{3}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{6}}-\frac{10b^{3}\ln\left(x\right)}{a^{6}}$	90
default	$-\frac{3b^{3}}{2a^{4}\left(a+b\,x^{\frac{1}{3}}\right)^{2}}-\frac{12b^{3}}{a^{5}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{a^{3}x}+\frac{9b}{2a^{4}x^{\frac{2}{3}}}-\frac{18b^{2}}{a^{5}x^{\frac{1}{3}}}+\frac{30b^{3}\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^{6}}-\frac{10b^{3}\ln\left(x\right)}{a^{6}}$	90

input
```
int(1/(a+b*x^(1/3))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/2*b^3/a^4/(a+b*x^(1/3))^2-12*b^3/a^5/(a+b*x^(1/3))-1/a^3/x+9/2*b/a^4/x^
(2/3)-18*b^2/a^5/x^(1/3)+30*b^3*ln(a+b*x^(1/3))/a^6-10*b^3*ln(x)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(89) = 178$.

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx = -\frac{20\,a^3b^6x^2+31\,a^6b^3x+2\,a^9-60\,(b^9x^3+2\,a^3b^6x^2+a^6b^3x)\log\left(bx^{\frac{1}{3}}+a\right)+60\,(b^9x^3+2\,a^3b^6x^2+a^6b^3}{2\,(a^6b^6x^3+2\,a^9b^3x^2}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^2,x, algorithm="fricas")
```

output
```
-1/2*(20*a^3*b^6*x^2 + 31*a^6*b^3*x + 2*a^9 - 60*(b^9*x^3 + 2*a^3*b^6*x^2
+ a^6*b^3*x)*log(b*x^(1/3) + a) + 60*(b^9*x^3 + 2*a^3*b^6*x^2 + a^6*b^3*x)
*log(x^(1/3)) + 3*(20*a*b^8*x^2 + 35*a^4*b^5*x + 12*a^7*b^2)*x^(2/3) - 3*(
10*a^2*b^7*x^2 + 16*a^5*b^4*x + 3*a^8*b)*x^(1/3))/(a^6*b^6*x^3 + 2*a^9*b^3
*x^2 + a^12*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(100) = 200$.

Time = 1.52 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.45

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^2} \\ -\frac{1}{a^3x} \\ -\frac{1}{2b^3x^2} \\ -\frac{2a^5x^{\frac{2}{3}}}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}} + \frac{5a^4bx}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}} - \frac{20a^3b^2x^{\frac{4}{3}}}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}} - \frac{20a^2b^3x^{\frac{5}{3}}\log(x)}{2a^8x^{\frac{5}{3}}+4a^7bx^2+2a^6b^2x^{\frac{7}{3}}} + \frac{60a^2b^3x^{\frac{5}{3}}\log}{2a^8x^{\frac{5}{3}}+4a^7b} \end{cases}$$

input
```
integrate(1/(a+b*x**(1/3))**3/x**2,x)
```

output
```
Piecewise((zoo/x**2, Eq(a, 0) & Eq(b, 0)), (-1/(a**3*x), Eq(b, 0)), (-1/(2
*b**3*x**2), Eq(a, 0)), (-2*a**5*x**(2/3)/(2*a**8*x**(5/3) + 4*a**7*b*x**2
 + 2*a**6*b**2*x**(7/3)) + 5*a**4*b*x/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2
*a**6*b**2*x**(7/3)) - 20*a**3*b**2*x**(4/3)/(2*a**8*x**(5/3) + 4*a**7*b*x
**2 + 2*a**6*b**2*x**(7/3)) - 20*a**2*b**3*x**(5/3)*log(x)/(2*a**8*x**(5/3
) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)) + 60*a**2*b**3*x**(5/3)*log(a/b
+ x**(1/3))/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)) - 90*
a**2*b**3*x**(5/3)/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)
) - 40*a*b**4*x**2*log(x)/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x
**(7/3)) + 120*a*b**4*x**2*log(a/b + x**(1/3))/(2*a**8*x**(5/3) + 4*a**7*b
*x**2 + 2*a**6*b**2*x**(7/3)) - 60*a*b**4*x**2/(2*a**8*x**(5/3) + 4*a**7*b
*x**2 + 2*a**6*b**2*x**(7/3)) - 20*b**5*x**(7/3)*log(x)/(2*a**8*x**(5/3) +
 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)) + 60*b**5*x**(7/3)*log(a/b + x**(1/
3))/(2*a**8*x**(5/3) + 4*a**7*b*x**2 + 2*a**6*b**2*x**(7/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx = -\frac{60\,b^4x^{\frac{4}{3}} + 90\,ab^3x + 20\,a^2b^2x^{\frac{2}{3}} - 5\,a^3bx^{\frac{1}{3}} + 2\,a^4}{2\left(a^5b^2x^{\frac{5}{3}} + 2\,a^6bx^{\frac{4}{3}} + a^7x\right)} + \frac{30\,b^3\log\left(bx^{\frac{1}{3}} + a\right)}{a^6} - \frac{10\,b^3\log\left(x\right)}{a^6}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^2,x, algorithm="maxima")
```

output
```
-1/2*(60*b^4*x^(4/3) + 90*a*b^3*x + 20*a^2*b^2*x^(2/3) - 5*a^3*b*x^(1/3) +
 2*a^4)/(a^5*b^2*x^(5/3) + 2*a^6*b*x^(4/3) + a^7*x) + 30*b^3*log(b*x^(1/3)
 + a)/a^6 - 10*b^3*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx = \frac{30\,b^3\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^6} - \frac{10\,b^3\log\left(|x|\right)}{a^6} - \frac{60\,ab^4x^{\frac{4}{3}}+90\,a^2b^3x+20\,a^3b^2x^{\frac{2}{3}}-5\,a^4bx^{\frac{1}{3}}+2\,a^5}{2\left(bx^{\frac{1}{3}}+a\right)^2a^6x}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^2,x, algorithm="giac")
```

output
```
30*b^3*log(abs(b*x^(1/3) + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/2*(60*a*b^
4*x^(4/3) + 90*a^2*b^3*x + 20*a^3*b^2*x^(2/3) - 5*a^4*b*x^(1/3) + 2*a^5)/(
(b*x^(1/3) + a)^2*a^6*x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx = \frac{60\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^6} - \frac{\frac{1}{a}-\frac{5\,b\,x^{1/3}}{2\,a^2}+\frac{45\,b^3\,x}{a^4}+\frac{10\,b^2\,x^{2/3}}{a^3}+\frac{30\,b^4\,x^{4/3}}{a^5}}{a^2\,x+b^2\,x^{5/3}+2\,a\,b\,x^{4/3}}$$

input
```
int(1/(x^2*(a + b*x^(1/3))^3),x)
```

output
```
(60*b^3*atanh((2*b*x^(1/3))/a + 1))/a^6 - (1/a - (5*b*x^(1/3))/(2*a^2) + (
45*b^3*x)/a^4 + (10*b^2*x^(2/3))/a^3 + (30*b^4*x^(4/3))/a^5)/(a^2*x + b^2*
x^(5/3) + 2*a*b*x^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.54

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^2}\,dx$$

$$= \frac{-60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)b^5 + 60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^5 - 20x^{\frac{2}{3}}a^3b^2 + 30x^{\frac{5}{3}}b^5 - 120x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}\right)a\,b^4 + 120x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}b\right.}{2a^6x\left(x^{\frac{2}{3}}b^2 + 2x^{\frac{1}{3}}ab + a\right.}$$

input
```
int(1/(a+b*x^(1/3))^3/x^2,x)
```

output
```
( - 60*x**(2/3)*log(x**(1/3))*b**5*x + 60*x**(2/3)*log(x**(1/3)*b + a)*b**
5*x - 20*x**(2/3)*a**3*b**2 + 30*x**(2/3)*b**5*x - 120*x**(1/3)*log(x**(1/
3))*a*b**4*x + 120*x**(1/3)*log(x**(1/3)*b + a)*a*b**4*x + 5*x**(1/3)*a**4
*b - 60*log(x**(1/3))*a**2*b**3*x + 60*log(x**(1/3)*b + a)*a**2*b**3*x - 2
*a**5 - 60*a**2*b**3*x)/(2*a**6*x*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2))
```

3.270 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3} dx = \frac{3b^6}{2a^7\left(a+b\sqrt[3]{x}\right)^2}+\frac{21b^6}{a^8\left(a+b\sqrt[3]{x}\right)}-\frac{1}{2a^3x^2}+\frac{9b}{5a^4x^{5/3}}-\frac{9b^2}{2a^5x^{4/3}}+\frac{10b^3}{a^6x}-\frac{45b^4}{2a^7x^{2/3}}+\frac{63b^5}{a^8\sqrt[3]{x}}-\frac{84b^6\log\left(a+b\sqrt[3]{x}\right)}{a^9}+\frac{28b^6\log(x)}{a^9}$$

output
```
3/2*b^6/a^7/(a+b*x^(1/3))^2+21*b^6/a^8/(a+b*x^(1/3))-1/2/a^3/x^2+9/5*b/a^4
/x^(5/3)-9/2*b^2/a^5/x^(4/3)+10*b^3/a^6/x-45/2*b^4/a^7/x^(2/3)+63*b^5/a^8/
x^(1/3)-84*b^6*ln(a+b*x^(1/3))/a^9+28*b^6*ln(x)/a^9
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3} dx = \frac{\frac{a\left(-5a^7+8a^6b\sqrt[3]{x}-14a^5b^2x^{2/3}+28a^4b^3x-70a^3b^4x^{4/3}+280a^2b^5x^{5/3}+1260ab^6x^2+840b^7x^{7/3}\right)}{\left(a+b\sqrt[3]{x}\right)^2x^2}-840b^6\log\left(a+b\sqrt[3]{x}\right)+280b^6\text{l}}{10a^9}$$

input
```
Integrate[1/((a + b*x^(1/3))^3*x^3),x]
```

output
```
((a*(-5*a^7 + 8*a^6*b*x^(1/3) - 14*a^5*b^2*x^(2/3) + 28*a^4*b^3*x - 70*a^3
*b^4*x^(4/3) + 280*a^2*b^5*x^(5/3) + 1260*a*b^6*x^2 + 840*b^7*x^(7/3)))/((
a + b*x^(1/3))^2*x^2) - 840*b^6*Log[a + b*x^(1/3)] + 280*b^6*Log[x])/(10*a
^9)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sqrt[3]{x}\right)^3} \, dx$$

$$\Big\downarrow \ 798$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 54$$

$$3\int \left(-\frac{28b^7}{a^9\left(a+b\sqrt[3]{x}\right)}-\frac{7b^7}{a^8\left(a+b\sqrt[3]{x}\right)^2}-\frac{b^7}{a^7\left(a+b\sqrt[3]{x}\right)^3}+\frac{28b^6}{a^9\sqrt[3]{x}}-\frac{21b^5}{a^8x^{2/3}}+\frac{15b^4}{a^7x}-\frac{10b^3}{a^6x^{4/3}}+\frac{6b^2}{a^5x^{5/3}}-\frac{3b}{a^4x^2}\right.$$

$$\Big\downarrow \ 2009$$

$$3\left(-\frac{28b^6\log\left(a+b\sqrt[3]{x}\right)}{a^9}+\frac{28b^6\log\left(\sqrt[3]{x}\right)}{a^9}+\frac{7b^6}{a^8\left(a+b\sqrt[3]{x}\right)}+\frac{21b^5}{a^8\sqrt[3]{x}}+\frac{b^6}{2a^7\left(a+b\sqrt[3]{x}\right)^2}-\frac{15b^4}{2a^7x^{2/3}}+\frac{10b^3}{3a^6x}-\frac{3}{2a^5}\right.$$

input
```
Int[1/((a + b*x^(1/3))^3*x^3),x]
```

output
```
3*(b^6/(2*a^7*(a + b*x^(1/3))^2) + (7*b^6)/(a^8*(a + b*x^(1/3))) - 1/(6*a^
3*x^2) + (3*b)/(5*a^4*x^(5/3)) - (3*b^2)/(2*a^5*x^(4/3)) + (10*b^3)/(3*a^6
*x) - (15*b^4)/(2*a^7*x^(2/3)) + (21*b^5)/(a^8*x^(1/3)) - (28*b^6*Log[a +
b*x^(1/3)])/a^9 + (28*b^6*Log[x^(1/3)])/a^9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{3b^6}{2a^7\left(a+b\,x^{\frac{1}{3}}\right)^2}+\frac{21b^6}{a^8\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{2a^3x^2}+\frac{9b}{5a^4x^{\frac{5}{3}}}-\frac{9b^2}{2a^5x^{\frac{4}{3}}}+\frac{10b^3}{a^6x}-\frac{45b^4}{2a^7x^{\frac{2}{3}}}+\frac{63b^5}{a^8x^{\frac{1}{3}}}-\frac{84b^6\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^9}$
default	$\frac{3b^6}{2a^7\left(a+b\,x^{\frac{1}{3}}\right)^2}+\frac{21b^6}{a^8\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{2a^3x^2}+\frac{9b}{5a^4x^{\frac{5}{3}}}-\frac{9b^2}{2a^5x^{\frac{4}{3}}}+\frac{10b^3}{a^6x}-\frac{45b^4}{2a^7x^{\frac{2}{3}}}+\frac{63b^5}{a^8x^{\frac{1}{3}}}-\frac{84b^6\ln\left(a+b\,x^{\frac{1}{3}}\right)}{a^9}$

input
```
int(1/(a+b*x^(1/3))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
3/2*b^6/a^7/(a+b*x^(1/3))^2+21*b^6/a^8/(a+b*x^(1/3))-1/2/a^3/x^2+9/5*b/a^4
/x^(5/3)-9/2*b^2/a^5/x^(4/3)+10*b^3/a^6/x-45/2*b^4/a^7/x^(2/3)+63*b^5/a^8/
x^(1/3)-84*b^6*ln(a+b*x^(1/3))/a^9+28*b^6*ln(x)/a^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.58

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx$$
$$= \frac{280\,a^3b^9x^3 + 420\,a^6b^6x^2 + 90\,a^9b^3x - 5\,a^{12} - 840\,(b^{12}x^4 + 2\,a^3b^9x^3 + a^6b^6x^2)\log\left(bx^{\frac{1}{3}} + a\right) + 840\,(b^{12}x^4}{}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^3,x, algorithm="fricas")
```

output
```
1/10*(280*a^3*b^9*x^3 + 420*a^6*b^6*x^2 + 90*a^9*b^3*x - 5*a^12 - 840*(b^1
2*x^4 + 2*a^3*b^9*x^3 + a^6*b^6*x^2)*log(b*x^(1/3) + a) + 840*(b^12*x^4 +
2*a^3*b^9*x^3 + a^6*b^6*x^2)*log(x^(1/3)) + 15*(56*a*b^11*x^3 + 98*a^4*b^8
*x^2 + 36*a^7*b^5*x - 3*a^10*b^2)*x^(2/3) - 3*(140*a^2*b^10*x^3 + 224*a^5*
b^7*x^2 + 63*a^8*b^4*x - 6*a^11*b)*x^(1/3))/(a^9*b^6*x^4 + 2*a^12*b^3*x^3
+ a^15*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(144) = 288.

Time = 3.62 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/3))**3/x**3,x)
```

output
```
Piecewise((zoo/x**3, Eq(a, 0) & Eq(b, 0)), (-1/(2*a**3*x**2), Eq(b, 0)), (
-1/(3*b**3*x**3), Eq(a, 0)), (-5*a**8*x**(2/3)/(10*a**11*x**(8/3) + 20*a**
10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 8*a**7*b*x/(10*a**11*x**(8/3) + 20*a
**10*b*x**3 + 10*a**9*b**2*x**(10/3)) - 14*a**6*b**2*x**(4/3)/(10*a**11*x*
*(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 28*a**5*b**3*x**(5/3)
/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) - 70*a**4*
b**4*x**2/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) +
 280*a**3*b**5*x**(7/3)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**
2*x**(10/3)) + 280*a**2*b**6*x**(8/3)*log(x)/(10*a**11*x**(8/3) + 20*a**10
*b*x**3 + 10*a**9*b**2*x**(10/3)) - 840*a**2*b**6*x**(8/3)*log(a/b + x**(1
/3))/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 1260
*a**2*b**6*x**(8/3)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x*
*(10/3)) + 560*a*b**7*x**3*log(x)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 1
0*a**9*b**2*x**(10/3)) - 1680*a*b**7*x**3*log(a/b + x**(1/3))/(10*a**11*x*
*(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 840*a*b**7*x**3/(10*a
**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3)) + 280*b**8*x**(1
0/3)*log(x)/(10*a**11*x**(8/3) + 20*a**10*b*x**3 + 10*a**9*b**2*x**(10/3))
 - 840*b**8*x**(10/3)*log(a/b + x**(1/3))/(10*a**11*x**(8/3) + 20*a**10*b*
x**3 + 10*a**9*b**2*x**(10/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx = \frac{840\,b^7x^{\frac{7}{3}} + 1260\,ab^6x^2 + 280\,a^2b^5x^{\frac{5}{3}} - 70\,a^3b^4x^{\frac{4}{3}} + 28\,a^4b^3x - 14\,a^5b^2x^{\frac{2}{3}} + 8\,a^6bx^{\frac{1}{3}} - 5\,a^7}{10\left(a^8b^2x^{\frac{8}{3}} + 2\,a^9bx^{\frac{7}{3}} + a^{10}x^2\right)} - \frac{84\,b^6\log\left(bx^{\frac{1}{3}} + a\right)}{a^9} + \frac{28\,b^6\log\left(x\right)}{a^9}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^3,x, algorithm="maxima")
```

output
```
1/10*(840*b^7*x^(7/3) + 1260*a*b^6*x^2 + 280*a^2*b^5*x^(5/3) - 70*a^3*b^4*
x^(4/3) + 28*a^4*b^3*x - 14*a^5*b^2*x^(2/3) + 8*a^6*b*x^(1/3) - 5*a^7)/(a^
8*b^2*x^(8/3) + 2*a^9*b*x^(7/3) + a^10*x^2) - 84*b^6*log(b*x^(1/3) + a)/a^
9 + 28*b^6*log(x)/a^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx = -\frac{84\,b^6\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{a^9}+\frac{28\,b^6\log\left(|x|\right)}{a^9} + \frac{840\,ab^7x^{\frac{7}{3}}+1260\,a^2b^6x^2+280\,a^3b^5x^{\frac{5}{3}}-70\,a^4b^4x^{\frac{4}{3}}+28\,a^5b^3x-14\,a^6b^2x^{\frac{2}{3}}+8\,a^7bx^{\frac{1}{3}}-5\,a^8}{10\left(bx^{\frac{1}{3}}+a\right)^2a^9x^2}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^3,x, algorithm="giac")
```

output
```
-84*b^6*log(abs(b*x^(1/3) + a))/a^9 + 28*b^6*log(abs(x))/a^9 + 1/10*(840*a
*b^7*x^(7/3) + 1260*a^2*b^6*x^2 + 280*a^3*b^5*x^(5/3) - 70*a^4*b^4*x^(4/3)
 + 28*a^5*b^3*x - 14*a^6*b^2*x^(2/3) + 8*a^7*b*x^(1/3) - 5*a^8)/((b*x^(1/3
) + a)^2*a^9*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx = \frac{\frac{4\,b\,x^{1/3}}{5\,a^2}-\frac{1}{2\,a}+\frac{14\,b^3\,x}{5\,a^4}-\frac{7\,b^2\,x^{2/3}}{5\,a^3}+\frac{126\,b^6\,x^2}{a^7}-\frac{7\,b^4\,x^{4/3}}{a^5}+\frac{28\,b^5\,x^{5/3}}{a^6}+\frac{84\,b^7\,x^{7/3}}{a^8}}{a^2\,x^2+b^2\,x^{8/3}+2\,a\,b\,x^{7/3}} - \frac{168\,b^6\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^9}$$

input
```
int(1/(x^3*(a + b*x^(1/3))^3),x)
```

output
```
((4*b*x^(1/3))/(5*a^2) - 1/(2*a) + (14*b^3*x)/(5*a^4) - (7*b^2*x^(2/3))/(5
*a^3) + (126*b^6*x^2)/a^7 - (7*b^4*x^(4/3))/a^5 + (28*b^5*x^(5/3))/a^6 + (
84*b^7*x^(7/3))/a^8)/(a^2*x^2 + b^2*x^(8/3) + 2*a*b*x^(7/3)) - (168*b^6*at
anh((2*b*x^(1/3))/a + 1))/a^9
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^3}\,dx$$
$$= \frac{840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}\right)b^8 - 840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^8 - 14x^{\frac{2}{3}}a^6b^2 + 280x^{\frac{5}{3}}a^3b^5 - 420x^{\frac{8}{3}}b^8 + 1680x^{\frac{7}{3}}\log\left(x^{\frac{1}{3}}\right)a\,b^7 -}{10}$$

input
```
int(1/(a+b*x^(1/3))^3/x^3,x)
```

output
```
(840*x**(2/3)*log(x**(1/3))*b**8*x**2 - 840*x**(2/3)*log(x**(1/3)*b + a)*b
**8*x**2 - 14*x**(2/3)*a**6*b**2 + 280*x**(2/3)*a**3*b**5*x - 420*x**(2/3)
*b**8*x**2 + 1680*x**(1/3)*log(x**(1/3))*a*b**7*x**2 - 1680*x**(1/3)*log(x
**(1/3)*b + a)*a*b**7*x**2 + 8*x**(1/3)*a**7*b - 70*x**(1/3)*a**4*b**4*x +
 840*log(x**(1/3))*a**2*b**6*x**2 - 840*log(x**(1/3)*b + a)*a**2*b**6*x**2
 - 5*a**8 + 28*a**5*b**3*x + 840*a**2*b**6*x**2)/(10*a**9*x**2*(x**(2/3)*b
**2 + 2*x**(1/3)*a*b + a**2))
```

3.271 $\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 183

$$\begin{aligned}\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4} \, dx = &-\frac{3b^9}{2a^{10}\left(a+b\sqrt[3]{x}\right)^2}-\frac{30b^9}{a^{11}\left(a+b\sqrt[3]{x}\right)}-\frac{1}{3a^3x^3}+\frac{9b}{8a^4x^{8/3}}\\&-\frac{18b^2}{7a^5x^{7/3}}+\frac{5b^3}{a^6x^2}-\frac{9b^4}{a^7x^{5/3}}+\frac{63b^5}{4a^8x^{4/3}}-\frac{28b^6}{a^9x}+\frac{54b^7}{a^{10}x^{2/3}}\\&-\frac{135b^8}{a^{11}\sqrt[3]{x}}+\frac{165b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{12}}-\frac{55b^9\log(x)}{a^{12}}\end{aligned}$$

output
```
-3/2*b^9/a^10/(a+b*x^(1/3))^2-30*b^9/a^11/(a+b*x^(1/3))-1/3/a^3/x^3+9/8*b/
a^4/x^(8/3)-18/7*b^2/a^5/x^(7/3)+5*b^3/a^6/x^2-9*b^4/a^7/x^(5/3)+63/4*b^5/
a^8/x^(4/3)-28*b^6/a^9/x+54*b^7/a^10/x^(2/3)-135*b^8/a^11/x^(1/3)+165*b^9*
ln(a+b*x^(1/3))/a^12-55*b^9*ln(x)/a^12
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx = -\frac{\frac{a\left(56a^{10}-77a^9b\sqrt[3]{x}+110a^8b^2x^{2/3}-165a^7b^3x+264a^6b^4x^{4/3}-462a^5b^5x^{5/3}+924a^4b^6x^2-2310a^3b^7x^{7/3}+9240a^2b^8x^{8/3}+41580ab^9x^3+277}{\left(a+b\sqrt[3]{x}\right)^2x^3}}{168a^{12}}$$

input
```
Integrate[1/((a + b*x^(1/3))^3*x^4),x]
```

output
```
-1/168*((a*(56*a^10 - 77*a^9*b*x^(1/3) + 110*a^8*b^2*x^(2/3) - 165*a^7*b^3
*x + 264*a^6*b^4*x^(4/3) - 462*a^5*b^5*x^(5/3) + 924*a^4*b^6*x^2 - 2310*a^
3*b^7*x^(7/3) + 9240*a^2*b^8*x^(8/3) + 41580*a*b^9*x^3 + 27720*b^10*x^(10/
3)))/((a + b*x^(1/3))^2*x^3) - 27720*b^9*Log[a + b*x^(1/3)] + 9240*b^9*Log
[x])/a^12
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+b\sqrt[3]{x}\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{55b^{10}}{a^{12}\left(a+b\sqrt[3]{x}\right)}+\frac{10b^{10}}{a^{11}\left(a+b\sqrt[3]{x}\right)^2}+\frac{b^{10}}{a^{10}\left(a+b\sqrt[3]{x}\right)^3}-\frac{55b^9}{a^{12}\sqrt[3]{x}}+\frac{45b^8}{a^{11}x^{2/3}}-\frac{36b^7}{a^{10}x}+\frac{28b^6}{a^9x^{4/3}}-\frac{21b^5}{a^8x^{5/3}}+\frac{15}{a^7}\right.$$

$\big\downarrow$ 2009

$$3\left(\frac{55b^9\log\left(a+b\sqrt[3]{x}\right)}{a^{12}}-\frac{55b^9\log\left(\sqrt[3]{x}\right)}{a^{12}}-\frac{10b^9}{a^{11}\left(a+b\sqrt[3]{x}\right)}-\frac{45b^8}{a^{11}\sqrt[3]{x}}-\frac{b^9}{2a^{10}\left(a+b\sqrt[3]{x}\right)^2}+\frac{18b^7}{a^{10}x^{2/3}}-\frac{28b^6}{3a^9x}+\frac{2}{4a^8}\right.$$

input
```
Int[1/((a + b*x^(1/3))^3*x^4),x]
```

output
```
3*(-1/2*b^9/(a^10*(a + b*x^(1/3))^2) - (10*b^9)/(a^11*(a + b*x^(1/3))) - 1
/(9*a^3*x^3) + (3*b)/(8*a^4*x^(8/3)) - (6*b^2)/(7*a^5*x^(7/3)) + (5*b^3)/(
3*a^6*x^2) - (3*b^4)/(a^7*x^(5/3)) + (21*b^5)/(4*a^8*x^(4/3)) - (28*b^6)/(
3*a^9*x) + (18*b^7)/(a^10*x^(2/3)) - (45*b^8)/(a^11*x^(1/3)) + (55*b^9*Log
[a + b*x^(1/3)])/a^12 - (55*b^9*Log[x^(1/3)])/a^12)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{3b^9}{2a^{10}\left(a+b\,x^{\frac{1}{3}}\right)^2}-\frac{30b^9}{a^{11}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{3a^3x^3}+\frac{9b}{8a^4x^{\frac{8}{3}}}-\frac{18b^2}{7a^5x^{\frac{7}{3}}}+\frac{5b^3}{a^6x^2}-\frac{9b^4}{a^7x^{\frac{5}{3}}}+\frac{63b^5}{4a^8x^{\frac{4}{3}}}-\frac{28b^6}{a^9x}+$
default	$-\frac{3b^9}{2a^{10}\left(a+b\,x^{\frac{1}{3}}\right)^2}-\frac{30b^9}{a^{11}\left(a+b\,x^{\frac{1}{3}}\right)}-\frac{1}{3a^3x^3}+\frac{9b}{8a^4x^{\frac{8}{3}}}-\frac{18b^2}{7a^5x^{\frac{7}{3}}}+\frac{5b^3}{a^6x^2}-\frac{9b^4}{a^7x^{\frac{5}{3}}}+\frac{63b^5}{4a^8x^{\frac{4}{3}}}-\frac{28b^6}{a^9x}+$

input
```
int(1/(a+b*x^(1/3))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-3/2*b^9/a^10/(a+b*x^(1/3))^2-30*b^9/a^11/(a+b*x^(1/3))-1/3/a^3/x^3+9/8*b/
a^4/x^(8/3)-18/7*b^2/a^5/x^(7/3)+5*b^3/a^6/x^2-9*b^4/a^7/x^(5/3)+63/4*b^5/
a^8/x^(4/3)-28*b^6/a^9/x+54*b^7/a^10/x^(2/3)-135*b^8/a^11/x^(1/3)+165*b^9*
ln(a+b*x^(1/3))/a^12-55*b^9*ln(x)/a^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3x^4}\,dx =$$

$$-\frac{9240\,a^3b^{12}x^4+13860\,a^6b^9x^3+3080\,a^9b^6x^2-728\,a^{12}b^3x+56\,a^{15}-27720\,(b^{15}x^5+2\,a^3b^{12}x^4+a^6b^9x^3}{}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^4,x, algorithm="fricas")
```

output
```
-1/168*(9240*a^3*b^12*x^4 + 13860*a^6*b^9*x^3 + 3080*a^9*b^6*x^2 - 728*a^1
2*b^3*x + 56*a^15 - 27720*(b^15*x^5 + 2*a^3*b^12*x^4 + a^6*b^9*x^3)*log(b*
x^(1/3) + a) + 27720*(b^15*x^5 + 2*a^3*b^12*x^4 + a^6*b^9*x^3)*log(x^(1/3)
) + 18*(1540*a*b^14*x^4 + 2695*a^4*b^11*x^3 + 990*a^7*b^8*x^2 - 99*a^10*b^
5*x + 24*a^13*b^2)*x^(2/3) - 63*(220*a^2*b^13*x^4 + 352*a^5*b^10*x^3 + 99*
a^8*b^7*x^2 - 18*a^11*b^4*x + 3*a^14*b)*x^(1/3))/(a^12*b^6*x^5 + 2*a^15*b^
3*x^4 + a^18*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(184) = 368.

Time = 7.79 (sec) , antiderivative size = 847, normalized size of antiderivative = 4.63

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**(1/3))**3/x**4,x)
```

output
```
Piecewise((zoo/x**4, Eq(a, 0) & Eq(b, 0)), (-1/(3*a**3*x**3), Eq(b, 0)), (
-1/(4*b**3*x**4), Eq(a, 0)), (-56*a**11*x**(2/3)/(168*a**14*x**(11/3) + 33
6*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 77*a**10*b*x/(168*a**14*x**(1
1/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) - 110*a**9*b**2*x**(4/
3)/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 1
65*a**8*b**3*x**(5/3)/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*
b**2*x**(13/3)) - 264*a**7*b**4*x**2/(168*a**14*x**(11/3) + 336*a**13*b*x*
*4 + 168*a**12*b**2*x**(13/3)) + 462*a**6*b**5*x**(7/3)/(168*a**14*x**(11/
3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) - 924*a**5*b**6*x**(8/3)
/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 231
0*a**4*b**7*x**3/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*
x**(13/3)) - 9240*a**3*b**8*x**(10/3)/(168*a**14*x**(11/3) + 336*a**13*b*x
**4 + 168*a**12*b**2*x**(13/3)) - 9240*a**2*b**9*x**(11/3)*log(x)/(168*a**
14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) + 27720*a**2*b
**9*x**(11/3)*log(a/b + x**(1/3))/(168*a**14*x**(11/3) + 336*a**13*b*x**4
+ 168*a**12*b**2*x**(13/3)) - 41580*a**2*b**9*x**(11/3)/(168*a**14*x**(11/
3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) - 18480*a*b**10*x**4*log
(x)/(168*a**14*x**(11/3) + 336*a**13*b*x**4 + 168*a**12*b**2*x**(13/3)) +
55440*a*b**10*x**4*log(a/b + x**(1/3))/(168*a**14*x**(11/3) + 336*a**13*b*
x**4 + 168*a**12*b**2*x**(13/3)) - 27720*a*b**10*x**4/(168*a**14*x**(11...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx = -\frac{27720\,b^{10}x^{\frac{10}{3}} + 41580\,ab^9x^3 + 9240\,a^2b^8x^{\frac{8}{3}} - 2310\,a^3b^7x^{\frac{7}{3}} + 924\,a^4b^6x^2 - 462\,a^5b^5x^{\frac{5}{3}} + 264\,a^6b^4x^{\frac{4}{3}} - 1}{168\left(a^{11}b^2x^{\frac{11}{3}} + 2\,a^{12}bx^{\frac{10}{3}} + a^{13}x^3\right)} + \frac{165\,b^9\log\left(bx^{\frac{1}{3}} + a\right)}{a^{12}} - \frac{55\,b^9\log\left(x\right)}{a^{12}}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^4,x, algorithm="maxima")
```

output
```
-1/168*(27720*b^10*x^(10/3) + 41580*a*b^9*x^3 + 9240*a^2*b^8*x^(8/3) - 231
0*a^3*b^7*x^(7/3) + 924*a^4*b^6*x^2 - 462*a^5*b^5*x^(5/3) + 264*a^6*b^4*x^
(4/3) - 165*a^7*b^3*x + 110*a^8*b^2*x^(2/3) - 77*a^9*b*x^(1/3) + 56*a^10)/
(a^11*b^2*x^(11/3) + 2*a^12*b*x^(10/3) + a^13*x^3) + 165*b^9*log(b*x^(1/3)
 + a)/a^12 - 55*b^9*log(x)/a^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx = \frac{165\,b^9\log\left(\left|bx^{\frac{1}{3}} + a\right|\right)}{a^{12}} - \frac{55\,b^9\log\left(|x|\right)}{a^{12}} - \frac{27720\,ab^{10}x^{\frac{10}{3}} + 41580\,a^2b^9x^3 + 9240\,a^3b^8x^{\frac{8}{3}} - 2310\,a^4b^7x^{\frac{7}{3}} + 924\,a^5b^6x^2 - 462\,a^6b^5x^{\frac{5}{3}} + 264\,a^7b^4x^{\frac{4}{3}} -}{168\left(bx^{\frac{1}{3}} + a\right)^2 a^{12}x^3}$$

input
```
integrate(1/(a+b*x^(1/3))^3/x^4,x, algorithm="giac")
```

output
```
165*b^9*log(abs(b*x^(1/3) + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/168*(27
720*a*b^10*x^(10/3) + 41580*a^2*b^9*x^3 + 9240*a^3*b^8*x^(8/3) - 2310*a^4*
b^7*x^(7/3) + 924*a^5*b^6*x^2 - 462*a^6*b^5*x^(5/3) + 264*a^7*b^4*x^(4/3)
- 165*a^8*b^3*x + 110*a^9*b^2*x^(2/3) - 77*a^10*b*x^(1/3) + 56*a^11)/((b*x
^(1/3) + a)^2*a^12*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx = \frac{330\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x^{1/3}}{a}+1\right)}{a^{12}} - \frac{\frac{1}{3\,a}-\frac{11\,b\,x^{1/3}}{24\,a^2}-\frac{55\,b^3\,x}{56\,a^4}+\frac{55\,b^2\,x^{2/3}}{84\,a^3}+\frac{11\,b^6\,x^2}{2\,a^7}+\frac{11\,b^4\,x^{4/3}}{7\,a^5}-\frac{11\,b^5\,x^{5/3}}{4\,a^6}+\frac{495\,b^9\,x^3}{2\,a^{10}}-\frac{55\,b^7\,x^{7/3}}{4\,a^8}+\frac{55\,b^8\,x^{8/3}}{a^9}+\frac{165\,b^{10}\,x^{10/3}}{a^{11}}}{a^2\,x^3+b^2\,x^{11/3}+2\,a\,b\,x^{10/3}}$$

input
```
int(1/(x^4*(a + b*x^(1/3))^3),x)
```

output
```
(330*b^9*atanh((2*b*x^(1/3))/a + 1))/a^12 - (1/(3*a) - (11*b*x^(1/3))/(24*
a^2) - (55*b^3*x)/(56*a^4) + (55*b^2*x^(2/3))/(84*a^3) + (11*b^6*x^2)/(2*a
^7) + (11*b^4*x^(4/3))/(7*a^5) - (11*b^5*x^(5/3))/(4*a^6) + (495*b^9*x^3)/
(2*a^10) - (55*b^7*x^(7/3))/(4*a^8) + (55*b^8*x^(8/3))/a^9 + (165*b^10*x^(
10/3))/a^11)/(a^2*x^3 + b^2*x^(11/3) + 2*a*b*x^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.25

$$\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^3 x^4}\,dx = \frac{-27720x^{\frac{11}{3}}\log\left(x^{\frac{1}{3}}\right)b^{11}+27720x^{\frac{11}{3}}\log\left(x^{\frac{1}{3}}b+a\right)b^{11}-110x^{\frac{2}{3}}a^9b^2+462x^{\frac{5}{3}}a^6b^5-9240x^{\frac{8}{3}}a^3b^8+13860x}{}$$

input
```
int(1/(a+b*x^(1/3))^3/x^4,x)
```

output
```
( - 27720*x**(2/3)*log(x**(1/3))*b**11*x**3 + 27720*x**(2/3)*log(x**(1/3)*
b + a)*b**11*x**3 - 110*x**(2/3)*a**9*b**2 + 462*x**(2/3)*a**6*b**5*x - 92
40*x**(2/3)*a**3*b**8*x**2 + 13860*x**(2/3)*b**11*x**3 - 55440*x**(1/3)*lo
g(x**(1/3))*a*b**10*x**3 + 55440*x**(1/3)*log(x**(1/3)*b + a)*a*b**10*x**3
 + 77*x**(1/3)*a**10*b - 264*x**(1/3)*a**7*b**4*x + 2310*x**(1/3)*a**4*b**
7*x**2 - 27720*log(x**(1/3))*a**2*b**9*x**3 + 27720*log(x**(1/3)*b + a)*a*
*2*b**9*x**3 - 56*a**11 + 165*a**8*b**3*x - 924*a**5*b**6*x**2 - 27720*a**
2*b**9*x**3)/(168*a**12*x**3*(x**(2/3)*b**2 + 2*x**(1/3)*a*b + a**2))
```

3.272 $\int \frac{1}{\sqrt{1+\sqrt[3]{x}}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}} \, dx = 6\sqrt{1+\sqrt[3]{x}} - 4\left(1+\sqrt[3]{x}\right)^{3/2} + \frac{6}{5}\left(1+\sqrt[3]{x}\right)^{5/2}$$

output
```
6*(1+x^(1/3))^(1/2)-4*(1+x^(1/3))^(3/2)+6/5*(1+x^(1/3))^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}} \, dx = \frac{2}{5}\sqrt{1+\sqrt[3]{x}}\left(8-4\sqrt[3]{x}+3x^{2/3}\right)$$

input
```
Integrate[1/Sqrt[1 + x^(1/3)],x]
```

output
```
(2*Sqrt[1 + x^(1/3)]*(8 - 4*x^(1/3) + 3*x^(2/3)))/5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt[3]{x}+1}}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{x^{2/3}}{\sqrt{\sqrt[3]{x}+1}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\left(\sqrt[3]{x}+1\right)^{3/2}-2\sqrt{\sqrt[3]{x}+1}+\frac{1}{\sqrt{\sqrt[3]{x}+1}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2}{5}\left(\sqrt[3]{x}+1\right)^{5/2}-\frac{4}{3}\left(\sqrt[3]{x}+1\right)^{3/2}+2\sqrt{\sqrt[3]{x}+1}\right)$$

input
```
Int[1/Sqrt[1 + x^(1/3)],x]
```

output
```
3*(2*Sqrt[1 + x^(1/3)] - (4*(1 + x^(1/3))^(3/2))/3 + (2*(1 + x^(1/3))^(5/2
))/5)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$6\sqrt{x^{\frac{1}{3}}+1}-4\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}}+\frac{6\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}}}{5}$	29
default	$6\sqrt{x^{\frac{1}{3}}+1}-4\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}}+\frac{6\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}}}{5}$	29
meijerg	$\frac{-\frac{16\sqrt{\pi}}{5}+\frac{\sqrt{\pi}\left(6x^{\frac{2}{3}}-8x^{\frac{1}{3}}+16\right)\sqrt{x^{\frac{1}{3}}+1}}{5}}{\sqrt{\pi}}$	36

input
```
int(1/(x^(1/3)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6*(x^(1/3)+1)^(1/2)-4*(x^(1/3)+1)^(3/2)+6/5*(x^(1/3)+1)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx=\frac{2}{5}\left(3\,x^{\frac{2}{3}}-4\,x^{\frac{1}{3}}+8\right)\sqrt{x^{\frac{1}{3}}+1}$$

input
```
integrate(1/(1+x^(1/3))^(1/2),x, algorithm="fricas")
```

output
```
2/5*(3*x^(2/3) - 4*x^(1/3) + 8)*sqrt(x^(1/3) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(36) = 72$.

Time = 0.86 (sec) , antiderivative size = 359, normalized size of antiderivative = 8.55

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx = \frac{6x^{\frac{14}{3}}\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} + \frac{10x^{\frac{13}{3}}\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} + \frac{30x^{\frac{11}{3}}\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} - \frac{48x^{\frac{11}{3}}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} + \frac{40x^{\frac{10}{3}}\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} - \frac{48x^{\frac{10}{3}}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} + \frac{10x^4\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} - \frac{16x^4}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} + \frac{16x^3\sqrt{\sqrt[3]{x}+1}}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3} - \frac{16x^3}{15x^{\frac{11}{3}}+15x^{\frac{10}{3}}+5x^4+5x^3}$$

input
```
integrate(1/(1+x**(1/3))**(1/2),x)
```

output
```
6*x**(14/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x
**3) + 10*x**(13/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x*
*4 + 5*x**3) + 30*x**(11/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3
) + 5*x**4 + 5*x**3) - 48*x**(11/3)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4
+ 5*x**3) + 40*x**(10/3)*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) +
 5*x**4 + 5*x**3) - 48*x**(10/3)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5
*x**3) + 10*x**4*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4
+ 5*x**3) - 16*x**4/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**3) + 16*x
**3*sqrt(x**(1/3) + 1)/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**3) - 1
6*x**3/(15*x**(11/3) + 15*x**(10/3) + 5*x**4 + 5*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx = \frac{6}{5}\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}} - 4\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}} + 6\sqrt{x^{\frac{1}{3}}+1}$$

input
```
integrate(1/(1+x^(1/3))^(1/2),x, algorithm="maxima")
```

output
```
6/5*(x^(1/3) + 1)^(5/2) - 4*(x^(1/3) + 1)^(3/2) + 6*sqrt(x^(1/3) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx = \frac{6}{5}\left(x^{\frac{1}{3}}+1\right)^{\frac{5}{2}} - 4\left(x^{\frac{1}{3}}+1\right)^{\frac{3}{2}} + 6\sqrt{x^{\frac{1}{3}}+1}$$

input
```
integrate(1/(1+x^(1/3))^(1/2),x, algorithm="giac")
```

output
```
6/5*(x^(1/3) + 1)^(5/2) - 4*(x^(1/3) + 1)^(3/2) + 6*sqrt(x^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ 4;\ -x^{1/3}\right)$$

input
```
int(1/(x^(1/3) + 1)^(1/2),x)
```

output
```
x*hypergeom([1/2, 3], 4, -x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx = \frac{2\sqrt{x^{\frac{1}{3}}+1}\left(3x^{\frac{2}{3}}-4x^{\frac{1}{3}}+8\right)}{5}$$

input

```
int(1/(1+x^(1/3))^(1/2),x)
```

output

```
(2*sqrt(x**(1/3) + 1)*(3*x**(2/3) - 4*x**(1/3) + 8))/5
```

3.273 $\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx=-\frac{2}{\sqrt{x}}+\frac{6}{\sqrt[6]{x}}+6\arctan\left(\sqrt[6]{x}\right)$$

output
```
-2/x^(1/2)+6/x^(1/6)+6*arctan(x^(1/6))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx=\frac{2\left(-1+3\sqrt[3]{x}\right)}{\sqrt{x}}+6\arctan\left(\sqrt[6]{x}\right)$$

input
```
Integrate[1/((1 + x^(1/3))*x^(3/2)),x]
```

output
```
(2*(-1 + 3*x^(1/3)))/Sqrt[x] + 6*ArcTan[x^(1/6)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {864, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt[3]{x}+1\right) x^{3/2}}dx$$
$$\Big\downarrow \text{864}$$
$$3 \int \frac{1}{\left(\sqrt[3]{x}+1\right) x^{5/6}}d\sqrt[3]{x}$$
$$\Big\downarrow \text{61}$$
$$3 \left(-\int \frac{1}{\left(\sqrt[3]{x}+1\right) \sqrt{x}}d\sqrt[3]{x}-\frac{2}{3 \sqrt{x}}\right)$$
$$\Big\downarrow \text{61}$$
$$3 \left(\int \frac{1}{\left(\sqrt[3]{x}+1\right) \sqrt[6]{x}}d\sqrt[3]{x}+\frac{2}{\sqrt[6]{x}}-\frac{2}{3 \sqrt{x}}\right)$$
$$\Big\downarrow \text{73}$$
$$3 \left(2 \int \frac{1}{x^{2/3}+1}d\sqrt[6]{x}+\frac{2}{\sqrt[6]{x}}-\frac{2}{3 \sqrt{x}}\right)$$
$$\Big\downarrow \text{216}$$
$$3 \left(2 \arctan \left(\sqrt[6]{x}\right)+\frac{2}{\sqrt[6]{x}}-\frac{2}{3 \sqrt{x}}\right)$$

input
```
Int[1/((1 + x^(1/3))*x^(3/2)),x]
```

output
```
3*(-2/(3*Sqrt[x]) + 2/x^(1/6) + 2*ArcTan[x^(1/6)])
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2}{\sqrt{x}}+\frac{6}{x^{\frac{1}{6}}}+6\arctan\left(x^{\frac{1}{6}}\right)$	18
default	$-\frac{2}{\sqrt{x}}+\frac{6}{x^{\frac{1}{6}}}+6\arctan\left(x^{\frac{1}{6}}\right)$	18
meijerg	$-\frac{2}{\sqrt{x}}+\frac{6}{x^{\frac{1}{6}}}+6\arctan\left(x^{\frac{1}{6}}\right)$	18

input
```
int(1/(x^(1/3)+1)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x^(1/2)+6/x^(1/6)+6*arctan(x^(1/6))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right) x^{3/2}}\, dx = \frac{2 \left(3\, x \arctan\left(x^{\frac{1}{6}}\right) + 3\, x^{\frac{5}{6}} - \sqrt{x}\right)}{x}$$

input
```
integrate(1/(1+x^(1/3))/x^(3/2),x, algorithm="fricas")
```

output
```
2*(3*x*arctan(x^(1/6)) + 3*x^(5/6) - sqrt(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right) x^{3/2}}\, dx = 6 \operatorname{atan}\left(\sqrt[6]{x}\right) - \frac{2}{\sqrt{x}} + \frac{6}{\sqrt[6]{x}}$$

input
```
integrate(1/(1+x**(1/3))/x**(3/2),x)
```

output
```
6*atan(x**(1/6)) - 2/sqrt(x) + 6/x**(1/6)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right) x^{3/2}}\, dx = \frac{2 \left(3\, x^{\frac{1}{3}} - 1\right)}{\sqrt{x}} + 6 \arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(1/(1+x^(1/3))/x^(3/2),x, algorithm="maxima")
```

output
```
2*(3*x^(1/3) - 1)/sqrt(x) + 6*arctan(x^(1/6))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx = \frac{2\left(3\,x^{\frac{1}{3}}-1\right)}{\sqrt{x}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(1/(1+x^(1/3))/x^(3/2),x, algorithm="giac")
```

output
```
2*(3*x^(1/3) - 1)/sqrt(x) + 6*arctan(x^(1/6))
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx = 6\,\mathrm{atan}\left(x^{1/6}\right) + \frac{6\,x^{1/3}-2}{\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(x^(1/3) + 1)),x)
```

output
```
6*atan(x^(1/6)) + (6*x^(1/3) - 2)/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(1+\sqrt[3]{x}\right)x^{3/2}}\,dx = \frac{6\sqrt{x}\,atan\left(x^{\frac{1}{6}}\right)+6x^{\frac{1}{3}}-2}{\sqrt{x}}$$

input
```
int(1/(1+x^(1/3))/x^(3/2),x)
```

output
```
(2*(3*sqrt(x)*atan(x**(1/6)) + 3*x**(1/3) - 1))/sqrt(x)
```

3.274 $\int \frac{x^{2/3}}{1+\sqrt[3]{x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}} dx = -3\sqrt[3]{x} + \frac{3x^{2/3}}{2} - x + \frac{3x^{4/3}}{4} + 3\log\left(1+\sqrt[3]{x}\right)$$

output
```
-3*x^(1/3)+3/2*x^(2/3)-x+3/4*x^(4/3)+3*ln(1+x^(1/3))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}} dx = \frac{1}{4}\sqrt[3]{x}\left(-12+6\sqrt[3]{x}-4x^{2/3}+3x\right) + 3\log\left(1+\sqrt[3]{x}\right)$$

input
```
Integrate[x^(2/3)/(1 + x^(1/3)),x]
```

output
```
(x^(1/3)*(-12 + 6*x^(1/3) - 4*x^(2/3) + 3*x))/4 + 3*Log[1 + x^(1/3)]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2/3}}{\sqrt[3]{x}+1}dx$$
$$\Big\downarrow \text{798}$$
$$3\int \frac{x^{4/3}}{\sqrt[3]{x}+1}d\sqrt[3]{x}$$
$$\Big\downarrow \text{49}$$
$$3\int \left(x-x^{2/3}+\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}+1}-1\right)d\sqrt[3]{x}$$
$$\Big\downarrow \text{2009}$$
$$3\left(\frac{x^{4/3}}{4}+\frac{x^{2/3}}{2}-\frac{x}{3}-\sqrt[3]{x}+\log\left(\sqrt[3]{x}+1\right)\right)$$

input
```
Int[x^(2/3)/(1 + x^(1/3)),x]
```

output
```
3*(-x^(1/3) + x^(2/3)/2 - x/3 + x^(4/3)/4 + Log[1 + x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$-3x^{\frac{1}{3}}+\frac{3x^{\frac{2}{3}}}{2}-x+\frac{3x^{\frac{4}{3}}}{4}+3\ln\left(x^{\frac{1}{3}}+1\right)$	28
default	$-3x^{\frac{1}{3}}+\frac{3x^{\frac{2}{3}}}{2}-x+\frac{3x^{\frac{4}{3}}}{4}+3\ln\left(x^{\frac{1}{3}}+1\right)$	28
meijerg	$-\frac{x^{\frac{1}{3}}\left(-15x+20x^{\frac{2}{3}}-30x^{\frac{1}{3}}+60\right)}{20}+3\ln\left(x^{\frac{1}{3}}+1\right)$	30
trager	$1-x+\left(-3+\frac{3x}{4}\right)x^{\frac{1}{3}}+\frac{3x^{\frac{2}{3}}}{2}+\ln\left(-3x^{\frac{2}{3}}-3x^{\frac{1}{3}}-x-1\right)$	36

input
```
int(x^(2/3)/(x^(1/3)+1),x,method=_RETURNVERBOSE)
```

output
```
-3*x^(1/3)+3/2*x^(2/3)-x+3/4*x^(4/3)+3*ln(x^(1/3)+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx=\frac{3}{4}\,(x-4)x^{\frac{1}{3}}-x+\frac{3}{2}\,x^{\frac{2}{3}}+3\,\log\left(x^{\frac{1}{3}}+1\right)$$

input
```
integrate(x^(2/3)/(1+x^(1/3)),x, algorithm="fricas")
```

output
```
3/4*(x - 4)*x^(1/3) - x + 3/2*x^(2/3) + 3*log(x^(1/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx = \frac{3x^{\frac{4}{3}}}{4} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} - x + 3\log\left(\sqrt[3]{x}+1\right)$$

input
```
integrate(x**(2/3)/(1+x**(1/3)),x)
```

output
```
3*x**(4/3)/4 + 3*x**(2/3)/2 - 3*x**(1/3) - x + 3*log(x**(1/3) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx = \frac{3}{4}\left(x^{\frac{1}{3}}+1\right)^{4} - 4\left(x^{\frac{1}{3}}+1\right)^{3} + 9\left(x^{\frac{1}{3}}+1\right)^{2} - 12\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}}+1\right) - 12$$

input
```
integrate(x^(2/3)/(1+x^(1/3)),x, algorithm="maxima")
```

output
```
3/4*(x^(1/3) + 1)^4 - 4*(x^(1/3) + 1)^3 + 9*(x^(1/3) + 1)^2 - 12*x^(1/3) +
 3*log(x^(1/3) + 1) - 12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx = \frac{3}{4}\,x^{\frac{4}{3}} - x + \frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}}+1\right)$$

input
```
integrate(x^(2/3)/(1+x^(1/3)),x, algorithm="giac")
```

output
```
3/4*x^(4/3) - x + 3/2*x^(2/3) - 3*x^(1/3) + 3*log(x^(1/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx = 3\,\ln\left(x^{1/3}+1\right) - x - 3\,x^{1/3} + \frac{3\,x^{2/3}}{2} + \frac{3\,x^{4/3}}{4}$$

input
```
int(x^(2/3)/(x^(1/3) + 1),x)
```

output
```
3*log(x^(1/3) + 1) - x - 3*x^(1/3) + (3*x^(2/3))/2 + (3*x^(4/3))/4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x^{2/3}}{1+\sqrt[3]{x}}\,dx = \frac{3x^{\frac{2}{3}}}{2} + \frac{3x^{\frac{4}{3}}}{4} - 3x^{\frac{1}{3}} + 3\log\left(x^{\frac{1}{3}}+1\right) - x$$

input
```
int(x^(2/3)/(1+x^(1/3)),x)
```

output
```
(6*x**(2/3) + 3*x**(1/3)*x - 12*x**(1/3) + 12*log(x**(1/3) + 1) - 4*x)/4
```

3.275 $\int \frac{1}{1+x^{2/3}} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{1}{1+x^{2/3}} \, dx = 3\sqrt[3]{x} - 3\arctan\left(\sqrt[3]{x}\right)$$

output
```
3*x^(1/3)-3*arctan(x^(1/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+x^{2/3}} \, dx = 3\sqrt[3]{x} - 3\arctan\left(\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + x^(2/3))^(-1),x]
```

output
```
3*x^(1/3) - 3*ArcTan[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {774, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{2/3}+1}dx$$
$$\Big\downarrow \text{774}$$
$$3\int \frac{x^{2/3}}{x^{2/3}+1}d\sqrt[3]{x}$$
$$\Big\downarrow \text{262}$$
$$3\left(\sqrt[3]{x}-\int \frac{1}{x^{2/3}+1}d\sqrt[3]{x}\right)$$
$$\Big\downarrow \text{216}$$
$$3\left(\sqrt[3]{x}-\arctan\left(\sqrt[3]{x}\right)\right)$$

input
```
Int[(1 + x^(2/3))^(-1),x]
```

output
```
3*(x^(1/3) - ArcTan[x^(1/3)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result
derivativedivides	$3x^{\frac{1}{3}}-3\arctan\left(x^{\frac{1}{3}}\right)$
meijerg	$3x^{\frac{1}{3}}-3\arctan\left(x^{\frac{1}{3}}\right)$
default	$\arctan\left(x\right)+3x^{\frac{1}{3}}-\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)-\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)-2\arctan\left(x^{\frac{1}{3}}\right)$
trager	$3x^{\frac{1}{3}}+\frac{3\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)^{2}x+4\,\mathrm{RootOf}\left(\_Z^{2}+1\right)x^{\frac{2}{3}}-\mathrm{RootOf}\left(\_Z^{2}+1\right)^{2}-2\,\mathrm{RootOf}\left(\_Z\right.}{\mathrm{RootOf}\left(\_Z^{2}+1\right)x-\mathrm{RootOf}\left(\_Z^{2}+1\right)+x+1}\right.}{2}$

input
```
int(1/(1+x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)-3*arctan(x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} - 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(1/(1+x^(2/3)),x, algorithm="fricas")
```

output
```
3*x^(1/3) - 3*arctan(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+x^{2/3}}\,dx = 3\sqrt[3]{x} - 3\,\mathrm{atan}\left(\sqrt[3]{x}\right)$$

input
```
integrate(1/(1+x**(2/3)),x)
```

output
```
3*x**(1/3) - 3*atan(x**(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} - 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(1/(1+x^(2/3)),x, algorithm="maxima")
```

output
```
3*x^(1/3) - 3*arctan(x^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} - 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(1/(1+x^(2/3)),x, algorithm="giac")
```

output
```
3*x^(1/3) - 3*arctan(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^{2/3}}\,dx = 3\,x^{1/3} - 3\,\mathrm{atan}\left(x^{1/3}\right)$$

input
```
int(1/(x^(2/3) + 1),x)
```

output
```
3*x^(1/3) - 3*atan(x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x^{2/3}}\,dx = -3atan\left(x^{\frac{1}{3}}\right) + 3x^{\frac{1}{3}}$$

input
```
int(1/(1+x^(2/3)),x)
```

output
```
3*( - atan(x**(1/3)) + x**(1/3))
```

3.276 $\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx=\frac{3}{2}\log\left(1+x^{2/3}\right)$$

output
```
3/2*ln(1+x^(2/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx=\frac{3}{2}\log\left(1+x^{2/3}\right)$$

input
```
Integrate[1/((1 + x^(2/3))*x^(1/3)),x]
```

output
```
(3*Log[1 + x^(2/3)])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x^{2/3}+1\right) \sqrt[3]{x}} \, dx$$
$$\Big\downarrow \; 792$$
$$\frac{3}{2} \log \left(x^{2/3}+1\right)$$

input
```
Int[1/((1 + x^(2/3))*x^(1/3)),x]
```

output
```
(3*Log[1 + x^(2/3)])/2
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{2}{3}}\right)}{2}$	9
default	$\frac{3\ln\left(1+x^{\frac{2}{3}}\right)}{2}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{2}{3}}\right)}{2}$	9
trager	$\frac{\ln\left(3x^{\frac{2}{3}}+3x^{\frac{4}{3}}+x^{2}+1\right)}{2}$	19

input
```
int(1/(1+x^(2/3))/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(1+x^(2/3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{(1+x^{2/3})\sqrt[3]{x}}\,dx = \frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(1+x^(2/3))/x^(1/3),x, algorithm="fricas")
```

output
```
3/2*log(x^(2/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx = \frac{3\log\left(x^{\frac{2}{3}}+1\right)}{2}$$

input
```
integrate(1/(1+x**(2/3))/x**(1/3),x)
```

output
```
3*log(x**(2/3) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx = \frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(1+x^(2/3))/x^(1/3),x, algorithm="maxima")
```

output
```
3/2*log(x^(2/3) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx = \frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

input
```
integrate(1/(1+x^(2/3))/x^(1/3),x, algorithm="giac")
```

output
```
3/2*log(x^(2/3) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx = \frac{3\,\ln\left(x^{2/3}+1\right)}{2}$$

input
```
int(1/(x^(1/3)*(x^(2/3) + 1)),x)
```

output
```
(3*log(x^(2/3) + 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx = \frac{3\log\left(x^{\frac{2}{3}}+1\right)}{2}$$

input
```
int(1/(1+x^(2/3))/x^(1/3),x)
```

output
```
(3*log(x**(2/3) + 1))/2
```

3.277 $\int \frac{1}{\left(1+x^{2/3}\right)x^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{1}{(1+x^{2/3})\,x^{2/3}}\,dx = 3\arctan\left(\sqrt[3]{x}\right)$$

output
```
3*arctan(x^(1/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^{2/3})\,x^{2/3}}\,dx = 3\arctan\left(\sqrt[3]{x}\right)$$

input
```
Integrate[1/((1 + x^(2/3))*x^(2/3)),x]
```

output
```
3*ArcTan[x^(1/3)]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {864, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x^{2/3}+1\right) x^{2/3}} \, dx$$

$$\Big\downarrow \text{ 864}$$

$$3 \int \frac{1}{x^{2/3}+1} d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 216}$$

$$3 \arctan\left(\sqrt[3]{x}\right)$$

input
```
Int[1/((1 + x^(2/3))*x^(2/3)),x]
```

output
```
3*ArcTan[x^(1/3)]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
derivativedivides	$3\arctan\left(x^{\frac{1}{3}}\right)$
default	$3\arctan\left(x^{\frac{1}{3}}\right)$
meijerg	$3\arctan\left(x^{\frac{1}{3}}\right)$
trager	$-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)^2 x+4\,\mathrm{RootOf}\left(\_Z^2+1\right)x^{\frac{2}{3}}-\mathrm{RootOf}\left(\_Z^2+1\right)^2-2\,\mathrm{RootOf}\left(\_Z^2+1\right)+}{\mathrm{RootOf}\left(\_Z^2+1\right)x-\mathrm{RootOf}\left(\_Z^2+1\right)+x+1}\right.}{2}$

input
```
int(1/x^(2/3)/(1+x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3*arctan(x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+x^{2/3}\right)x^{2/3}}\,dx = 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(1/(1+x^(2/3))/x^(2/3),x, algorithm="fricas")
```

output
```
3*arctan(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1+x^{2/3})\,x^{2/3}}\,dx = 3\,\mathrm{atan}\left(\sqrt[3]{x}\right)$$

input
```
integrate(1/(1+x**(2/3))/x**(2/3),x)
```

output
```
3*atan(x**(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x^{2/3})\,x^{2/3}}\,dx = 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(1/(1+x^(2/3))/x^(2/3),x, algorithm="maxima")
```

output
```
3*arctan(x^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x^{2/3})\,x^{2/3}}\,dx = 3\,\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(1/(1+x^(2/3))/x^(2/3),x, algorithm="giac")
```

output
```
3*arctan(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x^{2/3})\,x^{2/3}}\,dx = 3\,\mathrm{atan}\left(x^{1/3}\right)$$

input
```
int(1/(x^(2/3)*(x^(2/3) + 1)),x)
```

output
```
3*atan(x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x^{2/3})\,x^{2/3}}\,dx = 3atan\left(x^{\frac{1}{3}}\right)$$

input
```
int(1/(1+x^(2/3))/x^(2/3),x)
```

output
```
3*atan(x**(1/3))
```

3.278 $\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = \left(-1+x^{2/3}\right)^{3/2}$$

output
```
(-1+x^(2/3))^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = \left(-1+x^{2/3}\right)^{3/2}$$

input
```
Integrate[Sqrt[-1 + x^(2/3)]/x^(1/3),x]
```

output
```
(-1 + x^(2/3))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^{2/3}-1}}{\sqrt[3]{x}} \, dx$$

$$\Big\downarrow \text{793}$$

$$\left(x^{2/3}-1\right)^{3/2}$$

input
```
Int[Sqrt[-1 + x^(2/3)]/x^(1/3),x]
```

output
```
(-1 + x^(2/3))^(3/2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\left(-1+x^{\frac{2}{3}}\right)^{\frac{3}{2}}$	8
default	$\left(-1+x^{\frac{2}{3}}\right)^{\frac{3}{2}}$	8
meijerg	$\frac{3\sqrt{\operatorname{signum}\left(-1+x^{\frac{2}{3}}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2-2x^{\frac{2}{3}}\right)\sqrt{1-x^{\frac{2}{3}}}}{3}\right)}{4\sqrt{\pi}\sqrt{-\operatorname{signum}\left(-1+x^{\frac{2}{3}}\right)}}$	51

input
```
int((-1+x^(2/3))^(1/2)/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
(-1+x^(2/3))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx=\left(x^{\frac{2}{3}}-1\right)^{\frac{3}{2}}$$

input
```
integrate((-1+x^(2/3))^(1/2)/x^(1/3),x, algorithm="fricas")
```

output
```
(x^(2/3) - 1)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(8) = 16.

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}-1}-\sqrt{x^{\frac{2}{3}}-1}$$

input
```
integrate((-1+x**(2/3))**(1/2)/x**(1/3),x)
```

output
```
x**(2/3)*sqrt(x**(2/3) - 1) - sqrt(x**(2/3) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = \left(x^{\frac{2}{3}}-1\right)^{\frac{3}{2}}$$

input
```
integrate((-1+x^(2/3))^(1/2)/x^(1/3),x, algorithm="maxima")
```

output
```
(x^(2/3) - 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = \left(x^{\frac{2}{3}}-1\right)^{\frac{3}{2}}$$

input
```
integrate((-1+x^(2/3))^(1/2)/x^(1/3),x, algorithm="giac")
```

output
```
(x^(2/3) - 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = \left(x^{2/3}-1\right)^{3/2}$$

input
```
int((x^(2/3) - 1)^(1/2)/x^(1/3),x)
```

output
```
(x^(2/3) - 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{-1+x^{2/3}}}{\sqrt[3]{x}}\,dx = \sqrt{x^{\frac{2}{3}}-1}\left(x^{\frac{2}{3}}-1\right)$$

input
```
int((-1+x^(2/3))^(1/2)/x^(1/3),x)
```

output
```
sqrt(x**(2/3) - 1)*(x**(2/3) - 1)
```

3.279 $\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}} dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}} dx = \frac{3}{5}\left(1+x^{2/3}\right)^{5/2}$$

output
```
3/5*(1+x^(2/3))^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}} dx = \frac{3}{5}\left(1+x^{2/3}\right)^{5/2}$$

input
```
Integrate[(1 + x^(2/3))^(3/2)/x^(1/3),x]
```

output
```
(3*(1 + x^(2/3))^(5/2))/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^{2/3}+1\right)^{3/2}}{\sqrt[3]{x}} \, dx$$

$$\Big\downarrow \ 793$$

$$\frac{3}{5}\left(x^{2/3}+1\right)^{5/2}$$

input
```
Int[(1 + x^(2/3))^(3/2)/x^(1/3),x]
```

output
```
(3*(1 + x^(2/3))^(5/2))/5
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{3\left(1+x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{5}$	10
default	$\frac{3\left(1+x^{\frac{2}{3}}\right)^{\frac{5}{2}}}{5}$	10
meijerg	$\frac{-\frac{3\sqrt{\pi}}{5}+\frac{3\sqrt{\pi}\left(2x^{\frac{4}{3}}+4x^{\frac{2}{3}}+2\right)\sqrt{1+x^{\frac{2}{3}}}}{10}}{\sqrt{\pi}}$	36

input
```
int((1+x^(2/3))^(3/2)/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/5*(1+x^(2/3))^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.

Time = 0.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx = \frac{3}{5}\left(x^{\frac{4}{3}}+2\,x^{\frac{2}{3}}+1\right)\sqrt{x^{\frac{2}{3}}+1}$$

input
```
integrate((1+x^(2/3))^(3/2)/x^(1/3),x, algorithm="fricas")
```

output
```
3/5*(x^(4/3) + 2*x^(2/3) + 1)*sqrt(x^(2/3) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx = \frac{3x^{\frac{4}{3}}\sqrt{x^{\frac{2}{3}}+1}}{5} + \frac{6x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}}+1}}{5} + \frac{3\sqrt{x^{\frac{2}{3}}+1}}{5}$$

input
```
integrate((1+x**(2/3))**(3/2)/x**(1/3),x)
```

output
```
3*x**(4/3)*sqrt(x**(2/3) + 1)/5 + 6*x**(2/3)*sqrt(x**(2/3) + 1)/5 + 3*sqrt
(x**(2/3) + 1)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx = \frac{3}{5}\left(x^{\frac{2}{3}}+1\right)^{\frac{5}{2}}$$

input
```
integrate((1+x^(2/3))^(3/2)/x^(1/3),x, algorithm="maxima")
```

output
```
3/5*(x^(2/3) + 1)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx = \frac{3}{5}\left(x^{\frac{2}{3}}+1\right)^{\frac{5}{2}}$$

input
```
integrate((1+x^(2/3))^(3/2)/x^(1/3),x, algorithm="giac")
```

output
```
3/5*(x^(2/3) + 1)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx = \frac{3\left(x^{2/3}+1\right)^{5/2}}{5}$$

input
```
int((x^(2/3) + 1)^(3/2)/x^(1/3),x)
```

output
```
(3*(x^(2/3) + 1)^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{\left(1+x^{2/3}\right)^{3/2}}{\sqrt[3]{x}}\,dx = \frac{3\sqrt{x^{\frac{2}{3}}+1}\left(2x^{\frac{2}{3}}+x^{\frac{4}{3}}+1\right)}{5}$$

input
```
int((1+x^(2/3))^(3/2)/x^(1/3),x)
```

output
```
(3*sqrt(x**(2/3) + 1)*(2*x**(2/3) + x**(1/3)*x + 1))/5
```

3.280 $\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx = -6\sqrt[6]{x}+\frac{6x^{5/6}}{5}-\frac{3\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}+\frac{3\arctan\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[6]{x}}{1+\sqrt[3]{x}}\right)}{\sqrt{2}}$$

output
```
-6*x^(1/6)+6/5*x^(5/6)+3/2*arctan(-1+2^(1/2)*x^(1/6))*2^(1/2)+3/2*arctan(1
+2^(1/2)*x^(1/6))*2^(1/2)+3/2*arctanh(2^(1/2)*x^(1/6)/(1+x^(1/3)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx = \frac{6}{5}\left(-5+x^{2/3}\right)\sqrt[6]{x}+\frac{3\arctan\left(\frac{-1+\sqrt[3]{x}}{\sqrt{2}\sqrt[6]{x}}\right)}{\sqrt{2}}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[6]{x}}{1+\sqrt[3]{x}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x]/(1 + x^(2/3)),x]
```

output
```
(6*(-5 + x^(2/3))*x^(1/6))/5 + (3*ArcTan[(-1 + x^(1/3))/(Sqrt[2]*x^(1/6))]
)/Sqrt[2] + (3*ArcTanh[(Sqrt[2]*x^(1/6))/(1 + x^(1/3))])/Sqrt[2]
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.49, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {864, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x^{2/3}+1} \, dx$$

$$\Big\downarrow \text{864}$$

$$3 \int \frac{x^{7/6}}{x^{2/3}+1} d\sqrt[3]{x}$$

$$\Big\downarrow \text{262}$$

$$3 \left(\frac{2x^{5/6}}{5} - \int \frac{\sqrt{x}}{x^{2/3}+1} d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{262}$$

$$3 \left(\int \frac{1}{\left(x^{2/3}+1\right)\sqrt[6]{x}} d\sqrt[3]{x} + \frac{2x^{5/6}}{5} - 2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{266}$$

$$3 \left(2 \int \frac{1}{x^{4/3}+1} d\sqrt[6]{x} + \frac{2x^{5/6}}{5} - 2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{755}$$

$$3 \left(2 \left(\frac{1}{2} \int \frac{1-x^{2/3}}{x^{4/3}+1} d\sqrt[6]{x} + \frac{1}{2} \int \frac{x^{2/3}+1}{x^{4/3}+1} d\sqrt[6]{x}\right) + \frac{2x^{5/6}}{5} - 2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{1476}$$

$$3\left(2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}+\frac{1}{2}\int \frac{1}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}\right)+\frac{1}{2}\int \frac{1-x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}\right)+\frac{2x^{5/6}}{5}-2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{1082}$$

$$3\left(2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x^{2/3}-1}d\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x^{2/3}-1}d\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}\right)+\frac{2x^{5/6}}{5}-2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{217}$$

$$3\left(2\left(\frac{1}{2}\int \frac{1-x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}\right)\right)+\frac{2x^{5/6}}{5}-2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{1479}$$

$$3\left(2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt[6]{x}}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$3\left(2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[6]{x}}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}\right)\right)\right.$$

$$\Big\downarrow \text{27}$$

$$3\left(2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[6]{x}}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt[6]{x}+1}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$3\left(2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^{2/3}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x^{2/3}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}\right)\right)\right.$$

input
```
Int[Sqrt[x]/(1 + x^(2/3)),x]
```

output
```
3*(-2*x^(1/6) + (2*x^(5/6))/5 + 2*((-(ArcTan[1 - Sqrt[2]*x^(1/6)]/Sqrt[2])
 + ArcTan[1 + Sqrt[2]*x^(1/6)]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*x^(1/6)
+ x^(2/3)]/Sqrt[2] + Log[1 + Sqrt[2]*x^(1/6) + x^(2/3)]/(2*Sqrt[2]))/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5}-6x^{\frac{1}{6}}+\frac{3\sqrt{2}\left(\ln\left(\frac{x^{\frac{1}{3}}+\sqrt{2}\,x^{\frac{1}{6}}+1}{x^{\frac{1}{3}}-\sqrt{2}\,x^{\frac{1}{6}}+1}\right)+2\arctan\left(1+\sqrt{2}\,x^{\frac{1}{6}}\right)+2\arctan\left(-1+\sqrt{2}\,x^{\frac{1}{6}}\right)\right)}{4}$
default	$\frac{6x^{\frac{5}{6}}}{5}-6x^{\frac{1}{6}}+\frac{3\sqrt{2}\left(\ln\left(\frac{x^{\frac{1}{3}}+\sqrt{2}\,x^{\frac{1}{6}}+1}{x^{\frac{1}{3}}-\sqrt{2}\,x^{\frac{1}{6}}+1}\right)+2\arctan\left(1+\sqrt{2}\,x^{\frac{1}{6}}\right)+2\arctan\left(-1+\sqrt{2}\,x^{\frac{1}{6}}\right)\right)}{4}$
meijerg	$-\frac{2x^{\frac{1}{6}}\left(-9x^{\frac{2}{3}}+45\right)}{15}+\frac{3x^{\frac{1}{6}}\left(-\frac{\sqrt{2}\,\ln\left(x^{\frac{1}{3}}-\sqrt{2}\,x^{\frac{1}{6}}+1\right)}{2x^{\frac{1}{6}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^{\frac{1}{6}}}{2-\sqrt{2}\,x^{\frac{1}{6}}}\right)}{x^{\frac{1}{6}}}+\frac{\sqrt{2}\,\ln\left(x^{\frac{1}{3}}+\sqrt{2}\,x^{\frac{1}{6}}+1\right)}{2x^{\frac{1}{6}}}+\frac{\sqrt{2}\,\arctan(\ldots}{x^{\frac{1}{6}}}\ldots\right.}{2}$

input
```
int(x^(1/2)/(1+x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
6/5*x^(5/6)-6*x^(1/6)+3/4*2^(1/2)*(ln((x^(1/3)+2^(1/2)*x^(1/6)+1)/(x^(1/3)
-2^(1/2)*x^(1/6)+1))+2*arctan(1+2^(1/2)*x^(1/6))+2*arctan(-1+2^(1/2)*x^(1/
6)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx=\frac{3}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x^{\frac{1}{6}}+1\right)+\frac{3}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x^{\frac{1}{6}}-1\right)+\frac{3}{4}\,\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\,\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+\frac{6}{5}\,x^{\frac{5}{6}}-6\,x^{\frac{1}{6}}$$

input
```
integrate(x^(1/2)/(1+x^(2/3)),x, algorithm="fricas")
```

output
```
3/2*sqrt(2)*arctan(sqrt(2)*x^(1/6) + 1) + 3/2*sqrt(2)*arctan(sqrt(2)*x^(1/
6) - 1) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^(1/3) + 1) - 3/4*sqrt(2)*log
(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 6/5*x^(5/6) - 6*x^(1/6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx = \frac{27x^{\frac{5}{6}}\Gamma\left(\frac{9}{4}\right)}{10\Gamma\left(\frac{13}{4}\right)} - \frac{27\sqrt[6]{x}\Gamma\left(\frac{9}{4}\right)}{2\Gamma\left(\frac{13}{4}\right)} - \frac{27e^{-\frac{i\pi}{4}}\log\left(-\sqrt[6]{x}e^{\frac{i\pi}{4}}+1\right)\Gamma\left(\frac{9}{4}\right)}{8\Gamma\left(\frac{13}{4}\right)} + \frac{27ie^{-\frac{i\pi}{4}}\log\left(-\sqrt[6]{x}e^{\frac{3i\pi}{4}}+1\right)\Gamma\left(\frac{9}{4}\right)}{8\Gamma\left(\frac{13}{4}\right)} + \frac{27e^{-\frac{i\pi}{4}}\log\left(-\sqrt[6]{x}e^{\frac{5i\pi}{4}}+1\right)\Gamma\left(\frac{9}{4}\right)}{8\Gamma\left(\frac{13}{4}\right)} - \frac{27ie^{-\frac{i\pi}{4}}\log\left(-\sqrt[6]{x}e^{\frac{7i\pi}{4}}+1\right)\Gamma\left(\frac{9}{4}\right)}{8\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**(1/2)/(1+x**(2/3)),x)
```

output
```
27*x**(5/6)*gamma(9/4)/(10*gamma(13/4)) - 27*x**(1/6)*gamma(9/4)/(2*gamma(
13/4)) - 27*exp(-I*pi/4)*log(-x**(1/6)*exp_polar(I*pi/4) + 1)*gamma(9/4)/(
8*gamma(13/4)) + 27*I*exp(-I*pi/4)*log(-x**(1/6)*exp_polar(3*I*pi/4) + 1)*
gamma(9/4)/(8*gamma(13/4)) + 27*exp(-I*pi/4)*log(-x**(1/6)*exp_polar(5*I*p
i/4) + 1)*gamma(9/4)/(8*gamma(13/4)) - 27*I*exp(-I*pi/4)*log(-x**(1/6)*exp
_polar(7*I*pi/4) + 1)*gamma(9/4)/(8*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx = \frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right) + \frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right) + \frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) - \frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) + \frac{6}{5}x^{\frac{5}{6}} - 6x^{\frac{1}{6}}$$

input
```
integrate(x^(1/2)/(1+x^(2/3)),x, algorithm="maxima")
```

output
```
3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) + 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x
^(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 6/5*x^(5/6
) - 6*x^(1/6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx = \frac{3}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,x^{\frac{1}{6}}\right)\right) + \frac{3}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,x^{\frac{1}{6}}\right)\right) + \frac{3}{4}\,\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) - \frac{3}{4}\,\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) + \frac{6}{5}\,x^{\frac{5}{6}} - 6\,x^{\frac{1}{6}}$$

input
```
integrate(x^(1/2)/(1+x^(2/3)),x, algorithm="giac")
```

output
```
3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) + 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x
^(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 6/5*x^(5/6
) - 6*x^(1/6)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx = \frac{6\,x^{5/6}}{5} - 6\,x^{1/6} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

input
```
int(x^(1/2)/(x^(2/3) + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 + 3i/2) + 2^(1/2)*atan(2^(
1/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 - 3i/2) - 6*x^(1/6) + (6*x^(5/6))/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{x}}{1+x^{2/3}}\,dx = \frac{3\sqrt{2}\,atan\left(\frac{2x^{\frac{1}{6}}-\sqrt{2}}{\sqrt{2}}\right)}{2} + \frac{3\sqrt{2}\,atan\left(\frac{2x^{\frac{1}{6}}+\sqrt{2}}{\sqrt{2}}\right)}{2} + \frac{6x^{\frac{5}{6}}}{5} - 6x^{\frac{1}{6}} - \frac{3\sqrt{2}\log\left(-x^{\frac{1}{6}}\sqrt{2}+x^{\frac{1}{3}}+1\right)}{4} + \frac{3\sqrt{2}\log\left(x^{\frac{1}{6}}\sqrt{2}+x^{\frac{1}{3}}+1\right)}{4}$$

input
```
int(x^(1/2)/(1+x^(2/3)),x)
```

output
```
(3*(10*sqrt(2)*atan((2*x**(1/6) - sqrt(2))/sqrt(2)) + 10*sqrt(2)*atan((2*x
**(1/6) + sqrt(2))/sqrt(2)) + 8*x**(5/6) - 40*x**(1/6) - 5*sqrt(2)*log( -
x**(1/6)*sqrt(2) + x**(1/3) + 1) + 5*sqrt(2)*log(x**(1/6)*sqrt(2) + x**(1/
3) + 1)))/20
```

3.281 $\int \frac{\sqrt[3]{x}}{-1+x^{5/6}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt[3]{x}}{-1+x^{5/6}} \, dx &= 2\sqrt{x}+\frac{3}{5}\sqrt{10-2\sqrt{5}}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{10+2\sqrt{5}}}\right)\\ &-\frac{3}{5}\sqrt{10+2\sqrt{5}}\arctan\left(\frac{1+\sqrt{5}+4\sqrt[6]{x}}{\sqrt{10-2\sqrt{5}}}\right)+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)\\ &-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\sqrt[3]{x}\right)\\ &-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\left(1+\sqrt{5}\right)\sqrt[6]{x}+2\sqrt[3]{x}\right)\end{aligned}$$

output

```
2*x^(1/2)+3/5*(10-2*5^(1/2))^(1/2)*arctan((1-5^(1/2)+4*x^(1/6))/(10+2*5^(1
/2))^(1/2))-3/5*(10+2*5^(1/2))^(1/2)*arctan((1+5^(1/2)+4*x^(1/6))/(10-2*5^
(1/2))^(1/2))+6/5*ln(1-x^(1/6))-3/10*(5^(1/2)+1)*ln(2+(-5^(1/2)+1)*x^(1/6)
+2*x^(1/3))-3/10*(-5^(1/2)+1)*ln(2+(5^(1/2)+1)*x^(1/6)+2*x^(1/3))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx = 2\sqrt{x}+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\Bigg[1+\#1+\#1^2+\#1^3 +\#1^4\&,\frac{-\log\left(\sqrt[6]{x}-\#1\right)-2\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\Bigg]$$

input
```
Integrate[x^(1/3)/(-1 + x^(5/6)),x]
```

output
```
2*Sqrt[x] + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 + #1
^4 & , (-Log[x^(1/6) - #1] - 2*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*
#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {864, 25, 843, 823, 16, 27, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}}{x^{5/6}-1}\,dx$$

$$\Big\downarrow \ 864$$

$$6\int -\frac{x^{7/6}}{1-x^{5/6}}d\sqrt[6]{x}$$

$$\Big\downarrow \ 25$$

$$-6\int \frac{x^{7/6}}{1-x^{5/6}}d\sqrt[6]{x}$$

$$\Big\downarrow \ 843$$

$$6\left(\frac{\sqrt{x}}{3}-\int \frac{\sqrt[3]{x}}{1-x^{5/6}}d\sqrt[6]{x}\right)$$

$\Big\downarrow$ 823

$$6\left(-\frac{1}{5}\int \frac{1}{1-\sqrt[6]{x}}d\sqrt[6]{x}-\frac{2}{5}\int -\frac{-\left(\left(1+\sqrt{5}\right)\sqrt[6]{x}\right)+\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}-\frac{2}{5}\int -\frac{-\left(\left(1-\sqrt{5}\right)\sqrt[6]{x}\right)-\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1+\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}+\right.$$

$\Big\downarrow$ 16

$$6\left(-\frac{2}{5}\int -\frac{-\left(\left(1+\sqrt{5}\right)\sqrt[6]{x}\right)+\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}-\frac{2}{5}\int -\frac{-\left(\left(1-\sqrt{5}\right)\sqrt[6]{x}\right)-\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1+\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}+\frac{\sqrt{x}}{3}+\frac{1}{5}\log\left(1-\sqrt[6]{x}\right.\right.$$

$\Big\downarrow$ 27

$$6\left(\frac{1}{5}\int \frac{-\left(\left(1+\sqrt{5}\right)\sqrt[6]{x}\right)+\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}+\frac{1}{5}\int \frac{-\left(\left(1-\sqrt{5}\right)\sqrt[6]{x}\right)-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1+\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}+\frac{\sqrt{x}}{3}+\frac{1}{5}\log\left(1-\sqrt[6]{x}\right)\right)$$

$\Big\downarrow$ 1142

$$6\left(\frac{1}{5}\left(\sqrt{5}\int \frac{1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{4\sqrt[6]{x}-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}\right)+\frac{1}{5}\left(-\sqrt{5}\int \frac{}{2\sqrt[3]{x}}\right.\right.$$

$\Big\downarrow$ 1083

$$6\left(\frac{1}{5}\left(-2\sqrt{5}\int \frac{1}{-\sqrt[3]{x}-2\left(5+\sqrt{5}\right)}d\left(4\sqrt[6]{x}-\sqrt{5}+1\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{4\sqrt[6]{x}-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}\right)+\frac{1}{5}\left(2\right.\right.$$

$\Big\downarrow$ 217

$$6\left(\frac{1}{5}\left(\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{4\sqrt[6]{x}-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}\right)+\frac{1}{5}\left(-\frac{1}{4}\left(1-\sqrt{5}\right.\right.\right.$$

$\Big\downarrow$ 1103

$$6\left(\frac{1}{5}\left(\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\right)\right)+\frac{1}{5}\left(-\sqrt{\frac{10}{5-\sqrt{5}}}\,\mathrm{a}\right.\right.$$

input
```
Int[x^(1/3)/(-1 + x^(5/6)),x]
```

output
```
6*(Sqrt[x]/3 + Log[1 - x^(1/6)]/5 + (Sqrt[10/(5 + Sqrt[5])]*ArcTan[(1 - Sq
rt[5] + 4*x^(1/6))/Sqrt[2*(5 + Sqrt[5])]] - ((1 + Sqrt[5])*Log[2 + (1 - Sq
rt[5])*x^(1/6) + 2*x^(1/3)])/4)/5 + (-(Sqrt[10/(5 - Sqrt[5])]*ArcTan[(1 +
Sqrt[5] + 4*x^(1/6))/Sqrt[2*(5 - Sqrt[5])]]) - ((1 - Sqrt[5])*Log[2 + (1 +
 Sqrt[5])*x^(1/6) + 2*x^(1/3)])/4)/5)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 823
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*
k - 1)*m*(Pi/n)] + s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k
 - 1)*(Pi/n)]*x + s^2*x^2), x]; r^(m + 1)/(a*n*s^m)   Int[1/(r - s*x), x] -
 2*((-r)^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b
}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

method	result
meijerg	$-\frac{6(-1)^{\frac{2}{5}}\left(\frac{5\sqrt{x}\,(-1)^{\frac{3}{5}}}{3}+(-1)^{\frac{3}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sin\left(\frac{\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}{1-\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}\right)+\right.\right.}{5}$
derivativedivides	$2\sqrt{x}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}+\frac{3\left(\sqrt{5}-1\right)\ln\left(\sqrt{5}\,x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}+2\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+\sqrt{5}+4}{\sqrt{10-2}}\right.}{5\sqrt{10-2\sqrt{5}}}$
default	$2\sqrt{x}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}+\frac{3\left(\sqrt{5}-1\right)\ln\left(\sqrt{5}\,x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}+2\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+\sqrt{5}+4}{\sqrt{10-2}}\right.}{5\sqrt{10-2\sqrt{5}}}$

input
```
int(x^(1/3)/(-1+x^(5/6)),x,method=_RETURNVERBOSE)
```

output
```
-6/5*(-1)^(2/5)*(5/3*x^(1/2)*(-1)^(3/5)+(-1)^(3/5)*(ln(1-x^(1/6))-cos(1/5*
Pi)*ln(1-2*cos(2/5*Pi)*x^(1/6)+x^(1/3))+2*sin(1/5*Pi)*arctan(sin(2/5*Pi)*x
^(1/6)/(1-cos(2/5*Pi)*x^(1/6)))+cos(2/5*Pi)*ln(1+2*cos(1/5*Pi)*x^(1/6)+x^(
1/3))-2*sin(2/5*Pi)*arctan(sin(1/5*Pi)*x^(1/6)/(1+cos(1/5*Pi)*x^(1/6)))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx = -\frac{6}{5}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}\arctan\left(\frac{1}{10}\sqrt{\frac{1}{2}}\left(4\sqrt{5}x^{\frac{1}{6}}+\sqrt{5}+5\right)\sqrt{\sqrt{5}+5}\right) + \frac{3}{10}\left(\sqrt{5}-1\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+2\,x^{\frac{1}{3}}+2\right) - \frac{3}{10}\left(\sqrt{5}+1\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+2\,x^{\frac{1}{3}}+2\right) + \frac{2}{5}\sqrt{-\frac{9}{2}\sqrt{5}+\frac{45}{2}}\arctan\left(\frac{1}{30}\left(\sqrt{5}-5\right)\sqrt{-\frac{9}{2}\sqrt{5}+\frac{45}{2}} + \frac{2}{15}\sqrt{5}x^{\frac{1}{6}}\sqrt{-\frac{9}{2}\sqrt{5}+\frac{45}{2}}\right) + 2\sqrt{x} + \frac{6}{5}\log\left(x^{\frac{1}{6}}-1\right)$$

input
```
integrate(x^(1/3)/(-1+x^(5/6)),x, algorithm="fricas")
```

output
```
-6/5*sqrt(1/2)*sqrt(sqrt(5) + 5)*arctan(1/10*sqrt(1/2)*(4*sqrt(5)*x^(1/6)
+ sqrt(5) + 5)*sqrt(sqrt(5) + 5)) + 3/10*(sqrt(5) - 1)*log(x^(1/6)*(sqrt(5
) + 1) + 2*x^(1/3) + 2) - 3/10*(sqrt(5) + 1)*log(-x^(1/6)*(sqrt(5) - 1) +
2*x^(1/3) + 2) + 2/5*sqrt(-9/2*sqrt(5) + 45/2)*arctan(1/30*(sqrt(5) - 5)*s
qrt(-9/2*sqrt(5) + 45/2) + 2/15*sqrt(5)*x^(1/6)*sqrt(-9/2*sqrt(5) + 45/2))
 + 2*sqrt(x) + 6/5*log(x^(1/6) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(170) = 340$.

Time = 51.78 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt[3]{x}}{-1 + x^{5/6}}\, dx = 2\sqrt{x} + \frac{6\log\left(\sqrt[6]{x} - 1\right)}{5} - \frac{3\log\left(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} + \frac{3\sqrt{5}\log\left(8\sqrt[6]{x} + 8\sqrt{5}\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} - \frac{3\sqrt{5}\log\left(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} - \frac{3\log\left(-8\sqrt{5}\sqrt[6]{x} + 8\sqrt[6]{x} + 16\sqrt[3]{x} + 16\right)}{10} - \frac{3\sqrt{10}\sqrt{5-\sqrt{5}}\operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{5-\sqrt{5}}} + \frac{\sqrt{2}}{2\sqrt{5-\sqrt{5}}} + \frac{\sqrt{10}}{2\sqrt{5-\sqrt{5}}}\right)}{10} - \frac{3\sqrt{2}\sqrt{5-\sqrt{5}}\operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{5-\sqrt{5}}} + \frac{\sqrt{2}}{2\sqrt{5-\sqrt{5}}} + \frac{\sqrt{10}}{2\sqrt{5-\sqrt{5}}}\right)}{10} - \frac{3\sqrt{2}\sqrt{\sqrt{5}+5}\operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{\sqrt{5}+5}} - \frac{\sqrt{10}}{2\sqrt{\sqrt{5}+5}} + \frac{\sqrt{2}}{2\sqrt{\sqrt{5}+5}}\right)}{10} + \frac{3\sqrt{10}\sqrt{\sqrt{5}+5}\operatorname{atan}\left(\frac{2\sqrt{2}\sqrt[6]{x}}{\sqrt{\sqrt{5}+5}} - \frac{\sqrt{10}}{2\sqrt{\sqrt{5}+5}} + \frac{\sqrt{2}}{2\sqrt{\sqrt{5}+5}}\right)}{10}$$

input
```
integrate(x**(1/3)/(-1+x**(5/6)),x)
```

output
```
2*sqrt(x) + 6*log(x**(1/6) - 1)/5 - 3*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6)
+ 16*x**(1/3) + 16)/10 + 3*sqrt(5)*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6) + 1
6*x**(1/3) + 16)/10 - 3*sqrt(5)*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*
x**(1/3) + 16)/10 - 3*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3) +
 16)/10 - 3*sqrt(10)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1/6)/sqrt(5 - sq
rt(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5 - sqrt(5))))/1
0 - 3*sqrt(2)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1/6)/sqrt(5 - sqrt(5))
+ sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5 - sqrt(5))))/10 - 3*s
qrt(2)*sqrt(sqrt(5) + 5)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(
10)/(2*sqrt(sqrt(5) + 5)) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/10 + 3*sqrt(10)
*sqrt(sqrt(5) + 5)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(10)/(2
*sqrt(sqrt(5) + 5)) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/10
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(130) = 260$.

Time = 0.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx = -\frac{6}{5}\,(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right) - \frac{6\,\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\,\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4\,x^{\frac{1}{6}}}\right)}{5\,\sqrt{2\,\sqrt{5}-10}} + \frac{6\,\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4\,x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\,\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4\,x^{\frac{1}{6}}}\right)}{5\,\sqrt{-2\,\sqrt{5}-10}} + 2\,\sqrt{x} + \frac{6\,\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2\,x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{2}{5}}+(-1)^{\frac{2}{5}}\right)} - \frac{6\,\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2\,x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{2}{5}}-(-1)^{\frac{2}{5}}\right)}$$

input
```
integrate(x^(1/3)/(-1+x^(5/6)),x, algorithm="maxima")
```

output
```
-6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sq
rt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6
))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*
x^(1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^
(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5
)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))
/sqrt(-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5)
+ (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)
) - 6/5*log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x
^(1/3))/(sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx = \frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4\,x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4\,x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right) - \frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right) + 2\sqrt{x} - \frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right) + \frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate(x^(1/3)/(-1+x^(5/6)),x, algorithm="giac")
```

output
```
3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
 + 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-
2*sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) +
1) - 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x
) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx = \frac{6\ \ln\left(1296\,x^{1/6}-1296\right)}{5} - \ln\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)+\ln\left(750\right.$$

input
```
int(x^(1/3)/(x^(5/6) - 1),x)
```

output
```
(6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2)
- 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^
(1/2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(
1/2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/
2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))
/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5
^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1
/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

Reduce [F]

$$\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx = 2\sqrt{x}+\int \frac{1}{x^{\frac{4}{3}}-\sqrt{x}}dx$$

input
```
int(x^(1/3)/(-1+x^(5/6)),x)
```

output
```
2*sqrt(x) + int(1/(x**(1/3)*x - sqrt(x)),x)
```

3.282 $\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx=-\frac{1}{6}\sqrt{3-\frac{1}{\sqrt{x}}}\sqrt{x}+\sqrt{3-\frac{1}{\sqrt{x}}}x-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

output
```
-1/6*(3-1/x^(1/2))^(1/2)*x^(1/2)+(3-1/x^(1/2))^(1/2)*x-1/18*arctanh(1/3*(3
-1/x^(1/2))^(1/2)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx=\frac{1}{18}\left(-3\sqrt{3-\frac{1}{\sqrt{x}}}(\sqrt{x}-6x)-\sqrt{3}\operatorname{arctanh}\left(\sqrt{1-\frac{1}{3\sqrt{x}}}\right)\right)$$

input
```
Integrate[Sqrt[3 - 1/Sqrt[x]],x]
```

output
```
(-3*Sqrt[3 - 1/Sqrt[x]]*(Sqrt[x] - 6*x) - Sqrt[3]*ArcTanh[Sqrt[1 - 1/(3*Sq
rt[x])]]])/18
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {774, 798, 51, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}dx$$

$$\Big\downarrow \text{774}$$

$$2\int \sqrt{3-\frac{1}{\sqrt{x}}}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{51}$$

$$-2\left(-\frac{1}{4}\int \frac{1}{\sqrt{3-\frac{1}{\sqrt{x}}}x}d\frac{1}{\sqrt{x}}-\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{2x}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(\frac{1}{4}\left(\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{3\sqrt{x}}-\frac{1}{6}\int \frac{1}{\sqrt{3-\frac{1}{\sqrt{x}}}\sqrt{x}}d\frac{1}{\sqrt{x}}\right)-\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{2x}\right)$$

$$\Big\downarrow \text{73}$$

$$-2\left(\frac{1}{4}\left(\frac{1}{3}\int \frac{1}{3-x}d\sqrt{3-\frac{1}{\sqrt{x}}}+\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{3\sqrt{x}}\right)-\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{2x}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{1}{4}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{3\sqrt{x}}\right)-\frac{\sqrt{3-\frac{1}{\sqrt{x}}}}{2x}\right)$$

input

```
Int[Sqrt[3 - 1/Sqrt[x]],x]
```

output

```
-2*(-1/2*Sqrt[3 - 1/Sqrt[x]]/x + (Sqrt[3 - 1/Sqrt[x]]/(3*Sqrt[x]) + ArcTan
h[Sqrt[3 - 1/Sqrt[x]]/Sqrt[3]]/(3*Sqrt[3]))/4)
```

Defintions of rubi rules used

rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result	size
meijerg	$-\frac{i\sqrt{3}\,\sqrt{\operatorname{signum}\left(3\sqrt{x}-1\right)}\,\left(-\frac{i\sqrt{\pi}\,x^{\frac{1}{4}}\sqrt{3}\,\left(-18\sqrt{x}+3\right)\sqrt{-3\sqrt{x}+1}}{6}+\frac{i\sqrt{\pi}\,\arcsin\left(\sqrt{3}\,x^{\frac{1}{4}}\right)}{2}\right)}{9\sqrt{\pi}\,\sqrt{-\operatorname{signum}\left(3\sqrt{x}-1\right)}}$	75
derivativedivides	$-\frac{\sqrt{\frac{3\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\left(\ln\left(-\frac{\sqrt{3}}{6}+\sqrt{3}\,\sqrt{x}+\sqrt{3x-\sqrt{x}}\right)\sqrt{3}-36\sqrt{3x-\sqrt{x}}\,\sqrt{x}+6\sqrt{3x-\sqrt{x}}\right)}{36\sqrt{\left(3\sqrt{x}-1\right)\sqrt{x}}}$	91
default	$-\frac{\sqrt{\frac{3\sqrt{x}-1}{\sqrt{x}}}\,\sqrt{x}\,\left(\ln\left(-\frac{\sqrt{3}}{6}+\sqrt{3}\,\sqrt{x}+\sqrt{3x-\sqrt{x}}\right)\sqrt{3}-36\sqrt{3x-\sqrt{x}}\,\sqrt{x}+6\sqrt{3x-\sqrt{x}}\right)}{36\sqrt{\left(3\sqrt{x}-1\right)\sqrt{x}}}$	91

input
```
int((3-1/x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*I*3^(1/2)/Pi^(1/2)*signum(3*x^(1/2)-1)^(1/2)/(-signum(3*x^(1/2)-1))^(
1/2)*(-1/6*I*Pi^(1/2)*x^(1/4)*3^(1/2)*(-18*x^(1/2)+3)*(-3*x^(1/2)+1)^(1/2)
+1/2*I*Pi^(1/2)*arcsin(3^(1/2)*x^(1/4)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx = \frac{1}{6}\left(6\,x-\sqrt{x}\right)\sqrt{\frac{3\,x-\sqrt{x}}{x}} + \frac{1}{36}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{x}\sqrt{\frac{3\,x-\sqrt{x}}{x}}-6\,\sqrt{x}+1\right)$$

input
```
integrate((3-1/x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(6*x - sqrt(x))*sqrt((3*x - sqrt(x))/x) + 1/36*sqrt(3)*log(2*sqrt(3)*s
qrt(x)*sqrt((3*x - sqrt(x))/x) - 6*sqrt(x) + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx = \begin{cases} \frac{3x^{\frac{5}{4}}}{\sqrt{3\sqrt{x}-1}} - \frac{3x^{\frac{3}{4}}}{2\sqrt{3\sqrt{x}-1}} + \frac{\sqrt[4]{x}}{6\sqrt{3\sqrt{x}-1}} - \frac{\sqrt{3}\,\mathrm{acosh}\left(\sqrt{3}\,\sqrt[4]{x}\right)}{18} & \text{for } \left|\sqrt{x}\right| > \frac{1}{3} \\ -\frac{3ix^{\frac{5}{4}}}{\sqrt{1-3\sqrt{x}}} + \frac{3ix^{\frac{3}{4}}}{2\sqrt{1-3\sqrt{x}}} - \frac{i\sqrt[4]{x}}{6\sqrt{1-3\sqrt{x}}} + \frac{\sqrt{3}i\,\mathrm{asin}\left(\sqrt{3}\,\sqrt[4]{x}\right)}{18} & \text{otherwise} \end{cases}$$

input
```
integrate((3-1/x**(1/2))**(1/2),x)
```

output
```
Piecewise((3*x**(5/4)/sqrt(3*sqrt(x) - 1) - 3*x**(3/4)/(2*sqrt(3*sqrt(x) -
 1)) + x**(1/4)/(6*sqrt(3*sqrt(x) - 1)) - sqrt(3)*acosh(sqrt(3)*x**(1/4))/
18, Abs(sqrt(x)) > 1/3), (-3*I*x**(5/4)/sqrt(1 - 3*sqrt(x)) + 3*I*x**(3/4)
/(2*sqrt(1 - 3*sqrt(x))) - I*x**(1/4)/(6*sqrt(1 - 3*sqrt(x))) + sqrt(3)*I*
asin(sqrt(3)*x**(1/4))/18, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx = \frac{1}{36}\,\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{-\frac{1}{\sqrt{x}}+3}}{\sqrt{3}+\sqrt{-\frac{1}{\sqrt{x}}+3}}\right)+\frac{\left(-\frac{1}{\sqrt{x}}+3\right)^{\frac{3}{2}}+3\,\sqrt{-\frac{1}{\sqrt{x}}+3}}{6\left(\left(\frac{1}{\sqrt{x}}-3\right)^{2}+\frac{6}{\sqrt{x}}-9\right)}$$

input
```
integrate((3-1/x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
1/36*sqrt(3)*log(-(sqrt(3) - sqrt(-1/sqrt(x) + 3))/(sqrt(3) + sqrt(-1/sqrt
(x) + 3))) + 1/6*((-1/sqrt(x) + 3)^(3/2) + 3*sqrt(-1/sqrt(x) + 3))/((1/sqr
t(x) - 3)^2 + 6/sqrt(x) - 9)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx$$
$$=\frac{1}{36}\left(6\,\sqrt{3\,x-\sqrt{x}}(6\,\sqrt{x}-1)+\sqrt{3}\log\left(\left|-2\,\sqrt{3}\left(\sqrt{3}\sqrt{x}-\sqrt{3\,x-\sqrt{x}}\right)+1\right|\right)\right)\mathrm{sgn}(x)$$

input
```
integrate((3-1/x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/36*(6*sqrt(3*x - sqrt(x))*(6*sqrt(x) - 1) + sqrt(3)*log(abs(-2*sqrt(3)*(
sqrt(3)*sqrt(x) - sqrt(3*x - sqrt(x))) + 1)))*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx = \frac{4\,x\,\sqrt{3-\frac{1}{\sqrt{x}}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{2};\ \frac{5}{2};\ 3\,\sqrt{x}\right)}{3\,\sqrt{1-3\,\sqrt{x}}}$$

input
```
int((3 - 1/x^(1/2))^(1/2),x)
```

output
```
(4*x*(3 - 1/x^(1/2))^(1/2)*hypergeom([-1/2, 3/2], 5/2, 3*x^(1/2)))/(3*(1 -
 3*x^(1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \sqrt{3-\frac{1}{\sqrt{x}}}\,dx = x^{\frac{3}{4}}\sqrt{3\sqrt{x}-1} - \frac{x^{\frac{1}{4}}\sqrt{3\sqrt{x}-1}}{6} - \frac{\sqrt{3}\log\left(\sqrt{3\sqrt{x}-1}+x^{\frac{1}{4}}\sqrt{3}\right)}{18}$$

input
```
int((3-1/x^(1/2))^(1/2),x)
```

output
```
(18*x**(3/4)*sqrt(3*sqrt(x) - 1) - 3*x**(1/4)*sqrt(3*sqrt(x) - 1) - sqrt(3
)*log(sqrt(3*sqrt(x) - 1) + x**(1/4)*sqrt(3)))/18
```

3.283 $\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}} dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}} dx = -\frac{3}{2}\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}+\sqrt{1+\frac{1}{\sqrt{x}}}x+\frac{3}{2}\text{arctanh}\left(\sqrt{1+\frac{1}{\sqrt{x}}}\right)$$

output
```
-3/2*(1+1/x^(1/2))^(1/2)*x^(1/2)+(1+1/x^(1/2))^(1/2)*x+3/2*arctanh((1+1/x^
(1/2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}} dx = \frac{1}{2}\left(\sqrt{1+\frac{1}{\sqrt{x}}}(-3+2\sqrt{x})\sqrt{x}+3\text{arctanh}\left(\sqrt{1+\frac{1}{\sqrt{x}}}\right)\right)$$

input
```
Integrate[1/Sqrt[1 + 1/Sqrt[x]],x]
```

output
```
(Sqrt[1 + 1/Sqrt[x]]*(-3 + 2*Sqrt[x])*Sqrt[x] + 3*ArcTanh[Sqrt[1 + 1/Sqrt[
x]]])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {774, 798, 52, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{1}{\sqrt{x}}+1}}dx$$

$$\Big\downarrow \text{774}$$

$$2\int \frac{\sqrt{x}}{\sqrt{1+\frac{1}{\sqrt{x}}}}d\sqrt{x}$$

$$\Big\downarrow \text{798}$$

$$-2\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{52}$$

$$-2\left(-\frac{3}{4}\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}x}d\frac{1}{\sqrt{x}}-\frac{\sqrt{\frac{1}{\sqrt{x}}+1}}{2x}\right)$$

$$\Big\downarrow \text{52}$$

$$-2\left(-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}\sqrt{x}}d\frac{1}{\sqrt{x}}-\frac{\sqrt{\frac{1}{\sqrt{x}}+1}}{\sqrt{x}}\right)-\frac{\sqrt{\frac{1}{\sqrt{x}}+1}}{2x}\right)$$

$$\Big\downarrow \text{73}$$

$$-2\left(-\frac{3}{4}\left(-\int \frac{1}{x-1}d\sqrt{1+\frac{1}{\sqrt{x}}}-\frac{\sqrt{\frac{1}{\sqrt{x}}+1}}{\sqrt{x}}\right)-\frac{\sqrt{\frac{1}{\sqrt{x}}+1}}{2x}\right)$$

$$\Big\downarrow \text{220}$$

$$-2\left(-\frac{3}{4}\left(\text{arctanh}\left(\sqrt{\frac{1}{\sqrt{x}}+1}\right)-\frac{\sqrt{\frac{1}{\sqrt{x}}+1}}{\sqrt{x}}\right)-\frac{\sqrt{\frac{1}{\sqrt{x}}+1}}{2x}\right)$$

input
```
Int[1/Sqrt[1 + 1/Sqrt[x]],x]
```

output
```
-2*(-1/2*Sqrt[1 + 1/Sqrt[x]]/x - (3*(-(Sqrt[1 + 1/Sqrt[x]]/Sqrt[x]) + ArcT
anh[Sqrt[1 + 1/Sqrt[x]]]))/4)
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\, x^{\frac{1}{4}} (-10\sqrt{x}+15)\sqrt{1+\sqrt{x}}}{10}+\frac{3\sqrt{\pi}\, \operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{2}}{\sqrt{\pi}}$	38
default	$\frac{\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{x}\left(4\sqrt{x+\sqrt{x}}\,\sqrt{x}-6\sqrt{x+\sqrt{x}}+3\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)\right)}{4\sqrt{(1+\sqrt{x})\sqrt{x}}}$	65
derivativedivides	$\frac{(1+\sqrt{x})\left(4\sqrt{x+\sqrt{x}}\,\sqrt{x}-6\sqrt{x+\sqrt{x}}+3\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)\right)}{4\sqrt{\frac{1+\sqrt{x}}{\sqrt{x}}}\,\sqrt{(1+\sqrt{x})\sqrt{x}}}$	67

input
```
int(1/(1+1/x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/4)*(-10*x^(1/2)+15)*(1+x^(1/2))^(1/2)+3/4*
Pi^(1/2)*arcsinh(x^(1/4)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx = \frac{1}{2}\left(2\,x-3\,\sqrt{x}\right)\sqrt{\frac{x+\sqrt{x}}{x}}+\frac{3}{4}\,\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}+1\right) - \frac{3}{4}\,\log\left(\sqrt{\frac{x+\sqrt{x}}{x}}-1\right)$$

input
```
integrate(1/(1+1/x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*x - 3*sqrt(x))*sqrt((x + sqrt(x))/x) + 3/4*log(sqrt((x + sqrt(x))/x
) + 1) - 3/4*log(sqrt((x + sqrt(x))/x) - 1)
```

Sympy [A] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx = \frac{x^{\frac{5}{4}}}{\sqrt{\sqrt{x}+1}} - \frac{x^{\frac{3}{4}}}{2\sqrt{\sqrt{x}+1}} - \frac{3\sqrt[4]{x}}{2\sqrt{\sqrt{x}+1}} + \frac{3\operatorname{asinh}\left(\sqrt[4]{x}\right)}{2}$$

input
```
integrate(1/(1+1/x**(1/2))**(1/2),x)
```

output
```
x**(5/4)/sqrt(sqrt(x) + 1) - x**(3/4)/(2*sqrt(sqrt(x) + 1)) - 3*x**(1/4)/(
2*sqrt(sqrt(x) + 1)) + 3*asinh(x**(1/4))/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx = -\frac{3\left(\frac{1}{\sqrt{x}}+1\right)^{\frac{3}{2}} - 5\sqrt{\frac{1}{\sqrt{x}}+1}}{2\left(\left(\frac{1}{\sqrt{x}}+1\right)^{2} - \frac{2}{\sqrt{x}} - 1\right)} + \frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}+1\right) - \frac{3}{4}\log\left(\sqrt{\frac{1}{\sqrt{x}}+1}-1\right)$$

input
```
integrate(1/(1+1/x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(3*(1/sqrt(x) + 1)^(3/2) - 5*sqrt(1/sqrt(x) + 1))/((1/sqrt(x) + 1)^2
- 2/sqrt(x) - 1) + 3/4*log(sqrt(1/sqrt(x) + 1) + 1) - 3/4*log(sqrt(1/sqrt(
x) + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx = \frac{2\,\sqrt{x+\sqrt{x}}(2\,\sqrt{x}-3)-3\,\log\left(-2\,\sqrt{x+\sqrt{x}}+2\,\sqrt{x}+1\right)}{4\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(1+1/x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(x + sqrt(x))*(2*sqrt(x) - 3) - 3*log(-2*sqrt(x + sqrt(x)) + 2*
sqrt(x) + 1))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx = \frac{4\,x\,\sqrt{\sqrt{x}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\sqrt{x}\right)}{5\,\sqrt{\frac{1}{\sqrt{x}}+1}}$$

input
```
int(1/(1/x^(1/2) + 1)^(1/2),x)
```

output
```
(4*x*(x^(1/2) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -x^(1/2)))/(5*(1/x^(1/
2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}\,dx = x^{\frac{3}{4}}\sqrt{\sqrt{x}+1}-\frac{3x^{\frac{1}{4}}\sqrt{\sqrt{x}+1}}{2}+\frac{3\log\left(\sqrt{\sqrt{x}+1}+x^{\frac{1}{4}}\right)}{2}$$

input
```
int(1/(1+1/x^(1/2))^(1/2),x)
```

output
```
(2*x**(3/4)*sqrt(sqrt(x) + 1) - 3*x**(1/4)*sqrt(sqrt(x) + 1) + 3*log(sqrt(
sqrt(x) + 1) + x**(1/4)))/2
```

3.284 $\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx=\left(a+\frac{b}{x^{3/2}}\right)^{2/3} x-\frac{2b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}}{\sqrt[3]{a+\frac{b}{x^{3/2}}}\sqrt{x}}}{\sqrt{3}}\right)}{\sqrt{3}}+b^{2/3}\log\left(\sqrt[3]{a+\frac{b}{x^{3/2}}}-\frac{\sqrt[3]{b}}{\sqrt{x}}\right)$$

output
```
(a+b/x^(3/2))^(2/3)*x-2/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)/(a+b/x^(3/2))^(1
/3)/x^(1/2))*3^(1/2))*3^(1/2)+b^(2/3)*ln((a+b/x^(3/2))^(1/3)-b^(1/3)/x^(1/
2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(95) = 190.

Time = 6.02 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.95

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx =$$

$$-\frac{\left(a+\frac{b}{x^{3/2}}\right)^{2/3}\left(\sqrt[3]{b}-\sqrt[3]{b+ax^{3/2}}\right)\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b+ax^{3/2}}+\left(b+ax^{3/2}\right)^{2/3}\right)^2\left(3\left(b+ax^{3/2}\right)^{2/3}+2\sqrt{3}b^{2/}\right.}{3a\sqrt{x}\sqrt[3]{b+ax^{3/2}}\left(b+ax^{3/2}\right.}$$

input
```
Integrate[(a + b/x^(3/2))^(2/3),x]
```

output
```
-1/3*((a + b/x^(3/2))^(2/3)*(b^(1/3) - (b + a*x^(3/2))^(1/3))*(b^(2/3) + b
^(1/3)*(b + a*x^(3/2))^(1/3) + (b + a*x^(3/2))^(2/3))^2*(3*(b + a*x^(3/2))
^(2/3) + 2*Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b + a*x^(3/2))^(1/3))/b^(1/3))/
Sqrt[3]] + 2*b^(2/3)*Log[-b^(1/3) + (b + a*x^(3/2))^(1/3)] - b^(2/3)*Log[b
^(2/3) + b^(1/3)*(b + a*x^(3/2))^(1/3) + (b + a*x^(3/2))^(2/3)]))/(a*Sqrt[
x]*(b + a*x^(3/2))^(1/3)*(b + a*x^(3/2) + b^(2/3)*(b + a*x^(3/2))^(1/3) +
b^(1/3)*(b + a*x^(3/2))^(2/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {774, 858, 809, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx$$

$$\Big\downarrow \text{774}$$

$$2\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{858}$$

$$-2\int \frac{\left(bx^{3/2}+a\right)^{2/3}}{x^{3/2}}d\frac{1}{\sqrt{x}}$$

$$\Big\downarrow \text{809}$$

$$-2\left(b\int \frac{1}{\sqrt[3]{bx^{3/2}+a}}d\frac{1}{\sqrt{x}}-\frac{\left(a+bx^{3/2}\right)^{2/3}}{2x}\right)$$

$$\Big\downarrow \text{769}$$

$$-2\left(b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}}{\sqrt{x}\sqrt[3]{a+bx^{3/2}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^{3/2}}-\frac{\sqrt[3]{b}}{\sqrt{x}}\right)}{2\sqrt[3]{b}}\right)-\frac{\left(a+bx^{3/2}\right)^{2/3}}{2x}\right)$$

input
```
Int[(a + b/x^(3/2))^(2/3),x]
```

output
```
-2*(-1/2*(a + b*x^(3/2))^(2/3)/x + b*(ArcTan[(1 + (2*b^(1/3))/(Sqrt[x]*(a
+ b*x^(3/2))^(1/3)))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)/Sqrt[x]) +
 (a + b*x^(3/2))^(1/3)]/(2*b^(1/3))))
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}}dx$$

input
```
int((a+b/x^(3/2))^(2/3),x)
```

output
```
int((a+b/x^(3/2))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx = \text{Timed out}$$

input
```
integrate((a+b/x^(3/2))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.48

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx = -\frac{2a^{\frac{2}{3}}x\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{be^{i\pi}}{ax^{\frac{3}{2}}}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((a+b/x**(3/2))**(2/3),x)
```

output
```
-2*a**(2/3)*x*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*exp_polar(I*pi)/(a
*x**(3/2)))/(3*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx = \frac{2}{3}\sqrt{3}b^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sqrt{x}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right) + \left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}}x - \frac{1}{3}b^{\frac{2}{3}}\log\left(\left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}}x + \left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{1}{3}}b^{\frac{1}{3}}\sqrt{x}+b^{\frac{2}{3}}\right) + \frac{2}{3}b^{\frac{2}{3}}\log\left(\left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{1}{3}}\sqrt{x}-b^{\frac{1}{3}}\right)$$

input
```
integrate((a+b/x^(3/2))^(2/3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(2*(a + b/x^(3/2))^(1/3)*sqrt(x) +
b^(1/3))/b^(1/3)) + (a + b/x^(3/2))^(2/3)*x - 1/3*b^(2/3)*log((a + b/x^(3/
2))^(2/3)*x + (a + b/x^(3/2))^(1/3)*b^(1/3)*sqrt(x) + b^(2/3)) + 2/3*b^(2/
3)*log((a + b/x^(3/2))^(1/3)*sqrt(x) - b^(1/3))
```

Giac [F]

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx = \int \left(a+\frac{b}{x^{\frac{3}{2}}}\right)^{\frac{2}{3}} dx$$

input
```
integrate((a+b/x^(3/2))^(2/3),x, algorithm="giac")
```

output
```
integrate((a + b/x^(3/2))^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.39

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx = \frac{x\left(a+\frac{b}{x^{3/2}}\right)^{2/3} {}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\ \frac{1}{3};\ -\frac{b}{a\,x^{3/2}}\right)}{\left(\frac{b}{a\,x^{3/2}}+1\right)^{2/3}}$$

input
```
int((a + b/x^(3/2))^(2/3),x)
```

output
```
(x*(a + b/x^(3/2))^(2/3)*hypergeom([-2/3, -2/3], 1/3, -b/(a*x^(3/2))))/(b/
(a*x^(3/2)) + 1)^(2/3)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^{3/2}}\right)^{2/3} dx = \left(\sqrt{x}\,ax+b\right)^{\frac{2}{3}} - \left(\int \frac{\left(\sqrt{x}\,ax+b\right)^{\frac{2}{3}}}{a^2x^4-b^2x}dx\right)b^2 + \left(\int \frac{\sqrt{x}\left(\sqrt{x}\,ax+b\right)^{\frac{2}{3}}}{a^2x^3-b^2}dx\right)ab$$

input
```
int((a+b/x^(3/2))^(2/3),x)
```

output
```
(sqrt(x)*a*x + b)**(2/3) - int((sqrt(x)*a*x + b)**(2/3)/(a**2*x**4 - b**2*
x),x)*b**2 + int((sqrt(x)*(sqrt(x)*a*x + b)**(2/3))/(a**2*x**3 - b**2),x)*
a*b
```

3.285 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4 \, dx=\frac{3}{14} b x^{14/3}+\frac{a x^5}{5}$$

output
```
3/14*b*x^(14/3)+1/5*a*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4 \, dx=\frac{1}{70} \left(15 b+14 a \sqrt[3]{x}\right) x^{14/3}$$

input
```
Integrate[(a + b/x^(1/3))*x^4,x]
```

output
```
((15*b + 14*a*x^(1/3))*x^(14/3))/70
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{\sqrt[3]{x}}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a x^4+b x^{11/3}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a x^5}{5}+\frac{3}{14} b x^{14/3}$$

input
```
Int[(a + b/x^(1/3))*x^4,x]
```

output
```
(3*b*x^(14/3))/14 + (a*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{3b\,x^{\frac{14}{3}}}{14}+\frac{a\,x^{5}}{5}$	14
default	$\frac{3b\,x^{\frac{14}{3}}}{14}+\frac{a\,x^{5}}{5}$	14
trager	$\frac{a(x^{4}+x^{3}+x^{2}+x+1)(-1+x)}{5}+\frac{3b\,x^{\frac{14}{3}}}{14}$	26
orering	$\frac{13x^{5}\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{35}-\frac{3x^{2}\left(-\frac{b\,x^{\frac{8}{3}}}{3}+4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{3}\right)}{70}$	38

input
```
int((a+b/x^(1/3))*x^4,x,method=_RETURNVERBOSE)
```

output
```
3/14*b*x^(14/3)+1/5*a*x^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{4}\,dx=\frac{1}{5}\,ax^{5}+\frac{3}{14}\,bx^{\frac{14}{3}}$$

input
```
integrate((a+b/x^(1/3))*x^4,x, algorithm="fricas")
```

output
```
1/5*a*x^5 + 3/14*b*x^(14/3)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4 \, dx = \frac{ax^5}{5} + \frac{3bx^{\frac{14}{3}}}{14}$$

input
```
integrate((a+b/x**(1/3))*x**4,x)
```

output
```
a*x**5/5 + 3*b*x**(14/3)/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4 \, dx = \frac{1}{70} \left(14\,a + \frac{15\,b}{x^{\frac{1}{3}}}\right) x^5$$

input
```
integrate((a+b/x^(1/3))*x^4,x, algorithm="maxima")
```

output
```
1/70*(14*a + 15*b/x^(1/3))*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4 \, dx = \frac{1}{5}\,ax^5 + \frac{3}{14}\,bx^{\frac{14}{3}}$$

input
```
integrate((a+b/x^(1/3))*x^4,x, algorithm="giac")
```

output
```
1/5*a*x^5 + 3/14*b*x^(14/3)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4\,dx = \frac{a\,x^5}{5} + \frac{3\,b\,x^{14/3}}{14}$$

input
```
int(x^4*(a + b/x^(1/3)),x)
```

output
```
(a*x^5)/5 + (3*b*x^(14/3))/14
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^4\,dx = \frac{x^4\left(15x^{\frac{2}{3}}b+14ax\right)}{70}$$

input
```
int((a+b/x^(1/3))*x^4,x)
```

output
```
(x**4*(15*x**(2/3)*b + 14*a*x))/70
```

3.286 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^3 \, dx = \frac{3}{11} b x^{11/3} + \frac{a x^4}{4}$$

output
```
3/11*b*x^(11/3)+1/4*a*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^3 \, dx = \frac{3}{11} b x^{11/3} + \frac{a x^4}{4}$$

input
```
Integrate[(a + b/x^(1/3))*x^3,x]
```

output
```
(3*b*x^(11/3))/11 + (a*x^4)/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{\sqrt[3]{x}}\right)dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(ax^3+bx^{8/3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^4}{4}+\frac{3}{11}bx^{11/3}$$

input
```
Int[(a + b/x^(1/3))*x^3,x]
```

output
```
(3*b*x^(11/3))/11 + (a*x^4)/4
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{3b\,x^{\frac{11}{3}}}{11}+\frac{a\,x^{4}}{4}$	14
default	$\frac{3b\,x^{\frac{11}{3}}}{11}+\frac{a\,x^{4}}{4}$	14
trager	$\frac{a\left(x^{3}+x^{2}+x+1\right)\left(-1+x\right)}{4}+\frac{3b\,x^{\frac{11}{3}}}{11}$	23
orering	$\frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{4}}{11}-\frac{3x^{2}\left(-\frac{b\,x^{\frac{5}{3}}}{3}+3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{2}\right)}{44}$	38

input
```
int((a+b/x^(1/3))*x^3,x,method=_RETURNVERBOSE)
```

output
```
3/11*b*x^(11/3)+1/4*a*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3\,dx=\frac{1}{4}\,ax^4+\frac{3}{11}\,bx^{\frac{11}{3}}$$

input
```
integrate((a+b/x^(1/3))*x^3,x, algorithm="fricas")
```

output
```
1/4*a*x^4 + 3/11*b*x^(11/3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^3\,dx=\frac{ax^4}{4}+\frac{3bx^{\frac{11}{3}}}{11}$$

input
```
integrate((a+b/x**(1/3))*x**3,x)
```

output
```
a*x**4/4 + 3*b*x**(11/3)/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^3\,dx=\frac{1}{44}\left(11\,a+\frac{12\,b}{x^{\frac{1}{3}}}\right)x^4$$

input
```
integrate((a+b/x^(1/3))*x^3,x, algorithm="maxima")
```

output
```
1/44*(11*a + 12*b/x^(1/3))*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^3\,dx=\frac{1}{4}\,ax^4+\frac{3}{11}\,bx^{\frac{11}{3}}$$

input
```
integrate((a+b/x^(1/3))*x^3,x, algorithm="giac")
```

output
```
1/4*a*x^4 + 3/11*b*x^(11/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^3\,dx=\frac{a\,x^4}{4}+\frac{3\,b\,x^{11/3}}{11}$$

input
```
int(x^3*(a + b/x^(1/3)),x)
```

output
```
(a*x^4)/4 + (3*b*x^(11/3))/11
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^3\,dx=\frac{x^3\left(12x^{\frac{2}{3}}b+11ax\right)}{44}$$

input
```
int((a+b/x^(1/3))*x^3,x)
```

output
```
(x**3*(12*x**(2/3)*b + 11*a*x))/44
```

3.287 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^2 \, dx = \frac{3}{8} b x^{8/3} + \frac{a x^3}{3}$$

output
```
3/8*b*x^(8/3)+1/3*a*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^2 \, dx = \frac{3}{8} b x^{8/3} + \frac{a x^3}{3}$$

input
```
Integrate[(a + b/x^(1/3))*x^2,x]
```

output
```
(3*b*x^(8/3))/8 + (a*x^3)/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a + \frac{b}{\sqrt[3]{x}}\right) dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^2 + bx^{5/3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3} + \frac{3}{8}bx^{8/3}$$

input
```
Int[(a + b/x^(1/3))*x^2,x]
```

output
```
(3*b*x^(8/3))/8 + (a*x^3)/3
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{3b\,x^{\frac{8}{3}}}{8}+\frac{a\,x^{3}}{3}$	14
default	$\frac{3b\,x^{\frac{8}{3}}}{8}+\frac{a\,x^{3}}{3}$	14
trager	$\frac{a(x^{2}+x+1)(-1+x)}{3}+\frac{3b\,x^{\frac{8}{3}}}{8}$	20
orering	$\frac{7\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{3}}{12}-\frac{x^{2}\left(-\frac{b\,x^{\frac{2}{3}}}{3}+2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x\right)}{8}$	36

input
```
int((a+b/x^(1/3))*x^2,x,method=_RETURNVERBOSE)
```

output
```
3/8*b*x^(8/3)+1/3*a*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2}\,dx=\frac{1}{3}\,ax^{3}+\frac{3}{8}\,bx^{\frac{8}{3}}$$

input
```
integrate((a+b/x^(1/3))*x^2,x, algorithm="fricas")
```

output
```
1/3*a*x^3 + 3/8*b*x^(8/3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^2\, dx = \frac{ax^3}{3} + \frac{3bx^{\frac{8}{3}}}{8}$$

input
```
integrate((a+b/x**(1/3))*x**2,x)
```

output
```
a*x**3/3 + 3*b*x**(8/3)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^2\, dx = \frac{1}{24} \left(8\, a + \frac{9\, b}{x^{\frac{1}{3}}}\right) x^3$$

input
```
integrate((a+b/x^(1/3))*x^2,x, algorithm="maxima")
```

output
```
1/24*(8*a + 9*b/x^(1/3))*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x^2\, dx = \frac{1}{3}\, ax^3 + \frac{3}{8}\, bx^{\frac{8}{3}}$$

input
```
integrate((a+b/x^(1/3))*x^2,x, algorithm="giac")
```

output
```
1/3*a*x^3 + 3/8*b*x^(8/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^2\,dx=\frac{a\,x^3}{3}+\frac{3\,b\,x^{8/3}}{8}$$

input
```
int(x^2*(a + b/x^(1/3)),x)
```

output
```
(a*x^3)/3 + (3*b*x^(8/3))/8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)x^2\,dx=\frac{x^2\left(9x^{\frac{2}{3}}b+8ax\right)}{24}$$

input
```
int((a+b/x^(1/3))*x^2,x)
```

output
```
(x**2*(9*x**(2/3)*b + 8*a*x))/24
```

3.288 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x \, dx=\frac{3}{5}bx^{5/3}+\frac{ax^2}{2}$$

output
```
3/5*b*x^(5/3)+1/2*a*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x \, dx=\frac{3}{5}bx^{5/3}+\frac{ax^2}{2}$$

input
```
Integrate[(a + b/x^(1/3))*x,x]
```

output
```
(3*b*x^(5/3))/5 + (a*x^2)/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{\sqrt[3]{x}}\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax+bx^{2/3}\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}+\frac{3}{5}bx^{5/3}$$

input
```
Int[(a + b/x^(1/3))*x,x]
```

output
```
(3*b*x^(5/3))/5 + (a*x^2)/2
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{3b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{2}}{2}$	14
default	$\frac{3b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{2}}{2}$	14
trager	$\frac{(-1+x)a(1+x)}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$	17
orering	$\frac{4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{2}}{5}-\frac{3x^{2}\left(\frac{2b}{3x^{\frac{1}{3}}}+a\right)}{10}$	27

input
```
int((a+b/x^(1/3))*x,x,method=_RETURNVERBOSE)
```

output
```
3/5*b*x^(5/3)+1/2*a*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int\left(a+\frac{b}{\sqrt[3]{x}}\right)x\,dx=\frac{1}{2}\,ax^{2}+\frac{3}{5}\,bx^{\frac{5}{3}}$$

input
```
integrate((a+b/x^(1/3))*x,x, algorithm="fricas")
```

output
```
1/2*a*x^2 + 3/5*b*x^(5/3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x\, dx = \frac{ax^2}{2} + \frac{3bx^{\frac{5}{3}}}{5}$$

input
```
integrate((a+b/x**(1/3))*x,x)
```

output
```
a*x**2/2 + 3*b*x**(5/3)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x\, dx = \frac{1}{10} \left(5\,a + \frac{6\,b}{x^{\frac{1}{3}}}\right) x^2$$

input
```
integrate((a+b/x^(1/3))*x,x, algorithm="maxima")
```

output
```
1/10*(5*a + 6*b/x^(1/3))*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x\, dx = \frac{1}{2}\, ax^2 + \frac{3}{5}\, bx^{\frac{5}{3}}$$

input
```
integrate((a+b/x^(1/3))*x,x, algorithm="giac")
```

output
```
1/2*a*x^2 + 3/5*b*x^(5/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x\,dx = \frac{a\,x^2}{2}+\frac{3\,b\,x^{5/3}}{5}$$

input
```
int(x*(a + b/x^(1/3)),x)
```

output
```
(a*x^2)/2 + (3*b*x^(5/3))/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) x\,dx = \frac{x\left(6x^{\frac{2}{3}}b+5ax\right)}{10}$$

input
```
int((a+b/x^(1/3))*x,x)
```

output
```
(x*(6*x**(2/3)*b + 5*a*x))/10
```

3.289 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) \, dx=\frac{3}{2} b x^{2/3}+a x$$

output
```
3/2*b*x^(2/3)+a*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) \, dx=\frac{3}{2} b x^{2/3}+a x$$

input
```
Integrate[a + b/x^(1/3),x]
```

output
```
(3*b*x^(2/3))/2 + a*x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) dx$$

$$\Big\downarrow \ 2009$$

$$ax+\frac{3}{2}bx^{2/3}$$

input
```
Int[a + b/x^(1/3),x]
```

output
```
(3*b*x^(2/3))/2 + a*x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{3b\,x^{\frac{2}{3}}}{2}+ax$	11
default	$\frac{3b\,x^{\frac{2}{3}}}{2}+ax$	11
risch	$\frac{3b\,x^{\frac{2}{3}}}{2}+ax$	11
trager	$a(-1+x)+\frac{3b\,x^{\frac{2}{3}}}{2}$	13
orering	$\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x+\frac{b\,x^{\frac{2}{3}}}{2}$	17

input
```
int(a+b/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*b*x^(2/3)+a*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int\left(a+\frac{b}{\sqrt[3]{x}}\right)dx=ax+\frac{3}{2}bx^{\frac{2}{3}}$$

input
```
integrate(a+b/x^(1/3),x, algorithm="fricas")
```

output
```
a*x + 3/2*b*x^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) dx = ax + \frac{3bx^{\frac{2}{3}}}{2}$$

input
```
integrate(a+b/x**(1/3),x)
```

output
```
a*x + 3*b*x**(2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) dx = ax + \frac{3}{2} bx^{\frac{2}{3}}$$

input
```
integrate(a+b/x^(1/3),x, algorithm="maxima")
```

output
```
a*x + 3/2*b*x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right) dx = ax + \frac{3}{2} bx^{\frac{2}{3}}$$

input
```
integrate(a+b/x^(1/3),x, algorithm="giac")
```

output
```
a*x + 3/2*b*x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)\,dx=a\,x+\frac{3\,b\,x^{2/3}}{2}$$

input
```
int(a + b/x^(1/3),x)
```

output
```
a*x + (3*b*x^(2/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)\,dx=\frac{3x^{\frac{2}{3}}b}{2}+ax$$

input
```
int(a+b/x^(1/3),x)
```

output
```
(3*x**(2/3)*b + 2*a*x)/2
```

3.290 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x} \, dx = -\frac{3b}{\sqrt[3]{x}}+a\log(x)$$

output
```
-3*b/x^(1/3)+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x} \, dx = -\frac{3b}{\sqrt[3]{x}}+a\log(x)$$

input
```
Integrate[(a + b/x^(1/3))/x,x]
```

output
```
(-3*b)/x^(1/3) + a*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x}+\frac{b}{x^{4/3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)-\frac{3b}{\sqrt[3]{x}}$$

input
```
Int[(a + b/x^(1/3))/x,x]
```

output
```
(-3*b)/x^(1/3) + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{3b}{x^{\frac{1}{3}}}+a\ln\left(x\right)$	12
default	$-\frac{3b}{x^{\frac{1}{3}}}+a\ln\left(x\right)$	12
trager	$-\frac{3b}{x^{\frac{1}{3}}}-a\ln\left(\frac{1}{x}\right)$	15

input
```
int((a+b/x^(1/3))/x,x,method=_RETURNVERBOSE)
```

output
```
-3*b/x^(1/3)+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx=\frac{3\left(ax\log\left(x^{\frac{1}{3}}\right)-bx^{\frac{2}{3}}\right)}{x}$$

input
```
integrate((a+b/x^(1/3))/x,x, algorithm="fricas")
```

output
```
3*(a*x*log(x^(1/3)) - b*x^(2/3))/x
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx=a\log\left(x\right)-\frac{3b}{\sqrt[3]{x}}$$

input
```
integrate((a+b/x**(1/3))/x,x)
```

output
```
a*log(x) - 3*b/x**(1/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx = a\log(x) - \frac{3\,b}{x^{\frac{1}{3}}}$$

input
```
integrate((a+b/x^(1/3))/x,x, algorithm="maxima")
```

output
```
a*log(x) - 3*b/x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx = a\log(|x|) - \frac{3\,b}{x^{\frac{1}{3}}}$$

input
```
integrate((a+b/x^(1/3))/x,x, algorithm="giac")
```

output
```
a*log(abs(x)) - 3*b/x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx = 3\,a\,\ln\left(x^{1/3}\right) - \frac{3\,b}{x^{1/3}}$$

input
```
int((a + b/x^(1/3))/x,x)
```

output
```
3*a*log(x^(1/3)) - (3*b)/x^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x}\,dx = \frac{x^{\frac{1}{3}}\log(x)\,a-3b}{x^{\frac{1}{3}}}$$

input
```
int((a+b/x^(1/3))/x,x)
```

output
```
(x**(1/3)*log(x)*a - 3*b)/x**(1/3)
```

3.291 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2} \, dx = -\frac{3b}{4x^{4/3}} - \frac{a}{x}$$

output
```
-3/4*b/x^(4/3)-a/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2} \, dx = -\frac{3b}{4x^{4/3}} - \frac{a}{x}$$

input
```
Integrate[(a + b/x^(1/3))/x^2,x]
```

output
```
(-3*b)/(4*x^(4/3)) - a/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^2}+\frac{b}{x^{7/3}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{x}-\frac{3b}{4x^{4/3}}$$

input
```
Int[(a + b/x^(1/3))/x^2,x]
```

output
```
(-3*b)/(4*x^(4/3)) - a/x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{3b}{4x^{\frac{4}{3}}}-\frac{a}{x}$	14
default	$-\frac{3b}{4x^{\frac{4}{3}}}-\frac{a}{x}$	14
trager	$\frac{a(-1+x)}{x}-\frac{3b}{4x^{\frac{4}{3}}}$	16
orering	$-\frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{2x}-\frac{3\left(-\frac{b}{3x^{\frac{10}{3}}}-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{x^{3}}\right)x^{2}}{4}$	38

input
```
int((a+b/x^(1/3))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4*b/x^(4/3)-a/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx = -\frac{4\,ax+3\,bx^{\frac{2}{3}}}{4\,x^2}$$

input
```
integrate((a+b/x^(1/3))/x^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a*x + 3*b*x^(2/3))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx = -\frac{a}{x} - \frac{3b}{4x^{\frac{4}{3}}}$$

input
```
integrate((a+b/x**(1/3))/x**2,x)
```

output
```
-a/x - 3*b/(4*x**(4/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx = -\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4}{4\,b^3} + \frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a}{b^3} - \frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^2}{2\,b^3}$$

input
```
integrate((a+b/x^(1/3))/x^2,x, algorithm="maxima")
```

output
```
-3/4*(a + b/x^(1/3))^4/b^3 + 2*(a + b/x^(1/3))^3*a/b^3 - 3/2*(a + b/x^(1/3
))^2*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx = -\frac{4\,ax^{\frac{1}{3}} + 3\,b}{4\,x^{\frac{4}{3}}}$$

input
```
integrate((a+b/x^(1/3))/x^2,x, algorithm="giac")
```

output
```
-1/4*(4*a*x^(1/3) + 3*b)/x^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx=-\frac{a}{x}-\frac{3\,b}{4\,x^{4/3}}$$

input
```
int((a + b/x^(1/3))/x^2,x)
```

output
```
- a/x - (3*b)/(4*x^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^2}\,dx=\frac{-4x^{\frac{1}{3}}a-3b}{4x^{\frac{4}{3}}}$$

input
```
int((a+b/x^(1/3))/x^2,x)
```

output
```
( - 4*x**(1/3)*a - 3*b)/(4*x**(1/3)*x)
```

3.292 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx = -\frac{3b}{7x^{7/3}} - \frac{a}{2x^2}$$

output
```
-3/7*b/x^(7/3)-1/2*a/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx = \frac{-6b-7a\sqrt[3]{x}}{14x^{7/3}}$$

input
```
Integrate[(a + b/x^(1/3))/x^3,x]
```

output
```
(-6*b - 7*a*x^(1/3))/(14*x^(7/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x^{10/3}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{2x^2}-\frac{3b}{7x^{7/3}}$$

input
```
Int[(a + b/x^(1/3))/x^3,x]
```

output
```
(-3*b)/(7*x^(7/3)) - a/(2*x^2)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{3b}{7x^{\frac{7}{3}}}-\frac{a}{2x^{2}}$	14
default	$-\frac{3b}{7x^{\frac{7}{3}}}-\frac{a}{2x^{2}}$	14
trager	$\frac{(-1+x)a(1+x)}{2x^{2}}-\frac{3b}{7x^{\frac{7}{3}}}$	20
orering	$-\frac{8\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{7x^{2}}-\frac{3x^{2}\left(-\frac{b}{3x^{\frac{13}{3}}}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{x^{4}}\right)}{14}$	38

input
```
int((a+b/x^(1/3))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/7*b/x^(7/3)-1/2*a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^{3}}\,dx=-\frac{7\,ax+6\,bx^{\frac{2}{3}}}{14\,x^{3}}$$

input
```
integrate((a+b/x^(1/3))/x^3,x, algorithm="fricas")
```

output
```
-1/14*(7*a*x + 6*b*x^(2/3))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx = -\frac{a}{2x^2} - \frac{3b}{7x^{\frac{7}{3}}}$$

input
```
integrate((a+b/x**(1/3))/x**3,x)
```

output
```
-a/(2*x**2) - 3*b/(7*x**(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(13) = 26.

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.16

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx = -\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7}{7\,b^6} + \frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a}{2\,b^6} - \frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^2}{b^6} + \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^3}{2\,b^6} - \frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^4}{b^6} + \frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^5}{2\,b^6}$$

input
```
integrate((a+b/x^(1/3))/x^3,x, algorithm="maxima")
```

output
```
-3/7*(a + b/x^(1/3))^7/b^6 + 5/2*(a + b/x^(1/3))^6*a/b^6 - 6*(a + b/x^(1/3
))^5*a^2/b^6 + 15/2*(a + b/x^(1/3))^4*a^3/b^6 - 5*(a + b/x^(1/3))^3*a^4/b^
6 + 3/2*(a + b/x^(1/3))^2*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx = -\frac{7\,ax^{\frac{1}{3}}+6\,b}{14\,x^{\frac{7}{3}}}$$

input
```
integrate((a+b/x^(1/3))/x^3,x, algorithm="giac")
```

output
```
-1/14*(7*a*x^(1/3) + 6*b)/x^(7/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx = -\frac{a}{2\,x^2}-\frac{3\,b}{7\,x^{7/3}}$$

input
```
int((a + b/x^(1/3))/x^3,x)
```

output
```
- a/(2*x^2) - (3*b)/(7*x^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^3}\,dx = \frac{-7x^{\frac{1}{3}}a-6b}{14x^{\frac{7}{3}}}$$

input
```
int((a+b/x^(1/3))/x^3,x)
```

output
```
( - 7*x**(1/3)*a - 6*b)/(14*x**(1/3)*x**2)
```

3.293 $\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4} dx = -\frac{3b}{10x^{10/3}} - \frac{a}{3x^3}$$

output
```
-3/10*b/x^(10/3)-1/3*a/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4} dx = \frac{-9b-10a\sqrt[3]{x}}{30x^{10/3}}$$

input
```
Integrate[(a + b/x^(1/3))/x^4,x]
```

output
```
(-9*b - 10*a*x^(1/3))/(30*x^(10/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^{13/3}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{3x^3}-\frac{3b}{10x^{10/3}}$$

input
```
Int[(a + b/x^(1/3))/x^4,x]
```

output
```
(-3*b)/(10*x^(10/3)) - a/(3*x^3)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{3b}{10x^{\frac{10}{3}}}-\frac{a}{3x^{3}}$	14
default	$-\frac{3b}{10x^{\frac{10}{3}}}-\frac{a}{3x^{3}}$	14
trager	$\frac{a(x^{2}+x+1)(-1+x)}{3x^{3}}-\frac{3b}{10x^{\frac{10}{3}}}$	23
orering	$-\frac{11\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{15x^{3}}-\frac{x^{2}\left(-\frac{b}{3x^{\frac{16}{3}}}-\frac{4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{x^{5}}\right)}{10}$	38

input
```
int((a+b/x^(1/3))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-3/10*b/x^(10/3)-1/3*a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx=-\frac{10\,ax+9\,bx^{\frac{2}{3}}}{30\,x^4}$$

input
```
integrate((a+b/x^(1/3))/x^4,x, algorithm="fricas")
```

output
```
-1/30*(10*a*x + 9*b*x^(2/3))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx = -\frac{a}{3x^3} - \frac{3b}{10x^{\frac{10}{3}}}$$

input
```
integrate((a+b/x**(1/3))/x**4,x)
```

output
```
-a/(3*x**3) - 3*b/(10*x**(10/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 7.84

$$\begin{aligned}\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx = &-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{10}}{10\,b^9} + \frac{8\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{9}a}{3\,b^9} - \frac{21\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{8}a^2}{2\,b^9} \\ &+ \frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{7}a^3}{b^9} - \frac{35\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{6}a^4}{b^9} + \frac{168\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{5}a^5}{5\,b^9} \\ &- \frac{21\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{4}a^6}{b^9} + \frac{8\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{3}a^7}{b^9} - \frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}a^8}{2\,b^9}\end{aligned}$$

input
```
integrate((a+b/x^(1/3))/x^4,x, algorithm="maxima")
```

output
```
-3/10*(a + b/x^(1/3))^10/b^9 + 8/3*(a + b/x^(1/3))^9*a/b^9 - 21/2*(a + b/x
^(1/3))^8*a^2/b^9 + 24*(a + b/x^(1/3))^7*a^3/b^9 - 35*(a + b/x^(1/3))^6*a^
4/b^9 + 168/5*(a + b/x^(1/3))^5*a^5/b^9 - 21*(a + b/x^(1/3))^4*a^6/b^9 + 8
*(a + b/x^(1/3))^3*a^7/b^9 - 3/2*(a + b/x^(1/3))^2*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx = -\frac{10\,ax^{\frac{1}{3}}+9\,b}{30\,x^{\frac{10}{3}}}$$

input
```
integrate((a+b/x^(1/3))/x^4,x, algorithm="giac")
```

output
```
-1/30*(10*a*x^(1/3) + 9*b)/x^(10/3)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx = -\frac{a}{3\,x^3}-\frac{3\,b}{10\,x^{10/3}}$$

input
```
int((a + b/x^(1/3))/x^4,x)
```

output
```
- a/(3*x^3) - (3*b)/(10*x^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+\frac{b}{\sqrt[3]{x}}}{x^4}\,dx = \frac{-10x^{\frac{1}{3}}a-9b}{30x^{\frac{10}{3}}}$$

input
```
int((a+b/x^(1/3))/x^4,x)
```

output
```
( - 10*x**(1/3)*a - 9*b)/(30*x**(1/3)*x**3)
```

3.294 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx=\frac{3}{13}b^2x^{13/3}+\frac{3}{7}abx^{14/3}+\frac{a^2x^5}{5}$$

output
```
3/13*b^2*x^(13/3)+3/7*a*b*x^(14/3)+1/5*a^2*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx=\frac{1}{455}\left(105b^2+195ab\sqrt[3]{x}+91a^2x^{2/3}\right)x^{13/3}$$

input
```
Integrate[(a + b/x^(1/3))^2*x^4,x]
```

output
```
((105*b^2 + 195*a*b*x^(1/3) + 91*a^2*x^(2/3))*x^(13/3))/455
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx$$

$$\Big\downarrow \ 795$$

$$\int x^{10/3} \left(a \sqrt[3]{x}+b\right)^2 dx$$

$$\Big\downarrow \ 798$$

$$3 \int \left(\sqrt[3]{x} a+b\right)^2 x^4 d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3 \int \left(a^2 x^{14/3}+2 a b x^{13/3}+b^2 x^4\right) d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3 \left(\frac{a^2 x^5}{15}+\frac{1}{7} a b x^{14/3}+\frac{1}{13} b^2 x^{13/3}\right)$$

input
```
Int[(a + b/x^(1/3))^2*x^4,x]
```

output
```
3*((b^2*x^(13/3))/13 + (a*b*x^(14/3))/7 + (a^2*x^5)/15)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{3b^2x^{\frac{13}{3}}}{13}+\frac{3ab\,x^{\frac{14}{3}}}{7}+\frac{a^2x^5}{5}$
default	$\frac{3b^2x^{\frac{13}{3}}}{13}+\frac{3ab\,x^{\frac{14}{3}}}{7}+\frac{a^2x^5}{5}$
trager	$\frac{a^2(x^4+x^3+x^2+x+1)(-1+x)}{5}+\frac{3b^2x^{\frac{13}{3}}}{13}+\frac{3ab\,x^{\frac{14}{3}}}{7}$
orering	$\frac{47x^5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2}{91}-\frac{99x^2\left(-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{\frac{8}{3}}b}{3}+4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x^3\right)}{910}+\frac{9x^3\left(\frac{2x^{\frac{4}{3}}b^2}{9}-\frac{40\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{\frac{5}{3}}b}{9}+12\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x^2\right.}{910}$

input
```
int((a+b/x^(1/3))^2*x^4,x,method=_RETURNVERBOSE)
```

output
```
3/13*b^2*x^(13/3)+3/7*a*b*x^(14/3)+1/5*a^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx = \frac{1}{5} a^2 x^5 + \frac{3}{7} abx^{\frac{14}{3}} + \frac{3}{13} b^2 x^{\frac{13}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x^4,x, algorithm="fricas")
```

output
```
1/5*a^2*x^5 + 3/7*a*b*x^(14/3) + 3/13*b^2*x^(13/3)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx = \frac{a^2 x^5}{5} + \frac{3abx^{\frac{14}{3}}}{7} + \frac{3b^2 x^{\frac{13}{3}}}{13}$$

input
```
integrate((a+b/x**(1/3))**2*x**4,x)
```

output
```
a**2*x**5/5 + 3*a*b*x**(14/3)/7 + 3*b**2*x**(13/3)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx = \frac{1}{455} \left(91 a^2 + \frac{195 ab}{x^{\frac{1}{3}}} + \frac{105 b^2}{x^{\frac{2}{3}}}\right) x^5$$

input
```
integrate((a+b/x^(1/3))^2*x^4,x, algorithm="maxima")
```

output
```
1/455*(91*a^2 + 195*a*b/x^(1/3) + 105*b^2/x^(2/3))*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx = \frac{1}{5}\,a^2x^5 + \frac{3}{7}\,abx^{\frac{14}{3}} + \frac{3}{13}\,b^2x^{\frac{13}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x^4,x, algorithm="giac")
```

output
```
1/5*a^2*x^5 + 3/7*a*b*x^(14/3) + 3/13*b^2*x^(13/3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx = \frac{a^2\,x^5}{5} + \frac{3\,b^2\,x^{13/3}}{13} + \frac{3\,a\,b\,x^{14/3}}{7}$$

input
```
int(x^4*(a + b/x^(1/3))^2,x)
```

output
```
(a^2*x^5)/5 + (3*b^2*x^(13/3))/13 + (3*a*b*x^(14/3))/7
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4 \, dx = \frac{x^4\left(195x^{\frac{2}{3}}ab + 105x^{\frac{1}{3}}b^2 + 91a^2x\right)}{455}$$

input
```
int((a+b/x^(1/3))^2*x^4,x)
```

output
```
(x**4*(195*x**(2/3)*a*b + 105*x**(1/3)*b**2 + 91*a**2*x))/455
```

3.295 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3 \, dx = \frac{3}{10}b^2x^{10/3}+\frac{6}{11}abx^{11/3}+\frac{a^2x^4}{4}$$

output
```
3/10*b^2*x^(10/3)+6/11*a*b*x^(11/3)+1/4*a^2*x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3 \, dx = \frac{1}{220}\left(66b^2+120ab\sqrt[3]{x}+55a^2x^{2/3}\right)x^{10/3}$$

input
```
Integrate[(a + b/x^(1/3))^2*x^3,x]
```

output
```
((66*b^2 + 120*a*b*x^(1/3) + 55*a^2*x^(2/3))*x^(10/3))/220
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^2dx$$

$$\Big\downarrow \text{795}$$

$$\int x^{7/3}\left(a\sqrt[3]{x}+b\right)^2dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(\sqrt[3]{x}a+b\right)^2x^3d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(a^2x^{11/3}+2abx^{10/3}+b^2x^3\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^2x^4}{12}+\frac{2}{11}abx^{11/3}+\frac{1}{10}b^2x^{10/3}\right)$$

input
```
Int[(a + b/x^(1/3))^2*x^3,x]
```

output
```
3*((b^2*x^(10/3))/10 + (2*a*b*x^(11/3))/11 + (a^2*x^4)/12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{3b^2x^{\frac{10}{3}}}{10}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{a^2x^4}{4}$
default	$\frac{3b^2x^{\frac{10}{3}}}{10}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{a^2x^4}{4}$
trager	$\frac{a^2(x^3+x^2+x+1)(-1+x)}{4}+\frac{3b^2x^{\frac{10}{3}}}{10}+\frac{6ab\,x^{\frac{11}{3}}}{11}$
orering	$\frac{34\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x^4}{55}-\frac{9x^2\left(-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{\frac{5}{3}}b}{3}+3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x^2\right)}{55}+\frac{9x^3\left(\frac{2x^{\frac{1}{3}}b^2}{9}-\frac{28\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{\frac{2}{3}}b}{9}+6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x\right)}{440}$

input
```
int((a+b/x^(1/3))^2*x^3,x,method=_RETURNVERBOSE)
```

output
```
3/10*b^2*x^(10/3)+6/11*a*b*x^(11/3)+1/4*a^2*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3\,dx = \frac{1}{4}\,a^2x^4 + \frac{6}{11}\,abx^{\frac{11}{3}} + \frac{3}{10}\,b^2x^{\frac{10}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x^3,x, algorithm="fricas")
```

output
```
1/4*a^2*x^4 + 6/11*a*b*x^(11/3) + 3/10*b^2*x^(10/3)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3\,dx = \frac{a^2x^4}{4} + \frac{6abx^{\frac{11}{3}}}{11} + \frac{3b^2x^{\frac{10}{3}}}{10}$$

input
```
integrate((a+b/x**(1/3))**2*x**3,x)
```

output
```
a**2*x**4/4 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(10/3)/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3\,dx = \frac{1}{220}\left(55\,a^2 + \frac{120\,ab}{x^{\frac{1}{3}}} + \frac{66\,b^2}{x^{\frac{2}{3}}}\right)x^4$$

input
```
integrate((a+b/x^(1/3))^2*x^3,x, algorithm="maxima")
```

output
```
1/220*(55*a^2 + 120*a*b/x^(1/3) + 66*b^2/x^(2/3))*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3\,dx=\frac{1}{4}\,a^2x^4+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{10}\,b^2x^{\frac{10}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x^3,x, algorithm="giac")
```

output
```
1/4*a^2*x^4 + 6/11*a*b*x^(11/3) + 3/10*b^2*x^(10/3)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3\,dx=\frac{a^2\,x^4}{4}+\frac{3\,b^2\,x^{10/3}}{10}+\frac{6\,a\,b\,x^{11/3}}{11}$$

input
```
int(x^3*(a + b/x^(1/3))^2,x)
```

output
```
(a^2*x^4)/4 + (3*b^2*x^(10/3))/10 + (6*a*b*x^(11/3))/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3\,dx=\frac{x^3\left(120x^{\frac{2}{3}}ab+66x^{\frac{1}{3}}b^2+55a^2x\right)}{220}$$

input
```
int((a+b/x^(1/3))^2*x^3,x)
```

output
```
(x**3*(120*x**(2/3)*a*b + 66*x**(1/3)*b**2 + 55*a**2*x))/220
```

3.296 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2 \, dx = \frac{3}{7}b^2x^{7/3}+\frac{3}{4}abx^{8/3}+\frac{a^2x^3}{3}$$

output
```
3/7*b^2*x^(7/3)+3/4*a*b*x^(8/3)+1/3*a^2*x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2 \, dx = \frac{1}{84}\left(36b^2+63ab\sqrt[3]{x}+28a^2x^{2/3}\right)x^{7/3}$$

input
```
Integrate[(a + b/x^(1/3))^2*x^2,x]
```

output
```
((36*b^2 + 63*a*b*x^(1/3) + 28*a^2*x^(2/3))*x^(7/3))/84
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx$$

$$\Big\downarrow \; 795$$

$$\int x^{4/3} \left(a\sqrt[3]{x}+b\right)^2 dx$$

$$\Big\downarrow \; 798$$

$$3\int \left(\sqrt[3]{x}a+b\right)^2 x^2 d\sqrt[3]{x}$$

$$\Big\downarrow \; 49$$

$$3\int \left(a^2x^{8/3}+2abx^{7/3}+b^2x^2\right)d\sqrt[3]{x}$$

$$\Big\downarrow \; 2009$$

$$3\left(\frac{a^2x^3}{9}+\frac{1}{4}abx^{8/3}+\frac{1}{7}b^2x^{7/3}\right)$$

input
```
Int[(a + b/x^(1/3))^2*x^2,x]
```

output
```
3*((b^2*x^(7/3))/7 + (a*b*x^(8/3))/4 + (a^2*x^3)/9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{3b^{2}x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{8}{3}}}{4}+\frac{a^{2}x^{3}}{3}$	25
default	$\frac{3b^{2}x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{8}{3}}}{4}+\frac{a^{2}x^{3}}{3}$	25
trager	$\frac{a^{2}\left(x^{2}+x+1\right)\left(-1+x\right)}{3}+\frac{3b^{2}x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{8}{3}}}{4}$	31
orering	$\frac{16\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}x^{3}}{21}-\frac{15x^{2}\left(-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{\frac{2}{3}}b}{3}+2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}x\right)}{56}+\frac{3x^{3}\left(\frac{2b^{2}}{9x^{\frac{2}{3}}}-\frac{16\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{9x^{\frac{1}{3}}}+2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}\right)}{56}$	85

input
```
int((a+b/x^(1/3))^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
3/7*b^2*x^(7/3)+3/4*a*b*x^(8/3)+1/3*a^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2\,dx = \frac{1}{3}\,a^2x^3+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{7}\,b^2x^{\frac{7}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x^2,x, algorithm="fricas")
```

output
```
1/3*a^2*x^3 + 3/4*a*b*x^(8/3) + 3/7*b^2*x^(7/3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2\,dx = \frac{a^2x^3}{3}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^2x^{\frac{7}{3}}}{7}$$

input
```
integrate((a+b/x**(1/3))**2*x**2,x)
```

output
```
a**2*x**3/3 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(7/3)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2\,dx = \frac{1}{84}\left(28\,a^2+\frac{63\,ab}{x^{\frac{1}{3}}}+\frac{36\,b^2}{x^{\frac{2}{3}}}\right)x^3$$

input
```
integrate((a+b/x^(1/3))^2*x^2,x, algorithm="maxima")
```

output
```
1/84*(28*a^2 + 63*a*b/x^(1/3) + 36*b^2/x^(2/3))*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2\,dx = \frac{1}{3}\,a^2x^3 + \frac{3}{4}\,abx^{\frac{8}{3}} + \frac{3}{7}\,b^2x^{\frac{7}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x^2,x, algorithm="giac")
```

output
```
1/3*a^2*x^3 + 3/4*a*b*x^(8/3) + 3/7*b^2*x^(7/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2\,dx = \frac{a^2\,x^3}{3} + \frac{3\,b^2\,x^{7/3}}{7} + \frac{3\,a\,b\,x^{8/3}}{4}$$

input
```
int(x^2*(a + b/x^(1/3))^2,x)
```

output
```
(a^2*x^3)/3 + (3*b^2*x^(7/3))/7 + (3*a*b*x^(8/3))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2\,dx = \frac{x^2\left(63x^{\frac{2}{3}}ab + 36x^{\frac{1}{3}}b^2 + 28a^2x\right)}{84}$$

input
```
int((a+b/x^(1/3))^2*x^2,x)
```

output
```
(x**2*(63*x**(2/3)*a*b + 36*x**(1/3)*b**2 + 28*a**2*x))/84
```

3.297 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx=\frac{3}{4}b^2x^{4/3}+\frac{6}{5}abx^{5/3}+\frac{a^2x^2}{2}$$

output
```
3/4*b^2*x^(4/3)+6/5*a*b*x^(5/3)+1/2*a^2*x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx=\frac{1}{20}\left(15b^2+24ab\sqrt[3]{x}+10a^2x^{2/3}\right)x^{4/3}$$

input
```
Integrate[(a + b/x^(1/3))^2*x,x]
```

output
```
((15*b^2 + 24*a*b*x^(1/3) + 10*a^2*x^(2/3))*x^(4/3))/20
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx$$

$$\Big\downarrow \ 795$$

$$\int \sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)^2 dx$$

$$\Big\downarrow \ 798$$

$$3\int \left(\sqrt[3]{x}a+b\right)^2 x d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(x^{5/3}a^2+2bx^{4/3}a+b^2x\right) d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(\frac{a^2x^2}{6}+\frac{2}{5}abx^{5/3}+\frac{1}{4}b^2x^{4/3}\right)$$

input
```
Int[(a + b/x^(1/3))^2*x,x]
```

output
```
3*((b^2*x^(4/3))/4 + (2*a*b*x^(5/3))/5 + (a^2*x^2)/6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{3x^{\frac{4}{3}}b^2}{4}+\frac{6abx^{\frac{5}{3}}}{5}+\frac{a^2x^2}{2}$	25
default	$\frac{3x^{\frac{4}{3}}b^2}{4}+\frac{6abx^{\frac{5}{3}}}{5}+\frac{a^2x^2}{2}$	25
trager	$\frac{(-1+x)a^2(1+x)}{2}+\frac{3x^{\frac{4}{3}}b^2}{4}+\frac{6abx^{\frac{5}{3}}}{5}$	28
orering	$\frac{19\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x^2}{20}-\frac{9x^2\left(-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{3x^{\frac{1}{3}}}+\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2\right)}{20}+\frac{9x^3\left(\frac{2b^2}{9x^{\frac{5}{3}}}-\frac{4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{9x^{\frac{4}{3}}}\right)}{40}$	71

input
```
int((a+b/x^(1/3))^2*x,x,method=_RETURNVERBOSE)
```

output
```
3/4*x^(4/3)*b^2+6/5*a*b*x^(5/3)+1/2*a^2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx = \frac{1}{2}\,a^2x^2 + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{3}{4}\,b^2x^{\frac{4}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x,x, algorithm="fricas")
```

output
```
1/2*a^2*x^2 + 6/5*a*b*x^(5/3) + 3/4*b^2*x^(4/3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx = \frac{a^2x^2}{2} + \frac{6abx^{\frac{5}{3}}}{5} + \frac{3b^2x^{\frac{4}{3}}}{4}$$

input
```
integrate((a+b/x**(1/3))**2*x,x)
```

output
```
a**2*x**2/2 + 6*a*b*x**(5/3)/5 + 3*b**2*x**(4/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx = \frac{1}{20}\left(10\,a^2 + \frac{24\,ab}{x^{\frac{1}{3}}} + \frac{15\,b^2}{x^{\frac{2}{3}}}\right)x^2$$

input
```
integrate((a+b/x^(1/3))^2*x,x, algorithm="maxima")
```

output
```
1/20*(10*a^2 + 24*a*b/x^(1/3) + 15*b^2/x^(2/3))*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx=\frac{1}{2}\,a^2x^2+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{3}{4}\,b^2x^{\frac{4}{3}}$$

input
```
integrate((a+b/x^(1/3))^2*x,x, algorithm="giac")
```

output
```
1/2*a^2*x^2 + 6/5*a*b*x^(5/3) + 3/4*b^2*x^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx=\frac{a^2\,x^2}{2}+\frac{3\,b^2\,x^{4/3}}{4}+\frac{6\,a\,b\,x^{5/3}}{5}$$

input
```
int(x*(a + b/x^(1/3))^2,x)
```

output
```
(a^2*x^2)/2 + (3*b^2*x^(4/3))/4 + (6*a*b*x^(5/3))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x\,dx=\frac{x\left(24x^{\frac{2}{3}}ab+15x^{\frac{1}{3}}b^2+10a^2x\right)}{20}$$

input
```
int((a+b/x^(1/3))^2*x,x)
```

output
```
(x*(24*x**(2/3)*a*b + 15*x**(1/3)*b**2 + 10*a**2*x))/20
```

3.298 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx=\frac{\left(b+a\sqrt[3]{x}\right)^3}{a}$$

output
```
(b+a*x^(1/3))^3/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx=3b^2\sqrt[3]{x}+3abx^{2/3}+a^2x$$

input
```
Integrate[(a + b/x^(1/3))^2,x]
```

output
```
3*b^2*x^(1/3) + 3*a*b*x^(2/3) + a^2*x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{746\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx$$

$$\Big\downarrow \ 746$$

$$\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{a}$$

input
```
Int[(a + b/x^(1/3))^2,x]
```

output
```
((a + b/x^(1/3))^3*x)/a
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)^{3}}{a}$	14
default	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)^{3}}{a}$	14
trager	$a^{2}(-1+x)+3x^{\frac{1}{3}}b^{2}+3x^{\frac{2}{3}}ab$	24
orering	$\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}x-3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)x^{\frac{2}{3}}b+\frac{9x^{3}\left(\frac{2b^{2}}{9x^{\frac{8}{3}}}+\frac{8\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{9x^{\frac{7}{3}}}\right)}{2}$	53

input
```
int((a+b/x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
(b+a*x^(1/3))^3/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2\,dx=a^2x+3\,abx^{\frac{2}{3}}+3\,b^2x^{\frac{1}{3}}$$

input
```
integrate((a+b/x^(1/3))^2,x, algorithm="fricas")
```

output
```
a^2*x + 3*a*b*x^(2/3) + 3*b^2*x^(1/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx = a^2x + 3abx^{\frac{2}{3}} + 3b^2\sqrt[3]{x}$$

input
```
integrate((a+b/x**(1/3))**2,x)
```

output
```
a**2*x + 3*a*b*x**(2/3) + 3*b**2*x**(1/3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx = a^2x + 3\,abx^{\frac{2}{3}} + 3\,b^2x^{\frac{1}{3}}$$

input
```
integrate((a+b/x^(1/3))^2,x, algorithm="maxima")
```

output
```
a^2*x + 3*a*b*x^(2/3) + 3*b^2*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx = a^2x + 3\,abx^{\frac{2}{3}} + 3\,b^2x^{\frac{1}{3}}$$

input
```
integrate((a+b/x^(1/3))^2,x, algorithm="giac")
```

output
```
a^2*x + 3*a*b*x^(2/3) + 3*b^2*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx = a^2 x + 3 b^2 x^{1/3} + 3 a b x^{2/3}$$

input
```
int((a + b/x^(1/3))^2,x)
```

output
```
a^2*x + 3*b^2*x^(1/3) + 3*a*b*x^(2/3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^2 dx = 3x^{\frac{2}{3}}ab + 3x^{\frac{1}{3}}b^2 + a^2x$$

input
```
int((a+b/x^(1/3))^2,x)
```

output
```
3*x**(2/3)*a*b + 3*x**(1/3)*b**2 + a**2*x
```

3.299 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x} \, dx = -\frac{3b^2}{2x^{2/3}} - \frac{6ab}{\sqrt[3]{x}} + a^2 \log(x)$$

output
```
-3/2*b^2/x^(2/3)-6*a*b/x^(1/3)+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x} \, dx = -\frac{3b\left(b+4a\sqrt[3]{x}\right)}{2x^{2/3}} + a^2 \log(x)$$

input
```
Integrate[(a + b/x^(1/3))^2/x,x]
```

output
```
(-3*b*(b + 4*a*x^(1/3)))/(2*x^(2/3)) + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{5/3}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^2}{x}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^2}{\sqrt[3]{x}}+\frac{2ba}{x^{2/3}}+\frac{b^2}{x}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(a^2\log\left(\sqrt[3]{x}\right)-\frac{2ab}{\sqrt[3]{x}}-\frac{b^2}{2x^{2/3}}\right)$$

input
```
Int[(a + b/x^(1/3))^2/x,x]
```

output
```
3*(-1/2*b^2/x^(2/3) - (2*a*b)/x^(1/3) + a^2*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{3b^2}{2x^{\frac{2}{3}}}-\frac{6ab}{x^{\frac{1}{3}}}+a^2\ln\left(x\right)$	23
default	$-\frac{3b^2}{2x^{\frac{2}{3}}}-\frac{6ab}{x^{\frac{1}{3}}}+a^2\ln\left(x\right)$	23
trager	$-\frac{3b^2}{2x^{\frac{2}{3}}}-\frac{6ab}{x^{\frac{1}{3}}}-a^2\ln\left(\frac{1}{x}\right)$	26

input
```
int((a+b/x^(1/3))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-3/2*b^2/x^(2/3)-6*a*b/x^(1/3)+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx=\frac{3\left(2\,a^2x\log\left(x^{\frac{1}{3}}\right)-4\,abx^{\frac{2}{3}}-b^2x^{\frac{1}{3}}\right)}{2\,x}$$

input
```
integrate((a+b/x^(1/3))^2/x,x, algorithm="fricas")
```

output
```
3/2*(2*a^2*x*log(x^(1/3)) - 4*a*b*x^(2/3) - b^2*x^(1/3))/x
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx=a^2\log\left(x\right)-\frac{6ab}{\sqrt[3]{x}}-\frac{3b^2}{2x^{\frac{2}{3}}}$$

input
```
integrate((a+b/x**(1/3))**2/x,x)
```

output
```
a**2*log(x) - 6*a*b/x**(1/3) - 3*b**2/(2*x**(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx=a^2\log\left(x\right)-\frac{6\,ab}{x^{\frac{1}{3}}}-\frac{3\,b^2}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((a+b/x^(1/3))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) - 6*a*b/x^(1/3) - 3/2*b^2/x^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx = a^2\log\left(|x|\right) - \frac{3\left(4\,abx^{\frac{1}{3}}+b^2\right)}{2\,x^{\frac{2}{3}}}$$

input
```
integrate((a+b/x^(1/3))^2/x,x, algorithm="giac")
```

output
```
a^2*log(abs(x)) - 3/2*(4*a*b*x^(1/3) + b^2)/x^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx = 3\,a^2\,\ln\left(x^{1/3}\right) - \frac{\frac{3\,b^2}{2}+6\,a\,b\,x^{1/3}}{x^{2/3}}$$

input
```
int((a + b/x^(1/3))^2/x,x)
```

output
```
3*a^2*log(x^(1/3)) - ((3*b^2)/2 + 6*a*b*x^(1/3))/x^(2/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x}\,dx = \frac{2x^{\frac{2}{3}}\log(x)\,a^2-12x^{\frac{1}{3}}ab-3b^2}{2x^{\frac{2}{3}}}$$

input
```
int((a+b/x^(1/3))^2/x,x)
```

output
```
(2*x**(2/3)*log(x)*a**2 - 12*x**(1/3)*a*b - 3*b**2)/(2*x**(2/3))
```

3.300 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2} dx = -\frac{3b^2}{5x^{5/3}} - \frac{3ab}{2x^{4/3}} - \frac{a^2}{x}$$

output
```
-3/5*b^2/x^(5/3)-3/2*a*b/x^(4/3)-a^2/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2} dx = \frac{-6b^2 - 15ab\sqrt[3]{x} - 10a^2x^{2/3}}{10x^{5/3}}$$

input
```
Integrate[(a + b/x^(1/3))^2/x^2,x]
```

output
```
(-6*b^2 - 15*a*b*x^(1/3) - 10*a^2*x^(2/3))/(10*x^(5/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{8/3}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^2}{x^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^2}{x^{4/3}}+\frac{2ba}{x^{5/3}}+\frac{b^2}{x^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^2}{3x}-\frac{ab}{2x^{4/3}}-\frac{b^2}{5x^{5/3}}\right)$$

input
```
Int[(a + b/x^(1/3))^2/x^2,x]
```

output
```
3*(-1/5*b^2/x^(5/3) - (a*b)/(2*x^(4/3)) - a^2/(3*x))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{3b^2}{5x^{\frac{5}{3}}}-\frac{3ab}{2x^{\frac{4}{3}}}-\frac{a^2}{x}$	25
default	$-\frac{3b^2}{5x^{\frac{5}{3}}}-\frac{3ab}{2x^{\frac{4}{3}}}-\frac{a^2}{x}$	25
trager	$\frac{a^2(-1+x)}{x}-\frac{3b^2}{5x^{\frac{5}{3}}}-\frac{3ab}{2x^{\frac{4}{3}}}$	27
orering	$-\frac{23\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2}{5x}-\frac{63\left(-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{3x^{\frac{10}{3}}}-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2}{x^3}\right)x^2}{20}-\frac{9x^3\left(\frac{2b^2}{9x^{\frac{14}{3}}}+\frac{32\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{9x^{\frac{13}{3}}}+\frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2}{x^4}\right)}{20}$	90

input
```
int((a+b/x^(1/3))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/5*b^2/x^(5/3)-3/2*a*b/x^(4/3)-a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx = -\frac{10\,a^2x+15\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{10\,x^2}$$

input
```
integrate((a+b/x^(1/3))^2/x^2,x, algorithm="fricas")
```

output
```
-1/10*(10*a^2*x + 15*a*b*x^(2/3) + 6*b^2*x^(1/3))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx = -\frac{a^2}{x}-\frac{3ab}{2x^{\frac{4}{3}}}-\frac{3b^2}{5x^{\frac{5}{3}}}$$

input
```
integrate((a+b/x**(1/3))**2/x**2,x)
```

output
```
-a**2/x - 3*a*b/(2*x**(4/3)) - 3*b**2/(5*x**(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx = -\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5}{5\,b^3}+\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4a}{2\,b^3}-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3a^2}{b^3}$$

input
```
integrate((a+b/x^(1/3))^2/x^2,x, algorithm="maxima")
```

output
```
-3/5*(a + b/x^(1/3))^5/b^3 + 3/2*(a + b/x^(1/3))^4*a/b^3 - (a + b/x^(1/3))
^3*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx=-\frac{10\,a^2x^{\frac{2}{3}}+15\,abx^{\frac{1}{3}}+6\,b^2}{10\,x^{\frac{5}{3}}}$$

input
```
integrate((a+b/x^(1/3))^2/x^2,x, algorithm="giac")
```

output
```
-1/10*(10*a^2*x^(2/3) + 15*a*b*x^(1/3) + 6*b^2)/x^(5/3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx=-\frac{a^2}{x}-\frac{3\,b^2}{5\,x^{5/3}}-\frac{3\,a\,b}{2\,x^{4/3}}$$

input
```
int((a + b/x^(1/3))^2/x^2,x)
```

output
```
- a^2/x - (3*b^2)/(5*x^(5/3)) - (3*a*b)/(2*x^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^2}\,dx = \frac{-10x^{\frac{2}{3}}a^2 - 15x^{\frac{1}{3}}ab - 6b^2}{10x^{\frac{5}{3}}}$$

input
```
int((a+b/x^(1/3))^2/x^2,x)
```

output
```
( - 10*x**(2/3)*a**2 - 15*x**(1/3)*a*b - 6*b**2)/(10*x**(2/3)*x)
```

3.301 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3} \, dx = -\frac{3b^2}{8x^{8/3}}-\frac{6ab}{7x^{7/3}}-\frac{a^2}{2x^2}$$

output
```
-3/8*b^2/x^(8/3)-6/7*a*b/x^(7/3)-1/2*a^2/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3} \, dx = \frac{-21b^2-48ab\sqrt[3]{x}-28a^2x^{2/3}}{56x^{8/3}}$$

input
```
Integrate[(a + b/x^(1/3))^2/x^3,x]
```

output
```
(-21*b^2 - 48*a*b*x^(1/3) - 28*a^2*x^(2/3))/(56*x^(8/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{11/3}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^2}{x^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^2}{x^{7/3}}+\frac{2ba}{x^{8/3}}+\frac{b^2}{x^3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^2}{6x^2}-\frac{2ab}{7x^{7/3}}-\frac{b^2}{8x^{8/3}}\right)$$

input
```
Int[(a + b/x^(1/3))^2/x^3,x]
```

output
```
3*(-1/8*b^2/x^(8/3) - (2*a*b)/(7*x^(7/3)) - a^2/(6*x^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{3b^{2}}{8x^{\frac{8}{3}}}-\frac{6ab}{7x^{\frac{7}{3}}}-\frac{a^{2}}{2x^{2}}$	25
default	$-\frac{3b^{2}}{8x^{\frac{8}{3}}}-\frac{6ab}{7x^{\frac{7}{3}}}-\frac{a^{2}}{2x^{2}}$	25
trager	$\frac{(-1+x)a^{2}(1+x)}{2x^{2}}-\frac{3b^{2}}{8x^{\frac{8}{3}}}-\frac{6ab}{7x^{\frac{7}{3}}}$	31
orering	$-\frac{109\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}}{56x^{2}}-\frac{45x^{2}\left(-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{3x^{\frac{13}{3}}}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}}{x^{4}}\right)}{56}-\frac{9x^{3}\left(\frac{2b^{2}}{9x^{\frac{17}{3}}}+\frac{44\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{9x^{\frac{16}{3}}}+\frac{12\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}}{x^{5}}\right)}{112}$	90

input
```
int((a+b/x^(1/3))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*b^2/x^(8/3)-6/7*a*b/x^(7/3)-1/2*a^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx = -\frac{28\,a^2x + 48\,abx^{\frac{2}{3}} + 21\,b^2x^{\frac{1}{3}}}{56\,x^3}$$

input
```
integrate((a+b/x^(1/3))^2/x^3,x, algorithm="fricas")
```

output
```
-1/56*(28*a^2*x + 48*a*b*x^(2/3) + 21*b^2*x^(1/3))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx = -\frac{a^2}{2x^2} - \frac{6ab}{7x^{\frac{7}{3}}} - \frac{3b^2}{8x^{\frac{8}{3}}}$$

input
```
integrate((a+b/x**(1/3))**2/x**3,x)
```

output
```
-a**2/(2*x**2) - 6*a*b/(7*x**(7/3)) - 3*b**2/(8*x**(8/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.85

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx = -\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^8}{8\,b^6} + \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7 a}{7\,b^6} - \frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a^2}{b^6} + \frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^3}{b^6} - \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^4}{4\,b^6} + \frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^5}{b^6}$$

input
```
integrate((a+b/x^(1/3))^2/x^3,x, algorithm="maxima")
```

output
```
-3/8*(a + b/x^(1/3))^8/b^6 + 15/7*(a + b/x^(1/3))^7*a/b^6 - 5*(a + b/x^(1/
3))^6*a^2/b^6 + 6*(a + b/x^(1/3))^5*a^3/b^6 - 15/4*(a + b/x^(1/3))^4*a^4/b
^6 + (a + b/x^(1/3))^3*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx=-\frac{28\,a^2x^{\frac{2}{3}}+48\,abx^{\frac{1}{3}}+21\,b^2}{56\,x^{\frac{8}{3}}}$$

input
```
integrate((a+b/x^(1/3))^2/x^3,x, algorithm="giac")
```

output
```
-1/56*(28*a^2*x^(2/3) + 48*a*b*x^(1/3) + 21*b^2)/x^(8/3)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx=-\frac{a^2}{2\,x^2}-\frac{3\,b^2}{8\,x^{8/3}}-\frac{6\,a\,b}{7\,x^{7/3}}$$

input
```
int((a + b/x^(1/3))^2/x^3,x)
```

output
```
- a^2/(2*x^2) - (3*b^2)/(8*x^(8/3)) - (6*a*b)/(7*x^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^3}\,dx=\frac{-28x^{\frac{2}{3}}a^2-48x^{\frac{1}{3}}ab-21b^2}{56x^{\frac{8}{3}}}$$

input
```
int((a+b/x^(1/3))^2/x^3,x)
```

output
```
( - 28*x**(2/3)*a**2 - 48*x**(1/3)*a*b - 21*b**2)/(56*x**(2/3)*x**2)
```

3.302 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx=-\frac{3b^2}{11x^{11/3}}-\frac{3ab}{5x^{10/3}}-\frac{a^2}{3x^3}$$

output
```
-3/11*b^2/x^(11/3)-3/5*a*b/x^(10/3)-1/3*a^2/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx=\frac{-45b^2-99ab\sqrt[3]{x}-55a^2x^{2/3}}{165x^{11/3}}$$

input
```
Integrate[(a + b/x^(1/3))^2/x^4,x]
```

output
```
(-45*b^2 - 99*a*b*x^(1/3) - 55*a^2*x^(2/3))/(165*x^(11/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a\sqrt[3]{x}+b\right)^2}{x^{14/3}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^2}{x^4}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^2}{x^{10/3}}+\frac{2ba}{x^{11/3}}+\frac{b^2}{x^4}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^2}{9x^3}-\frac{ab}{5x^{10/3}}-\frac{b^2}{11x^{11/3}}\right)$$

input
```
Int[(a + b/x^(1/3))^2/x^4,x]
```

output
```
3*(-1/11*b^2/x^(11/3) - (a*b)/(5*x^(10/3)) - a^2/(9*x^3))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{3b^{2}}{11x^{\frac{11}{3}}}-\frac{3ab}{5x^{\frac{10}{3}}}-\frac{a^{2}}{3x^{3}}$	25
default	$-\frac{3b^{2}}{11x^{\frac{11}{3}}}-\frac{3ab}{5x^{\frac{10}{3}}}-\frac{a^{2}}{3x^{3}}$	25
trager	$\frac{a^{2}(x^{2}+x+1)(-1+x)}{3x^{3}}-\frac{3b^{2}}{11x^{\frac{11}{3}}}-\frac{3ab}{5x^{\frac{10}{3}}}$	34
orering	$-\frac{199\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}}{165x^{3}}-\frac{39x^{2}\left(-\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{3x^{\frac{16}{3}}}-\frac{4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}}{x^{5}}\right)}{110}-\frac{3x^{3}\left(\frac{2b^{2}}{9x^{\frac{20}{3}}}+\frac{56\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b}{9x^{\frac{19}{3}}}+\frac{20\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}}{x^{6}}\right)}{110}$	90

input
```
int((a+b/x^(1/3))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-3/11*b^2/x^(11/3)-3/5*a*b/x^(10/3)-1/3*a^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx = -\frac{55\,a^2x+99\,abx^{\frac{2}{3}}+45\,b^2x^{\frac{1}{3}}}{165\,x^4}$$

input
```
integrate((a+b/x^(1/3))^2/x^4,x, algorithm="fricas")
```

output
```
-1/165*(55*a^2*x + 99*a*b*x^(2/3) + 45*b^2*x^(1/3))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx = -\frac{a^2}{3x^3}-\frac{3ab}{5x^{\frac{10}{3}}}-\frac{3b^2}{11x^{\frac{11}{3}}}$$

input
```
integrate((a+b/x**(1/3))**2/x**4,x)
```

output
```
-a**2/(3*x**3) - 3*a*b/(5*x**(10/3)) - 3*b**2/(11*x**(11/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.38

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx = -\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{11}}{11\,b^9}+\frac{12\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{10}a}{5\,b^9}-\frac{28\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{9}a^2}{3\,b^9}+\frac{21\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{8}a^3}{b^9}-\frac{30\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{7}a^4}{b^9}+\frac{28\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{6}a^5}{b^9}-\frac{84\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{5}a^6}{5\,b^9}+\frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{4}a^7}{b^9}-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{3}a^8}{b^9}$$

input
```
integrate((a+b/x^(1/3))^2/x^4,x, algorithm="maxima")
```

output
```
-3/11*(a + b/x^(1/3))^11/b^9 + 12/5*(a + b/x^(1/3))^10*a/b^9 - 28/3*(a + b
/x^(1/3))^9*a^2/b^9 + 21*(a + b/x^(1/3))^8*a^3/b^9 - 30*(a + b/x^(1/3))^7*
a^4/b^9 + 28*(a + b/x^(1/3))^6*a^5/b^9 - 84/5*(a + b/x^(1/3))^5*a^6/b^9 +
6*(a + b/x^(1/3))^4*a^7/b^9 - (a + b/x^(1/3))^3*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx = -\frac{55\,a^2x^{\frac{2}{3}}+99\,abx^{\frac{1}{3}}+45\,b^2}{165\,x^{\frac{11}{3}}}$$

input
```
integrate((a+b/x^(1/3))^2/x^4,x, algorithm="giac")
```

output
```
-1/165*(55*a^2*x^(2/3) + 99*a*b*x^(1/3) + 45*b^2)/x^(11/3)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx = -\frac{a^2}{3\,x^3}-\frac{3\,b^2}{11\,x^{11/3}}-\frac{3\,a\,b}{5\,x^{10/3}}$$

input
```
int((a + b/x^(1/3))^2/x^4,x)
```

output
```
- a^2/(3*x^3) - (3*b^2)/(11*x^(11/3)) - (3*a*b)/(5*x^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}{x^4}\,dx = \frac{-55x^{\frac{2}{3}}a^2-99x^{\frac{1}{3}}ab-45b^2}{165x^{\frac{11}{3}}}$$

input
```
int((a+b/x^(1/3))^2/x^4,x)
```

output
```
( - 55*x**(2/3)*a**2 - 99*x**(1/3)*a*b - 45*b**2)/(165*x**(2/3)*x**3)
```

3.303 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4 \, dx = \frac{b^3x^4}{4}+\frac{9}{13}ab^2x^{13/3}+\frac{9}{14}a^2bx^{14/3}+\frac{a^3x^5}{5}$$

output
```
1/4*b^3*x^4+9/13*a*b^2*x^(13/3)+9/14*a^2*b*x^(14/3)+1/5*a^3*x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4 \, dx = \frac{455b^3x^4+1260ab^2x^{13/3}+1170a^2bx^{14/3}+364a^3x^5}{1820}$$

input
```
Integrate[(a + b/x^(1/3))^3*x^4,x]
```

output
```
(455*b^3*x^4 + 1260*a*b^2*x^(13/3) + 1170*a^2*b*x^(14/3) + 364*a^3*x^5)/18
20
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx$$

$$\Big\downarrow \text{795}$$

$$\int x^3 \left(a\sqrt[3]{x}+b\right)^3 dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(\sqrt[3]{x}a+b\right)^3 x^{11/3} d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(a^3x^{14/3}+3a^2bx^{13/3}+3ab^2x^4+b^3x^{11/3}\right) d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^3x^5}{15}+\frac{3}{14}a^2bx^{14/3}+\frac{3}{13}ab^2x^{13/3}+\frac{b^3x^4}{12}\right)$$

input
```
Int[(a + b/x^(1/3))^3*x^4,x]
```

output
```
3*((b^3*x^4)/12 + (3*a*b^2*x^(13/3))/13 + (3*a^2*b*x^(14/3))/14 + (a^3*x^5
)/15)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{b^3 x^4}{4}+\frac{9 a\, b^2 x^{\frac{13}{3}}}{13}+\frac{9 a^2 b\, x^{\frac{14}{3}}}{14}+\frac{a^3 x^5}{5}$
default	$\frac{b^3 x^4}{4}+\frac{9 a\, b^2 x^{\frac{13}{3}}}{13}+\frac{9 a^2 b\, x^{\frac{14}{3}}}{14}+\frac{a^3 x^5}{5}$
trager	$\frac{(4a^3x^4+4a^3x^3+5b^3x^3+4a^3x^2+5b^3x^2+4a^3x+5b^3x+4a^3+5b^3)(-1+x)}{20}+\frac{9 a\, b^2 x^{\frac{13}{3}}}{13}+\frac{9 a^2 b\, x^{\frac{14}{3}}}{14}$
orering	$\frac{x^5(188a^3x+199b^3)\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3}{364a^3x+364b^3}-\frac{9x^2(22a^3x+25b^3)\left(-\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 x^{\frac{8}{3}} b+4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 x^3\right)}{1820(a^3x+b^3)}+\frac{9x^3(4a^3x+5b^3)\left(\cdots\right.}{\cdots}$

input
```
int((a+b/x^(1/3))^3*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^3*x^4+9/13*a*b^2*x^(13/3)+9/14*a^2*b*x^(14/3)+1/5*a^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4\,dx = \frac{1}{5}\,a^3x^5 + \frac{9}{14}\,a^2bx^{\frac{14}{3}} + \frac{9}{13}\,ab^2x^{\frac{13}{3}} + \frac{1}{4}\,b^3x^4$$

input
```
integrate((a+b/x^(1/3))^3*x^4,x, algorithm="fricas")
```

output
```
1/5*a^3*x^5 + 9/14*a^2*b*x^(14/3) + 9/13*a*b^2*x^(13/3) + 1/4*b^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4\,dx = \frac{a^3x^5}{5} + \frac{9a^2bx^{\frac{14}{3}}}{14} + \frac{9ab^2x^{\frac{13}{3}}}{13} + \frac{b^3x^4}{4}$$

input
```
integrate((a+b/x**(1/3))**3*x**4,x)
```

output
```
a**3*x**5/5 + 9*a**2*b*x**(14/3)/14 + 9*a*b**2*x**(13/3)/13 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4\,dx = \frac{1}{1820}\left(364\,a^3 + \frac{1170\,a^2b}{x^{\frac{1}{3}}} + \frac{1260\,ab^2}{x^{\frac{2}{3}}} + \frac{455\,b^3}{x}\right)x^5$$

input
```
integrate((a+b/x^(1/3))^3*x^4,x, algorithm="maxima")
```

output
```
1/1820*(364*a^3 + 1170*a^2*b/x^(1/3) + 1260*a*b^2/x^(2/3) + 455*b^3/x)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4\,dx=\frac{1}{5}\,a^3x^5+\frac{9}{14}\,a^2bx^{\frac{14}{3}}+\frac{9}{13}\,ab^2x^{\frac{13}{3}}+\frac{1}{4}\,b^3x^4$$

input
```
integrate((a+b/x^(1/3))^3*x^4,x, algorithm="giac")
```

output
```
1/5*a^3*x^5 + 9/14*a^2*b*x^(14/3) + 9/13*a*b^2*x^(13/3) + 1/4*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4\,dx=\frac{a^3\,x^5}{5}+\frac{b^3\,x^4}{4}+\frac{9\,a\,b^2\,x^{13/3}}{13}+\frac{9\,a^2\,b\,x^{14/3}}{14}$$

input
```
int(x^4*(a + b/x^(1/3))^3,x)
```

output
```
(a^3*x^5)/5 + (b^3*x^4)/4 + (9*a*b^2*x^(13/3))/13 + (9*a^2*b*x^(14/3))/14
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4\,dx=\frac{x^4\left(1170x^{\frac{2}{3}}a^2b+1260x^{\frac{1}{3}}a\,b^2+364a^3x+455b^3\right)}{1820}$$

input
```
int((a+b/x^(1/3))^3*x^4,x)
```

output
```
(x**4*(1170*x**(2/3)*a**2*b + 1260*x**(1/3)*a*b**2 + 364*a**3*x + 455*b**3
))/1820
```

3.304 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx=\frac{b^3x^3}{3}+\frac{9}{10}ab^2x^{10/3}+\frac{9}{11}a^2bx^{11/3}+\frac{a^3x^4}{4}$$

output
```
1/3*b^3*x^3+9/10*a*b^2*x^(10/3)+9/11*a^2*b*x^(11/3)+1/4*a^3*x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx=\frac{1}{660}x^3\left(220b^3+594ab^2\sqrt[3]{x}+540a^2bx^{2/3}+165a^3x\right)$$

input
```
Integrate[(a + b/x^(1/3))^3*x^3,x]
```

output
```
(x^3*(220*b^3 + 594*a*b^2*x^(1/3) + 540*a^2*b*x^(2/3) + 165*a^3*x))/660
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^3dx$$

$$\Big\downarrow \text{795}$$

$$\int x^2\left(a\sqrt[3]{x}+b\right)^3dx$$

$$\Big\downarrow \text{798}$$

$$3\int \left(\sqrt[3]{x}a+b\right)^3x^{8/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(a^3x^{11/3}+3a^2bx^{10/3}+3ab^2x^3+b^3x^{8/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^3x^4}{12}+\frac{3}{11}a^2bx^{11/3}+\frac{3}{10}ab^2x^{10/3}+\frac{b^3x^3}{9}\right)$$

input
```
Int[(a + b/x^(1/3))^3*x^3,x]
```

output
```
3*((b^3*x^3)/9 + (3*a*b^2*x^(10/3))/10 + (3*a^2*b*x^(11/3))/11 + (a^3*x^4)
/12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{b^3x^3}{3}+\frac{9ab^2x^{\frac{10}{3}}}{10}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{a^3x^4}{4}$
default	$\frac{b^3x^3}{3}+\frac{9ab^2x^{\frac{10}{3}}}{10}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{a^3x^4}{4}$
trager	$\frac{(3a^3x^3+3a^3x^2+4b^3x^2+3a^3x+4b^3x+3a^3+4b^3)(-1+x)}{12}+\frac{9ab^2x^{\frac{10}{3}}}{10}+\frac{9a^2bx^{\frac{11}{3}}}{11}$
orering	$\frac{x^4(102a^3x+109b^3)\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3}{165a^3x+165b^3}-\frac{3x^2(6a^3x+7b^3)\left(-\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x^{\frac{5}{3}}b+3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3x^2\right)}{110(a^3x+b^3)}+\frac{3x^3(3a^3x+4b^3)\left(2\left(a+\right.\right.}{}$

input
```
int((a+b/x^(1/3))^3*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^3*x^3+9/10*a*b^2*x^(10/3)+9/11*a^2*b*x^(11/3)+1/4*a^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx = \frac{1}{4}\,a^3x^4+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x^(1/3))^3*x^3,x, algorithm="fricas")
```

output
```
1/4*a^3*x^4 + 9/11*a^2*b*x^(11/3) + 9/10*a*b^2*x^(10/3) + 1/3*b^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx = \frac{a^3x^4}{4}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{9ab^2x^{\frac{10}{3}}}{10}+\frac{b^3x^3}{3}$$

input
```
integrate((a+b/x**(1/3))**3*x**3,x)
```

output
```
a**3*x**4/4 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(10/3)/10 + b**3*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx = \frac{1}{660}\left(165\,a^3+\frac{540\,a^2b}{x^{\frac{1}{3}}}+\frac{594\,ab^2}{x^{\frac{2}{3}}}+\frac{220\,b^3}{x}\right)x^4$$

input
```
integrate((a+b/x^(1/3))^3*x^3,x, algorithm="maxima")
```

output
```
1/660*(165*a^3 + 540*a^2*b/x^(1/3) + 594*a*b^2/x^(2/3) + 220*b^3/x)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx=\frac{1}{4}\,a^3x^4+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{1}{3}\,b^3x^3$$

input
```
integrate((a+b/x^(1/3))^3*x^3,x, algorithm="giac")
```

output
```
1/4*a^3*x^4 + 9/11*a^2*b*x^(11/3) + 9/10*a*b^2*x^(10/3) + 1/3*b^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx=\frac{a^3\,x^4}{4}+\frac{b^3\,x^3}{3}+\frac{9\,a\,b^2\,x^{10/3}}{10}+\frac{9\,a^2\,b\,x^{11/3}}{11}$$

input
```
int(x^3*(a + b/x^(1/3))^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^3)/3 + (9*a*b^2*x^(10/3))/10 + (9*a^2*b*x^(11/3))/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3\,dx=\frac{x^3\left(540x^{\frac{2}{3}}a^2b+594x^{\frac{1}{3}}a\,b^2+165a^3x+220b^3\right)}{660}$$

input
```
int((a+b/x^(1/3))^3*x^3,x)
```

output
```
(x**3*(540*x**(2/3)*a**2*b + 594*x**(1/3)*a*b**2 + 165*a**3*x + 220*b**3))
/660
```

3.305 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2 \, dx = \frac{b^3x^2}{2} + \frac{9}{7}ab^2x^{7/3} + \frac{9}{8}a^2bx^{8/3} + \frac{a^3x^3}{3}$$

output
```
1/2*b^3*x^2+9/7*a*b^2*x^(7/3)+9/8*a^2*b*x^(8/3)+1/3*a^3*x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2 \, dx = \frac{1}{168}\left(84b^3x^2 + 216ab^2x^{7/3} + 189a^2bx^{8/3} + 56a^3x^3\right)$$

input
```
Integrate[(a + b/x^(1/3))^3*x^2,x]
```

output
```
(84*b^3*x^2 + 216*a*b^2*x^(7/3) + 189*a^2*b*x^(8/3) + 56*a^3*x^3)/168
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx$$

$$\Big\downarrow \ 795$$

$$\int x \left(a\sqrt[3]{x}+b\right)^3 dx$$

$$\Big\downarrow \ 798$$

$$3\int \left(\sqrt[3]{x}a+b\right)^3 x^{5/3} d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(x^{8/3}a^3+3bx^{7/3}a^2+3b^2x^2a+b^3x^{5/3}\right) d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(\frac{a^3x^3}{9}+\frac{3}{8}a^2bx^{8/3}+\frac{3}{7}ab^2x^{7/3}+\frac{b^3x^2}{6}\right)$$

input
```
Int[(a + b/x^(1/3))^3*x^2,x]
```

output
```
3*((b^3*x^2)/6 + (3*a*b^2*x^(7/3))/7 + (3*a^2*b*x^(8/3))/8 + (a^3*x^3)/9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{b^3x^2}{2}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{8}{3}}}{8}+\frac{a^3x^3}{3}$
default	$\frac{b^3x^2}{2}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{8}{3}}}{8}+\frac{a^3x^3}{3}$
trager	$\frac{(2a^3x^2+2a^3x+3b^3x+2a^3+3b^3)(-1+x)}{6}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{9a^2b\,x^{\frac{8}{3}}}{8}$
orering	$\frac{x^3(64a^3x+69b^3)\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3}{84a^3x+84b^3}-\frac{3x^2(5a^3x+6b^3)\left(-\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2x^{\frac{2}{3}}b+2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3x\right)}{56(a^3x+b^3)}+\frac{3x^3(2a^3x+3b^3)\left(\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right.}{3x^{\frac{2}{3}}}\right.}{11}$

input
```
int((a+b/x^(1/3))^3*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^3*x^2+9/7*a*b^2*x^(7/3)+9/8*a^2*b*x^(8/3)+1/3*a^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2\,dx = \frac{1}{3}\,a^3x^3 + \frac{9}{8}\,a^2bx^{\frac{8}{3}} + \frac{9}{7}\,ab^2x^{\frac{7}{3}} + \frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x^(1/3))^3*x^2,x, algorithm="fricas")
```

output
```
1/3*a^3*x^3 + 9/8*a^2*b*x^(8/3) + 9/7*a*b^2*x^(7/3) + 1/2*b^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2\,dx = \frac{a^3x^3}{3} + \frac{9a^2bx^{\frac{8}{3}}}{8} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{b^3x^2}{2}$$

input
```
integrate((a+b/x**(1/3))**3*x**2,x)
```

output
```
a**3*x**3/3 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(7/3)/7 + b**3*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2\,dx = \frac{1}{168}\left(56\,a^3 + \frac{189\,a^2b}{x^{\frac{1}{3}}} + \frac{216\,ab^2}{x^{\frac{2}{3}}} + \frac{84\,b^3}{x}\right)x^3$$

input
```
integrate((a+b/x^(1/3))^3*x^2,x, algorithm="maxima")
```

output
```
1/168*(56*a^3 + 189*a^2*b/x^(1/3) + 216*a*b^2/x^(2/3) + 84*b^3/x)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2\,dx = \frac{1}{3}\,a^3x^3 + \frac{9}{8}\,a^2bx^{\frac{8}{3}} + \frac{9}{7}\,ab^2x^{\frac{7}{3}} + \frac{1}{2}\,b^3x^2$$

input
```
integrate((a+b/x^(1/3))^3*x^2,x, algorithm="giac")
```

output
```
1/3*a^3*x^3 + 9/8*a^2*b*x^(8/3) + 9/7*a*b^2*x^(7/3) + 1/2*b^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2\,dx = \frac{a^3\,x^3}{3} + \frac{b^3\,x^2}{2} + \frac{9\,a\,b^2\,x^{7/3}}{7} + \frac{9\,a^2\,b\,x^{8/3}}{8}$$

input
```
int(x^2*(a + b/x^(1/3))^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^2)/2 + (9*a*b^2*x^(7/3))/7 + (9*a^2*b*x^(8/3))/8
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2\,dx = \frac{x^2\left(189x^{\frac{2}{3}}a^2b + 216x^{\frac{1}{3}}a\,b^2 + 56a^3x + 84b^3\right)}{168}$$

input
```
int((a+b/x^(1/3))^3*x^2,x)
```

output
```
(x**2*(189*x**(2/3)*a**2*b + 216*x**(1/3)*a*b**2 + 56*a**3*x + 84*b**3))/1
68
```

3.306 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = b^3x+\frac{9}{4}ab^2x^{4/3}+\frac{9}{5}a^2bx^{5/3}+\frac{a^3x^2}{2}$$

output
```
b^3*x+9/4*a*b^2*x^(4/3)+9/5*a^2*b*x^(5/3)+1/2*a^3*x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = \frac{1}{20}\left(20b^3x+45ab^2x^{4/3}+36a^2bx^{5/3}+10a^3x^2\right)$$

input
```
Integrate[(a + b/x^(1/3))^3*x,x]
```

output
```
(20*b^3*x + 45*a*b^2*x^(4/3) + 36*a^2*b*x^(5/3) + 10*a^3*x^2)/20
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {795, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx$$

$$\Big\downarrow \ 795$$

$$\int \left(a\sqrt[3]{x}+b\right)^3 dx$$

$$\Big\downarrow \ 774$$

$$3\int \left(\sqrt[3]{x}a+b\right)^3 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(x^{5/3}a^3+3bx^{4/3}a^2+3b^2xa+b^3x^{2/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(\frac{a^3x^2}{6}+\frac{3}{5}a^2bx^{5/3}+\frac{3}{4}ab^2x^{4/3}+\frac{b^3x}{3}\right)$$

input
```
Int[(a + b/x^(1/3))^3*x,x]
```

output
```
3*((b^3*x)/3 + (3*a*b^2*x^(4/3))/4 + (3*a^2*b*x^(5/3))/5 + (a^3*x^2)/6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

method	result
derivativedivides	$b^{3} x +\frac{9 x^{\frac{4}{3}} a \,b^{2}}{4}+\frac{9 a^{2} b \,x^{\frac{5}{3}}}{5}+\frac{a^{3} x^{2}}{2}$
default	$b^{3} x +\frac{9 x^{\frac{4}{3}} a \,b^{2}}{4}+\frac{9 a^{2} b \,x^{\frac{5}{3}}}{5}+\frac{a^{3} x^{2}}{2}$
trager	$\frac{\left(-1+x \right) \left(a^{3} x +a^{3}+2 b^{3}\right)}{2}+\frac{9 x^{\frac{4}{3}} a \,b^{2}}{4}+\frac{9 a^{2} b \,x^{\frac{5}{3}}}{5}$
orering	$\frac{x^{2} \left(19 a^{3} x +20 b^{3}\right) \left(a +\frac{b}{x^{\frac{1}{3}}}\right)^{3}}{20 a^{3} x +20 b^{3}}-\frac{9 x^{2} \left(-\frac{\left(a +\frac{b}{x^{\frac{1}{3}}}\right)^{2} b}{x^{\frac{1}{3}}}+\left(a +\frac{b}{x^{\frac{1}{3}}}\right)^{3}\right)}{20}+\frac{9 x^{3} \left(a^{3} x +2 b^{3}\right) \left(\frac{2 \left(a +\frac{b}{x^{\frac{1}{3}}}\right) b^{2}}{3 x^{\frac{5}{3}}}-\frac{2 \left(a +\frac{b}{x^{\frac{1}{3}}}\right)^{2}}{3 x^{\frac{4}{3}}}\right.}{40 \left(a^{3} x +b^{3}\right)}$

input
```
int((a+b/x^(1/3))^3*x,x,method=_RETURNVERBOSE)
```

output
```
b^3*x+9/4*x^(4/3)*a*b^2+9/5*a^2*b*x^(5/3)+1/2*a^3*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = \frac{1}{2}\,a^3x^2+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+b^3x$$

input
```
integrate((a+b/x^(1/3))^3*x,x, algorithm="fricas")
```

output
```
1/2*a^3*x^2 + 9/5*a^2*b*x^(5/3) + 9/4*a*b^2*x^(4/3) + b^3*x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = \frac{a^3x^2}{2}+\frac{9a^2bx^{\frac{5}{3}}}{5}+\frac{9ab^2x^{\frac{4}{3}}}{4}+b^3x$$

input
```
integrate((a+b/x**(1/3))**3*x,x)
```

output
```
a**3*x**2/2 + 9*a**2*b*x**(5/3)/5 + 9*a*b**2*x**(4/3)/4 + b**3*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = \frac{1}{20}\left(10\,a^3+\frac{36\,a^2b}{x^{\frac{1}{3}}}+\frac{45\,ab^2}{x^{\frac{2}{3}}}+\frac{20\,b^3}{x}\right)x^2$$

input
```
integrate((a+b/x^(1/3))^3*x,x, algorithm="maxima")
```

output
```
1/20*(10*a^3 + 36*a^2*b/x^(1/3) + 45*a*b^2/x^(2/3) + 20*b^3/x)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = \frac{1}{2}\,a^3x^2 + \frac{9}{5}\,a^2bx^{\frac{5}{3}} + \frac{9}{4}\,ab^2x^{\frac{4}{3}} + b^3x$$

input
```
integrate((a+b/x^(1/3))^3*x,x, algorithm="giac")
```

output
```
1/2*a^3*x^2 + 9/5*a^2*b*x^(5/3) + 9/4*a*b^2*x^(4/3) + b^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = b^3\,x + \frac{a^3\,x^2}{2} + \frac{9\,a\,b^2\,x^{4/3}}{4} + \frac{9\,a^2\,b\,x^{5/3}}{5}$$

input
```
int(x*(a + b/x^(1/3))^3,x)
```

output
```
b^3*x + (a^3*x^2)/2 + (9*a*b^2*x^(4/3))/4 + (9*a^2*b*x^(5/3))/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x\,dx = \frac{x\left(36x^{\frac{2}{3}}a^2b + 45x^{\frac{1}{3}}a\,b^2 + 10a^3x + 20b^3\right)}{20}$$

input
```
int((a+b/x^(1/3))^3*x,x)
```

output
```
(x*(36*x**(2/3)*a**2*b + 45*x**(1/3)*a*b**2 + 10*a**3*x + 20*b**3))/20
```

3.307 $\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = 9ab^2\sqrt[3]{x}+\frac{9}{2}a^2bx^{2/3}+a^3x+b^3\log(x)$$

output
```
9*a*b^2*x^(1/3)+9/2*a^2*b*x^(2/3)+a^3*x+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = 9ab^2\sqrt[3]{x}+\frac{9}{2}a^2bx^{2/3}+a^3x+b^3\log(x)$$

input
```
Integrate[(a + b/x^(1/3))^3,x]
```

output
```
9*a*b^2*x^(1/3) + (9*a^2*b*x^(2/3))/2 + a^3*x + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {774, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx$$

$$\Big\downarrow \text{ 774}$$

$$3\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^{2/3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 795}$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^3}{\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 49}$$

$$3\int \left(x^{2/3}a^3+3b\sqrt[3]{x}a^2+3b^2a+\frac{b^3}{\sqrt[3]{x}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$3\left(\frac{a^3x}{3}+\frac{3}{2}a^2bx^{2/3}+3ab^2\sqrt[3]{x}+b^3\log\left(\sqrt[3]{x}\right)\right)$$

input
```
Int[(a + b/x^(1/3))^3,x]
```

output
```
3*(3*a*b^2*x^(1/3) + (3*a^2*b*x^(2/3))/2 + (a^3*x)/3 + b^3*Log[x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$9a\,b^2x^{\frac{1}{3}}+\frac{9a^2b\,x^{\frac{2}{3}}}{2}+a^3x+b^3\ln\left(x\right)$	31
default	$9a\,b^2x^{\frac{1}{3}}+\frac{9a^2b\,x^{\frac{2}{3}}}{2}+a^3x+b^3\ln\left(x\right)$	31
trager	$a^3(-1+x)+9a\,b^2x^{\frac{1}{3}}+\frac{9a^2b\,x^{\frac{2}{3}}}{2}-b^3\ln\left(\frac{1}{x}\right)$	36

input
```
int((a+b/x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
9*a*b^2*x^(1/3)+9/2*a^2*b*x^(2/3)+a^3*x+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = a^3x+3b^3\log\left(x^{\frac{1}{3}}\right)+\frac{9}{2}a^2bx^{\frac{2}{3}}+9ab^2x^{\frac{1}{3}}$$

input
```
integrate((a+b/x^(1/3))^3,x, algorithm="fricas")
```

output
```
a^3*x + 3*b^3*log(x^(1/3)) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = a^3x+\frac{9a^2bx^{\frac{2}{3}}}{2}+9ab^2\sqrt[3]{x}+b^3\log(x)$$

input
```
integrate((a+b/x**(1/3))**3,x)
```

output
```
a**3*x + 9*a**2*b*x**(2/3)/2 + 9*a*b**2*x**(1/3) + b**3*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = a^3x+b^3\log(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}+9ab^2x^{\frac{1}{3}}$$

input
```
integrate((a+b/x^(1/3))^3,x, algorithm="maxima")
```

output
```
a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = a^3x + b^3\log\left(|x|\right) + \frac{9}{2}a^2bx^{\frac{2}{3}} + 9ab^2x^{\frac{1}{3}}$$

input
```
integrate((a+b/x^(1/3))^3,x, algorithm="giac")
```

output
```
a^3*x + b^3*log(abs(x)) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = 3b^3\ln\left(x^{1/3}\right) + a^3x + 9ab^2x^{1/3} + \frac{9a^2bx^{2/3}}{2}$$

input
```
int((a + b/x^(1/3))^3,x)
```

output
```
3*b^3*log(x^(1/3)) + a^3*x + 9*a*b^2*x^(1/3) + (9*a^2*b*x^(2/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{\sqrt[3]{x}}\right)^3 dx = \frac{9x^{\frac{2}{3}}a^2b}{2} + 9x^{\frac{1}{3}}ab^2 + 3\log\left(x^{\frac{1}{3}}\right)b^3 + a^3x$$

input
```
int((a+b/x^(1/3))^3,x)
```

output
```
(9*x**(2/3)*a**2*b + 18*x**(1/3)*a*b**2 + 6*log(x**(1/3))*b**3 + 2*a**3*x)
/2
```

3.308 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x} \, dx = -\frac{b^3}{x}-\frac{9ab^2}{2x^{2/3}}-\frac{9a^2b}{\sqrt[3]{x}}+a^3\log(x)$$

output
```
-b^3/x-9/2*a*b^2/x^(2/3)-9*a^2*b/x^(1/3)+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x} \, dx = -\frac{b\left(2b^2+9ab\sqrt[3]{x}+18a^2x^{2/3}\right)}{2x}+a^3\log(x)$$

input
```
Integrate[(a + b/x^(1/3))^3/x,x]
```

output
```
-1/2*(b*(2*b^2 + 9*a*b*x^(1/3) + 18*a^2*x^(2/3)))/x + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a\sqrt[3]{x}+b\right)^3}{x^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^3}{x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{a^3}{\sqrt[3]{x}}+\frac{3ba^2}{x^{2/3}}+\frac{3b^2a}{x}+\frac{b^3}{x^{4/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(a^3\log\left(\sqrt[3]{x}\right)-\frac{3a^2b}{\sqrt[3]{x}}-\frac{3ab^2}{2x^{2/3}}-\frac{b^3}{3x}\right)$$

input
```
Int[(a + b/x^(1/3))^3/x,x]
```

output
```
3*(-1/3*b^3/x - (3*a*b^2)/(2*x^(2/3)) - (3*a^2*b)/x^(1/3) + a^3*Log[x^(1/3
)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-\frac{b^{3}}{x}-\frac{9a\,b^{2}}{2x^{\frac{2}{3}}}-\frac{9a^{2}b}{x^{\frac{1}{3}}}+a^{3}\ln\left(x\right)$	34
default	$-\frac{b^{3}}{x}-\frac{9a\,b^{2}}{2x^{\frac{2}{3}}}-\frac{9a^{2}b}{x^{\frac{1}{3}}}+a^{3}\ln\left(x\right)$	34
trager	$\frac{b^{3}(-1+x)}{x}-\frac{9a\,b^{2}}{2x^{\frac{2}{3}}}-\frac{9a^{2}b}{x^{\frac{1}{3}}}-a^{3}\ln\left(\frac{1}{x}\right)$	39

input
```
int((a+b/x^(1/3))^3/x,x,method=_RETURNVERBOSE)
```

output
```
-b^3/x-9/2*a*b^2/x^(2/3)-9*a^2*b/x^(1/3)+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx = \frac{6\,a^3x\log\left(x^{\frac{1}{3}}\right)-18\,a^2bx^{\frac{2}{3}}-9\,ab^2x^{\frac{1}{3}}-2\,b^3}{2\,x}$$

input
```
integrate((a+b/x^(1/3))^3/x,x, algorithm="fricas")
```

output
```
1/2*(6*a^3*x*log(x^(1/3)) - 18*a^2*b*x^(2/3) - 9*a*b^2*x^(1/3) - 2*b^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx = a^3\log(x)-\frac{9a^2b}{\sqrt[3]{x}}-\frac{9ab^2}{2x^{\frac{2}{3}}}-\frac{b^3}{x}$$

input
```
integrate((a+b/x**(1/3))**3/x,x)
```

output
```
a**3*log(x) - 9*a**2*b/x**(1/3) - 9*a*b**2/(2*x**(2/3)) - b**3/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx = a^3\log(x)-\frac{9\,a^2b}{x^{\frac{1}{3}}}-\frac{9\,ab^2}{2\,x^{\frac{2}{3}}}-\frac{b^3}{x}$$

input
```
integrate((a+b/x^(1/3))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) - 9*a^2*b/x^(1/3) - 9/2*a*b^2/x^(2/3) - b^3/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx = a^3\log\left(|x|\right) - \frac{18\,a^2bx^{\frac{2}{3}} + 9\,ab^2x^{\frac{1}{3}} + 2\,b^3}{2\,x}$$

input
```
integrate((a+b/x^(1/3))^3/x,x, algorithm="giac")
```

output
```
a^3*log(abs(x)) - 1/2*(18*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3) + 2*b^3)/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx = 3\,a^3\,\ln\left(x^{1/3}\right) - \frac{b^3 + \frac{9\,a\,b^2\,x^{1/3}}{2} + 9\,a^2\,b\,x^{2/3}}{x}$$

input
```
int((a + b/x^(1/3))^3/x,x)
```

output
```
3*a^3*log(x^(1/3)) - (b^3 + (9*a*b^2*x^(1/3))/2 + 9*a^2*b*x^(2/3))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x}\,dx = \frac{-18x^{\frac{2}{3}}a^2b - 9x^{\frac{1}{3}}a\,b^2 + 2\log(x)\,a^3x - 2b^3}{2x}$$

input
```
int((a+b/x^(1/3))^3/x,x)
```

output
```
( - 18*x**(2/3)*a**2*b - 9*x**(1/3)*a*b**2 + 2*log(x)*a**3*x - 2*b**3)/(2*
x)
```

3.309 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2} dx = -\frac{b^3}{2x^2} - \frac{9ab^2}{5x^{5/3}} - \frac{9a^2b}{4x^{4/3}} - \frac{a^3}{x}$$

output
```
-1/2*b^3/x^2-9/5*a*b^2/x^(5/3)-9/4*a^2*b/x^(4/3)-a^3/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2} dx = \frac{-10b^3 - 36ab^2\sqrt[3]{x} - 45a^2bx^{2/3} - 20a^3x}{20x^2}$$

input
```
Integrate[(a + b/x^(1/3))^3/x^2,x]
```

output
```
(-10*b^3 - 36*a*b^2*x^(1/3) - 45*a^2*b*x^(2/3) - 20*a^3*x)/(20*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2} \, dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{\left(a\sqrt[3]{x}+b\right)^3}{x^3}dx$$

$$\Big\downarrow \; 798$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^3}{x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \; 53$$

$$3\int \left(\frac{a^3}{x^{4/3}}+\frac{3ba^2}{x^{5/3}}+\frac{3b^2a}{x^2}+\frac{b^3}{x^{7/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \; 2009$$

$$3\left(-\frac{a^3}{3x}-\frac{3a^2b}{4x^{4/3}}-\frac{3ab^2}{5x^{5/3}}-\frac{b^3}{6x^2}\right)$$

input
```
Int[(a + b/x^(1/3))^3/x^2,x]
```

output
```
3*(-1/6*b^3/x^2 - (3*a*b^2)/(5*x^(5/3)) - (3*a^2*b)/(4*x^(4/3)) - a^3/(3*x
))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{b^{3}}{2 x^{2}}-\frac{9 a\, b^{2}}{5 x^{\frac{5}{3}}}-\frac{9 a^{2} b}{4 x^{\frac{4}{3}}}-\frac{a^{3}}{x}$
default	$-\frac{b^{3}}{2 x^{2}}-\frac{9 a\, b^{2}}{5 x^{\frac{5}{3}}}-\frac{9 a^{2} b}{4 x^{\frac{4}{3}}}-\frac{a^{3}}{x}$
trager	$\frac{\left(-1+x \right) \left(2 a^{3} x +b^{3} x +b^{3}\right)}{2 x^{2}}-\frac{9 a\, b^{2}}{5 x^{\frac{5}{3}}}-\frac{9 a^{2} b}{4 x^{\frac{4}{3}}}$
orering	$-\frac{\left(23 a^{3} x +16 b^{3}\right) \left(a +\frac{b}{x^{\frac{1}{3}}}\right)^{3}}{5 x \left(a^{3} x +b^{3}\right)}-\frac{9 x^{2} \left(7 a^{3} x +4 b^{3}\right) \left(-\frac{\left(a +\frac{b}{x^{\frac{1}{3}}}\right)^{2} b}{x^{\frac{10}{3}}}-\frac{2 \left(a +\frac{b}{x^{\frac{1}{3}}}\right)^{3}}{x^{3}}\right)}{20 \left(a^{3} x +b^{3}\right)}-\frac{9 \left(2 a^{3} x +b^{3}\right) x^{3} \left(\frac{2 \left(a +\frac{b}{x^{\frac{1}{3}}}\right) b^{2}}{3 x^{\frac{14}{3}}} \cdots\right.}{40(a^{3} \cdots}$

input
```
int((a+b/x^(1/3))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/x^2-9/5*a*b^2/x^(5/3)-9/4*a^2*b/x^(4/3)-a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx=-\frac{20\,a^3x+45\,a^2bx^{\frac{2}{3}}+36\,ab^2x^{\frac{1}{3}}+10\,b^3}{20\,x^2}$$

input
```
integrate((a+b/x^(1/3))^3/x^2,x, algorithm="fricas")
```

output
```
-1/20*(20*a^3*x + 45*a^2*b*x^(2/3) + 36*a*b^2*x^(1/3) + 10*b^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx=-\frac{a^3}{x}-\frac{9a^2b}{4x^{\frac{4}{3}}}-\frac{9ab^2}{5x^{\frac{5}{3}}}-\frac{b^3}{2x^2}$$

input
```
integrate((a+b/x**(1/3))**3/x**2,x)
```

output
```
-a**3/x - 9*a**2*b/(4*x**(4/3)) - 9*a*b**2/(5*x**(5/3)) - b**3/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx=-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6}{2\,b^3}+\frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5a}{5\,b^3}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4a^2}{4\,b^3}$$

input
```
integrate((a+b/x^(1/3))^3/x^2,x, algorithm="maxima")
```

output
```
-1/2*(a + b/x^(1/3))^6/b^3 + 6/5*(a + b/x^(1/3))^5*a/b^3 - 3/4*(a + b/x^(1
/3))^4*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx = -\frac{20\,a^3x+45\,a^2bx^{\frac{2}{3}}+36\,ab^2x^{\frac{1}{3}}+10\,b^3}{20\,x^2}$$

input
```
integrate((a+b/x^(1/3))^3/x^2,x, algorithm="giac")
```

output
```
-1/20*(20*a^3*x + 45*a^2*b*x^(2/3) + 36*a*b^2*x^(1/3) + 10*b^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx = -\frac{20\,a^3\,x+10\,b^3+36\,a\,b^2\,x^{1/3}+45\,a^2\,b\,x^{2/3}}{20\,x^2}$$

input
```
int((a + b/x^(1/3))^3/x^2,x)
```

output
```
-(20*a^3*x + 10*b^3 + 36*a*b^2*x^(1/3) + 45*a^2*b*x^(2/3))/(20*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^2}\,dx = \frac{-45x^{\frac{2}{3}}a^2b-36x^{\frac{1}{3}}a\,b^2-20a^3x-10b^3}{20x^2}$$

input
```
int((a+b/x^(1/3))^3/x^2,x)
```

output
```
( - 45*x**(2/3)*a**2*b - 36*x**(1/3)*a*b**2 - 20*a**3*x - 10*b**3)/(20*x**
2)
```

3.310 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3} dx = -\frac{b^3}{3x^3} - \frac{9ab^2}{8x^{8/3}} - \frac{9a^2b}{7x^{7/3}} - \frac{a^3}{2x^2}$$

output
```
-1/3*b^3/x^3-9/8*a*b^2/x^(8/3)-9/7*a^2*b/x^(7/3)-1/2*a^3/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3} dx = \frac{-56b^3 - 189ab^2\sqrt[3]{x} - 216a^2bx^{2/3} - 84a^3x}{168x^3}$$

input
```
Integrate[(a + b/x^(1/3))^3/x^3,x]
```

output
```
(-56*b^3 - 189*a*b^2*x^(1/3) - 216*a^2*b*x^(2/3) - 84*a^3*x)/(168*x^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a \sqrt[3]{x}+b\right)^3}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$3 \int \frac{\left(\sqrt[3]{x} a+b\right)^3}{x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3 \int \left(\frac{a^3}{x^{7/3}}+\frac{3 b a^2}{x^{8/3}}+\frac{3 b^2 a}{x^3}+\frac{b^3}{x^{10/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3 \left(-\frac{a^3}{6 x^2}-\frac{3 a^2 b}{7 x^{7/3}}-\frac{3 a b^2}{8 x^{8/3}}-\frac{b^3}{9 x^3}\right)$$

input
```
Int[(a + b/x^(1/3))^3/x^3,x]
```

output
```
3*(-1/9*b^3/x^3 - (3*a*b^2)/(8*x^(8/3)) - (3*a^2*b)/(7*x^(7/3)) - a^3/(6*x
^2))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{b^{3}}{3x^{3}}-\frac{9a\,b^{2}}{8x^{\frac{8}{3}}}-\frac{9a^{2}b}{7x^{\frac{7}{3}}}-\frac{a^{3}}{2x^{2}}$
default	$-\frac{b^{3}}{3x^{3}}-\frac{9a\,b^{2}}{8x^{\frac{8}{3}}}-\frac{9a^{2}b}{7x^{\frac{7}{3}}}-\frac{a^{3}}{2x^{2}}$
trager	$\frac{(-1+x)(3a^{3}x^{2}+2b^{3}x^{2}+3a^{3}x+2b^{3}x+2b^{3})}{6x^{3}}-\frac{9a\,b^{2}}{8x^{\frac{8}{3}}}-\frac{9a^{2}b}{7x^{\frac{7}{3}}}$
orering	$-\frac{(327a^{3}x+272b^{3})\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{3}}{168x^{2}(a^{3}x+b^{3})}-\frac{3x^{2}(15a^{3}x+11b^{3})\left(-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{2}b}{x^{\frac{13}{3}}}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{3}}{x^{4}}\right)}{56(a^{3}x+b^{3})}-\frac{3(3a^{3}x+2b^{3})x^{3}\left(\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right) \cdots}{3x^{\frac{1}{3}} \cdots}\cdots\right)}{\cdots}$

input
```
int((a+b/x^(1/3))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^3/x^3-9/8*a*b^2/x^(8/3)-9/7*a^2*b/x^(7/3)-1/2*a^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx = -\frac{84\,a^3x+216\,a^2bx^{\frac{2}{3}}+189\,ab^2x^{\frac{1}{3}}+56\,b^3}{168\,x^3}$$

input
```
integrate((a+b/x^(1/3))^3/x^3,x, algorithm="fricas")
```

output
```
-1/168*(84*a^3*x + 216*a^2*b*x^(2/3) + 189*a*b^2*x^(1/3) + 56*b^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx = -\frac{a^3}{2x^2}-\frac{9a^2b}{7x^{\frac{7}{3}}}-\frac{9ab^2}{8x^{\frac{8}{3}}}-\frac{b^3}{3x^3}$$

input
```
integrate((a+b/x**(1/3))**3/x**3,x)
```

output
```
-a**3/(2*x**2) - 9*a**2*b/(7*x**(7/3)) - 9*a*b**2/(8*x**(8/3)) - b**3/(3*x
**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^9}{3\,b^6}+\frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^8 a}{8\,b^6}-\frac{30\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7 a^2}{7\,b^6}+\frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a^3}{b^6}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^4}{b^6}+\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^5}{4\,b^6}$$

input
```
integrate((a+b/x^(1/3))^3/x^3,x, algorithm="maxima")
```

output
```
-1/3*(a + b/x^(1/3))^9/b^6 + 15/8*(a + b/x^(1/3))^8*a/b^6 - 30/7*(a + b/x^
(1/3))^7*a^2/b^6 + 5*(a + b/x^(1/3))^6*a^3/b^6 - 3*(a + b/x^(1/3))^5*a^4/b
^6 + 3/4*(a + b/x^(1/3))^4*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx = -\frac{84\,a^3x+216\,a^2bx^{\frac{2}{3}}+189\,ab^2x^{\frac{1}{3}}+56\,b^3}{168\,x^3}$$

input
```
integrate((a+b/x^(1/3))^3/x^3,x, algorithm="giac")
```

output
```
-1/168*(84*a^3*x + 216*a^2*b*x^(2/3) + 189*a*b^2*x^(1/3) + 56*b^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx = -\frac{84\,a^3\,x+56\,b^3+189\,a\,b^2\,x^{1/3}+216\,a^2\,b\,x^{2/3}}{168\,x^3}$$

input
```
int((a + b/x^(1/3))^3/x^3,x)
```

output
```
-(84*a^3*x + 56*b^3 + 189*a*b^2*x^(1/3) + 216*a^2*b*x^(2/3))/(168*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^3}\,dx=\frac{-216x^{\frac{2}{3}}a^2b-189x^{\frac{1}{3}}a\,b^2-84a^3x-56b^3}{168x^3}$$

input
```
int((a+b/x^(1/3))^3/x^3,x)
```

output
```
( - 216*x**(2/3)*a**2*b - 189*x**(1/3)*a*b**2 - 84*a**3*x - 56*b**3)/(168*
x**3)
```

3.311 $\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4} dx = -\frac{b^3}{4x^4} - \frac{9ab^2}{11x^{11/3}} - \frac{9a^2b}{10x^{10/3}} - \frac{a^3}{3x^3}$$

output
```
-1/4*b^3/x^4-9/11*a*b^2/x^(11/3)-9/10*a^2*b/x^(10/3)-1/3*a^3/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4} dx = \frac{-165b^3 - 540ab^2\sqrt[3]{x} - 594a^2bx^{2/3} - 220a^3x}{660x^4}$$

input
```
Integrate[(a + b/x^(1/3))^3/x^4,x]
```

output
```
(-165*b^3 - 540*a*b^2*x^(1/3) - 594*a^2*b*x^(2/3) - 220*a^3*x)/(660*x^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{\left(a\sqrt[3]{x}+b\right)^3}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\left(\sqrt[3]{x}a+b\right)^3}{x^{13/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{53}$$

$$3\int \left(\frac{a^3}{x^{10/3}}+\frac{3ba^2}{x^{11/3}}+\frac{3b^2a}{x^4}+\frac{b^3}{x^{13/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^3}{9x^3}-\frac{3a^2b}{10x^{10/3}}-\frac{3ab^2}{11x^{11/3}}-\frac{b^3}{12x^4}\right)$$

input
```
Int[(a + b/x^(1/3))^3/x^4,x]
```

output
```
3*(-1/12*b^3/x^4 - (3*a*b^2)/(11*x^(11/3)) - (3*a^2*b)/(10*x^(10/3)) - a^3
/(9*x^3))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{b^3}{4x^4}-\frac{9a\,b^2}{11x^{\frac{11}{3}}}-\frac{9a^2b}{10x^{\frac{10}{3}}}-\frac{a^3}{3x^3}$
default	$-\frac{b^3}{4x^4}-\frac{9a\,b^2}{11x^{\frac{11}{3}}}-\frac{9a^2b}{10x^{\frac{10}{3}}}-\frac{a^3}{3x^3}$
trager	$\frac{(-1+x)(4a^3x^3+3b^3x^3+4a^3x^2+3b^3x^2+4a^3x+3b^3x+3b^3)}{12x^4}-\frac{9a\,b^2}{11x^{\frac{11}{3}}}-\frac{9a^2b}{10x^{\frac{10}{3}}}$
orering	$-\frac{(796a^3x+705b^3)\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3}{660x^3(a^3x+b^3)}-\frac{3x^2(26a^3x+21b^3)\left(-\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 b}{x^{\frac{16}{3}}}-\frac{4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3}{x^5}\right)}{220(a^3x+b^3)}-\frac{3(4a^3x+3b^3)x^3\left(\frac{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{3x^{\frac{2}{3}}}\right.}{}$

input
```
int((a+b/x^(1/3))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3/x^4-9/11*a*b^2/x^(11/3)-9/10*a^2*b/x^(10/3)-1/3*a^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx = -\frac{220\,a^3x+594\,a^2bx^{\frac{2}{3}}+540\,ab^2x^{\frac{1}{3}}+165\,b^3}{660\,x^4}$$

input
```
integrate((a+b/x^(1/3))^3/x^4,x, algorithm="fricas")
```

output
```
-1/660*(220*a^3*x + 594*a^2*b*x^(2/3) + 540*a*b^2*x^(1/3) + 165*b^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx = -\frac{a^3}{3x^3}-\frac{9a^2b}{10x^{\frac{10}{3}}}-\frac{9ab^2}{11x^{\frac{11}{3}}}-\frac{b^3}{4x^4}$$

input
```
integrate((a+b/x**(1/3))**3/x**4,x)
```

output
```
-a**3/(3*x**3) - 9*a**2*b/(10*x**(10/3)) - 9*a*b**2/(11*x**(11/3)) - b**3/
(4*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.17

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx = -\frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{12}}{4\,b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{11}a}{11\,b^9}-\frac{42\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{10}a^2}{5\,b^9}+\frac{56\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{9}a^3}{3\,b^9}-\frac{105\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{8}a^4}{4\,b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{7}a^5}{b^9}-\frac{14\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{6}a^6}{b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{5}a^7}{5\,b^9}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^{4}a^8}{4\,b^9}$$

input
```
integrate((a+b/x^(1/3))^3/x^4,x, algorithm="maxima")
```

output
```
-1/4*(a + b/x^(1/3))^12/b^9 + 24/11*(a + b/x^(1/3))^11*a/b^9 - 42/5*(a + b
/x^(1/3))^10*a^2/b^9 + 56/3*(a + b/x^(1/3))^9*a^3/b^9 - 105/4*(a + b/x^(1/
3))^8*a^4/b^9 + 24*(a + b/x^(1/3))^7*a^5/b^9 - 14*(a + b/x^(1/3))^6*a^6/b^
9 + 24/5*(a + b/x^(1/3))^5*a^7/b^9 - 3/4*(a + b/x^(1/3))^4*a^8/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx = -\frac{220\,a^3x+594\,a^2bx^{\frac{2}{3}}+540\,ab^2x^{\frac{1}{3}}+165\,b^3}{660\,x^4}$$

input
```
integrate((a+b/x^(1/3))^3/x^4,x, algorithm="giac")
```

output
```
-1/660*(220*a^3*x + 594*a^2*b*x^(2/3) + 540*a*b^2*x^(1/3) + 165*b^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx = -\frac{220\,a^3\,x+165\,b^3+540\,a\,b^2\,x^{1/3}+594\,a^2\,b\,x^{2/3}}{660\,x^4}$$

input
```
int((a + b/x^(1/3))^3/x^4,x)
```

output
```
-(220*a^3*x + 165*b^3 + 540*a*b^2*x^(1/3) + 594*a^2*b*x^(2/3))/(660*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}{x^4}\,dx = \frac{-594x^{\frac{2}{3}}a^2b-540x^{\frac{1}{3}}a\,b^2-220a^3x-165b^3}{660x^4}$$

input
```
int((a+b/x^(1/3))^3/x^4,x)
```

output
```
( - 594*x**(2/3)*a**2*b - 540*x**(1/3)*a*b**2 - 220*a**3*x - 165*b**3)/(66
0*x**4)
```

3.312 $\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}} dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}} dx = \frac{3b^8\sqrt[3]{x}}{a^9} - \frac{3b^7x^{2/3}}{2a^8} + \frac{b^6x}{a^7} - \frac{3b^5x^{4/3}}{4a^6} + \frac{3b^4x^{5/3}}{5a^5} - \frac{b^3x^2}{2a^4} + \frac{3b^2x^{7/3}}{7a^3} - \frac{3bx^{8/3}}{8a^2} + \frac{x^3}{3a} - \frac{3b^9\log\left(b+a\sqrt[3]{x}\right)}{a^{10}}$$

output
```
3*b^8*x^(1/3)/a^9-3/2*b^7*x^(2/3)/a^8+b^6*x/a^7-3/4*b^5*x^(4/3)/a^6+3/5*b^
4*x^(5/3)/a^5-1/2*b^3*x^2/a^4+3/7*b^2*x^(7/3)/a^3-3/8*b*x^(8/3)/a^2+1/3*x^
3/a-3*b^9*ln(b+a*x^(1/3))/a^10
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}} dx = \frac{2520b^8\sqrt[3]{x} - 1260ab^7x^{2/3} + 840a^2b^6x - 630a^3b^5x^{4/3} + 504a^4b^4x^{5/3} - 420a^5b^3x^2 + 360a^6b^2x^{7/3} - 315a^7b}{840a^9} - \frac{3b^9\log\left(b+a\sqrt[3]{x}\right)}{a^{10}}$$

input
```
Integrate[x^2/(a + b/x^(1/3)),x]
```

output
```
(2520*b^8*x^(1/3) - 1260*a*b^7*x^(2/3) + 840*a^2*b^6*x - 630*a^3*b^5*x^(4/
3) + 504*a^4*b^4*x^(5/3) - 420*a^5*b^3*x^2 + 360*a^6*b^2*x^(7/3) - 315*a^7
*b*x^(8/3) + 280*a^8*x^3)/(840*a^9) - (3*b^9*Log[b + a*x^(1/3)])/a^10
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{7/3}}{a\sqrt[3]{x}+b}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^3}{\sqrt[3]{x}a+b}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{b^9}{a^9\left(\sqrt[3]{x}a+b\right)}+\frac{b^8}{a^9}-\frac{\sqrt[3]{x}b^7}{a^8}+\frac{x^{2/3}b^6}{a^7}-\frac{xb^5}{a^6}+\frac{x^{4/3}b^4}{a^5}-\frac{x^{5/3}b^3}{a^4}+\frac{x^2b^2}{a^3}-\frac{x^{7/3}b}{a^2}+\frac{x^{8/3}}{a}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{10}}+\frac{b^8\sqrt[3]{x}}{a^9}-\frac{b^7x^{2/3}}{2a^8}+\frac{b^6x}{3a^7}-\frac{b^5x^{4/3}}{4a^6}+\frac{b^4x^{5/3}}{5a^5}-\frac{b^3x^2}{6a^4}+\frac{b^2x^{7/3}}{7a^3}-\frac{bx^{8/3}}{8a^2}+\frac{x^3}{9a}\right)$$

input
```
Int[x^2/(a + b/x^(1/3)),x]
```

output
```
3*((b^8*x^(1/3))/a^9 - (b^7*x^(2/3))/(2*a^8) + (b^6*x)/(3*a^7) - (b^5*x^(4
/3))/(4*a^6) + (b^4*x^(5/3))/(5*a^5) - (b^3*x^2)/(6*a^4) + (b^2*x^(7/3))/(
7*a^3) - (b*x^(8/3))/(8*a^2) + x^3/(9*a) - (b^9*Log[b + a*x^(1/3)])/a^10)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{a^{8} x^{3}}{3}-\frac{3 b \,x^{\frac{8}{3}} a^{7}}{8}+\frac{3 b^{2} x^{\frac{7}{3}} a^{6}}{7}-\frac{a^{5} b^{3} x^{2}}{2}+\frac{3 x^{\frac{5}{3}} a^{4} b^{4}}{5}-\frac{3 b^{5} x^{\frac{4}{3}} a^{3}}{4}+a^{2} b^{6} x -\frac{3 a \,b^{7} x^{\frac{2}{3}}}{2}+3 b^{8} x^{\frac{1}{3}}}{a^{9}}-\frac{3 b^{9} \ln \left(b +a \,x^{\frac{1}{3}}\right)}{a^{10}}$	110
default	$\frac{\frac{a^{8} x^{3}}{3}-\frac{3 b \,x^{\frac{8}{3}} a^{7}}{8}+\frac{3 b^{2} x^{\frac{7}{3}} a^{6}}{7}-\frac{a^{5} b^{3} x^{2}}{2}+\frac{3 x^{\frac{5}{3}} a^{4} b^{4}}{5}-\frac{3 b^{5} x^{\frac{4}{3}} a^{3}}{4}+a^{2} b^{6} x -\frac{3 a \,b^{7} x^{\frac{2}{3}}}{2}+3 b^{8} x^{\frac{1}{3}}}{a^{9}}-\frac{3 b^{9} \ln \left(b +a \,x^{\frac{1}{3}}\right)}{a^{10}}$	110

input
```
int(x^2/(a+b/x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/a^9*(1/9*a^8*x^3-1/8*b*x^(8/3)*a^7+1/7*b^2*x^(7/3)*a^6-1/6*a^5*b^3*x^2+1
/5*x^(5/3)*a^4*b^4-1/4*b^5*x^(4/3)*a^3+1/3*a^2*b^6*x-1/2*a*b^7*x^(2/3)+b^8
*x^(1/3))-3*b^9*ln(b+a*x^(1/3))/a^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx$$

$$= \frac{280\,a^9x^3 - 420\,a^6b^3x^2 + 840\,a^3b^6x - 2520\,b^9\log\left(ax^{\frac{1}{3}} + b\right) - 63\,(5\,a^8bx^2 - 8\,a^5b^4x + 20\,a^2b^7)x^{\frac{2}{3}} + 90\,(4}{840\,a^{10}}$$

input
```
integrate(x^2/(a+b/x^(1/3)),x, algorithm="fricas")
```

output
```
1/840*(280*a^9*x^3 - 420*a^6*b^3*x^2 + 840*a^3*b^6*x - 2520*b^9*log(a*x^(1
/3) + b) - 63*(5*a^8*b*x^2 - 8*a^5*b^4*x + 20*a^2*b^7)*x^(2/3) + 90*(4*a^7
*b^2*x^2 - 7*a^4*b^5*x + 28*a*b^8)*x^(1/3))/a^10
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx$$

$$= \begin{cases} \frac{x^3}{3a} - \frac{3bx^{\frac{8}{3}}}{8a^2} + \frac{3b^2x^{\frac{7}{3}}}{7a^3} - \frac{b^3x^2}{2a^4} + \frac{3b^4x^{\frac{5}{3}}}{5a^5} - \frac{3b^5x^{\frac{4}{3}}}{4a^6} + \frac{b^6x}{a^7} - \frac{3b^7x^{\frac{2}{3}}}{2a^8} + \frac{3b^8\sqrt[3]{x}}{a^9} - \frac{3b^9\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^{10}} & \text{for } a \neq 0 \\ \frac{3x^{\frac{10}{3}}}{10b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(a+b/x**(1/3)),x)
```

output
```
Piecewise((x**3/(3*a) - 3*b*x**(8/3)/(8*a**2) + 3*b**2*x**(7/3)/(7*a**3) -
 b**3*x**2/(2*a**4) + 3*b**4*x**(5/3)/(5*a**5) - 3*b**5*x**(4/3)/(4*a**6)
+ b**6*x/a**7 - 3*b**7*x**(2/3)/(2*a**8) + 3*b**8*x**(1/3)/a**9 - 3*b**9*l
og(x**(1/3) + b/a)/a**10, Ne(a, 0)), (3*x**(10/3)/(10*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx = -\frac{3\,b^9\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^{10}} - \frac{b^9\log\left(x\right)}{a^{10}} + \frac{\left(280\,a^8 - \frac{315\,a^7b}{x^{\frac{1}{3}}} + \frac{360\,a^6b^2}{x^{\frac{2}{3}}} - \frac{420\,a^5b^3}{x} + \frac{504\,a^4b^4}{x^{\frac{4}{3}}} - \frac{630\,a^3b^5}{x^{\frac{5}{3}}} + \frac{840\,a^2b^6}{x^2} - \frac{1260\,ab^7}{x^{\frac{7}{3}}} + \frac{2520\,b^8}{x^{\frac{8}{3}}}\right)x^3}{840\,a^9}$$

input
```
integrate(x^2/(a+b/x^(1/3)),x, algorithm="maxima")
```

output
```
-3*b^9*log(a + b/x^(1/3))/a^10 - b^9*log(x)/a^10 + 1/840*(280*a^8 - 315*a^
7*b/x^(1/3) + 360*a^6*b^2/x^(2/3) - 420*a^5*b^3/x + 504*a^4*b^4/x^(4/3) -
630*a^3*b^5/x^(5/3) + 840*a^2*b^6/x^2 - 1260*a*b^7/x^(7/3) + 2520*b^8/x^(8
/3))*x^3/a^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx = -\frac{3\,b^9\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^{10}} + \frac{280\,a^8x^3 - 315\,a^7bx^{\frac{8}{3}} + 360\,a^6b^2x^{\frac{7}{3}} - 420\,a^5b^3x^2 + 504\,a^4b^4x^{\frac{5}{3}} - 630\,a^3b^5x^{\frac{4}{3}} + 840\,a^2b^6x - 1260\,ab^7x^{\frac{2}{3}}}{840\,a^9}$$

input
```
integrate(x^2/(a+b/x^(1/3)),x, algorithm="giac")
```

output
```
-3*b^9*log(abs(a*x^(1/3) + b))/a^10 + 1/840*(280*a^8*x^3 - 315*a^7*b*x^(8/
3) + 360*a^6*b^2*x^(7/3) - 420*a^5*b^3*x^2 + 504*a^4*b^4*x^(5/3) - 630*a^3
*b^5*x^(4/3) + 840*a^2*b^6*x - 1260*a*b^7*x^(2/3) + 2520*b^8*x^(1/3))/a^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx=\frac{x^3}{3\,a}-\frac{3\,b\,x^{8/3}}{8\,a^2}+\frac{b^6\,x}{a^7}-\frac{3\,b^9\,\ln\left(b+a\,x^{1/3}\right)}{a^{10}}-\frac{b^3\,x^2}{2\,a^4}+\frac{3\,b^2\,x^{7/3}}{7\,a^3}+\frac{3\,b^4\,x^{5/3}}{5\,a^5}-\frac{3\,b^5\,x^{4/3}}{4\,a^6}-\frac{3\,b^7\,x^{2/3}}{2\,a^8}+\frac{3\,b^8\,x^{1/3}}{a^9}$$

input
```
int(x^2/(a + b/x^(1/3)),x)
```

output
```
x^3/(3*a) - (3*b*x^(8/3))/(8*a^2) + (b^6*x)/a^7 - (3*b^9*log(b + a*x^(1/3)
))/a^10 - (b^3*x^2)/(2*a^4) + (3*b^2*x^(7/3))/(7*a^3) + (3*b^4*x^(5/3))/(5
*a^5) - (3*b^5*x^(4/3))/(4*a^6) - (3*b^7*x^(2/3))/(2*a^8) + (3*b^8*x^(1/3)
)/a^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{a+\frac{b}{\sqrt[3]{x}}}\,dx$$

$$=\frac{-315x^{\frac{8}{3}}a^8b+504x^{\frac{5}{3}}a^5b^4-1260x^{\frac{2}{3}}a^2b^7+360x^{\frac{7}{3}}a^7b^2-630x^{\frac{4}{3}}a^4b^5+2520x^{\frac{1}{3}}a\,b^8-2520\log\left(x^{\frac{1}{3}}a+b\right)b^9}{840a^{10}}$$

input
```
int(x^2/(a+b/x^(1/3)),x)
```

output
```
( - 315*x**(2/3)*a**8*b*x**2 + 504*x**(2/3)*a**5*b**4*x - 1260*x**(2/3)*a*
*2*b**7 + 360*x**(1/3)*a**7*b**2*x**2 - 630*x**(1/3)*a**4*b**5*x + 2520*x*
*(1/3)*a*b**8 - 2520*log(x**(1/3)*a + b)*b**9 + 280*a**9*x**3 - 420*a**6*b
**3*x**2 + 840*a**3*b**6*x)/(840*a**10)
```

3.313 $\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}} \, dx = -\frac{3b^5\sqrt[3]{x}}{a^6}+\frac{3b^4x^{2/3}}{2a^5}-\frac{b^3x}{a^4}+\frac{3b^2x^{4/3}}{4a^3}-\frac{3bx^{5/3}}{5a^2}+\frac{x^2}{2a}+\frac{3b^6\log\left(b+a\sqrt[3]{x}\right)}{a^7}$$

output
```
-3*b^5*x^(1/3)/a^6+3/2*b^4*x^(2/3)/a^5-b^3*x/a^4+3/4*b^2*x^(4/3)/a^3-3/5*b
*x^(5/3)/a^2+1/2*x^2/a+3*b^6*ln(b+a*x^(1/3))/a^7
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}} \, dx = \frac{-60ab^5\sqrt[3]{x}+30a^2b^4x^{2/3}-20a^3b^3x+15a^4b^2x^{4/3}-12a^5bx^{5/3}+10a^6x^2+60b^6\log\left(b+a\sqrt[3]{x}\right)}{20a^7}$$

input
```
Integrate[x/(a + b/x^(1/3)),x]
```

output
```
(-60*a*b^5*x^(1/3) + 30*a^2*b^4*x^(2/3) - 20*a^3*b^3*x + 15*a^4*b^2*x^(4/3
) - 12*a^5*b*x^(5/3) + 10*a^6*x^2 + 60*b^6*Log[b + a*x^(1/3)])/(20*a^7)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^{4/3}}{a\sqrt[3]{x}+b}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^2}{\sqrt[3]{x}a+b}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{b^6}{a^6\left(\sqrt[3]{x}a+b\right)}-\frac{b^5}{a^6}+\frac{\sqrt[3]{x}b^4}{a^5}-\frac{x^{2/3}b^3}{a^4}+\frac{xb^2}{a^3}-\frac{x^{4/3}b}{a^2}+\frac{x^{5/3}}{a}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{b^6\log\left(a\sqrt[3]{x}+b\right)}{a^7}-\frac{b^5\sqrt[3]{x}}{a^6}+\frac{b^4x^{2/3}}{2a^5}-\frac{b^3x}{3a^4}+\frac{b^2x^{4/3}}{4a^3}-\frac{bx^{5/3}}{5a^2}+\frac{x^2}{6a}\right)$$

input
```
Int[x/(a + b/x^(1/3)),x]
```

output
```
3*(-((b^5*x^(1/3))/a^6) + (b^4*x^(2/3))/(2*a^5) - (b^3*x)/(3*a^4) + (b^2*x
^(4/3))/(4*a^3) - (b*x^(5/3))/(5*a^2) + x^2/(6*a) + (b^6*Log[b + a*x^(1/3)
])/a^7)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{a^5x^2}{2}-\frac{3bx^{\frac{5}{3}}a^4}{5}+\frac{3b^2x^{\frac{4}{3}}a^3}{4}-a^2b^3x+\frac{3x^{\frac{2}{3}}ab^4}{2}-3b^5x^{\frac{1}{3}}}{a^6}+\frac{3b^6\ln\left(b+a\,x^{\frac{1}{3}}\right)}{a^7}$	78
default	$\frac{\frac{a^5x^2}{2}-\frac{3bx^{\frac{5}{3}}a^4}{5}+\frac{3b^2x^{\frac{4}{3}}a^3}{4}-a^2b^3x+\frac{3x^{\frac{2}{3}}ab^4}{2}-3b^5x^{\frac{1}{3}}}{a^6}+\frac{3b^6\ln\left(b+a\,x^{\frac{1}{3}}\right)}{a^7}$	78

input
```
int(x/(a+b/x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/a^6*(1/6*a^5*x^2-1/5*b*x^(5/3)*a^4+1/4*b^2*x^(4/3)*a^3-1/3*a^2*b^3*x+1/2
*x^(2/3)*a*b^4-b^5*x^(1/3))+3*b^6*ln(b+a*x^(1/3))/a^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx = \frac{10\,a^6x^2-20\,a^3b^3x+60\,b^6\log\left(ax^{\frac{1}{3}}+b\right)-6\,(2\,a^5bx-5\,a^2b^4)x^{\frac{2}{3}}+15\,(a^4b^2x-4\,ab^5)x^{\frac{1}{3}}}{20\,a^7}$$

input
```
integrate(x/(a+b/x^(1/3)),x, algorithm="fricas")
```

output
```
1/20*(10*a^6*x^2 - 20*a^3*b^3*x + 60*b^6*log(a*x^(1/3) + b) - 6*(2*a^5*b*x
 - 5*a^2*b^4)*x^(2/3) + 15*(a^4*b^2*x - 4*a*b^5)*x^(1/3))/a^7
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx = \begin{cases} \frac{x^2}{2a}-\frac{3bx^{\frac{5}{3}}}{5a^2}+\frac{3b^2x^{\frac{4}{3}}}{4a^3}-\frac{b^3x}{a^4}+\frac{3b^4x^{\frac{2}{3}}}{2a^5}-\frac{3b^5\sqrt[3]{x}}{a^6}+\frac{3b^6\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^7} & \text{for } a\neq 0 \\ \frac{3x^{\frac{7}{3}}}{7b} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b/x**(1/3)),x)
```

output
```
Piecewise((x**2/(2*a) - 3*b*x**(5/3)/(5*a**2) + 3*b**2*x**(4/3)/(4*a**3) -
 b**3*x/a**4 + 3*b**4*x**(2/3)/(2*a**5) - 3*b**5*x**(1/3)/a**6 + 3*b**6*lo
g(x**(1/3) + b/a)/a**7, Ne(a, 0)), (3*x**(7/3)/(7*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx = \frac{3\,b^6\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^7} + \frac{b^6\log\left(x\right)}{a^7} + \frac{\left(10\,a^5 - \frac{12\,a^4b}{x^{\frac{1}{3}}} + \frac{15\,a^3b^2}{x^{\frac{2}{3}}} - \frac{20\,a^2b^3}{x} + \frac{30\,ab^4}{x^{\frac{4}{3}}} - \frac{60\,b^5}{x^{\frac{5}{3}}}\right)x^2}{20\,a^6}$$

input
```
integrate(x/(a+b/x^(1/3)),x, algorithm="maxima")
```

output
```
3*b^6*log(a + b/x^(1/3))/a^7 + b^6*log(x)/a^7 + 1/20*(10*a^5 - 12*a^4*b/x^
(1/3) + 15*a^3*b^2/x^(2/3) - 20*a^2*b^3/x + 30*a*b^4/x^(4/3) - 60*b^5/x^(5
/3))*x^2/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx = \frac{3\,b^6\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^7} + \frac{10\,a^5x^2 - 12\,a^4bx^{\frac{5}{3}} + 15\,a^3b^2x^{\frac{4}{3}} - 20\,a^2b^3x + 30\,ab^4x^{\frac{2}{3}} - 60\,b^5x^{\frac{1}{3}}}{20\,a^6}$$

input
```
integrate(x/(a+b/x^(1/3)),x, algorithm="giac")
```

output
```
3*b^6*log(abs(a*x^(1/3) + b))/a^7 + 1/20*(10*a^5*x^2 - 12*a^4*b*x^(5/3) +
15*a^3*b^2*x^(4/3) - 20*a^2*b^3*x + 30*a*b^4*x^(2/3) - 60*b^5*x^(1/3))/a^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx=\frac{x^2}{2\,a}-\frac{b^3\,x}{a^4}-\frac{3\,b\,x^{5/3}}{5\,a^2}+\frac{3\,b^6\,\ln\left(b+a\,x^{1/3}\right)}{a^7}+\frac{3\,b^2\,x^{4/3}}{4\,a^3}+\frac{3\,b^4\,x^{2/3}}{2\,a^5}-\frac{3\,b^5\,x^{1/3}}{a^6}$$

input
```
int(x/(a + b/x^(1/3)),x)
```

output
```
x^2/(2*a) - (b^3*x)/a^4 - (3*b*x^(5/3))/(5*a^2) + (3*b^6*log(b + a*x^(1/3)
))/a^7 + (3*b^2*x^(4/3))/(4*a^3) + (3*b^4*x^(2/3))/(2*a^5) - (3*b^5*x^(1/3
))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

$$\int \frac{x}{a+\frac{b}{\sqrt[3]{x}}}\,dx$$
$$=\frac{-12x^{\frac{5}{3}}a^5b+30x^{\frac{2}{3}}a^2b^4+15x^{\frac{4}{3}}a^4b^2-60x^{\frac{1}{3}}a\,b^5+60\log\left(x^{\frac{1}{3}}a+b\right)b^6+10a^6x^2-20a^3b^3x}{20a^7}$$

input
```
int(x/(a+b/x^(1/3)),x)
```

output
```
( - 12*x**(2/3)*a**5*b*x + 30*x**(2/3)*a**2*b**4 + 15*x**(1/3)*a**4*b**2*x
 - 60*x**(1/3)*a*b**5 + 60*log(x**(1/3)*a + b)*b**6 + 10*a**6*x**2 - 20*a*
*3*b**3*x)/(20*a**7)
```

3.314 $\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}} dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}} dx = \frac{3b^2\sqrt[3]{x}}{a^3} - \frac{3bx^{2/3}}{2a^2} + \frac{x}{a} - \frac{3b^3\log\left(b+a\sqrt[3]{x}\right)}{a^4}$$

output
```
3*b^2*x^(1/3)/a^3-3/2*b*x^(2/3)/a^2+x/a-3*b^3*ln(b+a*x^(1/3))/a^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}} dx = \frac{6ab^2\sqrt[3]{x} - 3a^2bx^{2/3} + 2a^3x - 6b^3\log\left(b+a\sqrt[3]{x}\right)}{2a^4}$$

input
```
Integrate[(a + b/x^(1/3))^(-1),x]
```

output
```
(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[b + a*x^(1/3)])/(
2*a^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {774, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}dx$$

$$\Big\downarrow \ 774$$

$$3\int \frac{x^{2/3}}{a+\frac{b}{\sqrt[3]{x}}}d\sqrt[3]{x}$$

$$\Big\downarrow \ 795$$

$$3\int \frac{x}{\sqrt[3]{x}a+b}d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(-\frac{b^3}{a^3\left(\sqrt[3]{x}a+b\right)}+\frac{b^2}{a^3}-\frac{\sqrt[3]{x}b}{a^2}+\frac{x^{2/3}}{a}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(-\frac{b^3\log\left(a\sqrt[3]{x}+b\right)}{a^4}+\frac{b^2\sqrt[3]{x}}{a^3}-\frac{bx^{2/3}}{2a^2}+\frac{x}{3a}\right)$$

input
```
Int[(a + b/x^(1/3))^(-1),x]
```

output
```
3*((b^2*x^(1/3))/a^3 - (b*x^(2/3))/(2*a^2) + x/(3*a) - (b^3*Log[b + a*x^(1
/3)])/a^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{a^2 x-\frac{3 x^{\frac{2}{3}} a b}{2}+3 x^{\frac{1}{3}} b^2}{a^3}-\frac{3 b^3 \ln\left(b+a\,x^{\frac{1}{3}}\right)}{a^4}$	44
default	$\frac{a^2 x-\frac{3 x^{\frac{2}{3}} a b}{2}+3 x^{\frac{1}{3}} b^2}{a^3}-\frac{3 b^3 \ln\left(b+a\,x^{\frac{1}{3}}\right)}{a^4}$	44

input
```
int(1/(a+b/x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/a^3*(1/3*a^2*x-1/2*x^(2/3)*a*b+x^(1/3)*b^2)-3*b^3*ln(b+a*x^(1/3))/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx = \frac{2\,a^3x - 6\,b^3\log\left(ax^{\frac{1}{3}}+b\right) - 3\,a^2bx^{\frac{2}{3}} + 6\,ab^2x^{\frac{1}{3}}}{2\,a^4}$$

input
```
integrate(1/(a+b/x^(1/3)),x, algorithm="fricas")
```

output
```
1/2*(2*a^3*x - 6*b^3*log(a*x^(1/3) + b) - 3*a^2*b*x^(2/3) + 6*a*b^2*x^(1/3
))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx = \begin{cases} \frac{x}{a} - \frac{3bx^{\frac{2}{3}}}{2a^2} + \frac{3b^2\sqrt[3]{x}}{a^3} - \frac{3b^3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^4} & \text{for } a \neq 0 \\ \frac{3x^{\frac{4}{3}}}{4b} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3)),x)
```

output
```
Piecewise((x/a - 3*b*x**(2/3)/(2*a**2) + 3*b**2*x**(1/3)/a**3 - 3*b**3*log
(x**(1/3) + b/a)/a**4, Ne(a, 0)), (3*x**(4/3)/(4*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx = -\frac{3\,b^3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^4} - \frac{b^3\log\left(x\right)}{a^4} + \frac{\left(2\,a^2 - \frac{3\,ab}{x^{\frac{1}{3}}} + \frac{6\,b^2}{x^{\frac{2}{3}}}\right)x}{2\,a^3}$$

input
```
integrate(1/(a+b/x^(1/3)),x, algorithm="maxima")
```

output
```
-3*b^3*log(a + b/x^(1/3))/a^4 - b^3*log(x)/a^4 + 1/2*(2*a^2 - 3*a*b/x^(1/3
) + 6*b^2/x^(2/3))*x/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx = -\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

input
```
integrate(1/(a+b/x^(1/3)),x, algorithm="giac")
```

output
```
-3*b^3*log(abs(a*x^(1/3) + b))/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*
x^(1/3))/a^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx = \frac{x}{a}-\frac{3\,b\,x^{2/3}}{2\,a^2}-\frac{3\,b^3\,\ln\left(b+a\,x^{1/3}\right)}{a^4}+\frac{3\,b^2\,x^{1/3}}{a^3}$$

input
```
int(1/(a + b/x^(1/3)),x)
```

output
```
x/a - (3*b*x^(2/3))/(2*a^2) - (3*b^3*log(b + a*x^(1/3)))/a^4 + (3*b^2*x^(1
/3))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+\frac{b}{\sqrt[3]{x}}}\,dx=\frac{-3x^{\frac{2}{3}}a^{2}b+6x^{\frac{1}{3}}a\,b^{2}-6\log\left(x^{\frac{1}{3}}a+b\right)b^{3}+2a^{3}x}{2a^{4}}$$

input
```
int(1/(a+b/x^(1/3)),x)
```

output
```
( - 3*x**(2/3)*a**2*b + 6*x**(1/3)*a*b**2 - 6*log(x**(1/3)*a + b)*b**3 + 2
*a**3*x)/(2*a**4)
```

3.315 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx=\frac{3\log\left(b+a\sqrt[3]{x}\right)}{a}$$

output
```
3*ln(b+a*x^(1/3))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx=\frac{3\log\left(b+a\sqrt[3]{x}\right)}{a}$$

input
```
Integrate[1/((a + b/x^(1/3))*x),x]
```

output
```
(3*Log[b + a*x^(1/3)])/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {795, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^{2/3}\left(a\sqrt[3]{x}+b\right)}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{3\log\left(a\sqrt[3]{x}+b\right)}{a}$$

input
```
Int[1/((a + b/x^(1/3))*x),x]
```

output
```
(3*Log[b + a*x^(1/3)])/a
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{3 \ln\left(b+a\,x^{\frac{1}{3}}\right)}{a}$	14
derivativedivides	$\frac{3 \ln\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a}+\frac{\ln\left(x\right)}{a}$	21

input
```
int(1/(a+b/x^(1/3))/x,x,method=_RETURNVERBOSE)
```

output
```
3*ln(b+a*x^(1/3))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx = \frac{3\,\log\left(ax^{\frac{1}{3}}+b\right)}{a}$$

input
```
integrate(1/(a+b/x^(1/3))/x,x, algorithm="fricas")
```

output
```
3*log(a*x^(1/3) + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx = \begin{cases} \frac{3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a} & \text{for}\ a \neq 0 \\ \frac{3\sqrt[3]{x}}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))/x,x)
```

output
```
Piecewise((3*log(x**(1/3) + b/a)/a, Ne(a, 0)), (3*x**(1/3)/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx=\frac{3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a}+\frac{\log(x)}{a}$$

input
```
integrate(1/(a+b/x^(1/3))/x,x, algorithm="maxima")
```

output
```
3*log(a + b/x^(1/3))/a + log(x)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx=\frac{3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a}$$

input
```
integrate(1/(a+b/x^(1/3))/x,x, algorithm="giac")
```

output
```
3*log(abs(a*x^(1/3) + b))/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx = \frac{3\,\ln\left(b+a\,x^{1/3}\right)}{a}$$

input
```
int(1/(x*(a + b/x^(1/3))),x)
```

output
```
(3*log(b + a*x^(1/3)))/a
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}\,dx = \frac{3\log\left(x^{\frac{1}{3}}a+b\right)}{a}$$

input
```
int(1/(a+b/x^(1/3))/x,x)
```

output
```
(3*log(x**(1/3)*a + b))/a
```

3.316 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2} dx = -\frac{3}{2bx^{2/3}}+\frac{3a}{b^2\sqrt[3]{x}}-\frac{3a^2\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^3}$$

output
```
-3/2/b/x^(2/3)+3*a/b^2/x^(1/3)-3*a^2*ln(a+b/x^(1/3))/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2} dx = -\frac{\frac{3b\left(b-2a\sqrt[3]{x}\right)}{x^{2/3}}+6a^2\log\left(b+a\sqrt[3]{x}\right)-2a^2\log(x)}{2b^3}$$

input
```
Integrate[1/((a + b/x^(1/3))*x^2),x]
```

output
```
-1/2*((3*b*(b - 2*a*x^(1/3)))/x^(2/3) + 6*a^2*Log[b + a*x^(1/3)] - 2*a^2*L
og[x])/b^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{\sqrt[3]{x}}\right)} \, dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{5/3} \left(a \sqrt[3]{x}+b\right)}dx$$

$$\Big\downarrow \text{798}$$

$$3 \int \frac{1}{\left(\sqrt[3]{x} a+b\right) x}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3 \int \left(-\frac{a^3}{b^3 \left(\sqrt[3]{x} a+b\right)}+\frac{a^2}{b^3 \sqrt[3]{x}}-\frac{a}{b^2 x^{2/3}}+\frac{1}{b x}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3 \left(-\frac{a^2 \log \left(a \sqrt[3]{x}+b\right)}{b^3}+\frac{a^2 \log \left(\sqrt[3]{x}\right)}{b^3}+\frac{a}{b^2 \sqrt[3]{x}}-\frac{1}{2 b x^{2/3}}\right)$$

input
```
Int[1/((a + b/x^(1/3))*x^2),x]
```

output
```
3*(-1/2*1/(b*x^(2/3)) + a/(b^2*x^(1/3)) - (a^2*Log[b + a*x^(1/3)])/b^3 + (
a^2*Log[x^(1/3)])/b^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{3}{2x^{\frac{2}{3}}b}+\frac{a^{2}\ln\left(x\right)}{b^{3}}+\frac{3a}{x^{\frac{1}{3}}b^{2}}-\frac{3a^{2}\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{3}}$	44
default	$-\frac{3}{2x^{\frac{2}{3}}b}+\frac{a^{2}\ln\left(x\right)}{b^{3}}+\frac{3a}{x^{\frac{1}{3}}b^{2}}-\frac{3a^{2}\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{3}}$	44

input
```
int(1/(a+b/x^(1/3))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/2/x^(2/3)/b+a^2/b^3*ln(x)+3*a/x^(1/3)/b^2-3*a^2/b^3*ln(b+a*x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx = -\frac{3\left(2\,a^2x\log\left(ax^{\frac{1}{3}}+b\right)-2\,a^2x\log\left(x^{\frac{1}{3}}\right)-2\,abx^{\frac{2}{3}}+b^2x^{\frac{1}{3}}\right)}{2\,b^3x}$$

input
```
integrate(1/(a+b/x^(1/3))/x^2,x, algorithm="fricas")
```

output
```
-3/2*(2*a^2*x*log(a*x^(1/3) + b) - 2*a^2*x*log(x^(1/3)) - 2*a*b*x^(2/3) +
b^2*x^(1/3))/(b^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for } a=0 \wedge b=0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for } a=0 \\ -\frac{1}{ax} & \text{for } b=0 \\ \frac{a^2\log(x)}{b^3}-\frac{3a^2\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{b^3}+\frac{3a}{b^2\sqrt[3]{x}}-\frac{3}{2bx^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))/x**2,x)
```

output
```
Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (-3/(2*b*x**(2/3)), Eq(a, 0
)), (-1/(a*x), Eq(b, 0)), (a**2*log(x)/b**3 - 3*a**2*log(x**(1/3) + b/a)/b
**3 + 3*a/(b**2*x**(1/3)) - 3/(2*b*x**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx = -\frac{3\,a^2\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3} - \frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2}{2\,b^3} + \frac{6\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a}{b^3}$$

input
```
integrate(1/(a+b/x^(1/3))/x^2,x, algorithm="maxima")
```

output
```
-3*a^2*log(a + b/x^(1/3))/b^3 - 3/2*(a + b/x^(1/3))^2/b^3 + 6*(a + b/x^(1/
3))*a/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx = -\frac{3\,a^2\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^3} + \frac{a^2\log\left(|x|\right)}{b^3} + \frac{3\left(2\,abx^{\frac{1}{3}}-b^2\right)}{2\,b^3x^{\frac{2}{3}}}$$

input
```
integrate(1/(a+b/x^(1/3))/x^2,x, algorithm="giac")
```

output
```
-3*a^2*log(abs(a*x^(1/3) + b))/b^3 + a^2*log(abs(x))/b^3 + 3/2*(2*a*b*x^(1
/3) - b^2)/(b^3*x^(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx = -\frac{\frac{3}{2\,b}-\frac{3\,a\,x^{1/3}}{b^2}}{x^{2/3}} - \frac{6\,a^2\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^3}$$

input
```
int(1/(x^2*(a + b/x^(1/3))),x)
```

output
```
- (3/(2*b) - (3*a*x^(1/3))/b^2)/x^(2/3) - (6*a^2*atanh((2*a*x^(1/3))/b + 1
))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^2}\,dx = \frac{3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)a^2-3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2+3x^{\frac{1}{3}}ab-\frac{3b^2}{2}}{x^{\frac{2}{3}}b^3}$$

input
```
int(1/(a+b/x^(1/3))/x^2,x)
```

output
```
(3*(2*x**(2/3)*log(x**(1/3))*a**2 - 2*x**(2/3)*log(x**(1/3)*a + b)*a**2 +
2*x**(1/3)*a*b - b**2))/(2*x**(2/3)*b**3)
```

3.317 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx=-\frac{3}{5bx^{5/3}}+\frac{3a}{4b^2x^{4/3}}-\frac{a^2}{b^3x}+\frac{3a^3}{2b^4x^{2/3}}-\frac{3a^4}{b^5\sqrt[3]{x}}+\frac{3a^5\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^6}$$

output
```
-3/5/b/x^(5/3)+3/4*a/b^2/x^(4/3)-a^2/b^3/x+3/2*a^3/b^4/x^(2/3)-3*a^4/b^5/x
^(1/3)+3*a^5*ln(a+b/x^(1/3))/b^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx$$
$$=\frac{\frac{b\left(-12b^4+15ab^3\sqrt[3]{x}-20a^2b^2x^{2/3}+30a^3bx-60a^4x^{4/3}\right)}{x^{5/3}}+60a^5\log\left(b+a\sqrt[3]{x}\right)-20a^5\log(x)}{20b^6}$$

input
```
Integrate[1/((a + b/x^(1/3))*x^3),x]
```

output
```
((b*(-12*b^4 + 15*a*b^3*x^(1/3) - 20*a^2*b^2*x^(2/3) + 30*a^3*b*x - 60*a^4
*x^(4/3)))/x^(5/3) + 60*a^5*Log[b + a*x^(1/3)] - 20*a^5*Log[x])/(20*b^6)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{\sqrt[3]{x}}\right)}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{8/3}\left(a\sqrt[3]{x}+b\right)}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)x^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{a^6}{b^6\left(\sqrt[3]{x}a+b\right)}-\frac{a^5}{b^6\sqrt[3]{x}}+\frac{a^4}{b^5x^{2/3}}-\frac{a^3}{b^4x}+\frac{a^2}{b^3x^{4/3}}-\frac{a}{b^2x^{5/3}}+\frac{1}{bx^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{a^5\log\left(a\sqrt[3]{x}+b\right)}{b^6}-\frac{a^5\log\left(\sqrt[3]{x}\right)}{b^6}-\frac{a^4}{b^5\sqrt[3]{x}}+\frac{a^3}{2b^4x^{2/3}}-\frac{a^2}{3b^3x}+\frac{a}{4b^2x^{4/3}}-\frac{1}{5bx^{5/3}}\right)$$

input
```
Int[1/((a + b/x^(1/3))*x^3),x]
```

output
```
3*(-1/5*1/(b*x^(5/3)) + a/(4*b^2*x^(4/3)) - a^2/(3*b^3*x) + a^3/(2*b^4*x^(
2/3)) - a^4/(b^5*x^(1/3)) + (a^5*Log[b + a*x^(1/3)])/b^6 - (a^5*Log[x^(1/3
)])/b^6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{3a^5\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^6}-\frac{3}{5b\,x^{\frac{5}{3}}}-\frac{a^2}{b^3x}-\frac{3a^4}{b^5x^{\frac{1}{3}}}+\frac{3a}{4b^2x^{\frac{4}{3}}}+\frac{3a^3}{2b^4x^{\frac{2}{3}}}-\frac{a^5\ln(x)}{b^6}$	78
default	$\frac{3a^5\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^6}-\frac{3}{5b\,x^{\frac{5}{3}}}-\frac{a^2}{b^3x}-\frac{3a^4}{b^5x^{\frac{1}{3}}}+\frac{3a}{4b^2x^{\frac{4}{3}}}+\frac{3a^3}{2b^4x^{\frac{2}{3}}}-\frac{a^5\ln(x)}{b^6}$	78

input
```
int(1/(a+b/x^(1/3))/x^3,x,method=_RETURNVERBOSE)
```

output
```
3*a^5/b^6*ln(b+a*x^(1/3))-3/5/b/x^(5/3)-a^2/b^3/x-3*a^4/b^5/x^(1/3)+3/4*a/
b^2/x^(4/3)+3/2*a^3/b^4/x^(2/3)-a^5/b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx = \frac{60\,a^5x^2\log\left(ax^{\frac{1}{3}}+b\right)-60\,a^5x^2\log\left(x^{\frac{1}{3}}\right)-20\,a^2b^3x-15\,(4\,a^4bx-ab^4)x^{\frac{2}{3}}+6\,(5\,a^3b^2x-2\,b^5)x^{\frac{1}{3}}}{20\,b^6x^2}$$

input
```
integrate(1/(a+b/x^(1/3))/x^3,x, algorithm="fricas")
```

output
```
1/20*(60*a^5*x^2*log(a*x^(1/3) + b) - 60*a^5*x^2*log(x^(1/3)) - 20*a^2*b^3
*x - 15*(4*a^4*b*x - a*b^4)*x^(2/3) + 6*(5*a^3*b^2*x - 2*b^5)*x^(1/3))/(b^
6*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a=0\wedge b=0 \\ -\frac{3}{5bx^{\frac{5}{3}}} & \text{for } a=0 \\ -\frac{1}{2ax^2} & \text{for } b=0 \\ -\frac{a^5\log(x)}{b^6}+\frac{3a^5\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{b^6}-\frac{3a^4}{b^5\sqrt[3]{x}}+\frac{3a^3}{2b^4x^{\frac{2}{3}}}-\frac{a^2}{b^3x}+\frac{3a}{4b^2x^{\frac{4}{3}}}-\frac{3}{5bx^{\frac{5}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))/x**3,x)
```

output
```
Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-3/(5*b*x**(5/3)), Eq(a, 0
)), (-1/(2*a*x**2), Eq(b, 0)), (-a**5*log(x)/b**6 + 3*a**5*log(x**(1/3) +
b/a)/b**6 - 3*a**4/(b**5*x**(1/3)) + 3*a**3/(2*b**4*x**(2/3)) - a**2/(b**3
*x) + 3*a/(4*b**2*x**(4/3)) - 3/(5*b*x**(5/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx = \frac{3\,a^5\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^6} - \frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5}{5\,b^6} + \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a}{4\,b^6} - \frac{10\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^2}{b^6} + \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^3}{b^6} - \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^4}{b^6}$$

input
```
integrate(1/(a+b/x^(1/3))/x^3,x, algorithm="maxima")
```

output
```
3*a^5*log(a + b/x^(1/3))/b^6 - 3/5*(a + b/x^(1/3))^5/b^6 + 15/4*(a + b/x^(
1/3))^4*a/b^6 - 10*(a + b/x^(1/3))^3*a^2/b^6 + 15*(a + b/x^(1/3))^2*a^3/b^
6 - 15*(a + b/x^(1/3))*a^4/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx = \frac{3\,a^5\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^6} - \frac{a^5\log\left(|x|\right)}{b^6} - \frac{60\,a^4bx^{\frac{4}{3}} - 30\,a^3b^2x + 20\,a^2b^3x^{\frac{2}{3}} - 15\,ab^4x^{\frac{1}{3}} + 12\,b^5}{20\,b^6x^{\frac{5}{3}}}$$

input
```
integrate(1/(a+b/x^(1/3))/x^3,x, algorithm="giac")
```

output
```
3*a^5*log(abs(a*x^(1/3) + b))/b^6 - a^5*log(abs(x))/b^6 - 1/20*(60*a^4*b*x
^(4/3) - 30*a^3*b^2*x + 20*a^2*b^3*x^(2/3) - 15*a*b^4*x^(1/3) + 12*b^5)/(b
^6*x^(5/3))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx=\frac{6\,a^5\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^6}-\frac{\frac{3}{5\,b}-\frac{3\,a\,x^{1/3}}{4\,b^2}-\frac{3\,a^3\,x}{2\,b^4}+\frac{a^2\,x^{2/3}}{b^3}+\frac{3\,a^4\,x^{4/3}}{b^5}}{x^{5/3}}$$

input
```
int(1/(x^3*(a + b/x^(1/3))),x)
```

output
```
(6*a^5*atanh((2*a*x^(1/3))/b + 1))/b^6 - (3/(5*b) - (3*a*x^(1/3))/(4*b^2)
- (3*a^3*x)/(2*b^4) + (a^2*x^(2/3))/b^3 + (3*a^4*x^(4/3))/b^5)/x^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^3}\,dx$$
$$=\frac{-60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)a^5+60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^5-20x^{\frac{2}{3}}a^2b^3-60x^{\frac{4}{3}}a^4b+15x^{\frac{1}{3}}a\,b^4+30a^3b^2x-12b^5}{20x^{\frac{5}{3}}b^6}$$

input
```
int(1/(a+b/x^(1/3))/x^3,x)
```

output
```
( - 60*x**(2/3)*log(x**(1/3))*a**5*x + 60*x**(2/3)*log(x**(1/3)*a + b)*a**
5*x - 20*x**(2/3)*a**2*b**3 - 60*x**(1/3)*a**4*b*x + 15*x**(1/3)*a*b**4 +
30*a**3*b**2*x - 12*b**5)/(20*x**(2/3)*b**6*x)
```

3.318 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx = -\frac{3}{8bx^{8/3}}+\frac{3a}{7b^2x^{7/3}}-\frac{a^2}{2b^3x^2}+\frac{3a^3}{5b^4x^{5/3}}-\frac{3a^4}{4b^5x^{4/3}} +\frac{a^5}{b^6x}-\frac{3a^6}{2b^7x^{2/3}}+\frac{3a^7}{b^8\sqrt[3]{x}}-\frac{3a^8\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^9}$$

output
```
-3/8/b/x^(8/3)+3/7*a/b^2/x^(7/3)-1/2*a^2/b^3/x^2+3/5*a^3/b^4/x^(5/3)-3/4*a
^4/b^5/x^(4/3)+a^5/b^6/x-3/2*a^6/b^7/x^(2/3)+3*a^7/b^8/x^(1/3)-3*a^8*ln(a+
b/x^(1/3))/b^9
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx = \frac{\frac{b\left(-105b^7+120ab^6\sqrt[3]{x}-140a^2b^5x^{2/3}+168a^3b^4x-210a^4b^3x^{4/3}+280a^5b^2x^{5/3}-420a^6bx^2+840a^7x^{7/3}\right)}{x^{8/3}}-840a^8\log\left(b+a\sqrt[3]{x}\right)+2\ldots}{280b^9}$$

input
```
Integrate[1/((a + b/x^(1/3))*x^4),x]
```

output
```
((b*(-105*b^7 + 120*a*b^6*x^(1/3) - 140*a^2*b^5*x^(2/3) + 168*a^3*b^4*x -
210*a^4*b^3*x^(4/3) + 280*a^5*b^2*x^(5/3) - 420*a^6*b*x^2 + 840*a^7*x^(7/3
)))/x^(8/3) - 840*a^8*Log[b + a*x^(1/3)] + 280*a^8*Log[x])/(280*b^9)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{11/3}\left(a\sqrt[3]{x}+b\right)}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)x^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(-\frac{a^9}{b^9\left(\sqrt[3]{x}a+b\right)}+\frac{a^8}{b^9\sqrt[3]{x}}-\frac{a^7}{b^8x^{2/3}}+\frac{a^6}{b^7x}-\frac{a^5}{b^6x^{4/3}}+\frac{a^4}{b^5x^{5/3}}-\frac{a^3}{b^4x^2}+\frac{a^2}{b^3x^{7/3}}-\frac{a}{b^2x^{8/3}}+\frac{1}{bx^3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{a^8\log\left(a\sqrt[3]{x}+b\right)}{b^9}+\frac{a^8\log\left(\sqrt[3]{x}\right)}{b^9}+\frac{a^7}{b^8\sqrt[3]{x}}-\frac{a^6}{2b^7x^{2/3}}+\frac{a^5}{3b^6x}-\frac{a^4}{4b^5x^{4/3}}+\frac{a^3}{5b^4x^{5/3}}-\frac{a^2}{6b^3x^2}+\frac{a}{7b^2x^{7/3}}-\frac{1}{8bx\ldots}\right.$$

input
```
Int[1/((a + b/x^(1/3))*x^4),x]
```

output
```
3*(-1/8*1/(b*x^(8/3)) + a/(7*b^2*x^(7/3)) - a^2/(6*b^3*x^2) + a^3/(5*b^4*x
^(5/3)) - a^4/(4*b^5*x^(4/3)) + a^5/(3*b^6*x) - a^6/(2*b^7*x^(2/3)) + a^7/
(b^8*x^(1/3)) - (a^8*Log[b + a*x^(1/3)])/b^9 + (a^8*Log[x^(1/3)])/b^9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{3}{8b\,x^{\frac{8}{3}}}-\frac{a^{2}}{2b^{3}x^{2}}-\frac{3a^{4}}{4b^{5}x^{\frac{4}{3}}}-\frac{3a^{6}}{2b^{7}x^{\frac{2}{3}}}+\frac{a^{8}\ln\left(x\right)}{b^{9}}+\frac{3a}{7b^{2}x^{\frac{7}{3}}}+\frac{3a^{3}}{5b^{4}x^{\frac{5}{3}}}+\frac{a^{5}}{b^{6}x}+\frac{3a^{7}}{b^{8}x^{\frac{1}{3}}}-\frac{3a^{8}\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{9}}$
default	$-\frac{3}{8b\,x^{\frac{8}{3}}}-\frac{a^{2}}{2b^{3}x^{2}}-\frac{3a^{4}}{4b^{5}x^{\frac{4}{3}}}-\frac{3a^{6}}{2b^{7}x^{\frac{2}{3}}}+\frac{a^{8}\ln\left(x\right)}{b^{9}}+\frac{3a}{7b^{2}x^{\frac{7}{3}}}+\frac{3a^{3}}{5b^{4}x^{\frac{5}{3}}}+\frac{a^{5}}{b^{6}x}+\frac{3a^{7}}{b^{8}x^{\frac{1}{3}}}-\frac{3a^{8}\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{9}}$

input
```
int(1/(a+b/x^(1/3))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-3/8/b/x^(8/3)-1/2*a^2/b^3/x^2-3/4*a^4/b^5/x^(4/3)-3/2*a^6/b^7/x^(2/3)+a^8
/b^9*ln(x)+3/7*a/b^2/x^(7/3)+3/5*a^3/b^4/x^(5/3)+a^5/b^6/x+3*a^7/b^8/x^(1/
3)-3*a^8/b^9*ln(b+a*x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{4}}\,dx = -\frac{840\,a^{8}x^{3}\log\left(ax^{\frac{1}{3}}+b\right)-840\,a^{8}x^{3}\log\left(x^{\frac{1}{3}}\right)-280\,a^{5}b^{3}x^{2}+140\,a^{2}b^{6}x-30\,(28\,a^{7}bx^{2}-7\,a^{4}b^{4}x+4\,ab}{280\,b^{9}x^{3}}$$

input
```
integrate(1/(a+b/x^(1/3))/x^4,x, algorithm="fricas")
```

output
```
-1/280*(840*a^8*x^3*log(a*x^(1/3) + b) - 840*a^8*x^3*log(x^(1/3)) - 280*a^
5*b^3*x^2 + 140*a^2*b^6*x - 30*(28*a^7*b*x^2 - 7*a^4*b^4*x + 4*a*b^7)*x^(2
/3) + 21*(20*a^6*b^2*x^2 - 8*a^3*b^5*x + 5*b^8)*x^(1/3))/(b^9*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{8}{3}}} & \text{for } a=0 \wedge b \\ -\frac{3}{8bx^{\frac{8}{3}}} & \text{for } a=0 \\ -\frac{1}{3ax^3} & \text{for } b=0 \\ \frac{a^8\log(x)}{b^9}-\frac{3a^8\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{b^9}+\frac{3a^7}{b^8\sqrt[3]{x}}-\frac{3a^6}{2b^7x^{\frac{2}{3}}}+\frac{a^5}{b^6x}-\frac{3a^4}{4b^5x^{\frac{4}{3}}}+\frac{3a^3}{5b^4x^{\frac{5}{3}}}-\frac{a^2}{2b^3x^2}+\frac{3a}{7b^2x^{\frac{7}{3}}}-\frac{3}{8bx^{\frac{8}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))/x**4,x)
```

output
```
Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (-3/(8*b*x**(8/3)), Eq(a, 0
)), (-1/(3*a*x**3), Eq(b, 0)), (a**8*log(x)/b**9 - 3*a**8*log(x**(1/3) + b
/a)/b**9 + 3*a**7/(b**8*x**(1/3)) - 3*a**6/(2*b**7*x**(2/3)) + a**5/(b**6*
x) - 3*a**4/(4*b**5*x**(4/3)) + 3*a**3/(5*b**4*x**(5/3)) - a**2/(2*b**3*x*
*2) + 3*a/(7*b**2*x**(7/3)) - 3/(8*b*x**(8/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx = -\frac{3\,a^8\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^9}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^8}{8\,b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7 a}{7\,b^9} - \frac{14\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a^2}{b^9}+\frac{168\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^3}{5\,b^9}-\frac{105\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^4}{2\,b^9} + \frac{56\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^5}{b^9}-\frac{42\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^6}{b^9}+\frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^7}{b^9}$$

input
```
integrate(1/(a+b/x^(1/3))/x^4,x, algorithm="maxima")
```

output
```
-3*a^8*log(a + b/x^(1/3))/b^9 - 3/8*(a + b/x^(1/3))^8/b^9 + 24/7*(a + b/x^
(1/3))^7*a/b^9 - 14*(a + b/x^(1/3))^6*a^2/b^9 + 168/5*(a + b/x^(1/3))^5*a^
3/b^9 - 105/2*(a + b/x^(1/3))^4*a^4/b^9 + 56*(a + b/x^(1/3))^3*a^5/b^9 - 4
2*(a + b/x^(1/3))^2*a^6/b^9 + 24*(a + b/x^(1/3))*a^7/b^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx = -\frac{3\,a^8\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^9}+\frac{a^8\log\left(|x|\right)}{b^9} + \frac{840\,a^7bx^{\frac{7}{3}}-420\,a^6b^2x^2+280\,a^5b^3x^{\frac{5}{3}}-210\,a^4b^4x^{\frac{4}{3}}+168\,a^3b^5x-140\,a^2b^6x^{\frac{2}{3}}+120\,ab^7x^{\frac{1}{3}}-105\,b^8}{280\,b^9x^{\frac{8}{3}}}$$

input
```
integrate(1/(a+b/x^(1/3))/x^4,x, algorithm="giac")
```

output
```
-3*a^8*log(abs(a*x^(1/3) + b))/b^9 + a^8*log(abs(x))/b^9 + 1/280*(840*a^7*
b*x^(7/3) - 420*a^6*b^2*x^2 + 280*a^5*b^3*x^(5/3) - 210*a^4*b^4*x^(4/3) +
168*a^3*b^5*x - 140*a^2*b^6*x^(2/3) + 120*a*b^7*x^(1/3) - 105*b^8)/(b^9*x^
(8/3))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx = -\frac{\frac{3}{8\,b}-\frac{3\,a\,x^{1/3}}{7\,b^2}-\frac{3\,a^3\,x}{5\,b^4}+\frac{a^2\,x^{2/3}}{2\,b^3}+\frac{3\,a^6\,x^2}{2\,b^7}+\frac{3\,a^4\,x^{4/3}}{4\,b^5}-\frac{a^5\,x^{5/3}}{b^6}-\frac{3\,a^7\,x^{7/3}}{b^8}}{x^{8/3}} - \frac{6\,a^8\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^9}$$

input
```
int(1/(x^4*(a + b/x^(1/3))),x)
```

output
```
- (3/(8*b) - (3*a*x^(1/3))/(7*b^2) - (3*a^3*x)/(5*b^4) + (a^2*x^(2/3))/(2*
b^3) + (3*a^6*x^2)/(2*b^7) + (3*a^4*x^(4/3))/(4*b^5) - (a^5*x^(5/3))/b^6 -
 (3*a^7*x^(7/3))/b^8)/x^(8/3) - (6*a^8*atanh((2*a*x^(1/3))/b + 1))/b^9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x^4}\,dx$$

$$= \frac{840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}\right)a^8 - 840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^8 + 280x^{\frac{5}{3}}a^5b^3 - 140x^{\frac{2}{3}}a^2b^6 + 840x^{\frac{7}{3}}a^7b - 210x^{\frac{4}{3}}a^4b^4 + 120x^{\frac{1}{3}}}{280x^{\frac{8}{3}}b^9}$$

input
```
int(1/(a+b/x^(1/3))/x^4,x)
```

output
```
(840*x**(2/3)*log(x**(1/3))*a**8*x**2 - 840*x**(2/3)*log(x**(1/3)*a + b)*a
**8*x**2 + 280*x**(2/3)*a**5*b**3*x - 140*x**(2/3)*a**2*b**6 + 840*x**(1/3
)*a**7*b*x**2 - 210*x**(1/3)*a**4*b**4*x + 120*x**(1/3)*a*b**7 - 420*a**6*
b**2*x**2 + 168*a**3*b**5*x - 105*b**8)/(280*x**(2/3)*b**9*x**2)
```

3.319 $\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 150

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} dx = -\frac{3b^{10}}{a^{11}\left(b+a\sqrt[3]{x}\right)} + \frac{27b^8\sqrt[3]{x}}{a^{10}} - \frac{12b^7x^{2/3}}{a^9} + \frac{7b^6x}{a^8} - \frac{9b^5x^{4/3}}{2a^7} + \frac{3b^4x^{5/3}}{a^6} - \frac{2b^3x^2}{a^5} + \frac{9b^2x^{7/3}}{7a^4} - \frac{3bx^{8/3}}{4a^3} + \frac{x^3}{3a^2} - \frac{30b^9\log\left(b+a\sqrt[3]{x}\right)}{a^{11}}$$

output
```
-3*b^10/a^11/(b+a*x^(1/3))+27*b^8*x^(1/3)/a^10-12*b^7*x^(2/3)/a^9+7*b^6*x/
a^8-9/2*b^5*x^(4/3)/a^7+3*b^4*x^(5/3)/a^6-2*b^3*x^2/a^5+9/7*b^2*x^(7/3)/a^
4-3/4*b*x^(8/3)/a^3+1/3*x^3/a^2-30*b^9*ln(b+a*x^(1/3))/a^11
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$$
$$= \frac{-252b^{10}+2268ab^9\sqrt[3]{x}+1260a^2b^8x^{2/3}-420a^3b^7x+210a^4b^6x^{4/3}-126a^5b^5x^{5/3}+84a^6b^4x^2-60a^7b^3x^{7/}}{84a^{11}\left(b+a\sqrt[3]{x}\right)}$$
$$-\frac{30b^9\log\left(b+a\sqrt[3]{x}\right)}{a^{11}}$$

input
```
Integrate[x^2/(a + b/x^(1/3))^2,x]
```

output
```
(-252*b^10 + 2268*a*b^9*x^(1/3) + 1260*a^2*b^8*x^(2/3) - 420*a^3*b^7*x + 2
10*a^4*b^6*x^(4/3) - 126*a^5*b^5*x^(5/3) + 84*a^6*b^4*x^2 - 60*a^7*b^3*x^(
7/3) + 45*a^8*b^2*x^(8/3) - 35*a^9*b*x^3 + 28*a^10*x^(10/3))/(84*a^11*(b +
 a*x^(1/3))) - (30*b^9*Log[b + a*x^(1/3)])/a^11
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}dx$$

$$\Big\downarrow \; 795$$

$$\int \frac{x^{8/3}}{\left(a\sqrt[3]{x}+b\right)^2}dx$$

$$\Big\downarrow \; 798$$

$$3\int \frac{x^{10/3}}{\left(\sqrt[3]{x}a+b\right)^2}d\sqrt[3]{x}$$

$\Big\downarrow$ 49

$$3\int \left(\frac{b^{10}}{a^{10}\left(\sqrt[3]{x}a+b\right)^2}-\frac{10b^9}{a^{10}\left(\sqrt[3]{x}a+b\right)}+\frac{9b^8}{a^{10}}-\frac{8\sqrt[3]{x}b^7}{a^9}+\frac{7x^{2/3}b^6}{a^8}-\frac{6xb^5}{a^7}+\frac{5x^{4/3}b^4}{a^6}-\frac{4x^{5/3}b^3}{a^5}+\frac{3x^2b^2}{a^4}-\frac{2x^7}{a}\right.$$

$\Big\downarrow$ 2009

$$3\left(-\frac{b^{10}}{a^{11}\left(a\sqrt[3]{x}+b\right)}-\frac{10b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{11}}+\frac{9b^8\sqrt[3]{x}}{a^{10}}-\frac{4b^7x^{2/3}}{a^9}+\frac{7b^6x}{3a^8}-\frac{3b^5x^{4/3}}{2a^7}+\frac{b^4x^{5/3}}{a^6}-\frac{2b^3x^2}{3a^5}+\frac{3b^2x^{7/3}}{7a^4}-\right.$$

input
```
Int[x^2/(a + b/x^(1/3))^2,x]
```

output
```
3*(-(b^10/(a^11*(b + a*x^(1/3)))) + (9*b^8*x^(1/3))/a^10 - (4*b^7*x^(2/3))
/a^9 + (7*b^6*x)/(3*a^8) - (3*b^5*x^(4/3))/(2*a^7) + (b^4*x^(5/3))/a^6 - (
2*b^3*x^2)/(3*a^5) + (3*b^2*x^(7/3))/(7*a^4) - (b*x^(8/3))/(4*a^3) + x^3/(
9*a^2) - (10*b^9*Log[b + a*x^(1/3)])/a^11)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{a^8x^3}{3}-\frac{3bx^{\frac{8}{3}}a^7}{4}+\frac{9b^2x^{\frac{7}{3}}a^6}{7}-2a^5b^3x^2+3x^{\frac{5}{3}}a^4b^4-\frac{9b^5x^{\frac{4}{3}}a^3}{2}+7a^2b^6x-12ab^7x^{\frac{2}{3}}+27b^8x^{\frac{1}{3}}}{a^{10}}-\frac{30b^9\ln\left(b+ax^{\frac{1}{3}}\right)}{a^{11}}-\frac{}{a^{11}}$
default	$\frac{\frac{a^8x^3}{3}-\frac{3bx^{\frac{8}{3}}a^7}{4}+\frac{9b^2x^{\frac{7}{3}}a^6}{7}-2a^5b^3x^2+3x^{\frac{5}{3}}a^4b^4-\frac{9b^5x^{\frac{4}{3}}a^3}{2}+7a^2b^6x-12ab^7x^{\frac{2}{3}}+27b^8x^{\frac{1}{3}}}{a^{10}}-\frac{30b^9\ln\left(b+ax^{\frac{1}{3}}\right)}{a^{11}}-\frac{}{a^{11}}$

input
```
int(x^2/(a+b/x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
3/a^10*(1/9*a^8*x^3-1/4*b*x^(8/3)*a^7+3/7*b^2*x^(7/3)*a^6-2/3*a^5*b^3*x^2+
x^(5/3)*a^4*b^4-3/2*b^5*x^(4/3)*a^3+7/3*a^2*b^6*x-4*a*b^7*x^(2/3)+9*b^8*x^
(1/3))-30*b^9*ln(b+a*x^(1/3))/a^11-3*b^10/a^11/(b+a*x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{28\,a^{12}x^4-140\,a^9b^3x^3+420\,a^6b^6x^2+588\,a^3b^9x-252\,b^{12}-2520\,(a^3b^9x+b^{12})\log\left(ax^{\frac{1}{3}}+b\right)-63\,(a^{11}b}{84\,(a^{14}x+a^{11}}$$

input
```
integrate(x^2/(a+b/x^(1/3))^2,x, algorithm="fricas")
```

output
```
1/84*(28*a^12*x^4 - 140*a^9*b^3*x^3 + 420*a^6*b^6*x^2 + 588*a^3*b^9*x - 25
2*b^12 - 2520*(a^3*b^9*x + b^12)*log(a*x^(1/3) + b) - 63*(a^11*b*x^3 - 3*a
^8*b^4*x^2 + 12*a^5*b^7*x + 20*a^2*b^10)*x^(2/3) + 18*(6*a^10*b^2*x^3 - 15
*a^7*b^5*x^2 + 105*a^4*b^8*x + 140*a*b^11)*x^(1/3))/(a^14*x + a^11*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(150) = 300$.

Time = 0.95 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.45

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \begin{cases} \frac{28a^{10}x^{\frac{10}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b} - \frac{35a^9bx^3}{84a^{12}\sqrt[3]{x}+84a^{11}b} + \frac{45a^8b^2x^{\frac{8}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b} - \frac{60a^7b^3x^{\frac{7}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b} + \frac{84a^6b^4x^2}{84a^{12}\sqrt[3]{x}+84a^{11}b} - \frac{126a^5b^5x^{\frac{5}{3}}}{84a^{12}\sqrt[3]{x}+84a^{11}b} \\ \frac{3x^{\frac{11}{3}}}{11b^2} \end{cases}$$

input
```
integrate(x**2/(a+b/x**(1/3))**2,x)
```

output
```
Piecewise((28*a**10*x**(10/3)/(84*a**12*x**(1/3) + 84*a**11*b) - 35*a**9*b
*x**3/(84*a**12*x**(1/3) + 84*a**11*b) + 45*a**8*b**2*x**(8/3)/(84*a**12*x
**(1/3) + 84*a**11*b) - 60*a**7*b**3*x**(7/3)/(84*a**12*x**(1/3) + 84*a**1
1*b) + 84*a**6*b**4*x**2/(84*a**12*x**(1/3) + 84*a**11*b) - 126*a**5*b**5*
x**(5/3)/(84*a**12*x**(1/3) + 84*a**11*b) + 210*a**4*b**6*x**(4/3)/(84*a**
12*x**(1/3) + 84*a**11*b) - 420*a**3*b**7*x/(84*a**12*x**(1/3) + 84*a**11*
b) + 1260*a**2*b**8*x**(2/3)/(84*a**12*x**(1/3) + 84*a**11*b) - 2520*a*b**
9*x**(1/3)*log(x**(1/3) + b/a)/(84*a**12*x**(1/3) + 84*a**11*b) - 2520*b**
10*log(x**(1/3) + b/a)/(84*a**12*x**(1/3) + 84*a**11*b) - 2520*b**10/(84*a
**12*x**(1/3) + 84*a**11*b), Ne(a, 0)), (3*x**(11/3)/(11*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{28\,a^9 - \frac{35\,a^8b}{x^{\frac{1}{3}}} + \frac{45\,a^7b^2}{x^{\frac{2}{3}}} - \frac{60\,a^6b^3}{x} + \frac{84\,a^5b^4}{x^{\frac{4}{3}}} - \frac{126\,a^4b^5}{x^{\frac{5}{3}}} + \frac{210\,a^3b^6}{x^2} - \frac{420\,a^2b^7}{x^{\frac{7}{3}}} + \frac{1260\,ab^8}{x^{\frac{8}{3}}} + \frac{2520\,b^9}{x^3}}{84\left(\frac{a^{11}}{x^3} + \frac{a^{10}b}{x^{\frac{10}{3}}}\right)} - \frac{30\,b^9\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^{11}} - \frac{10\,b^9\log(x)}{a^{11}}$$

input
```
integrate(x^2/(a+b/x^(1/3))^2,x, algorithm="maxima")
```

output
```
1/84*(28*a^9 - 35*a^8*b/x^(1/3) + 45*a^7*b^2/x^(2/3) - 60*a^6*b^3/x + 84*a
^5*b^4/x^(4/3) - 126*a^4*b^5/x^(5/3) + 210*a^3*b^6/x^2 - 420*a^2*b^7/x^(7/
3) + 1260*a*b^8/x^(8/3) + 2520*b^9/x^3)/(a^11/x^3 + a^10*b/x^(10/3)) - 30*
b^9*log(a + b/x^(1/3))/a^11 - 10*b^9*log(x)/a^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = -\frac{30\,b^9\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^{11}} - \frac{3\,b^{10}}{\left(ax^{\frac{1}{3}}+b\right)a^{11}} + \frac{28\,a^{16}x^3 - 63\,a^{15}bx^{\frac{8}{3}} + 108\,a^{14}b^2x^{\frac{7}{3}} - 168\,a^{13}b^3x^2 + 252\,a^{12}b^4x^{\frac{5}{3}} - 378\,a^{11}b^5x^{\frac{4}{3}} + 588\,a^{10}b^6x - 1008\,a^9}{84\,a^{18}}$$

input
```
integrate(x^2/(a+b/x^(1/3))^2,x, algorithm="giac")
```

output
```
-30*b^9*log(abs(a*x^(1/3) + b))/a^11 - 3*b^10/((a*x^(1/3) + b)*a^11) + 1/8
4*(28*a^16*x^3 - 63*a^15*b*x^(8/3) + 108*a^14*b^2*x^(7/3) - 168*a^13*b^3*x
^2 + 252*a^12*b^4*x^(5/3) - 378*a^11*b^5*x^(4/3) + 588*a^10*b^6*x - 1008*a
^9*b^7*x^(2/3) + 2268*a^8*b^8*x^(1/3))/a^18
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{x^3}{3\,a^2} - \frac{3\,b^{10}}{a\,\left(a^{10}\,b+a^{11}\,x^{1/3}\right)} - \frac{3\,b\,x^{8/3}}{4\,a^3} + \frac{7\,b^6\,x}{a^8} - \frac{30\,b^9\,\ln\left(b+a\,x^{1/3}\right)}{a^{11}} - \frac{2\,b^3\,x^2}{a^5} + \frac{9\,b^2\,x^{7/3}}{7\,a^4} + \frac{3\,b^4\,x^{5/3}}{a^6} - \frac{9\,b^5\,x^{4/3}}{2\,a^7} - \frac{12\,b^7\,x^{2/3}}{a^9} + \frac{27\,b^8\,x^{1/3}}{a^{10}}$$

input
```
int(x^2/(a + b/x^(1/3))^2,x)
```

output
```
x^3/(3*a^2) - (3*b^10)/(a*(a^10*b + a^11*x^(1/3))) - (3*b*x^(8/3))/(4*a^3)
 + (7*b^6*x)/a^8 - (30*b^9*log(b + a*x^(1/3)))/a^11 - (2*b^3*x^2)/a^5 + (9
*b^2*x^(7/3))/(7*a^4) + (3*b^4*x^(5/3))/a^6 - (9*b^5*x^(4/3))/(2*a^7) - (1
2*b^7*x^(2/3))/a^9 + (27*b^8*x^(1/3))/a^10
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$$

$$= \frac{45x^{\frac{8}{3}}a^8b^2 - 126x^{\frac{5}{3}}a^5b^5 + 1260x^{\frac{2}{3}}a^2b^8 - 2520x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a\,b^9 + 28x^{\frac{10}{3}}a^{10} - 60x^{\frac{7}{3}}a^7b^3 + 210x^{\frac{4}{3}}a^4b^6 +}{84a^{11}\left(x^{\frac{1}{3}}a+b\right)}$$

input
```
int(x^2/(a+b/x^(1/3))^2,x)
```

output
```
(45*x**(2/3)*a**8*b**2*x**2 - 126*x**(2/3)*a**5*b**5*x + 1260*x**(2/3)*a**
2*b**8 - 2520*x**(1/3)*log(x**(1/3)*a + b)*a*b**9 + 28*x**(1/3)*a**10*x**3
 - 60*x**(1/3)*a**7*b**3*x**2 + 210*x**(1/3)*a**4*b**6*x + 2520*x**(1/3)*a
*b**9 - 2520*log(x**(1/3)*a + b)*b**10 - 35*a**9*b*x**3 + 84*a**6*b**4*x**
2 - 420*a**3*b**7*x)/(84*a**11*(x**(1/3)*a + b))
```

3.320 $\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} dx = \frac{3b^7}{a^8\left(b+a\sqrt[3]{x}\right)} - \frac{18b^5\sqrt[3]{x}}{a^7} + \frac{15b^4x^{2/3}}{2a^6} - \frac{4b^3x}{a^5} + \frac{9b^2x^{4/3}}{4a^4} - \frac{6bx^{5/3}}{5a^3} + \frac{x^2}{2a^2} + \frac{21b^6\log\left(b+a\sqrt[3]{x}\right)}{a^8}$$

output
```
3*b^7/a^8/(b+a*x^(1/3))-18*b^5*x^(1/3)/a^7+15/2*b^4*x^(2/3)/a^6-4*b^3*x/a^
5+9/4*b^2*x^(4/3)/a^4-6/5*b*x^(5/3)/a^3+1/2*x^2/a^2+21*b^6*ln(b+a*x^(1/3))
/a^8
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$$
$$=\frac{60b^7-360ab^6\sqrt[3]{x}-210a^2b^5x^{2/3}+70a^3b^4x-35a^4b^3x^{4/3}+21a^5b^2x^{5/3}-14a^6bx^2+10a^7x^{7/3}}{20a^8\left(b+a\sqrt[3]{x}\right)}+\frac{21b^6\log\left(b+a\sqrt[3]{x}\right)}{a^8}$$

input
```
Integrate[x/(a + b/x^(1/3))^2,x]
```

output
```
(60*b^7 - 360*a*b^6*x^(1/3) - 210*a^2*b^5*x^(2/3) + 70*a^3*b^4*x - 35*a^4*
b^3*x^(4/3) + 21*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 10*a^7*x^(7/3))/(20*a^8*
(b + a*x^(1/3))) + (21*b^6*Log[b + a*x^(1/3)])/a^8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{x^{5/3}}{\left(a\sqrt[3]{x}+b\right)^2}dx$$
$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{7/3}}{\left(\sqrt[3]{x}a+b\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{b^7}{a^7\left(\sqrt[3]{x}a+b\right)^2}+\frac{7b^6}{a^7\left(\sqrt[3]{x}a+b\right)}-\frac{6b^5}{a^7}+\frac{5\sqrt[3]{x}b^4}{a^6}-\frac{4x^{2/3}b^3}{a^5}+\frac{3xb^2}{a^4}-\frac{2x^{4/3}b}{a^3}+\frac{x^{5/3}}{a^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{b^7}{a^8\left(a\sqrt[3]{x}+b\right)}+\frac{7b^6\log\left(a\sqrt[3]{x}+b\right)}{a^8}-\frac{6b^5\sqrt[3]{x}}{a^7}+\frac{5b^4x^{2/3}}{2a^6}-\frac{4b^3x}{3a^5}+\frac{3b^2x^{4/3}}{4a^4}-\frac{2bx^{5/3}}{5a^3}+\frac{x^2}{6a^2}\right)$$

input
```
Int[x/(a + b/x^(1/3))^2,x]
```

output
```
3*(b^7/(a^8*(b + a*x^(1/3))) - (6*b^5*x^(1/3))/a^7 + (5*b^4*x^(2/3))/(2*a^
6) - (4*b^3*x)/(3*a^5) + (3*b^2*x^(4/3))/(4*a^4) - (2*b*x^(5/3))/(5*a^3) +
 x^2/(6*a^2) + (7*b^6*Log[b + a*x^(1/3)])/a^8)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{a^5 x^2}{2}-\frac{6 b x^{\frac{5}{3}} a^4}{5}+\frac{9 b^2 x^{\frac{4}{3}} a^3}{4}-4 a^2 b^3 x+\frac{15 x^{\frac{2}{3}} a b^4}{2}-18 b^5 x^{\frac{1}{3}}}{a^7}+\frac{3 b^7}{a^8 \left(b+a x^{\frac{1}{3}}\right)}+\frac{21 b^6 \ln \left(b+a x^{\frac{1}{3}}\right)}{a^8}$	95
default	$\frac{\frac{a^5 x^2}{2}-\frac{6 b x^{\frac{5}{3}} a^4}{5}+\frac{9 b^2 x^{\frac{4}{3}} a^3}{4}-4 a^2 b^3 x+\frac{15 x^{\frac{2}{3}} a b^4}{2}-18 b^5 x^{\frac{1}{3}}}{a^7}+\frac{3 b^7}{a^8 \left(b+a x^{\frac{1}{3}}\right)}+\frac{21 b^6 \ln \left(b+a x^{\frac{1}{3}}\right)}{a^8}$	95

input
```
int(x/(a+b/x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
3/a^7*(1/6*a^5*x^2-2/5*b*x^(5/3)*a^4+3/4*b^2*x^(4/3)*a^3-4/3*a^2*b^3*x+5/2
*x^(2/3)*a*b^4-6*b^5*x^(1/3))+3*b^7/a^8/(b+a*x^(1/3))+21*b^6*ln(b+a*x^(1/3
))/a^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} \, dx$$
$$= \frac{10 a^9 x^3 - 70 a^6 b^3 x^2 - 80 a^3 b^6 x + 60 b^9 + 420 \left(a^3 b^6 x + b^9\right) \log\left(a x^{\frac{1}{3}} + b\right) - 6 \left(4 a^8 b x^2 - 21 a^5 b^4 x - 35 a^2\right.}{20 \left(a^{11} x + a^8 b^3\right)}$$

input
```
integrate(x/(a+b/x^(1/3))^2,x, algorithm="fricas")
```

output
```
1/20*(10*a^9*x^3 - 70*a^6*b^3*x^2 - 80*a^3*b^6*x + 60*b^9 + 420*(a^3*b^6*x
 + b^9)*log(a*x^(1/3) + b) - 6*(4*a^8*b*x^2 - 21*a^5*b^4*x - 35*a^2*b^7)*x
^(2/3) + 15*(3*a^7*b^2*x^2 - 21*a^4*b^5*x - 28*a*b^8)*x^(1/3))/(a^11*x + a
^8*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(110) = 220$.

Time = 0.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.45

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \begin{cases} \frac{10a^7x^{\frac{7}{3}}}{20a^9\sqrt[3]{x}+20a^8b} - \frac{14a^6bx^2}{20a^9\sqrt[3]{x}+20a^8b} + \frac{21a^5b^2x^{\frac{5}{3}}}{20a^9\sqrt[3]{x}+20a^8b} - \frac{35a^4b^3x^{\frac{4}{3}}}{20a^9\sqrt[3]{x}+20a^8b} + \frac{70a^3b^4x}{20a^9\sqrt[3]{x}+20a^8b} - \frac{210a^2b^5x^{\frac{2}{3}}}{20a^9\sqrt[3]{x}+20a^8b} + \frac{420ab^6 \cdots}{20 \cdots} \\ \frac{3x^{\frac{8}{3}}}{8b^2} \end{cases}$$

input
```
integrate(x/(a+b/x**(1/3))**2,x)
```

output
```
Piecewise((10*a**7*x**(7/3)/(20*a**9*x**(1/3) + 20*a**8*b) - 14*a**6*b*x**
2/(20*a**9*x**(1/3) + 20*a**8*b) + 21*a**5*b**2*x**(5/3)/(20*a**9*x**(1/3)
 + 20*a**8*b) - 35*a**4*b**3*x**(4/3)/(20*a**9*x**(1/3) + 20*a**8*b) + 70*
a**3*b**4*x/(20*a**9*x**(1/3) + 20*a**8*b) - 210*a**2*b**5*x**(2/3)/(20*a*
*9*x**(1/3) + 20*a**8*b) + 420*a*b**6*x**(1/3)*log(x**(1/3) + b/a)/(20*a**
9*x**(1/3) + 20*a**8*b) + 420*b**7*log(x**(1/3) + b/a)/(20*a**9*x**(1/3) +
 20*a**8*b) + 420*b**7/(20*a**9*x**(1/3) + 20*a**8*b), Ne(a, 0)), (3*x**(8
/3)/(8*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{10\,a^6 - \frac{14\,a^5b}{x^{\frac{1}{3}}} + \frac{21\,a^4b^2}{x^{\frac{2}{3}}} - \frac{35\,a^3b^3}{x} + \frac{70\,a^2b^4}{x^{\frac{4}{3}}} - \frac{210\,ab^5}{x^{\frac{5}{3}}} - \frac{420\,b^6}{x^2}}{20\left(\frac{a^8}{x^2} + \frac{a^7b}{x^{\frac{7}{3}}}\right)} + \frac{21\,b^6\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^8} + \frac{7\,b^6\log(x)}{a^8}$$

input
```
integrate(x/(a+b/x^(1/3))^2,x, algorithm="maxima")
```

output
```
1/20*(10*a^6 - 14*a^5*b/x^(1/3) + 21*a^4*b^2/x^(2/3) - 35*a^3*b^3/x + 70*a
^2*b^4/x^(4/3) - 210*a*b^5/x^(5/3) - 420*b^6/x^2)/(a^8/x^2 + a^7*b/x^(7/3)
) + 21*b^6*log(a + b/x^(1/3))/a^8 + 7*b^6*log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{21\,b^6\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8}+\frac{3\,b^7}{\left(ax^{\frac{1}{3}}+b\right)a^8} + \frac{10\,a^{10}x^2-24\,a^9bx^{\frac{5}{3}}+45\,a^8b^2x^{\frac{4}{3}}-80\,a^7b^3x+150\,a^6b^4x^{\frac{2}{3}}-360\,a^5b^5x^{\frac{1}{3}}}{20\,a^{12}}$$

input
```
integrate(x/(a+b/x^(1/3))^2,x, algorithm="giac")
```

output
```
21*b^6*log(abs(a*x^(1/3) + b))/a^8 + 3*b^7/((a*x^(1/3) + b)*a^8) + 1/20*(1
0*a^10*x^2 - 24*a^9*b*x^(5/3) + 45*a^8*b^2*x^(4/3) - 80*a^7*b^3*x + 150*a^
6*b^4*x^(2/3) - 360*a^5*b^5*x^(1/3))/a^12
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{x^2}{2\,a^2}+\frac{3\,b^7}{a\,\left(a^7\,b+a^8\,x^{1/3}\right)}-\frac{4\,b^3\,x}{a^5}-\frac{6\,b\,x^{5/3}}{5\,a^3} + \frac{21\,b^6\,\ln\left(b+a\,x^{1/3}\right)}{a^8}+\frac{9\,b^2\,x^{4/3}}{4\,a^4}+\frac{15\,b^4\,x^{2/3}}{2\,a^6}-\frac{18\,b^5\,x^{1/3}}{a^7}$$

input
```
int(x/(a + b/x^(1/3))^2,x)
```

output
```
x^2/(2*a^2) + (3*b^7)/(a*(a^7*b + a^8*x^(1/3))) - (4*b^3*x)/a^5 - (6*b*x^(
5/3))/(5*a^3) + (21*b^6*log(b + a*x^(1/3)))/a^8 + (9*b^2*x^(4/3))/(4*a^4)
+ (15*b^4*x^(2/3))/(2*a^6) - (18*b^5*x^(1/3))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx$$

$$= \frac{21x^{\frac{5}{3}}a^5b^2 - 210x^{\frac{2}{3}}a^2b^5 + 420x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a\,b^6 + 10x^{\frac{7}{3}}a^7 - 35x^{\frac{4}{3}}a^4b^3 - 420x^{\frac{1}{3}}a\,b^6 + 420\log\left(x^{\frac{1}{3}}a+b\right.}{20a^8\left(x^{\frac{1}{3}}a+b\right)}$$

input
```
int(x/(a+b/x^(1/3))^2,x)
```

output
```
(21*x**(2/3)*a**5*b**2*x - 210*x**(2/3)*a**2*b**5 + 420*x**(1/3)*log(x**(1
/3)*a + b)*a*b**6 + 10*x**(1/3)*a**7*x**2 - 35*x**(1/3)*a**4*b**3*x - 420*
x**(1/3)*a*b**6 + 420*log(x**(1/3)*a + b)*b**7 - 14*a**6*b*x**2 + 70*a**3*
b**4*x)/(20*a**8*(x**(1/3)*a + b))
```

3.321 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} dx = -\frac{3b^4}{a^5\left(b+a\sqrt[3]{x}\right)} + \frac{9b^2\sqrt[3]{x}}{a^4} - \frac{3bx^{2/3}}{a^3} + \frac{x}{a^2} - \frac{12b^3\log\left(b+a\sqrt[3]{x}\right)}{a^5}$$

output
```
-3*b^4/a^5/(b+a*x^(1/3))+9*b^2*x^(1/3)/a^4-3*b*x^(2/3)/a^3+x/a^2-12*b^3*ln
(b+a*x^(1/3))/a^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} dx = \frac{-3b^4+9ab^3\sqrt[3]{x}+6a^2b^2x^{2/3}-2a^3bx+a^4x^{4/3}}{a^5\left(b+a\sqrt[3]{x}\right)} - \frac{12b^3\log\left(b+a\sqrt[3]{x}\right)}{a^5}$$

input
```
Integrate[(a + b/x^(1/3))^(-2),x]
```

output
```
(-3*b^4 + 9*a*b^3*x^(1/3) + 6*a^2*b^2*x^(2/3) - 2*a^3*b*x + a^4*x^(4/3))/(
a^5*(b + a*x^(1/3))) - (12*b^3*Log[b + a*x^(1/3)])/a^5
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {774, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{x^{2/3}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{795}$$

$$3\int \frac{x^{4/3}}{\left(\sqrt[3]{x}a+b\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{b^4}{a^4\left(\sqrt[3]{x}a+b\right)^2}-\frac{4b^3}{a^4\left(\sqrt[3]{x}a+b\right)}+\frac{3b^2}{a^4}-\frac{2\sqrt[3]{x}b}{a^3}+\frac{x^{2/3}}{a^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{b^4}{a^5\left(a\sqrt[3]{x}+b\right)}-\frac{4b^3\log\left(a\sqrt[3]{x}+b\right)}{a^5}+\frac{3b^2\sqrt[3]{x}}{a^4}-\frac{bx^{2/3}}{a^3}+\frac{x}{3a^2}\right)$$

input
```
Int[(a + b/x^(1/3))^(-2),x]
```

output
```
3*(-(b^4/(a^5*(b + a*x^(1/3)))) + (3*b^2*x^(1/3))/a^4 - (b*x^(2/3))/a^3 +
x/(3*a^2) - (4*b^3*Log[b + a*x^(1/3)])/a^5)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{a^{2} x -3 x^{\frac{2}{3}} a b +9 x^{\frac{1}{3}} b^{2}}{a^{4}}-\frac{12 b^{3} \ln \left(b +a \,x^{\frac{1}{3}}\right)}{a^{5}}-\frac{3 b^{4}}{a^{5} \left(b +a \,x^{\frac{1}{3}}\right)}$	62
default	$\frac{a^{2} x -3 x^{\frac{2}{3}} a b +9 x^{\frac{1}{3}} b^{2}}{a^{4}}-\frac{12 b^{3} \ln \left(b +a \,x^{\frac{1}{3}}\right)}{a^{5}}-\frac{3 b^{4}}{a^{5} \left(b +a \,x^{\frac{1}{3}}\right)}$	62

input
```
int(1/(a+b/x^(1/3))^2,x,method=_RETURNVERBOSE)
```

output
```
3/a^4*(1/3*a^2*x-x^(2/3)*a*b+3*x^(1/3)*b^2)-12*b^3*ln(b+a*x^(1/3))/a^5-3*b
^4/a^5/(b+a*x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{a^6x^2+a^3b^3x-3\,b^6-12\,(a^3b^3x+b^6)\log\left(ax^{\frac{1}{3}}+b\right)-3\,(a^5bx+2\,a^2b^4)x^{\frac{2}{3}}+3\,(3\,a^4b^2x+4\,ab^5)x^{\frac{1}{3}}}{a^8x+a^5b^3}$$

input
```
integrate(1/(a+b/x^(1/3))^2,x, algorithm="fricas")
```

output
```
(a^6*x^2 + a^3*b^3*x - 3*b^6 - 12*(a^3*b^3*x + b^6)*log(a*x^(1/3) + b) - 3
*(a^5*b*x + 2*a^2*b^4)*x^(2/3) + 3*(3*a^4*b^2*x + 4*a*b^5)*x^(1/3))/(a^8*x
 + a^5*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(65) = 130.

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \begin{cases} \frac{a^4x^{\frac{4}{3}}}{a^6\sqrt[3]{x}+a^5b}-\frac{2a^3bx}{a^6\sqrt[3]{x}+a^5b}+\frac{6a^2b^2x^{\frac{2}{3}}}{a^6\sqrt[3]{x}+a^5b}-\frac{12ab^3\sqrt[3]{x}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^6\sqrt[3]{x}+a^5b}-\frac{12b^4\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^6\sqrt[3]{x}+a^5b}-\frac{12b^4}{a^6\sqrt[3]{x}+a^5b} & \text{for } a\neq 0 \\ \frac{3x^{\frac{5}{3}}}{5b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**2,x)
```

output
```
Piecewise((a**4*x**(4/3)/(a**6*x**(1/3) + a**5*b) - 2*a**3*b*x/(a**6*x**(1
/3) + a**5*b) + 6*a**2*b**2*x**(2/3)/(a**6*x**(1/3) + a**5*b) - 12*a*b**3*
x**(1/3)*log(x**(1/3) + b/a)/(a**6*x**(1/3) + a**5*b) - 12*b**4*log(x**(1/
3) + b/a)/(a**6*x**(1/3) + a**5*b) - 12*b**4/(a**6*x**(1/3) + a**5*b), Ne(
a, 0)), (3*x**(5/3)/(5*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{a^3 - \frac{2\,a^2 b}{x^{\frac{1}{3}}} + \frac{6\,ab^2}{x^{\frac{2}{3}}} + \frac{12\,b^3}{x}}{\frac{a^5}{x} + \frac{a^4 b}{x^{\frac{4}{3}}}} - \frac{12\,b^3 \log\left(a + \frac{b}{x^{\frac{1}{3}}}\right)}{a^5} - \frac{4\,b^3 \log(x)}{a^5}$$

input
```
integrate(1/(a+b/x^(1/3))^2,x, algorithm="maxima")
```

output
```
(a^3 - 2*a^2*b/x^(1/3) + 6*a*b^2/x^(2/3) + 12*b^3/x)/(a^5/x + a^4*b/x^(4/3
)) - 12*b^3*log(a + b/x^(1/3))/a^5 - 4*b^3*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = -\frac{12\,b^3 \log\left(\left|ax^{\frac{1}{3}} + b\right|\right)}{a^5} - \frac{3\,b^4}{\left(ax^{\frac{1}{3}} + b\right)a^5} + \frac{a^4 x - 3\,a^3 b x^{\frac{2}{3}} + 9\,a^2 b^2 x^{\frac{1}{3}}}{a^6}$$

input
```
integrate(1/(a+b/x^(1/3))^2,x, algorithm="giac")
```

output
```
-12*b^3*log(abs(a*x^(1/3) + b))/a^5 - 3*b^4/((a*x^(1/3) + b)*a^5) + (a^4*x
 - 3*a^3*b*x^(2/3) + 9*a^2*b^2*x^(1/3))/a^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{x}{a^2} - \frac{3\,b^4}{a\,\left(a^4\,b + a^5\,x^{1/3}\right)} - \frac{3\,b\,x^{2/3}}{a^3} - \frac{12\,b^3\,\ln\left(b + a\,x^{1/3}\right)}{a^5} + \frac{9\,b^2\,x^{1/3}}{a^4}$$

input
```
int(1/(a + b/x^(1/3))^2,x)
```

output
```
x/a^2 - (3*b^4)/(a*(a^4*b + a^5*x^(1/3))) - (3*b*x^(2/3))/a^3 - (12*b^3*lo
g(b + a*x^(1/3)))/a^5 + (9*b^2*x^(1/3))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}\,dx = \frac{6x^{\frac{2}{3}}a^2b^2-12x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a\,b^3+x^{\frac{4}{3}}a^4+12x^{\frac{1}{3}}a\,b^3-12\log\left(x^{\frac{1}{3}}a+b\right)b^4-2a^3bx}{a^5\left(x^{\frac{1}{3}}a+b\right)}$$

input
```
int(1/(a+b/x^(1/3))^2,x)
```

output
```
(6*x**(2/3)*a**2*b**2 - 12*x**(1/3)*log(x**(1/3)*a + b)*a*b**3 + x**(1/3)*
a**4*x + 12*x**(1/3)*a*b**3 - 12*log(x**(1/3)*a + b)*b**4 - 2*a**3*b*x)/(a
**5*(x**(1/3)*a + b))
```

3.322 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x} \, dx = \frac{3b}{a^2\left(b+a\sqrt[3]{x}\right)} + \frac{3\log\left(b+a\sqrt[3]{x}\right)}{a^2}$$

output
```
3*b/a^2/(b+a*x^(1/3))+3*ln(b+a*x^(1/3))/a^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x} \, dx = \frac{3\left(\frac{b}{b+a\sqrt[3]{x}} + \log\left(b+a\sqrt[3]{x}\right)\right)}{a^2}$$

input
```
Integrate[1/((a + b/x^(1/3))^2*x),x]
```

output
```
(3*(b/(b + a*x^(1/3)) + Log[b + a*x^(1/3)]))/a^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{\sqrt[3]{x}}{\left(\sqrt[3]{x}a+b\right)^2}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{1}{a\left(\sqrt[3]{x}a+b\right)}-\frac{b}{a\left(\sqrt[3]{x}a+b\right)^2}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{b}{a^2\left(a\sqrt[3]{x}+b\right)}+\frac{\log\left(a\sqrt[3]{x}+b\right)}{a^2}\right)$$

input
```
Int[1/((a + b/x^(1/3))^2*x),x]
```

output
```
3*(b/(a^2*(b + a*x^(1/3))) + Log[b + a*x^(1/3)]/a^2)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{3b}{a^{2}\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{3\ln\left(b+a\,x^{\frac{1}{3}}\right)}{a^{2}}$	30
derivativedivides	$\frac{3\ln\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^{2}}-\frac{3}{a\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}+\frac{\ln(x)}{a^{2}}$	35

input
```
int(1/(a+b/x^(1/3))^2/x,x,method=_RETURNVERBOSE)
```

output
```
3*b/a^2/(b+a*x^(1/3))+3*ln(b+a*x^(1/3))/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx = \frac{3\left(a^2bx^{\frac{2}{3}} - ab^2x^{\frac{1}{3}} + b^3 + (a^3x + b^3)\log\left(ax^{\frac{1}{3}} + b\right)\right)}{a^5x + a^2b^3}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x,x, algorithm="fricas")
```

output
```
3*(a^2*b*x^(2/3) - a*b^2*x^(1/3) + b^3 + (a^3*x + b^3)*log(a*x^(1/3) + b))
/(a^5*x + a^2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(29) = 58$.

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx = \begin{cases} \frac{3ax\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^3x+a^2bx^{\frac{2}{3}}} + \frac{3bx^{\frac{2}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{a^3x+a^2bx^{\frac{2}{3}}} + \frac{3bx^{\frac{2}{3}}}{a^3x+a^2bx^{\frac{2}{3}}} & \text{for } a \neq 0 \\ \frac{3x^{\frac{2}{3}}}{2b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**2/x,x)
```

output
```
Piecewise((3*a*x*log(x**(1/3) + b/a)/(a**3*x + a**2*b*x**(2/3)) + 3*b*x**(
2/3)*log(x**(1/3) + b/a)/(a**3*x + a**2*b*x**(2/3)) + 3*b*x**(2/3)/(a**3*x
 + a**2*b*x**(2/3)), Ne(a, 0)), (3*x**(2/3)/(2*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx = -\frac{3}{a^2+\frac{ab}{x^{\frac{1}{3}}}} + \frac{3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^2} + \frac{\log(x)}{a^2}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x,x, algorithm="maxima")
```

output
```
-3/(a^2 + a*b/x^(1/3)) + 3*log(a + b/x^(1/3))/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx = \frac{3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^2} + \frac{3b}{\left(ax^{\frac{1}{3}}+b\right)a^2}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x,x, algorithm="giac")
```

output
```
3*log(abs(a*x^(1/3) + b))/a^2 + 3*b/((a*x^(1/3) + b)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx = \frac{3\ln\left(b+a\,x^{1/3}\right)}{a^2} + \frac{3b}{a^2\,\left(b+a\,x^{1/3}\right)}$$

input
```
int(1/(x*(a + b/x^(1/3))^2),x)
```

output
```
(3*log(b + a*x^(1/3)))/a^2 + (3*b)/(a^2*(b + a*x^(1/3)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x}\,dx = \frac{3x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a-3x^{\frac{1}{3}}a+3\log\left(x^{\frac{1}{3}}a+b\right)b}{a^2\left(x^{\frac{1}{3}}a+b\right)}$$

input
```
int(1/(a+b/x^(1/3))^2/x,x)
```

output
```
(3*(x**(1/3)*log(x**(1/3)*a + b)*a - x**(1/3)*a + log(x**(1/3)*a + b)*b))/
(a**2*(x**(1/3)*a + b))
```

3.323 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2} \, dx = \frac{3a^2}{b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{3}{b^2\sqrt[3]{x}} + \frac{6a\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^3}$$

output
```
3*a^2/b^3/(a+b/x^(1/3))-3/b^2/x^(1/3)+6*a*ln(a+b/x^(1/3))/b^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2} \, dx = \frac{3\left(-\frac{b\left(2a+\frac{b}{\sqrt[3]{x}}\right)}{b+a\sqrt[3]{x}} + 2a\log\left(b+a\sqrt[3]{x}\right) - \frac{2}{3}a\log(x)\right)}{b^3}$$

input
```
Integrate[1/((a + b/x^(1/3))^2*x^2),x]
```

output
```
(3*(-((b*(2*a + b/x^(1/3)))/(b + a*x^(1/3))) + 2*a*Log[b + a*x^(1/3)] - (2
*a*Log[x])/3))/b^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{\sqrt[3]{x}}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{4/3} \left(a\sqrt[3]{x}+b\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)^2 x^{2/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{2a^2}{b^3 \left(\sqrt[3]{x}a+b\right)}+\frac{a^2}{b^2 \left(\sqrt[3]{x}a+b\right)^2}-\frac{2a}{b^3 \sqrt[3]{x}}+\frac{1}{b^2 x^{2/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{2a\log\left(a\sqrt[3]{x}+b\right)}{b^3}-\frac{2a\log\left(\sqrt[3]{x}\right)}{b^3}-\frac{a}{b^2 \left(a\sqrt[3]{x}+b\right)}-\frac{1}{b^2 \sqrt[3]{x}}\right)$$

input
```
Int[1/((a + b/x^(1/3))^2*x^2),x]
```

output
```
3*(-(a/(b^2*(b + a*x^(1/3)))) - 1/(b^2*x^(1/3)) + (2*a*Log[b + a*x^(1/3)])
/b^3 - (2*a*Log[x^(1/3)])/b^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$-\frac{3a}{b^{2}\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{6a\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{3}}-\frac{3}{x^{\frac{1}{3}}b^{2}}-\frac{2a\ln\left(x\right)}{b^{3}}$	47
default	$-\frac{3a}{b^{2}\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{6a\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{3}}-\frac{3}{x^{\frac{1}{3}}b^{2}}-\frac{2a\ln\left(x\right)}{b^{3}}$	47

input
```
int(1/(a+b/x^(1/3))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3*a/b^2/(b+a*x^(1/3))+6/b^3*a*ln(b+a*x^(1/3))-3/x^(1/3)/b^2-2/b^3*a*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(40) = 80$.

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.13

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx = \frac{3\left(a^2b^2x^{\frac{4}{3}} - ab^3x + 2\left(a^4x^2 + ab^3x\right)\log\left(ax^{\frac{1}{3}} + b\right) - 2\left(a^4x^2 + ab^3x\right)\log\left(x^{\frac{1}{3}}\right) - \left(2\,a^3bx + b^4\right)x^{\frac{2}{3}}\right)}{a^3b^3x^2 + b^6x}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^2,x, algorithm="fricas")
```

output
```
3*(a^2*b^2*x^(4/3) - a*b^3*x + 2*(a^4*x^2 + a*b^3*x)*log(a*x^(1/3) + b) -
2*(a^4*x^2 + a*b^3*x)*log(x^(1/3)) - (2*a^3*b*x + b^4)*x^(2/3))/(a^3*b^3*x
^2 + b^6*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(42) = 84$.

Time = 0.86 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.59

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a = 0 \wedge \\ -\frac{3}{b^2\sqrt[3]{x}} & \text{for } a = 0 \\ -\frac{1}{a^2x} & \text{for } b = 0 \\ -\frac{2a^2x^2\log(x)}{ab^3x^2+b^4x^{\frac{5}{3}}} + \frac{6a^2x^2\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{ab^3x^2+b^4x^{\frac{5}{3}}} - \frac{2abx^{\frac{5}{3}}\log(x)}{ab^3x^2+b^4x^{\frac{5}{3}}} + \frac{6abx^{\frac{5}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{ab^3x^2+b^4x^{\frac{5}{3}}} - \frac{6abx^{\frac{5}{3}}}{ab^3x^2+b^4x^{\frac{5}{3}}} - \frac{3b^2x^{\frac{4}{3}}}{ab^3x^2+b^4x^{\frac{5}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**2/x**2,x)
```

output
```
Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (-3/(b**2*x**(1/3)), Eq(a,
0)), (-1/(a**2*x), Eq(b, 0)), (-2*a**2*x**2*log(x)/(a*b**3*x**2 + b**4*x**
(5/3)) + 6*a**2*x**2*log(x**(1/3) + b/a)/(a*b**3*x**2 + b**4*x**(5/3)) - 2
*a*b*x**(5/3)*log(x)/(a*b**3*x**2 + b**4*x**(5/3)) + 6*a*b*x**(5/3)*log(x*
*(1/3) + b/a)/(a*b**3*x**2 + b**4*x**(5/3)) - 6*a*b*x**(5/3)/(a*b**3*x**2
+ b**4*x**(5/3)) - 3*b**2*x**(4/3)/(a*b**3*x**2 + b**4*x**(5/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx=\frac{6\,a\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3}-\frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3}+\frac{3\,a^2}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^3}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^2,x, algorithm="maxima")
```

output
```
6*a*log(a + b/x^(1/3))/b^3 - 3*(a + b/x^(1/3))/b^3 + 3*a^2/((a + b/x^(1/3)
)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx=\frac{6\,a\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^3}-\frac{2\,a\log\left(|x|\right)}{b^3}-\frac{3\left(2\,ax^{\frac{1}{3}}+b\right)}{\left(ax^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)b^2}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^2,x, algorithm="giac")
```

output
```
6*a*log(abs(a*x^(1/3) + b))/b^3 - 2*a*log(abs(x))/b^3 - 3*(2*a*x^(1/3) + b
)/((a*x^(2/3) + b*x^(1/3))*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx = \frac{12\,a\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^3} - \frac{\frac{3}{b}+\frac{6\,a\,x^{1/3}}{b^2}}{a\,x^{2/3}+b\,x^{1/3}}$$

input
```
int(1/(x^2*(a + b/x^(1/3))^2),x)
```

output
```
(12*a*atanh((2*a*x^(1/3))/b + 1))/b^3 - (3/b + (6*a*x^(1/3))/b^2)/(a*x^(2/
3) + b*x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.85

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^2}\,dx$$
$$= \frac{-6x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)a^2+6x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2+6x^{\frac{2}{3}}a^2-6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}\right)ab+6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)ab-3b^2}{x^{\frac{1}{3}}b^3\left(x^{\frac{1}{3}}a+b\right)}$$

input
```
int(1/(a+b/x^(1/3))^2/x^2,x)
```

output
```
(3*( - 2*x**(2/3)*log(x**(1/3))*a**2 + 2*x**(2/3)*log(x**(1/3)*a + b)*a**2
 + 2*x**(2/3)*a**2 - 2*x**(1/3)*log(x**(1/3))*a*b + 2*x**(1/3)*log(x**(1/3
)*a + b)*a*b - b**2))/(x**(1/3)*b**3*(x**(1/3)*a + b))
```

3.324 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3} \, dx = -\frac{3a^5}{b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{3}{4b^2x^{4/3}} + \frac{2a}{b^3x} - \frac{9a^2}{2b^4x^{2/3}} + \frac{12a^3}{b^5\sqrt[3]{x}} - \frac{15a^4\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^6}$$

output
```
-3*a^5/b^6/(a+b/x^(1/3))-3/4/b^2/x^(4/3)+2*a/b^3/x-9/2*a^2/b^4/x^(2/3)+12*
a^3/b^5/x^(1/3)-15*a^4*ln(a+b/x^(1/3))/b^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3} \, dx = \frac{\frac{b\left(-3b^4+5ab^3\sqrt[3]{x}-10a^2b^2x^{2/3}+30a^3bx+60a^4x^{4/3}\right)}{\left(b+a\sqrt[3]{x}\right)x^{4/3}}-60a^4\log\left(b+a\sqrt[3]{x}\right)+20a^4\log(x)}{4b^6}$$

input
```
Integrate[1/((a + b/x^(1/3))^2*x^3),x]
```

output
```
((b*(-3*b^4 + 5*a*b^3*x^(1/3) - 10*a^2*b^2*x^(2/3) + 30*a^3*b*x + 60*a^4*x
^(4/3)))/((b + a*x^(1/3))*x^(4/3)) - 60*a^4*Log[b + a*x^(1/3)] + 20*a^4*Lo
g[x])/(4*b^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} \, dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^{7/3}\left(a\sqrt[3]{x}+b\right)^2}dx$$

$$\Big\downarrow \text{ 798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)^2 x^{5/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(-\frac{5a^5}{b^6\left(\sqrt[3]{x}a+b\right)}-\frac{a^5}{b^5\left(\sqrt[3]{x}a+b\right)^2}+\frac{5a^4}{b^6\sqrt[3]{x}}-\frac{4a^3}{b^5x^{2/3}}+\frac{3a^2}{b^4x}-\frac{2a}{b^3x^{4/3}}+\frac{1}{b^2x^{5/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{5a^4\log\left(a\sqrt[3]{x}+b\right)}{b^6}+\frac{5a^4\log\left(\sqrt[3]{x}\right)}{b^6}+\frac{a^4}{b^5\left(a\sqrt[3]{x}+b\right)}+\frac{4a^3}{b^5\sqrt[3]{x}}-\frac{3a^2}{2b^4x^{2/3}}+\frac{2a}{3b^3x}-\frac{1}{4b^2x^{4/3}}\right)$$

input
```
Int[1/((a + b/x^(1/3))^2*x^3),x]
```

output
```
3*(a^4/(b^5*(b + a*x^(1/3))) - 1/(4*b^2*x^(4/3)) + (2*a)/(3*b^3*x) - (3*a^
2)/(2*b^4*x^(2/3)) + (4*a^3)/(b^5*x^(1/3)) - (5*a^4*Log[b + a*x^(1/3)])/b^
6 + (5*a^4*Log[x^(1/3)])/b^6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{3}{4b^2x^{\frac{4}{3}}}-\frac{9a^2}{2b^4x^{\frac{2}{3}}}+\frac{5a^4\ln(x)}{b^6}+\frac{12a^3}{b^5x^{\frac{1}{3}}}+\frac{2a}{b^3x}-\frac{15a^4\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^6}+\frac{3a^4}{b^5\left(b+a\,x^{\frac{1}{3}}\right)}$	84
default	$-\frac{3}{4b^2x^{\frac{4}{3}}}-\frac{9a^2}{2b^4x^{\frac{2}{3}}}+\frac{5a^4\ln(x)}{b^6}+\frac{12a^3}{b^5x^{\frac{1}{3}}}+\frac{2a}{b^3x}-\frac{15a^4\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^6}+\frac{3a^4}{b^5\left(b+a\,x^{\frac{1}{3}}\right)}$	84

input
```
int(1/(a+b/x^(1/3))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4/b^2/x^(4/3)-9/2*a^2/b^4/x^(2/3)+5*a^4/b^6*ln(x)+12*a^3/b^5/x^(1/3)+2*
a/b^3/x-15*a^4/b^6*ln(b+a*x^(1/3))+3*a^4/b^5/(b+a*x^(1/3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(73) = 146.

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2x^3}\,dx$$
$$=\frac{20\,a^4b^3x^2+8\,ab^6x-60\,(a^7x^3+a^4b^3x^2)\log\left(ax^{\frac{1}{3}}+b\right)+60\,(a^7x^3+a^4b^3x^2)\log\left(x^{\frac{1}{3}}\right)+3\,(20\,a^6bx^2+15}{4\,(a^3b^6x^3+b^9x^2)}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^3,x, algorithm="fricas")
```

output
```
1/4*(20*a^4*b^3*x^2 + 8*a*b^6*x - 60*(a^7*x^3 + a^4*b^3*x^2)*log(a*x^(1/3)
 + b) + 60*(a^7*x^3 + a^4*b^3*x^2)*log(x^(1/3)) + 3*(20*a^6*b*x^2 + 15*a^3
*b^4*x - b^7)*x^(2/3) - 6*(5*a^5*b^2*x^2 + 3*a^2*b^5*x)*x^(1/3))/(a^3*b^6*
x^3 + b^9*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(83) = 166$.

Time = 2.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.91

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} \\ -\frac{3}{4b^2x^{\frac{4}{3}}} \\ -\frac{1}{2a^2x^2} \\ \frac{20a^5x^3\log(x)}{4ab^6x^3+4b^7x^{\frac{8}{3}}} - \frac{60a^5x^3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{4ab^6x^3+4b^7x^{\frac{8}{3}}} + \frac{20a^4bx^{\frac{8}{3}}\log(x)}{4ab^6x^3+4b^7x^{\frac{8}{3}}} - \frac{60a^4bx^{\frac{8}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{4ab^6x^3+4b^7x^{\frac{8}{3}}} + \frac{60a^4bx^{\frac{8}{3}}}{4ab^6x^3+4b^7x^{\frac{8}{3}}} + \frac{30a^3b^2x^{\frac{7}{3}}}{4ab^6x^3+4b^7x^{\frac{8}{3}}} - \frac{}{4ab} \end{cases}$$

input

```
integrate(1/(a+b/x**(1/3))**2/x**3,x)
```

output

```
Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b**2*x**(4/3)), Eq(a
, 0)), (-1/(2*a**2*x**2), Eq(b, 0)), (20*a**5*x**3*log(x)/(4*a*b**6*x**3 +
 4*b**7*x**(8/3)) - 60*a**5*x**3*log(x**(1/3) + b/a)/(4*a*b**6*x**3 + 4*b*
*7*x**(8/3)) + 20*a**4*b*x**(8/3)*log(x)/(4*a*b**6*x**3 + 4*b**7*x**(8/3))
 - 60*a**4*b*x**(8/3)*log(x**(1/3) + b/a)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)
) + 60*a**4*b*x**(8/3)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)) + 30*a**3*b**2*x*
*(7/3)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)) - 10*a**2*b**3*x**2/(4*a*b**6*x**
3 + 4*b**7*x**(8/3)) + 5*a*b**4*x**(5/3)/(4*a*b**6*x**3 + 4*b**7*x**(8/3))
 - 3*b**5*x**(4/3)/(4*a*b**6*x**3 + 4*b**7*x**(8/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx = -\frac{15\,a^4\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^6} - \frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4}{4\,b^6} + \frac{5\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a}{b^6} - \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^2}{b^6} + \frac{30\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^3}{b^6} - \frac{3\,a^5}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^6}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^3,x, algorithm="maxima")
```

output
```
-15*a^4*log(a + b/x^(1/3))/b^6 - 3/4*(a + b/x^(1/3))^4/b^6 + 5*(a + b/x^(1
/3))^3*a/b^6 - 15*(a + b/x^(1/3))^2*a^2/b^6 + 30*(a + b/x^(1/3))*a^3/b^6 -
 3*a^5/((a + b/x^(1/3))*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx = -\frac{15\,a^4\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^6}+\frac{5\,a^4\log\left(|x|\right)}{b^6} + \frac{60\,a^4bx^{\frac{4}{3}}+30\,a^3b^2x-10\,a^2b^3x^{\frac{2}{3}}+5\,ab^4x^{\frac{1}{3}}-3\,b^5}{4\left(ax^{\frac{1}{3}}+b\right)b^6x^{\frac{4}{3}}}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^3,x, algorithm="giac")
```

output
```
-15*a^4*log(abs(a*x^(1/3) + b))/b^6 + 5*a^4*log(abs(x))/b^6 + 1/4*(60*a^4*
b*x^(4/3) + 30*a^3*b^2*x - 10*a^2*b^3*x^(2/3) + 5*a*b^4*x^(1/3) - 3*b^5)/(
(a*x^(1/3) + b)*b^6*x^(4/3))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx = \frac{\frac{5\,a\,x^{1/3}}{4\,b^2}-\frac{3}{4\,b}+\frac{15\,a^3\,x}{2\,b^4}-\frac{5\,a^2\,x^{2/3}}{2\,b^3}+\frac{15\,a^4\,x^{4/3}}{b^5}}{a\,x^{5/3}+b\,x^{4/3}} - \frac{30\,a^4\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^6}$$

input
```
int(1/(x^3*(a + b/x^(1/3))^2),x)
```

output
```
((5*a*x^(1/3))/(4*b^2) - 3/(4*b) + (15*a^3*x)/(2*b^4) - (5*a^2*x^(2/3))/(2
*b^3) + (15*a^4*x^(4/3))/b^5)/(a*x^(5/3) + b*x^(4/3)) - (30*a^4*atanh((2*a
*x^(1/3))/b + 1))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^3}\,dx = \frac{60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)a^5-60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^5-60x^{\frac{5}{3}}a^5-10x^{\frac{2}{3}}a^2b^3+60x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}\right)a^4b-60x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}a+b\right)}{4x^{\frac{4}{3}}b^6\left(x^{\frac{1}{3}}a+b\right)}$$

input
```
int(1/(a+b/x^(1/3))^2/x^3,x)
```

output
```
(60*x**(2/3)*log(x**(1/3))*a**5*x - 60*x**(2/3)*log(x**(1/3)*a + b)*a**5*x
 - 60*x**(2/3)*a**5*x - 10*x**(2/3)*a**2*b**3 + 60*x**(1/3)*log(x**(1/3))*
a**4*b*x - 60*x**(1/3)*log(x**(1/3)*a + b)*a**4*b*x + 5*x**(1/3)*a*b**4 +
30*a**3*b**2*x - 3*b**5)/(4*x**(1/3)*b**6*x*(x**(1/3)*a + b))
```

3.325 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4} \, dx = \frac{3a^8}{b^9\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{3}{7b^2x^{7/3}} + \frac{a}{b^3x^2} - \frac{9a^2}{5b^4x^{5/3}} + \frac{3a^3}{b^5x^{4/3}} - \frac{5a^4}{b^6x} + \frac{9a^5}{b^7x^{2/3}} - \frac{21a^6}{b^8\sqrt[3]{x}} + \frac{24a^7\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^9}$$

output
```
3*a^8/b^9/(a+b/x^(1/3))-3/7/b^2/x^(7/3)+a/b^3/x^2-9/5*a^2/b^4/x^(5/3)+3*a^
3/b^5/x^(4/3)-5*a^4/b^6/x+9*a^5/b^7/x^(2/3)-21*a^6/b^8/x^(1/3)+24*a^7*ln(a
+b/x^(1/3))/b^9
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4} \, dx = -\frac{\frac{b\left(15b^7-20ab^6\sqrt[3]{x}+28a^2b^5x^{2/3}-42a^3b^4x+70a^4b^3x^{4/3}-140a^5b^2x^{5/3}+420a^6bx^2+840a^7x^{7/3}\right)}{\left(b+a\sqrt[3]{x}\right)x^{7/3}}-840a^7\log\left(b+a\sqrt[3]{x}\right)+280a}{35b^9}$$

input
```
Integrate[1/((a + b/x^(1/3))^2*x^4),x]
```

output
```
-1/35*((b*(15*b^7 - 20*a*b^6*x^(1/3) + 28*a^2*b^5*x^(2/3) - 42*a^3*b^4*x +
 70*a^4*b^3*x^(4/3) - 140*a^5*b^2*x^(5/3) + 420*a^6*b*x^2 + 840*a^7*x^(7/3
)))/((b + a*x^(1/3))*x^(7/3)) - 840*a^7*Log[b + a*x^(1/3)] + 280*a^7*Log[x
])/b^9
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^{10/3}\left(a\sqrt[3]{x}+b\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)^2 x^{8/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{8a^8}{b^9\left(\sqrt[3]{x}a+b\right)}+\frac{a^8}{b^8\left(\sqrt[3]{x}a+b\right)^2}-\frac{8a^7}{b^9\sqrt[3]{x}}+\frac{7a^6}{b^8x^{2/3}}-\frac{6a^5}{b^7x}+\frac{5a^4}{b^6x^{4/3}}-\frac{4a^3}{b^5x^{5/3}}+\frac{3a^2}{b^4x^2}-\frac{2a}{b^3x^{7/3}}+\frac{1}{b^2x^{8/3}}\right)d$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{8a^7\log\left(a\sqrt[3]{x}+b\right)}{b^9}-\frac{8a^7\log\left(\sqrt[3]{x}\right)}{b^9}-\frac{a^7}{b^8\left(a\sqrt[3]{x}+b\right)}-\frac{7a^6}{b^8\sqrt[3]{x}}+\frac{3a^5}{b^7x^{2/3}}-\frac{5a^4}{3b^6x}+\frac{a^3}{b^5x^{4/3}}-\frac{3a^2}{5b^4x^{5/3}}+\frac{a}{3b^3x^2}-\right.$$

input
```
Int[1/((a + b/x^(1/3))^2*x^4),x]
```

output
```
3*(-(a^7/(b^8*(b + a*x^(1/3)))) - 1/(7*b^2*x^(7/3)) + a/(3*b^3*x^2) - (3*a
^2)/(5*b^4*x^(5/3)) + a^3/(b^5*x^(4/3)) - (5*a^4)/(3*b^6*x) + (3*a^5)/(b^7
*x^(2/3)) - (7*a^6)/(b^8*x^(1/3)) + (8*a^7*Log[b + a*x^(1/3)])/b^9 - (8*a^
7*Log[x^(1/3)])/b^9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{3}{7b^2x^{\frac{7}{3}}}-\frac{9a^2}{5b^4x^{\frac{5}{3}}}-\frac{5a^4}{b^6x}-\frac{8a^7\ln(x)}{b^9}-\frac{21a^6}{b^8x^{\frac{1}{3}}}+\frac{9a^5}{b^7x^{\frac{2}{3}}}+\frac{3a^3}{b^5x^{\frac{4}{3}}}+\frac{a}{b^3x^2}-\frac{3a^7}{b^8\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{24a^7\ln\left(b+a\right.}{b^9}$
default	$-\frac{3}{7b^2x^{\frac{7}{3}}}-\frac{9a^2}{5b^4x^{\frac{5}{3}}}-\frac{5a^4}{b^6x}-\frac{8a^7\ln(x)}{b^9}-\frac{21a^6}{b^8x^{\frac{1}{3}}}+\frac{9a^5}{b^7x^{\frac{2}{3}}}+\frac{3a^3}{b^5x^{\frac{4}{3}}}+\frac{a}{b^3x^2}-\frac{3a^7}{b^8\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{24a^7\ln\left(b+a\right.}{b^9}$

input
```
int(1/(a+b/x^(1/3))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-3/7/b^2/x^(7/3)-9/5*a^2/b^4/x^(5/3)-5*a^4/b^6/x-8/b^9*a^7*ln(x)-21*a^6/b^
8/x^(1/3)+9*a^5/b^7/x^(2/3)+3*a^3/b^5/x^(4/3)+a/b^3/x^2-3*a^7/b^8/(b+a*x^(
1/3))+24/b^9*a^7*ln(b+a*x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

$$\int\frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2x^4}\,dx=$$

$$-\frac{280\,a^7b^3x^3+140\,a^4b^6x^2-35\,ab^9x-840\,(a^{10}x^4+a^7b^3x^3)\log\left(ax^{\frac{1}{3}}+b\right)+840\,(a^{10}x^4+a^7b^3x^3)\log\left(\right.}{35\,(a^3b^9x^4+b}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^4,x, algorithm="fricas")
```

output
```
-1/35*(280*a^7*b^3*x^3 + 140*a^4*b^6*x^2 - 35*a*b^9*x - 840*(a^10*x^4 + a^
7*b^3*x^3)*log(a*x^(1/3) + b) + 840*(a^10*x^4 + a^7*b^3*x^3)*log(x^(1/3))
+ 15*(56*a^9*b*x^3 + 42*a^6*b^4*x^2 - 6*a^3*b^7*x + b^10)*x^(2/3) - 21*(20
*a^8*b^2*x^3 + 12*a^5*b^5*x^2 - 3*a^2*b^8*x)*x^(1/3))/(a^3*b^9*x^4 + b^12*
x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(121) = 242$.

Time = 4.62 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.58

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4} \, dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{3}}} \\ -\frac{3}{7b^2x^{\frac{7}{3}}} \\ -\frac{1}{3a^2x^3} \\ -\frac{280a^8x^4\log(x)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}} + \frac{840a^8x^4\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}} - \frac{280a^7bx^{\frac{11}{3}}\log(x)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}} + \frac{840a^7bx^{\frac{11}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}} - \frac{840a^7bx^{\frac{11}{3}}}{35ab^9x^4+35b^{10}x^{\frac{11}{3}}} - \frac{4}{35ab} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**2/x**4,x)
```

output
```
Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (-3/(7*b**2*x**(7/3)), Eq(a
, 0)), (-1/(3*a**2*x**3), Eq(b, 0)), (-280*a**8*x**4*log(x)/(35*a*b**9*x**
4 + 35*b**10*x**(11/3)) + 840*a**8*x**4*log(x**(1/3) + b/a)/(35*a*b**9*x**
4 + 35*b**10*x**(11/3)) - 280*a**7*b*x**(11/3)*log(x)/(35*a*b**9*x**4 + 35
*b**10*x**(11/3)) + 840*a**7*b*x**(11/3)*log(x**(1/3) + b/a)/(35*a*b**9*x*
*4 + 35*b**10*x**(11/3)) - 840*a**7*b*x**(11/3)/(35*a*b**9*x**4 + 35*b**10
*x**(11/3)) - 420*a**6*b**2*x**(10/3)/(35*a*b**9*x**4 + 35*b**10*x**(11/3)
) + 140*a**5*b**3*x**3/(35*a*b**9*x**4 + 35*b**10*x**(11/3)) - 70*a**4*b**
4*x**(8/3)/(35*a*b**9*x**4 + 35*b**10*x**(11/3)) + 42*a**3*b**5*x**(7/3)/(
35*a*b**9*x**4 + 35*b**10*x**(11/3)) - 28*a**2*b**6*x**2/(35*a*b**9*x**4 +
 35*b**10*x**(11/3)) + 20*a*b**7*x**(5/3)/(35*a*b**9*x**4 + 35*b**10*x**(1
1/3)) - 15*b**8*x**(4/3)/(35*a*b**9*x**4 + 35*b**10*x**(11/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4}\,dx = \frac{24\,a^7\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^9} - \frac{3\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7}{7\,b^9} + \frac{4\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a}{b^9} - \frac{84\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^2}{5\,b^9} + \frac{42\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^3}{b^9} - \frac{70\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^4}{b^9} + \frac{84\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^5}{b^9} - \frac{84\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^6}{b^9} + \frac{3\,a^8}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^9}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^4,x, algorithm="maxima")
```

output
```
24*a^7*log(a + b/x^(1/3))/b^9 - 3/7*(a + b/x^(1/3))^7/b^9 + 4*(a + b/x^(1/
3))^6*a/b^9 - 84/5*(a + b/x^(1/3))^5*a^2/b^9 + 42*(a + b/x^(1/3))^4*a^3/b^
9 - 70*(a + b/x^(1/3))^3*a^4/b^9 + 84*(a + b/x^(1/3))^2*a^5/b^9 - 84*(a +
b/x^(1/3))*a^6/b^9 + 3*a^8/((a + b/x^(1/3))*b^9)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4}\,dx = \frac{24\,a^7\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^9} - \frac{8\,a^7\log\left(|x|\right)}{b^9} - \frac{840\,a^7bx^{\frac{7}{3}} + 420\,a^6b^2x^2 - 140\,a^5b^3x^{\frac{5}{3}} + 70\,a^4b^4x^{\frac{4}{3}} - 42\,a^3b^5x + 28\,a^2b^6x^{\frac{2}{3}} - 20\,ab^7x^{\frac{1}{3}} + 15\,b^8}{35\left(ax^{\frac{1}{3}}+b\right)b^9x^{\frac{7}{3}}}$$

input
```
integrate(1/(a+b/x^(1/3))^2/x^4,x, algorithm="giac")
```

output
```
24*a^7*log(abs(a*x^(1/3) + b))/b^9 - 8*a^7*log(abs(x))/b^9 - 1/35*(840*a^7
*b*x^(7/3) + 420*a^6*b^2*x^2 - 140*a^5*b^3*x^(5/3) + 70*a^4*b^4*x^(4/3) -
42*a^3*b^5*x + 28*a^2*b^6*x^(2/3) - 20*a*b^7*x^(1/3) + 15*b^8)/((a*x^(1/3)
 + b)*b^9*x^(7/3))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4}\,dx = \frac{48\,a^7\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^9} - \frac{\frac{3}{7\,b}-\frac{4\,a\,x^{1/3}}{7\,b^2}-\frac{6\,a^3\,x}{5\,b^4}+\frac{4\,a^2\,x^{2/3}}{5\,b^3}+\frac{12\,a^6\,x^2}{b^7}+\frac{2\,a^4\,x^{4/3}}{b^5}-\frac{4\,a^5\,x^{5/3}}{b^6}+\frac{24\,a^7\,x^{7/3}}{b^8}}{a\,x^{8/3}+b\,x^{7/3}}$$

input
```
int(1/(x^4*(a + b/x^(1/3))^2),x)
```

output
```
(48*a^7*atanh((2*a*x^(1/3))/b + 1))/b^9 - (3/(7*b) - (4*a*x^(1/3))/(7*b^2)
 - (6*a^3*x)/(5*b^4) + (4*a^2*x^(2/3))/(5*b^3) + (12*a^6*x^2)/b^7 + (2*a^4
*x^(4/3))/b^5 - (4*a^5*x^(5/3))/b^6 + (24*a^7*x^(7/3))/b^8)/(a*x^(8/3) + b
*x^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^2 x^4}\,dx = \frac{-840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}\right)a^8+840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^8+840x^{\frac{8}{3}}a^8+140x^{\frac{5}{3}}a^5b^3-28x^{\frac{2}{3}}a^2b^6-840x^{\frac{7}{3}}\log\left(x^{\frac{1}{3}}\right)a^7b}{35x^{\frac{7}{3}}b^9\left(x^{\frac{1}{3}}a+b\right)}$$

input
```
int(1/(a+b/x^(1/3))^2/x^4,x)
```

output
```
( - 840*x**(2/3)*log(x**(1/3))*a**8*x**2 + 840*x**(2/3)*log(x**(1/3)*a + b
)*a**8*x**2 + 840*x**(2/3)*a**8*x**2 + 140*x**(2/3)*a**5*b**3*x - 28*x**(2
/3)*a**2*b**6 - 840*x**(1/3)*log(x**(1/3))*a**7*b*x**2 + 840*x**(1/3)*log(
x**(1/3)*a + b)*a**7*b*x**2 - 70*x**(1/3)*a**4*b**4*x + 20*x**(1/3)*a*b**7
 - 420*a**6*b**2*x**2 + 42*a**3*b**5*x - 15*b**8)/(35*x**(1/3)*b**9*x**2*(
x**(1/3)*a + b))
```

3.326 $\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 171

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{3b^{11}}{2a^{12}\left(b+a\sqrt[3]{x}\right)^2}-\frac{33b^{10}}{a^{12}\left(b+a\sqrt[3]{x}\right)}+\frac{135b^8\sqrt[3]{x}}{a^{11}}-\frac{54b^7x^{2/3}}{a^{10}}+\frac{28b^6x}{a^9}-\frac{63b^5x^{4/3}}{4a^8}+\frac{9b^4x^{5/3}}{a^7}-\frac{5b^3x^2}{a^6}+\frac{18b^2x^{7/3}}{7a^5}-\frac{9bx^{8/3}}{8a^4}+\frac{x^3}{3a^3}-\frac{165b^9\log\left(b+a\sqrt[3]{x}\right)}{a^{12}}$$

output
```
3/2*b^11/a^12/(b+a*x^(1/3))^2-33*b^10/a^12/(b+a*x^(1/3))+135*b^8*x^(1/3)/a
^11-54*b^7*x^(2/3)/a^10+28*b^6*x/a^9-63/4*b^5*x^(4/3)/a^8+9*b^4*x^(5/3)/a^
7-5*b^3*x^2/a^6+18/7*b^2*x^(7/3)/a^5-9/8*b*x^(8/3)/a^4+1/3*x^3/a^3-165*b^9
*ln(b+a*x^(1/3))/a^12
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3} \, dx$$
$$= \frac{-5292b^{11} + 17136ab^{10}\sqrt[3]{x} + 36288a^2b^9x^{2/3} + 9240a^3b^8x - 2310a^4b^7x^{4/3} + 924a^5b^6x^{5/3} - 462a^6b^5x^2 + 2}{168a^{12}\left(b+a\sqrt[3]{x}\right)^2}$$
$$- \frac{165b^9 \log \left(b+a\sqrt[3]{x}\right)}{a^{12}}$$

input
```
Integrate[x^2/(a + b/x^(1/3))^3,x]
```

output
```
(-5292*b^11 + 17136*a*b^10*x^(1/3) + 36288*a^2*b^9*x^(2/3) + 9240*a^3*b^8*
x - 2310*a^4*b^7*x^(4/3) + 924*a^5*b^6*x^(5/3) - 462*a^6*b^5*x^2 + 264*a^7
*b^4*x^(7/3) - 165*a^8*b^3*x^(8/3) + 110*a^9*b^2*x^3 - 77*a^10*b*x^(10/3)
+ 56*a^11*x^(11/3))/(168*a^12*(b + a*x^(1/3))^2) - (165*b^9*Log[b + a*x^(1
/3)])/a^12
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3} \, dx$$
$$\Big\downarrow \; 795$$
$$\int \frac{x^3}{\left(a\sqrt[3]{x}+b\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{11/3}}{\left(\sqrt[3]{x}a+b\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{b^{11}}{a^{11}\left(\sqrt[3]{x}a+b\right)^3}+\frac{11b^{10}}{a^{11}\left(\sqrt[3]{x}a+b\right)^2}-\frac{55b^9}{a^{11}\left(\sqrt[3]{x}a+b\right)}+\frac{45b^8}{a^{11}}-\frac{36\sqrt[3]{x}b^7}{a^{10}}+\frac{28x^{2/3}b^6}{a^9}-\frac{21xb^5}{a^8}+\frac{15x^{4/3}b^4}{a^7}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{b^{11}}{2a^{12}\left(a\sqrt[3]{x}+b\right)^2}-\frac{11b^{10}}{a^{12}\left(a\sqrt[3]{x}+b\right)}-\frac{55b^9\log\left(a\sqrt[3]{x}+b\right)}{a^{12}}+\frac{45b^8\sqrt[3]{x}}{a^{11}}-\frac{18b^7x^{2/3}}{a^{10}}+\frac{28b^6x}{3a^9}-\frac{21b^5x^{4/3}}{4a^8}+\frac{3b^4x}{a^7}\right.$$

input
```
Int[x^2/(a + b/x^(1/3))^3,x]
```

output
```
3*(b^11/(2*a^12*(b + a*x^(1/3))^2) - (11*b^10)/(a^12*(b + a*x^(1/3))) + (4
5*b^8*x^(1/3))/a^11 - (18*b^7*x^(2/3))/a^10 + (28*b^6*x)/(3*a^9) - (21*b^5
*x^(4/3))/(4*a^8) + (3*b^4*x^(5/3))/a^7 - (5*b^3*x^2)/(3*a^6) + (6*b^2*x^(
7/3))/(7*a^5) - (3*b*x^(8/3))/(8*a^4) + x^3/(9*a^3) - (55*b^9*Log[b + a*x^
(1/3)])/a^12)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{a^8x^3}{3}-\frac{9b\,x^{\frac{8}{3}}a^7}{8}+\frac{18b^2x^{\frac{7}{3}}a^6}{7}-5a^5b^3x^2+9x^{\frac{5}{3}}a^4b^4-\frac{63b^5x^{\frac{4}{3}}a^3}{4}+28a^2b^6x-54a\,b^7x^{\frac{2}{3}}+135b^8x^{\frac{1}{3}}}{a^{11}}+\frac{3b^{11}}{2a^{12}\left(b+a\,x^{\frac{1}{3}}\right)^2}-$
default	$\frac{\frac{a^8x^3}{3}-\frac{9b\,x^{\frac{8}{3}}a^7}{8}+\frac{18b^2x^{\frac{7}{3}}a^6}{7}-5a^5b^3x^2+9x^{\frac{5}{3}}a^4b^4-\frac{63b^5x^{\frac{4}{3}}a^3}{4}+28a^2b^6x-54a\,b^7x^{\frac{2}{3}}+135b^8x^{\frac{1}{3}}}{a^{11}}+\frac{3b^{11}}{2a^{12}\left(b+a\,x^{\frac{1}{3}}\right)^2}-$

input
```
int(x^2/(a+b/x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
3/a^11*(1/9*a^8*x^3-3/8*b*x^(8/3)*a^7+6/7*b^2*x^(7/3)*a^6-5/3*a^5*b^3*x^2+
3*x^(5/3)*a^4*b^4-21/4*b^5*x^(4/3)*a^3+28/3*a^2*b^6*x-18*a*b^7*x^(2/3)+45*
b^8*x^(1/3))+3/2*b^11/a^12/(b+a*x^(1/3))^2-33*b^10/a^12/(b+a*x^(1/3))-165*
b^9*ln(b+a*x^(1/3))/a^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$$
$$=\frac{56\,a^{15}x^5-728\,a^{12}b^3x^4+3080\,a^9b^6x^3+8568\,a^6b^9x^2-1344\,a^3b^{12}x-5292\,b^{15}-27720\,(a^6b^9x^2+2\,a^3b^{12}x}{}$$

input
```
integrate(x^2/(a+b/x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/168*(56*a^15*x^5 - 728*a^12*b^3*x^4 + 3080*a^9*b^6*x^3 + 8568*a^6*b^9*x^
2 - 1344*a^3*b^12*x - 5292*b^15 - 27720*(a^6*b^9*x^2 + 2*a^3*b^12*x + b^15
)*log(a*x^(1/3) + b) - 63*(3*a^14*b*x^4 - 18*a^11*b^4*x^3 + 99*a^8*b^7*x^2
 + 352*a^5*b^10*x + 220*a^2*b^13)*x^(2/3) + 18*(24*a^13*b^2*x^4 - 99*a^10*
b^5*x^3 + 990*a^7*b^8*x^2 + 2695*a^4*b^11*x + 1540*a*b^14)*x^(1/3))/(a^18*
x^2 + 2*a^15*b^3*x + a^12*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(170) = 340$.

Time = 1.21 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.65

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$$
$$= \begin{cases} \frac{56a^{11}x^{\frac{11}{3}}}{168a^{14}x^{\frac{2}{3}}+336a^{13}b\sqrt[3]{x}+168a^{12}b^2} - \frac{77a^{10}bx^{\frac{10}{3}}}{168a^{14}x^{\frac{2}{3}}+336a^{13}b\sqrt[3]{x}+168a^{12}b^2} + \frac{110a^9b^2x^3}{168a^{14}x^{\frac{2}{3}}+336a^{13}b\sqrt[3]{x}+168a^{12}b^2} - \frac{165a^8}{168a^{14}x^{\frac{2}{3}}+336a^{13}} \\ \frac{x^4}{4b^3} \end{cases}$$

input
```
integrate(x**2/(a+b/x**(1/3))**3,x)
```

output
```
Piecewise((56*a**11*x**(11/3)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) +
 168*a**12*b**2) - 77*a**10*b*x**(10/3)/(168*a**14*x**(2/3) + 336*a**13*b*
x**(1/3) + 168*a**12*b**2) + 110*a**9*b**2*x**3/(168*a**14*x**(2/3) + 336*
a**13*b*x**(1/3) + 168*a**12*b**2) - 165*a**8*b**3*x**(8/3)/(168*a**14*x**
(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) + 264*a**7*b**4*x**(7/3)/(1
68*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) - 462*a**6*b**5
*x**2/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) + 924*a
**5*b**6*x**(5/3)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b
**2) - 2310*a**4*b**7*x**(4/3)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3)
+ 168*a**12*b**2) + 9240*a**3*b**8*x/(168*a**14*x**(2/3) + 336*a**13*b*x**
(1/3) + 168*a**12*b**2) - 27720*a**2*b**9*x**(2/3)*log(x**(1/3) + b/a)/(16
8*a**14*x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) - 55440*a*b**10*
x**(1/3)*log(x**(1/3) + b/a)/(168*a**14*x**(2/3) + 336*a**13*b*x**(1/3) +
168*a**12*b**2) - 55440*a*b**10*x**(1/3)/(168*a**14*x**(2/3) + 336*a**13*b
*x**(1/3) + 168*a**12*b**2) - 27720*b**11*log(x**(1/3) + b/a)/(168*a**14*x
**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2) - 41580*b**11/(168*a**14*
x**(2/3) + 336*a**13*b*x**(1/3) + 168*a**12*b**2), Ne(a, 0)), (x**4/(4*b**
3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{56\,a^{10} - \frac{77\,a^9 b}{x^{\frac{1}{3}}} + \frac{110\,a^8 b^2}{x^{\frac{2}{3}}} - \frac{165\,a^7 b^3}{x} + \frac{264\,a^6 b^4}{x^{\frac{4}{3}}} - \frac{462\,a^5 b^5}{x^{\frac{5}{3}}} + \frac{924\,a^4 b^6}{x^2} - \frac{2310\,a^3 b^7}{x^{\frac{7}{3}}} + \frac{9240\,a^2 b^8}{x^{\frac{8}{3}}} + \frac{41580\,a b^9}{x^3} + \frac{27720\,b^{10}}{x^{\frac{10}{3}}}}{168\left(\frac{a^{13}}{x^3} + \frac{2\,a^{12} b}{x^{\frac{10}{3}}} + \frac{a^{11} b^2}{x^{\frac{11}{3}}}\right)} - \frac{165\,b^9 \log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^{12}} - \frac{55\,b^9 \log\left(x\right)}{a^{12}}$$

input
```
integrate(x^2/(a+b/x^(1/3))^3,x, algorithm="maxima")
```

output
```
1/168*(56*a^10 - 77*a^9*b/x^(1/3) + 110*a^8*b^2/x^(2/3) - 165*a^7*b^3/x +
264*a^6*b^4/x^(4/3) - 462*a^5*b^5/x^(5/3) + 924*a^4*b^6/x^2 - 2310*a^3*b^7
/x^(7/3) + 9240*a^2*b^8/x^(8/3) + 41580*a*b^9/x^3 + 27720*b^10/x^(10/3))/(
a^13/x^3 + 2*a^12*b/x^(10/3) + a^11*b^2/x^(11/3)) - 165*b^9*log(a + b/x^(1
/3))/a^12 - 55*b^9*log(x)/a^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = -\frac{165\,b^9\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^{12}} - \frac{3\left(22\,ab^{10}x^{\frac{1}{3}}+21\,b^{11}\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^{12}} + \frac{56\,a^{24}x^3-189\,a^{23}bx^{\frac{8}{3}}+432\,a^{22}b^2x^{\frac{7}{3}}-840\,a^{21}b^3x^2+1512\,a^{20}b^4x^{\frac{5}{3}}-2646\,a^{19}b^5x^{\frac{4}{3}}+4704\,a^{18}b^6x-90}{168\,a^{27}}$$

input
```
integrate(x^2/(a+b/x^(1/3))^3,x, algorithm="giac")
```

output
```
-165*b^9*log(abs(a*x^(1/3) + b))/a^12 - 3/2*(22*a*b^10*x^(1/3) + 21*b^11)/
((a*x^(1/3) + b)^2*a^12) + 1/168*(56*a^24*x^3 - 189*a^23*b*x^(8/3) + 432*a
^22*b^2*x^(7/3) - 840*a^21*b^3*x^2 + 1512*a^20*b^4*x^(5/3) - 2646*a^19*b^5
*x^(4/3) + 4704*a^18*b^6*x - 9072*a^17*b^7*x^(2/3) + 22680*a^16*b^8*x^(1/3
))/a^27
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{x^3}{3\,a^3} - \frac{\frac{63\,b^{11}}{2\,a}+33\,b^{10}\,x^{1/3}}{a^{11}\,b^2+a^{13}\,x^{2/3}+2\,a^{12}\,b\,x^{1/3}} - \frac{9\,b\,x^{8/3}}{8\,a^4} + \frac{28\,b^6\,x}{a^9} - \frac{165\,b^9\,\ln\left(b+a\,x^{1/3}\right)}{a^{12}} - \frac{5\,b^3\,x^2}{a^6} + \frac{18\,b^2\,x^{7/3}}{7\,a^5} + \frac{9\,b^4\,x^{5/3}}{a^7} - \frac{63\,b^5\,x^{4/3}}{4\,a^8} - \frac{54\,b^7\,x^{2/3}}{a^{10}} + \frac{135\,b^8\,x^{1/3}}{a^{11}}$$

input
```
int(x^2/(a + b/x^(1/3))^3,x)
```

output
```
x^3/(3*a^3) - ((63*b^11)/(2*a) + 33*b^10*x^(1/3))/(a^11*b^2 + a^13*x^(2/3)
 + 2*a^12*b*x^(1/3)) - (9*b*x^(8/3))/(8*a^4) + (28*b^6*x)/a^9 - (165*b^9*l
og(b + a*x^(1/3)))/a^12 - (5*b^3*x^2)/a^6 + (18*b^2*x^(7/3))/(7*a^5) + (9*
b^4*x^(5/3))/a^7 - (63*b^5*x^(4/3))/(4*a^8) - (54*b^7*x^(2/3))/a^10 + (135
*b^8*x^(1/3))/a^11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$$

$$= \frac{-27720x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2b^9+56x^{\frac{11}{3}}a^{11}-165x^{\frac{8}{3}}a^8b^3+924x^{\frac{5}{3}}a^5b^6+27720x^{\frac{2}{3}}a^2b^9-55440x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a\right.}{168}$$

input
```
int(x^2/(a+b/x^(1/3))^3,x)
```

output
```
( - 27720*x**(2/3)*log(x**(1/3)*a + b)*a**2*b**9 + 56*x**(2/3)*a**11*x**3
- 165*x**(2/3)*a**8*b**3*x**2 + 924*x**(2/3)*a**5*b**6*x + 27720*x**(2/3)*
a**2*b**9 - 55440*x**(1/3)*log(x**(1/3)*a + b)*a*b**10 - 77*x**(1/3)*a**10
*b*x**3 + 264*x**(1/3)*a**7*b**4*x**2 - 2310*x**(1/3)*a**4*b**7*x - 27720*
log(x**(1/3)*a + b)*b**11 + 110*a**9*b**2*x**3 - 462*a**6*b**5*x**2 + 9240
*a**3*b**8*x - 13860*b**11)/(168*a**12*(x**(2/3)*a**2 + 2*x**(1/3)*a*b + b
**2))
```

3.327 $\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = -\frac{3b^8}{2a^9\left(b+a\sqrt[3]{x}\right)^2}+\frac{24b^7}{a^9\left(b+a\sqrt[3]{x}\right)}-\frac{63b^5\sqrt[3]{x}}{a^8}+\frac{45b^4x^{2/3}}{2a^7}-\frac{10b^3x}{a^6}+\frac{9b^2x^{4/3}}{2a^5}-\frac{9bx^{5/3}}{5a^4}+\frac{x^2}{2a^3}+\frac{84b^6\log\left(b+a\sqrt[3]{x}\right)}{a^9}$$

output
```
-3/2*b^8/a^9/(b+a*x^(1/3))^2+24*b^7/a^9/(b+a*x^(1/3))-63*b^5*x^(1/3)/a^8+4
5/2*b^4*x^(2/3)/a^7-10*b^3*x/a^6+9/2*b^2*x^(4/3)/a^5-9/5*b*x^(5/3)/a^4+1/2
*x^2/a^3+84*b^6*ln(b+a*x^(1/3))/a^9
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3} dx = \frac{225b^8 - 390ab^7\sqrt[3]{x} - 1035a^2b^6x^{2/3} - 280a^3b^5x + 70a^4b^4x^{4/3} - 28a^5b^3x^{5/3} + 14a^6b^2x^2 - 8a^7bx^{7/3} + 5a^8x^{8/3}}{10a^9\left(b+a\sqrt[3]{x}\right)^2} + \frac{84b^6\log\left(b+a\sqrt[3]{x}\right)}{a^9}$$

input
```
Integrate[x/(a + b/x^(1/3))^3,x]
```

output
```
(225*b^8 - 390*a*b^7*x^(1/3) - 1035*a^2*b^6*x^(2/3) - 280*a^3*b^5*x + 70*a
^4*b^4*x^(4/3) - 28*a^5*b^3*x^(5/3) + 14*a^6*b^2*x^2 - 8*a^7*b*x^(7/3) + 5
*a^8*x^(8/3))/(10*a^9*(b + a*x^(1/3))^2) + (84*b^6*Log[b + a*x^(1/3)])/a^9
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {795, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3} dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{x^2}{\left(a\sqrt[3]{x}+b\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{x^{8/3}}{\left(\sqrt[3]{x}a+b\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(\frac{b^8}{a^8\left(\sqrt[3]{x}a+b\right)^3}-\frac{8b^7}{a^8\left(\sqrt[3]{x}a+b\right)^2}+\frac{28b^6}{a^8\left(\sqrt[3]{x}a+b\right)}-\frac{21b^5}{a^8}+\frac{15\sqrt[3]{x}b^4}{a^7}-\frac{10x^{2/3}b^3}{a^6}+\frac{6xb^2}{a^5}-\frac{3x^{4/3}b}{a^4}+\frac{x^{5/3}}{a^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{b^8}{2a^9\left(a\sqrt[3]{x}+b\right)^2}+\frac{8b^7}{a^9\left(a\sqrt[3]{x}+b\right)}+\frac{28b^6\log\left(a\sqrt[3]{x}+b\right)}{a^9}-\frac{21b^5\sqrt[3]{x}}{a^8}+\frac{15b^4x^{2/3}}{2a^7}-\frac{10b^3x}{3a^6}+\frac{3b^2x^{4/3}}{2a^5}-\frac{3bx^{5/3}}{5a^4}\right.$$

input
```
Int[x/(a + b/x^(1/3))^3,x]
```

output
```
3*(-1/2*b^8/(a^9*(b + a*x^(1/3))^2) + (8*b^7)/(a^9*(b + a*x^(1/3))) - (21*
b^5*x^(1/3))/a^8 + (15*b^4*x^(2/3))/(2*a^7) - (10*b^3*x)/(3*a^6) + (3*b^2*
x^(4/3))/(2*a^5) - (3*b*x^(5/3))/(5*a^4) + x^2/(6*a^3) + (28*b^6*Log[b + a
*x^(1/3)])/a^9)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{a^5x^2}{2}-\frac{9bx^{\frac{5}{3}}a^4}{5}+\frac{9b^2x^{\frac{4}{3}}a^3}{2}-10a^2b^3x+\frac{45x^{\frac{2}{3}}ab^4}{2}-63b^5x^{\frac{1}{3}}}{a^8}+\frac{24b^7}{a^9\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{84b^6\ln\left(b+a\,x^{\frac{1}{3}}\right)}{a^9}-\frac{3b^8}{2a^9\left(b+a\,x^{\frac{1}{3}}\right)^2}$
default	$\frac{\frac{a^5x^2}{2}-\frac{9bx^{\frac{5}{3}}a^4}{5}+\frac{9b^2x^{\frac{4}{3}}a^3}{2}-10a^2b^3x+\frac{45x^{\frac{2}{3}}ab^4}{2}-63b^5x^{\frac{1}{3}}}{a^8}+\frac{24b^7}{a^9\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{84b^6\ln\left(b+a\,x^{\frac{1}{3}}\right)}{a^9}-\frac{3b^8}{2a^9\left(b+a\,x^{\frac{1}{3}}\right)^2}$

input
```
int(x/(a+b/x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
3/a^8*(1/6*a^5*x^2-3/5*b*x^(5/3)*a^4+3/2*b^2*x^(4/3)*a^3-10/3*a^2*b^3*x+15
/2*x^(2/3)*a*b^4-21*b^5*x^(1/3))+24*b^7/a^9/(b+a*x^(1/3))+84*b^6*ln(b+a*x^
(1/3))/a^9-3/2*b^8/a^9/(b+a*x^(1/3))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.43

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{5\,a^{12}x^4-90\,a^9b^3x^3-195\,a^6b^6x^2+170\,a^3b^9x+225\,b^{12}+840\,(a^6b^6x^2+2\,a^3b^9x+b^{12})\log\left(ax^{\frac{1}{3}}+b\right)-3}{10\,(a^{15}x^2+2\,a}$$

input
```
integrate(x/(a+b/x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/10*(5*a^12*x^4 - 90*a^9*b^3*x^3 - 195*a^6*b^6*x^2 + 170*a^3*b^9*x + 225*
b^12 + 840*(a^6*b^6*x^2 + 2*a^3*b^9*x + b^12)*log(a*x^(1/3) + b) - 3*(6*a^
11*b*x^3 - 63*a^8*b^4*x^2 - 224*a^5*b^7*x - 140*a^2*b^10)*x^(2/3) + 15*(3*
a^10*b^2*x^3 - 36*a^7*b^5*x^2 - 98*a^4*b^8*x - 56*a*b^11)*x^(1/3))/(a^15*x
^2 + 2*a^12*b^3*x + a^9*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(131) = 262$.

Time = 0.55 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.68

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \begin{cases} \frac{5a^8x^{\frac{8}{3}}}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2} - \frac{8a^7bx^{\frac{7}{3}}}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2} + \frac{14a^6b^2x^2}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2} - \frac{28a^5b^3x^{\frac{5}{3}}}{10a^{11}x^{\frac{2}{3}}+20a^{10}b\sqrt[3]{x}+10a^9b^2} \\ \frac{x^3}{3b^3} \end{cases}$$

input
```
integrate(x/(a+b/x**(1/3))**3,x)
```

output
```
Piecewise((5*a**8*x**(8/3)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*a
**9*b**2) - 8*a**7*b*x**(7/3)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 1
0*a**9*b**2) + 14*a**6*b**2*x**2/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3)
 + 10*a**9*b**2) - 28*a**5*b**3*x**(5/3)/(10*a**11*x**(2/3) + 20*a**10*b*x*
*(1/3) + 10*a**9*b**2) + 70*a**4*b**4*x**(4/3)/(10*a**11*x**(2/3) + 20*a**
10*b*x**(1/3) + 10*a**9*b**2) - 280*a**3*b**5*x/(10*a**11*x**(2/3) + 20*a*
*10*b*x**(1/3) + 10*a**9*b**2) + 840*a**2*b**6*x**(2/3)*log(x**(1/3) + b/a
)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*a**9*b**2) + 1680*a*b**7*x
**(1/3)*log(x**(1/3) + b/a)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3) + 10*
a**9*b**2) + 1680*a*b**7*x**(1/3)/(10*a**11*x**(2/3) + 20*a**10*b*x**(1/3)
 + 10*a**9*b**2) + 840*b**8*log(x**(1/3) + b/a)/(10*a**11*x**(2/3) + 20*a*
*10*b*x**(1/3) + 10*a**9*b**2) + 1260*b**8/(10*a**11*x**(2/3) + 20*a**10*b
*x**(1/3) + 10*a**9*b**2), Ne(a, 0)), (x**3/(3*b**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{5\,a^7 - \frac{8\,a^6b}{x^{\frac{1}{3}}} + \frac{14\,a^5b^2}{x^{\frac{2}{3}}} - \frac{28\,a^4b^3}{x} + \frac{70\,a^3b^4}{x^{\frac{4}{3}}} - \frac{280\,a^2b^5}{x^{\frac{5}{3}}} - \frac{1260\,ab^6}{x^2} - \frac{840\,b^7}{x^{\frac{7}{3}}}}{10\left(\frac{a^{10}}{x^2} + \frac{2\,a^9b}{x^{\frac{7}{3}}} + \frac{a^8b^2}{x^{\frac{8}{3}}}\right)} + \frac{84\,b^6\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^9} + \frac{28\,b^6\log(x)}{a^9}$$

input
```
integrate(x/(a+b/x^(1/3))^3,x, algorithm="maxima")
```

output
```
1/10*(5*a^7 - 8*a^6*b/x^(1/3) + 14*a^5*b^2/x^(2/3) - 28*a^4*b^3/x + 70*a^3
*b^4/x^(4/3) - 280*a^2*b^5/x^(5/3) - 1260*a*b^6/x^2 - 840*b^7/x^(7/3))/(a^
10/x^2 + 2*a^9*b/x^(7/3) + a^8*b^2/x^(8/3)) + 84*b^6*log(a + b/x^(1/3))/a^
9 + 28*b^6*log(x)/a^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{84\,b^6\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^9}+\frac{3\left(16\,ab^7x^{\frac{1}{3}}+15\,b^8\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^9} + \frac{5\,a^{15}x^2-18\,a^{14}bx^{\frac{5}{3}}+45\,a^{13}b^2x^{\frac{4}{3}}-100\,a^{12}b^3x+225\,a^{11}b^4x^{\frac{2}{3}}-630\,a^{10}b^5x^{\frac{1}{3}}}{10\,a^{18}}$$

input
```
integrate(x/(a+b/x^(1/3))^3,x, algorithm="giac")
```

output
```
84*b^6*log(abs(a*x^(1/3) + b))/a^9 + 3/2*(16*a*b^7*x^(1/3) + 15*b^8)/((a*x
^(1/3) + b)^2*a^9) + 1/10*(5*a^15*x^2 - 18*a^14*b*x^(5/3) + 45*a^13*b^2*x^
(4/3) - 100*a^12*b^3*x + 225*a^11*b^4*x^(2/3) - 630*a^10*b^5*x^(1/3))/a^18
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{\frac{45\,b^8}{2\,a}+24\,b^7\,x^{1/3}}{a^8\,b^2+a^{10}\,x^{2/3}+2\,a^9\,b\,x^{1/3}}+\frac{x^2}{2\,a^3}-\frac{10\,b^3\,x}{a^6}-\frac{9\,b\,x^{5/3}}{5\,a^4} + \frac{84\,b^6\,\ln\left(b+a\,x^{1/3}\right)}{a^9}+\frac{9\,b^2\,x^{4/3}}{2\,a^5}+\frac{45\,b^4\,x^{2/3}}{2\,a^7}-\frac{63\,b^5\,x^{1/3}}{a^8}$$

input
```
int(x/(a + b/x^(1/3))^3,x)
```

output
```
((45*b^8)/(2*a) + 24*b^7*x^(1/3))/(a^8*b^2 + a^10*x^(2/3) + 2*a^9*b*x^(1/3
)) + x^2/(2*a^3) - (10*b^3*x)/a^6 - (9*b*x^(5/3))/(5*a^4) + (84*b^6*log(b
+ a*x^(1/3)))/a^9 + (9*b^2*x^(4/3))/(2*a^5) + (45*b^4*x^(2/3))/(2*a^7) - (
63*b^5*x^(1/3))/a^8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.12

$$\int \frac{x}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{840x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2b^6+5x^{\frac{8}{3}}a^8-28x^{\frac{5}{3}}a^5b^3-840x^{\frac{2}{3}}a^2b^6+1680x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)ab^7-8x^{\frac{7}{3}}a^7b+70x}{10a^9\left(x^{\frac{2}{3}}a^2+2x^{\frac{1}{3}}ab+b^2\right)}$$

input
```
int(x/(a+b/x^(1/3))^3,x)
```

output
```
(840*x**(2/3)*log(x**(1/3)*a + b)*a**2*b**6 + 5*x**(2/3)*a**8*x**2 - 28*x*
*(2/3)*a**5*b**3*x - 840*x**(2/3)*a**2*b**6 + 1680*x**(1/3)*log(x**(1/3)*a
 + b)*a*b**7 - 8*x**(1/3)*a**7*b*x**2 + 70*x**(1/3)*a**4*b**4*x + 840*log(
x**(1/3)*a + b)*b**8 + 14*a**6*b**2*x**2 - 280*a**3*b**5*x + 420*b**8)/(10
*a**9*(x**(2/3)*a**2 + 2*x**(1/3)*a*b + b**2))
```

3.328 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 90

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3} dx = \frac{3b^5}{2a^6\left(b+a\sqrt[3]{x}\right)^2} - \frac{15b^4}{a^6\left(b+a\sqrt[3]{x}\right)} + \frac{18b^2\sqrt[3]{x}}{a^5} - \frac{9bx^{2/3}}{2a^4} + \frac{x}{a^3} - \frac{30b^3\log\left(b+a\sqrt[3]{x}\right)}{a^6}$$

output
```
3/2*b^5/a^6/(b+a*x^(1/3))^2-15*b^4/a^6/(b+a*x^(1/3))+18*b^2*x^(1/3)/a^5-9/
2*b*x^(2/3)/a^4+x/a^3-30*b^3*ln(b+a*x^(1/3))/a^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3} dx = \frac{-27b^5+6ab^4\sqrt[3]{x}+63a^2b^3x^{2/3}+20a^3b^2x-5a^4bx^{4/3}+2a^5x^{5/3}}{2a^6\left(b+a\sqrt[3]{x}\right)^2} - \frac{30b^3\log\left(b+a\sqrt[3]{x}\right)}{a^6}$$

input
```
Integrate[(a + b/x^(1/3))^(-3),x]
```

output
```
(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b^2*x - 5*a^4*b*x
^(4/3) + 2*a^5*x^(5/3))/(2*a^6*(b + a*x^(1/3))^2) - (30*b^3*Log[b + a*x^(1
/3)])/a^6
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {774, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{x^{2/3}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{795}$$

$$3\int \frac{x^{5/3}}{\left(\sqrt[3]{x}a+b\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{b^5}{a^5\left(\sqrt[3]{x}a+b\right)^3}+\frac{5b^4}{a^5\left(\sqrt[3]{x}a+b\right)^2}-\frac{10b^3}{a^5\left(\sqrt[3]{x}a+b\right)}+\frac{6b^2}{a^5}-\frac{3\sqrt[3]{x}b}{a^4}+\frac{x^{2/3}}{a^3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{b^5}{2a^6\left(a\sqrt[3]{x}+b\right)^2}-\frac{5b^4}{a^6\left(a\sqrt[3]{x}+b\right)}-\frac{10b^3\log\left(a\sqrt[3]{x}+b\right)}{a^6}+\frac{6b^2\sqrt[3]{x}}{a^5}-\frac{3bx^{2/3}}{2a^4}+\frac{x}{3a^3}\right)$$

input
```
Int[(a + b/x^(1/3))^(-3),x]
```

output
```
3*(b^5/(2*a^6*(b + a*x^(1/3))^2) - (5*b^4)/(a^6*(b + a*x^(1/3))) + (6*b^2*
x^(1/3))/a^5 - (3*b*x^(2/3))/(2*a^4) + x/(3*a^3) - (10*b^3*Log[b + a*x^(1/
3)])/a^6)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{a^{2} x-\frac{9 x^{\frac{2}{3}} a b}{2}+18 x^{\frac{1}{3}} b^{2}}{a^{5}}-\frac{15 b^{4}}{a^{6} \left(b+a \,x^{\frac{1}{3}}\right)}-\frac{30 b^{3} \ln \left(b+a \,x^{\frac{1}{3}}\right)}{a^{6}}+\frac{3 b^{5}}{2 a^{6} \left(b+a \,x^{\frac{1}{3}}\right)^{2}}$	79
default	$\frac{a^{2} x-\frac{9 x^{\frac{2}{3}} a b}{2}+18 x^{\frac{1}{3}} b^{2}}{a^{5}}-\frac{15 b^{4}}{a^{6} \left(b+a \,x^{\frac{1}{3}}\right)}-\frac{30 b^{3} \ln \left(b+a \,x^{\frac{1}{3}}\right)}{a^{6}}+\frac{3 b^{5}}{2 a^{6} \left(b+a \,x^{\frac{1}{3}}\right)^{2}}$	79

input
```
int(1/(a+b/x^(1/3))^3,x,method=_RETURNVERBOSE)
```

output
```
3/a^5*(1/3*a^2*x-3/2*x^(2/3)*a*b+6*x^(1/3)*b^2)-15*b^4/a^6/(b+a*x^(1/3))-3
0*b^3*ln(b+a*x^(1/3))/a^6+3/2*b^5/a^6/(b+a*x^(1/3))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(76) = 152$.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{2\,a^9x^3+4\,a^6b^3x^2-34\,a^3b^6x-27\,b^9-60\,(a^6b^3x^2+2\,a^3b^6x+b^9)\log\left(ax^{\frac{1}{3}}+b\right)-3\,(3\,a^8bx^2+16\,a^5b^4x \cdots}{2\,(a^{12}x^2+2\,a^9b^3x+a^6b^6)}$$

input
```
integrate(1/(a+b/x^(1/3))^3,x, algorithm="fricas")
```

output
```
1/2*(2*a^9*x^3 + 4*a^6*b^3*x^2 - 34*a^3*b^6*x - 27*b^9 - 60*(a^6*b^3*x^2 +
 2*a^3*b^6*x + b^9)*log(a*x^(1/3) + b) - 3*(3*a^8*b*x^2 + 16*a^5*b^4*x + 1
0*a^2*b^7)*x^(2/3) + 3*(12*a^7*b^2*x^2 + 35*a^4*b^5*x + 20*a*b^8)*x^(1/3))
/(a^12*x^2 + 2*a^9*b^3*x + a^6*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(87) = 174$.

Time = 0.33 (sec) , antiderivative size = 362, normalized size of antiderivative = 4.02

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \begin{cases} \frac{2a^5x^{\frac{5}{3}}}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2} - \frac{5a^4bx^{\frac{4}{3}}}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2} + \frac{20a^3b^2x}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2} - \frac{60a^2b^3x^{\frac{2}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}+2a^6b^2} - \frac{120ab^4\sqrt[3]{x}\log\left(\cdots\right.}{2a^8x^{\frac{2}{3}}+4a^7b\sqrt[3]{x}\cdots} \\ \frac{x^2}{2b^3} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**3,x)
```

output
```
Piecewise((2*a**5*x**(5/3)/(2*a**8*x**(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b
**2) - 5*a**4*b*x**(4/3)/(2*a**8*x**(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**
2) + 20*a**3*b**2*x/(2*a**8*x**(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**2) -
60*a**2*b**3*x**(2/3)*log(x**(1/3) + b/a)/(2*a**8*x**(2/3) + 4*a**7*b*x**(
1/3) + 2*a**6*b**2) - 120*a*b**4*x**(1/3)*log(x**(1/3) + b/a)/(2*a**8*x**(
2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**2) - 120*a*b**4*x**(1/3)/(2*a**8*x**(
2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**2) - 60*b**5*log(x**(1/3) + b/a)/(2*a
**8*x**(2/3) + 4*a**7*b*x**(1/3) + 2*a**6*b**2) - 90*b**5/(2*a**8*x**(2/3)
 + 4*a**7*b*x**(1/3) + 2*a**6*b**2), Ne(a, 0)), (x**2/(2*b**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{2\,a^4-\frac{5\,a^3b}{x^{\frac{1}{3}}}+\frac{20\,a^2b^2}{x^{\frac{2}{3}}}+\frac{90\,ab^3}{x}+\frac{60\,b^4}{x^{\frac{4}{3}}}}{2\left(\frac{a^7}{x}+\frac{2\,a^6b}{x^{\frac{4}{3}}}+\frac{a^5b^2}{x^{\frac{5}{3}}}\right)} - \frac{30\,b^3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^6} - \frac{10\,b^3\log(x)}{a^6}$$

input
```
integrate(1/(a+b/x^(1/3))^3,x, algorithm="maxima")
```

output
```
1/2*(2*a^4 - 5*a^3*b/x^(1/3) + 20*a^2*b^2/x^(2/3) + 90*a*b^3/x + 60*b^4/x^
(4/3))/(a^7/x + 2*a^6*b/x^(4/3) + a^5*b^2/x^(5/3)) - 30*b^3*log(a + b/x^(1
/3))/a^6 - 10*b^3*log(x)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = -\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^6} - \frac{3\left(10\,ab^4x^{\frac{1}{3}}+9\,b^5\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^6} + \frac{2\,a^6x-9\,a^5bx^{\frac{2}{3}}+36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9}$$

input
```
integrate(1/(a+b/x^(1/3))^3,x, algorithm="giac")
```

output
```
-30*b^3*log(abs(a*x^(1/3) + b))/a^6 - 3/2*(10*a*b^4*x^(1/3) + 9*b^5)/((a*x
^(1/3) + b)^2*a^6) + 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/
a^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{x}{a^3} - \frac{\frac{27\,b^5}{2\,a} + 15\,b^4\,x^{1/3}}{a^5\,b^2 + a^7\,x^{2/3} + 2\,a^6\,b\,x^{1/3}} - \frac{9\,b\,x^{2/3}}{2\,a^4} - \frac{30\,b^3\,\ln\left(b + a\,x^{1/3}\right)}{a^6} + \frac{18\,b^2\,x^{1/3}}{a^5}$$

input
```
int(1/(a + b/x^(1/3))^3,x)
```

output
```
x/a^3 - ((27*b^5)/(2*a) + 15*b^4*x^(1/3))/(a^5*b^2 + a^7*x^(2/3) + 2*a^6*b
*x^(1/3)) - (9*b*x^(2/3))/(2*a^4) - (30*b^3*log(b + a*x^(1/3)))/a^6 + (18*
b^2*x^(1/3))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}\,dx = \frac{-60x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2b^3 + 2x^{\frac{5}{3}}a^5 + 60x^{\frac{2}{3}}a^2b^3 - 120x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a\,b^4 - 5x^{\frac{4}{3}}a^4b - 60\log\left(x^{\frac{1}{3}}a+b\right)b}{2a^6\left(x^{\frac{2}{3}}a^2 + 2x^{\frac{1}{3}}ab + b^2\right)}$$

input
```
int(1/(a+b/x^(1/3))^3,x)
```

output
```
( - 60*x**(2/3)*log(x**(1/3)*a + b)*a**2*b**3 + 2*x**(2/3)*a**5*x + 60*x**
(2/3)*a**2*b**3 - 120*x**(1/3)*log(x**(1/3)*a + b)*a*b**4 - 5*x**(1/3)*a**
4*b*x - 60*log(x**(1/3)*a + b)*b**5 + 20*a**3*b**2*x - 30*b**5)/(2*a**6*(x
**(2/3)*a**2 + 2*x**(1/3)*a*b + b**2))
```

3.329 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx=-\frac{3b^2}{2a^3\left(b+a\sqrt[3]{x}\right)^2}+\frac{6b}{a^3\left(b+a\sqrt[3]{x}\right)}+\frac{3\log\left(b+a\sqrt[3]{x}\right)}{a^3}$$

output
```
-3/2*b^2/a^3/(b+a*x^(1/3))^2+6*b/a^3/(b+a*x^(1/3))+3*ln(b+a*x^(1/3))/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx=\frac{3b\left(3b+4a\sqrt[3]{x}\right)}{2a^3\left(b+a\sqrt[3]{x}\right)^2}+\frac{3\log\left(b+a\sqrt[3]{x}\right)}{a^3}$$

input
```
Integrate[1/((a + b/x^(1/3))^3*x),x]
```

output
```
(3*b*(3*b + 4*a*x^(1/3)))/(2*a^3*(b + a*x^(1/3))^2) + (3*Log[b + a*x^(1/3)
])/a^3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}dx$$

$$\Big\downarrow \ 795$$

$$\int \frac{1}{\left(a\sqrt[3]{x}+b\right)^3}dx$$

$$\Big\downarrow \ 774$$

$$3\int \frac{x^{2/3}}{\left(\sqrt[3]{x}a+b\right)^3}d\sqrt[3]{x}$$

$$\Big\downarrow \ 49$$

$$3\int \left(\frac{b^2}{a^2\left(\sqrt[3]{x}a+b\right)^3}-\frac{2b}{a^2\left(\sqrt[3]{x}a+b\right)^2}+\frac{1}{a^2\left(\sqrt[3]{x}a+b\right)}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \ 2009$$

$$3\left(-\frac{b^2}{2a^3\left(a\sqrt[3]{x}+b\right)^2}+\frac{2b}{a^3\left(a\sqrt[3]{x}+b\right)}+\frac{\log\left(a\sqrt[3]{x}+b\right)}{a^3}\right)$$

input
```
Int[1/((a + b/x^(1/3))^3*x),x]
```

output
```
3*(-1/2*b^2/(a^3*(b + a*x^(1/3))^2) + (2*b)/(a^3*(b + a*x^(1/3))) + Log[b
+ a*x^(1/3)]/a^3)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{3\ln\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^3}-\frac{3}{a^2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}-\frac{3}{2a\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2}+\frac{\ln(x)}{a^3}$
default	$-\frac{b^6}{(a^3x+b^3)^2a^3}+\frac{2b^3}{a^3(a^3x+b^3)}+\frac{\ln(a^3x+b^3)}{a^3}-3a\,b^5\left(-\frac{\frac{-b^2x^{\frac{2}{3}}+\frac{b^3x^{\frac{1}{3}}}{a}+\frac{b^4}{2a^2}}{\left(x^{\frac{2}{3}}a^2-ab\,x^{\frac{1}{3}}+b^2\right)^2}+\frac{\ln\left(x^{\frac{2}{3}}a^2-ab\,x^{\frac{1}{3}}+b^2\right)}{2a}-\frac{\sqrt{3}\,\mathrm{ar}}{a}}{9a^2b^5}\right.$

input
```
int(1/(a+b/x^(1/3))^3/x,x,method=_RETURNVERBOSE)
```

output
```
3/a^3*ln(a+b/x^(1/3))-3/a^2/(a+b/x^(1/3))-3/2/a/(a+b/x^(1/3))^2+ln(x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(46) = 92.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.09

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx = \frac{3\left(6a^3b^3x+3b^6+2\left(a^6x^2+2a^3b^3x+b^6\right)\log\left(ax^{\frac{1}{3}}+b\right)+\left(4a^5bx+a^2b^4\right)x^{\frac{2}{3}}-\left(5a^4b^2x+2ab^5\right)x^{\frac{1}{3}}\right)}{2\left(a^9x^2+2a^6b^3x+a^3b^6\right)}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x,x, algorithm="fricas")
```

output
```
3/2*(6*a^3*b^3*x + 3*b^6 + 2*(a^6*x^2 + 2*a^3*b^3*x + b^6)*log(a*x^(1/3) +
 b) + (4*a^5*b*x + a^2*b^4)*x^(2/3) - (5*a^4*b^2*x + 2*a*b^5)*x^(1/3))/(a^
9*x^2 + 2*a^6*b^3*x + a^3*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(49) = 98.

Time = 0.72 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.44

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx = \begin{cases} \frac{6a^2x^{\frac{4}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}} + \frac{12abx\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}} + \frac{12abx}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}} + \frac{6b^2x^{\frac{2}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b^2x^{\frac{2}{3}}} + \frac{9b^2x^{\frac{2}{3}}}{2a^5x^{\frac{4}{3}}+4a^4bx+2a^3b} \\ \frac{x}{b^3} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**3/x,x)
```

output
```
Piecewise((6*a**2*x**(4/3)*log(x**(1/3) + b/a)/(2*a**5*x**(4/3) + 4*a**4*b
*x + 2*a**3*b**2*x**(2/3)) + 12*a*b*x*log(x**(1/3) + b/a)/(2*a**5*x**(4/3)
 + 4*a**4*b*x + 2*a**3*b**2*x**(2/3)) + 12*a*b*x/(2*a**5*x**(4/3) + 4*a**4
*b*x + 2*a**3*b**2*x**(2/3)) + 6*b**2*x**(2/3)*log(x**(1/3) + b/a)/(2*a**5
*x**(4/3) + 4*a**4*b*x + 2*a**3*b**2*x**(2/3)) + 9*b**2*x**(2/3)/(2*a**5*x
**(4/3) + 4*a**4*b*x + 2*a**3*b**2*x**(2/3)), Ne(a, 0)), (x/b**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx = -\frac{3\left(3\,a+\frac{2\,b}{x^{\frac{1}{3}}}\right)}{2\left(a^4+\frac{2\,a^3b}{x^{\frac{1}{3}}}+\frac{a^2b^2}{x^{\frac{2}{3}}}\right)}+\frac{3\,\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{a^3}+\frac{\log\left(x\right)}{a^3}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x,x, algorithm="maxima")
```

output
```
-3/2*(3*a + 2*b/x^(1/3))/(a^4 + 2*a^3*b/x^(1/3) + a^2*b^2/x^(2/3)) + 3*log
(a + b/x^(1/3))/a^3 + log(x)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx = \frac{3\,\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^3}+\frac{3\left(4\,bx^{\frac{1}{3}}+\frac{3\,b^2}{a}\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^2}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x,x, algorithm="giac")
```

output
```
3*log(abs(a*x^(1/3) + b))/a^3 + 3/2*(4*b*x^(1/3) + 3*b^2/a)/((a*x^(1/3) +
b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx = \frac{\frac{9\,b^2}{2\,a^3}+\frac{6\,b\,x^{1/3}}{a^2}}{b^2+a^2\,x^{2/3}+2\,a\,b\,x^{1/3}}+\frac{3\,\ln\left(b+a\,x^{1/3}\right)}{a^3}$$

input
```
int(1/(x*(a + b/x^(1/3))^3),x)
```

output
```
((9*b^2)/(2*a^3) + (6*b*x^(1/3))/a^2)/(b^2 + a^2*x^(2/3) + 2*a*b*x^(1/3))
+ (3*log(b + a*x^(1/3)))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x}\,dx = \frac{3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2-3x^{\frac{2}{3}}a^2+6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)ab+3\log\left(x^{\frac{1}{3}}a+b\right)b^2+\frac{3b^2}{2}}{a^3\left(x^{\frac{2}{3}}a^2+2x^{\frac{1}{3}}ab+b^2\right)}$$

input
```
int(1/(a+b/x^(1/3))^3/x,x)
```

output
```
(3*(2*x**(2/3)*log(x**(1/3)*a + b)*a**2 - 2*x**(2/3)*a**2 + 4*x**(1/3)*log
(x**(1/3)*a + b)*a*b + 2*log(x**(1/3)*a + b)*b**2 + b**2))/(2*a**3*(x**(2/
3)*a**2 + 2*x**(1/3)*a*b + b**2))
```

3.330 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2} \, dx = \frac{3a^2}{2b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} - \frac{6a}{b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^3}$$

output
```
3/2*a^2/b^3/(a+b/x^(1/3))^2-6*a/b^3/(a+b/x^(1/3))-3*ln(a+b/x^(1/3))/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2} \, dx = \frac{3\left(\frac{b\left(3b+2a\sqrt[3]{x}\right)}{\left(b+a\sqrt[3]{x}\right)^2} - 2\log\left(b+a\sqrt[3]{x}\right) + \frac{2\log(x)}{3}\right)}{2b^3}$$

input
```
Integrate[1/((a + b/x^(1/3))^3*x^2),x]
```

output
```
(3*((b*(3*b + 2*a*x^(1/3)))/(b + a*x^(1/3))^2 - 2*Log[b + a*x^(1/3)] + (2*
Log[x])/3))/(2*b^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{\sqrt[3]{x}}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x \left(a \sqrt[3]{x}+b\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3 \int \frac{1}{\left(\sqrt[3]{x} a+b\right)^3 \sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3 \int \left(-\frac{a}{b^3 \left(\sqrt[3]{x} a+b\right)}-\frac{a}{b^2 \left(\sqrt[3]{x} a+b\right)^2}-\frac{a}{b \left(\sqrt[3]{x} a+b\right)^3}+\frac{1}{b^3 \sqrt[3]{x}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3 \left(-\frac{\log \left(a \sqrt[3]{x}+b\right)}{b^3}+\frac{1}{b^2 \left(a \sqrt[3]{x}+b\right)}+\frac{1}{2 b \left(a \sqrt[3]{x}+b\right)^2}+\frac{\log \left(\sqrt[3]{x}\right)}{b^3}\right)$$

input
```
Int[1/((a + b/x^(1/3))^3*x^2),x]
```

output
```
3*(1/(2*b*(b + a*x^(1/3))^2) + 1/(b^2*(b + a*x^(1/3))) - Log[b + a*x^(1/3)
]/b^3 + Log[x^(1/3)]/b^3)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{3\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{3}}+\frac{3}{b^{2}\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{3}{2b\left(b+a\,x^{\frac{1}{3}}\right)^{2}}+\frac{\ln\left(x\right)}{b^{3}}$	49
default	$-\frac{3\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{3}}+\frac{3}{b^{2}\left(b+a\,x^{\frac{1}{3}}\right)}+\frac{3}{2b\left(b+a\,x^{\frac{1}{3}}\right)^{2}}+\frac{\ln\left(x\right)}{b^{3}}$	49

input
```
int(1/(a+b/x^(1/3))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-3/b^3*ln(b+a*x^(1/3))+3/b^2/(b+a*x^(1/3))+3/2/b/(b+a*x^(1/3))^2+1/b^3*ln(
x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(46) = 92$.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.39

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx = \frac{3\left(3\,b^6-2\,(a^6x^2+2\,a^3b^3x+b^6)\log\left(ax^{\frac{1}{3}}+b\right)+2\,(a^6x^2+2\,a^3b^3x+b^6)\log\left(x^{\frac{1}{3}}\right)+(2\,a^5bx+5\,a^2b^4)x^{\frac{2}{3}}\right.}{2\,(a^6b^3x^2+2\,a^3b^6x+b^9)}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^2,x, algorithm="fricas")
```

output
```
3/2*(3*b^6 - 2*(a^6*x^2 + 2*a^3*b^3*x + b^6)*log(a*x^(1/3) + b) + 2*(a^6*x
^2 + 2*a^3*b^3*x + b^6)*log(x^(1/3)) + (2*a^5*b*x + 5*a^2*b^4)*x^(2/3) - (
a^4*b^2*x + 4*a*b^5)*x^(1/3))/(a^6*b^3*x^2 + 2*a^3*b^6*x + b^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(49) = 98$.

Time = 1.46 (sec) , antiderivative size = 406, normalized size of antiderivative = 7.52

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx = \begin{cases} \tilde{\infty}\log(x) \\ \frac{\log(x)}{b^3} \\ -\frac{1}{a^3x} \\ \frac{2a^2x^{\frac{7}{3}}\log(x)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}} - \frac{6a^2x^{\frac{7}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}} + \frac{4abx^2\log(x)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}} - \frac{12abx^2\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^3x^{\frac{7}{3}}+4ab^4x^2+2b^5x^{\frac{5}{3}}} + \frac{6abx^2}{2a^2b^3x^{\frac{7}{3}}+4ab^4x}\end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**3/x**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0)), (log(x)/b**3, Eq(a, 0)), (-1/
(a**3*x), Eq(b, 0)), (2*a**2*x**(7/3)*log(x)/(2*a**2*b**3*x**(7/3) + 4*a*b
**4*x**2 + 2*b**5*x**(5/3)) - 6*a**2*x**(7/3)*log(x**(1/3) + b/a)/(2*a**2*
b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) + 4*a*b*x**2*log(x)/(2*a*
*2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) - 12*a*b*x**2*log(x**(
1/3) + b/a)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) + 6*a
*b*x**2/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) + 2*b**2*
x**(5/3)*log(x)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2 + 2*b**5*x**(5/3)) -
 6*b**2*x**(5/3)*log(x**(1/3) + b/a)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**2
 + 2*b**5*x**(5/3)) + 9*b**2*x**(5/3)/(2*a**2*b**3*x**(7/3) + 4*a*b**4*x**
2 + 2*b**5*x**(5/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx = -\frac{3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^3} - \frac{6\,a}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^3} + \frac{3\,a^2}{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 b^3}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^2,x, algorithm="maxima")
```

output
```
-3*log(a + b/x^(1/3))/b^3 - 6*a/((a + b/x^(1/3))*b^3) + 3/2*a^2/((a + b/x^
(1/3))^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx = -\frac{3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^3} + \frac{\log\left(|x|\right)}{b^3} + \frac{3\left(2\,abx^{\frac{1}{3}}+3\,b^2\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2 b^3}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^2,x, algorithm="giac")
```

output
```
-3*log(abs(a*x^(1/3) + b))/b^3 + log(abs(x))/b^3 + 3/2*(2*a*b*x^(1/3) + 3*
b^2)/((a*x^(1/3) + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx = \frac{\frac{9}{2\,b}+\frac{3\,a\,x^{1/3}}{b^2}}{b^2+a^2\,x^{2/3}+2\,a\,b\,x^{1/3}} - \frac{6\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^3}$$

input
```
int(1/(x^2*(a + b/x^(1/3))^3),x)
```

output
```
(9/(2*b) + (3*a*x^(1/3))/b^2)/(b^2 + a^2*x^(2/3) + 2*a*b*x^(1/3)) - (6*ata
nh((2*a*x^(1/3))/b + 1))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.13

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^2}\,dx$$
$$= \frac{3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)a^2 - 3x^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^2 - \frac{3x^{\frac{2}{3}}a^2}{2} + 6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}\right)ab - 6x^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}a+b\right)ab + 3\log\left(x^{\frac{1}{3}}\right)b^2}{b^3\left(x^{\frac{2}{3}}a^2+2x^{\frac{1}{3}}ab+b^2\right)}$$

input
```
int(1/(a+b/x^(1/3))^3/x^2,x)
```

output
```
(3*(2*x**(2/3)*log(x**(1/3))*a**2 - 2*x**(2/3)*log(x**(1/3)*a + b)*a**2 -
x**(2/3)*a**2 + 4*x**(1/3)*log(x**(1/3))*a*b - 4*x**(1/3)*log(x**(1/3)*a +
 b)*a*b + 2*log(x**(1/3))*b**2 - 2*log(x**(1/3)*a + b)*b**2 + 2*b**2))/(2*
b**3*(x**(2/3)*a**2 + 2*x**(1/3)*a*b + b**2))
```

3.331 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3} \, dx = -\frac{3a^5}{2b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} + \frac{15a^4}{b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{1}{b^3x} + \frac{9a}{2b^4x^{2/3}} - \frac{18a^2}{b^5\sqrt[3]{x}} + \frac{30a^3\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^6}$$

output
```
-3/2*a^5/b^6/(a+b/x^(1/3))^2+15*a^4/b^6/(a+b/x^(1/3))-1/b^3/x+9/2*a/b^4/x^
(2/3)-18*a^2/b^5/x^(1/3)+30*a^3*ln(a+b/x^(1/3))/b^6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx = -\frac{\frac{b\left(2b^4-5ab^3\sqrt[3]{x}+20a^2b^2x^{2/3}+90a^3bx+60a^4x^{4/3}\right)}{\left(b+a\sqrt[3]{x}\right)^2 x}-60a^3\log\left(b+a\sqrt[3]{x}\right)+20a^3\log(x)}{2b^6}$$

input
```
Integrate[1/((a + b/x^(1/3))^3*x^3),x]
```

output
```
-1/2*((b*(2*b^4 - 5*a*b^3*x^(1/3) + 20*a^2*b^2*x^(2/3) + 90*a^3*b*x + 60*a
^4*x^(4/3)))/((b + a*x^(1/3))^2*x) - 60*a^3*Log[b + a*x^(1/3)] + 20*a^3*Lo
g[x])/b^6
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}dx$$

$$\Big\downarrow \text{795}$$

$$\int \frac{1}{x^2\left(a\sqrt[3]{x}+b\right)^3}dx$$

$$\Big\downarrow \text{798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)^3 x^{4/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \color{blue}{54}$$

$$3\int \left(\frac{10a^4}{b^6\left(\sqrt[3]{x}a+b\right)}+\frac{4a^4}{b^5\left(\sqrt[3]{x}a+b\right)^2}+\frac{a^4}{b^4\left(\sqrt[3]{x}a+b\right)^3}-\frac{10a^3}{b^6\sqrt[3]{x}}+\frac{6a^2}{b^5x^{2/3}}-\frac{3a}{b^4x}+\frac{1}{b^3x^{4/3}}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \color{blue}{2009}$$

$$3\left(\frac{10a^3\log\left(a\sqrt[3]{x}+b\right)}{b^6}-\frac{10a^3\log\left(\sqrt[3]{x}\right)}{b^6}-\frac{4a^3}{b^5\left(a\sqrt[3]{x}+b\right)}-\frac{a^3}{2b^4\left(a\sqrt[3]{x}+b\right)^2}-\frac{6a^2}{b^5\sqrt[3]{x}}+\frac{3a}{2b^4x^{2/3}}-\frac{1}{3b^3x}\right)$$

input
```
Int[1/((a + b/x^(1/3))^3*x^3),x]
```

output
```
3*(-1/2*a^3/(b^4*(b + a*x^(1/3))^2) - (4*a^3)/(b^5*(b + a*x^(1/3))) - 1/(3
*b^3*x) + (3*a)/(2*b^4*x^(2/3)) - (6*a^2)/(b^5*x^(1/3)) + (10*a^3*Log[b +
a*x^(1/3)])/b^6 - (10*a^3*Log[x^(1/3)])/b^6)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{3a^{3}}{2b^{4}\left(b+a\,x^{\frac{1}{3}}\right)^{2}}+\frac{30a^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{6}}-\frac{12a^{3}}{b^{5}\left(b+a\,x^{\frac{1}{3}}\right)}-\frac{1}{b^{3}x}-\frac{10a^{3}\ln\left(x\right)}{b^{6}}-\frac{18a^{2}}{b^{5}x^{\frac{1}{3}}}+\frac{9a}{2b^{4}x^{\frac{2}{3}}}$	90
default	$-\frac{3a^{3}}{2b^{4}\left(b+a\,x^{\frac{1}{3}}\right)^{2}}+\frac{30a^{3}\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^{6}}-\frac{12a^{3}}{b^{5}\left(b+a\,x^{\frac{1}{3}}\right)}-\frac{1}{b^{3}x}-\frac{10a^{3}\ln\left(x\right)}{b^{6}}-\frac{18a^{2}}{b^{5}x^{\frac{1}{3}}}+\frac{9a}{2b^{4}x^{\frac{2}{3}}}$	90

input
```
int(1/(a+b/x^(1/3))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/2*a^3/b^4/(b+a*x^(1/3))^2+30/b^6*a^3*ln(b+a*x^(1/3))-12/b^5*a^3/(b+a*x^
(1/3))-1/b^3/x-10/b^6*a^3*ln(x)-18*a^2/b^5/x^(1/3)+9/2*a/b^4/x^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(79) = 158.

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.05

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^{3}x^{3}}\,dx = -\frac{20\,a^{6}b^{3}x^{2}+31\,a^{3}b^{6}x+2\,b^{9}-60\,(a^{9}x^{3}+2\,a^{6}b^{3}x^{2}+a^{3}b^{6}x)\log\left(ax^{\frac{1}{3}}+b\right)+60\,(a^{9}x^{3}+2\,a^{6}b^{3}x^{2}+a^{3}b^{}}{2\,(a^{6}b^{6}x^{3}+2\,a^{3}b^{9}x^{2}}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^3,x, algorithm="fricas")
```

output
```
-1/2*(20*a^6*b^3*x^2 + 31*a^3*b^6*x + 2*b^9 - 60*(a^9*x^3 + 2*a^6*b^3*x^2
+ a^3*b^6*x)*log(a*x^(1/3) + b) + 60*(a^9*x^3 + 2*a^6*b^3*x^2 + a^3*b^6*x)
*log(x^(1/3)) + 3*(20*a^8*b*x^2 + 35*a^5*b^4*x + 12*a^2*b^7)*x^(2/3) - 3*(
10*a^7*b^2*x^2 + 16*a^4*b^5*x + 3*a*b^8)*x^(1/3))/(a^6*b^6*x^3 + 2*a^3*b^9
*x^2 + b^12*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(88) = 176$.

Time = 4.20 (sec) , antiderivative size = 561, normalized size of antiderivative = 6.03

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x} \\ -\frac{1}{b^3x} \\ -\frac{1}{2a^3x^2} \\ -\frac{20a^5x^{\frac{10}{3}}\log(x)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}} + \frac{60a^5x^{\frac{10}{3}}\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}} - \frac{40a^4bx^3\log(x)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}} + \frac{120a^4bx^3\log\left(\sqrt[3]{x}+\frac{b}{a}\right)}{2a^2b^6x^{\frac{10}{3}}+4ab^7x^3+2b^8x^{\frac{8}{3}}} - \frac{6}{2a^2b^6x^{\frac{10}{3}}} \end{cases}$$

input
```
integrate(1/(a+b/x**(1/3))**3/x**3,x)
```

output
```
Piecewise((zoo/x, Eq(a, 0) & Eq(b, 0)), (-1/(b**3*x), Eq(a, 0)), (-1/(2*a*
*3*x**2), Eq(b, 0)), (-20*a**5*x**(10/3)*log(x)/(2*a**2*b**6*x**(10/3) + 4
*a*b**7*x**3 + 2*b**8*x**(8/3)) + 60*a**5*x**(10/3)*log(x**(1/3) + b/a)/(2
*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) - 40*a**4*b*x**3*l
og(x)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) + 120*a**4
*b*x**3*log(x**(1/3) + b/a)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**
8*x**(8/3)) - 60*a**4*b*x**3/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b*
*8*x**(8/3)) - 20*a**3*b**2*x**(8/3)*log(x)/(2*a**2*b**6*x**(10/3) + 4*a*b
**7*x**3 + 2*b**8*x**(8/3)) + 60*a**3*b**2*x**(8/3)*log(x**(1/3) + b/a)/(2
*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) - 90*a**3*b**2*x**
(8/3)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) - 20*a**2*
b**3*x**(7/3)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) +
5*a*b**4*x**2/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)) -
2*b**5*x**(5/3)/(2*a**2*b**6*x**(10/3) + 4*a*b**7*x**3 + 2*b**8*x**(8/3)),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx = \frac{30\,a^3\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^6} - \frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3}{b^6} + \frac{15\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a}{2\,b^6} - \frac{30\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^2}{b^6} + \frac{15\,a^4}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^6} - \frac{3\,a^5}{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 b^6}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^3,x, algorithm="maxima")
```

output
```
30*a^3*log(a + b/x^(1/3))/b^6 - (a + b/x^(1/3))^3/b^6 + 15/2*(a + b/x^(1/3
))^2*a/b^6 - 30*(a + b/x^(1/3))*a^2/b^6 + 15*a^4/((a + b/x^(1/3))*b^6) - 3
/2*a^5/((a + b/x^(1/3))^2*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx = \frac{30\,a^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^6} - \frac{10\,a^3\log\left(|x|\right)}{b^6} - \frac{60\,a^4bx^{\frac{4}{3}}+90\,a^3b^2x+20\,a^2b^3x^{\frac{2}{3}}-5\,ab^4x^{\frac{1}{3}}+2\,b^5}{2\left(ax^{\frac{1}{3}}+b\right)^2 b^6x}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^3,x, algorithm="giac")
```

output
```
30*a^3*log(abs(a*x^(1/3) + b))/b^6 - 10*a^3*log(abs(x))/b^6 - 1/2*(60*a^4*
b*x^(4/3) + 90*a^3*b^2*x + 20*a^2*b^3*x^(2/3) - 5*a*b^4*x^(1/3) + 2*b^5)/(
(a*x^(1/3) + b)^2*b^6*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx = \frac{60\,a^3\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^6} - \frac{\frac{1}{b}-\frac{5\,a\,x^{1/3}}{2\,b^2}+\frac{45\,a^3\,x}{b^4}+\frac{10\,a^2\,x^{2/3}}{b^3}+\frac{30\,a^4\,x^{4/3}}{b^5}}{b^2\,x+a^2\,x^{5/3}+2\,a\,b\,x^{4/3}}$$

input
```
int(1/(x^3*(a + b/x^(1/3))^3),x)
```

output
```
(60*a^3*atanh((2*a*x^(1/3))/b + 1))/b^6 - (1/b - (5*a*x^(1/3))/(2*b^2) + (
45*a^3*x)/b^4 + (10*a^2*x^(2/3))/b^3 + (30*a^4*x^(4/3))/b^5)/(b^2*x + a^2*
x^(5/3) + 2*a*b*x^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^3}\,dx = \frac{-60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}\right)a^5+60x^{\frac{5}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^5+30x^{\frac{5}{3}}a^5-20x^{\frac{2}{3}}a^2b^3-120x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}\right)a^4b+120x^{\frac{4}{3}}\log\left(x^{\frac{1}{3}}a\right.}{2b^6x\left(x^{\frac{2}{3}}a^2+2x^{\frac{1}{3}}ab+b\right.}$$

input
```
int(1/(a+b/x^(1/3))^3/x^3,x)
```

output
```
( - 60*x**(2/3)*log(x**(1/3))*a**5*x + 60*x**(2/3)*log(x**(1/3)*a + b)*a**
5*x + 30*x**(2/3)*a**5*x - 20*x**(2/3)*a**2*b**3 - 120*x**(1/3)*log(x**(1/
3))*a**4*b*x + 120*x**(1/3)*log(x**(1/3)*a + b)*a**4*b*x + 5*x**(1/3)*a*b*
*4 - 60*log(x**(1/3))*a**3*b**2*x + 60*log(x**(1/3)*a + b)*a**3*b**2*x - 6
0*a**3*b**2*x - 2*b**5)/(2*b**6*x*(x**(2/3)*a**2 + 2*x**(1/3)*a*b + b**2))
```

3.332 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4} \, dx = \frac{3a^8}{2b^9\left(a+\frac{b}{\sqrt[3]{x}}\right)^2} - \frac{24a^7}{b^9\left(a+\frac{b}{\sqrt[3]{x}}\right)} - \frac{1}{2b^3x^2} + \frac{9a}{5b^4x^{5/3}} - \frac{9a^2}{2b^5x^{4/3}} + \frac{10a^3}{b^6x} - \frac{45a^4}{2b^7x^{2/3}} + \frac{63a^5}{b^8\sqrt[3]{x}} - \frac{84a^6\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^9}$$

output
```
3/2*a^8/b^9/(a+b/x^(1/3))^2-24*a^7/b^9/(a+b/x^(1/3))-1/2/b^3/x^2+9/5*a/b^4
/x^(5/3)-9/2*a^2/b^5/x^(4/3)+10*a^3/b^6/x-45/2*a^4/b^7/x^(2/3)+63*a^5/b^8/
x^(1/3)-84*a^6*ln(a+b/x^(1/3))/b^9
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx$$
$$= \frac{\frac{b\left(-5b^7+8ab^6\sqrt[3]{x}-14a^2b^5x^{2/3}+28a^3b^4x-70a^4b^3x^{4/3}+280a^5b^2x^{5/3}+1260a^6bx^2+840a^7x^{7/3}\right)}{\left(b+a\sqrt[3]{x}\right)^2x^2}-840a^6\log\left(b+a\sqrt[3]{x}\right)+280a^6\,\mathrm{l}}{10b^9}$$

input
```
Integrate[1/((a + b/x^(1/3))^3*x^4),x]
```

output
```
((b*(-5*b^7 + 8*a*b^6*x^(1/3) - 14*a^2*b^5*x^(2/3) + 28*a^3*b^4*x - 70*a^4
*b^3*x^(4/3) + 280*a^5*b^2*x^(5/3) + 1260*a^6*b*x^2 + 840*a^7*x^(7/3)))/((
b + a*x^(1/3))^2*x^2) - 840*a^6*Log[b + a*x^(1/3)] + 280*a^6*Log[x])/(10*b
^9)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}dx$$
$$\Big\downarrow \text{ 795}$$
$$\int \frac{1}{x^3\left(a\sqrt[3]{x}+b\right)^3}dx$$
$$\Big\downarrow \text{ 798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)^3 x^{7/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(-\frac{28a^7}{b^9\left(\sqrt[3]{x}a+b\right)}-\frac{7a^7}{b^8\left(\sqrt[3]{x}a+b\right)^2}-\frac{a^7}{b^7\left(\sqrt[3]{x}a+b\right)^3}+\frac{28a^6}{b^9\sqrt[3]{x}}-\frac{21a^5}{b^8x^{2/3}}+\frac{15a^4}{b^7x}-\frac{10a^3}{b^6x^{4/3}}+\frac{6a^2}{b^5x^{5/3}}-\frac{3a}{b^4x^2}+\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(-\frac{28a^6\log\left(a\sqrt[3]{x}+b\right)}{b^9}+\frac{28a^6\log\left(\sqrt[3]{x}\right)}{b^9}+\frac{7a^6}{b^8\left(a\sqrt[3]{x}+b\right)}+\frac{a^6}{2b^7\left(a\sqrt[3]{x}+b\right)^2}+\frac{21a^5}{b^8\sqrt[3]{x}}-\frac{15a^4}{2b^7x^{2/3}}+\frac{10a^3}{3b^6x}-\frac{3a}{2b^5x}\right.$$

input
```
Int[1/((a + b/x^(1/3))^3*x^4),x]
```

output
```
3*(a^6/(2*b^7*(b + a*x^(1/3))^2) + (7*a^6)/(b^8*(b + a*x^(1/3))) - 1/(6*b^
3*x^2) + (3*a)/(5*b^4*x^(5/3)) - (3*a^2)/(2*b^5*x^(4/3)) + (10*a^3)/(3*b^6
*x) - (15*a^4)/(2*b^7*x^(2/3)) + (21*a^5)/(b^8*x^(1/3)) - (28*a^6*Log[b +
a*x^(1/3)])/b^9 + (28*a^6*Log[x^(1/3)])/b^9)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result
derivativedivides	$-\frac{1}{2b^3x^2}+\frac{28a^6\ln(x)}{b^9}+\frac{63a^5}{b^8x^{\frac{1}{3}}}-\frac{45a^4}{2b^7x^{\frac{2}{3}}}+\frac{10a^3}{b^6x}-\frac{9a^2}{2b^5x^{\frac{4}{3}}}+\frac{9a}{5b^4x^{\frac{5}{3}}}-\frac{84a^6\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^9}+\frac{21a^6}{b^8\left(b+a\,x^{\frac{1}{3}}\right)}$
default	$-\frac{1}{2b^3x^2}+\frac{28a^6\ln(x)}{b^9}+\frac{63a^5}{b^8x^{\frac{1}{3}}}-\frac{45a^4}{2b^7x^{\frac{2}{3}}}+\frac{10a^3}{b^6x}-\frac{9a^2}{2b^5x^{\frac{4}{3}}}+\frac{9a}{5b^4x^{\frac{5}{3}}}-\frac{84a^6\ln\left(b+a\,x^{\frac{1}{3}}\right)}{b^9}+\frac{21a^6}{b^8\left(b+a\,x^{\frac{1}{3}}\right)}$

input
```
int(1/(a+b/x^(1/3))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3/x^2+28/b^9*a^6*ln(x)+63*a^5/b^8/x^(1/3)-45/2*a^4/b^7/x^(2/3)+10*a
^3/b^6/x-9/2*a^2/b^5/x^(4/3)+9/5*a/b^4/x^(5/3)-84/b^9*a^6*ln(b+a*x^(1/3))+
21/b^8*a^6/(b+a*x^(1/3))+3/2*a^6/b^7/(b+a*x^(1/3))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(112) = 224.

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3x^4}\,dx$$
$$=\frac{280\,a^9b^3x^3+420\,a^6b^6x^2+90\,a^3b^9x-5\,b^{12}-840\,(a^{12}x^4+2\,a^9b^3x^3+a^6b^6x^2)\log\left(ax^{\frac{1}{3}}+b\right)+840\,(a^{12}x^4}{}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^4,x, algorithm="fricas")
```

output
```
1/10*(280*a^9*b^3*x^3 + 420*a^6*b^6*x^2 + 90*a^3*b^9*x - 5*b^12 - 840*(a^1
2*x^4 + 2*a^9*b^3*x^3 + a^6*b^6*x^2)*log(a*x^(1/3) + b) + 840*(a^12*x^4 +
2*a^9*b^3*x^3 + a^6*b^6*x^2)*log(x^(1/3)) + 15*(56*a^11*b*x^3 + 98*a^8*b^4
*x^2 + 36*a^5*b^7*x - 3*a^2*b^10)*x^(2/3) - 3*(140*a^10*b^2*x^3 + 224*a^7*
b^5*x^2 + 63*a^4*b^8*x - 6*a*b^11)*x^(1/3))/(a^6*b^9*x^4 + 2*a^3*b^12*x^3
+ b^15*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(133) = 266$.

Time = 7.71 (sec) , antiderivative size = 707, normalized size of antiderivative = 5.20

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**(1/3))**3/x**4,x)
```

output
```
Piecewise((zoo/x**2, Eq(a, 0) & Eq(b, 0)), (-1/(2*b**3*x**2), Eq(a, 0)), (
-1/(3*a**3*x**3), Eq(b, 0)), (280*a**8*x**(13/3)*log(x)/(10*a**2*b**9*x**(
13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) - 840*a**8*x**(13/3)*log(x**
(1/3) + b/a)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3
)) + 560*a**7*b*x**4*log(x)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10
*b**11*x**(11/3)) - 1680*a**7*b*x**4*log(x**(1/3) + b/a)/(10*a**2*b**9*x**
(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 840*a**7*b*x**4/(10*a**2*
b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 280*a**6*b**2*x**
(11/3)*log(x)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/
3)) - 840*a**6*b**2*x**(11/3)*log(x**(1/3) + b/a)/(10*a**2*b**9*x**(13/3)
+ 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 1260*a**6*b**2*x**(11/3)/(10*a**
2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) + 280*a**5*b**3*x
**(10/3)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) -
 70*a**4*b**4*x**3/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x*
*(11/3)) + 28*a**3*b**5*x**(8/3)/(10*a**2*b**9*x**(13/3) + 20*a*b**10*x**4
 + 10*b**11*x**(11/3)) - 14*a**2*b**6*x**(7/3)/(10*a**2*b**9*x**(13/3) + 2
0*a*b**10*x**4 + 10*b**11*x**(11/3)) + 8*a*b**7*x**2/(10*a**2*b**9*x**(13/
3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)) - 5*b**8*x**(5/3)/(10*a**2*b**9
*x**(13/3) + 20*a*b**10*x**4 + 10*b**11*x**(11/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx = -\frac{84\,a^6\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^9} - \frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6}{2\,b^9} + \frac{24\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a}{5\,b^9} - \frac{21\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^2}{b^9} + \frac{56\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^3}{b^9} - \frac{105\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^4}{b^9} + \frac{168\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^5}{b^9} - \frac{24\,a^7}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^9} + \frac{3\,a^8}{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 b^9}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^4,x, algorithm="maxima")
```

output
```
-84*a^6*log(a + b/x^(1/3))/b^9 - 1/2*(a + b/x^(1/3))^6/b^9 + 24/5*(a + b/x
^(1/3))^5*a/b^9 - 21*(a + b/x^(1/3))^4*a^2/b^9 + 56*(a + b/x^(1/3))^3*a^3/
b^9 - 105*(a + b/x^(1/3))^2*a^4/b^9 + 168*(a + b/x^(1/3))*a^5/b^9 - 24*a^7
/((a + b/x^(1/3))*b^9) + 3/2*a^8/((a + b/x^(1/3))^2*b^9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx = -\frac{84\,a^6\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^9} + \frac{28\,a^6\log\left(|x|\right)}{b^9} + \frac{840\,a^7bx^{\frac{7}{3}} + 1260\,a^6b^2x^2 + 280\,a^5b^3x^{\frac{5}{3}} - 70\,a^4b^4x^{\frac{4}{3}} + 28\,a^3b^5x - 14\,a^2b^6x^{\frac{2}{3}} + 8\,ab^7x^{\frac{1}{3}} - 5\,b^8}{10\left(ax^{\frac{1}{3}}+b\right)^2 b^9x^2}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^4,x, algorithm="giac")
```

output
```
-84*a^6*log(abs(a*x^(1/3) + b))/b^9 + 28*a^6*log(abs(x))/b^9 + 1/10*(840*a
^7*b*x^(7/3) + 1260*a^6*b^2*x^2 + 280*a^5*b^3*x^(5/3) - 70*a^4*b^4*x^(4/3)
 + 28*a^3*b^5*x - 14*a^2*b^6*x^(2/3) + 8*a*b^7*x^(1/3) - 5*b^8)/((a*x^(1/3
) + b)^2*b^9*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx = \frac{\frac{4\,a\,x^{1/3}}{5\,b^2}-\frac{1}{2\,b}+\frac{14\,a^3\,x}{5\,b^4}-\frac{7\,a^2\,x^{2/3}}{5\,b^3}+\frac{126\,a^6\,x^2}{b^7}-\frac{7\,a^4\,x^{4/3}}{b^5}+\frac{28\,a^5\,x^{5/3}}{b^6}+\frac{84\,a^7\,x^{7/3}}{b^8}}{a^2\,x^{8/3}+b^2\,x^2+2\,a\,b\,x^{7/3}} - \frac{168\,a^6\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^9}$$

input
```
int(1/(x^4*(a + b/x^(1/3))^3),x)
```

output
```
((4*a*x^(1/3))/(5*b^2) - 1/(2*b) + (14*a^3*x)/(5*b^4) - (7*a^2*x^(2/3))/(5
*b^3) + (126*a^6*x^2)/b^7 - (7*a^4*x^(4/3))/b^5 + (28*a^5*x^(5/3))/b^6 + (
84*a^7*x^(7/3))/b^8)/(a^2*x^(8/3) + b^2*x^2 + 2*a*b*x^(7/3)) - (168*a^6*at
anh((2*a*x^(1/3))/b + 1))/b^9
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^4}\,dx = \frac{840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}\right)a^8-840x^{\frac{8}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^8-420x^{\frac{8}{3}}a^8+280x^{\frac{5}{3}}a^5b^3-14x^{\frac{2}{3}}a^2b^6+1680x^{\frac{7}{3}}\log\left(x^{\frac{1}{3}}\right)a^7b-}{10}$$

input
```
int(1/(a+b/x^(1/3))^3/x^4,x)
```

output
```
(840*x**(2/3)*log(x**(1/3))*a**8*x**2 - 840*x**(2/3)*log(x**(1/3)*a + b)*a
**8*x**2 - 420*x**(2/3)*a**8*x**2 + 280*x**(2/3)*a**5*b**3*x - 14*x**(2/3)
*a**2*b**6 + 1680*x**(1/3)*log(x**(1/3))*a**7*b*x**2 - 1680*x**(1/3)*log(x
**(1/3)*a + b)*a**7*b*x**2 - 70*x**(1/3)*a**4*b**4*x + 8*x**(1/3)*a*b**7 +
 840*log(x**(1/3))*a**6*b**2*x**2 - 840*log(x**(1/3)*a + b)*a**6*b**2*x**2
 + 840*a**6*b**2*x**2 + 28*a**3*b**5*x - 5*b**8)/(10*b**9*x**2*(x**(2/3)*a
**2 + 2*x**(1/3)*a*b + b**2))
```

3.333 $\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 173

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5} \, dx = -\frac{3a^{11}}{2b^{12}\left(a+\frac{b}{\sqrt[3]{x}}\right)^2}+\frac{33a^{10}}{b^{12}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{1}{3b^3x^3}+\frac{9a}{8b^4x^{8/3}}-\frac{18a^2}{7b^5x^{7/3}}+\frac{5a^3}{b^6x^2}-\frac{9a^4}{b^7x^{5/3}}+\frac{63a^5}{4b^8x^{4/3}}-\frac{28a^6}{b^9x}+\frac{54a^7}{b^{10}x^{2/3}}-\frac{135a^8}{b^{11}\sqrt[3]{x}}+\frac{165a^9\log\left(a+\frac{b}{\sqrt[3]{x}}\right)}{b^{12}}$$

output

```
-3/2*a^11/b^12/(a+b/x^(1/3))^2+33*a^10/b^12/(a+b/x^(1/3))-1/3/b^3/x^3+9/8*
a/b^4/x^(8/3)-18/7*a^2/b^5/x^(7/3)+5*a^3/b^6/x^2-9*a^4/b^7/x^(5/3)+63/4*a^
5/b^8/x^(4/3)-28*a^6/b^9/x+54*a^7/b^10/x^(2/3)-135*a^8/b^11/x^(1/3)+165*a^
9*ln(a+b/x^(1/3))/b^12
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx = -\frac{\frac{b\left(56b^{10}-77ab^9\sqrt[3]{x}+110a^2b^8x^{2/3}-165a^3b^7x+264a^4b^6x^{4/3}-462a^5b^5x^{5/3}+924a^6b^4x^2-2310a^7b^3x^{7/3}+9240a^8b^2x^{8/3}+41580a^9bx^3+2772}{\left(b+a\sqrt[3]{x}\right)^2x^3}}{168b^{12}}$$

input
```
Integrate[1/((a + b/x^(1/3))^3*x^5),x]
```

output
```
-1/168*((b*(56*b^10 - 77*a*b^9*x^(1/3) + 110*a^2*b^8*x^(2/3) - 165*a^3*b^7
*x + 264*a^4*b^6*x^(4/3) - 462*a^5*b^5*x^(5/3) + 924*a^6*b^4*x^2 - 2310*a^
7*b^3*x^(7/3) + 9240*a^8*b^2*x^(8/3) + 41580*a^9*b*x^3 + 27720*a^10*x^(10/
3)))/((b + a*x^(1/3))^2*x^3) - 27720*a^9*Log[b + a*x^(1/3)] + 9240*a^9*Log
[x])/b^12
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {795, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+\frac{b}{\sqrt[3]{x}}\right)^3}dx$$

$$\Big\downarrow \text{ 795}$$

$$\int \frac{1}{x^4\left(a\sqrt[3]{x}+b\right)^3}dx$$

$$\Big\downarrow \text{ 798}$$

$$3\int \frac{1}{\left(\sqrt[3]{x}a+b\right)^3 x^{10/3}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{54}$$

$$3\int \left(\frac{55a^{10}}{b^{12}\left(\sqrt[3]{x}a+b\right)}+\frac{10a^{10}}{b^{11}\left(\sqrt[3]{x}a+b\right)^2}+\frac{a^{10}}{b^{10}\left(\sqrt[3]{x}a+b\right)^3}-\frac{55a^9}{b^{12}\sqrt[3]{x}}+\frac{45a^8}{b^{11}x^{2/3}}-\frac{36a^7}{b^{10}x}+\frac{28a^6}{b^9x^{4/3}}-\frac{21a^5}{b^8x^{5/3}}+\frac{15a}{b^7x}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{55a^9\log\left(a\sqrt[3]{x}+b\right)}{b^{12}}-\frac{55a^9\log\left(\sqrt[3]{x}\right)}{b^{12}}-\frac{10a^9}{b^{11}\left(a\sqrt[3]{x}+b\right)}-\frac{a^9}{2b^{10}\left(a\sqrt[3]{x}+b\right)^2}-\frac{45a^8}{b^{11}\sqrt[3]{x}}+\frac{18a^7}{b^{10}x^{2/3}}-\frac{28a^6}{3b^9x}+\frac{21}{4b^8}\right.$$

input
```
Int[1/((a + b/x^(1/3))^3*x^5),x]
```

output
```
3*(-1/2*a^9/(b^10*(b + a*x^(1/3))^2) - (10*a^9)/(b^11*(b + a*x^(1/3))) - 1
/(9*b^3*x^3) + (3*a)/(8*b^4*x^(8/3)) - (6*a^2)/(7*b^5*x^(7/3)) + (5*a^3)/(
3*b^6*x^2) - (3*a^4)/(b^7*x^(5/3)) + (21*a^5)/(4*b^8*x^(4/3)) - (28*a^6)/(
3*b^9*x) + (18*a^7)/(b^10*x^(2/3)) - (45*a^8)/(b^11*x^(1/3)) + (55*a^9*Log
[b + a*x^(1/3)])/b^12 - (55*a^9*Log[x^(1/3)])/b^12)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

method	result
derivativedivides	$-\frac{1}{3b^3x^3}-\frac{55a^9\ln(x)}{b^{12}}-\frac{135a^8}{b^{11}x^{\frac{1}{3}}}+\frac{54a^7}{b^{10}x^{\frac{2}{3}}}-\frac{28a^6}{b^9x}+\frac{63a^5}{4b^8x^{\frac{4}{3}}}-\frac{9a^4}{b^7x^{\frac{5}{3}}}+\frac{5a^3}{b^6x^2}-\frac{18a^2}{7b^5x^{\frac{7}{3}}}+\frac{9a}{8b^4x^{\frac{8}{3}}}-\frac{}{2b^{10}}$
default	$-\frac{1}{3b^3x^3}-\frac{55a^9\ln(x)}{b^{12}}-\frac{135a^8}{b^{11}x^{\frac{1}{3}}}+\frac{54a^7}{b^{10}x^{\frac{2}{3}}}-\frac{28a^6}{b^9x}+\frac{63a^5}{4b^8x^{\frac{4}{3}}}-\frac{9a^4}{b^7x^{\frac{5}{3}}}+\frac{5a^3}{b^6x^2}-\frac{18a^2}{7b^5x^{\frac{7}{3}}}+\frac{9a}{8b^4x^{\frac{8}{3}}}-\frac{}{2b^{10}}$

input
```
int(1/(a+b/x^(1/3))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^3/x^3-55/b^12*a^9*ln(x)-135*a^8/b^11/x^(1/3)+54*a^7/b^10/x^(2/3)-28
*a^6/b^9/x+63/4*a^5/b^8/x^(4/3)-9*a^4/b^7/x^(5/3)+5*a^3/b^6/x^2-18/7*a^2/b
^5/x^(7/3)+9/8*a/b^4/x^(8/3)-3/2*a^9/b^10/(b+a*x^(1/3))^2+165/b^12*a^9*ln(
b+a*x^(1/3))-30/b^11*a^9/(b+a*x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.52

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx =$$

$$-\frac{9240\,a^{12}b^3x^4+13860\,a^9b^6x^3+3080\,a^6b^9x^2-728\,a^3b^{12}x+56\,b^{15}-27720\,(a^{15}x^5+2\,a^{12}b^3x^4+a^9b^6x^3}{}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^5,x, algorithm="fricas")
```

output
```
-1/168*(9240*a^12*b^3*x^4 + 13860*a^9*b^6*x^3 + 3080*a^6*b^9*x^2 - 728*a^3
*b^12*x + 56*b^15 - 27720*(a^15*x^5 + 2*a^12*b^3*x^4 + a^9*b^6*x^3)*log(a*
x^(1/3) + b) + 27720*(a^15*x^5 + 2*a^12*b^3*x^4 + a^9*b^6*x^3)*log(x^(1/3)
) + 18*(1540*a^14*b*x^4 + 2695*a^11*b^4*x^3 + 990*a^8*b^7*x^2 - 99*a^5*b^1
0*x + 24*a^2*b^13)*x^(2/3) - 63*(220*a^13*b^2*x^4 + 352*a^10*b^5*x^3 + 99*
a^7*b^8*x^2 - 18*a^4*b^11*x + 3*a*b^14)*x^(1/3))/(a^6*b^12*x^5 + 2*a^3*b^1
5*x^4 + b^18*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(172) = 344$.

Time = 18.10 (sec) , antiderivative size = 848, normalized size of antiderivative = 4.90

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b/x**(1/3))**3/x**5,x)
```

output
```
Piecewise((zoo/x**3, Eq(a, 0) & Eq(b, 0)), (-1/(3*b**3*x**3), Eq(a, 0)), (
-1/(4*a**3*x**4), Eq(b, 0)), (-9240*a**11*x**(16/3)*log(x)/(168*a**2*b**12
*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) + 27720*a**11*x**(16/
3)*log(x**(1/3) + b/a)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*
b**14*x**(14/3)) - 18480*a**10*b*x**5*log(x)/(168*a**2*b**12*x**(16/3) + 3
36*a*b**13*x**5 + 168*b**14*x**(14/3)) + 55440*a**10*b*x**5*log(x**(1/3) +
 b/a)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3))
- 27720*a**10*b*x**5/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b*
*14*x**(14/3)) - 9240*a**9*b**2*x**(14/3)*log(x)/(168*a**2*b**12*x**(16/3)
 + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) + 27720*a**9*b**2*x**(14/3)*log
(x**(1/3) + b/a)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*
x**(14/3)) - 41580*a**9*b**2*x**(14/3)/(168*a**2*b**12*x**(16/3) + 336*a*b
**13*x**5 + 168*b**14*x**(14/3)) - 9240*a**8*b**3*x**(13/3)/(168*a**2*b**1
2*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) + 2310*a**7*b**4*x**
4/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) - 92
4*a**6*b**5*x**(11/3)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b
**14*x**(14/3)) + 462*a**5*b**6*x**(10/3)/(168*a**2*b**12*x**(16/3) + 336*
a*b**13*x**5 + 168*b**14*x**(14/3)) - 264*a**4*b**7*x**3/(168*a**2*b**12*x
**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) + 165*a**3*b**8*x**(8/3
)/(168*a**2*b**12*x**(16/3) + 336*a*b**13*x**5 + 168*b**14*x**(14/3)) -...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.14

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx = \frac{165\,a^9\log\left(a+\frac{b}{x^{\frac{1}{3}}}\right)}{b^{12}} - \frac{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^9}{3\,b^{12}} + \frac{33\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^8 a}{8\,b^{12}} - \frac{165\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^7 a^2}{7\,b^{12}} + \frac{165\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^6 a^3}{2\,b^{12}} - \frac{198\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^5 a^4}{b^{12}} + \frac{693\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^4 a^5}{2\,b^{12}} - \frac{462\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^3 a^6}{b^{12}} + \frac{495\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 a^7}{b^{12}} - \frac{495\left(a+\frac{b}{x^{\frac{1}{3}}}\right)a^8}{b^{12}} + \frac{33\,a^{10}}{\left(a+\frac{b}{x^{\frac{1}{3}}}\right)b^{12}} - \frac{3\,a^{11}}{2\left(a+\frac{b}{x^{\frac{1}{3}}}\right)^2 b^{12}}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^5,x, algorithm="maxima")
```

output
```
165*a^9*log(a + b/x^(1/3))/b^12 - 1/3*(a + b/x^(1/3))^9/b^12 + 33/8*(a + b
/x^(1/3))^8*a/b^12 - 165/7*(a + b/x^(1/3))^7*a^2/b^12 + 165/2*(a + b/x^(1/
3))^6*a^3/b^12 - 198*(a + b/x^(1/3))^5*a^4/b^12 + 693/2*(a + b/x^(1/3))^4*
a^5/b^12 - 462*(a + b/x^(1/3))^3*a^6/b^12 + 495*(a + b/x^(1/3))^2*a^7/b^12
 - 495*(a + b/x^(1/3))*a^8/b^12 + 33*a^10/((a + b/x^(1/3))*b^12) - 3/2*a^1
1/((a + b/x^(1/3))^2*b^12)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx = \frac{165\,a^9\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{b^{12}} - \frac{55\,a^9\log\left(|x|\right)}{b^{12}} - \frac{27720\,a^{10}bx^{\frac{10}{3}} + 41580\,a^9b^2x^3 + 9240\,a^8b^3x^{\frac{8}{3}} - 2310\,a^7b^4x^{\frac{7}{3}} + 924\,a^6b^5x^2 - 462\,a^5b^6x^{\frac{5}{3}} + 264\,a^4b^7x^{\frac{4}{3}} -}{168\left(ax^{\frac{1}{3}}+b\right)^2 b^{12}x^3}$$

input
```
integrate(1/(a+b/x^(1/3))^3/x^5,x, algorithm="giac")
```

output
```
165*a^9*log(abs(a*x^(1/3) + b))/b^12 - 55*a^9*log(abs(x))/b^12 - 1/168*(27
720*a^10*b*x^(10/3) + 41580*a^9*b^2*x^3 + 9240*a^8*b^3*x^(8/3) - 2310*a^7*
b^4*x^(7/3) + 924*a^6*b^5*x^2 - 462*a^5*b^6*x^(5/3) + 264*a^4*b^7*x^(4/3)
- 165*a^3*b^8*x + 110*a^2*b^9*x^(2/3) - 77*a*b^10*x^(1/3) + 56*b^11)/((a*x
^(1/3) + b)^2*b^12*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx = \frac{330\,a^9\,\mathrm{atanh}\left(\frac{2\,a\,x^{1/3}}{b}+1\right)}{b^{12}} - \frac{\frac{1}{3\,b}-\frac{11\,a\,x^{1/3}}{24\,b^2}-\frac{55\,a^3\,x}{56\,b^4}+\frac{55\,a^2\,x^{2/3}}{84\,b^3}+\frac{11\,a^6\,x^2}{2\,b^7}+\frac{11\,a^4\,x^{4/3}}{7\,b^5}-\frac{11\,a^5\,x^{5/3}}{4\,b^6}+\frac{495\,a^9\,x^3}{2\,b^{10}}-\frac{55\,a^7\,x^{7/3}}{4\,b^8}+\frac{55\,a^8\,x^{8/3}}{b^9}+\frac{165\,a^1}{b}}{a^2\,x^{11/3}+b^2\,x^3+2\,a\,b\,x^{10/3}}$$

input
```
int(1/(x^5*(a + b/x^(1/3))^3),x)
```

output
```
(330*a^9*atanh((2*a*x^(1/3))/b + 1))/b^12 - (1/(3*b) - (11*a*x^(1/3))/(24*
b^2) - (55*a^3*x)/(56*b^4) + (55*a^2*x^(2/3))/(84*b^3) + (11*a^6*x^2)/(2*b
^7) + (11*a^4*x^(4/3))/(7*b^5) - (11*a^5*x^(5/3))/(4*b^6) + (495*a^9*x^3)/
(2*b^10) - (55*a^7*x^(7/3))/(4*b^8) + (55*a^8*x^(8/3))/b^9 + (165*a^10*x^(
10/3))/b^11)/(a^2*x^(11/3) + b^2*x^3 + 2*a*b*x^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+\frac{b}{\sqrt[3]{x}}\right)^3 x^5}\,dx = \frac{-27720x^{\frac{11}{3}}\log\left(x^{\frac{1}{3}}\right)a^{11}+27720x^{\frac{11}{3}}\log\left(x^{\frac{1}{3}}a+b\right)a^{11}+13860x^{\frac{11}{3}}a^{11}-9240x^{\frac{8}{3}}a^8b^3+462x^{\frac{5}{3}}a^5b^6-110x}{}$$

input
```
int(1/(a+b/x^(1/3))^3/x^5,x)
```

output
```
( - 27720*x**(2/3)*log(x**(1/3))*a**11*x**3 + 27720*x**(2/3)*log(x**(1/3)*
a + b)*a**11*x**3 + 13860*x**(2/3)*a**11*x**3 - 9240*x**(2/3)*a**8*b**3*x*
*2 + 462*x**(2/3)*a**5*b**6*x - 110*x**(2/3)*a**2*b**9 - 55440*x**(1/3)*lo
g(x**(1/3))*a**10*b*x**3 + 55440*x**(1/3)*log(x**(1/3)*a + b)*a**10*b*x**3
 + 2310*x**(1/3)*a**7*b**4*x**2 - 264*x**(1/3)*a**4*b**7*x + 77*x**(1/3)*a
*b**10 - 27720*log(x**(1/3))*a**9*b**2*x**3 + 27720*log(x**(1/3)*a + b)*a*
*9*b**2*x**3 - 27720*a**9*b**2*x**3 - 924*a**6*b**5*x**2 + 165*a**3*b**8*x
 - 56*b**11)/(168*b**12*x**3*(x**(2/3)*a**2 + 2*x**(1/3)*a*b + b**2))
```

3.334 $\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}} \, dx = 3b^2\sqrt[3]{x} - \frac{3}{2}bx^{2/3} + x - 3b^3\log\left(b+\sqrt[3]{x}\right)$$

output
```
3*b^2*x^(1/3)-3/2*b*x^(2/3)+x-3*b^3*ln(b+x^(1/3))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}} \, dx = 3b^2\sqrt[3]{x} - \frac{3}{2}bx^{2/3} + x - 3b^3\log\left(b+\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + b/x^(1/3))^(-1),x]
```

output
```
3*b^2*x^(1/3) - (3*b*x^(2/3))/2 + x - 3*b^3*Log[b + x^(1/3)]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {774, 795, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{b}{\sqrt[3]{x}}+1}dx$$

$$\Big\downarrow \text{774}$$

$$3\int \frac{x^{2/3}}{\frac{b}{\sqrt[3]{x}}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \text{795}$$

$$3\int \frac{x}{b+\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$3\int \left(-\frac{b^3}{b+\sqrt[3]{x}}+b^2-\sqrt[3]{x}b+x^{2/3}\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(b^3\left(-\log\left(b+\sqrt[3]{x}\right)\right)+b^2\sqrt[3]{x}-\frac{1}{2}bx^{2/3}+\frac{x}{3}\right)$$

input
```
Int[(1 + b/x^(1/3))^(-1),x]
```

output
```
3*(b^2*x^(1/3) - (b*x^(2/3))/2 + x/3 - b^3*Log[b + x^(1/3)])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$3x^{\frac{1}{3}}b^2 - \frac{3bx^{\frac{2}{3}}}{2} + x - 3b^3\ln\left(b + x^{\frac{1}{3}}\right)$	28
default	$3x^{\frac{1}{3}}b^2 - \frac{3bx^{\frac{2}{3}}}{2} + x - 3b^3\ln\left(b + x^{\frac{1}{3}}\right)$	28

input
```
int(1/(1+b/x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)*b^2-3/2*b*x^(2/3)+x-3*b^3*ln(b+x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx = -3\,b^3\log\left(b+x^{\frac{1}{3}}\right)+3\,b^2x^{\frac{1}{3}}-\frac{3}{2}\,bx^{\frac{2}{3}}+x$$

input
```
integrate(1/(1+b/x^(1/3)),x, algorithm="fricas")
```

output
```
-3*b^3*log(b + x^(1/3)) + 3*b^2*x^(1/3) - 3/2*b*x^(2/3) + x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx = -3b^3\log\left(b+\sqrt[3]{x}\right)+3b^2\sqrt[3]{x}-\frac{3bx^{\frac{2}{3}}}{2}+x$$

input
```
integrate(1/(1+b/x**(1/3)),x)
```

output
```
-3*b**3*log(b + x**(1/3)) + 3*b**2*x**(1/3) - 3*b*x**(2/3)/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx = -b^3\log\left(x\right)-3\,b^3\log\left(\frac{b}{x^{\frac{1}{3}}}+1\right)+\frac{1}{2}\left(\frac{6\,b^2}{x^{\frac{2}{3}}}-\frac{3\,b}{x^{\frac{1}{3}}}+2\right)x$$

input
```
integrate(1/(1+b/x^(1/3)),x, algorithm="maxima")
```

output
```
-b^3*log(x) - 3*b^3*log(b/x^(1/3) + 1) + 1/2*(6*b^2/x^(2/3) - 3*b/x^(1/3)
+ 2)*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx = -3\,b^3\log\left(\left|b+x^{\frac{1}{3}}\right|\right)+3\,b^2x^{\frac{1}{3}}-\frac{3}{2}\,bx^{\frac{2}{3}}+x$$

input
```
integrate(1/(1+b/x^(1/3)),x, algorithm="giac")
```

output
```
-3*b^3*log(abs(b + x^(1/3))) + 3*b^2*x^(1/3) - 3/2*b*x^(2/3) + x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx = x-3\,b^3\,\ln\left(b+x^{1/3}\right)-\frac{3\,b\,x^{2/3}}{2}+3\,b^2\,x^{1/3}$$

input
```
int(1/(b/x^(1/3) + 1),x)
```

output
```
x - 3*b^3*log(b + x^(1/3)) - (3*b*x^(2/3))/2 + 3*b^2*x^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{1+\frac{b}{\sqrt[3]{x}}}\,dx = -\frac{3x^{\frac{2}{3}}b}{2}+3x^{\frac{1}{3}}b^2-3\log\left(x^{\frac{1}{3}}+b\right)b^3+x$$

input
```
int(1/(1+b/x^(1/3)),x)
```

output
```
( - 3*x**(2/3)*b + 6*x**(1/3)*b**2 - 6*log(x**(1/3) + b)*b**3 + 2*x)/2
```

3.335 $\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\frac{9}{25}\left(1+x^{5/3}\right)^{5/3}$$

output
```
9/25*(1+x^(5/3))^(5/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\frac{9}{25}\left(1+x^{5/3}\right)^{5/3}$$

input
```
Integrate[x^(2/3)*(1 + x^(5/3))^(2/3),x]
```

output
```
(9*(1 + x^(5/3))^(5/3))/25
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2/3}\left(x^{5/3}+1\right)^{2/3}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{9}{25}\left(x^{5/3}+1\right)^{5/3}$$

input
```
Int[x^(2/3)*(1 + x^(5/3))^(2/3),x]
```

output
```
(9*(1 + x^(5/3))^(5/3))/25
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{9\left(1+x^{\frac{5}{3}}\right)^{\frac{5}{3}}}{25}$	10
default	$\frac{9\left(1+x^{\frac{5}{3}}\right)^{\frac{5}{3}}}{25}$	10
meijerg	$\frac{3x^{\frac{5}{3}}\,\text{hypergeom}\left(\left[-\frac{2}{3},1\right],[2],-x^{\frac{5}{3}}\right)}{5}$	17

input
```
int(x^(2/3)*(1+x^(5/3))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
9/25*(1+x^(5/3))^(5/3)
```

Fricas [F(-1)]

Timed out.

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(x^(2/3)*(1+x^(5/3))^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\frac{9x^{\frac{5}{3}}\left(x^{\frac{5}{3}}+1\right)^{\frac{2}{3}}}{25}+\frac{9\left(x^{\frac{5}{3}}+1\right)^{\frac{2}{3}}}{25}$$

input
```
integrate(x**(2/3)*(1+x**(5/3))**(2/3),x)
```

output
```
9*x**(5/3)*(x**(5/3) + 1)**(2/3)/25 + 9*(x**(5/3) + 1)**(2/3)/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\frac{9}{25}\left(x^{\frac{5}{3}}+1\right)^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(1+x^(5/3))^(2/3),x, algorithm="maxima")
```

output
```
9/25*(x^(5/3) + 1)^(5/3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\frac{9}{25}\left(x^{\frac{5}{3}}+1\right)^{\frac{5}{3}}$$

input
```
integrate(x^(2/3)*(1+x^(5/3))^(2/3),x, algorithm="giac")
```

output
```
9/25*(x^(5/3) + 1)^(5/3)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\frac{9\,\left(x^{5/3}+1\right)^{5/3}}{25}$$

input
```
int(x^(2/3)*(x^(5/3) + 1)^(2/3),x)
```

output
```
(9*(x^(5/3) + 1)^(5/3))/25
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int x^{2/3}\left(1+x^{5/3}\right)^{2/3}\,dx=\frac{9\left(x^{\frac{5}{3}}+1\right)^{\frac{5}{3}}}{25}$$

input
```
int(x^(2/3)*(1+x^(5/3))^(2/3),x)
```

output
```
(9*(x**(2/3)*x + 1)**(2/3)*(x**(2/3)*x + 1))/25
```

3.336 $\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7} dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7} dx=-\frac{21}{260}\left(a^{10/3}-x^{10/3}\right)^{26/7}$$

output
```
-21/260*(a^(10/3)-x^(10/3))^(26/7)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7} dx=-\frac{21}{260}\left(a^{10/3}-x^{10/3}\right)^{26/7}$$

input
```
Integrate[x^(7/3)*(a^(10/3) - x^(10/3))^(19/7),x]
```

output
```
(-21*(a^(10/3) - x^(10/3))^(26/7))/260
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx$$

$$\Big\downarrow \; 793$$

$$-\frac{21}{260}\left(a^{10/3}-x^{10/3}\right)^{26/7}$$

input
```
Int[x^(7/3)*(a^(10/3) - x^(10/3))^(19/7),x]
```

output
```
(-21*(a^(10/3) - x^(10/3))^(26/7))/260
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{21\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{26}{7}}}{260}$	14
default	$-\frac{21\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{26}{7}}}{260}$	14

input
```
int(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x,method=_RETURNVERBOSE)
```

output
```
-21/260*(a^(10/3)-x^(10/3))^(26/7)
```

Fricas [F(-1)]

Timed out.

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx=\text{Timed out}$$

input
```
integrate(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx=\text{Timed out}$$

input
```
integrate(x**(7/3)*(a**(10/3)-x**(10/3))**(19/7),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx=-\frac{21}{260}\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{26}{7}}$$

input
```
integrate(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x, algorithm="maxima")
```

output
```
-21/260*(a^(10/3) - x^(10/3))^(26/7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx=-\frac{21}{260}\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{26}{7}}$$

input
```
integrate(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x, algorithm="giac")
```

output
```
-21/260*(a^(10/3) - x^(10/3))^(26/7)
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx=-\frac{21\left(a^{10/3}-x^{10/3}\right)^{26/7}}{260}$$

input
```
int(x^(7/3)*(a^(10/3) - x^(10/3))^(19/7),x)
```

output
```
-(21*(a^(10/3) - x^(10/3))^(26/7))/260
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int x^{7/3}\left(a^{10/3}-x^{10/3}\right)^{19/7}\,dx=\frac{-\frac{63x^{\frac{20}{3}}a^{\frac{20}{3}}}{130}-\frac{21a^{\frac{40}{3}}}{260}+\frac{21a^{\frac{10}{3}}x^{10}}{65}+\frac{21x^{\frac{10}{3}}a^{10}}{65}-\frac{21x^{\frac{40}{3}}}{260}}{\left(a^{\frac{10}{3}}-x^{\frac{10}{3}}\right)^{\frac{2}{7}}}$$

input
```
int(x^(7/3)*(a^(10/3)-x^(10/3))^(19/7),x)
```

output
```
(21*( - 6*x**(2/3)*a**(2/3)*a**6*x**6 - a**(1/3)*a**13 + 4*a**(1/3)*a**3*x
**10 + 4*x**(1/3)*a**10*x**3 - x**(1/3)*x**13))/(260*(a**(1/3)*a**3 - x**(
1/3)*x**3)**(2/7))
```

3.337 $\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx = \frac{5}{2}\sqrt{1+x^{4/5}}$$

output
```
5/2*(1+x^(4/5))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx = \frac{5}{2}\sqrt{1+x^{4/5}}$$

input
```
Integrate[1/(Sqrt[1 + x^(4/5)]*x^(1/5)),x]
```

output
```
(5*Sqrt[1 + x^(4/5)])/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^{4/5}+1}\sqrt[5]{x}}dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{5}{2}\sqrt{x^{4/5}+1}$$

input
```
Int[1/(Sqrt[1 + x^(4/5)]*x^(1/5)),x]
```

output
```
(5*Sqrt[1 + x^(4/5)])/2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{5\sqrt{1+x^{\frac{4}{5}}}}{2}$	10
default	$\frac{5\sqrt{1+x^{\frac{4}{5}}}}{2}$	10
meijerg	$\frac{-\frac{5\sqrt{\pi}}{2}+\frac{5\sqrt{\pi}\sqrt{1+x^{\frac{4}{5}}}}{2}}{\sqrt{\pi}}$	24

input
```
int(1/(1+x^(4/5))^(1/2)/x^(1/5),x,method=_RETURNVERBOSE)
```

output
```
5/2*(1+x^(4/5))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx=\text{Timed out}$$

input
```
integrate(1/(1+x^(4/5))^(1/2)/x^(1/5),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx=\frac{5\sqrt{x^{\frac{4}{5}}+1}}{2}$$

input
```
integrate(1/(1+x**(4/5))**(1/2)/x**(1/5),x)
```

output
```
5*sqrt(x**(4/5) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx = \frac{5}{2}\sqrt{x^{\frac{4}{5}}+1}$$

input
```
integrate(1/(1+x^(4/5))^(1/2)/x^(1/5),x, algorithm="maxima")
```

output
```
5/2*sqrt(x^(4/5) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx = \frac{5}{2}\sqrt{x^{\frac{4}{5}}+1}$$

input
```
integrate(1/(1+x^(4/5))^(1/2)/x^(1/5),x, algorithm="giac")
```

output
```
5/2*sqrt(x^(4/5) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx = \frac{5\sqrt{x^{4/5}+1}}{2}$$

input
```
int(1/(x^(1/5)*(x^(4/5) + 1)^(1/2)),x)
```

output
```
(5*(x^(4/5) + 1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{1+x^{4/5}}\sqrt[5]{x}}\,dx = \frac{5x^{\frac{2}{5}}\sqrt{x^{\frac{4}{5}}+1}+5x^{\frac{4}{5}}+5}{2\sqrt{x^{\frac{4}{5}}+1}+2x^{\frac{2}{5}}}$$

input
```
int(1/(1+x^(4/5))^(1/2)/x^(1/5),x)
```

output
```
(5*(x**(2/5)*sqrt(x**(4/5) + 1) + x**(4/5) + 1))/(2*(sqrt(x**(4/5) + 1) +
x**(2/5)))
```

3.338 $\int x^3(a+bx^n)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^3(a+bx^n)\,dx=\frac{ax^4}{4}+\frac{bx^{4+n}}{4+n}$$

output
```
1/4*a*x^4+b*x^(4+n)/(4+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^n)\,dx=\frac{ax^4}{4}+\frac{bx^{4+n}}{4+n}$$

input
```
Integrate[x^3*(a + b*x^n),x]
```

output
```
(a*x^4)/4 + (b*x^(4 + n))/(4 + n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^n)\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int (ax^3+bx^{n+3})\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^4}{4}+\frac{bx^{n+4}}{n+4}$$

input
```
Int[x^3*(a + b*x^n),x]
```

output
```
(a*x^4)/4 + (b*x^(4 + n))/(4 + n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{b\,x^{4} x^{n}}{4+n}+\frac{a\,x^{4}}{4}$	21
norman	$\frac{b\,x^{4} {\mathrm e}^{n \ln \left(x \right)}}{4+n}+\frac{a\,x^{4}}{4}$	23
parallelrisch	$\frac{4 x^{4} x^{n} b+x^{4} a n+4 a\,x^{4}}{4 n+16}$	30
orering	$\frac{x^{4} \left(n+7\right) \left(a+b\,x^{n}\right)}{4 n+16}-\frac{x^{2} \left(3 x^{2} \left(a+b\,x^{n}\right)+x^{2} b\,x^{n} n\right)}{4 \left(4+n\right)}$	54

input
```
int(x^3*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
b/(4+n)*x^4*x^n+1/4*a*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int x^3(a+bx^n)\,dx=\frac{4\,bx^4x^n+(an+4\,a)x^4}{4\,(n+4)}$$

input
```
integrate(x^3*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/4*(4*b*x^4*x^n + (a*n + 4*a)*x^4)/(n + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int x^3(a+bx^n)\,dx=\begin{cases}\frac{anx^4}{4n+16}+\frac{4ax^4}{4n+16}+\frac{4bx^4x^n}{4n+16} & \text{for } n\neq -4\\ \frac{ax^4}{4}+b\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(a+b*x**n),x)
```

output
```
Piecewise((a*n*x**4/(4*n + 16) + 4*a*x**4/(4*n + 16) + 4*b*x**4*x**n/(4*n
+ 16), Ne(n, -4)), (a*x**4/4 + b*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^3(a+bx^n)\,dx=\frac{1}{4}\,ax^4+\frac{bx^{n+4}}{n+4}$$

input
```
integrate(x^3*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + b*x^(n + 4)/(n + 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int x^3(a+bx^n)\,dx=\frac{4\,bx^4x^n+anx^4+4\,ax^4}{4\,(n+4)}$$

input
```
integrate(x^3*(a+b*x^n),x, algorithm="giac")
```

output
```
1/4*(4*b*x^4*x^n + a*n*x^4 + 4*a*x^4)/(n + 4)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x^3(a+bx^n)\,dx=\frac{a\,x^4}{4}+\frac{b\,x^n\,x^4}{n+4}$$

input
```
int(x^3*(a + b*x^n),x)
```

output
```
(a*x^4)/4 + (b*x^n*x^4)/(n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^3(a+bx^n)\,dx=\frac{x^4(4x^nb+an+4a)}{4n+16}$$

input
```
int(x^3*(a+b*x^n),x)
```

output
```
(x**4*(4*x**n*b + a*n + 4*a))/(4*(n + 4))
```

3.339 $\int x^2(a+bx^n)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^2(a+bx^n)\,dx=\frac{ax^3}{3}+\frac{bx^{3+n}}{3+n}$$

output
```
1/3*a*x^3+b*x^(3+n)/(3+n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^n)\,dx=\frac{ax^3}{3}+\frac{bx^{3+n}}{3+n}$$

input
```
Integrate[x^2*(a + b*x^n),x]
```

output
```
(a*x^3)/3 + (b*x^(3 + n))/(3 + n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^2+bx^{n+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^{n+3}}{n+3}$$

input
```
Int[x^2*(a + b*x^n),x]
```

output
```
(a*x^3)/3 + (b*x^(3 + n))/(3 + n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{b\,x^{3} x^{n}}{3+n}+\frac{a\,x^{3}}{3}$	21
norman	$\frac{b\,x^{3} {\mathrm e}^{n \ln\left(x \right)}}{3+n}+\frac{a\,x^{3}}{3}$	23
parallelrisch	$\frac{3 x^{3} x^{n} b +x^{3} a n +3 a\,x^{3}}{9+3 n}$	30
orering	$\frac{x^{3} \left(n +5\right) \left(a +b\,x^{n}\right)}{9+3 n}-\frac{x^{2} \left(2 x \left(a +b\,x^{n}\right)+x b\,x^{n} n \right)}{3 \left(3+n \right)}$	50

input
```
int(x^2*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
b/(3+n)*x^3*x^n+1/3*a*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int x^2(a+bx^n)\,dx=\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

input
```
integrate(x^2*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int x^2(a+bx^n)\,dx=\begin{cases}\frac{anx^3}{3n+9}+\frac{3ax^3}{3n+9}+\frac{3bx^3x^n}{3n+9} & \text{for}\ n\neq -3\\ \frac{ax^3}{3}+b\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a+b*x**n),x)
```

output
```
Piecewise((a*n*x**3/(3*n + 9) + 3*a*x**3/(3*n + 9) + 3*b*x**3*x**n/(3*n +
9), Ne(n, -3)), (a*x**3/3 + b*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^2(a+bx^n)\,dx = \frac{1}{3}\,ax^3 + \frac{bx^{n+3}}{n+3}$$

input
```
integrate(x^2*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + b*x^(n + 3)/(n + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int x^2(a+bx^n)\,dx = \frac{3\,bx^3x^n + anx^3 + 3\,ax^3}{3\,(n+3)}$$

input
```
integrate(x^2*(a+b*x^n),x, algorithm="giac")
```

output
```
1/3*(3*b*x^3*x^n + a*n*x^3 + 3*a*x^3)/(n + 3)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x^2(a+bx^n)\,dx=\frac{a\,x^3}{3}+\frac{b\,x^n\,x^3}{n+3}$$

input
```
int(x^2*(a + b*x^n),x)
```

output
```
(a*x^3)/3 + (b*x^n*x^3)/(n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^2(a+bx^n)\,dx=\frac{x^3(3x^nb+an+3a)}{3n+9}$$

input
```
int(x^2*(a+b*x^n),x)
```

output
```
(x**3*(3*x**n*b + a*n + 3*a))/(3*(n + 3))
```

3.340 $\int x(a+bx^n)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int x(a+bx^n)\,dx = \frac{ax^2}{2}+\frac{bx^{2+n}}{2+n}$$

output
```
1/2*a*x^2+b*x^(2+n)/(2+n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x(a+bx^n)\,dx = \frac{ax^2}{2}+\frac{bx^{2+n}}{2+n}$$

input
```
Integrate[x*(a + b*x^n),x]
```

output
```
(a*x^2)/2 + (b*x^(2 + n))/(2 + n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int (ax+bx^{n+1})\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}+\frac{bx^{n+2}}{n+2}$$

input
```
Int[x*(a + b*x^n),x]
```

output
```
(a*x^2)/2 + (b*x^(2 + n))/(2 + n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{b\,x^{2} x^{n}}{2+n}+\frac{a\,x^{2}}{2}$	21
norman	$\frac{b\,x^{2} {\mathrm e}^{n \ln \left(x \right)}}{2+n}+\frac{a\,x^{2}}{2}$	23
parallelrisch	$\frac{2 x^{2} x^{n} b +x^{2} a n +2 a\,x^{2}}{4+2 n}$	30
orering	$\frac{x^{2} \left(3+n \right) \left(a +b\,x^{n}\right)}{4+2 n}-\frac{x^{2} \left(a +b\,x^{n}+b\,x^{n} n \right)}{2 \left(2+n \right)}$	45

input
```
int(x*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
b/(2+n)*x^2*x^n+1/2*a*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int x(a+bx^n)\,dx=\frac{2\,bx^2x^n+(an+2\,a)x^2}{2\,(n+2)}$$

input
```
integrate(x*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15)=30$.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int x(a+bx^n)\,dx=\begin{cases}\frac{anx^2}{2n+4}+\frac{2ax^2}{2n+4}+\frac{2bx^2x^n}{2n+4} & \text{for } n\neq -2\\ \frac{ax^2}{2}+b\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*x**n),x)
```

output
```
Piecewise((a*n*x**2/(2*n + 4) + 2*a*x**2/(2*n + 4) + 2*b*x**2*x**n/(2*n +
4), Ne(n, -2)), (a*x**2/2 + b*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x(a+bx^n)\,dx = \frac{1}{2}\,ax^2 + \frac{bx^{n+2}}{n+2}$$

input
```
integrate(x*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + b*x^(n + 2)/(n + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int x(a+bx^n)\,dx = \frac{2\,bx^2x^n + anx^2 + 2\,ax^2}{2\,(n+2)}$$

input
```
integrate(x*(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*(2*b*x^2*x^n + a*n*x^2 + 2*a*x^2)/(n + 2)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x(a+bx^n)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^n\,x^2}{n+2}$$

input
```
int(x*(a + b*x^n),x)
```

output
```
(a*x^2)/2 + (b*x^n*x^2)/(n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x(a+bx^n)\,dx=\frac{x^2(2x^nb+an+2a)}{2n+4}$$

input
```
int(x*(a+b*x^n),x)
```

output
```
(x**2*(2*x**n*b + a*n + 2*a))/(2*(n + 2))
```

3.341 $\int (a + bx^n) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (a + bx^n) \, dx = ax + \frac{bx^{1+n}}{1+n}$$

output
```
a*x+b*x^(1+n)/(1+n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + bx^n) \, dx = ax + \frac{bx^{1+n}}{1+n}$$

input
```
Integrate[a + b*x^n,x]
```

output
```
a*x + (b*x^(1 + n))/(1 + n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx^n) \, dx$$

$$\Big\downarrow \; 2009$$

$$ax + \frac{bx^{n+1}}{n+1}$$

input
```
Int[a + b*x^n,x]
```

output
```
a*x + (b*x^(1 + n))/(1 + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
risch	$ax + \frac{bx\,x^n}{1+n}$	16
parallelrisch	$ax + \frac{bx\,x^n}{1+n}$	16
default	$ax + \frac{b\,x^{1+n}}{1+n}$	17
parts	$ax + \frac{b\,x^{1+n}}{1+n}$	17
norman	$ax + \frac{bx\,e^{n\ln(x)}}{1+n}$	18
orering	$x(a + b\,x^n) - \frac{xb\,x^n n}{1+n}$	24

input
```
int(a+b*x^n,x,method=_RETURNVERBOSE)
```

output
```
a*x+b/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a + bx^n)\ dx = \frac{bxx^n + (an + a)x}{n + 1}$$

input
```
integrate(a+b*x^n,x, algorithm="fricas")
```

output
```
(b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (a + bx^n)\, dx = ax + b\left(\begin{cases} \frac{x^{n+1}}{n+1} & \text{for}\, n \neq -1 \\ \log{(x)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*x**n,x)
```

output
```
a*x + b*Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + bx^n)\, dx = ax + \frac{bx^{n+1}}{n+1}$$

input
```
integrate(a+b*x^n,x, algorithm="maxima")
```

output
```
a*x + b*x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + bx^n)\, dx = ax + \frac{bx^{n+1}}{n+1}$$

input
```
integrate(a+b*x^n,x, algorithm="giac")
```

output
```
a*x + b*x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (a + bx^n) \, dx = a\,x + \frac{b\,x\,x^n}{n+1}$$

input
```
int(a + b*x^n,x)
```

output
```
a*x + (b*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (a + bx^n) \, dx = \frac{x(x^n b + an + a)}{n+1}$$

input
```
int(a+b*x^n,x)
```

output
```
(x*(x**n*b + a*n + a))/(n + 1)
```

3.342 $\int \frac{a+bx^n}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^n}{x} \, dx = \frac{bx^n}{n} + a\log(x)$$

output
```
b*x^n/n+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{a+bx^n}{x} \, dx = \frac{bx^n}{n} + \frac{a\log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)/x,x]
```

output
```
(b*x^n)/n + (a*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{x} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a}{x}+bx^{n-1}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a\log(x)+\frac{bx^n}{n}$$

input
```
Int[(a + b*x^n)/x,x]
```

output
```
(b*x^n)/n + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{b\,x^{n}}{n}+a\ln\left(x\right)$	14
norman	$a\ln\left(x\right)+\frac{b\,{\mathrm e}^{n\ln\left(x\right)}}{n}$	16
parallelrisch	$\frac{a\ln\left(x\right)n+b\,x^{n}}{n}$	16
derivativedivides	$\frac{b\,x^{n}+a\ln\left(x^{n}\right)}{n}$	17
default	$\frac{b\,x^{n}+a\ln\left(x^{n}\right)}{n}$	17

input
```
int((a+b*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
b*x^n/n+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{a+bx^n}{x}\,dx=\frac{an\log\left(x\right)+bx^n}{n}$$

input
```
integrate((a+b*x^n)/x,x, algorithm="fricas")
```

output
```
(a*n*log(x) + b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^n}{x}\,dx = \begin{cases} a\log(x) + \frac{bx^n}{n} & \text{for } n \neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)/x,x)
```

output
```
Piecewise((a*log(x) + b*x**n/n, Ne(n, 0)), ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{a+bx^n}{x}\,dx = \frac{bx^n}{n} + \frac{a\log(x^n)}{n}$$

input
```
integrate((a+b*x^n)/x,x, algorithm="maxima")
```

output
```
b*x^n/n + a*log(x^n)/n
```

Giac [F]

$$\int \frac{a+bx^n}{x}\,dx = \int \frac{bx^n+a}{x}\,dx$$

input
```
integrate((a+b*x^n)/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^n}{x}\,dx = a\ \ln(x) + \frac{b\,x^n}{n}$$

input
```
int((a + b*x^n)/x,x)
```

output
```
a*log(x) + (b*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{a+bx^n}{x}\,dx = \frac{x^n b + \log(x)\,an}{n}$$

input
```
int((a+b*x^n)/x,x)
```

output
```
(x**n*b + log(x)*a*n)/n
```

3.343 $\int \frac{a+bx^n}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{a+bx^n}{x^2} dx = -\frac{a}{x} - \frac{bx^{-1+n}}{1-n}$$

output
```
-a/x-b*x^(-1+n)/(1-n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^n}{x^2} dx = -\frac{a}{x} - \frac{bx^{-1+n}}{1-n}$$

input
```
Integrate[(a + b*x^n)/x^2,x]
```

output
```
-(a/x) - (b*x^(-1 + n))/(1 - n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{x^2}\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a}{x^2}+bx^{n-2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{x}-\frac{bx^{n-1}}{1-n}$$

input
```
Int[(a + b*x^n)/x^2,x]
```

output
```
-(a/x) - (b*x^(-1 + n))/(1 - n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{\frac{b\,{\mathrm e}^{n\ln\left(x\right)}}{-1+n}-a}{x}$	21
risch	$-\frac{a}{x}+\frac{b\,x^{n}}{\left(-1+n\right)x}$	21
parallelrisch	$\frac{b\,x^{n}-a n+a}{\left(-1+n\right)x}$	21
orering	$-\frac{\left(-3+n\right)\left(a+b\,x^{n}\right)}{x\left(-1+n\right)}+\frac{x^{2}\left(\frac{b\,x^{n}n}{x^{3}}-\frac{2\left(a+b\,x^{n}\right)}{x^{3}}\right)}{-1+n}$	53

input
```
int((a+b*x^n)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(b/(-1+n)*exp(n*ln(x))-a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^n}{x^2}\,dx=-\frac{an-bx^n-a}{(n-1)x}$$

input
```
integrate((a+b*x^n)/x^2,x, algorithm="fricas")
```

output
```
-(a*n - b*x^n - a)/((n - 1)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{a+bx^n}{x^2}\,dx = \begin{cases} -\frac{an}{nx-x} + \frac{a}{nx-x} + \frac{bx^n}{nx-x} & \text{for } n \neq 1 \\ -\frac{a}{x} + b\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)/x**2,x)
```

output
```
Piecewise((-a*n/(n*x - x) + a/(n*x - x) + b*x**n/(n*x - x), Ne(n, 1)), (-a
/x + b*log(x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^n}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-2>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{a+bx^n}{x^2}\,dx = \int \frac{bx^n+a}{x^2}\,dx$$

input
```
integrate((a+b*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^n}{x^2}\,dx = \frac{b\,x^n}{x\,(n-1)} - \frac{a}{x}$$

input
```
int((a + b*x^n)/x^2,x)
```

output
```
(b*x^n)/(x*(n - 1)) - a/x
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^n}{x^2}\,dx = \frac{x^n b-an+a}{x\,(n-1)}$$

input
```
int((a+b*x^n)/x^2,x)
```

output
```
(x**n*b - a*n + a)/(x*(n - 1))
```

3.344 $\int \frac{a+bx^n}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{a+bx^n}{x^3} dx = -\frac{a}{2x^2} - \frac{bx^{-2+n}}{2-n}$$

output
```
-1/2*a/x^2-b*x^(-2+n)/(2-n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^n}{x^3} dx = -\frac{a - \frac{2bx^n}{-2+n}}{2x^2}$$

input
```
Integrate[(a + b*x^n)/x^3,x]
```

output
```
-1/2*(a - (2*b*x^n)/(-2 + n))/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{x^3} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a}{x^3}+bx^{n-3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{2x^2}-\frac{bx^{n-2}}{2-n}$$

input
```
Int[(a + b*x^n)/x^3,x]
```

output
```
-1/2*a/x^2 - (b*x^(-2 + n))/(2 - n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
norman	$\frac{\frac{b\,{\mathrm e}^{n\ln\left(x\right)}}{n-2}-\frac{a}{2}}{x^{2}}$	21
risch	$-\frac{a}{2x^{2}}+\frac{b\,x^{n}}{\left(n-2\right)x^{2}}$	21
parallelrisch	$\frac{2b\,x^{n}-an+2a}{2\left(n-2\right)x^{2}}$	25
orering	$-\frac{\left(-5+n\right)\left(a+b\,x^{n}\right)}{2x^{2}\left(n-2\right)}+\frac{x^{2}\left(\frac{b\,x^{n}n}{x^{4}}-\frac{3\left(a+b\,x^{n}\right)}{x^{4}}\right)}{-4+2n}$	54

input
```
int((a+b*x^n)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(b/(n-2)*exp(n*ln(x))-1/2*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^n}{x^3}\,dx=-\frac{an-2\,bx^n-2\,a}{2\,(n-2)x^2}$$

input
```
integrate((a+b*x^n)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \frac{a+bx^n}{x^3}\,dx = \begin{cases} -\frac{an}{2nx^2-4x^2} + \frac{2a}{2nx^2-4x^2} + \frac{2bx^n}{2nx^2-4x^2} & \text{for } n \neq 2 \\ -\frac{a}{2x^2} + b\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)/x**3,x)
```

output
```
Piecewise((-a*n/(2*n*x**2 - 4*x**2) + 2*a/(2*n*x**2 - 4*x**2) + 2*b*x**n/(
2*n*x**2 - 4*x**2), Ne(n, 2)), (-a/(2*x**2) + b*log(x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^n}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{a+bx^n}{x^3}\,dx = \int \frac{bx^n+a}{x^3}\,dx$$

input
```
integrate((a+b*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^n}{x^3}\,dx = \frac{b\,x^n}{x^2\,(n-2)} - \frac{a}{2\,x^2}$$

input
```
int((a + b*x^n)/x^3,x)
```

output
```
(b*x^n)/(x^2*(n - 2)) - a/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^n}{x^3}\,dx = \frac{2x^n b - an + 2a}{2x^2\,(n-2)}$$

input
```
int((a+b*x^n)/x^3,x)
```

output
```
(2*x**n*b - a*n + 2*a)/(2*x**2*(n - 2))
```

3.345 $\int x^3(a+bx^n)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int x^3(a+bx^n)^2\,dx=\frac{a^2x^4}{4}+\frac{b^2x^{2(2+n)}}{2(2+n)}+\frac{2abx^{4+n}}{4+n}$$

output
```
1/4*a^2*x^4+b^2*x^(4+2*n)/(4+2*n)+2*a*b*x^(4+n)/(4+n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x^3(a+bx^n)^2\,dx=\frac{1}{4}x^4\left(a^2+\frac{8abx^n}{4+n}+\frac{2b^2x^{2n}}{2+n}\right)$$

input
```
Integrate[x^3*(a + b*x^n)^2,x]
```

output
```
(x^4*(a^2 + (8*a*b*x^n)/(4 + n) + (2*b^2*x^(2*n))/(2 + n)))/4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^3+2abx^{n+3}+b^2x^{2n+3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^4}{4}+\frac{2abx^{n+4}}{n+4}+\frac{b^2x^{2(n+2)}}{2(n+2)}$$

input
```
Int[x^3*(a + b*x^n)^2,x]
```

output
```
(a^2*x^4)/4 + (b^2*x^(2*(2 + n)))/(2*(2 + n)) + (2*a*b*x^(4 + n))/(4 + n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result
risch	$\frac{a^{2} x^{4}}{4}+\frac{b^{2} x^{4} x^{2 n}}{4+2 n}+\frac{2 a b \,x^{4} x^{n}}{4+n}$
norman	$\frac{a^{2} x^{4}}{4}+\frac{b^{2} x^{4} {\mathrm e}^{2 n \ln \left(x \right)}}{4+2 n}+\frac{2 a b \,x^{4} {\mathrm e}^{n \ln \left(x \right)}}{4+n}$
parallelrisch	$\frac{2 x^{4} x^{2 n} b^{2} n +8 b^{2} x^{4} x^{2 n}+8 x^{4} x^{n} a b n +x^{4} a^{2} n^{2}+16 x^{4} x^{n} a b +6 x^{4} a^{2} n +8 a^{2} x^{4}}{4 \left(2+n \right) \left(4+n \right)}$
orering	$\frac{x^{4} \left(2 n^{2}+21 n +37\right) \left(a +b \,x^{n}\right)^{2}}{8 \left(2+n \right) \left(4+n \right)}-\frac{3 \left(3+n \right) x^{2} \left(3 x^{2} \left(a +b \,x^{n}\right)^{2}+2 x^{2} \left(a +b \,x^{n}\right) b \,x^{n} n \right)}{8 \left(2+n \right) \left(4+n \right)}+\frac{x^{3} \left(6 x \left(a +b \,x^{n}\right)^{2}+10 x \left(a +b \,x^{n}\right) b \,x^{n} n +\right.}{8 n^{2}+48 n}$

input
```
int(x^3*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+1/2*b^2/(2+n)*x^4*(x^n)^2+2*a*b/(4+n)*x^4*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

$$\int x^3(a+bx^n)^2\,dx=\frac{2\,(b^2n+4\,b^2)x^4x^{2\,n}+8\,(abn+2\,ab)x^4x^n+(a^2n^2+6\,a^2n+8\,a^2)x^4}{4\,(n^2+6\,n+8)}$$

input
```
integrate(x^3*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*n + 4*b^2)*x^4*x^(2*n) + 8*(a*b*n + 2*a*b)*x^4*x^n + (a^2*n^2
+ 6*a^2*n + 8*a^2)*x^4)/(n^2 + 6*n + 8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(36) = 72$.

Time = 0.37 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.59

$$\int x^3(a+bx^n)^2\,dx = \begin{cases} \frac{a^2x^4}{4} + 2ab\log(x) - \frac{b^2}{4x^4} & \text{for } n = - \\ \frac{a^2x^4}{4} + abx^2 + b^2\log(x) & \text{for } n = - \\ \frac{a^2n^2x^4}{4n^2+24n+32} + \frac{6a^2nx^4}{4n^2+24n+32} + \frac{8a^2x^4}{4n^2+24n+32} + \frac{8abnx^4x^n}{4n^2+24n+32} + \frac{16abx^4x^n}{4n^2+24n+32} + \frac{2b^2nx^4x^{2n}}{4n^2+24n+32} + \frac{8b^2x^4x^{2n}}{4n^2+24n+32} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x**4/4 + 2*a*b*log(x) - b**2/(4*x**4), Eq(n, -4)), (a**2*x
**4/4 + a*b*x**2 + b**2*log(x), Eq(n, -2)), (a**2*n**2*x**4/(4*n**2 + 24*n
 + 32) + 6*a**2*n*x**4/(4*n**2 + 24*n + 32) + 8*a**2*x**4/(4*n**2 + 24*n +
 32) + 8*a*b*n*x**4*x**n/(4*n**2 + 24*n + 32) + 16*a*b*x**4*x**n/(4*n**2 +
 24*n + 32) + 2*b**2*n*x**4*x**(2*n)/(4*n**2 + 24*n + 32) + 8*b**2*x**4*x*
*(2*n)/(4*n**2 + 24*n + 32), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x^3(a+bx^n)^2\,dx = \frac{1}{4}a^2x^4 + \frac{b^2x^{2n+4}}{2(n+2)} + \frac{2abx^{n+4}}{n+4}$$

input
```
integrate(x^3*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/4*a^2*x^4 + 1/2*b^2*x^(2*n + 4)/(n + 2) + 2*a*b*x^(n + 4)/(n + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(40) = 80$.

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int x^3(a+bx^n)^2\,dx$$
$$= \frac{2\,b^2nx^4x^{2\,n} + 8\,abnx^4x^n + a^2n^2x^4 + 8\,b^2x^4x^{2\,n} + 16\,abx^4x^n + 6\,a^2nx^4 + 8\,a^2x^4}{4\,(n^2+6\,n+8)}$$

input
```
integrate(x^3*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/4*(2*b^2*n*x^4*x^(2*n) + 8*a*b*n*x^4*x^n + a^2*n^2*x^4 + 8*b^2*x^4*x^(2*
n) + 16*a*b*x^4*x^n + 6*a^2*n*x^4 + 8*a^2*x^4)/(n^2 + 6*n + 8)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int x^3(a+bx^n)^2\,dx = \frac{a^2\,x^4}{4} + \frac{b^2\,x^{2\,n}\,x^4}{2\,n+4} + \frac{2\,a\,b\,x^n\,x^4}{n+4}$$

input
```
int(x^3*(a + b*x^n)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^(2*n)*x^4)/(2*n + 4) + (2*a*b*x^n*x^4)/(n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int x^3(a+bx^n)^2\,dx = \frac{x^4(2x^{2n}b^2n + 8x^{2n}b^2 + 8x^nabn + 16x^nab + a^2n^2 + 6a^2n + 8a^2)}{4n^2+24n+32}$$

input
```
int(x^3*(a+b*x^n)^2,x)
```

output
```
(x**4*(2*x**(2*n)*b**2*n + 8*x**(2*n)*b**2 + 8*x**n*a*b*n + 16*x**n*a*b +
a**2*n**2 + 6*a**2*n + 8*a**2))/(4*(n**2 + 6*n + 8))
```

3.346 $\int x^2(a+bx^n)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^2(a+bx^n)^2\,dx=\frac{a^2x^3}{3}+\frac{2abx^{3+n}}{3+n}+\frac{b^2x^{3+2n}}{3+2n}$$

output
```
1/3*a^2*x^3+2*a*b*x^(3+n)/(3+n)+b^2*x^(3+2*n)/(3+2*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^n)^2\,dx=\frac{1}{3}x^3\left(a^2+\frac{6abx^n}{3+n}+\frac{3b^2x^{2n}}{3+2n}\right)$$

input
```
Integrate[x^2*(a + b*x^n)^2,x]
```

output
```
(x^3*(a^2 + (6*a*b*x^n)/(3 + n) + (3*b^2*x^(2*n))/(3 + 2*n)))/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^n)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^2 + 2abx^{n+2} + b^2x^{2(n+1)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^3}{3} + \frac{2abx^{n+3}}{n+3} + \frac{b^2x^{2n+3}}{2n+3}$$

input
```
Int[x^2*(a + b*x^n)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^(3 + n))/(3 + n) + (b^2*x^(3 + 2*n))/(3 + 2*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result
risch	$\frac{b^{2} x^{3} x^{2 n}}{2 n+3}+\frac{a^{2} x^{3}}{3}+\frac{2 a b \,x^{3} x^{n}}{3+n}$
norman	$\frac{b^{2} x^{3} {\mathrm e}^{2 n \ln \left(x \right)}}{2 n+3}+\frac{a^{2} x^{3}}{3}+\frac{2 a b \,x^{3} {\mathrm e}^{n \ln \left(x \right)}}{3+n}$
parallelrisch	$\frac{3 x^{3} x^{2 n} b^{2} n+9 b^{2} x^{3} x^{2 n}+12 x^{3} x^{n} a b n+2 x^{3} a^{2} n^{2}+18 x^{3} x^{n} a b+9 x^{3} a^{2} n+9 a^{2} x^{3}}{3 \left(2 n+3\right) \left(3+n \right)}$
orering	$\frac{x^{3} \left(2 n^{2}+15 n+19\right) \left(a+b \,x^{n}\right)^{2}}{3 \left(2 n+3\right) \left(3+n \right)}-\frac{x^{2} \left(2+n \right) \left(2 x \left(a+b \,x^{n}\right)^{2}+2 x \left(a+b \,x^{n}\right) b \,x^{n} n \right)}{\left(2 n+3\right) \left(3+n \right)}+\frac{x^{3} \left(2 \left(a+b \,x^{n}\right)^{2}+6 \left(a+b \,x^{n}\right) b \,x^{n} n+2 b^{2} x^{2 n}\right.}{6 n^{2}+27 n+27}$

input
```
int(x^2*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(2*n+3)*x^3*(x^n)^2+1/3*a^2*x^3+2*a*b/(3+n)*x^3*x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.81

$$\int x^2(a+bx^n)^2\,dx = \frac{3\,(b^2n+3\,b^2)x^3x^{2\,n}+6\,(2\,abn+3\,ab)x^3x^n+(2\,a^2n^2+9\,a^2n+9\,a^2)x^3}{3\,(2\,n^2+9\,n+9)}$$

input
```
integrate(x^2*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(b^2*n + 3*b^2)*x^3*x^(2*n) + 6*(2*a*b*n + 3*a*b)*x^3*x^n + (2*a^2*
n^2 + 9*a^2*n + 9*a^2)*x^3)/(2*n^2 + 9*n + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(36) = 72$.

Time = 0.46 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.91

$$\int x^2(a+bx^n)^2\,dx = \begin{cases} \frac{a^2x^3}{3} + 2ab\log(x) - \frac{b^2}{3x^3} & \text{for } n = - \\ \frac{a^2x^3}{3} + \frac{4abx^{\frac{3}{2}}}{3} + b^2\log(x) & \text{for } n = - \\ \frac{2a^2n^2x^3}{6n^2+27n+27} + \frac{9a^2nx^3}{6n^2+27n+27} + \frac{9a^2x^3}{6n^2+27n+27} + \frac{12abnx^3x^n}{6n^2+27n+27} + \frac{18abx^3x^n}{6n^2+27n+27} + \frac{3b^2nx^3x^{2n}}{6n^2+27n+27} + \frac{9b^2x^3x^{2n}}{6n^2+27n+27} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x**3/3 + 2*a*b*log(x) - b**2/(3*x**3), Eq(n, -3)), (a**2*x
**3/3 + 4*a*b*x**(3/2)/3 + b**2*log(x), Eq(n, -3/2)), (2*a**2*n**2*x**3/(6
*n**2 + 27*n + 27) + 9*a**2*n*x**3/(6*n**2 + 27*n + 27) + 9*a**2*x**3/(6*n
**2 + 27*n + 27) + 12*a*b*n*x**3*x**n/(6*n**2 + 27*n + 27) + 18*a*b*x**3*x
**n/(6*n**2 + 27*n + 27) + 3*b**2*n*x**3*x**(2*n)/(6*n**2 + 27*n + 27) + 9
*b**2*x**3*x**(2*n)/(6*n**2 + 27*n + 27), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^2(a+bx^n)^2\,dx = \frac{1}{3}a^2x^3 + \frac{b^2x^{2n+3}}{2n+3} + \frac{2abx^{n+3}}{n+3}$$

input
```
integrate(x^2*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + b^2*x^(2*n + 3)/(2*n + 3) + 2*a*b*x^(n + 3)/(n + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.12

$$\int x^2(a+bx^n)^2\,dx = \frac{3\,b^2nx^3x^{2\,n}+12\,abnx^3x^n+2\,a^2n^2x^3+9\,b^2x^3x^{2\,n}+18\,abx^3x^n+9\,a^2nx^3+9\,a^2x^3}{3\,(2\,n^2+9\,n+9)}$$

input
```
integrate(x^2*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/3*(3*b^2*n*x^3*x^(2*n) + 12*a*b*n*x^3*x^n + 2*a^2*n^2*x^3 + 9*b^2*x^3*x^
(2*n) + 18*a*b*x^3*x^n + 9*a^2*n*x^3 + 9*a^2*x^3)/(2*n^2 + 9*n + 9)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^n)^2\,dx = \frac{a^2\,x^3}{3}+\frac{b^2\,x^{2\,n}\,x^3}{2\,n+3}+\frac{2\,a\,b\,x^n\,x^3}{n+3}$$

input
```
int(x^2*(a + b*x^n)^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^(2*n)*x^3)/(2*n + 3) + (2*a*b*x^n*x^3)/(n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int x^2(a+bx^n)^2\,dx = \frac{x^3(3x^{2n}b^2n+9x^{2n}b^2+12x^nabn+18x^nab+2a^2n^2+9a^2n+9a^2)}{6n^2+27n+27}$$

input
```
int(x^2*(a+b*x^n)^2,x)
```

output
```
(x**3*(3*x**(2*n)*b**2*n + 9*x**(2*n)*b**2 + 12*x**n*a*b*n + 18*x**n*a*b +
 2*a**2*n**2 + 9*a**2*n + 9*a**2))/(3*(2*n**2 + 9*n + 9))
```

3.347 $\int x(a+bx^n)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int x(a+bx^n)^2\,dx=\frac{a^2x^2}{2}+\frac{b^2x^{2(1+n)}}{2(1+n)}+\frac{2abx^{2+n}}{2+n}$$

output
```
1/2*a^2*x^2+b^2*x^(2+2*n)/(2+2*n)+2*a*b*x^(2+n)/(2+n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x(a+bx^n)^2\,dx=\frac{1}{2}x^2\left(a^2+\frac{4abx^n}{2+n}+\frac{b^2x^{2n}}{1+n}\right)$$

input
```
Integrate[x*(a + b*x^n)^2,x]
```

output
```
(x^2*(a^2 + (4*a*b*x^n)/(2 + n) + (b^2*x^(2*n))/(1 + n)))/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x+2abx^{n+1}+b^2x^{2n+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^2}{2}+\frac{2abx^{n+2}}{n+2}+\frac{b^2x^{2(n+1)}}{2(n+1)}$$

input
```
Int[x*(a + b*x^n)^2,x]
```

output
```
(a^2*x^2)/2 + (b^2*x^(2*(1 + n)))/(2*(1 + n)) + (2*a*b*x^(2 + n))/(2 + n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{a^{2} x^{2}}{2}+\frac{b^{2} x^{2} x^{2 n}}{2+2 n}+\frac{2 a b \,x^{2} x^{n}}{2+n}$	43
norman	$\frac{a^{2} x^{2}}{2}+\frac{b^{2} x^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{2+2 n}+\frac{2 a b \,x^{2} {\mathrm e}^{n \ln \left(x \right)}}{2+n}$	47
parallelrisch	$\frac{x^{2} x^{2 n} b^{2} n +2 b^{2} x^{2} x^{2 n}+4 x^{2} x^{n} a b n +x^{2} a^{2} n^{2}+4 x^{2} x^{n} a b +3 x^{2} a^{2} n +2 a^{2} x^{2}}{2 \left(1+n \right) \left(2+n \right)}$	88
orering	$\frac{x^{2} \left(2 n +7\right) \left(a +b \,x^{n}\right)^{2}}{8+4 n}-\frac{3 x^{2} \left(\left(a +b \,x^{n}\right)^{2}+2 \left(a +b \,x^{n}\right) b \,x^{n} n \right)}{4 \left(2+n \right)}+\frac{x^{3} \left(\frac{2 \left(a +b \,x^{n}\right) b \,x^{n} n}{x}+\frac{2 b^{2} x^{2 n} n^{2}}{x}+\frac{2 \left(a +b \,x^{n}\right) b \,x^{n} n^{2}}{x}\right)}{4 n^{2}+12 n +8}$	128

input
```
int(x*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+1/2*b^2/(1+n)*x^2*(x^n)^2+2*a*b/(2+n)*x^2*x^n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

$$\int x(a+bx^n)^2\,dx=\frac{(b^2n+2\,b^2)x^2x^{2\,n}+4\,(abn+ab)x^2x^n+(a^2n^2+3\,a^2n+2\,a^2)x^2}{2\,(n^2+3\,n+2)}$$

input
```
integrate(x*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/2*((b^2*n + 2*b^2)*x^2*x^(2*n) + 4*(a*b*n + a*b)*x^2*x^n + (a^2*n^2 + 3*
a^2*n + 2*a^2)*x^2)/(n^2 + 3*n + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(36) = 72$.

Time = 0.31 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.57

$$\int x(a+bx^n)^2\,dx$$
$$= \begin{cases} \frac{a^2x^2}{2} + 2ab\log(x) - \frac{b^2}{2x^2} & \text{for } n = -2 \\ \frac{a^2x^2}{2} + 2abx + b^2\log(x) & \text{for } n = -1 \\ \frac{a^2n^2x^2}{2n^2+6n+4} + \frac{3a^2nx^2}{2n^2+6n+4} + \frac{2a^2x^2}{2n^2+6n+4} + \frac{4abnx^2x^n}{2n^2+6n+4} + \frac{4abx^2x^n}{2n^2+6n+4} + \frac{b^2nx^2x^{2n}}{2n^2+6n+4} + \frac{2b^2x^2x^{2n}}{2n^2+6n+4} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x**2/2 + 2*a*b*log(x) - b**2/(2*x**2), Eq(n, -2)), (a**2*x
**2/2 + 2*a*b*x + b**2*log(x), Eq(n, -1)), (a**2*n**2*x**2/(2*n**2 + 6*n +
 4) + 3*a**2*n*x**2/(2*n**2 + 6*n + 4) + 2*a**2*x**2/(2*n**2 + 6*n + 4) +
4*a*b*n*x**2*x**n/(2*n**2 + 6*n + 4) + 4*a*b*x**2*x**n/(2*n**2 + 6*n + 4)
+ b**2*n*x**2*x**(2*n)/(2*n**2 + 6*n + 4) + 2*b**2*x**2*x**(2*n)/(2*n**2 +
 6*n + 4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x(a+bx^n)^2\,dx = \frac{1}{2}a^2x^2 + \frac{b^2x^{2n+2}}{2(n+1)} + \frac{2abx^{n+2}}{n+2}$$

input
```
integrate(x*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/2*b^2*x^(2*n + 2)/(n + 1) + 2*a*b*x^(n + 2)/(n + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.98

$$\int x(a+bx^n)^2\,dx = \frac{b^2nx^2x^{2\,n}+4\,abnx^2x^n+a^2n^2x^2+2\,b^2x^2x^{2\,n}+4\,abx^2x^n+3\,a^2nx^2+2\,a^2x^2}{2\,(n^2+3\,n+2)}$$

input
```
integrate(x*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*n*x^2*x^(2*n) + 4*a*b*n*x^2*x^n + a^2*n^2*x^2 + 2*b^2*x^2*x^(2*n)
 + 4*a*b*x^2*x^n + 3*a^2*n*x^2 + 2*a^2*x^2)/(n^2 + 3*n + 2)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int x(a+bx^n)^2\,dx = \frac{a^2\,x^2}{2}+\frac{b^2\,x^{2\,n}\,x^2}{2\,n+2}+\frac{2\,a\,b\,x^n\,x^2}{n+2}$$

input
```
int(x*(a + b*x^n)^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^(2*n)*x^2)/(2*n + 2) + (2*a*b*x^n*x^2)/(n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int x(a+bx^n)^2\,dx = \frac{x^2(x^{2n}b^2n+2x^{2n}b^2+4x^nabn+4x^nab+a^2n^2+3a^2n+2a^2)}{2n^2+6n+4}$$

input
```
int(x*(a+b*x^n)^2,x)
```

output
```
(x**2*(x**(2*n)*b**2*n + 2*x**(2*n)*b**2 + 4*x**n*a*b*n + 4*x**n*a*b + a**
2*n**2 + 3*a**2*n + 2*a**2))/(2*(n**2 + 3*n + 2))
```

3.348 $\int (a + bx^n)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int (a + bx^n)^2 \, dx = a^2x + \frac{2abx^{1+n}}{1+n} + \frac{b^2x^{1+2n}}{1+2n}$$

output
```
a^2*x+2*a*b*x^(1+n)/(1+n)+b^2*x^(1+2*n)/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a + bx^n)^2 \, dx = x\left(a^2 + \frac{2abx^n}{1+n} + \frac{b^2x^{2n}}{1+2n}\right)$$

input
```
Integrate[(a + b*x^n)^2,x]
```

output
```
x*(a^2 + (2*a*b*x^n)/(1 + n) + (b^2*x^(2*n))/(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2\, dx$$

$$\Big\downarrow \text{775}$$

$$\int \left(a^2+2abx^n+b^2x^{2n}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{2abx^{n+1}}{n+1}+\frac{b^2x^{2n+1}}{2n+1}$$

input
```
Int[(a + b*x^n)^2,x]
```

output
```
a^2*x + (2*a*b*x^(1 + n))/(1 + n) + (b^2*x^(1 + 2*n))/(1 + 2*n)
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
risch	$a^2x+\frac{b^2x\,x^{2n}}{1+2n}+\frac{2abx\,x^n}{1+n}$	37
norman	$a^2x+\frac{b^2x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{2abx\,\mathrm{e}^{n\ln(x)}}{1+n}$	41
parallelrisch	$\frac{x\,x^{2n}b^2n+b^2x\,x^{2n}+4x\,x^n abn+2x\,a^2n^2+2x\,x^n ab+3x\,a^2n+a^2x}{(1+2n)(1+n)}$	74
orering	$x(a+b\,x^n)^2-\frac{6x\,n^2(a+b\,x^n)b\,x^n}{2n^2+3n+1}+\frac{x^3\left(\frac{2b^2x^{2n}n^2}{x^2}+\frac{2(a+b\,x^n)b\,x^n n^2}{x^2}-\frac{2(a+b\,x^n)b\,x^n n}{x^2}\right)}{2n^2+3n+1}$	111

input
```
int((a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+b^2/(1+2*n)*x*(x^n)^2+2*a*b/(1+n)*x*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (a+bx^n)^2\,dx=\frac{(b^2n+b^2)xx^{2n}+2\,(2\,abn+ab)xx^n+(2\,a^2n^2+3\,a^2n+a^2)x}{2\,n^2+3\,n+1}$$

input
```
integrate((a+b*x^n)^2,x, algorithm="fricas")
```

output
```
((b^2*n + b^2)*x*x^(2*n) + 2*(2*a*b*n + a*b)*x*x^n + (2*a^2*n^2 + 3*a^2*n
+ a^2)*x)/(2*n^2 + 3*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.79

$$\int (a+bx^n)^2\,dx$$
$$= \begin{cases} a^2x + 2ab\log(x) - \frac{b^2}{x} & \text{for } n = -1 \\ a^2x + 4ab\sqrt{x} + b^2\log(x) & \text{for } n = -\frac{1}{2} \\ \frac{2a^2n^2x}{2n^2+3n+1} + \frac{3a^2nx}{2n^2+3n+1} + \frac{a^2x}{2n^2+3n+1} + \frac{4abnxx^n}{2n^2+3n+1} + \frac{2abxx^n}{2n^2+3n+1} + \frac{b^2nxx^{2n}}{2n^2+3n+1} + \frac{b^2xx^{2n}}{2n^2+3n+1} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*log(x) - b**2/x, Eq(n, -1)), (a**2*x + 4*a*b*sqr
t(x) + b**2*log(x), Eq(n, -1/2)), (2*a**2*n**2*x/(2*n**2 + 3*n + 1) + 3*a*
*2*n*x/(2*n**2 + 3*n + 1) + a**2*x/(2*n**2 + 3*n + 1) + 4*a*b*n*x*x**n/(2*
n**2 + 3*n + 1) + 2*a*b*x*x**n/(2*n**2 + 3*n + 1) + b**2*n*x*x**(2*n)/(2*n
**2 + 3*n + 1) + b**2*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^2\,dx = a^2x + \frac{b^2x^{2n+1}}{2n+1} + \frac{2abx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2,x, algorithm="maxima")
```

output
```
a^2*x + b^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int (a + bx^n)^2 \, dx = \frac{2\,a^2n^2x + b^2nxx^{2\,n} + 4\,abnxx^n + 3\,a^2nx + b^2xx^{2\,n} + 2\,abxx^n + a^2x}{2\,n^2 + 3\,n + 1}$$

input
```
integrate((a+b*x^n)^2,x, algorithm="giac")
```

output
```
(2*a^2*n^2*x + b^2*n*x*x^(2*n) + 4*a*b*n*x*x^n + 3*a^2*n*x + b^2*x*x^(2*n)
 + 2*a*b*x*x^n + a^2*x)/(2*n^2 + 3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (a + bx^n)^2 \, dx = a^2\,x + \frac{b^2\,x\,x^{2\,n}}{2\,n + 1} + \frac{2\,a\,b\,x\,x^n}{n + 1}$$

input
```
int((a + b*x^n)^2,x)
```

output
```
a^2*x + (b^2*x*x^(2*n))/(2*n + 1) + (2*a*b*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int (a + bx^n)^2 \, dx = \frac{x(x^{2n}b^2n + x^{2n}b^2 + 4x^nabn + 2x^nab + 2a^2n^2 + 3a^2n + a^2)}{2n^2 + 3n + 1}$$

input
```
int((a+b*x^n)^2,x)
```

output
```
(x*(x**(2*n)*b**2*n + x**(2*n)*b**2 + 4*x**n*a*b*n + 2*x**n*a*b + 2*a**2*n
**2 + 3*a**2*n + a**2))/(2*n**2 + 3*n + 1)
```

3.349 $\int \frac{(a+bx^n)^2}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{(a+bx^n)^2}{x} dx = \frac{2abx^n}{n} + \frac{b^2x^{2n}}{2n} + a^2 \log(x)$$

output
```
2*a*b*x^n/n+1/2*b^2*x^(2*n)/n+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^2}{x} dx = \frac{bx^n(4a+bx^n)}{2n} + \frac{a^2 \log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)^2/x,x]
```

output
```
(b*x^n*(4*a + b*x^n))/(2*n) + (a^2*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{x} \, dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int x^{-n}(bx^n+a)^2 \, dx^n}{n}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(a^2x^{-n}+b^2x^n+2ab\right) dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2\log(x^n)+2abx^n+\frac{1}{2}b^2x^{2n}}{n}$$

input
```
Int[(a + b*x^n)^2/x,x]
```

output
```
(2*a*b*x^n + (b^2*x^(2*n))/2 + a^2*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\frac{b^2 x^{2n}}{2}+2ab\,x^n+a^2\ln(x^n)}{n}$	31
default	$\frac{\frac{b^2 x^{2n}}{2}+2ab\,x^n+a^2\ln(x^n)}{n}$	31
risch	$\frac{2ab\,x^n}{n}+\frac{b^2x^{2n}}{2n}+a^2\ln(x)$	31
parallelrisch	$\frac{b^2x^{2n}+2a^2\ln(x)n+4ab\,x^n}{2n}$	31
norman	$a^2\ln(x)+\frac{b^2\mathrm{e}^{2n\ln(x)}}{2n}+\frac{2ab\,\mathrm{e}^{n\ln(x)}}{n}$	35

input
```
int((a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/2*b^2*(x^n)^2+2*a*b*x^n+a^2*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{2\,a^2n\log(x)+b^2x^{2\,n}+4\,abx^n}{2\,n}$$

input
```
integrate((a+b*x^n)^2/x,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*n*log(x) + b^2*x^(2*n) + 4*a*b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^n)^2}{x}\,dx = \begin{cases} a^2\log(x) + \frac{2abx^n}{n} + \frac{b^2x^{2n}}{2n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**2/x,x)
```

output
```
Piecewise((a**2*log(x) + 2*a*b*x**n/n + b**2*x**(2*n)/(2*n), Ne(n, 0)), ((
a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{a^2\log(x^n)}{n} + \frac{b^2x^{2n} + 4\,abx^n}{2\,n}$$

input
```
integrate((a+b*x^n)^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x^n)/n + 1/2*(b^2*x^(2*n) + 4*a*b*x^n)/n
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{x}\,dx = \int \frac{(bx^n+a)^2}{x}\,dx$$

input
```
integrate((a+b*x^n)^2/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{b^2\,x^{2\,n}+4\,a\,b\,x^n+2\,a^2\,n\,\ln(x)}{2\,n}$$

input
```
int((a + b*x^n)^2/x,x)
```

output
```
(b^2*x^(2*n) + 4*a*b*x^n + 2*a^2*n*log(x))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{x^{2n}b^2+4x^nab+2\log(x)\,a^2n}{2n}$$

input
```
int((a+b*x^n)^2/x,x)
```

output
```
(x**(2*n)*b**2 + 4*x**n*a*b + 2*log(x)*a**2*n)/(2*n)
```

3.350 $\int \frac{(a+bx^n)^2}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{(a+bx^n)^2}{x^2}\,dx=-\frac{a^2}{x}-\frac{2abx^{-1+n}}{1-n}-\frac{b^2x^{-1+2n}}{1-2n}$$

output
```
-a^2/x-2*a*b*x^(-1+n)/(1-n)-b^2*x^(-1+2*n)/(1-2*n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^n)^2}{x^2}\,dx=\frac{-a^2+\frac{2abx^n}{-1+n}+\frac{b^2x^{2n}}{-1+2n}}{x}$$

input
```
Integrate[(a + b*x^n)^2/x^2,x]
```

output
```
(-a^2 + (2*a*b*x^n)/(-1 + n) + (b^2*x^(2*n))/(-1 + 2*n))/x
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{x^2} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a^2}{x^2}+2abx^{n-2}+b^2x^{2(n-1)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2}{x}-\frac{2abx^{n-1}}{1-n}-\frac{b^2x^{2n-1}}{1-2n}$$

input
```
Int[(a + b*x^n)^2/x^2,x]
```

output
```
-(a^2/x) - (2*a*b*x^(-1 + n))/(1 - n) - (b^2*x^(-1 + 2*n))/(1 - 2*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\frac{b^{2} e^{2 n \ln \left(x \right)}}{2 n -1}-a^{2}+\frac{2 a b \,e^{n \ln \left(x \right)}}{-1+n}}{x}$
risch	$-\frac{a^{2}}{x}+\frac{b^{2} x^{2 n}}{\left(2 n -1\right) x}+\frac{2 a b \,x^{n}}{\left(-1+n \right) x}$
parallelrisch	$\frac{b^{2} n \,x^{2 n}-b^{2} x^{2 n}+4 a b \,x^{n} n -2 a^{2} n^{2}-2 a b \,x^{n}+3 a^{2} n -a^{2}}{x \left(2 n -1\right) \left(-1+n \right)}$
orering	$-\frac{\left(2 n -7\right) \left(a +b \,x^{n}\right)^{2}}{x \left(2 n -1\right)}+\frac{3 x^{2} \left(n -2\right) \left(\frac{2 \left(a +b \,x^{n}\right) b \,x^{n} n}{x^{3}}-\frac{2 \left(a +b \,x^{n}\right)^{2}}{x^{3}}\right)}{2 n^{2}-3 n +1}-\frac{x^{3} \left(\frac{2 b^{2} x^{2 n} n^{2}}{x^{4}}-\frac{10 \left(a +b \,x^{n}\right) b \,x^{n} n}{x^{4}}+\frac{2 \left(a +b \,x^{n}\right) b \,x^{n} n^{2}}{x^{4}}+\right.}{2 n^{2}-3 n +1}$

input
```
int((a+b*x^n)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2/(2*n-1)*exp(n*ln(x))^2-a^2+2*a*b/(-1+n)*exp(n*ln(x)))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx^n)^2}{x^2}\,dx=-\frac{2\,a^2n^2-3\,a^2n+a^2-(b^2n-b^2)x^{2\,n}-2\,(2\,abn-ab)x^n}{(2\,n^2-3\,n+1)x}$$

input
```
integrate((a+b*x^n)^2/x^2,x, algorithm="fricas")
```

output
```
-(2*a^2*n^2 - 3*a^2*n + a^2 - (b^2*n - b^2)*x^(2*n) - 2*(2*a*b*n - a*b)*x^
n)/((2*n^2 - 3*n + 1)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.32

$$\int \frac{(a+bx^n)^2}{x^2}\,dx = \begin{cases} -\frac{a^2}{x} - \frac{4ab}{\sqrt{x}} + b^2\log(x) & \text{for } n \\ -\frac{a^2}{x} + 2ab\log(x) + b^2x & \text{for } n \\ -\frac{2a^2n^2}{2n^2x-3nx+x} + \frac{3a^2n}{2n^2x-3nx+x} - \frac{a^2}{2n^2x-3nx+x} + \frac{4abnx^n}{2n^2x-3nx+x} - \frac{2abx^n}{2n^2x-3nx+x} + \frac{b^2nx^{2n}}{2n^2x-3nx+x} - \frac{b^2x^{2n}}{2n^2x-3nx+x} & \text{other} \end{cases}$$

input
```
integrate((a+b*x**n)**2/x**2,x)
```

output
```
Piecewise((-a**2/x - 4*a*b/sqrt(x) + b**2*log(x), Eq(n, 1/2)), (-a**2/x +
2*a*b*log(x) + b**2*x, Eq(n, 1)), (-2*a**2*n**2/(2*n**2*x - 3*n*x + x) + 3
*a**2*n/(2*n**2*x - 3*n*x + x) - a**2/(2*n**2*x - 3*n*x + x) + 4*a*b*n*x**
n/(2*n**2*x - 3*n*x + x) - 2*a*b*x**n/(2*n**2*x - 3*n*x + x) + b**2*n*x**(
2*n)/(2*n**2*x - 3*n*x + x) - b**2*x**(2*n)/(2*n**2*x - 3*n*x + x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-2>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{x^2}\,dx = \int \frac{(bx^n+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^2}{x^2}\,dx = \frac{b^2\,x^{2\,n}}{x\,(2\,n-1)} - \frac{a^2}{x} + \frac{2\,a\,b\,x^n}{x\,(n-1)}$$

input
```
int((a + b*x^n)^2/x^2,x)
```

output
```
(b^2*x^(2*n))/(x*(2*n - 1)) - a^2/x + (2*a*b*x^n)/(x*(n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^n)^2}{x^2}\,dx = \frac{x^{2n}b^2n - x^{2n}b^2 + 4x^nabn - 2x^nab - 2a^2n^2 + 3a^2n - a^2}{x\,(2n^2-3n+1)}$$

input
```
int((a+b*x^n)^2/x^2,x)
```

output
```
(x**(2*n)*b**2*n - x**(2*n)*b**2 + 4*x**n*a*b*n - 2*x**n*a*b - 2*a**2*n**2
 + 3*a**2*n - a**2)/(x*(2*n**2 - 3*n + 1))
```

3.351 $\int \frac{(a+bx^n)^2}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = -\frac{a^2}{2x^2} - \frac{b^2x^{-2(1-n)}}{2(1-n)} - \frac{2abx^{-2+n}}{2-n}$$

output
```
-1/2*a^2/x^2-1/2*b^2/(1-n)/(x^(2-2*n))-2*a*b*x^(-2+n)/(2-n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = \frac{-a^2+\frac{4abx^n}{-2+n}+\frac{b^2x^{2n}}{-1+n}}{2x^2}$$

input
```
Integrate[(a + b*x^n)^2/x^3,x]
```

output
```
(-a^2 + (4*a*b*x^n)/(-2 + n) + (b^2*x^(2*n))/(-1 + n))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^2}{x^3}+2abx^{n-3}+b^2x^{2n-3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2x^2}-\frac{2abx^{n-2}}{2-n}-\frac{b^2x^{-2(1-n)}}{2(1-n)}$$

input
```
Int[(a + b*x^n)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 - b^2/(2*(1 - n)*x^(2*(1 - n))) - (2*a*b*x^(-2 + n))/(2 - n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result
norman	$\frac{-\frac{a^{2}}{2}+\frac{b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{-2+2 n}+\frac{2 a b \,{\mathrm e}^{n \ln \left(x \right)}}{n -2}}{x^{2}}$
risch	$-\frac{a^{2}}{2 x^{2}}+\frac{b^{2} x^{2 n}}{2 \left(-1+n \right) x^{2}}+\frac{2 a b \,x^{n}}{\left(n -2\right) x^{2}}$
parallelrisch	$\frac{b^{2} n \,x^{2 n}-2 b^{2} x^{2 n}+4 a b \,x^{n} n -a^{2} n^{2}-4 a b \,x^{n}+3 a^{2} n -2 a^{2}}{2 x^{2} \left(-1+n \right) \left(n -2\right)}$
orering	$-\frac{\left(2 n^{2}-15 n +19\right) \left(a +b \,x^{n}\right)^{2}}{4 x^{2} \left(n -2\right) \left(-1+n \right)}+\frac{3 x^{2} \left(-3+n \right) \left(\frac{2 \left(a +b \,x^{n}\right) b \,x^{n} n}{x^{4}}-\frac{3 \left(a +b \,x^{n}\right)^{2}}{x^{4}}\right)}{4 \left(n -2\right) \left(-1+n \right)}-\frac{x^{3} \left(\frac{2 b^{2} x^{2 n} n^{2}}{x^{5}}-\frac{14 \left(a +b \,x^{n}\right) b \,x^{n} n}{x^{5}}+\frac{2 (a+b x^{n}}{x^{5}}}{4 \left(n^{2}-3 n +2\right)}$

input
```
int((a+b*x^n)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a^2+1/2*b^2/(-1+n)*exp(n*ln(x))^2+2*a*b/(n-2)*exp(n*ln(x)))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = -\frac{a^2n^2 - 3\,a^2n + 2\,a^2 - (b^2n - 2\,b^2)x^{2\,n} - 4\,(abn - ab)x^n}{2\,(n^2 - 3\,n + 2)x^2}$$

input
```
integrate((a+b*x^n)^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*n^2 - 3*a^2*n + 2*a^2 - (b^2*n - 2*b^2)*x^(2*n) - 4*(a*b*n - a*b
)*x^n)/((n^2 - 3*n + 2)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.90

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = \begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x) \\ -\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2} \\ -\frac{a^2n^2}{2n^2x^2-6nx^2+4x^2} + \frac{3a^2n}{2n^2x^2-6nx^2+4x^2} - \frac{2a^2}{2n^2x^2-6nx^2+4x^2} + \frac{4abnx^n}{2n^2x^2-6nx^2+4x^2} - \frac{4abx^n}{2n^2x^2-6nx^2+4x^2} + \frac{b^2nx^{2n}}{2n^2x^2-6nx^2+4x^2} - \end{cases}$$

input
```
integrate((a+b*x**n)**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(n, 1)), (-a**2/(2*x*
*2) + 2*a*b*log(x) + b**2*x**2/2, Eq(n, 2)), (-a**2*n**2/(2*n**2*x**2 - 6*
n*x**2 + 4*x**2) + 3*a**2*n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) - 2*a**2/(2*
n**2*x**2 - 6*n*x**2 + 4*x**2) + 4*a*b*n*x**n/(2*n**2*x**2 - 6*n*x**2 + 4*
x**2) - 4*a*b*x**n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) + b**2*n*x**(2*n)/(2*
n**2*x**2 - 6*n*x**2 + 4*x**2) - 2*b**2*x**(2*n)/(2*n**2*x**2 - 6*n*x**2 +
 4*x**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = \int \frac{(bx^n+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = \frac{b^2\,x^{2\,n}}{x^2\,(2\,n-2)} - \frac{a^2}{2\,x^2} + \frac{2\,a\,b\,x^n}{x^2\,(n-2)}$$

input
```
int((a + b*x^n)^2/x^3,x)
```

output
```
(b^2*x^(2*n))/(x^2*(2*n - 2)) - a^2/(2*x^2) + (2*a*b*x^n)/(x^2*(n - 2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^n)^2}{x^3}\,dx = \frac{x^{2n}b^2n-2x^{2n}b^2+4x^nabn-4x^nab-a^2n^2+3a^2n-2a^2}{2x^2\,(n^2-3n+2)}$$

input
```
int((a+b*x^n)^2/x^3,x)
```

output
```
(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 4*x**n*a*b*n - 4*x**n*a*b - a**2*n**2
 + 3*a**2*n - 2*a**2)/(2*x**2*(n**2 - 3*n + 2))
```

3.352 $\int x^3(a+bx^n)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int x^3(a+bx^n)^3\,dx=\frac{a^3x^4}{4}+\frac{3ab^2x^{2(2+n)}}{2(2+n)}+\frac{3a^2bx^{4+n}}{4+n}+\frac{b^3x^{4+3n}}{4+3n}$$

output
```
1/4*a^3*x^4+3*a*b^2*x^(4+2*n)/(4+2*n)+3*a^2*b*x^(4+n)/(4+n)+b^3*x^(4+3*n)/
(4+3*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int x^3(a+bx^n)^3\,dx=\frac{1}{4}x^4\left(a^3+\frac{12a^2bx^n}{4+n}+\frac{6ab^2x^{2n}}{2+n}+\frac{4b^3x^{3n}}{4+3n}\right)$$

input
```
Integrate[x^3*(a + b*x^n)^3,x]
```

output
```
(x^4*(a^3 + (12*a^2*b*x^n)/(4 + n) + (6*a*b^2*x^(2*n))/(2 + n) + (4*b^3*x^
(3*n))/(4 + 3*n)))/4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^3+3a^2bx^{n+3}+3ab^2x^{2n+3}+b^3x^{3(n+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^4}{4}+\frac{3a^2bx^{n+4}}{n+4}+\frac{3ab^2x^{2(n+2)}}{2(n+2)}+\frac{b^3x^{3n+4}}{3n+4}$$

input
```
Int[x^3*(a + b*x^n)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a*b^2*x^(2*(2 + n)))/(2*(2 + n)) + (3*a^2*b*x^(4 + n))/(4
 + n) + (b^3*x^(4 + 3*n))/(4 + 3*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

method	result
risch	$\frac{a^3x^4}{4}+\frac{b^3x^4x^{3n}}{4+3n}+\frac{3a\,b^2x^4x^{2n}}{2(2+n)}+\frac{3a^2b\,x^4x^n}{4+n}$
parallelrisch	$\frac{4x^4x^{3n}b^3n^2+24x^4x^{3n}b^3n+18x^4x^{2n}a\,b^2n^2+32b^3x^4x^{3n}+96x^4x^{2n}a\,b^2n+36x^4x^n a^2b\,n^2+3x^4a^3n^3+96a\,b^2x^4x^{2n}+120x^4x^n a}{4(4+3n)(2+n)(4+n)}$
orering	$\frac{x^4(6n^3+77n^2+222n+175)(a+b\,x^n)^3}{24n^3+176n^2+384n+256}-\frac{x^2(11n^2+54n+55)\left(3x^2(a+b\,x^n)^3+3x^2(a+b\,x^n)^2b\,x^n n\right)}{8(3n^3+22n^2+48n+32)}+\frac{x^3(3n+5)\left(6x(a+b\right.}{}$

input
```
int(x^3*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*x^4+b^3/(4+3*n)*x^4*(x^n)^3+3/2*a*b^2*x^4/(2+n)*(x^n)^2+3*a^2*b/(4
+n)*x^4*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(61) = 122.

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.23

$$\int x^3(a+bx^n)^3\,dx$$
$$=\frac{4\,(b^3n^2+6\,b^3n+8\,b^3)x^4x^{3\,n}+6\,(3\,ab^2n^2+16\,ab^2n+16\,ab^2)x^4x^{2\,n}+12\,(3\,a^2bn^2+10\,a^2bn+8\,a^2b)x^4x^n}{4\,(3\,n^3+22\,n^2+48\,n+32)}$$

input
```
integrate(x^3*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/4*(4*(b^3*n^2 + 6*b^3*n + 8*b^3)*x^4*x^(3*n) + 6*(3*a*b^2*n^2 + 16*a*b^2
*n + 16*a*b^2)*x^4*x^(2*n) + 12*(3*a^2*b*n^2 + 10*a^2*b*n + 8*a^2*b)*x^4*x
^n + (3*a^3*n^3 + 22*a^3*n^2 + 48*a^3*n + 32*a^3)*x^4)/(3*n^3 + 22*n^2 + 4
8*n + 32)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(56) = 112$.

Time = 1.19 (sec) , antiderivative size = 507, normalized size of antiderivative = 7.80

$$\int x^3(a+bx^n)^3\,dx$$

$$= \begin{cases} \frac{a^3x^4}{4} + 3a^2b\log(x) - \frac{3ab^2}{4x^4} - \frac{b^3}{8x^8} \\ \frac{a^3x^4}{4} + \frac{3a^2bx^2}{2} + 3ab^2\log(x) - \frac{b^3}{2x^2} \\ \frac{a^3x^4}{4} + \frac{9a^2bx^{\frac{8}{3}}}{8} + \frac{9ab^2x^{\frac{4}{3}}}{4} + b^3\log(x) \\ \frac{3a^3n^3x^4}{12n^3+88n^2+192n+128} + \frac{22a^3n^2x^4}{12n^3+88n^2+192n+128} + \frac{48a^3nx^4}{12n^3+88n^2+192n+128} + \frac{32a^3x^4}{12n^3+88n^2+192n+128} + \frac{36a^2bn^2x^4x^n}{12n^3+88n^2+192n+128} \end{cases}$$

input

```
integrate(x**3*(a+b*x**n)**3,x)
```

output

```
Piecewise((a**3*x**4/4 + 3*a**2*b*log(x) - 3*a*b**2/(4*x**4) - b**3/(8*x**
8), Eq(n, -4)), (a**3*x**4/4 + 3*a**2*b*x**2/2 + 3*a*b**2*log(x) - b**3/(2
*x**2), Eq(n, -2)), (a**3*x**4/4 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(4/3)
/4 + b**3*log(x), Eq(n, -4/3)), (3*a**3*n**3*x**4/(12*n**3 + 88*n**2 + 192
*n + 128) + 22*a**3*n**2*x**4/(12*n**3 + 88*n**2 + 192*n + 128) + 48*a**3*
n*x**4/(12*n**3 + 88*n**2 + 192*n + 128) + 32*a**3*x**4/(12*n**3 + 88*n**2
 + 192*n + 128) + 36*a**2*b*n**2*x**4*x**n/(12*n**3 + 88*n**2 + 192*n + 12
8) + 120*a**2*b*n*x**4*x**n/(12*n**3 + 88*n**2 + 192*n + 128) + 96*a**2*b*
x**4*x**n/(12*n**3 + 88*n**2 + 192*n + 128) + 18*a*b**2*n**2*x**4*x**(2*n)
/(12*n**3 + 88*n**2 + 192*n + 128) + 96*a*b**2*n*x**4*x**(2*n)/(12*n**3 +
88*n**2 + 192*n + 128) + 96*a*b**2*x**4*x**(2*n)/(12*n**3 + 88*n**2 + 192*
n + 128) + 4*b**3*n**2*x**4*x**(3*n)/(12*n**3 + 88*n**2 + 192*n + 128) + 2
4*b**3*n*x**4*x**(3*n)/(12*n**3 + 88*n**2 + 192*n + 128) + 32*b**3*x**4*x*
*(3*n)/(12*n**3 + 88*n**2 + 192*n + 128), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x^3(a+bx^n)^3\,dx = \frac{1}{4}a^3x^4 + \frac{b^3x^{3n+4}}{3n+4} + \frac{3ab^2x^{2n+4}}{2(n+2)} + \frac{3a^2bx^{n+4}}{n+4}$$

input
```
integrate(x^3*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/4*a^3*x^4 + b^3*x^(3*n + 4)/(3*n + 4) + 3/2*a*b^2*x^(2*n + 4)/(n + 2) +
3*a^2*b*x^(n + 4)/(n + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(61) = 122.

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.89

$$\int x^3(a+bx^n)^3\,dx = \frac{4b^3n^2x^4x^{3n} + 18ab^2n^2x^4x^{2n} + 36a^2bn^2x^4x^n + 3a^3n^3x^4 + 24b^3nx^4x^{3n} + 96ab^2nx^4x^{2n} + 120a^2bnx^4x^n}{4(3n^3 + 22n^2 + 48n + 32)}$$

input
```
integrate(x^3*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/4*(4*b^3*n^2*x^4*x^(3*n) + 18*a*b^2*n^2*x^4*x^(2*n) + 36*a^2*b*n^2*x^4*x
^n + 3*a^3*n^3*x^4 + 24*b^3*n*x^4*x^(3*n) + 96*a*b^2*n*x^4*x^(2*n) + 120*a
^2*b*n*x^4*x^n + 22*a^3*n^2*x^4 + 32*b^3*x^4*x^(3*n) + 96*a*b^2*x^4*x^(2*n
) + 96*a^2*b*x^4*x^n + 48*a^3*n*x^4 + 32*a^3*x^4)/(3*n^3 + 22*n^2 + 48*n +
 32)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int x^3(a+bx^n)^3\,dx=\frac{a^3\,x^4}{4}+\frac{b^3\,x^{3\,n}\,x^4}{3\,n+4}+\frac{3\,a\,b^2\,x^{2\,n}\,x^4}{2\,n+4}+\frac{3\,a^2\,b\,x^n\,x^4}{n+4}$$

input
```
int(x^3*(a + b*x^n)^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^(3*n)*x^4)/(3*n + 4) + (3*a*b^2*x^(2*n)*x^4)/(2*n + 4
) + (3*a^2*b*x^n*x^4)/(n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.32

$$\int x^3(a+bx^n)^3\,dx$$
$$=\frac{x^4(4x^{3n}b^3n^2+24x^{3n}b^3n+32x^{3n}b^3+18x^{2n}a\,b^2n^2+96x^{2n}a\,b^2n+96x^{2n}a\,b^2+36x^na^2b\,n^2+120x^na^2bn-}{12n^3+88n^2+192n+128}$$

input
```
int(x^3*(a+b*x^n)^3,x)
```

output
```
(x**4*(4*x**(3*n)*b**3*n**2 + 24*x**(3*n)*b**3*n + 32*x**(3*n)*b**3 + 18*x
**(2*n)*a*b**2*n**2 + 96*x**(2*n)*a*b**2*n + 96*x**(2*n)*a*b**2 + 36*x**n*
a**2*b*n**2 + 120*x**n*a**2*b*n + 96*x**n*a**2*b + 3*a**3*n**3 + 22*a**3*n
**2 + 48*a**3*n + 32*a**3))/(4*(3*n**3 + 22*n**2 + 48*n + 32))
```

3.353 $\int x^2(a+bx^n)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int x^2(a+bx^n)^3\,dx=\frac{a^3x^3}{3}+\frac{b^3x^{3(1+n)}}{3(1+n)}+\frac{3a^2bx^{3+n}}{3+n}+\frac{3ab^2x^{3+2n}}{3+2n}$$

output
```
1/3*a^3*x^3+b^3*x^(3+3*n)/(3+3*n)+3*a^2*b*x^(3+n)/(3+n)+3*a*b^2*x^(3+2*n)/
(3+2*n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int x^2(a+bx^n)^3\,dx=\frac{1}{3}x^3\left(a^3+\frac{9a^2bx^n}{3+n}+\frac{9ab^2x^{2n}}{3+2n}+\frac{b^3x^{3n}}{1+n}\right)$$

input
```
Integrate[x^2*(a + b*x^n)^3,x]
```

output
```
(x^3*(a^3 + (9*a^2*b*x^n)/(3 + n) + (9*a*b^2*x^(2*n))/(3 + 2*n) + (b^3*x^(
3*n))/(1 + n)))/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^2+3a^2bx^{n+2}+3ab^2x^{2(n+1)}+b^3x^{3n+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3}{3}+\frac{3a^2bx^{n+3}}{n+3}+\frac{3ab^2x^{2n+3}}{2n+3}+\frac{b^3x^{3(n+1)}}{3(n+1)}$$

input
```
Int[x^2*(a + b*x^n)^3,x]
```

output
```
(a^3*x^3)/3 + (b^3*x^(3*(1 + n)))/(3*(1 + n)) + (3*a^2*b*x^(3 + n))/(3 + n
) + (3*a*b^2*x^(3 + 2*n))/(3 + 2*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

method	result
risch	$\frac{a^3x^3}{3}+\frac{b^3x^3x^{3n}}{3+3n}+\frac{3a\,b^2x^3x^{2n}}{2n+3}+\frac{3a^2b\,x^3x^n}{3+n}$
norman	$\frac{a^3x^3}{3}+\frac{b^3x^3\mathrm{e}^{3n\ln(x)}}{3+3n}+\frac{3a\,b^2x^3\mathrm{e}^{2n\ln(x)}}{2n+3}+\frac{3a^2b\,x^3\mathrm{e}^{n\ln(x)}}{3+n}$
parallelrisch	$\frac{2x^3x^{3n}b^3n^2+9x^3x^{3n}b^3n+9x^3x^{2n}a\,b^2n^2+9b^3x^3x^{3n}+36x^3x^{2n}a\,b^2n+18x^3x^na^2b\,n^2+2x^3a^3n^3+27a\,b^2x^3x^{2n}+45x^3x^na^2bn+}{3(1+n)(2n+3)(3+n)}$
orering	$\frac{x^3(6n^2+49n+65)(a+b\,x^n)^3}{18n^2+81n+81}-\frac{x^2(11n+25)\left(2x(a+b\,x^n)^3+3x(a+b\,x^n)^2b\,x^nn\right)}{9(2n^2+9n+9)}+\frac{2x^3\left(2(a+b\,x^n)^3+9(a+b\,x^n)^2b\,x^nn+6\right.}{3(2n^2}$

input
```
int(x^2*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+1/3*b^3/(1+n)*x^3*(x^n)^3+3*a*b^2/(2*n+3)*x^3*(x^n)^2+3*a^2*b/
(3+n)*x^3*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(62) = 124$.

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.18

$$\int x^2(a+bx^n)^3\,dx$$
$$=\frac{(2\,b^3n^2+9\,b^3n+9\,b^3)x^3x^{3\,n}+9\,(ab^2n^2+4\,ab^2n+3\,ab^2)x^3x^{2\,n}+9\,(2\,a^2bn^2+5\,a^2bn+3\,a^2b)x^3x^n+(2\,a}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

input
```
integrate(x^2*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n
+ 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (
2*a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(56) = 112$.

Time = 0.62 (sec) , antiderivative size = 500, normalized size of antiderivative = 7.58

$$\int x^2(a+bx^n)^3\,dx$$
$$= \begin{cases} \frac{a^3x^3}{3}+3a^2b\log{(x)}-\frac{ab^2}{x^3}-\frac{b^3}{6x^6} \\ \frac{a^3x^3}{3}+2a^2bx^{\frac{3}{2}}+3ab^2\log{(x)}-\frac{2b^3}{3x^{\frac{3}{2}}} \\ \frac{a^3x^3}{3}+\frac{3a^2bx^2}{2}+3ab^2x+b^3\log{(x)} \\ \frac{2a^3n^3x^3}{6n^3+33n^2+54n+27}+\frac{11a^3n^2x^3}{6n^3+33n^2+54n+27}+\frac{18a^3nx^3}{6n^3+33n^2+54n+27}+\frac{9a^3x^3}{6n^3+33n^2+54n+27}+\frac{18a^2bn^2x^3x^n}{6n^3+33n^2+54n+27}+\frac{45a^2bnx^3x}{6n^3+33n^2+54} \end{cases}$$

input
```
integrate(x**2*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*x**3/3 + 3*a**2*b*log(x) - a*b**2/x**3 - b**3/(6*x**6), Eq
(n, -3)), (a**3*x**3/3 + 2*a**2*b*x**(3/2) + 3*a*b**2*log(x) - 2*b**3/(3*x
**(3/2)), Eq(n, -3/2)), (a**3*x**3/3 + 3*a**2*b*x**2/2 + 3*a*b**2*x + b**3
*log(x), Eq(n, -1)), (2*a**3*n**3*x**3/(6*n**3 + 33*n**2 + 54*n + 27) + 11
*a**3*n**2*x**3/(6*n**3 + 33*n**2 + 54*n + 27) + 18*a**3*n*x**3/(6*n**3 +
33*n**2 + 54*n + 27) + 9*a**3*x**3/(6*n**3 + 33*n**2 + 54*n + 27) + 18*a**
2*b*n**2*x**3*x**n/(6*n**3 + 33*n**2 + 54*n + 27) + 45*a**2*b*n*x**3*x**n/
(6*n**3 + 33*n**2 + 54*n + 27) + 27*a**2*b*x**3*x**n/(6*n**3 + 33*n**2 + 5
4*n + 27) + 9*a*b**2*n**2*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 3
6*a*b**2*n*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 27*a*b**2*x**3*x
**(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 2*b**3*n**2*x**3*x**(3*n)/(6*n**3
 + 33*n**2 + 54*n + 27) + 9*b**3*n*x**3*x**(3*n)/(6*n**3 + 33*n**2 + 54*n
+ 27) + 9*b**3*x**3*x**(3*n)/(6*n**3 + 33*n**2 + 54*n + 27), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int x^2(a+bx^n)^3\,dx=\frac{1}{3}a^3x^3+\frac{b^3x^{3n+3}}{3(n+1)}+\frac{3ab^2x^{2n+3}}{2n+3}+\frac{3a^2bx^{n+3}}{n+3}$$

input
```
integrate(x^2*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 + 1/3*b^3*x^(3*n + 3)/(n + 1) + 3*a*b^2*x^(2*n + 3)/(2*n + 3)
+ 3*a^2*b*x^(n + 3)/(n + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(62) = 124.

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.85

$$\int x^2(a+bx^n)^3\,dx$$
$$=\frac{2b^3n^2x^3x^{3n}+9ab^2n^2x^3x^{2n}+18a^2bn^2x^3x^n+2a^3n^3x^3+9b^3nx^3x^{3n}+36ab^2nx^3x^{2n}+45a^2bnx^3x^n+}{3(2n^3+11n^2+18n+9)}$$

input
```
integrate(x^2*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/3*(2*b^3*n^2*x^3*x^(3*n) + 9*a*b^2*n^2*x^3*x^(2*n) + 18*a^2*b*n^2*x^3*x^
n + 2*a^3*n^3*x^3 + 9*b^3*n*x^3*x^(3*n) + 36*a*b^2*n*x^3*x^(2*n) + 45*a^2*
b*n*x^3*x^n + 11*a^3*n^2*x^3 + 9*b^3*x^3*x^(3*n) + 27*a*b^2*x^3*x^(2*n) +
27*a^2*b*x^3*x^n + 18*a^3*n*x^3 + 9*a^3*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^n)^3\,dx=\frac{a^3\,x^3}{3}+\frac{b^3\,x^{3\,n}\,x^3}{3\,n+3}+\frac{3\,a\,b^2\,x^{2\,n}\,x^3}{2\,n+3}+\frac{3\,a^2\,b\,x^n\,x^3}{n+3}$$

input
```
int(x^2*(a + b*x^n)^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^(3*n)*x^3)/(3*n + 3) + (3*a*b^2*x^(2*n)*x^3)/(2*n + 3
) + (3*a^2*b*x^n*x^3)/(n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.29

$$\int x^2(a+bx^n)^3\,dx$$
$$=\frac{x^3(2x^{3n}b^3n^2+9x^{3n}b^3n+9x^{3n}b^3+9x^{2n}a\,b^2n^2+36x^{2n}a\,b^2n+27x^{2n}a\,b^2+18x^na^2b\,n^2+45x^na^2bn+27x}{6n^3+33n^2+54n+27}$$

input
```
int(x^2*(a+b*x^n)^3,x)
```

output
```
(x**3*(2*x**(3*n)*b**3*n**2 + 9*x**(3*n)*b**3*n + 9*x**(3*n)*b**3 + 9*x**(
2*n)*a*b**2*n**2 + 36*x**(2*n)*a*b**2*n + 27*x**(2*n)*a*b**2 + 18*x**n*a**
2*b*n**2 + 45*x**n*a**2*b*n + 27*x**n*a**2*b + 2*a**3*n**3 + 11*a**3*n**2
+ 18*a**3*n + 9*a**3))/(3*(2*n**3 + 11*n**2 + 18*n + 9))
```

3.354 $\int x(a+bx^n)^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int x(a+bx^n)^3 \, dx = \frac{a^3x^2}{2} + \frac{3ab^2x^{2(1+n)}}{2(1+n)} + \frac{3a^2bx^{2+n}}{2+n} + \frac{b^3x^{2+3n}}{2+3n}$$

output
```
1/2*a^3*x^2+3*a*b^2*x^(2+2*n)/(2+2*n)+3*a^2*b*x^(2+n)/(2+n)+b^3*x^(2+3*n)/
(2+3*n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int x(a+bx^n)^3 \, dx = \frac{1}{2}x^2\left(a^3 + \frac{6a^2bx^n}{2+n} + \frac{3ab^2x^{2n}}{1+n} + \frac{2b^3x^{3n}}{2+3n}\right)$$

input
```
Integrate[x*(a + b*x^n)^3,x]
```

output
```
(x^2*(a^3 + (6*a^2*b*x^n)/(2 + n) + (3*a*b^2*x^(2*n))/(1 + n) + (2*b^3*x^(
3*n))/(2 + 3*n)))/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^3 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x+3a^2bx^{n+1}+3ab^2x^{2n+1}+b^3x^{3n+1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^2}{2}+\frac{3a^2bx^{n+2}}{n+2}+\frac{3ab^2x^{2(n+1)}}{2(n+1)}+\frac{b^3x^{3n+2}}{3n+2}$$

input
```
Int[x*(a + b*x^n)^3,x]
```

output
```
(a^3*x^2)/2 + (3*a*b^2*x^(2*(1 + n)))/(2*(1 + n)) + (3*a^2*b*x^(2 + n))/(2
 + n) + (b^3*x^(2 + 3*n))/(2 + 3*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

method	result
risch	$\frac{b^3x^2x^{3n}}{2+3n}+\frac{a^3x^2}{2}+\frac{3a\,b^2x^2x^{2n}}{2(1+n)}+\frac{3a^2b\,x^2x^n}{2+n}$
norman	$\frac{b^3x^2\mathrm{e}^{3n\ln(x)}}{2+3n}+\frac{a^3x^2}{2}+\frac{3a\,b^2x^2\mathrm{e}^{2n\ln(x)}}{2(1+n)}+\frac{3a^2b\,x^2\mathrm{e}^{n\ln(x)}}{2+n}$
parallelrisch	$\frac{2x^2x^{3n}b^3n^2+6x^2x^{3n}b^3n+9x^2x^{2n}a\,b^2n^2+4b^3x^2x^{3n}+24x^2x^{2n}a\,b^2n+18x^2x^na^2b\,n^2+3x^2a^3n^3+12a\,b^2x^2x^{2n}+30x^2x^na^2bn+}{2(2+3n)(1+n)(2+n)}$
orering	$\frac{3x^2(2n^2+9n+5)(a+b\,x^n)^3}{4(3n^2+8n+4)}-\frac{(11n+7)x^2\left((a+b\,x^n)^3+3(a+b\,x^n)^2b\,x^nn\right)}{4(3n^2+8n+4)}+\frac{x^3(1+3n)\left(\frac{3(a+b\,x^n)^2b\,x^nn}{x}+\frac{6(a+b\,x^n)b^2x^{2}}{x}\right.}{6n^3+22n^2+24n+8}$

input
```
int(x*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3/(2+3*n)*x^2*(x^n)^3+1/2*a^3*x^2+3/2*a*b^2/(1+n)*x^2*(x^n)^2+3*a^2*b/(2
+n)*x^2*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(61) = 122.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.23

$$\int x(a+bx^n)^3\,dx$$
$$=\frac{2\,(b^3n^2+3\,b^3n+2\,b^3)x^2x^{3\,n}+3\,(3\,ab^2n^2+8\,ab^2n+4\,ab^2)x^2x^{2\,n}+6\,(3\,a^2bn^2+5\,a^2bn+2\,a^2b)x^2x^n+(3}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

input
```
integrate(x*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*
n + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n +
 (3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n +
 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(56) = 112$.

Time $= 0.49$ (sec) , antiderivative size $= 500$, normalized size of antiderivative $= 7.69$

$$\int x(a+bx^n)^3\,dx$$

$$= \begin{cases} \frac{a^3x^2}{2} + 3a^2b\log{(x)} - \frac{3ab^2}{2x^2} - \frac{b^3}{4x^4} \\ \frac{a^3x^2}{2} + 3a^2bx + 3ab^2\log{(x)} - \frac{b^3}{x} \\ \frac{a^3x^2}{2} + \frac{9a^2bx^{\frac{4}{3}}}{4} + \frac{9ab^2x^{\frac{2}{3}}}{2} + b^3\log{(x)} \\ \frac{3a^3n^3x^2}{6n^3+22n^2+24n+8} + \frac{11a^3n^2x^2}{6n^3+22n^2+24n+8} + \frac{12a^3nx^2}{6n^3+22n^2+24n+8} + \frac{4a^3x^2}{6n^3+22n^2+24n+8} + \frac{18a^2bn^2x^2x^n}{6n^3+22n^2+24n+8} + \frac{30a^2bnx^2x^n}{6n^3+22n^2+24n+8} \end{cases}$$

input
```
integrate(x*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*x**2/2 + 3*a**2*b*log(x) - 3*a*b**2/(2*x**2) - b**3/(4*x**
4), Eq(n, -2)), (a**3*x**2/2 + 3*a**2*b*x + 3*a*b**2*log(x) - b**3/x, Eq(n
, -1)), (a**3*x**2/2 + 9*a**2*b*x**(4/3)/4 + 9*a*b**2*x**(2/3)/2 + b**3*lo
g(x), Eq(n, -2/3)), (3*a**3*n**3*x**2/(6*n**3 + 22*n**2 + 24*n + 8) + 11*a
**3*n**2*x**2/(6*n**3 + 22*n**2 + 24*n + 8) + 12*a**3*n*x**2/(6*n**3 + 22*
n**2 + 24*n + 8) + 4*a**3*x**2/(6*n**3 + 22*n**2 + 24*n + 8) + 18*a**2*b*n
**2*x**2*x**n/(6*n**3 + 22*n**2 + 24*n + 8) + 30*a**2*b*n*x**2*x**n/(6*n**
3 + 22*n**2 + 24*n + 8) + 12*a**2*b*x**2*x**n/(6*n**3 + 22*n**2 + 24*n + 8
) + 9*a*b**2*n**2*x**2*x**(2*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 24*a*b**2*
n*x**2*x**(2*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 12*a*b**2*x**2*x**(2*n)/(6
*n**3 + 22*n**2 + 24*n + 8) + 2*b**3*n**2*x**2*x**(3*n)/(6*n**3 + 22*n**2
+ 24*n + 8) + 6*b**3*n*x**2*x**(3*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 4*b**
3*x**2*x**(3*n)/(6*n**3 + 22*n**2 + 24*n + 8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x(a+bx^n)^3\,dx = \frac{1}{2}\,a^3x^2 + \frac{b^3x^{3\,n+2}}{3\,n+2} + \frac{3\,ab^2x^{2\,n+2}}{2\,(n+1)} + \frac{3\,a^2bx^{n+2}}{n+2}$$

input
```
integrate(x*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/2*a^3*x^2 + b^3*x^(3*n + 2)/(3*n + 2) + 3/2*a*b^2*x^(2*n + 2)/(n + 1) +
3*a^2*b*x^(n + 2)/(n + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(61) = 122.

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.89

$$\int x(a+bx^n)^3\,dx$$
$$= \frac{2\,b^3n^2x^2x^{3\,n} + 9\,ab^2n^2x^2x^{2\,n} + 18\,a^2bn^2x^2x^n + 3\,a^3n^3x^2 + 6\,b^3nx^2x^{3\,n} + 24\,ab^2nx^2x^{2\,n} + 30\,a^2bnx^2x^n + 1}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

input
```
integrate(x*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/2*(2*b^3*n^2*x^2*x^(3*n) + 9*a*b^2*n^2*x^2*x^(2*n) + 18*a^2*b*n^2*x^2*x^
n + 3*a^3*n^3*x^2 + 6*b^3*n*x^2*x^(3*n) + 24*a*b^2*n*x^2*x^(2*n) + 30*a^2*
b*n*x^2*x^n + 11*a^3*n^2*x^2 + 4*b^3*x^2*x^(3*n) + 12*a*b^2*x^2*x^(2*n) +
12*a^2*b*x^2*x^n + 12*a^3*n*x^2 + 4*a^3*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int x(a+bx^n)^3\,dx=\frac{a^3\,x^2}{2}+\frac{b^3\,x^{3\,n}\,x^2}{3\,n+2}+\frac{3\,a\,b^2\,x^{2\,n}\,x^2}{2\,n+2}+\frac{3\,a^2\,b\,x^n\,x^2}{n+2}$$

input
```
int(x*(a + b*x^n)^3,x)
```

output
```
(a^3*x^2)/2 + (b^3*x^(3*n)*x^2)/(3*n + 2) + (3*a*b^2*x^(2*n)*x^2)/(2*n + 2
) + (3*a^2*b*x^n*x^2)/(n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.32

$$\int x(a+bx^n)^3\,dx$$
$$=\frac{x^2(2x^{3n}b^3n^2+6x^{3n}b^3n+4x^{3n}b^3+9x^{2n}a\,b^2n^2+24x^{2n}a\,b^2n+12x^{2n}a\,b^2+18x^na^2b\,n^2+30x^na^2bn+12x}{6n^3+22n^2+24n+8}$$

input
```
int(x*(a+b*x^n)^3,x)
```

output
```
(x**2*(2*x**(3*n)*b**3*n**2 + 6*x**(3*n)*b**3*n + 4*x**(3*n)*b**3 + 9*x**(
2*n)*a*b**2*n**2 + 24*x**(2*n)*a*b**2*n + 12*x**(2*n)*a*b**2 + 18*x**n*a**
2*b*n**2 + 30*x**n*a**2*b*n + 12*x**n*a**2*b + 3*a**3*n**3 + 11*a**3*n**2
+ 12*a**3*n + 4*a**3))/(2*(3*n**3 + 11*n**2 + 12*n + 4))
```

3.355 $\int (a+bx^n)^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 60

$$\int (a+bx^n)^3 \, dx = a^3x + \frac{3a^2bx^{1+n}}{1+n} + \frac{3ab^2x^{1+2n}}{1+2n} + \frac{b^3x^{1+3n}}{1+3n}$$

output
```
a^3*x+3*a^2*b*x^(1+n)/(1+n)+3*a*b^2*x^(1+2*n)/(1+2*n)+b^3*x^(1+3*n)/(1+3*n
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^n)^3 \, dx = x\left(a^3 + \frac{3a^2bx^n}{1+n} + \frac{3ab^2x^{2n}}{1+2n} + \frac{b^3x^{3n}}{1+3n}\right)$$

input
```
Integrate[(a + b*x^n)^3,x]
```

output
```
x*(a^3 + (3*a^2*b*x^n)/(1 + n) + (3*a*b^2*x^(2*n))/(1 + 2*n) + (b^3*x^(3*n
))/(1 + 3*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {775, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^3\,dx$$

$$\Big\downarrow \text{775}$$

$$\int \left(a^3+3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+\frac{3a^2bx^{n+1}}{n+1}+\frac{3ab^2x^{2n+1}}{2n+1}+\frac{b^3x^{3n+1}}{3n+1}$$

input
```
Int[(a + b*x^n)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^(1 + n))/(1 + n) + (3*a*b^2*x^(1 + 2*n))/(1 + 2*n) + (b
^3*x^(1 + 3*n))/(1 + 3*n)
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

method	result
risch	$a^{3} x +\frac{b^{3} x \,x^{3 n}}{1+3 n}+\frac{3 a \,b^{2} x \,x^{2 n}}{1+2 n}+\frac{3 a^{2} b x \,x^{n}}{1+n}$
norman	$a^{3} x +\frac{b^{3} x \,{\mathrm e}^{3 n \ln \left(x \right)}}{1+3 n}+\frac{3 a \,b^{2} x \,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}+\frac{3 a^{2} b x \,{\mathrm e}^{n \ln \left(x \right)}}{1+n}$
parallelrisch	$\frac{2 x \,x^{3 n} b^{3} n^{2}+3 x \,x^{3 n} b^{3} n +9 x \,x^{2 n} a \,b^{2} n^{2}+b^{3} x \,x^{3 n}+12 x \,x^{2 n} a \,b^{2} n +18 x \,x^{n} a^{2} b \,n^{2}+6 x \,a^{3} n^{3}+3 a \,b^{2} x \,x^{2 n}+15 x \,x^{n} a^{2} b n +11 x \,a^{3} n^{2}}{\left(1+3 n \right) \left(1+2 n \right) \left(1+n \right)}$
orering	$x \left(a +b \,x^{n}\right)^{3}-\frac{3 x \left(11 n^{2}+1\right) \left(a +b \,x^{n}\right)^{2} b \,x^{n} n}{\left(2 n^{2}+3 n +1\right) \left(1+3 n \right)}+\frac{2 x^{3} \left(-1+3 n \right) \left(\frac{6 \left(a +b \,x^{n}\right) b^{2} x^{2 n} n^{2}}{x^{2}}+\frac{3 \left(a +b \,x^{n}\right)^{2} b \,x^{n} n^{2}}{x^{2}}-\frac{3 \left(a +b \,x^{n}\right)^{2} b \,x^{n} n}{x^{2}}\right.}{\left(2 n^{2}+3 n +1\right) \left(1+3 n \right)}$

input
```
int((a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+b^3/(1+3*n)*x*(x^n)^3+3*a*b^2/(1+2*n)*x*(x^n)^2+3*a^2*b/(1+n)*x*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(60) = 120.

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.17

$$\int (a+bx^n)^3\,dx = \frac{(2\,b^3n^2+3\,b^3n+b^3)xx^{3\,n}+3\,(3\,ab^2n^2+4\,ab^2n+ab^2)xx^{2\,n}+3\,(6\,a^2bn^2+5\,a^2bn+a^2b)xx^n+(6\,a^3n^3+}{6\,n^3+11\,n^2+6\,n+1}$$

input
```
integrate((a+b*x^n)^3,x, algorithm="fricas")
```

output
```
((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^
2)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11
*a^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(53) = 106$.

Time = 0.44 (sec) , antiderivative size = 469, normalized size of antiderivative = 7.82

$$\int (a+bx^n)^3\,dx = \begin{cases} a^3x + 3a^2b\log(x) - \frac{3ab^2}{x} - \frac{b^3}{2x^2} \\ a^3x + 6a^2b\sqrt{x} + 3ab^2\log(x) - \frac{2b^3}{\sqrt{x}} \\ a^3x + \frac{9a^2bx^{\frac{2}{3}}}{2} + 9ab^2\sqrt[3]{x} + b^3\log(x) \\ \frac{6a^3n^3x}{6n^3+11n^2+6n+1} + \frac{11a^3n^2x}{6n^3+11n^2+6n+1} + \frac{6a^3nx}{6n^3+11n^2+6n+1} + \frac{a^3x}{6n^3+11n^2+6n+1} + \frac{18a^2bn^2xx^n}{6n^3+11n^2+6n+1} + \frac{15a^2bnxx^n}{6n^3+11n^2+6n+1} + \frac{}{6n^3} \end{cases}$$

input
```
integrate((a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*log(x) - 3*a*b**2/x - b**3/(2*x**2), Eq(n, -1
)), (a**3*x + 6*a**2*b*sqrt(x) + 3*a*b**2*log(x) - 2*b**3/sqrt(x), Eq(n, -
1/2)), (a**3*x + 9*a**2*b*x**(2/3)/2 + 9*a*b**2*x**(1/3) + b**3*log(x), Eq
(n, -1/3)), (6*a**3*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a**3*n**2*x/(
6*n**3 + 11*n**2 + 6*n + 1) + 6*a**3*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*
*3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 18*a**2*b*n**2*x*x**n/(6*n**3 + 11*n**
2 + 6*n + 1) + 15*a**2*b*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a**2*b*
x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 9*a*b**2*n**2*x*x**(2*n)/(6*n**3 + 1
1*n**2 + 6*n + 1) + 12*a*b**2*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) +
3*a*b**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b**3*n**2*x*x**(3*n)/
(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**3*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n
 + 1) + b**3*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^3\,dx = a^3x + \frac{b^3x^{3n+1}}{3n+1} + \frac{3ab^2x^{2n+1}}{2n+1} + \frac{3a^2bx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^3,x, algorithm="maxima")
```

output
```
a^3*x + b^3*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*x^(2*n + 1)/(2*n + 1) + 3*a^2*
b*x^(n + 1)/(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(60) = 120$.

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.65

$$\int (a + bx^n)^3 \, dx$$
$$= \frac{6\,a^3n^3x + 2\,b^3n^2xx^{3\,n} + 9\,ab^2n^2xx^{2\,n} + 18\,a^2bn^2xx^n + 11\,a^3n^2x + 3\,b^3nxx^{3\,n} + 12\,ab^2nxx^{2\,n} + 15\,a^2bnxx}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

input
```
integrate((a+b*x^n)^3,x, algorithm="giac")
```

output
```
(6*a^3*n^3*x + 2*b^3*n^2*x*x^(3*n) + 9*a*b^2*n^2*x*x^(2*n) + 18*a^2*b*n^2*
x*x^n + 11*a^3*n^2*x + 3*b^3*n*x*x^(3*n) + 12*a*b^2*n*x*x^(2*n) + 15*a^2*b
*n*x*x^n + 6*a^3*n*x + b^3*x*x^(3*n) + 3*a*b^2*x*x^(2*n) + 3*a^2*b*x*x^n +
 a^3*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int (a + bx^n)^3 \, dx = a^3\,x + \frac{b^3\,x\,x^{3\,n}}{3\,n+1} + \frac{3\,a\,b^2\,x\,x^{2\,n}}{2\,n+1} + \frac{3\,a^2\,b\,x\,x^n}{n+1}$$

input
```
int((a + b*x^n)^3,x)
```

output
```
a^3*x + (b^3*x*x^(3*n))/(3*n + 1) + (3*a*b^2*x*x^(2*n))/(2*n + 1) + (3*a^2
*b*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.43

$$\int (a+bx^n)^3\, dx$$
$$= \frac{x(2x^{3n}b^3n^2+3x^{3n}b^3n+x^{3n}b^3+9x^{2n}a\,b^2n^2+12x^{2n}a\,b^2n+3x^{2n}a\,b^2+18x^na^2b\,n^2+15x^na^2bn+3x^na^2b}{6n^3+11n^2+6n+1}$$

input
```
int((a+b*x^n)^3,x)
```

output
```
(x*(2*x**(3*n)*b**3*n**2 + 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 9*x**(2*n)*
a*b**2*n**2 + 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n*
*2 + 15*x**n*a**2*b*n + 3*x**n*a**2*b + 6*a**3*n**3 + 11*a**3*n**2 + 6*a**
3*n + a**3))/(6*n**3 + 11*n**2 + 6*n + 1)
```

3.356 $\int \frac{(a+bx^n)^3}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{(a+bx^n)^3}{x} dx = \frac{3a^2bx^n}{n} + \frac{3ab^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n} + a^3\log(x)$$

output
```
3*a^2*b*x^n/n+3/2*a*b^2*x^(2*n)/n+1/3*b^3*x^(3*n)/n+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n)^3}{x} dx = \frac{bx^n(18a^2 + 9abx^n + 2b^2x^{2n})}{6n} + \frac{a^3\log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)^3/x,x]
```

output
```
(b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)))/(6*n) + (a^3*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}(bx^n+a)^3 \, dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^3x^{-n}+3ab^2x^n+b^3x^{2n}+3a^2b\right) dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\log\left(x^n\right)+3a^2bx^n+\frac{3}{2}ab^2x^{2n}+\frac{1}{3}b^3x^{3n}}{n}$$

input
```
Int[(a + b*x^n)^3/x,x]
```

output
```
(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/2 + (b^3*x^(3*n))/3 + a^3*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{b^3 x^{3n}}{3}+\frac{3a\,b^2 x^{2n}}{2}+3a^2 b\,x^n+a^3\ln(x^n)}{n}$	44
default	$\frac{\frac{b^3 x^{3n}}{3}+\frac{3a\,b^2 x^{2n}}{2}+3a^2 b\,x^n+a^3\ln(x^n)}{n}$	44
parallelrisch	$\frac{2b^3 x^{3n}+9a\,b^2 x^{2n}+6a^3\ln(x)n+18a^2 b\,x^n}{6n}$	45
risch	$\frac{3a^2 b\,x^n}{n}+\frac{3a\,b^2 x^{2n}}{2n}+\frac{b^3 x^{3n}}{3n}+a^3\ln(x)$	47
norman	$a^3\ln(x)+\frac{b^3 e^{3n\ln(x)}}{3n}+\frac{3a\,b^2 e^{2n\ln(x)}}{2n}+\frac{3a^2 b\,e^{n\ln(x)}}{n}$	53

input
```
int((a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/3*b^3*(x^n)^3+3/2*a*b^2*(x^n)^2+3*a^2*b*x^n+a^3*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^n)^3}{x}\,dx = \frac{6\,a^3 n\log(x)+2\,b^3 x^{3\,n}+9\,ab^2 x^{2\,n}+18\,a^2 b x^n}{6\,n}$$

input
```
integrate((a+b*x^n)^3/x,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)^3}{x}\,dx = \begin{cases} a^3\log(x) + \frac{3a^2bx^n}{n} + \frac{3ab^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**3/x,x)
```

output
```
Piecewise((a**3*log(x) + 3*a**2*b*x**n/n + 3*a*b**2*x**(2*n)/(2*n) + b**3*
x**(3*n)/(3*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^n)^3}{x}\,dx = \frac{a^3\log(x^n)}{n} + \frac{2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

input
```
integrate((a+b*x^n)^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x^n)/n + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Giac [F]

$$\int \frac{(a+bx^n)^3}{x}\,dx = \int \frac{(bx^n+a)^3}{x}\,dx$$

input
```
integrate((a+b*x^n)^3/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n)^3}{x}\,dx = a^3\,\ln(x)+\frac{b^3\,x^{3\,n}}{3\,n}+\frac{3\,a^2\,b\,x^n}{n}+\frac{3\,a\,b^2\,x^{2\,n}}{2\,n}$$

input
```
int((a + b*x^n)^3/x,x)
```

output
```
a^3*log(x) + (b^3*x^(3*n))/(3*n) + (3*a^2*b*x^n)/n + (3*a*b^2*x^(2*n))/(2*
n)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^n)^3}{x}\,dx = \frac{2x^{3n}b^3+9x^{2n}a\,b^2+18x^n a^2 b+6\log(x)\,a^3 n}{6n}$$

input
```
int((a+b*x^n)^3/x,x)
```

output
```
(2*x**(3*n)*b**3 + 9*x**(2*n)*a*b**2 + 18*x**n*a**2*b + 6*log(x)*a**3*n)/(
6*n)
```

3.357 $\int \frac{(a+bx^n)^3}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{(a+bx^n)^3}{x^2} dx = -\frac{a^3}{x} - \frac{3a^2bx^{-1+n}}{1-n} - \frac{3ab^2x^{-1+2n}}{1-2n} - \frac{b^3x^{-1+3n}}{1-3n}$$

output
```
-a^3/x-3*a^2*b*x^(-1+n)/(1-n)-3*a*b^2*x^(-1+2*n)/(1-2*n)-b^3*x^(-1+3*n)/(1
-3*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^n)^3}{x^2} dx = \frac{-a^3 + \frac{3a^2bx^n}{-1+n} + \frac{3ab^2x^{2n}}{-1+2n} + \frac{b^3x^{3n}}{-1+3n}}{x}$$

input
```
Integrate[(a + b*x^n)^3/x^2,x]
```

output
```
(-a^3 + (3*a^2*b*x^n)/(-1 + n) + (3*a*b^2*x^(2*n))/(-1 + 2*n) + (b^3*x^(3*
n))/(-1 + 3*n))/x
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3}{x^2} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^3}{x^2}+3a^2bx^{n-2}+3ab^2x^{2(n-1)}+b^3x^{3n-2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{x}-\frac{3a^2bx^{n-1}}{1-n}-\frac{3ab^2x^{2n-1}}{1-2n}-\frac{b^3x^{3n-1}}{1-3n}$$

input
```
Int[(a + b*x^n)^3/x^2,x]
```

output
```
-(a^3/x) - (3*a^2*b*x^(-1 + n))/(1 - n) - (3*a*b^2*x^(-1 + 2*n))/(1 - 2*n)
 - (b^3*x^(-1 + 3*n))/(1 - 3*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\frac{b^3 e^{3n\ln(x)}}{-1+3n}-a^3+\frac{3a\,b^2 e^{2n\ln(x)}}{2n-1}+\frac{3a^2 b\,e^{n\ln(x)}}{-1+n}}{x}$
risch	$-\frac{a^3}{x}+\frac{b^3 x^{3n}}{(-1+3n)x}+\frac{3a\,b^2 x^{2n}}{(2n-1)x}+\frac{3a^2 b\,x^n}{(-1+n)x}$
parallelrisch	$\frac{2x^{3n}b^3n^2-3x^{3n}b^3n+9x^{2n}a\,b^2n^2+b^3x^{3n}-12x^{2n}a\,b^2n+18x^na^2b\,n^2-6a^3n^3+3a\,b^2x^{2n}-15x^na^2bn+11a^3n^2+3a^2b\,x^n-6a^3n}{x(-1+3n)(2n-1)(-1+n)}$
orering	$-\frac{3(2n^2-9n+5)(a+b\,x^n)^3}{x(-1+3n)(2n-1)}+\frac{(11n-25)x^2\left(\frac{3(a+b\,x^n)^2b\,x^n n}{x^3}-\frac{2(a+b\,x^n)^3}{x^3}\right)}{6n^2-5n+1}-\frac{2x^3(-5+3n)\left(\frac{6b^2x^{2n}n^2(a+b\,x^n)}{x^4}+\frac{3(a+b\,x}{}\right.}{6n^3-11}$

input
```
int((a+b*x^n)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
(b^3/(-1+3*n)*exp(n*ln(x))^3-a^3+3*a*b^2/(2*n-1)*exp(n*ln(x))^2+3*a^2*b/(-
1+n)*exp(n*ln(x)))/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(63) = 126.

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx^n)^3}{x^2}\,dx = -\frac{6\,a^3n^3-11\,a^3n^2+6\,a^3n-a^3-(2\,b^3n^2-3\,b^3n+b^3)x^{3\,n}-3\,(3\,ab^2n^2-4\,ab^2n+ab^2)x^{2\,n}-3\,(6\,a^2bn^2}{(6\,n^3-11\,n^2+6\,n-1)x}$$

input
```
integrate((a+b*x^n)^3/x^2,x, algorithm="fricas")
```

output
```
-(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(
3*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^
2*b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(54) = 108$.

Time = 0.41 (sec) , antiderivative size = 508, normalized size of antiderivative = 7.70

$$\int \frac{(a+bx^n)^3}{x^2}\,dx$$
$$= \begin{cases} -\frac{a^3}{x} - \frac{9a^2b}{2x^{\frac{2}{3}}} - \frac{9ab^2}{\sqrt[3]{x}} + b^3\log(x) \\ -\frac{a^3}{x} - \frac{6a^2b}{\sqrt{x}} + 3ab^2\log(x) + 2b^3\sqrt{x} \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} \\ -\frac{6a^3n^3}{6n^3x-11n^2x+6nx-x} + \frac{11a^3n^2}{6n^3x-11n^2x+6nx-x} - \frac{6a^3n}{6n^3x-11n^2x+6nx-x} + \frac{a^3}{6n^3x-11n^2x+6nx-x} + \frac{18a^2bn^2x^n}{6n^3x-11n^2x+6nx-x} - \frac{}{6n^3} \end{cases}$$

input
```
integrate((a+b*x**n)**3/x**2,x)
```

output
```
Piecewise((-a**3/x - 9*a**2*b/(2*x**(2/3)) - 9*a*b**2/x**(1/3) + b**3*log(
x), Eq(n, 1/3)), (-a**3/x - 6*a**2*b/sqrt(x) + 3*a*b**2*log(x) + 2*b**3*sq
rt(x), Eq(n, 1/2)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2,
 Eq(n, 1)), (-6*a**3*n**3/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 11*a**3*n**
2/(6*n**3*x - 11*n**2*x + 6*n*x - x) - 6*a**3*n/(6*n**3*x - 11*n**2*x + 6*
n*x - x) + a**3/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 18*a**2*b*n**2*x**n/(
6*n**3*x - 11*n**2*x + 6*n*x - x) - 15*a**2*b*n*x**n/(6*n**3*x - 11*n**2*x
 + 6*n*x - x) + 3*a**2*b*x**n/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 9*a*b**
2*n**2*x**(2*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) - 12*a*b**2*n*x**(2*n)/
(6*n**3*x - 11*n**2*x + 6*n*x - x) + 3*a*b**2*x**(2*n)/(6*n**3*x - 11*n**2
*x + 6*n*x - x) + 2*b**3*n**2*x**(3*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x)
- 3*b**3*n*x**(3*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) + b**3*x**(3*n)/(6*
n**3*x - 11*n**2*x + 6*n*x - x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^3}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^3/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-2>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)^3}{x^2}\,dx = \int \frac{(bx^n+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^3}{x^2}\,dx = \frac{b^3\,x^{3\,n}}{x\,(3\,n-1)} - \frac{a^3}{x} + \frac{3\,a\,b^2\,x^{2\,n}}{x\,(2\,n-1)} + \frac{3\,a^2\,b\,x^n}{x\,(n-1)}$$

input
```
int((a + b*x^n)^3/x^2,x)
```

output
```
(b^3*x^(3*n))/(x*(3*n - 1)) - a^3/x + (3*a*b^2*x^(2*n))/(x*(2*n - 1)) + (3
*a^2*b*x^n)/(x*(n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx^n)^3}{x^2}\,dx = \frac{2x^{3n}b^3n^2 - 3x^{3n}b^3n + x^{3n}b^3 + 9x^{2n}a\,b^2n^2 - 12x^{2n}a\,b^2n + 3x^{2n}a\,b^2 + 18x^na^2b\,n^2 - 15x^na^2bn + 3x^na^2b -}{x\,(6n^3 - 11n^2 + 6n - 1)}$$

input
```
int((a+b*x^n)^3/x^2,x)
```

output
```
(2*x**(3*n)*b**3*n**2 - 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 9*x**(2*n)*a*b
**2*n**2 - 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n**2
- 15*x**n*a**2*b*n + 3*x**n*a**2*b - 6*a**3*n**3 + 11*a**3*n**2 - 6*a**3*n
 + a**3)/(x*(6*n**3 - 11*n**2 + 6*n - 1))
```

3.358 $\int \frac{(a+bx^n)^3}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{(a+bx^n)^3}{x^3} dx = -\frac{a^3}{2x^2} - \frac{3ab^2x^{-2(1-n)}}{2(1-n)} - \frac{3a^2bx^{-2+n}}{2-n} - \frac{b^3x^{-2+3n}}{2-3n}$$

output
```
-1/2*a^3/x^2-3/2*a*b^2/(1-n)/(x^(2-2*n))-3*a^2*b*x^(-2+n)/(2-n)-b^3*x^(-2+
3*n)/(2-3*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^3}{x^3} dx = \frac{-a^3 + \frac{6a^2bx^n}{-2+n} + \frac{3ab^2x^{2n}}{-1+n} + \frac{2b^3x^{3n}}{-2+3n}}{2x^2}$$

input
```
Integrate[(a + b*x^n)^3/x^3,x]
```

output
```
(-a^3 + (6*a^2*b*x^n)/(-2 + n) + (3*a*b^2*x^(2*n))/(-1 + n) + (2*b^3*x^(3*
n))/(-2 + 3*n))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3}{x^3}dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^3}+3a^2bx^{n-3}+3ab^2x^{2n-3}+b^3x^{3(n-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{2x^2}-\frac{3a^2bx^{n-2}}{2-n}-\frac{3ab^2x^{-2(1-n)}}{2(1-n)}-\frac{b^3x^{3n-2}}{2-3n}$$

input
```
Int[(a + b*x^n)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 - (3*a*b^2)/(2*(1 - n)*x^(2*(1 - n))) - (3*a^2*b*x^(-2 + n))/
(2 - n) - (b^3*x^(-2 + 3*n))/(2 - 3*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{a^{3}}{2x^{2}}+\frac{b^{3}x^{3n}}{(-2+3n)x^{2}}+\frac{3a\,b^{2}x^{2n}}{2(-1+n)x^{2}}+\frac{3a^{2}b\,x^{n}}{(n-2)x^{2}}$
parallelrisch	$\frac{2x^{3n}b^{3}n^{2}-6x^{3n}b^{3}n+9x^{2n}a\,b^{2}n^{2}+4b^{3}x^{3n}-24x^{2n}a\,b^{2}n+18x^{n}a^{2}b\,n^{2}-3a^{3}n^{3}+12a\,b^{2}x^{2n}-30x^{n}a^{2}bn+11a^{3}n^{2}+12a^{2}b\,x^{n}-12}{2x^{2}(-2+3n)(-1+n)(n-2)}$
orering	$-\frac{(6n^{2}-49n+65)(a+b\,x^{n})^{3}}{4x^{2}(3n^{2}-8n+4)}+\frac{x^{2}(11n^{2}-54n+55)\left(\frac{3(a+b\,x^{n})^{2}b\,x^{n}n}{x^{4}}-\frac{3(a+b\,x^{n})^{3}}{x^{4}}\right)}{12n^{3}-44n^{2}+48n-16}-\frac{x^{3}(-7+3n)\left(\frac{6b^{2}x^{2n}n^{2}(a+b\,x^{n})}{x^{5}}+\right.}{2(}$

input
```
int((a+b*x^n)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2+1/(-2+3*n)*b^3/x^2*(x^n)^3+3/2/(-1+n)*a*b^2/x^2*(x^n)^2+3/(n-
2)*a^2*b/x^2*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(63) = 126.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^n)^3}{x^3}\,dx = -\frac{3\,a^3n^3-11\,a^3n^2+12\,a^3n-4\,a^3-2\,(b^3n^2-3\,b^3n+2\,b^3)x^{3\,n}-3\,(3\,ab^2n^2-8\,ab^2n+4\,ab^2)x^{2\,n}-6\,(3}{2\,(3\,n^3-11\,n^2+12\,n-4)x^2}$$

input
```
integrate((a+b*x^n)^3/x^3,x, algorithm="fricas")
```

output
```
-1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2
*b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b
*n^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(58) = 116$.

Time = 0.57 (sec) , antiderivative size = 627, normalized size of antiderivative = 8.71

$$\int \frac{(a+bx^n)^3}{x^3}\,dx = \begin{cases} -\frac{a^3}{2x^2} - \frac{9a^2b}{4x^{\frac{4}{3}}} - \frac{9ab^2}{2x^{\frac{2}{3}}} + b^3\log(x) \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x \\ -\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} \\ -\frac{3a^3n^3}{6n^3x^2-22n^2x^2+24nx^2-8x^2} + \frac{11a^3n^2}{6n^3x^2-22n^2x^2+24nx^2-8x^2} - \frac{12a^3n}{6n^3x^2-22n^2x^2+24nx^2-8x^2} + \frac{4a^3}{6n^3x^2-22n^2x^2+24nx^2-8x^2} + \frac{}{6n} \end{cases}$$

input
```
integrate((a+b*x**n)**3/x**3,x)
```

output
```
Piecewise((-a**3/(2*x**2) - 9*a**2*b/(4*x**(4/3)) - 9*a*b**2/(2*x**(2/3))
+ b**3*log(x), Eq(n, 2/3)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x)
 + b**3*x, Eq(n, 1)), (-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2
+ b**3*x**4/4, Eq(n, 2)), (-3*a**3*n**3/(6*n**3*x**2 - 22*n**2*x**2 + 24*n
*x**2 - 8*x**2) + 11*a**3*n**2/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8
*x**2) - 12*a**3*n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 4*a
**3/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 18*a**2*b*n**2*x**
n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) - 30*a**2*b*n*x**n/(6*
n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 12*a**2*b*x**n/(6*n**3*x*
*2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 9*a*b**2*n**2*x**(2*n)/(6*n**3*x
**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) - 24*a*b**2*n*x**(2*n)/(6*n**3*x*
*2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 12*a*b**2*x**(2*n)/(6*n**3*x**2
- 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 2*b**3*n**2*x**(3*n)/(6*n**3*x**2 -
 22*n**2*x**2 + 24*n*x**2 - 8*x**2) - 6*b**3*n*x**(3*n)/(6*n**3*x**2 - 22*
n**2*x**2 + 24*n*x**2 - 8*x**2) + 4*b**3*x**(3*n)/(6*n**3*x**2 - 22*n**2*x
**2 + 24*n*x**2 - 8*x**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^3}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^3/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)^3}{x^3}\,dx = \int \frac{(bx^n+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n)^3}{x^3}\,dx = \frac{b^3\,x^{3\,n}}{x^2\,\left(3\,n-2\right)} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^{2\,n}}{x^2\,\left(2\,n-2\right)} + \frac{3\,a^2\,b\,x^n}{x^2\,\left(n-2\right)}$$

input
```
int((a + b*x^n)^3/x^3,x)
```

output
```
(b^3*x^(3*n))/(x^2*(3*n - 2)) - a^3/(2*x^2) + (3*a*b^2*x^(2*n))/(x^2*(2*n
- 2)) + (3*a^2*b*x^n)/(x^2*(n - 2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx^n)^3}{x^3}\,dx = \frac{2x^{3n}b^3n^2-6x^{3n}b^3n+4x^{3n}b^3+9x^{2n}a\,b^2n^2-24x^{2n}a\,b^2n+12x^{2n}a\,b^2+18x^{n}a^2b\,n^2-30x^{n}a^2bn+12x^{n}a}{2x^2\left(3n^3-11n^2+12n-4\right)}$$

input
```
int((a+b*x^n)^3/x^3,x)
```

output
```
(2*x**(3*n)*b**3*n**2 - 6*x**(3*n)*b**3*n + 4*x**(3*n)*b**3 + 9*x**(2*n)*a
*b**2*n**2 - 24*x**(2*n)*a*b**2*n + 12*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n*
*2 - 30*x**n*a**2*b*n + 12*x**n*a**2*b - 3*a**3*n**3 + 11*a**3*n**2 - 12*a
**3*n + 4*a**3)/(2*x**2*(3*n**3 - 11*n**2 + 12*n - 4))
```

3.359 $\int \frac{x}{a+bx^n} dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{x}{a+bx^n} dx = \frac{x^2 \text{Hypergeometric2F1}\left(1, \frac{2}{n}, \frac{2+n}{n}, -\frac{bx^n}{a}\right)}{2a}$$

output
```
1/2*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx^n} dx = \frac{x^2 \text{Hypergeometric2F1}\left(1, \frac{2}{n}, 1+\frac{2}{n}, -\frac{bx^n}{a}\right)}{2a}$$

input
```
Integrate[x/(a + b*x^n),x]
```

output
```
(x^2*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^n}\,dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a}$$

input
```
Int[x/(a + b*x^n),x]
```

output
```
(x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{a+b\,x^{n}}dx$$

input
```
int(x/(a+b*x^n),x)
```

output
```
int(x/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{x}{a+bx^n}\,dx = \int \frac{x}{bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(x/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{x}{a+bx^n}\,dx = \frac{2a^{\frac{2}{n}}a^{-1-\frac{2}{n}}x^2\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{n^2\Gamma\left(1+\frac{2}{n}\right)}$$

input
```
integrate(x/(a+b*x**n),x)
```

output
```
2*a**(2/n)*a**(-1 - 2/n)*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*g
amma(2/n)/(n**2*gamma(1 + 2/n))
```

Maxima [F]

$$\int \frac{x}{a+bx^n}\,dx = \int \frac{x}{bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x}{a+bx^n}\,dx=\int \frac{x}{bx^n+a}\,dx$$

input
```
integrate(x/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+bx^n}\,dx=\int \frac{x}{a+b\,x^n}\,dx$$

input
```
int(x/(a + b*x^n),x)
```

output
```
int(x/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{x}{a+bx^n}\,dx=\int \frac{x}{x^n b+a}dx$$

input
```
int(x/(a+b*x^n),x)
```

output
```
int(x/(x**n*b + a),x)
```

3.360 $\int \frac{1}{a+bx^n} dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^n} dx = \frac{x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

output
```
x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^n} dx = \frac{x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

input
```
Integrate[(a + b*x^n)^(-1),x]
```

output
```
(x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^n} \, dx$$

$$\Big\downarrow \text{778}$$

$$\frac{x \operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

input
```
Int[(a + b*x^n)^(-1),x]
```

output
```
(x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{a+b\,x^n}dx$$

input
```
int(1/(a+b*x^n),x)
```

output
```
int(1/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{1}{a+bx^n}\,dx=\int \frac{1}{bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \frac{1}{a+bx^n}\,dx=\frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate(1/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(
1/n)/(n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{1}{a+bx^n}\,dx=\int \frac{1}{bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(1/(b*x^n + a), x)
```

Giac [F]

$$\int \frac{1}{a+bx^n}\,dx = \int \frac{1}{bx^n+a}\,dx$$

input
```
integrate(1/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{a+bx^n}\,dx = \frac{x\,{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a}$$

input
```
int(1/(a + b*x^n),x)
```

output
```
(x*hypergeom([1, 1/n], 1/n + 1, -(b*x^n)/a))/a
```

Reduce [F]

$$\int \frac{1}{a+bx^n}\,dx = \int \frac{1}{x^n b+a}dx$$

input
```
int(1/(a+b*x^n),x)
```

output
```
int(1/(x**n*b + a),x)
```

3.361 $\int \frac{1}{x(a+bx^n)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

output
```
ln(x)/a-ln(a+b*x^n)/a/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx=\frac{\log\left(x^n\right)-\log\left(an(a+bx^n)\right)}{an}$$

input
```
Integrate[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b x^n)} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{b x^n+a} dx^n}{n}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int x^{-n} dx^n}{a} - \frac{b \int \frac{1}{b x^n+a} dx^n}{a}}{n}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(x^n)}{a} - \frac{b \int \frac{1}{b x^n+a} dx^n}{a}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(x^n)}{a} - \frac{\log(a+b x^n)}{a}}{n}$$

input
```
Int[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n]/a - Log[a + b*x^n]/a)/n
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(a+b\,x^{n})}{an}$	23
norman	$\frac{\ln(x)}{a}-\frac{\ln\left(a+b\,{\mathrm e}^{n\ln(x)}\right)}{an}$	26
risch	$\frac{\ln(x)}{a}-\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{an}$	26
derivativedivides	$\frac{\frac{\ln(x^{n})}{a}-\frac{\ln(a+b\,x^{n})}{a}}{n}$	27
default	$\frac{\frac{\ln(x^{n})}{a}-\frac{\ln(a+b\,x^{n})}{a}}{n}$	27

input
```
int(1/x/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(a+b*x^n))/a/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^n)}\,dx = \frac{n\log(x)-\log(bx^n+a)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(15) = 30.

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{1}{x(a+bx^n)}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{x^{-n}}{bn} & \text{for } a=0 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(b*n*x**n), Eq
(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log
(a/b + x**n)/(a*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a+bx^n)}\,dx = -\frac{\log(bx^n+a)}{an} + \frac{\log(x^n)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="maxima")
```

output
```
-log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

Giac [F]

$$\int \frac{1}{x(a+bx^n)}\,dx = \int \frac{1}{(bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^n)}\,dx = -\frac{\ln(a+b\,x^n)-n\,\ln(x)}{a\,n}$$

input
```
int(1/(x*(a + b*x^n)),x)
```

output
```
-(log(a + b*x^n) - n*log(x))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx=\frac{-\log(x^n b+a)+\log(x)\,n}{an}$$

input
```
int(1/x/(a+b*x^n),x)
```

output
```
( - log(x**n*b + a) + log(x)*n)/(a*n)
```

3.362 $\int \frac{1}{x^2(a+bx^n)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{1}{x^2\,(a+bx^n)}\,dx=-\frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax}$$

output
```
-hypergeom([1, -1/n],[-(1-n)/n],-b*x^n/a)/a/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\,(a+bx^n)}\,dx=-\frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{ax}$$

input
```
Integrate[1/(x^2*(a + b*x^n)),x]
```

output
```
-(Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)]/(a*x))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n)} dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax}$$

input
```
Int[1/(x^2*(a + b*x^n)),x]
```

output
```
-(Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)]/(a*x))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2 (a+b\,x^n)} dx$$

input
```
int(1/x^2/(a+b*x^n),x)
```

output
```
int(1/x^2/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a + bx^n)} \, dx = \int \frac{1}{(bx^n + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*x^n + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^2 (a + bx^n)} \, dx = -\frac{a^{-\frac{1}{n}} a^{-1+\frac{1}{n}} \Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{e^{i\pi}}{n}\right) \Gamma\left(-\frac{1}{n}\right)}{n^2 x \Gamma\left(1 - \frac{1}{n}\right)}$$

input
```
integrate(1/x**2/(a+b*x**n),x)
```

output
```
-a**(-1 + 1/n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*ga
mma(-1/n)/(a**(1/n)*n**2*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n)} \, dx = \int \frac{1}{(bx^n + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+bx^n\right)}\,dx=\int \frac{1}{(bx^n+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^n\right)}\,dx=\int \frac{1}{x^2\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/(x^2*(a + b*x^n)),x)
```

output
```
int(1/(x^2*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(a+bx^n\right)}\,dx=\int \frac{1}{x^n b\,x^2+a\,x^2}dx$$

input
```
int(1/x^2/(a+b*x^n),x)
```

output
```
int(1/(x**n*b*x**2 + a*x**2),x)
```

3.363 $\int \frac{1}{x^3(a+bx^n)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{1}{x^3\left(a+bx^n\right)}\,dx=-\frac{\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2}$$

output
```
-1/2*hypergeom([1, -2/n],[-(2-n)/n],-b*x^n/a)/a/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\left(a+bx^n\right)}\,dx=-\frac{\text{Hypergeometric2F1}\left(1,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2ax^2}$$

input
```
Integrate[1/(x^3*(a + b*x^n)),x]
```

output
```
-1/2*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)]/(a*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + bx^n)} \, dx$$

$$\Big\downarrow \; 888$$

$$-\frac{\text{Hypergeometric2F1}\left(1, -\frac{2}{n}, -\frac{2-n}{n}, -\frac{bx^n}{a}\right)}{2ax^2}$$

input
```
Int[1/(x^3*(a + b*x^n)),x]
```

output
```
-1/2*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)]/(a*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^3 (a + b\,x^n)} dx$$

input
```
int(1/x^3/(a+b*x^n),x)
```

output
```
int(1/x^3/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a + bx^n)} \, dx = \int \frac{1}{(bx^n + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*x^n + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^3 (a + bx^n)} \, dx = -\frac{2a^{-\frac{2}{n}} a^{-1+\frac{2}{n}} \Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{2e^{i\pi}}{n}\right) \Gamma\left(-\frac{2}{n}\right)}{n^2 x^2 \Gamma\left(1 - \frac{2}{n}\right)}$$

input
```
integrate(1/x**3/(a+b*x**n),x)
```

output
```
-2*a**(-1 + 2/n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n
)*gamma(-2/n)/(a**(2/n)*n**2*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n)} \, dx = \int \frac{1}{(bx^n + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^n)} \, dx = \int \frac{1}{(bx^n + a)x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n)} \, dx = \int \frac{1}{x^3 (a + b\,x^n)} \, dx$$

input
```
int(1/(x^3*(a + b*x^n)),x)
```

output
```
int(1/(x^3*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a + bx^n)} \, dx = \int \frac{1}{x^n b\,x^3 + a\,x^3} dx$$

input
```
int(1/x^3/(a+b*x^n),x)
```

output
```
int(1/(x**n*b*x**3 + a*x**3),x)
```

3.364 $\int \frac{x}{(a+bx^n)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{x}{(a+bx^n)^2} \, dx = \frac{x^2 \text{Hypergeometric2F1}\left(2, \frac{2}{n}, \frac{2+n}{n}, -\frac{bx^n}{a}\right)}{2a^2}$$

output
```
1/2*x^2*hypergeom([2, 2/n],[(2+n)/n],-b*x^n/a)/a^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^n)^2} \, dx = \frac{x^2 \text{Hypergeometric2F1}\left(2, \frac{2}{n}, 1+\frac{2}{n}, -\frac{bx^n}{a}\right)}{2a^2}$$

input
```
Integrate[x/(a + b*x^n)^2,x]
```

output
```
(x^2*Hypergeometric2F1[2, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 888$$

$$\frac{x^2 \,\text{Hypergeometric2F1}\left(2,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^2}$$

input
```
Int[x/(a + b*x^n)^2,x]
```

output
```
(x^2*Hypergeometric2F1[2, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{(a+b\,x^n)^2}dx$$

input
```
int(x/(a+b*x^n)^2,x)
```

output
```
int(x/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^n)^2}\,dx = \int \frac{x}{(bx^n+a)^2}\,dx$$

input
```
integrate(x/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 335, normalized size of antiderivative = 10.15

$$\begin{aligned}\int \frac{x}{(a+bx^n)^2}\,dx &= \frac{2aa^{\frac{2}{n}}a^{-2-\frac{2}{n}}nx^2\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{an^3\Gamma\left(1+\frac{2}{n}\right)+bn^3x^n\Gamma\left(1+\frac{2}{n}\right)} \\ &\quad+ \frac{2aa^{\frac{2}{n}}a^{-2-\frac{2}{n}}nx^2\Gamma\left(\frac{2}{n}\right)}{an^3\Gamma\left(1+\frac{2}{n}\right)+bn^3x^n\Gamma\left(1+\frac{2}{n}\right)} \\ &\quad- \frac{4aa^{\frac{2}{n}}a^{-2-\frac{2}{n}}x^2\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{an^3\Gamma\left(1+\frac{2}{n}\right)+bn^3x^n\Gamma\left(1+\frac{2}{n}\right)} \\ &\quad+ \frac{2a^{\frac{2}{n}}a^{-2-\frac{2}{n}}bnx^2x^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{an^3\Gamma\left(1+\frac{2}{n}\right)+bn^3x^n\Gamma\left(1+\frac{2}{n}\right)} \\ &\quad- \frac{4a^{\frac{2}{n}}a^{-2-\frac{2}{n}}bx^2x^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{an^3\Gamma\left(1+\frac{2}{n}\right)+bn^3x^n\Gamma\left(1+\frac{2}{n}\right)}\end{aligned}$$

input
```
integrate(x/(a+b*x**n)**2,x)
```

output
```
2*a*a**(2/n)*a**(-2 - 2/n)*n*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/
n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gamma(1 + 2/n)) + 2*a*a
**(2/n)*a**(-2 - 2/n)*n*x**2*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x*
*n*gamma(1 + 2/n)) - 4*a*a**(2/n)*a**(-2 - 2/n)*x**2*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gamm
a(1 + 2/n)) + 2*a**(2/n)*a**(-2 - 2/n)*b*n*x**2*x**n*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gamm
a(1 + 2/n)) - 4*a**(2/n)*a**(-2 - 2/n)*b*x**2*x**n*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gamma(
1 + 2/n))
```

Maxima [F]

$$\int \frac{x}{(a + bx^n)^2} \, dx = \int \frac{x}{(bx^n + a)^2} \, dx$$

input
```
integrate(x/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(n - 2)*integrate(x/(a*b*n*x^n + a^2*n), x) + x^2/(a*b*n*x^n + a^2*n)
```

Giac [F]

$$\int \frac{x}{(a + bx^n)^2} \, dx = \int \frac{x}{(bx^n + a)^2} \, dx$$

input
```
integrate(x/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)^2}\,dx = \int \frac{x}{(a+b\,x^n)^2}\,dx$$

input
```
int(x/(a + b*x^n)^2,x)
```

output
```
int(x/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^n)^2}\,dx = \int \frac{x}{x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int(x/(a+b*x^n)^2,x)
```

output
```
int(x/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.365 $\int \frac{1}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{(a+bx^n)^2} dx = \frac{x \,\text{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)}{a^2}$$

output
```
x*hypergeom([2, 1/n],[1+1/n],-b*x^n/a)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^n)^2} dx = \frac{x \,\text{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)}{a^2}$$

input
```
Integrate[(a + b*x^n)^(-2),x]
```

output
```
(x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \ 778$$

$$\frac{x \,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

input
```
Int[(a + b*x^n)^(-2),x]
```

output
```
(x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^2
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)^2}dx$$

input
```
int(1/(a+b*x^n)^2,x)
```

output
```
int(1/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2}\,dx$$

input
```
integrate(1/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 318, normalized size of antiderivative = 13.25

$$\int \frac{1}{(a+bx^n)^2}\,dx = \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate(1/(a+b*x**n)**2,x)
```

output
```
a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1/n)
*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1
 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1
, 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a
**(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n
)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**(1/
n)*a**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma
(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2} dx = \int \frac{1}{(bx^n+a)^2} dx$$

input
```
integrate(1/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + x/(a*b*n*x^n + a^2*n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2} dx = \int \frac{1}{(bx^n+a)^2} dx$$

input
```
integrate(1/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx^n)^2}\,dx = \frac{x\,{}_2F_1\left(2,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^2}$$

input
```
int(1/(a + b*x^n)^2,x)
```

output
```
(x*hypergeom([2, 1/n], 1/n + 1, -(b*x^n)/a))/a^2
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^2}\,dx = \int \frac{1}{x^{2n}b^2+2x^n ab+a^2}dx$$

input
```
int(1/(a+b*x^n)^2,x)
```

output
```
int(1/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.366 $\int \frac{1}{x(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx=\frac{1}{an\left(a+bx^n\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^n\right)}{a^2n}$$

output
```
1/a/n/(a+b*x^n)+ln(x)/a^2-ln(a+b*x^n)/a^2/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx=\frac{1}{an\left(a+bx^n\right)}+\frac{\log\left(x^n\right)}{a^2n}-\frac{\log\left(a+bx^n\right)}{a^2n}$$

input
```
Integrate[1/(x*(a + b*x^n)^2),x]
```

output
```
1/(a*n*(a + b*x^n)) + Log[x^n]/(a^2*n) - Log[a + b*x^n]/(a^2*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b x^n)^2} \, dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{x^{-n}}{(b x^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \; 54$$

$$\frac{\int \left(\frac{x^{-n}}{a^2}-\frac{b}{a^2 (b x^n+a)}-\frac{b}{a (b x^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\log (a+b x^n)}{a^2}+\frac{\log (x^n)}{a^2}+\frac{1}{a (a+b x^n)}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^2),x]
```

output
```
(1/(a*(a + b*x^n)) + Log[x^n]/a^2 - Log[a + b*x^n]/a^2)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\,x^n)}{a^2}+\frac{1}{a(a+b\,x^n)}+\frac{\ln(x^n)}{a^2}}{n}$	40
default	$\frac{-\frac{\ln(a+b\,x^n)}{a^2}+\frac{1}{a(a+b\,x^n)}+\frac{\ln(x^n)}{a^2}}{n}$	40
risch	$\frac{\ln(x)}{a^2}+\frac{1}{an(a+b\,x^n)}-\frac{\ln\left(x^n+\frac{a}{b}\right)}{a^2n}$	42
norman	$\frac{-\frac{b\,\mathrm{e}^{n\ln(x)}\ln(x)}{a^2n}+\frac{\ln(x)}{a}+\frac{b\ln(x)\mathrm{e}^{n\ln(x)}}{a^2}}{a+b\,\mathrm{e}^{n\ln(x)}}-\frac{\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{a^2n}$	65
parallelrisch	$\frac{\ln(x)x^nb^2n+\ln(x)abn-\ln(a+b\,x^n)x^nb^2-\ln(a+b\,x^n)ab+ab}{a^2bn(a+b\,x^n)}$	68

input
```
int(1/x/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/a^2*ln(a+b*x^n)+1/a/(a+b*x^n)+1/a^2*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx=\frac{bnx^n\log\left(x\right)+an\log\left(x\right)-\left(bx^n+a\right)\log\left(bx^n+a\right)+a}{a^2bnx^n+a^3n}$$

input
```
integrate(1/x/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
(b*n*x^n*log(x) + a*n*log(x) - (b*x^n + a)*log(b*x^n + a) + a)/(a^2*b*n*x^
n + a^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(31) = 62$.

Time = 0.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.10

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx = \begin{cases} \tilde{\infty}\log\left(x\right) & \text{for}\: a=0\wedge b=0\wedge n=0 \\ \frac{\log\left(x\right)}{a^2} & \text{for}\: b=0 \\ -\frac{x^{-2n}}{2b^2n} & \text{for}\: a=0 \\ \tilde{\infty}\log\left(x\right) & \text{for}\: b=-ax^{-n} \\ \frac{\log\left(x\right)}{\left(a+b\right)^2} & \text{for}\: n=0 \\ \frac{an\log\left(x\right)}{a^3n+a^2bnx^n} - \frac{a\log\left(\frac{a}{b}+x^n\right)}{a^3n+a^2bnx^n} + \frac{a}{a^3n+a^2bnx^n} + \frac{bnx^n\log\left(x\right)}{a^3n+a^2bnx^n} - \frac{bx^n\log\left(\frac{a}{b}+x^n\right)}{a^3n+a^2bnx^n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a**2, Eq(b
, 0)), (-1/(2*b**2*n*x**(2*n)), Eq(a, 0)), (zoo*log(x), Eq(b, -a/x**n)), (
log(x)/(a + b)**2, Eq(n, 0)), (a*n*log(x)/(a**3*n + a**2*b*n*x**n) - a*log
(a/b + x**n)/(a**3*n + a**2*b*n*x**n) + a/(a**3*n + a**2*b*n*x**n) + b*n*x
**n*log(x)/(a**3*n + a**2*b*n*x**n) - b*x**n*log(a/b + x**n)/(a**3*n + a**
2*b*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+bx^{n}\right)^{2}}\,dx=\frac{1}{abnx^{n}+a^{2}n}-\frac{\log\left(bx^{n}+a\right)}{a^{2}n}+\frac{\log\left(x^{n}\right)}{a^{2}n}$$

input
```
integrate(1/x/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/(a*b*n*x^n + a^2*n) - log(b*x^n + a)/(a^2*n) + log(x^n)/(a^2*n)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^{n}\right)^{2}}\,dx=\int \frac{1}{\left(bx^{n}+a\right)^{2}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^{n}\right)^{2}}\,dx=\frac{\ln\left(x\right)}{a^{2}}+\frac{1}{a\,n\,\left(a+b\,x^{n}\right)}-\frac{\ln\left(a+b\,x^{n}\right)}{a^{2}\,n}$$

input
```
int(1/(x*(a + b*x^n)^2),x)
```

output
```
log(x)/a^2 + 1/(a*n*(a + b*x^n)) - log(a + b*x^n)/(a^2*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

$$\int \frac{1}{x(a+bx^n)^2}\,dx = \frac{-x^n\log(x^nb+a)\,b+x^n\log(x)\,bn-x^nb-\log(x^nb+a)\,a+\log(x)\,an}{a^2n\,(x^nb+a)}$$

input
```
int(1/x/(a+b*x^n)^2,x)
```

output
```
( - x**n*log(x**n*b + a)*b + x**n*log(x)*b*n - x**n*b - log(x**n*b + a)*a
+ log(x)*a*n)/(a**2*n*(x**n*b + a))
```

3.367 $\int \frac{1}{x^2(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{1}{x^2 (a+bx^n)^2}\,dx = -\frac{\text{Hypergeometric2F1}\left(2,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2x}$$

output
```
-hypergeom([2, -1/n],[-(1-n)/n],-b*x^n/a)/a^2/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (a+bx^n)^2}\,dx = -\frac{\text{Hypergeometric2F1}\left(2,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2x}$$

input
```
Integrate[1/(x^2*(a + b*x^n)^2),x]
```

output
```
-(Hypergeometric2F1[2, -n^(-1), 1 - n^(-1), -((b*x^n)/a)]/(a^2*x))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n)^2} \, dx$$

$$\Big\downarrow \; 888$$

$$-\frac{\text{Hypergeometric2F1}\left(2,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2 x}$$

input
```
Int[1/(x^2*(a + b*x^n)^2),x]
```

output
```
-(Hypergeometric2F1[2, -n^(-1), -((1 - n)/n), -((b*x^n)/a)]/(a^2*x))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2 (a+b\,x^n)^2} dx$$

input
```
int(1/x^2/(a+b*x^n)^2,x)
```

output
```
int(1/x^2/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a+bx^n\right)^2}\,dx=\int \frac{1}{\left(bx^n+a\right)^2x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x^n + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 367, normalized size of antiderivative = 10.79

$$\int \frac{1}{x^2\left(a+bx^n\right)^2}\,dx=-\frac{aa^{-2+\frac{1}{n}}n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{aa^{\frac{1}{n}}n^3x\Gamma\left(1-\frac{1}{n}\right)+a^{\frac{1}{n}}bn^3xx^n\Gamma\left(1-\frac{1}{n}\right)}-\frac{aa^{-2+\frac{1}{n}}n\Gamma\left(-\frac{1}{n}\right)}{aa^{\frac{1}{n}}n^3x\Gamma\left(1-\frac{1}{n}\right)+a^{\frac{1}{n}}bn^3xx^n\Gamma\left(1-\frac{1}{n}\right)}-\frac{aa^{-2+\frac{1}{n}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{aa^{\frac{1}{n}}n^3x\Gamma\left(1-\frac{1}{n}\right)+a^{\frac{1}{n}}bn^3xx^n\Gamma\left(1-\frac{1}{n}\right)}-\frac{a^{-2+\frac{1}{n}}bnx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{aa^{\frac{1}{n}}n^3x\Gamma\left(1-\frac{1}{n}\right)+a^{\frac{1}{n}}bn^3xx^n\Gamma\left(1-\frac{1}{n}\right)}-\frac{a^{-2+\frac{1}{n}}bx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{aa^{\frac{1}{n}}n^3x\Gamma\left(1-\frac{1}{n}\right)+a^{\frac{1}{n}}bn^3xx^n\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate(1/x**2/(a+b*x**n)**2,x)
```

output
```
-a*a**(-2 + 1/n)*n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n
)*gamma(-1/n)/(a*a**(1/n)*n**3*x*gamma(1 - 1/n) + a**(1/n)*b*n**3*x*x**n*g
amma(1 - 1/n)) - a*a**(-2 + 1/n)*n*gamma(-1/n)/(a*a**(1/n)*n**3*x*gamma(1
- 1/n) + a**(1/n)*b*n**3*x*x**n*gamma(1 - 1/n)) - a*a**(-2 + 1/n)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a*a**(1/n)*n
**3*x*gamma(1 - 1/n) + a**(1/n)*b*n**3*x*x**n*gamma(1 - 1/n)) - a**(-2 + 1
/n)*b*n*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamm
a(-1/n)/(a*a**(1/n)*n**3*x*gamma(1 - 1/n) + a**(1/n)*b*n**3*x*x**n*gamma(1
 - 1/n)) - a**(-2 + 1/n)*b*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_
polar(I*pi)/n)*gamma(-1/n)/(a*a**(1/n)*n**3*x*gamma(1 - 1/n) + a**(1/n)*b*
n**3*x*x**n*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n)^2} \, dx = \int \frac{1}{(bx^n + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(n + 1)*integrate(1/(a*b*n*x^2*x^n + a^2*n*x^2), x) + 1/(a*b*n*x*x^n + a^2
*n*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^n)^2} \, dx = \int \frac{1}{(bx^n + a)^2 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^n\right)^2}\,dx=\int \frac{1}{x^2\,\left(a+b\,x^n\right)^2}\,dx$$

input
```
int(1/(x^2*(a + b*x^n)^2),x)
```

output
```
int(1/(x^2*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(a+bx^n\right)^2}\,dx=\int \frac{1}{x^{2n}b^2x^2+2x^nab\,x^2+a^2x^2}dx$$

input
```
int(1/x^2/(a+b*x^n)^2,x)
```

output
```
int(1/(x**(2*n)*b**2*x**2 + 2*x**n*a*b*x**2 + a**2*x**2),x)
```

3.368 $\int \frac{1}{x^3(a+bx^n)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{1}{x^3 (a+bx^n)^2} dx = -\frac{\text{Hypergeometric2F1}\left(2, -\frac{2}{n}, -\frac{2-n}{n}, -\frac{bx^n}{a}\right)}{2a^2x^2}$$

output
```
-1/2*hypergeom([2, -2/n],[-(2-n)/n],-b*x^n/a)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (a+bx^n)^2} dx = -\frac{\text{Hypergeometric2F1}\left(2, -\frac{2}{n}, 1-\frac{2}{n}, -\frac{bx^n}{a}\right)}{2a^2x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^n)^2),x]
```

output
```
-1/2*Hypergeometric2F1[2, -2/n, 1 - 2/n, -((b*x^n)/a)]/(a^2*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{\text{Hypergeometric2F1}\left(2,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2x^2}$$

input
```
Int[1/(x^3*(a + b*x^n)^2),x]
```

output
```
-1/2*Hypergeometric2F1[2, -2/n, -((2 - n)/n), -((b*x^n)/a)]/(a^2*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^3 (a+b\,x^n)^2}dx$$

input
```
int(1/x^3/(a+b*x^n)^2,x)
```

output
```
int(1/x^3/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a+bx^n)^2}\,dx = \int \frac{1}{(bx^n+a)^2 x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x^n + a^2*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 400, normalized size of antiderivative = 11.11

$$\int \frac{1}{x^3 (a+bx^n)^2}\,dx = -\frac{2aa^{-2+\frac{2}{n}}n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2e^{i\pi}}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{aa^{\frac{2}{n}}n^3x^2\Gamma\left(1-\frac{2}{n}\right)+a^{\frac{2}{n}}bn^3x^2x^n\Gamma\left(1-\frac{2}{n}\right)} - \frac{2aa^{-2+\frac{2}{n}}n\Gamma\left(-\frac{2}{n}\right)}{aa^{\frac{2}{n}}n^3x^2\Gamma\left(1-\frac{2}{n}\right)+a^{\frac{2}{n}}bn^3x^2x^n\Gamma\left(1-\frac{2}{n}\right)} - \frac{4aa^{-2+\frac{2}{n}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2e^{i\pi}}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{aa^{\frac{2}{n}}n^3x^2\Gamma\left(1-\frac{2}{n}\right)+a^{\frac{2}{n}}bn^3x^2x^n\Gamma\left(1-\frac{2}{n}\right)} - \frac{2a^{-2+\frac{2}{n}}bnx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2e^{i\pi}}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{aa^{\frac{2}{n}}n^3x^2\Gamma\left(1-\frac{2}{n}\right)+a^{\frac{2}{n}}bn^3x^2x^n\Gamma\left(1-\frac{2}{n}\right)} - \frac{4a^{-2+\frac{2}{n}}bx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2e^{i\pi}}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{aa^{\frac{2}{n}}n^3x^2\Gamma\left(1-\frac{2}{n}\right)+a^{\frac{2}{n}}bn^3x^2x^n\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate(1/x**3/(a+b*x**n)**2,x)
```

output
```
-2*a*a**(-2 + 2/n)*n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*p
i)/n)*gamma(-2/n)/(a*a**(2/n)*n**3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**3*x
**2*x**n*gamma(1 - 2/n)) - 2*a*a**(-2 + 2/n)*n*gamma(-2/n)/(a*a**(2/n)*n**
3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**3*x**2*x**n*gamma(1 - 2/n)) - 4*a*a*
*(-2 + 2/n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gam
ma(-2/n)/(a*a**(2/n)*n**3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**3*x**2*x**n*
gamma(1 - 2/n)) - 2*a**(-2 + 2/n)*b*n*x**n*lerchphi(b*x**n*exp_polar(I*pi)
/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a*a**(2/n)*n**3*x**2*gamma(1 - 2/
n) + a**(2/n)*b*n**3*x**2*x**n*gamma(1 - 2/n)) - 4*a**(-2 + 2/n)*b*x**n*le
rchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a*a*
*(2/n)*n**3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**3*x**2*x**n*gamma(1 - 2/n)
)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^n)^2}\, dx = \int \frac{1}{(bx^n+a)^2 x^3}\, dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(n + 2)*integrate(1/(a*b*n*x^3*x^n + a^2*n*x^3), x) + 1/(a*b*n*x^2*x^n + a
^2*n*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^n)^2}\, dx = \int \frac{1}{(bx^n+a)^2 x^3}\, dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^n)^2} \, dx = \int \frac{1}{x^3 \left(a+b\,x^n\right)^2} \, dx$$

input
```
int(1/(x^3*(a + b*x^n)^2),x)
```

output
```
int(1/(x^3*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^n)^2} \, dx = \int \frac{1}{x^{2n} b^2 x^3 + 2x^n ab\, x^3 + a^2 x^3} dx$$

input
```
int(1/x^3/(a+b*x^n)^2,x)
```

output
```
int(1/(x**(2*n)*b**2*x**3 + 2*x**n*a*b*x**3 + a**2*x**3),x)
```

3.369 $\int \frac{x}{(a+bx^n)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{x}{(a+bx^n)^3} \, dx = \frac{x^2 \operatorname{Hypergeometric2F1}\left(3, \frac{2}{n}, \frac{2+n}{n}, -\frac{bx^n}{a}\right)}{2a^3}$$

output
```
1/2*x^2*hypergeom([3, 2/n],[(2+n)/n],-b*x^n/a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^n)^3} \, dx = \frac{x^2 \operatorname{Hypergeometric2F1}\left(3, \frac{2}{n}, 1+\frac{2}{n}, -\frac{bx^n}{a}\right)}{2a^3}$$

input
```
Integrate[x/(a + b*x^n)^3,x]
```

output
```
(x^2*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)^3} dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2 \text{Hypergeometric2F1}\left(3, \frac{2}{n}, \frac{n+2}{n}, -\frac{bx^n}{a}\right)}{2a^3}$$

input
```
Int[x/(a + b*x^n)^3,x]
```

output
```
(x^2*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{(a+b\,x^n)^3}dx$$

input
```
int(x/(a+b*x^n)^3,x)
```

output
```
int(x/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^n)^3}\,dx = \int \frac{x}{(bx^n+a)^3}\,dx$$

input
```
integrate(x/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(x/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 1210, normalized size of antiderivative = 36.67

$$\int \frac{x}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*x**n)**3,x)
```

output
```
2*a**2*a**(2/n)*a**(-3 - 2/n)*n**2*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, 2/n)*gamma(2/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 +
2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n)) + 3*a**2*a**(2/n)*a**(-3 - 2/n)*
n**2*x**2*gamma(2/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 +
 2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n)) - 6*a**2*a**(2/n)*a**(-3 - 2/n)
*n*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*n**4*g
amma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x**(2*n)*gamma(
1 + 2/n)) - 2*a**2*a**(2/n)*a**(-3 - 2/n)*n*x**2*gamma(2/n)/(a**2*n**4*gam
ma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x**(2*n)*gamma(1
+ 2/n)) + 4*a**2*a**(2/n)*a**(-3 - 2/n)*x**2*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 2/n)*gamma(2/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma
(1 + 2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n)) + 4*a*a**(2/n)*a**(-3 - 2/n
)*b*n**2*x**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(
a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x**(
2*n)*gamma(1 + 2/n)) + 2*a*a**(2/n)*a**(-3 - 2/n)*b*n**2*x**2*x**n*gamma(2
/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4
*x**(2*n)*gamma(1 + 2/n)) - 12*a*a**(2/n)*a**(-3 - 2/n)*b*n*x**2*x**n*lerc
hphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*n**4*gamma(1 + 2/n
) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n)) -
2*a*a**(2/n)*a**(-3 - 2/n)*b*n*x**2*x**n*gamma(2/n)/(a**2*n**4*gamma(1 ...
```

Maxima [F]

$$\int \frac{x}{(a+bx^n)^3}\,dx = \int \frac{x}{(bx^n+a)^3}\,dx$$

input
```
integrate(x/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n -
1)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4
*n^2)
```

Giac [F]

$$\int \frac{x}{(a+bx^n)^3}\,dx=\int \frac{x}{(bx^n+a)^3}\,dx$$

input
```
integrate(x/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)^3}\,dx=\int \frac{x}{(a+b\,x^n)^3}\,dx$$

input
```
int(x/(a + b*x^n)^3,x)
```

output
```
int(x/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^n)^3}\,dx=\int \frac{x}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2 b+a^3}dx$$

input
```
int(x/(a+b*x^n)^3,x)
```

output
```
int(x/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.370 $\int \frac{1}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{(a+bx^n)^3} dx = \frac{x \text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

output
```
x*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^n)^3} dx = \frac{x \text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

input
```
Integrate[(a + b*x^n)^(-3),x]
```

output
```
(x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^3} dx$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x \,\text{Hypergeometric2F1}\left(3, \frac{1}{n}, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)}{a^3}$$

input
```
Int[(a + b*x^n)^(-3),x]
```

output
```
(x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a^3
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)^3}dx$$

input
```
int(1/(a+b*x^n)^3,x)
```

output
```
int(1/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 1226, normalized size of antiderivative = 51.08

$$\int \frac{1}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*x**n)**3,x)
```

output
```
2*a**2*a**(1/n)*a**(-3 - 1/n)*n**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1
/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 3*a**2*a**(1/n)*a**(-3 - 1/n)
*n**2*x*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 +
 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) - 3*a**2*a**(1/n)*a**(-3 - 1/
n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n**4*
gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gam
ma(1 + 1/n)) - a**2*a**(1/n)*a**(-3 - 1/n)*n*x*gamma(1/n)/(2*a**2*n**4*gam
ma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(
1 + 1/n)) + a**2*a**(1/n)*a**(-3 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(
1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 4*a*a**(1/n)*a**(-3 - 1/
n)*b*n**2*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*
a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x*
*(2*n)*gamma(1 + 1/n)) + 2*a*a**(1/n)*a**(-3 - 1/n)*b*n**2*x*x**n*gamma(1/
n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n
**4*x**(2*n)*gamma(1 + 1/n)) - 6*a*a**(1/n)*a**(-3 - 1/n)*b*n*x*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/
n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n))
 - a*a**(1/n)*a**(-3 - 1/n)*b*n*x*x**n*gamma(1/n)/(2*a**2*n**4*gamma(1 ...
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^3}\,dx=\int \frac{1}{(bx^n+a)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*
n - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4
*n^2)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^3}\,dx$$

input
```
integrate(1/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(-3), x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx^n)^3}\,dx = \frac{x\,{}_2\mathrm{F}_1\left(3,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^3}$$

input
```
int(1/(a + b*x^n)^3,x)
```

output
```
(x*hypergeom([3, 1/n], 1/n + 1, -(b*x^n)/a))/a^3
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^3}\,dx = \int \frac{1}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2 b+a^3}dx$$

input
```
int(1/(a+b*x^n)^3,x)
```

output
```
int(1/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.371 $\int \frac{1}{x(a+bx^n)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{1}{x(a+bx^n)^3} dx = \frac{1}{2an(a+bx^n)^2} + \frac{1}{a^2n(a+bx^n)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx^n)}{a^3n}$$

output
```
1/2/a/n/(a+b*x^n)^2+1/a^2/n/(a+b*x^n)+ln(x)/a^3-ln(a+b*x^n)/a^3/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx^n)^3} dx = \frac{\frac{a(3a+2bx^n)}{(a+bx^n)^2} + 2\log(x^n) - 2\log(a+bx^n)}{2a^3n}$$

input
```
Integrate[1/(x*(a + b*x^n)^3),x]
```

output
```
((a*(3*a + 2*b*x^n))/(a + b*x^n)^2 + 2*Log[x^n] - 2*Log[a + b*x^n])/(2*a^3
*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^n)^3}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{(bx^n+a)^3}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-n}}{a^3}-\frac{b}{a^3(bx^n+a)}-\frac{b}{a^2(bx^n+a)^2}-\frac{b}{a(bx^n+a)^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log(a+bx^n)}{a^3}+\frac{\log(x^n)}{a^3}+\frac{1}{a^2(a+bx^n)}+\frac{1}{2a(a+bx^n)^2}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^3),x]
```

output
```
(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + Log[x^n]/a^3 - Log[a + b*x^
n]/a^3)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\ln(x)}{a^3}+\frac{2b\,x^n+3a}{2a^2 n\left(a+b\,x^n\right)^2}-\frac{\ln\left(x^n+\frac{a}{b}\right)}{a^3 n}$
derivativedivides	$\frac{-\frac{\ln(a+b\,x^n)}{a^3}+\frac{1}{a^2(a+b\,x^n)}+\frac{1}{2a(a+b\,x^n)^2}+\frac{\ln(x^n)}{a^3}}{n}$
default	$\frac{-\frac{\ln(a+b\,x^n)}{a^3}+\frac{1}{a^2(a+b\,x^n)}+\frac{1}{2a(a+b\,x^n)^2}+\frac{\ln(x^n)}{a^3}}{n}$
norman	$\frac{\frac{\ln(x)}{a}+\frac{2b\ln(x)\mathrm{e}^{n\ln(x)}}{a^2}-\frac{2b\,\mathrm{e}^{n\ln(x)}}{a^2 n}+\frac{b^2\ln(x)\mathrm{e}^{2n\ln(x)}}{a^3}-\frac{3b^2\mathrm{e}^{2n\ln(x)}}{2a^3 n}}{\left(a+b\,\mathrm{e}^{n\ln(x)}\right)^2}-\frac{\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{a^3 n}$
parallelrisch	$\frac{2x^{2n}\ln(x)b^2n+4x^n\ln(x)abn-2\ln(a+b\,x^n)x^{2n}b^2+2a^2\ln(x)n-4\ln(a+b\,x^n)x^n ab-3b^2x^{2n}-2\ln(a+b\,x^n)a^2-4ab\,x^n}{2a^3 n(a+b\,x^n)^2}$

input
```
int(1/x/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^3+1/2*(2*b*x^n+3*a)/a^2/n/(a+b*x^n)^2-1/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.83

$$\int \frac{1}{x\left(a+bx^n\right)^3}\,dx$$
$$=\frac{2\,b^2nx^{2\,n}\log(x)+2\,a^2n\log(x)+3\,a^2+2\,(2\,abn\log(x)+ab)x^n-2\,(b^2x^{2\,n}+2\,abx^n+a^2)\log(bx^n+a)}{2\,(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n)}$$

input
```
integrate(1/x/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
 a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x
^(2*n) + 2*a^4*b*n*x^n + a^5*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(48) = 96$.

Time = 1.44 (sec) , antiderivative size = 415, normalized size of antiderivative = 7.16

$$\int \frac{1}{x(a+bx^n)^3}\,dx$$
$$= \begin{cases} \tilde{\infty}\log(x) \\ \frac{\log(x)}{a^3} \\ -\frac{x^{-3n}}{3b^3n} \\ \tilde{\infty}\log(x) \\ \frac{\log(x)}{(a+b)^3} \\ \frac{2a^2n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{3a^2}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{4abnx^n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{4a}{2a^5n+4} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a**3, Eq(b
, 0)), (-1/(3*b**3*n*x**(3*n)), Eq(a, 0)), (zoo*log(x), Eq(b, -a/x**n)), (
log(x)/(a + b)**3, Eq(n, 0)), (2*a**2*n*log(x)/(2*a**5*n + 4*a**4*b*n*x**n
 + 2*a**3*b**2*n*x**(2*n)) - 2*a**2*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n
*x**n + 2*a**3*b**2*n*x**(2*n)) + 3*a**2/(2*a**5*n + 4*a**4*b*n*x**n + 2*a
**3*b**2*n*x**(2*n)) + 4*a*b*n*x**n*log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2
*a**3*b**2*n*x**(2*n)) - 4*a*b*x**n*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n
*x**n + 2*a**3*b**2*n*x**(2*n)) + 2*a*b*x**n/(2*a**5*n + 4*a**4*b*n*x**n +
 2*a**3*b**2*n*x**(2*n)) + 2*b**2*n*x**(2*n)*log(x)/(2*a**5*n + 4*a**4*b*n
*x**n + 2*a**3*b**2*n*x**(2*n)) - 2*b**2*x**(2*n)*log(a/b + x**n)/(2*a**5*
n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a+bx^n)^3}\,dx = \frac{2\,bx^n+3\,a}{2\,(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n)} - \frac{\log(bx^n+a)}{a^3n} + \frac{\log(x^n)}{a^3n}$$

input
```
integrate(1/x/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) - log(b*x^
n + a)/(a^3*n) + log(x^n)/(a^3*n)
```

Giac [F]

$$\int \frac{1}{x(a+bx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^3x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^3*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{x(a+bx^n)^3}\,dx = \frac{\ln(x)}{a^3} + \frac{1}{a^2\,n\,(a+b\,x^n)} - \frac{\ln(a+b\,x^n)}{a^3\,n} + \frac{1}{2\,a\,n\,(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)}$$

input
```
int(1/(x*(a + b*x^n)^3),x)
```

output
```
log(x)/a^3 + 1/(a^2*n*(a + b*x^n)) - log(a + b*x^n)/(a^3*n) + 1/(2*a*n*(a^
2 + b^2*x^(2*n) + 2*a*b*x^n))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\left(a+bx^n\right)^3}\,dx$$
$$= \frac{-2x^{2n}\log(x^nb+a)\,b^2+2x^{2n}\log(x)\,b^2n-x^{2n}b^2-4x^n\log(x^nb+a)\,ab+4x^n\log(x)\,abn-2\log(x^nb+a)\,a}{2a^3n\left(x^{2n}b^2+2x^nab+a^2\right)}$$

input
```
int(1/x/(a+b*x^n)^3,x)
```

output
```
( - 2*x**(2*n)*log(x**n*b + a)*b**2 + 2*x**(2*n)*log(x)*b**2*n - x**(2*n)*
b**2 - 4*x**n*log(x**n*b + a)*a*b + 4*x**n*log(x)*a*b*n - 2*log(x**n*b + a
)*a**2 + 2*log(x)*a**2*n + 2*a**2)/(2*a**3*n*(x**(2*n)*b**2 + 2*x**n*a*b +
 a**2))
```

3.372 $\int \frac{1}{x^2(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{1}{x^2\left(a+bx^n\right)^3}\,dx=-\frac{\text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3x}$$

output
```
-hypergeom([3, -1/n],[-(1-n)/n],-b*x^n/a)/a^3/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(a+bx^n\right)^3}\,dx=-\frac{\text{Hypergeometric2F1}\left(3,-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3x}$$

input
```
Integrate[1/(x^2*(a + b*x^n)^3),x]
```

output
```
-(Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)]/(a^3*x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n)^3} \, dx$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\text{Hypergeometric2F1}\left(3,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^3 x}$$

input
```
Int[1/(x^2*(a + b*x^n)^3),x]
```

output
```
-(Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)/a)]/(a^3*x))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2 (a+b\,x^n)^3}dx$$

input
```
int(1/x^2/(a+b*x^n)^3,x)
```

output
```
int(1/x^2/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a + bx^n)^3} \, dx = \int \frac{1}{(bx^n + a)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^2*x^(3*n) + 3*a*b^2*x^2*x^(2*n) + 3*a^2*b*x^2*x^n + a^3*
x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 1435, normalized size of antiderivative = 42.21

$$\int \frac{1}{x^2 (a + bx^n)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(a+b*x**n)**3,x)
```

output
```
-2*a**2*a**(-3 + 1/n)*n**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar
(I*pi)/n)*gamma(-1/n)/(2*a**2*a**(1/n)*n**4*x*gamma(1 - 1/n) + 4*a*a**(1/n
)*b*n**4*x*x**n*gamma(1 - 1/n) + 2*a**(1/n)*b**2*n**4*x*x**(2*n)*gamma(1 -
 1/n)) - 3*a**2*a**(-3 + 1/n)*n**2*gamma(-1/n)/(2*a**2*a**(1/n)*n**4*x*gam
ma(1 - 1/n) + 4*a*a**(1/n)*b*n**4*x*x**n*gamma(1 - 1/n) + 2*a**(1/n)*b**2*
n**4*x*x**(2*n)*gamma(1 - 1/n)) - 3*a**2*a**(-3 + 1/n)*n*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(2*a**2*a**(1/n)*n**4*
x*gamma(1 - 1/n) + 4*a*a**(1/n)*b*n**4*x*x**n*gamma(1 - 1/n) + 2*a**(1/n)*
b**2*n**4*x*x**(2*n)*gamma(1 - 1/n)) - a**2*a**(-3 + 1/n)*n*gamma(-1/n)/(2
*a**2*a**(1/n)*n**4*x*gamma(1 - 1/n) + 4*a*a**(1/n)*b*n**4*x*x**n*gamma(1
- 1/n) + 2*a**(1/n)*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n)) - a**2*a**(-3 + 1
/n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(
2*a**2*a**(1/n)*n**4*x*gamma(1 - 1/n) + 4*a*a**(1/n)*b*n**4*x*x**n*gamma(1
 - 1/n) + 2*a**(1/n)*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n)) - 4*a*a**(-3 + 1
/n)*b*n**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*g
amma(-1/n)/(2*a**2*a**(1/n)*n**4*x*gamma(1 - 1/n) + 4*a*a**(1/n)*b*n**4*x*
x**n*gamma(1 - 1/n) + 2*a**(1/n)*b**2*n**4*x*x**(2*n)*gamma(1 - 1/n)) - 2*
a*a**(-3 + 1/n)*b*n**2*x**n*gamma(-1/n)/(2*a**2*a**(1/n)*n**4*x*gamma(1 -
1/n) + 4*a*a**(1/n)*b*n**4*x*x**n*gamma(1 - 1/n) + 2*a**(1/n)*b**2*n**4*x*
x**(2*n)*gamma(1 - 1/n)) - 6*a*a**(-3 + 1/n)*b*n*x**n*lerchphi(b*x**n*e...
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n)^3} \, dx = \int \frac{1}{(bx^n + a)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/
2*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x
^n + a^4*n^2*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^n)^3} \, dx = \int \frac{1}{(bx^n+a)^3 x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^n)^3} \, dx = \int \frac{1}{x^2 \left(a+b\,x^n\right)^3} \, dx$$

input
```
int(1/(x^2*(a + b*x^n)^3),x)
```

output
```
int(1/(x^2*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^n)^3} \, dx = \int \frac{1}{x^{3n} b^3 x^2 + 3x^{2n} a\, b^2 x^2 + 3x^n a^2 b\, x^2 + a^3 x^2} dx$$

input
```
int(1/x^2/(a+b*x^n)^3,x)
```

output
```
int(1/(x**(3*n)*b**3*x**2 + 3*x**(2*n)*a*b**2*x**2 + 3*x**n*a**2*b*x**2 +
a**3*x**2),x)
```

3.373 $\int \frac{1}{x^3(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{1}{x^3\left(a+bx^n\right)^3}\,dx=-\frac{\text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3x^2}$$

output
```
-1/2*hypergeom([3, -2/n],[-(2-n)/n],-b*x^n/a)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\left(a+bx^n\right)^3}\,dx=-\frac{\text{Hypergeometric2F1}\left(3,-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^n)^3),x]
```

output
```
-1/2*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)]/(a^3*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n)^3} \, dx$$
$$\Big\downarrow \text{ 888}$$
$$-\frac{\text{Hypergeometric2F1}\left(3,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3x^2}$$

input
```
Int[1/(x^3*(a + b*x^n)^3),x]
```

output
```
-1/2*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/a)]/(a^3*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^3 (a+b\,x^n)^3}dx$$

input
```
int(1/x^3/(a+b*x^n)^3,x)
```

output
```
int(1/x^3/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a + bx^n)^3} dx = \int \frac{1}{(bx^n + a)^3 x^3} dx$$

input
```
integrate(1/x^3/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^3*x^(3*n) + 3*a*b^2*x^3*x^(2*n) + 3*a^2*b*x^3*x^n + a^3*
x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 1479, normalized size of antiderivative = 41.08

$$\int \frac{1}{x^3 (a + bx^n)^3} dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(a+b*x**n)**3,x)
```

output
```
-2*a**2*a**(-3 + 2/n)*n**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_pol
ar(I*pi)/n)*gamma(-2/n)/(a**2*a**(2/n)*n**4*x**2*gamma(1 - 2/n) + 2*a*a**(
2/n)*b*n**4*x**2*x**n*gamma(1 - 2/n) + a**(2/n)*b**2*n**4*x**2*x**(2*n)*ga
mma(1 - 2/n)) - 3*a**2*a**(-3 + 2/n)*n**2*gamma(-2/n)/(a**2*a**(2/n)*n**4*
x**2*gamma(1 - 2/n) + 2*a*a**(2/n)*b*n**4*x**2*x**n*gamma(1 - 2/n) + a**(2
/n)*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n)) - 6*a**2*a**(-3 + 2/n)*n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**2*a*
*(2/n)*n**4*x**2*gamma(1 - 2/n) + 2*a*a**(2/n)*b*n**4*x**2*x**n*gamma(1 -
2/n) + a**(2/n)*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n)) - 2*a**2*a**(-3 +
2/n)*n*gamma(-2/n)/(a**2*a**(2/n)*n**4*x**2*gamma(1 - 2/n) + 2*a*a**(2/n)*
b*n**4*x**2*x**n*gamma(1 - 2/n) + a**(2/n)*b**2*n**4*x**2*x**(2*n)*gamma(1
 - 2/n)) - 4*a**2*a**(-3 + 2/n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*ex
p_polar(I*pi)/n)*gamma(-2/n)/(a**2*a**(2/n)*n**4*x**2*gamma(1 - 2/n) + 2*a
*a**(2/n)*b*n**4*x**2*x**n*gamma(1 - 2/n) + a**(2/n)*b**2*n**4*x**2*x**(2*
n)*gamma(1 - 2/n)) - 4*a*a**(-3 + 2/n)*b*n**2*x**n*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a**2*a**(2/n)*n**4*x**2*g
amma(1 - 2/n) + 2*a*a**(2/n)*b*n**4*x**2*x**n*gamma(1 - 2/n) + a**(2/n)*b*
*2*n**4*x**2*x**(2*n)*gamma(1 - 2/n)) - 2*a*a**(-3 + 2/n)*b*n**2*x**n*gamm
a(-2/n)/(a**2*a**(2/n)*n**4*x**2*gamma(1 - 2/n) + 2*a*a**(2/n)*b*n**4*x**2
*x**n*gamma(1 - 2/n) + a**(2/n)*b**2*n**4*x**2*x**(2*n)*gamma(1 - 2/n))...
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n)^3} \, dx = \int \frac{1}{(bx^n + a)^3 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2
*b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x
^n + a^4*n^2*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^n)^3} \, dx = \int \frac{1}{(bx^n+a)^3 x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^n)^3} \, dx = \int \frac{1}{x^3 (a+b\,x^n)^3} \, dx$$

input
```
int(1/(x^3*(a + b*x^n)^3),x)
```

output
```
int(1/(x^3*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^n)^3} \, dx = \int \frac{1}{x^{3n} b^3 x^3 + 3x^{2n} a\, b^2 x^3 + 3x^n a^2 b\, x^3 + a^3 x^3} dx$$

input
```
int(1/x^3/(a+b*x^n)^3,x)
```

output
```
int(1/(x**(3*n)*b**3*x**3 + 3*x**(2*n)*a*b**2*x**3 + 3*x**n*a**2*b*x**3 +
a**3*x**3),x)
```

3.374 $\int x^{-1+4n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int x^{-1+4n}(a+bx^n)\,dx=\frac{ax^{4n}}{4n}+\frac{bx^{5n}}{5n}$$

output
```
1/4*a*x^(4*n)/n+1/5*b*x^(5*n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1+4n}(a+bx^n)\,dx=\frac{x^{4n}(5a+4bx^n)}{20n}$$

input
```
Integrate[x^(-1 + 4*n)*(a + b*x^n),x]
```

output
```
(x^(4*n)*(5*a + 4*b*x^n))/(20*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{4n-1}+bx^{5n-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^{4n}}{4n}+\frac{bx^{5n}}{5n}$$

input
```
Int[x^(-1 + 4*n)*(a + b*x^n),x]
```

output
```
(a*x^(4*n))/(4*n) + (b*x^(5*n))/(5*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{a\,x^{4n}}{4n}+\frac{b\,x^{5n}}{5n}$	24
norman	$\frac{a\,{\mathrm e}^{4n\ln(x)}}{4n}+\frac{b\,{\mathrm e}^{5n\ln(x)}}{5n}$	28
parallelrisch	$\frac{4x\,x^{n}x^{-1+4n}b+5x\,x^{-1+4n}a}{20n}$	32
orering	$\frac{x(-1+9n)x^{-1+4n}(a+b\,x^{n})}{20n^{2}}-\frac{x^{2}\left(\frac{x^{-1+4n}(-1+4n)(a+b\,x^{n})}{x}+\frac{x^{-1+4n}b\,x^{n}n}{x}\right)}{20n^{2}}$	75

input
```
int(x^(-1+4*n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/4*a/n*(x^n)^4+1/5*b/n*(x^n)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1+4n}(a+bx^n)\,dx=\frac{4\,bx^{5\,n}+5\,ax^{4\,n}}{20\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/20*(4*b*x^(5*n) + 5*a*x^(4*n))/n
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int x^{-1+4n}(a+bx^n)\,dx=\begin{cases}\frac{axx^{4n-1}}{4n}+\frac{bxx^{n}x^{4n-1}}{5n} & \text{for}\ n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+4*n)*(a+b*x**n),x)
```

output
```
Piecewise((a*x*x**(4*n - 1)/(4*n) + b*x*x**n*x**(4*n - 1)/(5*n), Ne(n, 0))
, ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^{-1+4n}(a+bx^n)\ dx = \frac{bx^{5\,n}}{5\,n} + \frac{ax^{4\,n}}{4\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/5*b*x^(5*n)/n + 1/4*a*x^(4*n)/n
```

Giac [F]

$$\int x^{-1+4n}(a+bx^n)\ dx = \int (bx^n+a)x^{4\,n-1}\,dx$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)*x^(4*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^{-1+4n}(a+bx^n)\ dx = \frac{x^{4\,n}\,(5\,a+4\,b\,x^n)}{20\,n}$$

input
```
int(x^(4*n - 1)*(a + b*x^n),x)
```

output
```
(x^(4*n)*(5*a + 4*b*x^n))/(20*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^{-1+4n}(a+bx^n)\ dx = \frac{x^{4n}(4x^nb+5a)}{20n}$$

input
```
int(x^(-1+4*n)*(a+b*x^n),x)
```

output
```
(x**(4*n)*(4*x**n*b + 5*a))/(20*n)
```

3.375 $\int x^{-1+3n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int x^{-1+3n}(a+bx^n)\,dx=\frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}$$

output
```
1/3*a*x^(3*n)/n+1/4*b*x^(4*n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1+3n}(a+bx^n)\,dx=\frac{x^{3n}(4a+3bx^n)}{12n}$$

input
```
Integrate[x^(-1 + 3*n)*(a + b*x^n),x]
```

output
```
(x^(3*n)*(4*a + 3*b*x^n))/(12*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{3n-1}+bx^{4n-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}$$

input
```
Int[x^(-1 + 3*n)*(a + b*x^n),x]
```

output
```
(a*x^(3*n))/(3*n) + (b*x^(4*n))/(4*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{a\,x^{3n}}{3n}+\frac{b\,x^{4n}}{4n}$	24
norman	$\frac{a\,\mathrm{e}^{3n\ln(x)}}{3n}+\frac{b\,\mathrm{e}^{4n\ln(x)}}{4n}$	28
parallelrisch	$\frac{3x\,x^{n}x^{-1+3n}b+4x\,x^{-1+3n}a}{12n}$	32
orering	$\frac{x(-1+7n)x^{-1+3n}(a+b\,x^{n})}{12n^{2}}-\frac{x^{2}\left(\frac{x^{-1+3n}(-1+3n)(a+b\,x^{n})}{x}+\frac{x^{-1+3n}b\,x^{n}n}{x}\right)}{12n^{2}}$	75

input
```
int(x^(-1+3*n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/n*(x^n)^3+1/4*b/n*(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1+3n}(a+bx^n)\,dx=\frac{3\,bx^{4\,n}+4\,ax^{3\,n}}{12\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b*x^(4*n) + 4*a*x^(3*n))/n
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int x^{-1+3n}(a+bx^n)\,dx=\begin{cases}\frac{axx^{3n-1}}{3n}+\frac{bxx^{n}x^{3n-1}}{4n} & \text{for } n\neq 0\\ (a+b)\log{(x)} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n),x)
```

output
```
Piecewise((a*x*x**(3*n - 1)/(3*n) + b*x*x**n*x**(3*n - 1)/(4*n), Ne(n, 0))
, ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^{-1+3n}(a+bx^n)\,dx=\frac{bx^{4\,n}}{4\,n}+\frac{ax^{3\,n}}{3\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/4*b*x^(4*n)/n + 1/3*a*x^(3*n)/n
```

Giac [F]

$$\int x^{-1+3n}(a+bx^n)\,dx=\int (bx^n+a)x^{3\,n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)*x^(3*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^{-1+3n}(a+bx^n)\,dx=\frac{x^{3\,n}\,\left(4\,a+3\,b\,x^n\right)}{12\,n}$$

input
```
int(x^(3*n - 1)*(a + b*x^n),x)
```

output
```
(x^(3*n)*(4*a + 3*b*x^n))/(12*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^{-1+3n}(a+bx^n)\,dx=\frac{x^{3n}(3x^nb+4a)}{12n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n),x)
```

output
```
(x**(3*n)*(3*x**n*b + 4*a))/(12*n)
```

3.376 $\int x^{-1+2n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int x^{-1+2n}(a+bx^n)\,dx=\frac{ax^{2n}}{2n}+\frac{bx^{3n}}{3n}$$

output
```
1/2*a*x^(2*n)/n+1/3*b*x^(3*n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1+2n}(a+bx^n)\,dx=\frac{x^{2n}(3a+2bx^n)}{6n}$$

input
```
Integrate[x^(-1 + 2*n)*(a + b*x^n),x]
```

output
```
(x^(2*n)*(3*a + 2*b*x^n))/(6*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{2n-1}+bx^{3n-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^{2n}}{2n}+\frac{bx^{3n}}{3n}$$

input
```
Int[x^(-1 + 2*n)*(a + b*x^n),x]
```

output
```
(a*x^(2*n))/(2*n) + (b*x^(3*n))/(3*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{a\,x^{2n}}{2n}+\frac{b\,x^{3n}}{3n}$	24
norman	$\frac{a\,\mathrm{e}^{2n\ln(x)}}{2n}+\frac{b\,\mathrm{e}^{3n\ln(x)}}{3n}$	28
parallelrisch	$\frac{2x\,x^{n}x^{2n-1}b+3x\,x^{2n-1}a}{6n}$	32
orering	$\frac{x(-1+5n)x^{2n-1}(a+b\,x^{n})}{6n^{2}}-\frac{x^{2}\left(\frac{x^{2n-1}(2n-1)(a+b\,x^{n})}{x}+\frac{x^{2n-1}b\,x^{n}n}{x}\right)}{6n^{2}}$	75

input
```
int(x^(2*n-1)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/n*(x^n)^2+1/3*b/n*(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1+2n}(a+bx^n)\,dx=\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int x^{-1+2n}(a+bx^n)\,dx=\begin{cases}\frac{axx^{2n-1}}{2n}+\frac{bxx^nx^{2n-1}}{3n} & \text{for}\ n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n),x)
```

output
```
Piecewise((a*x*x**(2*n - 1)/(2*n) + b*x*x**n*x**(2*n - 1)/(3*n), Ne(n, 0))
, ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int x^{-1+2n}(a+bx^n)\,dx = \frac{bx^{3n}}{3n} + \frac{ax^{2n}}{2n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/3*b*x^(3*n)/n + 1/2*a*x^(2*n)/n
```

Giac [F]

$$\int x^{-1+2n}(a+bx^n)\,dx = \int (bx^n+a)x^{2n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)*x^(2*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^{-1+2n}(a+bx^n)\,dx=\frac{x^{2\,n}\,\left(3\,a+2\,b\,x^n\right)}{6\,n}$$

input
```
int(x^(2*n - 1)*(a + b*x^n),x)
```

output
```
(x^(2*n)*(3*a + 2*b*x^n))/(6*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^{-1+2n}(a+bx^n)\,dx=\frac{x^{2n}(2x^n b+3a)}{6n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n),x)
```

output
```
(x**(2*n)*(2*x**n*b + 3*a))/(6*n)
```

3.377 $\int x^{-1+n}(a+bx^n)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{(a+bx^n)^2}{2bn}$$

output
```
1/2*(a+b*x^n)^2/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{(a+bx^n)^2}{2bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n),x]
```

output
```
(a + b*x^n)^2/(2*b*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(ax^{n-1}+bx^{2n-1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ax^n}{n}+\frac{bx^{2n}}{2n}$$

input
```
Int[x^(-1 + n)*(a + b*x^n),x]
```

output
```
(a*x^n)/n + (b*x^(2*n))/(2*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{a\,x^{n}}{n}+\frac{b\,x^{2n}}{2n}$	21
norman	$\frac{a\,{\mathrm e}^{n\ln(x)}}{n}+\frac{b\,{\mathrm e}^{2n\ln(x)}}{2n}$	25
parallelrisch	$\frac{x\,x^{n}x^{-1+n}b+2x\,x^{-1+n}a}{2n}$	27
orering	$\frac{x(-1+3n)x^{-1+n}(a+b\,x^{n})}{2n^{2}}-\frac{x^{2}\left(\frac{x^{-1+n}(-1+n)(a+b\,x^{n})}{x}+\frac{x^{-1+n}b\,x^{n}n}{x}\right)}{2n^{2}}$	67

input
```
int(x^(-1+n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
a/n*x^n+1/2*b/n*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^{n})\,dx=\frac{bx^{2\,n}+2\,ax^{n}}{2\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(b*x^(2*n) + 2*a*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int x^{-1+n}(a+bx^{n})\,dx=\begin{cases}\frac{axx^{n-1}}{n}+\frac{bxx^{n}x^{n-1}}{2n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n),x)
```

output
```
Piecewise((a*x*x**(n - 1)/n + b*x*x**n*x**(n - 1)/(2*n), Ne(n, 0)), ((a +
b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)\,dx = \frac{(bx^n+a)^2}{2\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(b*x^n + a)^2/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)\,dx = \frac{bx^{2\,n}+2\,ax^n}{2\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*(b*x^(2*n) + 2*a*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{x^n\left(a+\frac{bx^n}{2}\right)}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n),x)
```

output
```
(x^n*(a + (b*x^n)/2))/n
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{x^n(x^nb+2a)}{2n}$$

input
```
int(x^(-1+n)*(a+b*x^n),x)
```

output
```
(x**n*(x**n*b + 2*a))/(2*n)
```

3.378 $\int \frac{a+bx^n}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^n}{x} \, dx = \frac{bx^n}{n} + a\log(x)$$

output
```
b*x^n/n+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{a+bx^n}{x} \, dx = \frac{bx^n}{n} + \frac{a\log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)/x,x]
```

output
```
(b*x^n)/n + (a*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^n}{x}\,dx$$
$$\Big\downarrow \; 802$$
$$\int \left(\frac{a}{x}+bx^{n-1}\right)dx$$
$$\Big\downarrow \; 2009$$
$$a\log(x)+\frac{bx^n}{n}$$

input
```
Int[(a + b*x^n)/x,x]
```

output
```
(b*x^n)/n + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{b\,x^{n}}{n}+a\ln\left(x\right)$	14
norman	$a\ln\left(x\right)+\frac{b\,{\mathrm e}^{n\ln\left(x\right)}}{n}$	16
parallelrisch	$\frac{a\ln\left(x\right)n+b\,x^{n}}{n}$	16
derivativedivides	$\frac{b\,x^{n}+a\ln\left(x^{n}\right)}{n}$	17
default	$\frac{b\,x^{n}+a\ln\left(x^{n}\right)}{n}$	17

input
```
int((a+b*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
b*x^n/n+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{a+bx^n}{x}\,dx=\frac{an\log\left(x\right)+bx^n}{n}$$

input
```
integrate((a+b*x^n)/x,x, algorithm="fricas")
```

output
```
(a*n*log(x) + b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^n}{x}\,dx = \begin{cases} a\log(x) + \frac{bx^n}{n} & \text{for } n \neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)/x,x)
```

output
```
Piecewise((a*log(x) + b*x**n/n, Ne(n, 0)), ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{a+bx^n}{x}\,dx = \frac{bx^n}{n} + \frac{a\log(x^n)}{n}$$

input
```
integrate((a+b*x^n)/x,x, algorithm="maxima")
```

output
```
b*x^n/n + a*log(x^n)/n
```

Giac [F]

$$\int \frac{a+bx^n}{x}\,dx = \int \frac{bx^n+a}{x}\,dx$$

input
```
integrate((a+b*x^n)/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^n}{x} \, dx = a \, \ln(x) + \frac{b \, x^n}{n}$$

input
```
int((a + b*x^n)/x,x)
```

output
```
a*log(x) + (b*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{a+bx^n}{x} \, dx = \frac{x^n b + \log(x) \, an}{n}$$

input
```
int((a+b*x^n)/x,x)
```

output
```
(x**n*b + log(x)*a*n)/n
```

3.379 $\int x^{-1-n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{-1-n}(a+bx^n)\,dx=-\frac{ax^{-n}}{n}+b\log(x)$$

output
```
-a/n/(x^n)+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^{-1-n}(a+bx^n)\,dx=-\frac{ax^{-n}}{n}+\frac{b\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - n)*(a + b*x^n),x]
```

output
```
-(a/(n*x^n)) + (b*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{-n-1}+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\log(x)-\frac{ax^{-n}}{n}$$

input
```
Int[x^(-1 - n)*(a + b*x^n),x]
```

output
```
-(a/(n*x^n)) + b*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{a\,x^{-n}}{n}+b\ln\left(x\right)$	17
norman	$\left(b\ln\left(x\right){\mathrm e}^{n\ln\left(x\right)}-\frac{a}{n}\right){\mathrm e}^{-n\ln\left(x\right)}$	25

input
```
int(x^(-1-n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-a/n/(x^n)+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^{-1-n}(a+bx^n)\,dx=\frac{bnx^n\log(x)-a}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
(b*n*x^n*log(x) - a)/(n*x^n)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^{-1-n}(a+bx^n)\,dx=\begin{cases}-\frac{ax^{-n}}{n}+\frac{b\log(x^n)}{n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1-n)*(a+b*x**n),x)
```

output
```
Piecewise((-a/(n*x**n) + b*log(x**n)/n, Ne(n, 0)), ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^{-1-n}(a+bx^n)\,dx = b\log(x) - \frac{a}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
b*log(x) - a/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^{-1-n}(a+bx^n)\,dx = \frac{bnx^n\log(x) - a}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n),x, algorithm="giac")
```

output
```
(b*n*x^n*log(x) - a)/(n*x^n)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int x^{-1-n}(a+bx^n)\,dx = \begin{cases} \ln(x)\ (a+b) & \text{if } n=0 \\ b\ \ln(x) - \frac{a}{n\,x^n} & \text{if } n\neq 0 \end{cases}$$

input
```
int((a + b*x^n)/x^(n + 1),x)
```

output
```
piecewise(n == 0, log(x)*(a + b), n ~= 0, b*log(x) - a/(n*x^n))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^{-1-n}(a+bx^n)\,dx=\frac{x^n\log(x)\,bn-a}{x^n n}$$

input
```
int(x^(-1-n)*(a+b*x^n),x)
```

output
```
(x**n*log(x)*b*n - a)/(x**n*n)
```

3.380 $\int x^{-1-2n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int x^{-1-2n}(a+bx^n)\,dx=-\frac{ax^{-2n}}{2n}-\frac{bx^{-n}}{n}$$

output
```
-1/2*a/n/(x^(2*n))-b/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int x^{-1-2n}(a+bx^n)\,dx=\frac{x^{-2n}(-a-2bx^n)}{2n}$$

input
```
Integrate[x^(-1 - 2*n)*(a + b*x^n),x]
```

output
```
(-a - 2*b*x^n)/(2*n*x^(2*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(ax^{-2n-1}+bx^{-n-1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ax^{-2n}}{2n}-\frac{bx^{-n}}{n}$$

input
```
Int[x^(-1 - 2*n)*(a + b*x^n),x]
```

output
```
-1/2*a/(n*x^(2*n)) - b/(n*x^n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{b\,x^{-n}}{n}-\frac{a\,x^{-2n}}{2n}$	24
norman	$\left(-\frac{a}{2n}-\frac{b\,\mathrm{e}^{n\ln(x)}}{n}\right)\mathrm{e}^{-2n\ln(x)}$	27
parallelrisch	$\frac{-2x\,x^{n}x^{-2n-1}b-x\,x^{-2n-1}a}{2n}$	32
orering	$-\frac{x(1+3n)x^{-2n-1}(a+b\,x^{n})}{2n^{2}}-\frac{x^{2}\left(\frac{x^{-2n-1}(-2n-1)(a+b\,x^{n})}{x}+\frac{x^{-2n-1}b\,x^{n}n}{x}\right)}{2n^{2}}$	75

input
```
int(x^(-2*n-1)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-b/n/(x^n)-1/2*a/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int x^{-1-2n}(a+bx^n)\,dx=-\frac{2\,bx^n+a}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x^n + a)/(n*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int x^{-1-2n}(a+bx^n)\,dx = \begin{cases} -\frac{axx^{-2n-1}}{2n} - \frac{bxx^nx^{-2n-1}}{n} & \text{for } n \neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-2*n)*(a+b*x**n),x)
```

output
```
Piecewise((-a*x*x**(-2*n - 1)/(2*n) - b*x*x**n*x**(-2*n - 1)/n, Ne(n, 0)),
 ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-1-2n}(a+bx^n)\,dx = -\frac{a}{2\,nx^{2\,n}} - \frac{b}{nx^n}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
-1/2*a/(n*x^(2*n)) - b/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int x^{-1-2n}(a+bx^n)\,dx = -\frac{2\,bx^n + a}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
-1/2*(2*b*x^n + a)/(n*x^(2*n))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int x^{-1-2n}(a+bx^n)\,dx=-\frac{a+2\,b\,x^n}{2\,n\,x^{2\,n}}$$

input
```
int((a + b*x^n)/x^(2*n + 1),x)
```

output
```
-(a + 2*b*x^n)/(2*n*x^(2*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int x^{-1-2n}(a+bx^n)\,dx=\frac{-2x^nb-a}{2x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a+b*x^n),x)
```

output
```
( - 2*x**n*b - a)/(2*x**(2*n)*n)
```

3.381 $\int x^{-1-3n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int x^{-1-3n}(a+bx^n)\,dx=-\frac{ax^{-3n}}{3n}-\frac{bx^{-2n}}{2n}$$

output
```
-1/3*a/n/(x^(3*n))-1/2*b/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-3n}(a+bx^n)\,dx=\frac{x^{-3n}(-2a-3bx^n)}{6n}$$

input
```
Integrate[x^(-1 - 3*n)*(a + b*x^n),x]
```

output
```
(-2*a - 3*b*x^n)/(6*n*x^(3*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{-3n-1}+bx^{-2n-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ax^{-3n}}{3n}-\frac{bx^{-2n}}{2n}$$

input
```
Int[x^(-1 - 3*n)*(a + b*x^n),x]
```

output
```
-1/3*a/(n*x^(3*n)) - b/(2*n*x^(2*n))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{b\,x^{-2n}}{2n}-\frac{a\,x^{-3n}}{3n}$	24
norman	$\left(-\frac{a}{3n}-\frac{b\,\mathrm{e}^{n\ln(x)}}{2n}\right)\mathrm{e}^{-3n\ln(x)}$	27
parallelrisch	$-\frac{3x\,x^{n}x^{-1-3n}b+2x\,x^{-1-3n}a}{6n}$	32
orering	$-\frac{x(5n+1)x^{-1-3n}(a+b\,x^{n})}{6n^{2}}-\frac{x^{2}\left(\frac{x^{-1-3n}(-1-3n)(a+b\,x^{n})}{x}+\frac{x^{-1-3n}b\,x^{n}n}{x}\right)}{6n^{2}}$	75

input
```
int(x^(-1-3*n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/n/(x^n)^2-1/3*a/n/(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-3n}(a+bx^n)\;dx=-\frac{3\,bx^n+2\,a}{6\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/6*(3*b*x^n + 2*a)/(n*x^(3*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^{-1-3n}(a+bx^n)\,dx = \begin{cases} -\frac{axx^{-3n-1}}{3n} - \frac{bxx^nx^{-3n-1}}{2n} & \text{for } n \neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-3*n)*(a+b*x**n),x)
```

output
```
Piecewise((-a*x*x**(-3*n - 1)/(3*n) - b*x*x**n*x**(-3*n - 1)/(2*n), Ne(n,
0)), ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^{-1-3n}(a+bx^n)\,dx = -\frac{a}{3\,nx^{3\,n}} - \frac{b}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
-1/3*a/(n*x^(3*n)) - 1/2*b/(n*x^(2*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-3n}(a+bx^n)\,dx = -\frac{3\,bx^n + 2\,a}{6\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
-1/6*(3*b*x^n + 2*a)/(n*x^(3*n))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-3n}(a+bx^n)\,dx=-\frac{2\,a+3\,b\,x^n}{6\,n\,x^{3\,n}}$$

input
```
int((a + b*x^n)/x^(3*n + 1),x)
```

output
```
-(2*a + 3*b*x^n)/(6*n*x^(3*n))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-3n}(a+bx^n)\,dx=\frac{-3x^nb-2a}{6x^{3n}n}$$

input
```
int(x^(-1-3*n)*(a+b*x^n),x)
```

output
```
( - 3*x**n*b - 2*a)/(6*x**(3*n)*n)
```

3.382 $\int x^{-1-4n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int x^{-1-4n}(a+bx^n)\,dx=-\frac{ax^{-4n}}{4n}-\frac{bx^{-3n}}{3n}$$

output
```
-1/4*a/n/(x^(4*n))-1/3*b/n/(x^(3*n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-4n}(a+bx^n)\,dx=\frac{x^{-4n}(-3a-4bx^n)}{12n}$$

input
```
Integrate[x^(-1 - 4*n)*(a + b*x^n),x]
```

output
```
(-3*a - 4*b*x^n)/(12*n*x^(4*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-4n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(ax^{-4n-1}+bx^{-3n-1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ax^{-4n}}{4n}-\frac{bx^{-3n}}{3n}$$

input
```
Int[x^(-1 - 4*n)*(a + b*x^n),x]
```

output
```
-1/4*a/(n*x^(4*n)) - b/(3*n*x^(3*n))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{b\,x^{-3n}}{3n}-\frac{a\,x^{-4n}}{4n}$	24
norman	$\left(-\frac{a}{4n}-\frac{b\,{\mathrm e}^{n\ln(x)}}{3n}\right){\mathrm e}^{-4n\ln(x)}$	27
parallelrisch	$-\frac{4x\,x^{n}x^{-4n-1}b+3x\,x^{-4n-1}a}{12n}$	32
orering	$-\frac{x(7n+1)x^{-4n-1}(a+b\,x^{n})}{12n^{2}}-\frac{x^{2}\left(\frac{x^{-4n-1}(-4n-1)(a+b\,x^{n})}{x}+\frac{x^{-4n-1}b\,x^{n}n}{x}\right)}{12n^{2}}$	75

input
```
int(x^(-4*n-1)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b/n/(x^n)^3-1/4*a/n/(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-4n}(a+bx^n)\;dx=-\frac{4\,bx^n+3\,a}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/12*(4*b*x^n + 3*a)/(n*x^(4*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^{-1-4n}(a+bx^n)\,dx = \begin{cases} -\frac{axx^{-4n-1}}{4n} - \frac{bxx^nx^{-4n-1}}{3n} & \text{for } n \neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-4*n)*(a+b*x**n),x)
```

output
```
Piecewise((-a*x*x**(-4*n - 1)/(4*n) - b*x*x**n*x**(-4*n - 1)/(3*n), Ne(n,
0)), ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^{-1-4n}(a+bx^n)\,dx = -\frac{a}{4\,nx^{4\,n}} - \frac{b}{3\,nx^{3\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
-1/4*a/(n*x^(4*n)) - 1/3*b/(n*x^(3*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-4n}(a+bx^n)\,dx = -\frac{4\,bx^n + 3\,a}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
-1/12*(4*b*x^n + 3*a)/(n*x^(4*n))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-4n}(a+bx^n)\,dx = -\frac{3\,a+4\,b\,x^n}{12\,n\,x^{4\,n}}$$

input
```
int((a + b*x^n)/x^(4*n + 1),x)
```

output
```
-(3*a + 4*b*x^n)/(12*n*x^(4*n))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-4n}(a+bx^n)\,dx = \frac{-4x^n b-3a}{12x^{4n}n}$$

input
```
int(x^(-1-4*n)*(a+b*x^n),x)
```

output
```
( - 4*x**n*b - 3*a)/(12*x**(4*n)*n)
```

3.383 $\int x^{-1-5n}(a+bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int x^{-1-5n}(a+bx^n)\,dx=-\frac{ax^{-5n}}{5n}-\frac{bx^{-4n}}{4n}$$

output
```
-1/5*a/n/(x^(5*n))-1/4*b/n/(x^(4*n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-5n}(a+bx^n)\,dx=\frac{x^{-5n}(-4a-5bx^n)}{20n}$$

input
```
Integrate[x^(-1 - 5*n)*(a + b*x^n),x]
```

output
```
(-4*a - 5*b*x^n)/(20*n*x^(5*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-5n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{-5n-1}+bx^{-4n-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ax^{-5n}}{5n}-\frac{bx^{-4n}}{4n}$$

input
```
Int[x^(-1 - 5*n)*(a + b*x^n),x]
```

output
```
-1/5*a/(n*x^(5*n)) - b/(4*n*x^(4*n))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{b\,x^{-4n}}{4n}-\frac{a\,x^{-5n}}{5n}$	24
norman	$\left(-\frac{a}{5n}-\frac{b\,\mathrm{e}^{n\ln(x)}}{4n}\right)\mathrm{e}^{-5n\ln(x)}$	27
parallelrisch	$-\frac{5x\,x^{n}x^{-1-5n}b+4x\,x^{-1-5n}a}{20n}$	32
orering	$-\frac{x(9n+1)x^{-1-5n}(a+b\,x^{n})}{20n^{2}}-\frac{x^{2}\left(\frac{x^{-1-5n}(-1-5n)(a+b\,x^{n})}{x}+\frac{x^{-1-5n}b\,x^{n}n}{x}\right)}{20n^{2}}$	75

input
```
int(x^(-1-5*n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b/n/(x^n)^4-1/5*a/n/(x^n)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-5n}(a+bx^n)\,dx=-\frac{5\,bx^n+4\,a}{20\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/20*(5*b*x^n + 4*a)/(n*x^(5*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^{-1-5n}(a+bx^n)\,dx = \begin{cases} -\frac{axx^{-5n-1}}{5n} - \frac{bxx^nx^{-5n-1}}{4n} & \text{for } n \neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-5*n)*(a+b*x**n),x)
```

output
```
Piecewise((-a*x*x**(-5*n - 1)/(5*n) - b*x*x**n*x**(-5*n - 1)/(4*n), Ne(n,
0)), ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^{-1-5n}(a+bx^n)\,dx = -\frac{a}{5\,nx^{5\,n}} - \frac{b}{4\,nx^{4\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
-1/5*a/(n*x^(5*n)) - 1/4*b/(n*x^(4*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-5n}(a+bx^n)\,dx = -\frac{5\,bx^n + 4\,a}{20\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
-1/20*(5*b*x^n + 4*a)/(n*x^(5*n))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-5n}(a+bx^n)\,dx=-\frac{4\,a+5\,b\,x^n}{20\,n\,x^{5\,n}}$$

input
```
int((a + b*x^n)/x^(5*n + 1),x)
```

output
```
-(4*a + 5*b*x^n)/(20*n*x^(5*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int x^{-1-5n}(a+bx^n)\,dx=\frac{-5x^nb-4a}{20x^{5n}n}$$

input
```
int(x^(-1-5*n)*(a+b*x^n),x)
```

output
```
( - 5*x**n*b - 4*a)/(20*x**(5*n)*n)
```

3.384 $\int x^{-1+4n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int x^{-1+4n}(a+bx^n)^2\,dx=\frac{a^2x^{4n}}{4n}+\frac{2abx^{5n}}{5n}+\frac{b^2x^{6n}}{6n}$$

output
```
1/4*a^2*x^(4*n)/n+2/5*a*b*x^(5*n)/n+1/6*b^2*x^(6*n)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1+4n}(a+bx^n)^2\,dx=\frac{x^{4n}(15a^2+24abx^n+10b^2x^{2n})}{60n}$$

input
```
Integrate[x^(-1 + 4*n)*(a + b*x^n)^2,x]
```

output
```
(x^(4*n)*(15*a^2 + 24*a*b*x^n + 10*b^2*x^(2*n)))/(60*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{3n}(bx^n+a)^2\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^2x^{3n}+2abx^{4n}+b^2x^{5n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}a^2x^{4n}+\frac{2}{5}abx^{5n}+\frac{1}{6}b^2x^{6n}}{n}$$

input
```
Int[x^(-1 + 4*n)*(a + b*x^n)^2,x]
```

output
```
((a^2*x^(4*n))/4 + (2*a*b*x^(5*n))/5 + (b^2*x^(6*n))/6)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result
risch	$\frac{a^2x^{4n}}{4n}+\frac{2ab\,x^{5n}}{5n}+\frac{b^2x^{6n}}{6n}$
parallelrisch	$\frac{10x\,x^{2n}x^{-1+4n}b^2+24x\,x^nx^{-1+4n}ab+15x\,x^{-1+4n}a^2}{60n}$
orering	$\frac{x(74n^2-15n+1)x^{-1+4n}(a+b\,x^n)^2}{120n^3}-\frac{x^2(-1+5n)\left(\frac{x^{-1+4n}(-1+4n)(a+b\,x^n)^2}{x}+\frac{2x^{-1+4n}(a+b\,x^n)b\,x^nn}{x}\right)}{40n^3}+\frac{x^3\left(\frac{x^{-1+4n}(\cdots}{}\right.}{}$

input
```
int(x^(-1+4*n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2/n*(x^n)^6+2/5*a*b/n*(x^n)^5+1/4*a^2/n*(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1+4n}(a+bx^n)^2\,dx=\frac{10\,b^2x^{6\,n}+24\,abx^{5\,n}+15\,a^2x^{4\,n}}{60\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/60*(10*b^2*x^(6*n) + 24*a*b*x^(5*n) + 15*a^2*x^(4*n))/n
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int x^{-1+4n}(a+bx^n)^2\,dx = \begin{cases} \frac{a^2xx^{4n-1}}{4n} + \frac{2abxx^nx^{4n-1}}{5n} + \frac{b^2xx^{2n}x^{4n-1}}{6n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x*x**(4*n - 1)/(4*n) + 2*a*b*x*x**n*x**(4*n - 1)/(5*n) + b
**2*x*x**(2*n)*x**(4*n - 1)/(6*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x^{-1+4n}(a+bx^n)^2\,dx = \frac{b^2x^{6n}}{6n} + \frac{2abx^{5n}}{5n} + \frac{a^2x^{4n}}{4n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^(6*n)/n + 2/5*a*b*x^(5*n)/n + 1/4*a^2*x^(4*n)/n
```

Giac [F]

$$\int x^{-1+4n}(a+bx^n)^2\,dx = \int (bx^n+a)^2x^{4n-1}\,dx$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2*x^(4*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int x^{-1+4n}(a+bx^n)^2\,dx=\frac{x^{4\,n}\,\left(15\,a^2+10\,b^2\,x^{2\,n}+24\,a\,b\,x^n\right)}{60\,n}$$

input
```
int(x^(4*n - 1)*(a + b*x^n)^2,x)
```

output
```
(x^(4*n)*(15*a^2 + 10*b^2*x^(2*n) + 24*a*b*x^n))/(60*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int x^{-1+4n}(a+bx^n)^2\,dx=\frac{x^{4n}(10x^{2n}b^2+24x^nab+15a^2)}{60n}$$

input
```
int(x^(-1+4*n)*(a+b*x^n)^2,x)
```

output
```
(x**(4*n)*(10*x**(2*n)*b**2 + 24*x**n*a*b + 15*a**2))/(60*n)
```

3.385 $\int x^{-1+3n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int x^{-1+3n}(a+bx^n)^2\,dx=\frac{a^2x^{3n}}{3n}+\frac{abx^{4n}}{2n}+\frac{b^2x^{5n}}{5n}$$

output
```
1/3*a^2*x^(3*n)/n+1/2*a*b*x^(4*n)/n+1/5*b^2*x^(5*n)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1+3n}(a+bx^n)^2\,dx=\frac{x^{3n}(10a^2+15abx^n+6b^2x^{2n})}{30n}$$

input
```
Integrate[x^(-1 + 3*n)*(a + b*x^n)^2,x]
```

output
```
(x^(3*n)*(10*a^2 + 15*a*b*x^n + 6*b^2*x^(2*n)))/(30*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}(a+bx^n)^2 \, dx$$
$$\Big\downarrow \; 798$$
$$\frac{\int x^{2n}(bx^n+a)^2 \, dx^n}{n}$$
$$\Big\downarrow \; 49$$
$$\frac{\int \left(a^2x^{2n}+2abx^{3n}+b^2x^{4n}\right) dx^n}{n}$$
$$\Big\downarrow \; 2009$$
$$\frac{\frac{1}{3}a^2x^{3n}+\frac{1}{2}abx^{4n}+\frac{1}{5}b^2x^{5n}}{n}$$

input
```
Int[x^(-1 + 3*n)*(a + b*x^n)^2,x]
```

output
```
((a^2*x^(3*n))/3 + (a*b*x^(4*n))/2 + (b^2*x^(5*n))/5)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result
risch	$\frac{a^{2} x^{3 n}}{3 n}+\frac{a b \,x^{4 n}}{2 n}+\frac{b^{2} x^{5 n}}{5 n}$
norman	$\frac{a^{2} {\mathrm e}^{3 n \ln \left(x \right)}}{3 n}+\frac{b^{2} {\mathrm e}^{5 n \ln \left(x \right)}}{5 n}+\frac{a b \,{\mathrm e}^{4 n \ln \left(x \right)}}{2 n}$
parallelrisch	$\frac{6 x \,x^{2 n} x^{-1+3 n} b^{2}+15 x \,x^{n} x^{-1+3 n} a b +10 x \,x^{-1+3 n} a^{2}}{30 n}$
orering	$\frac{x \left(47 n^{2}-12 n +1\right) x^{-1+3 n} \left(a +b \,x^{n}\right)^{2}}{60 n^{3}}-\frac{x^{2} \left(-1+4 n \right) \left(\frac{x^{-1+3 n} \left(-1+3 n \right) \left(a +b \,x^{n}\right)^{2}}{x}+\frac{2 x^{-1+3 n} \left(a +b \,x^{n}\right) b \,x^{n} n}{x}\right)}{20 n^{3}}+\frac{x^{3} \left(\frac{x^{-1+3 n}(\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^(-1+3*n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2/n*(x^n)^3+1/5*b^2/n*(x^n)^5+1/2*a*b/n*(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1+3n}(a+bx^n)^2\,dx=\frac{6\,b^2x^{5\,n}+15\,abx^{4\,n}+10\,a^2x^{3\,n}}{30\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/30*(6*b^2*x^(5*n) + 15*a*b*x^(4*n) + 10*a^2*x^(3*n))/n
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int x^{-1+3n}(a+bx^n)^2\,dx = \begin{cases} \frac{a^2xx^{3n-1}}{3n} + \frac{abxx^nx^{3n-1}}{2n} + \frac{b^2xx^{2n}x^{3n-1}}{5n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x*x**(3*n - 1)/(3*n) + a*b*x*x**n*x**(3*n - 1)/(2*n) + b**
2*x*x**(2*n)*x**(3*n - 1)/(5*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x^{-1+3n}(a+bx^n)^2\,dx = \frac{b^2x^{5n}}{5n} + \frac{abx^{4n}}{2n} + \frac{a^2x^{3n}}{3n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^(5*n)/n + 1/2*a*b*x^(4*n)/n + 1/3*a^2*x^(3*n)/n
```

Giac [F]

$$\int x^{-1+3n}(a+bx^n)^2\,dx = \int (bx^n+a)^2x^{3n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2*x^(3*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int x^{-1+3n}(a+bx^n)^2\,dx=\frac{x^{3\,n}\,\left(10\,a^2+6\,b^2\,x^{2\,n}+15\,a\,b\,x^n\right)}{30\,n}$$

input
```
int(x^(3*n - 1)*(a + b*x^n)^2,x)
```

output
```
(x^(3*n)*(10*a^2 + 6*b^2*x^(2*n) + 15*a*b*x^n))/(30*n)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int x^{-1+3n}(a+bx^n)^2\,dx=\frac{x^{3n}(6x^{2n}b^2+15x^nab+10a^2)}{30n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^2,x)
```

output
```
(x**(3*n)*(6*x**(2*n)*b**2 + 15*x**n*a*b + 10*a**2))/(30*n)
```

3.386 $\int x^{-1+2n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int x^{-1+2n}(a+bx^n)^2\,dx=\frac{a^2x^{2n}}{2n}+\frac{2abx^{3n}}{3n}+\frac{b^2x^{4n}}{4n}$$

output
```
1/2*a^2*x^(2*n)/n+2/3*a*b*x^(3*n)/n+1/4*b^2*x^(4*n)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1+2n}(a+bx^n)^2\,dx=\frac{x^{2n}(6a^2+8abx^n+3b^2x^{2n})}{12n}$$

input
```
Integrate[x^(-1 + 2*n)*(a + b*x^n)^2,x]
```

output
```
(x^(2*n)*(6*a^2 + 8*a*b*x^n + 3*b^2*x^(2*n)))/(12*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(a+bx^n)^2\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^n(bx^n+a)^2\,dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(a^2x^n+2abx^{2n}+b^2x^{3n}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{2}a^2x^{2n}+\frac{2}{3}abx^{3n}+\frac{1}{4}b^2x^{4n}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a + b*x^n)^2,x]
```

output
```
((a^2*x^(2*n))/2 + (2*a*b*x^(3*n))/3 + (b^2*x^(4*n))/4)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result
risch	$\frac{a^2 x^{2n}}{2n}+\frac{2ab\,x^{3n}}{3n}+\frac{b^2 x^{4n}}{4n}$
norman	$\frac{a^2 \mathrm{e}^{2n\ln(x)}}{2n}+\frac{b^2 \mathrm{e}^{4n\ln(x)}}{4n}+\frac{2ab\,\mathrm{e}^{3n\ln(x)}}{3n}$
parallelrisch	$\frac{3x\,x^{2n}x^{2n-1}b^2+8x\,x^n x^{2n-1}ab+6x\,x^{2n-1}a^2}{12n}$
orering	$\frac{x(26n^2-9n+1)x^{2n-1}(a+b\,x^n)^2}{24n^3}-\frac{x^2(-1+3n)\left(\frac{x^{2n-1}(2n-1)(a+b\,x^n)^2}{x}+\frac{2x^{2n-1}(a+b\,x^n)b\,x^n n}{x}\right)}{8n^3}+\frac{x^3\left(\frac{x^{2n-1}(2n-1)^2(a}{x^2}\right.}{}$

input
```
int(x^(2*n-1)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2/n*(x^n)^2+1/4*b^2/n*(x^n)^4+2/3*a*b/n*(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1+2n}(a+bx^n)^2\,dx=\frac{3\,b^2x^{4\,n}+8\,abx^{3\,n}+6\,a^2x^{2\,n}}{12\,n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*x^(4*n) + 8*a*b*x^(3*n) + 6*a^2*x^(2*n))/n
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int x^{-1+2n}(a+bx^n)^2\,dx=\begin{cases}\frac{a^2xx^{2n-1}}{2n}+\frac{2abxx^nx^{2n-1}}{3n}+\frac{b^2xx^{2n}x^{2n-1}}{4n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x*x**(2*n - 1)/(2*n) + 2*a*b*x*x**n*x**(2*n - 1)/(3*n) + b
**2*x*x**(2*n)*x**(2*n - 1)/(4*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x^{-1+2n}(a+bx^n)^2\,dx=\frac{b^2x^{4\,n}}{4\,n}+\frac{2\,abx^{3\,n}}{3\,n}+\frac{a^2x^{2\,n}}{2\,n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^(4*n)/n + 2/3*a*b*x^(3*n)/n + 1/2*a^2*x^(2*n)/n
```

Giac [F]

$$\int x^{-1+2n}(a+bx^n)^2\,dx=\int(bx^n+a)^2x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2*x^(2*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int x^{-1+2n}(a+bx^n)^2\,dx = \frac{x^{2\,n}\,\left(6\,a^2+3\,b^2\,x^{2\,n}+8\,a\,b\,x^n\right)}{12\,n}$$

input
```
int(x^(2*n - 1)*(a + b*x^n)^2,x)
```

output
```
(x^(2*n)*(6*a^2 + 3*b^2*x^(2*n) + 8*a*b*x^n))/(12*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int x^{-1+2n}(a+bx^n)^2\,dx = \frac{x^{2n}(3x^{2n}b^2+8x^nab+6a^2)}{12n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^2,x)
```

output
```
(x**(2*n)*(3*x**(2*n)*b**2 + 8*x**n*a*b + 6*a**2))/(12*n)
```

3.387 $\int x^{-1+n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{(a+bx^n)^3}{3bn}$$

output
```
1/3*(a+b*x^n)^3/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{(a+bx^n)^3}{3bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^2,x]
```

output
```
(a + b*x^n)^3/(3*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{(a+bx^n)^3}{3bn}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^2,x]
```

output
```
(a + b*x^n)^3/(3*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

method	result
risch	$\frac{a^2 x^n}{n}+\frac{ab\,x^{2n}}{n}+\frac{b^2 x^{3n}}{3n}$
norman	$\frac{a^2 \mathrm{e}^{n\ln(x)}}{n}+\frac{ab\,\mathrm{e}^{2n\ln(x)}}{n}+\frac{b^2 \mathrm{e}^{3n\ln(x)}}{3n}$
parallelrisch	$\frac{x\,x^{2n} x^{-1+n} b^2+3x\,x^n x^{-1+n} ab+3x\,x^{-1+n} a^2}{3n}$
orering	$\frac{x(11n^2-6n+1)x^{-1+n}(a+b\,x^n)^2}{6n^3}-\frac{x^2(2n-1)\left(\frac{x^{-1+n}(-1+n)(a+b\,x^n)^2}{x}+\frac{2x^{-1+n}(a+b\,x^n)b\,x^n n}{x}\right)}{2n^3}+\frac{x^3\left(\frac{x^{-1+n}(-1+n)^2(}{x^2}\right.}{}$

input
```
int(x^(-1+n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/n*x^n+a*b/n*(x^n)^2+1/3*b^2/n*(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{b^2x^{3\,n}+3\,abx^{2\,n}+3\,a^2x^n}{3\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^(3*n) + 3*a*b*x^(2*n) + 3*a^2*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int x^{-1+n}(a+bx^n)^2\,dx=\begin{cases}\frac{a^2xx^{n-1}}{n}+\frac{abxx^nx^{n-1}}{n}+\frac{b^2xx^{2n}x^{n-1}}{3n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x*x**(n - 1)/n + a*b*x*x**n*x**(n - 1)/n + b**2*x*x**(2*n)
*x**(n - 1)/(3*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{(bx^n+a)^3}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/3*(b*x^n + a)^3/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{b^2x^{3\,n}+3\,abx^{2\,n}+3\,a^2x^n}{3\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*x^(3*n) + 3*a*b*x^(2*n) + 3*a^2*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{x^n\,\left(a^2+\frac{b^2\,x^{2\,n}}{3}+a\,b\,x^n\right)}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^2,x)
```

output
```
(x^n*(a^2 + (b^2*x^(2*n))/3 + a*b*x^n))/n
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{x^n(x^{2n}b^2+3x^nab+3a^2)}{3n}$$

input
```
int(x^(-1+n)*(a+b*x^n)^2,x)
```

output
```
(x**n*(x**(2*n)*b**2 + 3*x**n*a*b + 3*a**2))/(3*n)
```

3.388 $\int \frac{(a+bx^n)^2}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{(a+bx^n)^2}{x} \, dx = \frac{2abx^n}{n} + \frac{b^2x^{2n}}{2n} + a^2\log(x)$$

output
```
2*a*b*x^n/n+1/2*b^2*x^(2*n)/n+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^2}{x} \, dx = \frac{bx^n(4a+bx^n)}{2n} + \frac{a^2\log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)^2/x,x]
```

output
```
(b*x^n*(4*a + b*x^n))/(2*n) + (a^2*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2}{x}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-n}(bx^n+a)^2dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int (a^2x^{-n}+b^2x^n+2ab)\,dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2\log(x^n)+2abx^n+\frac{1}{2}b^2x^{2n}}{n}$$

input
```
Int[(a + b*x^n)^2/x,x]
```

output
```
(2*a*b*x^n + (b^2*x^(2*n))/2 + a^2*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{\frac{b^2 x^{2n}}{2}+2ab\,x^n+a^2\ln(x^n)}{n}$	31
default	$\frac{\frac{b^2 x^{2n}}{2}+2ab\,x^n+a^2\ln(x^n)}{n}$	31
risch	$\frac{2ab\,x^n}{n}+\frac{b^2x^{2n}}{2n}+a^2\ln(x)$	31
parallelrisch	$\frac{b^2x^{2n}+2a^2\ln(x)n+4ab\,x^n}{2n}$	31
norman	$a^2\ln(x)+\frac{b^2\mathrm{e}^{2n\ln(x)}}{2n}+\frac{2ab\,\mathrm{e}^{n\ln(x)}}{n}$	35

input
```
int((a+b*x^n)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/2*b^2*(x^n)^2+2*a*b*x^n+a^2*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{2\,a^2n\log(x)+b^2x^{2\,n}+4\,abx^n}{2\,n}$$

input
```
integrate((a+b*x^n)^2/x,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*n*log(x) + b^2*x^(2*n) + 4*a*b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^n)^2}{x}\,dx = \begin{cases} a^2\log(x) + \frac{2abx^n}{n} + \frac{b^2x^{2n}}{2n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**2/x,x)
```

output
```
Piecewise((a**2*log(x) + 2*a*b*x**n/n + b**2*x**(2*n)/(2*n), Ne(n, 0)), ((
a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{a^2\log(x^n)}{n} + \frac{b^2x^{2n} + 4\,abx^n}{2\,n}$$

input
```
integrate((a+b*x^n)^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x^n)/n + 1/2*(b^2*x^(2*n) + 4*a*b*x^n)/n
```

Giac [F]

$$\int \frac{(a+bx^n)^2}{x}\,dx = \int \frac{(bx^n+a)^2}{x}\,dx$$

input
```
integrate((a+b*x^n)^2/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{b^2\,x^{2\,n}+4\,a\,b\,x^n+2\,a^2\,n\,\ln(x)}{2\,n}$$

input
```
int((a + b*x^n)^2/x,x)
```

output
```
(b^2*x^(2*n) + 4*a*b*x^n + 2*a^2*n*log(x))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^2}{x}\,dx = \frac{x^{2n}b^2+4x^n ab+2\log(x)\,a^2n}{2n}$$

input
```
int((a+b*x^n)^2/x,x)
```

output
```
(x**(2*n)*b**2 + 4*x**n*a*b + 2*log(x)*a**2*n)/(2*n)
```

3.389 $\int x^{-1-n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int x^{-1-n}(a+bx^n)^2\,dx=-\frac{a^2x^{-n}}{n}+\frac{b^2x^n}{n}+2ab\log(x)$$

output
```
-a^2/n/(x^n)+b^2*x^n/n+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int x^{-1-n}(a+bx^n)^2\,dx=-\frac{a^2x^{-n}-b^2x^n-2ab\log\left(x^n\right)}{n}$$

input
```
Integrate[x^(-1 - n)*(a + b*x^n)^2,x]
```

output
```
-((a^2/x^n - b^2*x^n - 2*a*b*Log[x^n])/n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-2n}(bx^n+a)^2\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^2x^{-2n}+2abx^{-n}+b^2\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2x^{-n}+2ab\log\left(x^n\right)+b^2x^n}{n}$$

input
```
Int[x^(-1 - n)*(a + b*x^n)^2,x]
```

output
```
(-(a^2/x^n) + b^2*x^n + 2*a*b*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{a^{2} x^{-n}}{n}+\frac{b^{2} x^{n}}{n}+2 a b \ln \left(x \right)$	31
norman	$\left(\frac{b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{n}+2 a b \ln \left(x \right) {\mathrm e}^{n \ln \left(x \right)}-\frac{a^{2}}{n}\right) {\mathrm e}^{-n \ln \left(x \right)}$	43

input
```
int(x^(-1-n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/n/(x^n)+b^2*x^n/n+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x^{-1-n}(a+bx^n)^2\,dx=\frac{2\,abnx^n\log(x)+b^2x^{2\,n}-a^2}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
(2*a*b*n*x^n*log(x) + b^2*x^(2*n) - a^2)/(n*x^n)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x^{-1-n}(a+bx^n)^2\,dx = \begin{cases} -\frac{a^2x^{-n}}{n} + \frac{2ab\log(x^n)}{n} + \frac{b^2x^n}{n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((-a**2/(n*x**n) + 2*a*b*log(x**n)/n + b**2*x**n/n, Ne(n, 0)), ((
a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^{-1-n}(a+bx^n)^2\,dx = 2\,ab\log(x) + \frac{b^2x^n}{n} - \frac{a^2}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
2*a*b*log(x) + b^2*x^n/n - a^2/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x^{-1-n}(a+bx^n)^2\,dx = \frac{2\,abnx^n\log(x) + b^2x^{2\,n} - a^2}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
(2*a*b*n*x^n*log(x) + b^2*x^(2*n) - a^2)/(n*x^n)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^{-1-n}(a+bx^n)^2 \, dx = 2\,a\,b\,\ln\left(x\right)+\frac{b^2\,x^n}{n}-\frac{a^2}{n\,x^n}$$

input
```
int((a + b*x^n)^2/x^(n + 1),x)
```

output
```
2*a*b*log(x) + (b^2*x^n)/n - a^2/(n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x^{-1-n}(a+bx^n)^2 \, dx = \frac{x^{2n}b^2+2x^n\log(x)\,abn-a^2}{x^n n}$$

input
```
int(x^(-1-n)*(a+b*x^n)^2,x)
```

output
```
(x**(2*n)*b**2 + 2*x**n*log(x)*a*b*n - a**2)/(x**n*n)
```

3.390 $\int x^{-1-2n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int x^{-1-2n}(a+bx^n)^2\,dx=-\frac{a^2x^{-2n}}{2n}-\frac{2abx^{-n}}{n}+b^2\log(x)$$

output
```
-1/2*a^2/n/(x^(2*n))-2*a*b/n/(x^n)+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int x^{-1-2n}(a+bx^n)^2\,dx=-\frac{ax^{-2n}(a+4bx^n)}{2n}+\frac{b^2\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 2*n)*(a + b*x^n)^2,x]
```

output
```
-1/2*(a*(a + 4*b*x^n))/(n*x^(2*n)) + (b^2*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-3n}(bx^n+a)^2\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^2x^{-3n}+2abx^{-2n}+b^2x^{-n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^2x^{-2n}-2abx^{-n}+b^2\log\left(x^n\right)}{n}$$

input
```
Int[x^(-1 - 2*n)*(a + b*x^n)^2,x]
```

output
```
(-1/2*a^2/x^(2*n) - (2*a*b)/x^n + b^2*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
risch	$b^2 \ln(x) - \frac{2ab\,x^{-n}}{n} - \frac{a^2 x^{-2n}}{2n}$	33
norman	$\left(b^2 \ln(x)\, \mathrm{e}^{2n\ln(x)} - \frac{a^2}{2n} - \frac{2ab\,\mathrm{e}^{n\ln(x)}}{n}\right) \mathrm{e}^{-2n\ln(x)}$	43

input
```
int(x^(-2*n-1)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*ln(x)-2*a*b/n/(x^n)-1/2*a^2/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int x^{-1-2n}(a+bx^n)^2\,dx = \frac{2\,b^2nx^{2\,n}\log(x) - 4\,abx^n - a^2}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) - 4*a*b*x^n - a^2)/(n*x^(2*n))
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int x^{-1-2n}(a+bx^n)^2\,dx=\begin{cases}-\frac{a^2x^{-2n}}{2n}-\frac{2abx^{-n}}{n}+\frac{b^2\log(x^n)}{n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1-2*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((-a**2/(2*n*x**(2*n)) - 2*a*b/(n*x**n) + b**2*log(x**n)/n, Ne(n,
 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x^{-1-2n}(a+bx^n)^2\,dx=b^2\log(x)-\frac{a^2}{2\,nx^{2\,n}}-\frac{2\,ab}{nx^n}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
b^2*log(x) - 1/2*a^2/(n*x^(2*n)) - 2*a*b/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int x^{-1-2n}(a+bx^n)^2\,dx=\frac{2\,b^2nx^{2\,n}\log(x)-4\,abx^n-a^2}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) - 4*a*b*x^n - a^2)/(n*x^(2*n))
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x^{-1-2n}(a+bx^n)^2\,dx = b^2\,\ln(x) - \frac{a^2}{2\,n\,x^{2\,n}} - \frac{2\,a\,b}{n\,x^n}$$

input
```
int((a + b*x^n)^2/x^(2*n + 1),x)
```

output
```
b^2*log(x) - a^2/(2*n*x^(2*n)) - (2*a*b)/(n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int x^{-1-2n}(a+bx^n)^2\,dx = \frac{2x^{2n}\log(x)\,b^2n - 4x^nab - a^2}{2x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a+b*x^n)^2,x)
```

output
```
(2*x**(2*n)*log(x)*b**2*n - 4*x**n*a*b - a**2)/(2*x**(2*n)*n)
```

3.391 $\int x^{-1-3n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int x^{-1-3n}(a+bx^n)^2\,dx = -\frac{x^{-3n}(a+bx^n)^3}{3an}$$

output
```
-1/3*(a+b*x^n)^3/a/n/(x^(3*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x^{-1-3n}(a+bx^n)^2\,dx = \frac{x^{-3n}(-a^2-3abx^n-3b^2x^{2n})}{3n}$$

input
```
Integrate[x^(-1 - 3*n)*(a + b*x^n)^2,x]
```

output
```
(-a^2 - 3*a*b*x^n - 3*b^2*x^(2*n))/(3*n*x^(3*n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{x^{-3n}(a+bx^n)^3}{3an}$$

input
```
Int[x^(-1 - 3*n)*(a + b*x^n)^2,x]
```

output
```
-1/3*(a + b*x^n)^3/(a*n*x^(3*n))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{b^{2} x^{-n}}{n}-\frac{a b \,x^{-2 n}}{n}-\frac{a^{2} x^{-3 n}}{3 n}$
norman	$\left(-\frac{a^{2}}{3 n}-\frac{b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{n}-\frac{a b \,{\mathrm e}^{n \ln \left(x \right)}}{n}\right) {\mathrm e}^{-3 n \ln \left(x \right)}$
parallelrisch	$\frac{-3 x \,x^{2 n} x^{-1-3 n} b^{2}-3 x \,x^{n} x^{-1-3 n} a b -x \,x^{-1-3 n} a^{2}}{3 n}$
orering	$-\frac{x \left(11 n^{2}+6 n +1\right) x^{-1-3 n} \left(a +b \,x^{n}\right)^{2}}{6 n^{3}}-\frac{x^{2} \left(1+2 n \right) \left(\frac{x^{-1-3 n} \left(-1-3 n \right) \left(a +b \,x^{n}\right)^{2}}{x}+\frac{2 x^{-1-3 n} \left(a +b \,x^{n}\right) b \,x^{n} n}{x}\right)}{2 n^{3}}-\frac{x^{3} \left(\frac{x^{-1-3 n} (}{}\right.}{}$

input
```
int(x^(-1-3*n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2/n/(x^n)-a*b/n/(x^n)^2-1/3*a^2/n/(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int x^{-1-3n}(a+bx^n)^2\,dx=-\frac{3\,b^2x^{2\,n}+3\,abx^n+a^2}{3\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*b^2*x^(2*n) + 3*a*b*x^n + a^2)/(n*x^(3*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int x^{-1-3n}(a+bx^n)^2\,dx = \begin{cases} -\frac{a^2xx^{-3n-1}}{3n} - \frac{abxx^nx^{-3n-1}}{n} - \frac{b^2xx^{2n}x^{-3n-1}}{n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-3*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((-a**2*x*x**(-3*n - 1)/(3*n) - a*b*x*x**n*x**(-3*n - 1)/n - b**2
*x*x**(2*n)*x**(-3*n - 1)/n, Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int x^{-1-3n}(a+bx^n)^2\,dx = -\frac{a^2}{3\,nx^{3\,n}} - \frac{ab}{nx^{2\,n}} - \frac{b^2}{nx^n}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/3*a^2/(n*x^(3*n)) - a*b/(n*x^(2*n)) - b^2/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int x^{-1-3n}(a+bx^n)^2\,dx = -\frac{3\,b^2x^{2\,n} + 3\,abx^n + a^2}{3\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*x^(2*n) + 3*a*b*x^n + a^2)/(n*x^(3*n))
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int x^{-1-3n}(a+bx^n)^2\,dx=-\frac{a^2+3\,b^2\,x^{2\,n}+3\,a\,b\,x^n}{3\,n\,x^{3\,n}}$$

input
```
int((a + b*x^n)^2/x^(3*n + 1),x)
```

output
```
-(a^2 + 3*b^2*x^(2*n) + 3*a*b*x^n)/(3*n*x^(3*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x^{-1-3n}(a+bx^n)^2\,dx=\frac{-3x^{2n}b^2-3x^nab-a^2}{3x^{3n}n}$$

input
```
int(x^(-1-3*n)*(a+b*x^n)^2,x)
```

output
```
( - 3*x**(2*n)*b**2 - 3*x**n*a*b - a**2)/(3*x**(3*n)*n)
```

3.392 $\int x^{-1-4n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int x^{-1-4n}(a+bx^n)^2\,dx=-\frac{a^2x^{-4n}}{4n}-\frac{2abx^{-3n}}{3n}-\frac{b^2x^{-2n}}{2n}$$

output
```
-1/4*a^2/n/(x^(4*n))-2/3*a*b/n/(x^(3*n))-1/2*b^2/n/(x^(2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-4n}(a+bx^n)^2\,dx=\frac{x^{-4n}(-3a^2-8abx^n-6b^2x^{2n})}{12n}$$

input
```
Integrate[x^(-1 - 4*n)*(a + b*x^n)^2,x]
```

output
```
(-3*a^2 - 8*a*b*x^n - 6*b^2*x^(2*n))/(12*n*x^(4*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-4n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-5n}(bx^n+a)^2\,dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(a^2x^{-5n}+2abx^{-4n}+b^2x^{-3n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}a^2x^{-4n}-\frac{2}{3}abx^{-3n}-\frac{1}{2}b^2x^{-2n}}{n}$$

input
```
Int[x^(-1 - 4*n)*(a + b*x^n)^2,x]
```

output
```
(-1/4*a^2/x^(4*n) - (2*a*b)/(3*x^(3*n)) - b^2/(2*x^(2*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{b^2 x^{-2n}}{2n}-\frac{2ab\,x^{-3n}}{3n}-\frac{a^2 x^{-4n}}{4n}$
norman	$\left(-\frac{a^2}{4n}-\frac{b^2 \mathrm{e}^{2n\ln(x)}}{2n}-\frac{2ab\,\mathrm{e}^{n\ln(x)}}{3n}\right)\mathrm{e}^{-4n\ln(x)}$
parallelrisch	$\frac{-6x\,x^{2n}x^{-4n-1}b^2-8x\,x^n x^{-4n-1}ab-3x\,x^{-4n-1}a^2}{12n}$
orering	$-\frac{x(26n^2+9n+1)x^{-4n-1}(a+b\,x^n)^2}{24n^3}-\frac{x^2(1+3n)\left(\frac{x^{-4n-1}(-4n-1)(a+b\,x^n)^2}{x}+\frac{2x^{-4n-1}(a+b\,x^n)b\,x^n n}{x}\right)}{8n^3}-\frac{x^3\left(\frac{x^{-4n-1}(\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^(-4*n-1)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/n/(x^n)^2-2/3*a*b/n/(x^n)^3-1/4*a^2/n/(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-4n}(a+bx^n)^2\,dx=-\frac{6\,b^2x^{2\,n}+8\,abx^n+3\,a^2}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*x^(2*n) + 8*a*b*x^n + 3*a^2)/(n*x^(4*n))
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int x^{-1-4n}(a+bx^n)^2\,dx = \begin{cases} -\frac{a^2xx^{-4n-1}}{4n} - \frac{2abxx^nx^{-4n-1}}{3n} - \frac{b^2xx^{2n}x^{-4n-1}}{2n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-4*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((-a**2*x*x**(-4*n - 1)/(4*n) - 2*a*b*x*x**n*x**(-4*n - 1)/(3*n)
- b**2*x*x**(2*n)*x**(-4*n - 1)/(2*n), Ne(n, 0)), ((a + b)**2*log(x), True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^{-1-4n}(a+bx^n)^2\,dx = -\frac{a^2}{4\,nx^{4\,n}} - \frac{2\,ab}{3\,nx^{3\,n}} - \frac{b^2}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2/(n*x^(4*n)) - 2/3*a*b/(n*x^(3*n)) - 1/2*b^2/(n*x^(2*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-4n}(a+bx^n)^2\,dx = -\frac{6\,b^2x^{2\,n} + 8\,abx^n + 3\,a^2}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
-1/12*(6*b^2*x^(2*n) + 8*a*b*x^n + 3*a^2)/(n*x^(4*n))
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-4n}(a+bx^n)^2\,dx = -\frac{3\,a^2+6\,b^2\,x^{2\,n}+8\,a\,b\,x^n}{12\,n\,x^{4\,n}}$$

input
```
int((a + b*x^n)^2/x^(4*n + 1),x)
```

output
```
-(3*a^2 + 6*b^2*x^(2*n) + 8*a*b*x^n)/(12*n*x^(4*n))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-4n}(a+bx^n)^2\,dx = \frac{-6x^{2n}b^2-8x^nab-3a^2}{12x^{4n}n}$$

input
```
int(x^(-1-4*n)*(a+b*x^n)^2,x)
```

output
```
( - 6*x**(2*n)*b**2 - 8*x**n*a*b - 3*a**2)/(12*x**(4*n)*n)
```

3.393 $\int x^{-1-5n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int x^{-1-5n}(a+bx^n)^2\,dx=-\frac{a^2x^{-5n}}{5n}-\frac{abx^{-4n}}{2n}-\frac{b^2x^{-3n}}{3n}$$

output
```
-1/5*a^2/n/(x^(5*n))-1/2*a*b/n/(x^(4*n))-1/3*b^2/n/(x^(3*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-5n}(a+bx^n)^2\,dx=\frac{x^{-5n}(-6a^2-15abx^n-10b^2x^{2n})}{30n}$$

input
```
Integrate[x^(-1 - 5*n)*(a + b*x^n)^2,x]
```

output
```
(-6*a^2 - 15*a*b*x^n - 10*b^2*x^(2*n))/(30*n*x^(5*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-5n-1}(a+bx^n)^2\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-6n}(bx^n+a)^2\,dx^n}{n}$$
$$\Big\downarrow \text{53}$$
$$\frac{\int \left(a^2x^{-6n}+2abx^{-5n}+b^2x^{-4n}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{1}{5}a^2x^{-5n}-\frac{1}{2}abx^{-4n}-\frac{1}{3}b^2x^{-3n}}{n}$$

input
```
Int[x^(-1 - 5*n)*(a + b*x^n)^2,x]
```

output
```
(-1/5*a^2/x^(5*n) - (a*b)/(2*x^(4*n)) - b^2/(3*x^(3*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{b^{2} x^{-3 n}}{3 n}-\frac{a b\, x^{-4 n}}{2 n}-\frac{a^{2} x^{-5 n}}{5 n}$
norman	$\left(-\frac{a^{2}}{5 n}-\frac{b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{3 n}-\frac{a b\, {\mathrm e}^{n \ln \left(x \right)}}{2 n}\right) {\mathrm e}^{-5 n \ln \left(x \right)}$
parallelrisch	$-\frac{10 x \,x^{2 n} x^{-1-5 n} b^{2}+15 x \,x^{n} x^{-1-5 n} a b +6 x \,x^{-1-5 n} a^{2}}{30 n}$
orering	$-\frac{x \left(47 n^{2}+12 n +1\right) x^{-1-5 n} \left(a +b \,x^{n}\right)^{2}}{60 n^{3}}-\frac{x^{2} \left(1+4 n \right) \left(\frac{x^{-1-5 n} \left(-1-5 n \right) \left(a +b \,x^{n}\right)^{2}}{x}+\frac{2 x^{-1-5 n} \left(a +b \,x^{n}\right) b \,x^{n} n}{x}\right)}{20 n^{3}}-\frac{x^{3} \left(\frac{x^{-1-5n}}{}\right.}{}$

input
```
int(x^(-1-5*n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2/n/(x^n)^3-1/2*a*b/n/(x^n)^4-1/5*a^2/n/(x^n)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-5n}(a+bx^n)^2\,dx=-\frac{10\,b^2x^{2\,n}+15\,abx^n+6\,a^2}{30\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-1/30*(10*b^2*x^(2*n) + 15*a*b*x^n + 6*a^2)/(n*x^(5*n))
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int x^{-1-5n}(a+bx^n)^2\,dx=\begin{cases}-\frac{a^2xx^{-5n-1}}{5n}-\frac{abxx^nx^{-5n-1}}{2n}-\frac{b^2xx^{2n}x^{-5n-1}}{3n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1-5*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((-a**2*x*x**(-5*n - 1)/(5*n) - a*b*x*x**n*x**(-5*n - 1)/(2*n) -
b**2*x*x**(2*n)*x**(-5*n - 1)/(3*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^{-1-5n}(a+bx^n)^2\,dx=-\frac{a^2}{5\,nx^{5\,n}}-\frac{ab}{2\,nx^{4\,n}}-\frac{b^2}{3\,nx^{3\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/5*a^2/(n*x^(5*n)) - 1/2*a*b/(n*x^(4*n)) - 1/3*b^2/(n*x^(3*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-5n}(a+bx^n)^2\,dx=-\frac{10\,b^2x^{2\,n}+15\,abx^n+6\,a^2}{30\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
-1/30*(10*b^2*x^(2*n) + 15*a*b*x^n + 6*a^2)/(n*x^(5*n))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-5n}(a+bx^n)^2\,dx=-\frac{6\,a^2+10\,b^2\,x^{2\,n}+15\,a\,b\,x^n}{30\,n\,x^{5\,n}}$$

input
```
int((a + b*x^n)^2/x^(5*n + 1),x)
```

output
```
-(6*a^2 + 10*b^2*x^(2*n) + 15*a*b*x^n)/(30*n*x^(5*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-5n}(a+bx^n)^2\,dx=\frac{-10x^{2n}b^2-15x^nab-6a^2}{30x^{5n}n}$$

input
```
int(x^(-1-5*n)*(a+b*x^n)^2,x)
```

output
```
( - 10*x**(2*n)*b**2 - 15*x**n*a*b - 6*a**2)/(30*x**(5*n)*n)
```

3.394 $\int x^{-1-6n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int x^{-1-6n}(a+bx^n)^2\,dx=-\frac{a^2x^{-6n}}{6n}-\frac{2abx^{-5n}}{5n}-\frac{b^2x^{-4n}}{4n}$$

output
```
-1/6*a^2/n/(x^(6*n))-2/5*a*b/n/(x^(5*n))-1/4*b^2/n/(x^(4*n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-6n}(a+bx^n)^2\,dx=\frac{x^{-6n}(-10a^2-24abx^n-15b^2x^{2n})}{60n}$$

input
```
Integrate[x^(-1 - 6*n)*(a + b*x^n)^2,x]
```

output
```
(-10*a^2 - 24*a*b*x^n - 15*b^2*x^(2*n))/(60*n*x^(6*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-6n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int x^{-7n}(bx^n+a)^2\,dx^n}{n}$$

$$\Big\downarrow \ 53$$

$$\frac{\int \left(a^2x^{-7n}+2abx^{-6n}+b^2x^{-5n}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{6}a^2x^{-6n}-\frac{2}{5}abx^{-5n}-\frac{1}{4}b^2x^{-4n}}{n}$$

input
```
Int[x^(-1 - 6*n)*(a + b*x^n)^2,x]
```

output
```
(-1/6*a^2/x^(6*n) - (2*a*b)/(5*x^(5*n)) - b^2/(4*x^(4*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{b^{2} x^{-4 n}}{4 n}-\frac{2 a b\, x^{-5 n}}{5 n}-\frac{a^{2} x^{-6 n}}{6 n}$
parallelrisch	$-\frac{15 x\, x^{2 n} x^{-1-6 n} b^{2}+24 x\, x^{n} x^{-1-6 n} a b+10 x\, x^{-1-6 n} a^{2}}{60 n}$
orering	$-\frac{x \left(74 n^{2}+15 n+1\right) x^{-1-6 n} \left(a+b\, x^{n}\right)^{2}}{120 n^{3}}-\frac{x^{2} \left(5 n+1\right) \left(\frac{x^{-1-6 n} \left(-1-6 n\right) \left(a+b\, x^{n}\right)^{2}}{x}+\frac{2 x^{-1-6 n} \left(a+b\, x^{n}\right) b\, x^{n} n}{x}\right)}{40 n^{3}}-x^{3}\left(\frac{x^{-1-6 n}}{}\cdots\right.$

input
```
int(x^(-1-6*n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^2/n/(x^n)^4-2/5*a*b/n/(x^n)^5-1/6*a^2/n/(x^n)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-6n}(a+bx^n)^2\,dx=-\frac{15\,b^2x^{2\,n}+24\,abx^n+10\,a^2}{60\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-1/60*(15*b^2*x^(2*n) + 24*a*b*x^n + 10*a^2)/(n*x^(6*n))
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int x^{-1-6n}(a+bx^n)^2\,dx = \begin{cases} -\frac{a^2xx^{-6n-1}}{6n} - \frac{2abxx^nx^{-6n-1}}{5n} - \frac{b^2xx^{2n}x^{-6n-1}}{4n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-6*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((-a**2*x*x**(-6*n - 1)/(6*n) - 2*a*b*x*x**n*x**(-6*n - 1)/(5*n)
- b**2*x*x**(2*n)*x**(-6*n - 1)/(4*n), Ne(n, 0)), ((a + b)**2*log(x), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^{-1-6n}(a+bx^n)^2\,dx = -\frac{a^2}{6\,nx^{6\,n}} - \frac{2\,ab}{5\,nx^{5\,n}} - \frac{b^2}{4\,nx^{4\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/6*a^2/(n*x^(6*n)) - 2/5*a*b/(n*x^(5*n)) - 1/4*b^2/(n*x^(4*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-6n}(a+bx^n)^2\,dx = -\frac{15\,b^2x^{2\,n} + 24\,abx^n + 10\,a^2}{60\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
-1/60*(15*b^2*x^(2*n) + 24*a*b*x^n + 10*a^2)/(n*x^(6*n))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-6n}(a+bx^n)^2\,dx=-\frac{10\,a^2+15\,b^2\,x^{2\,n}+24\,a\,b\,x^n}{60\,n\,x^{6\,n}}$$

input
```
int((a + b*x^n)^2/x^(6*n + 1),x)
```

output
```
-(10*a^2 + 15*b^2*x^(2*n) + 24*a*b*x^n)/(60*n*x^(6*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int x^{-1-6n}(a+bx^n)^2\,dx=\frac{-15x^{2n}b^2-24x^nab-10a^2}{60x^{6n}n}$$

input
```
int(x^(-1-6*n)*(a+b*x^n)^2,x)
```

output
```
( - 15*x**(2*n)*b**2 - 24*x**n*a*b - 10*a**2)/(60*x**(6*n)*n)
```

3.395 $\int x^{-1+4n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int x^{-1+4n}(a+bx^n)^3\,dx=\frac{a^3x^{4n}}{4n}+\frac{3a^2bx^{5n}}{5n}+\frac{ab^2x^{6n}}{2n}+\frac{b^3x^{7n}}{7n}$$

output
```
1/4*a^3*x^(4*n)/n+3/5*a^2*b*x^(5*n)/n+1/2*a*b^2*x^(6*n)/n+1/7*b^3*x^(7*n)/
n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1+4n}(a+bx^n)^3\,dx=\frac{x^{4n}(35a^3+84a^2bx^n+70ab^2x^{2n}+20b^3x^{3n})}{140n}$$

input
```
Integrate[x^(-1 + 4*n)*(a + b*x^n)^3,x]
```

output
```
(x^(4*n)*(35*a^3 + 84*a^2*b*x^n + 70*a*b^2*x^(2*n) + 20*b^3*x^(3*n)))/(140
*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{3n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^3x^{3n}+3a^2bx^{4n}+3ab^2x^{5n}+b^3x^{6n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}a^3x^{4n}+\frac{3}{5}a^2bx^{5n}+\frac{1}{2}ab^2x^{6n}+\frac{1}{7}b^3x^{7n}}{n}$$

input
```
Int[x^(-1 + 4*n)*(a + b*x^n)^3,x]
```

output
```
((a^3*x^(4*n))/4 + (3*a^2*b*x^(5*n))/5 + (a*b^2*x^(6*n))/2 + (b^3*x^(7*n))
/7)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
risch	$\frac{a^3 x^{4n}}{4n}+\frac{3a^2 b\, x^{5n}}{5n}+\frac{a\, b^2 x^{6n}}{2n}+\frac{b^3 x^{7n}}{7n}$
parallelrisch	$\frac{20x\, x^{3n} x^{-1+4n} b^3+70x\, x^{2n} x^{-1+4n} a\, b^2+84x\, x^n x^{-1+4n} a^2 b+35x\, x^{-1+4n} a^3}{140n}$
orering	$\frac{x(638n^3-179n^2+22n-1)x^{-1+4n}(a+b\,x^n)^3}{840n^4}-\frac{x^2(179n^2-66n+7)\left(\frac{x^{-1+4n}(-1+4n)(a+b\,x^n)^3}{x}+\frac{3x^{-1+4n}(a+b\,x^n)^2 b\,x^n n}{x}\right)}{840n^4}$

input
```
int(x^(-1+4*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3/n*(x^n)^7+1/2*a*b^2/n*(x^n)^6+3/5*a^2*b/n*(x^n)^5+1/4*a^3/n*(x^n)^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1+4n}(a+bx^n)^3\,dx=\frac{20\,b^3x^{7\,n}+70\,ab^2x^{6\,n}+84\,a^2bx^{5\,n}+35\,a^3x^{4\,n}}{140\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/140*(20*b^3*x^(7*n) + 70*a*b^2*x^(6*n) + 84*a^2*b*x^(5*n) + 35*a^3*x^(4*
n))/n
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int x^{-1+4n}(a+bx^n)^3\,dx = \begin{cases} \frac{a^3xx^{4n-1}}{4n} + \frac{3a^2bxx^nx^{4n-1}}{5n} + \frac{ab^2xx^{2n}x^{4n-1}}{2n} + \frac{b^3xx^{3n}x^{4n-1}}{7n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*x*x**(4*n - 1)/(4*n) + 3*a**2*b*x*x**n*x**(4*n - 1)/(5*n)
+ a*b**2*x*x**(2*n)*x**(4*n - 1)/(2*n) + b**3*x*x**(3*n)*x**(4*n - 1)/(7*n
), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^{-1+4n}(a+bx^n)^3\,dx = \frac{b^3x^{7n}}{7n} + \frac{ab^2x^{6n}}{2n} + \frac{3a^2bx^{5n}}{5n} + \frac{a^3x^{4n}}{4n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*x^(7*n)/n + 1/2*a*b^2*x^(6*n)/n + 3/5*a^2*b*x^(5*n)/n + 1/4*a^3*x^
(4*n)/n
```

Giac [F]

$$\int x^{-1+4n}(a+bx^n)^3\,dx = \int (bx^n+a)^3x^{4n-1}\,dx$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3*x^(4*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^{-1+4n}(a+bx^n)^3\,dx=\frac{a^3\,x^{4\,n}}{4\,n}+\frac{b^3\,x^{7\,n}}{7\,n}+\frac{3\,a^2\,b\,x^{5\,n}}{5\,n}+\frac{a\,b^2\,x^{6\,n}}{2\,n}$$

input
```
int(x^(4*n - 1)*(a + b*x^n)^3,x)
```

output
```
(a^3*x^(4*n))/(4*n) + (b^3*x^(7*n))/(7*n) + (3*a^2*b*x^(5*n))/(5*n) + (a*b
^2*x^(6*n))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int x^{-1+4n}(a+bx^n)^3\,dx=\frac{x^{4n}(20x^{3n}b^3+70x^{2n}a\,b^2+84x^na^2b+35a^3)}{140n}$$

input
```
int(x^(-1+4*n)*(a+b*x^n)^3,x)
```

output
```
(x**(4*n)*(20*x**(3*n)*b**3 + 70*x**(2*n)*a*b**2 + 84*x**n*a**2*b + 35*a**
3))/(140*n)
```

3.396 $\int x^{-1+3n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int x^{-1+3n}(a+bx^n)^3\,dx=\frac{a^3x^{3n}}{3n}+\frac{3a^2bx^{4n}}{4n}+\frac{3ab^2x^{5n}}{5n}+\frac{b^3x^{6n}}{6n}$$

output
```
1/3*a^3*x^(3*n)/n+3/4*a^2*b*x^(4*n)/n+3/5*a*b^2*x^(5*n)/n+1/6*b^3*x^(6*n)/
n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1+3n}(a+bx^n)^3\,dx=\frac{x^{3n}(20a^3+45a^2bx^n+36ab^2x^{2n}+10b^3x^{3n})}{60n}$$

input
```
Integrate[x^(-1 + 3*n)*(a + b*x^n)^3,x]
```

output
```
(x^(3*n)*(20*a^3 + 45*a^2*b*x^n + 36*a*b^2*x^(2*n) + 10*b^3*x^(3*n)))/(60*
n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{2n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^3x^{2n}+3a^2bx^{3n}+3ab^2x^{4n}+b^3x^{5n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}a^3x^{3n}+\frac{3}{4}a^2bx^{4n}+\frac{3}{5}ab^2x^{5n}+\frac{1}{6}b^3x^{6n}}{n}$$

input
```
Int[x^(-1 + 3*n)*(a + b*x^n)^3,x]
```

output
```
((a^3*x^(3*n))/3 + (3*a^2*b*x^(4*n))/4 + (3*a*b^2*x^(5*n))/5 + (b^3*x^(6*n
))/6)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
risch	$\frac{a^{3} x^{3 n}}{3 n}+\frac{3 a^{2} b\, x^{4 n}}{4 n}+\frac{3 a\, b^{2} x^{5 n}}{5 n}+\frac{b^{3} x^{6 n}}{6 n}$
parallelrisch	$\frac{10 x\, x^{3 n} x^{-1+3 n} b^{3}+36 x\, x^{2 n} x^{-1+3 n} a\, b^{2}+45 x\, x^{n} x^{-1+3 n} a^{2} b+20 x\, x^{-1+3 n} a^{3}}{60 n}$
orering	$\frac{x \left(342 n^{3}-119 n^{2}+18 n-1\right) x^{-1+3 n} \left(a+b\, x^{n}\right)^{3}}{360 n^{4}}-\frac{x^{2} \left(119 n^{2}-54 n+7\right) \left(\frac{x^{-1+3 n} \left(-1+3 n\right) \left(a+b\, x^{n}\right)^{3}}{x}+\frac{3 x^{-1+3 n} \left(a+b\, x^{n}\right)^{2} b\, x^{n} n}{x}\right)}{360 n^{4}}$

input
```
int(x^(-1+3*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^3/n*(x^n)^6+3/5*a*b^2/n*(x^n)^5+3/4*a^2*b/n*(x^n)^4+1/3*a^3/n*(x^n)^
3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1+3n}(a+bx^n)^3\,dx=\frac{10\,b^3x^{6\,n}+36\,ab^2x^{5\,n}+45\,a^2bx^{4\,n}+20\,a^3x^{3\,n}}{60\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/60*(10*b^3*x^(6*n) + 36*a*b^2*x^(5*n) + 45*a^2*b*x^(4*n) + 20*a^3*x^(3*n
))/n
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int x^{-1+3n}(a+bx^n)^3\,dx = \begin{cases} \frac{a^3xx^{3n-1}}{3n} + \frac{3a^2bxx^nx^{3n-1}}{4n} + \frac{3ab^2xx^{2n}x^{3n-1}}{5n} + \frac{b^3xx^{3n}x^{3n-1}}{6n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*x*x**(3*n - 1)/(3*n) + 3*a**2*b*x*x**n*x**(3*n - 1)/(4*n)
+ 3*a*b**2*x*x**(2*n)*x**(3*n - 1)/(5*n) + b**3*x*x**(3*n)*x**(3*n - 1)/(6
*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^{-1+3n}(a+bx^n)^3\,dx = \frac{b^3x^{6\,n}}{6\,n} + \frac{3\,ab^2x^{5\,n}}{5\,n} + \frac{3\,a^2bx^{4\,n}}{4\,n} + \frac{a^3x^{3\,n}}{3\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/6*b^3*x^(6*n)/n + 3/5*a*b^2*x^(5*n)/n + 3/4*a^2*b*x^(4*n)/n + 1/3*a^3*x^
(3*n)/n
```

Giac [F]

$$\int x^{-1+3n}(a+bx^n)^3\,dx=\int (bx^n+a)^3x^{3\,n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3*x^(3*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^{-1+3n}(a+bx^n)^3\,dx=\frac{a^3\,x^{3\,n}}{3\,n}+\frac{b^3\,x^{6\,n}}{6\,n}+\frac{3\,a^2\,b\,x^{4\,n}}{4\,n}+\frac{3\,a\,b^2\,x^{5\,n}}{5\,n}$$

input
```
int(x^(3*n - 1)*(a + b*x^n)^3,x)
```

output
```
(a^3*x^(3*n))/(3*n) + (b^3*x^(6*n))/(6*n) + (3*a^2*b*x^(4*n))/(4*n) + (3*a
*b^2*x^(5*n))/(5*n)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int x^{-1+3n}(a+bx^n)^3\,dx=\frac{x^{3n}(10x^{3n}b^3+36x^{2n}a\,b^2+45x^na^2b+20a^3)}{60n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^3,x)
```

output
```
(x**(3*n)*(10*x**(3*n)*b**3 + 36*x**(2*n)*a*b**2 + 45*x**n*a**2*b + 20*a**
3))/(60*n)
```

3.397 $\int x^{-1+2n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int x^{-1+2n}(a+bx^n)^3\,dx = -\frac{a(a+bx^n)^4}{4b^2n}+\frac{(a+bx^n)^5}{5b^2n}$$

output
```
-1/4*a*(a+b*x^n)^4/b^2/n+1/5*(a+b*x^n)^5/b^2/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int x^{-1+2n}(a+bx^n)^3\,dx = \frac{x^{2n}(10a^3+20a^2bx^n+15ab^2x^{2n}+4b^3x^{3n})}{20n}$$

input
```
Integrate[x^(-1 + 2*n)*(a + b*x^n)^3,x]
```

output
```
(x^(2*n)*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^(2*n) + 4*b^3*x^(3*n)))/(20*n
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^n (bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^4}{b}-\frac{a(bx^n+a)^3}{b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+bx^n)^5}{5b^2}-\frac{a(a+bx^n)^4}{4b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a + b*x^n)^3,x]
```

output
```
(-1/4*(a*(a + b*x^n)^4)/b^2 + (a + b*x^n)^5/(5*b^2))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

method	result
risch	$\frac{a^{2} b\, x^{3 n}}{n}+\frac{a^{3} x^{2 n}}{2 n}+\frac{b^{3} x^{5 n}}{5 n}+\frac{3 a\, b^{2} x^{4 n}}{4 n}$
norman	$\frac{a^{2} b\, {\mathrm e}^{3 n \ln \left(x \right)}}{n}+\frac{a^{3} {\mathrm e}^{2 n \ln \left(x \right)}}{2 n}+\frac{b^{3} {\mathrm e}^{5 n \ln \left(x \right)}}{5 n}+\frac{3 a\, b^{2} {\mathrm e}^{4 n \ln \left(x \right)}}{4 n}$
parallelrisch	$\frac{4 x\, x^{3 n} x^{2 n -1} b^{3}+15 x\, x^{2 n} x^{2 n -1} a\, b^{2}+20 x\, x^{n} x^{2 n -1} a^{2} b +10 x\, x^{2 n -1} a^{3}}{20 n}$
orering	$\frac{x \left(154 n^{3}-71 n^{2}+14 n -1\right) x^{2 n -1} \left(a +b\, x^{n}\right)^{3}}{120 n^{4}}-\frac{x^{2} \left(71 n^{2}-42 n +7\right) \left(\frac{x^{2 n -1} \left(2 n -1\right) \left(a +b\, x^{n}\right)^{3}}{x}+\frac{3 x^{2 n -1} \left(a +b\, x^{n}\right)^{2} b\, x^{n} n}{x}\right)}{120 n^{4}}+\frac{x^{3}(}{}$

input
```
int(x^(2*n-1)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*b/n*(x^n)^3+1/2*a^3/n*(x^n)^2+1/5*b^3/n*(x^n)^5+3/4*a*b^2/n*(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int x^{-1+2n}(a+bx^n)^3\,dx=\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n)
)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int x^{-1+2n}(a+bx^n)^3\,dx = \begin{cases} \frac{a^3xx^{2n-1}}{2n} + \frac{a^2bxx^nx^{2n-1}}{n} + \frac{3ab^2xx^{2n}x^{2n-1}}{4n} + \frac{b^3xx^{3n}x^{2n-1}}{5n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*x*x**(2*n - 1)/(2*n) + a**2*b*x*x**n*x**(2*n - 1)/n + 3*a*
b**2*x*x**(2*n)*x**(2*n - 1)/(4*n) + b**3*x*x**(3*n)*x**(2*n - 1)/(5*n), N
e(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int x^{-1+2n}(a+bx^n)^3\,dx = \frac{b^3x^{5n}}{5n} + \frac{3ab^2x^{4n}}{4n} + \frac{a^2bx^{3n}}{n} + \frac{a^3x^{2n}}{2n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/5*b^3*x^(5*n)/n + 3/4*a*b^2*x^(4*n)/n + a^2*b*x^(3*n)/n + 1/2*a^3*x^(2*n
)/n
```

Giac [F]

$$\int x^{-1+2n}(a+bx^n)^3\,dx = \int (bx^n+a)^3x^{2n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3*x^(2*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int x^{-1+2n}(a+bx^n)^3\,dx=\frac{a^3\,x^{2\,n}}{2\,n}+\frac{b^3\,x^{5\,n}}{5\,n}+\frac{a^2\,b\,x^{3\,n}}{n}+\frac{3\,a\,b^2\,x^{4\,n}}{4\,n}$$

input
```
int(x^(2*n - 1)*(a + b*x^n)^3,x)
```

output
```
(a^3*x^(2*n))/(2*n) + (b^3*x^(5*n))/(5*n) + (a^2*b*x^(3*n))/n + (3*a*b^2*x
^(4*n))/(4*n)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int x^{-1+2n}(a+bx^n)^3\,dx=\frac{x^{2n}(4x^{3n}b^3+15x^{2n}a\,b^2+20x^na^2b+10a^3)}{20n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^3,x)
```

output
```
(x**(2*n)*(4*x**(3*n)*b**3 + 15*x**(2*n)*a*b**2 + 20*x**n*a**2*b + 10*a**3
))/(20*n)
```

3.398 $\int x^{-1+n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)^3\,dx=\frac{(a+bx^n)^4}{4bn}$$

output
```
1/4*(a+b*x^n)^4/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^3\,dx=\frac{(a+bx^n)^4}{4bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^3,x]
```

output
```
(a + b*x^n)^4/(4*b*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{(a+bx^n)^4}{4bn}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^3,x]
```

output
```
(a + b*x^n)^4/(4*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(17) = 34.

Time = 0.66 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

method	result
risch	$\frac{a^3x^n}{n}+\frac{a\,b^2x^{3n}}{n}+\frac{b^3x^{4n}}{4n}+\frac{3a^2b\,x^{2n}}{2n}$
norman	$\frac{a^3\mathrm{e}^{n\ln(x)}}{n}+\frac{a\,b^2\mathrm{e}^{3n\ln(x)}}{n}+\frac{b^3\mathrm{e}^{4n\ln(x)}}{4n}+\frac{3a^2b\,\mathrm{e}^{2n\ln(x)}}{2n}$
parallelrisch	$\frac{x\,x^{3n}x^{-1+n}b^3+4x\,x^{2n}x^{-1+n}a\,b^2+6x\,x^nx^{-1+n}a^2b+4x\,x^{-1+n}a^3}{4n}$
orering	$\frac{x(50n^3-35n^2+10n-1)x^{-1+n}(a+b\,x^n)^3}{24n^4}-\frac{x^2(35n^2-30n+7)\left(\frac{x^{-1+n}(-1+n)(a+b\,x^n)^3}{x}+\frac{3x^{-1+n}(a+b\,x^n)^2b\,x^nn}{x}\right)}{24n^4}+\frac{x^3}{}$

input
```
int(x^(-1+n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/n*x^n+a*b^2/n*(x^n)^3+1/4*b^3/n*(x^n)^4+3/2*a^2*b/n*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int x^{-1+n}(a+bx^n)^3\,dx=\frac{b^3x^{4\,n}+4\,ab^2x^{3\,n}+6\,a^2bx^{2\,n}+4\,a^3x^n}{4\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/4*(b^3*x^(4*n) + 4*a*b^2*x^(3*n) + 6*a^2*b*x^(2*n) + 4*a^3*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(12) = 24$.

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.00

$$\int x^{-1+n}(a+bx^n)^3\,dx=\begin{cases}\frac{a^3xx^{n-1}}{n}+\frac{3a^2bxx^nx^{n-1}}{2n}+\frac{ab^2xx^{2n}x^{n-1}}{n}+\frac{b^3xx^{3n}x^{n-1}}{4n} & \text{for } n\neq 0\\ (a+b)^3\log{(x)} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*x*x**(n - 1)/n + 3*a**2*b*x*x**n*x**(n - 1)/(2*n) + a*b**2
*x*x**(2*n)*x**(n - 1)/n + b**3*x*x**(3*n)*x**(n - 1)/(4*n), Ne(n, 0)), ((
a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)^3\,dx = \frac{(bx^n+a)^4}{4\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/4*(b*x^n + a)^4/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(17) = 34.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int x^{-1+n}(a+bx^n)^3\,dx = \frac{b^3x^{4\,n}+4\,ab^2x^{3\,n}+6\,a^2bx^{2\,n}+4\,a^3x^n}{4\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/4*(b^3*x^(4*n) + 4*a*b^2*x^(3*n) + 6*a^2*b*x^(2*n) + 4*a^3*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int x^{-1+n}(a+bx^n)^3\,dx=\frac{a^3\,x^n}{n}+\frac{b^3\,x^{4\,n}}{4\,n}+\frac{3\,a^2\,b\,x^{2\,n}}{2\,n}+\frac{a\,b^2\,x^{3\,n}}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^3,x)
```

output
```
(a^3*x^n)/n + (b^3*x^(4*n))/(4*n) + (3*a^2*b*x^(2*n))/(2*n) + (a*b^2*x^(3*
n))/n
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int x^{-1+n}(a+bx^n)^3\,dx=\frac{x^n(x^{3n}b^3+4x^{2n}a\,b^2+6x^na^2b+4a^3)}{4n}$$

input
```
int(x^(-1+n)*(a+b*x^n)^3,x)
```

output
```
(x**n*(x**(3*n)*b**3 + 4*x**(2*n)*a*b**2 + 6*x**n*a**2*b + 4*a**3))/(4*n)
```

3.399 $\int \frac{(a+bx^n)^3}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{(a+bx^n)^3}{x} dx = \frac{3a^2bx^n}{n} + \frac{3ab^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n} + a^3 \log(x)$$

output
```
3*a^2*b*x^n/n+3/2*a*b^2*x^(2*n)/n+1/3*b^3*x^(3*n)/n+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n)^3}{x} dx = \frac{bx^n(18a^2 + 9abx^n + 2b^2x^{2n})}{6n} + \frac{a^3 \log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)^3/x,x]
```

output
```
(b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)))/(6*n) + (a^3*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3}{x} \, dx$$
$$\Big\downarrow \text{ 798}$$
$$\frac{\int x^{-n}(bx^n+a)^3 \, dx^n}{n}$$
$$\Big\downarrow \text{ 49}$$
$$\frac{\int \left(a^3x^{-n}+3ab^2x^n+b^3x^{2n}+3a^2b\right) dx^n}{n}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{a^3\log(x^n)+3a^2bx^n+\frac{3}{2}ab^2x^{2n}+\frac{1}{3}b^3x^{3n}}{n}$$

input
```
Int[(a + b*x^n)^3/x,x]
```

output
```
(3*a^2*b*x^n + (3*a*b^2*x^(2*n))/2 + (b^3*x^(3*n))/3 + a^3*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{b^3 x^{3n}}{3}+\frac{3a\,b^2 x^{2n}}{2}+3a^2 b\,x^n+a^3\ln\left(x^n\right)}{n}$	44
default	$\frac{\frac{b^3 x^{3n}}{3}+\frac{3a\,b^2 x^{2n}}{2}+3a^2 b\,x^n+a^3\ln\left(x^n\right)}{n}$	44
parallelrisch	$\frac{2b^3 x^{3n}+9a\,b^2 x^{2n}+6a^3\ln\left(x\right)n+18a^2 b\,x^n}{6n}$	45
risch	$\frac{3a^2 b\,x^n}{n}+\frac{3a\,b^2 x^{2n}}{2n}+\frac{b^3 x^{3n}}{3n}+a^3\ln\left(x\right)$	47
norman	$a^3\ln\left(x\right)+\frac{b^3 \mathrm{e}^{3n\ln\left(x\right)}}{3n}+\frac{3a\,b^2 \mathrm{e}^{2n\ln\left(x\right)}}{2n}+\frac{3a^2 b\,\mathrm{e}^{n\ln\left(x\right)}}{n}$	53

input
```
int((a+b*x^n)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/3*b^3*(x^n)^3+3/2*a*b^2*(x^n)^2+3*a^2*b*x^n+a^3*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^n)^3}{x}\,dx=\frac{6\,a^3n\log\left(x\right)+2\,b^3x^{3\,n}+9\,ab^2x^{2\,n}+18\,a^2bx^n}{6\,n}$$

input
```
integrate((a+b*x^n)^3/x,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)^3}{x}\,dx = \begin{cases} a^3\log(x) + \frac{3a^2bx^n}{n} + \frac{3ab^2x^{2n}}{2n} + \frac{b^3x^{3n}}{3n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**3/x,x)
```

output
```
Piecewise((a**3*log(x) + 3*a**2*b*x**n/n + 3*a*b**2*x**(2*n)/(2*n) + b**3*
x**(3*n)/(3*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^n)^3}{x}\,dx = \frac{a^3\log(x^n)}{n} + \frac{2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

input
```
integrate((a+b*x^n)^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x^n)/n + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Giac [F]

$$\int \frac{(a+bx^n)^3}{x}\,dx = \int \frac{(bx^n+a)^3}{x}\,dx$$

input
```
integrate((a+b*x^n)^3/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n)^3}{x}\,dx = a^3\,\ln(x) + \frac{b^3\,x^{3\,n}}{3\,n} + \frac{3\,a^2\,b\,x^n}{n} + \frac{3\,a\,b^2\,x^{2\,n}}{2\,n}$$

input
```
int((a + b*x^n)^3/x,x)
```

output
```
a^3*log(x) + (b^3*x^(3*n))/(3*n) + (3*a^2*b*x^n)/n + (3*a*b^2*x^(2*n))/(2*
n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^n)^3}{x}\,dx = \frac{2x^{3n}b^3 + 9x^{2n}a\,b^2 + 18x^n a^2 b + 6\log(x)\,a^3 n}{6n}$$

input
```
int((a+b*x^n)^3/x,x)
```

output
```
(2*x**(3*n)*b**3 + 9*x**(2*n)*a*b**2 + 18*x**n*a**2*b + 6*log(x)*a**3*n)/(
6*n)
```

3.400 $\int x^{-1-n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int x^{-1-n}(a+bx^n)^3\,dx=-\frac{a^3x^{-n}}{n}+\frac{3ab^2x^n}{n}+\frac{b^3x^{2n}}{2n}+3a^2b\log(x)$$

output
```
-a^3/n/(x^n)+3*a*b^2*x^n/n+1/2*b^3*x^(2*n)/n+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int x^{-1-n}(a+bx^n)^3\,dx=-\frac{2a^3x^{-n}-6ab^2x^n-b^3x^{2n}-6a^2b\log(x^n)}{2n}$$

input
```
Integrate[x^(-1 - n)*(a + b*x^n)^3,x]
```

output
```
-1/2*((2*a^3)/x^n - 6*a*b^2*x^n - b^3*x^(2*n) - 6*a^2*b*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-2n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^3x^{-2n}+3a^2bx^{-n}+b^3x^n+3ab^2\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^3x^{-n}+3a^2b\log\left(x^n\right)+3ab^2x^n+\frac{1}{2}b^3x^{2n}}{n}$$

input
```
Int[x^(-1 - n)*(a + b*x^n)^3,x]
```

output
```
(-(a^3/x^n) + 3*a*b^2*x^n + (b^3*x^(2*n))/2 + 3*a^2*b*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

method	result	size
risch	$3a^{2}b\ln\left(x\right)+\frac{b^{3}x^{2n}}{2n}+\frac{3a\,b^{2}x^{n}}{n}-\frac{a^{3}x^{-n}}{n}$	48
norman	$\left(3a^{2}b\ln\left(x\right){\mathrm e}^{n\ln\left(x\right)}-\frac{a^{3}}{n}+\frac{b^{3}{\mathrm e}^{3n\ln\left(x\right)}}{2n}+\frac{3a\,b^{2}{\mathrm e}^{2n\ln\left(x\right)}}{n}\right){\mathrm e}^{-n\ln\left(x\right)}$	62

input
```
int(x^(-1-n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
3*a^2*b*ln(x)+1/2*b^3/n*(x^n)^2+3*a*b^2*x^n/n-a^3/n/(x^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int x^{-1-n}(a+bx^n)^3\,dx=\frac{6\,a^{2}bnx^{n}\log\left(x\right)+b^{3}x^{3\,n}+6\,ab^{2}x^{2\,n}-2\,a^{3}}{2\,nx^{n}}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(6*a^2*b*n*x^n*log(x) + b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^3)/(n*x^n)
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int x^{-1-n}(a+bx^n)^3\,dx = \begin{cases} -\frac{a^3x^{-n}}{n} + \frac{3a^2b\log(x^n)}{n} + \frac{3ab^2x^n}{n} + \frac{b^3x^{2n}}{2n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((-a**3/(n*x**n) + 3*a**2*b*log(x**n)/n + 3*a*b**2*x**n/n + b**3*
x**(2*n)/(2*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int x^{-1-n}(a+bx^n)^3\,dx = 3\,a^2b\log(x) + \frac{b^3x^{2\,n}}{2\,n} + \frac{3\,ab^2x^n}{n} - \frac{a^3}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
3*a^2*b*log(x) + 1/2*b^3*x^(2*n)/n + 3*a*b^2*x^n/n - a^3/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int x^{-1-n}(a+bx^n)^3\,dx = \frac{6\,a^2bnx^n\log(x) + b^3x^{3\,n} + 6\,ab^2x^{2\,n} - 2\,a^3}{2\,nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/2*(6*a^2*b*n*x^n*log(x) + b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^3)/(n*x^n)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int x^{-1-n}(a+bx^n)^3 \, dx = 3\,a^2\,b\,\ln(x) - \frac{a^3}{n\,x^n} + \frac{b^3\,x^{2\,n}}{2\,n} + \frac{3\,a\,b^2\,x^n}{n}$$

input
```
int((a + b*x^n)^3/x^(n + 1),x)
```

output
```
3*a^2*b*log(x) - a^3/(n*x^n) + (b^3*x^(2*n))/(2*n) + (3*a*b^2*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int x^{-1-n}(a+bx^n)^3 \, dx = \frac{x^{3n}b^3 + 6x^{2n}a\,b^2 + 6x^n\log(x)\,a^2bn - 2a^3}{2x^n n}$$

input
```
int(x^(-1-n)*(a+b*x^n)^3,x)
```

output
```
(x**(3*n)*b**3 + 6*x**(2*n)*a*b**2 + 6*x**n*log(x)*a**2*b*n - 2*a**3)/(2*x
**n*n)
```

3.401 $\int x^{-1-2n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int x^{-1-2n}(a+bx^n)^3\,dx=-\frac{a^3x^{-2n}}{2n}-\frac{3a^2bx^{-n}}{n}+\frac{b^3x^n}{n}+3ab^2\log(x)$$

output
```
-1/2*a^3/n/(x^(2*n))-3*a^2*b/n/(x^n)+b^3*x^n/n+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int x^{-1-2n}(a+bx^n)^3\,dx=-\frac{a^3x^{-2n}+6a^2bx^{-n}-2b^3x^n-6ab^2\log(x^n)}{2n}$$

input
```
Integrate[x^(-1 - 2*n)*(a + b*x^n)^3,x]
```

output
```
-1/2*(a^3/x^(2*n) + (6*a^2*b)/x^n - 2*b^3*x^n - 6*a*b^2*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-3n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^3x^{-3n}+3a^2bx^{-2n}+3ab^2x^{-n}+b^3\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^3x^{-2n}-3a^2bx^{-n}+3ab^2\log\left(x^n\right)+b^3x^n}{n}$$

input
```
Int[x^(-1 - 2*n)*(a + b*x^n)^3,x]
```

output
```
(-1/2*a^3/x^(2*n) - (3*a^2*b)/x^n + b^3*x^n + 3*a*b^2*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

method	result	size
risch	$3a\,b^2 \ln(x) + \frac{b^3 x^n}{n} - \frac{3a^2 b\,x^{-n}}{n} - \frac{a^3 x^{-2n}}{2n}$	47
norman	$\left(\frac{b^3 e^{3n\ln(x)}}{n} + 3a\,b^2 \ln(x)\,e^{2n\ln(x)} - \frac{a^3}{2n} - \frac{3a^2 b\,e^{n\ln(x)}}{n}\right) e^{-2n\ln(x)}$	61

input
```
int(x^(-2*n-1)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
3*a*b^2*ln(x)+b^3*x^n/n-3*a^2*b/n/(x^n)-1/2*a^3/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^{-1-2n}(a+bx^n)^3\,dx = \frac{6\,ab^2nx^{2\,n}\log(x) + 2\,b^3x^{3\,n} - 6\,a^2bx^n - a^3}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(6*a*b^2*n*x^(2*n)*log(x) + 2*b^3*x^(3*n) - 6*a^2*b*x^n - a^3)/(n*x^(2
*n))
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^{-1-2n}(a+bx^n)^3\,dx = \begin{cases} -\frac{a^3x^{-2n}}{2n} - \frac{3a^2bx^{-n}}{n} + \frac{3ab^2\log(x^n)}{n} + \frac{b^3x^n}{n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-2*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((-a**3/(2*n*x**(2*n)) - 3*a**2*b/(n*x**n) + 3*a*b**2*log(x**n)/n
 + b**3*x**n/n, Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^{-1-2n}(a+bx^n)^3\,dx = 3\,ab^2\log(x) + \frac{b^3x^n}{n} - \frac{a^3}{2\,nx^{2\,n}} - \frac{3\,a^2b}{nx^n}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
3*a*b^2*log(x) + b^3*x^n/n - 1/2*a^3/(n*x^(2*n)) - 3*a^2*b/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^{-1-2n}(a+bx^n)^3\,dx = \frac{6\,ab^2nx^{2\,n}\log(x) + 2\,b^3x^{3\,n} - 6\,a^2bx^n - a^3}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/2*(6*a*b^2*n*x^(2*n)*log(x) + 2*b^3*x^(3*n) - 6*a^2*b*x^n - a^3)/(n*x^(2
*n))
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^{-1-2n}(a+bx^n)^3\,dx=\frac{b^3\,x^n}{n}+3\,a\,b^2\,\ln(x)-\frac{a^3}{2\,n\,x^{2\,n}}-\frac{3\,a^2\,b}{n\,x^n}$$

input
```
int((a + b*x^n)^3/x^(2*n + 1),x)
```

output
```
(b^3*x^n)/n + 3*a*b^2*log(x) - a^3/(2*n*x^(2*n)) - (3*a^2*b)/(n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^{-1-2n}(a+bx^n)^3\,dx=\frac{2x^{3n}b^3+6x^{2n}\log(x)\,a\,b^2n-6x^na^2b-a^3}{2x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a+b*x^n)^3,x)
```

output
```
(2*x**(3*n)*b**3 + 6*x**(2*n)*log(x)*a*b**2*n - 6*x**n*a**2*b - a**3)/(2*x
**(2*n)*n)
```

3.402 $\int x^{-1-3n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int x^{-1-3n}(a+bx^n)^3\,dx=-\frac{a^3x^{-3n}}{3n}-\frac{3a^2bx^{-2n}}{2n}-\frac{3ab^2x^{-n}}{n}+b^3\log(x)$$

output
```
-1/3*a^3/n/(x^(3*n))-3/2*a^2*b/n/(x^(2*n))-3*a*b^2/n/(x^n)+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int x^{-1-3n}(a+bx^n)^3\,dx=-\frac{ax^{-3n}(2a^2+9abx^n+18b^2x^{2n})}{6n}+\frac{b^3\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 3*n)*(a + b*x^n)^3,x]
```

output
```
-1/6*(a*(2*a^2 + 9*a*b*x^n + 18*b^2*x^(2*n)))/(n*x^(3*n)) + (b^3*Log[x^n])
/n
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-4n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^3x^{-4n}+3a^2bx^{-3n}+3ab^2x^{-2n}+b^3x^{-n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^3x^{-3n}-\frac{3}{2}a^2bx^{-2n}-3ab^2x^{-n}+b^3\log(x^n)}{n}$$

input
```
Int[x^(-1 - 3*n)*(a + b*x^n)^3,x]
```

output
```
(-1/3*a^3/x^(3*n) - (3*a^2*b)/(2*x^(2*n)) - (3*a*b^2)/x^n + b^3*Log[x^n])/
n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

method	result	size
risch	$b^3 \ln(x) - \frac{3a\,b^2 x^{-n}}{n} - \frac{3a^2 b\,x^{-2n}}{2n} - \frac{a^3 x^{-3n}}{3n}$	49
norman	$\left(b^3 \ln(x)\,\mathrm{e}^{3n\ln(x)} - \frac{a^3}{3n} - \frac{3a\,b^2 \mathrm{e}^{2n\ln(x)}}{n} - \frac{3a^2 b\,\mathrm{e}^{n\ln(x)}}{2n}\right)\mathrm{e}^{-3n\ln(x)}$	61

input
```
int(x^(-1-3*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3*ln(x)-3*a*b^2/n/(x^n)-3/2*a^2*b/n/(x^n)^2-1/3*a^3/n/(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int x^{-1-3n}(a+bx^n)^3\,dx = \frac{6\,b^3nx^{3\,n}\log(x) - 18\,ab^2x^{2\,n} - 9\,a^2bx^n - 2\,a^3}{6\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/6*(6*b^3*n*x^(3*n)*log(x) - 18*a*b^2*x^(2*n) - 9*a^2*b*x^n - 2*a^3)/(n*x
^(3*n))
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int x^{-1-3n}(a+bx^n)^3\,dx=\begin{cases}-\frac{a^3x^{-3n}}{3n}-\frac{3a^2bx^{-2n}}{2n}-\frac{3ab^2x^{-n}}{n}+\frac{b^3\log(x^n)}{n} & \text{for } n\neq 0\\ (a+b)^3\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1-3*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((-a**3/(3*n*x**(3*n)) - 3*a**2*b/(2*n*x**(2*n)) - 3*a*b**2/(n*x*
*n) + b**3*log(x**n)/n, Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x^{-1-3n}(a+bx^n)^3\,dx=b^3\log(x)-\frac{a^3}{3\,nx^{3\,n}}-\frac{3\,a^2b}{2\,nx^{2\,n}}-\frac{3\,ab^2}{nx^n}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
b^3*log(x) - 1/3*a^3/(n*x^(3*n)) - 3/2*a^2*b/(n*x^(2*n)) - 3*a*b^2/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int x^{-1-3n}(a+bx^n)^3\,dx=\frac{6\,b^3nx^{3\,n}\log(x)-18\,ab^2x^{2\,n}-9\,a^2bx^n-2\,a^3}{6\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/6*(6*b^3*n*x^(3*n)*log(x) - 18*a*b^2*x^(2*n) - 9*a^2*b*x^n - 2*a^3)/(n*x
^(3*n))
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x^{-1-3n}(a+bx^n)^3\,dx = b^3\ln(x) - \frac{a^3}{3\,n\,x^{3\,n}} - \frac{3\,a\,b^2}{n\,x^n} - \frac{3\,a^2\,b}{2\,n\,x^{2\,n}}$$

input
```
int((a + b*x^n)^3/x^(3*n + 1),x)
```

output
```
b^3*log(x) - a^3/(3*n*x^(3*n)) - (3*a*b^2)/(n*x^n) - (3*a^2*b)/(2*n*x^(2*n
))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int x^{-1-3n}(a+bx^n)^3\,dx = \frac{6x^{3n}\log(x)\,b^3n - 18x^{2n}a\,b^2 - 9x^na^2b - 2a^3}{6x^{3n}n}$$

input
```
int(x^(-1-3*n)*(a+b*x^n)^3,x)
```

output
```
(6*x**(3*n)*log(x)*b**3*n - 18*x**(2*n)*a*b**2 - 9*x**n*a**2*b - 2*a**3)/(
6*x**(3*n)*n)
```

3.403 $\int x^{-1-4n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int x^{-1-4n}(a+bx^n)^3\,dx = -\frac{x^{-4n}(a+bx^n)^4}{4an}$$

output
```
-1/4*(a+b*x^n)^4/a/n/(x^(4*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^{-1-4n}(a+bx^n)^3\,dx = \frac{x^{-4n}(-a^3-4a^2bx^n-6ab^2x^{2n}-4b^3x^{3n})}{4n}$$

input
```
Integrate[x^(-1 - 4*n)*(a + b*x^n)^3,x]
```

output
```
(-a^3 - 4*a^2*b*x^n - 6*a*b^2*x^(2*n) - 4*b^3*x^(3*n))/(4*n*x^(4*n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-4n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{x^{-4n}(a+bx^n)^4}{4an}$$

input
```
Int[x^(-1 - 4*n)*(a + b*x^n)^3,x]
```

output
```
-1/4*(a + b*x^n)^4/(a*n*x^(4*n))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

method	result
risch	$-\frac{b^{3} x^{-n}}{n}-\frac{3 a \,b^{2} x^{-2 n}}{2 n}-\frac{a^{2} b \,x^{-3 n}}{n}-\frac{a^{3} x^{-4 n}}{4 n}$
norman	$\left(-\frac{a^{3}}{4 n}-\frac{b^{3} {\mathrm e}^{3 n \ln \left(x \right)}}{n}-\frac{3 a \,b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{2 n}-\frac{a^{2} b \,{\mathrm e}^{n \ln \left(x \right)}}{n}\right) {\mathrm e}^{-4 n \ln \left(x \right)}$
parallelrisch	$\frac{-4 x \,x^{3 n} x^{-4 n -1} b^{3}-6 x \,x^{2 n} x^{-4 n -1} a \,b^{2}-4 x \,x^{n} x^{-4 n -1} a^{2} b -x \,x^{-4 n -1} a^{3}}{4 n}$
orering	$-\frac{x \left(50 n^{3}+35 n^{2}+10 n +1\right) x^{-4 n -1} \left(a +b \,x^{n}\right)^{3}}{24 n^{4}}-\frac{x^{2} \left(35 n^{2}+30 n +7\right) \left(\frac{x^{-4 n -1} \left(-4 n -1\right) \left(a +b \,x^{n}\right)^{3}}{x}+\frac{3 x^{-4 n -1} \left(a +b \,x^{n}\right)^{2} b \,x^{n} n}{x}\right)}{24 n^{4}}$

input
```
int(x^(-4*n-1)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^3/n/(x^n)-3/2*a*b^2/n/(x^n)^2-a^2*b/n/(x^n)^3-1/4*a^3/n/(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int x^{-1-4n}(a+bx^n)^3\,dx=-\frac{4\,b^3x^{3\,n}+6\,ab^2x^{2\,n}+4\,a^2bx^n+a^3}{4\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*x^(3*n) + 6*a*b^2*x^(2*n) + 4*a^2*b*x^n + a^3)/(n*x^(4*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(19) = 38.

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.83

$$\int x^{-1-4n}(a+bx^n)^3\,dx = \begin{cases} -\frac{a^3xx^{-4n-1}}{4n}-\frac{a^2bxx^nx^{-4n-1}}{n}-\frac{3ab^2xx^{2n}x^{-4n-1}}{2n}-\frac{b^3xx^{3n}x^{-4n-1}}{n} & \text{for } n\neq 0\\ (a+b)^3\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1-4*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((-a**3*x*x**(-4*n - 1)/(4*n) - a**2*b*x*x**n*x**(-4*n - 1)/n - 3
*a*b**2*x*x**(2*n)*x**(-4*n - 1)/(2*n) - b**3*x*x**(3*n)*x**(-4*n - 1)/n,
Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int x^{-1-4n}(a+bx^n)^3\,dx = -\frac{a^3}{4\,nx^{4\,n}} - \frac{a^2b}{nx^{3\,n}} - \frac{3\,ab^2}{2\,nx^{2\,n}} - \frac{b^3}{nx^n}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/4*a^3/(n*x^(4*n)) - a^2*b/(n*x^(3*n)) - 3/2*a*b^2/(n*x^(2*n)) - b^3/(n*
x^n)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int x^{-1-4n}(a+bx^n)^3\,dx = -\frac{4\,b^3x^{3\,n} + 6\,ab^2x^{2\,n} + 4\,a^2bx^n + a^3}{4\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
-1/4*(4*b^3*x^(3*n) + 6*a*b^2*x^(2*n) + 4*a^2*b*x^n + a^3)/(n*x^(4*n))
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int x^{-1-4n}(a+bx^n)^3\,dx=-\frac{a^3}{4\,n\,x^{4\,n}}-\frac{b^3}{n\,x^n}-\frac{3\,a\,b^2}{2\,n\,x^{2\,n}}-\frac{a^2\,b}{n\,x^{3\,n}}$$

input
```
int((a + b*x^n)^3/x^(4*n + 1),x)
```

output
```
- a^3/(4*n*x^(4*n)) - b^3/(n*x^n) - (3*a*b^2)/(2*n*x^(2*n)) - (a^2*b)/(n*x
^(3*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^{-1-4n}(a+bx^n)^3\,dx=\frac{-4x^{3n}b^3-6x^{2n}a\,b^2-4x^na^2b-a^3}{4x^{4n}n}$$

input
```
int(x^(-1-4*n)*(a+b*x^n)^3,x)
```

output
```
( - 4*x**(3*n)*b**3 - 6*x**(2*n)*a*b**2 - 4*x**n*a**2*b - a**3)/(4*x**(4*n
)*n)
```

3.404 $\int x^{-1-5n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int x^{-1-5n}(a+bx^n)^3\,dx=-\frac{x^{-5n}(a+bx^n)^4}{5an}+\frac{bx^{-4n}(a+bx^n)^4}{20a^2n}$$

output
```
-1/5*(a+b*x^n)^4/a/n/(x^(5*n))+1/20*b*(a+b*x^n)^4/a^2/n/(x^(4*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int x^{-1-5n}(a+bx^n)^3\,dx=\frac{x^{-5n}(-4a^3-15a^2bx^n-20ab^2x^{2n}-10b^3x^{3n})}{20n}$$

input
```
Integrate[x^(-1 - 5*n)*(a + b*x^n)^3,x]
```

output
```
(-4*a^3 - 15*a^2*b*x^n - 20*a*b^2*x^(2*n) - 10*b^3*x^(3*n))/(20*n*x^(5*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-5n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-6n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{b\int x^{-5n}(bx^n+a)^3dx^n}{5a}-\frac{x^{-5n}(a+bx^n)^4}{5a}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{bx^{-4n}(a+bx^n)^4}{20a^2}-\frac{x^{-5n}(a+bx^n)^4}{5a}}{n}$$

input
```
Int[x^(-1 - 5*n)*(a + b*x^n)^3,x]
```

output
```
(-1/5*(a + b*x^n)^4/(a*x^(5*n)) + (b*(a + b*x^n)^4)/(20*a^2*x^(4*n)))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{b^{3} x^{-2 n}}{2 n}-\frac{a\,b^{2} x^{-3 n}}{n}-\frac{3 a^{2} b\,x^{-4 n}}{4 n}-\frac{a^{3} x^{-5 n}}{5 n}$
norman	$\left(-\frac{a^{3}}{5 n}-\frac{b^{3} {\mathrm e}^{3 n \ln \left(x \right)}}{2 n}-\frac{a\,b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{n}-\frac{3 a^{2} b\,{\mathrm e}^{n \ln \left(x \right)}}{4 n}\right) {\mathrm e}^{-5 n \ln \left(x \right)}$
parallelrisch	$\frac{-10 x\,x^{3 n} x^{-1-5 n} b^{3}-20 x\,x^{2 n} x^{-1-5 n} a\,b^{2}-15 x\,x^{n} x^{-1-5 n} a^{2} b-4 x\,x^{-1-5 n} a^{3}}{20 n}$
orering	$-\frac{x \left(154 n^{3}+71 n^{2}+14 n+1\right) x^{-1-5 n} \left(a+b\,x^{n}\right)^{3}}{120 n^{4}}-\frac{x^{2} \left(71 n^{2}+42 n+7\right) \left(\frac{x^{-1-5 n} \left(-1-5 n \right) \left(a+b\,x^{n}\right)^{3}}{x}+\frac{3 x^{-1-5 n} \left(a+b\,x^{n}\right)^{2} b\,x^{n} n}{x}\right)}{120 n^{4}}$

input

```
int(x^(-1-5*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*b^3/n/(x^n)^2-a*b^2/n/(x^n)^3-3/4*a^2*b/n/(x^n)^4-1/5*a^3/n/(x^n)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int x^{-1-5n}(a+bx^n)^3\,dx = -\frac{10\,b^3x^{3\,n}+20\,ab^2x^{2\,n}+15\,a^2bx^n+4\,a^3}{20\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/20*(10*b^3*x^(3*n) + 20*a*b^2*x^(2*n) + 15*a^2*b*x^n + 4*a^3)/(n*x^(5*n
))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(39) = 78.

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.88

$$\int x^{-1-5n}(a+bx^n)^3\,dx$$
$$= \begin{cases} -\frac{a^3xx^{-5n-1}}{5n}-\frac{3a^2bxx^nx^{-5n-1}}{4n}-\frac{ab^2xx^{2n}x^{-5n-1}}{n}-\frac{b^3xx^{3n}x^{-5n-1}}{2n} & \text{for } n\neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-5*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((-a**3*x*x**(-5*n - 1)/(5*n) - 3*a**2*b*x*x**n*x**(-5*n - 1)/(4*
n) - a*b**2*x*x**(2*n)*x**(-5*n - 1)/n - b**3*x*x**(3*n)*x**(-5*n - 1)/(2*
n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int x^{-1-5n}(a+bx^n)^3\,dx = -\frac{a^3}{5\,nx^{5\,n}} - \frac{3\,a^2b}{4\,nx^{4\,n}} - \frac{ab^2}{nx^{3\,n}} - \frac{b^3}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/5*a^3/(n*x^(5*n)) - 3/4*a^2*b/(n*x^(4*n)) - a*b^2/(n*x^(3*n)) - 1/2*b^3
/(n*x^(2*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int x^{-1-5n}(a+bx^n)^3\,dx = -\frac{10\,b^3x^{3\,n} + 20\,ab^2x^{2\,n} + 15\,a^2bx^n + 4\,a^3}{20\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
-1/20*(10*b^3*x^(3*n) + 20*a*b^2*x^(2*n) + 15*a^2*b*x^n + 4*a^3)/(n*x^(5*n
))
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int x^{-1-5n}(a+bx^n)^3\,dx = -\frac{a^3}{5\,n\,x^{5\,n}} - \frac{b^3}{2\,n\,x^{2\,n}} - \frac{a\,b^2}{n\,x^{3\,n}} - \frac{3\,a^2\,b}{4\,n\,x^{4\,n}}$$

input
```
int((a + b*x^n)^3/x^(5*n + 1),x)
```

output
```
- a^3/(5*n*x^(5*n)) - b^3/(2*n*x^(2*n)) - (a*b^2)/(n*x^(3*n)) - (3*a^2*b)/
(4*n*x^(4*n))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int x^{-1-5n}(a+bx^n)^3\,dx=\frac{-10x^{3n}b^3-20x^{2n}a\,b^2-15x^na^2b-4a^3}{20x^{5n}n}$$

input
```
int(x^(-1-5*n)*(a+b*x^n)^3,x)
```

output
```
( - 10*x**(3*n)*b**3 - 20*x**(2*n)*a*b**2 - 15*x**n*a**2*b - 4*a**3)/(20*x
**(5*n)*n)
```

3.405 $\int x^{-1-6n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int x^{-1-6n}(a+bx^n)^3\,dx=-\frac{a^3x^{-6n}}{6n}-\frac{3a^2bx^{-5n}}{5n}-\frac{3ab^2x^{-4n}}{4n}-\frac{b^3x^{-3n}}{3n}$$

output
```
-1/6*a^3/n/(x^(6*n))-3/5*a^2*b/n/(x^(5*n))-3/4*a*b^2/n/(x^(4*n))-1/3*b^3/n
/(x^(3*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-6n}(a+bx^n)^3\,dx=\frac{x^{-6n}(-10a^3-36a^2bx^n-45ab^2x^{2n}-20b^3x^{3n})}{60n}$$

input
```
Integrate[x^(-1 - 6*n)*(a + b*x^n)^3,x]
```

output
```
(-10*a^3 - 36*a^2*b*x^n - 45*a*b^2*x^(2*n) - 20*b^3*x^(3*n))/(60*n*x^(6*n)
)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-6n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-7n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(a^3x^{-7n}+3a^2bx^{-6n}+3ab^2x^{-5n}+b^3x^{-4n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{6}a^3x^{-6n}-\frac{3}{5}a^2bx^{-5n}-\frac{3}{4}ab^2x^{-4n}-\frac{1}{3}b^3x^{-3n}}{n}$$

input
```
Int[x^(-1 - 6*n)*(a + b*x^n)^3,x]
```

output
```
(-1/6*a^3/x^(6*n) - (3*a^2*b)/(5*x^(5*n)) - (3*a*b^2)/(4*x^(4*n)) - b^3/(3
*x^(3*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{b^{3} x^{-3 n}}{3 n}-\frac{3 a \,b^{2} x^{-4 n}}{4 n}-\frac{3 a^{2} b \,x^{-5 n}}{5 n}-\frac{a^{3} x^{-6 n}}{6 n}$
parallelrisch	$\frac{-20 x \,x^{3 n} x^{-1-6 n} b^{3}-45 x \,x^{2 n} x^{-1-6 n} a \,b^{2}-36 x \,x^{n} x^{-1-6 n} a^{2} b -10 x \,x^{-1-6 n} a^{3}}{60 n}$
orering	$-\frac{x \left(342 n^{3}+119 n^{2}+18 n +1\right) x^{-1-6 n} \left(a +b \,x^{n}\right)^{3}}{360 n^{4}}-\frac{x^{2} \left(119 n^{2}+54 n +7\right) \left(\frac{x^{-1-6 n} \left(-1-6 n \right) \left(a +b \,x^{n}\right)^{3}}{x}+\frac{3 x^{-1-6 n} \left(a +b \,x^{n}\right)^{2} b \,x^{n} \cdots}{x}\cdots\right)}{360 n^{4}}$

input
```
int(x^(-1-6*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^3/n/(x^n)^3-3/4*a*b^2/n/(x^n)^4-3/5*a^2*b/n/(x^n)^5-1/6*a^3/n/(x^n)
^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-6n}(a+bx^n)^3\,dx = -\frac{20\,b^3x^{3\,n}+45\,ab^2x^{2\,n}+36\,a^2bx^n+10\,a^3}{60\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/60*(20*b^3*x^(3*n) + 45*a*b^2*x^(2*n) + 36*a^2*b*x^n + 10*a^3)/(n*x^(6*
n))
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int x^{-1-6n}(a+bx^n)^3\,dx = \begin{cases} -\frac{a^3xx^{-6n-1}}{6n} - \frac{3a^2bxx^nx^{-6n-1}}{5n} - \frac{3ab^2xx^{2n}x^{-6n-1}}{4n} - \frac{b^3xx^{3n}x^{-6n-1}}{3n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-6*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((-a**3*x*x**(-6*n - 1)/(6*n) - 3*a**2*b*x*x**n*x**(-6*n - 1)/(5*
n) - 3*a*b**2*x*x**(2*n)*x**(-6*n - 1)/(4*n) - b**3*x*x**(3*n)*x**(-6*n -
1)/(3*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x^{-1-6n}(a+bx^n)^3\,dx = -\frac{a^3}{6\,nx^{6\,n}} - \frac{3\,a^2b}{5\,nx^{5\,n}} - \frac{3\,ab^2}{4\,nx^{4\,n}} - \frac{b^3}{3\,nx^{3\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/6*a^3/(n*x^(6*n)) - 3/5*a^2*b/(n*x^(5*n)) - 3/4*a*b^2/(n*x^(4*n)) - 1/3
*b^3/(n*x^(3*n))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-6n}(a+bx^n)^3\,dx=-\frac{20\,b^3x^{3\,n}+45\,ab^2x^{2\,n}+36\,a^2bx^n+10\,a^3}{60\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
-1/60*(20*b^3*x^(3*n) + 45*a*b^2*x^(2*n) + 36*a^2*b*x^n + 10*a^3)/(n*x^(6*
n))
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x^{-1-6n}(a+bx^n)^3\,dx=-\frac{a^3}{6\,n\,x^{6\,n}}-\frac{b^3}{3\,n\,x^{3\,n}}-\frac{3\,a\,b^2}{4\,n\,x^{4\,n}}-\frac{3\,a^2\,b}{5\,n\,x^{5\,n}}$$

input
```
int((a + b*x^n)^3/x^(6*n + 1),x)
```

output
```
- a^3/(6*n*x^(6*n)) - b^3/(3*n*x^(3*n)) - (3*a*b^2)/(4*n*x^(4*n)) - (3*a^2
*b)/(5*n*x^(5*n))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-6n}(a+bx^n)^3\,dx=\frac{-20x^{3n}b^3-45x^{2n}a\,b^2-36x^na^2b-10a^3}{60x^{6n}n}$$

input
```
int(x^(-1-6*n)*(a+b*x^n)^3,x)
```

output
```
( - 20*x**(3*n)*b**3 - 45*x**(2*n)*a*b**2 - 36*x**n*a**2*b - 10*a**3)/(60*
x**(6*n)*n)
```

3.406 $\int x^{-1-7n}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int x^{-1-7n}(a+bx^n)^3\,dx=-\frac{a^3x^{-7n}}{7n}-\frac{a^2bx^{-6n}}{2n}-\frac{3ab^2x^{-5n}}{5n}-\frac{b^3x^{-4n}}{4n}$$

output
```
-1/7*a^3/n/(x^(7*n))-1/2*a^2*b/n/(x^(6*n))-3/5*a*b^2/n/(x^(5*n))-1/4*b^3/n
/(x^(4*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-7n}(a+bx^n)^3\,dx=\frac{x^{-7n}(-20a^3-70a^2bx^n-84ab^2x^{2n}-35b^3x^{3n})}{140n}$$

input
```
Integrate[x^(-1 - 7*n)*(a + b*x^n)^3,x]
```

output
```
(-20*a^3 - 70*a^2*b*x^n - 84*a*b^2*x^(2*n) - 35*b^3*x^(3*n))/(140*n*x^(7*n
))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-7n-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-8n}(bx^n+a)^3\,dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(a^3x^{-8n}+3a^2bx^{-7n}+3ab^2x^{-6n}+b^3x^{-5n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{7}a^3x^{-7n}-\frac{1}{2}a^2bx^{-6n}-\frac{3}{5}ab^2x^{-5n}-\frac{1}{4}b^3x^{-4n}}{n}$$

input
```
Int[x^(-1 - 7*n)*(a + b*x^n)^3,x]
```

output
```
(-1/7*a^3/x^(7*n) - (a^2*b)/(2*x^(6*n)) - (3*a*b^2)/(5*x^(5*n)) - b^3/(4*x
^(4*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{b^{3} x^{-4 n}}{4 n}-\frac{3 a \,b^{2} x^{-5 n}}{5 n}-\frac{a^{2} b \,x^{-6 n}}{2 n}-\frac{a^{3} x^{-7 n}}{7 n}$
parallelrisch	$-\frac{35 x \,x^{3 n} x^{-1-7 n} b^{3}+84 x \,x^{2 n} x^{-1-7 n} a \,b^{2}+70 x \,x^{n} x^{-1-7 n} a^{2} b +20 x \,x^{-1-7 n} a^{3}}{140 n}$
orering	$-\frac{x \left(638 n^{3}+179 n^{2}+22 n +1\right) x^{-1-7 n} \left(a +b \,x^{n}\right)^{3}}{840 n^{4}}-\frac{x^{2} \left(179 n^{2}+66 n +7\right) \left(\frac{x^{-1-7 n} \left(-1-7 n \right) \left(a +b \,x^{n}\right)^{3}}{x}+\frac{3 x^{-1-7 n} \left(a +b \,x^{n}\right)^{2} b \,x^{n}}{x}\right.}{840 n^{4}}$

input
```
int(x^(-1-7*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3/n/(x^n)^4-3/5*a*b^2/n/(x^n)^5-1/2*a^2*b/n/(x^n)^6-1/7*a^3/n/(x^n)
^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-7n}(a+bx^n)^3\,dx=-\frac{35\,b^3x^{3\,n}+84\,ab^2x^{2\,n}+70\,a^2bx^n+20\,a^3}{140\,nx^{7\,n}}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/140*(35*b^3*x^(3*n) + 84*a*b^2*x^(2*n) + 70*a^2*b*x^n + 20*a^3)/(n*x^(7
*n))
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int x^{-1-7n}(a+bx^n)^3\,dx = \begin{cases} -\frac{a^3xx^{-7n-1}}{7n} - \frac{a^2bxx^nx^{-7n-1}}{2n} - \frac{3ab^2xx^{2n}x^{-7n-1}}{5n} - \frac{b^3xx^{3n}x^{-7n-1}}{4n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input

```
integrate(x**(-1-7*n)*(a+b*x**n)**3,x)
```

output

```
Piecewise((-a**3*x*x**(-7*n - 1)/(7*n) - a**2*b*x*x**n*x**(-7*n - 1)/(2*n)
 - 3*a*b**2*x*x**(2*n)*x**(-7*n - 1)/(5*n) - b**3*x*x**(3*n)*x**(-7*n - 1)
/(4*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x^{-1-7n}(a+bx^n)^3\,dx = -\frac{a^3}{7\,nx^{7\,n}} - \frac{a^2b}{2\,nx^{6\,n}} - \frac{3\,ab^2}{5\,nx^{5\,n}} - \frac{b^3}{4\,nx^{4\,n}}$$

input

```
integrate(x^(-1-7*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output

```
-1/7*a^3/(n*x^(7*n)) - 1/2*a^2*b/(n*x^(6*n)) - 3/5*a*b^2/(n*x^(5*n)) - 1/4
*b^3/(n*x^(4*n))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-7n}(a+bx^n)^3\,dx = -\frac{35\,b^3x^{3\,n}+84\,ab^2x^{2\,n}+70\,a^2bx^n+20\,a^3}{140\,nx^{7\,n}}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
-1/140*(35*b^3*x^(3*n) + 84*a*b^2*x^(2*n) + 70*a^2*b*x^n + 20*a^3)/(n*x^(7
*n))
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x^{-1-7n}(a+bx^n)^3\,dx = -\frac{a^3}{7\,n\,x^{7\,n}}-\frac{b^3}{4\,n\,x^{4\,n}}-\frac{3\,a\,b^2}{5\,n\,x^{5\,n}}-\frac{a^2\,b}{2\,n\,x^{6\,n}}$$

input
```
int((a + b*x^n)^3/x^(7*n + 1),x)
```

output
```
- a^3/(7*n*x^(7*n)) - b^3/(4*n*x^(4*n)) - (3*a*b^2)/(5*n*x^(5*n)) - (a^2*b
)/(2*n*x^(6*n))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int x^{-1-7n}(a+bx^n)^3\,dx = \frac{-35x^{3n}b^3-84x^{2n}a\,b^2-70x^na^2b-20a^3}{140x^{7n}n}$$

input
```
int(x^(-1-7*n)*(a+b*x^n)^3,x)
```

output
```
( - 35*x**(3*n)*b**3 - 84*x**(2*n)*a*b**2 - 70*x**n*a**2*b - 20*a**3)/(140
*x**(7*n)*n)
```

3.407 $\int x^{-1+4n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int x^{-1+4n}(a+bx^n)^5\,dx=-\frac{a^3(a+bx^n)^6}{6b^4n}+\frac{3a^2(a+bx^n)^7}{7b^4n}-\frac{3a(a+bx^n)^8}{8b^4n}+\frac{(a+bx^n)^9}{9b^4n}$$

output
```
-1/6*a^3*(a+b*x^n)^6/b^4/n+3/7*a^2*(a+b*x^n)^7/b^4/n-3/8*a*(a+b*x^n)^8/b^4
/n+1/9*(a+b*x^n)^9/b^4/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int x^{-1+4n}(a+bx^n)^5\,dx=\frac{x^{4n}(126a^5+504a^4bx^n+840a^3b^2x^{2n}+720a^2b^3x^{3n}+315ab^4x^{4n}+56b^5x^{5n})}{504n}$$

input
```
Integrate[x^(-1 + 4*n)*(a + b*x^n)^5,x]
```

output
```
(x^(4*n)*(126*a^5 + 504*a^4*b*x^n + 840*a^3*b^2*x^(2*n) + 720*a^2*b^3*x^(3
*n) + 315*a*b^4*x^(4*n) + 56*b^5*x^(5*n)))/(504*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{3n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^8}{b^3}-\frac{3a(bx^n+a)^7}{b^3}+\frac{3a^2(bx^n+a)^6}{b^3}-\frac{a^3(bx^n+a)^5}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^3(a+bx^n)^6}{6b^4}+\frac{3a^2(a+bx^n)^7}{7b^4}+\frac{(a+bx^n)^9}{9b^4}-\frac{3a(a+bx^n)^8}{8b^4}}{n}$$

input
```
Int[x^(-1 + 4*n)*(a + b*x^n)^5,x]
```

output
```
(-1/6*(a^3*(a + b*x^n)^6)/b^4 + (3*a^2*(a + b*x^n)^7)/(7*b^4) - (3*a*(a +
b*x^n)^8)/(8*b^4) + (a + b*x^n)^9/(9*b^4))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

method	result
risch	$\frac{b^5 x^{9n}}{9n}+\frac{5a\,b^4 x^{8n}}{8n}+\frac{10a^2 b^3 x^{7n}}{7n}+\frac{5a^3 b^2 x^{6n}}{3n}+\frac{a^4 b\,x^{5n}}{n}+\frac{a^5 x^{4n}}{4n}$
parallelrisch	$\frac{56x\,x^{5n} x^{-1+4n} b^5+315x\,x^{4n} x^{-1+4n} a\,b^4+720x\,x^{3n} x^{-1+4n} a^2 b^3+840x\,x^{2n} x^{-1+4n} a^3 b^2+504x\,x^{n} x^{-1+4n} a^4 b+126x\,x^{-1+4n}}{504n}$
orering	Expression too large to display

input
```
int(x^(-1+4*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^5/n*(x^n)^9+5/8*a*b^4/n*(x^n)^8+10/7*a^2*b^3/n*(x^n)^7+5/3*a^3*b^2/n
*(x^n)^6+a^4*b/n*(x^n)^5+1/4*a^5/n*(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int x^{-1+4n}(a+bx^n)^5\,dx$$
$$=\frac{56\,b^5x^{9\,n}+315\,ab^4x^{8\,n}+720\,a^2b^3x^{7\,n}+840\,a^3b^2x^{6\,n}+504\,a^4bx^{5\,n}+126\,a^5x^{4\,n}}{504\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/504*(56*b^5*x^(9*n) + 315*a*b^4*x^(8*n) + 720*a^2*b^3*x^(7*n) + 840*a^3*
b^2*x^(6*n) + 504*a^4*b*x^(5*n) + 126*a^5*x^(4*n))/n
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int x^{-1+4n}(a+bx^n)^5\,dx$$
$$= \begin{cases} \frac{a^5xx^{4n-1}}{4n} + \frac{a^4bxx^nx^{4n-1}}{n} + \frac{5a^3b^2xx^{2n}x^{4n-1}}{3n} + \frac{10a^2b^3xx^{3n}x^{4n-1}}{7n} + \frac{5ab^4xx^{4n}x^{4n-1}}{8n} + \frac{b^5xx^{5n}x^{4n-1}}{9n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((a**5*x*x**(4*n - 1)/(4*n) + a**4*b*x*x**n*x**(4*n - 1)/n + 5*a*
*3*b**2*x*x**(2*n)*x**(4*n - 1)/(3*n) + 10*a**2*b**3*x*x**(3*n)*x**(4*n -
1)/(7*n) + 5*a*b**4*x*x**(4*n)*x**(4*n - 1)/(8*n) + b**5*x*x**(5*n)*x**(4*
n - 1)/(9*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int x^{-1+4n}(a+bx^n)^5\,dx = \frac{b^5x^{9\,n}}{9\,n} + \frac{5\,ab^4x^{8\,n}}{8\,n} + \frac{10\,a^2b^3x^{7\,n}}{7\,n} + \frac{5\,a^3b^2x^{6\,n}}{3\,n} + \frac{a^4bx^{5\,n}}{n} + \frac{a^5x^{4\,n}}{4\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
1/9*b^5*x^(9*n)/n + 5/8*a*b^4*x^(8*n)/n + 10/7*a^2*b^3*x^(7*n)/n + 5/3*a^3
*b^2*x^(6*n)/n + a^4*b*x^(5*n)/n + 1/4*a^5*x^(4*n)/n
```

Giac [F]

$$\int x^{-1+4n}(a+bx^n)^5\,dx=\int (bx^n+a)^5x^{4\,n-1}\,dx$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^5*x^(4*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int x^{-1+4n}(a+bx^n)^5\,dx=\frac{a^5\,x^{4\,n}}{4\,n}+\frac{b^5\,x^{9\,n}}{9\,n}+\frac{5\,a^3\,b^2\,x^{6\,n}}{3\,n}+\frac{10\,a^2\,b^3\,x^{7\,n}}{7\,n}+\frac{a^4\,b\,x^{5\,n}}{n}+\frac{5\,a\,b^4\,x^{8\,n}}{8\,n}$$

input
```
int(x^(4*n - 1)*(a + b*x^n)^5,x)
```

output
```
(a^5*x^(4*n))/(4*n) + (b^5*x^(9*n))/(9*n) + (5*a^3*b^2*x^(6*n))/(3*n) + (1
0*a^2*b^3*x^(7*n))/(7*n) + (a^4*b*x^(5*n))/n + (5*a*b^4*x^(8*n))/(8*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int x^{-1+4n}(a+bx^n)^5\,dx=\frac{x^{4n}(56x^{5n}b^5+315x^{4n}a\,b^4+720x^{3n}a^2b^3+840x^{2n}a^3b^2+504x^na^4b+126a^5)}{504n}$$

input
```
int(x^(-1+4*n)*(a+b*x^n)^5,x)
```

output
```
(x**(4*n)*(56*x**(5*n)*b**5 + 315*x**(4*n)*a*b**4 + 720*x**(3*n)*a**2*b**3
 + 840*x**(2*n)*a**3*b**2 + 504*x**n*a**4*b + 126*a**5))/(504*n)
```

3.408 $\int x^{-1+3n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int x^{-1+3n}(a+bx^n)^5\,dx=\frac{a^2(a+bx^n)^6}{6b^3n}-\frac{2a(a+bx^n)^7}{7b^3n}+\frac{(a+bx^n)^8}{8b^3n}$$

output
```
1/6*a^2*(a+b*x^n)^6/b^3/n-2/7*a*(a+b*x^n)^7/b^3/n+1/8*(a+b*x^n)^8/b^3/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int x^{-1+3n}(a+bx^n)^5\,dx =\frac{x^{3n}(56a^5+210a^4bx^n+336a^3b^2x^{2n}+280a^2b^3x^{3n}+120ab^4x^{4n}+21b^5x^{5n})}{168n}$$

input
```
Integrate[x^(-1 + 3*n)*(a + b*x^n)^5,x]
```

output
```
(x^(3*n)*(56*a^5 + 210*a^4*b*x^n + 336*a^3*b^2*x^(2*n) + 280*a^2*b^3*x^(3*
n) + 120*a*b^4*x^(4*n) + 21*b^5*x^(5*n)))/(168*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{2n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^7}{b^2}-\frac{2a(bx^n+a)^6}{b^2}+\frac{a^2(bx^n+a)^5}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2(a+bx^n)^6}{6b^3}+\frac{(a+bx^n)^8}{8b^3}-\frac{2a(a+bx^n)^7}{7b^3}}{n}$$

input
```
Int[x^(-1 + 3*n)*(a + b*x^n)^5,x]
```

output
```
((a^2*(a + b*x^n)^6)/(6*b^3) - (2*a*(a + b*x^n)^7)/(7*b^3) + (a + b*x^n)^8
/(8*b^3))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

method	result
risch	$\frac{b^5 x^{8n}}{8n}+\frac{5a\,b^4 x^{7n}}{7n}+\frac{5a^2 b^3 x^{6n}}{3n}+\frac{2a^3 b^2 x^{5n}}{n}+\frac{5a^4 b\,x^{4n}}{4n}+\frac{a^5 x^{3n}}{3n}$
parallelrisch	$\frac{21x\,x^{5n}x^{-1+3n}b^5+120x\,x^{4n}x^{-1+3n}a\,b^4+280x\,x^{3n}x^{-1+3n}a^2b^3+336x\,x^{2n}x^{-1+3n}a^3b^2+210x\,x^{n}x^{-1+3n}a^4b+56x\,x^{-1+3n}a}{168n}$
orering	Expression too large to display

input
```
int(x^(-1+3*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^5/n*(x^n)^8+5/7*a*b^4/n*(x^n)^7+5/3*a^2*b^3/n*(x^n)^6+2*a^3*b^2/n*(x
^n)^5+5/4*a^4*b/n*(x^n)^4+1/3*a^5/n*(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int x^{-1+3n}(a+bx^n)^5\,dx$$
$$=\frac{21\,b^5x^{8\,n}+120\,ab^4x^{7\,n}+280\,a^2b^3x^{6\,n}+336\,a^3b^2x^{5\,n}+210\,a^4bx^{4\,n}+56\,a^5x^{3\,n}}{168\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/168*(21*b^5*x^(8*n) + 120*a*b^4*x^(7*n) + 280*a^2*b^3*x^(6*n) + 336*a^3*
b^2*x^(5*n) + 210*a^4*b*x^(4*n) + 56*a^5*x^(3*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(51) = 102$.

Time = 0.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.23

$$\int x^{-1+3n}(a+bx^n)^5\,dx = \begin{cases} \frac{a^5xx^{3n-1}}{3n} + \frac{5a^4bxx^nx^{3n-1}}{4n} + \frac{2a^3b^2xx^{2n}x^{3n-1}}{n} + \frac{5a^2b^3xx^{3n}x^{3n-1}}{3n} + \frac{5ab^4xx^{4n}x^{3n-1}}{7n} + \frac{b^5xx^{5n}x^{3n-1}}{8n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((a**5*x*x**(3*n - 1)/(3*n) + 5*a**4*b*x*x**n*x**(3*n - 1)/(4*n)
+ 2*a**3*b**2*x*x**(2*n)*x**(3*n - 1)/n + 5*a**2*b**3*x*x**(3*n)*x**(3*n -
 1)/(3*n) + 5*a*b**4*x*x**(4*n)*x**(3*n - 1)/(7*n) + b**5*x*x**(5*n)*x**(3
*n - 1)/(8*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int x^{-1+3n}(a+bx^n)^5\,dx = \frac{b^5x^{8n}}{8n} + \frac{5ab^4x^{7n}}{7n} + \frac{5a^2b^3x^{6n}}{3n} + \frac{2a^3b^2x^{5n}}{n} + \frac{5a^4bx^{4n}}{4n} + \frac{a^5x^{3n}}{3n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
1/8*b^5*x^(8*n)/n + 5/7*a*b^4*x^(7*n)/n + 5/3*a^2*b^3*x^(6*n)/n + 2*a^3*b^
2*x^(5*n)/n + 5/4*a^4*b*x^(4*n)/n + 1/3*a^5*x^(3*n)/n
```

Giac [F]

$$\int x^{-1+3n}(a+bx^n)^5\,dx=\int (bx^n+a)^5x^{3\,n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^5*x^(3*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int x^{-1+3n}(a+bx^n)^5\,dx=\frac{a^5\,x^{3\,n}}{3\,n}+\frac{b^5\,x^{8\,n}}{8\,n}+\frac{2\,a^3\,b^2\,x^{5\,n}}{n}+\frac{5\,a^2\,b^3\,x^{6\,n}}{3\,n}+\frac{5\,a^4\,b\,x^{4\,n}}{4\,n}+\frac{5\,a\,b^4\,x^{7\,n}}{7\,n}$$

input
```
int(x^(3*n - 1)*(a + b*x^n)^5,x)
```

output
```
(a^5*x^(3*n))/(3*n) + (b^5*x^(8*n))/(8*n) + (2*a^3*b^2*x^(5*n))/n + (5*a^2
*b^3*x^(6*n))/(3*n) + (5*a^4*b*x^(4*n))/(4*n) + (5*a*b^4*x^(7*n))/(7*n)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int x^{-1+3n}(a+bx^n)^5\,dx=\frac{x^{3n}(21x^{5n}b^5+120x^{4n}a\,b^4+280x^{3n}a^2b^3+336x^{2n}a^3b^2+210x^na^4b+56a^5)}{168n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^5,x)
```

output
```
(x**(3*n)*(21*x**(5*n)*b**5 + 120*x**(4*n)*a*b**4 + 280*x**(3*n)*a**2*b**3
 + 336*x**(2*n)*a**3*b**2 + 210*x**n*a**4*b + 56*a**5))/(168*n)
```

3.409 $\int x^{-1+2n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int x^{-1+2n}(a+bx^n)^5\,dx=-\frac{a(a+bx^n)^6}{6b^2n}+\frac{(a+bx^n)^7}{7b^2n}$$

output
```
-1/6*a*(a+b*x^n)^6/b^2/n+1/7*(a+b*x^n)^7/b^2/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

$$\int x^{-1+2n}(a+bx^n)^5\,dx =\frac{x^{2n}(21a^5+70a^4bx^n+105a^3b^2x^{2n}+84a^2b^3x^{3n}+35ab^4x^{4n}+6b^5x^{5n})}{42n}$$

input
```
Integrate[x^(-1 + 2*n)*(a + b*x^n)^5,x]
```

output
```
(x^(2*n)*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2*x^(2*n) + 84*a^2*b^3*x^(3*n)
 + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n)))/(42*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^n(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^6}{b}-\frac{a(bx^n+a)^5}{b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+bx^n)^7}{7b^2}-\frac{a(a+bx^n)^6}{6b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a + b*x^n)^5,x]
```

output
```
(-1/6*(a*(a + b*x^n)^6)/b^2 + (a + b*x^n)^7/(7*b^2))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(36) = 72$.

Time = 1.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

method	result	size
risch	$\frac{b^5 x^{7n}}{7n}+\frac{5a\,b^4 x^{6n}}{6n}+\frac{2a^2 b^3 x^{5n}}{n}+\frac{5a^3 b^2 x^{4n}}{2n}+\frac{5a^4 b\,x^{3n}}{3n}+\frac{a^5 x^{2n}}{2n}$	88
parallelrisch	$\frac{6x\,x^{5n}x^{2n-1}b^5+35x\,x^{4n}x^{2n-1}a\,b^4+84x\,x^{3n}x^{2n-1}a^2b^3+105x\,x^{2n}x^{2n-1}a^3b^2+70x\,x^{n}x^{2n-1}a^4b+21x\,x^{2n-1}a^5}{42n}$	116
orering	Expression too large to display	2807

input
```
int(x^(2*n-1)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^5/n*(x^n)^7+5/6*a*b^4/n*(x^n)^6+2*a^2*b^3/n*(x^n)^5+5/2*a^3*b^2/n*(x
^n)^4+5/3*a^4*b/n*(x^n)^3+1/2*a^5/n*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

$$\int x^{-1+2n}(a+bx^n)^5\,dx = \frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*
x^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(31) = 62$.

Time = 0.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.45

$$\int x^{-1+2n}(a+bx^n)^5\,dx = \begin{cases} \frac{a^5xx^{2n-1}}{2n} + \frac{5a^4bxx^nx^{2n-1}}{3n} + \frac{5a^3b^2xx^{2n}x^{2n-1}}{2n} + \frac{2a^2b^3xx^{3n}x^{2n-1}}{n} + \frac{5ab^4xx^{4n}x^{2n-1}}{6n} + \frac{b^5xx^{5n}x^{2n-1}}{7n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((a**5*x*x**(2*n - 1)/(2*n) + 5*a**4*b*x*x**n*x**(2*n - 1)/(3*n)
+ 5*a**3*b**2*x*x**(2*n)*x**(2*n - 1)/(2*n) + 2*a**2*b**3*x*x**(3*n)*x**(2
*n - 1)/n + 5*a*b**4*x*x**(4*n)*x**(2*n - 1)/(6*n) + b**5*x*x**(5*n)*x**(2
*n - 1)/(7*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int x^{-1+2n}(a+bx^n)^5\,dx = \frac{b^5x^{7n}}{7n} + \frac{5ab^4x^{6n}}{6n} + \frac{2a^2b^3x^{5n}}{n} + \frac{5a^3b^2x^{4n}}{2n} + \frac{5a^4bx^{3n}}{3n} + \frac{a^5x^{2n}}{2n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
1/7*b^5*x^(7*n)/n + 5/6*a*b^4*x^(6*n)/n + 2*a^2*b^3*x^(5*n)/n + 5/2*a^3*b^
2*x^(4*n)/n + 5/3*a^4*b*x^(3*n)/n + 1/2*a^5*x^(2*n)/n
```

Giac [F]

$$\int x^{-1+2n}(a+bx^n)^5\,dx=\int (bx^n+a)^5x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^5*x^(2*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int x^{-1+2n}(a+bx^n)^5\,dx=\frac{a^5\,x^{2\,n}}{2\,n}+\frac{b^5\,x^{7\,n}}{7\,n}+\frac{5\,a^3\,b^2\,x^{4\,n}}{2\,n}+\frac{2\,a^2\,b^3\,x^{5\,n}}{n}+\frac{5\,a^4\,b\,x^{3\,n}}{3\,n}+\frac{5\,a\,b^4\,x^{6\,n}}{6\,n}$$

input
```
int(x^(2*n - 1)*(a + b*x^n)^5,x)
```

output
```
(a^5*x^(2*n))/(2*n) + (b^5*x^(7*n))/(7*n) + (5*a^3*b^2*x^(4*n))/(2*n) + (2
*a^2*b^3*x^(5*n))/n + (5*a^4*b*x^(3*n))/(3*n) + (5*a*b^4*x^(6*n))/(6*n)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int x^{-1+2n}(a+bx^n)^5\,dx=\frac{x^{2n}(6x^{5n}b^5+35x^{4n}a\,b^4+84x^{3n}a^2b^3+105x^{2n}a^3b^2+70x^na^4b+21a^5)}{42n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^5,x)
```

output
```
(x**(2*n)*(6*x**(5*n)*b**5 + 35*x**(4*n)*a*b**4 + 84*x**(3*n)*a**2*b**3 +
105*x**(2*n)*a**3*b**2 + 70*x**n*a**4*b + 21*a**5))/(42*n)
```

3.410 $\int x^{-1+n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)^5\,dx=\frac{(a+bx^n)^6}{6bn}$$

output
```
1/6*(a+b*x^n)^6/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^5\,dx=\frac{(a+bx^n)^6}{6bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^5,x]
```

output
```
(a + b*x^n)^6/(6*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^5\,dx$$
$$\Big\downarrow \text{793}$$
$$\frac{(a+bx^n)^6}{6bn}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^5,x]
```

output
```
(a + b*x^n)^6/(6*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(17) = 34.

Time = 1.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.42

method	result	size
risch	$\frac{b^5x^{6n}}{6n}+\frac{a\,b^4x^{5n}}{n}+\frac{5a^2b^3x^{4n}}{2n}+\frac{10a^3b^2x^{3n}}{3n}+\frac{5a^4b\,x^{2n}}{2n}+\frac{a^5x^n}{n}$	84
parallelrisch	$\frac{x\,x^{5n}x^{-1+n}b^5+6x\,x^{4n}x^{-1+n}a\,b^4+15x\,x^{3n}x^{-1+n}a^2b^3+20x\,x^{2n}x^{-1+n}a^3b^2+15x\,x^nx^{-1+n}a^4b+6x\,x^{-1+n}a^5}{6n}$	103
orering	Expression too large to display	2535

input
```
int(x^(-1+n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^5/n*(x^n)^6+a*b^4/n*(x^n)^5+5/2*a^2*b^3/n*(x^n)^4+10/3*a^3*b^2/n*(x^
n)^3+5/2*a^4*b/n*(x^n)^2+a^5/n*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.74

$$\int x^{-1+n}(a+bx^n)^5\,dx = \frac{b^5x^{6\,n}+6\,ab^4x^{5\,n}+15\,a^2b^3x^{4\,n}+20\,a^3b^2x^{3\,n}+15\,a^4bx^{2\,n}+6\,a^5x^n}{6\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/6*(b^5*x^(6*n) + 6*a*b^4*x^(5*n) + 15*a^2*b^3*x^(4*n) + 20*a^3*b^2*x^(3*
n) + 15*a^4*b*x^(2*n) + 6*a^5*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(12) = 24$.

Time = 0.86 (sec) , antiderivative size = 124, normalized size of antiderivative = 6.53

$$\int x^{-1+n}(a+bx^n)^5\,dx = \begin{cases} \frac{a^5xx^{n-1}}{n}+\frac{5a^4bxx^nx^{n-1}}{2n}+\frac{10a^3b^2xx^{2n}x^{n-1}}{3n}+\frac{5a^2b^3xx^{3n}x^{n-1}}{2n}+\frac{ab^4xx^{4n}x^{n-1}}{n}+\frac{b^5xx^{5n}x^{n-1}}{6n} & \text{for}\ n\neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((a**5*x*x**(n - 1)/n + 5*a**4*b*x*x**n*x**(n - 1)/(2*n) + 10*a**
3*b**2*x*x**(2*n)*x**(n - 1)/(3*n) + 5*a**2*b**3*x*x**(3*n)*x**(n - 1)/(2*
n) + a*b**4*x*x**(4*n)*x**(n - 1)/n + b**5*x*x**(5*n)*x**(n - 1)/(6*n), Ne
(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)^5\,dx=\frac{(bx^n+a)^6}{6\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
1/6*(b*x^n + a)^6/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(17) = 34.

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.74

$$\int x^{-1+n}(a+bx^n)^5\,dx=\frac{b^5x^{6\,n}+6\,ab^4x^{5\,n}+15\,a^2b^3x^{4\,n}+20\,a^3b^2x^{3\,n}+15\,a^4bx^{2\,n}+6\,a^5x^n}{6\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
1/6*(b^5*x^(6*n) + 6*a*b^4*x^(5*n) + 15*a^2*b^3*x^(4*n) + 20*a^3*b^2*x^(3*
n) + 15*a^4*b*x^(2*n) + 6*a^5*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.37

$$\int x^{-1+n}(a+bx^n)^5\,dx = \frac{a^5\,x^n}{n}+\frac{b^5\,x^{6\,n}}{6\,n}+\frac{10\,a^3\,b^2\,x^{3\,n}}{3\,n}+\frac{5\,a^2\,b^3\,x^{4\,n}}{2\,n}+\frac{5\,a^4\,b\,x^{2\,n}}{2\,n}+\frac{a\,b^4\,x^{5\,n}}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^5,x)
```

output
```
(a^5*x^n)/n + (b^5*x^(6*n))/(6*n) + (10*a^3*b^2*x^(3*n))/(3*n) + (5*a^2*b^
3*x^(4*n))/(2*n) + (5*a^4*b*x^(2*n))/(2*n) + (a*b^4*x^(5*n))/n
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int x^{-1+n}(a+bx^n)^5\,dx = \frac{x^n(x^{5n}b^5+6x^{4n}a\,b^4+15x^{3n}a^2b^3+20x^{2n}a^3b^2+15x^na^4b+6a^5)}{6n}$$

input
```
int(x^(-1+n)*(a+b*x^n)^5,x)
```

output
```
(x**n*(x**(5*n)*b**5 + 6*x**(4*n)*a*b**4 + 15*x**(3*n)*a**2*b**3 + 20*x**(
2*n)*a**3*b**2 + 15*x**n*a**4*b + 6*a**5))/(6*n)
```

3.411 $\int \frac{(a+bx^n)^5}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{(a+bx^n)^5}{x} \, dx = \frac{5a^4bx^n}{n} + \frac{5a^3b^2x^{2n}}{n} + \frac{10a^2b^3x^{3n}}{3n} + \frac{5ab^4x^{4n}}{4n} + \frac{b^5x^{5n}}{5n} + a^5\log(x)$$

output
```
5*a^4*b*x^n/n+5*a^3*b^2*x^(2*n)/n+10/3*a^2*b^3*x^(3*n)/n+5/4*a*b^4*x^(4*n)
/n+1/5*b^5*x^(5*n)/n+a^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^n)^5}{x} \, dx = \frac{bx^n(300a^4 + 300a^3bx^n + 200a^2b^2x^{2n} + 75ab^3x^{3n} + 12b^4x^{4n})}{60n} + \frac{a^5\log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)^5/x,x]
```

output
```
(b*x^n*(300*a^4 + 300*a^3*b*x^n + 200*a^2*b^2*x^(2*n) + 75*a*b^3*x^(3*n) +
 12*b^4*x^(4*n)))/(60*n) + (a^5*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^5}{x}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^5x^{-n}+10a^3b^2x^n+10a^2b^3x^{2n}+5ab^4x^{3n}+b^5x^{4n}+5a^4b\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\log(x^n)+5a^4bx^n+5a^3b^2x^{2n}+\frac{10}{3}a^2b^3x^{3n}+\frac{5}{4}ab^4x^{4n}+\frac{1}{5}b^5x^{5n}}{n}$$

input
```
Int[(a + b*x^n)^5/x,x]
```

output
```
(5*a^4*b*x^n + 5*a^3*b^2*x^(2*n) + (10*a^2*b^3*x^(3*n))/3 + (5*a*b^4*x^(4*
n))/4 + (b^5*x^(5*n))/5 + a^5*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{b^5 x^{5n}}{5}+\frac{5 a\, b^4 x^{4n}}{4}+\frac{10 a^2 b^3 x^{3n}}{3}+5 a^3 b^2 x^{2n}+5 a^4 b\, x^n+a^5 \ln(x^n)}{n}$	70
default	$\frac{\frac{b^5 x^{5n}}{5}+\frac{5 a\, b^4 x^{4n}}{4}+\frac{10 a^2 b^3 x^{3n}}{3}+5 a^3 b^2 x^{2n}+5 a^4 b\, x^n+a^5 \ln(x^n)}{n}$	70
parallelrisch	$\frac{12 b^5 x^{5n}+75 a\, b^4 x^{4n}+200 a^2 b^3 x^{3n}+300 a^3 b^2 x^{2n}+60 a^5 \ln(x) n+300 a^4 b\, x^n}{60 n}$	71
risch	$\frac{5 a^4 b\, x^n}{n}+\frac{5 a^3 b^2 x^{2n}}{n}+\frac{10 a^2 b^3 x^{3n}}{3n}+\frac{5 a\, b^4 x^{4n}}{4n}+\frac{b^5 x^{5n}}{5n}+a^5 \ln\left(x\right)$	79
norman	$a^5 \ln\left(x\right)+\frac{b^5 e^{5n \ln(x)}}{5n}+\frac{5 a\, b^4 e^{4n \ln(x)}}{4n}+\frac{10 a^2 b^3 e^{3n \ln(x)}}{3n}+\frac{5 a^3 b^2 e^{2n \ln(x)}}{n}+\frac{5 a^4 b\, e^{n \ln(x)}}{n}$	89

input
```
int((a+b*x^n)^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/5*b^5*(x^n)^5+5/4*a*b^4*(x^n)^4+10/3*a^2*b^3*(x^n)^3+5*a^3*b^2*(x^n
)^2+5*a^4*b*x^n+a^5*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^5}{x}\,dx = \frac{60\,a^5 n\log\left(x\right)+12\,b^5 x^{5\,n}+75\,ab^4 x^{4\,n}+200\,a^2 b^3 x^{3\,n}+300\,a^3 b^2 x^{2\,n}+300\,a^4 b x^n}{60\,n}$$

input
```
integrate((a+b*x^n)^5/x,x, algorithm="fricas")
```

output
```
1/60*(60*a^5*n*log(x) + 12*b^5*x^(5*n) + 75*a*b^4*x^(4*n) + 200*a^2*b^3*x^
(3*n) + 300*a^3*b^2*x^(2*n) + 300*a^4*b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^n)^5}{x}\,dx = \begin{cases} a^5\log(x) + \frac{5a^4bx^n}{n} + \frac{5a^3b^2x^{2n}}{n} + \frac{10a^2b^3x^{3n}}{3n} + \frac{5ab^4x^{4n}}{4n} + \frac{b^5x^{5n}}{5n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**5/x,x)
```

output
```
Piecewise((a**5*log(x) + 5*a**4*b*x**n/n + 5*a**3*b**2*x**(2*n)/n + 10*a**
2*b**3*x**(3*n)/(3*n) + 5*a*b**4*x**(4*n)/(4*n) + b**5*x**(5*n)/(5*n), Ne(
n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^n)^5}{x}\,dx = \frac{a^5\log(x^n)}{n} + \frac{12\,b^5x^{5\,n} + 75\,ab^4x^{4\,n} + 200\,a^2b^3x^{3\,n} + 300\,a^3b^2x^{2\,n} + 300\,a^4bx^n}{60\,n}$$

input
```
integrate((a+b*x^n)^5/x,x, algorithm="maxima")
```

output
```
a^5*log(x^n)/n + 1/60*(12*b^5*x^(5*n) + 75*a*b^4*x^(4*n) + 200*a^2*b^3*x^(
3*n) + 300*a^3*b^2*x^(2*n) + 300*a^4*b*x^n)/n
```

Giac [F]

$$\int \frac{(a+bx^n)^5}{x}\,dx = \int \frac{(bx^n+a)^5}{x}\,dx$$

input
```
integrate((a+b*x^n)^5/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^5/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^n)^5}{x}\,dx = a^5 \ln(x) + \frac{b^5 x^{5n}}{5n} + \frac{5a^3 b^2 x^{2n}}{n} + \frac{10a^2 b^3 x^{3n}}{3n} + \frac{5a^4 b x^n}{n} + \frac{5a b^4 x^{4n}}{4n}$$

input
```
int((a + b*x^n)^5/x,x)
```

output
```
a^5*log(x) + (b^5*x^(5*n))/(5*n) + (5*a^3*b^2*x^(2*n))/n + (10*a^2*b^3*x^(
3*n))/(3*n) + (5*a^4*b*x^n)/n + (5*a*b^4*x^(4*n))/(4*n)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^n)^5}{x}\,dx = \frac{12x^{5n}b^5 + 75x^{4n}a b^4 + 200x^{3n}a^2b^3 + 300x^{2n}a^3b^2 + 300x^n a^4 b + 60\log(x)\,a^5 n}{60n}$$

input
```
int((a+b*x^n)^5/x,x)
```

output
```
(12*x**(5*n)*b**5 + 75*x**(4*n)*a*b**4 + 200*x**(3*n)*a**2*b**3 + 300*x**(
2*n)*a**3*b**2 + 300*x**n*a**4*b + 60*log(x)*a**5*n)/(60*n)
```

3.412 $\int x^{-1-n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int x^{-1-n}(a+bx^n)^5\,dx=-\frac{a^5x^{-n}}{n}+\frac{10a^3b^2x^n}{n}+\frac{5a^2b^3x^{2n}}{n}+\frac{5ab^4x^{3n}}{3n}+\frac{b^5x^{4n}}{4n}+5a^4b\log(x)$$

output
```
-a^5/n/(x^n)+10*a^3*b^2*x^n/n+5*a^2*b^3*x^(2*n)/n+5/3*a*b^4*x^(3*n)/n+1/4*
b^5*x^(4*n)/n+5*a^4*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int x^{-1-n}(a+bx^n)^5\,dx =-\frac{12a^5x^{-n}-120a^3b^2x^n-60a^2b^3x^{2n}-20ab^4x^{3n}-3b^5x^{4n}-60a^4b\log(x^n)}{12n}$$

input
```
Integrate[x^(-1 - n)*(a + b*x^n)^5,x]
```

output
```
-1/12*((12*a^5)/x^n - 120*a^3*b^2*x^n - 60*a^2*b^3*x^(2*n) - 20*a*b^4*x^(3
*n) - 3*b^5*x^(4*n) - 60*a^4*b*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(a+bx^n)^5\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-2n}(bx^n+a)^5\,dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(a^5x^{-2n}+5a^4bx^{-n}+10a^2b^3x^n+5ab^4x^{2n}+b^5x^{3n}+10a^3b^2\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-a^5x^{-n}+5a^4b\log\left(x^n\right)+10a^3b^2x^n+5a^2b^3x^{2n}+\frac{5}{3}ab^4x^{3n}+\frac{1}{4}b^5x^{4n}}{n}$$

input
```
Int[x^(-1 - n)*(a + b*x^n)^5,x]
```

output
```
(-(a^5/x^n) + 10*a^3*b^2*x^n + 5*a^2*b^3*x^(2*n) + (5*a*b^4*x^(3*n))/3 + (
b^5*x^(4*n))/4 + 5*a^4*b*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

method	result	size
risch	$5a^4 b\ln\left(x\right)+\frac{b^5 x^{4n}}{4n}+\frac{5a\,b^4 x^{3n}}{3n}+\frac{5a^2 b^3 x^{2n}}{n}+\frac{10a^3 b^2 x^n}{n}-\frac{a^5 x^{-n}}{n}$	80
norman	$\left(5a^4 b\ln\left(x\right){\mathrm e}^{n\ln\left(x\right)}-\frac{a^5}{n}+\frac{b^5 {\mathrm e}^{5n\ln\left(x\right)}}{4n}+\frac{5a\,b^4 {\mathrm e}^{4n\ln\left(x\right)}}{3n}+\frac{5a^2 b^3 {\mathrm e}^{3n\ln\left(x\right)}}{n}+\frac{10a^3 b^2 {\mathrm e}^{2n\ln\left(x\right)}}{n}\right){\mathrm e}^{-n\ln\left(x\right)}$	98

input
```
int(x^(-1-n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
5*a^4*b*ln(x)+1/4*b^5/n*(x^n)^4+5/3*a*b^4/n*(x^n)^3+5*a^2*b^3/n*(x^n)^2+10
*a^3*b^2*x^n/n-a^5/n/(x^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^{-1-n}(a+bx^n)^5\,dx = \frac{60\,a^4bnx^n\log\left(x\right)+3\,b^5x^{5\,n}+20\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}+120\,a^3b^2x^{2\,n}-12\,a^5}{12\,nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/12*(60*a^4*b*n*x^n*log(x) + 3*b^5*x^(5*n) + 20*a*b^4*x^(4*n) + 60*a^2*b^
3*x^(3*n) + 120*a^3*b^2*x^(2*n) - 12*a^5)/(n*x^n)
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int x^{-1-n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5x^{-n}}{n} + \frac{5a^4b\log(x^n)}{n} + \frac{10a^3b^2x^n}{n} + \frac{5a^2b^3x^{2n}}{n} + \frac{5ab^4x^{3n}}{3n} + \frac{b^5x^{4n}}{4n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5/(n*x**n) + 5*a**4*b*log(x**n)/n + 10*a**3*b**2*x**n/n + 5
*a**2*b**3*x**(2*n)/n + 5*a*b**4*x**(3*n)/(3*n) + b**5*x**(4*n)/(4*n), Ne(
n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int x^{-1-n}(a+bx^n)^5\,dx = 5\,a^4b\log(x) + \frac{b^5x^{4\,n}}{4\,n} + \frac{5\,ab^4x^{3\,n}}{3\,n} + \frac{5\,a^2b^3x^{2\,n}}{n} + \frac{10\,a^3b^2x^n}{n} - \frac{a^5}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
5*a^4*b*log(x) + 1/4*b^5*x^(4*n)/n + 5/3*a*b^4*x^(3*n)/n + 5*a^2*b^3*x^(2*
n)/n + 10*a^3*b^2*x^n/n - a^5/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^{-1-n}(a+bx^n)^5\,dx = \frac{60\,a^4bnx^n\log(x)+3\,b^5x^{5\,n}+20\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}+120\,a^3b^2x^{2\,n}-12\,a^5}{12\,nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
1/12*(60*a^4*b*n*x^n*log(x) + 3*b^5*x^(5*n) + 20*a*b^4*x^(4*n) + 60*a^2*b^
3*x^(3*n) + 120*a^3*b^2*x^(2*n) - 12*a^5)/(n*x^n)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int x^{-1-n}(a+bx^n)^5\,dx = 5\,a^4\,b\,\ln(x)-\frac{a^5}{n\,x^n}+\frac{b^5\,x^{4\,n}}{4\,n}+\frac{5\,a^2\,b^3\,x^{2\,n}}{n}+\frac{5\,a\,b^4\,x^{3\,n}}{3\,n}+\frac{10\,a^3\,b^2\,x^n}{n}$$

input
```
int((a + b*x^n)^5/x^(n + 1),x)
```

output
```
5*a^4*b*log(x) - a^5/(n*x^n) + (b^5*x^(4*n))/(4*n) + (5*a^2*b^3*x^(2*n))/n
 + (5*a*b^4*x^(3*n))/(3*n) + (10*a^3*b^2*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int x^{-1-n}(a+bx^n)^5\,dx = \frac{3x^{5n}b^5+20x^{4n}a\,b^4+60x^{3n}a^2b^3+120x^{2n}a^3b^2+60x^n\log(x)\,a^4bn-12a^5}{12x^nn}$$

input
```
int(x^(-1-n)*(a+b*x^n)^5,x)
```

output
```
(3*x**(5*n)*b**5 + 20*x**(4*n)*a*b**4 + 60*x**(3*n)*a**2*b**3 + 120*x**(2*
n)*a**3*b**2 + 60*x**n*log(x)*a**4*b*n - 12*a**5)/(12*x**n*n)
```

3.413 $\int x^{-1-2n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int x^{-1-2n}(a+bx^n)^5\,dx = -\frac{a^5x^{-2n}}{2n}-\frac{5a^4bx^{-n}}{n}+\frac{10a^2b^3x^n}{n}+\frac{5ab^4x^{2n}}{2n}+\frac{b^5x^{3n}}{3n}+10a^3b^2\log(x)$$

output
```
-1/2*a^5/n/(x^(2*n))-5*a^4*b/n/(x^n)+10*a^2*b^3*x^n/n+5/2*a*b^4*x^(2*n)/n+
1/3*b^5*x^(3*n)/n+10*a^3*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}(a+bx^n)^5\,dx = \frac{-3a^5x^{-2n}-30a^4bx^{-n}+60a^2b^3x^n+15ab^4x^{2n}+2b^5x^{3n}+60a^3b^2\log(x^n)}{6n}$$

input
```
Integrate[x^(-1 - 2*n)*(a + b*x^n)^5,x]
```

output
```
((-3*a^5)/x^(2*n) - (30*a^4*b)/x^n + 60*a^2*b^3*x^n + 15*a*b^4*x^(2*n) + 2
*b^5*x^(3*n) + 60*a^3*b^2*Log[x^n])/(6*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int x^{-3n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(a^5x^{-3n}+5a^4bx^{-2n}+10a^3b^2x^{-n}+5ab^4x^n+b^5x^{2n}+10a^2b^3\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{2}a^5x^{-2n}-5a^4bx^{-n}+10a^3b^2\log\left(x^n\right)+10a^2b^3x^n+\frac{5}{2}ab^4x^{2n}+\frac{1}{3}b^5x^{3n}}{n}$$

input
```
Int[x^(-1 - 2*n)*(a + b*x^n)^5,x]
```

output
```
(-1/2*a^5/x^(2*n) - (5*a^4*b)/x^n + 10*a^2*b^3*x^n + (5*a*b^4*x^(2*n))/2 +
 (b^5*x^(3*n))/3 + 10*a^3*b^2*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result	size
risch	$10a^{3}b^{2}\ln\left(x\right)+\frac{b^{5}x^{3n}}{3n}+\frac{5a\,b^{4}x^{2n}}{2n}+\frac{10a^{2}b^{3}x^{n}}{n}-\frac{5a^{4}b\,x^{-n}}{n}-\frac{a^{5}x^{-2n}}{2n}$	80
norman	$\left(10a^{3}b^{2}\ln\left(x\right){\mathrm e}^{2n\ln\left(x\right)}-\frac{a^{5}}{2n}+\frac{b^{5}{\mathrm e}^{5n\ln\left(x\right)}}{3n}+\frac{5a\,b^{4}{\mathrm e}^{4n\ln\left(x\right)}}{2n}+\frac{10a^{2}b^{3}{\mathrm e}^{3n\ln\left(x\right)}}{n}-\frac{5a^{4}b\,{\mathrm e}^{n\ln\left(x\right)}}{n}\right){\mathrm e}^{-2n\ln\left(x\right)}$	98

input
```
int(x^(-2*n-1)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
10*a^3*b^2*ln(x)+1/3*b^5/n*(x^n)^3+5/2*a*b^4/n*(x^n)^2+10*a^2*b^3*x^n/n-5*
a^4*b/n/(x^n)-1/2*a^5/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1-2n}(a+bx^n)^5\,dx = \frac{60\,a^{3}b^{2}nx^{2\,n}\log\left(x\right)+2\,b^{5}x^{5\,n}+15\,ab^{4}x^{4\,n}+60\,a^{2}b^{3}x^{3\,n}-30\,a^{4}bx^{n}-3\,a^{5}}{6\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/6*(60*a^3*b^2*n*x^(2*n)*log(x) + 2*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 60*a
^2*b^3*x^(3*n) - 30*a^4*b*x^n - 3*a^5)/(n*x^(2*n))
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int x^{-1-2n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5x^{-2n}}{2n} - \frac{5a^4bx^{-n}}{n} + \frac{10a^3b^2\log{(x^n)}}{n} + \frac{10a^2b^3x^n}{n} + \frac{5ab^4x^{2n}}{2n} + \frac{b^5x^{3n}}{3n} & \text{for}\ n \neq 0 \\ (a+b)^5\log{(x)} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-2*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5/(2*n*x**(2*n)) - 5*a**4*b/(n*x**n) + 10*a**3*b**2*log(x**
n)/n + 10*a**2*b**3*x**n/n + 5*a*b**4*x**(2*n)/(2*n) + b**5*x**(3*n)/(3*n)
, Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{-1-2n}(a+bx^n)^5\,dx = 10\,a^3b^2\log{(x)} + \frac{b^5x^{3\,n}}{3\,n} + \frac{5\,ab^4x^{2\,n}}{2\,n} + \frac{10\,a^2b^3x^n}{n} - \frac{a^5}{2\,nx^{2\,n}} - \frac{5\,a^4b}{nx^n}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
10*a^3*b^2*log(x) + 1/3*b^5*x^(3*n)/n + 5/2*a*b^4*x^(2*n)/n + 10*a^2*b^3*x
^n/n - 1/2*a^5/(n*x^(2*n)) - 5*a^4*b/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1-2n}(a+bx^n)^5\,dx = \frac{60\,a^3b^2nx^{2\,n}\log(x)+2\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+60\,a^2b^3x^{3\,n}-30\,a^4bx^n-3\,a^5}{6\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
1/6*(60*a^3*b^2*n*x^(2*n)*log(x) + 2*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 60*a
^2*b^3*x^(3*n) - 30*a^4*b*x^n - 3*a^5)/(n*x^(2*n))
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{-1-2n}(a+bx^n)^5\,dx = \frac{b^5\,x^{3\,n}}{3\,n}-\frac{a^5}{2\,n\,x^{2\,n}}+10\,a^3\,b^2\,\ln(x)-\frac{5\,a^4\,b}{n\,x^n}+\frac{5\,a\,b^4\,x^{2\,n}}{2\,n}+\frac{10\,a^2\,b^3\,x^n}{n}$$

input
```
int((a + b*x^n)^5/x^(2*n + 1),x)
```

output
```
(b^5*x^(3*n))/(3*n) - a^5/(2*n*x^(2*n)) + 10*a^3*b^2*log(x) - (5*a^4*b)/(n
*x^n) + (5*a*b^4*x^(2*n))/(2*n) + (10*a^2*b^3*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1-2n}(a+bx^n)^5\,dx = \frac{2x^{5n}b^5+15x^{4n}a\,b^4+60x^{3n}a^2b^3+60x^{2n}\log(x)\,a^3b^2n-30x^na^4b-3a^5}{6x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a+b*x^n)^5,x)
```

output
```
(2*x**(5*n)*b**5 + 15*x**(4*n)*a*b**4 + 60*x**(3*n)*a**2*b**3 + 60*x**(2*n
)*log(x)*a**3*b**2*n - 30*x**n*a**4*b - 3*a**5)/(6*x**(2*n)*n)
```

3.414 $\int x^{-1-3n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int x^{-1-3n}(a+bx^n)^5\,dx=-\frac{a^5x^{-3n}}{3n}-\frac{5a^4bx^{-2n}}{2n}-\frac{10a^3b^2x^{-n}}{n}+\frac{5ab^4x^n}{n}+\frac{b^5x^{2n}}{2n}+10a^2b^3\log(x)$$

output
```
-1/3*a^5/n/(x^(3*n))-5/2*a^4*b/n/(x^(2*n))-10*a^3*b^2/n/(x^n)+5*a*b^4*x^n/
n+1/2*b^5*x^(2*n)/n+10*a^2*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1-3n}(a+bx^n)^5\,dx=\frac{x^{-3n}(-2a^5-15a^4bx^n-60a^3b^2x^{2n}+30ab^4x^{4n}+3b^5x^{5n})}{6n}+\frac{10a^2b^3\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 3*n)*(a + b*x^n)^5,x]
```

output
```
(-2*a^5 - 15*a^4*b*x^n - 60*a^3*b^2*x^(2*n) + 30*a*b^4*x^(4*n) + 3*b^5*x^(
5*n))/(6*n*x^(3*n)) + (10*a^2*b^3*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-4n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^5x^{-4n}+5a^4bx^{-3n}+10a^3b^2x^{-2n}+10a^2b^3x^{-n}+b^5x^n+5ab^4\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^5x^{-3n}-\frac{5}{2}a^4bx^{-2n}-10a^3b^2x^{-n}+10a^2b^3\log\left(x^n\right)+5ab^4x^n+\frac{1}{2}b^5x^{2n}}{n}$$

input
```
Int[x^(-1 - 3*n)*(a + b*x^n)^5,x]
```

output
```
(-1/3*a^5/x^(3*n) - (5*a^4*b)/(2*x^(2*n)) - (10*a^3*b^2)/x^n + 5*a*b^4*x^n
 + (b^5*x^(2*n))/2 + 10*a^2*b^3*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result	size
risch	$10a^2b^3\ln(x)+\frac{b^5x^{2n}}{2n}+\frac{5a\,b^4x^n}{n}-\frac{10a^3b^2x^{-n}}{n}-\frac{5a^4b\,x^{-2n}}{2n}-\frac{a^5x^{-3n}}{3n}$	80
norman	$\left(10a^2b^3\ln(x)\,e^{3n\ln(x)}-\frac{a^5}{3n}+\frac{b^5e^{5n\ln(x)}}{2n}+\frac{5a\,b^4e^{4n\ln(x)}}{n}-\frac{10a^3b^2e^{2n\ln(x)}}{n}-\frac{5a^4b\,e^{n\ln(x)}}{2n}\right)e^{-3n\ln(x)}$	98

input
```
int(x^(-1-3*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
10*a^2*b^3*ln(x)+1/2*b^5/n*(x^n)^2+5*a*b^4*x^n/n-10*a^3*b^2/n/(x^n)-5/2*a^
4*b/n/(x^n)^2-1/3*a^5/n/(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1-3n}(a+bx^n)^5\,dx$$
$$=\frac{60\,a^2b^3nx^{3\,n}\log(x)+3\,b^5x^{5\,n}+30\,ab^4x^{4\,n}-60\,a^3b^2x^{2\,n}-15\,a^4bx^n-2\,a^5}{6\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/6*(60*a^2*b^3*n*x^(3*n)*log(x) + 3*b^5*x^(5*n) + 30*a*b^4*x^(4*n) - 60*a
^3*b^2*x^(2*n) - 15*a^4*b*x^n - 2*a^5)/(n*x^(3*n))
```

Sympy [A] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int x^{-1-3n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5x^{-3n}}{3n} - \frac{5a^4bx^{-2n}}{2n} - \frac{10a^3b^2x^{-n}}{n} + \frac{10a^2b^3\log(x^n)}{n} + \frac{5ab^4x^n}{n} + \frac{b^5x^{2n}}{2n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-3*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5/(3*n*x**(3*n)) - 5*a**4*b/(2*n*x**(2*n)) - 10*a**3*b**2/(
n*x**n) + 10*a**2*b**3*log(x**n)/n + 5*a*b**4*x**n/n + b**5*x**(2*n)/(2*n)
, Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int x^{-1-3n}(a+bx^n)^5\,dx = 10\,a^2b^3\log(x) + \frac{b^5x^{2\,n}}{2\,n} + \frac{5\,ab^4x^n}{n} - \frac{a^5}{3\,nx^{3\,n}} - \frac{5\,a^4b}{2\,nx^{2\,n}} - \frac{10\,a^3b^2}{nx^n}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
10*a^2*b^3*log(x) + 1/2*b^5*x^(2*n)/n + 5*a*b^4*x^n/n - 1/3*a^5/(n*x^(3*n)
) - 5/2*a^4*b/(n*x^(2*n)) - 10*a^3*b^2/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1-3n}(a+bx^n)^5\,dx = \frac{60\,a^2b^3nx^{3\,n}\log(x)+3\,b^5x^{5\,n}+30\,ab^4x^{4\,n}-60\,a^3b^2x^{2\,n}-15\,a^4bx^n-2\,a^5}{6\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
1/6*(60*a^2*b^3*n*x^(3*n)*log(x) + 3*b^5*x^(5*n) + 30*a*b^4*x^(4*n) - 60*a
^3*b^2*x^(2*n) - 15*a^4*b*x^n - 2*a^5)/(n*x^(3*n))
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int x^{-1-3n}(a+bx^n)^5\,dx = \frac{b^5\,x^{2\,n}}{2\,n}-\frac{a^5}{3\,n\,x^{3\,n}}+10\,a^2\,b^3\,\ln(x)-\frac{10\,a^3\,b^2}{n\,x^n}+\frac{5\,a\,b^4\,x^n}{n}-\frac{5\,a^4\,b}{2\,n\,x^{2\,n}}$$

input
```
int((a + b*x^n)^5/x^(3*n + 1),x)
```

output
```
(b^5*x^(2*n))/(2*n) - a^5/(3*n*x^(3*n)) + 10*a^2*b^3*log(x) - (10*a^3*b^2)
/(n*x^n) + (5*a*b^4*x^n)/n - (5*a^4*b)/(2*n*x^(2*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1-3n}(a+bx^n)^5\,dx = \frac{3x^{5n}b^5+30x^{4n}a\,b^4+60x^{3n}\log(x)\,a^2b^3n-60x^{2n}a^3b^2-15x^na^4b-2a^5}{6x^{3n}n}$$

input
```
int(x^(-1-3*n)*(a+b*x^n)^5,x)
```

output
```
(3*x**(5*n)*b**5 + 30*x**(4*n)*a*b**4 + 60*x**(3*n)*log(x)*a**2*b**3*n - 6
0*x**(2*n)*a**3*b**2 - 15*x**n*a**4*b - 2*a**5)/(6*x**(3*n)*n)
```

3.415 $\int x^{-1-4n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int x^{-1-4n}(a+bx^n)^5\,dx=-\frac{a^5x^{-4n}}{4n}-\frac{5a^4bx^{-3n}}{3n}-\frac{5a^3b^2x^{-2n}}{n}-\frac{10a^2b^3x^{-n}}{n}+\frac{b^5x^n}{n}+5ab^4\log(x)$$

output
```
-1/4*a^5/n/(x^(4*n))-5/3*a^4*b/n/(x^(3*n))-5*a^3*b^2/n/(x^(2*n))-10*a^2*b^
3/n/(x^n)+b^5*x^n/n+5*a*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int x^{-1-4n}(a+bx^n)^5\,dx=\frac{x^{-4n}(-3a^5-20a^4bx^n-60a^3b^2x^{2n}-120a^2b^3x^{3n}+12b^5x^{5n})}{12n}+\frac{5ab^4\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 4*n)*(a + b*x^n)^5,x]
```

output
```
(-3*a^5 - 20*a^4*b*x^n - 60*a^3*b^2*x^(2*n) - 120*a^2*b^3*x^(3*n) + 12*b^5
*x^(5*n))/(12*n*x^(4*n)) + (5*a*b^4*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-4n-1}(a+bx^n)^5\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-5n}(bx^n+a)^5\,dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(a^5x^{-5n}+5a^4bx^{-4n}+10a^3b^2x^{-3n}+10a^2b^3x^{-2n}+5ab^4x^{-n}+b^5\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{1}{4}a^5x^{-4n}-\frac{5}{3}a^4bx^{-3n}-5a^3b^2x^{-2n}-10a^2b^3x^{-n}+5ab^4\log\left(x^n\right)+b^5x^n}{n}$$

input
```
Int[x^(-1 - 4*n)*(a + b*x^n)^5,x]
```

output
```
(-1/4*a^5/x^(4*n) - (5*a^4*b)/(3*x^(3*n)) - (5*a^3*b^2)/x^(2*n) - (10*a^2*
b^3)/x^n + b^5*x^n + 5*a*b^4*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
risch	$5a\,b^{4}\ln\left(x\right)+\frac{b^{5}x^{n}}{n}-\frac{10a^{2}b^{3}x^{-n}}{n}-\frac{5a^{3}b^{2}x^{-2n}}{n}-\frac{5a^{4}b\,x^{-3n}}{3n}-\frac{a^{5}x^{-4n}}{4n}$	79
norman	$\left(\frac{b^{5}\mathrm{e}^{5n\ln\left(x\right)}}{n}+5a\,b^{4}\ln\left(x\right)\mathrm{e}^{4n\ln\left(x\right)}-\frac{a^{5}}{4n}-\frac{10a^{2}b^{3}\mathrm{e}^{3n\ln\left(x\right)}}{n}-\frac{5a^{3}b^{2}\mathrm{e}^{2n\ln\left(x\right)}}{n}-\frac{5a^{4}b\,\mathrm{e}^{n\ln\left(x\right)}}{3n}\right)\mathrm{e}^{-4n\ln\left(x\right)}$	97

input
```
int(x^(-4*n-1)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
5*a*b^4*ln(x)+b^5*x^n/n-10*a^2*b^3/n/(x^n)-5*a^3*b^2/n/(x^n)^2-5/3*a^4*b/n
/(x^n)^3-1/4*a^5/n/(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int x^{-1-4n}(a+bx^n)^5\,dx = \frac{60\,ab^{4}nx^{4\,n}\log\left(x\right)+12\,b^{5}x^{5\,n}-120\,a^{2}b^{3}x^{3\,n}-60\,a^{3}b^{2}x^{2\,n}-20\,a^{4}bx^{n}-3\,a^{5}}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/12*(60*a*b^4*n*x^(4*n)*log(x) + 12*b^5*x^(5*n) - 120*a^2*b^3*x^(3*n) - 6
0*a^3*b^2*x^(2*n) - 20*a^4*b*x^n - 3*a^5)/(n*x^(4*n))
```

Sympy [A] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int x^{-1-4n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5x^{-4n}}{4n} - \frac{5a^4bx^{-3n}}{3n} - \frac{5a^3b^2x^{-2n}}{n} - \frac{10a^2b^3x^{-n}}{n} + \frac{5ab^4\log(x^n)}{n} + \frac{b^5x^n}{n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-4*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5/(4*n*x**(4*n)) - 5*a**4*b/(3*n*x**(3*n)) - 5*a**3*b**2/(n
*x**(2*n)) - 10*a**2*b**3/(n*x**n) + 5*a*b**4*log(x**n)/n + b**5*x**n/n, N
e(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int x^{-1-4n}(a+bx^n)^5\,dx = 5\,ab^4\log(x) + \frac{b^5x^n}{n} - \frac{a^5}{4\,nx^{4\,n}} - \frac{5\,a^4b}{3\,nx^{3\,n}} - \frac{5\,a^3b^2}{nx^{2\,n}} - \frac{10\,a^2b^3}{nx^n}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
5*a*b^4*log(x) + b^5*x^n/n - 1/4*a^5/(n*x^(4*n)) - 5/3*a^4*b/(n*x^(3*n)) -
 5*a^3*b^2/(n*x^(2*n)) - 10*a^2*b^3/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int x^{-1-4n}(a+bx^n)^5\,dx = \frac{60\,ab^4nx^{4\,n}\log(x)+12\,b^5x^{5\,n}-120\,a^2b^3x^{3\,n}-60\,a^3b^2x^{2\,n}-20\,a^4bx^n-3\,a^5}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
1/12*(60*a*b^4*n*x^(4*n)*log(x) + 12*b^5*x^(5*n) - 120*a^2*b^3*x^(3*n) - 6
0*a^3*b^2*x^(2*n) - 20*a^4*b*x^n - 3*a^5)/(n*x^(4*n))
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int x^{-1-4n}(a+bx^n)^5\,dx = \frac{b^5\,x^n}{n}+5\,a\,b^4\,\ln(x)-\frac{a^5}{4\,n\,x^{4\,n}}-\frac{10\,a^2\,b^3}{n\,x^n}-\frac{5\,a^3\,b^2}{n\,x^{2\,n}}-\frac{5\,a^4\,b}{3\,n\,x^{3\,n}}$$

input
```
int((a + b*x^n)^5/x^(4*n + 1),x)
```

output
```
(b^5*x^n)/n + 5*a*b^4*log(x) - a^5/(4*n*x^(4*n)) - (10*a^2*b^3)/(n*x^n) -
(5*a^3*b^2)/(n*x^(2*n)) - (5*a^4*b)/(3*n*x^(3*n))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int x^{-1-4n}(a+bx^n)^5\,dx = \frac{12x^{5n}b^5+60x^{4n}\log(x)\,a\,b^4n-120x^{3n}a^2b^3-60x^{2n}a^3b^2-20x^na^4b-3a^5}{12x^{4n}n}$$

input
```
int(x^(-1-4*n)*(a+b*x^n)^5,x)
```

output
```
(12*x**(5*n)*b**5 + 60*x**(4*n)*log(x)*a*b**4*n - 120*x**(3*n)*a**2*b**3 -
60*x**(2*n)*a**3*b**2 - 20*x**n*a**4*b - 3*a**5)/(12*x**(4*n)*n)
```

3.416 $\int x^{-1-5n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int x^{-1-5n}(a+bx^n)^5\,dx=-\frac{a^5x^{-5n}}{5n}-\frac{5a^4bx^{-4n}}{4n}-\frac{10a^3b^2x^{-3n}}{3n}-\frac{5a^2b^3x^{-2n}}{n}-\frac{5ab^4x^{-n}}{n}+b^5\log(x)$$

output
```
-1/5*a^5/n/(x^(5*n))-5/4*a^4*b/n/(x^(4*n))-10/3*a^3*b^2/n/(x^(3*n))-5*a^2*
b^3/n/(x^(2*n))-5*a*b^4/n/(x^n)+b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int x^{-1-5n}(a+bx^n)^5\,dx=-\frac{ax^{-5n}(12a^4+75a^3bx^n+200a^2b^2x^{2n}+300ab^3x^{3n}+300b^4x^{4n})}{60n}+\frac{b^5\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 5*n)*(a + b*x^n)^5,x]
```

output
```
-1/60*(a*(12*a^4 + 75*a^3*b*x^n + 200*a^2*b^2*x^(2*n) + 300*a*b^3*x^(3*n)
+ 300*b^4*x^(4*n)))/(n*x^(5*n)) + (b^5*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-5n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-6n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^5x^{-6n}+5a^4bx^{-5n}+10a^3b^2x^{-4n}+10a^2b^3x^{-3n}+5ab^4x^{-2n}+b^5x^{-n}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{5}a^5x^{-5n}-\frac{5}{4}a^4bx^{-4n}-\frac{10}{3}a^3b^2x^{-3n}-5a^2b^3x^{-2n}-5ab^4x^{-n}+b^5\log\left(x^n\right)}{n}$$

input
```
Int[x^(-1 - 5*n)*(a + b*x^n)^5,x]
```

output
```
(-1/5*a^5/x^(5*n) - (5*a^4*b)/(4*x^(4*n)) - (10*a^3*b^2)/(3*x^(3*n)) - (5*
a^2*b^3)/x^(2*n) - (5*a*b^4)/x^n + b^5*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result	size
risch	$b^5 \ln(x) - \frac{5 a b^4 x^{-n}}{n} - \frac{5 a^2 b^3 x^{-2n}}{n} - \frac{10 a^3 b^2 x^{-3n}}{3n} - \frac{5 a^4 b x^{-4n}}{4n} - \frac{a^5 x^{-5n}}{5n}$	81
norman	$\left(b^5 \ln(x) e^{5n \ln(x)} - \frac{a^5}{5n} - \frac{5 a b^4 e^{4n \ln(x)}}{n} - \frac{5 a^2 b^3 e^{3n \ln(x)}}{n} - \frac{10 a^3 b^2 e^{2n \ln(x)}}{3n} - \frac{5 a^4 b e^{n \ln(x)}}{4n}\right) e^{-5n \ln(x)}$	97

input
```
int(x^(-1-5*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
b^5*ln(x)-5*a*b^4/n/(x^n)-5*a^2*b^3/n/(x^n)^2-10/3*a^3*b^2/n/(x^n)^3-5/4*a
^4*b/n/(x^n)^4-1/5*a^5/n/(x^n)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int x^{-1-5n}(a+bx^n)^5 \, dx = \frac{60 b^5 n x^{5n} \log(x) - 300 a b^4 x^{4n} - 300 a^2 b^3 x^{3n} - 200 a^3 b^2 x^{2n} - 75 a^4 b x^n - 12 a^5}{60 n x^{5n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
1/60*(60*b^5*n*x^(5*n)*log(x) - 300*a*b^4*x^(4*n) - 300*a^2*b^3*x^(3*n) -
200*a^3*b^2*x^(2*n) - 75*a^4*b*x^n - 12*a^5)/(n*x^(5*n))
```

Sympy [A] (verification not implemented)

Time = 7.87 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int x^{-1-5n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5x^{-5n}}{5n} - \frac{5a^4bx^{-4n}}{4n} - \frac{10a^3b^2x^{-3n}}{3n} - \frac{5a^2b^3x^{-2n}}{n} - \frac{5ab^4x^{-n}}{n} + \frac{b^5\log(x^n)}{n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-5*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5/(5*n*x**(5*n)) - 5*a**4*b/(4*n*x**(4*n)) - 10*a**3*b**2/(
3*n*x**(3*n)) - 5*a**2*b**3/(n*x**(2*n)) - 5*a*b**4/(n*x**n) + b**5*log(x*
*n)/n, Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int x^{-1-5n}(a+bx^n)^5\,dx = b^5\log(x) - \frac{a^5}{5\,nx^{5\,n}} - \frac{5\,a^4b}{4\,nx^{4\,n}} - \frac{10\,a^3b^2}{3\,nx^{3\,n}} - \frac{5\,a^2b^3}{nx^{2\,n}} - \frac{5\,ab^4}{nx^n}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
b^5*log(x) - 1/5*a^5/(n*x^(5*n)) - 5/4*a^4*b/(n*x^(4*n)) - 10/3*a^3*b^2/(n
*x^(3*n)) - 5*a^2*b^3/(n*x^(2*n)) - 5*a*b^4/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int x^{-1-5n}(a+bx^n)^5\,dx$$
$$=\frac{60\,b^5nx^{5\,n}\log{(x)}-300\,ab^4x^{4\,n}-300\,a^2b^3x^{3\,n}-200\,a^3b^2x^{2\,n}-75\,a^4bx^n-12\,a^5}{60\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
1/60*(60*b^5*n*x^(5*n)*log(x) - 300*a*b^4*x^(4*n) - 300*a^2*b^3*x^(3*n) -
200*a^3*b^2*x^(2*n) - 75*a^4*b*x^n - 12*a^5)/(n*x^(5*n))
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int x^{-1-5n}(a+bx^n)^5\,dx=b^5\,\ln{(x)}-\frac{a^5}{5\,n\,x^{5\,n}}-\frac{5\,a^2\,b^3}{n\,x^{2\,n}}-\frac{10\,a^3\,b^2}{3\,n\,x^{3\,n}}-\frac{5\,a\,b^4}{n\,x^n}-\frac{5\,a^4\,b}{4\,n\,x^{4\,n}}$$

input
```
int((a + b*x^n)^5/x^(5*n + 1),x)
```

output
```
b^5*log(x) - a^5/(5*n*x^(5*n)) - (5*a^2*b^3)/(n*x^(2*n)) - (10*a^3*b^2)/(3
*n*x^(3*n)) - (5*a*b^4)/(n*x^n) - (5*a^4*b)/(4*n*x^(4*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int x^{-1-5n}(a+bx^n)^5\,dx$$
$$=\frac{60x^{5n}\log(x)\,b^5n-300x^{4n}a\,b^4-300x^{3n}a^2b^3-200x^{2n}a^3b^2-75x^na^4b-12a^5}{60x^{5n}n}$$

input
```
int(x^(-1-5*n)*(a+b*x^n)^5,x)
```

output
```
(60*x**(5*n)*log(x)*b**5*n - 300*x**(4*n)*a*b**4 - 300*x**(3*n)*a**2*b**3
- 200*x**(2*n)*a**3*b**2 - 75*x**n*a**4*b - 12*a**5)/(60*x**(5*n)*n)
```

3.417 $\int x^{-1-6n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int x^{-1-6n}(a+bx^n)^5\,dx=-\frac{x^{-6n}(a+bx^n)^6}{6an}$$

output
```
-1/6*(a+b*x^n)^6/a/n/(x^(6*n))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 74 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.08

$$\int x^{-1-6n}(a+bx^n)^5\,dx =\frac{x^{-6n}(-a^5-6a^4bx^n-15a^3b^2x^{2n}-20a^2b^3x^{3n}-15ab^4x^{4n}-6b^5x^{5n})}{6n}$$

input
```
Integrate[x^(-1 - 6*n)*(a + b*x^n)^5,x]
```

output
```
(-a^5 - 6*a^4*b*x^n - 15*a^3*b^2*x^(2*n) - 20*a^2*b^3*x^(3*n) - 15*a*b^4*x
^(4*n) - 6*b^5*x^(5*n))/(6*n*x^(6*n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-6n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{x^{-6n}(a+bx^n)^6}{6an}$$

input
```
Int[x^(-1 - 6*n)*(a + b*x^n)^5,x]
```

output
```
-1/6*(a + b*x^n)^6/(a*n*x^(6*n))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(24) = 48$.

Time = 1.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.67

method	result	si
risch	$-\frac{b^5 x^{-n}}{n}-\frac{5a\,b^4 x^{-2n}}{2n}-\frac{10a^2b^3x^{-3n}}{3n}-\frac{5a^3b^2x^{-4n}}{2n}-\frac{a^4b\,x^{-5n}}{n}-\frac{a^5x^{-6n}}{6n}$	88
parallelrisch	$\frac{-6x\,x^{5n}x^{-1-6n}b^5-15x\,x^{4n}x^{-1-6n}a\,b^4-20x\,x^{3n}x^{-1-6n}a^2b^3-15x\,x^{2n}x^{-1-6n}a^3b^2-6x\,x^{n}x^{-1-6n}a^4b-x\,x^{-1-6n}a^5}{6n}$	11
orering	Expression too large to display	28

input
```
int(x^(-1-6*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
-b^5/n/(x^n)-5/2*a*b^4/n/(x^n)^2-10/3*a^2*b^3/n/(x^n)^3-5/2*a^3*b^2/n/(x^n
)^4-a^4*b/n/(x^n)^5-1/6*a^5/n/(x^n)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(24) = 48.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int x^{-1-6n}(a+bx^n)^5\,dx = -\frac{6\,b^5x^{5\,n}+15\,ab^4x^{4\,n}+20\,a^2b^3x^{3\,n}+15\,a^3b^2x^{2\,n}+6\,a^4bx^n+a^5}{6\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
-1/6*(6*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 20*a^2*b^3*x^(3*n) + 15*a^3*b^2*x
^(2*n) + 6*a^4*b*x^n + a^5)/(n*x^(6*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(19) = 38.

Time = 0.91 (sec) , antiderivative size = 146, normalized size of antiderivative = 6.08

$$\int x^{-1-6n}(a+bx^n)^5\,dx$$
$$= \begin{cases} -\frac{a^5xx^{-6n-1}}{6n} - \frac{a^4bxx^nx^{-6n-1}}{n} - \frac{5a^3b^2xx^{2n}x^{-6n-1}}{2n} - \frac{10a^2b^3xx^{3n}x^{-6n-1}}{3n} - \frac{5ab^4xx^{4n}x^{-6n-1}}{2n} - \frac{b^5xx^{5n}x^{-6n-1}}{n} & \text{for } n \neq \\ (a+b)^5\log(x) & \text{otherwis} \end{cases}$$

input
```
integrate(x**(-1-6*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5*x*x**(-6*n - 1)/(6*n) - a**4*b*x*x**n*x**(-6*n - 1)/n - 5
*a**3*b**2*x*x**(2*n)*x**(-6*n - 1)/(2*n) - 10*a**2*b**3*x*x**(3*n)*x**(-6
*n - 1)/(3*n) - 5*a*b**4*x*x**(4*n)*x**(-6*n - 1)/(2*n) - b**5*x*x**(5*n)*
x**(-6*n - 1)/n, Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int x^{-1-6n}(a+bx^n)^5\,dx = -\frac{a^5}{6\,nx^{6\,n}} - \frac{a^4b}{nx^{5\,n}} - \frac{5\,a^3b^2}{2\,nx^{4\,n}} - \frac{10\,a^2b^3}{3\,nx^{3\,n}} - \frac{5\,ab^4}{2\,nx^{2\,n}} - \frac{b^5}{nx^n}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
-1/6*a^5/(n*x^(6*n)) - a^4*b/(n*x^(5*n)) - 5/2*a^3*b^2/(n*x^(4*n)) - 10/3*
a^2*b^3/(n*x^(3*n)) - 5/2*a*b^4/(n*x^(2*n)) - b^5/(n*x^n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(24) = 48$.

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int x^{-1-6n}(a+bx^n)^5\,dx = -\frac{6\,b^5x^{5\,n} + 15\,ab^4x^{4\,n} + 20\,a^2b^3x^{3\,n} + 15\,a^3b^2x^{2\,n} + 6\,a^4bx^n + a^5}{6\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
-1/6*(6*b^5*x^(5*n) + 15*a*b^4*x^(4*n) + 20*a^2*b^3*x^(3*n) + 15*a^3*b^2*x
^(2*n) + 6*a^4*b*x^n + a^5)/(n*x^(6*n))
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int x^{-1-6n}(a+bx^n)^5\,dx = -\frac{a^5}{6\,n\,x^{6\,n}} - \frac{b^5}{n\,x^n} - \frac{10\,a^2\,b^3}{3\,n\,x^{3\,n}} - \frac{5\,a^3\,b^2}{2\,n\,x^{4\,n}} - \frac{5\,a\,b^4}{2\,n\,x^{2\,n}} - \frac{a^4\,b}{n\,x^{5\,n}}$$

input
```
int((a + b*x^n)^5/x^(6*n + 1),x)
```

output
```
- a^5/(6*n*x^(6*n)) - b^5/(n*x^n) - (10*a^2*b^3)/(3*n*x^(3*n)) - (5*a^3*b^
2)/(2*n*x^(4*n)) - (5*a*b^4)/(2*n*x^(2*n)) - (a^4*b)/(n*x^(5*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.08

$$\int x^{-1-6n}(a+bx^n)^5\,dx = \frac{-6x^{5n}b^5 - 15x^{4n}a\,b^4 - 20x^{3n}a^2b^3 - 15x^{2n}a^3b^2 - 6x^na^4b - a^5}{6x^{6n}n}$$

input
```
int(x^(-1-6*n)*(a+b*x^n)^5,x)
```

output
```
( - 6*x**(5*n)*b**5 - 15*x**(4*n)*a*b**4 - 20*x**(3*n)*a**2*b**3 - 15*x**(
2*n)*a**3*b**2 - 6*x**n*a**4*b - a**5)/(6*x**(6*n)*n)
```

3.418 $\int x^{-1-7n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int x^{-1-7n}(a+bx^n)^5\,dx=-\frac{x^{-7n}(a+bx^n)^6}{7an}+\frac{bx^{-6n}(a+bx^n)^6}{42a^2n}$$

output
```
-1/7*(a+b*x^n)^6/a/n/(x^(7*n))+1/42*b*(a+b*x^n)^6/a^2/n/(x^(6*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int x^{-1-7n}(a+bx^n)^5\,dx =\frac{x^{-7n}(-6a^5-35a^4bx^n-84a^3b^2x^{2n}-105a^2b^3x^{3n}-70ab^4x^{4n}-21b^5x^{5n})}{42n}$$

input
```
Integrate[x^(-1 - 7*n)*(a + b*x^n)^5,x]
```

output
```
(-6*a^5 - 35*a^4*b*x^n - 84*a^3*b^2*x^(2*n) - 105*a^2*b^3*x^(3*n) - 70*a*b
^4*x^(4*n) - 21*b^5*x^(5*n))/(42*n*x^(7*n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-7n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-8n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{b\int x^{-7n}(bx^n+a)^5dx^n}{7a}-\frac{x^{-7n}(a+bx^n)^6}{7a}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{bx^{-6n}(a+bx^n)^6}{42a^2}-\frac{x^{-7n}(a+bx^n)^6}{7a}}{n}$$

input
```
Int[x^(-1 - 7*n)*(a + b*x^n)^5,x]
```

output
```
(-1/7*(a + b*x^n)^6/(a*x^(7*n)) + (b*(a + b*x^n)^6)/(42*a^2*x^(6*n)))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.76

method	result
risch	$-\frac{b^{5} x^{-2 n}}{2 n}-\frac{5 a \,b^{4} x^{-3 n}}{3 n}-\frac{5 a^{2} b^{3} x^{-4 n}}{2 n}-\frac{2 a^{3} b^{2} x^{-5 n}}{n}-\frac{5 a^{4} b \,x^{-6 n}}{6 n}-\frac{a^{5} x^{-7 n}}{7 n}$
parallelrisch	$\frac{-21 x \,x^{5 n} x^{-1-7 n} b^{5}-70 x \,x^{4 n} x^{-1-7 n} a \,b^{4}-105 x \,x^{3 n} x^{-1-7 n} a^{2} b^{3}-84 x \,x^{2 n} x^{-1-7 n} a^{3} b^{2}-35 x \,x^{n} x^{-1-7 n} a^{4} b-6 x \,x^{-1-7 n} a^{5}}{42 n}$
orering	Expression too large to display

input
```
int(x^(-1-7*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^5/n/(x^n)^2-5/3*a*b^4/n/(x^n)^3-5/2*a^2*b^3/n/(x^n)^4-2*a^3*b^2/n/(
x^n)^5-5/6*a^4*b/n/(x^n)^6-1/7*a^5/n/(x^n)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int x^{-1-7n}(a+bx^n)^5\,dx = -\frac{21\,b^{5}x^{5\,n}+70\,ab^{4}x^{4\,n}+105\,a^{2}b^{3}x^{3\,n}+84\,a^{3}b^{2}x^{2\,n}+35\,a^{4}bx^{n}+6\,a^{5}}{42\,nx^{7\,n}}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
-1/42*(21*b^5*x^(5*n) + 70*a*b^4*x^(4*n) + 105*a^2*b^3*x^(3*n) + 84*a^3*b^
2*x^(2*n) + 35*a^4*b*x^n + 6*a^5)/(n*x^(7*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(39) = 78$.

Time = 0.89 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.00

$$\int x^{-1-7n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5xx^{-7n-1}}{7n} - \frac{5a^4bxx^nx^{-7n-1}}{6n} - \frac{2a^3b^2xx^{2n}x^{-7n-1}}{n} - \frac{5a^2b^3xx^{3n}x^{-7n-1}}{2n} - \frac{5ab^4xx^{4n}x^{-7n-1}}{3n} - \frac{b^5xx^{5n}x^{-7n-1}}{2n} & \text{for } n \neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-7*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5*x*x**(-7*n - 1)/(7*n) - 5*a**4*b*x*x**n*x**(-7*n - 1)/(6*
n) - 2*a**3*b**2*x*x**(2*n)*x**(-7*n - 1)/n - 5*a**2*b**3*x*x**(3*n)*x**(-
7*n - 1)/(2*n) - 5*a*b**4*x*x**(4*n)*x**(-7*n - 1)/(3*n) - b**5*x*x**(5*n)
*x**(-7*n - 1)/(2*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.98

$$\int x^{-1-7n}(a+bx^n)^5\,dx = -\frac{a^5}{7\,nx^{7\,n}} - \frac{5\,a^4b}{6\,nx^{6\,n}} - \frac{2\,a^3b^2}{nx^{5\,n}} - \frac{5\,a^2b^3}{2\,nx^{4\,n}} - \frac{5\,ab^4}{3\,nx^{3\,n}} - \frac{b^5}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
-1/7*a^5/(n*x^(7*n)) - 5/6*a^4*b/(n*x^(6*n)) - 2*a^3*b^2/(n*x^(5*n)) - 5/2
*a^2*b^3/(n*x^(4*n)) - 5/3*a*b^4/(n*x^(3*n)) - 1/2*b^5/(n*x^(2*n))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int x^{-1-7n}(a+bx^n)^5\,dx = -\frac{21\,b^5x^{5\,n}+70\,ab^4x^{4\,n}+105\,a^2b^3x^{3\,n}+84\,a^3b^2x^{2\,n}+35\,a^4bx^n+6\,a^5}{42\,nx^{7\,n}}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
-1/42*(21*b^5*x^(5*n) + 70*a*b^4*x^(4*n) + 105*a^2*b^3*x^(3*n) + 84*a^3*b^
2*x^(2*n) + 35*a^4*b*x^n + 6*a^5)/(n*x^(7*n))
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.98

$$\int x^{-1-7n}(a+bx^n)^5\,dx = -\frac{a^5}{7\,n\,x^{7\,n}}-\frac{b^5}{2\,n\,x^{2\,n}}-\frac{5\,a^2\,b^3}{2\,n\,x^{4\,n}}-\frac{2\,a^3\,b^2}{n\,x^{5\,n}}-\frac{5\,a\,b^4}{3\,n\,x^{3\,n}}-\frac{5\,a^4\,b}{6\,n\,x^{6\,n}}$$

input
```
int((a + b*x^n)^5/x^(7*n + 1),x)
```

output
```
- a^5/(7*n*x^(7*n)) - b^5/(2*n*x^(2*n)) - (5*a^2*b^3)/(2*n*x^(4*n)) - (2*a
^3*b^2)/(n*x^(5*n)) - (5*a*b^4)/(3*n*x^(3*n)) - (5*a^4*b)/(6*n*x^(6*n))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int x^{-1-7n}(a+bx^n)^5\,dx = \frac{-21x^{5n}b^5-70x^{4n}a\,b^4-105x^{3n}a^2b^3-84x^{2n}a^3b^2-35x^na^4b-6a^5}{42x^{7n}n}$$

input
```
int(x^(-1-7*n)*(a+b*x^n)^5,x)
```

output
```
( - 21*x**(5*n)*b**5 - 70*x**(4*n)*a*b**4 - 105*x**(3*n)*a**2*b**3 - 84*x*
*(2*n)*a**3*b**2 - 35*x**n*a**4*b - 6*a**5)/(42*x**(7*n)*n)
```

3.419 $\int x^{-1-8n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int x^{-1-8n}(a+bx^n)^5\,dx=-\frac{x^{-8n}(a+bx^n)^6}{8an}+\frac{bx^{-7n}(a+bx^n)^6}{28a^2n}-\frac{b^2x^{-6n}(a+bx^n)^6}{168a^3n}$$

output
```
-1/8*(a+b*x^n)^6/a/n/(x^(8*n))+1/28*b*(a+b*x^n)^6/a^2/n/(x^(7*n))-1/168*b^
2*(a+b*x^n)^6/a^3/n/(x^(6*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int x^{-1-8n}(a+bx^n)^5\,dx =\frac{x^{-8n}(-21a^5-120a^4bx^n-280a^3b^2x^{2n}-336a^2b^3x^{3n}-210ab^4x^{4n}-56b^5x^{5n})}{168n}$$

input
```
Integrate[x^(-1 - 8*n)*(a + b*x^n)^5,x]
```

output
```
(-21*a^5 - 120*a^4*b*x^n - 280*a^3*b^2*x^(2*n) - 336*a^2*b^3*x^(3*n) - 210
*a*b^4*x^(4*n) - 56*b^5*x^(5*n))/(168*n*x^(8*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-8n-1}(a+bx^n)^5\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-9n}(bx^n+a)^5\,dx^n}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{b\int x^{-8n}(bx^n+a)^5dx^n}{4a}-\frac{x^{-8n}(a+bx^n)^6}{8a}}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{b\left(-\frac{b\int x^{-7n}(bx^n+a)^5dx^n}{7a}-\frac{x^{-7n}(a+bx^n)^6}{7a}\right)}{4a}-\frac{x^{-8n}(a+bx^n)^6}{8a}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{-\frac{b\left(\frac{bx^{-6n}(a+bx^n)^6}{42a^2}-\frac{x^{-7n}(a+bx^n)^6}{7a}\right)}{4a}-\frac{x^{-8n}(a+bx^n)^6}{8a}}{n}$$

input

```
Int[x^(-1 - 8*n)*(a + b*x^n)^5,x]
```

output

```
(-1/8*(a + b*x^n)^6/(a*x^(8*n)) - (b*(-1/7*(a + b*x^n)^6/(a*x^(7*n)) + (b*
(a + b*x^n)^6)/(42*a^2*x^(6*n))))/(4*a))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{b^{5} x^{-3 n}}{3 n}-\frac{5 a\, b^{4} x^{-4 n}}{4 n}-\frac{2 a^{2} b^{3} x^{-5 n}}{n}-\frac{5 a^{3} b^{2} x^{-6 n}}{3 n}-\frac{5 a^{4} b\, x^{-7 n}}{7 n}-\frac{a^{5} x^{-8 n}}{8 n}$
parallelrisch	$\frac{-56 x\, x^{5 n} x^{-1-8 n} b^{5}-210 x\, x^{4 n} x^{-1-8 n} a\, b^{4}-336 x\, x^{3 n} x^{-1-8 n} a^{2} b^{3}-280 x\, x^{2 n} x^{-1-8 n} a^{3} b^{2}-120 x\, x^{n} x^{-1-8 n} a^{4} b-21 x\, x^{-1-8 n}}{168 n}$
orering	Expression too large to display

input
```
int(x^(-1-8*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^5/n/(x^n)^3-5/4*a*b^4/n/(x^n)^4-2*a^2*b^3/n/(x^n)^5-5/3*a^3*b^2/n/(
x^n)^6-5/7*a^4*b/n/(x^n)^7-1/8*a^5/n/(x^n)^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int x^{-1-8n}(a+bx^n)^5\,dx = -\frac{56\,b^5x^{5\,n}+210\,ab^4x^{4\,n}+336\,a^2b^3x^{3\,n}+280\,a^3b^2x^{2\,n}+120\,a^4bx^n+21\,a^5}{168\,nx^{8\,n}}$$

input
```
integrate(x^(-1-8*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
-1/168*(56*b^5*x^(5*n) + 210*a*b^4*x^(4*n) + 336*a^2*b^3*x^(3*n) + 280*a^3
*b^2*x^(2*n) + 120*a^4*b*x^n + 21*a^5)/(n*x^(8*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(63) = 126.

Time = 0.93 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int x^{-1-8n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5xx^{-8n-1}}{8n} - \frac{5a^4bxx^nx^{-8n-1}}{7n} - \frac{5a^3b^2xx^{2n}x^{-8n-1}}{3n} - \frac{2a^2b^3xx^{3n}x^{-8n-1}}{n} - \frac{5ab^4xx^{4n}x^{-8n-1}}{4n} - \frac{b^5xx^{5n}x^{-8n-1}}{3n} & \text{for } n \neq 0 \\ (a+b)^5\log{(x)} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-8*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5*x*x**(-8*n - 1)/(8*n) - 5*a**4*b*x*x**n*x**(-8*n - 1)/(7*
n) - 5*a**3*b**2*x*x**(2*n)*x**(-8*n - 1)/(3*n) - 2*a**2*b**3*x*x**(3*n)*x
**(-8*n - 1)/n - 5*a*b**4*x*x**(4*n)*x**(-8*n - 1)/(4*n) - b**5*x*x**(5*n)
*x**(-8*n - 1)/(3*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int x^{-1-8n}(a+bx^n)^5\,dx = -\frac{a^5}{8\,nx^{8\,n}} - \frac{5\,a^4b}{7\,nx^{7\,n}} - \frac{5\,a^3b^2}{3\,nx^{6\,n}} - \frac{2\,a^2b^3}{nx^{5\,n}} - \frac{5\,ab^4}{4\,nx^{4\,n}} - \frac{b^5}{3\,nx^{3\,n}}$$

input
```
integrate(x^(-1-8*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
-1/8*a^5/(n*x^(8*n)) - 5/7*a^4*b/(n*x^(7*n)) - 5/3*a^3*b^2/(n*x^(6*n)) - 2
*a^2*b^3/(n*x^(5*n)) - 5/4*a*b^4/(n*x^(4*n)) - 1/3*b^5/(n*x^(3*n))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int x^{-1-8n}(a+bx^n)^5\,dx = -\frac{56\,b^5x^{5\,n} + 210\,ab^4x^{4\,n} + 336\,a^2b^3x^{3\,n} + 280\,a^3b^2x^{2\,n} + 120\,a^4bx^n + 21\,a^5}{168\,nx^{8\,n}}$$

input
```
integrate(x^(-1-8*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
-1/168*(56*b^5*x^(5*n) + 210*a*b^4*x^(4*n) + 336*a^2*b^3*x^(3*n) + 280*a^3
*b^2*x^(2*n) + 120*a^4*b*x^n + 21*a^5)/(n*x^(8*n))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int x^{-1-8n}(a+bx^n)^5\,dx = -\frac{a^5}{8\,n\,x^{8\,n}} - \frac{b^5}{3\,n\,x^{3\,n}} - \frac{2\,a^2\,b^3}{n\,x^{5\,n}} - \frac{5\,a^3\,b^2}{3\,n\,x^{6\,n}} - \frac{5\,a\,b^4}{4\,n\,x^{4\,n}} - \frac{5\,a^4\,b}{7\,n\,x^{7\,n}}$$

input
```
int((a + b*x^n)^5/x^(8*n + 1),x)
```

output
```
- a^5/(8*n*x^(8*n)) - b^5/(3*n*x^(3*n)) - (2*a^2*b^3)/(n*x^(5*n)) - (5*a^3
*b^2)/(3*n*x^(6*n)) - (5*a*b^4)/(4*n*x^(4*n)) - (5*a^4*b)/(7*n*x^(7*n))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int x^{-1-8n}(a+bx^n)^5\,dx = \frac{-56x^{5n}b^5-210x^{4n}a\,b^4-336x^{3n}a^2b^3-280x^{2n}a^3b^2-120x^na^4b-21a^5}{168x^{8n}n}$$

input
```
int(x^(-1-8*n)*(a+b*x^n)^5,x)
```

output
```
( - 56*x**(5*n)*b**5 - 210*x**(4*n)*a*b**4 - 336*x**(3*n)*a**2*b**3 - 280*
x**(2*n)*a**3*b**2 - 120*x**n*a**4*b - 21*a**5)/(168*x**(8*n)*n)
```

3.420 $\int x^{-1-9n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int x^{-1-9n}(a+bx^n)^5\,dx=-\frac{a^5x^{-9n}}{9n}-\frac{5a^4bx^{-8n}}{8n}-\frac{10a^3b^2x^{-7n}}{7n}-\frac{5a^2b^3x^{-6n}}{3n}-\frac{ab^4x^{-5n}}{n}-\frac{b^5x^{-4n}}{4n}$$

output
```
-1/9*a^5/n/(x^(9*n))-5/8*a^4*b/n/(x^(8*n))-10/7*a^3*b^2/n/(x^(7*n))-5/3*a^
2*b^3/n/(x^(6*n))-a*b^4/n/(x^(5*n))-1/4*b^5/n/(x^(4*n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int x^{-1-9n}(a+bx^n)^5\,dx=\frac{x^{-9n}(-56a^5-315a^4bx^n-720a^3b^2x^{2n}-840a^2b^3x^{3n}-504ab^4x^{4n}-126b^5x^{5n})}{504n}$$

input
```
Integrate[x^(-1 - 9*n)*(a + b*x^n)^5,x]
```

output
```
(-56*a^5 - 315*a^4*b*x^n - 720*a^3*b^2*x^(2*n) - 840*a^2*b^3*x^(3*n) - 504
*a*b^4*x^(4*n) - 126*b^5*x^(5*n))/(504*n*x^(9*n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-9n-1}(a+bx^n)^5\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-10n}(bx^n+a)^5\,dx^n}{n}$$
$$\Big\downarrow \text{53}$$
$$\frac{\int \left(a^5x^{-10n}+5a^4bx^{-9n}+10a^3b^2x^{-8n}+10a^2b^3x^{-7n}+5ab^4x^{-6n}+b^5x^{-5n}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{1}{9}a^5x^{-9n}-\frac{5}{8}a^4bx^{-8n}-\frac{10}{7}a^3b^2x^{-7n}-\frac{5}{3}a^2b^3x^{-6n}-ab^4x^{-5n}-\frac{1}{4}b^5x^{-4n}}{n}$$

input
```
Int[x^(-1 - 9*n)*(a + b*x^n)^5,x]
```

output
```
(-1/9*a^5/x^(9*n) - (5*a^4*b)/(8*x^(8*n)) - (10*a^3*b^2)/(7*x^(7*n)) - (5*
a^2*b^3)/(3*x^(6*n)) - (a*b^4)/x^(5*n) - b^5/(4*x^(4*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{b^{5} x^{-4 n}}{4 n}-\frac{a\,b^{4} x^{-5 n}}{n}-\frac{5 a^{2} b^{3} x^{-6 n}}{3 n}-\frac{10 a^{3} b^{2} x^{-7 n}}{7 n}-\frac{5 a^{4} b\,x^{-8 n}}{8 n}-\frac{a^{5} x^{-9 n}}{9 n}$
parallelrisch	$\frac{-126 x\,x^{5 n} x^{-1-9 n} b^{5}-504 x\,x^{4 n} x^{-1-9 n} a\,b^{4}-840 x\,x^{3 n} x^{-1-9 n} a^{2} b^{3}-720 x\,x^{2 n} x^{-1-9 n} a^{3} b^{2}-315 x\,x^{n} x^{-1-9 n} a^{4} b-56 x\,x^{-1-9}}{504 n}$
orering	Expression too large to display

input
```
int(x^(-1-9*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^5/n/(x^n)^4-a*b^4/n/(x^n)^5-5/3*a^2*b^3/n/(x^n)^6-10/7*a^3*b^2/n/(x
^n)^7-5/8*a^4*b/n/(x^n)^8-1/9*a^5/n/(x^n)^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int x^{-1-9n}(a+bx^n)^5\,dx = -\frac{126\,b^5x^{5\,n}+504\,ab^4x^{4\,n}+840\,a^2b^3x^{3\,n}+720\,a^3b^2x^{2\,n}+315\,a^4bx^n+56\,a^5}{504\,nx^{9\,n}}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
-1/504*(126*b^5*x^(5*n) + 504*a*b^4*x^(4*n) + 840*a^2*b^3*x^(3*n) + 720*a^
3*b^2*x^(2*n) + 315*a^4*b*x^n + 56*a^5)/(n*x^(9*n))
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int x^{-1-9n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5xx^{-9n-1}}{9n}-\frac{5a^4bxx^nx^{-9n-1}}{8n}-\frac{10a^3b^2xx^{2n}x^{-9n-1}}{7n}-\frac{5a^2b^3xx^{3n}x^{-9n-1}}{3n}-\frac{ab^4xx^{4n}x^{-9n-1}}{n}-\frac{b^5xx^{5n}x^{-9n-1}}{4n} & \text{for } n\neq 0 \\ (a+b)^5\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-9*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5*x*x**(-9*n - 1)/(9*n) - 5*a**4*b*x*x**n*x**(-9*n - 1)/(8*
n) - 10*a**3*b**2*x*x**(2*n)*x**(-9*n - 1)/(7*n) - 5*a**2*b**3*x*x**(3*n)*
x**(-9*n - 1)/(3*n) - a*b**4*x*x**(4*n)*x**(-9*n - 1)/n - b**5*x*x**(5*n)*
x**(-9*n - 1)/(4*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int x^{-1-9n}(a+bx^n)^5\,dx = -\frac{a^5}{9\,nx^{9\,n}} - \frac{5\,a^4b}{8\,nx^{8\,n}} - \frac{10\,a^3b^2}{7\,nx^{7\,n}} - \frac{5\,a^2b^3}{3\,nx^{6\,n}} - \frac{ab^4}{nx^{5\,n}} - \frac{b^5}{4\,nx^{4\,n}}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
-1/9*a^5/(n*x^(9*n)) - 5/8*a^4*b/(n*x^(8*n)) - 10/7*a^3*b^2/(n*x^(7*n)) -
5/3*a^2*b^3/(n*x^(6*n)) - a*b^4/(n*x^(5*n)) - 1/4*b^5/(n*x^(4*n))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int x^{-1-9n}(a+bx^n)^5\,dx = -\frac{126\,b^5x^{5\,n} + 504\,ab^4x^{4\,n} + 840\,a^2b^3x^{3\,n} + 720\,a^3b^2x^{2\,n} + 315\,a^4bx^n + 56\,a^5}{504\,nx^{9\,n}}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
-1/504*(126*b^5*x^(5*n) + 504*a*b^4*x^(4*n) + 840*a^2*b^3*x^(3*n) + 720*a^
3*b^2*x^(2*n) + 315*a^4*b*x^n + 56*a^5)/(n*x^(9*n))
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02

$$\int x^{-1-9n}(a+bx^n)^5\,dx = -\frac{a^5}{9\,n\,x^{9\,n}} - \frac{b^5}{4\,n\,x^{4\,n}} - \frac{5\,a^2\,b^3}{3\,n\,x^{6\,n}} - \frac{10\,a^3\,b^2}{7\,n\,x^{7\,n}} - \frac{a\,b^4}{n\,x^{5\,n}} - \frac{5\,a^4\,b}{8\,n\,x^{8\,n}}$$

input
```
int((a + b*x^n)^5/x^(9*n + 1),x)
```

output
```
- a^5/(9*n*x^(9*n)) - b^5/(4*n*x^(4*n)) - (5*a^2*b^3)/(3*n*x^(6*n)) - (10*
a^3*b^2)/(7*n*x^(7*n)) - (a*b^4)/(n*x^(5*n)) - (5*a^4*b)/(8*n*x^(8*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int x^{-1-9n}(a+bx^n)^5\,dx$$
$$=\frac{-126x^{5n}b^5-504x^{4n}a\,b^4-840x^{3n}a^2b^3-720x^{2n}a^3b^2-315x^na^4b-56a^5}{504x^{9n}n}$$

input
```
int(x^(-1-9*n)*(a+b*x^n)^5,x)
```

output
```
( - 126*x**(5*n)*b**5 - 504*x**(4*n)*a*b**4 - 840*x**(3*n)*a**2*b**3 - 720
*x**(2*n)*a**3*b**2 - 315*x**n*a**4*b - 56*a**5)/(504*x**(9*n)*n)
```

3.421 $\int x^{-1-10n}(a+bx^n)^5\,dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int x^{-1-10n}(a+bx^n)^5\,dx=-\frac{a^5x^{-10n}}{10n}-\frac{5a^4bx^{-9n}}{9n}-\frac{5a^3b^2x^{-8n}}{4n}-\frac{10a^2b^3x^{-7n}}{7n}-\frac{5ab^4x^{-6n}}{6n}-\frac{b^5x^{-5n}}{5n}$$

output
```
-1/10*a^5/n/(x^(10*n))-5/9*a^4*b/n/(x^(9*n))-5/4*a^3*b^2/n/(x^(8*n))-10/7*
a^2*b^3/n/(x^(7*n))-5/6*a*b^4/n/(x^(6*n))-1/5*b^5/n/(x^(5*n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

$$\int x^{-1-10n}(a+bx^n)^5\,dx=\frac{x^{-10n}(-126a^5-700a^4bx^n-1575a^3b^2x^{2n}-1800a^2b^3x^{3n}-1050ab^4x^{4n}-252b^5x^{5n})}{1260n}$$

input
```
Integrate[x^(-1 - 10*n)*(a + b*x^n)^5,x]
```

output
```
(-126*a^5 - 700*a^4*b*x^n - 1575*a^3*b^2*x^(2*n) - 1800*a^2*b^3*x^(3*n) -
1050*a*b^4*x^(4*n) - 252*b^5*x^(5*n))/(1260*n*x^(10*n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-10n-1}(a+bx^n)^5\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-11n}(bx^n+a)^5\,dx^n}{n}$$
$$\Big\downarrow \text{53}$$
$$\frac{\int \left(a^5x^{-11n}+5a^4bx^{-10n}+10a^3b^2x^{-9n}+10a^2b^3x^{-8n}+5ab^4x^{-7n}+b^5x^{-6n}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{1}{10}a^5x^{-10n}-\frac{5}{9}a^4bx^{-9n}-\frac{5}{4}a^3b^2x^{-8n}-\frac{10}{7}a^2b^3x^{-7n}-\frac{5}{6}ab^4x^{-6n}-\frac{1}{5}b^5x^{-5n}}{n}$$

input
```
Int[x^(-1 - 10*n)*(a + b*x^n)^5,x]
```

output
```
(-1/10*a^5/x^(10*n) - (5*a^4*b)/(9*x^(9*n)) - (5*a^3*b^2)/(4*x^(8*n)) - (1
0*a^2*b^3)/(7*x^(7*n)) - (5*a*b^4)/(6*x^(6*n)) - b^5/(5*x^(5*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{b^5x^{-5n}}{5n}-\frac{5a\,b^4x^{-6n}}{6n}-\frac{10a^2b^3x^{-7n}}{7n}-\frac{5a^3b^2x^{-8n}}{4n}-\frac{5a^4b\,x^{-9n}}{9n}-\frac{a^5x^{-10n}}{10n}$
parallelrisch	$\frac{-252x\,x^{5n}x^{-1-10n}b^5-1050x\,x^{4n}x^{-1-10n}a\,b^4-1800x\,x^{3n}x^{-1-10n}a^2b^3-1575x\,x^{2n}x^{-1-10n}a^3b^2-700x\,x^{n}x^{-1-10n}a^4b-1}{1260n}$
orering	Expression too large to display

input
```
int(x^(-1-10*n)*(a+b*x^n)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^5/n/(x^n)^5-5/6*a*b^4/n/(x^n)^6-10/7*a^2*b^3/n/(x^n)^7-5/4*a^3*b^2/
n/(x^n)^8-5/9*a^4*b/n/(x^n)^9-1/10*a^5/n/(x^n)^10
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

$$\int x^{-1-10n}(a+bx^n)^5\,dx = -\frac{252\,b^5x^{5\,n}+1050\,ab^4x^{4\,n}+1800\,a^2b^3x^{3\,n}+1575\,a^3b^2x^{2\,n}+700\,a^4bx^n+126\,a^5}{1260\,nx^{10\,n}}$$

input
```
integrate(x^(-1-10*n)*(a+b*x^n)^5,x, algorithm="fricas")
```

output
```
-1/1260*(252*b^5*x^(5*n) + 1050*a*b^4*x^(4*n) + 1800*a^2*b^3*x^(3*n) + 157
5*a^3*b^2*x^(2*n) + 700*a^4*b*x^n + 126*a^5)/(n*x^(10*n))
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.53

$$\int x^{-1-10n}(a+bx^n)^5\,dx = \begin{cases} -\frac{a^5xx^{-10n-1}}{10n} - \frac{5a^4bxx^nx^{-10n-1}}{9n} - \frac{5a^3b^2xx^{2n}x^{-10n-1}}{4n} - \frac{10a^2b^3xx^{3n}x^{-10n-1}}{7n} - \frac{5ab^4xx^{4n}x^{-10n-1}}{6n} - \frac{b^5xx^{5n}x^{-10n-1}}{5n} & \text{for} \\ (a+b)^5\log(x) & \text{oth} \end{cases}$$

input
```
integrate(x**(-1-10*n)*(a+b*x**n)**5,x)
```

output
```
Piecewise((-a**5*x*x**(-10*n - 1)/(10*n) - 5*a**4*b*x*x**n*x**(-10*n - 1)/
(9*n) - 5*a**3*b**2*x*x**(2*n)*x**(-10*n - 1)/(4*n) - 10*a**2*b**3*x*x**(3
*n)*x**(-10*n - 1)/(7*n) - 5*a*b**4*x*x**(4*n)*x**(-10*n - 1)/(6*n) - b**5
*x*x**(5*n)*x**(-10*n - 1)/(5*n), Ne(n, 0)), ((a + b)**5*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x^{-1-10n}(a+bx^n)^5\,dx = -\frac{a^5}{10\,nx^{10\,n}} - \frac{5\,a^4b}{9\,nx^{9\,n}} - \frac{5\,a^3b^2}{4\,nx^{8\,n}} - \frac{10\,a^2b^3}{7\,nx^{7\,n}} - \frac{5\,ab^4}{6\,nx^{6\,n}} - \frac{b^5}{5\,nx^{5\,n}}$$

input
```
integrate(x^(-1-10*n)*(a+b*x^n)^5,x, algorithm="maxima")
```

output
```
-1/10*a^5/(n*x^(10*n)) - 5/9*a^4*b/(n*x^(9*n)) - 5/4*a^3*b^2/(n*x^(8*n)) -
 10/7*a^2*b^3/(n*x^(7*n)) - 5/6*a*b^4/(n*x^(6*n)) - 1/5*b^5/(n*x^(5*n))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

$$\int x^{-1-10n}(a+bx^n)^5\,dx$$
$$= -\frac{252\,b^5x^{5\,n} + 1050\,ab^4x^{4\,n} + 1800\,a^2b^3x^{3\,n} + 1575\,a^3b^2x^{2\,n} + 700\,a^4bx^n + 126\,a^5}{1260\,nx^{10\,n}}$$

input
```
integrate(x^(-1-10*n)*(a+b*x^n)^5,x, algorithm="giac")
```

output
```
-1/1260*(252*b^5*x^(5*n) + 1050*a*b^4*x^(4*n) + 1800*a^2*b^3*x^(3*n) + 157
5*a^3*b^2*x^(2*n) + 700*a^4*b*x^n + 126*a^5)/(n*x^(10*n))
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x^{-1-10n}(a+bx^n)^5\,dx = -\frac{a^5}{10\,n\,x^{10\,n}} - \frac{b^5}{5\,n\,x^{5\,n}} - \frac{10\,a^2\,b^3}{7\,n\,x^{7\,n}} - \frac{5\,a^3\,b^2}{4\,n\,x^{8\,n}} - \frac{5\,a\,b^4}{6\,n\,x^{6\,n}} - \frac{5\,a^4\,b}{9\,n\,x^{9\,n}}$$

input
```
int((a + b*x^n)^5/x^(10*n + 1),x)
```

output
```
- a^5/(10*n*x^(10*n)) - b^5/(5*n*x^(5*n)) - (10*a^2*b^3)/(7*n*x^(7*n)) - (
5*a^3*b^2)/(4*n*x^(8*n)) - (5*a*b^4)/(6*n*x^(6*n)) - (5*a^4*b)/(9*n*x^(9*n
))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

$$\int x^{-1-10n}(a+bx^n)^5\,dx = \frac{-252x^{5n}b^5-1050x^{4n}a\,b^4-1800x^{3n}a^2b^3-1575x^{2n}a^3b^2-700x^na^4b-126a^5}{1260x^{10n}n}$$

input
```
int(x^(-1-10*n)*(a+b*x^n)^5,x)
```

output
```
( - 252*x**(5*n)*b**5 - 1050*x**(4*n)*a*b**4 - 1800*x**(3*n)*a**2*b**3 - 1
575*x**(2*n)*a**3*b**2 - 700*x**n*a**4*b - 126*a**5)/(1260*x**(10*n)*n)
```

3.422 $\int x^{-1+9n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 151

$$\int x^{-1+9n}(a+bx^n)^8\,dx=\frac{a^8x^{9n}}{9n}+\frac{4a^7bx^{10n}}{5n}+\frac{28a^6b^2x^{11n}}{11n}+\frac{14a^5b^3x^{12n}}{3n}+\frac{70a^4b^4x^{13n}}{13n}+\frac{4a^3b^5x^{14n}}{n}+\frac{28a^2b^6x^{15n}}{15n}+\frac{ab^7x^{16n}}{2n}+\frac{b^8x^{17n}}{17n}$$

output
```
1/9*a^8*x^(9*n)/n+4/5*a^7*b*x^(10*n)/n+28/11*a^6*b^2*x^(11*n)/n+14/3*a^5*b
^3*x^(12*n)/n+70/13*a^4*b^4*x^(13*n)/n+4*a^3*b^5*x^(14*n)/n+28/15*a^2*b^6*
x^(15*n)/n+1/2*a*b^7*x^(16*n)/n+1/17*b^8*x^(17*n)/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1+9n}(a+bx^n)^8\,dx=\frac{x^{9n}(24310a^8+175032a^7bx^n+556920a^6b^2x^{2n}+1021020a^5b^3x^{3n}+1178100a^4b^4x^{4n}+875160a^3b^5x^{5n}+\cdots}{218790n}$$

input
```
Integrate[x^(-1 + 9*n)*(a + b*x^n)^8,x]
```

output
```
(x^(9*n)*(24310*a^8 + 175032*a^7*b*x^n + 556920*a^6*b^2*x^(2*n) + 1021020*
a^5*b^3*x^(3*n) + 1178100*a^4*b^4*x^(4*n) + 875160*a^3*b^5*x^(5*n) + 40840
8*a^2*b^6*x^(6*n) + 109395*a*b^7*x^(7*n) + 12870*b^8*x^(8*n)))/(218790*n)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{9n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{8n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^8x^{8n}+8a^7bx^{9n}+28a^6b^2x^{10n}+56a^5b^3x^{11n}+70a^4b^4x^{12n}+56a^3b^5x^{13n}+28a^2b^6x^{14n}+8ab^7x^{15n}+b^8x^{16n}\right)}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{9}a^8x^{9n}+\frac{4}{5}a^7bx^{10n}+\frac{28}{11}a^6b^2x^{11n}+\frac{14}{3}a^5b^3x^{12n}+\frac{70}{13}a^4b^4x^{13n}+4a^3b^5x^{14n}+\frac{28}{15}a^2b^6x^{15n}+\frac{1}{2}ab^7x^{16n}+\frac{1}{17}b^8x^{17n}}{n}$$

input
```
Int[x^(-1 + 9*n)*(a + b*x^n)^8,x]
```

output
```
((a^8*x^(9*n))/9 + (4*a^7*b*x^(10*n))/5 + (28*a^6*b^2*x^(11*n))/11 + (14*a
^5*b^3*x^(12*n))/3 + (70*a^4*b^4*x^(13*n))/13 + 4*a^3*b^5*x^(14*n) + (28*a
^2*b^6*x^(15*n))/15 + (a*b^7*x^(16*n))/2 + (b^8*x^(17*n))/17)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.53 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

method	result
risch	$\frac{a^8 x^{9n}}{9n}+\frac{4a^7 b\,x^{10n}}{5n}+\frac{28a^6 b^2 x^{11n}}{11n}+\frac{14a^5 b^3 x^{12n}}{3n}+\frac{70a^4 b^4 x^{13n}}{13n}+\frac{4a^3 b^5 x^{14n}}{n}+\frac{28a^2 b^6 x^{15n}}{15n}+\frac{a\,b^7 x^{16n}}{2n}+\frac{b^8 x^{1}}{17}$
parallelrisch	$\frac{12870x\,x^{8n} x^{-1+9n} b^8+109395x\,x^{7n} x^{-1+9n} a\,b^7+408408x\,x^{6n} x^{-1+9n} a^2 b^6+875160x\,x^{5n} x^{-1+9n} a^3 b^5+1178100x\,x^{4n} x^{-1+9}}{218790n}$
orering	Expression too large to display

input
```
int(x^(-1+9*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/17*b^8/n*(x^n)^17+1/2*a*b^7/n*(x^n)^16+28/15*a^2*b^6/n*(x^n)^15+4*a^3*b^
5/n*(x^n)^14+70/13*a^4*b^4/n*(x^n)^13+14/3*a^5*b^3/n*(x^n)^12+28/11*a^6*b^
2/n*(x^n)^11+4/5*a^7*b/n*(x^n)^10+1/9*a^8/n*(x^n)^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1+9n}(a+bx^n)^8\,dx$$
$$=\frac{12870\,b^8x^{17\,n}+109395\,ab^7x^{16\,n}+408408\,a^2b^6x^{15\,n}+875160\,a^3b^5x^{14\,n}+1178100\,a^4b^4x^{13\,n}+1021020\,a^5b}{218790\,n}$$

input
```
integrate(x^(-1+9*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/218790*(12870*b^8*x^(17*n) + 109395*a*b^7*x^(16*n) + 408408*a^2*b^6*x^(1
5*n) + 875160*a^3*b^5*x^(14*n) + 1178100*a^4*b^4*x^(13*n) + 1021020*a^5*b^
3*x^(12*n) + 556920*a^6*b^2*x^(11*n) + 175032*a^7*b*x^(10*n) + 24310*a^8*x
^(9*n))/n
```

Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.40

$$\int x^{-1+9n}(a+bx^n)^8\,dx$$
$$=\begin{cases}\frac{a^8xx^{9n-1}}{9n}+\frac{4a^7bxx^nx^{9n-1}}{5n}+\frac{28a^6b^2xx^{2n}x^{9n-1}}{11n}+\frac{14a^5b^3xx^{3n}x^{9n-1}}{3n}+\frac{70a^4b^4xx^{4n}x^{9n-1}}{13n}+\frac{4a^3b^5xx^{5n}x^{9n-1}}{n}+\frac{28a^2b^6xx^{6n}x^9}{15n}\\(a+b)^8\log{(x)}\end{cases}$$

input
```
integrate(x**(-1+9*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(9*n - 1)/(9*n) + 4*a**7*b*x*x**n*x**(9*n - 1)/(5*n)
+ 28*a**6*b**2*x*x**(2*n)*x**(9*n - 1)/(11*n) + 14*a**5*b**3*x*x**(3*n)*x*
*(9*n - 1)/(3*n) + 70*a**4*b**4*x*x**(4*n)*x**(9*n - 1)/(13*n) + 4*a**3*b*
*5*x*x**(5*n)*x**(9*n - 1)/n + 28*a**2*b**6*x*x**(6*n)*x**(9*n - 1)/(15*n)
 + a*b**7*x*x**(7*n)*x**(9*n - 1)/(2*n) + b**8*x*x**(8*n)*x**(9*n - 1)/(17
*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int x^{-1+9n}(a+bx^n)^8\,dx=\frac{b^8x^{17\,n}}{17\,n}+\frac{ab^7x^{16\,n}}{2\,n}+\frac{28\,a^2b^6x^{15\,n}}{15\,n}+\frac{4\,a^3b^5x^{14\,n}}{n}+\frac{70\,a^4b^4x^{13\,n}}{13\,n}+\frac{14\,a^5b^3x^{12\,n}}{3\,n}+\frac{28\,a^6b^2x^{11\,n}}{11\,n}+\frac{4\,a^7bx^{10\,n}}{5\,n}+\frac{a^8x^{9\,n}}{9\,n}$$

input
```
integrate(x^(-1+9*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/17*b^8*x^(17*n)/n + 1/2*a*b^7*x^(16*n)/n + 28/15*a^2*b^6*x^(15*n)/n + 4*
a^3*b^5*x^(14*n)/n + 70/13*a^4*b^4*x^(13*n)/n + 14/3*a^5*b^3*x^(12*n)/n +
28/11*a^6*b^2*x^(11*n)/n + 4/5*a^7*b*x^(10*n)/n + 1/9*a^8*x^(9*n)/n
```

Giac [F]

$$\int x^{-1+9n}(a+bx^n)^8\,dx=\int (bx^n+a)^8x^{9\,n-1}\,dx$$

input
```
integrate(x^(-1+9*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(9*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int x^{-1+9n}(a+bx^n)^8\,dx=\frac{a^8\,x^{9\,n}}{9\,n}+\frac{b^8\,x^{17\,n}}{17\,n}+\frac{28\,a^6\,b^2\,x^{11\,n}}{11\,n}+\frac{14\,a^5\,b^3\,x^{12\,n}}{3\,n}+\frac{70\,a^4\,b^4\,x^{13\,n}}{13\,n}+\frac{4\,a^3\,b^5\,x^{14\,n}}{n}+\frac{28\,a^2\,b^6\,x^{15\,n}}{15\,n}+\frac{4\,a^7\,b\,x^{10\,n}}{5\,n}+\frac{a\,b^7\,x^{16\,n}}{2\,n}$$

input
```
int(x^(9*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(9*n))/(9*n) + (b^8*x^(17*n))/(17*n) + (28*a^6*b^2*x^(11*n))/(11*n)
 + (14*a^5*b^3*x^(12*n))/(3*n) + (70*a^4*b^4*x^(13*n))/(13*n) + (4*a^3*b^5
*x^(14*n))/n + (28*a^2*b^6*x^(15*n))/(15*n) + (4*a^7*b*x^(10*n))/(5*n) + (
a*b^7*x^(16*n))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int x^{-1+9n}(a+bx^n)^8\,dx$$
$$=\frac{x^{9n}(12870x^{8n}b^8+109395x^{7n}a\,b^7+408408x^{6n}a^2b^6+875160x^{5n}a^3b^5+1178100x^{4n}a^4b^4+1021020x^{3n}a^5b}{218790n}$$

input
```
int(x^(-1+9*n)*(a+b*x^n)^8,x)
```

output
```
(x**(9*n)*(12870*x**(8*n)*b**8 + 109395*x**(7*n)*a*b**7 + 408408*x**(6*n)*
a**2*b**6 + 875160*x**(5*n)*a**3*b**5 + 1178100*x**(4*n)*a**4*b**4 + 10210
20*x**(3*n)*a**5*b**3 + 556920*x**(2*n)*a**6*b**2 + 175032*x**n*a**7*b + 2
4310*a**8))/(218790*n)
```

3.423 $\int x^{-1+8n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 151

$$\int x^{-1+8n}(a+bx^n)^8\,dx=\frac{a^8x^{8n}}{8n}+\frac{8a^7bx^{9n}}{9n}+\frac{14a^6b^2x^{10n}}{5n}+\frac{56a^5b^3x^{11n}}{11n}+\frac{35a^4b^4x^{12n}}{6n}+\frac{56a^3b^5x^{13n}}{13n}+\frac{2a^2b^6x^{14n}}{n}+\frac{8ab^7x^{15n}}{15n}+\frac{b^8x^{16n}}{16n}$$

output
```
1/8*a^8*x^(8*n)/n+8/9*a^7*b*x^(9*n)/n+14/5*a^6*b^2*x^(10*n)/n+56/11*a^5*b^
3*x^(11*n)/n+35/6*a^4*b^4*x^(12*n)/n+56/13*a^3*b^5*x^(13*n)/n+2*a^2*b^6*x^
(14*n)/n+8/15*a*b^7*x^(15*n)/n+1/16*b^8*x^(16*n)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1+8n}(a+bx^n)^8\,dx$$
$$=\frac{x^{8n}(12870a^8+91520a^7bx^n+288288a^6b^2x^{2n}+524160a^5b^3x^{3n}+600600a^4b^4x^{4n}+443520a^3b^5x^{5n}+2059}{102960n}$$

input
```
Integrate[x^(-1 + 8*n)*(a + b*x^n)^8,x]
```

output
```
(x^(8*n)*(12870*a^8 + 91520*a^7*b*x^n + 288288*a^6*b^2*x^(2*n) + 524160*a^
5*b^3*x^(3*n) + 600600*a^4*b^4*x^(4*n) + 443520*a^3*b^5*x^(5*n) + 205920*a
^2*b^6*x^(6*n) + 54912*a*b^7*x^(7*n) + 6435*b^8*x^(8*n)))/(102960*n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{8n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{7n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^8x^{7n}+8a^7bx^{8n}+28a^6b^2x^{9n}+56a^5b^3x^{10n}+70a^4b^4x^{11n}+56a^3b^5x^{12n}+28a^2b^6x^{13n}+8ab^7x^{14n}+b^8x^{15n}\right) d}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}a^8x^{8n}+\frac{8}{9}a^7bx^{9n}+\frac{14}{5}a^6b^2x^{10n}+\frac{56}{11}a^5b^3x^{11n}+\frac{35}{6}a^4b^4x^{12n}+\frac{56}{13}a^3b^5x^{13n}+2a^2b^6x^{14n}+\frac{8}{15}ab^7x^{15n}+\frac{1}{16}b^8x^{16n}}{n}$$

input
```
Int[x^(-1 + 8*n)*(a + b*x^n)^8,x]
```

output
```
((a^8*x^(8*n))/8 + (8*a^7*b*x^(9*n))/9 + (14*a^6*b^2*x^(10*n))/5 + (56*a^5
*b^3*x^(11*n))/11 + (35*a^4*b^4*x^(12*n))/6 + (56*a^3*b^5*x^(13*n))/13 + 2
*a^2*b^6*x^(14*n) + (8*a*b^7*x^(15*n))/15 + (b^8*x^(16*n))/16)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.92 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

method	result
risch	$\frac{a^{8} x^{8 n}}{8 n}+\frac{8 a^{7} b\, x^{9 n}}{9 n}+\frac{14 a^{6} b^{2} x^{10 n}}{5 n}+\frac{56 a^{5} b^{3} x^{11 n}}{11 n}+\frac{35 a^{4} b^{4} x^{12 n}}{6 n}+\frac{56 a^{3} b^{5} x^{13 n}}{13 n}+\frac{2 a^{2} b^{6} x^{14 n}}{n}+\frac{8 a\, b^{7} x^{15 n}}{15 n}+\frac{b^{8} x^{1}}{16}$
parallelrisch	$\frac{6435 x\, x^{8 n} x^{-1+8 n} b^{8}+54912 x\, x^{7 n} x^{-1+8 n} a\, b^{7}+205920 x\, x^{6 n} x^{-1+8 n} a^{2} b^{6}+443520 x\, x^{5 n} x^{-1+8 n} a^{3} b^{5}+600600 x\, x^{4 n} x^{-1+8 n} a^{4}}{102960 n}$
orering	Expression too large to display

input
```
int(x^(-1+8*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/16*b^8/n*(x^n)^16+8/15*a*b^7/n*(x^n)^15+2*a^2*b^6/n*(x^n)^14+56/13*a^3*b
^5/n*(x^n)^13+35/6*a^4*b^4/n*(x^n)^12+56/11*a^5*b^3/n*(x^n)^11+14/5*a^6*b^
2/n*(x^n)^10+8/9*a^7*b/n*(x^n)^9+1/8*a^8/n*(x^n)^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1+8n}(a+bx^n)^8\,dx = \frac{6435\,b^8x^{16\,n}+54912\,ab^7x^{15\,n}+205920\,a^2b^6x^{14\,n}+443520\,a^3b^5x^{13\,n}+600600\,a^4b^4x^{12\,n}+524160\,a^5b^3x^{11\,n}}{102960\,n}$$

input
```
integrate(x^(-1+8*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/102960*(6435*b^8*x^(16*n) + 54912*a*b^7*x^(15*n) + 205920*a^2*b^6*x^(14*
n) + 443520*a^3*b^5*x^(13*n) + 600600*a^4*b^4*x^(12*n) + 524160*a^5*b^3*x^
(11*n) + 288288*a^6*b^2*x^(10*n) + 91520*a^7*b*x^(9*n) + 12870*a^8*x^(8*n)
)/n
```

Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.42

$$\int x^{-1+8n}(a+bx^n)^8\,dx = \begin{cases} \frac{a^8xx^{8n-1}}{8n}+\frac{8a^7bxx^nx^{8n-1}}{9n}+\frac{14a^6b^2xx^{2n}x^{8n-1}}{5n}+\frac{56a^5b^3xx^{3n}x^{8n-1}}{11n}+\frac{35a^4b^4xx^{4n}x^{8n-1}}{6n}+\frac{56a^3b^5xx^{5n}x^{8n-1}}{13n}+\frac{2a^2b^6xx^{6n}x^{8}}{n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+8*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(8*n - 1)/(8*n) + 8*a**7*b*x*x**n*x**(8*n - 1)/(9*n)
+ 14*a**6*b**2*x*x**(2*n)*x**(8*n - 1)/(5*n) + 56*a**5*b**3*x*x**(3*n)*x**
(8*n - 1)/(11*n) + 35*a**4*b**4*x*x**(4*n)*x**(8*n - 1)/(6*n) + 56*a**3*b*
*5*x*x**(5*n)*x**(8*n - 1)/(13*n) + 2*a**2*b**6*x*x**(6*n)*x**(8*n - 1)/n
+ 8*a*b**7*x*x**(7*n)*x**(8*n - 1)/(15*n) + b**8*x*x**(8*n)*x**(8*n - 1)/(
16*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int x^{-1+8n}(a+bx^n)^8\,dx = \frac{b^8x^{16\,n}}{16\,n} + \frac{8\,ab^7x^{15\,n}}{15\,n} + \frac{2\,a^2b^6x^{14\,n}}{n} + \frac{56\,a^3b^5x^{13\,n}}{13\,n} + \frac{35\,a^4b^4x^{12\,n}}{6\,n} + \frac{56\,a^5b^3x^{11\,n}}{11\,n} + \frac{14\,a^6b^2x^{10\,n}}{5\,n} + \frac{8\,a^7bx^{9\,n}}{9\,n} + \frac{a^8x^{8\,n}}{8\,n}$$

input
```
integrate(x^(-1+8*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/16*b^8*x^(16*n)/n + 8/15*a*b^7*x^(15*n)/n + 2*a^2*b^6*x^(14*n)/n + 56/13
*a^3*b^5*x^(13*n)/n + 35/6*a^4*b^4*x^(12*n)/n + 56/11*a^5*b^3*x^(11*n)/n +
 14/5*a^6*b^2*x^(10*n)/n + 8/9*a^7*b*x^(9*n)/n + 1/8*a^8*x^(8*n)/n
```

Giac [F]

$$\int x^{-1+8n}(a+bx^n)^8\,dx = \int (bx^n+a)^8x^{8\,n-1}\,dx$$

input
```
integrate(x^(-1+8*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(8*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int x^{-1+8n}(a+bx^n)^8\,dx = \frac{a^8\,x^{8\,n}}{8\,n} + \frac{b^8\,x^{16\,n}}{16\,n} + \frac{14\,a^6\,b^2\,x^{10\,n}}{5\,n} + \frac{56\,a^5\,b^3\,x^{11\,n}}{11\,n} + \frac{35\,a^4\,b^4\,x^{12\,n}}{6\,n} + \frac{56\,a^3\,b^5\,x^{13\,n}}{13\,n} + \frac{2\,a^2\,b^6\,x^{14\,n}}{n} + \frac{8\,a^7\,b\,x^{9\,n}}{9\,n} + \frac{8\,a\,b^7\,x^{15\,n}}{15\,n}$$

input
```
int(x^(8*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(8*n))/(8*n) + (b^8*x^(16*n))/(16*n) + (14*a^6*b^2*x^(10*n))/(5*n)
+ (56*a^5*b^3*x^(11*n))/(11*n) + (35*a^4*b^4*x^(12*n))/(6*n) + (56*a^3*b^5
*x^(13*n))/(13*n) + (2*a^2*b^6*x^(14*n))/n + (8*a^7*b*x^(9*n))/(9*n) + (8*
a*b^7*x^(15*n))/(15*n)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int x^{-1+8n}(a+bx^n)^8\,dx$$
$$=\frac{x^{8n}(6435x^{8n}b^8+54912x^{7n}a\,b^7+205920x^{6n}a^2b^6+443520x^{5n}a^3b^5+600600x^{4n}a^4b^4+524160x^{3n}a^5b^3+2}{102960n}$$

input
```
int(x^(-1+8*n)*(a+b*x^n)^8,x)
```

output
```
(x**(8*n)*(6435*x**(8*n)*b**8 + 54912*x**(7*n)*a*b**7 + 205920*x**(6*n)*a*
*2*b**6 + 443520*x**(5*n)*a**3*b**5 + 600600*x**(4*n)*a**4*b**4 + 524160*x
**(3*n)*a**5*b**3 + 288288*x**(2*n)*a**6*b**2 + 91520*x**n*a**7*b + 12870*
a**8))/(102960*n)
```

3.424 $\int x^{-1+7n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\int x^{-1+7n}(a+bx^n)^8\,dx=\frac{a^6(a+bx^n)^9}{9b^7n}-\frac{3a^5(a+bx^n)^{10}}{5b^7n}+\frac{15a^4(a+bx^n)^{11}}{11b^7n}-\frac{5a^3(a+bx^n)^{12}}{3b^7n}+\frac{15a^2(a+bx^n)^{13}}{13b^7n}-\frac{3a(a+bx^n)^{14}}{7b^7n}+\frac{(a+bx^n)^{15}}{15b^7n}$$

output
```
1/9*a^6*(a+b*x^n)^9/b^7/n-3/5*a^5*(a+b*x^n)^10/b^7/n+15/11*a^4*(a+b*x^n)^1
1/b^7/n-5/3*a^3*(a+b*x^n)^12/b^7/n+15/13*a^2*(a+b*x^n)^13/b^7/n-3/7*a*(a+b
*x^n)^14/b^7/n+1/15*(a+b*x^n)^15/b^7/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1+7n}(a+bx^n)^8\,dx=\frac{x^{7n}(6435a^8+45045a^7bx^n+140140a^6b^2x^{2n}+252252a^5b^3x^{3n}+286650a^4b^4x^{4n}+210210a^3b^5x^{5n}+97020}{45045n}$$

input
```
Integrate[x^(-1 + 7*n)*(a + b*x^n)^8,x]
```

output
```
(x^(7*n)*(6435*a^8 + 45045*a^7*b*x^n + 140140*a^6*b^2*x^(2*n) + 252252*a^5
*b^3*x^(3*n) + 286650*a^4*b^4*x^(4*n) + 210210*a^3*b^5*x^(5*n) + 97020*a^2
*b^6*x^(6*n) + 25740*a*b^7*x^(7*n) + 3003*b^8*x^(8*n)))/(45045*n)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{6n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^{14}}{b^6}-\frac{6a(bx^n+a)^{13}}{b^6}+\frac{15a^2(bx^n+a)^{12}}{b^6}-\frac{20a^3(bx^n+a)^{11}}{b^6}+\frac{15a^4(bx^n+a)^{10}}{b^6}-\frac{6a^5(bx^n+a)^9}{b^6}+\frac{a^6(bx^n+a)^8}{b^6}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^6(a+bx^n)^9}{9b^7}-\frac{3a^5(a+bx^n)^{10}}{5b^7}+\frac{15a^4(a+bx^n)^{11}}{11b^7}-\frac{5a^3(a+bx^n)^{12}}{3b^7}+\frac{15a^2(a+bx^n)^{13}}{13b^7}+\frac{(a+bx^n)^{15}}{15b^7}-\frac{3a(a+bx^n)^{14}}{7b^7}}{n}$$

input
```
Int[x^(-1 + 7*n)*(a + b*x^n)^8,x]
```

output
```
((a^6*(a + b*x^n)^9)/(9*b^7) - (3*a^5*(a + b*x^n)^10)/(5*b^7) + (15*a^4*(a
 + b*x^n)^11)/(11*b^7) - (5*a^3*(a + b*x^n)^12)/(3*b^7) + (15*a^2*(a + b*x
^n)^13)/(13*b^7) - (3*a*(a + b*x^n)^14)/(7*b^7) + (a + b*x^n)^15/(15*b^7))
/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

method	result
risch	$\frac{b^{8} x^{15 n}}{15 n}+\frac{4 a\, b^{7} x^{14 n}}{7 n}+\frac{28 a^{2} b^{6} x^{13 n}}{13 n}+\frac{14 a^{3} b^{5} x^{12 n}}{3 n}+\frac{70 a^{4} b^{4} x^{11 n}}{11 n}+\frac{28 a^{5} b^{3} x^{10 n}}{5 n}+\frac{28 a^{6} b^{2} x^{9 n}}{9 n}+\frac{a^{7} b\, x^{8 n}}{n}+\frac{a^{8} x^{7}}{7 n}$
parallelrisch	$\frac{3003 x\, x^{8 n} x^{-1+7 n} b^{8}+25740 x\, x^{7 n} x^{-1+7 n} a\, b^{7}+97020 x\, x^{6 n} x^{-1+7 n} a^{2} b^{6}+210210 x\, x^{5 n} x^{-1+7 n} a^{3} b^{5}+286650 x\, x^{4 n} x^{-1+7 n} a^{4} b}{45045 n}$
orering	Expression too large to display

input
```
int(x^(-1+7*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/15*b^8/n*(x^n)^15+4/7*a*b^7/n*(x^n)^14+28/13*a^2*b^6/n*(x^n)^13+14/3*a^3
*b^5/n*(x^n)^12+70/11*a^4*b^4/n*(x^n)^11+28/5*a^5*b^3/n*(x^n)^10+28/9*a^6*
b^2/n*(x^n)^9+a^7*b/n*(x^n)^8+1/7*a^8/n*(x^n)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1+7n}(a+bx^n)^8\,dx = \frac{3003\,b^8x^{15\,n}+25740\,ab^7x^{14\,n}+97020\,a^2b^6x^{13\,n}+210210\,a^3b^5x^{12\,n}+286650\,a^4b^4x^{11\,n}+252252\,a^5b^3x^{10\,n}}{45045\,n}$$

input
```
integrate(x^(-1+7*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/45045*(3003*b^8*x^(15*n) + 25740*a*b^7*x^(14*n) + 97020*a^2*b^6*x^(13*n)
 + 210210*a^3*b^5*x^(12*n) + 286650*a^4*b^4*x^(11*n) + 252252*a^5*b^3*x^(1
0*n) + 140140*a^6*b^2*x^(9*n) + 45045*a^7*b*x^(8*n) + 6435*a^8*x^(7*n))/n
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.41

$$\int x^{-1+7n}(a+bx^n)^8\,dx = \begin{cases} \frac{a^8xx^{7n-1}}{7n}+\frac{a^7bxx^nx^{7n-1}}{n}+\frac{28a^6b^2xx^{2n}x^{7n-1}}{9n}+\frac{28a^5b^3xx^{3n}x^{7n-1}}{5n}+\frac{70a^4b^4xx^{4n}x^{7n-1}}{11n}+\frac{14a^3b^5xx^{5n}x^{7n-1}}{3n}+\frac{28a^2b^6xx^{6n}x^{7}}{13n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+7*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(7*n - 1)/(7*n) + a**7*b*x*x**n*x**(7*n - 1)/n + 28*a
**6*b**2*x*x**(2*n)*x**(7*n - 1)/(9*n) + 28*a**5*b**3*x*x**(3*n)*x**(7*n -
 1)/(5*n) + 70*a**4*b**4*x*x**(4*n)*x**(7*n - 1)/(11*n) + 14*a**3*b**5*x*x
**(5*n)*x**(7*n - 1)/(3*n) + 28*a**2*b**6*x*x**(6*n)*x**(7*n - 1)/(13*n) +
 4*a*b**7*x*x**(7*n)*x**(7*n - 1)/(7*n) + b**8*x*x**(8*n)*x**(7*n - 1)/(15
*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int x^{-1+7n}(a+bx^n)^8\,dx = \frac{b^8x^{15\,n}}{15\,n} + \frac{4\,ab^7x^{14\,n}}{7\,n} + \frac{28\,a^2b^6x^{13\,n}}{13\,n} + \frac{14\,a^3b^5x^{12\,n}}{3\,n} + \frac{70\,a^4b^4x^{11\,n}}{11\,n} + \frac{28\,a^5b^3x^{10\,n}}{5\,n} + \frac{28\,a^6b^2x^{9\,n}}{9\,n} + \frac{a^7bx^{8\,n}}{n} + \frac{a^8x^{7\,n}}{7\,n}$$

input
```
integrate(x^(-1+7*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/15*b^8*x^(15*n)/n + 4/7*a*b^7*x^(14*n)/n + 28/13*a^2*b^6*x^(13*n)/n + 14
/3*a^3*b^5*x^(12*n)/n + 70/11*a^4*b^4*x^(11*n)/n + 28/5*a^5*b^3*x^(10*n)/n
 + 28/9*a^6*b^2*x^(9*n)/n + a^7*b*x^(8*n)/n + 1/7*a^8*x^(7*n)/n
```

Giac [F]

$$\int x^{-1+7n}(a+bx^n)^8\,dx = \int (bx^n+a)^8x^{7\,n-1}\,dx$$

input
```
integrate(x^(-1+7*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(7*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int x^{-1+7n}(a+bx^n)^8\,dx = \frac{a^8\,x^{7\,n}}{7\,n} + \frac{b^8\,x^{15\,n}}{15\,n} + \frac{28\,a^6\,b^2\,x^{9\,n}}{9\,n} + \frac{28\,a^5\,b^3\,x^{10\,n}}{5\,n} + \frac{70\,a^4\,b^4\,x^{11\,n}}{11\,n} + \frac{14\,a^3\,b^5\,x^{12\,n}}{3\,n} + \frac{28\,a^2\,b^6\,x^{13\,n}}{13\,n} + \frac{a^7\,b\,x^{8\,n}}{n} + \frac{4\,a\,b^7\,x^{14\,n}}{7\,n}$$

input
```
int(x^(7*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(7*n))/(7*n) + (b^8*x^(15*n))/(15*n) + (28*a^6*b^2*x^(9*n))/(9*n) +
 (28*a^5*b^3*x^(10*n))/(5*n) + (70*a^4*b^4*x^(11*n))/(11*n) + (14*a^3*b^5*
x^(12*n))/(3*n) + (28*a^2*b^6*x^(13*n))/(13*n) + (a^7*b*x^(8*n))/n + (4*a*
b^7*x^(14*n))/(7*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int x^{-1+7n}(a+bx^n)^8\,dx$$
$$=\frac{x^{7n}(3003x^{8n}b^8+25740x^{7n}a\,b^7+97020x^{6n}a^2b^6+210210x^{5n}a^3b^5+286650x^{4n}a^4b^4+252252x^{3n}a^5b^3+14}{45045n}$$

input
```
int(x^(-1+7*n)*(a+b*x^n)^8,x)
```

output
```
(x**(7*n)*(3003*x**(8*n)*b**8 + 25740*x**(7*n)*a*b**7 + 97020*x**(6*n)*a**
2*b**6 + 210210*x**(5*n)*a**3*b**5 + 286650*x**(4*n)*a**4*b**4 + 252252*x*
*(3*n)*a**5*b**3 + 140140*x**(2*n)*a**6*b**2 + 45045*x**n*a**7*b + 6435*a*
*8))/(45045*n)
```

3.425 $\int x^{-1+6n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 128

$$\int x^{-1+6n}(a+bx^n)^8\,dx=-\frac{a^5(a+bx^n)^9}{9b^6n}+\frac{a^4(a+bx^n)^{10}}{2b^6n}-\frac{10a^3(a+bx^n)^{11}}{11b^6n}+\frac{5a^2(a+bx^n)^{12}}{6b^6n}-\frac{5a(a+bx^n)^{13}}{13b^6n}+\frac{(a+bx^n)^{14}}{14b^6n}$$

output
```
-1/9*a^5*(a+b*x^n)^9/b^6/n+1/2*a^4*(a+b*x^n)^10/b^6/n-10/11*a^3*(a+b*x^n)^
11/b^6/n+5/6*a^2*(a+b*x^n)^12/b^6/n-5/13*a*(a+b*x^n)^13/b^6/n+1/14*(a+b*x^
n)^14/b^6/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int x^{-1+6n}(a+bx^n)^8\,dx=\frac{x^{6n}(3003a^8+20592a^7bx^n+63063a^6b^2x^{2n}+112112a^5b^3x^{3n}+126126a^4b^4x^{4n}+91728a^3b^5x^{5n}+42042a^2}{18018n}$$

input
```
Integrate[x^(-1 + 6*n)*(a + b*x^n)^8,x]
```

output
```
(x^(6*n)*(3003*a^8 + 20592*a^7*b*x^n + 63063*a^6*b^2*x^(2*n) + 112112*a^5*
b^3*x^(3*n) + 126126*a^4*b^4*x^(4*n) + 91728*a^3*b^5*x^(5*n) + 42042*a^2*b
^6*x^(6*n) + 11088*a*b^7*x^(7*n) + 1287*b^8*x^(8*n)))/(18018*n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{6n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{5n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^{13}}{b^5}-\frac{5a(bx^n+a)^{12}}{b^5}+\frac{10a^2(bx^n+a)^{11}}{b^5}-\frac{10a^3(bx^n+a)^{10}}{b^5}+\frac{5a^4(bx^n+a)^9}{b^5}-\frac{a^5(bx^n+a)^8}{b^5}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^5(a+bx^n)^9}{9b^6}+\frac{a^4(a+bx^n)^{10}}{2b^6}-\frac{10a^3(a+bx^n)^{11}}{11b^6}+\frac{5a^2(a+bx^n)^{12}}{6b^6}+\frac{(a+bx^n)^{14}}{14b^6}-\frac{5a(a+bx^n)^{13}}{13b^6}}{n}$$

input
```
Int[x^(-1 + 6*n)*(a + b*x^n)^8,x]
```

output
```
(-1/9*(a^5*(a + b*x^n)^9)/b^6 + (a^4*(a + b*x^n)^10)/(2*b^6) - (10*a^3*(a
+ b*x^n)^11)/(11*b^6) + (5*a^2*(a + b*x^n)^12)/(6*b^6) - (5*a*(a + b*x^n)^
13)/(13*b^6) + (a + b*x^n)^14/(14*b^6))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.63 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06

method	result
risch	$\frac{b^8 x^{14n}}{14n}+\frac{8a\,b^7 x^{13n}}{13n}+\frac{7a^2 b^6 x^{12n}}{3n}+\frac{56a^3 b^5 x^{11n}}{11n}+\frac{7a^4 b^4 x^{10n}}{n}+\frac{56a^5 b^3 x^{9n}}{9n}+\frac{7a^6 b^2 x^{8n}}{2n}+\frac{8a^7 b\,x^{7n}}{7n}+\frac{a^8 x^{6n}}{6n}$
parallelrisch	$\frac{1287x\,x^{8n}x^{-1+6n}b^8+11088x\,x^{7n}x^{-1+6n}a\,b^7+42042x\,x^{6n}x^{-1+6n}a^2b^6+91728x\,x^{5n}x^{-1+6n}a^3b^5+126126x\,x^{4n}x^{-1+6n}a^4b^4}{18018n}$
orering	Expression too large to display

input
```
int(x^(-1+6*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/14*b^8/n*(x^n)^14+8/13*a*b^7/n*(x^n)^13+7/3*a^2*b^6/n*(x^n)^12+56/11*a^3
*b^5/n*(x^n)^11+7*a^4*b^4/n*(x^n)^10+56/9*a^5*b^3/n*(x^n)^9+7/2*a^6*b^2/n*
(x^n)^8+8/7*a^7*b/n*(x^n)^7+1/6*a^8/n*(x^n)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int x^{-1+6n}(a+bx^n)^8\,dx = \frac{1287\,b^8x^{14\,n}+11088\,ab^7x^{13\,n}+42042\,a^2b^6x^{12\,n}+91728\,a^3b^5x^{11\,n}+126126\,a^4b^4x^{10\,n}+112112\,a^5b^3x^{9\,n}+}{18018\,n}$$

input
```
integrate(x^(-1+6*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/18018*(1287*b^8*x^(14*n) + 11088*a*b^7*x^(13*n) + 42042*a^2*b^6*x^(12*n)
 + 91728*a^3*b^5*x^(11*n) + 126126*a^4*b^4*x^(10*n) + 112112*a^5*b^3*x^(9*
n) + 63063*a^6*b^2*x^(8*n) + 20592*a^7*b*x^(7*n) + 3003*a^8*x^(6*n))/n
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.67

$$\int x^{-1+6n}(a+bx^n)^8\,dx = \begin{cases} \frac{a^8xx^{6n-1}}{6n}+\frac{8a^7bxx^nx^{6n-1}}{7n}+\frac{7a^6b^2xx^{2n}x^{6n-1}}{2n}+\frac{56a^5b^3xx^{3n}x^{6n-1}}{9n}+\frac{7a^4b^4xx^{4n}x^{6n-1}}{n}+\frac{56a^3b^5xx^{5n}x^{6n-1}}{11n}+\frac{7a^2b^6xx^{6n}x^{6n-}}{3n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+6*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(6*n - 1)/(6*n) + 8*a**7*b*x*x**n*x**(6*n - 1)/(7*n)
+ 7*a**6*b**2*x*x**(2*n)*x**(6*n - 1)/(2*n) + 56*a**5*b**3*x*x**(3*n)*x**(
6*n - 1)/(9*n) + 7*a**4*b**4*x*x**(4*n)*x**(6*n - 1)/n + 56*a**3*b**5*x*x*
*(5*n)*x**(6*n - 1)/(11*n) + 7*a**2*b**6*x*x**(6*n)*x**(6*n - 1)/(3*n) + 8
*a*b**7*x*x**(7*n)*x**(6*n - 1)/(13*n) + b**8*x*x**(8*n)*x**(6*n - 1)/(14*
n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int x^{-1+6n}(a+bx^n)^8\,dx = \frac{b^8x^{14\,n}}{14\,n} + \frac{8\,ab^7x^{13\,n}}{13\,n} + \frac{7\,a^2b^6x^{12\,n}}{3\,n} + \frac{56\,a^3b^5x^{11\,n}}{11\,n} + \frac{7\,a^4b^4x^{10\,n}}{n} + \frac{56\,a^5b^3x^{9\,n}}{9\,n} + \frac{7\,a^6b^2x^{8\,n}}{2\,n} + \frac{8\,a^7bx^{7\,n}}{7\,n} + \frac{a^8x^{6\,n}}{6\,n}$$

input
```
integrate(x^(-1+6*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/14*b^8*x^(14*n)/n + 8/13*a*b^7*x^(13*n)/n + 7/3*a^2*b^6*x^(12*n)/n + 56/
11*a^3*b^5*x^(11*n)/n + 7*a^4*b^4*x^(10*n)/n + 56/9*a^5*b^3*x^(9*n)/n + 7/
2*a^6*b^2*x^(8*n)/n + 8/7*a^7*b*x^(7*n)/n + 1/6*a^8*x^(6*n)/n
```

Giac [F]

$$\int x^{-1+6n}(a+bx^n)^8\,dx = \int (bx^n+a)^8x^{6\,n-1}\,dx$$

input
```
integrate(x^(-1+6*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(6*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int x^{-1+6n}(a+bx^n)^8\,dx = \frac{a^8\,x^{6\,n}}{6\,n} + \frac{b^8\,x^{14\,n}}{14\,n} + \frac{7\,a^6\,b^2\,x^{8\,n}}{2\,n} + \frac{56\,a^5\,b^3\,x^{9\,n}}{9\,n} + \frac{7\,a^4\,b^4\,x^{10\,n}}{n} + \frac{56\,a^3\,b^5\,x^{11\,n}}{11\,n} + \frac{7\,a^2\,b^6\,x^{12\,n}}{3\,n} + \frac{8\,a^7\,b\,x^{7\,n}}{7\,n} + \frac{8\,a\,b^7\,x^{13\,n}}{13\,n}$$

input
```
int(x^(6*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(6*n))/(6*n) + (b^8*x^(14*n))/(14*n) + (7*a^6*b^2*x^(8*n))/(2*n) +
(56*a^5*b^3*x^(9*n))/(9*n) + (7*a^4*b^4*x^(10*n))/n + (56*a^3*b^5*x^(11*n)
)/(11*n) + (7*a^2*b^6*x^(12*n))/(3*n) + (8*a^7*b*x^(7*n))/(7*n) + (8*a*b^7
*x^(13*n))/(13*n)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int x^{-1+6n}(a+bx^n)^8\,dx$$
$$=\frac{x^{6n}(1287x^{8n}b^8+11088x^{7n}a\,b^7+42042x^{6n}a^2b^6+91728x^{5n}a^3b^5+126126x^{4n}a^4b^4+112112x^{3n}a^5b^3+630}{18018n}$$

input
```
int(x^(-1+6*n)*(a+b*x^n)^8,x)
```

output
```
(x**(6*n)*(1287*x**(8*n)*b**8 + 11088*x**(7*n)*a*b**7 + 42042*x**(6*n)*a**
2*b**6 + 91728*x**(5*n)*a**3*b**5 + 126126*x**(4*n)*a**4*b**4 + 112112*x**
(3*n)*a**5*b**3 + 63063*x**(2*n)*a**6*b**2 + 20592*x**n*a**7*b + 3003*a**8
))/(18018*n)
```

3.426 $\int x^{-1+5n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 106

$$\int x^{-1+5n}(a+bx^n)^8\,dx=\frac{a^4(a+bx^n)^9}{9b^5n}-\frac{2a^3(a+bx^n)^{10}}{5b^5n}+\frac{6a^2(a+bx^n)^{11}}{11b^5n}-\frac{a(a+bx^n)^{12}}{3b^5n}+\frac{(a+bx^n)^{13}}{13b^5n}$$

output
```
1/9*a^4*(a+b*x^n)^9/b^5/n-2/5*a^3*(a+b*x^n)^10/b^5/n+6/11*a^2*(a+b*x^n)^11
/b^5/n-1/3*a*(a+b*x^n)^12/b^5/n+1/13*(a+b*x^n)^13/b^5/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int x^{-1+5n}(a+bx^n)^8\,dx=\frac{x^{5n}(1287a^8+8580a^7bx^n+25740a^6b^2x^{2n}+45045a^5b^3x^{3n}+50050a^4b^4x^{4n}+36036a^3b^5x^{5n}+16380a^2b^6x}{6435n}$$

input
```
Integrate[x^(-1 + 5*n)*(a + b*x^n)^8,x]
```

output
```
(x^(5*n)*(1287*a^8 + 8580*a^7*b*x^n + 25740*a^6*b^2*x^(2*n) + 45045*a^5*b^
3*x^(3*n) + 50050*a^4*b^4*x^(4*n) + 36036*a^3*b^5*x^(5*n) + 16380*a^2*b^6*
x^(6*n) + 4290*a*b^7*x^(7*n) + 495*b^8*x^(8*n)))/(6435*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{4n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^{12}}{b^4}-\frac{4a(bx^n+a)^{11}}{b^4}+\frac{6a^2(bx^n+a)^{10}}{b^4}-\frac{4a^3(bx^n+a)^9}{b^4}+\frac{a^4(bx^n+a)^8}{b^4}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^4(a+bx^n)^9}{9b^5}-\frac{2a^3(a+bx^n)^{10}}{5b^5}+\frac{6a^2(a+bx^n)^{11}}{11b^5}+\frac{(a+bx^n)^{13}}{13b^5}-\frac{a(a+bx^n)^{12}}{3b^5}}{n}$$

input
```
Int[x^(-1 + 5*n)*(a + b*x^n)^8,x]
```

output
```
((a^4*(a + b*x^n)^9)/(9*b^5) - (2*a^3*(a + b*x^n)^10)/(5*b^5) + (6*a^2*(a
+ b*x^n)^11)/(11*b^5) - (a*(a + b*x^n)^12)/(3*b^5) + (a + b*x^n)^13/(13*b^
5))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

method	result
risch	$\frac{b^8 x^{13n}}{13n}+\frac{2a\,b^7 x^{12n}}{3n}+\frac{28a^2b^6x^{11n}}{11n}+\frac{28a^3b^5x^{10n}}{5n}+\frac{70a^4b^4x^{9n}}{9n}+\frac{7a^5b^3x^{8n}}{n}+\frac{4a^6b^2x^{7n}}{n}+\frac{4a^7b\,x^{6n}}{3n}+\frac{a^8x^{5n}}{5n}$
parallelrisch	$\frac{495x\,x^{8n}x^{-1+5n}b^8+4290x\,x^{7n}x^{-1+5n}a\,b^7+16380x\,x^{6n}x^{-1+5n}a^2b^6+36036x\,x^{5n}x^{-1+5n}a^3b^5+50050x\,x^{4n}x^{-1+5n}a^4b^4+45}{6435n}$
orering	Expression too large to display

input
```
int(x^(-1+5*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^8/n*(x^n)^13+2/3*a*b^7/n*(x^n)^12+28/11*a^2*b^6/n*(x^n)^11+28/5*a^3
*b^5/n*(x^n)^10+70/9*a^4*b^4/n*(x^n)^9+7*a^5*b^3/n*(x^n)^8+4*a^6*b^2/n*(x^
n)^7+4/3*a^7*b/n*(x^n)^6+1/5*a^8/n*(x^n)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int x^{-1+5n}(a+bx^n)^8\,dx = \frac{495\,b^8x^{13\,n}+4290\,ab^7x^{12\,n}+16380\,a^2b^6x^{11\,n}+36036\,a^3b^5x^{10\,n}+50050\,a^4b^4x^{9\,n}+45045\,a^5b^3x^{8\,n}+2574\cdots}{6435\,n}$$

input
```
integrate(x^(-1+5*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/6435*(495*b^8*x^(13*n) + 4290*a*b^7*x^(12*n) + 16380*a^2*b^6*x^(11*n) +
36036*a^3*b^5*x^(10*n) + 50050*a^4*b^4*x^(9*n) + 45045*a^5*b^3*x^(8*n) + 2
5740*a^6*b^2*x^(7*n) + 8580*a^7*b*x^(6*n) + 1287*a^8*x^(5*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(90) = 180$.

Time = 2.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.00

$$\int x^{-1+5n}(a+bx^n)^8\,dx = \begin{cases} \frac{a^8xx^{5n-1}}{5n}+\frac{4a^7bxx^nx^{5n-1}}{3n}+\frac{4a^6b^2xx^{2n}x^{5n-1}}{n}+\frac{7a^5b^3xx^{3n}x^{5n-1}}{n}+\frac{70a^4b^4xx^{4n}x^{5n-1}}{9n}+\frac{28a^3b^5xx^{5n}x^{5n-1}}{5n}+\frac{28a^2b^6xx^{6n}x^{5n}\cdots}{11n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+5*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(5*n - 1)/(5*n) + 4*a**7*b*x*x**n*x**(5*n - 1)/(3*n)
+ 4*a**6*b**2*x*x**(2*n)*x**(5*n - 1)/n + 7*a**5*b**3*x*x**(3*n)*x**(5*n -
 1)/n + 70*a**4*b**4*x*x**(4*n)*x**(5*n - 1)/(9*n) + 28*a**3*b**5*x*x**(5*
n)*x**(5*n - 1)/(5*n) + 28*a**2*b**6*x*x**(6*n)*x**(5*n - 1)/(11*n) + 2*a*
b**7*x*x**(7*n)*x**(5*n - 1)/(3*n) + b**8*x*x**(8*n)*x**(5*n - 1)/(13*n),
Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27

$$\int x^{-1+5n}(a+bx^n)^8\,dx=\frac{b^8x^{13n}}{13n}+\frac{2ab^7x^{12n}}{3n}+\frac{28a^2b^6x^{11n}}{11n}+\frac{28a^3b^5x^{10n}}{5n}+\frac{70a^4b^4x^{9n}}{9n}+\frac{7a^5b^3x^{8n}}{n}+\frac{4a^6b^2x^{7n}}{n}+\frac{4a^7bx^{6n}}{3n}+\frac{a^8x^{5n}}{5n}$$

input
```
integrate(x^(-1+5*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/13*b^8*x^(13*n)/n + 2/3*a*b^7*x^(12*n)/n + 28/11*a^2*b^6*x^(11*n)/n + 28
/5*a^3*b^5*x^(10*n)/n + 70/9*a^4*b^4*x^(9*n)/n + 7*a^5*b^3*x^(8*n)/n + 4*a
^6*b^2*x^(7*n)/n + 4/3*a^7*b*x^(6*n)/n + 1/5*a^8*x^(5*n)/n
```

Giac [F]

$$\int x^{-1+5n}(a+bx^n)^8\,dx=\int (bx^n+a)^8x^{5n-1}\,dx$$

input
```
integrate(x^(-1+5*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(5*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27

$$\int x^{-1+5n}(a+bx^n)^8\,dx=\frac{a^8\,x^{5n}}{5n}+\frac{b^8\,x^{13n}}{13n}+\frac{4\,a^6\,b^2\,x^{7n}}{n}+\frac{7\,a^5\,b^3\,x^{8n}}{n}+\frac{70\,a^4\,b^4\,x^{9n}}{9n}+\frac{28\,a^3\,b^5\,x^{10n}}{5n}+\frac{28\,a^2\,b^6\,x^{11n}}{11n}+\frac{4\,a^7\,b\,x^{6n}}{3n}+\frac{2\,a\,b^7\,x^{12n}}{3n}$$

input
```
int(x^(5*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(5*n))/(5*n) + (b^8*x^(13*n))/(13*n) + (4*a^6*b^2*x^(7*n))/n + (7*a
^5*b^3*x^(8*n))/n + (70*a^4*b^4*x^(9*n))/(9*n) + (28*a^3*b^5*x^(10*n))/(5*
n) + (28*a^2*b^6*x^(11*n))/(11*n) + (4*a^7*b*x^(6*n))/(3*n) + (2*a*b^7*x^(
12*n))/(3*n)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05

$$\int x^{-1+5n}(a+bx^n)^8\,dx$$
$$= \frac{x^{5n}(495x^{8n}b^8+4290x^{7n}a\,b^7+16380x^{6n}a^2b^6+36036x^{5n}a^3b^5+50050x^{4n}a^4b^4+45045x^{3n}a^5b^3+25740x^2}{6435n}$$

input
```
int(x^(-1+5*n)*(a+b*x^n)^8,x)
```

output
```
(x**(5*n)*(495*x**(8*n)*b**8 + 4290*x**(7*n)*a*b**7 + 16380*x**(6*n)*a**2*
b**6 + 36036*x**(5*n)*a**3*b**5 + 50050*x**(4*n)*a**4*b**4 + 45045*x**(3*n
)*a**5*b**3 + 25740*x**(2*n)*a**6*b**2 + 8580*x**n*a**7*b + 1287*a**8))/(6
435*n)
```

3.427 $\int x^{-1+4n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int x^{-1+4n}(a+bx^n)^8\,dx=-\frac{a^3(a+bx^n)^9}{9b^4n}+\frac{3a^2(a+bx^n)^{10}}{10b^4n}-\frac{3a(a+bx^n)^{11}}{11b^4n}+\frac{(a+bx^n)^{12}}{12b^4n}$$

output
```
-1/9*a^3*(a+b*x^n)^9/b^4/n+3/10*a^2*(a+b*x^n)^10/b^4/n-3/11*a*(a+b*x^n)^11
/b^4/n+1/12*(a+b*x^n)^12/b^4/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int x^{-1+4n}(a+bx^n)^8\,dx$$
$$=\frac{x^{4n}(495a^8+3168a^7bx^n+9240a^6b^2x^{2n}+15840a^5b^3x^{3n}+17325a^4b^4x^{4n}+12320a^3b^5x^{5n}+5544a^2b^6x^{6n}}{1980n}$$

input
```
Integrate[x^(-1 + 4*n)*(a + b*x^n)^8,x]
```

output
```
(x^(4*n)*(495*a^8 + 3168*a^7*b*x^n + 9240*a^6*b^2*x^(2*n) + 15840*a^5*b^3*
x^(3*n) + 17325*a^4*b^4*x^(4*n) + 12320*a^3*b^5*x^(5*n) + 5544*a^2*b^6*x^(
6*n) + 1440*a*b^7*x^(7*n) + 165*b^8*x^(8*n)))/(1980*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int x^{3n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(\frac{(bx^n+a)^{11}}{b^3}-\frac{3a(bx^n+a)^{10}}{b^3}+\frac{3a^2(bx^n+a)^9}{b^3}-\frac{a^3(bx^n+a)^8}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{a^3(a+bx^n)^9}{9b^4}+\frac{3a^2(a+bx^n)^{10}}{10b^4}+\frac{(a+bx^n)^{12}}{12b^4}-\frac{3a(a+bx^n)^{11}}{11b^4}}{n}$$

input
```
Int[x^(-1 + 4*n)*(a + b*x^n)^8,x]
```

output
```
(-1/9*(a^3*(a + b*x^n)^9)/b^4 + (3*a^2*(a + b*x^n)^10)/(10*b^4) - (3*a*(a
+ b*x^n)^11)/(11*b^4) + (a + b*x^n)^12/(12*b^4))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 8.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

method	result
risch	$\frac{b^8x^{12n}}{12n}+\frac{8a\,b^7x^{11n}}{11n}+\frac{14a^2b^6x^{10n}}{5n}+\frac{56a^3b^5x^{9n}}{9n}+\frac{35a^4b^4x^{8n}}{4n}+\frac{8a^5b^3x^{7n}}{n}+\frac{14a^6b^2x^{6n}}{3n}+\frac{8a^7b\,x^{5n}}{5n}+\frac{a^8x^{4n}}{4n}$
parallelrisch	$\frac{165x\,x^{8n}x^{-1+4n}b^8+1440x\,x^{7n}x^{-1+4n}a\,b^7+5544x\,x^{6n}x^{-1+4n}a^2b^6+12320x\,x^{5n}x^{-1+4n}a^3b^5+17325x\,x^{4n}x^{-1+4n}a^4b^4+158}{1980n}$
orering	Expression too large to display

input
```
int(x^(-1+4*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^8/n*(x^n)^12+8/11*a*b^7/n*(x^n)^11+14/5*a^2*b^6/n*(x^n)^10+56/9*a^3
*b^5/n*(x^n)^9+35/4*a^4*b^4/n*(x^n)^8+8*a^5*b^3/n*(x^n)^7+14/3*a^6*b^2/n*(
x^n)^6+8/5*a^7*b/n*(x^n)^5+1/4*a^8/n*(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int x^{-1+4n}(a+bx^n)^8\,dx = \frac{165\,b^8x^{12\,n}+1440\,ab^7x^{11\,n}+5544\,a^2b^6x^{10\,n}+12320\,a^3b^5x^{9\,n}+17325\,a^4b^4x^{8\,n}+15840\,a^5b^3x^{7\,n}+9240\,a^6}{1980\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/1980*(165*b^8*x^(12*n) + 1440*a*b^7*x^(11*n) + 5544*a^2*b^6*x^(10*n) + 1
2320*a^3*b^5*x^(9*n) + 17325*a^4*b^4*x^(8*n) + 15840*a^5*b^3*x^(7*n) + 924
0*a^6*b^2*x^(6*n) + 3168*a^7*b*x^(5*n) + 495*a^8*x^(4*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(71) = 142$.

Time = 2.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.55

$$\int x^{-1+4n}(a+bx^n)^8\,dx$$
$$= \begin{cases} \frac{a^8xx^{4n-1}}{4n} + \frac{8a^7bxx^nx^{4n-1}}{5n} + \frac{14a^6b^2xx^{2n}x^{4n-1}}{3n} + \frac{8a^5b^3xx^{3n}x^{4n-1}}{n} + \frac{35a^4b^4xx^{4n}x^{4n-1}}{4n} + \frac{56a^3b^5xx^{5n}x^{4n-1}}{9n} + \frac{14a^2b^6xx^{6n}x^{4}}{5n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+4*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(4*n - 1)/(4*n) + 8*a**7*b*x*x**n*x**(4*n - 1)/(5*n)
+ 14*a**6*b**2*x*x**(2*n)*x**(4*n - 1)/(3*n) + 8*a**5*b**3*x*x**(3*n)*x**(
4*n - 1)/n + 35*a**4*b**4*x*x**(4*n)*x**(4*n - 1)/(4*n) + 56*a**3*b**5*x*x
**(5*n)*x**(4*n - 1)/(9*n) + 14*a**2*b**6*x*x**(6*n)*x**(4*n - 1)/(5*n) +
8*a*b**7*x*x**(7*n)*x**(4*n - 1)/(11*n) + b**8*x*x**(8*n)*x**(4*n - 1)/(12
*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.61

$$\int x^{-1+4n}(a+bx^n)^8\,dx = \frac{b^8x^{12\,n}}{12\,n} + \frac{8\,ab^7x^{11\,n}}{11\,n} + \frac{14\,a^2b^6x^{10\,n}}{5\,n} + \frac{56\,a^3b^5x^{9\,n}}{9\,n} + \frac{35\,a^4b^4x^{8\,n}}{4\,n} + \frac{8\,a^5b^3x^{7\,n}}{n} + \frac{14\,a^6b^2x^{6\,n}}{3\,n} + \frac{8\,a^7bx^{5\,n}}{5\,n} + \frac{a^8x^{4\,n}}{4\,n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/12*b^8*x^(12*n)/n + 8/11*a*b^7*x^(11*n)/n + 14/5*a^2*b^6*x^(10*n)/n + 56
/9*a^3*b^5*x^(9*n)/n + 35/4*a^4*b^4*x^(8*n)/n + 8*a^5*b^3*x^(7*n)/n + 14/3
*a^6*b^2*x^(6*n)/n + 8/5*a^7*b*x^(5*n)/n + 1/4*a^8*x^(4*n)/n
```

Giac [F]

$$\int x^{-1+4n}(a+bx^n)^8\,dx=\int (bx^n+a)^8x^{4\,n-1}\,dx$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(4*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.61

$$\int x^{-1+4n}(a+bx^n)^8\,dx=\frac{a^8\,x^{4\,n}}{4\,n}+\frac{b^8\,x^{12\,n}}{12\,n}+\frac{14\,a^6\,b^2\,x^{6\,n}}{3\,n}+\frac{8\,a^5\,b^3\,x^{7\,n}}{n}+\frac{35\,a^4\,b^4\,x^{8\,n}}{4\,n}+\frac{56\,a^3\,b^5\,x^{9\,n}}{9\,n}+\frac{14\,a^2\,b^6\,x^{10\,n}}{5\,n}+\frac{8\,a^7\,b\,x^{5\,n}}{5\,n}+\frac{8\,a\,b^7\,x^{11\,n}}{11\,n}$$

input
```
int(x^(4*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(4*n))/(4*n) + (b^8*x^(12*n))/(12*n) + (14*a^6*b^2*x^(6*n))/(3*n) +
 (8*a^5*b^3*x^(7*n))/n + (35*a^4*b^4*x^(8*n))/(4*n) + (56*a^3*b^5*x^(9*n))
/(9*n) + (14*a^2*b^6*x^(10*n))/(5*n) + (8*a^7*b*x^(5*n))/(5*n) + (8*a*b^7*
x^(11*n))/(11*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.32

$$\int x^{-1+4n}(a+bx^n)^8 \, dx$$
$$= \frac{x^{4n}(165x^{8n}b^8 + 1440x^{7n}a\,b^7 + 5544x^{6n}a^2b^6 + 12320x^{5n}a^3b^5 + 17325x^{4n}a^4b^4 + 15840x^{3n}a^5b^3 + 9240x^{2n}a}{1980n}$$

input
```
int(x^(-1+4*n)*(a+b*x^n)^8,x)
```

output
```
(x**(4*n)*(165*x**(8*n)*b**8 + 1440*x**(7*n)*a*b**7 + 5544*x**(6*n)*a**2*b
**6 + 12320*x**(5*n)*a**3*b**5 + 17325*x**(4*n)*a**4*b**4 + 15840*x**(3*n)
*a**5*b**3 + 9240*x**(2*n)*a**6*b**2 + 3168*x**n*a**7*b + 495*a**8))/(1980
*n)
```

3.428 $\int x^{-1+3n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int x^{-1+3n}(a+bx^n)^8\,dx=\frac{a^2(a+bx^n)^9}{9b^3n}-\frac{a(a+bx^n)^{10}}{5b^3n}+\frac{(a+bx^n)^{11}}{11b^3n}$$

output
```
1/9*a^2*(a+b*x^n)^9/b^3/n-1/5*a*(a+b*x^n)^10/b^3/n+1/11*(a+b*x^n)^11/b^3/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int x^{-1+3n}(a+bx^n)^8\,dx$$
$$=\frac{x^{3n}(165a^8+990a^7bx^n+2772a^6b^2x^{2n}+4620a^5b^3x^{3n}+4950a^4b^4x^{4n}+3465a^3b^5x^{5n}+1540a^2b^6x^{6n}+396\cdots}{495n}$$

input
```
Integrate[x^(-1 + 3*n)*(a + b*x^n)^8,x]
```

output
```
(x^(3*n)*(165*a^8 + 990*a^7*b*x^n + 2772*a^6*b^2*x^(2*n) + 4620*a^5*b^3*x^
(3*n) + 4950*a^4*b^4*x^(4*n) + 3465*a^3*b^5*x^(5*n) + 1540*a^2*b^6*x^(6*n)
 + 396*a*b^7*x^(7*n) + 45*b^8*x^(8*n)))/(495*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{2n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^{10}}{b^2}-\frac{2a(bx^n+a)^9}{b^2}+\frac{a^2(bx^n+a)^8}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2(a+bx^n)^9}{9b^3}+\frac{(a+bx^n)^{11}}{11b^3}-\frac{a(a+bx^n)^{10}}{5b^3}}{n}$$

input
```
Int[x^(-1 + 3*n)*(a + b*x^n)^8,x]
```

output
```
((a^2*(a + b*x^n)^9)/(9*b^3) - (a*(a + b*x^n)^10)/(5*b^3) + (a + b*x^n)^11
/(11*b^3))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(56) = 112$.

Time = 8.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.19

method	result
risch	$\frac{b^8 x^{11n}}{11n} + \frac{4a\,b^7 x^{10n}}{5n} + \frac{28a^2 b^6 x^{9n}}{9n} + \frac{7a^3 b^5 x^{8n}}{n} + \frac{10a^4 b^4 x^{7n}}{n} + \frac{28a^5 b^3 x^{6n}}{3n} + \frac{28a^6 b^2 x^{5n}}{5n} + \frac{2a^7 b\,x^{4n}}{n} + \frac{a^8 x^{3n}}{3n}$
parallelrisch	$\frac{45x\,x^{8n} x^{-1+3n} b^8 + 396x\,x^{7n} x^{-1+3n} a\,b^7 + 1540x\,x^{6n} x^{-1+3n} a^2 b^6 + 3465x\,x^{5n} x^{-1+3n} a^3 b^5 + 4950x\,x^{4n} x^{-1+3n} a^4 b^4 + 4620x\,x}{495n}$
orering	Expression too large to display

input
```
int(x^(-1+3*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^8/n*(x^n)^11+4/5*a*b^7/n*(x^n)^10+28/9*a^2*b^6/n*(x^n)^9+7*a^3*b^5/
n*(x^n)^8+10*a^4*b^4/n*(x^n)^7+28/3*a^5*b^3/n*(x^n)^6+28/5*a^6*b^2/n*(x^n)
^5+2*a^7*b/n*(x^n)^4+1/3*a^8/n*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(56) = 112$.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int x^{-1+3n}(a+bx^n)^8\,dx$$
$$= \frac{45\,b^8 x^{11\,n} + 396\,ab^7 x^{10\,n} + 1540\,a^2 b^6 x^{9\,n} + 3465\,a^3 b^5 x^{8\,n} + 4950\,a^4 b^4 x^{7\,n} + 4620\,a^5 b^3 x^{6\,n} + 2772\,a^6 b^2 x^{5\,n}}{495\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/495*(45*b^8*x^(11*n) + 396*a*b^7*x^(10*n) + 1540*a^2*b^6*x^(9*n) + 3465*
a^3*b^5*x^(8*n) + 4950*a^4*b^4*x^(7*n) + 4620*a^5*b^3*x^(6*n) + 2772*a^6*b
^2*x^(5*n) + 990*a^7*b*x^(4*n) + 165*a^8*x^(3*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(49) = 98.

Time = 2.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int x^{-1+3n}(a+bx^n)^8\,dx$$
$$= \begin{cases} \frac{a^8xx^{3n-1}}{3n} + \frac{2a^7bxx^nx^{3n-1}}{n} + \frac{28a^6b^2xx^{2n}x^{3n-1}}{5n} + \frac{28a^5b^3xx^{3n}x^{3n-1}}{3n} + \frac{10a^4b^4xx^{4n}x^{3n-1}}{n} + \frac{7a^3b^5xx^{5n}x^{3n-1}}{n} + \frac{28a^2b^6xx^{6n}x^{3}}{9n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(3*n - 1)/(3*n) + 2*a**7*b*x*x**n*x**(3*n - 1)/n + 28
*a**6*b**2*x*x**(2*n)*x**(3*n - 1)/(5*n) + 28*a**5*b**3*x*x**(3*n)*x**(3*n
 - 1)/(3*n) + 10*a**4*b**4*x*x**(4*n)*x**(3*n - 1)/n + 7*a**3*b**5*x*x**(5
*n)*x**(3*n - 1)/n + 28*a**2*b**6*x*x**(6*n)*x**(3*n - 1)/(9*n) + 4*a*b**7
*x*x**(7*n)*x**(3*n - 1)/(5*n) + b**8*x*x**(8*n)*x**(3*n - 1)/(11*n), Ne(n
, 0)), ((a + b)**8*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(56) = 112.

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.18

$$\int x^{-1+3n}(a+bx^n)^8\,dx = \frac{b^8x^{11\,n}}{11\,n} + \frac{4\,ab^7x^{10\,n}}{5\,n} + \frac{28\,a^2b^6x^{9\,n}}{9\,n} + \frac{7\,a^3b^5x^{8\,n}}{n} + \frac{10\,a^4b^4x^{7\,n}}{n} + \frac{28\,a^5b^3x^{6\,n}}{3\,n} + \frac{28\,a^6b^2x^{5\,n}}{5\,n} + \frac{2\,a^7bx^{4\,n}}{n} + \frac{a^8x^{3\,n}}{3\,n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/11*b^8*x^(11*n)/n + 4/5*a*b^7*x^(10*n)/n + 28/9*a^2*b^6*x^(9*n)/n + 7*a^
3*b^5*x^(8*n)/n + 10*a^4*b^4*x^(7*n)/n + 28/3*a^5*b^3*x^(6*n)/n + 28/5*a^6
*b^2*x^(5*n)/n + 2*a^7*b*x^(4*n)/n + 1/3*a^8*x^(3*n)/n
```

Giac [F]

$$\int x^{-1+3n}(a+bx^n)^8 \, dx = \int (bx^n+a)^8 x^{3n-1} \, dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(3*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.18

$$\int x^{-1+3n}(a+bx^n)^8 \, dx = \frac{a^8 x^{3n}}{3n} + \frac{b^8 x^{11n}}{11n} + \frac{28a^6b^2x^{5n}}{5n} + \frac{28a^5b^3x^{6n}}{3n} + \frac{10a^4b^4x^{7n}}{n} + \frac{7a^3b^5x^{8n}}{n} + \frac{28a^2b^6x^{9n}}{9n} + \frac{2a^7bx^{4n}}{n} + \frac{4ab^7x^{10n}}{5n}$$

input
```
int(x^(3*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(3*n))/(3*n) + (b^8*x^(11*n))/(11*n) + (28*a^6*b^2*x^(5*n))/(5*n) +
 (28*a^5*b^3*x^(6*n))/(3*n) + (10*a^4*b^4*x^(7*n))/n + (7*a^3*b^5*x^(8*n))
/n + (28*a^2*b^6*x^(9*n))/(9*n) + (2*a^7*b*x^(4*n))/n + (4*a*b^7*x^(10*n))
/(5*n)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.79

$$\int x^{-1+3n}(a+bx^n)^8\,dx = \frac{x^{3n}(45x^{8n}b^8+396x^{7n}a\,b^7+1540x^{6n}a^2b^6+3465x^{5n}a^3b^5+4950x^{4n}a^4b^4+4620x^{3n}a^5b^3+2772x^{2n}a^6b^2+}{495n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^8,x)
```

output
```
(x**(3*n)*(45*x**(8*n)*b**8 + 396*x**(7*n)*a*b**7 + 1540*x**(6*n)*a**2*b**
6 + 3465*x**(5*n)*a**3*b**5 + 4950*x**(4*n)*a**4*b**4 + 4620*x**(3*n)*a**5
*b**3 + 2772*x**(2*n)*a**6*b**2 + 990*x**n*a**7*b + 165*a**8))/(495*n)
```

3.429 $\int x^{-1+2n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int x^{-1+2n}(a+bx^n)^8\,dx=-\frac{a(a+bx^n)^9}{9b^2n}+\frac{(a+bx^n)^{10}}{10b^2n}$$

output
```
-1/9*a*(a+b*x^n)^9/b^2/n+1/10*(a+b*x^n)^10/b^2/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(40) = 80.

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.82

$$\int x^{-1+2n}(a+bx^n)^8\,dx$$
$$=\frac{x^{2n}(45a^8+240a^7bx^n+630a^6b^2x^{2n}+1008a^5b^3x^{3n}+1050a^4b^4x^{4n}+720a^3b^5x^{5n}+315a^2b^6x^{6n}+80ab^7x^7}{90n}$$

input
```
Integrate[x^(-1 + 2*n)*(a + b*x^n)^8,x]
```

output
```
(x^(2*n)*(45*a^8 + 240*a^7*b*x^n + 630*a^6*b^2*x^(2*n) + 1008*a^5*b^3*x^(3
*n) + 1050*a^4*b^4*x^(4*n) + 720*a^3*b^5*x^(5*n) + 315*a^2*b^6*x^(6*n) + 8
0*a*b^7*x^(7*n) + 9*b^8*x^(8*n)))/(90*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^n(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(bx^n+a)^9}{b}-\frac{a(bx^n+a)^8}{b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+bx^n)^{10}}{10b^2}-\frac{a(a+bx^n)^9}{9b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a + b*x^n)^8,x]
```

output
```
(-1/9*(a*(a + b*x^n)^9)/b^2 + (a + b*x^n)^10/(10*b^2))/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(36) = 72$.

Time = 7.96 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.40

method	result
risch	$\frac{b^{8} x^{10 n}}{10 n}+\frac{8 a \,b^{7} x^{9 n}}{9 n}+\frac{7 a^{2} b^{6} x^{8 n}}{2 n}+\frac{8 a^{3} b^{5} x^{7 n}}{n}+\frac{35 a^{4} b^{4} x^{6 n}}{3 n}+\frac{56 a^{5} b^{3} x^{5 n}}{5 n}+\frac{7 a^{6} b^{2} x^{4 n}}{n}+\frac{8 a^{7} b \,x^{3 n}}{3 n}+\frac{a^{8} x^{2 n}}{2 n}$
parallelrisch	$\frac{9 x \,x^{8 n} x^{2 n -1} b^{8}+80 x \,x^{7 n} x^{2 n -1} a \,b^{7}+315 x \,x^{6 n} x^{2 n -1} a^{2} b^{6}+720 x \,x^{5 n} x^{2 n -1} a^{3} b^{5}+1050 x \,x^{4 n} x^{2 n -1} a^{4} b^{4}+1008 x \,x^{3 n} x^{2 n -1} a^{5} b}{90 n}$
orering	Expression too large to display

input
```
int(x^(2*n-1)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^8/n*(x^n)^10+8/9*a*b^7/n*(x^n)^9+7/2*a^2*b^6/n*(x^n)^8+8*a^3*b^5/n*
(x^n)^7+35/3*a^4*b^4/n*(x^n)^6+56/5*a^5*b^3/n*(x^n)^5+7*a^6*b^2/n*(x^n)^4+
8/3*a^7*b/n*(x^n)^3+1/2*a^8/n*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(36) = 72$.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.82

$$\int x^{-1+2n}(a+bx^n)^8\,dx$$
$$= \frac{9\,b^8x^{10\,n} + 80\,ab^7x^{9\,n} + 315\,a^2b^6x^{8\,n} + 720\,a^3b^5x^{7\,n} + 1050\,a^4b^4x^{6\,n} + 1008\,a^5b^3x^{5\,n} + 630\,a^6b^2x^{4\,n} + 240\,a}{90\,n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/90*(9*b^8*x^(10*n) + 80*a*b^7*x^(9*n) + 315*a^2*b^6*x^(8*n) + 720*a^3*b^
5*x^(7*n) + 1050*a^4*b^4*x^(6*n) + 1008*a^5*b^3*x^(5*n) + 630*a^6*b^2*x^(4
*n) + 240*a^7*b*x^(3*n) + 45*a^8*x^(2*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(31) = 62$.

Time = 2.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 5.30

$$\int x^{-1+2n}(a+bx^n)^8\,dx$$
$$= \begin{cases} \frac{a^8xx^{2n-1}}{2n} + \frac{8a^7bxx^nx^{2n-1}}{3n} + \frac{7a^6b^2xx^{2n}x^{2n-1}}{n} + \frac{56a^5b^3xx^{3n}x^{2n-1}}{5n} + \frac{35a^4b^4xx^{4n}x^{2n-1}}{3n} + \frac{8a^3b^5xx^{5n}x^{2n-1}}{n} + \frac{7a^2b^6xx^{6n}x^{2n-}}{2n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(2*n - 1)/(2*n) + 8*a**7*b*x*x**n*x**(2*n - 1)/(3*n)
+ 7*a**6*b**2*x*x**(2*n)*x**(2*n - 1)/n + 56*a**5*b**3*x*x**(3*n)*x**(2*n
- 1)/(5*n) + 35*a**4*b**4*x*x**(4*n)*x**(2*n - 1)/(3*n) + 8*a**3*b**5*x*x*
*(5*n)*x**(2*n - 1)/n + 7*a**2*b**6*x*x**(6*n)*x**(2*n - 1)/(2*n) + 8*a*b*
*7*x*x**(7*n)*x**(2*n - 1)/(9*n) + b**8*x*x**(8*n)*x**(2*n - 1)/(10*n), Ne
(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.38

$$\int x^{-1+2n}(a+bx^n)^8\,dx = \frac{b^8x^{10\,n}}{10\,n} + \frac{8\,ab^7x^{9\,n}}{9\,n} + \frac{7\,a^2b^6x^{8\,n}}{2\,n} + \frac{8\,a^3b^5x^{7\,n}}{n} + \frac{35\,a^4b^4x^{6\,n}}{3\,n} + \frac{56\,a^5b^3x^{5\,n}}{5\,n} + \frac{7\,a^6b^2x^{4\,n}}{n} + \frac{8\,a^7bx^{3\,n}}{3\,n} + \frac{a^8x^{2\,n}}{2\,n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/10*b^8*x^(10*n)/n + 8/9*a*b^7*x^(9*n)/n + 7/2*a^2*b^6*x^(8*n)/n + 8*a^3*
b^5*x^(7*n)/n + 35/3*a^4*b^4*x^(6*n)/n + 56/5*a^5*b^3*x^(5*n)/n + 7*a^6*b^
2*x^(4*n)/n + 8/3*a^7*b*x^(3*n)/n + 1/2*a^8*x^(2*n)/n
```

Giac [F]

$$\int x^{-1+2n}(a+bx^n)^8\,dx = \int (bx^n+a)^8x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8*x^(2*n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.38

$$\int x^{-1+2n}(a+bx^n)^8\,dx = \frac{a^8\,x^{2\,n}}{2\,n} + \frac{b^8\,x^{10\,n}}{10\,n} + \frac{7\,a^6\,b^2\,x^{4\,n}}{n} + \frac{56\,a^5\,b^3\,x^{5\,n}}{5\,n} + \frac{35\,a^4\,b^4\,x^{6\,n}}{3\,n} + \frac{8\,a^3\,b^5\,x^{7\,n}}{n} + \frac{7\,a^2\,b^6\,x^{8\,n}}{2\,n} + \frac{8\,a^7\,b\,x^{3\,n}}{3\,n} + \frac{8\,a\,b^7\,x^{9\,n}}{9\,n}$$

input
```
int(x^(2*n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^(2*n))/(2*n) + (b^8*x^(10*n))/(10*n) + (7*a^6*b^2*x^(4*n))/n + (56*
a^5*b^3*x^(5*n))/(5*n) + (35*a^4*b^4*x^(6*n))/(3*n) + (8*a^3*b^5*x^(7*n))/
n + (7*a^2*b^6*x^(8*n))/(2*n) + (8*a^7*b*x^(3*n))/(3*n) + (8*a*b^7*x^(9*n)
)/(9*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.78

$$\int x^{-1+2n}(a+bx^n)^8\,dx$$
$$= \frac{x^{2n}(9x^{8n}b^8 + 80x^{7n}a\,b^7 + 315x^{6n}a^2b^6 + 720x^{5n}a^3b^5 + 1050x^{4n}a^4b^4 + 1008x^{3n}a^5b^3 + 630x^{2n}a^6b^2 + 240x^{n}}{90n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^8,x)
```

output
```
(x**(2*n)*(9*x**(8*n)*b**8 + 80*x**(7*n)*a*b**7 + 315*x**(6*n)*a**2*b**6 +
 720*x**(5*n)*a**3*b**5 + 1050*x**(4*n)*a**4*b**4 + 1008*x**(3*n)*a**5*b**
3 + 630*x**(2*n)*a**6*b**2 + 240*x**n*a**7*b + 45*a**8))/(90*n)
```

3.430 $\int x^{-1+n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)^8\,dx=\frac{(a+bx^n)^9}{9bn}$$

output
```
1/9*(a+b*x^n)^9/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^8\,dx=\frac{(a+bx^n)^9}{9bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^8,x]
```

output
```
(a + b*x^n)^9/(9*b*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \ 793$$

$$\frac{(a+bx^n)^9}{9bn}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^8,x]
```

output
```
(a + b*x^n)^9/(9*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(17) = 34.

Time = 8.38 (sec) , antiderivative size = 132, normalized size of antiderivative = 6.95

method	result
risch	$\frac{b^{8} x^{9 n}}{9 n}+\frac{a\,b^{7} x^{8 n}}{n}+\frac{4 a^{2} b^{6} x^{7 n}}{n}+\frac{28 a^{3} b^{5} x^{6 n}}{3 n}+\frac{14 a^{4} b^{4} x^{5 n}}{n}+\frac{14 a^{5} b^{3} x^{4 n}}{n}+\frac{28 a^{6} b^{2} x^{3 n}}{3 n}+\frac{4 a^{7} b\,x^{2 n}}{n}+\frac{a^{8} x^{n}}{n}$
parallelrisch	$\frac{x\,x^{8 n} x^{-1+n} b^{8}+9 x\,x^{7 n} x^{-1+n} a\,b^{7}+36 x\,x^{6 n} x^{-1+n} a^{2} b^{6}+84 x\,x^{5 n} x^{-1+n} a^{3} b^{5}+126 x\,x^{4 n} x^{-1+n} a^{4} b^{4}+126 x\,x^{3 n} x^{-1+n} a^{5} b^{3}+8}{9 n}$
orering	Expression too large to display

input
```
int(x^(-1+n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^8/n*(x^n)^9+a*b^7/n*(x^n)^8+4*a^2*b^6/n*(x^n)^7+28/3*a^3*b^5/n*(x^n)
^6+14*a^4*b^4/n*(x^n)^5+14*a^5*b^3/n*(x^n)^4+28/3*a^6*b^2/n*(x^n)^3+4*a^7*
b/n*(x^n)^2+a^8/n*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.79

$$\int x^{-1+n}(a+bx^n)^8\,dx$$
$$= \frac{b^8x^{9n}+9\,ab^7x^{8n}+36\,a^2b^6x^{7n}+84\,a^3b^5x^{6n}+126\,a^4b^4x^{5n}+126\,a^5b^3x^{4n}+84\,a^6b^2x^{3n}+36\,a^7bx^{2n}+9}{9\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/9*(b^8*x^(9*n) + 9*a*b^7*x^(8*n) + 36*a^2*b^6*x^(7*n) + 84*a^3*b^5*x^(6*
n) + 126*a^4*b^4*x^(5*n) + 126*a^5*b^3*x^(4*n) + 84*a^6*b^2*x^(3*n) + 36*a
^7*b*x^(2*n) + 9*a^8*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(12) = 24$.

Time = 2.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.95

$$\int x^{-1+n}(a+bx^n)^8\,dx$$
$$= \begin{cases} \frac{a^8xx^{n-1}}{n}+\frac{4a^7bxx^nx^{n-1}}{n}+\frac{28a^6b^2xx^{2n}x^{n-1}}{3n}+\frac{14a^5b^3xx^{3n}x^{n-1}}{n}+\frac{14a^4b^4xx^{4n}x^{n-1}}{n}+\frac{28a^3b^5xx^{5n}x^{n-1}}{3n}+\frac{4a^2b^6xx^{6n}x^{n-1}}{n}+ \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((a**8*x*x**(n - 1)/n + 4*a**7*b*x*x**n*x**(n - 1)/n + 28*a**6*b*
*2*x*x**(2*n)*x**(n - 1)/(3*n) + 14*a**5*b**3*x*x**(3*n)*x**(n - 1)/n + 14
*a**4*b**4*x*x**(4*n)*x**(n - 1)/n + 28*a**3*b**5*x*x**(5*n)*x**(n - 1)/(3
*n) + 4*a**2*b**6*x*x**(6*n)*x**(n - 1)/n + a*b**7*x*x**(7*n)*x**(n - 1)/n
 + b**8*x*x**(8*n)*x**(n - 1)/(9*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)^8\,dx=\frac{(bx^n+a)^9}{9\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
1/9*(b*x^n + a)^9/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.79

$$\int x^{-1+n}(a+bx^n)^8\,dx$$
$$=\frac{b^8x^{9\,n}+9\,ab^7x^{8\,n}+36\,a^2b^6x^{7\,n}+84\,a^3b^5x^{6\,n}+126\,a^4b^4x^{5\,n}+126\,a^5b^3x^{4\,n}+84\,a^6b^2x^{3\,n}+36\,a^7bx^{2\,n}+9}{9\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/9*(b^8*x^(9*n) + 9*a*b^7*x^(8*n) + 36*a^2*b^6*x^(7*n) + 84*a^3*b^5*x^(6*
n) + 126*a^4*b^4*x^(5*n) + 126*a^5*b^3*x^(4*n) + 84*a^6*b^2*x^(3*n) + 36*a
^7*b*x^(2*n) + 9*a^8*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 131, normalized size of antiderivative = 6.89

$$\int x^{-1+n}(a+bx^n)^8\,dx = \frac{a^8\,x^n}{n}+\frac{b^8\,x^{9\,n}}{9\,n}+\frac{28\,a^6\,b^2\,x^{3\,n}}{3\,n}+\frac{14\,a^5\,b^3\,x^{4\,n}}{n}+\frac{14\,a^4\,b^4\,x^{5\,n}}{n}+\frac{28\,a^3\,b^5\,x^{6\,n}}{3\,n}+\frac{4\,a^2\,b^6\,x^{7\,n}}{n}+\frac{4\,a^7\,b\,x^{2\,n}}{n}+\frac{a\,b^7\,x^{8\,n}}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^8,x)
```

output
```
(a^8*x^n)/n + (b^8*x^(9*n))/(9*n) + (28*a^6*b^2*x^(3*n))/(3*n) + (14*a^5*b
^3*x^(4*n))/n + (14*a^4*b^4*x^(5*n))/n + (28*a^3*b^5*x^(6*n))/(3*n) + (4*a
^2*b^6*x^(7*n))/n + (4*a^7*b*x^(2*n))/n + (a*b^7*x^(8*n))/n
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 5.68

$$\int x^{-1+n}(a+bx^n)^8\,dx = \frac{x^n(x^{8n}b^8+9x^{7n}a\,b^7+36x^{6n}a^2b^6+84x^{5n}a^3b^5+126x^{4n}a^4b^4+126x^{3n}a^5b^3+84x^{2n}a^6b^2+36x^na^7b+9a^8)}{9n}$$

input
```
int(x^(-1+n)*(a+b*x^n)^8,x)
```

output
```
(x**n*(x**(8*n)*b**8 + 9*x**(7*n)*a*b**7 + 36*x**(6*n)*a**2*b**6 + 84*x**(
5*n)*a**3*b**5 + 126*x**(4*n)*a**4*b**4 + 126*x**(3*n)*a**5*b**3 + 84*x**(
2*n)*a**6*b**2 + 36*x**n*a**7*b + 9*a**8))/(9*n)
```

3.431 $\int \frac{(a+bx^n)^8}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 138

$$\int \frac{(a+bx^n)^8}{x} \, dx = \frac{8a^7bx^n}{n} + \frac{14a^6b^2x^{2n}}{n} + \frac{56a^5b^3x^{3n}}{3n} + \frac{35a^4b^4x^{4n}}{2n} + \frac{56a^3b^5x^{5n}}{5n} + \frac{14a^2b^6x^{6n}}{3n} + \frac{8ab^7x^{7n}}{7n} + \frac{b^8x^{8n}}{8n} + a^8\log(x)$$

output
```
8*a^7*b*x^n/n+14*a^6*b^2*x^(2*n)/n+56/3*a^5*b^3*x^(3*n)/n+35/2*a^4*b^4*x^(
4*n)/n+56/5*a^3*b^5*x^(5*n)/n+14/3*a^2*b^6*x^(6*n)/n+8/7*a*b^7*x^(7*n)/n+1
/8*b^8*x^(8*n)/n+a^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^n)^8}{x} \, dx = \frac{bx^n(6720a^7 + 11760a^6bx^n + 15680a^5b^2x^{2n} + 14700a^4b^3x^{3n} + 9408a^3b^4x^{4n} + 3920a^2b^5x^{5n} + 960ab^6x^{6n} + }{840n} + \frac{a^8\log(x^n)}{n}$$

input
```
Integrate[(a + b*x^n)^8/x,x]
```

output
```
(b*x^n*(6720*a^7 + 11760*a^6*b*x^n + 15680*a^5*b^2*x^(2*n) + 14700*a^4*b^3
*x^(3*n) + 9408*a^3*b^4*x^(4*n) + 3920*a^2*b^5*x^(5*n) + 960*a*b^6*x^(6*n)
 + 105*b^7*x^(7*n)))/(840*n) + (a^8*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^8}{x} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int x^{-n}(bx^n+a)^8 \, dx^n}{n}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(a^8x^{-n}+28a^6b^2x^n+56a^5b^3x^{2n}+70a^4b^4x^{3n}+56a^3b^5x^{4n}+28a^2b^6x^{5n}+8ab^7x^{6n}+b^8x^{7n}+8a^7b\right) dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^8\log(x^n)+8a^7bx^n+14a^6b^2x^{2n}+\frac{56}{3}a^5b^3x^{3n}+\frac{35}{2}a^4b^4x^{4n}+\frac{56}{5}a^3b^5x^{5n}+\frac{14}{3}a^2b^6x^{6n}+\frac{8}{7}ab^7x^{7n}+\frac{1}{8}b^8x^{8n}}{n}$$

input
```
Int[(a + b*x^n)^8/x,x]
```

output
```
(8*a^7*b*x^n + 14*a^6*b^2*x^(2*n) + (56*a^5*b^3*x^(3*n))/3 + (35*a^4*b^4*x
^(4*n))/2 + (56*a^3*b^5*x^(5*n))/5 + (14*a^2*b^6*x^(6*n))/3 + (8*a*b^7*x^(
7*n))/7 + (b^8*x^(8*n))/8 + a^8*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 1.93 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{b^{8} x^{8 n}}{8}+\frac{8 a \,b^{7} x^{7 n}}{7}+\frac{14 a^{2} b^{6} x^{6 n}}{3}+\frac{56 a^{3} b^{5} x^{5 n}}{5}+\frac{35 a^{4} b^{4} x^{4 n}}{2}+\frac{56 a^{5} b^{3} x^{3 n}}{3}+14 a^{6} b^{2} x^{2 n}+8 a^{7} b \,x^{n}+a^{8} \ln \left(x^{n}\right)}{n}$
default	$\frac{\frac{b^{8} x^{8 n}}{8}+\frac{8 a \,b^{7} x^{7 n}}{7}+\frac{14 a^{2} b^{6} x^{6 n}}{3}+\frac{56 a^{3} b^{5} x^{5 n}}{5}+\frac{35 a^{4} b^{4} x^{4 n}}{2}+\frac{56 a^{5} b^{3} x^{3 n}}{3}+14 a^{6} b^{2} x^{2 n}+8 a^{7} b \,x^{n}+a^{8} \ln \left(x^{n}\right)}{n}$
parallelrisch	$\frac{105 b^{8} x^{8 n}+960 a \,b^{7} x^{7 n}+3920 a^{2} b^{6} x^{6 n}+9408 a^{3} b^{5} x^{5 n}+14700 a^{4} b^{4} x^{4 n}+15680 a^{5} b^{3} x^{3 n}+11760 a^{6} b^{2} x^{2 n}+840 a^{8} \ln \left(x \right) n+}{840 n}$
risch	$\frac{8 a^{7} b \,x^{n}}{n}+\frac{14 a^{6} b^{2} x^{2 n}}{n}+\frac{56 a^{5} b^{3} x^{3 n}}{3 n}+\frac{35 a^{4} b^{4} x^{4 n}}{2 n}+\frac{56 a^{3} b^{5} x^{5 n}}{5 n}+\frac{14 a^{2} b^{6} x^{6 n}}{3 n}+\frac{8 a \,b^{7} x^{7 n}}{7 n}+\frac{b^{8} x^{8 n}}{8 n}+a^{8}$

input
```
int((a+b*x^n)^8/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/8*b^8*(x^n)^8+8/7*a*b^7*(x^n)^7+14/3*a^2*b^6*(x^n)^6+56/5*a^3*b^5*(
x^n)^5+35/2*a^4*b^4*(x^n)^4+56/3*a^5*b^3*(x^n)^3+14*a^6*b^2*(x^n)^2+8*a^7*
b*x^n+a^8*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^n)^8}{x}\,dx$$
$$= \frac{840\,a^8 n\log(x) + 105\,b^8x^{8\,n} + 960\,ab^7x^{7\,n} + 3920\,a^2b^6x^{6\,n} + 9408\,a^3b^5x^{5\,n} + 14700\,a^4b^4x^{4\,n} + 15680\,a^5b^3x}{840\,n}$$

input
```
integrate((a+b*x^n)^8/x,x, algorithm="fricas")
```

output
```
1/840*(840*a^8*n*log(x) + 105*b^8*x^(8*n) + 960*a*b^7*x^(7*n) + 3920*a^2*b
^6*x^(6*n) + 9408*a^3*b^5*x^(5*n) + 14700*a^4*b^4*x^(4*n) + 15680*a^5*b^3*
x^(3*n) + 11760*a^6*b^2*x^(2*n) + 6720*a^7*b*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^n)^8}{x}\,dx$$
$$= \begin{cases} a^8\log(x) + \frac{8a^7bx^n}{n} + \frac{14a^6b^2x^{2n}}{n} + \frac{56a^5b^3x^{3n}}{3n} + \frac{35a^4b^4x^{4n}}{2n} + \frac{56a^3b^5x^{5n}}{5n} + \frac{14a^2b^6x^{6n}}{3n} + \frac{8ab^7x^{7n}}{7n} + \frac{b^8x^{8n}}{8n} & \text{for } n \neq 0 \\ (a+b)^8\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**8/x,x)
```

output
```
Piecewise((a**8*log(x) + 8*a**7*b*x**n/n + 14*a**6*b**2*x**(2*n)/n + 56*a*
*5*b**3*x**(3*n)/(3*n) + 35*a**4*b**4*x**(4*n)/(2*n) + 56*a**3*b**5*x**(5*
n)/(5*n) + 14*a**2*b**6*x**(6*n)/(3*n) + 8*a*b**7*x**(7*n)/(7*n) + b**8*x*
*(8*n)/(8*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^n)^8}{x}\,dx = \frac{a^8\log(x^n)}{n} + \frac{105\,b^8x^{8\,n} + 960\,ab^7x^{7\,n} + 3920\,a^2b^6x^{6\,n} + 9408\,a^3b^5x^{5\,n} + 14700\,a^4b^4x^{4\,n} + 15680\,a^5b^3x^{3\,n} + 11760\,a^6b}{840\,n}$$

input
```
integrate((a+b*x^n)^8/x,x, algorithm="maxima")
```

output
```
a^8*log(x^n)/n + 1/840*(105*b^8*x^(8*n) + 960*a*b^7*x^(7*n) + 3920*a^2*b^6
*x^(6*n) + 9408*a^3*b^5*x^(5*n) + 14700*a^4*b^4*x^(4*n) + 15680*a^5*b^3*x^
(3*n) + 11760*a^6*b^2*x^(2*n) + 6720*a^7*b*x^n)/n
```

Giac [F]

$$\int \frac{(a+bx^n)^8}{x}\,dx = \int \frac{(bx^n+a)^8}{x}\,dx$$

input
```
integrate((a+b*x^n)^8/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^8/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^n)^8}{x}\,dx = a^8\,\ln(x) + \frac{b^8\,x^{8\,n}}{8\,n} + \frac{14\,a^6\,b^2\,x^{2\,n}}{n} + \frac{56\,a^5\,b^3\,x^{3\,n}}{3\,n} + \frac{35\,a^4\,b^4\,x^{4\,n}}{2\,n} + \frac{56\,a^3\,b^5\,x^{5\,n}}{5\,n} + \frac{14\,a^2\,b^6\,x^{6\,n}}{3\,n} + \frac{8\,a^7\,b\,x^n}{n} + \frac{8\,a\,b^7\,x^{7\,n}}{7\,n}$$

input
```
int((a + b*x^n)^8/x,x)
```

output
```
a^8*log(x) + (b^8*x^(8*n))/(8*n) + (14*a^6*b^2*x^(2*n))/n + (56*a^5*b^3*x^
(3*n))/(3*n) + (35*a^4*b^4*x^(4*n))/(2*n) + (56*a^3*b^5*x^(5*n))/(5*n) + (
14*a^2*b^6*x^(6*n))/(3*n) + (8*a^7*b*x^n)/n + (8*a*b^7*x^(7*n))/(7*n)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^n)^8}{x}\,dx$$
$$= \frac{105x^{8n}b^8 + 960x^{7n}a\,b^7 + 3920x^{6n}a^2b^6 + 9408x^{5n}a^3b^5 + 14700x^{4n}a^4b^4 + 15680x^{3n}a^5b^3 + 11760x^{2n}a^6b^2 +}{840n}$$

input
```
int((a+b*x^n)^8/x,x)
```

output
```
(105*x**(8*n)*b**8 + 960*x**(7*n)*a*b**7 + 3920*x**(6*n)*a**2*b**6 + 9408*
x**(5*n)*a**3*b**5 + 14700*x**(4*n)*a**4*b**4 + 15680*x**(3*n)*a**5*b**3 +
 11760*x**(2*n)*a**6*b**2 + 6720*x**n*a**7*b + 840*log(x)*a**8*n)/(840*n)
```

3.432 $\int x^{-1-n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 135

$$\int x^{-1-n}(a+bx^n)^8\,dx = -\frac{a^8x^{-n}}{n}+\frac{28a^6b^2x^n}{n}+\frac{28a^5b^3x^{2n}}{n}+\frac{70a^4b^4x^{3n}}{3n}+\frac{14a^3b^5x^{4n}}{n}+\frac{28a^2b^6x^{5n}}{5n}+\frac{4ab^7x^{6n}}{3n}+\frac{b^8x^{7n}}{7n}+8a^7b\log(x)$$

output
```
-a^8/n/(x^n)+28*a^6*b^2*x^n/n+28*a^5*b^3*x^(2*n)/n+70/3*a^4*b^4*x^(3*n)/n+
14*a^3*b^5*x^(4*n)/n+28/5*a^2*b^6*x^(5*n)/n+4/3*a*b^7*x^(6*n)/n+1/7*b^8*x^
(7*n)/n+8*a^7*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

$$\int x^{-1-n}(a+bx^n)^8\,dx = \frac{-105a^8x^{-n}+2940a^6b^2x^n+2940a^5b^3x^{2n}+2450a^4b^4x^{3n}+1470a^3b^5x^{4n}+588a^2b^6x^{5n}+140ab^7x^{6n}+15}{105n}$$

input
```
Integrate[x^(-1 - n)*(a + b*x^n)^8,x]
```

output
```
((-105*a^8)/x^n + 2940*a^6*b^2*x^n + 2940*a^5*b^3*x^(2*n) + 2450*a^4*b^4*x
^(3*n) + 1470*a^3*b^5*x^(4*n) + 588*a^2*b^6*x^(5*n) + 140*a*b^7*x^(6*n) +
15*b^8*x^(7*n) + 840*a^7*b*Log[x^n])/(105*n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(a+bx^n)^8\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-2n}(bx^n+a)^8\,dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(a^8x^{-2n}+8a^7bx^{-n}+56a^5b^3x^n+70a^4b^4x^{2n}+56a^3b^5x^{3n}+28a^2b^6x^{4n}+8ab^7x^{5n}+b^8x^{6n}+28a^6b^2\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-a^8x^{-n}+8a^7b\log(x^n)+28a^6b^2x^n+28a^5b^3x^{2n}+\frac{70}{3}a^4b^4x^{3n}+14a^3b^5x^{4n}+\frac{28}{5}a^2b^6x^{5n}+\frac{4}{3}ab^7x^{6n}+\frac{1}{7}b^8x^{7n}}{n}$$

input
```
Int[x^(-1 - n)*(a + b*x^n)^8,x]
```

output
```
(-(a^8/x^n) + 28*a^6*b^2*x^n + 28*a^5*b^3*x^(2*n) + (70*a^4*b^4*x^(3*n))/3
 + 14*a^3*b^5*x^(4*n) + (28*a^2*b^6*x^(5*n))/5 + (4*a*b^7*x^(6*n))/3 + (b^
8*x^(7*n))/7 + 8*a^7*b*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

method	result	s
risch	$8a^7 b\ln(x)+\frac{b^8x^{7n}}{7n}+\frac{4a\,b^7x^{6n}}{3n}+\frac{28a^2b^6x^{5n}}{5n}+\frac{14a^3b^5x^{4n}}{n}+\frac{70a^4b^4x^{3n}}{3n}+\frac{28a^5b^3x^{2n}}{n}+\frac{28a^6b^2x^{n}}{n}-\frac{a^8x^{-n}}{n}$	1

input
```
int(x^(-1-n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
8*a^7*b*ln(x)+1/7*b^8/n*(x^n)^7+4/3*a*b^7/n*(x^n)^6+28/5*a^2*b^6/n*(x^n)^5
+14*a^3*b^5/n*(x^n)^4+70/3*a^4*b^4/n*(x^n)^3+28*a^5*b^3/n*(x^n)^2+28*a^6*b
^2*x^n/n-a^8/n/(x^n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int x^{-1-n}(a+bx^n)^8\,dx$$
$$=\frac{840\,a^7bnx^n\log(x)+15\,b^8x^{8\,n}+140\,ab^7x^{7\,n}+588\,a^2b^6x^{6\,n}+1470\,a^3b^5x^{5\,n}+2450\,a^4b^4x^{4\,n}+2940\,a^5b^3x^{3}}{105\,nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/105*(840*a^7*b*n*x^n*log(x) + 15*b^8*x^(8*n) + 140*a*b^7*x^(7*n) + 588*a
^2*b^6*x^(6*n) + 1470*a^3*b^5*x^(5*n) + 2450*a^4*b^4*x^(4*n) + 2940*a^5*b^
3*x^(3*n) + 2940*a^6*b^2*x^(2*n) - 105*a^8)/(n*x^n)
```

Sympy [A] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int x^{-1-n}(a+bx^n)^8\,dx$$
$$= \begin{cases} -\frac{a^8x^{-n}}{n} + \frac{8a^7b\log(x^n)}{n} + \frac{28a^6b^2x^n}{n} + \frac{28a^5b^3x^{2n}}{n} + \frac{70a^4b^4x^{3n}}{3n} + \frac{14a^3b^5x^{4n}}{n} + \frac{28a^2b^6x^{5n}}{5n} + \frac{4ab^7x^{6n}}{3n} + \frac{b^8x^{7n}}{7n} & \text{for } n \neq \\ (a+b)^8\log(x) & \text{otherw} \end{cases}$$

input
```
integrate(x**(-1-n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(n*x**n) + 8*a**7*b*log(x**n)/n + 28*a**6*b**2*x**n/n + 2
8*a**5*b**3*x**(2*n)/n + 70*a**4*b**4*x**(3*n)/(3*n) + 14*a**3*b**5*x**(4*
n)/n + 28*a**2*b**6*x**(5*n)/(5*n) + 4*a*b**7*x**(6*n)/(3*n) + b**8*x**(7*
n)/(7*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.94

$$\int x^{-1-n}(a+bx^n)^8\,dx = 8\,a^7b\log(x) + \frac{b^8x^{7\,n}}{7\,n} + \frac{4\,ab^7x^{6\,n}}{3\,n} + \frac{28\,a^2b^6x^{5\,n}}{5\,n} + \frac{14\,a^3b^5x^{4\,n}}{n} + \frac{70\,a^4b^4x^{3\,n}}{3\,n} + \frac{28\,a^5b^3x^{2\,n}}{n} + \frac{28\,a^6b^2x^n}{n} - \frac{a^8}{nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
8*a^7*b*log(x) + 1/7*b^8*x^(7*n)/n + 4/3*a*b^7*x^(6*n)/n + 28/5*a^2*b^6*x^
(5*n)/n + 14*a^3*b^5*x^(4*n)/n + 70/3*a^4*b^4*x^(3*n)/n + 28*a^5*b^3*x^(2*
n)/n + 28*a^6*b^2*x^n/n - a^8/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int x^{-1-n}(a+bx^n)^8\,dx$$
$$=\frac{840\,a^7bnx^n\log(x)+15\,b^8x^{8\,n}+140\,ab^7x^{7\,n}+588\,a^2b^6x^{6\,n}+1470\,a^3b^5x^{5\,n}+2450\,a^4b^4x^{4\,n}+2940\,a^5b^3x^3}{105\,nx^n}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/105*(840*a^7*b*n*x^n*log(x) + 15*b^8*x^(8*n) + 140*a*b^7*x^(7*n) + 588*a
^2*b^6*x^(6*n) + 1470*a^3*b^5*x^(5*n) + 2450*a^4*b^4*x^(4*n) + 2940*a^5*b^
3*x^(3*n) + 2940*a^6*b^2*x^(2*n) - 105*a^8)/(n*x^n)
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.94

$$\int x^{-1-n}(a+bx^n)^8\,dx=8\,a^7\,b\,\ln(x)-\frac{a^8}{n\,x^n}+\frac{b^8\,x^{7\,n}}{7\,n}+\frac{28\,a^5\,b^3\,x^{2\,n}}{n}+\frac{70\,a^4\,b^4\,x^{3\,n}}{3\,n}+\frac{14\,a^3\,b^5\,x^{4\,n}}{n}+\frac{28\,a^2\,b^6\,x^{5\,n}}{5\,n}+\frac{4\,a\,b^7\,x^{6\,n}}{3\,n}+\frac{28\,a^6\,b^2\,x^n}{n}$$

input
```
int((a + b*x^n)^8/x^(n + 1),x)
```

output
```
8*a^7*b*log(x) - a^8/(n*x^n) + (b^8*x^(7*n))/(7*n) + (28*a^5*b^3*x^(2*n))/
n + (70*a^4*b^4*x^(3*n))/(3*n) + (14*a^3*b^5*x^(4*n))/n + (28*a^2*b^6*x^(5
*n))/(5*n) + (4*a*b^7*x^(6*n))/(3*n) + (28*a^6*b^2*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int x^{-1-n}(a+bx^n)^8\,dx$$
$$=\frac{15x^{8n}b^8+140x^{7n}a\,b^7+588x^{6n}a^2b^6+1470x^{5n}a^3b^5+2450x^{4n}a^4b^4+2940x^{3n}a^5b^3+2940x^{2n}a^6b^2+840x}{105x^n n}$$

input
```
int(x^(-1-n)*(a+b*x^n)^8,x)
```

output
```
(15*x**(8*n)*b**8 + 140*x**(7*n)*a*b**7 + 588*x**(6*n)*a**2*b**6 + 1470*x*
*(5*n)*a**3*b**5 + 2450*x**(4*n)*a**4*b**4 + 2940*x**(3*n)*a**5*b**3 + 294
0*x**(2*n)*a**6*b**2 + 840*x**n*log(x)*a**7*b*n - 105*a**8)/(105*x**n*n)
```

3.433 $\int x^{-1-2n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 135

$$\int x^{-1-2n}(a+bx^n)^8\,dx = -\frac{a^8x^{-2n}}{2n}-\frac{8a^7bx^{-n}}{n}+\frac{56a^5b^3x^n}{n}+\frac{35a^4b^4x^{2n}}{n}+\frac{56a^3b^5x^{3n}}{3n}+\frac{7a^2b^6x^{4n}}{n}+\frac{8ab^7x^{5n}}{5n}+\frac{b^8x^{6n}}{6n}+28a^6b^2\log(x)$$

output
```
-1/2*a^8/n/(x^(2*n))-8*a^7*b/n/(x^n)+56*a^5*b^3*x^n/n+35*a^4*b^4*x^(2*n)/n
+56/3*a^3*b^5*x^(3*n)/n+7*a^2*b^6*x^(4*n)/n+8/5*a*b^7*x^(5*n)/n+1/6*b^8*x^
(6*n)/n+28*a^6*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

$$\int x^{-1-2n}(a+bx^n)^8\,dx = \frac{-15a^8x^{-2n}-240a^7bx^{-n}+1680a^5b^3x^n+1050a^4b^4x^{2n}+560a^3b^5x^{3n}+210a^2b^6x^{4n}+48ab^7x^{5n}+5b^8x^{6n}}{30n}$$

input
```
Integrate[x^(-1 - 2*n)*(a + b*x^n)^8,x]
```

output
```
((-15*a^8)/x^(2*n) - (240*a^7*b)/x^n + 1680*a^5*b^3*x^n + 1050*a^4*b^4*x^(
2*n) + 560*a^3*b^5*x^(3*n) + 210*a^2*b^6*x^(4*n) + 48*a*b^7*x^(5*n) + 5*b^
8*x^(6*n) + 840*a^6*b^2*Log[x^n])/(30*n)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int x^{-3n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(a^8x^{-3n}+8a^7bx^{-2n}+28a^6b^2x^{-n}+70a^4b^4x^n+56a^3b^5x^{2n}+28a^2b^6x^{3n}+8ab^7x^{4n}+b^8x^{5n}+56a^5b^3\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{2}a^8x^{-2n}-8a^7bx^{-n}+28a^6b^2\log(x^n)+56a^5b^3x^n+35a^4b^4x^{2n}+\frac{56}{3}a^3b^5x^{3n}+7a^2b^6x^{4n}+\frac{8}{5}ab^7x^{5n}+\frac{1}{6}b^8x^{6n}}{n}$$

input
```
Int[x^(-1 - 2*n)*(a + b*x^n)^8,x]
```

output
```
(-1/2*a^8/x^(2*n) - (8*a^7*b)/x^n + 56*a^5*b^3*x^n + 35*a^4*b^4*x^(2*n) +
(56*a^3*b^5*x^(3*n))/3 + 7*a^2*b^6*x^(4*n) + (8*a*b^7*x^(5*n))/5 + (b^8*x^
(6*n))/6 + 28*a^6*b^2*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

method	result
risch	$28a^6b^2\ln(x)+\frac{b^8x^{6n}}{6n}+\frac{8a\,b^7x^{5n}}{5n}+\frac{7a^2b^6x^{4n}}{n}+\frac{56a^3b^5x^{3n}}{3n}+\frac{35a^4b^4x^{2n}}{n}+\frac{56a^5b^3x^n}{n}-\frac{8a^7b\,x^{-n}}{n}-\frac{a^8x^{-2n}}{2n}$

input
```
int(x^(-2*n-1)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
28*a^6*b^2*ln(x)+1/6*b^8/n*(x^n)^6+8/5*a*b^7/n*(x^n)^5+7*a^2*b^6/n*(x^n)^4
+56/3*a^3*b^5/n*(x^n)^3+35*a^4*b^4/n*(x^n)^2+56*a^5*b^3*x^n/n-8*a^7*b/n/(x
^n)-1/2*a^8/n/(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}(a+bx^n)^8\,dx$$
$$=\frac{840\,a^6b^2nx^{2\,n}\log(x)+5\,b^8x^{8\,n}+48\,ab^7x^{7\,n}+210\,a^2b^6x^{6\,n}+560\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}+1680\,a^5b^3x^{3\,n}}{30\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/30*(840*a^6*b^2*n*x^(2*n)*log(x) + 5*b^8*x^(8*n) + 48*a*b^7*x^(7*n) + 21
0*a^2*b^6*x^(6*n) + 560*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 1680*a^5*
b^3*x^(3*n) - 240*a^7*b*x^n - 15*a^8)/(n*x^(2*n))
```

Sympy [A] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int x^{-1-2n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8x^{-2n}}{2n} - \frac{8a^7bx^{-n}}{n} + \frac{28a^6b^2\log(x^n)}{n} + \frac{56a^5b^3x^n}{n} + \frac{35a^4b^4x^{2n}}{n} + \frac{56a^3b^5x^{3n}}{3n} + \frac{7a^2b^6x^{4n}}{n} + \frac{8ab^7x^{5n}}{5n} + \frac{b^8x^{6n}}{6n} & \text{for } n \neq \\ (a+b)^8\log(x) & \text{otherw} \end{cases}$$

input
```
integrate(x**(-1-2*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(2*n*x**(2*n)) - 8*a**7*b/(n*x**n) + 28*a**6*b**2*log(x**
n)/n + 56*a**5*b**3*x**n/n + 35*a**4*b**4*x**(2*n)/n + 56*a**3*b**5*x**(3*
n)/(3*n) + 7*a**2*b**6*x**(4*n)/n + 8*a*b**7*x**(5*n)/(5*n) + b**8*x**(6*n
)/(6*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96

$$\int x^{-1-2n}(a+bx^n)^8\,dx = 28\,a^6b^2\log(x) + \frac{b^8x^{6\,n}}{6\,n} + \frac{8\,ab^7x^{5\,n}}{5\,n} + \frac{7\,a^2b^6x^{4\,n}}{n} + \frac{56\,a^3b^5x^{3\,n}}{3\,n} + \frac{35\,a^4b^4x^{2\,n}}{n} + \frac{56\,a^5b^3x^n}{n} - \frac{a^8}{2\,nx^{2\,n}} - \frac{8\,a^7b}{nx^n}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
28*a^6*b^2*log(x) + 1/6*b^8*x^(6*n)/n + 8/5*a*b^7*x^(5*n)/n + 7*a^2*b^6*x^
(4*n)/n + 56/3*a^3*b^5*x^(3*n)/n + 35*a^4*b^4*x^(2*n)/n + 56*a^5*b^3*x^n/n
 - 1/2*a^8/(n*x^(2*n)) - 8*a^7*b/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}(a+bx^n)^8\,dx = \frac{840\,a^6b^2nx^{2\,n}\log(x)+5\,b^8x^{8\,n}+48\,ab^7x^{7\,n}+210\,a^2b^6x^{6\,n}+560\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}+1680\,a^5b^3x^{3\,n}}{30\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/30*(840*a^6*b^2*n*x^(2*n)*log(x) + 5*b^8*x^(8*n) + 48*a*b^7*x^(7*n) + 21
0*a^2*b^6*x^(6*n) + 560*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 1680*a^5*
b^3*x^(3*n) - 240*a^7*b*x^n - 15*a^8)/(n*x^(2*n))
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96

$$\int x^{-1-2n}(a+bx^n)^8\,dx = \frac{b^8\,x^{6\,n}}{6\,n} - \frac{a^8}{2\,n\,x^{2\,n}} + 28\,a^6\,b^2\,\ln(x) + \frac{35\,a^4\,b^4\,x^{2\,n}}{n} + \frac{56\,a^3\,b^5\,x^{3\,n}}{3\,n} + \frac{7\,a^2\,b^6\,x^{4\,n}}{n} - \frac{8\,a^7\,b}{n\,x^n} + \frac{8\,a\,b^7\,x^{5\,n}}{5\,n} + \frac{56\,a^5\,b^3\,x^n}{n}$$

input
```
int((a + b*x^n)^8/x^(2*n + 1),x)
```

output
```
(b^8*x^(6*n))/(6*n) - a^8/(2*n*x^(2*n)) + 28*a^6*b^2*log(x) + (35*a^4*b^4*
x^(2*n))/n + (56*a^3*b^5*x^(3*n))/(3*n) + (7*a^2*b^6*x^(4*n))/n - (8*a^7*b
)/(n*x^n) + (8*a*b^7*x^(5*n))/(5*n) + (56*a^5*b^3*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-2n}(a+bx^n)^8\,dx$$
$$= \frac{5x^{8n}b^8 + 48x^{7n}a\,b^7 + 210x^{6n}a^2b^6 + 560x^{5n}a^3b^5 + 1050x^{4n}a^4b^4 + 1680x^{3n}a^5b^3 + 840x^{2n}\log(x)\,a^6b^2n - 24}{30x^{2n}n}$$

input
```
int(x^(-1-2*n)*(a+b*x^n)^8,x)
```

output
```
(5*x**(8*n)*b**8 + 48*x**(7*n)*a*b**7 + 210*x**(6*n)*a**2*b**6 + 560*x**(5
*n)*a**3*b**5 + 1050*x**(4*n)*a**4*b**4 + 1680*x**(3*n)*a**5*b**3 + 840*x*
*(2*n)*log(x)*a**6*b**2*n - 240*x**n*a**7*b - 15*a**8)/(30*x**(2*n)*n)
```

3.434 $\int x^{-1-3n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int x^{-1-3n}(a+bx^n)^8\,dx=-\frac{a^8x^{-3n}}{3n}-\frac{4a^7bx^{-2n}}{n}-\frac{28a^6b^2x^{-n}}{n}+\frac{70a^4b^4x^n}{n}+\frac{28a^3b^5x^{2n}}{n}+\frac{28a^2b^6x^{3n}}{3n}+\frac{2ab^7x^{4n}}{n}+\frac{b^8x^{5n}}{5n}+56a^5b^3\log(x)$$

output
```
-1/3*a^8/n/(x^(3*n))-4*a^7*b/n/(x^(2*n))-28*a^6*b^2/n/(x^n)+70*a^4*b^4*x^n
/n+28*a^3*b^5*x^(2*n)/n+28/3*a^2*b^6*x^(3*n)/n+2*a*b^7*x^(4*n)/n+1/5*b^8*x
^(5*n)/n+56*a^5*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-3n}(a+bx^n)^8\,dx=\frac{x^{-3n}(-5a^8-60a^7bx^n-420a^6b^2x^{2n}+1050a^4b^4x^{4n}+420a^3b^5x^{5n}+140a^2b^6x^{6n}+30ab^7x^{7n}+3b^8x^{8n})}{15n}+\frac{56a^5b^3\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 3*n)*(a + b*x^n)^8,x]
```

output
```
(-5*a^8 - 60*a^7*b*x^n - 420*a^6*b^2*x^(2*n) + 1050*a^4*b^4*x^(4*n) + 420*
a^3*b^5*x^(5*n) + 140*a^2*b^6*x^(6*n) + 30*a*b^7*x^(7*n) + 3*b^8*x^(8*n))/
(15*n*x^(3*n)) + (56*a^5*b^3*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-4n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^8x^{-4n}+8a^7bx^{-3n}+28a^6b^2x^{-2n}+56a^5b^3x^{-n}+56a^3b^5x^n+28a^2b^6x^{2n}+8ab^7x^{3n}+b^8x^{4n}+70a^4b^4\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^8x^{-3n}-4a^7bx^{-2n}-28a^6b^2x^{-n}+56a^5b^3\log(x^n)+70a^4b^4x^n+28a^3b^5x^{2n}+\frac{28}{3}a^2b^6x^{3n}+2ab^7x^{4n}+\frac{1}{5}b^8x^{5n}}{n}$$

input
```
Int[x^(-1 - 3*n)*(a + b*x^n)^8,x]
```

output
```
(-1/3*a^8/x^(3*n) - (4*a^7*b)/x^(2*n) - (28*a^6*b^2)/x^n + 70*a^4*b^4*x^n
+ 28*a^3*b^5*x^(2*n) + (28*a^2*b^6*x^(3*n))/3 + 2*a*b^7*x^(4*n) + (b^8*x^(
5*n))/5 + 56*a^5*b^3*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

method	result
risch	$56a^5b^3\ln(x)+\frac{b^8x^{5n}}{5n}+\frac{2a\,b^7x^{4n}}{n}+\frac{28a^2b^6x^{3n}}{3n}+\frac{28a^3b^5x^{2n}}{n}+\frac{70a^4b^4x^n}{n}-\frac{28a^6b^2x^{-n}}{n}-\frac{4a^7b\,x^{-2n}}{n}-\frac{a^8x^{-3n}}{3n}$

input
```
int(x^(-1-3*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
56*a^5*b^3*ln(x)+1/5*b^8/n*(x^n)^5+2*a*b^7/n*(x^n)^4+28/3*a^2*b^6/n*(x^n)^
3+28*a^3*b^5/n*(x^n)^2+70*a^4*b^4*x^n/n-28*a^6*b^2/n/(x^n)-4*a^7*b/n/(x^n)
^2-1/3*a^8/n/(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-3n}(a+bx^n)^8\,dx = \frac{840\,a^5b^3nx^{3\,n}\log(x)+3\,b^8x^{8\,n}+30\,ab^7x^{7\,n}+140\,a^2b^6x^{6\,n}+420\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}-420\,a^6b^2x^{2\,n}}{15\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/15*(840*a^5*b^3*n*x^(3*n)*log(x) + 3*b^8*x^(8*n) + 30*a*b^7*x^(7*n) + 14
0*a^2*b^6*x^(6*n) + 420*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) - 420*a^6*b
^2*x^(2*n) - 60*a^7*b*x^n - 5*a^8)/(n*x^(3*n))
```

Sympy [A] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int x^{-1-3n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8x^{-3n}}{3n} - \frac{4a^7bx^{-2n}}{n} - \frac{28a^6b^2x^{-n}}{n} + \frac{56a^5b^3\log{(x^n)}}{n} + \frac{70a^4b^4x^n}{n} + \frac{28a^3b^5x^{2n}}{n} + \frac{28a^2b^6x^{3n}}{3n} + \frac{2ab^7x^{4n}}{n} + \frac{b^8x^{5n}}{5n} & \text{for } n \\ (a+b)^8\log{(x)} & \text{other} \end{cases}$$

input
```
integrate(x**(-1-3*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(3*n*x**(3*n)) - 4*a**7*b/(n*x**(2*n)) - 28*a**6*b**2/(n*
x**n) + 56*a**5*b**3*log(x**n)/n + 70*a**4*b**4*x**n/n + 28*a**3*b**5*x**(
2*n)/n + 28*a**2*b**6*x**(3*n)/(3*n) + 2*a*b**7*x**(4*n)/n + b**8*x**(5*n)
/(5*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int x^{-1-3n}(a+bx^n)^8\,dx = 56\,a^5b^3\log{(x)} + \frac{b^8x^{5\,n}}{5\,n} + \frac{2\,ab^7x^{4\,n}}{n} + \frac{28\,a^2b^6x^{3\,n}}{3\,n} + \frac{28\,a^3b^5x^{2\,n}}{n} + \frac{70\,a^4b^4x^n}{n} - \frac{a^8}{3\,nx^{3\,n}} - \frac{4\,a^7b}{nx^{2\,n}} - \frac{28\,a^6b^2}{nx^n}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
56*a^5*b^3*log(x) + 1/5*b^8*x^(5*n)/n + 2*a*b^7*x^(4*n)/n + 28/3*a^2*b^6*x
^(3*n)/n + 28*a^3*b^5*x^(2*n)/n + 70*a^4*b^4*x^n/n - 1/3*a^8/(n*x^(3*n)) -
 4*a^7*b/(n*x^(2*n)) - 28*a^6*b^2/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-3n}(a+bx^n)^8\,dx$$
$$= \frac{840\,a^5b^3nx^{3\,n}\log{(x)} + 3\,b^8x^{8\,n} + 30\,ab^7x^{7\,n} + 140\,a^2b^6x^{6\,n} + 420\,a^3b^5x^{5\,n} + 1050\,a^4b^4x^{4\,n} - 420\,a^6b^2x^{2\,n} -}{15\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/15*(840*a^5*b^3*n*x^(3*n)*log(x) + 3*b^8*x^(8*n) + 30*a*b^7*x^(7*n) + 14
0*a^2*b^6*x^(6*n) + 420*a^3*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) - 420*a^6*b
^2*x^(2*n) - 60*a^7*b*x^n - 5*a^8)/(n*x^(3*n))
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int x^{-1-3n}(a+bx^n)^8\,dx = \frac{b^8\,x^{5\,n}}{5\,n} - \frac{a^8}{3\,n\,x^{3\,n}} + 56\,a^5\,b^3\,\ln{(x)} - \frac{28\,a^6\,b^2}{n\,x^n} + \frac{28\,a^3\,b^5\,x^{2\,n}}{n} + \frac{28\,a^2\,b^6\,x^{3\,n}}{3\,n} - \frac{4\,a^7\,b}{n\,x^{2\,n}} + \frac{2\,a\,b^7\,x^{4\,n}}{n} + \frac{70\,a^4\,b^4\,x^n}{n}$$

input
```
int((a + b*x^n)^8/x^(3*n + 1),x)
```

output
```
(b^8*x^(5*n))/(5*n) - a^8/(3*n*x^(3*n)) + 56*a^5*b^3*log(x) - (28*a^6*b^2)
/(n*x^n) + (28*a^3*b^5*x^(2*n))/n + (28*a^2*b^6*x^(3*n))/(3*n) - (4*a^7*b)
/(n*x^(2*n)) + (2*a*b^7*x^(4*n))/n + (70*a^4*b^4*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-3n}(a+bx^n)^8\,dx$$
$$=\frac{3x^{8n}b^8+30x^{7n}a\,b^7+140x^{6n}a^2b^6+420x^{5n}a^3b^5+1050x^{4n}a^4b^4+840x^{3n}\log(x)\,a^5b^3n-420x^{2n}a^6b^2-60x}{15x^{3n}n}$$

input
```
int(x^(-1-3*n)*(a+b*x^n)^8,x)
```

output
```
(3*x**(8*n)*b**8 + 30*x**(7*n)*a*b**7 + 140*x**(6*n)*a**2*b**6 + 420*x**(5
*n)*a**3*b**5 + 1050*x**(4*n)*a**4*b**4 + 840*x**(3*n)*log(x)*a**5*b**3*n
- 420*x**(2*n)*a**6*b**2 - 60*x**n*a**7*b - 5*a**8)/(15*x**(3*n)*n)
```

3.435 $\int x^{-1-4n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 135

$$\int x^{-1-4n}(a+bx^n)^8\,dx = -\frac{a^8x^{-4n}}{4n}-\frac{8a^7bx^{-3n}}{3n}-\frac{14a^6b^2x^{-2n}}{n}-\frac{56a^5b^3x^{-n}}{n}+\frac{56a^3b^5x^n}{n}+\frac{14a^2b^6x^{2n}}{n}+\frac{8ab^7x^{3n}}{3n}+\frac{b^8x^{4n}}{4n}+70a^4b^4\log(x)$$

output
```
-1/4*a^8/n/(x^(4*n))-8/3*a^7*b/n/(x^(3*n))-14*a^6*b^2/n/(x^(2*n))-56*a^5*b
^3/n/(x^n)+56*a^3*b^5*x^n/n+14*a^2*b^6*x^(2*n)/n+8/3*a*b^7*x^(3*n)/n+1/4*b
^8*x^(4*n)/n+70*a^4*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-4n}(a+bx^n)^8\,dx = \frac{x^{-4n}(-3a^8-32a^7bx^n-168a^6b^2x^{2n}-672a^5b^3x^{3n}+672a^3b^5x^{5n}+168a^2b^6x^{6n}+32ab^7x^{7n}+3b^8x^{8n})}{12n}+\frac{70a^4b^4\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 4*n)*(a + b*x^n)^8,x]
```

output
```
(-3*a^8 - 32*a^7*b*x^n - 168*a^6*b^2*x^(2*n) - 672*a^5*b^3*x^(3*n) + 672*a
^3*b^5*x^(5*n) + 168*a^2*b^6*x^(6*n) + 32*a*b^7*x^(7*n) + 3*b^8*x^(8*n))/(
12*n*x^(4*n)) + (70*a^4*b^4*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-4n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-5n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^8x^{-5n}+8a^7bx^{-4n}+28a^6b^2x^{-3n}+56a^5b^3x^{-2n}+70a^4b^4x^{-n}+28a^2b^6x^n+8ab^7x^{2n}+b^8x^{3n}+56a^3b^5\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}a^8x^{-4n}-\frac{8}{3}a^7bx^{-3n}-14a^6b^2x^{-2n}-56a^5b^3x^{-n}+70a^4b^4\log(x^n)+56a^3b^5x^n+14a^2b^6x^{2n}+\frac{8}{3}ab^7x^{3n}+\frac{1}{4}b^8x}{n}$$

input
```
Int[x^(-1 - 4*n)*(a + b*x^n)^8,x]
```

output
```
(-1/4*a^8/x^(4*n) - (8*a^7*b)/(3*x^(3*n)) - (14*a^6*b^2)/x^(2*n) - (56*a^5
*b^3)/x^n + 56*a^3*b^5*x^n + 14*a^2*b^6*x^(2*n) + (8*a*b^7*x^(3*n))/3 + (b
^8*x^(4*n))/4 + 70*a^4*b^4*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

method	result
risch	$70a^4b^4\ln(x)+\frac{b^8x^{4n}}{4n}+\frac{8a\,b^7x^{3n}}{3n}+\frac{14a^2b^6x^{2n}}{n}+\frac{56a^3b^5x^n}{n}-\frac{56a^5b^3x^{-n}}{n}-\frac{14a^6b^2x^{-2n}}{n}-\frac{8a^7b\,x^{-3n}}{3n}-\frac{a^8x^{-4n}}{4n}$

input
```
int(x^(-4*n-1)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
70*a^4*b^4*ln(x)+1/4*b^8/n*(x^n)^4+8/3*a*b^7/n*(x^n)^3+14*a^2*b^6/n*(x^n)^
2+56*a^3*b^5*x^n/n-56*a^5*b^3/n/(x^n)-14*a^6*b^2/n/(x^n)^2-8/3*a^7*b/n/(x^
n)^3-1/4*a^8/n/(x^n)^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-4n}(a+bx^n)^8\,dx$$
$$=\frac{840\,a^4b^4nx^{4\,n}\log(x)+3\,b^8x^{8\,n}+32\,ab^7x^{7\,n}+168\,a^2b^6x^{6\,n}+672\,a^3b^5x^{5\,n}-672\,a^5b^3x^{3\,n}-168\,a^6b^2x^{2\,n}-\cdots}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/12*(840*a^4*b^4*n*x^(4*n)*log(x) + 3*b^8*x^(8*n) + 32*a*b^7*x^(7*n) + 16
8*a^2*b^6*x^(6*n) + 672*a^3*b^5*x^(5*n) - 672*a^5*b^3*x^(3*n) - 168*a^6*b^
2*x^(2*n) - 32*a^7*b*x^n - 3*a^8)/(n*x^(4*n))
```

Sympy [A] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int x^{-1-4n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8x^{-4n}}{4n} - \frac{8a^7bx^{-3n}}{3n} - \frac{14a^6b^2x^{-2n}}{n} - \frac{56a^5b^3x^{-n}}{n} + \frac{70a^4b^4\log(x^n)}{n} + \frac{56a^3b^5x^n}{n} + \frac{14a^2b^6x^{2n}}{n} + \frac{8ab^7x^{3n}}{3n} + \frac{b^8x^{4n}}{4n} & \text{for } \\ (a+b)^8\log(x) & \text{oth} \end{cases}$$

input
```
integrate(x**(-1-4*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(4*n*x**(4*n)) - 8*a**7*b/(3*n*x**(3*n)) - 14*a**6*b**2/(
n*x**(2*n)) - 56*a**5*b**3/(n*x**n) + 70*a**4*b**4*log(x**n)/n + 56*a**3*b
**5*x**n/n + 14*a**2*b**6*x**(2*n)/n + 8*a*b**7*x**(3*n)/(3*n) + b**8*x**(
4*n)/(4*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int x^{-1-4n}(a+bx^n)^8\,dx = 70\,a^4b^4\log(x) + \frac{b^8x^{4\,n}}{4\,n} + \frac{8\,ab^7x^{3\,n}}{3\,n} + \frac{14\,a^2b^6x^{2\,n}}{n} + \frac{56\,a^3b^5x^n}{n} - \frac{a^8}{4\,nx^{4\,n}} - \frac{8\,a^7b}{3\,nx^{3\,n}} - \frac{14\,a^6b^2}{nx^{2\,n}} - \frac{56\,a^5b^3}{nx^n}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
70*a^4*b^4*log(x) + 1/4*b^8*x^(4*n)/n + 8/3*a*b^7*x^(3*n)/n + 14*a^2*b^6*x
^(2*n)/n + 56*a^3*b^5*x^n/n - 1/4*a^8/(n*x^(4*n)) - 8/3*a^7*b/(n*x^(3*n))
- 14*a^6*b^2/(n*x^(2*n)) - 56*a^5*b^3/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-4n}(a+bx^n)^8\,dx$$
$$= \frac{840\,a^4b^4nx^{4\,n}\log{(x)} + 3\,b^8x^{8\,n} + 32\,ab^7x^{7\,n} + 168\,a^2b^6x^{6\,n} + 672\,a^3b^5x^{5\,n} - 672\,a^5b^3x^{3\,n} - 168\,a^6b^2x^{2\,n} -}{12\,nx^{4\,n}}$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/12*(840*a^4*b^4*n*x^(4*n)*log(x) + 3*b^8*x^(8*n) + 32*a*b^7*x^(7*n) + 16
8*a^2*b^6*x^(6*n) + 672*a^3*b^5*x^(5*n) - 672*a^5*b^3*x^(3*n) - 168*a^6*b^
2*x^(2*n) - 32*a^7*b*x^n - 3*a^8)/(n*x^(4*n))
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int x^{-1-4n}(a+bx^n)^8\,dx = \frac{b^8\,x^{4\,n}}{4\,n} - \frac{a^8}{4\,n\,x^{4\,n}} + 70\,a^4\,b^4\,\ln{(x)} - \frac{56\,a^5\,b^3}{n\,x^n} + \frac{14\,a^2\,b^6\,x^{2\,n}}{n} - \frac{14\,a^6\,b^2}{n\,x^{2\,n}} + \frac{8\,a\,b^7\,x^{3\,n}}{3\,n} - \frac{8\,a^7\,b}{3\,n\,x^{3\,n}} + \frac{56\,a^3\,b^5\,x^n}{n}$$

input
```
int((a + b*x^n)^8/x^(4*n + 1),x)
```

output
```
(b^8*x^(4*n))/(4*n) - a^8/(4*n*x^(4*n)) + 70*a^4*b^4*log(x) - (56*a^5*b^3)
/(n*x^n) + (14*a^2*b^6*x^(2*n))/n - (14*a^6*b^2)/(n*x^(2*n)) + (8*a*b^7*x^
(3*n))/(3*n) - (8*a^7*b)/(3*n*x^(3*n)) + (56*a^3*b^5*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-4n}(a+bx^n)^8\,dx$$
$$=\frac{3x^{8n}b^8+32x^{7n}a\,b^7+168x^{6n}a^2b^6+672x^{5n}a^3b^5+840x^{4n}\log(x)\,a^4b^4n-672x^{3n}a^5b^3-168x^{2n}a^6b^2-32x}{12x^{4n}n}$$

input
```
int(x^(-1-4*n)*(a+b*x^n)^8,x)
```

output
```
(3*x**(8*n)*b**8 + 32*x**(7*n)*a*b**7 + 168*x**(6*n)*a**2*b**6 + 672*x**(5
*n)*a**3*b**5 + 840*x**(4*n)*log(x)*a**4*b**4*n - 672*x**(3*n)*a**5*b**3 -
 168*x**(2*n)*a**6*b**2 - 32*x**n*a**7*b - 3*a**8)/(12*x**(4*n)*n)
```

3.436 $\int x^{-1-5n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 133

$$\int x^{-1-5n}(a+bx^n)^8\,dx=-\frac{a^8x^{-5n}}{5n}-\frac{2a^7bx^{-4n}}{n}-\frac{28a^6b^2x^{-3n}}{3n}-\frac{28a^5b^3x^{-2n}}{n}-\frac{70a^4b^4x^{-n}}{n}+\frac{28a^2b^6x^n}{n}+\frac{4ab^7x^{2n}}{n}+\frac{b^8x^{3n}}{3n}+56a^3b^5\log(x)$$

output
```
-1/5*a^8/n/(x^(5*n))-2*a^7*b/n/(x^(4*n))-28/3*a^6*b^2/n/(x^(3*n))-28*a^5*b
^3/n/(x^(2*n))-70*a^4*b^4/n/(x^n)+28*a^2*b^6*x^n/n+4*a*b^7*x^(2*n)/n+1/3*b
^8*x^(3*n)/n+56*a^3*b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-5n}(a+bx^n)^8\,dx=\frac{x^{-5n}(-3a^8-30a^7bx^n-140a^6b^2x^{2n}-420a^5b^3x^{3n}-1050a^4b^4x^{4n}+420a^2b^6x^{6n}+60ab^7x^{7n}+5b^8x^{8n})}{15n}+\frac{56a^3b^5\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 5*n)*(a + b*x^n)^8,x]
```

output
```
(-3*a^8 - 30*a^7*b*x^n - 140*a^6*b^2*x^(2*n) - 420*a^5*b^3*x^(3*n) - 1050*
a^4*b^4*x^(4*n) + 420*a^2*b^6*x^(6*n) + 60*a*b^7*x^(7*n) + 5*b^8*x^(8*n))/
(15*n*x^(5*n)) + (56*a^3*b^5*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-5n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-6n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^8x^{-6n}+8a^7bx^{-5n}+28a^6b^2x^{-4n}+56a^5b^3x^{-3n}+70a^4b^4x^{-2n}+56a^3b^5x^{-n}+8ab^7x^n+b^8x^{2n}+28a^2b^6\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{5}a^8x^{-5n}-2a^7bx^{-4n}-\frac{28}{3}a^6b^2x^{-3n}-28a^5b^3x^{-2n}-70a^4b^4x^{-n}+56a^3b^5\log\left(x^n\right)+28a^2b^6x^n+4ab^7x^{2n}+\frac{1}{3}b^8}{n}$$

input
```
Int[x^(-1 - 5*n)*(a + b*x^n)^8,x]
```

output
```
(-1/5*a^8/x^(5*n) - (2*a^7*b)/x^(4*n) - (28*a^6*b^2)/(3*x^(3*n)) - (28*a^5
*b^3)/x^(2*n) - (70*a^4*b^4)/x^n + 28*a^2*b^6*x^n + 4*a*b^7*x^(2*n) + (b^8
*x^(3*n))/3 + 56*a^3*b^5*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

method	result
risch	$56a^3b^5\ln(x)+\frac{b^8x^{3n}}{3n}+\frac{4a\,b^7x^{2n}}{n}+\frac{28a^2b^6x^n}{n}-\frac{70a^4b^4x^{-n}}{n}-\frac{28a^5b^3x^{-2n}}{n}-\frac{28a^6b^2x^{-3n}}{3n}-\frac{2a^7b\,x^{-4n}}{n}-\frac{a^8x^{-}}{5n}$

input
```
int(x^(-1-5*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
56*a^3*b^5*ln(x)+1/3*b^8/n*(x^n)^3+4*a*b^7/n*(x^n)^2+28*a^2*b^6*x^n/n-70*a
^4*b^4/n/(x^n)-28*a^5*b^3/n/(x^n)^2-28/3*a^6*b^2/n/(x^n)^3-2*a^7*b/n/(x^n)
^4-1/5*a^8/n/(x^n)^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-5n}(a+bx^n)^8\,dx$$
$$=\frac{840\,a^3b^5nx^{5\,n}\log(x)+5\,b^8x^{8\,n}+60\,ab^7x^{7\,n}+420\,a^2b^6x^{6\,n}-1050\,a^4b^4x^{4\,n}-420\,a^5b^3x^{3\,n}-140\,a^6b^2x^{2\,n}}{15\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/15*(840*a^3*b^5*n*x^(5*n)*log(x) + 5*b^8*x^(8*n) + 60*a*b^7*x^(7*n) + 42
0*a^2*b^6*x^(6*n) - 1050*a^4*b^4*x^(4*n) - 420*a^5*b^3*x^(3*n) - 140*a^6*b
^2*x^(2*n) - 30*a^7*b*x^n - 3*a^8)/(n*x^(5*n))
```

Sympy [A] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int x^{-1-5n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8x^{-5n}}{5n} - \frac{2a^7bx^{-4n}}{n} - \frac{28a^6b^2x^{-3n}}{3n} - \frac{28a^5b^3x^{-2n}}{n} - \frac{70a^4b^4x^{-n}}{n} + \frac{56a^3b^5\log(x^n)}{n} + \frac{28a^2b^6x^n}{n} + \frac{4ab^7x^{2n}}{n} + \frac{b^8x^{3n}}{3n} & \text{fo} \\ (a+b)^8\log(x) & \text{ot} \end{cases}$$

input
```
integrate(x**(-1-5*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(5*n*x**(5*n)) - 2*a**7*b/(n*x**(4*n)) - 28*a**6*b**2/(3*
n*x**(3*n)) - 28*a**5*b**3/(n*x**(2*n)) - 70*a**4*b**4/(n*x**n) + 56*a**3*
b**5*log(x**n)/n + 28*a**2*b**6*x**n/n + 4*a*b**7*x**(2*n)/n + b**8*x**(3*
n)/(3*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int x^{-1-5n}(a+bx^n)^8\,dx = 56\,a^3b^5\log(x) + \frac{b^8x^{3\,n}}{3\,n} + \frac{4\,ab^7x^{2\,n}}{n} + \frac{28\,a^2b^6x^n}{n} - \frac{a^8}{5\,nx^{5\,n}} - \frac{2\,a^7b}{nx^{4\,n}} - \frac{28\,a^6b^2}{3\,nx^{3\,n}} - \frac{28\,a^5b^3}{nx^{2\,n}} - \frac{70\,a^4b^4}{nx^n}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
56*a^3*b^5*log(x) + 1/3*b^8*x^(3*n)/n + 4*a*b^7*x^(2*n)/n + 28*a^2*b^6*x^n
/n - 1/5*a^8/(n*x^(5*n)) - 2*a^7*b/(n*x^(4*n)) - 28/3*a^6*b^2/(n*x^(3*n))
- 28*a^5*b^3/(n*x^(2*n)) - 70*a^4*b^4/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-5n}(a+bx^n)^8\,dx = \frac{840\,a^3b^5nx^{5\,n}\log\left(x\right)+5\,b^8x^{8\,n}+60\,ab^7x^{7\,n}+420\,a^2b^6x^{6\,n}-1050\,a^4b^4x^{4\,n}-420\,a^5b^3x^{3\,n}-140\,a^6b^2x^{2\,n}}{15\,nx^{5\,n}}$$

input
```
integrate(x^(-1-5*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/15*(840*a^3*b^5*n*x^(5*n)*log(x) + 5*b^8*x^(8*n) + 60*a*b^7*x^(7*n) + 42
0*a^2*b^6*x^(6*n) - 1050*a^4*b^4*x^(4*n) - 420*a^5*b^3*x^(3*n) - 140*a^6*b
^2*x^(2*n) - 30*a^7*b*x^n - 3*a^8)/(n*x^(5*n))
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int x^{-1-5n}(a+bx^n)^8\,dx = \frac{b^8\,x^{3\,n}}{3\,n}-\frac{a^8}{5\,n\,x^{5\,n}}+56\,a^3\,b^5\,\ln\left(x\right)-\frac{70\,a^4\,b^4}{n\,x^n}-\frac{28\,a^5\,b^3}{n\,x^{2\,n}}-\frac{28\,a^6\,b^2}{3\,n\,x^{3\,n}}+\frac{4\,a\,b^7\,x^{2\,n}}{n}-\frac{2\,a^7\,b}{n\,x^{4\,n}}+\frac{28\,a^2\,b^6\,x^n}{n}$$

input
```
int((a + b*x^n)^8/x^(5*n + 1),x)
```

output
```
(b^8*x^(3*n))/(3*n) - a^8/(5*n*x^(5*n)) + 56*a^3*b^5*log(x) - (70*a^4*b^4)
/(n*x^n) - (28*a^5*b^3)/(n*x^(2*n)) - (28*a^6*b^2)/(3*n*x^(3*n)) + (4*a*b^
7*x^(2*n))/n - (2*a^7*b)/(n*x^(4*n)) + (28*a^2*b^6*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-5n}(a+bx^n)^8\,dx$$
$$= \frac{5x^{8n}b^8 + 60x^{7n}a\,b^7 + 420x^{6n}a^2b^6 + 840x^{5n}\log(x)\,a^3b^5n - 1050x^{4n}a^4b^4 - 420x^{3n}a^5b^3 - 140x^{2n}a^6b^2 - 30x}{15x^{5n}n}$$

input
```
int(x^(-1-5*n)*(a+b*x^n)^8,x)
```

output
```
(5*x**(8*n)*b**8 + 60*x**(7*n)*a*b**7 + 420*x**(6*n)*a**2*b**6 + 840*x**(5
*n)*log(x)*a**3*b**5*n - 1050*x**(4*n)*a**4*b**4 - 420*x**(3*n)*a**5*b**3
- 140*x**(2*n)*a**6*b**2 - 30*x**n*a**7*b - 3*a**8)/(15*x**(5*n)*n)
```

3.437 $\int x^{-1-6n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 135

$$\int x^{-1-6n}(a+bx^n)^8\,dx = -\frac{a^8x^{-6n}}{6n}-\frac{8a^7bx^{-5n}}{5n}-\frac{7a^6b^2x^{-4n}}{n}-\frac{56a^5b^3x^{-3n}}{3n}-\frac{35a^4b^4x^{-2n}}{n}-\frac{56a^3b^5x^{-n}}{n}+\frac{8ab^7x^n}{n}+\frac{b^8x^{2n}}{2n}+28a^2b^6\log(x)$$

output
```
-1/6*a^8/n/(x^(6*n))-8/5*a^7*b/n/(x^(5*n))-7*a^6*b^2/n/(x^(4*n))-56/3*a^5*
b^3/n/(x^(3*n))-35*a^4*b^4/n/(x^(2*n))-56*a^3*b^5/n/(x^n)+8*a*b^7*x^n/n+1/
2*b^8*x^(2*n)/n+28*a^2*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-6n}(a+bx^n)^8\,dx = \frac{x^{-6n}(-5a^8-48a^7bx^n-210a^6b^2x^{2n}-560a^5b^3x^{3n}-1050a^4b^4x^{4n}-1680a^3b^5x^{5n}+240ab^7x^{7n}+15b^8x^{8n}}{30n}+\frac{28a^2b^6\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 6*n)*(a + b*x^n)^8,x]
```

output
```
(-5*a^8 - 48*a^7*b*x^n - 210*a^6*b^2*x^(2*n) - 560*a^5*b^3*x^(3*n) - 1050*
a^4*b^4*x^(4*n) - 1680*a^3*b^5*x^(5*n) + 240*a*b^7*x^(7*n) + 15*b^8*x^(8*n
))/(30*n*x^(6*n)) + (28*a^2*b^6*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-6n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-7n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^8x^{-7n}+8a^7bx^{-6n}+28a^6b^2x^{-5n}+56a^5b^3x^{-4n}+70a^4b^4x^{-3n}+56a^3b^5x^{-2n}+28a^2b^6x^{-n}+b^8x^n+8ab^7\right)dx}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{6}a^8x^{-6n}-\frac{8}{5}a^7bx^{-5n}-7a^6b^2x^{-4n}-\frac{56}{3}a^5b^3x^{-3n}-35a^4b^4x^{-2n}-56a^3b^5x^{-n}+28a^2b^6\log(x^n)+8ab^7x^n+\frac{1}{2}b^8}{n}$$

input
```
Int[x^(-1 - 6*n)*(a + b*x^n)^8,x]
```

output
```
(-1/6*a^8/x^(6*n) - (8*a^7*b)/(5*x^(5*n)) - (7*a^6*b^2)/x^(4*n) - (56*a^5*
b^3)/(3*x^(3*n)) - (35*a^4*b^4)/x^(2*n) - (56*a^3*b^5)/x^n + 8*a*b^7*x^n +
 (b^8*x^(2*n))/2 + 28*a^2*b^6*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

method	result
risch	$28a^2b^6\ln(x)+\frac{b^8x^{2n}}{2n}+\frac{8a\,b^7x^n}{n}-\frac{56a^3b^5x^{-n}}{n}-\frac{35a^4b^4x^{-2n}}{n}-\frac{56a^5b^3x^{-3n}}{3n}-\frac{7a^6b^2x^{-4n}}{n}-\frac{8a^7b\,x^{-5n}}{5n}-\frac{a^8x^{-}}{6n}$

input
```
int(x^(-1-6*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
28*a^2*b^6*ln(x)+1/2*b^8/n*(x^n)^2+8*a*b^7*x^n/n-56*a^3*b^5/n/(x^n)-35*a^4
*b^4/n/(x^n)^2-56/3*a^5*b^3/n/(x^n)^3-7*a^6*b^2/n/(x^n)^4-8/5*a^7*b/n/(x^n
)^5-1/6*a^8/n/(x^n)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-6n}(a+bx^n)^8\,dx$$
$$=\frac{840\,a^2b^6nx^{6\,n}\log(x)+15\,b^8x^{8\,n}+240\,ab^7x^{7\,n}-1680\,a^3b^5x^{5\,n}-1050\,a^4b^4x^{4\,n}-560\,a^5b^3x^{3\,n}-210\,a^6b^2x}{30\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/30*(840*a^2*b^6*n*x^(6*n)*log(x) + 15*b^8*x^(8*n) + 240*a*b^7*x^(7*n) -
1680*a^3*b^5*x^(5*n) - 1050*a^4*b^4*x^(4*n) - 560*a^5*b^3*x^(3*n) - 210*a^
6*b^2*x^(2*n) - 48*a^7*b*x^n - 5*a^8)/(n*x^(6*n))
```

Sympy [A] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\int x^{-1-6n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8x^{-6n}}{6n} - \frac{8a^7bx^{-5n}}{5n} - \frac{7a^6b^2x^{-4n}}{n} - \frac{56a^5b^3x^{-3n}}{3n} - \frac{35a^4b^4x^{-2n}}{n} - \frac{56a^3b^5x^{-n}}{n} + 28a^2b^6\log(x) + \frac{8ab^7x^n}{n} + \frac{b^8x^{2n}}{2n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-6*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(6*n*x**(6*n)) - 8*a**7*b/(5*n*x**(5*n)) - 7*a**6*b**2/(n
*x**(4*n)) - 56*a**5*b**3/(3*n*x**(3*n)) - 35*a**4*b**4/(n*x**(2*n)) - 56*
a**3*b**5/(n*x**n) + 28*a**2*b**6*log(x) + 8*a*b**7*x**n/n + b**8*x**(2*n)
/(2*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int x^{-1-6n}(a+bx^n)^8\,dx = 28\,a^2b^6\log(x) + \frac{b^8x^{2\,n}}{2\,n} + \frac{8\,ab^7x^n}{n} - \frac{a^8}{6\,nx^{6\,n}} - \frac{8\,a^7b}{5\,nx^{5\,n}} - \frac{7\,a^6b^2}{nx^{4\,n}} - \frac{56\,a^5b^3}{3\,nx^{3\,n}} - \frac{35\,a^4b^4}{nx^{2\,n}} - \frac{56\,a^3b^5}{nx^n}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
28*a^2*b^6*log(x) + 1/2*b^8*x^(2*n)/n + 8*a*b^7*x^n/n - 1/6*a^8/(n*x^(6*n)
) - 8/5*a^7*b/(n*x^(5*n)) - 7*a^6*b^2/(n*x^(4*n)) - 56/3*a^5*b^3/(n*x^(3*n
)) - 35*a^4*b^4/(n*x^(2*n)) - 56*a^3*b^5/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-6n}(a+bx^n)^8\,dx = \frac{840\,a^2b^6nx^{6\,n}\log(x)+15\,b^8x^{8\,n}+240\,ab^7x^{7\,n}-1680\,a^3b^5x^{5\,n}-1050\,a^4b^4x^{4\,n}-560\,a^5b^3x^{3\,n}-210\,a^6b^2x}{30\,nx^{6\,n}}$$

input
```
integrate(x^(-1-6*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/30*(840*a^2*b^6*n*x^(6*n)*log(x) + 15*b^8*x^(8*n) + 240*a*b^7*x^(7*n) -
1680*a^3*b^5*x^(5*n) - 1050*a^4*b^4*x^(4*n) - 560*a^5*b^3*x^(3*n) - 210*a^
6*b^2*x^(2*n) - 48*a^7*b*x^n - 5*a^8)/(n*x^(6*n))
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int x^{-1-6n}(a+bx^n)^8\,dx = \frac{b^8\,x^{2\,n}}{2\,n}-\frac{a^8}{6\,n\,x^{6\,n}}+28\,a^2\,b^6\,\ln(x)-\frac{56\,a^3\,b^5}{n\,x^n}-\frac{35\,a^4\,b^4}{n\,x^{2\,n}}-\frac{56\,a^5\,b^3}{3\,n\,x^{3\,n}}-\frac{7\,a^6\,b^2}{n\,x^{4\,n}}+\frac{8\,a\,b^7\,x^n}{n}-\frac{8\,a^7\,b}{5\,n\,x^{5\,n}}$$

input
```
int((a + b*x^n)^8/x^(6*n + 1),x)
```

output
```
(b^8*x^(2*n))/(2*n) - a^8/(6*n*x^(6*n)) + 28*a^2*b^6*log(x) - (56*a^3*b^5)
/(n*x^n) - (35*a^4*b^4)/(n*x^(2*n)) - (56*a^5*b^3)/(3*n*x^(3*n)) - (7*a^6*
b^2)/(n*x^(4*n)) + (8*a*b^7*x^n)/n - (8*a^7*b)/(5*n*x^(5*n))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86

$$\int x^{-1-6n}(a+bx^n)^8\,dx$$
$$= \frac{15x^{8n}b^8 + 240x^{7n}a\,b^7 + 840x^{6n}\log(x)\,a^2b^6n - 1680x^{5n}a^3b^5 - 1050x^{4n}a^4b^4 - 560x^{3n}a^5b^3 - 210x^{2n}a^6b^2 -}{30x^{6n}n}$$

input
```
int(x^(-1-6*n)*(a+b*x^n)^8,x)
```

output
```
(15*x**(8*n)*b**8 + 240*x**(7*n)*a*b**7 + 840*x**(6*n)*log(x)*a**2*b**6*n
- 1680*x**(5*n)*a**3*b**5 - 1050*x**(4*n)*a**4*b**4 - 560*x**(3*n)*a**5*b*
*3 - 210*x**(2*n)*a**6*b**2 - 48*x**n*a**7*b - 5*a**8)/(30*x**(6*n)*n)
```

3.438 $\int x^{-1-7n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int x^{-1-7n}(a+bx^n)^8\,dx = -\frac{a^8x^{-7n}}{7n}-\frac{4a^7bx^{-6n}}{3n}-\frac{28a^6b^2x^{-5n}}{5n}-\frac{14a^5b^3x^{-4n}}{n}-\frac{70a^4b^4x^{-3n}}{3n}-\frac{28a^3b^5x^{-2n}}{n}-\frac{28a^2b^6x^{-n}}{n}+\frac{b^8x^n}{n}+8ab^7\log(x)$$

output
```
-1/7*a^8/n/(x^(7*n))-4/3*a^7*b/n/(x^(6*n))-28/5*a^6*b^2/n/(x^(5*n))-14*a^5
*b^3/n/(x^(4*n))-70/3*a^4*b^4/n/(x^(3*n))-28*a^3*b^5/n/(x^(2*n))-28*a^2*b^
6/n/(x^n)+b^8*x^n/n+8*a*b^7*ln(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-7n}(a+bx^n)^8\,dx = \frac{x^{-7n}(-15a^8-140a^7bx^n-588a^6b^2x^{2n}-1470a^5b^3x^{3n}-2450a^4b^4x^{4n}-2940a^3b^5x^{5n}-2940a^2b^6x^{6n}+10}{105n}+\frac{8ab^7\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 7*n)*(a + b*x^n)^8,x]
```

output
```
(-15*a^8 - 140*a^7*b*x^n - 588*a^6*b^2*x^(2*n) - 1470*a^5*b^3*x^(3*n) - 24
50*a^4*b^4*x^(4*n) - 2940*a^3*b^5*x^(5*n) - 2940*a^2*b^6*x^(6*n) + 105*b^8
*x^(8*n))/(105*n*x^(7*n)) + (8*a*b^7*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-7n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int x^{-8n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(a^8x^{-8n}+8a^7bx^{-7n}+28a^6b^2x^{-6n}+56a^5b^3x^{-5n}+70a^4b^4x^{-4n}+56a^3b^5x^{-3n}+28a^2b^6x^{-2n}+8ab^7x^{-n}+b^8\right)}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{7}a^8x^{-7n}-\frac{4}{3}a^7bx^{-6n}-\frac{28}{5}a^6b^2x^{-5n}-14a^5b^3x^{-4n}-\frac{70}{3}a^4b^4x^{-3n}-28a^3b^5x^{-2n}-28a^2b^6x^{-n}+8ab^7\log(x^n)+}{n}$$

input
```
Int[x^(-1 - 7*n)*(a + b*x^n)^8,x]
```

output
```
(-1/7*a^8/x^(7*n) - (4*a^7*b)/(3*x^(6*n)) - (28*a^6*b^2)/(5*x^(5*n)) - (14
*a^5*b^3)/x^(4*n) - (70*a^4*b^4)/(3*x^(3*n)) - (28*a^3*b^5)/x^(2*n) - (28*
a^2*b^6)/x^n + b^8*x^n + 8*a*b^7*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

method	result
risch	$8a\,b^7\ln\left(x\right)+\frac{b^8x^n}{n}-\frac{28a^2b^6x^{-n}}{n}-\frac{28a^3b^5x^{-2n}}{n}-\frac{70a^4b^4x^{-3n}}{3n}-\frac{14a^5b^3x^{-4n}}{n}-\frac{28a^6b^2x^{-5n}}{5n}-\frac{4a^7b\,x^{-6n}}{3n}-\frac{a^8}{}$

input
```
int(x^(-1-7*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
8*a*b^7*ln(x)+b^8*x^n/n-28*a^2*b^6/n/(x^n)-28*a^3*b^5/n/(x^n)^2-70/3*a^4*b
^4/n/(x^n)^3-14*a^5*b^3/n/(x^n)^4-28/5*a^6*b^2/n/(x^n)^5-4/3*a^7*b/n/(x^n)
^6-1/7*a^8/n/(x^n)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-7n}(a+bx^n)^8\,dx = \frac{840\,ab^7nx^{7\,n}\log\left(x\right)+105\,b^8x^{8\,n}-2940\,a^2b^6x^{6\,n}-2940\,a^3b^5x^{5\,n}-2450\,a^4b^4x^{4\,n}-1470\,a^5b^3x^{3\,n}-588\,a^{6}}{105\,nx^{7\,n}}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/105*(840*a*b^7*n*x^(7*n)*log(x) + 105*b^8*x^(8*n) - 2940*a^2*b^6*x^(6*n)
 - 2940*a^3*b^5*x^(5*n) - 2450*a^4*b^4*x^(4*n) - 1470*a^5*b^3*x^(3*n) - 58
8*a^6*b^2*x^(2*n) - 140*a^7*b*x^n - 15*a^8)/(n*x^(7*n))
```

Sympy [A] (verification not implemented)

Time = 22.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int x^{-1-7n}(a+bx^n)^8\,dx$$
$$=\begin{cases} -\frac{a^8x^{-7n}}{7n}-\frac{4a^7bx^{-6n}}{3n}-\frac{28a^6b^2x^{-5n}}{5n}-\frac{14a^5b^3x^{-4n}}{n}-\frac{70a^4b^4x^{-3n}}{3n}-\frac{28a^3b^5x^{-2n}}{n}-\frac{28a^2b^6x^{-n}}{n}+\frac{8ab^7\log(x^n)}{n}+\frac{b^8x^n}{n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-7*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(7*n*x**(7*n)) - 4*a**7*b/(3*n*x**(6*n)) - 28*a**6*b**2/(
5*n*x**(5*n)) - 14*a**5*b**3/(n*x**(4*n)) - 70*a**4*b**4/(3*n*x**(3*n)) -
28*a**3*b**5/(n*x**(2*n)) - 28*a**2*b**6/(n*x**n) + 8*a*b**7*log(x**n)/n +
 b**8*x**n/n, Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03

$$\int x^{-1-7n}(a+bx^n)^8\,dx=8\,ab^7\log(x)+\frac{b^8x^n}{n}-\frac{a^8}{7\,nx^{7\,n}}-\frac{4\,a^7b}{3\,nx^{6\,n}}-\frac{28\,a^6b^2}{5\,nx^{5\,n}}-\frac{14\,a^5b^3}{nx^{4\,n}}-\frac{70\,a^4b^4}{3\,nx^{3\,n}}-\frac{28\,a^3b^5}{nx^{2\,n}}-\frac{28\,a^2b^6}{nx^n}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
8*a*b^7*log(x) + b^8*x^n/n - 1/7*a^8/(n*x^(7*n)) - 4/3*a^7*b/(n*x^(6*n)) -
 28/5*a^6*b^2/(n*x^(5*n)) - 14*a^5*b^3/(n*x^(4*n)) - 70/3*a^4*b^4/(n*x^(3*
n)) - 28*a^3*b^5/(n*x^(2*n)) - 28*a^2*b^6/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-7n}(a+bx^n)^8\,dx = \frac{840\,ab^7nx^{7\,n}\log\left(x\right)+105\,b^8x^{8\,n}-2940\,a^2b^6x^{6\,n}-2940\,a^3b^5x^{5\,n}-2450\,a^4b^4x^{4\,n}-1470\,a^5b^3x^{3\,n}-588\,a^6}{105\,nx^{7\,n}}$$

input
```
integrate(x^(-1-7*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/105*(840*a*b^7*n*x^(7*n)*log(x) + 105*b^8*x^(8*n) - 2940*a^2*b^6*x^(6*n)
 - 2940*a^3*b^5*x^(5*n) - 2450*a^4*b^4*x^(4*n) - 1470*a^5*b^3*x^(3*n) - 58
8*a^6*b^2*x^(2*n) - 140*a^7*b*x^n - 15*a^8)/(n*x^(7*n))
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03

$$\int x^{-1-7n}(a+bx^n)^8\,dx = \frac{b^8\,x^n}{n}+8\,a\,b^7\,\ln\left(x\right)-\frac{a^8}{7\,n\,x^{7\,n}}-\frac{28\,a^2\,b^6}{n\,x^n}-\frac{28\,a^3\,b^5}{n\,x^{2\,n}} -\frac{70\,a^4\,b^4}{3\,n\,x^{3\,n}}-\frac{14\,a^5\,b^3}{n\,x^{4\,n}}-\frac{28\,a^6\,b^2}{5\,n\,x^{5\,n}}-\frac{4\,a^7\,b}{3\,n\,x^{6\,n}}$$

input
```
int((a + b*x^n)^8/x^(7*n + 1),x)
```

output
```
(b^8*x^n)/n + 8*a*b^7*log(x) - a^8/(7*n*x^(7*n)) - (28*a^2*b^6)/(n*x^n) -
(28*a^3*b^5)/(n*x^(2*n)) - (70*a^4*b^4)/(3*n*x^(3*n)) - (14*a^5*b^3)/(n*x^
(4*n)) - (28*a^6*b^2)/(5*n*x^(5*n)) - (4*a^7*b)/(3*n*x^(6*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int x^{-1-7n}(a+bx^n)^8\,dx$$
$$=\frac{105x^{8n}b^8+840x^{7n}\log(x)\,a\,b^7n-2940x^{6n}a^2b^6-2940x^{5n}a^3b^5-2450x^{4n}a^4b^4-1470x^{3n}a^5b^3-588x^{2n}a^6b}{105x^{7n}n}$$

input
```
int(x^(-1-7*n)*(a+b*x^n)^8,x)
```

output
```
(105*x**(8*n)*b**8 + 840*x**(7*n)*log(x)*a*b**7*n - 2940*x**(6*n)*a**2*b**
6 - 2940*x**(5*n)*a**3*b**5 - 2450*x**(4*n)*a**4*b**4 - 1470*x**(3*n)*a**5
*b**3 - 588*x**(2*n)*a**6*b**2 - 140*x**n*a**7*b - 15*a**8)/(105*x**(7*n)*
n)
```

3.439 $\int x^{-1-8n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int x^{-1-8n}(a+bx^n)^8\,dx = -\frac{a^8x^{-8n}}{8n} - \frac{8a^7bx^{-7n}}{7n} - \frac{14a^6b^2x^{-6n}}{3n} - \frac{56a^5b^3x^{-5n}}{5n} - \frac{35a^4b^4x^{-4n}}{2n} - \frac{56a^3b^5x^{-3n}}{3n} - \frac{14a^2b^6x^{-2n}}{n} - \frac{8ab^7x^{-n}}{n} + b^8\log(x)$$

output
```
-1/8*a^8/n/(x^(8*n))-8/7*a^7*b/n/(x^(7*n))-14/3*a^6*b^2/n/(x^(6*n))-56/5*a
^5*b^3/n/(x^(5*n))-35/2*a^4*b^4/n/(x^(4*n))-56/3*a^3*b^5/n/(x^(3*n))-14*a^
2*b^6/n/(x^(2*n))-8*a*b^7/n/(x^n)+b^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int x^{-1-8n}(a+bx^n)^8\,dx = -\frac{ax^{-8n}(105a^7 + 960a^6bx^n + 3920a^5b^2x^{2n} + 9408a^4b^3x^{3n} + 14700a^3b^4x^{4n} + 15680a^2b^5x^{5n} + 11760ab^6x^{6n}}{840n} + \frac{b^8\log(x^n)}{n}$$

input
```
Integrate[x^(-1 - 8*n)*(a + b*x^n)^8,x]
```

output
```
-1/840*(a*(105*a^7 + 960*a^6*b*x^n + 3920*a^5*b^2*x^(2*n) + 9408*a^4*b^3*x
^(3*n) + 14700*a^3*b^4*x^(4*n) + 15680*a^2*b^5*x^(5*n) + 11760*a*b^6*x^(6*
n) + 6720*b^7*x^(7*n)))/(n*x^(8*n)) + (b^8*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-8n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-9n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(a^8x^{-9n}+8a^7bx^{-8n}+28a^6b^2x^{-7n}+56a^5b^3x^{-6n}+70a^4b^4x^{-5n}+56a^3b^5x^{-4n}+28a^2b^6x^{-3n}+8ab^7x^{-2n}+b^8\right.}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{8}a^8x^{-8n}-\frac{8}{7}a^7bx^{-7n}-\frac{14}{3}a^6b^2x^{-6n}-\frac{56}{5}a^5b^3x^{-5n}-\frac{35}{2}a^4b^4x^{-4n}-\frac{56}{3}a^3b^5x^{-3n}-14a^2b^6x^{-2n}-8ab^7x^{-n}+b^8}{n}$$

input
```
Int[x^(-1 - 8*n)*(a + b*x^n)^8,x]
```

output
```
(-1/8*a^8/x^(8*n) - (8*a^7*b)/(7*x^(7*n)) - (14*a^6*b^2)/(3*x^(6*n)) - (56
*a^5*b^3)/(5*x^(5*n)) - (35*a^4*b^4)/(2*x^(4*n)) - (56*a^3*b^5)/(3*x^(3*n)
) - (14*a^2*b^6)/x^(2*n) - (8*a*b^7)/x^n + b^8*Log[x^n])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

method	result
risch	$b^8 \ln(x) - \frac{8a\,b^7 x^{-n}}{n} - \frac{14a^2b^6x^{-2n}}{n} - \frac{56a^3b^5x^{-3n}}{3n} - \frac{35a^4b^4x^{-4n}}{2n} - \frac{56a^5b^3x^{-5n}}{5n} - \frac{14a^6b^2x^{-6n}}{3n} - \frac{8a^7b\,x^{-7n}}{7n} - \frac{a}{}$

input
```
int(x^(-1-8*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
b^8*ln(x)-8*a*b^7/n/(x^n)-14*a^2*b^6/n/(x^n)^2-56/3*a^3*b^5/n/(x^n)^3-35/2
*a^4*b^4/n/(x^n)^4-56/5*a^5*b^3/n/(x^n)^5-14/3*a^6*b^2/n/(x^n)^6-8/7*a^7*b
/n/(x^n)^7-1/8*a^8/n/(x^n)^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int x^{-1-8n}(a+bx^n)^8\,dx = \frac{840\,b^8nx^{8\,n}\log(x) - 6720\,ab^7x^{7\,n} - 11760\,a^2b^6x^{6\,n} - 15680\,a^3b^5x^{5\,n} - 14700\,a^4b^4x^{4\,n} - 9408\,a^5b^3x^{3\,n} - 3}{840\,nx^{8\,n}}$$

input
```
integrate(x^(-1-8*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
1/840*(840*b^8*n*x^(8*n)*log(x) - 6720*a*b^7*x^(7*n) - 11760*a^2*b^6*x^(6*
n) - 15680*a^3*b^5*x^(5*n) - 14700*a^4*b^4*x^(4*n) - 9408*a^5*b^3*x^(3*n)
- 3920*a^6*b^2*x^(2*n) - 960*a^7*b*x^n - 105*a^8)/(n*x^(8*n))
```

Sympy [A] (verification not implemented)

Time = 35.77 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int x^{-1-8n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8x^{-8n}}{8n} - \frac{8a^7bx^{-7n}}{7n} - \frac{14a^6b^2x^{-6n}}{3n} - \frac{56a^5b^3x^{-5n}}{5n} - \frac{35a^4b^4x^{-4n}}{2n} - \frac{56a^3b^5x^{-3n}}{3n} - \frac{14a^2b^6x^{-2n}}{n} - \frac{8ab^7x^{-n}}{n} + \frac{b^8\log(x^n)}{n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-8*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8/(8*n*x**(8*n)) - 8*a**7*b/(7*n*x**(7*n)) - 14*a**6*b**2/(
3*n*x**(6*n)) - 56*a**5*b**3/(5*n*x**(5*n)) - 35*a**4*b**4/(2*n*x**(4*n))
- 56*a**3*b**5/(3*n*x**(3*n)) - 14*a**2*b**6/(n*x**(2*n)) - 8*a*b**7/(n*x*
*n) + b**8*log(x**n)/n, Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int x^{-1-8n}(a+bx^n)^8\,dx = b^8\log(x) - \frac{a^8}{8\,nx^{8\,n}} - \frac{8\,a^7b}{7\,nx^{7\,n}} - \frac{14\,a^6b^2}{3\,nx^{6\,n}} - \frac{56\,a^5b^3}{5\,nx^{5\,n}} - \frac{35\,a^4b^4}{2\,nx^{4\,n}} - \frac{56\,a^3b^5}{3\,nx^{3\,n}} - \frac{14\,a^2b^6}{nx^{2\,n}} - \frac{8\,ab^7}{nx^n}$$

input
```
integrate(x^(-1-8*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
b^8*log(x) - 1/8*a^8/(n*x^(8*n)) - 8/7*a^7*b/(n*x^(7*n)) - 14/3*a^6*b^2/(n
*x^(6*n)) - 56/5*a^5*b^3/(n*x^(5*n)) - 35/2*a^4*b^4/(n*x^(4*n)) - 56/3*a^3
*b^5/(n*x^(3*n)) - 14*a^2*b^6/(n*x^(2*n)) - 8*a*b^7/(n*x^n)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int x^{-1-8n}(a+bx^n)^8\,dx = \frac{840\,b^8nx^{8\,n}\log\left(x\right) - 6720\,ab^7x^{7\,n} - 11760\,a^2b^6x^{6\,n} - 15680\,a^3b^5x^{5\,n} - 14700\,a^4b^4x^{4\,n} - 9408\,a^5b^3x^{3\,n} - 3}{840\,nx^{8\,n}}$$

input
```
integrate(x^(-1-8*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
1/840*(840*b^8*n*x^(8*n)*log(x) - 6720*a*b^7*x^(7*n) - 11760*a^2*b^6*x^(6*
n) - 15680*a^3*b^5*x^(5*n) - 14700*a^4*b^4*x^(4*n) - 9408*a^5*b^3*x^(3*n)
- 3920*a^6*b^2*x^(2*n) - 960*a^7*b*x^n - 105*a^8)/(n*x^(8*n))
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int x^{-1-8n}(a+bx^n)^8\,dx = b^8\,\ln\left(x\right) - \frac{a^8}{8\,n\,x^{8\,n}} - \frac{14\,a^2\,b^6}{n\,x^{2\,n}} - \frac{56\,a^3\,b^5}{3\,n\,x^{3\,n}} - \frac{35\,a^4\,b^4}{2\,n\,x^{4\,n}} - \frac{56\,a^5\,b^3}{5\,n\,x^{5\,n}} - \frac{14\,a^6\,b^2}{3\,n\,x^{6\,n}} - \frac{8\,a\,b^7}{n\,x^n} - \frac{8\,a^7\,b}{7\,n\,x^{7\,n}}$$

input
```
int((a + b*x^n)^8/x^(8*n + 1),x)
```

output
```
b^8*log(x) - a^8/(8*n*x^(8*n)) - (14*a^2*b^6)/(n*x^(2*n)) - (56*a^3*b^5)/(
3*n*x^(3*n)) - (35*a^4*b^4)/(2*n*x^(4*n)) - (56*a^5*b^3)/(5*n*x^(5*n)) - (
14*a^6*b^2)/(3*n*x^(6*n)) - (8*a*b^7)/(n*x^n) - (8*a^7*b)/(7*n*x^(7*n))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int x^{-1-8n}(a+bx^n)^8\,dx$$
$$= \frac{840x^{8n}\log(x)\,b^8n - 6720x^{7n}a\,b^7 - 11760x^{6n}a^2b^6 - 15680x^{5n}a^3b^5 - 14700x^{4n}a^4b^4 - 9408x^{3n}a^5b^3 - 3920}{840x^{8n}n}$$

input
```
int(x^(-1-8*n)*(a+b*x^n)^8,x)
```

output
```
(840*x**(8*n)*log(x)*b**8*n - 6720*x**(7*n)*a*b**7 - 11760*x**(6*n)*a**2*b
**6 - 15680*x**(5*n)*a**3*b**5 - 14700*x**(4*n)*a**4*b**4 - 9408*x**(3*n)*
a**5*b**3 - 3920*x**(2*n)*a**6*b**2 - 960*x**n*a**7*b - 105*a**8)/(840*x**
(8*n)*n)
```

3.440 $\int x^{-1-9n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int x^{-1-9n}(a+bx^n)^8\,dx=-\frac{x^{-9n}(a+bx^n)^9}{9an}$$

output
```
-1/9*(a+b*x^n)^9/a/n/(x^(9*n))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(24) = 48.

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.71

$$\int x^{-1-9n}(a+bx^n)^8\,dx$$
$$=\frac{x^{-9n}(-a^8-9a^7bx^n-36a^6b^2x^{2n}-84a^5b^3x^{3n}-126a^4b^4x^{4n}-126a^3b^5x^{5n}-84a^2b^6x^{6n}-36ab^7x^{7n}-9b^8}{9n}$$

input
```
Integrate[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

output
```
(-a^8 - 9*a^7*b*x^n - 36*a^6*b^2*x^(2*n) - 84*a^5*b^3*x^(3*n) - 126*a^4*b^
4*x^(4*n) - 126*a^3*b^5*x^(5*n) - 84*a^2*b^6*x^(6*n) - 36*a*b^7*x^(7*n) -
9*b^8*x^(8*n))/(9*n*x^(9*n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-9n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{x^{-9n}(a+bx^n)^9}{9an}$$

input
```
Int[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

output
```
-1/9*(a + b*x^n)^9/(a*n*x^(9*n))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(24) = 48.

Time = 7.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.67

method	result
risch	$-\frac{b^8x^{-n}}{n}-\frac{4a\,b^7x^{-2n}}{n}-\frac{28a^2b^6x^{-3n}}{3n}-\frac{14a^3b^5x^{-4n}}{n}-\frac{14a^4b^4x^{-5n}}{n}-\frac{28a^5b^3x^{-6n}}{3n}-\frac{4a^6b^2x^{-7n}}{n}-\frac{a^7b\,x^{-8n}}{n}$
parallelrisch	$\frac{-9x\,x^{8n}x^{-1-9n}b^8-36x\,x^{7n}x^{-1-9n}a\,b^7-84x\,x^{6n}x^{-1-9n}a^2b^6-126x\,x^{5n}x^{-1-9n}a^3b^5-126x\,x^{4n}x^{-1-9n}a^4b^4-84x\,x^{3n}x^{-1-}}{9n}$
orering	Expression too large to display

input
```
int(x^(-1-9*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-b^8/n/(x^n)-4*a*b^7/n/(x^n)^2-28/3*a^2*b^6/n/(x^n)^3-14*a^3*b^5/n/(x^n)^4
-14*a^4*b^4/n/(x^n)^5-28/3*a^5*b^3/n/(x^n)^6-4*a^6*b^2/n/(x^n)^7-a^7*b/n/(
x^n)^8-1/9*a^8/n/(x^n)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.62

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{9\,b^8x^{8\,n}+36\,ab^7x^{7\,n}+84\,a^2b^6x^{6\,n}+126\,a^3b^5x^{5\,n}+126\,a^4b^4x^{4\,n}+84\,a^5b^3x^{3\,n}+36\,a^6b^2x^{2\,n}+9\,a^7bx^n+}{9\,nx^{9\,n}}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*
x^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) +
9*a^7*b*x^n + a^8)/(n*x^(9*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(19) = 38$.

Time = 2.47 (sec) , antiderivative size = 221, normalized size of antiderivative = 9.21

$$\int x^{-1-9n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8xx^{-9n-1}}{9n} - \frac{a^7bxx^nx^{-9n-1}}{n} - \frac{4a^6b^2xx^{2n}x^{-9n-1}}{n} - \frac{28a^5b^3xx^{3n}x^{-9n-1}}{3n} - \frac{14a^4b^4xx^{4n}x^{-9n-1}}{n} - \frac{14a^3b^5xx^{5n}x^{-9n-1}}{n} - 28 \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-9*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-9*n - 1)/(9*n) - a**7*b*x*x**n*x**(-9*n - 1)/n - 4
*a**6*b**2*x*x**(2*n)*x**(-9*n - 1)/n - 28*a**5*b**3*x*x**(3*n)*x**(-9*n -
 1)/(3*n) - 14*a**4*b**4*x*x**(4*n)*x**(-9*n - 1)/n - 14*a**3*b**5*x*x**(5
*n)*x**(-9*n - 1)/n - 28*a**2*b**6*x*x**(6*n)*x**(-9*n - 1)/(3*n) - 4*a*b*
*7*x*x**(7*n)*x**(-9*n - 1)/n - b**8*x*x**(8*n)*x**(-9*n - 1)/n, Ne(n, 0))
, ((a + b)**8*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.29

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{a^8}{9\,nx^{9\,n}} - \frac{a^7b}{nx^{8\,n}} - \frac{4\,a^6b^2}{nx^{7\,n}} - \frac{28\,a^5b^3}{3\,nx^{6\,n}} - \frac{14\,a^4b^4}{nx^{5\,n}} - \frac{14\,a^3b^5}{nx^{4\,n}} - \frac{28\,a^2b^6}{3\,nx^{3\,n}} - \frac{4\,ab^7}{nx^{2\,n}} - \frac{b^8}{nx^n}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/9*a^8/(n*x^(9*n)) - a^7*b/(n*x^(8*n)) - 4*a^6*b^2/(n*x^(7*n)) - 28/3*a^
5*b^3/(n*x^(6*n)) - 14*a^4*b^4/(n*x^(5*n)) - 14*a^3*b^5/(n*x^(4*n)) - 28/3
*a^2*b^6/(n*x^(3*n)) - 4*a*b^7/(n*x^(2*n)) - b^8/(n*x^n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.62

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{9\,b^8x^{8\,n} + 36\,ab^7x^{7\,n} + 84\,a^2b^6x^{6\,n} + 126\,a^3b^5x^{5\,n} + 126\,a^4b^4x^{4\,n} + 84\,a^5b^3x^{3\,n} + 36\,a^6b^2x^{2\,n} + 9\,a^7bx^n +}{9\,nx^{9\,n}}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*
x^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) +
9*a^7*b*x^n + a^8)/(n*x^(9*n))
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.29

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{a^8}{9\,n\,x^{9\,n}} - \frac{b^8}{n\,x^n} - \frac{28\,a^2\,b^6}{3\,n\,x^{3\,n}} - \frac{14\,a^3\,b^5}{n\,x^{4\,n}} - \frac{14\,a^4\,b^4}{n\,x^{5\,n}} - \frac{28\,a^5\,b^3}{3\,n\,x^{6\,n}} - \frac{4\,a^6\,b^2}{n\,x^{7\,n}} - \frac{4\,a\,b^7}{n\,x^{2\,n}} - \frac{a^7\,b}{n\,x^{8\,n}}$$

input
```
int((a + b*x^n)^8/x^(9*n + 1),x)
```

output
```
- a^8/(9*n*x^(9*n)) - b^8/(n*x^n) - (28*a^2*b^6)/(3*n*x^(3*n)) - (14*a^3*b
^5)/(n*x^(4*n)) - (14*a^4*b^4)/(n*x^(5*n)) - (28*a^5*b^3)/(3*n*x^(6*n)) -
(4*a^6*b^2)/(n*x^(7*n)) - (4*a*b^7)/(n*x^(2*n)) - (a^7*b)/(n*x^(8*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.71

$$\int x^{-1-9n}(a+bx^n)^8\,dx = \frac{-9x^{8n}b^8-36x^{7n}a\,b^7-84x^{6n}a^2b^6-126x^{5n}a^3b^5-126x^{4n}a^4b^4-84x^{3n}a^5b^3-36x^{2n}a^6b^2-9x^na^7b-a^8}{9x^{9n}n}$$

input
```
int(x^(-1-9*n)*(a+b*x^n)^8,x)
```

output
```
( - 9*x**(8*n)*b**8 - 36*x**(7*n)*a*b**7 - 84*x**(6*n)*a**2*b**6 - 126*x**
(5*n)*a**3*b**5 - 126*x**(4*n)*a**4*b**4 - 84*x**(3*n)*a**5*b**3 - 36*x**(
2*n)*a**6*b**2 - 9*x**n*a**7*b - a**8)/(9*x**(9*n)*n)
```

3.441 $\int x^{-1-10n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int x^{-1-10n}(a+bx^n)^8\,dx=-\frac{x^{-10n}(a+bx^n)^9}{10an}+\frac{bx^{-9n}(a+bx^n)^9}{90a^2n}$$

output
```
-1/10*(a+b*x^n)^9/a/n/(x^(10*n))+1/90*b*(a+b*x^n)^9/a^2/n/(x^(9*n))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(50) = 100.

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.26

$$\int x^{-1-10n}(a+bx^n)^8\,dx$$
$$=\frac{x^{-10n}(-9a^8-80a^7bx^n-315a^6b^2x^{2n}-720a^5b^3x^{3n}-1050a^4b^4x^{4n}-1008a^3b^5x^{5n}-630a^2b^6x^{6n}-240ab}{90n}$$

input
```
Integrate[x^(-1 - 10*n)*(a + b*x^n)^8,x]
```

output
```
(-9*a^8 - 80*a^7*b*x^n - 315*a^6*b^2*x^(2*n) - 720*a^5*b^3*x^(3*n) - 1050*
a^4*b^4*x^(4*n) - 1008*a^3*b^5*x^(5*n) - 630*a^2*b^6*x^(6*n) - 240*a*b^7*x
^(7*n) - 45*b^8*x^(8*n))/(90*n*x^(10*n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-10n-1}(a+bx^n)^8\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int x^{-11n}(bx^n+a)^8\,dx^n}{n}$$
$$\Big\downarrow \text{55}$$
$$\frac{-\frac{b\int x^{-10n}(bx^n+a)^8dx^n}{10a}-\frac{x^{-10n}(a+bx^n)^9}{10a}}{n}$$
$$\Big\downarrow \text{48}$$
$$\frac{\frac{bx^{-9n}(a+bx^n)^9}{90a^2}-\frac{x^{-10n}(a+bx^n)^9}{10a}}{n}$$

input
```
Int[x^(-1 - 10*n)*(a + b*x^n)^8,x]
```

output
```
(-1/10*(a + b*x^n)^9/(a*x^(10*n)) + (b*(a + b*x^n)^9)/(90*a^2*x^(9*n)))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(50) = 100$.

Time = 7.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.72

method	result
risch	$-\frac{b^{8} x^{-2 n}}{2 n}-\frac{8 a\, b^{7} x^{-3 n}}{3 n}-\frac{7 a^{2} b^{6} x^{-4 n}}{n}-\frac{56 a^{3} b^{5} x^{-5 n}}{5 n}-\frac{35 a^{4} b^{4} x^{-6 n}}{3 n}-\frac{8 a^{5} b^{3} x^{-7 n}}{n}-\frac{7 a^{6} b^{2} x^{-8 n}}{2 n}-\frac{8 a^{7} b\, x^{-9 n}}{9 n}-$
parallelrisch	$\frac{-45 x\, x^{8 n} x^{-1-10 n} b^{8}-240 x\, x^{7 n} x^{-1-10 n} a\, b^{7}-630 x\, x^{6 n} x^{-1-10 n} a^{2} b^{6}-1008 x\, x^{5 n} x^{-1-10 n} a^{3} b^{5}-1050 x\, x^{4 n} x^{-1-10 n} a^{4} b^{4}-72}{90 n}$
orering	Expression too large to display

input
```
int(x^(-1-10*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^8/n/(x^n)^2-8/3*a*b^7/n/(x^n)^3-7*a^2*b^6/n/(x^n)^4-56/5*a^3*b^5/n/
(x^n)^5-35/3*a^4*b^4/n/(x^n)^6-8*a^5*b^3/n/(x^n)^7-7/2*a^6*b^2/n/(x^n)^8-8
/9*a^7*b/n/(x^n)^9-1/10*a^8/n/(x^n)^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(50) = 100$.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.26

$$\int x^{-1-10n}(a+bx^n)^8\,dx = -\frac{45\,b^8x^{8\,n}+240\,ab^7x^{7\,n}+630\,a^2b^6x^{6\,n}+1008\,a^3b^5x^{5\,n}+1050\,a^4b^4x^{4\,n}+720\,a^5b^3x^{3\,n}+315\,a^6b^2x^{2\,n}+8}{90\,nx^{10\,n}}$$

input
```
integrate(x^(-1-10*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/90*(45*b^8*x^(8*n) + 240*a*b^7*x^(7*n) + 630*a^2*b^6*x^(6*n) + 1008*a^3
*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 720*a^5*b^3*x^(3*n) + 315*a^6*b^2*x^
(2*n) + 80*a^7*b*x^n + 9*a^8)/(n*x^(10*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(39) = 78$.

Time = 2.55 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.60

$$\int x^{-1-10n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8xx^{-10n-1}}{10n} - \frac{8a^7bxx^nx^{-10n-1}}{9n} - \frac{7a^6b^2xx^{2n}x^{-10n-1}}{2n} - \frac{8a^5b^3xx^{3n}x^{-10n-1}}{n} - \frac{35a^4b^4xx^{4n}x^{-10n-1}}{3n} - \frac{56a^3b^5xx^{5n}x^{-10n-1}}{5n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-10*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-10*n - 1)/(10*n) - 8*a**7*b*x*x**n*x**(-10*n - 1)/
(9*n) - 7*a**6*b**2*x*x**(2*n)*x**(-10*n - 1)/(2*n) - 8*a**5*b**3*x*x**(3*
n)*x**(-10*n - 1)/n - 35*a**4*b**4*x*x**(4*n)*x**(-10*n - 1)/(3*n) - 56*a*
*3*b**5*x*x**(5*n)*x**(-10*n - 1)/(5*n) - 7*a**2*b**6*x*x**(6*n)*x**(-10*n
 - 1)/n - 8*a*b**7*x*x**(7*n)*x**(-10*n - 1)/(3*n) - b**8*x*x**(8*n)*x**(-
10*n - 1)/(2*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(50) = 100$.

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int x^{-1-10n}(a+bx^n)^8\,dx = -\frac{a^8}{10\,nx^{10\,n}} - \frac{8\,a^7b}{9\,nx^{9\,n}} - \frac{7\,a^6b^2}{2\,nx^{8\,n}} - \frac{8\,a^5b^3}{nx^{7\,n}} - \frac{35\,a^4b^4}{3\,nx^{6\,n}} - \frac{56\,a^3b^5}{5\,nx^{5\,n}} - \frac{7\,a^2b^6}{nx^{4\,n}} - \frac{8\,ab^7}{3\,nx^{3\,n}} - \frac{b^8}{2\,nx^{2\,n}}$$

input
```
integrate(x^(-1-10*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/10*a^8/(n*x^(10*n)) - 8/9*a^7*b/(n*x^(9*n)) - 7/2*a^6*b^2/(n*x^(8*n)) -
 8*a^5*b^3/(n*x^(7*n)) - 35/3*a^4*b^4/(n*x^(6*n)) - 56/5*a^3*b^5/(n*x^(5*n
)) - 7*a^2*b^6/(n*x^(4*n)) - 8/3*a*b^7/(n*x^(3*n)) - 1/2*b^8/(n*x^(2*n))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(50) = 100$.

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.26

$$\int x^{-1-10n}(a+bx^n)^8\,dx = -\frac{45\,b^8x^{8\,n} + 240\,ab^7x^{7\,n} + 630\,a^2b^6x^{6\,n} + 1008\,a^3b^5x^{5\,n} + 1050\,a^4b^4x^{4\,n} + 720\,a^5b^3x^{3\,n} + 315\,a^6b^2x^{2\,n} + 8}{90\,nx^{10\,n}}$$

input
```
integrate(x^(-1-10*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/90*(45*b^8*x^(8*n) + 240*a*b^7*x^(7*n) + 630*a^2*b^6*x^(6*n) + 1008*a^3
*b^5*x^(5*n) + 1050*a^4*b^4*x^(4*n) + 720*a^5*b^3*x^(3*n) + 315*a^6*b^2*x^
(2*n) + 80*a^7*b*x^n + 9*a^8)/(n*x^(10*n))
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int x^{-1-10n}(a+bx^n)^8\,dx = -\frac{a^8}{10\,n\,x^{10\,n}}-\frac{b^8}{2\,n\,x^{2\,n}}-\frac{7\,a^2\,b^6}{n\,x^{4\,n}}-\frac{56\,a^3\,b^5}{5\,n\,x^{5\,n}} -\frac{35\,a^4\,b^4}{3\,n\,x^{6\,n}}-\frac{8\,a^5\,b^3}{n\,x^{7\,n}}-\frac{7\,a^6\,b^2}{2\,n\,x^{8\,n}}-\frac{8\,a\,b^7}{3\,n\,x^{3\,n}}-\frac{8\,a^7\,b}{9\,n\,x^{9\,n}}$$

input
```
int((a + b*x^n)^8/x^(10*n + 1),x)
```

output
```
- a^8/(10*n*x^(10*n)) - b^8/(2*n*x^(2*n)) - (7*a^2*b^6)/(n*x^(4*n)) - (56*
a^3*b^5)/(5*n*x^(5*n)) - (35*a^4*b^4)/(3*n*x^(6*n)) - (8*a^5*b^3)/(n*x^(7*
n)) - (7*a^6*b^2)/(2*n*x^(8*n)) - (8*a*b^7)/(3*n*x^(3*n)) - (8*a^7*b)/(9*n
*x^(9*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.26

$$\int x^{-1-10n}(a+bx^n)^8\,dx = \frac{-45x^{8n}b^8-240x^{7n}a\,b^7-630x^{6n}a^2b^6-1008x^{5n}a^3b^5-1050x^{4n}a^4b^4-720x^{3n}a^5b^3-315x^{2n}a^6b^2-80x^na}{90x^{10n}n}$$

input
```
int(x^(-1-10*n)*(a+b*x^n)^8,x)
```

output
```
( - 45*x**(8*n)*b**8 - 240*x**(7*n)*a*b**7 - 630*x**(6*n)*a**2*b**6 - 1008
*x**(5*n)*a**3*b**5 - 1050*x**(4*n)*a**4*b**4 - 720*x**(3*n)*a**5*b**3 - 3
15*x**(2*n)*a**6*b**2 - 80*x**n*a**7*b - 9*a**8)/(90*x**(10*n)*n)
```

3.442 $\int x^{-1-11n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int x^{-1-11n}(a+bx^n)^8\,dx=-\frac{x^{-11n}(a+bx^n)^9}{11an}+\frac{bx^{-10n}(a+bx^n)^9}{55a^2n}-\frac{b^2x^{-9n}(a+bx^n)^9}{495a^3n}$$

output
```
-1/11*(a+b*x^n)^9/a/n/(x^(11*n))+1/55*b*(a+b*x^n)^9/a^2/n/(x^(10*n))-1/495
*b^2*(a+b*x^n)^9/a^3/n/(x^(9*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int x^{-1-11n}(a+bx^n)^8\,dx$$
$$=\frac{x^{-11n}(-45a^8-396a^7bx^n-1540a^6b^2x^{2n}-3465a^5b^3x^{3n}-4950a^4b^4x^{4n}-4620a^3b^5x^{5n}-2772a^2b^6x^{6n}-}{495n}$$

input
```
Integrate[x^(-1 - 11*n)*(a + b*x^n)^8,x]
```

output
```
(-45*a^8 - 396*a^7*b*x^n - 1540*a^6*b^2*x^(2*n) - 3465*a^5*b^3*x^(3*n) - 4
950*a^4*b^4*x^(4*n) - 4620*a^3*b^5*x^(5*n) - 2772*a^2*b^6*x^(6*n) - 990*a*
b^7*x^(7*n) - 165*b^8*x^(8*n))/(495*n*x^(11*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-11n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-12n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{2b\int x^{-11n}(bx^n+a)^8dx^n}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{2b\left(-\frac{b\int x^{-10n}(bx^n+a)^8dx^n}{10a}-\frac{x^{-10n}(a+bx^n)^9}{10a}\right)}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{-\frac{2b\left(\frac{bx^{-9n}(a+bx^n)^9}{90a^2}-\frac{x^{-10n}(a+bx^n)^9}{10a}\right)}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}}{n}$$

input
```
Int[x^(-1 - 11*n)*(a + b*x^n)^8,x]
```

output
```
(-1/11*(a + b*x^n)^9/(a*x^(11*n)) - (2*b*(-1/10*(a + b*x^n)^9/(a*x^(10*n))
 + (b*(a + b*x^n)^9)/(90*a^2*x^(9*n))))/(11*a))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 7.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

method	result
risch	$-\frac{b^{8} x^{-3 n}}{3 n}-\frac{2 a \,b^{7} x^{-4 n}}{n}-\frac{28 a^{2} b^{6} x^{-5 n}}{5 n}-\frac{28 a^{3} b^{5} x^{-6 n}}{3 n}-\frac{10 a^{4} b^{4} x^{-7 n}}{n}-\frac{7 a^{5} b^{3} x^{-8 n}}{n}-\frac{28 a^{6} b^{2} x^{-9 n}}{9 n}-\frac{4 a^{7} b \,x^{-10}}{5 n}$
parallelrisch	$\frac{-165 x \,x^{8 n} x^{-1-11 n} b^{8}-990 x \,x^{7 n} x^{-1-11 n} a \,b^{7}-2772 x \,x^{6 n} x^{-1-11 n} a^{2} b^{6}-4620 x \,x^{5 n} x^{-1-11 n} a^{3} b^{5}-4950 x \,x^{4 n} x^{-1-11 n} a^{4} b^{4}-}{495 n}$
orering	Expression too large to display

input
```
int(x^(-1-11*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^8/n/(x^n)^3-2*a*b^7/n/(x^n)^4-28/5*a^2*b^6/n/(x^n)^5-28/3*a^3*b^5/n
/(x^n)^6-10*a^4*b^4/n/(x^n)^7-7*a^5*b^3/n/(x^n)^8-28/9*a^6*b^2/n/(x^n)^9-4
/5*a^7*b/n/(x^n)^10-1/11*a^8/n/(x^n)^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int x^{-1-11n}(a+bx^n)^8\,dx = -\frac{165\,b^8x^{8\,n}+990\,ab^7x^{7\,n}+2772\,a^2b^6x^{6\,n}+4620\,a^3b^5x^{5\,n}+4950\,a^4b^4x^{4\,n}+3465\,a^5b^3x^{3\,n}+1540\,a^6b^2x^2}{495\,nx^{11\,n}}$$

input
```
integrate(x^(-1-11*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/495*(165*b^8*x^(8*n) + 990*a*b^7*x^(7*n) + 2772*a^2*b^6*x^(6*n) + 4620*
a^3*b^5*x^(5*n) + 4950*a^4*b^4*x^(4*n) + 3465*a^5*b^3*x^(3*n) + 1540*a^6*b
^2*x^(2*n) + 396*a^7*b*x^n + 45*a^8)/(n*x^(11*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(63) = 126$.

Time = 2.41 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.96

$$\int x^{-1-11n}(a+bx^n)^8\,dx$$
$$= \begin{cases} -\frac{a^8xx^{-11n-1}}{11n} - \frac{4a^7bxx^nx^{-11n-1}}{5n} - \frac{28a^6b^2xx^{2n}x^{-11n-1}}{9n} - \frac{7a^5b^3xx^{3n}x^{-11n-1}}{n} - \frac{10a^4b^4xx^{4n}x^{-11n-1}}{n} - \frac{28a^3b^5xx^{5n}x^{-11n-1}}{3n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-11*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-11*n - 1)/(11*n) - 4*a**7*b*x*x**n*x**(-11*n - 1)/
(5*n) - 28*a**6*b**2*x*x**(2*n)*x**(-11*n - 1)/(9*n) - 7*a**5*b**3*x*x**(3
*n)*x**(-11*n - 1)/n - 10*a**4*b**4*x*x**(4*n)*x**(-11*n - 1)/n - 28*a**3*
b**5*x*x**(5*n)*x**(-11*n - 1)/(3*n) - 28*a**2*b**6*x*x**(6*n)*x**(-11*n -
 1)/(5*n) - 2*a*b**7*x*x**(7*n)*x**(-11*n - 1)/n - b**8*x*x**(8*n)*x**(-11
*n - 1)/(3*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int x^{-1-11n}(a+bx^n)^8\,dx = -\frac{a^8}{11\,nx^{11\,n}} - \frac{4\,a^7b}{5\,nx^{10\,n}} - \frac{28\,a^6b^2}{9\,nx^{9\,n}} - \frac{7\,a^5b^3}{nx^{8\,n}} - \frac{10\,a^4b^4}{nx^{7\,n}} - \frac{28\,a^3b^5}{3\,nx^{6\,n}} - \frac{28\,a^2b^6}{5\,nx^{5\,n}} - \frac{2\,ab^7}{nx^{4\,n}} - \frac{b^8}{3\,nx^{3\,n}}$$

input
```
integrate(x^(-1-11*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/11*a^8/(n*x^(11*n)) - 4/5*a^7*b/(n*x^(10*n)) - 28/9*a^6*b^2/(n*x^(9*n))
 - 7*a^5*b^3/(n*x^(8*n)) - 10*a^4*b^4/(n*x^(7*n)) - 28/3*a^3*b^5/(n*x^(6*n
)) - 28/5*a^2*b^6/(n*x^(5*n)) - 2*a*b^7/(n*x^(4*n)) - 1/3*b^8/(n*x^(3*n))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int x^{-1-11n}(a+bx^n)^8\,dx = -\frac{165\,b^8x^{8\,n} + 990\,ab^7x^{7\,n} + 2772\,a^2b^6x^{6\,n} + 4620\,a^3b^5x^{5\,n} + 4950\,a^4b^4x^{4\,n} + 3465\,a^5b^3x^{3\,n} + 1540\,a^6b^2x^2}{495\,nx^{11\,n}}$$

input
```
integrate(x^(-1-11*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/495*(165*b^8*x^(8*n) + 990*a*b^7*x^(7*n) + 2772*a^2*b^6*x^(6*n) + 4620*
a^3*b^5*x^(5*n) + 4950*a^4*b^4*x^(4*n) + 3465*a^5*b^3*x^(3*n) + 1540*a^6*b
^2*x^(2*n) + 396*a^7*b*x^n + 45*a^8)/(n*x^(11*n))
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int x^{-1-11n}(a+bx^n)^8\,dx = -\frac{a^8}{11\,n\,x^{11\,n}} - \frac{b^8}{3\,n\,x^{3\,n}} - \frac{28\,a^2\,b^6}{5\,n\,x^{5\,n}} - \frac{28\,a^3\,b^5}{3\,n\,x^{6\,n}} - \frac{10\,a^4\,b^4}{n\,x^{7\,n}} - \frac{7\,a^5\,b^3}{n\,x^{8\,n}} - \frac{28\,a^6\,b^2}{9\,n\,x^{9\,n}} - \frac{2\,a\,b^7}{n\,x^{4\,n}} - \frac{4\,a^7\,b}{5\,n\,x^{10\,n}}$$

input
```
int((a + b*x^n)^8/x^(11*n + 1),x)
```

output
```
- a^8/(11*n*x^(11*n)) - b^8/(3*n*x^(3*n)) - (28*a^2*b^6)/(5*n*x^(5*n)) - (
28*a^3*b^5)/(3*n*x^(6*n)) - (10*a^4*b^4)/(n*x^(7*n)) - (7*a^5*b^3)/(n*x^(8
*n)) - (28*a^6*b^2)/(9*n*x^(9*n)) - (2*a*b^7)/(n*x^(4*n)) - (4*a^7*b)/(5*n
*x^(10*n))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int x^{-1-11n}(a+bx^n)^8\,dx = \frac{-165x^{8n}b^8 - 990x^{7n}a\,b^7 - 2772x^{6n}a^2b^6 - 4620x^{5n}a^3b^5 - 4950x^{4n}a^4b^4 - 3465x^{3n}a^5b^3 - 1540x^{2n}a^6b^2 - 3}{495x^{11n}n}$$

input
```
int(x^(-1-11*n)*(a+b*x^n)^8,x)
```

output
```
( - 165*x**(8*n)*b**8 - 990*x**(7*n)*a*b**7 - 2772*x**(6*n)*a**2*b**6 - 46
20*x**(5*n)*a**3*b**5 - 4950*x**(4*n)*a**4*b**4 - 3465*x**(3*n)*a**5*b**3
- 1540*x**(2*n)*a**6*b**2 - 396*x**n*a**7*b - 45*a**8)/(495*x**(11*n)*n)
```

3.443 $\int x^{-1-12n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int x^{-1-12n}(a+bx^n)^8\,dx = -\frac{x^{-12n}(a+bx^n)^9}{12an}+\frac{bx^{-11n}(a+bx^n)^9}{44a^2n} -\frac{b^2x^{-10n}(a+bx^n)^9}{220a^3n}+\frac{b^3x^{-9n}(a+bx^n)^9}{1980a^4n}$$

output
```
-1/12*(a+b*x^n)^9/a/n/(x^(12*n))+1/44*b*(a+b*x^n)^9/a^2/n/(x^(11*n))-1/220
*b^2*(a+b*x^n)^9/a^3/n/(x^(10*n))+1/1980*b^3*(a+b*x^n)^9/a^4/n/(x^(9*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int x^{-1-12n}(a+bx^n)^8\,dx = \frac{x^{-12n}(-165a^8-1440a^7bx^n-5544a^6b^2x^{2n}-12320a^5b^3x^{3n}-17325a^4b^4x^{4n}-15840a^3b^5x^{5n}-9240a^2b^6\cdots}{1980n}$$

input
```
Integrate[x^(-1 - 12*n)*(a + b*x^n)^8,x]
```

output
```
(-165*a^8 - 1440*a^7*b*x^n - 5544*a^6*b^2*x^(2*n) - 12320*a^5*b^3*x^(3*n)
- 17325*a^4*b^4*x^(4*n) - 15840*a^3*b^5*x^(5*n) - 9240*a^2*b^6*x^(6*n) - 3
168*a*b^7*x^(7*n) - 495*b^8*x^(8*n))/(1980*n*x^(12*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {798, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-12n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-13n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{b\int x^{-12n}(bx^n+a)^8dx^n}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{b\left(-\frac{2b\int x^{-11n}(bx^n+a)^8dx^n}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}\right)}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{b\left(-\frac{2b\left(-\frac{b\int x^{-10n}(bx^n+a)^8dx^n}{10a}-\frac{x^{-10n}(a+bx^n)^9}{10a}\right)}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}\right)}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{-\frac{b\left(-\frac{2b\left(\frac{bx^{-9n}(a+bx^n)^9}{90a^2}-\frac{x^{-10n}(a+bx^n)^9}{10a}\right)}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}\right)}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}}{n}$$

input
```
Int[x^(-1 - 12*n)*(a + b*x^n)^8,x]
```

output
```
(-1/12*(a + b*x^n)^9/(a*x^(12*n)) - (b*(-1/11*(a + b*x^n)^9/(a*x^(11*n)) -
 (2*b*(-1/10*(a + b*x^n)^9/(a*x^(10*n)) + (b*(a + b*x^n)^9)/(90*a^2*x^(9*n
))))/(11*a)))/(4*a))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{b^{8} x^{-4 n}}{4 n}-\frac{8 a \,b^{7} x^{-5 n}}{5 n}-\frac{14 a^{2} b^{6} x^{-6 n}}{3 n}-\frac{8 a^{3} b^{5} x^{-7 n}}{n}-\frac{35 a^{4} b^{4} x^{-8 n}}{4 n}-\frac{56 a^{5} b^{3} x^{-9 n}}{9 n}-\frac{14 a^{6} b^{2} x^{-10 n}}{5 n}-\frac{8 a^{7} b \,x^{-1}}{11 n}$
parallelrisch	$\frac{-495 x \,x^{8 n} x^{-1-12 n} b^{8}-3168 x \,x^{7 n} x^{-1-12 n} a \,b^{7}-9240 x \,x^{6 n} x^{-1-12 n} a^{2} b^{6}-15840 x \,x^{5 n} x^{-1-12 n} a^{3} b^{5}-17325 x \,x^{4 n} x^{-1-12 n} a^{4}}{1980 n}$
orering	Expression too large to display

input
```
int(x^(-1-12*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^8/n/(x^n)^4-8/5*a*b^7/n/(x^n)^5-14/3*a^2*b^6/n/(x^n)^6-8*a^3*b^5/n/
(x^n)^7-35/4*a^4*b^4/n/(x^n)^8-56/9*a^5*b^3/n/(x^n)^9-14/5*a^6*b^2/n/(x^n)
^10-8/11*a^7*b/n/(x^n)^11-1/12*a^8/n/(x^n)^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int x^{-1-12n}(a+bx^n)^8\,dx = -\frac{495\,b^8x^{8\,n}+3168\,ab^7x^{7\,n}+9240\,a^2b^6x^{6\,n}+15840\,a^3b^5x^{5\,n}+17325\,a^4b^4x^{4\,n}+12320\,a^5b^3x^{3\,n}+5544\,a^6}{1980\,nx^{12\,n}}$$

input
```
integrate(x^(-1-12*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/1980*(495*b^8*x^(8*n) + 3168*a*b^7*x^(7*n) + 9240*a^2*b^6*x^(6*n) + 158
40*a^3*b^5*x^(5*n) + 17325*a^4*b^4*x^(4*n) + 12320*a^5*b^3*x^(3*n) + 5544*
a^6*b^2*x^(2*n) + 1440*a^7*b*x^n + 165*a^8)/(n*x^(12*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(87) = 174.

Time = 2.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.22

$$\int x^{-1-12n}(a+bx^n)^8\,dx = \begin{cases} -\frac{a^8xx^{-12n-1}}{12n} - \frac{8a^7bxx^nx^{-12n-1}}{11n} - \frac{14a^6b^2xx^{2n}x^{-12n-1}}{5n} - \frac{56a^5b^3xx^{3n}x^{-12n-1}}{9n} - \frac{35a^4b^4xx^{4n}x^{-12n-1}}{4n} - \frac{8a^3b^5xx^{5n}x^{-12n-1}}{n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-12*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-12*n - 1)/(12*n) - 8*a**7*b*x*x**n*x**(-12*n - 1)/
(11*n) - 14*a**6*b**2*x*x**(2*n)*x**(-12*n - 1)/(5*n) - 56*a**5*b**3*x*x**
(3*n)*x**(-12*n - 1)/(9*n) - 35*a**4*b**4*x*x**(4*n)*x**(-12*n - 1)/(4*n)
- 8*a**3*b**5*x*x**(5*n)*x**(-12*n - 1)/n - 14*a**2*b**6*x*x**(6*n)*x**(-1
2*n - 1)/(3*n) - 8*a*b**7*x*x**(7*n)*x**(-12*n - 1)/(5*n) - b**8*x*x**(8*n
)*x**(-12*n - 1)/(4*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.47

$$\int x^{-1-12n}(a+bx^n)^8\,dx = -\frac{a^8}{12\,nx^{12\,n}} - \frac{8\,a^7b}{11\,nx^{11\,n}} - \frac{14\,a^6b^2}{5\,nx^{10\,n}} - \frac{56\,a^5b^3}{9\,nx^{9\,n}} - \frac{35\,a^4b^4}{4\,nx^{8\,n}} - \frac{8\,a^3b^5}{nx^{7\,n}} - \frac{14\,a^2b^6}{3\,nx^{6\,n}} - \frac{8\,ab^7}{5\,nx^{5\,n}} - \frac{b^8}{4\,nx^{4\,n}}$$

input
```
integrate(x^(-1-12*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/12*a^8/(n*x^(12*n)) - 8/11*a^7*b/(n*x^(11*n)) - 14/5*a^6*b^2/(n*x^(10*n
)) - 56/9*a^5*b^3/(n*x^(9*n)) - 35/4*a^4*b^4/(n*x^(8*n)) - 8*a^3*b^5/(n*x^
(7*n)) - 14/3*a^2*b^6/(n*x^(6*n)) - 8/5*a*b^7/(n*x^(5*n)) - 1/4*b^8/(n*x^(
4*n))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int x^{-1-12n}(a+bx^n)^8\,dx = -\frac{495\,b^8x^{8\,n} + 3168\,ab^7x^{7\,n} + 9240\,a^2b^6x^{6\,n} + 15840\,a^3b^5x^{5\,n} + 17325\,a^4b^4x^{4\,n} + 12320\,a^5b^3x^{3\,n} + 5544\,a^6}{1980\,nx^{12\,n}}$$

input
```
integrate(x^(-1-12*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/1980*(495*b^8*x^(8*n) + 3168*a*b^7*x^(7*n) + 9240*a^2*b^6*x^(6*n) + 158
40*a^3*b^5*x^(5*n) + 17325*a^4*b^4*x^(4*n) + 12320*a^5*b^3*x^(3*n) + 5544*
a^6*b^2*x^(2*n) + 1440*a^7*b*x^n + 165*a^8)/(n*x^(12*n))
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.47

$$\int x^{-1-12n}(a+bx^n)^8\,dx = -\frac{a^8}{12\,n\,x^{12\,n}}-\frac{b^8}{4\,n\,x^{4\,n}}-\frac{14\,a^2\,b^6}{3\,n\,x^{6\,n}}-\frac{8\,a^3\,b^5}{n\,x^{7\,n}}-\frac{35\,a^4\,b^4}{4\,n\,x^{8\,n}}-\frac{56\,a^5\,b^3}{9\,n\,x^{9\,n}}-\frac{14\,a^6\,b^2}{5\,n\,x^{10\,n}}-\frac{8\,a\,b^7}{5\,n\,x^{5\,n}}-\frac{8\,a^7\,b}{11\,n\,x^{11\,n}}$$

input
```
int((a + b*x^n)^8/x^(12*n + 1),x)
```

output
```
- a^8/(12*n*x^(12*n)) - b^8/(4*n*x^(4*n)) - (14*a^2*b^6)/(3*n*x^(6*n)) - (
8*a^3*b^5)/(n*x^(7*n)) - (35*a^4*b^4)/(4*n*x^(8*n)) - (56*a^5*b^3)/(9*n*x^
(9*n)) - (14*a^6*b^2)/(5*n*x^(10*n)) - (8*a*b^7)/(5*n*x^(5*n)) - (8*a^7*b)
/(11*n*x^(11*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int x^{-1-12n}(a+bx^n)^8\,dx = \frac{-495x^{8n}b^8-3168x^{7n}a\,b^7-9240x^{6n}a^2b^6-15840x^{5n}a^3b^5-17325x^{4n}a^4b^4-12320x^{3n}a^5b^3-5544x^{2n}a^6b}{1980x^{12n}n}$$

input
```
int(x^(-1-12*n)*(a+b*x^n)^8,x)
```

output
```
( - 495*x**(8*n)*b**8 - 3168*x**(7*n)*a*b**7 - 9240*x**(6*n)*a**2*b**6 - 1
5840*x**(5*n)*a**3*b**5 - 17325*x**(4*n)*a**4*b**4 - 12320*x**(3*n)*a**5*b
**3 - 5544*x**(2*n)*a**6*b**2 - 1440*x**n*a**7*b - 165*a**8)/(1980*x**(12*
n)*n)
```

3.444 $\int x^{-1-13n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 131

$$\int x^{-1-13n}(a+bx^n)^8\,dx = -\frac{x^{-13n}(a+bx^n)^9}{13an}+\frac{bx^{-12n}(a+bx^n)^9}{39a^2n}-\frac{b^2x^{-11n}(a+bx^n)^9}{143a^3n}+\frac{b^3x^{-10n}(a+bx^n)^9}{715a^4n}-\frac{b^4x^{-9n}(a+bx^n)^9}{6435a^5n}$$

output
```
-1/13*(a+b*x^n)^9/a/n/(x^(13*n))+1/39*b*(a+b*x^n)^9/a^2/n/(x^(12*n))-1/143
*b^2*(a+b*x^n)^9/a^3/n/(x^(11*n))+1/715*b^3*(a+b*x^n)^9/a^4/n/(x^(10*n))-1
/6435*b^4*(a+b*x^n)^9/a^5/n/(x^(9*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int x^{-1-13n}(a+bx^n)^8\,dx = \frac{x^{-13n}(-495a^8-4290a^7bx^n-16380a^6b^2x^{2n}-36036a^5b^3x^{3n}-50050a^4b^4x^{4n}-45045a^3b^5x^{5n}-25740a^2}{6435n}$$

input
```
Integrate[x^(-1 - 13*n)*(a + b*x^n)^8,x]
```

output
```
(-495*a^8 - 4290*a^7*b*x^n - 16380*a^6*b^2*x^(2*n) - 36036*a^5*b^3*x^(3*n)
 - 50050*a^4*b^4*x^(4*n) - 45045*a^3*b^5*x^(5*n) - 25740*a^2*b^6*x^(6*n) -
 8580*a*b^7*x^(7*n) - 1287*b^8*x^(8*n))/(6435*n*x^(13*n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {798, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-13n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-14n}(bx^n+a)^8dx^n}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{4b\int x^{-13n}(bx^n+a)^8dx^n}{13a}-\frac{x^{-13n}(a+bx^n)^9}{13a}}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{4b\left(-\frac{b\int x^{-12n}(bx^n+a)^8dx^n}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}\right)}{13a}-\frac{x^{-13n}(a+bx^n)^9}{13a}}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{4b\left(-\frac{b\left(-\frac{2b\int x^{-11n}(bx^n+a)^8dx^n}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}\right)}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}\right)}{13a}-\frac{x^{-13n}(a+bx^n)^9}{13a}}{n}$$

$$\Big\downarrow \text{55}$$

$$\frac{-\frac{4b\left(-\frac{b\left(-\frac{2b\left(-\frac{b\int x^{-10n}(bx^n+a)^8dx^n}{10a}-\frac{x^{-10n}(a+bx^n)^9}{10a}\right)}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}\right)}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}\right)}{13a}-\frac{x^{-13n}(a+bx^n)^9}{13a}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{-\frac{4b\left(-\frac{b\left(-\frac{2b\left(\frac{bx^{-9n}(a+bx^n)^9}{90a^2}-\frac{x^{-10n}(a+bx^n)^9}{10a}\right)}{11a}-\frac{x^{-11n}(a+bx^n)^9}{11a}\right)}{4a}-\frac{x^{-12n}(a+bx^n)^9}{12a}\right)}{13a}-\frac{x^{-13n}(a+bx^n)^9}{13a}}{n}$$

input
```
Int[x^(-1 - 13*n)*(a + b*x^n)^8,x]
```

output
```
(-1/13*(a + b*x^n)^9/(a*x^(13*n)) - (4*b*(-1/12*(a + b*x^n)^9/(a*x^(12*n))
 - (b*(-1/11*(a + b*x^n)^9/(a*x^(11*n)) - (2*b*(-1/10*(a + b*x^n)^9/(a*x^(
10*n)) + (b*(a + b*x^n)^9)/(90*a^2*x^(9*n))))/(11*a)))/(4*a)))/(13*a))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 7.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{b^8 x^{-5n}}{5n} - \frac{4a\,b^7 x^{-6n}}{3n} - \frac{4a^2 b^6 x^{-7n}}{n} - \frac{7a^3 b^5 x^{-8n}}{n} - \frac{70a^4 b^4 x^{-9n}}{9n} - \frac{28a^5 b^3 x^{-10n}}{5n} - \frac{28a^6 b^2 x^{-11n}}{11n} - \frac{2a^7 b\, x^{-1}}{3n}$
parallelrisch	$\frac{-1287x\,x^{8n}x^{-1-13n}b^8-8580x\,x^{7n}x^{-1-13n}a\,b^7-25740x\,x^{6n}x^{-1-13n}a^2b^6-45045x\,x^{5n}x^{-1-13n}a^3b^5-50050x\,x^{4n}x^{-1-13n}}{6435n}$
orering	Expression too large to display

input
```
int(x^(-1-13*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^8/n/(x^n)^5-4/3*a*b^7/n/(x^n)^6-4*a^2*b^6/n/(x^n)^7-7*a^3*b^5/n/(x^
n)^8-70/9*a^4*b^4/n/(x^n)^9-28/5*a^5*b^3/n/(x^n)^10-28/11*a^6*b^2/n/(x^n)^
11-2/3*a^7*b/n/(x^n)^12-1/13*a^8/n/(x^n)^13
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int x^{-1-13n}(a+bx^n)^8\,dx =$$
$$-\frac{1287\,b^8x^{8\,n}+8580\,ab^7x^{7\,n}+25740\,a^2b^6x^{6\,n}+45045\,a^3b^5x^{5\,n}+50050\,a^4b^4x^{4\,n}+36036\,a^5b^3x^{3\,n}+1638}{6435\,nx^{13\,n}}$$

input
```
integrate(x^(-1-13*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/6435*(1287*b^8*x^(8*n) + 8580*a*b^7*x^(7*n) + 25740*a^2*b^6*x^(6*n) + 4
5045*a^3*b^5*x^(5*n) + 50050*a^4*b^4*x^(4*n) + 36036*a^5*b^3*x^(3*n) + 163
80*a^6*b^2*x^(2*n) + 4290*a^7*b*x^n + 495*a^8)/(n*x^(13*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(110) = 220$.

Time = 2.47 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.76

$$\int x^{-1-13n}(a+bx^n)^8\,dx$$
$$= \begin{cases} -\frac{a^8xx^{-13n-1}}{13n} - \frac{2a^7bxx^nx^{-13n-1}}{3n} - \frac{28a^6b^2xx^{2n}x^{-13n-1}}{11n} - \frac{28a^5b^3xx^{3n}x^{-13n-1}}{5n} - \frac{70a^4b^4xx^{4n}x^{-13n-1}}{9n} - \frac{7a^3b^5xx^{5n}x^{-13n-1}}{n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-13*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-13*n - 1)/(13*n) - 2*a**7*b*x*x**n*x**(-13*n - 1)/
(3*n) - 28*a**6*b**2*x*x**(2*n)*x**(-13*n - 1)/(11*n) - 28*a**5*b**3*x*x**
(3*n)*x**(-13*n - 1)/(5*n) - 70*a**4*b**4*x*x**(4*n)*x**(-13*n - 1)/(9*n)
- 7*a**3*b**5*x*x**(5*n)*x**(-13*n - 1)/n - 4*a**2*b**6*x*x**(6*n)*x**(-13
*n - 1)/n - 4*a*b**7*x*x**(7*n)*x**(-13*n - 1)/(3*n) - b**8*x*x**(8*n)*x**
(-13*n - 1)/(5*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int x^{-1-13n}(a+bx^n)^8\,dx = -\frac{a^8}{13\,nx^{13\,n}} - \frac{2\,a^7b}{3\,nx^{12\,n}} - \frac{28\,a^6b^2}{11\,nx^{11\,n}} - \frac{28\,a^5b^3}{5\,nx^{10\,n}} - \frac{70\,a^4b^4}{9\,nx^{9\,n}} - \frac{7\,a^3b^5}{nx^{8\,n}} - \frac{4\,a^2b^6}{nx^{7\,n}} - \frac{4\,ab^7}{3\,nx^{6\,n}} - \frac{b^8}{5\,nx^{5\,n}}$$

input
```
integrate(x^(-1-13*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/13*a^8/(n*x^(13*n)) - 2/3*a^7*b/(n*x^(12*n)) - 28/11*a^6*b^2/(n*x^(11*n
)) - 28/5*a^5*b^3/(n*x^(10*n)) - 70/9*a^4*b^4/(n*x^(9*n)) - 7*a^3*b^5/(n*x
^(8*n)) - 4*a^2*b^6/(n*x^(7*n)) - 4/3*a*b^7/(n*x^(6*n)) - 1/5*b^8/(n*x^(5*
n))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int x^{-1-13n}(a+bx^n)^8\,dx = -\frac{1287\,b^8x^{8\,n}+8580\,ab^7x^{7\,n}+25740\,a^2b^6x^{6\,n}+45045\,a^3b^5x^{5\,n}+50050\,a^4b^4x^{4\,n}+36036\,a^5b^3x^{3\,n}+1638}{6435\,nx^{13\,n}}$$

input
```
integrate(x^(-1-13*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/6435*(1287*b^8*x^(8*n) + 8580*a*b^7*x^(7*n) + 25740*a^2*b^6*x^(6*n) + 4
5045*a^3*b^5*x^(5*n) + 50050*a^4*b^4*x^(4*n) + 36036*a^5*b^3*x^(3*n) + 163
80*a^6*b^2*x^(2*n) + 4290*a^7*b*x^n + 495*a^8)/(n*x^(13*n))
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int x^{-1-13n}(a+bx^n)^8\,dx = -\frac{a^8}{13\,n\,x^{13\,n}}-\frac{b^8}{5\,n\,x^{5\,n}}-\frac{4\,a^2\,b^6}{n\,x^{7\,n}}-\frac{7\,a^3\,b^5}{n\,x^{8\,n}}-\frac{70\,a^4\,b^4}{9\,n\,x^{9\,n}}-\frac{28\,a^5\,b^3}{5\,n\,x^{10\,n}}-\frac{28\,a^6\,b^2}{11\,n\,x^{11\,n}}-\frac{4\,a\,b^7}{3\,n\,x^{6\,n}}-\frac{2\,a^7\,b}{3\,n\,x^{12\,n}}$$

input
```
int((a + b*x^n)^8/x^(13*n + 1),x)
```

output
```
- a^8/(13*n*x^(13*n)) - b^8/(5*n*x^(5*n)) - (4*a^2*b^6)/(n*x^(7*n)) - (7*a
^3*b^5)/(n*x^(8*n)) - (70*a^4*b^4)/(9*n*x^(9*n)) - (28*a^5*b^3)/(5*n*x^(10
*n)) - (28*a^6*b^2)/(11*n*x^(11*n)) - (4*a*b^7)/(3*n*x^(6*n)) - (2*a^7*b)/
(3*n*x^(12*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int x^{-1-13n}(a+bx^n)^8\,dx$$
$$= \frac{-1287x^{8n}b^8 - 8580x^{7n}a\,b^7 - 25740x^{6n}a^2b^6 - 45045x^{5n}a^3b^5 - 50050x^{4n}a^4b^4 - 36036x^{3n}a^5b^3 - 16380x^{2n}}{6435x^{13n}n}$$

input
```
int(x^(-1-13*n)*(a+b*x^n)^8,x)
```

output
```
( - 1287*x**(8*n)*b**8 - 8580*x**(7*n)*a*b**7 - 25740*x**(6*n)*a**2*b**6 -
 45045*x**(5*n)*a**3*b**5 - 50050*x**(4*n)*a**4*b**4 - 36036*x**(3*n)*a**5
*b**3 - 16380*x**(2*n)*a**6*b**2 - 4290*x**n*a**7*b - 495*a**8)/(6435*x**(
13*n)*n)
```

3.445 $\int x^{-1-14n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 151

$$\int x^{-1-14n}(a+bx^n)^8\,dx=-\frac{a^8x^{-14n}}{14n}-\frac{8a^7bx^{-13n}}{13n}-\frac{7a^6b^2x^{-12n}}{3n}-\frac{56a^5b^3x^{-11n}}{11n}-\frac{7a^4b^4x^{-10n}}{n}-\frac{56a^3b^5x^{-9n}}{9n}-\frac{7a^2b^6x^{-8n}}{2n}-\frac{8ab^7x^{-7n}}{7n}-\frac{b^8x^{-6n}}{6n}$$

output
```
-1/14*a^8/n/(x^(14*n))-8/13*a^7*b/n/(x^(13*n))-7/3*a^6*b^2/n/(x^(12*n))-56
/11*a^5*b^3/n/(x^(11*n))-7*a^4*b^4/n/(x^(10*n))-56/9*a^3*b^5/n/(x^(9*n))-7
/2*a^2*b^6/n/(x^(8*n))-8/7*a*b^7/n/(x^(7*n))-1/6*b^8/n/(x^(6*n))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-14n}(a+bx^n)^8\,dx=\frac{x^{-14n}(-1287a^8-11088a^7bx^n-42042a^6b^2x^{2n}-91728a^5b^3x^{3n}-126126a^4b^4x^{4n}-112112a^3b^5x^{5n}-630}{18018n}$$

input
```
Integrate[x^(-1 - 14*n)*(a + b*x^n)^8,x]
```

output
```
(-1287*a^8 - 11088*a^7*b*x^n - 42042*a^6*b^2*x^(2*n) - 91728*a^5*b^3*x^(3*
n) - 126126*a^4*b^4*x^(4*n) - 112112*a^3*b^5*x^(5*n) - 63063*a^2*b^6*x^(6*
n) - 20592*a*b^7*x^(7*n) - 3003*b^8*x^(8*n))/(18018*n*x^(14*n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-14n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int x^{-15n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \ 53$$

$$\frac{\int \left(a^8x^{-15n}+8a^7bx^{-14n}+28a^6b^2x^{-13n}+56a^5b^3x^{-12n}+70a^4b^4x^{-11n}+56a^3b^5x^{-10n}+28a^2b^6x^{-9n}+8ab^7x^{-8n}\right.}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{14}a^8x^{-14n}-\frac{8}{13}a^7bx^{-13n}-\frac{7}{3}a^6b^2x^{-12n}-\frac{56}{11}a^5b^3x^{-11n}-7a^4b^4x^{-10n}-\frac{56}{9}a^3b^5x^{-9n}-\frac{7}{2}a^2b^6x^{-8n}-\frac{8}{7}ab^7x^{-7n}}{n}$$

input
```
Int[x^(-1 - 14*n)*(a + b*x^n)^8,x]
```

output
```
(-1/14*a^8/x^(14*n) - (8*a^7*b)/(13*x^(13*n)) - (7*a^6*b^2)/(3*x^(12*n)) -
 (56*a^5*b^3)/(11*x^(11*n)) - (7*a^4*b^4)/x^(10*n) - (56*a^3*b^5)/(9*x^(9*
n)) - (7*a^2*b^6)/(2*x^(8*n)) - (8*a*b^7)/(7*x^(7*n)) - b^8/(6*x^(6*n)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{b^{8} x^{-6 n}}{6 n}-\frac{8 a\, b^{7} x^{-7 n}}{7 n}-\frac{7 a^{2} b^{6} x^{-8 n}}{2 n}-\frac{56 a^{3} b^{5} x^{-9 n}}{9 n}-\frac{7 a^{4} b^{4} x^{-10 n}}{n}-\frac{56 a^{5} b^{3} x^{-11 n}}{11 n}-\frac{7 a^{6} b^{2} x^{-12 n}}{3 n}-\frac{8 a^{7} b\, x^{-1}}{13 n}$
parallelrisch	$\frac{-3003 x\, x^{8 n} x^{-1-14 n} b^{8}-20592 x\, x^{7 n} x^{-1-14 n} a\, b^{7}-63063 x\, x^{6 n} x^{-1-14 n} a^{2} b^{6}-112112 x\, x^{5 n} x^{-1-14 n} a^{3} b^{5}-126126 x\, x^{4 n} x^{-1-}}{18018 n}$
orering	Expression too large to display

input
```
int(x^(-1-14*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/6*b^8/n/(x^n)^6-8/7*a*b^7/n/(x^n)^7-7/2*a^2*b^6/n/(x^n)^8-56/9*a^3*b^5/
n/(x^n)^9-7*a^4*b^4/n/(x^n)^10-56/11*a^5*b^3/n/(x^n)^11-7/3*a^6*b^2/n/(x^n
)^12-8/13*a^7*b/n/(x^n)^13-1/14*a^8/n/(x^n)^14
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-14n}(a+bx^n)^8\,dx = -\frac{3003\,b^8x^{8\,n}+20592\,ab^7x^{7\,n}+63063\,a^2b^6x^{6\,n}+112112\,a^3b^5x^{5\,n}+126126\,a^4b^4x^{4\,n}+91728\,a^5b^3x^{3\,n}+4}{18018\,nx^{14\,n}}$$

input
```
integrate(x^(-1-14*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/18018*(3003*b^8*x^(8*n) + 20592*a*b^7*x^(7*n) + 63063*a^2*b^6*x^(6*n) +
 112112*a^3*b^5*x^(5*n) + 126126*a^4*b^4*x^(4*n) + 91728*a^5*b^3*x^(3*n) +
 42042*a^6*b^2*x^(2*n) + 11088*a^7*b*x^n + 1287*a^8)/(n*x^(14*n))
```

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.53

$$\int x^{-1-14n}(a+bx^n)^8\,dx$$
$$= \begin{cases} -\frac{a^8xx^{-14n-1}}{14n}-\frac{8a^7bxx^nx^{-14n-1}}{13n}-\frac{7a^6b^2xx^{2n}x^{-14n-1}}{3n}-\frac{56a^5b^3xx^{3n}x^{-14n-1}}{11n}-\frac{7a^4b^4xx^{4n}x^{-14n-1}}{n}-\frac{56a^3b^5xx^{5n}x^{-14n-1}}{9n} \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-14*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-14*n - 1)/(14*n) - 8*a**7*b*x*x**n*x**(-14*n - 1)/
(13*n) - 7*a**6*b**2*x*x**(2*n)*x**(-14*n - 1)/(3*n) - 56*a**5*b**3*x*x**(
3*n)*x**(-14*n - 1)/(11*n) - 7*a**4*b**4*x*x**(4*n)*x**(-14*n - 1)/n - 56*
a**3*b**5*x*x**(5*n)*x**(-14*n - 1)/(9*n) - 7*a**2*b**6*x*x**(6*n)*x**(-14
*n - 1)/(2*n) - 8*a*b**7*x*x**(7*n)*x**(-14*n - 1)/(7*n) - b**8*x*x**(8*n)
*x**(-14*n - 1)/(6*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int x^{-1-14n}(a+bx^n)^8\,dx = -\frac{a^8}{14\,nx^{14\,n}} - \frac{8\,a^7b}{13\,nx^{13\,n}} - \frac{7\,a^6b^2}{3\,nx^{12\,n}} - \frac{56\,a^5b^3}{11\,nx^{11\,n}} - \frac{7\,a^4b^4}{nx^{10\,n}} - \frac{56\,a^3b^5}{9\,nx^{9\,n}} - \frac{7\,a^2b^6}{2\,nx^{8\,n}} - \frac{8\,ab^7}{7\,nx^{7\,n}} - \frac{b^8}{6\,nx^{6\,n}}$$

input
```
integrate(x^(-1-14*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/14*a^8/(n*x^(14*n)) - 8/13*a^7*b/(n*x^(13*n)) - 7/3*a^6*b^2/(n*x^(12*n)
) - 56/11*a^5*b^3/(n*x^(11*n)) - 7*a^4*b^4/(n*x^(10*n)) - 56/9*a^3*b^5/(n*
x^(9*n)) - 7/2*a^2*b^6/(n*x^(8*n)) - 8/7*a*b^7/(n*x^(7*n)) - 1/6*b^8/(n*x^
(6*n))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-14n}(a+bx^n)^8\,dx = -\frac{3003\,b^8x^{8\,n} + 20592\,ab^7x^{7\,n} + 63063\,a^2b^6x^{6\,n} + 112112\,a^3b^5x^{5\,n} + 126126\,a^4b^4x^{4\,n} + 91728\,a^5b^3x^{3\,n} + 4}{18018\,nx^{14\,n}}$$

input
```
integrate(x^(-1-14*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/18018*(3003*b^8*x^(8*n) + 20592*a*b^7*x^(7*n) + 63063*a^2*b^6*x^(6*n) +
 112112*a^3*b^5*x^(5*n) + 126126*a^4*b^4*x^(4*n) + 91728*a^5*b^3*x^(3*n) +
 42042*a^6*b^2*x^(2*n) + 11088*a^7*b*x^n + 1287*a^8)/(n*x^(14*n))
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int x^{-1-14n}(a+bx^n)^8\,dx = -\frac{a^8}{14\,n\,x^{14\,n}} - \frac{b^8}{6\,n\,x^{6\,n}} - \frac{7\,a^2\,b^6}{2\,n\,x^{8\,n}} - \frac{56\,a^3\,b^5}{9\,n\,x^{9\,n}} - \frac{7\,a^4\,b^4}{n\,x^{10\,n}} - \frac{56\,a^5\,b^3}{11\,n\,x^{11\,n}} - \frac{7\,a^6\,b^2}{3\,n\,x^{12\,n}} - \frac{8\,a\,b^7}{7\,n\,x^{7\,n}} - \frac{8\,a^7\,b}{13\,n\,x^{13\,n}}$$

input
```
int((a + b*x^n)^8/x^(14*n + 1),x)
```

output
```
- a^8/(14*n*x^(14*n)) - b^8/(6*n*x^(6*n)) - (7*a^2*b^6)/(2*n*x^(8*n)) - (5
6*a^3*b^5)/(9*n*x^(9*n)) - (7*a^4*b^4)/(n*x^(10*n)) - (56*a^5*b^3)/(11*n*x
^(11*n)) - (7*a^6*b^2)/(3*n*x^(12*n)) - (8*a*b^7)/(7*n*x^(7*n)) - (8*a^7*b
)/(13*n*x^(13*n))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-14n}(a+bx^n)^8\,dx$$
$$= \frac{-3003x^{8n}b^8 - 20592x^{7n}a\,b^7 - 63063x^{6n}a^2b^6 - 112112x^{5n}a^3b^5 - 126126x^{4n}a^4b^4 - 91728x^{3n}a^5b^3 - 42042}{18018x^{14n}n}$$

input
```
int(x^(-1-14*n)*(a+b*x^n)^8,x)
```

output
```
( - 3003*x**(8*n)*b**8 - 20592*x**(7*n)*a*b**7 - 63063*x**(6*n)*a**2*b**6
- 112112*x**(5*n)*a**3*b**5 - 126126*x**(4*n)*a**4*b**4 - 91728*x**(3*n)*a
**5*b**3 - 42042*x**(2*n)*a**6*b**2 - 11088*x**n*a**7*b - 1287*a**8)/(1801
8*x**(14*n)*n)
```

3.446 $\int x^{-1-15n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 151

$$\int x^{-1-15n}(a+bx^n)^8\,dx = -\frac{a^8x^{-15n}}{15n}-\frac{4a^7bx^{-14n}}{7n}-\frac{28a^6b^2x^{-13n}}{13n}-\frac{14a^5b^3x^{-12n}}{3n}-\frac{70a^4b^4x^{-11n}}{11n}-\frac{28a^3b^5x^{-10n}}{5n}-\frac{28a^2b^6x^{-9n}}{9n}-\frac{ab^7x^{-8n}}{n}-\frac{b^8x^{-7n}}{7n}$$

output
```
-1/15*a^8/n/(x^(15*n))-4/7*a^7*b/n/(x^(14*n))-28/13*a^6*b^2/n/(x^(13*n))-1
4/3*a^5*b^3/n/(x^(12*n))-70/11*a^4*b^4/n/(x^(11*n))-28/5*a^3*b^5/n/(x^(10*
n))-28/9*a^2*b^6/n/(x^(9*n))-a*b^7/n/(x^(8*n))-1/7*b^8/n/(x^(7*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-15n}(a+bx^n)^8\,dx = \frac{x^{-15n}(-3003a^8-25740a^7bx^n-97020a^6b^2x^{2n}-210210a^5b^3x^{3n}-286650a^4b^4x^{4n}-252252a^3b^5x^{5n}-14}{45045n}$$

input
```
Integrate[x^(-1 - 15*n)*(a + b*x^n)^8,x]
```

output
```
(-3003*a^8 - 25740*a^7*b*x^n - 97020*a^6*b^2*x^(2*n) - 210210*a^5*b^3*x^(3
*n) - 286650*a^4*b^4*x^(4*n) - 252252*a^3*b^5*x^(5*n) - 140140*a^2*b^6*x^(
6*n) - 45045*a*b^7*x^(7*n) - 6435*b^8*x^(8*n))/(45045*n*x^(15*n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-15n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-16n}(bx^n+a)^8\,dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int (a^8x^{-16n}+8a^7bx^{-15n}+28a^6b^2x^{-14n}+56a^5b^3x^{-13n}+70a^4b^4x^{-12n}+56a^3b^5x^{-11n}+28a^2b^6x^{-10n}+8ab^7x^{-9}}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{15}a^8x^{-15n}-\frac{4}{7}a^7bx^{-14n}-\frac{28}{13}a^6b^2x^{-13n}-\frac{14}{3}a^5b^3x^{-12n}-\frac{70}{11}a^4b^4x^{-11n}-\frac{28}{5}a^3b^5x^{-10n}-\frac{28}{9}a^2b^6x^{-9n}-ab^7x^{-8r}}{n}$$

input
```
Int[x^(-1 - 15*n)*(a + b*x^n)^8,x]
```

output
```
(-1/15*a^8/x^(15*n) - (4*a^7*b)/(7*x^(14*n)) - (28*a^6*b^2)/(13*x^(13*n))
- (14*a^5*b^3)/(3*x^(12*n)) - (70*a^4*b^4)/(11*x^(11*n)) - (28*a^3*b^5)/(5
*x^(10*n)) - (28*a^2*b^6)/(9*x^(9*n)) - (a*b^7)/x^(8*n) - b^8/(7*x^(7*n)))
/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 7.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{b^{8} x^{-7 n}}{7 n}-\frac{a\,b^{7} x^{-8 n}}{n}-\frac{28 a^{2} b^{6} x^{-9 n}}{9 n}-\frac{28 a^{3} b^{5} x^{-10 n}}{5 n}-\frac{70 a^{4} b^{4} x^{-11 n}}{11 n}-\frac{14 a^{5} b^{3} x^{-12 n}}{3 n}-\frac{28 a^{6} b^{2} x^{-13 n}}{13 n}-\frac{4 a^{7} b\,x}{7}$
parallelrisch	$\frac{-6435 x\,x^{8 n} x^{-1-15 n} b^{8}-45045 x\,x^{7 n} x^{-1-15 n} a\,b^{7}-140140 x\,x^{6 n} x^{-1-15 n} a^{2} b^{6}-252252 x\,x^{5 n} x^{-1-15 n} a^{3} b^{5}-286650 x\,x^{4 n} x^{-1}}{45045 n}$
orering	Expression too large to display

input
```
int(x^(-1-15*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*b^8/n/(x^n)^7-a*b^7/n/(x^n)^8-28/9*a^2*b^6/n/(x^n)^9-28/5*a^3*b^5/n/(
x^n)^10-70/11*a^4*b^4/n/(x^n)^11-14/3*a^5*b^3/n/(x^n)^12-28/13*a^6*b^2/n/(
x^n)^13-4/7*a^7*b/n/(x^n)^14-1/15*a^8/n/(x^n)^15
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-15n}(a+bx^n)^8\,dx = -\frac{6435\,b^8x^{8\,n}+45045\,ab^7x^{7\,n}+140140\,a^2b^6x^{6\,n}+252252\,a^3b^5x^{5\,n}+286650\,a^4b^4x^{4\,n}+210210\,a^5b^3x^{3\,n}+\cdots}{45045\,nx^{15\,n}}$$

input
```
integrate(x^(-1-15*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/45045*(6435*b^8*x^(8*n) + 45045*a*b^7*x^(7*n) + 140140*a^2*b^6*x^(6*n)
+ 252252*a^3*b^5*x^(5*n) + 286650*a^4*b^4*x^(4*n) + 210210*a^5*b^3*x^(3*n)
 + 97020*a^6*b^2*x^(2*n) + 25740*a^7*b*x^n + 3003*a^8)/(n*x^(15*n))
```

Sympy [A] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.52

$$\int x^{-1-15n}(a+bx^n)^8\,dx$$
$$= \begin{cases} -\frac{a^8xx^{-15n-1}}{15n}-\frac{4a^7bxx^nx^{-15n-1}}{7n}-\frac{28a^6b^2xx^{2n}x^{-15n-1}}{13n}-\frac{14a^5b^3xx^{3n}x^{-15n-1}}{3n}-\frac{70a^4b^4xx^{4n}x^{-15n-1}}{11n}-\frac{28a^3b^5xx^{5n}x^{-15n-1}}{5n}\cdots \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-15*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-15*n - 1)/(15*n) - 4*a**7*b*x*x**n*x**(-15*n - 1)/
(7*n) - 28*a**6*b**2*x*x**(2*n)*x**(-15*n - 1)/(13*n) - 14*a**5*b**3*x*x**
(3*n)*x**(-15*n - 1)/(3*n) - 70*a**4*b**4*x*x**(4*n)*x**(-15*n - 1)/(11*n)
 - 28*a**3*b**5*x*x**(5*n)*x**(-15*n - 1)/(5*n) - 28*a**2*b**6*x*x**(6*n)*
x**(-15*n - 1)/(9*n) - a*b**7*x*x**(7*n)*x**(-15*n - 1)/n - b**8*x*x**(8*n
)*x**(-15*n - 1)/(7*n), Ne(n, 0)), ((a + b)**8*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int x^{-1-15n}(a+bx^n)^8\,dx = -\frac{a^8}{15\,nx^{15\,n}} - \frac{4\,a^7b}{7\,nx^{14\,n}} - \frac{28\,a^6b^2}{13\,nx^{13\,n}} - \frac{14\,a^5b^3}{3\,nx^{12\,n}} - \frac{70\,a^4b^4}{11\,nx^{11\,n}} - \frac{28\,a^3b^5}{5\,nx^{10\,n}} - \frac{28\,a^2b^6}{9\,nx^{9\,n}} - \frac{ab^7}{nx^{8\,n}} - \frac{b^8}{7\,nx^{7\,n}}$$

input
```
integrate(x^(-1-15*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/15*a^8/(n*x^(15*n)) - 4/7*a^7*b/(n*x^(14*n)) - 28/13*a^6*b^2/(n*x^(13*n
)) - 14/3*a^5*b^3/(n*x^(12*n)) - 70/11*a^4*b^4/(n*x^(11*n)) - 28/5*a^3*b^5
/(n*x^(10*n)) - 28/9*a^2*b^6/(n*x^(9*n)) - a*b^7/(n*x^(8*n)) - 1/7*b^8/(n*
x^(7*n))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-15n}(a+bx^n)^8\,dx = -\frac{6435\,b^8x^{8\,n} + 45045\,ab^7x^{7\,n} + 140140\,a^2b^6x^{6\,n} + 252252\,a^3b^5x^{5\,n} + 286650\,a^4b^4x^{4\,n} + 210210\,a^5b^3x^{3\,n} +}{45045\,nx^{15\,n}}$$

input
```
integrate(x^(-1-15*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/45045*(6435*b^8*x^(8*n) + 45045*a*b^7*x^(7*n) + 140140*a^2*b^6*x^(6*n)
+ 252252*a^3*b^5*x^(5*n) + 286650*a^4*b^4*x^(4*n) + 210210*a^5*b^3*x^(3*n)
 + 97020*a^6*b^2*x^(2*n) + 25740*a^7*b*x^n + 3003*a^8)/(n*x^(15*n))
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int x^{-1-15n}(a+bx^n)^8\,dx = -\frac{a^8}{15\,n\,x^{15\,n}}-\frac{b^8}{7\,n\,x^{7\,n}}-\frac{28\,a^2\,b^6}{9\,n\,x^{9\,n}}-\frac{28\,a^3\,b^5}{5\,n\,x^{10\,n}}-\frac{70\,a^4\,b^4}{11\,n\,x^{11\,n}}-\frac{14\,a^5\,b^3}{3\,n\,x^{12\,n}}-\frac{28\,a^6\,b^2}{13\,n\,x^{13\,n}}-\frac{a\,b^7}{n\,x^{8\,n}}-\frac{4\,a^7\,b}{7\,n\,x^{14\,n}}$$

input
```
int((a + b*x^n)^8/x^(15*n + 1),x)
```

output
```
- a^8/(15*n*x^(15*n)) - b^8/(7*n*x^(7*n)) - (28*a^2*b^6)/(9*n*x^(9*n)) - (
28*a^3*b^5)/(5*n*x^(10*n)) - (70*a^4*b^4)/(11*n*x^(11*n)) - (14*a^5*b^3)/(
3*n*x^(12*n)) - (28*a^6*b^2)/(13*n*x^(13*n)) - (a*b^7)/(n*x^(8*n)) - (4*a^
7*b)/(7*n*x^(14*n))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^{-1-15n}(a+bx^n)^8\,dx = \frac{-6435x^{8n}b^8-45045x^{7n}a\,b^7-140140x^{6n}a^2b^6-252252x^{5n}a^3b^5-286650x^{4n}a^4b^4-210210x^{3n}a^5b^3-970}{45045x^{15n}n}$$

input
```
int(x^(-1-15*n)*(a+b*x^n)^8,x)
```

output
```
( - 6435*x**(8*n)*b**8 - 45045*x**(7*n)*a*b**7 - 140140*x**(6*n)*a**2*b**6
 - 252252*x**(5*n)*a**3*b**5 - 286650*x**(4*n)*a**4*b**4 - 210210*x**(3*n)
*a**5*b**3 - 97020*x**(2*n)*a**6*b**2 - 25740*x**n*a**7*b - 3003*a**8)/(45
045*x**(15*n)*n)
```

3.447 $\int x^{-1+n}(a+bx^n)^{16}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)^{16}\,dx=\frac{(a+bx^n)^{17}}{17bn}$$

output
```
1/17*(a+b*x^n)^17/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^{16}\,dx=\frac{(a+bx^n)^{17}}{17bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^16,x]
```

output
```
(a + b*x^n)^17/(17*b*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^{16}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{(a+bx^n)^{17}}{17bn}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^16,x]
```

output
```
(a + b*x^n)^17/(17*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(17) = 34.

Time = 381.25 (sec) , antiderivative size = 260, normalized size of antiderivative = 13.68

method	result
risch	$\frac{b^{16}x^{17n}}{17n}+\frac{b^{15}a\,x^{16n}}{n}+\frac{8b^{14}a^2x^{15n}}{n}+\frac{40b^{13}a^3x^{14n}}{n}+\frac{140b^{12}a^4x^{13n}}{n}+\frac{364b^{11}a^5x^{12n}}{n}+\frac{728b^{10}a^6x^{11n}}{n}+\frac{1144b^9}{n}$
parallelrisch	$b^{16}x^{-1+n}x^{16n}x+17a\,b^{15}x^{-1+n}x^{15n}x+136a^2b^{14}x^{-1+n}x^{14n}x+680a^3b^{13}x^{-1+n}x^{13n}x+2380a^4b^{12}x^{-1+n}x^{12n}x+6188a^5b^{11}$
orering	Expression too large to display

input
```
int(x^(-1+n)*(a+b*x^n)^16,x,method=_RETURNVERBOSE)
```

output
```
1/17*b^16/n*(x^n)^17+b^15*a/n*(x^n)^16+8*b^14*a^2/n*(x^n)^15+40*b^13*a^3/n
*(x^n)^14+140*b^12*a^4/n*(x^n)^13+364*b^11*a^5/n*(x^n)^12+728*b^10*a^6/n*(
x^n)^11+1144*b^9*a^7/n*(x^n)^10+1430*a^8*b^8/n*(x^n)^9+1430*a^9*b^7/n*(x^n
)^8+1144*a^10*b^6/n*(x^n)^7+728*a^11*b^5/n*(x^n)^6+364*a^12*b^4/n*(x^n)^5+
140*a^13*b^3/n*(x^n)^4+40*a^14*b^2/n*(x^n)^3+8*a^15*b/n*(x^n)^2+a^16/n*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 11.26

$$\int x^{-1+n}(a+bx^n)^{16}\,dx$$
$$= \frac{b^{16}x^{17\,n}+17\,ab^{15}x^{16\,n}+136\,a^2b^{14}x^{15\,n}+680\,a^3b^{13}x^{14\,n}+2380\,a^4b^{12}x^{13\,n}+6188\,a^5b^{11}x^{12\,n}+12376\,a^6b^{10}}{}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^16,x, algorithm="fricas")
```

output
```
1/17*(b^16*x^(17*n) + 17*a*b^15*x^(16*n) + 136*a^2*b^14*x^(15*n) + 680*a^3
*b^13*x^(14*n) + 2380*a^4*b^12*x^(13*n) + 6188*a^5*b^11*x^(12*n) + 12376*a
^6*b^10*x^(11*n) + 19448*a^7*b^9*x^(10*n) + 24310*a^8*b^8*x^(9*n) + 24310*
a^9*b^7*x^(8*n) + 19448*a^10*b^6*x^(7*n) + 12376*a^11*b^5*x^(6*n) + 6188*a
^12*b^4*x^(5*n) + 2380*a^13*b^3*x^(4*n) + 680*a^14*b^2*x^(3*n) + 136*a^15*
b*x^(2*n) + 17*a^16*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(12) = 24$.

Time = 26.74 (sec) , antiderivative size = 362, normalized size of antiderivative = 19.05

$$\int x^{-1+n}(a+bx^n)^{16}\,dx$$
$$=\begin{cases}\frac{a^{16}xx^{n-1}}{n}+\frac{8a^{15}bxx^nx^{n-1}}{n}+\frac{40a^{14}b^2xx^{2n}x^{n-1}}{n}+\frac{140a^{13}b^3xx^{3n}x^{n-1}}{n}+\frac{364a^{12}b^4xx^{4n}x^{n-1}}{n}+\frac{728a^{11}b^5xx^{5n}x^{n-1}}{n}+\frac{1144a^{10}b^6}{} \\ (a+b)^{16}\log(x)\end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**16,x)
```

output
```
Piecewise((a**16*x*x**(n - 1)/n + 8*a**15*b*x*x**n*x**(n - 1)/n + 40*a**14
*b**2*x*x**(2*n)*x**(n - 1)/n + 140*a**13*b**3*x*x**(3*n)*x**(n - 1)/n + 3
64*a**12*b**4*x*x**(4*n)*x**(n - 1)/n + 728*a**11*b**5*x*x**(5*n)*x**(n -
1)/n + 1144*a**10*b**6*x*x**(6*n)*x**(n - 1)/n + 1430*a**9*b**7*x*x**(7*n)
*x**(n - 1)/n + 1430*a**8*b**8*x*x**(8*n)*x**(n - 1)/n + 1144*a**7*b**9*x*
x**(9*n)*x**(n - 1)/n + 728*a**6*b**10*x*x**(10*n)*x**(n - 1)/n + 364*a**5
*b**11*x*x**(11*n)*x**(n - 1)/n + 140*a**4*b**12*x*x**(12*n)*x**(n - 1)/n
+ 40*a**3*b**13*x*x**(13*n)*x**(n - 1)/n + 8*a**2*b**14*x*x**(14*n)*x**(n
- 1)/n + a*b**15*x*x**(15*n)*x**(n - 1)/n + b**16*x*x**(16*n)*x**(n - 1)/(
17*n), Ne(n, 0)), ((a + b)**16*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)^{16}\,dx=\frac{(bx^n+a)^{17}}{17\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^16,x, algorithm="maxima")
```

output
```
1/17*(b*x^n + a)^17/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 11.26

$$\int x^{-1+n}(a+bx^n)^{16}\,dx$$
$$= \frac{b^{16}x^{17n} + 17\,ab^{15}x^{16n} + 136\,a^2b^{14}x^{15n} + 680\,a^3b^{13}x^{14n} + 2380\,a^4b^{12}x^{13n} + 6188\,a^5b^{11}x^{12n} + 12376\,a^6b^{10}}{}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^16,x, algorithm="giac")
```

output
```
1/17*(b^16*x^(17*n) + 17*a*b^15*x^(16*n) + 136*a^2*b^14*x^(15*n) + 680*a^3
*b^13*x^(14*n) + 2380*a^4*b^12*x^(13*n) + 6188*a^5*b^11*x^(12*n) + 12376*a
^6*b^10*x^(11*n) + 19448*a^7*b^9*x^(10*n) + 24310*a^8*b^8*x^(9*n) + 24310*
a^9*b^7*x^(8*n) + 19448*a^10*b^6*x^(7*n) + 12376*a^11*b^5*x^(6*n) + 6188*a
^12*b^4*x^(5*n) + 2380*a^13*b^3*x^(4*n) + 680*a^14*b^2*x^(3*n) + 136*a^15*
b*x^(2*n) + 17*a^16*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 259, normalized size of antiderivative = 13.63

$$\int x^{-1+n}(a+bx^n)^{16}\,dx = \frac{a^{16}\,x^n}{n} + \frac{b^{16}\,x^{17\,n}}{17\,n} + \frac{40\,a^{14}\,b^2\,x^{3\,n}}{n} + \frac{140\,a^{13}\,b^3\,x^{4\,n}}{n} + \frac{364\,a^{12}\,b^4\,x^{5\,n}}{n} + \frac{728\,a^{11}\,b^5\,x^{6\,n}}{n} + \frac{1144\,a^{10}\,b^6\,x^{7\,n}}{n} + \frac{1430\,a^9\,b^7\,x^{8\,n}}{n} + \frac{1430\,a^8\,b^8\,x^{9\,n}}{n} + \frac{1144\,a^7\,b^9\,x^{10\,n}}{n} + \frac{728\,a^6\,b^{10}\,x^{11\,n}}{n} + \frac{364\,a^5\,b^{11}\,x^{12\,n}}{n} + \frac{140\,a^4\,b^{12}\,x^{13\,n}}{n} + \frac{40\,a^3\,b^{13}\,x^{14\,n}}{n} + \frac{8\,a^2\,b^{14}\,x^{15\,n}}{n} + \frac{8\,a^{15}\,b\,x^{2\,n}}{n} + \frac{a\,b^{15}\,x^{16\,n}}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^16,x)
```

output
```
(a^16*x^n)/n + (b^16*x^(17*n))/(17*n) + (40*a^14*b^2*x^(3*n))/n + (140*a^1
3*b^3*x^(4*n))/n + (364*a^12*b^4*x^(5*n))/n + (728*a^11*b^5*x^(6*n))/n + (
1144*a^10*b^6*x^(7*n))/n + (1430*a^9*b^7*x^(8*n))/n + (1430*a^8*b^8*x^(9*n
))/n + (1144*a^7*b^9*x^(10*n))/n + (728*a^6*b^10*x^(11*n))/n + (364*a^5*b^
11*x^(12*n))/n + (140*a^4*b^12*x^(13*n))/n + (40*a^3*b^13*x^(14*n))/n + (8
*a^2*b^14*x^(15*n))/n + (8*a^15*b*x^(2*n))/n + (a*b^15*x^(16*n))/n
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 212, normalized size of antiderivative = 11.16

$$\int x^{-1+n}(a+bx^n)^{16}\,dx$$
$$= \frac{x^n(x^{16n}b^{16} + 17x^{15n}a\,b^{15} + 136x^{14n}a^2b^{14} + 680x^{13n}a^3b^{13} + 2380x^{12n}a^4b^{12} + 6188x^{11n}a^5b^{11} + 12376x^{10n}a^{6}}{}$$

input
```
int(x^(-1+n)*(a+b*x^n)^16,x)
```

output
```
(x**n*(x**(16*n)*b**16 + 17*x**(15*n)*a*b**15 + 136*x**(14*n)*a**2*b**14 +
 680*x**(13*n)*a**3*b**13 + 2380*x**(12*n)*a**4*b**12 + 6188*x**(11*n)*a**
5*b**11 + 12376*x**(10*n)*a**6*b**10 + 19448*x**(9*n)*a**7*b**9 + 24310*x*
*(8*n)*a**8*b**8 + 24310*x**(7*n)*a**9*b**7 + 19448*x**(6*n)*a**10*b**6 +
12376*x**(5*n)*a**11*b**5 + 6188*x**(4*n)*a**12*b**4 + 2380*x**(3*n)*a**13
*b**3 + 680*x**(2*n)*a**14*b**2 + 136*x**n*a**15*b + 17*a**16))/(17*n)
```

3.448 $\int x^{12}(a+bx^{13})^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

output
```
1/169*(b*x^13+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}$$

input
```
Integrate[x^12*(a + b*x^13)^12,x]
```

output
```
(a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*
x^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^9
1)/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)
/13 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx$$
$$\Big\downarrow \; 793$$
$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

input
```
Int[x^12*(a + b*x^13)^12,x]
```

output
```
(a + b*x^13)^13/(169*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{13}+a\right)^{13}}{169 b}$
gosper	$\frac{22}{13}a^{3} b^{9} x^{130}+\frac{1}{13}a\,b^{11} x^{156}+\frac{55}{13}a^{9} b^{3} x^{52}+\frac{1}{169}b^{12} x^{169}+\frac{22}{13}a^{10} b^{2} x^{39}+\frac{55}{13}a^{4} b^{8} x^{117}+\frac{132}{13}a^{7} b^{5} x^{78}$
parallelrisch	$\frac{22}{13}a^{3} b^{9} x^{130}+\frac{1}{13}a\,b^{11} x^{156}+\frac{55}{13}a^{9} b^{3} x^{52}+\frac{1}{169}b^{12} x^{169}+\frac{22}{13}a^{10} b^{2} x^{39}+\frac{55}{13}a^{4} b^{8} x^{117}+\frac{132}{13}a^{7} b^{5} x^{78}$
orering	$\frac{x^{13}\left(b^{12} x^{156}+13 a\,b^{11} x^{143}+78 a^{2} b^{10} x^{130}+286 a^{3} b^{9} x^{117}+715 a^{4} b^{8} x^{104}+1287 a^{5} b^{7} x^{91}+1716 a^{6} b^{6} x^{78}+1716 a^{7} b^{5} x^{65}+1287 a^{8} b^{4}\right.}{169}$
risch	$\frac{b^{12} x^{169}}{169}+\frac{a\,b^{11} x^{156}}{13}+\frac{6 a^{2} b^{10} x^{143}}{13}+\frac{22 a^{3} b^{9} x^{130}}{13}+\frac{55 a^{4} b^{8} x^{117}}{13}+\frac{99 a^{5} b^{7} x^{104}}{13}+\frac{132 a^{6} b^{6} x^{91}}{13}+\frac{132 a^{7} b^{5} x^{78}}{13}+$

input
```
int(x^12*(b*x^13+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/169*(b*x^13+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

output
```
1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9
*x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 +
 132/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^1
0*b^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}(a+bx^{13})^{12}\,dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^2x^{39}}{13} + \frac{55a^9b^3x^{52}}{13} + \frac{99a^8b^4x^{65}}{13} + \frac{132a^7b^5x^{78}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

input
```
integrate(x**12*(b*x**13+a)**12,x)
```

output
```
a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**
3*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**
6*x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**
9*x**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/16
9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}(a+bx^{13})^{12}\,dx = \frac{(bx^{13}+a)^{13}}{169\,b}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
```

output
```
1/169*(b*x^13 + a)^13/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}(a+bx^{13})^{12}\,dx=\frac{(bx^{13}+a)^{13}}{169\,b}$$

input
```
integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

output
```
1/169*(b*x^13 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}(a+bx^{13})^{12}\,dx=\frac{(b\,x^{13}+a)^{13}}{169\,b}$$

input
```
int(x^12*(a + b*x^13)^12,x)
```

output
```
(a + b*x^13)^13/(169*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{12}(a+bx^{13})^{12}\,dx$$
$$=\frac{x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+17}{169}$$

input
```
int(x^12*(b*x^13+a)^12,x)
```

output
```
(x**13*(13*a**12 + 78*a**11*b*x**13 + 286*a**10*b**2*x**26 + 715*a**9*b**3
*x**39 + 1287*a**8*b**4*x**52 + 1716*a**7*b**5*x**65 + 1716*a**6*b**6*x**7
8 + 1287*a**5*b**7*x**91 + 715*a**4*b**8*x**104 + 286*a**3*b**9*x**117 + 7
8*a**2*b**10*x**130 + 13*a*b**11*x**143 + b**12*x**156))/169
```

3.449 $\int x^{24}(a+bx^{25})^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

output
```
1/325*(b*x^25+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}$$

input
```
Integrate[x^24*(a + b*x^25)^12,x]
```

output
```
(a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*
x^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x
^175)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^25
0)/25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{24}(a+bx^{25})^{12}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{(a+bx^{25})^{13}}{325b}$$

input
```
Int[x^24*(a + b*x^25)^12,x]
```

output
```
(a + b*x^25)^13/(325*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{(b\,x^{25}+a)^{13}}{325b}$
gosper	$\frac{1}{25}a\,b^{11}x^{300}+\frac{1}{25}a^{12}x^{25}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{11}{5}a^{4}b^{8}x^{225}$
parallelrisch	$\frac{1}{25}a\,b^{11}x^{300}+\frac{1}{25}a^{12}x^{25}+\frac{6}{25}a^{11}b\,x^{50}+\frac{132}{25}a^{7}b^{5}x^{150}+\frac{99}{25}a^{8}b^{4}x^{125}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{11}{5}a^{4}b^{8}x^{225}$
orering	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^{2}b^{10}x^{250}+286a^{3}b^{9}x^{225}+715a^{4}b^{8}x^{200}+1287a^{5}b^{7}x^{175}+1716a^{6}b^{6}x^{150}+1716a^{7}b^{5}x^{125}+1287a}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+$

input
```
int(x^24*(b*x^25+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/325*(b*x^25+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=&\ \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}\\&+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}\\&+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}\\&+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

output
```
1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9
*x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 +
 132/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a
^10*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}(a+bx^{25})^{12}\,dx = \frac{a^{12}x^{25}}{25} + \frac{6a^{11}bx^{50}}{25} + \frac{22a^{10}b^2x^{75}}{25} + \frac{11a^9b^3x^{100}}{5} + \frac{99a^8b^4x^{125}}{25} + \frac{132a^7b^5x^{150}}{25} + \frac{132a^6b^6x^{175}}{25} + \frac{99a^5b^7x^{200}}{25} + \frac{11a^4b^8x^{225}}{5} + \frac{22a^3b^9x^{250}}{25} + \frac{6a^2b^{10}x^{275}}{25} + \frac{ab^{11}x^{300}}{25} + \frac{b^{12}x^{325}}{325}$$

input
```
integrate(x**24*(b*x**25+a)**12,x)
```

output
```
a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**
3*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b
**6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b
**9*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/
325
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}(a+bx^{25})^{12}\,dx = \frac{(bx^{25}+a)^{13}}{325\,b}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

output
```
1/325*(b*x^25 + a)^13/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

input
```
integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

output
```
1/325*(b*x^25 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(b\,x^{25}+a\right)^{13}}{325\,b}$$

input
```
int(x^24*(a + b*x^25)^12,x)
```

output
```
(a + b*x^25)^13/(325*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx$$
$$=\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1}{325}$$

input
```
int(x^24*(b*x^25+a)^12,x)
```

output
```
(x**25*(13*a**12 + 78*a**11*b*x**25 + 286*a**10*b**2*x**50 + 715*a**9*b**3
*x**75 + 1287*a**8*b**4*x**100 + 1716*a**7*b**5*x**125 + 1716*a**6*b**6*x*
*150 + 1287*a**5*b**7*x**175 + 715*a**4*b**8*x**200 + 286*a**3*b**9*x**225
 + 78*a**2*b**10*x**250 + 13*a*b**11*x**275 + b**12*x**300))/325
```

3.450 $\int x^{36}(a+bx^{37})^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

output
```
1/481*(b*x^37+a)^13/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=&\ \frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}\\&+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}\\&+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}\end{aligned}$$

input
```
Integrate[x^36*(a + b*x^37)^12,x]
```

output
```
(a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3
*x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6
*x^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x
^370)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx$$
$$\Big\downarrow \; 793$$
$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

input
```
Int[x^36*(a + b*x^37)^12,x]
```

output
```
(a + b*x^37)^13/(481*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{37}+a\right)^{13}}{481 b}$
gosper	$\frac{132}{37} a^{7} b^{5} x^{222}+\frac{22}{37} a^{10} b^{2} x^{111}+\frac{99}{37} a^{5} b^{7} x^{296}+\frac{6}{37} a^{11} b\,x^{74}+\frac{132}{37} a^{6} b^{6} x^{259}+\frac{1}{481} b^{12} x^{481}+\frac{55}{37} a^{9} b^{3} x^{1}$
parallelrisch	$\frac{132}{37} a^{7} b^{5} x^{222}+\frac{22}{37} a^{10} b^{2} x^{111}+\frac{99}{37} a^{5} b^{7} x^{296}+\frac{6}{37} a^{11} b\,x^{74}+\frac{132}{37} a^{6} b^{6} x^{259}+\frac{1}{481} b^{12} x^{481}+\frac{55}{37} a^{9} b^{3} x^{1}$
orering	$\frac{x^{37} \left(b^{12} x^{444}+13 a\,b^{11} x^{407}+78 a^{2} b^{10} x^{370}+286 a^{3} b^{9} x^{333}+715 a^{4} b^{8} x^{296}+1287 a^{5} b^{7} x^{259}+1716 a^{6} b^{6} x^{222}+1716 a^{7} b^{5} x^{185}+1287 a\right.}{481}$
risch	$\frac{b^{12} x^{481}}{481}+\frac{a\,b^{11} x^{444}}{37}+\frac{6 a^{2} b^{10} x^{407}}{37}+\frac{22 a^{3} b^{9} x^{370}}{37}+\frac{55 a^{4} b^{8} x^{333}}{37}+\frac{99 a^{5} b^{7} x^{296}}{37}+\frac{132 a^{6} b^{6} x^{259}}{37}+\frac{132 a^{7} b^{5} x^{222}}{37}+$

input
```
int(x^36*(b*x^37+a)^12,x,method=_RETURNVERBOSE)
```

output
```
1/481*(b*x^37+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=&\ \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}\\&+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}\\&+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}\\&+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
```

output
```
1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9
*x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259
+ 132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37
*a^10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

input
```
integrate(x**36*(b*x**37+a)**12,x)
```

output
```
a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b*
*3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6
*b**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**
3*b**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**4
81/481
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

output
```
1/481*(b*x^37 + a)^13/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

input
```
integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

output
```
1/481*(b*x^37 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(b\,x^{37}+a\right)^{13}}{481\,b}$$

input
```
int(x^36*(a + b*x^37)^12,x)
```

output
```
(a + b*x^37)^13/(481*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx$$
$$=\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1}{481}$$

input
```
int(x^36*(b*x^37+a)^12,x)
```

output
```
(x**37*(13*a**12 + 78*a**11*b*x**37 + 286*a**10*b**2*x**74 + 715*a**9*b**3
*x**111 + 1287*a**8*b**4*x**148 + 1716*a**7*b**5*x**185 + 1716*a**6*b**6*x
**222 + 1287*a**5*b**7*x**259 + 715*a**4*b**8*x**296 + 286*a**3*b**9*x**33
3 + 78*a**2*b**10*x**370 + 13*a*b**11*x**407 + b**12*x**444))/481
```

3.451 $\int x^{12m}(a+bx^{1+12m})^{12}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

output
```
1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b+156bm}$$

input
```
Integrate[x^(12*m)*(a + b*x^(1 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12m}\left(a+bx^{12m+1}\right)^{12}\,dx$$
$$\Big\downarrow \; 793$$
$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

input
```
Int[x^(12*m)*(a + b*x^(1 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(25) = 50$.

Time = 84.52 (sec) , antiderivative size = 288, normalized size of antiderivative = 10.67

method	result
parallelrisch	$\frac{b^{12}x^{12+144m}x^{12m}x+13a\,b^{11}x^{11+132m}x^{12m}x+78a^{2}b^{10}x^{10+120m}x^{12m}x+286a^{3}b^{9}x^{9+108m}x^{12m}x+715a^{4}b^{8}x^{8+96m}x^{12m}x+}{}$
risch	$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{1+12m}+\frac{6a^{2}b^{10}x^{11}x^{132m}}{1+12m}+\frac{22a^{3}b^{9}x^{10}x^{120m}}{1+12m}+\frac{55a^{4}b^{8}x^{9}x^{108m}}{1+12m}+\frac{99a^{5}b^{7}x^{8}x^{96m}}{1+12m}+\frac{132}{}$
orering	Expression too large to display

input
```
int(x^(12*m)*(a+b*x^(1+12*m))^12,x,method=_RETURNVERBOSE)
```

output
```
1/13*(b^12*(x^(1+12*m))^12*x^(12*m)*x+13*a*b^11*(x^(1+12*m))^11*x^(12*m)*x
+78*a^2*b^10*(x^(1+12*m))^10*x^(12*m)*x+286*a^3*b^9*(x^(1+12*m))^9*x^(12*m
)*x+715*a^4*b^8*(x^(1+12*m))^8*x^(12*m)*x+1287*a^5*b^7*(x^(1+12*m))^7*x^(1
2*m)*x+1716*a^6*b^6*(x^(1+12*m))^6*x^(12*m)*x+1716*a^7*b^5*(x^(1+12*m))^5*
x^(12*m)*x+1287*a^8*b^4*(x^(1+12*m))^4*x^(12*m)*x+715*a^9*b^3*(x^(1+12*m))
^3*x^(12*m)*x+286*a^10*b^2*(x^(1+12*m))^2*x^(12*m)*x+78*a^11*b*x^(1+12*m)*
x^(12*m)*x+13*a^12*x^(12*m)*x)/(1+12*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 7.19

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx = \frac{b^{12}x^{156\,m+13}+13\,ab^{11}x^{144\,m+12}+78\,a^2b^{10}x^{132\,m+11}+286\,a^3b^9x^{120\,m+10}+715\,a^4b^8x^{108\,m+9}+1287\,a^5b^7x^{96}}{}$$

input
```
integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="fricas")
```

output
```
1/13*(b^12*x^(156*m + 13) + 13*a*b^11*x^(144*m + 12) + 78*a^2*b^10*x^(132*
m + 11) + 286*a^3*b^9*x^(120*m + 10) + 715*a^4*b^8*x^(108*m + 9) + 1287*a^
5*b^7*x^(96*m + 8) + 1716*a^6*b^6*x^(84*m + 7) + 1716*a^7*b^5*x^(72*m + 6)
 + 1287*a^8*b^4*x^(60*m + 5) + 715*a^9*b^3*x^(48*m + 4) + 286*a^10*b^2*x^(
36*m + 3) + 78*a^11*b*x^(24*m + 2) + 13*a^12*x^(12*m + 1))/(12*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(19) = 38$.

Time = 7.81 (sec) , antiderivative size = 345, normalized size of antiderivative = 12.78

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx$$
$$= \begin{cases} \frac{13a^{12}xx^{12m}}{156m+13} + \frac{78a^{11}bxx^{12m}x^{12m+1}}{156m+13} + \frac{286a^{10}b^2xx^{12m}x^{24m+2}}{156m+13} + \frac{715a^9b^3xx^{12m}x^{36m+3}}{156m+13} + \frac{1287a^8b^4xx^{12m}x^{48m+4}}{156m+13} + \frac{1716a^7b^5xx^{1}}{156m} \\ (a+b)^{12}\log(x) \end{cases}$$

input
```
integrate(x**(12*m)*(a+b*x**(1+12*m))**12,x)
```

output
```
Piecewise((13*a**12*x*x**(12*m)/(156*m + 13) + 78*a**11*b*x*x**(12*m)*x**(
12*m + 1)/(156*m + 13) + 286*a**10*b**2*x*x**(12*m)*x**(24*m + 2)/(156*m +
 13) + 715*a**9*b**3*x*x**(12*m)*x**(36*m + 3)/(156*m + 13) + 1287*a**8*b*
*4*x*x**(12*m)*x**(48*m + 4)/(156*m + 13) + 1716*a**7*b**5*x*x**(12*m)*x**
(60*m + 5)/(156*m + 13) + 1716*a**6*b**6*x*x**(12*m)*x**(72*m + 6)/(156*m
+ 13) + 1287*a**5*b**7*x*x**(12*m)*x**(84*m + 7)/(156*m + 13) + 715*a**4*b
**8*x*x**(12*m)*x**(96*m + 8)/(156*m + 13) + 286*a**3*b**9*x*x**(12*m)*x**
(108*m + 9)/(156*m + 13) + 78*a**2*b**10*x*x**(12*m)*x**(120*m + 10)/(156*
m + 13) + 13*a*b**11*x*x**(12*m)*x**(132*m + 11)/(156*m + 13) + b**12*x*x*
*(12*m)*x**(144*m + 12)/(156*m + 13), Ne(m, -1/12)), ((a + b)**12*log(x),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx = \frac{\left(bx^{12\,m+1}+a\right)^{13}}{13\,b(12\,m+1)}$$

input
```
integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="maxima")
```

output
```
1/13*(b*x^(12*m + 1) + a)^13/(b*(12*m + 1))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{\left(bx^{12\,m+1}+a\right)^{13}}{13\,b(12\,m+1)}$$

input
```
integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="giac")
```

output
```
1/13*(b*x^(12*m + 1) + a)^13/(b*(12*m + 1))
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1}+\frac{132\,a^7\,b^5\,x^{72\,m}\,x^6}{12\,m+1}+\frac{132\,a^6\,b^6\,x^{84\,m}\,x^7}{12\,m+1}+\frac{99\,a^5\,b^7\,x^{96\,m}\,x^8}{12\,m+1}+\frac{55\,a^4\,b^8\,x^{108\,m}\,x^9}{12\,m+1}+\frac{22\,a^3\,b^9\,x^{120\,m}\,x^{10}}{12\,m+1}+\frac{6\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{12\,m+1}$$

input
```
int(x^(12*m)*(a + b*x^(12*m + 1))^12,x)
```

output
```
(b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^1
1*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^
10*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (
99*a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1
) + (132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*
m + 1) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^1
0)/(12*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 7.48

$$\int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx$$
$$=\frac{x^{12m}x(x^{144m}b^{12}x^{12}+13x^{132m}a\,b^{11}x^{11}+78x^{120m}a^2b^{10}x^{10}+286x^{108m}a^3b^9x^9+715x^{96m}a^4b^8x^8+1287x^{84m}}{}$$

input
```
int(x^(12*m)*(a+b*x^(1+12*m))^12,x)
```

output
```
(x**(12*m)*x*(x**(144*m)*b**12*x**12 + 13*x**(132*m)*a*b**11*x**11 + 78*x*
*(120*m)*a**2*b**10*x**10 + 286*x**(108*m)*a**3*b**9*x**9 + 715*x**(96*m)*
a**4*b**8*x**8 + 1287*x**(84*m)*a**5*b**7*x**7 + 1716*x**(72*m)*a**6*b**6*
x**6 + 1716*x**(60*m)*a**7*b**5*x**5 + 1287*x**(48*m)*a**8*b**4*x**4 + 715
*x**(36*m)*a**9*b**3*x**3 + 286*x**(24*m)*a**10*b**2*x**2 + 78*x**(12*m)*a
**11*b*x + 13*a**12))/(13*(12*m + 1))
```

3.452 $\int x^{12+12(-1+m)}(a+bx^{1+12m})^{12}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

output
```
1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b+156bm}$$

input
```
Integrate[x^(12 + 12*(-1 + m))*(a + b*x^(1 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12(m-1)+12}\left(a+bx^{12m+1}\right)^{12}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

input
```
Int[x^(12 + 12*(-1 + m))*(a + b*x^(1 + 12*m))^12,x]
```

output
```
(a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(25) = 50.

Time = 85.91 (sec) , antiderivative size = 288, normalized size of antiderivative = 10.67

method	result
parallelrisch	$\frac{b^{12}x^{12+144m}x^{12m}x+13a\,b^{11}x^{11+132m}x^{12m}x+78a^2b^{10}x^{10+120m}x^{12m}x+286a^3b^9x^{9+108m}x^{12m}x+715a^4b^8x^{8+96m}x^{12m}x+}{}$
risch	$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{1+12m}+\frac{6a^2b^{10}x^{11}x^{132m}}{1+12m}+\frac{22a^3b^9x^{10}x^{120m}}{1+12m}+\frac{55a^4b^8x^9x^{108m}}{1+12m}+\frac{99a^5b^7x^8x^{96m}}{1+12m}+$ 132
orering	Expression too large to display

input
```
int(x^(12*m)*(a+b*x^(1+12*m))^12,x,method=_RETURNVERBOSE)
```

output
```
1/13*(b^12*(x^(1+12*m))^12*x^(12*m)*x+13*a*b^11*(x^(1+12*m))^11*x^(12*m)*x
+78*a^2*b^10*(x^(1+12*m))^10*x^(12*m)*x+286*a^3*b^9*(x^(1+12*m))^9*x^(12*m
)*x+715*a^4*b^8*(x^(1+12*m))^8*x^(12*m)*x+1287*a^5*b^7*(x^(1+12*m))^7*x^(1
2*m)*x+1716*a^6*b^6*(x^(1+12*m))^6*x^(12*m)*x+1716*a^7*b^5*(x^(1+12*m))^5*
x^(12*m)*x+1287*a^8*b^4*(x^(1+12*m))^4*x^(12*m)*x+715*a^9*b^3*(x^(1+12*m))
^3*x^(12*m)*x+286*a^10*b^2*(x^(1+12*m))^2*x^(12*m)*x+78*a^11*b*x^(1+12*m)*
x^(12*m)*x+13*a^12*x^(12*m)*x)/(1+12*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 7.19

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx = \frac{b^{12}x^{156\,m+13}+13\,ab^{11}x^{144\,m+12}+78\,a^2b^{10}x^{132\,m+11}+286\,a^3b^9x^{120\,m+10}+715\,a^4b^8x^{108\,m+9}+1287\,a^5b^7x^{96}}{}$$

input
```
integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="fricas")
```

output
```
1/13*(b^12*x^(156*m + 13) + 13*a*b^11*x^(144*m + 12) + 78*a^2*b^10*x^(132*
m + 11) + 286*a^3*b^9*x^(120*m + 10) + 715*a^4*b^8*x^(108*m + 9) + 1287*a^
5*b^7*x^(96*m + 8) + 1716*a^6*b^6*x^(84*m + 7) + 1716*a^7*b^5*x^(72*m + 6)
 + 1287*a^8*b^4*x^(60*m + 5) + 715*a^9*b^3*x^(48*m + 4) + 286*a^10*b^2*x^(
36*m + 3) + 78*a^11*b*x^(24*m + 2) + 13*a^12*x^(12*m + 1))/(12*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(19) = 38.

Time = 7.82 (sec) , antiderivative size = 345, normalized size of antiderivative = 12.78

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx$$
$$=\begin{cases}\frac{13a^{12}xx^{12m}}{156m+13}+\frac{78a^{11}bxx^{12m}x^{12m+1}}{156m+13}+\frac{286a^{10}b^2xx^{12m}x^{24m+2}}{156m+13}+\frac{715a^9b^3xx^{12m}x^{36m+3}}{156m+13}+\frac{1287a^8b^4xx^{12m}x^{48m+4}}{156m+13}+\frac{1716a^7b^5xx^{1}}{156m}\\ (a+b)^{12}\log(x)\end{cases}$$

input
```
integrate(x**(12*m)*(a+b*x**(1+12*m))**12,x)
```

output
```
Piecewise((13*a**12*x*x**(12*m)/(156*m + 13) + 78*a**11*b*x*x**(12*m)*x**(
12*m + 1)/(156*m + 13) + 286*a**10*b**2*x*x**(12*m)*x**(24*m + 2)/(156*m +
 13) + 715*a**9*b**3*x*x**(12*m)*x**(36*m + 3)/(156*m + 13) + 1287*a**8*b*
*4*x*x**(12*m)*x**(48*m + 4)/(156*m + 13) + 1716*a**7*b**5*x*x**(12*m)*x**
(60*m + 5)/(156*m + 13) + 1716*a**6*b**6*x*x**(12*m)*x**(72*m + 6)/(156*m
+ 13) + 1287*a**5*b**7*x*x**(12*m)*x**(84*m + 7)/(156*m + 13) + 715*a**4*b
**8*x*x**(12*m)*x**(96*m + 8)/(156*m + 13) + 286*a**3*b**9*x*x**(12*m)*x**
(108*m + 9)/(156*m + 13) + 78*a**2*b**10*x*x**(12*m)*x**(120*m + 10)/(156*
m + 13) + 13*a*b**11*x*x**(12*m)*x**(132*m + 11)/(156*m + 13) + b**12*x*x*
*(12*m)*x**(144*m + 12)/(156*m + 13), Ne(m, -1/12)), ((a + b)**12*log(x),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{\left(bx^{12\,m+1}+a\right)^{13}}{13\,b(12\,m+1)}$$

input
```
integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="maxima")
```

output
```
1/13*(b*x^(12*m + 1) + a)^13/(b*(12*m + 1))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx=\frac{\left(bx^{12\,m+1}+a\right)^{13}}{13\,b(12\,m+1)}$$

input
```
integrate(x^(12*m)*(a+b*x^(1+12*m))^12,x, algorithm="giac")
```

output
```
1/13*(b*x^(12*m + 1) + a)^13/(b*(12*m + 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\begin{aligned}\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx=&\ \frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}\\&+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}\\&+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1}\\&+\frac{132\,a^7\,b^5\,x^{72\,m}\,x^6}{12\,m+1}+\frac{132\,a^6\,b^6\,x^{84\,m}\,x^7}{12\,m+1}\\&+\frac{99\,a^5\,b^7\,x^{96\,m}\,x^8}{12\,m+1}+\frac{55\,a^4\,b^8\,x^{108\,m}\,x^9}{12\,m+1}\\&+\frac{22\,a^3\,b^9\,x^{120\,m}\,x^{10}}{12\,m+1}+\frac{6\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{12\,m+1}\end{aligned}$$

input
```
int(x^(12*m)*(a + b*x^(12*m + 1))^12,x)
```

output
```
(b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^1
1*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^
10*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (
99*a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1
) + (132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*
m + 1) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^1
0)/(12*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 7.48

$$\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx = \frac{x^{12m}x(x^{144m}b^{12}x^{12}+13x^{132m}a\,b^{11}x^{11}+78x^{120m}a^2b^{10}x^{10}+286x^{108m}a^3b^9x^9+715x^{96m}a^4b^8x^8+1287x^{84m}}{}$$

input
```
int(x^(12*m)*(a+b*x^(1+12*m))^12,x)
```

output
```
(x**(12*m)*x*(x**(144*m)*b**12*x**12 + 13*x**(132*m)*a*b**11*x**11 + 78*x*
*(120*m)*a**2*b**10*x**10 + 286*x**(108*m)*a**3*b**9*x**9 + 715*x**(96*m)*
a**4*b**8*x**8 + 1287*x**(84*m)*a**5*b**7*x**7 + 1716*x**(72*m)*a**6*b**6*
x**6 + 1716*x**(60*m)*a**7*b**5*x**5 + 1287*x**(48*m)*a**8*b**4*x**4 + 715
*x**(36*m)*a**9*b**3*x**3 + 286*x**(24*m)*a**10*b**2*x**2 + 78*x**(12*m)*a
**11*b*x + 13*a**12))/(13*(12*m + 1))
```

3.453 $\int \frac{x^{-1+5n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \frac{x^{-1+5n}}{a+bx^n} dx = -\frac{a^3x^n}{b^4n} + \frac{a^2x^{2n}}{2b^3n} - \frac{ax^{3n}}{3b^2n} + \frac{x^{4n}}{4bn} + \frac{a^4\log(a+bx^n)}{b^5n}$$

output
```
-a^3*x^n/b^4/n+1/2*a^2*x^(2*n)/b^3/n-1/3*a*x^(3*n)/b^2/n+1/4*x^(4*n)/b/n+a
^4*ln(a+b*x^n)/b^5/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+5n}}{a+bx^n} dx = \frac{bx^n(-12a^3+6a^2bx^n-4ab^2x^{2n}+3b^3x^{3n})+12a^4\log(a+bx^n)}{12b^5n}$$

input
```
Integrate[x^(-1 + 5*n)/(a + b*x^n),x]
```

output
```
(b*x^n*(-12*a^3 + 6*a^2*b*x^n - 4*a*b^2*x^(2*n) + 3*b^3*x^(3*n)) + 12*a^4*
Log[a + b*x^n])/(12*b^5*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5n-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{x^{4n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(\frac{a^2x^n}{b^3}-\frac{ax^{2n}}{b^2}+\frac{x^{3n}}{b}+\frac{a^4}{b^4(bx^n+a)}-\frac{a^3}{b^4}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a^4\log(a+bx^n)}{b^5}-\frac{a^3x^n}{b^4}+\frac{a^2x^{2n}}{2b^3}-\frac{ax^{3n}}{3b^2}+\frac{x^{4n}}{4b}}{n}$$

input
```
Int[x^(-1 + 5*n)/(a + b*x^n),x]
```

output
```
(-((a^3*x^n)/b^4) + (a^2*x^(2*n))/(2*b^3) - (a*x^(3*n))/(3*b^2) + x^(4*n)/
(4*b) + (a^4*Log[a + b*x^n])/b^5)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{x^{4n}}{4bn}-\frac{a\,x^{3n}}{3b^{2}n}+\frac{a^{2}x^{2n}}{2b^{3}n}-\frac{a^{3}x^{n}}{b^{4}n}+\frac{a^{4}\ln\left(x^{n}+\frac{a}{b}\right)}{b^{5}n}$	79
norman	$\frac{{\mathrm e}^{4n\ln\left(x\right)}}{4bn}-\frac{a\,{\mathrm e}^{3n\ln\left(x\right)}}{3b^{2}n}+\frac{a^{2}{\mathrm e}^{2n\ln\left(x\right)}}{2b^{3}n}-\frac{a^{3}{\mathrm e}^{n\ln\left(x\right)}}{b^{4}n}+\frac{a^{4}\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{b^{5}n}$	87

input
```
int(x^(-1+5*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/n*(x^n)^4-1/3*a/b^2/n*(x^n)^3+1/2*a^2/b^3/n*(x^n)^2-a^3*x^n/b^4/n+a^
4/b^5/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+5n}}{a+bx^n}\,dx=\frac{3\,b^4x^{4\,n}-4\,ab^3x^{3\,n}+6\,a^2b^2x^{2\,n}-12\,a^3bx^n+12\,a^4\log\left(bx^n+a\right)}{12\,b^5n}$$

input
```
integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^(4*n) - 4*a*b^3*x^(3*n) + 6*a^2*b^2*x^(2*n) - 12*a^3*b*x^n +
 12*a^4*log(b*x^n + a))/(b^5*n)
```

Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{x^{-1+5n}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{5n-1}}{5an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{a^4\log\left(\frac{a}{b}+x^n\right)}{b^5n} - \frac{a^3x^n}{b^4n} + \frac{a^2x^{2n}}{2b^3n} - \frac{ax^{3n}}{3b^2n} + \frac{x^{4n}}{4bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+5*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(5*n - 1)/(5*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (a**4*log(a/b + x**n)/(b**5*n) - a**3*x**
n/(b**4*n) + a**2*x**(2*n)/(2*b**3*n) - a*x**(3*n)/(3*b**2*n) + x**(4*n)/(
4*b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+5n}}{a+bx^n}\,dx = \frac{a^4\log\left(\frac{bx^n+a}{b}\right)}{b^5n} + \frac{3\,b^3x^{4\,n} - 4\,ab^2x^{3\,n} + 6\,a^2bx^{2\,n} - 12\,a^3x^n}{12\,b^4n}$$

input
```
integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
a^4*log((b*x^n + a)/b)/(b^5*n) + 1/12*(3*b^3*x^(4*n) - 4*a*b^2*x^(3*n) + 6
*a^2*b*x^(2*n) - 12*a^3*x^n)/(b^4*n)
```

Giac [F]

$$\int \frac{x^{-1+5n}}{a+bx^n}\,dx = \int \frac{x^{5\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(5*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+5n}}{a+bx^n}\,dx = \int \frac{x^{5\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(5*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(5*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+5n}}{a+bx^n}\,dx = \frac{3x^{4n}b^4-4x^{3n}a\,b^3+6x^{2n}a^2b^2-12x^na^3b+12\log(x^nb+a)\,a^4}{12b^5n}$$

input
```
int(x^(-1+5*n)/(a+b*x^n),x)
```

output
```
(3*x**(4*n)*b**4 - 4*x**(3*n)*a*b**3 + 6*x**(2*n)*a**2*b**2 - 12*x**n*a**3
*b + 12*log(x**n*b + a)*a**4)/(12*b**5*n)
```

3.454 $\int \frac{x^{-1+4n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \frac{x^{-1+4n}}{a+bx^n} dx = \frac{a^2x^n}{b^3n} - \frac{ax^{2n}}{2b^2n} + \frac{x^{3n}}{3bn} - \frac{a^3\log(a+bx^n)}{b^4n}$$

output
```
a^2*x^n/b^3/n-1/2*a*x^(2*n)/b^2/n+1/3*x^(3*n)/b/n-a^3*ln(a+b*x^n)/b^4/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+4n}}{a+bx^n} dx = \frac{bx^n(6a^2-3abx^n+2b^2x^{2n})-6a^3\log(a+bx^n)}{6b^4n}$$

input
```
Integrate[x^(-1 + 4*n)/(a + b*x^n),x]
```

output
```
(b*x^n*(6*a^2 - 3*a*b*x^n + 2*b^2*x^(2*n)) - 6*a^3*Log[a + b*x^n])/(6*b^4*
n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{x^{3n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(-\frac{ax^n}{b^2}+\frac{x^{2n}}{b}-\frac{a^3}{b^3(bx^n+a)}+\frac{a^2}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{a^3\log(a+bx^n)}{b^4}+\frac{a^2x^n}{b^3}-\frac{ax^{2n}}{2b^2}+\frac{x^{3n}}{3b}}{n}$$

input
```
Int[x^(-1 + 4*n)/(a + b*x^n),x]
```

output
```
((a^2*x^n)/b^3 - (a*x^(2*n))/(2*b^2) + x^(3*n)/(3*b) - (a^3*Log[a + b*x^n]
)/b^4)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{x^{3n}}{3bn}-\frac{a\,x^{2n}}{2b^{2}n}+\frac{a^{2}x^{n}}{b^{3}n}-\frac{a^{3}\ln\left(x^{n}+\frac{a}{b}\right)}{b^{4}n}$	63
norman	$\frac{a^{2}\mathrm{e}^{n\ln(x)}}{b^{3}n}+\frac{\mathrm{e}^{3n\ln(x)}}{3bn}-\frac{a\,\mathrm{e}^{2n\ln(x)}}{2b^{2}n}-\frac{a^{3}\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{b^{4}n}$	69

input
```
int(x^(-1+4*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/n*(x^n)^3-1/2*a/b^2/n*(x^n)^2+a^2*x^n/b^3/n-a^3/b^4/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+4n}}{a+bx^n}\,dx=\frac{2\,b^3x^{3\,n}-3\,ab^2x^{2\,n}+6\,a^2bx^n-6\,a^3\log\left(bx^n+a\right)}{6\,b^4n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^(3*n) - 3*a*b^2*x^(2*n) + 6*a^2*b*x^n - 6*a^3*log(b*x^n + a))
/(b^4*n)
```

Sympy [A] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+4n}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{xx^{4n-1}}{4an} & \text{for } b = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ -\frac{a^3\log\left(\frac{a}{b}+x^n\right)}{b^4n} + \frac{a^2x^n}{b^3n} - \frac{ax^{2n}}{2b^2n} + \frac{x^{3n}}{3bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(4*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (-a**3*log(a/b + x**n)/(b**4*n) + a**2*x*
*n/(b**3*n) - a*x**(2*n)/(2*b**2*n) + x**(3*n)/(3*b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1+4n}}{a+bx^n}\,dx = -\frac{a^3\log\left(\frac{bx^n+a}{b}\right)}{b^4n} + \frac{2\,b^2x^{3\,n} - 3\,abx^{2\,n} + 6\,a^2x^n}{6\,b^3n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-a^3*log((b*x^n + a)/b)/(b^4*n) + 1/6*(2*b^2*x^(3*n) - 3*a*b*x^(2*n) + 6*a
^2*x^n)/(b^3*n)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{a+bx^n}\,dx=\int \frac{x^{4\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{a+bx^n}\,dx=\int \frac{x^{4\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(4*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(4*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+4n}}{a+bx^n}\,dx=\frac{2x^{3n}b^3-3x^{2n}a\,b^2+6x^na^2b-6\log(x^nb+a)\,a^3}{6b^4n}$$

input
```
int(x^(-1+4*n)/(a+b*x^n),x)
```

output
```
(2*x**(3*n)*b**3 - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b - 6*log(x**n*b + a)*a
**3)/(6*b**4*n)
```

3.455 $\int \frac{x^{-1+3n}}{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx=-\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}+\frac{a^2\log(a+bx^n)}{b^3n}$$

output
```
-a*x^n/b^2/n+1/2*x^(2*n)/b/n+a^2*ln(a+b*x^n)/b^3/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx=\frac{bx^n(-2a+bx^n)+2a^2\log(a+bx^n)}{2b^3n}$$

input
```
Integrate[x^(-1 + 3*n)/(a + b*x^n),x]
```

output
```
(b*x^n*(-2*a + b*x^n) + 2*a^2*Log[a + b*x^n])/(2*b^3*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{2n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{x^n}{b}+\frac{a^2}{b^2(bx^n+a)}-\frac{a}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\log(a+bx^n)}{b^3}-\frac{ax^n}{b^2}+\frac{x^{2n}}{2b}}{n}$$

input
```
Int[x^(-1 + 3*n)/(a + b*x^n),x]
```

output
```
(-((a*x^n)/b^2) + x^(2*n)/(2*b) + (a^2*Log[a + b*x^n])/b^3)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{x^{2n}}{2bn}-\frac{a\,x^{n}}{b^{2}n}+\frac{a^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{b^{3}n}$	47
norman	$\frac{{\mathrm e}^{2n\ln\left(x\right)}}{2bn}-\frac{a\,{\mathrm e}^{n\ln\left(x\right)}}{b^{2}n}+\frac{a^{2}\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{b^{3}n}$	51

input
```
int(x^(-1+3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/n*(x^n)^2-a*x^n/b^2/n+a^2/b^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx=\frac{b^2x^{2\,n}-2\,abx^n+2\,a^2\log\left(bx^n+a\right)}{2\,b^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^(2*n) - 2*a*b*x^n + 2*a^2*log(b*x^n + a))/(b^3*n)
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx = \begin{cases} \frac{\log{(x)}}{a} & \text{for}\ b=0 \wedge n=0 \\ \frac{xx^{3n-1}}{3an} & \text{for}\ b=0 \\ \frac{\log{(x)}}{a+b} & \text{for}\ n=0 \\ \frac{a^2\log{\left(\frac{a}{b}+x^n\right)}}{b^3n} - \frac{ax^n}{b^2n} + \frac{x^{2n}}{2bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(3*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (a**2*log(a/b + x**n)/(b**3*n) - a*x**n/(
b**2*n) + x**(2*n)/(2*b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx = \frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3n} + \frac{bx^{2n}-2\,ax^n}{2\,b^2n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
a^2*log((b*x^n + a)/b)/(b^3*n) + 1/2*(b*x^(2*n) - 2*a*x^n)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx = \int \frac{x^{3\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx = \int \frac{x^{3\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(3*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(3*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+3n}}{a+bx^n}\,dx = \frac{x^{2n}b^2-2x^nab+2\log(x^nb+a)\,a^2}{2b^3n}$$

input
```
int(x^(-1+3*n)/(a+b*x^n),x)
```

output
```
(x**(2*n)*b**2 - 2*x**n*a*b + 2*log(x**n*b + a)*a**2)/(2*b**3*n)
```

3.456 $\int \frac{x^{-1+2n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{x^{-1+2n}}{a+bx^n} \, dx = \frac{x^n}{bn} - \frac{a \log(a+bx^n)}{b^2n}$$

output
```
x^n/b/n-a*ln(a+b*x^n)/b^2/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+2n}}{a+bx^n} \, dx = \frac{bx^n - a \log(bn(a+bx^n))}{b^2n}$$

input
```
Integrate[x^(-1 + 2*n)/(a + b*x^n),x]
```

output
```
(b*x^n - a*Log[b*n*(a + b*x^n)])/(b^2*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^n}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{1}{b}-\frac{a}{b(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{x^n}{b}-\frac{a\log(a+bx^n)}{b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a + b*x^n),x]
```

output
```
(x^n/b - (a*Log[a + b*x^n])/b^2)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{x^n}{bn} - \frac{a \ln\left(x^n + \frac{a}{b}\right)}{b^2 n}$	31
norman	$\frac{\mathrm{e}^{n \ln(x)}}{bn} - \frac{a \ln\left(a + b\,\mathrm{e}^{n \ln(x)}\right)}{b^2 n}$	33

input
```
int(x^(2*n-1)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
x^n/b/n-a/b^2/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+2n}}{a+bx^n}\,dx = \frac{bx^n - a\log\left(bx^n + a\right)}{b^2 n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
(b*x^n - a*log(b*x^n + a))/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.

Time = 1.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{x^{-1+2n}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{2n-1}}{2an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ -\frac{a\log\left(\frac{a}{b}+x^n\right)}{b^2n}+\frac{x^n}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)/(2*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (-a*log(a/b + x**n)/(b**2*n) + x**n/(b*n)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+2n}}{a+bx^n}\,dx = \frac{x^n}{bn}-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{a+bx^n}\,dx=\int \frac{x^{2\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{a+bx^n}\,dx=\int \frac{x^{2\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(2*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(2*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+2n}}{a+bx^n}\,dx=\frac{x^n b-\log(x^n b+a)\,a}{b^2 n}$$

input
```
int(x^(-1+2*n)/(a+b*x^n),x)
```

output
```
(x**n*b - log(x**n*b + a)*a)/(b**2*n)
```

3.457 $\int \frac{x^{-1+n}}{a+bx^n} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^{-1+n}}{a+bx^n} \, dx = \frac{\log(a+bx^n)}{bn}$$

output
```
ln(a+b*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n} \, dx = \frac{\log(a+bx^n)}{bn}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n),x]
```

output
```
Log[a + b*x^n]/(b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{a+bx^n} \, dx$$

$$\Big\downarrow \text{792}$$

$$\frac{\log(a+bx^n)}{bn}$$

input
```
Int[x^(-1 + n)/(a + b*x^n),x]
```

output
```
Log[a + b*x^n]/(b*n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{\ln\left(a+b\,{\mathrm e}^{n \ln\left(x \right)}\right)}{b n}$	18
risch	$\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{b n}$	18

input
```
int(x^(-1+n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/b/n*ln(a+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(bx^n+a)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
log(b*x^n + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.91 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{n-1}}{an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(a*n), Eq(b, 0)),
 (log(x)/(a + b), Eq(n, 0)), (log(a/b + x**n)/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(bx^n+a)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
log(b*x^n + a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(|bx^n+a|)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
log(abs(b*x^n + a))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\ln(a+b\,x^n)}{b\,n}$$

input
```
int(x^(n - 1)/(a + b*x^n),x)
```

output
```
log(a + b*x^n)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(x^n b + a)}{bn}$$

input
```
int(x^(-1+n)/(a+b*x^n),x)
```

output
```
log(x**n*b + a)/(b*n)
```

3.458 $\int \frac{1}{x(a+bx^n)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

output
```
ln(x)/a-ln(a+b*x^n)/a/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx=\frac{\log\left(x^n\right)-\log\left(an(a+bx^n)\right)}{an}$$

input
```
Integrate[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b x^n)} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int x^{-n}dx^n}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\log(a+bx^n)}{a}}{n}$$

input
```
Int[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n]/a - Log[a + b*x^n]/a)/n
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(a+b\,x^{n})}{an}$	23
norman	$\frac{\ln(x)}{a}-\frac{\ln\left(a+b\,{\mathrm e}^{n\ln(x)}\right)}{an}$	26
risch	$\frac{\ln(x)}{a}-\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{an}$	26
derivativedivides	$\frac{\frac{\ln(x^{n})}{a}-\frac{\ln(a+b\,x^{n})}{a}}{n}$	27
default	$\frac{\frac{\ln(x^{n})}{a}-\frac{\ln(a+b\,x^{n})}{a}}{n}$	27

input
```
int(1/x/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(a+b*x^n))/a/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^n)}\,dx=\frac{n\log(x)-\log(bx^n+a)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{1}{x(a+bx^n)}\,dx=\begin{cases}\tilde{\infty}\log(x) & \text{for } a=0\wedge b=0\wedge n=0\\ -\frac{x^{-n}}{bn} & \text{for } a=0\\ \frac{\log(x)}{a} & \text{for } b=0\\ \frac{\log(x)}{a+b} & \text{for } n=0\\ \frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(b*n*x**n), Eq
(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log
(a/b + x**n)/(a*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx = -\frac{\log\left(bx^n+a\right)}{an}+\frac{\log\left(x^n\right)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="maxima")
```

output
```
-log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx = \int \frac{1}{(bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx = -\frac{\ln\left(a+b\,x^n\right)-n\,\ln\left(x\right)}{a\,n}$$

input
```
int(1/(x*(a + b*x^n)),x)
```

output
```
-(log(a + b*x^n) - n*log(x))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\,(a+bx^n)}\,dx = \frac{-\log(x^n b + a) + \log(x)\,n}{an}$$

input
```
int(1/x/(a+b*x^n),x)
```

output
```
( - log(x**n*b + a) + log(x)*n)/(a*n)
```

3.459 $\int \frac{x^{-1-n}}{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx=-\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n\right)}{a^2n}$$

output
```
-1/a/n/(x^n)-b*ln(x)/a^2+b*ln(a+b*x^n)/a^2/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx=-\frac{ax^{-n}+b\log\left(x^n\right)-b\log\left(a+bx^n\right)}{a^2n}$$

input
```
Integrate[x^(-1 - n)/(a + b*x^n),x]
```

output
```
-((a/x^n + b*Log[x^n] - b*Log[a + b*x^n])/(a^2*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{a+bx^n}dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{x^{-2n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \ 54$$

$$\frac{\int \left(\frac{x^{-2n}}{a}-\frac{bx^{-n}}{a^2}+\frac{b^2}{a^2(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b\log(x^n)}{a^2}+\frac{b\log(a+bx^n)}{a^2}-\frac{x^{-n}}{a}}{n}$$

input
```
Int[x^(-1 - n)/(a + b*x^n),x]
```

output
```
(-(1/(a*x^n)) - (b*Log[x^n])/a^2 + (b*Log[a + b*x^n])/a^2)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{x^{-n}}{an}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(x^{n}+\frac{a}{b}\right)}{a^{2}n}$	41
norman	$\left(-\frac{1}{an}-\frac{b\ln\left(x\right)\mathrm{e}^{n\ln\left(x\right)}}{a^{2}}\right)\mathrm{e}^{-n\ln\left(x\right)}+\frac{b\ln\left(a+b\,\mathrm{e}^{n\ln\left(x\right)}\right)}{a^{2}n}$	50

input
```
int(x^(-1-n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/a/n/(x^n)-b*ln(x)/a^2+b/a^2/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx=-\frac{bnx^n\log\left(x\right)-bx^n\log\left(bx^n+a\right)+a}{a^2nx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-(b*n*x^n*log(x) - b*x^n*log(b*x^n + a) + a)/(a^2*n*x^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(31) = 62$.

Time = 2.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-n}x^{-n-1}}{2bn} & \text{for } a=0 \\ -\frac{xx^{-n-1}}{an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ -\frac{x^{-n}}{an} - \frac{b\log(x^n)}{a^2n} + \frac{b\log\left(\frac{a}{b}+x^n\right)}{a^2n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(2
*b*n*x**n), Eq(a, 0)), (-x*x**(-n - 1)/(a*n), Eq(b, 0)), (log(x)/(a + b),
Eq(n, 0)), (-1/(a*n*x**n) - b*log(x**n)/(a**2*n) + b*log(a/b + x**n)/(a**2
*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = -\frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{bx^n+a}{b}\right)}{a^2n} - \frac{1}{anx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*log(x)/a^2 + b*log((b*x^n + a)/b)/(a^2*n) - 1/(a*n*x^n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \int \frac{x^{-n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \int \frac{1}{x^{n+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \frac{x^n\log(x^nb+a)\,b-x^n\log(x)\,bn-a}{x^na^2n}$$

input
```
int(x^(-1-n)/(a+b*x^n),x)
```

output
```
(x**n*log(x**n*b + a)*b - x**n*log(x)*b*n - a)/(x**n*a**2*n)
```

3.460 $\int \frac{x^{-1-2n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^{-1-2n}}{a+bx^n} dx = -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx^n)}{a^3n}$$

output
```
-1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+b^2*ln(x)/a^3-b^2*ln(a+b*x^n)/a^3/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1-2n}}{a+bx^n} dx = -\frac{ax^{-2n}(a-2bx^n) - 2b^2 \log(x^n) + 2b^2 \log(a+bx^n)}{2a^3n}$$

input
```
Integrate[x^(-1 - 2*n)/(a + b*x^n),x]
```

output
```
-1/2*((a*(a - 2*b*x^n))/x^(2*n) - 2*b^2*Log[x^n] + 2*b^2*Log[a + b*x^n])/(
a^3*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{a+bx^n} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-3n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-3n}}{a}-\frac{bx^{-2n}}{a^2}+\frac{b^2x^{-n}}{a^3}-\frac{b^3}{a^3(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\log(x^n)}{a^3}-\frac{b^2\log(a+bx^n)}{a^3}+\frac{bx^{-n}}{a^2}-\frac{x^{-2n}}{2a}}{n}$$

input
```
Int[x^(-1 - 2*n)/(a + b*x^n),x]
```

output
```
(-1/2*1/(a*x^(2*n)) + b/(a^2*x^n) + (b^2*Log[x^n])/a^3 - (b^2*Log[a + b*x^
n])/a^3)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{b\,x^{-n}}{a^{2} n}-\frac{x^{-2 n}}{2 a n}+\frac{b^{2} \ln \left(x \right)}{a^{3}}-\frac{b^{2} \ln \left(x^{n}+\frac{a}{b}\right)}{a^{3} n}$	58
norman	$\left(\frac{b\,{\mathrm e}^{n \ln \left(x \right)}}{a^{2} n}-\frac{1}{2 a n}+\frac{b^{2} \ln \left(x \right) {\mathrm e}^{2 n \ln \left(x \right)}}{a^{3}}\right) {\mathrm e}^{-2 n \ln \left(x \right)}-\frac{b^{2} \ln \left(a +b\,{\mathrm e}^{n \ln \left(x \right)}\right)}{a^{3} n}$	69

input
```
int(x^(-2*n-1)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
b/a^2/n/(x^n)-1/2/a/n/(x^n)^2+b^2*ln(x)/a^3-b^2/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1-2n}}{a+bx^n}\,dx = \frac{2\,b^{2}nx^{2\,n}\log\left(x\right) - 2\,b^{2}x^{2\,n}\log\left(bx^{n} + a\right) + 2\,abx^{n} - a^{2}}{2\,a^{3}nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) - 2*b^2*x^(2*n)*log(b*x^n + a) + 2*a*b*x^n - a
^2)/(a^3*n*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(48) = 96$.

Time = 3.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1-2n}}{a+bx^n}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-n}x^{-2n-1}}{3bn} & \text{for } a=0 \\ -\frac{xx^{-2n-1}}{2an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b^2\log(x^n)}{a^3n}-\frac{b^2\log\left(\frac{a}{b}+x^n\right)}{a^3n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-2*n)/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-2*n - 1)/
(3*b*n*x**n), Eq(a, 0)), (-x*x**(-2*n - 1)/(2*a*n), Eq(b, 0)), (log(x)/(a
+ b), Eq(n, 0)), (-1/(2*a*n*x**(2*n)) + b/(a**2*n*x**n) + b**2*log(x**n)/(
a**3*n) - b**2*log(a/b + x**n)/(a**3*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1-2n}}{a+bx^n}\,dx = \frac{b^2\log(x)}{a^3} - \frac{b^2\log\left(\frac{bx^n+a}{b}\right)}{a^3n} + \frac{2\,bx^n-a}{2\,a^2nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*log(x)/a^3 - b^2*log((b*x^n + a)/b)/(a^3*n) + 1/2*(2*b*x^n - a)/(a^2*n
*x^(2*n))
```

Giac [F]

$$\int \frac{x^{-1-2n}}{a+bx^n}\,dx = \int \frac{x^{-2\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{a+bx^n}\,dx = \int \frac{1}{x^{2\,n+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(2*n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1-2n}}{a+bx^n}\,dx = \frac{-2x^{2n}\log(x^nb+a)\,b^2+2x^{2n}\log(x)\,b^2n+2x^nab-a^2}{2x^{2n}a^3n}$$

input
```
int(x^(-1-2*n)/(a+b*x^n),x)
```

output
```
( - 2*x**(2*n)*log(x**n*b + a)*b**2 + 2*x**(2*n)*log(x)*b**2*n + 2*x**n*a*
b - a**2)/(2*x**(2*n)*a**3*n)
```

3.461 $\int \frac{x^{-1-3n}}{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx = -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{b^2x^{-n}}{a^3n}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx^n)}{a^4n}$$

output
```
-1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))-b^2/a^3/n/(x^n)-b^3*ln(x)/a^4+b^3
*ln(a+b*x^n)/a^4/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx = -\frac{ax^{-3n}(2a^2-3abx^n+6b^2x^{2n})+6b^3\log(x^n)-6b^3\log(a+bx^n)}{6a^4n}$$

input
```
Integrate[x^(-1 - 3*n)/(a + b*x^n),x]
```

output
```
-1/6*((a*(2*a^2 - 3*a*b*x^n + 6*b^2*x^(2*n)))/x^(3*n) + 6*b^3*Log[x^n] - 6
*b^3*Log[a + b*x^n])/(a^4*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{a+bx^n} \, dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{-4n}}{bx^n+a}dx^n}{n}$$
$$\Big\downarrow \text{54}$$
$$\frac{\int \left(\frac{x^{-4n}}{a}-\frac{bx^{-3n}}{a^2}+\frac{b^2x^{-2n}}{a^3}-\frac{b^3x^{-n}}{a^4}+\frac{b^4}{a^4(bx^n+a)}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{b^3\log(x^n)}{a^4}+\frac{b^3\log(a+bx^n)}{a^4}-\frac{b^2x^{-n}}{a^3}+\frac{bx^{-2n}}{2a^2}-\frac{x^{-3n}}{3a}}{n}$$

input
```
Int[x^(-1 - 3*n)/(a + b*x^n),x]
```

output
```
(-1/3*1/(a*x^(3*n)) + b/(2*a^2*x^(2*n)) - b^2/(a^3*x^n) - (b^3*Log[x^n])/a
^4 + (b^3*Log[a + b*x^n])/a^4)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{b^{2} x^{-n}}{a^{3} n}+\frac{b\, x^{-2 n}}{2 a^{2} n}-\frac{x^{-3 n}}{3 a n}-\frac{b^{3} \ln \left(x \right)}{a^{4}}+\frac{b^{3} \ln \left(x^{n}+\frac{a}{b}\right)}{a^{4} n}$	75
norman	$\left(-\frac{1}{3 a n}+\frac{b\,{\mathrm e}^{n \ln \left(x \right)}}{2 a^{2} n}-\frac{b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{a^{3} n}-\frac{b^{3} \ln \left(x \right) {\mathrm e}^{3 n \ln \left(x \right)}}{a^{4}}\right) {\mathrm e}^{-3 n \ln \left(x \right)}+\frac{b^{3} \ln \left(a +b\,{\mathrm e}^{n \ln \left(x \right)}\right)}{a^{4} n}$	88

input
```
int(x^(-1-3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-b^2/a^3/n/(x^n)+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^n)^3-b^3*ln(x)/a^4+b^3/a^4
/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx=-\frac{6\,b^{3}nx^{3\,n}\log\left(x\right)-6\,b^{3}x^{3\,n}\log\left(bx^{n}+a\right)+6\,ab^{2}x^{2\,n}-3\,a^{2}bx^{n}+2\,a^{3}}{6\,a^{4}nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/6*(6*b^3*n*x^(3*n)*log(x) - 6*b^3*x^(3*n)*log(b*x^n + a) + 6*a*b^2*x^(2
*n) - 3*a^2*b*x^n + 2*a^3)/(a^4*n*x^(3*n))
```

Sympy [A] (verification not implemented)

Time = 7.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0\wedge b=0\wedge n=0 \\ -\frac{xx^{-n}x^{-3n-1}}{4bn} & \text{for } a=0 \\ -\frac{xx^{-3n-1}}{3an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{b^2x^{-n}}{a^3n}-\frac{b^3\log(x^n)}{a^4n}+\frac{b^3\log\left(\frac{a}{b}+x^n\right)}{a^4n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-3*n)/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-3*n - 1)/
(4*b*n*x**n), Eq(a, 0)), (-x*x**(-3*n - 1)/(3*a*n), Eq(b, 0)), (log(x)/(a
+ b), Eq(n, 0)), (-1/(3*a*n*x**(3*n)) + b/(2*a**2*n*x**(2*n)) - b**2/(a**3
*n*x**n) - b**3*log(x**n)/(a**4*n) + b**3*log(a/b + x**n)/(a**4*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx = -\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(\frac{bx^n+a}{b}\right)}{a^4n}-\frac{6\,b^2x^{2\,n}-3\,abx^n+2\,a^2}{6\,a^3nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b^3*log(x)/a^4 + b^3*log((b*x^n + a)/b)/(a^4*n) - 1/6*(6*b^2*x^(2*n) - 3*
a*b*x^n + 2*a^2)/(a^3*n*x^(3*n))
```

Giac [F]

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx=\int \frac{x^{-3\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx=\int \frac{1}{x^{3\,n+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/(x^(3*n + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(3*n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1-3n}}{a+bx^n}\,dx=\frac{6x^{3n}\log(x^nb+a)\,b^3-6x^{3n}\log(x)\,b^3n-6x^{2n}a\,b^2+3x^na^2b-2a^3}{6x^{3n}a^4n}$$

input
```
int(x^(-1-3*n)/(a+b*x^n),x)
```

output
```
(6*x**(3*n)*log(x**n*b + a)*b**3 - 6*x**(3*n)*log(x)*b**3*n - 6*x**(2*n)*a
*b**2 + 3*x**n*a**2*b - 2*a**3)/(6*x**(3*n)*a**4*n)
```

3.462 $\int \frac{x^{4+5(-1+n)}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n} dx = -\frac{a^3x^n}{b^4n} + \frac{a^2x^{2n}}{2b^3n} - \frac{ax^{3n}}{3b^2n} + \frac{x^{4n}}{4bn} + \frac{a^4\log(a+bx^n)}{b^5n}$$

output
```
-a^3*x^n/b^4/n+1/2*a^2*x^(2*n)/b^3/n-1/3*a*x^(3*n)/b^2/n+1/4*x^(4*n)/b/n+a
^4*ln(a+b*x^n)/b^5/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n} dx = \frac{bx^n(-12a^3+6a^2bx^n-4ab^2x^{2n}+3b^3x^{3n})+12a^4\log(a+bx^n)}{12b^5n}$$

input
```
Integrate[x^(4 + 5*(-1 + n))/(a + b*x^n),x]
```

output
```
(b*x^n*(-12*a^3 + 6*a^2*b*x^n - 4*a*b^2*x^(2*n) + 3*b^3*x^(3*n)) + 12*a^4*
Log[a + b*x^n])/(12*b^5*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5(n-1)+4}}{a+bx^n}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^{4n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(\frac{a^2x^n}{b^3}-\frac{ax^{2n}}{b^2}+\frac{x^{3n}}{b}+\frac{a^4}{b^4(bx^n+a)}-\frac{a^3}{b^4}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^4\log(a+bx^n)}{b^5}-\frac{a^3x^n}{b^4}+\frac{a^2x^{2n}}{2b^3}-\frac{ax^{3n}}{3b^2}+\frac{x^{4n}}{4b}}{n}$$

input
```
Int[x^(4 + 5*(-1 + n))/(a + b*x^n),x]
```

output
```
(-((a^3*x^n)/b^4) + (a^2*x^(2*n))/(2*b^3) - (a*x^(3*n))/(3*b^2) + x^(4*n)/
(4*b) + (a^4*Log[a + b*x^n])/b^5)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{x^{4n}}{4bn}-\frac{a\,x^{3n}}{3b^{2}n}+\frac{a^{2}x^{2n}}{2b^{3}n}-\frac{a^{3}x^{n}}{b^{4}n}+\frac{a^{4}\ln\left(x^{n}+\frac{a}{b}\right)}{b^{5}n}$	79
norman	$\frac{{\rm e}^{4n\ln(x)}}{4bn}-\frac{a\,{\rm e}^{3n\ln(x)}}{3b^{2}n}+\frac{a^{2}{\rm e}^{2n\ln(x)}}{2b^{3}n}-\frac{a^{3}{\rm e}^{n\ln(x)}}{b^{4}n}+\frac{a^{4}\ln\left(a+b\,{\rm e}^{n\ln(x)}\right)}{b^{5}n}$	87

input
```
int(x^(-1+5*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/n*(x^n)^4-1/3*a/b^2/n*(x^n)^3+1/2*a^2/b^3/n*(x^n)^2-a^3*x^n/b^4/n+a^
4/b^5/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx=\frac{3\,b^4x^{4\,n}-4\,ab^3x^{3\,n}+6\,a^2b^2x^{2\,n}-12\,a^3bx^n+12\,a^4\log\left(bx^n+a\right)}{12\,b^5n}$$

input
```
integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^(4*n) - 4*a*b^3*x^(3*n) + 6*a^2*b^2*x^(2*n) - 12*a^3*b*x^n +
 12*a^4*log(b*x^n + a))/(b^5*n)
```

Sympy [A] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{5n-1}}{5an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{a^4\log\left(\frac{a}{b}+x^n\right)}{b^5n} - \frac{a^3x^n}{b^4n} + \frac{a^2x^{2n}}{2b^3n} - \frac{ax^{3n}}{3b^2n} + \frac{x^{4n}}{4bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+5*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(5*n - 1)/(5*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (a**4*log(a/b + x**n)/(b**5*n) - a**3*x**
n/(b**4*n) + a**2*x**(2*n)/(2*b**3*n) - a*x**(3*n)/(3*b**2*n) + x**(4*n)/(
4*b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx = \frac{a^4\log\left(\frac{bx^n+a}{b}\right)}{b^5n} + \frac{3\,b^3x^{4\,n} - 4\,ab^2x^{3\,n} + 6\,a^2bx^{2\,n} - 12\,a^3x^n}{12\,b^4n}$$

input
```
integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
a^4*log((b*x^n + a)/b)/(b^5*n) + 1/12*(3*b^3*x^(4*n) - 4*a*b^2*x^(3*n) + 6
*a^2*b*x^(2*n) - 12*a^3*x^n)/(b^4*n)
```

Giac [F]

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx = \int \frac{x^{5\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+5*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(5*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx = \int \frac{x^{5\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(5*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(5*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^{4+5(-1+n)}}{a+bx^n}\,dx = \frac{3x^{4n}b^4-4x^{3n}a\,b^3+6x^{2n}a^2b^2-12x^na^3b+12\log(x^nb+a)\,a^4}{12b^5n}$$

input
```
int(x^(-1+5*n)/(a+b*x^n),x)
```

output
```
(3*x**(4*n)*b**4 - 4*x**(3*n)*a*b**3 + 6*x**(2*n)*a**2*b**2 - 12*x**n*a**3
*b + 12*log(x**n*b + a)*a**4)/(12*b**5*n)
```

3.463 $\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx=\frac{a^2x^n}{b^3n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn}-\frac{a^3\log(a+bx^n)}{b^4n}$$

output
```
a^2*x^n/b^3/n-1/2*a*x^(2*n)/b^2/n+1/3*x^(3*n)/b/n-a^3*ln(a+b*x^n)/b^4/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx=\frac{bx^n(6a^2-3abx^n+2b^2x^{2n})-6a^3\log(a+bx^n)}{6b^4n}$$

input
```
Integrate[x^(3 + 4*(-1 + n))/(a + b*x^n),x]
```

output
```
(b*x^n*(6*a^2 - 3*a*b*x^n + 2*b^2*x^(2*n)) - 6*a^3*Log[a + b*x^n])/(6*b^4*
n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4(n-1)+3}}{a+bx^n}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{x^{3n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(-\frac{ax^n}{b^2}+\frac{x^{2n}}{b}-\frac{a^3}{b^3(bx^n+a)}+\frac{a^2}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{a^3\log(a+bx^n)}{b^4}+\frac{a^2x^n}{b^3}-\frac{ax^{2n}}{2b^2}+\frac{x^{3n}}{3b}}{n}$$

input
```
Int[x^(3 + 4*(-1 + n))/(a + b*x^n),x]
```

output
```
((a^2*x^n)/b^3 - (a*x^(2*n))/(2*b^2) + x^(3*n)/(3*b) - (a^3*Log[a + b*x^n]
)/b^4)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{x^{3n}}{3bn}-\frac{a\,x^{2n}}{2b^{2}n}+\frac{a^{2}x^{n}}{b^{3}n}-\frac{a^{3}\ln\left(x^{n}+\frac{a}{b}\right)}{b^{4}n}$	63
norman	$\frac{a^{2}\mathrm{e}^{n\ln(x)}}{b^{3}n}+\frac{\mathrm{e}^{3n\ln(x)}}{3bn}-\frac{a\,\mathrm{e}^{2n\ln(x)}}{2b^{2}n}-\frac{a^{3}\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{b^{4}n}$	69

input
```
int(x^(-1+4*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/n*(x^n)^3-1/2*a/b^2/n*(x^n)^2+a^2*x^n/b^3/n-a^3/b^4/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx=\frac{2\,b^3x^{3\,n}-3\,ab^2x^{2\,n}+6\,a^2bx^n-6\,a^3\log\left(bx^n+a\right)}{6\,b^4n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^(3*n) - 3*a*b^2*x^(2*n) + 6*a^2*b*x^n - 6*a^3*log(b*x^n + a))
/(b^4*n)
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{4n-1}}{4an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ -\frac{a^3\log\left(\frac{a}{b}+x^n\right)}{b^4n}+\frac{a^2x^n}{b^3n}-\frac{ax^{2n}}{2b^2n}+\frac{x^{3n}}{3bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(4*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (-a**3*log(a/b + x**n)/(b**4*n) + a**2*x*
*n/(b**3*n) - a*x**(2*n)/(2*b**2*n) + x**(3*n)/(3*b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx = -\frac{a^3\log\left(\frac{bx^n+a}{b}\right)}{b^4n}+\frac{2\,b^2x^{3\,n}-3\,abx^{2\,n}+6\,a^2x^n}{6\,b^3n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-a^3*log((b*x^n + a)/b)/(b^4*n) + 1/6*(2*b^2*x^(3*n) - 3*a*b*x^(2*n) + 6*a
^2*x^n)/(b^3*n)
```

Giac [F]

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx=\int \frac{x^{4\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx=\int \frac{x^{4\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(4*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(4*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{x^{3+4(-1+n)}}{a+bx^n}\,dx=\frac{2x^{3n}b^3-3x^{2n}a\,b^2+6x^n a^2 b-6\log(x^n b+a)\,a^3}{6b^4n}$$

input
```
int(x^(-1+4*n)/(a+b*x^n),x)
```

output
```
(2*x**(3*n)*b**3 - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b - 6*log(x**n*b + a)*a
**3)/(6*b**4*n)
```

3.464 $\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx = -\frac{ax^n}{b^2n}+\frac{x^{2n}}{2bn}+\frac{a^2\log(a+bx^n)}{b^3n}$$

output
```
-a*x^n/b^2/n+1/2*x^(2*n)/b/n+a^2*ln(a+b*x^n)/b^3/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx = \frac{bx^n(-2a+bx^n)+2a^2\log(a+bx^n)}{2b^3n}$$

input
```
Integrate[x^(2 + 3*(-1 + n))/(a + b*x^n),x]
```

output
```
(b*x^n*(-2*a + b*x^n) + 2*a^2*Log[a + b*x^n])/(2*b^3*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3(n-1)+2}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{2n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{x^n}{b}+\frac{a^2}{b^2(bx^n+a)}-\frac{a}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\log(a+bx^n)}{b^3}-\frac{ax^n}{b^2}+\frac{x^{2n}}{2b}}{n}$$

input
```
Int[x^(2 + 3*(-1 + n))/(a + b*x^n),x]
```

output
```
(-((a*x^n)/b^2) + x^(2*n)/(2*b) + (a^2*Log[a + b*x^n])/b^3)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{x^{2n}}{2bn}-\frac{a\,x^{n}}{b^{2}n}+\frac{a^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{b^{3}n}$	47
norman	$\frac{\mathrm{e}^{2n\ln(x)}}{2bn}-\frac{a\,\mathrm{e}^{n\ln(x)}}{b^{2}n}+\frac{a^{2}\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{b^{3}n}$	51

input
```
int(x^(-1+3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/n*(x^n)^2-a*x^n/b^2/n+a^2/b^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx=\frac{b^2x^{2\,n}-2\,abx^n+2\,a^2\log\left(bx^n+a\right)}{2\,b^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^(2*n) - 2*a*b*x^n + 2*a^2*log(b*x^n + a))/(b^3*n)
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{3n-1}}{3an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{a^2\log\left(\frac{a}{b}+x^n\right)}{b^3n} - \frac{ax^n}{b^2n} + \frac{x^{2n}}{2bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(3*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (a**2*log(a/b + x**n)/(b**3*n) - a*x**n/(
b**2*n) + x**(2*n)/(2*b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx = \frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3n} + \frac{bx^{2\,n} - 2\,ax^n}{2\,b^2n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
a^2*log((b*x^n + a)/b)/(b^3*n) + 1/2*(b*x^(2*n) - 2*a*x^n)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx = \int \frac{x^{3\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx = \int \frac{x^{3\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(3*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(3*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{2+3(-1+n)}}{a+bx^n}\,dx = \frac{x^{2n}b^2 - 2x^n ab + 2\log(x^n b + a)\,a^2}{2b^3 n}$$

input
```
int(x^(-1+3*n)/(a+b*x^n),x)
```

output
```
(x**(2*n)*b**2 - 2*x**n*a*b + 2*log(x**n*b + a)*a**2)/(2*b**3*n)
```

3.465 $\int \frac{x^{1+2(-1+n)}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n} dx = \frac{x^n}{bn} - \frac{a\log(a+bx^n)}{b^2n}$$

output
```
x^n/b/n-a*ln(a+b*x^n)/b^2/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n} dx = \frac{bx^n - a\log(bn(a+bx^n))}{b^2n}$$

input
```
Integrate[x^(1 + 2*(-1 + n))/(a + b*x^n),x]
```

output
```
(b*x^n - a*Log[b*n*(a + b*x^n)])/(b^2*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2(n-1)+1}}{a+bx^n}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^n}{bx^n+a}dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(\frac{1}{b}-\frac{a}{b(bx^n+a)}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{x^n}{b}-\frac{a\log(a+bx^n)}{b^2}}{n}$$

input
```
Int[x^(1 + 2*(-1 + n))/(a + b*x^n),x]
```

output
```
(x^n/b - (a*Log[a + b*x^n])/b^2)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{x^n}{bn} - \frac{a\ln\left(x^n+\frac{a}{b}\right)}{b^2 n}$	31
norman	$\frac{e^{n\ln(x)}}{bn} - \frac{a\ln\left(a+b\,e^{n\ln(x)}\right)}{b^2 n}$	33

input
```
int(x^(2*n-1)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
x^n/b/n-a/b^2/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx = \frac{bx^n - a\log\left(bx^n+a\right)}{b^2 n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
(b*x^n - a*log(b*x^n + a))/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.96 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{xx^{2n-1}}{2an} & \text{for } b = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ -\frac{a\log\left(\frac{a}{b}+x^n\right)}{b^2n} + \frac{x^n}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)/(2*a*n), Eq(b,
0)), (log(x)/(a + b), Eq(n, 0)), (-a*log(a/b + x**n)/(b**2*n) + x**n/(b*n)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx = \frac{x^n}{bn} - \frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx=\int \frac{x^{2\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx=\int \frac{x^{2\,n-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(2*n - 1)/(a + b*x^n),x)
```

output
```
int(x^(2*n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^{1+2(-1+n)}}{a+bx^n}\,dx=\frac{x^nb-\log(x^nb+a)\,a}{b^2n}$$

input
```
int(x^(-1+2*n)/(a+b*x^n),x)
```

output
```
(x**n*b - log(x**n*b + a)*a)/(b**2*n)
```

3.466 $\int \frac{x^{-1+n}}{a+bx^n} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^{-1+n}}{a+bx^n} dx = \frac{\log(a+bx^n)}{bn}$$

output
```
ln(a+b*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n} dx = \frac{\log(a+bx^n)}{bn}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n),x]
```

output
```
Log[a + b*x^n]/(b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{\log(a+bx^n)}{bn}$$

input
```
Int[x^(-1 + n)/(a + b*x^n),x]
```

output
```
Log[a + b*x^n]/(b*n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{bn}$	18
risch	$\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{bn}$	18

input
```
int(x^(-1+n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/b/n*ln(a+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n} dx = \frac{\log(bx^n+a)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
log(b*x^n + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{x^{-1+n}}{a+bx^n} dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{n-1}}{an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(a*n), Eq(b, 0)),
 (log(x)/(a + b), Eq(n, 0)), (log(a/b + x**n)/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log\left(bx^n+a\right)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
log(b*x^n + a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log\left(|bx^n+a|\right)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
log(abs(b*x^n + a))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\ln\left(a+b\,x^n\right)}{b\,n}$$

input
```
int(x^(n - 1)/(a + b*x^n),x)
```

output
```
log(a + b*x^n)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx=\frac{\log(x^nb+a)}{bn}$$

input
```
int(x^(-1+n)/(a+b*x^n),x)
```

output
```
log(x**n*b + a)/(b*n)
```

3.467 $\int \frac{1}{x(a+bx^n)} dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x(a+bx^n)} dx = \frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

output
```
ln(x)/a-ln(a+b*x^n)/a/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(a+bx^n)} dx = \frac{\log(x^n) - \log(an(a+bx^n))}{an}$$

input
```
Integrate[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^n)}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int x^{-n}dx^n}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\log(a+bx^n)}{a}}{n}$$

input
```
Int[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n]/a - Log[a + b*x^n]/a)/n
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{n\ln\left(x\right)-\ln\left(a+b\,x^{n}\right)}{an}$	23
norman	$\frac{\ln\left(x\right)}{a}-\frac{\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{an}$	26
risch	$\frac{\ln\left(x\right)}{a}-\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{an}$	26
derivativedivides	$\frac{\frac{\ln\left(x^{n}\right)}{a}-\frac{\ln\left(a+b\,x^{n}\right)}{a}}{n}$	27
default	$\frac{\frac{\ln\left(x^{n}\right)}{a}-\frac{\ln\left(a+b\,x^{n}\right)}{a}}{n}$	27

input
```
int(1/x/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(a+b*x^n))/a/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx=\frac{n\log\left(x\right)-\log\left(bx^n+a\right)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx=\begin{cases}\tilde{\infty}\log\left(x\right) & \text{for } a=0\wedge b=0\wedge n=0\\ -\frac{x^{-n}}{bn} & \text{for } a=0\\ \frac{\log\left(x\right)}{a} & \text{for } b=0\\ \frac{\log\left(x\right)}{a+b} & \text{for } n=0\\ \frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(b*n*x**n), Eq
(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log
(a/b + x**n)/(a*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx = -\frac{\log\left(bx^n+a\right)}{an}+\frac{\log\left(x^n\right)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="maxima")
```

output
```
-log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx = \int \frac{1}{(bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^n\right)}\,dx = -\frac{\ln\left(a+b\,x^n\right)-n\,\ln\left(x\right)}{a\,n}$$

input
```
int(1/(x*(a + b*x^n)),x)
```

output
```
-(log(a + b*x^n) - n*log(x))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\,(a+bx^n)}\,dx = \frac{-\log(x^n b+a)+\log(x)\,n}{an}$$

input
```
int(1/x/(a+b*x^n),x)
```

output
```
( - log(x**n*b + a) + log(x)*n)/(a*n)
```

3.468 $\int \frac{x^{-1-n}}{a+bx^n} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{x^{-1-n}}{a+bx^n} \, dx = -\frac{x^{-n}}{an} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^n)}{a^2n}$$

output
```
-1/a/n/(x^n)-b*ln(x)/a^2+b*ln(a+b*x^n)/a^2/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1-n}}{a+bx^n} \, dx = -\frac{ax^{-n} + b\log(x^n) - b\log(a+bx^n)}{a^2n}$$

input
```
Integrate[x^(-1 - n)/(a + b*x^n),x]
```

output
```
-((a/x^n + b*Log[x^n] - b*Log[a + b*x^n])/(a^2*n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{a+bx^n} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-2n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-2n}}{a}-\frac{bx^{-n}}{a^2}+\frac{b^2}{a^2(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b\log(x^n)}{a^2}+\frac{b\log(a+bx^n)}{a^2}-\frac{x^{-n}}{a}}{n}$$

input
```
Int[x^(-1 - n)/(a + b*x^n),x]
```

output
```
(-(1/(a*x^n)) - (b*Log[x^n])/a^2 + (b*Log[a + b*x^n])/a^2)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{x^{-n}}{an}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(x^n+\frac{a}{b}\right)}{a^2n}$	41
norman	$\left(-\frac{1}{an}-\frac{b\ln(x)\mathrm{e}^{n\ln(x)}}{a^2}\right)\mathrm{e}^{-n\ln(x)}+\frac{b\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{a^2n}$	50

input
```
int(x^(-1-n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/a/n/(x^n)-b*ln(x)/a^2+b/a^2/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx=-\frac{bnx^n\log(x)-bx^n\log(bx^n+a)+a}{a^2nx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-(b*n*x^n*log(x) - b*x^n*log(b*x^n + a) + a)/(a^2*n*x^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(31) = 62$.

Time = 1.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \begin{cases} \tilde{\infty}\log{(x)} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{xx^{-n}x^{-n-1}}{2bn} & \text{for } a = 0 \\ -\frac{xx^{-n-1}}{an} & \text{for } b = 0 \\ \frac{\log{(x)}}{a+b} & \text{for } n = 0 \\ -\frac{x^{-n}}{an} - \frac{b\log{(x^n)}}{a^2n} + \frac{b\log{\left(\frac{a}{b}+x^n\right)}}{a^2n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(2
*b*n*x**n), Eq(a, 0)), (-x*x**(-n - 1)/(a*n), Eq(b, 0)), (log(x)/(a + b),
Eq(n, 0)), (-1/(a*n*x**n) - b*log(x**n)/(a**2*n) + b*log(a/b + x**n)/(a**2
*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = -\frac{b\log{(x)}}{a^2} + \frac{b\log{\left(\frac{bx^n+a}{b}\right)}}{a^2n} - \frac{1}{anx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*log(x)/a^2 + b*log((b*x^n + a)/b)/(a^2*n) - 1/(a*n*x^n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \int \frac{x^{-n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \int \frac{1}{x^{n+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \frac{x^n\log(x^nb+a)\,b-x^n\log(x)\,bn-a}{x^na^2n}$$

input
```
int(x^(-1-n)/(a+b*x^n),x)
```

output
```
(x**n*log(x**n*b + a)*b - x**n*log(x)*b*n - a)/(x**n*a**2*n)
```

3.469 $\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx = -\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^n\right)}{a^3n}$$

output
```
-1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+b^2*ln(x)/a^3-b^2*ln(a+b*x^n)/a^3/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx = -\frac{ax^{-2n}(a-2bx^n)-2b^2\log\left(x^n\right)+2b^2\log\left(a+bx^n\right)}{2a^3n}$$

input
```
Integrate[x^(-3 - 2*(-1 + n))/(a + b*x^n),x]
```

output
```
-1/2*((a*(a - 2*b*x^n))/x^(2*n) - 2*b^2*Log[x^n] + 2*b^2*Log[a + b*x^n])/(
a^3*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2(n-1)-3}}{a+bx^n}dx$$

$$\Big\downarrow \textbf{798}$$

$$\frac{\int \frac{x^{-3n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \textbf{54}$$

$$\frac{\int \left(\frac{x^{-3n}}{a}-\frac{bx^{-2n}}{a^2}+\frac{b^2x^{-n}}{a^3}-\frac{b^3}{a^3(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{b^2\log(x^n)}{a^3}-\frac{b^2\log(a+bx^n)}{a^3}+\frac{bx^{-n}}{a^2}-\frac{x^{-2n}}{2a}}{n}$$

input
```
Int[x^(-3 - 2*(-1 + n))/(a + b*x^n),x]
```

output
```
(-1/2*1/(a*x^(2*n)) + b/(a^2*x^n) + (b^2*Log[x^n])/a^3 - (b^2*Log[a + b*x^
n])/a^3)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{b\,x^{-n}}{a^{2} n}-\frac{x^{-2 n}}{2 a n}+\frac{b^{2} \ln \left(x \right)}{a^{3}}-\frac{b^{2} \ln \left(x^{n}+\frac{a}{b}\right)}{a^{3} n}$	58
norman	$\left(\frac{b\,{\mathrm e}^{n \ln \left(x \right)}}{a^{2} n}-\frac{1}{2 a n}+\frac{b^{2} \ln \left(x \right) {\mathrm e}^{2 n \ln \left(x \right)}}{a^{3}}\right) {\mathrm e}^{-2 n \ln \left(x \right)}-\frac{b^{2} \ln \left(a +b\,{\mathrm e}^{n \ln \left(x \right)}\right)}{a^{3} n}$	69

input
```
int(x^(-2*n-1)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
b/a^2/n/(x^n)-1/2/a/n/(x^n)^2+b^2*ln(x)/a^3-b^2/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx=\frac{2\,b^{2}nx^{2\,n}\log\left(x\right)-2\,b^{2}x^{2\,n}\log\left(bx^{n}+a\right)+2\,abx^{n}-a^{2}}{2\,a^{3}nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) - 2*b^2*x^(2*n)*log(b*x^n + a) + 2*a*b*x^n - a
^2)/(a^3*n*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(48) = 96$.

Time = 3.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-n}x^{-2n-1}}{3bn} & \text{for } a=0 \\ -\frac{xx^{-2n-1}}{2an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{b^2\log(x^n)}{a^3n} - \frac{b^2\log\left(\frac{a}{b}+x^n\right)}{a^3n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-2*n)/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-2*n - 1)/
(3*b*n*x**n), Eq(a, 0)), (-x*x**(-2*n - 1)/(2*a*n), Eq(b, 0)), (log(x)/(a
+ b), Eq(n, 0)), (-1/(2*a*n*x**(2*n)) + b/(a**2*n*x**n) + b**2*log(x**n)/(
a**3*n) - b**2*log(a/b + x**n)/(a**3*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx = \frac{b^2\log(x)}{a^3} - \frac{b^2\log\left(\frac{bx^n+a}{b}\right)}{a^3n} + \frac{2\,bx^n-a}{2\,a^2nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*log(x)/a^3 - b^2*log((b*x^n + a)/b)/(a^3*n) + 1/2*(2*b*x^n - a)/(a^2*n
*x^(2*n))
```

Giac [F]

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx=\int \frac{x^{-2\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx=\int \frac{1}{x^{2\,n+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(2*n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^{-3-2(-1+n)}}{a+bx^n}\,dx=\frac{-2x^{2n}\log(x^nb+a)\,b^2+2x^{2n}\log(x)\,b^2n+2x^nab-a^2}{2x^{2n}a^3n}$$

input
```
int(x^(-1-2*n)/(a+b*x^n),x)
```

output
```
( - 2*x**(2*n)*log(x**n*b + a)*b**2 + 2*x**(2*n)*log(x)*b**2*n + 2*x**n*a*
b - a**2)/(2*x**(2*n)*a**3*n)
```

3.470 $\int \frac{x^{-4-3(-1+n)}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n} dx = -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{b^2x^{-n}}{a^3n} - \frac{b^3\log(x)}{a^4} + \frac{b^3\log(a+bx^n)}{a^4n}$$

output
```
-1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))-b^2/a^3/n/(x^n)-b^3*ln(x)/a^4+b^3
*ln(a+b*x^n)/a^4/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n} dx = -\frac{ax^{-3n}(2a^2-3abx^n+6b^2x^{2n})+6b^3\log(x^n)-6b^3\log(a+bx^n)}{6a^4n}$$

input
```
Integrate[x^(-4 - 3*(-1 + n))/(a + b*x^n),x]
```

output
```
-1/6*((a*(2*a^2 - 3*a*b*x^n + 6*b^2*x^(2*n)))/x^(3*n) + 6*b^3*Log[x^n] - 6
*b^3*Log[a + b*x^n])/(a^4*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3(n-1)-4}}{a+bx^n}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-4n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-4n}}{a}-\frac{bx^{-3n}}{a^2}+\frac{b^2x^{-2n}}{a^3}-\frac{b^3x^{-n}}{a^4}+\frac{b^4}{a^4(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3\log(x^n)}{a^4}+\frac{b^3\log(a+bx^n)}{a^4}-\frac{b^2x^{-n}}{a^3}+\frac{bx^{-2n}}{2a^2}-\frac{x^{-3n}}{3a}}{n}$$

input
```
Int[x^(-4 - 3*(-1 + n))/(a + b*x^n),x]
```

output
```
(-1/3*1/(a*x^(3*n)) + b/(2*a^2*x^(2*n)) - b^2/(a^3*x^n) - (b^3*Log[x^n])/a
^4 + (b^3*Log[a + b*x^n])/a^4)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{b^{2} x^{-n}}{a^{3} n}+\frac{b\,x^{-2 n}}{2 a^{2} n}-\frac{x^{-3 n}}{3 a n}-\frac{b^{3} \ln \left(x \right)}{a^{4}}+\frac{b^{3} \ln \left(x^{n}+\frac{a}{b}\right)}{a^{4} n}$	75
norman	$\left(-\frac{1}{3 a n}+\frac{b\,{\mathrm e}^{n \ln \left(x \right)}}{2 a^{2} n}-\frac{b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{a^{3} n}-\frac{b^{3} \ln \left(x \right) {\mathrm e}^{3 n \ln \left(x \right)}}{a^{4}}\right) {\mathrm e}^{-3 n \ln \left(x \right)}+\frac{b^{3} \ln \left(a +b\,{\mathrm e}^{n \ln \left(x \right)}\right)}{a^{4} n}$	88

input
```
int(x^(-1-3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-b^2/a^3/n/(x^n)+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^n)^3-b^3*ln(x)/a^4+b^3/a^4
/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx = -\frac{6\,b^{3} n x^{3\,n} \log\left(x\right) - 6\,b^{3} x^{3\,n} \log\left(b x^{n} + a\right) + 6\,a b^{2} x^{2\,n} - 3\,a^{2} b x^{n} + 2\,a^{3}}{6\,a^{4} n x^{3\,n}}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/6*(6*b^3*n*x^(3*n)*log(x) - 6*b^3*x^(3*n)*log(b*x^n + a) + 6*a*b^2*x^(2
*n) - 3*a^2*b*x^n + 2*a^3)/(a^4*n*x^(3*n))
```

Sympy [A] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-n}x^{-3n-1}}{4bn} & \text{for } a=0 \\ -\frac{xx^{-3n-1}}{3an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ -\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{b^2x^{-n}}{a^3n}-\frac{b^3\log(x^n)}{a^4n}+\frac{b^3\log\left(\frac{a}{b}+x^n\right)}{a^4n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-3*n)/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-3*n - 1)/
(4*b*n*x**n), Eq(a, 0)), (-x*x**(-3*n - 1)/(3*a*n), Eq(b, 0)), (log(x)/(a
+ b), Eq(n, 0)), (-1/(3*a*n*x**(3*n)) + b/(2*a**2*n*x**(2*n)) - b**2/(a**3
*n*x**n) - b**3*log(x**n)/(a**4*n) + b**3*log(a/b + x**n)/(a**4*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx = -\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(\frac{bx^n+a}{b}\right)}{a^4n}-\frac{6\,b^2x^{2\,n}-3\,abx^n+2\,a^2}{6\,a^3nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b^3*log(x)/a^4 + b^3*log((b*x^n + a)/b)/(a^4*n) - 1/6*(6*b^2*x^(2*n) - 3*
a*b*x^n + 2*a^2)/(a^3*n*x^(3*n))
```

Giac [F]

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx = \int \frac{x^{-3\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx = \int \frac{1}{x^{3\,n+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(3*n + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(3*n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{x^{-4-3(-1+n)}}{a+bx^n}\,dx = \frac{6x^{3n}\log(x^nb+a)\,b^3-6x^{3n}\log(x)\,b^3n-6x^{2n}a\,b^2+3x^na^2b-2a^3}{6x^{3n}a^4n}$$

input
```
int(x^(-1-3*n)/(a+b*x^n),x)
```

output
```
(6*x**(3*n)*log(x**n*b + a)*b**3 - 6*x**(3*n)*log(x)*b**3*n - 6*x**(2*n)*a
*b**2 + 3*x**n*a**2*b - 2*a**3)/(6*x**(3*n)*a**4*n)
```

3.471 $\int \frac{x^{-1+5n}}{2+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx = -\frac{8x^n}{b^4n}+\frac{2x^{2n}}{b^3n}-\frac{2x^{3n}}{3b^2n}+\frac{x^{4n}}{4bn}+\frac{16\log(2+bx^n)}{b^5n}$$

output
```
-8*x^n/b^4/n+2*x^(2*n)/b^3/n-2/3*x^(3*n)/b^2/n+1/4*x^(4*n)/b/n+16*ln(2+b*x
^n)/b^5/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx = \frac{bx^n(-96+24bx^n-8b^2x^{2n}+3b^3x^{3n})+192\log(2+bx^n)}{12b^5n}$$

input
```
Integrate[x^(-1 + 5*n)/(2 + b*x^n),x]
```

output
```
(b*x^n*(-96 + 24*b*x^n - 8*b^2*x^(2*n) + 3*b^3*x^(3*n)) + 192*Log[2 + b*x^
n])/(12*b^5*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5n-1}}{bx^n+2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{4n}}{bx^n+2}dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(\frac{4x^n}{b^3}-\frac{2x^{2n}}{b^2}+\frac{x^{3n}}{b}+\frac{16}{b^4(bx^n+2)}-\frac{8}{b^4}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{16\log(bx^n+2)}{b^5}-\frac{8x^n}{b^4}+\frac{2x^{2n}}{b^3}-\frac{2x^{3n}}{3b^2}+\frac{x^{4n}}{4b}}{n}$$

input
```
Int[x^(-1 + 5*n)/(2 + b*x^n),x]
```

output
```
((-8*x^n)/b^4 + (2*x^(2*n))/b^3 - (2*x^(3*n))/(3*b^2) + x^(4*n)/(4*b) + (1
6*Log[2 + b*x^n])/b^5)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

method	result
risch	$\frac{x^{4n}}{4bn}-\frac{2x^{3n}}{3b^{2}n}+\frac{2x^{2n}}{b^{3}n}-\frac{8x^{n}}{b^{4}n}+\frac{16\ln\left(x^{n}+\frac{2}{b}\right)}{b^{5}n}$
norman	$-\frac{8\,\mathrm{e}^{n\ln(x)}}{b^{4}n}+\frac{2\,\mathrm{e}^{2n\ln(x)}}{b^{3}n}-\frac{2\,\mathrm{e}^{3n\ln(x)}}{3b^{2}n}+\frac{\mathrm{e}^{4n\ln(x)}}{4bn}+\frac{16\ln\left(2+b\,\mathrm{e}^{n\ln(x)}\right)}{b^{5}n}$
meijerg	$\frac{16i(-1)^{-\frac{5\,\mathrm{csgn}(ib)}{2}-\frac{5\,\mathrm{csgn}(ix^{n})}{2}+\frac{5\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ib)}{2}}\left(\frac{ix^{n}b(-1)^{\frac{5\,\mathrm{csgn}(ib)}{2}+\frac{5\,\mathrm{csgn}(ix^{n})}{2}-\frac{5\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ib)}{2}}\left(-\frac{15ix^{3n}b^{3}(-1)^{\frac{3\,\mathrm{csgn}(i}{2}}}{\dots}\right.}{\dots}\right.}{\dots}$

input
```
int(x^(-1+5*n)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/n*(x^n)^4-2/3/b^2/n*(x^n)^3+2/b^3/n*(x^n)^2-8*x^n/b^4/n+16/b^5/n*ln(
x^n+2/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx=\frac{3\,b^4x^{4\,n}-8\,b^3x^{3\,n}+24\,b^2x^{2\,n}-96\,bx^n+192\,\log\left(bx^n+2\right)}{12\,b^5n}$$

input
```
integrate(x^(-1+5*n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b^4*x^(4*n) - 8*b^3*x^(3*n) + 24*b^2*x^(2*n) - 96*b*x^n + 192*log(
b*x^n + 2))/(b^5*n)
```

Sympy [A] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx = \begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge n = 0 \\ \frac{xx^{5n-1}}{10n} & \text{for } b = 0 \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ \frac{x^{4n}}{4bn} - \frac{2x^{3n}}{3b^2n} + \frac{2x^{2n}}{b^3n} - \frac{8x^n}{b^4n} + \frac{16\log\left(x^n+\frac{2}{b}\right)}{b^5n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+5*n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (x*x**(5*n - 1)/(10*n), Eq(b, 0
)), (log(x)/(b + 2), Eq(n, 0)), (x**(4*n)/(4*b*n) - 2*x**(3*n)/(3*b**2*n)
+ 2*x**(2*n)/(b**3*n) - 8*x**n/(b**4*n) + 16*log(x**n + 2/b)/(b**5*n), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx = \frac{3\,b^3x^{4\,n} - 8\,b^2x^{3\,n} + 24\,bx^{2\,n} - 96\,x^n}{12\,b^4n} + \frac{16\,\log\left(\frac{bx^n+2}{b}\right)}{b^5n}$$

input
```
integrate(x^(-1+5*n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
1/12*(3*b^3*x^(4*n) - 8*b^2*x^(3*n) + 24*b*x^(2*n) - 96*x^n)/(b^4*n) + 16*
log((b*x^n + 2)/b)/(b^5*n)
```

Giac [F]

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx = \int \frac{x^{5\,n-1}}{bx^n+2}\,dx$$

input
```
integrate(x^(-1+5*n)/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(5*n - 1)/(b*x^n + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx = \int \frac{x^{5\,n-1}}{b\,x^n+2}\,dx$$

input
```
int(x^(5*n - 1)/(b*x^n + 2),x)
```

output
```
int(x^(5*n - 1)/(b*x^n + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{x^{-1+5n}}{2+bx^n}\,dx = \frac{3x^{4n}b^4-8x^{3n}b^3+24x^{2n}b^2-96x^nb+192\log(x^nb+2)}{12b^5n}$$

input
```
int(x^(-1+5*n)/(2+b*x^n),x)
```

output
```
(3*x**(4*n)*b**4 - 8*x**(3*n)*b**3 + 24*x**(2*n)*b**2 - 96*x**n*b + 192*lo
g(x**n*b + 2))/(12*b**5*n)
```

3.472 $\int \frac{x^{-1+4n}}{2+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx = \frac{4x^n}{b^3n} - \frac{x^{2n}}{b^2n} + \frac{x^{3n}}{3bn} - \frac{8\log(2+bx^n)}{b^4n}$$

output
```
4*x^n/b^3/n-x^(2*n)/b^2/n+1/3*x^(3*n)/b/n-8*ln(2+b*x^n)/b^4/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx = \frac{bx^n(12-3bx^n+b^2x^{2n}) - 24\log(2+bx^n)}{3b^4n}$$

input
```
Integrate[x^(-1 + 4*n)/(2 + b*x^n),x]
```

output
```
(b*x^n*(12 - 3*b*x^n + b^2*x^(2*n)) - 24*Log[2 + b*x^n])/(3*b^4*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{bx^n+2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{3n}}{bx^n+2}dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(-\frac{2x^n}{b^2}+\frac{x^{2n}}{b}-\frac{8}{b^3(bx^n+2)}+\frac{4}{b^3}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{8\log(bx^n+2)}{b^4}+\frac{4x^n}{b^3}-\frac{x^{2n}}{b^2}+\frac{x^{3n}}{3b}}{n}$$

input
```
Int[x^(-1 + 4*n)/(2 + b*x^n),x]
```

output
```
((4*x^n)/b^3 - x^(2*n)/b^2 + x^(3*n)/(3*b) - (8*Log[2 + b*x^n])/b^4)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x^{3n}}{3bn}-\frac{x^{2n}}{b^2n}+\frac{4x^n}{b^3n}-\frac{8\ln\left(x^n+\frac{2}{b}\right)}{b^4n}$
norman	$\frac{4\,\mathrm{e}^{n\ln(x)}}{b^3n}-\frac{\mathrm{e}^{2n\ln(x)}}{b^2n}+\frac{\mathrm{e}^{3n\ln(x)}}{3bn}-\frac{8\ln\left(2+b\,\mathrm{e}^{n\ln(x)}\right)}{b^4n}$
meijerg	$\frac{-\frac{ix^nb\left(-(-1)^{\mathrm{csgn}(ib)+\mathrm{csgn}(ix^n)+\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ib)}x^{2n}b^2+3ix^nb(-1)^{\frac{\mathrm{csgn}(ib)}{2}+\frac{\mathrm{csgn}(ix^n)}{2}-\frac{\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ib)}{2}}+12\right)(-1)^{-\frac{3\,\mathrm{csgn}(ib)}{2}-\frac{3\,\mathrm{cs}}{}}}{3}}{b^4n}$

input
```
int(x^(-1+4*n)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/n*(x^n)^3-1/b^2/n*(x^n)^2+4*x^n/b^3/n-8/b^4/n*ln(x^n+2/b)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx=\frac{b^3x^{3\,n}-3\,b^2x^{2\,n}+12\,bx^n-24\,\log\left(bx^n+2\right)}{3\,b^4n}$$

input
```
integrate(x^(-1+4*n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
1/3*(b^3*x^(3*n) - 3*b^2*x^(2*n) + 12*b*x^n - 24*log(b*x^n + 2))/(b^4*n)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx = \begin{cases} \frac{\log(x)}{2} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{4n-1}}{8n} & \text{for } b=0 \\ \frac{\log(x)}{b+2} & \text{for } n=0 \\ \frac{x^{3n}}{3bn} - \frac{x^{2n}}{b^2n} + \frac{4x^n}{b^3n} - \frac{8\log\left(x^n+\frac{2}{b}\right)}{b^4n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(8*n), Eq(b, 0)
), (log(x)/(b + 2), Eq(n, 0)), (x**(3*n)/(3*b*n) - x**(2*n)/(b**2*n) + 4*x
**n/(b**3*n) - 8*log(x**n + 2/b)/(b**4*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx = \frac{b^2x^{3n} - 3\,bx^{2n} + 12\,x^n}{3\,b^3n} - \frac{8\,\log\left(\frac{bx^n+2}{b}\right)}{b^4n}$$

input
```
integrate(x^(-1+4*n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
1/3*(b^2*x^(3*n) - 3*b*x^(2*n) + 12*x^n)/(b^3*n) - 8*log((b*x^n + 2)/b)/(b
^4*n)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx = \int \frac{x^{4\,n-1}}{bx^n+2}\,dx$$

input
```
integrate(x^(-1+4*n)/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(b*x^n + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx = \int \frac{x^{4\,n-1}}{b\,x^n+2}\,dx$$

input
```
int(x^(4*n - 1)/(b*x^n + 2),x)
```

output
```
int(x^(4*n - 1)/(b*x^n + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+4n}}{2+bx^n}\,dx = \frac{x^{3n}b^3-3x^{2n}b^2+12x^nb-24\log(x^nb+2)}{3b^4n}$$

input
```
int(x^(-1+4*n)/(2+b*x^n),x)
```

output
```
(x**(3*n)*b**3 - 3*x**(2*n)*b**2 + 12*x**n*b - 24*log(x**n*b + 2))/(3*b**4
*n)
```

3.473 $\int \frac{x^{-1+3n}}{2+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx = -\frac{2x^n}{b^2n}+\frac{x^{2n}}{2bn}+\frac{4\log(2+bx^n)}{b^3n}$$

output
```
-2*x^n/b^2/n+1/2*x^(2*n)/b/n+4*ln(2+b*x^n)/b^3/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx = \frac{bx^n(-4+bx^n)+8\log(2+bx^n)}{2b^3n}$$

input
```
Integrate[x^(-1 + 3*n)/(2 + b*x^n),x]
```

output
```
(b*x^n*(-4 + b*x^n) + 8*Log[2 + b*x^n])/(2*b^3*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{bx^n+2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{2n}}{bx^n+2}dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(\frac{x^n}{b}+\frac{4}{b^2(bx^n+2)}-\frac{2}{b^2}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{4\log(bx^n+2)}{b^3}-\frac{2x^n}{b^2}+\frac{x^{2n}}{2b}}{n}$$

input
```
Int[x^(-1 + 3*n)/(2 + b*x^n),x]
```

output
```
((-2*x^n)/b^2 + x^(2*n)/(2*b) + (4*Log[2 + b*x^n])/b^3)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result
risch	$\frac{x^{2n}}{2bn}-\frac{2x^{n}}{b^{2}n}+\frac{4\ln\left(x^{n}+\frac{2}{b}\right)}{b^{3}n}$
norman	$-\frac{2\,\mathrm{e}^{n\ln(x)}}{b^{2}n}+\frac{\mathrm{e}^{2n\ln(x)}}{2bn}+\frac{4\ln\left(2+b\,\mathrm{e}^{n\ln(x)}\right)}{b^{3}n}$
meijerg	$-\frac{4i(-1)^{-\frac{3\,\mathrm{csgn}(ib)}{2}-\frac{3\,\mathrm{csgn}(ix^{n})}{2}+\frac{3\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ib)}{2}}\left(\frac{ix^{n}b(-1)^{\frac{5\,\mathrm{csgn}(ib)}{2}+\frac{5\,\mathrm{csgn}(ix^{n})}{2}-\frac{\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ib)}{2}}\left(3ix^{n}b(-1)^{\frac{\mathrm{csgn}(ib)}{2}+\mathrm{csg}}\right.}{12}\right.}{b^{3}n}$

input
```
int(x^(-1+3*n)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/n*(x^n)^2-2*x^n/b^2/n+4/b^3/n*ln(x^n+2/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx=\frac{b^2x^{2\,n}-4\,bx^n+8\,\log\left(bx^n+2\right)}{2\,b^3n}$$

input
```
integrate(x^(-1+3*n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^(2*n) - 4*b*x^n + 8*log(b*x^n + 2))/(b^3*n)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx = \begin{cases} \frac{\log(x)}{2} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{3n-1}}{6n} & \text{for } b=0 \\ \frac{\log(x)}{b+2} & \text{for } n=0 \\ \frac{x^{2n}}{2bn} - \frac{2x^n}{b^2n} + \frac{4\log\left(x^n+\frac{2}{b}\right)}{b^3n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(6*n), Eq(b, 0)
), (log(x)/(b + 2), Eq(n, 0)), (x**(2*n)/(2*b*n) - 2*x**n/(b**2*n) + 4*log
(x**n + 2/b)/(b**3*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx = \frac{bx^{2n}-4x^n}{2b^2n} + \frac{4\log\left(\frac{bx^n+2}{b}\right)}{b^3n}$$

input
```
integrate(x^(-1+3*n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(b*x^(2*n) - 4*x^n)/(b^2*n) + 4*log((b*x^n + 2)/b)/(b^3*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx = \int \frac{x^{3\,n-1}}{bx^n+2}\,dx$$

input
```
integrate(x^(-1+3*n)/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(b*x^n + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx = \int \frac{x^{3\,n-1}}{b\,x^n+2}\,dx$$

input
```
int(x^(3*n - 1)/(b*x^n + 2),x)
```

output
```
int(x^(3*n - 1)/(b*x^n + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+3n}}{2+bx^n}\,dx = \frac{x^{2n}b^2-4x^nb+8\log(x^nb+2)}{2b^3n}$$

input
```
int(x^(-1+3*n)/(2+b*x^n),x)
```

output
```
(x**(2*n)*b**2 - 4*x**n*b + 8*log(x**n*b + 2))/(2*b**3*n)
```

3.474 $\int \frac{x^{-1+2n}}{2+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx=\frac{x^n}{bn}-\frac{2\log(2+bx^n)}{b^2n}$$

output
```
x^n/b/n-2*ln(2+b*x^n)/b^2/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx=\frac{bx^n-2\log(bn(2+bx^n))}{b^2n}$$

input
```
Integrate[x^(-1 + 2*n)/(2 + b*x^n),x]
```

output
```
(b*x^n - 2*Log[b*n*(2 + b*x^n)])/(b^2*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{bx^n+2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^n}{bx^n+2}dx^n}{n}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(\frac{1}{b}-\frac{2}{b(bx^n+2)}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{x^n}{b}-\frac{2\log(bx^n+2)}{b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)/(2 + b*x^n),x]
```

output
```
(x^n/b - (2*Log[2 + b*x^n])/b^2)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

method	result
risch	$\frac{x^{n}}{b n}-\frac{2 \ln \left(x^{n}+\frac{2}{b}\right)}{b^{2} n}$
norman	$\frac{{\mathrm e}^{n \ln \left(x \right)}}{b n}-\frac{2 \ln \left(2+b \,{\mathrm e}^{n \ln \left(x \right)}\right)}{b^{2} n}$
meijerg	$-\frac{2 \left(-1\right)^{\mathrm{csgn}\left(i b \right)+\mathrm{csgn}\left(i x^{n}\right)+\mathrm{csgn}\left(i x^{n}\right) \mathrm{csgn}\left(i b \right)} \left(-\frac{i x^{n} b \left(-1\right)^{\frac{\mathrm{csgn}\left(i b \right)}{2}+\frac{\mathrm{csgn}\left(i x^{n}\right)}{2}+\frac{3\, \mathrm{csgn}\left(i x^{n}\right) \mathrm{csgn}\left(i b \right)}{2}}}{2}-\left(-1\right)^{2\, \mathrm{csgn}\left(i b \right)} \left(-1\right)^{2\, \mathrm{csgn}\left(i x^{n}\right)} \left(-\right.}{b^{2} n}$

input
```
int(x^(2*n-1)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
x^n/b/n-2/b^2/n*ln(x^n+2/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx = \frac{bx^n - 2\,\log\left(bx^n + 2\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
(b*x^n - 2*log(b*x^n + 2))/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx = \begin{cases} \frac{\log(x)}{2} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{2n-1}}{4n} & \text{for } b=0 \\ \frac{\log(x)}{b+2} & \text{for } n=0 \\ \frac{x^n}{bn} - \frac{2\log\left(x^n+\frac{2}{b}\right)}{b^2n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)/(4*n), Eq(b, 0)
), (log(x)/(b + 2), Eq(n, 0)), (x**n/(b*n) - 2*log(x**n + 2/b)/(b**2*n), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx = \frac{x^n}{bn} - \frac{2\log\left(\frac{bx^n+2}{b}\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
x^n/(b*n) - 2*log((b*x^n + 2)/b)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx = \int \frac{x^{2\,n-1}}{bx^n+2}\,dx$$

input
```
integrate(x^(-1+2*n)/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(b*x^n + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx = \int \frac{x^{2\,n-1}}{b\,x^n+2}\,dx$$

input
```
int(x^(2*n - 1)/(b*x^n + 2),x)
```

output
```
int(x^(2*n - 1)/(b*x^n + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+2n}}{2+bx^n}\,dx = \frac{x^n b - 2\log(x^n b + 2)}{b^2 n}$$

input
```
int(x^(-1+2*n)/(2+b*x^n),x)
```

output
```
(x**n*b - 2*log(x**n*b + 2))/(b**2*n)
```

3.475 $\int \frac{x^{-1+n}}{2+bx^n} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^{-1+n}}{2+bx^n} dx = \frac{\log(2+bx^n)}{bn}$$

output
```
ln(2+b*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{2+bx^n} dx = \frac{\log(2+bx^n)}{bn}$$

input
```
Integrate[x^(-1 + n)/(2 + b*x^n),x]
```

output
```
Log[2 + b*x^n]/(b*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{792\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{bx^n+2} \, dx$$

$$\Big\downarrow \; 792$$

$$\frac{\log(bx^n+2)}{bn}$$

input
```
Int[x^(-1 + n)/(2 + b*x^n),x]
```

output
```
Log[2 + b*x^n]/(b*n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{\ln\left(2+b\,{\mathrm e}^{n \ln(x)}\right)}{b n}$	18
risch	$\frac{\ln\left(x^{n}+\frac{2}{b}\right)}{b n}$	18
meijerg	$-\frac{i (-1)^{-\frac{\operatorname{csgn}(i b)}{2}-\frac{\operatorname{csgn}(i x^{n})}{2}+\frac{\operatorname{csgn}(i x^{n}) \operatorname{csgn}(i b)}{2}-\frac{-1+n}{n}-\frac{1}{n}} \ln\left(1-\frac{i x^{n} b (-1)^{\frac{\operatorname{csgn}(i b)}{2}+\frac{\operatorname{csgn}(i x^{n})}{2}-\frac{\operatorname{csgn}(i x^{n}) \operatorname{csgn}(i b)}{2}}}{2}\right)}{b n}$	99

input
```
int(x^(-1+n)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/b/n*ln(2+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{2+bx^n}\,dx = \frac{\log(bx^n+2)}{bn}$$

input
```
integrate(x^(-1+n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
log(b*x^n + 2)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{x^{-1+n}}{2+bx^n}\,dx = \begin{cases} \frac{\log(x)}{2} & \text{for}\: b=0 \wedge n=0 \\ \frac{xx^{n-1}}{2n} & \text{for}\: b=0 \\ \frac{\log(x)}{b+2} & \text{for}\: n=0 \\ \frac{\log\left(x^n+\frac{2}{b}\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(2*n), Eq(b, 0)),
 (log(x)/(b + 2), Eq(n, 0)), (log(x**n + 2/b)/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{2+bx^n}\,dx = \frac{\log(bx^n+2)}{bn}$$

input
```
integrate(x^(-1+n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
log(b*x^n + 2)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+n}}{2+bx^n}\,dx = \frac{\log(|bx^n+2|)}{bn}$$

input
```
integrate(x^(-1+n)/(2+b*x^n),x, algorithm="giac")
```

output
```
log(abs(b*x^n + 2))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{2+bx^n}\,dx = \frac{\ln(b\,x^n+2)}{b\,n}$$

input
```
int(x^(n - 1)/(b*x^n + 2),x)
```

output
```
log(b*x^n + 2)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{2+bx^n}\,dx = \frac{\log(x^n b+2)}{bn}$$

input
```
int(x^(-1+n)/(2+b*x^n),x)
```

output
```
log(x**n*b + 2)/(b*n)
```

3.476 $\int \frac{1}{x(2+bx^n)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x(2+bx^n)}\,dx = \frac{\log(x)}{2} - \frac{\log(2+bx^n)}{2n}$$

output
```
1/2*ln(x)-1/2*ln(2+b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(2+bx^n)}\,dx = \frac{\log(x^n) - \log(n(2+bx^n))}{2n}$$

input
```
Integrate[1/(x*(2 + b*x^n)),x]
```

output
```
(Log[x^n] - Log[n*(2 + b*x^n)])/(2*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (bx^n+2)} \, dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{x^{-n}}{bx^n+2}dx^n}{n}$$

$$\Big\downarrow \; 47$$

$$\frac{\frac{\int x^{-n}dx^n}{2}-\frac{1}{2}b\int \frac{1}{bx^n+2}dx^n}{n}$$

$$\Big\downarrow \; 14$$

$$\frac{\frac{\log(x^n)}{2}-\frac{1}{2}b\int \frac{1}{bx^n+2}dx^n}{n}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\log(x^n)}{2}-\frac{1}{2}\log(bx^n+2)}{n}$$

input
```
Int[1/(x*(2 + b*x^n)),x]
```

output
```
(Log[x^n]/2 - Log[2 + b*x^n]/2)/n
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\ln(x)}{2}-\frac{\ln\left(2+b\,e^{n\ln(x)}\right)}{2n}$
risch	$\frac{\ln(x)}{2}-\frac{\ln\left(x^{n}+\frac{2}{b}\right)}{2n}$
parallelrisch	$\frac{n\ln(x)-\ln(2+b\,x^{n})}{2n}$
derivativedivides	$\frac{-\frac{\ln(2+b\,x^{n})}{2}+\frac{\ln(x^{n})}{2}}{n}$
default	$\frac{-\frac{\ln(2+b\,x^{n})}{2}+\frac{\ln(x^{n})}{2}}{n}$
meijerg	$\frac{n\ln(x)-\ln(2)+\ln(b)+i\left(\frac{\operatorname{csgn}(ib)}{2}+\frac{\operatorname{csgn}(ix^{n})}{2}-\frac{\operatorname{csgn}(ix^{n})\operatorname{csgn}(ib)}{2}-\frac{1}{2}\right)\pi-\ln\left(1-\frac{ix^{n}b(-1)^{\frac{\operatorname{csgn}(ib)}{2}+\frac{\operatorname{csgn}(ix^{n})}{2}-\frac{\operatorname{csgn}(ix^{n})}{2}}}{2}\right.}{2n}$

input
```
int(1/x/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x)-1/2/n*ln(2+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(2+bx^n)} \, dx = \frac{n \log(x) - \log(bx^n + 2)}{2n}$$

input
```
integrate(1/x/(2+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(n*log(x) - log(b*x^n + 2))/n
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{x(2+bx^n)} \, dx = \begin{cases} \frac{\log(x)}{2} & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \frac{\log(x)}{b+2} & \text{for } n = 0 \\ \frac{\log(x)}{2} - \frac{\log\left(x^n + \frac{2}{b}\right)}{2n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(b + 2), E
q(n, 0)), (log(x)/2 - log(x**n + 2/b)/(2*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(2+bx^n\right)}\,dx = -\frac{\log\left(bx^n+2\right)}{2\,n} + \frac{\log\left(x^n\right)}{2\,n}$$

input
```
integrate(1/x/(2+b*x^n),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^n + 2)/n + 1/2*log(x^n)/n
```

Giac [F]

$$\int \frac{1}{x\left(2+bx^n\right)}\,dx = \int \frac{1}{(bx^n+2)x}\,dx$$

input
```
integrate(1/x/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + 2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(2+bx^n\right)}\,dx = \frac{\ln\left(x\right)}{2} - \frac{\ln\left(b\,x^n+2\right)}{2\,n}$$

input
```
int(1/(x*(b*x^n + 2)),x)
```

output
```
log(x)/2 - log(b*x^n + 2)/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(2+bx^n\right)}\,dx=\frac{-\log(x^nb+2)+\log(x)\,n}{2n}$$

input
```
int(1/x/(2+b*x^n),x)
```

output
```
( - log(x**n*b + 2) + log(x)*n)/(2*n)
```

3.477 $\int \frac{x^{-1-n}}{2+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{x^{-1-n}}{2+bx^n} dx = -\frac{x^{-n}}{2n} - \frac{1}{4}b\log(x) + \frac{b\log(2+bx^n)}{4n}$$

output
```
-1/2/n/(x^n)-1/4*b*ln(x)+1/4*b*ln(2+b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1-n}}{2+bx^n} dx = -\frac{2x^{-n} + b\log(x^n) - b\log(n(2+bx^n))}{4n}$$

input
```
Integrate[x^(-1 - n)/(2 + b*x^n),x]
```

output
```
-1/4*(2/x^n + b*Log[x^n] - b*Log[n*(2 + b*x^n)])/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{bx^n+2}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{-2n}}{bx^n+2}dx^n}{n}$$
$$\Big\downarrow \text{54}$$
$$\frac{\int \left(\frac{x^{-2n}}{2}-\frac{bx^{-n}}{4}+\frac{b^2}{4(bx^n+2)}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{1}{4}b\log(x^n)+\frac{1}{4}b\log(bx^n+2)-\frac{x^{-n}}{2}}{n}$$

input
```
Int[x^(-1 - n)/(2 + b*x^n),x]
```

output
```
(-1/2*1/x^n - (b*Log[x^n])/4 + (b*Log[2 + b*x^n])/4)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{x^{-n}}{2n}-\frac{b\ln\left(x\right)}{4}+\frac{b\ln\left(x^{n}+\frac{2}{b}\right)}{4n}$
norman	$\left(-\frac{b\ln\left(x\right){\mathrm e}^{n\ln\left(x\right)}}{4}-\frac{1}{2n}\right){\mathrm e}^{-n\ln\left(x\right)}+\frac{b\ln\left(2+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{4n}$
meijerg	$-\frac{i b \left(-1\right)^{\frac{\mathrm{csgn}\left(i b\right)}{2}+\frac{\mathrm{csgn}\left(i x^{n}\right)}{2}-\frac{\mathrm{csgn}\left(i x^{n}\right)\mathrm{csgn}\left(i b\right)}{2}}\left(-\left(-1\right)^{\frac{\left(\frac{-1-n}{n}+\frac{1}{n}\right)\mathrm{csgn}\left(i b\right)}{2}+\frac{\left(\frac{-1-n}{n}+\frac{1}{n}\right)\mathrm{csgn}\left(i x^{n}\right)}{2}-\frac{\left(\frac{-1-n}{n}+\frac{1}{n}\right)\mathrm{csgn}\left(i x^{n}\right)\mathrm{csgn}\left(i b\right)}{2}}\right.}{}$

input
```
int(x^(-1-n)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2/n/(x^n)-1/4*b*ln(x)+1/4*b/n*ln(x^n+2/b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1-n}}{2+bx^n}\,dx=-\frac{bnx^n\log\left(x\right)-bx^n\log\left(bx^n+2\right)+2}{4\,nx^n}$$

input
```
integrate(x^(-1-n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
-1/4*(b*n*x^n*log(x) - b*x^n*log(b*x^n + 2) + 2)/(n*x^n)
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{x^{-1-n}}{2+bx^n}\,dx=\begin{cases}\frac{\log(x)}{2} & \text{for } b=0\wedge n=0\\ -\frac{xx^{-n-1}}{2n} & \text{for } b=0\\ \frac{\log(x)}{b+2} & \text{for } n=0\\ -\frac{b\log(x^n)}{4n}+\frac{b\log\left(x^n+\frac{2}{b}\right)}{4n}-\frac{x^{-n}}{2n} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1-n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(2*n), Eq(b, 0)
), (log(x)/(b + 2), Eq(n, 0)), (-b*log(x**n)/(4*n) + b*log(x**n + 2/b)/(4*
n) - 1/(2*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1-n}}{2+bx^n}\,dx=-\frac{1}{4}\,b\log(x)+\frac{b\log\left(\frac{bx^n+2}{b}\right)}{4\,n}-\frac{1}{2\,nx^n}$$

input
```
integrate(x^(-1-n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
-1/4*b*log(x) + 1/4*b*log((b*x^n + 2)/b)/n - 1/2/(n*x^n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{2+bx^n}\,dx = \int \frac{x^{-n-1}}{bx^n+2}\,dx$$

input
```
integrate(x^(-1-n)/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(b*x^n + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{2+bx^n}\,dx = \int \frac{1}{x^{n+1}\,(b\,x^n+2)}\,dx$$

input
```
int(1/(x^(n + 1)*(b*x^n + 2)),x)
```

output
```
int(1/(x^(n + 1)*(b*x^n + 2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1-n}}{2+bx^n}\,dx = \frac{x^n \log(x^n b+2)\,b - x^n \log(x)\,bn - 2}{4x^n n}$$

input
```
int(x^(-1-n)/(2+b*x^n),x)
```

output
```
(x**n*log(x**n*b + 2)*b - x**n*log(x)*b*n - 2)/(4*x**n*n)
```

3.478 $\int \frac{x^{-1-2n}}{2+bx^n} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{x^{-1-2n}}{2+bx^n} \, dx = -\frac{x^{-2n}}{4n} + \frac{bx^{-n}}{4n} + \frac{1}{8}b^2\log(x) - \frac{b^2\log(2+bx^n)}{8n}$$

output
```
-1/4/n/(x^(2*n))+1/4*b/n/(x^n)+1/8*b^2*ln(x)-1/8*b^2*ln(2+b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1-2n}}{2+bx^n} \, dx = -\frac{x^{-2n}(2-2bx^n) - b^2\log(x^n) + b^2\log(n(2+bx^n))}{8n}$$

input
```
Integrate[x^(-1 - 2*n)/(2 + b*x^n),x]
```

output
```
-1/8*((2 - 2*b*x^n)/x^(2*n) - b^2*Log[x^n] + b^2*Log[n*(2 + b*x^n)])/n
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{bx^n+2} \, dx$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{x^{-3n}}{bx^n+2}dx^n}{n}$$
$$\Big\downarrow \text{54}$$
$$\frac{\int \left(\frac{x^{-3n}}{2}-\frac{1}{4}bx^{-2n}+\frac{1}{8}b^2x^{-n}-\frac{b^3}{8(bx^n+2)}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{8}b^2\log(x^n)-\frac{1}{8}b^2\log(bx^n+2)+\frac{bx^{-n}}{4}-\frac{1}{4}x^{-2n}}{n}$$

input
```
Int[x^(-1 - 2*n)/(2 + b*x^n),x]
```

output
```
(-1/4*1/x^(2*n) + b/(4*x^n) + (b^2*Log[x^n])/8 - (b^2*Log[2 + b*x^n])/8)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result
risch	$\frac{b\,x^{-n}}{4n}-\frac{x^{-2n}}{4n}+\frac{b^2\ln(x)}{8}-\frac{b^2\ln\left(x^n+\frac{2}{b}\right)}{8n}$
norman	$\left(\frac{b^2\ln(x){\rm e}^{2n\ln(x)}}{8}-\frac{1}{4n}+\frac{b\,{\rm e}^{n\ln(x)}}{4n}\right){\rm e}^{-2n\ln(x)}-\frac{b^2\ln\left(2+b\,{\rm e}^{n\ln(x)}\right)}{8n}$
meijerg	$-\frac{(-1)^{\operatorname{csgn}(ib)+\operatorname{csgn}(ix^n)+\operatorname{csgn}(ix^n)\operatorname{csgn}(ib)}b^2\left(-(-1)^{\frac{\left(\frac{-2n-1}{n}+\frac{1}{n}\right)\operatorname{csgn}(ib)}{2}+\frac{\left(\frac{-2n-1}{n}+\frac{1}{n}\right)\operatorname{csgn}(ix^n)}{2}-\frac{\frac{\left(\frac{-2n-1}{n}+\frac{1}{n}\right)\operatorname{csgn}(ix^n)\operatorname{csgn}(i}{2}}{2}}\right.}{}$

input
```
int(x^(-2*n-1)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/4*b/n/(x^n)-1/4/n/(x^n)^2+1/8*b^2*ln(x)-1/8*b^2/n*ln(x^n+2/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int\frac{x^{-1-2n}}{2+bx^n}\,dx=\frac{b^2nx^{2\,n}\log\left(x\right)-b^2x^{2\,n}\log\left(bx^n+2\right)+2\,bx^n-2}{8\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
1/8*(b^2*n*x^(2*n)*log(x) - b^2*x^(2*n)*log(b*x^n + 2) + 2*b*x^n - 2)/(n*x
^(2*n))
```

Sympy [A] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{x^{-1-2n}}{2+bx^n}\,dx = \begin{cases} \frac{\log(x)}{2} & \text{for } b=0 \wedge n=0 \\ -\frac{xx^{-2n-1}}{4n} & \text{for } b=0 \\ \frac{\log(x)}{b+2} & \text{for } n=0 \\ \frac{b^2\log(x^n)}{8n} - \frac{b^2\log\left(x^n+\frac{2}{b}\right)}{8n} + \frac{bx^{-n}}{4n} - \frac{x^{-2n}}{4n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-2*n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (-x*x**(-2*n - 1)/(4*n), Eq(b,
0)), (log(x)/(b + 2), Eq(n, 0)), (b**2*log(x**n)/(8*n) - b**2*log(x**n + 2
/b)/(8*n) + b/(4*n*x**n) - 1/(4*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1-2n}}{2+bx^n}\,dx = \frac{1}{8}\,b^2\log(x) - \frac{b^2\log\left(\frac{bx^n+2}{b}\right)}{8\,n} + \frac{bx^n-1}{4\,nx^{2\,n}}$$

input
```
integrate(x^(-1-2*n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
1/8*b^2*log(x) - 1/8*b^2*log((b*x^n + 2)/b)/n + 1/4*(b*x^n - 1)/(n*x^(2*n)
)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{2+bx^n}\,dx = \int \frac{x^{-2n-1}}{bx^n+2}\,dx$$

input
```
integrate(x^(-1-2*n)/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(b*x^n + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{2+bx^n}\,dx = \int \frac{1}{x^{2n+1}\,(b\,x^n+2)}\,dx$$

input
```
int(1/(x^(2*n + 1)*(b*x^n + 2)),x)
```

output
```
int(1/(x^(2*n + 1)*(b*x^n + 2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{x^{-1-2n}}{2+bx^n}\,dx = \frac{-x^{2n}\log(x^n b+2)\,b^2+x^{2n}\log(x)\,b^2 n+2x^n b-2}{8x^{2n}n}$$

input
```
int(x^(-1-2*n)/(2+b*x^n),x)
```

output
```
( - x**(2*n)*log(x**n*b + 2)*b**2 + x**(2*n)*log(x)*b**2*n + 2*x**n*b - 2)
/(8*x**(2*n)*n)
```

3.479 $\int \frac{x^{-1-3n}}{2+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx=-\frac{x^{-3n}}{6n}+\frac{bx^{-2n}}{8n}-\frac{b^2x^{-n}}{8n}-\frac{1}{16}b^3\log(x)+\frac{b^3\log(2+bx^n)}{16n}$$

output
```
-1/6/n/(x^(3*n))+1/8*b/n/(x^(2*n))-1/8*b^2/n/(x^n)-1/16*b^3*ln(x)+1/16*b^3
*ln(2+b*x^n)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx=-\frac{x^{-3n}(8-6bx^n+6b^2x^{2n})+3b^3\log(x^n)-3b^3\log(n(2+bx^n))}{48n}$$

input
```
Integrate[x^(-1 - 3*n)/(2 + b*x^n),x]
```

output
```
-1/48*((8 - 6*b*x^n + 6*b^2*x^(2*n))/x^(3*n) + 3*b^3*Log[x^n] - 3*b^3*Log[
n*(2 + b*x^n)])/n
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{bx^n+2} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^{-4n}}{bx^n+2}dx^n}{n}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(\frac{x^{-4n}}{2}-\frac{1}{4}bx^{-3n}+\frac{1}{8}b^2x^{-2n}-\frac{1}{16}b^3x^{-n}+\frac{b^4}{16(bx^n+2)}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{16}b^3\log(x^n)+\frac{1}{16}b^3\log(bx^n+2)-\frac{1}{8}b^2x^{-n}+\frac{1}{8}bx^{-2n}-\frac{1}{6}x^{-3n}}{n}$$

input
```
Int[x^(-1 - 3*n)/(2 + b*x^n),x]
```

output
```
(-1/6*1/x^(3*n) + b/(8*x^(2*n)) - b^2/(8*x^n) - (b^3*Log[x^n])/16 + (b^3*L
og[2 + b*x^n])/16)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{b^{2} x^{-n}}{8 n}+\frac{b\,x^{-2 n}}{8 n}-\frac{x^{-3 n}}{6 n}-\frac{b^{3} \ln \left(x \right)}{16}+\frac{b^{3} \ln \left(x^{n}+\frac{2}{b}\right)}{16 n}$
norman	$\left(-\frac{b^{3} \ln \left(x \right) {\mathrm e}^{3 n \ln \left(x \right)}}{16}-\frac{1}{6 n}+\frac{b\,{\mathrm e}^{n \ln \left(x \right)}}{8 n}-\frac{b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{8 n}\right) {\mathrm e}^{-3 n \ln \left(x \right)}+\frac{b^{3} \ln \left(2+b\,{\mathrm e}^{n \ln \left(x \right)}\right)}{16 n}$
meijerg	$\frac{i b^{3} \left(-1\right)^{\frac{3\,\mathrm{csgn}\left(i b \right)}{2}+\frac{3\,\mathrm{csgn}\left(i x^{n}\right)}{2}-\frac{3\,\mathrm{csgn}\left(i x^{n}\right) \mathrm{csgn}\left(i b \right)}{2}} \left(-\frac{\left(-1\right)^{\frac{\left(\frac{-1-3 n}{n}+\frac{1}{n}\right) \mathrm{csgn}\left(i b \right)}{2}+\frac{\left(\frac{-1-3 n}{n}+\frac{1}{n}\right) \mathrm{csgn}\left(i x^{n}\right)}{2}-\frac{\left(\frac{-1-3 n}{n}+\frac{1}{n}\right) \mathrm{csgn}\left(i x^{n}\right) \cdots}{2}} \cdots}{3} \cdots\right.}{\cdots}$

input
```
int(x^(-1-3*n)/(2+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^2/n/(x^n)+1/8*b/n/(x^n)^2-1/6/n/(x^n)^3-1/16*b^3*ln(x)+1/16*b^3/n*l
n(x^n+2/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx=-\frac{3\,b^{3}nx^{3\,n}\log\left(x\right)-3\,b^{3}x^{3\,n}\log\left(bx^{n}+2\right)+6\,b^{2}x^{2\,n}-6\,bx^{n}+8}{48\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)/(2+b*x^n),x, algorithm="fricas")
```

output
```
-1/48*(3*b^3*n*x^(3*n)*log(x) - 3*b^3*x^(3*n)*log(b*x^n + 2) + 6*b^2*x^(2*
n) - 6*b*x^n + 8)/(n*x^(3*n))
```

Sympy [A] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx = \begin{cases} \frac{\log(x)}{2} & \text{for } b=0 \wedge n=0 \\ -\frac{xx^{-3n-1}}{6n} & \text{for } b=0 \\ \frac{\log(x)}{b+2} & \text{for } n=0 \\ -\frac{b^3\log(x^n)}{16n} + \frac{b^3\log\left(x^n+\frac{2}{b}\right)}{16n} - \frac{b^2x^{-n}}{8n} + \frac{bx^{-2n}}{8n} - \frac{x^{-3n}}{6n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-3*n)/(2+b*x**n),x)
```

output
```
Piecewise((log(x)/2, Eq(b, 0) & Eq(n, 0)), (-x*x**(-3*n - 1)/(6*n), Eq(b,
0)), (log(x)/(b + 2), Eq(n, 0)), (-b**3*log(x**n)/(16*n) + b**3*log(x**n +
 2/b)/(16*n) - b**2/(8*n*x**n) + b/(8*n*x**(2*n)) - 1/(6*n*x**(3*n)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx = -\frac{1}{16}\,b^3\log(x) + \frac{b^3\log\left(\frac{bx^n+2}{b}\right)}{16\,n} - \frac{3\,b^2x^{2\,n} - 3\,bx^n + 4}{24\,nx^{3\,n}}$$

input
```
integrate(x^(-1-3*n)/(2+b*x^n),x, algorithm="maxima")
```

output
```
-1/16*b^3*log(x) + 1/16*b^3*log((b*x^n + 2)/b)/n - 1/24*(3*b^2*x^(2*n) - 3
*b*x^n + 4)/(n*x^(3*n))
```

Giac [F]

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx = \int \frac{x^{-3\,n-1}}{bx^n+2}\,dx$$

input
```
integrate(x^(-1-3*n)/(2+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(b*x^n + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx = \int \frac{1}{x^{3\,n+1}\,(b\,x^n+2)}\,dx$$

input
```
int(1/(x^(3*n + 1)*(b*x^n + 2)),x)
```

output
```
int(1/(x^(3*n + 1)*(b*x^n + 2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{x^{-1-3n}}{2+bx^n}\,dx = \frac{3x^{3n}\log(x^nb+2)\,b^3-3x^{3n}\log(x)\,b^3n-6x^{2n}b^2+6x^nb-8}{48x^{3n}n}$$

input
```
int(x^(-1-3*n)/(2+b*x^n),x)
```

output
```
(3*x**(3*n)*log(x**n*b + 2)*b**3 - 3*x**(3*n)*log(x)*b**3*n - 6*x**(2*n)*b
**2 + 6*x**n*b - 8)/(48*x**(3*n)*n)
```

3.480 $\int \frac{x^{-1+4n}}{(a+bx^n)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2} \, dx = -\frac{2ax^n}{b^3n} + \frac{x^{2n}}{2b^2n} + \frac{a^3}{b^4n(a+bx^n)} + \frac{3a^2\log(a+bx^n)}{b^4n}$$

output
```
-2*a*x^n/b^3/n+1/2*x^(2*n)/b^2/n+a^3/b^4/n/(a+b*x^n)+3*a^2*ln(a+b*x^n)/b^4
/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2} \, dx = \frac{2a^3 - 4a^2bx^n - 3ab^2x^{2n} + b^3x^{3n}}{2b^4n(a+bx^n)} + \frac{3a^2\log(a+bx^n)}{b^4n}$$

input
```
Integrate[x^(-1 + 4*n)/(a + b*x^n)^2,x]
```

output
```
(2*a^3 - 4*a^2*b*x^n - 3*a*b^2*x^(2*n) + b^3*x^(3*n))/(2*b^4*n*(a + b*x^n)
) + (3*a^2*Log[a + b*x^n])/(b^4*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{3n}}{(bx^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{x^n}{b^2}+\frac{3a^2}{b^3(bx^n+a)}-\frac{a^3}{b^3(bx^n+a)^2}-\frac{2a}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3}{b^4(a+bx^n)}+\frac{3a^2\log(a+bx^n)}{b^4}-\frac{2ax^n}{b^3}+\frac{x^{2n}}{2b^2}}{n}$$

input
```
Int[x^(-1 + 4*n)/(a + b*x^n)^2,x]
```

output
```
((-2*a*x^n)/b^3 + x^(2*n)/(2*b^2) + a^3/(b^4*(a + b*x^n)) + (3*a^2*Log[a +
 b*x^n])/b^4)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{x^{2n}}{2b^2n}-\frac{2a\,x^n}{b^3n}+\frac{a^3}{b^4n(a+b\,x^n)}+\frac{3a^2\ln\left(x^n+\frac{a}{b}\right)}{b^4n}$	67
norman	$\frac{\frac{3a^3}{b^4n}+\frac{\mathrm{e}^{3n\ln(x)}}{2bn}-\frac{3a\,\mathrm{e}^{2n\ln(x)}}{2b^2n}}{a+b\,\mathrm{e}^{n\ln(x)}}+\frac{3a^2\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{b^4n}$	78

input
```
int(x^(-1+4*n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2/n*(x^n)^2-2*a*x^n/b^3/n+a^3/b^4/n/(a+b*x^n)+3*a^2/b^4/n*ln(x^n+a/b
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx=\frac{b^3x^{3\,n}-3\,ab^2x^{2\,n}-4\,a^2bx^n+2\,a^3+6\,(a^2bx^n+a^3)\log\left(bx^n+a\right)}{2\,(b^5nx^n+ab^4n)}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^(3*n) - 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 2*a^3 + 6*(a^2*b*x^n +
a^3)*log(b*x^n + a))/(b^5*n*x^n + a*b^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(56) = 112.

Time = 11.85 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.70

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx = \begin{cases} \frac{\log(x)}{a^2} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{4n-1}}{4a^2n} & \text{for } b=0 \\ \frac{\log(x)}{(a+b)^2} & \text{for } n=0 \\ \frac{6a^3\log\left(\frac{a}{b}+x^n\right)}{2ab^4n+2b^5nx^n} + \frac{6a^3}{2ab^4n+2b^5nx^n} + \frac{6a^2bx^n\log\left(\frac{a}{b}+x^n\right)}{2ab^4n+2b^5nx^n} - \frac{3ab^2x^{2n}}{2ab^4n+2b^5nx^n} + \frac{b^3x^{3n}}{2ab^4n+2b^5nx^n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+4*n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((log(x)/a**2, Eq(b, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(4*a**2*n),
Eq(b, 0)), (log(x)/(a + b)**2, Eq(n, 0)), (6*a**3*log(a/b + x**n)/(2*a*b**
4*n + 2*b**5*n*x**n) + 6*a**3/(2*a*b**4*n + 2*b**5*n*x**n) + 6*a**2*b*x**n
*log(a/b + x**n)/(2*a*b**4*n + 2*b**5*n*x**n) - 3*a*b**2*x**(2*n)/(2*a*b**
4*n + 2*b**5*n*x**n) + b**3*x**(3*n)/(2*a*b**4*n + 2*b**5*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx = \frac{b^3x^{3n} - 3\,ab^2x^{2n} - 4\,a^2bx^n + 2\,a^3}{2\,(b^5nx^n + ab^4n)} + \frac{3\,a^2\log\left(\frac{bx^n+a}{b}\right)}{b^4n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*x^(3*n) - 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 2*a^3)/(b^5*n*x^n + a*b
^4*n) + 3*a^2*log((b*x^n + a)/b)/(b^4*n)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx = \int \frac{x^{4\,n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx = \int \frac{x^{4\,n-1}}{(a+b\,x^n)^2}\,dx$$

input
```
int(x^(4*n - 1)/(a + b*x^n)^2,x)
```

output
```
int(x^(4*n - 1)/(a + b*x^n)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+4n}}{(a+bx^n)^2}\,dx = \frac{x^{3n}b^3-3x^{2n}a\,b^2+6x^n\log(x^nb+a)\,a^2b-6x^na^2b+6\log(x^nb+a)\,a^3}{2b^4n\,(x^nb+a)}$$

input
```
int(x^(-1+4*n)/(a+b*x^n)^2,x)
```

output
```
(x**(3*n)*b**3 - 3*x**(2*n)*a*b**2 + 6*x**n*log(x**n*b + a)*a**2*b - 6*x**
n*a**2*b + 6*log(x**n*b + a)*a**3)/(2*b**4*n*(x**n*b + a))
```

3.481 $\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx=\frac{x^n}{b^2n}-\frac{a^2}{b^3n\,(a+bx^n)}-\frac{2a\log\left(a+bx^n\right)}{b^3n}$$

output
```
x^n/b^2/n-a^2/b^3/n/(a+b*x^n)-2*a*ln(a+b*x^n)/b^3/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx=\frac{-a^2+abx^n+b^2x^{2n}}{b^3n\,(a+bx^n)}-\frac{2a\log\left(a+bx^n\right)}{b^3n}$$

input
```
Integrate[x^(-1 + 3*n)/(a + b*x^n)^2,x]
```

output
```
(-a^2 + a*b*x^n + b^2*x^(2*n))/(b^3*n*(a + b*x^n)) - (2*a*Log[a + b*x^n])/
(b^3*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^2} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{a^2}{b^2(bx^n+a)^2}-\frac{2a}{b^2(bx^n+a)}+\frac{1}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2}{b^3(a+bx^n)}-\frac{2a\log(a+bx^n)}{b^3}+\frac{x^n}{b^2}}{n}$$

input
```
Int[x^(-1 + 3*n)/(a + b*x^n)^2,x]
```

output
```
(x^n/b^2 - a^2/(b^3*(a + b*x^n)) - (2*a*Log[a + b*x^n])/b^3)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

method	result	size
risch	$\frac{x^{n}}{b^{2} n}-\frac{a^{2}}{b^{3} n \left(a+b\,x^{n}\right)}-\frac{2 a \ln \left(x^{n}+\frac{a}{b}\right)}{b^{3} n}$	51
norman	$\frac{\frac{\mathrm{e}^{2 n \ln \left(x \right)}}{b n}-\frac{2 a^{2}}{b^{3} n}}{a +b\,\mathrm{e}^{n \ln \left(x \right)}}-\frac{2 a \ln \left(a +b\,\mathrm{e}^{n \ln \left(x \right)}\right)}{b^{3} n}$	59

input
```
int(x^(-1+3*n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
x^n/b^2/n-a^2/b^3/n/(a+b*x^n)-2*a/b^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx = \frac{b^2x^{2\,n} + abx^n - a^2 - 2\left(abx^n + a^2\right)\log\left(bx^n + a\right)}{b^4nx^n + ab^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
(b^2*x^(2*n) + a*b*x^n - a^2 - 2*(a*b*x^n + a^2)*log(b*x^n + a))/(b^4*n*x^
n + a*b^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(39) = 78$.

Time = 5.84 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.77

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx = \begin{cases} \frac{\log(x)}{a^2} & \text{for } b = 0 \wedge n = 0 \\ \frac{xx^{3n-1}}{3a^2n} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ -\frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{ab^3n+b^4nx^n} - \frac{2a^2}{ab^3n+b^4nx^n} - \frac{2abx^n\log\left(\frac{a}{b}+x^n\right)}{ab^3n+b^4nx^n} + \frac{b^2x^{2n}}{ab^3n+b^4nx^n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((log(x)/a**2, Eq(b, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(3*a**2*n),
Eq(b, 0)), (log(x)/(a + b)**2, Eq(n, 0)), (-2*a**2*log(a/b + x**n)/(a*b**3
*n + b**4*n*x**n) - 2*a**2/(a*b**3*n + b**4*n*x**n) - 2*a*b*x**n*log(a/b +
 x**n)/(a*b**3*n + b**4*n*x**n) + b**2*x**(2*n)/(a*b**3*n + b**4*n*x**n),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx = \frac{b^2x^{2n} + abx^n - a^2}{b^4nx^n + ab^3n} - \frac{2a\log\left(\frac{bx^n+a}{b}\right)}{b^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(b^2*x^(2*n) + a*b*x^n - a^2)/(b^4*n*x^n + a*b^3*n) - 2*a*log((b*x^n + a)/
b)/(b^3*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx = \int \frac{x^{3n-1}}{(a+b\,x^n)^2}\,dx$$

input
```
int(x^(3*n - 1)/(a + b*x^n)^2,x)
```

output
```
int(x^(3*n - 1)/(a + b*x^n)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2}\,dx = \frac{x^{2n}b^2 - 2x^n \log(x^n b+a)\,ab + 2x^n ab - 2\log(x^n b+a)\,a^2}{b^3 n\,(x^n b+a)}$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^2,x)
```

output
```
(x**(2*n)*b**2 - 2*x**n*log(x**n*b + a)*a*b + 2*x**n*a*b - 2*log(x**n*b +
a)*a**2)/(b**3*n*(x**n*b + a))
```

3.482 $\int \frac{x^{-1+2n}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2} dx = \frac{a}{b^2n(a+bx^n)} + \frac{\log(a+bx^n)}{b^2n}$$

output
```
a/b^2/n/(a+b*x^n)+ln(a+b*x^n)/b^2/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2} dx = \frac{a}{b^2n(a+bx^n)} + \frac{\log(abn+b^2nx^n)}{b^2n}$$

input
```
Integrate[x^(-1 + 2*n)/(a + b*x^n)^2,x]
```

output
```
a/(b^2*n*(a + b*x^n)) + Log[a*b*n + b^2*n*x^n]/(b^2*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{1}{b(bx^n+a)}-\frac{a}{b(bx^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a}{b^2(a+bx^n)}+\frac{\log(a+bx^n)}{b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)/(a + b*x^n)^2,x]
```

output
```
(a/(b^2*(a + b*x^n)) + Log[a + b*x^n]/b^2)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{a}{b^2 n(a+b\,x^n)}+\frac{\ln\left(x^n+\frac{a}{b}\right)}{b^2 n}$	36
norman	$\frac{a}{b^2 n\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}+\frac{\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{b^2 n}$	38

input
```
int(x^(2*n-1)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a/b^2/n/(a+b*x^n)+1/b^2/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx=\frac{(bx^n+a)\log(bx^n+a)+a}{b^3nx^n+ab^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
((b*x^n + a)*log(b*x^n + a) + a)/(b^3*n*x^n + a*b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(26) = 52.

Time = 6.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx = \begin{cases} \frac{\log(x)}{a^2} & \text{for } b = 0 \wedge n = 0 \\ \frac{xx^{2n-1}}{2a^2n} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b)^2} & \text{for } n = 0 \\ \frac{a\log\left(\frac{a}{b}+x^n\right)}{ab^2n+b^3nx^n} + \frac{a}{ab^2n+b^3nx^n} + \frac{bx^n\log\left(\frac{a}{b}+x^n\right)}{ab^2n+b^3nx^n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((log(x)/a**2, Eq(b, 0) & Eq(n, 0)), (x*x**(2*n - 1)/(2*a**2*n),
Eq(b, 0)), (log(x)/(a + b)**2, Eq(n, 0)), (a*log(a/b + x**n)/(a*b**2*n + b
**3*n*x**n) + a/(a*b**2*n + b**3*n*x**n) + b*x**n*log(a/b + x**n)/(a*b**2*
n + b**3*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx = \frac{a}{b^3nx^n+ab^2n} + \frac{\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
a/(b^3*n*x^n + a*b^2*n) + log((b*x^n + a)/b)/(b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)^2}\,dx$$

input
```
int(x^(2*n - 1)/(a + b*x^n)^2,x)
```

output
```
int(x^(2*n - 1)/(a + b*x^n)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2}\,dx = \frac{x^n \log(x^n b+a)\,b - x^n b + \log(x^n b+a)\,a}{b^2 n\,(x^n b+a)}$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^2,x)
```

output
```
(x**n*log(x**n*b + a)*b - x**n*b + log(x**n*b + a)*a)/(b**2*n*(x**n*b + a)
)
```

3.483 $\int \frac{x^{-1+n}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{x^{-1+n}}{(a+bx^n)^2} dx = -\frac{1}{bn(a+bx^n)}$$

output
```
-1/b/n/(a+b*x^n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2} dx = -\frac{1}{bn(a+bx^n)}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n)^2,x]
```

output
```
-(1/(b*n*(a + b*x^n)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a+bx^n)^2}\,dx$$
$$\Big\downarrow \; 793$$
$$-\frac{1}{bn\,(a+bx^n)}$$

input
```
Int[x^(-1 + n)/(a + b*x^n)^2,x]
```

output
```
-(1/(b*n*(a + b*x^n)))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{1}{bn(a+b\,x^{n})}$	18
parallelrisch	$\frac{x\,x^{-1+n}}{an(a+b\,x^{n})}$	23
norman	$\frac{\mathrm{e}^{n\ln(x)}}{an\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}$	24

input
```
int(x^(-1+n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/n/(a+b*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{1}{b^2nx^n+abn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*n*x^n + a*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(12) = 24$.

Time = 0.92 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.82

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-2n}x^{n-1}}{b^2n} & \text{for } a=0 \\ \frac{\tilde{\infty}xx^{n-1}}{n} & \text{for } b=-ax^{-n} \\ \frac{\log(x)}{(a+b)^2} & \text{for } n=0 \\ \frac{xx^{n-1}}{a^2n+abnx^n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(n - 1)/(b*
*2*n*x**(2*n)), Eq(a, 0)), (zoo*x*x**(n - 1)/n, Eq(b, -a/x**n)), (log(x)/(
a + b)**2, Eq(n, 0)), (x*x**(n - 1)/(a**2*n + a*b*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{1}{(bx^n+a)bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/((b*x^n + a)*b*n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{1}{(bx^n+a)bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
-1/((b*x^n + a)*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{a}{b\,(a^2\,n+a\,b\,n\,x^n)}$$

input
```
int(x^(n - 1)/(a + b*x^n)^2,x)
```

output
```
-a/(b*(a^2*n + a*b*n*x^n))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = \frac{x^n}{an\,(x^nb+a)}$$

input
```
int(x^(-1+n)/(a+b*x^n)^2,x)
```

output
```
x**n/(a*n*(x**n*b + a))
```

3.484 $\int \frac{1}{x(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx=\frac{1}{an\left(a+bx^n\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^n\right)}{a^2n}$$

output
```
1/a/n/(a+b*x^n)+ln(x)/a^2-ln(a+b*x^n)/a^2/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx=\frac{1}{an\left(a+bx^n\right)}+\frac{\log\left(x^n\right)}{a^2n}-\frac{\log\left(a+bx^n\right)}{a^2n}$$

input
```
Integrate[1/(x*(a + b*x^n)^2),x]
```

output
```
1/(a*n*(a + b*x^n)) + Log[x^n]/(a^2*n) - Log[a + b*x^n]/(a^2*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b x^n)^2} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{(b x^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-n}}{a^2}-\frac{b}{a^2 (b x^n+a)}-\frac{b}{a (b x^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log(a+b x^n)}{a^2}+\frac{\log(x^n)}{a^2}+\frac{1}{a(a+b x^n)}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^2),x]
```

output
```
(1/(a*(a + b*x^n)) + Log[x^n]/a^2 - Log[a + b*x^n]/a^2)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\,x^n)}{a^2}+\frac{1}{a(a+b\,x^n)}+\frac{\ln(x^n)}{a^2}}{n}$	40
default	$\frac{-\frac{\ln(a+b\,x^n)}{a^2}+\frac{1}{a(a+b\,x^n)}+\frac{\ln(x^n)}{a^2}}{n}$	40
risch	$\frac{\ln(x)}{a^2}+\frac{1}{an(a+b\,x^n)}-\frac{\ln(x^n+\frac{a}{b})}{a^2n}$	42
norman	$\frac{-\frac{b\,\mathrm{e}^{n\ln(x)}\ln(x)}{a^2n}+\frac{\ln(x)}{a}+\frac{b\ln(x)\mathrm{e}^{n\ln(x)}}{a^2}}{a+b\,\mathrm{e}^{n\ln(x)}}-\frac{\ln(a+b\,\mathrm{e}^{n\ln(x)})}{a^2n}$	65
parallelrisch	$\frac{\ln(x)x^n b^2 n+\ln(x)abn-\ln(a+b\,x^n)x^n b^2-\ln(a+b\,x^n)ab+ab}{a^2bn(a+b\,x^n)}$	68

input
```
int(1/x/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/a^2*ln(a+b*x^n)+1/a/(a+b*x^n)+1/a^2*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx=\frac{bnx^n\log\left(x\right)+an\log\left(x\right)-\left(bx^n+a\right)\log\left(bx^n+a\right)+a}{a^2bnx^n+a^3n}$$

input
```
integrate(1/x/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
(b*n*x^n*log(x) + a*n*log(x) - (b*x^n + a)*log(b*x^n + a) + a)/(a^2*b*n*x^
n + a^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(31) = 62$.

Time = 0.73 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.10

$$\int \frac{1}{x\left(a+bx^n\right)^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ \frac{\log(x)}{a^2} & \text{for } b=0 \\ -\frac{x^{-2n}}{2b^2n} & \text{for } a=0 \\ \tilde{\infty}\log(x) & \text{for } b=-ax^{-n} \\ \frac{\log(x)}{(a+b)^2} & \text{for } n=0 \\ \frac{an\log(x)}{a^3n+a^2bnx^n} - \frac{a\log\left(\frac{a}{b}+x^n\right)}{a^3n+a^2bnx^n} + \frac{a}{a^3n+a^2bnx^n} + \frac{bnx^n\log(x)}{a^3n+a^2bnx^n} - \frac{bx^n\log\left(\frac{a}{b}+x^n\right)}{a^3n+a^2bnx^n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a**2, Eq(b
, 0)), (-1/(2*b**2*n*x**(2*n)), Eq(a, 0)), (zoo*log(x), Eq(b, -a/x**n)), (
log(x)/(a + b)**2, Eq(n, 0)), (a*n*log(x)/(a**3*n + a**2*b*n*x**n) - a*log
(a/b + x**n)/(a**3*n + a**2*b*n*x**n) + a/(a**3*n + a**2*b*n*x**n) + b*n*x
**n*log(x)/(a**3*n + a**2*b*n*x**n) - b*x**n*log(a/b + x**n)/(a**3*n + a**
2*b*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+bx^{n}\right)^{2}}\,dx=\frac{1}{abnx^{n}+a^{2}n}-\frac{\log\left(bx^{n}+a\right)}{a^{2}n}+\frac{\log\left(x^{n}\right)}{a^{2}n}$$

input
```
integrate(1/x/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/(a*b*n*x^n + a^2*n) - log(b*x^n + a)/(a^2*n) + log(x^n)/(a^2*n)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^{n}\right)^{2}}\,dx=\int \frac{1}{\left(bx^{n}+a\right)^{2}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^{n}\right)^{2}}\,dx=\frac{\ln\left(x\right)}{a^{2}}+\frac{1}{a\,n\,\left(a+b\,x^{n}\right)}-\frac{\ln\left(a+b\,x^{n}\right)}{a^{2}\,n}$$

input
```
int(1/(x*(a + b*x^n)^2),x)
```

output
```
log(x)/a^2 + 1/(a*n*(a + b*x^n)) - log(a + b*x^n)/(a^2*n)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

$$\int \frac{1}{x\,(a+bx^n)^2}\,dx$$
$$= \frac{-x^n\log(x^nb+a)\,b+x^n\log(x)\,bn-x^nb-\log(x^nb+a)\,a+\log(x)\,an}{a^2n\,(x^nb+a)}$$

input
```
int(1/x/(a+b*x^n)^2,x)
```

output
```
( - x**n*log(x**n*b + a)*b + x**n*log(x)*b*n - x**n*b - log(x**n*b + a)*a
+ log(x)*a*n)/(a**2*n*(x**n*b + a))
```

3.485 $\int \frac{x^{-1-n}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^{-1-n}}{(a+bx^n)^2} dx = -\frac{x^{-n}}{a^2n} - \frac{b}{a^2n(a+bx^n)} - \frac{2b\log(x)}{a^3} + \frac{2b\log(a+bx^n)}{a^3n}$$

output
```
-1/a^2/n/(x^n)-b/a^2/n/(a+b*x^n)-2*b*ln(x)/a^3+2*b*ln(a+b*x^n)/a^3/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1-n}}{(a+bx^n)^2} dx = \frac{\frac{a(-2b-ax^{-n})}{a+bx^n} - 2b\log(x^n) + 2b\log(a+bx^n)}{a^3n}$$

input
```
Integrate[x^(-1 - n)/(a + b*x^n)^2,x]
```

output
```
((a*(-2*b - a/x^n))/(a + b*x^n) - 2*b*Log[x^n] + 2*b*Log[a + b*x^n])/(a^3*
n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-2n}}{(bx^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-2n}}{a^2}-\frac{2bx^{-n}}{a^3}+\frac{2b^2}{a^3(bx^n+a)}+\frac{b^2}{a^2(bx^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2b\log(x^n)}{a^3}+\frac{2b\log(a+bx^n)}{a^3}-\frac{b}{a^2(a+bx^n)}-\frac{x^{-n}}{a^2}}{n}$$

input
```
Int[x^(-1 - n)/(a + b*x^n)^2,x]
```

output
```
(-(1/(a^2*x^n)) - b/(a^2*(a + b*x^n)) - (2*b*Log[x^n])/a^3 + (2*b*Log[a +
b*x^n])/a^3)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{x^{-n}}{a^{2} n}-\frac{2 b \ln \left(x \right)}{a^{3}}-\frac{b}{a^{2} n \left(a +b \,x^{n}\right)}+\frac{2 b \ln \left(x^{n}+\frac{a}{b}\right)}{a^{3} n}$	60
norman	$\frac{\left(\frac{2 b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}{a^{3} n}-\frac{1}{a n}-\frac{2 b \ln \left(x \right) {\mathrm e}^{n \ln \left(x \right)}}{a^{2}}-\frac{2 b^{2} \ln \left(x \right) {\mathrm e}^{2 n \ln \left(x \right)}}{a^{3}}\right) {\mathrm e}^{-n \ln \left(x \right)}}{a +b \,{\mathrm e}^{n \ln \left(x \right)}}+\frac{2 b \ln \left(a +b \,{\mathrm e}^{n \ln \left(x \right)}\right)}{a^{3} n}$	97

input
```
int(x^(-1-n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/n/(x^n)-2*b*ln(x)/a^3-b/a^2/n/(a+b*x^n)+2*b/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx$$
$$= -\frac{2\,b^2nx^{2\,n}\log{(x)} + a^2 + 2\,(abn\log{(x)} + ab)x^n - 2\,(b^2x^{2\,n} + abx^n)\log{(bx^n + a)}}{a^3bnx^{2\,n} + a^4nx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-(2*b^2*n*x^(2*n)*log(x) + a^2 + 2*(a*b*n*log(x) + a*b)*x^n - 2*(b^2*x^(2*
n) + a*b*x^n)*log(b*x^n + a))/(a^3*b*n*x^(2*n) + a^4*n*x^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(49) = 98.

Time = 8.87 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.60

$$\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for} \\ -\frac{xx^{-n-1}}{a^2n} & \text{for} \\ -\frac{xx^{-2n}x^{-n-1}}{3b^2n} & \text{for} \\ \frac{\tilde{\infty}xx^{-n-1}}{n} & \text{for} \\ \frac{\log(x)}{(a+b)^2} & \text{for} \\ -\frac{a^2}{a^4nx^n+a^3bnx^{2n}} - \frac{2abx^n\log(x^n)}{a^4nx^n+a^3bnx^{2n}} + \frac{2abx^n\log\left(\frac{a}{b}+x^n\right)}{a^4nx^n+a^3bnx^{2n}} - \frac{2abx^n}{a^4nx^n+a^3bnx^{2n}} - \frac{2b^2x^{2n}\log(x^n)}{a^4nx^n+a^3bnx^{2n}} + \frac{2b^2x^{2n}\log\left(\frac{a}{b}+x^n\right)}{a^4nx^n+a^3bnx^{2n}} & \text{oth} \end{cases}$$

input
```
integrate(x**(-1-n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(a
**2*n), Eq(b, 0)), (-x*x**(-n - 1)/(3*b**2*n*x**(2*n)), Eq(a, 0)), (zoo*x*
x**(-n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)**2, Eq(n, 0)), (-a**2/(a**
4*n*x**n + a**3*b*n*x**(2*n)) - 2*a*b*x**n*log(x**n)/(a**4*n*x**n + a**3*b
*n*x**(2*n)) + 2*a*b*x**n*log(a/b + x**n)/(a**4*n*x**n + a**3*b*n*x**(2*n)
) - 2*a*b*x**n/(a**4*n*x**n + a**3*b*n*x**(2*n)) - 2*b**2*x**(2*n)*log(x**
n)/(a**4*n*x**n + a**3*b*n*x**(2*n)) + 2*b**2*x**(2*n)*log(a/b + x**n)/(a*
*4*n*x**n + a**3*b*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx = -\frac{2\,bx^n+a}{a^2bnx^{2\,n}+a^3nx^n} - \frac{2\,b\log(x)}{a^3} + \frac{2\,b\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(2*b*x^n + a)/(a^2*b*n*x^(2*n) + a^3*n*x^n) - 2*b*log(x)/a^3 + 2*b*log((b
*x^n + a)/b)/(a^3*n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx = \int \frac{x^{-n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx = \int \frac{1}{x^{n+1}\,(a+b\,x^n)^2}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n)^2),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int \frac{x^{-1-n}}{(a+bx^n)^2}\,dx = \frac{2x^{2n}\log(x^nb+a)\,b^2-2x^{2n}\log(x)\,b^2n+2x^{2n}b^2+2x^n\log(x^nb+a)\,ab-2x^n\log(x)\,abn-a^2}{x^na^3n\,(x^nb+a)}$$

input
```
int(x^(-1-n)/(a+b*x^n)^2,x)
```

output
```
(2*x**(2*n)*log(x**n*b + a)*b**2 - 2*x**(2*n)*log(x)*b**2*n + 2*x**(2*n)*b
**2 + 2*x**n*log(x**n*b + a)*a*b - 2*x**n*log(x)*a*b*n - a**2)/(x**n*a**3*
n*(x**n*b + a))
```

3.486 $\int \frac{x^{-1-2n}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2} dx = -\frac{x^{-2n}}{2a^2n} + \frac{2bx^{-n}}{a^3n} + \frac{b^2}{a^3n(a+bx^n)} + \frac{3b^2\log(x)}{a^4} - \frac{3b^2\log(a+bx^n)}{a^4n}$$

output
```
-1/2/a^2/n/(x^(2*n))+2*b/a^3/n/(x^n)+b^2/a^3/n/(a+b*x^n)+3*b^2*ln(x)/a^4-3
*b^2*ln(a+b*x^n)/a^4/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2} dx = \frac{\frac{6ab^2-a^3x^{-2n}+3a^2bx^{-n}}{a+bx^n} + 6b^2\log(x^n) - 6b^2\log(a+bx^n)}{2a^4n}$$

input
```
Integrate[x^(-1 - 2*n)/(a + b*x^n)^2,x]
```

output
```
((6*a*b^2 - a^3/x^(2*n) + (3*a^2*b)/x^n)/(a + b*x^n) + 6*b^2*Log[x^n] - 6*
b^2*Log[a + b*x^n])/(2*a^4*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^{-3n}}{(bx^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(\frac{x^{-3n}}{a^2}-\frac{2bx^{-2n}}{a^3}+\frac{3b^2x^{-n}}{a^4}-\frac{3b^3}{a^4(bx^n+a)}-\frac{b^3}{a^3(bx^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3b^2\log(x^n)}{a^4}-\frac{3b^2\log(a+bx^n)}{a^4}+\frac{b^2}{a^3(a+bx^n)}+\frac{2bx^{-n}}{a^3}-\frac{x^{-2n}}{2a^2}}{n}$$

input
```
Int[x^(-1 - 2*n)/(a + b*x^n)^2,x]
```

output
```
(-1/2*1/(a^2*x^(2*n)) + (2*b)/(a^3*x^n) + b^2/(a^3*(a + b*x^n)) + (3*b^2*L
og[x^n])/a^4 - (3*b^2*Log[a + b*x^n])/a^4)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{2b\,x^{-n}}{a^{3}n}-\frac{x^{-2n}}{2a^{2}n}+\frac{3b^{2}\ln\left(x\right)}{a^{4}}+\frac{b^{2}}{a^{3}n\left(a+b\,x^{n}\right)}-\frac{3b^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{a^{4}n}$	79
norman	$\frac{\left(-\frac{3b^{3}{\mathrm e}^{3n\ln\left(x\right)}}{a^{4}n}-\frac{1}{2an}+\frac{3b\,{\mathrm e}^{n\ln\left(x\right)}}{2a^{2}n}+\frac{3b^{2}\ln\left(x\right){\mathrm e}^{2n\ln\left(x\right)}}{a^{3}}+\frac{3b^{3}\ln\left(x\right){\mathrm e}^{3n\ln\left(x\right)}}{a^{4}}\right){\mathrm e}^{-2n\ln\left(x\right)}}{a+b\,{\mathrm e}^{n\ln\left(x\right)}}-\frac{3b^{2}\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{a^{4}n}$	117

input
```
int(x^(-2*n-1)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
2*b/a^3/n/(x^n)-1/2/a^2/n/(x^n)^2+3*b^2*ln(x)/a^4+b^2/a^3/n/(a+b*x^n)-3*b^
2/a^4/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.35

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx = \frac{6\,b^3nx^{3\,n}\log(x)+3\,a^2bx^n-a^3+6\,(ab^2n\log(x)+ab^2)x^{2\,n}-6\,(b^3x^{3\,n}+ab^2x^{2\,n})\log(bx^n+a)}{2\,(a^4bnx^{3\,n}+a^5nx^{2\,n})}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/2*(6*b^3*n*x^(3*n)*log(x) + 3*a^2*b*x^n - a^3 + 6*(a*b^2*n*log(x) + a*b^
2)*x^(2*n) - 6*(b^3*x^(3*n) + a*b^2*x^(2*n))*log(b*x^n + a))/(a^4*b*n*x^(3
*n) + a^5*n*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(70) = 140.

Time = 22.00 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.42

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\log(x) \\ -\frac{xx^{-2n-1}}{2a^2n} \\ -\frac{xx^{-2n}x^{-2n-1}}{4b^2n} \\ \frac{\tilde{\infty}xx^{-2n-1}}{n} \\ \frac{\log(x)}{(a+b)^2} \\ -\frac{a^3}{2a^5nx^{2n}+2a^4bnx^{3n}} + \frac{3a^2bx^n}{2a^5nx^{2n}+2a^4bnx^{3n}} + \frac{6ab^2x^{2n}\log(x^n)}{2a^5nx^{2n}+2a^4bnx^{3n}} - \frac{6ab^2x^{2n}\log\left(\frac{a}{b}+x^n\right)}{2a^5nx^{2n}+2a^4bnx^{3n}} + \frac{6ab^2x^{2n}}{2a^5nx^{2n}+2a^4bnx^{3n}} + \frac{6b^3x^{3n}\,\mathrm{lo}}{2a^5nx^{2n}+2} \end{cases}$$

input
```
integrate(x**(-1-2*n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-2*n - 1)/
(2*a**2*n), Eq(b, 0)), (-x*x**(-2*n - 1)/(4*b**2*n*x**(2*n)), Eq(a, 0)), (
zoo*x*x**(-2*n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)**2, Eq(n, 0)), (-a
**3/(2*a**5*n*x**(2*n) + 2*a**4*b*n*x**(3*n)) + 3*a**2*b*x**n/(2*a**5*n*x*
*(2*n) + 2*a**4*b*n*x**(3*n)) + 6*a*b**2*x**(2*n)*log(x**n)/(2*a**5*n*x**(
2*n) + 2*a**4*b*n*x**(3*n)) - 6*a*b**2*x**(2*n)*log(a/b + x**n)/(2*a**5*n*
x**(2*n) + 2*a**4*b*n*x**(3*n)) + 6*a*b**2*x**(2*n)/(2*a**5*n*x**(2*n) + 2
*a**4*b*n*x**(3*n)) + 6*b**3*x**(3*n)*log(x**n)/(2*a**5*n*x**(2*n) + 2*a**
4*b*n*x**(3*n)) - 6*b**3*x**(3*n)*log(a/b + x**n)/(2*a**5*n*x**(2*n) + 2*a
**4*b*n*x**(3*n)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx = \int \frac{x^{-2n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx = \int \frac{1}{x^{2n+1}\,(a+b\,x^n)^2}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a + b*x^n)^2),x)
```

output
```
int(1/(x^(2*n + 1)*(a + b*x^n)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.45

$$\int \frac{x^{-1-2n}}{(a+bx^n)^2}\,dx = \frac{-6x^{3n}\log(x^nb+a)\,b^3+6x^{3n}\log(x)\,b^3n-6x^{3n}b^3-6x^{2n}\log(x^nb+a)\,a\,b^2+6x^{2n}\log(x)\,a\,b^2n+3x^na^2b-}{2x^{2n}a^4n\,(x^nb+a)}$$

input
```
int(x^(-1-2*n)/(a+b*x^n)^2,x)
```

output
```
( - 6*x**(3*n)*log(x**n*b + a)*b**3 + 6*x**(3*n)*log(x)*b**3*n - 6*x**(3*n
)*b**3 - 6*x**(2*n)*log(x**n*b + a)*a*b**2 + 6*x**(2*n)*log(x)*a*b**2*n +
3*x**n*a**2*b - a**3)/(2*x**(2*n)*a**4*n*(x**n*b + a))
```

3.487 $\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx = -\frac{x^{-3n}}{3a^2n}+\frac{bx^{-2n}}{a^3n}-\frac{3b^2x^{-n}}{a^4n}-\frac{b^3}{a^4n\,(a+bx^n)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx^n)}{a^5n}$$

output
```
-1/3/a^2/n/(x^(3*n))+b/a^3/n/(x^(2*n))-3*b^2/a^4/n/(x^n)-b^3/a^4/n/(a+b*x^
n)-4*b^3*ln(x)/a^5+4*b^3*ln(a+b*x^n)/a^5/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx = -\frac{\frac{ax^{-3n}(a^3-2a^2bx^n+6ab^2x^{2n}+12b^3x^{3n})}{a+bx^n}+12b^3\log(x^n)-12b^3\log(a+bx^n)}{3a^5n}$$

input
```
Integrate[x^(-1 - 3*n)/(a + b*x^n)^2,x]
```

output
```
-1/3*((a*(a^3 - 2*a^2*b*x^n + 6*a*b^2*x^(2*n) + 12*b^3*x^(3*n)))/(x^(3*n)*
(a + b*x^n)) + 12*b^3*Log[x^n] - 12*b^3*Log[a + b*x^n])/(a^5*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-4n}}{(bx^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-4n}}{a^2}-\frac{2bx^{-3n}}{a^3}+\frac{3b^2x^{-2n}}{a^4}-\frac{4b^3x^{-n}}{a^5}+\frac{4b^4}{a^5(bx^n+a)}+\frac{b^4}{a^4(bx^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4b^3\log(x^n)}{a^5}+\frac{4b^3\log(a+bx^n)}{a^5}-\frac{b^3}{a^4(a+bx^n)}-\frac{3b^2x^{-n}}{a^4}+\frac{bx^{-2n}}{a^3}-\frac{x^{-3n}}{3a^2}}{n}$$

input
```
Int[x^(-1 - 3*n)/(a + b*x^n)^2,x]
```

output
```
(-1/3*1/(a^2*x^(3*n)) + b/(a^3*x^(2*n)) - (3*b^2)/(a^4*x^n) - b^3/(a^4*(a
+ b*x^n)) - (4*b^3*Log[x^n])/a^5 + (4*b^3*Log[a + b*x^n])/a^5)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result	size
risch	$-\frac{3b^2x^{-n}}{a^4n}+\frac{b\,x^{-2n}}{a^3n}-\frac{x^{-3n}}{3a^2n}-\frac{4b^3\ln(x)}{a^5}-\frac{b^3}{a^4n(a+b\,x^n)}+\frac{4b^3\ln\left(x^n+\frac{a}{b}\right)}{a^5n}$	95
norman	$\frac{\left(\frac{4b^4\mathrm{e}^{4n\ln(x)}}{a^5n}-\frac{1}{3an}+\frac{2b\,\mathrm{e}^{n\ln(x)}}{3a^2n}-\frac{2b^2\mathrm{e}^{2n\ln(x)}}{a^3n}-\frac{4b^3\ln(x)\mathrm{e}^{3n\ln(x)}}{a^4}-\frac{4b^4\ln(x)\mathrm{e}^{4n\ln(x)}}{a^5}\right)\mathrm{e}^{-3n\ln(x)}}{a+b\,\mathrm{e}^{n\ln(x)}}+\frac{4b^3\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{a^5n}$	135

input
```
int(x^(-1-3*n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-3*b^2/a^4/n/(x^n)+b/a^3/n/(x^n)^2-1/3/a^2/n/(x^n)^3-4*b^3*ln(x)/a^5-b^3/a
^4/n/(a+b*x^n)+4*b^3/a^5/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int\frac{x^{-1-3n}}{(a+bx^n)^2}\,dx=$$
$$-\frac{12\,b^4nx^{4\,n}\log(x)+6\,a^2b^2x^{2\,n}-2\,a^3bx^n+a^4+12\,(ab^3n\log(x)+ab^3)x^{3\,n}-12\,(b^4x^{4\,n}+ab^3x^{3\,n})\log(}{3\,(a^5bnx^{4\,n}+a^6nx^{3\,n})}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-1/3*(12*b^4*n*x^(4*n)*log(x) + 6*a^2*b^2*x^(2*n) - 2*a^3*b*x^n + a^4 + 12
*(a*b^3*n*log(x) + a*b^3)*x^(3*n) - 12*(b^4*x^(4*n) + a*b^3*x^(3*n))*log(b
*x^n + a))/(a^5*b*n*x^(4*n) + a^6*n*x^(3*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(83) = 166$.

Time = 46.18 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.09

$$\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx = \begin{cases} \tilde{\infty}\log(x) \\ -\frac{xx^{-3n-1}}{3a^2n} \\ -\frac{xx^{-2n}x^{-3n-1}}{5b^2n} \\ \frac{\tilde{\infty}xx^{-3n-1}}{n} \\ \frac{\log(x)}{(a+b)^2} \\ -\frac{a^4}{3a^6nx^{3n}+3a^5bnx^{4n}} + \frac{2a^3bx^n}{3a^6nx^{3n}+3a^5bnx^{4n}} - \frac{6a^2b^2x^{2n}}{3a^6nx^{3n}+3a^5bnx^{4n}} - \frac{12ab^3x^{3n}\log(x^n)}{3a^6nx^{3n}+3a^5bnx^{4n}} + \frac{12ab^3x^{3n}\log\left(\frac{a}{b}+x^n\right)}{3a^6nx^{3n}+3a^5bnx^{4n}} - \frac{12ab^3}{3a^6nx^{3n}+}\end{cases}$$

input
```
integrate(x**(-1-3*n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-3*n - 1)/
(3*a**2*n), Eq(b, 0)), (-x*x**(-3*n - 1)/(5*b**2*n*x**(2*n)), Eq(a, 0)), (
zoo*x*x**(-3*n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)**2, Eq(n, 0)), (-a
**4/(3*a**6*n*x**(3*n) + 3*a**5*b*n*x**(4*n)) + 2*a**3*b*x**n/(3*a**6*n*x*
*(3*n) + 3*a**5*b*n*x**(4*n)) - 6*a**2*b**2*x**(2*n)/(3*a**6*n*x**(3*n) +
3*a**5*b*n*x**(4*n)) - 12*a*b**3*x**(3*n)*log(x**n)/(3*a**6*n*x**(3*n) + 3
*a**5*b*n*x**(4*n)) + 12*a*b**3*x**(3*n)*log(a/b + x**n)/(3*a**6*n*x**(3*n
) + 3*a**5*b*n*x**(4*n)) - 12*a*b**3*x**(3*n)/(3*a**6*n*x**(3*n) + 3*a**5*
b*n*x**(4*n)) - 12*b**4*x**(4*n)*log(x**n)/(3*a**6*n*x**(3*n) + 3*a**5*b*n
*x**(4*n)) + 12*b**4*x**(4*n)*log(a/b + x**n)/(3*a**6*n*x**(3*n) + 3*a**5*
b*n*x**(4*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx = -\frac{12\,b^3x^{3\,n}+6\,ab^2x^{2\,n}-2\,a^2bx^n+a^3}{3\,(a^4bnx^{4\,n}+a^5nx^{3\,n})}-\frac{4\,b^3\log{(x)}}{a^5}+\frac{4\,b^3\log{\left(\frac{bx^n+a}{b}\right)}}{a^5n}$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/3*(12*b^3*x^(3*n) + 6*a*b^2*x^(2*n) - 2*a^2*b*x^n + a^3)/(a^4*b*n*x^(4*
n) + a^5*n*x^(3*n)) - 4*b^3*log(x)/a^5 + 4*b^3*log((b*x^n + a)/b)/(a^5*n)
```

Giac [F]

$$\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx = \int \frac{x^{-3\,n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx = \int \frac{1}{x^{3\,n+1}\,(a+b\,x^n)^2}\,dx$$

input
```
int(1/(x^(3*n + 1)*(a + b*x^n)^2),x)
```

output
```
int(1/(x^(3*n + 1)*(a + b*x^n)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \frac{x^{-1-3n}}{(a+bx^n)^2}\,dx$$
$$= \frac{12x^{4n}\log(x^nb+a)\,b^4 - 12x^{4n}\log(x)\,b^4n + 12x^{4n}b^4 + 12x^{3n}\log(x^nb+a)\,a\,b^3 - 12x^{3n}\log(x)\,a\,b^3n - 6x^{2n}a}{3x^{3n}a^5n\,(x^nb+a)}$$

input
```
int(x^(-1-3*n)/(a+b*x^n)^2,x)
```

output
```
(12*x**(4*n)*log(x**n*b + a)*b**4 - 12*x**(4*n)*log(x)*b**4*n + 12*x**(4*n
)*b**4 + 12*x**(3*n)*log(x**n*b + a)*a*b**3 - 12*x**(3*n)*log(x)*a*b**3*n
- 6*x**(2*n)*a**2*b**2 + 2*x**n*a**3*b - a**4)/(3*x**(3*n)*a**5*n*(x**n*b
+ a))
```

3.488 $\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx=\frac{x^n}{b^3n}+\frac{a^3}{2b^4n(a+bx^n)^2}-\frac{3a^2}{b^4n(a+bx^n)}-\frac{3a\log(a+bx^n)}{b^4n}$$

output
```
x^n/b^3/n+1/2*a^3/b^4/n/(a+b*x^n)^2-3*a^2/b^4/n/(a+b*x^n)-3*a*ln(a+b*x^n)/
b^4/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx=\frac{\frac{-5a^3-4a^2bx^n+4ab^2x^{2n}+2b^3x^{3n}}{(a+bx^n)^2}-6a\log(a+bx^n)}{2b^4n}$$

input
```
Integrate[x^(-1 + 4*n)/(a + b*x^n)^3,x]
```

output
```
((-5*a^3 - 4*a^2*b*x^n + 4*a*b^2*x^(2*n) + 2*b^3*x^(3*n))/(a + b*x^n)^2 -
6*a*Log[a + b*x^n])/(2*b^4*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{(a+bx^n)^3} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{3n}}{(bx^n+a)^3}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\frac{a^3}{b^3(bx^n+a)^3}+\frac{3a^2}{b^3(bx^n+a)^2}-\frac{3a}{b^3(bx^n+a)}+\frac{1}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3}{2b^4(a+bx^n)^2}-\frac{3a^2}{b^4(a+bx^n)}-\frac{3a\log(a+bx^n)}{b^4}+\frac{x^n}{b^3}}{n}$$

input
```
Int[x^(-1 + 4*n)/(a + b*x^n)^3,x]
```

output
```
(x^n/b^3 + a^3/(2*b^4*(a + b*x^n)^2) - (3*a^2)/(b^4*(a + b*x^n)) - (3*a*Lo
g[a + b*x^n])/b^4)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{x^{n}}{b^{3} n}-\frac{a^{2} \left(6 b \,x^{n}+5 a \right)}{2 b^{4} n \left(a +b \,x^{n}\right)^{2}}-\frac{3 a \ln \left(x^{n}+\frac{a}{b}\right)}{b^{4} n}$	61
norman	$\frac{\frac{{\mathrm e}^{3 n \ln \left(x \right)}}{b n}-\frac{9 a^{3}}{2 b^{4} n}-\frac{6 a^{2} {\mathrm e}^{n \ln \left(x \right)}}{b^{3} n}}{\left(a +b \,{\mathrm e}^{n \ln \left(x \right)}\right)^{2}}-\frac{3 a \ln \left(a +b \,{\mathrm e}^{n \ln \left(x \right)}\right)}{b^{4} n}$	75

input
```
int(x^(-1+4*n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
x^n/b^3/n-1/2*a^2*(6*b*x^n+5*a)/b^4/n/(a+b*x^n)^2-3*a/b^4/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx = \frac{2\,b^3x^{3\,n}+4\,ab^2x^{2\,n}-4\,a^2bx^n-5\,a^3-6\,(ab^2x^{2\,n}+2\,a^2bx^n+a^3)\log\left(bx^n+a\right)}{2\,(b^6nx^{2\,n}+2\,ab^5nx^n+a^2b^4n)}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*x^(3*n) + 4*a*b^2*x^(2*n) - 4*a^2*b*x^n - 5*a^3 - 6*(a*b^2*x^(2
*n) + 2*a^2*b*x^n + a^3)*log(b*x^n + a))/(b^6*n*x^(2*n) + 2*a*b^5*n*x^n +
a^2*b^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(60) = 120$.

Time = 19.47 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.41

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx$$
$$= \begin{cases} \frac{\log(x)}{a^3} \\ \frac{xx^{4n-1}}{4a^3n} \\ \frac{\log(x)}{(a+b)^3} \\ -\frac{6a^3\log\left(\frac{a}{b}+x^n\right)}{2a^2b^4n+4ab^5nx^n+2b^6nx^{2n}} - \frac{9a^3}{2a^2b^4n+4ab^5nx^n+2b^6nx^{2n}} - \frac{12a^2bx^n\log\left(\frac{a}{b}+x^n\right)}{2a^2b^4n+4ab^5nx^n+2b^6nx^{2n}} - \frac{12a^2bx^n}{2a^2b^4n+4ab^5nx^n+2b^6nx^{2n}} - \frac{6a}{2a^2b^4} \end{cases}$$

input
```
integrate(x**(-1+4*n)/(a+b*x**n)**3,x)
```

output
```
Piecewise((log(x)/a**3, Eq(b, 0) & Eq(n, 0)), (x*x**(4*n - 1)/(4*a**3*n),
Eq(b, 0)), (log(x)/(a + b)**3, Eq(n, 0)), (-6*a**3*log(a/b + x**n)/(2*a**2
*b**4*n + 4*a*b**5*n*x**n + 2*b**6*n*x**(2*n)) - 9*a**3/(2*a**2*b**4*n + 4
*a*b**5*n*x**n + 2*b**6*n*x**(2*n)) - 12*a**2*b*x**n*log(a/b + x**n)/(2*a*
*2*b**4*n + 4*a*b**5*n*x**n + 2*b**6*n*x**(2*n)) - 12*a**2*b*x**n/(2*a**2*
b**4*n + 4*a*b**5*n*x**n + 2*b**6*n*x**(2*n)) - 6*a*b**2*x**(2*n)*log(a/b
+ x**n)/(2*a**2*b**4*n + 4*a*b**5*n*x**n + 2*b**6*n*x**(2*n)) + 2*b**3*x**
(3*n)/(2*a**2*b**4*n + 4*a*b**5*n*x**n + 2*b**6*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx = \frac{2\,b^3x^{3\,n} + 4\,ab^2x^{2\,n} - 4\,a^2bx^n - 5\,a^3}{2\,(b^6nx^{2\,n} + 2\,ab^5nx^n + a^2b^4n)} - \frac{3\,a\log\left(\frac{bx^n+a}{b}\right)}{b^4n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/2*(2*b^3*x^(3*n) + 4*a*b^2*x^(2*n) - 4*a^2*b*x^n - 5*a^3)/(b^6*n*x^(2*n)
 + 2*a*b^5*n*x^n + a^2*b^4*n) - 3*a*log((b*x^n + a)/b)/(b^4*n)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx = \int \frac{x^{4n-1}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx = \int \frac{x^{4n-1}}{(a+b\,x^n)^3}\,dx$$

input
```
int(x^(4*n - 1)/(a + b*x^n)^3,x)
```

output
```
int(x^(4*n - 1)/(a + b*x^n)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.51

$$\int \frac{x^{-1+4n}}{(a+bx^n)^3}\,dx = \frac{2x^{3n}b^3 - 6x^{2n}\log(x^n b+a)\,a\,b^2 + 6x^{2n}a\,b^2 - 12x^n\log(x^n b+a)\,a^2b - 6\log(x^n b+a)\,a^3 - 3a^3}{2b^4n\,(x^{2n}b^2+2x^n ab+a^2)}$$

input
```
int(x^(-1+4*n)/(a+b*x^n)^3,x)
```

output
```
(2*x**(3*n)*b**3 - 6*x**(2*n)*log(x**n*b + a)*a*b**2 + 6*x**(2*n)*a*b**2 -
 12*x**n*log(x**n*b + a)*a**2*b - 6*log(x**n*b + a)*a**3 - 3*a**3)/(2*b**4
*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.489 $\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx=-\frac{a^2}{2b^3n\,(a+bx^n)^2}+\frac{2a}{b^3n\,(a+bx^n)}+\frac{\log(a+bx^n)}{b^3n}$$

output
```
-1/2*a^2/b^3/n/(a+b*x^n)^2+2*a/b^3/n/(a+b*x^n)+ln(a+b*x^n)/b^3/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx=\frac{\frac{a(3a+4bx^n)}{(a+bx^n)^2}+2\log(b^2n(a+bx^n))}{2b^3n}$$

input
```
Integrate[x^(-1 + 3*n)/(a + b*x^n)^3,x]
```

output
```
((a*(3*a + 4*b*x^n))/(a + b*x^n)^2 + 2*Log[b^2*n*(a + b*x^n)])/(2*b^3*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^3}dx^n}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{a^2}{b^2(bx^n+a)^3}-\frac{2a}{b^2(bx^n+a)^2}+\frac{1}{b^2(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2}{2b^3(a+bx^n)^2}+\frac{2a}{b^3(a+bx^n)}+\frac{\log(a+bx^n)}{b^3}}{n}$$

input
```
Int[x^(-1 + 3*n)/(a + b*x^n)^3,x]
```

output
```
(-1/2*a^2/(b^3*(a + b*x^n)^2) + (2*a)/(b^3*(a + b*x^n)) + Log[a + b*x^n]/b
^3)/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{a(4b\,x^{n}+3a)}{2b^{3}n(a+b\,x^{n})^{2}}+\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{b^{3}n}$	47
norman	$\frac{\frac{3a^{2}}{2b^{3}n}+\frac{2a\,\mathrm{e}^{n\ln(x)}}{b^{2}n}}{\left(a+b\,\mathrm{e}^{n\ln(x)}\right)^{2}}+\frac{\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{b^{3}n}$	57

input
```
int(x^(-1+3*n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*(4*b*x^n+3*a)/b^3/n/(a+b*x^n)^2+1/b^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx=\frac{4\,abx^n+3\,a^2+2\,(b^2x^{2\,n}+2\,abx^n+a^2)\log\left(bx^n+a\right)}{2\,(b^5nx^{2\,n}+2\,ab^4nx^n+a^2b^3n)}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*x^n + 3*a^2 + 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))
/(b^5*n*x^(2*n) + 2*a*b^4*n*x^n + a^2*b^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(46) = 92$.

Time = 17.39 (sec) , antiderivative size = 260, normalized size of antiderivative = 4.64

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx = \begin{cases} \frac{\log(x)}{a^3} \\ \frac{xx^{3n-1}}{3a^3n} \\ \frac{\log(x)}{(a+b)^3} \\ \frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{3a^2}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{4abx^n\log\left(\frac{a}{b}+x^n\right)}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{4abx^n}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{2b^2x}{2a^2b^3n+} \end{cases}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**3,x)
```

output
```
Piecewise((log(x)/a**3, Eq(b, 0) & Eq(n, 0)), (x*x**(3*n - 1)/(3*a**3*n),
Eq(b, 0)), (log(x)/(a + b)**3, Eq(n, 0)), (2*a**2*log(a/b + x**n)/(2*a**2*
b**3*n + 4*a*b**4*n*x**n + 2*b**5*n*x**(2*n)) + 3*a**2/(2*a**2*b**3*n + 4*
a*b**4*n*x**n + 2*b**5*n*x**(2*n)) + 4*a*b*x**n*log(a/b + x**n)/(2*a**2*b*
*3*n + 4*a*b**4*n*x**n + 2*b**5*n*x**(2*n)) + 4*a*b*x**n/(2*a**2*b**3*n +
4*a*b**4*n*x**n + 2*b**5*n*x**(2*n)) + 2*b**2*x**(2*n)*log(a/b + x**n)/(2*
a**2*b**3*n + 4*a*b**4*n*x**n + 2*b**5*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx = \frac{4\,abx^n+3\,a^2}{2\,(b^5nx^{2\,n}+2\,ab^4nx^n+a^2b^3n)} + \frac{\log\left(\frac{bx^n+a}{b}\right)}{b^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/2*(4*a*b*x^n + 3*a^2)/(b^5*n*x^(2*n) + 2*a*b^4*n*x^n + a^2*b^3*n) + log(
(b*x^n + a)/b)/(b^3*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx = \int \frac{x^{3n-1}}{(a+b\,x^n)^3}\,dx$$

input
```
int(x^(3*n - 1)/(a + b*x^n)^3,x)
```

output
```
int(x^(3*n - 1)/(a + b*x^n)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.61

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3}\,dx = \frac{2x^{2n}\log(x^nb+a)\,b^2-2x^{2n}b^2+4x^n\log(x^nb+a)\,ab+2\log(x^nb+a)\,a^2+a^2}{2b^3n\,(x^{2n}b^2+2x^nab+a^2)}$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^3,x)
```

output
```
(2*x**(2*n)*log(x**n*b + a)*b**2 - 2*x**(2*n)*b**2 + 4*x**n*log(x**n*b + a
)*a*b + 2*log(x**n*b + a)*a**2 + a**2)/(2*b**3*n*(x**(2*n)*b**2 + 2*x**n*a
*b + a**2))
```

3.490 $\int \frac{x^{-1+2n}}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3} dx = \frac{x^{2n}}{2an(a+bx^n)^2}$$

output
```
1/2*x^(2*n)/a/n/(a+b*x^n)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3} dx = \frac{-a-2bx^n}{2b^2n(a+bx^n)^2}$$

input
```
Integrate[x^(-1 + 2*n)/(a + b*x^n)^3,x]
```

output
```
(-a - 2*b*x^n)/(2*b^2*n*(a + b*x^n)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \text{ 796}$$

$$\frac{x^{2n}}{2an\,(a+bx^n)^2}$$

input
```
Int[x^(-1 + 2*n)/(a + b*x^n)^3,x]
```

output
```
x^(2*n)/(2*a*n*(a + b*x^n)^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{2b\,x^{n}+a}{2b^{2}n\left(a+b\,x^{n}\right)^{2}}$	26
parallelrisch	$\frac{x^{2n-1}x}{2an\left(a+b\,x^{n}\right)^{2}}$	26
norman	$\frac{-\frac{{\mathrm e}^{n\ln\left(x\right)}}{bn}-\frac{a}{2b^{2}n}}{\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)^{2}}$	36

input
```
int(x^(2*n-1)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*x^n+a)/b^2/n/(a+b*x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx = -\frac{2\,bx^n + a}{2\,(b^4nx^{2\,n} + 2\,ab^3nx^n + a^2b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(17) = 34.

Time = 1.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.67

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{xx^{-3n}x^{2n-1}}{b^3n} & \text{for } a = 0 \\ \frac{\tilde{\infty}xx^{2n-1}}{n} & \text{for } b = -ax^{-n} \\ \frac{\log(x)}{(a+b)^3} & \text{for } n = 0 \\ \frac{xx^{2n-1}}{2a^3n+4a^2bnx^n+2ab^2nx^{2n}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(2*n - 1)/(
b**3*n*x**(3*n)), Eq(a, 0)), (zoo*x*x**(2*n - 1)/n, Eq(b, -a/x**n)), (log(
x)/(a + b)**3, Eq(n, 0)), (x*x**(2*n - 1)/(2*a**3*n + 4*a**2*b*n*x**n + 2*
a*b**2*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx = -\frac{2\,bx^n + a}{2\,(b^4nx^{2\,n} + 2\,ab^3nx^n + a^2b^2n)}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(b*x^n + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx = \frac{x^{2\,n}}{2\,\left(a^3\,n + 2\,a^2\,b\,n\,x^n + a\,b^2\,n\,x^{2\,n}\right)}$$

input
```
int(x^(2*n - 1)/(a + b*x^n)^3,x)
```

output
```
x^(2*n)/(2*(a^3*n + 2*a^2*b*n*x^n + a*b^2*n*x^(2*n)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3}\,dx = \frac{x^{2n}}{2an\,(x^{2n}b^2+2x^nab+a^2)}$$

input

```
int(x^(-1+2*n)/(a+b*x^n)^3,x)
```

output

```
x**(2*n)/(2*a*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.491 $\int \frac{x^{-1+n}}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{x^{-1+n}}{(a+bx^n)^3} dx = -\frac{1}{2bn(a+bx^n)^2}$$

output
```
-1/2/b/n/(a+b*x^n)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^3} dx = -\frac{1}{2bn(a+bx^n)^2}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n)^3,x]
```

output
```
-1/2*1/(b*n*(a + b*x^n)^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a+bx^n)^3}\,dx$$
$$\Big\downarrow \ 793$$
$$-\frac{1}{2bn\,(a+bx^n)^2}$$

input
```
Int[x^(-1 + n)/(a + b*x^n)^3,x]
```

output
```
-1/2*1/(b*n*(a + b*x^n)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{1}{2bn\left(a+b\,x^{n}\right)^{2}}$	18
norman	$-\frac{1}{2bn\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)^{2}}$	20
parallelrisch	$\frac{b\,x^{-1+n}x\,x^{n}+2x^{-1+n}ax}{2a^{2}n\left(a+b\,x^{n}\right)^{2}}$	39

input
```
int(x^(-1+n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/n/(a+b*x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,(b^3nx^{2\,n}+2\,ab^2nx^n+a^2bn)}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/2/(b^3*n*x^(2*n) + 2*a*b^2*n*x^n + a^2*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(15) = 30.

Time = 1.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 7.16

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-3n}x^{n-1}}{2b^3n} & \text{for } a=0 \\ \frac{\tilde{\infty}xx^{n-1}}{n} & \text{for } b=-ax^{-n} \\ \frac{\log(x)}{(a+b)^3} & \text{for } n=0 \\ \frac{2axx^{n-1}}{2a^4n+4a^3bnx^n+2a^2b^2nx^{2n}}+\frac{bxx^nx^{n-1}}{2a^4n+4a^3bnx^n+2a^2b^2nx^{2n}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(n - 1)/(2*
b**3*n*x**(3*n)), Eq(a, 0)), (zoo*x*x**(n - 1)/n, Eq(b, -a/x**n)), (log(x)
/(a + b)**3, Eq(n, 0)), (2*a*x*x**(n - 1)/(2*a**4*n + 4*a**3*b*n*x**n + 2*
a**2*b**2*n*x**(2*n)) + b*x*x**n*x**(n - 1)/(2*a**4*n + 4*a**3*b*n*x**n +
2*a**2*b**2*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,(bx^n+a)^2bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/2/((b*x^n + a)^2*b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,(bx^n+a)^2bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
-1/2/((b*x^n + a)^2*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,b^3\,n\,x^{2\,n}+2\,a^2\,b\,n+4\,a\,b^2\,n\,x^n}$$

input
```
int(x^(n - 1)/(a + b*x^n)^3,x)
```

output
```
-1/(2*b^3*n*x^(2*n) + 2*a^2*b*n + 4*a*b^2*n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2bn\,(x^{2n}b^2+2x^n ab+a^2)}$$

input
```
int(x^(-1+n)/(a+b*x^n)^3,x)
```

output
```
( - 1)/(2*b*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.492 $\int \frac{1}{x(a+bx^n)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{1}{x(a+bx^n)^3} dx = \frac{1}{2an(a+bx^n)^2} + \frac{1}{a^2n(a+bx^n)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx^n)}{a^3n}$$

output
```
1/2/a/n/(a+b*x^n)^2+1/a^2/n/(a+b*x^n)+ln(x)/a^3-ln(a+b*x^n)/a^3/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+bx^n)^3} dx = \frac{\frac{a(3a+2bx^n)}{(a+bx^n)^2} + 2\log(x^n) - 2\log(a+bx^n)}{2a^3n}$$

input
```
Integrate[1/(x*(a + b*x^n)^3),x]
```

output
```
((a*(3*a + 2*b*x^n))/(a + b*x^n)^2 + 2*Log[x^n] - 2*Log[a + b*x^n])/(2*a^3
*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b x^n)^3} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{(b x^n+a)^3}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-n}}{a^3}-\frac{b}{a^3 (b x^n+a)}-\frac{b}{a^2 (b x^n+a)^2}-\frac{b}{a (b x^n+a)^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log(a+b x^n)}{a^3}+\frac{\log(x^n)}{a^3}+\frac{1}{a^2 (a+b x^n)}+\frac{1}{2 a (a+b x^n)^2}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^3),x]
```

output
```
(1/(2*a*(a + b*x^n)^2) + 1/(a^2*(a + b*x^n)) + Log[x^n]/a^3 - Log[a + b*x^
n]/a^3)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\ln(x)}{a^3}+\frac{2b\,x^n+3a}{2a^2n(a+b\,x^n)^2}-\frac{\ln\left(x^n+\frac{a}{b}\right)}{a^3n}$
derivativedivides	$\frac{-\frac{\ln(a+b\,x^n)}{a^3}+\frac{1}{a^2(a+b\,x^n)}+\frac{1}{2a(a+b\,x^n)^2}+\frac{\ln(x^n)}{a^3}}{n}$
default	$\frac{-\frac{\ln(a+b\,x^n)}{a^3}+\frac{1}{a^2(a+b\,x^n)}+\frac{1}{2a(a+b\,x^n)^2}+\frac{\ln(x^n)}{a^3}}{n}$
norman	$\frac{\frac{\ln(x)}{a}+\frac{2b\ln(x)e^{n\ln(x)}}{a^2}-\frac{2b\,e^{n\ln(x)}}{a^2n}+\frac{b^2\ln(x)e^{2n\ln(x)}}{a^3}-\frac{3b^2e^{2n\ln(x)}}{2a^3n}}{\left(a+b\,e^{n\ln(x)}\right)^2}-\frac{\ln\left(a+b\,e^{n\ln(x)}\right)}{a^3n}$
parallelrisch	$\frac{2x^{2n}\ln(x)b^2n+4x^n\ln(x)abn-2\ln(a+b\,x^n)x^{2n}b^2+2a^2\ln(x)n-4\ln(a+b\,x^n)x^nab-3b^2x^{2n}-2\ln(a+b\,x^n)a^2-4ab\,x^n}{2a^3n(a+b\,x^n)^2}$

input
```
int(1/x/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^3+1/2*(2*b*x^n+3*a)/a^2/n/(a+b*x^n)^2-1/a^3/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.83

$$\int\frac{1}{x\,(a+bx^n)^3}\,dx$$
$$=\frac{2\,b^2nx^{2\,n}\log(x)+2\,a^2n\log(x)+3\,a^2+2\,(2\,abn\log(x)+ab)x^n-2\,(b^2x^{2\,n}+2\,abx^n+a^2)\log(bx^n+a)}{2\,(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n)}$$

input
```
integrate(1/x/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
 a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x
^(2*n) + 2*a^4*b*n*x^n + a^5*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(48) = 96.

Time = 1.17 (sec) , antiderivative size = 415, normalized size of antiderivative = 7.16

$$\int \frac{1}{x(a+bx^n)^3}\,dx$$
$$= \begin{cases} \tilde{\infty}\log(x) \\ \frac{\log(x)}{a^3} \\ -\frac{x^{-3n}}{3b^3n} \\ \tilde{\infty}\log(x) \\ \frac{\log(x)}{(a+b)^3} \\ \frac{2a^2n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{3a^2}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} + \frac{4abnx^n\log(x)}{2a^5n+4a^4bnx^n+2a^3b^2nx^{2n}} - \frac{4a}{2a^5n+4} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a**3, Eq(b
, 0)), (-1/(3*b**3*n*x**(3*n)), Eq(a, 0)), (zoo*log(x), Eq(b, -a/x**n)), (
log(x)/(a + b)**3, Eq(n, 0)), (2*a**2*n*log(x)/(2*a**5*n + 4*a**4*b*n*x**n
 + 2*a**3*b**2*n*x**(2*n)) - 2*a**2*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n
*x**n + 2*a**3*b**2*n*x**(2*n)) + 3*a**2/(2*a**5*n + 4*a**4*b*n*x**n + 2*a
**3*b**2*n*x**(2*n)) + 4*a*b*n*x**n*log(x)/(2*a**5*n + 4*a**4*b*n*x**n + 2
*a**3*b**2*n*x**(2*n)) - 4*a*b*x**n*log(a/b + x**n)/(2*a**5*n + 4*a**4*b*n
*x**n + 2*a**3*b**2*n*x**(2*n)) + 2*a*b*x**n/(2*a**5*n + 4*a**4*b*n*x**n +
 2*a**3*b**2*n*x**(2*n)) + 2*b**2*n*x**(2*n)*log(x)/(2*a**5*n + 4*a**4*b*n
*x**n + 2*a**3*b**2*n*x**(2*n)) - 2*b**2*x**(2*n)*log(a/b + x**n)/(2*a**5*
n + 4*a**4*b*n*x**n + 2*a**3*b**2*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a+bx^n)^3}\,dx = \frac{2\,bx^n+3\,a}{2\,(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n)} - \frac{\log(bx^n+a)}{a^3n} + \frac{\log(x^n)}{a^3n}$$

input
```
integrate(1/x/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) - log(b*x^
n + a)/(a^3*n) + log(x^n)/(a^3*n)
```

Giac [F]

$$\int \frac{1}{x(a+bx^n)^3}\,dx = \int \frac{1}{(bx^n+a)^3x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^3*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{x(a+bx^n)^3}\,dx = \frac{\ln(x)}{a^3} + \frac{1}{a^2\,n\,(a+b\,x^n)} - \frac{\ln(a+b\,x^n)}{a^3\,n} + \frac{1}{2\,a\,n\,(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n)}$$

input
```
int(1/(x*(a + b*x^n)^3),x)
```

output
```
log(x)/a^3 + 1/(a^2*n*(a + b*x^n)) - log(a + b*x^n)/(a^3*n) + 1/(2*a*n*(a^
2 + b^2*x^(2*n) + 2*a*b*x^n))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\,(a+bx^n)^3}\,dx$$
$$= \frac{-2x^{2n}\log(x^n b+a)\,b^2+2x^{2n}\log(x)\,b^2n-x^{2n}b^2-4x^n\log(x^n b+a)\,ab+4x^n\log(x)\,abn-2\log(x^n b+a)\,a}{2a^3n\,(x^{2n}b^2+2x^nab+a^2)}$$

input
```
int(1/x/(a+b*x^n)^3,x)
```

output
```
( - 2*x**(2*n)*log(x**n*b + a)*b**2 + 2*x**(2*n)*log(x)*b**2*n - x**(2*n)*
b**2 - 4*x**n*log(x**n*b + a)*a*b + 4*x**n*log(x)*a*b*n - 2*log(x**n*b + a
)*a**2 + 2*log(x)*a**2*n + 2*a**2)/(2*a**3*n*(x**(2*n)*b**2 + 2*x**n*a*b +
 a**2))
```

3.493 $\int \frac{x^{-1-n}}{(a+bx^n)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{x^{-1-n}}{(a+bx^n)^3} \, dx = -\frac{x^{-n}}{a^3n} - \frac{b}{2a^2n(a+bx^n)^2} - \frac{2b}{a^3n(a+bx^n)} - \frac{3b\log(x)}{a^4} + \frac{3b\log(a+bx^n)}{a^4n}$$

output
```
-1/a^3/n/(x^n)-1/2*b/a^2/n/(a+b*x^n)^2-2*b/a^3/n/(a+b*x^n)-3*b*ln(x)/a^4+3
*b*ln(a+b*x^n)/a^4/n
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1-n}}{(a+bx^n)^3} \, dx = -\frac{\frac{ax^{-n}(2a^2+9abx^n+6b^2x^{2n})}{(a+bx^n)^2} + 6b\log(x^n) - 6b\log(a+bx^n)}{2a^4n}$$

input
```
Integrate[x^(-1 - n)/(a + b*x^n)^3,x]
```

output
```
-1/2*((a*(2*a^2 + 9*a*b*x^n + 6*b^2*x^(2*n)))/(x^n*(a + b*x^n)^2) + 6*b*Lo
g[x^n] - 6*b*Log[a + b*x^n])/(a^4*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-2n}}{(bx^n+a)^3}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-2n}}{a^3}-\frac{3bx^{-n}}{a^4}+\frac{3b^2}{a^4(bx^n+a)}+\frac{2b^2}{a^3(bx^n+a)^2}+\frac{b^2}{a^2(bx^n+a)^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3b\log(x^n)}{a^4}+\frac{3b\log(a+bx^n)}{a^4}-\frac{2b}{a^3(a+bx^n)}-\frac{x^{-n}}{a^3}-\frac{b}{2a^2(a+bx^n)^2}}{n}$$

input
```
Int[x^(-1 - n)/(a + b*x^n)^3,x]
```

output
```
(-(1/(a^3*x^n)) - b/(2*a^2*(a + b*x^n)^2) - (2*b)/(a^3*(a + b*x^n)) - (3*b
*Log[x^n])/a^4 + (3*b*Log[a + b*x^n])/a^4)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{x^{-n}}{a^3 n}-\frac{3b\ln(x)}{a^4}-\frac{b(4b\,x^n+5a)}{2a^3n(a+b\,x^n)^2}+\frac{3b\ln\left(x^n+\frac{a}{b}\right)}{a^4 n}$	70
norman	$\frac{\left(-\frac{1}{an}-\frac{3b\ln(x)e^{n\ln(x)}}{a^2}-\frac{6b^2\ln(x)e^{2n\ln(x)}}{a^3}+\frac{6b^2e^{2n\ln(x)}}{a^3n}-\frac{3b^3\ln(x)e^{3n\ln(x)}}{a^4}+\frac{9b^3e^{3n\ln(x)}}{2a^4n}\right)e^{-n\ln(x)}}{\left(a+b\,e^{n\ln(x)}\right)^2}+\frac{3b\ln\left(a+b\,e^{n\ln(x)}\right)}{a^4 n}$	132

input
```
int(x^(-1-n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3/n/(x^n)-3*b*ln(x)/a^4-1/2*b*(4*b*x^n+5*a)/a^3/n/(a+b*x^n)^2+3*b/a^4
/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.81

$$\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx =$$
$$-\frac{6\,b^3nx^{3\,n}\log(x)+2\,a^3+6\,(2\,ab^2n\log(x)+ab^2)x^{2\,n}+3\,(2\,a^2bn\log(x)+3\,a^2b)x^n-6\,(b^3x^{3\,n}+2\,ab^2x}{2\,(a^4b^2nx^{3\,n}+2\,a^5bnx^{2\,n}+a^6nx^n)}$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/2*(6*b^3*n*x^(3*n)*log(x) + 2*a^3 + 6*(2*a*b^2*n*log(x) + a*b^2)*x^(2*n
) + 3*(2*a^2*b*n*log(x) + 3*a^2*b)*x^n - 6*(b^3*x^(3*n) + 2*a*b^2*x^(2*n)
+ a^2*b*x^n)*log(b*x^n + a))/(a^4*b^2*n*x^(3*n) + 2*a^5*b*n*x^(2*n) + a^6*
n*x^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(68) = 136$.

Time = 26.12 (sec) , antiderivative size = 547, normalized size of antiderivative = 7.10

$$\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx = \begin{cases} \tilde{\infty}\log(x) \\ -\frac{xx^{-n-1}}{a^3n} \\ -\frac{xx^{-3n}x^{-n-1}}{4b^3n} \\ \frac{\tilde{\infty}xx^{-n-1}}{n} \\ \frac{\log(x)}{(a+b)^3} \\ -\frac{2a^3}{2a^6nx^n+4a^5bnx^{2n}+2a^4b^2nx^{3n}} - \frac{6a^2bx^n\log(x^n)}{2a^6nx^n+4a^5bnx^{2n}+2a^4b^2nx^{3n}} + \frac{6a^2bx^n\log\left(\frac{a}{b}+x^n\right)}{2a^6nx^n+4a^5bnx^{2n}+2a^4b^2nx^{3n}} - \frac{9a^2bx^n}{2a^6nx^n+4a^5bnx^{2n}+2a^4b^2} \end{cases}$$

input
```
integrate(x**(-1-n)/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(a
**3*n), Eq(b, 0)), (-x*x**(-n - 1)/(4*b**3*n*x**(3*n)), Eq(a, 0)), (zoo*x*
x**(-n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)**3, Eq(n, 0)), (-2*a**3/(2
*a**6*n*x**n + 4*a**5*b*n*x**(2*n) + 2*a**4*b**2*n*x**(3*n)) - 6*a**2*b*x*
*n*log(x**n)/(2*a**6*n*x**n + 4*a**5*b*n*x**(2*n) + 2*a**4*b**2*n*x**(3*n)
) + 6*a**2*b*x**n*log(a/b + x**n)/(2*a**6*n*x**n + 4*a**5*b*n*x**(2*n) + 2
*a**4*b**2*n*x**(3*n)) - 9*a**2*b*x**n/(2*a**6*n*x**n + 4*a**5*b*n*x**(2*n
) + 2*a**4*b**2*n*x**(3*n)) - 12*a*b**2*x**(2*n)*log(x**n)/(2*a**6*n*x**n
+ 4*a**5*b*n*x**(2*n) + 2*a**4*b**2*n*x**(3*n)) + 12*a*b**2*x**(2*n)*log(a
/b + x**n)/(2*a**6*n*x**n + 4*a**5*b*n*x**(2*n) + 2*a**4*b**2*n*x**(3*n))
- 6*a*b**2*x**(2*n)/(2*a**6*n*x**n + 4*a**5*b*n*x**(2*n) + 2*a**4*b**2*n*x
**(3*n)) - 6*b**3*x**(3*n)*log(x**n)/(2*a**6*n*x**n + 4*a**5*b*n*x**(2*n)
+ 2*a**4*b**2*n*x**(3*n)) + 6*b**3*x**(3*n)*log(a/b + x**n)/(2*a**6*n*x**n
 + 4*a**5*b*n*x**(2*n) + 2*a**4*b**2*n*x**(3*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx = -\frac{6\,b^2x^{2\,n}+9\,abx^n+2\,a^2}{2\,(a^3b^2nx^{3\,n}+2\,a^4bnx^{2\,n}+a^5nx^n)}-\frac{3\,b\log{(x)}}{a^4}+\frac{3\,b\log{\left(\frac{bx^n+a}{b}\right)}}{a^4n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/2*(6*b^2*x^(2*n) + 9*a*b*x^n + 2*a^2)/(a^3*b^2*n*x^(3*n) + 2*a^4*b*n*x^
(2*n) + a^5*n*x^n) - 3*b*log(x)/a^4 + 3*b*log((b*x^n + a)/b)/(a^4*n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx = \int \frac{x^{-n-1}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx=\int \frac{1}{x^{n+1}\,(a+b\,x^n)^3}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n)^3),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{x^{-1-n}}{(a+bx^n)^3}\,dx = \frac{6x^{3n}\log(x^nb+a)\,b^3-6x^{3n}\log(x)\,b^3n+3x^{3n}b^3+12x^{2n}\log(x^nb+a)\,a\,b^2-12x^{2n}\log(x)\,a\,b^2n+6x^n\log(x}{2x^na^4n\,(x^{2n}b^2+2x^nab+a^2)}$$

input
```
int(x^(-1-n)/(a+b*x^n)^3,x)
```

output
```
(6*x**(3*n)*log(x**n*b + a)*b**3 - 6*x**(3*n)*log(x)*b**3*n + 3*x**(3*n)*b
**3 + 12*x**(2*n)*log(x**n*b + a)*a*b**2 - 12*x**(2*n)*log(x)*a*b**2*n + 6
*x**n*log(x**n*b + a)*a**2*b - 6*x**n*log(x)*a**2*b*n - 6*x**n*a**2*b - 2*
a**3)/(2*x**n*a**4*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.494 $\int \frac{x^{-1-2n}}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3} dx = -\frac{x^{-2n}}{2a^3n} + \frac{3bx^{-n}}{a^4n} + \frac{b^2}{2a^3n(a+bx^n)^2} + \frac{3b^2}{a^4n(a+bx^n)} + \frac{6b^2\log(x)}{a^5} - \frac{6b^2\log(a+bx^n)}{a^5n}$$

output
```
-1/2/a^3/n/(x^(2*n))+3*b/a^4/n/(x^n)+1/2*b^2/a^3/n/(a+b*x^n)^2+3*b^2/a^4/n
/(a+b*x^n)+6*b^2*ln(x)/a^5-6*b^2*ln(a+b*x^n)/a^5/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3} dx = \frac{\frac{ax^{-2n}(-a^3+4a^2bx^n+18ab^2x^{2n}+12b^3x^{3n})}{(a+bx^n)^2} + 12b^2\log(x^n) - 12b^2\log(a+bx^n)}{2a^5n}$$

input
```
Integrate[x^(-1 - 2*n)/(a + b*x^n)^3,x]
```

output
```
((a*(-a^3 + 4*a^2*b*x^n + 18*a*b^2*x^(2*n) + 12*b^3*x^(3*n)))/(x^(2*n)*(a
+ b*x^n)^2) + 12*b^2*Log[x^n] - 12*b^2*Log[a + b*x^n])/(2*a^5*n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{(a+bx^n)^3}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-3n}}{(bx^n+a)^3}dx^n}{n}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{x^{-3n}}{a^3}-\frac{3bx^{-2n}}{a^4}+\frac{6b^2x^{-n}}{a^5}-\frac{6b^3}{a^5(bx^n+a)}-\frac{3b^3}{a^4(bx^n+a)^2}-\frac{b^3}{a^3(bx^n+a)^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{6b^2\log(x^n)}{a^5}-\frac{6b^2\log(a+bx^n)}{a^5}+\frac{3b^2}{a^4(a+bx^n)}+\frac{3bx^{-n}}{a^4}+\frac{b^2}{2a^3(a+bx^n)^2}-\frac{x^{-2n}}{2a^3}}{n}$$

input
```
Int[x^(-1 - 2*n)/(a + b*x^n)^3,x]
```

output
```
(-1/2*1/(a^3*x^(2*n)) + (3*b)/(a^4*x^n) + b^2/(2*a^3*(a + b*x^n)^2) + (3*b
^2)/(a^4*(a + b*x^n)) + (6*b^2*Log[x^n])/a^5 - (6*b^2*Log[a + b*x^n])/a^5)
/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result
risch	$\frac{3b\,x^{-n}}{a^{4}n}-\frac{x^{-2n}}{2a^{3}n}+\frac{6b^{2}\ln\left(x\right)}{a^{5}}+\frac{b^{2}\left(6b\,x^{n}+7a\right)}{2a^{4}n\left(a+b\,x^{n}\right)^{2}}-\frac{6b^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{a^{5}n}$
norman	$\frac{\left(\frac{9b^{2}{\mathrm e}^{2n\ln\left(x\right)}}{a^{3}n}-\frac{1}{2an}+\frac{6b^{2}\ln\left(x\right){\mathrm e}^{2n\ln\left(x\right)}}{a^{3}}+\frac{2b\,{\mathrm e}^{n\ln\left(x\right)}}{a^{2}n}+\frac{12b^{3}\ln\left(x\right){\mathrm e}^{3n\ln\left(x\right)}}{a^{4}}+\frac{6b^{4}\ln\left(x\right){\mathrm e}^{4n\ln\left(x\right)}}{a^{5}}+\frac{6b^{3}{\mathrm e}^{3n\ln\left(x\right)}}{a^{4}n}\right){\mathrm e}^{-2n\ln\left(x\right)}}{\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)^{2}}-\frac{6b^{2}\ln\left(a+b\right.}{a^{5}n}$

input
```
int(x^(-2*n-1)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
3*b/a^4/n/(x^n)-1/2/a^3/n/(x^n)^2+6*b^2*ln(x)/a^5+1/2*b^2*(6*b*x^n+7*a)/a^
4/n/(a+b*x^n)^2-6*b^2/a^5/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.58

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx$$
$$= \frac{12\,b^4nx^{4\,n}\log(x) + 4\,a^3bx^n - a^4 + 12\,(2\,ab^3n\log(x) + ab^3)x^{3\,n} + 6\,(2\,a^2b^2n\log(x) + 3\,a^2b^2)x^{2\,n} - 12\,(b^4}{2\,(a^5b^2nx^{4\,n} + 2\,a^6bnx^{3\,n} + a^7nx^{2\,n})}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(12*b^4*n*x^(4*n)*log(x) + 4*a^3*b*x^n - a^4 + 12*(2*a*b^3*n*log(x) +
a*b^3)*x^(3*n) + 6*(2*a^2*b^2*n*log(x) + 3*a^2*b^2)*x^(2*n) - 12*(b^4*x^(4
*n) + 2*a*b^3*x^(3*n) + a^2*b^2*x^(2*n))*log(b*x^n + a))/(a^5*b^2*n*x^(4*n
) + 2*a^6*b*n*x^(3*n) + a^7*n*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(90) = 180.

Time = 48.62 (sec) , antiderivative size = 629, normalized size of antiderivative = 6.23

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx$$
$$= \begin{cases} \tilde{\infty}\log(x) \\ -\frac{xx^{-2n-1}}{2a^3n} \\ -\frac{xx^{-3n}x^{-2n-1}}{5b^3n} \\ \frac{\tilde{\infty}xx^{-2n-1}}{n} \\ \frac{\log(x)}{(a+b)^3} \\ -\frac{a^4}{2a^7nx^{2n}+4a^6bnx^{3n}+2a^5b^2nx^{4n}} + \frac{4a^3bx^n}{2a^7nx^{2n}+4a^6bnx^{3n}+2a^5b^2nx^{4n}} + \frac{12a^2b^2x^{2n}\log(x^n)}{2a^7nx^{2n}+4a^6bnx^{3n}+2a^5b^2nx^{4n}} - \frac{12a^2b^2x^{2n}\log(\frac{a}{b}+}{2a^7nx^{2n}+4a^6bnx^{3n}+2a}\end{cases}$$

input
```
integrate(x**(-1-2*n)/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-2*n - 1)/
(2*a**3*n), Eq(b, 0)), (-x*x**(-2*n - 1)/(5*b**3*n*x**(3*n)), Eq(a, 0)), (
zoo*x*x**(-2*n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)**3, Eq(n, 0)), (-a
**4/(2*a**7*n*x**(2*n) + 4*a**6*b*n*x**(3*n) + 2*a**5*b**2*n*x**(4*n)) + 4
*a**3*b*x**n/(2*a**7*n*x**(2*n) + 4*a**6*b*n*x**(3*n) + 2*a**5*b**2*n*x**(
4*n)) + 12*a**2*b**2*x**(2*n)*log(x**n)/(2*a**7*n*x**(2*n) + 4*a**6*b*n*x*
*(3*n) + 2*a**5*b**2*n*x**(4*n)) - 12*a**2*b**2*x**(2*n)*log(a/b + x**n)/(
2*a**7*n*x**(2*n) + 4*a**6*b*n*x**(3*n) + 2*a**5*b**2*n*x**(4*n)) + 18*a**
2*b**2*x**(2*n)/(2*a**7*n*x**(2*n) + 4*a**6*b*n*x**(3*n) + 2*a**5*b**2*n*x
**(4*n)) + 24*a*b**3*x**(3*n)*log(x**n)/(2*a**7*n*x**(2*n) + 4*a**6*b*n*x*
*(3*n) + 2*a**5*b**2*n*x**(4*n)) - 24*a*b**3*x**(3*n)*log(a/b + x**n)/(2*a
**7*n*x**(2*n) + 4*a**6*b*n*x**(3*n) + 2*a**5*b**2*n*x**(4*n)) + 12*a*b**3
*x**(3*n)/(2*a**7*n*x**(2*n) + 4*a**6*b*n*x**(3*n) + 2*a**5*b**2*n*x**(4*n
)) + 12*b**4*x**(4*n)*log(x**n)/(2*a**7*n*x**(2*n) + 4*a**6*b*n*x**(3*n) +
 2*a**5*b**2*n*x**(4*n)) - 12*b**4*x**(4*n)*log(a/b + x**n)/(2*a**7*n*x**(
2*n) + 4*a**6*b*n*x**(3*n) + 2*a**5*b**2*n*x**(4*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx = \frac{12\,b^3x^{3\,n}+18\,ab^2x^{2\,n}+4\,a^2bx^n-a^3}{2\,(a^4b^2nx^{4\,n}+2\,a^5bnx^{3\,n}+a^6nx^{2\,n})}+\frac{6\,b^2\log{(x)}}{a^5}-\frac{6\,b^2\log{\left(\frac{bx^n+a}{b}\right)}}{a^5n}$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/2*(12*b^3*x^(3*n) + 18*a*b^2*x^(2*n) + 4*a^2*b*x^n - a^3)/(a^4*b^2*n*x^(
4*n) + 2*a^5*b*n*x^(3*n) + a^6*n*x^(2*n)) + 6*b^2*log(x)/a^5 - 6*b^2*log((
b*x^n + a)/b)/(a^5*n)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx = \int \frac{x^{-2\,n-1}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx = \int \frac{1}{x^{2\,n+1}\,(a+b\,x^n)^3}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a + b*x^n)^3),x)
```

output
```
int(1/(x^(2*n + 1)*(a + b*x^n)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1-2n}}{(a+bx^n)^3}\,dx$$
$$= \frac{-12x^{4n}\log(x^nb+a)\,b^4+12x^{4n}\log(x)\,b^4n-6x^{4n}b^4-24x^{3n}\log(x^nb+a)\,a\,b^3+24x^{3n}\log(x)\,a\,b^3n-12x^{2}}{2x^{2n}a^5n\,(x^{2n}b^2+2x^nab+a^2)}$$

input
```
int(x^(-1-2*n)/(a+b*x^n)^3,x)
```

output
```
( - 12*x**(4*n)*log(x**n*b + a)*b**4 + 12*x**(4*n)*log(x)*b**4*n - 6*x**(4
*n)*b**4 - 24*x**(3*n)*log(x**n*b + a)*a*b**3 + 24*x**(3*n)*log(x)*a*b**3*
n - 12*x**(2*n)*log(x**n*b + a)*a**2*b**2 + 12*x**(2*n)*log(x)*a**2*b**2*n
 + 12*x**(2*n)*a**2*b**2 + 4*x**n*a**3*b - a**4)/(2*x**(2*n)*a**5*n*(x**(2
*n)*b**2 + 2*x**n*a*b + a**2))
```

3.495 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = -\frac{2x^{-n/2}}{an}+\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}$$

output
```
-2/a/n/(x^(1/2*n))+2*b^(1/2)*arctan(a^(1/2)/b^(1/2)/(x^(1/2*n)))/a^(3/2)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = -\frac{2x^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[x^(-1 - n/2)/(a + b*x^n),x]
```

output
```
(-2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^n)/a)])/(a*n*x^(n/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{2}-1}}{a+bx^n}dx$$

$$\Big\downarrow \ 868$$

$$-\frac{2\int \frac{1}{bx^n+a}dx^{-n/2}}{n}$$

$$\Big\downarrow \ 772$$

$$-\frac{2\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/2}}{n}$$

$$\Big\downarrow \ 262$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/2}}{a}\right)}{n}$$

$$\Big\downarrow \ 218$$

$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{n}$$

input
```
Int[x^(-1 - n/2)/(a + b*x^n),x]
```

output
```
(-2*(1/(a*x^(n/2)) - (Sqrt[b]*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2))])/a^(3/2)))
/n
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
risch	$-\frac{2x^{-\frac{n}{2}}}{an}+\frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-ab}}{b}\right)}{a^{2}n}-\frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-ab}}{b}\right)}{a^{2}n}$	79

input
```
int(x^(-1-1/2*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-2/a/n/(x^(1/2*n))+1/a^2*(-a*b)^(1/2)/n*ln(x^(1/2*n)-1/b*(-a*b)^(1/2))-1/a
^2*(-a*b)^(1/2)/n*ln(x^(1/2*n)+1/b*(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.62

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \left[-\frac{2\,xx^{-\frac{1}{2}\,n-1}-\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2x^{-n-2}+2\,axx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{b}{a}}-b}{ax^2x^{-n-2}+b}\right)}{an},\ -\frac{2\left(xx^{-\frac{1}{2}\,n-1}-\sqrt{\frac{b}{a}}\arctan\left(\frac{axx^{-\frac{1}{2}\,n-1}\sqrt{\frac{b}{a}}}{b}\right)\right)}{an}\right]$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
[-(2*x*x^(-1/2*n - 1) - sqrt(-b/a)*log((a*x^2*x^(-n - 2) + 2*a*x*x^(-1/2*n
 - 1)*sqrt(-b/a) - b)/(a*x^2*x^(-n - 2) + b)))/(a*n), -2*(x*x^(-1/2*n - 1)
 - sqrt(b/a)*arctan(a*x*x^(-1/2*n - 1)*sqrt(b/a)/b))/(a*n)]
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = -\frac{2x^{-\frac{n}{2}}}{an}-\frac{2\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

input
```
integrate(x**(-1-1/2*n)/(a+b*x**n),x)
```

output
```
-2/(a*n*x**(n/2)) - 2*sqrt(b)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(a**(3/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(1/2*n)/(a*b*x*x^n + a^2*x), x) - 2/(a*n*x^(1/2*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-1/2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{n}{2}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(n/2 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(n/2 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n} dx = \int \frac{1}{x^{\frac{3n}{2}}bx + x^{\frac{n}{2}}ax} dx$$

input
```
int(x^(-1-1/2*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((3*n)/2)*b*x + x**(n/2)*a*x),x)
```

3.496 $\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n} dx = -\frac{3x^{-2n/3}}{2an} + \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}n} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{a^{5/3}n} + \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{5/3}n}$$

output
```
-3/2/a/n/(x^(2/3*n))+3^(1/2)*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3*
n))*3^(1/2)/a^(1/3))/a^(5/3)/n-b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3*n))/a^(5/
3)/n+1/2*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x^(1/3*n)+b^(2/3)*x^(2/3*n))/a
^(5/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n} dx = -\frac{3x^{-2n/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{1}{3},-\frac{bx^n}{a}\right)}{2an}$$

input
```
Integrate[x^(-1 - (2*n)/3)/(a + b*x^n),x]
```

output
```
(-3*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^n)/a)])/(2*a*n*x^((2*n)/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {886, 868, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{2n}{3}-1}}{a+bx^n}dx$$

$$\Big\downarrow \ 886$$

$$-\frac{b\int \frac{x^{\frac{n-3}{3}}}{bx^n+a}dx}{a}-\frac{3x^{-2n/3}}{2an}$$

$$\Big\downarrow \ 868$$

$$-\frac{3b\int \frac{1}{bx^n+a}dx^{n/3}}{an}-\frac{3x^{-2n/3}}{2an}$$

$$\Big\downarrow \ 750$$

$$-\frac{3b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x^{n/3}+\sqrt[3]{a}}dx^{n/3}}{3a^{2/3}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

$$\Big\downarrow \ 16$$

$$-\frac{3b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

$$\Big\downarrow \ 1142$$

$$-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

$\downarrow$ 25

$$-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

$\downarrow$ 27

$$-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

$\downarrow$ 1082

$$-\frac{3b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{3\int\frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

$\downarrow$ 217

$$-\frac{3b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

$\Big\downarrow$ 1103

$$-\frac{3b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}$$

input
```
Int[x^(-1 - (2*n)/3)/(a + b*x^n),x]
```

output
```
-3/(2*a*n*x^((2*n)/3)) - (3*b*(Log[a^(1/3) + b^(1/3)*x^(n/3)]/(3*a^(2/3)*b
^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(n/3))/a^(1/3))/Sqrt[3]])/b
^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(n/3) + b^(2/3)*x^((2*n)/3)]/(2*
b^(1/3)))/(3*a^(2/3))))/(a*n)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.75 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{3x^{-\frac{2n}{3}}}{2an}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{5}n^{3}\_Z^{3}+b^{2}\right)}\_R\ln\left(x^{\frac{n}{3}}-\frac{a^{2}n\_R}{b}\right)\right)$	54

input
```
int(x^(-1-2/3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-3/2/a/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-a^2*n/b*_R),_R=RootOf(_Z^3*a^5*
n^3+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = -\frac{3\,xx^{-\frac{2}{3}\,n-1}-2\,\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}a\sqrt{x}x^{-\frac{1}{3}\,n-\frac{1}{2}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+\sqrt{3}b}{3\,b}\right)+\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(-\frac{a\sqrt{x}x^{-\frac{1}{3}\,n-\frac{1}{2}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-b}{x}\right.}{2\,an}$$

input
```
integrate(x^(-1-2/3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/2*(3*x*x^(-2/3*n - 1) - 2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3
)*a*sqrt(x)*x^(-1/3*n - 1/2)*(-b^2/a^2)^(1/3) + sqrt(3)*b)/b) + (-b^2/a^2)
^(1/3)*log(-(a*sqrt(x)*x^(-1/3*n - 1/2)*(-b^2/a^2)^(2/3) - b*x*x^(-2/3*n -
 1) + b*(-b^2/a^2)^(1/3))/x) - 2*(-b^2/a^2)^(1/3)*log((b*x*x^(-1/3*n - 1/2
) + a*sqrt(x)*(-b^2/a^2)^(2/3))/x))/(a*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.20

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \frac{x^{-\frac{2n}{3}}\Gamma\left(-\frac{2}{3}\right)}{an\Gamma\left(\frac{1}{3}\right)} - \frac{2b^{\frac{2}{3}}e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)} + \frac{2b^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)} - \frac{2b^{\frac{2}{3}}e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(x**(-1-2/3*n)/(a+b*x**n),x)
```

output
```
gamma(-2/3)/(a*n*x**(2*n/3)*gamma(1/3)) - 2*b**(2/3)*exp(-I*pi/3)*log(1 -
b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*n*ga
mma(1/3)) + 2*b**(2/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))
*gamma(-2/3)/(3*a**(5/3)*n*gamma(1/3)) - 2*b**(2/3)*exp(I*pi/3)*log(1 - b*
*(1/3)*x**(n/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*n*ga
mma(1/3))
```

Maxima [F]

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{2}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-2/3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(1/3*n)/(a*b*x*x^n + a^2*x), x) - 3/2/(a*n*x^(2/3*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx=\int \frac{x^{-\frac{2}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-2/3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-2/3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx=\int \frac{1}{x^{\frac{2\,n}{3}+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/(x^((2*n)/3 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^((2*n)/3 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{2n}{3}}}{a+bx^n}\,dx=\int \frac{1}{x^{\frac{5n}{3}}bx+x^{\frac{2n}{3}}ax}dx$$

input
```
int(x^(-1-2/3*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((5*n)/3)*b*x + x**((2*n)/3)*a*x),x)
```

3.497 $\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 175

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = -\frac{4x^{-3n/4}}{3an}+\frac{\sqrt{2}b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{a^{7/4}n}-\frac{\sqrt{2}b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{a^{7/4}n}-\frac{\sqrt{2}b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{n/4}}{\sqrt{a}+\sqrt{b}x^{n/2}}\right)}{a^{7/4}n}$$

output
```
-4/3/a/n/(x^(3/4*n))-2^(1/2)*b^(3/4)*arctan(-1+2^(1/2)*b^(1/4)*x^(1/4*n)/a
^(1/4))/a^(7/4)/n-2^(1/2)*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/4*n)/a^(1/
4))/a^(7/4)/n-2^(1/2)*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/4*n)/(a
^(1/2)+b^(1/2)*x^(1/2*n)))/a^(7/4)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = -\frac{4x^{-3n/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{bx^n}{a}\right)}{3an}$$

input
```
Integrate[x^(-1 - (3*n)/4)/(a + b*x^n),x]
```

output
```
(-4*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^n)/a)])/(3*a*n*x^((3*n)/4))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {886, 868, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{3n}{4}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{b\int \frac{x^{\frac{n-4}{4}}}{bx^n+a}dx}{a}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \text{868}$$

$$-\frac{4b\int \frac{1}{bx^n+a}dx^{n/4}}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \text{755}$$

$$-\frac{4b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^{n/2}+\sqrt{a}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{4b\left(\frac{\frac{\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{4b\left(\frac{\frac{\int \frac{1}{-x^{n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x^{n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \text{217}$$

$$-\frac{4b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{4b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x^{n/4}}{\sqrt[4]{b}\left(-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{n/4}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x^{n/4}}{\sqrt[4]{b}\left(-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{n/4}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \; \color{blue}{27}$$

$$-\frac{4b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x^{n/4}}{-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x^{n/4}+\sqrt[4]{a}}{\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

$$\Big\downarrow \; \color{blue}{1103}$$

$$-\frac{4b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{n/4}+\sqrt{a}+\sqrt{b}x^{n/2}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{n/4}+\sqrt{a}+\sqrt{b}x^{n/2}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}$$

input
```
Int[x^(-1 - (3*n)/4)/(a + b*x^n),x]
```

output
```
-4/(3*a*n*x^((3*n)/4)) - (4*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x^(n/4))/a^(
1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x^(n/4))/a^
(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*x^(n/4) + Sqrt[b]*x^(n/2)]/(Sqrt[2]*a^(1/4)*b^(1/4)) + L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x^(n/4) + Sqrt[b]*x^(n/2)]/(2*Sqrt[2]
*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(a*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.31

method	result	size
risch	$-\frac{4x^{-\frac{3n}{4}}}{3an}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}n^{4}\_Z^{4}+b^{3}\right)}\_R\ln\left(x^{\frac{n}{4}}-\frac{a^{2}n\_R}{b}\right)\right)$	54

input
```
int(x^(-1-3/4*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-4/3/a/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-a^2*n/b*_R),_R=RootOf(_Z^4*a^7*
n^4+b^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx$$
$$= \frac{3\,an\left(-\frac{b^3}{a^7n^4}\right)^{\frac{1}{4}}\log\left(\frac{a^5n^3x^{\frac{2}{3}}\left(-\frac{b^3}{a^7n^4}\right)^{\frac{3}{4}}+b^2xx^{-\frac{1}{4}\,n-\frac{1}{3}}}{x}\right)-3\,an\left(-\frac{b^3}{a^7n^4}\right)^{\frac{1}{4}}\log\left(-\frac{a^5n^3x^{\frac{2}{3}}\left(-\frac{b^3}{a^7n^4}\right)^{\frac{3}{4}}-b^2xx^{-\frac{1}{4}\,n-\frac{1}{3}}}{x}\right)-}{}$$

input
```
integrate(x^(-1-3/4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/3*(3*a*n*(-b^3/(a^7*n^4))^(1/4)*log((a^5*n^3*x^(2/3)*(-b^3/(a^7*n^4))^(3
/4) + b^2*x*x^(-1/4*n - 1/3))/x) - 3*a*n*(-b^3/(a^7*n^4))^(1/4)*log(-(a^5*
n^3*x^(2/3)*(-b^3/(a^7*n^4))^(3/4) - b^2*x*x^(-1/4*n - 1/3))/x) - 3*I*a*n*
(-b^3/(a^7*n^4))^(1/4)*log((I*a^5*n^3*x^(2/3)*(-b^3/(a^7*n^4))^(3/4) + b^2
*x*x^(-1/4*n - 1/3))/x) + 3*I*a*n*(-b^3/(a^7*n^4))^(1/4)*log((-I*a^5*n^3*x
^(2/3)*(-b^3/(a^7*n^4))^(3/4) + b^2*x*x^(-1/4*n - 1/3))/x) - 4*x*x^(-3/4*n
 - 1))/(a*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.53

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \frac{x^{-\frac{3n}{4}}\Gamma\left(-\frac{3}{4}\right)}{an\Gamma\left(\frac{1}{4}\right)} - \frac{3b^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} + \frac{3ib^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} + \frac{3b^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{5i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} - \frac{3ib^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{7i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(x**(-1-3/4*n)/(a+b*x**n),x)
```

output
```
gamma(-3/4)/(a*n*x**(3*n/4)*gamma(1/4)) - 3*b**(3/4)*exp(-I*pi/4)*log(1 -
b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*n*ga
mma(1/4)) + 3*I*b**(3/4)*exp(-I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(
3*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*n*gamma(1/4)) + 3*b**(3/4)*exp
(-I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-3
/4)/(4*a**(7/4)*n*gamma(1/4)) - 3*I*b**(3/4)*exp(-I*pi/4)*log(1 - b**(1/4)
*x**(n/4)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*n*gamma(1/
4))
```

Maxima [F]

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{3}{4}n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-3/4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(1/4*n)/(a*b*x*x^n + a^2*x), x) - 4/3/(a*n*x^(3/4*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{3}{4}n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-3/4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-3/4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{3n}{4}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^((3*n)/4 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^((3*n)/4 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{7n}{4}}bx + x^{\frac{3n}{4}}ax}\,dx$$

input
```
int(x^(-1-3/4*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((7*n)/4)*b*x + x**((3*n)/4)*a*x),x)
```

3.498 $\int \frac{x^{-1-n}}{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx=-\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n\right)}{a^2n}$$

output
```
-1/a/n/(x^n)-b*ln(x)/a^2+b*ln(a+b*x^n)/a^2/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx=-\frac{ax^{-n}+b\log\left(x^n\right)-b\log\left(a+bx^n\right)}{a^2n}$$

input
```
Integrate[x^(-1 - n)/(a + b*x^n),x]
```

output
```
-((a/x^n + b*Log[x^n] - b*Log[a + b*x^n])/(a^2*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{x^{-2n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \ 54$$

$$\frac{\int \left(\frac{x^{-2n}}{a}-\frac{bx^{-n}}{a^2}+\frac{b^2}{a^2(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b\log(x^n)}{a^2}+\frac{b\log(a+bx^n)}{a^2}-\frac{x^{-n}}{a}}{n}$$

input
```
Int[x^(-1 - n)/(a + b*x^n),x]
```

output
```
(-(1/(a*x^n)) - (b*Log[x^n])/a^2 + (b*Log[a + b*x^n])/a^2)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{x^{-n}}{an}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(x^n+\frac{a}{b}\right)}{a^2n}$	41
norman	$\left(-\frac{1}{an}-\frac{b\ln(x)\mathrm{e}^{n\ln(x)}}{a^2}\right)\mathrm{e}^{-n\ln(x)}+\frac{b\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{a^2n}$	50

input
```
int(x^(-1-n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/a/n/(x^n)-b*ln(x)/a^2+b/a^2/n*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx=-\frac{bnx^n\log(x)-bx^n\log(bx^n+a)+a}{a^2nx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-(b*n*x^n*log(x) - b*x^n*log(b*x^n + a) + a)/(a^2*n*x^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(31) = 62$.

Time = 1.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \begin{cases} \tilde{\infty}\log{(x)} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{xx^{-n}x^{-n-1}}{2bn} & \text{for } a = 0 \\ -\frac{xx^{-n-1}}{an} & \text{for } b = 0 \\ \frac{\log{(x)}}{a+b} & \text{for } n = 0 \\ -\frac{x^{-n}}{an} - \frac{b\log{(x^n)}}{a^2n} + \frac{b\log{\left(\frac{a}{b}+x^n\right)}}{a^2n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-n)/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(2
*b*n*x**n), Eq(a, 0)), (-x*x**(-n - 1)/(a*n), Eq(b, 0)), (log(x)/(a + b),
Eq(n, 0)), (-1/(a*n*x**n) - b*log(x**n)/(a**2*n) + b*log(a/b + x**n)/(a**2
*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = -\frac{b\log{(x)}}{a^2} + \frac{b\log{\left(\frac{bx^n+a}{b}\right)}}{a^2n} - \frac{1}{anx^n}$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*log(x)/a^2 + b*log((b*x^n + a)/b)/(a^2*n) - 1/(a*n*x^n)
```

Giac [F]

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \int \frac{x^{-n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \int \frac{1}{x^{n+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-n}}{a+bx^n}\,dx = \frac{x^n\log(x^nb+a)\,b - x^n\log(x)\,bn - a}{x^na^2n}$$

input
```
int(x^(-1-n)/(a+b*x^n),x)
```

output
```
(x**n*log(x**n*b + a)*b - x**n*log(x)*b*n - a)/(x**n*a**2*n)
```

3.499 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx=-\frac{2x^{-n/2}}{an}+\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}n}$$

output
```
-2/a/n/(x^(1/2*n))+2*b^(1/2)*arctan(a^(1/2)/b^(1/2)/(x^(1/2*n)))/a^(3/2)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx=-\frac{2x^{-n/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[x^(-1 - n/2)/(a + b*x^n),x]
```

output
```
(-2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^n)/a)])/(a*n*x^(n/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{2}-1}}{a+bx^n}dx$$
$$\Big\downarrow \ 868$$
$$-\frac{2\int \frac{1}{bx^n+a}dx^{-n/2}}{n}$$
$$\Big\downarrow \ 772$$
$$-\frac{2\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/2}}{n}$$
$$\Big\downarrow \ 262$$
$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/2}}{a}\right)}{n}$$
$$\Big\downarrow \ 218$$
$$-\frac{2\left(\frac{x^{-n/2}}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{n}$$

input
```
Int[x^(-1 - n/2)/(a + b*x^n),x]
```

output
```
(-2*(1/(a*x^(n/2)) - (Sqrt[b]*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2))])/a^(3/2)))
/n
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
risch	$-\frac{2x^{-\frac{n}{2}}}{an}+\frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-ab}}{b}\right)}{a^{2}n}-\frac{\sqrt{-ab}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-ab}}{b}\right)}{a^{2}n}$	79

input
```
int(x^(-1-1/2*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-2/a/n/(x^(1/2*n))+1/a^2*(-a*b)^(1/2)/n*ln(x^(1/2*n)-1/b*(-a*b)^(1/2))-1/a
^2*(-a*b)^(1/2)/n*ln(x^(1/2*n)+1/b*(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.62

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \left[-\frac{2\,xx^{-\frac{1}{2}\,n-1} - \sqrt{-\frac{b}{a}}\log\left(\frac{ax^2x^{-n-2}+2\,axx^{-\frac{1}{2}\,n-1}\sqrt{-\frac{b}{a}}-b}{ax^2x^{-n-2}+b}\right)}{an}, -\frac{2\left(xx^{-\frac{1}{2}\,n-1} - \sqrt{\frac{b}{a}}\arctan\left(\frac{axx^{-\frac{1}{2}\,n-1}\sqrt{\frac{b}{a}}}{b}\right)\right)}{an}\right]$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
[-(2*x*x^(-1/2*n - 1) - sqrt(-b/a)*log((a*x^2*x^(-n - 2) + 2*a*x*x^(-1/2*n
 - 1)*sqrt(-b/a) - b)/(a*x^2*x^(-n - 2) + b)))/(a*n), -2*(x*x^(-1/2*n - 1)
 - sqrt(b/a)*arctan(a*x*x^(-1/2*n - 1)*sqrt(b/a)/b))/(a*n)]
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = -\frac{2x^{-\frac{n}{2}}}{an} - \frac{2\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

input
```
integrate(x**(-1-1/2*n)/(a+b*x**n),x)
```

output
```
-2/(a*n*x**(n/2)) - 2*sqrt(b)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(a**(3/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(1/2*n)/(a*b*x*x^n + a^2*x), x) - 2/(a*n*x^(1/2*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-1/2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{n}{2}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(n/2 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(n/2 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{3n}{2}}bx + x^{\frac{n}{2}}ax}dx$$

input
```
int(x^(-1-1/2*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((3*n)/2)*b*x + x**(n/2)*a*x),x)
```

3.500 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx=-\frac{3x^{-n/3}}{an}-\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{a^{4/3}n}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x^{-n/3}\right)}{a^{4/3}n}-\frac{\sqrt[3]{b}\log\left(b^{2/3}+a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}\right)}{2a^{4/3}n}$$

output
```
-3/a/n/(x^(1/3*n))-3^(1/2)*b^(1/3)*arctan(1/3*(b^(1/3)-2*a^(1/3)/(x^(1/3*n
)))*3^(1/2)/b^(1/3))/a^(4/3)/n+b^(1/3)*ln(b^(1/3)+a^(1/3)/(x^(1/3*n)))/a^(
4/3)/n-1/2*b^(1/3)*ln(b^(2/3)+a^(2/3)/(x^(2/3*n))-a^(1/3)*b^(1/3)/(x^(1/3*
n)))/a^(4/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx=-\frac{3x^{-n/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},-\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[x^(-1 - n/3)/(a + b*x^n),x]
```

output
```
(-3*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^n)/a)])/(a*n*x^(n/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {868, 772, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{3}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \; 868$$

$$-\frac{3\int \frac{1}{bx^n+a}dx^{-n/3}}{n}$$

$$\Big\downarrow \; 772$$

$$-\frac{3\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/3}}{n}$$

$$\Big\downarrow \; 843$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/3}}{a}\right)}{n}$$

$$\Big\downarrow \; 750$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}}dx^{-n/3}}{3b^{2/3}}\right)}{a}\right)}{n}$$

$$\downarrow 16$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{n}$$

$$\downarrow 1142$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{n}$$

$$\downarrow 25$$

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{\int \frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{n}$$

$\downarrow$ 27

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{n}$$

$\downarrow$ 1082

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{3\int \frac{1}{-x^{-2n/3}-3}d\left(1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{n}$$

$\downarrow$ 217

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{n}$$

$\downarrow$ 1103

$$-\frac{3\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{n}$$

input
```
Int[x^(-1 - n/3)/(a + b*x^n),x]
```

output
```
(-3*(1/(a*x^(n/3)) - (b*(Log[b^(1/3) + a^(1/3)/x^(n/3)]/(3*a^(1/3)*b^(2/3)
) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3))/(b^(1/3)*x^(n/3)))/Sqrt[3]])/a^(1/
3)) - Log[b^(2/3) + a^(2/3)/x^((2*n)/3) - (a^(1/3)*b^(1/3))/x^(n/3)]/(2*a^
(1/3)))/(3*b^(2/3))))/a))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

method	result	size
risch	$-\frac{3x^{-\frac{n}{3}}}{an}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{4}n^{3}\_Z^{3}-b\right)}\_R\ln\left(x^{\frac{n}{3}}+\frac{a^{3}n^{2}\_R^{2}}{b}\right)\right)$	57

input
```
int(x^(-1-1/3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+a^3*n^2/b*_R^2),_R=RootOf(_Z^3*a^4*
n^3-b))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx = -\frac{6\,xx^{-\frac{1}{3}\,n-1} - 2\,\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}axx^{-\frac{1}{3}\,n-1}\left(\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right) - 2\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}\,n-1}+\left(\frac{b}{a}\right)^{\frac{1}{3}}}{x}\right) + \left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\right.}{2\,an}$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/2*(6*x*x^(-1/3*n - 1) - 2*sqrt(3)*(b/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x
*x^(-1/3*n - 1)*(b/a)^(2/3) - sqrt(3)*b)/b) - 2*(b/a)^(1/3)*log((x*x^(-1/3
*n - 1) + (b/a)^(1/3))/x) + (b/a)^(1/3)*log((x^2*x^(-2/3*n - 2) - x*x^(-1/
3*n - 1)*(b/a)^(1/3) + (b/a)^(2/3))/x^2))/(a*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.20

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \frac{x^{-\frac{n}{3}}\Gamma\left(-\frac{1}{3}\right)}{an\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}n\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}n\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}n\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(x**(-1-1/3*n)/(a+b*x**n),x)
```

output
```
gamma(-1/3)/(a*n*x**(n/3)*gamma(2/3)) - b**(1/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*n*gamm
a(2/3)) - b**(1/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gam
ma(-1/3)/(3*a**(4/3)*n*gamma(2/3)) - b**(1/3)*exp(2*I*pi/3)*log(1 - b**(1/
3)*x**(n/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*n*gamma(
2/3))
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{1}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(2/3*n)/(a*b*x*x^n + a^2*x), x) - 3/(a*n*x^(1/3*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{1}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-1/3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{n}{3}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(n/3 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(n/3 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{4n}{3}}bx+x^{\frac{n}{3}}ax}dx$$

input
```
int(x^(-1-1/3*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((4*n)/3)*b*x + x**(n/3)*a*x),x)
```

3.501 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 172

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n} dx = -\frac{4x^{-n/4}}{an} - \frac{\sqrt{2}\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{5/4}n} + \frac{\sqrt{2}\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{5/4}n} + \frac{\sqrt{2}\sqrt[4]{b}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}}{\sqrt{b}+\sqrt{a}x^{-n/2}}\right)}{a^{5/4}n}$$

output
```
-4/a/n/(x^(1/4*n))-2^(1/2)*b^(1/4)*arctan(1-2^(1/2)*a^(1/4)/b^(1/4)/(x^(1/
4*n)))/a^(5/4)/n+2^(1/2)*b^(1/4)*arctan(1+2^(1/2)*a^(1/4)/b^(1/4)/(x^(1/4*
n)))/a^(5/4)/n+2^(1/2)*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)/(x^(1/4*n))
/(b^(1/2)+a^(1/2)/(x^(1/2*n))))/a^(5/4)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n} dx = -\frac{4x^{-n/4}\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[x^(-1 - n/4)/(a + b*x^n),x]
```

output
```
(-4*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^n)/a)])/(a*n*x^(n/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {868, 772, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{4}-1}}{a+bx^n}dx$$

$$\Big\downarrow \ 868$$

$$-\frac{4\int \frac{1}{bx^n+a}dx^{-n/4}}{n}$$

$$\Big\downarrow \ 772$$

$$-\frac{4\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/4}}{n}$$

$$\Big\downarrow \ 843$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/4}}{a}\right)}{n}$$

$$\Big\downarrow \ 755$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{a}x^{-n/2}+\sqrt{b}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{n}$$

$$\Big\downarrow \ 1476$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}+\frac{\int\frac{1}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}}{2\sqrt{b}}\right)}{a}\right)}{n}$$

$\Big\downarrow$ 1082

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int\frac{1}{-x^{-n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x^{-n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{n}$$

$\Big\downarrow$ 217

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{n}$$

$\Big\downarrow$ 1479

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\cdots\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots\right)}{a}\cdots\right)}{n}$$

$\downarrow$ 25

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\cdots}{2\sqrt{b}}\cdots\right)}{a}\cdots\right)}{n}$$

$\downarrow$ 27

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{2}\sqrt{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{b}}\cdots\right)}{a}\cdots\right)}{n}$$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{4\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}+\sqrt{a}x^{-n/2}+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}+\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\cdots\right)}{a}\cdots\right)}{n}$$

input
```
Int[x^(-1 - n/4)/(a + b*x^n),x]
```

output
```
(-4*(1/(a*x^(n/4)) - (b*((-(ArcTan[1 - (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/4))
]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/
4))]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] + Sqrt[a]/
x^(n/2) - (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)) + L
og[Sqrt[b] + Sqrt[a]/x^(n/2) + (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)]/(2*Sqrt[
2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

method	result	size
risch	$-\frac{4x^{-\frac{n}{4}}}{an}+\left(\sum_{\_R=\mathrm{RootOf}\left(a^{5}n^{4}\_Z^{4}+b\right)}\_R\ln\left(x^{\frac{n}{4}}-\frac{a^{4}n^{3}\_R^{3}}{b}\right)\right)$	56

input
```
int(x^(-1-1/4*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)-a^4*n^3/b*_R^3),_R=RootOf(_Z^4*a^5*
n^4+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \frac{an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(\frac{an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+xx^{-\frac{1}{4}n-1}}{x}\right)-an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(-\frac{an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}-xx^{-\frac{1}{4}n-1}}{x}\right)+i\,an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log}{an}$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
(a*n*(-b/(a^5*n^4))^(1/4)*log((a*n*(-b/(a^5*n^4))^(1/4) + x*x^(-1/4*n - 1)
)/x) - a*n*(-b/(a^5*n^4))^(1/4)*log(-(a*n*(-b/(a^5*n^4))^(1/4) - x*x^(-1/4
*n - 1))/x) + I*a*n*(-b/(a^5*n^4))^(1/4)*log((I*a*n*(-b/(a^5*n^4))^(1/4) +
 x*x^(-1/4*n - 1))/x) - I*a*n*(-b/(a^5*n^4))^(1/4)*log((-I*a*n*(-b/(a^5*n^
4))^(1/4) + x*x^(-1/4*n - 1))/x) - 4*x*x^(-1/4*n - 1))/(a*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx &= \frac{x^{-\frac{n}{4}}\Gamma\left(-\frac{1}{4}\right)}{an\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt[4]{b}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)} \\ &- \frac{i\sqrt[4]{b}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{\sqrt[4]{b}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{5i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{i\sqrt[4]{b}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{7i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)}\end{aligned}$$

input
```
integrate(x**(-1-1/4*n)/(a+b*x**n),x)
```

output
```
gamma(-1/4)/(a*n*x**(n/4)*gamma(3/4)) - b**(1/4)*exp(-3*I*pi/4)*log(1 - b*
*(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*n*gamm
a(3/4)) - I*b**(1/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*
I*pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*n*gamma(3/4)) + b**(1/4)*exp(-3*
I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-1/4
)/(4*a**(5/4)*n*gamma(3/4)) + I*b**(1/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x
**(n/4)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*n*gamma(3/4)
)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx=\int \frac{x^{-\frac{1}{4}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(3/4*n)/(a*b*x*x^n + a^2*x), x) - 4/(a*n*x^(1/4*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx=\int \frac{x^{-\frac{1}{4}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-1/4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-1/4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{n}{4}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^(n/4 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^(n/4 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{5n}{4}}bx+x^{\frac{n}{4}}ax}dx$$

input
```
int(x^(-1-1/4*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((5*n)/4)*b*x + x**(n/4)*a*x),x)
```

3.502 $\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = -\frac{2x^{-3n/2}}{3an} + \frac{2bx^{-n/2}}{a^2n} - \frac{2b^{3/2}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}n}$$

output
```
-2/3/a/n/(x^(3/2*n))+2*b/a^2/n/(x^(1/2*n))-2*b^(3/2)*arctan(a^(1/2)/b^(1/2
)/(x^(1/2*n)))/a^(5/2)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = -\frac{2x^{-3n/2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{bx^n}{a}\right)}{3an}$$

input
```
Integrate[x^(-1 - (3*n)/2)/(a + b*x^n),x]
```

output
```
(-2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^n)/a)])/(3*a*n*x^((3*n)/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {886, 868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{3n}{2}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{ 886}$$

$$-\frac{b\int \frac{x^{-\frac{n}{2}-1}}{bx^n+a}dx}{a}-\frac{2x^{-3n/2}}{3an}$$

$$\Big\downarrow \text{ 868}$$

$$\frac{2b\int \frac{1}{bx^n+a}dx^{-n/2}}{an}-\frac{2x^{-3n/2}}{3an}$$

$$\Big\downarrow \text{ 772}$$

$$\frac{2b\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/2}}{an}-\frac{2x^{-3n/2}}{3an}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2b\left(\frac{x^{-n/2}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/2}}{a}\right)}{an}-\frac{2x^{-3n/2}}{3an}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2b\left(\frac{x^{-n/2}}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{an}-\frac{2x^{-3n/2}}{3an}$$

input
```
Int[x^(-1 - (3*n)/2)/(a + b*x^n),x]
```

output
```
-2/(3*a*n*x^((3*n)/2)) + (2*b*(1/(a*x^(n/2)) - (Sqrt[b]*ArcTan[Sqrt[a]/(Sq
rt[b]*x^(n/2))])/a^(3/2)))/(a*n)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{2b\,x^{-\frac{n}{2}}}{a^{2}n}-\frac{2x^{-\frac{3n}{2}}}{3an}+\frac{\sqrt{-ab}\,b\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-ab}}{b}\right)}{a^{3}n}-\frac{\sqrt{-ab}\,b\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-ab}}{b}\right)}{a^{3}n}$	97

input
```
int(x^(-1-3/2*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
2*b/a^2/n/(x^(1/2*n))-2/3/a/n/(x^(1/2*n))^3+1/a^3*(-a*b)^(1/2)*b/n*ln(x^(1
/2*n)+1/b*(-a*b)^(1/2))-1/a^3*(-a*b)^(1/2)*b/n*ln(x^(1/2*n)-1/b*(-a*b)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.72

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx$$

$$= \left[-\frac{2\,axx^{-\frac{3}{2}\,n-1}-3\,b\sqrt{-\frac{b}{a}}\log\left(-\frac{2\,a^3x^{\frac{5}{3}}x^{-\frac{5}{2}\,n-\frac{5}{3}}\sqrt{-\frac{b}{a}}-a^3x^2x^{-3\,n-2}-2\,a^2bxx^{-\frac{3}{2}\,n-1}\sqrt{-\frac{b}{a}}+2\,a^2bx^{\frac{4}{3}}x^{-2\,n-\frac{4}{3}}+2\,ab^2x^{\frac{1}{3}}x^{-\frac{1}{2}\,n}}{a^3x^2x^{-3\,n-2}+b^3}}{3\,a^2n}\right.$$

$$\left.-\frac{2\left(axx^{-\frac{3}{2}\,n-1}+3\,b\sqrt{\frac{b}{a}}\arctan\left(\frac{ax^{\frac{1}{3}}x^{-\frac{1}{2}\,n-\frac{1}{3}}\sqrt{\frac{b}{a}}}{b}\right)-3\,bx^{\frac{1}{3}}x^{-\frac{1}{2}\,n-\frac{1}{3}}\right)}{3\,a^2n}\right]$$

input
```
integrate(x^(-1-3/2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
[-1/3*(2*a*x*x^(-3/2*n - 1) - 3*b*sqrt(-b/a)*log(-(2*a^3*x^(5/3)*x^(-5/2*n
 - 5/3)*sqrt(-b/a) - a^3*x^2*x^(-3*n - 2) - 2*a^2*b*x*x^(-3/2*n - 1)*sqrt(
-b/a) + 2*a^2*b*x^(4/3)*x^(-2*n - 4/3) + 2*a*b^2*x^(1/3)*x^(-1/2*n - 1/3)*
sqrt(-b/a) - 2*a*b^2*x^(2/3)*x^(-n - 2/3) + b^3)/(a^3*x^2*x^(-3*n - 2) + b
^3)) - 6*b*x^(1/3)*x^(-1/2*n - 1/3))/(a^2*n), -2/3*(a*x*x^(-3/2*n - 1) + 3
*b*sqrt(b/a)*arctan(a*x^(1/3)*x^(-1/2*n - 1/3)*sqrt(b/a)/b) - 3*b*x^(1/3)*
x^(-1/2*n - 1/3))/(a^2*n)]
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = -\frac{2x^{-\frac{3n}{2}}}{3an} + \frac{2bx^{-\frac{n}{2}}}{a^2n} + \frac{2b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

input
```
integrate(x**(-1-3/2*n)/(a+b*x**n),x)
```

output
```
-2/(3*a*n*x**(3*n/2)) + 2*b/(a**2*n*x**(n/2)) + 2*b**(3/2)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(a**(5/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{3}{2}n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-3/2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*integrate(x^(1/2*n)/(a^2*b*x*x^n + a^3*x), x) + 2/3*(3*b*x^n - a)/(a^2*n*x^(3/2*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{3}{2}n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-3/2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-3/2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{3\,n}{2}+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/(x^((3*n)/2 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^((3*n)/2 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{5n}{2}}bx+x^{\frac{3n}{2}}ax}dx$$

input
```
int(x^(-1-3/2*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((5*n)/2)*b*x + x**((3*n)/2)*a*x),x)
```

3.503 $\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 176

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx=-\frac{3x^{-4n/3}}{4an}+\frac{3bx^{-n/3}}{a^2n}+\frac{\sqrt{3}b^{4/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{a^{7/3}n}-\frac{b^{4/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x^{-n/3}\right)}{a^{7/3}n}+\frac{b^{4/3}\log\left(b^{2/3}+a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}\right)}{2a^{7/3}n}$$

output
```
-3/4/a/n/(x^(4/3*n))+3*b/a^2/n/(x^(1/3*n))+3^(1/2)*b^(4/3)*arctan(1/3*(b^(
1/3)-2*a^(1/3)/(x^(1/3*n)))*3^(1/2)/b^(1/3))/a^(7/3)/n-b^(4/3)*ln(b^(1/3)+
a^(1/3)/(x^(1/3*n)))/a^(7/3)/n+1/2*b^(4/3)*ln(b^(2/3)+a^(2/3)/(x^(2/3*n))-
a^(1/3)*b^(1/3)/(x^(1/3*n)))/a^(7/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx=-\frac{3x^{-4n/3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},1,-\frac{1}{3},-\frac{bx^n}{a}\right)}{4an}$$

input
```
Integrate[x^(-1 - (4*n)/3)/(a + b*x^n),x]
```

output
```
(-3*Hypergeometric2F1[-4/3, 1, -1/3, -((b*x^n)/a)])/(4*a*n*x^((4*n)/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {886, 868, 772, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{4n}{3}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{886}$$

$$-\frac{b\int \frac{x^{-\frac{n}{3}-1}}{bx^n+a}dx}{a}-\frac{3x^{-4n/3}}{4an}$$

$$\Big\downarrow \text{868}$$

$$\frac{3b\int \frac{1}{bx^n+a}dx^{-n/3}}{an}-\frac{3x^{-4n/3}}{4an}$$

$$\Big\downarrow \text{772}$$

$$\frac{3b\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/3}}{an}-\frac{3x^{-4n/3}}{4an}$$

$$\Big\downarrow \text{843}$$

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/3}}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}$$

$\Big\downarrow$ 750

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}}dx^{-n/3}}{3b^{2/3}}\right)}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}$$

$\Big\downarrow$ 16

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}$$

$\Big\downarrow$ 1142

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int \frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\int -\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}$$

$$\frac{3x^{-4n/3}}{4an}$$

$$\Big\downarrow \text{25}$$

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}$$

$$\frac{3x^{-4n/3}}{4an}$$

$$\Big\downarrow \text{27}$$

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}$$

$$\frac{3x^{-4n/3}}{4an}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{3\int \frac{1}{-x^{-2n/3}-3}d\left(1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}$$

$\Big\downarrow$ 217

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}$$

$\Big\downarrow$ 1103

$$\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}$$

input
```
Int[x^(-1 - (4*n)/3)/(a + b*x^n),x]
```

output
```
-3/(4*a*n*x^((4*n)/3)) + (3*b*(1/(a*x^(n/3)) - (b*(Log[b^(1/3) + a^(1/3)/x
^(n/3)]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3))/(b^(1/3)
*x^(n/3)))/Sqrt[3]])/a^(1/3)) - Log[b^(2/3) + a^(2/3)/x^((2*n)/3) - (a^(1/
3)*b^(1/3))/x^(n/3)]/(2*a^(1/3)))/(3*b^(2/3))))/a))/(a*n)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{3b\,x^{-\frac{n}{3}}}{a^{2} n}-\frac{3 x^{-\frac{4 n}{3}}}{4 a n}+\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7} n^{3} \_Z^{3}+b^{4}\right)}\_R\ln\left(x^{\frac{n}{3}}+\frac{a^{5} n^{2} \_R^{2}}{b^{3}}\right)\right)$	73

input
```
int(x^(-1-4/3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
3*b/a^2/n/(x^(1/3*n))-3/4/a/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+a^5*n^2/b^
3*_R^2),_R=RootOf(_Z^3*a^7*n^3+b^4))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx =$$

$$-\frac{3\,axx^{-\frac{4}{3}\,n-1} - 4\,\sqrt{3}b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax^{\frac{1}{4}}x^{-\frac{1}{3}\,n-\frac{1}{4}}\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right) + 2\,b\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\frac{x^{\frac{3}{4}}x^{-\frac{1}{3}\,n-\frac{1}{4}}\left(-\frac{b}{a}\right)^{\frac{1}{3}}+xx^{-\cdots}}{x}\cdots\right.}{4\,a^2n}$$

input
```
integrate(x^(-1-4/3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/4*(3*a*x*x^(-4/3*n - 1) - 4*sqrt(3)*b*(-b/a)^(1/3)*arctan(1/3*(2*sqrt(3
)*a*x^(1/4)*x^(-1/3*n - 1/4)*(-b/a)^(2/3) - sqrt(3)*b)/b) + 2*b*(-b/a)^(1/
3)*log((x^(3/4)*x^(-1/3*n - 1/4)*(-b/a)^(1/3) + x*x^(-2/3*n - 1/2) + sqrt(
x)*(-b/a)^(2/3))/x) - 4*b*(-b/a)^(1/3)*log((x*x^(-1/3*n - 1/4) - x^(3/4)*(
-b/a)^(1/3))/x) - 12*b*x^(1/4)*x^(-1/3*n - 1/4))/(a^2*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.31

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \frac{x^{-\frac{4n}{3}}\Gamma\left(-\frac{4}{3}\right)}{an\Gamma\left(-\frac{1}{3}\right)} - \frac{4bx^{-\frac{n}{3}}\Gamma\left(-\frac{4}{3}\right)}{a^2n\Gamma\left(-\frac{1}{3}\right)} + \frac{4b^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)} + \frac{4b^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)} + \frac{4b^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(x**(-1-4/3*n)/(a+b*x**n),x)
```

output
```
gamma(-4/3)/(a*n*x**(4*n/3)*gamma(-1/3)) - 4*b*gamma(-4/3)/(a**2*n*x**(n/3
)*gamma(-1/3)) + 4*b**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_p
olar(I*pi/3)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*n*gamma(-1/3)) + 4*b**(4/3)
*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(-4/3)/(3*a**(7/
3)*n*gamma(-1/3)) + 4*b**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp
_polar(5*I*pi/3)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*n*gamma(-1/3))
```

Maxima [F]

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{4}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-4/3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*integrate(x^(2/3*n)/(a^2*b*x*x^n + a^3*x), x) + 3/4*(4*b*x^n - a)/(a^2
*n*x^(4/3*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{4}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-4/3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-4/3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{4\,n}{3}+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/(x^((4*n)/3 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^((4*n)/3 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{7n}{3}}bx+x^{\frac{4n}{3}}ax}dx$$

input
```
int(x^(-1-4/3*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((7*n)/3)*b*x + x**((4*n)/3)*a*x),x)
```

3.504 $\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 191

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n} dx = -\frac{4x^{-5n/4}}{5an} + \frac{4bx^{-n/4}}{a^2n} + \frac{\sqrt{2}b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{9/4}n} - \frac{\sqrt{2}b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{9/4}n} - \frac{\sqrt{2}b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}}{\sqrt{b}+\sqrt{a}x^{-n/2}}\right)}{a^{9/4}n}$$

output
```
-4/5/a/n/(x^(5/4*n))+4*b/a^2/n/(x^(1/4*n))+2^(1/2)*b^(5/4)*arctan(1-2^(1/2
)*a^(1/4)/b^(1/4)/(x^(1/4*n)))/a^(9/4)/n-2^(1/2)*b^(5/4)*arctan(1+2^(1/2)*
a^(1/4)/b^(1/4)/(x^(1/4*n)))/a^(9/4)/n-2^(1/2)*b^(5/4)*arctanh(2^(1/2)*a^(
1/4)*b^(1/4)/(x^(1/4*n))/(b^(1/2)+a^(1/2)/(x^(1/2*n))))/a^(9/4)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.18

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx=-\frac{4x^{-5n/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\frac{bx^n}{a}\right)}{5an}$$

input
```
Integrate[x^(-1 - (5*n)/4)/(a + b*x^n),x]
```

output
```
(-4*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^n)/a)])/(5*a*n*x^((5*n)/4))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.45, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {886, 868, 772, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{5n}{4}-1}}{a+bx^n}dx$$

$$\Big\downarrow \; 886$$

$$-\frac{b\int \frac{x^{-\frac{n}{4}-1}}{bx^n+a}dx}{a}-\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \; 868$$

$$\frac{4b\int \frac{1}{bx^n+a}dx^{-n/4}}{an}-\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \; 772$$

$$\frac{4b\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/4}}{an}-\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \; 843$$

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\int\frac{1}{ax^{-n}+b}dx^{-n/4}}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}$$

$\big\downarrow$ 755

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{a}x^{-n/2}+\sqrt{b}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}$$

$\big\downarrow$ 1476

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}+\frac{\int\frac{1}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}}{2\sqrt{b}}\right)}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}$$

$\big\downarrow$ 1082

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int\frac{1}{-x^{-n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x^{-n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \text{217}$$

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \text{1479}$$

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots}{\cdots}\right)}{a}\right)}{an}$$

$$\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \text{25}$$

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\cdots}{2\sqrt{b}}\cdots\right)}{a}\cdots\right)}{an}\cdots$$

$$\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \ 27$$

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{2}\sqrt{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{b}}\cdots\right)}{a}\cdots\right)}{an}\cdots$$

$$\frac{4x^{-5n/4}}{5an}$$

$$\Big\downarrow \ 1103$$

$$\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}+\sqrt{a}x^{-n/2}+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}+\sqrt{}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{a}}{an}$$

$$\frac{4x^{-5n/4}}{5an}$$

input
```
Int[x^(-1 - (5*n)/4)/(a + b*x^n),x]
```

output
```
-4/(5*a*n*x^((5*n)/4)) + (4*b*(1/(a*x^(n/4)) - (b*((-(ArcTan[1 - (Sqrt[2]*
a^(1/4))/(b^(1/4)*x^(n/4))]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[
2]*a^(1/4))/(b^(1/4)*x^(n/4))]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) + (-
1/2*Log[Sqrt[b] + Sqrt[a]/x^(n/2) - (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)]/(Sq
rt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[b] + Sqrt[a]/x^(n/2) + (Sqrt[2]*a^(1/4)*
b^(1/4))/x^(n/4)]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/(a*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.66 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{4b\,x^{-\frac{n}{4}}}{a^{2}n}-\frac{4x^{-\frac{5n}{4}}}{5an}+\left(\sum_{\_R=\operatorname{RootOf}\left(a^{9}n^{4}\_Z^{4}+b^{5}\right)}\_R\ln\left(x^{\frac{n}{4}}+\frac{a^{7}n^{3}\_R^{3}}{b^{4}}\right)\right)$	73

input
```
int(x^(-1-5/4*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
4*b/a^2/n/(x^(1/4*n))-4/5/a/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+a^7*n^3/b^
4*_R^3),_R=RootOf(_Z^4*a^9*n^4+b^5))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.37

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx =$$

$$-\frac{5\,a^{2}n\left(-\frac{b^{5}}{a^{9}n^{4}}\right)^{\frac{1}{4}}\log\left(\frac{a^{2}nx^{\frac{4}{5}}\left(-\frac{b^{5}}{a^{9}n^{4}}\right)^{\frac{1}{4}}+bxx^{-\frac{1}{4}n-\frac{1}{5}}}{x}\right)-5\,a^{2}n\left(-\frac{b^{5}}{a^{9}n^{4}}\right)^{\frac{1}{4}}\log\left(-\frac{a^{2}nx^{\frac{4}{5}}\left(-\frac{b^{5}}{a^{9}n^{4}}\right)^{\frac{1}{4}}-bxx^{-\frac{1}{4}n-\frac{1}{5}}}{x}\right)+}{}$$

input
```
integrate(x^(-1-5/4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/5*(5*a^2*n*(-b^5/(a^9*n^4))^(1/4)*log((a^2*n*x^(4/5)*(-b^5/(a^9*n^4))^(
1/4) + b*x*x^(-1/4*n - 1/5))/x) - 5*a^2*n*(-b^5/(a^9*n^4))^(1/4)*log(-(a^2
*n*x^(4/5)*(-b^5/(a^9*n^4))^(1/4) - b*x*x^(-1/4*n - 1/5))/x) + 5*I*a^2*n*(
-b^5/(a^9*n^4))^(1/4)*log((I*a^2*n*x^(4/5)*(-b^5/(a^9*n^4))^(1/4) + b*x*x^
(-1/4*n - 1/5))/x) - 5*I*a^2*n*(-b^5/(a^9*n^4))^(1/4)*log((-I*a^2*n*x^(4/5
)*(-b^5/(a^9*n^4))^(1/4) + b*x*x^(-1/4*n - 1/5))/x) + 4*a*x*x^(-5/4*n - 1)
 - 20*b*x^(1/5)*x^(-1/4*n - 1/5))/(a^2*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.62

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \frac{x^{-\frac{5n}{4}}\Gamma\left(-\frac{5}{4}\right)}{an\Gamma\left(-\frac{1}{4}\right)} - \frac{5bx^{-\frac{n}{4}}\Gamma\left(-\frac{5}{4}\right)}{a^2n\Gamma\left(-\frac{1}{4}\right)} + \frac{5b^{\frac{5}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)} + \frac{5ib^{\frac{5}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)} - \frac{5b^{\frac{5}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{5i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)} - \frac{5ib^{\frac{5}{4}}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{7i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(x**(-1-5/4*n)/(a+b*x**n),x)
```

output
```
gamma(-5/4)/(a*n*x**(5*n/4)*gamma(-1/4)) - 5*b*gamma(-5/4)/(a**2*n*x**(n/4
)*gamma(-1/4)) + 5*b**(5/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_p
olar(I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*n*gamma(-1/4)) + 5*I*b**(5/
4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*
gamma(-5/4)/(4*a**(9/4)*n*gamma(-1/4)) - 5*b**(5/4)*exp(-3*I*pi/4)*log(1 -
 b**(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*n
*gamma(-1/4)) - 5*I*b**(5/4)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_
polar(7*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*n*gamma(-1/4))
```

Maxima [F]

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{5}{4}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-5/4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*integrate(x^(3/4*n)/(a^2*b*x*x^n + a^3*x), x) + 4/5*(5*b*x^n - a)/(a^2
*n*x^(5/4*n))
```

Giac [F]

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \int \frac{x^{-\frac{5}{4}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1-5/4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^(-5/4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{5\,n}{4}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/(x^((5*n)/4 + 1)*(a + b*x^n)),x)
```

output
```
int(1/(x^((5*n)/4 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \int \frac{1}{x^{\frac{9n}{4}}bx+x^{\frac{5n}{4}}ax}dx$$

input
```
int(x^(-1-5/4*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((9*n)/4)*b*x + x**((5*n)/4)*a*x),x)
```

3.505 $\int x\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x\sqrt{a+bx^n}\,dx=\frac{x^2(a+bx^n)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a}$$

output
```
1/2*x^2*(a+b*x^n)^(3/2)*hypergeom([1, 3/2+2/n],[(2+n)/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x\sqrt{a+bx^n}\,dx=\frac{x^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x*Sqrt[a + b*x^n],x]
```

output
```
(x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 2/n, 1 + 2/n, -((b*x^n)/a)])/
(2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{a+bx^n}\int x\sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[x*Sqrt[a + b*x^n],x]
```

output
```
(x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 2/n, (2 + n)/n, -((b*x^n)/a)]
)/(2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x\sqrt{a+b\,x^{n}}dx$$

input
```
int(x*(a+b*x^n)^(1/2),x)
```

output
```
int(x*(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+bx^n}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x\sqrt{a+bx^n}\,dx=\frac{a^{\frac{2}{n}}a^{\frac{1}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**(1/2),x)
```

output
```
a**(2/n)*a**(1/2 - 2/n)*x**2*gamma(2/n)*hyper((-1/2, 2/n), (1 + 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n))
```

Maxima [F]

$$\int x\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^n}\,dx=\int x\,\sqrt{a+b\,x^n}\,dx$$

input
```
int(x*(a + b*x^n)^(1/2),x)
```

output
```
int(x*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+bx^n}\,dx = \frac{2\sqrt{x^n b+a}\,x^2+\left(\int \frac{\sqrt{x^n b+a}\,x}{x^n bn+4x^n b+an+4a}dx\right)a\,n^2+4\left(\int \frac{\sqrt{x^n b+a}\,x}{x^n bn+4x^n b+an+4a}dx\right)an}{n+4}$$

input
```
int(x*(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*x**2 + int((sqrt(x**n*b + a)*x)/(x**n*b*n + 4*x**n*b +
 a*n + 4*a),x)*a*n**2 + 4*int((sqrt(x**n*b + a)*x)/(x**n*b*n + 4*x**n*b +
a*n + 4*a),x)*a*n)/(n + 4)
```

3.506 $\int \sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \sqrt{a+bx^n}\,dx = \frac{x(a+bx^n)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

output
```
x*(a+b*x^n)^(3/2)*hypergeom([1, 3/2+1/n],[1+1/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \sqrt{a+bx^n}\,dx = \frac{x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a
)])/Sqrt[1 + (b*x^n)/a]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\sqrt{a+bx^n}\int \sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a
)])/Sqrt[1 + (b*x^n)/a]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \sqrt{a+b\,x^{n}}dx$$

input
```
int((a+b*x^n)^(1/2),x)
```

output
```
int((a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \sqrt{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{\frac{1}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2),x)
```

output
```
a**(1/n)*a**(1/2 - 1/n)*x*gamma(1/n)*hyper((-1/2, 1/n), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}\,dx$$

input
```
integrate((a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}\,dx$$

input
```
integrate((a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \sqrt{a+bx^n}\,dx = \frac{x\sqrt{a+b\,x^n}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\sqrt{\frac{b\,x^n}{a}+1}}$$

input
```
int((a + b*x^n)^(1/2),x)
```

output
```
(x*(a + b*x^n)^(1/2)*hypergeom([-1/2, 1/n], 1/n + 1, -(b*x^n)/a))/((b*x^n)
/a + 1)^(1/2)
```

Reduce [F]

$$\int \sqrt{a+bx^n}\,dx = \frac{2\sqrt{x^n b+a}\,x+\left(\int \frac{\sqrt{x^n b+a}}{x^n bn+2x^n b+an+2a}dx\right)a\,n^2+2\left(\int \frac{\sqrt{x^n b+a}}{x^n bn+2x^n b+an+2a}dx\right)an}{n+2}$$

input
```
int((a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*x + int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n +
2*a),x)*a*n**2 + 2*int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n + 2*a),
x)*a*n)/(n + 2)
```

3.507 $\int \frac{\sqrt{a+bx^n}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{\sqrt{a+bx^n}}{x} \, dx = \frac{2\sqrt{a+bx^n}}{n} - \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*(a+b*x^n)^(1/2)/n-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^n}}{x} \, dx = \frac{2\left(\sqrt{a+bx^n} - \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Integrate[Sqrt[a + b*x^n]/x,x]
```

output
```
(2*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]]))/n
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{x} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[a + b*x^n]/x,x]
```

output
```
(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{n}$	36
default	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{n}$	36
risch	$\frac{2\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{n}-\frac{2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{\sqrt{a}}\right)}{n}$	42

input
```
int((a+b*x^n)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+\sqrt{bx^n+a}\right)}{n}\right]$$

input
```
integrate((a+b*x^n)^(1/2)/x,x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^
n + a))/n, 2*(sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + sqrt(b*x^n + a))
/n]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 1.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = -\frac{2\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}$$

input
```
integrate((a+b*x**n)**(1/2)/x,x)
```

output
```
-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/n + 2*a/(sqrt(b)*n*x**(n/2)*s
qrt(a/(b*x**n) + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a/(b*x**n) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\sqrt{bx^n+a}}{n}$$

input
```
integrate((a+b*x^n)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
*sqrt(b*x^n + a)/n
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \int \frac{\sqrt{bx^n+a}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \int \frac{\sqrt{a+b\,x^n}}{x}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x,x)
```

output
```
int((a + b*x^n)^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \frac{2\sqrt{x^n b+a}+\left(\int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx\right)an}{n}$$

input
```
int((a+b*x^n)^(1/2)/x,x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*a*n)/n
```

3.508 $\int \frac{\sqrt{a+bx^n}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx=-\frac{(a+bx^n)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax}$$

output
```
-(a+b*x^n)^(3/2)*hypergeom([1, 3/2-1/n],[-(1-n)/n],-b*x^n/a)/a/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx=-\frac{\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^n]/x^2,x]
```

output
```
-((Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -n^(-1), 1 - n^(-1), -((b*x^n)/
a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\int \frac{\sqrt{\frac{bx^n}{a}+1}}{x^2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[Sqrt[a + b*x^n]/x^2,x]
```

output
```
-((Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -n^(-1), -((1 - n)/n), -((b*x^n
)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}}{x^{2}}dx$$

input
```
int((a+b*x^n)^(1/2)/x^2,x)
```

output
```
int((a+b*x^n)^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx = \frac{a^{-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)/x**2,x)
```

output
```
a**(1/2 + 1/n)*gamma(-1/n)*hyper((-1/2, -1/n), (1 - 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx = \int \frac{\sqrt{bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx = \int \frac{\sqrt{bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx = \int \frac{\sqrt{a+b\,x^n}}{x^2}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x^2,x)
```

output
```
int((a + b*x^n)^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}}{x^2}\,dx = \frac{2\sqrt{x^n b+a}+\left(\int \frac{\sqrt{x^n b+a}}{x^n bn\,x^2-2x^n b\,x^2+an\,x^2-2a\,x^2}dx\right)a\,n^2x-2\left(\int \frac{\sqrt{x^n b+a}}{x^n bn\,x^2-2x^n b\,x^2+an\,x^2-2a\,x^2}dx\right)anx}{x\,(n-2)}$$

input
```
int((a+b*x^n)^(1/2)/x^2,x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*n*x**2 - 2*x**n*b*x**2
+ a*n*x**2 - 2*a*x**2),x)*a*n**2*x - 2*int(sqrt(x**n*b + a)/(x**n*b*n*x**2
 - 2*x**n*b*x**2 + a*n*x**2 - 2*a*x**2),x)*a*n*x)/(x*(n - 2))
```

3.509 $\int \frac{\sqrt{a+bx^n}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx = -\frac{(a+bx^n)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2}$$

output
```
-1/2*(a+b*x^n)^(3/2)*hypergeom([1, 3/2-2/n],[-(2-n)/n],-b*x^n/a)/a/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx = -\frac{\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^n]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -2/n, 1 - 2/n, -((b*x^n)/a)]
)/(x^2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx$$

$$\Big\downarrow \ 889$$

$$\frac{\sqrt{a+bx^n}\int \frac{\sqrt{\frac{bx^n}{a}+1}}{x^3}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \ 888$$

$$-\frac{\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[Sqrt[a + b*x^n]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -2/n, -((2 - n)/n), -((b*x^n
)/a)])/(x^2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}}{x^{3}}dx$$

input
```
int((a+b*x^n)^(1/2)/x^3,x)
```

output
```
int((a+b*x^n)^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx = \frac{a^{-\frac{2}{n}}a^{\frac{1}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)/x**3,x)
```

output
```
a**(1/2 + 2/n)*gamma(-2/n)*hyper((-1/2, -2/n), (1 - 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx = \int \frac{\sqrt{bx^n+a}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx = \int \frac{\sqrt{bx^n+a}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx=\int \frac{\sqrt{a+b\,x^n}}{x^3}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x^3,x)
```

output
```
int((a + b*x^n)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}}{x^3}\,dx = \frac{2\sqrt{x^n b+a}+\left(\int \frac{\sqrt{x^n b+a}}{x^n bn\,x^3-4x^n b\,x^3+an\,x^3-4a\,x^3}dx\right)a\,n^2x^2-4\left(\int \frac{\sqrt{x^n b+a}}{x^n bn\,x^3-4x^n b\,x^3+an\,x^3-4a\,x^3}dx\right)an\,x^2}{x^2\,(n-4)}$$

input
```
int((a+b*x^n)^(1/2)/x^3,x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*n*x**3 - 4*x**n*b*x**3
+ a*n*x**3 - 4*a*x**3),x)*a*n**2*x**2 - 4*int(sqrt(x**n*b + a)/(x**n*b*n*x
**3 - 4*x**n*b*x**3 + a*n*x**3 - 4*a*x**3),x)*a*n*x**2)/(x**2*(n - 4))
```

3.510 $\int x(a+bx^n)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x(a+bx^n)^{3/2}\,dx=\frac{x^2(a+bx^n)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2}+\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a}$$

output
```
1/2*x^2*(a+b*x^n)^(5/2)*hypergeom([1, 5/2+2/n],[(2+n)/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int x(a+bx^n)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x*(a + b*x^n)^(3/2),x]
```

output
```
(a*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 2/n, 1 + 2/n, -((b*x^n)/a)]
)/(2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^{3/2}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\int x\left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{ax^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[x*(a + b*x^n)^(3/2),x]
```

output
```
(a*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x(a+b\,x^{n})^{\frac{3}{2}}\,dx$$

input
```
int(x*(a+b*x^n)^(3/2),x)
```

output
```
int(x*(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+bx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x(a+bx^n)^{3/2}\,dx = \frac{a^{\frac{2}{n}}a^{\frac{3}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**(3/2),x)
```

output
```
a**(2/n)*a**(3/2 - 2/n)*x**2*gamma(2/n)*hyper((-3/2, 2/n), (1 + 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n))
```

Maxima [F]

$$\int x(a+bx^n)^{3/2}\,dx=\int (bx^n+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+bx^n)^{3/2}\,dx=\int (bx^n+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n)^{3/2}\,dx=\int x\,\left(a+b\,x^n\right)^{3/2}\,dx$$

input
```
int(x*(a + b*x^n)^(3/2),x)
```

output
```
int(x*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int x(a+bx^n)^{3/2}\,dx=\frac{2x^n\sqrt{x^nb+a}\,bn\,x^2+8x^n\sqrt{x^nb+a}\,b\,x^2+8\sqrt{x^nb+a}\,an\,x^2+8\sqrt{x^nb+a}\,a\,x^2+9\left(\int\frac{}{3x^nbn^2+}\right.}{}$$

input
```
int(x*(a+b*x^n)^(3/2),x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n*x**2 + 8*x**n*sqrt(x**n*b + a)*b*x**2 + 8*sqr
t(x**n*b + a)*a*n*x**2 + 8*sqrt(x**n*b + a)*a*x**2 + 9*int((sqrt(x**n*b +
a)*x)/(3*x**n*b*n**2 + 16*x**n*b*n + 16*x**n*b + 3*a*n**2 + 16*a*n + 16*a)
,x)*a**2*n**4 + 48*int((sqrt(x**n*b + a)*x)/(3*x**n*b*n**2 + 16*x**n*b*n +
 16*x**n*b + 3*a*n**2 + 16*a*n + 16*a),x)*a**2*n**3 + 48*int((sqrt(x**n*b
+ a)*x)/(3*x**n*b*n**2 + 16*x**n*b*n + 16*x**n*b + 3*a*n**2 + 16*a*n + 16*
a),x)*a**2*n**2)/(3*n**2 + 16*n + 16)
```

3.511 $\int (a+bx^n)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int (a+bx^n)^{3/2}\,dx=\frac{x(a+bx^n)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2}+\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

output
```
x*(a+b*x^n)^(5/2)*hypergeom([1, 5/2+1/n],[1+1/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+bx^n)^{3/2}\,dx=\frac{ax\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(a + b*x^n)^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/Sqrt[1 + (b*x^n)/a]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^{3/2}\,dx$$

$$\Big\downarrow \text{779}$$

$$\frac{a\sqrt{a+bx^n}\int \left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{778}$$

$$\frac{ax\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/Sqrt[1 + (b*x^n)/a]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^{n})^{\frac{3}{2}}\,dx$$

input
```
int((a+b*x^n)^(3/2),x)
```

output
```
int((a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+bx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+bx^n)^{3/2}\,dx = \frac{a^{\frac{1}{n}}a^{\frac{3}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2),x)
```

output
```
a**(1/n)*a**(3/2 - 1/n)*x*gamma(1/n)*hyper((-3/2, 1/n), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^{3/2}\,dx = \int (bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int (a+bx^n)^{3/2}\,dx = \int (bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int (a+bx^n)^{3/2}\,dx=\frac{x\,{\left(a+b\,x^n\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(-\frac{3}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{{\left(\frac{b\,x^n}{a}+1\right)}^{3/2}}$$

input
```
int((a + b*x^n)^(3/2),x)
```

output
```
(x*(a + b*x^n)^(3/2)*hypergeom([-3/2, 1/n], 1/n + 1, -(b*x^n)/a))/((b*x^n)
/a + 1)^(3/2)
```

Reduce [F]

$$\int (a+bx^n)^{3/2}\,dx=\frac{2x^n\sqrt{x^nb+a}\,bnx+4x^n\sqrt{x^nb+a}\,bx+8\sqrt{x^nb+a}\,anx+4\sqrt{x^nb+a}\,ax+9\left(\int\frac{\sqrt{}}{3x^nbn^2+8x^nbn\cdots}\right.\cdots}{}$$

input
```
int((a+b*x^n)^(3/2),x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n*x + 4*x**n*sqrt(x**n*b + a)*b*x + 8*sqrt(x**n
*b + a)*a*n*x + 4*sqrt(x**n*b + a)*a*x + 9*int(sqrt(x**n*b + a)/(3*x**n*b*
n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**4 + 24*i
nt(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*
a*n + 4*a),x)*a**2*n**3 + 12*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*
b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**2)/(3*n**2 + 8*n + 4)
```

3.512 $\int \frac{(a+bx^n)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx=\frac{2a\sqrt{a+bx^n}}{n}+\frac{2(a+bx^n)^{3/2}}{3n}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*a*(a+b*x^n)^(1/2)/n+2/3*(a+b*x^n)^(3/2)/n-2*a^(3/2)*arctanh((a+b*x^n)^(1
/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx=\frac{2\sqrt{a+bx^n}(4a+bx^n)-6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3n}$$

input
```
Integrate[(a + b*x^n)^(3/2)/x,x]
```

output
```
(2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[
a]])/(3*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}(bx^n+a)^{3/2}\,dx^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int x^{-n}\sqrt{bx^n+a}dx^n+\frac{2}{3}(a+bx^n)^{3/2}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^n)^{3/2}}{n}$$

input
```
Int[(a + b*x^n)^(3/2)/x,x]
```

output
```
((2*(a + b*x^n)^(3/2))/3 + a*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x^n]/Sqrt[a]]))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{n}$	48
default	$\frac{\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^{n}}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{n}$	48
risch	$\frac{2\left(b\,{\mathrm e}^{n\ln\left(x\right)}+4a\right)\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{3n}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{\sqrt{a}}\right)}{n}$	53

input
```
int((a+b*x^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/3*(a+b*x^n)^(3/2)+2*a*(a+b*x^n)^(1/2)-2*a^(3/2)*arctanh((a+b*x^n)^(
1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx = \left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(bx^n+4\,a)\sqrt{bx^n+a}}{3\,n}, \frac{2\left(3\,\sqrt{-a}a\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)\right.}{3\,n}\right.$$

input
```
integrate((a+b*x^n)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*
x^n + 4*a)*sqrt(b*x^n + a))/n, 2/3*(3*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*x^
n + a)) + (b*x^n + 4*a)*sqrt(b*x^n + a))/n]
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx = \frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n} - \frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n} + \frac{2\sqrt{a}bx^n\sqrt{1+\frac{bx^n}{a}}}{3n}$$

input
```
integrate((a+b*x**n)**(3/2)/x,x)
```

output
```
8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2
)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx = \frac{a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\left((bx^n+a)^{\frac{3}{2}}+3\sqrt{bx^n+a}\,a\right)}{3\,n}$$

input
```
integrate((a+b*x^n)^(3/2)/x,x, algorithm="maxima")
```

output
```
a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
/3*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx = \int \frac{(a+b\,x^n)^{3/2}}{x}\,dx$$

input
```
int((a + b*x^n)^(3/2)/x,x)
```

output
```
int((a + b*x^n)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}}{x}\,dx = \frac{2x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,a+3\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)a^2n}{3n}$$

input
```
int((a+b*x^n)^(3/2)/x,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b + a)*a + 3*int(sqrt(x**n*b + a)
/(x**n*b*x + a*x),x)*a**2*n)/(3*n)
```

3.513 $\int \frac{(a+bx^n)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^n)^{3/2}}{x^2} dx = -\frac{(a+bx^n)^{5/2}\text{Hypergeometric2F1}\left(1,\frac{5}{2}-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax}$$

output
```
-(a+b*x^n)^(5/2)*hypergeom([1, 5/2-1/n],[-(1-n)/n],-b*x^n/a)/a/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^n)^{3/2}}{x^2} dx = -\frac{a\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(a + b*x^n)^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -n^(-1), 1 - n^(-1), -((b*x^n
)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{a\sqrt{a+bx^n}\int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{x^2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{a\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -n^(-1), -((1 - n)/n), -((b*x
^n)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^n)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a+b*x^n)^(3/2)/x^2,x)
```

output
```
int((a+b*x^n)^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx = \frac{a^{-\frac{1}{n}}a^{\frac{3}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)/x**2,x)
```

output
```
a**(3/2 + 1/n)*gamma(-1/n)*hyper((-3/2, -1/n), (1 - 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx=\int \frac{(a+b\,x^n)^{3/2}}{x^2}\,dx$$

input
```
int((a + b*x^n)^(3/2)/x^2,x)
```

output
```
int((a + b*x^n)^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}}{x^2}\,dx=\frac{2x^n\sqrt{x^nb+a}\,bn-4x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,an-4\sqrt{x^nb+a}\,a+9\Big(\int \frac{}{3x^nbn^2x^2-8x^n}$$

input
```
int((a+b*x^n)^(3/2)/x^2,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n - 4*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b +
 a)*a*n - 4*sqrt(x**n*b + a)*a + 9*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**
2 - 8*x**n*b*n*x**2 + 4*x**n*b*x**2 + 3*a*n**2*x**2 - 8*a*n*x**2 + 4*a*x**
2),x)*a**2*n**4*x - 24*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**2 - 8*x**n*b
*n*x**2 + 4*x**n*b*x**2 + 3*a*n**2*x**2 - 8*a*n*x**2 + 4*a*x**2),x)*a**2*n
**3*x + 12*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**2 - 8*x**n*b*n*x**2 + 4*
x**n*b*x**2 + 3*a*n**2*x**2 - 8*a*n*x**2 + 4*a*x**2),x)*a**2*n**2*x)/(x*(3
*n**2 - 8*n + 4))
```

3.514 $\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = -\frac{(a+bx^n)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2}-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2}$$

output
```
-1/2*(a+b*x^n)^(5/2)*hypergeom([1, 5/2-2/n],[-(2-n)/n],-b*x^n/a)/a/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(a + b*x^n)^(3/2)/x^3,x]
```

output
```
-1/2*(a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -2/n, 1 - 2/n, -((b*x^n)/a
)])/(x^2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{a\sqrt{a+bx^n} \int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{x^3}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{a\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(3/2)/x^3,x]
```

output
```
-1/2*(a*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -2/n, -((2 - n)/n), -((b*x
^n)/a)])/(x^2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int((a+b*x^n)^(3/2)/x^3,x)
```

output
```
int((a+b*x^n)^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = \frac{a^{-\frac{2}{n}}a^{\frac{3}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)/x**3,x)
```

output
```
a**(3/2 + 2/n)*gamma(-2/n)*hyper((-3/2, -2/n), (1 - 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = \int \frac{(a+b\,x^n)^{3/2}}{x^3}\,dx$$

input
```
int((a + b*x^n)^(3/2)/x^3,x)
```

output
```
int((a + b*x^n)^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}}{x^3}\,dx = \frac{2x^n\sqrt{x^nb+a}\,bn-8x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,an-8\sqrt{x^nb+a}\,a+9\left(\int \frac{}{3x^nbn^2x^3-16x}\right.}{}$$

input
```
int((a+b*x^n)^(3/2)/x^3,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n - 8*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b +
 a)*a*n - 8*sqrt(x**n*b + a)*a + 9*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**
3 - 16*x**n*b*n*x**3 + 16*x**n*b*x**3 + 3*a*n**2*x**3 - 16*a*n*x**3 + 16*a
*x**3),x)*a**2*n**4*x**2 - 48*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**3 - 1
6*x**n*b*n*x**3 + 16*x**n*b*x**3 + 3*a*n**2*x**3 - 16*a*n*x**3 + 16*a*x**3
),x)*a**2*n**3*x**2 + 48*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**3 - 16*x**
n*b*n*x**3 + 16*x**n*b*x**3 + 3*a*n**2*x**3 - 16*a*n*x**3 + 16*a*x**3),x)*
a**2*n**2*x**2)/(x**2*(3*n**2 - 16*n + 16))
```

3.515 $\int x(a+bx^n)^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x(a+bx^n)^{5/2}\,dx=\frac{x^2(a+bx^n)^{7/2}\,\text{Hypergeometric2F1}\left(1,\frac{7}{2}+\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a}$$

output
```
1/2*x^2*(a+b*x^n)^(7/2)*hypergeom([1, 7/2+2/n],[(2+n)/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int x(a+bx^n)^{5/2}\,dx=\frac{a^2x^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x*(a + b*x^n)^(5/2),x]
```

output
```
(a^2*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, 2/n, 1 + 2/n, -((b*x^n)/a
)])/(2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^{5/2}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{a^2\sqrt{a+bx^n}\int x\left(\frac{bx^n}{a}+1\right)^{5/2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{a^2x^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[x*(a + b*x^n)^(5/2),x]
```

output
```
(a^2*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, 2/n, (2 + n)/n, -((b*x^n)
/a)])/(2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x(a+b\,x^n)^{\frac{5}{2}}\,dx$$

input
```
int(x*(a+b*x^n)^(5/2),x)
```

output
```
int(x*(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+bx^n)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x(a+bx^n)^{5/2}\,dx = \frac{a^{\frac{2}{n}}a^{\frac{5}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**(5/2),x)
```

output
```
a**(2/n)*a**(5/2 - 2/n)*x**2*gamma(2/n)*hyper((-5/2, 2/n), (1 + 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n))
```

Maxima [F]

$$\int x(a+bx^n)^{5/2}\,dx=\int (bx^n+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(5/2)*x, x)
```

Giac [F]

$$\int x(a+bx^n)^{5/2}\,dx=\int (bx^n+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n)^{5/2}\,dx=\int x\,(a+b\,x^n)^{5/2}\,dx$$

input
```
int(x*(a + b*x^n)^(5/2),x)
```

output
```
int(x*(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int x(a+bx^n)^{5/2}\,dx=\frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2x^2+32x^{2n}\sqrt{x^nb+a}\,b^2n\,x^2+32x^{2n}\sqrt{x^nb+a}\,b^2x^2+22x^n\sqrt{x^nb+a}\,ab\,n^2}{}$$

input
```
int(x*(a+b*x^n)^(5/2),x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2*x**2 + 32*x**(2*n)*sqrt(x**n*b + a)
*b**2*n*x**2 + 32*x**(2*n)*sqrt(x**n*b + a)*b**2*x**2 + 22*x**n*sqrt(x**n*
b + a)*a*b*n**2*x**2 + 104*x**n*sqrt(x**n*b + a)*a*b*n*x**2 + 64*x**n*sqrt
(x**n*b + a)*a*b*x**2 + 46*sqrt(x**n*b + a)*a**2*n**2*x**2 + 72*sqrt(x**n*
b + a)*a**2*n*x**2 + 32*sqrt(x**n*b + a)*a**2*x**2 + 225*int((sqrt(x**n*b
 + a)*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a
*n**3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**6 + 1380*int((sqrt(x**n*b +
 a)*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a*
n**3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**5 + 2160*int((sqrt(x**n*b +
a)*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a*n
**3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**4 + 960*int((sqrt(x**n*b + a)
*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a*n**
3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**3)/(15*n**3 + 92*n**2 + 144*n +
 64)
```

3.516 $\int (a+bx^n)^{5/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int (a+bx^n)^{5/2}\,dx=\frac{x(a+bx^n)^{7/2}\,\text{Hypergeometric2F1}\left(1,\frac{7}{2}+\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

output
```
x*(a+b*x^n)^(7/2)*hypergeom([1, 7/2+1/n],[1+1/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int (a+bx^n)^{5/2}\,dx=\frac{a^2x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(a + b*x^n)^(5/2),x]
```

output
```
(a^2*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, n^(-1), 1 + n^(-1), -((b*x^
n)/a)])/Sqrt[1 + (b*x^n)/a]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^{5/2}\,dx$$

$$\Big\downarrow \text{779}$$

$$\frac{a^2\sqrt{a+bx^n}\int \left(\frac{bx^n}{a}+1\right)^{5/2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{778}$$

$$\frac{a^2x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(5/2),x]
```

output
```
(a^2*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, n^(-1), 1 + n^(-1), -((b*x^
n)/a)])/Sqrt[1 + (b*x^n)/a]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^{n})^{\frac{5}{2}}\,dx$$

input
```
int((a+b*x^n)^(5/2),x)
```

output
```
int((a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+bx^n)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int (a+bx^n)^{5/2}\,dx = \frac{a^{\frac{1}{n}}a^{\frac{5}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(5/2),x)
```

output
```
a**(1/n)*a**(5/2 - 1/n)*x*gamma(1/n)*hyper((-5/2, 1/n), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^{5/2}\,dx=\int (bx^n+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int (a+bx^n)^{5/2}\,dx=\int (bx^n+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int (a+bx^n)^{5/2}\,dx=\frac{x\,(a+b\,x^n)^{5/2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a}+1\right)^{5/2}}$$

input
```
int((a + b*x^n)^(5/2),x)
```

output
```
(x*(a + b*x^n)^(5/2)*hypergeom([-5/2, 1/n], 1/n + 1, -(b*x^n)/a))/((b*x^n)
/a + 1)^(5/2)
```

Reduce [F]

$$\int (a+bx^n)^{5/2}\,dx=\frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2x+16x^{2n}\sqrt{x^nb+a}\,b^2nx+8x^{2n}\sqrt{x^nb+a}\,b^2x+22x^n\sqrt{x^nb+a}\,ab\,n^2x+}{}$$

input
```
int((a+b*x^n)^(5/2),x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2*x + 16*x**(2*n)*sqrt(x**n*b + a)*b*
*2*n*x + 8*x**(2*n)*sqrt(x**n*b + a)*b**2*x + 22*x**n*sqrt(x**n*b + a)*a*b
*n**2*x + 52*x**n*sqrt(x**n*b + a)*a*b*n*x + 16*x**n*sqrt(x**n*b + a)*a*b*
x + 46*sqrt(x**n*b + a)*a**2*n**2*x + 36*sqrt(x**n*b + a)*a**2*n*x + 8*sqr
t(x**n*b + a)*a**2*x + 225*int(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*
b*n**2 + 36*x**n*b*n + 8*x**n*b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)
*a**3*n**6 + 690*int(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 3
6*x**n*b*n + 8*x**n*b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*n**5
 + 540*int(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n
 + 8*x**n*b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*n**4 + 120*int
(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*
b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*n**3)/(15*n**3 + 46*n**2
 + 36*n + 8)
```

3.517 $\int \frac{(a+bx^n)^{5/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{(a+bx^n)^{5/2}}{x} dx = \frac{2a^2\sqrt{a+bx^n}}{n} + \frac{2a(a+bx^n)^{3/2}}{3n} + \frac{2(a+bx^n)^{5/2}}{5n} - \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*a^2*(a+b*x^n)^(1/2)/n+2/3*a*(a+b*x^n)^(3/2)/n+2/5*(a+b*x^n)^(5/2)/n-2*a^
(5/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^n)^{5/2}}{x} dx = \frac{2\sqrt{a+bx^n}(23a^2+11abx^n+3b^2x^{2n}) - 30a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{15n}$$

input
```
Integrate[(a + b*x^n)^(5/2)/x,x]
```

output
```
(2*Sqrt[a + b*x^n]*(23*a^2 + 11*a*b*x^n + 3*b^2*x^(2*n)) - 30*a^(5/2)*ArcT
anh[Sqrt[a + b*x^n]/Sqrt[a]])/(15*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}(bx^n+a)^{5/2}\,dx^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int x^{-n}(bx^n+a)^{3/2}\,dx^n+\frac{2}{5}(a+bx^n)^{5/2}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\int x^{-n}\sqrt{bx^n+a}dx^n+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\left(a\left(a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{a\left(a\left(\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\left(a\left(2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}}{n}$$

input
```
Int[(a + b*x^n)^(5/2)/x,x]
```

output
```
((2*(a + b*x^n)^(5/2))/5 + a*((2*(a + b*x^n)^(3/2))/3 + a*(2*Sqrt[a + b*x^
n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{2(a+b\,x^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b\,x^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b\,x^n}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{n}$	62
default	$\frac{\frac{2(a+b\,x^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b\,x^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b\,x^n}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{n}$	62
risch	$\frac{2\left(3b^2\mathrm{e}^{2n\ln(x)}+11ab\,\mathrm{e}^{n\ln(x)}+23a^2\right)\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{15n}-\frac{2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)}{n}$	69

input
```
int((a+b*x^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/5*(a+b*x^n)^(5/2)+2/3*a*(a+b*x^n)^(3/2)+2*a^2*(a+b*x^n)^(1/2)-2*a^(
5/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx^n)^{5/2}}{x}\,dx = \left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(3\,b^2x^{2\,n}+11\,abx^n+23\,a^2)\sqrt{bx^n+a}}{15\,n}, \frac{2\left(15\sqrt{-a}\right.}{}\right.$$

input
```
integrate((a+b*x^n)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*a^(5/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(
3*b^2*x^(2*n) + 11*a*b*x^n + 23*a^2)*sqrt(b*x^n + a))/n, 2/15*(15*sqrt(-a)
*a^2*arctan(sqrt(-a)/sqrt(b*x^n + a)) + (3*b^2*x^(2*n) + 11*a*b*x^n + 23*a
^2)*sqrt(b*x^n + a))/n]
```

Sympy [A] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^n)^{5/2}}{x}\,dx = \frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx^n}{a}}}{15n} + \frac{a^{\frac{5}{2}}\log\left(\frac{bx^n}{a}\right)}{n} - \frac{2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n} + \frac{22a^{\frac{3}{2}}bx^n\sqrt{1+\frac{bx^n}{a}}}{15n} + \frac{2\sqrt{a}b^2x^{2n}\sqrt{1+\frac{bx^n}{a}}}{5n}$$

input
```
integrate((a+b*x**n)**(5/2)/x,x)
```

output
```
46*a**(5/2)*sqrt(1 + b*x**n/a)/(15*n) + a**(5/2)*log(b*x**n/a)/n - 2*a**(5
/2)*log(sqrt(1 + b*x**n/a) + 1)/n + 22*a**(3/2)*b*x**n*sqrt(1 + b*x**n/a)/
(15*n) + 2*sqrt(a)*b**2*x**(2*n)*sqrt(1 + b*x**n/a)/(5*n)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^{5/2}}{x}\,dx = \frac{a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\left(3\,(bx^n+a)^{\frac{5}{2}} + 5\,(bx^n+a)^{\frac{3}{2}}a + 15\,\sqrt{bx^n+a}a^2\right)}{15\,n}$$

input
```
integrate((a+b*x^n)^(5/2)/x,x, algorithm="maxima")
```

output
```
a^(5/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
/15*(3*(b*x^n + a)^(5/2) + 5*(b*x^n + a)^(3/2)*a + 15*sqrt(b*x^n + a)*a^2)
/n
```

Giac [F]

$$\int \frac{(a+bx^n)^{5/2}}{x}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{5/2}}{x}\,dx = \int \frac{(a+b\,x^n)^{5/2}}{x}\,dx$$

input
```
int((a + b*x^n)^(5/2)/x,x)
```

output
```
int((a + b*x^n)^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{5/2}}{x}\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2+22x^n\sqrt{x^nb+a}\,ab+46\sqrt{x^nb+a}\,a^2+15\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)a^3n}{15n}$$

input
```
int((a+b*x^n)^(5/2)/x,x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a*b + 46*sqrt
(x**n*b + a)*a**2 + 15*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*a**3*n)/(1
5*n)
```

3.518 $\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx = -\frac{(a+bx^n)^{7/2}\,\text{Hypergeometric2F1}\left(1,\frac{7}{2}-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax}$$

output
```
-(a+b*x^n)^(7/2)*hypergeom([1, 7/2-1/n],[-(1-n)/n],-b*x^n/a)/a/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx = -\frac{a^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(a + b*x^n)^(5/2)/x^2,x]
```

output
```
-((a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -n^(-1), 1 - n^(-1), -((b*x
^n)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{5/2}}{x^2}dx$$
$$\Big\downarrow \text{889}$$
$$\frac{a^2\sqrt{a+bx^n}\int \frac{\left(\frac{bx^n}{a}+1\right)^{5/2}}{x^2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$
$$\Big\downarrow \text{888}$$
$$-\frac{a^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(5/2)/x^2,x]
```

output
```
-((a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -n^(-1), -((1 - n)/n), -((b
*x^n)/a)])/(x*Sqrt[1 + (b*x^n)/a]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{\frac{5}{2}}}{x^{2}}dx$$

input
```
int((a+b*x^n)^(5/2)/x^2,x)
```

output
```
int((a+b*x^n)^(5/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx = \frac{a^{-\frac{1}{n}}a^{\frac{5}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(5/2)/x**2,x)
```

output
```
a**(5/2 + 1/n)*gamma(-1/n)*hyper((-5/2, -1/n), (1 - 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(5/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx=\int \frac{{\left(a+b\,x^n\right)}^{5/2}}{x^2}\,dx$$

input
```
int((a + b*x^n)^(5/2)/x^2,x)
```

output
```
int((a + b*x^n)^(5/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{5/2}}{x^2}\,dx=\frac{6x^{2n}\sqrt{x^n b+a}\,b^2n^2-16x^{2n}\sqrt{x^n b+a}\,b^2n+8x^{2n}\sqrt{x^n b+a}\,b^2+22x^n\sqrt{x^n b+a}\,ab\,n^2-}{}$$

input
```
int((a+b*x^n)^(5/2)/x^2,x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2 - 16*x**(2*n)*sqrt(x**n*b + a)*b**2
*n + 8*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a*b*n**2
- 52*x**n*sqrt(x**n*b + a)*a*b*n + 16*x**n*sqrt(x**n*b + a)*a*b + 46*sqrt(
x**n*b + a)*a**2*n**2 - 36*sqrt(x**n*b + a)*a**2*n + 8*sqrt(x**n*b + a)*a*
*2 + 225*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**2 - 46*x**n*b*n**2*x**2 +
 36*x**n*b*n*x**2 - 8*x**n*b*x**2 + 15*a*n**3*x**2 - 46*a*n**2*x**2 + 36*a
*n*x**2 - 8*a*x**2),x)*a**3*n**6*x - 690*int(sqrt(x**n*b + a)/(15*x**n*b*n
**3*x**2 - 46*x**n*b*n**2*x**2 + 36*x**n*b*n*x**2 - 8*x**n*b*x**2 + 15*a*n
**3*x**2 - 46*a*n**2*x**2 + 36*a*n*x**2 - 8*a*x**2),x)*a**3*n**5*x + 540*i
nt(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**2 - 46*x**n*b*n**2*x**2 + 36*x**n*b
*n*x**2 - 8*x**n*b*x**2 + 15*a*n**3*x**2 - 46*a*n**2*x**2 + 36*a*n*x**2 -
8*a*x**2),x)*a**3*n**4*x - 120*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**2 -
 46*x**n*b*n**2*x**2 + 36*x**n*b*n*x**2 - 8*x**n*b*x**2 + 15*a*n**3*x**2 -
 46*a*n**2*x**2 + 36*a*n*x**2 - 8*a*x**2),x)*a**3*n**3*x)/(x*(15*n**3 - 46
*n**2 + 36*n - 8))
```

3.519 $\int \frac{(a+bx^n)^{5/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^n)^{5/2}}{x^3} dx = -\frac{(a+bx^n)^{7/2} \text{Hypergeometric2F1}\left(1, \frac{7}{2}-\frac{2}{n}, -\frac{2-n}{n}, -\frac{bx^n}{a}\right)}{2ax^2}$$

output
```
-1/2*(a+b*x^n)^(7/2)*hypergeom([1, 7/2-2/n],[-(2-n)/n],-b*x^n/a)/a/x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^n)^{5/2}}{x^3} dx = -\frac{a^2\sqrt{a+bx^n} \text{Hypergeometric2F1}\left(-\frac{5}{2}, -\frac{2}{n}, 1-\frac{2}{n}, -\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(a + b*x^n)^(5/2)/x^3,x]
```

output
```
-1/2*(a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -2/n, 1 - 2/n, -((b*x^n)
/a)])/(x^2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{a^2\sqrt{a+bx^n}\int \frac{\left(\frac{bx^n}{a}+1\right)^{5/2}}{x^3}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{a^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(a + b*x^n)^(5/2)/x^3,x]
```

output
```
-1/2*(a^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -2/n, -((2 - n)/n), -((b
*x^n)/a)])/(x^2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(a+b\,x^n)^{\frac{5}{2}}}{x^3}dx$$

input
```
int((a+b*x^n)^(5/2)/x^3,x)
```

output
```
int((a+b*x^n)^(5/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx = \frac{a^{-\frac{2}{n}}a^{\frac{5}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**(5/2)/x**3,x)
```

output
```
a**(5/2 + 2/n)*gamma(-2/n)*hyper((-5/2, -2/n), (1 - 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(5/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx=\int \frac{(a+b\,x^n)^{5/2}}{x^3}\,dx$$

input
```
int((a + b*x^n)^(5/2)/x^3,x)
```

output
```
int((a + b*x^n)^(5/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{5/2}}{x^3}\,dx=\frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2-32x^{2n}\sqrt{x^nb+a}\,b^2n+32x^{2n}\sqrt{x^nb+a}\,b^2+22x^n\sqrt{x^nb+a}\,ab\,n^2}{}$$

input
```
int((a+b*x^n)^(5/2)/x^3,x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2 - 32*x**(2*n)*sqrt(x**n*b + a)*b**2
*n + 32*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a*b*n**2
 - 104*x**n*sqrt(x**n*b + a)*a*b*n + 64*x**n*sqrt(x**n*b + a)*a*b + 46*sqr
t(x**n*b + a)*a**2*n**2 - 72*sqrt(x**n*b + a)*a**2*n + 32*sqrt(x**n*b + a)
*a**2 + 225*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**3 - 92*x**n*b*n**2*x**
3 + 144*x**n*b*n*x**3 - 64*x**n*b*x**3 + 15*a*n**3*x**3 - 92*a*n**2*x**3 +
 144*a*n*x**3 - 64*a*x**3),x)*a**3*n**6*x**2 - 1380*int(sqrt(x**n*b + a)/(
15*x**n*b*n**3*x**3 - 92*x**n*b*n**2*x**3 + 144*x**n*b*n*x**3 - 64*x**n*b*
x**3 + 15*a*n**3*x**3 - 92*a*n**2*x**3 + 144*a*n*x**3 - 64*a*x**3),x)*a**3
*n**5*x**2 + 2160*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**3 - 92*x**n*b*n*
*2*x**3 + 144*x**n*b*n*x**3 - 64*x**n*b*x**3 + 15*a*n**3*x**3 - 92*a*n**2*
x**3 + 144*a*n*x**3 - 64*a*x**3),x)*a**3*n**4*x**2 - 960*int(sqrt(x**n*b +
 a)/(15*x**n*b*n**3*x**3 - 92*x**n*b*n**2*x**3 + 144*x**n*b*n*x**3 - 64*x*
*n*b*x**3 + 15*a*n**3*x**3 - 92*a*n**2*x**3 + 144*a*n*x**3 - 64*a*x**3),x)
*a**3*n**3*x**2)/(x**2*(15*n**3 - 92*n**2 + 144*n - 64))
```

3.520 $\int \frac{x}{\sqrt{a+bx^n}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{x}{\sqrt{a+bx^n}} \, dx = \frac{x^2\sqrt{a+bx^n} \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a}$$

output
```
1/2*x^2*(a+b*x^n)^(1/2)*hypergeom([1, 1/2+2/n],[(2+n)/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{x}{\sqrt{a+bx^n}} \, dx = \frac{x^2\sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}$$

input
```
Integrate[x/Sqrt[a + b*x^n],x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2/n, 1 + 2/n, -((b*x^n)/a)
])/(2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}$$

input
```
Int[x/Sqrt[a + b*x^n],x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2/n, (2 + n)/n, -((b*x^n)/
a)])/(2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x/(a+b*x^n)^(1/2),x)
```

output
```
int(x/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{a+bx^n}}\,dx = \frac{a^{\frac{2}{n}}a^{-\frac{1}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}$$

input
```
integrate(x/(a+b*x**n)**(1/2),x)
```

output
```
a**(2/n)*a**(-1/2 - 2/n)*x**2*gamma(2/n)*hyper((1/2, 2/n), (1 + 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n))
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^n}}\,dx=\int \frac{x}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^n}}\,dx=\int \frac{x}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^n}}\,dx = \int \frac{x}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x/(a + b*x^n)^(1/2),x)
```

output
```
int(x/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+bx^n}}\,dx = \int \frac{\sqrt{x^nb+a}\,x}{x^nb+a}dx$$

input
```
int(x/(a+b*x^n)^(1/2),x)
```

output
```
int((sqrt(x**n*b + a)*x)/(x**n*b + a),x)
```

3.521 $\int \frac{1}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx=\frac{x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

output
```
x*(a+b*x^n)^(1/2)*hypergeom([1, 1/2+1/n],[1+1/n],-b*x^n/a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx=\frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

input
```
Integrate[1/Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/Sqrt[a + b*x^n]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx$$
$$\Big\downarrow \text{779}$$
$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$
$$\Big\downarrow \text{778}$$
$$\frac{x\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

input
```
Int[1/Sqrt[a + b*x^n],x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/Sqrt[a + b*x^n]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(1/(a+b*x^n)^(1/2),x)
```

output
```
int(1/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{1}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate(1/(a+b*x**n)**(1/2),x)
```

output
```
a**(1/n)*a**(-1/2 - 1/n)*x*gamma(1/n)*hyper((1/2, 1/n), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(1/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx = \frac{x\,\sqrt{\frac{b\,x^n}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\sqrt{a+b\,x^n}}$$

input
```
int(1/(a + b*x^n)^(1/2),x)
```

output
```
(x*((b*x^n)/a + 1)^(1/2)*hypergeom([1/2, 1/n], 1/n + 1, -(b*x^n)/a))/(a +
b*x^n)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^n}}\,dx = \int \frac{\sqrt{x^nb+a}}{x^nb+a}dx$$

input
```
int(1/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b + a),x)
```

3.522 $\int \frac{1}{x\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \ 73$$

$$\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{bn}$$

$$\Big\downarrow \ 221$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	23
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	23

input
```
int(1/x/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)}{\sqrt{a}n}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)}{an}\right]$$

input
```
integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*n), 2*sqrt(-a
)*arctan(sqrt(-a)/sqrt(b*x^n + a))/(a*n)]
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = -\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}n}$$

input
```
integrate(1/x/(a+b*x**n)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(sqrt(a)*n)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*n)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx=\int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx$$

input
```
int(1/x/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)
```

3.523 $\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx=-\frac{\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax}$$

output
```
-(a+b*x^n)^(1/2)*hypergeom([1, 1/2-1/n],[-(1-n)/n],-b*x^n/a)/a/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx=-\frac{\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^n]),x]
```

output
```
-((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -n^(-1), 1 - n^(-1), -((b*x^
n)/a)])/(x*Sqrt[a + b*x^n]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{x^2\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^n]),x]
```

output
```
-((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -n^(-1), -((1 - n)/n), -((b*
x^n)/a)])/(x*Sqrt[a + b*x^n]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{a+b\,x^n}}dx$$

input
```
int(1/x^2/(a+b*x^n)^(1/2),x)
```

output
```
int(1/x^2/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx = \frac{a^{-\frac{1}{n}}a^{-\frac{1}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate(1/x**2/(a+b*x**n)**(1/2),x)
```

output
```
a**(-1/2 + 1/n)*gamma(-1/n)*hyper((1/2, -1/n), (1 - 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx=\int \frac{1}{x^2\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^2*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^n}}\,dx=\int \frac{\sqrt{x^nb+a}}{x^nb\,x^2+a\,x^2}dx$$

input
```
int(1/x^2/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x**2 + a*x**2),x)
```

3.524 $\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx=-\frac{\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2}$$

output
```
-1/2*(a+b*x^n)^(1/2)*hypergeom([1, 1/2-2/n],[-(2-n)/n],-b*x^n/a)/a/x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx=-\frac{\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^n]),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2/n, 1 - 2/n, -((b*x^n)/
a)])/(x^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{x^3\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^n]),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2/n, -((2 - n)/n), -((b*
x^n)/a)])/(x^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{a+b\,x^n}}dx$$

input
```
int(1/x^3/(a+b*x^n)^(1/2),x)
```

output
```
int(1/x^3/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx = \frac{a^{-\frac{2}{n}}a^{-\frac{1}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate(1/x**3/(a+b*x**n)**(1/2),x)
```

output
```
a**(-1/2 + 2/n)*gamma(-2/n)*hyper((1/2, -2/n), (1 - 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^n + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx=\int \frac{1}{x^3\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^3*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a+bx^n}}\,dx=\int \frac{\sqrt{x^n b+a}}{x^n b\,x^3+a\,x^3}dx$$

input
```
int(1/x^3/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x**3 + a*x**3),x)
```

3.525 $\int \frac{x}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx=\frac{x^2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2}+\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}$$

output
```
1/2*x^2*hypergeom([1, -1/2+2/n],[(2+n)/n],-b*x^n/a)/a/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx=\frac{x^2\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2}{n},1+\frac{2}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}$$

input
```
Integrate[x/(a + b*x^n)^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 2/n, 1 + 2/n, -((b*x^n)/a)
])/(2*a*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}$$

input
```
Int[x/(a + b*x^n)^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 2/n, (2 + n)/n, -((b*x^n)/
a)])/(2*a*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{(a+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int(x/(a+b*x^n)^(3/2),x)
```

output
```
int(x/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx = \frac{a^{\frac{2}{n}}a^{-\frac{3}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}$$

input
```
integrate(x/(a+b*x**n)**(3/2),x)
```

output
```
a**(2/n)*a**(-3/2 - 2/n)*x**2*gamma(2/n)*hyper((3/2, 2/n), (1 + 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n))
```

Maxima [F]

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx = \int \frac{x}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx = \int \frac{x}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx = \int \frac{x}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int(x/(a + b*x^n)^(3/2),x)
```

output
```
int(x/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{x^n b+a}\,x}{x^{2n}b^2+2x^n ab+a^2}dx$$

input
```
int(x/(a+b*x^n)^(3/2),x)
```

output
```
int((sqrt(x**n*b + a)*x)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.526 $\int \frac{1}{(a+bx^n)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(a+bx^n)^{3/2}} dx = \frac{x \,\text{Hypergeometric2F1}\left(1,-\frac{1}{2}+\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{a+bx^n}}$$

output
```
x*hypergeom([1, -1/2+1/n],[1+1/n],-b*x^n/a)/a/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+bx^n)^{3/2}} dx = \frac{x\sqrt{1+\frac{bx^n}{a}} \,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{a+bx^n}}$$

input
```
Integrate[(a + b*x^n)^(-3/2),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^{3/2}} dx$$
$$\Big\downarrow \text{779}$$
$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}$$
$$\Big\downarrow \text{778}$$
$$\frac{x\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a\sqrt{a+bx^n}}$$

input
```
Int[(a + b*x^n)^(-3/2),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(a+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*x^n)^(3/2),x)
```

output
```
int(1/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+bx^n)^{3/2}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{3}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate(1/(a+b*x**n)**(3/2),x)
```

output
```
a**(1/n)*a**(-3/2 - 1/n)*x*gamma(1/n)*hyper((3/2, 1/n), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^{3/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^{3/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx^n)^{3/2}}\,dx = \frac{x\left(\frac{b\,x^n}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(a+b\,x^n\right)^{3/2}}$$

input
```
int(1/(a + b*x^n)^(3/2),x)
```

output
```
(x*((b*x^n)/a + 1)^(3/2)*hypergeom([3/2, 1/n], 1/n + 1, -(b*x^n)/a))/(a +
b*x^n)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{x^n b+a}}{x^{2n}b^2+2x^n ab+a^2}dx$$

input
```
int(1/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.527 $\int \frac{1}{x(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx=\frac{2}{an\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

output
```
2/a/n/(a+b*x^n)^(1/2)-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx=\frac{2}{an\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

input
```
Integrate[1/(x*(a + b*x^n)^(3/2)),x]
```

output
```
2/(a*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{(bx^n+a)^{3/2}}dx^n}{n}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{a}+\frac{2}{a\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{ab}+\frac{2}{a\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2}{a\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2))/n
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	39
default	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	39

input
```
int(1/x/(a+b*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/a/(a+b*x^n)^(1/2)-2/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.94

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx = \left[\frac{\left(\sqrt{a}bx^n+a^{\frac{3}{2}}\right)\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\sqrt{bx^n+a}a}{a^2bnx^n+a^3n}, \frac{2\left(\left(\sqrt{-a}bx^n+\sqrt{-a}a\right)\text{a}\right.}{a^2bn}\right.$$

input
```
integrate(1/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
[((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/
x^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*n*x^n + a^3*n), 2*((sqrt(-a)*b*x^n + sq
rt(-a)*a)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + sqrt(b*x^n + a)*a)/(a^2*b*n*x
^n + a^3*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(39) = 78.

Time = 1.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.83

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx = \frac{2a^3\sqrt{1+\frac{bx^n}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^3\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}+\frac{a^2bx^n\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}-\frac{2a^2bx^n\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}$$

input
```
integrate(1/x/(a+b*x**n)**(3/2),x)
```

output
```
2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x*
*n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) + 1
)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*n
 + a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/2
)*n + a**(7/2)*b*n*x**n)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx = \frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n} + \frac{2}{\sqrt{bx^n+a}an}$$

input
```
integrate(1/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n) +
 2/(sqrt(b*x^n + a)*a*n)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{1}{x\left(a+b\,x^n\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*x^n)^(3/2)),x)
```

output
```
int(1/(x*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{\sqrt{x^nb+a}}{x^{2n}b^2x+2x^nabx+a^2x}dx$$

input
```
int(1/x/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x + 2*x**n*a*b*x + a**2*x),x)
```

3.528 $\int \frac{1}{x^2(a+bx^n)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}} \, dx = -\frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{2}-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}$$

output
```
-hypergeom([1, -1/2-1/n],[-(1-n)/n],-b*x^n/a)/a/x/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}} \, dx = -\frac{\sqrt{1+\frac{bx^n}{a}}\, \text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}$$

input
```
Integrate[1/(x^2*(a + b*x^n)^(3/2)),x]
```

output
```
-((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -n^(-1), 1 - n^(-1), -((b*x^
n)/a)])/(a*x*Sqrt[a + b*x^n]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1} \int \frac{1}{x^2 \left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1} \, \text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}$$

input
```
Int[1/(x^2*(a + b*x^n)^(3/2)),x]
```

output
```
-((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -n^(-1), -((1 - n)/n), -((b*
x^n)/a)])/(a*x*Sqrt[a + b*x^n]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2\left(a+b\,x^n\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a+b*x^n)^(3/2),x)
```

output
```
int(1/x^2/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(a+bx^n\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}}\,dx = \frac{a^{-\frac{1}{n}}a^{-\frac{3}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate(1/x**2/(a+b*x**n)**(3/2),x)
```

output
```
a**(-3/2 + 1/n)*gamma(-1/n)*hyper((3/2, -1/n), (1 - 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}} \, dx = \int \frac{1}{x^2 \left(a+b\,x^n\right)^{3/2}} \,dx$$

input
```
int(1/(x^2*(a + b*x^n)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^n)^{3/2}} \, dx = \int \frac{\sqrt{x^n b+a}}{x^{2n} b^2 x^2+2 x^n a b\, x^2+a^2 x^2} dx$$

input
```
int(1/x^2/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x**2 + 2*x**n*a*b*x**2 + a**2*x**2),x)
```

3.529 $\int \frac{1}{x^3(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^3 (a+bx^n)^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{2}-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}$$

output
```
-1/2*hypergeom([1, -1/2-2/n],[-(2-n)/n],-b*x^n/a)/a/x^2/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3 (a+bx^n)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}$$

input
```
Integrate[1/(x^3*(a + b*x^n)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -2/n, 1 - 2/n, -((b*x^n)/
a)])/(a*x^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{x^3\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}$$

input
```
Int[1/(x^3*(a + b*x^n)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -2/n, -((2 - n)/n), -((b*
x^n)/a)])/(a*x^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(a+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^3/(a+b*x^n)^(3/2),x)
```

output
```
int(1/x^3/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx^n)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a+bx^n)^{3/2}}\,dx = \frac{a^{-\frac{2}{n}} a^{-\frac{3}{2}+\frac{2}{n}} \Gamma\left(-\frac{2}{n}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, -\frac{2}{n} \\ 1-\frac{2}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{nx^2 \Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate(1/x**3/(a+b*x**n)**(3/2),x)
```

output
```
a**(-3/2 + 2/n)*gamma(-2/n)*hyper((3/2, -2/n), (1 - 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^n)^{3/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^n)^{3/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n)^{3/2}}\, dx = \int \frac{1}{x^3 (a + b\, x^n)^{3/2}}\, dx$$

input
```
int(1/(x^3*(a + b*x^n)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a + bx^n)^{3/2}}\, dx = \int \frac{\sqrt{x^n b + a}}{x^{2n} b^2 x^3 + 2x^n ab\, x^3 + a^2 x^3} dx$$

input
```
int(1/x^3/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x**3 + 2*x**n*a*b*x**3 + a**2*x**3),x)
```

3.530 $\int \frac{x}{(a+bx^n)^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{x}{(a+bx^n)^{5/2}} dx = \frac{x^2 \operatorname{Hypergeometric2F1}\left(1, -\frac{3}{2}+\frac{2}{n}, \frac{2+n}{n}, -\frac{bx^n}{a}\right)}{2a(a+bx^n)^{3/2}}$$

output
```
1/2*x^2*hypergeom([1, -3/2+2/n],[(2+n)/n],-b*x^n/a)/a/(a+b*x^n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{x}{(a+bx^n)^{5/2}} dx = \frac{x^2 \sqrt{1+\frac{bx^n}{a}} \operatorname{Hypergeometric2F1}\left(\frac{5}{2}, \frac{2}{n}, 1+\frac{2}{n}, -\frac{bx^n}{a}\right)}{2a^2\sqrt{a+bx^n}}$$

input
```
Integrate[x/(a + b*x^n)^(5/2),x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 2/n, 1 + 2/n, -((b*x^n)/a)
])/(2*a^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \ 889$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x}{\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^2\sqrt{a+bx^n}}$$

input
```
Int[x/(a + b*x^n)^(5/2),x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 2/n, (2 + n)/n, -((b*x^n)/
a)])/(2*a^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\left(a+b\,x^{n}\right)^{\frac{5}{2}}}dx$$

input
```
int(x/(a+b*x^n)^(5/2),x)
```

output
```
int(x/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{x}{(a+bx^n)^{5/2}}\,dx = \frac{a^{\frac{2}{n}}a^{-\frac{5}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}$$

input
```
integrate(x/(a+b*x**n)**(5/2),x)
```

output
```
a**(2/n)*a**(-5/2 - 2/n)*x**2*gamma(2/n)*hyper((5/2, 2/n), (1 + 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n))
```

Maxima [F]

$$\int \frac{x}{(a+bx^n)^{5/2}}\,dx = \int \frac{x}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^n)^{5/2}}\,dx = \int \frac{x}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)^{5/2}}\,dx = \int \frac{x}{(a+b\,x^n)^{5/2}}\,dx$$

input
```
int(x/(a + b*x^n)^(5/2),x)
```

output
```
int(x/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^n)^{5/2}}\,dx = \int \frac{\sqrt{x^n b+a}\,x}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2 b+a^3}dx$$

input
```
int(x/(a+b*x^n)^(5/2),x)
```

output
```
int((sqrt(x**n*b + a)*x)/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*
b + a**3),x)
```

3.531 $\int \frac{1}{(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \frac{x\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2}+\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(a+bx^n)^{3/2}}$$

output
```
x*hypergeom([1, -3/2+1/n],[1+1/n],-b*x^n/a)/a/(a+b*x^n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2\sqrt{a+bx^n}}$$

input
```
Integrate[(a + b*x^n)^(-5/2),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^{5/2}} \, dx$$

$$\Big\downarrow \; 779$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}$$

$$\Big\downarrow \; 778$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2\sqrt{a+bx^n}}$$

input
```
Int[(a + b*x^n)^(-5/2),x]
```

output
```
(x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*x^n)^(5/2),x)
```

output
```
int(1/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \frac{a^{\frac{1}{n}}a^{-\frac{5}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate(1/(a+b*x**n)**(5/2),x)
```

output
```
a**(1/n)*a**(-5/2 - 1/n)*x*gamma(1/n)*hyper((5/2, 1/n), (1 + 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \frac{x\,\left(\frac{b\,x^n}{a}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{(a+b\,x^n)^{5/2}}$$

input
```
int(1/(a + b*x^n)^(5/2),x)
```

output
```
(x*((b*x^n)/a + 1)^(5/2)*hypergeom([5/2, 1/n], 1/n + 1, -(b*x^n)/a))/(a +
b*x^n)^(5/2)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^{5/2}}\,dx = \int \frac{\sqrt{x^n b+a}}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2 b+a^3}dx$$

input
```
int(1/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b +
a**3),x)
```

3.532 $\int \frac{1}{x(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{1}{x(a+bx^n)^{5/2}}\,dx = \frac{2}{3an(a+bx^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+bx^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

output
```
2/3/a/n/(a+b*x^n)^(3/2)+2/a^2/n/(a+b*x^n)^(1/2)-2*arctanh((a+b*x^n)^(1/2)/
a^(1/2))/a^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(a+bx^n)^{5/2}}\,dx = \frac{2(a+3(a+bx^n))}{3a^2n(a+bx^n)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

input
```
Integrate[1/(x*(a + b*x^n)^(5/2)),x]
```

output
```
(2*(a + 3*(a + b*x^n)))/(3*a^2*n*(a + b*x^n)^(3/2)) - (2*ArcTanh[Sqrt[a +
b*x^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b x^n)^{5/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{(b x^n+a)^{5/2}}dx^n}{n}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{x^{-n}}{(b x^n+a)^{3/2}}dx^n}{a}+\frac{2}{3 a (a+b x^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\frac{\int \frac{x^{-n}}{\sqrt{b x^n+a}}dx^n}{a}+\frac{2}{a \sqrt{a+b x^n}}}{a}+\frac{2}{3 a (a+b x^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{2 \int \frac{1}{\frac{x^{2 n}}{b}-\frac{a}{b}}d\sqrt{b x^n+a}}{a b}+\frac{2}{a \sqrt{a+b x^n}}}{a}+\frac{2}{3 a (a+b x^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{2}{a \sqrt{a+b x^n}}-\frac{2 \text{arctanh}\left(\frac{\sqrt{a+b x^n}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3 a (a+b x^n)^{3/2}}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^(5/2)),x]
```

output
```
(2/(3*a*(a + b*x^n)^(3/2)) + (2/(a*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a +
b*x^n]/Sqrt[a]])/a^(3/2))/a)/n
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{a+b\,x^{n}}}+\frac{2}{3a\left(a+b\,x^{n}\right)^{\frac{3}{2}}}}{n}$	53
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{a+b\,x^{n}}}+\frac{2}{3a\left(a+b\,x^{n}\right)^{\frac{3}{2}}}}{n}$	53

input
```
int(1/x/(a+b*x^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-2/a^(5/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))+2/a^2/(a+b*x^n)^(1/2)+2/3
/a/(a+b*x^n)^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.29

$$\int \frac{1}{x\left(a+bx^n\right)^{5/2}}\,dx = \left[\frac{3\left(\sqrt{a}b^2x^{2n}+2a^{\frac{3}{2}}bx^n+a^{\frac{5}{2}}\right)\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\left(3abx^n+4a^2\right)\sqrt{bx^n+}}{3\left(a^3b^2nx^{2n}+2a^4bnx^n+a^5n\right)}\right.$$

input
```
integrate(1/x/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(sqrt(a)*b^2*x^(2*n) + 2*a^(3/2)*b*x^n + a^(5/2))*log((b*x^n - 2*s
qrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(3*a*b*x^n + 4*a^2)*sqrt(b*x^n + a)
)/(a^3*b^2*n*x^(2*n) + 2*a^4*b*n*x^n + a^5*n), 2/3*(3*(sqrt(-a)*b^2*x^(2*n
) + 2*sqrt(-a)*a*b*x^n + sqrt(-a)*a^2)*arctan(sqrt(-a)/sqrt(b*x^n + a)) +
(3*a*b*x^n + 4*a^2)*sqrt(b*x^n + a))/(a^3*b^2*n*x^(2*n) + 2*a^4*b*n*x^n +
a^5*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(58) = 116$.

Time = 2.86 (sec) , antiderivative size = 860, normalized size of antiderivative = 12.46

$$\int \frac{1}{x\left(a+bx^n\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b*x**n)**(5/2),x)
```

output
```
8*a**7*sqrt(1 + b*x**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15
/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) + 3*a**7*log(b*x**n/a)/
(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a*
*(13/2)*b**3*n*x**(3*n)) - 6*a**7*log(sqrt(1 + b*x**n/a) + 1)/(3*a**(19/2)
*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3
*n*x**(3*n)) + 14*a**6*b*x**n*sqrt(1 + b*x**n/a)/(3*a**(19/2)*n + 9*a**(17
/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n))
+ 9*a**6*b*x**n*log(b*x**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a*
*(15/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) - 18*a**6*b*x**n*lo
g(sqrt(1 + b*x**n/a) + 1)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15
/2)*b**2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) + 6*a**5*b**2*x**(2*n)*
sqrt(1 + b*x**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**
2*n*x**(2*n) + 3*a**(13/2)*b**3*n*x**(3*n)) + 9*a**5*b**2*x**(2*n)*log(b*x
**n/a)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n)
 + 3*a**(13/2)*b**3*n*x**(3*n)) - 18*a**5*b**2*x**(2*n)*log(sqrt(1 + b*x**
n/a) + 1)/(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2
*n) + 3*a**(13/2)*b**3*n*x**(3*n)) + 3*a**4*b**3*x**(3*n)*log(b*x**n/a)/(3
*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3*a**(
13/2)*b**3*n*x**(3*n)) - 6*a**4*b**3*x**(3*n)*log(sqrt(1 + b*x**n/a) + 1)/
(3*a**(19/2)*n + 9*a**(17/2)*b*n*x**n + 9*a**(15/2)*b**2*n*x**(2*n) + 3...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^{n}\right)^{5/2}}\,dx=\frac{\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}n}+\frac{2\left(3\,bx^{n}+4\,a\right)}{3\left(bx^{n}+a\right)^{\frac{3}{2}}a^{2}n}$$

input
```
integrate(1/x/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(5/2)*n) +
 2/3*(3*b*x^n + 4*a)/((b*x^n + a)^(3/2)*a^2*n)
```

Giac [F]

$$\int \frac{1}{x(a+bx^n)^{5/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^n)^{5/2}}\,dx = \int \frac{1}{x\,(a+b\,x^n)^{5/2}}\,dx$$

input
```
int(1/(x*(a + b*x^n)^(5/2)),x)
```

output
```
int(1/(x*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x(a+bx^n)^{5/2}}\,dx = \int \frac{\sqrt{x^n b+a}}{x^{3n}b^3x+3x^{2n}a\,b^2x+3x^n a^2bx+a^3x}dx$$

input
```
int(1/x/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(3*n)*b**3*x + 3*x**(2*n)*a*b**2*x + 3*x**n*a**2*
b*x + a**3*x),x)
```

3.533 $\int \frac{1}{x^2(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,-\frac{3}{2}-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax (a+bx^n)^{3/2}}$$

output
```
-hypergeom([1, -3/2-1/n],[-(1-n)/n],-b*x^n/a)/a/x/(a+b*x^n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}}\,dx = -\frac{\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{1}{n},1-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2 x\sqrt{a+bx^n}}$$

input
```
Integrate[1/(x^2*(a + b*x^n)^(5/2)),x]
```

output
```
-((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -n^(-1), 1 - n^(-1), -((b*x^
n)/a)])/(a^2*x*Sqrt[a + b*x^n]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{x^2\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2 x\sqrt{a+bx^n}}$$

input
```
Int[1/(x^2*(a + b*x^n)^(5/2)),x]
```

output
```
-((Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -n^(-1), -((1 - n)/n), -((b*
x^n)/a)])/(a^2*x*Sqrt[a + b*x^n]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(a +b \,x^{n}\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^2/(a+b*x^n)^(5/2),x)
```

output
```
int(1/x^2/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}}\,dx = \frac{a^{-\frac{1}{n}}a^{-\frac{5}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},-\frac{1}{n}\\ 1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate(1/x**2/(a+b*x**n)**(5/2),x)
```

output
```
a**(-5/2 + 1/n)*gamma(-1/n)*hyper((5/2, -1/n), (1 - 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}} \, dx = \int \frac{1}{x^2 \left(a+b\,x^n\right)^{5/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^n)^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^n)^{5/2}} \, dx = \int \frac{\sqrt{x^n b+a}}{x^{3n} b^3 x^2 + 3x^{2n} a\, b^2 x^2 + 3x^n a^2 b\, x^2 + a^3 x^2} dx$$

input
```
int(1/x^2/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(3*n)*b**3*x**2 + 3*x**(2*n)*a*b**2*x**2 + 3*x**n
*a**2*b*x**2 + a**3*x**2),x)
```

3.534 $\int \frac{1}{x^3(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,-\frac{3}{2}-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2 (a+bx^n)^{3/2}}$$

output
```
-1/2*hypergeom([1, -3/2-2/n],[-(2-n)/n],-b*x^n/a)/a/x^2/(a+b*x^n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx = -\frac{\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2}{n},1-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^2x^2\sqrt{a+bx^n}}$$

input
```
Integrate[1/(x^3*(a + b*x^n)^(5/2)),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -2/n, 1 - 2/n, -((b*x^n)/
a)])/(a^2*x^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}} \, dx$$
$$\Big\downarrow \ 889$$
$$\frac{\sqrt{\frac{bx^n}{a}+1} \int \frac{1}{x^3 \left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}$$
$$\Big\downarrow \ 888$$
$$-\frac{\sqrt{\frac{bx^n}{a}+1} \,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2x^2\sqrt{a+bx^n}}$$

input
```
Int[1/(x^3*(a + b*x^n)^(5/2)),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -2/n, -((2 - n)/n), -((b*
x^n)/a)])/(a^2*x^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(a +b \,x^{n}\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x^3/(a+b*x^n)^(5/2),x)
```

output
```
int(1/x^3/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx = \frac{a^{-\frac{2}{n}} a^{-\frac{5}{2}+\frac{2}{n}} \Gamma\left(-\frac{2}{n}\right) {}_2F_1\left(\left.\begin{matrix}\frac{5}{2}, -\frac{2}{n}\\ 1-\frac{2}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate(1/x**3/(a+b*x**n)**(5/2),x)
```

output
```
a**(-5/2 + 2/n)*gamma(-2/n)*hyper((5/2, -2/n), (1 - 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{5}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx = \int \frac{1}{(bx^n+a)^{\frac{5}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx=\int \frac{1}{x^3\,{\left(a+b\,x^n\right)}^{5/2}} \,d x$$

input
```
int(1/(x^3*(a + b*x^n)^(5/2)),x)
```

output
```
int(1/(x^3*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^n)^{5/2}}\,dx=\int \frac{\sqrt{x^{n} b +a}}{x^{3 n} b^{3} x^{3}+3 x^{2 n} a \,b^{2} x^{3}+3 x^{n} a^{2} b \,x^{3}+a^{3} x^{3}}d x$$

input
```
int(1/x^3/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(3*n)*b**3*x**3 + 3*x**(2*n)*a*b**2*x**3 + 3*x**n
*a**2*b*x**3 + a**3*x**3),x)
```

3.535 $\int x^{-1+4n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx=-\frac{2a^3(a+bx^n)^{3/2}}{3b^4n}+\frac{6a^2(a+bx^n)^{5/2}}{5b^4n}-\frac{6a(a+bx^n)^{7/2}}{7b^4n}+\frac{2(a+bx^n)^{9/2}}{9b^4n}$$

output
```
-2/3*a^3*(a+b*x^n)^(3/2)/b^4/n+6/5*a^2*(a+b*x^n)^(5/2)/b^4/n-6/7*a*(a+b*x^
n)^(7/2)/b^4/n+2/9*(a+b*x^n)^(9/2)/b^4/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx=\frac{2(a+bx^n)^{3/2}\left(-16a^3+24a^2bx^n-30ab^2x^{2n}+35b^3x^{3n}\right)}{315b^4n}$$

input
```
Integrate[x^(-1 + 4*n)*Sqrt[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)^(3/2)*(-16*a^3 + 24*a^2*b*x^n - 30*a*b^2*x^(2*n) + 35*b^3*x
^(3*n)))/(315*b^4*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int x^{3n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \ 53$$

$$\frac{\int \left(\frac{(bx^n+a)^{7/2}}{b^3}-\frac{3a(bx^n+a)^{5/2}}{b^3}+\frac{3a^2(bx^n+a)^{3/2}}{b^3}-\frac{a^3\sqrt{bx^n+a}}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{2a^3(a+bx^n)^{3/2}}{3b^4}+\frac{6a^2(a+bx^n)^{5/2}}{5b^4}+\frac{2(a+bx^n)^{9/2}}{9b^4}-\frac{6a(a+bx^n)^{7/2}}{7b^4}}{n}$$

input
```
Int[x^(-1 + 4*n)*Sqrt[a + b*x^n],x]
```

output
```
((-2*a^3*(a + b*x^n)^(3/2))/(3*b^4) + (6*a^2*(a + b*x^n)^(5/2))/(5*b^4) -
(6*a*(a + b*x^n)^(7/2))/(7*b^4) + (2*(a + b*x^n)^(9/2))/(9*b^4))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{2\left(-35x^{4n}b^{4}-5a\,x^{3n}b^{3}+6a^{2}x^{2n}b^{2}-8a^{3}x^{n}b+16a^{4}\right)\sqrt{a+b\,x^{n}}}{315b^{4}n}$	67

input
```
int(x^(-1+4*n)*(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-35*(x^n)^4*b^4-5*a*(x^n)^3*b^3+6*a^2*(x^n)^2*b^2-8*a^3*x^n*b+16*a
^4)*(a+b*x^n)^(1/2)/b^4/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx=\frac{2\left(35\,b^4x^{4\,n}+5\,ab^3x^{3\,n}-6\,a^2b^2x^{2\,n}+8\,a^3bx^n-16\,a^4\right)\sqrt{bx^n+a}}{315\,b^4n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*x^(4*n) + 5*a*b^3*x^(3*n) - 6*a^2*b^2*x^(2*n) + 8*a^3*b*x^n
- 16*a^4)*sqrt(b*x^n + a)/(b^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2572 vs. 2(82) = 164.

Time = 5.16 (sec) , antiderivative size = 2572, normalized size of antiderivative = 27.96

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx = \text{Too large to display}$$

input
```
integrate(x**(-1+4*n)*(a+b*x**n)**(1/2),x)
```

output
```
-32*a**(29/2)*b**(23/2)*x**(23*n/2)*sqrt(a/(b*x**n) + 1)/(315*a**(21/2)*b*
*15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b**17*
n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19*n*x*
*(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**(17*n
)) - 176*a**(27/2)*b**(25/2)*x**(25*n/2)*sqrt(a/(b*x**n) + 1)/(315*a**(21/
2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17/2)*b
**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*b**19
*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*n*x**
(17*n)) - 396*a**(25/2)*b**(27/2)*x**(27*n/2)*sqrt(a/(b*x**n) + 1)/(315*a*
*(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a**(17
/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(13/2)*
b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b**21*
n*x**(17*n)) - 462*a**(23/2)*b**(29/2)*x**(29*n/2)*sqrt(a/(b*x**n) + 1)/(3
15*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4725*a
**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*a**(1
3/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a**(9/2)*b
**21*n*x**(17*n)) - 210*a**(21/2)*b**(31/2)*x**(31*n/2)*sqrt(a/(b*x**n) +
1)/(315*a**(21/2)*b**15*n*x**(11*n) + 1890*a**(19/2)*b**16*n*x**(12*n) + 4
725*a**(17/2)*b**17*n*x**(13*n) + 6300*a**(15/2)*b**18*n*x**(14*n) + 4725*
a**(13/2)*b**19*n*x**(15*n) + 1890*a**(11/2)*b**20*n*x**(16*n) + 315*a*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.72

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx = \frac{2\,(35\,b^4x^{4\,n}+5\,ab^3x^{3\,n}-6\,a^2b^2x^{2\,n}+8\,a^3bx^n-16\,a^4)\sqrt{bx^n+a}}{315\,b^4n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^(4*n) + 5*a*b^3*x^(3*n) - 6*a^2*b^2*x^(2*n) + 8*a^3*b*x^n
- 16*a^4)*sqrt(b*x^n + a)/(b^4*n)
```

Giac [F]

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}x^{4\,n-1}\,dx$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(4*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx = \int x^{4\,n-1}\,\sqrt{a+b\,x^n}\,dx$$

input
```
int(x^(4*n - 1)*(a + b*x^n)^(1/2),x)
```

output
```
int(x^(4*n - 1)*(a + b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int x^{-1+4n}\sqrt{a+bx^n}\,dx = \frac{2\sqrt{x^n b+a}\,(35x^{4n}b^4+5x^{3n}a\,b^3-6x^{2n}a^2b^2+8x^n a^3b-16a^4)}{315b^4n}$$

input
```
int(x^(-1+4*n)*(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(35*x**(4*n)*b**4 + 5*x**(3*n)*a*b**3 - 6*x**(2*n)*a**
2*b**2 + 8*x**n*a**3*b - 16*a**4))/(315*b**4*n)
```

3.536 $\int x^{-1+3n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx=\frac{2a^2(a+bx^n)^{3/2}}{3b^3n}-\frac{4a(a+bx^n)^{5/2}}{5b^3n}+\frac{2(a+bx^n)^{7/2}}{7b^3n}$$

output
```
2/3*a^2*(a+b*x^n)^(3/2)/b^3/n-4/5*a*(a+b*x^n)^(5/2)/b^3/n+2/7*(a+b*x^n)^(7
/2)/b^3/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx=\frac{2(a+bx^n)^{3/2}\left(8a^2-12abx^n+15b^2x^{2n}\right)}{105b^3n}$$

input
```
Integrate[x^(-1 + 3*n)*Sqrt[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)^(3/2)*(8*a^2 - 12*a*b*x^n + 15*b^2*x^(2*n)))/(105*b^3*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{2n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{(bx^n+a)^{5/2}}{b^2}-\frac{2a(bx^n+a)^{3/2}}{b^2}+\frac{a^2\sqrt{bx^n+a}}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2a^2(a+bx^n)^{3/2}}{3b^3}+\frac{2(a+bx^n)^{7/2}}{7b^3}-\frac{4a(a+bx^n)^{5/2}}{5b^3}}{n}$$

input
```
Int[x^(-1 + 3*n)*Sqrt[a + b*x^n],x]
```

output
```
((2*a^2*(a + b*x^n)^(3/2))/(3*b^3) - (4*a*(a + b*x^n)^(5/2))/(5*b^3) + (2*
(a + b*x^n)^(7/2))/(7*b^3))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{2\left(15b^{3}x^{3n}+3a\,b^{2}x^{2n}-4a^{2}b\,x^{n}+8a^{3}\right)\sqrt{a+b\,x^{n}}}{105b^{3}n}$	54

input
```
int(x^(-1+3*n)*(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*(x^n)^3*b^3+3*a*(x^n)^2*b^2-4*a^2*b*x^n+8*a^3)*(a+b*x^n)^(1/2)/b
^3/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx = \frac{2\,(15\,b^3x^{3\,n}+3\,ab^2x^{2\,n}-4\,a^2bx^n+8\,a^3)\sqrt{bx^n+a}}{105\,b^3n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*x^(3*n) + 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 8*a^3)*sqrt(b*x^n
+ a)/(b^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(60) = 120.

Time = 3.08 (sec) , antiderivative size = 1015, normalized size of antiderivative = 14.93

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx = \text{Too large to display}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**(1/2),x)
```

output
```
16*a**(19/2)*b**(9/2)*x**(9*n/2)*sqrt(a/(b*x**n) + 1)/(105*a**(13/2)*b**7*
n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n)
+ 105*a**(7/2)*b**10*n*x**(7*n)) + 40*a**(17/2)*b**(11/2)*x**(11*n/2)*sqrt
(a/(b*x**n) + 1)/(105*a**(13/2)*b**7*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**
(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a**(7/2)*b**10*n*x**(7*n)) + 30
*a**(15/2)*b**(13/2)*x**(13*n/2)*sqrt(a/(b*x**n) + 1)/(105*a**(13/2)*b**7*
n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n)
+ 105*a**(7/2)*b**10*n*x**(7*n)) + 40*a**(13/2)*b**(15/2)*x**(15*n/2)*sqrt
(a/(b*x**n) + 1)/(105*a**(13/2)*b**7*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**
(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a**(7/2)*b**10*n*x**(7*n)) + 10
0*a**(11/2)*b**(17/2)*x**(17*n/2)*sqrt(a/(b*x**n) + 1)/(105*a**(13/2)*b**7
*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n)
 + 105*a**(7/2)*b**10*n*x**(7*n)) + 96*a**(9/2)*b**(19/2)*x**(19*n/2)*sqrt
(a/(b*x**n) + 1)/(105*a**(13/2)*b**7*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**
(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) + 105*a**(7/2)*b**10*n*x**(7*n)) + 30
*a**(7/2)*b**(21/2)*x**(21*n/2)*sqrt(a/(b*x**n) + 1)/(105*a**(13/2)*b**7*n
*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*n) +
 105*a**(7/2)*b**10*n*x**(7*n)) - 16*a**10*b**4*x**(4*n)/(105*a**(13/2)*b*
*7*n*x**(4*n) + 315*a**(11/2)*b**8*n*x**(5*n) + 315*a**(9/2)*b**9*n*x**(6*
n) + 105*a**(7/2)*b**10*n*x**(7*n)) - 48*a**9*b**5*x**(5*n)/(105*a**(13...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx = \frac{2\,(15\,b^3x^{3\,n}+3\,ab^2x^{2\,n}-4\,a^2bx^n+8\,a^3)\sqrt{bx^n+a}}{105\,b^3n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*b^3*x^(3*n) + 3*a*b^2*x^(2*n) - 4*a^2*b*x^n + 8*a^3)*sqrt(b*x^n
+ a)/(b^3*n)
```

Giac [F]

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}x^{3\,n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx = \int x^{3\,n-1}\,\sqrt{a+b\,x^n}\,dx$$

input
```
int(x^(3*n - 1)*(a + b*x^n)^(1/2),x)
```

output
```
int(x^(3*n - 1)*(a + b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int x^{-1+3n}\sqrt{a+bx^n}\,dx = \frac{2\sqrt{x^n b+a}\,(15x^{3n}b^3+3x^{2n}a\,b^2-4x^n a^2 b+8a^3)}{105b^3 n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(15*x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 - 4*x**n*a**2*b
+ 8*a**3))/(105*b**3*n)
```

3.537 $\int x^{-1+2n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx=-\frac{2a(a+bx^n)^{3/2}}{3b^2n}+\frac{2(a+bx^n)^{5/2}}{5b^2n}$$

output
```
-2/3*a*(a+b*x^n)^(3/2)/b^2/n+2/5*(a+b*x^n)^(5/2)/b^2/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx=\frac{2(a+bx^n)^{3/2}\,(-2a+3bx^n)}{15b^2n}$$

input
```
Integrate[x^(-1 + 2*n)*Sqrt[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)^(3/2)*(-2*a + 3*b*x^n))/(15*b^2*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^n\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{(bx^n+a)^{3/2}}{b}-\frac{a\sqrt{bx^n+a}}{b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(a+bx^n)^{5/2}}{5b^2}-\frac{2a(a+bx^n)^{3/2}}{3b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)*Sqrt[a + b*x^n],x]
```

output
```
((-2*a*(a + b*x^n)^(3/2))/(3*b^2) + (2*(a + b*x^n)^(5/2))/(5*b^2))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{2\left(-3 b^{2} x^{2 n}-a b\, x^{n}+2 a^{2}\right) \sqrt{a +b\, x^{n}}}{15 b^{2} n}$	41

input
```
int(x^(2*n-1)*(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*b^2*(x^n)^2-a*b*x^n+2*a^2)*(a+b*x^n)^(1/2)/b^2/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx = \frac{2\,{\left(3\,b^{2} x^{2\,n} + a b x^{n} - 2\,a^{2}\right)} \sqrt{b x^{n} + a}}{15\,b^{2} n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^(2*n) + a*b*x^n - 2*a^2)*sqrt(b*x^n + a)/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(37) = 74$.

Time = 1.91 (sec) , antiderivative size = 338, normalized size of antiderivative = 7.68

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx = -\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}x^{\frac{3n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n+15a^{\frac{5}{2}}b^4nx^{2n}} - \frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}x^{\frac{5n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n+15a^{\frac{5}{2}}b^4nx^{2n}} + \frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}x^{\frac{7n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n+15a^{\frac{5}{2}}b^4nx^{2n}} + \frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}x^{\frac{9n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^{\frac{7}{2}}b^3nx^n+15a^{\frac{5}{2}}b^4nx^{2n}} + \frac{4a^6bx^n}{15a^{\frac{7}{2}}b^3nx^n+15a^{\frac{5}{2}}b^4nx^{2n}} + \frac{4a^5b^2x^{2n}}{15a^{\frac{7}{2}}b^3nx^n+15a^{\frac{5}{2}}b^4nx^{2n}}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**(1/2),x)
```

output
```
-4*a**(11/2)*b**(3/2)*x**(3*n/2)*sqrt(a/(b*x**n) + 1)/(15*a**(7/2)*b**3*n*
x**n + 15*a**(5/2)*b**4*n*x**(2*n)) - 2*a**(9/2)*b**(5/2)*x**(5*n/2)*sqrt(
a/(b*x**n) + 1)/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n)) +
8*a**(7/2)*b**(7/2)*x**(7*n/2)*sqrt(a/(b*x**n) + 1)/(15*a**(7/2)*b**3*n*x*
*n + 15*a**(5/2)*b**4*n*x**(2*n)) + 6*a**(5/2)*b**(9/2)*x**(9*n/2)*sqrt(a/
(b*x**n) + 1)/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n)) + 4*
a**6*b*x**n/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n)) + 4*a*
*5*b**2*x**(2*n)/(15*a**(7/2)*b**3*n*x**n + 15*a**(5/2)*b**4*n*x**(2*n))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx = \frac{2\,(3\,b^2x^{2\,n}+abx^n-2\,a^2)\sqrt{bx^n+a}}{15\,b^2n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^2*x^(2*n) + a*b*x^n - 2*a^2)*sqrt(b*x^n + a)/(b^2*n)
```

Giac [F]

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}\,x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx=\int x^{2\,n-1}\,\sqrt{a+b\,x^n}\,dx$$

input
```
int(x^(2*n - 1)*(a + b*x^n)^(1/2),x)
```

output
```
int(x^(2*n - 1)*(a + b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int x^{-1+2n}\sqrt{a+bx^n}\,dx=\frac{2\sqrt{x^nb+a}\,(3x^{2n}b^2+x^nab-2a^2)}{15b^2n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(3*x**(2*n)*b**2 + x**n*a*b - 2*a**2))/(15*b**2*n)
```

3.538 $\int x^{-1+n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2(a+bx^n)^{3/2}}{3bn}$$

output
```
2/3*(a+b*x^n)^(3/2)/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2(a+bx^n)^{3/2}}{3bn}$$

input
```
Integrate[x^(-1 + n)*Sqrt[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)^(3/2))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{2(a+bx^n)^{3/2}}{3bn}$$

input
```
Int[x^(-1 + n)*Sqrt[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)^(3/2))/(3*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{2(a+b\,x^n)^{\frac{3}{2}}}{3bn}$	18

input
```
int(x^(-1+n)*(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a+b*x^n)^(3/2)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(15) = 30.

Time = 1.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3bn}+\frac{2\sqrt{a}x^n\sqrt{1+\frac{bx^n}{a}}}{3n}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**(1/2),x)
```

output
```
2*a**(3/2)*sqrt(1 + b*x**n/a)/(3*b*n) + 2*sqrt(a)*x**n*sqrt(1 + b*x**n/a)/
(3*n)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\,{\left(a+b\,x^n\right)}^{3/2}}{3\,b\,n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^(1/2),x)
```

output
```
(2*(a + b*x^n)^(3/2))/(3*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2\sqrt{x^nb+a}\,(x^nb+a)}{3bn}$$

input
```
int(x^(-1+n)*(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(x**n*b + a))/(3*b*n)
```

3.539 $\int \frac{\sqrt{a+bx^n}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{\sqrt{a+bx^n}}{x} \, dx = \frac{2\sqrt{a+bx^n}}{n} - \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*(a+b*x^n)^(1/2)/n-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^n}}{x} \, dx = \frac{2\left(\sqrt{a+bx^n} - \sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Integrate[Sqrt[a + b*x^n]/x,x]
```

output
```
(2*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]]))/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[a + b*x^n]/x,x]
```

output
```
(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{n}$	36
default	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{n}$	36
risch	$\frac{2\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{\sqrt{a}}\right)}{n}$	42

input
```
int((a+b*x^n)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+\sqrt{bx^n+a}\right)}{n}\right]$$

input
```
integrate((a+b*x^n)^(1/2)/x,x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^
n + a))/n, 2*(sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + sqrt(b*x^n + a))
/n]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.93 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = -\frac{2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}$$

input
```
integrate((a+b*x**n)**(1/2)/x,x)
```

output
```
-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/n + 2*a/(sqrt(b)*n*x**(n/2)*s
qrt(a/(b*x**n) + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a/(b*x**n) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\sqrt{bx^n+a}}{n}$$

input
```
integrate((a+b*x^n)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
*sqrt(b*x^n + a)/n
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \int \frac{\sqrt{bx^n+a}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \int \frac{\sqrt{a+b\,x^n}}{x}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x,x)
```

output
```
int((a + b*x^n)^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}}{x}\,dx = \frac{2\sqrt{x^n b+a}+\left(\int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx\right)an}{n}$$

input
```
int((a+b*x^n)^(1/2)/x,x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*a*n)/n
```

3.540 $\int x^{-1-n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx=-\frac{x^{-n}\sqrt{a+bx^n}}{n}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-(a+b*x^n)^(1/2)/n/(x^n)-b*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx=-\frac{x^{-n}\sqrt{a+bx^n}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}}}{n}$$

input
```
Integrate[x^(-1 - n)*Sqrt[a + b*x^n],x]
```

output
```
-((Sqrt[a + b*x^n]/x^n + (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/Sqrt[a])/n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int x^{-2n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{1}{2}b\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n - x^{-n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \; 73$$

$$\frac{\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a} - x^{-n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{x^{-n}\left(-\sqrt{a+bx^n}\right) - \frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}}}{n}$$

input
```
Int[x^(-1 - n)*Sqrt[a + b*x^n],x]
```

output
```
(-(Sqrt[a + b*x^n]/x^n) - (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/Sqrt[a])/n
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-1-n}\sqrt{a+b\,x^{n}}dx$$

input
```
int(x^(-1-n)*(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-n)*(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.27

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx$$
$$=\left[\frac{\sqrt{a}bx^n\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)-2\sqrt{bx^n+a}a}{2anx^n},\frac{\sqrt{-a}bx^n\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)-\sqrt{bx^n+a}a}{anx^n}\right]$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x^n*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) - 2
*sqrt(b*x^n + a)*a)/(a*n*x^n), (sqrt(-a)*b*x^n*arctan(sqrt(-a)/sqrt(b*x^n
+ a)) - sqrt(b*x^n + a)*a)/(a*n*x^n)]
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx=-\frac{\sqrt{b}x^{-\frac{n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{n}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}n}$$

input
```
integrate(x**(-1-n)*(a+b*x**n)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**n) + 1)/(n*x**(n/2)) - b*asinh(sqrt(a)/(sqrt(b)*x**(
n/2)))/(sqrt(a)*n)
```

Maxima [F]

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-n - 1), x)
```

Giac [F]

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}x^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx=\int \frac{\sqrt{a+b\,x^n}}{x^{n+1}}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x^(n + 1),x)
```

output
```
int((a + b*x^n)^(1/2)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}\sqrt{a+bx^n}\,dx = \frac{-2\sqrt{x^n b+a}+x^n\left(\int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx\right)bn}{2x^n n}$$

input
```
int(x^(-1-n)*(a+b*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(x**n*b + a) + x**n*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*b*n
)/(2*x**n*n)
```

3.541 $\int x^{-1-2n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx=-\frac{x^{-2n}\sqrt{a+bx^n}}{2n}-\frac{bx^{-n}\sqrt{a+bx^n}}{4an}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{3/2}n}$$

output
```
-1/2*(a+b*x^n)^(1/2)/n/(x^(2*n))-1/4*b*(a+b*x^n)^(1/2)/a/n/(x^n)+1/4*b^2*a
rctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx=-\frac{x^{-2n}\sqrt{a+bx^n}(2a+bx^n)}{4an}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{3/2}n}$$

input
```
Integrate[x^(-1 - 2*n)*Sqrt[a + b*x^n],x]
```

output
```
-1/4*(Sqrt[a + b*x^n]*(2*a + b*x^n))/(a*n*x^(2*n)) + (b^2*ArcTanh[Sqrt[a +
 b*x^n]/Sqrt[a]])/(4*a^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-3n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{4}b\int \frac{x^{-2n}}{\sqrt{bx^n+a}}dx^n-\frac{1}{2}x^{-2n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{1}{4}b\left(-\frac{b\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{2a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)-\frac{1}{2}x^{-2n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)-\frac{1}{2}x^{-2n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)-\frac{1}{2}x^{-2n}\sqrt{a+bx^n}}{n}$$

input

```
Int[x^(-1 - 2*n)*Sqrt[a + b*x^n],x]
```

output
```
(-1/2*Sqrt[a + b*x^n]/x^(2*n) + (b*(-(Sqrt[a + b*x^n]/(a*x^n)) + (b*ArcTan
h[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2)))/4)/n
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-2n-1}\sqrt{a+b\,x^{n}}dx$$

input
```
int(x^(-2*n-1)*(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-2*n-1)*(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx = \left[\frac{\sqrt{a}b^2x^{2\,n}\log\left(\frac{bx^n+2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)-2\,(abx^n+2\,a^2)\sqrt{bx^n+a}}{8\,a^2nx^{2\,n}}, -\frac{\sqrt{-a}b^2x^{2\,n}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+(abx^n+2\,a^2)\sqrt{bx^n+a}}{4\,a^2nx^{2\,n}}\right]$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b^2*x^(2*n)*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^
n) - 2*(a*b*x^n + 2*a^2)*sqrt(b*x^n + a))/(a^2*n*x^(2*n)), -1/4*(sqrt(-a)*
b^2*x^(2*n)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + (a*b*x^n + 2*a^2)*sqrt(b*x^
n + a))/(a^2*n*x^(2*n))]
```

Sympy [A] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx = -\frac{ax^{-\frac{5n}{2}}}{2\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{3\sqrt{b}x^{-\frac{3n}{2}}}{4n\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{b^{\frac{3}{2}}x^{-\frac{n}{2}}}{4an\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{4a^{\frac{3}{2}}n}$$

input
```
integrate(x**(-1-2*n)*(a+b*x**n)**(1/2),x)
```

output
```
-a/(2*sqrt(b)*n*x**(5*n/2)*sqrt(a/(b*x**n) + 1)) - 3*sqrt(b)/(4*n*x**(3*n/
2)*sqrt(a/(b*x**n) + 1)) - b**(3/2)/(4*a*n*x**(n/2)*sqrt(a/(b*x**n) + 1))
+ b**2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(4*a**(3/2)*n)
```

Maxima [F]

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-2*n - 1), x)
```

Giac [F]

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}x^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx=\int \frac{\sqrt{a+b\,x^n}}{x^{2\,n+1}}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x^(2*n + 1),x)
```

output
```
int((a + b*x^n)^(1/2)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}\sqrt{a+bx^n}\,dx=\frac{-2x^n\sqrt{x^nb+a}\,b-4\sqrt{x^nb+a}\,a-x^{2n}\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)b^2n}{8x^{2n}an}$$

input
```
int(x^(-1-2*n)*(a+b*x^n)^(1/2),x)
```

output
```
( - 2*x**n*sqrt(x**n*b + a)*b - 4*sqrt(x**n*b + a)*a - x**(2*n)*int(sqrt(x
**n*b + a)/(x**n*b*x + a*x),x)*b**2*n)/(8*x**(2*n)*a*n)
```

3.542 $\int x^{-1-3n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx=-\frac{x^{-3n}\sqrt{a+bx^n}}{3n}-\frac{bx^{-2n}\sqrt{a+bx^n}}{12an}+\frac{b^2x^{-n}\sqrt{a+bx^n}}{8a^2n}-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{5/2}n}$$

output
```
-1/3*(a+b*x^n)^(1/2)/n/(x^(3*n))-1/12*b*(a+b*x^n)^(1/2)/a/n/(x^(2*n))+1/8*
b^2*(a+b*x^n)^(1/2)/a^2/n/(x^n)-1/8*b^3*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a
^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx=\frac{\sqrt{a}x^{-3n}\sqrt{a+bx^n}(-8a^2-2abx^n+3b^2x^{2n})-3b^3\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{24a^{5/2}n}$$

input
```
Integrate[x^(-1 - 3*n)*Sqrt[a + b*x^n],x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^n]*(-8*a^2 - 2*a*b*x^n + 3*b^2*x^(2*n)))/x^(3*n) -
3*b^3*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(24*a^(5/2)*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {798, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-4n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{6}b\int \frac{x^{-3n}}{\sqrt{bx^n+a}}dx^n-\frac{1}{3}x^{-3n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{1}{6}b\left(-\frac{3b\int \frac{x^{-2n}}{\sqrt{bx^n+a}}dx^n}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)-\frac{1}{3}x^{-3n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{2a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)-\frac{1}{3}x^{-3n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)-\frac{1}{3}x^{-3n}\sqrt{a+bx^n}}{n}$$

$\downarrow$ 221

$$\frac{\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)-\frac{1}{3}x^{-3n}\sqrt{a+bx^n}}{n}$$

input
```
Int[x^(-1 - 3*n)*Sqrt[a + b*x^n],x]
```

output
```
(-1/3*Sqrt[a + b*x^n]/x^(3*n) + (b*(-1/2*Sqrt[a + b*x^n]/(a*x^(2*n)) - (3*
b*(-(Sqrt[a + b*x^n]/(a*x^n)) + (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/
2)))/(4*a)))/6)/n
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-1-3n}\sqrt{a+b\,x^{n}}dx$$

input
```
int(x^(-1-3*n)*(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-3*n)*(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.59

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx$$
$$=\left[\frac{3\sqrt{a}b^3x^{3\,n}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\left(3\,ab^2x^{2\,n}-2\,a^2bx^n-8\,a^3\right)\sqrt{bx^n+a}}{48\,a^3nx^{3\,n}},\ \frac{3\sqrt{-a}b^3x^{3\,n}\arctan\left(\frac{\sqrt{}}{\sqrt{bx}}\right.}{}\right.$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*sqrt(a)*b^3*x^(3*n)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)
/x^n) + 2*(3*a*b^2*x^(2*n) - 2*a^2*b*x^n - 8*a^3)*sqrt(b*x^n + a))/(a^3*n*
x^(3*n)), 1/24*(3*sqrt(-a)*b^3*x^(3*n)*arctan(sqrt(-a)/sqrt(b*x^n + a)) +
(3*a*b^2*x^(2*n) - 2*a^2*b*x^n - 8*a^3)*sqrt(b*x^n + a))/(a^3*n*x^(3*n))]
```

Sympy [A] (verification not implemented)

Time = 17.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx = -\frac{ax^{-\frac{7n}{2}}}{3\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}} - \frac{5\sqrt{b}x^{-\frac{5n}{2}}}{12n\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{b^{\frac{3}{2}}x^{-\frac{3n}{2}}}{24an\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{b^{\frac{5}{2}}x^{-\frac{n}{2}}}{8a^2n\sqrt{\frac{ax^{-n}}{b}+1}} - \frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{8a^{\frac{5}{2}}n}$$

input
```
integrate(x**(-1-3*n)*(a+b*x**n)**(1/2),x)
```

output
```
-a/(3*sqrt(b)*n*x**(7*n/2)*sqrt(a/(b*x**n) + 1)) - 5*sqrt(b)/(12*n*x**(5*n
/2)*sqrt(a/(b*x**n) + 1)) + b**(3/2)/(24*a*n*x**(3*n/2)*sqrt(a/(b*x**n) +
1)) + b**(5/2)/(8*a**2*n*x**(n/2)*sqrt(a/(b*x**n) + 1)) - b**3*asinh(sqrt(
a)/(sqrt(b)*x**(n/2)))/(8*a**(5/2)*n)
```

Maxima [F]

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx = \int \sqrt{bx^n+a}\,x^{-3\,n-1}\,dx$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-3*n - 1), x)
```

Giac [F]

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}x^{-3\,n-1}\,dx$$

input
```
integrate(x^(-1-3*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx=\int \frac{\sqrt{a+b\,x^n}}{x^{3\,n+1}}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x^(3*n + 1),x)
```

output
```
int((a + b*x^n)^(1/2)/x^(3*n + 1), x)
```

Reduce [F]

$$\int x^{-1-3n}\sqrt{a+bx^n}\,dx$$
$$=\frac{6x^{2n}\sqrt{x^nb+a}\,b^2-4x^n\sqrt{x^nb+a}\,ab-16\sqrt{x^nb+a}\,a^2+3x^{3n}\left(\int\frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)b^3n}{48x^{3n}a^2n}$$

input
```
int(x^(-1-3*n)*(a+b*x^n)^(1/2),x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2 - 4*x**n*sqrt(x**n*b + a)*a*b - 16*sqrt(
x**n*b + a)*a**2 + 3*x**(3*n)*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*b**
3*n)/(48*x**(3*n)*a**2*n)
```

3.543 $\int x^{-1-4n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx=-\frac{x^{-4n}\sqrt{a+bx^n}}{4n}-\frac{bx^{-3n}\sqrt{a+bx^n}}{24an}+\frac{5b^2x^{-2n}\sqrt{a+bx^n}}{96a^2n}-\frac{5b^3x^{-n}\sqrt{a+bx^n}}{64a^3n}+\frac{5b^4\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{64a^{7/2}n}$$

output
```
-1/4*(a+b*x^n)^(1/2)/n/(x^(4*n))-1/24*b*(a+b*x^n)^(1/2)/a/n/(x^(3*n))+5/96
*b^2*(a+b*x^n)^(1/2)/a^2/n/(x^(2*n))-5/64*b^3*(a+b*x^n)^(1/2)/a^3/n/(x^n)+
5/64*b^4*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(7/2)/n
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx=\frac{-\sqrt{a}x^{-4n}\sqrt{a+bx^n}(48a^3+8a^2bx^n-10ab^2x^{2n}+15b^3x^{3n})+15b^4\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{192a^{7/2}n}$$

input
```
Integrate[x^(-1 - 4*n)*Sqrt[a + b*x^n],x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^n]*(48*a^3 + 8*a^2*b*x^n - 10*a*b^2*x^(2*n) + 15*
b^3*x^(3*n)))/x^(4*n)) + 15*b^4*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(192*a^(
7/2)*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {798, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-4n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-5n}\sqrt{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{8}b\int \frac{x^{-4n}}{\sqrt{bx^n+a}}dx^n-\frac{1}{4}x^{-4n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{1}{8}b\left(-\frac{5b\int \frac{x^{-3n}}{\sqrt{bx^n+a}}dx^n}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)-\frac{1}{4}x^{-4n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int \frac{x^{-2n}}{\sqrt{bx^n+a}}dx^n}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)-\frac{1}{4}x^{-4n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{2a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)-\frac{1}{4}x^{-4n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)-\frac{1}{4}x^{-4n}\sqrt{a+bx^n}}{n}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)-\frac{1}{4}x^{-4n}\sqrt{a+bx^n}}{n}$$

input
```
Int[x^(-1 - 4*n)*Sqrt[a + b*x^n],x]
```

output
```
(-1/4*Sqrt[a + b*x^n]/x^(4*n) + (b*(-1/3*Sqrt[a + b*x^n]/(a*x^(3*n)) - (5*
b*(-1/2*Sqrt[a + b*x^n]/(a*x^(2*n)) - (3*b*(-(Sqrt[a + b*x^n]/(a*x^n)) + (
b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a)))/8)/n
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-4n-1}\sqrt{a+b\,x^{n}}dx$$

input
```
int(x^(-4*n-1)*(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-4*n-1)*(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.45

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx = \left[\frac{15\sqrt{a}b^4x^{4n}\log\left(\frac{bx^n+2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)-2\left(15ab^3x^{3n}-10a^2b^2x^{2n}+8a^3bx^n+48a^4\right)\sqrt{bx^n+a}}{384a^4nx^{4n}},\ -\frac{15\sqrt{-a}b^4x^{4n}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+\left(15ab^3x^{3n}-10a^2b^2x^{2n}+8a^3bx^n+48a^4\right)\sqrt{bx^n+a}}{192a^4nx^{4n}}\right]$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(15*sqrt(a)*b^4*x^(4*n)*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*
a)/x^n) - 2*(15*a*b^3*x^(3*n) - 10*a^2*b^2*x^(2*n) + 8*a^3*b*x^n + 48*a^4)
*sqrt(b*x^n + a))/(a^4*n*x^(4*n)), -1/192*(15*sqrt(-a)*b^4*x^(4*n)*arctan(
sqrt(-a)/sqrt(b*x^n + a)) + (15*a*b^3*x^(3*n) - 10*a^2*b^2*x^(2*n) + 8*a^3
*b*x^n + 48*a^4)*sqrt(b*x^n + a))/(a^4*n*x^(4*n))]
```

Sympy [A] (verification not implemented)

Time = 54.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.25

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx = -\frac{ax^{-\frac{9n}{2}}}{4\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{7\sqrt{b}x^{-\frac{7n}{2}}}{24n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{b^{\frac{3}{2}}x^{-\frac{5n}{2}}}{96an\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{5b^{\frac{5}{2}}x^{-\frac{3n}{2}}}{192a^2n\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{5b^{\frac{7}{2}}x^{-\frac{n}{2}}}{64a^3n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{64a^{\frac{7}{2}}n}$$

input
```
integrate(x**(-1-4*n)*(a+b*x**n)**(1/2),x)
```

output
```
-a/(4*sqrt(b)*n*x**(9*n/2)*sqrt(a/(b*x**n) + 1)) - 7*sqrt(b)/(24*n*x**(7*n
/2)*sqrt(a/(b*x**n) + 1)) + b**(3/2)/(96*a*n*x**(5*n/2)*sqrt(a/(b*x**n) +
1)) - 5*b**(5/2)/(192*a**2*n*x**(3*n/2)*sqrt(a/(b*x**n) + 1)) - 5*b**(7/2)
/(64*a**3*n*x**(n/2)*sqrt(a/(b*x**n) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)
*x**(n/2)))/(64*a**(7/2)*n)
```

Maxima [F]

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}x^{-4\,n-1}\,dx$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-4*n - 1), x)
```

Giac [F]

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx=\int \sqrt{bx^n+a}x^{-4\,n-1}\,dx$$

input
```
integrate(x^(-1-4*n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(-4*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx=\int \frac{\sqrt{a+b\,x^n}}{x^{4\,n+1}}\,dx$$

input
```
int((a + b*x^n)^(1/2)/x^(4*n + 1),x)
```

output
```
int((a + b*x^n)^(1/2)/x^(4*n + 1), x)
```

Reduce [F]

$$\int x^{-1-4n}\sqrt{a+bx^n}\,dx$$
$$=\frac{-30x^{3n}\sqrt{x^nb+a}\,b^3+20x^{2n}\sqrt{x^nb+a}\,a\,b^2-16x^n\sqrt{x^nb+a}\,a^2b-96\sqrt{x^nb+a}\,a^3-15x^{4n}\left(\int\frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)}{384x^{4n}a^3n}$$

input
```
int(x^(-1-4*n)*(a+b*x^n)^(1/2),x)
```

output
```
( - 30*x**(3*n)*sqrt(x**n*b + a)*b**3 + 20*x**(2*n)*sqrt(x**n*b + a)*a*b**
2 - 16*x**n*sqrt(x**n*b + a)*a**2*b - 96*sqrt(x**n*b + a)*a**3 - 15*x**(4*
n)*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*b**4*n)/(384*x**(4*n)*a**3*n)
```

3.544 $\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx=-\frac{2a^3\sqrt{a+bx^n}}{b^4n}+\frac{2a^2(a+bx^n)^{3/2}}{b^4n}-\frac{6a(a+bx^n)^{5/2}}{5b^4n}+\frac{2(a+bx^n)^{7/2}}{7b^4n}$$

output
```
-2*a^3*(a+b*x^n)^(1/2)/b^4/n+2*a^2*(a+b*x^n)^(3/2)/b^4/n-6/5*a*(a+b*x^n)^(
5/2)/b^4/n+2/7*(a+b*x^n)^(7/2)/b^4/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx=\frac{2\sqrt{a+bx^n}(-16a^3+8a^2bx^n-6ab^2x^{2n}+5b^3x^{3n})}{35b^4n}$$

input
```
Integrate[x^(-1 + 4*n)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a + b*x^n]*(-16*a^3 + 8*a^2*b*x^n - 6*a*b^2*x^(2*n) + 5*b^3*x^(3*n
)))/(35*b^4*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n-1}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{3n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(-\frac{a^3}{b^3\sqrt{bx^n+a}}+\frac{3\sqrt{bx^n+a}a^2}{b^3}-\frac{3(bx^n+a)^{3/2}a}{b^3}+\frac{(bx^n+a)^{5/2}}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2a^3\sqrt{a+bx^n}}{b^4}+\frac{2a^2(a+bx^n)^{3/2}}{b^4}+\frac{2(a+bx^n)^{7/2}}{7b^4}-\frac{6a(a+bx^n)^{5/2}}{5b^4}}{n}$$

input
```
Int[x^(-1 + 4*n)/Sqrt[a + b*x^n],x]
```

output
```
((-2*a^3*Sqrt[a + b*x^n])/b^4 + (2*a^2*(a + b*x^n)^(3/2))/b^4 - (6*a*(a +
b*x^n)^(5/2))/(5*b^4) + (2*(a + b*x^n)^(7/2))/(7*b^4))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

method	result	size
risch	$-\frac{2\left(-5 b^{3} x^{3 n}+6 a\,b^{2} x^{2 n}-8 a^{2} b\,x^{n}+16 a^{3}\right) \sqrt{a +b\,x^{n}}}{35 b^{4} n}$	54

input
```
int(x^(-1+4*n)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-5*(x^n)^3*b^3+6*a*(x^n)^2*b^2-8*a^2*b*x^n+16*a^3)*(a+b*x^n)^(1/2)/
b^4/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx = \frac{2\,{\left(5\,b^{3} x^{3\,n} - 6\,a b^{2} x^{2\,n} + 8\,a^{2} b x^{n} - 16\,a^{3}\right)} \sqrt{b x^{n} + a}}{35\,b^{4} n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^(3*n) - 6*a*b^2*x^(2*n) + 8*a^2*b*x^n - 16*a^3)*sqrt(b*x^n +
 a)/(b^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2422 vs. 2(78) = 156.

Time = 3.55 (sec) , antiderivative size = 2422, normalized size of antiderivative = 27.52

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx = \text{Too large to display}$$

input
```
integrate(x**(-1+4*n)/(a+b*x**n)**(1/2),x)
```

output
```
-32*a**(25/2)*b**(23/2)*x**(23*n/2)*sqrt(a/(b*x**n) + 1)/(35*a**(19/2)*b**
15*n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x
**(13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*
n) + 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 176
*a**(23/2)*b**(25/2)*x**(25*n/2)*sqrt(a/(b*x**n) + 1)/(35*a**(19/2)*b**15*
n*x**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(
13*n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n)
+ 210*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 396*a*
*(21/2)*b**(27/2)*x**(27*n/2)*sqrt(a/(b*x**n) + 1)/(35*a**(19/2)*b**15*n*x
**(11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*
n) + 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 2
10*a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 462*a**(1
9/2)*b**(29/2)*x**(29*n/2)*sqrt(a/(b*x**n) + 1)/(35*a**(19/2)*b**15*n*x**(
11*n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n)
+ 700*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*
a**(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 280*a**(17/2
)*b**(31/2)*x**(31*n/2)*sqrt(a/(b*x**n) + 1)/(35*a**(19/2)*b**15*n*x**(11*
n) + 210*a**(17/2)*b**16*n*x**(12*n) + 525*a**(15/2)*b**17*n*x**(13*n) + 7
00*a**(13/2)*b**18*n*x**(14*n) + 525*a**(11/2)*b**19*n*x**(15*n) + 210*a**
(9/2)*b**20*n*x**(16*n) + 35*a**(7/2)*b**21*n*x**(17*n)) - 42*a**(15/2)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx = \frac{2\,(5\,b^4x^{4\,n} - ab^3x^{3\,n} + 2\,a^2b^2x^{2\,n} - 8\,a^3bx^n - 16\,a^4)}{35\,\sqrt{bx^n+a}b^4n}$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*b^4*x^(4*n) - a*b^3*x^(3*n) + 2*a^2*b^2*x^(2*n) - 8*a^3*b*x^n - 16
*a^4)/(sqrt(b*x^n + a)*b^4*n)
```

Giac [F]

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{4\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+4*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(4*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{4\,n-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(4*n - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(4*n - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{x^{-1+4n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^n b+a}\,(5x^{3n}b^3-6x^{2n}a\,b^2+8x^n a^2 b-16a^3)}{35b^4 n}$$

input
```
int(x^(-1+4*n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(5*x**(3*n)*b**3 - 6*x**(2*n)*a*b**2 + 8*x**n*a**2*b -
 16*a**3))/(35*b**4*n)
```

3.545 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx=\frac{2a^2\sqrt{a+bx^n}}{b^3n}-\frac{4a(a+bx^n)^{3/2}}{3b^3n}+\frac{2(a+bx^n)^{5/2}}{5b^3n}$$

output
```
2*a^2*(a+b*x^n)^(1/2)/b^3/n-4/3*a*(a+b*x^n)^(3/2)/b^3/n+2/5*(a+b*x^n)^(5/2
)/b^3/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx=\frac{2\sqrt{a+bx^n}(8a^2-4abx^n+3b^2x^{2n})}{15b^3n}$$

input
```
Integrate[x^(-1 + 3*n)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a + b*x^n]*(8*a^2 - 4*a*b*x^n + 3*b^2*x^(2*n)))/(15*b^3*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{2n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{a^2}{b^2\sqrt{bx^n+a}}-\frac{2\sqrt{bx^n+a}a}{b^2}+\frac{(bx^n+a)^{3/2}}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2a^2\sqrt{a+bx^n}}{b^3}+\frac{2(a+bx^n)^{5/2}}{5b^3}-\frac{4a(a+bx^n)^{3/2}}{3b^3}}{n}$$

input
```
Int[x^(-1 + 3*n)/Sqrt[a + b*x^n],x]
```

output
```
((2*a^2*Sqrt[a + b*x^n])/b^3 - (4*a*(a + b*x^n)^(3/2))/(3*b^3) + (2*(a + b
*x^n)^(5/2))/(5*b^3))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{2\left(3b^{2}x^{2n}-4ab\,x^{n}+8a^{2}\right)\sqrt{a+b\,x^{n}}}{15b^{3}n}$	41

input
```
int(x^(-1+3*n)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*(x^n)^2-4*a*b*x^n+8*a^2)*(a+b*x^n)^(1/2)/b^3/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx = \frac{2\,(3\,b^2x^{2\,n}-4\,abx^n+8\,a^2)\sqrt{bx^n+a}}{15\,b^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^(2*n) - 4*a*b*x^n + 8*a^2)*sqrt(b*x^n + a)/(b^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(58) = 116$.

Time = 2.18 (sec) , antiderivative size = 916, normalized size of antiderivative = 13.88

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx = \text{Too large to display}$$

input

```
integrate(x**(-1+3*n)/(a+b*x**n)**(1/2),x)
```

output

```
16*a**(15/2)*b**(9/2)*x**(9*n/2)*sqrt(a/(b*x**n) + 1)/(15*a**(11/2)*b**7*n
*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15
*a**(5/2)*b**10*n*x**(7*n)) + 40*a**(13/2)*b**(11/2)*x**(11*n/2)*sqrt(a/(b
*x**n) + 1)/(15*a**(11/2)*b**7*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) +
45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*b**10*n*x**(7*n)) + 30*a**(11/2)
*b**(13/2)*x**(13*n/2)*sqrt(a/(b*x**n) + 1)/(15*a**(11/2)*b**7*n*x**(4*n)
+ 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*
b**10*n*x**(7*n)) + 10*a**(9/2)*b**(15/2)*x**(15*n/2)*sqrt(a/(b*x**n) + 1)
/(15*a**(11/2)*b**7*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)
*b**9*n*x**(6*n) + 15*a**(5/2)*b**10*n*x**(7*n)) + 10*a**(7/2)*b**(17/2)*x
**(17*n/2)*sqrt(a/(b*x**n) + 1)/(15*a**(11/2)*b**7*n*x**(4*n) + 45*a**(9/2
)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n) + 15*a**(5/2)*b**10*n*x**(
7*n)) + 6*a**(5/2)*b**(19/2)*x**(19*n/2)*sqrt(a/(b*x**n) + 1)/(15*a**(11/2
)*b**7*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6
*n) + 15*a**(5/2)*b**10*n*x**(7*n)) - 16*a**8*b**4*x**(4*n)/(15*a**(11/2)*
b**7*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n
) + 15*a**(5/2)*b**10*n*x**(7*n)) - 48*a**7*b**5*x**(5*n)/(15*a**(11/2)*b*
*7*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n)
+ 15*a**(5/2)*b**10*n*x**(7*n)) - 48*a**6*b**6*x**(6*n)/(15*a**(11/2)*b**7
*n*x**(4*n) + 45*a**(9/2)*b**8*n*x**(5*n) + 45*a**(7/2)*b**9*n*x**(6*n)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx = \frac{2\,(3\,b^3x^{3\,n} - ab^2x^{2\,n} + 4\,a^2bx^n + 8\,a^3)}{15\,\sqrt{bx^n+a}b^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^3*x^(3*n) - a*b^2*x^(2*n) + 4*a^2*b*x^n + 8*a^3)/(sqrt(b*x^n + a
)*b^3*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(3*n - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(3*n - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^n b+a}\,(3x^{2n}b^2-4x^n ab+8a^2)}{15b^3n}$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(3*x**(2*n)*b**2 - 4*x**n*a*b + 8*a**2))/(15*b**3*n)
```

3.546 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = -\frac{2a\sqrt{a+bx^n}}{b^2n}+\frac{2(a+bx^n)^{3/2}}{3b^2n}$$

output
```
-2*a*(a+b*x^n)^(1/2)/b^2/n+2/3*(a+b*x^n)^(3/2)/b^2/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = \frac{2(-2a+bx^n)\sqrt{a+bx^n}}{3b^2n}$$

input
```
Integrate[x^(-1 + 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(2*(-2*a + b*x^n)*Sqrt[a + b*x^n])/(3*b^2*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{x^n}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{\sqrt{bx^n+a}}{b}-\frac{a}{b\sqrt{bx^n+a}}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2(a+bx^n)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx^n}}{b^2}}{n}$$

input
```
Int[x^(-1 + 2*n)/Sqrt[a + b*x^n],x]
```

output
```
((-2*a*Sqrt[a + b*x^n])/b^2 + (2*(a + b*x^n)^(3/2))/(3*b^2))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{2(-b\,x^{n}+2a)\sqrt{a+b\,x^{n}}}{3b^{2}n}$	28

input
```
int(x^(2*n-1)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*x^n+2*a)*(a+b*x^n)^(1/2)/b^2/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = \frac{2\,\sqrt{bx^n+a}(bx^n-2\,a)}{3\,b^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^n + a)*(b*x^n - 2*a)/(b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(36) = 72$.

Time = 1.43 (sec) , antiderivative size = 275, normalized size of antiderivative = 6.55

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = -\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}x^{\frac{3n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}} - \frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}x^{\frac{5n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}} + \frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}x^{\frac{7n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}} + \frac{4a^4bx^n}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}} + \frac{4a^3b^2x^{2n}}{3a^{\frac{5}{2}}b^3nx^n+3a^{\frac{3}{2}}b^4nx^{2n}}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-4*a**(7/2)*b**(3/2)*x**(3*n/2)*sqrt(a/(b*x**n) + 1)/(3*a**(5/2)*b**3*n*x*
*n + 3*a**(3/2)*b**4*n*x**(2*n)) - 2*a**(5/2)*b**(5/2)*x**(5*n/2)*sqrt(a/(
b*x**n) + 1)/(3*a**(5/2)*b**3*n*x**n + 3*a**(3/2)*b**4*n*x**(2*n)) + 2*a**
(3/2)*b**(7/2)*x**(7*n/2)*sqrt(a/(b*x**n) + 1)/(3*a**(5/2)*b**3*n*x**n + 3
*a**(3/2)*b**4*n*x**(2*n)) + 4*a**4*b*x**n/(3*a**(5/2)*b**3*n*x**n + 3*a**
(3/2)*b**4*n*x**(2*n)) + 4*a**3*b**2*x**(2*n)/(3*a**(5/2)*b**3*n*x**n + 3*
a**(3/2)*b**4*n*x**(2*n))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = \frac{2\left(b^2x^{2n}-abx^n-2a^2\right)}{3\sqrt{bx^n+a}b^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^(2*n) - a*b*x^n - 2*a^2)/(sqrt(b*x^n + a)*b^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(2*n - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^nb+a}\,(x^nb-2a)}{3b^2n}$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(x**n*b - 2*a))/(3*b**2*n)
```

3.547 $\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx=\frac{2\sqrt{a+bx^n}}{bn}$$

output
```
2*(a+b*x^n)^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx=\frac{2\sqrt{a+bx^n}}{bn}$$

input
```
Integrate[x^(-1 + n)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a + b*x^n])/(b*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \ 793$$

$$\frac{2\sqrt{a+bx^n}}{bn}$$

input
```
Int[x^(-1 + n)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a + b*x^n])/(b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{2\sqrt{a+b\,x^{n}}}{bn}$	18

input
```
int(x^(-1+n)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a+b*x^n)^(1/2)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{bx^n+a}}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^n + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(14) = 28.

Time = 0.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \begin{cases} \frac{\log(x)}{\sqrt{a}} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{n-1}}{\sqrt{a}n} & \text{for } b=0 \\ \frac{\log(x)}{\sqrt{a+b}} & \text{for } n=0 \\ \frac{2\sqrt{a+bx^n}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**(1/2),x)
```

output
```
Piecewise((log(x)/sqrt(a), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(sqrt(a)*n)
, Eq(b, 0)), (log(x)/sqrt(a + b), Eq(n, 0)), (2*sqrt(a + b*x**n)/(b*n), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{bx^n+a}}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x^n + a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{bx^n+a}}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x^n + a)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{a+b\,x^n}}{b\,n}$$

input
```
int(x^(n - 1)/(a + b*x^n)^(1/2),x)
```

output
```
(2*(a + b*x^n)^(1/2))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^n b+a}}{bn}$$

input
```
int(x^(-1+n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a))/(b*n)
```

3.548 $\int \frac{1}{x\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{bn}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[a + b*x^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	23
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$	23

input
```
int(1/x/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \left[\frac{\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)}{\sqrt{a}n}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)}{an}\right]$$

input
```
integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*n), 2*sqrt(-a
)*arctan(sqrt(-a)/sqrt(b*x^n + a))/(a*n)]
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = -\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}n}$$

input
```
integrate(1/x/(a+b*x**n)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(sqrt(a)*n)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*n)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^n + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a+bx^n}}\,dx = \int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx$$

input
```
int(1/x/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)
```

3.549 $\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-n}\sqrt{a+bx^n}}{an}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

output
```
-(a+b*x^n)^(1/2)/a/n/(x^n)+b*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-n}\sqrt{a+bx^n}}{an}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

input
```
Integrate[x^(-1 - n)/Sqrt[a + b*x^n],x]
```

output
```
-(Sqrt[a + b*x^n]/(a*n*x^n)) + (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/
2)*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n-1}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{x^{-2n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \; 52$$

$$\frac{-\frac{b\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{2a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}}{n}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^{-n}\sqrt{a+bx^n}}{a}}{n}$$

input
```
Int[x^(-1 - n)/Sqrt[a + b*x^n],x]
```

output
```
(-(Sqrt[a + b*x^n]/(a*x^n)) + (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2)
)/n
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1-n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-1-n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-n)/(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.19

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = \left[\frac{\sqrt{a}bx^n\log\left(\frac{bx^n+2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)-2\sqrt{bx^n+a}a}{2a^2nx^n}, -\frac{\sqrt{-a}bx^n\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+\sqrt{bx^n+a}a}{a^2nx^n}\right]$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*b*x^n*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) - 2
*sqrt(b*x^n + a)*a)/(a^2*n*x^n), -(sqrt(-a)*b*x^n*arctan(sqrt(-a)/sqrt(b*x
^n + a)) + sqrt(b*x^n + a)*a)/(a^2*n*x^n)]
```

Sympy [A] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = -\frac{\sqrt{b}x^{-\frac{n}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{an}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}n}$$

input
```
integrate(x**(-1-n)/(a+b*x**n)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**n) + 1)/(a*n*x**(n/2)) + b*asinh(sqrt(a)/(sqrt(b)*x*
*(n/2)))/(a**(3/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{x^{n+1}\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^(n + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^(n + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n}}{\sqrt{a+bx^n}}\,dx = \frac{-2\sqrt{x^n b+a}-x^n\left(\int \frac{\sqrt{x^n b+a}}{x^n bx+ax}dx\right)bn}{2x^n an}$$

input
```
int(x^(-1-n)/(a+b*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(x**n*b + a) - x**n*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*b*n
)/(2*x**n*a*n)
```

3.550 $\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-2n}\sqrt{a+bx^n}}{2an}+\frac{3bx^{-n}\sqrt{a+bx^n}}{4a^2n}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{5/2}n}$$

output
```
-1/2*(a+b*x^n)^(1/2)/a/n/(x^(2*n))+3/4*b*(a+b*x^n)^(1/2)/a^2/n/(x^n)-3/4*b
^2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = \frac{\sqrt{a}x^{-2n}\sqrt{a+bx^n}(-2a+3bx^n)-3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{4a^{5/2}n}$$

input
```
Integrate[x^(-1 - 2*n)/Sqrt[a + b*x^n],x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^n]*(-2*a + 3*b*x^n))/x^(2*n) - 3*b^2*ArcTanh[Sqrt[a
 + b*x^n]/Sqrt[a]])/(4*a^(5/2)*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-2n-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-3n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{3b\int \frac{x^{-2n}}{\sqrt{bx^n+a}}dx^n}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{3b\left(-\frac{b\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{2a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}}{n}$$

input
```
Int[x^(-1 - 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(-1/2*Sqrt[a + b*x^n]/(a*x^(2*n)) - (3*b*(-(Sqrt[a + b*x^n]/(a*x^n)) + (b*
ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2)))/(4*a))/n
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-2n-1}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-2*n-1)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-2*n-1)/(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.77

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = \left[\frac{3\,\sqrt{a}b^2x^{2\,n}\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(3\,abx^n-2\,a^2)\sqrt{bx^n+a}}{8\,a^3nx^{2\,n}}, \frac{3\,\sqrt{-a}b^2x^{2\,n}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+(3\,ab\right.$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*x^(2*n)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/
x^n) + 2*(3*a*b*x^n - 2*a^2)*sqrt(b*x^n + a))/(a^3*n*x^(2*n)), 1/4*(3*sqrt
(-a)*b^2*x^(2*n)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + (3*a*b*x^n - 2*a^2)*sq
rt(b*x^n + a))/(a^3*n*x^(2*n))]
```

Sympy [A] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-\frac{5n}{2}}}{2\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{\sqrt{b}x^{-\frac{3n}{2}}}{4an\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{3b^{\frac{3}{2}}x^{-\frac{n}{2}}}{4a^2n\sqrt{\frac{ax^{-n}}{b}+1}} - \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{4a^{\frac{5}{2}}n}$$

input
```
integrate(x**(-1-2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-1/(2*sqrt(b)*n*x**(5*n/2)*sqrt(a/(b*x**n) + 1)) + sqrt(b)/(4*a*n*x**(3*n/
2)*sqrt(a/(b*x**n) + 1)) + 3*b**(3/2)/(4*a**2*n*x**(n/2)*sqrt(a/(b*x**n) +
 1)) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(4*a**(5/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-2n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-2n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{x^{2n+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^(2*n + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^(2*n + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-2n}}{\sqrt{a+bx^n}}\,dx = \frac{6x^n\sqrt{x^nb+a}\,b-4\sqrt{x^nb+a}\,a+3x^{2n}\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)b^2n}{8x^{2n}a^2n}$$

input
```
int(x^(-1-2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(6*x**n*sqrt(x**n*b + a)*b - 4*sqrt(x**n*b + a)*a + 3*x**(2*n)*int(sqrt(x*
*n*b + a)/(x**n*b*x + a*x),x)*b**2*n)/(8*x**(2*n)*a**2*n)
```

3.551 $\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-3n}\sqrt{a+bx^n}}{3an}+\frac{5bx^{-2n}\sqrt{a+bx^n}}{12a^2n} - \frac{5b^2x^{-n}\sqrt{a+bx^n}}{8a^3n}+\frac{5b^3\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{8a^{7/2}n}$$

output
```
-1/3*(a+b*x^n)^(1/2)/a/n/(x^(3*n))+5/12*b*(a+b*x^n)^(1/2)/a^2/n/(x^(2*n))-
5/8*b^2*(a+b*x^n)^(1/2)/a^3/n/(x^n)+5/8*b^3*arctanh((a+b*x^n)^(1/2)/a^(1/2
))/a^(7/2)/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = \frac{\sqrt{a}x^{-3n}\sqrt{a+bx^n}(-8a^2+10abx^n-15b^2x^{2n})+15b^3\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{24a^{7/2}n}$$

input
```
Integrate[x^(-1 - 3*n)/Sqrt[a + b*x^n],x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^n]*(-8*a^2 + 10*a*b*x^n - 15*b^2*x^(2*n)))/x^(3*n)
+ 15*b^3*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(24*a^(7/2)*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {798, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-3n-1}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{x^{-4n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \; 52$$

$$\frac{-\frac{5b\int \frac{x^{-3n}}{\sqrt{bx^n+a}}dx^n}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}}{n}$$

$$\Big\downarrow \; 52$$

$$\frac{-\frac{5b\left(-\frac{3b\int \frac{x^{-2n}}{\sqrt{bx^n+a}}dx^n}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}}{n}$$

$$\Big\downarrow \; 52$$

$$\frac{-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{2a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}}{n}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{5b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{5b\left(-\frac{3b\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}}{n}$$

input
```
Int[x^(-1 - 3*n)/Sqrt[a + b*x^n],x]
```

output
```
(-1/3*Sqrt[a + b*x^n]/(a*x^(3*n)) - (5*b*(-1/2*Sqrt[a + b*x^n]/(a*x^(2*n))
 - (3*b*(-(Sqrt[a + b*x^n]/(a*x^n)) + (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])
/a^(3/2)))/(4*a)))/(6*a))/n
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1-3n}}{\sqrt{a+b\,x^n}}dx$$

input
```
int(x^(-1-3*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-3*n)/(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.55

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = \left[\frac{15\,\sqrt{a}b^3x^{3\,n}\log\left(\frac{bx^n+2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)-2\,(15\,ab^2x^{2\,n}-10\,a^2bx^n+8\,a^3)\sqrt{bx^n+a}}{48\,a^4nx^{3\,n}}, -\frac{15\,\sqrt{-a}b^3x^{3\,n}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+(15\,ab^2x^{2\,n}-10\,a^2bx^n+8\,a^3)\sqrt{bx^n+a}}{24\,a^4nx^{3\,n}}\right]$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*sqrt(a)*b^3*x^(3*n)*log((b*x^n + 2*sqrt(b*x^n + a)*sqrt(a) + 2*a
)/x^n) - 2*(15*a*b^2*x^(2*n) - 10*a^2*b*x^n + 8*a^3)*sqrt(b*x^n + a))/(a^4
*n*x^(3*n)), -1/24*(15*sqrt(-a)*b^3*x^(3*n)*arctan(sqrt(-a)/sqrt(b*x^n + a
)) + (15*a*b^2*x^(2*n) - 10*a^2*b*x^n + 8*a^3)*sqrt(b*x^n + a))/(a^4*n*x^(
3*n))]
```

Sympy [A] (verification not implemented)

Time = 29.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-\frac{7n}{2}}}{3\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{\sqrt{b}x^{-\frac{5n}{2}}}{12an\sqrt{\frac{ax^{-n}}{b}+1}} - \frac{5b^{\frac{3}{2}}x^{-\frac{3n}{2}}}{24a^2n\sqrt{\frac{ax^{-n}}{b}+1}} - \frac{5b^{\frac{5}{2}}x^{-\frac{n}{2}}}{8a^3n\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{8a^{\frac{7}{2}}n}$$

input
```
integrate(x**(-1-3*n)/(a+b*x**n)**(1/2),x)
```

output
```
-1/(3*sqrt(b)*n*x**(7*n/2)*sqrt(a/(b*x**n) + 1)) + sqrt(b)/(12*a*n*x**(5*n
/2)*sqrt(a/(b*x**n) + 1)) - 5*b**(3/2)/(24*a**2*n*x**(3*n/2)*sqrt(a/(b*x**
n) + 1)) - 5*b**(5/2)/(8*a**3*n*x**(n/2)*sqrt(a/(b*x**n) + 1)) + 5*b**3*as
inh(sqrt(a)/(sqrt(b)*x**(n/2)))/(8*a**(7/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-3n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-3*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-3\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-3*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-3*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{x^{3\,n+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^(3*n + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^(3*n + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-3n}}{\sqrt{a+bx^n}}\,dx = \frac{-30x^{2n}\sqrt{x^nb+a}\,b^2+20x^n\sqrt{x^nb+a}\,ab-16\sqrt{x^nb+a}\,a^2-15x^{3n}\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)b^3n}{48x^{3n}a^3n}$$

input
```
int(x^(-1-3*n)/(a+b*x^n)^(1/2),x)
```

output
```
( - 30*x**(2*n)*sqrt(x**n*b + a)*b**2 + 20*x**n*sqrt(x**n*b + a)*a*b - 16*
sqrt(x**n*b + a)*a**2 - 15*x**(3*n)*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),
x)*b**3*n)/(48*x**(3*n)*a**3*n)
```

3.552 $\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-4n}\sqrt{a+bx^n}}{4an}+\frac{7bx^{-3n}\sqrt{a+bx^n}}{24a^2n}-\frac{35b^2x^{-2n}\sqrt{a+bx^n}}{96a^3n}+\frac{35b^3x^{-n}\sqrt{a+bx^n}}{64a^4n}-\frac{35b^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{64a^{9/2}n}$$

output
```
-1/4*(a+b*x^n)^(1/2)/a/n/(x^(4*n))+7/24*b*(a+b*x^n)^(1/2)/a^2/n/(x^(3*n))-
35/96*b^2*(a+b*x^n)^(1/2)/a^3/n/(x^(2*n))+35/64*b^3*(a+b*x^n)^(1/2)/a^4/n/
(x^n)-35/64*b^4*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(9/2)/n
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx = \frac{\sqrt{a}x^{-4n}\sqrt{a+bx^n}(-48a^3+56a^2bx^n-70ab^2x^{2n}+105b^3x^{3n})-105b^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{192a^{9/2}n}$$

input
```
Integrate[x^(-1 - 4*n)/Sqrt[a + b*x^n],x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^n]*(-48*a^3 + 56*a^2*b*x^n - 70*a*b^2*x^(2*n) + 105
*b^3*x^(3*n)))/x^(4*n) - 105*b^4*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(192*a^
(9/2)*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {798, 52, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-4n-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{x^{-5n}}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{7b\int \frac{x^{-4n}}{\sqrt{bx^n+a}}dx^n}{8a}-\frac{x^{-4n}\sqrt{a+bx^n}}{4a}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{7b\left(-\frac{5b\int \frac{x^{-3n}}{\sqrt{bx^n+a}}dx^n}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)}{8a}-\frac{x^{-4n}\sqrt{a+bx^n}}{4a}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{x^{-2n}}{\sqrt{bx^n+a}}dx^n}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)}{8a}-\frac{x^{-4n}\sqrt{a+bx^n}}{4a}}{n}$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{2a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)}{8a}-\frac{x^{-4n}\sqrt{a+bx^n}}{4a}}{n}$$

$\downarrow$ 73

$$\frac{-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{a}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)}{8a}-\frac{x^{-4n}\sqrt{a+bx^n}}{4a}}{n}$$

$\downarrow$ 221

$$\frac{-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{x^{-n}\sqrt{a+bx^n}}{a}\right)}{4a}-\frac{x^{-2n}\sqrt{a+bx^n}}{2a}\right)}{6a}-\frac{x^{-3n}\sqrt{a+bx^n}}{3a}\right)}{8a}-\frac{x^{-4n}\sqrt{a+bx^n}}{4a}}{n}$$

input
```
Int[x^(-1 - 4*n)/Sqrt[a + b*x^n],x]
```

output
```
(-1/4*Sqrt[a + b*x^n]/(a*x^(4*n)) - (7*b*(-1/3*Sqrt[a + b*x^n]/(a*x^(3*n))
 - (5*b*(-1/2*Sqrt[a + b*x^n]/(a*x^(2*n)) - (3*b*(-(Sqrt[a + b*x^n]/(a*x^n
)) + (b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a))
/n
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-4n-1}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-4*n-1)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-4*n-1)/(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.42

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx$$
$$= \left[\frac{105\,\sqrt{a}b^4x^{4\,n}\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(105\,ab^3x^{3\,n}-70\,a^2b^2x^{2\,n}+56\,a^3bx^n-48\,a^4)\sqrt{bx^n+a}}{384\,a^5nx^{4\,n}},\ 105\right.$$

input
```
integrate(x^(-1-4*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(105*sqrt(a)*b^4*x^(4*n)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2
*a)/x^n) + 2*(105*a*b^3*x^(3*n) - 70*a^2*b^2*x^(2*n) + 56*a^3*b*x^n - 48*a
^4)*sqrt(b*x^n + a))/(a^5*n*x^(4*n)), 1/192*(105*sqrt(-a)*b^4*x^(4*n)*arct
an(sqrt(-a)/sqrt(b*x^n + a)) + (105*a*b^3*x^(3*n) - 70*a^2*b^2*x^(2*n) + 5
6*a^3*b*x^n - 48*a^4)*sqrt(b*x^n + a))/(a^5*n*x^(4*n))]
```

Sympy [A] (verification not implemented)

Time = 95.59 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.26

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{-\frac{9n}{2}}}{4\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{\sqrt{b}x^{-\frac{7n}{2}}}{24an\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{7b^{\frac{3}{2}}x^{-\frac{5n}{2}}}{96a^2n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{35b^{\frac{5}{2}}x^{-\frac{3n}{2}}}{192a^3n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{35b^{\frac{7}{2}}x^{-\frac{n}{2}}}{64a^4n\sqrt{\frac{ax^{-n}}{b}+1}}-\frac{35b^4\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{64a^{\frac{9}{2}}n}$$

input
```
integrate(x**(-1-4*n)/(a+b*x**n)**(1/2),x)
```

output
```
-1/(4*sqrt(b)*n*x**(9*n/2)*sqrt(a/(b*x**n) + 1)) + sqrt(b)/(24*a*n*x**(7*n
/2)*sqrt(a/(b*x**n) + 1)) - 7*b**(3/2)/(96*a**2*n*x**(5*n/2)*sqrt(a/(b*x**
n) + 1)) + 35*b**(5/2)/(192*a**3*n*x**(3*n/2)*sqrt(a/(b*x**n) + 1)) + 35*b
**(7/2)/(64*a**4*n*x**(n/2)*sqrt(a/(b*x**n) + 1)) - 35*b**4*asinh(sqrt(a)/
(sqrt(b)*x**(n/2)))/(64*a**(9/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-4\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-4*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-4*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-4\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-4*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-4*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{x^{4\,n+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^(4*n + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^(4*n + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-4n}}{\sqrt{a+bx^n}}\,dx$$
$$= \frac{210x^{3n}\sqrt{x^nb+a}\,b^3-140x^{2n}\sqrt{x^nb+a}\,a\,b^2+112x^n\sqrt{x^nb+a}\,a^2b-96\sqrt{x^nb+a}\,a^3+105x^{4n}\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}d\right.}{384x^{4n}a^4n}$$

input
```
int(x^(-1-4*n)/(a+b*x^n)^(1/2),x)
```

output
```
(210*x**(3*n)*sqrt(x**n*b + a)*b**3 - 140*x**(2*n)*sqrt(x**n*b + a)*a*b**2
 + 112*x**n*sqrt(x**n*b + a)*a**2*b - 96*sqrt(x**n*b + a)*a**3 + 105*x**(4
*n)*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*b**4*n)/(384*x**(4*n)*a**4*n)
```

3.553 $\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx=\frac{3\sqrt[3]{a+bx^n}}{n}-\frac{\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^n}}{\sqrt{3}\sqrt[3]{a}}\right)}{n}-\frac{1}{2}\sqrt[3]{a}\log(x)+\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2n}$$

output
```
3*(a+b*x^n)^(1/3)/n-3^(1/2)*a^(1/3)*arctan(1/3*(a^(1/3)+2*(a+b*x^n)^(1/3))
*3^(1/2)/a^(1/3))/n-1/2*a^(1/3)*ln(x)+3/2*a^(1/3)*ln(a^(1/3)-(a+b*x^n)^(1/
3))/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx$$

$$=\frac{6\sqrt[3]{a+bx^n}-2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^n}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)-\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+\right.}{2n}$$

input
```
Integrate[(a + b*x^n)^(1/3)/x,x]
```

output
```
(6*(a + b*x^n)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x^n)^(1/3))
/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[a^(1/3) - (a + b*x^n)^(1/3)] - a^(1/3)*
Log[a^(2/3) + a^(1/3)*(a + b*x^n)^(1/3) + (a + b*x^n)^(2/3)])/(2*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^n}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{-n}\sqrt[3]{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{x^{-n}}{(bx^n+a)^{2/3}}dx^n+3\sqrt[3]{a+bx^n}}{n}$$

$$\Big\downarrow \text{69}$$

$$\frac{a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^n+a}}d\sqrt[3]{bx^n+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^{2n}+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^n+a}}d\sqrt[3]{bx^n+a}}{2\sqrt[3]{a}}-\frac{\log(x^n)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^n}}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\left(-\frac{3\int \frac{1}{x^{2n}+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^n+a}}d\sqrt[3]{bx^n+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2a^{2/3}}-\frac{\log(x^n)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^n}}{n}$$

$$\Big\downarrow \text{1082}$$

$$\frac{a\left(\frac{3\int \frac{1}{-x^{2n}-3}d\left(\frac{2\sqrt[3]{bx^n+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2a^{2/3}}-\frac{\log(x^n)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^n}}{n}$$

$$\Big\downarrow \text{217}$$

$$\frac{a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^n}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^n}\right)}{2a^{2/3}}-\frac{\log(x^n)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^n}}{n}$$

input
```
Int[(a + b*x^n)^(1/3)/x,x]
```

output
```
(3*(a + b*x^n)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^n)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(2/3)) - Log[x^n]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*
x^n)^(1/3)])/(2*a^(2/3))))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{3(a+b\,x^{n})^{\frac{1}{3}}+3\left(\frac{\ln\left((a+b\,x^{n})^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((a+b\,x^{n})^{\frac{2}{3}}+a^{\frac{1}{3}}(a+b\,x^{n})^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(a+b\,x^{n})^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right)a}{n}$
default	$\frac{3(a+b\,x^{n})^{\frac{1}{3}}+3\left(\frac{\ln\left((a+b\,x^{n})^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((a+b\,x^{n})^{\frac{2}{3}}+a^{\frac{1}{3}}(a+b\,x^{n})^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(a+b\,x^{n})^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}\right)a}{n}$

input
```
int((a+b*x^n)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(3*(a+b*x^n)^(1/3)+3*(1/3/a^(2/3)*ln((a+b*x^n)^(1/3)-a^(1/3))-1/6/a^(2
/3)*ln((a+b*x^n)^(2/3)+a^(1/3)*(a+b*x^n)^(1/3)+a^(2/3))-1/3/a^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(a+b*x^n)^(1/3)+1)))*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx = -\frac{2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^n+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{1}{3}}\log\left((bx^n+a)^{\frac{2}{3}}+(bx^n+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{1}{3}}\log\left((bx^n+\right.}{2n}$$

input
```
integrate((a+b*x^n)^(1/3)/x,x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^n + a)^(1/3)*a^(2/3) +
sqrt(3)*a)/a) + a^(1/3)*log((b*x^n + a)^(2/3) + (b*x^n + a)^(1/3)*a^(1/3)
+ a^(2/3)) - 2*a^(1/3)*log((b*x^n + a)^(1/3) - a^(1/3)) - 6*(b*x^n + a)^(1
/3))/n
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx = -\frac{\sqrt[3]{b}x^{\frac{n}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ax^{-n}e^{i\pi}}{b}\right)}{n\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((a+b*x**n)**(1/3)/x,x)
```

output
```
-b**(1/3)*x**(n/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*p
i)/(b*x**n))/(n*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx = -\frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^n+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{n} - \frac{a^{\frac{1}{3}}\log\left((bx^n+a)^{\frac{2}{3}}+(bx^n+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,n} + \frac{a^{\frac{1}{3}}\log\left((bx^n+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{n} + \frac{3\,(bx^n+a)^{\frac{1}{3}}}{n}$$

input
```
integrate((a+b*x^n)^(1/3)/x,x, algorithm="maxima")
```

output
```
-sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^n + a)^(1/3) + a^(1/3))/a^(1/3
))/n - 1/2*a^(1/3)*log((b*x^n + a)^(2/3) + (b*x^n + a)^(1/3)*a^(1/3) + a^(
2/3))/n + a^(1/3)*log((b*x^n + a)^(1/3) - a^(1/3))/n + 3*(b*x^n + a)^(1/3)
/n
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx = \int \frac{(bx^n+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(1/3)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx = \int \frac{(a+b\,x^n)^{1/3}}{x}\,dx$$

input
```
int((a + b*x^n)^(1/3)/x,x)
```

output
```
int((a + b*x^n)^(1/3)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^n}}{x}\,dx = \frac{3(x^n b+a)^{\frac{1}{3}} + \left(\int \frac{(x^n b+a)^{\frac{1}{3}}}{x^n bx+ax}dx\right)an}{n}$$

input
```
int((a+b*x^n)^(1/3)/x,x)
```

output
```
(3*(x**n*b + a)**(1/3) + int((x**n*b + a)**(1/3)/(x**n*b*x + a*x),x)*a*n)/
n
```

3.554 $\int x^{-1+n}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{(a+bx^n)^3}{3bn}$$

output
```
1/3*(a+b*x^n)^3/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{(a+bx^n)^3}{3bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^2,x]
```

output
```
(a + b*x^n)^3/(3*b*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \textcolor{blue}{793}$$

$$\frac{(a+bx^n)^3}{3bn}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^2,x]
```

output
```
(a + b*x^n)^3/(3*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

method	result
risch	$\frac{a^{2} x^{n}}{n}+\frac{a b\,x^{2 n}}{n}+\frac{b^{2} x^{3 n}}{3 n}$
norman	$\frac{a^{2} {\mathrm e}^{n \ln \left(x \right)}}{n}+\frac{a b\,{\mathrm e}^{2 n \ln \left(x \right)}}{n}+\frac{b^{2} {\mathrm e}^{3 n \ln \left(x \right)}}{3 n}$
parallelrisch	$\frac{x\,x^{2 n} x^{-1+n} b^{2}+3 x\,x^{n} x^{-1+n} a b +3 x\,x^{-1+n} a^{2}}{3 n}$
orering	$\frac{x \left(11 n^{2}-6 n +1\right) x^{-1+n} \left(a +b\,x^{n}\right)^{2}}{6 n^{3}}-\frac{x^{2} \left(2 n -1\right) \left(\frac{x^{-1+n} \left(-1+n \right) \left(a +b\,x^{n}\right)^{2}}{x}+\frac{2 x^{-1+n} \left(a +b\,x^{n}\right) b\,x^{n} n}{x}\right)}{2 n^{3}}+\frac{x^{3} \left(\frac{x^{-1+n} \left(-1+n \right)^{2} (}{x^{2}}\right.}{}$

input
```
int(x^(-1+n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/n*x^n+a*b/n*(x^n)^2+1/3*b^2/n*(x^n)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{b^2x^{3\,n}+3\,abx^{2\,n}+3\,a^2x^n}{3\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^(3*n) + 3*a*b*x^(2*n) + 3*a^2*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int x^{-1+n}(a+bx^n)^2\,dx=\begin{cases}\frac{a^2xx^{n-1}}{n}+\frac{abxx^nx^{n-1}}{n}+\frac{b^2xx^{2n}x^{n-1}}{3n} & \text{for}\ n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*x*x**(n - 1)/n + a*b*x*x**n*x**(n - 1)/n + b**2*x*x**(2*n)
*x**(n - 1)/(3*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{(bx^n+a)^3}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
1/3*(b*x^n + a)^3/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{b^2x^{3\,n}+3\,abx^{2\,n}+3\,a^2x^n}{3\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/3*(b^2*x^(3*n) + 3*a*b*x^(2*n) + 3*a^2*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{x^n\left(a^2+\frac{b^2\,x^{2\,n}}{3}+a\,b\,x^n\right)}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^2,x)
```

output
```
(x^n*(a^2 + (b^2*x^(2*n))/3 + a*b*x^n))/n
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int x^{-1+n}(a+bx^n)^2\,dx=\frac{x^n(x^{2n}b^2+3x^nab+3a^2)}{3n}$$

input
```
int(x^(-1+n)*(a+b*x^n)^2,x)
```

output
```
(x**n*(x**(2*n)*b**2 + 3*x**n*a*b + 3*a**2))/(3*n)
```

3.555 $\int x^{-1+n}(a+bx^n)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{(a+bx^n)^2}{2bn}$$

output
```
1/2*(a+b*x^n)^2/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{(a+bx^n)^2}{2bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n),x]
```

output
```
(a + b*x^n)^2/(2*b*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{n-1}+bx^{2n-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^n}{n}+\frac{bx^{2n}}{2n}$$

input
```
Int[x^(-1 + n)*(a + b*x^n),x]
```

output
```
(a*x^n)/n + (b*x^(2*n))/(2*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{a\,x^{n}}{n}+\frac{b\,x^{2n}}{2n}$	21
norman	$\frac{a\,\mathrm{e}^{n\ln(x)}}{n}+\frac{b\,\mathrm{e}^{2n\ln(x)}}{2n}$	25
parallelrisch	$\frac{b\,x^{-1+n}x\,x^{n}+2x^{-1+n}ax}{2n}$	27
orering	$\frac{x(-1+3n)x^{-1+n}(a+b\,x^{n})}{2n^{2}}-\frac{x^{2}\left(\frac{x^{-1+n}(-1+n)(a+b\,x^{n})}{x}+\frac{x^{-1+n}b\,x^{n}n}{x}\right)}{2n^{2}}$	67

input
```
int(x^(-1+n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
a/n*x^n+1/2*b/n*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{bx^{2\,n}+2\,ax^n}{2\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(b*x^(2*n) + 2*a*x^n)/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int x^{-1+n}(a+bx^n)\,dx=\begin{cases}\frac{axx^{n-1}}{n}+\frac{bxx^nx^{n-1}}{2n} & \text{for } n\neq 0\\ (a+b)\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n),x)
```

output
```
Piecewise((a*x*x**(n - 1)/n + b*x*x**n*x**(n - 1)/(2*n), Ne(n, 0)), ((a +
b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)\ dx = \frac{(bx^n+a)^2}{2\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(b*x^n + a)^2/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)\ dx = \frac{bx^{2\,n}+2\,ax^n}{2\,n}$$

input
```
integrate(x^(-1+n)*(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*(b*x^(2*n) + 2*a*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{x^n\left(a+\frac{bx^n}{2}\right)}{n}$$

input
```
int(x^(n - 1)*(a + b*x^n),x)
```

output
```
(x^n*(a + (b*x^n)/2))/n
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int x^{-1+n}(a+bx^n)\,dx=\frac{x^n(x^nb+2a)}{2n}$$

input
```
int(x^(-1+n)*(a+b*x^n),x)
```

output
```
(x**n*(x**n*b + 2*a))/(2*n)
```

3.556 $\int \frac{x^{-1+n}}{a+bx^n} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^{-1+n}}{a+bx^n} \, dx = \frac{\log(a+bx^n)}{bn}$$

output
```
ln(a+b*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n} \, dx = \frac{\log(a+bx^n)}{bn}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n),x]
```

output
```
Log[a + b*x^n]/(b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \; 792$$

$$\frac{\log(a+bx^n)}{bn}$$

input
```
Int[x^(-1 + n)/(a + b*x^n),x]
```

output
```
Log[a + b*x^n]/(b*n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{bn}$	18
risch	$\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{bn}$	18

input
```
int(x^(-1+n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/b/n*ln(a+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(bx^n+a)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
log(b*x^n + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 1.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{n-1}}{an} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(a*n), Eq(b, 0)),
 (log(x)/(a + b), Eq(n, 0)), (log(a/b + x**n)/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log\left(bx^n+a\right)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
log(b*x^n + a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log\left(|bx^n+a|\right)}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
log(abs(b*x^n + a))/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\ln\left(a+b\,x^n\right)}{b\,n}$$

input
```
int(x^(n - 1)/(a + b*x^n),x)
```

output
```
log(a + b*x^n)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{a+bx^n}\,dx = \frac{\log(x^n b + a)}{bn}$$

input
```
int(x^(-1+n)/(a+b*x^n),x)
```

output
```
log(x**n*b + a)/(b*n)
```

3.557 $\int \frac{x^{-1+n}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{x^{-1+n}}{(a+bx^n)^2} dx = -\frac{1}{bn(a+bx^n)}$$

output
```
-1/b/n/(a+b*x^n)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2} dx = -\frac{1}{bn(a+bx^n)}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n)^2,x]
```

output
```
-(1/(b*n*(a + b*x^n)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 793$$

$$-\frac{1}{bn\,(a+bx^n)}$$

input
```
Int[x^(-1 + n)/(a + b*x^n)^2,x]
```

output
```
-(1/(b*n*(a + b*x^n)))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{1}{bn(a+b\,x^{n})}$	18
parallelrisch	$\frac{x\,x^{-1+n}}{an(a+b\,x^{n})}$	23
norman	$\frac{{\mathrm e}^{n\ln(x)}}{an\left(a+b\,{\mathrm e}^{n\ln(x)}\right)}$	24

input
```
int(x^(-1+n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/n/(a+b*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{1}{b^2nx^n+abn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*n*x^n + a*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(12) = 24.

Time = 0.86 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.82

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-2n}x^{n-1}}{b^2n} & \text{for } a=0 \\ \frac{\tilde{\infty}xx^{n-1}}{n} & \text{for } b=-ax^{-n} \\ \frac{\log(x)}{(a+b)^2} & \text{for } n=0 \\ \frac{xx^{n-1}}{a^2n+abnx^n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(n - 1)/(b*
*2*n*x**(2*n)), Eq(a, 0)), (zoo*x*x**(n - 1)/n, Eq(b, -a/x**n)), (log(x)/(
a + b)**2, Eq(n, 0)), (x*x**(n - 1)/(a**2*n + a*b*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{1}{(bx^n+a)bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-1/((b*x^n + a)*b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{1}{(bx^n+a)bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
-1/((b*x^n + a)*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{a}{b\,\left(a^2\,n+a\,b\,n\,x^n\right)}$$

input
```
int(x^(n - 1)/(a + b*x^n)^2,x)
```

output
```
-a/(b*(a^2*n + a*b*n*x^n))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{x^{-1+n}}{(a+bx^n)^2}\,dx = \frac{x^n}{an\,(x^n b+a)}$$

input
```
int(x^(-1+n)/(a+b*x^n)^2,x)
```

output
```
x**n/(a*n*(x**n*b + a))
```

3.558 $\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx=-\frac{1}{2bn\,(a+bx^n)^2}$$

output
```
-1/2/b/n/(a+b*x^n)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx=-\frac{1}{2bn\,(a+bx^n)^2}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n)^3,x]
```

output
```
-1/2*1/(b*n*(a + b*x^n)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a+bx^n)^3}\,dx$$
$$\Big\downarrow \; 793$$
$$-\frac{1}{2bn\,(a+bx^n)^2}$$

input
```
Int[x^(-1 + n)/(a + b*x^n)^3,x]
```

output
```
-1/2*1/(b*n*(a + b*x^n)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{1}{2bn(a+b\,x^{n})^{2}}$	18
norman	$-\frac{1}{2bn\left(a+b\,{\mathrm e}^{n\ln(x)}\right)^{2}}$	20
parallelrisch	$\frac{b\,x^{-1+n}x\,x^{n}+2x^{-1+n}ax}{2a^{2}n(a+b\,x^{n})^{2}}$	39

input
```
int(x^(-1+n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/n/(a+b*x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,(b^3nx^{2\,n}+2\,ab^2nx^n+a^2bn)}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
-1/2/(b^3*n*x^(2*n) + 2*a*b^2*n*x^n + a^2*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(15) = 30.

Time = 1.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 7.16

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0\wedge b=0\wedge n=0 \\ -\frac{xx^{-3n}x^{n-1}}{2b^3n} & \text{for } a=0 \\ \frac{\tilde{\infty}xx^{n-1}}{n} & \text{for } b=-ax^{-n} \\ \frac{\log(x)}{(a+b)^3} & \text{for } n=0 \\ \frac{2axx^{n-1}}{2a^4n+4a^3bnx^n+2a^2b^2nx^{2n}} + \frac{bxx^nx^{n-1}}{2a^4n+4a^3bnx^n+2a^2b^2nx^{2n}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(n - 1)/(2*
b**3*n*x**(3*n)), Eq(a, 0)), (zoo*x*x**(n - 1)/n, Eq(b, -a/x**n)), (log(x)
/(a + b)**3, Eq(n, 0)), (2*a*x*x**(n - 1)/(2*a**4*n + 4*a**3*b*n*x**n + 2*
a**2*b**2*n*x**(2*n)) + b*x*x**n*x**(n - 1)/(2*a**4*n + 4*a**3*b*n*x**n +
2*a**2*b**2*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,(bx^n+a)^2bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-1/2/((b*x^n + a)^2*b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,(bx^n+a)^2bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
-1/2/((b*x^n + a)^2*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2\,b^3\,n\,x^{2\,n}+2\,a^2\,b\,n+4\,a\,b^2\,n\,x^n}$$

input
```
int(x^(n - 1)/(a + b*x^n)^3,x)
```

output
```
-1/(2*b^3*n*x^(2*n) + 2*a^2*b*n + 4*a*b^2*n*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{x^{-1+n}}{(a+bx^n)^3}\,dx = -\frac{1}{2bn\,(x^{2n}b^2+2x^nab+a^2)}$$

input
```
int(x^(-1+n)/(a+b*x^n)^3,x)
```

output
```
( - 1)/(2*b*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.559 $\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx=\frac{2a^3x^{n/2}}{n}+\frac{2a^2bx^{3n/2}}{n}+\frac{6ab^2x^{5n/2}}{5n}+\frac{2b^3x^{7n/2}}{7n}$$

output
```
2*a^3*x^(1/2*n)/n+2*a^2*b*x^(3/2*n)/n+6/5*a*b^2*x^(5/2*n)/n+2/7*b^3*x^(7/2
*n)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx=\frac{2x^{n/2}(35a^3+35a^2bx^n+21ab^2x^{2n}+5b^3x^{3n})}{35n}$$

input
```
Integrate[x^(-1 + n/2)*(a + b*x^n)^3,x]
```

output
```
(2*x^(n/2)*(35*a^3 + 35*a^2*b*x^n + 21*a*b^2*x^(2*n) + 5*b^3*x^(3*n)))/(35
*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{2}-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^{\frac{n-2}{2}}+3a^2bx^{\frac{3n}{2}-1}+3ab^2x^{\frac{5n}{2}-1}+b^3x^{\frac{7n}{2}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3x^{n/2}}{n}+\frac{2a^2bx^{3n/2}}{n}+\frac{6ab^2x^{5n/2}}{5n}+\frac{2b^3x^{7n/2}}{7n}$$

input
```
Int[x^(-1 + n/2)*(a + b*x^n)^3,x]
```

output
```
(2*a^3*x^(n/2))/n + (2*a^2*b*x^((3*n)/2))/n + (6*a*b^2*x^((5*n)/2))/(5*n)
+ (2*b^3*x^((7*n)/2))/(7*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

method	result
risch	$\frac{2a^3x^{\frac{n}{2}}}{n}+\frac{2a^2b\,x^{\frac{3n}{2}}}{n}+\frac{6a\,b^2x^{\frac{5n}{2}}}{5n}+\frac{2b^3x^{\frac{7n}{2}}}{7n}$
parallelrisch	$\frac{10x\,x^{3n}x^{-1+\frac{n}{2}}b^3+42x\,x^{2n}x^{-1+\frac{n}{2}}a\,b^2+70x\,x^{n}x^{-1+\frac{n}{2}}a^2b+70x\,x^{-1+\frac{n}{2}}a^3}{35n}$
orering	$\frac{8x(44n^3-43n^2+16n-2)x^{-1+\frac{n}{2}}(a+b\,x^n)^3}{105n^4}-\frac{8x^2(43n^2-48n+14)\left(\frac{x^{-1+\frac{n}{2}}\left(-1+\frac{n}{2}\right)(a+b\,x^n)^3}{x}+\frac{3x^{-1+\frac{n}{2}}(a+b\,x^n)^2b\,x^n n}{x}\right)}{105n^4}$

input
```
int(x^(-1+1/2*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
2/7*b^3/n*(x^(1/2*n))^7+6/5*a*b^2/n*(x^(1/2*n))^5+2*a^2*b/n*(x^(1/2*n))^3+
2*a^3*x^(1/2*n)/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx = \frac{2\left(5\,b^3x^7x^{\frac{7}{2}\,n-7}+21\,ab^2x^5x^{\frac{5}{2}\,n-5}+35\,a^2bx^3x^{\frac{3}{2}\,n-3}+35\,a^3xx^{\frac{1}{2}\,n-1}\right)}{35\,n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^7*x^(7/2*n - 7) + 21*a*b^2*x^5*x^(5/2*n - 5) + 35*a^2*b*x^3*
x^(3/2*n - 3) + 35*a^3*x*x^(1/2*n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx = \begin{cases} \frac{2a^3xx^{\frac{n}{2}-1}}{n} + \frac{2a^2bxx^nx^{\frac{n}{2}-1}}{n} + \frac{6ab^2xx^{2n}x^{\frac{n}{2}-1}}{5n} + \frac{2b^3xx^{3n}x^{\frac{n}{2}-1}}{7n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+1/2*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((2*a**3*x*x**(n/2 - 1)/n + 2*a**2*b*x*x**n*x**(n/2 - 1)/n + 6*a*
b**2*x*x**(2*n)*x**(n/2 - 1)/(5*n) + 2*b**3*x*x**(3*n)*x**(n/2 - 1)/(7*n),
 Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx = \frac{2\,b^3x^{\frac{7}{2}\,n}}{7\,n} + \frac{6\,ab^2x^{\frac{5}{2}\,n}}{5\,n} + \frac{2\,a^2bx^{\frac{3}{2}\,n}}{n} + \frac{2\,a^3x^{\frac{1}{2}\,n}}{n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
2/7*b^3*x^(7/2*n)/n + 6/5*a*b^2*x^(5/2*n)/n + 2*a^2*b*x^(3/2*n)/n + 2*a^3*
x^(1/2*n)/n
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx=\frac{2\left(5\,b^3x^{3\,n}x^{\frac{1}{2}\,n}+21\,ab^2x^{2\,n}x^{\frac{1}{2}\,n}+35\,a^2bx^{\frac{1}{2}\,n}x^n+35\,a^3\sqrt{x^n}\right)}{35\,n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
2/35*(5*b^3*x^(3*n)*x^(1/2*n) + 21*a*b^2*x^(2*n)*x^(1/2*n) + 35*a^2*b*x^(1
/2*n)*x^n + 35*a^3*sqrt(x^n))/n
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx=x^{\frac{n}{2}-1}\left(\frac{2\,a^3\,x}{n}+\frac{2\,b^3\,x\,x^{3\,n}}{7\,n}+\frac{2\,a^2\,b\,x\,x^n}{n}+\frac{6\,a\,b^2\,x\,x^{2\,n}}{5\,n}\right)$$

input
```
int(x^(n/2 - 1)*(a + b*x^n)^3,x)
```

output
```
x^(n/2 - 1)*((2*a^3*x)/n + (2*b^3*x*x^(3*n))/(7*n) + (2*a^2*b*x*x^n)/n + (
6*a*b^2*x*x^(2*n))/(5*n))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^3\,dx=\frac{2x^{\frac{n}{2}}(5x^{3n}b^3+21x^{2n}a\,b^2+35x^na^2b+35a^3)}{35n}$$

input
```
int(x^(-1+1/2*n)*(a+b*x^n)^3,x)
```

output
```
(2*x**(n/2)*(5*x**(3*n)*b**3 + 21*x**(2*n)*a*b**2 + 35*x**n*a**2*b + 35*a*
*3))/(35*n)
```

3.560 $\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx=\frac{2a^2x^{n/2}}{n}+\frac{4abx^{3n/2}}{3n}+\frac{2b^2x^{5n/2}}{5n}$$

output
```
2*a^2*x^(1/2*n)/n+4/3*a*b*x^(3/2*n)/n+2/5*b^2*x^(5/2*n)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx=\frac{2x^{n/2}(15a^2+10abx^n+3b^2x^{2n})}{15n}$$

input
```
Integrate[x^(-1 + n/2)*(a + b*x^n)^2,x]
```

output
```
(2*x^(n/2)*(15*a^2 + 10*a*b*x^n + 3*b^2*x^(2*n)))/(15*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{2}-1}(a+bx^n)^2\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^{\frac{n-2}{2}}+2abx^{\frac{3n}{2}-1}+b^2x^{\frac{5n}{2}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2x^{n/2}}{n}+\frac{4abx^{3n/2}}{3n}+\frac{2b^2x^{5n/2}}{5n}$$

input
```
Int[x^(-1 + n/2)*(a + b*x^n)^2,x]
```

output
```
(2*a^2*x^(n/2))/n + (4*a*b*x^((3*n)/2))/(3*n) + (2*b^2*x^((5*n)/2))/(5*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2a^2x^{\frac{n}{2}}}{n}+\frac{4ab\,x^{\frac{3n}{2}}}{3n}+\frac{2b^2x^{\frac{5n}{2}}}{5n}$
norman	$\frac{2a^2\mathrm{e}^{\frac{n\ln(x)}{2}}}{n}+\frac{2b^2\mathrm{e}^{\frac{5n\ln(x)}{2}}}{5n}+\frac{4ab\,\mathrm{e}^{\frac{3n\ln(x)}{2}}}{3n}$
parallelrisch	$\frac{6x\,x^{2n}x^{-1+\frac{n}{2}}b^2+20x\,x^nx^{-1+\frac{n}{2}}ab+30x\,x^{-1+\frac{n}{2}}a^2}{15n}$
orering	$\frac{2x(23n^2-18n+4)x^{-1+\frac{n}{2}}(a+b\,x^n)^2}{15n^3}-\frac{4x^2(-2+3n)\left(\frac{x^{-1+\frac{n}{2}}\left(-1+\frac{n}{2}\right)(a+b\,x^n)^2}{x}+\frac{2x^{-1+\frac{n}{2}}(a+b\,x^n)b\,x^nn}{x}\right)}{5n^3}+\frac{8x^3\left(\frac{x^{-1+}}{}\right.}{}$

input
```
int(x^(-1+1/2*n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
2*a^2*x^(1/2*n)/n+2/5*b^2/n*(x^(1/2*n))^5+4/3*a*b/n*(x^(1/2*n))^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx=\frac{2\left(3\,b^2x^5x^{\frac{5}{2}\,n-5}+10\,abx^3x^{\frac{3}{2}\,n-3}+15\,a^2xx^{\frac{1}{2}\,n-1}\right)}{15\,n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^5*x^(5/2*n - 5) + 10*a*b*x^3*x^(3/2*n - 3) + 15*a^2*x*x^(1/2
*n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx=\begin{cases}\frac{2a^2xx^{\frac{n}{2}-1}}{n}+\frac{4abxx^nx^{\frac{n}{2}-1}}{3n}+\frac{2b^2xx^{2n}x^{\frac{n}{2}-1}}{5n} & \text{for } n\neq 0\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+1/2*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((2*a**2*x*x**(n/2 - 1)/n + 4*a*b*x*x**n*x**(n/2 - 1)/(3*n) + 2*b
**2*x*x**(2*n)*x**(n/2 - 1)/(5*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx=\frac{2\,b^2x^{\frac{5}{2}\,n}}{5\,n}+\frac{4\,abx^{\frac{3}{2}\,n}}{3\,n}+\frac{2\,a^2x^{\frac{1}{2}\,n}}{n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
2/5*b^2*x^(5/2*n)/n + 4/3*a*b*x^(3/2*n)/n + 2*a^2*x^(1/2*n)/n
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx=\frac{2\left(3\,b^2x^{2\,n}x^{\frac{1}{2}\,n}+10\,abx^{\frac{1}{2}\,n}x^n+15\,a^2\sqrt{x^n}\right)}{15\,n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
2/15*(3*b^2*x^(2*n)*x^(1/2*n) + 10*a*b*x^(1/2*n)*x^n + 15*a^2*sqrt(x^n))/n
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx = x^{\frac{n}{2}-1}\left(\frac{2\,a^2\,x}{n}+\frac{2\,b^2\,x\,x^{2\,n}}{5\,n}+\frac{4\,a\,b\,x\,x^n}{3\,n}\right)$$

input
```
int(x^(n/2 - 1)*(a + b*x^n)^2,x)
```

output
```
x^(n/2 - 1)*((2*a^2*x)/n + (2*b^2*x*x^(2*n))/(5*n) + (4*a*b*x*x^n)/(3*n))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int x^{-1+\frac{n}{2}}(a+bx^n)^2\,dx = \frac{2x^{\frac{n}{2}}(3x^{2n}b^2+10x^nab+15a^2)}{15n}$$

input
```
int(x^(-1+1/2*n)*(a+b*x^n)^2,x)
```

output
```
(2*x**(n/2)*(3*x**(2*n)*b**2 + 10*x**n*a*b + 15*a**2))/(15*n)
```

3.561 $\int x^{-1+\frac{n}{2}}(a+bx^n)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\,dx=\frac{2ax^{n/2}}{n}+\frac{2bx^{3n/2}}{3n}$$

output
```
2*a*x^(1/2*n)/n+2/3*b*x^(3/2*n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\,dx=\frac{2x^{n/2}(3a+bx^n)}{3n}$$

input
```
Integrate[x^(-1 + n/2)*(a + b*x^n),x]
```

output
```
(2*x^(n/2)*(3*a + b*x^n))/(3*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{2}-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{\frac{n-2}{2}}+bx^{\frac{3n}{2}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ax^{n/2}}{n}+\frac{2bx^{3n/2}}{3n}$$

input
```
Int[x^(-1 + n/2)*(a + b*x^n),x]
```

output
```
(2*a*x^(n/2))/n + (2*b*x^((3*n)/2))/(3*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{2a\,x^{\frac{n}{2}}}{n}+\frac{2b\,x^{\frac{3n}{2}}}{3n}$	26
norman	$\frac{2a\,{\mathrm e}^{\frac{n\ln\left(x\right)}{2}}}{n}+\frac{2b\,{\mathrm e}^{\frac{3n\ln\left(x\right)}{2}}}{3n}$	28
parallelrisch	$\frac{2x\,x^{n} x^{-1+\frac{n}{2}} b+6x\,x^{-1+\frac{n}{2}} a}{3n}$	32
orering	$\frac{4x(2n-1)x^{-1+\frac{n}{2}} \left(a+b\,x^{n}\right)}{3n^{2}}-\frac{4x^{2} \left(\frac{x^{-1+\frac{n}{2}} \left(-1+\frac{n}{2}\right) \left(a+b\,x^{n}\right)}{x}+\frac{x^{-1+\frac{n}{2}} b\,x^{n} n}{x}\right)}{3n^{2}}$	75

input
```
int(x^(-1+1/2*n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
2*a*x^(1/2*n)/n+2/3*b/n*(x^(1/2*n))^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\,dx=\frac{2\left(bx^3x^{\frac{3}{2}\,n-3}+3\,axx^{\frac{1}{2}\,n-1}\right)}{3\,n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
2/3*(b*x^3*x^(3/2*n - 3) + 3*a*x*x^(1/2*n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\,dx=\begin{cases}\frac{2axx^{\frac{n}{2}-1}}{n}+\frac{2bxx^nx^{\frac{n}{2}-1}}{3n} & \text{for } n\neq 0\\ (a+b)\log{(x)} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+1/2*n)*(a+b*x**n),x)
```

output
```
Piecewise((2*a*x*x**(n/2 - 1)/n + 2*b*x*x**n*x**(n/2 - 1)/(3*n), Ne(n, 0))
, ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\,dx=\frac{2\,bx^{\frac{3}{2}\,n}}{3\,n}+\frac{2\,ax^{\frac{1}{2}\,n}}{n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
2/3*b*x^(3/2*n)/n + 2*a*x^(1/2*n)/n
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\,dx=\frac{2\left(bx^{\frac{1}{2}\,n}x^n+3\,a\sqrt{x^n}\right)}{3\,n}$$

input
```
integrate(x^(-1+1/2*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
2/3*(b*x^(1/2*n)*x^n + 3*a*sqrt(x^n))/n
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\ dx=\frac{2\,x^{n/2}\,(3\,a+b\,x^n)}{3\,n}$$

input
```
int(x^(n/2 - 1)*(a + b*x^n),x)
```

output
```
(2*x^(n/2)*(3*a + b*x^n))/(3*n)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int x^{-1+\frac{n}{2}}(a+bx^n)\ dx=\frac{2x^{\frac{n}{2}}(x^nb+3a)}{3n}$$

input
```
int(x^(-1+1/2*n)*(a+b*x^n),x)
```

output
```
(2*x**(n/2)*(x**n*b + 3*a))/(3*n)
```

3.562 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

output
```
2*arctan(1/a^(1/2)*b^(1/2)*x^(1/2*n))/a^(1/2)/b^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

input
```
Integrate[x^(-1 + n/2)/(a + b*x^n),x]
```

output
```
(2*ArcTan[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {868, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{ 868}$$

$$\frac{2\int \frac{1}{bx^n+a}dx^{n/2}}{n}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

input
```
Int[x^(-1 + n/2)/(a + b*x^n),x]
```

output
```
(2*ArcTan[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*n)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.63 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

method	result	size
risch	$-\frac{\ln\left(x^{\frac{n}{2}}-\frac{a}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,n}+\frac{\ln\left(x^{\frac{n}{2}}+\frac{a}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,n}$	54

input
```
int(x^(-1+1/2*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/(-a*b)^(1/2)/n*ln(x^(1/2*n)-1/(-a*b)^(1/2)*a)+1/(-a*b)^(1/2)/n*ln(x^(1/
2*n)+1/(-a*b)^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.88

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx=\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2x^{n-2}-2\sqrt{-ab}xx^{\frac{1}{2}n-1}-a}{bx^2x^{n-2}+a}\right)}{abn},\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}xx^{\frac{1}{2}n-1}}{a}\right)}{abn}\right]$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
[-sqrt(-a*b)*log((b*x^2*x^(n - 2) - 2*sqrt(-a*b)*x*x^(1/2*n - 1) - a)/(b*x
^2*x^(n - 2) + a))/(a*b*n), 2*sqrt(a*b)*arctan(sqrt(a*b)*x*x^(1/2*n - 1)/a
)/(a*b*n)]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

input
```
integrate(x**(-1+1/2*n)/(a+b*x**n),x)
```

output
```
2*atan(sqrt(b)*x**(n/2)/sqrt(a))/(sqrt(a)*sqrt(b)*n)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{x^{\frac{1}{2}n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(x^(1/2*n - 1)/(b*x^n + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx = \frac{2\,\arctan\left(\frac{b\sqrt{x^n}}{\sqrt{ab}}\right)}{\sqrt{ab}n}$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
2*arctan(b*sqrt(x^n)/sqrt(a*b))/(sqrt(a*b)*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{x^{\frac{n}{2}-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(n/2 - 1)/(a + b*x^n),x)
```

output
```
int(x^(n/2 - 1)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{x^{\frac{n}{2}}}{x^n bx+ax}dx$$

input
```
int(x^(-1+1/2*n)/(a+b*x^n),x)
```

output
```
int(x**(n/2)/(x**n*b*x + a*x),x)
```

3.563 $\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2} dx = \frac{x^{n/2}}{an(a+bx^n)} + \frac{\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}n}$$

output
```
x^(1/2*n)/a/n/(a+b*x^n)+arctan(1/a^(1/2)*b^(1/2)*x^(1/2*n))/a^(3/2)/b^(1/2
)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2} dx = \frac{\frac{ax^{n/2}}{a+bx^n} + \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}}}{a^2n}$$

input
```
Integrate[x^(-1 + n/2)/(a + b*x^n)^2,x]
```

output
```
((a*x^(n/2))/(a + b*x^n) + (Sqrt[a]*ArcTan[(Sqrt[b]*x^(n/2))/Sqrt[a]])/Sqr
t[b])/(a^2*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {868, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{868}$$

$$\frac{2\int \frac{1}{(bx^n+a)^2}dx^{n/2}}{n}$$

$$\Big\downarrow \text{215}$$

$$\frac{2\left(\frac{\int \frac{1}{bx^n+a}dx^{n/2}}{2a}+\frac{x^{n/2}}{2a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x^{n/2}}{2a(a+bx^n)}\right)}{n}$$

input
```
Int[x^(-1 + n/2)/(a + b*x^n)^2,x]
```

output
```
(2*(x^(n/2)/(2*a*(a + b*x^n)) + ArcTan[(Sqrt[b]*x^(n/2))/Sqrt[a]]/(2*a^(3/
2)*Sqrt[b])))/n
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

method	result	size
risch	$\frac{x^{\frac{n}{2}}}{an(a+b\,x^{n})}-\frac{\ln\left(x^{\frac{n}{2}}-\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,na}+\frac{\ln\left(x^{\frac{n}{2}}+\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,na}$	86

input
```
int(x^(-1+1/2*n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/n*x^(1/2*n)/(a+b*(x^(1/2*n))^2)-1/2/(-a*b)^(1/2)/n/a*ln(x^(1/2*n)-1/(-
a*b)^(1/2)*a)+1/2/(-a*b)^(1/2)/n/a*ln(x^(1/2*n)+1/(-a*b)^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2}\,dx = \left[\frac{2\,abxx^{\frac{1}{2}\,n-1}-\left(\sqrt{-ab}bx^2x^{n-2}+\sqrt{-ab}a\right)\log\left(\frac{bx^2x^{n-2}-2\sqrt{-ab}xx^{\frac{1}{2}\,n-1}-a}{bx^2x^{n-2}+a}\right)}{2\left(a^2b^2nx^2x^{n-2}+a^3bn\right)}, \frac{abxx^{\frac{1}{2}\,n-1}+\left(\sqrt{ab}bx^2x^{n-2}+\sqrt{ \right.}{a^2b^2nx^2x^{n-2}}\right.$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*x*x^(1/2*n - 1) - (sqrt(-a*b)*b*x^2*x^(n - 2) + sqrt(-a*b)*a)*
log((b*x^2*x^(n - 2) - 2*sqrt(-a*b)*x*x^(1/2*n - 1) - a)/(b*x^2*x^(n - 2)
+ a)))/(a^2*b^2*n*x^2*x^(n - 2) + a^3*b*n), (a*b*x*x^(1/2*n - 1) + (sqrt(a
*b)*b*x^2*x^(n - 2) + sqrt(a*b)*a)*arctan(sqrt(a*b)*x*x^(1/2*n - 1)/a))/(a
^2*b^2*n*x^2*x^(n - 2) + a^3*b*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(42) = 84.

Time = 1.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.86

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2}\,dx = \frac{a^{\frac{3}{2}}x^{\frac{n}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^3\sqrt{b}nx^{\frac{n}{2}}+a^2b^{\frac{3}{2}}nx^{\frac{3n}{2}}}+\frac{\sqrt{a}bx^{\frac{3n}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^3\sqrt{b}nx^{\frac{n}{2}}+a^2b^{\frac{3}{2}}nx^{\frac{3n}{2}}}+\frac{a\sqrt{b}x^n}{a^3\sqrt{b}nx^{\frac{n}{2}}+a^2b^{\frac{3}{2}}nx^{\frac{3n}{2}}}$$

input
```
integrate(x**(-1+1/2*n)/(a+b*x**n)**2,x)
```

output
```
a**(3/2)*x**(n/2)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(a**3*sqrt(b)*n*x**(n/2)
+ a**2*b**(3/2)*n*x**(3*n/2)) + sqrt(a)*b*x**(3*n/2)*atan(sqrt(b)*x**(n/2)
/sqrt(a))/(a**3*sqrt(b)*n*x**(n/2) + a**2*b**(3/2)*n*x**(3*n/2)) + a*sqrt(
b)*x**n/(a**3*sqrt(b)*n*x**(n/2) + a**2*b**(3/2)*n*x**(3*n/2))
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2}\,dx = \int \frac{x^{\frac{1}{2}\,n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
x^(1/2*n)/(a*b*n*x^n + a^2*n) + integrate(1/2*x^(1/2*n)/(a*b*x*x^n + a^2*x
), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2}\,dx = \frac{\frac{\arctan\left(\frac{b\sqrt{x^n}}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{\sqrt{x^n}}{(bx^n+a)a}}{n}$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
(arctan(b*sqrt(x^n)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x^n)/((b*x^n + a)*a))/
n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2}\,dx = \int \frac{x^{\frac{n}{2}-1}}{(a+b\,x^n)^2}\,dx$$

input
```
int(x^(n/2 - 1)/(a + b*x^n)^2,x)
```

output
```
int(x^(n/2 - 1)/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^2}\,dx = \int \frac{x^{\frac{n}{2}}}{x^{2n}b^2x + 2x^n abx + a^2x}dx$$

input
```
int(x^(-1+1/2*n)/(a+b*x^n)^2,x)
```

output
```
int(x**(n/2)/(x**(2*n)*b**2*x + 2*x**n*a*b*x + a**2*x),x)
```

3.564 $\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx = \frac{x^{n/2}}{2an\,(a+bx^n)^2} + \frac{3x^{n/2}}{4a^2n\,(a+bx^n)} + \frac{3\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}n}$$

output
```
1/2*x^(1/2*n)/a/n/(a+b*x^n)^2+3/4*x^(1/2*n)/a^2/n/(a+b*x^n)+3/4*arctan(1/a
^(1/2)*b^(1/2)*x^(1/2*n))/a^(5/2)/b^(1/2)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx = \frac{2x^{n/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},-\frac{bx^n}{a}\right)}{a^3n}$$

input
```
Integrate[x^(-1 + n/2)/(a + b*x^n)^3,x]
```

output
```
(2*x^(n/2)*Hypergeometric2F1[1/2, 3, 3/2, -((b*x^n)/a)])/(a^3*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {868, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{(a+bx^n)^3}dx$$

$$\Big\downarrow \text{868}$$

$$\frac{2\int \frac{1}{(bx^n+a)^3}dx^{n/2}}{n}$$

$$\Big\downarrow \text{215}$$

$$\frac{2\left(\frac{3\int \frac{1}{(bx^n+a)^2}dx^{n/2}}{4a}+\frac{x^{n/2}}{4a(a+bx^n)^2}\right)}{n}$$

$$\Big\downarrow \text{215}$$

$$\frac{2\left(\frac{3\left(\frac{\int \frac{1}{bx^n+a}dx^{n/2}}{2a}+\frac{x^{n/2}}{2a(a+bx^n)}\right)}{4a}+\frac{x^{n/2}}{4a(a+bx^n)^2}\right)}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x^{n/2}}{2a(a+bx^n)}\right)}{4a}+\frac{x^{n/2}}{4a(a+bx^n)^2}\right)}{n}$$

input
```
Int[x^(-1 + n/2)/(a + b*x^n)^3,x]
```

output
```
(2*(x^(n/2)/(4*a*(a + b*x^n)^2) + (3*(x^(n/2)/(2*a*(a + b*x^n)) + ArcTan[(
Sqrt[b]*x^(n/2))/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)))/n
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{x^{\frac{n}{2}} (3b\,x^{n}+5a)}{4a^{2} n (a+b\,x^{n})^{2}}-\frac{3\ln\left(x^{\frac{n}{2}}-\frac{a}{\sqrt{-ab}}\right)}{8\sqrt{-ab}\,n\,a^{2}}+\frac{3\ln\left(x^{\frac{n}{2}}+\frac{a}{\sqrt{-ab}}\right)}{8\sqrt{-ab}\,n\,a^{2}}$	101

input
```
int(x^(-1+1/2*n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(1/2*n)*(3*b*(x^(1/2*n))^2+5*a)/a^2/n/(a+b*(x^(1/2*n))^2)^2-3/8/(-a*
b)^(1/2)/n/a^2*ln(x^(1/2*n)-1/(-a*b)^(1/2)*a)+3/8/(-a*b)^(1/2)/n/a^2*ln(x^
(1/2*n)+1/(-a*b)^(1/2)*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(69) = 138$.

Time = 0.09 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.57

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx$$
$$= \left[\frac{6\,ab^2x^3x^{\frac{3}{2}\,n-3} + 10\,a^2bxx^{\frac{1}{2}\,n-1} - 3\left(\sqrt{-ab}b^2x^4x^{2\,n-4} + 2\,\sqrt{-ab}abx^2x^{n-2} + \sqrt{-ab}a^2\right)\log\left(\frac{bx^2x^{n-2}-2\,\sqrt{-ab}}{bx^2x^{n-2}}\right.}{8\left(a^3b^3nx^4x^{2\,n-4} + 2\,a^4b^2nx^2x^{n-2} + a^5bn\right)}\right.$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
[1/8*(6*a*b^2*x^3*x^(3/2*n - 3) + 10*a^2*b*x*x^(1/2*n - 1) - 3*(sqrt(-a*b)
*b^2*x^4*x^(2*n - 4) + 2*sqrt(-a*b)*a*b*x^2*x^(n - 2) + sqrt(-a*b)*a^2)*lo
g((b*x^2*x^(n - 2) - 2*sqrt(-a*b)*x*x^(1/2*n - 1) - a)/(b*x^2*x^(n - 2) +
a)))/(a^3*b^3*n*x^4*x^(2*n - 4) + 2*a^4*b^2*n*x^2*x^(n - 2) + a^5*b*n), 1/
4*(3*a*b^2*x^3*x^(3/2*n - 3) + 5*a^2*b*x*x^(1/2*n - 1) + 3*(sqrt(a*b)*b^2*
x^4*x^(2*n - 4) + 2*sqrt(a*b)*a*b*x^2*x^(n - 2) + sqrt(a*b)*a^2)*arctan(sq
rt(a*b)*x*x^(1/2*n - 1)/a))/(a^3*b^3*n*x^4*x^(2*n - 4) + 2*a^4*b^2*n*x^2*x
^(n - 2) + a^5*b*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(70) = 140$.

Time = 3.18 (sec) , antiderivative size = 683, normalized size of antiderivative = 7.67

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx = \frac{3a^{\frac{11}{2}}x^{\frac{n}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4a^8\sqrt{b}nx^{\frac{n}{2}}+12a^7b^{\frac{3}{2}}nx^{\frac{3n}{2}}+12a^6b^{\frac{5}{2}}nx^{\frac{5n}{2}}+4a^5b^{\frac{7}{2}}nx^{\frac{7n}{2}}} + \frac{9a^{\frac{9}{2}}bx^{\frac{3n}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4a^8\sqrt{b}nx^{\frac{n}{2}}+12a^7b^{\frac{3}{2}}nx^{\frac{3n}{2}}+12a^6b^{\frac{5}{2}}nx^{\frac{5n}{2}}+4a^5b^{\frac{7}{2}}nx^{\frac{7n}{2}}} + \frac{9a^{\frac{7}{2}}b^2x^{\frac{5n}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4a^8\sqrt{b}nx^{\frac{n}{2}}+12a^7b^{\frac{3}{2}}nx^{\frac{3n}{2}}+12a^6b^{\frac{5}{2}}nx^{\frac{5n}{2}}+4a^5b^{\frac{7}{2}}nx^{\frac{7n}{2}}} + \frac{3a^{\frac{5}{2}}b^3x^{\frac{7n}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4a^8\sqrt{b}nx^{\frac{n}{2}}+12a^7b^{\frac{3}{2}}nx^{\frac{3n}{2}}+12a^6b^{\frac{5}{2}}nx^{\frac{5n}{2}}+4a^5b^{\frac{7}{2}}nx^{\frac{7n}{2}}} + \frac{5a^5\sqrt{b}x^n}{4a^8\sqrt{b}nx^{\frac{n}{2}}+12a^7b^{\frac{3}{2}}nx^{\frac{3n}{2}}+12a^6b^{\frac{5}{2}}nx^{\frac{5n}{2}}+4a^5b^{\frac{7}{2}}nx^{\frac{7n}{2}}} + \frac{8a^4b^{\frac{3}{2}}x^{2n}}{4a^8\sqrt{b}nx^{\frac{n}{2}}+12a^7b^{\frac{3}{2}}nx^{\frac{3n}{2}}+12a^6b^{\frac{5}{2}}nx^{\frac{5n}{2}}+4a^5b^{\frac{7}{2}}nx^{\frac{7n}{2}}} + \frac{3a^3b^{\frac{5}{2}}x^{3n}}{4a^8\sqrt{b}nx^{\frac{n}{2}}+12a^7b^{\frac{3}{2}}nx^{\frac{3n}{2}}+12a^6b^{\frac{5}{2}}nx^{\frac{5n}{2}}+4a^5b^{\frac{7}{2}}nx^{\frac{7n}{2}}}$$

input
```
integrate(x**(-1+1/2*n)/(a+b*x**n)**3,x)
```

output
```
3*a**(11/2)*x**(n/2)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(4*a**8*sqrt(b)*n*x**(
n/2) + 12*a**7*b**(3/2)*n*x**(3*n/2) + 12*a**6*b**(5/2)*n*x**(5*n/2) + 4*a
**5*b**(7/2)*n*x**(7*n/2)) + 9*a**(9/2)*b*x**(3*n/2)*atan(sqrt(b)*x**(n/2)
/sqrt(a))/(4*a**8*sqrt(b)*n*x**(n/2) + 12*a**7*b**(3/2)*n*x**(3*n/2) + 12*
a**6*b**(5/2)*n*x**(5*n/2) + 4*a**5*b**(7/2)*n*x**(7*n/2)) + 9*a**(7/2)*b*
*2*x**(5*n/2)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(4*a**8*sqrt(b)*n*x**(n/2) +
12*a**7*b**(3/2)*n*x**(3*n/2) + 12*a**6*b**(5/2)*n*x**(5*n/2) + 4*a**5*b**
(7/2)*n*x**(7*n/2)) + 3*a**(5/2)*b**3*x**(7*n/2)*atan(sqrt(b)*x**(n/2)/sqr
t(a))/(4*a**8*sqrt(b)*n*x**(n/2) + 12*a**7*b**(3/2)*n*x**(3*n/2) + 12*a**6
*b**(5/2)*n*x**(5*n/2) + 4*a**5*b**(7/2)*n*x**(7*n/2)) + 5*a**5*sqrt(b)*x*
*n/(4*a**8*sqrt(b)*n*x**(n/2) + 12*a**7*b**(3/2)*n*x**(3*n/2) + 12*a**6*b*
*(5/2)*n*x**(5*n/2) + 4*a**5*b**(7/2)*n*x**(7*n/2)) + 8*a**4*b**(3/2)*x**(
2*n)/(4*a**8*sqrt(b)*n*x**(n/2) + 12*a**7*b**(3/2)*n*x**(3*n/2) + 12*a**6*
b**(5/2)*n*x**(5*n/2) + 4*a**5*b**(7/2)*n*x**(7*n/2)) + 3*a**3*b**(5/2)*x*
*(3*n)/(4*a**8*sqrt(b)*n*x**(n/2) + 12*a**7*b**(3/2)*n*x**(3*n/2) + 12*a**
6*b**(5/2)*n*x**(5*n/2) + 4*a**5*b**(7/2)*n*x**(7*n/2))
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx = \int \frac{x^{\frac{1}{2}n-1}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/4*(3*b*x^(3/2*n) + 5*a*x^(1/2*n))/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a
^4*n) + 3*integrate(1/8*x^(1/2*n)/(a^2*b*x*x^n + a^3*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx = \frac{\frac{3\arctan\left(\frac{b\sqrt{x^n}}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^{\frac{1}{2}n}x^n+5\,a\sqrt{x^n}}{(bx^n+a)^2a^2}}{4\,n}$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/4*(3*arctan(b*sqrt(x^n)/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^(1/2*n)*x^n
+ 5*a*sqrt(x^n))/((b*x^n + a)^2*a^2))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx = \int \frac{x^{\frac{n}{2}-1}}{(a+b\,x^n)^3}\,dx$$

input
```
int(x^(n/2 - 1)/(a + b*x^n)^3,x)
```

output
```
int(x^(n/2 - 1)/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{(a+bx^n)^3}\,dx = \int \frac{x^{\frac{n}{2}}}{x^{3n}b^3x+3x^{2n}a\,b^2x+3x^na^2bx+a^3x}dx$$

input
```
int(x^(-1+1/2*n)/(a+b*x^n)^3,x)
```

output
```
int(x**(n/2)/(x**(3*n)*b**3*x + 3*x**(2*n)*a*b**2*x + 3*x**n*a**2*b*x + a*
*3*x),x)
```

3.565 $\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx=\frac{3a^3x^{n/3}}{n}+\frac{9a^2bx^{4n/3}}{4n}+\frac{9ab^2x^{7n/3}}{7n}+\frac{3b^3x^{10n/3}}{10n}$$

output
```
3*a^3*x^(1/3*n)/n+9/4*a^2*b*x^(4/3*n)/n+9/7*a*b^2*x^(7/3*n)/n+3/10*b^3*x^(
10/3*n)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx=\frac{3x^{n/3}(140a^3+105a^2bx^n+60ab^2x^{2n}+14b^3x^{3n})}{140n}$$

input
```
Integrate[x^(-1 + n/3)*(a + b*x^n)^3,x]
```

output
```
(3*x^(n/3)*(140*a^3 + 105*a^2*b*x^n + 60*a*b^2*x^(2*n) + 14*b^3*x^(3*n)))/
(140*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{3}-1}(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^{\frac{n-3}{3}}+3a^2bx^{\frac{4n}{3}-1}+3ab^2x^{\frac{7n}{3}-1}+b^3x^{\frac{10n}{3}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^3x^{n/3}}{n}+\frac{9a^2bx^{4n/3}}{4n}+\frac{9ab^2x^{7n/3}}{7n}+\frac{3b^3x^{10n/3}}{10n}$$

input
```
Int[x^(-1 + n/3)*(a + b*x^n)^3,x]
```

output
```
(3*a^3*x^(n/3))/n + (9*a^2*b*x^((4*n)/3))/(4*n) + (9*a*b^2*x^((7*n)/3))/(7
*n) + (3*b^3*x^((10*n)/3))/(10*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

method	result
risch	$\frac{3a^3x^{\frac{n}{3}}}{n}+\frac{9a^2b\,x^{\frac{4n}{3}}}{4n}+\frac{9a\,b^2x^{\frac{7n}{3}}}{7n}+\frac{3b^3x^{\frac{10n}{3}}}{10n}$
parallelrisch	$\frac{42x\,x^{3n}x^{-1+\frac{n}{3}}b^3+180x\,x^{2n}x^{-1+\frac{n}{3}}a\,b^2+315x\,x^{n}x^{-1+\frac{n}{3}}a^2b+420x\,x^{-1+\frac{n}{3}}a^3}{140n}$
orering	$\frac{3x(418n^3-477n^2+198n-27)x^{-1+\frac{n}{3}}(a+b\,x^n)^3}{280n^4}-\frac{27x^2(53n^2-66n+21)\left(\frac{x^{-1+\frac{n}{3}}\left(-1+\frac{n}{3}\right)(a+b\,x^n)^3}{x}+\frac{3x^{-1+\frac{n}{3}}(a+b\,x^n)^2b}{x}\right.}{280n^4}$

input
```
int(x^(-1+1/3*n)*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
3/10*b^3/n*(x^(1/3*n))^10+9/7*a*b^2/n*(x^(1/3*n))^7+9/4*a^2*b/n*(x^(1/3*n)
)^4+3*a^3*x^(1/3*n)/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx$$
$$=\frac{3\left(14\,b^3x^{10}x^{\frac{10}{3}\,n-10}+60\,ab^2x^7x^{\frac{7}{3}\,n-7}+105\,a^2bx^4x^{\frac{4}{3}\,n-4}+140\,a^3xx^{\frac{1}{3}\,n-1}\right)}{140\,n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
3/140*(14*b^3*x^10*x^(10/3*n - 10) + 60*a*b^2*x^7*x^(7/3*n - 7) + 105*a^2*
b*x^4*x^(4/3*n - 4) + 140*a^3*x*x^(1/3*n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx = \begin{cases} \frac{3a^3xx^{\frac{n}{3}-1}}{n} + \frac{9a^2bxx^nx^{\frac{n}{3}-1}}{4n} + \frac{9ab^2xx^{2n}x^{\frac{n}{3}-1}}{7n} + \frac{3b^3xx^{3n}x^{\frac{n}{3}-1}}{10n} & \text{for } n \neq 0 \\ (a+b)^3\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+1/3*n)*(a+b*x**n)**3,x)
```

output
```
Piecewise((3*a**3*x*x**(n/3 - 1)/n + 9*a**2*b*x*x**n*x**(n/3 - 1)/(4*n) +
9*a*b**2*x*x**(2*n)*x**(n/3 - 1)/(7*n) + 3*b**3*x*x**(3*n)*x**(n/3 - 1)/(1
0*n), Ne(n, 0)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx = \frac{3\,b^3x^{\frac{10}{3}\,n}}{10\,n} + \frac{9\,ab^2x^{\frac{7}{3}\,n}}{7\,n} + \frac{9\,a^2bx^{\frac{4}{3}\,n}}{4\,n} + \frac{3\,a^3x^{\frac{1}{3}\,n}}{n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
3/10*b^3*x^(10/3*n)/n + 9/7*a*b^2*x^(7/3*n)/n + 9/4*a^2*b*x^(4/3*n)/n + 3*
a^3*x^(1/3*n)/n
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx = \frac{3\left(14\,b^3x^{3\,n}(x^n)^{\frac{1}{3}}+60\,ab^2x^{2\,n}(x^n)^{\frac{1}{3}}+105\,a^2b(x^n)^{\frac{4}{3}}+140\,a^3x^{\frac{1}{3}\,n}\right)}{140\,n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
3/140*(14*b^3*x^(3*n)*(x^n)^(1/3) + 60*a*b^2*x^(2*n)*(x^n)^(1/3) + 105*a^2
*b*(x^n)^(4/3) + 140*a^3*x^(1/3*n))/n
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx = x^{\frac{n}{3}-1}\left(\frac{3\,a^3\,x}{n}+\frac{3\,b^3\,x\,x^{3\,n}}{10\,n}+\frac{9\,a^2\,b\,x\,x^n}{4\,n}+\frac{9\,a\,b^2\,x\,x^{2\,n}}{7\,n}\right)$$

input
```
int(x^(n/3 - 1)*(a + b*x^n)^3,x)
```

output
```
x^(n/3 - 1)*((3*a^3*x)/n + (3*b^3*x*x^(3*n))/(10*n) + (9*a^2*b*x*x^n)/(4*n
) + (9*a*b^2*x*x^(2*n))/(7*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^3\,dx = \frac{3x^{\frac{n}{3}}(14x^{3n}b^3+60x^{2n}a\,b^2+105x^na^2b+140a^3)}{140n}$$

input
```
int(x^(-1+1/3*n)*(a+b*x^n)^3,x)
```

output
```
(3*x**(n/3)*(14*x**(3*n)*b**3 + 60*x**(2*n)*a*b**2 + 105*x**n*a**2*b + 140
*a**3))/(140*n)
```

3.566 $\int x^{-1+\frac{n}{3}}(a+bx^n)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\,dx=\frac{3a^2x^{n/3}}{n}+\frac{3abx^{4n/3}}{2n}+\frac{3b^2x^{7n/3}}{7n}$$

output
```
3*a^2*x^(1/3*n)/n+3/2*a*b*x^(4/3*n)/n+3/7*b^2*x^(7/3*n)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\,dx=\frac{3x^{n/3}(14a^2+7abx^n+2b^2x^{2n})}{14n}$$

input
```
Integrate[x^(-1 + n/3)*(a + b*x^n)^2,x]
```

output
```
(3*x^(n/3)*(14*a^2 + 7*a*b*x^n + 2*b^2*x^(2*n)))/(14*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{3}-1}(a+bx^n)^2\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(a^2x^{\frac{n-3}{3}}+2abx^{\frac{4n}{3}-1}+b^2x^{\frac{7n}{3}-1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3a^2x^{n/3}}{n}+\frac{3abx^{4n/3}}{2n}+\frac{3b^2x^{7n/3}}{7n}$$

input
```
Int[x^(-1 + n/3)*(a + b*x^n)^2,x]
```

output
```
(3*a^2*x^(n/3))/n + (3*a*b*x^((4*n)/3))/(2*n) + (3*b^2*x^((7*n)/3))/(7*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
risch	$\frac{3a^2x^{\frac{n}{3}}}{n}+\frac{3ab\,x^{\frac{4n}{3}}}{2n}+\frac{3b^2x^{\frac{7n}{3}}}{7n}$
parallelrisch	$\frac{6x\,x^{2n}x^{-1+\frac{n}{3}}b^2+21x\,x^n x^{-1+\frac{n}{3}}ab+42x\,x^{-1+\frac{n}{3}}a^2}{14n}$
orering	$\frac{9x(13n^2-12n+3)x^{-1+\frac{n}{3}}(a+b\,x^n)^2}{28n^3}-\frac{27x^2(4n-3)\left(\frac{x^{-1+\frac{n}{3}}\left(-1+\frac{n}{3}\right)(a+b\,x^n)^2}{x}+\frac{2x^{-1+\frac{n}{3}}(a+b\,x^n)b\,x^n n}{x}\right)}{28n^3}+\frac{27x^3\left(\frac{x^{-1+}}{}\right.}{}$

input
```
int(x^(-1+1/3*n)*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
3/7*b^2/n*(x^(1/3*n))^7+3/2*a*b/n*(x^(1/3*n))^4+3*a^2*x^(1/3*n)/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\,dx=\frac{3\left(2\,b^2x^7x^{\frac{7}{3}\,n-7}+7\,abx^4x^{\frac{4}{3}\,n-4}+14\,a^2xx^{\frac{1}{3}\,n-1}\right)}{14\,n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
3/14*(2*b^2*x^7*x^(7/3*n - 7) + 7*a*b*x^4*x^(4/3*n - 4) + 14*a^2*x*x^(1/3*
n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\,dx = \begin{cases} \frac{3a^2xx^{\frac{n}{3}-1}}{n} + \frac{3abxx^nx^{\frac{n}{3}-1}}{2n} + \frac{3b^2xx^{2n}x^{\frac{n}{3}-1}}{7n} & \text{for } n \neq 0 \\ (a+b)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+1/3*n)*(a+b*x**n)**2,x)
```

output
```
Piecewise((3*a**2*x*x**(n/3 - 1)/n + 3*a*b*x*x**n*x**(n/3 - 1)/(2*n) + 3*b
**2*x*x**(2*n)*x**(n/3 - 1)/(7*n), Ne(n, 0)), ((a + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\,dx = \frac{3\,b^2x^{\frac{7}{3}\,n}}{7\,n} + \frac{3\,abx^{\frac{4}{3}\,n}}{2\,n} + \frac{3\,a^2x^{\frac{1}{3}\,n}}{n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
3/7*b^2*x^(7/3*n)/n + 3/2*a*b*x^(4/3*n)/n + 3*a^2*x^(1/3*n)/n
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\,dx = \frac{3\left(2\,b^2x^{2\,n}(x^n)^{\frac{1}{3}} + 7\,ab(x^n)^{\frac{4}{3}} + 14\,a^2x^{\frac{1}{3}\,n}\right)}{14\,n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
3/14*(2*b^2*x^(2*n)*(x^n)^(1/3) + 7*a*b*(x^n)^(4/3) + 14*a^2*x^(1/3*n))/n
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\, dx = x^{\frac{n}{3}-1}\left(\frac{3\,a^2\,x}{n}+\frac{3\,b^2\,x\,x^{2\,n}}{7\,n}+\frac{3\,a\,b\,x\,x^n}{2\,n}\right)$$

input
```
int(x^(n/3 - 1)*(a + b*x^n)^2,x)
```

output
```
x^(n/3 - 1)*((3*a^2*x)/n + (3*b^2*x*x^(2*n))/(7*n) + (3*a*b*x*x^n)/(2*n))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int x^{-1+\frac{n}{3}}(a+bx^n)^2\, dx = \frac{3x^{\frac{n}{3}}(2x^{2n}b^2+7x^nab+14a^2)}{14n}$$

input
```
int(x^(-1+1/3*n)*(a+b*x^n)^2,x)
```

output
```
(3*x**(n/3)*(2*x**(2*n)*b**2 + 7*x**n*a*b + 14*a**2))/(14*n)
```

3.567 $\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx=\frac{3ax^{n/3}}{n}+\frac{3bx^{4n/3}}{4n}$$

output
```
3*a*x^(1/3*n)/n+3/4*b*x^(4/3*n)/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx=\frac{3x^{n/3}(4a+bx^n)}{4n}$$

input
```
Integrate[x^(-1 + n/3)*(a + b*x^n),x]
```

output
```
(3*x^(n/3)*(4*a + b*x^n))/(4*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{3}-1}(a+bx^n)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{\frac{n-3}{3}}+bx^{\frac{4n}{3}-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ax^{n/3}}{n}+\frac{3bx^{4n/3}}{4n}$$

input
```
Int[x^(-1 + n/3)*(a + b*x^n),x]
```

output
```
(3*a*x^(n/3))/n + (3*b*x^((4*n)/3))/(4*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{3a\,x^{\frac{n}{3}}}{n}+\frac{3b\,x^{\frac{4n}{3}}}{4n}$	26
norman	$\frac{3a\,\mathrm{e}^{\frac{n\ln(x)}{3}}}{n}+\frac{3b\,\mathrm{e}^{\frac{4n\ln(x)}{3}}}{4n}$	28
parallelrisch	$\frac{3x\,x^{n}x^{-1+\frac{n}{3}}b+12x\,x^{-1+\frac{n}{3}}a}{4n}$	32
orering	$\frac{3x(-3+5n)x^{-1+\frac{n}{3}}(a+b\,x^{n})}{4n^{2}}-\frac{9x^{2}\left(\frac{x^{-1+\frac{n}{3}}\left(-1+\frac{n}{3}\right)(a+b\,x^{n})}{x}+\frac{x^{-1+\frac{n}{3}}b\,x^{n}n}{x}\right)}{4n^{2}}$	75

input
```
int(x^(-1+1/3*n)*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
3*a*x^(1/3*n)/n+3/4*b/n*(x^(1/3*n))^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^{-1+\frac{n}{3}}(a+bx^{n})\,dx=\frac{3\left(bx^{4}x^{\frac{4}{3}\,n-4}+4\,axx^{\frac{1}{3}\,n-1}\right)}{4\,n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n),x, algorithm="fricas")
```

output
```
3/4*(b*x^4*x^(4/3*n - 4) + 4*a*x*x^(1/3*n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx = \begin{cases} \frac{3axx^{\frac{n}{3}-1}}{n} + \frac{3bxx^nx^{\frac{n}{3}-1}}{4n} & \text{for } n \neq 0 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+1/3*n)*(a+b*x**n),x)
```

output
```
Piecewise((3*a*x*x**(n/3 - 1)/n + 3*b*x*x**n*x**(n/3 - 1)/(4*n), Ne(n, 0))
, ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx = \frac{3\,bx^{\frac{4}{3}\,n}}{4\,n} + \frac{3\,ax^{\frac{1}{3}\,n}}{n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n),x, algorithm="maxima")
```

output
```
3/4*b*x^(4/3*n)/n + 3*a*x^(1/3*n)/n
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx = \frac{3\left(b(x^n)^{\frac{4}{3}} + 4\,ax^{\frac{1}{3}\,n}\right)}{4\,n}$$

input
```
integrate(x^(-1+1/3*n)*(a+b*x^n),x, algorithm="giac")
```

output
```
3/4*(b*(x^n)^(4/3) + 4*a*x^(1/3*n))/n
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx=\frac{3\,x^{n/3}\,(4\,a+b\,x^n)}{4\,n}$$

input
```
int(x^(n/3 - 1)*(a + b*x^n),x)
```

output
```
(3*x^(n/3)*(4*a + b*x^n))/(4*n)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int x^{-1+\frac{n}{3}}(a+bx^n)\,dx=\frac{3x^{\frac{n}{3}}(x^nb+4a)}{4n}$$

input
```
int(x^(-1+1/3*n)*(a+b*x^n),x)
```

output
```
(3*x**(n/3)*(x**n*b + 4*a))/(4*n)
```

3.568 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n} dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}n} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{2/3}\sqrt[3]{b}n}$$

output
```
-3^(1/2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3*n))*3^(1/2)/a^(1/3))/a^(2/3)
/b^(1/3)/n+ln(a^(1/3)+b^(1/3)*x^(1/3*n))/a^(2/3)/b^(1/3)/n-1/2*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x^(1/3*n)+b^(2/3)*x^(2/3*n))/a^(2/3)/b^(1/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.22

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n} dx = \frac{3x^{n/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},1,\frac{4}{3},-\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[x^(-1 + n/3)/(a + b*x^n),x]
```

output
```
(3*x^(n/3)*Hypergeometric2F1[1/3, 1, 4/3, -((b*x^n)/a)])/(a*n)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {868, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{868}$$

$$\frac{3\int \frac{1}{bx^n+a}dx^{n/3}}{n}$$

$$\Big\downarrow \text{750}$$

$$\frac{3\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x^{n/3}+\sqrt[3]{a}}dx^{n/3}}{3a^{2/3}}\right)}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{n}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{n}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{n}$$

$$\Big\downarrow \; 1082$$

$$\frac{3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{3\int \frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{n}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{n}$$

input
```
Int[x^(-1 + n/3)/(a + b*x^n),x]
```

output
```
(3*(Log[a^(1/3) + b^(1/3)*x^(n/3)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*x^(n/3))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(
1/3)*b^(1/3)*x^(n/3) + b^(2/3)*x^((2*n)/3)]/(2*b^(1/3)))/(3*a^(2/3))))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

method	result	size
risch	$\sum_{\_R=\operatorname{RootOf}(a^{2} b\,n^{3} \_Z^{3}-1)} \_R \ln\left(a n \_R +x^{\frac{n}{3}}\right)$	31

input
```
int(x^(-1+1/3*n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(a*n*_R+x^(1/3*n)),_R=RootOf(_Z^3*a^2*b*n^3-1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.80

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n}\,dx = \left[\frac{\sqrt{3}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3x^{n-3}-3\,(a^2b)^{\frac{1}{3}}axx^{\frac{1}{3}\,n-1}-a^2+\sqrt{3}\left(2\,abx^2x^{\frac{2}{3}\,n-2}+(a^2b)^{\frac{2}{3}}xx^{\frac{1}{3}\,n-1}-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3x^{n-3}+a}\right)+2\,(\right.}{2\,a^2bn}\right.$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3*x^(n - 3) - 3*(a^2
*b)^(1/3)*a*x*x^(1/3*n - 1) - a^2 + sqrt(3)*(2*a*b*x^2*x^(2/3*n - 2) + (a^
2*b)^(2/3)*x*x^(1/3*n - 1) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x
^3*x^(n - 3) + a)) + 2*(a^2*b)^(2/3)*log((a*b*x*x^(1/3*n - 1) + (a^2*b)^(2
/3))/x) - (a^2*b)^(2/3)*log((a*b*x^2*x^(2/3*n - 2) - (a^2*b)^(2/3)*x*x^(1/
3*n - 1) + (a^2*b)^(1/3)*a)/x^2))/(a^2*b*n), 1/2*(2*sqrt(3)*a*b*sqrt((a^2*
b)^(1/3)/b)*arctan(1/3*sqrt(3)*(2*(a^2*b)^(2/3)*x*x^(1/3*n - 1) - (a^2*b)^
(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(a^2*b)^(2/3)*log((a*b*x*x^(1/3*n
- 1) + (a^2*b)^(2/3))/x) - (a^2*b)^(2/3)*log((a*b*x^2*x^(2/3*n - 2) - (a^2
*b)^(2/3)*x*x^(1/3*n - 1) + (a^2*b)^(1/3)*a)/x^2))/(a^2*b*n)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n}\,dx = -\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}n\Gamma\left(\frac{4}{3}\right)} + \frac{\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}n\Gamma\left(\frac{4}{3}\right)} - \frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\sqrt[3]{b}n\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(x**(-1+1/3*n)/(a+b*x**n),x)
```

output
```
-exp(-I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(
1/3)/(3*a**(2/3)*b**(1/3)*n*gamma(4/3)) + log(1 - b**(1/3)*x**(n/3)*exp_po
lar(I*pi)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*b**(1/3)*n*gamma(4/3)) - exp(I*
pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(1/3)/(
3*a**(2/3)*b**(1/3)*n*gamma(4/3))
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{x^{\frac{1}{3}n-1}}{bx^n+a}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(x^(1/3*n - 1)/(b*x^n + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n}\,dx = -\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}n}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}n}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}}{2n}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
-1/2*(2*(-a/b)^(1/3)*log(abs(x^(1/3*n) - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-a/b)^(1/3))/(-a/b)^(1/3))/(
a*b) - (-a*b^2)^(1/3)*log(x^(1/3*n)*(-a/b)^(1/3) + (x^n)^(2/3) + (-a/b)^(2
/3))/(a*b))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{x^{\frac{n}{3}-1}}{a+b\,x^n}\,dx$$

input
```
int(x^(n/3 - 1)/(a + b*x^n),x)
```

output
```
int(x^(n/3 - 1)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{x^{\frac{n}{3}}}{x^n bx+ax}dx$$

input
```
int(x^(-1+1/3*n)/(a+b*x^n),x)
```

output
```
int(x**(n/3)/(x**n*b*x + a*x),x)
```

3.569 $\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 169

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2}\,dx = \frac{x^{n/3}}{an(a+bx^n)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}n} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{5/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{3a^{5/3}\sqrt[3]{b}n}$$

output
```
x^(1/3*n)/a/n/(a+b*x^n)-2/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3*n))*3^(1/
2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(1/3)/n+2/3*ln(a^(1/3)+b^(1/3)*x^(1/3*n))/a^
(5/3)/b^(1/3)/n-1/3*ln(a^(2/3)-a^(1/3)*b^(1/3)*x^(1/3*n)+b^(2/3)*x^(2/3*n)
)/a^(5/3)/b^(1/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.19

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2}\,dx = \frac{3x^{n/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},2,\frac{4}{3},-\frac{bx^n}{a}\right)}{a^2n}$$

input
```
Integrate[x^(-1 + n/3)/(a + b*x^n)^2,x]
```

output
```
(3*x^(n/3)*Hypergeometric2F1[1/3, 2, 4/3, -((b*x^n)/a)])/(a^2*n)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {868, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{868}$$

$$\frac{3\int \frac{1}{(bx^n+a)^2}dx^{n/3}}{n}$$

$$\Big\downarrow \text{749}$$

$$\frac{3\left(\frac{2\int \frac{1}{bx^n+a}dx^{n/3}}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{750}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x^{n/3}+\sqrt[3]{a}}dx^{n/3}}{3a^{2/3}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{3\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{3\int \frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{3\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{n}$$

input
```
Int[x^(-1 + n/3)/(a + b*x^n)^2,x]
```

output
```
(3*(x^(n/3)/(3*a*(a + b*x^n)) + (2*(Log[a^(1/3) + b^(1/3)*x^(n/3)]/(3*a^(2
/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(n/3))/a^(1/3))/Sqrt[3
]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(n/3) + b^(2/3)*x^((2*n)/3)
]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{x^{\frac{n}{3}}}{an(a+b\,x^{n})}+\left(\sum_{\_R=\operatorname{RootOf}\left(27a^{5}b\,n^{3}\_Z^{3}-8\right)}\_R\ln\left(x^{\frac{n}{3}}+\frac{3a^{2}n\_R}{2}\right)\right)$	61

input
```
int(x^(-1+1/3*n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a/n*x^(1/3*n)/((x^(1/3*n))^3*b+a)+sum(_R*ln(x^(1/3*n)+3/2*a^2*n*_R),_R=R
ootOf(27*_Z^3*a^5*b*n^3-8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(124) = 248$.

Time = 0.09 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.40

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
[1/3*(3*a^2*b*x*x^(1/3*n - 1) + 3*sqrt(1/3)*(a*b^2*x^3*x^(n - 3) + a^2*b)*
sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3*x^(n - 3) - 3*(a^2*b)^(1/3)*a*x*x^(1
/3*n - 1) - a^2 + 3*sqrt(1/3)*(2*a*b*x^2*x^(2/3*n - 2) + (a^2*b)^(2/3)*x*x
^(1/3*n - 1) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3*x^(n - 3) +
 a)) + 2*((a^2*b)^(2/3)*b*x^3*x^(n - 3) + (a^2*b)^(2/3)*a)*log((a*b*x*x^(1
/3*n - 1) + (a^2*b)^(2/3))/x) - ((a^2*b)^(2/3)*b*x^3*x^(n - 3) + (a^2*b)^(
2/3)*a)*log((a*b*x^2*x^(2/3*n - 2) - (a^2*b)^(2/3)*x*x^(1/3*n - 1) + (a^2*
b)^(1/3)*a)/x^2))/(a^3*b^2*n*x^3*x^(n - 3) + a^4*b*n), 1/3*(3*a^2*b*x*x^(1
/3*n - 1) + 6*sqrt(1/3)*(a*b^2*x^3*x^(n - 3) + a^2*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x*x^(1/3*n - 1) - (a^2*b)^(1/3)*a)*sqr
t((a^2*b)^(1/3)/b)/a^2) + 2*((a^2*b)^(2/3)*b*x^3*x^(n - 3) + (a^2*b)^(2/3)
*a)*log((a*b*x*x^(1/3*n - 1) + (a^2*b)^(2/3))/x) - ((a^2*b)^(2/3)*b*x^3*x^
(n - 3) + (a^2*b)^(2/3)*a)*log((a*b*x^2*x^(2/3*n - 2) - (a^2*b)^(2/3)*x*x^
(1/3*n - 1) + (a^2*b)^(1/3)*a)/x^2))/(a^3*b^2*n*x^3*x^(n - 3) + a^4*b*n)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.44

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x**(-1+1/3*n)/(a+b*x**n)**2,x)
```

output
```
-2*a**(4/3)*x**(2*n/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3
))*gamma(1/3)/(9*a**3*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/3) + 9*a**
2*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3)) + 2*a**(4/3)*x**(2*n/3)*ex
p(I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(1/3)/(
9*a**3*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/3) + 9*a**2*b**(4/3)*n*x*
*(5*n/3)*exp(I*pi/3)*gamma(4/3)) - 2*a**(4/3)*x**(2*n/3)*exp(2*I*pi/3)*log
(1 - b**(1/3)*x**(n/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b*
*(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/3) + 9*a**2*b**(4/3)*n*x**(5*n/3)*
exp(I*pi/3)*gamma(4/3)) - 2*a**(1/3)*b*x**(5*n/3)*log(1 - b**(1/3)*x**(n/3
)*exp_polar(I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(1/3)*n*x**(2*n/3)*exp
(I*pi/3)*gamma(4/3) + 9*a**2*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3))
 + 2*a**(1/3)*b*x**(5*n/3)*exp(I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar
(I*pi)/a**(1/3))*gamma(1/3)/(9*a**3*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamm
a(4/3) + 9*a**2*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3)) - 2*a**(1/3)
*b*x**(5*n/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(5*I*pi/3)/
a**(1/3))*gamma(1/3)/(9*a**3*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/3)
+ 9*a**2*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3)) + 3*a*b**(1/3)*x**n
*exp(I*pi/3)*gamma(1/3)/(9*a**3*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/
3) + 9*a**2*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3))
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2}\,dx = \int \frac{x^{\frac{1}{3}n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
x^(1/3*n)/(a*b*n*x^n + a^2*n) + 2*integrate(1/3*x^(1/3*n)/(a*b*x*x^n + a^2
*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2}\,dx = -\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}n}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{3x^{\frac{1}{3}n}}{(bx^n+a)a}-\frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}n}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+(x^n)^{\frac{2}{3}}+\right.}{a^2b}}{3n}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
-1/3*(2*(-a/b)^(1/3)*log(abs(x^(1/3*n) - (-a/b)^(1/3)))/a^2 - 3*x^(1/3*n)/
((b*x^n + a)*a) - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n)
 + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*b^2)^(1/3)*log(x^(1/3*n)*(-a/
b)^(1/3) + (x^n)^(2/3) + (-a/b)^(2/3))/(a^2*b))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2}\,dx = \int \frac{x^{\frac{n}{3}-1}}{(a+b\,x^n)^2}\,dx$$

input
```
int(x^(n/3 - 1)/(a + b*x^n)^2,x)
```

output
```
int(x^(n/3 - 1)/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^2}\,dx = \int \frac{x^{\frac{n}{3}}}{x^{2n}b^2x + 2x^n abx + a^2x}dx$$

input
```
int(x^(-1+1/3*n)/(a+b*x^n)^2,x)
```

output
```
int(x**(n/3)/(x**(2*n)*b**2*x + 2*x**n*a*b*x + a**2*x),x)
```

3.570 $\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx=\frac{x^{n/3}}{2an(a+bx^n)^2}+\frac{5x^{n/3}}{6a^2n(a+bx^n)}-\frac{5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}n}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{9a^{8/3}\sqrt[3]{b}n}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{18a^{8/3}\sqrt[3]{b}n}$$

output
```
1/2*x^(1/3*n)/a/n/(a+b*x^n)^2+5/6*x^(1/3*n)/a^2/n/(a+b*x^n)-5/9*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x^(1/3*n))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/b^(1/3)/n+
5/9*ln(a^(1/3)+b^(1/3)*x^(1/3*n))/a^(8/3)/b^(1/3)/n-5/18*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x^(1/3*n)+b^(2/3)*x^(2/3*n))/a^(8/3)/b^(1/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.16

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx = \frac{3x^{n/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},3,\frac{4}{3},-\frac{bx^n}{a}\right)}{a^3n}$$

input
```
Integrate[x^(-1 + n/3)/(a + b*x^n)^3,x]
```

output
```
(3*x^(n/3)*Hypergeometric2F1[1/3, 3, 4/3, -((b*x^n)/a)])/(a^3*n)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {868, 749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{(a+bx^n)^3}dx$$

$$\Big\downarrow \ 868$$

$$\frac{3\int \frac{1}{(bx^n+a)^3}dx^{n/3}}{n}$$

$$\Big\downarrow \ 749$$

$$\frac{3\left(\frac{5\int \frac{1}{(bx^n+a)^2}dx^{n/3}}{6a}+\frac{x^{n/3}}{6a(a+bx^n)^2}\right)}{n}$$

$$\Big\downarrow \ 749$$

$$\frac{3\left(\frac{5\left(\frac{2\int \frac{1}{bx^n+a}dx^{n/3}}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{6a}+\frac{x^{n/3}}{6a(a+bx^n)^2}\right)}{n}$$

$\downarrow$ 750

$$\frac{3\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x^{n/3}+\sqrt[3]{a}}dx^{n/3}}{3a^{2/3}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{6a}+\frac{x^{n/3}}{6a(a+bx^n)^2}\right)}{n}$$

$\downarrow$ 16

$$\frac{3\left(\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{6a}+\frac{x^{n/3}}{6a(a+bx^n)^2}\right)}{n}$$

$\downarrow$ 1142

$$3\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^{n})}\right)}{6a}+\right.}{n}$$

$$\Big\downarrow 25$$

$$3\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^{n})}\right)}{6a}+\right.}{n}$$

$\downarrow$ 27

$$\frac{3\left(\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{6a}+$$

$$n$$

$\downarrow$ 1082

$$\frac{3\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{3\int \frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{6a}+\frac{x^{n/3}}{6a(a+bx^n)^2}\right)}{n}$$

$\downarrow$ 217

$$\frac{3\left(\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{6a}+\frac{x^{n/3}}{6a(a+bx^n)^2}\right)}{n}$$

$\downarrow$ 1103

$$\frac{3\left(\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x^{n/3}}{3a(a+bx^n)}\right)}{6a}+\frac{x^{n/3}}{6a(a+bx^n)^2}\right)}{n}$$

input
```
Int[x^(-1 + n/3)/(a + b*x^n)^3,x]
```

output
```
(3*(x^(n/3)/(6*a*(a + b*x^n)^2) + (5*(x^(n/3)/(3*a*(a + b*x^n)) + (2*(Log[
a^(1/3) + b^(1/3)*x^(n/3)]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (
2*b^(1/3)*x^(n/3))/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(
1/3)*x^(n/3) + b^(2/3)*x^((2*n)/3)]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a)))/(6*
a)))/n
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{x^{\frac{n}{3}} (5b\,x^{n}+8a)}{6a^{2}n(a+b\,x^{n})^{2}}+\left(\sum_{\_R=\mathrm{RootOf}\left(729a^{8}b\,n^{3}\_Z^{3}-125\right)}\_R\ln\left(x^{\frac{n}{3}}+\frac{9a^{3}n\_R}{5}\right)\right)$	76

input
```
int(x^(-1+1/3*n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^(1/3*n)*(5*(x^(1/3*n))^3*b+8*a)/a^2/n/((x^(1/3*n))^3*b+a)^2+sum(_R*l
n(x^(1/3*n)+9/5*a^3*n*_R),_R=RootOf(729*_Z^3*a^8*b*n^3-125))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(147) = 294$.

Time = 0.09 (sec) , antiderivative size = 785, normalized size of antiderivative = 3.92

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
[1/18*(15*a^2*b^2*x^4*x^(4/3*n - 4) + 24*a^3*b*x*x^(1/3*n - 1) + 15*sqrt(1
/3)*(a*b^3*x^6*x^(2*n - 6) + 2*a^2*b^2*x^3*x^(n - 3) + a^3*b)*sqrt(-(a^2*b
)^(1/3)/b)*log((2*a*b*x^3*x^(n - 3) - 3*(a^2*b)^(1/3)*a*x*x^(1/3*n - 1) -
a^2 + 3*sqrt(1/3)*(2*a*b*x^2*x^(2/3*n - 2) + (a^2*b)^(2/3)*x*x^(1/3*n - 1)
 - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3*x^(n - 3) + a)) + 10*((
a^2*b)^(2/3)*b^2*x^6*x^(2*n - 6) + 2*(a^2*b)^(2/3)*a*b*x^3*x^(n - 3) + (a^
2*b)^(2/3)*a^2)*log((a*b*x*x^(1/3*n - 1) + (a^2*b)^(2/3))/x) - 5*((a^2*b)^
(2/3)*b^2*x^6*x^(2*n - 6) + 2*(a^2*b)^(2/3)*a*b*x^3*x^(n - 3) + (a^2*b)^(2
/3)*a^2)*log((a*b*x^2*x^(2/3*n - 2) - (a^2*b)^(2/3)*x*x^(1/3*n - 1) + (a^2
*b)^(1/3)*a)/x^2))/(a^4*b^3*n*x^6*x^(2*n - 6) + 2*a^5*b^2*n*x^3*x^(n - 3)
+ a^6*b*n), 1/18*(15*a^2*b^2*x^4*x^(4/3*n - 4) + 24*a^3*b*x*x^(1/3*n - 1)
+ 30*sqrt(1/3)*(a*b^3*x^6*x^(2*n - 6) + 2*a^2*b^2*x^3*x^(n - 3) + a^3*b)*s
qrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x*x^(1/3*n - 1) - (
a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 10*((a^2*b)^(2/3)*b^2*x^6*x^(
2*n - 6) + 2*(a^2*b)^(2/3)*a*b*x^3*x^(n - 3) + (a^2*b)^(2/3)*a^2)*log((a*b
*x*x^(1/3*n - 1) + (a^2*b)^(2/3))/x) - 5*((a^2*b)^(2/3)*b^2*x^6*x^(2*n - 6
) + 2*(a^2*b)^(2/3)*a*b*x^3*x^(n - 3) + (a^2*b)^(2/3)*a^2)*log((a*b*x^2*x^
(2/3*n - 2) - (a^2*b)^(2/3)*x*x^(1/3*n - 1) + (a^2*b)^(1/3)*a)/x^2))/(a^4*
b^3*n*x^6*x^(2*n - 6) + 2*a^5*b^2*n*x^3*x^(n - 3) + a^6*b*n)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 2531, normalized size of antiderivative = 12.66

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(-1+1/3*n)/(a+b*x**n)**3,x)
```

output
```
-10*a**(13/3)*x**(2*n/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1
/3))*gamma(1/3)/(54*a**7*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/3) + 16
2*a**6*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3) + 162*a**5*b**(7/3)*n*
x**(8*n/3)*exp(I*pi/3)*gamma(4/3) + 54*a**4*b**(10/3)*n*x**(11*n/3)*exp(I*
pi/3)*gamma(4/3)) + 10*a**(13/3)*x**(2*n/3)*exp(I*pi/3)*log(1 - b**(1/3)*x
**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(1/3)/(54*a**7*b**(1/3)*n*x**(2*n/3
)*exp(I*pi/3)*gamma(4/3) + 162*a**6*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamm
a(4/3) + 162*a**5*b**(7/3)*n*x**(8*n/3)*exp(I*pi/3)*gamma(4/3) + 54*a**4*b
**(10/3)*n*x**(11*n/3)*exp(I*pi/3)*gamma(4/3)) - 10*a**(13/3)*x**(2*n/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma
(1/3)/(54*a**7*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/3) + 162*a**6*b**
(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3) + 162*a**5*b**(7/3)*n*x**(8*n/3)
*exp(I*pi/3)*gamma(4/3) + 54*a**4*b**(10/3)*n*x**(11*n/3)*exp(I*pi/3)*gamm
a(4/3)) - 30*a**(10/3)*b*x**(5*n/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*
pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(1/3)*n*x**(2*n/3)*exp(I*pi/3)*gamm
a(4/3) + 162*a**6*b**(4/3)*n*x**(5*n/3)*exp(I*pi/3)*gamma(4/3) + 162*a**5*
b**(7/3)*n*x**(8*n/3)*exp(I*pi/3)*gamma(4/3) + 54*a**4*b**(10/3)*n*x**(11*
n/3)*exp(I*pi/3)*gamma(4/3)) + 30*a**(10/3)*b*x**(5*n/3)*exp(I*pi/3)*log(1
 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(1/3)/(54*a**7*b**(1/3
)*n*x**(2*n/3)*exp(I*pi/3)*gamma(4/3) + 162*a**6*b**(4/3)*n*x**(5*n/3)*...
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx = \int \frac{x^{\frac{1}{3}n-1}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/6*(5*b*x^(4/3*n) + 8*a*x^(1/3*n))/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a
^4*n) + 5*integrate(1/9*x^(1/3*n)/(a^2*b*x*x^n + a^3*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx = -\frac{\frac{10\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}n}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^3}-\frac{10\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}n}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3b}-\frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^3b}}{18n}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
-1/18*(10*(-a/b)^(1/3)*log(abs(x^(1/3*n) - (-a/b)^(1/3)))/a^3 - 10*sqrt(3)
*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^3*b) - 5*(-a*b^2)^(1/3)*log(x^(1/3*n)*(-a/b)^(1/3) + (x^n)^(2/3) +
(-a/b)^(2/3))/(a^3*b) - 3*(5*b*(x^n)^(4/3) + 8*a*x^(1/3*n))/((b*x^n + a)^2
*a^2))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx = \int \frac{x^{\frac{n}{3}-1}}{(a+b\,x^n)^3}\,dx$$

input
```
int(x^(n/3 - 1)/(a + b*x^n)^3,x)
```

output
```
int(x^(n/3 - 1)/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+bx^n)^3}\,dx = \int \frac{x^{\frac{n}{3}}}{x^{3n}b^3x + 3x^{2n}a\,b^2x + 3x^na^2bx + a^3x}dx$$

input
```
int(x^(-1+1/3*n)/(a+b*x^n)^3,x)
```

output
```
int(x**(n/3)/(x**(3*n)*b**3*x + 3*x**(2*n)*a*b**2*x + 3*x**n*a**2*b*x + a*
*3*x),x)
```

3.571 $\int \frac{x^m}{a+bx^{1+m}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{x^m}{a+bx^{1+m}}\,dx=\frac{\log\left(a+bx^{1+m}\right)}{b(1+m)}$$

output
```
ln(a+b*x^(1+m))/b/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{x^m}{a+bx^{1+m}}\,dx=\frac{\log\left((b+bm)\left(a+bx^{1+m}\right)\right)}{b(1+m)}$$

input
```
Integrate[x^m/(a + b*x^(1 + m)),x]
```

output
```
Log[(b + b*m)*(a + b*x^(1 + m))]/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+bx^{m+1}}dx$$

$$\Big\downarrow \text{792}$$

$$\frac{\log\left(a+bx^{m+1}\right)}{b(m+1)}$$

input
```
Int[x^m/(a + b*x^(1 + m)),x]
```

output
```
Log[a + b*x^(1 + m)]/(b*(1 + m))
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
norman	$\frac{\ln\left(a+bx\,{\mathrm e}^{m\ln\left(x\right)}\right)}{b\left(1+m\right)}$	21
risch	$\frac{\ln\left(x\right)}{b}-\frac{m\ln\left(x\right)}{b\left(1+m\right)}+\frac{\ln\left(x^{m}+\frac{a}{xb}\right)}{b\left(1+m\right)}$	43

input
```
int(x^m/(a+b*x^(1+m)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(1+m)*ln(a+b*x*exp(m*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{x^m}{a+bx^{1+m}}\,dx = \frac{\log\left(bx^{m+1}+a\right)}{bm+b}$$

input
```
integrate(x^m/(a+b*x^(1+m)),x, algorithm="fricas")
```

output
```
log(b*x^(m + 1) + a)/(b*m + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{x^m}{a+bx^{1+m}}\,dx = \begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge m = -1 \\ \frac{xx^m}{a(m+1)} & \text{for } b = 0 \\ \frac{\log(x)}{a+b} & \text{for } m = -1 \\ \frac{\log(x)}{bm+b} + \frac{\log\left(\frac{a}{bx}+x^m\right)}{bm+b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m/(a+b*x**(1+m)),x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(m, -1)), (x*x**m/(a*(m + 1)), Eq(b, 0))
, (log(x)/(a + b), Eq(m, -1)), (log(x)/(b*m + b) + log(a/(b*x) + x**m)/(b*
m + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{a+bx^{1+m}}\,dx = \frac{\log\left(bx^{m+1}+a\right)}{b(m+1)}$$

input
```
integrate(x^m/(a+b*x^(1+m)),x, algorithm="maxima")
```

output
```
log(b*x^(m + 1) + a)/(b*(m + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{a+bx^{1+m}}\,dx = \frac{\log\left(|bx^{m+1}+a|\right)}{bm+b}$$

input
```
integrate(x^m/(a+b*x^(1+m)),x, algorithm="giac")
```

output
```
log(abs(b*x^(m + 1) + a))/(b*m + b)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{a+bx^{1+m}}\,dx = \frac{\ln\left(a+b\,x^{m+1}\right)}{b\,\left(m+1\right)}$$

input
```
int(x^m/(a + b*x^(m + 1)),x)
```

output
```
log(a + b*x^(m + 1))/(b*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{x^m}{a + b x^{1+m}} \, dx = \frac{\log(x^m b x + a)}{b (m + 1)}$$

input
```
int(x^m/(a+b*x^(1+m)),x)
```

output
```
log(x**m*b*x + a)/(b*(m + 1))
```

3.572 $\int x^m(a+bx^{1+m})^n\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int x^m(a+bx^{1+m})^n\,dx = \frac{(a+bx^{1+m})^{1+n}}{b(1+m)(1+n)}$$

output
```
(a+b*x^(1+m))^(1+n)/b/(1+m)/(1+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^m(a+bx^{1+m})^n\,dx = \frac{(a+bx^{1+m})^{1+n}}{b(1+m)(1+n)}$$

input
```
Integrate[x^m*(a + b*x^(1 + m))^n,x]
```

output
```
(a + b*x^(1 + m))^(1 + n)/(b*(1 + m)*(1 + n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a + bx^{m+1})^n \, dx$$

$$\Big\downarrow \text{793}$$

$$\frac{(a + bx^{m+1})^{n+1}}{b(m+1)(n+1)}$$

input
```
Int[x^m*(a + b*x^(1 + m))^n,x]
```

output
```
(a + b*x^(1 + m))^(1 + n)/(b*(1 + m)*(1 + n))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result	size
risch	$\frac{(x^m bx+a)(x^m bx+a)^n}{b(mn+m+n+1)}$	32
norman	$\frac{a\,\mathrm{e}^{n\ln\left(a+bx\,\mathrm{e}^{m\ln(x)}\right)}}{b(mn+m+n+1)}+\frac{x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{n\ln\left(a+bx\,\mathrm{e}^{m\ln(x)}\right)}}{mn+m+n+1}$	60

input
```
int(x^m*(a+b*x^(1+m))^n,x,method=_RETURNVERBOSE)
```

output
```
(x^m*b*x+a)/b/(m*n+m+n+1)*(x^m*b*x+a)^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^m\left(a+bx^{1+m}\right)^n\,dx=\frac{(bx^{m+1}+a)(bx^{m+1}+a)^n}{bm+(bm+b)n+b}$$

input
```
integrate(x^m*(a+b*x^(1+m))^n,x, algorithm="fricas")
```

output
```
(b*x^(m + 1) + a)*(b*x^(m + 1) + a)^n/(b*m + (b*m + b)*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(19) = 38.

Time = 10.59 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.04

$$\int x^m\left(a+bx^{1+m}\right)^n\,dx=\begin{cases}\frac{\log(x)}{a} & \text{for}\ b=0\wedge m=-1\wedge n=-1\\ \frac{a^nxx^m}{m+1} & \text{for}\ b=0\\ (a+b)^n\log(x) & \text{for}\ m=-1\\ \frac{\log(x)}{bm+b}+\frac{\log\left(\frac{a}{bx}+x^m\right)}{bm+b} & \text{for}\ n=-1\\ \frac{a\left(a+bx^{m+1}\right)^n}{bmn+bm+bn+b}+\frac{bxx^m\left(a+bx^{m+1}\right)^n}{bmn+bm+bn+b} & \text{otherwise}\end{cases}$$

input
```
integrate(x**m*(a+b*x**(1+m))**n,x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(m, -1) & Eq(n, -1)), (a**n*x*x**m/(m +
1), Eq(b, 0)), ((a + b)**n*log(x), Eq(m, -1)), (log(x)/(b*m + b) + log(a/(
b*x) + x**m)/(b*m + b), Eq(n, -1)), (a*(a + b*x**(m + 1))**n/(b*m*n + b*m
+ b*n + b) + b*x*x**m*(a + b*x**(m + 1))**n/(b*m*n + b*m + b*n + b), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^m (a + bx^{1+m})^n \, dx = \frac{(bx^{m+1} + a)^{n+1}}{b(m+1)(n+1)}$$

input
```
integrate(x^m*(a+b*x^(1+m))^n,x, algorithm="maxima")
```

output
```
(b*x^(m + 1) + a)^(n + 1)/(b*(m + 1)*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^m (a + bx^{1+m})^n \, dx = \frac{(bx^{m+1} + a)^{n+1}}{(bm + b)(n+1)}$$

input
```
integrate(x^m*(a+b*x^(1+m))^n,x, algorithm="giac")
```

output
```
(b*x^(m + 1) + a)^(n + 1)/((b*m + b)*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^m \left(a + bx^{1+m}\right)^n \, dx = \frac{\left(a + b\,x^{m+1}\right)^{n+1}}{b\,\left(m+1\right)\,\left(n+1\right)}$$

input
```
int(x^m*(a + b*x^(m + 1))^n,x)
```

output
```
(a + b*x^(m + 1))^(n + 1)/(b*(m + 1)*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x^m \left(a + bx^{1+m}\right)^n \, dx = \frac{\left(x^m bx + a\right)^n \left(x^m bx + a\right)}{b\,(mn + m + n + 1)}$$

input
```
int(x^m*(a+b*x^(1+m))^n,x)
```

output
```
((x**m*b*x + a)**n*(x**m*b*x + a))/(b*(m*n + m + n + 1))
```

3.573 $\int x^m(a+bx^n)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int x^m(a+bx^n)^3\,dx=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{1+m+n}}{1+m+n}+\frac{3ab^2x^{1+m+2n}}{1+m+2n}+\frac{b^3x^{1+m+3n}}{1+m+3n}$$

output
```
a^3*x^(1+m)/(1+m)+3*a^2*b*x^(1+m+n)/(1+m+n)+3*a*b^2*x^(1+m+2*n)/(1+m+2*n)+
b^3*x^(1+m+3*n)/(1+m+3*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int x^m(a+bx^n)^3\,dx=x^{1+m}\left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3ab^2x^{2n}}{1+m+2n}+\frac{b^3x^{3n}}{1+m+3n}\right)$$

input
```
Integrate[x^m*(a + b*x^n)^3,x]
```

output
```
x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*b^2*x^(2*n))/(1
+ m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(a+bx^n)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^m+3a^2bx^{m+n}+3ab^2x^{m+2n}+b^3x^{m+3n}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+n+1}}{m+n+1}+\frac{3ab^2x^{m+2n+1}}{m+2n+1}+\frac{b^3x^{m+3n+1}}{m+3n+1}$$

input
```
Int[x^m*(a + b*x^n)^3,x]
```

output
```
(a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(1 + m + n))/(1 + m + n) + (3*a*b^2*x
^(1 + m + 2*n))/(1 + m + 2*n) + (b^3*x^(1 + m + 3*n))/(1 + m + 3*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

method	result
risch	$\frac{a^{3}x\,x^{m}}{1+m}+\frac{b^{3}x\,x^{m}x^{3n}}{1+m+3n}+\frac{3a\,b^{2}x\,x^{m}x^{2n}}{1+m+2n}+\frac{3a^{2}bx\,x^{m}x^{n}}{1+n+m}$
norman	$\frac{a^{3}x\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b^{3}x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{3n\ln(x)}}{1+m+3n}+\frac{3a\,b^{2}x\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{2n\ln(x)}}{1+m+2n}+\frac{3a^{2}bx\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{n\ln(x)}}{1+n+m}$
parallelrisch	$b^{3}x\,x^{m}x^{3n}+x\,x^{m}a^{3}m^{3}+6x\,x^{m}a^{3}n^{3}+3x\,x^{m}a^{3}m^{2}+12x\,x^{m}x^{2n}a\,b^{2}m^{2}n+9x\,x^{m}x^{2n}a\,b^{2}m\,n^{2}+24x\,x^{m}x^{2n}a\,b^{2}mn+3x\,x^{m}x^{n}$
orering	$\frac{x(4m^{3}+18m^{2}n+22m\,n^{2}+6n^{3}+6m^{2}+18mn+11n^{2}+4m+6n+1)x^{m}(a+b\,x^{n})^{3}}{m^{4}+6m^{3}n+11m^{2}n^{2}+6m\,n^{3}+4m^{3}+18m^{2}n+22m\,n^{2}+6n^{3}+6m^{2}+18mn+11n^{2}+4m+6n+1}-\frac{x^{2}(6m^{2}+18m}{m^{4}+6m^{3}n+11m^{2}n^{2}+6m}$

input
```
int(x^m*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/(1+m)*x*x^m+b^3/(1+m+3*n)*x*x^m*(x^n)^3+3*a*b^2/(1+m+2*n)*x*x^m*(x^n)^
2+3*a^2*b/(1+n+m)*x*x^m*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(75) = 150$.

Time = 0.08 (sec) , antiderivative size = 362, normalized size of antiderivative = 4.83

$$\int x^m(a+bx^n)^3\,dx$$
$$=\frac{(b^3m^3+3\,b^3m^2+3\,b^3m+b^3+2\,(b^3m+b^3)n^2+3\,(b^3m^2+2\,b^3m+b^3)n)xx^mx^{3\,n}+3\,(ab^2m^3+3\,ab^2m^2}{}$$

input
```
integrate(x^m*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
 2*b^3*m + b^3)*n)*x*x^m*x^(3*n) + 3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m
+ a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n)*x
*x^m*x^(2*n) + 3*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m
 + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x*x^m*x^n + (a^3*m^3
+ 6*a^3*n^3 + 3*a^3*m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^
2 + 2*a^3*m + a^3)*n)*x*x^m)/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m
+ 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3546 vs. $2(70) = 140$.

Time = 3.03 (sec) , antiderivative size = 3546, normalized size of antiderivative = 47.28

$$\int x^m (a + bx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(a+b*x**n)**3,x)
```

output
```
Piecewise(((a + b)**3*log(x), Eq(m, -1) & Eq(n, 0)), (a**3*log(x) + 3*a**2
*b*x**n/n + 3*a*b**2*x**(2*n)/(2*n) + b**3*x**(3*n)/(3*n), Eq(m, -1)), (-a
**3/(3*n*x**(3*n)) - 3*a**2*b/(2*n*x**(2*n)) - 3*a*b**2/(n*x**n) + b**3*lo
g(x**n)/n, Eq(m, -3*n - 1)), (-a**3/(2*n*x**(2*n)) - 3*a**2*b/(n*x**n) + 3
*a*b**2*log(x**n)/n + b**3*x**n/n, Eq(m, -2*n - 1)), (-a**3/(n*x**n) + 3*a
**2*b*log(x**n)/n + 3*a*b**2*x**n/n + b**3*x**(2*n)/(2*n), Eq(m, -n - 1)),
 (a**3*m**3*x*x**m/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2 + 18*m**2*n +
6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n**2 + 6*n + 1)
 + 6*a**3*m**2*n*x*x**m/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2 + 18*m**2
*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n**2 + 6*n
 + 1) + 3*a**3*m**2*x*x**m/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2 + 18*m
**2*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n**2 +
6*n + 1) + 11*a**3*m*n**2*x*x**m/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2
+ 18*m**2*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n
**2 + 6*n + 1) + 12*a**3*m*n*x*x**m/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n*
*2 + 18*m**2*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 1
1*n**2 + 6*n + 1) + 3*a**3*m*x*x**m/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n*
*2 + 18*m**2*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 1
1*n**2 + 6*n + 1) + 6*a**3*n**3*x*x**m/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2
*n**2 + 18*m**2*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^m(a+bx^n)^3\,dx=\frac{b^3x^{m+3\,n+1}}{m+3\,n+1}+\frac{3\,ab^2x^{m+2\,n+1}}{m+2\,n+1}+\frac{3\,a^2bx^{m+n+1}}{m+n+1}+\frac{a^3x^{m+1}}{m+1}$$

input
```
integrate(x^m*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
b^3*x^(m + 3*n + 1)/(m + 3*n + 1) + 3*a*b^2*x^(m + 2*n + 1)/(m + 2*n + 1)
+ 3*a^2*b*x^(m + n + 1)/(m + n + 1) + a^3*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 622, normalized size of antiderivative = 8.29

$$\int x^m (a + bx^n)^3 \, dx = \frac{b^3 m^3 x x^m x^{3n} + 3 b^3 m^2 n x x^m x^{3n} + 2 b^3 m n^2 x x^m x^{3n} + 3 a b^2 m^3 x x^m x^{2n} + 12 a b^2 m^2 n x x^m x^{2n} + 9 a b^2 m n^2 x}{}$$

input
```
integrate(x^m*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
(b^3*m^3*x*x^m*x^(3*n) + 3*b^3*m^2*n*x*x^m*x^(3*n) + 2*b^3*m*n^2*x*x^m*x^(
3*n) + 3*a*b^2*m^3*x*x^m*x^(2*n) + 12*a*b^2*m^2*n*x*x^m*x^(2*n) + 9*a*b^2*
m*n^2*x*x^m*x^(2*n) + 3*a^2*b*m^3*x*x^m*x^n + 15*a^2*b*m^2*n*x*x^m*x^n + 1
8*a^2*b*m*n^2*x*x^m*x^n + a^3*m^3*x*x^m + 6*a^3*m^2*n*x*x^m + 11*a^3*m*n^2
*x*x^m + 6*a^3*n^3*x*x^m + 3*b^3*m^2*x*x^m*x^(3*n) + 6*b^3*m*n*x*x^m*x^(3*
n) + 2*b^3*n^2*x*x^m*x^(3*n) + 9*a*b^2*m^2*x*x^m*x^(2*n) + 24*a*b^2*m*n*x*
x^m*x^(2*n) + 9*a*b^2*n^2*x*x^m*x^(2*n) + 9*a^2*b*m^2*x*x^m*x^n + 30*a^2*b
*m*n*x*x^m*x^n + 18*a^2*b*n^2*x*x^m*x^n + 3*a^3*m^2*x*x^m + 12*a^3*m*n*x*x
^m + 11*a^3*n^2*x*x^m + 3*b^3*m*x*x^m*x^(3*n) + 3*b^3*n*x*x^m*x^(3*n) + 9*
a*b^2*m*x*x^m*x^(2*n) + 12*a*b^2*n*x*x^m*x^(2*n) + 9*a^2*b*m*x*x^m*x^n + 1
5*a^2*b*n*x*x^m*x^n + 3*a^3*m*x*x^m + 6*a^3*n*x*x^m + b^3*x*x^m*x^(3*n) +
3*a*b^2*x*x^m*x^(2*n) + 3*a^2*b*x*x^m*x^n + a^3*x*x^m)/(m^4 + 6*m^3*n + 11
*m^2*n^2 + 6*m*n^3 + 4*m^3 + 18*m^2*n + 22*m*n^2 + 6*n^3 + 6*m^2 + 18*m*n
+ 11*n^2 + 4*m + 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x^m (a + bx^n)^3 \, dx = \frac{a^3 \, x \, x^m}{m+1} + \frac{b^3 \, x \, x^m \, x^{3\,n}}{m + 3\,n + 1} + \frac{3\,a^2 \, b \, x \, x^m \, x^n}{m + n + 1} + \frac{3\,a\,b^2 \, x \, x^m \, x^{2\,n}}{m + 2\,n + 1}$$

input
```
int(x^m*(a + b*x^n)^3,x)
```

output

```
(a^3*x*x^m)/(m + 1) + (b^3*x*x^m*x^(3*n))/(m + 3*n + 1) + (3*a^2*b*x*x^m*x
^n)/(m + n + 1) + (3*a*b^2*x*x^m*x^(2*n))/(m + 2*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 477, normalized size of antiderivative = 6.36

$$\int x^m(a+bx^n)^3\,dx$$
$$=\frac{x^m x(a^3+2x^{3n}b^3n^2+3x^{3n}b^3n+3x^{2n}a\,b^2+3x^n a^2b+9x^{2n}a\,b^2n^2+12x^{2n}a\,b^2n+18x^n a^2b\,n^2+15x^n a^2bn}{}$$

input

```
int(x^m*(a+b*x^n)^3,x)
```

output

```
(x**m*x*(x**(3*n)*b**3*m**3 + 3*x**(3*n)*b**3*m**2*n + 3*x**(3*n)*b**3*m**
2 + 2*x**(3*n)*b**3*m*n**2 + 6*x**(3*n)*b**3*m*n + 3*x**(3*n)*b**3*m + 2*x
**(3*n)*b**3*n**2 + 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 3*x**(2*n)*a*b**2*
m**3 + 12*x**(2*n)*a*b**2*m**2*n + 9*x**(2*n)*a*b**2*m**2 + 9*x**(2*n)*a*b
**2*m*n**2 + 24*x**(2*n)*a*b**2*m*n + 9*x**(2*n)*a*b**2*m + 9*x**(2*n)*a*b
**2*n**2 + 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b*m**3 +
 15*x**n*a**2*b*m**2*n + 9*x**n*a**2*b*m**2 + 18*x**n*a**2*b*m*n**2 + 30*x
**n*a**2*b*m*n + 9*x**n*a**2*b*m + 18*x**n*a**2*b*n**2 + 15*x**n*a**2*b*n
+ 3*x**n*a**2*b + a**3*m**3 + 6*a**3*m**2*n + 3*a**3*m**2 + 11*a**3*m*n**2
 + 12*a**3*m*n + 3*a**3*m + 6*a**3*n**3 + 11*a**3*n**2 + 6*a**3*n + a**3))
/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2 + 18*m**2*n + 6*m**2 + 6*m*n**3
+ 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n**2 + 6*n + 1)
```

3.574 $\int x^m (a+bx^n)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x^m (a+bx^n)^2 \, dx = \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{1+m+n}}{1+m+n} + \frac{b^2 x^{1+m+2n}}{1+m+2n}$$

output
```
a^2*x^(1+m)/(1+m)+2*a*b*x^(1+m+n)/(1+m+n)+b^2*x^(1+m+2*n)/(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int x^m (a+bx^n)^2 \, dx = x^{1+m}\left(\frac{a^2}{1+m} + \frac{2abx^n}{1+m+n} + \frac{b^2 x^{2n}}{1+m+2n}\right)$$

input
```
Integrate[x^m*(a + b*x^n)^2,x]
```

output
```
x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + (b^2*x^(2*n))/(1 + m + 
2*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a+bx^n)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^m+2abx^{m+n}+b^2x^{m+2n}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+n+1}}{m+n+1}+\frac{b^2x^{m+2n+1}}{m+2n+1}$$

input
```
Int[x^m*(a + b*x^n)^2,x]
```

output
```
(a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(1 + m + n))/(1 + m + n) + (b^2*x^(1 +
m + 2*n))/(1 + m + 2*n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

method	result
risch	$\frac{a^2 x\, x^m}{1+m}+\frac{b^2 x\, x^m x^{2n}}{1+m+2n}+\frac{2abx\, x^m x^n}{1+n+m}$
norman	$\frac{a^2 x\, \mathrm{e}^{m\ln(x)}}{1+m}+\frac{b^2 x\, \mathrm{e}^{m\ln(x)} \mathrm{e}^{2n\ln(x)}}{1+m+2n}+\frac{2abx\, \mathrm{e}^{m\ln(x)} \mathrm{e}^{n\ln(x)}}{1+n+m}$
parallelrisch	$\frac{b^2 x\, x^m x^{2n}+x\, x^m a^2 m^2+2x\, x^m a^2 n^2+x\, x^m x^{2n} b^2 mn+2x\, x^m x^n ab\, m^2+2x\, x^m a^2 m+3x\, x^m a^2 n+x\, x^m x^{2n} b^2 m^2+2x\, x^m x^{2n} b^2}{(1+m)(1+m+2n)(1+n+m}$
orering	$\frac{x(3m^2+6mn+2n^2+3m+3n+1)x^m(a+b\,x^n)^2}{(m^2+2mn+2m+2n+1)(1+n+m)}-\frac{3x^2(m+n)\left(\frac{x^m m(a+b\,x^n)^2}{x}+\frac{2x^m(a+b\,x^n)b\,x^n n}{x}\right)}{(m^2+2mn+2m+2n+1)(1+n+m)}+\frac{x^3\left(\frac{x^m m^2(a+b\,x^n)}{x^2}\right.}{}$

input
```
int(x^m*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*x^m+b^2/(1+m+2*n)*x*x^m*(x^n)^2+2*a*b/(1+n+m)*x*x^m*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(51) = 102.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.96

$$\int x^m (a+bx^n)^2 \, dx$$
$$= \frac{(b^2m^2 + 2\,b^2m + b^2 + (b^2m + b^2)n)xx^mx^{2\,n} + 2\,(abm^2 + 2\,abm + ab + 2\,(abm + ab)n)xx^mx^n + (a^2m^2 +}{m^3 + 2\,(m+1)n^2 + 3\,m^2 + 3\,(m^2 + 2\,m + 1)n + 3\,m + 1}$$

input
```
integrate(x^m*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 2*b^2*m + b^2 + (b^2*m + b^2)*n)*x*x^m*x^(2*n) + 2*(a*b*m^2 +
2*a*b*m + a*b + 2*(a*b*m + a*b)*n)*x*x^m*x^n + (a^2*m^2 + 2*a^2*n^2 + 2*a^
2*m + a^2 + 3*(a^2*m + a^2)*n)*x*x^m)/(m^3 + 2*(m + 1)*n^2 + 3*m^2 + 3*(m^
2 + 2*m + 1)*n + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(46) = 92$.

Time = 1.34 (sec) , antiderivative size = 988, normalized size of antiderivative = 19.37

$$\int x^m (a + bx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(a+b*x**n)**2,x)
```

output
```
Piecewise(((a + b)**2*log(x), Eq(m, -1) & Eq(n, 0)), (a**2*log(x) + 2*a*b*
x**n/n + b**2*x**(2*n)/(2*n), Eq(m, -1)), (-a**2/(2*n*x**(2*n)) - 2*a*b/(n
*x**n) + b**2*log(x**n)/n, Eq(m, -2*n - 1)), (-a**2/(n*x**n) + 2*a*b*log(x
**n)/n + b**2*x**n/n, Eq(m, -n - 1)), (a**2*m**2*x*x**m/(m**3 + 3*m**2*n +
 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 3*a**2*m*n*x*x**m/(
m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*
a**2*m*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2
+ 3*n + 1) + 2*a**2*n**2*x*x**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m
*n + 3*m + 2*n**2 + 3*n + 1) + 3*a**2*n*x*x**m/(m**3 + 3*m**2*n + 3*m**2 +
 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + a**2*x*x**m/(m**3 + 3*m**2*n
 + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*b*m**2*x*x**m
*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n +
1) + 4*a*b*m*n*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n +
3*m + 2*n**2 + 3*n + 1) + 4*a*b*m*x*x**m*x**n/(m**3 + 3*m**2*n + 3*m**2 +
2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 4*a*b*n*x*x**m*x**n/(m**3 + 3
*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*b*x*x*
*m*x**n/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n
+ 1) + b**2*m**2*x*x**m*x**(2*n)/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*
m*n + 3*m + 2*n**2 + 3*n + 1) + b**2*m*n*x*x**m*x**(2*n)/(m**3 + 3*m**2*n
+ 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b**2*m*x*x**m...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^m (a+bx^n)^2 \, dx = \frac{b^2 x^{m+2n+1}}{m+2n+1} + \frac{2\,abx^{m+n+1}}{m+n+1} + \frac{a^2 x^{m+1}}{m+1}$$

input
```
integrate(x^m*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
b^2*x^(m + 2*n + 1)/(m + 2*n + 1) + 2*a*b*x^(m + n + 1)/(m + n + 1) + a^2*
x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.67

$$\int x^m (a+bx^n)^2 \, dx$$
$$= \frac{b^2m^2xx^mx^{2n} + b^2mnxx^mx^{2n} + 2\,abm^2xx^mx^n + 4\,abmnxx^mx^n + a^2m^2xx^m + 3\,a^2mnxx^m + 2\,a^2n^2xx^m}{m^3 + 3\,m^2n + 2\,mn^2}$$

input
```
integrate(x^m*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
(b^2*m^2*x*x^m*x^(2*n) + b^2*m*n*x*x^m*x^(2*n) + 2*a*b*m^2*x*x^m*x^n + 4*a
*b*m*n*x*x^m*x^n + a^2*m^2*x*x^m + 3*a^2*m*n*x*x^m + 2*a^2*n^2*x*x^m + 2*b
^2*m*x*x^m*x^(2*n) + b^2*n*x*x^m*x^(2*n) + 4*a*b*m*x*x^m*x^n + 4*a*b*n*x*x
^m*x^n + 2*a^2*m*x*x^m + 3*a^2*n*x*x^m + b^2*x*x^m*x^(2*n) + 2*a*b*x*x^m*x
^n + a^2*x*x^m)/(m^3 + 3*m^2*n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3
*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int x^m (a+bx^n)^2 \, dx = \frac{a^2\,x\,x^m}{m+1} + \frac{b^2\,x\,x^m\,x^{2\,n}}{m+2\,n+1} + \frac{2\,a\,b\,x\,x^m\,x^n}{m+n+1}$$

input
```
int(x^m*(a + b*x^n)^2,x)
```

output
```
(a^2*x*x^m)/(m + 1) + (b^2*x*x^m*x^(2*n))/(m + 2*n + 1) + (2*a*b*x*x^m*x^n
)/(m + n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.47

$$\int x^m (a+bx^n)^2 \, dx$$
$$= \frac{x^m x(x^{2n}b^2m^2 + x^{2n}b^2mn + 2x^{2n}b^2m + x^{2n}b^2n + x^{2n}b^2 + 2x^n ab\, m^2 + 4x^n abmn + 4x^n abm + 4x^n abn + 2}{m^3 + 3m^2n + 2m\, n^2 + 3m^2 + 6mn + 2n^2 + 3m + 3n}$$

input
```
int(x^m*(a+b*x^n)^2,x)
```

output
```
(x**m*x*(x**(2*n)*b**2*m**2 + x**(2*n)*b**2*m*n + 2*x**(2*n)*b**2*m + x**(
2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m**2 + 4*x**n*a*b*m*n + 4*x**n*a*
b*m + 4*x**n*a*b*n + 2*x**n*a*b + a**2*m**2 + 3*a**2*m*n + 2*a**2*m + 2*a*
*2*n**2 + 3*a**2*n + a**2))/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n +
 3*m + 2*n**2 + 3*n + 1)
```

3.575 $\int x^m(a+bx^n)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int x^m(a+bx^n)\,dx=\frac{ax^{1+m}}{1+m}+\frac{bx^{1+m+n}}{1+m+n}$$

output
```
a*x^(1+m)/(1+m)+b*x^(1+m+n)/(1+m+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^m(a+bx^n)\,dx=\frac{ax^{1+m}}{1+m}+\frac{bx^{1+m+n}}{1+m+n}$$

input
```
Integrate[x^m*(a + b*x^n),x]
```

output
```
(a*x^(1 + m))/(1 + m) + (b*x^(1 + m + n))/(1 + m + n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a + bx^n) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int (ax^m + bx^{m+n}) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^{m+1}}{m+1} + \frac{bx^{m+n+1}}{m+n+1}$$

input
```
Int[x^m*(a + b*x^n),x]
```

output
```
(a*x^(1 + m))/(1 + m) + (b*x^(1 + m + n))/(1 + m + n)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{ax\,x^{m}}{1+m}+\frac{bx\,x^{m}x^{n}}{1+n+m}$	28
norman	$\frac{ax\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{bx\,\mathrm{e}^{m\ln(x)}\mathrm{e}^{n\ln(x)}}{1+n+m}$	34
parallelrisch	$\frac{x\,x^{m}x^{n}bm+x\,x^{m}x^{n}b+x\,x^{m}am+x\,x^{m}an+x\,x^{m}a}{(1+m)(1+n+m)}$	53
orering	$\frac{x(2m+n+1)x^{m}(a+b\,x^{n})}{m^{2}+mn+2m+n+1}-\frac{x^{2}\left(\frac{x^{m}m(a+b\,x^{n})}{x}+\frac{x^{m}b\,x^{n}n}{x}\right)}{m^{2}+mn+2m+n+1}$	81

input
```
int(x^m*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*x^m+b/(1+n+m)*x*x^m*x^n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int x^m(a+bx^n)\ dx=\frac{(bm+b)xx^mx^n+(am+an+a)xx^m}{m^2+(m+1)n+2\,m+1}$$

input
```
integrate(x^m*(a+b*x^n),x, algorithm="fricas")
```

output
```
((b*m + b)*x*x^m*x^n + (a*m + a*n + a)*x*x^m)/(m^2 + (m + 1)*n + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(22) = 44.

Time = 0.61 (sec) , antiderivative size = 151, normalized size of antiderivative = 5.59

$$\int x^m(a+bx^n)\,dx$$
$$= \begin{cases} (a+b)\log(x) & \text{for } m=-1 \,/ \\ a\log(x)+\frac{bx^n}{n} & \text{for } m=-1 \\ -\frac{ax^{-n}}{n}+\frac{b\log(x^n)}{n} & \text{for } m=-n\ - \\ \frac{amxx^m}{m^2+mn+2m+n+1}+\frac{anxx^m}{m^2+mn+2m+n+1}+\frac{axx^m}{m^2+mn+2m+n+1}+\frac{bmxx^mx^n}{m^2+mn+2m+n+1}+\frac{bxx^mx^n}{m^2+mn+2m+n+1} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(a+b*x**n),x)
```

output
```
Piecewise(((a + b)*log(x), Eq(m, -1) & Eq(n, 0)), (a*log(x) + b*x**n/n, Eq
(m, -1)), (-a/(n*x**n) + b*log(x**n)/n, Eq(m, -n - 1)), (a*m*x*x**m/(m**2
+ m*n + 2*m + n + 1) + a*n*x*x**m/(m**2 + m*n + 2*m + n + 1) + a*x*x**m/(m
**2 + m*n + 2*m + n + 1) + b*m*x*x**m*x**n/(m**2 + m*n + 2*m + n + 1) + b*
x*x**m*x**n/(m**2 + m*n + 2*m + n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^m(a+bx^n)\,dx = \frac{bx^{m+n+1}}{m+n+1}+\frac{ax^{m+1}}{m+1}$$

input
```
integrate(x^m*(a+b*x^n),x, algorithm="maxima")
```

output
```
b*x^(m + n + 1)/(m + n + 1) + a*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int x^m(a+bx^n)\,dx = \frac{bmxx^mx^n + amxx^m + anxx^m + bxx^mx^n + axx^m}{m^2 + mn + 2\,m + n + 1}$$

input
```
integrate(x^m*(a+b*x^n),x, algorithm="giac")
```

output
```
(b*m*x*x^m*x^n + a*m*x*x^m + a*n*x*x^m + b*x*x^m*x^n + a*x*x^m)/(m^2 + m*n
 + 2*m + n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^m(a+bx^n)\,dx = \frac{a\,x\,x^m}{m+1} + \frac{b\,x\,x^m\,x^n}{m+n+1}$$

input
```
int(x^m*(a + b*x^n),x)
```

output
```
(a*x*x^m)/(m + 1) + (b*x*x^m*x^n)/(m + n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int x^m(a+bx^n)\,dx = \frac{x^m x(x^n bm + x^n b + am + an + a)}{m^2 + mn + 2m + n + 1}$$

input
```
int(x^m*(a+b*x^n),x)
```

output
```
(x**m*x*(x**n*b*m + x**n*b + a*m + a*n + a))/(m**2 + m*n + 2*m + n + 1)
```

3.576 $\int \frac{x^m}{a+bx^n} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^m}{a+bx^n} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x^m}{a+bx^n} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, 1+\frac{1+m}{n}, -\frac{bx^n}{a}\right)}{a(1+m)}$$

input
```
Integrate[x^m/(a + b*x^n),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b*x^n)/a)])/(
a*(1 + m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+bx^n} \, dx$$

$$\Big\downarrow \; 888$$

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(1, \frac{m+1}{n}, \frac{m+n+1}{n}, -\frac{bx^n}{a}\right)}{a(m+1)}$$

input
```
Int[x^m/(a + b*x^n),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(
a*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{a+b\,x^n}dx$$

input
```
int(x^m/(a+b*x^n),x)
```

output
```
int(x^m/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{x^m}{a+bx^n}\,dx=\int \frac{x^m}{bx^n+a}\,dx$$

input
```
integrate(x^m/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(x^m/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.32

$$\int \frac{x^m}{a+bx^n}\,dx=\frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-1-\frac{1}{n}}mx^{m+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}+\frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-1-\frac{1}{n}}x^{m+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate(x**m/(a+b*x**n),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*m*x**(m + 1)*lerchphi(b*x**n*exp_polar(
I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n)) + a**(
m/n + 1/n)*a**(-m/n - 1 - 1/n)*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n))
```

Maxima [F]

$$\int \frac{x^m}{a+bx^n}\,dx = \int \frac{x^m}{bx^n+a}\,dx$$

input
```
integrate(x^m/(a+b*x^n),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^m}{a+bx^n}\,dx = \int \frac{x^m}{bx^n+a}\,dx$$

input
```
integrate(x^m/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^n}\,dx = \int \frac{x^m}{a+b\,x^n}\,dx$$

input
```
int(x^m/(a + b*x^n),x)
```

output
```
int(x^m/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{x^m}{a+bx^n}\,dx = \int \frac{x^m}{x^n b + a}dx$$

input
```
int(x^m/(a+b*x^n),x)
```

output
```
int(x**m/(x**n*b + a),x)
```

3.577 $\int \frac{x^m}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^m}{(a+bx^n)^2} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right)}{a^2(1+m)}$$

output
```
x^(1+m)*hypergeom([2, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x^m}{(a+bx^n)^2} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{n}, 1+\frac{1+m}{n}, -\frac{bx^n}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[x^m/(a + b*x^n)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/n, 1 + (1 + m)/n, -((b*x^n)/a)])/(
a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(2,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^2(m+1)}$$

input
```
Int[x^m/(a + b*x^n)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(
a^2*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(a+b\,x^n)^2}dx$$

input
```
int(x^m/(a+b*x^n)^2,x)
```

output
```
int(x^m/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^m}{(a+bx^n)^2}\,dx = \int \frac{x^m}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral(x^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 1068, normalized size of antiderivative = 26.70

$$\int \frac{x^m}{(a+bx^n)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**m/(a+b*x**n)**2,x)
```

output
```
-a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*m**2*x**(m + 1)*lerchphi(b*x**n*exp_
polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n)
 + b*n**3*x**n*gamma(m/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n
)*m*n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/
n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n))
+ a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*m*n*x**(m + 1)*gamma(m/n + 1/n)/(a*
n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) - 2*a*a**(m/
n + 1/n)*a**(-m/n - 2 - 1/n)*m*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x*
*n*gamma(m/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*n*x**(m +
1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n*
*3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) + a*a**(m/n +
1/n)*a**(-m/n - 2 - 1/n)*n*x**(m + 1)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n +
 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) - a*a**(m/n + 1/n)*a**(-m/n
- 2 - 1/n)*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gam
ma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1
/n)) - a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*b*m**2*x**n*x**(m + 1)*lerchphi(
b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n
 + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) + a**(m/n + 1/n)*a**(-m/n
- 2 - 1/n)*b*m*n*x**n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, ...
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^n)^2}\,dx = \int \frac{x^m}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(m - n + 1)*integrate(x^m/(a*b*n*x^n + a^2*n), x) + x*x^m/(a*b*n*x^n + a^
2*n)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^n)^2}\,dx = \int \frac{x^m}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^n)^2}\,dx = \int \frac{x^m}{(a+b\,x^n)^2}\,dx$$

input
```
int(x^m/(a + b*x^n)^2,x)
```

output
```
int(x^m/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^n)^2}\,dx = \int \frac{x^m}{x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int(x^m/(a+b*x^n)^2,x)
```

output
```
int(x**m/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.578 $\int \frac{x^m}{(a+bx^n)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^m}{(a+bx^n)^3} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(3, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right)}{a^3(1+m)}$$

output
```
x^(1+m)*hypergeom([3, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x^m}{(a+bx^n)^3} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(3, \frac{1+m}{n}, 1+\frac{1+m}{n}, -\frac{bx^n}{a}\right)}{a^3(1+m)}$$

input
```
Integrate[x^m/(a + b*x^n)^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, (1 + m)/n, 1 + (1 + m)/n, -((b*x^n)/a)])/(
a^3*(1 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \; 888$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(3,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^3(m+1)}$$

input
```
Int[x^m/(a + b*x^n)^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(
a^3*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(a+b\,x^n)^3}dx$$

input
```
int(x^m/(a+b*x^n)^3,x)
```

output
```
int(x^m/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^m}{(a+bx^n)^3}\,dx = \int \frac{x^m}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral(x^m/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 4539, normalized size of antiderivative = 113.48

$$\int \frac{x^m}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**m/(a+b*x**n)**3,x)
```

output
```
a**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/n)*m**3*x**(m + 1)*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**2*n**4*gamma(m/n + 1
 + 1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamm
a(m/n + 1 + 1/n)) - 3*a**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/n)*m**2*n*x**(m
 + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2
*a**2*n**4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2
*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) - a**2*a**(m/n + 1/n)*a**(-m/n -
 3 - 1/n)*m**2*n*x**(m + 1)*gamma(m/n + 1/n)/(2*a**2*n**4*gamma(m/n + 1 +
1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(m
/n + 1 + 1/n)) + 3*a**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/n)*m**2*x**(m + 1)
*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**2
*n**4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*b**2
*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) + 2*a**2*a**(m/n + 1/n)*a**(-m/n - 3
- 1/n)*m*n**2*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*
gamma(m/n + 1/n)/(2*a**2*n**4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma
(m/n + 1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) + 3*a**2*a**(
m/n + 1/n)*a**(-m/n - 3 - 1/n)*m*n**2*x**(m + 1)*gamma(m/n + 1/n)/(2*a**2*
n**4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*b**2*
n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) - 6*a**2*a**(m/n + 1/n)*a**(-m/n - 3 -
 1/n)*m*n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*g...
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^n)^3}\,dx=\int \frac{x^m}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*integrate(1/2*x^m/(a^2*b*n^2*x^n + a
^3*n^2), x) - 1/2*(a*(m - 3*n + 1)*x*x^m + b*(m - 2*n + 1)*x*e^(m*log(x) +
 n*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^n)^3}\, dx = \int \frac{x^m}{(bx^n+a)^3}\, dx$$

input
```
integrate(x^m/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^n)^3}\, dx = \int \frac{x^m}{(a+b\,x^n)^3}\, dx$$

input
```
int(x^m/(a + b*x^n)^3,x)
```

output
```
int(x^m/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^n)^3}\, dx = \int \frac{x^m}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2 b+a^3}dx$$

input
```
int(x^m/(a+b*x^n)^3,x)
```

output
```
int(x**m/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3),x)
```

3.579 $\int \frac{x^m}{(a+bx^n)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(10,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^{10}(1+m)}$$

output
```
x^(1+m)*hypergeom([10, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^10/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(10,\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{a^{10}(1+m)}$$

input
```
Integrate[x^m/(a + b*x^n)^10,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[10, (1 + m)/n, 1 + (1 + m)/n, -((b*x^n)/a)])/
(a^10*(1 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(10,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^{10}(m+1)}$$

input
```
Int[x^m/(a + b*x^n)^10,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[10, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/
(a^10*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(a+b\,x^n)^{10}}dx$$

input
```
int(x^m/(a+b*x^n)^10,x)
```

output
```
int(x^m/(a+b*x^n)^10,x)
```

Fricas [F]

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx = \int \frac{x^m}{(bx^n+a)^{10}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^10,x, algorithm="fricas")
```

output
```
integral(x^m/(b^10*x^(10*n) + 10*a*b^9*x^(9*n) + 45*a^2*b^8*x^(8*n) + 120*
a^3*b^7*x^(7*n) + 210*a^4*b^6*x^(6*n) + 252*a^5*b^5*x^(5*n) + 210*a^6*b^4*
x^(4*n) + 120*a^7*b^3*x^(3*n) + 45*a^8*b^2*x^(2*n) + 10*a^9*b*x^n + a^10),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a+b*x**n)**10,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx = \int \frac{x^m}{(bx^n+a)^{10}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^10,x, algorithm="maxima")
```

output
```
-(m^9 - 9*m^8*(5*n - 1) - 362880*n^9 + 6*(145*n^2 - 60*n + 6)*m^7 + 102657
6*n^8 - 42*(225*n^3 - 145*n^2 + 30*n - 2)*m^6 - 1172700*n^7 + 21*(3013*n^4
 - 2700*n^3 + 870*n^2 - 120*n + 6)*m^5 + 723680*n^6 - 21*(12825*n^5 - 1506
5*n^4 + 6750*n^3 - 1450*n^2 + 150*n - 6)*m^4 - 269325*n^5 + 2*(361840*n^6
- 538650*n^5 + 316365*n^4 - 94500*n^3 + 15225*n^2 - 1260*n + 42)*m^3 + 632
73*n^4 - 6*(195450*n^7 - 361840*n^6 + 269325*n^5 - 105455*n^4 + 23625*n^3
- 3045*n^2 + 210*n - 6)*m^2 - 9450*n^3 + 3*(342192*n^8 - 781800*n^7 + 7236
80*n^6 - 359100*n^5 + 105455*n^4 - 18900*n^3 + 2030*n^2 - 120*n + 3)*m + 8
70*n^2 - 45*n + 1)*integrate(1/362880*x^m/(a^9*b*n^9*x^n + a^10*n^9), x) +
 1/362880*((m^8 - m^7*(45*n - 8) + 1026576*n^8 + (870*n^2 - 315*n + 28)*m^
6 - 1172700*n^7 - (9450*n^3 - 5220*n^2 + 945*n - 56)*m^5 + 723680*n^6 + (6
3273*n^4 - 47250*n^3 + 13050*n^2 - 1575*n + 70)*m^4 - 269325*n^5 - (269325
*n^5 - 253092*n^4 + 94500*n^3 - 17400*n^2 + 1575*n - 56)*m^3 + 63273*n^4 +
 (723680*n^6 - 807975*n^5 + 379638*n^4 - 94500*n^3 + 13050*n^2 - 945*n + 2
8)*m^2 - 9450*n^3 - (1172700*n^7 - 1447360*n^6 + 807975*n^5 - 253092*n^4 +
 47250*n^3 - 5220*n^2 + 315*n - 8)*m + 870*n^2 - 45*n + 1)*a^8*x*x^m + (m^
8 - 4*m^7*(11*n - 2) + 362880*n^8 + 14*(59*n^2 - 22*n + 2)*m^6 - 663696*n^
7 - 28*(308*n^3 - 177*n^2 + 33*n - 2)*m^5 + 509004*n^6 + 7*(7807*n^4 - 616
0*n^3 + 1770*n^2 - 220*n + 10)*m^4 - 214676*n^5 - 28*(7667*n^5 - 7807*n^4
+ 3080*n^3 - 590*n^2 + 55*n - 2)*m^3 + 54649*n^4 + 2*(254502*n^6 - 3220...
```

Giac [F]

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx=\int \frac{x^m}{(bx^n+a)^{10}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^10,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^n + a)^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx = \int \frac{x^m}{(a+b\,x^n)^{10}}\,dx$$

input
```
int(x^m/(a + b*x^n)^10,x)
```

output
```
int(x^m/(a + b*x^n)^10, x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^n)^{10}}\,dx$$
$$= \int \frac{x^m}{x^{10n}b^{10} + 10x^{9n}a\,b^9 + 45x^{8n}a^2b^8 + 120x^{7n}a^3b^7 + 210x^{6n}a^4b^6 + 252x^{5n}a^5b^5 + 210x^{4n}a^6b^4 + 120x^{3n}a^7b}$$

input
```
int(x^m/(a+b*x^n)^10,x)
```

output
```
int(x**m/(x**(10*n)*b**10 + 10*x**(9*n)*a*b**9 + 45*x**(8*n)*a**2*b**8 + 1
20*x**(7*n)*a**3*b**7 + 210*x**(6*n)*a**4*b**6 + 252*x**(5*n)*a**5*b**5 +
210*x**(4*n)*a**6*b**4 + 120*x**(3*n)*a**7*b**3 + 45*x**(2*n)*a**8*b**2 +
10*x**n*a**9*b + a**10),x)
```

3.580 $\int x^m(a+bx^{2+2m})^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int x^m\left(a+bx^{2+2m}\right)^3\,dx=\frac{a^3x^{1+m}}{1+m}+\frac{a^2bx^{3(1+m)}}{1+m}+\frac{3ab^2x^{5(1+m)}}{5(1+m)}+\frac{b^3x^{7(1+m)}}{7(1+m)}$$

output
```
a^3*x^(1+m)/(1+m)+a^2*b*x^(3+3*m)/(1+m)+3*a*b^2*x^(5+5*m)/(5+5*m)+b^3*x^(7
+7*m)/(7+7*m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int x^m\left(a+bx^{2+2m}\right)^3\,dx=\frac{35a^3x^{1+m}+35a^2bx^{3+3m}+21ab^2x^{5+5m}+5b^3x^{7+7m}}{35+35m}$$

input
```
Integrate[x^m*(a + b*x^(2 + 2*m))^3,x]
```

output
```
(35*a^3*x^(1 + m) + 35*a^2*b*x^(3 + 3*m) + 21*a*b^2*x^(5 + 5*m) + 5*b^3*x^
(7 + 7*m))/(35 + 35*m)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^{2m+2}\right)^3 \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(a^3x^m+3a^2bx^{3m+2}+3ab^2x^{5m+4}+b^3x^{7m+6}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3x^{m+1}}{m+1}+\frac{a^2bx^{3(m+1)}}{m+1}+\frac{3ab^2x^{5(m+1)}}{5(m+1)}+\frac{b^3x^{7(m+1)}}{7(m+1)}$$

input
```
Int[x^m*(a + b*x^(2 + 2*m))^3,x]
```

output
```
(a^3*x^(1 + m))/(1 + m) + (a^2*b*x^(3*(1 + m)))/(1 + m) + (3*a*b^2*x^(5*(1
 + m)))/(5*(1 + m)) + (b^3*x^(7*(1 + m)))/(7*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

method	result
risch	$\frac{b^{3} x^{7} x^{7 m}}{7+7 m}+\frac{3 a \,b^{2} x^{5} x^{5 m}}{5 \left(1+m \right)}+\frac{a^{2} b \,x^{3} x^{3 m}}{1+m}+\frac{a^{3} x \,x^{m}}{1+m}$
parallelrisch	$\frac{5 x \,x^{m} x^{6+6 m} b^{3}+21 x \,x^{m} x^{4+4 m} a \,b^{2}+35 x \,x^{m} x^{2+2 m} a^{2} b +35 x \,x^{m} a^{3}}{35+35 m}$
orering	$\frac{x \left(176 m^{3}+442 m^{2}+372 m +105\right) x^{m} \left(a +b \,x^{2+2 m}\right)^{3}}{105 m^{4}+420 m^{3}+630 m^{2}+420 m +105}-\frac{x^{2} \left(86 m^{2}+124 m +45\right) \left(\frac{x^{m} m \left(a +b \,x^{2+2 m}\right)^{3}}{x}+\frac{3 x^{m} \left(a +b \,x^{2+2 m}\right)^{2} b \,x^{2+2}}{x}\right.}{105 \left(m^{4}+4 m^{3}+6 m^{2}+4 m +1\right)}$

input
```
int(x^m*(a+b*x^(2+2*m))^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*x^7/(1+m)*(x^m)^7+3/5*a*b^2*x^5/(1+m)*(x^m)^5+a^2*b*x^3/(1+m)*(x^m
)^3+a^3/(1+m)*x*x^m
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int x^m \left(a+bx^{2+2m}\right)^3 \, dx = \frac{5\, b^3 x^7 x^{7\, m} + 21\, a b^2 x^5 x^{5\, m} + 35\, a^2 b x^3 x^{3\, m} + 35\, a^3 x x^m}{35\, (m+1)}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^3,x, algorithm="fricas")
```

output
```
1/35*(5*b^3*x^7*x^(7*m) + 21*a*b^2*x^5*x^(5*m) + 35*a^2*b*x^3*x^(3*m) + 35
*a^3*x*x^m)/(m + 1)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int x^m \left(a + bx^{2+2m}\right)^3 \, dx = \begin{cases} \frac{35a^3xx^m}{35m+35} + \frac{35a^2bxx^mx^{2m+2}}{35m+35} + \frac{21ab^2xx^mx^{4m+4}}{35m+35} + \frac{5b^3xx^mx^{6m+6}}{35m+35} & \text{for } m \neq -1 \\ (a+b)^3 \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(a+b*x**(2+2*m))**3,x)
```

output
```
Piecewise((35*a**3*x*x**m/(35*m + 35) + 35*a**2*b*x*x**m*x**(2*m + 2)/(35*
m + 35) + 21*a*b**2*x*x**m*x**(4*m + 4)/(35*m + 35) + 5*b**3*x*x**m*x**(6*
m + 6)/(35*m + 35), Ne(m, -1)), ((a + b)**3*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int x^m \left(a + bx^{2+2m}\right)^3 \, dx = \frac{b^3x^{7\,m+7}}{7\,(m+1)} + \frac{3\,ab^2x^{5\,m+5}}{5\,(m+1)} + \frac{a^2bx^{3\,m+3}}{m+1} + \frac{a^3x^{m+1}}{m+1}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^3,x, algorithm="maxima")
```

output
```
1/7*b^3*x^(7*m + 7)/(m + 1) + 3/5*a*b^2*x^(5*m + 5)/(m + 1) + a^2*b*x^(3*m
 + 3)/(m + 1) + a^3*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int x^m \left(a+bx^{2+2m}\right)^3 \, dx = \frac{5\,b^3x^7x^{7\,m}+21\,ab^2x^5x^{5\,m}+35\,a^2bx^3x^{3\,m}+35\,a^3xx^m}{35\,(m+1)}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^3,x, algorithm="giac")
```

output
```
1/35*(5*b^3*x^7*x^(7*m) + 21*a*b^2*x^5*x^(5*m) + 35*a^2*b*x^3*x^(3*m) + 35
*a^3*x*x^m)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int x^m \left(a+bx^{2+2m}\right)^3 \, dx = \frac{a^3\,x\,x^m}{m+1}+\frac{b^3\,x^{7\,m}\,x^7}{7\,m+7}+\frac{a^2\,b\,x^{3\,m}\,x^3}{m+1}+\frac{3\,a\,b^2\,x^{5\,m}\,x^5}{5\,m+5}$$

input
```
int(x^m*(a + b*x^(2*m + 2))^3,x)
```

output
```
(a^3*x*x^m)/(m + 1) + (b^3*x^(7*m)*x^7)/(7*m + 7) + (a^2*b*x^(3*m)*x^3)/(m
 + 1) + (3*a*b^2*x^(5*m)*x^5)/(5*m + 5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int x^m \left(a+bx^{2+2m}\right)^3 \, dx = \frac{x^m x(5x^{6m}b^3x^6+21x^{4m}a\,b^2x^4+35x^{2m}a^2b\,x^2+35a^3)}{35m+35}$$

input
```
int(x^m*(a+b*x^(2+2*m))^3,x)
```

output
```
(x**m*x*(5*x**(6*m)*b**3*x**6 + 21*x**(4*m)*a*b**2*x**4 + 35*x**(2*m)*a**2
*b*x**2 + 35*a**3))/(35*(m + 1))
```

3.581 $\int x^m (a+bx^{2+2m})^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int x^m \left(a+bx^{2+2m}\right)^2 \, dx = \frac{a^2x^{1+m}}{1+m} + \frac{2abx^{3(1+m)}}{3(1+m)} + \frac{b^2x^{5(1+m)}}{5(1+m)}$$

output
```
a^2*x^(1+m)/(1+m)+2*a*b*x^(3+3*m)/(3+3*m)+b^2*x^(5+5*m)/(5+5*m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int x^m \left(a+bx^{2+2m}\right)^2 \, dx = \frac{15a^2x^{1+m} + 10abx^{3+3m} + 3b^2x^{5+5m}}{15+15m}$$

input
```
Integrate[x^m*(a + b*x^(2 + 2*m))^2,x]
```

output
```
(15*a^2*x^(1 + m) + 10*a*b*x^(3 + 3*m) + 3*b^2*x^(5 + 5*m))/(15 + 15*m)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^{2m+2}\right)^2 \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(a^2x^m+2abx^{3m+2}+b^2x^{5m+4}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{3(m+1)}}{3(m+1)}+\frac{b^2x^{5(m+1)}}{5(m+1)}$$

input
```
Int[x^m*(a + b*x^(2 + 2*m))^2,x]
```

output
```
(a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3*(1 + m)))/(3*(1 + m)) + (b^2*x^(5*(1
 + m)))/(5*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result
risch	$\frac{b^2 x^5 x^{5m}}{5+5m}+\frac{2ab\,x^3 x^{3m}}{3(1+m)}+\frac{a^2 x\,x^m}{1+m}$
parallelrisch	$\frac{3x\,x^m x^{4+4m} b^2+10x\,x^m x^{2+2m} ab+15x\,x^m a^2}{15+15m}$
orering	$\frac{x(23m^2+37m+15)x^m\left(a+b\,x^{2+2m}\right)^2}{15(m^2+2m+1)(1+m)}-\frac{x^2(3m+2)\left(\frac{x^m m\left(a+b\,x^{2+2m}\right)^2}{x}+\frac{2x^m\left(a+b\,x^{2+2m}\right)b\,x^{2+2m}(2+2m)}{x}\right)}{5(m^2+2m+1)(1+m)}+\frac{x^3\left(\frac{x^m}{\cdots}\right.}{\cdots}$

input
```
int(x^m*(a+b*x^(2+2*m))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*x^5/(1+m)*(x^m)^5+2/3*a*b*x^3/(1+m)*(x^m)^3+a^2/(1+m)*x*x^m
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int x^m\left(a+bx^{2+2m}\right)^2\,dx=\frac{3\,b^2x^5x^{5\,m}+10\,abx^3x^{3\,m}+15\,a^2xx^m}{15\,(m+1)}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^2,x, algorithm="fricas")
```

output
```
1/15*(3*b^2*x^5*x^(5*m) + 10*a*b*x^3*x^(3*m) + 15*a^2*x*x^m)/(m + 1)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31

$$\int x^m\left(a+bx^{2+2m}\right)^2\,dx=\begin{cases}\frac{15a^2xx^m}{15m+15}+\frac{10abxx^mx^{2m+2}}{15m+15}+\frac{3b^2xx^mx^{4m+4}}{15m+15} & \text{for } m\neq -1\\ (a+b)^2\log(x) & \text{otherwise}\end{cases}$$

input
```
integrate(x**m*(a+b*x**(2+2*m))**2,x)
```

output
```
Piecewise((15*a**2*x*x**m/(15*m + 15) + 10*a*b*x*x**m*x**(2*m + 2)/(15*m +
 15) + 3*b**2*x*x**m*x**(4*m + 4)/(15*m + 15), Ne(m, -1)), ((a + b)**2*log
(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int x^m \left(a+bx^{2+2m}\right)^2 \, dx = \frac{b^2x^{5\,m+5}}{5\,(m+1)} + \frac{2\,abx^{3\,m+3}}{3\,(m+1)} + \frac{a^2x^{m+1}}{m+1}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^(5*m + 5)/(m + 1) + 2/3*a*b*x^(3*m + 3)/(m + 1) + a^2*x^(m + 1)/
(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int x^m \left(a+bx^{2+2m}\right)^2 \, dx = \frac{3\,b^2x^5x^{5\,m} + 10\,abx^3x^{3\,m} + 15\,a^2xx^m}{15\,(m+1)}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^2,x, algorithm="giac")
```

output
```
1/15*(3*b^2*x^5*x^(5*m) + 10*a*b*x^3*x^(3*m) + 15*a^2*x*x^m)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

$$\int x^m \left(a+bx^{2+2m}\right)^2 \, dx = \frac{x^{m+1} \left(\frac{b^2 x^{4m+4}}{5} + a^2 + \frac{2ab x^{2m+2}}{3}\right)}{m+1}$$

input
```
int(x^m*(a + b*x^(2*m + 2))^2,x)
```

output
```
(x^(m + 1)*((b^2*x^(4*m + 4))/5 + a^2 + (2*a*b*x^(2*m + 2))/3))/(m + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int x^m \left(a+bx^{2+2m}\right)^2 \, dx = \frac{x^m x(3x^{4m} b^2 x^4 + 10x^{2m} ab\, x^2 + 15a^2)}{15m+15}$$

input
```
int(x^m*(a+b*x^(2+2*m))^2,x)
```

output
```
(x**m*x*(3*x**(4*m)*b**2*x**4 + 10*x**(2*m)*a*b*x**2 + 15*a**2))/(15*(m +
1))
```

3.582 $\int x^m(a+bx^{2+2m})\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x^m\left(a+bx^{2+2m}\right)\,dx=\frac{ax^{1+m}}{1+m}+\frac{bx^{3(1+m)}}{3(1+m)}$$

output
```
a*x^(1+m)/(1+m)+b*x^(3+3*m)/(3+3*m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^m\left(a+bx^{2+2m}\right)\,dx=\frac{3ax^{1+m}+bx^{3+3m}}{3+3m}$$

input
```
Integrate[x^m*(a + b*x^(2 + 2*m)),x]
```

output
```
(3*a*x^(1 + m) + b*x^(3 + 3*m))/(3 + 3*m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^{2m+2}\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^m+bx^{3m+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{3(m+1)}}{3(m+1)}$$

input
```
Int[x^m*(a + b*x^(2 + 2*m)),x]
```

output
```
(a*x^(1 + m))/(1 + m) + (b*x^(3*(1 + m)))/(3*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{a x \,x^{m}}{1+m}+\frac{b \,x^{3} x^{3 m}}{3+3 m}$	29
parallelrisch	$\frac{x \,x^{m} x^{2+2 m} b +3 x \,x^{m} a}{3+3 m}$	29
norman	$\frac{a x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b \,x^{3} {\mathrm e}^{3 m \ln \left(x \right)}}{3+3 m}$	33
orering	$\frac{x \left(4 m +3\right) x^{m} \left(a +b \,x^{2+2 m}\right)}{3 m^{2}+6 m +3}-\frac{x^{2} \left(\frac{x^{m} m \left(a +b \,x^{2+2 m}\right)}{x}+\frac{x^{m} b \,x^{2+2 m} \left(2+2 m \right)}{x}\right)}{3 \left(m^{2}+2 m +1\right)}$	89

input
```
int(x^m*(a+b*x^(2+2*m)),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*x^m+1/3*b/(1+m)*x^3*(x^m)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int x^m \left(a+bx^{2+2m}\right)\,dx=\frac{bx^3x^{3\,m}+3\,axx^m}{3\,(m+1)}$$

input
```
integrate(x^m*(a+b*x^(2+2*m)),x, algorithm="fricas")
```

output
```
1/3*(b*x^3*x^(3*m) + 3*a*x*x^m)/(m + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int x^m \left(a + bx^{2+2m}\right) \, dx = \begin{cases} \frac{3axx^m}{3m+3} + \frac{bxx^m x^{2m+2}}{3m+3} & \text{for } m \neq -1 \\ (a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(a+b*x**(2+2*m)),x)
```

output
```
Piecewise((3*a*x*x**m/(3*m + 3) + b*x*x**m*x**(2*m + 2)/(3*m + 3), Ne(m, -
1)), ((a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int x^m \left(a + bx^{2+2m}\right) \, dx = \frac{bx^{3m+3}}{3(m+1)} + \frac{ax^{m+1}}{m+1}$$

input
```
integrate(x^m*(a+b*x^(2+2*m)),x, algorithm="maxima")
```

output
```
1/3*b*x^(3*m + 3)/(m + 1) + a*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int x^m \left(a + bx^{2+2m}\right) \, dx = \frac{bx^3 x^{3m} + 3\,axx^m}{3(m+1)}$$

input
```
integrate(x^m*(a+b*x^(2+2*m)),x, algorithm="giac")
```

output
```
1/3*(b*x^3*x^(3*m) + 3*a*x*x^m)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int x^m \left(a+bx^{2+2m}\right) \, dx = \frac{x^{m+1} \left(a+\frac{b\,x^{2\,m+2}}{3}\right)}{m+1}$$

input
```
int(x^m*(a + b*x^(2*m + 2)),x)
```

output
```
(x^(m + 1)*(a + (b*x^(2*m + 2))/3))/(m + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^m \left(a+bx^{2+2m}\right) \, dx = \frac{x^m x(x^{2m} b\,x^2+3a)}{3m+3}$$

input
```
int(x^m*(a+b*x^(2+2*m)),x)
```

output
```
(x**m*x*(x**(2*m)*b*x**2 + 3*a))/(3*(m + 1))
```

3.583 $\int \frac{x^m}{a+bx^{2+2m}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(1+m)}$$

output
```
arctan(b^(1/2)*x^(1+m)/a^(1/2))/a^(1/2)/b^(1/2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(1+m)}$$

input
```
Integrate[x^m/(a + b*x^(2 + 2*m)),x]
```

output
```
ArcTan[(Sqrt[b]*x^(1 + m))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*(1 + m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {868, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+bx^{2m+2}}\,dx$$

$$\Big\downarrow \text{868}$$

$$\frac{\int \frac{1}{bx^{2m+2}+a}dx^{m+1}}{m+1}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x^{m+1}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(m+1)}$$

input
```
Int[x^m/(a + b*x^(2 + 2*m)),x]
```

output
```
ArcTan[(Sqrt[b]*x^(1 + m))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*(1 + m))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(25) = 50.

Time = 0.66 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

method	result	size
risch	$-\frac{\ln\left(x^{m}-\frac{a}{x\sqrt{-ab}}\right)}{2\sqrt{-ab}\,(1+m)}+\frac{\ln\left(x^{m}+\frac{a}{x\sqrt{-ab}}\right)}{2\sqrt{-ab}\,(1+m)}$	61

input
```
int(x^m/(a+b*x^(2+2*m)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-a*b)^(1/2)/(1+m)*ln(x^m-a/x/(-a*b)^(1/2))+1/2/(-a*b)^(1/2)/(1+m)*ln
(x^m+a/x/(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.76

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx=\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2x^{2\,m}-2\,\sqrt{-ab}xx^m-a}{bx^2x^{2\,m}+a}\right)}{2\,(abm+ab)},\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}xx^m}{a}\right)}{abm+ab}\right]$$

input
```
integrate(x^m/(a+b*x^(2+2*m)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2*x^(2*m) - 2*sqrt(-a*b)*x*x^m - a)/(b*x^2*x^(2*
m) + a))/(a*b*m + a*b), sqrt(a*b)*arctan(sqrt(a*b)*x*x^m/a)/(a*b*m + a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(29) = 58$.

Time = 6.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.76

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge m=-1 \\ \frac{xx^m}{a(m+1)} & \text{for } b=0 \\ -\frac{xx^mx^{-2m-2}}{b(m+1)} & \text{for } a=0 \\ \frac{\log(x)}{a+b} & \text{for } m=-1 \\ \frac{\log\left(x^m-\frac{\sqrt{-\frac{a}{b}}}{x}\right)}{2bm\sqrt{-\frac{a}{b}}+2b\sqrt{-\frac{a}{b}}} - \frac{\log\left(x^m+\frac{\sqrt{-\frac{a}{b}}}{x}\right)}{2bm\sqrt{-\frac{a}{b}}+2b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m/(a+b*x**(2+2*m)),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(m, -1)), (x*x**m/(a*(m + 1
)), Eq(b, 0)), (-x*x**m*x**(-2*m - 2)/(b*(m + 1)), Eq(a, 0)), (log(x)/(a +
 b), Eq(m, -1)), (log(x**m - sqrt(-a/b)/x)/(2*b*m*sqrt(-a/b) + 2*b*sqrt(-a
/b)) - log(x**m + sqrt(-a/b)/x)/(2*b*m*sqrt(-a/b) + 2*b*sqrt(-a/b)), True)
)
```

Maxima [F]

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx = \int \frac{x^m}{bx^{2m+2}+a}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m)),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a), x)
```

Giac [F]

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx = \int \frac{x^m}{bx^{2\,m+2}+a}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m)),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx = \int \frac{x^m}{a+b\,x^{2\,m+2}}\,dx$$

input
```
int(x^m/(a + b*x^(2*m + 2)),x)
```

output
```
int(x^m/(a + b*x^(2*m + 2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^m}{a+bx^{2+2m}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{x^m bx}{\sqrt{b}\,\sqrt{a}}\right)}{ab\,(m+1)}$$

input
```
int(x^m/(a+b*x^(2+2*m)),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((x**m*b*x)/(sqrt(b)*sqrt(a))))/(a*b*(m + 1))
```

3.584 $\int \frac{x^m}{\left(a+bx^{2+2m}\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{x^m}{\left(a+bx^{2+2m}\right)^2}\,dx=\frac{x^{1+m}}{2a(1+m)\left(a+bx^{2(1+m)}\right)}+\frac{\arctan\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}(1+m)}$$

output
```
1/2*x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))+1/2*arctan(b^(1/2)*x^(1+m)/a^(1/2))/a^
(3/2)/b^(1/2)/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{x^m}{\left(a+bx^{2+2m}\right)^2}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2+2m},1+\frac{1+m}{2+2m},-\frac{bx^{2+2m}}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[x^m/(a + b*x^(2 + 2*m))^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/(2 + 2*m), 1 + (1 + m)/(2 + 2*m),
-((b*x^(2 + 2*m))/a)])/(a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {868, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^{2m+2})^2}dx$$

$$\Big\downarrow \text{868}$$

$$\frac{\int \frac{1}{(bx^{2m+2}+a)^2}dx^{m+1}}{m+1}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{bx^{2m+2}+a}dx^{m+1}}{2a}+\frac{x^{m+1}}{2a(a+bx^{2m+2})}}{m+1}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x^{m+1}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x^{m+1}}{2a(a+bx^{2m+2})}}{m+1}$$

input
```
Int[x^m/(a + b*x^(2 + 2*m))^2,x]
```

output
```
(x^(1 + m)/(2*a*(a + b*x^(2 + 2*m))) + ArcTan[(Sqrt[b]*x^(1 + m))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]))/(1 + m)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{x\,x^{m}}{2(1+m)a(a+b\,x^{2}x^{2m})}-\frac{\ln\left(x^{m}-\frac{a}{x\sqrt{-ab}}\right)}{4\sqrt{-ab}\,(1+m)a}+\frac{\ln\left(x^{m}+\frac{a}{x\sqrt{-ab}}\right)}{4\sqrt{-ab}\,(1+m)a}$	95

input
```
int(x^m/(a+b*x^(2+2*m))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(1+m)/a*x^m/(a+b*x^2*(x^m)^2)-1/4/(-a*b)^(1/2)/(1+m)/a*ln(x^m-a/x/(-
a*b)^(1/2))+1/4/(-a*b)^(1/2)/(1+m)/a*ln(x^m+a/x/(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.09

$$\int \frac{x^m}{(a+bx^{2+2m})^2}\,dx$$
$$= \left[\frac{2\,abxx^m - \left(\sqrt{-ab}bx^2x^{2\,m} + \sqrt{-ab}a\right)\log\left(\frac{bx^2x^{2\,m}-2\,\sqrt{-ab}xx^m-a}{bx^2x^{2\,m}+a}\right)}{4\left(a^3bm + a^3b + (a^2b^2m + a^2b^2)x^2x^{2\,m}\right)}, \frac{abxx^m + \left(\sqrt{ab}bx^2x^{2\,m} + \sqrt{ab}a\right)\text{arcta}}{2\left(a^3bm + a^3b + (a^2b^2m + a^2b^2)x\right.}\right.$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x*x^m - (sqrt(-a*b)*b*x^2*x^(2*m) + sqrt(-a*b)*a)*log((b*x^2*x
^(2*m) - 2*sqrt(-a*b)*x*x^m - a)/(b*x^2*x^(2*m) + a)))/(a^3*b*m + a^3*b +
(a^2*b^2*m + a^2*b^2)*x^2*x^(2*m)), 1/2*(a*b*x*x^m + (sqrt(a*b)*b*x^2*x^(2
*m) + sqrt(a*b)*a)*arctan(sqrt(a*b)*x*x^m/a))/(a^3*b*m + a^3*b + (a^2*b^2*
m + a^2*b^2)*x^2*x^(2*m))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^{2+2m})^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a+b*x**(2+2*m))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^2}\,dx = \int \frac{x^m}{(bx^{2m+2}+a)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^2,x, algorithm="maxima")
```

output
```
1/2*x*x^m/(a*b*(m + 1)*x^2*x^(2*m) + a^2*(m + 1)) + integrate(1/2*x^m/(a*b
*x^2*x^(2*m) + a^2), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^2}\,dx = \int \frac{x^m}{(bx^{2m+2}+a)^2}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^2,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^{2+2m})^2}\,dx = \int \frac{x^m}{(a+b\,x^{2\,m+2})^2}\,dx$$

input
```
int(x^m/(a + b*x^(2*m + 2))^2,x)
```

output
```
int(x^m/(a + b*x^(2*m + 2))^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{x^m}{(a+bx^{2+2m})^2}\,dx = \frac{x^{2m}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{x^m bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{x^m bx}{\sqrt{b}\,\sqrt{a}}\right)a+x^m abx}{2a^2b\,(x^{2m}bm\,x^2+x^{2m}b\,x^2+am+a)}$$

input
```
int(x^m/(a+b*x^(2+2*m))^2,x)
```

output
```
(x**(2*m)*sqrt(b)*sqrt(a)*atan((x**m*b*x)/(sqrt(b)*sqrt(a)))*b*x**2 + sqrt
(b)*sqrt(a)*atan((x**m*b*x)/(sqrt(b)*sqrt(a)))*a + x**m*a*b*x)/(2*a**2*b*(
x**(2*m)*b*m*x**2 + x**(2*m)*b*x**2 + a*m + a))
```

3.585 $\int \frac{x^m}{(a+bx^{2+2m})^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{x^m}{(a+bx^{2+2m})^3}\,dx = \frac{x^{1+m}}{4a(1+m)\left(a+bx^{2(1+m)}\right)^2} + \frac{3x^{1+m}}{8a^2(1+m)\left(a+bx^{2(1+m)}\right)} + \frac{3\arctan\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}(1+m)}$$

output
```
1/4*x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^2+3/8*x^(1+m)/a^2/(1+m)/(a+b*x^(2+2*m)
)+3/8*arctan(b^(1/2)*x^(1+m)/a^(1/2))/a^(5/2)/b^(1/2)/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int \frac{x^m}{(a+bx^{2+2m})^3}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},-\frac{bx^{2+2m}}{a}\right)}{a^3(1+m)}$$

input
```
Integrate[x^m/(a + b*x^(2 + 2*m))^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1/2, 3, 3/2, -((b*x^(2 + 2*m))/a)])/(a^3*(1 +
m))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {868, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^{2m+2})^3}\,dx$$

$$\Big\downarrow \; 868$$

$$\frac{\int \frac{1}{(bx^{2m+2}+a)^3}dx^{m+1}}{m+1}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{3\int \frac{1}{\left(bx^{2m+2}+a\right)^2}dx^{m+1}}{4a}+\frac{x^{m+1}}{4a(a+bx^{2m+2})^2}}{m+1}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{bx^{2m+2}+a}dx^{m+1}}{2a}+\frac{x^{m+1}}{2a\left(a+bx^{2m+2}\right)}\right)}{4a}+\frac{x^{m+1}}{4a(a+bx^{2m+2})^2}}{m+1}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x^{m+1}}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x^{m+1}}{2a\left(a+bx^{2m+2}\right)}\right)}{4a}+\frac{x^{m+1}}{4a(a+bx^{2m+2})^2}}{m+1}$$

input
```
Int[x^m/(a + b*x^(2 + 2*m))^3,x]
```

output
```
(x^(1 + m)/(4*a*(a + b*x^(2 + 2*m))^2) + (3*(x^(1 + m)/(2*a*(a + b*x^(2 +
2*m))) + ArcTan[(Sqrt[b]*x^(1 + m))/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/
(1 + m)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{x\,x^{m}\left(3 b\,x^{2} x^{2 m}+5 a\right)}{8(1+m)a^{2}\left(a+b\,x^{2} x^{2 m}\right)^{2}}-\frac{3\ln\left(x^{m}-\frac{a}{x\sqrt{-a b}}\right)}{16\sqrt{-a b}\,(1+m)a^{2}}+\frac{3\ln\left(x^{m}+\frac{a}{x\sqrt{-a b}}\right)}{16\sqrt{-a b}\,(1+m)a^{2}}$	110

input
```
int(x^m/(a+b*x^(2+2*m))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*x^m*(3*b*x^2*(x^m)^2+5*a)/(1+m)/a^2/(a+b*x^2*(x^m)^2)^2-3/16/(-a*b)^
(1/2)/(1+m)/a^2*ln(x^m-a/x/(-a*b)^(1/2))+3/16/(-a*b)^(1/2)/(1+m)/a^2*ln(x^
m+a/x/(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.46

$$\int \frac{x^m}{(a+bx^{2+2m})^3} dx = \left[\frac{6\,ab^2x^3x^{3\,m} + 10\,a^2bxx^m - 3\left(\sqrt{-ab}b^2x^4x^{4\,m} + 2\,\sqrt{-ab}abx^2x^{2\,m} + \sqrt{-ab}a^2\right)\log\left(\frac{bx^2x^{2\,m} - 2\,\sqrt{-ab}xx^m - a}{bx^2x^{2\,m} + a}\right)}{16\left(a^5bm + a^5b + (a^3b^3m + a^3b^3)x^4x^{4\,m} + 2\,(a^4b^2m + a^4b^2)x^2x^{2\,m}\right)}\right.$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*b^2*x^3*x^(3*m) + 10*a^2*b*x*x^m - 3*(sqrt(-a*b)*b^2*x^4*x^(4*m
) + 2*sqrt(-a*b)*a*b*x^2*x^(2*m) + sqrt(-a*b)*a^2)*log((b*x^2*x^(2*m) - 2*
sqrt(-a*b)*x*x^m - a)/(b*x^2*x^(2*m) + a)))/(a^5*b*m + a^5*b + (a^3*b^3*m
+ a^3*b^3)*x^4*x^(4*m) + 2*(a^4*b^2*m + a^4*b^2)*x^2*x^(2*m)), 1/8*(3*a*b^
2*x^3*x^(3*m) + 5*a^2*b*x*x^m + 3*(sqrt(a*b)*b^2*x^4*x^(4*m) + 2*sqrt(a*b)
*a*b*x^2*x^(2*m) + sqrt(a*b)*a^2)*arctan(sqrt(a*b)*x*x^m/a))/(a^5*b*m + a^
5*b + (a^3*b^3*m + a^3*b^3)*x^4*x^(4*m) + 2*(a^4*b^2*m + a^4*b^2)*x^2*x^(2
*m))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^{2+2m})^3} dx = \text{Timed out}$$

input
```
integrate(x**m/(a+b*x**(2+2*m))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^3}\,dx = \int \frac{x^m}{(bx^{2m+2}+a)^3}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^3,x, algorithm="maxima")
```

output
```
1/8*(3*b*x^3*x^(3*m) + 5*a*x*x^m)/(a^2*b^2*(m + 1)*x^4*x^(4*m) + 2*a^3*b*(
m + 1)*x^2*x^(2*m) + a^4*(m + 1)) + 3*integrate(1/8*x^m/(a^2*b*x^2*x^(2*m)
 + a^3), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^3}\,dx = \int \frac{x^m}{(bx^{2m+2}+a)^3}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^3,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^{2+2m})^3}\,dx = \int \frac{x^m}{(a+b\,x^{2\,m+2})^3}\,dx$$

input
```
int(x^m/(a + b*x^(2*m + 2))^3,x)
```

output
```
int(x^m/(a + b*x^(2*m + 2))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.87

$$\int \frac{x^m}{(a+bx^{2+2m})^3}\,dx$$
$$= \frac{3x^{4m}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{x^m bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4+6x^{2m}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{x^m bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{x^m bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+3x^{3m}a\,b^2x^3}{8a^3b\,(x^{4m}b^2m\,x^4+x^{4m}b^2x^4+2x^{2m}abm\,x^2+2x^{2m}ab\,x^2+a^2m+a^2)}$$

input
```
int(x^m/(a+b*x^(2+2*m))^3,x)
```

output
```
(3*x**(4*m)*sqrt(b)*sqrt(a)*atan((x**m*b*x)/(sqrt(b)*sqrt(a)))*b**2*x**4 +
 6*x**(2*m)*sqrt(b)*sqrt(a)*atan((x**m*b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 +
3*sqrt(b)*sqrt(a)*atan((x**m*b*x)/(sqrt(b)*sqrt(a)))*a**2 + 3*x**(3*m)*a*b
**2*x**3 + 5*x**m*a**2*b*x)/(8*a**3*b*(x**(4*m)*b**2*m*x**4 + x**(4*m)*b**
2*x**4 + 2*x**(2*m)*a*b*m*x**2 + 2*x**(2*m)*a*b*x**2 + a**2*m + a**2))
```

3.586 $\int x^{-1+n}(a+bx^n)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx=\frac{2(a+bx^n)^{5/2}}{5bn}$$

output
```
2/5*(a+b*x^n)^(5/2)/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx=\frac{2(a+bx^n)^{5/2}}{5bn}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^(3/2),x]
```

output
```
(2*(a + b*x^n)^(5/2))/(5*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^{3/2}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{2(a+bx^n)^{5/2}}{5bn}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^(3/2),x]
```

output
```
(2*(a + b*x^n)^(5/2))/(5*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.58 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

method	result	size
risch	$\frac{2\left(a^{2}+2 a b\,x^{n}+b^{2} x^{2 n}\right)\sqrt{a+b\,x^{n}}}{5 b n}$	38

input
```
int(x^(-1+n)*(a+b*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a^2+2*a*b*x^n+b^2*(x^n)^2)*(a+b*x^n)^(1/2)/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx = \frac{2\,(b^2x^{2\,n}+2\,abx^n+a^2)\sqrt{bx^n+a}}{5\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
2/5*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*sqrt(b*x^n + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(15) = 30$.

Time = 11.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.38

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx = \begin{cases} a^{\frac{3}{2}}\log\left(x\right) & \text{for}\ b=0\wedge n=0 \\ \frac{a^{\frac{3}{2}}xx^{n-1}}{n} & \text{for}\ b=0 \\ (a+b)^{\frac{3}{2}}\log\left(x\right) & \text{for}\ n=0 \\ \frac{2a^2\sqrt{a+bx^n}}{5bn}+\frac{4ax^n\sqrt{a+bx^n}}{5n}+\frac{2bx^{2n}\sqrt{a+bx^n}}{5n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**(3/2),x)
```

output
```
Piecewise((a**(3/2)*log(x), Eq(b, 0) & Eq(n, 0)), (a**(3/2)*x*x**(n - 1)/n
, Eq(b, 0)), ((a + b)**(3/2)*log(x), Eq(n, 0)), (2*a**2*sqrt(a + b*x**n)/(
5*b*n) + 4*a*x**n*sqrt(a + b*x**n)/(5*n) + 2*b*x**(2*n)*sqrt(a + b*x**n)/(
5*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx=\frac{2\,(bx^n+a)^{\frac{5}{2}}}{5\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b*x^n + a)^(5/2)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.38

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx=\frac{2\left(3\,(bx^n+a)^{\frac{5}{2}}-10\,(bx^n+a)^{\frac{3}{2}}a+30\,\sqrt{bx^n+a}a^2+10\left((bx^n+a)^{\frac{3}{2}}-3\,\sqrt{bx^n+a}a\right)a\right)}{15\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
2/15*(3*(b*x^n + a)^(5/2) - 10*(b*x^n + a)^(3/2)*a + 30*sqrt(b*x^n + a)*a^
2 + 10*((b*x^n + a)^(3/2) - 3*sqrt(b*x^n + a)*a)*a)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx=\frac{2\,(a+b\,x^n)^{5/2}}{5\,b\,n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^(3/2),x)
```

output
```
(2*(a + b*x^n)^(5/2))/(5*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int x^{-1+n}(a+bx^n)^{3/2}\,dx=\frac{2\sqrt{x^nb+a}\,(x^{2n}b^2+2x^nab+a^2)}{5bn}$$

input
```
int(x^(-1+n)*(a+b*x^n)^(3/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))/(5*b*n)
```

3.587 $\int x^{-1+n}\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2(a+bx^n)^{3/2}}{3bn}$$

output
```
2/3*(a+b*x^n)^(3/2)/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2(a+bx^n)^{3/2}}{3bn}$$

input
```
Integrate[x^(-1 + n)*Sqrt[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)^(3/2))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}\sqrt{a+bx^n}\,dx$$

$$\Big\downarrow \text{793}$$

$$\frac{2(a+bx^n)^{3/2}}{3bn}$$

input
```
Int[x^(-1 + n)*Sqrt[a + b*x^n],x]
```

output
```
(2*(a + b*x^n)^(3/2))/(3*b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{2(a+b\,x^{n})^{\frac{3}{2}}}{3bn}$	18

input
```
int(x^(-1+n)*(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a+b*x^n)^(3/2)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(15) = 30.

Time = 1.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx=\frac{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3bn}+\frac{2\sqrt{a}x^n\sqrt{1+\frac{bx^n}{a}}}{3n}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**(1/2),x)
```

output
```
2*a**(3/2)*sqrt(1 + b*x**n/a)/(3*b*n) + 2*sqrt(a)*x**n*sqrt(1 + b*x**n/a)/
(3*n)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
2/3*(b*x^n + a)^(3/2)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\,{\left(a+b\,x^n\right)}^{3/2}}{3\,b\,n}$$

input
```
int(x^(n - 1)*(a + b*x^n)^(1/2),x)
```

output
```
(2*(a + b*x^n)^(3/2))/(3*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^{-1+n}\sqrt{a+bx^n}\,dx = \frac{2\sqrt{x^n b+a}\,(x^n b+a)}{3bn}$$

input
```
int(x^(-1+n)*(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(x**n*b + a))/(3*b*n)
```

3.588 $\int \frac{x^{-1+n}}{\sqrt{a+bx^n}} dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}} dx = \frac{2\sqrt{a+bx^n}}{bn}$$

output
```
2*(a+b*x^n)^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}} dx = \frac{2\sqrt{a+bx^n}}{bn}$$

input
```
Integrate[x^(-1 + n)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a + b*x^n])/(b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{2\sqrt{a+bx^n}}{bn}$$

input
```
Int[x^(-1 + n)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a + b*x^n])/(b*n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{2\sqrt{a+b\,x^{n}}}{bn}$	18

input
```
int(x^(-1+n)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a+b*x^n)^(1/2)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{bx^n+a}}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^n + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.

Time = 1.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \begin{cases} \frac{\log(x)}{\sqrt{a}} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{n-1}}{\sqrt{a}n} & \text{for } b=0 \\ \frac{\log(x)}{\sqrt{a+b}} & \text{for } n=0 \\ \frac{2\sqrt{a+bx^n}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**(1/2),x)
```

output
```
Piecewise((log(x)/sqrt(a), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(sqrt(a)*n)
, Eq(b, 0)), (log(x)/sqrt(a + b), Eq(n, 0)), (2*sqrt(a + b*x**n)/(b*n), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{bx^n+a}}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x^n + a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{bx^n+a}}{bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b*x^n + a)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{a+b\,x^n}}{b\,n}$$

input
```
int(x^(n - 1)/(a + b*x^n)^(1/2),x)
```

output
```
(2*(a + b*x^n)^(1/2))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^nb+a}}{bn}$$

input
```
int(x^(-1+n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a))/(b*n)
```

3.589 $\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = -\frac{2}{bn\sqrt{a+bx^n}}$$

output
```
-2/b/n/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = -\frac{2}{bn\sqrt{a+bx^n}}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n)^(3/2),x]
```

output
```
-2/(b*n*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a+bx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{2}{bn\sqrt{a+bx^n}}$$

input
```
Int[x^(-1 + n)/(a + b*x^n)^(3/2),x]
```

output
```
-2/(b*n*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [F]

$$\int \frac{x^{-1+n}}{(a+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int(x^(-1+n)/(a+b*x^n)^(3/2),x)
```

output
```
int(x^(-1+n)/(a+b*x^n)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = -\frac{2\sqrt{bx^n+a}}{b^2nx^n+abn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^n + a)/(b^2*n*x^n + a*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(15) = 30.

Time = 2.95 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = \begin{cases} \frac{\log(x)}{a^{\frac{3}{2}}} & \text{for}\ b=0\wedge n=0 \\ \frac{xx^{n-1}}{a^{\frac{3}{2}}n} & \text{for}\ b=0 \\ \frac{\log(x)}{(a+b)^{\frac{3}{2}}} & \text{for}\ n=0 \\ -\frac{2}{bn\sqrt{a+bx^n}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**(3/2),x)
```

output
```
Piecewise((log(x)/a**(3/2), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(a**(3/2)*
n), Eq(b, 0)), (log(x)/(a + b)**(3/2), Eq(n, 0)), (-2/(b*n*sqrt(a + b*x**n
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = -\frac{2}{\sqrt{bx^n+a}bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(b*x^n + a)*b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = -\frac{2}{\sqrt{bx^n+a}bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
-2/(sqrt(b*x^n + a)*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = -\frac{2}{b\,n\,\sqrt{a+b\,x^n}}$$

input
```
int(x^(n - 1)/(a + b*x^n)^(3/2),x)
```

output
```
-2/(b*n*(a + b*x^n)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{x^{-1+n}}{(a+bx^n)^{3/2}}\,dx = -\frac{2\sqrt{x^n b+a}}{bn\,(x^n b+a)}$$

input
```
int(x^(-1+n)/(a+b*x^n)^(3/2),x)
```

output
```
( - 2*sqrt(x**n*b + a))/(b*n*(x**n*b + a))
```

3.590 $\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}} \, dx = -\frac{2}{3bn(a+bx^n)^{3/2}}$$

output
```
-2/3/b/n/(a+b*x^n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}} \, dx = -\frac{2}{3bn(a+bx^n)^{3/2}}$$

input
```
Integrate[x^(-1 + n)/(a + b*x^n)^(5/2),x]
```

output
```
-2/(3*b*n*(a + b*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{(a+bx^n)^{5/2}} \, dx$$
$$\Big\downarrow \; 793$$
$$-\frac{2}{3bn (a+bx^n)^{3/2}}$$

input
```
Int[x^(-1 + n)/(a + b*x^n)^(5/2),x]
```

output
```
-2/(3*b*n*(a + b*x^n)^(3/2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [F]

$$\int \frac{x^{-1+n}}{(a+b\,x^{n})^{\frac{5}{2}}}dx$$

input
```
int(x^(-1+n)/(a+b*x^n)^(5/2),x)
```

output
```
int(x^(-1+n)/(a+b*x^n)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}}\,dx = -\frac{2\sqrt{bx^n+a}}{3\left(b^3nx^{2n}+2\,ab^2nx^n+a^2bn\right)}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^n + a)/(b^3*n*x^(2*n) + 2*a*b^2*n*x^n + a^2*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(17) = 34$.

Time = 19.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}}\,dx = \begin{cases} \frac{\log(x)}{a^{\frac{5}{2}}} & \text{for } b=0 \wedge n=0 \\ \frac{xx^{n-1}}{a^{\frac{5}{2}}n} & \text{for } b=0 \\ \frac{\log(x)}{(a+b)^{\frac{5}{2}}} & \text{for } n=0 \\ -\frac{2}{3abn\sqrt{a+bx^n}+3b^2nx^n\sqrt{a+bx^n}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)/(a+b*x**n)**(5/2),x)
```

output
```
Piecewise((log(x)/a**(5/2), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(a**(5/2)*
n), Eq(b, 0)), (log(x)/(a + b)**(5/2), Eq(n, 0)), (-2/(3*a*b*n*sqrt(a + b*
x**n) + 3*b**2*n*x**n*sqrt(a + b*x**n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}}\,dx = -\frac{2}{3\,(bx^n+a)^{\frac{3}{2}}bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
-2/3/((b*x^n + a)^(3/2)*b*n)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}}\,dx = -\frac{2}{3\,(bx^n+a)^{\frac{3}{2}}bn}$$

input
```
integrate(x^(-1+n)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
-2/3/((b*x^n + a)^(3/2)*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}}\,dx = -\frac{2}{3\,b\,n\,(a+b\,x^n)^{3/2}}$$

input
```
int(x^(n - 1)/(a + b*x^n)^(5/2),x)
```

output
```
-2/(3*b*n*(a + b*x^n)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{x^{-1+n}}{(a+bx^n)^{5/2}}\,dx = -\frac{2\sqrt{x^n b+a}}{3bn\left(x^{2n}b^2+2x^n ab+a^2\right)}$$

input
```
int(x^(-1+n)/(a+b*x^n)^(5/2),x)
```

output
```
( - 2*sqrt(x**n*b + a))/(3*b*n*(x**(2*n)*b**2 + 2*x**n*a*b + a**2))
```

3.591 $\int x^m (a+bx^n)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int x^m (a+bx^n)^{3/2}\,dx = \frac{x^{1+m}(a+bx^n)^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2}+\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*(a+b*x^n)^(5/2)*hypergeom([1, 5/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a
/(1+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int x^m (a+bx^n)^{3/2}\,dx = \frac{ax^{1+m}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{(1+m)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x^m*(a + b*x^n)^(3/2),x]
```

output
```
(a*x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, (1 + m)/n, 1 + (1 + m
)/n, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m(a+bx^n)^{3/2}\,dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{a\sqrt{a+bx^n}\int x^m\left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{ax^{m+1}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{(m+1)\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[x^m*(a + b*x^n)^(3/2),x]
```

output
```
(a*x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, (1 + m)/n, (1 + m + n
)/n, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^m (a+b\,x^n)^{\frac{3}{2}}\,dx$$

input
```
int(x^m*(a+b*x^n)^(3/2),x)
```

output
```
int(x^m*(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m (a+bx^n)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int x^m (a+bx^n)^{3/2}\,dx = \frac{a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}+\frac{3}{2}-\frac{1}{n}} x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{3}{2}, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix}\right| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate(x**m*(a+b*x**n)**(3/2),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + 3/2 - 1/n)*x**(m + 1)*gamma(m/n + 1/n)*hyper((-3
/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n +
 1 + 1/n))
```

Maxima [F]

$$\int x^m (a+bx^n)^{3/2}\,dx = \int (bx^n+a)^{\frac{3}{2}} x^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(3/2)*x^m, x)
```

Giac [F]

$$\int x^m (a+bx^n)^{3/2}\,dx = \int (bx^n+a)^{\frac{3}{2}} x^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (a+bx^n)^{3/2}\,dx = \int x^m\,(a+b\,x^n)^{3/2}\,dx$$

input
```
int(x^m*(a + b*x^n)^(3/2),x)
```

output
```
int(x^m*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int x^m(a+bx^n)^{3/2}\,dx = \frac{4x^{m+n}\sqrt{x^nb+a}\,bmx + 2x^{m+n}\sqrt{x^nb+a}\,bnx + 4x^{m+n}\sqrt{x^nb+a}\,bx + 4x^m\sqrt{x^nb+a}\,amx + 8}{}$$

input
```
int(x^m*(a+b*x^n)^(3/2),x)
```

output

```
(4*x**(m + n)*sqrt(x**n*b + a)*b*m*x + 2*x**(m + n)*sqrt(x**n*b + a)*b*n*x
 + 4*x**(m + n)*sqrt(x**n*b + a)*b*x + 4*x**m*sqrt(x**n*b + a)*a*m*x + 8*x
**m*sqrt(x**n*b + a)*a*n*x + 4*x**m*sqrt(x**n*b + a)*a*x + 12*int((x**m*sq
rt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2
 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n +
 4*a),x)*a**2*m**2*n**2 + 24*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 +
8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m*
*2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*m*n**3 + 24*int((x*
*m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b
*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*
a*n + 4*a),x)*a**2*m*n**2 + 9*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 +
 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m
**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**4 + 24*int((x**
m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*
n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a
*n + 4*a),x)*a**2*n**3 + 12*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8
*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**
2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**2)/(4*m**2 + 8*m*
n + 8*m + 3*n**2 + 8*n + 4)
```

3.592 $\int x^m\sqrt{a+bx^n}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int x^m\sqrt{a+bx^n}\,dx=\frac{x^{1+m}(a+bx^n)^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*(a+b*x^n)^(3/2)*hypergeom([1, 3/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a
/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int x^m\sqrt{a+bx^n}\,dx=\frac{x^{1+m}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{(1+m)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x^m*Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, (1 + m)/n, 1 + (1 + m)/
n, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{a+bx^n} \, dx$$

$$\Big\downarrow \ 889$$

$$\frac{\sqrt{a+bx^n} \int x^m \sqrt{\frac{bx^n}{a}+1} dx}{\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^{m+1}\sqrt{a+bx^n} \, \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{(m+1)\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[x^m*Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, (1 + m)/n, (1 + m + n)/
n, -((b*x^n)/a)])/((1 + m)*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^m \sqrt{a+b\,x^n}dx$$

input
```
int(x^m*(a+b*x^n)^(1/2),x)
```

output
```
int(x^m*(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m\sqrt{a+bx^n}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int x^m\sqrt{a+bx^n}\,dx = \frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}+\frac{1}{2}-\frac{1}{n}}x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate(x**m*(a+b*x**n)**(1/2),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + 1/2 - 1/n)*x**(m + 1)*gamma(m/n + 1/n)*hyper((-1
/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n +
 1 + 1/n))
```

Maxima [F]

$$\int x^m\sqrt{a+bx^n}\,dx=\int\sqrt{bx^n+a}x^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^n + a)*x^m, x)
```

Giac [F]

$$\int x^m\sqrt{a+bx^n}\,dx=\int\sqrt{bx^n+a}x^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m\sqrt{a+bx^n}\,dx=\int x^m\,\sqrt{a+b\,x^n}\,dx$$

input
```
int(x^m*(a + b*x^n)^(1/2),x)
```

output
```
int(x^m*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int x^m\sqrt{a+bx^n}\,dx = \frac{2x^m\sqrt{x^nb+a}\,x+2\left(\int\frac{x^m\sqrt{x^nb+a}}{2x^nbm+x^nbn+2x^nb+2am+an+2a}dx\right)amn+\left(\int\frac{x^m\sqrt{x^nb+a}}{2x^nbm+x^nbn+2x^nb+2am+an+2a}dx\right)a\,n^2+2}{2m+n+2}$$

input
```
int(x^m*(a+b*x^n)^(1/2),x)
```

output
```
(2*x**m*sqrt(x**n*b + a)*x + 2*int((x**m*sqrt(x**n*b + a))/(2*x**n*b*m + x
**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*a*m*n + int((x**m*sqrt(x**n*b +
 a))/(2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*a*n**2 + 2*
int((x**m*sqrt(x**n*b + a))/(2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a*
n + 2*a),x)*a*n)/(2*m + n + 2)
```

3.593 $\int \frac{x^m}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx=\frac{x^{1+m}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*(a+b*x^n)^(1/2)*hypergeom([1, 1/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a
/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx=\frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{(1+m)\sqrt{a+bx^n}}$$

input
```
Integrate[x^m/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, 1 + (1 +
m)/n, -((b*x^n)/a)])/((1 + m)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^m}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{(m+1)\sqrt{a+bx^n}}$$

input
```
Int[x^m/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m)/n, (1 + m +
n)/n, -((b*x^n)/a)])/((1 + m)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{a+b\,x^n}}dx$$

input
```
int(x^m/(a+b*x^n)^(1/2),x)
```

output
```
int(x^m/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx = \frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-\frac{1}{2}-\frac{1}{n}}x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate(x**m/(a+b*x**n)**(1/2),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n - 1/2 - 1/n)*x**(m + 1)*gamma(m/n + 1/n)*hyper((1/
2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n +
1 + 1/n))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx = \int \frac{x^m}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx = \int \frac{x^m}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx = \int \frac{x^m}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^m/(a + b*x^n)^(1/2),x)
```

output
```
int(x^m/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}}\,dx = \int \frac{x^m\sqrt{x^n b+a}}{x^n b+a}dx$$

input
```
int(x^m/(a+b*x^n)^(1/2),x)
```

output
```
int((x**m*sqrt(x**n*b + a))/(x**n*b + a),x)
```

3.594 $\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2}+\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)\sqrt{a+bx^n}}$$

output
```
x^(1+m)*hypergeom([1, -1/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(1+m)/(a+b*x^n
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{a(1+m)\sqrt{a+bx^n}}$$

input
```
Integrate[x^m/(a + b*x^n)^(3/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, (1 + m)/n, 1 + (1 +
m)/n, -((b*x^n)/a)])/(a*(1 + m)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^m}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a(m+1)\sqrt{a+bx^n}}$$

input
```
Int[x^m/(a + b*x^n)^(3/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, (1 + m)/n, (1 + m +
n)/n, -((b*x^n)/a)])/(a*(1 + m)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(a+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a+b*x^n)^(3/2),x)
```

output
```
int(x^m/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-\frac{3}{2}-\frac{1}{n}}x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate(x**m/(a+b*x**n)**(3/2),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n - 3/2 - 1/n)*x**(m + 1)*gamma(m/n + 1/n)*hyper((3/
2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n +
1 + 1/n))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \int \frac{x^m}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \int \frac{x^m}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \int \frac{x^m}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int(x^m/(a + b*x^n)^(3/2),x)
```

output
```
int(x^m/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^n)^{3/2}}\,dx = \int \frac{x^m\sqrt{x^nb+a}}{x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int(x^m/(a+b*x^n)^(3/2),x)
```

output
```
int((x**m*sqrt(x**n*b + a))/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.595 $\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2}+\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)\,(a+bx^n)^{3/2}}$$

output
```
x^(1+m)*hypergeom([1, -3/2+(1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(1+m)/(a+b*x^n
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx = \frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{n},1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{a^2(1+m)\sqrt{a+bx^n}}$$

input
```
Integrate[x^m/(a + b*x^n)^(5/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, (1 + m)/n, 1 + (1 +
m)/n, -((b*x^n)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \; 889$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^m}{\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}$$

$$\Big\downarrow \; 888$$

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^2(m+1)\sqrt{a+bx^n}}$$

input
```
Int[x^m/(a + b*x^n)^(5/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, (1 + m)/n, (1 + m +
n)/n, -((b*x^n)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(a+b\,x^n)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a+b*x^n)^(5/2),x)
```

output
```
int(x^m/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx = \frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-\frac{5}{2}-\frac{1}{n}}x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate(x**m/(a+b*x**n)**(5/2),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n - 5/2 - 1/n)*x**(m + 1)*gamma(m/n + 1/n)*hyper((5/
2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n +
1 + 1/n))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx = \int \frac{x^m}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx = \int \frac{x^m}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx=\int \frac{x^m}{{\left(a+b\,x^n\right)}^{5/2}} \,d x$$

input
```
int(x^m/(a + b*x^n)^(5/2),x)
```

output
```
int(x^m/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^n)^{5/2}}\,dx=\int \frac{x^m \sqrt{x^n b+a}}{x^{3n} b^3+3x^{2n} a\,b^2+3x^n a^2 b+a^3}d x$$

input
```
int(x^m/(a+b*x^n)^(5/2),x)
```

output
```
int((x**m*sqrt(x**n*b + a))/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a*
*2*b + a**3),x)
```

3.596 $\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{2(2+n)}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(5+\frac{8}{n}\right),3+\frac{4}{n},-\frac{bx^n}{a}\right)}{2a(2+n)}$$

output
```
1/2*x^(4+2*n)*(a+b*x^n)^(1/2)*hypergeom([1, 5/2+4/n],[3+4/n],-b*x^n/a)/a/(
2+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{4+2n}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},2+\frac{4}{n},3+\frac{4}{n},-\frac{bx^n}{a}\right)}{2(2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(3 + 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(4 + 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2 + 4/n, 3 + 4/n,
-((b*x^n)/a)])/(2*(2 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n+3}}{\sqrt{a+bx^n}}dx$$
$$\Big\downarrow \ 889$$
$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{2n+3}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$
$$\Big\downarrow \ 888$$
$$\frac{x^{2(n+2)}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},2\left(1+\frac{2}{n}\right),3+\frac{4}{n},-\frac{bx^n}{a}\right)}{2(n+2)\sqrt{a+bx^n}}$$

input
```
Int[x^(3 + 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(2*(2 + n))*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2*(1 + 2/n), 3 +
 4/n, -((b*x^n)/a)])/(2*(2 + n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{2n+3}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(2*n+3)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(2*n+3)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(3+2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \frac{a^{-\frac{5}{2}-\frac{4}{n}}a^{2+\frac{4}{n}}x^{2n+4}\Gamma\left(2+\frac{4}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},2+\frac{4}{n}\\3+\frac{4}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{4}{n}\right)}$$

input
```
integrate(x**(3+2*n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-5/2 - 4/n)*a**(2 + 4/n)*x**(2*n + 4)*gamma(2 + 4/n)*hyper((1/2, 2 + 4
/n), (3 + 4/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(3 + 4/n))
```

Maxima [F]

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{2n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3+2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(2*n + 3)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{2n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3+2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n + 3)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{2\,n+3}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(2*n + 3)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(2*n + 3)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{3+2n}}{\sqrt{a+bx^n}}\,dx = \frac{2x^n\sqrt{x^nb+a}\,bn\,x^4+16x^n\sqrt{x^nb+a}\,b\,x^4-4\sqrt{x^nb+a}\,an\,x^4-16\sqrt{x^nb+a}\,a\,x^4+48\left(\int \frac{\sqrt{x^n}}{3x^nb\,n^2+32x^nbn+64}\right.}{}$$

input
```
int(x^(3+2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*(x**n*sqrt(x**n*b + a)*b*n*x**4 + 8*x**n*sqrt(x**n*b + a)*b*x**4 - 2*sq
rt(x**n*b + a)*a*n*x**4 - 8*sqrt(x**n*b + a)*a*x**4 + 24*int((sqrt(x**n*b
+ a)*x**3)/(3*x**n*b*n**2 + 32*x**n*b*n + 64*x**n*b + 3*a*n**2 + 32*a*n +
64*a),x)*a**2*n**3 + 352*int((sqrt(x**n*b + a)*x**3)/(3*x**n*b*n**2 + 32*x
**n*b*n + 64*x**n*b + 3*a*n**2 + 32*a*n + 64*a),x)*a**2*n**2 + 1536*int((s
qrt(x**n*b + a)*x**3)/(3*x**n*b*n**2 + 32*x**n*b*n + 64*x**n*b + 3*a*n**2
+ 32*a*n + 64*a),x)*a**2*n + 2048*int((sqrt(x**n*b + a)*x**3)/(3*x**n*b*n*
*2 + 32*x**n*b*n + 64*x**n*b + 3*a*n**2 + 32*a*n + 64*a),x)*a**2))/(b**2*(
3*n**2 + 32*n + 64))
```

3.597 $\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{4+n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(3+\frac{8}{n}\right),2\left(1+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a(4+n)}$$

output
```
x^(4+n)*(a+b*x^n)^(1/2)*hypergeom([1, 3/2+4/n],[2+4/n],-b*x^n/a)/a/(4+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{4+n}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4+n}{n},1+\frac{4+n}{n},-\frac{bx^n}{a}\right)}{(4+n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(3 + n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(4 + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (4 + n)/n, 1 + (4 +
n)/n, -((b*x^n)/a)])/((4 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n+3}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{n+3}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{n+4}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+4}{n},2\left(1+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{(n+4)\sqrt{a+bx^n}}$$

input
```
Int[x^(3 + n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(4 + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (4 + n)/n, 2*(1 + 2/
n), -((b*x^n)/a)])/((4 + n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{3+n}}{\sqrt{a+b\,x^{n}}}dx$$

input

```
int(x^(3+n)/(a+b*x^n)^(1/2),x)
```

output

```
int(x^(3+n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^(3+n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx = \frac{a^{-\frac{3}{2}-\frac{4}{n}}a^{1+\frac{4}{n}}x^{n+4}\Gamma\left(1+\frac{4}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{4}{n}\\2+\frac{4}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{4}{n}\right)}$$

input
```
integrate(x**(3+n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-3/2 - 4/n)*a**(1 + 4/n)*x**(n + 4)*gamma(1 + 4/n)*hyper((1/2, 1 + 4/n
), (2 + 4/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 4/n))
```

Maxima [F]

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3+n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(n + 3)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3+n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(n + 3)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{n+3}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(n + 3)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(n + 3)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{3+n}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^n b+a}\,x^4-8\left(\int \frac{\sqrt{x^n b+a}\,x^3}{x^n bn+8x^n b+an+8a}dx\right)an-64\left(\int \frac{\sqrt{x^n b+a}\,x^3}{x^n bn+8x^n b+an+8a}dx\right)a}{b\,(n+8)}$$

input
```
int(x^(3+n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*(sqrt(x**n*b + a)*x**4 - 4*int((sqrt(x**n*b + a)*x**3)/(x**n*b*n + 8*x*
*n*b + a*n + 8*a),x)*a*n - 32*int((sqrt(x**n*b + a)*x**3)/(x**n*b*n + 8*x*
*n*b + a*n + 8*a),x)*a))/(b*(n + 8))
```

3.598 $\int \frac{x^{3-n}}{\sqrt{a+bx^n}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}} \, dx = \frac{x^{4-n}\sqrt{a+bx^n}\, \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(-1+\frac{8}{n}\right), \frac{4}{n}, -\frac{bx^n}{a}\right)}{a(4-n)}$$

output
```
x^(4-n)*(a+b*x^n)^(1/2)*hypergeom([1, -1/2+4/n],[4/n],-b*x^n/a)/a/(4-n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}} \, dx = -\frac{x^{4-n}\sqrt{1+\frac{bx^n}{a}}\, \text{Hypergeometric2F1}\left(\frac{1}{2}, -1+\frac{4}{n}, \frac{4}{n}, -\frac{bx^n}{a}\right)}{(-4+n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(3 - n)/Sqrt[a + b*x^n],x]
```

output
```
-((x^(4 - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -1 + 4/n, 4/n, -((
b*x^n)/a)])/((-4 + n)*Sqrt[a + b*x^n]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{3-n}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{4-n}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{n}-1,\frac{4}{n},-\frac{bx^n}{a}\right)}{(4-n)\sqrt{a+bx^n}}$$

input
```
Int[x^(3 - n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(4 - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -1 + 4/n, 4/n, -((b*
x^n)/a)])/((4 - n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{3-n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(3-n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(3-n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(3-n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx = \frac{a^{-1+\frac{4}{n}}a^{\frac{1}{2}-\frac{4}{n}}b^{-1+\frac{4}{n}}b^{1-\frac{4}{n}}x^{4-n}\Gamma\left(-1+\frac{4}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},-1+\frac{4}{n}\\ \frac{4}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{4}{n}\right)}$$

input
```
integrate(x**(3-n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-1 + 4/n)*a**(1/2 - 4/n)*b**(-1 + 4/n)*b**(1 - 4/n)*x**(4 - n)*gamma(-
1 + 4/n)*hyper((1/2, -1 + 4/n), (4/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma
(4/n))
```

Maxima [F]

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3-n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-n + 3)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3-n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-n + 3)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{3-n}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(3 - n)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(3 - n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{3-n}}{\sqrt{a+bx^n}}\,dx = \frac{-2\sqrt{x^nb+a}\,x^4-x^n\left(\int \frac{\sqrt{x^nb+a}\,x^3}{x^nbn-4x^nb+an-4a}dx\right)b\,n^2+12x^n\left(\int \frac{\sqrt{x^nb+a}\,x^3}{x^nbn-4x^nb+an-4a}dx\right)bn-32x^n\left(\int \frac{\sqrt{x^nb+a}\,x^3}{x^nbn-4x^nb+an-4a}dx\right)b}{2x^na\,(n-4)}$$

input
```
int(x^(3-n)/(a+b*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(x**n*b + a)*x**4 - x**n*int((sqrt(x**n*b + a)*x**3)/(x**n*b*n -
 4*x**n*b + a*n - 4*a),x)*b*n**2 + 12*x**n*int((sqrt(x**n*b + a)*x**3)/(x*
*n*b*n - 4*x**n*b + a*n - 4*a),x)*b*n - 32*x**n*int((sqrt(x**n*b + a)*x**3
)/(x**n*b*n - 4*x**n*b + a*n - 4*a),x)*b)/(2*x**n*a*(n - 4))
```

3.599 $\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{4-2n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-3+\frac{8}{n}\right),-1+\frac{4}{n},-\frac{bx^n}{a}\right)}{2a(2-n)}$$

output
```
1/2*x^(4-2*n)*(a+b*x^n)^(1/2)*hypergeom([1, -3/2+4/n],[-1+4/n],-b*x^n/a)/a
/(2-n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = -\frac{x^{4-2n}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-2+\frac{4}{n},-1+\frac{4}{n},-\frac{bx^n}{a}\right)}{2(-2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(3 - 2*n)/Sqrt[a + b*x^n],x]
```

output
```
-1/2*(x^(4 - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2 + 4/n, -1
+ 4/n, -((b*x^n)/a)])/((-2 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{3-2n}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{4-2n}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-2\left(1-\frac{2}{n}\right),\frac{4}{n}-1,-\frac{bx^n}{a}\right)}{2(2-n)\sqrt{a+bx^n}}$$

input
```
Int[x^(3 - 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(4 - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2*(1 - 2/n), -1 +
 4/n, -((b*x^n)/a)])/(2*(2 - n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{3-2n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(3-2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(3-2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(3-2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = \frac{a^{-2+\frac{4}{n}}a^{\frac{3}{2}-\frac{4}{n}}b^{-2+\frac{4}{n}}b^{2-\frac{4}{n}}x^{4-2n}\Gamma\left(-2+\frac{4}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},-2+\frac{4}{n}\\-1+\frac{4}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(-1+\frac{4}{n}\right)}$$

input
```
integrate(x**(3-2*n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-2 + 4/n)*a**(3/2 - 4/n)*b**(-2 + 4/n)*b**(2 - 4/n)*x**(4 - 2*n)*gamma
(-2 + 4/n)*hyper((1/2, -2 + 4/n), (-1 + 4/n,), b*x**n*exp_polar(I*pi)/a)/(
n*gamma(-1 + 4/n))
```

Maxima [F]

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-2\,n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3-2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-2*n + 3)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-2\,n+3}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(3-2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-2*n + 3)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{3-2\,n}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(3 - 2*n)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(3 - 2*n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{3-2n}}{\sqrt{a+bx^n}}\,dx = \frac{6x^n\sqrt{x^nb+a}\,bn\,x^4 - 16x^n\sqrt{x^nb+a}\,b\,x^4 - 4\sqrt{x^nb+a}\,an\,x^4 + 16\sqrt{x^nb+a}\,a\,x^4 + 3x^{2n}\left(\int \frac{\sqrt{x^n b}}{x^n b\,n^2-6x^n bn+8x}\right.}{}$$

input
```
int(x^(3-2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(6*x**n*sqrt(x**n*b + a)*b*n*x**4 - 16*x**n*sqrt(x**n*b + a)*b*x**4 - 4*sq
rt(x**n*b + a)*a*n*x**4 + 16*sqrt(x**n*b + a)*a*x**4 + 3*x**(2*n)*int((sqr
t(x**n*b + a)*x**3)/(x**n*b*n**2 - 6*x**n*b*n + 8*x**n*b + a*n**2 - 6*a*n
+ 8*a),x)*b**2*n**4 - 50*x**(2*n)*int((sqrt(x**n*b + a)*x**3)/(x**n*b*n**2
 - 6*x**n*b*n + 8*x**n*b + a*n**2 - 6*a*n + 8*a),x)*b**2*n**3 + 280*x**(2*
n)*int((sqrt(x**n*b + a)*x**3)/(x**n*b*n**2 - 6*x**n*b*n + 8*x**n*b + a*n*
*2 - 6*a*n + 8*a),x)*b**2*n**2 - 640*x**(2*n)*int((sqrt(x**n*b + a)*x**3)/
(x**n*b*n**2 - 6*x**n*b*n + 8*x**n*b + a*n**2 - 6*a*n + 8*a),x)*b**2*n + 5
12*x**(2*n)*int((sqrt(x**n*b + a)*x**3)/(x**n*b*n**2 - 6*x**n*b*n + 8*x**n
*b + a*n**2 - 6*a*n + 8*a),x)*b**2)/(8*x**(2*n)*a**2*(n**2 - 6*n + 8))
```

3.600 $\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{1+m+2n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(5+\frac{2(1+m)}{n}\right),\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{a(1+m+2n)}$$

output
```
x^(1+m+2*n)*(a+b*x^n)^(1/2)*hypergeom([1, 5/2+(1+m)/n],[(1+m+3*n)/n],-b*x^
n/a)/a/(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{1+m+2n}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+2n}{n},1+\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{(1+m+2n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(m + 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m + 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/
n, 1 + (1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{m+2n}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m+2n+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2n+1}{n},\frac{m+3n+1}{n},-\frac{bx^n}{a}\right)}{(m+2n+1)\sqrt{a+bx^n}}$$

input
```
Int[x^(m + 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m + 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + 2*n)/
n, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1 + m + 2*n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{m+2n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(m+2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(m+2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(m+2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \frac{a^{-\frac{m}{n}-\frac{5}{2}-\frac{1}{n}}a^{\frac{m}{n}+2+\frac{1}{n}}x^{m+2n+1}\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}+2+\frac{1}{n}\\ \frac{m}{n}+3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}$$

input
```
integrate(x**(m+2*n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-m/n - 5/2 - 1/n)*a**(m/n + 2 + 1/n)*x**(m + 2*n + 1)*gamma(m/n + 2 +
1/n)*hyper((1/2, m/n + 2 + 1/n), (m/n + 3 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(m/n + 3 + 1/n))
```

Maxima [F]

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m+2n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m+2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m + 2*n)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m+2n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m+2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m + 2*n)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m+2\,n}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(m + 2*n)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(m + 2*n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{m+2n}}{\sqrt{a+bx^n}}\,dx = \text{Too large to display}$$

input
```
int(x^(m+2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*(2*x**(m + n)*sqrt(x**n*b + a)*b*m*x + x**(m + n)*sqrt(x**n*b + a)*b*n*
x + 2*x**(m + n)*sqrt(x**n*b + a)*b*x - 2*x**m*sqrt(x**n*b + a)*a*m*x - 2*
x**m*sqrt(x**n*b + a)*a*n*x - 2*x**m*sqrt(x**n*b + a)*a*x + 8*int((x**m*sq
rt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2
 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n +
 4*a),x)*a**2*m**4 + 24*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**
n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 +
8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*m**3*n + 32*int((x**m*sq
rt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2
 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n +
 4*a),x)*a**2*m**3 + 22*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**
n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 +
8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*m**2*n**2 + 72*int((x**m
*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n
**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*
n + 4*a),x)*a**2*m**2*n + 48*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 +
8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m*
*2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*m**2 + 6*int((x**m*
sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n*
*2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*...
```

3.601 $\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx=\frac{x^{1+m+n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{a(1+m+n)}$$

output
```
x^(1+m+n)*(a+b*x^n)^(1/2)*hypergeom([1, 1/2+(1+m+n)/n],[(1+m+2*n)/n],-b*x^
n/a)/a/(1+m+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx=\frac{x^{1+m+n}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m+n}{n},1+\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{(1+m+n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(m + n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, 1
 + (1 + m + n)/n, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{m+n}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m+n+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+n+1}{n},\frac{m+2n+1}{n},-\frac{bx^n}{a}\right)}{(m+n+1)\sqrt{a+bx^n}}$$

input
```
Int[x^(m + n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m + n)/n, (
1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{m+n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(m+n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(m+n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(m+n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx = \frac{a^{-\frac{m}{n}-\frac{3}{2}-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}x^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}+1+\frac{1}{n}\\\frac{m}{n}+2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate(x**(m+n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-m/n - 3/2 - 1/n)*a**(m/n + 1 + 1/n)*x**(m + n + 1)*gamma(m/n + 1 + 1/
n)*hyper((1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b*x**n*exp_polar(I*pi)/a)
/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m+n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m+n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m + n)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m+n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m+n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m + n)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{m+n}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(m + n)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(m + n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{m+n}}{\sqrt{a+bx^n}}\,dx$$
$$=\frac{2x^m\sqrt{x^nb+a}\,x-4\left(\int \frac{x^m\sqrt{x^nb+a}}{2x^nbm+x^nbn+2x^nb+2am+an+2a}dx\right)a\,m^2-2\left(\int \frac{x^m\sqrt{x^nb+a}}{2x^nbm+x^nbn+2x^nb+2am+an+2a}dx\right)amn-$$

input
```
int(x^(m+n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*(x**m*sqrt(x**n*b + a)*x - 2*int((x**m*sqrt(x**n*b + a))/(2*x**n*b*m +
x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*a*m**2 - int((x**m*sqrt(x**n*b
 + a))/(2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*a*m*n - 4
*int((x**m*sqrt(x**n*b + a))/(2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a
*n + 2*a),x)*a*m - int((x**m*sqrt(x**n*b + a))/(2*x**n*b*m + x**n*b*n + 2*
x**n*b + 2*a*m + a*n + 2*a),x)*a*n - 2*int((x**m*sqrt(x**n*b + a))/(2*x**n
*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*a))/(b*(2*m + n + 2))
```

3.602 $\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{1+m-n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-1+\frac{2(1+m)}{n}\right),\frac{1+m}{n},-\frac{bx^n}{a}\right)}{a(1+m-n)}$$

output
```
x^(1+m-n)*(a+b*x^n)^(1/2)*hypergeom([1, -1/2+(1+m)/n],[(1+m)/n],-b*x^n/a)/
a/(1+m-n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{1+m-n}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m-n}{n},\frac{1+m}{n},-\frac{bx^n}{a}\right)}{(1+m-n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(m - n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - n)/n, (
1 + m)/n, -((b*x^n)/a)])/((1 + m - n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{m-n}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m-n+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-n+1}{n},\frac{m+1}{n},-\frac{bx^n}{a}\right)}{(m-n+1)\sqrt{a+bx^n}}$$

input
```
Int[x^(m - n)/Sqrt[a + b*x^n],x]
```

output

```
(x^(1 + m - n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - n)/n, (
1 + m)/n, -((b*x^n)/a)])/((1 + m - n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{m-n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(m-n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(m-n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(m-n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx=\frac{a^{-\frac{m}{n}+\frac{1}{2}-\frac{1}{n}}a^{\frac{m}{n}-1+\frac{1}{n}}x^{m-n+1}\Gamma\left(\frac{m}{n}-1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}-1+\frac{1}{n}\\ \frac{m}{n}+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}$$

input
```
integrate(x**(m-n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-m/n + 1/2 - 1/n)*a**(m/n - 1 + 1/n)*x**(m - n + 1)*gamma(m/n - 1 + 1/
n)*hyper((1/2, m/n - 1 + 1/n), (m/n + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*
gamma(m/n + 1/n))
```

Maxima [F]

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{m-n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m-n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m - n)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{m-n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m-n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m - n)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m-n}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(m - n)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(m - n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{m-n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^m\sqrt{x^nb+a}}{x^{2n}b+x^na}dx$$

input
```
int(x^(m-n)/(a+b*x^n)^(1/2),x)
```

output
```
int((x**m*sqrt(x**n*b + a))/(x**(2*n)*b + x**n*a),x)
```

3.603 $\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{1+m-2n}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(-3+\frac{2(1+m)}{n}\right),\frac{1+m-n}{n},-\frac{bx^n}{a}\right)}{a(1+m-2n)}$$

output
```
x^(1+m-2*n)*(a+b*x^n)^(1/2)*hypergeom([1, -3/2+(1+m)/n],[(1+m-n)/n],-b*x^n
/a)/a/(1+m-2*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx = \frac{x^{1+m-2n}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m-2n}{n},1+\frac{1+m-2n}{n},-\frac{bx^n}{a}\right)}{(1+m-2n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^(m - 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - 2*n)/
n, 1 + (1 + m - 2*n)/n, -((b*x^n)/a)])/((1 + m - 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^{m-2n}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m-2n+1}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-2n+1}{n},\frac{m-n+1}{n},-\frac{bx^n}{a}\right)}{(m-2n+1)\sqrt{a+bx^n}}$$

input
```
Int[x^(m - 2*n)/Sqrt[a + b*x^n],x]
```

output
```
(x^(1 + m - 2*n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, (1 + m - 2*n)/
n, (1 + m - n)/n, -((b*x^n)/a)])/((1 + m - 2*n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{m-2n}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(m-2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(m-2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(m-2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx = \frac{a^{-\frac{m}{n}+\frac{3}{2}-\frac{1}{n}}a^{\frac{m}{n}-2+\frac{1}{n}}x^{m-2n+1}\Gamma\left(\frac{m}{n}-2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}-2+\frac{1}{n}\\ \frac{m}{n}-1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}-1+\frac{1}{n}\right)}$$

input
```
integrate(x**(m-2*n)/(a+b*x**n)**(1/2),x)
```

output
```
a**(-m/n + 3/2 - 1/n)*a**(m/n - 2 + 1/n)*x**(m - 2*n + 1)*gamma(m/n - 2 +
1/n)*hyper((1/2, m/n - 2 + 1/n), (m/n - 1 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(m/n - 1 + 1/n))
```

Maxima [F]

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m-2n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m-2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m - 2*n)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{m-2n}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(m-2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m - 2*n)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{m-2\,n}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(m - 2*n)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(m - 2*n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{m-2n}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^m\sqrt{x^nb+a}}{x^{3n}b+x^{2n}a}dx$$

input
```
int(x^(m-2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((x**m*sqrt(x**n*b + a))/(x**(3*n)*b + x**(2*n)*a),x)
```

3.604 $\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)\,dx$

Optimal result

Integrand size = 42, antiderivative size = 15

$$\int \left(-\frac{bnx^{-1+m+n}}{2\,(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)\,dx=\frac{x^m}{\sqrt{a+bx^n}}$$

output
```
x^m/(a+b*x^n)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.40

$$\int \left(-\frac{bnx^{-1+m+n}}{2\,(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)\,dx=\frac{x^m\sqrt{1+\frac{bx^n}{a}}\left(2a(m+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{n},\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(2m-n)x^n\,\text{Hyper}\right.}{2a(m+n)\sqrt{a+bx^n}}$$

input
```
Integrate[-1/2*(b*n*x^(-1 + m + n))/(a + b*x^n)^(3/2) + (m*x^(-1 + m))/Sqr
t[a + b*x^n],x]
```

output
```
(x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/
n, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m
/n, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 8.40, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{mx^{m-1}}{\sqrt{a+bx^n}}-\frac{bnx^{m+n-1}}{2\left(a+bx^n\right)^{3/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}-\frac{bnx^{m+n}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+n}{n},\frac{m}{n}+2,-\frac{bx^n}{a}\right)}{2a(m+n)\sqrt{a+bx^n}}$$

input
```
Int[-1/2*(b*n*x^(-1 + m + n))/(a + b*x^n)^(3/2) + (m*x^(-1 + m))/Sqrt[a +
b*x^n],x]
```

output
```
(x^m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, m/n, (m + n)/n, -((b*x^n)/
a)])/Sqrt[a + b*x^n] - (b*n*x^(m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F
1[3/2, (m + n)/n, 2 + m/n, -((b*x^n)/a)])/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(-\frac{bn\,x^{-1+m+n}}{2\left(a+b\,x^{n}\right)^{\frac{3}{2}}}+\frac{m\,x^{m-1}}{\sqrt{a+b\,x^{n}}}\right)dx$$

input
```
int(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(m-1)/(a+b*x^n)^(1/2),x)
```

output
```
int(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(m-1)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{bnx^{-1+m+n}}{2\left(a+bx^{n}\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^{n}}}\right)dx=\text{Exception raised: TypeError}$$

input
```
integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x
, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 7.60

$$\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\frac{a^{\frac{m}{n}}a^{-\frac{m}{n}-\frac{1}{2}}mx^m\Gamma\left(\frac{m}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}\\\frac{m}{n}+1\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1\right)}-\frac{a^{-\frac{m}{n}-\frac{5}{2}}a^{\frac{m}{n}+1}bx^{m+n}\Gamma\left(\frac{m}{n}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\\frac{m}{n}+2\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{n}+2\right)}$$

input
```
integrate(-1/2*b*n*x**(-1+m+n)/(a+b*x**n)**(3/2)+m*x**(-1+m)/(a+b*x**n)**(
1/2),x)
```

output
```
a**(m/n)*a**(-m/n - 1/2)*m*x**m*gamma(m/n)*hyper((1/2, m/n), (m/n + 1,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1)) - a**(-m/n - 5/2)*a**(m/n + 1)
*b*x**(m + n)*gamma(m/n + 1)*hyper((3/2, m/n + 1), (m/n + 2,), b*x**n*exp_
polar(I*pi)/a)/(2*gamma(m/n + 2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\frac{x^m}{\sqrt{bx^n+a}}$$

input
```
integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x
, algorithm="maxima")
```

output
```
x^m/sqrt(b*x^n + a)
```

Giac [F]

$$\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\int -\frac{bnx^{m+n-1}}{2(bx^n+a)^{\frac{3}{2}}}+\frac{mx^{m-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x
, algorithm="giac")
```

output
```
integrate(-1/2*b*n*x^(m + n - 1)/(b*x^n + a)^(3/2) + m*x^(m - 1)/sqrt(b*x^
n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n-1}}{2\,(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(
3/2)),x)
```

output
```
int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\frac{x^m\sqrt{x^nb+a}}{x^nb+a}$$

input
```
int(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x)
```

output
```
(x**m*sqrt(x**n*b + a))/(x**n*b + a)
```

3.605 $\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}} dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}} dx = \frac{5a^2x^{n/2}\sqrt{a+bx^n}}{8b^3n} - \frac{5ax^{3n/2}\sqrt{a+bx^n}}{12b^2n} + \frac{x^{5n/2}\sqrt{a+bx^n}}{3bn} - \frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{8b^{7/2}n}$$

output
```
5/8*a^2*x^(1/2*n)*(a+b*x^n)^(1/2)/b^3/n-5/12*a*x^(3/2*n)*(a+b*x^n)^(1/2)/b
^2/n+1/3*x^(5/2*n)*(a+b*x^n)^(1/2)/b/n-5/8*a^3*arctanh(b^(1/2)*x^(1/2*n)/(
a+b*x^n)^(1/2))/b^(7/2)/n
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}} dx = \frac{\sqrt{a+bx^n}\left(\sqrt{b}x^{n/2}(15a^2-10abx^n+8b^2x^{2n}) - \frac{15a^{5/2}\text{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^n}{a}}}\right)}{24b^{7/2}n}$$

input
```
Integrate[x^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(Sqrt[a + b*x^n]*(Sqrt[b]*x^(n/2)*(15*a^2 - 10*a*b*x^n + 8*b^2*x^(2*n)) -
(15*a^(5/2)*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]])/Sqrt[1 + (b*x^n)/a]))/(24*
b^(7/2)*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {880, 252, 252, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{7n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{880}$$

$$\frac{2a^3\int \frac{x^{3n}}{(bx^n+a)^3\left(1-\frac{bx^n}{bx^n+a}\right)^4}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{n}$$

$$\Big\downarrow \text{252}$$

$$\frac{2a^3\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\int \frac{x^{2n}}{(bx^n+a)^2\left(1-\frac{bx^n}{bx^n+a}\right)^3}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{6b}\right)}{n}$$

$$\Big\downarrow \text{252}$$

$$\frac{2a^3\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\int \frac{x^n}{(bx^n+a)\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{4b}\right)}{6b}\right)}{n}$$

$$\Big\downarrow \text{252}$$

$$\frac{2a^3\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\int \frac{1}{1-\frac{bx^n}{bx^n+a}}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{2b}\right)}{4b}\right)}{6b}\right)}{n}$$

$\Big\downarrow$ 219

$$\frac{2a^3\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{2b^{3/2}}\right)}{4b}\right)}{6b}\right)}{n}$$

input

```
Int[x^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

output

```
(2*a^3*(x^((5*n)/2)/(6*b*(a + b*x^n)^(5/2)*(1 - (b*x^n)/(a + b*x^n))^3) -
(5*(x^((3*n)/2)/(4*b*(a + b*x^n)^(3/2)*(1 - (b*x^n)/(a + b*x^n))^2) - (3*(
x^(n/2)/(2*b*Sqrt[a + b*x^n]*(1 - (b*x^n)/(a + b*x^n))) - ArcTanh[(Sqrt[b]
*x^(n/2))/Sqrt[a + b*x^n]]/(2*b^(3/2))))/(4*b)))/(6*b)))/n
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 880
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Simp[k*(a^(p + Simplify[(m + 1)/n])/n)   Subst[Int[x^(k*Simplify
[(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/
(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[p + Simpli
fy[(m + 1)/n]] && LtQ[-1, p, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{{\mathrm e}^{\frac{n \ln \left(x \right)}{2}} \left(8 \,{\mathrm e}^{2 n \ln \left(x \right)} b^{2}-10 a b \,{\mathrm e}^{n \ln \left(x \right)}+15 a^{2}\right) \sqrt{a +b \,{\mathrm e}^{n \ln \left(x \right)}}}{24 b^{3} n}-\frac{5 a^{3} \ln \left(\sqrt{b}\, {\mathrm e}^{\frac{n \ln \left(x \right)}{2}}+\sqrt{a +b \,{\mathrm e}^{n \ln \left(x \right)}}\right)}{8 b^{\frac{7}{2}} n}$	98

input
```
int(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*exp(1/2*n*ln(x))*(8*exp(1/2*n*ln(x))^4*b^2-10*a*exp(1/2*n*ln(x))^2*b+
15*a^2)*(a+b*exp(1/2*n*ln(x))^2)^(1/2)/b^3/n-5/8*a^3/b^(7/2)/n*ln(b^(1/2)*
exp(1/2*n*ln(x))+(a+b*exp(1/2*n*ln(x))^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$$
$$= \left[\frac{15\,a^3\sqrt{b}\log\left(2\,\sqrt{bx^n+a}\sqrt{b}x^{\frac{1}{2}\,n}-2\,bx^n-a\right)+2\left(8\,b^3x^{\frac{5}{2}\,n}-10\,ab^2x^{\frac{3}{2}\,n}+15\,a^2bx^{\frac{1}{2}\,n}\right)\sqrt{bx^n+a}}{48\,b^4n}, \frac{15\,a^3}{}\right.$$

input
```
integrate(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*log(2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n) - 2*b*x^n -
a) + 2*(8*b^3*x^(5/2*n) - 10*a*b^2*x^(3/2*n) + 15*a^2*b*x^(1/2*n))*sqrt(b*
x^n + a))/(b^4*n), 1/24*(15*a^3*sqrt(-b)*arctan(sqrt(b*x^n + a)*sqrt(-b)/(
b*x^(1/2*n))) + (8*b^3*x^(5/2*n) - 10*a*b^2*x^(3/2*n) + 15*a^2*b*x^(1/2*n)
)*sqrt(b*x^n + a))/(b^4*n)]
```

Sympy [A] (verification not implemented)

Time = 10.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{5a^{\frac{5}{2}}x^{\frac{n}{2}}}{8b^3n\sqrt{1+\frac{bx^n}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3n}{2}}}{24b^2n\sqrt{1+\frac{bx^n}{a}}}-\frac{\sqrt{a}x^{\frac{5n}{2}}}{12bn\sqrt{1+\frac{bx^n}{a}}} - \frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}n}+\frac{x^{\frac{7n}{2}}}{3\sqrt{a}n\sqrt{1+\frac{bx^n}{a}}}$$

input
```
integrate(x**(-1+7/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
5*a**(5/2)*x**(n/2)/(8*b**3*n*sqrt(1 + b*x**n/a)) + 5*a**(3/2)*x**(3*n/2)/
(24*b**2*n*sqrt(1 + b*x**n/a)) - sqrt(a)*x**(5*n/2)/(12*b*n*sqrt(1 + b*x**
n/a)) - 5*a**3*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(8*b**(7/2)*n) + x**(7*n/2)
/(3*sqrt(a)*n*sqrt(1 + b*x**n/a))
```

Maxima [F]

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{7}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{7}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{7\,n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^((7*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^((7*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{7n}{2}}\sqrt{x^n b+a}}{x^n bx+ax}dx$$

input
```
int(x^(-1+7/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((x**((7*n)/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x)
```

3.606 $\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=-\frac{3ax^{n/2}\sqrt{a+bx^n}}{4b^2n}+\frac{x^{3n/2}\sqrt{a+bx^n}}{2bn}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}n}$$

output
```
-3/4*a*x^(1/2*n)*(a+b*x^n)^(1/2)/b^2/n+1/2*x^(3/2*n)*(a+b*x^n)^(1/2)/b/n+3
/4*a^2*arctanh(b^(1/2)*x^(1/2*n)/(a+b*x^n)^(1/2))/b^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=\frac{\sqrt{b}x^{n/2}(-3a^2-abx^n+2b^2x^{2n})+3a^{5/2}\sqrt{1+\frac{bx^n}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{4b^{5/2}n\sqrt{a+bx^n}}$$

input
```
Integrate[x^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(Sqrt[b]*x^(n/2)*(-3*a^2 - a*b*x^n + 2*b^2*x^(2*n)) + 3*a^(5/2)*Sqrt[1 + (
b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(4*b^(5/2)*n*Sqrt[a + b*x^n]
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {880, 252, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{5n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{880}$$

$$\frac{2a^2\int \frac{x^{2n}}{(bx^n+a)^2\left(1-\frac{bx^n}{bx^n+a}\right)^3}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{n}$$

$$\Big\downarrow \text{252}$$

$$\frac{2a^2\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\int \frac{x^n}{(bx^n+a)\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{4b}\right)}{n}$$

$$\Big\downarrow \text{252}$$

$$\frac{2a^2\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\int \frac{1}{1-\frac{bx^n}{bx^n+a}}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{2b}\right)}{4b}\right)}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a^2\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{2b^{3/2}}\right)}{4b}\right)}{n}$$

input
```
Int[x^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*a^2*(x^((3*n)/2)/(4*b*(a + b*x^n)^(3/2)*(1 - (b*x^n)/(a + b*x^n))^2) -
(3*(x^(n/2)/(2*b*Sqrt[a + b*x^n]*(1 - (b*x^n)/(a + b*x^n))) - ArcTanh[(Sqr
t[b]*x^(n/2))/Sqrt[a + b*x^n]]/(2*b^(3/2))))/(4*b)))/n
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 880
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Simp[k*(a^(p + Simplify[(m + 1)/n])/n)   Subst[Int[x^(k*Simplify
[(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/
(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[p + Simpli
fy[(m + 1)/n]] && LtQ[-1, p, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{e^{\frac{n\ln(x)}{2}}\left(-2b\,e^{n\ln(x)}+3a\right)\sqrt{a+b\,e^{n\ln(x)}}}{4b^2n}+\frac{3a^2\ln\left(\sqrt{b}\,e^{\frac{n\ln(x)}{2}}+\sqrt{a+b\,e^{n\ln(x)}}\right)}{4b^{\frac{5}{2}}n}$	82

input
```
int(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(1/2*n*ln(x))*(-2*b*exp(1/2*n*ln(x))^2+3*a)*(a+b*exp(1/2*n*ln(x))^
2)^(1/2)/b^2/n+3/4*a^2/b^(5/2)/n*ln(b^(1/2)*exp(1/2*n*ln(x))+(a+b*exp(1/2*
n*ln(x))^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.58

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(-2\sqrt{bx^n+a}\sqrt{b}x^{\frac{1}{2}\,n}-2\,bx^n-a\right)+2\left(2\,b^2x^{\frac{3}{2}\,n}-3\,abx^{\frac{1}{2}\,n}\right)\sqrt{bx^n+a}}{8\,b^3n},\ -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-b}}{bx^{\frac{1}{2}\,n}}\right)-\left(2\,b^2x^{\frac{3}{2}\,n}-3\,abx^{\frac{1}{2}\,n}\right)\sqrt{bx^n+a}}{4\,b^3n}\right]$$

input
```
integrate(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*log(-2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n) - 2*b*x^n - a
) + 2*(2*b^2*x^(3/2*n) - 3*a*b*x^(1/2*n))*sqrt(b*x^n + a))/(b^3*n), -1/4*(
3*a^2*sqrt(-b)*arctan(sqrt(b*x^n + a)*sqrt(-b)/(b*x^(1/2*n))) - (2*b^2*x^(
3/2*n) - 3*a*b*x^(1/2*n))*sqrt(b*x^n + a))/(b^3*n)]
```

Sympy [A] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{3a^{\frac{3}{2}}x^{\frac{n}{2}}}{4b^2n\sqrt{1+\frac{bx^n}{a}}} - \frac{\sqrt{a}x^{\frac{3n}{2}}}{4bn\sqrt{1+\frac{bx^n}{a}}} + \frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}n} + \frac{x^{\frac{5n}{2}}}{2\sqrt{a}n\sqrt{1+\frac{bx^n}{a}}}$$

input
```
integrate(x**(-1+5/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-3*a**(3/2)*x**(n/2)/(4*b**2*n*sqrt(1 + b*x**n/a)) - sqrt(a)*x**(3*n/2)/(4
*b*n*sqrt(1 + b*x**n/a)) + 3*a**2*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(4*b**(5
/2)*n) + x**(5*n/2)/(2*sqrt(a)*n*sqrt(1 + b*x**n/a))
```

Maxima [F]

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{\frac{5\,n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^((5*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^((5*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=\int \frac{x^{\frac{5n}{2}}\sqrt{x^nb+a}}{x^nbx+ax}dx$$

input
```
int(x^(-1+5/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((x**((5*n)/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x)
```

3.607 $\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{x^{n/2}\sqrt{a+bx^n}}{bn} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}n}$$

output
```
x^(1/2*n)*(a+b*x^n)^(1/2)/b/n-a*arctanh(b^(1/2)*x^(1/2*n)/(a+b*x^n)^(1/2))
/b^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\sqrt{b}x^{n/2}(a+bx^n) - a^{3/2}\sqrt{1+\frac{bx^n}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{b^{3/2}n\sqrt{a+bx^n}}$$

input
```
Integrate[x^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(Sqrt[b]*x^(n/2)*(a + b*x^n) - a^(3/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b
]*x^(n/2))/Sqrt[a]])/(b^(3/2)*n*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {880, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{3n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 880}$$

$$\frac{2a\int \frac{x^n}{(bx^n+a)\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{n}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{2a\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\int \frac{1}{1-\frac{bx^n}{bx^n+a}}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{2b}\right)}{n}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2a\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{2b^{3/2}}\right)}{n}$$

input
```
Int[x^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*a*(x^(n/2)/(2*b*Sqrt[a + b*x^n]*(1 - (b*x^n)/(a + b*x^n))) - ArcTanh[(S
qrt[b]*x^(n/2))/Sqrt[a + b*x^n]]/(2*b^(3/2))))/n
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 880
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Simp[k*(a^(p + Simplify[(m + 1)/n])/n)   Subst[Int[x^(k*Simplify
[(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/
(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[p + Simpli
fy[(m + 1)/n]] && LtQ[-1, p, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{e^{\frac{n\ln(x)}{2}}\sqrt{a+b\,e^{n\ln(x)}}}{bn}-\frac{a\ln\left(\sqrt{b}\,e^{\frac{n\ln(x)}{2}}+\sqrt{a+b\,e^{n\ln(x)}}\right)}{b^{\frac{3}{2}}n}$	64

input
```
int(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/n*exp(1/2*n*ln(x))*(a+b*exp(1/2*n*ln(x))^2)^(1/2)-a/b^(3/2)/n*ln(b^(1/
2)*exp(1/2*n*ln(x))+(a+b*exp(1/2*n*ln(x))^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \left[\frac{2\sqrt{bx^n+a}bx^{\frac{1}{2}n}+a\sqrt{b}\log\left(2\sqrt{bx^n+a}\sqrt{b}x^{\frac{1}{2}n}-2bx^n-a\right)}{2b^2n}, \frac{\sqrt{bx^n+a}bx^{\frac{1}{2}n}+a\sqrt{-b}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-b}}{bx^{\frac{1}{2}n}}\right)}{b^2n}\right]$$

input
```
integrate(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(b*x^n + a)*b*x^(1/2*n) + a*sqrt(b)*log(2*sqrt(b*x^n + a)*sqrt
(b)*x^(1/2*n) - 2*b*x^n - a))/(b^2*n), (sqrt(b*x^n + a)*b*x^(1/2*n) + a*sq
rt(-b)*arctan(sqrt(b*x^n + a)*sqrt(-b)/(b*x^(1/2*n))))/(b^2*n)]
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\sqrt{a}x^{\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}}{bn} - \frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}n}$$

input
```
integrate(x**(-1+3/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
sqrt(a)*x**(n/2)*sqrt(1 + b*x**n/a)/(b*n) - a*asinh(sqrt(b)*x**(n/2)/sqrt(
a))/(b**(3/2)*n)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{3\,n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^((3*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^((3*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{3n}{2}}\sqrt{x^n b+a}}{x^n bx+ax}dx$$

input
```
int(x^(-1+3/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((x**((3*n)/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x)
```

3.608 $\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}n}$$

output
```
2*arctanh(b^(1/2)*x^(1/2*n)/(a+b*x^n)^(1/2))/b^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{a}\sqrt{1+\frac{bx^n}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{a+bx^n}}$$

input
```
Integrate[x^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a]*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]])/(Sqrt[b
]*n*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {868, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{868}$$

$$\frac{2\int \frac{1}{\sqrt{bx^n+a}}dx^{n/2}}{n}$$

$$\Big\downarrow \text{224}$$

$$\frac{2\int \frac{1}{1-bx^n}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{n}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}n}$$

input
```
Int[x^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]])/(Sqrt[b]*n)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^{n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^{n}}}\,dx = \frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}n}$$

input
```
integrate(x**(-1+1/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
2*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(sqrt(b)*n)
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(1/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(27) = 54$.

Time $= 0.45$ (sec) , antiderivative size $= 89$, normalized size of antiderivative $= 2.54$

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{\frac{a\log\left(\sqrt{-2\sqrt{bx^n+a}\sqrt{b}|x|^{\frac{1}{2}n}\cos\left(-\frac{1}{4}\pi n\mathrm{sgn}(x)+\frac{1}{4}\pi n-\pi\left\lfloor-\frac{1}{4}n\mathrm{sgn}(x)+\frac{1}{4}n+\frac{1}{2}\right\rfloor\right)+bx^n+b|x|^n+a}\right)}{\sqrt{b}}-\sqrt{bx^n+a}\sqrt{x^n}}{n}$$

input
```
integrate(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
-(a*log(sqrt(-2*sqrt(b*x^n + a)*sqrt(b)*abs(x)^(1/2*n)*cos(-1/4*pi*n*sgn(x
) + 1/4*pi*n - pi*floor(-1/4*n*sgn(x) + 1/4*n + 1/2)) + b*x^n + b*abs(x)^n
 + a))/sqrt(b) - sqrt(b*x^n + a)*sqrt(x^n))/n
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(x^(n/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int(x^(n/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{\frac{n}{2}}\sqrt{x^n b+a}}{x^n bx+ax}dx$$

input
```
int(x^(-1+1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((x**(n/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x)
```

3.609 $\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx=-\frac{2x^{-n/2}\sqrt{a+bx^n}}{an}$$

output
```
-2*(a+b*x^n)^(1/2)/a/n/(x^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx=-\frac{2x^{-n/2}\sqrt{a+bx^n}}{an}$$

input
```
Integrate[x^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(a*n*x^(n/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{2x^{-n/2}\sqrt{a+bx^n}}{an}$$

input
```
Int[x^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(a*n*x^(n/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{ax^{-n}}{b}+1}}{an}$$

input
```
integrate(x**(-1-1/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x**n) + 1)/(a*n)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{1}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\,\sqrt{a+b\,x^n}}{a\,n\,x^{n/2}}$$

input
```
int(1/(x^(n/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
-(2*(a + b*x^n)^(1/2))/(a*n*x^(n/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\sqrt{x^n b+a}}{x^{\frac{n}{2}}an}$$

input
```
int(x^(-1-1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(x**n*b + a))/(x**(n/2)*a*n)
```

3.610 $\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}} dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}} dx = -\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an} + \frac{4bx^{-n/2}\sqrt{a+bx^n}}{3a^2n}$$

output
```
-2/3*(a+b*x^n)^(1/2)/a/n/(x^(3/2*n))+4/3*b*(a+b*x^n)^(1/2)/a^2/n/(x^(1/2*n
))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}} dx = -\frac{2x^{-3n/2}(a-2bx^n)\sqrt{a+bx^n}}{3a^2n}$$

input
```
Integrate[x^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*(a - 2*b*x^n)*Sqrt[a + b*x^n])/(3*a^2*n*x^((3*n)/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{3n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{2b\int \frac{x^{-\frac{n}{2}-1}}{\sqrt{bx^n+a}}dx}{3a}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}$$

$$\Big\downarrow \text{796}$$

$$\frac{4bx^{-n/2}\sqrt{a+bx^n}}{3a^2n}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}$$

input
```
Int[x^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(3*a*n*x^((3*n)/2)) + (4*b*Sqrt[a + b*x^n])/(3*a^2*n*
x^(n/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\sqrt{b}x^{-n}\sqrt{\frac{ax^{-n}}{b}+1}}{3an} + \frac{4b^{\frac{3}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^2n}$$

input
```
integrate(x**(-1-3/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x**n) + 1)/(3*a*n*x**n) + 4*b**(3/2)*sqrt(a/(b*x**n)
+ 1)/(3*a**2*n)
```

Maxima [F]

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{x^{\frac{3\,n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^((3*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^((3*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{x^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^n b+a}\,(2x^n b-a)}{3x^{\frac{3n}{2}}a^2 n}$$

input
```
int(x^(-1-3/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(2*x**n*b - a))/(3*x**((3*n)/2)*a**2*n)
```

3.611 $\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}} dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}} \, dx = -\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an} + \frac{8bx^{-3n/2}\sqrt{a+bx^n}}{15a^2n} - \frac{16b^2x^{-n/2}\sqrt{a+bx^n}}{15a^3n}$$

output
```
-2/5*(a+b*x^n)^(1/2)/a/n/(x^(5/2*n))+8/15*b*(a+b*x^n)^(1/2)/a^2/n/(x^(3/2*
n))-16/15*b^2*(a+b*x^n)^(1/2)/a^3/n/(x^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.57

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}} \, dx = -\frac{2x^{-5n/2}\sqrt{a+bx^n}(3a^2-4abx^n+8b^2x^{2n})}{15a^3n}$$

input
```
Integrate[x^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n]*(3*a^2 - 4*a*b*x^n + 8*b^2*x^(2*n)))/(15*a^3*n*x^((5*n
)/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{5n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{4b\int \frac{x^{-\frac{3n}{2}-1}}{\sqrt{bx^n+a}}dx}{5a}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}$$

$$\Big\downarrow \text{803}$$

$$-\frac{4b\left(-\frac{2b\int \frac{x^{-\frac{n}{2}-1}}{\sqrt{bx^n+a}}dx}{3a}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}\right)}{5a}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}$$

$$\Big\downarrow \text{796}$$

$$-\frac{4b\left(\frac{4bx^{-n/2}\sqrt{a+bx^n}}{3a^2n}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}\right)}{5a}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}$$

input
```
Int[x^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(5*a*n*x^((5*n)/2)) - (4*b*((-2*Sqrt[a + b*x^n])/(3*a
*n*x^((3*n)/2)) + (4*b*Sqrt[a + b*x^n])/(3*a^2*n*x^(n/2))))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(76) = 152$.

Time = 0.82 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.98

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{4a^3b^{\frac{11}{2}}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{6a^2b^{\frac{13}{2}}x^{2n}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{24ab^{\frac{15}{2}}x^{3n}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{16b^{\frac{17}{2}}x^{4n}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}}$$

input
```
integrate(x**(-1-5/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-6*a**4*b**(9/2)*sqrt(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b
**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 4*a**3*b**(11/2)*x**n*sqrt(a/(
b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n) + 15*a**3*
b**6*n*x**(4*n)) - 6*a**2*b**(13/2)*x**(2*n)*sqrt(a/(b*x**n) + 1)/(15*a**5
*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 24
*a*b**(15/2)*x**(3*n)*sqrt(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a
**4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 16*b**(17/2)*x**(4*n)*sqr
t(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n) + 15*
a**3*b**6*n*x**(4*n))
```

Maxima [F]

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{5}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{5}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{x^{\frac{5\,n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^((5*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^((5*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{\sqrt{x^n b+a}}{x^{\frac{7n}{2}} bx + x^{\frac{5n}{2}} ax}dx$$

input
```
int(x^(-1-5/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**((7*n)/2)*b*x + x**((5*n)/2)*a*x),x)
```

3.612 $\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}} dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}} dx = -\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an} + \frac{12bx^{-5n/2}\sqrt{a+bx^n}}{35a^2n} - \frac{16b^2x^{-3n/2}\sqrt{a+bx^n}}{35a^3n} + \frac{32b^3x^{-n/2}\sqrt{a+bx^n}}{35a^4n}$$

output
```
-2/7*(a+b*x^n)^(1/2)/a/n/(x^(7/2*n))+12/35*b*(a+b*x^n)^(1/2)/a^2/n/(x^(5/2
*n))-16/35*b^2*(a+b*x^n)^(1/2)/a^3/n/(x^(3/2*n))+32/35*b^3*(a+b*x^n)^(1/2)
/a^4/n/(x^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}} dx = -\frac{2x^{-7n/2}\sqrt{a+bx^n}(5a^3-6a^2bx^n+8ab^2x^{2n}-16b^3x^{3n})}{35a^4n}$$

input
```
Integrate[x^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n]*(5*a^3 - 6*a^2*b*x^n + 8*a*b^2*x^(2*n) - 16*b^3*x^(3*n
)))/(35*a^4*n*x^((7*n)/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-\frac{7n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{6b\int \frac{x^{-\frac{5n}{2}-1}}{\sqrt{bx^n+a}}dx}{7a}-\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}$$

$$\Big\downarrow \text{803}$$

$$-\frac{6b\left(-\frac{4b\int \frac{x^{-\frac{3n}{2}-1}}{\sqrt{bx^n+a}}dx}{5a}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}\right)}{7a}-\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}$$

$$\Big\downarrow \text{803}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{x^{-\frac{n}{2}-1}}{\sqrt{bx^n+a}}dx}{3a}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}\right)}{5a}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}\right)}{7a}-\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}$$

$$\Big\downarrow \text{796}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{4bx^{-n/2}\sqrt{a+bx^n}}{3a^2n}-\frac{2x^{-3n/2}\sqrt{a+bx^n}}{3an}\right)}{5a}-\frac{2x^{-5n/2}\sqrt{a+bx^n}}{5an}\right)}{7a}-\frac{2x^{-7n/2}\sqrt{a+bx^n}}{7an}$$

input
```
Int[x^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(7*a*n*x^((7*n)/2)) - (6*b*((-2*Sqrt[a + b*x^n])/(5*a
*n*x^((5*n)/2)) - (4*b*((-2*Sqrt[a + b*x^n])/(3*a*n*x^((3*n)/2)) + (4*b*Sq
rt[a + b*x^n])/(3*a^2*n*x^(n/2))))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(105) = 210.

Time = 1.09 (sec) , antiderivative size = 605, normalized size of antiderivative = 5.04

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} - \frac{18a^5b^{\frac{21}{2}}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} - \frac{10a^4b^{\frac{23}{2}}x^{2n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{10a^3b^{\frac{25}{2}}x^{3n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{60a^2b^{\frac{27}{2}}x^{4n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{80ab^{\frac{29}{2}}x^{5n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{32b^{\frac{31}{2}}x^{6n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}}$$

input
```
integrate(x**(-1-7/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-10*a**6*b**(19/2)*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**
6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n))
 - 18*a**5*b**(21/2)*x**n*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*x**(3*n) +
105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x*
*(6*n)) - 10*a**4*b**(23/2)*x**(2*n)*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*
x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4
*b**12*n*x**(6*n)) + 10*a**3*b**(25/2)*x**(3*n)*sqrt(a/(b*x**n) + 1)/(35*a
**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n
) + 35*a**4*b**12*n*x**(6*n)) + 60*a**2*b**(27/2)*x**(4*n)*sqrt(a/(b*x**n)
 + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**1
1*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n)) + 80*a*b**(29/2)*x**(5*n)*sqrt(a/
(b*x**n) + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a
**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n)) + 32*b**(31/2)*x**(6*n)*s
qrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) +
 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n))
```

Maxima [F]

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{7}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{x^{-\frac{7}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{x^{\frac{7\,n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/(x^((7*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/(x^((7*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{\sqrt{x^n b+a}}{x^{\frac{9n}{2}}bx+x^{\frac{7n}{2}}ax}dx$$

input
```
int(x^(-1-7/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**((9*n)/2)*b*x + x**((7*n)/2)*a*x),x)
```

3.613 $\int x^m(a+bx^{2+2m})^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int x^m\left(a+bx^{2+2m}\right)^{5/2}\,dx=\frac{5a^2x^{1+m}\sqrt{a+bx^{2(1+m)}}}{16(1+m)}+\frac{5ax^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{24(1+m)}+\frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{5/2}}{6(1+m)}+\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a+bx^{2(1+m)}}}\right)}{16\sqrt{b}(1+m)}$$

output
```
5*a^2*x^(1+m)*(a+b*x^(2+2*m))^(1/2)/(16+16*m)+5*a*x^(1+m)*(a+b*x^(2+2*m))^
(3/2)/(24+24*m)+x^(1+m)*(a+b*x^(2+2*m))^(5/2)/(6+6*m)+5/16*a^3*arctanh(b^(
1/2)*x^(1+m)/(a+b*x^(2+2*m))^(1/2))/b^(1/2)/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.65

$$\int x^m\left(a+bx^{2+2m}\right)^{5/2}\,dx=\frac{a^2x^{1+m}\sqrt{a+bx^{2+2m}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{2+2m},1+\frac{1+m}{2+2m},-\frac{bx^{2+2m}}{a}\right)}{(1+m)\sqrt{1+\frac{bx^{2+2m}}{a}}}$$

input
```
Integrate[x^m*(a + b*x^(2 + 2*m))^(5/2),x]
```

output
```
(a^2*x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)]*Hypergeometric2F1[-5/2, (1 + m)/(2
+ 2*m), 1 + (1 + m)/(2 + 2*m), -((b*x^(2 + 2*m))/a)])/((1 + m)*Sqrt[1 + (b
*x^(2 + 2*m))/a])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {868, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^{2m+2}\right)^{5/2} dx$$

$$\Big\downarrow \text{868}$$

$$\frac{\int \left(bx^{2m+2}+a\right)^{5/2}dx^{m+1}}{m+1}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{5}{6}a\int \left(bx^{2m+2}+a\right)^{3/2}dx^{m+1}+\frac{1}{6}x^{m+1}\left(a+bx^{2m+2}\right)^{5/2}}{m+1}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^{2m+2}+a}dx^{m+1}+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}\right)+\frac{1}{6}x^{m+1}\left(a+bx^{2m+2}\right)^{5/2}}{m+1}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^{2m+2}+a}}dx^{m+1}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}\right)+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}\right)+\frac{1}{6}x^{m+1}\left(a+bx^{2m+2}\right)^{5/2}}{m+1}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-bx^{2m+2}}d\frac{x^{m+1}}{\sqrt{bx^{2m+2}+a}}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}\right)+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}\right)+\frac{1}{6}x^{m+1}\left(a+bx^{2m+2}\right)^{5/2}}{m+1}$$

$\downarrow$ 219

$$\frac{\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{m+1}}{\sqrt{a+bx^{2m+2}}}\right)}{2\sqrt{b}}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}\right)+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}\right)+\frac{1}{6}x^{m+1}\left(a+bx^{2m+2}\right)^{5/2}}{m+1}$$

input
```
Int[x^m*(a + b*x^(2 + 2*m))^(5/2),x]
```

output
```
((x^(1 + m)*(a + b*x^(2 + 2*m))^(5/2))/6 + (5*a*((x^(1 + m)*(a + b*x^(2 +
2*m))^(3/2))/4 + (3*a*((x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)])/2 + (a*ArcTanh[
(Sqrt[b]*x^(1 + m))/Sqrt[a + b*x^(2 + 2*m)]])/(2*Sqrt[b])))/4))/6)/(1 + m)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [F]

$$\int x^{m} \left(a+b\,x^{2+2 m}\right)^{\frac{5}{2}}d x$$

input
```
int(x^m*(a+b*x^(2+2*m))^(5/2),x)
```

output
```
int(x^m*(a+b*x^(2+2*m))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \left(a+bx^{2+2m}\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 53.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int x^m \left(a+bx^{2+2m}\right)^{5/2} dx = \frac{\sqrt{\pi}\sqrt{a}a^{-\frac{m}{2m+2}+\frac{5}{2}-\frac{1}{2m+2}}x^{m+1}{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\frac{bx^{2m+2}e^{i\pi}}{a}\right)}{2m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

input
```
integrate(x**m*(a+b*x**(2+2*m))**(5/2),x)
```

output
```
sqrt(pi)*sqrt(a)*a**(-m/(2*m + 2) + 5/2 - 1/(2*m + 2))*x**(m + 1)*hyper((-
5/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**(2*m + 2)*exp_polar(I*pi
)/a)/(2*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*gamma(m/(2*m + 2) + 1 +
 1/(2*m + 2)))
```

Maxima [F]

$$\int x^m\left(a+bx^{2+2m}\right)^{5/2}\,dx=\int\left(bx^{2\,m+2}+a\right)^{\frac{5}{2}}x^m\,dx$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(2*m + 2) + a)^(5/2)*x^m, x)
```

Giac [F]

$$\int x^m\left(a+bx^{2+2m}\right)^{5/2}\,dx=\int\left(bx^{2\,m+2}+a\right)^{\frac{5}{2}}x^m\,dx$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^(2*m + 2) + a)^(5/2)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m\left(a+bx^{2+2m}\right)^{5/2}\,dx=\int x^m\,\left(a+b\,x^{2\,m+2}\right)^{5/2}\,dx$$

input
```
int(x^m*(a + b*x^(2*m + 2))^(5/2),x)
```

output
```
int(x^m*(a + b*x^(2*m + 2))^(5/2), x)
```

Reduce [F]

$$\int x^m \left(a + b x^{2+2m}\right)^{5/2} dx = \frac{8x^{5m}\sqrt{x^{2m} b\, x^2 + a}\, b^2 x^5 + 26x^{3m}\sqrt{x^{2m} b\, x^2 + a}\, ab\, x^3 + 33x^m\sqrt{x^{2m} b\, x^2 + a}\, a^2 x + 15\left(\int \frac{x^{m}\cdots}{\cdots}\right.}{48m + 48}$$

input
```
int(x^m*(a+b*x^(2+2*m))^(5/2),x)
```

output
```
(8*x**(5*m)*sqrt(x**(2*m)*b*x**2 + a)*b**2*x**5 + 26*x**(3*m)*sqrt(x**(2*m
)*b*x**2 + a)*a*b*x**3 + 33*x**m*sqrt(x**(2*m)*b*x**2 + a)*a**2*x + 15*int
((x**m*sqrt(x**(2*m)*b*x**2 + a))/(x**(2*m)*b*x**2 + a),x)*a**3*m + 15*int
((x**m*sqrt(x**(2*m)*b*x**2 + a))/(x**(2*m)*b*x**2 + a),x)*a**3)/(48*(m + 
1))
```

3.614 $\int x^m(a+bx^{2+2m})^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int x^m\left(a+bx^{2+2m}\right)^{3/2}\,dx=\frac{3ax^{1+m}\sqrt{a+bx^{2(1+m)}}}{8(1+m)}+\frac{x^{1+m}\left(a+bx^{2(1+m)}\right)^{3/2}}{4(1+m)}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a+bx^{2(1+m)}}}\right)}{8\sqrt{b}(1+m)}$$

output
```
3*a*x^(1+m)*(a+b*x^(2+2*m))^(1/2)/(8+8*m)+x^(1+m)*(a+b*x^(2+2*m))^(3/2)/(4
+4*m)+3/8*a^2*arctanh(b^(1/2)*x^(1+m)/(a+b*x^(2+2*m))^(1/2))/b^(1/2)/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int x^m\left(a+bx^{2+2m}\right)^{3/2}\,dx=\frac{ax^{1+m}\sqrt{a+bx^{2+2m}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2+2m},1+\frac{1+m}{2+2m},-\frac{bx^{2+2m}}{a}\right)}{(1+m)\sqrt{1+\frac{bx^{2+2m}}{a}}}$$

input
```
Integrate[x^m*(a + b*x^(2 + 2*m))^(3/2),x]
```

output
```
(a*x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)]*Hypergeometric2F1[-3/2, (1 + m)/(2 +
2*m), 1 + (1 + m)/(2 + 2*m), -((b*x^(2 + 2*m))/a)])/((1 + m)*Sqrt[1 + (b*x
^(2 + 2*m))/a])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {868, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^{2m+2}\right)^{3/2} dx$$

$$\Big\downarrow \text{868}$$

$$\frac{\int \left(bx^{2m+2}+a\right)^{3/2}dx^{m+1}}{m+1}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}a\int \sqrt{bx^{2m+2}+a}dx^{m+1}+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}}{m+1}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^{2m+2}+a}}dx^{m+1}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}\right)+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}}{m+1}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-bx^{2m+2}}d\frac{x^{m+1}}{\sqrt{bx^{2m+2}+a}}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}\right)+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}}{m+1}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{m+1}}{\sqrt{a+bx^{2m+2}}}\right)}{2\sqrt{b}}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}\right)+\frac{1}{4}x^{m+1}\left(a+bx^{2m+2}\right)^{3/2}}{m+1}$$

input
```
Int[x^m*(a + b*x^(2 + 2*m))^(3/2),x]
```

output
```
((x^(1 + m)*(a + b*x^(2 + 2*m))^(3/2))/4 + (3*a*((x^(1 + m)*Sqrt[a + b*x^(
2 + 2*m)])/2 + (a*ArcTanh[(Sqrt[b]*x^(1 + m))/Sqrt[a + b*x^(2 + 2*m)]])/(2
*Sqrt[b])))/4)/(1 + m)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [F]

$$\int x^{m} \left(a + b\, x^{2+2 m}\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a+b*x^(2+2*m))^(3/2),x)
```

output
```
int(x^m*(a+b*x^(2+2*m))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \left(a + bx^{2+2m}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int x^m \left(a+bx^{2+2m}\right)^{3/2} dx = \frac{\sqrt{\pi}\sqrt{a}a^{-\frac{m}{2m+2}+\frac{3}{2}-\frac{1}{2m+2}}x^{m+1}{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{2} \\ \frac{m}{2m+2}+1+\frac{1}{2m+2} \end{matrix}\middle| \frac{bx^{2m+2}e^{i\pi}}{a}\right)}{2m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

input
```
integrate(x**m*(a+b*x**(2+2*m))**(3/2),x)
```

output
```
sqrt(pi)*sqrt(a)*a**(-m/(2*m + 2) + 3/2 - 1/(2*m + 2))*x**(m + 1)*hyper((-
3/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**(2*m + 2)*exp_polar(I*pi
)/a)/(2*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*gamma(m/(2*m + 2) + 1 +
 1/(2*m + 2)))
```

Maxima [F]

$$\int x^m \left(a+bx^{2+2m}\right)^{3/2} dx = \int \left(bx^{2m+2}+a\right)^{\frac{3}{2}} x^m dx$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(2*m + 2) + a)^(3/2)*x^m, x)
```

Giac [F]

$$\int x^m \left(a+bx^{2+2m}\right)^{3/2} dx = \int \left(bx^{2m+2}+a\right)^{\frac{3}{2}} x^m dx$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^(2*m + 2) + a)^(3/2)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(a+bx^{2+2m}\right)^{3/2} dx = \int x^m \left(a+b\,x^{2m+2}\right)^{3/2} dx$$

input
```
int(x^m*(a + b*x^(2*m + 2))^(3/2),x)
```

output
```
int(x^m*(a + b*x^(2*m + 2))^(3/2), x)
```

Reduce [F]

$$\int x^m \left(a+bx^{2+2m}\right)^{3/2}\,dx=\frac{2x^{3m}\sqrt{x^{2m}b\,x^2+a}\,b\,x^3+5x^m\sqrt{x^{2m}b\,x^2+a}\,ax+3\left(\int \frac{x^m\sqrt{x^{2m}b\,x^2+a}}{x^{2m}b\,x^2+a}dx\right)a^2m+3\left(\int \frac{x^m\sqrt{x}}{x^{2m}}\right.}{8m+8}$$

input
```
int(x^m*(a+b*x^(2+2*m))^(3/2),x)
```

output
```
(2*x**(3*m)*sqrt(x**(2*m)*b*x**2 + a)*b*x**3 + 5*x**m*sqrt(x**(2*m)*b*x**2
 + a)*a*x + 3*int((x**m*sqrt(x**(2*m)*b*x**2 + a))/(x**(2*m)*b*x**2 + a),x
)*a**2*m + 3*int((x**m*sqrt(x**(2*m)*b*x**2 + a))/(x**(2*m)*b*x**2 + a),x)
*a**2)/(8*(m + 1))
```

3.615 $\int x^m \sqrt{a+bx^{2+2m}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int x^m \sqrt{a+bx^{2+2m}}\,dx=\frac{x^{1+m}\sqrt{a+bx^{2(1+m)}}}{2(1+m)}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a+bx^{2(1+m)}}}\right)}{2\sqrt{b}(1+m)}$$

output
```
x^(1+m)*(a+b*x^(2+2*m))^(1/2)/(2+2*m)+1/2*a*arctanh(b^(1/2)*x^(1+m)/(a+b*x
^(2+2*m))^(1/2))/b^(1/2)/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int x^m \sqrt{a+bx^{2+2m}}\,dx =\frac{x^{1+m}\sqrt{a+bx^{2+2m}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2+2m},1+\frac{1+m}{2+2m},-\frac{bx^{2+2m}}{a}\right)}{(1+m)\sqrt{1+\frac{bx^{2+2m}}{a}}}$$

input
```
Integrate[x^m*Sqrt[a + b*x^(2 + 2*m)],x]
```

output
```
(x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)]*Hypergeometric2F1[-1/2, (1 + m)/(2 + 2*
m), 1 + (1 + m)/(2 + 2*m), -((b*x^(2 + 2*m))/a)])/((1 + m)*Sqrt[1 + (b*x^(
2 + 2*m))/a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {868, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{a+bx^{2m+2}}\,dx$$

$$\Big\downarrow \text{868}$$

$$\frac{\int \sqrt{bx^{2m+2}+a}dx^{m+1}}{m+1}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{2}a\int \frac{1}{\sqrt{bx^{2m+2}+a}}dx^{m+1}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}}{m+1}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}a\int \frac{1}{1-bx^{2m+2}}d\frac{x^{m+1}}{\sqrt{bx^{2m+2}+a}}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}}{m+1}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^{m+1}}{\sqrt{a+bx^{2m+2}}}\right)}{2\sqrt{b}}+\frac{1}{2}x^{m+1}\sqrt{a+bx^{2m+2}}}{m+1}$$

input
```
Int[x^m*Sqrt[a + b*x^(2 + 2*m)],x]
```

output
```
((x^(1 + m)*Sqrt[a + b*x^(2 + 2*m)])/2 + (a*ArcTanh[(Sqrt[b]*x^(1 + m))/Sq
rt[a + b*x^(2 + 2*m)]])/(2*Sqrt[b]))/(1 + m)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [F]

$$\int x^m \sqrt{a + b\,x^{2+2m}}dx$$

input
```
int(x^m*(a+b*x^(2+2*m))^(1/2),x)
```

output
```
int(x^m*(a+b*x^(2+2*m))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sqrt{a + bx^{2+2m}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int x^m\sqrt{a+bx^{2+2m}}\,dx = \frac{\sqrt{\pi}\sqrt{a}a^{-\frac{m}{2m+2}+\frac{1}{2}-\frac{1}{2m+2}}x^{m+1}{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\frac{bx^{2m+2}e^{i\pi}}{a}\right)}{2m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

input
```
integrate(x**m*(a+b*x**(2+2*m))**(1/2),x)
```

output
```
sqrt(pi)*sqrt(a)*a**(-m/(2*m + 2) + 1/2 - 1/(2*m + 2))*x**(m + 1)*hyper((-
1/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**(2*m + 2)*exp_polar(I*pi
)/a)/(2*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*gamma(m/(2*m + 2) + 1 +
 1/(2*m + 2)))
```

Maxima [F]

$$\int x^m\sqrt{a+bx^{2+2m}}\,dx = \int\sqrt{bx^{2m+2}+a}\,x^m\,dx$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^(2*m + 2) + a)*x^m, x)
```

Giac [F]

$$\int x^m\sqrt{a+bx^{2+2m}}\,dx=\int\sqrt{bx^{2\,m+2}+a}\,x^m\,dx$$

input
```
integrate(x^m*(a+b*x^(2+2*m))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^(2*m + 2) + a)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m\sqrt{a+bx^{2+2m}}\,dx=\int x^m\,\sqrt{a+b\,x^{2\,m+2}}\,dx$$

input
```
int(x^m*(a + b*x^(2*m + 2))^(1/2),x)
```

output
```
int(x^m*(a + b*x^(2*m + 2))^(1/2), x)
```

Reduce [F]

$$\int x^m\sqrt{a+bx^{2+2m}}\,dx$$
$$=\frac{x^m\sqrt{x^{2m}b\,x^2+a}\,x+\left(\int\frac{x^m\sqrt{x^{2m}b\,x^2+a}}{x^{2m}b\,x^2+a}dx\right)am+\left(\int\frac{x^m\sqrt{x^{2m}b\,x^2+a}}{x^{2m}b\,x^2+a}dx\right)a}{2m+2}$$

input
```
int(x^m*(a+b*x^(2+2*m))^(1/2),x)
```

output
```
(x**m*sqrt(x**(2*m)*b*x**2 + a)*x + int((x**m*sqrt(x**(2*m)*b*x**2 + a))/(
x**(2*m)*b*x**2 + a),x)*a*m + int((x**m*sqrt(x**(2*m)*b*x**2 + a))/(x**(2*
m)*b*x**2 + a),x)*a)/(2*(m + 1))
```

3.616 $\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a+bx^{2(1+m)}}}\right)}{\sqrt{b}(1+m)}$$

output
```
arctanh(b^(1/2)*x^(1+m)/(a+b*x^(2+2*m))^(1/2))/b^(1/2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx=\frac{\sqrt{a}\sqrt{1+\frac{bx^{2+2m}}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^{1+m}}{\sqrt{a}}\right)}{\sqrt{b}(1+m)\sqrt{a+bx^{2+2m}}}$$

input
```
Integrate[x^m/Sqrt[a + b*x^(2 + 2*m)],x]
```

output
```
(Sqrt[a]*Sqrt[1 + (b*x^(2 + 2*m))/a]*ArcSinh[(Sqrt[b]*x^(1 + m))/Sqrt[a]])
/(Sqrt[b]*(1 + m)*Sqrt[a + b*x^(2 + 2*m)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {868, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx^{2m+2}}}dx$$

$$\Big\downarrow \text{ 868}$$

$$\frac{\int \frac{1}{\sqrt{bx^{2m+2}+a}}dx^{m+1}}{m+1}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\int \frac{1}{1-bx^{2m+2}}d\frac{x^{m+1}}{\sqrt{bx^{2m+2}+a}}}{m+1}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{m+1}}{\sqrt{a+bx^{2m+2}}}\right)}{\sqrt{b}(m+1)}$$

input
```
Int[x^m/Sqrt[a + b*x^(2 + 2*m)],x]
```

output
```
ArcTanh[(Sqrt[b]*x^(1 + m))/Sqrt[a + b*x^(2 + 2*m)]]/(Sqrt[b]*(1 + m))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{x^m}{\sqrt{a+b\,x^{2+2m}}}dx$$

input
```
int(x^m/(a+b*x^(2+2*m))^(1/2),x)
```

output
```
int(x^m/(a+b*x^(2+2*m))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.82

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx = \frac{\sqrt{\pi}\sqrt{a}a^{-\frac{m}{2m+2}-\frac{1}{2}-\frac{1}{2m+2}}x^{m+1}{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\frac{bx^{2m+2}e^{i\pi}}{a}\right)}{2m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

input
```
integrate(x**m/(a+b*x**(2+2*m))**(1/2),x)
```

output
```
sqrt(pi)*sqrt(a)*a**(-m/(2*m + 2) - 1/2 - 1/(2*m + 2))*x**(m + 1)*hyper((1
/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**(2*m + 2)*exp_polar(I*pi)
/a)/(2*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*gamma(m/(2*m + 2) + 1 +
1/(2*m + 2)))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx = \int \frac{x^m}{\sqrt{bx^{2\,m+2}+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*x^(2*m + 2) + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx = \int \frac{x^m}{\sqrt{bx^{2\,m+2}+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x^(2*m + 2) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx = \int \frac{x^m}{\sqrt{a+b\,x^{2\,m+2}}}\,dx$$

input
```
int(x^m/(a + b*x^(2*m + 2))^(1/2),x)
```

output
```
int(x^m/(a + b*x^(2*m + 2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx^{2+2m}}}\,dx = \int \frac{x^m\sqrt{x^{2m}b\,x^2+a}}{x^{2m}b\,x^2+a}dx$$

input
```
int(x^m/(a+b*x^(2+2*m))^(1/2),x)
```

output
```
int((x**m*sqrt(x**(2*m)*b*x**2 + a))/(x**(2*m)*b*x**2 + a),x)
```

3.617 $\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \frac{x^{1+m}}{a(1+m)\sqrt{a+bx^{2(1+m)}}}$$

output
```
x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \frac{x^{1+m}}{a(1+m)\sqrt{a+bx^{2+2m}}}$$

input
```
Integrate[x^m/(a + b*x^(2 + 2*m))^(3/2),x]
```

output
```
x^(1 + m)/(a*(1 + m)*Sqrt[a + b*x^(2 + 2*m)])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^{2m+2})^{3/2}}\,dx$$

$$\Big\downarrow \ 796$$

$$\frac{x^{m+1}}{a(m+1)\sqrt{a+bx^{2(m+1)}}}$$

input
```
Int[x^m/(a + b*x^(2 + 2*m))^(3/2),x]
```

output
```
x^(1 + m)/(a*(1 + m)*Sqrt[a + b*x^(2*(1 + m))])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^m}{(a+b\,x^{2+2m})^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a+b*x^(2+2*m))^(3/2),x)
```

output
```
int(x^m/(a+b*x^(2+2*m))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \frac{\sqrt{bx^2x^{2\,m}+a}\,xx^m}{(abm+ab)x^2x^{2\,m}+a^2m+a^2}$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2*x^(2*m) + a)*x*x^m/((a*b*m + a*b)*x^2*x^(2*m) + a^2*m + a^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.69

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \frac{\sqrt{\pi}\sqrt{a}a^{-\frac{m}{2m+2}-\frac{3}{2}-\frac{1}{2m+2}}x^{m+1}{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\frac{bx^{2m+2}e^{i\pi}}{a}\right)}{2m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

input
```
integrate(x**m/(a+b*x**(2+2*m))**(3/2),x)
```

output
```
sqrt(pi)*sqrt(a)*a**(-m/(2*m + 2) - 3/2 - 1/(2*m + 2))*x**(m + 1)*hyper((3
/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**(2*m + 2)*exp_polar(I*pi)
/a)/(2*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*gamma(m/(2*m + 2) + 1 +
1/(2*m + 2)))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \int \frac{x^m}{(bx^{2\,m+2}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \int \frac{x^m}{(bx^{2\,m+2}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \frac{x^{m+1}}{a\,\sqrt{a+b\,x^{2\,m+2}}\,(m+1)}$$

input
```
int(x^m/(a + b*x^(2*m + 2))^(3/2),x)
```

output
```
x^(m + 1)/(a*(a + b*x^(2*m + 2))^(1/2)*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{x^m}{(a+bx^{2+2m})^{3/2}}\,dx = \frac{x^m\sqrt{x^{2m}b\,x^2+a}\,x}{a\,(x^{2m}bm\,x^2+x^{2m}b\,x^2+am+a)}$$

input
```
int(x^m/(a+b*x^(2+2*m))^(3/2),x)
```

output
```
(x**m*sqrt(x**(2*m)*b*x**2 + a)*x)/(a*(x**(2*m)*b*m*x**2 + x**(2*m)*b*x**2
 + a*m + a))
```

3.618 $\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{5/2}}\,dx=\frac{x^{1+m}}{3a(1+m)\left(a+bx^{2(1+m)}\right)^{3/2}}+\frac{2x^{1+m}}{3a^2(1+m)\sqrt{a+bx^{2(1+m)}}}$$

output
```
1/3*x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^(3/2)+2/3*x^(1+m)/a^2/(1+m)/(a+b*x^(2+
2*m))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{x^m}{\left(a+bx^{2+2m}\right)^{5/2}}\,dx=\frac{x^{1+m}\left(3a+2bx^{2+2m}\right)}{3a^2(1+m)\left(a+bx^{2+2m}\right)^{3/2}}$$

input
```
Integrate[x^m/(a + b*x^(2 + 2*m))^(5/2),x]
```

output
```
(x^(1 + m)*(3*a + 2*b*x^(2 + 2*m)))/(3*a^2*(1 + m)*(a + b*x^(2 + 2*m))^(3/
2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\left(a+bx^{2m+2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2b\int \frac{x^{3m+2}}{\left(bx^{2(m+1)}+a\right)^{5/2}}dx}{a}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{3/2}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{2bx^{3(m+1)}}{3a^2(m+1)\left(a+bx^{2(m+1)}\right)^{3/2}}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{3/2}}$$

input
```
Int[x^m/(a + b*x^(2 + 2*m))^(5/2),x]
```

output
```
x^(1 + m)/(a*(1 + m)*(a + b*x^(2*(1 + m)))^(3/2)) + (2*b*x^(3*(1 + m)))/(3
*a^2*(1 + m)*(a + b*x^(2*(1 + m)))^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^m}{\left(a+b\,x^{2+2m}\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a+b*x^(2+2*m))^(5/2),x)
```

output
```
int(x^m/(a+b*x^(2+2*m))^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{x^m}{(a+bx^{2+2m})^{5/2}}\,dx = \frac{(2\,bx^3x^{3\,m}+3\,axx^m)\sqrt{bx^2x^{2\,m}+a}}{3\,((a^2b^2m+a^2b^2)x^4x^{4\,m}+a^4m+a^4+2\,(a^3bm+a^3b)x^2x^{2\,m})}$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*x^3*x^(3*m) + 3*a*x*x^m)*sqrt(b*x^2*x^(2*m) + a)/((a^2*b^2*m + a^
2*b^2)*x^4*x^(4*m) + a^4*m + a^4 + 2*(a^3*b*m + a^3*b)*x^2*x^(2*m))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{x^m}{(a+bx^{2+2m})^{5/2}}\,dx = \frac{\sqrt{\pi}\sqrt{a}a^{-\frac{m}{2m+2}-\frac{5}{2}-\frac{1}{2m+2}}x^{m+1}{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\frac{bx^{2m+2}e^{i\pi}}{a}\right)}{2m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

input
```
integrate(x**m/(a+b*x**(2+2*m))**(5/2),x)
```

output
```
sqrt(pi)*sqrt(a)*a**(-m/(2*m + 2) - 5/2 - 1/(2*m + 2))*x**(m + 1)*hyper((5
/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**(2*m + 2)*exp_polar(I*pi)
/a)/(2*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*gamma(m/(2*m + 2) + 1 +
1/(2*m + 2)))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^{5/2}}\,dx = \int \frac{x^m}{(bx^{2\,m+2}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^{5/2}}\,dx = \int \frac{x^m}{(bx^{2\,m+2}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \frac{x^m}{(a+bx^{2+2m})^{5/2}}\,dx = \frac{x^{m+1}\left(a+\frac{2\,b\,x^{2\,m+2}}{3}\right)}{a^2\,\left(a+b\,x^{2\,m+2}\right)^{3/2}\,(m+1)}$$

input
```
int(x^m/(a + b*x^(2*m + 2))^(5/2),x)
```

output
```
(x^(m + 1)*(a + (2*b*x^(2*m + 2))/3))/(a^2*(a + b*x^(2*m + 2))^(3/2)*(m +
1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{x^m}{(a+bx^{2+2m})^{5/2}}\,dx = \frac{x^m\sqrt{x^{2m}b\,x^2+a}\,x(2x^{2m}b\,x^2+3a)}{3a^2\left(x^{4m}b^2m\,x^4+x^{4m}b^2x^4+2x^{2m}abm\,x^2+2x^{2m}ab\,x^2+a^2m+a^2\right)}$$

input
```
int(x^m/(a+b*x^(2+2*m))^(5/2),x)
```

output
```
(x**m*sqrt(x**(2*m)*b*x**2 + a)*x*(2*x**(2*m)*b*x**2 + 3*a))/(3*a**2*(x**(
4*m)*b**2*m*x**4 + x**(4*m)*b**2*x**4 + 2*x**(2*m)*a*b*m*x**2 + 2*x**(2*m)
*a*b*x**2 + a**2*m + a**2))
```

3.619 $\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \frac{x^{1+m}}{5a(1+m)\left(a+bx^{2(1+m)}\right)^{5/2}} + \frac{4x^{1+m}}{15a^2(1+m)\left(a+bx^{2(1+m)}\right)^{3/2}} + \frac{8x^{1+m}}{15a^3(1+m)\sqrt{a+bx^{2(1+m)}}}$$

output
```
1/5*x^(1+m)/a/(1+m)/(a+b*x^(2+2*m))^(5/2)+4/15*x^(1+m)/a^2/(1+m)/(a+b*x^(2
+2*m))^(3/2)+8/15*x^(1+m)/a^3/(1+m)/(a+b*x^(2+2*m))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \frac{x^{1+m}(15a^2+20abx^{2+2m}+8b^2x^{4+4m})}{15a^3(1+m)\left(a+bx^{2+2m}\right)^{5/2}}$$

input
```
Integrate[x^m/(a + b*x^(2 + 2*m))^(7/2),x]
```

output
```
(x^(1 + m)*(15*a^2 + 20*a*b*x^(2 + 2*m) + 8*b^2*x^(4 + 4*m)))/(15*a^3*(1 +
 m)*(a + b*x^(2 + 2*m))^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\left(a+bx^{2m+2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\int \frac{x^{3m+2}}{\left(bx^{2(m+1)}+a\right)^{7/2}}dx}{a}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\left(\frac{2b\int \frac{x^{5m+4}}{\left(bx^{2(m+1)}+a\right)^{7/2}}dx}{3a}+\frac{x^{3(m+1)}}{3a(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}\right)}{a}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4b\left(\frac{2bx^{5(m+1)}}{15a^2(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}+\frac{x^{3(m+1)}}{3a(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}\right)}{a}+\frac{x^{m+1}}{a(m+1)\left(a+bx^{2(m+1)}\right)^{5/2}}$$

input
```
Int[x^m/(a + b*x^(2 + 2*m))^(7/2),x]
```

output
```
x^(1 + m)/(a*(1 + m)*(a + b*x^(2*(1 + m)))^(5/2)) + (4*b*(x^(3*(1 + m))/(3
*a*(1 + m)*(a + b*x^(2*(1 + m)))^(5/2)) + (2*b*x^(5*(1 + m)))/(15*a^2*(1 +
 m)*(a + b*x^(2*(1 + m)))^(5/2))))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^m}{\left(a+b\,x^{2+2m}\right)^{\frac{7}{2}}}dx$$

input
```
int(x^m/(a+b*x^(2+2*m))^(7/2),x)
```

output
```
int(x^m/(a+b*x^(2+2*m))^(7/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.39

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \frac{(8\,b^2x^5x^{5\,m}+20\,abx^3x^{3\,m}+15\,a^2xx^m)\sqrt{bx^2x^{2\,m}+a}}{15\,((a^3b^3m+a^3b^3)x^6x^{6\,m}+a^6m+a^6+3\,(a^4b^2m+a^4b^2)x^4x^{4\,m}+3\,(a^5bm+a^5b)$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(7/2),x, algorithm="fricas")
```

output
```
1/15*(8*b^2*x^5*x^(5*m) + 20*a*b*x^3*x^(3*m) + 15*a^2*x*x^m)*sqrt(b*x^2*x^
(2*m) + a)/((a^3*b^3*m + a^3*b^3)*x^6*x^(6*m) + a^6*m + a^6 + 3*(a^4*b^2*m
 + a^4*b^2)*x^4*x^(4*m) + 3*(a^5*b*m + a^5*b)*x^2*x^(2*m))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 57.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \frac{\sqrt{\pi}\sqrt{a}a^{-\frac{m}{2m+2}-\frac{7}{2}-\frac{1}{2m+2}}x^{m+1}{}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{1}{2}\\ \frac{m}{2m+2}+1+\frac{1}{2m+2}\end{matrix}\middle|\frac{bx^{2m+2}e^{i\pi}}{a}\right)}{2m\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)+2\Gamma\left(\frac{m}{2m+2}+1+\frac{1}{2m+2}\right)}$$

input
```
integrate(x**m/(a+b*x**(2+2*m))**(7/2),x)
```

output
```
sqrt(pi)*sqrt(a)*a**(-m/(2*m + 2) - 7/2 - 1/(2*m + 2))*x**(m + 1)*hyper((7
/2, 1/2), (m/(2*m + 2) + 1 + 1/(2*m + 2),), b*x**(2*m + 2)*exp_polar(I*pi)
/a)/(2*m*gamma(m/(2*m + 2) + 1 + 1/(2*m + 2)) + 2*gamma(m/(2*m + 2) + 1 +
1/(2*m + 2)))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \int \frac{x^m}{(bx^{2\,m+2}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(7/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \int \frac{x^m}{(bx^{2\,m+2}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2+2*m))^(7/2),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^(2*m + 2) + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \frac{x^{m+1}\left(\frac{8\,b^2\,x^{4\,m+4}}{15}+a^2+\frac{4\,a\,b\,x^{2\,m+2}}{3}\right)}{a^3\,\left(a+b\,x^{2\,m+2}\right)^{5/2}\,(m+1)}$$

input
```
int(x^m/(a + b*x^(2*m + 2))^(7/2),x)
```

output
```
(x^(m + 1)*((8*b^2*x^(4*m + 4))/15 + a^2 + (4*a*b*x^(2*m + 2))/3))/(a^3*(a
 + b*x^(2*m + 2))^(5/2)*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int \frac{x^m}{(a+bx^{2+2m})^{7/2}}\,dx = \frac{x^m\sqrt{x^{2m}b\,x^2+a}\,x(8x^{4m}b^2x^4+20x^{2m}ab\,x^2+15a^2)}{15a^3\,(x^{6m}b^3m\,x^6+x^{6m}b^3x^6+3x^{4m}a\,b^2m\,x^4+3x^{4m}a\,b^2x^4+3x^{2m}a^2bm\,x^2+3x^{2m}}$$

input
```
int(x^m/(a+b*x^(2+2*m))^(7/2),x)
```

output
```
(x**m*sqrt(x**(2*m)*b*x**2 + a)*x*(8*x**(4*m)*b**2*x**4 + 20*x**(2*m)*a*b*
x**2 + 15*a**2))/(15*a**3*(x**(6*m)*b**3*m*x**6 + x**(6*m)*b**3*x**6 + 3*x
**(4*m)*a*b**2*m*x**4 + 3*x**(4*m)*a*b**2*x**4 + 3*x**(2*m)*a**2*b*m*x**2
+ 3*x**(2*m)*a**2*b*x**2 + a**3*m + a**3))
```

3.620 $\int x^m \sqrt{1+x^{1+m}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int x^m \sqrt{1+x^{1+m}}\,dx = \frac{2(1+x^{1+m})^{3/2}}{3(1+m)}$$

output
```
2*(1+x^(1+m))^(3/2)/(3+3*m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{1+x^{1+m}}\,dx = \frac{2(1+x^{1+m})^{3/2}}{3(1+m)}$$

input
```
Integrate[x^m*Sqrt[1 + x^(1 + m)],x]
```

output
```
(2*(1 + x^(1 + m))^(3/2))/(3*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{x^{m+1}+1} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{2\left(x^{m+1}+1\right)^{3/2}}{3(m+1)}$$

input
```
Int[x^m*Sqrt[1 + x^(1 + m)],x]
```

output
```
(2*(1 + x^(1 + m))^(3/2))/(3*(1 + m))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{2(1+x\,x^{m})^{\frac{3}{2}}}{3(1+m)}$	17
meijerg	$-\frac{\frac{2(-1)^{-\frac{m}{1+m}-\frac{1}{1+m}}\Gamma\left(\frac{1}{2}-\frac{m}{1+m}-\frac{1}{1+m}\right)}{3\Gamma\left(2-\frac{m}{1+m}-\frac{1}{1+m}\right)}-\frac{(-1)^{-\frac{m}{1+m}-\frac{1}{1+m}}\Gamma\left(\frac{1}{2}-\frac{m}{1+m}-\frac{1}{1+m}\right)\left(2+2x^{1+m}\right)\sqrt{1+x^{1+m}}}{3\Gamma\left(2-\frac{m}{1+m}-\frac{1}{1+m}\right)}}{2\sqrt{\pi}\,(1+m)}$	146

input
```
int(x^m*(1+x^(1+m))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x*x^m)^(3/2)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^m\sqrt{1+x^{1+m}}\,dx=\frac{2\left(x^{m+1}+1\right)^{\frac{3}{2}}}{3\left(m+1\right)}$$

input
```
integrate(x^m*(1+x^(1+m))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^(m + 1) + 1)^(3/2)/(m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.

Time = 0.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int x^m\sqrt{1+x^{1+m}}\,dx=\begin{cases}\frac{2x^{m+1}\sqrt{x^{m+1}+1}}{3m+3}+\frac{2\sqrt{x^{m+1}+1}}{3m+3} & \text{for}\ m\neq-1\\ \sqrt{2}\log\left(x\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**m*(1+x**(1+m))**(1/2),x)
```

output
```
Piecewise((2*x**(m + 1)*sqrt(x**(m + 1) + 1)/(3*m + 3) + 2*sqrt(x**(m + 1)
 + 1)/(3*m + 3), Ne(m, -1)), (sqrt(2)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^m\sqrt{1+x^{1+m}}\,dx=\frac{2\,(x^{m+1}+1)^{\frac{3}{2}}}{3\,(m+1)}$$

input
```
integrate(x^m*(1+x^(1+m))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^(m + 1) + 1)^(3/2)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^m\sqrt{1+x^{1+m}}\,dx=\frac{2\,(x^{m+1}+1)^{\frac{3}{2}}}{3\,(m+1)}$$

input
```
integrate(x^m*(1+x^(1+m))^(1/2),x, algorithm="giac")
```

output
```
2/3*(x^(m + 1) + 1)^(3/2)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x^m\sqrt{1+x^{1+m}}\,dx=\frac{2\,(x^{m+1}+1)^{3/2}}{3\,(m+1)}$$

input
```
int(x^m*(x^(m + 1) + 1)^(1/2),x)
```

output
```
(2*(x^(m + 1) + 1)^(3/2))/(3*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int x^m\sqrt{1+x^{1+m}}\,dx=\frac{2\sqrt{x^m x+1}\,(x^m x+1)}{3m+3}$$

input
```
int(x^m*(1+x^(1+m))^(1/2),x)
```

output
```
(2*sqrt(x**m*x + 1)*(x**m*x + 1))/(3*(m + 1))
```

3.621 $\int x^m \sqrt{a^2+x^{1+m}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 22

$$\int x^m \sqrt{a^2+x^{1+m}}\,dx = \frac{2(a^2+x^{1+m})^{3/2}}{3(1+m)}$$

output
```
2*(a^2+x^(1+m))^(3/2)/(3+3*m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{a^2+x^{1+m}}\,dx = \frac{2(a^2+x^{1+m})^{3/2}}{3(1+m)}$$

input
```
Integrate[x^m*Sqrt[a^2 + x^(1 + m)],x]
```

output
```
(2*(a^2 + x^(1 + m))^(3/2))/(3*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{a^2+x^{m+1}}\,dx$$

$$\Big\downarrow \ 793$$

$$\frac{2\left(a^2+x^{m+1}\right)^{3/2}}{3(m+1)}$$

input
```
Int[x^m*Sqrt[a^2 + x^(1 + m)],x]
```

output
```
(2*(a^2 + x^(1 + m))^(3/2))/(3*(1 + m))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{2\left(a^2+x\,x^m\right)^{\frac{3}{2}}}{3(1+m)}$	19

input
```
int(x^m*(a^2+x^(1+m))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a^2+x*x^m)^(3/2)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int x^m \sqrt{a^2 + x^{1+m}}\, dx = \frac{2\left(a^2 + x^{m+1}\right)^{\frac{3}{2}}}{3\,(m+1)}$$

input
```
integrate(x^m*(a^2+x^(1+m))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(a^2 + x^(m + 1))^(3/2)/(m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(17) = 34.

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int x^m \sqrt{a^2 + x^{1+m}}\, dx = \begin{cases} \frac{2a^2\sqrt{a^2+x^{m+1}}}{3m+3} + \frac{2x^{m+1}\sqrt{a^2+x^{m+1}}}{3m+3} & \text{for}\ m \neq -1 \\ \sqrt{a^2+1}\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(a**2+x**(1+m))**(1/2),x)
```

output
```
Piecewise((2*a**2*sqrt(a**2 + x**(m + 1))/(3*m + 3) + 2*x**(m + 1)*sqrt(a*
*2 + x**(m + 1))/(3*m + 3), Ne(m, -1)), (sqrt(a**2 + 1)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int x^m \sqrt{a^2 + x^{1+m}}\, dx = \frac{2\left(a^2 + x^{m+1}\right)^{\frac{3}{2}}}{3\,(m+1)}$$

input
```
integrate(x^m*(a^2+x^(1+m))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(a^2 + x^(m + 1))^(3/2)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int x^m \sqrt{a^2 + x^{1+m}}\, dx = \frac{2\left(a^2 + x^{m+1}\right)^{\frac{3}{2}}}{3\,(m+1)}$$

input
```
integrate(x^m*(a^2+x^(1+m))^(1/2),x, algorithm="giac")
```

output
```
2/3*(a^2 + x^(m + 1))^(3/2)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^m \sqrt{a^2 + x^{1+m}}\, dx = \frac{2\left(x^{m+1} + a^2\right)^{3/2}}{3\,(m+1)}$$

input
```
int(x^m*(x^(m + 1) + a^2)^(1/2),x)
```

output
```
(2*(x^(m + 1) + a^2)^(3/2))/(3*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int x^m\sqrt{a^2+x^{1+m}}\,dx=\frac{2\sqrt{x^mx+a^2}\,(x^mx+a^2)}{3m+3}$$

input

```
int(x^m*(a^2+x^(1+m))^(1/2),x)
```

output

```
(2*sqrt(x**m*x + a**2)*(x**m*x + a**2))/(3*(m + 1))
```

3.622 $\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx=\frac{x^{1+m}\sqrt{1+\frac{bx^{-2+m}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1+m}{2-m},\frac{1-2m}{2-m},-\frac{bx^{-2+m}}{a}\right)}{(1+m)\sqrt{a+bx^{-2+m}}}$$

output
```
x^(1+m)*(1+b*x^(-2+m)/a)^(1/2)*hypergeom([1/2, -(1+m)/(2-m)],[(1-2*m)/(2-m
)],-b*x^(-2+m)/a)/(1+m)/(a+b*x^(-2+m))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx = \frac{x^{1+m}\sqrt{1+\frac{bx^{-2+m}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{-2+m},1+\frac{1+m}{-2+m},-\frac{bx^{-2+m}}{a}\right)}{(1+m)\sqrt{a+bx^{-2+m}}}$$

input
```
Integrate[x^m/Sqrt[a + b*x^(-2 + m)],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^(-2 + m))/a]*Hypergeometric2F1[1/2, (1 + m)/(-2 +
 m), 1 + (1 + m)/(-2 + m), -((b*x^(-2 + m))/a)])/((1 + m)*Sqrt[a + b*x^(-2
 + m)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx^{m-2}}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^{m-2}}{a}+1}\int \frac{x^m}{\sqrt{\frac{bx^{m-2}}{a}+1}}dx}{\sqrt{a+bx^{m-2}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^{m+1}\sqrt{\frac{bx^{m-2}}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m+1}{2-m},1-\frac{m+1}{2-m},-\frac{bx^{m-2}}{a}\right)}{(m+1)\sqrt{a+bx^{m-2}}}$$

input
```
Int[x^m/Sqrt[a + b*x^(-2 + m)],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^(-2 + m))/a]*Hypergeometric2F1[1/2, -((1 + m)/(2
- m)), 1 - (1 + m)/(2 - m), -((b*x^(-2 + m))/a)])/((1 + m)*Sqrt[a + b*x^(-
2 + m)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{a+b\,x^{-2+m}}}dx$$

input
```
int(x^m/(a+b*x^(-2+m))^(1/2),x)
```

output
```
int(x^m/(a+b*x^(-2+m))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*x^(-2+m))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx = \frac{a^{\frac{m}{m-2}+\frac{1}{m-2}}a^{-\frac{m}{m-2}-\frac{1}{2}-\frac{1}{m-2}}x^{m+1}\Gamma\left(\frac{m}{m-2}+\frac{1}{m-2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{m-2}+\frac{1}{m-2}\\ \frac{m}{m-2}+1+\frac{1}{m-2}\end{matrix}\middle|\frac{bx^{m-2}e^{i\pi}}{a}\right)}{m\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)-2\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)}$$

input
```
integrate(x**m/(a+b*x**(-2+m))**(1/2),x)
```

output
```
a**(m/(m - 2) + 1/(m - 2))*a**(-m/(m - 2) - 1/2 - 1/(m - 2))*x**(m + 1)*ga
mma(m/(m - 2) + 1/(m - 2))*hyper((1/2, m/(m - 2) + 1/(m - 2)), (m/(m - 2)
+ 1 + 1/(m - 2),), b*x**(m - 2)*exp_polar(I*pi)/a)/(m*gamma(m/(m - 2) + 1
+ 1/(m - 2)) - 2*gamma(m/(m - 2) + 1 + 1/(m - 2)))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx = \int \frac{x^m}{\sqrt{bx^{m-2}+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^(-2+m))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*x^(m - 2) + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx = \int \frac{x^m}{\sqrt{bx^{m-2}+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^(-2+m))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x^(m - 2) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx = \int \frac{x^m}{\sqrt{a+b\,x^{m-2}}}\,dx$$

input
```
int(x^m/(a + b*x^(m - 2))^(1/2),x)
```

output
```
int(x^m/(a + b*x^(m - 2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx^{-2+m}}}\,dx = \int \frac{x^m\sqrt{x^m b+a\,x^2}\,x}{x^m b+a\,x^2}\,dx$$

input
```
int(x^m/(a+b*x^(-2+m))^(1/2),x)
```

output
```
int((x**m*sqrt(x**m*b + a*x**2)*x)/(x**m*b + a*x**2),x)
```

3.623 $\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = \frac{x^{1+m}\sqrt{1+\frac{bx^{2-m}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2-m},\frac{3}{2-m},-\frac{bx^{2-m}}{a}\right)}{(1+m)\sqrt{a+bx^{2-m}}}$$

output
```
x^(1+m)*(1+b*x^(2-m)/a)^(1/2)*hypergeom([1/2, (1+m)/(2-m)],[3/(2-m)],-b*x^
(2-m)/a)/(1+m)/(a+b*x^(2-m))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = \frac{x^{1+m}\sqrt{1+\frac{bx^{2-m}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2-m},-\frac{3}{-2+m},-\frac{bx^{2-m}}{a}\right)}{(1+m)\sqrt{a+bx^{2-m}}}$$

input
```
Integrate[x^m/Sqrt[a + b*x^(2 - m)],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^(2 - m))/a]*Hypergeometric2F1[1/2, (1 + m)/(2 - m
), -3/(-2 + m), -((b*x^(2 - m))/a)])/((1 + m)*Sqrt[a + b*x^(2 - m)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^{2-m}}{a}+1}\int \frac{x^m}{\sqrt{\frac{bx^{2-m}}{a}+1}}dx}{\sqrt{a+bx^{2-m}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}\sqrt{\frac{bx^{2-m}}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2-m},\frac{3}{2-m},-\frac{bx^{2-m}}{a}\right)}{(m+1)\sqrt{a+bx^{2-m}}}$$

input
```
Int[x^m/Sqrt[a + b*x^(2 - m)],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^(2 - m))/a]*Hypergeometric2F1[1/2, (1 + m)/(2 - m
), 3/(2 - m), -((b*x^(2 - m))/a)])/((1 + m)*Sqrt[a + b*x^(2 - m)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{a+b\,x^{2-m}}}dx$$

input
```
int(x^m/(a+b*x^(2-m))^(1/2),x)
```

output
```
int(x^m/(a+b*x^(2-m))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*x^(2-m))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.58

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = -\frac{a^{-\frac{m}{2-m}-\frac{1}{2}-\frac{1}{2-m}}x^{m+1}\Gamma\left(-\frac{m}{m-2}-\frac{1}{m-2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},-\frac{m}{m-2}-\frac{1}{m-2}\\-\frac{m}{m-2}+1-\frac{1}{m-2}\end{matrix}\middle|\frac{bx^{2-m}e^{i\pi}}{a}\right)}{a^{\frac{m}{m-2}+\frac{1}{m-2}}m\Gamma\left(-\frac{m}{m-2}+1-\frac{1}{m-2}\right)-2a^{\frac{m}{m-2}+\frac{1}{m-2}}\Gamma\left(-\frac{m}{m-2}+1-\frac{1}{m-2}\right)}$$

input
```
integrate(x**m/(a+b*x**(2-m))**(1/2),x)
```

output
```
-a**(-m/(2 - m) - 1/2 - 1/(2 - m))*x**(m + 1)*gamma(-m/(m - 2) - 1/(m - 2)
)*hyper((1/2, -m/(m - 2) - 1/(m - 2)), (-m/(m - 2) + 1 - 1/(m - 2),), b*x*
*(2 - m)*exp_polar(I*pi)/a)/(a**(m/(m - 2) + 1/(m - 2))*m*gamma(-m/(m - 2)
 + 1 - 1/(m - 2)) - 2*a**(m/(m - 2) + 1/(m - 2))*gamma(-m/(m - 2) + 1 - 1/
(m - 2)))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = \int \frac{x^m}{\sqrt{bx^{-m+2}+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2-m))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*x^(-m + 2) + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = \int \frac{x^m}{\sqrt{bx^{-m+2}+a}}\,dx$$

input
```
integrate(x^m/(a+b*x^(2-m))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x^(-m + 2) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = \int \frac{x^m}{\sqrt{a+b\,x^{2-m}}}\,dx$$

input
```
int(x^m/(a + b*x^(2 - m))^(1/2),x)
```

output
```
int(x^m/(a + b*x^(2 - m))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx^{2-m}}}\,dx = \int \frac{x^{\frac{3m}{2}}}{\sqrt{x^m a+b\,x^2}}dx$$

input
```
int(x^m/(a+b*x^(2-m))^(1/2),x)
```

output
```
int(x**((3*m)/2)/sqrt(x**m*a + b*x**2),x)
```

3.624 $\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right) dx$

Optimal result

Integrand size = 45, antiderivative size = 26

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right) dx=\frac{2x^3\sqrt{a+bx^{-2+m}}}{b(4+m)}$$

output
```
2*x^3*(a+b*x^(-2+m))^(1/2)/b/(4+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.73 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.92

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right) dx$$
$$=\frac{x\sqrt{1+\frac{bx^{-2+m}}{a}}\left(2a(1+m)x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{-2+m},\frac{1+m}{-2+m},-\frac{bx^{-2+m}}{a}\right)+b(4+m)x^m\,\text{Hypergeom}\right.}{b(1+m)(4+m)\sqrt{a+bx^{-2+m}}}$$

input
```
Integrate[(6*a*x^2)/(b*(4 + m)*Sqrt[a + b*x^(-2 + m)]) + x^m/Sqrt[a + b*x^
(-2 + m)],x]
```

output
```
(x*Sqrt[1 + (b*x^(-2 + m))/a]*(2*a*(1 + m)*x^2*Hypergeometric2F1[1/2, 3/(-
2 + m), (1 + m)/(-2 + m), -((b*x^(-2 + m))/a)] + b*(4 + m)*x^m*Hypergeomet
ric2F1[1/2, (1 + m)/(-2 + m), (-1 + 2*m)/(-2 + m), -((b*x^(-2 + m))/a)]))/
(b*(1 + m)*(4 + m)*Sqrt[a + b*x^(-2 + m)])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 6.15, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^m}{\sqrt{a+bx^{m-2}}}+\frac{6ax^2}{b(m+4)\sqrt{a+bx^{m-2}}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+1}\sqrt{\frac{bx^{m-2}}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m+1}{2-m},\frac{1-2m}{2-m},-\frac{bx^{m-2}}{a}\right)}{(m+1)\sqrt{a+bx^{m-2}}}+\frac{2ax^3\sqrt{\frac{bx^{m-2}}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{3}{2-m},-\frac{m+1}{2-m},-\frac{bx^{m-2}}{a}\right)}{b(m+4)\sqrt{a+bx^{m-2}}}$$

input
```
Int[(6*a*x^2)/(b*(4 + m)*Sqrt[a + b*x^(-2 + m)]) + x^m/Sqrt[a + b*x^(-2 +
m)],x]
```

output
```
(2*a*x^3*Sqrt[1 + (b*x^(-2 + m))/a]*Hypergeometric2F1[1/2, -3/(2 - m), -((
1 + m)/(2 - m)), -((b*x^(-2 + m))/a)])/(b*(4 + m)*Sqrt[a + b*x^(-2 + m)])
+ (x^(1 + m)*Sqrt[1 + (b*x^(-2 + m))/a]*Hypergeometric2F1[1/2, -((1 + m)/(
2 - m)), (1 - 2*m)/(2 - m), -((b*x^(-2 + m))/a)])/((1 + m)*Sqrt[a + b*x^(-
2 + m)])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

method	result	size
risch	$\frac{2x\left(a\,x^{2}+b\,x^{m}\right)}{b\left(4+m\right)\sqrt{\frac{a\,x^{2}+b\,x^{m}}{x^{2}}}}$	40

input
```
int(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
2*x*(a*x^2+b*x^m)/b/(4+m)/((a*x^2+b*x^m)/x^2)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int\left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right)dx=\text{Exception raised: TypeError}$$

input
```
integrate(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x,
 algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 7.46

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right)\,dx = \frac{6aa^{-\frac{1}{2}-\frac{3}{m-2}}a^{\frac{3}{m-2}}x^3\Gamma\left(\frac{3}{m-2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{m-2}\\1+\frac{3}{m-2}\end{matrix}\middle|\frac{bx^{m-2}e^{i\pi}}{a}\right)}{b\left(m+4\right)\left(m\Gamma\left(1+\frac{3}{m-2}\right)-2\Gamma\left(1+\frac{3}{m-2}\right)\right)} + \frac{a^{\frac{m}{m-2}+\frac{1}{m-2}}a^{-\frac{m}{m-2}-\frac{1}{2}-\frac{1}{m-2}}x^{m+1}\Gamma\left(\frac{m}{m-2}+\frac{1}{m-2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{m-2}+\frac{1}{m-2}\\\frac{m}{m-2}+1+\frac{1}{m-2}\end{matrix}\middle|\frac{bx^{m-2}e^{i\pi}}{a}\right)}{m\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)-2\Gamma\left(\frac{m}{m-2}+1+\frac{1}{m-2}\right)}$$

input
```
integrate(6*a*x**2/b/(4+m)/(a+b*x**(-2+m))**(1/2)+x**m/(a+b*x**(-2+m))**(1
/2),x)
```

output
```
6*a*a**(-1/2 - 3/(m - 2))*a**(3/(m - 2))*x**3*gamma(3/(m - 2))*hyper((1/2,
 3/(m - 2)), (1 + 3/(m - 2),), b*x**(m - 2)*exp_polar(I*pi)/a)/(b*(m + 4)*
(m*gamma(1 + 3/(m - 2)) - 2*gamma(1 + 3/(m - 2)))) + a**(m/(m - 2) + 1/(m
- 2))*a**(-m/(m - 2) - 1/2 - 1/(m - 2))*x**(m + 1)*gamma(m/(m - 2) + 1/(m
- 2))*hyper((1/2, m/(m - 2) + 1/(m - 2)), (m/(m - 2) + 1 + 1/(m - 2),), b*
x**(m - 2)*exp_polar(I*pi)/a)/(m*gamma(m/(m - 2) + 1 + 1/(m - 2)) - 2*gamm
a(m/(m - 2) + 1 + 1/(m - 2)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right)\,dx = \frac{2\left(ax^4+bx^2x^m\right)}{\sqrt{ax^2+bx^m}b(m+4)}$$

input
```
integrate(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x,
 algorithm="maxima")
```

output
```
2*(a*x^4 + b*x^2*x^m)/(sqrt(a*x^2 + b*x^m)*b*(m + 4))
```

Giac [F]

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right)dx = \int \frac{x^m}{\sqrt{bx^{m-2}+a}}+\frac{6\,ax^2}{\sqrt{bx^{m-2}+a}b(m+4)}\,dx$$

input
```
integrate(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x,
 algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x^(m - 2) + a) + 6*a*x^2/(sqrt(b*x^(m - 2) + a)*b*(m
+ 4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right)dx = \int \frac{x^m}{\sqrt{a+b\,x^{m-2}}}+\frac{6\,a\,x^2}{b\,\sqrt{a+b\,x^{m-2}}\,\left(m+4\right)}\,dx$$

input
```
int(x^m/(a + b*x^(m - 2))^(1/2) + (6*a*x^2)/(b*(a + b*x^(m - 2))^(1/2)*(m
+ 4)),x)
```

output
```
int(x^m/(a + b*x^(m - 2))^(1/2) + (6*a*x^2)/(b*(a + b*x^(m - 2))^(1/2)*(m
+ 4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \left(\frac{6ax^2}{b(4+m)\sqrt{a+bx^{-2+m}}}+\frac{x^m}{\sqrt{a+bx^{-2+m}}}\right) dx = \frac{2\sqrt{x^m b+a\,x^2}\,x^2}{b\,(m+4)}$$

input
```
int(6*a*x^2/b/(4+m)/(a+b*x^(-2+m))^(1/2)+x^m/(a+b*x^(-2+m))^(1/2),x)
```

output
```
(2*sqrt(x**m*b + a*x**2)*x**2)/(b*(m + 4))
```

3.625 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 15

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+bx^n}}$$

output
```
x^m/(a+b*x^n)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.40

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx=\frac{x^m\sqrt{1+\frac{bx^n}{a}}\left(2a(m+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{n},\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(\right.}{2a(m+n)\sqrt{a+}}$$

input
```
Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

output
```
(x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/
n, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m
/n, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {27, 951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-1}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{x^{m-1}(b(2m-n)x^n+2am)}{\left(bx^n+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{951}$$

$$\frac{x^m}{\sqrt{a+bx^n}}$$

input
```
Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

output
```
x^m/Sqrt[a + b*x^n]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{x^{m-1}(2am+b(2m-n)\,x^{n})}{2\left(a+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/2*x^(m-1)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int(1/2*x^(m-1)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx = \frac{xx^{m-1}}{\sqrt{bx^n+a}}$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm=
"fricas")
```

output
```
x*x^(m - 1)/sqrt(b*x^n + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 8.13

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx = \frac{aa^{\frac{m}{n}}a^{-\frac{m}{n}-\frac{3}{2}}mx^m\Gamma\left(\frac{m}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1\right)} + \frac{a^{-\frac{m}{n}-\frac{5}{2}}a^{\frac{m}{n}+1}bx^{m+n}(2m-n)\Gamma\left(\frac{m}{n}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{2n\Gamma\left(\frac{m}{n}+2\right)}$$

input
```
integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
a*a**(m/n)*a**(-m/n - 3/2)*m*x**m*gamma(m/n)*hyper((3/2, m/n), (m/n + 1,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1)) + a**(-m/n - 5/2)*a**(m/n +
1)*b*x**(m + n)*(2*m - n)*gamma(m/n + 1)*hyper((3/2, m/n + 1), (m/n + 2,),
 b*x**n*exp_polar(I*pi)/a)/(2*n*gamma(m/n + 2))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx = \frac{x^m}{\sqrt{bx^n+a}}$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm=
"maxima")
```

output
```
x^m/sqrt(b*x^n + a)
```

Giac [F]

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx=\int \frac{(b(2\,m-n)x^n+2\,am)x^{m-1}}{2\,(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+b\,x^n}}$$

input
```
int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)),x)
```

output
```
x^m/(a + b*x^n)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx=\frac{x^m\sqrt{x^nb+a}}{x^nb+a}$$

input
```
int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
(x**m*sqrt(x**n*b + a))/(x**n*b + a)
```

3.626 $\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right) dx$

Optimal result

Integrand size = 42, antiderivative size = 15

$$\int \left(-\frac{bnx^{-1+m+n}}{2\,(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right) dx=\frac{x^m}{\sqrt{a+bx^n}}$$

output
```
x^m/(a+b*x^n)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.40

$$\int \left(-\frac{bnx^{-1+m+n}}{2\,(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right) dx=\frac{x^m\sqrt{1+\frac{bx^n}{a}}\left(2a(m+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{n},\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(2m-n)x^n\,\text{Hyper}\right.}{2a(m+n)\sqrt{a+bx^n}}$$

input
```
Integrate[-1/2*(b*n*x^(-1 + m + n))/(a + b*x^n)^(3/2) + (m*x^(-1 + m))/Sqr
t[a + b*x^n],x]
```

output
```
(x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/
n, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m
/n, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 126, normalized size of antiderivative = 8.40, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{mx^{m-1}}{\sqrt{a+bx^n}}-\frac{bnx^{m+n-1}}{2\,(a+bx^n)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^m\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{n},\frac{m+n}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}-\frac{bnx^{m+n}\sqrt{\frac{bx^n}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+n}{n},\frac{m}{n}+2,-\frac{bx^n}{a}\right)}{2a(m+n)\sqrt{a+bx^n}}$$

input
```
Int[-1/2*(b*n*x^(-1 + m + n))/(a + b*x^n)^(3/2) + (m*x^(-1 + m))/Sqrt[a +
b*x^n],x]
```

output
```
(x^m*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, m/n, (m + n)/n, -((b*x^n)/
a)])/Sqrt[a + b*x^n] - (b*n*x^(m + n)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F
1[3/2, (m + n)/n, 2 + m/n, -((b*x^n)/a)])/(2*a*(m + n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(-\frac{bn\,x^{-1+m+n}}{2\,(a+b\,x^{n})^{\frac{3}{2}}}+\frac{m\,x^{m-1}}{\sqrt{a+b\,x^{n}}}\right)dx$$

input
```
int(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(m-1)/(a+b*x^n)^(1/2),x)
```

output
```
int(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(m-1)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{bnx^{-1+m+n}}{2\,(a+bx^{n})^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^{n}}}\right)dx=\text{Exception raised: TypeError}$$

input
```
integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x
, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 7.60

$$\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\frac{a^{\frac{m}{n}}a^{-\frac{m}{n}-\frac{1}{2}}mx^m\Gamma\left(\frac{m}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1\right)}-\frac{a^{-\frac{m}{n}-\frac{5}{2}}a^{\frac{m}{n}+1}bx^{m+n}\Gamma\left(\frac{m}{n}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{n}+2\right)}$$

input
```
integrate(-1/2*b*n*x**(-1+m+n)/(a+b*x**n)**(3/2)+m*x**(-1+m)/(a+b*x**n)**(
1/2),x)
```

output
```
a**(m/n)*a**(-m/n - 1/2)*m*x**m*gamma(m/n)*hyper((1/2, m/n), (m/n + 1,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1)) - a**(-m/n - 5/2)*a**(m/n + 1)
*b*x**(m + n)*gamma(m/n + 1)*hyper((3/2, m/n + 1), (m/n + 2,), b*x**n*exp_
polar(I*pi)/a)/(2*gamma(m/n + 2))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \left(-\frac{bnx^{-1+m+n}}{2(a+bx^n)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\frac{x^m}{\sqrt{bx^n+a}}$$

input
```
integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x
, algorithm="maxima")
```

output
```
x^m/sqrt(b*x^n + a)
```

Giac [F]

$$\int \left(-\frac{bnx^{-1+m+n}}{2\left(a+bx^n\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\int -\frac{bnx^{m+n-1}}{2\left(bx^n+a\right)^{\frac{3}{2}}}+\frac{mx^{m-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x
, algorithm="giac")
```

output
```
integrate(-1/2*b*n*x^(m + n - 1)/(b*x^n + a)^(3/2) + m*x^(m - 1)/sqrt(b*x^
n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{bnx^{-1+m+n}}{2\left(a+bx^n\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x^n}}-\frac{b\,n\,x^{m+n-1}}{2\left(a+b\,x^n\right)^{3/2}}\,dx$$

input
```
int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(
3/2)),x)
```

output
```
int((m*x^(m - 1))/(a + b*x^n)^(1/2) - (b*n*x^(m + n - 1))/(2*(a + b*x^n)^(
3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \left(-\frac{bnx^{-1+m+n}}{2\left(a+bx^n\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^n}}\right)dx=\frac{x^m\sqrt{x^nb+a}}{x^nb+a}$$

input
```
int(-1/2*b*n*x^(-1+m+n)/(a+b*x^n)^(3/2)+m*x^(-1+m)/(a+b*x^n)^(1/2),x)
```

output
```
(x**m*sqrt(x**n*b + a))/(x**n*b + a)
```

3.627 $\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x^{1+m}}{\sqrt[3]{a+bx^{3(1+m)}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}(1+m)}-\frac{\log\left(\sqrt[3]{b}x^{1+m}-\sqrt[3]{a+bx^{3(1+m)}}\right)}{2\sqrt[3]{b}(1+m)}$$

output
```
1/3*arctan(1/3*(1+2*b^(1/3)*x^(1+m)/(a+b*x^(3+3*m))^(1/3))*3^(1/2))*3^(1/2
)/b^(1/3)/(1+m)-1/2*ln(b^(1/3)*x^(1+m)-(a+b*x^(3+3*m))^(1/3))/b^(1/3)/(1+m
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx=\frac{x^{1+m}\sqrt[3]{1+\frac{bx^{3+3m}}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^{3+3m}}{a}\right)}{(1+m)\sqrt[3]{a+bx^{3+3m}}}$$

input
```
Integrate[x^m/(a + b*x^(3*(1 + m)))^(1/3),x]
```

output
```
(x^(1 + m)*(1 + (b*x^(3 + 3*m))/a)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3,
-((b*x^(3 + 3*m))/a)])/((1 + m)*(a + b*x^(3 + 3*m))^(1/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {868, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(m+1)}}}dx$$

$$\Big\downarrow \text{ 868}$$

$$\frac{\int \frac{1}{\sqrt[3]{bx^{3m+3}+a}}dx^{m+1}}{m+1}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x^{m+1}}{\sqrt[3]{a+bx^{3m+3}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^{3m+3}}-\sqrt[3]{b}x^{m+1}\right)}{2\sqrt[3]{b}}}{m+1}$$

input
```
Int[x^m/(a + b*x^(3*(1 + m)))^(1/3),x]
```

output
```
(ArcTan[(1 + (2*b^(1/3)*x^(1 + m))/(a + b*x^(3 + 3*m))^(1/3))/Sqrt[3]]/(Sq
rt[3]*b^(1/3)) - Log[-(b^(1/3)*x^(1 + m)) + (a + b*x^(3 + 3*m))^(1/3)]/(2*
b^(1/3)))/(1 + m)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{x^m}{(a+b\,x^{3+3m})^{\frac{1}{3}}}dx$$

input
```
int(x^m/(a+b*x^(3+3*m))^(1/3),x)
```

output
```
int(x^m/(a+b*x^(3+3*m))^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a+b*x^(3+3*m))^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx=\frac{\sqrt[3]{a}a^{-\frac{m}{3m+3}-\frac{1}{3}-\frac{1}{3m+3}}x^{m+1}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{m}{3m+3}+1+\frac{1}{3m+3}\end{matrix}\middle|\frac{bx^{3m+3}e^{i\pi}}{a}\right)}{3m\Gamma\left(\frac{m}{3m+3}+1+\frac{1}{3m+3}\right)+3\Gamma\left(\frac{m}{3m+3}+1+\frac{1}{3m+3}\right)}$$

input
```
integrate(x**m/(a+b*x**(3+3*m))**(1/3),x)
```

output
```
a**(1/3)*a**(-m/(3*m + 3) - 1/3 - 1/(3*m + 3))*x**(m + 1)*gamma(1/3)*hyper
((1/3, 1/3), (m/(3*m + 3) + 1 + 1/(3*m + 3),), b*x**(3*m + 3)*exp_polar(I*
pi)/a)/(3*m*gamma(m/(3*m + 3) + 1 + 1/(3*m + 3)) + 3*gamma(m/(3*m + 3) + 1
 + 1/(3*m + 3)))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx=\int \frac{x^m}{(bx^{3\,m+3}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(3+3*m))^(1/3),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^(3*m + 3) + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx = \int \frac{x^m}{(bx^{3\,m+3}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^m/(a+b*x^(3+3*m))^(1/3),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^(3*m + 3) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx = \int \frac{x^m}{\left(a+b\,x^{3\,m+3}\right)^{1/3}}\,dx$$

input
```
int(x^m/(a + b*x^(3*m + 3))^(1/3),x)
```

output
```
int(x^m/(a + b*x^(3*m + 3))^(1/3), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt[3]{a+bx^{3(1+m)}}}\,dx = \int \frac{x^m}{(x^{3m}b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^m/(a+b*x^(3+3*m))^(1/3),x)
```

output
```
int(x**m/(x**(3*m)*b*x**3 + a)**(1/3),x)
```

3.628 $\int x^m \left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int x^m \left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx = \frac{x^{1+m}\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3}}{1+m} - \frac{2b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x^{\frac{1}{2}(-1-m)}}{\sqrt[3]{a+bx^{-\frac{3}{2}(1+m)}}}}{\sqrt{3}}\right)}{\sqrt{3}(1+m)} + \frac{b^{2/3}\log\left(\sqrt[3]{b}x^{\frac{1}{2}(-1-m)}-\sqrt[3]{a+bx^{-\frac{3}{2}(1+m)}}\right)}{1+m}$$

output
```
x^(1+m)*(a+b/(x^(3/2+3/2*m)))^(2/3)/(1+m)-2/3*b^(2/3)*arctan(1/3*(1+2*b^(1
/3)*x^(-1/2-1/2*m)/(a+b/(x^(3/2+3/2*m)))^(1/3))*3^(1/2))*3^(1/2)/(1+m)+b^(
2/3)*ln(b^(1/3)*x^(-1/2-1/2*m)-(a+b/(x^(3/2+3/2*m)))^(1/3))/(1+m)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 347 vs. $2(139) = 278$.

Time = 0.67 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.50

$$\int x^m \left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx =$$

$$-\frac{x^{\frac{1}{2}(-1-m)}\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3}\left(\sqrt[3]{b}-\sqrt[3]{b+ax^{\frac{3(1+m)}{2}}}\right)\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b+ax^{\frac{3(1+m)}{2}}}+\left(b+ax^{\frac{3(1+m)}{2}}\right)^{2/3}\right)^2}{3a(1+m)\sqrt[3]{b}\cdots}$$

input
```
Integrate[x^m*(a + b/x^((3*(1 + m))/2))^(2/3),x]
```

output
```
-1/3*(x^((-1 - m)/2)*(a + b/x^((3*(1 + m))/2))^(2/3)*(b^(1/3) - (b + a*x^(
(3*(1 + m))/2))^(1/3))*(b^(2/3) + b^(1/3)*(b + a*x^((3*(1 + m))/2))^(1/3)
+ (b + a*x^((3*(1 + m))/2))^(2/3))^2*(3*(b + a*x^((3*(1 + m))/2))^(2/3) +
2*Sqrt[3]*b^(2/3)*ArcTan[(b^(1/3) + 2*(b + a*x^((3*(1 + m))/2))^(1/3))/(Sq
rt[3]*b^(1/3))] + 2*b^(2/3)*Log[b^(1/3) - (b + a*x^((3*(1 + m))/2))^(1/3)]
 - b^(2/3)*Log[b^(2/3) + b^(1/3)*(b + a*x^((3*(1 + m))/2))^(1/3) + (b + a*
x^((3*(1 + m))/2))^(2/3)]))/(a*(1 + m)*(b + a*x^((3*(1 + m))/2))^(1/3)*(b
+ a*x^((3*(1 + m))/2) + b^(2/3)*(b + a*x^((3*(1 + m))/2))^(1/3) + b^(1/3)*
(b + a*x^((3*(1 + m))/2))^(2/3)))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {872, 868, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^{-\frac{3}{2}(m+1)}\right)^{2/3} dx$$

$$\Big\downarrow \text{872}$$

$$b\int \frac{x^{m-\frac{3(m+1)}{2}}}{\sqrt[3]{bx^{-\frac{3}{2}(m+1)}+a}}dx+\frac{x^{m+1}\left(a+bx^{-\frac{3}{2}(m+1)}\right)^{2/3}}{m+1}$$

$$\Big\downarrow \text{868}$$

$$\frac{x^{m+1}\left(a+bx^{-\frac{3}{2}(m+1)}\right)^{2/3}}{m+1}-\frac{2b\int \frac{1}{\sqrt[3]{bx^{\frac{3}{2}(-m-1)}+a}}dx^{\frac{1}{2}(-m-1)}}{m+1}$$

$$\Big\downarrow \text{769}$$

$$\frac{x^{m+1}\left(a+bx^{-\frac{3}{2}(m+1)}\right)^{2/3}}{m+1}-\frac{2b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x^{\frac{1}{2}(-m-1)}}{\sqrt[3]{a+bx^{\frac{3}{2}(-m-1)}}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^{\frac{3}{2}(-m-1)}}-\sqrt[3]{b}x^{\frac{1}{2}(-m-1)}\right)}{2\sqrt[3]{b}}\right)}{m+1}$$

input

```
Int[x^m*(a + b/x^((3*(1 + m))/2))^(2/3),x]
```

output

```
(x^(1 + m)*(a + b/x^((3*(1 + m))/2))^(2/3))/(1 + m) - (2*b*(ArcTan[(1 + (2
*b^(1/3)*x^((-1 - m)/2))/(a + b*x^((3*(-1 - m))/2))^(1/3))/Sqrt[3]]/(Sqrt[
3]*b^(1/3)) - Log[-(b^(1/3)*x^((-1 - m)/2)) + (a + b*x^((3*(-1 - m))/2))^(
1/3)]/(2*b^(1/3))))/(1 + m)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 872
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[x^(m + n)*(a + b*x^
n)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && Gt
Q[p, 0]
```

Maple [F]

$$\int x^{m}\left(a+b\,x^{-\frac{3m}{2}-\frac{3}{2}}\right)^{\frac{2}{3}}dx$$

input
```
int(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x)
```

output
```
int(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.62 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int x^m\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx = -\frac{2a^{\frac{2m}{3m+3}+\frac{2}{3}+\frac{2}{3m+3}}x^{m+1}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\-\frac{2m}{3m+3}+1-\frac{2}{3m+3}\end{matrix}\middle|\frac{bx^{-\frac{3m}{2}-\frac{3}{2}}e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}m\Gamma\left(-\frac{2m}{3m+3}+1-\frac{2}{3m+3}\right)+3a^{\frac{2}{3}}\Gamma\left(-\frac{2m}{3m+3}+1-\frac{2}{3m+3}\right)}$$

input
```
integrate(x**m*(a+b/(x**(3/2+3/2*m)))**(2/3),x)
```

output
```
-2*a**(2*m/(3*m + 3) + 2/3 + 2/(3*m + 3))*x**(m + 1)*gamma(-2/3)*hyper((-2
/3, -2/3), (-2*m/(3*m + 3) + 1 - 2/(3*m + 3),), b*x**(-3*m/2 - 3/2)*exp_po
lar(I*pi)/a)/(3*a**(2/3)*m*gamma(-2*m/(3*m + 3) + 1 - 2/(3*m + 3)) + 3*a**
(2/3)*gamma(-2*m/(3*m + 3) + 1 - 2/(3*m + 3)))
```

Maxima [F]

$$\int x^m\left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx = \int \left(a+\frac{b}{x^{\frac{3}{2}m+\frac{3}{2}}}\right)^{\frac{2}{3}}x^m\,dx$$

input
```
integrate(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^(-3/2*m - 3/2) + a)^(2/3)*x^m, x)
```

Giac [F]

$$\int x^m \left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx = \int \left(a+\frac{b}{x^{\frac{3}{2}m+\frac{3}{2}}}\right)^{\frac{2}{3}} x^m \, dx$$

input
```
integrate(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x, algorithm="giac")
```

output
```
integrate((a + b/x^(3/2*m + 3/2))^(2/3)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx = \int x^m \left(a+\frac{b}{x^{\frac{3m}{2}+\frac{3}{2}}}\right)^{2/3} dx$$

input
```
int(x^m*(a + b/x^((3*m)/2 + 3/2))^(2/3),x)
```

output
```
int(x^m*(a + b/x^((3*m)/2 + 3/2))^(2/3), x)
```

Reduce [F]

$$\int x^m \left(a+bx^{-\frac{3}{2}(1+m)}\right)^{2/3} dx = \frac{\left(x^{\frac{3m}{2}+\frac{1}{2}}ax+b\right)^{\frac{2}{3}} + \left(\int \frac{\left(x^{\frac{3m}{2}+\frac{1}{2}}ax+b\right)^{\frac{2}{3}}}{x^{\frac{3m}{2}+\frac{1}{2}}a\,x^2+bx}dx\right) bm + \left(\int \frac{\left(x^{\frac{3m}{2}+\frac{1}{2}}ax+b\right)^{\frac{2}{3}}}{x^{\frac{3m}{2}+\frac{1}{2}}a\,x^2+bx}dx\right) b}{m+1}$$

input
```
int(x^m*(a+b/(x^(3/2+3/2*m)))^(2/3),x)
```

output
```
((x**((3*m + 1)/2)*a*x + b)**(2/3) + int((x**((3*m + 1)/2)*a*x + b)**(2/3)
/(x**((3*m + 1)/2)*a*x**2 + b*x),x)*b*m + int((x**((3*m + 1)/2)*a*x + b)**
(2/3)/(x**((3*m + 1)/2)*a*x**2 + b*x),x)*b)/(m + 1)
```

3.629 $\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a+bx^n}}}{\sqrt{3}}\right)}{\sqrt[3]{b}n}-\frac{3\log\left(\sqrt[3]{b}x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2\sqrt[3]{b}n}$$

output
```
3^(1/2)*arctan(1/3*(1+2*b^(1/3)*x^(1/3*n)/(a+b*x^n)^(1/3))*3^(1/2))/b^(1/3
)/n-3/2*ln(b^(1/3)*x^(1/3*n)-(a+b*x^n)^(1/3))/b^(1/3)/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx=\frac{3x^{n/3}\sqrt[3]{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^n}{a}\right)}{n\sqrt[3]{a+bx^n}}$$

input
```
Integrate[x^(-1 + n/3)/(a + b*x^n)^(1/3),x]
```

output
```
(3*x^(n/3)*(1 + (b*x^n)/a)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, -((b*x^n
)/a)])/(n*(a + b*x^n)^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {868, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{\frac{n}{3}-1}}{\sqrt[3]{a+bx^n}}dx$$

$$\Big\downarrow \text{ 868}$$

$$\frac{3\int \frac{1}{\sqrt[3]{bx^n+a}}dx^{n/3}}{n}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a+bx^n}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^n}-\sqrt[3]{b}x^{n/3}\right)}{2\sqrt[3]{b}}\right)}{n}$$

input
```
Int[x^(-1 + n/3)/(a + b*x^n)^(1/3),x]
```

output
```
(3*(ArcTan[(1 + (2*b^(1/3)*x^(n/3))/(a + b*x^n)^(1/3))/Sqrt[3]]/(Sqrt[3]*b
^(1/3)) - Log[-(b^(1/3)*x^(n/3)) + (a + b*x^n)^(1/3)]/(2*b^(1/3))))/n
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Maple [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{(a+b\,x^{n})^{\frac{1}{3}}}dx$$

input
```
int(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x)
```

output
```
int(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx = \frac{x^{\frac{n}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt[3]{a}n\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(x**(-1+1/3*n)/(a+b*x**n)**(1/3),x)
```

output
```
x**(n/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**n*exp_polar(I*pi)/a)/(a
**(1/3)*n*gamma(4/3))
```

Maxima [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{x^{\frac{1}{3}n-1}}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x^(1/3*n - 1)/(b*x^n + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx = \int \frac{x^{\frac{1}{3}n-1}}{(bx^n+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^(1/3*n - 1)/(b*x^n + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx=\int \frac{x^{\frac{n}{3}-1}}{\left(a+b\,x^n\right)^{1/3}}\,dx$$

input
```
int(x^(n/3 - 1)/(a + b*x^n)^(1/3),x)
```

output
```
int(x^(n/3 - 1)/(a + b*x^n)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^{-1+\frac{n}{3}}}{\sqrt[3]{a+bx^n}}\,dx=\left(\int \frac{x^{\frac{4n}{3}}}{x^n\left(x^nb+a\right)^{\frac{1}{3}}bx+\left(x^nb+a\right)^{\frac{1}{3}}ax}dx\right)b+\left(\int \frac{x^{\frac{n}{3}}}{x^n\left(x^nb+a\right)^{\frac{1}{3}}bx+\left(x^nb+a\right)^{\frac{1}{3}}ax}dx\right)a$$

input
```
int(x^(-1+1/3*n)/(a+b*x^n)^(1/3),x)
```

output
```
int(x**((4*n)/3)/(x**n*(x**n*b + a)**(1/3)*b*x + (x**n*b + a)**(1/3)*a*x),
x)*b + int(x**(n/3)/(x**n*(x**n*b + a)**(1/3)*b*x + (x**n*b + a)**(1/3)*a*
x),x)*a
```

3.630 $\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx = -\frac{3x^{-2n/3}(a+bx^n)^{2/3}}{2n} + \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a+bx^n}}}{\sqrt{3}}\right)}{n} - \frac{3b^{2/3}\log\left(\sqrt[3]{b}x^{n/3}-\sqrt[3]{a+bx^n}\right)}{2n}$$

output
```
-3/2*(a+b*x^n)^(2/3)/n/(x^(2/3*n))+3^(1/2)*b^(2/3)*arctan(1/3*(1+2*b^(1/3)
*x^(1/3*n)/(a+b*x^n)^(1/3))*3^(1/2))/n-3/2*b^(2/3)*ln(b^(1/3)*x^(1/3*n)-(a
+b*x^n)^(1/3))/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx = -\frac{3x^{-2n/3}(a+bx^n)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{3},\frac{1}{3},-\frac{bx^n}{a}\right)}{2n\left(1+\frac{bx^n}{a}\right)^{2/3}}$$

input
```
Integrate[x^(-1 - (2*n)/3)*(a + b*x^n)^(2/3),x]
```

output
```
(-3*(a + b*x^n)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, -((b*x^n)/a)])/(2
*n*x^((2*n)/3)*(1 + (b*x^n)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {872, 868, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{2n}{3}-1}(a+bx^n)^{2/3}\,dx$$

$$\Big\downarrow \text{872}$$

$$b\int \frac{x^{\frac{n-3}{3}}}{\sqrt[3]{bx^n+a}}dx-\frac{3x^{-2n/3}(a+bx^n)^{2/3}}{2n}$$

$$\Big\downarrow \text{868}$$

$$\frac{3b\int \frac{1}{\sqrt[3]{bx^n+a}}dx^{n/3}}{n}-\frac{3x^{-2n/3}(a+bx^n)^{2/3}}{2n}$$

$$\Big\downarrow \text{769}$$

$$\frac{3b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a+bx^n}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^n}-\sqrt[3]{b}x^{n/3}\right)}{2\sqrt[3]{b}}\right)}{n}-\frac{3x^{-2n/3}(a+bx^n)^{2/3}}{2n}$$

input
```
Int[x^(-1 - (2*n)/3)*(a + b*x^n)^(2/3),x]
```

output
```
(-3*(a + b*x^n)^(2/3))/(2*n*x^((2*n)/3)) + (3*b*(ArcTan[(1 + (2*b^(1/3)*x^
(n/3))/(a + b*x^n)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x^(n/
3)) + (a + b*x^n)^(1/3)]/(2*b^(1/3))))/n
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 872
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[x^(m + n)*(a + b*x^
n)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && Gt
Q[p, 0]
```

Maple [F]

$$\int x^{-1-\frac{2n}{3}} \left(a+b\,x^{n}\right)^{\frac{2}{3}}d x$$

input
```
int(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x)
```

output
```
int(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.40

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx = \frac{a^{\frac{2}{3}}x^{-\frac{2n}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(x**(-1-2/3*n)*(a+b*x**n)**(2/3),x)
```

output
```
a**(2/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*x**n*exp_polar(I*pi)/a)
/(n*x**(2*n/3)*gamma(1/3))
```

Maxima [F]

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx=\int (bx^n+a)^{\frac{2}{3}}x^{-\frac{2}{3}n-1}\,dx$$

input
```
integrate(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(2/3)*x^(-2/3*n - 1), x)
```

Giac [F]

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx=\int (bx^n+a)^{\frac{2}{3}}x^{-\frac{2}{3}n-1}\,dx$$

input
```
integrate(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(2/3)*x^(-2/3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx=\int \frac{(a+b\,x^n)^{2/3}}{x^{\frac{2\,n}{3}+1}}\,dx$$

input
```
int((a + b*x^n)^(2/3)/x^((2*n)/3 + 1),x)
```

output
```
int((a + b*x^n)^(2/3)/x^((2*n)/3 + 1), x)
```

Reduce [F]

$$\int x^{-1-\frac{2n}{3}}(a+bx^n)^{2/3}\,dx=\int\frac{(x^nb+a)^{\frac{2}{3}}}{x^{\frac{2n}{3}}x}dx$$

input
```
int(x^(-1-2/3*n)*(a+b*x^n)^(2/3),x)
```

output
```
int((x**n*b + a)**(2/3)/(x**((2*n)/3)*x),x)
```

3.631 $\int x^m(a+bx^n)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int x^m(a+bx^n)^p\,dx = \frac{x^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m}$$

output
```
x^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/(1+m)/((
1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int x^m(a+bx^n)^p\,dx = \frac{x^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{1+m}$$

input
```
Integrate[x^m*(a + b*x^n)^p,x]
```

output
```
(x^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, 1 + (1 + m)/n, -
((b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int x^m \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{m+1}$$

input
```
Int[x^m*(a + b*x^n)^p,x]
```

output
```
(x^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n, -
((b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^m(a+b\,x^n)^p\,dx$$

input
```
int(x^m*(a+b*x^n)^p,x)
```

output
```
int(x^m*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^m(a+bx^n)^p\,dx=\int(bx^n+a)^px^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.63 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int x^m(a+bx^n)^p\,dx=\frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}+p-\frac{1}{n}}x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate(x**m*(a+b*x**n)**p,x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*x**(m + 1)*gamma(m/n + 1/n)*hyper((-p,
m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1 +
 1/n))
```

Maxima [F]

$$\int x^m(a+bx^n)^p\,dx=\int (bx^n+a)^px^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^m, x)
```

Giac [F]

$$\int x^m(a+bx^n)^p\,dx=\int (bx^n+a)^px^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m(a+bx^n)^p\,dx=\int x^m\,(a+b\,x^n)^p\,dx$$

input
```
int(x^m*(a + b*x^n)^p,x)
```

output
```
int(x^m*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int x^m(a+bx^n)^p\,dx = \frac{x^m(x^nb+a)^p\,x+\left(\int \frac{x^m(x^nb+a)^p}{x^nbm+x^nbnp+x^nb+am+anp+a}dx\right)amnp+\left(\int \frac{x^m(x^nb+a)^p}{x^nbm+x^nbnp+x^nb+am+anp+a}dx\right)a\,n^2p^2+\left(\int\right.}{np+m+1}$$

input
```
int(x^m*(a+b*x^n)^p,x)
```

output
```
(x**m*(x**n*b + a)**p*x + int((x**m*(x**n*b + a)**p)/(x**n*b*m + x**n*b*n*
p + x**n*b + a*m + a*n*p + a),x)*a*m*n*p + int((x**m*(x**n*b + a)**p)/(x**
n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*n**2*p**2 + int((x**m*
(x**n*b + a)**p)/(x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*n
*p)/(m + n*p + 1)
```

3.632 $\int x^{-1+n}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(a+bx^n)^{1+p}}{bn(1+p)}$$

output
```
(a+b*x^n)^(p+1)/b/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(a+bx^n)^{1+p}}{bn(1+p)}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
(a + b*x^n)^(1 + p)/(b*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{793}$$

$$\frac{(a+bx^n)^{p+1}}{bn(p+1)}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
(a + b*x^n)^(1 + p)/(b*n*(1 + p))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
risch	$\frac{(a+b\,x^{n})(a+b\,x^{n})^{p}}{b(p+1)n}$	29

input
```
int(x^(-1+n)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*x^n)/b/(p+1)/n*(a+b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(bx^n+a)(bx^n+a)^p}{bnp+bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
(b*x^n + a)*(b*x^n + a)^p/(b*n*p + b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(15) = 30$.

Time = 11.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int x^{-1+n}(a+bx^n)^p\,dx=\begin{cases}\frac{\log(x)}{a} & \text{for } b=0\wedge n=0\wedge p=-1\\ \frac{a^pxx^{n-1}}{n} & \text{for } b=0\\ (a+b)^p\log(x) & \text{for } n=0\\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{for } p=-1\\ \frac{a(a+bx^n)^p}{bnp+bn}+\frac{bx^n(a+bx^n)^p}{bnp+bn} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**p,x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0) & Eq(p, -1)), (a**p*x*x**(n - 1)/
n, Eq(b, 0)), ((a + b)**p*log(x), Eq(n, 0)), (log(a/b + x**n)/(b*n), Eq(p,
 -1)), (a*(a + b*x**n)**p/(b*n*p + b*n) + b*x**n*(a + b*x**n)**p/(b*n*p +
b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx = \frac{(bx^n+a)^{p+1}}{bn(p+1)}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
(b*x^n + a)^(p + 1)/(b*n*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx = \frac{(bx^n+a)^{p+1}}{bn(p+1)}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
(b*x^n + a)^(p + 1)/(b*n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx = \frac{(a+b\,x^n)^{p+1}}{b\,n\,(p+1)}$$

input
```
int(x^(n - 1)*(a + b*x^n)^p,x)
```

output
```
(a + b*x^n)^(p + 1)/(b*n*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(x^nb+a)^p\,(x^nb+a)}{bn\,(p+1)}$$

input
```
int(x^(-1+n)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**n*b + a))/(b*n*(p + 1))
```

3.633 $\int x^m (bx^n)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int x^m (bx^n)^p \, dx = \frac{x^{1+m} (bx^n)^p}{1+m+np}$$

output
```
x^(1+m)*(b*x^n)^p/(n*p+m+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^m (bx^n)^p \, dx = \frac{x^{1+m} (bx^n)^p}{1+m+np}$$

input
```
Integrate[x^m*(b*x^n)^p,x]
```

output
```
(x^(1 + m)*(b*x^n)^p)/(1 + m + n*p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (bx^n)^p \, dx$$

$$\Big\downarrow \text{ 23}$$

$$x^{-np}(bx^n)^p \int x^{m+np}dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}(bx^n)^p}{m+np+1}$$

input
```
Int[x^m*(b*x^n)^p,x]
```

output
```
(x^(1 + m)*(b*x^n)^p)/(1 + m + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{x\,x^{m} \left(b\,x^{n}\right)^{p}}{n p +m +1}$	21
orering	$\frac{x\,x^{m} \left(b\,x^{n}\right)^{p}}{n p +m +1}$	21
gosper	$\frac{x^{1+m} \left(b\,x^{n}\right)^{p}}{n p +m +1}$	22

input
```
int(x^m*(b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x/(n*p+m+1)*x^m*(b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^m (bx^n)^p \, dx = \frac{xx^m e^{(np\log(x)+p\log(b))}}{np+m+1}$$

input
```
integrate(x^m*(b*x^n)^p,x, algorithm="fricas")
```

output
```
x*x^m*e^(n*p*log(x) + p*log(b))/(n*p + m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.

Time = 1.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int x^m (bx^n)^p \, dx = \begin{cases} \frac{xx^m(bx^n)^p}{m+np+1} & \text{for } m \neq -np-1 \\ xx^{-np-1}(bx^n)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(b*x**n)**p,x)
```

output
```
Piecewise((x*x**m*(b*x**n)**p/(m + n*p + 1), Ne(m, -n*p - 1)), (x*x**(-n*p
 - 1)*(b*x**n)**p*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x^m (bx^n)^p \, dx = \frac{b^p x e^{(m \log(x) + p \log(x^n))}}{np + m + 1}$$

input
```
integrate(x^m*(b*x^n)^p,x, algorithm="maxima")
```

output
```
b^p*x*e^(m*log(x) + p*log(x^n))/(n*p + m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int x^m (bx^n)^p \, dx = \frac{x x^m e^{(np \log(x) + p \log(b))}}{np + m + 1}$$

input
```
integrate(x^m*(b*x^n)^p,x, algorithm="giac")
```

output
```
x*x^m*e^(n*p*log(x) + p*log(b))/(n*p + m + 1)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^m (bx^n)^p \, dx = \frac{x^{m+1} (b\, x^n)^p}{m + n\, p + 1}$$

input
```
int(x^m*(b*x^n)^p,x)
```

output
```
(x^(m + 1)*(b*x^n)^p)/(m + n*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int x^m (bx^n)^p \, dx = \frac{x^{np+m} b^p x}{np + m + 1}$$

input
```
int(x^m*(b*x^n)^p,x)
```

output
```
(x**(m + n*p)*b**p*x)/(m + n*p + 1)
```

3.634 $\int x^2(bx^n)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int x^2(bx^n)^p \, dx = \frac{x^3(bx^n)^p}{3+np}$$

output
```
x^3*(b*x^n)^p/(n*p+3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2(bx^n)^p \, dx = \frac{x^3(bx^n)^p}{3+np}$$

input
```
Integrate[x^2*(b*x^n)^p,x]
```

output
```
(x^3*(b*x^n)^p)/(3 + n*p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (bx^n)^p \, dx$$

$$\Big\downarrow \ 23$$

$$x^{-np}(bx^n)^p \int x^{np+2}dx$$

$$\Big\downarrow \ 15$$

$$\frac{x^3(bx^n)^p}{np+3}$$

input
```
Int[x^2*(b*x^n)^p,x]
```

output
```
(x^3*(b*x^n)^p)/(3 + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^3(b\,x^n)^p}{np+3}$	19
parallelrisch	$\frac{x^3(b\,x^n)^p}{np+3}$	19
orering	$\frac{x^3(b\,x^n)^p}{np+3}$	19

input
```
int(x^2*(b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x^3*(b*x^n)^p/(n*p+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2(bx^n)^p\,dx = \frac{x^3e^{(np\log(x)+p\log(b))}}{np+3}$$

input
```
integrate(x^2*(b*x^n)^p,x, algorithm="fricas")
```

output
```
x^3*e^(n*p*log(x) + p*log(b))/(n*p + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x^2(bx^n)^p\,dx = \begin{cases} \frac{x^3(bx^n)^p}{np+3} & \text{for } n \neq -\frac{3}{p} \\ x^3\left(bx^{-\frac{3}{p}}\right)^p\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**n)**p,x)
```

output
```
Piecewise((x**3*(b*x**n)**p/(n*p + 3), Ne(n, -3/p)), (x**3*(b/x**(3/p))**p
*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^2(bx^n)^p\,dx = \frac{b^p x^3 (x^n)^p}{np+3}$$

input
```
integrate(x^2*(b*x^n)^p,x, algorithm="maxima")
```

output
```
b^p*x^3*(x^n)^p/(n*p + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2(bx^n)^p\,dx = \frac{x^3 e^{(np\log(x)+p\log(b))}}{np+3}$$

input
```
integrate(x^2*(b*x^n)^p,x, algorithm="giac")
```

output
```
x^3*e^(n*p*log(x) + p*log(b))/(n*p + 3)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2(bx^n)^p \, dx = \frac{x^3 \, (b \, x^n)^p}{n \, p + 3}$$

input
```
int(x^2*(b*x^n)^p,x)
```

output
```
(x^3*(b*x^n)^p)/(n*p + 3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x^2(bx^n)^p \, dx = \frac{x^{np} b^p x^3}{np + 3}$$

input
```
int(x^2*(b*x^n)^p,x)
```

output
```
(x**(n*p)*b**p*x**3)/(n*p + 3)
```

3.635 $\int x(bx^n)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int x(bx^n)^p \, dx = \frac{x^2(bx^n)^p}{2+np}$$

output
```
x^2*(b*x^n)^p/(n*p+2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x(bx^n)^p \, dx = \frac{x^2(bx^n)^p}{2+np}$$

input
```
Integrate[x*(b*x^n)^p,x]
```

output
```
(x^2*(b*x^n)^p)/(2 + n*p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(bx^n)^p\,dx$$

$$\Big\downarrow \text{23}$$

$$x^{-np}(bx^n)^p\int x^{np+1}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2(bx^n)^p}{np+2}$$

input
```
Int[x*(b*x^n)^p,x]
```

output
```
(x^2*(b*x^n)^p)/(2 + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^2(b\,x^n)^p}{np+2}$	19
parallelrisch	$\frac{x^2(b\,x^n)^p}{np+2}$	19
orering	$\frac{x^2(b\,x^n)^p}{np+2}$	19

input
```
int(x*(b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x^2*(b*x^n)^p/(n*p+2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x(bx^n)^p\,dx = \frac{x^2 e^{(np\log(x)+p\log(b))}}{np+2}$$

input
```
integrate(x*(b*x^n)^p,x, algorithm="fricas")
```

output
```
x^2*e^(n*p*log(x) + p*log(b))/(n*p + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x(bx^n)^p\,dx = \begin{cases} \frac{x^2(bx^n)^p}{np+2} & \text{for } n \neq -\frac{2}{p} \\ x^2\left(bx^{-\frac{2}{p}}\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**n)**p,x)
```

output
```
Piecewise((x**2*(b*x**n)**p/(n*p + 2), Ne(n, -2/p)), (x**2*(b/x**(2/p))**p
*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x(bx^n)^p\,dx = \frac{b^p x^2 (x^n)^p}{np+2}$$

input
```
integrate(x*(b*x^n)^p,x, algorithm="maxima")
```

output
```
b^p*x^2*(x^n)^p/(n*p + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x(bx^n)^p\,dx = \frac{x^2 e^{(np\log(x)+p\log(b))}}{np+2}$$

input
```
integrate(x*(b*x^n)^p,x, algorithm="giac")
```

output
```
x^2*e^(n*p*log(x) + p*log(b))/(n*p + 2)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x(bx^n)^p\,dx=\frac{x^2\,{\left(b\,x^n\right)}^p}{n\,p+2}$$

input
```
int(x*(b*x^n)^p,x)
```

output
```
(x^2*(b*x^n)^p)/(n*p + 2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int x(bx^n)^p\,dx=\frac{x^{np}b^p x^2}{np+2}$$

input
```
int(x*(b*x^n)^p,x)
```

output
```
(x**(n*p)*b**p*x**2)/(n*p + 2)
```

3.636 $\int (bx^n)^p \, dx$

Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (bx^n)^p \, dx = \frac{x(bx^n)^p}{1+np}$$

output
```
x*(b*x^n)^p/(n*p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (bx^n)^p \, dx = \frac{x(bx^n)^p}{1+np}$$

input
```
Integrate[(b*x^n)^p,x]
```

output
```
(x*(b*x^n)^p)/(1 + n*p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^n)^p \, dx$$

$$\Big\downarrow \; 20$$

$$x^{-np}(bx^n)^p \int x^{np}dx$$

$$\Big\downarrow \; 15$$

$$\frac{x(bx^n)^p}{np+1}$$

input
```
Int[(b*x^n)^p,x]
```

output
```
(x*(b*x^n)^p)/(1 + n*p)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x(b\,x^{n})^{p}}{np+1}$	17
parallelrisch	$\frac{x(b\,x^{n})^{p}}{np+1}$	17
orering	$\frac{x(b\,x^{n})^{p}}{np+1}$	17
norman	$\frac{x\,{\mathrm e}^{p\ln\left(b\,{\mathrm e}^{n\ln(x)}\right)}}{np+1}$	21

input
```
int((b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
x*(b*x^n)^p/(n*p+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (bx^n)^p \,dx = \frac{xe^{(np\log(x)+p\log(b))}}{np+1}$$

input
```
integrate((b*x^n)^p,x, algorithm="fricas")
```

output
```
x*e^(n*p*log(x) + p*log(b))/(n*p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int (bx^n)^p \, dx = \begin{cases} \frac{x(bx^n)^p}{np+1} & \text{for } n \neq -\frac{1}{p} \\ x\left(bx^{-\frac{1}{p}}\right)^p \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**n)**p,x)
```

output
```
Piecewise((x*(b*x**n)**p/(n*p + 1), Ne(n, -1/p)), (x*(b/x**(1/p))**p*log(x
), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (bx^n)^p \, dx = \frac{b^p x (x^n)^p}{np+1}$$

input
```
integrate((b*x^n)^p,x, algorithm="maxima")
```

output
```
b^p*x*(x^n)^p/(n*p + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (bx^n)^p \, dx = \frac{xe^{(np\log(x)+p\log(b))}}{np+1}$$

input
```
integrate((b*x^n)^p,x, algorithm="giac")
```

output
```
x*e^(n*p*log(x) + p*log(b))/(n*p + 1)
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (bx^n)^p \, dx = \frac{x\,(b\,x^n)^p}{n\,p+1}$$

input
```
int((b*x^n)^p,x)
```

output
```
(x*(b*x^n)^p)/(n*p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (bx^n)^p \, dx = \frac{x^{np} b^p x}{np+1}$$

input
```
int((b*x^n)^p,x)
```

output
```
(x**(n*p)*b**p*x)/(n*p + 1)
```

3.637 $\int \frac{(bx^n)^p}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{(bx^n)^p}{x} dx = \frac{(bx^n)^p}{np}$$

output
```
(b*x^n)^p/n/p
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx^n)^p}{x} dx = \frac{(bx^n)^p}{np}$$

input
```
Integrate[(b*x^n)^p/x,x]
```

output
```
(b*x^n)^p/(n*p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {21, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n)^p}{x} \, dx$$

$$\Big\downarrow \; 21$$

$$\frac{b \int (bx^n)^{p-1} \, dx^n}{n}$$

$$\Big\downarrow \; 17$$

$$\frac{(bx^n)^p}{np}$$

input
```
Int[(b*x^n)^p/x,x]
```

output
```
(b*x^n)^p/(n*p)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 21
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(n*a^(Simplify[(
m + 1)/n] - 1))   Subst[Int[(a*x)^(Simplify[(m + 1)/n] + p - 1), x], x, x^n
], x] /; FreeQ[{a, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{(b\,x^n)^p}{np}$	15
derivativedivides	$\frac{(b\,x^n)^p}{np}$	15
default	$\frac{(b\,x^n)^p}{np}$	15
parallelrisch	$\frac{(b\,x^n)^p}{np}$	15
orering	$\frac{(b\,x^n)^p}{np}$	15

input
```
int((b*x^n)^p/x,x,method=_RETURNVERBOSE)
```

output
```
(b*x^n)^p/n/p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{(bx^n)^p}{x}\,dx = \frac{e^{(np\log(x)+p\log(b))}}{np}$$

input
```
integrate((b*x^n)^p/x,x, algorithm="fricas")
```

output
```
e^(n*p*log(x) + p*log(b))/(n*p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{(bx^n)^p}{x}\,dx = \begin{cases} \log(x) & \text{for } n = 0 \wedge p = 0 \\ b^p \log(x) & \text{for } n = 0 \\ \log(x) & \text{for } p = 0 \\ \frac{(bx^n)^p}{np} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**n)**p/x,x)
```

output
```
Piecewise((log(x), Eq(n, 0) & Eq(p, 0)), (b**p*log(x), Eq(n, 0)), (log(x),
 Eq(p, 0)), ((b*x**n)**p/(n*p), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{(bx^n)^p}{x}\,dx = \frac{b^p(x^n)^p}{np}$$

input
```
integrate((b*x^n)^p/x,x, algorithm="maxima")
```

output
```
b^p*(x^n)^p/(n*p)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx^n)^p}{x}\,dx = \frac{(bx^n)^p}{np}$$

input
```
integrate((b*x^n)^p/x,x, algorithm="giac")
```

output
```
(b*x^n)^p/(n*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^n)^p}{x}\,dx = \int \frac{(b\,x^n)^p}{x}\,dx$$

input
```
int((b*x^n)^p/x,x)
```

output
```
int((b*x^n)^p/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{(bx^n)^p}{x}\,dx = \frac{x^{np}b^p}{np}$$

input
```
int((b*x^n)^p/x,x)
```

output
```
(x**(n*p)*b**p)/(n*p)
```

3.638 $\int \frac{(bx^n)^p}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{(bx^n)^p}{x^2}\,dx = -\frac{(bx^n)^p}{(1-np)x}$$

output
```
-(b*x^n)^p/(-n*p+1)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(bx^n)^p}{x^2}\,dx = \frac{(bx^n)^p}{(-1+np)x}$$

input
```
Integrate[(b*x^n)^p/x^2,x]
```

output
```
(b*x^n)^p/((-1 + n*p)*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n)^p}{x^2} \, dx$$
$$\big\downarrow \; 23$$
$$x^{-np}(bx^n)^p \int x^{np-2}dx$$
$$\big\downarrow \; 15$$
$$-\frac{(bx^n)^p}{x(1-np)}$$

input
```
Int[(b*x^n)^p/x^2,x]
```

output
```
-((b*x^n)^p/((1 - n*p)*x))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{(b\,x^{n})^{p}}{x(np-1)}$	19
parallelrisch	$\frac{(b\,x^{n})^{p}}{x(np-1)}$	19
orering	$\frac{(b\,x^{n})^{p}}{x(np-1)}$	19

input
```
int((b*x^n)^p/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x/(n*p-1)*(b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(bx^n)^p}{x^2}\,dx = \frac{e^{(np\log(x)+p\log(b))}}{(np-1)x}$$

input
```
integrate((b*x^n)^p/x^2,x, algorithm="fricas")
```

output
```
e^(n*p*log(x) + p*log(b))/((n*p - 1)*x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{(bx^n)^p}{x^2}\,dx = \begin{cases} \frac{(bx^n)^p}{npx-x} & \text{for}\ n \neq \frac{1}{p} \\ \frac{\left(bx^{\frac{1}{p}}\right)^p \log(x)}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**n)**p/x**2,x)
```

output
```
Piecewise(((b*x**n)**p/(n*p*x - x), Ne(n, 1/p)), ((b*x**(1/p))**p*log(x)/x
, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(bx^n)^p}{x^2} dx = \frac{b^p(x^n)^p}{(np-1)x}$$

input
```
integrate((b*x^n)^p/x^2,x, algorithm="maxima")
```

output
```
b^p*(x^n)^p/((n*p - 1)*x)
```

Giac [F]

$$\int \frac{(bx^n)^p}{x^2} dx = \int \frac{(bx^n)^p}{x^2} dx$$

input
```
integrate((b*x^n)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^n)^p}{x^2} dx = \int \frac{(b\,x^n)^p}{x^2} dx$$

input
```
int((b*x^n)^p/x^2,x)
```

output
```
int((b*x^n)^p/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(bx^n)^p}{x^2} \, dx = \frac{x^{np}b^p}{x\,(np-1)}$$

input
```
int((b*x^n)^p/x^2,x)
```

output
```
(x**(n*p)*b**p)/(x*(n*p - 1))
```

3.639 $\int \frac{(bx^n)^p}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{(bx^n)^p}{x^3} dx = -\frac{(bx^n)^p}{(2-np)x^2}$$

output
```
-(b*x^n)^p/(-n*p+2)/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(bx^n)^p}{x^3} dx = \frac{(bx^n)^p}{(-2+np)x^2}$$

input
```
Integrate[(b*x^n)^p/x^3,x]
```

output
```
(b*x^n)^p/((-2 + n*p)*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n)^p}{x^3} dx$$

$$\Big\downarrow \text{23}$$

$$x^{-np}(bx^n)^p \int x^{np-3}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{(bx^n)^p}{x^2(2-np)}$$

input
```
Int[(b*x^n)^p/x^3,x]
```

output
```
-((b*x^n)^p/((2 - n*p)*x^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{(b\,x^{n})^{p}}{x^{2}(np-2)}$	19
parallelrisch	$\frac{(b\,x^{n})^{p}}{x^{2}(np-2)}$	19
orering	$\frac{(b\,x^{n})^{p}}{x^{2}(np-2)}$	19

input
```
int((b*x^n)^p/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/x^2/(n*p-2)*(b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(bx^n)^p}{x^3}\,dx = \frac{e^{(np\log(x)+p\log(b))}}{(np-2)x^2}$$

input
```
integrate((b*x^n)^p/x^3,x, algorithm="fricas")
```

output
```
e^(n*p*log(x) + p*log(b))/((n*p - 2)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(bx^n)^p}{x^3}\,dx = \begin{cases} \frac{(bx^n)^p}{npx^2-2x^2} & \text{for } n \neq \frac{2}{p} \\ \frac{\left(bx^{\frac{2}{p}}\right)^p \log(x)}{x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**n)**p/x**3,x)
```

output
```
Piecewise(((b*x**n)**p/(n*p*x**2 - 2*x**2), Ne(n, 2/p)), ((b*x**(2/p))**p*
log(x)/x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(bx^n)^p}{x^3} \, dx = \frac{b^p (x^n)^p}{(np-2)x^2}$$

input
```
integrate((b*x^n)^p/x^3,x, algorithm="maxima")
```

output
```
b^p*(x^n)^p/((n*p - 2)*x^2)
```

Giac [F]

$$\int \frac{(bx^n)^p}{x^3} \, dx = \int \frac{(bx^n)^p}{x^3} \, dx$$

input
```
integrate((b*x^n)^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^n)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^n)^p}{x^3} \, dx = \int \frac{(b\,x^n)^p}{x^3} \, dx$$

input
```
int((b*x^n)^p/x^3,x)
```

output
```
int((b*x^n)^p/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(bx^n)^p}{x^3} \, dx = \frac{x^{np} b^p}{x^2 (np - 2)}$$

input
```
int((b*x^n)^p/x^3,x)
```

output
```
(x**(n*p)*b**p)/(x**2*(n*p - 2))
```

3.640 $\int \frac{(bx^n)^p}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{(bx^n)^p}{x^4} dx = -\frac{(bx^n)^p}{(3-np)x^3}$$

output
```
-(b*x^n)^p/(-n*p+3)/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(bx^n)^p}{x^4} dx = \frac{(bx^n)^p}{(-3+np)x^3}$$

input
```
Integrate[(b*x^n)^p/x^4,x]
```

output
```
(b*x^n)^p/((-3 + n*p)*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {23, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n)^p}{x^4} dx$$

$$\Big\downarrow \; 23$$

$$x^{-np}(bx^n)^p \int x^{np-4}dx$$

$$\Big\downarrow \; 15$$

$$-\frac{(bx^n)^p}{x^3(3-np)}$$

input
```
Int[(b*x^n)^p/x^4,x]
```

output
```
-((b*x^n)^p/((3 - n*p)*x^3))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 23
```
Int[(x_)^(m_.)*((a_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)
   Int[x^(m + n*p), x], x] /; FreeQ[{a, m, n, p}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{(b\,x^{n})^{p}}{x^{3}(np-3)}$	19
parallelrisch	$\frac{(b\,x^{n})^{p}}{x^{3}(np-3)}$	19
orering	$\frac{(b\,x^{n})^{p}}{x^{3}(np-3)}$	19

input
```
int((b*x^n)^p/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/x^3/(n*p-3)*(b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(bx^n)^p}{x^4}\,dx = \frac{e^{(np\log(x)+p\log(b))}}{(np-3)x^3}$$

input
```
integrate((b*x^n)^p/x^4,x, algorithm="fricas")
```

output
```
e^(n*p*log(x) + p*log(b))/((n*p - 3)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(bx^n)^p}{x^4}\,dx = \begin{cases} \frac{(bx^n)^p}{npx^3-3x^3} & \text{for } n \neq \frac{3}{p} \\ \frac{\left(bx^{\frac{3}{p}}\right)^p \log(x)}{x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**n)**p/x**4,x)
```

output
```
Piecewise((((b*x**n)**p/(n*p*x**3 - 3*x**3), Ne(n, 3/p)), ((b*x**(3/p))**p*
log(x)/x**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(bx^n)^p}{x^4} dx = \frac{b^p(x^n)^p}{(np-3)x^3}$$

input
```
integrate((b*x^n)^p/x^4,x, algorithm="maxima")
```

output
```
b^p*(x^n)^p/((n*p - 3)*x^3)
```

Giac [F]

$$\int \frac{(bx^n)^p}{x^4} dx = \int \frac{(bx^n)^p}{x^4} dx$$

input
```
integrate((b*x^n)^p/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^n)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx^n)^p}{x^4}\,dx = \int \frac{(b\,x^n)^p}{x^4}\,dx$$

input
```
int((b*x^n)^p/x^4,x)
```

output
```
int((b*x^n)^p/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(bx^n)^p}{x^4}\,dx = \frac{x^{np}b^p}{x^3\,(np-3)}$$

input
```
int((b*x^n)^p/x^4,x)
```

output
```
(x**(n*p)*b**p)/(x**3*(n*p - 3))
```

3.641 $\int x^{-1+n}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(a+bx^n)^{1+p}}{bn(1+p)}$$

output
```
(a+b*x^n)^(p+1)/b/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(a+bx^n)^{1+p}}{bn(1+p)}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
(a + b*x^n)^(1 + p)/(b*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{(a+bx^n)^{p+1}}{bn(p+1)}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
(a + b*x^n)^(1 + p)/(b*n*(1 + p))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
risch	$\frac{(a+b\,x^n)(a+b\,x^n)^p}{b(p+1)n}$	29

input
```
int(x^(-1+n)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*x^n)/b/(p+1)/n*(a+b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^{-1+n}(a+bx^n)^p\,dx = \frac{(bx^n+a)(bx^n+a)^p}{bnp+bn}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
(b*x^n + a)*(b*x^n + a)^p/(b*n*p + b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(15) = 30.

Time = 11.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.39

$$\int x^{-1+n}(a+bx^n)^p\,dx = \begin{cases} \frac{\log{(x)}}{a} & \text{for}\: b=0 \wedge n=0 \wedge p=-1 \\ \frac{a^p x x^{n-1}}{n} & \text{for}\: b=0 \\ (a+b)^p \log{(x)} & \text{for}\: n=0 \\ \frac{\log{(\frac{a}{b}+x^n)}}{bn} & \text{for}\: p=-1 \\ \frac{a(a+bx^n)^p}{bnp+bn} + \frac{bx^n(a+bx^n)^p}{bnp+bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**p,x)
```

output
```
Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0) & Eq(p, -1)), (a**p*x*x**(n - 1)/
n, Eq(b, 0)), ((a + b)**p*log(x), Eq(n, 0)), (log(a/b + x**n)/(b*n), Eq(p,
 -1)), (a*(a + b*x**n)**p/(b*n*p + b*n) + b*x**n*(a + b*x**n)**p/(b*n*p +
b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(bx^n+a)^{p+1}}{bn(p+1)}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
(b*x^n + a)^(p + 1)/(b*n*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(bx^n+a)^{p+1}}{bn(p+1)}$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
(b*x^n + a)^(p + 1)/(b*n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(a+b\,x^n)^{p+1}}{b\,n\,(p+1)}$$

input
```
int(x^(n - 1)*(a + b*x^n)^p,x)
```

output
```
(a + b*x^n)^(p + 1)/(b*n*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int x^{-1+n}(a+bx^n)^p\,dx=\frac{(x^nb+a)^p\,(x^nb+a)}{bn\,(p+1)}$$

input
```
int(x^(-1+n)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**n*b + a))/(b*n*(p + 1))
```

3.642 $\int x^{-1+2n}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int x^{-1+2n}(a+bx^n)^p\,dx=-\frac{a(a+bx^n)^{1+p}}{b^2n(1+p)}+\frac{(a+bx^n)^{2+p}}{b^2n(2+p)}$$

output
```
-a*(a+b*x^n)^(p+1)/b^2/n/(p+1)+(a+b*x^n)^(2+p)/b^2/n/(2+p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int x^{-1+2n}(a+bx^n)^p\,dx=\frac{(a+bx^n)^{1+p}(-a+b(1+p)x^n)}{b^2n(1+p)(2+p)}$$

input
```
Integrate[x^(-1 + 2*n)*(a + b*x^n)^p,x]
```

output
```
((a + b*x^n)^(1 + p)*(-a + b*(1 + p)*x^n))/(b^2*n*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^n(bx^n+a)^p\,dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{(bx^n+a)^{p+1}}{b}-\frac{a(bx^n+a)^p}{b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+bx^n)^{p+2}}{b^2(p+2)}-\frac{a(a+bx^n)^{p+1}}{b^2(p+1)}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a + b*x^n)^p,x]
```

output
```
(-((a*(a + b*x^n)^(1 + p))/(b^2*(1 + p))) + (a + b*x^n)^(2 + p)/(b^2*(2 +
p)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{\left(-b^{2} p\,x^{2 n}-a p\,x^{n} b-b^{2} x^{2 n}+a^{2}\right) \left(a+b\,x^{n}\right)^{p}}{\left(p+1\right) \left(2+p\right) n\,b^{2}}$	61

input
```
int(x^(2*n-1)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
-(-b^2*p*(x^n)^2-a*p*x^n*b-b^2*(x^n)^2+a^2)/(p+1)/(2+p)/n/b^2*(a+b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int x^{-1+2n}(a+bx^n)^p\,dx=\frac{(abpx^n-a^2+(b^2p+b^2)x^{2\,n})(bx^n+a)^p}{b^2np^2+3\,b^2np+2\,b^2n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
(a*b*p*x^n - a^2 + (b^2*p + b^2)*x^(2*n))*(b*x^n + a)^p/(b^2*n*p^2 + 3*b^2
*n*p + 2*b^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(37) = 74$.

Time = 31.50 (sec) , antiderivative size = 274, normalized size of antiderivative = 5.59

$$\int x^{-1+2n}(a+bx^n)^p\,dx$$
$$= \begin{cases} \frac{a^p x x^{2n-1}}{2n} & \text{for } b=0 \\ (a+b)^p \log(x) & \text{for } n=0 \\ \frac{a\log\left(\frac{a}{b}+x^n\right)}{ab^2n+b^3nx^n} + \frac{a}{ab^2n+b^3nx^n} + \frac{bx^n\log\left(\frac{a}{b}+x^n\right)}{ab^2n+b^3nx^n} & \text{for } p=-2 \\ -\frac{a\log\left(\frac{a}{b}+x^n\right)}{b^2n} + \frac{x^n}{bn} & \text{for } p=-1 \\ -\frac{a^2(a+bx^n)^p}{b^2np^2+3b^2np+2b^2n} + \frac{abpx^n(a+bx^n)^p}{b^2np^2+3b^2np+2b^2n} + \frac{b^2px^{2n}(a+bx^n)^p}{b^2np^2+3b^2np+2b^2n} + \frac{b^2x^{2n}(a+bx^n)^p}{b^2np^2+3b^2np+2b^2n} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**p,x)
```

output
```
Piecewise((a**p*x*x**(2*n - 1)/(2*n), Eq(b, 0)), ((a + b)**p*log(x), Eq(n,
 0)), (a*log(a/b + x**n)/(a*b**2*n + b**3*n*x**n) + a/(a*b**2*n + b**3*n*x
**n) + b*x**n*log(a/b + x**n)/(a*b**2*n + b**3*n*x**n), Eq(p, -2)), (-a*lo
g(a/b + x**n)/(b**2*n) + x**n/(b*n), Eq(p, -1)), (-a**2*(a + b*x**n)**p/(b
**2*n*p**2 + 3*b**2*n*p + 2*b**2*n) + a*b*p*x**n*(a + b*x**n)**p/(b**2*n*p
**2 + 3*b**2*n*p + 2*b**2*n) + b**2*p*x**(2*n)*(a + b*x**n)**p/(b**2*n*p**
2 + 3*b**2*n*p + 2*b**2*n) + b**2*x**(2*n)*(a + b*x**n)**p/(b**2*n*p**2 +
3*b**2*n*p + 2*b**2*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int x^{-1+2n}(a+bx^n)^p\,dx = \frac{(b^2(p+1)x^{2n} + abpx^n - a^2)(bx^n+a)^p}{(p^2+3p+2)b^2n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
(b^2*(p + 1)*x^(2*n) + a*b*p*x^n - a^2)*(b*x^n + a)^p/((p^2 + 3*p + 2)*b^2
*n)
```

Giac [F]

$$\int x^{-1+2n}(a+bx^n)^p\,dx=\int (bx^n+a)^p x^{2n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}(a+bx^n)^p\,dx=\int x^{2n-1}\,(a+b\,x^n)^p\,dx$$

input
```
int(x^(2*n - 1)*(a + b*x^n)^p,x)
```

output
```
int(x^(2*n - 1)*(a + b*x^n)^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int x^{-1+2n}(a+bx^n)^p\,dx=\frac{(x^nb+a)^p\,(x^{2n}b^2p+x^{2n}b^2+x^nabp-a^2)}{b^2n\,(p^2+3p+2)}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**(2*n)*b**2*p + x**(2*n)*b**2 + x**n*a*b*p - a**2))/(b
**2*n*(p**2 + 3*p + 2))
```

3.643 $\int x^{-1+3n}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int x^{-1+3n}(a+bx^n)^p\,dx=\frac{a^2(a+bx^n)^{1+p}}{b^3n(1+p)}-\frac{2a(a+bx^n)^{2+p}}{b^3n(2+p)}+\frac{(a+bx^n)^{3+p}}{b^3n(3+p)}$$

output
```
a^2*(a+b*x^n)^(p+1)/b^3/n/(p+1)-2*a*(a+b*x^n)^(2+p)/b^3/n/(2+p)+(a+b*x^n)^
(3+p)/b^3/n/(3+p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int x^{-1+3n}(a+bx^n)^p\,dx=\frac{(a+bx^n)^{1+p}\left(2a^2-2ab(1+p)x^n+b^2(2+3p+p^2)\,x^{2n}\right)}{b^3n(1+p)(2+p)(3+p)}$$

input
```
Integrate[x^(-1 + 3*n)*(a + b*x^n)^p,x]
```

output
```
((a + b*x^n)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^n + b^2*(2 + 3*p + p^2)*x^(2
*n)))/(b^3*n*(1 + p)*(2 + p)*(3 + p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{\int x^{2n}(bx^n+a)^p\,dx^n}{n}$$

$$\Big\downarrow \ 53$$

$$\frac{\int \left(\frac{a^2(bx^n+a)^p}{b^2}-\frac{2a(bx^n+a)^{p+1}}{b^2}+\frac{(bx^n+a)^{p+2}}{b^2}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a^2(a+bx^n)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx^n)^{p+2}}{b^3(p+2)}+\frac{(a+bx^n)^{p+3}}{b^3(p+3)}}{n}$$

input
```
Int[x^(-1 + 3*n)*(a + b*x^n)^p,x]
```

output
```
((a^2*(a + b*x^n)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x^n)^(2 + p))/(b^3*
(2 + p)) + (a + b*x^n)^(3 + p)/(b^3*(3 + p)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

method	result	size
risch	$\frac{\left(b^{3} p^{2} x^{3 n}+a\,b^{2} p^{2} x^{2 n}+3 b^{3} p\,x^{3 n}+a p\,x^{2 n} b^{2}+2 b^{3} x^{3 n}-2 a^{2} p\,x^{n} b+2 a^{3}\right) \left(a+b\,x^{n}\right)^{p}}{\left(2+p\right) \left(3+p\right) \left(p+1\right) n\,b^{3}}$	105

input
```
int(x^(-1+3*n)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
(b^3*p^2*(x^n)^3+a*b^2*p^2*(x^n)^2+3*b^3*p*(x^n)^3+a*p*(x^n)^2*b^2+2*(x^n)
^3*b^3-2*a^2*p*x^n*b+2*a^3)/(2+p)/(3+p)/(p+1)/n/b^3*(a+b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.44

$$\int x^{-1+3n}(a+bx^n)^p\,dx = -\frac{\left(2\,a^{2} b p x^{n} - 2\,a^{3} - \left(b^{3} p^{2} + 3\,b^{3} p + 2\,b^{3}\right)x^{3\,n} - \left(a b^{2} p^{2} + a b^{2} p\right)x^{2\,n}\right)\left(b x^{n} + a\right)^{p}}{b^{3} n p^{3} + 6\,b^{3} n p^{2} + 11\,b^{3} n p + 6\,b^{3} n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
-(2*a^2*b*p*x^n - 2*a^3 - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^(3*n) - (a*b^2*p^2
 + a*b^2*p)*x^(2*n))*(b*x^n + a)^p/(b^3*n*p^3 + 6*b^3*n*p^2 + 11*b^3*n*p +
 6*b^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(61) = 122$.

Time = 95.97 (sec) , antiderivative size = 770, normalized size of antiderivative = 10.27

$$\int x^{-1+3n}(a+bx^n)^p\,dx$$
$$= \begin{cases} \frac{a^pxx^{3n-1}}{3n} \\ (a+b)^p\log(x) \\ \frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{3a^2}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{4abx^n\log\left(\frac{a}{b}+x^n\right)}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{4abx^n}{2a^2b^3n+4ab^4nx^n+2b^5nx^{2n}} + \frac{2b^2x}{2a^2b^3n+} \\ -\frac{2a^2\log\left(\frac{a}{b}+x^n\right)}{ab^3n+b^4nx^n} - \frac{2a^2}{ab^3n+b^4nx^n} - \frac{2abx^n\log\left(\frac{a}{b}+x^n\right)}{ab^3n+b^4nx^n} + \frac{b^2x^{2n}}{ab^3n+b^4nx^n} \\ \frac{a^2\log\left(\frac{a}{b}+x^n\right)}{b^3n} - \frac{ax^n}{b^2n} + \frac{x^{2n}}{2bn} \\ \frac{2a^3(a+bx^n)^p}{b^3np^3+6b^3np^2+11b^3np+6b^3n} - \frac{2a^2bpx^n(a+bx^n)^p}{b^3np^3+6b^3np^2+11b^3np+6b^3n} + \frac{ab^2p^2x^{2n}(a+bx^n)^p}{b^3np^3+6b^3np^2+11b^3np+6b^3n} + \frac{ab^2px^{2n}(a+bx^n)^p}{b^3np^3+6b^3np^2+11b^3np+6b^3n} + \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**p,x)
```

output
```
Piecewise((a**p*x*x**(3*n - 1)/(3*n), Eq(b, 0)), ((a + b)**p*log(x), Eq(n,
 0)), (2*a**2*log(a/b + x**n)/(2*a**2*b**3*n + 4*a*b**4*n*x**n + 2*b**5*n*
x**(2*n)) + 3*a**2/(2*a**2*b**3*n + 4*a*b**4*n*x**n + 2*b**5*n*x**(2*n)) +
 4*a*b*x**n*log(a/b + x**n)/(2*a**2*b**3*n + 4*a*b**4*n*x**n + 2*b**5*n*x*
*(2*n)) + 4*a*b*x**n/(2*a**2*b**3*n + 4*a*b**4*n*x**n + 2*b**5*n*x**(2*n))
 + 2*b**2*x**(2*n)*log(a/b + x**n)/(2*a**2*b**3*n + 4*a*b**4*n*x**n + 2*b*
*5*n*x**(2*n)), Eq(p, -3)), (-2*a**2*log(a/b + x**n)/(a*b**3*n + b**4*n*x*
*n) - 2*a**2/(a*b**3*n + b**4*n*x**n) - 2*a*b*x**n*log(a/b + x**n)/(a*b**3
*n + b**4*n*x**n) + b**2*x**(2*n)/(a*b**3*n + b**4*n*x**n), Eq(p, -2)), (a
**2*log(a/b + x**n)/(b**3*n) - a*x**n/(b**2*n) + x**(2*n)/(2*b*n), Eq(p, -
1)), (2*a**3*(a + b*x**n)**p/(b**3*n*p**3 + 6*b**3*n*p**2 + 11*b**3*n*p +
6*b**3*n) - 2*a**2*b*p*x**n*(a + b*x**n)**p/(b**3*n*p**3 + 6*b**3*n*p**2 +
 11*b**3*n*p + 6*b**3*n) + a*b**2*p**2*x**(2*n)*(a + b*x**n)**p/(b**3*n*p*
*3 + 6*b**3*n*p**2 + 11*b**3*n*p + 6*b**3*n) + a*b**2*p*x**(2*n)*(a + b*x*
*n)**p/(b**3*n*p**3 + 6*b**3*n*p**2 + 11*b**3*n*p + 6*b**3*n) + b**3*p**2*
x**(3*n)*(a + b*x**n)**p/(b**3*n*p**3 + 6*b**3*n*p**2 + 11*b**3*n*p + 6*b*
*3*n) + 3*b**3*p*x**(3*n)*(a + b*x**n)**p/(b**3*n*p**3 + 6*b**3*n*p**2 + 1
1*b**3*n*p + 6*b**3*n) + 2*b**3*x**(3*n)*(a + b*x**n)**p/(b**3*n*p**3 + 6*
b**3*n*p**2 + 11*b**3*n*p + 6*b**3*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int x^{-1+3n}(a+bx^n)^p\,dx = \frac{((p^2+3\,p+2)b^3x^{3\,n}+(p^2+p)ab^2x^{2\,n}-2\,a^2bpx^n+2\,a^3)(bx^n+a)^p}{(p^3+6\,p^2+11\,p+6)b^3n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*b^3*x^(3*n) + (p^2 + p)*a*b^2*x^(2*n) - 2*a^2*b*p*x^n + 2
*a^3)*(b*x^n + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3*n)
```

Giac [F]

$$\int x^{-1+3n}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{3\,n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+3n}(a+bx^n)^p\,dx = \int x^{3\,n-1}\,(a+b\,x^n)^p\,dx$$

input
```
int(x^(3*n - 1)*(a + b*x^n)^p,x)
```

output
```
int(x^(3*n - 1)*(a + b*x^n)^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int x^{-1+3n}(a+bx^n)^p\,dx = \frac{(x^nb+a)^p\,(x^{3n}b^3p^2+3x^{3n}b^3p+2x^{3n}b^3+x^{2n}a\,b^2p^2+x^{2n}a\,b^2p-2x^na^2bp+2a^3)}{b^3n\,(p^3+6p^2+11p+6)}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**(3*n)*b**3*p**2 + 3*x**(3*n)*b**3*p + 2*x**(3*n)*b**3
 + x**(2*n)*a*b**2*p**2 + x**(2*n)*a*b**2*p - 2*x**n*a**2*b*p + 2*a**3))/(
b**3*n*(p**3 + 6*p**2 + 11*p + 6))
```

3.644 $\int x^{-1+4n}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 103

$$\int x^{-1+4n}(a+bx^n)^p\,dx=-\frac{a^3(a+bx^n)^{1+p}}{b^4n(1+p)}+\frac{3a^2(a+bx^n)^{2+p}}{b^4n(2+p)}-\frac{3a(a+bx^n)^{3+p}}{b^4n(3+p)}+\frac{(a+bx^n)^{4+p}}{b^4n(4+p)}$$

output
```
-a^3*(a+b*x^n)^(p+1)/b^4/n/(p+1)+3*a^2*(a+b*x^n)^(2+p)/b^4/n/(2+p)-3*a*(a+
b*x^n)^(3+p)/b^4/n/(3+p)+(a+b*x^n)^(4+p)/b^4/n/(4+p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int x^{-1+4n}(a+bx^n)^p\,dx=\frac{(a+bx^n)^{1+p}\left(-\frac{a^3}{1+p}+\frac{3a^2(a+bx^n)}{2+p}-\frac{3a(a+bx^n)^2}{3+p}+\frac{(a+bx^n)^3}{4+p}\right)}{b^4n}$$

input
```
Integrate[x^(-1 + 4*n)*(a + b*x^n)^p,x]
```

output
```
((a + b*x^n)^(1 + p)*(-(a^3/(1 + p)) + (3*a^2*(a + b*x^n))/(2 + p) - (3*a*
(a + b*x^n)^2)/(3 + p) + (a + b*x^n)^3/(4 + p)))/(b^4*n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{4n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{\int x^{3n}(bx^n+a)^p\,dx^n}{n}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(-\frac{a^3(bx^n+a)^p}{b^3}+\frac{3a^2(bx^n+a)^{p+1}}{b^3}-\frac{3a(bx^n+a)^{p+2}}{b^3}+\frac{(bx^n+a)^{p+3}}{b^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^3(a+bx^n)^{p+1}}{b^4(p+1)}+\frac{3a^2(a+bx^n)^{p+2}}{b^4(p+2)}-\frac{3a(a+bx^n)^{p+3}}{b^4(p+3)}+\frac{(a+bx^n)^{p+4}}{b^4(p+4)}}{n}$$

input
```
Int[x^(-1 + 4*n)*(a + b*x^n)^p,x]
```

output
```
(-((a^3*(a + b*x^n)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^n)^(2 + p))/
(b^4*(2 + p)) - (3*a*(a + b*x^n)^(3 + p))/(b^4*(3 + p)) + (a + b*x^n)^(4 +
 p)/(b^4*(4 + p)))/n
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{\left(-b^4p^3x^{4n}-a\,b^3p^3x^{3n}-6b^4p^2x^{4n}-3a\,b^3p^2x^{3n}-11b^4p\,x^{4n}+3a^2b^2p^2x^{2n}-2ap\,x^{3n}b^3-6x^{4n}b^4+3a^2p\,x^{2n}b^2-6a^3p\,x^nb+6a^4\right)(a+}{(3+p)(4+p)(2+p)(p+1)n\,b^4}$

input
```
int(x^(-1+4*n)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
-(-b^4*p^3*(x^n)^4-a*b^3*p^3*(x^n)^3-6*b^4*p^2*(x^n)^4-3*a*b^3*p^2*(x^n)^3
-11*b^4*p*(x^n)^4+3*a^2*b^2*p^2*(x^n)^2-2*a*p*(x^n)^3*b^3-6*(x^n)^4*b^4+3*
a^2*p*(x^n)^2*b^2-6*a^3*p*x^n*b+6*a^4)/(3+p)/(4+p)/(2+p)/(p+1)/n/b^4*(a+b*
x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.53

$$\int x^{-1+4n}(a+bx^n)^p\,dx$$
$$=\frac{(6a^3bpx^n-6a^4+(b^4p^3+6b^4p^2+11b^4p+6b^4)x^{4n}+(ab^3p^3+3ab^3p^2+2ab^3p)x^{3n}-3(a^2b^2p^2+a^2b^2}{b^4np^4+10b^4np^3+35b^4np^2+50b^4np+24b^4n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
(6*a^3*b*p*x^n - 6*a^4 + (b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^(4*n)
+ (a*b^3*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^(3*n) - 3*(a^2*b^2*p^2 + a^2*b^2
*p)*x^(2*n))*(b*x^n + a)^p/(b^4*n*p^4 + 10*b^4*n*p^3 + 35*b^4*n*p^2 + 50*b
^4*n*p + 24*b^4*n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+4n}(a+bx^n)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(-1+4*n)*(a+b*x**n)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int x^{-1+4n}(a+bx^n)^p\,dx$$
$$=\frac{((p^3+6p^2+11p+6)b^4x^{4n}+(p^3+3p^2+2p)ab^3x^{3n}-3(p^2+p)a^2b^2x^{2n}+6a^3bpx^n-6a^4)(bx^n+a)^p}{(p^4+10p^3+35p^2+50p+24)b^4n}$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*b^4*x^(4*n) + (p^3 + 3*p^2 + 2*p)*a*b^3*x^(3*n)
- 3*(p^2 + p)*a^2*b^2*x^(2*n) + 6*a^3*b*p*x^n - 6*a^4)*(b*x^n + a)^p/((p^4
 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4*n)
```

Giac [F]

$$\int x^{-1+4n}(a+bx^n)^p\,dx=\int(bx^n+a)^p x^{4n-1}\,dx$$

input
```
integrate(x^(-1+4*n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(4*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+4n}(a+bx^n)^p\,dx=\int x^{4n-1}\,(a+b\,x^n)^p\,dx$$

input
```
int(x^(4*n - 1)*(a + b*x^n)^p,x)
```

output
```
int(x^(4*n - 1)*(a + b*x^n)^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.62

$$\int x^{-1+4n}(a+bx^n)^p\,dx$$
$$=\frac{(x^nb+a)^p\,(x^{4n}b^4p^3+6x^{4n}b^4p^2+11x^{4n}b^4p+6x^{4n}b^4+x^{3n}a\,b^3p^3+3x^{3n}a\,b^3p^2+2x^{3n}a\,b^3p-3x^{2n}a^2b^2p^2}{b^4n\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int(x^(-1+4*n)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**(4*n)*b**4*p**3 + 6*x**(4*n)*b**4*p**2 + 11*x**(4*n)*
b**4*p + 6*x**(4*n)*b**4 + x**(3*n)*a*b**3*p**3 + 3*x**(3*n)*a*b**3*p**2 +
 2*x**(3*n)*a*b**3*p - 3*x**(2*n)*a**2*b**2*p**2 - 3*x**(2*n)*a**2*b**2*p
+ 6*x**n*a**3*b*p - 6*a**4))/(b**4*n*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24
))
```

3.645 $\int x^{-1-n-np}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int x^{-1-n-np}(a+bx^n)^p\,dx=-\frac{x^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)}$$

output
```
-(a+b*x^n)^(p+1)/a/n/(p+1)/(x^(n*(p+1)))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int x^{-1-n-np}(a+bx^n)^p\,dx=-\frac{x^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)}$$

input
```
Integrate[x^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*n*(1 + p)*x^(n*(1 + p))))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n(-p)-n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{x^{-n(p+1)}(a+bx^n)^{p+1}}{an(p+1)}$$

input
```
Int[x^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*n*(1 + p)*x^(n*(1 + p))))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(33) = 66$.

Time = 0.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

method	result	size
parallelrisch	$-\frac{x\,x^{n} x^{-np-n-1} (a+b\,x^{n})^{p} b^{2}+x\,x^{-np-n-1} (a+b\,x^{n})^{p} a b}{b n (p+1) a}$	70

input
```
int(x^(-n*p-n-1)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
-(x*x^n*x^(-n*p-n-1)*(a+b*x^n)^p*b^2+x*x^(-n*p-n-1)*(a+b*x^n)^p*a*b)/b/n/(
p+1)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int x^{-1-n-np}(a+bx^n)^p\,dx=-\frac{(bxx^{-np-n-1}x^n+axx^{-np-n-1})(bx^n+a)^p}{anp+an}$$

input
```
integrate(x^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
-(b*x*x^(-n*p - n - 1)*x^n + a*x*x^(-n*p - n - 1))*(b*x^n + a)^p/(a*n*p +
a*n)
```

Sympy [A] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int x^{-1-n-np}(a+bx^n)^p\,dx=\frac{a^pa^{-p-1}b^{p+1}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-1)}{n\Gamma\left(-p\right)}$$

input
```
integrate(x**(-n*p-n-1)*(a+b*x**n)**p,x)
```

output
```
a**p*a**(-p - 1)*b**(p + 1)*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 1)/(n*gam
ma(-p))
```

Maxima [F]

$$\int x^{-1-n-np}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-np-n-1}\,dx$$

input
```
integrate(x^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - n - 1), x)
```

Giac [F]

$$\int x^{-1-n-np}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-np-n-1}\,dx$$

input
```
integrate(x^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n-np}(a+bx^n)^p\,dx = \int \frac{(a+b\,x^n)^p}{x^{n+n\,p+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(n + n*p + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n + n*p + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int x^{-1-n-np}(a+bx^n)^p \, dx = -\frac{(x^n b+a)^p (x^n b+a)}{x^{np+n} a n (p+1)}$$

input
```
int(x^(-n*p-n-1)*(a+b*x^n)^p,x)
```

output
```
( - (x**n*b + a)**p*(x**n*b + a))/(x**(n*p + n)*a*n*(p + 1))
```

3.646 $\int x^{-1-9n}(a+bx^n)^8\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int x^{-1-9n}(a+bx^n)^8\,dx=-\frac{x^{-9n}(a+bx^n)^9}{9an}$$

output
```
-1/9*(a+b*x^n)^9/a/n/(x^(9*n))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(24) = 48.

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.71

$$\int x^{-1-9n}(a+bx^n)^8\,dx$$
$$=\frac{x^{-9n}(-a^8-9a^7bx^n-36a^6b^2x^{2n}-84a^5b^3x^{3n}-126a^4b^4x^{4n}-126a^3b^5x^{5n}-84a^2b^6x^{6n}-36ab^7x^{7n}-9b^8}{9n}$$

input
```
Integrate[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

output
```
(-a^8 - 9*a^7*b*x^n - 36*a^6*b^2*x^(2*n) - 84*a^5*b^3*x^(3*n) - 126*a^4*b^
4*x^(4*n) - 126*a^3*b^5*x^(5*n) - 84*a^2*b^6*x^(6*n) - 36*a*b^7*x^(7*n) -
9*b^8*x^(8*n))/(9*n*x^(9*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-9n-1}(a+bx^n)^8\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{x^{-9n}(a+bx^n)^9}{9an}$$

input
```
Int[x^(-1 - 9*n)*(a + b*x^n)^8,x]
```

output
```
-1/9*(a + b*x^n)^9/(a*n*x^(9*n))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(24) = 48.

Time = 7.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.67

method	result
risch	$-\frac{b^8x^{-n}}{n}-\frac{4a\,b^7x^{-2n}}{n}-\frac{28a^2b^6x^{-3n}}{3n}-\frac{14a^3b^5x^{-4n}}{n}-\frac{14a^4b^4x^{-5n}}{n}-\frac{28a^5b^3x^{-6n}}{3n}-\frac{4a^6b^2x^{-7n}}{n}-\frac{a^7b\,x^{-8n}}{n}$
parallelrisch	$\frac{-9x\,x^{8n}x^{-1-9n}b^8-36x\,x^{7n}x^{-1-9n}a\,b^7-84x\,x^{6n}x^{-1-9n}a^2b^6-126x\,x^{5n}x^{-1-9n}a^3b^5-126x\,x^{4n}x^{-1-9n}a^4b^4-84x\,x^{3n}x^{-1-}}{9n}$
orering	Expression too large to display

input
```
int(x^(-1-9*n)*(a+b*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-b^8/n/(x^n)-4*a*b^7/n/(x^n)^2-28/3*a^2*b^6/n/(x^n)^3-14*a^3*b^5/n/(x^n)^4
-14*a^4*b^4/n/(x^n)^5-28/3*a^5*b^3/n/(x^n)^6-4*a^6*b^2/n/(x^n)^7-a^7*b/n/(
x^n)^8-1/9*a^8/n/(x^n)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.62

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{9\,b^8x^{8\,n}+36\,ab^7x^{7\,n}+84\,a^2b^6x^{6\,n}+126\,a^3b^5x^{5\,n}+126\,a^4b^4x^{4\,n}+84\,a^5b^3x^{3\,n}+36\,a^6b^2x^{2\,n}+9\,a^7bx^n+}{9\,nx^{9\,n}}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="fricas")
```

output
```
-1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*
x^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) +
9*a^7*b*x^n + a^8)/(n*x^(9*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(19) = 38$.

Time = 2.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 9.21

$$\int x^{-1-9n}(a+bx^n)^8\,dx$$
$$= \begin{cases} -\frac{a^8xx^{-9n-1}}{9n} - \frac{a^7bxx^nx^{-9n-1}}{n} - \frac{4a^6b^2xx^{2n}x^{-9n-1}}{n} - \frac{28a^5b^3xx^{3n}x^{-9n-1}}{3n} - \frac{14a^4b^4xx^{4n}x^{-9n-1}}{n} - \frac{14a^3b^5xx^{5n}x^{-9n-1}}{n} - 28a \\ (a+b)^8\log(x) \end{cases}$$

input
```
integrate(x**(-1-9*n)*(a+b*x**n)**8,x)
```

output
```
Piecewise((-a**8*x*x**(-9*n - 1)/(9*n) - a**7*b*x*x**n*x**(-9*n - 1)/n - 4
*a**6*b**2*x*x**(2*n)*x**(-9*n - 1)/n - 28*a**5*b**3*x*x**(3*n)*x**(-9*n -
 1)/(3*n) - 14*a**4*b**4*x*x**(4*n)*x**(-9*n - 1)/n - 14*a**3*b**5*x*x**(5
*n)*x**(-9*n - 1)/n - 28*a**2*b**6*x*x**(6*n)*x**(-9*n - 1)/(3*n) - 4*a*b*
*7*x*x**(7*n)*x**(-9*n - 1)/n - b**8*x*x**(8*n)*x**(-9*n - 1)/n, Ne(n, 0))
, ((a + b)**8*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.29

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{a^8}{9\,nx^{9\,n}} - \frac{a^7b}{nx^{8\,n}} - \frac{4\,a^6b^2}{nx^{7\,n}} - \frac{28\,a^5b^3}{3\,nx^{6\,n}} - \frac{14\,a^4b^4}{nx^{5\,n}} - \frac{14\,a^3b^5}{nx^{4\,n}} - \frac{28\,a^2b^6}{3\,nx^{3\,n}} - \frac{4\,ab^7}{nx^{2\,n}} - \frac{b^8}{nx^n}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="maxima")
```

output
```
-1/9*a^8/(n*x^(9*n)) - a^7*b/(n*x^(8*n)) - 4*a^6*b^2/(n*x^(7*n)) - 28/3*a^
5*b^3/(n*x^(6*n)) - 14*a^4*b^4/(n*x^(5*n)) - 14*a^3*b^5/(n*x^(4*n)) - 28/3
*a^2*b^6/(n*x^(3*n)) - 4*a*b^7/(n*x^(2*n)) - b^8/(n*x^n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(24) = 48$.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.62

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{9\,b^8x^{8\,n}+36\,ab^7x^{7\,n}+84\,a^2b^6x^{6\,n}+126\,a^3b^5x^{5\,n}+126\,a^4b^4x^{4\,n}+84\,a^5b^3x^{3\,n}+36\,a^6b^2x^{2\,n}+9\,a^7bx^n+}{9\,nx^{9\,n}}$$

input
```
integrate(x^(-1-9*n)*(a+b*x^n)^8,x, algorithm="giac")
```

output
```
-1/9*(9*b^8*x^(8*n) + 36*a*b^7*x^(7*n) + 84*a^2*b^6*x^(6*n) + 126*a^3*b^5*
x^(5*n) + 126*a^4*b^4*x^(4*n) + 84*a^5*b^3*x^(3*n) + 36*a^6*b^2*x^(2*n) +
9*a^7*b*x^n + a^8)/(n*x^(9*n))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.29

$$\int x^{-1-9n}(a+bx^n)^8\,dx = -\frac{a^8}{9\,n\,x^{9\,n}}-\frac{b^8}{n\,x^n}-\frac{28\,a^2\,b^6}{3\,n\,x^{3\,n}}-\frac{14\,a^3\,b^5}{n\,x^{4\,n}}-\frac{14\,a^4\,b^4}{n\,x^{5\,n}}-\frac{28\,a^5\,b^3}{3\,n\,x^{6\,n}}-\frac{4\,a^6\,b^2}{n\,x^{7\,n}}-\frac{4\,a\,b^7}{n\,x^{2\,n}}-\frac{a^7\,b}{n\,x^{8\,n}}$$

input
```
int((a + b*x^n)^8/x^(9*n + 1),x)
```

output
```
- a^8/(9*n*x^(9*n)) - b^8/(n*x^n) - (28*a^2*b^6)/(3*n*x^(3*n)) - (14*a^3*b
^5)/(n*x^(4*n)) - (14*a^4*b^4)/(n*x^(5*n)) - (28*a^5*b^3)/(3*n*x^(6*n)) -
(4*a^6*b^2)/(n*x^(7*n)) - (4*a*b^7)/(n*x^(2*n)) - (a^7*b)/(n*x^(8*n))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.71

$$\int x^{-1-9n}(a+bx^n)^8\,dx$$
$$=\frac{-9x^{8n}b^8-36x^{7n}a\,b^7-84x^{6n}a^2b^6-126x^{5n}a^3b^5-126x^{4n}a^4b^4-84x^{3n}a^5b^3-36x^{2n}a^6b^2-9x^na^7b-a^8}{9x^{9n}n}$$

input
```
int(x^(-1-9*n)*(a+b*x^n)^8,x)
```

output
```
( - 9*x**(8*n)*b**8 - 36*x**(7*n)*a*b**7 - 84*x**(6*n)*a**2*b**6 - 126*x**
(5*n)*a**3*b**5 - 126*x**(4*n)*a**4*b**4 - 84*x**(3*n)*a**5*b**3 - 36*x**(
2*n)*a**6*b**2 - 9*x**n*a**7*b - a**8)/(9*x**(9*n)*n)
```

3.647 $\int x^{-4-3p}(a+bx^3)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int x^{-4-3p}(a+bx^3)^p\,dx = -\frac{x^{-3(1+p)}(a+bx^3)^{1+p}}{3a(1+p)}$$

output
```
-1/3*(b*x^3+a)^(p+1)/a/(p+1)/(x^(3*p+3))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int x^{-4-3p}(a+bx^3)^p\,dx = \frac{x^{-3-3p}(a+bx^3)^{1+p}}{a(-3-3p)}$$

input
```
Integrate[x^(-4 - 3*p)*(a + b*x^3)^p,x]
```

output
```
(x^(-3 - 3*p)*(a + b*x^3)^(1 + p))/(a*(-3 - 3*p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-3p-4}\left(a+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{x^{-3(p+1)}\left(a+bx^3\right)^{p+1}}{3a(p+1)}$$

input
```
Int[x^(-4 - 3*p)*(a + b*x^3)^p,x]
```

output
```
-1/3*(a + b*x^3)^(1 + p)/(a*(1 + p)*x^(3*(1 + p)))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
gosper	$-\frac{x^{-3-3p}\left(b\,x^{3}+a\right)^{p+1}}{3a\left(p+1\right)}$	29
orering	$-\frac{x\left(b\,x^{3}+a\right)x^{-4-3p}\left(b\,x^{3}+a\right)^{p}}{3a\left(p+1\right)}$	35

input
```
int(x^(-4-3*p)*(b*x^3+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^(-3-3*p)/a/(p+1)*(b*x^3+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x^{-4-3p}(a+bx^3)^p \, dx = -\frac{(bx^4+ax)(bx^3+a)^p x^{-3p-4}}{3(ap+a)}$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
-1/3*(b*x^4 + a*x)*(b*x^3 + a)^p*x^(-3*p - 4)/(a*p + a)
```

Sympy [A] (verification not implemented)

Time = 110.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x^{-4-3p}(a+bx^3)^p \, dx = \frac{a^p x^{-3p-3}\left(1+\frac{bx^3}{a}\right)^{p+1}\Gamma(-p-1)}{3\Gamma(-p)}$$

input
```
integrate(x**(-4-3*p)*(b*x**3+a)**p,x)
```

output
```
a**p*x**(-3*p - 3)*(1 + b*x**3/a)**(p + 1)*gamma(-p - 1)/(3*gamma(-p))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int x^{-4-3p}\left(a+bx^3\right)^p\,dx=-\frac{(bx^3+a)e^{\left(p\log\left(bx^3+a\right)-3\,p\log(x)\right)}}{3\,a(p+1)x^3}$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
-1/3*(b*x^3 + a)*e^(p*log(b*x^3 + a) - 3*p*log(x))/(a*(p + 1)*x^3)
```

Giac [F]

$$\int x^{-4-3p}\left(a+bx^3\right)^p\,dx=\int\left(bx^3+a\right)^p x^{-3\,p-4}\,dx$$

input
```
integrate(x^(-4-3*p)*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*x^(-3*p - 4), x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int x^{-4-3p}\left(a+bx^3\right)^p\,dx=-\left(b\,x^3+a\right)^p\left(\frac{x}{3\,x^{3\,p+4}\,(p+1)}+\frac{b\,x^4}{3\,a\,x^{3\,p+4}\,(p+1)}\right)$$

input
```
int((a + b*x^3)^p/x^(3*p + 4),x)
```

output
```
-(a + b*x^3)^p*(x/(3*x^(3*p + 4)*(p + 1)) + (b*x^4)/(3*a*x^(3*p + 4)*(p +
1)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x^{-4-3p}(a+bx^3)^p\,dx = -\frac{(b\,x^3+a)^p\,(b\,x^3+a)}{3x^{3p}a\,x^3\,(p+1)}$$

input
```
int(x^(-4-3*p)*(b*x^3+a)^p,x)
```

output
```
( - (a + b*x**3)**p*(a + b*x**3))/(3*x**(3*p)*a*x**3*(p + 1))
```

3.648 $\int \frac{(a+bx^3)^8}{x^{28}} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^3)^8}{x^{28}} dx = -\frac{(a+bx^3)^9}{27ax^{27}}$$

output
```
-1/27*(b*x^3+a)^9/a/x^27
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(19) = 38.

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 5.68

$$\int \frac{(a+bx^3)^8}{x^{28}} dx = -\frac{a^8}{27x^{27}} - \frac{a^7b}{3x^{24}} - \frac{4a^6b^2}{3x^{21}} - \frac{28a^5b^3}{9x^{18}} - \frac{14a^4b^4}{3x^{15}} - \frac{14a^3b^5}{3x^{12}} - \frac{28a^2b^6}{9x^9} - \frac{4ab^7}{3x^6} - \frac{b^8}{3x^3}$$

input
```
Integrate[(a + b*x^3)^8/x^28,x]
```

output
```
-1/27*a^8/x^27 - (a^7*b)/(3*x^24) - (4*a^6*b^2)/(3*x^21) - (28*a^5*b^3)/(9
*x^18) - (14*a^4*b^4)/(3*x^15) - (14*a^3*b^5)/(3*x^12) - (28*a^2*b^6)/(9*x
^9) - (4*a*b^7)/(3*x^6) - b^8/(3*x^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{28}} \, dx$$
$$\Big\downarrow \; 796$$
$$-\frac{(a+bx^3)^9}{27ax^{27}}$$

input
```
Int[(a + b*x^3)^8/x^28,x]
```

output
```
-1/27*(a + b*x^3)^9/(a*x^27)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.79

method	result	size
gosper	$-\frac{9b^8x^{24}+36a\,b^7x^{21}+84a^2b^6x^{18}+126a^3b^5x^{15}+126a^4b^4x^{12}+84a^5b^3x^9+36a^6b^2x^6+9a^7b\,x^3+a^8}{27x^{27}}$	91
default	$-\frac{b^8}{3x^3}-\frac{4a^6b^2}{3x^{21}}-\frac{14a^4b^4}{3x^{15}}-\frac{a^7b}{3x^{24}}-\frac{a^8}{27x^{27}}-\frac{4a\,b^7}{3x^6}-\frac{28a^5b^3}{9x^{18}}-\frac{28a^2b^6}{9x^9}-\frac{14a^3b^5}{3x^{12}}$	91
orering	$-\frac{9b^8x^{24}+36a\,b^7x^{21}+84a^2b^6x^{18}+126a^3b^5x^{15}+126a^4b^4x^{12}+84a^5b^3x^9+36a^6b^2x^6+9a^7b\,x^3+a^8}{27x^{27}}$	91
norman	$\frac{-\frac{1}{27}a^8-\frac{4}{3}a\,b^7x^{21}-\frac{4}{3}a^6b^2x^6-\frac{1}{3}a^7b\,x^3-\frac{1}{3}b^8x^{24}-\frac{28}{9}a^5b^3x^9-\frac{14}{3}a^4b^4x^{12}-\frac{28}{9}a^2b^6x^{18}-\frac{14}{3}a^3b^5x^{15}}{x^{27}}$	92
risch	$\frac{-\frac{1}{27}a^8-\frac{4}{3}a\,b^7x^{21}-\frac{4}{3}a^6b^2x^6-\frac{1}{3}a^7b\,x^3-\frac{1}{3}b^8x^{24}-\frac{28}{9}a^5b^3x^9-\frac{14}{3}a^4b^4x^{12}-\frac{28}{9}a^2b^6x^{18}-\frac{14}{3}a^3b^5x^{15}}{x^{27}}$	92
parallelrisch	$\frac{-9b^8x^{24}-36a\,b^7x^{21}-84a^2b^6x^{18}-126a^3b^5x^{15}-126a^4b^4x^{12}-84a^5b^3x^9-36a^6b^2x^6-9a^7b\,x^3-a^8}{27x^{27}}$	93

input
```
int((b*x^3+a)^8/x^28,x,method=_RETURNVERBOSE)
```

output
```
-1/27*(9*b^8*x^24+36*a*b^7*x^21+84*a^2*b^6*x^18+126*a^3*b^5*x^15+126*a^4*b
^4*x^12+84*a^5*b^3*x^9+36*a^6*b^2*x^6+9*a^7*b*x^3+a^8)/x^27
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

input
```
integrate((b*x^3+a)^8/x^28,x, algorithm="fricas")
```

output
```
-1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 1
26*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^2
7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(15) = 30$.

Time = 0.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.11

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = \frac{-a^8-9a^7bx^3-36a^6b^2x^6-84a^5b^3x^9-126a^4b^4x^{12}-126a^3b^5x^{15}-84a^2b^6x^{18}-36ab^7x^{21}-9b^8x^{24}}{27x^{27}}$$

input
```
integrate((b*x**3+a)**8/x**28,x)
```

output
```
(-a**8 - 9*a**7*b*x**3 - 36*a**6*b**2*x**6 - 84*a**5*b**3*x**9 - 126*a**4*
b**4*x**12 - 126*a**3*b**5*x**15 - 84*a**2*b**6*x**18 - 36*a*b**7*x**21 -
9*b**8*x**24)/(27*x**27)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

input
```
integrate((b*x^3+a)^8/x^28,x, algorithm="maxima")
```

output
```
-1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 1
26*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^2
7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

input
```
integrate((b*x^3+a)^8/x^28,x, algorithm="giac")
```

output
```
-1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 1
26*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^2
7
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{\frac{a^8}{27}+\frac{a^7\,b\,x^3}{3}+\frac{4\,a^6\,b^2\,x^6}{3}+\frac{28\,a^5\,b^3\,x^9}{9}+\frac{14\,a^4\,b^4\,x^{12}}{3}+\frac{14\,a^3\,b^5\,x^{15}}{3}+\frac{28\,a^2\,b^6\,x^{18}}{9}+\frac{4\,a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{3}}{x^{27}}$$

input
```
int((a + b*x^3)^8/x^28,x)
```

output
```
-(a^8/27 + (b^8*x^24)/3 + (a^7*b*x^3)/3 + (4*a*b^7*x^21)/3 + (4*a^6*b^2*x^
6)/3 + (28*a^5*b^3*x^9)/9 + (14*a^4*b^4*x^12)/3 + (14*a^3*b^5*x^15)/3 + (2
8*a^2*b^6*x^18)/9)/x^27
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = \frac{-9b^8x^{24}-36a\,b^7x^{21}-84a^2b^6x^{18}-126a^3b^5x^{15}-126a^4b^4x^{12}-84a^5b^3x^9-36a^6b^2x^6-9a^7b\,x^3-a^8}{27x^{27}}$$

input
```
int((b*x^3+a)^8/x^28,x)
```

output
```
( - a**8 - 9*a**7*b*x**3 - 36*a**6*b**2*x**6 - 84*a**5*b**3*x**9 - 126*a**
4*b**4*x**12 - 126*a**3*b**5*x**15 - 84*a**2*b**6*x**18 - 36*a*b**7*x**21
- 9*b**8*x**24)/(27*x**27)
```

3.649 $\int \frac{1}{x(a+bx^n)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x(a+bx^n)}\,dx = \frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

output
```
ln(x)/a-ln(a+b*x^n)/a/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(a+bx^n)}\,dx = \frac{\log(x^n) - \log(an(a+bx^n))}{an}$$

input
```
Integrate[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^n\right)}dx$$

$$\Big\downarrow \; 798$$

$$\frac{\int \frac{x^{-n}}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \; 47$$

$$\frac{\frac{\int x^{-n}dx^n}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \; 14$$

$$\frac{\frac{\log(x^n)}{a}-\frac{b\int \frac{1}{bx^n+a}dx^n}{a}}{n}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\log(x^n)}{a}-\frac{\log(a+bx^n)}{a}}{n}$$

input
```
Int[1/(x*(a + b*x^n)),x]
```

output
```
(Log[x^n]/a - Log[a + b*x^n]/a)/n
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(a+b\,x^{n})}{an}$	23
norman	$\frac{\ln(x)}{a}-\frac{\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{an}$	26
risch	$\frac{\ln(x)}{a}-\frac{\ln\left(x^{n}+\frac{a}{b}\right)}{an}$	26
derivativedivides	$\frac{-\frac{\ln(a+b\,x^{n})}{a}+\frac{\ln(x^{n})}{a}}{n}$	27
default	$\frac{-\frac{\ln(a+b\,x^{n})}{a}+\frac{\ln(x^{n})}{a}}{n}$	27

input
```
int(1/x/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
(n*ln(x)-ln(a+b*x^n))/a/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^n)}\,dx = \frac{n\log(x) - \log(bx^n + a)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(15) = 30.

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{1}{x(a+bx^n)}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{x^{-n}}{bn} & \text{for } a = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a+b} & \text{for } n = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^n\right)}{an} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(b*n*x**n), Eq
(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a - log
(a/b + x**n)/(a*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a+bx^n)}\,dx = -\frac{\log(bx^n+a)}{an} + \frac{\log(x^n)}{an}$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="maxima")
```

output
```
-log(b*x^n + a)/(a*n) + log(x^n)/(a*n)
```

Giac [F]

$$\int \frac{1}{x(a+bx^n)}\,dx = \int \frac{1}{(bx^n+a)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^n)}\,dx = -\frac{\ln(a+b\,x^n)-n\,\ln(x)}{a\,n}$$

input
```
int(1/(x*(a + b*x^n)),x)
```

output
```
-(log(a + b*x^n) - n*log(x))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^n)}\,dx = \frac{-\log(x^n b+a)+\log(x)\,n}{an}$$

input
```
int(1/x/(a+b*x^n),x)
```

output
```
( - log(x**n*b + a) + log(x)*n)/(a*n)
```

3.650 $\int \frac{1}{x(a+bx^3)} dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x(a+bx^3)} dx = \frac{\log(x)}{a} - \frac{\log(a+bx^3)}{3a}$$

output
```
ln(x)/a-1/3*ln(b*x^3+a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx^3)} dx = \frac{\log(x)}{a} - \frac{\log(a+bx^3)}{3a}$$

input
```
Integrate[1/(x*(a + b*x^3)),x]
```

output
```
Log[x]/a - Log[a + b*x^3]/(3*a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+b x^3)} \, dx$$
$$\Big\downarrow \ 798$$
$$\frac{1}{3} \int \frac{1}{x^3 (b x^3+a)}dx^3$$
$$\Big\downarrow \ 47$$
$$\frac{1}{3} \left(\frac{\int \frac{1}{x^3}dx^3}{a}-\frac{b \int \frac{1}{b x^3+a}dx^3}{a}\right)$$
$$\Big\downarrow \ 14$$
$$\frac{1}{3} \left(\frac{\log \left(x^3\right)}{a}-\frac{b \int \frac{1}{b x^3+a}dx^3}{a}\right)$$
$$\Big\downarrow \ 16$$
$$\frac{1}{3} \left(\frac{\log \left(x^3\right)}{a}-\frac{\log \left(a+b x^3\right)}{a}\right)$$

input
```
Int[1/(x*(a + b*x^3)),x]
```

output
```
(Log[x^3]/a - Log[a + b*x^3]/a)/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^{3}+a)}{3a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^{3}+a)}{3a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^{3}+a)}{3a}$	21
parallelrisch	$\frac{3\ln(x)-\ln(b\,x^{3}+a)}{3a}$	21

input
```
int(1/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/3*ln(b*x^3+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a+bx^3)}\,dx = -\frac{\log(bx^3+a) - 3\log(x)}{3a}$$

input
```
integrate(1/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{x(a+bx^3)}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^3\right)}{3a}$$

input
```
integrate(1/x/(b*x**3+a),x)
```

output
```
log(x)/a - log(a/b + x**3)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(a+bx^3)}\,dx = -\frac{\log(bx^3+a)}{3a} + \frac{\log(x^3)}{3a}$$

input
```
integrate(1/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*log(b*x^3 + a)/a + 1/3*log(x^3)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx = -\frac{\log\left(\left|bx^3+a\right|\right)}{3\,a} + \frac{\log\left(\left|x\right|\right)}{a}$$

input
```
integrate(1/x/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx = -\frac{\ln\left(b\,x^3+a\right)-3\,\ln\left(x\right)}{3\,a}$$

input
```
int(1/(x*(a + b*x^3)),x)
```

output
```
-(log(a + b*x^3) - 3*log(x))/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{1}{x\left(a+bx^3\right)}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)+3\log(x)}{3a}$$

input
```
int(1/x/(b*x^3+a),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - log(a**(1/3) + b
**(1/3)*x) + 3*log(x))/(3*a)
```

3.651 $\int \frac{1}{x(a+bx^{-n})}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{x\left(a+bx^{-n}\right)}\,dx=\frac{\log\left(b+ax^n\right)}{an}$$

output
```
ln(b+a*x^n)/a/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^{-n}\right)}\,dx=\frac{\log\left(b+ax^n\right)}{an}$$

input
```
Integrate[1/(x*(a + b/x^n)),x]
```

output
```
Log[b + a*x^n]/(a*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {795, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^{-n}\right)}dx$$
$$\Big\downarrow \text{795}$$
$$\int \frac{x^{n-1}}{ax^n+b}dx$$
$$\Big\downarrow \text{792}$$
$$\frac{\log\left(ax^n+b\right)}{an}$$

input
```
Int[1/(x*(a + b/x^n)),x]
```

output
```
Log[b + a*x^n]/(a*n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\ln(b+a\,x^{n})}{an}$	16
default	$\frac{\ln(b+a\,x^{n})}{an}$	16
parallelrisch	$\frac{\ln(b+a\,x^{n})}{an}$	16
norman	$\frac{\ln\left(a\,\mathrm{e}^{n\ln(x)}+b\right)}{an}$	18
risch	$\frac{\ln\left(x^{n}+\frac{b}{a}\right)}{an}$	18

input
```
int(1/x/(a+b/(x^n)),x,method=_RETURNVERBOSE)
```

output
```
ln(b+a*x^n)/a/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^{-n}\right)}\,dx=\frac{\log\left(ax^{n}+b\right)}{an}$$

input
```
integrate(1/x/(a+b/(x^n)),x, algorithm="fricas")
```

output
```
log(a*x^n + b)/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{1}{x\,(a+bx^{-n})}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ \frac{x^n}{bn} & \text{for } a=0 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log(x)}{a} + \frac{\log\left(\frac{a}{b}+x^{-n}\right)}{an} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b/(x**n)),x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (x**n/(b*n), Eq(a,
 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a + log(a/
b + x**(-n))/(a*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{1}{x\,(a+bx^{-n})}\,dx = \frac{\log\left(a+\frac{b}{x^n}\right)}{an} - \frac{\log\left(\frac{1}{x^n}\right)}{an}$$

input
```
integrate(1/x/(a+b/(x^n)),x, algorithm="maxima")
```

output
```
log(a + b/x^n)/(a*n) - log(1/(x^n))/(a*n)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^{-n}\right)}\,dx = \int \frac{1}{\left(a+\frac{b}{x^n}\right)x}\,dx$$

input
```
integrate(1/x/(a+b/(x^n)),x, algorithm="giac")
```

output
```
integrate(1/((a + b/x^n)*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1}{x\left(a+bx^{-n}\right)}\,dx = \frac{\ln\left(a+\frac{b}{x^n}\right)+n\,\ln\left(x\right)}{a\,n}$$

input
```
int(1/(x*(a + b/x^n)),x)
```

output
```
(log(a + b/x^n) + n*log(x))/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^{-n}\right)}\,dx = \frac{\log(x^n a+b)}{an}$$

input
```
int(1/x/(a+b/(x^n)),x)
```

output
```
log(x**n*a + b)/(a*n)
```

3.652 $\int (cx)^m (a+bx^n)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (cx)^m (a+bx^n)^2 \, dx = \frac{a^2(cx)^{1+m}}{c(1+m)} + \frac{2abx^n(cx)^{1+m}}{c(1+m+n)} + \frac{b^2x^{2n}(cx)^{1+m}}{c(1+m+2n)}$$

output
```
a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*x^n*(c*x)^(1+m)/c/(1+m+n)+b^2*x^(2*n)*(c*x)^
(1+m)/c/(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

$$\int (cx)^m (a+bx^n)^2 \, dx = x(cx)^m \left(\frac{a^2}{1+m} + \frac{2abx^n}{1+m+n} + \frac{b^2x^{2n}}{1+m+2n}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^n)^2,x]
```

output
```
x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + (b^2*x^(2*n))/(1 + m +
2*n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(a+bx^n\right)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2(cx)^m+2abx^n(cx)^m+b^2x^{2n}(cx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2abx^{n+1}(cx)^m}{m+n+1}+\frac{b^2x^{2n+1}(cx)^m}{m+2n+1}$$

input
```
Int[(c*x)^m*(a + b*x^n)^2,x]
```

output
```
(2*a*b*x^(1 + n)*(c*x)^m)/(1 + m + n) + (b^2*x^(1 + 2*n)*(c*x)^m)/(1 + m +
 2*n) + (a^2*(c*x)^(1 + m))/(c*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.87

method	result
risch	$\frac{x\left(b^{2} m^{2} x^{2 n}+m n\, b^{2} x^{2 n}+2 a b\, x^{n} m^{2}+4 x^{n} a b m n+2 m\, b^{2} x^{2 n}+b^{2} x^{2 n} n+a^{2} m^{2}+3 a^{2} m n+2 a^{2} n^{2}+4 a b m\, x^{n}+4 a b\, x^{n} n+b^{2} x^{2 n}+2 m}{(1+m)(1+n+m)(1+m+2 n)}$
parallelrisch	$\frac{4 x\, x^{n} (c x)^{m} a b m n+x\, x^{2 n} (c x)^{m} b^{2}+x (c x)^{m} a^{2} m^{2}+2 x (c x)^{m} a^{2} n^{2}+2 x (c x)^{m} a^{2} m+3 x (c x)^{m} a^{2} n+x\, x^{2 n} (c x)^{m} b^{2} m n+2 x\, x^{n} (c x)}{(1+n}$
orering	$\frac{x\left(3 m^{2}+6 m n+2 n^{2}+3 m+3 n+1\right) (c x)^{m} \left(a+b\, x^{n}\right)^{2}}{\left(m^{2}+2 m n+2 m+2 n+1\right)(1+n+m)}-\frac{3 x^{2} (m+n) \left(\frac{(c x)^{m} m \left(a+b\, x^{n}\right)^{2}}{x}+\frac{2 (c x)^{m} \left(a+b\, x^{n}\right) b\, x^{n} n}{x}\right)}{\left(m^{2}+2 m n+2 m+2 n+1\right)(1+n+m)}+\frac{x^{3}\left(\frac{(c x)^{m}}{}\right.}{}$

input
```
int((c*x)^m*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*(x^n)^2+m*n*b^2*(x^n)^2+2*a*b*x^n*m^2+4*x^n*a*b*m*n+2*m*b^2*(x^
n)^2+b^2*(x^n)^2*n+a^2*m^2+3*a^2*m*n+2*a^2*n^2+4*a*b*m*x^n+4*a*b*x^n*n+b^2
*(x^n)^2+2*m*a^2+3*a^2*n+2*a*b*x^n+a^2)/(1+m)/(1+n+m)/(1+m+2*n)*x^m*c^m*ex
p(1/2*I*csgn(I*c*x)*Pi*m*(csgn(I*c*x)-csgn(I*x))*(-csgn(I*c*x)+csgn(I*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(70) = 140$.

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int (cx)^m (a+bx^n)^2\, dx = \frac{(b^2m^2 + 2\,b^2m + b^2 + (b^2m + b^2)n)xx^{2\,n}e^{(m\log(c)+m\log(x))} + 2\,(abm^2 + 2\,abm + ab + 2\,(abm + ab)n)xx^{n}e}{m^3 + 2\,(m+1)n^2 + 3\,m^2 + 3\,(m^2 + 2\,m}$$

input
```
integrate((c*x)^m*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 2*b^2*m + b^2 + (b^2*m + b^2)*n)*x*x^(2*n)*e^(m*log(c) + m*log
(x)) + 2*(a*b*m^2 + 2*a*b*m + a*b + 2*(a*b*m + a*b)*n)*x*x^n*e^(m*log(c) +
 m*log(x)) + (a^2*m^2 + 2*a^2*n^2 + 2*a^2*m + a^2 + 3*(a^2*m + a^2)*n)*x*e
^(m*log(c) + m*log(x)))/(m^3 + 2*(m + 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n
 + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(60) = 120$.

Time = 1.67 (sec) , antiderivative size = 1148, normalized size of antiderivative = 16.40

$$\int (cx)^m (a + bx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(a+b*x**n)**2,x)
```

output
```
Piecewise(((a + b)**2*log(x)/c, Eq(m, -1) & Eq(n, 0)), ((a**2*log(x) + 2*a
*b*x**n/n + b**2*x**(2*n)/(2*n))/c, Eq(m, -1)), (a**2*Piecewise((0**(-2*n
- 1)*x, Eq(c, 0)), (Piecewise((-1/(2*n*(c*x)**(2*n)), Ne(n, 0)), (log(c*x)
, True))/c, True)) + 2*a*b*Piecewise((-x*x**n*(c*x)**(-2*n - 1)/n, Ne(n, 0
)), (x*x**n*(c*x)**(-2*n - 1)*log(x), True)) + b**2*x*x**(2*n)*(c*x)**(-2*
n - 1)*log(x), Eq(m, -2*n - 1)), (a**2*Piecewise((0**(-n - 1)*x, Eq(c, 0))
, (Piecewise((-1/(n*(c*x)**n), Ne(n, 0)), (log(c*x), True))/c, True)) + 2*
a*b*x*x**n*(c*x)**(-n - 1)*log(x) + b**2*Piecewise((x*x**(2*n)*(c*x)**(-n
- 1)/n, Ne(n, 0)), (x*x**(2*n)*(c*x)**(-n - 1)*log(x), True)), Eq(m, -n -
1)), (a**2*m**2*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n +
3*m + 2*n**2 + 3*n + 1) + 3*a**2*m*n*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2
+ 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a**2*m*x*(c*x)**m/(m**3 +
 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a**2*n
**2*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2
 + 3*n + 1) + 3*a**2*n*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6
*m*n + 3*m + 2*n**2 + 3*n + 1) + a**2*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2
 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*b*m**2*x*x**n*(c*x)**m
/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) +
4*a*b*m*n*x*x**n*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3
*m + 2*n**2 + 3*n + 1) + 4*a*b*m*x*x**n*(c*x)**m/(m**3 + 3*m**2*n + 3*m...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (cx)^m (a+bx^n)^2 \, dx = \frac{b^2 c^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{2\,abc^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

input

```
integrate((c*x)^m*(a+b*x^n)^2,x, algorithm="maxima")
```

output

```
b^2*c^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*c^m*x*e^(m*log(x
) + n*log(x))/(m + n + 1) + (c*x)^(m + 1)*a^2/(c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(70) = 140.

Time = 0.13 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.76

$$\int (cx)^m (a+bx^n)^2 \, dx = \text{Too large to display}$$

input

```
integrate((c*x)^m*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
(b^2*m^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + b^2*m*n*x*x^(2*n)*e^(m*log(c)
 + m*log(x)) + 2*a*b*m^2*x*x^n*e^(m*log(c) + m*log(x)) + b^2*m^2*x*x^n*e^(
m*log(c) + m*log(x)) + 4*a*b*m*n*x*x^n*e^(m*log(c) + m*log(x)) + b^2*m*n*x
*x^n*e^(m*log(c) + m*log(x)) + a^2*m^2*x*e^(m*log(c) + m*log(x)) + 2*a*b*m
^2*x*e^(m*log(c) + m*log(x)) + b^2*m^2*x*e^(m*log(c) + m*log(x)) + 3*a^2*m
*n*x*e^(m*log(c) + m*log(x)) + 4*a*b*m*n*x*e^(m*log(c) + m*log(x)) + b^2*m
*n*x*e^(m*log(c) + m*log(x)) + 2*a^2*n^2*x*e^(m*log(c) + m*log(x)) + 2*b^2
*m*x*x^(2*n)*e^(m*log(c) + m*log(x)) + b^2*n*x*x^(2*n)*e^(m*log(c) + m*log
(x)) + 4*a*b*m*x*x^n*e^(m*log(c) + m*log(x)) + 2*b^2*m*x*x^n*e^(m*log(c) +
 m*log(x)) + 4*a*b*n*x*x^n*e^(m*log(c) + m*log(x)) + b^2*n*x*x^n*e^(m*log(
c) + m*log(x)) + 2*a^2*m*x*e^(m*log(c) + m*log(x)) + 4*a*b*m*x*e^(m*log(c)
 + m*log(x)) + 2*b^2*m*x*e^(m*log(c) + m*log(x)) + 3*a^2*n*x*e^(m*log(c) +
 m*log(x)) + 4*a*b*n*x*e^(m*log(c) + m*log(x)) + b^2*n*x*e^(m*log(c) + m*l
og(x)) + b^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(c) +
 m*log(x)) + b^2*x*x^n*e^(m*log(c) + m*log(x)) + a^2*x*e^(m*log(c) + m*log
(x)) + 2*a*b*x*e^(m*log(c) + m*log(x)) + b^2*x*e^(m*log(c) + m*log(x)))/(m
^3 + 3*m^2*n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int (cx)^m (a+bx^n)^2\,dx = (c\,x)^m\left(\frac{a^2\,x}{m+1}+\frac{b^2\,x\,x^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{2\,a\,b\,x\,x^n\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

input
```
int((c*x)^m*(a + b*x^n)^2,x)
```

output
```
(c*x)^m*((a^2*x)/(m + 1) + (b^2*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n
+ m^2 + 2*n^2 + 1) + (2*a*b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2
+ 2*n^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.57

$$\int (cx)^m (a+bx^n)^2 \, dx$$
$$= \frac{x^m c^m x(x^{2n}b^2m^2 + x^{2n}b^2mn + 2x^{2n}b^2m + x^{2n}b^2n + x^{2n}b^2 + 2x^n ab\, m^2 + 4x^n abmn + 4x^n abm + 4x^n abn}{m^3 + 3m^2n + 2m\, n^2 + 3m^2 + 6mn + 2n^2 + 3m + 3}$$

input
```
int((c*x)^m*(a+b*x^n)^2,x)
```

output
```
(x**m*c**m*x*(x**(2*n)*b**2*m**2 + x**(2*n)*b**2*m*n + 2*x**(2*n)*b**2*m +
 x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m**2 + 4*x**n*a*b*m*n + 4*x*
*n*a*b*m + 4*x**n*a*b*n + 2*x**n*a*b + a**2*m**2 + 3*a**2*m*n + 2*a**2*m +
 2*a**2*n**2 + 3*a**2*n + a**2))/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*
m*n + 3*m + 2*n**2 + 3*n + 1)
```

3.653 $\int (cx)^m \left(a+bx^3\right)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int (cx)^m \left(a+bx^3\right)^2 \, dx = \frac{a^2(cx)^{1+m}}{c(1+m)} + \frac{2ab(cx)^{4+m}}{c^4(4+m)} + \frac{b^2(cx)^{7+m}}{c^7(7+m)}$$

output
```
a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(4+m)/c^4/(4+m)+b^2*(c*x)^(7+m)/c^7/(7
+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (cx)^m \left(a+bx^3\right)^2 \, dx = x(cx)^m \left(\frac{a^2}{1+m} + \frac{2abx^3}{4+m} + \frac{b^2x^6}{7+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^3)^2,x]
```

output
```
x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + (b^2*x^6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^2 (cx)^m \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2(cx)^m + \frac{2ab(cx)^{m+3}}{c^3} + \frac{b^2(cx)^{m+6}}{c^6}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(cx)^{m+1}}{c(m+1)} + \frac{2ab(cx)^{m+4}}{c^4(m+4)} + \frac{b^2(cx)^{m+7}}{c^7(m+7)}$$

input
```
Int[(c*x)^m*(a + b*x^3)^2,x]
```

output
```
(a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(4 + m))/(c^4*(4 + m)) + (b
^2*(c*x)^(7 + m))/(c^7*(7 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
norman	$\frac{a^{2} x \,{\mathrm e}^{m \ln \left(c x \right)}}{1+m}+\frac{b^{2} x^{7} {\mathrm e}^{m \ln \left(c x \right)}}{7+m}+\frac{2 a b \,x^{4} {\mathrm e}^{m \ln \left(c x \right)}}{4+m}$
gosper	$\frac{x \left(b^{2} m^{2} x^{6}+5 m \,x^{6} b^{2}+4 b^{2} x^{6}+2 a b \,m^{2} x^{3}+16 m \,x^{3} a b +14 a b \,x^{3}+a^{2} m^{2}+11 m \,a^{2}+28 a^{2}\right) \left(c x \right)^{m}}{\left(7+m \right) \left(4+m \right) \left(1+m \right)}$
risch	$\frac{x \left(b^{2} m^{2} x^{6}+5 m \,x^{6} b^{2}+4 b^{2} x^{6}+2 a b \,m^{2} x^{3}+16 m \,x^{3} a b +14 a b \,x^{3}+a^{2} m^{2}+11 m \,a^{2}+28 a^{2}\right) \left(c x \right)^{m}}{\left(7+m \right) \left(4+m \right) \left(1+m \right)}$
orering	$\frac{x \left(b^{2} m^{2} x^{6}+5 m \,x^{6} b^{2}+4 b^{2} x^{6}+2 a b \,m^{2} x^{3}+16 m \,x^{3} a b +14 a b \,x^{3}+a^{2} m^{2}+11 m \,a^{2}+28 a^{2}\right) \left(c x \right)^{m}}{\left(7+m \right) \left(4+m \right) \left(1+m \right)}$
parallelrisch	$\frac{x^{7} \left(c x \right)^{m} b^{2} m^{2}+5 x^{7} \left(c x \right)^{m} b^{2} m +4 x^{7} \left(c x \right)^{m} b^{2}+2 x^{4} \left(c x \right)^{m} a b \,m^{2}+16 x^{4} \left(c x \right)^{m} a b m +14 x^{4} \left(c x \right)^{m} a b +x \left(c x \right)^{m} a^{2} m^{2}+11 x \left(c x \right)^{m} a^{2} m}{\left(7+m \right) \left(4+m \right) \left(1+m \right)}$

input
```
int((c*x)^m*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*exp(m*ln(c*x))+b^2/(7+m)*x^7*exp(m*ln(c*x))+2*a*b/(4+m)*x^4*ex
p(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int (cx)^m \left(a+bx^3\right)^2 \, dx = \frac{((b^2m^2 + 5\,b^2m + 4\,b^2)x^7 + 2\,(abm^2 + 8\,abm + 7\,ab)x^4 + (a^2m^2 + 11\,a^2m + 28\,a^2)x)(cx)^m}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate((c*x)^m*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 5*b^2*m + 4*b^2)*x^7 + 2*(a*b*m^2 + 8*a*b*m + 7*a*b)*x^4 + (a^
2*m^2 + 11*a^2*m + 28*a^2)*x)*(c*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(49) = 98$.

Time = 0.53 (sec) , antiderivative size = 337, normalized size of antiderivative = 5.81

$$\int (cx)^m \left(a+bx^3\right)^2 dx$$
$$= \begin{cases} \frac{-\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\log(x)}{c^7} \\ \frac{-\frac{a^2}{3x^3}+2ab\log(x)+\frac{b^2x^3}{3}}{c^4} \\ \frac{a^2\log(x)+\frac{2abx^3}{3}+\frac{b^2x^6}{6}}{c} \\ \frac{a^2m^2x(cx)^m}{m^3+12m^2+39m+28}+\frac{11a^2mx(cx)^m}{m^3+12m^2+39m+28}+\frac{28a^2x(cx)^m}{m^3+12m^2+39m+28}+\frac{2abm^2x^4(cx)^m}{m^3+12m^2+39m+28}+\frac{16abmx^4(cx)^m}{m^3+12m^2+39m+28}+\frac{14abx^4}{m^3+12m^2} \end{cases}$$

input
```
integrate((c*x)**m*(b*x**3+a)**2,x)
```

output
```
Piecewise(((-a**2/(6*x**6) - 2*a*b/(3*x**3) + b**2*log(x))/c**7, Eq(m, -7)
), ((-a**2/(3*x**3) + 2*a*b*log(x) + b**2*x**3/3)/c**4, Eq(m, -4)), ((a**2
*log(x) + 2*a*b*x**3/3 + b**2*x**6/6)/c, Eq(m, -1)), (a**2*m**2*x*(c*x)**m
/(m**3 + 12*m**2 + 39*m + 28) + 11*a**2*m*x*(c*x)**m/(m**3 + 12*m**2 + 39*
m + 28) + 28*a**2*x*(c*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 2*a*b*m**2*x**
4*(c*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 16*a*b*m*x**4*(c*x)**m/(m**3 + 1
2*m**2 + 39*m + 28) + 14*a*b*x**4*(c*x)**m/(m**3 + 12*m**2 + 39*m + 28) +
b**2*m**2*x**7*(c*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 5*b**2*m*x**7*(c*x)
**m/(m**3 + 12*m**2 + 39*m + 28) + 4*b**2*x**7*(c*x)**m/(m**3 + 12*m**2 +
39*m + 28), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(a+bx^3\right)^2 dx = \frac{b^2c^mx^7x^m}{m+7}+\frac{2\,abc^mx^4x^m}{m+4}+\frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
b^2*c^m*x^7*x^m/(m + 7) + 2*a*b*c^m*x^4*x^m/(m + 4) + (c*x)^(m + 1)*a^2/(c
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

$$\int (cx)^m \left(a+bx^3\right)^2 \, dx$$
$$= \frac{(cx)^m b^2m^2x^7 + 5\,(cx)^m b^2mx^7 + 4\,(cx)^m b^2x^7 + 2\,(cx)^m abm^2x^4 + 16\,(cx)^m abmx^4 + 14\,(cx)^m abx^4 + (c}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate((c*x)^m*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
((c*x)^m*b^2*m^2*x^7 + 5*(c*x)^m*b^2*m*x^7 + 4*(c*x)^m*b^2*x^7 + 2*(c*x)^m
*a*b*m^2*x^4 + 16*(c*x)^m*a*b*m*x^4 + 14*(c*x)^m*a*b*x^4 + (c*x)^m*a^2*m^2
*x + 11*(c*x)^m*a^2*m*x + 28*(c*x)^m*a^2*x)/(m^3 + 12*m^2 + 39*m + 28)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (cx)^m \left(a+bx^3\right)^2 \, dx = (c\,x)^m \left(\frac{a^2\,x\,(m^2+11\,m+28)}{m^3+12\,m^2+39\,m+28} + \frac{b^2\,x^7\,(m^2+5\,m+4)}{m^3+12\,m^2+39\,m+28} + \frac{2\,a\,b\,x^4\,(m^2+8\,m+7)}{m^3+12\,m^2+39\,m+28}\right)$$

input
```
int((c*x)^m*(a + b*x^3)^2,x)
```

output
```
(c*x)^m*((a^2*x*(11*m + m^2 + 28))/(39*m + 12*m^2 + m^3 + 28) + (b^2*x^7*(
5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (2*a*b*x^4*(8*m + m^2 + 7))/(
39*m + 12*m^2 + m^3 + 28))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int (cx)^m \left(a+bx^3\right)^2 \, dx = \frac{x^m c^m x(b^2m^2x^6+5b^2m\,x^6+4b^2x^6+2ab\,m^2x^3+16abm\,x^3+14ab\,x^3+a^2m^2+11a^2m+28a^2)}{m^3+12m^2+39m+28}$$

input
```
int((c*x)^m*(b*x^3+a)^2,x)
```

output
```
(x**m*c**m*x*(a**2*m**2 + 11*a**2*m + 28*a**2 + 2*a*b*m**2*x**3 + 16*a*b*m
*x**3 + 14*a*b*x**3 + b**2*m**2*x**6 + 5*b**2*m*x**6 + 4*b**2*x**6))/(m**3
 + 12*m**2 + 39*m + 28)
```

3.654 $\int (cx)^m \left(a+bx^2\right)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int (cx)^m \left(a+bx^2\right)^2 \, dx = \frac{a^2(cx)^{1+m}}{c(1+m)} + \frac{2ab(cx)^{3+m}}{c^3(3+m)} + \frac{b^2(cx)^{5+m}}{c^5(5+m)}$$

output
```
a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(3+m)/c^3/(3+m)+b^2*(c*x)^(5+m)/c^5/(5
+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (cx)^m \left(a+bx^2\right)^2 \, dx = x(cx)^m \left(\frac{a^2}{1+m} + \frac{2abx^2}{3+m} + \frac{b^2x^4}{5+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^2,x]
```

output
```
x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 (cx)^m \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2(cx)^m+\frac{2ab(cx)^{m+2}}{c^2}+\frac{b^2(cx)^{m+4}}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2ab(cx)^{m+3}}{c^3(m+3)}+\frac{b^2(cx)^{m+5}}{c^5(m+5)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^2,x]
```

output
```
(a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(3 + m))/(c^3*(3 + m)) + (b
^2*(c*x)^(5 + m))/(c^5*(5 + m))
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
norman	$\frac{a^{2} x \,{\mathrm e}^{m \ln \left(c x \right)}}{1+m}+\frac{b^{2} x^{5} {\mathrm e}^{m \ln \left(c x \right)}}{5+m}+\frac{2 a b \,x^{3} {\mathrm e}^{m \ln \left(c x \right)}}{3+m}$
gosper	$\frac{x \left(b^{2} m^{2} x^{4}+4 m \,x^{4} b^{2}+2 a b \,m^{2} x^{2}+3 b^{2} x^{4}+12 m \,x^{2} a b +a^{2} m^{2}+10 a b \,x^{2}+8 m \,a^{2}+15 a^{2}\right) \left(c x \right)^{m}}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$
risch	$\frac{x \left(b^{2} m^{2} x^{4}+4 m \,x^{4} b^{2}+2 a b \,m^{2} x^{2}+3 b^{2} x^{4}+12 m \,x^{2} a b +a^{2} m^{2}+10 a b \,x^{2}+8 m \,a^{2}+15 a^{2}\right) \left(c x \right)^{m}}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$
orering	$\frac{x \left(b^{2} m^{2} x^{4}+4 m \,x^{4} b^{2}+2 a b \,m^{2} x^{2}+3 b^{2} x^{4}+12 m \,x^{2} a b +a^{2} m^{2}+10 a b \,x^{2}+8 m \,a^{2}+15 a^{2}\right) \left(c x \right)^{m}}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$
parallelrisch	$\frac{x^{5} \left(c x \right)^{m} b^{2} m^{2}+4 x^{5} \left(c x \right)^{m} b^{2} m +3 x^{5} \left(c x \right)^{m} b^{2}+2 x^{3} \left(c x \right)^{m} a b \,m^{2}+12 x^{3} \left(c x \right)^{m} a b m +10 x^{3} \left(c x \right)^{m} a b +x \left(c x \right)^{m} a^{2} m^{2}+8 x \left(c x \right)^{m} a^{2} m}{\left(5+m \right) \left(3+m \right) \left(1+m \right)}$

input
```
int((c*x)^m*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*exp(m*ln(c*x))+b^2/(5+m)*x^5*exp(m*ln(c*x))+2*a*b/(3+m)*x^3*ex
p(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int (cx)^m \left(a+bx^2\right)^2 \, dx = \frac{((b^2m^2+4\,b^2m+3\,b^2)x^5+2\,(abm^2+6\,abm+5\,ab)x^3+(a^2m^2+8\,a^2m+15\,a^2)x)(cx)^m}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^
2*m^2 + 8*a^2*m + 15*a^2)*x)*(c*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(49) = 98$.

Time = 0.37 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.69

$$\int (cx)^m \left(a+bx^2\right)^2 dx$$
$$= \begin{cases} \frac{-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log{(x)}}{c^5} \\ \frac{-\frac{a^2}{2x^2}+2ab\log{(x)}+\frac{b^2x^2}{2}}{c^3} \\ \frac{a^2\log{(x)}+abx^2+\frac{b^2x^4}{4}}{c} \\ \frac{a^2m^2x(cx)^m}{m^3+9m^2+23m+15}+\frac{8a^2mx(cx)^m}{m^3+9m^2+23m+15}+\frac{15a^2x(cx)^m}{m^3+9m^2+23m+15}+\frac{2abm^2x^3(cx)^m}{m^3+9m^2+23m+15}+\frac{12abmx^3(cx)^m}{m^3+9m^2+23m+15}+\frac{10abx^3(cx)^m}{m^3+9m^2+23m+} \end{cases}$$

input
```
integrate((c*x)**m*(b*x**2+a)**2,x)
```

output
```
Piecewise(((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/c**5, Eq(m, -5)), ((-
a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/c**3, Eq(m, -3)), ((a**2*log(x
) + a*b*x**2 + b**2*x**4/4)/c, Eq(m, -1)), (a**2*m**2*x*(c*x)**m/(m**3 + 9
*m**2 + 23*m + 15) + 8*a**2*m*x*(c*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 15*
a**2*x*(c*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 2*a*b*m**2*x**3*(c*x)**m/(m*
*3 + 9*m**2 + 23*m + 15) + 12*a*b*m*x**3*(c*x)**m/(m**3 + 9*m**2 + 23*m +
15) + 10*a*b*x**3*(c*x)**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*m**2*x**5*(c
*x)**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*x**5*(c*x)**m/(m**3 + 9*m**2
 + 23*m + 15) + 3*b**2*x**5*(c*x)**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(a+bx^2\right)^2 dx = \frac{b^2c^mx^5x^m}{m+5} + \frac{2\,abc^mx^3x^m}{m+3} + \frac{(cx)^{m+1}a^2}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
b^2*c^m*x^5*x^m/(m + 5) + 2*a*b*c^m*x^3*x^m/(m + 3) + (c*x)^(m + 1)*a^2/(c
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

$$\int (cx)^m \left(a + bx^2\right)^2 dx = \frac{(cx)^m b^2 m^2 x^5 + 4\,(cx)^m b^2 m x^5 + 2\,(cx)^m abm^2 x^3 + 3\,(cx)^m b^2 x^5 + 12\,(cx)^m abmx^3 + (cx)^m a^2 m^2 x + 10}{m^3 + 9\,m^2 + 23\,m + 15}$$

input
```
integrate((c*x)^m*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
((c*x)^m*b^2*m^2*x^5 + 4*(c*x)^m*b^2*m*x^5 + 2*(c*x)^m*a*b*m^2*x^3 + 3*(c*
x)^m*b^2*x^5 + 12*(c*x)^m*a*b*m*x^3 + (c*x)^m*a^2*m^2*x + 10*(c*x)^m*a*b*x
^3 + 8*(c*x)^m*a^2*m*x + 15*(c*x)^m*a^2*x)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (cx)^m \left(a + bx^2\right)^2 dx = (c\,x)^m \left(\frac{a^2\,x\,(m^2 + 8\,m + 15)}{m^3 + 9\,m^2 + 23\,m + 15} + \frac{b^2\,x^5\,(m^2 + 4\,m + 3)}{m^3 + 9\,m^2 + 23\,m + 15} + \frac{2\,a\,b\,x^3\,(m^2 + 6\,m + 5)}{m^3 + 9\,m^2 + 23\,m + 15}\right)$$

input
```
int((c*x)^m*(a + b*x^2)^2,x)
```

output
```
(c*x)^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*
m + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*
m + 9*m^2 + m^3 + 15))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int (cx)^m \left(a+bx^2\right)^2 \, dx = \frac{x^m c^m x(b^2m^2x^4+4b^2m\,x^4+2ab\,m^2x^2+3b^2x^4+12abm\,x^2+a^2m^2+10ab\,x^2+8a^2m+15a^2)}{m^3+9m^2+23m+15}$$

input
```
int((c*x)^m*(b*x^2+a)^2,x)
```

output
```
(x**m*c**m*x*(a**2*m**2 + 8*a**2*m + 15*a**2 + 2*a*b*m**2*x**2 + 12*a*b*m*
x**2 + 10*a*b*x**2 + b**2*m**2*x**4 + 4*b**2*m*x**4 + 3*b**2*x**4))/(m**3
+ 9*m**2 + 23*m + 15)
```

3.655 $\int (cx)^m (a+bx)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int (cx)^m (a+bx)^2 \, dx = \frac{a^2 (cx)^{1+m}}{c(1+m)} + \frac{2ab(cx)^{2+m}}{c^2(2+m)} + \frac{b^2 (cx)^{3+m}}{c^3(3+m)}$$

output
```
a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(2+m)/c^2/(2+m)+b^2*(c*x)^(3+m)/c^3/(3
+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int (cx)^m (a+bx)^2 \, dx = x(cx)^m \left(\frac{a^2}{1+m} + \frac{2abx}{2+m} + \frac{b^2 x^2}{3+m} \right)$$

input
```
Integrate[(c*x)^m*(a + b*x)^2,x]
```

output
```
x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(cx)^m\,dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(a^2(cx)^m+\frac{2ab(cx)^{m+1}}{c}+\frac{b^2(cx)^{m+2}}{c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2ab(cx)^{m+2}}{c^2(m+2)}+\frac{b^2(cx)^{m+3}}{c^3(m+3)}$$

input
```
Int[(c*x)^m*(a + b*x)^2,x]
```

output
```
(a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(2 + m))/(c^2*(2 + m)) + (b
^2*(c*x)^(3 + m))/(c^3*(3 + m))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
norman	$\frac{a^{2} x \,{\mathrm e}^{m \ln \left(c x \right)}}{1+m}+\frac{b^{2} x^{3} {\mathrm e}^{m \ln \left(c x \right)}}{3+m}+\frac{2 a b \,x^{2} {\mathrm e}^{m \ln \left(c x \right)}}{2+m}$
gosper	$\frac{x \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +3 b^{2} m \,x^{2}+a^{2} m^{2}+8 m x a b +2 b^{2} x^{2}+5 m \,a^{2}+6 a b x +6 a^{2}\right) \left(c x \right)^{m}}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$
risch	$\frac{x \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +3 b^{2} m \,x^{2}+a^{2} m^{2}+8 m x a b +2 b^{2} x^{2}+5 m \,a^{2}+6 a b x +6 a^{2}\right) \left(c x \right)^{m}}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$
orering	$\frac{x \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +3 b^{2} m \,x^{2}+a^{2} m^{2}+8 m x a b +2 b^{2} x^{2}+5 m \,a^{2}+6 a b x +6 a^{2}\right) \left(c x \right)^{m}}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$
parallelrisch	$\frac{x^{3} \left(c x \right)^{m} b^{2} m^{2}+3 x^{3} \left(c x \right)^{m} b^{2} m +2 x^{2} \left(c x \right)^{m} a b \,m^{2}+2 x^{3} \left(c x \right)^{m} b^{2}+8 x^{2} \left(c x \right)^{m} a b m +x \left(c x \right)^{m} a^{2} m^{2}+6 x^{2} \left(c x \right)^{m} a b +5 x \left(c x \right)^{m} a^{2} m +6}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$

input
```
int((c*x)^m*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*exp(m*ln(c*x))+b^2/(3+m)*x^3*exp(m*ln(c*x))+2*a*b/(2+m)*x^2*ex
p(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int (cx)^m(a+bx)^2\,dx = \frac{((b^2m^2+3\,b^2m+2\,b^2)x^3+2\,(abm^2+4\,abm+3\,ab)x^2+(a^2m^2+5\,a^2m+6\,a^2)x)(cx)^m}{m^3+6\,m^2+11\,m+6}$$

input
```
integrate((c*x)^m*(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^
2*m^2 + 5*a^2*m + 6*a^2)*x)*(c*x)^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.57

$$\int (cx)^m (a+bx)^2 \, dx = \begin{cases} \frac{-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)}{c^3} \\ \frac{-\frac{a^2}{x} + 2ab\log(x) + b^2 x}{c^2} \\ \frac{a^2 \log(x) + 2abx + \frac{b^2x^2}{2}}{c} \\ \frac{a^2m^2x(cx)^m}{m^3+6m^2+11m+6} + \frac{5a^2mx(cx)^m}{m^3+6m^2+11m+6} + \frac{6a^2x(cx)^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2(cx)^m}{m^3+6m^2+11m+6} + \frac{8abmx^2(cx)^m}{m^3+6m^2+11m+6} + \frac{6abx^2(cx)^m}{m^3+6m^2+11m+6} + \end{cases}$$

input
```
integrate((c*x)**m*(b*x+a)**2,x)
```

output
```
Piecewise(((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x))/c**3, Eq(m, -3)), ((-a
**2/x + 2*a*b*log(x) + b**2*x)/c**2, Eq(m, -2)), ((a**2*log(x) + 2*a*b*x +
 b**2*x**2/2)/c, Eq(m, -1)), (a**2*m**2*x*(c*x)**m/(m**3 + 6*m**2 + 11*m +
 6) + 5*a**2*m*x*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a**2*x*(c*x)**m/(
m**3 + 6*m**2 + 11*m + 6) + 2*a*b*m**2*x**2*(c*x)**m/(m**3 + 6*m**2 + 11*m
 + 6) + 8*a*b*m*x**2*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 6*a*b*x**2*(c*x
)**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3*(c*x)**m/(m**3 + 6*m**2 +
 11*m + 6) + 3*b**2*m*x**3*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6) + 2*b**2*x*
*3*(c*x)**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (cx)^m (a+bx)^2 \, dx = \frac{b^2c^mx^3x^m}{m+3} + \frac{2\,abc^mx^2x^m}{m+2} + \frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x+a)^2,x, algorithm="maxima")
```

output
```
b^2*c^m*x^3*x^m/(m + 3) + 2*a*b*c^m*x^2*x^m/(m + 2) + (c*x)^(m + 1)*a^2/(c
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

$$\int (cx)^m (a+bx)^2\,dx = \frac{(cx)^m b^2m^2x^3 + 2\,(cx)^m abm^2x^2 + 3\,(cx)^m b^2mx^3 + (cx)^m a^2m^2x + 8\,(cx)^m abmx^2 + 2\,(cx)^m b^2x^3 + 5\,(c}{m^3 + 6\,m^2 + 11\,m + 6}$$

input
```
integrate((c*x)^m*(b*x+a)^2,x, algorithm="giac")
```

output
```
((c*x)^m*b^2*m^2*x^3 + 2*(c*x)^m*a*b*m^2*x^2 + 3*(c*x)^m*b^2*m*x^3 + (c*x)
^m*a^2*m^2*x + 8*(c*x)^m*a*b*m*x^2 + 2*(c*x)^m*b^2*x^3 + 5*(c*x)^m*a^2*m*x
 + 6*(c*x)^m*a*b*x^2 + 6*(c*x)^m*a^2*x)/(m^3 + 6*m^2 + 11*m + 6)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (cx)^m (a+bx)^2\,dx = (c\,x)^m \left(\frac{a^2\,x\,(m^2+5\,m+6)}{m^3+6\,m^2+11\,m+6} + \frac{b^2\,x^3\,(m^2+3\,m+2)}{m^3+6\,m^2+11\,m+6} + \frac{2\,a\,b\,x^2\,(m^2+4\,m+3)}{m^3+6\,m^2+11\,m+6}\right)$$

input
```
int((c*x)^m*(a + b*x)^2,x)
```

output
```
(c*x)^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m
+ m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m +
 6*m^2 + m^3 + 6))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int (cx)^m (a+bx)^2 \, dx$$
$$= \frac{x^m c^m x (b^2 m^2 x^2 + 2ab\, m^2 x + 3b^2 m\, x^2 + a^2 m^2 + 8abmx + 2b^2 x^2 + 5a^2 m + 6abx + 6a^2)}{m^3 + 6m^2 + 11m + 6}$$

input
```
int((c*x)^m*(b*x+a)^2,x)
```

output
```
(x**m*c**m*x*(a**2*m**2 + 5*a**2*m + 6*a**2 + 2*a*b*m**2*x + 8*a*b*m*x + 6
*a*b*x + b**2*m**2*x**2 + 3*b**2*m*x**2 + 2*b**2*x**2))/(m**3 + 6*m**2 + 1
1*m + 6)
```

3.656 $\int \left(a+\frac{b}{x}\right)^2 (cx)^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m \, dx = -\frac{b^2 c (cx)^{-1+m}}{1-m} + \frac{2ab(cx)^m}{m} + \frac{a^2 (cx)^{1+m}}{c(1+m)}$$

output
```
-b^2*c*(c*x)^(-1+m)/(1-m)+2*a*b*(c*x)^m/m+a^2*(c*x)^(1+m)/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m \, dx = (cx)^m \left(\frac{2ab}{m} + \frac{b^2}{(-1+m)x} + \frac{a^2 x}{1+m}\right)$$

input
```
Integrate[(a + b/x)^2*(c*x)^m,x]
```

output
```
(c*x)^m*((2*a*b)/m + b^2/((-1 + m)*x) + (a^2*x)/(1 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(a^2(cx)^m+2abc(cx)^{m-1}+b^2c^2(cx)^{m-2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(cx)^{m+1}}{c(m+1)}+\frac{2ab(cx)^m}{m}-\frac{b^2c(cx)^{m-1}}{1-m}$$

input
```
Int[(a + b/x)^2*(c*x)^m,x]
```

output
```
-((b^2*c*(c*x)^(-1 + m))/(1 - m)) + (2*a*b*(c*x)^m)/m + (a^2*(c*x)^(1 + m)
)/(c*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

method	result	size
norman	$\frac{\frac{b^2 e^{m \ln(cx)}}{m-1}+\frac{a^2 x^2 e^{m \ln(cx)}}{1+m}+\frac{2abx\, e^{m \ln(cx)}}{m}}{x}$	56
gosper	$\frac{(cx)^m (a^2x^2m^2-a^2x^2m+2ab\, m^2x+b^2m^2-2abx+b^2m)}{x(1+m)m(m-1)}$	68
risch	$\frac{(cx)^m (a^2x^2m^2-a^2x^2m+2ab\, m^2x+b^2m^2-2abx+b^2m)}{x(1+m)m(m-1)}$	68
orering	$\frac{(a^2x^2m^2-a^2x^2m+2ab\, m^2x+b^2m^2-2abx+b^2m)x\left(a+\frac{b}{x}\right)^2 (cx)^m}{(1+m)m(m-1)(ax+b)^2}$	82
parallelrisch	$\frac{x^2(cx)^m a^2m^2-x^2(cx)^m a^2m+2x(cx)^m ab\, m^2+(cx)^m b^2m^2-2x(cx)^m ab+(cx)^m b^2m}{x(1+m)m(m-1)}$	93

input
```
int((a+b/x)^2*(c*x)^m,x,method=_RETURNVERBOSE)
```

output
```
(b^2/(m-1)*exp(m*ln(c*x))+a^2/(1+m)*x^2*exp(m*ln(c*x))+2*a*b/m*x*exp(m*ln(
c*x)))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m\,dx = \frac{(b^2m^2+b^2m+(a^2m^2-a^2m)x^2+2\,(abm^2-ab)x)(cx)^m}{(m^3-m)x}$$

input
```
integrate((a+b/x)^2*(c*x)^m,x, algorithm="fricas")
```

output
```
(b^2*m^2 + b^2*m + (a^2*m^2 - a^2*m)*x^2 + 2*(a*b*m^2 - a*b)*x)*(c*x)^m/((
m^3 - m)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(41) = 82$.

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.69

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m\,dx = \begin{cases} \frac{a^2\log(x)-\frac{2ab}{x}-\frac{b^2}{2x^2}}{c} & \text{for } m=-1 \\ a^2x+2ab\log(x)-\frac{b^2}{x} & \text{for } m=0 \\ c\left(\frac{a^2x^2}{2}+2abx+b^2\log(x)\right) & \text{for } m=1 \\ \frac{a^2m^2x^2(cx)^m}{m^3x-mx}-\frac{a^2mx^2(cx)^m}{m^3x-mx}+\frac{2abm^2x(cx)^m}{m^3x-mx}-\frac{2abx(cx)^m}{m^3x-mx}+\frac{b^2m^2(cx)^m}{m^3x-mx}+\frac{b^2m(cx)^m}{m^3x-mx} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x)**2*(c*x)**m,x)
```

output
```
Piecewise(((a**2*log(x) - 2*a*b/x - b**2/(2*x**2))/c, Eq(m, -1)), (a**2*x
+ 2*a*b*log(x) - b**2/x, Eq(m, 0)), (c*(a**2*x**2/2 + 2*a*b*x + b**2*log(x
)), Eq(m, 1)), (a**2*m**2*x**2*(c*x)**m/(m**3*x - m*x) - a**2*m*x**2*(c*x)
**m/(m**3*x - m*x) + 2*a*b*m**2*x*(c*x)**m/(m**3*x - m*x) - 2*a*b*x*(c*x)*
*m/(m**3*x - m*x) + b**2*m**2*(c*x)**m/(m**3*x - m*x) + b**2*m*(c*x)**m/(m
**3*x - m*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m\,dx = \frac{2\,abc^mx^m}{m}+\frac{b^2c^mx^m}{(m-1)x}+\frac{(cx)^{m+1}\,a^2}{c(m+1)}$$

input
```
integrate((a+b/x)^2*(c*x)^m,x, algorithm="maxima")
```

output
```
2*a*b*c^m*x^m/m + b^2*c^m*x^m/((m - 1)*x) + (c*x)^(m + 1)*a^2/(c*(m + 1))
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m\,dx=\int (cx)^m\left(a+\frac{b}{x}\right)^2\,dx$$

input
```
integrate((a+b/x)^2*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*(a + b/x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m\,dx=\frac{b^2\,(c\,x)^m}{x\,(m-1)}+\frac{a\,(c\,x)^m\,(2\,b+2\,b\,m+a\,m\,x)}{m\,(m+1)}$$

input
```
int((c*x)^m*(a + b/x)^2,x)
```

output
```
(b^2*(c*x)^m)/(x*(m - 1)) + (a*(c*x)^m*(2*b + 2*b*m + a*m*x))/(m*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x}\right)^2 (cx)^m\,dx=\frac{x^m c^m(a^2m^2x^2-a^2m\,x^2+2ab\,m^2x+b^2m^2-2abx+b^2m)}{mx\,(m^2-1)}$$

input
```
int((a+b/x)^2*(c*x)^m,x)
```

output
```
(x**m*c**m*(a**2*m**2*x**2 - a**2*m*x**2 + 2*a*b*m**2*x - 2*a*b*x + b**2*m
**2 + b**2*m))/(m*x*(m**2 - 1))
```

3.657 $\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m \, dx = -\frac{b^2c^3(cx)^{-3+m}}{3-m}-\frac{2abc(cx)^{-1+m}}{1-m}+\frac{a^2(cx)^{1+m}}{c(1+m)}$$

output
```
-b^2*c^3*(c*x)^(-3+m)/(3-m)-2*a*b*c*(c*x)^(-1+m)/(1-m)+a^2*(c*x)^(1+m)/c/(
1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m \, dx = \frac{(cx)^m \left(b^2(-1+m^2)+2ab(-3-2m+m^2)\,x^2+a^2(3-4m+m^2)\,x^4\right)}{(-3+m)(-1+m)(1+m)x^3}$$

input
```
Integrate[(a + b/x^2)^2*(c*x)^m,x]
```

output
```
((c*x)^m*(b^2*(-1 + m^2) + 2*a*b*(-3 - 2*m + m^2)*x^2 + a^2*(3 - 4*m + m^2
)*x^4))/((-3 + m)*(-1 + m)*(1 + m)*x^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(a^2 (cx)^m+2 a b c^2 (cx)^{m-2}+b^2 c^4 (cx)^{m-4}\right) \, dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2 (cx)^{m+1}}{c (m+1)}-\frac{2 a b c (cx)^{m-1}}{1-m}-\frac{b^2 c^3 (cx)^{m-3}}{3-m}$$

input
```
Int[(a + b/x^2)^2*(c*x)^m,x]
```

output
```
-((b^2*c^3*(c*x)^(-3 + m))/(3 - m)) - (2*a*b*c*(c*x)^(-1 + m))/(1 - m) + (
a^2*(c*x)^(1 + m))/(c*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

method	result	size
norman	$\frac{\frac{b^2 e^{m\ln(cx)}}{-3+m}+\frac{a^2 x^4 e^{m\ln(cx)}}{1+m}+\frac{2ab\,x^2 e^{m\ln(cx)}}{m-1}}{x^3}$	60
gosper	$\frac{(cx)^m\left(a^2m^2x^4-4a^2m\,x^4+3a^2x^4+2ab\,m^2x^2-4m\,x^2ab-6ab\,x^2+b^2m^2-b^2\right)}{x^3(1+m)(m-1)(-3+m)}$	90
risch	$\frac{(cx)^m\left(a^2m^2x^4-4a^2m\,x^4+3a^2x^4+2ab\,m^2x^2-4m\,x^2ab-6ab\,x^2+b^2m^2-b^2\right)}{x^3(1+m)(m-1)(-3+m)}$	90
orering	$\frac{\left(a^2m^2x^4-4a^2m\,x^4+3a^2x^4+2ab\,m^2x^2-4m\,x^2ab-6ab\,x^2+b^2m^2-b^2\right)x\left(a+\frac{b}{x^2}\right)^2(cx)^m}{(1+m)(m-1)(-3+m)(a\,x^2+b)^2}$	106
parallelrisch	$\frac{x^4(cx)^m a^2m^2-4x^4(cx)^m a^2m+3x^4(cx)^m a^2+2x^2(cx)^m ab\,m^2-4x^2(cx)^m abm-6x^2(cx)^m ab+(cx)^m b^2m^2-b^2(cx)^m}{x^3(1+m)(m-1)(-3+m)}$	125

input
```
int((a+b/x^2)^2*(c*x)^m,x,method=_RETURNVERBOSE)
```

output
```
(b^2/(-3+m)*exp(m*ln(c*x))+a^2/(1+m)*x^4*exp(m*ln(c*x))+2*a*b/(m-1)*x^2*ex
p(m*ln(c*x)))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33

$$\int\left(a+\frac{b}{x^2}\right)^2(cx)^m\,dx = \frac{((a^2m^2-4\,a^2m+3\,a^2)x^4+b^2m^2+2\,(abm^2-2\,abm-3\,ab)x^2-b^2)(cx)^m}{(m^3-3\,m^2-m+3)x^3}$$

input
```
integrate((a+b/x^2)^2*(c*x)^m,x, algorithm="fricas")
```

output
```
((a^2*m^2 - 4*a^2*m + 3*a^2)*x^4 + b^2*m^2 + 2*(a*b*m^2 - 2*a*b*m - 3*a*b)
*x^2 - b^2)*(c*x)^m/((m^3 - 3*m^2 - m + 3)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(48) = 96$.

Time = 0.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 6.36

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m\,dx = \begin{cases} \frac{a^2\log(x)-\frac{ab}{x^2}-\frac{b^2}{4x^4}}{c} \\ c\left(\frac{a^2x^2}{2}+2ab\log(x)-\frac{b^2}{2x^2}\right) \\ c^3\left(\frac{a^2x^4}{4}+abx^2+b^2\log(x)\right) \\ \frac{a^2m^2x^4(cx)^m}{m^3x^3-3m^2x^3-mx^3+3x^3}-\frac{4a^2mx^4(cx)^m}{m^3x^3-3m^2x^3-mx^3+3x^3}+\frac{3a^2x^4(cx)^m}{m^3x^3-3m^2x^3-mx^3+3x^3}+\frac{2abm^2x^2(cx)^m}{m^3x^3-3m^2x^3-mx^3+3x^3}-\frac{4abmx^2(c}{m^3x^3-3m^2x^3-} \end{cases}$$

input
```
integrate((a+b/x**2)**2*(c*x)**m,x)
```

output
```
Piecewise(((a**2*log(x) - a*b/x**2 - b**2/(4*x**4))/c, Eq(m, -1)), (c*(a**
2*x**2/2 + 2*a*b*log(x) - b**2/(2*x**2)), Eq(m, 1)), (c**3*(a**2*x**4/4 +
a*b*x**2 + b**2*log(x)), Eq(m, 3)), (a**2*m**2*x**4*(c*x)**m/(m**3*x**3 -
3*m**2*x**3 - m*x**3 + 3*x**3) - 4*a**2*m*x**4*(c*x)**m/(m**3*x**3 - 3*m**
2*x**3 - m*x**3 + 3*x**3) + 3*a**2*x**4*(c*x)**m/(m**3*x**3 - 3*m**2*x**3
- m*x**3 + 3*x**3) + 2*a*b*m**2*x**2*(c*x)**m/(m**3*x**3 - 3*m**2*x**3 - m
*x**3 + 3*x**3) - 4*a*b*m*x**2*(c*x)**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3
+ 3*x**3) - 6*a*b*x**2*(c*x)**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3 + 3*x**3
) + b**2*m**2*(c*x)**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3 + 3*x**3) - b**2*
(c*x)**m/(m**3*x**3 - 3*m**2*x**3 - m*x**3 + 3*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m\,dx = \frac{2\,abc^mx^m}{(m-1)x}+\frac{(cx)^{m+1}a^2}{c(m+1)}+\frac{b^2c^mx^m}{(m-3)x^3}$$

input
```
integrate((a+b/x^2)^2*(c*x)^m,x, algorithm="maxima")
```

output
```
2*a*b*c^m*x^m/((m - 1)*x) + (c*x)^(m + 1)*a^2/(c*(m + 1)) + b^2*c^m*x^m/((
m - 3)*x^3)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m\,dx=\int (cx)^m\left(a+\frac{b}{x^2}\right)^2\,dx$$

input
```
integrate((a+b/x^2)^2*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*(a + b/x^2)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m\,dx = \frac{(c\,x)^m\,\left(-a^2\,m^2\,x^4+4\,a^2\,m\,x^4-3\,a^2\,x^4-2\,a\,b\,m^2\,x^2+4\,a\,b\,m\,x^2+6\,a\,b\,x^2-b^2\,m^2+b^2\right)}{x^3\,\left(-m^3+3\,m^2+m-3\right)}$$

input
```
int((c*x)^m*(a + b/x^2)^2,x)
```

output
```
((c*x)^m*(b^2 - b^2*m^2 - 3*a^2*x^4 + 4*a^2*m*x^4 - a^2*m^2*x^4 + 6*a*b*x^
2 + 4*a*b*m*x^2 - 2*a*b*m^2*x^2))/(x^3*(m + 3*m^2 - m^3 - 3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.48

$$\int \left(a+\frac{b}{x^2}\right)^2 (cx)^m \, dx = \frac{x^m c^m (a^2m^2x^4 - 4a^2m\,x^4 + 3a^2x^4 + 2ab\,m^2x^2 - 4abm\,x^2 - 6ab\,x^2 + b^2m^2 - b^2)}{x^3\,(m^3 - 3m^2 - m + 3)}$$

input
```
int((a+b/x^2)^2*(c*x)^m,x)
```

output
```
(x**m*c**m*(a**2*m**2*x**4 - 4*a**2*m*x**4 + 3*a**2*x**4 + 2*a*b*m**2*x**2
 - 4*a*b*m*x**2 - 6*a*b*x**2 + b**2*m**2 - b**2))/(x**3*(m**3 - 3*m**2 - m
 + 3))
```

3.658 $\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m \, dx = -\frac{b^2c^5(cx)^{-5+m}}{5-m} - \frac{2abc^2(cx)^{-2+m}}{2-m} + \frac{a^2(cx)^{1+m}}{c(1+m)}$$

output
```
-b^2*c^5*(c*x)^(-5+m)/(5-m)-2*a*b*c^2*(c*x)^(-2+m)/(2-m)+a^2*(c*x)^(1+m)/c
/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m \, dx = \frac{(cx)^m \left(b^2(-2-m+m^2)+2ab(-5-4m+m^2)\,x^3+a^2(10-7m+m^2)\,x^6\right)}{(-5+m)(-2+m)(1+m)x^5}$$

input
```
Integrate[(a + b/x^3)^2*(c*x)^m,x]
```

output
```
((c*x)^m*(b^2*(-2 - m + m^2) + 2*a*b*(-5 - 4*m + m^2)*x^3 + a^2*(10 - 7*m
+ m^2)*x^6))/((-5 + m)*(-2 + m)*(1 + m)*x^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2 (cx)^m+2abc^3 (cx)^{m-3}+b^2 c^6 (cx)^{m-6}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 (cx)^{m+1}}{c(m+1)}-\frac{2abc^2 (cx)^{m-2}}{2-m}-\frac{b^2 c^5 (cx)^{m-5}}{5-m}$$

input
```
Int[(a + b/x^3)^2*(c*x)^m,x]
```

output
```
-((b^2*c^5*(c*x)^(-5 + m))/(5 - m)) - (2*a*b*c^2*(c*x)^(-2 + m))/(2 - m) +
 (a^2*(c*x)^(1 + m))/(c*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
norman	$\frac{\frac{b^{2} e^{m \ln(cx)}}{-5+m}+\frac{a^{2} x^{6} e^{m \ln(cx)}}{1+m}+\frac{2 a b \,x^{3} e^{m \ln(cx)}}{-2+m}}{x^{5}}$
gosper	$\frac{(cx)^{m} (a^{2} m^{2} x^{6}-7 a^{2} m \,x^{6}+10 a^{2} x^{6}+2 a b \,m^{2} x^{3}-8 m \,x^{3} a b-10 a b \,x^{3}+b^{2} m^{2}-b^{2} m-2 b^{2})}{x^{5} (1+m) (-2+m) (-5+m)}$
risch	$\frac{(cx)^{m} (a^{2} m^{2} x^{6}-7 a^{2} m \,x^{6}+10 a^{2} x^{6}+2 a b \,m^{2} x^{3}-8 m \,x^{3} a b-10 a b \,x^{3}+b^{2} m^{2}-b^{2} m-2 b^{2})}{x^{5} (1+m) (-2+m) (-5+m)}$
orering	$\frac{(a^{2} m^{2} x^{6}-7 a^{2} m \,x^{6}+10 a^{2} x^{6}+2 a b \,m^{2} x^{3}-8 m \,x^{3} a b-10 a b \,x^{3}+b^{2} m^{2}-b^{2} m-2 b^{2}) x \left(a+\frac{b}{x^{3}}\right)^{2} (cx)^{m}}{(1+m) (-2+m) (-5+m) (a \,x^{3}+b)^{2}}$
parallelrisch	$\frac{x^{6} (cx)^{m} a^{2} m^{2}-7 x^{6} (cx)^{m} a^{2} m+10 x^{6} (cx)^{m} a^{2}+2 x^{3} (cx)^{m} a b \,m^{2}-8 x^{3} (cx)^{m} a b m-10 x^{3} (cx)^{m} a b+(cx)^{m} b^{2} m^{2}-(cx)^{m} b^{2} m-2 b}{x^{5} (1+m) (-2+m) (-5+m)}$

input
```
int((a+b/x^3)^2*(c*x)^m,x,method=_RETURNVERBOSE)
```

output
```
(b^2/(-5+m)*exp(m*ln(c*x))+a^2/(1+m)*x^6*exp(m*ln(c*x))+2*a*b/(-2+m)*x^3*e
xp(m*ln(c*x)))/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m \, dx = \frac{((a^2m^2 - 7\,a^2m + 10\,a^2)x^6 + b^2m^2 + 2\,(abm^2 - 4\,abm - 5\,ab)x^3 - b^2m - 2\,b^2)(cx)^m}{(m^3 - 6\,m^2 + 3\,m + 10)x^5}$$

input
```
integrate((a+b/x^3)^2*(c*x)^m,x, algorithm="fricas")
```

output
```
((a^2*m^2 - 7*a^2*m + 10*a^2)*x^6 + b^2*m^2 + 2*(a*b*m^2 - 4*a*b*m - 5*a*b
)*x^3 - b^2*m - 2*b^2)*(c*x)^m/((m^3 - 6*m^2 + 3*m + 10)*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(49) = 98$.

Time = 0.49 (sec) , antiderivative size = 449, normalized size of antiderivative = 7.13

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m\,dx$$
$$= \begin{cases} \frac{a^2\log(x)-\frac{2ab}{3x^3}-\frac{b^2}{6x^6}}{c} \\ c^2\left(\frac{a^2x^3}{3}+2ab\log(x)-\frac{b^2}{3x^3}\right) \\ c^5\left(\frac{a^2x^6}{6}+\frac{2abx^3}{3}+b^2\log(x)\right) \\ \frac{a^2m^2x^6(cx)^m}{m^3x^5-6m^2x^5+3mx^5+10x^5}-\frac{7a^2mx^6(cx)^m}{m^3x^5-6m^2x^5+3mx^5+10x^5}+\frac{10a^2x^6(cx)^m}{m^3x^5-6m^2x^5+3mx^5+10x^5}+\frac{2abm^2x^3(cx)^m}{m^3x^5-6m^2x^5+3mx^5+10x^5}-\frac{}{m^3x^5} \end{cases}$$

input
```
integrate((a+b/x**3)**2*(c*x)**m,x)
```

output
```
Piecewise(((a**2*log(x) - 2*a*b/(3*x**3) - b**2/(6*x**6))/c, Eq(m, -1)), (
c**2*(a**2*x**3/3 + 2*a*b*log(x) - b**2/(3*x**3)), Eq(m, 2)), (c**5*(a**2*
x**6/6 + 2*a*b*x**3/3 + b**2*log(x)), Eq(m, 5)), (a**2*m**2*x**6*(c*x)**m/
(m**3*x**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) - 7*a**2*m*x**6*(c*x)**m/(m
**3*x**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) + 10*a**2*x**6*(c*x)**m/(m**3
*x**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) + 2*a*b*m**2*x**3*(c*x)**m/(m**3
*x**5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) - 8*a*b*m*x**3*(c*x)**m/(m**3*x*
*5 - 6*m**2*x**5 + 3*m*x**5 + 10*x**5) - 10*a*b*x**3*(c*x)**m/(m**3*x**5 -
 6*m**2*x**5 + 3*m*x**5 + 10*x**5) + b**2*m**2*(c*x)**m/(m**3*x**5 - 6*m**
2*x**5 + 3*m*x**5 + 10*x**5) - b**2*m*(c*x)**m/(m**3*x**5 - 6*m**2*x**5 +
3*m*x**5 + 10*x**5) - 2*b**2*(c*x)**m/(m**3*x**5 - 6*m**2*x**5 + 3*m*x**5
+ 10*x**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m\,dx=\frac{(cx)^{m+1}\,a^2}{c(m+1)}+\frac{2\,abc^mx^m}{(m-2)x^2}+\frac{b^2c^mx^m}{(m-5)x^5}$$

input
```
integrate((a+b/x^3)^2*(c*x)^m,x, algorithm="maxima")
```

output
```
(c*x)^(m + 1)*a^2/(c*(m + 1)) + 2*a*b*c^m*x^m/((m - 2)*x^2) + b^2*c^m*x^m/
((m - 5)*x^5)
```

Giac [F]

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m\,dx=\int (cx)^m\left(a+\frac{b}{x^3}\right)^2\,dx$$

input
```
integrate((a+b/x^3)^2*(c*x)^m,x, algorithm="giac")
```

output
```
integrate((c*x)^m*(a + b/x^3)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m\,dx= -\frac{(c\,x)^m\,\left(-a^2\,m^2\,x^6+7\,a^2\,m\,x^6-10\,a^2\,x^6-2\,a\,b\,m^2\,x^3+8\,a\,b\,m\,x^3+10\,a\,b\,x^3-b^2\,m^2+b^2\,m+2\,b^2\right)}{x^5\,\left(m^3-6\,m^2+3\,m+10\right)}$$

input
```
int((c*x)^m*(a + b/x^3)^2,x)
```

output
```
-((c*x)^m*(b^2*m + 2*b^2 - b^2*m^2 - 10*a^2*x^6 + 7*a^2*m*x^6 - a^2*m^2*x^
6 + 10*a*b*x^3 + 8*a*b*m*x^3 - 2*a*b*m^2*x^3))/(x^5*(3*m - 6*m^2 + m^3 + 1
0))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int \left(a+\frac{b}{x^3}\right)^2 (cx)^m \, dx = \frac{x^m c^m (a^2 m^2 x^6 - 7a^2 m \, x^6 + 10a^2 x^6 + 2ab \, m^2 x^3 - 8abm \, x^3 - 10ab \, x^3 + b^2 m^2 - b^2 m - 2b^2)}{x^5 \left(m^3 - 6m^2 + 3m + 10\right)}$$

input
```
int((a+b/x^3)^2*(c*x)^m,x)
```

output
```
(x**m*c**m*(a**2*m**2*x**6 - 7*a**2*m*x**6 + 10*a**2*x**6 + 2*a*b*m**2*x**
3 - 8*a*b*m*x**3 - 10*a*b*x**3 + b**2*m**2 - b**2*m - 2*b**2))/(x**5*(m**3
 - 6*m**2 + 3*m + 10))
```

3.659 $\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 222

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = -\frac{3(cx)^{-2n/3}}{2acn} + \frac{\sqrt{3}b^{2/3}x^{2n/3}(cx)^{-2n/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}cn} - \frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{a^{5/3}cn} + \frac{b^{2/3}x^{2n/3}(cx)^{-2n/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2a^{5/3}cn}$$

output

```
-3/2/a/c/n/((c*x)^(2/3*n))+3^(1/2)*b^(2/3)*x^(2/3*n)*arctan(1/3*(a^(1/3)-2
*b^(1/3)*x^(1/3*n))*3^(1/2)/a^(1/3))/a^(5/3)/c/n/((c*x)^(2/3*n))-b^(2/3)*x
^(2/3*n)*ln(a^(1/3)+b^(1/3)*x^(1/3*n))/a^(5/3)/c/n/((c*x)^(2/3*n))+1/2*b^(
2/3)*x^(2/3*n)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x^(1/3*n)+b^(2/3)*x^(2/3*n))/a^(
5/3)/c/n/((c*x)^(2/3*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.18

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = -\frac{3x(cx)^{-1-\frac{2n}{3}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{1}{3},-\frac{bx^n}{a}\right)}{2an}$$

input
```
Integrate[(c*x)^(-1 - (2*n)/3)/(a + b*x^n),x]
```

output
```
(-3*x*(c*x)^(-1 - (2*n)/3)*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^n)/a)])/
(2*a*n)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.81, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {887, 886, 868, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{2n}{3}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \; 887$$

$$\frac{x^{2n/3}(cx)^{-2n/3}\int \frac{x^{-\frac{2n}{3}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \; 886$$

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{b\int \frac{x^{\frac{n-3}{3}}}{bx^n+a}dx}{a}-\frac{3x^{-2n/3}}{2an}\right)}{c}$$

$$\Big\downarrow \; 868$$

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\int \frac{1}{bx^n+a}dx^{n/3}}{an}-\frac{3x^{-2n/3}}{2an}\right)}{c}$$

$\Big\downarrow$ 750

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x^{n/3}+\sqrt[3]{a}}dx^{n/3}}{3a^{2/3}}\right)}{an}-\frac{3x^{-2n/3}}{2an}\right)}{c}$$

$\Big\downarrow$ 16

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}\right)}{c}$$

$\Big\downarrow$ 1142

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right.}{an}\right.}{c}$$

$\Big\downarrow$ 25

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}\right.}{c}$$

$\Big\downarrow$ 27

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right.}{an}\right.}{c}$$

$\Big\downarrow$ 1082

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}+\frac{3\int\frac{1}{-x^{2n/3}-3}d\left(1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}\right.}{c}$$

$\Big\downarrow$ 217

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x^{n/3}}{-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}+a^{2/3}}dx^{n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}\right)}{c}$$

$\downarrow$ 1103

$$\frac{x^{2n/3}(cx)^{-2n/3}\left(-\frac{3b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^{n/3}+b^{2/3}x^{2n/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{n/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{n/3}\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{an}-\frac{3x^{-2n/3}}{2an}\right)}{c}$$

input
```
Int[(c*x)^(-1 - (2*n)/3)/(a + b*x^n),x]
```

output
```
(x^((2*n)/3)*(-3/(2*a*n*x^((2*n)/3)) - (3*b*(Log[a^(1/3) + b^(1/3)*x^(n/3)
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(n/3))/a^(1/3
))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(n/3) + b^(2/3)*x^
((2*n)/3)]/(2*b^(1/3)))/(3*a^(2/3))))/(a*n)))/(c*(c*x)^((2*n)/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 887
```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[c^IntPart[
m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m/(a + b*x^n), x], x] /; FreeQ
[{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n
] || SumSimplerQ[m, -n])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+b\,x^n}dx$$

input
```
int((c*x)^(-1-2/3*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-2/3*n)/(a+b*x^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.48

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = -\frac{3\,xe^{\left(-\frac{1}{3}\,(2\,n+3)\log(c)-\frac{1}{3}\,(2\,n+3)\log(x)\right)} - 2\,\sqrt{3}\left(-\frac{b^2c^{-2\,n-3}}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}a\left(-\frac{b^2c^{-2\,n-3}}{a^2}\right)^{\frac{1}{3}}\sqrt{x}e^{\left(-\frac{1}{6}\,(2\,n+3)\log(c)-\frac{1}{6}\,(2\,n-}}{3\,bc^{-n-\frac{3}{2}}}}{}$$

input
```
integrate((c*x)^(-1-2/3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/2*(3*x*e^(-1/3*(2*n + 3)*log(c) - 1/3*(2*n + 3)*log(x)) - 2*sqrt(3)*(-b
^2*c^(-2*n - 3)/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*(-b^2*c^(-2*n - 3)/a^2)
^(1/3)*sqrt(x)*e^(-1/6*(2*n + 3)*log(c) - 1/6*(2*n + 3)*log(x)) + sqrt(3)*
b*c^(-n - 3/2))/(b*c^(-n - 3/2))) - 2*(-b^2*c^(-2*n - 3)/a^2)^(1/3)*log((b
*c^(-n - 3/2)*x*e^(-1/6*(2*n + 3)*log(c) - 1/6*(2*n + 3)*log(x)) + a*(-b^2
*c^(-2*n - 3)/a^2)^(2/3)*sqrt(x))/x) + (-b^2*c^(-2*n - 3)/a^2)^(1/3)*log((
b*c^(-n - 3/2)*x*e^(-1/3*(2*n + 3)*log(c) - 1/3*(2*n + 3)*log(x)) - a*(-b^
2*c^(-2*n - 3)/a^2)^(2/3)*sqrt(x)*e^(-1/6*(2*n + 3)*log(c) - 1/6*(2*n + 3)
*log(x)) - b*c^(-n - 3/2)*(-b^2*c^(-2*n - 3)/a^2)^(1/3))/x))/(a*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.05

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \frac{c^{-\frac{2n}{3}-1}x^{-\frac{2n}{3}}\Gamma\left(-\frac{2}{3}\right)}{an\Gamma\left(\frac{1}{3}\right)} - \frac{2b^{\frac{2}{3}}c^{-\frac{2n}{3}-1}e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)} + \frac{2b^{\frac{2}{3}}c^{-\frac{2n}{3}-1}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)} - \frac{2b^{\frac{2}{3}}c^{-\frac{2n}{3}-1}e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{2}{3}\right)}{3a^{\frac{5}{3}}n\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((c*x)**(-1-2/3*n)/(a+b*x**n),x)
```

output
```
c**(-2*n/3 - 1)*gamma(-2/3)/(a*n*x**(2*n/3)*gamma(1/3)) - 2*b**(2/3)*c**(-
2*n/3 - 1)*exp(-I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/
3))*gamma(-2/3)/(3*a**(5/3)*n*gamma(1/3)) + 2*b**(2/3)*c**(-2*n/3 - 1)*log
(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*n
*gamma(1/3)) - 2*b**(2/3)*c**(-2*n/3 - 1)*exp(I*pi/3)*log(1 - b**(1/3)*x**
(n/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(-2/3)/(3*a**(5/3)*n*gamma(1/3))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{2}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-2/3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(1/3*n)/(a*b*c^(2/3*n + 1)*x*x^n + a^2*c^(2/3*n + 1)*x), x)
 - 3/2*c^(-2/3*n - 1)/(a*n*x^(2/3*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{2}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-2/3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-2/3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \int \frac{1}{(c\,x)^{\frac{2\,n}{3}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/((c*x)^((2*n)/3 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^((2*n)/3 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{2n}{3}}}{a+bx^n}\,dx = \frac{\int \frac{1}{x^{\frac{5n}{3}}bx+x^{\frac{2n}{3}}ax}dx}{c^{\frac{2n}{3}}c}$$

input
```
int((c*x)^(-1-2/3*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((5*n)/3)*b*x + x**((2*n)/3)*a*x),x)/(c**((2*n)/3)*c)
```

3.660 $\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 237

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n} \, dx = -\frac{4(cx)^{-3n/4}}{3acn} + \frac{\sqrt{2}b^{3/4}x^{3n/4}(cx)^{-3n/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{a^{7/4}cn} - \frac{\sqrt{2}b^{3/4}x^{3n/4}(cx)^{-3n/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{a^{7/4}cn} - \frac{\sqrt{2}b^{3/4}x^{3n/4}(cx)^{-3n/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{n/4}}{\sqrt{a}+\sqrt{b}x^{n/2}}\right)}{a^{7/4}cn}$$

output
```
-4/3/a/c/n/((c*x)^(3/4*n))-2^(1/2)*b^(3/4)*x^(3/4*n)*arctan(-1+2^(1/2)*b^(
1/4)*x^(1/4*n)/a^(1/4))/a^(7/4)/c/n/((c*x)^(3/4*n))-2^(1/2)*b^(3/4)*x^(3/4
*n)*arctan(1+2^(1/2)*b^(1/4)*x^(1/4*n)/a^(1/4))/a^(7/4)/c/n/((c*x)^(3/4*n)
)-2^(1/2)*b^(3/4)*x^(3/4*n)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/4*n)/(a^(
1/2)+b^(1/2)*x^(1/2*n)))/a^(7/4)/c/n/((c*x)^(3/4*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = -\frac{4x(cx)^{-1-\frac{3n}{4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{bx^n}{a}\right)}{3an}$$

input
```
Integrate[(c*x)^(-1 - (3*n)/4)/(a + b*x^n),x]
```

output
```
(-4*x*(c*x)^(-1 - (3*n)/4)*Hypergeometric2F1[-3/4, 1, 1/4, -((b*x^n)/a)])/
(3*a*n)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {887, 886, 868, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{3n}{4}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{887}$$

$$\frac{x^{3n/4}(cx)^{-3n/4}\int \frac{x^{-\frac{3n}{4}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{886}$$

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{b\int \frac{x^{\frac{n-4}{4}}}{bx^n+a}dx}{a}-\frac{4x^{-3n/4}}{3an}\right)}{c}$$

$$\Big\downarrow \text{868}$$

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\int \frac{1}{bx^n+a}dx^{n/4}}{an}-\frac{4x^{-3n/4}}{3an}\right)}{c}$$

$\Big\downarrow$ 755

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^{n/2}+\sqrt{a}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}\right)}{c}$$

$\Big\downarrow$ 1476

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{\frac{\int \frac{1}{-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}\right)}{c}$$

$\Big\downarrow$ 1082

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{\frac{\int \frac{1}{-x^{n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x^{n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}\right)}{c}$$

$\Big\downarrow$ 217

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^{n/2}}{bx^n+a}dx^{n/4}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}-\frac{4x^{-3n/4}}{3an}\right)}{c}$$

$\Big\downarrow$ 1479

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x^{n/4}}{\sqrt[4]{b}\left(-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{n/4}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots}{2\cdots}\cdots\right)}{an}\cdots\right)}{c}$$

$\Big\downarrow$ 25

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x^{n/4}}{\sqrt[4]{b}\left(-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x^{n/4}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\cdots}{2\sqrt{a}}\right)}{an}\right.}{c}$$

$\downarrow$ 27

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x^{n/4}}{-\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}+\sqrt[4]{a}}{\frac{\sqrt{2}\sqrt[4]{a}x^{n/4}}{\sqrt[4]{b}}+x^{n/2}+\frac{\sqrt{a}}{\sqrt{b}}}dx^{n/4}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}\right.}{c}$$

$\downarrow$ 1103

$$\frac{x^{3n/4}(cx)^{-3n/4}\left(-\frac{4b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x^{n/4}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{n/4}+\sqrt{a}+\sqrt{b}x^{n/2}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{n/4}+\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{an}\cdots\right)}{c}$$

input
```
Int[(c*x)^(-1 - (3*n)/4)/(a + b*x^n),x]
```

output
```
(x^((3*n)/4)*(-4/(3*a*n*x^((3*n)/4)) - (4*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4
)*x^(n/4))/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/
4)*x^(n/4))/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x^(n/4) + Sqrt[b]*x^(n/2)]/(Sqrt[2]*a^(1/4
)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x^(n/4) + Sqrt[b]*x^(n/
2)]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(a*n)))/(c*(c*x)^((3*n)/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 887
```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[c^IntPart[
m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m/(a + b*x^n), x], x] /; FreeQ
[{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n
] || SumSimplerQ[m, -n])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(-1-3/4*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-3/4*n)/(a+b*x^n),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.70

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^{n}}\,dx = \frac{3\,an\left(-\frac{b^{3}c^{-3\,n-4}}{a^{7}n^{4}}\right)^{\frac{1}{4}}\log\left(\frac{a^{5}n^{3}x^{\frac{2}{3}}\left(-\frac{b^{3}c^{-3\,n-4}}{a^{7}n^{4}}\right)^{\frac{3}{4}}+b^{2}c^{-2\,n-\frac{8}{3}}xe^{\left(-\frac{1}{12}\,(3\,n+4)\log(c)-\frac{1}{12}\,(3\,n+4)\log(x)\right)}}{x}\right)-3\,an\left(-\frac{b^{3}c^{-3\,n-4}}{a^{7}n^{4}}\right)^{\frac{1}{4}}}{}$$

input
```
integrate((c*x)^(-1-3/4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/3*(3*a*n*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(1/4)*log((a^5*n^3*x^(2/3)*(-b^3*
c^(-3*n - 4)/(a^7*n^4))^(3/4) + b^2*c^(-2*n - 8/3)*x*e^(-1/12*(3*n + 4)*lo
g(c) - 1/12*(3*n + 4)*log(x)))/x) - 3*a*n*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(1
/4)*log(-(a^5*n^3*x^(2/3)*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(3/4) - b^2*c^(-2*
n - 8/3)*x*e^(-1/12*(3*n + 4)*log(c) - 1/12*(3*n + 4)*log(x)))/x) - 3*I*a*
n*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(1/4)*log((I*a^5*n^3*x^(2/3)*(-b^3*c^(-3*n
 - 4)/(a^7*n^4))^(3/4) + b^2*c^(-2*n - 8/3)*x*e^(-1/12*(3*n + 4)*log(c) -
1/12*(3*n + 4)*log(x)))/x) + 3*I*a*n*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(1/4)*l
og((-I*a^5*n^3*x^(2/3)*(-b^3*c^(-3*n - 4)/(a^7*n^4))^(3/4) + b^2*c^(-2*n -
 8/3)*x*e^(-1/12*(3*n + 4)*log(c) - 1/12*(3*n + 4)*log(x)))/x) - 4*x*e^(-1
/4*(3*n + 4)*log(c) - 1/4*(3*n + 4)*log(x)))/(a*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.34

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \frac{c^{-\frac{3n}{4}-1}x^{-\frac{3n}{4}}\Gamma\left(-\frac{3}{4}\right)}{an\Gamma\left(\frac{1}{4}\right)} - \frac{3b^{\frac{3}{4}}c^{-\frac{3n}{4}-1}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} + \frac{3ib^{\frac{3}{4}}c^{-\frac{3n}{4}-1}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} + \frac{3b^{\frac{3}{4}}c^{-\frac{3n}{4}-1}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{5i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)} - \frac{3ib^{\frac{3}{4}}c^{-\frac{3n}{4}-1}e^{-\frac{i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{7i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{3}{4}\right)}{4a^{\frac{7}{4}}n\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((c*x)**(-1-3/4*n)/(a+b*x**n),x)
```

output
```
c**(-3*n/4 - 1)*gamma(-3/4)/(a*n*x**(3*n/4)*gamma(1/4)) - 3*b**(3/4)*c**(-
3*n/4 - 1)*exp(-I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/
4))*gamma(-3/4)/(4*a**(7/4)*n*gamma(1/4)) + 3*I*b**(3/4)*c**(-3*n/4 - 1)*e
xp(-I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(
-3/4)/(4*a**(7/4)*n*gamma(1/4)) + 3*b**(3/4)*c**(-3*n/4 - 1)*exp(-I*pi/4)*
log(1 - b**(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**
(7/4)*n*gamma(1/4)) - 3*I*b**(3/4)*c**(-3*n/4 - 1)*exp(-I*pi/4)*log(1 - b*
*(1/4)*x**(n/4)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-3/4)/(4*a**(7/4)*n*ga
mma(1/4))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{3}{4}n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-3/4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(1/4*n)/(a*b*c^(3/4*n + 1)*x*x^n + a^2*c^(3/4*n + 1)*x), x)
 - 4/3*c^(-3/4*n - 1)/(a*n*x^(3/4*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{3}{4}n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-3/4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-3/4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx=\int \frac{1}{{\left(c\,x\right)}^{\frac{3\,n}{4}+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/((c*x)^((3*n)/4 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^((3*n)/4 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{3n}{4}}}{a+bx^n}\,dx=\frac{\int \frac{1}{x^{\frac{7n}{4}}bx+x^{\frac{3n}{4}}ax}dx}{c^{\frac{3n}{4}}c}$$

input
```
int((c*x)^(-1-3/4*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((7*n)/4)*b*x + x**((3*n)/4)*a*x),x)/(c**((3*n)/4)*c)
```

3.661 $\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx=-\frac{(cx)^{-n}}{acn}-\frac{bx^n(cx)^{-n}\log(x)}{a^2c}+\frac{bx^n(cx)^{-n}\log(a+bx^n)}{a^2cn}$$

output
```
-1/a/c/n/((c*x)^n)-b*x^n*ln(x)/a^2/c/((c*x)^n)+b*x^n*ln(a+b*x^n)/a^2/c/n/(
(c*x)^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx=-\frac{(cx)^{-n}\left(a+bx^n\log(x^n)-bx^n\log(a+bx^n)\right)}{a^2cn}$$

input
```
Integrate[(c*x)^(-1 - n)/(a + b*x^n),x]
```

output
```
-((a + b*x^n*Log[x^n] - b*x^n*Log[a + b*x^n])/(a^2*c*n*(c*x)^n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {800, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-n-1}}{a+bx^n} \, dx$$

$$\Big\downarrow \text{ 800}$$

$$\frac{x^n (cx)^{-n} \int \frac{x^{-n-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{x^n (cx)^{-n} \int \frac{x^{-2n}}{bx^n+a}dx^n}{cn}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{x^n (cx)^{-n} \int \left(\frac{x^{-2n}}{a} - \frac{bx^{-n}}{a^2} + \frac{b^2}{a^2(bx^n+a)}\right) dx^n}{cn}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^n (cx)^{-n} \left(-\frac{b \log(x^n)}{a^2} + \frac{b \log(a+bx^n)}{a^2} - \frac{x^{-n}}{a}\right)}{cn}$$

input
```
Int[(c*x)^(-1 - n)/(a + b*x^n),x]
```

output
```
(x^n*(-(1/(a*x^n)) - (b*Log[x^n])/a^2 + (b*Log[a + b*x^n])/a^2))/(c*n*(c*x
)^n)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(cx)^{-1-n}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(-1-n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-n)/(a+b*x^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx = -\frac{bc^{-n-1}nx^n\log{(x)} - bc^{-n-1}x^n\log{(bx^n+a)} + ac^{-n-1}}{a^2nx^n}$$

input
```
integrate((c*x)^(-1-n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-(b*c^(-n - 1)*n*x^n*log(x) - b*c^(-n - 1)*x^n*log(b*x^n + a) + a*c^(-n -
1))/(a^2*n*x^n)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx = -\frac{c^{-n-1}x^{-n}}{an} + \frac{bc^{-n-1}\log\left(\frac{ax^{-n}}{b}+1\right)}{a^2n}$$

input
```
integrate((c*x)**(-1-n)/(a+b*x**n),x)
```

output
```
-c**(-n - 1)/(a*n*x**n) + b*c**(-n - 1)*log(a/(b*x**n) + 1)/(a**2*n)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx = -\frac{bc^{-n-1}\log(x)}{a^2} + \frac{bc^{-n-1}\log\left(\frac{bx^n+a}{b}\right)}{a^2n} - \frac{c^{-n-1}}{anx^n}$$

input
```
integrate((c*x)^(-1-n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*c^(-n - 1)*log(x)/a^2 + b*c^(-n - 1)*log((b*x^n + a)/b)/(a^2*n) - c^(-n
 - 1)/(a*n*x^n)
```

Giac [F]

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx = \int \frac{(cx)^{-n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx = \int \frac{1}{(c\,x)^{n+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/((c*x)^(n + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^(n + 1)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{(cx)^{-1-n}}{a+bx^n}\,dx = \frac{x^n \log(x^n b+a)\,b - x^n \log(x)\,bn - a}{x^n c^n a^2 cn}$$

input
```
int((c*x)^(-1-n)/(a+b*x^n),x)
```

output
```
(x**n*log(x**n*b + a)*b - x**n*log(x)*b*n - a)/(x**n*c**n*a**2*c*n)
```

3.662 $\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n} dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n} dx = -\frac{2(cx)^{-n/2}}{acn} + \frac{2\sqrt{b}x^{n/2}(cx)^{-n/2}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}cn}$$

output
```
-2/a/c/n/((c*x)^(1/2*n))+2*b^(1/2)*x^(1/2*n)*arctan(a^(1/2)/b^(1/2)/(x^(1/
2*n)))/a^(3/2)/c/n/((c*x)^(1/2*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n} dx = -\frac{2x(cx)^{-1-\frac{n}{2}} \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[(c*x)^(-1 - n/2)/(a + b*x^n),x]
```

output
```
(-2*x*(c*x)^(-1 - n/2)*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^n)/a)])/(a*n
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {870, 868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{n}{2}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{870}$$

$$\frac{x^{n/2}(cx)^{-n/2}\int \frac{x^{-\frac{n}{2}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{868}$$

$$-\frac{2x^{n/2}(cx)^{-n/2}\int \frac{1}{bx^n+a}dx^{-n/2}}{cn}$$

$$\Big\downarrow \text{772}$$

$$-\frac{2x^{n/2}(cx)^{-n/2}\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/2}}{cn}$$

$$\Big\downarrow \text{262}$$

$$-\frac{2x^{n/2}(cx)^{-n/2}\left(\frac{x^{-n/2}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/2}}{a}\right)}{cn}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2x^{n/2}(cx)^{-n/2}\left(\frac{x^{-n/2}}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{cn}$$

input
```
Int[(c*x)^(-1 - n/2)/(a + b*x^n),x]
```

output
```
(-2*x^(n/2)*(1/(a*x^(n/2)) - (Sqrt[b]*ArcTan[Sqrt[a]/(Sqrt[b]*x^(n/2))])/a
^(3/2)))/(c*n*(c*x)^(n/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 870
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(-1-1/2*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-1/2*n)/(a+b*x^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.20

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \left[-\frac{2\,xe^{\left(-\frac{1}{2}\,(n+2)\log(c)-\frac{1}{2}\,(n+2)\log(x)\right)}-\sqrt{-\frac{bc^{-n-2}}{a}}\log\left(\frac{ax^2e^{\left(-(n+2)\log(c)-(n+2)\log(x)\right)}+2\,a\sqrt{-\frac{bc^{-n-2}}{a}}xe^{\left(-\frac{1}{2}\,(n+2)\log(c)-\frac{1}{2}\right.}}{ax^2e^{\left(-(n+2)\log(c)-(n+2)\log(x)\right)}+bc^{-n-2}}\right.}{an}, -\frac{2\left(xe^{\left(-\frac{1}{2}\,(n+2)\log(c)-\frac{1}{2}\,(n+2)\log(x)\right)}-\sqrt{\frac{bc^{-n-2}}{a}}\arctan\left(\frac{a\sqrt{\frac{bc^{-n-2}}{a}}xe^{\left(-\frac{1}{2}\,(n+2)\log(c)-\frac{1}{2}\,(n+2)\log(x)\right)}}{bc^{-n-2}}\right)\right)}{an}\right]$$

input
```
integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
[-(2*x*e^(-1/2*(n + 2)*log(c) - 1/2*(n + 2)*log(x)) - sqrt(-b*c^(-n - 2)/a
)*log((a*x^2*e^(-(n + 2)*log(c) - (n + 2)*log(x)) + 2*a*sqrt(-b*c^(-n - 2)
/a)*x*e^(-1/2*(n + 2)*log(c) - 1/2*(n + 2)*log(x)) - b*c^(-n - 2))/(a*x^2*
e^(-(n + 2)*log(c) - (n + 2)*log(x)) + b*c^(-n - 2))))/(a*n), -2*(x*e^(-1/
2*(n + 2)*log(c) - 1/2*(n + 2)*log(x)) - sqrt(b*c^(-n - 2)/a)*arctan(a*sqr
t(b*c^(-n - 2)/a)*x*e^(-1/2*(n + 2)*log(c) - 1/2*(n + 2)*log(x))/(b*c^(-n
- 2))))/(a*n)]
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx = -\frac{2c^{-\frac{n}{2}-1}x^{-\frac{n}{2}}}{an} - \frac{2\sqrt{b}c^{-\frac{n}{2}-1}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

input
```
integrate((c*x)**(-1-1/2*n)/(a+b*x**n),x)
```

output
```
-2*c**(-n/2 - 1)/(a*n*x**(n/2)) - 2*sqrt(b)*c**(-n/2 - 1)*atan(sqrt(b)*x**
(n/2)/sqrt(a))/(a**(3/2)*n)
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{1}{2}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(1/2*n)/(a*b*c^(1/2*n + 1)*x*x^n + a^2*c^(1/2*n + 1)*x), x)
 - 2*c^(-1/2*n - 1)/(a*n*x^(1/2*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{1}{2}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-1/2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-1/2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \int \frac{1}{{\left(c\,x\right)}^{\frac{n}{2}+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/((c*x)^(n/2 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^(n/2 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{a+bx^n}\,dx = \frac{\int \frac{1}{x^{\frac{3n}{2}}bx+x^{\frac{n}{2}}ax}dx}{c^{\frac{n}{2}}c}$$

input
```
int((c*x)^(-1-1/2*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((3*n)/2)*b*x + x**(n/2)*a*x),x)/(c**(n/2)*c)
```

3.663 $\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n} dx$

Optimal result

Integrand size = 21, antiderivative size = 220

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n} dx = -\frac{3(cx)^{-n/3}}{acn} - \frac{\sqrt{3}\sqrt[3]{b}x^{n/3}(cx)^{-n/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{a^{4/3}cn} + \frac{\sqrt[3]{b}x^{n/3}(cx)^{-n/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x^{-n/3}\right)}{a^{4/3}cn} - \frac{\sqrt[3]{b}x^{n/3}(cx)^{-n/3}\log\left(b^{2/3}+a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}\right)}{2a^{4/3}cn}$$

output
```
-3/a/c/n/((c*x)^(1/3*n))-3^(1/2)*b^(1/3)*x^(1/3*n)*arctan(1/3*(b^(1/3)-2*a
^(1/3)/(x^(1/3*n)))*3^(1/2)/b^(1/3))/a^(4/3)/c/n/((c*x)^(1/3*n))+b^(1/3)*x
^(1/3*n)*ln(b^(1/3)+a^(1/3)/(x^(1/3*n)))/a^(4/3)/c/n/((c*x)^(1/3*n))-1/2*b
^(1/3)*x^(1/3*n)*ln(b^(2/3)+a^(2/3)/(x^(2/3*n))-a^(1/3)*b^(1/3)/(x^(1/3*n)
))/a^(4/3)/c/n/((c*x)^(1/3*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.17

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx=-\frac{3x(cx)^{-1-\frac{n}{3}}\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},-\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[(c*x)^(-1 - n/3)/(a + b*x^n),x]
```

output
```
(-3*x*(c*x)^(-1 - n/3)*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^n)/a)])/(a*n
)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.80, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {870, 868, 772, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{n}{3}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{870}$$

$$\frac{x^{n/3}(cx)^{-n/3}\int \frac{x^{-\frac{n}{3}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{868}$$

$$-\frac{3x^{n/3}(cx)^{-n/3}\int \frac{1}{bx^n+a}dx^{-n/3}}{cn}$$

$$\Big\downarrow \text{772}$$

$$-\frac{3x^{n/3}(cx)^{-n/3}\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/3}}{cn}$$

$$\Big\downarrow \text{843}$$

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\int\frac{1}{ax^{-n}+b}dx^{-n/3}}{a}\right)}{cn}$$

$\downarrow$ 750

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\int\frac{1}{\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}}dx^{-n/3}}{3b^{2/3}}\right)}{a}\right)}{cn}$$

$\downarrow$ 16

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{cn}$$

$\downarrow$ 1142

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{\cdots}\right.}{\cdots}\right)}{a}\right)}{cn}$$

$\downarrow$ 25

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{\cdots}\right.}{3\cdots}\cdots\right.}{a}\cdots\right.}{cn}$$

$$\Big\downarrow \; 27$$

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{\cdots}\right.}{\cdots}\cdots\right.}{a}\cdots\right.}{cn}$$

$$\Big\downarrow \; 1082$$

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{3\int\frac{1}{-x^{-2n/3}-3}d\left(1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\cdots\right.}{3\sqrt[3]{a}b^{2/3}}\cdots\right.}{a}\cdots\right.}{cn}$$

$$\Big\downarrow \; 217$$

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{cn}$$

$\Big\downarrow$ 1103

$$-\frac{3x^{n/3}(cx)^{-n/3}\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{cn}$$

input
```
Int[(c*x)^(-1 - n/3)/(a + b*x^n),x]
```

output
```
(-3*x^(n/3)*(1/(a*x^(n/3)) - (b*(Log[b^(1/3) + a^(1/3)/x^(n/3)]/(3*a^(1/3)
*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(1/3))/(b^(1/3)*x^(n/3)))/Sqrt[3]
])/a^(1/3)) - Log[b^(2/3) + a^(2/3)/x^((2*n)/3) - (a^(1/3)*b^(1/3))/x^(n/3
)]/(2*a^(1/3)))/(3*b^(2/3))))/a))/(c*n*(c*x)^(n/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !IntegerQ[n]
```

rule 870
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] && !Intege
rQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(-1-1/3*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-1/3*n)/(a+b*x^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx = -\frac{6\,xe^{\left(-\frac{1}{3}\,(n+3)\log(c)-\frac{1}{3}\,(n+3)\log(x)\right)} - 2\,\sqrt{3}\left(\frac{bc^{-n-3}}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}a\left(\frac{bc^{-n-3}}{a}\right)^{\frac{2}{3}}xe^{\left(-\frac{1}{3}\,(n+3)\log(c)-\frac{1}{3}\,(n+3)\log(x)\right)}-\sqrt{3}bc^{}}{3\,bc^{-n-3}}\right.}{}$$

input

```
integrate((c*x)^(-1-1/3*n)/(a+b*x^n),x, algorithm="fricas")
```

output

```
-1/2*(6*x*e^(-1/3*(n + 3)*log(c) - 1/3*(n + 3)*log(x)) - 2*sqrt(3)*(b*c^(-
n - 3)/a)^(1/3)*arctan(1/3*(2*sqrt(3)*a*(b*c^(-n - 3)/a)^(2/3)*x*e^(-1/3*(
n + 3)*log(c) - 1/3*(n + 3)*log(x)) - sqrt(3)*b*c^(-n - 3))/(b*c^(-n - 3))
) - 2*(b*c^(-n - 3)/a)^(1/3)*log((x*e^(-1/3*(n + 3)*log(c) - 1/3*(n + 3)*l
og(x)) + (b*c^(-n - 3)/a)^(1/3))/x) + (b*c^(-n - 3)/a)^(1/3)*log((x^2*e^(-
2/3*(n + 3)*log(c) - 2/3*(n + 3)*log(x)) - (b*c^(-n - 3)/a)^(1/3)*x*e^(-1/
3*(n + 3)*log(c) - 1/3*(n + 3)*log(x)) + (b*c^(-n - 3)/a)^(2/3))/x^2))/(a*
n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.01

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \frac{c^{-\frac{n}{3}-1}x^{-\frac{n}{3}}\Gamma\left(-\frac{1}{3}\right)}{an\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}c^{-\frac{n}{3}-1}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}n\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}c^{-\frac{n}{3}-1}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}n\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}c^{-\frac{n}{3}-1}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}n\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((c*x)**(-1-1/3*n)/(a+b*x**n),x)
```

output
```
c**(-n/3 - 1)*gamma(-1/3)/(a*n*x**(n/3)*gamma(2/3)) - b**(1/3)*c**(-n/3 -
1)*exp(-2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*ga
mma(-1/3)/(3*a**(4/3)*n*gamma(2/3)) - b**(1/3)*c**(-n/3 - 1)*log(1 - b**(1
/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*n*gamma(2/3
)) - b**(1/3)*c**(-n/3 - 1)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_po
lar(5*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*n*gamma(2/3))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{1}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-1/3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(2/3*n)/(a*b*c^(1/3*n + 1)*x*x^n + a^2*c^(1/3*n + 1)*x), x)
 - 3*c^(-1/3*n - 1)/(a*n*x^(1/3*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{1}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-1/3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-1/3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \int \frac{1}{(c\,x)^{\frac{n}{3}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/((c*x)^(n/3 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^(n/3 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{n}{3}}}{a+bx^n}\,dx = \frac{\int \frac{1}{x^{\frac{4n}{3}}bx+x^{\frac{n}{3}}ax}dx}{c^{\frac{n}{3}}c}$$

input
```
int((c*x)^(-1-1/3*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((4*n)/3)*b*x + x**(n/3)*a*x),x)/(c**(n/3)*c)
```

3.664 $\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n} dx$

Optimal result

Integrand size = 21, antiderivative size = 234

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n} dx = -\frac{4(cx)^{-n/4}}{acn} - \frac{\sqrt{2}\sqrt[4]{b}x^{n/4}(cx)^{-n/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{5/4}cn} + \frac{\sqrt{2}\sqrt[4]{b}x^{n/4}(cx)^{-n/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{5/4}cn} + \frac{\sqrt{2}\sqrt[4]{b}x^{n/4}(cx)^{-n/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}}{\sqrt{b}+\sqrt{a}x^{-n/2}}\right)}{a^{5/4}cn}$$

output
```
-4/a/c/n/((c*x)^(1/4*n))-2^(1/2)*b^(1/4)*x^(1/4*n)*arctan(1-2^(1/2)*a^(1/4
)/b^(1/4)/(x^(1/4*n)))/a^(5/4)/c/n/((c*x)^(1/4*n))+2^(1/2)*b^(1/4)*x^(1/4*
n)*arctan(1+2^(1/2)*a^(1/4)/b^(1/4)/(x^(1/4*n)))/a^(5/4)/c/n/((c*x)^(1/4*n
))+2^(1/2)*b^(1/4)*x^(1/4*n)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)/(x^(1/4*n))/(
b^(1/2)+a^(1/2)/(x^(1/2*n))))/a^(5/4)/c/n/((c*x)^(1/4*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx = -\frac{4x(cx)^{-1-\frac{n}{4}}\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{bx^n}{a}\right)}{an}$$

input
```
Integrate[(c*x)^(-1 - n/4)/(a + b*x^n),x]
```

output
```
(-4*x*(c*x)^(-1 - n/4)*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^n)/a)])/(a*n
)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {870, 868, 772, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{n}{4}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{870}$$

$$\frac{x^{n/4}(cx)^{-n/4}\int \frac{x^{-\frac{n}{4}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{868}$$

$$-\frac{4x^{n/4}(cx)^{-n/4}\int \frac{1}{bx^n+a}dx^{-n/4}}{cn}$$

$$\Big\downarrow \text{772}$$

$$-\frac{4x^{n/4}(cx)^{-n/4}\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/4}}{cn}$$

$$\Big\downarrow \text{843}$$

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\int\frac{1}{ax^{-n}+b}dx^{-n/4}}{a}\right)}{cn}$$

$\downarrow$ 755

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{a}x^{-n/2}+\sqrt{b}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{cn}$$

$\downarrow$ 1476

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}+\frac{\int\frac{1}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}}{2\sqrt{b}}\right)}{a}\right)}{cn}$$

$\downarrow$ 1082

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int\frac{1}{-x^{-n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x^{-n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{cn}$$

$\downarrow$ 217

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{cn}$$

$\downarrow$ 1479

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\arctan\left(\cdots\right.}{\cdots}\cdots\right)}{a}\right)}{cn}$$

$\downarrow$ 25

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots\right)}{a}\cdots\right)}{cn}$$

$$\Big\downarrow \; 27$$

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{2}\sqrt{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\cdots}{2\sqrt{\cdots}}\cdots\right)}{a}\cdots\right)}{cn}$$

$$\Big\downarrow \; 1103$$

$$-\frac{4x^{n/4}(cx)^{-n/4}\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}+\sqrt{a}x^{-n/2}+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\right.}{}}{2\sqrt{b}}\right.}{a}\right.}{cn}$$

input
```
Int[(c*x)^(-1 - n/4)/(a + b*x^n),x]
```

output
```
(-4*x^(n/4)*(1/(a*x^(n/4)) - (b*((-(ArcTan[1 - (Sqrt[2]*a^(1/4))/(b^(1/4)*
x^(n/4))]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*a^(1/4))/(b^(1/
4)*x^(n/4))]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) + (-1/2*Log[Sqrt[b] +
Sqrt[a]/x^(n/2) - (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)]/(Sqrt[2]*a^(1/4)*b^(1
/4)) + Log[Sqrt[b] + Sqrt[a]/x^(n/2) + (Sqrt[2]*a^(1/4)*b^(1/4))/x^(n/4)]/
(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/(c*n*(c*x)^(n/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 870
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Intege
rQ[n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+b\,x^n}dx$$

input
```
int((c*x)^(-1-1/4*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-1/4*n)/(a+b*x^n),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.27

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx$$
$$= \frac{an\left(-\frac{bc^{-n-4}}{a^5n^4}\right)^{\frac{1}{4}}\log\left(\frac{an\left(-\frac{bc^{-n-4}}{a^5n^4}\right)^{\frac{1}{4}}+xe^{\left(-\frac{1}{4}(n+4)\log(c)-\frac{1}{4}(n+4)\log(x)\right)}}{x}\right)-an\left(-\frac{bc^{-n-4}}{a^5n^4}\right)^{\frac{1}{4}}\log\left(-\frac{an\left(-\frac{bc^{-n-4}}{a^5n^4}\right)^{\frac{1}{4}}-xe^{\left(\cdots\right.}}{\cdots}\right.}{\cdots}$$

input
```
integrate((c*x)^(-1-1/4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
(a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4)*log((a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1
/4) + x*e^(-1/4*(n + 4)*log(c) - 1/4*(n + 4)*log(x)))/x) - a*n*(-b*c^(-n -
 4)/(a^5*n^4))^(1/4)*log(-(a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4) - x*e^(-1/4
*(n + 4)*log(c) - 1/4*(n + 4)*log(x)))/x) + I*a*n*(-b*c^(-n - 4)/(a^5*n^4)
)^(1/4)*log((I*a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4) + x*e^(-1/4*(n + 4)*log
(c) - 1/4*(n + 4)*log(x)))/x) - I*a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4)*log(
(-I*a*n*(-b*c^(-n - 4)/(a^5*n^4))^(1/4) + x*e^(-1/4*(n + 4)*log(c) - 1/4*(
n + 4)*log(x)))/x) - 4*x*e^(-1/4*(n + 4)*log(c) - 1/4*(n + 4)*log(x)))/(a*
n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.32

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \frac{c^{-\frac{n}{4}-1}x^{-\frac{n}{4}}\Gamma\left(-\frac{1}{4}\right)}{an\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt[4]{b}c^{-\frac{n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)} - \frac{i\sqrt[4]{b}c^{-\frac{n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt[4]{b}c^{-\frac{n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{5i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)} + \frac{i\sqrt[4]{b}c^{-\frac{n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{7i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{1}{4}\right)}{4a^{\frac{5}{4}}n\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((c*x)**(-1-1/4*n)/(a+b*x**n),x)
```

output
```
c**(-n/4 - 1)*gamma(-1/4)/(a*n*x**(n/4)*gamma(3/4)) - b**(1/4)*c**(-n/4 -
1)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*ga
mma(-1/4)/(4*a**(5/4)*n*gamma(3/4)) - I*b**(1/4)*c**(-n/4 - 1)*exp(-3*I*pi
/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(-1/4)/(4
*a**(5/4)*n*gamma(3/4)) + b**(1/4)*c**(-n/4 - 1)*exp(-3*I*pi/4)*log(1 - b*
*(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*n*ga
mma(3/4)) + I*b**(1/4)*c**(-n/4 - 1)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n
/4)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-1/4)/(4*a**(5/4)*n*gamma(3/4))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{1}{4}n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-1/4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-b*integrate(x^(3/4*n)/(a*b*c^(1/4*n + 1)*x*x^n + a^2*c^(1/4*n + 1)*x), x)
 - 4*c^(-1/4*n - 1)/(a*n*x^(1/4*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{1}{4}n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-1/4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-1/4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \int \frac{1}{(c\,x)^{\frac{n}{4}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/((c*x)^(n/4 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^(n/4 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{n}{4}}}{a+bx^n}\,dx = \frac{\int \frac{1}{x^{\frac{5n}{4}}bx+x^{\frac{n}{4}}ax}dx}{c^{\frac{n}{4}}c}$$

input
```
int((c*x)^(-1-1/4*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((5*n)/4)*b*x + x**(n/4)*a*x),x)/(c**(n/4)*c)
```

3.665 $\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx=-\frac{2(cx)^{-3n/2}}{3acn}+\frac{2bx^n(cx)^{-3n/2}}{a^2cn}-\frac{2b^{3/2}x^{3n/2}(cx)^{-3n/2}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{5/2}cn}$$

output
```
-2/3/a/c/n/((c*x)^(3/2*n))+2*b*x^n/a^2/c/n/((c*x)^(3/2*n))-2*b^(3/2)*x^(3/
2*n)*arctan(a^(1/2)/b^(1/2)/(x^(1/2*n)))/a^(5/2)/c/n/((c*x)^(3/2*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx=-\frac{2x(cx)^{-1-\frac{3n}{2}}\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{bx^n}{a}\right)}{3an}$$

input
```
Integrate[(c*x)^(-1 - (3*n)/2)/(a + b*x^n),x]
```

output
```
(-2*x*(c*x)^(-1 - (3*n)/2)*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^n)/a)])
/(3*a*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {887, 886, 868, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{3n}{2}-1}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{887}$$

$$\frac{x^{3n/2}(cx)^{-3n/2}\int \frac{x^{-\frac{3n}{2}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{886}$$

$$\frac{x^{3n/2}(cx)^{-3n/2}\left(-\frac{b\int \frac{x^{-\frac{n}{2}-1}}{bx^n+a}dx}{a}-\frac{2x^{-3n/2}}{3an}\right)}{c}$$

$$\Big\downarrow \text{868}$$

$$\frac{x^{3n/2}(cx)^{-3n/2}\left(\frac{2b\int \frac{1}{bx^n+a}dx^{-n/2}}{an}-\frac{2x^{-3n/2}}{3an}\right)}{c}$$

$$\Big\downarrow \text{772}$$

$$\frac{x^{3n/2}(cx)^{-3n/2}\left(\frac{2b\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/2}}{an}-\frac{2x^{-3n/2}}{3an}\right)}{c}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^{3n/2}(cx)^{-3n/2}\left(\frac{2b\left(\frac{x^{-n/2}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/2}}{a}\right)}{an}-\frac{2x^{-3n/2}}{3an}\right)}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^{3n/2}(cx)^{-3n/2}\left(\frac{2b\left(\frac{x^{-n/2}}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}x^{-n/2}}{\sqrt{b}}\right)}{a^{3/2}}\right)}{an}-\frac{2x^{-3n/2}}{3an}\right)}{c}$$

input
```
Int[(c*x)^(-1 - (3*n)/2)/(a + b*x^n),x]
```

output
```
(x^((3*n)/2)*(-2/(3*a*n*x^((3*n)/2)) + (2*b*(1/(a*x^(n/2)) - (Sqrt[b]*ArcT
an[Sqrt[a]/(Sqrt[b]*x^(n/2))])/a^(3/2)))/(a*n)))/(c*(c*x)^((3*n)/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 887
```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[c^IntPart[
m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m/(a + b*x^n), x], x] /; FreeQ
[{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n
] || SumSimplerQ[m, -n])
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(-1-3/2*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-3/2*n)/(a+b*x^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 504, normalized size of antiderivative = 5.04

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)^(-1-3/2*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
[1/3*(3*b*c^(-n - 2/3)*sqrt(-b*c^(-n - 2/3)/a)*log(-(2*a^2*b*c^(-n - 2/3)*
x^(4/3)*e^(-2/3*(3*n + 2)*log(c) - 2/3*(3*n + 2)*log(x)) - a^3*x^2*e^(-(3*
n + 2)*log(c) - (3*n + 2)*log(x)) - 2*a*b^2*c^(-2*n - 4/3)*x^(2/3)*e^(-1/3
*(3*n + 2)*log(c) - 1/3*(3*n + 2)*log(x)) + b^3*c^(-3*n - 2) - 2*(a^2*b*c^
(-n - 2/3)*x*e^(-1/2*(3*n + 2)*log(c) - 1/2*(3*n + 2)*log(x)) - a^3*x^(5/3
)*e^(-5/6*(3*n + 2)*log(c) - 5/6*(3*n + 2)*log(x)) - a*b^2*c^(-2*n - 4/3)*
x^(1/3)*e^(-1/6*(3*n + 2)*log(c) - 1/6*(3*n + 2)*log(x)))*sqrt(-b*c^(-n -
2/3)/a))/(a^3*x^2*e^(-(3*n + 2)*log(c) - (3*n + 2)*log(x)) + b^3*c^(-3*n -
 2))) + 6*b*c^(-n - 2/3)*x^(1/3)*e^(-1/6*(3*n + 2)*log(c) - 1/6*(3*n + 2)*
log(x)) - 2*a*x*e^(-1/2*(3*n + 2)*log(c) - 1/2*(3*n + 2)*log(x)))/(a^2*n),
 -2/3*(3*b*c^(-n - 2/3)*sqrt(b*c^(-n - 2/3)/a)*arctan(a*sqrt(b*c^(-n - 2/3
)/a)*x^(1/3)*e^(-1/6*(3*n + 2)*log(c) - 1/6*(3*n + 2)*log(x))/(b*c^(-n - 2
/3))) - 3*b*c^(-n - 2/3)*x^(1/3)*e^(-1/6*(3*n + 2)*log(c) - 1/6*(3*n + 2)*
log(x)) + a*x*e^(-1/2*(3*n + 2)*log(c) - 1/2*(3*n + 2)*log(x)))/(a^2*n)]
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = -\frac{2c^{-\frac{3n}{2}-1}x^{-\frac{3n}{2}}}{3an} + \frac{2bc^{-\frac{3n}{2}-1}x^{-\frac{n}{2}}}{a^2n} + \frac{2b^{\frac{3}{2}}c^{-\frac{3n}{2}-1}\operatorname{atan}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

input
```
integrate((c*x)**(-1-3/2*n)/(a+b*x**n),x)
```

output
```
-2*c**(-3*n/2 - 1)/(3*a*n*x**(3*n/2)) + 2*b*c**(-3*n/2 - 1)/(a**2*n*x**(n/
2)) + 2*b**(3/2)*c**(-3*n/2 - 1)*atan(sqrt(b)*x**(n/2)/sqrt(a))/(a**(5/2)*
n)
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{3}{2}n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-3/2*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*integrate(x^(1/2*n)/(a^2*b*c^(3/2*n + 1)*x*x^n + a^3*c^(3/2*n + 1)*x),
 x) + 2/3*(3*b*x^n - a)*c^(-3/2*n - 1)/(a^2*n*x^(3/2*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{3}{2}n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-3/2*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-3/2*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \int \frac{1}{(c\,x)^{\frac{3\,n}{2}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{a+bx^n}\,dx = \frac{\int \frac{1}{x^{\frac{5n}{2}}bx+x^{\frac{3n}{2}}ax}dx}{c^{\frac{3n}{2}}c}$$

input
```
int((c*x)^(-1-3/2*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((5*n)/2)*b*x + x**((3*n)/2)*a*x),x)/(c**((3*n)/2)*c)
```

3.666 $\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 246

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = -\frac{3(cx)^{-4n/3}}{4acn}+\frac{3bx^n(cx)^{-4n/3}}{a^2cn} + \frac{\sqrt{3}b^{4/3}x^{4n/3}(cx)^{-4n/3}\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{a^{7/3}cn} - \frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(\sqrt[3]{b}+\sqrt[3]{a}x^{-n/3}\right)}{a^{7/3}cn} + \frac{b^{4/3}x^{4n/3}(cx)^{-4n/3}\log\left(b^{2/3}+a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}\right)}{2a^{7/3}cn}$$

output
```
-3/4/a/c/n/((c*x)^(4/3*n))+3*b*x^n/a^2/c/n/((c*x)^(4/3*n))+3^(1/2)*b^(4/3)
*x^(4/3*n)*arctan(1/3*(b^(1/3)-2*a^(1/3)/(x^(1/3*n)))*3^(1/2)/b^(1/3))/a^(
7/3)/c/n/((c*x)^(4/3*n))-b^(4/3)*x^(4/3*n)*ln(b^(1/3)+a^(1/3)/(x^(1/3*n)))
/a^(7/3)/c/n/((c*x)^(4/3*n))+1/2*b^(4/3)*x^(4/3*n)*ln(b^(2/3)+a^(2/3)/(x^(
2/3*n))-a^(1/3)*b^(1/3)/(x^(1/3*n)))/a^(7/3)/c/n/((c*x)^(4/3*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx=-\frac{3x(cx)^{-1-\frac{4n}{3}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},1,-\frac{1}{3},-\frac{bx^n}{a}\right)}{4an}$$

input
```
Integrate[(c*x)^(-1 - (4*n)/3)/(a + b*x^n),x]
```

output
```
(-3*x*(c*x)^(-1 - (4*n)/3)*Hypergeometric2F1[-4/3, 1, -1/3, -((b*x^n)/a)])
/(4*a*n)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.80, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {887, 886, 868, 772, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{4n}{3}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{887}$$

$$\frac{x^{4n/3}(cx)^{-4n/3}\int \frac{x^{-\frac{4n}{3}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{886}$$

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(-\frac{b\int \frac{x^{-\frac{n}{3}-1}}{bx^n+a}dx}{a}-\frac{3x^{-4n/3}}{4an}\right)}{c}$$

$$\Big\downarrow \text{868}$$

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\int \frac{1}{bx^n+a}dx^{-n/3}}{an}-\frac{3x^{-4n/3}}{4an}\right)}{c}$$

$\Big\downarrow$ 772

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/3}}{an}-\frac{3x^{-4n/3}}{4an}\right)}{c}$$

$\Big\downarrow$ 843

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/3}}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}\right)}{c}$$

$\Big\downarrow$ 750

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}}dx^{-n/3}}{3b^{2/3}}\right)}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}\right)}{c}$$

$\Big\downarrow$ 16

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}-\frac{3x^{-4n/3}}{4an}\right)}{c}$$

$$\Big\downarrow \text{1142}$$

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\int-\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{\cdots}\right.}{\cdots}\right.}{a}\right.}{an}\right.}{c}$$

$\downarrow$ 25

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{\int\frac{\sqrt[3]{a}\left(\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}\right)}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{2\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{\cdots}\right.}{3\cdots}\right.}{a}\right.}{an}\right.}{c}$$

$\downarrow$ 27

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{b}\int\frac{1}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{\cdots}\right.}{\cdots}\right.}{a}\right.}{an}\right.}{c}$$

$\downarrow$ 1082

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}+\frac{3\int\frac{1}{-x^{-2n/3}-3}d\left(1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\cdots\right.}{3\sqrt[3]{a}b^{2/3}}\right.}{a}\right.}{an}\right.}{c}$$

$\downarrow$ 217

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{b}-2\sqrt[3]{a}x^{-n/3}}{a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}}dx^{-n/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}\right)}{c}$$

$\downarrow$ 1103

$$\frac{x^{4n/3}(cx)^{-4n/3}\left(\frac{3b\left(\frac{x^{-n/3}}{a}-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}x^{-2n/3}-\sqrt[3]{a}\sqrt[3]{b}x^{-n/3}+b^{2/3}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{a}x^{-n/3}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{3b^{2/3}}+\frac{\log\left(\sqrt[3]{a}x^{-n/3}+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}\right)}{an}-\right.}{c}$$

input
```
Int[(c*x)^(-1 - (4*n)/3)/(a + b*x^n),x]
```

output
```
(x^((4*n)/3)*(-3/(4*a*n*x^((4*n)/3)) + (3*b*(1/(a*x^(n/3)) - (b*(Log[b^(1/
3) + a^(1/3)/x^(n/3)]/(3*a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*a^(
1/3))/(b^(1/3)*x^(n/3)))/Sqrt[3]])/a^(1/3)) - Log[b^(2/3) + a^(2/3)/x^((2*
n)/3) - (a^(1/3)*b^(1/3))/x^(n/3)]/(2*a^(1/3)))/(3*b^(2/3))))/a))/(a*n)))/
(c*(c*x)^((4*n)/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 887
```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[c^IntPart[
m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m/(a + b*x^n), x], x] /; FreeQ
[{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n
] || SumSimplerQ[m, -n])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+b\,x^n}dx$$

input
```
int((c*x)^(-1-4/3*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-4/3*n)/(a+b*x^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.43

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \frac{4\sqrt{3}bc^{-n-\frac{3}{4}}\left(-\frac{bc^{-n-\frac{3}{4}}}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}a\left(-\frac{bc^{-n-\frac{3}{4}}}{a}\right)^{\frac{2}{3}}x^{\frac{1}{4}}e^{\left(-\frac{1}{12}\,(4\,n+3)\log(c)-\frac{1}{12}\,(4\,n+3)\log(x)\right)}-\sqrt{3}bc^{-n-\frac{3}{4}}}{3\,bc^{-n-\frac{3}{4}}}\right)-2\,bc^{-n-\frac{3}{4}}}{}$$

input
```
integrate((c*x)^(-1-4/3*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(3)*b*c^(-n - 3/4)*(-b*c^(-n - 3/4)/a)^(1/3)*arctan(1/3*(2*sqrt
(3)*a*(-b*c^(-n - 3/4)/a)^(2/3)*x^(1/4)*e^(-1/12*(4*n + 3)*log(c) - 1/12*(
4*n + 3)*log(x)) - sqrt(3)*b*c^(-n - 3/4))/(b*c^(-n - 3/4))) - 2*b*c^(-n -
 3/4)*(-b*c^(-n - 3/4)/a)^(1/3)*log(((-b*c^(-n - 3/4)/a)^(1/3)*x^(3/4)*e^(
-1/12*(4*n + 3)*log(c) - 1/12*(4*n + 3)*log(x)) + x*e^(-1/6*(4*n + 3)*log(
c) - 1/6*(4*n + 3)*log(x)) + (-b*c^(-n - 3/4)/a)^(2/3)*sqrt(x))/x) + 4*b*c
^(-n - 3/4)*(-b*c^(-n - 3/4)/a)^(1/3)*log((x*e^(-1/12*(4*n + 3)*log(c) - 1
/12*(4*n + 3)*log(x)) - (-b*c^(-n - 3/4)/a)^(1/3)*x^(3/4))/x) + 12*b*c^(-n
 - 3/4)*x^(1/4)*e^(-1/12*(4*n + 3)*log(c) - 1/12*(4*n + 3)*log(x)) - 3*a*x
*e^(-1/3*(4*n + 3)*log(c) - 1/3*(4*n + 3)*log(x)))/(a^2*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.14

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \frac{c^{-\frac{4n}{3}-1}x^{-\frac{4n}{3}}\Gamma\left(-\frac{4}{3}\right)}{an\Gamma\left(-\frac{1}{3}\right)} - \frac{4bc^{-\frac{4n}{3}-1}x^{-\frac{n}{3}}\Gamma\left(-\frac{4}{3}\right)}{a^2n\Gamma\left(-\frac{1}{3}\right)} + \frac{4b^{\frac{4}{3}}c^{-\frac{4n}{3}-1}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)} + \frac{4b^{\frac{4}{3}}c^{-\frac{4n}{3}-1}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)} + \frac{4b^{\frac{4}{3}}c^{-\frac{4n}{3}-1}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{n}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{4}{3}\right)}{3a^{\frac{7}{3}}n\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((c*x)**(-1-4/3*n)/(a+b*x**n),x)
```

output
```
c**(-4*n/3 - 1)*gamma(-4/3)/(a*n*x**(4*n/3)*gamma(-1/3)) - 4*b*c**(-4*n/3
- 1)*gamma(-4/3)/(a**2*n*x**(n/3)*gamma(-1/3)) + 4*b**(4/3)*c**(-4*n/3 - 1
)*exp(-2*I*pi/3)*log(1 - b**(1/3)*x**(n/3)*exp_polar(I*pi/3)/a**(1/3))*gam
ma(-4/3)/(3*a**(7/3)*n*gamma(-1/3)) + 4*b**(4/3)*c**(-4*n/3 - 1)*log(1 - b
**(1/3)*x**(n/3)*exp_polar(I*pi)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*n*gamma
(-1/3)) + 4*b**(4/3)*c**(-4*n/3 - 1)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(n/
3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(-4/3)/(3*a**(7/3)*n*gamma(-1/3))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{4}{3}n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-4/3*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*integrate(x^(2/3*n)/(a^2*b*c^(4/3*n + 1)*x*x^n + a^3*c^(4/3*n + 1)*x),
 x) + 3/4*(4*b*x^n - a)*c^(-4/3*n - 1)/(a^2*n*x^(4/3*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx=\int \frac{(cx)^{-\frac{4}{3}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-4/3*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-4/3*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx=\int \frac{1}{{(c\,x)}^{\frac{4\,n}{3}+1}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int(1/((c*x)^((4*n)/3 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^((4*n)/3 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{4n}{3}}}{a+bx^n}\,dx=\frac{\int \frac{1}{x^{\frac{7n}{3}}bx+x^{\frac{4n}{3}}ax}dx}{c^{\frac{4n}{3}}c}$$

input
```
int((c*x)^(-1-4/3*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((7*n)/3)*b*x + x**((4*n)/3)*a*x),x)/(c**((4*n)/3)*c)
```

3.667 $\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n} dx$

Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n} dx = -\frac{4(cx)^{-5n/4}}{5acn} + \frac{4bx^n(cx)^{-5n/4}}{a^2cn} + \frac{\sqrt{2}b^{5/4}x^{5n/4}(cx)^{-5n/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{9/4}cn} - \frac{\sqrt{2}b^{5/4}x^{5n/4}(cx)^{-5n/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{a^{9/4}cn} - \frac{\sqrt{2}b^{5/4}x^{5n/4}(cx)^{-5n/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}}{\sqrt{b}+\sqrt{a}x^{-n/2}}\right)}{a^{9/4}cn}$$

output
```
-4/5/a/c/n/((c*x)^(5/4*n))+4*b*x^n/a^2/c/n/((c*x)^(5/4*n))+2^(1/2)*b^(5/4)
*x^(5/4*n)*arctan(1-2^(1/2)*a^(1/4)/b^(1/4)/(x^(1/4*n)))/a^(9/4)/c/n/((c*x
)^(5/4*n))-2^(1/2)*b^(5/4)*x^(5/4*n)*arctan(1+2^(1/2)*a^(1/4)/b^(1/4)/(x^(
1/4*n)))/a^(9/4)/c/n/((c*x)^(5/4*n))-2^(1/2)*b^(5/4)*x^(5/4*n)*arctanh(2^(
1/2)*a^(1/4)*b^(1/4)/(x^(1/4*n))/(b^(1/2)+a^(1/2)/(x^(1/2*n))))/a^(9/4)/c/
n/((c*x)^(5/4*n))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.15

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = -\frac{4x(cx)^{-1-\frac{5n}{4}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\frac{bx^n}{a}\right)}{5an}$$

input
```
Integrate[(c*x)^(-1 - (5*n)/4)/(a + b*x^n),x]
```

output
```
(-4*x*(c*x)^(-1 - (5*n)/4)*Hypergeometric2F1[-5/4, 1, -1/4, -((b*x^n)/a)])
/(5*a*n)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {887, 886, 868, 772, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{5n}{4}-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{ 887}$$

$$\frac{x^{5n/4}(cx)^{-5n/4}\int \frac{x^{-\frac{5n}{4}-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{ 886}$$

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(-\frac{b\int \frac{x^{-\frac{n}{4}-1}}{bx^n+a}dx}{a}-\frac{4x^{-5n/4}}{5an}\right)}{c}$$

$$\Big\downarrow \text{ 868}$$

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\int \frac{1}{bx^n+a}dx^{-n/4}}{an}-\frac{4x^{-5n/4}}{5an}\right)}{c}$$

$$\Big\downarrow \; 772$$

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\int \frac{x^{-n}}{ax^{-n}+b}dx^{-n/4}}{an}-\frac{4x^{-5n/4}}{5an}\right)}{c}$$

$$\Big\downarrow \; 843$$

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\int \frac{1}{ax^{-n}+b}dx^{-n/4}}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}\right)}{c}$$

$$\Big\downarrow \; 755$$

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{a}x^{-n/2}+\sqrt{b}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}\right)}{c}$$

$$\Big\downarrow \; 1476$$

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\int\frac{1}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}+\frac{\int\frac{1}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}}}{2\sqrt{b}}\right)}{a}\right)}{an}\right)}{c}$$

↓ 1082

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{1}{-x^{-n/2}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x^{-n/2}-1}d\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\int \frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}\right)}{a}\right)}{an}-\ldots\right.}{c}$$

$\downarrow$ 217

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\int\frac{\sqrt{b}-\sqrt{a}x^{-n/2}}{ax^{-n}+b}dx^{-n/4}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{an}-\frac{4x^{-5n/4}}{5an}\right)}{c}$$

$\downarrow$ 1479

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\arctan\left(\cdots\right.}{\cdots}\right.}{a}\right.}{an}\right.}{c}$$

$\downarrow$ 25

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{a}\left(x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}\right)}{\sqrt[4]{a}\left(x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}\right)}dx^{-n/4}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)\cdots}{\sqrt{2}\cdots}\cdots\right)}{a}\cdots\right)}{an}\cdots\right)}{c}\cdots$$

$\Big\downarrow$ 27

$$x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{b}-2\sqrt[4]{a}x^{-n/4}}{x^{-n/2}-\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{2}\sqrt{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}+\sqrt[4]{b}}{x^{-n/2}+\frac{\sqrt{2}\sqrt[4]{b}x^{-n/4}}{\sqrt[4]{a}}+\frac{\sqrt{b}}{\sqrt{a}}}dx^{-n/4}}{2\sqrt{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\cdots}{2\sqrt{\cdots}}\right)}{a}\right)}{an}\cdots\right)}{c}$$

$\downarrow$ 1103

$$\frac{x^{5n/4}(cx)^{-5n/4}\left(\frac{4b\left(\frac{x^{-n/4}}{a}-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{a}x^{-n/4}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x^{-n/4}+\sqrt{a}x^{-n/2}+\sqrt{b}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\cdots\right.}{\cdots}}{2\sqrt{b}}\right)}{a}\right)}{an}\cdots\right)}{c}$$

input
```
Int[(c*x)^(-1 - (5*n)/4)/(a + b*x^n),x]
```

output
```
(x^((5*n)/4)*(-4/(5*a*n*x^((5*n)/4)) + (4*b*(1/(a*x^(n/4)) - (b*((-(ArcTan
[1 - (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/4))]/(Sqrt[2]*a^(1/4)*b^(1/4))) + Arc
Tan[1 + (Sqrt[2]*a^(1/4))/(b^(1/4)*x^(n/4))]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2
*Sqrt[b]) + (-1/2*Log[Sqrt[b] + Sqrt[a]/x^(n/2) - (Sqrt[2]*a^(1/4)*b^(1/4)
)/x^(n/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[b] + Sqrt[a]/x^(n/2) + (Sq
rt[2]*a^(1/4)*b^(1/4))/x^(n/4)]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))
/a))/(a*n)))/(c*(c*x)^((5*n)/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 886
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Simp[b/a   Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a
, b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

rule 887
```
Int[((c_)*(x_))^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[c^IntPart[
m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m/(a + b*x^n), x], x] /; FreeQ
[{a, b, c, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && (SumSimplerQ[m, n
] || SumSimplerQ[m, -n])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+b\,x^n}dx$$

input
```
int((c*x)^(-1-5/4*n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1-5/4*n)/(a+b*x^n),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.65

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = -\frac{5\,a^2n\left(-\frac{b^5c^{-5\,n-4}}{a^9n^4}\right)^{\frac{1}{4}}\log\left(\frac{a^2nx^{\frac{4}{5}}\left(-\frac{b^5c^{-5\,n-4}}{a^9n^4}\right)^{\frac{1}{4}}+bc^{-n-\frac{4}{5}}xe^{\left(-\frac{1}{20}\,(5\,n+4)\log(c)-\frac{1}{20}\,(5\,n+4)\log(x)\right)}}{x}\right)-5\,a^2n\left(-\frac{b^5c^{-5\,n-4}}{a^9n^4}\right)}{}$$

input
```
integrate((c*x)^(-1-5/4*n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
-1/5*(5*a^2*n*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4)*log((a^2*n*x^(4/5)*(-b^5
*c^(-5*n - 4)/(a^9*n^4))^(1/4) + b*c^(-n - 4/5)*x*e^(-1/20*(5*n + 4)*log(c
) - 1/20*(5*n + 4)*log(x)))/x) - 5*a^2*n*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/
4)*log(-(a^2*n*x^(4/5)*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4) - b*c^(-n - 4/5
)*x*e^(-1/20*(5*n + 4)*log(c) - 1/20*(5*n + 4)*log(x)))/x) + 5*I*a^2*n*(-b
^5*c^(-5*n - 4)/(a^9*n^4))^(1/4)*log((I*a^2*n*x^(4/5)*(-b^5*c^(-5*n - 4)/(
a^9*n^4))^(1/4) + b*c^(-n - 4/5)*x*e^(-1/20*(5*n + 4)*log(c) - 1/20*(5*n +
 4)*log(x)))/x) - 5*I*a^2*n*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4)*log((-I*a^
2*n*x^(4/5)*(-b^5*c^(-5*n - 4)/(a^9*n^4))^(1/4) + b*c^(-n - 4/5)*x*e^(-1/2
0*(5*n + 4)*log(c) - 1/20*(5*n + 4)*log(x)))/x) - 20*b*c^(-n - 4/5)*x^(1/5
)*e^(-1/20*(5*n + 4)*log(c) - 1/20*(5*n + 4)*log(x)) + 4*a*x*e^(-1/4*(5*n
+ 4)*log(c) - 1/4*(5*n + 4)*log(x)))/(a^2*n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.42

$$
\begin{aligned}
\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx &= \frac{c^{-\frac{5n}{4}-1}x^{-\frac{5n}{4}}\Gamma\left(-\frac{5}{4}\right)}{an\Gamma\left(-\frac{1}{4}\right)} - \frac{5bc^{-\frac{5n}{4}-1}x^{-\frac{n}{4}}\Gamma\left(-\frac{5}{4}\right)}{a^2n\Gamma\left(-\frac{1}{4}\right)} \\
&+ \frac{5b^{\frac{5}{4}}c^{-\frac{5n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)} \\
&+ \frac{5ib^{\frac{5}{4}}c^{-\frac{5n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{3i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)} \\
&- \frac{5b^{\frac{5}{4}}c^{-\frac{5n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{5i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)} \\
&- \frac{5ib^{\frac{5}{4}}c^{-\frac{5n}{4}-1}e^{-\frac{3i\pi}{4}}\log\left(1-\frac{\sqrt[4]{b}x^{\frac{n}{4}}e^{\frac{7i\pi}{4}}}{\sqrt[4]{a}}\right)\Gamma\left(-\frac{5}{4}\right)}{4a^{\frac{9}{4}}n\Gamma\left(-\frac{1}{4}\right)}
\end{aligned}
$$

input
```
integrate((c*x)**(-1-5/4*n)/(a+b*x**n),x)
```

output
```
c**(-5*n/4 - 1)*gamma(-5/4)/(a*n*x**(5*n/4)*gamma(-1/4)) - 5*b*c**(-5*n/4
- 1)*gamma(-5/4)/(a**2*n*x**(n/4)*gamma(-1/4)) + 5*b**(5/4)*c**(-5*n/4 - 1
)*exp(-3*I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(I*pi/4)/a**(1/4))*gam
ma(-5/4)/(4*a**(9/4)*n*gamma(-1/4)) + 5*I*b**(5/4)*c**(-5*n/4 - 1)*exp(-3*
I*pi/4)*log(1 - b**(1/4)*x**(n/4)*exp_polar(3*I*pi/4)/a**(1/4))*gamma(-5/4
)/(4*a**(9/4)*n*gamma(-1/4)) - 5*b**(5/4)*c**(-5*n/4 - 1)*exp(-3*I*pi/4)*l
og(1 - b**(1/4)*x**(n/4)*exp_polar(5*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(
9/4)*n*gamma(-1/4)) - 5*I*b**(5/4)*c**(-5*n/4 - 1)*exp(-3*I*pi/4)*log(1 -
b**(1/4)*x**(n/4)*exp_polar(7*I*pi/4)/a**(1/4))*gamma(-5/4)/(4*a**(9/4)*n*
gamma(-1/4))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{5}{4}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-5/4*n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
b^2*integrate(x^(3/4*n)/(a^2*b*c^(5/4*n + 1)*x*x^n + a^3*c^(5/4*n + 1)*x),
 x) + 4/5*(5*b*x^n - a)*c^(-5/4*n - 1)/(a^2*n*x^(5/4*n))
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \int \frac{(cx)^{-\frac{5}{4}\,n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1-5/4*n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(-5/4*n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \int \frac{1}{(c\,x)^{\frac{5\,n}{4}+1}\,(a+b\,x^n)}\,dx$$

input
```
int(1/((c*x)^((5*n)/4 + 1)*(a + b*x^n)),x)
```

output
```
int(1/((c*x)^((5*n)/4 + 1)*(a + b*x^n)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{5n}{4}}}{a+bx^n}\,dx = \frac{\int \frac{1}{x^{\frac{9n}{4}}bx+x^{\frac{5n}{4}}ax}dx}{c^{\frac{5n}{4}}c}$$

input
```
int((c*x)^(-1-5/4*n)/(a+b*x^n),x)
```

output
```
int(1/(x**((9*n)/4)*b*x + x**((5*n)/4)*a*x),x)/(c**((5*n)/4)*c)
```

3.668 $\int \frac{(cx)^{4+n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx=\frac{(cx)^{5+n}\,\text{Hypergeometric2F1}\left(1,\frac{5+n}{n},2+\frac{5}{n},-\frac{bx^n}{a}\right)}{ac(5+n)}$$

output
```
(c*x)^(5+n)*hypergeom([1, (5+n)/n],[2+5/n],-b*x^n/a)/a/c/(5+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx=\frac{x(cx)^{4+n}\,\text{Hypergeometric2F1}\left(1,\frac{5+n}{n},1+\frac{5+n}{n},-\frac{bx^n}{a}\right)}{a(5+n)}$$

input
```
Integrate[(c*x)^(4 + n)/(a + b*x^n),x]
```

output
```
(x*(c*x)^(4 + n)*Hypergeometric2F1[1, (5 + n)/n, 1 + (5 + n)/n, -((b*x^n)/
a)])/(a*(5 + n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n+4}}{a+bx^n}\,dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{n+5}\,\text{Hypergeometric2F1}\left(1,\frac{n+5}{n},2+\frac{5}{n},-\frac{bx^n}{a}\right)}{ac(n+5)}$$

input
```
Int[(c*x)^(4 + n)/(a + b*x^n),x]
```

output
```
((c*x)^(5 + n)*Hypergeometric2F1[1, (5 + n)/n, 2 + 5/n, -((b*x^n)/a)])/(a*
c*(5 + n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{4+n}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(4+n)/(a+b*x^n),x)
```

output
```
int((c*x)^(4+n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx=\int \frac{(cx)^{n+4}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(4+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((c*x)^(n + 4)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.66

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx=\frac{a^{-2-\frac{5}{n}}a^{1+\frac{5}{n}}c^{n+4}x^{n+5}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{5}{n}\right)\Gamma\left(1+\frac{5}{n}\right)}{n\Gamma\left(2+\frac{5}{n}\right)}+\frac{5a^{-2-\frac{5}{n}}a^{1+\frac{5}{n}}c^{n+4}x^{n+5}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{5}{n}\right)\Gamma\left(1+\frac{5}{n}\right)}{n^2\Gamma\left(2+\frac{5}{n}\right)}$$

input
```
integrate((c*x)**(4+n)/(a+b*x**n),x)
```

output
```
a**(-2 - 5/n)*a**(1 + 5/n)*c**(n + 4)*x**(n + 5)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1 + 5/n)*gamma(1 + 5/n)/(n*gamma(2 + 5/n)) + 5*a**(-2 - 5/n)*
a**(1 + 5/n)*c**(n + 4)*x**(n + 5)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1
 + 5/n)*gamma(1 + 5/n)/(n**2*gamma(2 + 5/n))
```

Maxima [F]

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+4}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(4+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
1/5*c^(n + 4)*x^5/b - a*c^(n + 4)*integrate(x^4/(b^2*x^n + a*b), x)
```

Giac [F]

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+4}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(4+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(n + 4)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx = \int \frac{(c\,x)^{n+4}}{a+b\,x^n}\,dx$$

input
```
int((c*x)^(n + 4)/(a + b*x^n),x)
```

output
```
int((c*x)^(n + 4)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(cx)^{4+n}}{a+bx^n}\,dx = \frac{c^n c^4 \left(-5\left(\int \frac{x^4}{x^n b+a}dx\right)a+x^5\right)}{5b}$$

input
```
int((c*x)^(4+n)/(a+b*x^n),x)
```

output
```
(c**n*c**4*( - 5*int(x**4/(x**n*b + a),x)*a + x**5))/(5*b)
```

3.669 $\int \frac{(cx)^{3+n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{(cx)^{3+n}}{a+bx^n} dx = \frac{(cx)^{4+n} \text{Hypergeometric2F1}\left(1, \frac{4+n}{n}, 2\left(1+\frac{2}{n}\right), -\frac{bx^n}{a}\right)}{ac(4+n)}$$

output
```
(c*x)^(4+n)*hypergeom([1, (4+n)/n],[2+4/n],-b*x^n/a)/a/c/(4+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{(cx)^{3+n}}{a+bx^n} dx = \frac{x(cx)^{3+n} \text{Hypergeometric2F1}\left(1, \frac{4+n}{n}, 1+\frac{4+n}{n}, -\frac{bx^n}{a}\right)}{a(4+n)}$$

input
```
Integrate[(c*x)^(3 + n)/(a + b*x^n),x]
```

output
```
(x*(c*x)^(3 + n)*Hypergeometric2F1[1, (4 + n)/n, 1 + (4 + n)/n, -((b*x^n)/
a)])/(a*(4 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n+3}}{a+bx^n}\,dx$$
$$\Big\downarrow \; 888$$
$$\frac{(cx)^{n+4}\,\text{Hypergeometric2F1}\left(1,\frac{n+4}{n},2\left(1+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{ac(n+4)}$$

input
```
Int[(c*x)^(3 + n)/(a + b*x^n),x]
```

output
```
((c*x)^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((b*x^n)/a)])
/(a*c*(4 + n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{3+n}}{a+b\,x^n}dx$$

input
```
int((c*x)^(3+n)/(a+b*x^n),x)
```

output
```
int((c*x)^(3+n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^{3+n}}{a+bx^n}\,dx=\int \frac{(cx)^{n+3}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(3+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((c*x)^(n + 3)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int \frac{(cx)^{3+n}}{a+bx^n}\,dx=\frac{a^{-2-\frac{4}{n}}a^{1+\frac{4}{n}}c^{n+3}x^{n+4}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{n\Gamma\left(2+\frac{4}{n}\right)}+\frac{4a^{-2-\frac{4}{n}}a^{1+\frac{4}{n}}c^{n+3}x^{n+4}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{n^2\Gamma\left(2+\frac{4}{n}\right)}$$

input
```
integrate((c*x)**(3+n)/(a+b*x**n),x)
```

output
```
a**(-2 - 4/n)*a**(1 + 4/n)*c**(n + 3)*x**(n + 4)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1 + 4/n)*gamma(1 + 4/n)/(n*gamma(2 + 4/n)) + 4*a**(-2 - 4/n)*
a**(1 + 4/n)*c**(n + 3)*x**(n + 4)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1
 + 4/n)*gamma(1 + 4/n)/(n**2*gamma(2 + 4/n))
```

Maxima [F]

$$\int \frac{(cx)^{3+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+3}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(3+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
1/4*c^(n + 3)*x^4/b - a*c^(n + 3)*integrate(x^3/(b^2*x^n + a*b), x)
```

Giac [F]

$$\int \frac{(cx)^{3+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+3}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(3+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(n + 3)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3+n}}{a+bx^n}\,dx = \int \frac{(c\,x)^{n+3}}{a+b\,x^n}\,dx$$

input
```
int((c*x)^(n + 3)/(a + b*x^n),x)
```

output
```
int((c*x)^(n + 3)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(cx)^{3+n}}{a+bx^n}\,dx = \frac{c^n c^3 \left(-4\left(\int \frac{x^3}{x^n b+a}dx\right)a + x^4\right)}{4b}$$

input
```
int((c*x)^(3+n)/(a+b*x^n),x)
```

output
```
(c**n*c**3*( - 4*int(x**3/(x**n*b + a),x)*a + x**4))/(4*b)
```

3.670 $\int \frac{(cx)^{2+n}}{a+bx^n} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(cx)^{2+n}}{a+bx^n} \, dx = \frac{(cx)^{3+n} \text{Hypergeometric2F1}\left(1, \frac{3+n}{n}, 2+\frac{3}{n}, -\frac{bx^n}{a}\right)}{ac(3+n)}$$

output
```
(c*x)^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-b*x^n/a)/a/c/(3+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^{2+n}}{a+bx^n} \, dx = \frac{x(cx)^{2+n} \text{Hypergeometric2F1}\left(1, \frac{3+n}{n}, 1+\frac{3+n}{n}, -\frac{bx^n}{a}\right)}{a(3+n)}$$

input
```
Integrate[(c*x)^(2 + n)/(a + b*x^n),x]
```

output
```
(x*(c*x)^(2 + n)*Hypergeometric2F1[1, (3 + n)/n, 1 + (3 + n)/n, -((b*x^n)/
a)])/(a*(3 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n+2}}{a+bx^n}\,dx$$
$$\Big\downarrow \text{ 888}$$
$$\frac{(cx)^{n+3}\,\text{Hypergeometric2F1}\left(1,\frac{n+3}{n},2+\frac{3}{n},-\frac{bx^n}{a}\right)}{ac(n+3)}$$

input
```
Int[(c*x)^(2 + n)/(a + b*x^n),x]
```

output
```
((c*x)^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((b*x^n)/a)])/(a*
c*(3 + n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{2+n}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(2+n)/(a+b*x^n),x)
```

output
```
int((c*x)^(2+n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^{2+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+2}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(2+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((c*x)^(n + 2)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.66

$$\int \frac{(cx)^{2+n}}{a+bx^n}\,dx = \frac{a^{-2-\frac{3}{n}}a^{1+\frac{3}{n}}c^{n+2}x^{n+3}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{3a^{-2-\frac{3}{n}}a^{1+\frac{3}{n}}c^{n+2}x^{n+3}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n^2\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate((c*x)**(2+n)/(a+b*x**n),x)
```

output
```
a**(-2 - 3/n)*a**(1 + 3/n)*c**(n + 2)*x**(n + 3)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1 + 3/n)*gamma(1 + 3/n)/(n*gamma(2 + 3/n)) + 3*a**(-2 - 3/n)*
a**(1 + 3/n)*c**(n + 2)*x**(n + 3)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1
 + 3/n)*gamma(1 + 3/n)/(n**2*gamma(2 + 3/n))
```

Maxima [F]

$$\int \frac{(cx)^{2+n}}{a+bx^n}\,dx=\int \frac{(cx)^{n+2}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(2+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
1/3*c^(n + 2)*x^3/b - a*c^(n + 2)*integrate(x^2/(b^2*x^n + a*b), x)
```

Giac [F]

$$\int \frac{(cx)^{2+n}}{a+bx^n}\,dx=\int \frac{(cx)^{n+2}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(2+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(n + 2)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{2+n}}{a+bx^n}\,dx=\int \frac{(c\,x)^{n+2}}{a+b\,x^n}\,dx$$

input
```
int((c*x)^(n + 2)/(a + b*x^n),x)
```

output
```
int((c*x)^(n + 2)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(cx)^{2+n}}{a+bx^n}\,dx = \frac{c^n c^2 \left(-3\left(\int \frac{x^2}{x^n b+a}dx\right)a+x^3\right)}{3b}$$

input
```
int((c*x)^(2+n)/(a+b*x^n),x)
```

output
```
(c**n*c**2*( - 3*int(x**2/(x**n*b + a),x)*a + x**3))/(3*b)
```

3.671 $\int \frac{(cx)^{1+n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(cx)^{1+n}}{a+bx^n} dx = \frac{(cx)^{2+n} \text{Hypergeometric2F1}\left(1, \frac{2+n}{n}, 2\left(1+\frac{1}{n}\right), -\frac{bx^n}{a}\right)}{ac(2+n)}$$

output
```
(c*x)^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-b*x^n/a)/a/c/(2+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(cx)^{1+n}}{a+bx^n} dx = \frac{cx^2(cx)^n \text{Hypergeometric2F1}\left(1, \frac{2+n}{n}, 1+\frac{2+n}{n}, -\frac{bx^n}{a}\right)}{a(2+n)}$$

input
```
Integrate[(c*x)^(1 + n)/(a + b*x^n),x]
```

output
```
(c*x^2*(c*x)^n*Hypergeometric2F1[1, (2 + n)/n, 1 + (2 + n)/n, -((b*x^n)/a)
])/(a*(2 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n+1}}{a+bx^n} \, dx$$
$$\Big\downarrow \text{ 888}$$
$$\frac{(cx)^{n+2} \text{Hypergeometric2F1}\left(1,\frac{n+2}{n},2\left(1+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{ac(n+2)}$$

input
```
Int[(c*x)^(1 + n)/(a + b*x^n),x]
```

output
```
((c*x)^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((b*x^n)/a
)])/(a*c*(2 + n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{1+n}}{a+b\,x^n}dx$$

input
```
int((c*x)^(1+n)/(a+b*x^n),x)
```

output
```
int((c*x)^(1+n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^{1+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(1+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((c*x)^(n + 1)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.66

$$\int \frac{(cx)^{1+n}}{a+bx^n}\,dx = \frac{a^{-2-\frac{2}{n}}a^{1+\frac{2}{n}}c^{n+1}x^{n+2}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + \frac{2a^{-2-\frac{2}{n}}a^{1+\frac{2}{n}}c^{n+1}x^{n+2}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n^2\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate((c*x)**(1+n)/(a+b*x**n),x)
```

output
```
a**(-2 - 2/n)*a**(1 + 2/n)*c**(n + 1)*x**(n + 2)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1 + 2/n)*gamma(1 + 2/n)/(n*gamma(2 + 2/n)) + 2*a**(-2 - 2/n)*
a**(1 + 2/n)*c**(n + 1)*x**(n + 2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1
 + 2/n)*gamma(1 + 2/n)/(n**2*gamma(2 + 2/n))
```

Maxima [F]

$$\int \frac{(cx)^{1+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(1+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-a*c^(n + 1)*integrate(x/(b^2*x^n + a*b), x) + 1/2*c^(n + 1)*x^2/b
```

Giac [F]

$$\int \frac{(cx)^{1+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n+1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(1+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(n + 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{1+n}}{a+bx^n}\,dx = \int \frac{(c\,x)^{n+1}}{a+b\,x^n}\,dx$$

input
```
int((c*x)^(n + 1)/(a + b*x^n),x)
```

output
```
int((c*x)^(n + 1)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(cx)^{1+n}}{a+bx^n}\,dx = \frac{c^n c\left(-2\left(\int \frac{x}{x^n b+a}dx\right)a + x^2\right)}{2b}$$

input
```
int((c*x)^(1+n)/(a+b*x^n),x)
```

output
```
(c**n*c*( - 2*int(x/(x***n*b + a),x)*a + x**2))/(2*b)
```

3.672 $\int \frac{(cx)^n}{a+bx^n} dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{(cx)^n}{a+bx^n} dx = \frac{(cx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{bx^n}{a}\right)}{ac(1+n)}$$

output
```
(c*x)^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-b*x^n/a)/a/c/(1+n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{(cx)^n}{a+bx^n} dx = \frac{x(cx)^n\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(1+n)}$$

input
```
Integrate[(c*x)^n/(a + b*x^n),x]
```

output
```
(x*(c*x)^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((b*x^n)/a)])/(a*
(1 + n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^n}{a+bx^n} \, dx$$

$$\Big\downarrow \ 888$$

$$\frac{(cx)^{n+1} \text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{bx^n}{a}\right)}{ac(n+1)}$$

input
```
Int[(c*x)^n/(a + b*x^n),x]
```

output
```
((c*x)^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((b*x^n)/a)])
/(a*c*(1 + n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^n}{a+b\,x^n}dx$$

input
```
int((c*x)^n/(a+b*x^n),x)
```

output
```
int((c*x)^n/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^n}{a+bx^n}\,dx = \int \frac{(cx)^n}{bx^n+a}\,dx$$

input
```
integrate((c*x)^n/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((c*x)^n/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{(cx)^n}{a+bx^n}\,dx = -\frac{a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}c^n x\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**n/(a+b*x**n),x)
```

output
```
-a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*c**n*x*lerchphi(a*exp_polar(I*pi)/(b*
x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{(cx)^n}{a+bx^n}\,dx=\int \frac{(cx)^n}{bx^n+a}\,dx$$

input
```
integrate((c*x)^n/(a+b*x^n),x, algorithm="maxima")
```

output
```
-a*c^n*integrate(1/(b^2*x^n + a*b), x) + c^n*x/b
```

Giac [F]

$$\int \frac{(cx)^n}{a+bx^n}\,dx=\int \frac{(cx)^n}{bx^n+a}\,dx$$

input
```
integrate((c*x)^n/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^n/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^n}{a+bx^n}\,dx=\int \frac{(c\,x)^n}{a+b\,x^n}\,dx$$

input
```
int((c*x)^n/(a + b*x^n),x)
```

output
```
int((c*x)^n/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(cx)^n}{a+bx^n}\,dx = \frac{c^n\left(-\left(\int \frac{1}{x^n b+a}dx\right)a+x\right)}{b}$$

input
```
int((c*x)^n/(a+b*x^n),x)
```

output
```
(c**n*( - int(1/(x**n*b + a),x)*a + x))/b
```

3.673 $\int \frac{(cx)^{-1+n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{(cx)^{-1+n}}{a+bx^n} dx = \frac{x^{-n}(cx)^n \log(a+bx^n)}{bcn}$$

output
```
(c*x)^n*ln(a+b*x^n)/b/c/n/(x^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(cx)^{-1+n}}{a+bx^n} dx = \frac{x^{1-n}(cx)^{-1+n} \log(a+bx^n)}{bn}$$

input
```
Integrate[(c*x)^(-1 + n)/(a + b*x^n),x]
```

output
```
(x^(1 - n)*(c*x)^(-1 + n)*Log[a + b*x^n])/(b*n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {800, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n-1}}{a+bx^n} \, dx$$

$$\Big\downarrow \text{800}$$

$$\frac{x^{-n}(cx)^n \int \frac{x^{n-1}}{bx^n+a}dx}{c}$$

$$\Big\downarrow \text{792}$$

$$\frac{x^{-n}(cx)^n \log(a+bx^n)}{bcn}$$

input
```
Int[(c*x)^(-1 + n)/(a + b*x^n),x]
```

output
```
((c*x)^n*Log[a + b*x^n])/(b*c*n*x^n)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(cx)^{-1+n}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(-1+n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-1+n)/(a+b*x^n),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx = \frac{c^{n-1}\log\left(bx^n+a\right)}{bn}$$

input
```
integrate((c*x)^(-1+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
c^(n - 1)*log(b*x^n + a)/(b*n)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx = \frac{c^{n-1}\log\left(1+\frac{bx^n}{a}\right)}{bn}$$

input
```
integrate((c*x)**(-1+n)/(a+b*x**n),x)
```

output
```
c**(n - 1)*log(1 + b*x**n/a)/(b*n)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx = \frac{c^{n-1}\log\left(\frac{bx^n+a}{b}\right)}{bn}$$

input
```
integrate((c*x)^(-1+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
c^(n - 1)*log((b*x^n + a)/b)/(b*n)
```

Giac [F]

$$\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n-1}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-1+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(n - 1)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx = \int \frac{(c\,x)^{n-1}}{a+b\,x^n}\,dx$$

input
```
int((c*x)^(n - 1)/(a + b*x^n),x)
```

output
```
int((c*x)^(n - 1)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{(cx)^{-1+n}}{a+bx^n}\,dx=\frac{c^n\log(x^nb+a)}{bcn}$$

input
```
int((c*x)^(-1+n)/(a+b*x^n),x)
```

output
```
(c**n*log(x**n*b + a))/(b*c*n)
```

3.674 $\int \frac{(cx)^{-2+n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \frac{(cx)^{-2+n}}{a+bx^n} dx = -\frac{(cx)^{-1+n} \text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},-\frac{bx^n}{a}\right)}{ac(1-n)}$$

output
```
-(c*x)^(-1+n)*hypergeom([1, -(1-n)/n],[2-1/n],-b*x^n/a)/a/c/(1-n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^{-2+n}}{a+bx^n} dx = \frac{x(cx)^{-2+n} \text{Hypergeometric2F1}\left(1,\frac{-1+n}{n},1+\frac{-1+n}{n},-\frac{bx^n}{a}\right)}{a(-1+n)}$$

input
```
Integrate[(c*x)^(-2 + n)/(a + b*x^n),x]
```

output
```
(x*(c*x)^(-2 + n)*Hypergeometric2F1[1, (-1 + n)/n, 1 + (-1 + n)/n, -((b*x^
n)/a)])/(a*(-1 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n-2}}{a+bx^n}\,dx$$
$$\Big\downarrow \ 888$$
$$-\frac{(cx)^{n-1}\,\text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},-\frac{bx^n}{a}\right)}{ac(1-n)}$$

input
```
Int[(c*x)^(-2 + n)/(a + b*x^n),x]
```

output
```
-(((c*x)^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((b*x^n)
/a)])/(a*c*(1 - n)))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{n-2}}{a+b\,x^{n}}dx$$

input
```
int((c*x)^(n-2)/(a+b*x^n),x)
```

output
```
int((c*x)^(n-2)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n-2}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-2+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((c*x)^(n - 2)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{1-\frac{1}{n}}b^{-\frac{1}{n}}b^{-1+\frac{1}{n}}c^{n-2}\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{1}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{an^2x\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate((c*x)**(-2+n)/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(1 - 1/n)*b**(-1 + 1/n)*c**(n - 2)*lerchphi(a*exp_polar(I*pi)/
(b*x**n), 1, 1/n)*gamma(-1/n)/(a*b**(1/n)*n**2*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n-2}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-2+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-a*c^n*integrate(1/(b^2*c^2*x^2*x^n + a*b*c^2*x^2), x) - c^(n - 2)/(b*x)
```

Giac [F]

$$\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n-2}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-2+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(n - 2)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx = \int \frac{(c\,x)^{n-2}}{a+b\,x^n}\,dx$$

input
```
int((c*x)^(n - 2)/(a + b*x^n),x)
```

output
```
int((c*x)^(n - 2)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(cx)^{-2+n}}{a+bx^n}\,dx = -\frac{c^n\left(\left(\int \frac{1}{x^n b\,x^2+a\,x^2}dx\right)ax+1\right)}{b\,c^2x}$$

input
```
int((c*x)^(-2+n)/(a+b*x^n),x)
```

output
```
( - c**n*(int(1/(x**n*b*x**2 + a*x**2),x)*a*x + 1))/(b*c**2*x)
```

3.675 $\int \frac{(cx)^{-3+n}}{a+bx^n} dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx=-\frac{(cx)^{-2+n}\,\text{Hypergeometric2F1}\left(1,-\frac{2-n}{n},2\left(1-\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{ac(2-n)}$$

output
```
-(c*x)^(-2+n)*hypergeom([1, -(2-n)/n],[2-2/n],-b*x^n/a)/a/c/(2-n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx=\frac{x(cx)^{-3+n}\,\text{Hypergeometric2F1}\left(1,\frac{-2+n}{n},1+\frac{-2+n}{n},-\frac{bx^n}{a}\right)}{a(-2+n)}$$

input
```
Integrate[(c*x)^(-3 + n)/(a + b*x^n),x]
```

output
```
(x*(c*x)^(-3 + n)*Hypergeometric2F1[1, (-2 + n)/n, 1 + (-2 + n)/n, -((b*x^
n)/a)])/(a*(-2 + n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n-3}}{a+bx^n}\,dx$$
$$\Big\downarrow \ 888$$
$$-\frac{(cx)^{n-2}\,\text{Hypergeometric2F1}\left(1,-\frac{2-n}{n},2\left(1-\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{ac(2-n)}$$

input
```
Int[(c*x)^(-3 + n)/(a + b*x^n),x]
```

output
```
-(((c*x)^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)), -((b*
x^n)/a)])/(a*c*(2 - n)))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{-3+n}}{a+b\,x^n}dx$$

input
```
int((c*x)^(-3+n)/(a+b*x^n),x)
```

output
```
int((c*x)^(-3+n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n-3}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-3+n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((c*x)^(n - 3)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx = \frac{2a^{\frac{2}{n}}a^{1-\frac{2}{n}}b^{-\frac{2}{n}}b^{-1+\frac{2}{n}}c^{n-3}\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{2}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{an^2x^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate((c*x)**(-3+n)/(a+b*x**n),x)
```

output
```
2*a**(2/n)*a**(1 - 2/n)*b**(-1 + 2/n)*c**(n - 3)*lerchphi(a*exp_polar(I*pi
)/(b*x**n), 1, 2/n)*gamma(-2/n)/(a*b**(2/n)*n**2*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n-3}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-3+n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-a*c^n*integrate(1/(b^2*c^3*x^3*x^n + a*b*c^3*x^3), x) - 1/2*c^(n - 3)/(b*
x^2)
```

Giac [F]

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx = \int \frac{(cx)^{n-3}}{bx^n+a}\,dx$$

input
```
integrate((c*x)^(-3+n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((c*x)^(n - 3)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx = \int \frac{(c\,x)^{n-3}}{a+b\,x^n}\,dx$$

input
```
int((c*x)^(n - 3)/(a + b*x^n),x)
```

output
```
int((c*x)^(n - 3)/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{(cx)^{-3+n}}{a+bx^n}\,dx = \frac{c^n\left(-2\left(\int \frac{1}{x^n b\,x^3+a\,x^3}dx\right)a\,x^2-1\right)}{2b\,c^3x^2}$$

input
```
int((c*x)^(-3+n)/(a+b*x^n),x)
```

output
```
(c**n*( - 2*int(1/(x**n*b*x**3 + a*x**3),x)*a*x**2 - 1))/(2*b*c**3*x**2)
```

3.676 $\int \frac{(cx)^{-1+n}}{(a+bx^n)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2} \, dx = \frac{(cx)^n}{acn\,(a+bx^n)}$$

output
```
(c*x)^n/a/c/n/(a+b*x^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2} \, dx = -\frac{x^{1-n}(cx)^{-1+n}}{bn\,(a+bx^n)}$$

input
```
Integrate[(c*x)^(-1 + n)/(a + b*x^n)^2,x]
```

output
```
-((x^(1 - n)*(c*x)^(-1 + n))/(b*n*(a + b*x^n)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{n-1}}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \; 796$$

$$\frac{(cx)^n}{acn\,(a+bx^n)}$$

input
```
Int[(c*x)^(-1 + n)/(a + b*x^n)^2,x]
```

output
```
(c*x)^n/(a*c*n*(a + b*x^n))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{x(cx)^{-1+n}}{an(a+b\,x^{n})}$	25
risch	$\frac{x\,{\mathrm e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(icx)^{3}\pi+i\,\mathrm{csgn}(icx)^{2}\,\mathrm{csgn}(ic)\pi+i\,\mathrm{csgn}(icx)^{2}\,\mathrm{csgn}(ix)\pi-i\,\mathrm{csgn}(icx)\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix)\pi+2\ln(x)+2\ln(c)\right)}{2}}}{an(a+b\,x^{n})}$	99

input
```
int((c*x)^(-1+n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(c*x)^(-1+n)/a/n/(a+b*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{c^{n-1}}{b^2nx^n+abn}$$

input
```
integrate((c*x)^(-1+n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
-c^(n - 1)/(b^2*n*x^n + a*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

Time = 0.82 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.04

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx = \begin{cases} \frac{\tilde{\infty}\log(x)}{c} & \text{for}\ a=0 \wedge b=0 \wedge n=0 \\ -\frac{xx^{-2n}(cx)^{n-1}}{b^2n} & \text{for}\ a=0 \\ \frac{\tilde{\infty}x(cx)^{n-1}}{n} & \text{for}\ b=-ax^{-n} \\ \frac{\log(x)}{c(a+b)^2} & \text{for}\ n=0 \\ \frac{x(cx)^{n-1}}{a^2n+abnx^n} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**(-1+n)/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*log(x)/c, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*(c*x)**(n -
1)/(b**2*n*x**(2*n)), Eq(a, 0)), (zoo*x*(c*x)**(n - 1)/n, Eq(b, -a/x**n)),
 (log(x)/(c*(a + b)**2), Eq(n, 0)), (x*(c*x)**(n - 1)/(a**2*n + a*b*n*x**n
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx = -\frac{c^n}{b^2cnx^n + abcn}$$

input
```
integrate((c*x)^(-1+n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-c^n/(b^2*c*n*x^n + a*b*c*n)
```

Giac [F]

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx = \int \frac{(cx)^{n-1}}{(bx^n+a)^2}\,dx$$

input
```
integrate((c*x)^(-1+n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((c*x)^(n - 1)/(b*x^n + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx = \frac{x\,(c\,x)^{n-1}}{a\,b\,n\,\left(x^n+\frac{a}{b}\right)}$$

input
```
int((c*x)^(n - 1)/(a + b*x^n)^2,x)
```

output
```
(x*(c*x)^(n - 1))/(a*b*n*(x^n + a/b))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{(cx)^{-1+n}}{(a+bx^n)^2}\,dx = \frac{x^n c^n}{acn\,(x^n b+a)}$$

input
```
int((c*x)^(-1+n)/(a+b*x^n)^2,x)
```

output
```
(x**n*c**n)/(a*c*n*(x**n*b + a))
```

3.677 $\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{5a^2x^{-3n}(cx)^{7n/2}\sqrt{a+bx^n}}{8b^3cn} - \frac{5ax^{-2n}(cx)^{7n/2}\sqrt{a+bx^n}}{12b^2cn} + \frac{x^{-n}(cx)^{7n/2}\sqrt{a+bx^n}}{3bcn} - \frac{5a^3x^{-7n/2}(cx)^{7n/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{8b^{7/2}cn}$$

output

```
5/8*a^2*(c*x)^(7/2*n)*(a+b*x^n)^(1/2)/b^3/c/n/(x^(3*n))-5/12*a*(c*x)^(7/2*
n)*(a+b*x^n)^(1/2)/b^2/c/n/(x^(2*n))+1/3*(c*x)^(7/2*n)*(a+b*x^n)^(1/2)/b/c
/n/(x^n)-5/8*a^3*(c*x)^(7/2*n)*arctanh(b^(1/2)*x^(1/2*n)/(a+b*x^n)^(1/2))/
b^(7/2)/c/n/(x^(7/2*n))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{x^{-7n/2}(cx)^{7n/2}\sqrt{a+bx^n}\left(\sqrt{b}x^{n/2}\sqrt{1+\frac{bx^n}{a}}(15a^2-10abx^n+8b^2x^{2n}) - 15a^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)\right)}{24b^{7/2}cn\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(c*x)^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
((c*x)^((7*n)/2)*Sqrt[a + b*x^n]*(Sqrt[b]*x^(n/2)*Sqrt[1 + (b*x^n)/a]*(15*
a^2 - 10*a*b*x^n + 8*b^2*x^(2*n)) - 15*a^(5/2)*ArcSinh[(Sqrt[b]*x^(n/2))/S
qrt[a]]))/(24*b^(7/2)*c*n*x^((7*n)/2)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {883, 880, 252, 252, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{7n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{883}$$

$$\frac{x^{-7n/2}(cx)^{7n/2}\int \frac{x^{\frac{7n}{2}-1}}{\sqrt{bx^n+a}}dx}{c}$$

$$\Big\downarrow \text{880}$$

$$\frac{2a^3x^{-7n/2}(cx)^{7n/2}\int \frac{x^{3n}}{(bx^n+a)^3\left(1-\frac{bx^n}{bx^n+a}\right)^4}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{cn}$$

$$\Big\downarrow \text{252}$$

$$\frac{2a^3x^{-7n/2}(cx)^{7n/2}\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\int \frac{x^{2n}}{(bx^n+a)^2\left(1-\frac{bx^n}{bx^n+a}\right)^3}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{6b}\right)}{cn}$$

$$\Big\downarrow \text{252}$$

$$\frac{2a^3x^{-7n/2}(cx)^{7n/2}\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\int\frac{x^n}{(bx^n+a)\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{4b}\right)}{6b}\right)}{cn}$$

$\Big\downarrow$ 252

$$\frac{2a^3x^{-7n/2}(cx)^{7n/2}\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\int\frac{1}{1-\frac{bx^n}{bx^n+a}}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{2b}\right)}{4b}\right)}{6b}\right)}{cn}$$

$\Big\downarrow$ 219

$$\frac{2a^3x^{-7n/2}(cx)^{7n/2}\left(\frac{x^{5n/2}}{6b(a+bx^n)^{5/2}\left(1-\frac{bx^n}{a+bx^n}\right)^3}-\frac{5\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{2b^{3/2}}\right)}{4b}\right)}{6b}\right)}{cn}$$

input
```
Int[(c*x)^(-1 + (7*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*a^3*(c*x)^((7*n)/2)*(x^((5*n)/2)/(6*b*(a + b*x^n)^(5/2)*(1 - (b*x^n)/(a
 + b*x^n))^3) - (5*(x^((3*n)/2)/(4*b*(a + b*x^n)^(3/2)*(1 - (b*x^n)/(a + b
*x^n))^2) - (3*(x^(n/2)/(2*b*Sqrt[a + b*x^n]*(1 - (b*x^n)/(a + b*x^n))) -
ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]]/(2*b^(3/2))))/(4*b)))/(6*b)))/(
c*n*x^((7*n)/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 880
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Simp[k*(a^(p + Simplify[(m + 1)/n])/n)   Subst[Int[x^(k*Simplify
[(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/
(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[p + Simpli
fy[(m + 1)/n]] && LtQ[-1, p, 0]
```

rule 883
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p]]
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+b\,x^n}}dx$$

input
```
int((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.33

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \left[\frac{15\,a^3\sqrt{b}c^{\frac{7}{2}\,n-1}\log\left(2\,\sqrt{bx^n+a}\sqrt{b}x^{\frac{1}{2}\,n}-2\,bx^n-a\right)+2\left(8\,b^3c^{\frac{7}{2}\,n-1}x^{\frac{5}{2}\,n}-10\,ab^2c^{\frac{7}{2}\,n-1}x^{\frac{3}{2}\,n}+15\,a^2bc^{\frac{7}{2}\,n-}\right.}{48\,b^4n}\right.$$

input
```
integrate((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^3*sqrt(b)*c^(7/2*n - 1)*log(2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n
) - 2*b*x^n - a) + 2*(8*b^3*c^(7/2*n - 1)*x^(5/2*n) - 10*a*b^2*c^(7/2*n -
1)*x^(3/2*n) + 15*a^2*b*c^(7/2*n - 1)*x^(1/2*n))*sqrt(b*x^n + a))/(b^4*n),
 1/24*(15*a^3*sqrt(-b)*c^(7/2*n - 1)*arctan(sqrt(b*x^n + a)*sqrt(-b)/(b*x^
(1/2*n))) + (8*b^3*c^(7/2*n - 1)*x^(5/2*n) - 10*a*b^2*c^(7/2*n - 1)*x^(3/2
*n) + 15*a^2*b*c^(7/2*n - 1)*x^(1/2*n))*sqrt(b*x^n + a))/(b^4*n)]
```

Sympy [A] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{5a^{\frac{5}{2}}c^{\frac{7n}{2}-1}x^{\frac{n}{2}}}{8b^3n\sqrt{1+\frac{bx^n}{a}}} + \frac{5a^{\frac{3}{2}}c^{\frac{7n}{2}-1}x^{\frac{3n}{2}}}{24b^2n\sqrt{1+\frac{bx^n}{a}}} - \frac{\sqrt{a}c^{\frac{7n}{2}-1}x^{\frac{5n}{2}}}{12bn\sqrt{1+\frac{bx^n}{a}}} - \frac{5a^3c^{\frac{7n}{2}-1}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}n} + \frac{c^{\frac{7n}{2}-1}x^{\frac{7n}{2}}}{3\sqrt{a}n\sqrt{1+\frac{bx^n}{a}}}$$

input
```
integrate((c*x)**(-1+7/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
5*a**(5/2)*c**(7*n/2 - 1)*x**(n/2)/(8*b**3*n*sqrt(1 + b*x**n/a)) + 5*a**(3
/2)*c**(7*n/2 - 1)*x**(3*n/2)/(24*b**2*n*sqrt(1 + b*x**n/a)) - sqrt(a)*c**
(7*n/2 - 1)*x**(5*n/2)/(12*b*n*sqrt(1 + b*x**n/a)) - 5*a**3*c**(7*n/2 - 1)
*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(8*b**(7/2)*n) + c**(7*n/2 - 1)*x**(7*n/2
)/(3*sqrt(a)*n*sqrt(1 + b*x**n/a))
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{7}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(c\,x)^{\frac{7\,n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((c*x)^((7*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int((c*x)^((7*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1+\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{c^{\frac{7n}{2}}\left(\int \frac{x^{\frac{7n}{2}}\sqrt{x^n b+a}}{x^n bx+ax}dx\right)}{c}$$

input
```
int((c*x)^(-1+7/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(c**((7*n)/2)*int((x**((7*n)/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x))/c
```

3.678 $\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{3ax^{-2n}(cx)^{5n/2}\sqrt{a+bx^n}}{4b^2cn}+\frac{x^{-n}(cx)^{5n/2}\sqrt{a+bx^n}}{2bcn}+\frac{3a^2x^{-5n/2}(cx)^{5n/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{4b^{5/2}cn}$$

output
```
-3/4*a*(c*x)^(5/2*n)*(a+b*x^n)^(1/2)/b^2/c/n/(x^(2*n))+1/2*(c*x)^(5/2*n)*(
a+b*x^n)^(1/2)/b/c/n/(x^n)+3/4*a^2*(c*x)^(5/2*n)*arctanh(b^(1/2)*x^(1/2*n)
/(a+b*x^n)^(1/2))/b^(5/2)/c/n/(x^(5/2*n))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{ax^{-5n/2}(cx)^{5n/2}\sqrt{1+\frac{bx^n}{a}}\left(\sqrt{b}x^{n/2}(-3a+2bx^n)\sqrt{1+\frac{bx^n}{a}}+3a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)\right)}{4b^{5/2}cn\sqrt{a+bx^n}}$$

input
```
Integrate[(c*x)^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(a*(c*x)^((5*n)/2)*Sqrt[1 + (b*x^n)/a]*(Sqrt[b]*x^(n/2)*(-3*a + 2*b*x^n)*S
qrt[1 + (b*x^n)/a] + 3*a^(3/2)*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]]))/(4*b^(
5/2)*c*n*x^((5*n)/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {883, 880, 252, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{5n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{ 883}$$

$$\frac{x^{-5n/2}(cx)^{5n/2}\int \frac{x^{\frac{5n}{2}-1}}{\sqrt{bx^n+a}}dx}{c}$$

$$\Big\downarrow \text{ 880}$$

$$\frac{2a^2x^{-5n/2}(cx)^{5n/2}\int \frac{x^{2n}}{(bx^n+a)^2\left(1-\frac{bx^n}{bx^n+a}\right)^3}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{cn}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{2a^2x^{-5n/2}(cx)^{5n/2}\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\int \frac{x^n}{(bx^n+a)\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{4b}\right)}{cn}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{2a^2x^{-5n/2}(cx)^{5n/2}\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\int\frac{1}{1-\frac{bx^n}{bx^n+a}}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{2b}\right)}{4b}\right)}{cn}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2a^2x^{-5n/2}(cx)^{5n/2}\left(\frac{x^{3n/2}}{4b(a+bx^n)^{3/2}\left(1-\frac{bx^n}{a+bx^n}\right)^2}-\frac{3\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{2b^{3/2}}\right)}{4b}\right)}{cn}$$

input

```
Int[(c*x)^(-1 + (5*n)/2)/Sqrt[a + b*x^n],x]
```

output

```
(2*a^2*(c*x)^((5*n)/2)*(x^((3*n)/2)/(4*b*(a + b*x^n)^(3/2)*(1 - (b*x^n)/(a
 + b*x^n))^2) - (3*(x^(n/2)/(2*b*Sqrt[a + b*x^n]*(1 - (b*x^n)/(a + b*x^n))
) - ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]]/(2*b^(3/2))))/(4*b)))/(c*n*
x^((5*n)/2))
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 880
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Simp[k*(a^(p + Simplify[(m + 1)/n])/n)   Subst[Int[x^(k*Simplify
[(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/
(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[p + Simpli
fy[(m + 1)/n]] && LtQ[-1, p, 0]
```

rule 883
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p]]
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+b\,x^n}}dx$$

input
```
int((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.44

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \left[\frac{3\,a^2\sqrt{b}c^{\frac{5}{2}\,n-1}\log\left(-2\,\sqrt{bx^n+a}\sqrt{b}x^{\frac{1}{2}\,n}-2\,bx^n-a\right)+2\left(2\,b^2c^{\frac{5}{2}\,n-1}x^{\frac{3}{2}\,n}-3\,abc^{\frac{5}{2}\,n-1}x^{\frac{1}{2}\,n}\right)\sqrt{bx^n+a}}{8\,b^3n}, -\frac{3\,a^2\sqrt{-b}c^{\frac{5}{2}\,n-1}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-b}}{bx^{\frac{1}{2}\,n}}\right)-\left(2\,b^2c^{\frac{5}{2}\,n-1}x^{\frac{3}{2}\,n}-3\,abc^{\frac{5}{2}\,n-1}x^{\frac{1}{2}\,n}\right)\sqrt{bx^n+a}}{4\,b^3n}\right]$$

input
```
integrate((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*a^2*sqrt(b)*c^(5/2*n - 1)*log(-2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n)
 - 2*b*x^n - a) + 2*(2*b^2*c^(5/2*n - 1)*x^(3/2*n) - 3*a*b*c^(5/2*n - 1)*x
^(1/2*n))*sqrt(b*x^n + a))/(b^3*n), -1/4*(3*a^2*sqrt(-b)*c^(5/2*n - 1)*arc
tan(sqrt(b*x^n + a)*sqrt(-b)/(b*x^(1/2*n))) - (2*b^2*c^(5/2*n - 1)*x^(3/2*
n) - 3*a*b*c^(5/2*n - 1)*x^(1/2*n))*sqrt(b*x^n + a))/(b^3*n)]
```

Sympy [A] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{3a^{\frac{3}{2}}c^{\frac{5n}{2}-1}x^{\frac{n}{2}}}{4b^2n\sqrt{1+\frac{bx^n}{a}}} - \frac{\sqrt{a}c^{\frac{5n}{2}-1}x^{\frac{3n}{2}}}{4bn\sqrt{1+\frac{bx^n}{a}}} + \frac{3a^2c^{\frac{5n}{2}-1}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}n} + \frac{c^{\frac{5n}{2}-1}x^{\frac{5n}{2}}}{2\sqrt{a}n\sqrt{1+\frac{bx^n}{a}}}$$

input
```
integrate((c*x)**(-1+5/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-3*a**(3/2)*c**(5*n/2 - 1)*x**(n/2)/(4*b**2*n*sqrt(1 + b*x**n/a)) - sqrt(a
)*c**(5*n/2 - 1)*x**(3*n/2)/(4*b*n*sqrt(1 + b*x**n/a)) + 3*a**2*c**(5*n/2
- 1)*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(4*b**(5/2)*n) + c**(5*n/2 - 1)*x**(5
*n/2)/(2*sqrt(a)*n*sqrt(1 + b*x**n/a))
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{5}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(c\,x)^{\frac{5\,n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((c*x)^((5*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int((c*x)^((5*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1+\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{c^{\frac{5n}{2}}\left(\int \frac{x^{\frac{5n}{2}}\sqrt{x^n b+a}}{x^n bx+ax}dx\right)}{c}$$

input
```
int((c*x)^(-1+5/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(c**((5*n)/2)*int((x**((5*n)/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x))/c
```

3.679 $\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx=\frac{x^{-n}(cx)^{3n/2}\sqrt{a+bx^n}}{bcn}-\frac{ax^{-3n/2}(cx)^{3n/2}\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{b^{3/2}cn}$$

output
```
(c*x)^(3/2*n)*(a+b*x^n)^(1/2)/b/c/n/(x^n)-a*(c*x)^(3/2*n)*arctanh(b^(1/2)*
x^(1/2*n)/(a+b*x^n)^(1/2))/b^(3/2)/c/n/(x^(3/2*n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx=\frac{ax^{1-\frac{3n}{2}}(cx)^{-1+\frac{3n}{2}}\sqrt{1+\frac{bx^n}{a}}\left(\sqrt{b}x^{n/2}\sqrt{\frac{a+bx^n}{a}}-\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)\right)}{b^{3/2}n\sqrt{a+bx^n}}$$

input
```
Integrate[(c*x)^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(a*x^(1 - (3*n)/2)*(c*x)^(-1 + (3*n)/2)*Sqrt[1 + (b*x^n)/a]*(Sqrt[b]*x^(n/
2)*Sqrt[(a + b*x^n)/a] - Sqrt[a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a]]))/(b^(
3/2)*n*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {883, 880, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{3n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{883}$$

$$\frac{x^{-3n/2}(cx)^{3n/2}\int \frac{x^{\frac{3n}{2}-1}}{\sqrt{bx^n+a}}dx}{c}$$

$$\Big\downarrow \text{880}$$

$$\frac{2ax^{-3n/2}(cx)^{3n/2}\int \frac{x^n}{(bx^n+a)\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{cn}$$

$$\Big\downarrow \text{252}$$

$$\frac{2ax^{-3n/2}(cx)^{3n/2}\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\int \frac{1}{1-\frac{bx^n}{bx^n+a}}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{2b}\right)}{cn}$$

$$\Big\downarrow \text{219}$$

$$\frac{2ax^{-3n/2}(cx)^{3n/2}\left(\frac{x^{n/2}}{2b\sqrt{a+bx^n}\left(1-\frac{bx^n}{a+bx^n}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{2b^{3/2}}\right)}{cn}$$

input
```
Int[(c*x)^(-1 + (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*a*(c*x)^((3*n)/2)*(x^(n/2)/(2*b*Sqrt[a + b*x^n]*(1 - (b*x^n)/(a + b*x^n
))) - ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]]/(2*b^(3/2))))/(c*n*x^((3*
n)/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 880
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[p]}, Simp[k*(a^(p + Simplify[(m + 1)/n])/n)   Subst[Int[x^(k*Simplify
[(m + 1)/n] - 1)/(1 - b*x^k)^(p + Simplify[(m + 1)/n] + 1), x], x, x^(n/k)/
(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[p + Simpli
fy[(m + 1)/n]] && LtQ[-1, p, 0]
```

rule 883
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p]]
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+b\,x^n}}dx$$

input
```
int((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.62

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$$
$$= \left[\frac{2\sqrt{bx^n+a}bc^{\frac{3}{2}n-1}x^{\frac{1}{2}n}+a\sqrt{b}c^{\frac{3}{2}n-1}\log\left(2\sqrt{bx^n+a}\sqrt{b}x^{\frac{1}{2}n}-2bx^n-a\right)}{2b^2n}, \frac{\sqrt{bx^n+a}bc^{\frac{3}{2}n-1}x^{\frac{1}{2}n}+a\sqrt{-}}{b^2}\right.$$

input
```
integrate((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(b*x^n + a)*b*c^(3/2*n - 1)*x^(1/2*n) + a*sqrt(b)*c^(3/2*n - 1
)*log(2*sqrt(b*x^n + a)*sqrt(b)*x^(1/2*n) - 2*b*x^n - a))/(b^2*n), (sqrt(b
*x^n + a)*b*c^(3/2*n - 1)*x^(1/2*n) + a*sqrt(-b)*c^(3/2*n - 1)*arctan(sqrt
(b*x^n + a)*sqrt(-b)/(b*x^(1/2*n))))/(b^2*n)]
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\sqrt{a}c^{\frac{3n}{2}-1}x^{\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}}{bn} - \frac{ac^{\frac{3n}{2}-1}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}n}$$

input
```
integrate((c*x)**(-1+3/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
sqrt(a)*c**(3*n/2 - 1)*x**(n/2)*sqrt(1 + b*x**n/a)/(b*n) - a*c**(3*n/2 - 1
)*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(b**(3/2)*n)
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(c\,x)^{\frac{3\,n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((c*x)^((3*n)/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int((c*x)^((3*n)/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1+\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{c^{\frac{3n}{2}}\left(\int \frac{x^{\frac{3n}{2}}\sqrt{x^nb+a}}{x^nbx+ax}dx\right)}{c}$$

input
```
int((c*x)^(-1+3/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(c**((3*n)/2)*int((x**((3*n)/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x))/c
```

3.680 $\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{2x^{-n/2}(cx)^{n/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}cn}$$

output
```
2*(c*x)^(1/2*n)*arctanh(b^(1/2)*x^(1/2*n)/(a+b*x^n)^(1/2))/b^(1/2)/c/n/(x^
(1/2*n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{a}x^{-n/2}(cx)^{n/2}\sqrt{1+\frac{bx^n}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}cn\sqrt{a+bx^n}}$$

input
```
Integrate[(c*x)^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*Sqrt[a]*(c*x)^(n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[
a]])/(Sqrt[b]*c*n*x^(n/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {870, 868, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{\frac{n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{870}$$

$$\frac{x^{-n/2}(cx)^{n/2}\int \frac{x^{\frac{n-2}{2}}}{\sqrt{bx^n+a}}dx}{c}$$

$$\Big\downarrow \text{868}$$

$$\frac{2x^{-n/2}(cx)^{n/2}\int \frac{1}{\sqrt{bx^n+a}}dx^{n/2}}{cn}$$

$$\Big\downarrow \text{224}$$

$$\frac{2x^{-n/2}(cx)^{n/2}\int \frac{1}{1-bx^n}d\frac{x^{n/2}}{\sqrt{bx^n+a}}}{cn}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x^{-n/2}(cx)^{n/2}\text{arctanh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a+bx^n}}\right)}{\sqrt{b}cn}$$

input
```
Int[(c*x)^(-1 + n/2)/Sqrt[a + b*x^n],x]
```

output
```
(2*(c*x)^(n/2)*ArcTanh[(Sqrt[b]*x^(n/2))/Sqrt[a + b*x^n]])/(Sqrt[b]*c*n*x^
(n/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 870
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !Intege
rQ[n]
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{2c^{\frac{n}{2}-1}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}n}$$

input
```
integrate((c*x)**(-1+1/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
2*c**(n/2 - 1)*asinh(sqrt(b)*x**(n/2)/sqrt(a))/(sqrt(b)*n)
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(c\,x)^{\frac{n}{2}-1}}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((c*x)^(n/2 - 1)/(a + b*x^n)^(1/2),x)
```

output
```
int((c*x)^(n/2 - 1)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1+\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{c^{\frac{n}{2}}\left(\int \frac{x^{\frac{n}{2}}\sqrt{x^n b+a}}{x^n bx+ax}dx\right)}{c}$$

input
```
int((c*x)^(-1+1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(c**(n/2)*int((x**(n/2)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x))/c
```

3.681 $\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2(cx)^{-n/2}\sqrt{a+bx^n}}{acn}$$

output
```
-2*(a+b*x^n)^(1/2)/a/c/n/((c*x)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2x(cx)^{-1-\frac{n}{2}}\sqrt{a+bx^n}}{an}$$

input
```
Integrate[(c*x)^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*x*(c*x)^(-1 - n/2)*Sqrt[a + b*x^n])/(a*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \ 796$$

$$-\frac{2(cx)^{-n/2}\sqrt{a+bx^n}}{acn}$$

input
```
Int[(c*x)^(-1 - n/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^(n/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\sqrt{b}c^{-\frac{n}{2}-1}\sqrt{\frac{ax^{-n}}{b}+1}}{an}$$

input
```
integrate((c*x)**(-1-1/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-2*sqrt(b)*c**(-n/2 - 1)*sqrt(a/(b*x**n) + 1)/(a*n)
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{-\frac{1}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{-\frac{1}{2}n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(-1/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\,x\,\sqrt{a+b\,x^n}}{a\,n\,(c\,x)^{\frac{n}{2}+1}}$$

input
```
int(1/((c*x)^(n/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
-(2*x*(a + b*x^n)^(1/2))/(a*n*(c*x)^(n/2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(cx)^{-1-\frac{n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\sqrt{x^n b+a}}{x^{\frac{n}{2}}c^{\frac{n}{2}}acn}$$

input
```
int((c*x)^(-1-1/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
( - 2*sqrt(x**n*b + a))/(x**(n/2)*c**(n/2)*a*c*n)
```

3.682 $\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx=-\frac{2(cx)^{-3n/2}\sqrt{a+bx^n}}{3acn}+\frac{4bx^n(cx)^{-3n/2}\sqrt{a+bx^n}}{3a^2cn}$$

output
```
-2/3*(a+b*x^n)^(1/2)/a/c/n/((c*x)^(3/2*n))+4/3*b*x^n*(a+b*x^n)^(1/2)/a^2/c
/n/((c*x)^(3/2*n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx=-\frac{2(cx)^{-3n/2}\left(a-2bx^n\right)\sqrt{a+bx^n}}{3a^2cn}$$

input
```
Integrate[(c*x)^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*(a - 2*b*x^n)*Sqrt[a + b*x^n])/(3*a^2*c*n*(c*x)^((3*n)/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{3n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{2\int (cx)^{-\frac{3n}{2}-1}\sqrt{bx^n+a}dx}{a}-\frac{2(cx)^{-3n/2}\sqrt{a+bx^n}}{acn}$$

$$\Big\downarrow \text{796}$$

$$\frac{4(cx)^{-3n/2}(a+bx^n)^{3/2}}{3a^2cn}-\frac{2(cx)^{-3n/2}\sqrt{a+bx^n}}{acn}$$

input
```
Int[(c*x)^(-1 - (3*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^((3*n)/2)) + (4*(a + b*x^n)^(3/2))/(3*a^
2*c*n*(c*x)^((3*n)/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{2\sqrt{b}c^{-\frac{3n}{2}-1}x^{-n}\sqrt{\frac{ax^{-n}}{b}+1}}{3an}+\frac{4b^{\frac{3}{2}}c^{-\frac{3n}{2}-1}\sqrt{\frac{ax^{-n}}{b}+1}}{3a^2n}$$

input
```
integrate((c*x)**(-1-3/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-2*sqrt(b)*c**(-3*n/2 - 1)*sqrt(a/(b*x**n) + 1)/(3*a*n*x**n) + 4*b**(3/2)*
c**(-3*n/2 - 1)*sqrt(a/(b*x**n) + 1)/(3*a**2*n)
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{-\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{-\frac{3}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(-3/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{(c\,x)^{\frac{3\,n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/((c*x)^((3*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{(cx)^{-1-\frac{3n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^n b+a}\,(2x^n b-a)}{3x^{\frac{3n}{2}}c^{\frac{3n}{2}}a^2cn}$$

input
```
int((c*x)^(-1-3/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*(2*x**n*b - a))/(3*x**((3*n)/2)*c**((3*n)/2)*a**2*c*n)
```

3.683 $\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{5acn}+\frac{8bx^n(cx)^{-5n/2}\sqrt{a+bx^n}}{15a^2cn}-\frac{16b^2x^{2n}(cx)^{-5n/2}\sqrt{a+bx^n}}{15a^3cn}$$

output
```
-2/5*(a+b*x^n)^(1/2)/a/c/n/((c*x)^(5/2*n))+8/15*b*x^n*(a+b*x^n)^(1/2)/a^2/
c/n/((c*x)^(5/2*n))-16/15*b^2*x^(2*n)*(a+b*x^n)^(1/2)/a^3/c/n/((c*x)^(5/2*
n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}(3a^2-4abx^n+8b^2x^{2n})}{15a^3cn}$$

input
```
Integrate[(c*x)^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n]*(3*a^2 - 4*a*b*x^n + 8*b^2*x^(2*n)))/(15*a^3*c*n*(c*x)
^((5*n)/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{5n}{2}-1}}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \ 805$$

$$-\frac{4\int (cx)^{-\frac{5n}{2}-1}\sqrt{bx^n+a}dx}{a}-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}$$

$$\Big\downarrow \ 805$$

$$-\frac{4\left(-\frac{2\int (cx)^{-\frac{5n}{2}-1}(bx^n+a)^{3/2}dx}{3a}-\frac{2(cx)^{-5n/2}(a+bx^n)^{3/2}}{3acn}\right)}{a}-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}$$

$$\Big\downarrow \ 796$$

$$-\frac{4\left(\frac{4(cx)^{-5n/2}(a+bx^n)^{5/2}}{15a^2cn}-\frac{2(cx)^{-5n/2}(a+bx^n)^{3/2}}{3acn}\right)}{a}-\frac{2(cx)^{-5n/2}\sqrt{a+bx^n}}{acn}$$

input
```
Int[(c*x)^(-1 - (5*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^((5*n)/2)) - (4*((-2*(a + b*x^n)^(3/2))/
(3*a*c*n*(c*x)^((5*n)/2)) + (4*(a + b*x^n)^(5/2))/(15*a^2*c*n*(c*x)^((5*n)
/2))))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(97) = 194$.

Time = 0.98 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.62

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{6a^4b^{\frac{9}{2}}c^{-\frac{5n}{2}-1}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{4a^3b^{\frac{11}{2}}c^{-\frac{5n}{2}-1}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{6a^2b^{\frac{13}{2}}c^{-\frac{5n}{2}-1}x^{2n}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{24ab^{\frac{15}{2}}c^{-\frac{5n}{2}-1}x^{3n}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}} - \frac{16b^{\frac{17}{2}}c^{-\frac{5n}{2}-1}x^{4n}\sqrt{\frac{ax^{-n}}{b}+1}}{15a^5b^4nx^{2n}+30a^4b^5nx^{3n}+15a^3b^6nx^{4n}}$$

input
```
integrate((c*x)**(-1-5/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-6*a**4*b**(9/2)*c**(-5*n/2 - 1)*sqrt(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(
2*n) + 30*a**4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 4*a**3*b**(11/
2)*c**(-5*n/2 - 1)*x**n*sqrt(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30
*a**4*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 6*a**2*b**(13/2)*c**(-5
*n/2 - 1)*x**(2*n)*sqrt(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4
*b**5*n*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 24*a*b**(15/2)*c**(-5*n/2 -
1)*x**(3*n)*sqrt(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n
*x**(3*n) + 15*a**3*b**6*n*x**(4*n)) - 16*b**(17/2)*c**(-5*n/2 - 1)*x**(4*
n)*sqrt(a/(b*x**n) + 1)/(15*a**5*b**4*n*x**(2*n) + 30*a**4*b**5*n*x**(3*n)
 + 15*a**3*b**6*n*x**(4*n))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=\int \frac{(cx)^{-\frac{5}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=\int \frac{(cx)^{-\frac{5}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(-5/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx=\int \frac{1}{(c\,x)^{\frac{5\,n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/((c*x)^((5*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/((c*x)^((5*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{5n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\int \frac{\sqrt{x^n b+a}}{x^{\frac{7n}{2}}bx+x^{\frac{5n}{2}}ax}dx}{c^{\frac{5n}{2}}c}$$

input
```
int((c*x)^(-1-5/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**((7*n)/2)*b*x + x**((5*n)/2)*a*x),x)/(c**((5*n)/2
)*c)
```

3.684 $\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}} dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}} dx = -\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{7acn} + \frac{12bx^n(cx)^{-7n/2}\sqrt{a+bx^n}}{35a^2cn} - \frac{16b^2x^{2n}(cx)^{-7n/2}\sqrt{a+bx^n}}{35a^3cn} + \frac{32b^3x^{3n}(cx)^{-7n/2}\sqrt{a+bx^n}}{35a^4cn}$$

output
```
-2/7*(a+b*x^n)^(1/2)/a/c/n/((c*x)^(7/2*n))+12/35*b*x^n*(a+b*x^n)^(1/2)/a^2
/c/n/((c*x)^(7/2*n))-16/35*b^2*x^(2*n)*(a+b*x^n)^(1/2)/a^3/c/n/((c*x)^(7/2
*n))+32/35*b^3*x^(3*n)*(a+b*x^n)^(1/2)/a^4/c/n/((c*x)^(7/2*n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}} dx = -\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}(5a^3-6a^2bx^n+8ab^2x^{2n}-16b^3x^{3n})}{35a^4cn}$$

input
```
Integrate[(c*x)^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n]*(5*a^3 - 6*a^2*b*x^n + 8*a*b^2*x^(2*n) - 16*b^3*x^(3*n
)))/(35*a^4*c*n*(c*x)^((7*n)/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {805, 805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{-\frac{7n}{2}-1}}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{6\int (cx)^{-\frac{7n}{2}-1}\sqrt{bx^n+a}dx}{a}-\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}$$

$$\Big\downarrow \text{805}$$

$$-\frac{6\left(-\frac{4\int (cx)^{-\frac{7n}{2}-1}(bx^n+a)^{3/2}dx}{3a}-\frac{2(cx)^{-7n/2}(a+bx^n)^{3/2}}{3acn}\right)}{a}-\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}$$

$$\Big\downarrow \text{805}$$

$$-\frac{6\left(-\frac{4\left(-\frac{2\int (cx)^{-\frac{7n}{2}-1}(bx^n+a)^{5/2}dx}{5a}-\frac{2(cx)^{-7n/2}(a+bx^n)^{5/2}}{5acn}\right)}{3a}-\frac{2(cx)^{-7n/2}(a+bx^n)^{3/2}}{3acn}\right)}{a}-\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}$$

$$\Big\downarrow \text{796}$$

$$-\frac{6\left(-\frac{4\left(\frac{4(cx)^{-7n/2}(a+bx^n)^{7/2}}{35a^2cn}-\frac{2(cx)^{-7n/2}(a+bx^n)^{5/2}}{5acn}\right)}{3a}-\frac{2(cx)^{-7n/2}(a+bx^n)^{3/2}}{3acn}\right)}{a}-\frac{2(cx)^{-7n/2}\sqrt{a+bx^n}}{acn}$$

input
```
Int[(c*x)^(-1 - (7*n)/2)/Sqrt[a + b*x^n],x]
```

output
```
(-2*Sqrt[a + b*x^n])/(a*c*n*(c*x)^((7*n)/2)) - (6*((-2*(a + b*x^n)^(3/2))/
(3*a*c*n*(c*x)^((7*n)/2)) - (4*((-2*(a + b*x^n)^(5/2))/(5*a*c*n*(c*x)^((7*
n)/2)) + (4*(a + b*x^n)^(7/2))/(35*a^2*c*n*(c*x)^((7*n)/2))))/(3*a)))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(134) = 268$.

Time = 1.18 (sec) , antiderivative size = 677, normalized size of antiderivative = 4.42

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = -\frac{10a^6b^{\frac{19}{2}}c^{-\frac{7n}{2}-1}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} - \frac{18a^5b^{\frac{21}{2}}c^{-\frac{7n}{2}-1}x^n\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} - \frac{10a^4b^{\frac{23}{2}}c^{-\frac{7n}{2}-1}x^{2n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{10a^3b^{\frac{25}{2}}c^{-\frac{7n}{2}-1}x^{3n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{60a^2b^{\frac{27}{2}}c^{-\frac{7n}{2}-1}x^{4n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{80ab^{\frac{29}{2}}c^{-\frac{7n}{2}-1}x^{5n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}} + \frac{32b^{\frac{31}{2}}c^{-\frac{7n}{2}-1}x^{6n}\sqrt{\frac{ax^{-n}}{b}+1}}{35a^7b^9nx^{3n}+105a^6b^{10}nx^{4n}+105a^5b^{11}nx^{5n}+35a^4b^{12}nx^{6n}}$$

input
```
integrate((c*x)**(-1-7/2*n)/(a+b*x**n)**(1/2),x)
```

output
```
-10*a**6*b**(19/2)*c**(-7*n/2 - 1)*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*x*
*(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b
**12*n*x**(6*n)) - 18*a**5*b**(21/2)*c**(-7*n/2 - 1)*x**n*sqrt(a/(b*x**n)
+ 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11
*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n)) - 10*a**4*b**(23/2)*c**(-7*n/2 - 1
)*x**(2*n)*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*
n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n)) + 10*a*
*3*b**(25/2)*c**(-7*n/2 - 1)*x**(3*n)*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n
*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**
4*b**12*n*x**(6*n)) + 60*a**2*b**(27/2)*c**(-7*n/2 - 1)*x**(4*n)*sqrt(a/(b
*x**n) + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**
5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n)) + 80*a*b**(29/2)*c**(-7*n/2
 - 1)*x**(5*n)*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*x**(3*n) + 105*a**6*b*
*10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4*b**12*n*x**(6*n)) + 3
2*b**(31/2)*c**(-7*n/2 - 1)*x**(6*n)*sqrt(a/(b*x**n) + 1)/(35*a**7*b**9*n*
x**(3*n) + 105*a**6*b**10*n*x**(4*n) + 105*a**5*b**11*n*x**(5*n) + 35*a**4
*b**12*n*x**(6*n))
```

Maxima [F]

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{-\frac{7}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{(cx)^{-\frac{7}{2}\,n-1}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(-7/2*n - 1)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \int \frac{1}{(c\,x)^{\frac{7\,n}{2}+1}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int(1/((c*x)^((7*n)/2 + 1)*(a + b*x^n)^(1/2)),x)
```

output
```
int(1/((c*x)^((7*n)/2 + 1)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1-\frac{7n}{2}}}{\sqrt{a+bx^n}}\,dx = \frac{\int \frac{\sqrt{x^n b+a}}{x^{\frac{9n}{2}}bx+x^{\frac{7n}{2}}ax}dx}{c^{\frac{7n}{2}}c}$$

input
```
int((c*x)^(-1-7/2*n)/(a+b*x^n)^(1/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**((9*n)/2)*b*x + x**((7*n)/2)*a*x),x)/(c**((7*n)/2
)*c)
```

3.685 $\int (cx)^m (a+bx^n)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int (cx)^m (a+bx^n)^p \, dx = \frac{(cx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c(1+m)}$$

output

```
(c*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/(1
+m)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int (cx)^m (a+bx^n)^p \, dx = \frac{x(cx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{1+m}$$

input

```
Integrate[(c*x)^m*(a + b*x^n)^p,x]
```

output
```
(x*(c*x)^m*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, 1 + (1 + m)/n, -
((b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (cx)^m \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)}$$

input
```
Int[(c*x)^m*(a + b*x^n)^p,x]
```

output
```
((c*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/
n, -((b*x^n)/a)])/(c*(1 + m)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m (a + b\, x^n)^p\, dx$$

input
```
int((c*x)^m*(a+b*x^n)^p,x)
```

output
```
int((c*x)^m*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (cx)^m (a + bx^n)^p\ dx = \int (bx^n + a)^p (cx)^m\ dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.92 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int (cx)^m (a+bx^n)^p \, dx = \frac{a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}+p-\frac{1}{n}} c^m x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**m*(a+b*x**n)**p,x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*c**m*x**(m + 1)*gamma(m/n + 1/n)*hyper(
(-p, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n
+ 1 + 1/n))
```

Maxima [F]

$$\int (cx)^m (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m (a+bx^n)^p \, dx = \int (c\,x)^m \,(a+b\,x^n)^p \, dx$$

input
```
int((c*x)^m*(a + b*x^n)^p,x)
```

output
```
int((c*x)^m*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int (cx)^m (a+bx^n)^p \, dx$$
$$= \frac{c^m \left(x^m (x^n b+a)^p x + \left(\int \frac{x^m (x^n b+a)^p}{x^n bm+x^n bnp+x^n b+am+anp+a} dx \right) amnp + \left(\int \frac{x^m (x^n b+a)^p}{x^n bm+x^n bnp+x^n b+am+anp+a} dx \right) a\,n^2 p^2 -}{np+m+1}$$

input
```
int((c*x)^m*(a+b*x^n)^p,x)
```

output
```
(c**m*(x**m*(x**n*b + a)**p*x + int((x**m*(x**n*b + a)**p)/(x**n*b*m + x**
n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*m*n*p + int((x**m*(x**n*b + a)**p
)/(x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*n**2*p**2 + int(
(x**m*(x**n*b + a)**p)/(x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),
x)*a*n*p))/(m + n*p + 1)
```

3.686 $\int (cx)^{-1+n} (a + bx^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int (cx)^{-1+n} (a + bx^n)^p \, dx = \frac{x^{-n}(cx)^n (a + bx^n)^{1+p}}{bcn(1 + p)}$$

output
```
(c*x)^n*(a+b*x^n)^(p+1)/b/c/n/(p+1)/(x^n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int (cx)^{-1+n} (a + bx^n)^p \, dx = \frac{x^{-n}(cx)^n (a + bx^n)^{1+p}}{bcn + bcnp}$$

input
```
Integrate[(c*x)^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
((c*x)^n*(a + b*x^n)^(1 + p))/((b*c*n + b*c*n*p)*x^n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {800, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{n-1}(a+bx^n)^p\,dx$$
$$\Big\downarrow \text{800}$$
$$\frac{x^{-n}(cx)^n\int x^{n-1}(bx^n+a)^p\,dx}{c}$$
$$\Big\downarrow \text{793}$$
$$\frac{x^{-n}(cx)^n(a+bx^n)^{p+1}}{bcn(p+1)}$$

input
```
Int[(c*x)^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
((c*x)^n*(a + b*x^n)^(1 + p))/(b*c*n*(1 + p)*x^n)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 800
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int (cx)^{-1+n} (a + b\,x^n)^p \, dx$$

input
```
int((c*x)^(-1+n)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(-1+n)*(a+b*x^n)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int (cx)^{-1+n} (a + bx^n)^p \, dx = \frac{(bc^{n-1}x^n + ac^{n-1})(bx^n + a)^p}{bnp + bn}$$

input
```
integrate((c*x)^(-1+n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
(b*c^(n - 1)*x^n + a*c^(n - 1))*(b*x^n + a)^p/(b*n*p + b*n)
```

Sympy [A] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int (cx)^{-1+n} (a + bx^n)^p \, dx = \frac{a^{p+1}c^{n-1}\left(1 + \frac{bx^n}{a}\right)^{p+1}}{bnp + bn}$$

input
```
integrate((c*x)**(-1+n)*(a+b*x**n)**p,x)
```

output
```
a**(p + 1)*c**(n - 1)*(1 + b*x**n/a)**(p + 1)/(b*n*p + b*n)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int (cx)^{-1+n} (a + bx^n)^p \, dx = \frac{(bc^n x^n + ac^n)(bx^n + a)^p}{bcn(p + 1)}$$

input
```
integrate((c*x)^(-1+n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
(b*c^n*x^n + a*c^n)*(b*x^n + a)^p/(b*c*n*(p + 1))
```

Giac [F]

$$\int (cx)^{-1+n} (a + bx^n)^p \, dx = \int (bx^n + a)^p (cx)^{n-1} \, dx$$

input
```
integrate((c*x)^(-1+n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1+n} (a + bx^n)^p \, dx = \int (c\,x)^{n-1}\,(a + b\,x^n)^p \,dx$$

input
```
int((c*x)^(n - 1)*(a + b*x^n)^p,x)
```

output
```
int((c*x)^(n - 1)*(a + b*x^n)^p, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int (cx)^{-1+n} (a+bx^n)^p \, dx = \frac{c^n (x^n b+a)^p (x^n b+a)}{bcn (p+1)}$$

input
```
int((c*x)^(-1+n)*(a+b*x^n)^p,x)
```

output
```
(c**n*(x**n*b + a)**p*(x**n*b + a))/(b*c*n*(p + 1))
```

3.687 $\int (cx)^{3n} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int (cx)^{3n} (a+bx^n)^p \, dx = \frac{(cx)^{1+3n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(3+\frac{1}{n},-p,4+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(1+3n)}$$

output
```
(c*x)^(1+3*n)*(a+b*x^n)^p*hypergeom([-p, 3+1/n],[4+1/n],-b*x^n/a)/c/(1+3*n
)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int (cx)^{3n} (a+bx^n)^p \, dx = \frac{x(cx)^{3n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(3+\frac{1}{n},-p,4+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+3n}$$

input
```
Integrate[(c*x)^(3*n)*(a + b*x^n)^p,x]
```

output
```
(x*(c*x)^(3*n)*(a + b*x^n)^p*Hypergeometric2F1[3 + n^(-1), -p, 4 + n^(-1),
 -((b*x^n)/a)])/((1 + 3*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3n} (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{ 889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (cx)^{3n} \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(cx)^{3n+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(3+\frac{1}{n},-p,4+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(3n+1)}$$

input
```
Int[(c*x)^(3*n)*(a + b*x^n)^p,x]
```

output
```
((c*x)^(1 + 3*n)*(a + b*x^n)^p*Hypergeometric2F1[3 + n^(-1), -p, 4 + n^(-1
), -((b*x^n)/a)])/(c*(1 + 3*n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{3n} (a + b\,x^n)^p\,dx$$

input
```
int((c*x)^(3*n)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(3*n)*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (cx)^{3n} (a + bx^n)^p \, dx = \int (bx^n + a)^p (cx)^{3\,n} \, dx$$

input
```
integrate((c*x)^(3*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(c*x)^(3*n), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int (cx)^{3n} (a+bx^n)^p \, dx = \frac{a^{3+\frac{1}{n}} a^{p-3-\frac{1}{n}} c^{3n} x^{3n+1} \Gamma\left(3+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 3+\frac{1}{n} \\ 4+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**(3*n)*(a+b*x**n)**p,x)
```

output
```
a**(3 + 1/n)*a**(p - 3 - 1/n)*c**(3*n)*x**(3*n + 1)*gamma(3 + 1/n)*hyper((
-p, 3 + 1/n), (4 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(4 + 1/n))
```

Maxima [F]

$$\int (cx)^{3n} (a+bx^n)^p \, dx = \int (bx^n + a)^p (cx)^{3n} \, dx$$

input
```
integrate((c*x)^(3*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(3*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int (cx)^{3n} (a+bx^n)^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(3*n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,2,4,4]%%%}+%%%{4,[2,0,6,4,2,3,4]%%%}+%%%{6,[2,0
,6,4,2,2,
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3n} (a + bx^n)^p \, dx = \int (c\,x)^{3\,n} (a + b\,x^n)^p \, dx$$

input
```
int((c*x)^(3*n)*(a + b*x^n)^p,x)
```

output
```
int((c*x)^(3*n)*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int (cx)^{3n} (a + bx^n)^p \, dx = \text{too large to display}$$

input
```
int((c*x)^(3*n)*(a+b*x^n)^p,x)
```

output
```
(c**(3*n)*(x**(3*n)*(x**n*b + a)**p*b**3*n**3*p**3*x + 3*x**(3*n)*(x**n*b
+ a)**p*b**3*n**3*p**2*x + 2*x**(3*n)*(x**n*b + a)**p*b**3*n**3*p*x + 3*x*
*(3*n)*(x**n*b + a)**p*b**3*n**2*p**2*x + 6*x**(3*n)*(x**n*b + a)**p*b**3*
n**2*p*x + 2*x**(3*n)*(x**n*b + a)**p*b**3*n**2*x + 3*x**(3*n)*(x**n*b + a
)**p*b**3*n*p*x + 3*x**(3*n)*(x**n*b + a)**p*b**3*n*x + x**(3*n)*(x**n*b +
 a)**p*b**3*x + x**(2*n)*(x**n*b + a)**p*a*b**2*n**3*p**3*x + x**(2*n)*(x*
*n*b + a)**p*a*b**2*n**3*p**2*x + 2*x**(2*n)*(x**n*b + a)**p*a*b**2*n**2*p
**2*x + x**(2*n)*(x**n*b + a)**p*a*b**2*n**2*p*x + x**(2*n)*(x**n*b + a)**
p*a*b**2*n*p*x - 2*x**n*(x**n*b + a)**p*a**2*b*n**3*p**2*x - x**n*(x**n*b
+ a)**p*a**2*b*n**2*p**2*x - 2*x**n*(x**n*b + a)**p*a**2*b*n**2*p*x - x**n
*(x**n*b + a)**p*a**2*b*n*p*x + 2*(x**n*b + a)**p*a**3*n**3*p*x + 3*(x**n*
b + a)**p*a**3*n**2*p*x + (x**n*b + a)**p*a**3*n*p*x - 2*int((x**n*b + a)*
*p/(x**n*b*n**4*p**4 + 6*x**n*b*n**4*p**3 + 11*x**n*b*n**4*p**2 + 6*x**n*b
*n**4*p + 4*x**n*b*n**3*p**3 + 18*x**n*b*n**3*p**2 + 22*x**n*b*n**3*p + 6*
x**n*b*n**3 + 6*x**n*b*n**2*p**2 + 18*x**n*b*n**2*p + 11*x**n*b*n**2 + 4*x
**n*b*n*p + 6*x**n*b*n + x**n*b + a*n**4*p**4 + 6*a*n**4*p**3 + 11*a*n**4*
p**2 + 6*a*n**4*p + 4*a*n**3*p**3 + 18*a*n**3*p**2 + 22*a*n**3*p + 6*a*n**
3 + 6*a*n**2*p**2 + 18*a*n**2*p + 11*a*n**2 + 4*a*n*p + 6*a*n + a),x)*a**4
*n**7*p**5 - 12*int((x**n*b + a)**p/(x**n*b*n**4*p**4 + 6*x**n*b*n**4*p**3
 + 11*x**n*b*n**4*p**2 + 6*x**n*b*n**4*p + 4*x**n*b*n**3*p**3 + 18*x**n...
```

3.688 $\int (cx)^{2n} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int (cx)^{2n} (a+bx^n)^p \, dx = \frac{(cx)^{1+2n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2+\frac{1}{n},-p,3+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(1+2n)}$$

output
```
(c*x)^(1+2*n)*(a+b*x^n)^p*hypergeom([-p, 2+1/n],[3+1/n],-b*x^n/a)/c/(1+2*n
)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int (cx)^{2n} (a+bx^n)^p \, dx = \frac{x(cx)^{2n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2+\frac{1}{n},-p,3+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+2n}$$

input
```
Integrate[(c*x)^(2*n)*(a + b*x^n)^p,x]
```

output
```
(x*(c*x)^(2*n)*(a + b*x^n)^p*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1),
 -((b*x^n)/a)])/((1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{2n} (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (cx)^{2n} \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{2n+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(2+\frac{1}{n},-p,3+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(2n+1)}$$

input
```
Int[(c*x)^(2*n)*(a + b*x^n)^p,x]
```

output
```
((c*x)^(1 + 2*n)*(a + b*x^n)^p*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1
), -((b*x^n)/a)])/(c*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{2n} (a + b\,x^{n})^{p}\,dx$$

input
```
int((c*x)^(2*n)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(2*n)*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (cx)^{2n} (a + bx^n)^p \, dx = \int (bx^n + a)^p (cx)^{2\,n} \, dx$$

input
```
integrate((c*x)^(2*n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(c*x)^(2*n), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int (cx)^{2n}(a+bx^n)^p\,dx = \frac{a^{2+\frac{1}{n}}a^{p-2-\frac{1}{n}}c^{2n}x^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**(2*n)*(a+b*x**n)**p,x)
```

output
```
a**(2 + 1/n)*a**(p - 2 - 1/n)*c**(2*n)*x**(2*n + 1)*gamma(2 + 1/n)*hyper((
-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n))
```

Maxima [F]

$$\int (cx)^{2n}(a+bx^n)^p\,dx = \int (bx^n+a)^p(cx)^{2n}\,dx$$

input
```
integrate((c*x)^(2*n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int (cx)^{2n}(a+bx^n)^p\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^(2*n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3]%%%}+%%%{-3,[1,0,4,3,1,2,3]%%%}+%%%{-3,[
1,0,4,3,1
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{2n} (a + bx^n)^p \, dx = \int (c\,x)^{2\,n} \, (a + b\,x^n)^p \, dx$$

input
```
int((c*x)^(2*n)*(a + b*x^n)^p,x)
```

output
```
int((c*x)^(2*n)*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int (cx)^{2n} (a + bx^n)^p \, dx = \text{too large to display}$$

input
```
int((c*x)^(2*n)*(a+b*x^n)^p,x)
```

output
```
(c**(2*n)*(x**(2*n)*(x***n*b + a)**p*b**2*n**2*p**2*x + x**(2*n)*(x***n*b +
a)**p*b**2*n**2*p*x + 2*x**(2*n)*(x***n*b + a)**p*b**2*n*p*x + x**(2*n)*(x*
*n*b + a)**p*b**2*n*x + x**(2*n)*(x***n*b + a)**p*b**2*x + x***n*(x***n*b + a
)**p*a*b*n**2*p**2*x + x***n*(x***n*b + a)**p*a*b*n*p*x - (x***n*b + a)**p*a*
*2*n**2*p*x - (x***n*b + a)**p*a**2*n*p*x + int((x***n*b + a)**p/(x***n*b*n**
3*p**3 + 3*x***n*b*n**3*p**2 + 2*x***n*b*n**3*p + 3*x***n*b*n**2*p**2 + 6*x**
n*b*n**2*p + 2*x***n*b*n**2 + 3*x***n*b*n*p + 3*x***n*b*n + x***n*b + a*n**3*p
**3 + 3*a*n**3*p**2 + 2*a*n**3*p + 3*a*n**2*p**2 + 6*a*n**2*p + 2*a*n**2 +
 3*a*n*p + 3*a*n + a),x)*a**3*n**5*p**4 + 3*int((x***n*b + a)**p/(x***n*b*n*
*3*p**3 + 3*x***n*b*n**3*p**2 + 2*x***n*b*n**3*p + 3*x***n*b*n**2*p**2 + 6*x*
*n*b*n**2*p + 2*x***n*b*n**2 + 3*x***n*b*n*p + 3*x***n*b*n + x***n*b + a*n**3*
p**3 + 3*a*n**3*p**2 + 2*a*n**3*p + 3*a*n**2*p**2 + 6*a*n**2*p + 2*a*n**2
+ 3*a*n*p + 3*a*n + a),x)*a**3*n**5*p**3 + 2*int((x***n*b + a)**p/(x***n*b*n
**3*p**3 + 3*x***n*b*n**3*p**2 + 2*x***n*b*n**3*p + 3*x***n*b*n**2*p**2 + 6*x
**n*b*n**2*p + 2*x***n*b*n**2 + 3*x***n*b*n*p + 3*x***n*b*n + x***n*b + a*n**3
*p**3 + 3*a*n**3*p**2 + 2*a*n**3*p + 3*a*n**2*p**2 + 6*a*n**2*p + 2*a*n**2
 + 3*a*n*p + 3*a*n + a),x)*a**3*n**5*p**2 + int((x***n*b + a)**p/(x***n*b*n*
*3*p**3 + 3*x***n*b*n**3*p**2 + 2*x***n*b*n**3*p + 3*x***n*b*n**2*p**2 + 6*x*
*n*b*n**2*p + 2*x***n*b*n**2 + 3*x***n*b*n*p + 3*x***n*b*n + x***n*b + a*n**3*
p**3 + 3*a*n**3*p**2 + 2*a*n**3*p + 3*a*n**2*p**2 + 6*a*n**2*p + 2*a*n*...
```

3.689 $\int (cx)^n (a+bx^n)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int (cx)^n (a+bx^n)^p \, dx = \frac{(cx)^{1+n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(1+n)}$$

output
```
(c*x)^(1+n)*(a+b*x^n)^p*hypergeom([-p, 1+1/n],[2+1/n],-b*x^n/a)/c/(1+n)/((
1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int (cx)^n (a+bx^n)^p \, dx = \frac{x(cx)^n (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+n}$$

input
```
Integrate[(c*x)^n*(a + b*x^n)^p,x]
```

output
```
(x*(c*x)^n*(a + b*x^n)^p*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1), -((
b*x^n)/a)])/((1 + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^n (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (cx)^n \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{n+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(n+1)}$$

input
```
Int[(c*x)^n*(a + b*x^n)^p,x]
```

output
```
((c*x)^(1 + n)*(a + b*x^n)^p*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1),
 -((b*x^n)/a)])/(c*(1 + n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^n (a+b\,x^n)^p\,dx$$

input
```
int((c*x)^n*(a+b*x^n)^p,x)
```

output
```
int((c*x)^n*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (cx)^n (a+bx^n)^p\,dx=\int (bx^n+a)^p(cx)^n\,dx$$

input
```
integrate((c*x)^n*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(c*x)^n, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int (cx)^n (a+bx^n)^p \, dx = \frac{a^{1+\frac{1}{n}} a^{p-1-\frac{1}{n}} c^n x^{n+1} \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**n*(a+b*x**n)**p,x)
```

output
```
a**(1 + 1/n)*a**(p - 1 - 1/n)*c**n*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1
+ 1/n), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int (cx)^n (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^n \, dx$$

input
```
integrate((c*x)^n*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int (cx)^n (a+bx^n)^p \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^n*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,0,2,1,0,1,2]%%%}+%%%{-1,[0,0,2,1,0,0,2]%%%} / %%%{1,
[0,0,3,2,
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^n (a+bx^n)^p \, dx = \int (c\,x)^n\,(a+b\,x^n)^p \, dx$$

input
```
int((c*x)^n*(a + b*x^n)^p,x)
```

output
```
int((c*x)^n*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int (cx)^n (a+bx^n)^p \, dx$$
$$= \frac{c^n \left(x^n (x^n b+a)^p bnpx + x^n (x^n b+a)^p bx + (x^n b+a)^p anpx - \left(\int \frac{(x^n b+a)^p}{x^n b n^2 p^2+x^n b n^2 p+2x^n bnp+x^n bn+x^n b+a n^2 p^2+a\right.\right.}{}$$

input
```
int((c*x)^n*(a+b*x^n)^p,x)
```

output
```
(c**n*(x***n*(x***n*b + a)**p*b*n*p*x + x***n*(x***n*b + a)**p*b*x + (x***n*b +
 a)**p*a*n*p*x - int((x***n*b + a)**p/(x***n*b*n**2*p**2 + x***n*b*n**2*p + 2
*x***n*b*n*p + x***n*b*n + x***n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n +
 a),x)*a**2*n**3*p**3 - int((x***n*b + a)**p/(x***n*b*n**2*p**2 + x***n*b*n**
2*p + 2*x***n*b*n*p + x***n*b*n + x***n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p
+ a*n + a),x)*a**2*n**3*p**2 - 2*int((x***n*b + a)**p/(x***n*b*n**2*p**2 + x
**n*b*n**2*p + 2*x***n*b*n*p + x***n*b*n + x***n*b + a*n**2*p**2 + a*n**2*p +
 2*a*n*p + a*n + a),x)*a**2*n**2*p**2 - int((x***n*b + a)**p/(x***n*b*n**2*p
**2 + x***n*b*n**2*p + 2*x***n*b*n*p + x***n*b*n + x***n*b + a*n**2*p**2 + a*n
**2*p + 2*a*n*p + a*n + a),x)*a**2*n**2*p - int((x***n*b + a)**p/(x***n*b*n*
*2*p**2 + x***n*b*n**2*p + 2*x***n*b*n*p + x***n*b*n + x***n*b + a*n**2*p**2 +
 a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n*p))/(b*(n**2*p**2 + n**2*p + 2*n*
p + n + 1))
```

3.690 $\int (a+bx^n)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 46

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

output
```
x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p \, dx = x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Integrate[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Int[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^{n})^{p}\,dx$$

input
```
int((a+b*x^n)^p,x)
```

output
```
int((a+b*x^n)^p,x)
```

Fricas [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p \, dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p,x)
```

output
```
a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p, x)
```

Giac [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int (a+bx^n)^p \, dx = \frac{x\,(a+b\,x^n)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a}+1\right)^p}$$

input
```
int((a + b*x^n)^p,x)
```

output
```
(x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1
)^p
```

Reduce [F]

$$\int (a+bx^n)^p \, dx$$
$$= \frac{(x^n b+a)^p x + \left(\int \frac{(x^n b+a)^p}{x^n bnp+x^n b+anp+a}dx\right) a\, n^2 p^2 + \left(\int \frac{(x^n b+a)^p}{x^n bnp+x^n b+anp+a}dx\right) anp}{np+1}$$

input
```
int((a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*x + int((x**n*b + a)**p/(x**n*b*n*p + x**n*b + a*n*p + a)
,x)*a*n**2*p**2 + int((x**n*b + a)**p/(x**n*b*n*p + x**n*b + a*n*p + a),x)
*a*n*p)/(n*p + 1)
```

3.691 $\int (cx)^{-n} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int (cx)^{-n} (a+bx^n)^p \, dx = \frac{(cx)^{1-n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-1+\frac{1}{n},-p,\frac{1}{n},-\frac{bx^n}{a}\right)}{c(1-n)}$$

output
```
(c*x)^(1-n)*(a+b*x^n)^p*hypergeom([-p, -1+1/n],[1/n],-b*x^n/a)/c/(1-n)/((1
+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int (cx)^{-n} (a+bx^n)^p \, dx = -\frac{x(cx)^{-n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-1+\frac{1}{n},-p,\frac{1}{n},-\frac{bx^n}{a}\right)}{-1+n}$$

input
```
Integrate[(a + b*x^n)^p/(c*x)^n,x]
```

output
```
-((x*(a + b*x^n)^p*Hypergeometric2F1[-1 + n^(-1), -p, n^(-1), -((b*x^n)/a)
])/((-1 + n)*(c*x)^n*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-n} (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (cx)^{-n} \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{1-n} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n}-1,-p,\frac{1}{n},-\frac{bx^n}{a}\right)}{c(1-n)}$$

input
```
Int[(a + b*x^n)^p/(c*x)^n,x]
```

output
```
((c*x)^(1 - n)*(a + b*x^n)^p*Hypergeometric2F1[-1 + n^(-1), -p, n^(-1), -(
(b*x^n)/a)])/(c*(1 - n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^{n})^{p}\,(cx)^{-n}\,dx$$

input
```
int((a+b*x^n)^p/((c*x)^n),x)
```

output
```
int((a+b*x^n)^p/((c*x)^n),x)
```

Fricas [F]

$$\int (cx)^{-n}\,(a+bx^n)^p\,dx=\int \frac{(bx^n+a)^p}{(cx)^n}\,dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(c*x)^n, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int (cx)^{-n}(a+bx^n)^p\,dx = \frac{a^{-1+\frac{1}{n}}a^{p+1-\frac{1}{n}}b^{-1+\frac{1}{n}}b^{1-\frac{1}{n}}c^{-n}x^{1-n}\Gamma\left(-1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,-1+\frac{1}{n}\\ \frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p/((c*x)**n),x)
```

output
```
a**(-1 + 1/n)*a**(p + 1 - 1/n)*b**(-1 + 1/n)*b**(1 - 1/n)*x**(1 - n)*gamma
(-1 + 1/n)*hyper((-p, -1 + 1/n), (1/n,), b*x**n*exp_polar(I*pi)/a)/(c**n*n
*gamma(1/n))
```

Maxima [F]

$$\int (cx)^{-n}(a+bx^n)^p\,dx = \int \frac{(bx^n+a)^p}{(cx)^n}\,dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(c*x)^n, x)
```

Giac [F]

$$\int (cx)^{-n}\left(a+bx^n\right)^p\,dx=\int \frac{\left(bx^n+a\right)^p}{\left(cx\right)^n}\,dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(c*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-n}\left(a+bx^n\right)^p\,dx=\int \frac{\left(a+b\,x^n\right)^p}{\left(c\,x\right)^n}\,dx$$

input
```
int((a + b*x^n)^p/(c*x)^n,x)
```

output
```
int((a + b*x^n)^p/(c*x)^n, x)
```

Reduce [F]

$$\int (cx)^{-n}\left(a+bx^n\right)^p\,dx$$
$$=\frac{\left(x^n b+a\right)^p x+x^n\left(\int \frac{\left(x^n b+a\right)^p}{x^{2n} bnp-x^{2n} bn+x^{2n} b+x^n anp-x^n an+x^n a}dx\right)a\,n^2p^2-x^n\left(\int \frac{\left(x^n b+a\right)^p}{x^{2n} bnp-x^{2n} bn+x^{2n} b+x^n anp-x^n an+x^n}\right.}{x^n c^n\left(np-n+1\right)}$$

input
```
int((a+b*x^n)^p/((c*x)^n),x)
```

output
```
((x**n*b + a)**p*x + x**n*int((x**n*b + a)**p/(x**(2*n)*b*n*p - x**(2*n)*b
*n + x**(2*n)*b + x**n*a*n*p - x**n*a*n + x**n*a),x)*a*n**2*p**2 - x**n*in
t((x**n*b + a)**p/(x**(2*n)*b*n*p - x**(2*n)*b*n + x**(2*n)*b + x**n*a*n*p
 - x**n*a*n + x**n*a),x)*a*n**2*p + x**n*int((x**n*b + a)**p/(x**(2*n)*b*n
*p - x**(2*n)*b*n + x**(2*n)*b + x**n*a*n*p - x**n*a*n + x**n*a),x)*a*n*p)
/(x**n*c**n*(n*p - n + 1))
```

3.692 $\int (cx)^{-2n} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int (cx)^{-2n} (a+bx^n)^p \, dx = \frac{(cx)^{1-2n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-2+\frac{1}{n},-p,-1+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(1-2n)}$$

output
```
(c*x)^(1-2*n)*(a+b*x^n)^p*hypergeom([-p, -2+1/n],[-1+1/n],-b*x^n/a)/c/(1-2
*n)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int (cx)^{-2n} (a+bx^n)^p \, dx = -\frac{x(cx)^{-2n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-2+\frac{1}{n},-p,-1+\frac{1}{n},-\frac{bx^n}{a}\right)}{-1+2n}$$

input
```
Integrate[(a + b*x^n)^p/(c*x)^(2*n),x]
```

output
```
-((x*(a + b*x^n)^p*Hypergeometric2F1[-2 + n^(-1), -p, -1 + n^(-1), -((b*x^
n)/a)])/((-1 + 2*n)*(c*x)^(2*n)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2n} (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{ 889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (cx)^{-2n} \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(cx)^{1-2n} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n}-2,-p,\frac{1}{n}-1,-\frac{bx^n}{a}\right)}{c(1-2n)}$$

input
```
Int[(a + b*x^n)^p/(c*x)^(2*n),x]
```

output
```
((c*x)^(1 - 2*n)*(a + b*x^n)^p*Hypergeometric2F1[-2 + n^(-1), -p, -1 + n^(
-1), -((b*x^n)/a)])/(c*(1 - 2*n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^{n})^{p} \,(cx)^{-2n}\,dx$$

input
```
int((a+b*x^n)^p/((c*x)^(2*n)),x)
```

output
```
int((a+b*x^n)^p/((c*x)^(2*n)),x)
```

Fricas [F]

$$\int (cx)^{-2n}(a+bx^n)^p\,dx=\int \frac{(bx^n+a)^p}{(cx)^{2\,n}}\,dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^(2*n)),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(c*x)^(2*n), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.72 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int (cx)^{-2n} (a+bx^n)^p \, dx = \frac{a^{-2+\frac{1}{n}} a^{p+2-\frac{1}{n}} b^{-2+\frac{1}{n}} b^{2-\frac{1}{n}} c^{-2n} x^{1-2n} \Gamma\left(-2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, -2+\frac{1}{n} \\ -1+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(-1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p/((c*x)**(2*n)),x)
```

output
```
a**(-2 + 1/n)*a**(p + 2 - 1/n)*b**(-2 + 1/n)*b**(2 - 1/n)*x**(1 - 2*n)*gam
ma(-2 + 1/n)*hyper((-p, -2 + 1/n), (-1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/
(c**(2*n)*n*gamma(-1 + 1/n))
```

Maxima [F]

$$\int (cx)^{-2n} (a+bx^n)^p \, dx = \int \frac{(bx^n+a)^p}{(cx)^{2n}} \, dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^(2*n)),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(c*x)^(2*n), x)
```

Giac [F]

$$\int (cx)^{-2n} (a+bx^n)^p \, dx = \int \frac{(bx^n+a)^p}{(cx)^{2n}} \, dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^(2*n)),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(c*x)^(2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-2n} (a+bx^n)^p \, dx = \int \frac{(a+b\,x^n)^p}{(c\,x)^{2\,n}} \, dx$$

input
```
int((a + b*x^n)^p/(c*x)^(2*n),x)
```

output
```
int((a + b*x^n)^p/(c*x)^(2*n), x)
```

Reduce [F]

$$\int (cx)^{-2n} (a+bx^n)^p \, dx$$
$$= \frac{(x^n b+a)^p x + x^{2n}\left(\int \frac{(x^n b+a)^p}{x^{3n}bnp-2x^{3n}bn+x^{3n}b+x^{2n}anp-2x^{2n}an+x^{2n}a}dx\right) a\,n^2p^2 - 2x^{2n}\left(\int \frac{(x^n b+a)^p}{x^{3n}bnp-2x^{3n}bn+x^{3n}b+x^{2n}anp\cdots}\right.}{x^{2n}c^{2n}\,(np-2n+1)}$$

input
```
int((a+b*x^n)^p/((c*x)^(2*n)),x)
```

output
```
((x**n*b + a)**p*x + x**(2*n)*int((x**n*b + a)**p/(x**(3*n)*b*n*p - 2*x**(
3*n)*b*n + x**(3*n)*b + x**(2*n)*a*n*p - 2*x**(2*n)*a*n + x**(2*n)*a),x)*a
*n**2*p**2 - 2*x**(2*n)*int((x**n*b + a)**p/(x**(3*n)*b*n*p - 2*x**(3*n)*b
*n + x**(3*n)*b + x**(2*n)*a*n*p - 2*x**(2*n)*a*n + x**(2*n)*a),x)*a*n**2*
p + x**(2*n)*int((x**n*b + a)**p/(x**(3*n)*b*n*p - 2*x**(3*n)*b*n + x**(3*
n)*b + x**(2*n)*a*n*p - 2*x**(2*n)*a*n + x**(2*n)*a),x)*a*n*p)/(x**(2*n)*c
**(2*n)*(n*p - 2*n + 1))
```

3.693 $\int (cx)^{-3n} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int (cx)^{-3n} (a+bx^n)^p \, dx = \frac{(cx)^{1-3n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-3+\frac{1}{n},-p,-2+\frac{1}{n},-\frac{bx^n}{a}\right)}{c(1-3n)}$$

output
```
(c*x)^(1-3*n)*(a+b*x^n)^p*hypergeom([-p, -3+1/n],[-2+1/n],-b*x^n/a)/c/(1-3
*n)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int (cx)^{-3n} (a+bx^n)^p \, dx = -\frac{x(cx)^{-3n} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-3+\frac{1}{n},-p,-2+\frac{1}{n},-\frac{bx^n}{a}\right)}{-1+3n}$$

input
```
Integrate[(a + b*x^n)^p/(c*x)^(3*n),x]
```

output
```
-((x*(a + b*x^n)^p*Hypergeometric2F1[-3 + n^(-1), -p, -2 + n^(-1), -((b*x^
n)/a)])/((-1 + 3*n)*(c*x)^(3*n)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-3n} (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (cx)^{-3n} \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{1-3n} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n}-3,-p,\frac{1}{n}-2,-\frac{bx^n}{a}\right)}{c(1-3n)}$$

input
```
Int[(a + b*x^n)^p/(c*x)^(3*n),x]
```

output
```
((c*x)^(1 - 3*n)*(a + b*x^n)^p*Hypergeometric2F1[-3 + n^(-1), -p, -2 + n^(
-1), -((b*x^n)/a)])/(c*(1 - 3*n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^{n})^{p}\,(cx)^{-3n}\,dx$$

input
```
int((a+b*x^n)^p/((c*x)^(3*n)),x)
```

output
```
int((a+b*x^n)^p/((c*x)^(3*n)),x)
```

Fricas [F]

$$\int (cx)^{-3n}\,(a+bx^n)^p\;dx=\int \frac{(bx^n+a)^p}{(cx)^{3\,n}}\,dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^(3*n)),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p/(c*x)^(3*n), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int (cx)^{-3n}(a+bx^n)^p\,dx = \frac{a^{-3+\frac{1}{n}}a^{p+3-\frac{1}{n}}b^{-3+\frac{1}{n}}b^{3-\frac{1}{n}}c^{-3n}x^{1-3n}\Gamma\left(-3+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,-3+\frac{1}{n}\\ -2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(-2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p/((c*x)**(3*n)),x)
```

output
```
a**(-3 + 1/n)*a**(p + 3 - 1/n)*b**(-3 + 1/n)*b**(3 - 1/n)*x**(1 - 3*n)*gam
ma(-3 + 1/n)*hyper((-p, -3 + 1/n), (-2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/
(c**(3*n)*n*gamma(-2 + 1/n))
```

Maxima [F]

$$\int (cx)^{-3n}(a+bx^n)^p\,dx = \int \frac{(bx^n+a)^p}{(cx)^{3n}}\,dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^(3*n)),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p/(c*x)^(3*n), x)
```

Giac [F]

$$\int (cx)^{-3n} (a+bx^n)^p \, dx = \int \frac{(bx^n+a)^p}{(cx)^{3n}} \, dx$$

input
```
integrate((a+b*x^n)^p/((c*x)^(3*n)),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p/(c*x)^(3*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-3n} (a+bx^n)^p \, dx = \int \frac{(a+b\,x^n)^p}{(c\,x)^{3\,n}} \, dx$$

input
```
int((a + b*x^n)^p/(c*x)^(3*n),x)
```

output
```
int((a + b*x^n)^p/(c*x)^(3*n), x)
```

Reduce [F]

$$\int (cx)^{-3n} (a+bx^n)^p \, dx$$
$$= \frac{(x^n b+a)^p x + x^{3n} \left(\int \frac{(x^n b+a)^p}{x^{4n} bnp - 3x^{4n} bn + x^{4n} b + x^{3n} anp - 3x^{3n} an + x^{3n} a} dx\right) a\, n^2 p^2 - 3x^{3n} \left(\int \frac{(x^n b+a)^p}{x^{4n} bnp - 3x^{4n} bn + x^{4n} b + x^{3n} anp} \right.}{x^{3n} c^{3n} (np-3n+1)}$$

input
```
int((a+b*x^n)^p/((c*x)^(3*n)),x)
```

output
```
((x**n*b + a)**p*x + x**(3*n)*int((x**n*b + a)**p/(x**(4*n)*b*n*p - 3*x**(
4*n)*b*n + x**(4*n)*b + x**(3*n)*a*n*p - 3*x**(3*n)*a*n + x**(3*n)*a),x)*a
*n**2*p**2 - 3*x**(3*n)*int((x**n*b + a)**p/(x**(4*n)*b*n*p - 3*x**(4*n)*b
*n + x**(4*n)*b + x**(3*n)*a*n*p - 3*x**(3*n)*a*n + x**(3*n)*a),x)*a*n**2*
p + x**(3*n)*int((x**n*b + a)**p/(x**(4*n)*b*n*p - 3*x**(4*n)*b*n + x**(4*
n)*b + x**(3*n)*a*n*p - 3*x**(3*n)*a*n + x**(3*n)*a),x)*a*n*p)/(x**(3*n)*c
**(3*n)*(n*p - 3*n + 1))
```

3.694 $\int (cx)^{-1+n-np} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \frac{(cx)^{n-np} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{cn(1-p)}$$

output
```
(c*x)^(-n*p+n)*(a+b*x^n)^p*hypergeom([-p, 1-p],[2-p],-b*x^n/a)/c/n/(1-p)/(
(1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \frac{(cx)^{n-np} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{cn-cnp}$$

input
```
Integrate[(c*x)^(-1 + n - n*p)*(a + b*x^n)^p,x]
```

output
```
((c*x)^(n - n*p)*(a + b*x^n)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^
n)/a)])/((c*n - c*n*p)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {883, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{n(-p)+n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{883}$$

$$\frac{x^{-n(1-p)}(cx)^{n-np}\int x^{-pn+n-1}(bx^n+a)^p\,dx}{c}$$

$$\Big\downarrow \text{882}$$

$$\frac{ax^{-n(1-p)-np}(cx)^{n-np}\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^n}{bx^n+a}}{cn}$$

$$\Big\downarrow \text{74}$$

$$\frac{ax^{-(n(1-p))-np+n}(cx)^{n-np}(a+bx^n)^{p-1}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^n}{bx^n+a}\right)}{cn(1-p)}$$

input
```
Int[(c*x)^(-1 + n - n*p)*(a + b*x^n)^p,x]
```

output
```
(a*x^(n - n*(1 - p) - n*p)*(c*x)^(n - n*p)*(a + b*x^n)^(-1 + p)*Hypergeome
tric2F1[2, 1 - p, 2 - p, (b*x^n)/(a + b*x^n)])/(c*n*(1 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 883
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p]]
```

Maple [F]

$$\int (cx)^{-np+n-1} (a+b\,x^n)^p\,dx$$

input
```
int((c*x)^(-n*p+n-1)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(-n*p+n-1)*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^{-np+n-1} \, dx$$

input
```
integrate((c*x)^(-n*p+n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(c*x)^(-n*p + n - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \frac{a^{1-p}a^{2p-1}b^{1-p}b^{p-1}c^{-np+n-1}x^{-np+n}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma(2-p)}$$

input
```
integrate((c*x)**(-n*p+n-1)*(a+b*x**n)**p,x)
```

output
```
a**(1 - p)*a**(2*p - 1)*b**(1 - p)*b**(p - 1)*c**(-n*p + n - 1)*x**(-n*p +
 n)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), b*x**n*exp_polar(I*pi)/a)/(n
*gamma(2 - p))
```

Maxima [F]

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \int (bx^n + a)^p (cx)^{-np+n-1} \, dx$$

input
```
integrate((c*x)^(-n*p+n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p + n - 1), x)
```

Giac [F]

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^{-np+n-1} \, dx$$

input
```
integrate((c*x)^(-n*p+n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p + n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \int (c\,x)^{n-n\,p-1}\,(a+b\,x^n)^p \, dx$$

input
```
int((c*x)^(n - n*p - 1)*(a + b*x^n)^p,x)
```

output
```
int((c*x)^(n - n*p - 1)*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int (cx)^{-1+n-np} (a+bx^n)^p \, dx = \frac{c^n \left(x^n (x^n b+a)^p + x^{np} \left(\int \frac{x^n (x^n b+a)^p}{x^{np+n} bx + x^{np} ax} dx\right) anp\right)}{x^{np} c^{np} cn}$$

input
```
int((c*x)^(-n*p+n-1)*(a+b*x^n)^p,x)
```

output
```
(c**n*(x**n*(x**n*b + a)**p + x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p
 + n)*b*x + x**(n*p)*a*x),x)*a*n*p))/(x**(n*p)*c**(n*p)*c*n)
```

3.695 $\int (cx)^{-1-np} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 63

$$\int (cx)^{-1-np} (a+bx^n)^p \, dx = -\frac{(cx)^{-np} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{cnp}$$

output
```
-(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/c/n/p/((c*x)^(n*p))/((1+b*
x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int (cx)^{-1-np} (a+bx^n)^p \, dx = -\frac{x(cx)^{-1-np} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}$$

input
```
Integrate[(c*x)^(-1 - n*p)*(a + b*x^n)^p,x]
```

output
```
-((x*(c*x)^(-1 - n*p)*(a + b*x^n)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*
x^n)/a)])/(n*p*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {883, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-np-1} (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{ 883}$$

$$\frac{x^{np}(cx)^{-np} \int x^{-np-1}(bx^n+a)^p \, dx}{c}$$

$$\Big\downarrow \text{ 882}$$

$$\frac{(cx)^{-np} \left(\frac{x^n}{a+bx^n}\right)^p (a+bx^n)^p \int \frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-1}}{1-\frac{bx^n}{bx^n+a}} d\frac{x^n}{bx^n+a}}{cn}$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{(cx)^{-np} (a+bx^n)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{bx^n}{bx^n+a}\right)}{cnp}$$

input
```
Int[(c*x)^(-1 - n*p)*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[1, -p, 1 - p, (b*x^n)/(a + b*x^n)])/(c*
n*p*(c*x)^(n*p)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 883
```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a + b*x^n)^p, x], x] /
; FreeQ[{a, b, c, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p]]
```

Maple [F]

$$\int (cx)^{-np-1} (a+b\,x^{n})^{p}\,dx$$

input
```
int((c*x)^(-n*p-1)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(-n*p-1)*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (cx)^{-1-np} (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^{-np-1} \, dx$$

input
```
integrate((c*x)^(-n*p-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(c*x)^(-n*p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int (cx)^{-1-np} (a+bx^n)^p \, dx = \frac{a^p c^{-np-1} x^{-np} \Gamma(-p) \, {}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma(1-p)}$$

input
```
integrate((c*x)**(-n*p-1)*(a+b*x**n)**p,x)
```

output
```
a**p*c**(-n*p - 1)*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**n*exp_polar(I*
pi)/a)/(n*x**(n*p)*gamma(1 - p))
```

Maxima [F]

$$\int (cx)^{-1-np} (a+bx^n)^p \, dx = \int (bx^n + a)^p (cx)^{-np-1} \, dx$$

input
```
integrate((c*x)^(-n*p-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 1), x)
```

Giac [F]

$$\int (cx)^{-1-np}\left(a+bx^n\right)^p\,dx=\int (bx^n+a)^p(cx)^{-np-1}\,dx$$

input
```
integrate((c*x)^(-n*p-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-np}\left(a+bx^n\right)^p\,dx=\int \frac{\left(a+b\,x^n\right)^p}{\left(c\,x\right)^{n\,p+1}}\,dx$$

input
```
int((a + b*x^n)^p/(c*x)^(n*p + 1),x)
```

output
```
int((a + b*x^n)^p/(c*x)^(n*p + 1), x)
```

Reduce [F]

$$\int (cx)^{-1-np}\left(a+bx^n\right)^p\,dx=\frac{\int \frac{(x^nb+a)^p}{x^{np}x}dx}{c^{np}c}$$

input
```
int((c*x)^(-n*p-1)*(a+b*x^n)^p,x)
```

output
```
int((x**n*b + a)**p/(x**(n*p)*x),x)/(c**(n*p)*c)
```

3.696 $\int (cx)^{-1-n-np} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 37

$$\int (cx)^{-1-n-np} (a+bx^n)^p \, dx = -\frac{(cx)^{-n(1+p)} (a+bx^n)^{1+p}}{acn(1+p)}$$

output
```
-(a+b*x^n)^(p+1)/a/c/n/(p+1)/((c*x)^(n*(p+1)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (cx)^{-1-n-np} (a+bx^n)^p \, dx = -\frac{x(cx)^{-1-n(1+p)} (a+bx^n)^{1+p}}{an(1+p)}$$

input
```
Integrate[(c*x)^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((x*(c*x)^(-1 - n*(1 + p))*(a + b*x^n)^(1 + p))/(a*n*(1 + p)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{n(-p)-n-1} (a+bx^n)^p \, dx$$

$$\Big\downarrow \; 796$$

$$-\frac{(cx)^{-n(p+1)} (a+bx^n)^{p+1}}{acn(p+1)}$$

input
```
Int[(c*x)^(-1 - n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(1 + p))))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.00

method	result	size
parallelrisch	$-\frac{x\,x^{n} \left(a+b\,x^{n}\right)^{p} \left(c x\right)^{-n p-n-1} b^{2}+x \left(a+b\,x^{n}\right)^{p} \left(c x\right)^{-n p-n-1} a b}{n \left(p+1\right) b a}$	74

input
```
int((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
-(x*x^n*(a+b*x^n)^p*(c*x)^(-n*p-n-1)*b^2+x*(a+b*x^n)^p*(c*x)^(-n*p-n-1)*a*
b)/n/(p+1)/b/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int (cx)^{-1-n-np} (a+bx^n)^p \, dx = -\frac{\left(bxx^n e^{(-(np+n+1)\log(c)-(np+n+1)\log(x))} + axe^{(-(np+n+1)\log(c)-(np+n+1)\log(x))}\right)(bx^n+a)^p}{anp+an}$$

input
```
integrate((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
-(b*x*x^n*e^(-(n*p + n + 1)*log(c) - (n*p + n + 1)*log(x)) + a*x*e^(-(n*p
+ n + 1)*log(c) - (n*p + n + 1)*log(x)))*(b*x^n + a)^p/(a*n*p + a*n)
```

Sympy [A] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int (cx)^{-1-n-np} (a+bx^n)^p \, dx = \frac{a^p a^{-p-1} b^{p+1} c^{-np-n-1} \left(\frac{ax^{-n}}{b}+1\right)^{p+1} \Gamma(-p-1)}{n\Gamma(-p)}$$

input
```
integrate((c*x)**(-n*p-n-1)*(a+b*x**n)**p,x)
```

output
```
a**p*a**(-p - 1)*b**(p + 1)*c**(-n*p - n - 1)*(a/(b*x**n) + 1)**(p + 1)*ga
mma(-p - 1)/(n*gamma(-p))
```

Maxima [F]

$$\int (cx)^{-1-n-np}\left(a+bx^n\right)^p\,dx=\int (bx^n+a)^p(cx)^{-np-n-1}\,dx$$

input
```
integrate((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - n - 1), x)
```

Giac [F]

$$\int (cx)^{-1-n-np}\left(a+bx^n\right)^p\,dx=\int (bx^n+a)^p(cx)^{-np-n-1}\,dx$$

input
```
integrate((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-n-np}\left(a+bx^n\right)^p\,dx=\int \frac{(a+b\,x^n)^p}{(c\,x)^{n+n\,p+1}}\,dx$$

input
```
int((a + b*x^n)^p/(c*x)^(n + n*p + 1),x)
```

output
```
int((a + b*x^n)^p/(c*x)^(n + n*p + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int (cx)^{-1-n-np}(a+bx^n)^p\,dx = -\frac{(x^n b+a)^p\,(x^n b+a)}{x^{np+n}c^{np+n}acn\,(p+1)}$$

input
```
int((c*x)^(-n*p-n-1)*(a+b*x^n)^p,x)
```

output
```
( - (x**n*b + a)**p*(x**n*b + a))/(x**(n*p + n)*c**(n*p + n)*a*c*n*(p + 1)
)
```

3.697 $\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx = -\frac{(cx)^{-n(2+p)} (a+bx^n)^{1+p}}{acn(2+p)} + \frac{bx^n (cx)^{-n(2+p)} (a+bx^n)^{1+p}}{a^2 cn(1+p)(2+p)}$$

output
```
-(a+b*x^n)^(p+1)/a/c/n/(2+p)/((c*x)^(n*(2+p)))+b*x^n*(a+b*x^n)^(p+1)/a^2/c
/n/(p+1)/(2+p)/((c*x)^(n*(2+p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx = -\frac{x (cx)^{-1-n(2+p)} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-2-p,-p,-1-p,-\frac{bx^n}{a}\right)}{n(2+p)}$$

input
```
Integrate[(c*x)^(-1 - 2*n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((x*(c*x)^(-1 - n*(2 + p))*(a + b*x^n)^p*Hypergeometric2F1[-2 - p, -p, -1
 - p, -((b*x^n)/a)])/(n*(2 + p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{n(-p)-2n-1} (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{\int (cx)^{-n(p+2)-1} (bx^n+a)^{p+1} dx}{a(p+1)} - \frac{(cx)^{-n(p+2)} (a+bx^n)^{p+1}}{acn(p+1)}$$

$$\Big\downarrow \text{796}$$

$$\frac{(cx)^{-n(p+2)} (a+bx^n)^{p+2}}{a^2cn(p+1)(p+2)} - \frac{(cx)^{-n(p+2)} (a+bx^n)^{p+1}}{acn(p+1)}$$

input
```
Int[(c*x)^(-1 - 2*n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(2 + p)))) + (a + b*x^n)^(2
+ p)/(a^2*c*n*(1 + p)*(2 + p)*(c*x)^(n*(2 + p)))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [F]

$$\int (cx)^{-np-2n-1} (a+b\,x^{n})^{p}\, dx$$

input
```
int((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.73

$$\int (cx)^{-1-2n-np}(a+bx^n)^p\,dx = -\frac{\left(abpxx^n e^{(-(np+2\,n+1)\log(c)-(np+2\,n+1)\log(x))} - b^2 x x^{2\,n} e^{(-(np+2\,n+1)\log(c)-(np+2\,n+1)\log(x))} + (a^2p+a^2)xe^{(-(\cdots}\right.}{a^2np^2+3\,a^2np+2\,a^2n}$$

input
```
integrate((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
-(a*b*p*x*x^n*e^(-(n*p + 2*n + 1)*log(c) - (n*p + 2*n + 1)*log(x)) - b^2*x
*x^(2*n)*e^(-(n*p + 2*n + 1)*log(c) - (n*p + 2*n + 1)*log(x)) + (a^2*p + a
^2)*x*e^(-(n*p + 2*n + 1)*log(c) - (n*p + 2*n + 1)*log(x)))*(b*x^n + a)^p/
(a^2*n*p^2 + 3*a^2*n*p + 2*a^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(63) = 126$.

Time = 5.71 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.07

$$\int (cx)^{-1-2n-np}\left(a+bx^n\right)^p\,dx = -\frac{aa^pa^{-p-2}b^{p+2}c^{-np-2n-1}px^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{bn\Gamma\left(-p\right)} - \frac{aa^pa^{-p-2}b^{p+2}c^{-np-2n-1}x^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{bn\Gamma\left(-p\right)} + \frac{a^pa^{-p-2}b^{p+2}c^{-np-2n-1}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{n\Gamma\left(-p\right)}$$

input
```
integrate((c*x)**(-n*p-2*n-1)*(a+b*x**n)**p,x)
```

output
```
-a*a**p*a**(-p - 2)*b**(p + 2)*c**(-n*p - 2*n - 1)*p*(a/(b*x**n) + 1)**(p
+ 1)*gamma(-p - 2)/(b*n*x**n*gamma(-p)) - a*a**p*a**(-p - 2)*b**(p + 2)*c*
*(-n*p - 2*n - 1)*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 2)/(b*n*x**n*gamma(
-p)) + a**p*a**(-p - 2)*b**(p + 2)*c**(-n*p - 2*n - 1)*(a/(b*x**n) + 1)**(
p + 1)*gamma(-p - 2)/(n*gamma(-p))
```

Maxima [F]

$$\int (cx)^{-1-2n-np}\left(a+bx^n\right)^p\,dx = \int (bx^n+a)^p(cx)^{-np-2n-1}\,dx$$

input
```
integrate((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 2*n - 1), x)
```

Giac [F]

$$\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^{-np-2n-1} \, dx$$

input
```
integrate((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx = \int \frac{(a+b\,x^n)^p}{(c\,x)^{2\,n+n\,p+1}} \, dx$$

input
```
int((a + b*x^n)^p/(c*x)^(2*n + n*p + 1),x)
```

output
```
int((a + b*x^n)^p/(c*x)^(2*n + n*p + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int (cx)^{-1-2n-np} (a+bx^n)^p \, dx = \frac{(x^n b+a)^p (x^{2n} b^2 - x^n abp - a^2 p - a^2)}{x^{np+2n} c^{np+2n} a^2 cn (p^2+3p+2)}$$

input
```
int((c*x)^(-n*p-2*n-1)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**(2*n)*b**2 - x**n*a*b*p - a**2*p - a**2))/(x**(n*p +
2*n)*c**(n*p + 2*n)*a**2*c*n*(p**2 + 3*p + 2))
```

3.698 $\int (cx)^{-1-3n-np} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 139

$$\int (cx)^{-1-3n-np} (a+bx^n)^p \, dx = -\frac{(cx)^{-n(3+p)} (a+bx^n)^{1+p}}{acn(3+p)} + \frac{2bx^n (cx)^{-n(3+p)} (a+bx^n)^{1+p}}{a^2cn(2+p)(3+p)} - \frac{2b^2x^{2n} (cx)^{-n(3+p)} (a+bx^n)^{1+p}}{a^3cn(1+p)(2+p)(3+p)}$$

output
```
-(a+b*x^n)^(p+1)/a/c/n/(3+p)/((c*x)^(n*(3+p)))+2*b*x^n*(a+b*x^n)^(p+1)/a^2
/c/n/(2+p)/(3+p)/((c*x)^(n*(3+p)))-2*b^2*x^(2*n)*(a+b*x^n)^(p+1)/a^3/c/n/(
p+1)/(2+p)/(3+p)/((c*x)^(n*(3+p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.50

$$\int (cx)^{-1-3n-np} (a+bx^n)^p \, dx = -\frac{x(cx)^{-1-n(3+p)} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-3-p,-p,-2-p,-\frac{bx^n}{a}\right)}{n(3+p)}$$

input
```
Integrate[(c*x)^(-1 - 3*n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((x*(c*x)^(-1 - n*(3 + p))*(a + b*x^n)^p*Hypergeometric2F1[-3 - p, -p, -2
 - p, -((b*x^n)/a)])/(n*(3 + p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{n(-p)-3n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{2\int (cx)^{-n(p+3)-1}(bx^n+a)^{p+1}dx}{a(p+1)}-\frac{(cx)^{-n(p+3)}(a+bx^n)^{p+1}}{acn(p+1)}$$

$$\Big\downarrow \text{805}$$

$$-\frac{2\left(-\frac{\int (cx)^{-n(p+3)-1}(bx^n+a)^{p+2}dx}{a(p+2)}-\frac{(cx)^{-n(p+3)}(a+bx^n)^{p+2}}{acn(p+2)}\right)}{a(p+1)}-\frac{(cx)^{-n(p+3)}(a+bx^n)^{p+1}}{acn(p+1)}$$

$$\Big\downarrow \text{796}$$

$$-\frac{2\left(\frac{(cx)^{-n(p+3)}(a+bx^n)^{p+3}}{a^2cn(p+2)(p+3)}-\frac{(cx)^{-n(p+3)}(a+bx^n)^{p+2}}{acn(p+2)}\right)}{a(p+1)}-\frac{(cx)^{-n(p+3)}(a+bx^n)^{p+1}}{acn(p+1)}$$

input
```
Int[(c*x)^(-1 - 3*n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(3 + p)))) - (2*(-((a + b*x^
n)^(2 + p)/(a*c*n*(2 + p)*(c*x)^(n*(3 + p)))) + (a + b*x^n)^(3 + p)/(a^2*c
*n*(2 + p)*(3 + p)*(c*x)^(n*(3 + p)))))/(a*(1 + p))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [F]

$$\int (cx)^{-np-3n-1}\left(a+b\,x^{n}\right)^{p}dx$$

input
```
int((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.55

$$\int (cx)^{-1-3n-np}\left(a+bx^n\right)^p\,dx = \frac{\left(2\,ab^2pxx^{2\,n}e^{(-(np+3\,n+1)\log(c)-(np+3\,n+1)\log(x))} - 2\,b^3xx^{3\,n}e^{(-(np+3\,n+1)\log(c)-(np+3\,n+1)\log(x))} - (a^2bp^2 + a^2b\right.}{a^3np^3 + 6\,a^3np^2}$$

input
```
integrate((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
(2*a*b^2*p*x*x^(2*n)*e^(-(n*p + 3*n + 1)*log(c) - (n*p + 3*n + 1)*log(x))
- 2*b^3*x*x^(3*n)*e^(-(n*p + 3*n + 1)*log(c) - (n*p + 3*n + 1)*log(x)) - (
a^2*b*p^2 + a^2*b*p)*x*x^n*e^(-(n*p + 3*n + 1)*log(c) - (n*p + 3*n + 1)*lo
g(x)) - (a^3*p^2 + 3*a^3*p + 2*a^3)*x*e^(-(n*p + 3*n + 1)*log(c) - (n*p +
3*n + 1)*log(x)))*(b*x^n + a)^p/(a^3*n*p^3 + 6*a^3*n*p^2 + 11*a^3*n*p + 6*
a^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(110) = 220$.

Time = 6.25 (sec) , antiderivative size = 566, normalized size of antiderivative = 4.07

$$\begin{aligned}
&\int (cx)^{-1-3n-np}(a+bx^n)^p\,dx \\
&= \frac{a^2a^pa^{-p-3}b^{p+3}c^{-np-3n-1}p^2\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma(-p)+2abnx^n\Gamma(-p)+b^2nx^{2n}\Gamma(-p)} \\
&\quad+\frac{3a^2a^pa^{-p-3}b^{p+3}c^{-np-3n-1}p\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma(-p)+2abnx^n\Gamma(-p)+b^2nx^{2n}\Gamma(-p)} \\
&\quad+\frac{2a^2a^pa^{-p-3}b^{p+3}c^{-np-3n-1}\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma(-p)+2abnx^n\Gamma(-p)+b^2nx^{2n}\Gamma(-p)} \\
&\quad-\frac{2aa^pa^{-p-3}bb^{p+3}c^{-np-3n-1}px^n\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma(-p)+2abnx^n\Gamma(-p)+b^2nx^{2n}\Gamma(-p)} \\
&\quad-\frac{2aa^pa^{-p-3}bb^{p+3}c^{-np-3n-1}x^n\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma(-p)+2abnx^n\Gamma(-p)+b^2nx^{2n}\Gamma(-p)} \\
&\quad+\frac{2a^pa^{-p-3}b^2b^{p+3}c^{-np-3n-1}x^{2n}\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma(-p)+2abnx^n\Gamma(-p)+b^2nx^{2n}\Gamma(-p)}
\end{aligned}$$

input
```
integrate((c*x)**(-n*p-3*n-1)*(a+b*x**n)**p,x)
```

output
```
a**2*a**p*a**(-p - 3)*b**(p + 3)*c**(-n*p - 3*n - 1)*p**2*(a/(b*x**n) + 1)
**(p + 3)*gamma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*
n*x**(2*n)*gamma(-p)) + 3*a**2*a**p*a**(-p - 3)*b**(p + 3)*c**(-n*p - 3*n
- 1)*p*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n
*x**n*gamma(-p) + b**2*n*x**(2*n)*gamma(-p)) + 2*a**2*a**p*a**(-p - 3)*b**
(p + 3)*c**(-n*p - 3*n - 1)*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**2*
n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n*x**(2*n)*gamma(-p)) - 2*a*a*
*p*a**(-p - 3)*b*b**(p + 3)*c**(-n*p - 3*n - 1)*p*x**n*(a/(b*x**n) + 1)**(
p + 3)*gamma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n*x
**(2*n)*gamma(-p)) - 2*a*a**p*a**(-p - 3)*b*b**(p + 3)*c**(-n*p - 3*n - 1)
*x**n*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*
x**n*gamma(-p) + b**2*n*x**(2*n)*gamma(-p)) + 2*a**p*a**(-p - 3)*b**2*b**(
p + 3)*c**(-n*p - 3*n - 1)*x**(2*n)*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3
)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n*x**(2*n)*gamma(-p))
```

Maxima [F]

$$\int (cx)^{-1-3n-np} (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^{-np-3n-1} \, dx$$

input
```
integrate((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 3*n - 1), x)
```

Giac [F]

$$\int (cx)^{-1-3n-np} (a+bx^n)^p \, dx = \int (bx^n+a)^p (cx)^{-np-3n-1} \, dx$$

input
```
integrate((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-3n-np}(a+bx^n)^p\,dx=\int \frac{(a+b\,x^n)^p}{(c\,x)^{3\,n+n\,p+1}}\,dx$$

input
```
int((a + b*x^n)^p/(c*x)^(3*n + n*p + 1),x)
```

output
```
int((a + b*x^n)^p/(c*x)^(3*n + n*p + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int (cx)^{-1-3n-np}(a+bx^n)^p\,dx = \frac{(x^nb+a)^p\,(-2x^{3n}b^3+2x^{2n}a\,b^2p-x^na^2b\,p^2-x^na^2bp-a^3p^2-3a^3p-2a^3)}{x^{np+3n}c^{np+3n}a^3cn\,(p^3+6p^2+11p+6)}$$

input
```
int((c*x)^(-n*p-3*n-1)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*( - 2*x**(3*n)*b**3 + 2*x**(2*n)*a*b**2*p - x**n*a**2*b*p
**2 - x**n*a**2*b*p - a**3*p**2 - 3*a**3*p - 2*a**3))/(x**(n*p + 3*n)*c**(
n*p + 3*n)*a**3*c*n*(p**3 + 6*p**2 + 11*p + 6))
```

3.699 $\int (cx)^{-1-4n-np} (a+bx^n)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 199

$$\int (cx)^{-1-4n-np} (a+bx^n)^p \, dx = -\frac{(cx)^{-n(4+p)} (a+bx^n)^{1+p}}{acn(4+p)} + \frac{3bx^n (cx)^{-n(4+p)} (a+bx^n)^{1+p}}{a^2 cn(3+p)(4+p)} - \frac{6b^2 x^{2n} (cx)^{-n(4+p)} (a+bx^n)^{1+p}}{a^3 cn(2+p)(3+p)(4+p)} + \frac{6b^3 x^{3n} (cx)^{-n(4+p)} (a+bx^n)^{1+p}}{a^4 cn(1+p)(2+p)(3+p)(4+p)}$$

output
```
-(a+b*x^n)^(p+1)/a/c/n/(4+p)/((c*x)^(n*(4+p)))+3*b*x^n*(a+b*x^n)^(p+1)/a^2
/c/n/(3+p)/(4+p)/((c*x)^(n*(4+p)))-6*b^2*x^(2*n)*(a+b*x^n)^(p+1)/a^3/c/n/(
2+p)/(3+p)/(4+p)/((c*x)^(n*(4+p)))+6*b^3*x^(3*n)*(a+b*x^n)^(p+1)/a^4/c/n/(
p+1)/(2+p)/(3+p)/(4+p)/((c*x)^(n*(4+p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.35

$$\int (cx)^{-1-4n-np}(a+bx^n)^p\,dx = -\frac{x(cx)^{-1-n(4+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-4-p,-p,-3-p,-\frac{bx^n}{a}\right)}{n(4+p)}$$

input
```
Integrate[(c*x)^(-1 - 4*n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((x*(c*x)^(-1 - n*(4 + p))*(a + b*x^n)^p*Hypergeometric2F1[-4 - p, -p, -3
 - p, -((b*x^n)/a)])/(n*(4 + p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {805, 805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{n(-p)-4n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{3\int (cx)^{-n(p+4)-1}(bx^n+a)^{p+1}dx}{a(p+1)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+1}}{acn(p+1)}$$

$$\Big\downarrow \text{805}$$

$$-\frac{3\left(-\frac{2\int (cx)^{-n(p+4)-1}(bx^n+a)^{p+2}dx}{a(p+2)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+2}}{acn(p+2)}\right)}{a(p+1)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+1}}{acn(p+1)}$$

$$\Big\downarrow \text{805}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int (cx)^{-n(p+4)-1}(bx^n+a)^{p+3}dx}{a(p+3)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+3}}{acn(p+3)}\right)}{a(p+2)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+2}}{acn(p+2)}\right)}{a(p+1)}-\frac{(cx)^{-n(p+4)}\left(a+bx^n\right)^{p+1}}{acn(p+1)}$$

$$\Big\downarrow \text{796}$$

$$-\frac{3\left(-\frac{2\left(\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+4}}{a^2cn(p+3)(p+4)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+3}}{acn(p+3)}\right)}{a(p+2)}-\frac{(cx)^{-n(p+4)}(a+bx^n)^{p+2}}{acn(p+2)}\right)}{a(p+1)}-\frac{(cx)^{-n(p+4)}\left(a+bx^n\right)^{p+1}}{acn(p+1)}$$

input
```
Int[(c*x)^(-1 - 4*n - n*p)*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*c*n*(1 + p)*(c*x)^(n*(4 + p)))) - (3*(-((a + b*x^
n)^(2 + p)/(a*c*n*(2 + p)*(c*x)^(n*(4 + p)))) - (2*(-((a + b*x^n)^(3 + p)/
(a*c*n*(3 + p)*(c*x)^(n*(4 + p)))) + (a + b*x^n)^(4 + p)/(a^2*c*n*(3 + p)*
(4 + p)*(c*x)^(n*(4 + p)))))/(a*(2 + p))))/(a*(1 + p))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [F]

$$\int (cx)^{-np-4n-1} (a + b x^n)^p \, dx$$

input
```
int((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x)
```

output
```
int((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.48

$$\int (cx)^{-1-4n-np} (a+bx^n)^p \, dx = -\frac{\left(6\,ab^3pxx^{3\,n}e^{(-(np+4\,n+1)\log(c)-(np+4\,n+1)\log(x))} - 6\,b^4xx^{4\,n}e^{(-(np+4\,n+1)\log(c)-(np+4\,n+1)\log(x))} - 3\,(a^2b^2p^2\right.}{}$$

input
```
integrate((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
-(6*a*b^3*p*x*x^(3*n)*e^(-(n*p + 4*n + 1)*log(c) - (n*p + 4*n + 1)*log(x))
 - 6*b^4*x*x^(4*n)*e^(-(n*p + 4*n + 1)*log(c) - (n*p + 4*n + 1)*log(x)) -
3*(a^2*b^2*p^2 + a^2*b^2*p)*x*x^(2*n)*e^(-(n*p + 4*n + 1)*log(c) - (n*p +
4*n + 1)*log(x)) + (a^3*b*p^3 + 3*a^3*b*p^2 + 2*a^3*b*p)*x*x^n*e^(-(n*p +
4*n + 1)*log(c) - (n*p + 4*n + 1)*log(x)) + (a^4*p^3 + 6*a^4*p^2 + 11*a^4*
p + 6*a^4)*x*e^(-(n*p + 4*n + 1)*log(c) - (n*p + 4*n + 1)*log(x)))*(b*x^n
+ a)^p/(a^4*n*p^4 + 10*a^4*n*p^3 + 35*a^4*n*p^2 + 50*a^4*n*p + 24*a^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. 2(160) = 320.

Time = 6.71 (sec) , antiderivative size = 1168, normalized size of antiderivative = 5.87

$$\int (cx)^{-1-4n-np} (a+bx^n)^p \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**(-n*p-4*n-1)*(a+b*x**n)**p,x)
```

output
```
-a**3*a**p*a**(-p - 4)*b**(p + 4)*c**(-n*p - 4*n - 1)*p**3*(a/(b*x**n) + 1
)**(p + 4)*gamma(-p - 4)/(a**3*n*gamma(-p) + 3*a**2*b*n*x**n*gamma(-p) + 3
*a*b**2*n*x**(2*n)*gamma(-p) + b**3*n*x**(3*n)*gamma(-p)) - 6*a**3*a**p*a*
*(-p - 4)*b**(p + 4)*c**(-n*p - 4*n - 1)*p**2*(a/(b*x**n) + 1)**(p + 4)*ga
mma(-p - 4)/(a**3*n*gamma(-p) + 3*a**2*b*n*x**n*gamma(-p) + 3*a*b**2*n*x**
(2*n)*gamma(-p) + b**3*n*x**(3*n)*gamma(-p)) - 11*a**3*a**p*a**(-p - 4)*b*
*(p + 4)*c**(-n*p - 4*n - 1)*p*(a/(b*x**n) + 1)**(p + 4)*gamma(-p - 4)/(a*
*3*n*gamma(-p) + 3*a**2*b*n*x**n*gamma(-p) + 3*a*b**2*n*x**(2*n)*gamma(-p)
 + b**3*n*x**(3*n)*gamma(-p)) - 6*a**3*a**p*a**(-p - 4)*b**(p + 4)*c**(-n*
p - 4*n - 1)*(a/(b*x**n) + 1)**(p + 4)*gamma(-p - 4)/(a**3*n*gamma(-p) + 3
*a**2*b*n*x**n*gamma(-p) + 3*a*b**2*n*x**(2*n)*gamma(-p) + b**3*n*x**(3*n)
*gamma(-p)) + 3*a**2*a**p*a**(-p - 4)*b*b**(p + 4)*c**(-n*p - 4*n - 1)*p**
2*x**n*(a/(b*x**n) + 1)**(p + 4)*gamma(-p - 4)/(a**3*n*gamma(-p) + 3*a**2*
b*n*x**n*gamma(-p) + 3*a*b**2*n*x**(2*n)*gamma(-p) + b**3*n*x**(3*n)*gamma
(-p)) + 9*a**2*a**p*a**(-p - 4)*b*b**(p + 4)*c**(-n*p - 4*n - 1)*p*x**n*(a
/(b*x**n) + 1)**(p + 4)*gamma(-p - 4)/(a**3*n*gamma(-p) + 3*a**2*b*n*x**n*
gamma(-p) + 3*a*b**2*n*x**(2*n)*gamma(-p) + b**3*n*x**(3*n)*gamma(-p)) + 6
*a**2*a**p*a**(-p - 4)*b*b**(p + 4)*c**(-n*p - 4*n - 1)*x**n*(a/(b*x**n) +
 1)**(p + 4)*gamma(-p - 4)/(a**3*n*gamma(-p) + 3*a**2*b*n*x**n*gamma(-p) +
 3*a*b**2*n*x**(2*n)*gamma(-p) + b**3*n*x**(3*n)*gamma(-p)) - 6*a*a**p*...
```

Maxima [F]

$$\int (cx)^{-1-4n-np}(a+bx^n)^p\,dx = \int (bx^n+a)^p(cx)^{-np-4n-1}\,dx$$

input
```
integrate((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 4*n - 1), x)
```

Giac [F]

$$\int (cx)^{-1-4n-np}\left(a+bx^n\right)^p\,dx=\int (bx^n+a)^p(cx)^{-np-4\,n-1}\,dx$$

input
```
integrate((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(c*x)^(-n*p - 4*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-4n-np}\left(a+bx^n\right)^p\,dx=\int \frac{(a+b\,x^n)^p}{(c\,x)^{4\,n+n\,p+1}}\,dx$$

input
```
int((a + b*x^n)^p/(c*x)^(4*n + n*p + 1),x)
```

output
```
int((a + b*x^n)^p/(c*x)^(4*n + n*p + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

$$\int (cx)^{-1-4n-np}\left(a+bx^n\right)^p\,dx$$
$$=\frac{(x^nb+a)^p\,(6x^{4n}b^4-6x^{3n}a\,b^3p+3x^{2n}a^2b^2p^2+3x^{2n}a^2b^2p-x^na^3b\,p^3-3x^na^3b\,p^2-2x^na^3bp-a^4p^3-6a}{x^{np+4n}c^{np+4n}a^4cn\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int((c*x)^(-n*p-4*n-1)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(6*x**(4*n)*b**4 - 6*x**(3*n)*a*b**3*p + 3*x**(2*n)*a**2*
b**2*p**2 + 3*x**(2*n)*a**2*b**2*p - x**n*a**3*b*p**3 - 3*x**n*a**3*b*p**2
 - 2*x**n*a**3*b*p - a**4*p**3 - 6*a**4*p**2 - 11*a**4*p - 6*a**4))/(x**(n
*p + 4*n)*c**(n*p + 4*n)*a**4*c*n*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.700 $\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx=-\frac{x^{-2n(1+p)}(a+bx^n)^{1+2p}}{2an(1+p)}+\frac{bx^{-n(1+2p)}(a+bx^n)^{1+2p}}{2a^2n(1+p)(1+2p)}$$

output
```
-1/2*(a+b*x^n)^(1+2*p)/a/n/(p+1)/(x^(2*n*(p+1)))+1/2*b*(a+b*x^n)^(1+2*p)/a
^2/n/(p+1)/(1+2*p)/(x^(n*(1+2*p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx=-\frac{x^{-2n(1+p)}(a+bx^n)^{2p}\left(1+\frac{bx^n}{a}\right)^{-2p}\text{Hypergeometric2F1}\left(-2p,-2(1+p),1-2(1+p),-\frac{bx^n}{a}\right)}{2n(1+p)}$$

input
```
Integrate[x^(-1 - 2*n*(1 + p))*(a + b*x^n)^(2*p),x]
```

output
```
-1/2*((a + b*x^n)^(2*p)*Hypergeometric2F1[-2*p, -2*(1 + p), 1 - 2*(1 + p),
 -((b*x^n)/a)])/(n*(1 + p)*x^(2*n*(1 + p))*(1 + (b*x^n)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n(p+1)-1}(a+bx^n)^{2p}\,dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{b\int x^{-2(p+1)n+n-1}(bx^n+a)^{2p}dx}{2a(p+1)}-\frac{x^{-2n(p+1)}(a+bx^n)^{2p+1}}{2an(p+1)}$$

$$\Big\downarrow \text{796}$$

$$\frac{bx^{-n(2p+1)}(a+bx^n)^{2p+1}}{2a^2n(p+1)(2p+1)}-\frac{x^{-2n(p+1)}(a+bx^n)^{2p+1}}{2an(p+1)}$$

input
```
Int[x^(-1 - 2*n*(1 + p))*(a + b*x^n)^(2*p),x]
```

output
```
-1/2*(a + b*x^n)^(1 + 2*p)/(a*n*(1 + p)*x^(2*n*(1 + p))) + (b*(a + b*x^n)^
(1 + 2*p))/(2*a^2*n*(1 + p)*(1 + 2*p)*x^(n*(1 + 2*p)))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{-1-2n(p+1)}(a+b\,x^n)^{2p}\,dx$$

input
```
int(x^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x)
```

output
```
int(x^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx = -\frac{\left(2\,abpxx^{-2\,np-2\,n-1}x^n - b^2xx^{-2\,np-2\,n-1}x^{2\,n} + (2\,a^2p+a^2)xx^{-2\,np-2\,n-1}\right)(bx^n+a)^{2\,p}}{2\,(2\,a^2np^2+3\,a^2np+a^2n)}$$

input
```
integrate(x^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*p*x*x^(-2*n*p - 2*n - 1)*x^n - b^2*x*x^(-2*n*p - 2*n - 1)*x^(2
*n) + (2*a^2*p + a^2)*x*x^(-2*n*p - 2*n - 1))*(b*x^n + a)^(2*p)/(2*a^2*n*p
^2 + 3*a^2*n*p + a^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(65) = 130.

Time = 28.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx = -\frac{2b^{2p+2}px^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{2p+1}\Gamma(-2p-2)}{abn\Gamma(-2p)} - \frac{b^{2p+2}x^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{2p+1}\Gamma(-2p-2)}{abn\Gamma(-2p)} + \frac{b^{2p+2}\left(\frac{ax^{-n}}{b}+1\right)^{2p+1}\Gamma(-2p-2)}{a^2n\Gamma(-2p)}$$

input
```
integrate(x**(-1-2*n*(p+1))*(a+b*x**n)**(2*p),x)
```

output
```
-2*b**(2*p + 2)*p*(a/(b*x**n) + 1)**(2*p + 1)*gamma(-2*p - 2)/(a*b*n*x**n*
gamma(-2*p)) - b**(2*p + 2)*(a/(b*x**n) + 1)**(2*p + 1)*gamma(-2*p - 2)/(a
*b*n*x**n*gamma(-2*p)) + b**(2*p + 2)*(a/(b*x**n) + 1)**(2*p + 1)*gamma(-2
*p - 2)/(a**2*n*gamma(-2*p))
```

Maxima [F]

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx = \int (bx^n+a)^{2p}x^{-2n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(2*p)*x^(-2*n*(p + 1) - 1), x)
```

Giac [F]

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx=\int (bx^n+a)^{2p}x^{-2n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(2*p)*x^(-2*n*(p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx=\int \frac{(a+b\,x^n)^{2\,p}}{x^{2\,n\,(p+1)+1}}\,dx$$

input
```
int((a + b*x^n)^(2*p)/x^(2*n*(p + 1) + 1),x)
```

output
```
int((a + b*x^n)^(2*p)/x^(2*n*(p + 1) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^{-1-2n(1+p)}(a+bx^n)^{2p}\,dx=\frac{(x^nb+a)^{2p}\left(x^{2n}b^2-2x^nabp-2a^2p-a^2\right)}{2x^{2np+2n}a^2n\left(2p^2+3p+1\right)}$$

input
```
int(x^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x)
```

output
```
((x**n*b + a)**(2*p)*(x**(2*n)*b**2 - 2*x**n*a*b*p - 2*a**2*p - a**2))/(2*
x**(2*n*p + 2*n)*a**2*n*(2*p**2 + 3*p + 1))
```

3.701 $\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p} \, dx = -\frac{(cx)^{-2n(1+p)} (a+bx^n)^{1+2p}}{2acn(1+p)} + \frac{bx^n (cx)^{-2n(1+p)} (a+bx^n)^{1+2p}}{2a^2cn(1+p)(1+2p)}$$

output
```
-1/2*(a+b*x^n)^(1+2*p)/a/c/n/(p+1)/((c*x)^(2*n*(p+1)))+1/2*b*x^n*(a+b*x^n)
^(1+2*p)/a^2/c/n/(p+1)/(1+2*p)/((c*x)^(2*n*(p+1)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p} \, dx = -\frac{x(cx)^{-1-2n(1+p)} (a+bx^n)^{2p} \left(1+\frac{bx^n}{a}\right)^{-2p} \text{Hypergeometric2F1}\left(-2p,-2(1+p),1-2(1+p),-\frac{bx^n}{a}\right)}{2n(1+p)}$$

input
```
Integrate[(c*x)^(-1 - 2*n*(1 + p))*(a + b*x^n)^(2*p),x]
```

output
```
-1/2*(x*(c*x)^(-1 - 2*n*(1 + p))*(a + b*x^n)^(2*p)*Hypergeometric2F1[-2*p,
 -2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p
))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{-2n(p+1)-1} (a+bx^n)^{2p} \, dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{\int (cx)^{-2n(p+1)-1} (bx^n+a)^{2p+1} dx}{a(2p+1)} - \frac{(cx)^{-2n(p+1)} (a+bx^n)^{2p+1}}{acn(2p+1)}$$

$$\Big\downarrow \text{796}$$

$$\frac{(cx)^{-2n(p+1)} (a+bx^n)^{2(p+1)}}{2a^2cn(p+1)(2p+1)} - \frac{(cx)^{-2n(p+1)} (a+bx^n)^{2p+1}}{acn(2p+1)}$$

input
```
Int[(c*x)^(-1 - 2*n*(1 + p))*(a + b*x^n)^(2*p),x]
```

output
```
(a + b*x^n)^(2*(1 + p))/(2*a^2*c*n*(1 + p)*(1 + 2*p)*(c*x)^(2*n*(1 + p)))
- (a + b*x^n)^(1 + 2*p)/(a*c*n*(1 + 2*p)*(c*x)^(2*n*(1 + p)))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [F]

$$\int (cx)^{-1-2n(p+1)} (a+b\,x^n)^{2p}\,dx$$

input
```
int((c*x)^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x)
```

output
```
int((c*x)^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.64

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p}\,dx = -\frac{\left(2\,abpxx^n e^{\left(-(2\,np+2\,n+1)\log(c)-(2\,np+2\,n+1)\log(x)\right)} - b^2 x x^{2\,n} e^{\left(-(2\,np+2\,n+1)\log(c)-(2\,np+2\,n+1)\log(x)\right)} + (2\,a^2p +$$
$$2\left(2\,a^2np^2+3\,a^2np+a^2n\right)$$

input
```
integrate((c*x)^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(c) - (2*n*p + 2*n + 1)*log(x
)) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(c) - (2*n*p + 2*n + 1)*log(x)
) + (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(c) - (2*n*p + 2*n + 1)*log
(x)))*(b*x^n + a)^(2*p)/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(75) = 150$.

Time = 34.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.83

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p}\,dx = -\frac{2b^{2p+2}c^{-2np-2n-1}px^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{2p+1}\Gamma(-2p-2)}{abn\Gamma(-2p)} - \frac{b^{2p+2}c^{-2np-2n-1}x^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{2p+1}\Gamma(-2p-2)}{abn\Gamma(-2p)} + \frac{b^{2p+2}c^{-2np-2n-1}\left(\frac{ax^{-n}}{b}+1\right)^{2p+1}\Gamma(-2p-2)}{a^2n\Gamma(-2p)}$$

input
```
integrate((c*x)**(-1-2*n*(p+1))*(a+b*x**n)**(2*p),x)
```

output
```
-2*b**(2*p + 2)*c**(-2*n*p - 2*n - 1)*p*(a/(b*x**n) + 1)**(2*p + 1)*gamma(
-2*p - 2)/(a*b*n*x**n*gamma(-2*p)) - b**(2*p + 2)*c**(-2*n*p - 2*n - 1)*(a
/(b*x**n) + 1)**(2*p + 1)*gamma(-2*p - 2)/(a*b*n*x**n*gamma(-2*p)) + b**(2
*p + 2)*c**(-2*n*p - 2*n - 1)*(a/(b*x**n) + 1)**(2*p + 1)*gamma(-2*p - 2)/
(a**2*n*gamma(-2*p))
```

Maxima [F]

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p}\,dx = \int (bx^n+a)^{2p}(cx)^{-2n(p+1)-1}\,dx$$

input
```
integrate((c*x)^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^(2*p)*(c*x)^(-2*n*(p + 1) - 1), x)
```

Giac [F]

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p} \, dx = \int (bx^n+a)^{2\,p}(cx)^{-2\,n(p+1)-1} \, dx$$

input
```
integrate((c*x)^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(2*p)*(c*x)^(-2*n*(p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p} \, dx = \int \frac{(a+b\,x^n)^{2\,p}}{(c\,x)^{2\,n\,(p+1)+1}} \, dx$$

input
```
int((a + b*x^n)^(2*p)/(c*x)^(2*n*(p + 1) + 1),x)
```

output
```
int((a + b*x^n)^(2*p)/(c*x)^(2*n*(p + 1) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int (cx)^{-1-2n(1+p)} (a+bx^n)^{2p} \, dx = \frac{(x^n b+a)^{2p} (x^{2n} b^2 - 2x^n abp - 2a^2 p - a^2)}{2x^{2np+2n} c^{2np+2n} a^2 cn (2p^2+3p+1)}$$

input
```
int((c*x)^(-1-2*n*(p+1))*(a+b*x^n)^(2*p),x)
```

output
```
((x**n*b + a)**(2*p)*(x**(2*n)*b**2 - 2*x**n*a*b*p - 2*a**2*p - a**2))/(2*
x**(2*n*p + 2*n)*c**(2*n*p + 2*n)*a**2*c*n*(2*p**2 + 3*p + 1))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file